Chapter 8
Risk and Rates of Return

Learning Objectives

After reading this chapter, students should be able to:

- Explain the difference between stand-alone risk and risk in a portfolio context.
- Describe how risk aversion affects a stock's required rate of return.
- Discuss the difference between diversifiable risk and market risk, and explain how each type of risk affects well-diversified investors.
- Describe what the CAPM is and illustrate how it can be used to estimate a stock's required rate of return.
- Discuss how changes in the general stock and bond markets could lead to changes in the required rate of return on a firm's stock.
- Discuss how changes in a firm's operations might lead to changes in the required rate of return on the firm's stock.

Lecture Suggestions

Risk analysis is an important topic, but it is difficult to teach at the introductory level. We just try to give students an intuitive overview of how risk can be defined and measured, and leave a technical treatment to advanced courses. Our primary goals are to be sure students understand (1) that investment risk is the uncertainty about returns on an asset, (2) the concept of portfolio risk, and (3) the effects of risk on required rates of return.

What we cover, and the way we cover it, can be seen by scanning the slides and Integrated Case solution for Chapter 8, which appears at the end of this chapter's solutions. For other suggestions about the lecture, please see the "Lecture Suggestions" in Chapter 2, where we describe how we conduct our classes.

DAYS ON CHAPTER: 3 OF 56 DAYS (50-minute periods)

Answers to End-of-Chapter Questions

8-1 **a.** The portfolio would be free of default risk and liquidity risk, but inflation could erode the portfolio's purchasing power. If the actual inflation rate is greater than that expected, interest rates in general will rise to incorporate a larger inflation premium (IP) and—as we saw in Chapter 7—the value of the portfolio would decline.

b. Ivan would be subject to reinvestment risk. Ivan might expect to "roll over" the Treasury bills at a constant (or even increasing) rate of interest, but if interest rates fall, his investment income will decrease.

c. A U.S. government-backed bond that provided interest with constant purchasing power (that is, an indexed bond) would be close to riskless. The U.S. Treasury currently issues indexed bonds.

8-2 **a.** The probability distribution for complete certainty is a vertical line.

b. The probability distribution for total uncertainty is the X-axis from $-\infty$ to $+\infty$.

8-3 **a.** The expected return on a life insurance policy is calculated just as for a common stock. Each outcome is multiplied by its probability of occurrence, and then these products are summed. Therefore, the 1-year term policy pays $10,000 at death, and the probability of the policyholder's death in that year is 2%. Then, there is a 98% probability of zero return and a 2% probability of $10,000:

$$\text{Expected return} = 0.98(\$0) + 0.02(\$10{,}000) = \$200.$$

This expected return could be compared to the premium paid. Generally, the premium will be larger because of sales and administrative costs, and insurance company profits, indicating a negative expected rate of return on the investment in the policy.

b. There is a perfect negative correlation between the returns on the life insurance policy and the returns on the policyholder's human capital. In fact, these events (death and future lifetime earnings capacity) are mutually exclusive.

c. People are generally risk averse. Therefore, they are willing to pay a premium to decrease the uncertainty of their future cash flows. A life insurance policy guarantees an income (the face value of the policy) to the policyholder's beneficiaries when the policyholder's future earnings capacity drops to zero.

8-4 Yes, if the portfolio's beta is equal to zero. In practice, however, it may be impossible to find individual stocks that have a nonpositive beta. In this case it would also be impossible to have a stock portfolio with a zero beta. Even if such a portfolio could be constructed, investors would probably be better off just purchasing Treasury bills, or other zero beta investments.

8-5 Security A is less risky if held in a diversified portfolio because of its negative correlation with other stocks. In a single-asset portfolio, Security A would be more risky because $\sigma_A > \sigma_B$ and $CV_A > CV_B$.

8-6 No. For a stock to have a negative beta, its returns would have to logically be expected to go up in the future when other stocks' returns were falling. Just because in one year the stock's return increases when the market declined doesn't mean the stock has a negative beta. A stock in a given year may move counter to the overall market, even though the stock's beta is positive.

8-7 The risk premium on a high-beta stock would increase more than that on a low-beta stock.

RP_j = Risk Premium for Stock j = $(r_M - r_{RF})b_j$.

If risk aversion increases, the slope of the SML will increase, and so will the market risk premium $(r_M - r_{RF})$. The product $(r_M - r_{RF})b_j$ is the risk premium of the j^{th} stock. If b_j is low (say, 0.5), then the product will be small; RP_j will increase by only half the increase in RP_M. However, if b_j is large (say, 2.0), then its risk premium will rise by twice the increase in RP_M.

8-8 According to the Security Market Line (SML) equation, an increase in beta will increase a company's expected return by an amount equal to the market risk premium times the change in beta. For example, assume that the risk-free rate is 6%, and the market risk premium is 5%. If the company's beta doubles from 0.8 to 1.6 its expected return increases from 10% to 14%. Therefore, in general, a company's expected return will not double when its beta doubles.

8-9 **a.** A decrease in risk aversion will decrease the return an investor will require on stocks. Thus, prices on stocks will increase because the cost of equity will decline.

b. With a decline in risk aversion, the risk premium will decline as compared to the historical difference between returns on stocks and bonds.

c. The implication of using the SML equation with historical risk premiums (which would be higher than the "current" risk premium) is that the CAPM estimated required return would actually be higher than what would be reflected if the more current risk premium were used.

8-10 According to the Security Market Line (SML) equation, an increase in beta will increase a company's expected return by an amount equal to the market risk premium times the change in beta. For example, assume that the risk-free rate is 4%, and the market risk premium is 6%. If the company's beta increases 50% from 0.8 to 1.2 its expected return increases from 8.8% to 11.2%. Therefore, the company's expected return will not increase by 50%.

Solutions to End-of-Chapter Problems

8-1 $\hat{r}$ = (0.1)(-50%) + (0.2)(-5%) + (0.4)(16%) + (0.2)(25%) + (0.1)(60%)
= 11.40%.

σ^2 = (-50% – 11.40%)^2(0.1) + (-5% – 11.40%)^2(0.2) + (16% – 11.40%)^2(0.4)
+ (25% – 11.40%)^2(0.2) + (60% – 11.40%)^2(0.1)

σ^2 = 712.44; σ = 26.69%.

$$CV = \frac{26.69\%}{11.40\%} = 2.34.$$

8-2

	Investment	Beta
	\$50,000	0.95
	30,000	1.5
Total	\$80,000	

b_p = (\$50,000/\$80,000)(0.95) + (\$30,000/\$80,000)(1.5) = 1.16.

8-3 r_{RF} = 5%; r_M = 8.5%; b = 0.9; r = ?

r = r_{RF} + (r_M – r_{RF})b
= 5% + (8.5% – 5%)0.9
= 8.15%.

8-4 r_{RF} = 6%; RP_M = 7%; r_M = ?

r_M = 6% + (7%)1 = 13%.

r =? when b = 1.5?

r = 6% + 7%(1.5) = 16.5%.

