CHAPTER 2

2.1 (a) An ideal voltage source model implies constant voltage regardless of the external connections. This could result in current approaching infinity in the case of a short circuit and therefore would require infinite power to supplied by the source. Similarly, an ideal current source implies constant current regardless of external connections. This cannot possibly be true in the case of an open circuit.

(b) An appropriately sized resistor in series with the ideal voltage source, or in parallel with the ideal current source, will provide a more accurate model of a physical power supply.

2.2 The operational amplifiers must be operating in their linear amplification range. This means that the output voltage can never exceed the power supply voltage. Usually, the ideal op-amp assumptions are used to develop a linear model.

2.3 (a) $\begin{bmatrix} R_1+R_2 & -R_2 \\ -R_2 & R_2+R_3+L_1 s \end{bmatrix}\begin{bmatrix} I_1(s) \\ I_2(s) \end{bmatrix} = \begin{bmatrix} V_1(s) \\ 0 \end{bmatrix}, V_2(s) = R_3 I_3(s)$

$$I_2(s) = \frac{\begin{vmatrix} R_1+R_2 & V\ (s) \\ -R_2 & 0 \end{vmatrix}}{\begin{vmatrix} R_1+R_2 & -R_2 \\ -R_2 & R_2+R_3+L_1 s \end{vmatrix}} = \frac{R\ V_1(s)}{\left(R_1+R_2\right)L_1 s + R_1R_2 + R_1R_3 + R_2R_3}$$

$$\frac{V_2(s)}{V_1(s)} = \frac{R_2R_3}{\left(R_1+R_2\right)L_1 s + R_1R_2 + R_1R_3 + R_2R_3}$$

(b) Replace R_3 with

$$\frac{(R_1+R_2)R_3L_2 s}{R_3 + L_2 s}$$

$$\frac{V_2(s)}{V_1(s)} = \frac{R_2R_3L_2 s}{L_1L_2(R_1+R_2)s^2 + [R_3(L_1+L_2)(R_1+R_2) + R_1R_2L_2]s + R_1R_2R_3}$$

(c)

$$V_1(s) = 10/s,$$

$$v_{2ss} = \lim_{s\to 0} \frac{s(10/s)R_2R_3}{\left(R_1+R_2\right)L_1 s + R_1R_2 + R_1R_3 + R_2R_3} = \frac{10R_2R_3}{R_1R_2 + R_1R_3 + R_2R_3}$$

2.4 (a)

$$\frac{V_2(s)-V_i(s)}{10/s} + \frac{V_2(s)-V_i(s)}{10} + \frac{V_2(s)}{5} = 0, \quad V_2(s)\left(s/10 + 1/10 + 1/5\right) = V_1(s)\left(s/10 + 1/10\right) \Rightarrow \frac{V_2(s)}{V_1(s)} = \frac{s+1}{s+3}$$

(b)

$$\frac{V_2(s)-V_i(s)}{10/s} + \frac{V_2(s)-V_i(s)}{10} + \frac{V_2(s)}{5} + \frac{V_2(s)}{2/s} = 0, \quad V_2(s)\left(s/10 + 1/10 + 1/5 + s/2\right) = V_1(s)\left(s/10 + 1/10\right) \Rightarrow \frac{V_2(s)}{V_1(s)} = \frac{s+1}{6s+3}$$

(c) $(a) V_2(s) = \dfrac{10(s+1)}{s(s+3)}, \ \lim_{s\to 0} sV_2(s) = 10/3 \quad (b) V_2(s) = \dfrac{10(s+1)}{s(6s+3)}, \quad \lim_{s\to 0} sV_2(s) = 10/3$

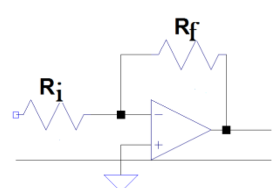
2.5

$$(a) v_o(t) = v_i(t) \Rightarrow \frac{V_o(s)}{V_i(s)} = 1. \quad (b) v_o(t) = \left(\frac{R_f}{R_i} + 1\right) v_i(t) \Rightarrow \frac{V_o(s)}{V_i(s)} = \frac{R_f}{R_i} + 1$$

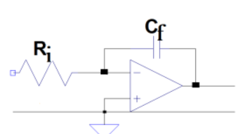
$$(c) v_o(t) = -R_f C \frac{dv_i(t)}{dt} \qquad (d) v_o(t) = \frac{-1}{RC} \int_{-\infty}^{t} v_i(\tau) d\tau$$

2.6 From the solution to Problem 2.5(a), The gain of the first op-amp stage is one. For the second stage,

$v_{o2} = \dfrac{-10K}{1K} e_s - \dfrac{10K}{50K}(-12) = -10e_s + 2.4$. For the third stage, $v_o = \dfrac{-2K}{1K} v_{o2} = 20e_s - 4.8$.

