

a. $\begin{bmatrix} 3 & 2 & 0 \\ 2 & 4 & 5 \\ 0 & 5 & 6 \end{bmatrix}$ 3 x 3, square, symmetric

b. $\begin{bmatrix} x \\ x^2 \\ x^3 \\ x^4 \end{bmatrix}$ 4 x 1 column

c. $\begin{bmatrix} 4 & 0 \\ 0 & 8 \end{bmatrix}$ 2 x 2, square, diagonal

d. $[1 \ y \ y^2 \ y^3]$ 1 x 4, row

e. $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ 3 x 3, square, diagonal, identity

f. $\begin{bmatrix} 3 & -1 & 0 & 0 & 0 \\ 2 & 0 & 6 & 0 & 0 \\ 0 & 4 & 1 & 4 & 0 \\ 0 & 0 & 5 & 4 & 2 \\ 0 & 0 & 0 & 7 & 8 \end{bmatrix}$ 5 x 5, square, banded

g. $\begin{bmatrix} 1 & 2 & 2 & 2 \\ 0 & 1 & 3 & 3 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ 4 x 4, square, upper triangular

h. $\begin{bmatrix} c_1 & 0 & 0 & 0 \\ 0 & c_2 & 0 & 0 \\ 0 & 0 & c_3 & 0 \\ 0 & 0 & 0 & c_4 \end{bmatrix}$ 4 x 4, square, diagonal

$$\text{a. } [A] + [B] = \begin{bmatrix} 4 & 2 & 1 \\ 7 & 0 & -7 \\ 1 & -5 & 3 \end{bmatrix} + \begin{bmatrix} 1 & 2 & -1 \\ 5 & 3 & 3 \\ 4 & 5 & -7 \end{bmatrix} = \begin{bmatrix} 5 & 4 & 0 \\ 12 & 3 & -4 \\ 5 & 0 & -4 \end{bmatrix}$$

$$\text{b. } [A] - [B] = \begin{bmatrix} 4 & 2 & 1 \\ 7 & 0 & -7 \\ 1 & -5 & 3 \end{bmatrix} - \begin{bmatrix} 1 & 2 & -1 \\ 5 & 3 & 3 \\ 4 & 5 & -7 \end{bmatrix} = \begin{bmatrix} 3 & 0 & 2 \\ 2 & -3 & -10 \\ -3 & -10 & 10 \end{bmatrix}$$

$$\text{c. } 3[A] = 3 \begin{bmatrix} 4 & 2 & 1 \\ 7 & 0 & -7 \\ 1 & -5 & 3 \end{bmatrix} = \begin{bmatrix} 12 & 6 & 3 \\ 21 & 0 & -21 \\ 3 & -15 & 9 \end{bmatrix}$$

$$\text{d. } [A][B] = \begin{bmatrix} 4 & 2 & 1 \\ 7 & 0 & -7 \\ 1 & -5 & 3 \end{bmatrix} \begin{bmatrix} 1 & 2 & -1 \\ 5 & 3 & 3 \\ 4 & 5 & -7 \end{bmatrix} = \begin{bmatrix} 18 & 19 & -5 \\ -21 & -21 & 42 \\ -12 & 2 & -37 \end{bmatrix}$$

$$\text{e. } [A]\{C\} = \begin{bmatrix} 4 & 2 & 1 \\ 7 & 0 & -7 \\ 1 & -5 & 3 \end{bmatrix} \begin{Bmatrix} 1 \\ -2 \\ 4 \end{Bmatrix} = \begin{Bmatrix} 4 \\ -21 \\ 23 \end{Bmatrix}$$

$$\text{f. } [A]^2 = \begin{bmatrix} 4 & 2 & 1 \\ 7 & 0 & -7 \\ 1 & -5 & 3 \end{bmatrix} \begin{bmatrix} 4 & 2 & 1 \\ 7 & 0 & -7 \\ 1 & -5 & 3 \end{bmatrix} = \begin{bmatrix} 31 & 3 & -7 \\ 21 & 49 & -14 \\ -28 & -13 & 45 \end{bmatrix}$$

$$\text{g. } [I][A] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 4 & 2 & 1 \\ 7 & 0 & -7 \\ 1 & -5 & 3 \end{bmatrix} = \begin{bmatrix} 4 & 2 & 1 \\ 7 & 0 & -7 \\ 1 & -5 & 3 \end{bmatrix}$$

$$[A][I] = \begin{bmatrix} 4 & 2 & 1 \\ 7 & 0 & -7 \\ 1 & -5 & 3 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 4 & 2 & 1 \\ 7 & 0 & -7 \\ 1 & -5 & 3 \end{bmatrix}$$

2.3

$$[A_{11}] = \begin{bmatrix} 5 & 7 & 2 \\ 3 & 8 & -3 \end{bmatrix}$$

$$[A_{12}] = \begin{bmatrix} 0 & 3 & 5 \\ -5 & 0 & 8 \end{bmatrix}$$

$$[A_{21}] = \begin{bmatrix} 1 & 4 & 0 \\ 0 & 10 & 5 \\ 2 & -5 & 9 \end{bmatrix}$$

$$[A_{22}] = \begin{bmatrix} 7 & 15 & 9 \\ 12 & 3 & -1 \\ 2 & 18 & -10 \end{bmatrix}$$

$$\{B_{11}\} = \begin{Bmatrix} 2 \\ 8 \\ -5 \end{Bmatrix}$$

$$[B_{12}] = \begin{bmatrix} 10 & 0 \\ 7 & 5 \\ 2 & -4 \end{bmatrix}$$

$$\{B_{21}\} = \begin{Bmatrix} 4 \\ 3 \\ 1 \end{Bmatrix}$$

$$[B_{22}] = \begin{bmatrix} 8 & 13 \\ 12 & 0 \\ 5 & 7 \end{bmatrix}$$

$$[A_{11}]\{B_{11}\} + [A_{12}]\{B_{21}\} = \begin{Bmatrix} 70 \\ 73 \end{Bmatrix}$$