8-5 **a.** r = 11%; r_{RF} = 7%; RP_M = 4%.

r = r_{RF} + (r_M – r_{RF})b
11% = 7% + 4%b
4% = 4%b
b = 1.

b. r_{RF} = 7%; RP_M = 6%; b = 1.

r = r_{RF} + (r_M – r_{RF})b
= 7% + (6%)1
= 13%.

8-6 **a.** $\hat{r} = \sum_{i=1}^{N} P_i r_i$.

$\hat{r}_Y = 0.1(-35\%) + 0.2(0\%) + 0.4(20\%) + 0.2(25\%) + 0.1(45\%)$
$= 14\%$ versus 12% for X.

b. $\sigma = \sqrt{\sum_{i=1}^{N} (r_i - \hat{r})^2 P_i}$.

$\sigma_X^2 = (-10\% - 12\%)^2(0.1) + (2\% - 12\%)^2(0.2) + (12\% - 12\%)^2(0.4)$
$+ (20\% - 12\%)^2(0.2) + (38\% - 12\%)^2(0.1) = 148.8.$

$\sigma_X = 12.20\%$ versus 20.35% for Y.

$CV_X = \sigma_X / \hat{r}_X = 12.20\%/12\% = 1.02$, while

$CV_Y = 20.35\%/14\% = 1.45$.

If Stock Y is less highly correlated with the market than X, then it might have a lower beta than Stock X, and hence be less risky in a portfolio sense.

8-7 Portfolio beta = $\frac{\$400{,}000}{\$4{,}000{,}000}(1.50) + \frac{\$600{,}000}{\$4{,}000{,}000}(-0.50) + \frac{\$1{,}000{,}000}{\$4{,}000{,}000}(1.25) + \frac{\$2{,}000{,}000}{\$4{,}000{,}000}(0.75)$

$b_p = (0.1)(1.5) + (0.15)(-0.50) + (0.25)(1.25) + (0.5)(0.75)$
$= 0.15 - 0.075 + 0.3125 + 0.375 = 0.7625.$

$r_p = r_{RF} + (r_M - r_{RF})(b_p) = 6\% + (14\% - 6\%)(0.7625) = 12.1\%.$

Alternative solution: First, calculate the return for each stock using the CAPM equation $[r_{RF} + (r_M - r_{RF})b]$, and then calculate the weighted average of these returns.

$r_{RF} = 6\%$ and $(r_M - r_{RF}) = 8\%$.

Stock	Investment	Beta	$r = r_{RF} + (r_M - r_{RF})b$	Weight
A	$ 400,000	1.50	18%	0.10
B	600,000	(0.50)	2	0.15
C	1,000,000	1.25	16	0.25
D	2,000,000	0.75	12	0.50
Total	$4,000,000			1.00

$r_p = 18\%(0.10) + 2\%(0.15) + 16\%(0.25) + 12\%(0.50) = 12.1\%.$

8-8 In equilibrium:

$r_J = \hat{r}_J = 10.5\%.$

$r_J = r_{RF} + (r_M - r_{RF})b$
$10.5\% = 4\% + (8.5\% - 4\%)b$
$b = 1.44.$

8-9 We know that $b_I = 1.30$, $b_J = 0.60$, $r_M = 11\%$, $r_{RF} = 5\%$.

$r_i = r_{RF} + (r_M - r_{RF})b_i = 5\% + (11\% - 5\%)b_i$.

$r_I = 5\% + 6\%(1.30) = 12.8\%$
$r_J = 5\% + 6\%(0.60) = \underline{8.6}$
$\underline{\underline{4.2}}\%$

8-10 An index fund will have a beta of 1.0. If r_M is 11.0% (given in the problem) and the risk-free rate is 4%, you can calculate the market risk premium (RP_M) calculated as $r_M - r_{RF}$ as follows:

$$r = r_{RF} + (RP_M)b$$
$$11.0\% = 4\% + (RP_M)1.0$$
$$RP_M = 7.0\%.$$

Now, you can use the RP_M, the r_{RF}, and the two stocks' betas to calculate their required returns.

Bradford:

$$\begin{aligned} r_B &= r_{RF} + (RP_M)b \\ &= 4\% + (7.0\%)1.2 \\ &= 4\% + 8.4\% \\ &= 12.4\%. \end{aligned}$$

Farley:

$$\begin{aligned} r_F &= r_{RF} + (RP_M)b \\ &= 4\% + (7.0\%)0.7 \\ &= 4\% + 4.9\% \\ &= 8.9\%. \end{aligned}$$

The difference in their required returns is:

$12.4\% - 8.9\% = 3.5\%$.

8-11 $r_{RF} = r^* + IP = 2.5\% + 3.5\% = 6\%$.

$r = 6\% + (6.5\%)1.7 = 17.05\%$.

8-12 **a.** $r_i = r_{RF} + (r_M - r_{RF})b_i = 9\% + (14\% - 9\%)1.3 = 15.5\%$.

b. 1. r_{RF} increases to 10%:

r_M increases by 1 percentage point, from 14% to 15%.

$r_i = r_{RF} + (r_M - r_{RF})b_i = 10\% + (15\% - 10\%)1.3 = 16.5\%$.

2. r_{RF} decreases to 8%:

r_M decreases by 1%, from 14% to 13%.

$r_i = r_{RF} + (r_M - r_{RF})b_i = 8\% + (13\% - 8\%)1.3 = 14.5\%$.

c. 1. r_M increases to 16%:

$r_i = r_{RF} + (r_M - r_{RF})b_i = 9\% + (16\% - 9\%)1.3 = 18.1\%.$

2. r_M decreases to 13%:

$r_i = r_{RF} + (r_M - r_{RF})b_i = 9\% + (13\% - 9\%)1.3 = 14.2\%.$

8-13 **a.** Using Stock X (or any stock):

$$\begin{aligned} 9\% &= r_{RF} + (r_M - r_{RF})b_X \\ 9\% &= 5.5\% + (r_M - r_{RF})0.8 \\ (r_M - r_{RF}) &= 4.375\%. \end{aligned}$$

b. $b_Q = {}^1/_3(0.8) + {}^1/_3(1.2) + {}^1/_3(1.6)$
$b_Q = 0.2667 + 0.4000 + 0.5333$
$b_Q = 1.2.$

c. $r_Q = 5.5\% + 4.375\%(1.2)$
$r_Q = 10.75\%.$

d. Since the returns on the 3 stocks included in Portfolio Q are not perfectly positively correlated, one would expect the standard deviation of the portfolio to be less than 15%.

8-14

$$\begin{aligned} \text{Old portfolio beta} &= \frac{\$90,000}{\$100,000}(b) + \frac{\$10,000}{\$100,000}(0.9) \\ 1.4 &= 0.9b + 0.09 \\ 1.31 &= 0.9b \\ 1.456 &= b. \end{aligned}$$

New portfolio beta = $0.9(1.456) + 0.1(1.6) = 1.4704 \approx 1.47.$

8-15 $b_{RMI} = 1.75$; $b_{TMY} = 0.8$. No changes occur.

