Chapter 2

Random Variables, Distributions, and   
Expectations

2.1 Discrete; continuous; continuous; discrete; discrete; continuous.

2.2 A table of sample space and assigned values of the random variable is shown next.

Sample Space *x*

*NNN* 0

*NNB* 1

*NBN* 1

*BNN* 1

*NBB* 2

*BNB* 2

*BBN* 2

*BBB* 3

2.3 A table of sample space and assigned values of the random variable is shown next.

Sample Space *w*

*HHH* 3

*HHT* 1

*HTH* 1

*THH* 1

*HTT −*1

*THT −*1

*TTH −*1

*TTT −*3

2.4 *S* = *{HHH,THHH,HTHHH,TTHHH,TTTHHH,HTTHHH, THTHHH,   
 HHTHHH}*; The sample space is discrete.

Copyright ©2013 Pearson Education, Inc.   
 15

16 *Chapter 2 Random Variables, Distributions, and Expectations*

∑

2.5

(a) *c* = 1*/*30 since 1 =

(b) *c* = 1*/*10 since   
 )(

*c*(*x*2 + 4) = 30*c*.

*x*=0

)

) ) )]

∑

1=

(2 3 [(2)(3

*c* =*c*

(2)(3 (2)(3

+ + = 10*c.*

*x*

*x*=0

∫*∞*

2.6 (a) *P*(*X >* 200) =

3*−x* 0 3

20000 10000

*dx* = *−*

1 2 2 1

 *∞*



 =1

200 (*x*+100)3

∫120

(b) *P*(80 *< X <* 200) =

(*x*+100)2

20000

*dx* = *−*

200

10000

9*.*

120



 =~~1000~~ 0*.*1020.

∫1

80

(*x*+100)3 (*x*+100)2 9801 =

80

∫1*.*2 1 ( )1*.*2

 

2.7 (a) *P*(*X <* 1*.*2) =

0 *x*

*dx* + (2 *− x*) *dx* =*~~x~~*~~2~~

1 2

+ 2*x −~~x~~*~~2~~ = 0*.*68.

2

0 1

∫1  1

(b) *P*(0*.*5 *< X <* 1) =

0*.*5 *x*



*dx* =*~~x~~*~~2~~ = 0*.*375.

2

0*.*5

2.8 (a) *P*(0 *< X <* 1) =

∫1

0

2(*x*+2)

5

1



*dx* =~~(~~*~~x~~*~~+2)2~~ = 1.

5

(b) *P*(1*/*4 *< X <* 1*/*2) =

∫1*/*2

1*/*4

2(*x*+2)

5

0

1*/*2



*dx* =~~(~~*~~x~~*~~+2)2~~ = 19*/*80.

5

1*/*4

(2)(

2.9 We can select *x* defective sets from 2, and 3 *− x* good sets from 5 in

5

)

ways. A

random selection of 3 from 7 sets can be made in

(2)( )

(7)

3

*x* 3*−x*

ways. Therefore,

5

*x* 3*−x*

*f*(*x*) = (7) *, x* = 0*,*1*,*2*.*

3

In tabular form

*x* 0 1 2

*f* (*x*) 2/7 4/7 1/7

The following is a probability histogram:

4/7

3/7

2/7

1/7

1 2 3

*x*

Copyright ©2013 Pearson Education, Inc.

3

2

*f(x)*

*Solutions for Exercises in Chapter 2* 17

2.10 (a) *P*(*T* = 5) = *F*(5) *− F*(4) = 3*/*4 *−* 1*/*2 = 1*/*4.

(b) *P*(*T >* 3) = 1 *− F*(3) = 1 *−* 1*/*2 = 1*/*2.

(c) *P* (1*.*4 *< T <* 6) = *F* (6) *− F* (1*.*4) = 3*/*4 *−* 1*/*4 = 1*/*2.

