Chapter 5
Time Value of Money

Learning Objectives

After reading this chapter, students should be able to:

- Explain how the time value of money works and discuss why it is such an important concept in finance.
- Calculate the present value and future value of lump sums.
- Identify the different types of annuities, calculate the present value and future value of both an ordinary annuity and an annuity due, and calculate the relevant annuity payments.
- Calculate the present value and future value of an uneven cash flow stream. You will use this knowledge in later chapters that show how to value common stocks and corporate projects.
- Explain the difference between nominal, periodic, and effective interest rates. An understanding of these concepts is necessary when comparing rates of returns on alternative investments.
- Discuss the basics of loan amortization and develop a loan amortization schedule that you might use when considering an auto loan or home mortgage loan.

Lecture Suggestions

We regard Chapter 5 as the most important chapter in the book, so we spend a good bit of time on it. We approach time value in three ways. First, we try to get students to understand the basic concepts by use of time lines and simple logic. Second, we explain how the basic formulas follow the logic set forth in the time lines. Third, we show how financial calculators and spreadsheets can be used to solve various time value problems in an efficient manner. Once we have been through the basics, we have students work problems and become proficient with the calculations and also get an idea about the sensitivity of output, such as present or future value, to changes in input variables, such as the interest rate or number of payments.

Some instructors prefer to take a strictly analytical approach and have students focus on the formulas themselves. The argument is made that students treat their calculators as "black boxes," and that they do not understand where their answers are coming from or what they mean. We disagree. We think that our approach shows students the logic behind the calculations as well as alternative approaches, and because calculators are so efficient, students can actually see the significance of what they are doing better if they use a calculator. We also think it is important to teach students how to use the type of technology (calculators and spreadsheets) they must use when they venture out into the real world.

In the past, the biggest stumbling block to many of our students has been time value, and the biggest problem was that they did not know how to use their calculator. Since time value is the foundation for many of the concepts that follow, this chapter is near the beginning of the text. This gives students more time to become comfortable with the concepts and the tools (formulas, calculators, and spreadsheets) covered in this chapter. Therefore, we strongly encourage students to get a calculator, learn to use it, and bring it to class so they can work problems with us as we go through the lectures. Our urging, plus the fact that we can now provide relatively brief, course-specific manuals for the leading calculators, has reduced if not eliminated the problem.

Our research suggests that the best calculator for the money for most students is the HP-10BII. Finance and accounting majors might be better off with a more powerful calculator, such as the HP-17BII. We recommend these two for people who do not already have a calculator, but we tell them that any financial calculator that has an IRR function will do.

We also tell students that it is essential that they work lots of problems, including the end-of-chapter problems. We emphasize that this chapter is critical, so they should invest the time now to get the material down. We stress that they simply cannot do well with the material that follows without having this material down cold. Bond and stock valuation, cost of capital, and capital budgeting make little sense, and one certainly cannot work problems in these areas, without understanding time value of money first.

We base our lecture on the integrated case. The case goes systematically through the key points in the chapter, and within a context that helps students see the real world relevance of the material in the chapter. We ask the students to read the chapter, and also to "look over" the case before class. However, our class consists of about 1,000 students, many of whom view the lecture on TV, so we cannot count on them to prepare for class. For this reason, we designed our lectures to be useful to both prepared and unprepared students.

Since we have easy access to computer projection equipment, we generally use the *PowerPoint* slides as the core of our lectures. We <u>strongly</u> suggest to our students that they print a copy of the *PowerPoint* slides for the chapter from the website and bring it to class. This will provide them with a hard copy of our lecture, and they can take notes in the space provided. Students can then concentrate on the lecture rather than on taking notes.

We do not stick strictly to the slide show—we go to the board frequently to present somewhat different examples, to help answer questions, and the like. We like the spontaneity and change of pace trips to the board provide, and, of course, use of the board provides needed flexibility. Also, if we feel that we have covered a topic adequately at the board, we then click quickly through one or more slides.

The lecture notes we take to class consist of our own marked-up copy of the *PowerPoint* slides, with notes on the comments we want to say about each slide. If we want to bring up some current event, provide an additional example, or the like, we use post-it notes attached at the proper spot. The advantages of this system are (1) that we have a carefully structured lecture that is easy for us to prepare (now that we have it done) and for students to follow, and (2) that both we and the students always know exactly where we are. The students also appreciate the fact that our lectures are closely coordinated with both the text and our exams.

The slides contain the essence of the solution to each part of the integrated case, but we also provide more in-depth solutions in this *Instructor's Manual*. It is not essential, but you might find it useful to read through the detailed solution. Also, we put a copy of the solution on reserve in the library for interested students, but most find that they do not need it.

Finally, we remind students again, at the start of the lecture on Chapter 5, that they should bring a printout of the *PowerPoint* slides to class, for otherwise they will find it difficult to take notes. We also repeat our request that they get a financial calculator and our brief manual for it that can be found on the website, and bring the calculator to class so they can work through calculations as we cover them in the lecture.

DAYS ON CHAPTER: 4 OF 56 DAYS (50-minute periods)

Answers to End-of-Chapter Questions

5-1 The opportunity cost is the rate of interest one could earn on an alternative investment with a risk equal to the risk of the investment in question. This is the value of I in the TVM equations, and it is shown on the top of a time line, between the first and second tick marks. It is not a single rate—the opportunity cost rate varies depending on the riskiness and maturity of an investment, and it also varies from year to year depending on inflationary expectations (see Chapter 6).

5-2 True. The second series is an uneven cash flow stream, but it contains an annuity of $400 for 8 years. The series could also be thought of as a $100 annuity for 10 years plus an additional payment of $100 in Year 2, plus additional payments of $300 in Years 3 through 10.

5-3 True, because of compounding effects—growth on growth. The following example demonstrates the point. The annual growth rate is I in the following equation:

$$\$1(1 + I)^{10} = \$2.$$

We can find I in the equation above as follows:

Using a financial calculator input N = 10, PV = -1, PMT = 0, FV = 2, and I/YR = ? Solving for I/YR you obtain 7.18%.

Viewed another way, if earnings had grown at the rate of 10% per year for 10 years, then EPS would have increased from $1.00 to $2.59, found as follows: Using a financial calculator, input N = 10, I/YR = 10, PV = -1, PMT = 0, and FV = ?. Solving for FV you obtain $2.59. This formulation recognizes the "interest on interest" phenomenon.

5-4 For the same stated rate, daily compounding is best. You would earn more "interest on interest."

5-5 False. One can find the present value of an embedded annuity and add this PV to the PVs of the other individual cash flows to determine the present value of the cash flow stream.

5-6 The concept of a perpetuity implies that payments will be received forever. FV (Perpetuity) = PV (Perpetuity)$(1 + I)^{\infty} = \infty$.

5-7 The annual percentage rate (APR) is the periodic rate times the number of periods per year. It is also called the nominal, or stated, rate. With the "Truth in Lending" law, Congress required that financial institutions disclose the APR so the rate charged would be more "transparent" to consumers. The APR is equal to the effective annual rate only when compounding occurs annually. If more frequent compounding occurs, the effective rate is always greater than the annual percentage rate. Nominal rates can be compared with one another, but only if the instruments being compared use the same number of compounding periods per year. If this is not the case, then the instruments being compared should be put on an effective annual rate basis for comparisons.

5-8 A loan amortization schedule is a table showing precisely how a loan will be repaid. It gives the required payment on each payment date and a breakdown of the payment, showing how much is interest and how much is repayment of principal. These schedules can be used for any loans that are paid off in installments over time such as automobile loans, home mortgage loans, student loans, and many business loans.

5-9 When the cash flow is paid many years later, the present value of this cash flow is almost zero as it is heavily discounted. Hence, each additional cash flow contributes to an every decreasing, miniscule amount to the present value of the perpetuity, and hence does not increase it materially.

Solutions to End-of-Chapter Problems

5-1

0 — 10% — 1 — 2 — 3 — 4 — 5

PV = 10,000 $\quad$ FV_5 = ?

FV_5 = \$10,000$(1.10)^5$
= \$10,000(1.61051) = \$16,105.10.

Alternatively, with a financial calculator enter the following: N = 5, I/YR = 10, PV = -10000, and PMT = 0. Solve for FV = \$16,105.10.

5-2

0 — 7% — 5 — 10 — 15 — 20

PV = ? $\quad$ FV_{20} = 5,000

With a financial calculator enter the following: N = 20, I/YR = 7, PMT = 0, and FV = 5000. Solve for PV = \$1,292.10.

5-3

0 — I/YR = ? — 18

PV = 250,000 $\quad$ FV_{18} = 1,000,000

With a financial calculator enter the following: N = 18, PV = -250000, PMT = 0, and FV = 1000000. Solve for I/YR = 8.01% ≈ 8%.

5-4

0 — 6.5% — N = ?

PV = 1 $\quad$ FV_N = 2

\$2 = \$1$(1.065)^N$.

With a financial calculator enter the following: I/YR = 6.5, PV = -1, PMT = 0, and FV = 2. Solve for N = 11.01 ≈ 11 years.

5-5

0 — 12% — 1 — 2 — • • • — N – 2 — N – 1 — N

PV = 42,180.53 $\quad$ 5,000 $\quad$ 5,000 $\quad$ 5,000 $\quad$ 5,000 $\quad$ FV = 250,000

Using your financial calculator, enter the following data: I/YR = 12; PV = -42180.53; PMT = -5000; FV = 250000; N = ? Solve for N = 11. It will take 11 years to accumulate \$250,000.

5-6 Ordinary annuity:

0	1	2	3	4	5
7%					
	300	300	300	300	300
					FVA_5 = ?

With a financial calculator enter the following: N = 5, I/YR = 7, PV = 0, and PMT = 300. Solve for FV = $1,725.22.

Annuity due:

0	1	2	3	4	5
7%					
300	300	300	300	300	

With a financial calculator, switch to "BEG" and enter the following: N = 5, I/YR = 7, PV = 0, and PMT = 300. Solve for FV = $1,845.99. Don't forget to switch back to "END" mode.

5-7

0	1	2	3	4	5	6
8%						
	100	100	100	200	300	500
PV = ?						FV = ?