$r_{RF} = 5\%$. Decreases by 2% to 3%.

$r_M = 11.5\%$. Falls to 8.5%.

Now SML: $r_i = r_{RF} + (r_M - r_{RF})b_i$.

$r_{RMI} = 3\% + (8.5\% - 3\%)1.7 = 3\% + 5.5\%(1.7) =$	12.35%
$r_{TMY} = 3\% + (8.5\% - 3\%)0.8 = 3\% + 5.5\%(0.8) =$	7.4%
Difference	4.95%

8-16 Step 1: Determine the market risk premium from the CAPM:

$$\begin{aligned} 0.13 &= 0.05 + (r_M - r_{RF})1.5 \\ (r_M - r_{RF}) &= 0.053. \end{aligned}$$

Step 2: Calculate the beta of the new portfolio:

($2,000,000/$12,000,000)(1.1) + ($10,000,000/$12,000,000)(1.5) = 1.4333.

Step 3: Calculate the required return on the new portfolio:

$5\% + (5.3\%)(1.4333) = 12.60\%$.

8-17 After additional investments are made, for the entire fund to have an expected return of 13%, the portfolio must have a beta of 1.5455 as shown below:

$$13\% = 4.5\% + (5.5\%)b$$
$$b = 1.5455.$$

Since the fund's beta is a weighted average of the betas of all the individual investments, we can calculate the required beta on the additional investment as follows:

$$1.5455 = \frac{(\$20{,}000{,}000)(1.5)}{\$25{,}000{,}000} + \frac{\$5{,}000{,}000X}{\$25{,}000{,}000}$$
$$1.5455 = 1.2 + 0.2X$$
$$0.3455 = 0.2X$$
$$X = 1.7275.$$

8-18 **a.** (\$1 million)(0.5) + (\$0)(0.5) = \$0.5 million.

b. You would probably take the sure \$0.5 million.

c. Risk averter.

d. 1. (\$1.15 million)(0.5) + (\$0)(0.5) = \$575,000, or an expected profit of \$75,000.

2. \$75,000/\$500,000 = 15%.

3. This depends on the individual's degree of risk aversion.

4. Again, this depends on the individual.

5. The situation would be unchanged if the stocks' returns were perfectly positively correlated. Otherwise, the stock portfolio would have the same expected return as the single stock (15%) but a lower standard deviation. If the correlation coefficient between each pair of stocks was a negative one, the portfolio would be virtually riskless. Since ρ for stocks is generally in the range of +0.35, investing in a portfolio of stocks would definitely be an improvement over investing in the single stock.

8-19 $\hat{r}_X = 9\%$; $b_X = 0.8$; $\sigma_X = 30\%$.

$\hat{r}_Y = 14\%$; $b_Y = 1.3$; $\sigma_Y = 20\%$.

$r_{RF} = 6\%$; $RP_M = 6.5\%$.

a. $CV_X = 30\%/9\% = 3.33$. $CV_Y = 20\%/14\% = 1.43$.

b. For diversified investors the relevant risk is measured by beta. Therefore, the stock with the higher beta is more risky. Stock Y has the higher beta so it is more risky than Stock X.

c. $r_X = 5\% + 6.5\%(0.8)$
$= 10.2\%.$

$r_Y = 5\% + 6.5\%(1.3)$
$= 13.45\%.$

d. $r_X = 10.2\%$; $\hat{r}_X = 9\%$.
$r_Y = 13.45\%$; $\hat{r}_Y = 14\%$.

Stock Y would be most attractive to a diversified investor since its expected return of 14% is greater than its required return of 13.45%.

e. $b_p = (\$8{,}000/\$10{,}000)0.8 + (\$2{,}000/\$10{,}000)1.3$
$= 0.64 + 0.26$
$= 0.9.$

$r_p = 5\% + 6.5\%(0.9)$
$= 10.85\%.$

f. If RP_M increases from 6.5% to 7%, the stock with the highest beta will have the largest increase in its required return. Therefore, Stock Y will have the greatest increase.

Check:

$r_X = 5\% + 7\%(0.8)$
$= 10.6\%.$ Increase 10.2% to 10.6%.

$r_Y = 5\% + 7\%(1.3)$
$= 14.1\%.$ Increase 13.45% to 14.1%.

8-20 The answers to a, b, c, and d are given below:

	r_A	r_B	Portfolio
2007	(18.00%)	(14.50%)	(16.25%)
2008	33.00	21.80	27.40
2009	15.00	30.50	22.75
2010	(0.50)	(7.60)	(4.05)
2011	27.00	26.30	26.65
Mean	11.30	11.30	11.30
Std. Dev.	20.79	20.78	20.13
Coef. Var.	1.84	1.84	1.78

e. A risk-averse investor would choose the portfolio over either Stock A or Stock B alone, since the portfolio offers the same expected return but with less risk. This result occurs because returns on A and B are not perfectly positively correlated ($r_{AB} = 0.88$).

8-21 **a.** $\hat{r}_M = 0.1(-28\%) + 0.2(0\%) + 0.4(12\%) + 0.2(30\%) + 0.1(50\%) = 13\%.$

$r_{RF} = 6\%.$ (given)

Therefore, the SML equation is:

$r_i = r_{RF} + (r_M – r_{RF})b_i = 6\% + (13\% – 6\%)b_i = 6\% + (7\%)b_i$.

b. First, determine the fund's beta, b_F. The weights are the percentage of funds invested in each stock:

A = \$160/\$500 = 0.32.
B = \$120/\$500 = 0.24.
C = \$80/\$500 = 0.16.
D = \$80/\$500 = 0.16.
E = \$60/\$500 = 0.12.

$b_F = 0.32(0.5) + 0.24(1.2) + 0.16(1.8) + 0.16(1.0) + 0.12(1.6)$
$= 0.16 + 0.288 + 0.288 + 0.16 + 0.192 = 1.088$.

Next, use $b_F = 1.088$ in the SML determined in Part a:

$\hat{r}_F = 6\% + (13\% – 6\%)1.088 = 6\% + 7.616\% = 13.616\%$.

c. r_N = Required rate of return on new stock = 6% + (7%)1.5 = 16.5%.

An expected return of 15% on the new stock is below the 16.5% required rate of return on an investment with a risk of b = 1.5. Since $r_N = 16.5\% > \hat{r}_N = 15\%$, the new stock should <u>not</u> be purchased. The expected rate of return that would make the fund indifferent to purchasing the stock is 16.5%.

Comprehensive/Spreadsheet Problem

Note to Instructors:
The solution to this problem is <u>not</u> provided to students at the back of their text. Instructors can access the *Excel* file on the textbook's website.

8-22 a.

	Bartman	Reynolds	Index
2011	24.7%	-1.1%	32.8%
2010	-4.2%	13.2%	1.2%
2009	62.8%	-10.0%	34.9%
2008	2.9%	-0.4%	14.8%
2007	61.0%	11.7%	19.0%
Avg Returns	29.4%	2.7%	20.6%

b.