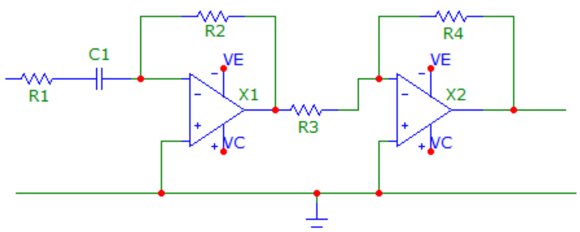


2.7 (a) $\dfrac{-Z_f}{Z_i} = -10, \Rightarrow Z_f = 10Z_i$, let $R_i = 10K$, then $R_f = 100K$.



(b) $\dfrac{-Z_f}{Z_i} = \dfrac{-10}{s} = \dfrac{1/sC}{R}$, let $R = 10K$, then $C = 10\mu F$.

(c)

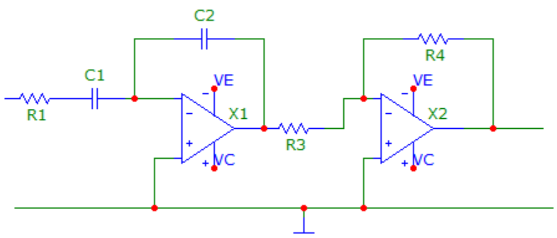


Let $R_3 = R_4 = 100K$. Gain = -1 for second stage. For first stage,

$$G_1(s) = \frac{-R_2}{R_1 + \frac{1}{sC_1}} = \frac{-\left(\frac{R_2}{R_1}\right)s}{s + \frac{1}{R_1C_1}} = \frac{10s}{s+10} \Rightarrow R_1C_1 = 10^{-1}, \quad R_2 = 10R_1$$

Let $R_1 = 10K$, then $R_2 = 100K$, and $C_1 = 10\mu F$.

2.7 (d)

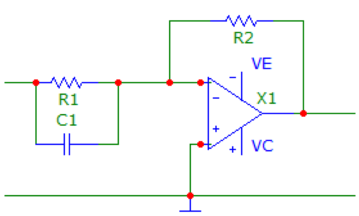


Let $R_3 = = R_4 = 100K$. Gain = -1 for second stage. For first stage,

$$G_1(s) = \frac{-\frac{1}{sC_2}}{R_1 + \frac{1}{sC_1}} = \frac{-\left(\frac{1}{R_1C_2}\right)}{s + \frac{1}{R_1C_1}} = \frac{-10}{s+10} \Rightarrow R_1C = R_1C_2 = 10^{-1},$$

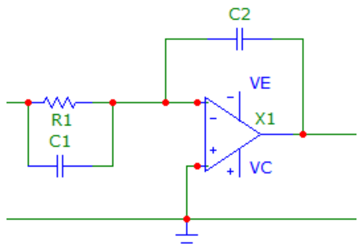
Let $R_1 = 10K$, then $C_1 = C_2 = 10\mu F$.

(e)



$$\frac{-Z_f}{Z_i} = \frac{-R_2(1+R_1C_1s)}{R_1} = -\left(\frac{R_2}{R_1} + R_2C_1s\right) = -(10s+1) \Rightarrow R_fC_1 = 10, \frac{R_f}{R_i} = 1, \quad \text{let } R_2 = R_1 = 100K, C_1 = 100\mu F.$$

2.7 (f)



$$\frac{-Z_f}{Z_i} = -\left(\frac{C_1}{C_2} + \frac{1}{R_1 C_2 s}\right) = -\left(10 + \frac{0.1}{s}\right) \Rightarrow \frac{C_1}{C_2} = 10, \quad R_1C_2 = 10 \quad \text{Let } R_1 = 1M\Omega, \text{ then } C_2 = 10\mu F, \text{ and } C_1 = 100\mu F.$$