Using a financial calculator, enter the following: CF_0 = 0; CF_1 = 100; N_j = 3; CF_4 = 200 (Note calculator will show CF_2 on screen.); CF_5 = 300 (Note calculator will show CF_3 on screen.); CF_6 = 500 (Note calculator will show CF_4 on screen.); and I/YR = 8. Solve for NPV = $923.98.

To solve for the FV of the cash flow stream with a calculator that doesn't have the NFV key, do the following: Enter N = 6, I/YR = 8, PV = -923.98, and PMT = 0. Solve for FV = $1,466.24. You can check this as follows:

0	1	2	3	4	5	6
8%						
	100	100	100	200	300	500
					× (1.08) →	324.00
				× $(1.08)^2$ →		233.28
			× $(1.08)^3$ →			125.97
		× $(1.08)^4$ →				136.05
	× $(1.08)^5$ →					146.93
						$1,466.23

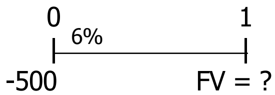
5-8 Using a financial calculator, enter the following: N = 60, I/YR = 1, PV = -20000, and FV = 0. Solve for PMT = $444.89.

$$\begin{aligned} \text{EAR} &= \left(1+\frac{I_{NOM}}{M}\right)^{M} - 1.0 \\ &= (1.01)^{12} - 1.0 \\ &= 12.68\%. \end{aligned}$$

Alternatively, using a financial calculator, enter the following: NOM% = 12 and P/YR = 12. Solve for EFF% = 12.6825%. Remember to change back to P/YR = 1 on your calculator.

5-9 Using the financial calculator, we find the number of years for the deposit to triple as follows: Number of years (N) =22.5: Input I/YR = 5%, PV = –100, PMT = 0, FV = 300.

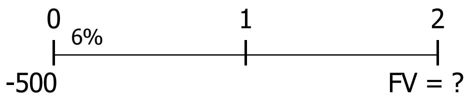
5-10 a.



$\$500(1.06) = \$530.00.$

Using a financial calculator, enter N = 1, I/YR = 6, PV = -500, PMT = 0, and FV = ? Solve for FV = $530.00.

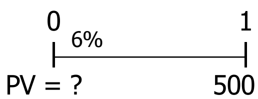
b.



$\$500(1.06)^2 = \$561.80.$

Using a financial calculator, enter N = 2, I/YR = 6, PV = -500, PMT = 0, and FV = ? Solve for FV = $561.80.

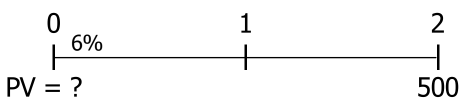
c.



$\$500(1/1.06) = \$471.70.$

Using a financial calculator, enter N = 1, I/YR = 6, PMT = 0, and FV = 500, and PV = ? Solve for PV = $471.70.

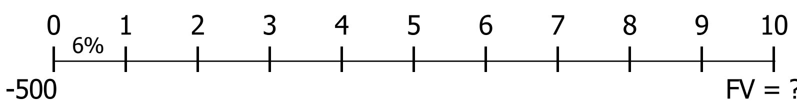
d.



$\$500(1/1.06)^2 = \$445.00.$

Using a financial calculator, enter N = 2, I/YR = 6, PMT = 0, FV = 500, and PV = ? Solve for PV = $445.00.

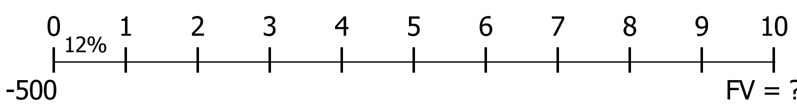
5-11 a.



$\$500(1.06)^{10} = \$895.42.$

Using a financial calculator, enter N = 10, I/YR = 6, PV = -500, PMT = 0, and FV = ? Solve for FV = $895.42.

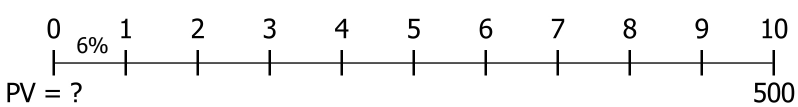
b.



$\$500(1.12)^{10} = \$1,552.92.$

Using a financial calculator, enter N = 10, I/YR = 12, PV = -500, PMT = 0, and FV = ? Solve for FV = $1,552.92.

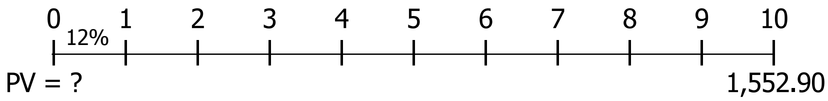
c.



$\$500/(1.06)^{10} = \$279.20.$

Using a financial calculator, enter N = 10, I/YR = 6, PMT = 0, FV = 500, and PV = ? Solve for PV = $279.20.

d.



$\$1{,}552.90/(1.12)^{10} = \$499.99$.

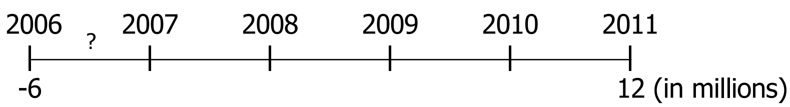
Using a financial calculator, enter N = 10, I/YR = 12, PMT = 0, FV = 1552.90, and PV = ? Solve for PV = $499.99.

$\$1{,}552.90/(1.06)^{10} = \$867.13$.

Using a financial calculator, enter N = 10, I/YR = 6, PMT = 0, FV = 1552.90, and PV = ? Solve for PV = $867.13.

e. The present value is the value today of a sum of money to be received in the future. For example, the value today of $1,552.90 to be received 10 years in the future is about $500 at an interest rate of 12%, but it is approximately $867 if the interest rate is 6%. Therefore, if you had $500 today and invested it at 12%, you would end up with $1,552.90 in 10 years. The present value depends on the interest rate because the interest rate determines the amount of interest you forgo by not having the money today.

5-12 **a.**



With a calculator, enter N = 5, PV = -6, PMT = 0, FV = 12, and then solve for I/YR = 14.87%.

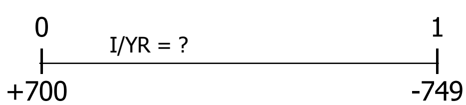
b. The calculation described in the quotation fails to consider the compounding effect of interest. It can be demonstrated to be incorrect as follows:

$\$6{,}000{,}000(1.20)^5 = \$6{,}000{,}000(2.48832) = \$14{,}929{,}920$,

which is greater than $12 million. Thus, the annual growth rate is less than 20%; in fact, it is about 15%, as shown in Part a.

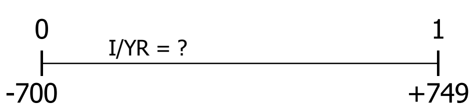
5-13 These problems can all be solved using a financial calculator by entering the known values shown on the time lines and then pressing the I/YR button.

a.



With a financial calculator, enter: N = 1, PV = 700, PMT = 0, and FV = -749. I/YR = 7%.

b.



With a financial calculator, enter: N = 1, PV = -700, PMT = 0, and FV = 749. I/YR = 7%.

c.

0	I/YR = ?	10
+85,000		-201,229

With a financial calculator, enter: N = 10, PV = 85000, PMT = 0, and FV = -201229. I/YR = 9%.

d.

0	1	2	3	4	5
I/YR = ?					
+9,000	-2,684.80	-2,684.80	-2,684.80	-2,684.80	-2,684.80

With a financial calculator, enter: N = 5, PV = 9000, PMT = -2684.80, and FV = 0. I/YR = 15%.

5-14 **a.**

7%	?
-200	400

With a financial calculator, enter I/YR = 7, PV = -200, PMT = 0, and FV = 400. Then press the N key to find N = 10.24. Override I/YR with the other values to find N = 7.27, 4.19, and 1.00.

b.

10%	?
-200	400

Enter: I/YR = 10, PV = -200, PMT = 0, and FV = 400.
N = 7.27.

c.

18%	?
-200	400

Enter: I/YR = 18, PV = -200, PMT = 0, and FV = 400.
N = 4.19.

d.

100%	?
-200	400

Enter: I/YR = 100, PV = -200, PMT = 0, and FV = 400.
N = 1.00.

5-15 **a.**

0	1	2	3	4	5	6	7	8	9	10
10%										
	400	400	400	400	400	400	400	400	400	400
										FV = ?

With a financial calculator, enter N = 10, I/YR = 10, PV = 0, and PMT = -400. Then press the FV key to find FV = $6,374.97.

b.

0	1	2	3	4	5
5%					
	200	200	200	200	200
					FV = ?

With a financial calculator, enter N = 5, I/YR = 5, PV = 0, and PMT = -200. Then press the FV key to find FV = $1,105.13.

c.

0	1	2	3	4	5
0%					
	400	400	400	400	400
					FV = ?

With a financial calculator, enter N = 5, I/YR = 0, PV = 0, and PMT = -400. Then press the FV key to find FV = $2,000.

d. To solve Part d using a financial calculator, repeat the procedures discussed in Parts a, b, and c, but first switch the calculator to "BEG" mode. Make sure you switch the calculator back to "END" mode after working the problem.

1.

0	1	2	3	4	5	6	7	8	9	10
10%										
400	400	400	400	400	400	400	400	400	400	FV = ?

With a financial calculator on BEG, enter: N = 10, I/YR = 10, PV = 0, and PMT = -400. FV = $7,012.47.

2.

0	1	2	3	4	5
5%					
200	200	200	200	200	FV = ?

With a financial calculator on BEG, enter: N = 5, I/YR = 5, PV = 0, and PMT = -200. FV = $1,160.38.

3.

0	1	2	3	4	5
0%					
400	400	400	400	400	FV = ?

With a financial calculator on BEG, enter: N = 5, I/YR = 0, PV = 0, and PMT = -400. FV = $2,000.

5-16 a.

0	1	2	3	4	5	6	7	8	9	10
10%										
PV = ?	400	400	400	400	400	400	400	400	400	400

With a financial calculator, simply enter the known values and then press the key for the unknown. Enter: N = 10, I/YR = 10, PMT = -400, and FV = 0. PV = $2,457.83.

b.

