**Solutions Manual**

***Essentials of Corporate Finance***

**Ross, Westerfield, and Jordan**

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Prepared by

Brad Jordan

University of Kentucky

Joe Smolira

Belmont University

***CHAPTER 1***

**INTRODUCTION TO CORPORATE FINANCE**

**Answers to Concepts Review and Critical Thinking Questions**

**1.** Capital budgeting (deciding on whether to expand a manufacturing plant), capital structure (deciding whether to issue new equity and use the proceeds to retire outstanding debt), and working capital management (modifying the firm’s credit collection policy with its customers).

**2.** Disadvantages: unlimited liability, limited life, difficulty in transferring ownership, hard to raise capital funds. Some advantages: simpler, less regulation, the owners are also the managers, sometimes personal tax rates are better than corporate tax rates.

**3.** The primary disadvantage of the corporate form is the double taxation to shareholders of distributed earnings and dividends. Some advantages include: limited liability, ease of transferability, ability to raise capital, and unlimited life.

**4.** The treasurer’s office and the controller’s office are the two primary organizational groups that report directly to the chief financial officer. The controller’s office handles cost and financial accounting, tax management, and management information systems. The treasurer’s office is responsible for cash and credit management, capital budgeting, and financial planning. Therefore, the study of corporate finance is concentrated within the functions of the treasurer’s office.

**5.** To maximize the current market value (share price) of the equity of the firm (whether it’s publicly traded or not).

**6.** In the corporate form of ownership, the shareholders are the owners of the firm. The shareholders elect the directors of the corporation, who in turn appoint the firm’s management. This separation of ownership from control in the corporate form of organization is what causes agency problems to exist. Management may act in its own or someone else’s best interests, rather than those of the shareholders. If such events occur, they may contradict the goal of maximizing the share price of the equity of the firm.

**7.** A primary market transaction.

**8.** In auction markets like the NYSE, brokers and agents meet at a physical location (the exchange) to buy and sell their assets. Dealer markets like NASDAQ represent dealers operating in dispersed locales who buy and sell assets themselves, usually communicating with other dealers electronically or literally over the counter.

**9.** Since such organizations frequently pursue social or political missions, many different goals are conceivable. One goal that is often cited is revenue minimization; i.e., providing their goods and services to society at the lowest possible cost. Another approach might be to observe that even a not-for-profit business has equity. Thus, an appropriate goal would be to maximize the value of the equity.

**10.** An argument can be made either way. At one extreme, we could argue that in a market economy, all of these things are priced. This implies an optimal level of ethical and/or illegal behavior and the framework of stock valuation explicitly includes these. At the other extreme, we could argue that these are non-economic phenomena and are best handled through the political process. The following is a classic (and highly relevant) thought question that illustrates this debate: “A firm has estimated that the cost of improving the safety of one of its products is $30 million. However, the firm believes that improving the safety of the product will only save $20 million in product liability claims. What should the firm do?”

**11.** The goal will be the same, but the best course of action toward that goal may require adjustments due to different social, political, and economic climates.

**12.** The goal of management should be to maximize the share price for the current shareholders. If management believes that it can improve the profitability of the firm so that the share price will exceed $35, then they should fight the offer from the outside company. If management believes that this bidder or other unidentified bidders will actually pay more than $35 per share to acquire the company, then they should still fight the offer. However, if the current management cannot increase the value of the firm beyond the bid price, and no other higher bids come in, then management is not acting in the interests of the shareholders by fighting the offer. Since current managers often lose their jobs when the corporation is acquired, poorly monitored managers have an incentive to fight corporate takeovers in situations such as this.

**13.** We would expect agency problems to be less severe in other countries, primarily due to the relatively small percentage of individual ownership. Fewer individual owners should reduce the number of diverse opinions concerning corporate goals. The high percentage of institutional ownership might lead to a higher degree of agreement between owners and managers on decisions concerning risky projects. In addition, institutions may be able to implement more effective monitoring mechanisms than can individual owners, given institutions’ deeper resources and experiences with their own management. The increase in institutional ownership of stock in the United States and the growing activism of these large shareholder groups may lead to a reduction in agency problems for U.S. corporations and a more efficient market for corporate control.

**14.** How much is too much? Who is worth more, Michael Fries or LeBron James? The simplest answer is that there is a market for executives just as there is for all types of labor. Executive compensation is the price that clears the market. The same is true for athletes and performers. Having said that, one aspect of executive compensation deserves comment. A primary reason executive compensation has grown so dramatically is that companies have increasingly moved to stock-based compensation. Such movement is obviously consistent with the attempt to better align stockholder and management interests. In recent years, stock prices have soared, so management has cleaned up. It is sometimes argued that much of this reward is simply due to rising stock prices in general, not managerial performance. Perhaps in the future, executive compensation will be designed to reward only differential performance, i.e., stock price increases in excess of general market increases.

**15.** The biggest reason that a company would “go dark” is because of the increased audit costs associated with Sarbanes-Oxley compliance. A company should always do a cost-benefit analysis, and it may be the case that the costs of complying with Sarbox outweigh the benefits. Of course, the company could always be trying to hide financial issues of the company! This is also one of the costs of going dark: Investors surely believe that some companies are going dark to avoid the increased scrutiny from Sarbox. This taints other companies that go dark just to avoid compliance costs. This is similar to the lemon problem with used automobiles: Buyers tend to underpay because they know a certain percentage of used cars are lemons. So, investors will tend to pay less for the company stock than they otherwise would. It is important to note that even if the company delists, its stock is still likely traded, but on the over-the-counter market pink sheets rather than on an organized exchange. This adds another cost since the stock is likely to be less liquid now. All else the same, investors pay less for an asset with less liquidity. Overall, the cost to the company is likely a reduced market value. Whether delisting is good or bad for investors depends on the individual circumstances of the company. It is also important to remember that there are already many small companies that file only limited financial information.

**CHAPTER 2**

***WORKING WITH FINANCIAL STATEMENTS***

**Answers to Concepts Review and Critical Thinking Questions**

**1.** Liquidity measures how quickly and easily an asset can be converted to cash without significant loss in value. It’s desirable for firms to have high liquidity so that they can more safely meet short-term creditor demands. However, liquidity also has an opportunity cost. Firms generally reap higher returns by investing in illiquid, productive assets. It’s up to the firm’s financial management staff to find a reasonable compromise between these opposing needs.

**2.** The recognition and matching principles in financial accounting call for revenues, and the costs associated with producing those revenues, to be “booked” when the revenue process is essentially complete, not necessarily when the cash is collected or bills are paid. Note that this way is not necessarily correct; it’s the way accountants have chosen to do it.

**3.** Historical costs can be objectively and precisely measured, whereas market values can be difficult to estimate, and different analysts would come up with different numbers. Thus, there is a tradeoff between relevance (market values) and objectivity (book values).

**4.** Depreciation is a non-cash deduction that reflects adjustments made in asset book values in accordance with the matching principle in financial accounting. Interest expense is a cash outlay, but it’s a financing cost, not an operating cost.

**5.** Market values can never be negative. Imagine a share of stock selling for –$20. This would mean that if you placed an order for 100 shares, you would get the stock along with a check for $2,000. How many shares do you want to buy? More generally, because of corporate and individual bankruptcy laws, net worth for a person or a corporation cannot be negative, implying that liabilities cannot exceed assets in market value.

**6.** For a successful company that is rapidly expanding, capital outlays would typically be large, possibly leading to negative cash flow from assets. In general, what matters is whether the money is spent wisely, not whether cash flow from assets is positive or negative.

**7.** It’s probably not a good sign for an established company, but it would be fairly ordinary for a start-up, so it depends.

1. For example, if a company were to become more efficient in inventory management, the amount of inventory needed would decline. The same might be true if it becomes better at collecting its receivables. In general, anything that leads to a decline in ending NWC relative to beginning NWC would have this effect. Negative net capital spending would mean more long-lived assets were liquidated than purchased.

**9.** If a company raises more money from selling stock than it pays in dividends in a particular period, its cash flow to stockholders will be negative. If a company borrows more than it pays in interest, its cash flow to creditors will be negative.

**10.** The adjustments discussed were purely accounting changes; they had no cash flow or market value consequences unless the new accounting information caused stockholders to revalue the company.

**Solutions to Questions and Problems**

*NOTE: All end-of-chapter problems were solved using a spreadsheet. Many problems require multiple steps. Due to space and readability constraints, when these intermediate steps are included in this solutions manual, rounding may appear to have occurred. However, the final answer for each problem is found without rounding during any step in the problem.*

*Basic*

**1.** The balance sheet for the company will look like this:

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | Balance sheet | | | | |
|  | Current assets | $2,030 |  | Current liabilities | $1,640 |
|  | Net fixed assets | 9,780 |  | Long-term debt | 4,490 |
|  |  |  |  | Owners’ equity | 5,680 |
|  |  |  |  |  |  |
|  | Total assets | $11,810 |  | Total liabilities and owners’ equity | $11,810 |

The owners’ equity is a plug variable. We know that total assets must equal total liabilities and owners’ equity. Total liabilities and owners’ equity is the sum of all debt and equity, so if we subtract debt from total liabilities and owners’ equity, the remainder must be the equity balance, so:

Owners’ equity = Total liabilities and owners’ equity – Current liabilities – Long-term debt

Owners’ equity = $11,810 – 1,640 – 4,490

Owners’ equity = $5,680

Net working capital is current assets minus current liabilities, so:

NWC = Current assets – Current liabilities

NWC = $2,030 – 1,640

NWC = $390

**2.** The income statement starts with revenues and subtracts costs to arrive at EBIT. We then subtract out interest to get taxable income, and then subtract taxes to arrive at net income. Doing so, we get:

Income Statement

Sales $634,000

Costs 328,000

Depreciation 73,000

EBIT $233,000

Interest 38,000

Taxable income $195,000

Taxes 68,250

Net income $126,750

**3.** The dividends paid plus the addition to retained earnings must equal net income, so:

Net income = Dividends + Addition to retained earnings

Addition to retained earnings = $126,750 – 43,000

Addition to retained earnings = $83,750

**4.** Earnings per share is the net income divided by the shares outstanding, so:

EPS = Net income / Shares outstanding

EPS = $126,750 / 35,000

EPS = $3.62 per share

And dividends per share are the total dividends paid divided by the shares outstanding, so:

DPS = Dividends / Shares outstanding

DPS = $43,000 / 35,000

DPS = $1.23 per share

**5.** Using Table 2.3, we can see the marginal tax schedule. The first $50,000 of income is taxed at 15 percent, the next $25,000 is taxed at 25 percent, the next $25,000 is taxed at 34 percent, and the next $143,000 is taxed at 39 percent. So, the total taxes for the company will be:

Taxes = .15($50,000) + .25($25,000) + .34($25,000) + .39($243,000 – 100,000)

Taxes = $78,020

**6.** The average tax rate is the total taxes paid divided by taxable income, so:

Average tax rate = Total tax / Taxable income

Average tax rate = $78,020 / $243,000

Average tax rate = .3211, or 32.11%

The marginal tax rate is the tax rate on the next dollar of income. The company has net income of $243,000 and the 39 percent tax bracket is applicable to a net income up to $335,000, so the marginal tax rate is 39 percent.

**7.** To calculate the OCF, we first need to construct an income statement. The income statement starts with revenues and subtracts costs to arrive at EBIT. We then subtract out interest to get taxable income, and then subtract taxes to arrive at net income. Doing so, we get:

Income Statement

Sales $38,530

Costs 12,750

Depreciation 2,550

EBIT $23,230

Interest 1,850

Taxable income $21,380

Taxes (35%) 7,483

Net income $13.897

Now we can calculate the OCF, which is:

OCF = EBIT + Depreciation – Taxes

OCF = $23,230 + 2,550 – 7,483

OCF = $18,297

**8.** Net capital spending is the increase in fixed assets, plus depreciation. Using this relationship, we find:

Net capital spending = NFAend – NFAbeg + Depreciation

Net capital spending = $2,134,000 – 1,975,000 + 325,000

Net capital spending = $484,000

**9.** The change in net working capital is the end of period net working capital minus the beginning of period net working capital, so:

Change in NWC = NWCend – NWCbeg

Change in NWC = (CAend – CLend) – (CAbeg – CLbeg)

Change in NWC = ($1,685 – 1,305) – (1,530 – 1,270)

Change in NWC = $120

**10.** The cash flow to creditors is the interest paid, minus any net new borrowing, so:

Cash flow to creditors = Interest paid – Net new borrowing

Cash flow to creditors = Interest paid – (LTDend – LTDbeg)

Cash flow to creditors = $102,800 – ($1,551,000 – 1,410,000)

Cash flow to creditors = –$38,200

**11.** The cash flow to stockholders is the dividends paid minus any new equity raised. So, the cash flow to stockholders is: (Note that APIS is the additional paid-in surplus.)

Cash flow to stockholders = Dividends paid – Net new equity

Cash flow to stockholders = Dividends paid – [(Commonend + APISend) – (Commonbeg + APISbeg)]

Cash flow to stockholders = $148,500 – [($148,000 + 2,618,000) – ($130,000 + 2,332,000)]

Cash flow to stockholders = –$155,500

**12.** We know that cash flow from assets is equal to cash flow to creditors plus cash flow to stockholders. So, cash flow from assets is:

Cash flow from assets = Cash flow to creditors + Cash flow to stockholders

Cash flow from assets = –$38,200 – 155,500

Cash flow from assets = –$193,700

We also know that cash flow from assets is equal to the operating cash flow minus the change in net working capital and the net capital spending. We can use this relationship to find the operating cash flow. Doing so, we find:

Cash flow from assets = OCF – Change in NWC – Net capital spending

–$193,700 = OCF – (–$115,000) – (705,000)

OCF = –$193,700 – 115,000 + 705,000

OCF = $396,300

*Intermediate*

**13.** To find the book value of current assets, we use: NWC = CA – CL. Rearranging to solve for current assets, we get:

CA = NWC + CL = $220,000 + 850,000 = $1,070,000

The market value of current assets and fixed assets is given, so:

Book value CA = $1,070,000 NWC = $1,050,000

Book value NFA = $3,300,000 Market value NFA = $4,800,000

Book value assets = $4,370,000 Total = $5,850,000

**14.** *a.* To calculate the OCF, we first need to construct an income statement. The income statement starts with revenues and subtracts costs to arrive at EBIT. We then subtract out interest to get taxable income, and then subtract taxes to arrive at net income. Doing so, we get:

Income Statement

Sales $173,000

Costs 91,400

Other Expenses 5,100

Depreciation 12,100

EBIT $64,400

Interest 8,900

Taxable income $55,500

Taxes 21,090

Net income $34,410

Dividends $9,700

Addition to retained earnings 24,710

Dividends paid plus addition to retained earnings must equal net income, so:

Net income = Dividends + Addition to retained earnings

Addition to retained earnings = $34,410 – 9,700

Addition to retained earnings = $24,710

So, the operating cash flow is:

OCF = EBIT + Depreciation – Taxes

OCF = $64,400 + 12,100 – 21,090

OCF = $55,410

*b.* The cash flow to creditors is the interest paid, minus any new borrowing. Since the company redeemed long-term debt, the net new borrowing is negative. So, the cash flow to creditors is:

Cash flow to creditors = Interest paid – Net new borrowing

Cash flow to creditors = $8,900 – (–$4,000)

Cash flow to creditors = $12,900

*c.* The cash flow to stockholders is the dividends paid minus any new equity. So, the cash flow to stockholders is:

Cash flow to stockholders = Dividends paid – Net new equity

Cash flow to stockholders = $9,700 – 2,900

Cash flow to stockholders = $6,800

*d.* In this case, to find the addition to NWC, we need to find the cash flow from assets. We can then use the cash flow from assets equation to find the change in NWC. We know that cash flow from assets is equal to cash flow to creditors plus cash flow to stockholders. So, cash flow from assets is:

Cash flow from assets = Cash flow to creditors + Cash flow to stockholders

Cash flow from assets = $12,900 + 6,800

Cash flow from assets = $19,700

Net capital spending is equal to depreciation plus the increase in fixed assets, so:

Net capital spending = Depreciation + Increase in fixed assets

Net capital spending = $12,100 + 23,140

Net capital spending = $35,240

Now we can use the cash flow from assets equation to find the change in NWC. Doing so, we find:

Cash flow from assets = OCF – Change in NWC – Net capital spending

$19,700 = $55,410 – Change in NWC – $35,240

Change in NWC = $470

**15.** Here we need to work the income statement backward. Starting with net income, we know that net income is:

Net income = Dividends + Addition to retained earnings

Net income = $2,170 + 3,500

Net income = $5,670

Net income is also the taxable income, minus the taxable income times the tax rate, or:

Net income = Taxable income – (Taxable income)(Tax rate)

Net income = Taxable income(1 – Tax rate)

We can rearrange this equation and solve for the taxable income as:

Taxable income = Net income / (1 – Tax rate)

Taxable income = $5,670 / (1 – .40)

Taxable income = $9,450

EBIT minus interest equals taxable income, so rearranging this relationship, we find:

EBIT = Taxable income + Interest

EBIT = $9,450 + 1,980

EBIT = $11,430

Now that we have the EBIT, we know that sales minus costs minus depreciation equals EBIT. Solving this equation for EBIT, we find:

EBIT = Sales – Costs – Depreciation

$11,430 = $67,000 – 49,200 – Depreciation

Depreciation = $6,370

**16.** We can fill in the balance sheet with the numbers we are given. The balance sheet will be:

Balance Sheet

Cash $197,000 Accounts payable $288,000

Accounts receivable 265,000 Notes payable 194,000

Inventory 563,000 Current liabilities $482,000

Current assets $1,025,000 Long-term debt 1,490,000

Total liabilities $2,072,000

Tangible net fixed assets $5,150,000

Intangible net fixed assets 863,000 Common stock ??

Accumulated retained earnings 4,586,000

Total assets $7,038,000 Total liabilities & owners’ equity $7,038,000

Total liabilities and owners’ equity is:

TL & OE = CL + LTD + Common stock + Retained earnings

Solving for this equation for common stock gives us:

Common stock = $7,038,000 – 4,586,000 – 2,072,000

Common stock = $380,000

**17.** Owners’ equity is the maximum of total assets minus total liabilities, or zero. Although the book value of owners’ equity can be negative, the market value of owners’ equity cannot be negative, so:

Owners’ equity = Max [(TA – TL), 0]

*a.* If total assets are $9,300, the owners’ equity is:

Owners’ equity = Max[($9,300 – 8,400), 0]

Owners’ equity = $900

*b.* If total assets are $6,900, the owners’ equity is:

Owners’ equity = Max[($6,900 – 8,400), 0]

Owners’ equity = $0

**18.** *a.* Using Table 2.3, we can see the marginal tax schedule. For Corporation Growth, the first $50,000 of income is taxed at 15 percent, the next $25,000 is taxed at 25 percent, and the next $1,500 is taxed at 34 percent. So, the total taxes for the company will be:

TaxesGrowth = .15($50,000) + .25($25,000) + .34($1,500)

TaxesGrowth = $14,260

For Corporation Income, the first $50,000 of income is taxed at 15 percent, the next $25,000 is taxed at 25 percent, the next $25,000 is taxed at 34 percent, the next $235,000 is taxed at 39 percent, and the next $7,315,000 is taxed at 34 percent. So, the total taxes for the company will be:

TaxesIncome = .15($50,000) + .25($25,000) + .34($25,000) + .39($235,000)

+ .34($7,315,000)

TaxesIncome = $2,601,000

*b.* The marginal tax rate is the tax rate on the next $1 of earnings. Each firm has a marginal tax rate of 34% on the next $10,000 of taxable income, despite their different average tax rates, so both firms will pay an additional $3,400 in taxes.

**19.** *a.* The income statement starts with revenues and subtracts costs to arrive at EBIT. We then subtract interest to get taxable income, and then subtract taxes to arrive at net income. Doing so, we get:

Income Statement

Sales $2,350,000

Cost of goods sold 1,925,000

Admin expenses 530,000

Depreciation 420,000

EBIT $ 105,000

Interest 245,000

Taxable income –$140,000

Taxes (35%) 0

Net income –$140,000

The taxes are zero since we are ignoring any carryback or carryforward provisions.

*b.* The operating cash flow for the year was:

OCF = EBIT + Depreciation – Taxes

OCF = $105,000 + 420,000 – 0

OCF = $525,000

*c.* Net income was negative because of the tax deductibility of depreciation and interest expense. However, the actual cash flow from operations was positive because depreciation is a non-cash expense and interest is a financing, not an operating, expense.

**20.** A firm can still pay out dividends if net income is negative; it just has to be sure there is sufficient cash flow to make the dividend payments. The assumptions made in the question are:

Change in NWC = Net capital spending = Net new equity = 0

To find the new long-term debt, we first need to find the cash flow from assets. The cash flow from assets is:

Cash flow from assets = OCF – Change in NWC – Net capital spending

Cash flow from assets = $525,000 – 0 – 0

Cash flow from assets = $525,000

We can also find the cash flow to stockholders, which is:

Cash flow to stockholders = Dividends – Net new equity

Cash flow to stockholders = $395,000 – 0

Cash flow to stockholders = $395,000

Now we can use the cash flow from assets equation to find the cash flow to creditors. Doing so, we get:

Cash flow from assets = Cash flow to creditors + Cash flow to stockholders

$525,000 = Cash flow to creditors + $395,000

Cash flow to creditors = $130,000

Now we can use the cash flow to creditors equation to find:

Cash flow to creditors = Interest – Net new long-term debt

$130,000 = $245,000 – Net new long-term debt

Net new long-term debt = $115,000

**21.** *a.* To calculate the OCF, we first need to construct an income statement. The income statement starts with revenues and subtracts costs to arrive at EBIT. We then subtract out interest to get taxable income, and then subtract taxes to arrive at net income. Doing so, we get:

Income Statement

Sales $28,476

Cost of goods sold 20,136

Depreciation 3,408

EBIT $ 4,932

Interest 497

Taxable income $ 4,435

Taxes (40%) 1,774

Net income $ 2,661

*b.* The operating cash flow for the year was:

OCF = EBIT + Depreciation – Taxes

OCF = $4,932 + 3,408 – 1,774

OCF = $6,566

*c.* To calculate the cash flow from assets, we also need the change in net working capital and net capital spending. The change in net working capital was:

Change in NWC = NWCend – NWCbeg

Change in NWC = (CAend – CLend) – (CAbeg – CLbeg)

Change in NWC = ($4,234 – 2,981) – ($3,528 – 3,110)

Change in NWC = $835

And the net capital spending was:

Net capital spending = NFAend – NFAbeg + Depreciation

Net capital spending = $22,608 – 19,872 + 3,408

Net capital spending = $6,144

So, the cash flow from assets was:

Cash flow from assets = OCF – Change in NWC – Net capital spending

Cash flow from assets = $6,566 – 835 – 6,144

Cash flow from assets = –$413

The cash flow from assets can be positive or negative, since it represents whether the firm raised funds or distributed funds on a net basis. In this problem, even though net income and OCF are positive, the firm invested heavily in fixed assets and net working capital; it had to raise a net $413 in funds from its stockholders and creditors to make these investments.

*d.* The cash flow to creditors was:

Cash flow to creditors = Interest – Net new LTD

Cash flow to creditors = $497 – 0

Cash flow to creditors = $497

Rearranging the cash flow from assets equation, we can calculate the cash flow to stockholders as:

Cash flow from assets = Cash flow to stockholders + Cash flow to creditors

–$413 = Cash flow to stockholders + $497

Cash flow to stockholders = –$910

Now we can use the cash flow to stockholders equation to find the net new equity as:

Cash flow to stockholders = Dividends – Net new equity

–$910 = $739 – Net new equity

Net new equity = $1,649

The firm had positive earnings in an accounting sense (NI > 0) and had positive cash flow from operations. The firm invested $835 in new net working capital and $6,144 in new fixed assets. The firm had to raise $413 from its stakeholders to support this new investment. It accomplished this by raising $1,649 in the form of new equity. After paying out $739 in the form of dividends to shareholders and $497 in the form of interest to creditors, $413 was left to just meet the firm’s cash flow needs for investment.

**22.** *a.* To calculate owners’ equity, we first need total liabilities and owners’ equity. From the balance sheet relationship we know that this is equal to total assets. We are given the necessary information to calculate total assets. Total assets are current assets plus fixed assets, so:

Total assets = Current assets + Fixed assets = Total liabilities and owners’ equity

For 2015, we get:

Total assets = $2,718 + 12,602

Total assets = $15,320

Now, we can solve for owners’ equity as:

Total liabilities and owners’ equity = Current liabilities + Long-term debt + Owners’ equity

$15,320 = $1,174 + 6,873 + Owners’ equity

Owners’ equity = $7,273

For 2016, we get:

Total assets = $2,881 + 13,175

Total assets = $16,056

Now we can solve for owners’ equity as:

Total liabilities and owners’ equity = Current liabilities + Long-term debt + Owners’ equity

$16,056 = $1,726 + 8,019 + Owners’ equity

Owners’ equity = $6,311

*b.* The change in net working capital was:

Change in NWC = NWCend – NWCbeg

Change in NWC = (CAend – CLend) – (CAbeg – CLbeg)

Change in NWC = ($2,881 – 1,726) – ($2,718 – 1,174)

Change in NWC = –$389

*c.* To find the amount of fixed assets the company sold, we need to find the net capital spending. The net capital spending was:

Net capital spending = NFAend – NFAbeg + Depreciation

Net capital spending = $13,175 – 12,602 + 3,434

Net capital spending = $4,007

To find the fixed assets sold, we can also calculate net capital spending as:

Net capital spending = Fixed assets bought – Fixed assets sold

$4,007 = $7,160 – Fixed assets sold

Fixed assets sold = $3,153

To calculate the cash flow from assets, we first need to calculate the operating cash flow. For the operating cash flow, we need the income statement. So, the income statement for the year is:

Income Statement

Sales $40,664

Costs 20,393

Depreciation 3,434

EBIT $16,837

Interest 638

Taxable income $16,199

Taxes (40%) 6,480

Net income $ 9,719

Now we can calculate the operating cash flow, which is:

OCF = EBIT + Depreciation – Taxes

OCF = $16,837 + 3,434 – 6,480

OCF = $13,791

And the cash flow from assets is:

Cash flow from assets = OCF – Change in NWC – Net capital spending.

Cash flow from assets = $13,791 – (–$389) – 4,007

Cash flow from assets = $10,173

*d.* To find the cash flow to creditors, we first need to find the net new borrowing. The net new borrowing is the difference between the ending long-term debt and the beginning long-term debt, so:

Net new borrowing = LTDEnding – LTDBeginnning

Net new borrowing = $8,019 – 6,873

Net new borrowing = $1,146

So, the cash flow to creditors is:

Cash flow to creditors = Interest – Net new borrowing

Cash flow to creditors = $638 – 1,146

Cash flow to creditors = –$508

The net new borrowing is also the difference between the debt issued and the debt retired. We know the amount the company issued during the year, so we can find the amount the company retired. The amount of debt retired was:

Net new borrowing = Debt issued – Debt retired

$1,146 = $2,155 – Debt retired

Debt retired = $1,009

**23.** To construct the cash flow identity, we will begin with cash flow from assets. Cash flow from assets is:

Cash flow from assets = OCF – Change in NWC – Net capital spending

So, the operating cash flow is:

OCF = EBIT + Depreciation – Taxes

OCF = $103,562 + 69,038 – 27,703

OCF = $144,897

Next, we will calculate the change in net working capital, which is:

Change in NWC = NWCend – NWCbeg

Change in NWC = (CAend – CLend) – (CAbeg – CLbeg)

Change in NWC = ($73,571 – 34,127) – ($58,325 – 30,352)

Change in NWC = $11,471

Now, we can calculate the capital spending. The capital spending is:

Net capital spending = NFAend – NFAbeg + Depreciation

Net capital spending = $513,980 – 435,670 + 69,038

Net capital spending = $147,348

Now, we have the cash flow from assets, which is:

Cash flow from assets = OCF – Change in NWC – Net capital spending

Cash flow from assets = $144,897 – 11,471 – 147,348

Cash flow from assets = –$13,922

The company’s assets generated an outflow of $13,922. The cash flow from operations was $144,897, and the company spent $11,471 on net working capital and $147,348 on fixed assets.

The cash flow to creditors is:

Cash flow to creditors = Interest paid – New long-term debt

Cash flow to creditors = Interest paid – (Long-term debtend – Long-term debtbeg)

Cash flow to creditors = $24,410 – ($192,300 – 173,100)

Cash flow to creditors = $5,210

The cash flow to stockholders is a little trickier in this problem. First, we need to calculate the new equity sold. The equity balance increased during the year. The only way to increase the equity balance is retained earnings or sell equity. To calculate the new equity sold, we can use the following equation:

New equity = Ending equity – Beginning equity – Addition to retained earnings

New equity = $361,124 – 290,543 – 35,249

New equity = $35,332

What happened was the equity account increased by $70,581. Of this increase, $35,249 came from addition to retained earnings, so the remainder must have been the sale of new equity. Now we can calculate the cash flow to stockholders as:

Cash flow to stockholders = Dividends paid – Net new equity

Cash flow to stockholders = $16,200 – 35,332

Cash flow to stockholders = –$19,132

The company paid $5,210 to creditors and raised $19,132 from stockholders.

Finally, the cash flow identity is:

Cash flow from assets = Cash flow to creditors + Cash flow to stockholders

–$13,922 = $5,210 + –$19,132

The cash flow identity balances, which is what we expect.

*Challenge*

**24.** Net capital spending = NFAend – NFAbeg + Depreciation

= (NFAend – NFAbeg) + (Depreciation + ADbeg) – ADbeg

= (NFAend – NFAbeg)+ ADend – ADbeg

= (NFAend + ADend) – (NFAbeg + ADbeg)

= FAend – FAbeg

**25.** *a.* The tax bubble causes average tax rates to catch up to marginal tax rates, thus eliminating the tax advantage of low marginal rates for high-income corporations.

*b.* Taxes = .15($50K) + .25($25K) + .34($25K) + .39($235K) = $113.9K

Average tax rate = $113.9K / $335K = 34%

The marginal tax rate on the next dollar of income is 34 percent.

For corporate taxable income levels of $335K to $10M, average tax rates are equal to marginal tax rates.

Taxes = .34($10M) + .35($5M) + .38($3.333M) = $6,416,667

Average tax rate = $6,416,667 / $18,333,334 = 35%

The marginal tax rate on the next dollar of income is 35 percent. For corporate taxable income levels over $18,333,334, average tax rates are again equal to marginal tax rates.

*c.* At the end of the “tax bubble”, the marginal tax rate on the next dollar should equal the average tax rate on all preceding dollars. Since the upper threshold of the bubble bracket is now $200,000, the marginal tax rate on dollar $200,001 should be 34 percent, and the total tax paid on the first $200,000 should be $200,000(.34). So, we get:

Taxes = .34($200K) = $68K = .15($50K) + .25($25K) + .34($25K) + X($100K)

X($100K) = $68K – 22.25K = $45.75K

X = $45.75K / $100K

X = 45.75%

**CHAPTER 3**

***WORKING WITH FINANCIAL STATEMENTS***

**Answers to Concepts Review and Critical Thinking Questions**

**1.** *a.* If inventory is purchased with cash, then there is no change in the current ratio. If inventory is purchased on credit, then there is a decrease in the current ratio if it was initially greater than 1.0.

*b.* Reducing accounts payable with cash increases the current ratio if it was initially greater than 1.0.

*c.* Reducing short-term debt with cash increases the current ratio if it was initially greater than 1.0.

*d.* As long-term debt approaches maturity, the principal repayment and the remaining interest expense become current liabilities. Thus, if debt is paid off with cash, the current ratio increases if it was initially greater than 1.0. If the debt has not yet become a current liability, then paying it off will reduce the current ratio since current liabilities are not affected.

*e.* Reduction of accounts receivables and an increase in cash leaves the current ratio unchanged.

*f.* Inventory sold at cost reduces inventory and raises cash, so the current ratio is unchanged.

*g.*  Inventory sold for a profit raises cash in excess of the inventory recorded at cost, so the current ratio increases.

**2.** The firm has increased inventory relative to other current assets; therefore, assuming current liability levels remain mostly unchanged, liquidity has potentially decreased.

**3.** A current ratio of .50 means that the firm has twice as much in current liabilities as it does in current assets; the firm potentially has poor liquidity. If pressed by its short-term creditors and suppliers for immediate payment, the firm might have a difficult time meeting its obligations. A current ratio of 1.50 means the firm has 50% more current assets than it does current liabilities. This probably represents an improvement in liquidity; short-term obligations can generally be met completely with a safety factor built in. A current ratio of 15.0, however, might be excessive. Any excess funds sitting in current assets generally earn little or no return. These excess funds might be put to better use by investing in productive long-term assets or distributing the funds to shareholders.

**4.**  *a.* Quick ratio provides a measure of the short-term liquidity of the firm, after removing the effects of inventory, generally the least liquid of the firm’s current assets.

*b.* Cash ratio represents the ability of the firm to completely pay off its current liabilities balance with its most liquid asset (cash).

*c.* The capital intensity ratio tells us the dollar amount investment in assets needed to generate one dollar in sales.

*d.* Total asset turnover measures how much in sales is generated by each dollar of firm assets.

*e.* Equity multiplier represents the degree of leverage for an equity investor of the firm; it measures the dollar worth of firm assets each equity dollar has a claim to.

*f.* Times interest earned ratio provides a relative measure of how well the firm’s operating earnings can cover current interest obligations.

*g.* Profit margin is the accounting measure of bottom-line profit per dollar of sales.

*h.* Return on assets is a measure of bottom-line profit per dollar of total assets.

i*.* Return on equity is a measure of bottom-line profit per dollar of equity.

*j.* Price–earnings ratio reflects how much value per share the market places on a dollar of accounting earnings for a firm.

**5.** Common size financial statements express all balance sheet accounts as a percentage of total assets and all income statement accounts as a percentage of total sales. Using these percentage values rather than nominal dollar values facilitates comparisons between firms of different size or business type.

**6.** Peer group analysis involves comparing the financial ratios and operating performance of a particular firm to a set of peer group firms in the same industry or line of business. Comparing a firm to its peers allows the financial manager to evaluate whether some aspects of the firm’s operations, finances, or investment activities are out of line with the norm, thereby providing some guidance on appropriate actions to take to adjust these ratios, if appropriate. An aspirant group would be a set of firms whose performance the company in question would like to emulate. The financial manager often uses the financial ratios of aspirant groups as the target ratios for his or her firm; some managers are evaluated by how well they match the performance of an identified aspirant group.

**7.** Return on equity is probably the most important accounting ratio that measures the bottom-line performance of the firm with respect to the equity shareholders. The Du Pont identity emphasizes the role of a firm’s profitability, asset utilization efficiency, and financial leverage in achieving a ROE figure. For example, a firm with ROE of 20% would seem to be doing well, but this figure may be misleading if it were a marginally profitable (low profit margin) and highly levered (high equity multiplier). If the firm’s margins were to erode slightly, the ROE would be heavily impacted.

**8.** The book-to-bill ratio is intended to measure whether demand is growing or falling. It is closely followed because it is a barometer for the entire high-tech industry where levels of revenues and earnings have been relatively volatile.

**9.** If a company is growing by opening new stores, then presumably total revenues would be rising. Comparing total sales at two different points in time might be misleading. Same-store sales control for this by only looking at revenues of stores open within a specific period.

**10.** *a.* For an electric utility such as Con Ed, expressing costs on a per kilowatt-hour basis would be a way of comparing costs with other utilities of different sizes.

*b.* For a retailer such as JC Penney, expressing sales on a per square foot basis would be useful in comparing revenue production against other retailers.

*c.* For an airline such as Delta, expressing costs on a per passenger mile basis allows for comparisons with other airlines by examining how much it costs to fly one passenger one mile.

*d.* For an on-line service such as Google or Yahoo!, using a per web hit basis for costs would allow for comparisons with similar services.

*e.* For a hospital such as Holy Cross, revenues and costs expressed on a per bed basis would be useful.

*f.* For a college textbook publisher such as McGraw-Hill Higher Education, the leading publisher of finance textbooks for the college market, the obvious standardization would be per book sold.

**11.** As with any ratio analysis, the ratios themselves do not necessarily indicate a problem, but simply indicate that something is different and it is up to us to determine if a problem exists. If the cost of goods sold as a percentage of sales is increasing, we would expect that EBIT as a percentage of sales would decrease, all else constant. An increase in the cost of goods sold as a percentage of sales occurs because the cost of raw materials or other inventory is increasing at a faster rate than the sales price.

This may be a bad sign since the contribution of each sales dollar to net income and cash flow is lower. However, when a new product, for example, the HDTV, enters the market, the price of one unit will often be high relative to the cost of goods sold per unit, and demand, therefore sales, initially small. As the product market becomes more developed, price of the product generally drops, and sales increase as more competition enters the market. In this case, the increase in cost of goods sold as a percentage of sales is to be expected. The maker or seller expects to boost sales at a faster rate than its cost of goods sold increases. In this case, a good practice would be to examine the common-size income statements to see if this is an industry-wide occurrence.

**12.** If we assume that the cause is negative, the two reasons for the trend of increasing cost of goods sold as a percentage of sales are that costs are becoming too high or the sales price is not increasing fast enough. If the cause is an increase in the cost of goods sold, the manager should look at possible actions to control costs. If costs can be lowered by seeking lower cost suppliers of similar or higher quality, the cost of goods sold as a percentage of sales should decrease. Another alternative is to increase the sales price to cover the increase in the cost of goods sold. Depending on the industry, this may be difficult or impossible. For example, if the company sells most of its products under a long-term contract that has a fixed price, it may not be able to increase the sales price and will be forced to look for other cost-cutting possibilities. Additionally, if the market is competitive, the company might also be unable to increase the sales price.

**Solutions to Questions and Problems**

*NOTE: All end-of-chapter problems were solved using a spreadsheet. Many problems require multiple steps. Due to space and readability constraints, when these intermediate steps are included in this solutions manual, rounding may appear to have occurred. However, the final answer for each problem is found without rounding during any step in the problem.*

*Basic*

**1.** To find the current assets, we must use the net working capital equation. Doing so, we find:

NWC = Current assets – Current liabilities

$1,965 = Current assets – $5,460

Current assets = $7,425

Now, use this number to calculate the current ratio and the quick ratio. The current ratio is:

Current ratio = Current assets / Current liabilities

Current ratio = $7,425 / $5,460

Current ratio = 1.36 times

And the quick ratio is:

Quick ratio = (Current assets – Inventory) / Current liabilities

Quick ratio = ($7,425 – 2,170) / $5,460

Quick ratio = .96 times

**2.** To find the return on assets and return on equity, we need net income. We can calculate the net income using the profit margin. Doing so, we find the net income is:

Profit margin = Net income / Sales

.07 = Net income / $13,500,000

Net income = $945,000

Now we can calculate the return on assets as:

ROA = Net income / Total assets

ROA = $945,000 / $8,700,000

ROA = .1086, or 10.86%

We do not have the equity for the company, but we know that equity must be equal to total assets minus total debt, so the ROE is:

ROE = Net income / (Total assets – Total debt)

ROE = $945,000 / ($8,700,000 – 4,100,000)

ROE = .2054, or 20.54%

**3.** The receivables turnover for the company was:

Receivables turnover = Credit sales / Receivables

Receivables turnover = $6,787,626 / $583,174

Receivables turnover = 11.64 times

Using the receivables turnover, we can calculate the days’ sales in receivables as:

Days’ sales in receivables = 365 days / Receivables turnover

Days’ sales in receivables = 365 days / 11.64

Days’ sales in receivables = 31.36 days

The average collection period, which is the same as the days’ sales in receivables, was 31.36 days.

**4.** The inventory turnover for the company was:

Inventory turnover = COGS / Inventory

Inventory turnover = $8,543,132 / $527,156

Inventory turnover = 16.21 times

Using the inventory turnover, we can calculate the days’ sales in inventory as:

Days’ sales in inventory = 365 days / Inventory turnover

Days’ sales in inventory = 365 days / 16.21

Days’ sales in inventory = 22.52 days

On average, a unit of inventory sat on the shelf 22.52 days before it was sold.

**5.** To find the debt–equity ratio using the total debt ratio, we need to rearrange the total debt ratio equation. We must realize that the total assets are equal to total debt plus total equity. Doing so, we find:

Total debt ratio = Total debt / Total assets

.19 = Total debt / (Total debt + Total equity)

.81(Total debt) = .19(Total equity)

Total debt / Total equity = .19 / .81

Debt–equity ratio = .23

And the equity multiplier is one plus the debt–equity ratio, so:

Equity multiplier = 1 + D/E

Equity multiplier = 1 + .23

Equity multiplier = 1.23

**6.** We need to calculate the net income before we calculate the earnings per share. The sum of dividends and addition to retained earnings must equal net income, so net income must have been:

Net income = Addition to retained earnings + Dividends

Net income = $534,000 + 185,000

Net income = $719,000

So, the earnings per share were:

EPS = Net income / Shares outstanding

EPS = $719,000 / 365,000

EPS = $1.97 per share

The dividends per share were:

Dividends per share = Total dividends / Shares outstanding

Dividends per share = $185,000 / 365,000

Dividends per share = $.51 per share

The book value per share was:

Book value per share = Total equity / Shares outstanding

Book value per share = $7,450,000 / 365,000

Book value per share = $20.41 per share

The market-to-book ratio is:

Market-to-book ratio = Share price / Book value per share

Market-to-book ratio = $49 / $20.41

Market-to-book ratio = 2.40 times

The PE ratio is:

PE ratio = Share price / EPS

PE ratio = $49 / $1.97

PE ratio = 24.87 times

Sales per share are:

Sales per share = Total sales / Shares outstanding

Sales per share = $15,400,000 / 365,000

Sales per share = $42.19

The P/S ratio is:

P/S ratio = Share price / Sales per share

P/S ratio = $49 / $42.19

P/S ratio = 1.16 times

**7.** With the information given, we must use the Du Pont identity to calculate return on equity. Doing so, we find:

ROE = (Profit margin)(Total asset turnover)(Equity multiplier)

ROE = (.052)(1.65)(1.83)

ROE = .1570, or 15.70%

**8.** We can use the Du Pont identity and solve for the equity multiplier. With the equity multiplier we can find the debt–equity ratio. Doing so we find:

ROE = (Profit margin)(Total asset turnover)(Equity multiplier)

.1720 = (.076)(1.73)(Equity multiplier)

Equity multiplier = 1.31

Now, using the equation for the equity multiplier, we get:

Equity multiplier = 1 + Debt–equity ratio

1.31 = 1 + Debt–equity ratio

Debt–equity ratio = .31

**9.** To find the days’ sales in payables, we first need to find the payables turnover. The payables turnover was:

Payables turnover = Cost of goods sold / Payables balance

Payables turnover = $87,386 / $19,472

Payables turnover = 4.49 times

Now, we can use the payables turnover to find the days’ sales in payables as:

Days’ sales in payables = 365 days / Payables turnover

Days’ sales in payables = 365 days / 4.49

Days’ sales in payables = 81.33 days

The company left its bills to suppliers outstanding for 81.33 days on average. A large value for this ratio could imply that either (1) the company is having liquidity problems, making it difficult to pay off its short-term obligations, or (2) that the company has successfully negotiated lenient credit terms from its suppliers.

