Chapter 1

Lesson 1-1 Discretionary and Essential Expenses

Check Your Understanding (Example 1)

Add the book prices and divide by the number of prices being compared.

$$\frac{128 + 118 + 96 + 102 + 100 + 118 + 118 + 102}{8} = 110.25$$

The mean of the textbook prices is $110.25.

Check Your Understanding (Example 2)

All parts of the sigma notation must be complete. Since there are 12 bills for the year, the index must go from 1 to 12. The fraction 1/12 before the sigma notation indicates a division of the sum by 12 which is the definition of the average for this example.

$$\bar{x} = \frac{1}{12} \sum_{i=1}^{12} x_i = \$29$$

Extend Your Understanding (Example 2)

The second through seventh month covers a 6-month period. The index goes from 2 to 12 and the sum is multiplied by 1/6 in the average formula shown here.

$$\bar{x} = \frac{1}{6} \sum_{i=2}^{7} x_i$$

Check Your Understanding (Example 3)

Answers will vary. Explanation should include the fact that as long as the number of scores less than $798 is equal to the number of scores greater than $798, the median can be the same and the scores can be different.

Check Your Understanding (Example 4)

Answers will vary. The two middle scores must have a mean of 817. The number of scores below the median must equal the number of scores above the median.

Check Your Understanding (Example 5)

Answers will vary. 56 must appear the most in the data set in order to be the mode.

Check Your Understanding (Example 6)

Add all of the frequencies for prices below $320.

2 + 4 + 1 + 2 + 5 + 6 = 20

20

Check Your Understanding (Example 7)

There are 30 pieces of data in this data set. The chart arranges the data in numerical order. If the chart was written as a list, it would look like {250, 250, 275, 275, 275, 275, 280, …}. The price with the highest frequency (appears the most in the list), is the mode. The answer is $315.

Applications

1. Whether an item or service falls under the category of essential or discretionary, that expense is wasted unless it is something that you have need of or use for.

2a. $$\frac{65 + 50 + 44 + 86 + 90 + 50 + 35 + 110 + 70 + 50 + 35 + 60 + 56}{13} \approx \$61.62$$

2b. Arrange the fees in numerical order: 35, 35, 44, 50, 50, 50, 56, 60, 65, 70, 86, 90, 110. There are an odd number of fees. The median is the 7th fee in the numerical list, $56.

2c. $50 appears the most in the list. It is the mode.

3a. $$\frac{699 + 599 + 699 + 680 + 590 + 720 + 650 + 800}{8} \approx \$679.63$$

3b. Arrange the prices in numerical order : 590, 599, 650, 680, 699, 699, 720, 800. There are an even number of prices. The median is the average of the 4th and 5th prices, $689.50.

3c. $699 appears the most in the list. It is the mode.

4. Solve the equation where x represents 7th car price.

(15500 + 18800 + 16900 + 19900 + 18000 + 21000 + x)/7 = 18500

15500 + 18800 + 16900 + 19900 + 18000 + 21000 + x = 129500

110100 + x = 128500

x = $19,400

5a. (2800 + 1990 + 2005 + 2400 + 1860 + 2200 + 2000)/7 ≈ $2,179.29

5b. Slightly higher, about $70 higher

5c. Let *x* represent Ashley's amount.

(2800 + 1990 + 2005 + 2400 + 1860 + 2200 + 2000 + x)/8 = 2110

2800 + 1990 + 2005 + 2400 + 1860 + 2200 + 2000 + x = 16880

15255 + x = 16880

x = $1,625

6. All parts of the sigma notation must be complete. Since there are 12 bills for the year, the index must go from 1 to 12. The fraction 1/12 before the sigma notation indicates a division of the sum by 12 which is the definition of the average for this example.

$$\bar{x} = \frac{1}{12}\sum_{i=1}^{12} x_i = \$47.33$$

7. All parts of the sigma notation must be complete. Since there are 6 bills for the first half-year period, the index must go from 1 to 6. The fraction 1/6 before the sigma notation indicates a division of the sum by 6 which is the definition of the average for this example.

$$\bar{x} = \frac{1}{6}\sum_{i=1}^{6} x_i = \$43.00$$

8. All parts of the sigma notation must be complete. Since there are 8 bills for the period from April to November with April the 4th month and November the 11th month, the index must go from 4 to 11. The fraction 1/8 before the sigma notation indicates a division of the sum by 8 which is the definition of the average for this example.

$$\bar{x} = \frac{1}{8}\sum_{i=4}^{11} x_i = \$50.25$$

9. All parts of the sigma notation must be complete. It appears that July, August, and September have the highest consecutive bills. Since there are 3 bills for the period July to September with July the 7th month and September the 9th month, the index must go from 7 to 9. The fraction 1/3 before the sigma notation indicates a division of the sum by 3 which is the definition of the average for this example.

$$\bar{x} = \frac{1}{3}\sum_{i=7}^{9} x_i$$

10. Since there are 7 days in the week with Monday the 1st day and Friday the 7th day and the index goes from 1 to 7, this expression asks for the average of the daily cell phone usage in minutes. 43 is the average daily number of minutes used over the week.

11. Since the index goes from 2 to 5 which covers the 4-day period from Tuesday through Friday inclusive, this expression is asking for the average of the daily cell phone usage in minutes for those 4 days. 36 is the average daily number of minutes used from Tuesday through Friday.

12. It appears that the lowest daily minutes used over a 4-day period were from Wednesday through Saturday inclusive.

$$\bar{x} = \frac{1}{4}\sum_{i=3}^{6} x_i = \$33.75$$

13a. Count the number of seats in each seating category. These will be the frequencies for those categories.

Seat Type	Price	Number of Seats
First Class	$850	16
Deluxe Premium	$540	25
Preferred Plus	$400	33
Preferred	$320	16
Economy	$280	59

13b. 16 × 850 + 25 × 540 + 33 × 400 + 16 × 320 + 59 × 280 = $61,940

13c. Mean = (16 × 850 + 25 × 540 + 33 × 400 + 16 × 320 + 59 × 280)/149 ≈ $415.70

Median = $320; Use the frequency table. There are an odd number of seat prices for this flight. The median will occur at the middle price which is the 75th price when you count down from the highest or up from the lowest.

Mode = $280. The price of $280 has the highest frequency.