0	1	2	3	4	5
5%					
PV = ?	200	200	200	200	200

With a financial calculator, enter: N = 5, I/YR = 5, PMT = -200, and FV = 0. PV = $865.90.

c.

0	1	2	3	4	5
0%					
PV = ?	400	400	400	400	400

With a financial calculator, enter: N = 5, I/YR = 0, PMT = -400, and FV = 0. PV = $2,000.00.

d. 1.

0	1	2	3	4	5	6	7	8	9	10
10%										
400	400	400	400	400	400	400	400	400	400	
PV = ?										

With a financial calculator on BEG, enter: N = 10, I/YR = 10, PMT = -400, and FV = 0. PV = $2,703.61.

2.

0	1	2	3	4	5
5%					
200	200	200	200	200	
PV = ?					

With a financial calculator on BEG, enter: N = 5, I/YR = 5, PMT = -200, and FV = 0. PV = $909.19.

3.

0	1	2	3	4	5
0%					
400	400	400	400	400	
PV = ?					

With a financial calculator on BEG, enter: N = 5, I/YR = 0, PMT = -400, and FV = 0. PV = $2,000.00.

5-17 $PV_{7\%}$ = $100/0.07 = $1,428.57. $PV_{14\%}$ = $100/0.14 = $714.29.

When the interest rate is doubled, the PV of the perpetuity is halved.

5-18

0	1	2	3	4	• • •	30
I/YR = ?						
85,000	-8,273.59	-8,273.59	-8,273.59	-8,273.59		-8,273.59

With a calculator, enter N = 30, PV = 85000, PMT = -8273.59, FV = 0, and then solve for I/YR = 9%.

5-19 **a.**

Cash Stream A

0	1	2	3	4	5
8%					
PV = ?	100	400	400	400	300

Cash Stream B

0	1	2	3	4	5
8%					
PV = ?	300	400	400	400	100

With a financial calculator, simply enter the cash flows (be sure to enter CF_0 = 0), enter I/YR = 8, and press the NPV key to find NPV = PV = $1,251.25 for the first problem. Override I/YR = 8 with I/YR = 0 to find the next PV for Cash Stream A. Enter the cash flows for Cash Stream B, enter I/YR = 8, and press the NPV key to find NPV = PV = $1,300.32. Override I/YR = 8 with I/YR = 0 to find the next PV for Cash Stream B.

b. PV_A = $100 + $400 + $400 + $400 + $300 = $1,600.

PV_B = $300 + $400 + $400 + $400 + $100 = $1,600.

5-20 **a.** Begin with a time line:

40	41		64	65
9%		• • •		
	5,000		5,000	5,000
				FV = ?

Using a financial calculator input the following: N = 25, I/YR = 9, PV = 0, PMT = 5000, and solve for FV = \$423,504.48.

b.

40	41		69	70
9%		• • •		
	5,000		5,000	5,000
				FV = ?

Using a financial calculator input the following: N = 30, I/YR = 9, PV = 0, PMT = 5000, and solve for FV = \$681,537.69.

c. 1.

65	66	67		84	85
9%			• • •		
423,504.48	PMT	PMT		PMT	PMT

Using a financial calculator, input the following: N = 20, I/YR = 9, PV = -423504.48, FV = 0, and solve for PMT = \$46,393.42.

2.

70	71	72		84	85
9%			• • •		
681,537.69	PMT	PMT		PMT	PMT

Using a financial calculator, input the following: N = 15, I/YR = 9, PV = -681537.69, FV = 0, and solve for PMT = \$84,550.80.

5-21 Contract 1: $$PV = \frac{\$3{,}000{,}000}{1.10} + \frac{\$3{,}000{,}000}{(1.10)^2} + \frac{\$3{,}000{,}000}{(1.10)^3} + \frac{\$3{,}000{,}000}{(1.10)^4}$$
$$= \$2{,}727{,}272.73 + \$2{,}479{,}338.84 + \$2{,}253{,}944.40 + \$2{,}049{,}040.37$$
$$= \$9{,}509{,}596.34.$$

Using your financial calculator, enter the following data: CF_0 = 0; $CF_{1\text{-}4}$ = 3000000; I/YR = 10; NPV = ? Solve for NPV = \$9,509,596.34.

Contract 2: $$PV = \frac{\$2{,}000{,}000}{1.10} + \frac{\$3{,}000{,}000}{(1.10)^2} + \frac{\$4{,}000{,}000}{(1.10)^3} + \frac{\$5{,}000{,}000}{(1.10)^4}$$
$$= \$1{,}818{,}181.82 + \$2{,}479{,}338.84 + \$3{,}005{,}259.20 + \$3{,}415{,}067.28$$
$$= \$10{,}717{,}847.14.$$

Alternatively, using your financial calculator, enter the following data: CF_0 = 0; CF_1 = 2000000; CF_2 = 3000000; CF_3 = 4000000; CF_4 = 5000000; I/YR = 10; NPV = ? Solve for NPV = \$10,717,847.14.

Contract 3: $$PV = \frac{\$7{,}000{,}000}{1.10} + \frac{\$1{,}000{,}000}{(1.10)^2} + \frac{\$1{,}000{,}000}{(1.10)^3} + \frac{\$1{,}000{,}000}{(1.10)^4}$$
$$= \$6{,}363{,}636.36 + \$826{,}446.28 + \$751{,}314.80 + \$683{,}013.46$$
$$= \$8{,}624{,}410.90.$$

Alternatively, using your financial calculator, enter the following data: $CF_0 = 0$; $CF_1 = 7000000$; $CF_2 = 1000000$; $CF_3 = 1000000$; $CF_4 = 1000000$; I/YR = 10; NPV = ? Solve for NPV = $8,624,410.90.

Contract 2 gives the quarterback the highest present value; therefore, he should accept Contract 2.

5-22 **a.** If Wang Yun expects a 7% annual return on her investments:

1 payment	10 payments	30 payments
	N = 10	N = 30
	I/YR = 7	I/YR = 7
	PMT = 9500000	PMT = 5500000
	FV = 0	FV = 0
PV = $61,000,000	PV = $66,724,025	PV = $68,249,727

Wang Yun should accept the 30-year payment option as it carries the highest present value ($68,249,727).

b. If Wang Yun expects an 8% annual return on her investments:

1 payment	10 payments	30 payments
	N = 10	N = 30
	I/YR = 8	I/YR = 8
	PMT = 9500000	PMT = 5500000
	FV = 0	FV = 0
PV = $61,000,000	PV = $63,745,773	PV = $61,917,808

Wang Yun should accept the 10-year payment option as it carries the highest present value ($63,745,773).

c. If Wang Yun expects a 9% annual return on her investments:

1 payment	10 payments	30 payments
	N = 10	N = 30
	I/YR = 9	I/YR = 9
	PMT = 9500000	PMT = 5500000
	FV = 0	FV = 0
PV = $61,000,000	PV = $60,967,748	PV = $56,505,097

Wang Yun should accept the lump-sum payment option as it carries the highest present value ($61,000,000).

d. The higher the interest rate, the more useful it is to get money rapidly, because it can be invested at those high rates and earn lots more money. So, cash comes fastest with #1, slowest with #3, so the higher the rate, the more the choice is tilted toward #1. You can also think about this another way. The higher the discount rate, the more distant cash flows are penalized, so again, #3 looks worst at high rates, #1 best at high rates.

5-23 **a.** This can be done with a calculator by specifying an interest rate of 5% per period for 20 periods with 1 payment per period.

N = $10 \times 2 = 20$, I/YR = 10/2 = 5, PV = -10000, FV = 0. Solve for PMT = $802.43.

b. Set up an amortization table:

Period	Beginning Balance	Payment	Interest	Payment of Principal	Ending Balance
1	$10,000.00	$802.43	$500.00	$302.43	$9,697.57
2	9,697.57	802.43	484.88	317.55	9,380.02
			$984.88		

Because the mortgage balance declines with each payment, the portion of the payment that is applied to interest declines, while the portion of the payment that is applied to principal increases. The total payment remains constant over the life of the mortgage.

c. Jan must report interest of $984.88 on Schedule B for the first year. Her interest income will decline in each successive year for the reason explained in Part b.

d. Interest is calculated on the beginning balance for each period, as this is the amount the lender has loaned and the borrower has borrowed. As the loan is amortized (paid off), the beginning balance, hence the interest charge, declines and the repayment of principal increases.

5-24 a.

```
0  12%  1     2     3     4     5
|-------|-----|-----|-----|-----|
-500                          FV = ?
```

With a financial calculator, enter N = 5, I/YR = 12, PV = -500, and PMT = 0, and then press FV to obtain FV = $881.17.

b.

```
0  6%  1    2    3    4    5    6    7    8    9    10
|------|----|----|----|----|----|----|----|----|----|
-500                                            FV = ?
```

With a financial calculator, enter N = 10, I/YR = 6, PV = -500, and PMT = 0, and then press FV to obtain FV = $895.42.

Alternatively, $FV_N = PV\left(1+\frac{I_{NOM}}{M}\right)^{NM} = \$500\left(1+\frac{0.12}{2}\right)^{5(2)}$

$= \$500(1.06)^{10} = \$895.42.$

c.

```
0  3%  4     8     12    16    20
|------|-----|-----|-----|-----|
-500                         FV = ?
```

With a financial calculator, enter N = 20, I/YR = 3, PV = -500, and PMT = 0, and then press FV to obtain FV = $903.06.

Alternatively, $FV_N = \$500\left(1+\frac{0.12}{4}\right)^{5(4)} = \$500(1.03)^{20} = \$903.06.$

d.

```
0  1%  12    24    36    48    60
|------|-----|-----|-----|-----|
-500                         FV = ?
```

With a financial calculator, enter N = 60, I/YR = 1, PV = -500, and PMT = 0, and then press FV to obtain FV = \$908.35.

Alternatively, $FV_N = \$500\left(1+\frac{0.12}{12}\right)^{5(12)} = \$500(1.01)^{60} = \$908.35.$

e.

```
0          365                 1,825
|  0.0329% |------ • • • -------|
-500                           FV = ?
```

With a financial calculator, enter N = 1825, I/YR = 12/365 = 0.032877, PV = -500, and PMT = 0, and then press FV to obtain FV = \$910.97.