**10.** With the information provided, we need to calculate the return on equity using an extended return on equity equation. We first need to find the equity multiplier, which is:

Equity multiplier = 1 + Debt–equity ratio

Equity multiplier = 1 + .75

Equity multiplier = 1.75

Now we can calculate the return on equity as:

ROE = (ROA)(Equity multiplier)

ROE = .069(1.75)

ROE = .1208, or 12.08%

The return on equity equation we used was an abbreviated version of the Du Pont identity. If we multiply the profit margin and total asset turnover ratios from the Du Pont identity, we get:

(Net income / Sales)(Sales / Total assets) = Net income / Total assets = ROA

With the return on equity, we can calculate the net income as:

ROE = Net income / Total equity

.1208 = Net income / $815,000

Net income = $98,411

**11.** To find the internal growth rate, we need the plowback, or retention, ratio. The plowback ratio is:

*b* = 1 – .25

*b* = .75

Now, we can use the internal growth rate equation to find:

Internal growth rate = [(ROA)(*b*)] / [1 – (ROA)(*b*)]

Internal growth rate = [.072(.75)] / [1 – .072(.75)]

Internal growth rate = .0571, or 5.71%

**12.** To find the sustainable growth rate we need the plowback, or retention, ratio. The plowback ratio is:

*b* = 1 – .20

*b* = .80

Now, we can use the sustainable growth rate equation to find:

Sustainable growth rate = [(ROE)(*b*)] / [1 – (ROE)(*b*)]

Sustainable growth rate = [.168(.80)] / [1 – .168(.80)]

Sustainable growth rate = .1553, or 15.53%

**13.** We need the return on equity to calculate the sustainable growth rate. To calculate return on equity, we need to realize that the total asset turnover is the inverse of the capital intensity ratio and the equity multiplier is one plus the debt–equity ratio. So, the return on equity is:

ROE = (Profit margin)(Total asset turnover)(Equity multiplier)

ROE = (.081)(1/.45)(1 + .55)

ROE = .2790, or 27.90%

Next we need the plowback ratio. The plowback ratio is one minus the payout ratio. We can calculate the payout ratio as the dividends divided by net income, so the plowback ratio is:

*b* = 1 – ($65,000 / $120,000)

*b* = .46

Now we can use the sustainable growth rate equation to find:

Sustainable growth rate = [(ROE)(*b*)] / [1 – (ROE)(*b*)]

Sustainable growth rate = [.2790(.46)] / [1 – .2790(.46)]

Sustainable growth rate = .1466, or 14.66%

**14.** We need the return on equity to calculate the sustainable growth rate. Using the Du Pont identity, the return on equity is:

ROE = (Profit margin)(Total asset turnover)(Equity multiplier)

ROE = (.057)(2.80)(1.47)

ROE = .2346, or 23.46%

To find the sustainable growth rate, we need the plowback, or retention, ratio. The plowback ratio is:

*b* = 1 – .55

*b* = .45

Now, we can use the sustainable growth rate equation to find:

Sustainable growth rate = [(ROE)(*b*)] / [1 – (ROE)(*b*)]

Sustainable growth rate = [.2346(.45)] / [1 – .2346(.45)]

Sustainable growth rate = .1180, or 11.80%

**15.** To calculate the common-size balance sheet, we divide each asset account by total assets, and each liability and equity account by total liabilities and equity. For example, the common-size cash percentage for 2015 is:

Cash percentage = Cash / Total assets

Cash percentage = $19,256 / $880,664

Cash percentage = .0219, or 2.19%

Repeating this procedure for each account, we get:

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  |  |  |  |  |  | 2015 |  |  | 2016 |  |
|  |  |  |  |  | Assets |  |  |  |  |  |  |
|  | Current assets | | | |  |  |  |  |  |  |  |
|  |  | Cash | |  |  |  | $19,256 | 2.19% |  | $21,946 | 2.21% |
|  |  | Accounts receivable | | | |  | 46,396 | 5.27% |  | 54,486 | 5.50% |
|  |  | Inventory | | |  |  | 109,626 | 12.45% |  | 129,253 | 13.04% |
|  |  |  | Total | |  |  | $175,278 | 19.90% |  | $205,685 | 20.76% |
|  | Fixed assets | | | |  |  |  |  |  |  |  |
|  |  | Net plant and equipment | | | |  | $705,386 | 80.10% |  | $785,205 | 79.24% |
|  | Total assets | | | |  |  | $880,664 | 100% |  | $990,890 | 100% |
|  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  | Liabilities and owners' equity | | | | |  |  |  |  |
|  | Current liabilities | | | | |  |  |  |  |  |  |
|  |  | Accounts payable | | | |  | $171,531 | 19.48% |  | $153,984 | 15.54% |
|  |  | Notes payable | | | |  | 79,218 | 9.00% |  | 107,606 | 10.86% |
|  |  |  | Total | |  |  | $250,749 | 28.47% |  | $261,590 | 26.40% |
|  | Long-term debt | | | |  |  | $255,000 | 28.96% |  | $278,500 | 28.11% |
|  | Owners' equity | | | |  |  |  |  |  |  |  |
|  |  | Common stock and paid-in surplus | | | | | $160,000 | 18.17% |  | $170,000 | 17.16% |
|  |  | Accumulated retained earnings | | | | | 214,915 | 24.40% |  | 280,800 | 28.34% |
|  |  |  | Total | |  |  | $374,915 | 42.57% |  | $450,800 | 45.49% |
|  | Total liabilities and owners' equity | | | | | | $880,664 | 100% |  | $990,890 | 100% |

**16.** *a.* The current ratio is calculated as:

Curent ratio = Current assets / Current liabilities

Current ratio2015 = $175,278 / $250,749

Current ratio2015 = .70 times

Current ratio2016 = $205,685 / $261,590

Current ratio2016 = .79 times

*b.* The quick ratio is calculated as:

Quick ratio = (Current assets – Inventory) / Current liabilities

Quick ratio2015 = ($175,278 – 109,626) / $250,749

Quick ratio2015 = .26 times

Quick ratio2016 = ($205,685 – 129,253) / $261,590

Quick ratio2016 = .29 times

*c.* The cash ratio is calculated as:

Cash ratio = Cash / Current liabilities

Cash ratio2015 = $19,256 / $250,749

Cash ratio2015 = .08 times

Cash ratio2016 = $21,946 / $261,590

Cash ratio2016 = .08 times

*d.* The debt–equity ratio is calculated as:

Debt–equity ratio = Total debt / Total equity

Debt–equity ratio = (Current liabilities + Long-term debt) / Total equity

Debt–equity ratio2015 = ($250,749 + 255,000) / $374,915

Debt–equity ratio2015 = 1.35 times

Debt–equity ratio2016 = ($261,590 + 278,500) / $450,800

Debt–equity ratio2016 = 1.20 times

And the equity multiplier is:

Equity multiplier = 1 + Debt–equity ratio

Equity multiplier2015 = 1 + 1.35

Equity multiplier2015 = 2.35 times

Equity multiplier2016 = 1 + 1.20

Equity multiplier2016 = 2.20 times

*e.* The total debt ratio is calculated as:

Total debt ratio = Total debt / Total assets

Total debt ratio = (Current liabilities + Long-term debt) / Total assets

Total debt ratio2015 = ($250,749 + 255,000) / $880,664

Total debt ratio2015 = .57 times

Total debt ratio2016 = ($261,590 + 278,500) / $990,890

Total debt ratio2016 = .55 times

**17.** Using the Du Pont identity to calculate ROE, we get:

ROE = (Profit margin)(Total asset turnover)(Equity multiplier)

ROE = (Net income / Sales)(Sales / Total assets)(Total asset / Total equity)

ROE = ($89,351 / $2,945,376)($2,945,376 / $990,890)($990,890 / $450,800)

ROE =.1982, or 19.82%

**18.** One equation to calculate ROA is:

ROA = (Profit margin)(Total asset turnover)

We can solve this equation to find total asset turnover as:

.081 = .058(Total asset turnover)

Total asset turnover = 1.40 times

Now, solve the ROE equation to find the equity multiplier, which is:

ROE = (ROA)(Equity multiplier)

.16 = .081(Equity multiplier)

Equity multiplier = 1.98 times

**19.** To calculate the ROA, we first need to find the net income. Using the profit margin equation, we find:

Profit margin = Net income / Sales

.058 = Net income / $14,500,000

Net income = $841,000

Now we can calculate ROA as:

ROA = Net income / Total assets

ROA = $841,000 / $11,200,000

ROA = .0751, or 7.51%

**20.** To calculate the internal growth rate, we need to find the ROA and the plowback ratio. The ROA for the company is:

ROA = Net income / Total assets

ROA = $15,112 / $120,400

ROA = .1255, or 12.55%

And the plowback ratio is:

*b* = 1 – .40

*b* = .60

Now, we can use the internal growth rate equation to find:

Internal growth rate = [(ROA)(*b*)] / [1 – (ROA)(*b*)]

Internal growth rate = [.1255(.60)] / [1 – .1255(.60)]

Internal growth rate = .0814, or 8.14%

**21.** To calculate the sustainable growth rate, we need to find the ROE and the plowback ratio. The ROE for the company is:

ROE = Net income / Equity

ROE = $15,112 / $82,000

ROE = .1843, or 18.43%

Using the plowback ratio we calculated in the previous problem, we find the sustainable growth rate is:

Sustainable growth rate = [(ROE)(*b*)] / [1 – (ROE)(*b*)]

Sustainable growth rate = [(.1843)(.60)] / [1 – (.1843)(.60)]

Sustainable growth rate = .1243, or 12.43%

**22.** The total asset turnover is:

Total asset turnover = Sales / Total assets

Total asset turnover = $13,800,000 / $7,100,000

Total asset turnover = 1.94 times

If the target total asset turnover is 2.75 times, we can use the total asset turnover equation to solve for the necessary sales level. The new sales level will be:

Total asset turnover = Sales / Total assets

2.75 = Sales / $7,100,000

Sales = $19,525,000

**23.** To find the ROE, we need the equity balance. Since we have the total debt, if we can find the total assets we can calculate the equity. Using the total debt ratio, we find total assets as:

Debt ratio = Total debt / Total assets

.75 = $353,000 / Total assets

Total assets = $470,667

Total liabilities and equity is equal to total assets. Using this relationship, we find:

Total liabilities and equity = Total debt + Total equity

$470,667 = $353,000 + Total equity

Total equity = $117,667

Now, we can calculate the ROE as:

ROE = Net income / Total equity

ROE = $18,750 / $117,667

ROE = .1593, or 15.93%

**24.** The earnings per share are:

EPS = Net income / Shares

EPS = $8,400,000 / 4,600,000

EPS = $1.83

The price–earnings ratio is:

PE = Price / EPS

PE = $58 / $1.83

PE = 31.76 times

The sales per share are:

Sales per share = Sales / Shares

Sales per share = $52,800,000 / 4,600,000

Sales per share = $11.48

The price–sales ratio is:

P/S = Price / Sales per share

P/S = $58 / $11.48

P/S = 5.05 times

The book value per share is:

Book value per share = Book value of equity / Shares

Book value per share = $25,300,000 / 4,600,000

Book value per share = $5.50 per share

And the market-to-book ratio is:

Market-to-book = Market value per share / Book value per share

Market-to-book = $58 / $5.50

Market-to-book = 10.55 times

**25.** To find the profit margin, we need the net income and sales. We can use the total asset turnover to find the sales and the return on assets to find the net income. Beginning with the total asset turnover, we find sales are:

Total asset turnover = Sales / Total assets

1.80 = Sales / $7,450,000

Sales = $13,410,000

And the net income is:

ROA = Net income / Total assets

.0825 = Net income / $7,450,000

Net income = $614,625

Now we can find the profit margin, which is:

Profit margin = Net income / Sales

Profit margin = $614,625 / $13,410,000

Profit margin = .0458, or 4.58%

**26.** First, we need the enterprise value, which is:

Enterprise value = Market capitalization + Debt – Cash

Enterprise value = $635,000 + 215,000 – 39,000

Enterprise value = $811,000

And EBITDA is:

EBITDA = EBIT + Depreciation & Amortization

EBITDA = $96,400 + 168,000

EBITDA = $264,400

So, the enterprise value–EBITDA multiple is:

Enterprise value–EBITDA multiple = $811,000 / $264,400

Enterprise value–EBITDA multiple = 3.07 times

*Intermediate*

**27.** We can rearrange the Du Pont identity to calculate the profit margin. So, we need the equity multiplier and the total asset turnover. The equity multiplier is:

Equity multiplier = 1 + Debt–equity ratio

Equity multiplier = 1 + .25

Equity multiplier = 1.25

And the total asset turnover is:

Total asset turnover = Sales / Total assets

Total asset turnover = $10,570 / $4,670

Total asset turnover = 2.26 times

Now, we can use the Du Pont identity to find total sales as:

ROE = (Profit margin)(Total asset turnover)(Equity multiplier)

.15 = (PM)(2.26)(1.25)

Profit margin = .0530, or 5.30%

Rearranging the profit margin ratio, we can find the net income, which is:

Profit margin = Net income / Sales

.0530 = Net income / $10,570

Net income = $560.40

**28.** This is a multi-step problem in which we need to calculate several ratios to find the fixed assets. If we know total assets and current assets, we can calculate the fixed assets. Using the current ratio to find the current assets, we get:

Current ratio = Current assets / Current liabilities

1.30 = Current assets / $2,435

Current assets = $3,165.50

Now, we are going to use the profit margin to find the net income and use the net income to find the equity. Doing so, we get:

Profit margin = Net income / Sales

.09 = Net income / $11,610

Net income = $1,044.90

And using this net income figure in the return on equity equation to find the equity, we get:

ROE = Net income / Total equity

.128 = $1,044.90 / Total equity

Total equity = $8,163.28

Now, we can use the long-term debt ratio to find the total long-term debt. The equation is:

Long-term debt ratio = Long-term debt / (Long-term debt + Total equity)

Inverting both sides we get:

1 / Long-term debt ratio = 1 + (Total equity / Long-term debt)

1 / .45 = 1 + (Total equity / Long-term debt)

Total equity / Long-term debt = 1.222

$8,163.28 / Long-term debt = 1.222

Long-term debt = $6,679.05

Now, we can calculate the total debt as:

Total debt = Current liabilities + Long-term debt

Total debt = $2,435 + 6,679.05

Total debt = $9,114.05

This allows us to calculate the total assets as:

Total assets = Total debt + Total equity

Total assets = $9,114.05 + 8,163.28

Total assets = $17,277.33

Finally, we can calculate the net fixed assets as:

Net fixed assets = Total assets – Current assets

Net fixed assets = $17,277.33 – 3,165.50

Net fixed assets = $14,111.83

**29.** The child’s profit margin is:

Profit margin = Net income / Sales

Profit margin = $2 / $50

Profit margin = .04, or 4%

And the store’s profit margin is:

Profit margin = Net income / Sales

Profit margin = $15,800,000 / $790,000,000

Profit margin = .02, or 2%

The advertisement is referring to the store’s profit margin, but a more appropriate earnings measure for the firm’s owners is the return on equity. The store’s return on equity is:

ROE = Net income / Total equity

ROE = Net income / (Total assets – Total debt)

ROE = $15,800,000 / ($365,000,000 – 229,800,000)

ROE = .1169, or 11.69%

**30.** To calculate the profit margin, we first need to calculate the sales. Using the days’ sales in receivables, we find the receivables turnover is:

Days’ sales in receivables = 365 days / Receivables turnover

29.38 days = 365 days / Receivables turnover

Receivables turnover = 12.42 times

Now, we can use the receivables turnover to calculate the sales as:

Receivables turnover = Sales / Receivables

12.42 = Sales / $145,350

Sales = $1,805,744

So, the profit margin is:

Profit margin = Net income / Sales

Profit margin = $162,840 / $1,805,744

Profit margin = .0902, or 9.02%

The total asset turnover is:

Total asset turnover = Sales / Total assets

Total asset turnover = $1,805,744 / $794,350

Total asset turnover = 2.27 times

We need to use the Du Pont identity to calculate the return on equity. Using this relationship, we get:

ROE = (Profit margin)(Total asset turnover)(1 + Debt–equity ratio)

ROE = (.0902)(2.27)(1 + .25)

ROE = .2562, or 25.62%

**31.** Here, we need to work the income statement backward to find the EBIT. Starting at the bottom of the income statement, we know that the taxes are the taxable income times the tax rate. The net income is the taxable income minus taxes. Rearranging this equation, we get:

Net income = Taxable income – (TC)(Taxable income)

Net income = (1 – TC)(Taxable income)

Using this relationship we find the taxable income is:

Net income = (1 – TC)(Taxable income)

$8,417 = (1 – .34)(Taxable income)

Taxable income = $12,753.03

Now, we can calculate the EBIT as:

Taxable income = EBIT – Interest

$12,753.03 = EBIT – $4,632

EBIT = $17,385.03

So, the cash coverage ratio is:

Cash coverage ratio = (EBIT + Depreciation expense) / Interest

Cash coverage ratio = ($17,385.03 + 5,105) / $4,632

Cash coverage ratio = 4.86 times

**32.** To find the times interest earned, we need the EBIT and interest expense. EBIT is sales minus costs minus depreciation, so:

EBIT = Sales – Costs – Depreciation

EBIT = $534,000 – 241,680 – 60,400

EBIT = $231,920

Now, we need the interest expense. We know the EBIT, so if we find the taxable income (EBT), the difference between these two is the interest expense. To find EBT, we must work backward through the income statement. We need total dividends paid. We can use the dividends per share equation to find the total dividends. Doing so, we find:

DPS = Dividends / Shares

$1.35 = Dividends / 20,000

Dividends = $27,000

Net income is the sum of dividends and addition to retained earnings, so:

Net income = Dividends + Addition to retained earnings

Net income = $27,000 + 72,800

Net income = $99,800

We know that the taxes are the taxable income times the tax rate. The net income is the taxable income minus taxes. Rearranging this equation, we get:

Net income = Taxable income – (TC)(EBT)

Net income = (1 – TC)(EBT)

$99,800 = (1 – .34)(EBT)

EBT = $151,212

Now, we can use the income statement relationship:

EBT = EBIT – Interest

$151,212 = $231,920 – Interest

Interest = $80,708

So, the times interest earned ratio is:

Times interest earned = EBIT / Interest

Times interest earned = $231,920 / $80,708

Times interest earned = 2.87 times

**33.** To find the return on equity, we need the net income and total equity. We can use the total debt ratio to find the total assets as:

Total debt ratio = Total debt / Total assets

.37 = $673,000 / Total assets

Total assets = $1,818,919

Using the balance sheet relationship that total assets is equal to total liabilities and equity, we find the total equity is:

Total assets = Total debt + Equity

$1,818,919 = $673,000 + Equity

Equity = $1,145,919

We have the return on equity and the equity. We can use the return on equity equation to find net income is:

ROE = Net income / Equity

.1290 = Net income / $1,145,919

Net income = $147,824

We have all the information necessary to calculate the ROA. Doing so, we find the ROA is:

ROA = Net income / Total assets

ROA = $147,824 / $1,818,919

ROA = .0813, or 8.13%

**34.** The currency is generally irrelevant in calculating any financial ratio. The company’s profit margin is:

Profit margin = Net income / Sales

Profit margin = –£27,860 / £512,621

Profit margin = –.0543, or –5.43%

As long as both net income and sales are measured in the same currency, there is no problem; in fact, except for some market value ratios like EPS and BVPS, none of the financial ratios discussed in the text are measured in terms of currency. This is one reason why financial ratio analysis is widely used in international finance to compare the business operations of firms and/or divisions across national economic borders.

We can use the profit margin we previously calculated and the dollar sales to calculate the net income. Doing so, we get:

Profit margin = Net income / Sales

–.0543 = Net income / $765,828

Net income = –$41,621.33

**35.** Here, we need to calculate several ratios given the financial statements. The ratios are:

*Short-term solvency ratios:*

Current ratio = Current assets / Current liabilities

Current ratio2015 = $28,666 / $6,319

Current ratio2015 = 4.54 times

Current ratio2016 = $32,409 / $7,427

Current ratio2016 = 4.36 times

Quick ratio = (Current assets – Inventory) / Current liabilities

Quick ratio2015 = ($28,666 – 17,357) / $6,319

Quick ratio2015 = 1.79 times

Quick ratio2016 = ($32,409 – 19,350) / $7,427

Quick ratio2016 = 1.76 times

Cash ratio = Cash / Current liabilities

Cash ratio2015 = $4,607 / $6,319

Cash ratio2015 = .73 times

Cash ratio2016 = $4,910 / $7,427

Cash ratio2016 = .66 times

*Asset utilization ratios:*

Total asset turnover = Sales / Total assets

Total asset turnover = $205,227 / $109,219

Total asset turnover = 1.88 times

Inventory turnover = COGS / Inventory

Inventory turnover = $138,383 / $19,350

Inventory turnover = 7.15 times

Receivables turnover = Sales / Receivables

Receivables turnover = $205,227 / $8,149

Receivables turnover = 25.18 times

*Long-term solvency ratios:*

Total debt ratio = (Current liabilities + Long-term debt) / Total assets

Total debt ratio2015 = ($6,319 + 22,500) / $87,354

Total debt ratio2015 = .33 times

Total debt ratio2016 = ($7,427 + 19,000) / $109,219

Total debt ratio2016 = .24 times

Debt–equity ratio = (Current liabilities + Long-term debt) / Total equity

Debt–equity ratio2015 = ($6,319 + 22,500) / $58,535

Debt–equity ratio2015 = .49 times

Debt–equity ratio2016 = ($7,427 + 19,000) / $82,792

Debt–equity ratio2016 = .32 times

Equity multiplier = 1 + D/E ratio

Equity multiplier2015 = 1 + .49

Equity multiplier2015 = 1.49 times

Equity multiplier2016 = 1 + .32

Equity multiplier2016 = 1.32 times

Times interest earned = EBIT / Interest

Times interest earned = $60,934 / $1,617

Times interest earned = 37.68 times

Cash coverage ratio = (EBIT + Depreciation) / Interest

Cash coverage ratio = ($60,934 + 5,910) / $1,617

Cash coverage ratio = 41.34 times

*Profitability ratios:*

Profit margin = Net income / Sales

Profit margin = $38,557 / $205,227

Profit margin = .1879, or 18.79%

Return on assets = Net income / Total assets

Return on assets = $38,557 / $109,219

Return on assets = .3530, or 35.30%

Return on equity = Net income / Total equity

Return on equity = $38,557 / $82,792

Return on equity = .4657, or 46.57%

**36.** The Du Pont identity is:

ROE = (PM)(Total asset turnover)(Equity multiplier)

ROE = (Net income / Sales)(Sales / Total assets)(Total assets / Total equity)

ROE = ($38,557 / $205,227)($205,227 / $109,219)($109,219 / $82,792)

ROE = .4657, or 46.57%

**37.** To find the price–earnings ratio we first need the earnings per share. The earnings per share are:

EPS = Net income / Shares outstanding

EPS = $38,557 / 10,000 shares

EPS = $3.86

So, the price–earnings ratio is:

PE ratio = Share price / EPS

PE ratio = $73 / $3.86

PE ratio = 18.93 times

The sales per share are:

Sales per share = Sales / Shares outstanding

Sales per share = $205,227 / 10,000

Sales per share = $20.52

So, the price–sales ratio is:

P/S ratio = Share price / Sales per share

P/S ratio = $73 / $20.52

P/S ratio = 3.56 times

The dividends per share are:

Dividends per share = Total dividends /Shares outstanding

Dividends per share = $14,300 / 10,000 shares

Dividends per share = $1.43 per share

To find the market-to-book ratio, we first need the book value per share. The book value per share is:

Book value per share = Total equity / Shares outstanding

Book value per share = $82,792 / 10,000 shares

Book value per share = $8.28 per share

So, the market-to-book ratio is:

Market-to-book ratio = Share price / Book value per share

Market-to-book ratio = $73 / $8.28

Market-to-book ratio = 8.82 times

**38.** The current ratio appears to be relatively high when compared to the median; however, it is below the upper quartile, meaning that at least 25 percent of firms in the industry have a higher current ratio. Overall, it does not appear that the current ratio is out of line with the industry. The total asset turnover is low when compared to the industry. In fact, the total asset turnover is in the lower quartile. This means that the company does not use assets as efficiently overall or that the company has newer assets than the industry. This would mean that the assets have not been depreciated, which would mean a higher book value and a lower total asset turnover. The debt–equity ratio is in line with the industry, between the median and the lower quartile. The profit margin is in the upper quartile. The company may be better at controlling costs, or has a better product which enables it to charge a premium price. It could also be negative in that the company may have too high of a margin on its sales, which could reduce overall net income.

**39.** To find the profit margin, we can solve the Du Pont identity. First, we need to find the retention ratio. The retention ratio for the company is:

*b* = 1 – .30

*b* = .70

Now, we can use the sustainable growth rate equation to find the ROE. Doing so, we find:

Sustainable growth rate = [(ROE)(*b*)] / [1 – (ROE)(*b*)]

.08 = [ROE(.70)] / [1 – ROE(.70)]

ROE = .1058, or 10.58%

Now, we can use the Du Pont identity. We are given the total asset to sales ratio, which is the inverse of the total asset turnover, and the equity multiplier is one plus the debt–equity ratio. Solving the Du Pont identity for the profit margin, we find:

ROE = (Profit margin)(Total asset turnover)(Equity multiplier)

.1058 = (Profit margin)(1 / 1.35)(1 + .85)

Profit margin = .0772, or 7.72%

**40.** The earnings per share is the net income divided by the shares outstanding. Since all numbers are in millions, the earnings per share for Abercrombie & Fitch was:

EPS = $51.821 / 68.98

EPS = $.75

And the earnings per share for American Eagle Outfitters was:

EPS = $80.322 / 195.43

EPS = $.41

The market-to-book ratio is the stock price divided by the book value per share. To find the book value per share, we divide the total equity by the shares outstanding. The book value per share and market-to-book ratio for Abercrombie & Fitch was:

Book value per share = $1,389.701 / 68.98

Book value per share = $20.15

Market-to-book = $24.81 / $20.15

Market-to-book = 1.23 times

And the market-to-book ratio for American Eagle Outfitters was:

Book value per share = $1,139.746 / 195.43

Book value per share = $5.83

Market-to-book = $13.84 / $5.83

Market-to-book = 2.37 times

And the price–earnings ratio for Abercrombie & Fitch was:

PE = $24.81 / $.75

PE = 33.03 times

And for American Eagle Outfitters, the PE was:

PE = $13.84 / $.41

PE = 33.67 times

**41.** To find the total asset turnover, we can solve the ROA equation. First, we need to find the retention ratio. The retention ratio for the company is:

*b* = 1 – .25

*b* = .75

Now, we can use the internal growth rate equation to find the ROA. Doing so, we find:

Internal growth rate = [(ROA)(*b*)] / [1 – (ROA)(*b*)]

.068 = [ROA(.75)] / [1 – ROA(.75)]

ROA = .0849, or 8.49%

Now, we can use the ROA equation to find the total asset turnover is:

ROA = (PM)(TAT)

.0849 = (.073)TAT

Total asset turnover = 1.16 times

**42.** To calculate the sustainable growth rate, we need to calculate the return on equity. We can use the Du Pont identity to calculate the return on equity if we can find the equity multiplier. Using the total debt ratio, we can find the debt–equity ratio is:

Total debt ratio = Total debt / Total assets

.30 = Total debt / Total assets

1 / .30 = Total assets / Total debt

1 / .30 = (Total debt + Total equity) / Total debt

1 / .30 = 1 + Total equity / Total debt

Total equity / Total debt = (1 / .30) – 1

Total debt / Total equity = 1 / [(1 /.30) – 1]

Total debt / Total equity = .43

Debt–equity ratio = .43

So, the equity multiplier is:

Equity multiplier = 1 + Debt–equity ratio

Equity multiplier = 1 + .43

Equity multiplier = 1.43 times

Using the Du Pont identity, the ROE is:

ROE = (Profit margin)(Total asset turnover)(Equity multiplier)

ROE = (.0520)(1.90)(1.43)

ROE = .1411, or 14.11%

To calculate the sustainable growth rate, we also need the retention ratio. The retention ratio is:

*b* = 1 – .15

*b* = .85

Now we can calculate the sustainable growth rate as:

Sustainable growth rate = [(ROE)(*b*)] / [1 – (ROE)(*b*)]

Sustainable growth rate = [.1411(.85)] / [1 – .1411(.85)]

Sustainable growth rate = .1363, or 13.63%

And the return on assets is:

ROA = (Profit margin)(Total asset turnover)

ROA = (.0520)(1.90)

ROA = .0988, or 9.88%

**43.** To find the sustainable growth rate, we need the retention ratio and the return on equity. The payout ratio is the dividend payment divided by net income, so:

*b* = 1 – ($8,100 / $19,000)

*b* = .5737

And the return on equity is:

ROE = Net income / Total equity

ROE = $19,000 / $91,000

ROE = .2088, or 20.88%

So, the sustainable growth rate is:

Sustainable growth rate = [(ROE)(*b*)] / [1 – (ROE)(*b*)]

Sustainable growth rate = [.2088(.5737)] / [1 – .2088(.5737)]

Sustainable growth rate = .1361, or 13.61%

The total assets of the company are equal to the total debt plus the total equity. The total assets will increase at the sustainable growth rate, so the total assets next year will be:

New total assets = (1 + Sustainable growth rate)(Total assets)

New total assets = (1 + .1361)($67,000 + 91,000)

New total assets = $179,500.62

We can find the new total debt amount by multiplying the new total assets by the total debt ratio. Doing so, we find the new total debt is:

New total debt = [Total debt / (Total debt + Total equity)](New total assets)

New total debt = [$67,000 / ($67,000 + 91,000)]($179,500.62)

New total debt = $76,117.35

The additional borrowing is the difference between the new total debt and the current total debt, so:

Additional borrowing = New total debt – Current total debt

Additional borrowing = $76,117.35 – 67,000

Additional borrowing = $9,117.35

The growth rate that could be achieved with no outside financing at all is the internal growth rate. To find the internal growth rate we first need the return on assets, which is:

ROA = Net income / Total assets

ROA = $19,000 / ($67,000 + 91,000)

ROA = .1203, or 12.03%

So, the internal growth rate is:

Internal growth rate = [(ROA)(*b*)] / [1 – (ROA)(*b*)]

Internal growth rate = [(.1203)(.5737)] / [1 – (.1203)(.5737)]

Internal growth rate = .0741, or 7.41%

**44.** We can find the payout ratio from the sustainable growth rate formula. First, we need the return on equity. Using the Du Pont identity, we find the return on equity is:

ROE = (Profit margin)(Total asset turnover)(Equity multiplier)

ROE = (.06)(1.10)(1 + .35)

ROE = .0891, or 8.91%

Now we can use the sustainable growth rate equation to find the retention ratio, which is:

Sustainable growth rate = [(ROE)(*b*)] / [1 – (ROE)(*b*)]

.13 = [.0891(*b*)] / [1 – .0891(*b*)]

*b* = 1.29

So, the payout ratio is:

Payout ratio = 1 – *b*

Payout ratio = 1 – 1.29

Payout ratio = –.29, or –29%

This is a dividend payout ratio of –29%, which is impossible; the growth rate is not consistent with the other constraints. The lowest possible payout rate is zero, which corresponds to retention ratio of one, or total earnings retention. The maximum sustainable growth rate for this company is:

Sustainable growth rate = [(ROE)(*b*)] / [1 – (ROE)(*b*)]

Sustainable growth rate = [.0891(1)] / [1 – .0891(1)]

Sustainable growth rate = .0978, or 9.78%

**45.** Using the beginning of period total assets, the ROA is:

ROABegin = $1,233 / $14,013

ROABegin = .0880, or 8.80%

Using the end of period total assets, the ROA is:

ROAEnd = $1,233 / $15,256

ROAEnd = .0808, or 8.08%

The ROE using beginning of period equity is:

ROEBegin = $1,233 / $3,986

ROEBegin = .3093, or 30.93%

The ROE using the end of period equity is:

ROEEnd = $1,233 / $4,995

ROEEnd = .2468, or 24.68%

The retention ratio, which is one minus the dividend payout ratio, is:

*b* = 1 – Dividends/Net income

*b* = 1 – $251 / $1,233

*b* = .7964, or 79.64%

With the growth rate equations, we need to use the ROA and ROE based on the end of period assets or equity, so the internal growth rate is:

Internal growth rate = [(ROA)(*b*)] / [1 – (ROA)(*b*)]

Internal growth rate = [(.0808)(.7964)] / [1 – (.0808)(.7964)]

Internal growth rate = .0688, or 6.88%

And the sustainable growth rate is:

Sustainable growth rate = [(ROE)(*b*)] / [1 – (ROE)(*b*)]

Sustainable growth rate = [(.2468)(.7964)] / [1 – (.2468)(.7964)]

Sustainable growth rate = .2447, or 24.47%

Using ROA × b and end of period assets to find the internal growth rate, we find:

Internal growth rate = ROAEnd × *b*

Internal growth rate = .0808 × .7964

Internal growth rate = .0644, or 6.44%

And, using ROE × *b* and the end of period equity to find the sustainable growth rate, we find:

Sustainable growth rate = ROEEnd × *b*

Sustainable growth rate = .2468 × .7964

Sustainable growth rate = .1966, or 19.66%

Using ROA × *b* and beginning of period assets to find the internal growth rate, we find:

Internal growth rate = ROABegin × *b*

Internal growth rate = .0880 × .7964

Internal growth rate = .0701, or 7.01%

And, using ROE × *b* and the beginning of period equity to find the sustainable growth rate, we find:

Sustainable growth rate = ROEBegin × *b*

Sustainable growth rate = .3093 × .7964

Sustainable growth rate = .2464, or 24.64%

|  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | **46.** |  |  |  |  |  |  | Return on |  |  |  |  |
|  |  |  |  |  |  |  |  | equity |  |  |  |  |
|  |  |  |  |  |  |  |  | 58.20% |  |  |  |  |
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|  |  |  |  |  |  |  | Return on | *multiplied* | Equity |  |  |  |
|  |  |  |  |  |  |  | assets | *By* | multiplier |  |  |  |
|  |  |  |  |  |  |  | 15.04% |  | 3.869 |  |  |  |
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|  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  | Profit margin |  | *multiplied by* | | | Total asset turnover |  |  |
|  |  |  |  |  | 11.41% |  |  |  |  | 1.318 |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  | Net income | *divided by* |  | Sales |  | Sales | *divided by* | Total assets |  |
|  |  |  |  | $846.912 |  |  | $7,421.768 |  | $7,421.768 |  | $5,629.516 |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  | *subtracted from* |  |  |  |  |  |  |  |
|  |  |  |  | Total costs |  |  | Sales |  | Fixed assets | *plus* | Current assets |  |
|  |  |  |  | $6,574.856 |  |  | $7,421.768 |  | $3,382.469 |  | $2,247.047 |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  | Cash |  |
|  | Cost of goods sold | | |  | Depreciation |  |  |  |  |  | $471.985 |  |
|  | $4,085.602 | | |  | $176.312 |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  | Accounts rec. |  | Inventory |
|  | Other expenses | | |  | Interest |  |  |  |  | $697.455 |  | $1,077.607 |
|  | $1,770.279 | | |  | $83.532 |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  | Taxes |  |  |  |  |  |  |  |
|  |  |  |  |  | $459.131 |  |  |  |  |  |  |  |

**CHAPTER 4**

***INTRODUCTION TO VALUATION: THE TIME VALUE OF MONEY***

**Answers to Concepts Review and Critical Thinking Questions**

**1.** Compounding refers to the growth of a dollar amount through time via reinvestment of interest earned. It is also the process of determining the future value of an investment. Discounting is the process of determining the value today of an amount to be received in the future.

**2.** Future values grow (assuming a positive rate of return); present values shrink.

**3.** The future value rises (assuming a positive rate of return); the present value falls.

**4.** It depends. The large deposit will have a larger future value for some period, but after time, the smaller deposit with the larger interest rate will eventually become larger. The length of time for the smaller deposit to overtake the larger deposit depends on the amount deposited in each account and the interest rates.

**5.** It would appear to be both deceptive and unethical to run such an ad without a disclaimer or explanation.

**6.** It’s a reflection of the time value of money. TMCC gets to use the $24,099. If TMCC uses it wisely, it will be worth more than $100,000 in thirty years.

**7.** This will probably make the security less desirable. TMCC will only repurchase the security prior to maturity if it to its advantage, i.e. interest rates decline. Given the drop in interest rates needed to make this viable for TMCC, it is unlikely the company will repurchase the security. This is an example of a “call” feature. Such features are discussed at length in a later chapter.

**8.** The key considerations would be: (1) Is the rate of return implicit in the offer attractive relative to other, similar risk investments? and (2) How risky is the investment; i.e., how certain are we that we will actually get the $100,000? Thus, our answer does depend on who is making the promise to repay.

**9.** The Treasury security would have a somewhat higher price because the Treasury is the strongest of all borrowers.

**10.** The price would be higher because, as time passes, the price of the security will tend to rise toward $100,000. This rise is just a reflection of the time value of money. As time passes, the time until receipt of the $100,000 grows shorter, and the present value rises. In 2018, the price will probably be higher for the same reason. We cannot be sure, however, because interest rates could be much higher, or TMCC’s financial position could deteriorate. Either event would tend to depress the security’s price.

**Solutions to Questions and Problems**

*NOTE: All end-of-chapter problems were solved using a spreadsheet. Many problems require multiple steps. Due to space and readability constraints, when these intermediate steps are included in this solutions manual, rounding may appear to have occurred. However, the final answer for each problem is found without rounding during any step in the problem.*

*Basic*

**1.** The time line for the cash flows is:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | |  | |  | |  | |  | |  | |  | |  | |  | |  | | 10 | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| $8,100 | |  | |  | |  | |  | |  | |  | |  | |  | |  | | FV | |

The simple interest per year is:

$8,100 × .06 = $486

So, after 10 years, you will have:

$486 × 10 = $4,860 in interest.

The total balance will be $8,100 + 4,860 = $12,960

With compound interest, we use the future value formula:

FV = PV(1 +*r*)*t*

FV = $8,100(1.06)10

FV = $14,505.87

The difference is:

$14,505.87 – 12,960 = $1,545.87

**2.** To find the FV of a lump sum, we use:

FV = PV(1 + *r*)*t*

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | |  | |  | |  | |  | |  | |  | |  | |  | |  | | 7 | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| $3,150 | |  | |  | |  | |  | |  | |  | |  | |  | |  | | FV | |

FV = $3,150(1.13)7 = $7,410.71

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | |  | |  | |  | |  | |  | |  | |  | |  | |  | | 16 | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| $8,453 | |  | |  | |  | |  | |  | |  | |  | |  | |  | | FV | |

FV = $8,453(1.07)16 = $24,954.64

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | |  | |  | |  | |  | |  | |  | |  | |  | |  | | 19 | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| $89,305 | |  | |  | |  | |  | |  | |  | |  | |  | |  | | FV | |

FV = $89,305(1.09)19 = $459,176.06

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | |  | |  | |  | |  | |  | |  | |  | |  | |  | | 26 | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| $227,382 | |  | |  | |  | |  | |  | |  | |  | |  | |  | | FV | |

FV = $227,382(1.05)26 = $808,495.97

**3.** To find the PV of a lump sum, we use:

PV = FV / (1 + *r*)*t*

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | |  | |  | |  | |  | |  | |  | |  | |  | |  | | 15 | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| PV | |  | |  | |  | |  | |  | |  | |  | |  | |  | | $17,328 | |

PV = $17,328 / (1.07)15 = $6,280.46

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | |  | |  | |  | |  | |  | |  | |  | |  | |  | | 8 | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| PV | |  | |  | |  | |  | |  | |  | |  | |  | |  | | $41,517 | |

PV = $41,517 / (1.11)8 = $18,015.33

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | |  | |  | |  | |  | |  | |  | |  | |  | |  | | 13 | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| PV | |  | |  | |  | |  | |  | |  | |  | |  | |  | | $790,382 | |

PV = $790,382 / (1.10)13 = $228,945.51

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | |  | |  | |  | |  | |  | |  | |  | |  | |  | | 25 | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| PV | |  | |  | |  | |  | |  | |  | |  | |  | |  | | $647,816 | |

PV = $647,816 / (1.13)25 = $30,153.40

**4.** To answer this question, we can use either the FV or the PV formula. Both will give the same answer since they are the inverse of each other. We will use the FV formula, that is:

FV = PV(1 + *r*)*t*

Solving for *r*, we get:

*r* = (FV / PV)1 / *t* – 1

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | |  | |  | |  | |  | |  | |  | |  | |  | |  | | 11 | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| –$715 | |  | |  | |  | |  | |  | |  | |  | |  | |  | | $1,381 | |

FV = $1,381 = $715(1 + *r*)11

*r* = ($1,381 / $715)1/11 – 1

*r* = .0617, or 6.17%

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | |  | |  | |  | |  | |  | |  | |  | |  | |  | | 8 | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| –$905 | |  | |  | |  | |  | |  | |  | |  | |  | |  | | $1,718 | |

FV = $1,718 = $905(1 + *r*)8

*r* = ($1,718 / $905)1/8 – 1

*r* = .0834, or 8.34%

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | |  | |  | |  | |  | |  | |  | |  | |  | |  | | 23 | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| –$15,000 | |  | |  | |  | |  | |  | |  | |  | |  | |  | | $141,832 | |

FV = $141,832 = $15,000(1 + *r*)23

*r* = ($141,832 / $15,000)1/23 – 1

*r* = .1026, or 10.26%

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | |  | |  | |  | |  | |  | |  | |  | |  | |  | | 16 | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| –$70,300 | |  | |  | |  | |  | |  | |  | |  | |  | |  | | $312,815 | |

FV = $312,815 = $70,300(1 + *r*)16

*r* = ($312,815 / $70,300)1/16 – 1

*r* = .0978, or 9.78%

**5.** To answer this question, we can use either the FV or the PV formula. Both will give the same answer since they are the inverse of each other. We will use the FV formula, that is:

FV = PV(1 + *r*)*t*

Solving for *t*, we get:

*t* = ln(FV / PV) / ln(1 + *r*)

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | |  | |  | |  | |  | |  | |  | |  | |  | |  | | *t* | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| –$195 | |  | |  | |  | |  | |  | |  | |  | |  | |  | | $873 | |

FV = $873 = $195 (1.09)*t*

*t* = ln($873 / $195) / ln 1.09

*t* = 17.39 years

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | |  | |  | |  | |  | |  | |  | |  | |  | |  | | *t* | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| –$2,105 | |  | |  | |  | |  | |  | |  | |  | |  | |  | | $3,500 | |

FV = $3,500 = $2,105(1.07)*t*

*t* = ln($3,500 / $2,105) / ln 1.07

*t* = 7.51 years

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | |  | |  | |  | |  | |  | |  | |  | |  | |  | | *t* | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| –$47,800 | |  | |  | |  | |  | |  | |  | |  | |  | |  | | $326,500 | |

FV = $326,500 = $47,800(1.12)*t*

*t* = ln($326,500 / $47,800) / ln 1.12

*t* = 16.95 years

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | |  | |  | | |  | | |  | | |  | | |  | | |  | | |  | | |  | | | *t* | | |
|  |  |  |  | |  |  | |  |  | |  |  | |  |  | |  |  | |  |  | |  |  | |  |  | |  |  | |
|  |  |  | |  |  | |  |  | |  |  | |  |  | |  |  | |  |  | |  |  | |  |  | |  |
| –$38,650 | |  | |  | | |  | | |  | | |  | | |  | | |  | | |  | | |  | | | $213,380 | | |

FV = $213,380 = $38,650(1.19)*t*

*t* = ln($213,380 / $38,650) / ln 1.19

*t* = 9.82 years

**6.** The time line is:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | |  | |  | |  | |  | |  | |  | |  | |  | |  | | 18 | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| –$53,000 | |  | |  | |  | |  | |  | |  | |  | |  | |  | | $295,000 | |

To answer this question, we can use either the FV or the PV formula. Both will give the same answer since they are the inverse of each other. We will use the FV formula, that is:

FV = PV(1 + *r*)*t*

Solving for *r*, we get:

*r* = (FV / PV)1 / *t* – 1

*r* = ($295,000 / $53,000)1/18 – 1

*r* = .1001, or 10.01%

**7.** To find the length of time for money to double, triple, etc., the present value and future value are irrelevant as long as the future value is twice the present value for doubling, three times as large for tripling, etc. To answer this question, we can use either the FV or the PV formula. Both will give the same answer since they are the inverse of each other. We will use the FV formula, that is:

FV = PV(1 + *r*)*t*

Solving for *t*, we get:

*t* = ln(FV / PV) / ln(1 + *r*)

The length of time to double your money is:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | |  | |  | |  | |  | |  | |  | |  | |  | |  | | *t* | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| –$1 | |  | |  | |  | |  | |  | |  | |  | |  | |  | | $2 | |

FV = $2 = $1(1.047)*t*

*t* = ln 2 / ln 1.047

*t* = 15.09 years

The length of time to quadruple your money is:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | |  | |  | |  | |  | |  | |  | |  | |  | |  | | *t* | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| –$1 | |  | |  | |  | |  | |  | |  | |  | |  | |  | | $4 | |

FV = $4 = $1(1.047)*t*

*t* = ln 4 / ln 1.047

*t* = 30.18 years

Notice that the length of time to quadruple your money is twice as long as the time needed to double your money. This is an important concept of time value of money.