2.8

(a) $A = 2 - 2B,\ B = 8A + 18, \therefore A + 2B = 2, \text{ and } -8A + B = 18$

$$\begin{bmatrix} 1 & 2 \\ -8 & 1 \end{bmatrix} \begin{bmatrix} A \\ B \end{bmatrix} =$$

$$(b) \begin{bmatrix} A \\ B \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ -8 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 2 \\ 18 \end{bmatrix} = \frac{\begin{bmatrix} 1 & -2 \\ 8 & 1 \end{bmatrix}}{\begin{vmatrix} 1 & 2 \\ -8 & 1 \end{vmatrix}} \begin{bmatrix} 2 \\ 18 \end{bmatrix} = \begin{bmatrix} -2 \\ 2 \end{bmatrix}$$

$$(c) A = \frac{\begin{vmatrix} 2 & 2 \\ 18 & 1 \end{vmatrix}}{\begin{vmatrix} 1 & 2 \\ -8 & 1 \end{vmatrix}} = \frac{-34}{17}, \quad B = \frac{\begin{vmatrix} 1 & 2 \\ -8 & 18 \end{vmatrix}}{\begin{vmatrix} 1 & 2 \\ -8 & 1 \end{vmatrix}} = \frac{34}{17}$$

$(d) >> M = [1,2;-8,1], V = [2;18]$

$>> AB = inv(M)*V$

2.9 (a)

Figure P2.9(a): $C = G_1G_2E + G_2HF$

Figure P2.9(b): $C = G_aE + G_aG_bF$

$\therefore G_a = G_1G_2$ and $G_b = H/G_1$

(b)

Figure P2.9(c): $C = G_cE + G_dF$

$\therefore G_c = G_1G_2$ and $G_d = G_2H$

2.10 (a)

Figure P2.10(a): $C = G_1G_2E,\ \ F = G_1HE$

Figure P2.10(b): $C = G_aE, \quad F = G_aG_bE$

$\therefore G_a = G_1G_2$ and $G_b = H/G_2$

(b)

Figure P2.10(c): $C = G_cG_dE, \quad F = G_cE$

$\therefore G_c = G_1H$ and $G_d = G_2/H$

2.11 (**a**) $A = 4 - 2B - 3C, \quad B = 3A, \quad C = 1 + A - B$

(b) $\Delta = 1 - (-6 + 9 - 3) = 1, \quad A = \frac{(4)(1)}{1} + \frac{(-3)(1)(1)}{1} = 1$

$$B = \frac{(3)(1)(4)}{1} + \frac{(1)(-3)(3)}{1} = 3, \quad C = \frac{(4)(1)(1)}{1} + \frac{(4)(1)(3)(-1)}{1} + \frac{(1)(1)[1-(-6)]}{1} = -1$$