(d) *P*(*T ≤* 5*|T ≥* 2) =*~~P~~*~~(2~~*~~≤T≤~~*~~5)~~=~~3~~*~~/~~*~~4~~*~~−~~*~~1~~*~~/~~*~~4~~ =2

*P* (*T ≥*2) 1*−*1*/*4

2.11 The c.d.f. of *X* is   
 ⎧

⎪0*,* for *x <* 0*,*

3*.*

⎪

⎪0*.*41*,* for 0 *≤ x <* 1*,*

⎪   
⎨

*F* (*x*) =

0*.*78*,* for 1 *≤ x <* 2*,* ⎪0*.*94*,* for 2 *≤ x <* 3*,*

⎪

⎪0*.*99*,* for 3 *≤ x <* 4*,*

⎪

⎩

1*,* for *x ≥* 4*.*

2.12 (a) *P*(*X <* 0*.*2) = *F*(0*.*2) = 1 *− e−*1*.*6 = 0*.*7981;

(b) *f*(*x*) = *F′*(*x*) = 8*e−*8*x*. Therefore, *P*(*X <* 0*.*2) = 8

0*.*7981.

2.13 The c.d.f. of *X* is   
 ⎧

⎪0*,* for *x <* 0*,*

∫0*.*2

0

*e−*8*x dx* = *−e−*8*x|*0*.*2 =

0

⎨

*F* (*x*) =

2*/*7*,* for 0 *≤ x <* 1*,* ⎪6*/*7*,* for 1 *≤ x <* 2*,*

⎩

1*,* for *x ≥* 2*.*

(a) *P*(*X* = 1) = *P*(*X ≤* 1) *− P*(*X ≤* 0) = 6*/*7 *−* 2*/*7 = 4*/*7;

(b) *P*(0 *< X ≤* 2) = *P*(*X ≤* 2) *− P*(*X ≤* 0) = 1 *−* 2*/*7 = 5*/*7.

2.14 A graph of the c.d.f. is shown next.

1

6/7   
5/7   
4/7   
3/7   
2/7   
1/7

2.15 (a) 1 = *k*

∫1

0

0 1 2

*x*



*√~~x~~ dx* =~~2~~*~~k~~*1 Therefore, *k* =3

3 *x*3*/*2 0 = ~~3~~ . 2.

Copyright ©2013 Pearson Education, Inc.

*F(x)*

2*k*

18 *Chapter 2 Random Variables, Distributions, and Expectations*

∫*x √* 

(b) For 0 *≤ x <* 1, *F*(*x*) =3

2

Hence,

*t dt* = *t*3*/*2*x x*3*/*2.

0 0 =

⎧

⎨0*, x<*0

*F* (*x*) = *x*3*/*2*,* 0*≤x<*1

⎩

1*, x≥*1

*P*(0*.*3 *< X <* 0*.*6) = *F*(0*.*6) *− F*(0*.*3) = (0*.*6)3*/*2 *−* (0*.*3)3*/*2 = 0*.*3004.

2.16 Denote by *X* the number of spades int he three draws. Let *S* and *N* stand for a spade   
 and not a spade, respectively. Then

*P* (*X* = 0) = *P* (*N N N* ) = (39*/*52)(38*/*51)(37*/*50) = 703*/*1700,

*P* (*X* = 1) = *P* (*SN N* ) + *P* (*N SN* ) + *P* (*N N S*) = 3(13*/*52)(39*/*51)(38*/*50) = 741*/*1700, *P* (*X* = 3) = *P* (*SSS*) = (13*/*52)(12*/*51)(11*/*50) = 11*/*850, and

*P* (*X* = 2) = 1 *−* 703*/*1700 *−* 741*/*1700 *−* 11*/*850 = 117*/*850. The probability mass function for *X* is then

*x* 0 1 2 3

*f* (*x*) 703*/*1700 741*/*1700 117*/*850 11*/*850

2.17 Let *T* be the total value of the three coins. Let *D* and *N* stand for a dime and nickel,

respectively. Since we are selecting without replacement, the sample space containing

elements for which *t* = 20*,* 25, and 30 cents corresponding to the selecting of 2 nickels

and 1 dime, 1 nickel and 2 dimes, and 3 dimes. Therefore, *P* (*T* = 20) =   
 (21)(4 2)

(22)(41)

(63)

=1 5,

*P* (*T* = 25) =   
*P* (*T* = 30) =

(63)

(43)

=3 5,

(63)= 5 ,

and the probability distribution in tabular form is

*t* 20 25 30

*P*(*T* = *t*) 1/5 3/5 1/5

As a probability histogram

3/5

2/5

1/5

20 25 30

*x*

Copyright ©2013 Pearson Education, Inc.