14a. There are 20 prices of cases sold. The index must go from 1 to 20 and the sum of the prices multiplied by 1/20.

$$\bar{x} = \frac{1}{20}\sum_{i=1}^{20} x_i = \$40.34$$

14b. Make a list of the case prices in numerical order. Count the number of occurrences in the list for each price. These numbers will be the frequencies for the respective prices.

b.

Price	Frequency
8.99	2
19.99	3
29.99	2
35.99	4
39.99	1
49.99	5
54.99	1
79.99	1
99.99	1

14c. $(2 \times 8.99 + 3 \times 19.99 + 2 \times 29.99 + 4 \times 35.99 + 39.99 + 5 \times 49.99 + 54.99 + 79.99 + 99.99)/20 = \$40.34$

14d. Median = \$35.99; There are 20 prices. The median will be the average of the 10th and 11th price. Mode = \$49.99; This is the price that appears the most in the list.

15. $$\frac{4.10(3) + 4.85(2) + 8x + 12 + 12.5(2)}{3 + 2 + x + 1 + 2}$$

$$= \frac{59 + 8x}{8 + x}$$

Lesson 1-2   Travel Expenses

Check Your Understanding (Example 1)

The cumulative frequency entry on the line $250 \leq m < 500$ indicates that 584,730,000 trips each less than 500 miles were taken.

Check Your Understanding (Example 2)

This represents two intervals on the chart. Add the relative frequencies for \$700–\$799.99 and \$800–\$899.99: 0.095 + 0.056 = 0.151. This represents 15.1% of the total bookings.

Check Your Understanding (Example 3)

= C2*100 or = B2/B12*100 The relative frequency in the chart is expressed as a decimal. It can be converted to a percent by multiplying by 100.

Check Your Understanding (Example 4)

54.5% of the fares were at or below \$499.99.

Extend Your Understanding (Example 4)

100(136 + 80 + 96)/1434 = 21.8%

Check Your Understanding (Example 5)

100(24/80) = 30%

Applications 1-2

1. Travel makes you richer by broadening your horizons and exposing you to new experiences. The quote starts out by speaking of a monetary expense. But making you "richer" does not necessarily mean monetarily richer.

2a.

Price Interval	Frequency	Cumulative Frequency
\$20–\$29.99	8	8
\$30–\$39.99	10	18
\$40–\$49.99	6	24
\$50–\$59.99	2	26
\$60–\$69.99	9	35
\$70–\$79.99	5	40

2b. Using the cumulative frequency entry for the \$30–\$39.99 category, there are 18 prices at or below \$39.99.

2c. Since there are a total of 40 prices, 18 of which are at or below \$39.99, there are 40 – 18, or 22 prices at or above \$40.

2d. There are 35 prices below \$70, 8 of which are at or below \$29.99. So, 35 – 8, or 27 prices are between \$29.99 and \$70.

3a.

Price Interval	Frequency	Cumulative Frequency	Relative Frequency
\$20–\$29.99	8	8	0.200
\$30–\$39.99	10	18	0.250
\$40–\$49.99	6	24	0.150
\$50–\$59.99	2	26	0.050
\$60–\$69.99	9	35	0.225
\$70–\$79.99	5	40	0.125

3b. Examine the relative frequency entries. The ranges $20–$29.99, $30–$39.99, and $60–$60.99 have a relative frequency greater than 0.18 and less than 0.27.

3c. Convert the relative frequency of 0.125 to a percent. 12.5% of the bus fares fell in this range.

4a.–4c.

| | 4a. | 4b. | 4c. |
2015 Month	# of Vehicles	Relative Frequency	Cumulative Frequency	Relative Cumulative Frequency
April	127,355	0.150	127,355	0.150
May	140,226	0.165	267,581	0.314
June	135,000	0.159	402,581	0.473
July	145,000	0.170	547,581	0.643
August	175,000	0.206	722,581	0.849
September	128,850	0.151	851,431	1.000

4d. $(127355 + 140226 + 135000 + 145000 + 175000 + 128850)/6 \approx 141{,}905$

4e. Arrange the vehicle numbers in numerical order: 127355, 128850, 135000, 140226, 145000, 175000. There are an even number of entries in the list. The median will be the average of the 3rd and 4th entry. $(135000 + 140226)/2 = 137{,}613$

4f. The total number of vehicles over the 6-month period is 851,431. The percentage of vehicles for July is $100(145000/851431) \approx 17\%$

4g. 47.3% — in the relative cumulative frequency column for June.

5a. The total of the entries in the frequency column is 80

5b. $3/80 \approx 0.038$

5c. $0.250; 20/80 = 0.250$

5d. $28; 4 + 2 + 3 + 12 + 7 = 28$

5e. $66; 4 + 2 + 3 + 12 + 7 + 2 + 4 + 20 + 5 + 7 = 66$ or $59 + 7 = 66$

5f. $0.425; 34/80 = 0.425$

5g. $1.000; 80/80 = 1.000$

6a. $12 + x; 5 + x + 3 + 4 = 12 + x$

6b. $\dfrac{5}{12+x}$ Frequency for $190 divided by the total $12 + x$

6c. $8 + x; 5 + x + 3 = 8 + x$

6d. $\dfrac{(8+x)}{(12+x)}; (5 + x + 3)/(5 + x + 3 + 4) = (8 + x)/(12 + x)$

7a. C9= SUM(C2:C8)

7b. D3 = C3/C9

7c. D7 = C7/C9

7d. E4 = SUM(C2:C4) or = E3 + C4

7e. F5 = E5/C9

8a. If C6 is decreased by 2, then the total in C9 will also be decreased by 2. Because every entry in the D column is dependent on the amount in C9, all cells will change in value. To see the change in the spreadsheet, rounding would have to be set to 3 decimal places.

8b. The frequencies in cells C2 through C5 would remain unchanged. Therefore, the cumulative frequencies in cells E2 through E5 also would remain unchanged. The cumulative frequencies in cells E6 through E8 would change.

8c. If C6 is decreased by 2, then the total in C9 also will be decreased by 2. The entries in cells F2 through F7 would change because those entries are dependent on the total in C9. To see the change in the spreadsheet, rounding would have to be set to 3 decimal places. The entry in cell F8 would not change since it represents 100% of the rooms.