(c)

```
>> M=[1 2 3;3 -1 0;1 -1 -1], V=[4;0;-1]
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2.12 (a) $\Delta = 1 - (-G_1 - G_2 - G_4 - G_3 G_4 - G_1 G_2 G_3 G_4) + (G_1 G_2 + G_1 G_4 + G_2 G_4) - (-G_1 G_2 G_4)$

$\Delta = 1 + G_1 + G_2 + G_4 + G_3 G_4 + G_1 G_2 + G_1 G_4 + G_2 G_4 + G_1 G_2 G_4 + G_1 G_2 G_3 G_4$

$M_1 = G_1 G_2 G_3 G_4, \quad \Delta_1 = 1.$

$M_2 = G_3 G_4, \quad \Delta_2 = 1 + G_1$

$M_3 = G_1, \quad \Delta_3 = 1 + G_4$

$M_4 = -1, \quad \Delta_4 = 1 + G_1 + G_4 + G_1 G_4$

$M_5 = -G_3 G_4 G_1, \quad \Delta_5 = 1$

$$\frac{D}{R} = \frac{G_1 G_2 G_3 G_4 + G_3 G_4 - G_4 - 1}{\Delta}$$

(b)

$$B = -R - G_2 B + G_1 A$$
$$C = G_3 R + G_2 G_3 B - G_4 C$$
$$D = B + G_4 C$$

2.13. (a) $\frac{C(s)}{R(s)} = \frac{G_1 G_2 G_3 + 2G_2 G_3(1+G_1)}{1-(-G_1 - G_2 - G_3 - G_1 G_2 G_3) + G_1 G_2 + G_1 G_3} = \frac{3G_1 G_2 G_3 + 2G_2 G_3}{1 + G_1 + G_2 + G_3 + G_1 G_2 G_3 + G_1 G_2 + G_1 G_3}$

(b) $A = R - G_1 A - C, \quad B = G_2(2R + G_1 A - B) - C, \quad C = G_3 B$

$$C = \frac{\begin{vmatrix} 1+G_1 & 0 & 1 \\ -G_1 G_2 & 1+G_2 & 2G_2 \\ 0 & -G_3 & 0 \end{vmatrix}}{\begin{vmatrix} 1+G_1 & 0 & 1 \\ -G_1 G_2 & 1+G_2 & 1 \\ 0 & -G_3 & 1 \end{vmatrix}} = \frac{3G_1 G_2 G_3 + 2G_2 G_3}{1 + G_1 + G_2 + G_3 + G_1 G_2 G_3 + G_1 G_2 + G_1 G_3}$$

2.14 (a)

$$\Delta = 1 - [1s^{-1} - 2s^{-1} - 8s^{-1} - 3] + [2s^{-2} + 6s^{-1}] = 4 + 17s^{-1} + 2s^{-2}$$
$$M_1 = 3, \quad \Delta_1 = 1 + 2s^{-1}$$
$$M_2 = 8s^{-1} \quad \Delta_2 = 1$$
$$T(s) = \frac{3(1+2s^{-1}) + 8s^{-1}}{\Delta} = \frac{3s^2 14s}{4s^2 + 17s + 2}$$

(b)

$$A(s) = \frac{R(s) - C(s)}{1 + s^{-1}}; \quad B(s) = \frac{2A(s)}{1 + s^{-1}}; \quad C(s) = 4s^{-1} B(s) + 3A(s)$$

2.15

$$\Delta = 1-[-s^{-1}-2s^{-1}-3s^{-1}-4s^{-3}]+2s^{-2}+6s^{-2} = 1+6s^{-1}+9s^{-2}+4s^{-3}$$

$$M_1 = 2s^{-3}, \quad \Delta_1 = 1; \quad M_2 = s^{-2}, \quad \Delta_2 = 1+3s^{-1}$$

$$T(s) = \frac{s+5}{s^3+6s^2+9s+4}$$

2.16 (a)

$$\Delta = 1-[-G_2H_1 - G_1G_2H_2]; \quad M_1 = G_1G_3, \quad \Delta_1 = 1+G_2H_1; \quad M_2 = G_1G_2, \quad \Delta_2 = 1$$

$$T(s) = \frac{G_1G_2 + G_1G_3 + G_1G_2G_3H_1}{1+G_2H_1+G_1G_2H_2}$$

(b)

$$T(s) = \frac{\dfrac{6(s+2)}{s+8}\left(\dfrac{2}{s(s+1)}+\dfrac{4}{s}\right)}{1+\left(\dfrac{6(s+2)}{s+8}\right)\left(\dfrac{2}{s(s+1)}\right)} = \frac{24s^2+84s+72}{s^3+9s^2+20s+24}$$