*f(x)*

*Solutions for Exercises in Chapter 2* 19

(10)

2.18 There are

4

ways of selecting any 4 CDs from 10. We can select *x* jazz CDs from 5   
 (5)( )

and 4 *− x* from the remaining CDs in   
 (5)( )

5

*x* 4*−x*

ways. Hence

5

4*−x*

*f* (*x*) =*x*(10) *, x* = 0*,*1*,*2*,*3*,*4*.*

4

∫*x*

2.19 (a) For *x ≥* 0, *F*(*x*) =

0

1

xp(*−t/*2000) *dt* = *−* exp(*−t/*2000)*|x*

~~2000~~ e 0

= 1 *−* exp(*−x/*2000). So

{

*F* (*x*) =

0*, x <* 0*,*

1 *−* exp(*−x/*2000)*, x ≥* 0*.*

(b) *P*(*X >* 1000) = 1 *− F*(1000) = 1 *−* [1 *−* exp(*−*1000*/*2000)] = 0*.*6065.

(c) *P* (*X <* 2000) = *F* (2000) = 1 *−* exp(*−*2000*/*2000) = 0*.*6321.

2.20 (a) *f*(*x*) *≥* 0 and

∫26*.*252 

26*.*25

23*.*75 5 *dx*= 5*t* 23*.*75 =~~2~~*~~.~~*~~5~~ =

1.

∫24

(b) *P*(*X <* 24) =

2

*dx* =2(24 *−* 23*.*75) = 0*.*1.

23*.*75 5 5

∫26*.*252

(c) *P* (*X >* 26) = 26 5 *dx*= 5(26*.*25*−*26)=0*.*1.Itisnotextremelyrare.

∫*∞* *∞*

2.21 (a) *f*(*x*) *≥* 0 and

1

3*x−*4 *dx* = *−*3*~~x−~~*~~3~~

3

∫*x*



 = 1. So, this is a valid density function.

1

(b) For *x ≥* 1, *F*(*x*) =

1

3*t−*4 *dt* = 1 *− x−*3. So,

{

0*, x <* 1*,*

*F* (*x*) =

1*−x−*3*, x ≥* 1*.*

(c) *P* (*X >* 4) = 1 *− F* (4) = 4*−*3 = 0*.*0156.

∫ (

1

2.22 (a) 1 = *k −*1(3*−x*2) *dx* = *k* 3*x −~~x~~*~~3~~3

∫*x*

)1





*−*1

=~~16~~3 *k*.So,*k*= ~~16~~ .

( )

(b) For *−*1 *≤ x <* 1, *F*(*x*) =~~3~~ *dt* = 3*t −*1 *x*

( ) (9 )16(1*−*1(3*−t*2))

So, *P*

*X <*1

2

=1 *−*~~1~~

2 *−* 16 2 16

(1)3 3*t*3 *−*1 =2 ~~16~~ *x−* ~~13~~6.

=~~99~~

2 128 .

(c) *P* (*|X| <* 0*.*8) = *P* (*X < −*0*.*8) + *P* (*X >* 0*.*8) = *F* (*−*0*.*8) + 1 *− F* (0*.*8)(1 ) (1 )

=1+

2.23

2.24

2160*.*8+ ~~16~~0*.*83 *−* 2 160*.*8*−* ~~16~~0*.*83 = 0*.*164.

∫*y*

(a) For *y ≥* 0, *F*(*y*) =14 0 *e−t/*4 *dy*=1*−ey/*4. So, *P*(*Y >* 6) = *e−*6*/*4 = 0*.*2231. This

probability certainly cannot be considered as “unlikely.”

(b) *P*(*Y ≤* 1) = 1 *− e−*1*/*4 = 0*.*2212, which is not so small either.