$$[A_{21}]\{B_{11}\} + [A_{22}]\{B_{21}\} = \begin{Bmatrix} 116 \\ 111 \\ -29 \end{Bmatrix}$$

$$[A_{11}][B_{12}] + [A_{12}][B_{22}] = \begin{bmatrix} 164 & 62 \\ 80 & 43 \end{bmatrix}$$

$$[A_{21}][B_{12}] + [A_{22}][B_{22}] = \begin{bmatrix} 319 & 174 \\ 207 & 179 \\ 185 & -105 \end{bmatrix}$$

$$[A][B] = \begin{bmatrix} 70 & 164 & 62 \\ 73 & 80 & 43 \\ 116 & 319 & 174 \\ 111 & 207 & 179 \\ -29 & 185 & -105 \end{bmatrix}$$

2.4

a.

$$[A] = \begin{bmatrix} 1 & 4 & 2 \\ 8 & 3 & 6 \\ 7 & 1 & -2 \end{bmatrix}$$

$$[A]^T = \begin{bmatrix} 1 & 8 & 7 \\ 4 & 3 & 1 \\ 2 & 6 & -2 \end{bmatrix}$$

$$[B] = \begin{bmatrix} 0 & 5 & -1 \\ -3 & 1 & 7 \\ 2 & 4 & -4 \end{bmatrix}$$

$$[B]^T = \begin{bmatrix} 0 & -3 & 2 \\ 5 & 1 & 4 \\ -1 & 7 & -4 \end{bmatrix}$$

b.

$$\left[\begin{bmatrix} 1 & 4 & 2 \\ 8 & 3 & 6 \\ 7 & 1 & -2 \end{bmatrix} + \begin{bmatrix} 0 & 5 & -1 \\ -3 & 1 & 7 \\ 2 & 4 & -4 \end{bmatrix} \right]^T = \begin{bmatrix} 1 & 9 & 1 \\ 5 & 4 & 13 \\ 9 & 5 & -6 \end{bmatrix}^T = \begin{bmatrix} 1 & 5 & 9 \\ 9 & 4 & 5 \\ 1 & 13 & -6 \end{bmatrix}$$

$$[A]^T + [B]^T = \begin{bmatrix} 1 & 8 & 7 \\ 4 & 3 & 1 \\ 2 & 6 & -2 \end{bmatrix} + \begin{bmatrix} 0 & -3 & 2 \\ 5 & 1 & 4 \\ -1 & 7 & -4 \end{bmatrix} = \begin{bmatrix} 1 & 5 & 9 \\ 9 & 4 & 5 \\ 1 & 13 & -6 \end{bmatrix}$$

c.

$$([A][B])^T = \left(\begin{bmatrix} 1 & 4 & 2 \\ 8 & 3 & 6 \\ 7 & 1 & -2 \end{bmatrix} \begin{bmatrix} 0 & 5 & -1 \\ -3 & 1 & 7 \\ 2 & 4 & -4 \end{bmatrix} \right)^T = \begin{bmatrix} -8 & 17 & 19 \\ 3 & 67 & -11 \\ -7 & 28 & 8 \end{bmatrix}^T = \begin{bmatrix} -8 & 3 & -7 \\ 17 & 67 & 28 \\ 19 & -11 & 8 \end{bmatrix}$$

$$[B]^T[A]^T = \begin{bmatrix} 0 & -3 & 2 \\ 5 & 1 & 4 \\ -1 & 7 & -4 \end{bmatrix} \begin{bmatrix} 1 & 8 & 7 \\ 4 & 3 & 1 \\ 2 & 6 & -2 \end{bmatrix} = \begin{bmatrix} -8 & 3 & -7 \\ 17 & 67 & 28 \\ 19 & -11 & 8 \end{bmatrix}$$

2.5

a.

$$\begin{vmatrix} 2 & 10 & 0 \\ 16 & 6 & 14 \\ 12 & -4 & 18 \end{vmatrix} = (2)(6)(18) + (10)(14)(12) + (0)(16)(-4) \\ - (10)(16)(18) - (2)(14)(-4) - (6)(6)(12)$$

$$\underline{\underline{\det(A) = -872}} \quad \leftarrow$$

$$\begin{vmatrix} 2 & 10 & 0 \\ 16 & 6 & 14 \\ 12 & -4 & 18 \end{vmatrix} = 2 \begin{vmatrix} 6 & 14 \\ -4 & 18 \end{vmatrix} - 10 \begin{vmatrix} 16 & 14 \\ 12 & 18 \end{vmatrix} + 0 \begin{vmatrix} 16 & 6 \\ 12 & -4 \end{vmatrix} \\ = 2[(6)(18) - (14)(-4)] - 10[(16)(18) - (14)(12)] + 0$$

$$\underline{\underline{\det(A) = -872}} \quad \leftarrow$$

matrix [B] is singular because elements of second row and first row are linearly dependent.

$$\underline{\underline{\det(B) = 0}} \quad \leftarrow$$

This result can be shown by direct expansion as well.

b.