(c)

$$T(s) = \frac{24s+36}{s^2+8s+12}$$

2.17 (a) $M\ddot{x}_1 = -B(\dot{x}_1-\dot{x}_2); \quad 0 = -B(\dot{x}_2-\dot{x}_1)-Kx_2$

(b) $M\ddot{x}+K_2x_1+B(\dot{x}_1-\dot{x}_2) = 0; \quad B(\dot{x}_2-\dot{x}_1)+K_1x_2 = 0$

(c) $M_1\ddot{x}_1+B_1\dot{x}_1+K(x_1-x_2) = 0; \quad K(x_2-x_1) = f(t); \quad \therefore M_1\ddot{x}_1+B_1\dot{x}_1 = f(t)$

(d) $M_1\ddot{x}_1+b_1(\dot{x}_1-\dot{x}_2)+k_1x_1+k_2(x_1-x_2); \quad M_2\ddot{x}_2+b_1(\dot{x}_2-\dot{x}_1)+k_2(x_2-x_1) = F$

2.18 (a)

$$(Ms^2+Bs)X_1(s)-BsX_2(s) = F(s); \quad -BsX_1(s)+(Bs+K)X_2(s) = 0$$

$$\begin{bmatrix} Ms^2+Bs & -Bs \\ -Bs & Bs+K \end{bmatrix}\begin{bmatrix} X_1(s) \\ X_2(s) \end{bmatrix} = \begin{bmatrix} F(s) \\ 0 \end{bmatrix}; \quad X_1(s) = \frac{\begin{vmatrix} F(s) & -Bs \\ 0 & Bs+K \end{vmatrix}}{\begin{vmatrix} Ms^2+Bs & -Bs \\ -Bs & Bs+K \end{vmatrix}}$$

$$\frac{X_1(s)}{F(s)} = \frac{Bs+K}{MBs^3+MKs^2+BKs}$$

(b)

$$(Ms^2+Bs+K_2)X_1(s)-BsX_2(s)=F(s); \quad -BsX_1(s)+(Bs+K_1)X_2(s)=0$$

$$X_1(s)=\frac{\begin{vmatrix}F(s) & -Bs\\ 0 & Bs+K_1\end{vmatrix}}{\begin{vmatrix}Ms^2+Bs+K_2 & -Bs\\ -Bs & Bs+K_1\end{vmatrix}}; \quad \frac{X_1(s)}{F(s)}=\frac{Bs+K_1}{MBs^3+MK_1s^2+B(K_1+K_2)s+K_1K_2}$$

(c) $M_1s^2X_1(s)+cX_1(s)=F(s); \quad \dfrac{X_1(s)}{F(s)}=\dfrac{1}{M_1s^2+B_1s}=\dfrac{2}{s(s+4)}$

(d)

$$(m_1s^2+b_1s+k_1+k_2)X_1(s)-(b_1s+k_2)X_2(s)=0$$
$$-(b_1s+k_2)X_1(s)+(m_2s^2+b_1s+k_2)X_2(s)=F(s);$$

$$X_1(s)=\frac{\begin{vmatrix}0 & -(b_1s+k_2)\\ F(s) & (m_2s^2+b_1s+k_2)\end{vmatrix}}{\begin{vmatrix}(m_1s^2+b_1s+k_1+k_2) & -(b_1s+k_2)\\ -(b_1s+k_2) & (m_2s^2+b_1s+k_2)\end{vmatrix}};$$

$$\frac{X_1(s)}{F(s)}=\frac{b_1s+k_2}{m_1m_2s^4+b_1(m_1+m_2)s^3+[k_1m_2+k_2(m_1+m_2)]s^2+b_1k_1s+k_2^2}$$

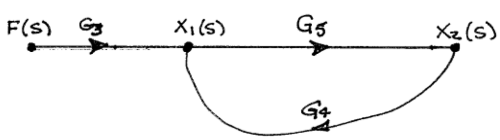
2.19 (a) $M\ddot{y}+(B_1+B_2)\dot{y}+Ky=f$

(b) $[Ms^2+(B_1+B_2)s+K]Y(s)=F(s); \quad \dfrac{Y(s)}{F(s)}=\dfrac{1}{Ms^2+(B_1+B_2)s+K}$

2.20 (a)

$$X_2(s)=\frac{M_1s^2+Bs+K_1}{Bs+K_1}X_1(s)=G_5X_1(s)$$

$$X_1(s)=\frac{-1}{Bs+K_1}F(s)+\frac{M_2s^2+Bs+K_1+K_2}{Bs+K_1}X_2(s)=G_3F(s)+G_4X_2(s)$$



(b) See Example 2.11. –

2.21 (a) $4\ddot{x}_1+2\dot{x}_1+5x_1+2(x_1-x_2)=0; \quad 6\ddot{x}_2+3\dot{x}_2+7x_2+2(x_2-x_1)=f$