Alternatively, $FV_N = \$500\left(1+\frac{0.12}{365}\right)^{5(365)} = \$500(1.00032877)^{1,825} = \$910.97.$

f. The FVs increase because as the compounding periods increase, interest is earned on interest more frequently.

5-25 a.

```
0    2    4    6    8    10
| 6% |----|----|----|----|
PV = ?                   500
```

With a financial calculator, enter N = 10, I/YR = 6, PMT = 0, and FV = 500, and then press PV to obtain PV = \$279.20.

Alternatively, $PV = FV_N\left(\frac{1}{1+I_{NOM}/M}\right)^{NM} = \$500\left(\frac{1}{1+0.12/2}\right)^{5(2)}$

$= \$500\left(\frac{1}{1.06}\right)^{10} = \$279.20.$

b.

```
0    4    8    12   16   20
| 3% |----|----|----|----|
PV = ?                   500
```

With a financial calculator, enter N = 20, I/YR = 3, PMT = 0, and FV = 500, and then press PV to obtain PV = \$276.84.

Alternatively, $PV = \$500\left(\frac{1}{1+0.12/4}\right)^{4(5)} = \$500\left(\frac{1}{1.03}\right)^{20} = \$276.84.$

c.

```
0    1    2          12
| 1% |----|-- • • • --|
PV = ?               500
```

With a financial calculator, enter N = 12, I/YR = 1, PMT = 0, and FV = 500, and then press PV to obtain PV = \$443.72.

Alternatively, $PV = \$500\left(\frac{1}{1+0.12/12}\right)^{12(1)} = \$500\left(\frac{1}{1.01}\right)^{12} = \$443.72.$

d. The PVs for Parts a and b decline as periods/year increases. This occurs because, with more frequent compounding, a smaller initial amount (PV) is required to get to \$500 after 5 years. For Part c, even though there are 12 periods/year, compounding occurs over only 1 year, so the PV is larger.

5-26 **a.**

0 (6%)	1	2	3	•••	9	10
	-400	-400	-400		-400	-400
						FV = ?

Enter N = 5 × 2 = 10, I/YR = 12/2 = 6, PV = 0, PMT = -400, and then press FV to get FV = \$5,272.32.

b. Now the number of periods is calculated as N = 5 × 4 = 20, I/YR = 12/4 = 3, PV = 0, and PMT = -200. The calculator solution is \$5,374.07. The solution assumes that the nominal interest rate is compounded at the annuity period.

c. The annuity in Part b earns more because the money is on deposit for a longer period of time and thus earns more interest. Also, because compounding is more frequent, more interest is earned on interest.

5-27 Using the information given in the problem, you can solve for the maximum car price attainable.

Financed for 48 months	Financed for 60 months
N = 48	N = 60
I/YR = 1 (12%/12 = 1%)	I/YR = 1
PMT = -350	PMT = -350
FV = 0	FV = 0
PV = 13,290.89	PV = 15,734.26

You must add the value of the down payment to the present value of the car payments. If financed for 48 months, you can afford a car valued up to \$17,290.89 (\$13,290.89 + \$4,000). If financing for 60 months, you can afford a car valued up to \$19,734.26 (\$15,734.26 + \$4,000).

5-28

$$\text{EAR} = (1 + \text{Periodic Rate})^{M} - 1\#$$

$$4.04\% = (1 + \text{Periodic Rate})^{2} - 1$$

Periodic Rate = 2%

PV = 100,000

0	1	2	3	4	5	6	7	8	9	10
	PMT	PMT	PMT	PMT	PMT	PMT	PMT	PMT	PMT	PMT

Using the financial calculator, we calculate the semiannual loan payment (PMT) as follows: PMT =\$11,132.65: Input N = 10, I/YR = 2%, PV = –100,000, FV = 0.

5-29 **a.** Bank A: I_{NOM} = Effective annual rate = 4%.

Bank B:

$$\text{Effective annual rate} = \left(1 + \frac{0.035}{365}\right)^{365} - 1.0 = (1.000096)^{365} - 1.0$$

$$= 1.035618 - 1.0 = 0.035618 = 3.5618\%.$$

With a financial calculator, you can use the interest rate conversion feature to obtain the same answer. You would choose Bank A because its EAR is higher.

b. If funds must be left on deposit until the end of the compounding period (1 year for Bank A and 1 day for Bank B), and you think there is a high probability that you will make a withdrawal during the year, then Bank B might be preferable. For example, if the withdrawal is made after 6 months, you would earn nothing on the Bank A account but $(1.000096)^{365/2} - 1.0 = 1.765\%$ on the Bank B account.

Ten or more years ago, most banks were set up as described above, but now virtually all are computerized and pay interest from the day of deposit to the day of withdrawal, provided at least $1 is in the account at the end of the period.

5-30 Here you want to have an effective annual rate on the credit extended that is 2% more than what the bank is charging you, so you can cover overhead. First, we must find the EAR = EFF% on the bank loan. Enter NOM% = 6, P/YR = 12, and press EFF% to get EAR = 6.1678%.

So, to cover overhead you need to charge customers a nominal rate so that the corresponding EAR = 8.1678%. To find this nominal rate, enter EFF% = 8.1678, P/YR = 12, and press NOM% to get $I_{NOM} = 7.8771\%$. (Customers will be required to pay monthly, so P/YR = 12.)

Alternative solution: We need to find the effective annual rate (EAR) the bank is charging first. Then, we can add 2% to this EAR to calculate the nominal rate that you should quote your customers.

Bank EAR: $EAR = (1 + I_{NOM}/M)^M - 1 = (1 + 0.06/12)^{12} - 1 = 6.1678\%$.

So, the EAR you want to earn on your receivables is 8.1678%.

Nominal rate you should quote customers:

$$\begin{aligned} 8.1678\% &= (1 + I_{NOM}/12)^{12} - 1 \\ 1.081678 &= (1 + I_{NOM}/12)^{12} \\ 1.006564 &= 1 + I_{NOM}/12 \\ I_{NOM} &= 0.006564(12) = 7.8771\%. \end{aligned}$$

5-31 $I_{NOM} = 12\%$, daily compounding 360-day year.
Cost per day = 0.12/360 = 0.0003333 = 0.03333%.
Customers' credit period = 90 days.

If you loaned $1, after 90 days a customer would owe you $(1 + 0.12/360)^{90} \times \$1 = \$1.030449$. So, the required markup would be 3.0449% or approximately 3%.

5-32 **a.** Using the information given in the problem, you can solve for the length of time required to reach $1 million.

Shan Chen: I/YR = 6; PV = 30000; PMT = 5000; FV = -1000000; and then solve for N = 38.742182. Therefore, Shan Chen will be 25 + 38.74 = 63.74 years old when she becomes a millionaire.

Shan Xia: I/YR = 20; PV = 30000; PMT = 5000; FV = -1000000; and then solve for N = 16.043713. Therefore, Shan Xia will be 25 + 16.04 = 41.04 years old when she becomes a millionaire.

b. Using the 16.043713 year target, you can solve for the required payment:
N = 16.043713; I/YR = 6; PV = 30000; FV = -1000000; then solve for PMT = $35,825.33.

If Shan Chen wishes to reach the investment goal at the same time as Shan Xia, she will need to contribute $35,825.33 per year.

c. Shan Chen is investing in a relatively safe fund, so there is a good chance that she will achieve her goal, albeit slowly. Shan Xia is investing in a very risky fund, so while she might do quite well and become a millionaire shortly, there is also a good chance that she will lose her entire investment. High expected returns in the market are almost <u>always</u> accompanied by a lot of risk. We couldn't say whether Shan Chen is rational or irrational, just that she seems to have less tolerance for risk than Shan Xia does.

5-33 **a.**

0 (5%)	1	2	3	4
PV = ?	-10,000	-10,000	-10,000	-10,000

With a calculator, enter N = 4, I/YR = 5, PMT = -10000, and FV = 0. Then press PV to get PV = $35,459.51.

b. At this point, we have a 3-year, 5% annuity whose value is $27,232.48. You can also think of the problem as follows:

$35,459.51(1.05) – $10,000 = $27,232.49.

5-34

0 (8%)	1	2	3	4	5	6
	1,500	1,500	1,500	1,500	1,500	?
						FV = 10,000

With a financial calculator, get a "ballpark" estimate of the number of years it will take to reach your goal, by entering I/YR = 8, PV = 0, PMT = -1500, and FV = 10000, and solving for N = 5.55 years. This answer assumes that a payment of $1,500 will be made 55/100th of the way through Year 5.

Now find the FV of $1,500 for 5 years at 8% as follows: N = 5, I/YR = 8, PV = 0, PMT = -1500, and solve for FV = $8,799.90. Compound this value for 1 year at 8% to obtain the value in the account after 6 years and before the last payment is made; it is $8,799.90(1.08) = $9,503.89. Thus, you will have to make a payment of $10,000 – $9,503.89 = $496.11 at Year 6.

Alternative solution: $10,000 is needed 6 years from today. The plan is to deposit $1,500 annually in an 8% interest account, with the first payment to be made one year from today. The last deposit will be for less than $1,500 if less is needed to realize $10,000 in 6 years.

Calculate how large last payment will be:

N = 6; I/YR = 8; PV = 0; PMT = -1500; and solve for FV = $11,003.89.

If the last payment is $1,500, then the account will contain $11,003.89 – $10,000 = $1,003.89 too much. Thus, the last payment should be reduced by this excess amount:

Last payment = $1,500 – $1,003.89 = $496.11.

5-35 Begin with a time line:

0	7%	1	2	3
		5,000	5,500	6,050
				FV = ?

Use a financial calculator to calculate the present value of the cash flows and then determine the future value of this present value amount:

Step 1: CF_0 = 0, CF_1 = 5000, CF_2 = 5500, CF_3 = 6050, I/YR = 7. Solve for NPV = $14,415.41.

Step 2: Input the following data: N = 3, I/YR = 7, PV = -14415.41, PMT = 0, and solve for FV = $17,659.50.

5-36 **a.** With a financial calculator, enter N = 3, I/YR = 10, PV = -25000, and FV = 0, and solve for PMT = $10,052.87. Then go through the amortization procedure as described in your calculator manual to get the entries for the amortization table.