9. $42.5\%; 100(34/80) = 42.5\%$

10. $75\%; 100(75/100) = 75\%$

11a. 30%. Approximately 30% of fares listed are at or below $119.

11b. 91.1%. Approximately 91.1% of the fares listed are at or below $272.

11c. 173. Since the fare of $272 has a percentile rank of 91%, check the fare numerically below it. There are 74 fares listed at or below $173. $100(74/90) \approx 82\%$

12a. $\dfrac{100a}{a+b+c+d+e+f}$

12b. $\dfrac{100(a+b)}{a+b+c+d+e+f}$

12c. $\dfrac{100(a+b+c)}{a+b+c+d+e+f}$

12d. $\dfrac{100(a+b+c+d)}{a+b+c+d+e+f}$

12e. $\dfrac{100(a+b+c+d+e)}{a+b+c+d+e+f}$

12f. 100%

Lesson 1-3 Entertainment Expenses

Check Your Understanding (Example 1)

60. Subtract the lowest number, 8, from the highest number, 68. The range is the difference between the highest and lowest data points.

Extend Your Understanding (Example 1)

The range would not change, since 34 and 43 were neither the maximum or minimum pieces of data.

Check Your Understanding (Example 2)

The mean is 35.4. The sum of the deviations is (8 − 35.4) + (12 − 35.4) + (34 − 35.4) + (55 − 35.4) + (68 − 35.4) = 0, and when divided by 5 the mean deviation is 0. The mean deviation for any distribution is always 0.

Extend Your Understanding (Example 2)

20.88. The mean is 35.4. The five absolute value deviations from the mean are 27.4, 23.4, 1.4, 19.6, and 32.6. The sum of these deviations is 104.4, so dividing by 5 gives mean absolute deviation 20.88. The mean absolute deviation must be greater than or equal to 0.

Check Your Understanding (Example 3)

63.44. The mean is 19.4 The data is substituted into the variance formula $\dfrac{\sum_{i=1}^{5}(x_i-\bar{x})^2}{5}$ where $n = 5$. The expression becomes $(11-19.4)^2$ + $(13-19.4)^2$ + $(15-19.4)^2$ + $(28-19.4)^2$ + $(30-19.4)^2$ and this can be plugged into a calculator in one keystroke sequence and then divided by 5; students do not need to get each separate addend's value.

Check Your Understanding (Example 4)

Number	$(x - \bar{x})$	$(x - \bar{x})^2$
11	−8.4	70.56
13	−6.4	40.96
15	−4.4	19.36
28	8.6	73.96
30	10.6	112.36

Add the last column. Divide by 5 to get the average. Then take the square root of the result. The standard deviation is 7.96 to the nearest hundredth.

Check Your Understanding (Example 5)

$5.57. The total frequency is 30. The mean is 12. Note that there are three different donations, but they all must be weighted by the number of people who made that donation. Students can use their calculator or make a table manually to find the variance:

Number (x_i)	Frequency (f)	$(x_i - \bar{x})$	$(x_i - \bar{x})^2$	$(x_i - \bar{x})^2 f$
5	6	−7	49	294
10	15	−2	4	60
20	9	8	64	576

The standard deviation formula is $\sqrt{\dfrac{\sum_{i=1}^{n}(x_i-\bar{x})^2}{n}}$

294 + 60 + 576 = 930.

930 ÷ 30 = 31 and the square root of 31 to the nearest hundredth is 5.57.

Extend Your Understanding (Example 5)

That column's sum is not the mean deviation since it does not include the frequency. That column is only the sum of 6 deviations, and there are 80 deviations since the frequency is 80.

Applications

1. Answers may vary. The gist of the quote is that sometimes, when entertainment venues get crowded, some people avoid going to them because they don't like the crowds. The charm in the expression is that it is oxymoronic to say "nobody goes there" if the venue is actually very crowded.

2. 13.964. To find the standard deviation given the variance, take the square root of the variance.

3a. $112.80. The 10 monthly bill amounts are added and divided by 10 to find the mean.

3b. $62. The highest bill is $150 and the lowest is $88 and the difference, which is the range, is 62.

3c. 456.76. The variance formula is $\dfrac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}$. The mean is 112.80, and $n = 10$. The variance is found by substitution.

3d. $21.37. The standard deviation is the square root of the variance, and the square root of 456.76, rounded to the nearest hundredth, is 21.37.

4a. $202.80. The costs of the five games are added and divided by 5 to find the mean.

4b. $104. The highest game cost is $266 and the least expensive game cost is $162, and the range is the difference, which is $104.

4c. 1,182.96. The variance formula is $\dfrac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}$. The mean is 202.80, and $n = 5$. The variance is found by substitution.

4d. $34.39. The standard deviation is the square root of the variance and the square root of 1,182.96, rounded to the nearest hundredth, is 34.39.

4e. $40.56. The mean cost, $202.80, is divided by 5 to find the cost per person.

5a. 4,133, 2,369, 1,295, 928, and 679. To view these large numbers as "batches" of 1,000, remove the three zeroes at the right end of each number.

5b. 1,880,800. The five numbers are added and the sum is divided by 5.

5c. 3,454,000. The highest number is 4,133,000 and the lowest number is 679,000. The range is the difference between these numbers, so the range is 3,454,000.

5d. 1,265,390. The standard deviation formula is

$$\sqrt{\dfrac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}}$$. The mean is 1,880,800 and $n = 5$. The standard deviation is found by substituting into the formula.

6. 25. The variance is the square of the standard deviation, and $5^2 = 25$.

7. The customers, because the standard deviation of 2 pounds is relatively more compared to the mean of 60 pounds than compared to a mean of 400 pounds. Although both distributions have a standard deviation of 2, they are not dispersed the same. Standard deviations need to be viewed with respect to their means.

8a. 40. The range is the difference between the minimum and maximum number. Since the minimum is 23 and the range is 17, 17 can be added to 23 to find the maximum, which is 40.

8b. $M - r$. The minimum score can be found by subtracting the range from the maximum score.

9. $82.30. If 25 members spend an average of $82 each then the total spent by all 25 members is found by multiplying 25 and 82. That product is 2,050. They then can add the 26th person's ticket price, 90, to the total to get 2,140. Divide the new total by 26 and round to the nearest cent to compute the new mean.

10a. $1,192. This is found by adding the four expenses in the cost column.