$$\underline{\underline{\det([A]^T) = \det(A) = -872}} \quad \leftarrow$$

2.5
Cont.

$$c. \det(5[A]) = \begin{vmatrix} 10 & 50 & 0 \\ 80 & 30 & 70 \\ 60 & -20 & 90 \end{vmatrix} = (10)(30)(90) + (50)(70)(60) + 0 \\ - (50)(80)(90) - (10)(70)(-20) - 0$$

$$\underline{\underline{\det(5[A]) = -109000}}$$

Since matrix $[A]$ is 3×3 , alternatively,

$$\det(5[A]) = 5^3 \det(A) = (125)(-872) = \underline{\underline{-109000}}$$

2.6

$$\det(A) = \begin{vmatrix} 0 & 5 & 0 \\ 8 & 3 & 7 \\ 9 & -2 & 9 \end{vmatrix} = (0)(3)(9) + (5)(7)(9) + (0)(8)(-2) \\ - (5)(8)(9) - (0)(7)(-2) - (0)(3)(9)$$

$$\underline{\underline{\det(A) = -45}} \quad \leftarrow$$

$$\underline{\underline{\det([A]^T) = \det(A) = -45}} \quad \leftarrow$$

2.7

$$\begin{bmatrix} 10875000 & -1812500 & 0 \\ -1812500 & 6343750 & -4531250 \\ 0 & -4531250 & 4531250 \end{bmatrix} \begin{Bmatrix} u_2 \\ u_3 \\ u_4 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 800 \end{Bmatrix}$$

Following the steps discussed in Section 2.7, we get

$$\begin{Bmatrix} u_2 \\ u_3 \\ u_4 \end{Bmatrix} = 10^{-3} \begin{Bmatrix} 0.0883 \\ 0.5297 \\ 0.7062 \end{Bmatrix}$$

2.8

$$\begin{bmatrix} 0 & 5 & 0 \\ 8 & 3 & 7 \\ 9 & -2 & 9 \end{bmatrix}$$

Because of the zero elements in Row 1, the lower triangular matrix will not have a triangular form, instead it becomes

$$[L] = \begin{bmatrix} 0 & 1.0000 & 0 \\ 0.8889 & 0.9556 & 1.0000 \\ 1.0000 & 0 & 0 \end{bmatrix}$$

$$[U] = \begin{bmatrix} 9 & -2 & 9 \\ 0 & 5 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

Check:

$$[L][U] = \begin{bmatrix} 0 & 1.0000 & 0 \\ 0.8889 & 0.9556 & 1.0000 \\ 1.0000 & 0 & 0 \end{bmatrix} \begin{bmatrix} 9 & -2 & 9 \\ 0 & 5 & 0 \\ 0 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 0 & 5 & 0 \\ 8 & 3 & 7 \\ 9 & -2 & 9 \end{bmatrix}$$

2.9

$$[A] = \begin{bmatrix} 10875000 & -1812500 & 0 \\ -1812500 & 6343750 & -4531250 \\ 0 & -4531250 & 4531250 \end{bmatrix}$$

$$\{b\} = \begin{Bmatrix} 0 \\ 0 \\ 800 \end{Bmatrix}$$

$$[L] = \begin{bmatrix} 1.0000 & 0 & 0 \\ -0.1667 & 1.0000 & 0 \\ 0 & -0.7500 & 1.0000 \end{bmatrix}$$

$$[u] = 10^7 \begin{bmatrix} 1.0875 & -0.1812 & 0 \\ 0 & 0.6042 & -0.4531 \\ 0 & 0 & 0.1133 \end{bmatrix}$$

$$\{z\} = [L]^{-1} \{b\} = \begin{bmatrix} 1.0000 & 0 & 0 \\ -0.1667 & 1.0000 & 0 \\ 0 & -0.7500 & 1.0000 \end{bmatrix}^{-1} \begin{Bmatrix} 0 \\ 0 \\ 800 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 800 \end{Bmatrix}$$

$$\{U\} = [u]^{-1} \{z\} = 10^7 \begin{bmatrix} 1.0875 & -0.1812 & 0 \\ 0 & 0.6042 & -0.4531 \\ 0 & 0 & 0.1133 \end{bmatrix}^{-1} \begin{Bmatrix} 0 \\ 0 \\ 800 \end{Bmatrix}$$

$$\{U\} = 10^{-3} \begin{Bmatrix} 0.0883 \\ 0.5297 \\ 0.7062 \end{Bmatrix}$$

Note the difference between **u** denoting upper triangular matrix and **U** denoting the displacement results

2.10

$$[A] = \begin{bmatrix} 10875000 & -1812500 & 0 \\ -1812500 & 6343750 & -4531250 \\ 0 & -4531250 & 4531250 \end{bmatrix}$$

$$\{b\} = \begin{Bmatrix} 0 \\ 0 \\ 800 \end{Bmatrix}$$

$$[A]^{-1} = 10^{-6} \begin{bmatrix} 0.1103 & 0.1103 & 0.1103 \\ 0.1103 & 0.6621 & 0.6621 \\ 0.1103 & 0.6621 & 0.8828 \end{bmatrix}$$

$$\begin{Bmatrix} u_2 \\ u_3 \\ u_4 \end{Bmatrix} = [A]^{-1} \{b\} = 10^{-6} \begin{bmatrix} 0.1103 & 0.1103 & 0.1103 \\ 0.1103 & 0.6621 & 0.6621 \\ 0.1103 & 0.6621 & 0.8828 \end{bmatrix} \begin{Bmatrix} 0 \\ 0 \\ 800 \end{Bmatrix} = 10^{-3} \begin{Bmatrix} 0.0883 \\ 0.5297 \\ 0.7062 \end{Bmatrix}$$