Year	Beginning Balance	Payment	Interest	Repayment of Principal	Remaining Balance
1	$25,000.00	$10,052.87	$2,500.00	$7,552.87	$17,447.13
2	17,447.13	10,052.87	1,744.71	8,308.16	9,138.97
3	9,138.97	10,052.87	913.90	9,138.97	0
		$30,158.61	$5,158.61	$25,000.00	

b.

	% Interest	% Principal
Year 1:	$2,500/$10,052.87 = 24.87%	$7,552.87/$10,052.87 = 75.13%
Year 2:	$1,744.71/$10,052.87 = 17.36%	$8,308.16/$10,052.87 = 82.64%
Year 3:	$913.90/$10,052.87 = 9.09%	$9,138.97/$10,052.87 = 90.91%

These percentages change over time because, even though the total payment is constant, the amount of interest paid each year is declining as the balance declines.

5-37 **a.** Using a financial calculator, enter N = 3, I/YR = 7, PV = -90000, and FV = 0, then solve for PMT = $34,294.65.

3-year amortization schedule:

Period	Beginning Balance	Payment	Interest	Principal Repayment	Ending Balance
1	$90,000.00	$34,294.65	$6,300.00	$27,994.65	$62,005.35
2	62,005.35	34,294.65	4,340.37	29,954.28	32,051.07
3	32,051.07	34,294.65	2,243.58	32,051.07	0

No. Each payment would be $34,294.65, which is significantly larger than the $7,500 payments that could be paid (affordable).

b. Using a financial calculator, enter N = 30, I/YR = 7, PV = -90000, and FV = 0, then solve for PMT = $7,252.78.

Yes. Each payment would now be $7,252.78, which is less than the $7,500 payment given in the problem that could be made (affordable).

c. 30-year amortization with balloon payment at end of Year 3:

Period	Beginning Balance	Payment	Interest	Principal Repayment	Ending Balance
1	$90,000.00	$7,252.78	$6,300.00	$ 952.78	$89,047.22
2	89,047.22	7,252.78	6,233.31	1,019.47	88,027.75
3	88,027.75	7,252.78	6,161.94	1,090.84	86,936.91

The loan balance at the end of Year 3 is $86,936.91 and the balloon payment is $86,936.91 + $7,252.78 = $94,189.69.

5-38 **a.** Begin with a time line:

6-mos.	0	1	2	3	4	5	6
Years	0		1		2		3
	2%						
		1,000	1,000	1,000	1,000	1,000	FVA = ?

Since the first payment is made 6 months from today, we have a 5-period ordinary annuity. The applicable interest rate is 4%/2 = 2%. First, we find the FVA of the ordinary annuity in period 5 by entering the following data in the financial calculator: N = 5, I/YR = 4/2 = 2, PV = 0, and PMT = -1000. We find FVA_5 = $5,204.04. Now, we must compound this amount for 1 semiannual period at 2%.

$5,204.04(1.02) = $5,308.12.

b. Here's the time line:

Qtrs	0	1	2	3	4
	1%				
		PMT = ?	PMT = ?		FV = 10,000

Required value of annuity = $9,802.96 ← FV = 10,000

Step 1: Discount the $10,000 back 2 quarters to find the required value of the 2-period annuity at the end of Quarter 2, so that its FV at the end of the 4^{th} quarter is $10,000.

Using a financial calculator enter N = 2, I/YR = 1, PMT = 0, FV = 10000, and solve for PV = $9,802.96.

Step 2: Now you can determine the required payment of the 2-period annuity with a FV of $9,802.96.

Using a financial calculator, enter N = 2, I/YR = 1, PV = 0, FV = 9802.96, and solve for PMT = $4,877.09.

5-39 **a.** Using the information given in the problem, you can solve for the length of time required to pay off the card.

I/YR = 1.5 (18%/12); PV = 350; PMT = -10; FV = 0; and then solve for N = 50 months.

b. If Tang Zhe makes monthly payments of $30, we can solve for the length of time required before the account is paid in full.

I/YR = 1.5; PV = 350; PMT = -30; FV = 0; and then solve for N = 12.921 ≈ 13 months.

With $30 monthly payments, Tang Zhe will only need 13 months to pay off the account.

c. Total payments @ $10.month: $50 \times \$10 = \$500.00$
Total payments @ $30/month: $12.921 \times \$30 = \underline{\ \ 387.62}$
Extra interest $= \underline{\underline{\$112.38}}$

5-40

	0	1	2	3
12/31/10	12/31/11	12/31/12	12/31/13	12/31/14
7%				
34,000.00	35,020.00	36,070.60	37,152.72	38,267.30
	100,000.00	↑		
	<u>20,000.00</u>			
	155,020.00	Payment will be made		

Step 1: Calculate salary amounts (2010-2014):

2010: $34,000
2011: $34,000(1.03) = $35,020.00
2012: $35,020(1.03) = $36,070.60
2013: $36,070.60(1.03) = $37,152.72
2014: $37,152.72(1.03) = $38,267.30

Step 2: Compound back pay, pain and suffering, and legal costs to 12/31/12 payment date:

$\$34{,}000(1.07)^2 + \$155{,}020(1.07)^1$
$\$38{,}926.60 + \$165{,}871.40 = \$204{,}798.00.$

Step 3: Discount future salary back to 12/31/12 payment date:

$\$36{,}070.60 + \$37{,}152.72/(1.07)^1 + \$38{,}267.30/(1.07)^2$
$\$36{,}070.60 + \$34{,}722.17 + \$33{,}424.14 = \$104{,}216.91.$

Step 4: City must write check for $204,798.00 + $104,216.91 = $309,014.91 ≈ $309,015.

5-41

1. Will save for 10 years, then receive payments for 25 years. How much must he deposit at the end of each of the next 10 years?

2. Wants payments of $40,000 per year in today's dollars for first payment only. Real income will decline. Inflation will be 5%. Therefore, to find the inflated fixed payments, we have this time line:

0	5	10
5%		
40,000		FV = ?

Enter N = 10, I/YR = 5, PV = -40000, PMT = 0, and press FV to get FV = $65,155.79.

3. He now has $100,000 in an account that pays 8%, annual compounding. We need to find the FV of the $100,000 after 10 years. Enter N = 10, I/YR = 8, PV = -100000, PMT = 0, and solve for FV = $215,892.50.

4. He wants to withdraw, or have payments of, $65,155.79 per year for 25 years, with the first payment made at the beginning of the first retirement year. So, we have a 25-year annuity due with PMT = 65,155.79, at an interest rate of 8%. Set the calculator to "BEG" mode, then enter N = 25, I/YR = 8, PMT = 65155.79, FV = 0, and solve for PV = $751,165.35. This amount must be on hand to make the 25 payments.

5. Since the original $100,000, which grows to $215,892.50, will be available, we must save enough to accumulate $751,165.35 - $215,892.50 = $535,272.85.

   So, the time line looks like this:

```
                                    Retires
 50      51       52             59       60       61             83       84       85
   8%
 |-------|--------|----- • • • ---|--------|--------|---- • • • ----|--------|--------|
$100,000 PMT     PMT             PMT      PMT
                                  -65,155.79 -65,155.79        -65,155.79 -65,155.79
                                 + 215,892.50
                                 - 751,165.35 = PVA(due)
          Need to accumulate -$535,272.85 = FVA10
```

6. The $535,272.85 is the FV of a 10-year ordinary annuity. The payments will be deposited in the bank and earn 8% interest. Therefore, set the calculator to "END" mode and enter N = 10, I/YR = 8, PV = 0, FV = 535272.85, and solve for PMT = $36,949.61 ≈ $36,950.

5-42 Step 1: Determine the annual cost of college. The current cost is $15,000 per year, but that is escalating at a 5% inflation rate:

College Year	Current Cost	Years from Now	Inflation Adjustment	Cash Required
1	$15,000	5	$(1.05)^5$	$19,144.22
2	15,000	6	$(1.05)^6$	20,101.43
3	15,000	7	$(1.05)^7$	21,106.51
4	15,000	8	$(1.05)^8$	22,161.83

Now put these costs on a time line:

```
13      14      15      16      17      18       19       20       21
|-------|-------|-------|-------|-------|--------|--------|--------|
                                     -19,144  –20,101  –21,107  –22,162
```

How much must be accumulated by age 18 to provide these payments at ages 18 through 21 if the funds are invested in an account paying 6%, compounded annually?

With a financial calculator enter: CF_0 = 19144, CF_1 = 20101, CF_2 = 21107, CF_3 = 22162, and I/YR = 6. Solve for NPV = $75,500.00.

Thus, the father must accumulate $75,500 by the time his daughter reaches age 18.

Step 2: The daughter has $7,500 now (age 13) to help achieve that goal. Five years hence, that $7,500, when invested at 6%, will be worth $10,037: $\$7,500(1.06)^5 = \$10,036.69 \approx$ $10,037.

Step 3: The father needs to accumulate only $75,500 – $10,037 = $65,463. The key to completing the problem at this point is to realize the series of deposits represent an ordinary annuity rather than an annuity due, despite the fact the first payment is made at the beginning of the first year. The reason it is not an annuity due is there is no interest paid on the last payment that occurs when the daughter is 18.

Using a financial calculator, N = 6, I/YR = 6, PV = 0, and FV = -65463. PMT = $9,384.95 ≈ $9,385.

Comprehensive/Spreadsheet Problem

Note to Instructors:
The solution to this problem is not provided to students at the back of their text. Instructors can access the *Excel* file on the textbook's website or the Instructor's Resource CD.

5-43 **a.**

Inputs:	PV =	$1,000
	I =	10%
	N =	5
Formula:	FV = PV(1+I)^N =	$1,610.51
Wizard (FV):		$1,610.51

b.

Years (D11)	Interest Rate (D10)		
$ 1,610.51	0%	5%	20%
0	$1,000.00	$1,000.00	$1,000.00
1	$1,000.00	$1,050.00	$1,200.00
2	$1,000.00	$1,102.50	$1,440.00
3	$1,000.00	$1,157.63	$1,728.00
4	$1,000.00	$1,215.51	$2,073.60
5	$1,000.00	$1,276.28	$2,488.32

c.