∫1

(a) *f*(*y*) *≥* 0 and 0 5(1*−y*)4 *dy*=*−*(1*−y*)5*|*0 = 1. So, this is a valid density

function.

Copyright ©2013 Pearson Education, Inc.

3

1

1

20 *Chapter 2 Random Variables, Distributions, and Expectations*

(b) *P*(*Y <* 0*.*1) = *−* (1 *− y*)5*|*0*.*1 = 1 *−* (1 *−* 0*.*1)5 = 0*.*4095.

0

(c) *P* (*Y >* 0*.*5) = (1 *−* 0*.*5)5 = 0*.*03125.

2.25

2.26

∫1

(a) Using integral by parts and setting 1 = *k* 0 *y*4(1*−y*)3 *dy*,weobtain*k*=280.

(b) For 0 *≤ y <* 1, *F*(*y*) = 56*y*5(1 *− y*)3 + 28*y*6(1 *− y*)2 + 8*y*7(1 *− y*) + *y*8. So,

*P* (*Y ≤* 0*.*5) = 0*.*3633.

(c) Using the cdf in (b), *P* (*Y >* 0*.*8) = 0*.*0563.

(a) The event *Y* = *y* means that among 5 selected, exactly *y* tubes meet the spec-  
 ification (*M* ) and 5 *− y* (*M′*) does not. The probability for one combination of

such a situation is (0*.*99)*y*(1 *−* 0*.*99)5*−y* if we assume independence among the

5!

tubes. Since there are *~~y~~*~~!(5~~*~~−y~~*~~)!~~ p ermutations of getting *y M* s and 5 *− y M′*s, the   
probability of this event (*Y* = *y*) would be what it is specified in the problem.

(b) Three out of 5 is outside of specification means that *Y* = 2. *P* (*Y* = 2) = 9*.*8*×*10*−*6 which is extremely small. So, the conjecture is false.

2.27

∑

(a) *P*(*X >* 8) = 1 *− P*(*X ≤* 8) = *e−*6~~6~~*~~x~~*

*x*!

( )

60

=1*−e−*6 +~~61~~ +*···*+~~68~~ = 0*.*1528.

0! 1! 8!

*x*=0

(b) *P*(*X* = 2) = *e−*6~~62~~ = 0*.*0446.

2!

∫*x*

2.28 For 0 *< x <* 1, *F*(*x*) = 2 0 (1 *− t*) *dt* = *−* (1 *− t*)2*|x*0 = 1 *−* (1 *− x*)2.

(a) *P*(*X ≤* 1*/*3) = 1 *−* (1 *−* 1*/*3)2 = 5*/*9.

(b) *P*(*X >* 0*.*5) = (1 *−* 1*/*2)2 = 1*/*4.

(c) *P* (*X <* 0*.*75 *| X ≥* 0*.*5) =*~~P~~*~~(0~~*~~.~~*~~5~~*~~≤X<~~*~~0~~*~~.~~*~~75)~~

*P* (*X≥*0*.*5)

∑ ∑ ∑

= ~~(~~~~1~~*~~−~~*~~0~~*~~.~~*~~5)2~~*~~−~~*~~(1~~*~~−~~*~~0~~*~~.~~*~~75)2~~ =3

(1*−*0*.*5)2 4.

2.29 (a) *f* (*x, y*) = *c*

*x*=0 *y*=0

∑ *xy* = 36*c* = 1. Hence *c* = 1*/*36.

*x*=0 *y*=0

(b)

∑∑ ∑

*f* (*x, y*) = *c* ∑ *|x − y|* = 15*c* = 1. Hence *c* = 1*/*15.

*x y x y*

2.30 The joint probability distribution of (*X,Y* ) is

*x*

*f* (*x, y*) 0 1 2 3

0 0 1/30 2/30 3/30

*y* 1 1/30 2/30 3/30 4/30

2 2/30 3/30 4/30 5/30

(a) *P*(*X ≤* 2*,Y* = 1) = *f*(0*,*1) + *f*(1*,*1) + *f*(2*,*1) = 1*/*30 + 2*/*30 + 3*/*30 = 1*/*5.