10b. $119.20. This is found by dividing the total cost by 10.

10c. 0. The standard deviation measures dispersion from the mean. If all of Ralph's paychecks are the same, then that number is the mean and the deviations of each paycheck from the mean are all 0.

11a. 20. The range is found by subtracting the minimum number, 30, from the maximum number, 50.

11b. The women did. In general, their deviations from the mean were higher than the men's.

12a. 5. The median is the middle number, after the numbers are arranged in order.

12b. 5.2. The mean is the sum of the five numbers divided by 5.

12c. 2.14. The standard deviation formula is

$$\sqrt{\dfrac{\sum\limits_{i=1}^{n}(x_i - \overline{x})^2}{n}}$$. The mean is 5.2 and $n = 5$. The standard deviation is found by substituting into the formula.

12d. Mean = 163.6; median = 5; standard deviation = 318.20. The mean is found by find the new sum, 818, by 5. The median is found by looking for the middle number after the numbers are ordered. Since 8 was the original distribution's maximum, and 8 is being replaced by 800, the middle number does not change. The standard deviation formula is

$$\sqrt{\dfrac{\sum\limits_{i=1}^{n}(x_i - \overline{x})^2}{n}}$$. The mean is now 163.6 and $n = 5$. The standard deviation is found by substituting into the formula.

12e. Median. The median is resistant to outliers because outliers occur at the ends of a distribution, and the median is the middle number.

13a. Theme park admissions because 3.99 < 4.

13b. The theme park admissions are more spread out because their standard deviation is greater relative to its mean than the home stereo prices.

14a. 0. If all numbers are the same, the maximum and minimum numbers are equal, so the range is 0.

14b. 0. The standard deviation measure deviation for the mean, and if all numbers are the same, the mean is that same number and the deviations are all 0.

14c. Yes. If the range is 0, then all the numbers are the same. In that case, all of the mean deviations are 0, and the standard deviation is, as a result, 0.

14d. The range is not as good a measure of dispersion since there is an extreme number and no other numbers near it, and the range ignores all the intermediate values.

Lesson 1-4 Vacation Expenses

Check Your Understanding (Example 1)

–0.67. A height of 54 represents 2/3 of a standard deviation below the mean.

Check Your Understanding (Example 2)

$13.80. The z-score formula has four variables in it. Given any three of them, the fourth variable can be solved for.

$$z = \dfrac{x - \mu}{\sigma}$$

Substitute: $1.5 = \dfrac{x - 12}{1.20}$

$(1.5)(1.20) = x - 12$

$1.8 = x - 12$

$x = 13.80$

Check Your Understanding (Example 3)

9.18%. Convert 60 to a z-score using the formula $z = \dfrac{x - \mu}{\sigma}$. Use substitution to find that the z-score for a raw score of 60 is $z = \dfrac{60 - 56}{3}$. To the nearest hundredth, this is equal to 1.33. Look at the row 1.3 on the z-score table and go to the 0.03 column to create 1.33. The table entry, .9082, is the area below the z-score of 1.33. Subtract .9082 from 1 to find the answer 9.18%. A common error is that students give the percent *below* 60 instead of *above* 60. Using a calculator the answer is 9.12%, and this is due to the ability of the calculator to not round the original z-score. These differences are inconsequential.

Extend Your Understanding (Example 3)

Convert 51 to a z-score using the formula $z = \dfrac{x - \mu}{\sigma}$. Use substitution to find that the z-score for a raw score of 51 is $z = \dfrac{51 - 56}{3}$. To the nearest hundredth, this is equal to –1.67. Look at the row –1.6 on the z-score table and go to the 0.07 column to create –1.67. The table entry, .0475, is the area below the z-score of –1.67. Subtract .0475 from 1 to find the answer 95.25%.

Check Your Understanding (Example 4)

21%. Convert 1,000 and 2,000 to z-scores using the formula $z = \dfrac{x - \mu}{\sigma}$. Use substitution to find that the z-score for a raw score of 1,000 is

$z = \dfrac{1000 - 2313}{390}$. To the nearest hundredth, this is equal to –3.37. The area below this z-score is, from the table, .0004. Use substitution to find that the z-score for a raw score of 2,000 is $z = \dfrac{2000 - 2313}{390}$.

This is equal to –0.80. The area below this z-score is .2119. Subtract the smaller area from the larger area.

.2119 – .0004 = .2115

To the nearest percent, this is 21%.

Check Your Understanding (Example 5)

74. This rating represents the 25th percentile. Look in the body of the z-score table for the number closest to .2500. That area is .2514, and it is the percent below a z-score of –0.67. Use the z-score formula to convert this to a raw score.

Substitute: $-0.67 = \dfrac{x - 78}{6.5}$

(–0.67)(6.5) = x – 78

–4.355 = x – 78

x = 73.645, which, when rounded to the nearest integer, is 74.

Applications

1. Answers may vary. Famous author Robert Louis Stevenson feels that the journey is as or more important than the destination. Travel on its own can be very interesting and exciting, and that doesn't necessarily depend on the actual destination.

2a. –0.83. Convert 197 to a z-score using the formula $z = \dfrac{x - \mu}{\sigma}$. Use substitution to find that the z-score for a raw score of 197 is

$z = \dfrac{197 - 247}{60}$. To the nearest hundredth, this is equal to –0.83.

2b. 0.5. Convert 277 to a z-score using the formula $z = \dfrac{x - \mu}{\sigma}$. Use substitution to find that the z-score for a raw score of 277 is

$z = \dfrac{277 - 247}{60}$. To the nearest hundredth, this is equal to 0.5.

2c. 1.05. Convert 310 to a z-score using the formula $z = \dfrac{x - \mu}{\sigma}$. Use substitution to find that the z-score for a raw score of 277 is

$z = \dfrac{310 - 247}{60}$. To the nearest hundredth, this is equal to 1.05.

3. $337. Use the z-score formula to convert this to a raw score.

Substitute: $1.5 = \dfrac{x - 247}{60}$

(1.5)(60) = x – 247

90 = x – 247

x = 337

4a. 9.12%. Convert 167 to a z-score using the formula $z = \dfrac{x - \mu}{\sigma}$. Use substitution to find that the z-score for a raw score of 167 is

$z = \dfrac{167 - 247}{60}$. To the nearest hundredth, this is equal to –1.33. Look at the row –1.3 on the z-score table and go to the 0.03 column to create –1.33. The table entry, .0918, is the area below the z-score of –1.33.