2.11

(a) Using Gaussian method

$$\begin{aligned}x_1 + x_2 + x_3 &= 6 \\2x_1 + 5x_2 + x_3 &= 15 \\-3x_1 + x_2 + 5x_3 &= 14\end{aligned}$$

$$\left\{ \begin{array}{l} 2x_1 + 5x_2 + x_3 = 15 \\ -2x_1 - 2x_2 - 2x_3 = -12 \end{array} \right. \\ \hline 3x_2 - x_3 = 3$$

$$\left\{ \begin{array}{l} -3x_1 + x_2 + 5x_3 = 14 \\ 3x_1 + 3x_2 + 3x_3 = 18 \end{array} \right. \\ \hline 4x_2 + 8x_3 = 32$$

$$\left\{ \begin{array}{l} 3x_2 - x_3 = 3 \\ 4x_2 + 8x_3 = 32 \end{array} \right. \rightarrow \begin{array}{l} x_2 - \frac{1}{3}x_3 = 1 \\ 4x_2 + 8x_3 = 32 \end{array}$$

$$\left\{ \begin{array}{l} 4x_2 + 8x_3 = 32 \\ -4x_2 + \frac{4}{3}x_3 = -4 \end{array} \right.$$

$$\frac{28}{3}x_3 = 28 \rightarrow \underline{\underline{x_3 = 3}} \quad \leftarrow$$

$$x_2 = 1 + \frac{1}{3}x_3 = 1 + \frac{1}{3}(3) = 2 \quad \underline{\underline{x_2 = 2}} \quad \leftarrow$$

$$x_1 = 6 - x_2 - x_3 = 6 - 2 - 3 = 1 \quad \underline{\underline{x_1 = 1}} \quad \leftarrow$$

2.11
Cont.

(b) Using the LU decomposition method

$$[A] = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 5 & 1 \\ -3 & 1 & 5 \end{bmatrix} \quad \{b\} = \begin{Bmatrix} 6 \\ 15 \\ 14 \end{Bmatrix}$$

$$u_{11} = a_{11} = 1 \quad u_{12} = a_{12} = 1 \quad u_{13} = a_{13} = 1$$

$$l_{21} = \frac{a_{21}}{u_{11}} = \frac{2}{1} = 2 \quad l_{31} = \frac{a_{31}}{u_{11}} = \frac{-3}{1} = -3$$

$$u_{22} = a_{22} - l_{21}u_{12} = 5 - (2)(1) = 3$$

$$u_{23} = a_{23} - l_{21}u_{13} = 1 - (2)(1) = -1$$

$$l_{32} = \frac{a_{32} - l_{31}u_{12}}{u_{22}} = \frac{1 - (-3)(1)}{3} = \frac{4}{3}$$

$$u_{33} = a_{33} - (l_{31}u_{13} + l_{32}u_{23}) = 5 - [(-3)(1) + (\frac{4}{3})(-1)] = \frac{28}{3}$$

∴

$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & 5 & 1 \\ -3 & 1 & 5 \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -3 & \frac{4}{3} & 1 \end{bmatrix}}_{[L]} \underbrace{\begin{bmatrix} 1 & 1 & 1 \\ 0 & 3 & -1 \\ 0 & 0 & \frac{28}{3} \end{bmatrix}}_{[U]}$$

$$[L]\{z\} = \{b\} \quad \rightarrow \quad \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -3 & \frac{4}{3} & 1 \end{bmatrix} \begin{Bmatrix} z_1 \\ z_2 \\ z_3 \end{Bmatrix} = \begin{Bmatrix} 6 \\ 15 \\ 14 \end{Bmatrix}$$

$$\{z\} = \begin{Bmatrix} 6 \\ 3 \\ 28 \end{Bmatrix}$$

$$[U]\{x\} = \{z\} \quad \rightarrow \quad \begin{bmatrix} 1 & 1 & 1 \\ 0 & 3 & -1 \\ 0 & 0 & \frac{28}{3} \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{Bmatrix} 6 \\ 3 \\ 28 \end{Bmatrix}$$

$$\{x\} = \begin{Bmatrix} 1 \\ 2 \\ 3 \end{Bmatrix} \quad \leftarrow$$

2.11
Cont.

(c) by finding the inverse of the coefficient matrix

$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & 5 & 1 \\ -3 & 1 & 5 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{Bmatrix} 6 \\ 15 \\ 14 \end{Bmatrix}$$

$$\begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 5 & 1 \\ -3 & 1 & 5 \end{bmatrix}^{-1} \begin{Bmatrix} 6 \\ 15 \\ 14 \end{Bmatrix}$$

$$\begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{bmatrix} 0.8571 & -0.1429 & -0.1429 \\ -0.4643 & 0.2857 & 0.0357 \\ 0.6071 & -0.1429 & 0.1071 \end{bmatrix} \begin{Bmatrix} 6 \\ 15 \\ 14 \end{Bmatrix}$$

$$\begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{Bmatrix} 1 \\ 2 \\ 3 \end{Bmatrix} \quad \leftarrow$$

$$[A]^{-1} = \begin{bmatrix} \frac{1}{5} & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{80} & 0 \\ 0 & 0 & 0 & \frac{1}{4} \end{bmatrix} \quad \leftarrow$$

$$[B]^{-1} = \begin{bmatrix} 0.8571 & -0.1429 & -0.1429 \\ -0.4643 & 0.2857 & 0.0357 \\ 0.6071 & -0.1429 & 0.1071 \end{bmatrix} \quad \leftarrow$$

$$[C] = \begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix}$$

$$\begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix} \begin{bmatrix} X_{11} & X_{12} \\ X_{21} & X_{22} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad [C]^{-1}$$

$$\begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix} \begin{Bmatrix} X_{11} \\ X_{21} \end{Bmatrix} = \begin{Bmatrix} 1 \\ 0 \end{Bmatrix} \quad \rightarrow \quad X_{11} = \frac{K_{22}}{K_{11}K_{22} - K_{12}K_{21}}$$