$\left(4s^2+2s+7\right)X_1(s)-2X_2(s)=0; \quad \left(6s^2+3s+9\right)X_2(s)-2X_1(s)=F(s)$

(b)

$$X_1(s)=\frac{\begin{vmatrix}0 & -2\\ F(s) & 6s^2+3s+9\end{vmatrix}}{\begin{vmatrix}4s^2+2s+7 & -2\\ -2 & 6s^2+3s+9\end{vmatrix}}; \quad \frac{X_1(s)}{F(s)}=\frac{2}{24s^4+24s^3+84s^2+39s+59}$$

(c) $$X_2(s)=\frac{\begin{vmatrix}4s^2+2s+7 & 0\\ -2 & F(s)\end{vmatrix}}{\begin{vmatrix}4s^2+2s+7 & -2\\ -2 & 6s^2+3s+9\end{vmatrix}};\quad \frac{X_2(s)}{F(s)}=\frac{4s^2+2s+7}{24s^4+24s^3+84s^2+39s+59}$$

2.22 (a) $\tau(t)=J_1\ddot{\theta}_1+B_2\dot{\theta}_1+B_1(\dot{\theta}_1-\dot{\theta}_2);\quad 0=J_2\ddot{\theta}_2+B_1(\dot{\theta}_2-\dot{\theta}_1)$

(b)

$$\begin{bmatrix}J_1s^2+(B_1+B_2)s & -B_1s\\ -B_1s & J_2s^2+B_1s\end{bmatrix}\begin{bmatrix}\Theta_1(s)\\ \Theta_2(s)\end{bmatrix}=\begin{bmatrix}\mathrm{T}(s)\\ 0\end{bmatrix}$$

$$\Theta_2(s)=\frac{\begin{vmatrix}J_1s^2+(B_1+B_2)s & \mathrm{T}(s)\\ -B_1s & 0\end{vmatrix}}{\begin{vmatrix}J_1s^2+(B_1+B_2)s & -B_1s\\ -B_1s & J_2s^2+B_1s\end{vmatrix}};\quad \frac{\Theta_2(s)}{\mathrm{T}(s)}=\frac{B_1}{J_1J_2s^3+(J_1B_1+J_2B_1+J_2B_2)s^2+B_1B_2s}$$

2.23 (a) $J\ddot{\theta}+(B_1+B_2)\dot{\theta}=\tau$ (b) $\dfrac{\Theta(s)}{\mathrm{T}(s)}=\dfrac{1}{Js^2+(B_1+B_2)s}$

2.24 (a) From (2-51): Note that $\Omega(s)=s\Theta(s)$.

$$G_\Omega=\frac{\Omega(s)}{E_a(s)}=\frac{0.005}{7\times10^{-12}s^2+3.519\times10^{-7}s+1.289\times10^{-4}}=\frac{714285714.2857}{(s+4.99\times10^4)\,(s+369)}$$

(b) $\Omega(s)=\left(\dfrac{6}{s}\right)G_\Omega(s);\quad \omega_{ss}=\lim_{s\to0}s\Omega(s)=\dfrac{714285714.2857}{369(4.99\times10^4)}=232.7535\text{(rad/s)}$

2.25 (a)

$$e_m=K_m\frac{d\theta_m}{dt};\quad e_a=R_mi_a+L_m\frac{di_a}{dt}+e_m;\quad \tau=K_\tau i_a;$$

$$\tau=J_m\frac{d^2\theta_m}{dt^2}+B_m\frac{d\theta_m}{dt}+K(\theta_m-\theta_L);\quad 0=J_L\frac{d^2\theta_L}{dt^2}+B\frac{d\theta_L}{dt}+K(\theta_L-\theta_m)$$

(b)

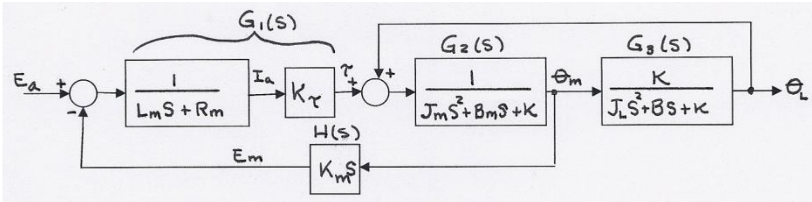
$$E_m(s)=K_ms\Theta_m(s);$$

$$E_a(s)=R_mI_a(s)+L_msI_a(s)+E_m(s)\Rightarrow I_a(s)=\frac{E_a(s)-E_m(s)}{L_ms+R_m};$$

$$\mathrm{T}(s)=K_\tau I_a(s)$$

$$\mathrm{T}(s)=J_ms^2\Theta_m(s)+B_ms\Theta_m(s)+K(\Theta_m(s)-\Theta_L(s))\Rightarrow\Theta_m(s)=\frac{\mathrm{T}(s)+K\Theta_L(s)}{J_ms^2+B_ms+K}$$