4b. 97.72%. Convert 367 to a z-score using the formula $z = \dfrac{x - \mu}{\sigma}$. Use substitution to find that the z-score for a raw score of 167 is

$z = \dfrac{367 - 247}{60}$. This is equal to 2. Look at the row 2.0 on the z-score table and go to the 0.00 column to create 2.00. The table entry, .9772, is the area below the z-score of 2.

4c. 50%. Since $247 is the mean, it is also the median, and occurs in the middle. Therefore, 50% of the numbers are below and 50% are above.

4d. 4.27%. Convert 350 to a z-score using the formula $z = \dfrac{x - \mu}{\sigma}$. Use substitution to find that the z-score for a raw score of 167 is $z = \dfrac{350 - 247}{60}$. This is equal to 1.72 to the nearest hundredth. Look at the row 1.7 on the z-score table and go to the 0.02 column to create 1.72. The table entry, .0973, is the area below the z-score of 1.72, and this must be subtracted from 1 to get the area above.

4e. 0.13%. Convert 67 to a z-score using the formula $z = \dfrac{x - \mu}{\sigma}$. Use substitution to find that the z-score for a raw score of 167 is $z = \dfrac{67 - 247}{60}$. This is equal to –3. Look at the row –3.0 on the z-score table and go to the 0.00 column to create –3.00. The table entry, .0013, is the area below the z-score of –3.

4f. 59.47%. Convert 200 and 300 to z-scores using the formula $z = \dfrac{x - \mu}{\sigma}$. Use substitution to find that the z-score for a raw score of 200 is $z = \dfrac{200 - 247}{60}$. To the nearest hundredth, this is equal to –0.78. The area below this z-score is, from the table, .2177. Use substitution to find that the z-score for a raw score of 300 is $z = \dfrac{300 - 247}{60}$. This is equal to –0.88 to the nearest hundredth. The area below this z-score is .8106. Subtract the smaller area from the larger area.

4g. 2.44%. Convert 360 and 400 to z-scores using the formula $z = \dfrac{x - \mu}{\sigma}$. Use substitution to find that the z-score for a raw score of 360 is $z = \dfrac{360 - 247}{60}$. To the nearest hundredth, this is equal to 1.88. The area below this z-score is, from the table, .9699. Use substitution to find that the z-score for a raw score of 400 is $z = \dfrac{400 - 247}{60}$. This is equal to 2.55. The area below this z-score is .9946. Subtract the smaller area from the larger area.

4h. 50%. The median is the middle number, and 50% of the numbers in the distribution lie below the median, and 50% lie above.

4i. 50%. In a normal distribution, the mean is equal to the median, and the median is the middle number, and 50% of the numbers in the distribution lie below the median, and 50% lie above.

5a. $2,470. The mean for one day, $247, is multiplied by 10.

5b. $600. The standard deviation is multiplied by 10, since multiplication changes the deviations from the mean.

5c. 360,000. The variance is the square of the standard deviation, and $600^2 = 360,000$.

6a. 0.65. Convert 400 to a z-score using the formula $z = \dfrac{x - \mu}{\sigma}$. Use substitution to find that the z-score for a raw score of 400 is $z = \dfrac{400 - 348}{80}$. To the nearest hundredth, this is equal to –0.65.

6b. 84.13%. Convert 268 to a z-score using the formula $z = \dfrac{x - \mu}{\sigma}$. Use substitution to find that the z-score for a raw score of 268 is $z = \dfrac{268 - 348}{80}$. This is equal to –1. Look at the row –1.0 on the z-score table and go to the 0.00 column to create –1.00. The table entry, .1587, is the area below the z-score of –1.00, and this must be subtracted from 1 to get the area above.

7a. 7.78%. Convert 1,200 to a z-score using the formula $z = \dfrac{x - \mu}{\sigma}$. Use substitution to find that the z-score for a raw score of 268 is $z = \dfrac{1200 - 1755}{390}$. This is equal to –1.42 to the nearest hundredth. Look at the row –1.4 on the z-score table and go to the 0.02 column to create –1.42. The table entry, .0778, is the area below the z-score of –1.42.

7b. $936. Use the z-score formula to convert this to a raw score.

Substitute: $-2.1 = \dfrac{x - 1755}{390}$

(–2.1)(390) = x – 1755

–819 = x – 1755

x = 936

7c. Lower. The z-score measures the number of standard deviations from the mean, and if the z-score stays the same and the standard deviation increases, the score is farther from the mean. Since the z-score is negative, it is lower.

8a. 2.28%. Convert 64 to a z-score using the formula $z = \dfrac{x - \mu}{\sigma}$. Use substitution to find that the z-score for a raw score of 64 is $z = \dfrac{64 - 69}{2.5}$. This is equal to –2. Look at the row –2.0 on the z-score table and go to the 0.00 column to create –2.00. The table entry, .0228, is the area below the z-score of –2.00.

8b. 71.6 inches. The z-score for the top 15% is the same as the z-score for the bottom 85%, and the table uses the scores below each z-score to do computations. Look in the body of the table for the closest number to .8500. It is .8508, which has a z-score of 1.04. Use the z-score formula to convert this to a raw score.

Substitute: $1.04 = \dfrac{x - 69}{2.5}$

(1.04)(2.5) = x – 69

2.6 = x – 69

x = 71.6

9. Each raw score needs to be converted to a z-score.

Substitute Lisa's score: $z = \dfrac{89 - 88}{12}$

z = 0.08, and this represents a score above the mean at Royal Ranch.

Substitute Angela's score: $z = \dfrac{90 - 97}{14}$

z = –0.5, and this is below the mean.

Although Lisa had the lower raw score, Angela is the better golfer. Her score is 0.5 standard deviations below the mean, while Lisa's score is higher than the mean for her course.

10a. 100-pound weights.

10b. The ping-pong ball distribution is more dispersed; the standard deviation is much greater relative to the mean of 2.7 than the standard deviation of the weights is to its mean.

11. $1,945.24. Go to the body of the z-score table and find the number closest to .8000. This is .7995, which corresponds to a z-score of 0.84.

