Chapter 7
Interest Rates

Learning Objectives

After reading this chapter, students should be able to:

- List the various factors that influence the cost of money.
- Discuss how market interest rates are affected by borrowers' need for capital, expected inflation, different securities' risks, and securities' liquidity.
- Explain what the yield curve is, what determines its shape, and how you can use the yield curve to help forecast future interest rates.

Lecture Suggestions

Chapter 7 is important because it lays the groundwork for the following chapters. Additionally, students have a curiosity about interest rates, so this chapter stimulates their interest in the course.

What we cover, and the way we cover it, can be seen by scanning the slides and Integrated Case solution for Chapter 7, which appears at the end of this chapter's solutions. For other suggestions about the lecture, please see the "Lecture Suggestions" in Chapter 2, where we describe how we conduct our classes.

DAYS ON CHAPTER: 1 OF 56 DAYS (50-minute periods)

Answers to End-of-Chapter Questions

7-1 Regional mortgage rate differentials do exist, depending on supply/demand conditions in the different regions. However, relatively high rates in one region would attract capital from other regions, and the end result would be a differential that was just sufficient to cover the costs of effecting the transfer (perhaps ½ of one percentage point). Differentials are more likely in the residential mortgage market than the business loan market, and not at all likely for the large, nationwide firms, which will do their borrowing in the lowest-cost money centers thereby quickly equalizing rates for large corporate loans. Interest rates are more competitive, making it easier for small borrowers, and borrowers in rural areas, to obtain lower cost loans.

7-2 Short-term interest rates are more volatile because (1) the Fed operates mainly in the short-term sector, hence Federal Reserve intervention has its major effect here, and (2) long-term interest rates reflect the average expected inflation rate over the next 20 to 30 years, and this average does not change as radically as year-to-year expectations.

7-3 Interest rates will fall as the recession takes hold because (1) business borrowings will decrease and (2) the Fed will increase the money supply to stimulate the economy. Thus, it would be better to borrow short-term now, and then to convert to long-term when rates have reached a cyclical low. Note, though, that this answer requires interest rate forecasting, which is extremely difficult to do with better than 50% accuracy.

7-4 **a.** If transfers between the two markets are costly, interest rates would be different in the two areas. Area Y, with the relatively young population, would have less in savings accumulation and stronger loan demand. Area O, with the relatively old population, would have more savings accumulation and weaker loan demand as the members of the older population have already purchased their houses and are less consumption oriented. Thus, supply/demand equilibrium would be at a higher rate of interest in Area Y.

b. Yes. Nationwide branching, and so forth, would reduce the cost of financial transfers between the areas. Thus, funds would flow from Area O with excess relative supply to Area Y with excess relative demand. This flow would increase the interest rate in Area O and decrease the interest rate in Y until the rates were roughly equal, the difference being the transfer cost.

7-5 A significant increase in productivity would raise the rate of return on producers' investment, thus causing the demand curve (see Figure 7-1 in the textbook) to shift to the right. This would increase the amount of savings and investment in the economy, thus causing all interest rates to rise.

7-6 **a.** The immediate effect on the yield curve would be to lower interest rates in the short-term end of the market, since the Fed deals primarily in that market segment. However, people would expect higher future inflation, which would raise long-term rates. The result would be a much steeper yield curve.

b. If the policy is maintained, the expanded money supply will result in increased rates of inflation and increased inflationary expectations. This will cause investors to increase the inflation premium on all debt securities, and the entire yield curve would rise; that is, all rates would be higher.

7-7 **a.** S&Ls would have a higher level of net income with a "normal" yield curve. In this situation their liabilities (deposits), which are short-term, would have a lower cost than the returns being generated by their assets (mortgages), which are long-term. Thus, they would have a positive "spread."

b. It depends on the situation. A sharp increase in inflation would increase interest rates along the entire yield curve. If the increase were large, short-term interest rates might be boosted above the long-term interest rates that prevailed prior to the inflation increase. Then, since the bulk of the fixed-rate mortgages were initiated when interest rates were lower, the deposits (liabilities) of the S&Ls would cost more than the returns being provided on the assets. If this situation continued for any length of time, the equity (reserves) of the S&Ls would be drained to the point that only a "bailout" would prevent bankruptcy. This has indeed happened in the United States. Thus, in this situation the S&L industry would be better off selling their mortgages to federal agencies and collecting servicing fees rather than holding the mortgages they originated.

7-8 Treasury bonds, along with all other bonds, are available to investors as an alternative investment to common stocks. An increase in the return on Treasury bonds would increase the appeal of these bonds relative to common stocks, and some investors would sell their stocks to buy T-bonds. This would cause stock prices, in general, to fall. Another way to view this is that a relatively riskless investment (T-bonds) has increased its return by 4 percentage points. The return demanded on riskier investments (stocks) would also increase, thus driving down stock prices. The exact relationship will be discussed in Chapter 8 (with respect to risk) and Chapters 9 and 10 (with respect to price).

7-9 A trade deficit occurs when the U.S. buys more than it sells. In other words, a trade deficit occurs when the U.S. imports more than it exports. When trade deficits occur, they must be financed, and the main source of financing is debt. Therefore, the larger the U.S. trade deficit, the more the U.S. must borrow, and as the U.S. increases its borrowing, this drives up interest rates.