Inputs:	FV =	$1,000
	I =	10%
	N =	5
Formula:	PV = FV/(1+I)^N =	$ 620.92
Wizard (PV):		$ 620.92

d.

Inputs:	PV =	-$1,000
	FV =	$2,000
	I =	?
	N =	5
Wizard (Rate):		14.87%

e.

Inputs:	PV =	-5.3	
	FV =	10.6	
	I = growth rate	2%	
	N =	?	
Wizard (NPER):		35.00	= Years to double.

f. Inputs:

PMT	$	(1,000)
N		5
I		15%

PV: Use function wizard (PV) — **PV = $3,352.16**

FV: Use function wizard (FV) — **FV = $6,742.38**

g. For the PV, each payment would be received one period sooner, hence would be discounted back one less year. This would make the PV larger. We can find the PV of the annuity due by finding the PV of an ordinary annuity and then multiplying it by (1 + I).

PV annuity due = $3,352.16 × 1.15 = **$3,854.98**

Exactly the same adjustment is made to find the FV of the annuity due.

FV annuity due = $6,742.38 × 1.15 = **$7,753.74**

h. Part a. FV with semiannual compounding:

		Orig. Inputs:	New Inputs:
Inputs:	PV =	$1,000	$1,000
	I =	10%	5%
	N =	5	10
Formula:	FV = PV(1+I)^N =	$ 1,610.51	$ 1,628.89
Wizard (FV):		$ 1,610.51	$ 1,628.89

Part c. PV with semiannual compounding:

		Orig. Inputs:	New Inputs:
Inputs:	FV =	$1,000	$1,000
	I =	10%	5%
	N =	5	10
Formula:	FV = FV/(1+I)^N =	$ 620.92	$ 613.91
Wizard (PV):		$ 620.92	$ 613.91

i. Inputs:

N	10
I	8%
PV	-$1,000

PMT: Use function wizard (PMT) — **PMT = $149.03**

PMT (Due): Use function wizard (PMT) — **PMT = $137.99**

j.

Year	Payment
1	100
2	200
3	400

Rate = 8%

To find the PV, use the NPV function: **PV = $581.59**

Year	Payment	x	(1 + I)^(N – t)	=	FV
1	100		1.1664		116.64
2	200		1.0800		216.00
3	400		1.0000		400.00

Sum of FV's =	732.64

An alternative procedure for finding the FV would be to find the PV of the series using the NPV function, then compound that amount for 3 years at 8%, as is done below:

PV =	$581.59
FV of PV =	$732.64

k. (1)

	A	B	C	D	E
(i) EAR	6.00%	6.09%	6.14%	6.17%	6.18%
(ii) Deposit $5,000. What is FV_1?	$5,300	$5,305	$5,307	$5,308	$5,309
(iii) Deposit $5,000. What is FV_2?	$5,618	$5,628	$5,632	$5,636	$5,637

(2) Would they be equally able to attract funds? No. People would prefer more compounding to less.
(i) What nominal rate would cause all banks to provide same EAR as Bank A?

	A	B	C	D	E
I_{NOM}	6.00%	5.91%	5.87%	5.84%	5.83%

Each of these nominal rates based on the frequency of compounding will provide an EAR of 6%.

(3) You need $5,000 at the end of the year. How much do you need to deposit annually for A, semiannually, for B, etc. beginning today, to have $5,000 at the end of the year?

	A	B	C	D	E
PMT	$4,716.98	$2,391.31	$1,204.16	$403.32	$13.29

(4) Even if the banks provided the same EAR, would a rational investor be indifferent between the banks? Probably not. An investor would probably prefer the bank that compounded more frequently.

l.

Original amount of mortgage:	$15,000
Term to maturity:	4
Interest rate:	8%

Annual payment (use PMT function): ($4,528.81)

Year	Beginning Balance	Payment	Interest	Principal	Ending Balance
1	$ 15,000.00	$ 4,528.81	$1,200.00	$3,328.81	$ 11,671.19
2	$ 11,671.19	$ 4,528.81	$ 933.70	$3,595.12	$ 8,076.07
3	$ 8,076.07	$ 4,528.81	$ 646.09	$3,882.73	$ 4,193.34
4	$ 4,193.34	$ 4,528.81	$ 335.47	$4,193.34	$ 0.00

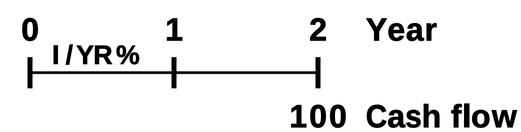
Integrated Case

5-44
Kwantung Commercial Bank
Time Value of Money Analysis

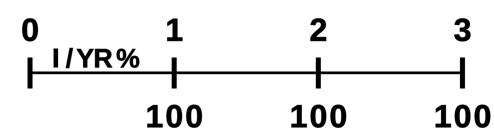
You have applied for a job with a local bank. As part of its evaluation process, you must take an examination on time value of money analysis covering the following questions.

A.	**Draw time lines for (1) a $100 lump sum cash flow at the end of Year 2; (2) an ordinary annuity of $100 per year for 3 years; and (3) an uneven cash flow stream of -$50, $100, $75, and $50 at the end of Years 0 through 3.**

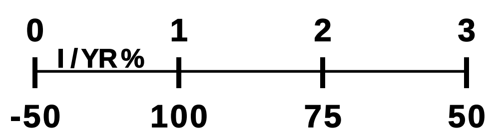
ANSWER: [Show S5-1 through S5-4 here.] A <u>time line</u> is a graphical representation that is used to show the timing of cash flows. The tick marks represent end of periods (often years), so time 0 is today; Time 1 is the end of the first year, or 1 year from today; and so on.



Lump sum



Annuity



Uneven cash flow stream

A <u>lump sum</u> is a single flow; for example, a $100 inflow in Year 2, as shown in the top time line.

An <u>annuity</u> is a series of equal cash flows occurring over equal intervals, as illustrated in the middle time line.

An __uneven cash flow stream__ is an irregular series of cash flows that do not constitute an annuity, as in the lower time line. -50 represents a cash __outflow__ rather than a receipt or inflow.

B.	**(1)**	**What's the future value of \$100 after 3 years if it earns 10%, annual compounding?**

ANSWER: [Show S5-5 through S5-7 here.] Show dollars corresponding to question mark, calculated as follows:

0	1	2	3
10%			
100			FV = ?

After 1 year:

$$FV_1 = PV + I_1 = PV + PV(I) = PV(1 + I) = \$100(1.10) = \$110.00.$$

Similarly:

$$\begin{aligned} FV_2 &= FV_1 + I_2 = FV_1 + FV_1(I) \\ &= FV_1(1 + I) = \$110(1.10) = \$121.00 \\ &= PV(1 + I)(1 + I) = PV(1 + I)^2. \end{aligned}$$

$$\begin{aligned} FV_3 &= FV_2 + I_3 = FV_2 + FV_2(I) \\ &= FV_2(1 + I) = \$121(1.10) = \$133.10 \\ &= PV(1 + I)^2(1 + I) = PV(1 + I)^3. \end{aligned}$$

In general, we see that:

$$FV_N = PV(1 + I)^N,$$

So $FV_3 = \$100(1.10)^3 = \$100(1.3310) = \$133.10.$

Note that this equation has 4 variables: FV_N, PV, I/YR, and N. Here, we know all except FV_N, so we solve for FV_N. We will, however, often solve for one of the other three variables. __By far, the easiest way to__

work all time value problems is with a financial calculator. Just plug in any three of the four values and find the fourth.

Finding future values (moving to the right along the time line) is called compounding. Note that there are 3 ways of finding FV_3:

- **Regular calculator:**
  1. **\$100(1.10)(1.10)(1.10) = \$133.10.**
  2. **\$100$(1.10)^3$ = \$133.10.**

- **Financial calculator:**
  This is especially efficient for more complex problems, including exam problems. Input the following values: N = 3, I/YR = 10, PV = -100, PMT = 0, and solve for FV = \$133.10.

- **Spreadsheet:**
  Spreadsheet programs are ideally suited for solving time value of money problems. The spreadsheet can be set up using the specific FV spreadsheet function or by entering a FV formula/equation.

B.	**(2)**	**What's the present value of \$100 to be received in 3 years if the interest rate is 10%, annual compounding?**

Answer: **[Show S5-8 through S5-10 here.] Finding present values, or discounting (moving to the left along the time line), is the reverse of compounding, and the basic present value equation is the reciprocal of the compounding equation:**

```
      0          1          2          3
      |   10%    |          |          |
      |----------|----------|----------|
   PV = ?                             100
```

$FV_N = PV(1 + I)^N$ transforms to:

$$PV = \frac{FV_N}{(1+I)^N} = FV_N\left(\frac{1}{1+I}\right)^N = FV_N\,(1+I)^{-N}$$

Thus:

$$PV = \$100\left(\frac{1}{1.10}\right)^3 = \$100(0.7513) = \$75.13.$$

The same methods (regular calculator, financial calculator, and spreadsheet program) used for finding future values are also used to find present values, which is called discounting.

Using a financial calculator input: N = 3, I/YR = 10, PMT = 0, FV = 100, and then solve for PV = $75.13.

C.	**What annual interest rate would cause $100 to grow to $125.97 in 3 years?**

ANSWER: [Show S5-11 here.]

0	1	2	3
-100			125.97
	$\$100(1 + I)$	$\$100(1 + I)^2$	$\$100(1 + I)^3$

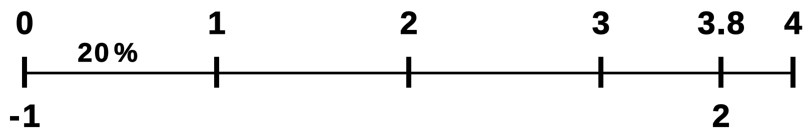
$$FV = \$100(1 + I)^3 = \$125.97.$$

Using a financial calculator; enter N = 3, PV = -100, PMT = 0, FV = 125.97, then press the I/YR button to find I/YR = 8%.

Calculators can find interest rates quite easily, even when periods and/or interest rates are not whole numbers, and when uneven cash flow streams are involved. (With uneven cash flows, we must use the "CF_j" calculator key, and the interest rate is called the IRR, or "internal rate of return;" we will use this feature in capital budgeting.)