**8.** The time line is:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | |  | |  | |  | |  | |  | |  | |  | |  | |  | | 140 | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| –$20 | |  | |  | |  | |  | |  | |  | |  | |  | |  | | $15,000 | |

To answer this question, we can use either the FV or the PV formula. Both will give the same answer since they are the inverse of each other. We will use the FV formula, that is:

FV = PV(1 + *r*)*t*

Solving for *r*, we get:

*r* = (FV / PV)1 / *t* – 1

*r* = ($15,000 / $20)1/140 – 1

*r* = .0484, or 4.84%

**9.** The time line is:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | |  | |  | |  | |  | |  | |  | |  | |  | |  | | *t* | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| –$35,000 | |  | |  | |  | |  | |  | |  | |  | |  | |  | | $150,000 | |

To answer this question, we can use either the FV or the PV formula. Both will give the same answer since they are the inverse of each other. We will use the FV formula, that is:

FV = PV(1 + *r*)*t*

Solving for *t*, we get:

*t* = ln(FV / PV) / ln(1 + *r*)

FV = $150,000 = $35,000(1.021)*t*

*t* = ln($150,000 / $35,000) / ln 1.021

*t* = 70.02 years

**10.** The time line is:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | |  | |  | |  | |  | |  | |  | |  | |  | |  | | 25 | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| PV | |  | |  | |  | |  | |  | |  | |  | |  | |  | | $730,000,000 | |

To find the PV of a lump sum, we use:

PV = FV / (1 + *r*)*t*

PV = $730,000,000 / (1.055)25

PV = $191,430,603.85

**11.** The time line is:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | |  | |  | |  | |  | |  | |  | |  | |  | |  | | 80 | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| PV | |  | |  | |  | |  | |  | |  | |  | |  | |  | | $1,000,000 | |

To find the PV of a lump sum, we use:

PV = FV / (1 + *r*)*t*

PV = $1,000,000 / (1.0725)80

PV = $3,700.12

**12.** The time line is:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | |  | |  | |  | |  | |  | |  | |  | |  | |  | | 111 | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 50 | |  | |  | |  | |  | |  | |  | |  | |  | |  | | FV | |

To find the FV of a lump sum, we use:

FV = PV(1 + *r*)*t*

FV = $50(1.043)111

FV = $5,352.15

**13.** The time line is:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | |  | |  | |  | |  | |  | |  | |  | |  | |  | | 119 | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| $150 | |  | |  | |  | |  | |  | |  | |  | |  | |  | | $1,620,000 | |

To answer this question, we can use either the FV or the PV formula. Both will give the same answer since they are the inverse of each other. We will use the FV formula, that is:

FV = PV(1 + *r*)*t*

Solving for *r*, we get:

*r* = (FV / PV)1 / *t* – 1

*r* = ($1,620,000 / $150)1/119 – 1

*r* = .0812, or 8.12%

To find the FV of the first prize in 2045, we use:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | |  | |  | |  | |  | |  | |  | |  | |  | |  | | 31 | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| $1,620,000 | |  | |  | |  | |  | |  | |  | |  | |  | |  | | FV | |

FV = PV(1 + *r*)*t*

FV = $1,620,000(1.0812)31

FV = $18,207,019.19

**14.** The time line is:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | |  | |  | |  | |  | |  | |  | |  | |  | |  | | 76 | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| –$.10 | |  | |  | |  | |  | |  | |  | |  | |  | |  | | $3,207,852 | |

To answer this question, we can use either the FV or the PV formula. Both will give the same answer since they are the inverse of each other. We will use the FV formula, that is:

FV = PV(1 + *r*)*t*

Solving for *r*, we get:

*r* = (FV / PV)1 / *t* – 1

*r* = ($3,207,852 / $.10)1/76 – 1

*r* = .2554, or 25.54%

**15.** The time line is:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | |  | |  | |  | |  | |  | |  | |  | |  | |  | | 4 | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| –$12,377,500 | |  | |  | |  | |  | |  | |  | |  | |  | |  | | $10,311,500 | |

To answer this question, we can use either the FV or the PV formula. Both will give the same answer since they are the inverse of each other. We will use the FV formula, that is:

FV = PV(1 + *r*)*t*

Solving for *r*, we get:

*r* = (FV / PV)1 / *t* – 1

*r* = ($10,311,500 / $12,377,500)1/4 – 1

*r* = –.0446, or –4.46%

*Intermediate*

**16.** To answer this question, we can use either the FV or the PV formula. Both will give the same answer since they are the inverse of each other. We will use the FV formula, that is:

FV = PV(1 + *r*)*t*

Solving for *r*, we get:

*r* = (FV / PV)1 / *t* – 1

*a.* The time line is:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | |  | |  | |  | |  | |  | |  | |  | |  | |  | | 20 | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| –$50 | |  | |  | |  | |  | |  | |  | |  | |  | |  | | $100 | |

*r* = (FV / PV)1 / *t* – 1

*r* = ($100 / $50)1/20 – 1

*r* = .0353, or 3.53%

*b.* The time line is:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | |  | |  | |  | |  | |  | |  | |  | |  | |  | | 10 | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| –$50 | |  | |  | |  | |  | |  | |  | |  | |  | |  | | FV | |

FV = PV(1 + *r*)*t*

FV = $50(1 + .001)10

FV = $50.50

*c.* The time line is:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | |  | |  | |  | |  | |  | |  | |  | |  | |  | | 10 | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| –$50.50 | |  | |  | |  | |  | |  | |  | |  | |  | |  | | $100 | |

*r* = (FV / PV)1 / *t* – 1

*r* = ($100 / $50.50)1/10 – 1

*r* = .0707, or 7.07%

**17.** The time line is:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | |  | |  | |  | |  | |  | |  | |  | |  | |  | | 10 | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| PV | |  | |  | |  | |  | |  | |  | |  | |  | |  | | $150,000 | |

To find the PV of a lump sum, we use:

PV = FV / (1 + *r*)*t*

PV = $150,000 / (1.106)10

PV = $54,769.65

**18.** To find the FV of a lump sum, we use:

FV = PV(1 + *r*)*t*

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | |  | |  | |  | |  | |  | |  | |  | |  | |  | | 45 | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| $5,000 | |  | |  | |  | |  | |  | |  | |  | |  | |  | | FV | |

FV = $5,000(1.0975)45

FV = $328,983.26

If you wait 10 years, the value of your deposit at your retirement will be:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | |  | |  | |  | |  | |  | |  | |  | |  | |  | | 35 | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| $5,000 | |  | |  | |  | |  | |  | |  | |  | |  | |  | | FV | |

FV = $5,000(1.0975)35

FV = $129,756.32

Better start early!

**19.** The time line is:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | |  | | 2 | |  | |  | |  | |  | |  | |  | |  | | 8 | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  | |  | | $13,000 | |  | |  | |  | |  | |  | |  | |  | | FV | |

Even though we need to calculate the value in eight years, you will only have the money for six years, so we need to use six years as the number of periods. To find the FV of a lump sum, we use:

FV = PV(1 + *r*)*t*

FV = $13,000(1.075)6

FV = $20,062.92

**20.** The time line is:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | |  | | 2 | |  | |  | |  | |  | |  | |  | |  | | *t* | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  | |  | | $30,000 | |  | |  | |  | |  | |  | |  | |  | | $150,000 | |

To answer this question, we can use either the FV or the PV formula. Both will give the same answer since they are the inverse of each other. We will use the FV formula, that is:

FV = PV(1 + *r*)*t*

$150,000 = $30,000(1.09)*t*

*t* = ln($150,000 / $30,000) / ln 1.09

*t* = 18.68 years

From now, you’ll wait 2 + 18.68 = 20.68 years

**21.** To find the FV of a lump sum, we use:

FV = PV(1 + *r*)*t*

In Regency Bank, you will have:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | |  | |  | |  | |  | |  | |  | |  | |  | |  | | 240 | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| $6,150 | |  | |  | |  | |  | |  | |  | |  | |  | |  | | FV | |

FV = $6,150(1.01)240

FV = $66,989.20

And in King Bank, you will have:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | |  | |  | |  | |  | |  | |  | |  | |  | |  | | 20 | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| $6,150 | |  | |  | |  | |  | |  | |  | |  | |  | |  | | FV | |

FV = $6,150(1.12)20

FV = $59,324.70

**22.** The time line is:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | |  | |  | |  | |  | |  | |  | |  | |  | |  | | *t* | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| –$1 | |  | |  | |  | |  | |  | |  | |  | |  | |  | | $3 | |

To find the length of time for money to double, triple, etc., the present value and future value are irrelevant as long as the future value is twice the present value for doubling, three times as large for tripling, etc. We also need to be careful about the number of periods. Since the length of the compounding is three months and we have 24 months, there are eight compounding periods. To answer this question, we can use either the FV or the PV formula. Both will give the same answer since they are the inverse of each other. We will use the FV formula, that is:

FV = PV(1 + *r*)*t*

Solving for *r*, we get:

*r* = (FV / PV)1 / *t* – 1

*r* = ($3 / $1)1/8 – 1

*r* = .1472, or 14.72%

**23.** The time line is:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | |  | |  | |  | |  | |  | |  | |  | |  | |  | | *t* | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| –$1,800 | |  | |  | |  | |  | |  | |  | |  | |  | |  | | $3,100 | |

To answer this question, we can use either the FV or the PV formula. Both will give the same answer since they are the inverse of each other. We will use the FV formula, that is:

FV = PV(1 + *r*)*t*

$3,100 = $1,800(1.0027)*t*

*t* = ln($3,100 / $1,800) / ln 1.0027

*t* = 201.61 months

**24.** The time line is:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | |  | |  | |  | |  | |  | |  | |  | |  | |  | | 120 | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| PV | |  | |  | |  | |  | |  | |  | |  | |  | |  | | $85,000 | |

To find the PV of a lump sum, we use:

PV = FV / (1 + *r*)*t*

PV = $85,000 / (1.0065)120

PV = $39,062.98

**25.** The time line is:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | |  | |  | |  | |  | |  | |  | |  | |  | |  | | 45 | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| PV | |  | |  | |  | |  | |  | |  | |  | |  | |  | | $1,000,000 | |

To find the PV of a lump sum, we use:

PV = FV / (1 + *r*)*t*

So, if you can earn 11 percent, you will need to invest:

PV = $1,000,000 / (1.11)45

PV = $9,129.90

And if you can earn 5.5 percent, you will need to invest:

PV = $1,000,000 / (1.055)45

PV = $89,875.09

*Challenge*

**26.** The time line is:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | |  | |  | |  | |  | |  | |  | |  | |  | |  | | 40 | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| –$20,000 | |  | |  | |  | |  | |  | |  | |  | |  | |  | | FV | |

In this case, we have an investment that earns two different interest rates. We will calculate the value of the investment at the end of the first 20 years then use this value with the second interest rate to find the final value at the end of 40 years. Using the future value equation, at the end of the first 20 years, the account will be worth:

Value in 20 years = PV(1 + *r*)*t*

Value in 20 years = $20,000(1.07)20

Value in 20 years = $77,393.69

Now we can find out how much this will be worth 20 years later at the end of the investment. Using the future value equation, we find:

Value in 40 years = PV(1 + *r*)*t*

Value in 40 years = $77,393.69(1.11)20

Value in 40 years = $623,972.03

It is irrelevant which interest rate is offered when as long as each interest rate is offered for 20 years. We can find the value of the initial investment in 40 years with the following:

FV = PV(1 + *r*1)*t* (1 + *r*2)*t*

FV = $20,000(1.11)20(1.07)20

FV = $623,972.03

With the commutative property of multiplication, it does not matter which order the interest rates occur, the final value will always be the same.

**Calculator Solutions**

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **1.** |  | | |  | | |  | | |  | | |  | | |
| Enter | 10 | | | 6% | | | ±$8,100 | | |  | | |  | | |
|  |  | **N** |  |  | **I/Y** |  |  | **PV** |  |  | **PMT** |  |  | **FV** |  | |
| Solve for |  | | |  | | |  | | |  | | | $14,505.87 | | |

$14,505.87 – [$8,100 + 10($486)] = $1,545.87

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **2.** |  | | |  | | |  | | |  | | |  | | |
| Enter | 7 | | | 13% | | | ±$3,150 | | |  | | |  | | |
|  |  | **N** |  |  | **I/Y** |  |  | **PV** |  |  | **PMT** |  |  | **FV** |  | |
| Solve for |  | | |  | | |  | | |  | | | $7,410.71 | | |

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| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  | | |  | | |  | | |  | | |  | | |
| Enter | 16 | | | 7% | | | ±$8,453 | | |  | | |  | | |
|  |  | **N** |  |  | **I/Y** |  |  | **PV** |  |  | **PMT** |  |  | **FV** |  | |
| Solve for |  | | |  | | |  | | |  | | | $24,954.64 | | |

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|  |  | | |  | | |  | | |  | | |  | | |
| Enter | 19 | | | 9% | | | ±$89,305 | | |  | | |  | | |
|  |  | **N** |  |  | **I/Y** |  |  | **PV** |  |  | **PMT** |  |  | **FV** |  | |
| Solve for |  | | |  | | |  | | |  | | | $459,176.06 | | |

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|  |  | | |  | | |  | | |  | | |  | | |
| Enter | 26 | | | 5% | | | ±$227,382 | | |  | | |  | | |
|  |  | **N** |  |  | **I/Y** |  |  | **PV** |  |  | **PMT** |  |  | **FV** |  | |
| Solve for |  | | |  | | |  | | |  | | | $808,495.97 | | |

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| **3.** |  | | |  | | |  | | |  | | |  | | |
| Enter | 15 | | | 7% | | |  | | |  | | | $17,328 | | |
|  |  | **N** |  |  | **I/Y** |  |  | **PV** |  |  | **PMT** |  |  | **FV** |  | |
| Solve for |  | | |  | | | –$6,280.46 | | |  | | |  | | |

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|  |  | | |  | | |  | | |  | | |  | | |
| Enter | 8 | | | 11% | | |  | | |  | | | $41,517 | | |
|  |  | **N** |  |  | **I/Y** |  |  | **PV** |  |  | **PMT** |  |  | **FV** |  | |
| Solve for |  | | |  | | | –$18,015.33 | | |  | | |  | | |

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|  |  | | |  | | |  | | |  | | |  | | |
| Enter | 13 | | | 10% | | |  | | |  | | | $790,382 | | |
|  |  | **N** |  |  | **I/Y** |  |  | **PV** |  |  | **PMT** |  |  | **FV** |  | |
| Solve for |  | | |  | | | –$228,945.51 | | |  | | |  | | |

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|  |  | | |  | | |  | | |  | | |  | | |
| Enter | 25 | | | 13% | | |  | | |  | | | $647,816 | | |
|  |  | **N** |  |  | **I/Y** |  |  | **PV** |  |  | **PMT** |  |  | **FV** |  | |
| Solve for |  | | |  | | | –$30,513.40 | | |  | | |  | | |

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| **4.** |  | | |  | | |  | | |  | | |  | | |
| Enter | 11 | | |  | | | ±$715 | | |  | | | $1,381 | | |
|  |  | **N** |  |  | **I/Y** |  |  | **PV** |  |  | **PMT** |  |  | **FV** |  | |
| Solve for |  | | | 6.17% | | |  | | |  | | |  | | |

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|  |  | | |  | | |  | | |  | | |  | | |
| Enter | 8 | | |  | | | ±$905 | | |  | | | $1,718 | | |
|  |  | **N** |  |  | **I/Y** |  |  | **PV** |  |  | **PMT** |  |  | **FV** |  | |
| Solve for |  | | | 8.34% | | |  | | |  | | |  | | |

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|  |  | | |  | | |  | | |  | | |  | | |
| Enter | 23 | | |  | | | ±$15,000 | | |  | | | $141,832 | | |
|  |  | **N** |  |  | **I/Y** |  |  | **PV** |  |  | **PMT** |  |  | **FV** |  | |
| Solve for |  | | | 10.26% | | |  | | |  | | |  | | |

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|  |  | | |  | | |  | | |  | | |  | | |
| Enter | 16 | | |  | | | ±$70,300 | | |  | | | $312,815 | | |
|  |  | **N** |  |  | **I/Y** |  |  | **PV** |  |  | **PMT** |  |  | **FV** |  | |
| Solve for |  | | | 9.78% | | |  | | |  | | |  | | |

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| **5.** |  | | |  | | |  | | |  | | |  | | |
| Enter |  | | | 9% | | | ±$195 | | |  | | | $873 | | |
|  |  | **N** |  |  | **I/Y** |  |  | **PV** |  |  | **PMT** |  |  | **FV** |  | |
| Solve for | 17.39 | | |  | | |  | | |  | | |  | | |

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|  |  | | |  | | |  | | |  | | |  | | |
| Enter |  | | | 7% | | | ±$2,105 | | |  | | | $3,500 | | |
|  |  | **N** |  |  | **I/Y** |  |  | **PV** |  |  | **PMT** |  |  | **FV** |  | |
| Solve for | 7.51 | | |  | | |  | | |  | | |  | | |

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|  |  | | |  | | |  | | |  | | |  | | |
| Enter |  | | | 12% | | | ±$47,800 | | |  | | | $326,500 | | |
|  |  | **N** |  |  | **I/Y** |  |  | **PV** |  |  | **PMT** |  |  | **FV** |  | |
| Solve for | 16.95 | | |  | | |  | | |  | | |  | | |

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|  |  | | |  | | |  | | |  | | |  | | |
| Enter |  | | | 19% | | | ±$38,650 | | |  | | | $213,380 | | |
|  |  | **N** |  |  | **I/Y** |  |  | **PV** |  |  | **PMT** |  |  | **FV** |  | |
| Solve for | 9.82 | | |  | | |  | | |  | | |  | | |

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| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **6.** |  | | |  | | |  | | |  | | |  | | |
| Enter | 18 | | |  | | | ±$53,000 | | |  | | | $295,000 | | |
|  |  | **N** |  |  | **I/Y** |  |  | **PV** |  |  | **PMT** |  |  | **FV** |  | |
| Solve for |  | | | 10.01% | | |  | | |  | | |  | | |

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| **7.** |  | | |  | | |  | | |  | | |  | | |
| Enter |  | | | 4.7% | | | ±$1 | | |  | | | $2 | | |
|  |  | **N** |  |  | **I/Y** |  |  | **PV** |  |  | **PMT** |  |  | **FV** |  | |
| Solve for | 15.09 | | |  | | |  | | |  | | |  | | |

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|  |  | | |  | | |  | | |  | | |  | | |
| Enter |  | | | 4.7% | | | ±$1 | | |  | | | $4 | | |
|  |  | **N** |  |  | **I/Y** |  |  | **PV** |  |  | **PMT** |  |  | **FV** |  | |
| Solve for | 30.18 | | |  | | |  | | |  | | |  | | |

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| **8.** |  | | |  | | |  | | |  | | |  | | |
| Enter | 140 | | |  | | | ±$20 | | |  | | | $15,000 | | |
|  |  | **N** |  |  | **I/Y** |  |  | **PV** |  |  | **PMT** |  |  | **FV** |  | |
| Solve for |  | | | 4.84% | | |  | | |  | | |  | | |

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| **9.** |  | | |  | | |  | | |  | | |  | | |
| Enter |  | | | 2.1% | | | ±$35,000 | | |  | | | $150,000 | | |
|  |  | **N** |  |  | **I/Y** |  |  | **PV** |  |  | **PMT** |  |  | **FV** |  | |
| Solve for | 70.02 | | |  | | |  | | |  | | |  | | |

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| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **10.** |  | | |  | | |  | | |  | | |  | | |
| Enter | 25 | | | 5.5% | | |  | | |  | | | $730,000,000 | | |
|  |  | **N** |  |  | **I/Y** |  |  | **PV** |  |  | **PMT** |  |  | **FV** |  | |
| Solve for |  | | |  | | | –$191,430,604 | | |  | | |  | | |

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| **11.** |  | | |  | | |  | | |  | | |  | | |
| Enter | 80 | | | 7.25% | | |  | | |  | | | $1,000,000 | | |
|  |  | **N** |  |  | **I/Y** |  |  | **PV** |  |  | **PMT** |  |  | **FV** |  | |
| Solve for |  | | |  | | | –$3,700.12 | | |  | | |  | | |

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| **12.** |  | | |  | | |  | | |  | | |  | | |
| Enter | 111 | | | 4.3% | | | ±$50 | | |  | | |  | | |
|  |  | **N** |  |  | **I/Y** |  |  | **PV** |  |  | **PMT** |  |  | **FV** |  | |
| Solve for |  | | |  | | |  | | |  | | | $5,352.15 | | |

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| **13.** |  | | |  | | |  | | |  | | |  | | |
| Enter | 119 | | |  | | | ±$150 | | |  | | | $1,620,000 | | |
|  |  | **N** |  |  | **I/Y** |  |  | **PV** |  |  | **PMT** |  |  | **FV** |  | |
| Solve for |  | | | 8.12% | | |  | | |  | | |  | | |

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|  |  | | |  | | |  | | |  | | |  | | |
| Enter | 31 | | | 8.12% | | | ±$1,620,000 | | |  | | |  | | |
|  |  | **N** |  |  | **I/Y** |  |  | **PV** |  |  | **PMT** |  |  | **FV** |  | |
| Solve for |  | | |  | | |  | | |  | | | $18,207,019.19 | | |

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| **14.** |  | | |  | | |  | | |  | | |  | | |
| Enter | 76 | | |  | | | ±$.10 | | |  | | | $3,207,852 | | |
|  |  | **N** |  |  | **I/Y** |  |  | **PV** |  |  | **PMT** |  |  | **FV** |  | |
| Solve for |  | | | 25.54% | | |  | | |  | | |  | | |

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| **15.** |  | | |  | | |  | | |  | | |  | | |
| Enter | 4 | | |  | | | ±$12,377,500 | | |  | | | $10,311,500 | | |
|  |  | **N** |  |  | **I/Y** |  |  | **PV** |  |  | **PMT** |  |  | **FV** |  | |
| Solve for |  | | | –4.46% | | |  | | |  | | |  | | |

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| **16.** *a.* |  | | |  | | |  | | |  | | |  | | |
| Enter | 20 | | |  | | | ±$50 | | |  | | | $100 | | |
|  |  | **N** |  |  | **I/Y** |  |  | **PV** |  |  | **PMT** |  |  | **FV** |  | |
| Solve for |  | | | 3.53% | | |  | | |  | | |  | | |

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| *b.* |  | | |  | | |  | | |  | | |  | | |
| Enter | 10 | | | .10% | | | ±$50 | | |  | | |  | | |
|  |  | **N** |  |  | **I/Y** |  |  | **PV** |  |  | **PMT** |  |  | **FV** |  | |
| Solve for |  | | |  | | |  | | |  | | | $50.50 | | |

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| *c****.*** |  | | |  | | |  | | |  | | |  | | |
| Enter | 10 | | |  | | | ±$50.50 | | |  | | | $100 | | |
|  |  | **N** |  |  | **I/Y** |  |  | **PV** |  |  | **PMT** |  |  | **FV** |  | |
| Solve for |  | | | 7.07% | | |  | | |  | | |  | | |

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| **17.** |  | | |  | | |  | | |  | | |  | | |
| Enter | 10 | | | 10.60% | | |  | | |  | | | $150,000 | | |
|  |  | **N** |  |  | **I/Y** |  |  | **PV** |  |  | **PMT** |  |  | **FV** |  | |
| Solve for |  | | |  | | | –$54,769.65 | | |  | | |  | | |

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| **18.** |  | | |  | | |  | | |  | | |  | | |
| Enter | 45 | | | 9.75% | | | ±$5,000 | | |  | | |  | | |
|  |  | **N** |  |  | **I/Y** |  |  | **PV** |  |  | **PMT** |  |  | **FV** |  | |
| Solve for |  | | |  | | |  | | |  | | | $328,983.26 | | |

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|  |  | | |  | | |  | | |  | | |  | | |
| Enter | 35 | | | 9.75% | | | ±$5,000 | | |  | | |  | | |
|  |  | **N** |  |  | **I/Y** |  |  | **PV** |  |  | **PMT** |  |  | **FV** |  | |
| Solve for |  | | |  | | |  | | |  | | | $129,756.32 | | |

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| **19.** |  | | |  | | |  | | |  | | |  | | |
| Enter | 6 | | | 7.5% | | | ±$13,000 | | |  | | |  | | |
|  |  | **N** |  |  | **I/Y** |  |  | **PV** |  |  | **PMT** |  |  | **FV** |  | |
| Solve for |  | | |  | | |  | | |  | | | $20,062.92 | | |

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **20.** |  | | |  | | |  | | |  | | |  | | |
| Enter |  | | | 9% | | | ±$30,000 | | |  | | | $150,000 | | |
|  |  | **N** |  |  | **I/Y** |  |  | **PV** |  |  | **PMT** |  |  | **FV** |  | |
| Solve for | 18.68 | | |  | | |  | | |  | | |  | | |

You must wait 2 + 18.68 = 20.68 years.

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **21.** |  | | |  | | |  | | |  | | |  | | |
| Enter | 240 | | | 1% | | | ±$6,150 | | |  | | |  | | |
|  |  | **N** |  |  | **I/Y** |  |  | **PV** |  |  | **PMT** |  |  | **FV** |  | |
| Solve for |  | | |  | | |  | | |  | | | $66,989.20 | | |

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| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  | | |  | | |  | | |  | | |  | | |
| Enter | 20 | | | 12% | | | ±$6,150 | | |  | | |  | | |
|  |  | **N** |  |  | **I/Y** |  |  | **PV** |  |  | **PMT** |  |  | **FV** |  | |
| Solve for |  | | |  | | |  | | |  | | | $59,324.70 | | |

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| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **22.** |  | | |  | | |  | | |  | | |  | | |
| Enter | 8 | | |  | | | ±$1 | | |  | | | $3 | | |
|  |  | **N** |  |  | **I/Y** |  |  | **PV** |  |  | **PMT** |  |  | **FV** |  | |
| Solve for |  | | | 14.72% | | |  | | |  | | |  | | |

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| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **23.** |  | | |  | | |  | | |  | | |  | | |
| Enter |  | | | .27% | | | ±$1,800 | | |  | | | $3,100 | | |
|  |  | **N** |  |  | **I/Y** |  |  | **PV** |  |  | **PMT** |  |  | **FV** |  | |
| Solve for | 201.61 | | |  | | |  | | |  | | |  | | |

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **24.** |  | | |  | | |  | | |  | | |  | | |
| Enter | 120 | | | .65% | | |  | | |  | | | $85,000 | | |
|  |  | **N** |  |  | **I/Y** |  |  | **PV** |  |  | **PMT** |  |  | **FV** |  | |
| Solve for |  | | |  | | | –$39,062.98 | | |  | | |  | | |

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| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **25.** |  | | |  | | |  | | |  | | |  | | |
| Enter | 45 | | | 11% | | |  | | |  | | | $1,000,000 | | |
|  |  | **N** |  |  | **I/Y** |  |  | **PV** |  |  | **PMT** |  |  | **FV** |  | |
| Solve for |  | | |  | | | –$9,129.90 | | |  | | |  | | |

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|  |  | | |  | | |  | | |  | | |  | | |
| Enter | 45 | | | 5.5% | | |  | | |  | | | $1,000,000 | | |
|  |  | **N** |  |  | **I/Y** |  |  | **PV** |  |  | **PMT** |  |  | **FV** |  | |
| Solve for |  | | |  | | | –$89,875.09 | | |  | | |  | | |

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| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **26.** |  | | |  | | |  | | |  | | |  | | |
| Enter | 20 | | | 7% | | | ±$20,000 | | |  | | |  | | |
|  |  | **N** |  |  | **I/Y** |  |  | **PV** |  |  | **PMT** |  |  | **FV** |  | |
| Solve for |  | | |  | | |  | | |  | | | $77,393.69 | | |

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| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  | | |  | | |  | | |  | | |  | | |
| Enter | 20 | | | 11% | | | ±$77,393.69 | | |  | | |  | | |
|  |  | **N** |  |  | **I/Y** |  |  | **PV** |  |  | **PMT** |  |  | **FV** |  | |
| Solve for |  | | |  | | |  | | |  | | | $623,972.03 | | |

**CHAPTER 5**

***DISCOUNTED CASH FLOW VALUATION***

**Answers to Concepts Review and Critical Thinking Questions**

**1.** Assuming positive cash flows and a positive interest rate, both the present and the future value will rise.

**2.** Assuming positive cash flows and a positive interest rate, the present value will fall, and the future value will rise.

**3.** It’s deceptive, but very common. The deception is particularly irritating given that such lotteries are usually government sponsored!

**4.** The most important consideration is the interest rate the lottery uses to calculate the lump sum option. If you can earn an interest rate that is higher than you are being offered, you can create larger annuity payments. Of course, taxes are also a consideration, as well as how badly you really need $5 million today.

**5.** If the total amount of money is fixed, you want as much as possible as soon as possible. The team (or, more accurately, the team owner) wants just the opposite.

**6.** The better deal is the one with equal installments.

**7.** Yes, they should. APRs generally don’t provide the relevant rate. The only advantage is that they are easier to compute, but, with modern computing equipment, that advantage is not very important.

**8.** A freshman does. The reason is that the freshman gets to use the money for much longer before interest starts to accrue.

**9.** The subsidy is the present value (on the day the loan is made) of the interest that would have accrued up until the time it actually begins to accrue.

**10.** The problem is that the subsidy makes it easier to repay the loan, not obtain it. However, the ability to repay the loan depends on future employment, not current need. For example, consider a student who is currently needy, but is preparing for a career in a high-paying area (such as corporate finance!). Should this student receive the subsidy? How about a student who is currently not needy, but is preparing for a relatively low-paying job (such as becoming a college professor)?

**Solutions to Questions and Problems**

*NOTE: All end-of-chapter problems were solved using a spreadsheet. Many problems require multiple steps. Due to space and readability constraints, when these intermediate steps are included in this solutions manual, rounding may appear to have occurred. However, the final answer for each problem is found without rounding during any step in the problem.*

*Basic*

**1.** The time line is:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | | 1 | | 2 | | 3 | | 4 | |  | |  | |  | |  | |  | |  | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| PV | | $680 | | $490 | | $975 | | $1,160 | |  | |  | |  | |  | |  | |  | |

To solve this problem, we must find the PV of each cash flow and add them. To find the PV of a lump sum, we use:

PV = FV / (1 + *r*)*t*

PV@10% = $680 / 1.10 + $490 / 1.102 + $975 / 1.103 + $1,160 / 1.104 = $2,547.97

PV@18% = $680 / 1.18 + $490 / 1.182 + $975 / 1.183 + $1,160 / 1.184 = $2,119.91

PV@24% = $680 / 1.24 + $490 / 1.242 + $975 / 1.243 + $1,160 / 1.244 = $1,869.09

**2.** The times lines are:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | | 1 | | 2 | | 3 | | 4 | | 5 | | 6 | | 7 | | 8 | | 9 | |  | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| PV | | $3,400 | | $3,400 | | $3,400 | | $3,400 | | $3,400 | | $3,400 | | $3,400 | | $3,400 | | $3,400 | |  | |

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | | 1 | | 2 | | 3 | | 4 | | 5 | |  | |  | |  | |  | |  | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| PV | | $5,200 | | $5,200 | | $5,200 | | $5,200 | | $5,200 | |  | |  | |  | |  | |  | |

To find the PVA, we use the equation:

PVA = *C*({1 – [1/(1 + *r*)*t*]} / *r* )

At an interest rate of 6 percent:

X@6%: PVA = $3,400{[1 – (1/1.06)9 ] / .06 } = $23,125.75

Y@6%: PVA = $5,200{[1 – (1/1.06)5 ] / .06 } = $21,904.29

And at an interest rate of 22 percent:

X@22%: PVA = $3,400{[1 – (1/1.22)9 ] / .22 } = $12,873.37

Y@22%: PVA = $5,200{[1 – (1/1.22)5 ] / .22 } = $14,890.93

Notice that the PV of Investment X has a greater PV at a 6 percent interest rate, but a lower PV at a 22 percent interest rate. The reason is that X has greater total cash flows. At a lower interest rate, the total cash flow is more important since the cost of waiting (the interest rate) is not as great. At a higher interest rate, Y is more valuable since it has larger annual payments. At a higher interest rate, getting these payments early are more important since the cost of waiting (the interest rate) is so much greater.

**3.** The time line is:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | | 1 | | 2 | | 3 | | 4 | |  | |  | |  | |  | |  | |  | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  | | $985 | | $1,160 | | $1,325 | | $1,495 | |  | |  | |  | |  | |  | |  | |

To solve this problem, we must find the FV of each cash flow and sum. To find the FV of a lump sum, we use:

FV = PV(1 + *r*)*t*

FV@8% = $985(1.08)3 + $1,160(1.08)2 + $1,325(1.08) + $1,495 = $5,519.84

FV@11% = $985(1.11)3 + $1,160(1.11)2 + $1,325(1.11) + $1,495 = $5,742.10

FV@24% = $985(1.24)3 + $1,160(1.24)2 + $1,325(1.24) + $1,495 = $6,799.64

Notice, since we are finding the value at Year 4, the cash flow at Year 4 is added to the FV of the other cash flows. In other words, we do not need to compound this cash flow.

**4.** To find the PVA, we use the equation:

PVA = *C*({1 – [1/(1 + *r*)*t*]} / *r*)

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | | 1 | |  | |  | |  | | **…** |  | |  | |  | |  | | 15 | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| PV | | $5,450 | | $5,450 | | $5,450 | | $5,450 | | $5,450 | | $5,450 | | $5,450 | | $5,450 | | $5,450 | |

PVA@15 yrs: PVA = $5,450{[1 – (1/1.08)15] / .08} = $46,649.16

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | | 1 | |  | |  | |  | | **…** |  | |  | |  | |  | | 40 | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| PV | | $5,450 | | $5,450 | | $5,450 | | $5,450 | | $5,450 | | $5,450 | | $5,450 | | $5,450 | | $5,450 | |

PVA@40 yrs: PVA = $5,450{[1 – (1/1.08)40] / .08} = $64,989.14

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | | 1 | |  | |  | |  | | **…** |  | |  | |  | |  | | 75 | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| PV | | $5,450 | | $5,450 | | $5,450 | | $5,450 | | $5,450 | | $5,450 | | $5,450 | | $5,450 | | $5,450 | |

PVA@75 yrs: PVA = $5,450{[1 – (1/1.08)75] / .08} = $67,912.91

To find the PV of a perpetuity, we use the equation:

PV = *C* / *r*

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | | 1 | |  | |  | |  | | **…** |  | |  | |  | |  | | ∞ | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| PV | | $5,450 | | $5,450 | | $5,450 | | $5,450 | | $5,450 | | $5,450 | | $5,450 | | $5,450 | | $5,450 | |

PV = $5,450 / .08

PV = $68,125

Notice that as the length of the annuity payments increases, the present value of the annuity approaches the present value of the perpetuity. The present value of the 75-year annuity and the present value of the perpetuity imply that the value today of all perpetuity payments beyond 75 years is only $212.09.

**5.** Here we have the PVA, the length of the annuity, and the interest rate. We want to calculate the annuity payment. Using the PVA equation and solving for the payment in each case, we find:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | | 1 | | 2 | | 3 | | 4 | | 5 | | 6 | |  | |  | |  | |  | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| $24,500 | | *C* | | *C* | | *C* | | *C* | | *C* | | *C* | |  | |  | |  | |  | |

PVA = *C*({1 – [1 / (1 + *r*)*t*]} / *r*))

$24,500 = $*C*{[1 – (1 / 1.11)6] / .11}

*C* = $24,500 / 4.23054

*C* = $5,791.23

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | | 1 | | 2 | | 3 | | 4 | | 5 | | 6 | | 7 | | 8 | |  | |  | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| $19,700 | | *C* | | *C* | | *C* | | *C* | | *C* | | *C* | | *C* | | *C* | |  | |  | |

PVA = *C*({1 – [1 / (1 + *r*)*t*]} / *r* ))

$19,700 = $*C*{[1 – (1 / 1.07)8] / .07}

*C* = $19,700 / 5.97130

*C* = $3,299.11

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | | 1 | |  | |  | |  | | **…** |  | |  | |  | |  | | 15 | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| $136,400 | | *C* | | *C* | | *C* | | *C* | | *C* | | *C* | | *C* | | *C* | | *C* | |

PVA = *C*({1 – [1 / (1 + *r*)*t*]} / *r* ))

$136,400 = $*C*{[1 – (1 / 1.08)15] / .08}

*C* = $136,400 / 8.55948

*C* = $15,935.55

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | | 1 | |  | |  | |  | | **…** |  | |  | |  | |  | | 20 | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| $286,650 | | *C* | | *C* | | *C* | | *C* | | *C* | | *C* | | *C* | | *C* | | *C* | |

PVA = *C*({1 – [1 / (1 + *r*)*t*]} / *r* ))

$286,650 = $*C*{[1 – (1 / 1 .06)20] / .06}

*C* = $286,650 / 11.46992

*C* = $24,904.27

**6.** Here we need to find the present value of an annuity. Using the PVA equation, we find:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | | 1 | | 2 | | 3 | | 4 | | 5 | | 6 | | 7 | |  | |  | |  | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| PVA | | $2,100 | | $2,100 | | $2,100 | | $2,100 | | $2,100 | | $2,100 | | $2,100 | |  | |  | |  | |

PVA = *C*({1 – [1 / (1 + *r*)*t*]} / *r*)

PVA = $2,100{[1 – (1 / 1.05)7] / .05}

PVA = $12,151.38

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | | 1 | | 2 | | 3 | | 4 | | 5 | | 6 | | 7 | | 8 | | 9 | |  | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| PVA | | $1,095 | | $1,095 | | $1,095 | | $1,095 | | $1,095 | | $1,095 | | $1,095 | | $1,095 | | $1,095 | |  | |

PVA = *C*({1 – [1 / (1 + *r*)*t*]} / *r*)

PVA = $1,095{[1 – (1 / 1.10)9 ] / .10}

PVA = $6,306.13

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | | 1 | |  | |  | |  | | **…** |  | |  | |  | |  | | 18 | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| PVA | | $11,000 | | $11,000 | | $11,000 | | $11,000 | | $11,000 | | $11,000 | | $11,000 | | $11,000 | | $11,000 | |

PVA = *C*({1 – [1 / (1 + *r*)*t*]} / *r*)

PVA = $11,000{[1 – (1 / 1.08)18 ] / .08}

PVA = $103,090.76

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | | 1 | |  | |  | |  | | **…** |  | |  | |  | |  | | 28 | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| PVA | | $30,000 | | $30,000 | | $30,000 | | $30,000 | | $30,000 | | $30,000 | | $30,000 | | $30,000 | | $30,000 | |

PVA = *C*({1 – [1 / (1 + *r*)*t*]} / *r*)

PVA = $30,000{[1 – (1 / 1.14)28] / .14}

PVA = $208,819.87

**7.** Here we have the FVA, the length of the annuity, and the interest rate. We want to calculate the annuity payment. Using the FVA equation:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | | 1 | |  | |  | |  | |  | |  | |  | | 8 | |  | |  | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  | |  | |  | |  | |  | |  | |  | |  | | $30,000 | |  | |  | |
|  | | *C* | | *C* | | *C* | | *C* | | *C* | | *C* | | *C* | | *C* | |  | |  | |

FVA = *C*{[(1 + *r*)*t* – 1] / *r*}

$30,000 = $*C*[(1.058 – 1) / .05]

*C* = $30,000 / 9.54911

*C* = $3,141.65

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | | 1 | |  | |  | |  | | **…** |  | |  | |  | |  | | 40 | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  | |  | |  | |  | |  | |  | |  | |  | |  | | $1,200,000 | |
|  | | *C* | | *C* | | *C* | | *C* | |  | *C* | | *C* | | *C* | | *C* | | *C* | |

FVA = *C*{[(1 + *r*)*t* – 1] / *r*}

$1,200,000 = $*C*[(1.0740 – 1) / .07]

*C* = $1,200,000 / 199.63511

*C* = $6,010.97

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | | 1 | |  | |  | |  | | **…** |  | |  | |  | |  | | 25 | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  | |  | |  | |  | |  | |  | |  | |  | |  | | $625,000 | |
|  | | *C* | | *C* | | *C* | | *C* | |  | *C* | | *C* | | *C* | | *C* | | *C* | |

FVA = *C*{[(1 + *r*)*t* – 1] / *r*}

$625,000 = $*C*[(1.0825 – 1) / .08]

*C* = $625,000 / 73.10594

*C* = $8,549.24

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | | 1 | |  | |  | |  | | **…** |  | |  | |  | |  | | 13 | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  | |  | |  | |  | |  | |  | |  | |  | |  | | $125,000 | |
|  | | *C* | | *C* | | *C* | | *C* | |  | *C* | | *C* | | *C* | | *C* | | *C* | |

FVA = *C*{[(1 + *r*)*t* – 1] / *r*}

$125,000 = $*C*[(1.0413 – 1) / .04]

*C* = $125,000 / 16.62684

*C* = $7,517.97

**8.** Here, we need to find the future value of an annuity. Using the FVA equation, we find:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | | 1 | |  | |  | |  | |  | |  | |  | |  | |  | | 10 | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  | |  | |  | |  | |  | |  | |  | |  | |  | |  | | FVA | |
|  | | $1,900 | | $1,900 | | $1,900 | | $1,900 | | $1,900 | | $1,900 | | $1,900 | | $1,900 | | $1,900 | | $1,900 | |

FVA = *C*{[(1 + *r*)*t* – 1] / *r*}

FVA = $1,900[(1.0810 – 1) / .08]

FVA = $27,524.47

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | | 1 | |  | |  | |  | | **…** |  | |  | |  | |  | | 40 | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  | |  | |  | |  | |  | |  | |  | |  | |  | | FVA | |
|  | | $6,000 | | $6,000 | | $6,000 | | $6,000 | |  | $6,000 | | $6,000 | | $6,000 | | $6,000 | | $6,000 | |

FVA = *C*{[(1 + *r*)*t* – 1] / *r*}

FVA = $6,000[(1.0940 – 1) / .09]

FVA = $2,027,294.67

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | | 1 | |  | |  | |  | |  | |  | |  | |  | | 9 | |  | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  | |  | |  | |  | |  | |  | |  | |  | |  | | FVA | |  | |
|  | | $2,950 | | $2,950 | | $2,950 | | $2,950 | | $2,950 | | $2,950 | | $2,950 | | $2,950 | | $2,950 | |  | |

FVA = *C*{[(1 + *r*)*t* – 1] / *r*}

FVA = $2,950[(1.069 – 1) / .06]

FVA = $33,899.38

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | | 1 | |  | |  | |  | | **…** |  | |  | |  | |  | | 30 | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  | |  | |  | |  | |  | |  | |  | |  | |  | | FVA | |
|  | | $6,400 | | $6,400 | | $6,400 | | $6,400 | |  | $6,400 | | $6,400 | | $6,400 | | $6,400 | | $6,400 | |

FVA = *C*{[(1 + *r*)*t* – 1] / *r*}

FVA = $6,400[(1.1030 – 1) / .10]

FVA = $1,052,761.75

**9.** Here we need to find the FVA. The equation to find the FVA is:

FVA = *C*{[(1 + *r*)*t* – 1] / *r*}

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | | 1 | |  | |  | |  | | **…** |  | |  | |  | |  | | 20 | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  | |  | |  | |  | |  | |  | |  | |  | |  | | FVA | |
|  | | $5,000 | | $5,000 | | $5,000 | | $5,000 | |  | $5,000 | | $5,000 | | $5,000 | | $5,000 | | $5,000 | |

FVA for 20 years = $5,000[(1.09620 – 1) / .096]

FVA for 20 years = $273,685.74

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | | 1 | |  | |  | |  | | **…** |  | |  | |  | |  | | 40 | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  | |  | |  | |  | |  | |  | |  | |  | |  | | FVA | |
|  | | $5,000 | | $5,000 | | $5,000 | | $5,000 | |  | $5,000 | | $5,000 | | $5,000 | | $5,000 | | $5,000 | |

FVA for 40 years = $5,000[(1.09640 – 1) / .096]

FVA for 40 years = $1,985,526.07

Notice that because of exponential growth, doubling the number of periods does not merely double the FVA.