	Bartman	Reynolds	Index
Standard deviation of return	31.5%	9.7%	13.8%

On a stand-alone basis, it would appear that Bartman is the most risky, Reynolds the least risky.

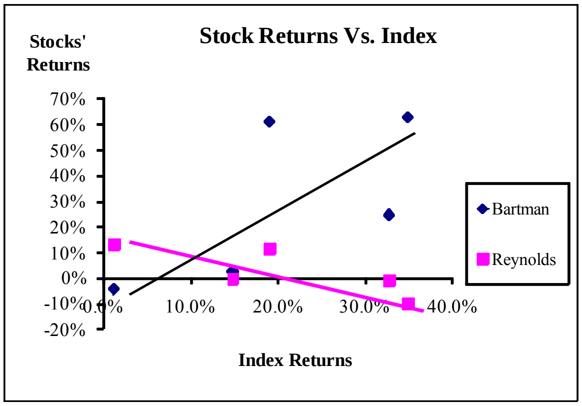
c. Divide the standard deviation by the average return:

	Bartman	Reynolds	Index
Coefficient of Variation	1.07	3.63	0.67

Reynolds now looks most risky, because its risk (SD) per unit of return is highest.

d.

Year	Index	Bartman	Reynolds
2011	32.8%	24.7%	-1.1%
2010	1.2%	-4.2%	13.2%
2009	34.9%	62.8%	-10.0%
2008	14.8%	2.9%	-0.4%
2007	19.0%	61.0%	11.7%



It is clear that Bartman moves with the market and Reynolds moves counter to the market. So, Bartman has a positive beta and Reynolds a negative one.

e. Bartman's calculations:

SUMMARY OUTPUT

Bartman's beta = 1.539

Regression Statistics	
Multiple R	0.67547
R Square	0.45626
Adjusted R Square	0.27502
Standard Error	0.26812
Observations	5

ANOVA

	df	SS	MS	F	Significance F
Regression	1	0.18097	0.18097	2.51737	0.21079
Residual	3	0.21567	0.07189		
Total	4	0.39664			

	Coefficients	Standard Error	t Stat	P-value	Lower 95%	Upper 95%	Lower 95.0%	Upper 95.0%
Intercept	-0.02203	0.23270	-0.09466	0.93055	-0.76259	0.71853	-0.76259	0.71853
X Variable 1	1.53931	0.97018	1.58662	0.21079	-1.54824	4.62687	-1.54824	4.62687

RESIDUAL OUTPUT

Observation	Predicted Y	Residuals
1	0.48227	-0.23481
2	-0.00328	-0.03854
3	0.51531	0.11260
4	0.20651	-0.17794
5	0.27115	0.33868

Reynolds' calculations:

SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.79735
R Square	0.63576
Adjusted R Square	0.51435
Standard Error	0.06769
Observations	5

Reynolds' beta = -0.560

ANOVA

	df	SS	MS	F	Significance F
Regression	1	0.02399	0.02399	5.23641	0.10612
Residual	3	0.01374	0.00458		
Total	4	0.03774			

	Coefficients	Standard Error	t Stat	P-value	Lower 95%	Upper 95%	Lower 95.0%	Upper 95.0%
Intercept	0.14196	0.05874	2.41659	0.09446	-0.04499	0.32892	-0.04499	0.32892
X Variable 1	-0.56046	0.24492	-2.28832	0.10612	-1.33991	0.21899	-1.33991	0.21899

RESIDUAL OUTPUT

Observation	Predicted Y	Residuals
1	-0.04165	0.03113
2	0.13514	-0.00283
3	-0.05368	-0.04676
4	0.05875	-0.06292
5	0.03522	0.08138

Note that these betas are consistent with the scatter diagrams we constructed earlier. Reynolds' beta suggests that it is less risky than average in a CAPM sense, whereas Bartman is more risky than average.

f.

Market return =		11.000%				
Risk-free rate =		6.040%				
Required return	=	Risk-free rate	+	Market risk premium	×	Beta
Bartman:						
Required return	=	6.040%	+	4.960%	×	1.539
Required return	=	13.675%				
Reynolds:						
Required return	=	6.040%	+	4.960%	×	-0.560
Required return	=	3.260%				

This suggests that Reynolds' stock is like an insurance policy that has a low expected return, but it will pay off in the event of a market decline. Actually, it is hard to find negative-beta stocks, so we would not be inclined to believe the Reynolds' data.

g. The beta of a portfolio is simply a weighted average of the betas of the stocks in the portfolio, so this portfolio's beta would be:

Portfolio beta = 0.489

Required return on portfolio	=	Risk-free rate	+	Market risk premium	×	Beta
Required return	=	6.040%	+	4.960%	×	0.489
Portfolio required return	=	8.468%				

h.

	Beta	Portfolio Weight
Bartman	1.539	25%
Stock A	0.769	15%
Stock B	0.985	40%
Stock C	1.423	20%
		100%
Portfolio Beta =	1.179	

Required return on portfolio	=	Risk-free rate	+	Market risk premium	×	Beta
Required return on portfolio	=	6.04%	+	4.96%	×	1.179
Required return on portfolio	=	11.887%				

Integrated Case

8-23
China Development Industrial Bank
Risk and Return

Assume that you recently graduated with a major in finance. You just landed a job as a financial planner with China Development Industrial Bank (CDIB), a large financial services corporation. Your first assignment is to invest $100,000 for a client. Because the funds are to be invested in a business at the end of 1 year, you have been instructed to plan for a 1-year holding period. Further, your boss has restricted you to the investment alternatives in the following table, shown with their probabilities and associated outcomes. (For now, disregard the items at the bottom of the data; you will fill in the blanks later.)

Returns on Alternative Investments

		Estimated Rate of Return					
State of the Economy	Prob.	T-Bills	High Tech	Collec-tions	U.S. Rubber	Market Portfolio	2-Stock Portfolio
Recession	0.1	5.5%	-27.0%	27.0%	6.0%[a]	-17.0%	0.0%
Below Avg.	0.2	5.5	-7.0	13.0	-14.0	-3.0	
Average	0.4	5.5	15.0	0.0	3.0	10.0	7.5
Above Avg.	0.2	5.5	30.0	-11.0	41.0	25.0	
Boom	0.1	5.5	45.0	-21.0	26.0	38.0	12.0
r-hat ($\hat{r}$)				1.0%	9.8%	10.5%	
Std. dev. (σ)		0.0		13.2	18.8	15.2	3.4
Coeff. of Var. (CV)				13.2	1.9	1.4	0.5
beta (b)				-0.87	0.88		

Note:
[a] The estimated returns of U.S. Rubber do not always move in the same direction as the overall economy. For example, when the economy is below average, consumers purchase fewer tires than they would if the economy were stronger. However, if the economy is in a flat-out recession, a large number of consumers who were planning to purchase a new car may choose to wait and instead purchase new tires for the car that they currently own. Under these circumstances, we would expect U.S. Rubber's stock price to be higher if there is a recession than if the economy was just below average.