Substitute: $0.84 = \dfrac{x - 1600}{411}$

(0.84)(411) = x – 1600

345.24 = x – 1600

x = $1,945.24

Lesson 1-5 Personal Expenses

Check Your Understanding (Example 1)

About $50. Go up from the 250 mark on the *x*-axis to where you might expect a *y*-value to lie.

Check Your Understanding (Example 2)

Typically, sales of cold beverages will increase as the temperature increases, so the correlation between temperature and cold water sales will be positive. This correlation is probably causal, so temperature is the explanatory variable and cold water bottle sales is the response variable.

Check Your Understanding (Example 3)

Answers will vary. Sample answer: amount spent on snacks and the amount spent on music downloads. They may be correlated, but the lurking variable for both might be the amount of one's disposable income.

Check Your Understanding (Example 4)

A positive correlation since the tax on a given item is a function of the price of that item. As the cell phone price increases, the tax for the cell phone will also increase.

Check Your Understanding (Example 5)

$Y = -11.9x + 67.7$

Check Your Understanding (Example 6)

$0.58. The slope of the line can be interpreted as an increase of $0.29 in clothing expense for every dollar increase in income. Therefore, there is a $0.58 increase in clothing expense for a 2 dollar increase in income.

Extend Your Understanding (Example 6)

Interpolation, since $280 is within the domain.

Check Your Understanding (Example 7)

$0.58. Students can use the equation to show the $0.58 increase between two incomes that are $2 apart. For example, using $x = 302$, the clothing expense is estimated to be 65.56%. Using $x = 304$, the clothing expense is estimated to be $66.14. There is a difference of $0.58 here.
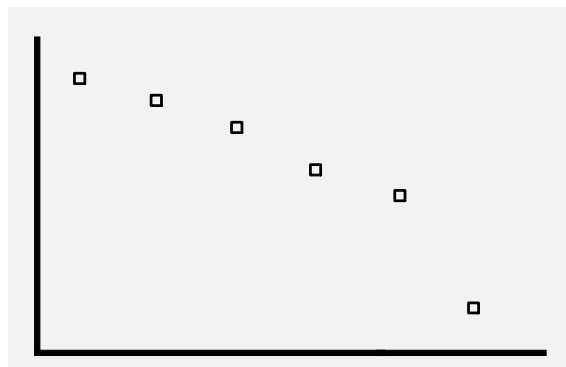
Check Your Understanding (Example 8)

$r = -0.998$. This represents a strong negative correlation.

Extend Your Understanding (Example 8)

Both correlations are weak since their absolute values are less than 0.3. The first has a weak negative correlation and the second has a weak positive correlation.
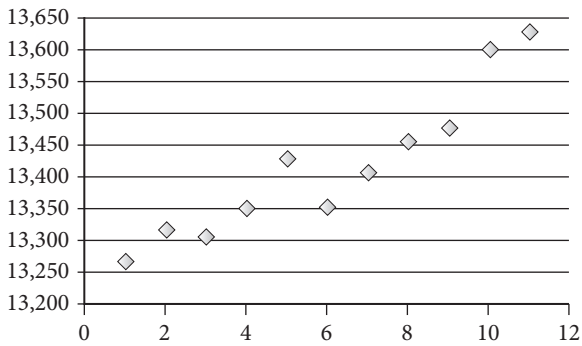
Applications

1. As you have seen in this chapter, past data can yield important information about trends and help us make predictions about present and future occurrences. Scatter plots and regression analysis help us do just that!

2. Negative. As the days increase, the amount in the envelope decreases.

3a. positive

3b. negative

3c. no

3d. negative

3e. negative

3f. no

4a. As *x* decreases, *y* increases. Or, as the price increases, the number of downloads decreases. Both mean the same thing.

4b. There is a negative correlation.

4c. Approximately 1,911,000 songs. The average of $0.59 and $0.49 is $0.54 and the average of 1,877 and 1,944 is 1,911.

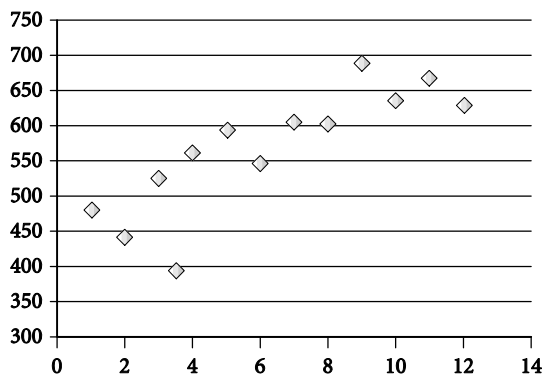5a. As the months increased, the disposable income increased.

5b. Positive correlation

5c. The plots show an increase in the disposable income. The income in December might be $13,640B.

6a. Overall, there was an increase in spending but some individual years showed a decrease.

6b. There is a positive correlation.

7a. Graph A shows a moderate correlation which could be $r = 0.49$. The points on graph B appear to lie very close a straight line. This implies a strong correlation.

7b. Neither, since there is no causal relationship.

8a. $r = 0.21$ weak positive

8b. $r = -0.87$ strong negative

8c. $r = 0.55$ moderate positive

8d. $r = -0.099$ weak negative

8e. $r = 0.99$ strong positive

8f. $r = -0.49$ moderate negative

9. c

10. Yes, if the points are linear, it is possible that the regression line can go through each of them. If the points are not linear, it is possible that the regression line can go through none of them.

11a. $y = -0.19x + 6.36$

11b. $-0.19/1$; This is the x-coefficient in the regression equation.

11c. Single lens price per boxes purchased.

11d. $2.18; Use $y = -0.19(22) + 6.36$. $y = 2.18$

11e. Interpolation because 22 is in the domain.

11f. $r = -0.99$

11g. Strong negative correlation.

12a. $y = -1380.57x + 2634.90$

12b. $-1380.57/1$

12c. Downloads per dollar.

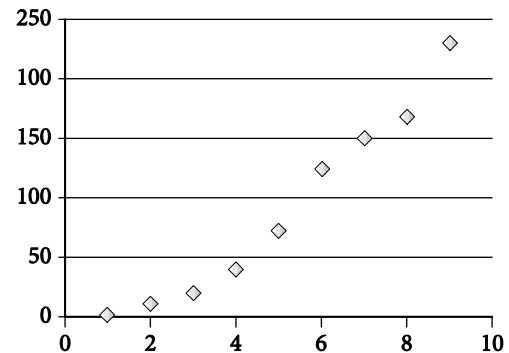12d. 2014; $y = -1380.57(0.45) + 2634.90$. $y \approx 2014$

12e. Interpolation. $0.45 is in the domain.