**10.** The time line is:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | | 1 | |  | |  | |  | | **…** |  | |  | |  | |  | | ∞ | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| PV | | $30,000 | | $30,000 | | $30,000 | | $30,000 | | $30,000 | | $30,000 | | $30,000 | | $30,000 | | $30,000 | |

This cash flow is a perpetuity. To find the PV of a perpetuity, we use the equation:

PV = *C* / *r*

PV = $30,000 / .05

PV = $600,000.00

**11.** The time line is:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | | 1 | |  | |  | |  | | **…** |  | |  | |  | |  | | ∞ | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| –$645,000 | | $30,000 | | $30,000 | | $30,000 | | $30,000 | | $30,000 | | $30,000 | | $30,000 | | $30,000 | | $30,000 | |

Here we need to find the interest rate that equates the perpetuity cash flows with the PV of the cash flows. Using the PV of a perpetuity equation:

PV = *C* / *r*

$645,000 = $30,000 / *r*

We can now solve for the interest rate as follows:

*r* = $30,000 / $645,000

*r* = .0465, or 4.65%

**12.** For discrete compounding, to find the EAR, we use the equation:

EAR = [1 + (APR / *m*)]*m* – 1

EAR = [1 + (.10 / 4)]4 – 1 = .1038, or 10.38%

EAR = [1 + (.17 / 12)]12 – 1 = .1839, or 18.39%

EAR = [1 + (.13 / 365)]365 – 1 = .1388, or 13.88%

EAR = [1 + (.09 / 2)]2 – 1 = .0920, or 9.20%

**13.** Here we are given the EAR and need to find the APR. Using the equation for discrete compounding:

EAR = [1 + (APR / *m*)]*m* – 1

We can now solve for the APR. Doing so, we get:

APR = *m*[(1 + EAR)1/*m* – 1]

EAR = .14 = [1 + (APR / 2)]2 – 1 APR = 2[(1.14)1/2 – 1] = .1354, or 13.54%

EAR = .09 = [1 + (APR / 12)]12 – 1 APR = 12[(1.09)1/12 – 1] = .0865, or 8.65%

EAR = .08 = [1 + (APR / 52)]52 – 1 APR = 52[(1.08)1/52 – 1] = .0770, or 7.70%

EAR = .13 = [1 + (APR / 365)]365 – 1 APR = 365[(1.13)1/365 – 1] = .1222, or 12.22%

**14.** For discrete compounding, to find the EAR, we use the equation:

EAR = [1 + (APR / *m*)]*m* – 1

So, for each bank, the EAR is:

First National: EAR = [1 + (.101 / 12)]12 – 1 = .1058, or 10.58%

First United: EAR = [1 + (.103 / 2)]2 – 1 = .1057, or 10.57%

For a borrower, First United would be preferred since the EAR of the loan is lower. Notice that the higher APR does not necessarily mean the higher EAR. The number of compounding periods within a year will also affect the EAR.

**15.** The reported rate is the APR, so we need to convert the EAR to an APR as follows:

EAR = [1 + (APR / *m*)]*m* – 1

APR = *m*[(1 + EAR)1/*m* – 1]

APR = 365[(1.142)1/365 – 1]

APR = .1328, or 13.28%

This is deceptive because the borrower is actually paying annualized interest of 14.2 percent per year, not the 13.28 percent reported on the loan contract.

**16.** The time line is:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | | 1 | |  | |  | |  | | **…** |  | |  | |  | |  | | 32 | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| $1,345 | |  | |  | |  | |  | |  | |  | |  | |  | | FV | |

For this problem, we need to find the FV of a lump sum using the equation:

FV = PV(1 + *r*)*t*

It is important to note that compounding occurs semiannually. To account for this, we will divide the interest rate by two (the number of compounding periods in a year), and multiply the number of periods by two. Doing so, we get:

FV = $1,345[1 + (.084 / 2)]32

FV = $5,017.52

**17.** For this problem, we need to find the FV of a lump sum using the equation:

FV = PV(1 + *r*)*t*

It is important to note that compounding occurs daily. To account for this, we will divide the interest rate by 365 (the number of days in a year, ignoring leap year), and multiply the number of periods by 365. Doing so, we get:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | | 1 | |  | |  | |  | | **…** |  | |  | |  | |  | | 5(365) | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| $3,650 | |  | |  | |  | |  | |  | |  | |  | |  | | FV | |

FV in 5 years = $3,650[1 + (.047 / 365)]5(365) = $4,616.85

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | | 1 | |  | |  | |  | | **…** |  | |  | |  | |  | | 10(365) | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| $3,650 | |  | |  | |  | |  | |  | |  | |  | |  | | FV | |

FV in 10 years = $3,650[1 + (.047 / 365)]10(365) = $5,839.80

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | | 1 | |  | |  | |  | | **…** |  | |  | |  | |  | | 20(365) | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| $3,650 | |  | |  | |  | |  | |  | |  | |  | |  | | FV | |

FV in 20 years = $3,650[1 + (.047 / 365)]20(365) = $9,343.37

**18.** The time line is:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | | 1 | |  | |  | |  | | **…** |  | |  | |  | |  | | 9(365) | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| PV | |  | |  | |  | |  | |  | |  | |  | |  | | $65,000 | |

For this problem, we simply need to find the PV of a lump sum using the equation:

PV = FV / (1 + *r*)*t*

It is important to note that compounding occurs on a daily basis. To account for this, we will divide the interest rate by 365 (the number of days in a year, ignoring leap year), and multiply the number of periods by 365. Doing so, we get:

PV = $65,000 / [(1 + .055 / 365)9(365)]

PV = $39,623.59

**19.** The APR is simply the interest rate per period times the number of periods in a year. In this case, the interest rate is 13.7 percent per month, and there are 12 months in a year, so we get:

APR = 12(13.7%)

APR = 164.40%

To find the EAR, we use the EAR formula:

EAR = [1 + (APR / *m*)]*m* – 1

EAR = (1 + .137)12 – 1

EAR = 3.6679, or 366.79%

Notice that we didn’t need to divide the APR by the number of compounding periods per year. We do this division to get the interest rate per period, but in this problem we are already given the interest rate per period.

**20.** The time line is:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | | 1 | |  | |  | |  | | **…** |  | |  | |  | |  | | 60 | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| $73,400 | | *C* | | *C* | | *C* | | *C* | | *C* | | *C* | | *C* | | *C* | | *C* | |

We first need to find the annuity payment. We have the PVA, the length of the annuity, and the interest rate. Using the PVA equation:

PVA = *C*({1 – [1 / (1 + *r*)*t*]} / *r* )

$73,400 = $*C*[1 – {1 / [1 + (.051/12)]60} / (.051/12)]

Solving for the payment, we get:

*C* = $73,400 / 52.86227

*C* = $1,388.51

To find the EAR, we use the EAR equation:

EAR = [1 + (APR / *m*)]*m* – 1

EAR = [1 + (.051 / 12)]12 – 1

EAR = .0522, or 5.22%

**21.** The time line is:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | | 1 | |  | |  | |  | | **…** |  | |  | |  | |  | | *t* | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| –$14,480 | | $400 | | $400 | | $400 | | $400 | | $400 | | $400 | | $400 | | $400 | | $400 | |

Here we need to find the length of an annuity. We know the interest rate, the PV, and the payments. Using the PVA equation:

PVA = *C*({1 – [1/(1 + *r*)*t*]} / *r*)

$14,480 = $400{[1 – (1/1.014)*t* ] / .014}

Now we solve for *t*:

1/1.014*t* = 1 – [($14,480)(.014) / ($400)]

1.014*t* = 1 / .4932

*t* = ln 2.0276 / ln 1.014

*t* = 50.84 months

**22.** The time line is:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | | 1 | |  | |  | |  | | **…** |  | |  | |  | |  | | *t* | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| –$4 | |  | |  | |  | |  | |  | |  | |  | |  | | $5 | |

Here we are trying to find the interest rate when we know the PV and FV. Using the FV equation:

FV = PV(1 + *r*)

$5 = $4(1 + *r*)

*r* = $5/$4 – 1

*r* = .2500, or 25.00% per week

The interest rate is 25 percent per week. To find the APR, we multiply this rate by the number of weeks in a year, so:

APR = 52(25%) = 1,300.00%

And using the equation to find the EAR, we find:

EAR = [1 + (APR / *m*)]*m* – 1

EAR = [1 + .25]52 – 1

EAR = 109,475.4425 or 10,947,544.25%

**23.** The time line is:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | | 1 | |  | |  | |  | | **…** |  | |  | |  | |  | | ∞ | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| –$425,000 | | $3,300 | | $3,300 | | $3,300 | | $3,300 | | $3,300 | | $3,300 | | $3,300 | | $3,300 | | $3,300 | |

Here we need to find the interest rate that equates the perpetuity cash flows with the PV of the cash flows. Using the PV of a perpetuity equation:

PV = *C* / *r*

$425,000 = $3,300 / *r*

We can now solve for the interest rate as follows:

*r* = $3,300 / $425,000

*r* = .0078, or .78% per month

The interest rate is .78 percent per month. To find the APR, we multiply this rate by the number of months in a year, so:

APR = 12(.78%)

APR = 9.32%

And using the equation to find the EAR, we find:

EAR = [1 + (APR / *m*)]*m* – 1

EAR = [1 + .0078]12 – 1

EAR = .0973, or 9.73%

**24.** The time line is:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | | 1 | |  | |  | |  | | **…** |  | |  | |  | |  | | 420 | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  | |  | |  | |  | |  | |  | |  | |  | |  | | FVA | |
|  | | $500 | | $500 | | $500 | | $500 | |  | $500 | | $500 | | $500 | | $500 | | $500 | |

This problem requires us to find the FVA. The equation to find the FVA is:

FVA = *C*{[(1 + *r*)*t* – 1] / *r*}

FVA = $500[{[1 + (.098 / 12)]420 – 1} / (.098 / 12)]

FVA = $1,803,031.33

**25.** In the previous problem, the compounding period is monthly. This assumption still holds. Since the cash flows are annual, we need to use the EAR to calculate the future value of annual cash flows. It is important to remember that you have to make sure the compounding periods of the interest rate matches with the cash flows. In this case, we have annual cash flows, so we need the EAR since it is the true annual interest rate you will earn. So, finding the EAR:

EAR = [1 + (APR / *m*)]*m* – 1

EAR = [1 + (.098 / 12)]12 – 1

EAR = .1025, or 10.25%

The time line is:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | | 1 | |  | |  | |  | | **…** |  | |  | |  | |  | | 35 | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  | |  | |  | |  | |  | |  | |  | |  | |  | | FVA | |
|  | | $6,000 | | $6,000 | | $6,000 | | $6,000 | |  | $6,000 | | $6,000 | | $6,000 | | $6,000 | | $6,000 | |

Using the FVA equation, we get:

FVA = *C*{[(1 + *r*)*t* – 1] / *r*}

FVA = $6,000[(1.102535 – 1) / .1025]

FVA = $1,723,472.13

**26.** The time line is:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | | 1 | |  | |  | |  | | **…** |  | |  | |  | |  | | 16 | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| PVA | | $2,500 | | $2,500 | | $2,500 | | $2,500 | | $2,500 | | $2,500 | | $2,500 | | $2,500 | | $2,500 | |

The cash flows are an annuity with four payments per year for four years, or 16 payments. We can use the PVA equation:

PVA = *C*({1 – [1/(1 + *r*)*t*]} / *r* ))

PVA = $2,500{[1 – (1 / 1.0038)16] / .0038}

PVA = $38,736.93

**27.** The time line is:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | | 1 | | 2 | | 3 | | 4 | |  | |  | |  | |  | |  | |  | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| PV | | $1,200 | | $1,100 | | $800 | | $600 | |  | |  | |  | |  | |  | |  | |

To solve this problem, we must find the PV of each cash flow and add them. To find the PV of a lump sum, we use:

PV = FV / (1 + *r*)*t*

PV = $1,200 / 1.0675 + $1,100 / 1.06752 + $800 / 1.06753 + $600 / 1.06754

PV = $3,209.09

**28.** The time line is:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | | 1 | | 2 | | 3 | | 4 | |  | |  | |  | |  | |  | |  | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| PV | | $1,400 | | $1,900 | | $3,400 | | $4,300 | |  | |  | |  | |  | |  | |  | |

To solve this problem, we must find the PV of each cash flow and add them. To find the PV of a lump sum, we use:

PV = FV / (1 + *r*)*t*

PV = $1,400 / 1.0713 + $1,900 / 1.07132 + $3,400 / 1.07133 + $4,300 / 1.07134

PV = $8,992.21

*Intermediate*

**29.** The total interest paid by First Simple Bank is the interest rate per period times the number of periods. In other words, the interest by First Simple Bank paid over 10 years will be:

.058(10) = .58

First Complex Bank pays compound interest, so the interest paid by this bank will be the FV factor of $1, or:

(1 + *r*)10

Setting the two equal, we get:

(.058)(10) = (1 + *r*)10 – 1

*r* = 1.581/10 – 1

*r* = .0468, or 4.68%

**30.** The time line is:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | | 1 | |  | |  | |  | | **…** |  | |  | |  | |  | | 60 | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| $58,600 | |  | |  | |  | |  | |  | |  | |  | |  | |  | |
| *C* | | *C* | | *C* | | *C* | | *C* | |  | *C* | | *C* | | *C* | | *C* | |  | |

We need to use the PVA due equation, which is:

PVAdue = (1 + *r*)PVA

Using this equation:

PVAdue = $58,600 = [1 + (.052 / 12)] × *C*[{1 – 1 / [1 + (.052 / 12)]60]} / (.052 / 12)

$58,347.16 = $*C*{1 – [1 / (1 + .052 / 12)60]} / (.052 / 12)

*C* = $1,106.44

Notice, to find the payment for the PVA due, we find the PV of an ordinary annuity, then compound this amount forward one period.

**31.** The time line is:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | | 1 | |  | |  | |  | | **…** |  | |  | |  | |  | | 12 | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| –$10,000 | |  | |  | |  | |  | |  | |  | |  | |  | | FV | |

Here we need to find the FV of a lump sum, with a changing interest rate. We must do this problem in two parts.After the first six months, the balance will be:

FV = $10,000[1 + (.016 / 12)]6

FV = $10,080.27

This is the balance in six months. The FV in another six months will be:

FV = $10,080.27[1 + (.179 / 12)]6

FV = $11,016.77

The problem asks for the interest accrued, so, to find the interest, we subtract the beginning balance from the principal. The interest accrued is:

Interest = $11,016.77 – 10,000.00

Interest = $1,016.77

**32.** We will calculate the time we must wait if we deposit in the bank that pays simple interest. The interest amount we will receive each year in this bank will be:

Interest = $85,000(.048)

Interest = $4,080 per year

The deposit will have to increase by the difference between the amount we need by the amount we originally deposit divided by the interest earned per year, so the number of years it will take in the bank that pays simple interest is:

Years to wait = ($235,000 – 85,000) / $4,080

Years to wait = 36.76 years

To find the number of years it will take in the bank that pays compound interest, we can use the future value equation for a lump sum and solve for the periods. Doing so, we find:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | | 1 | |  | |  | |  | | **…** |  | |  | |  | |  | | *t* | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| –$85,000 | |  | |  | |  | |  | |  | |  | |  | |  | | $235,000 | |

FV = PV(1 + *r*)*t*

$235,000 = $85,000[1 + (.048 / 12)]*t*

*t* = 254.74 months, or 21.23 years

**33.** The time line is:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | | 1 | |  | |  | |  | | **…** |  | |  | |  | |  | | 12 | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| –$1 | |  | |  | |  | |  | |  | |  | |  | |  | | FV | |

Here we need to find the future value of a lump sum. We need to make sure to use the correct number of periods. So, the future value after one year will be:

FV = PV(1 + *r*)*t*

FV = $1(1.0129)12

FV = $1.17

And the future value after two years will be:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | | 1 | |  | |  | |  | | **…** |  | |  | |  | |  | | 24 | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| –$1 | |  | |  | |  | |  | |  | |  | |  | |  | | FV | |

FV = PV(1 + *r*)*t*

FV = $1(1.0129)24

FV = $1.36

**34.** The time line is:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | | 1 | |  | |  | |  | | **…** |  | |  | |  | |  | | 31 | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| –£440 | | £60 | | £60 | | £60 | | £60 | | £60 | | £60 | | £60 | | £60 | | £60 | |

Here we are given the PVA, number of periods, and the amount of the annuity. We need to solve for the interest rate. Even though the currency is pounds and not dollars, we can still use the same time value equations. Using the PVA equation:

PVA = *C*({1 – [1/(1 + *r*)*t*]} / *r* )

£440 = £60[{1 – [1 / (1 + *r*)]31}/ *r*]

To find the interest rate, we need to solve this equation on a financial calculator, using a spreadsheet, or by trial and error. If you use trial and error, remember that increasing the interest rate decreases the PVA, and decreasing the interest rate increases the PVA. Using a spreadsheet, we find:

*r* = 13.36%

Not bad for an English Literature major!

**35.** Here we need to compare two cash flows. The only way to compare cash flows is to find the value of the cash flows at a common time, so we will find the present value of each cash flow stream. Since the cash flows are monthly, we need to use the monthly interest rate, which is:

Monthly rate = .07 / 12

Monthly rate = .0058 or .58%

The value today of the $6,700 monthly salary is:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | | 1 | |  | |  | |  | | **…** |  | |  | |  | |  | | 24 | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| PVA | | $6,700 | | $6,700 | | $6,700 | | $6,700 | | $6,700 | | $6,700 | | $6,700 | | $6,700 | | $6,700 | |

PVA = *C*({1 – [1/(1 + *r*)*t*]} / *r* )

PVA = $6,700{[1 – (1 / 1.0058)24] / .0058}

PVA = $149,645.17

To find the value of the second option, we find the present value of the monthly payments and add the bonus. We can add the bonus since it is paid today. So:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | | 1 | |  | |  | |  | | **…** |  | |  | |  | |  | | 24 | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| $25,000 | | $5,400 | | $5,400 | | $5,400 | | $5,400 | | $5,400 | | $5,400 | | $5,400 | | $5,400 | | $5,400 | |

PVA = *C*({1 – [1/(1 + *r*)*t*]} / *r* )*)*

PVA = $5,400{[1 – (1/1.0058)24] / .0058}

PVA = $120,609.54

So, the total value of the second option is:

Value of second option = $120,609.54 + 25,000

Value of second option = $145,609.54

The difference in the value of the two options today is:

Difference in value today = $149,645.17 – 145,609.54

Difference in value today = $4,035.63

What if we found the future value of the two cash flows? For the monthly salary, the future value will be:

FVA = *C*{[(1 + *r*)*t* – 1] / *r*}

FVA = $6,700{[(1 + .0058*)*24 – 1] / .0058}

FVA = $172,062.91

To find the future value of the second option we also need to find the future value of the bonus as well. So, the future value of this option is:

FV = *C*{[(1 + *r*)*t* – 1] / *r*} + PV(1 + *r*)*t*

FV = $5,400{[(1 + .0058*)*24 – 1] / .0058} + $25,000(1 + .0058)24

FV = $167,422.72

So, the first option is still the better choice. The difference between the future values of the two options is:

Difference in future value = $172,062.91 – 167,422.72

Difference in future value = $4,640.19

No matter when you compare two cash flows, the cash flow with the greatest value in one period will always have the greatest value in any other period. Here’s a question for you: What is the future value of $4,035.63 (the difference in the cash flows at time zero) in 24 months at an interest rate of .58 percent per month? With no calculations, you know the future value must be $4,640.19, the difference in the cash flows at the same time!

**36.** The time line is:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | | 1 | |  | |  | |  | | **…** |  | |  | |  | |  | | 20 | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| PVA | | $17,500 | | $17,500 | | $17,500 | | $17,500 | | $17,500 | | $17,500 | | $17,500 | | $17,500 | | $17,500 | |

The cash flows are an annuity, so we can use the present value of an annuity equation. Doing so, we find:

PVA = *C*(1 – [1 / (1 + *r*)*t*]} / *r*))

PVA = $17,500[1 – (1 / 1.08)20 / .08]

PVA = $171,817.58

**37.** The investment we should choose is the investment with the higher rate of return. We will use the future value equation to find the interest rate for each option. Doing so, we find the return for Investment G is:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | |  | |  | |  | |  | |  | |  | |  | |  | |  | | 6 | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| –$39,000 | |  | |  | |  | |  | |  | |  | |  | |  | |  | | $75,000 | |

FV = PV(1 + *r*)*t*

$75,000 = $39,000(1 + *r*)6

*r* = ($75,000 / $39,000)1/6 – 1

*r* = .1151, or 11.51%

And, the return for Investment H is:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | |  | |  | |  | |  | |  | |  | |  | |  | |  | | 9 | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| –$39,000 | |  | |  | |  | |  | |  | |  | |  | |  | |  | | $105,000 | |

FV = PV(1 + *r*)*t*

$105,000 = $39,000(1 + *r*)9

*r* = ($105,000 / $39,000)1/9 – 1

*r* = .1163, or 11.63%

So, we should choose Investment H.

**38.** The time line is:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | | 1 | | 2 | | 3 | | 4 | | 5 | | 6 | | 7 | | 8 | | 9 | | 10 | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| PVA | | $7,300 | | $7,300 | | $7,300 | | $7,300 | | $7,300 | | $7,300 | | $7,300 | | $7,300 | | $7,300 | | $7,300 | |

The present value of an annuity falls as *r* increases, and the present value of an annuity rises as *r* decreases. The future value of an annuity rises as *r* increases, and the future value of an annuity falls as *r* decreases.

Here we need to calculate the present value of an annuity for different interest rates. Using the present value of an annuity equation and an interest rate of 10 percent, we get:

PVA = *C*({1 – [1 / (1 + *r*)*t*]} / *r*))

PVA = $7,300{[1 – (1 / 1.10)10] / .10}

PVA = $44,855.34

At an interest rate of 5 percent, the present value of the annuity is:

PVA = *C*({1 – [1 / (1 + *r*)*t*]} / *r*))

PVA = $7,300{[1 – (1 / 1.05)10] / .05}

PVA = $56,368.66

And, at an interest rate of 15 percent, the present value of the annuity is:

PVA = *C*({1 – [1 / (1 + *r*)*t*]} / *r*))

PVA = $7,300{[1 – (1 / 1.15)10] / .15}

PVA = $36,637.01

**39.** The time line is:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | | 1 | | 2 | | 3 | | 4 | | 5 | | 6 | | 7 | | **…** | *t* | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  | |  | |  | |  | |  | |  | |  | |  | | –$50,000 | |
|  | | $250 | | $250 | | $250 | | $250 | | $250 | | $250 | | $250 | |  | $250 | |

Here we are given the future value of an annuity, the interest rate, and the number of payments. We need to find the number of periods of the annuity payments. So, we can solve the future value of an annuity equation for the number of periods as follows:

FVA = *C*{[(1 + *r*)*t* – 1] / *r*}

$50,000 = $250[{[1 + (.08 / 12)]*t* – 1} / (.08 / 12)]

200 = {[1 + (.08 / 12)]*t* – 1} / (.08 / 12)

1.333 = (1 + .00667)*t* – 1

2.333 = (1.00667)*t*

ln 2.333 = *t* (ln 1.00667)

*t* = ln 2.333 / ln 1.00667

*t* = 127.52 payments

**40.** The time line is:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | | 1 | |  | |  | |  | | **…** |  | |  | |  | |  | | 60 | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| –$80,000 | | $1,650 | | $1,650 | | $1,650 | | $1,650 | | $1,650 | | $1,650 | | $1,650 | | $1,650 | | $1,650 | |

Here we are given the PVA, number of periods, and the amount of the annuity. We need to solve for the interest rate. Using the PVA equation:

PVA = ({1 – [1/(1 + *r*)*t*]} / *r* ))

$80,000 = $1,650[{1 – [1 / (1 + *r*)]60}/ *r*]

To find the interest rate, we need to solve this equation on a financial calculator, using a spreadsheet, or by trial and error. If you use trial and error, remember that increasing the interest rate decreases the PVA, and decreasing the interest rate increases the PVA. Using a spreadsheet, we find:

*r* = .00727, or .727%

This is the monthly interest rate. To find the APR with a monthly interest rate, we simply multiply the monthly rate by 12, so the APR is:

APR = .00727 × 12

APR = .0872, or 8.72%

**41.** The time line is:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | | 1 | | 2 | | 3 | | 4 | | 5 | |  | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |
| PV | | $3,400,000 | | $12,400,000 | | $12,400,000 | | $13,400,000 | | $13,400,000 | |  | |

To solve this problem, we must find the PV of each cash flow and add them. To find the PV of a lump sum, we use:

PV = FV / (1 + *r*)*t*

PV = $3,400,000 / 1.11 + $12,400,000 / 1.112 + $12,400,000 / 1.113

+ $13,400,000 / 1.114 + $13,400,000 / 1.115

PV = $38,973,197.21

**42.** The time line is:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | | 1 | | 2 | | 3 | | 4 | | 5 | | 6 | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |
| $5,100,000 | | $985,000 | | $28,585,000 | | $15,085,000 | | $22,085,000 | | $24,085,000 | | $18,360,000 | |

To solve this problem, we must find the PV of each cash flow and add them. To find the PV of a lump sum, we use:

PV = FV / (1 + *r*)*t*

PV = $5,100,000 + $985,000 / 1.11 + $28,585,000 / 1.112 + $15,085,000 / 1.113 + $22,085,000 / 1.114 + $24,085,000 / 1.115 + $18,360,000 / 1.116

PV = $78,874,991.18

**43.** Here we are given the PVA, number of periods, and the amount of the annuity. We need to solve for the interest rate. First, we need to find the amount borrowed since it is only 80 percent of the building value. So, the amount borrowed is:

Amount borrowed = .80($2,950,000)

Amount borrowed = $2,360,000

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | | 1 | |  | |  | |  | | **…** |  | |  | |  | |  | | 360 | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| –$2,360,000 | | $14,300 | | $14,300 | | $14,300 | | $14,300 | | $14,300 | | $14,300 | | $14,300 | | $14,300 | | $14,300 | |

Now we can use the PVA equation:

PVA = *C*({1 – [1 / (1 + *r*)*t*]} / *r*))

$2,360,000 = $14,300[{1 – [1 / (1 + *r*)]360}/ *r*]

To find the interest rate, we need to solve this equation on a financial calculator, using a spreadsheet, or by trial and error. If you use trial and error, remember that increasing the interest rate decreases the PVA, and decreasing the interest rate increases the PVA. Using a spreadsheet, we find:

*r* = .00508, or .508%

This is the monthly interest rate. To find the APR with a monthly interest rate, we simply multiply the monthly rate by 12, so the APR is:

APR = .00508 × 12

APR = .0610, or 6.10%

And the EAR is:

EAR = [1 + (APR / *m*)]*m* – 1

EAR = [1 + .00508]12 – 1

EAR = .0627, or 6.27%

**44.** Here, we have two cash flow streams that will be combined in the future. In essence, we have three time lines. We will start with the time lines for the savings period, which are:

Bond account:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | | 1 | | 2 | | 3 | | 4 | | 5 | | 6 | | 7 | | 8 | | 9 | | 10 | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  | $75,000 | | $6,000 | | $6,000 | | $6,000 | | $6,000 | | $6,000 | | $6,000 | | $6,000 | | $6,000 | | $6,000 | | $6,000 | |

Stock account:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | |  | |  | |  | |  | |  | |  | |  | |  | |  | | 10 | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  | $300,000 | |  | |  | |  | |  | |  | |  | |  | |  | |  | |  | |

To find the withdrawal amount, we need to know the present value, as well as the interest rate and periods, which are given. The present value of the retirement account is the future value of the stock and bond account. We need to find the future value of each account and add the future values together. For the bond account the future value is the value of the current savings plus the value of the annual deposits. So, the future value of the bond account will be:

FV = *C*{[(1 + *r*)*t* – 1] / *r*} + PV(1 + *r*)*t*

FV = $6,000{[(1 + .07)10 – 1] / .07} + $75,000(1 + .07)10

FV = $230,435.04

The total value of the stock account at retirement will be the future value of a lump sum, so:

FV = PV(1 + *r*)*t*

FV = $300,000(1 + .105)10

FV = $814,224.25

The total value of the account at retirement will be:

Total value at retirement = $230,435.04 + 814,224.25

Total value at retirement = $1,044,659.29

So, at retirement, we have:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | | 1 | |  | |  | |  | | **…** |  | |  | |  | |  | | 25 | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| –$1,044,659.29 | | *C* | | *C* | | *C* | | *C* | | *C* | | *C* | | *C* | | *C* | | *C* | |

This amount is the present value of the annual withdrawals. Now we can use the present value of an annuity equation to find the annuity amount. Doing so, we find the annual withdrawal will be:

PVA = *C*({1 – [1 / (1 + *r*)*t*]} / *r*))

$1,044,659.29 = *C*[{1 – [1 / (1 + .0625)]25} / .0625]

*C* = $83,671.59

**45.** The time line is:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | | 1 | |  | |  | |  | | **…** |  | |  | |  | |  | | 60 | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| –$24,500 | | $465 | | $465 | | $465 | | $465 | | $465 | | $465 | | $465 | | $465 | | $465 | |

Here we are given the PVA for an annuity due, number of periods, and the amount of the annuity. We need to solve for the interest rate. Using the PVA equation:

PVA = *C*[{1 – [1 / (1 + *r*)]*t*}/ *r*](1 + *r*)

$24,500 = $465[{1 – [1 / (1 + *r*)]60}/ *r*](1 + *r*)

To find the interest rate, we need to solve this equation on a financial calculator, using a spreadsheet, or by trial and error. If you use trial and error, remember that increasing the interest rate decreases the PVA, and decreasing the interest rate increases the PVA. Using a spreadsheet, we find:

*r* = .00452, or .452%

This is the monthly interest rate. To find the APR with a monthly interest rate, we simply multiply the monthly rate by 12, so the APR is:

APR = .00452 × 12

APR = .0542, or 5.42%

**46.** *a.* If the payments are in the form of an ordinary annuity, the present value will be:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | | 1 | | 2 | | 3 | | 4 | | 5 | |  | |  | |  | |  | |  | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| PVA | | $13,500 | | $13,500 | | $13,500 | | $13,500 | | $13,500 | |  | |  | |  | |  | |  | |

PVA = *C*({1 – [1/(1 + *r*)*t*]} / *r*))

PVA = $13,500[{1 – [1 / (1 + .075)]5}/ .075]

PVA = $54,619.45

If the payments are an annuity due, the present value will be:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | | 1 | | 2 | | 3 | | 4 | | 5 | |  | |  | |  | |  | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| PVADue | |  | |  | |  | |  | |  | |  | |  | |  | |  | |
| $13,500 | | $13,500 | | $13,500 | | $13,500 | | $13,500 | |  | |  | |  | |  | |  | |

PVAdue = (1 + *r*) PVA

PVAdue = (1 + .075)$54,619.45

PVAdue = $58,715.90

*b.* We can find the future value of the ordinary annuity as:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | | 1 | | 2 | | 3 | | 4 | | 5 | |  | |  | |  | |  | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  | |  | |  | |  | |  | | FVA | |  | |  | |  | |  | |
|  | | $13,500 | | $13,500 | | $13,500 | | $13,500 | | $13,500 | |  | |  | |  | |  | |

FVA = *C*{[(1 + *r*)*t* – 1] / *r*}

FVA = $13,500{[(1 + .075)5 – 1] / .075}

FVA = $78,413.28

If the payments are an annuity due, the future value will be:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | | 1 | | 2 | | 3 | | 4 | | 5 | |  | |  | |  | |  | |  | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| $13,500 | | $13,500 | | $13,500 | | $13,500 | | $13,500 | | FVADue | |  | |  | |  | |  | |  | |

FVAdue = (1 + *r*) FVA

FVAdue = (1 + .075)$78,413.28

FVAdue = $84,294.27

*c.* Assuming a positive interest rate, the present value of an annuity due will always be larger than the present value of an ordinary annuity. Each cash flow in an annuity due is received one period earlier, which means there is one period less to discount each cash flow. Assuming a positive interest rate, the future value of an annuity due will always be higher than the future value of an ordinary annuity. Since each cash flow is made one period sooner, each cash flow receives one extra period of compounding.

**47.** Here, we need to find the difference between the present value of an annuity and the present value of a perpetuity. The annuity time line is:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | | 1 | |  | |  | |  | | **…** |  | |  | |  | |  | | 30 | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| PVA | | $12,700 | | $12,700 | | $12,700 | | $12,700 | | $12,700 | | $12,700 | | $12,700 | | $12,700 | | $12,700 | |

PVA = *C*({1 – [1 / (1 + *r*)*t*]} / *r*))

PVA = $12,700{[1 – (1 / 1.054)30] / .054}

PVA = $186,634.92

And the present value of the perpetuity is:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | | 1 | |  | |  | |  | | **…** |  | |  | |  | |  | | ∞ | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| PV | | $12,700 | | $12,700 | | $12,700 | | $12,700 | | $12,700 | | $12,700 | | $12,700 | | $12,700 | | $12,700 | |

PV = *C* / *r*

PV = $12,700 / .054

PV = $235,185.19

So, the difference in the present values is:

Difference = $235,185.19 – 185,634.92

Difference = $48,550.27

There is another common way to answer this question. We need to recognize that the difference in the cash flows is a perpetuity of $12,700 beginning 31 years from now. We can find the present value of this perpetuity and the solution will be the difference in the cash flows. So, we can find the present value of this perpetuity as:

PVP = *C* / *r*

PVP = $12,700 / .054

PVP = $235,185.19

This is the present value 30 years from now, one period before the first cash flows. We can now find the present value of this lump sum as:

PV = FV / (1 + *r*)*t*

PV = $235,185.19 / (1 + .054)30

PV = $48,550.27

This is the same answer we calculated before.

**48.** The time line is:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | | 1 | |  | | **…** |  | | 18 | | 19 | |  | | **…** |  | | 30 | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  | |  | |  | |  | |  | | $7,750 | | $7,750 | | $7,750 | | $7,750 | |

Here we need to find the present value of an annuity at several different times. The annuity has semiannual payments, so we need the semiannual interest rate. The semiannual interest rate is:

Semiannual rate = .09 / 2

Semiannual rate = .045

Now, we can use the present value of an annuity equation. Doing so, we get:

PVA = *C*({1 – [1/(1 + *r*)*t*]} / *r*))

PVA = $7,750{[1 – (1 / 1.045)12] / .045}

PVA = $70,669.00

This is the present value one period before the first payment. The first payment occurs nine and one-half years from now, so this is the value of the annuity nine years from now. Since the interest rate is semiannual, we must also be careful to use the number of semiannual periods. The value of the annuity five years from now is:

PV = FV / (1 + *r*)*t*

PV = $70,669.00 / (1 + .045)8

PV = $49,693.39

And the value of the annuity three years from now is:

PV = FV / (1 + *r*)*t*

PV = $70,669.00 / (1 + .045)12

PV = $41,670.96

And the value of the annuity today is:

PV = FV / (1 + *r*)*t*

PV = $70,669.00 / (1 + .045)18

PV = $31,998.95

**49.** The time line is:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | | 1 | |  | |  | |  | |  | | 6 | |  | | **…** |  | | 20 | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| PV | |  | |  | |  | |  | |  | | $2,150 | | $2,150 | | $2,150 | | $2,150 | |

Since the first payment is received six years from today and the last payment is received 20 years from now, there are 15 payments. We can use the present value of an annuity formula, which will give us the present value 5 years from now, one period before the first payment. So, the present value of the annuity in 5five years is:

PVA = *C*({1 – [1 / (1 + *r*)*t*]} / *r*)

PVA = $2,150{[1 – (1 / 1.069)15] / .069}

PVA = $19,706.30

And using the present value equation for a lump sum, the present value of the annuity today is:

PV = FV / (1 + *r*)*t*

PV = $19,706.30 / (1 + .069)5

PV = $14,116.16

**50.** The time line is:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | | 1 | |  | |  | |  | | **…** |  | |  | |  | |  | | 120 | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| PVA | | $1,450 | | $1,450 | | $1,450 | | $1,450 | | $1,450 | | $1,450 | | $1,450 | | $1,450 | | $1,450 | |

Here, we have an annuity with two different interest rates. To answer this question, we simply need to find the present value in multiple steps. The present value of the last six years’ payments at a 7 percent interest rate is:

PVA = *C*({1 – [1 / (1 + *r*)*t*]} / *r*))

PVA = $1,450[{1 – 1 / [1 + (.07 / 12)]72} / (.07 / 12)]

PVA = $85,048.94

We can now discount this value back to time zero. We must be sure to use the number of months as the periods since interest is compounded monthly. We also need to use the interest rate that applies during the first four years. Doing so, we find:

PV = FV / (1 + *r*)*t*

PV = $85,048.95 / (1 + .09 / 12)48

PV = $59,416.39

Now we can find the present value of the annuity payments for the first four years. The present value of these payments is:

PVA = *C*({1 – [1/(1 + *r*)*t*]} / *r* )

PVA = $1,450[{1 – 1 / [1 + (.09 / 12)]48} / (.09 / 12)]

PVA = $58,267.93

So, the total present value of the cash flows is:

PV = $59,416.39 + 58,267.93

PV = $117,684.33

**51.** The time line is:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | | 1 | |  | |  | |  | | **…** |  | |  | |  | |  | | 120 | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| PVA | | $1,145 | | $1,145 | | $1,145 | | $1,145 | | $1,145 | | $1,145 | | $1,145 | | $1,145 | | $1,145 | |

To answer this question we need to find the future value of the annuity, and then find the present value that makes the lump sum investment equivalent. We also need to make sure to use the number of months as the number of periods. So, the future value of the annuity is:

FVA = *C*{[(1 + *r*)*t* – 1] / *r*}

FVA = $1,145{[(1 + .07 / 12)120 – 1] / (.07 / 12)}

FVA = $198,182.10

Now we can find the present value that would permit the lump sum investment to be equal to this future value. This investment has annual compounding, so the number of periods is the number of years. So, the present value we would need to deposit is:

PV = FV / (1 + *r*)*t*

PV = $198,182.10 / (1 + .09)10

PV = $83,714.26

**52.** The time line is:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | | 1 | |  | | **…** |  | | 19 | | 20 | |  | | **…** |  | | ∞ | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  | |  | |  | |  | |  | | $5,000 | | $5,000 | | $5,000 | | $5,000 | |

Here we need to find the present value of a perpetuity at a date before the perpetuity begins. We will begin by find the present value of the perpetuity. Doing so, we find:

PV = *C* / *r*

PV = $5,000 / .0635

PV = $78,740.16

This is the present value of the perpetuity at Year 19, one period before the payments begin. So, using the present value of a lump sum equation to find the value at Year 7, we find:

PV = FV / (1 + *r*)*t*

PV = $78,740.16 / (1 + .0635)12

PV = $37,613.73

**53.** The time line is:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | | 1 | |  | |  | |  | | **…** |  | |  | |  | |  | | 12 | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| –$25,000 | | $2,441.67 | | $2,441.67 | | $2,441.67 | | $2,441.67 | | $2,441.67 | | $2,441.67 | | $2,441.67 | | $2,441.67 | | $2,441.67 | |

Here we are given the PVA, number of periods, and the amount of the annuity. We need to solve for the interest rate. We must be careful to use the cash flows of the loan. Using the present value of an annuity equation, we find:

PVA = *C*({1 – [1/(1 + *r*)*t*]} / *r*)

$25,000 = $2,441.67[{1 – [1 / (1 + *r*)]12}/ *r*]

To find the interest rate, we need to solve this equation on a financial calculator, using a spreadsheet, or by trial and error. If you use trial and error, remember that increasing the interest rate lowers the PVA, and increasing the interest rate decreases the PVA. Using a spreadsheet, we find:

*r* = .02530, or 2.530%

This is the monthly interest rate. To find the APR with a monthly interest rate, we simply multiply the monthly rate by 12, so the APR is:

APR = .02530 × 12

APR = .3037, or 30.37%

And the EAR is:

EAR = [1 + (APR / *m*)]*m* – 1

EAR = [1 + .02530]12 – 1

EAR = .3497, or 34.97%

**54.** The time line is:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | | 1 | | 2 | | 3 | | 4 | | 5 | |  | |  | |  | |  | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  | |  | |  | |  | |  | | FV | |  | |  | |  | |  | |
|  | |  | | $15,000 | | $24,000 | |  | | $33,000 | |  | |  | |  | |  | |

To solve this problem, we must find the FV of each cash flow and add them. To find the FV of a lump sum, we use:

FV = PV(1 + *r*)*t*

FV = $15,000(1.058)3 + $24,000(1.058)2 + $33,000

FV = $77,629.04

Notice, since we are finding the value at Year 5, the cash flow at Year 5 is simply added to the FV of the other cash flows. In other words, we do not need to compound this cash flow. To find the value in Year 10, we need to find the future value of this lump sum. Doing so, we find:

FV = PV(1 + *r*)*t*

FV = $77,629.04(1.058)5

FV = $102,908.81

**55.** The payment for a loan repaid with equal payments is the annuity payment with the loan value as the PV of the annuity. So, the loan payment will be:

PVA = *C*({1 – [1/(1 + *r*)*t*]} / *r* )

$54,000 = *C*{[1 – 1 / (1 + .08)3] / .08}

*C* = $20,953.81

The interest payment is the beginning balance times the interest rate for the period, and the principal payment is the total payment minus the interest payment. The ending balance is the beginning balance minus the principal payment. The ending balance for a period is the beginning balance for the next period. The amortization table for an equal payment is:

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  | Year | Beginning Balance | Total Payment | Interest Payment | Principal Payment | Ending  Balance |
|  | 1 | $54,000.00 | $20,953.81 | $4,320.00 | $16,633.81 | $37,366.19 |
|  | 2 | 37,366.19 | 20,953.81 | 2,989.30 | 17,964.51 | 19,401.68 |
|  | 3 | 19,401.68 | 20,953.81 | 1,552.13 | 19,401.68 | 0 |

In the third year, $1,552.13 of interest is paid.

Total interest over life of the loan = $4,320 + 2,989.30 + 1,552.13

Total interest over life of the loan = $8,861.43

**56.** This amortization table calls for equal principal payments of $18,000 per year. The interest payment is the beginning balance times the interest rate for the period, and the total payment is the principal payment plus the interest payment. The ending balance for a period is the beginning balance for the next period. The amortization table for an equal principal reduction is:

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  | Year | Beginning Balance | Total Payment | Interest Payment | Principal Payment | Ending  Balance |
|  | 1 | $54,000.00 | $22,320.00 | $4,320.00 | $18,000.00 | $36,000.00 |
|  | 2 | 36,000.00 | 20,880.00 | 2,880.00 | 18,000.00 | 18,000.00 |
|  | 3 | 18,000.00 | 19,440.00 | 1,440.00 | 18,000.00 | 0 |

In the third year, $1,440 of interest is paid.

Total interest over life of the loan = $4,320 + 2,880 + 1,440

Total interest over life of the loan = $8,640

Notice that the total payments for the equal principal reduction loan are lower. This is because more principal is repaid early in the loan, which reduces the total interest expense over the life of the loan.

*Challenge*

**57.** The time line is:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | |  | |  | |  | |  | |  | |  | |  | |  | |  | | 1 | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| –$15,570 | |  | |  | |  | |  | |  | |  | |  | |  | |  | | $18,000 | |

To find the APR and EAR, we need to use the actual cash flows of the loan. In other words, the interest rate quoted in the problem is only relevant to determine the total interest under the terms given. The cash flows of the loan are the $18,000 you must repay in one year, and the $15,570 you borrow today. The interest rate of the loan is:

$18,000 = $15,570(1 + *r*)

*r* = ($18,000 / $15,570) – 1

*r* = .1561, or 15.61%

Because of the discount, you only get the use of $15,570 and the interest you pay on that amount is 15.61%, not 13.5%.