(b) *P*(*X >* 2*,Y ≤* 1) = *f*(3*,*0) + *f*(3*,*1) = 3*/*30 + 4*/*30 = 7*/*30.

(c) *P* (*X > Y* ) = *f* (1*,* 0) + *f* (2*,* 0) + *f* (3*,* 0) + *f* (2*,* 1) + *f* (3*,* 1) + *f* (3*,* 2)   
 = 1*/*30 + 2*/*30 + 3*/*30 + 3*/*30 + 4*/*30 + 5*/*30 = 3*/*5.

Copyright ©2013 Pearson Education, Inc.

8

3

3

3

3

*Solutions for Exercises in Chapter 2* 21

(d) *P*(*X* + *Y* = 4) = *f*(2*,*2) + *f*(3*,*1) = 4*/*30 + 4*/*30 = 4*/*15.

(e) The possible outcomes of *X* are 0, 1, 2, and 3, and the possible outcomes of   
 *Y* are 0, 1, and 2. The marginal distribution of *X* can be calculated such as   
 *fX*(0) = 1*/*30 + 2*/*30 = 1*/*10. Finally, we have the distribution tables.

*x* 0 1 2 3 *y* 0 1 2

*fX*(*x*) 1/10 1/5 3/10 4/10 *fY* (*y*) 1/5 1/3 7/15

2.31 (a) We can select *x* oranges from 3, *y* apples from 2, and 4 *− x − y* bananas from 3(3)(2)( ) (8)

in

3

*x y* 4*−x−y*

ways. A random selection of 4 pieces of fruit can be made in

4

ways. Therefore,

(3)(2)(

3

)

*f*(*x,y*) =*xy*(8)

4*−x−y*

*, x* = 0*,*1*,*2*,*3; *y* = 0*,*1*,*2; 1 *≤ x* + *y ≤* 4*.*

4

Hence, we have the following joint probability table with the marginal distributions on the last row and last column.

*x*

*f* (*x, y*) 0 1 2 3 *fY* (*y*)

0 0 3/70 9/70 3/70 3/14

*y* 1 2/70 18/70 18/70 2/70 8/14

2 3/70 9/70 3/70 0 3/14

*fX*(*x*) 1/14 6/14 6/14 1/14

(b) *P*[(*X,Y* ) *∈ A*] = *P*(*X* + *Y ≤* 2) = *f*(1*,*0) + *f*(2*,*0) + *f*(0*,* 1) + *f*(1*,* 1) + *f*(0*,* 2)

= 3*/*70 + 9*/*70 + 2*/*70 + 18*/*70 + 3*/*70 = 1*/*2.

(c) *P* (*Y* = 0*|X* = 2) =

*P*(*X* = 2*,Y* = 0)   
 *P*(*X* = 2)

9*/*70

=

6*/*14 10.

(d) We know from (c) that *P*(*Y* = 0*|X* = 2) = 3*/*10, and we can calculate

*P*(*Y* = 1*|X* = 2) =

∫1

18*/*70 3

=

6*/*14 5*,*and*P*(*Y*=2*|X*=2)=6*/*14 10*.*

2.32 (a) *g*(*x*) =2 (*x* + 2*y*) *dy* =2(*x* + 1), for 0 *≤ x ≤* 1.

3

(b) *h*(*y*) =2

0 3

∫1 (*x* + 2*y*) *dx* =1(1 + 4*y*), for 0 *≤ y ≤* 1.

3 0

(c) *P* (*X <* 1*/*2) =2

3

∫1*/*2 (*x* + 1) *dx* =~~5~~

3 0

2.33 (a) *P*(*X* + *Y ≤* 1*/*2) =

∫1*−x*

12 .

∫1*/*2 ∫1*/*2*−y* 0 0 24*xy dx dy* = 12

∫1*/*2 (1 )2

0 2 *−y y dy* =~~1~~ 16 .

(b) *g*(*x*) = 24*xy dy* = 12*x*(1 *− x*)2, for 0 *≤ x <* 1.