12f. -0.90

12g. Strong negative.

13a. (1, 1.4), (2, 11.6), (3, 20.7), (4, 40.0), (5, 72.3), (6, 125.1), (7, 150.3), (8,169.2), (9, 231.2)

13b.

13c. $y = 28.94x - 53.38$

13d. $28.94/1$

13e. Millions of iPhones sold per year.

13f. 0.98 strong positive

13g. $236.02B extrapolation. 2016 (year 10) is not in the domain.

13h. 211.88M

Assessment

Really? Really! Revisited

a. 61,648.5 minutes; 168.9×365

b. 1,027.475 hours; 61648.5/60

c. 42.8 days per year; 1027.475/24

d. 3,531 days watching TV in an average lifetime.; 42.8×82.5

e. 9.7 years; 3531/365

f.

Leisure Activity	Average Minutes Spent Per Day	Average Time Spent Over the Course of 82.5 Years in Years
Watching TV	168.9	9.7
Socializing/Communicating	42.4	2.4
Playing Games/Computer Use	26.7	1.5
Reading	19.6	1.1
Sports/Exercise/Recreation	17.9	1.0
Relaxing/Thinking	16.6	0.95
All Other Leisure Activities	24.8	1.4

g. 35.4 years.; Add the average times in the chart to 25 and 4 years mentioned in the problem. That yields a total of 47.1 years. Subtract that amount from 82.5 to get 35.4 years of unaccounted for time.

Applications

1a.

Headphone Price	Frequency
$75	4
$89	1
$90	1
$95	2
$100	2
$110	2
$136	4
$180	6

1b. $123.32; $(75 \times 4 + 89 \times 1 + 90 \times 1 + 95 \times 2 + 100 \times 2 + 110 \times 2 + 136 \times 4 + 180 \times 6)/22 \approx$ $123.32

1c. Median = $110. There are an even number of prices in the list. The median will occur at the average of the 11th and 12th prices. Both of those prices are $110 so the median is $110. Mode = $180; $180 occurred the most in the list.

2a.
$$\bar{x} = \frac{1}{6} \sum_{i=1}^{6} x_i = \$98.5B$$

2b.
$$\bar{x} = \frac{1}{4} \sum_{i=3}^{6} x_i = \$104.3B$$

2c.
$$\bar{x} = \frac{1}{3} \sum_{i=2}^{4} x_i = \$89.3B$$

3a.

Amount	Frequency	Relative Frequency	Cumulative Frequency	Relative Cumulative Frequency
$0–$99.99	15	0.060	15	0.060
$100–$199.99	25	0.100	40	0.160
$200–$299.99	30	0.120	70	0.280
$300–$399.99	72	0.288	142	0.568
$400–$499.99	60	0.240	202	0.808
$500–$599.99	28	0.112	230	0.920
$600–$699.99	20	0.080	250	1.000
Total	250			

3b. 12%

3c. 202; (72 + 30 + 25 + 15)

3d. 56.8%; 100(72 + 30 + 25 + 15)/250

3e. 325: 48.6%; 100(7 + 4 + 24)/72; 350: 73.6%; 100(7 + 4 + 24 + 18)/72. These are the percentile ranks within the $300–$399.99 band.

4a. Write the spreadsheet formula that was used to calculate the amount in cell B7.

= sum (B2:B6)

4b. Write the spreadsheet formula that was used to calculate the amount in cell C3. = B3/B7.

4c. Write the spreadsheet formula that was used to calculate the amount in cell D5. = D4 + B5 or =sum(B2:B5).

4d. Write the spreadsheet formula that was used to calculate the amount in cell E6. = D6/B7.

4e. Describe what the amount in cell C5 means in the context of this problem. 3.5% of the reviews were "Poor."

4f. Describe what the amount in cell D4 means in the context of this problem. 1,083 of the reviews were "Average" or better.

4g. Describe what the amount in cell E3 means in the context of this problem. 81.7% of the reviews were "Very good" or better.

5a. How many students are in the Key Club? 48. This is found by adding the numbers in the frequency column.

5b. Mean = 8.3. This is found by adding all 48 scores and dividing by 48. The number of trips can be multiplied by the frequency column to make this more efficient to compute.

Median = 8. The median is the middle score. There are an even number of scores, so the median is the average of the 24th and 25th score. This occurs in the third row.

Mode = 9. The mode is the score with the highest frequency.

Range = 9. The range is the difference between the high and low scores.

Variance = 6.3. Use a table:

Number (x_i)	Frequency (f)	($x_i - \bar{x}$)	($x_i - \bar{x}$)^2	($x_i - \bar{x}$)$^2 f$
5	8	−3.3	10.89	87.12
6	7	−2.3	5.29	37.03
8	11	−0.3	0.09	0.99
9	15	0.7	0.49	7.35
12	3	3.7	13.69	41.07
14	4	5.7	32.49	129.96
Total	48	–	–	303.52

The data is substituted into the variance formula

$$\frac{\sum\limits_{i=1}^{n}(x_i - \bar{x})^2}{n}$$ where $n = 48$.

Standard deviation = 2.5. The standard deviation is the square root of the variance.

5c. 18%. Convert 8 to a z-score using the formula $z = \dfrac{x - \mu}{\sigma}$. Use substitution to find that the z-score for a raw score of 8 is $z = \dfrac{8 - 6.2}{2}$. This is equal to 0.9. Look at the row 0.9 on the z-score table and go to the 0.00 column to create 0.9. The table entry, .8159, is the area below the z-score of 0.09, and this must be subtracted from 1 to get the area above, which is 18% rounded to the nearest percent.