**58.** The time line is:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | –24 | | –23 | | **…** | –12 | | –11 | | **…** | 0 | | 1 | |  | | **…** | 60 | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  | | $3,583.33 | | $3,583.33 | | $3,833.33 | | $3,833.33 | | $4,083.33 | | $4,083.33 | | $4,083.33 | |
|  | |  | |  |  | |  | |  | $200,000 | |  | |  | |  |  | |
|  | |  | |  |  | |  | |  | $25,000 | |  | |  | |  |  | |

Here we have cash flows that would have occurred in the past and cash flows that would occur in the future. We need to bring both cash flows to today. Before we calculate the value of the cash flows today, we must adjust the interest rate so we have the effective monthly interest rate. Finding the APR with monthly compounding and dividing by 12 will give us the effective monthly rate. The APR with monthly compounding is:

APR = 12[(1.07)1/12 – 1]

APR = .0678, or 6.78%

To find the value today of the back pay from two years ago, we will find the FV of the annuity, and then find the FV of the lump sum. Doing so gives us:

FVA = ($43,000 / 12)[{[1 + (.0678 / 12)]12 – 1} / (.0678 / 12)]

FVA = $44,362.73

FV = PV(1 + *r*)*t*

FV = $44,362.73(1.07)

FV = $47,468.12

Notice we found the FV of the annuity with the effective monthly rate, and then found the FV of the lump sum with the EAR. Alternatively, we could have found the FV of the lump sum with the effective monthly rate as long as we used 12 periods. The answer would be the same either way.

Now, we need to find the value today of last year’s back pay:

FVA = ($46,000 / 12)[{[1 + (.0678 / 12)]12 – 1} / (.0678 / 12)]

FVA = $47,457.81

Next, we find the value today of the five year’s future salary:

PVA = ($49,000 / 12){[{1 – {1 / [1 + (.0678 / 12)]12(5)}] / (.0678 / 12)}

PVA = $207,276.79

The value today of the jury award is the sum of salaries, plus the compensation for pain and suffering, and court costs. The award should be for the amount of:

Award = $47,468.12 + 47,457.81 + 207,276.79 + 200,000 + 25,000

Award = $527,202.72

As the plaintiff, you would prefer a lower interest rate. In this problem, we are calculating both the PV and FV of annuities. A lower interest rate will decrease the FVA, but increase the PVA. So, by a lower interest rate, we are lowering the value of the back pay. But, we are also increasing the PV of the future salary. Since the future salary is larger and has a longer time, this is the more important cash flow to the plaintiff.

**59.** Again, to find the interest rate of a loan, we need to look at the cash flows of the loan. Since this loan is in the form of a lump sum, the amount you will repay is the FV of the principal amount, which will be:

Loan repayment amount = $15,000(1.12) = $16,800

The amount you will receive today is the principal amount of the loan times one minus the points.

Amount received = $15,000(1 – .02) = $14,700

So, the time line is:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | |  | |  | |  | |  | |  | |  | |  | |  | |  | | 1 | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| –$14,700 | |  | |  | |  | |  | |  | |  | |  | |  | |  | | $16,800 | |

Now, we find the interest rate for this PV and FV.

$16,800 = $14,700(1 + *r*)

*r* = ($16,800 / $14,700) – 1

*r* = .1429, or 14.29%

**60.** The time line is:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 | | 1 | | 2 | | 3 | | 4 | | 5 | | 6 | |  | | **…** | 65 | |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  | | $800 | | $800 | | $900 | | $900 | | $1,000 | | $1,000 | |  | | FV | |
|  | |  | |  | |  | |  | |  | |  | |  | |  |  | |

We need to find the FV of the premiums to compare with the cash payment promised at age 65. We

have to find the value of the premiums at Year 6 first since the interest rate changes at that time. So:

FV1 = $800(1.09)5 = $1,230.90

FV2 = $800(1.09)4 = $1,129.27

FV3 = $900(1.09)3 = $1,165.53

FV4 = $900(1.09)2 = $1,069.29

FV5 = $1,000(1.09)1 = $1,090.00

Value at Year 6 = $1,230.90 + 1,129.27 + 1,165.53 + 1,069.29 + 1,090 + 1,000

Value at Year 6 = $6,684.98

Finding the FV of this lump sum at the child’s 65th birthday:

FV = $6,684.98(1.055)59

FV = $157,396.57

The policy is not worth buying; the future value of the deposits is $157,396.57, but the policy contract will pay off $150,000. The premiums are worth $7,396.57 more than the policy payoff.

Note, we could also compare the PV of the two cash flows. The PV of the premiums is:

PV = $800 / 1.09 + $800 / 1.092 + $900 / 1.093 + $900 / 1.094 + $1,000 / 1.095 + $1,000 / 1.096

PV = $3,986.04

And the value today of the $150,000 at age 65 is:

PV = ($150,000 / 1.05559) / 1.096

PV = $3,798.72

The premiums still have the higher cash flow. At Time 0, the difference is $187.32. Whenever you are comparing two or more cash flow streams, the cash flow with the highest value at one time will have the highest value at any other time.

Here is a question for you: Suppose you invest $187.32, the difference in the cash flows at Time 0, for six years at a 9 percent interest rate, and then for 59 years at a 5.5 percent interest rate. How much will it be worth? Without doing calculations, you know it will be worth $7,396.57, the difference in the cash flows at Year 65!

**Calculator Solutions**

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| **1.** |  |  |  |  |  |  |
|  | **CFo** | $0 | **CFo** | $0 | **CFo** | $0 |
|  | **C01** | $680 | **C01** | $680 | **C01** | $680 |
|  | **F01** | 1 | **F01** | 1 | **F01** | 1 |
|  | **C02** | $490 | **C02** | $490 | **C02** | $490 |
|  | **F02** | 1 | **F02** | 1 | **F02** | 1 |
|  | **C03** | $975 | **C03** | $975 | **C03** | $975 |
|  | **F03** | 1 | **F03** | 1 | **F03** | 1 |
|  | **C04** | $1,160 | **C04** | $1,160 | **C04** | $1,160 |
|  | **F04** | 1 | **F04** | 1 | **F04** | 1 |
|  | I = 10 | | I = 18 | | I = 24 | |
|  | NPV CPT | | NPV CPT | | NPV CPT | |
|  | $2,547.97 | | $2,119.91 | | $1,869.09 | |

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **2.** |  | | |  | | |  | | |  | | |  | | |
| Enter | 9 | | | 6% | | |  | | | ±$3,400 | | |  | | |
|  |  | **N** |  |  | **I/Y** |  |  | **PV** |  |  | **PMT** |  |  | **FV** |  | |
| Solve for |  | | |  | | | $23,125.75 | | |  | | |  | | |

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  | | |  | | |  | | |  | | |  | | |
| Enter | 5 | | | 6% | | |  | | | ±$5,200 | | |  | | |
|  |  | **N** |  |  | **I/Y** |  |  | **PV** |  |  | **PMT** |  |  | **FV** |  | |
| Solve for |  | | |  | | | $21,904.29 | | |  | | |  | | |

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  | | |  | | |  | | |  | | |  | | |
| Enter | 9 | | | 22% | | |  | | | ±$3,400 | | |  | | |
|  |  | **N** |  |  | **I/Y** |  |  | **PV** |  |  | **PMT** |  |  | **FV** |  | |
| Solve for |  | | |  | | | $12,873.37 | | |  | | |  | | |

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|  |  | | |  | | |  | | |  | | |  | | |
| Enter | 5 | | | 22% | | |  | | | ±$5,200 | | |  | | |
|  |  | **N** |  |  | **I/Y** |  |  | **PV** |  |  | **PMT** |  |  | **FV** |  | |
| Solve for |  | | |  | | | $14,890.93 | | |  | | |  | | |

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| --- | --- | --- | --- | --- | --- | --- |
| **3.** |  |  |  |  |  |  |
|  | **CFo** | $0 | **CFo** | $0 | **CFo** | $0 |
|  | **C01** | $985 | **C01** | $985 | **C01** | $985 |
|  | **F01** | 1 | **F01** | 1 | **F01** | 1 |
|  | **C02** | $1,160 | **C02** | $1,160 | **C02** | $1,160 |
|  | **F02** | 1 | **F02** | 1 | **F02** | 1 |
|  | **C03** | $1,325 | **C03** | $1,325 | **C03** | $1,325 |
|  | **F03** | 1 | **F03** | 1 | **F03** | 1 |
|  | **C04** | $1,495 | **C04** | $1,495 | **C04** | $1,495 |
|  | **F04** | 1 | **F04** | 1 | **F04** | 1 |
|  | I = 8 | | I = 11 | | I = 24 | |
|  | NFV CPT | | NFV CPT | | NFV CPT | |
|  | $5,519.84 | | $5,742.10 | | $6,799.64 | |

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| **4.** |  | | |  | | |  | | |  | | |  | | |
| Enter | 15 | | | 8% | | |  | | | ±$5,450 | | |  | | |
|  |  | **N** |  |  | **I/Y** |  |  | **PV** |  |  | **PMT** |  |  | **FV** |  | |
| Solve for |  | | |  | | | $46,649.16 | | |  | | |  | | |

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| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  | | |  | | |  | | |  | | |  | | |
| Enter | 40 | | | 8% | | |  | | | ±$5,450 | | |  | | |
|  |  | **N** |  |  | **I/Y** |  |  | **PV** |  |  | **PMT** |  |  | **FV** |  | |
| Solve for |  | | |  | | | $64,989.14 | | |  | | |  | | |

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|  |  | | |  | | |  | | |  | | |  | | |
| Enter | 75 | | | 8% | | |  | | | ±$5,450 | | |  | | |
|  |  | **N** |  |  | **I/Y** |  |  | **PV** |  |  | **PMT** |  |  | **FV** |  | |
| Solve for |  | | |  | | | $67,912.91 | | |  | | |  | | |

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| **5.** |  | | |  | | |  | | |  | | |  | | |
| Enter | 6 | | | 11% | | | ±$24,500 | | |  | | |  | | |
|  |  | **N** |  |  | **I/Y** |  |  | **PV** |  |  | **PMT** |  |  | **FV** |  | |
| Solve for |  | | |  | | |  | | | $5,791.23 | | |  | | |

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|  |  | | |  | | |  | | |  | | |  | | |
| Enter | 8 | | | 7% | | | ±$19,700 | | |  | | |  | | |
|  |  | **N** |  |  | **I/Y** |  |  | **PV** |  |  | **PMT** |  |  | **FV** |  | |
| Solve for |  | | |  | | |  | | | $3,299.11 | | |  | | |

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| Enter | 15 | | | 8% | | | ±$136,400 | | |  | | |  | | |
|  |  | **N** |  |  | **I/Y** |  |  | **PV** |  |  | **PMT** |  |  | **FV** |  | |
| Solve for |  | | |  | | |  | | | $15,935.55 | | |  | | |

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|  |  | | |  | | |  | | |  | | |  | | |
| Enter | 20 | | | 6% | | | ±$285,650 | | |  | | |  | | |
|  |  | **N** |  |  | **I/Y** |  |  | **PV** |  |  | **PMT** |  |  | **FV** |  | |
| Solve for |  | | |  | | |  | | | $24,904.27 | | |  | | |

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| **6.** |  | | |  | | |  | | |  | | |  | | |
| Enter | 7 | | | 5% | | |  | | | ±$2,100 | | |  | | |
|  |  | **N** |  |  | **I/Y** |  |  | **PV** |  |  | **PMT** |  |  | **FV** |  | |
| Solve for |  | | |  | | | $12,151.38 | | |  | | |  | | |

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|  |  | | |  | | |  | | |  | | |  | | |
| Enter | 9 | | | 10% | | |  | | | ±$1,095 | | |  | | |
|  |  | **N** |  |  | **I/Y** |  |  | **PV** |  |  | **PMT** |  |  | **FV** |  | |
| Solve for |  | | |  | | | $6,306.13 | | |  | | |  | | |

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|  |  | | |  | | |  | | |  | | |  | | |
| Enter | 18 | | | 8% | | |  | | | ±$11,000 | | |  | | |
|  |  | **N** |  |  | **I/Y** |  |  | **PV** |  |  | **PMT** |  |  | **FV** |  | |
| Solve for |  | | |  | | | $103,090.76 | | |  | | |  | | |

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|  |  | | |  | | |  | | |  | | |  | | |
| Enter | 28 | | | 14% | | |  | | | ±$30,000 | | |  | | |
|  |  | **N** |  |  | **I/Y** |  |  | **PV** |  |  | **PMT** |  |  | **FV** |  | |
| Solve for |  | | |  | | | $208,819.87 | | |  | | |  | | |

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| **7.** |  | | |  | | |  | | |  | | |  | | |
| Enter | 8 | | | 5% | | |  | | |  | | | ±$30,000 | | |
|  |  | **N** |  |  | **I/Y** |  |  | **PV** |  |  | **PMT** |  |  | **FV** |  | |
| Solve for |  | | |  | | |  | | | $3,141.65 | | |  | | |

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|  |  | | |  | | |  | | |  | | |  | | |
| Enter | 40 | | | 7% | | |  | | |  | | | ±$1,200,000 | | |
|  |  | **N** |  |  | **I/Y** |  |  | **PV** |  |  | **PMT** |  |  | **FV** |  | |
| Solve for |  | | |  | | |  | | | $6,010.97 | | |  | | |

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|  |  | | |  | | |  | | |  | | |  | | |
| Enter | 25 | | | 8% | | |  | | |  | | | ±$625,000 | | |
|  |  | **N** |  |  | **I/Y** |  |  | **PV** |  |  | **PMT** |  |  | **FV** |  | |
| Solve for |  | | |  | | |  | | | $8,549.24 | | |  | | |

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|  |  | | |  | | |  | | |  | | |  | | |
| Enter | 13 | | | 4% | | |  | | |  | | | ±$125,000 | | |
|  |  | **N** |  |  | **I/Y** |  |  | **PV** |  |  | **PMT** |  |  | **FV** |  | |
| Solve for |  | | |  | | |  | | | $7,517.97 | | |  | | |

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| **8.** |  | | |  | | |  | | |  | | |  | | |
| Enter | 10 | | | 8% | | |  | | | ±$1,900 | | |  | | |
|  |  | **N** |  |  | **I/Y** |  |  | **PV** |  |  | **PMT** |  |  | **FV** |  | |
| Solve for |  | | |  | | |  | | |  | | | $27,524.47 | | |

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|  |  | | |  | | |  | | |  | | |  | | |
| Enter | 40 | | | 9% | | |  | | | ±$6,000 | | |  | | |
|  |  | **N** |  |  | **I/Y** |  |  | **PV** |  |  | **PMT** |  |  | **FV** |  | |
| Solve for |  | | |  | | |  | | |  | | | $2,027,294.67 | | |

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|  |  | | |  | | |  | | |  | | |  | | |
| Enter | 9 | | | 6% | | |  | | | ±$2,950 | | |  | | |
|  |  | **N** |  |  | **I/Y** |  |  | **PV** |  |  | **PMT** |  |  | **FV** |  | |
| Solve for |  | | |  | | |  | | |  | | | $33,899.38 | | |

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|  |  | | |  | | |  | | |  | | |  | | |
| Enter | 30 | | | 10% | | |  | | | ±$6,400 | | |  | | |
|  |  | **N** |  |  | **I/Y** |  |  | **PV** |  |  | **PMT** |  |  | **FV** |  | |
| Solve for |  | | |  | | |  | | |  | | | $1,052,761.75 | | |

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| **9.** |  | | |  | | |  | | |  | | |  | | |
| Enter | 20 | | | 9.6% | | | ±$5,000 | | |  | | |  | | |
|  |  | **N** |  |  | **I/Y** |  |  | **PV** |  |  | **PMT** |  |  | **FV** |  | |
| Solve for |  | | |  | | |  | | |  | | | $273,685.74 | | |

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|  |  | | |  | | |  | | |  | | |  | | |
| Enter | 40 | | | 9.6% | | | ±$5,000 | | |  | | |  | | |
|  |  | **N** |  |  | **I/Y** |  |  | **PV** |  |  | **PMT** |  |  | **FV** |  | |
| Solve for |  | | |  | | |  | | |  | | | $1,985,526.07 | | |

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| **12.** |  | | |  | | |  | | |  | | |  | | |
| Enter | 10% | | |  | | | 4 | | |  | | |  | | |
|  |  | **NOM** |  |  | **EFF** |  |  | **C/Y** |  |  |  |  |  |  |  | |
| Solve for |  | | | 10.38% | | |  | | |  | | |  | | |

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|  |  | | |  | | |  | | |  | | |  | | |
| Enter | 17% | | |  | | | 12 | | |  | | |  | | |
|  |  | **NOM** |  |  | **EFF** |  |  | **C/Y** |  |  |  |  |  |  |  | |
| Solve for |  | | | 18.39% | | |  | | |  | | |  | | |

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|  |  | | |  | | |  | | |  | | |  | | |
| Enter | 13% | | |  | | | 365 | | |  | | |  | | |
|  |  | **NOM** |  |  | **EFF** |  |  | **C/Y** |  |  |  |  |  |  |  | |
| Solve for |  | | | 13.88% | | |  | | |  | | |  | | |

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|  |  | | |  | | |  | | |  | | |  | | |
| Enter | 9% | | |  | | | 2 | | |  | | |  | | |
|  |  | **NOM** |  |  | **EFF** |  |  | **C/Y** |  |  |  |  |  |  |  | |
| Solve for |  | | | 9.20% | | |  | | |  | | |  | | |

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| **13.** |  | | |  | | |  | | |  | | |  | | |
| Enter |  | | | 14% | | | 2 | | |  | | |  | | |
|  |  | **NOM** |  |  | **EFF** |  |  | **C/Y** |  |  |  |  |  |  |  | |
| Solve for | 13.54% | | |  | | |  | | |  | | |  | | |

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| Enter |  | | | 9% | | | 12 | | |  | | |  | | |
|  |  | **NOM** |  |  | **EFF** |  |  | **C/Y** |  |  |  |  |  |  |  | |
| Solve for | 8.65% | | |  | | |  | | |  | | |  | | |

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| Enter |  | | | 8% | | | 52 | | |  | | |  | | |
|  |  | **NOM** |  |  | **EFF** |  |  | **C/Y** |  |  |  |  |  |  |  | |
| Solve for | 7.70% | | |  | | |  | | |  | | |  | | |

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| Enter |  | | | 13% | | | 365 | | |  | | |  | | |
|  |  | **NOM** |  |  | **EFF** |  |  | **C/Y** |  |  |  |  |  |  |  | |
| Solve for | 12.22% | | |  | | |  | | |  | | |  | | |

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| **14.** |  | | |  | | |  | | |  | | |  | | |
| Enter | 10.1% | | |  | | | 12 | | |  | | |  | | |
|  |  | **NOM** |  |  | **EFF** |  |  | **C/Y** |  |  |  |  |  |  |  | |
| Solve for |  | | | 10.58% | | |  | | |  | | |  | | |

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| Enter | 10.3% | | |  | | | 2 | | |  | | |  | | |
|  |  | **NOM** |  |  | **EFF** |  |  | **C/Y** |  |  |  |  |  |  |  | |
| Solve for |  | | | 10.57% | | |  | | |  | | |  | | |

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| **15.** |  | | |  | | |  | | |  | | |  | | |
| Enter |  | | | 14.2% | | | 365 | | |  | | |  | | |
|  |  | **NOM** |  |  | **EFF** |  |  | **C/Y** |  |  |  |  |  |  |  | |
| Solve for | 13.28% | | |  | | |  | | |  | | |  | | |

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| **16.** |  | | |  | | |  | | |  | | |  | | |
| Enter | 16 × 2 | | | 8.4% / 2 | | | ±$1,345 | | |  | | |  | | |
|  |  | **N** |  |  | **I/Y** |  |  | **PV** |  |  | **PMT** |  |  | **FV** |  | |
| Solve for |  | | |  | | |  | | |  | | | $5,017.52 | | |

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| **17.** |  | | |  | | |  | | |  | | |  | | |
| Enter | 5 × 365 | | | 4.7% / 365 | | | ±$3,650 | | |  | | |  | | |
|  |  | **N** |  |  | **I/Y** |  |  | **PV** |  |  | **PMT** |  |  | **FV** |  | |
| Solve for |  | | |  | | |  | | |  | | | $4,616.85 | | |

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|  |  | | |  | | |  | | |  | | |  | | |
| Enter | 10 × 365 | | | 4.7% / 365 | | | ±$3,650 | | |  | | |  | | |
|  |  | **N** |  |  | **I/Y** |  |  | **PV** |  |  | **PMT** |  |  | **FV** |  | |
| Solve for |  | | |  | | |  | | |  | | | $5,839.80 | | |

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|  |  | | |  | | |  | | |  | | |  | | |
| Enter | 20 × 365 | | | 4.7% / 365 | | | ±$3,650 | | |  | | |  | | |
|  |  | **N** |  |  | **I/Y** |  |  | **PV** |  |  | **PMT** |  |  | **FV** |  | |
| Solve for |  | | |  | | |  | | |  | | | $9,343.37 | | |

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| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **18.** |  | | |  | | |  | | |  | | |  | | |
| Enter | 9 × 365 | | | 5.5% / 365 | | |  | | |  | | | ±$65,000 | | |
|  |  | **N** |  |  | **I/Y** |  |  | **PV** |  |  | **PMT** |  |  | **FV** |  | |
| Solve for |  | | |  | | | $39,623.59 | | |  | | |  | | |

**19.** APR = 12(13.7%) = 164.4%

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  | | |  | | |  | | |  | | |  | | |
| Enter | 164.4% | | |  | | | 12 | | |  | | |  | | |
|  |  | **NOM** |  |  | **EFF** |  |  | **C/Y** |  |  |  |  |  |  |  | |
| Solve for |  | | | 366.79% | | |  | | |  | | |  | | |

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| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **20.** |  | | |  | | |  | | |  | | |  | | |
| Enter | 60 | | | 5.1% / 12 | | | ±$73,400 | | |  | | |  | | |
|  |  | **N** |  |  | **I/Y** |  |  | **PV** |  |  | **PMT** |  |  | **FV** |  | |
| Solve for |  | | |  | | |  | | | $1,388.51 | | |  | | |

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  | | |  | | |  | | |  | | |  | | |
| Enter | 5.1% | | |  | | | 12 | | |  | | |  | | |
|  |  | **NOM** |  |  | **EFF** |  |  | **C/Y** |  |  |  |  |  |  |  | |
| Solve for |  | | | 5.22% | | |  | | |  | | |  | | |

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **21.** |  | | |  | | |  | | |  | | |  | | |
| Enter |  | | | 1.4% | | | $14,480 | | | ±$400 | | |  | | |
|  |  | **N** |  |  | **I/Y** |  |  | **PV** |  |  | **PMT** |  |  | **FV** |  | |
| Solve for | 50.84 | | |  | | |  | | |  | | |  | | |

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **22.** |  | | |  | | |  | | |  | | |  | | |
| Enter | 1 | | |  | | | ±$4 | | |  | | | $5 | | |
|  |  | **N** |  |  | **I/Y** |  |  | **PV** |  |  | **PMT** |  |  | **FV** |  | |
| Solve for |  | | | 25% | | |  | | |  | | |  | | |

APR = 52(25%) = 1,300%

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  | | |  | | |  | | |  | | |  | | |
| Enter | 1,300% | | |  | | | 52 | | |  | | |  | | |
|  |  | **NOM** |  |  | **EFF** |  |  | **C/Y** |  |  |  |  |  |  |  | |
| Solve for |  | | | 10,947,544.25% | | |  | | |  | | |  | | |

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **24.** |  | | |  | | |  | | |  | | |  | | |
| Enter | 35 × 12 | | | 9.8% / 12 | | |  | | | ±$500 | | |  | | |
|  |  | **N** |  |  | **I/Y** |  |  | **PV** |  |  | **PMT** |  |  | **FV** |  | |
| Solve for |  | | |  | | |  | | |  | | | $1,803,031.33 | | |

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **25.** |  | | |  | | |  | | |  | | |  | | |
| Enter | 9.8% | | |  | | | 12 | | |  | | |  | | |
|  |  | **NOM** |  |  | **EFF** |  |  | **C/Y** |  |  |  |  |  |  |  | |
| Solve for |  | | | 10.25% | | |  | | |  | | |  | | |

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  | | |  | | |  | | |  | | |  | | |
| Enter | 35 | | | 10.25% | | |  | | | ±$6,000 | | |  | | |
|  |  | **N** |  |  | **I/Y** |  |  | **PV** |  |  | **PMT** |  |  | **FV** |  | |
| Solve for |  | | |  | | |  | | |  | | | $1,723,472.13 | | |

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **26.** |  | | |  | | |  | | |  | | |  | | |
| Enter | 4 × 4 | | | .38% | | |  | | | ±$2,500 | | |  | | |
|  |  | **N** |  |  | **I/Y** |  |  | **PV** |  |  | **PMT** |  |  | **FV** |  | |
| Solve for |  | | |  | | | $38,736.93 | | |  | | |  | | |

|  |  |  |
| --- | --- | --- |
| **27.** |  |  |
|  | **CFo** | $0 |
|  | **C01** | $1,200 |
|  | **F01** | 1 |
|  | **C02** | $1,100 |
|  | **F02** | 1 |
|  | **C03** | $800 |
|  | **F03** | 1 |
|  | **C04** | $600 |
|  | **F04** | 1 |
|  | I = 6.75% | |
|  | NPV CPT | |
|  | $3,209.09 | |

|  |  |  |
| --- | --- | --- |
| **28.** |  |  |
|  | **CFo** | $0 |
|  | **C01** | $1,400 |
|  | **F01** | 1 |
|  | **C02** | $1,900 |
|  | **F02** | 1 |
|  | **C03** | $3,400 |
|  | **F03** | 1 |
|  | **C04** | $4,300 |
|  | **F04** | 1 |
|  | I = 7.13% | |
|  | NPV CPT | |
|  | $8,992.21 | |

**29.** First Simple: $100(.058) = $5.80; 10 year investment = $100 + 10($5.80) = $158

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  | | |  | | |  | | |  | | |  | | |
| Enter | 10 | | |  | | | ±$100 | | |  | | | $158 | | |
|  |  | **N** |  |  | **I/Y** |  |  | **PV** |  |  | **PMT** |  |  | **FV** |  | |
| Solve for |  | | | 4.68% | | |  | | |  | | |  | | |

**30.** 2nd BGN 2nd SET

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  | | |  | | |  | | |  | | |  | | |
| Enter | 60 | | | 5.20% / 12 | | | ±$58,600 | | |  | | |  | | |
|  |  | **N** |  |  | **I/Y** |  |  | **PV** |  |  | **PMT** |  |  | **FV** |  | |
| Solve for |  | | |  | | |  | | | $1,106.44 | | |  | | |

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **31.** |  | | |  | | |  | | |  | | |  | | |
| Enter | 6 | | | 1.60% / 12 | | | ±$10,000 | | |  | | |  | | |
|  |  | **N** |  |  | **I/Y** |  |  | **PV** |  |  | **PMT** |  |  | **FV** |  | |
| Solve for |  | | |  | | |  | | |  | | | $10,080.27 | | |

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  | | |  | | |  | | |  | | |  | | |
| Enter | 6 | | | 17.9% / 12 | | | ±$10,080.27 | | |  | | |  | | |
|  |  | **N** |  |  | **I/Y** |  |  | **PV** |  |  | **PMT** |  |  | **FV** |  | |
| Solve for |  | | |  | | |  | | |  | | | $11,016.77 | | |

Interest = $11,016.77 – 10,000

Interest = $1,016.77

**32.** First: $85,000(.048) = $4,080 per year

($235,000 – 85,000) / $4,080 = 36.76 years

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Second: |  | | |  | | |  | | |  | | |  | | |
| Enter |  | | | 4.8% / 12 | | | ±$85,000 | | |  | | | $235,000 | | |
|  |  | **N** |  |  | **I/Y** |  |  | **PV** |  |  | **PMT** |  |  | **FV** |  | |
| Solve for | 254.74 | | |  | | |  | | |  | | |  | | |

254.74 / 12 = 21.23 years

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **33.** |  | | |  | | |  | | |  | | |  | | |
| Enter | 12 | | | 1.29% | | | ±$1 | | |  | | |  | | |
|  |  | **N** |  |  | **I/Y** |  |  | **PV** |  |  | **PMT** |  |  | **FV** |  | |
| Solve for |  | | |  | | |  | | |  | | | $1.17 | | |

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  | | |  | | |  | | |  | | |  | | |
| Enter | 24 | | | 1.29% | | | ±$1 | | |  | | |  | | |
|  |  | **N** |  |  | **I/Y** |  |  | **PV** |  |  | **PMT** |  |  | **FV** |  | |
| Solve for |  | | |  | | |  | | |  | | | $1.36 | | |

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **34.** |  | | |  | | |  | | |  | | |  | | |
| Enter | 31 | | |  | | | ±£440 | | | £60 | | |  | | |
|  |  | **N** |  |  | **I/Y** |  |  | **PV** |  |  | **PMT** |  |  | **FV** |  | |
| Solve for |  | | | 13.36% | | |  | | |  | | |  | | |

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **35.** |  | | |  | | |  | | |  | | |  | | |
| Enter | 24 | | | 7% /12 | | |  | | | ±$6,700 | | |  | | |
|  |  | **N** |  |  | **I/Y** |  |  | **PV** |  |  | **PMT** |  |  | **FV** |  | |
| Solve for |  | | |  | | | $149,645.17 | | |  | | |  | | |

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| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  | | |  | | |  | | |  | | |  | | |
| Enter | 24 | | | 7% / 12 | | |  | | | ±$5,400 | | |  | | |
|  |  | **N** |  |  | **I/Y** |  |  | **PV** |  |  | **PMT** |  |  | **FV** |  | |
| Solve for |  | | |  | | | $120,609.54 | | |  | | |  | | |

$120,609.54 + 25,000 = $145,609.54

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **36.** |  | | |  | | |  | | |  | | |  | | |
| Enter | 20 | | | 8% | | |  | | | ±$17,500 | | |  | | |
|  |  | **N** |  |  | **I/Y** |  |  | **PV** |  |  | **PMT** |  |  | **FV** |  | |
| Solve for |  | | |  | | | $171,817.58 | | |  | | |  | | |

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| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **37.** |  | | |  | | |  | | |  | | |  | | |
| Enter | 6 | | |  | | | ±$39,000 | | |  | | | $75,000 | | |
|  |  | **N** |  |  | **I/Y** |  |  | **PV** |  |  | **PMT** |  |  | **FV** |  | |
| Solve for |  | | | 11.51% | | |  | | |  | | |  | | |

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| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  | | |  | | |  | | |  | | |  | | |
| Enter | 9 | | |  | | | ±$39,000 | | |  | | | $105,000 | | |
|  |  | **N** |  |  | **I/Y** |  |  | **PV** |  |  | **PMT** |  |  | **FV** |  | |
| Solve for |  | | | 11.63% | | |  | | |  | | |  | | |

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **38.** |  | | |  | | |  | | |  | | |  | | |
| Enter | 10 | | | 10% | | |  | | | ±$7,300 | | |  | | |
|  |  | **N** |  |  | **I/Y** |  |  | **PV** |  |  | **PMT** |  |  | **FV** |  | |
| Solve for |  | | |  | | | $44,855.34 | | |  | | |  | | |

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  | | |  | | |  | | |  | | |  | | |
| Enter | 10 | | | 5% | | |  | | | ±$7,300 | | |  | | |
|  |  | **N** |  |  | **I/Y** |  |  | **PV** |  |  | **PMT** |  |  | **FV** |  | |
| Solve for |  | | |  | | | $56,368.66 | | |  | | |  | | |

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  | | |  | | |  | | |  | | |  | | |
| Enter | 10 | | | 15% | | |  | | | ±$7,300 | | |  | | |
|  |  | **N** |  |  | **I/Y** |  |  | **PV** |  |  | **PMT** |  |  | **FV** |  | |
| Solve for |  | | |  | | | $36,637.01 | | |  | | |  | | |

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **39.** |  | | |  | | |  | | |  | | |  | | |
| Enter |  | | | 8% / 12 | | |  | | | ±$250 | | | $50,000 | | |
|  |  | **N** |  |  | **I/Y** |  |  | **PV** |  |  | **PMT** |  |  | **FV** |  | |
| Solve for | 127.52 | | |  | | |  | | |  | | |  | | |

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **40.** |  | | |  | | |  | | |  | | |  | | |
| Enter | 60 | | |  | | | ±$80,000 | | | $1,650 | | |  | | |
|  |  | **N** |  |  | **I/Y** |  |  | **PV** |  |  | **PMT** |  |  | **FV** |  | |
| Solve for |  | | | .727% | | |  | | |  | | |  | | |

APR = .727%(12) = 8.72%

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| **41.** |  |  | **42.** | |  |  |
|  | **CF**o | 0 |  | | **CF**o | $5,100,000 |
|  | **C01** | $3,400,000 |  | | **C01** | $985,000 |
|  | **F01** | 1 |  | | **F01** | 1 |
|  | **C02** | $12,400,000 |  | | **C02** | $28,585,000 |
|  | **F02** | 1 |  | | **F02** | 1 |
|  | **C03** | $12,400,000 |  | | **C03** | $15,085,000 |
|  | **F03** | 3 |  | | **F03** | 1 |
|  | **C04** | $13,400,000 |  | | **C04** | $22,085,000 |
|  | **F04** | 1 |  | | **F04** | 2 |
|  | **C05** | $13,400,000 |  | | **C05** | $24,085,000 |
|  | **F05** | 1 |  | | **F05** | 1 |
|  | **C06** |  |  | | **C06** | $18,360,000 |
|  | **F06** |  |  | | **F06** | 1 |
|  | I = 11% | | |  | I = 11% | | |
|  | NPV CPT | | |  | NPV CPT | | |
|  | $38,973,197.21 | | |  | $78,874,991.18 | | |

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **43.** |  | | |  | | |  | | |  | | |  | | |
| Enter | 30 × 12 | | |  | | | .80($2,950,000) | | | ±$14,300 | | |  | | |
|  |  | **N** |  |  | **I/Y** |  |  | **PV** |  |  | **PMT** |  |  | **FV** |  | |
| Solve for |  | | | .508% | | |  | | |  | | |  | | |

APR = .508%(12) = 6.10%

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  | | |  | | |  | | |  | | |  | | |
| Enter | 6.10% | | |  | | | 12 | | |  | | |  | | |
|  |  | **NOM** |  |  | **EFF** |  |  | **C/Y** |  |  |  |  |  |  |  | |
| Solve for |  | | | 6.27% | | |  | | |  | | |  | | |

**44.** Future value of bond account:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  | | |  | | |  | | |  | | |  | | |
| Enter | 10 | | | 7% | | | ±$75,000 | | | ±$6,000 | | |  | | |
|  |  | **N** |  |  | **I/Y** |  |  | **PV** |  |  | **PMT** |  |  | **FV** |  | |
| Solve for |  | | |  | | |  | | |  | | | $230,435.04 | | |

Future value of stock account:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  | | |  | | |  | | |  | | |  | | |
| Enter | 10 | | | 10.5% | | | ±$300,000 | | |  | | |  | | |
|  |  | **N** |  |  | **I/Y** |  |  | **PV** |  |  | **PMT** |  |  | **FV** |  | |
| Solve for |  | | |  | | |  | | |  | | | $814,224.25 | | |

Future value of retirement account:

FV = $230,435.04 + 814,224.25

FV = $1,044,659.29

Annual withdrawal amount:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  | | |  | | |  | | |  | | |  | | |
| Enter | 25 | | | 6.25% | | | ±$1,044,659.29 | | |  | | |  | | |
|  |  | **N** |  |  | **I/Y** |  |  | **PV** |  |  | **PMT** |  |  | **FV** |  | |
| Solve for |  | | |  | | |  | | | $83,671.59 | | |  | | |

**45.** 2nd BGN 2nd SET

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  | | |  | | |  | | |  | | |  | | |
| Enter | 60 | | |  | | | ±$24,500 | | | $465 | | |  | | |
|  |  | **N** |  |  | **I/Y** |  |  | **PV** |  |  | **PMT** |  |  | **FV** |  | |
| Solve for |  | | | .452% | | |  | | |  | | |  | | |

APR = .452%(12) = 5.42%

**46.**

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| *a.* |  | | |  | | |  | | |  | | |  | | |
| Enter | 5 | | | 7.5% | | |  | | | ±$13,500 | | |  | | |
|  |  | **N** |  |  | **I/Y** |  |  | **PV** |  |  | **PMT** |  |  | **FV** |  | |
| Solve for |  | | |  | | | $54,619.45 | | |  | | |  | | |

2nd BGN 2nd SET

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  | | |  | | |  | | |  | | |  | | |
| Enter | 5 | | | 7.5% | | |  | | | ±$13,500 | | |  | | |
|  |  | **N** |  |  | **I/Y** |  |  | **PV** |  |  | **PMT** |  |  | **FV** |  | |
| Solve for |  | | |  | | | $58,715.90 | | |  | | |  | | |

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| *b.* |  | | |  | | |  | | |  | | |  | | |
| Enter | 5 | | | 7.5% | | |  | | | ±$13,500 | | |  | | |
|  |  | **N** |  |  | **I/Y** |  |  | **PV** |  |  | **PMT** |  |  | **FV** |  | |
| Solve for |  | | |  | | |  | | |  | | | $78,413.28 | | |

2nd BGN 2nd SET

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  | | |  | | |  | | |  | | |  | | |
| Enter | 5 | | | 7.5% | | |  | | | ±$13,500 | | |  | | |
|  |  | **N** |  |  | **I/Y** |  |  | **PV** |  |  | **PMT** |  |  | **FV** |  | |
| Solve for |  | | |  | | |  | | |  | | | $84,294.27 | | |

**47.** Present value of annuity:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  | | |  | | |  | | |  | | |  | | |
| Enter | 30 | | | 5.4% | | |  | | | ±$12,700 | | |  | | |
|  |  | **N** |  |  | **I/Y** |  |  | **PV** |  |  | **PMT** |  |  | **FV** |  | |
| Solve for |  | | |  | | | $186,634.92 | | |  | | |  | | |

And the present value of the perpetuity is:

PVP = *C* / *r*

PVP = $12,700 / .054

PVP = $235,185.19

So the difference in the present values is:

Difference = $235,185.19 – 186,634.92

Difference = $48,550.27

**48.** Value at *t* = 9

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  | | |  | | |  | | |  | | |  | | |
| Enter | 12 | | | 9% / 2 | | |  | | | ±$7,750 | | |  | | |
|  |  | **N** |  |  | **I/Y** |  |  | **PV** |  |  | **PMT** |  |  | **FV** |  | |
| Solve for |  | | |  | | | $70,669.00 | | |  | | |  | | |

Value at *t* = 5

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  | | |  | | |  | | |  | | |  | | |
| Enter | 4 × 2 | | | 9% / 2 | | |  | | |  | | | ±$70,669.00 | | |
|  |  | **N** |  |  | **I/Y** |  |  | **PV** |  |  | **PMT** |  |  | **FV** |  | |
| Solve for |  | | |  | | | $49,693.39 | | |  | | |  | | |

Value at *t* = 3

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  | | |  | | |  | | |  | | |  | | |
| Enter | 6 × 2 | | | 9% / 2 | | |  | | |  | | | ±$70,669.00 | | |
|  |  | **N** |  |  | **I/Y** |  |  | **PV** |  |  | **PMT** |  |  | **FV** |  | |
| Solve for |  | | |  | | | $41,670.96 | | |  | | |  | | |

Value today

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  | | |  | | |  | | |  | | |  | | |
| Enter | 9 × 2 | | | 9% / 2 | | |  | | |  | | | ±$70,669.00 | | |
|  |  | **N** |  |  | **I/Y** |  |  | **PV** |  |  | **PMT** |  |  | **FV** |  | |
| Solve for |  | | |  | | | $31,998.95 | | |  | | |  | | |

**49.** Value at *t* = 5

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  | | |  | | |  | | |  | | |  | | |
| Enter | 15 | | | 6.9% | | |  | | | ±$2,150 | | |  | | |
|  |  | **N** |  |  | **I/Y** |  |  | **PV** |  |  | **PMT** |  |  | **FV** |  | |
| Solve for |  | | |  | | | $19,706.30 | | |  | | |  | | |

Value today

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  | | |  | | |  | | |  | | |  | | |
| Enter | 5 | | | 6.9% | | |  | | |  | | | ±$19,706.30 | | |
|  |  | **N** |  |  | **I/Y** |  |  | **PV** |  |  | **PMT** |  |  | **FV** |  | |
| Solve for |  | | |  | | | $14,116.16 | | |  | | |  | | |

**50.**  Value at *t* = 4

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  | | |  | | |  | | |  | | |  | | |
| Enter | 6 × 12 | | | 7% / 12 | | |  | | | ±$1,450 | | |  | | |
|  |  | **N** |  |  | **I/Y** |  |  | **PV** |  |  | **PMT** |  |  | **FV** |  | |
| Solve for |  | | |  | | | $85,048.94 | | |  | | |  | | |

Value today

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  | | |  | | |  | | |  | | |  | | |
| Enter | 4 × 12 | | | 9% / 12 | | |  | | | ±$1,450 | | | ±$85,048.94 | | |
|  |  | **N** |  |  | **I/Y** |  |  | **PV** |  |  | **PMT** |  |  | **FV** |  | |
| Solve for |  | | |  | | | $117,684.33 | | |  | | |  | | |

**51.** FV of A

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  | | |  | | |  | | |  | | |  | | |
| Enter | 10 × 12 | | | 7% / 12 | | |  | | | ±$1,145 | | |  | | |
|  |  | **N** |  |  | **I/Y** |  |  | **PV** |  |  | **PMT** |  |  | **FV** |  | |
| Solve for |  | | |  | | |  | | |  | | | $198,182.10 | | |

Value to invest in B

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  | | |  | | |  | | |  | | |  | | |
| Enter | 10 | | | 9% | | |  | | |  | | | ±$198,182.10 | | |
|  |  | **N** |  |  | **I/Y** |  |  | **PV** |  |  | **PMT** |  |  | **FV** |  | |
| Solve for |  | | |  | | | $83,714.26 | | |  | | |  | | |

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **53.** |  | | |  | | |  | | |  | | |  | | |
| Enter | 12 | | |  | | | ±$25,000 | | | $2,441.67 | | |  | | |
|  |  | **N** |  |  | **I/Y** |  |  | **PV** |  |  | **PMT** |  |  | **FV** |  | |
| Solve for |  | | | 2.530% | | |  | | |  | | |  | | |

APR = 2.530%(12) = 30.37%

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  | | |  | | |  | | |  | | |  | | |
| Enter | 30.37% | | |  | | | 12 | | |  | | |  | | |
|  |  | **NOM** |  |  | **EFF** |  |  | **C/Y** |  |  |  |  |  |  |  | |
| Solve for |  | | | 34.97% | | |  | | |  | | |  | | |

|  |  |  |
| --- | --- | --- |
| **54.** |  |  |
|  | **CF**o | $0 |
|  | **C01** | $0 |
|  | **F01** | 1 |
|  | **C02** | $15,000 |
|  | **F02** | 1 |
|  | **C03** | $24,000 |
|  | **F03** | 1 |
|  | **C04** | $0 |
|  | **F04** | 1 |
|  | **C05** | $33,000 |
|  | **F05** | 1 |
|  | I = 5.8% | | |
|  | NPV NFV | | |
|  | $77,629.04 | | |

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Enter | 5 | | | 5.8% | | | ±$77,629.04 | | |  | | |  | | |
|  |  | **N** |  |  | **I/Y** |  |  | **PV** |  |  | **PMT** |  |  | **FV** |  | |
| Solve for |  | | |  | | |  | | |  | | | $102,908.81 | | |

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **57.** |  | | |  | | |  | | |  | | |  | | |
| Enter | 1 | | |  | | | ±$15,570 | | |  | | | $18,000 | | |
|  |  | **N** |  |  | **I/Y** |  |  | **PV** |  |  | **PMT** |  |  | **FV** |  | |
| Solve for |  | | | 15.61% | | |  | | |  | | |  | | |

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **58.** |  | | |  | | |  | | |  | | |  | | |
| Enter |  | | | 7% | | | 12 | | |  | | |  | | |
|  |  | **NOM** |  |  | **EFF** |  |  | **C/Y** |  |  |  |  |  |  |  | |
| Solve for | 6.78% | | |  | | |  | | |  | | |  | | |

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  | | |  | | |  | | |  | | |  | | |
| Enter | 12 | | | 6.78% / 12 | | |  | | | ±$43,000 / 12 | | |  | | |
|  |  | **N** |  |  | **I/Y** |  |  | **PV** |  |  | **PMT** |  |  | **FV** |  | |
| Solve for |  | | |  | | |  | | |  | | | $44,362.73 | | |

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  | | |  | | |  | | |  | | |  | | |
| Enter | 1 | | | 7% | | | ±$44,362.73 | | |  | | |  | | |
|  |  | **N** |  |  | **I/Y** |  |  | **PV** |  |  | **PMT** |  |  | **FV** |  | |
| Solve for |  | | |  | | |  | | |  | | | $47,468.12 | | |

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  | | |  | | |  | | |  | | |  | | |
| Enter | 12 | | | 6.78% / 12 | | |  | | | ±$46,000 / 12 | | |  | | |
|  |  | **N** |  |  | **I/Y** |  |  | **PV** |  |  | **PMT** |  |  | **FV** |  | |
| Solve for |  | | |  | | |  | | |  | | | $47,457.81 | | |

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  | | |  | | |  | | |  | | |  | | |
| Enter | 60 | | | 6.78% / 12 | | |  | | | ±$49,000 / 12 | | |  | | |
|  |  | **N** |  |  | **I/Y** |  |  | **PV** |  |  | **PMT** |  |  | **FV** |  | |
| Solve for |  | | |  | | | $207,276.79 | | |  | | |  | | |

Award = $47,468.12 + 47,457.81 + 207,276.79 + 200,000 + 25,000 = $527,202.72

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **59.** |  | | |  | | |  | | |  | | |  | | |
| Enter | 1 | | |  | | | $14,700 | | |  | | | ±$16,800 | | |
|  |  | **N** |  |  | **I/Y** |  |  | **PV** |  |  | **PMT** |  |  | **FV** |  | |
| Solve for |  | | | 14.29% | | |  | | |  | | |  | | |

**60.** Value at Year 6:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  | | |  | | |  | | |  | | |  | | |
| Enter | 5 | | | 9% | | | ±$800 | | |  | | |  | | |
|  |  | **N** |  |  | **I/Y** |  |  | **PV** |  |  | **PMT** |  |  | **FV** |  | |
| Solve for |  | | |  | | |  | | |  | | | $1,230.90 | | |

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  | | |  | | |  | | |  | | |  | | |
| Enter | 4 | | | 9% | | | ±$800 | | |  | | |  | | |
|  |  | **N** |  |  | **I/Y** |  |  | **PV** |  |  | **PMT** |  |  | **FV** |  | |
| Solve for |  | | |  | | |  | | |  | | | $1,129.27 | | |

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Enter | 3 | | | 9% | | | ±$900 | | |  | | |  | | |
|  |  | **N** |  |  | **I/Y** |  |  | **PV** |  |  | **PMT** |  |  | **FV** |  | |
| Solve for |  | | |  | | |  | | |  | | | $1,165.53 | | |

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  | | |  | | |  | | |  | | |  | | |
| Enter | 2 | | | 9% | | | ±$900 | | |  | | |  | | |
|  |  | **N** |  |  | **I/Y** |  |  | **PV** |  |  | **PMT** |  |  | **FV** |  | |
| Solve for |  | | |  | | |  | | |  | | | $1,069.29 | | |

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  | | |  | | |  | | |  | | |  | | |
| Enter | 1 | | | 9% | | | ±$1,000 | | |  | | |  | | |
|  |  | **N** |  |  | **I/Y** |  |  | **PV** |  |  | **PMT** |  |  | **FV** |  | |
| Solve for |  | | |  | | |  | | |  | | | $1,090 | | |

So, at Year 5, the value is: $1,230.90 + 1,129.27 + 1,165.53 + 1,069.29 + 1,090 + 1,000 = $6,684.98

At Year 65, the value is:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  | | |  | | |  | | |  | | |  | | |
| Enter | 59 | | | 5.5% | | | ±$6,684.98 | | |  | | |  | | |
|  |  | **N** |  |  | **I/Y** |  |  | **PV** |  |  | **PMT** |  |  | **FV** |  | |
| Solve for |  | | |  | | |  | | |  | | | $157,396.57 | | |

The policy is not worth buying; the future value of the deposits is $157,396.57 but the policy

contract will pay off $150,000.