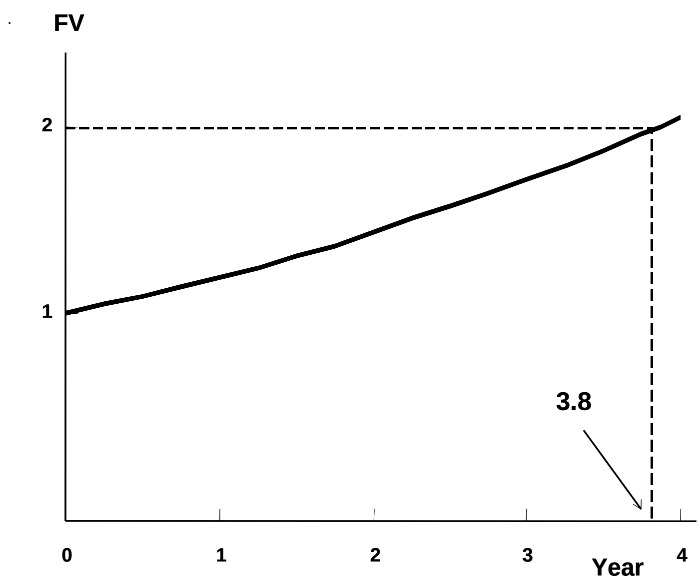
D.	**If a company's sales are growing at a rate of 20% annually, how long will it take sales to double?**

ANSWER: [Show S5-12 here.] We have this situation in time line format:



Say we want to find out how long it will take us to double our money at an interest rate of 20%. We can use any numbers, say $1 and $2, with this equation:

$$FV_N = \$2 = \$1(1 + I)^N$$
$$\$2 = \$1(1.20)^N.$$



We would plug I/YR = 20, PV = -1, PMT = 0, and FV = 2 into our calculator, and then press the N button to find the number of years it would take $1 (or any other beginning amount) to double when growth occurs at a 20% rate. The answer is 3.8 years, but some calculators will round this value up to the next highest whole number. The graph also shows what is happening.

Optional Question

A farmer can spend $60/acre to plant pine trees on some marginal land. The expected real rate of return is 4%, and the expected inflation rate is 6%. What is the expected value of the timber after 20 years?

ANSWER: $FV_{20} = \$60(1 + 0.04 + 0.06)^{20} = \$60(1.10)^{20} = \$403.65$ **per acre.**

We could have asked: How long would it take $60 to grow to $403.65, given the real rate of return of 4% and an inflation rate of 6%? Of course, the answer would be 20 years.

E. **What's the difference between an ordinary annuity and an annuity due? What type of annuity is shown here? How would you change it to the other type of annuity?**

0	1	2	3
0	100	100	100

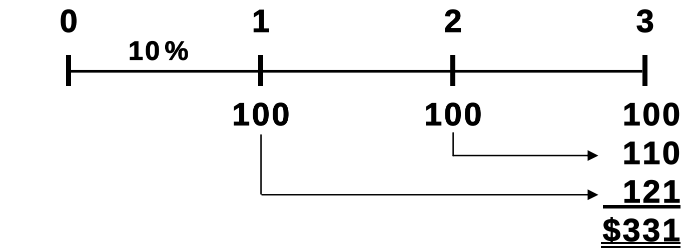
ANSWER: **[Show S5-13 here.] This is an ordinary annuity—it has its payments at the end of each period; that is, the first payment is made 1 period from today. Conversely, an annuity due has its first payment today. In other words, an <u>ordinary annuity has end-of-period payments</u>, while an <u>annuity due has beginning-of-period payments</u>.**

The annuity shown above is an ordinary annuity. To convert it to an annuity due, shift each payment to the left, so you end up with a payment under the 0 but none under the 3 as illustrated below.

0	1	2	3
100	100	100	

F.	**(1)**	**What is the future value of a 3-year, $100 ordinary annuity if the annual interest rate is 10%?**

ANSWER: [Show S5-14 here.]



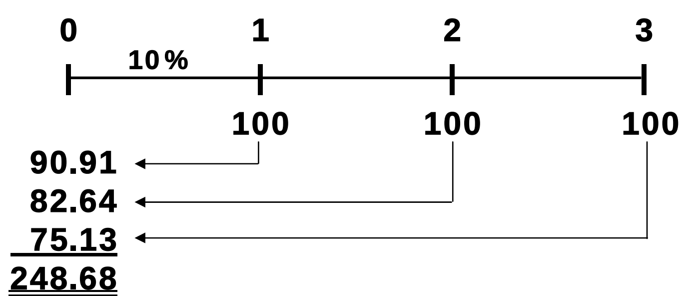
Go through the following discussion. One approach would be to treat each annuity flow as a lump sum. Here we have

$$\mathbf{FVA_N = \$100(1) + \$100(1.10) + \$100(1.10)^2}$$
$$\mathbf{= \$100[1 + (1.10) + (1.10)^2] = \$100(3.3100) = \$331.00.}$$

Future values of annuities may be calculated in 3 ways: (1) Treat the payments as lump sums. (2) Use a financial calculator. (3) Use a spreadsheet.

F.	**(2)**	**What is its present value?**

ANSWER: [Show S5-15 here.]



The present value of the annuity is $248.68. Here we used the lump sum approach, but the same result could be obtained by using a calculator. Input N = 3, I/YR = 10, PMT = 100, FV = 0, and press the PV button.

F. (3) What would the future and present values be if it was an annuity due?

ANSWER: [Show S5-16 and S5-17 here.] If the annuity were an annuity due, each payment would be shifted to the left, so each payment is compounded over an additional period or discounted back over one less period.

In our situation, the future value of the annuity due is $364.10:

$$FVA_{3\ Due} = \$331.00(1.10)^1 = \$364.10.$$

This same result could be obtained by using the time line:

$$\$133.10 + \$121.00 + \$110.00 = \$364.10.$$

The best way to work annuity due problems is to switch your calculator to "BEG" or beginning or "DUE" mode, and go through the normal process. Note that it's critical to remember to change back to "END" mode after working an annuity due problem with your calculator.

In our situation, the present value of the annuity due is $273.55:

$$PVA_{3\ Due} = \$248.68(1.10)^1 = \$273.55.$$

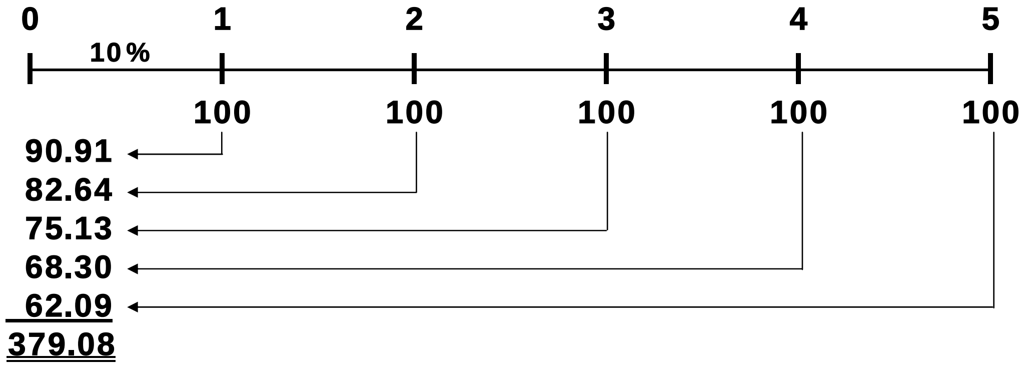
This same result could be obtained by using the time line:

$$\$100 + \$90.91 + \$82.64 = \$273.55.$$

G. A 5-year $100 ordinary annuity has an annual interest rate of 10%.

(1) What is its present value?

ANSWER: [Show S5-18 here.]



The present value of the annuity is $379.08. Here we used the lump sum approach, but the same result could be obtained by using a calculator. Input N = 5, I/YR = 10, PMT = 100, FV = 0, and press the PV button.

G.	(2)	What would the present value be if it was a 10-year annuity?

ANSWER: [Show S5-19 here.] The present value of the 10-year annuity is $614.46. To solve with a financial calculator, input N = 10, I/YR = 10, PMT = 100, FV = 0, and press the PV button.

G.	(3)	What would the present value be if it was a 25-year annuity?

ANSWER: The present value of the 25-year annuity is $907.70. To solve with a financial calculator, input N = 25, I/YR = 10, PMT = 100, FV = 0, and press the PV button.

G.	(4)	What would the present value be if this was a perpetuity?

ANSWER: The present value of the $100 perpetuity is $1,000. The PV is solved by dividing the annual payment by the interest rate: $100/0.10 = $1,000.

H. A 20-year-old student wants to save $3 a day for her retirement. Every day she places $3 in a drawer. At the end of each year, she invests the accumulated savings ($1,095) in a brokerage account with an expected annual return of 12%.

(1) If she keeps saving in this manner, how much will she have accumulated at age 65?

ANSWER: [Show S5-20 and S5-21 here.] If she begins saving today, and sticks to her plan, she will have saved $1,487,261.89 by the time she reaches 65. With a financial calculator, enter the following inputs: N = 45, I/YR = 12, PV = 0, PMT = -1095, then press the FV button to solve for $1,487,261.89.

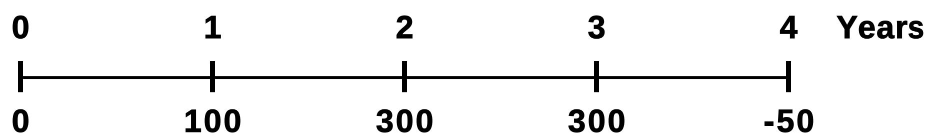
H. (2) If a 40-year-old investor began saving in this manner, how much would he have at age 65?

ANSWER: [Show S5-22 here.] This question demonstrates the power of compound interest and the importance of getting started on a regular savings program at an early age. The 40-year old investor will have saved only $146,000.59 by the time he reaches 65. With a financial calculator, enter the following inputs: N = 25, I/YR = 12, PV = 0, PMT = -1095, then press the FV button to solve for $146,000.59.