CDIB's economic forecasting staff has developed probability estimates for the state of the U.S. economy; and its security analysts have developed a sophisticated computer program, which was used to estimate the rate of return on each alternative under each state of the economy. High Tech Inc. is an electronics firm, Collections Inc. collects past-due debts, and U.S. Rubber manufactures tires and various other rubber and plastics products. CDIB also maintains a "market portfolio" that owns a market-weighted fraction of all publicly traded stocks; you can invest in that portfolio and thus obtain average stock market results. Given the situation described, answer the following questions.

A. (1) Why is the T-bill's return independent of the state of the economy? Do T-bills promise a completely risk-free return? Explain.

Answer: [Show S8-1 through S8-6 here.] The 5.5% T-bill return does not depend on the state of the economy because the Treasury must (and will) redeem the bills at par regardless of the state of the economy.

The T-bills are risk free in the default risk sense because the 5.5% return will be realized in all possible economic states. However, remember that this return is composed of the real risk-free rate, say 3%, plus an inflation premium, say 2.5%. Since there is uncertainty about inflation, it is unlikely that the realized <u>real</u> rate of return would equal the expected 3%. For example, if inflation averaged 3.5% over the year, then the realized real return would only be 5.5% – 3.5% = 2%, not the expected 3%. Thus, in terms of purchasing power, T-bills are not riskless.

Also, if you invested in a portfolio of T-bills, and rates then declined, your nominal income would fall; that is, T-bills are exposed to <u>reinvestment risk</u>. So, we conclude that there are no truly risk-free securities in the United States. If the Treasury sold inflation-indexed,

tax-exempt bonds, they would be truly riskless, but all actual securities are exposed to some type of risk.

A. (2) Why are High Tech's returns expected to move with the economy, whereas Collections' are expected to move counter to the economy?

Answer: [Show S8-7 here.] High Tech's returns move with, hence are positively correlated with, the economy, because the firm's sales, and hence profits, will generally experience the same type of ups and downs as the economy. If the economy is booming, so will High Tech. On the other hand, Collections is considered by many investors to be a hedge against both bad times and high inflation, so if the stock market crashes, investors in this stock should do relatively well. Stocks such as Collections are thus <u>negatively correlated</u> with (move counter to) the economy. (Note: In actuality, it is almost impossible to find stocks that are <u>expected</u> to move counter to the economy.)

B. Calculate the expected rate of return on each alternative and fill in the blanks on the row for $\hat{r}$ in the previous table.

Answer: [Show S8-8 and S8-9 here.] The expected rate of return, $\hat{r}$, is expressed as follows:

$$\hat{r} = \sum_{i=1}^{N} P_i r_i \text{ .}$$

Here P_i is the probability of occurrence of the ith state, r_i is the estimated rate of return for that state of the economy, and N is the number of states of the economy. Here is the calculation for High Tech:

$$\begin{aligned}\hat{r}_{\text{High Tech}} &= 0.1(-27.0\%) + 0.2(-7.0\%) + 0.4(15.0\%) + 0.2(30.0\%) \\ &\quad + 0.1(45.0\%) \\ &= 12.4\%.\end{aligned}$$

We use the same formula to calculate $\hat{r}$'s for the other alternatives:

$$\hat{r}_{\text{T-bills}} = 5.5\%.$$

$$\hat{r}_{\text{Collections}} = 1.0\%.$$

$$\hat{r}_{\text{U.S. Rubber}} = 9.8\%.$$

$$\hat{r}_{M} = 10.5\%.$$

C.	You should recognize that basing a decision solely on expected returns is appropriate only for risk-neutral individuals. Because your client, like most people, is risk-averse, the riskiness of each alternative is an important aspect of the decision. One possible measure of risk is the standard deviation of returns.
(1)	Calculate this value for each alternative and fill in the blank on the row for σ in the table.

Answer: [Show S8-10 and S8-11 here.] The standard deviation is calculated as follows:

$$\sigma = \sqrt{\sum_{i=1}^{N}(r_i - \hat{r})^2 P_i}.$$

$$\begin{aligned}\sigma_{\text{High Tech}} &= [(-27.0 - 12.4)^2(0.1) + (-7.0 - 12.4)^2(0.2) \\ &\quad + (15.0 - 12.4)^2(0.4) + (30.0 - 12.4)^2(0.2) \\ &\quad + (45.0 - 12.4)^2(0.1)]^{½} \\ &= \sqrt{401.4} = 20.0\%.\end{aligned}$$

Here are the standard deviations for the other alternatives:

$$\sigma_{\text{T-bills}} = 0.0\%.$$

$$\sigma_{\text{Collections}} = 13.2\%.$$

$$\sigma_{\text{U.S. Rubber}} = 18.8\%.$$

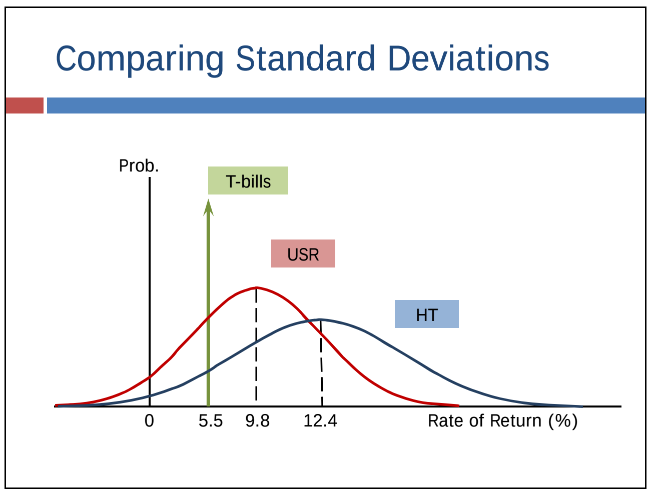
$$\sigma_{M} = 15.2\%.$$

C.	**(2)**	**What type of risk is measured by the standard deviation?**

Answer: **[Show S8-12 through S8-14 here.] The standard deviation is a measure of a security's (or a portfolio's) <u>stand-alone risk</u>. The larger the standard deviation, the higher the probability that actual realized returns will fall far below the expected return, and that losses rather than profits will be incurred.**

C.	**(3)**	**Draw a graph that shows <u>roughly</u> the shape of the probability distributions for High Tech, U.S. Rubber, and T-bills.**

Answer:



On the basis of these data, High Tech is the most risky investment, T-bills the least risky.