7-10 The yield on corporates is equal to:

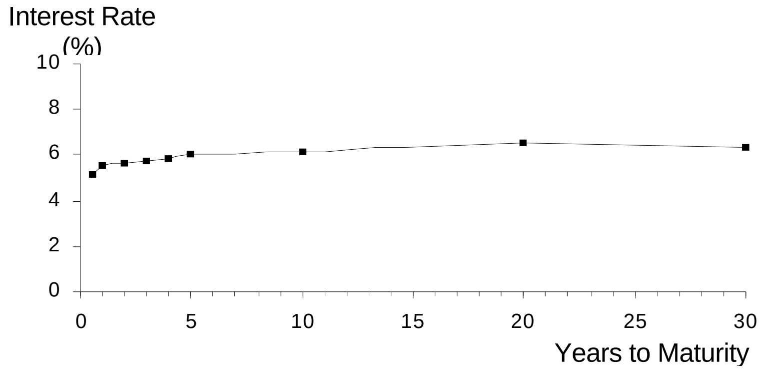
$$r_t = r^* + IP_t + MRP_t + DRP + LP.$$

Thus, a steeper corporate yield curve would indicate an increase in risk (as reflected by DRP and LP) and an increase in expected inflation (as reflected by IP_t). MRP_t is a premium that reflects interest rate risk and is higher on longer-term securities. Consequently, an increase in risk and expected inflation would impact MRP_t as well.

Solutions to End-of-Chapter Problems

7-1 **a.**

Term	Rate
6 months	5.1%
1 year	5.5
2 years	5.6
3 years	5.7
4 years	5.8
5 years	6.0
10 years	6.1
20 years	6.5
30 years	6.3



b. The yield curve shown is an upward sloping yield curve.

c. This yield curve tells us generally that either inflation is expected to increase or there is an increasing maturity risk premium.

d. Even though the borrower reinvests in increasing short-term rates, those rates are still below the long-term rate, but what makes the higher long-term rate attractive is the rollover risk that may possibly occur if the short-term rates go even higher than the long-term rate (and that *could* be for a long time!). This exposes you to rollover risk. If you borrow for 30 years outright you have locked in a 6.3% interest rate each year.

7-2 T-bill rate = r* + IP
5.5% = r* + 3.25%
r* = 2.25%.

7-3 $r^* = 3\%$; $I_1 = 2\%$; $I_2 = 4\%$; $I_3 = 4\%$; MRP = 0; $r_{T2} = ?$; $r_{T3} = ?$

r = r* + IP + DRP + LP + MRP.

Since these are Treasury securities, DRP = LP = 0.

$r_{T2} = r^* + IP_2$.
$IP_2 = (2\% + 4\%)/2 = 3\%$.
$r_{T2} = 3\% + 3\% = 6\%$.

$r_{T3} = r^* + IP_3$.
$IP_3 = (2\% + 4\% + 4\%)/3 = 3.33\%$.
$r_{T3} = 3\% + 3.33\% = 6.33\%$.

7-4 r_{T10} = 6%; r_{C10} = 8%; LP = 0.5%; DRP = ?

$r = r^* + IP + DRP + LP + MRP.$
$r_{T10} = 6\% = r^* + IP_{10} + MRP_{10}$; DRP = LP = 0.

$r_{C10} = 8\% = r^* + IP_{10} + DRP + 0.5\% + MRP_{10}.$

Because both bonds are 10-year bonds the inflation premium and maturity risk premium on both bonds are equal. The only difference between them is the liquidity and default risk premiums.

$r_{C10} = 8\% = r^* + IP_{10} + MRP_{10} + 0.5\% + DRP$. But we know from above that $r^* + IP_{10} + MRP_{10}$ = 6%; therefore,

$r_{C10} = 8\% = 6\% + 0.5\% + DRP$
$1.5\% = DRP.$

7-5 r^* = 3%; IP_2 = 3%; r_{T2} = 6.2%; MRP_2 = ?

$r_{T2} = r^* + IP_2 + MRP_2 = 6.2\%$
$r_{T2} = 3\% + 3\% + MRP_2 = 6.2\%$
$MRP_2 = 0.2\%.$

7-6 r^* = 5%; IP_4 = 16%; MRP = DRP = LP = 0; r_{RF4} = ?

$r_{RF4} = (1 + r^*)(1 + IP_4) – 1$
$= (1.05)(1.16) – 1$
$= 0.218 = 21.8\%.$

7-7 r_{T1} = 5%; ${}_1r_{T1}$ = 6%; r_{T2} = ?

$(1 + r_{T2})^2 = (1.05)(1.06)$
$(1 + r_{T2})^2 = 1.113$
$1 + r_{T2} = 1.055$
$r_{T2} = 5.5\%.$

7-8 Let X equal the yield on 2-year securities 4 years from now:

$(1.07)^4(1 + X)^2 = (1.075)^6$
$(1.3108)(1 + X)^2 = 1.5433$

$$1 + X = \left(\frac{1.5433}{1.3108}\right)^{1/2}$$

$X = 8.5\%.$

7-9 $r_7 = r^* + IP_7 + MRP_7 + DRP + LP.$
$r^* = 0.03.$
$IP_7 = [0.03 + 0.04 + (5)(0.035)]/7 = 0.035.$
$MRP_7 = 0.0005(t – 1) = 0.0005(6) = 0.003.$
DRP = 0.
LP = 0.

$r_{T7} = 0.03 + 0.035 + 0.003 = 0.068 = 6.8\%.$

7-10 Basic relevant equations:

$r_t = r^* + IP_t + DRP_t + MRP_t + IP_t$.

But here IP_t is the only premium, so $r_t = r^* + IP_t$.

IP_t = Avg. inflation = $(I_1 + I_2 + . . .)/N$.

We know that $I_1 = IP_1 = 3\%$ and $r^* = 2\%$. Therefore,

$r_{T1} = 2\% + 3\% = 5\%$. $r_{T3} = r_{T1} + 2\% = 5\% + 2\% = 7\%$. But,

$r_{T3} = r^* + IP_3 = 2\% + IP_3 = 7\%$, so

$IP_3 = 7\% – 2\% = 5\%$.