$$0=J_Ls^2\Theta_L(s)+Bs\Theta_L(s)+K(\Theta_L(s)-\Theta_m(s))\Rightarrow\Theta_L(s)=\frac{K\Theta_m(s)}{J_Ls^2+Bs+K}$$



(c)

$$\frac{\Theta_m(s)}{E_a(s)} = \frac{G_1G_2}{1+G_1G_2H-KG_2G_3}$$

(d)

$$\frac{\Theta_L(s)}{E_a(s)} = \frac{G_1G_2G_3}{1+G_1G_2H-KG_2G_3}$$

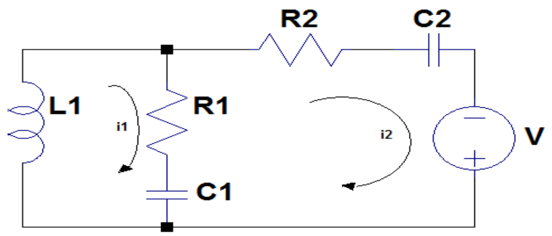
(e) From (c) and (d), or by inspection,

$$\frac{\Theta_L}{\Theta_m} = \frac{\Theta_L}{E_a}\left(\frac{\Theta_m}{E_a}\right)^{-1} = G_3$$

2.26 From Example 2.11 write analogous differential equation models.

$$M_1\ddot{x}_1 + B(\dot{x}_1-\dot{x}_2) + K_1(x_1-x_2) = 0 \quad \Leftrightarrow \quad L_1\frac{di_1}{dt} + R(i_1-i_2) + \frac{1}{C_1}\left[\int i_1dt - \int i_2dt\right] = 0$$

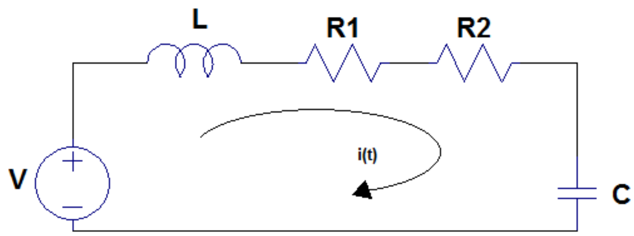
$$M_2\ddot{x}_2 + B(\dot{x}_2-\dot{x}_1) + K_1(x_2-x_1) = f \quad \Leftrightarrow \quad L_2\frac{di_2}{dt} + R(i_2-i_1) + \frac{1}{C_1}\left[\int i_2dt - \int i_1dt\right] + \frac{1}{C_2}\int i_2dt = v$$



2.27 (a)

$M\ddot{y} + (B_1+B_2)\dot{y} + Ky = f$. Let $i = \dot{y}$ and $v = f$.

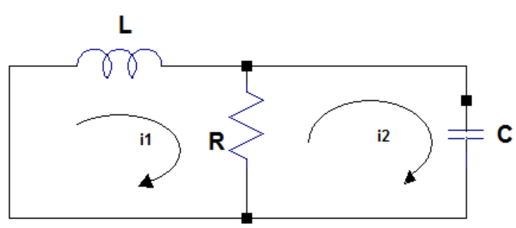
$M\frac{di}{dt} + (B_1+B_2)i + K\int idt = v$, let $L = M$, $R_1 = B_1$, $R_2 = B_2$, and $1/C = K$.



(b)

$M\ddot{x}_1 + B(\dot{x}_1-\dot{x}_2) = 0; \qquad B(\dot{x}_2-\dot{x}_1) + K_1x_2 = 0$

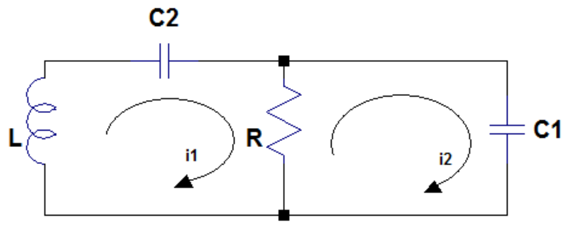
Let $i_1 = \dot{x}_1$, $i_2 = \dot{x}_2$, $1/C = K$, $L = M$, and $R = B$.



(c)

$$M\ddot{x}_1 + B(\dot{x}_1 - \dot{x}_2) + K_2 x_1 = 0 \quad \Rightarrow \quad L\frac{di_1}{dt} + R(i_1 - i_2) + \frac{1}{C_2}\int i_1 dt = 0$$

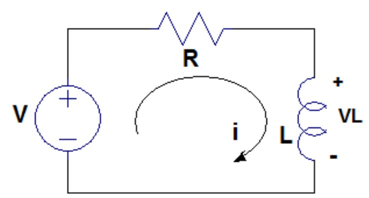
$$B(\dot{x}_2 - \dot{x}_1) + K_1 x_2 = 0 \quad \Rightarrow \quad R(i_2 - i_1) + \frac{1}{C_1}\int i_2 dt = 0$$



2.28 (a) From Problem 2.17(c) solution,

$$M_1\ddot{x}_1 + B_1\dot{x}_1 + K(x_1 - x_2) = 0; \quad K(x_2 - x_1) = f(t); \quad \therefore M_1\ddot{x}_1 + B_1\dot{x}_1 = f(t).$$

$$L = M_1, \quad R = B_1, \quad i = \dot{x}_1, \quad v = f(t), \quad \text{and} \quad v_L = M_1\ddot{x}_1$$

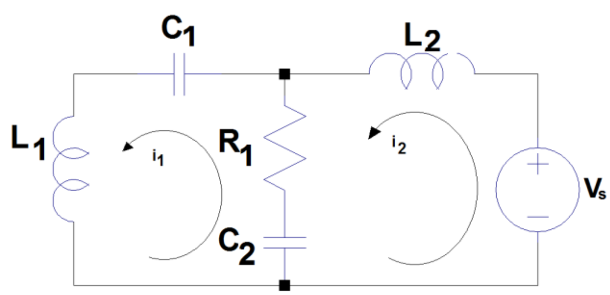


(b) From Problem 2.17(d) solution, let $i_1 = \dot{x}_1, \quad i_2 = \dot{x}_2, \quad v = F.$

$$M_1\ddot{x}_1 + b_1(\dot{x}_1 - \dot{x}_2) + (k_1 + k_2)x_1 - b_1\dot{x}_2 - k_2 x_2 = 0 \quad \Leftrightarrow \quad M_1\frac{di_1}{dt} + b_1(i_1 - i_2) + k_1\int i_1 dt + k_2\int [i_1 - i_2]dt = 0$$