D.	**Suppose you suddenly remembered that the coefficient of variation (CV) is generally regarded as being a better measure of stand-alone risk than the standard deviation when the alternatives being considered have widely differing expected returns. Calculate the missing CVs and fill in the blanks on the row for CV in the table. Does the CV produce the same risk rankings as the standard deviation? Explain.**

Answer: **[Show S8-15 through S8-18 here.] The coefficient of variation (CV) is a standardized measure of dispersion about the expected value; it shows the amount of risk per unit of return.**

$$CV = \sigma / \hat{r}.$$

$$CV_{T\text{-bills}} = 0.0\% / 5.5\% = 0.0.$$

$$CV_{High\ Tech} = 20.0\% / 12.4\% = 1.6.$$

$$CV_{Collections} = 13.2\% / 1.0\% = 13.2.$$

$$CV_{U.S.\ Rubber} = 18.8\% / 9.8\% = 1.9.$$

$$CV_{M} = 15.2\% / 10.5\% = 1.4.$$

When we measure risk per unit of return, Collections, with its low expected return, becomes the most risky stock. The CV is a better measure of an asset's stand-alone risk than σ because CV considers both the expected value and the dispersion of a distribution—a security with a low expected return and a low standard deviation could have a higher chance of a loss than one with a high σ but a high $\hat{r}$.

E. **Suppose you created a two-stock portfolio by investing $50,000 in High Tech and $50,000 in Collections.**

(1) **Calculate the expected return ($\hat{r}_p$), the standard deviation (σ_p), and the coefficient of variation (CV_p) for this portfolio and fill in the appropriate blanks in the table.**

Answer: **[Show S8-19 through S8-22 here.] To find the expected rate of return on the two-stock portfolio, we first calculate the rate of return on the portfolio in each state of the economy. Since we have half of our money in each stock, the portfolio's return will be a weighted average in each type of economy. For a recession, we have: $r_p = 0.5(-27\%) +$**

0.5(27%) = 0%. We would do similar calculations for the other states of the economy, and obtain these results:

State	**Portfolio**
Recession	**0.0%**
Below average	**3.0**
Average	**7.5**
Above average	**9.5**
Boom	**12.0**

Now we can multiply the probability times the outcome in each state of the economy to calculate the expected return on this two-stock portfolio, 6.7%.

Alternatively, we could apply this formula,

$$r = w_i \times r_i = 0.5(12.4\%) + 0.5(1.0\%) = 6.7\%,$$

which finds r as the weighted average of the expected returns of the individual securities in the portfolio.

It is tempting to find the standard deviation of the portfolio as the weighted average of the standard deviations of the individual securities, as follows:

$$\sigma_p \neq w_i(\sigma_i) + w_j(\sigma_j) = 0.5(20\%) + 0.5(13.2\%) = 16.6\%.$$

<u>However, this is not correct</u>—it is necessary to use a different formula, the one for σ that we used earlier, applied to the two-stock portfolio's returns.

The portfolio's σ depends jointly on (1) each security's σ and (2) the correlation between the securities' returns. The best way to approach the problem is to estimate the portfolio's risk and return in each state of the economy, and then to estimate σ_p with the σ formula. Given the distribution of returns for the portfolio, we can calculate the portfolio's σ and CV as shown below:

$$\sigma_p = [(0.0 - 6.7)^2(0.1) + (3.0 - 6.7)^2(0.2) + (7.5 - 6.7)^2(0.4) + (9.5 - 6.7)^2(0.2) + (12.0 - 6.7)^2(0.1)]^{1/2} = 3.4\%.$$

$$CV_p = 3.4\%/6.7\% = 0.51.$$

E. (2) How does the riskiness of this two-stock portfolio compare with the riskiness of the individual stocks if they were held in isolation?

Answer: [Show S8-23 through S8-27 here.] Using either σ or CV as our stand-alone risk measure, the stand-alone risk of the portfolio is significantly less than the stand-alone risk of the individual stocks. This is because the two stocks are negatively correlated—when High Tech is doing poorly, Collections is doing well, and vice versa. Combining the two stocks diversifies away some of the risk inherent in each stock if it were held in isolation, i.e., in a one-stock portfolio.

Optional Question

Does the expected rate of return on the portfolio depend on the percentage of the portfolio invested in each stock? What about the riskiness of the portfolio?

Answer: Using a spreadsheet model, it's easy to vary the composition of the portfolio to show the effect on the portfolio's expected rate of return and standard deviation:

	High Tech Plus Collections	
% in High Tech	$\hat{r}_p$	σ_p
0%	1.0%	13.2%
10	2.1	9.9
20	3.3	6.6
30	4.4	3.2
40	5.6	0.4
50	6.7	3.4
60	7.8	6.8
70	9.0	10.1
80	10.1	13.4
90	11.3	16.7
100	12.4	20.0

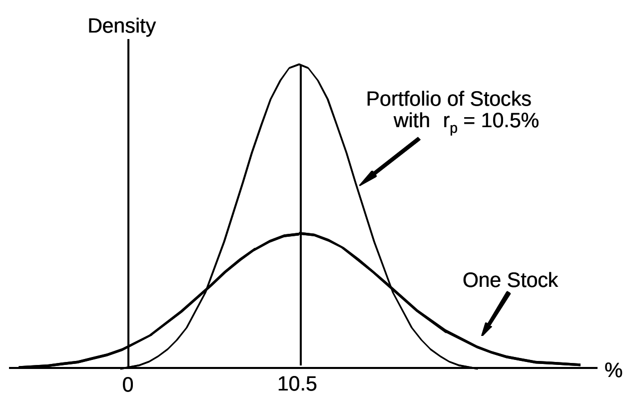
The expected rate of return on the portfolio is merely a linear combination of the two stock's expected rates of return. However, portfolio risk is another matter. σ_p begins to fall as High Tech and Collections are combined; it reaches near zero at 40% High Tech; and then it begins to rise. High Tech and Collections can be combined to form a near zero-risk portfolio because they are very close to being perfectly negatively correlated; their correlation coefficient is -0.9995. (Note: Unfortunately, we cannot find any actual stocks with r = -1.0.)

F. **Suppose an investor starts with a portfolio consisting of one randomly selected stock.**

(1) What would happen to the riskiness and to the expected return of the portfolio as more randomly selected stocks were added to the portfolio?

(2) What is the implication for investors? Draw a graph of the two portfolios to illustrate your answer.

Answer: [Show S8-28 here.]



The standard deviation gets smaller as more stocks are combined in the portfolio, while r_p (the portfolio's return) remains constant. Thus, by adding stocks to your portfolio, which initially started as a 1-stock portfolio, risk has been reduced.

In the real world, stocks are positively correlated with one another—if the economy does well, so do stocks in general, and vice versa. Correlation coefficients between stocks generally range in the vicinity of +0.35. A single stock selected at random would on average have a standard deviation of about 35%. As additional stocks are added to the portfolio, the portfolio's standard deviation decreases because the added stocks are not perfectly positively correlated. However, as more and more stocks are added, each new stock has less of a risk-reducing impact, and eventually adding additional stocks has virtually no effect on the portfolio's risk as measured by σ. In fact, σ stabilizes at about 20% when 40 or more randomly selected stocks are added. Thus, by combining stocks into well-diversified portfolios, investors can eliminate almost one-half the riskiness of holding individual stocks. (Note: It is not completely costless to diversify, so even the largest institutional investors hold less than all stocks. Even index funds generally hold a smaller portfolio that is highly correlated with an index such as the S&P 500 rather than holding all the stocks in the index.)