We also know that I_t = Constant after t = 1.

We can set up this table:

	r*	I	IP_t	$r = r^* + IP_t$
1	2%	3%	3%/1 = 3%	5%
2	2%	I	$(3\% + I)/2 = IP_2$	
3	2%	I	$(3\% + I + I)/3 = IP_3$	$r_{T3} = 7\%$, so $IP_3 = 7\% – 2\% = 5\%$.

$$IP_3 = (3\% + 2I)/3 = 5\%$$
$$2I = 12\%$$
$$I = 6\%.$$

7-11 We're given all the components to determine the yield on the bonds except the default risk premium (DRP) and MRP. Calculate the MRP as $0.1\%(5 – 1) = 0.4\%$. Now, we can solve for the DRP as follows:

$$7.75\% = 2.3\% + 2.5\% + 0.4\% + 1.0\% + DRP, \text{ or } DRP = 1.55\%.$$

7-12 First, calculate the inflation premiums for the next three and five years, respectively. They are $IP_3 = (2.5\% + 3.2\% + 3.6\%)/3 = 3.1\%$ and $IP_5 = (2.5\% + 3.2\% + 3.6\% + 3.6\% + 3.6\%)/5 = 3.3\%$. The real risk-free rate is given as 2.75%. Since the default and liquidity premiums are zero on Treasury bonds, we can now solve for the maturity risk premium. Thus, $6.25\% = 2.75\% + 3.1\% + MRP_3$, or $MRP_3 = 0.4\%$. Similarly, $6.8\% = 2.75\% + 3.3\% + MRP_5$, or $MRP_5 = 0.75\%$. Thus, $MRP_5 – MRP_3 = 0.75\% – 0.40\% = 0.35\%$.

7-13
$$r_{C8} = r^* + IP_8 + MRP_8 + DRP_8 + LP_8$$
$$8.3\% = 2.5\% + (2.8\% \times 4 + 3.75\% \times 4)/8 + 0.0\% + DRP_8 + 0.75\%$$
$$8.3\% = 2.5\% + 3.275\% + 0.0\% + DRP_8 + 0.75\%$$
$$8.3\% = 6.525\% + DRP_8$$
$$DRP_8 = 1.775\%.$$

7-14 **a.** $(1.045)^2 = (1.03)(1 + X)$
$1.092/1.03 = 1 + X$
$X = 6\%.$

b. For riskless bonds under the expectations theory, the interest rate for a bond of any maturity is $r_N = r^* +$ average inflation over N years. If $r^* = 1\%$, we can solve for IP_N:

Year 1: $r_1 = 1\% + I_1 = 3\%;$
$I_1 =$ expected inflation $= 3\% – 1\% = 2\%.$

Year 2: $r_1 = 1\% + I_2 = 6\%;$
$I_2 =$ expected inflation $= 6\% – 1\% = 5\%.$

Note also that the average inflation rate is (2% + 5%)/2 = 3.5%, which, when added to r* = 1%, produces the yield on a 2-year bond, 4.5%. Therefore, all of our results are consistent.

7-15 $r^* = 2\%; MRP = 0\%; r_{T1} = 5\%; r_{T2} = 7\%; X = ?$

X represents the one-year rate on a bond one year from now (Year 2).

$$(1.07)^2 = (1.05)(1 + X)$$
$$\frac{1.1449}{1.05} = 1 + X$$
$$X = 9\%.$$

$9\% = r^* + I_2$
$9\% = 2\% + I_2$
$7\% = I_2.$

The average interest rate during the 2-year period differs from the 1-year interest rate expected for Year 2 because of the inflation rate reflected in the two interest rates. The inflation rate reflected in the interest rate on any security is the average rate of inflation expected over the security's life.

7-16 $r_{RF6} = 20.84\%; MRP = DRP = LP = 0; r^* = 6\%; IP_6 = ?$

$r_{RF6} = (1 + r^*)(1 + IP_6) – 1$
$20.84\% = (1.06)(1 + IP_6) – 1$
$1.2084 = (1.06)(1 + IP_6)$
$1.14 = 1 + IP_6$
$0.14 = IP_6.$

7-17 $r_{T5} = 5.2\%; r_{T10} = 6.4\%; r_{C10} = 8.4\%; IP_{10} = 2.5\%; MRP = 0.$ For Treasury securities, DRP = LP = 0.

$DRP_5 + LP_5 = DRP_{10} + LP_{10}.$ $r_{C5} = ?$

$r_{T10} = r^* + IP_{10}$
$6.4\% = r^* + 2.5\%$
$r^* = 3.9\%.$

$r_{T5} = r^* + IP_5$
$5.2\% = 3.9\% + IP_5$
$1.3\% = IP_5.$

$r_{C10} = r^* + IP_{10} + DRP_{10} + LP_{10}$
$8.4\% = 3.9\% + 2.5\% + DRP_{10} + LP_{10}$
$2\% = DRP_{10} + LP_{10}.$

$r_{C5} = 3.9\% + 1.3\% + DRP_5 + LP_5$, but $DRP_5 + LP_5 = DRP_{10} + LP_{10} = 2\%$. So,

$r_{C5} = 3.9\% + 1.3\% + 2\%$
$= 7.2\%.$

7-18 a.