$$M_2\ddot{x}_2 + b_1(\dot{x}_2 - \dot{x}_1) + k_2(x_2 - x_1) = F \quad \Leftrightarrow \quad M_2\frac{di_2}{dt} + b_1(i_2 - i_1) + k_2\left[\int i_2 dt - \int i_1 dt\right] = v$$

Let $L_1 = M_1, \quad L_2 = M_2, \quad b_1 = R, \quad C_1 = 1/k_1, \quad C_2 = 1/k_2$



2.29 (a)

$$\tau_2 = K(\theta_2 - \theta_3); \qquad 0 = K(\theta_3 - \theta_2) + B\dot{\theta}_3 + J\ddot{\theta}_3$$

$$\theta_2 = \frac{r_1}{r_2}\theta_1, \quad \tau_2 = \frac{r_2}{r_1}\tau_1$$

$$\frac{r_2}{r_1}\tau_1 = K\left[\frac{r_1}{r_2}\theta_1 - \theta_3\right] \Rightarrow \tau_1 = K\left(\frac{r_1}{r_2}\right)^2\left(\theta_1 - \frac{r_2}{r_1}\theta_3\right)$$

$$0 = K(\theta_3 - \frac{r_1}{r_2}\theta_1) + B\dot{\theta}_3 + J\ddot{\theta}_3$$

(b)

$$\tau_1 = K_1(\theta_1 - \theta_3^{'}); \quad 0 = J_1\ddot{\theta}_3^{'} + B_1\dot{\theta}_3^{'} + K_1(\theta_3^{'} - \theta_1)$$

$$\therefore K_1 = \left(\frac{r_1}{r_2}\right)^2 K; \quad J_1 = \left(\frac{r_1}{r_2}\right)^2 J; \quad B_1 = \left(\frac{r_1}{r_2}\right)^2 B; \quad \theta_3^{'} = \frac{r_2}{r_1}\theta_3$$

2.30 $z_1 = n^2 z_2 \Rightarrow z_1 = (0.01)^2 100K\Omega = 10\Omega$

2.31 (a) $r_g = \frac{1}{100} r_m = 100\ (\text{rpm})$. The encoder produces 100 pulses per revolution of the gearbox output shaft. Therefore, the resolution is $360^\circ/100 = 3.6^\circ$ per pulse.

(b) $J_L = 0.01\ (\text{Kg}\cdot m^2) \quad J_m = (0.01)^2 J_L = 10^{-6} (\text{Kg}\cdot m^2)$

2.32 (a)

$$\frac{\Theta_L(s)}{E_a(s)} = \frac{1}{20s}\left[\frac{\frac{38}{(2s+21)(2s+1)}}{1+\frac{19}{(2s+21)(2s+1)}}\right] = \frac{1}{20s}\left[\frac{38}{4s^2+44s+40}\right] = \frac{0.475}{s(s+1)(s+10)}$$

(b)

$$\frac{\Theta_L(s)}{\Theta_C(s)} = \frac{20(K_P s + K_D)\frac{0.475}{s(s+1)(s+10)}}{1+\frac{20(K_P s + K_D)0.475}{s(s+1)(s+10)}} = \frac{9.5(K_P s + K_D)}{s^3+11s^2+(10+9.5K_P)s+9.5K_D}$$

(c)

$$\frac{E_a(s)}{\Theta_C(s)} = \frac{20(K_P s + K_D)}{1+\frac{20(K_P s + K_D)0.475}{s(s+1)(s+10)}} = \frac{20(K_P s + K_D)s(s+1)(s+10)}{s^3+11s^2+(10+9.5K_P)s+9.5K_D}$$

(d)

$$\frac{\Theta_L(s)}{E_a(s)} = \frac{\Theta_L(s)}{\Theta_C(s)}\frac{\Theta_C(s)}{E_a(s)} = \frac{0.475}{s(s+1)(s+10)}$$