The implication is clear: <u>Investors should hold well-diversified portfolios of stocks rather than individual stocks.</u> (In fact, individuals can hold diversified portfolios through mutual fund investments.) By doing so, they can eliminate about half of the riskiness inherent in individual stocks.

G.	**(1)**	**Should the effects of a portfolio impact the way investors think about the riskiness of individual stocks?**

Answer: **[Show S8-29 and S8-30 here.] Portfolio diversification does affect investors' views of risk. A stock's stand-alone risk as measured by its σ or CV, may be important to an undiversified investor, but it is not**

relevant to a well-diversified investor. A rational, risk-averse investor is more interested in the impact that the stock has on the riskiness of his or her portfolio than on the stock's stand-alone risk. Stand-alone risk is composed of diversifiable risk, which can be eliminated by holding the stock in a well-diversified portfolio, and the risk that remains is called market risk because it is present even when the entire market portfolio is held.

G. (2) If you decided to hold a 1-stock portfolio (and consequently were exposed to more risk than diversified investors), could you expect to be compensated for all of your risk; that is, could you earn a risk premium on the part of your risk that you could have eliminated by diversifying?

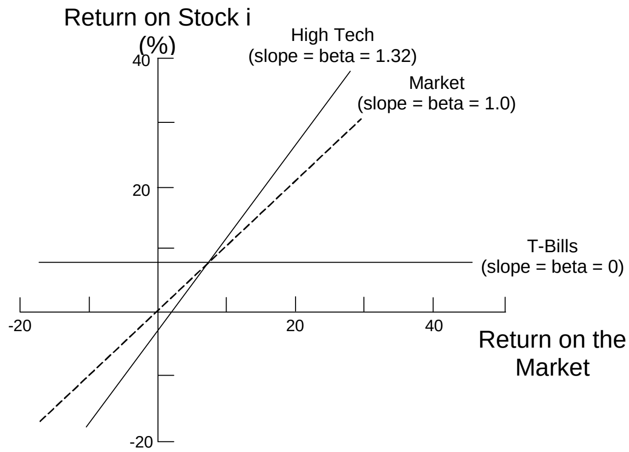
Answer: [Show S8-31 here.] If you hold a one-stock portfolio, you will be exposed to a high degree of risk, but you won't be compensated for it. If the return were high enough to compensate you for your high risk, it would be a bargain for more rational, diversified investors. They would start buying it, and these buy orders would drive the price up and the return down. Thus, you simply could not find stocks in the market with returns high enough to compensate you for the stock's diversifiable risk.

H. The expected rates of return and the beta coefficients of the alternatives supplied by CDIB's computer program are as follows:

Security	Return ($\hat{r}$)	Risk (Beta)
High Tech	12.4%	1.32
Market	10.5	1.00
U.S. Rubber	9.8	0.88
T-Bills	5.5	0.00
Collections	1.0	(0.87)

(1) What is a beta coefficient, and how are betas used in risk analysis?

Answer: [Show S8-32 through S8-38 here.]



(Draw the framework of the graph, put up the data, then plot the points for the market (45° line) and connect them, and then get the slope as $\Delta Y / \Delta X = 1.0$.) State that an average stock, by definition, moves with the market. Then do the same with High Tech and T-bills. Beta coefficients measure the relative volatility of a given stock vis-à-vis an average stock. The average stock's beta is 1.0. Most stocks have betas in the range of 0.5 to 1.5. Theoretically, betas can be negative, but in the real world they are generally positive.

Betas are calculated as the slope of the "characteristic" line, which is the regression line showing the relationship between a given stock and the general stock market. As explained in Web Appendix 8A, we could estimate the slopes, and then use the slopes as the betas. In practice, 5 years of monthly data, with 60 observations, would generally be used, and a computer would be used to obtain a least squares regression line.

H. (2) Do the expected returns appear to be related to each alternative's market risk?

Answer: [Show S8-39 here.] The expected returns are related to each alternative's market risk—that is, the higher the alternative's rate of return the higher its beta. Also, note that T-bills have zero risk.

H. (3) **Is it possible to choose among the alternatives on the basis of the information developed thus far? Use the data given at the start of the problem to construct a graph that shows how the T-bill's, High Tech's, and the market's beta coefficients are calculated. Then discuss what betas measure and how they are used in risk analysis.**

Answer: We do not yet have enough information to choose among the various alternatives. We need to know the required rates of return on these alternatives and compare them with their expected returns.

I. **The yield curve is currently flat; that is, long-term Treasury bonds also have a 5.5% yield. Consequently, CDIB assumes that the risk-free rate is 5.5%.**

(1) **Write out the Security Market Line (SML) equation, use it to calculate the required rate of return on each alternative, and graph the relationship between the expected and required rates of return.**

Answer: [Show S8-40 through S8-42 here.] Here is the SML equation:

$$r_i = r_{RF} + (r_M - r_{RF})b_i.$$

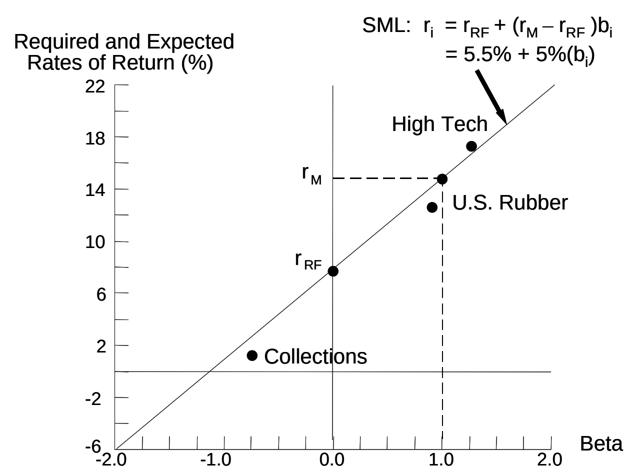
CDIB has estimated the risk-free rate to be r_{RF} = 5.5%. Further, our estimate of $r_M = \hat{r}_M$ is 10.5%. Thus, the required rates of return for the alternatives are as follows:

High Tech:	**5.5% + (10.5% – 5.5%)1.32 = 12.10%.**
Market:	**5.5% + (10.5% – 5.5%)1.00 = 10.50%.**
U.S. Rubber:	**5.5% + (10.5% – 5.5%)0.88 = 9.90%.**
T-bills:	**5.5% + (10.5% – 5.5%)0 = 5.50%.**
Collections:	**5.5% + (10.5% – 5.5%)-0.87 = 1.15%.**

I. (2) How do the expected rates of return compare with the required rates of return?

Answer: **[Show S8-43 and S8-44 here.] We have the following relationships:**

Security	Expected Return ($\hat{r}$)	Required Return (r)	Condition
High Tech	12.4%	12.1%	Undervalued: $\hat{r}$ > r
Market	10.5	10.5	Fairly valued (market equilibrium)
U.S. Rubber	9.8	9.9	Overvalued: r > $\hat{r}$
T-bills	5.5	5.5	Fairly valued
Collections	1.0	1.2	Overvalued: r > $\hat{r}$



(Note: The plot looks somewhat unusual in that the X-axis extends to the left of zero. We have a negative-beta stock, hence a required return that is less than the risk-free rate.) The T-bills and market portfolio plot on the SML, High Tech plots above it, and Collections and U.S. Rubber plot below it. Thus, the T-bills and the market

portfolio promise a fair return, High Tech is a good deal because its expected return is above its required return, and Collections and U.S. Rubber have expected returns below their required returns.