$$X_{21} = \frac{-K_{21}}{K_{11}K_{22} - K_{12}K_{21}} \quad \leftarrow$$

$$\begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix} \begin{Bmatrix} X_{12} \\ X_{22} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 1 \end{Bmatrix} \quad \rightarrow \quad X_{12} = \frac{-K_{12}}{K_{11}K_{22} - K_{12}K_{21}} \quad \leftarrow$$

$$X_{22} = \frac{K_{11}}{K_{11}K_{22} - K_{12}K_{21}}$$

2.13

For a 2x2 matrix:

$$\det(\alpha \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}) \stackrel{?}{=} \alpha^2 \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$$

$$\alpha \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \begin{bmatrix} \alpha a_{11} & \alpha a_{12} \\ \alpha a_{21} & \alpha a_{22} \end{bmatrix}$$

$$\det \begin{bmatrix} \alpha a_{11} & \alpha a_{12} \\ \alpha a_{21} & \alpha a_{22} \end{bmatrix} = (\alpha a_{11})(\alpha a_{22}) - (\alpha a_{12})(\alpha a_{21})$$

$$\det(\alpha \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}) = \alpha^2 (a_{11}a_{22} - a_{12}a_{21}) = \alpha^2 \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$$

For a 3x3 matrix:

Q.E.D.

$$\det(\alpha \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}) \stackrel{?}{=} \alpha^3 \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

$$\det \begin{bmatrix} \alpha a_{11} & \alpha a_{12} & \alpha a_{13} \\ \alpha a_{21} & \alpha a_{22} & \alpha a_{23} \\ \alpha a_{31} & \alpha a_{32} & \alpha a_{33} \end{bmatrix} = (\alpha a_{11})(\alpha a_{22})(\alpha a_{33}) + (\alpha a_{12})(\alpha a_{23})(\alpha a_{31}) + \dots - (\alpha a_{13})(\alpha a_{22})(\alpha a_{31})$$

$$\det(\alpha \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}) = \alpha^3 \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

Q.E.D.

In general for a nxn matrix, we have

$$\det(\alpha \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{bmatrix}) = \alpha^n \begin{vmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{vmatrix}$$

2.14

Using Equation (2.83), we have

$$\begin{bmatrix} -\omega^2 + \frac{2K}{m_1} & -\frac{K}{m_1} \\ -\frac{K}{m_2} & -\omega^2 + \frac{2K}{m_2} \end{bmatrix} \begin{Bmatrix} X_1 \\ X_2 \end{Bmatrix} = 0$$

$$\begin{vmatrix} -\omega^2 + \frac{2(100)}{0.1} & -\frac{100}{0.1} \\ -\frac{100}{0.2} & -\omega^2 + \frac{2(100)}{0.2} \end{vmatrix} = 0$$

$$\omega^4 - 3000\omega^2 + 1.5 \times 10^6 = 0$$

$$\omega_1^2 = 2366 \text{ (rad/s)}^2$$

$$\omega_2^2 = 634 \text{ (rad/s)}^2$$

$$\omega_1 = 48.6 \text{ rad/s}$$

$$\omega_2 = 25.2 \text{ rad/s}$$



$$(-2366 + 2000)X_1 - 1000X_2 = 0 \quad \rightarrow \quad \frac{X_2}{X_1} = -0.366$$

$$(-634 + 2000)X_1 - 1000X_2 = 0 \quad \rightarrow \quad \frac{X_2}{X_1} = 1.366$$

2.15

```
>> a=[4 2 1;7 0 -7;1 -5 3]
```

```
a =
```

```
4 2 1
7 0 -7
1 -5 3
```

```
>> b=[1 2 -1;5 3 3;4 5 -7]
```

```
b =
```

```
1 2 -1
5 3 3
4 5 -7
```

```
>> c=[1;-2;4]
```

```
c =
```

```
1
-2
4
```

```
>> a+b
```

```
ans =
```

```
5 4 0
12 3 -4
5 0 -4
```

```
>> a-b
```

```
ans =
```

```
3 0 2
2 -3 -10
-3 -10 10
```

2.15
Cont.

```
>> 3*a
```

```
ans =
```

```
12  6  3  
21  0 -21  
3 -15  9
```

```
>> a*b
```

```
ans =
```

```
18 19 -5  
-21 -21 42  
-12 2 -37
```

```
>> a*c
```

```
ans =
```

```
4  
-21  
23
```

```
>> a*a
```

```
ans =
```

```
31  3 -7  
21 49 -14  
-28 -13 45
```

```
>> i=[1 0 0;0 1 0;0 0 1]
```

```
i =
```

```
1  0  0  
0  1  0  
0  0  1
```

2.15
Cont.

```
>> i*a
```

```
ans =
```

```
 4  2  1  
 7  0 -7  
 1 -5  3
```

```
>> a*i
```

```
ans =
```

```
 4  2  1  
 7  0 -7  
 1 -5  3
```

```
>>
```

2.16

```
>> A=[1 4 2;8 3 6;7 1 -2]
```

```
A =
```

```
1 4 2
8 3 6
7 1 -2
```

```
>> B=[0 5 -1;-3 1 7;2 4 -4]
```

```
B =
```

```
0 5 -1
-3 1 7
2 4 -4
```

```
>> A'
```

```
ans =
```

```
1 8 7
4 3 1
2 6 -2
```

```
>> B'
```

```
ans =
```

```
0 -3 2
5 1 4
-1 7 -4
```

```
>> (A+B)'
```

```
ans =
```

```
1 5 9
9 4 5
1 13 -6
```