Years to Maturity	Real Risk-Free Rate (r*)	IP_t**	MRP	$r_T = r^* + IP_t + MRP_t$
1	2%	7.00%	0.2%	9.20%
2	2	6.00	0.4	8.40
3	2	5.00	0.6	7.60
4	2	4.50	0.8	7.30
5	2	4.20	1.0	7.20
10	2	3.60	1.0	6.60
20	2	3.30	1.0	6.30

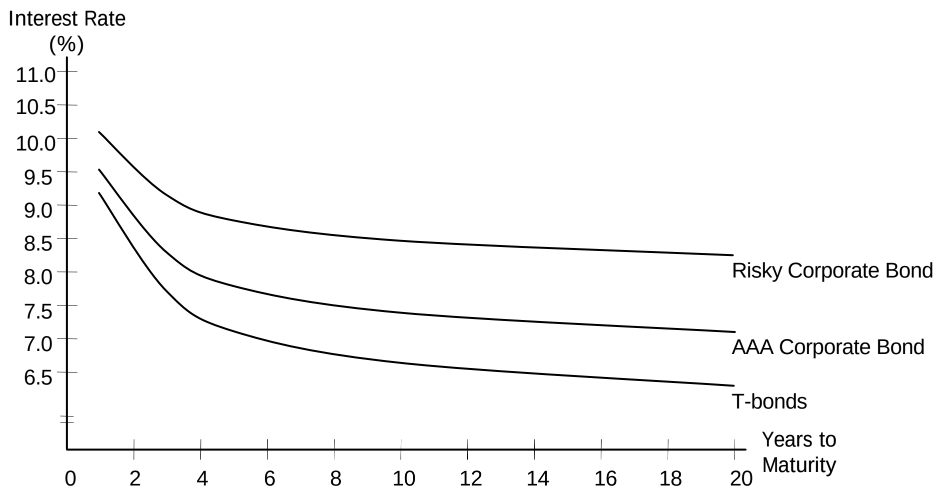
**The computation of the inflation premium is as follows:

Year	Expected Inflation	IP_t
1	7%	7.00%
2	5	6.00
3	3	5.00
4	3	4.50
5	3	4.20
10	3	3.60
20	3	3.30

For example, the calculation for IP_3 is as follows:

$$IP_3 = \frac{7\% + 5\% + 3\%}{3} = 5.00\%.$$

Thus, the yield curve would be as follows:



b. The interest rate on the AAA-rated corporate bonds has the same components as the Treasury securities, except that the AAA-rated corporate bonds have default risk, so a default risk premium must be included. Therefore,

$$r = r^* + IP_t + MRP_t + DRP_t.$$

For a strong company, the default risk premium is virtually zero for short-term bonds. However, as time to maturity increases, the probability of default, although still small, is sufficient to warrant a default premium. Thus, the yield risk curve for the AAA-rated corporate bonds will rise above the yield curve for the Treasury securities. In the graph, the default risk premium was assumed to be 1.0 percentage point on the 20-year AAA-rated corporate bonds. The return should equal 6.3% + 1% = 7.3%.

c. The lower-rated corporate bonds would have significantly more default risk than either Treasury securities or AAA-rated corporate bonds, and the risk of default would increase over time due to possible financial deterioration. In this example, the default risk premium was assumed to be 1.0 percentage point on the 1-year lower-rated corporate bonds and 2.0 percentage points on the 20-year lower-rated corporate bonds. The 20-year return should equal 6.3% + 2% = 8.3%.

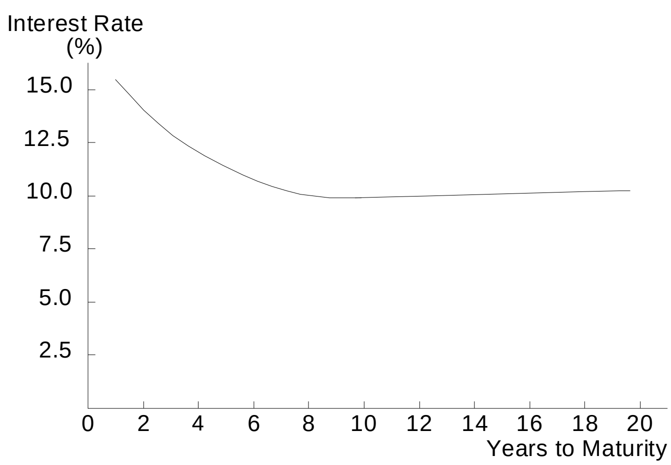
7-19 **a.** The average rate of inflation for the 5-year period is calculated as:

$$IP_5 = (0.13 + 0.09 + 0.07 + 0.06 + 0.06)/5 = 8.20\%.$$

b. $r_{T5} = r^* + IP_5 = 2\% + 8.2\% = 10.20\%.$

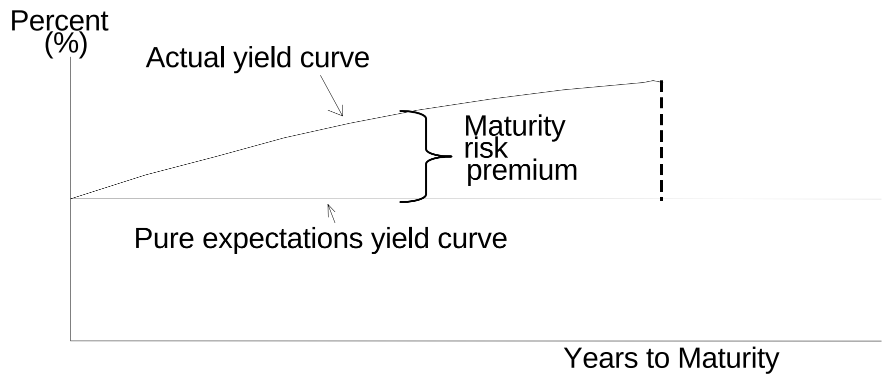
c. Here is the general situation:

Year	Expected Annual Inflation (I_t)	IP_t	r^*	MRP_t	r_t
1	13%	13.0%	2%	0.1%	15.1%
2	9	11.0	2	0.2	13.2
5	6	8.2	2	0.5	10.7
.	.	.	.	.	.
.	.	.	.	.	.
.	.	.	.	.	.
10	6	7.1	2	1.0	10.1
20	6	6.6	2	2.0	10.6



d. The "normal" yield curve is upward sloping because, in "normal" times, inflation is not expected to trend either up or down, so IP is the same for debt of all maturities, but the MRP increases with years, so the yield curve slopes up. During a recession, the yield curve typically slopes up especially steeply, because inflation and consequently short-term interest rates are currently low, yet people expect inflation and interest rates to rise as the economy comes out of the recession.

e. If inflation rates are expected to be constant, then the expectations theory holds that the yield curve should be horizontal. However, in this event it is likely that maturity risk premiums would be applied to long-term bonds because of the greater risks of holding long-term rather than short-term bonds:



If maturity risk premiums were added to the yield curve in Part e above, then the yield curve would be more nearly normal; that is, the long-term end of the curve would be raised. (The yield curve shown in this answer is upward sloping; the yield curve shown in Part c is downward sloping.)

Comprehensive/Spreadsheet Problem

Note to Instructors:
The solution to this problem is <u>not</u> provided to students at the back of their text. Instructors can access the *Excel* file on the textbook's website or the Instructor's Resource CD.

7-20 **a.** 1. This action will increase the supply of money; therefore, interest rates will decline.

2. This action will cause interest rates to increase.

3. The larger the federal deficit, other things held constant, the higher the level of interest rates.

4. This expectation will cause interest rates to increase.

b. <u>12-year Treasury Bond</u>

Real risk-free rate (r*):					4.000%
Maturity:	12				
Expected inflation:	for the next	2	years	=	2%
Expected inflation:	for the next	4	years	=	3%
Expected inflation:	for the next	6	years	=	4%
		12			
Inflation premium:	=((G27*D27)+(G28*D28)+(G29*D29))/D30				3.333%
Maturity risk premium:	= 0.02*(C26-1)%			=	0.220%
12-year Treasury yield	= r* + IP_{12} + MRP_{12} =				7.553%

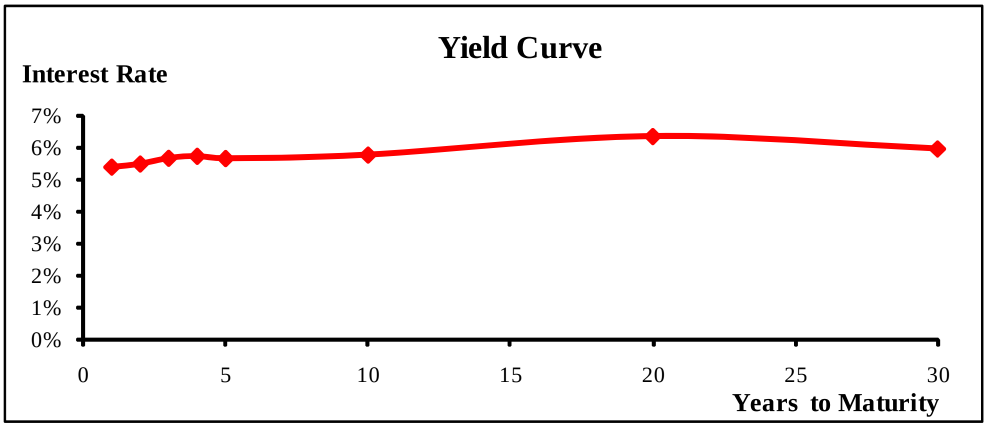
<u>7-year Corporate Bond</u>

Rating :	A				
Real risk-free rate (r*):					4.000%
Maturity:	7				
Expected inflation:	for the next	2	years	=	2%
Expected inflation:	for the next	4	years	=	3%
Expected inflation:	for the next	1	years	=	4%
		7			
Inflation premium:	=((G41*D41)+(G42*D42)+(G43*D43))/D44				2.857%
Maturity risk premium:	= 0.02*(C40-1)%			=	0.120%
Liquidity premium:	Given in problem				0.300%
Default risk premium:	=IF(B36=H46,I46-G48,IF(B36=H47,I47-G48,I48-G48))				0.680%

7-year Corporate yield = r* + IP_7 + MRP_7 + LP + DRP = **7.957%**

Yield Spread = Corporate – Treasury =		0.404%
Reconciliation:	Default premium	0.680%
	Liquidity premium	0.300%
	Inflation premium	-0.476%
	Maturity premium	-0.100%
		0.404%

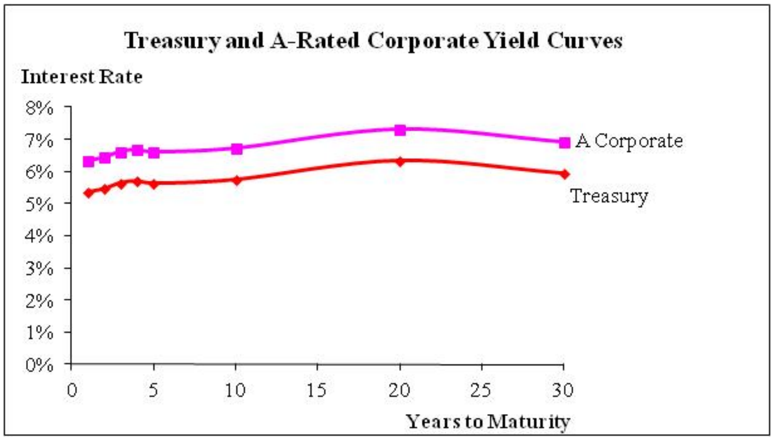
c.



d. The real risk-free rate would be the same for the corporate and treasury bonds. Similarly, without information to the contrary, we would assume that the maturity and inflation premiums would be the same for bonds with the same maturities. However, the corporate bond would have a liquidity premium and a default premium. If we assume that these premiums are constant across maturities, then we can use the LP and DRP as determined above and add them to the T-bond yields to find the corporate yields. This procedure was used in the table below.