I. (3) Does the fact that Collections has an expected return that is less than the T-bill rate make any sense? Explain.

Answer: Collections is an interesting stock. Its negative beta indicates negative market risk—including it in a portfolio of "normal" stocks will lower the portfolio's risk. Therefore, its required rate of return is below the risk-free rate. Basically, this means that Collections is a valuable security to rational, well-diversified investors. To see why, consider this question: Would any rational investor ever make an investment that has a negative expected return? The answer is "yes"—just think of the purchase of a life or fire insurance policy. The fire insurance policy has a negative expected return because of commissions and insurance company profits, but businesses buy fire insurance because they pay off at a time when normal operations are in bad shape. Life insurance is similar—it has a high return when work income ceases. A negative-beta stock is conceptually similar to an insurance policy.

I. (4) What would be the market risk and the required return of a 50-50 portfolio of High Tech and Collections? Of High Tech and U.S. Rubber?

Answer: [Show S8-45 and S8-46 here.] Note that the beta of a portfolio is simply the weighted average of the betas of the stocks in the portfolio. Thus, the beta of a portfolio with 50% High Tech and 50% Collections is:

$$b_p = \sum_{i=1}^{N} w_i b_i .$$

$$b_p = 0.5(b_{High\ Tech}) + 0.5(b_{Collections}) = 0.5(1.32) + 0.5(-0.87) = 0.225,$$

$$r_p = r_{RF} + (r_M - r_{RF})b_p = 5.5\% + (10.5\% - 5.5\%)(0.225) = 5.5\% + 5\%(0.225) = 6.63\% \approx 6.6\%.$$

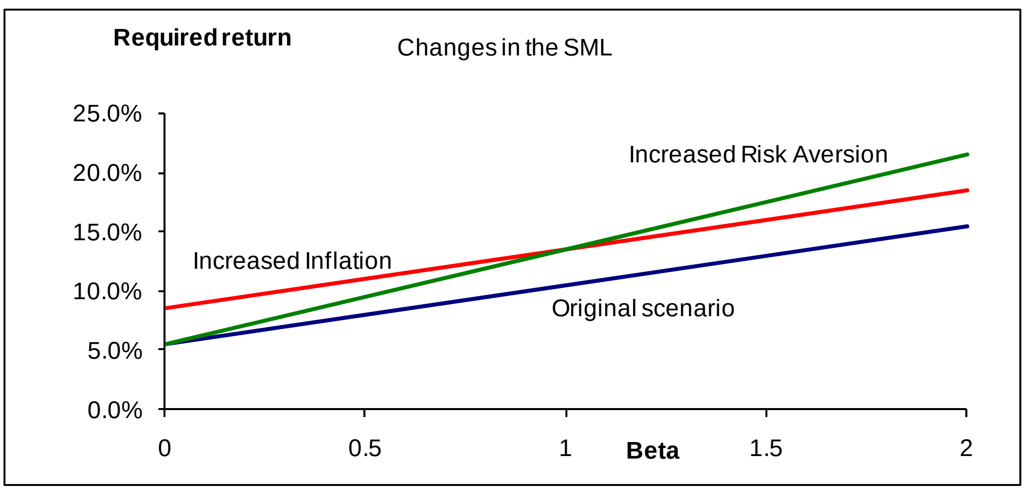
For a portfolio consisting of 50% High Tech plus 50% U.S. Rubber, the required return would be:

$$b_p = 0.5(1.32) + 0.5(0.88) = 1.10.$$

$$r_p = 5.5\% + 5\%(1.10) = 11.00\%.$$

J. (1) Suppose investors raised their inflation expectations by 3 percentage points over current estimates as reflected in the 5.5% risk-free rate. What effect would higher inflation have on the SML and on the returns required on high- and low-risk securities?

Answer: [Show S8-47 here.]



Here we have plotted the SML for betas ranging from 0 to 2.0. The base-case SML is based on r_{RF} = 5.5% and r_M = 10.5%. If inflation expectations increase by 3 percentage points, with no change in risk

aversion, then the entire SML is shifted upward (parallel to the base case SML) by 3 percentage points. Now, r_{RF} = 8.5%, r_M = 13.5%, and all securities' required returns rise by 3 percentage points. Note that the market risk premium, $r_M - r_{RF}$, remains at 5 percentage points.

J. (2) Suppose instead that investors' risk aversion increased enough to cause the market risk premium to increase by 3 percentage points. (Inflation remains constant.) What effect would this have on the SML and on returns of high- and low-risk securities?

Answer: [Show S8-48 through S8-50 here.] When investors' risk aversion increases, the SML is rotated upward about the Y-intercept (r_{RF}). r_{RF} remains at 5.5%, but now r_M increases to 13.5%, so the market risk premium increases to 8%. The required rate of return will rise sharply on high-risk (high-beta) stocks, but not as much on low-beta securities.

Optional Question

Financial managers are more concerned with investment decisions relating to real assets such as plant and equipment than with investments in financial assets such as securities. How does the analysis that we have gone through relate to real-asset investment decisions, especially corporate capital budgeting decisions?

Answer: There is a great deal of similarity between your financial asset decisions and a firm's capital budgeting decisions. Here is the linkage:

1. **A company may be thought of as a portfolio of assets. If the company diversifies its assets, and especially if it invests in some projects that tend to do well when others are doing badly, it can lower the variability of its returns.**

2. **Companies obtain their investment funds from investors, who buy the firm's stocks and bonds. When investors buy these securities, they require a risk premium that is based on the company's risk as they (investors) see it. Further, since investors in general hold well-diversified portfolios of stocks and bonds, the risk that is relevant to them is the security's market risk, not its stand-alone risk. Thus, investors view the firm's risk from a market-risk perspective.**

3. **Therefore, when a manager makes a decision to build a new plant, the riskiness of the investment in the plant that is relevant to the firm's investors (its owners) is its market risk, not its stand-alone risk. Accordingly, managers need to know how physical-asset investment decisions affect their firm's beta coefficient. A particular asset may look quite risky when viewed in isolation, but if its returns are negatively correlated with returns on the firm's other assets, the asset may really have low risk. We will discuss all this in more detail in our capital budgeting discussion.**