2.16
Cont.

>> A'+B'

ans =

1 5 9
9 4 5
1 13 -6

>> (A*B)'

ans =

-8 3 -7
17 67 28
19 -11 8

>> B'*A'

ans =

-8 3 -7
17 67 28
19 -11 8

>>

2.17

```
>> A=[2 10 0;16 6 14;12 -4 18]
```

```
A =
```

```
 2 10  0  
16  6 14  
12 -4 18
```

```
>> B=[2 10 0;4 20 0;12 -4 18]
```

```
B =
```

```
 2 10  0  
 4 20  0  
12 -4 18
```

```
>> det(A)
```

```
ans =
```

```
-872
```

```
>> det(B)
```

```
ans =
```

```
0
```

```
>> det((A)')
```

```
ans =
```

```
-872
```

```
>> det(5*(A))
```

```
ans =
```

```
-109000
```

```
>>
```

2.18

```
>> A=[0 5 0;8 3 7;9 -2 9]
```

```
A =
```

```
0 5 0  
8 3 7  
9 -2 9
```

```
>> det(A)
```

```
ans =
```

```
-45
```

```
>> det((A)')
```

```
ans =
```

```
-45
```

```
>>
```

2.19

```
>> A=[10875000 -1812500 0;-1812500 6343750 -4531250;0 -4531250 4531250]
```

```
A =
```

```
10875000 -1812500 0
-1812500 6343750 -4531250
0 -4531250 4531250
```

```
>> b=[0;0;800]
```

```
b =
```

```
0
0
800
```

```
>> x=A\b
```

```
x =
```

```
1.0e-003 *
```

```
0.0883
0.5297
0.7062
```

```
>>
```


2.20

```
>> A=[0 5 0;8 3 7;9 -2 9]
```

```
A =
```

```
0 5 0
8 3 7
9 -2 9
```

```
>> [l,u]=lu(A)
```

```
l =
```

```
0 1.0000 0
0.8889 0.9556 1.0000
1.0000 0 0
```

```
u =
```

```
9 -2 9
0 5 0
0 0 -1
```

```
>> l*u
```

```
ans =
```

```
0 5 0
8 3 7
9 -2 9
```

2.21

```
>> A=[10875000 -1812500 0;-1812500 6343750 -4531250;0 -4531250 4531250]
```

```
A =
```

```
10875000 -1812500 0
-1812500 6343750 -4531250
0 -4531250 4531250
```

```
>> b=[0;0;800]
```

```
b =
```

```
0
0
800
```

```
>> [l,u]=lu(A)
```

```
l =
```

```
1.0000 0 0
-0.1667 1.0000 0
0 -0.7500 1.0000
```

```
u =
```

```
1.0e+007 *
```

```
1.0875 -0.1812 0
0 0.6042 -0.4531
0 0 0.1133
```

```
>> z=inv(l)*b
```

```
z =
```

```
0
0
800
```

```
>> U=inv(u)*z
```

```
U =
```

```
1.0e-003 *
```

```
0.0883
0.5297
0.7062
```

Note the difference between **u** denoting upper triangular matrix and **U** denoting the displacement results

2.22

```
>> A=[10875000 -1812500 0;-1812500 6343750 -4531250;0 -4531250 4531250]
```

```
A =
```

```
10875000 -1812500 0
-1812500 6343750 -4531250
0 -4531250 4531250
```

```
>> b=[0;0;800]
```

```
b =
```

```
0
0
800
```

```
>> Ainverse=inv(A)
```

```
Ainverse =
```

```
1.0e-006 *
```

```
0.1103 0.1103 0.1103
0.1103 0.6621 0.6621
0.1103 0.6621 0.8828
```

```
>> u=Ainverse*b
```

```
u =
```

```
1.0e-003 *
```

```
0.0883
0.5297
0.7062
```

```
>>
```

2.23

```
>> A=[1 1 1;2 5 1;-3 1 5]
```

```
A =
```

```
1 1 1
2 5 1
-3 1 5
```

```
>> b=[6 15 14]
```

```
b =
```

```
6 15 14
```

```
>> b=[6;15;14]
```

```
b =
```

```
6
15
14
```

(a) using the Gaussian method

```
>> x=A\b
```

```
x =
```

```
1.0000
2.0000
3.0000
```

(b) using the LU decomposition method

```
>> [l,u]=lu(A)
```

```
l =
```

```
-0.3333 0.2353 1.0000
-0.6667 1.0000 0
1.0000 0 0
```

```
u =
```

```
-3.0000 1.0000 5.0000
0 5.6667 4.3333
0 0 1.6471
```

2.23
Cont.

```
>> z=inv(l)*b
```

```
z =
```

```
14.0000  
24.3333  
4.9412
```

```
>> x=inv(u)*z
```

```
x =
```

```
1.0000  
2.0000  
3.0000
```

(c) by finding the inverse of the coefficient matrix

```
>> x=inv(A)*b
```

```
x =
```

```
1.0000  
2.0000  
3.0000
```

```
>>
```

2.24

For example, consider the following 4 x 4 matrix, and $\alpha = 2$ and $\alpha = 3$.

```
>> A=[1 2 1 3;2 1 4 1;5 3 0 1;4 1 5 7]
```

A =

```
1 2 1 3
2 1 4 1
5 3 0 1
4 1 5 7
```

```
>> det(A)
```

ans =

219

```
>> det(2*A)
```

ans =

3504

Since matrix A is 4 x 4 then let us examine to see if $\det(2*A) = 2^4 * \det(A)$?

```
>> 2^4*det(A)
```

ans =

3504

```
>> det(3*A)
```

ans =

17739

Or is $\det(3*A) = 3^4 * \det(A)$?

```
>> 3^4*det(A)
```

ans =

17739

2.24
Cont.

Let us now consider the following 3 x 3 matrix, and $\alpha = 2$ and $\alpha = 3$.

```
>> B=[1 2 1;2 1 4;5 3 0]
```

```
B =
```

```
    1    2    1  
    2    1    4  
    5    3    0
```

```
>> det(B)
```

```
ans =
```

```
    29
```

```
>> det(2*B)
```

```
ans =
```

```
   232
```