Years	Treasury	A-Corporate	Spread	LP	DRP
1	5.37%	6.35%	0.98%	0.30%	0.68%
2	5.47%	6.45%	0.98%	0.30%	0.68%
3	5.65%	6.63%	0.98%	0.30%	0.68%
4	5.71%	6.69%	0.98%	0.30%	0.68%
5	5.64%	6.62%	0.98%	0.30%	0.68%
10	5.75%	6.73%	0.98%	0.30%	0.68%
20	6.33%	7.31%	0.98%	0.30%	0.68%
30	5.94%	6.92%	0.98%	0.30%	0.68%

Now we can graph the data in the first 3 columns of the above table to get the Treasury and corporate (A-rated) yield curves:



Note that if we constructed yield curves for corporate bonds with other ratings, the higher the rating, the lower the curves would be. Note too that the DRP for different ratings can change over time as investors' (1) risk aversion and (2) perceptions of risk change, and this can lead to different yield spreads and curve positions. Expectations for inflation can also change, and this will lead to upward or downward shifts in all the yield curves.

e. Short-term rates are more volatile than longer-term rates; therefore, the left side of the yield curve would be most volatile over time.

f. (1) **The 1-year rate, 1 year from now**

$$(1 + r_2)^2 = (1 + r_1) \times (1 + {}_1r_1)$$
$$1.1124 = 1.0537 \times (1 + {}_1r_1)$$
$$\boxed{5.57\% = {}_1r_1}$$

(2) **The 5-year rate, 5 years from now**

$$(1 + r_{10})^{10} = (1 + r_5)^5 \times (1 + {}_5r_5)^5$$
$$1.7491 = 1.3157 \times (1 + {}_5r_5)^5$$
$$1.3294 = (1 + {}_5r_5)^5$$
$$\boxed{5.86\% = {}_5r_5}$$

(3) **The 10-year rate, 10 years from now**

$$(1 + r_{20})^{20} = (1 + r_{10})^{10} \times (1 + {}_{10}r_{10})^{10}$$
$$3.4128 = 1.7491 \times (1 + {}_{10}r_{10})^{10}$$
$$1.9512 = (1 + {}_{10}r_{10})^{10}$$
$$\boxed{6.91\% = {}_{10}r_{10}}$$

(4) **The 10-year rate, 20 years from now**

$$(1 + r_{30})^{30} = (1 + r_{20})^{20} \times (1 + {}_{20}r_{10})^{10}$$
$$5.6468 = 3.4128 \times (1 + {}_{20}r_{10})^{10}$$
$$1.6546 = (1 + {}_{20}r_{10})^{10}$$
$$\boxed{5.16\% = {}_{20}r_{10}}$$

Integrated Case

7-21
Morton Handley & Company
Interest Rate Determination

Maria Juarez is a professional tennis player, and your firm manages her money. She has asked you to give her information about what determines the level of various interest rates. Your boss has prepared some questions for you to consider.

A.	**What are the four most fundamental factors that affect the cost of money, or the general level of interest rates, in the economy?**

Answer: [Show S7-1 and S7-2 here.] The four most fundamental factors affecting the cost of money are (1) production opportunities, (2) time preferences for consumption, (3) risk, and (4) inflation.

Production opportunities are the investment opportunities in productive (cash-generating) assets. Time preferences for consumption are the preferences of consumers for current consumption as opposed to saving for future consumption. Risk, in a financial market context, is the chance that an investment will provide a low or negative return. Inflation is the amount by which prices increase over time.

The interest rate paid to savers depends (1) on the rate of return producers expect to earn on invested capital, (2) on savers' time preferences for current versus future consumption, (3) on the riskiness of the loan, and (4) on the expected future rate of inflation. Producers' expected returns on their business investments set an upper limit to how much they can pay for savings, while consumers' time preferences for consumption

establish how much consumption they are willing to defer, hence how much they will save at different interest rates. Higher risk and higher inflation also lead to higher interest rates.

B.	**What is the real risk-free rate of interest (r*) and the nominal risk-free rate (r_{RF})? How are these two rates measured?**

Answer: **[Show S7-3 and S7-4 here.] Keep these equations in mind as we discuss interest rates. We will define the terms as we go along:**

$$r = r^* + IP + DRP + LP + MRP.$$

$$r_{RF} = r^* + IP.$$

The real risk-free rate, r*, is the rate that would exist on default-free securities in the absence of inflation.

The nominal risk-free rate, r_{RF}, is equal to the real risk-free rate plus an inflation premium, which is equal to the average rate of inflation expected over the life of the security.

There is no truly riskless security, but the closest thing is a short-term U.S. Treasury bill (T-bill), which is free of most risks. The real risk-free rate, r*, is estimated by subtracting the expected rate of inflation from the rate on short-term treasury securities. It is generally assumed that r* is in the range of 1 to 4 percentage points. The T-bond rate is used as a proxy for the long-term risk-free rate. However, we know that all long-term bonds contain interest rate risk, so the T-bond rate is not really riskless. It is, however, free of default risk.