**CHAPTER 6**

***INTEREST RATES AND BOND VALUTION***

**Answers to Concepts Review and Critical Thinking Questions**

**1.** No. As interest rates fluctuate, the value of a Treasury security will fluctuate. Long-term Treasury securities have substantial interest rate risk.

**2.** All else the same, the Treasury security will have lower coupons because of its lower default risk, so it will have greater interest rate risk.

**3.** No. If the bid were higher than the ask, the implication would be that a dealer was willing to sell a bond and immediately buy it back at a higher price. How many such transactions would you like to do?

**4.** Prices and yields move in opposite directions. Since the bid price must be lower, the bid yield must be higher.

**5.** There are two benefits. First, the company can take advantage of interest rate declines by calling in an issue and replacing it with a lower coupon issue. Second, a company might wish to eliminate a covenant for some reason. Calling the issue does this. The cost to the company is a higher coupon. A put provision is desirable from an investor’s standpoint, so it helps the company by reducing the coupon rate on the bond. The cost to the company is that it may have to buy back the bond at an unattractive price.

**6.** Bond issuers look at outstanding bonds of similar maturity and risk. The yields on such bonds are used to establish the coupon rate necessary for a particular issue to initially sell for par value. Bond issuers also ask potential purchasers what coupon rate would be necessary to attract them. The coupon rate is fixed and determines what the bond’s coupon payments will be. The required return is what investors actually demand on the issue, and it will fluctuate through time. The coupon rate and required return are equal only if the bond sells for exactly par.

**7.** Yes. Some investors have obligations that are denominated in dollars; i.e., they are nominal. Their primary concern is that an investment provides the needed nominal dollar amounts. Pension funds, for example, often must plan for pension payments many years in the future. If those payments are fixed in dollar terms, then it is the nominal return on an investment that is important.

**8.** Companies pay to have their bonds rated because unrated bonds can be difficult to sell; many large investors are prohibited from investing in unrated issues.

**9.** Junk bonds often are not rated because there would no point in an issuer paying a rating agency to assign its bonds a low rating (it’s like paying someone to kick you!).

**10.** Bond ratings have a subjective factor to them. Split ratings reflect a difference of opinion among credit agencies.

**11.** As a general constitutional principle, the federal government cannot tax the states without their consent if doing so would interfere with state government functions. At one time, this principle was thought to provide for the tax-exempt status of municipal interest payments. However, modern court rulings make it clear that Congress can revoke the municipal exemption, so the only basis now appears to be historical precedent. The fact that the states and the federal government do not tax each other’s securities is referred to as “reciprocal immunity.”

**12.** One measure of liquidity is the bid-ask spread. Liquid instruments have relatively small spreads. Looking at Figure 6.3, the bellwether bond has a spread of .0312 percent; it is one of the most liquid of all investments. Generally, liquidity declines after a bond is issued. Some older bonds will often have spreads as that are much wider.

**13.** Companies charge that bond rating agencies are pressuring them to pay for bond ratings. When a company pays for a rating, it has the opportunity to make its case for a particular rating. With an unsolicited rating, the company has no input.

**14.** A 100-year bond looks like a share of preferred stock. In particular, it is a loan with a life that almost certainly exceeds the life of the lender, assuming that the lender is an individual. With a junk bond, the credit risk can be so high that the borrower is almost certain to default, meaning that the creditors are very likely to end up as part owners of the business. In both cases, the “equity in disguise” has a significant tax advantage.

**15.** *a*. The bond price is the present value when discounting the future cash flows from a bond; YTM is the interest rate used in discounting the future cash flows (coupon payments and principal) back to their present values.

*b.* If the coupon rate is higher than the required return on a bond, the bond will sell at a premium, since it provides periodic income in the form of coupon payments in excess of that required by investors on other similar bonds. If the coupon rate is lower than the required return on a bond, the bond will sell at a discount, since it provides insufficient coupon payments compared to that required by investors on other similar bonds. For premium bonds, the coupon rate exceeds the YTM; for discount bonds, the YTM exceeds the coupon rate, and for bonds selling at par, the YTM is equal to the coupon rate.

*c.* Current yield is defined as the annual coupon payment divided by the current bond price. For premium bonds, the current yield exceeds the YTM, for discount bonds the current yield is less than the YTM, and for bonds selling at par value, the current yield is equal to the YTM. In all cases, the current yield plus the expected one-period capital gains yield of the bond must be equal to the required return.

**Solutions to Questions and Problems**

*NOTE: All end-of-chapter problems were solved using a spreadsheet. Many problems require multiple steps. Due to space and readability constraints, when these intermediate steps are included in this solutions manual, rounding may appear to have occurred. However, the final answer for each problem is found without rounding during any step in the problem.*

*Basic*

**1.** The yield to maturity is the required rate of return on a bond expressed as a nominal annual interest rate. For noncallable bonds, the yield to maturity and required rate of return are interchangeable terms. Unlike YTM and required return, the coupon rate is not a return used as the interest rate in bond cash flow valuation, but is a fixed percentage of par over the life of the bond used to set the coupon payment amount. For the example given, the coupon rate on the bond is still 10 percent, and the YTM is 8 percent.

**2.** Price and yield move in opposite directions; if interest rates rise, the price of the bond will fall. This is because the fixed coupon payments determined by the fixed coupon rate are not as valuable when interest rates rise–hence, the price of the bond decreases.

**3.** The price of any bond is the PV of the interest payment, plus the PV of the par value. Notice this problem assumes an annual coupon. The price of the bond will be:

*P* = $70({1 – [1/(1 + .084)]9} / .084) + $1,000[1 / (1 + .084)9]

*P* = $913.98

We would like to introduce shorthand notation here. Rather than write (or type, as the case may be) the entire equation for the PV of a lump sum, or the PVA equation, it is common to abbreviate the equations as:

PVIF*R,t* = 1 / (1 + *R*)*t*

which stands for Present Value Interest Factor

PVIFA*R,t*= ({1 – [1/(1 + *R*)]*t* } / *R*)

which stands for Present Value Interest Factor of an Annuity

These abbreviations are shorthand notation for the equations in which the interest rate and the number of periods are substituted into the equation and solved. We will use this shorthand notation in the remainder of the solutions key. The bond price equation for this problem would be:

*P* = $70(PVIFA8.4%,9) + $1,000(PVIF8.4%,9)

*P* = $913.98

**4.** Here, we need to find the YTM of a bond. The equation for the bond price is:

*P* = $961.50 = $70(PVIFA*R%*,9) + $1,000(PVIF*R*%,9)

Notice the equation cannot be solved directly for *R*. Using a spreadsheet, a financial calculator, or trial and error, we find:

*R* = YTM = 7.61%

If you are using trial and error to find the YTM of the bond, you might be wondering how to pick an interest rate to start the process. First, we know the YTM has to be higher than the coupon rate since the bond is a discount bond. That still leaves a lot of interest rates to check. One way to get a starting point is to use the following equation, which will give you an approximation of the YTM:

Approximate YTM = [Annual interest payment + (Par value – Price) / Years to maturity] /

[(Price + Par value) / 2]

Solving for this problem, we get:

Approximate YTM = [$70 + ($38.50 / 9)] / [($961.50 + 1,000) / 2]

Approximate YTM = .0757, or 7.57%

This is not the exact YTM, but it is close, and it will give you a place to start.

**5.** Here we need to find the coupon rate of the bond. All we need to do is to set up the bond pricing equation and solve for the coupon payment as follows:

*P* = $963 = *C*(PVIFA6.14%,12) + $1,000(PVIF6.14%,12)

Solving for the coupon payment, we get:

*C* = $56.95

The coupon payment is the coupon rate times par value. Using this relationship, we get:

Coupon rate = $56.95 / $1,000

Coupon rate = .0570, or 5.70%

**6.** To find the price of this bond, we need to realize that the maturity of the bond is 14 years. The bond was issued one year ago, with 15 years to maturity, so there are 14 years left on the bond. Also, the coupons are semiannual, so we need to use the semiannual interest rate and the number of semiannual periods. The price of the bond is:

*P* = $30.50(PVIFA2.65%,28) + $1,000(PVIF2.65%,28)

*P* = $1,078.37

**7.** Here, we are finding the YTM of a semiannual coupon bond. The bond price equation is:

*P* = $940 = $27(PVIFA*R%*,26) + $1,000(PVIF*R%*,26)

Since we cannot solve the equation directly for *R*, using a spreadsheet, a financial calculator, or trial and error, we find:

*R* = 3.037%

Since the coupon payments are semiannual, this is the semiannual interest rate. The YTM is the APR of the bond, so:

YTM = 23.037%

YTM = 6.07%

**8.** Here, we need to find the coupon rate of the bond. All we need to do is to set up the bond pricing equation and solve for the coupon payment as follows:

*P* = $945 = *C*(PVIFA3.1%,21) + $1,000(PVIF3.1%,21)

Solving for the coupon payment, we get:

*C* = $27.40

Since this is the semiannual payment, the annual coupon payment is:

2 × $27.40 = $54.80

And the coupon rate is the coupon payment divided by par value, so:

Coupon rate = $54.80 / $1,000

Coupon rate = .0548, or 5.48%

**9.** The approximate relationship between nominal interest rates (*R*), real interest rates (*r*), and inflation (*h*), is:

*R* = *r* + *h*

Approximate *r* = .045 – .016

Approximate *r* =.0285, or 2.85%

The Fisher equation, which shows the exact relationship between nominal interest rates, real interest rates, and inflation, is:

(1 + *R*) = (1 + *r*)(1 + *h*)

(1 + .045) = (1 + *r*)(1 + .016)

Exact *r* = [(1 + .045) / (1 + .016)] – 1

Exact *r* = .0281, or 2.81%

**10.** The Fisher equation, which shows the exact relationship between nominal interest rates, real interest rates, and inflation, is:

(1 + *R*) = (1 + *r*)(1 + *h*)

*R* = (1 + .019)(1 + .034) – 1

*R* = .0536, or 5.36%

**11.** The Fisher equation, which shows the exact relationship between nominal interest rates, real interest rates, and inflation, is:

(1 + *R*) = (1 + *r*)(1 + *h*)

*h* = [(1 + .13) / (1 + .08)] – 1

*h* = .0463, or 4.63%

**12.** The Fisher equation, which shows the exact relationship between nominal interest rates, real interest rates, and inflation, is:

(1 + *R*) = (1 + *r*)(1 + *h*)

*r* = [(1 + .117) / (1.023)] – 1

*r* = .0919, or 9.19%

**13.** The coupon rate, located in the second column of the quote is 6.125%. The bid price is:

Bid price = 146.1719%$1,000

Bid price = $1,461.719

The previous day’s ask price is found by:

Previous day’s asked price = Today’s asked price – Change

Previous day’s asked price = 146.2344 – (–.1172)

Previous day’s asked price = 146.3516

The previous day’s price in dollars was:

Previous day’s dollar price = (146.3516 / 100)$1,000

Previous day’s dollar price = $1,463.516

**14.** This is a premium bond because it sells for more than 100 percent of face value. The current yield is based on the asked price, so the current yield is:

Current yield = Annual coupon payment / Price

Current yield = $47.50 / $1,373.672

Current yield = .0346, or 3.46%

The YTM is located under the “Asked Yield” column, so the YTM is 2.521%.

The bid-ask spread is the difference between the bid price and the ask price, so:

Bid-Ask spread = 137.3672 – 137.3047

Bid-Ask spread = .0625% × $1,000

Bid-Ask spread = $.625

**15.** To find the price of a zero coupon bond, we need to find the value of the future cash flows. With a zero coupon bond, the only cash flow is the par value at maturity. We find the present value assuming semiannual compounding to keep the YTM of a zero coupon bond equivalent to the YTM of a coupon bond, so:

P = $10,000(PVIF2.45%,34)

P = $4,391.30

**16.** To find the price of this bond, we need to find the present value of the bond’s cash flows. So, the price of the bond is:

P = $49(PVIFA1.90%,26) + $2,000(PVIF1.90%,26)

P = $2,224.04

**17.** To find the price of this bond, we need to find the present value of the bond’s cash flows. So, the price of the bond is:

P = $92.50(PVIFA1.95%,32) + $5,000(PVIF1.95%,32)

P = $4,881.80

*Intermediate*

**18.** Here, we are finding the price of annual coupon bonds for various maturity lengths. The bond price equation is:

*P* = *C*(PVIFA*R%*,*t*) + $1,000(PVIF*R%*,*t*)

X: *P*0 = $42.50(PVIFA3.5%,13) + $1,000(PVIF3.5%,13) = $1,126.68

*P*1 = $42.50(PVIFA3.5%,12) + $1,000(PVIF3.5%,12) = $1,120.44

*P*3 = $42.50(PVIFA3.5%,10) + $1,000(PVIF3.5%,10) = $1,106.59

*P*8 = $42.50(PVIFA3.5%,5) + $1,000(PVIF3.5%,5) = $1,062.37

*P*12 = $42.50(PVIFA3.5%,1) + $1,000(PVIF3.5%,1) = $1,014.25

*P*13 = $1,000

Y: *P*0 = $35(PVIFA4.25%,13) + $1,000(PVIF4.25%,13) = $883.33

*P*1 = $35(PVIFA4.25%,12) + $1,000(PVIF4.25%,12) = $888.52

*P*3 = $35(PVIFA4.25%,10) + $1,000(PVIF4.25%,10) = $900.29

*P*8 = $35(PVIFA4.25%,5) + $1,000(PVIF4.25%,5) = $939.92

*P*12 = $35(PVIFA4.25%,1) + $1,000(PVIF4.25%,1) = $985.90

*P*13 = $1,000

All else held equal, the premium over par value for a premium bond declines as maturity approaches, and the discount from par value for a discount bond declines as maturity approaches. This is called “pull to par.” In both cases, the largest percentage price changes occur at the shortest maturity lengths.

Also, notice that the price of each bond when no time is left to maturity is the par value, even though the purchaser would receive the par value plus the coupon payment immediately. This is because we calculate the clean price of the bond.

**19.** Any bond that sells at par has a YTM equal to the coupon rate. Both bonds sell at par, so the initial YTM on both bonds is the coupon rate, 6.2 percent. If the YTM suddenly rises to 8.2 percent:

*P*Bill = $31(PVIFA4.1%,10) + $1,000(PVIF4.1%,10) = $919.29

*P*Ted = $31(PVIFA4.1%,50) + $1,000(PVIF4.1%,50) = $788.81

The percentage change in price is calculated as:

Percentage change in price = (New price – Original price) / Original price

Δ*P*Bill% = ($919.29 – 1,000) / $1,000 = –.0807, or –8.07%

Δ*P*Ted% = ($788.81 – 1,000) / $1,000 = –.2112, or –21.12%

If the YTM suddenly falls to 4.2 percent:

*P*Bill = $31(PVIFA2.1%,10) + $1,000(PVIF2.1%,10) = $1,089.36

*P*Ted = $31(PVIFA2.1%,50) + $1,000(PVIF2.1%,50) = $1,307.73

Δ*P*Bill% = ($1,089.36 – 1,000) / $1,000 = .0894, or +8.94%

Δ*P*Ted% = ($1,307.73 – 1,000) / $1,000 = .3077, or +30.77%

All else the same, the longer the maturity of a bond, the greater is its price sensitivity to changes in interest rates.

**20.** Initially, at a YTM of 8 percent, the prices of the two bonds are:

*P*J = $20(PVIFA4%,26) + $1,000(PVIF4%,26) = $680.34

*P*S = $70(PVIFA4%,26) + $1,000(PVIF4%,26) = $1,479.48

If the YTM rises from 8 percent to 10 percent:

*P*J = $20(PVIFA5%,26) + $1,000(PVIF5%,26) = $568.74

*P*S = $70(PVIFA5%,26) + $1,000(PVIF5%,26) = $1,287.50

The percentage change in price is calculated as:

Percentage change in price = (New price – Original price) / Original price

Δ*P*J% = ($568.74 – 680.34) / $680.34 = –.1640, or –16.40%

Δ*P*S% = ($1,287.50 – 1,479.48) / $1,479.48 = –.1298, or –12.98%

If the YTM declines from 8 percent to 6 percent:

*P*J = $20(PVIFA3%,26) + $1,000(PVIF3%,26) = $821.23

*P*S = $70(PVIFA3%,26) + $1,000(PVIF3%,26) = $1,715.07

Δ*P*J% = ($821.23 – 680.34) / $680.34 = .2071, or +20.71%

Δ*P*S% = ($1,715.07 – 1,479.48) / $1,479.48 = .1592, or +15.92%

All else the same, the lower the coupon rate on a bond, the greater is its price sensitivity to changes in interest rates.

**21.** The current yield is:

Current yield = Annual coupon payment / Price

Current yield = $63 / $970

Current yield =.0649, or 6.49%

The bond price equation for this bond is:

*P*0 = $970 = $31.50(PVIFA*R%*,44) + $1,000(PVIF*R%*,44)

Using a spreadsheet, financial calculator, or trial and error we find:

*R* = 3.280%

This is the semiannual interest rate, so the YTM is:

YTM = 2 × 3.280%

YTM = 6.56%

The effective annual yield is the same as the EAR, so using the EAR equation from the previous chapter:

Effective annual yield = (1 + .03280)2 – 1

Effective annual yield = .0667, or 6.67%

**22.** The company should set the coupon rate on its new bonds equal to the required return of the existing bonds. The required return can be observed in the market by finding the YTM on outstanding bonds of the company. So, the YTM on the bonds currently sold in the market is:

*P* = $1,074 = $28(PVIFA*R%*,50) + $1,000(PVIF*R%*,50)

Using a spreadsheet, financial calculator, or trial and error, we find:

*R* = 2.537%

This is the semiannual interest rate, so the YTM is:

YTM = 2 × 2.537%

YTM = 5.07%

**23.** Accrued interest is the coupon payment for the period times the fraction of the period that has passed since the last coupon payment. Since we have a semiannual coupon bond, the coupon payment per six months is one-half of the annual coupon payment. There are five months until the next coupon payment, so one month has passed since the last coupon payment. The accrued interest for the bond is:

Accrued interest = $57/2 × 1/6

Accrued interest = $4.75

And we calculate the clean price as:

Clean price = Dirty price – Accrued interest

Clean price = $1,043 – 4.75

Clean price = $1,038.25

**24.** Accrued interest is the coupon payment for the period times the fraction of the period that has passed since the last coupon payment. Since we have a semiannual coupon bond, the coupon payment per six months is one-half of the annual coupon payment. There are two months until the next coupon payment, so four months have passed since the last coupon payment. The accrued interest for the bond is:

Accrued interest = $59/2 × 4/6

Accrued interest = $19.67

And we calculate the dirty price as:

Dirty price = Clean price + Accrued interest

Dirty price = $993 + 19.67

Dirty price = $1,012.67

**25.** The bond has 15 years to maturity, so the bond price equation is:

*P* = $916.45 = $42.50(PVIFA*R%*,30) + $1,000(PVIF*R%*,30)

Using a spreadsheet, financial calculator, or trial and error, we find:

*R* = 4.780%

This is the semiannual interest rate, so the YTM is:

YTM = 2 × 4.780%

YTM = 9.56%

The current yield is the annual coupon payment divided by the bond price, so:

Current yield = $85 / $916.45

Current yield = .0927, or 9.27%

**26.** *a*. The coupon bonds have coupon rate of 6.8 percent which matches the 6.8 percent required return, so they will sell at par. The number of bonds that must be sold is the amount needed divided by the bond price, so:

Number of coupon bonds to sell = $35,000,000 / $1,000

Number of coupon bonds to sell = 35,000

The number of zero coupon bonds to sell would be:

Price of zero coupon bonds = $1,000 / 1.03440

Price of zero coupon bonds = $262.53

Number of zero coupon bonds to sell = $35,000,000 / $262.53

Number of zero coupon bonds to sell = 133,318.247

*b*. The repayment of the coupon bond will be the par value plus the last coupon payment times the number of bonds issued. So:

Coupon bonds repayment = 35,000($1,000) + 35,000($1,000)(.068 / 2)

Coupon bonds repayment = $36,190,000

The repayment of the zero coupon bond will be the par value times the number of bonds issued, so:

Zeroes repayment = 133,318.247($1,000)

Zeroes repayment = $133,318,247

*c*. The total coupon payment for the coupon bonds will be the number bonds times the coupon payment. For the cash flow of the coupon bonds, we need to account for the tax deductibility of the interest payments. To do this, we will multiply the total coupon payment times one minus the tax rate. So:

Coupon bonds: (35,000)($68)(1 – .35)

Coupon bonds: $1,547,000 cash outflow

Note that this is a cash outflow since the company is making the interest payment.

For the zero coupon bonds, the first year interest payment is the difference in the price of the zero at the end of the year and the beginning of the year. The price of the zeroes in one year will be:

*P*1 = $1,000 / 1.03438

*P*1 = $280.69

The Year 1 interest deduction per bond will be this price minus the price at the beginning of the year, which we found in part *b*, so:

Year 1 interest deduction per bond = $280.69 – 262.53

Year 1 interest deduction per bond = $18.16

The total cash flow for the zeroes will be the interest deduction for the year times the number of zeroes sold, times the tax rate. The cash flow for the zeroes in Year 1 will be:

Cash flows for zeroes in Year 1 = (133,318.247)($18.16)(.35)

Cash flows for zeroes in Year 1 = $847,161.00

Notice the cash flow for the zeroes is a cash inflow. This is because of the tax deductibility of the imputed interest expense. That is, the company gets to write off the interest expense for the year, even though the company did not have a cash flow for the interest expense. This reduces the company’s tax liability, which is a cash inflow.

During the life of the bond, the zero generates cash inflows to the firm in the form of the interest tax shield of debt. We should note an important point here: If you find the PV of the cash flows from the coupon bond and the zero coupon bond, they will be the same. This is because of the much larger repayment amount for the zeroes.

**27.** The maturity is indeterminate. A bond selling at par can have any length of maturity.

**28.** The bond asked price is 104.3850, so the dollar price is:

Dollar price = 104.3850% × $1,000

Dollar price = $1,043.850

So the bond price equation is:

*P* = $1,043.850 = $29.25(PVIFA*R%*,22) + $1,000(PVIF*R%*,22)

Using a spreadsheet, financial calculator, or trial and error, we find:

*R* = 2.659%

This is the semiannual interest rate, so the YTM is:

YTM = 2 × 2.659%

YTM = 5.32%

**29.** The coupon rate of the bond is 6.125 percent and the bond matures in 20 years. The bond coupon payments are semiannual, so the asked price is:

*P* = $30.625(PVIFA1.935%,40) + $1,000(PVIF1.935%,40)

*P* = $1,311.98

The bid-ask spread is .0628. We also know the bid price must be less than the asked price, so the bid price is:

Bid price = $1,311.98 – (.0628 / 100)($1,000)

Bid price = $1,311.35

**30.** Here, we need to find the coupon rate of the bond. The price of the bond is:

Dollar price = 103.8235% × $1,000

Dollar price = $1,038.235

So the bond price equation is:

*P* = $1,038.235 = *C*(PVIFA1.09%,8) + $1,000(PVIF1.09%,8)

Solving for the coupon payment, we get:

*C* = $15.92

Since this is the semiannual payment, the annual coupon payment is:

2 × $15.92 = $31.83

And the coupon rate is the coupon rate divided by par value, so:

Coupon rate = $31.83 / $1,000

Coupon rate = .0318, or 3.18%

**31.** Here we need to find the yield to maturity. The dollar price of the bond is:

Dollar price = 96.153% × $2,000

Dollar price = $1,923.06

So, the bond price equation is:

*P* = $1,923.06 = $54(PVIFA*R*%,8) + $2,000(PVIF*R*%,8)

Using a spreadsheet, financial calculator, or trial and error, we find:

*R* = 3.254%

This is the semiannual interest rate, so the YTM is:

YTM = 2 × 3.254%

YTM = 6.51%

**32.** The bond price equation is:

*P* = $71.25(PVIFA3.01%,10) + $2,000(PVIF3.01%,10)

*P* = $2,094.21

The current yield is the annual coupon payment divided by the bond price, so:

Current yield = $142.50 / $2,094.21

Current yield = .0680, or 6.80%

**33.** Here, we need to find the coupon rate of the bond. The dollar price of the bond is:

Dollar price = 94.735% × $2,000

Dollar price = $1,894.70

Now, we need to do is to set up the bond pricing equation and solve for the coupon payment as follows:

*P* = $1,894.70 = *C*(PVIFA3.425%,24) + $2,000(PVIF3.425%,24)

Solving for the coupon payment, we get:

*C* = $61.99

Since this is the semiannual payment, the annual coupon payment is:

2 × $61.99 = $123.99

And the coupon rate is the coupon rate divided by par value, so:

Coupon rate = $123.99 / $2,000

Coupon rate = .0620, or 6.20%

*Challenge*

**34.** To find the capital gains yield and the current yield, we need to find the price of the bond. The current price of Bond P and the price of Bond P in one year is:

P: *P*0 = $85(PVIFA7%,5) + $1,000(PVIF7%,5) = $1,061.50

*P*1 = $85(PVIFA7%,4) + $1,000(PVIF7%,4) = $1,050.81

Current yield = $85 / $1,061.50 = .0801, or 8.01%

The capital gains yield is:

Capital gains yield = (New price – Original price) / Original price

Capital gains yield = ($1,050.81 – 1,061.50) / $1,061.50 = –.0101, or –1.01%

The current price of Bond D and the price of Bond D in one year is:

D: *P*0 = $55(PVIFA7%,5) + $1,000(PVIF7%,5) = $938.50

*P*1 = $55(PVIFA7%,4) + $1,000(PVIF7%,4) = $949.19

Current yield = $55 / $938.50 = .0586, or 5.86%

Capital gains yield = ($949.19 – 938.50) / $938.50 = +.0114, or +1.14%

All else held constant, premium bonds pay high current income while having price depreciation as maturity nears; discount bonds do not pay high current income but have price appreciation as maturity nears. For either bond, the total return is still 7 percent, but this return is distributed differently between current income and capital gains.

**35.** *a*. The rate of return you expect to earn if you purchase a bond and hold it until maturity is the YTM. The bond price equation for this bond is:

*P*0 = $875 = $70(PVIFA*R%*,10) + $1,000(PVIF *R%*,10)

Using a spreadsheet, financial calculator, or trial and error we find:

*R* = YTM = 8.94%

*b*. To find our HPY, we need to find the price of the bond in two years. The price of the bond in two years, at the new interest rate, will be:

*P*2 = $70(PVIFA7.94%,8) + $1,000(PVIF7.94%,8) = $945.70

To calculate the HPY, we need to find the interest rate that equates the price we paid for the bond with the cash flows we received. The cash flows we received were $70 each year for two years, and the price of the bond when we sold it. The equation to find our HPY is:

$875 = $70(PVIFA*R%*,2) + $945.70(PVIF*R%*,2)

Solving for *R*, we get:

*R* = HPY = 11.81%

The realized HPY is greater than the expected YTM when the bond was purchased because interest rates dropped by 1 percent; bond prices rise when yields fall.

**Calculator Solutions**

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| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **3.** |  | | |  | | |  | | |  | | |  | | |
| Enter | 9 | | | 8.4% | | |  | | | ±$70 | | | ±$1,000 | | |
|  |  | **N** |  |  | **I/Y** |  |  | **PV** |  |  | **PMT** |  |  | **FV** |  | |
| Solve for |  | | |  | | | $913.98 | | |  | | |  | | |

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **4.** |  | | |  | | |  | | |  | | |  | | |
| Enter | 9 | | |  | | | ±$961.50 | | | $70 | | | $1,000 | | |
|  |  | **N** |  |  | **I/Y** |  |  | **PV** |  |  | **PMT** |  |  | **FV** |  | |
| Solve for |  | | | 7.61% | | |  | | |  | | |  | | |

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **5.** |  | | |  | | |  | | |  | | |  | | |
| Enter | 12 | | | 6.14% | | | ±$963 | | |  | | | $1,000 | | |
|  |  | **N** |  |  | **I/Y** |  |  | **PV** |  |  | **PMT** |  |  | **FV** |  | |
| Solve for |  | | |  | | |  | | | $56.95 | | |  | | |

Coupon rate = $56.95 / $1,000

Coupon rate = .0570, or 5.70%

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **6.** |  | | |  | | |  | | |  | | |  | | |
| Enter | 14 × 2 | | | 5.3% / 2 | | |  | | | ±$61 / 2 | | | ±$1,000 | | |
|  |  | **N** |  |  | **I/Y** |  |  | **PV** |  |  | **PMT** |  |  | **FV** |  | |
| Solve for |  | | |  | | | $1,078.37 | | |  | | |  | | |

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **7.** |  | | |  | | |  | | |  | | |  | | |
| Enter | 13 × 2 | | |  | | | ±940 | | | $54 / 2 | | | $1,000 | | |
|  |  | **N** |  |  | **I/Y** |  |  | **PV** |  |  | **PMT** |  |  | **FV** |  | |
| Solve for |  | | | 3.037% | | |  | | |  | | |  | | |

YTM = 3.037 × 2

YTM = 6.07%

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **8.** |  | | |  | | |  | | |  | | |  | | |
| Enter | 10.5 × 2 | | | 6.2% / 2 | | | ±$945 | | |  | | | $1,000 | | |
|  |  | **N** |  |  | **I/Y** |  |  | **PV** |  |  | **PMT** |  |  | **FV** |  | |
| Solve for |  | | |  | | |  | | | $27.40 | | |  | | |

Annual coupon = $27.40 × 2

Annual coupon = $54.80

Coupon rate = $54.80 / $1,000

Coupon rate = .0548, or 5.48%

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **15.** |  | | |  | | |  | | |  | | |  | | |
| Enter | 34 | | | 4.9% / 2 | | |  | | |  | | | ±$10,000 | | |
|  |  | **N** |  |  | **I/Y** |  |  | **PV** |  |  | **PMT** |  |  | **FV** |  | |
| Solve for |  | | |  | | | $4,391.30 | | |  | | |  | | |

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **16.** |  | | |  | | |  | | |  | | |  | | |
| Enter | 26 | | | 3.8% / 2 | | |  | | | ±$49 / 2 | | | ±$2,000 | | |
|  |  | **N** |  |  | **I/Y** |  |  | **PV** |  |  | **PMT** |  |  | **FV** |  | |
| Solve for |  | | |  | | | $2,224.04 | | |  | | |  | | |

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **17.** |  | | |  | | |  | | |  | | |  | | |
| Enter | 32 | | | 3.9% / 2 | | |  | | | ±$185 / 2 | | | ±$5,000 | | |
|  |  | **N** |  |  | **I/Y** |  |  | **PV** |  |  | **PMT** |  |  | **FV** |  | |
| Solve for |  | | |  | | | $4,881.80 | | |  | | |  | | |

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **18.** | Bond X | | |  | | |  | | |  | | |  | | |
| Enter | 13 | | | 7% / 2 | | |  | | | ±$42.50 | | | ±$1,000 | | |
|  |  | **N** |  |  | **I/Y** |  |  | **PV** |  |  | **PMT** |  |  | **FV** |  | |
| Solve for |  | | |  | | | $1,126.68 | | |  | | |  | | |

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  | | |  | | |  | | |  | | |  | | |
| Enter | 12 | | | 7% / 2 | | |  | | | ±$42.50 | | | ±$1,000 | | |
|  |  | **N** |  |  | **I/Y** |  |  | **PV** |  |  | **PMT** |  |  | **FV** |  | |
| Solve for |  | | |  | | | $1,120.44 | | |  | | |  | | |

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  | | |  | | |  | | |  | | |  | | |
| Enter | 10 | | | 7% / 2 | | |  | | | ±$42.50 | | | ±$1,000 | | |
|  |  | **N** |  |  | **I/Y** |  |  | **PV** |  |  | **PMT** |  |  | **FV** |  | |
| Solve for |  | | |  | | | $1,106.59 | | |  | | |  | | |

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  | | |  | | |  | | |  | | |  | | |
| Enter | 5 | | | 7% / 2 | | |  | | | ±$42.50 | | | ±$1,000 | | |
|  |  | **N** |  |  | **I/Y** |  |  | **PV** |  |  | **PMT** |  |  | **FV** |  | |
| Solve for |  | | |  | | | $1,062.37 | | |  | | |  | | |

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  | | |  | | |  | | |  | | |  | | |
| Enter | 1 | | | 7% / 2 | | |  | | | ±$42.50 | | | ±$1,000 | | |
|  |  | **N** |  |  | **I/Y** |  |  | **PV** |  |  | **PMT** |  |  | **FV** |  | |
| Solve for |  | | |  | | | $1,014.25 | | |  | | |  | | |

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | Bond Y | | |  | | |  | | |  | | |  | | |
| Enter | 13 | | | 8.5% / 2 | | |  | | | ±$35 | | | ±$1,000 | | |
|  |  | **N** |  |  | **I/Y** |  |  | **PV** |  |  | **PMT** |  |  | **FV** |  | |
| Solve for |  | | |  | | | $883.33 | | |  | | |  | | |

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  | | |  | | |  | | |  | | |  | | |
| Enter | 12 | | | 8.5% / 2 | | |  | | | ±$35 | | | ±$1,000 | | |
|  |  | **N** |  |  | **I/Y** |  |  | **PV** |  |  | **PMT** |  |  | **FV** |  | |
| Solve for |  | | |  | | | $888.52 | | |  | | |  | | |

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  | | |  | | |  | | |  | | |  | | |
| Enter | 10 | | | 8.5% / 2 | | |  | | | ±$35 | | | ±$1,000 | | |
|  |  | **N** |  |  | **I/Y** |  |  | **PV** |  |  | **PMT** |  |  | **FV** |  | |
| Solve for |  | | |  | | | $900.29 | | |  | | |  | | |

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  | | |  | | |  | | |  | | |  | | |
| Enter | 5 | | | 8.5% / 2 | | |  | | | ±$35 | | | ±$1,000 | | |
|  |  | **N** |  |  | **I/Y** |  |  | **PV** |  |  | **PMT** |  |  | **FV** |  | |
| Solve for |  | | |  | | | $939.92 | | |  | | |  | | |

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  | | |  | | |  | | |  | | |  | | |
| Enter | 1 | | | 8.5% / 2 | | |  | | | ±$35 | | | ±$1,000 | | |
|  |  | **N** |  |  | **I/Y** |  |  | **PV** |  |  | **PMT** |  |  | **FV** |  | |
| Solve for |  | | |  | | | $985.90 | | |  | | |  | | |

**19.** If both bonds sell at par, the initial YTM on both bonds is the coupon rate, 6.2 percent. If the YTM suddenly rises to 8.2 percent:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | PBill | | |  | | |  | | |  | | |  | | |
| Enter | 5× 2 | | | 8.2% / 2 | | |  | | | ±$62 / 2 | | | ±$1,000 | | |
|  |  | **N** |  |  | **I/Y** |  |  | **PV** |  |  | **PMT** |  |  | **FV** |  | |
| Solve for |  | | |  | | | $919.29 | | |  | | |  | | |

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | PTed | | |  | | |  | | |  | | |  | | |
| Enter | 25 × 2 | | | 8.2% / 2 | | |  | | | ±$62 / 2 | | | ±$1,000 | | |
|  |  | **N** |  |  | **I/Y** |  |  | **PV** |  |  | **PMT** |  |  | **FV** |  | |
| Solve for |  | | |  | | | $788.81 | | |  | | |  | | |

ΔPBill% = ($919.29 – 1,000) / $1,000 = –8.07%

ΔPTed% = ($788.81 – 1,000) / $1,000 = –21.12%

If the YTM suddenly falls to 4.2 percent:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | PBill | | |  | | |  | | |  | | |  | | |
| Enter | 5 × 2 | | | 4.2% / 2 | | |  | | | ±$62 / 2 | | | ±$1,000 | | |
|  |  | **N** |  |  | **I/Y** |  |  | **PV** |  |  | **PMT** |  |  | **FV** |  | |
| Solve for |  | | |  | | | $1,089.36 | | |  | | |  | | |

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | PTed | | |  | | |  | | |  | | |  | | |
| Enter | 25 × 2 | | | 4.2% / 2 | | |  | | | ±$62 / 2 | | | ±$1,000 | | |
|  |  | **N** |  |  | **I/Y** |  |  | **PV** |  |  | **PMT** |  |  | **FV** |  | |
| Solve for |  | | |  | | | $1,307.73 | | |  | | |  | | |

ΔPBill% = ($1,089.36 – 1000) / $1,000 = +8.94%

ΔPTed% = ($1,307.73 – 1000) / $1,000 = +30.77%

All else the same, the longer the maturity of a bond, the greater is its price sensitivity to changes in interest rates.

**20.** Initially, at a YTM of 8 percent, the prices of the two bonds are:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | PJ | | |  | | |  | | |  | | |  | | |
| Enter | 13 × 2 | | | 4% | | |  | | | ±$40 / 2 | | | ±$1,000 | | |
|  |  | **N** |  |  | **I/Y** |  |  | **PV** |  |  | **PMT** |  |  | **FV** |  | |
| Solve for |  | | |  | | | $680.34 | | |  | | |  | | |

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | PS | | |  | | |  | | |  | | |  | | |
| Enter | 13 × 2 | | | 4% | | |  | | | ±$140 / 2 | | | ±$1,000 | | |
|  |  | **N** |  |  | **I/Y** |  |  | **PV** |  |  | **PMT** |  |  | **FV** |  | |
| Solve for |  | | |  | | | $1,479.48 | | |  | | |  | | |

If the YTM rises from 8 percent to 10 percent:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | PJ | | |  | | |  | | |  | | |  | | |
| Enter | 13 × 2 | | | 5% | | |  | | | ±$40 / 2 | | | ±$1,000 | | |
|  |  | **N** |  |  | **I/Y** |  |  | **PV** |  |  | **PMT** |  |  | **FV** |  | |
| Solve for |  | | |  | | | $568.74 | | |  | | |  | | |

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | PS | | |  | | |  | | |  | | |  | | |
| Enter | 13 × 2 | | | 5% | | |  | | | ±$140 / 2 | | | ±$1,000 | | |
|  |  | **N** |  |  | **I/Y** |  |  | **PV** |  |  | **PMT** |  |  | **FV** |  | |
| Solve for |  | | |  | | | $1,287.50 | | |  | | |  | | |

ΔPJ% = ($568.74 – 680.34) / $680.34 = –16.40%

ΔPS% = ($1,287.50 – 1,479.48) / $1,479.48 = –12.98%

If the YTM declines from 8 percent to 6 percent:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | PJ | | |  | | |  | | |  | | |  | | |
| Enter | 13 × 2 | | | 3% | | |  | | | ±$40 / 2 | | | ±$1,000 | | |
|  |  | **N** |  |  | **I/Y** |  |  | **PV** |  |  | **PMT** |  |  | **FV** |  | |
| Solve for |  | | |  | | | $821.23 | | |  | | |  | | |

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | PS | | |  | | |  | | |  | | |  | | |
| Enter | 13 × 2 | | | 3% | | |  | | | ±$140 / 2 | | | ±$1,000 | | |
|  |  | **N** |  |  | **I/Y** |  |  | **PV** |  |  | **PMT** |  |  | **FV** |  | |
| Solve for |  | | |  | | | $1,715.07 | | |  | | |  | | |

ΔPJ% = ($821.23 – 680.34) / $680.34 = +20.71%

ΔPS% = ($1,715.07 – 1,479.48) / $1,479.48 = +15.92%

All else the same, the lower the coupon rate on a bond, the greater is its price sensitivity to changes in interest rates.