Is $\det(2*B) = 2^3 * \det(B)$?

```
>> 2^3*det(B)
```

```
ans =
```

```
   232
```

```
>> det(3*B)
```

```
ans =
```

```
   783
```

Or is $\det(3*B) = 3^3 * \det(B)$?

```
>> 3^3*det(B)
```

```
ans =
```

```
   783
```

```
>>
```

2.25

```
>> A=[7.11 -1.23 0 0 0;-1.23 1.99 -0.76 0 0;0 -0.76 0.851 -0.091 0;0 0 -0.091 2.311 -2.22;0 0 0 -2.22 3.69]
```

A =

```
7.1100 -1.2300 0 0 0
-1.2300 1.9900 -0.7600 0 0
0 -0.7600 0.8510 -0.0910 0
0 0 -0.0910 2.3110 -2.2200
0 0 0 -2.2200 3.6900
```

```
>> b=[5.88*20; 0; 0; 0; 1.47*70]
```

b =

```
117.6000
0
0
0
102.9000
```

```
>> T=A\b
```

T =

```
20.5898
23.4091
27.9719
66.0789
67.6410
```

```
>>
```


2.26

```
>> A=[7.11 -1.23 0 0 0;-1.23 1.99 -0.76 0 0;0 -0.76 0.851 -0.091 0;0 0 -0.091 2.311 -2.22;0 0 0 -2.22 3.69]
```

```
A =
```

```
7.1100 -1.2300 0 0 0
-1.2300 1.9900 -0.7600 0 0
0 -0.7600 0.8510 -0.0910 0
0 0 -0.0910 2.3110 -2.2200
0 0 0 -2.2200 3.6900
```

```
>> b=[5.88*20; 0; 0; 0; 1.47*70]
```

```
b =
```

```
117.6000
0
0
0
102.9000
```

```
>> Ainverse=inv(A)
```

```
Ainverse =
```

```
0.1681 0.1585 0.1430 0.0133 0.0080
0.1585 0.9160 0.8263 0.0771 0.0464
0.1430 0.8263 1.9323 0.1803 0.1085
0.0133 0.0771 0.1803 1.0421 0.6269
0.0080 0.0464 0.1085 0.6269 0.6482
```

```
>> T=Ainverse*b
```

```
T =
```

```
20.5898
23.4091
27.9719
66.0789
67.6410
```

2.27

```
>> A=[7.11 -1.23 0 0 0;-1.23 1.99 -0.76 0 0;0 -0.76 0.851 -0.091 0;0 0 -0.091 2.311 -2.22;0 0 0 -2.22 3.69]
```

A =

```
7.1100 -1.2300 0 0 0
-1.2300 1.9900 -0.7600 0 0
0 -0.7600 0.8510 -0.0910 0
0 0 -0.0910 2.3110 -2.2200
0 0 0 -2.2200 3.6900
```

```
>> b=[5.88*20; 0; 0; 0; 1.47*70]
```

b =

```
117.6000
0
0
0
102.9000
```

```
>> [l,u]=lu(A)
```

l =

```
1.0000 0 0 0 0
-0.1730 1.0000 0 0 0
0 -0.4276 1.0000 0 0
0 0 -0.1730 1.0000 0
0 0 0 -0.9672 1.0000
```

u =

```
7.1100 -1.2300 0 0 0
0 1.7772 -0.7600 0 0
0 0 0.5260 -0.0910 0
0 0 0 2.2953 -2.2200
0 0 0 0 1.5428
```

```
>> z=inv(l)*b
```

z =

```
117.6000
20.3443
8.6999
1.5051
104.3558
```

```
>> T=inv(u)*z
```

T =

```
20.5898
23.4091
27.9719
66.0789
67.6410
```

```
>>
```

```
>> A=[1 0 0 0 0;-0.0408 0.0888 -0.0408 0 0;0 -0.0408 0.0888 -0.0408 0;0 0 -0.0408 0.0888 -0.0408;  
0 0 0 -0.0408 0.04455]
```

A =

1.0000	0	0	0	0
-0.0408	0.0888	-0.0408	0	0
0	-0.0408	0.0888	-0.0408	0
0	0	-0.0408	0.0888	-0.0408
0	0	0	-0.0408	0.0445

```
>> b=[100;0.144;0.144;0.144;0.075]
```

b =

100.0000
0.1440
0.1440
0.1440
0.0750

```
>> T=A\b
```

T =

100.0000
75.0387
59.7901
51.5633
48.9064

```
>>
```

2.29

```
>> A=10^5*[7.2 0 0 0 -1.49 -1.49;0 7.2 0 -4.22 -1.49 -1.49;0 0 8.44 0 -4.22 0;0 -4.22 0 4.22 0 0;-1.49 -1.49 -4.22 0 5.71 1.49;-1.49 -1.49 0 0 1.49 1.49]
```

A =

720000	0	0	0	-149000	-149000
0	720000	0	-422000	-149000	-149000
0	0	844000	0	-422000	0
0	-422000	0	422000	0	0
-149000	-149000	-422000	0	571000	149000
-149000	-149000	0	0	149000	149000

```
>> b=[0;0;0;-500;0;-500]
```

b =

0
0
0
-500
0
-500

```
>> U=A\b
```

U =

-0.0036
-0.0103
0.0012
-0.0115
0.0024
-0.0195

```
>>
```

2.30

```
>> A=[2000 -1000;-500 1000]
```

```
A =
```

```
    2000   -1000  
   -500    1000
```

The eigenvalues are:

```
>> eig(A)
```

```
ans =
```

```
1.0e+003 *  
    2.3660  
    0.6340
```