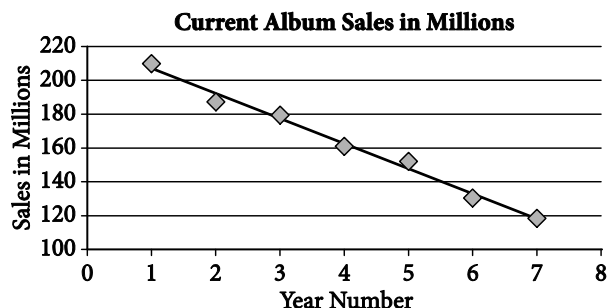5d. 38%. Convert 5 and 7 to z-scores using the formula $z = \dfrac{x - \mu}{\sigma}$. Use substitution to find that the z-score for a raw score of 5 is $z = \dfrac{5 - 6.2}{2}$. To the nearest hundredth, this is equal to –0.6. The area below this z-score is, from the table, .2743. Use substitution to find that the z-score for a raw score of 7 is $z = \dfrac{7 - 6.2}{2}$. This is equal to 0.4. The area below this z-score is .6554. Subtract the smaller area from the larger area.
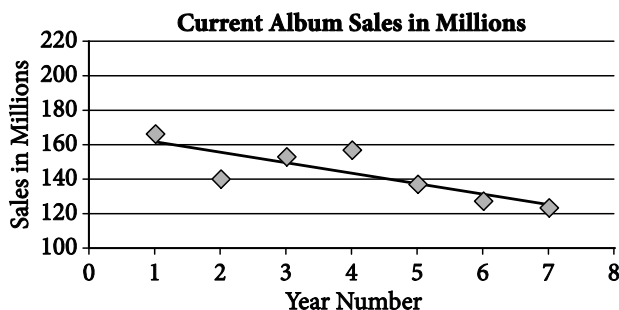
6a.

Current Album Sales in Millions

6b. $y = -14.83x + 221.97$

6c. −14.83/1

6d. Millions of album sales per year.

6e. 103.3 million; let $x = 8$. $Y = -14.83(8) + 221.97 \approx 103.3$

6f. Extrapolation, because 8 is outside of the domain.

6g. $r = -0.99$

6h. Strong negative.

7a.

Current Album Sales in Millions

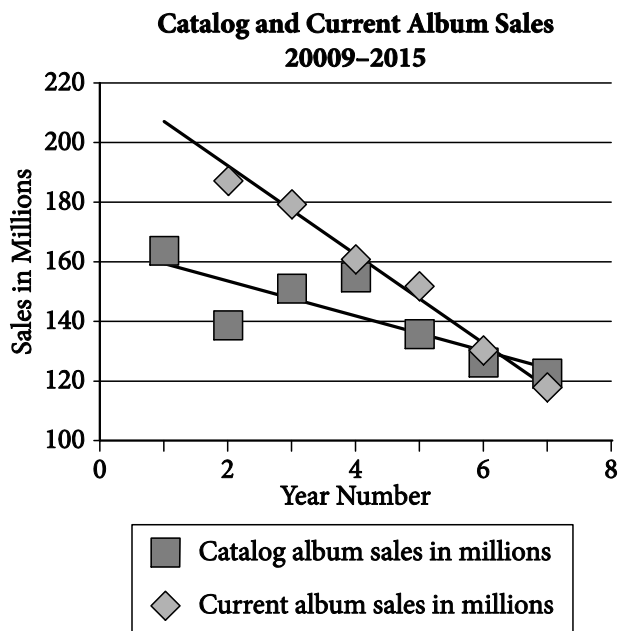7b. $y = -5.83x + 165.37$

7c. $-5.83/1$

7d. Millions of album sales per year.

7e. 118.73 million; let $x = 8$. $Y = -5.83(8) + 165.37 = 118.73$

7f. $r = -0.83$

7g. Strong negative

7h. Answer—Because the lines intersect, at some point in 2015 the catalog sales sold more than the current sales.

Catalog and Current Album Sales 20009–2015

■ Catalog album sales in millions
◇ Current album sales in millions

8a. 113,109. The five numbers in the attendance column are added and divided by 5.

8b. 4,597. This needs to be done with a hand calculator or online calculator. The numbers are too much large to do by hand. Students must learn how to find the standard deviation on the statistics menu.

8c. 3 Since the standard deviation is 4,597, the interval represented by one standard deviation above and below the mean is from 108,512 to 117,706. This includes three of the years.

8d. Yes. The points approximate a straight line.

8e. 0.994. This can only be done using the correlation coefficient item on the statistics menu of a calculator.

8f. $y = 3,230.5x + 77,574.3$ The regression equation feature of a calculator must be used.

8g. Answers vary. Students must substitute into the regression equation.

9a. 69%. Convert 1,000 and 2,000 to z-scores using the formula $z = \dfrac{x - \mu}{\sigma}$. Use substitution to find that the z-score for a raw score of 1,000 is $z = \dfrac{1000 - 1751}{421}$. To the nearest hundredth, this is equal to -1.78. The area below this z-score is, from the table, .0375. Use substitution to find that the z-score for a raw score of 2,000 is $z = \dfrac{2000 - 1751}{421}$. This is equal to 0.59. The area below this z-score is .7224. Subtract the smaller area from the larger area.

9b. 41. Convert 800 to a z-score using the formula $z = \dfrac{x - \mu}{\sigma}$. Use substitution to find that the z-score for a raw score of 800 is $z = \dfrac{800 - 1751}{421}$. To the nearest hundredth, this is equal to -2.26. The area below this z-score is, from the table, .0122. Convert this to 1.22% and find 1.22% of 3,408.

10. The answer to the current problem is 3,003 which is found by computing 14 C 8.

11a. 70

11b. = sum(B2:B6). This is the sum of the frequency column.

11c. Write the spreadsheet formula that was used to find the mean. = C8/B8. Since the scores must be weighted by their frequencies, the total C8 is divided by the total frequency in B8.

11d. Write the spreadsheet formula that was used to calculate the amount in cell D5.

=((A5 – C8 / B8)^2)*B5. These are the squared deviations from the mean, which must be weighted by the frequencies.

11e. Find the variance for the data set. Round to the nearest hundredth. 3.03. The entry in cell D8 is divided by the total frequency in B8.

12. 101. The top 5% is represented by a z-score equivalent to the z-score representing the bottom 95%. Look for the closest number to .9500 in the body of the table. It is in the middle of 1.64 and 1.65. We could use 1.645 or just round it to 1.65. Convert this z-score to a raw score using the formula.

Substitute: $1.65 = \dfrac{x - 66}{21}$

$(1.65)(21) = x - 66$

$34.65 = x - 66$

$x = 101$ to the nearest integer.