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **21.** |  | | |  | | |  | | |  | | |  | | |
| Enter | 22 × 2 | | |  | | | ±$970 | | | $63 / 2 | | | $1,000 | | |
|  |  | **N** |  |  | **I/Y** |  |  | **PV** |  |  | **PMT** |  |  | **FV** |  | |
| Solve for |  | | | 3.280% | | |  | | |  | | |  | | |

YTM = 2 × 3.280%

YTM = 6.56%

Effective annual yield:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  | | |  | | |  | | |  | | |  | | |
| Enter | 6.56% | | |  | | | 2 | | |  | | |  | | |
|  |  | **NOM** |  |  | **EFF** |  |  | **C/Y** |  |  |  |  |  |  |  | |
| Solve for |  | | | 6.67% | | |  | | |  | | |  | | |

**22.** The company should set the coupon rate on its new bonds equal to the required return; the required return can be observed in the market by finding the YTM on outstanding bonds of the company.

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  | | |  | | |  | | |  | | |  | | |
| Enter | 25 × 2 | | |  | | | ±$1,074 | | | $56 / 2 | | | $1,000 | | |
|  |  | **N** |  |  | **I/Y** |  |  | **PV** |  |  | **PMT** |  |  | **FV** |  | |
| Solve for |  | | | 2.537% | | |  | | |  | | |  | | |

YTM = 2 × 2.537%

YTM = 5.07%

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **25.** |  | | |  | | |  | | |  | | |  | | |
| Enter | 15 × 2 | | |  | | | ±$916.45 | | | $85 / 2 | | | $1,000 | | |
|  |  | **N** |  |  | **I/Y** |  |  | **PV** |  |  | **PMT** |  |  | **FV** |  | |
| Solve for |  | | | 4.780% | | |  | | |  | | |  | | |

YTM = 2 × 4.780%

YTM = 9.56%

**26.** *a.* The coupon bonds have a 6.8% coupon which matches the 6.8% required return, so they will sell at par. For the zeroes, the price is:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  | | |  | | |  | | |  | | |  | | |
| Enter | 20 × 2 | | | 6.8% / 2 | | |  | | |  | | | ±$1,000 | | |
|  |  | **N** |  |  | **I/Y** |  |  | **PV** |  |  | **PMT** |  |  | **FV** |  | |
| Solve for |  | | |  | | | $262.53 | | |  | | |  | | |

*c*. The price of the zeroes in one year will be:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  | | |  | | |  | | |  | | |  | | |
| Enter | 19 × 2 | | | 6.8% / 2 | | |  | | |  | | | ±$1,000 | | |
|  |  | **N** |  |  | **I/Y** |  |  | **PV** |  |  | **PMT** |  |  | **FV** |  | |
| Solve for |  | | |  | | | $280.69 | | |  | | |  | | |

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **28.** |  | | |  | | |  | | |  | | |  | | |
| Enter | 11 × 2 | | |  | | | ±$1,043.850 | | | $58.50 / 2 | | | $1,000 | | |
|  |  | **N** |  |  | **I/Y** |  |  | **PV** |  |  | **PMT** |  |  | **FV** |  | |
| Solve for |  | | | 2.659% | | |  | | |  | | |  | | |

YTM = 2 × 2.659%

YTM = 5.32%

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **29.** |  | | |  | | |  | | |  | | |  | | |
| Enter | 20 × 2 | | | 3.87% / 2 | | |  | | | ±$61.25 / 2 | | | ±$1,000 | | |
|  |  | **N** |  |  | **I/Y** |  |  | **PV** |  |  | **PMT** |  |  | **FV** |  | |
| Solve for |  | | |  | | | $1,311.98 | | |  | | |  | | |

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
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| **30.** |  | | |  | | |  | | |  | | |  | | |
| Enter | 4 × 2 | | | 2.18% / 2 | | | ±$1,038.235 | | |  | | | $1,000 | | |
|  |  | **N** |  |  | **I/Y** |  |  | **PV** |  |  | **PMT** |  |  | **FV** |  | |
| Solve for |  | | |  | | |  | | | $15.92 | | |  | | |

Annual coupon = $15.92 × 2

Annual coupon = $31.83

Coupon rate = $31.83 / $1,000

Coupon rate = .0318, or 3.18%

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **31.** |  | | |  | | |  | | |  | | |  | | |
| Enter | 4 × 2 | | |  | | | ±$1,923.06 | | | $54 | | | $2,000 | | |
|  |  | **N** |  |  | **I/Y** |  |  | **PV** |  |  | **PMT** |  |  | **FV** |  | |
| Solve for |  | | | 3.254% | | |  | | |  | | |  | | |

YTM = 2 × 3.254%

YTM = 6.51%

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **32.** |  | | |  | | |  | | |  | | |  | | |
| Enter | 5 × 2 | | | 6.02% / 2 | | |  | | | ±$71.25 | | | ±$2,000 | | |
|  |  | **N** |  |  | **I/Y** |  |  | **PV** |  |  | **PMT** |  |  | **FV** |  | |
| Solve for |  | | |  | | | $2,094.21 | | |  | | |  | | |

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| **33.** |  | | |  | | |  | | |  | | |  | | |
| Enter | 13 × 2 | | | 6.85% / 2 | | | ±$1,894.70 | | |  | | | $2,000 | | |
|  |  | **N** |  |  | **I/Y** |  |  | **PV** |  |  | **PMT** |  |  | **FV** |  | |
| Solve for |  | | |  | | |  | | | $61.99 | | |  | | |

Annual coupon = $61.99 × 2

Annual coupon = $123.99

Coupon rate = $123.99 / $2,000

Coupon rate = .0620, or 6.20%

**34.**

Bond P

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| P0 |  | | |  | | |  | | |  | | |  | | |
| Enter | 5 | | | 7% | | |  | | | ±$85 | | | ±$1,000 | | |
|  |  | **N** |  |  | **I/Y** |  |  | **PV** |  |  | **PMT** |  |  | **FV** |  | |
| Solve for |  | | |  | | | $1,061.50 | | |  | | |  | | |

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| P1 |  | | |  | | |  | | |  | | |  | | |
| Enter | 4 | | | 7% | | |  | | | ±$85 | | | ±$1,000 | | |
|  |  | **N** |  |  | **I/Y** |  |  | **PV** |  |  | **PMT** |  |  | **FV** |  | |
| Solve for |  | | |  | | | $1,050.81 | | |  | | |  | | |

Current yield = $85 / $1,061.50 = .0801, or 8.01%

Capital gains yield = ($1,050.81 – 1,061.50) / $1,061.50 = –1.01%

Bond D

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| P0 |  | | |  | | |  | | |  | | |  | | |
| Enter | 5 | | | 7% | | |  | | | ±$55 | | | ±$1,000 | | |
|  |  | **N** |  |  | **I/Y** |  |  | **PV** |  |  | **PMT** |  |  | **FV** |  | |
| Solve for |  | | |  | | | $938.50 | | |  | | |  | | |

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| P1 |  | | |  | | |  | | |  | | |  | | |
| Enter | 4 | | | 7% | | |  | | | ±$55 | | | ±$1,000 | | |
|  |  | **N** |  |  | **I/Y** |  |  | **PV** |  |  | **PMT** |  |  | **FV** |  | |
| Solve for |  | | |  | | | $949.19 | | |  | | |  | | |

Current yield = $55 / $938.50 = .0586, or 5.86%

Capital gains yield = ($949.19 – 938.50) / $938.50 = +1.14%

All else held constant, premium bonds pay high current income while having price depreciation as maturity nears; discount bonds do not pay high current income but have price appreciation as maturity nears. For either bond, the total return is still 7 percent, but this return is distributed differently between current income and capital gains.

**35.**

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| *a.* |  | | |  | | |  | | |  | | |  | | |
| Enter | 10 | | |  | | | ±$875 | | | $70 | | | $1,000 | | |
|  |  | **N** |  |  | **I/Y** |  |  | **PV** |  |  | **PMT** |  |  | **FV** |  | |
| Solve for |  | | | 8.94% | | |  | | |  | | |  | | |

This is the rate of return you expect to earn on your investment when you purchase the bond.

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| *b.* |  | | |  | | |  | | |  | | |  | | |
| Enter | 8 | | | 7.94% | | |  | | | ±$70 | | | ±$1,000 | | |
|  |  | **N** |  |  | **I/Y** |  |  | **PV** |  |  | **PMT** |  |  | **FV** |  | |
| Solve for |  | | |  | | | $945.70 | | |  | | |  | | |

The HPY is:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  | | |  | | |  | | |  | | |  | | |
| Enter | 2 | | |  | | | ±$875 | | | $70 | | | $945.70 | | |
|  |  | **N** |  |  | **I/Y** |  |  | **PV** |  |  | **PMT** |  |  | **FV** |  | |
| Solve for |  | | | 11.81% | | |  | | |  | | |  | | |

The realized HPY is greater than the expected YTM when the bond was purchased because interest rates dropped by 1 percent; bond prices rise when yields fall.

**CHAPTER 7**

***EQUITY MARKETS AND STOCK VALUATION***

**Answers to Concepts Review and Critical Thinking Questions**

**1.** The value of any investment depends on its cash flows; i.e., what investors will actually receive. The cash flows from a share of stock are the dividends.

**2.** Investors believe the company will eventually start paying dividends (or be sold to another company).

**3.** In general, companies that need the cash will often forgo dividends since dividends are a cash expense. Young, growing companies with profitable investment opportunities are one example; another example is a company in financial distress. This question is examined in depth in a later chapter.

**4.** The general method for valuing a share of stock is to find the present value of all expected future dividends. The dividend growth model presented in the text is only valid (i) if dividends are expected to occur forever; that is, the stock provides dividends in perpetuity, and (ii) if a constant growth rate of dividends occurs forever. A violation of the first assumption might be a company that is expected to cease operations and dissolve itself some finite number of years from now. The stock of such a company would be valued by the methods of this chapter by applying the general method of valuation. A violation of the second assumption might be a start-up firm that isn’t currently paying any dividends, but is expected to eventually start making dividend payments some number of years from now. This stock would also be valued by the general dividend valuation method of this chapter.

**5.** The common stock probably has a higher price because the dividend can grow, whereas it is fixed on the preferred. However, the preferred is less risky because of the dividend and liquidation preference, so it is possible the preferred could be worth more, depending on the circumstances.

**6.** The two components are the dividend yield and the capital gains yield. For most companies, the capital gains yield is larger. This is easy to see for companies that pay no dividends. For companies that do pay dividends, the dividend yields are rarely over five percent and are often much less.

**7.** The dividend growth model makes the implicit assumption that the stock price will grow at the same constant rate as the dividend. What this means is that if the cash flows on an investment grow at a constant rate through time, the value of that investment grows at the same rate as the cash flows.

**8.** For a particular year, this can (and often does) happen. Going back to the cash flow identity, the dividend payments depend on operating cash flow, capital spending, the change in net working capital, and the cash flow to creditors. The firm could have positive operating cash flow with negative earnings, sell fixed assets, reduce net working capital, or raise cash from creditors in order to pay dividends. While this is possible in the short term, as a practical matter over the longer term, the company would probably need to have a positive net income (at least on average) in order to maintain a dividend.

**9.** It wouldn’t seem to be. Investors who don’t like the voting features of a particular class of stock are under no obligation to buy it.

**10.** Investors buy such stock because they want it, recognizing that the shares have no voting power. Presumably, investors pay less for such shares than they would otherwise.

**11.** Presumably, the current stock value reflects the risk, timing, and magnitude of all future cash flows, both short-term *and* long-term. If this is correct, then the statement is false.

**12.** A reasonable limit for the growth rate is the growth rate of the economy, which in the U.S. has historically been about 3 to 3.5 percent (after accounting for inflation). As we will see in a later chapter, inflation has historically averaged about 3 percent, so 6 to 6.5 percent (before accounting for inflation) would be a reasonable limit.

**13.** In a declassified board, every board seat is up for election every year. This structure allows investors to vote out a director (and even the entire board) much more quickly if investors are dissatisfied. However, this structure also makes it more difficult to fight off a hostile takeover bid. In contrast, a classified board can more effectively negotiate on behalf of stockholders, perhaps securing better terms in a deal. Classified boards are also important for institutional memory. If an entire board were voted out in a single year, there would be no board members available to evaluate the company’s direction with regards to previous decisions.

**14.** The major difficulty in using price ratio analysis is determining the correct benchmark PE ratio. In a previous chapter, we showed how the sustainable growth rate is determined, and in a future chapter we will discuss the required return. Although not exact measures, the growth rate and required return have a solid economic basis. With the PE ratio, like any other ratio, it is difficult to determine what the ratio should be. Since a small difference in the PE ratio can have a significant effect on the calculated stock price, it is easy to arrive at an incorrect valuation.

**Solutions to Questions and Problems**

*NOTE: All end-of-chapter problems were solved using a spreadsheet. Many problems require multiple steps. Due to space and readability constraints, when these intermediate steps are included in this solutions manual, rounding may appear to have occurred. However, the final answer for each problem is found without rounding during any step in the problem.*

*Basic*

**1.** The constant dividend growth model is:

*Pt* = *Dt* × (1 + *g*) / (*R* – *g*)

So, the price of the stock today is:

*P*0 = *D*0 (1 + *g*) / (*R* – *g*)

*P*0 = $2.35(1.041) / (.104 – .041)

*P*0 = $38.83

The dividend at Year 4 is the dividend today times the FVIF for the growth rate in dividends and four years, so:

*P*3 = *D*3 (1 + *g*) / (*R* – *g*)

*P*3 = *D*0 (1 + *g*)4 / (*R* – *g*)

*P*3 = $2.35(1.041)4 / (.104 – .041)

*P*3 = $43.81

We can do the same thing to find the dividend in Year 16, which gives us the price in Year 15, so:

*P*15 = *D*15 (1 + *g*) / (*R* – *g*)

*P*15 = *D*0 (1 + *g*)16 / (*R* – *g*)

*P*15 = $2.35(1.041)16 / (.104 – .041)

*P*15 = $70.95

There is another feature of the constant dividend growth model: The stock price grows at the dividend growth rate. So, if we know the stock price today, we can find the future value for any time in the future we want to calculate the stock price. In this problem, we want to know the stock price in three years, and we have already calculated the stock price today. The stock price in three years will be:

*P*3 = *P*0(1 + *g*)3

*P*3 = $38.83(1 + .041)3

*P*3 = $43.81

And the stock price in 15 years will be:

*P*15 = *P*0(1 + *g*)15

*P*15 = $38.83(1 + .041)15

*P*15 = $70.95

**2.** We need to find the required return of the stock. Using the constant growth model, we can solve the equation for *R*. Doing so, we find:

*R* = (*D*1 / *P*0) + *g*

*R* = ($2.48 / $39.85) + .045

*R* = .1072, or 10.72%

**3.** The dividend yield is the dividend next year divided by the current price, so the dividend yield is:

Dividend yield = *D*1 / *P*0

Dividend yield = $2.48 / $39.85

Dividend yield = .0622, or 6.22%

The capital gains yield, or percentage increase in the stock price, is the same as the dividend growth rate, so:

Capital gains yield = 4.5%

**4.** Using the constant growth model, we find the price of the stock today is:

*P*0 = *D*1 / (*R* – *g*)

*P*0 = $3.65 / (.11 – .051)

*P*0 = $61.86

**5.** The required return of a stock is made up of two parts: The dividend yield and the capital gains yield. So, the required return of this stock is:

*R* = Dividend yield + Capital gains yield

*R* = .058 + .046

*R* = .1040, or 10.40%

**6.** We know the stock has a required return of 11.5 percent, and the dividend and capital gains yields are equal, so:

Dividend yield = 1/2(.115)

Dividend yield = .0575 = Capital gains yield

Now we know both the dividend yield and capital gains yield. The dividend is simply the stock price times the dividend yield, so:

*D*1 = .0575($67)

*D*1 = $3.85

This is the dividend next year. The question asks for the dividend this year. Using the relationship between the dividend this year and the dividend next year:

*D*1 = *D*0(1 + *g*)

We can solve for the dividend that was just paid:

$3.85 = *D*0(1 + .0575)

*D*0 = $3.85 / 1.0575

*D*0 = $3.64

**7.** The price of any financial instrument is the present value of the future cash flows. The future dividends of this stock are an annuity for 9 years, so the price of the stock is the present value of an annuity, which will be:

*P*0 = $13.50(PVIFA9.2%,9)

*P*0 = $80.28

**8.** The price of a share of preferred stock is the dividend divided by the required return. This is the same equation as the constant growth model, with a dividend growth rate of zero percent. Remember, most preferred stock pays a fixed dividend, so the growth rate is zero. This is a special case of the dividend growth model where the growth rate is zero, or the level perpetuity equation. Using this equation, we find the price per share of the preferred stock is:

*R* = *D* / *P*0

*R* = $3.45 / $77.32

*R* = .0446, or 4.46%

**9.** If the company uses straight voting, you will need to own one-half of the shares, plus one share, in order to guarantee enough votes to win the election. So, the number of shares needed to guarantee election under straight voting will be:

Shares needed = (450,000 shares / 2) + 1

Shares needed = 225,001

And the total cost to you will be the shares needed times the price per share, or:

Total cost = 225,001 × $34

Total cost = $7,650,034

If the company uses cumulative voting, you will need 1/(*N* + 1) percent of the stock (plus one share) to guarantee election, where N is the number of seats up for election. So, the percentage of the company’s stock you need will be:

Percent of stock needed = 1 / (*N* + 1)

Percent of stock needed = 1 / (4 + 1)

Percent of stock needed = .20, or 20%

So, the number of shares you need to purchase is:

Number of shares to purchase = (450,000 × .20) + 1

Number of shares to purchase = 90,001

And the total cost to you will be the shares needed times the price per share, or:

Total cost = 90,001 × $34

Total cost = $3,060,034

**10.** We need to find the growth rate of dividends. Using the constant growth model, we can solve the equation for *g*. Doing so, we find:

*g* = *R* – (*D*1 / *P*0)

*g* = .105 – ($4.25 / $73)

*g* = .0468, or 4.68%

**11.** Here, we have a stock that pays no dividends for 20 years. Once the stock begins paying dividends, it will have the same dividends forever, a preferred stock. We value the stock at that point, using the preferred stock equation. It is important to remember that the price we find will be the price one year before the first dividend, so:

*P*19 = *D*20 / *R*

*P*19 = $20 / .071

*P*19 = $281.69

The price of the stock today is simply the present value of the stock price in the future. We simply discount the future stock price at the required return. The price of the stock today will be:

*P*0 = $281.69 / 1.07119

*P*0 = $76.52

**12.** Here, we need to value a stock with two different required returns. Using the constant growth model and a required return of 12 percent, the stock price today is:

*P*0 = *D*1 / (*R* – *g*)

*P*0 = $3.14 / (.12 – .045)

*P*0 = $41.87

And the stock price today with a required return of 8 percent will be:

*P*0 = *D*1 / (*R* – *g*)

*P*0 = $3.14 / (.08 – .045)

*P*0 = $89.71

All else held constant, a higher required return means that the stock will sell for a lower price. Also, notice that the stock price is very sensitive to the required return. In this case, the required return fell by 1/3 but the stock price more than doubled.

**13.** Using the equation to calculate the price of a share of stock with the PE ratio:

P = Benchmark PE ratio × EPS

So, with a PE ratio of 18, we find:

P = 18($2.65)

P = $47.70

And with a PE ratio of 21, we find:

P = 21($2.65)

P = $55.65

**14.** First, we need to find the sales per share, which is:

Sales per share = Sales / Shares outstanding

Sales per share = $1,350,000 / 130,000

Sales per share = $10.38

Using the equation to calculate the price of a share of stock with the PS ratio:

P = Benchmark PS ratio × Sales per share

So, with a PS ratio of 4.8, we find:

P = 4.8($10.38)

P = $49.85

And with a PS ratio of 3.7, we find:

P = 3.7($10.38)

P = $38.42

*Intermediate*

**15.** Here, we have a stock that pays no dividends for nine years. Once the stock begins paying dividends, it will have a constant growth rate of dividends. We can use the constant growth model at that point. It is important to remember that the general constant dividend growth formula is:

*Pt* = [*Dt* × (1 + *g*)] / (*R* – *g*)

This means that since we will use the dividend in Year 10, we will be finding the stock price in Year 9. The dividend growth model is similar to the present value of an annuity and the present value of a perpetuity: The equation gives you the present value one period before the first payment. So, the price of the stock in Year 9 will be:

*P*9 = *D*10 / (*R* – *g*)

*P*9 = $19.00 / (.13 – .05)

*P*9 = $237.50

The price of the stock today is simply the PV of the stock price in the future. We simply discount the future stock price at the required return. The price of the stock today will be:

*P*0 = $237.50 / 1.139

*P*0 = $79.06

**16.** The price of a stock is the PV of the future dividends. This stock is paying four dividends, so the price of the stock is the PV of these dividends discounted at the required return. So, the price of the stock is:

*P*0 = $8 / 1.104 + $13 / 1.1042 + $18 / 1.1043 + $23 / 1.1044

*P*0 = $46.77

**17.** With nonconstant dividends, we find the price of the stock when the dividends level off at a constant growth rate, and then find the present value of the future stock price, plus the present value of all dividends during the nonconstant growth period. The stock begins constant growth after the fourth dividend is paid, so we can find the price of the stock at Year 4, when the constant dividend growth begins, as:

*P*4 = *D*4 (1 + *g*) / (*R* – *g*)

*P*4 = $3.25(1.05) / (.11 – .05)

*P*4 = $56.88

The price of the stock today is the present value of the first four dividends, plus the present value of the Year 4 stock price. So, the price of the stock today will be:

*P*0 = $6 / 1.11 + $12 / 1.112 + $17 / 1.113 + $3.25 / 1.114 + $56.88 / 1.114

*P*0 = $67.18

**18.** With nonconstant dividends, we find the price of the stock when the dividends level off at a constant growth rate, and then find the present value of the future stock price, plus the present value of all dividends during the nonconstant growth period. The stock begins constant growth after the third dividend is paid, so we can find the price of the stock in Year 3, when the constant dividend growth begins as:

*P*3 = *D*3 (1 + *g*2) / (*R* – *g*2)

*P*3 = *D*0 (1 + *g*1)3 (1 + *g*2) / (*R* – *g*2)

*P*3 = $2.50(1.25)3(1.06) / (.115 – .06)

*P*3 = $94.11

The price of the stock today is the present value of the first three dividends, plus the present value of the Year 3 stock price. The price of the stock today will be:

*P*0 = $2.50(1.25) / 1.115 + $2.50(1.25)2 / 1.1152 + $2.50(1.25)3 / 1.1153 + $94.11 / 1.1153

*P*0 = $77.35

**19.** The constant growth model can be applied even if the dividends are declining by a constant percentage just make sure to recognize the negative growth. So, the price of the stock today will be:

*P*0 = *D*0 (1 + *g*) / (*R* – *g*)

*P*0 = $16.30(1 – .045) / [(.12 – (–.045)]

*P*0 = $94.34

**20.** We are given the stock price, the dividend growth rate, and the required return, and are asked to find the dividend. Using the constant dividend growth model, we get:

*P*0 = *D*0 (1 + *g*) / (*R* – *g*)

Solving this equation for the dividend gives us:

*D*0 = *P*0 (*R* – *g*) / (1 + *g*)

*D*0 = $53.95(.103 – .049) / (1 + .049)

*D*0 = $2.78

**21.** The highest dividend yield will occur when the stock price is the lowest. So, using the 52-week low stock price, the highest dividend yield was:

Dividend yield = *D* / *P*Low

Dividend yield = $1.84 / $44.75

Dividend yield = .0411, or 4.11%

The lowest dividend yield occurred when the stock price was the highest, so:

Dividend yield = *D* / *P*High

Dividend yield = $1.84 / $55.75

Dividend yield = .0330, or 3.30%

**22.** With nonconstant dividends, we find the price of the stock when the dividends level off at a constant growth rate, and then find the present value of the future stock price, plus the present value of all dividends during the nonconstant growth period. Since the first dividend with constant growth is in Year 6, we can find the price of the stock in Year 5, one year before the constant dividend growth begins as:

*P*5 = *D*6 / (*R* – *g*)

*P*5 = *D*0 (1 + *g1*)5 (1 + *g2*) / (*R* – *g*)

*P*5 = $4.25(1.075)5(1.05) / (.10 – .05)

*P*5 = $128.13

The price of the stock today is the present value of the first five dividends, plus the present value of the Year 5 stock price. The price of the stock today will be:

*P*0 = $4.25(1.075) / 1.10 + $4.25(1.075)2 / 1.102 + $4.25(1.075)3 / 1.103 + $4.25(1.075)4 / 1.104

+ $4.25(1.075)5 / 1.105 + $128.13 / 1.105

*P*0 = $99.40

According to the constant growth model, the stock seems to be overvalued. The factors that would affect the stock price are both the nonconstant growth rate and the long-term growth rate, the length of the nonconstant growth, and the required return.

**23.** We need to find the required return of the stock. Using the constant growth model, we can solve the equation for *R*. Doing so, we find:

*R* = (*D*1 / *P*0) + *g*

*R* = [$.40(1 + .03) / $86.39] + .03

*R* = .0348, or 3.48%

The required return depends on the company and the industry. As we will see in a later chapter, this required return appears to be low relative to historic stock returns.

**24.** We need to find the required return of the stock. Using the constant growth model, we can solve the equation for *R*. Doing so, we find:

*R* = (*D*1 / *P*0) + *g*

*R* = [$.12(1 – .15) / $13.30] + (–.15)

*R* = –.1423, or –14.23%

Obviously, this number is incorrect. The required return can never be negative. Alcoa investors must believe that the dividend growth rate over the past 10 years is not indicative of future growth in dividends. For Alcoa, sales, cash flows, and EPS had all fallen during part of this period, in large part due to increased competition.

**25.** The annual dividend paid to stockholders is $.31, and the dividend yield is .90 percent. Using the equation for the dividend yield:

Dividend yield = Dividend / Stock price

We can plug the numbers in and solve for the stock price:

.009 = $.31 / *P*0

*P*0 = $.31 /. 009

*P*0 = $34.44

The dividend yield quoted in the newspaper is rounded. This means the price calculated using the dividend will be slightly different from the actual price. The required return for Tootsie Roll shareholders using the dividend discount model is:

*R* = (*D*1 / *P*0) + *g*

*R* = [$.31(1 + .055) / $33.30] + .055

*R* = .0648, or 6.48%

As we will see in a later, this required return appears to be low relative to historic stock returns.

**26.** *a.* Using the equation to calculate the price of a share of stock with the PE ratio:

P = Benchmark PE ratio × EPS

So, with a PE ratio of 19, we find:

P = 19($4.65)

P = $88.35

*b.*  First, we need to find the earnings per share next year, which will be:

EPS1 = EPS0(1 + *g*)

EPS1 = $4.65(1 + .07)

EPS1 = $4.98

Using the equation to calculate the price of a share of stock with the PE ratio:

P1 = Benchmark PE ratio × EPS1

P1 = 19($4.98)

P1 = $94.53

*c.* To find the implied return over the next year, we calculate the return as:

*R* = (P1 – P0) / P0

*R* = ($94.53 – 88.35) / $88.35

*R* = .07, or 7%

Notice that the return is the same as the growth rate in earnings. Assuming a stock pays no dividends and the PE ratio is constant, this will always be true when using price ratios to evaluate the price of a share of stock.

**27.** We need to find the PE ratio each year, which is:

PE1 = $53.18 / $3.15 = 16.88

PE2 = $62.14 / $3.35 = 18.55

PE3 = $73.21 / $3.60 = 20.34

PE4 = $70.21 / $3.85 = 18.24

So, the average PE is:

Average PE = (16.88 + 18.55 + 20.34 + 18.24) / 4

Average PE = 18.50

Next, we need to find the earnings per share next year, which will be:

EPS1 = EPS0(1 + *g*)

EPS1 = $3.85(1 + .08)

EPS1 = $4.16

Using the equation to calculate the price of a share of stock with the PE ratio:

*P*1 = Benchmark PE ratio × EPS1

*P*1 = 18.50($4.16)

*P*1 = $76.93

**28.** First, we need to find the earnings per share next year, which will be:

EPS1 = EPS0(1 + *g*)

EPS1 = $2.95(1 + .09)

EPS1 = $3.22

To find the high target stock price, we need to find the average high PE ratio each year, which is:

PE1 = $48.60 / $2.35 = 20.68

PE2 = $57.34 / $2.48 = 23.12

PE3 = $69.46 / $2.63 = 26.41

PE4 = $74.85 / $2.95 = 25.37

So, the average high PE is:

Average high PE = (20.68 + 23.12 + 26.41 + 25.37) / 4

Average high PE = 23.90

Using the equation to calculate the price of a share of stock with the PE ratio, the high target price is:

*P*1 = Benchmark PE ratio × EPS1

*P*1 = 23.90($3.22)

*P*1 = $76.84

To find the low target stock price, we need to find the average low PE ratio each year, which is:

PE1 = $37.25 / $2.35 = 15.85

PE2 = $42.18 / $2.48 = 17.01

PE3 = $55.85 / $2.63 = 21.24

PE4 = $63.18 / $2.95 = 21.42

So, the average low PE is:

Average low PE = (15.85 + 17.01 + 21.24 + 21.42) / 4

Average low PE = 18.88

Using the equation to calculate the price of a share of stock with the PE ratio, the low target price is:

*P*1 = Benchmark PE ratio × EPS1

*P*1 = 18.88($3.22)

*P*1 = $60.70

**29.** To find the target price in five years, we first need to find the EPS in five years, which will be:

EPS5 = EPS0(1 + *g*)5

EPS5 = $2.75(1 + .08)5

EPS5 = $4.04

So, the target stock price in five years is:

*P*5 = Benchmark PE ratio × EPS5

*P*5 = 21($4.04)

*P*5 = $84.85

**30.** We need to begin by finding the dividend for each year over the next five years, so:

*D*1 = $1.15(1 + .10) = $1.27

*D*2 = $1.15(1 + .10)2 = $1.39

*D*3 = $1.15(1 + .10)3 = $1.53

*D*4 = $1.15(1 + .10)4 = $1.69

*D*5 = $1.15(1 + .10)5 = $1.86

To find the EPS in Year 5, we can use the dividends and payout ratio, which gives us:

EPS5 = *D*5 / Payout ratio

EPS5 = $1.86 / .40

EPS5 = $4.64

So, the terminal stock price in Year 5 will be:

*P*5 = Benchmark PE ratio × EPS5

*P*5 = 21($4.64)

*P*5 = $97.40

The stock price today is the present value of the dividends for the next five years, plus the present value of the terminal stock price, discounted at the required return, or:

*P*0 = $1.27 / 1.11 + $1.39 / 1.112 + $1.53 / 1.113 + $1.69 / 1.114 + ($1.86 + 97.40) / 1.115

*P*0 = $63.41

*Challenge*

**31.** We are asked to find the dividend yield and capital gains yield for each of the stocks. All of the stocks have a required return of 17 percent, which is the sum of the dividend yield and the capital gains yield. To find the components of the total return, we need to find the stock price for each stock. Using this stock price and the dividend, we can calculate the dividend yield. The capital gains yield for each stock will be the total return (required return) minus the dividend yield.

W: *P*0 = *D*0(1 + *g*) / (*R* – *g*)

*P*0 = $2.40(1.08) / (.17 – .08)

*P*0 = $28.80

Dividend yield = *D*1/*P*0

Dividend yield = $2.40(1.08) / $28.80

Dividend yield = .09, or 9%

Capital gains yield = Total return – Dividend yield

Capital gains yield = .17 – .09

Capital gains yield = .08, or 8%

X: *P*0 = D0(1 + *g*) / (*R* – *g*)

*P*0 = $2.40 / (.17 – .00)

*P*0 = $14.12

Dividend yield = *D*1 / *P*0

Dividend yield = $2.40 / $14.12

Dividend yield = .17, or 17%

Capital gains yield = Total return – Dividend yield

Capital gains yield = .17 – .17

Capital gains yield = .00, or 0%

Y: *P*0 = *D*0(1 + *g*) / (*R* – *g*)

*P*0 = $2.40(1 – .05) / [.17 – (–.05)]

*P*0 = $10.36

Dividend yield = *D*1 / *P*0

Dividend yield = $2.40(1 – .05) / $10.36

Dividend yield = .22, or 22%

Capital gains yield = Total return – Dividend yield

Capital gains yield = .17 – .22

Capital gains yield = –.05, or –5%

Z: To find the price of Stock Z, we find the price of the stock when the dividends level off at a constant growth rate, and then find the present value of the future stock price, plus the present value of all dividends during the nonconstant growth period. The stock begins constant growth in Year 3, so we can find the price of the stock in Year 2, one year before the constant dividend growth begins, as:

*P*2 = *D*2 (1 + *g*2) / (*R* – *g*2)

*P*2 = *D*0 (1 + *g1*)2 (1 + *g2*) / (*R* – *g*2)

*P*2 = $2.40(1.20)2(1.12) / (.17 – .12)

*P*2 = $77.41

The price of the stock today is the present value of the first three dividends, plus the present value of the Year 3 stock price. The price of the stock today will be:

*P*0 = $2.40(1.20) / 1.17 + $2.40(1.20)2 / 1.172 + $77.41 / 1.172

*P*0 = $61.54

Dividend yield = *D*1 / *P*0

Dividend yield = $2.40(1.20) / $61.54

Dividend yield = .0468, or 4.68%

Capital gains yield = Total return – Dividend yield

Capital gains yield = .17 – .0468

Capital gains yield = .1232, or 12.32%

In all cases, the required return is 17 percent, but the return is distributed differently between current income and capital gains. High-growth stocks have an appreciable capital gains component but a relatively small current income yield; conversely, mature, negative-growth stocks provide a high current income but also price depreciation over time.

**32.** *a*. Using the constant growth model, the price of the stock paying annual dividends will be:

*P*0 = *D*0(1 + *g*) / (*R* – *g*)

*P*0 = $2.20(1.06) / (.12 – .06)

*P*0 = $38.87

*b*. If the company pays quarterly dividends instead of annual dividends, the quarterly dividend will be one-fourth of annual dividend, or:

Quarterly dividend = $2.20(1.06) / 4

Quarterly dividend = $.5830

To find the equivalent annual dividend, we must assume that the quarterly dividends are reinvested at the required return. We can then use this interest rate to find the equivalent annual dividend. In other words, when we receive the quarterly dividend, we reinvest it at the required return on the stock. So, the effective quarterly rate is:

Effective quarterly rate = 1.12.25 – 1

Effective quarterly rate = .0287

The effective annual dividend will be the FVA of the quarterly dividend payments at the effective quarterly required return. In this case, the effective annual dividend will be:

Effective *D*1 = $.5830(FVIFA2.87%,4)

Effective *D*1 = $2.43

Now, we can use the constant growth model to find the current stock price as:

*P*0 = $2.43 / (.12 – .06)

*P*0 = $40.57

Note that we cannot simply find the quarterly effective required return and growth rate to find the value of the stock. This would assume the dividends increased each quarter, not each year. Assuming you can reinvest the dividends at the required return of the stock, this model would be appropriate.

**CHAPTER 8**

***NET PRESENT VALUE AND OTHER INVESTMENT CRITERIA***

**Answers to Concepts Review and Critical Thinking Questions**

**1.** A payback period less than the project’s life means that the NPV is positive for a zero discount rate, but nothing more definitive can be said. For discount rates greater than zero, the payback period will still be less than the project’s life, but the NPV may be positive, zero, or negative, depending on whether the discount rate is less than, equal to, or greater than the IRR.

**2.** If a project has a positive NPV for a certain discount rate, then it will also have a positive NPV for a zero discount rate; thus the payback period must be less than the project life. If NPV is positive, then the present value of future cash inflows is greater than the initial investment cost; thus the PI must be greater than 1. If NPV is positive for a certain discount rate R, then it will be zero for some larger discount rate R\*; thus the IRR must be greater than the required return.

**3.** *a.* Payback period is simply the break-even point of a series of cash flows. To actually compute the payback period, it is assumed that any cash flow occurring during a given period is realized continuously throughout the period, and not at a single point in time. The payback is then the point in time for the series of cash flows when the initial cash outlays are fully recovered. Given some predetermined cutoff for the payback period, the decision rule is to accept projects that payback before this cutoff, and reject projects that take longer to payback.

*b.* The worst problem associated with payback period is that it ignores the time value of money. In addition, the selection of a hurdle point for payback period is an arbitrary exercise that lacks any steadfast rule or method. The payback period is biased towards short-term projects; it fully ignores any cash flows that occur after the cutoff point.

*c.* Despite its shortcomings, payback is often used because the analysis is straightforward and simple. Materiality considerations often warrant a payback analysis as sufficient; maintenance projects are another example where the detailed analysis of other methods is often not needed. Since payback is biased towards liquidity, it may be a useful and appropriate analysis method for short-term projects where cash management is most important.

**4.** *a.* The average accounting return is interpreted as an average measure of the accounting performance of a project over time, computed as some average profit measure due to the project divided by some average balance sheet value for the project. This text computes AAR as average net income with respect to average (total) book value. Given some predetermined cutoff for AAR, the decision rule is to accept projects with an AAR in excess of the target measure, and reject all other projects.

*b*. AAR is not a measure of cash flows and market value, but a measure of financial statement accounts that often bear little semblance to the relevant value of a project. In addition, the selection of a cutoff is arbitrary, and the time value of money is ignored. For a financial manager, both the reliance on accounting numbers rather than relevant market data and the exclusion of time value of money considerations are troubling. Despite these problems, AAR continues to be used in practice because (1) the accounting information is usually available, (2) analysts often use accounting ratios to analyze firm performance, and (3) managerial compensation is often tied to the attainment of certain target accounting ratio goals.

**5.** *a.* NPV is simply the sum of the present values of a project’s cash flows. NPV specifically measures, after considering the time value of money, the net increase or decrease in firm wealth due to the project. The decision rule is to accept projects that have a positive NPV, and reject projects with a negative NPV.

*b.* NPV is superior to the other methods of analysis presented in the text because it has no serious flaws. The method unambiguously ranks mutually exclusive projects, and can differentiate between projects of different scale and time horizon. The only drawback to NPV is that it relies on cash flow and discount rate values that are often estimates and not certain, but this is a problem shared by the other performance criteria as well. A project with NPV = $2,500 implies that the total shareholder wealth of the firm will increase by $2,500 if the project is accepted.

**6.** *a.* The IRR is the discount rate that causes the NPV of a series of cash flows to be equal to zero. IRR can thus be interpreted as a financial break-even rate of return; at the IRR discount rate, the net value of the project is zero. The IRR decision rule is to accept projects with IRRs greater than the discount rate, and to reject projects with IRRs less than the discount rate.

*b.* IRR is the interest rate that causes NPV for a series of cash flows to be zero. NPV is preferred in all situations to IRR; IRR can lead to ambiguous results if there are non-conventional cash flows, and also ambiguously ranks some mutually exclusive projects. However, for stand-alone projects with conventional cash flows, IRR and NPV are interchangeable techniques.

*c.* IRR is frequently used because it is easier for many financial managers and analysts to rate performance in relative terms, such as “12%”, than in absolute terms, such as “$46,000.” IRR may be a preferred method to NPV in situations where an appropriate discount rate is unknown or uncertain; in this situation, IRR would provide more information about the project than would NPV.

**7.** *a.* The profitability index is the present value of the future cash flows divided by the initial investment. As such, it is a benefit/cost ratio, providing a measure of the relative profitability of a project. The profitability index decision rule is to accept projects with a PI greater than one, and to reject projects with a PI less than one.

*b.* PI = ( NPV + cost ) / cost = 1 + ( NPV / cost ). If a firm has a basket of positive NPV projects and is subject to capital rationing, PI may provide a good ranking measure of the projects, indicating the “bang for the buck” of each particular project.

**8.** For a project with future cash flows that are an annuity:

Payback = I / C

And the IRR is:

0 = – I + C / IRR

Solving the IRR equation for IRR, we get:

IRR = C / I

Notice this is just the reciprocal of the payback. So:

IRR = 1 / Payback

For long-lived projects with relatively constant cash flows, the sooner the project pays back, the greater is the IRR.

**9.** There are a number of reasons. Two of the most important have to do with transportation costs and exchange rates. Manufacturing in the U.S. places the finished product much closer to the point of sale, resulting in significant savings in transportation costs. It also reduces inventories because goods spend less time in transit. Higher labor costs tend to offset these savings to some degree, at least compared to other possible manufacturing locations. Of great importance is the fact that manufacturing in the U.S. means that a much higher proportion of the costs are paid in dollars. Since sales are in dollars, the net effect is to immunize profits to a large extent against fluctuations in exchange rates. This issue is discussed in greater detail in the chapter on international finance.

**10.** The single biggest difficulty, by far, is coming up with reliable cash flow estimates. Determining an appropriate discount rate is also not a simple task. These issues are discussed in greater depth in the next several chapters. The payback approach is probably the simplest, followed by the AAR, but even these require revenue and cost projections. The discounted cash flow measures (NPV, IRR, and profitability index) are really only slightly more difficult in practice.

**11.** Yes, they are. Such entities generally need to allocate available capital efficiently, just as for-profits do. However, it is frequently the case that the “revenues” from not-for-profit ventures are not tangible. For example, charitable giving has real opportunity costs, but the benefits are generally hard to measure. To the extent that benefits are measurable, the question of an appropriate required return remains. Payback rules are commonly used in such cases. Finally, realistic cost/benefit analysis along the lines indicated should definitely be used by the U.S. government and would go a long way toward balancing the budget!

**12.** The yield to maturity is the internal rate of return on a bond. The two concepts are identical with the exception that YTM is applied to bonds and IRR is applied to capital budgeting.

**13.** The MIRR is calculated by finding the present value of all cash outflows, the future value of all cash inflows to the end of the project, and then calculating the IRR of the two cash flows. As a result, the cash flows have been discounted or compounded by one interest rate (the required return), and then the interest rate between the two remaining cash flows is calculated. As such, the MIRR is not a true interest rate. In contrast, consider the IRR. If you take the initial investment, and calculate the future value at the IRR, you can replicate the future cash flows of the project exactly.

**14.** The statement is incorrect. It is true that if you calculate the future value of all intermediate cash flows to the end of the project at the required return, then calculate the NPV of this future value and the initial investment, you will get the same NPV. However, NPV says nothing about reinvestment of intermediate cash flows. The NPV is the present value of the project cash flows. What is actually done with those cash flows once they are generated is not relevant. Put differently, the value of a project depends on the cash flows generated by the project, not on the future value of those cash flows. The fact that the reinvestment “works” only if you use the required return as the reinvestment rate is also irrelevant simply because reinvestment is not relevant in the first place to the value of the project.

One caveat: Our discussion here assumes that the cash flows are truly available once they are generated, meaning that it is up to firm management to decide what to do with the cash flows. In certain cases, there may be a requirement that the cash flows be reinvested. For example, in international investing, a company may be required to reinvest the cash flows in the country in which they are generated and not “repatriate” the money. Such funds are said to be “blocked” and reinvestment becomes relevant because the cash flows are not truly available.

**15.** The statement is incorrect. It is true that if you calculate the future value of all intermediate cash flows to the end of the project at the IRR, then calculate the IRR of this future value and the initial investment, you will get the same IRR. However, as in the previous question, what is done with the cash flows once they are generated does not affect the IRR. Consider the following example:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | C0 | C1 | C2 | IRR |
| Project A | –$100 | $10 | $110 | 10% |

Suppose this $100 is a deposit into a bank account. The IRR of the cash flows is 10 percent. Does the IRR change if the Year 1 cash flow is reinvested in the account, or if it is withdrawn and spent on pizza? No. Finally, consider the yield to maturity calculation on a bond. If you think about it, the YTM is the IRR on the bond, but no mention of a reinvestment assumption for the bond coupons is suggested. The reason is that reinvestment is irrelevant to the YTM calculation; in the same way, reinvestment is irrelevant in the IRR calculation. Our caveat about blocked funds applies here as well.