Note the natural frequencies of the system are equal to the square root of the eigenvalues.

```
>> sqrt(eig(A))
```

```
ans =
```

```
    48.6418  
    25.1789
```

The eigenvector and eigenvalues are given by:

```
>> [v,e]=eig(A)
```

```
v =
```

```
    0.9391    0.5907  
   -0.3437    0.8069
```

```
e =
```

```
1.0e+003 *  
    2.3660     0  
     0    0.6340
```

2.30
Cont.

Normalizing the eigenvector with respect to X_1 , we get:

>> -0.3437/0.9391

ans =

-0.3660

Therefore, the first mode is given by $X_2/X_1 = -0.3660$.

>> .8069/0.5907

ans =

1.3660

The second mode is then given by $X_2/X_1 = 1.3660$.

Problem 2-31

$$[K] = \begin{bmatrix} 7.11 & -1.23 & 0 & 0 & 0 \\ -1.23 & 1.99 & -0.76 & 0 & 0 \\ 0 & -0.76 & 0.851 & -0.091 & 0 \\ 0 & 0 & -0.091 & 2.31 & -2.22 \\ 0 & 0 & 0 & -2.22 & 3.69 \end{bmatrix}$$

$$[K]^{-1} = \begin{bmatrix} 0.1681 & 0.1585 & 0.143 & 0.0134 & 0.008 \\ 0.1585 & 0.9161 & 0.8264 & 0.0772 & 0.0464 \\ 0.143 & 0.8264 & 1.9324 & 0.1805 & 0.1086 \\ 0.0134 & 0.0772 & 0.1805 & 1.0431 & 0.6276 \\ 0.008 & 0.0464 & 0.1086 & 0.6276 & 0.6486 \end{bmatrix}$$

$$\{T\} = \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \end{bmatrix}$$

$$\{T\} = \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \end{bmatrix}$$

$$[F] = \begin{bmatrix} 117.6 \\ 0 \\ 0 \\ 0 \\ 102.9 \end{bmatrix}$$

$$= [K]^{-1} [F] = \begin{bmatrix} 20.59 \\ 23.41 \\ 27.98 \\ 66.15 \\ 67.68 \end{bmatrix}$$

Problem 2-32

$$[K] = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ -0.041 & 0.0888 & -0.0408 & 0 & 0 \\ 0 & -0.0408 & 0.0888 & -0.041 & 0 \\ 0 & 0 & -0.0408 & 0.0888 & -0.0408 \\ 0 & 0 & 0 & -0.041 & 0.04455 \end{bmatrix}$$

$$[K]^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0.688 & 16.8623 & 12.191 & 9.6701 & 8.85613 \\ 0.4974 & 12.1906 & 26.532 & 21.047 & 19.2751 \\ 0.3945 & 9.67011 & 21.047 & 36.137 & 33.0956 \\ 0.3613 & 8.85613 & 19.275 & 33.096 & 52.7564 \end{bmatrix}$$

$$\{T\} = \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \end{bmatrix}$$

$$[F] = \begin{bmatrix} 100 \\ 0.144 \\ 0.144 \\ 0.144 \\ 0.075 \end{bmatrix}$$

$$\{T\} = \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \end{bmatrix}$$

$$= [K]^{-1} [F] = \begin{bmatrix} 100.00 \\ 75.04 \\ 59.79 \\ 51.56 \\ 48.91 \end{bmatrix}$$

Problem 2-33

$$[K] = \begin{bmatrix} 7.2 & 0 & 0 & 0 & -1.49 & -1.49 \\ 0 & 7.2 & 0 & -4.22 & -1.49 & -1.49 \\ 0 & 0 & 8.44 & 0 & -4.22 & 0 \\ 0 & -4.22 & 0 & 4.22 & 0 & 0 \\ -1.49 & -1.49 & -4.22 & 0 & 5.71 & 1.49 \\ -1.49 & -1.49 & 0 & 0 & 1.49 & 1.49 \end{bmatrix}$$

$$\{T\} = \begin{bmatrix} U2x \\ U2y \\ U4x \\ U4y \\ U5x \\ U5y \end{bmatrix}$$

$$[F] = \begin{bmatrix} 0 \\ 0 \\ 0 \\ -0.005 \\ 0 \\ -0.005 \end{bmatrix}$$

$$[K]^{-1} = \begin{bmatrix} 0.23697 & 0.23697 & 0 & 0.237 & 0 & 0.473934 \\ 0.23697 & 0.90811 & 0 & 0.9081 & 0 & 1.145075 \\ 0 & -1E-17 & 0.237 & -1E-17 & 0.23697 & -0.23697 \\ 0.23697 & 0.90811 & 0 & 1.1451 & 0 & 1.145075 \\ 0 & -2E-17 & 0.237 & -2E-17 & 0.47393 & -0.47393 \\ 0.47393 & 1.14507 & -0.237 & 1.1451 & -0.4739 & 2.764083 \end{bmatrix}$$

$$\{U\} = \begin{bmatrix} U2x \\ U2y \\ U4x \\ U4y \\ U5x \\ U5y \end{bmatrix}$$

$$= [K]^{-1} [F] = \begin{bmatrix} -0.0036 \\ -0.0103 \\ 0.0012 \\ -0.0115 \\ 0.0024 \\ -0.0195 \end{bmatrix}$$