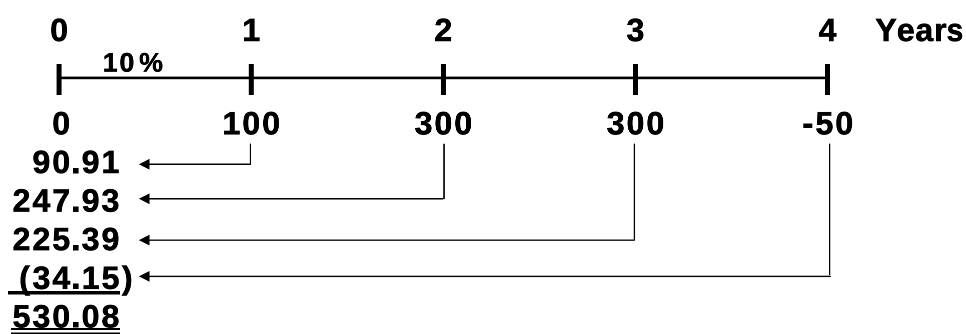
H. (3) How much would the 40-year-old investor have to save each year to accumulate the same amount at 65 as the 20-year-old investor?

ANSWER: [Show S5-23 here.] Again, this question demonstrates the power of compound interest and the importance of getting started on a regular savings program at an early age. The 40-year old investor will have to save $11,154.42 every year, or $30.56 per day, in order to have as much saved as the 20-year old investor by the time he reaches 65. With a financial calculator, enter the following inputs: N = 25, I/YR = 12, PV = 0, FV = 1487261.89, then press the PMT button to solve for $11,154.42.

I. **What is the present value of the following uneven cash flow stream? The annual interest rate is 10%.**



ANSWER: [Show S5-24 and S5-25 here.] Here we have an uneven cash flow stream. The most straightforward approach is to find the PVs of each cash flow and then sum them as shown below:



Note that the $50 Year 4 outflow remains an outflow even when discounted. There are numerous ways of finding the present value of an uneven cash flow stream. But by far the easiest way to deal with uneven cash flow streams is with a financial calculator. Calculators have a function that on the HP-17B is called “CFLO,” for “cash flow.” Other calculators could use other designations such as CF_0 and CF_j, but they explain how to use them in the manual. Anyway, you would input the cash flows, so they are in the calculator’s memory, then input the interest rate, I/YR, and then press the NPV or PV button to find the present value of the cash flow stream.

J.	**(1)**	**Will the future value be larger or smaller if we compound an initial amount more often than annually (e.g., semiannually, holding the stated (nominal) rate constant)? Why?**

ANSWER: **[Show S5-26 here.] Accounts that pay interest more frequently than once a year, for example, semiannually, quarterly, or daily, have future values that are higher because interest is earned on interest more often. Virtually all banks now pay interest daily on passbook and money fund accounts, so they use daily compounding.**

J.	**(2)**	**Define (a) the stated (or quoted or nominal) rate, (b) the periodic rate, and (c) the effective annual rate (EAR or EFF%).**

ANSWER: **[Show S5-27 and S5-28 here.] The <u>quoted</u>, or <u>nominal</u>, rate is merely the quoted percentage rate of return, the periodic rate is the rate charged by a lender or paid by a borrower each period (periodic rate = I_{NOM}/M), and the <u>effective annual rate (EAR)</u> is the rate of interest that would provide an identical future dollar value <u>under annual compounding</u>.**

J.	**(3)**	**What is the EAR corresponding to a nominal rate of 10% compounded semiannually? Compounded quarterly? Compounded daily?**

ANSWER: **[Show S5-29 through S5-31 here.] The effective annual rate for 10% semiannual compounding, is 10.25%:**

$$\text{EAR} = \text{Effective annual rate} = \left(\frac{1 + I_{NOM}}{M}\right)^{M} - 1.0.$$

If I_{NOM} = 10% and interest is compounded semiannually, then:

$$EAR = \left(1 + \frac{0.10}{2}\right)^2 - 1.0 = (1.05)^2 - 1.0$$

$$= 1.1025 - 1.0 = 0.1025 = 10.25\%.$$

For quarterly compounding, the effective annual rate is 10.38%:

$$(1.025)^4 - 1.0 = 1.1038 - 1.0 = 0.1038 = 10.38\%.$$

Daily compounding would produce an effective annual rate of 10.52%.

J.	(4)	What is the future value of $100 after 3 years under 10% semiannual compounding? Quarterly compounding?

ANSWER: [Show S5-32 here.] Under semiannual compounding, the $100 is compounded over 6 semiannual periods at a 5.0% periodic rate:

I_{NOM} = 10%.

$$FV_N = \left(1 + \frac{I_{NOM}}{M}\right)^{MN} = \$100\left(1 + \frac{0.10}{2}\right)^{2(3)}$$

$$= \$100(1.05)^6 = \$134.01.$$

Quarterly: $FV_N = \$100(1.025)^{12} = \$134.49$.

The return when using quarterly compounding is clearly higher.

Another approach here would be to use the effective annual rate and compound over annual periods:

Semiannually: $\$100(1.1025)^3 = \$134.01$.

Quarterly: $\$100(1.1038)^3 = \$134.49$.

K.	When will the EAR equal the nominal (quoted) rate?

ANSWER: **[Show S5-33 here.] If annual compounding is used, then the nominal rate will be equal to the effective annual rate. If more frequent compounding is used, the effective annual rate will be <u>above</u> the nominal rate.**

L. (1) What is the value at the end of Year 3 of the following cash flow stream if interest is 10% compounded semiannually? (Hint: You can use the EAR and treat the cash flows as an ordinary annuity or use the periodic rate and compound the cash flows individually.)

0		2		4		6	Periods
0		100		100		100	

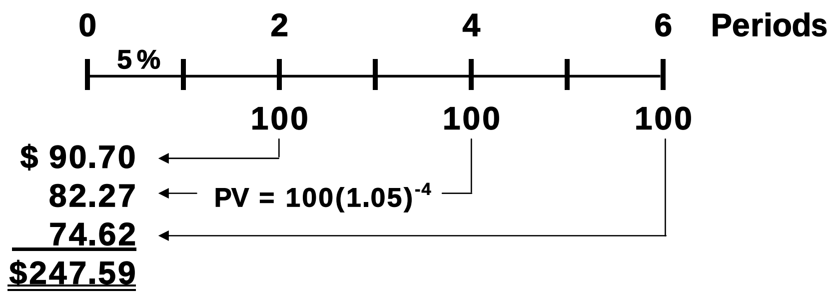
ANSWER: **[Show S5-34 through S5-36 here.]**

0		2		4		6	Periods
5%							
		100		100		100.00	
						110.25	$= \$100(1.05)^2$
						<u>121.55</u>	$= \$100(1.05)^4$
						<u>\$331.80</u>	

Here we have a different situation. The payments occur annually, but compounding occurs each 6 months. Thus, we <u>cannot</u> use normal annuity valuation techniques.

L.	(2)	What is the PV?

ANSWER: [Show S5-37 here.]



To use a financial calculator, input N = 3, I/YR = 10.25, PMT = 100, FV = 0, and then press the PV key to find PV = $247.59.

L.	(3)	What would be wrong with your answer to Parts L(1) and L(2) if you used the nominal rate, 10%, rather than the EAR or the periodic rate, $I_{NOM}/2 = 10\%/2 = 5\%$ to solve the problems?

ANSWER: I_{NOM} can be used in the calculations only when annual compounding occurs. If the nominal rate of 10% were used to discount the payment stream, the present value would be overstated by $272.32 – $247.59 = $24.73.

M.	(1)	Construct an amortization schedule for a $1,000, 10% annual interest loan with three equal installments.
	(2)	What is the annual interest expense for the borrower and the annual interest income for the lender during Year 2?

ANSWER: [Show S5-38 through S5-44 here.] To begin, note that the face amount of the loan, $1,000, is the present value of a 3-year annuity at a 10% rate:

0	1	2	3
10%			
-1,000	PMT	PMT	PMT

$$PVA_3 = PMT\left(\frac{1}{1+I}\right)^1 + PMT\left(\frac{1}{1+I}\right)^2 + PMT\left(\frac{1}{1+I}\right)^3$$

$$\$1,000 = PMT(1 + I)^{-1} + PMT(1 + I)^{-2} + PMT(1 + I)^{-3}.$$

We have an equation with only one unknown, so we can solve it to find PMT. The easy way is with a financial calculator. Input N = 3, I/YR = 10, PV = -1000, FV = 0, and then press the PMT button to get PMT = 402.1148036, rounded to $402.11.

Amortization Schedule:

Period	Beginning Balance	Payment	Interest	Payment of Principal	Ending Balance
1	$1,000.00	$402.11	$100.00	$302.11	$697.89
2	697.89	402.11	69.79	332.32	365.57
3	365.57	402.13*	36.56	365.57	0.00

*Due to rounding, the third payment was increased by $0.02 to cause the ending balance after the third year to equal $0.

Now make the following points regarding the amortization schedule:

- **The $402.11 annual payment includes both interest and principal. Interest in the first year is calculated as follows:**

  1st year interest = I × Beginning balance = 0.1 × $1,000 = $100.

- **The repayment of principal is the difference between the $402.11 annual payment and the interest payment:**

  1st year principal repayment = $402.11 – $100 = $302.11.

- **The loan balance at the end of the first year is:**

  1st year ending balance = Beginning balance – Principal repayment
  = \$1,000 – \$302.11 = \$697.89.

- **We would continue these steps in the following years.**

- **Notice that the interest each year declines because the beginning loan balance is declining. Since the payment is constant, but the interest component is declining, the principal repayment portion is increasing each year.**

- **The interest component is an expense that is deductible to a business or a homeowner, and it is taxable income to the lender. If you buy a house, you will get a schedule constructed like ours, but longer, with $30 \times 12 = 360$ monthly payments if you get a 30-year, fixed-rate mortgage.**

- **The payment may have to be increased by a few cents in the final year to take care of rounding errors and make the final payment produce a zero ending balance.**

- **The lender received a 10% rate of interest on the average amount of money that was invested each year, and the \$1,000 loan was paid off. This is what amortization schedules are designed to do.**

- **Most financial calculators have built-in amortization functions.**