C.	**Define the terms inflation premium (IP), default risk premium (DRP), liquidity premium (LP), and maturity risk premium (MRP). Which of these premiums is included in determining the interest rate on (1) short-term U.S. Treasury securities, (2) long-term U.S. Treasury securities, (3) short-term corporate securities, and (4) long-term corporate securities? Explain how the premiums would vary over time and among the different securities listed.**

Answer: **[Show S7-5 here.] The <u>inflation premium</u> (IP) is a premium added to the real risk-free rate of interest to compensate for expected inflation.**

The <u>default risk premium</u> (DRP) is a premium based on the probability that the issuer will default on the loan, and it is measured by the difference between the interest rate on a U.S. Treasury bond and a corporate bond of equal maturity, liquidity, and other features.

A <u>liquid asset</u> is one that can be sold at a predictable price on short notice; a <u>liquidity premium</u> is added to the rate of interest on securities that are not liquid.

The <u>maturity risk premium</u> (MRP) is a premium that reflects <u>interest rate risk</u>; longer-term securities have more interest rate risk (the risk of capital loss due to rising interest rates) than do shorter-term securities, and the MRP is added to reflect this risk.

1. **Short-term treasury securities include only an inflation premium.**

2. **Long-term treasury securities contain an inflation premium plus a maturity risk premium. Note that the inflation premium added to long-term securities will differ from that**

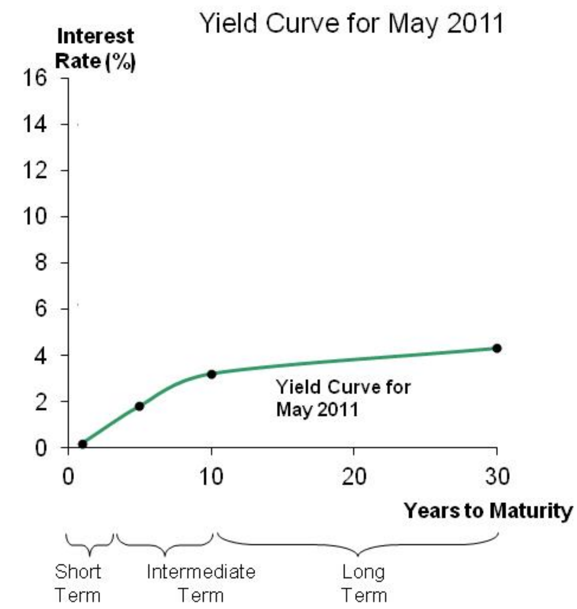
for short-term securities unless the rate of inflation is expected to remain constant.

3. **The rate on short-term corporate securities is equal to the real risk-free rate plus premiums for inflation, default risk, and liquidity. The size of the default and liquidity premiums will vary depending on the financial strength of the issuing corporation and its degree of liquidity, with larger corporations generally having greater liquidity because of more active trading.**

4. **The rate for long-term corporate securities also includes a premium for maturity risk. Thus, long-term corporate securities generally carry the highest yields of these four types of securities.**

D.	**What is the term structure of interest rates? What is a yield curve?**

Answer: **[Show S7-6 here. S7-6 shows a recent (May 2011) Treasury yield curve.] The term structure of interest rates is the relationship between interest rates, or yields, and maturities of securities. When this relationship is graphed, the resulting curve is called a yield curve. (Sketch out a yield curve on the board.)**



Years to Maturity	Yield
1	0.2%
5	1.8
10	3.2
30	4.3

E. **Suppose most investors expect the inflation rate to be 5% next year, 6% the following year, and 8% thereafter. The real risk-free rate is 3%. The maturity risk premium is zero for bonds that mature in 1 year or less and 0.1% for 2-year bonds; then the MRP increases by 0.1% per year thereafter for 20 years, after which it is stable. What is the interest rate on 1-, 10-, and 20-year Treasury bonds? Draw a yield curve with these data. What factors can explain why this constructed yield curve is upward-sloping?**

Answer: [Show S7-7 through S7-12 here.]

Step 1: Find the average expected inflation rate over Years 1 to 20:

Yr 1: IP = 5.0%.

Yr 10: IP = (5 + 6 + 8 + 8 + 8 + . . . + 8)/10 = 7.5%.

Yr 20: IP = (5 + 6 + 8 + 8 + . . . + 8)/20 = 7.75%.

Step 2: Find the maturity risk premium in each year:

Yr 1: MRP = 0.0%.

Yr 10: MRP = 0.1% $\times$ 9 = 0.9%.

Yr 20: MRP = 0.1% $\times$ 19 = 1.9%.

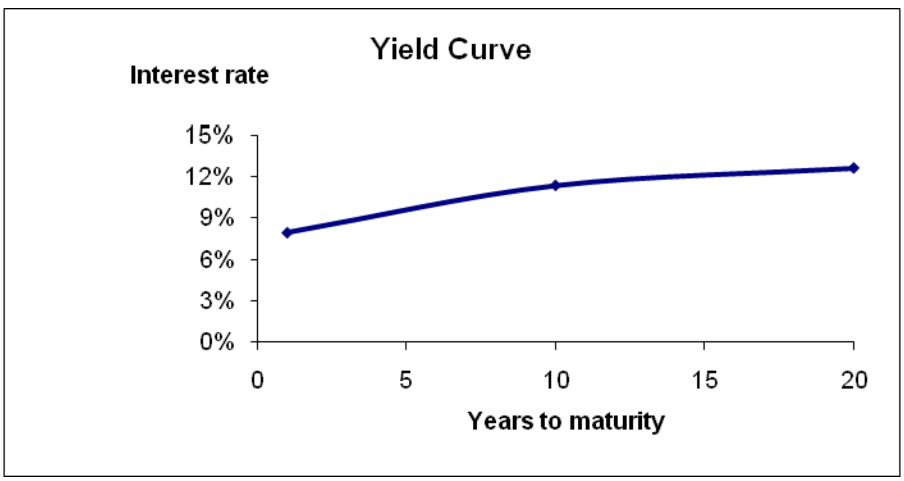
Step 3: Sum the IPs and MRPs, and add r* = 3%:

Yr 1: r_{RF} = 3% + 5.0% + 0.0% = 8.0%.

Yr 10: r_{RF} = 3% + 7.5% + 0.9% = 11.4%.

Yr 20: r_{RF} = 3% + 7.75% + 1.9% = 12.65%.

The shape of the yield curve depends primarily on two factors: (1) expectations about future inflation and (2) the relative riskiness of securities with different maturities.



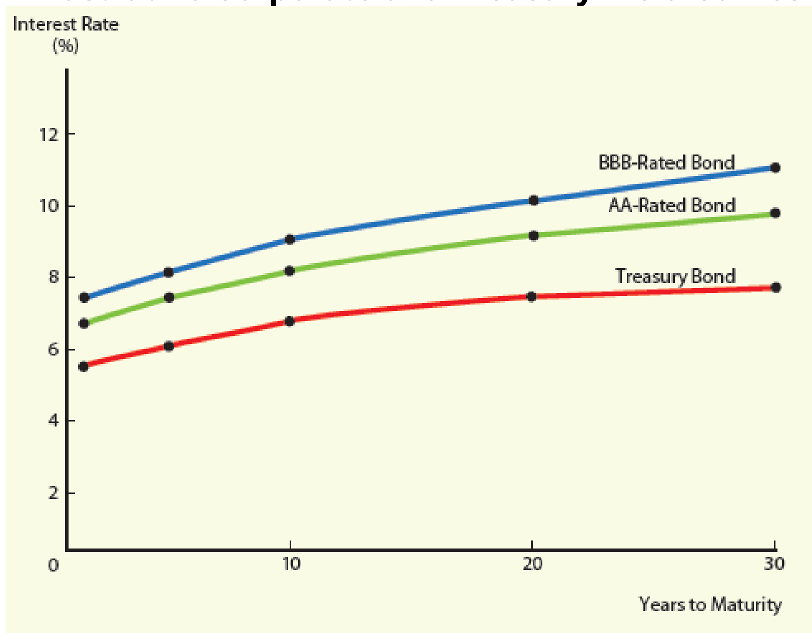
The constructed yield curve is upward sloping. This is due to increasing expected inflation and an increasing maturity risk premium.

F. At any given time, how would the yield curve facing a AAA-rated company compare with the yield curve for U.S. Treasury securities? At any given time, how would the yield curve facing a BB-rated company compare with the yield curve for U.S. Treasury

securities? Draw a graph to illustrate your answer.

Answer: **[Show S7-13 through S7-15 here.] (Curves for AAA-rated and BB-rated securities have been added to an illustrative yield curve to demonstrate that riskier securities require higher returns.) The yield curve normally slopes upward, indicating that short-term interest rates are lower than long-term interest rates. Yield curves can be drawn for government securities or for the securities of any corporation, but corporate yield curves will always lie above government yield curves, and the riskier the corporation, the higher its yield curve. The spread between a corporate yield curve and the Treasury curve widens as the corporate bond rating decreases, and is equal to the default risk premium plus the liquidity premium.**

Illustrative Corporate and Treasury Yield Curves



	Interest Rate		
	Treasury Bond	AA-Rated Bond	BBB-Rated Bond
1 year	5.5%	6.7%	7.4%
5 years	6.1	7.4	8.1
10 years	6.8	8.2	9.1
20 years	7.4	9.2	10.2
30 years	7.7	9.8	11.1

G. What is the pure expectations theory? What does the pure expectations theory imply about the term structure of interest rates?

Answer: [Show S7-16 and S7-17 here.] The pure expectations theory assumes that investors establish bond prices and interest rates strictly on the basis of expectations for interest rates. This means that they are indifferent with respect to maturity in the sense that they do not view long-term bonds as being riskier than short-term bonds. If this were true, then the maturity risk premium would be zero, and long-term interest rates would simply be a weighted average of current and expected future short-term interest rates.

If the pure expectations theory is correct, you can use the yield curve to "back out" expected future interest rates.

H. Suppose you observe the following term structure for Treasury securities:

Maturity	Yield
1 year	6.0%
2 years	6.2
3 years	6.4
4 years	6.5
5 years	6.5

Assume that the pure expectations theory of the term structure is correct. (This implies that you can use the yield curve provided to "back out" the market's expectations about future interest rates.) What does the market expect will be the interest rate on 1-year securities 1 year from now? What does the market expect will be the interest rate on 3-year securities 2 years from now?

Answer: [Show S7-18 through S7-22 here.] Calculation for r on 1-year securities one year from now:

$$(1.062)^2 = (1.06)(1 + X)$$
$$1.1278 = (1.06)(1 + X)$$
$$\frac{1.1278}{1.06} = 1 + X$$
$$6.4\% = X.$$

One year from now, 1-year securities will yield 6.4%.

Calculation for r on 3-year securities two years from now:

$$(1.065)^5 = (1.062)^2(1 + X)^3$$
$$\frac{(1.065)^5}{(1.062)^2} = (1 + X)^3$$
$$\frac{1.3701}{1.1278} = (1 + X)^3$$

$$(1.2148)^{1/3} - 1 = X$$
$$6.7\% = X.$$

Two years from now, 3-year securities will yield 6.7%.