**Solutions to Questions and Problems**

*Basic*

*NOTE: All end-of-chapter problems were solved using a spreadsheet. Many problems require multiple steps. Due to space and readability constraints, when these intermediate steps are included in this solutions manual, rounding may appear to have occurred. However, the final answer for each problem is found without rounding during any step in the problem.*

**1.** To calculate the payback period, we need to find the time it takes to recover the initial investment. After two years, the project has created:

$2,800 + 3,200 = $6,000

in cash flows. The project still needs to create another:

$6,700 – 6,000 = $700

in cash flows. During the third year, the cash flows from the project will be $2,200. So, the payback period will be 2 years, plus what we still need to make divided by what we will make during the third year. The payback period is:

Payback = 2 + ($700 / $2,200)

Payback = 2.32 years

**2.** To calculate the payback period, we need to find the time it takes to recover the initial investment. The cash flows in this problem are an annuity, so the calculation is simpler. If the initial cost is $3,100, the payback period is:

Payback = 3 + $295 / $935

Payback = 3.32 years

There is a shortcut to calculate payback period when the future cash flows are an annuity. Just divide the initial cost by the annual cash flow. For the $3,100 cost, the payback period is:

Payback = $3,100 / $935

Payback = 3.22 years

For an initial cost of $4,300, the payback period is:

Payback = $4,300 / $935

Payback = 4.60 years

The payback period for an initial cost of $7,900 is a little trickier. Notice that the total cash inflows after eight years will be:

Total cash inflows = 8($935)

Total cash inflows = $7,480

If the initial cost is $7,900, the project never pays back. Notice that if you use the shortcut for annuity cash flows, you get:

Payback = $7,900 / $935

Payback = 8.45 years

This answer does not make sense since the cash flows stop after eight years, so again, we must conclude the payback period is never.

**3.** Project A has cash flows of:

Cash flows = $23,000 + 28,000

Cash flows = $51,000

during the first two years. The cash flows are still short by $9,000 of recapturing the initial investment, so the payback for Project A is:

Payback = 2 + ($9,000 / $19,000)

Payback = 2.47 years

Project B has cash flows of:

Cash flows = $21,000 + 26,000 + 29,000

Cash flows = $76,000

during the first three years. The cash flows are still short by $29,000 of recapturing the initial investment, so the payback for Project B is:

Payback = 3 + ($29,000 / $260,000)

Payback = 3.11 years

Using the payback criterion and a cutoff of 3 years, accept Project A and reject Project B.

**4.** Our definition of AAR is the average net income divided by the average book value. The average net income for this project is:

Average net income = ($1,368,000 + 1,935,000 + 1,738,000 + 1,310,000) / 4

Average net income = $1,587,750

And the average book value is:

Average book value = ($12,500,000 + 0) / 2

Average book value = $6,250,000

So, the AAR for this project is:

AAR = Average net income / Average book value

AAR = $1,587,750 / $6,250,000

AAR = .2540, or 25.40%

**5.** The IRR is the interest rate that makes the NPV of the project equal to zero. So, the equation that defines the IRR for this project is:

0 = –$168,500 + $86,000 / (1+IRR) + $91,000 / (1+IRR)2 + $53,000 / (1+IRR)3

Using a spreadsheet, financial calculator, or trial and error to find the root of the equation, we find that:

IRR = 18.79%

Since the cash flows are conventional and the IRR is greater than the required return, we would accept the project.

**6.** The NPV of a project is the PV of the outflows minus by the PV of the inflows. The equation for the NPV of this project at a 9 percent required return is:

NPV = –$168,500 + $86,000 / 1.09 + $91,000 / 1.092 + $53,000 / 1.093

NPV = $27,917.69

At a 9 percent required return, the NPV is positive, so we would accept the project.

The equation for the NPV of the project at a 21 percent required return is:

NPV = –$168,500 + $86,000 / 1.21 + $91,000 / 1.212 + $53,000 / 1.213

NPV = –$5,354.28

At a 21 percent required return, the NPV is negative, so we would reject the project.

**7.** The NPV of a project is the PV of the outflows plus the PV of the inflows. Since the cash inflows are an annuity, the equation for the NPV of this project at an 8 percent required return is:

NPV = –$8,450 + $2,145(PVIFA8%, 8)

NPV = $3,876.54

At an 8 percent required return, the NPV is positive, so we would accept the project.

The equation for the NPV of the project at a 24 percent required return is:

NPV = –$8,450 + $2,145(PVIFA24%, 8)

NPV = –$1,111.48

At a 24 percent required return, the NPV is negative, so we would reject the project.

We would be indifferent to the project if the required return was equal to the IRR of the project, since at that required return the NPV is zero. The IRR of the project is:

0 = –$8,450 + $2,145(PVIFAIRR, 8)

IRR = .1913, or 19.13%

**8.** The IRR is the interest rate that makes the NPV of the project equal to zero. So, the equation that defines the IRR for this project is:

0 = –$19,400 + $9,800 / (1+IRR) + $11,300 / (1+IRR)2 + $6,900 / (1+IRR)3

Using a spreadsheet, financial calculator, or trial and error to find the root of the equation, we find that:

IRR = 22.09%

**9.** The NPV of a project is the PV of the outflows plus by the PV of the inflows. At a zero discount rate (and only at a zero discount rate), the cash flows can be added together across time. So, the NPV of the project at a zero percent required return is:

NPV = –$19,400 + 9,800 + 11,300 + 6,900

NPV = $8,600

The NPV at a 10 percent required return is:

NPV = –$19,400 + $9,800 / 1.10 + $11,300 / 1.102 + $6,900 / 1.103

NPV = $4,032.01

The NPV at a 20 percent required return is:

NPV = –$19,400 + $9,800 / 1.20 + $11,300 / 1.202 + $6,900 / 1.203

NPV = $606.94

And the NPV at a 30 percent required return is:

NPV = –$19,400 + $9,800 / 1.30 + $11,300 / 1.302 + $6,900 / 1.303

NPV = –$2,034.50

Notice that as the required return increases, the NPV of the project decreases. This will always be true for projects with conventional cash flows. Conventional cash flows are negative at the beginning of the project and positive throughout the rest of the project.

**10.** *a.* The IRR is the interest rate that makes the NPV of the project equal to zero. The equation for the IRR of Project A is:

0 = –$78,500 + $43,000 / (1 + IRR) + $29,000 / (1 + IRR)2 + $23,000 / (1 + IRR)3

+ $21,000 / (1 + IRR)4

Using a spreadsheet, financial calculator, or trial and error to find the root of the equation, we find that:

IRR = 20.70%

The equation for the IRR of Project B is:

0 = –$78,500 + $21,000 / (1 + IRR) + $28,000 / (1 + IRR)2 + $34,000 / (1 + IRR)3

+ $41,000 / (1 + IRR)4

Using a spreadsheet, financial calculator, or trial and error to find the root of the equation, we find that:

IRR = 18.73%

Examining the IRRs of the projects, we see that the IRRA is greater than the IRRB, so the IRR decision rule implies accepting Project A. This may not be a correct decision, however, because the IRR criterion has a ranking problem for mutually exclusive projects. To see if the IRR decision rule is correct or not, we need to evaluate the project NPVs.

*b.* The NPV of Project A is:

NPVA = –$78,500 + $43,000 / 1.11+ $29,000 / 1.112 + $23,000 / 1.113 + $21,000 / 1.114

NPVA = $14,426.54

And the NPV of Project B is:

NPVB = –$78,500 + $21,000 / 1.11 + $28,000 / 1.112 + $34,000 / 1.113 + $41,000 / 1.114

NPVB = $15,012.82

The NPVB is greater than the NPVA, so we should accept Project B.

*c.* To find the crossover rate, we subtract the cash flows from one project from the cash flows of the other project. Here, we will subtract the cash flows for Project B from the cash flows of Project A. Once we find these differential cash flows, we find the IRR. The equation for the crossover rate is:

0 = $22,000 / (1 + *R*) + $1,000 / (1 + *R*)2 – $11,000 / (1 + *R*)3 – $20,000 / (1 + *R*)4

Using a spreadsheet, financial calculator, or trial and error to find the root of the equation, we find that:

*R* = 12.21%

At discount rates above 12.21% choose Project A; for discount rates below 12.21% choose Project B; indifferent between A and B at a discount rate of 12.21%.

**11.** The IRR is the interest rate that makes the NPV of the project equal to zero. The equation to calculate the IRR of Project X is:

0 = –$23,400 + $13,100 / (1 + IRR) + $9,480 / (1 + IRR)2 + $7,890 / (1 + IRR)3

Using a spreadsheet, financial calculator, or trial and error to find the root of the equation, we find that:

IRR = 15.98%

For Project Y, the equation to find the IRR is:

0 = –$23,400 + $9,200 / (1 + IRR) + $10,620 / (1 + IRR)2 + $11,180 / (1 + IRR)3

Using a spreadsheet, financial calculator, or trial and error to find the root of the equation, we find that:

IRR = 14.95%

To find the crossover rate, we subtract the cash flows from one project from the cash flows of the other project, and find the IRR of the differential cash flows. We will subtract the cash flows from Project Y from the cash flows from Project X. It is irrelevant which cash flows we subtract from the other. Subtracting the cash flows, the equation to calculate the IRR for these differential cash flows is:

Crossover rate: 0 = $3,900 / (1 + *R*) – $1,140 / (1 + *R*)2 – $3,290 / (1 + *R*)3

Using a spreadsheet, financial calculator, or trial and error to find the root of the equation, we find that:

*R* = 7.62%

The table below shows the NPV of each project for different required returns. Notice that Project Y always has a higher NPV for discount rates below 7.62 percent, and always has a lower NPV for discount rates above 7.62 percent.

|  |  |  |  |
| --- | --- | --- | --- |
| R | $NPVX |  | $NPVY |
| 0% | $7,070.00 |  | $7,600.00 |
| 5% | 4,490.51 |  | 4,652.26 |
| 10% | 2,271.68 |  | 2,140.20 |
| 15% | 347.35 |  | –18.72 |
| 20% | –1,334.03 |  | –1,888.43 |
| 25% | –2,813.12 |  | –3,519.04 |

And the NPV profile is:

**12.** *a.* The equation for the NPV of the project is:

NPV = –$39,000,000 + $57,000,000 / 1.10 – $9,000,000 / 1.102

NPV = $5,380,165.29

The NPV is greater than 0, so we would accept the project.

*b.* The equation for the IRR of the project is:

0 = –$39,000,000 + $57,000,000 / (1 + IRR) – $9,000,000 / (1 + IRR)2

From Descartes’ rule of signs, we know there are two IRRs since the cash flows change signs twice. From trial and error, the two IRRs are:

IRR = 28.15%, –81.99%

When there are multiple IRRs, the IRR decision rule is ambiguous. Both IRRs are correct, that is, both interest rates make the NPV of the project equal to zero. If we are evaluating whether or not to accept this project, we would not want to use the IRR to make our decision.

**13.** The profitability index is defined as the PV of the future cash flows divided by the initial investment. The equation for the profitability index at a required return of 10 percent is:

PI = ($15,800 / 1.10 + $13,600 / 1.102 + $8,300 / 1.103) / $27,500

PI = 1.158

The equation for the profitability index at a required return of 15 percent is:

PI = ($15,800 / 1.15 + $13,600 / 1.152 + $8,300 / 1.153) / $27,500

PI = 1.072

The equation for the profitability index at a required return of 22 percent is:

PI = ($15,800 / 1.22 + $13,600 / 1.222 + $8,300 / 1.223) / $27,500

PI = .969

We would accept the project if the required return were 10 percent or 15 percent since the PI is greater than one. We would reject the project if the required return were 22 percent since the PI is less than one.

**14.** *a.* The profitability index is defined as the PV of the future cash flows divided by the initial investment. The equation for the profitability index for each project is:

PII = ($28,300 / 1.11 + $34,800 / 1.112 + $43,700 / 1.113) / $78,000

PII = 1.099

PIII = ($9,600 / 1.11 + $17,400 / 1.112 + $15,600 / 1.113) / $28,800

PIII = 1.187

The profitability index decision rule implies that we accept Project II, since PIII is greater than the PII.

*b.* The NPV of each project is:

NPVI = –$78,000 + $28,300 / 1.11 + $34,800 / 1.112 + $43,700 / 1.113

NPVI = $7,693.02

NPVII = –$28,800 + $9,600 / 1.11 + $17,400 / 1.112 + $15,600 / 1.113

NPVII = $5,377.46

The NPV decision rule implies accepting Project I, since the NPVI is greater than the NPVII.

*c.* Using the profitability index to compare mutually exclusive projects can be ambiguous when the magnitudes of the cash flows for the two projects are of different scale. In this problem, Project I is roughly 2.5 times as large as Project II and produces a larger NPV, yet the profitability index criterion implies that Project II is more acceptable.

**15.** *a*. The payback period for each project is:

A: 3 + ($110,000 / $325,000) = 3.34 years

B: 1 + ($18,300 / $19,900) = 1.92 years

The payback criterion implies accepting Project B, because it pays back sooner than Project A.

*b*. The NPV for each project is:

A: NPV = –$235,000 + $29,000 / 1.13 + $45,000 / 1.132 + $51,000 / 1.133 + $325,000 / 1.134

NPV = $60,579.46

B: NPV = –$47,000 + $28,700 / 1.13 + $19,900 / 1.132 + $17,300 / 1.133 + $16,200 / 1.134

NPV = $15,908.38

NPV criterion implies we accept Project A because Project A has a higher NPV than Project B.

*c.* The IRR for each project is:

A: $235,000 = $29,000 / (1 + IRR) + $45,000 / (1 + IRR)2 + $51,000 / (1 + IRR)3

+ $325,000 / (1 + IRR)4

Using a spreadsheet, financial calculator, or trial and error to find the root of the equation, we find that:

IRR = 21.02%

B: $47,000 = $28,700 / (1 + IRR) + $19,900 / (1 + IRR)2 + $17,300 / (1 + IRR)3

+ $16,200 / (1 + IRR)4

Using a spreadsheet, financial calculator, or trial and error to find the root of the equation, we find that:

IRR = 30.57%

IRR decision rule implies we accept Project B because IRR for B is greater than IRR for A.

*d.* The profitability index for each project is:

A: PI = ($29,000 / 1.13 + $45,000 / 1.132 + $51,000 / 1.133 + $325,000 / 1.134) / $235,000

PI = 1.258

B: PI = ($28,700 / 1.13 + $19,900 / 1.132 + $17,300 / 1.133 + $16,200 / 1.134) / $47,000

PI = 1.338

Profitability index criterion implies accept Project B because its PI is greater than Project A’s.

*e.* In this instance, the NPV criterion implies that you should accept Project A, while payback period, IRR, and the profitability index imply that you should accept Project B. The final decision should be based on the NPV since it does not have the ranking problem associated with the other capital budgeting techniques. Therefore, you should accept Project A.

**16.** *a.* The IRR for each project is:

M: $150,000 = $68,600 / (1 + IRR) + $76,800 / (1 + IRR)2 + $71,300 / (1 + IRR)3

+ $40,500 / (1 + IRR)4

Using a spreadsheet, financial calculator, or trial and error to find the root of the equation, we find that:

IRR = 27.82%

N: $372,000 = $159,300 / (1 + IRR) + $193,200 / (1 + IRR)2 + $154,800 / (1 + IRR)3

+ 110,400 / (1 + IRR)4

Using a spreadsheet, financial calculator, or trial and error to find the root of the equation, we find that:

IRR = 25.57%

IRR decision rule implies we accept Project M because IRR for M is greater than IRR for N.

*b.* The NPV for each project is:

M: NPV = –$150,000 + $68,600 / 1.15 + $76,800 / 1.152 + $71,300 / 1.153 + $40,500 / 1.154

NPV = $37,760.92

N: NPV = –$372,000 + $159,300 / 1.15 + $193,200 / 1.152 + $154,800 / 1.153

+ $110,400 / 1.154

NPV = $77,513.77

NPV criterion implies we accept Project N because Project N has a higher NPV than Project M.

*c.* Accept Project N since the NPV is higher. IRR cannot be used to rank mutually exclusive projects.

**17.** *a.* The profitability index for each project is:

Y: PI = ($19,800 / 1.12 + $17,500 / 1.122 + $20,700 / 1.123 + $14,600 / 1.124) / $43,400

PI = 1.282

Z: PI = ($32,000 / 1.12 + $30,100 / 1.122 + $29,500 / 1.123 + $27,300 / 1.124) / $78,000

PI = 1.166

Profitability index criterion implies accept Project Y because its PI is greater than Project Z’s.

*b.* The NPV for each project is:

Y: NPV = –$43,400 + $19,800 / 1.12 + $17,500 / 1.122 + $20,700 / 1.123 + $14,600 / 1.124

NPV = $12,241.88

Z: NPV = –$78,000 + $32,000 / 1.12 + $30,100 / 1.122 + $29,500 / 1.123 + $27,300 / 1.124

NPV = $12,914.13

NPV criterion implies we accept Project Z because Project Z has a higher NPV than Project Y.

*c.* Accept Project Z since the NPV is higher. The profitability index cannot be used to rank mutually exclusive projects.

**18.** To find the crossover rate, we subtract the cash flows from one project from the cash flows of the other project, and find the IRR of the differential cash flows. We will subtract the cash flows from Project J from the cash flows from Project I. It is irrelevant which cash flows we subtract from the other. Subtracting the cash flows, the equation to calculate the IRR for these differential cash flows is:

Crossover rate: 0 = $23,200 / (1 + *R*) + $5,600 / (1 + *R*)2 – $13,600 / (1 + *R*)3 – $26,200 / (1 + R)4

Using a spreadsheet, financial calculator, or trial and error to find the root of the equation, we find that:

*R* = 14.06%

At a lower interest rate, Project J is more valuable because of the higher total cash flows. At a higher interest rate, Project I becomes more valuable since the differential cash flows received in the first two years are larger than the cash flows for Project J.

**19.** If the payback period is exactly equal to the project’s life then the IRR must be equal to zero since the project pays back exactly the initial investment. If the project never pays back its initial investment, then the IRR of the project must negative.

**20.** At a zero discount rate (and only at a zero discount rate), the cash flows can be added together across time. So, the NPV of the project at a zero percent required return is:

NPV = –$745,382 + 265,381 + 304,172 + 225,153 + 208,614

NPV = $257,938

If the required return is infinite, future cash flows have no value. Even if the cash flow in one year is $1 trillion, at an infinite rate of interest, the value of this cash flow today is zero. So, if the future cash flows have no value today, the NPV of the project is simply the cash flow today. At an infinite interest rate:

NPV = –$745,382

The interest rate that makes the NPV of a project equal to zero is the IRR. The equation for the IRR of this project is:

0 = –$745,382 + 265,381 / (1 + IRR) + 304,172 / (1 + IRR)2 + 225,153 / (1 + IRR)3

+ 208,614 / (1 + IRR)4

Using a spreadsheet, financial calculator, or trial and error to find the root of the equation, we find that:

IRR = 13.79%

**21.** *a*. The payback period for each project is:

F: 2 + ($21,600 / $81,600) = 2.26 years

G: 3 + ($5,200 / $166,800) = 3.03 years

The payback criterion implies accepting Project F because it pays back sooner than Project G. Project G does not meet the minimum payback of three years.

*b*. The NPV for each project is:

F: NPV = –$180,000 + $93,600 / 1.10 + $64,800 / 1.102 + $81,600 / 1.103 + $72,000 / 1.104

+ $64,800 / 1.105

NPV = $109,364.59

G: NPV = –$280,000 + $64,800 / 1.10 + $86,400 / 1.102 + $123,600 / 1.103

+ $166,800 / 1.104 + $187,200 / 1.105

NPV = $173,339.68

NPV criterion implies we should accept Project G because Project G has a higher NPV than Project F.

*c.* Even though Project G does not meet the payback period of three years, it does provide the largest increase in shareholder wealth, therefore, choose Project G. Payback period should generally be ignored in this situation.

**22.** The MIRR for the project with all three approaches is:

*Discounting approach:*

In the discounting approach, we find the value of all cash outflows to Time 0, while any cash inflows remain at the time at which they occur. So, discounting the cash outflows to Time 0, we find:

Time 0 cash flow = –$27,500 – $4,050 / 1.105

Time 0 cash flow = –$30,014.73

So, the MIRR using the discounting approach is:

0 = –$30,014.73 + $10,430 / (1 + MIRR) + $13,850 / (1 + MIRR)2 + $11,270 / (1 + MIRR)3

+ $9,830 / (1 + MIRR)4

Using a spreadsheet, financial calculator, or trial and error to find the root of the equation, we find that:

MIRR = 19.21%

*Reinvestment approach:*

In the reinvestment approach, we find the future value of all cash except the initial cash flow at the end of the project. So, reinvesting the cash flows to Time 5, we find:

Time 5 cash flow = $10,430(1.104) + $13,850(1.103) + $11,270(1.102) + $9,830(1.10) – $4,050

Time 5 cash flow = $54,104.61

So, the MIRR using the reinvestment approach is:

0 = –$27,500 + $54,104.61 / (1 + MIRR)5

$54,104.61 / $27,500 = (1 + MIRR)5

MIRR = ($54,104.61 / $27,500)1/5 – 1

MIRR = .1449, or 14.49%

*Combination approach:*

In the combination approach, we find the value of all cash outflows at Time 0, and the value of all cash inflows at the end of the project. So, the value of the cash flows is:

Time 0 cash flow = –$27,500 – $4,050 / 1.105

Time 0 cash flow = –$30,014.73

Time 5 cash flow = $10,430(1.104) + $13,850(1.103) + $11,270(1.102) + $9,830(1.10)

Time 5 cash flow = $58,154.61

So, the MIRR using the discounting approach is:

0 = –$30,014.73 + $58,154.61 / (1 + MIRR)5

$58,154.61 / $30,014.73 = (1 + MIRR)5

MIRR = ($58,154.61 / $30,014.73)1/5 – 1

MIRR = .1414, or 14.14%

*Intermediate*

**23.** With different discounting and reinvestment rates, we need to make sure to use the appropriate interest rate. The MIRR for the project with all three approaches is:

*Discounting approach:*

In the discounting approach, we find the value of all cash outflows to Time 0 at the discount rate, while any cash inflows remain at the time at which they occur. So, discounting the cash outflows to Time 0, we find:

Time 0 cash flow = –$27,500 – $4,050 / 1.115

Time 0 cash flow = –$29,903.48

So, the MIRR using the discounting approach is:

0 = –$29,903.48 + $10,430 / (1 + MIRR) + $13,850 / (1 + MIRR)2 + $11,270 / (1 + MIRR)3

+ $9,830 / (1 + MIRR)4

Using a spreadsheet, financial calculator, or trial and error to find the root of the equation, we find that:

MIRR = 19.41%

*Reinvestment approach:*

In the reinvestment approach, we find the future value of all cash except the initial cash flow at the end of the project using the reinvestment rate. So, reinvesting the cash flows to Time 5, we find:

Time 5 cash flow = $10,430(1.084) + $13,850(1.083) + $11,270(1.082) + $9,830(1.08) – $4,050

Time 5 cash flow = $51,348.64

So, the MIRR using the reinvestment approach is:

0 = –$27,500 + $51,384.64 / (1 + MIRR)5

$51,348.64 / $27,500 = (1 + MIRR)5

MIRR = ($51,348.64 / $27,500)1/5 – 1

MIRR = .1330, or 13.30%

*Combination approach:*

In the combination approach, we find the value of all cash outflows at Time 0 using the discount rate, and the value of all cash inflows at the end of the project using the reinvestment rate. So, the value of the cash flows is:

Time 0 cash flow = –$27,500 – $4,050 / 1.115

Time 0 cash flow = –$29,903.48

Time 5 cash flow = $10,430(1.084) + $13,850(1.083) + $11,270(1.082) + $9,830(1.08)

Time 5 cash flow = $55,398.64

So, the MIRR using the combination approach is:

0 = –$29,903.48 + $55,398.64 / (1 + MIRR)5

$55,398.64 / $29,903.48 = (1 + MIRR)5

MIRR = ($55,398.64 / $29,903.48)1/5 – 1

MIRR = .1312, or 13.12%

**24.** To find the crossover rate, we subtract the cash flows from one project from the cash flows of the other project, and find the IRR of the differential cash flows. We will subtract the cash flows from Project S from the cash flows from Project R. It is irrelevant which cash flows we subtract from the other. Subtracting the cash flows, the equation to calculate the IRR for these differential cash flows is:

0 = $31,000 – $3,000 / (1 + *R*) – $1,000 / (1 + *R*)2 – $14,000 / (1 + *R*)3 – $21,000 / (1 + *R*)4

– $3,000 / (1 + *R*)5

Using a spreadsheet, financial calculator, or trial and error to find the root of the equation, we find that:

*R* = 9.24%

The NPV of the projects at the crossover rate must be equal. The NPV of each project at the crossover rate is:

R: NPV = –$45,000 + $17,000 / 1.0924 + $19,000 / 1.09242 + $21,000 / 1.09243

+ $9,000 / 1.09244 + $7,000 / 1.09245

NPV = $13,414.39

S: NPV = –$76,000 + $20,000 / 1.0924 + $20,000 / 1.09242 + $35,000 / 1.09243

+ $30,000 / 1.09244 + $10,000 / 1.09245

NPV = $13,414.39

**25.** The IRR of the project is:

$91,000 = $55,000 / (1 + IRR) + $46,000 / (1 + IRR)2

Using a spreadsheet, financial calculator, or trial and error to find the root of the equation, we find that:

IRR = 7.47%

At an interest rate of 10 percent, the NPV is:

NPV = $91,000 – $55,000 / 1.10 – $46,000 / 1.102

NPV = $2,983.47

At an interest rate of zero percent, we can add cash flows, so the NPV is:

NPV = $91,000 – $55,000 – $46,000

NPV = –$10,000.00

And at an interest rate of 24 percent, the NPV is:

NPV = $91,000 – $55,000/1.24 – $46,000/1.242

NPV = $16,728.41

The cash flows for the project are unconventional. Since the initial cash flow is positive and the remaining cash flows are negative, the decision rule for IRR is invalid in this case. The NPV profile is upward sloping, indicating that the project is more valuable when the interest rate increases.

**26.** By definition, the profitability index is:

PI = Discounted Value of Future Cash Flows / Initial Cost

But note that the discounted value of future cash flows is just the NPV overstated by the neglected initial costs, so:

NPV = Discounted Value of Future Cash Flows – Initial Cost

Discounted Value of Future Cash Flows = NPV + Initial Cost

Substituting, we get:

PI = (NPV + Initial Cost) / Initial Cost

PI = NPV / Initial Cost + 1

PI = NPV Index + 1

NPV Index = PI – 1

**27.** *a.* To have a payback equal to the project’s life, given *C* is a constant cash flow for N years:

*C* = I/N

*b.* To have a positive NPV, I < *C* (PVIFA*R*%, *N*). Thus, *C* > I / (PVIFA*R*%, *N*).

*c.* Benefits = *C* (PVIFA*R%, N*) = 2 × costs = 2I

*C* = 2I / (PVIFA*R%, N*)

*Challenge*

**28.** *a.* Here the cash inflows of the project go on forever, which is a perpetuity. Unlike ordinary perpetuity cash flows, the cash flows here grow at a constant rate forever, which is a growing perpetuity. If you remember back to the chapter on stock valuation, we presented a formula for valuing a stock with constant growth in dividends. This formula is actually the formula for a growing perpetuity, so we can use it here. The PV of the future cash flows from the project is:

PV of cash inflows = *C1* / (*R* – *g*)

PV of cash inflows = $135,000 / (.12 – .047)

PV of cash inflows = $1,849,315.07

NPV is the PV of the outflows plus by the PV of the inflows, so the NPV is:

NPV of the project = –$1,575,000 + 1,849,315.07

NPV of the project = $274,315.07

The NPV is positive, so we would accept the project.

*b.* Here we want to know the minimum growth rate in cash flows necessary to accept the project. The minimum growth rate is the growth rate at which we would have a zero NPV. The equation for a zero NPV, using the equation for the PV of a growing perpetuity, is:

0 = –$1,575,000 + $135,000 / (.12 – *g*)

Solving for *g*, we get:

*g* = .0343, or 3.43%

**29.** The IRR is the interest rate that makes the NPV of the project equal to zero. So, the IRR of the project is:

0 = $35,000 – $27,000 / (1 + IRR) + $29,000 / (1 + IRR)2

Even though it appears there are two IRRs, a spreadsheet, financial calculator, or trial and error will not give an answer. The reason is that there is no real IRR for this set of cash flows. If you examine the IRR equation, what we are really doing is solving for the roots of the equation. Going back to high school algebra, in this problem we are solving a quadratic equation. In case you don’t remember, the quadratic formula is:

x = 

In this case, the equation is:

x = 

The square root term works out to be:

729,000,000 – 4,060,000,000 = –3,331,000,000

The square root of a negative number is a complex number, so there is no real number solution, meaning the project has no real IRR.

**30.** First, we need to find the future value of the cash flows for the one year in which they are blocked by the government. So, reinvesting each cash inflow for one year, we find:

Year 2 cash flow = $303,800(1.04) = $315,952

Year 3 cash flow = $219,700(1.04) = $228,488

Year 4 cash flow = $320,000(1.04) = $332,800

Year 5 cash flow = $288,700(1.04) = $300,248

So, the NPV of the project is:

NPV = –$862,000 + $315,952 / 1.102 + $228,488 / 1.103 + $332,800 / 1.104 + $300,248 / 1.105

NPV = –$15,478.97

And the IRR of the project is:

0 = –$862,000 + $315,952 / (1 + IRR)2 – $228,488 / (1 + IRR)3 + $332,800 / (1 + IRR)4

+ $300,248 / (1 + IRR)5

Using a spreadsheet, financial calculator, or trial and error to find the root of the equation, we find that:

IRR = 9.42%

While this may look like a MIRR calculation, it is not an MIRR, rather it is a standard IRR calculation. Since the cash inflows are blocked by the government, they are not available to the company for a period of one year. Thus, all we are doing is calculating the IRR based on when the cash flows actually occur for the company.

**Calculator Solutions**

|  |  |  |
| --- | --- | --- |
| **5.** |  |  |
|  | **CF**o | –$168,500 |
|  | **C01** | $86,000 |
|  | **F01** | 1 |
|  | **C02** | $91,000 |
|  | **F02** | 1 |
|  | **C03** | $53,000 |
|  | **F03** | 1 |
|  | IRR CPT | |
|  | 18.79% | |

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **6.** |  |  |  |  |
|  | **CF**o | –$168,500 | **CF**o | –$168,500 |
|  | **C01** | $86,000 | **C01** | $86,000 |
|  | **F01** | 1 | **F01** | 1 |
|  | **C02** | $91,000 | **C02** | $91,000 |
|  | **F02** | 1 | **F02** | 1 |
|  | **C03** | $53,000 | **C03** | $53,000 |
|  | **F03** | 1 | **F03** | 1 |
|  | I = 9% | | I = 21% | |
|  | NPV CPT | | NPV CPT | |
|  | $27,917.69 | | –$5,354.28 | |

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| **7.** |  |  |  |  |  |  |
|  | **CF**o | –$8,450 | **CF**o | –$8,450 | **CF**o | –$8,450 |
|  | **C01** | $2,145 | **C01** | $2,145 | **C01** | $2,145 |
|  | **F01** | 8 | **F01** | 8 | **F01** | 8 |
|  | I = 8% | | I = 24% | | IRR CPT | |
|  | NPV CPT | | NPV CPT | | 19.13% | |
|  | $3,876.54 | | –$1,111.48 | |  | |

|  |  |  |
| --- | --- | --- |
| **8.** |  |  |
|  | **CF**o | –$19,400 |
|  | **C01** | $9,800 |
|  | **F01** | 1 |
|  | **C02** | $11,300 |
|  | **F02** | 1 |
|  | **C03** | $6,900 |
|  | **F03** | 1 |
|  | IRR CPT | |
|  | 22.09% | |

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **9.** |  |  |  |  |
|  | **CF**o | –$19,400 | **CF**o | –$19,400 |
|  | **C01** | $9,800 | **C01** | $9,800 |
|  | **F01** | 1 | **F01** | 1 |
|  | **C02** | $11,300 | **C02** | $11,300 |
|  | **F02** | 1 | **F02** | 1 |
|  | **C03** | $6,900 | **C03** | $6,900 |
|  | **F03** | 1 | **F03** | 1 |
|  | I = 0% | | I = 10% | |
|  | NPV CPT | | NPV CPT | |
|  | $8,600.00 | | $4,032.01 | |

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  |  |  |  |  |
|  | **CF**o | –$19,400 | **CF**o | –$19,400 |
|  | **C01** | $9,800 | **C01** | $9,800 |
|  | **F01** | 1 | **F01** | 1 |
|  | **C02** | $11,300 | **C02** | $11,300 |
|  | **F02** | 1 | **F02** | 1 |
|  | **C03** | $6,900 | **C03** | $6,900 |
|  | **F03** | 1 | **F03** | 1 |
|  | I = 20% | | I = 30% | |
|  | NPV CPT | | NPV CPT | |
|  | $606.94 | | –$2,034.50 | |

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **10.** | *CF (A)* |  |  |  |
|  | **Cf**o | –$78,500 | **CF**o | –$78,500 |
|  | **C01** | $43,000 | **C01** | $43,000 |
|  | **F01** | 1 | **F01** | 1 |
|  | **C02** | $29,000 | **C02** | $29,000 |
|  | **F02** | 1 | **F02** | 1 |
|  | **C03** | $23,000 | **C03** | $23,000 |
|  | **F03** | 1 | **F03** | 1 |
|  | **C04** | $21,000 | **C04** | $21,000 |
|  | **F04** | 1 | **F04** | 1 |
|  | CPT IRR | | I = 11% | |
|  | 20.70% | | NPV CPT | |
|  |  | | $14,426.54 | |

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | *CF (B)* |  |  |  |
|  | **CF**o | –$78,500 | **CF**o | –$78,500 |
|  | **C01** | $21,000 | **C01** | $21,000 |
|  | **F01** | 1 | **F01** | 1 |
|  | **C02** | $28,000 | **C02** | $28,000 |
|  | **F02** | 1 | **F02** | 1 |
|  | **C03** | $34,000 | **C03** | $34,000 |
|  | **F03** | 1 | **F03** | 1 |
|  | **C04** | $41,000 | **C04** | $41,000 |
|  | **F04** | 1 | **F04** | 1 |
|  | CPT IRR | | I = 11% | |
|  | 18.73% | | NPV CPT | |
|  |  | | $15,012.82 | |

Crossover rate:

|  |  |  |
| --- | --- | --- |
|  | **CF**o | $0 |
|  | **C01** | $22,000 |
|  | **F01** | 1 |
|  | **C02** | $1,000 |
|  | **F02** | 1 |
|  | **C03** | –$11,000 |
|  | **F03** | 1 |
|  | **C04** | –$20,000 |
|  | **F04** | 1 |
|  | CPT IRR | |
|  | 12.21% | |

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **11.** | *CF (X)* |  |  |  |
|  | **CF**o | –$23,400 | **CF**o | –$23,400 |
|  | **C01** | $13,100 | **C01** | $13,100 |
|  | **F01** | 1 | **F01** | 1 |
|  | **C02** | $9,480 | **C02** | $9,480 |
|  | **F02** | 1 | **F02** | 1 |
|  | **C03** | $7,890 | **C03** | $7,890 |
|  | **F03** | 1 | **F03** | 1 |
|  | I = 0% | | I = 25% | |
|  | NPV CPT | | NPV CPT | |
|  | $7,070 | | –$2,813.12 | |

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | *CF (Y)* |  |  |  |
|  | **CF**o | –$23,400 | **CF**o | –$23,400 |
|  | **C01** | $9,200 | **C01** | $9,200 |
|  | **F01** | 1 | **F01** | 1 |
|  | **C02** | $10,620 | **C02** | $10,620 |
|  | **F02** | 1 | **F02** | 1 |
|  | **C03** | $11,180 | **C03** | $11,180 |
|  | **F03** | 1 | **F03** | 1 |
|  | I = 0% | | I = 25% | |
|  | NPV CPT | | NPV CPT | |
|  | $7,600 | | –$3,519.04 | |

Crossover rate:

|  |  |  |
| --- | --- | --- |
|  | **CF**o | $0 |
|  | **C01** | $3,900 |
|  | **F01** | 1 |
|  | **C02** | –$1,140 |
|  | **F02** | 1 |
|  | **C03** | –$3,290 |
|  | **F03** | 1 |
|  | CPT IRR | |
|  | 7.62% | |

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **12.** |  |  |  |  |
|  | **CF**o | –$39,000,000 | **CF**o | –$39,000,000 |
|  | **C01** | $57,000,000 | **C01** | $57,000,000 |
|  | **F01** | 1 | **F01** | 1 |
|  | **C02** | –$9,000,000 | **C02** | –$9,000,000 |
|  | **F02** | 1 | **F02** | 1 |
|  | I = 10 | | IRR CPT | |
|  | NPV CPT | | 28.15% | |
|  | $5,380,165.29 | |  | |

NOTE: This is the only IRR the BA II Plus will calculate. The second IRR of –81.99% must be

calculated using another program, by hand, or trial and error.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| **13.** |  |  |  |  |  |  |
|  | **CF**o | $0 | **CF**o | $0 | **CF**o | $0 |
|  | **C01** | $15,800 | **C01** | $15,800 | **C01** | $15,800 |
|  | **F01** | 1 | **F01** | 1 | **F01** | 1 |
|  | **C02** | $13,600 | **C02** | $13,600 | **C02** | $13,600 |
|  | **F02** | 1 | **F02** | 1 | **F02** | 1 |
|  | **C03** | $8,300 | **C03** | $8,300 | **C03** | $8,300 |
|  | **F03** | 1 | **F03** | 1 | **F03** | 1 |
|  | I = 10% | | I = 15% | | I = 22% | |
|  | NPV CPT | | NPV CPT | | NPV CPT | |
|  | $31,839.22 | | $29,480.07 | | $26,659.02 | |

@10%: PI = $31,839.22 / $27,500 = 1.158

@15%: PI = $29,480.07 / $27,500 = 1.072

@22%: PI = $26,659.02 / $27,500 = .969

**14.** *a.* The profitability indexes are:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  |  | *CF (I)* |  | *CF (II)* |
|  | **CF**o | $0 | **CF**o | $0 |
|  | **C01** | $28,300 | **C01** | $9,600 |
|  | **F01** | 1 | **F01** | 1 |
|  | **C02** | $34,800 | **C02** | $17,400 |
|  | **F02** | 1 | **F02** | 1 |
|  | **C03** | $43,700 | **C03** | $15,600 |
|  | **F03** | 1 | **F03** | 1 |
|  | I = 11% | | I = 11% | |
|  | NPV CPT | | NPV CPT | |
|  | $85,693.02 | | $34,177.46 | |

PII = $85,693.02 / $78,000 = 1.099

PIII = $34,177.46 / $28,800 = 1.187

*b.* The NPV of each project is:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  |  | *CF (I)* |  | *CF (II)* |
|  | **CF**o | –$78,000 | **CF**o | –$28,800 |
|  | **C01** | $28,300 | **C01** | $9,600 |
|  | **F01** | 1 | **F01** | 1 |
|  | **C02** | $34,800 | **C02** | $17,400 |
|  | **F02** | 1 | **F02** | 1 |
|  | **C03** | $43,700 | **C03** | $15,600 |
|  | **F03** | 1 | **F03** | 1 |
|  | I = 11% | | I = 11% | |
|  | NPV CPT | | NPV CPT | |
|  | $7,693.02 | | $5,377.46 | |

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| **15.** | *CF (A)* |  |  |  |  |  |
|  | **CF**o | –$235,000 | **CF**o | –$235,000 | **CF**o | $0 |
|  | **C01** | $29,000 | **C01** | $29,000 | **C01** | $29,000 |
|  | **F01** | 1 | **F01** | 1 | **F01** | 1 |
|  | **C02** | $45,000 | **C02** | $45,000 | **C02** | $45,000 |
|  | **F02** | 1 | **F02** | 1 | **F02** | 1 |
|  | **C03** | $51,000 | **C03** | $51,000 | **C03** | $51,000 |
|  | **F03** | 1 | **F03** | 1 | **F03** | 1 |
|  | **C04** | $325,000 | **C04** | $325,000 | **C04** | $325,000 |
|  | **F04** | 1 | **F04** | 1 | **F04** | 1 |
|  | I = 13% | | IRR CPT | | I = 13% | |
|  | NPV CPT | | 21.02% | | NPV CPT | |
|  | $60,579.46 | |  | | $295,579.46 | |

PI = $295,579.46 / $235,000 = 1.258

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  | *CF (B)* |  |  |  |  |  |
|  | **CF**o | –$47,000 | **CF**o | –$47,000 | **CF**o | $0 |
|  | **C01** | $28,700 | **C01** | $28,700 | **C01** | $28,700 |
|  | **F01** | 1 | **F01** | 1 | **F01** | 1 |
|  | **C02** | $19,900 | **C02** | $19,900 | **C02** | $19,900 |
|  | **F02** | 1 | **F02** | 1 | **F02** | 1 |
|  | **C03** | $17,300 | **C03** | $17,300 | **C03** | $17,300 |
|  | **F03** | 1 | **F03** | 1 | **F03** | 1 |
|  | **C04** | $16,200 | **C04** | $16,200 | **C04** | $16,200 |
|  | **F04** | 1 | **F04** | 1 | **F04** | 1 |
|  | I = 13% | | IRR CPT | | I = 13% | |
|  | NPV CPT | | 30.57% | | NPV CPT | |
|  | $15,908.38 | |  | | $62,908.38 | |

PI = $62,908.38 / $47,000 = 1.338

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **16.** | *Project M* |  |  |  |
|  | **CF**o | –$150,000 | **CF**o | –$150,000 |
|  | **C01** | $68,600 | **C01** | $68,600 |
|  | **F01** | 1 | **F01** | 1 |
|  | **C02** | $76,800 | **C02** | $76,800 |
|  | **F02** | 1 | **F02** | 1 |
|  | **C03** | $71,300 | **C03** | $71,300 |
|  | **F03** | 1 | **F03** | 1 |
|  | **C04** | $40,500 | **C04** | $40,500 |
|  | **F04** | 1 | **F04** | 1 |
|  | CPT IRR | | I = 15% | |
|  | 27.82% | | NPV CPT | |
|  |  | | $37,760.92 | |

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | *Project N* |  |  |  |
|  | **CF**o | –$372,000 | **CF**o | –$372,000 |
|  | **C01** | $159,300 | **C01** | $159,300 |
|  | **F01** | 1 | **F01** | 1 |
|  | **C02** | $193,200 | **C02** | $193,200 |
|  | **F02** | 1 | **F02** | 1 |
|  | **C03** | $154,800 | **C03** | $154,800 |
|  | **F03** | 1 | **F03** | 1 |
|  | **C04** | $110,400 | **C04** | $110,400 |
|  | **F04** | 1 | **F04** | 1 |
|  | CPT IRR | | I = 15% | |
|  | 25.57% | | NPV CPT | |
|  |  | | $77,513.77 | |

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **17.** | *Project Y* |  |  |  |
|  | **CF**o | $0 | **CF**o | –$43,400 |
|  | **C01** | $19,800 | **C01** | $19,800 |
|  | **F01** | 1 | **F01** | 1 |
|  | **C02** | $17,500 | **C02** | $17,500 |
|  | **F02** | 1 | **F02** | 1 |
|  | **C03** | $20,700 | **C03** | $20,700 |
|  | **F03** | 1 | **F03** | 1 |
|  | **C04** | $14,600 | **C04** | $14,600 |
|  | **F04** | 1 | **F04** | 1 |
|  | I = 12% | | I = 12% | |
|  | NPV CPT | | NPV CPT | |
|  | $55,641.88 | | $12,241.88 | |

PI = $55,641.88 / $43,400 = 1.282

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | *Project Z* |  |  |  |
|  | **CF**o | $0 | **CF**o | –$78,000 |
|  | **C01** | $32,000 | **C01** | $32,000 |
|  | **F01** | 1 | **F01** | 1 |
|  | **C02** | $30,100 | **C02** | $30,100 |
|  | **F02** | 1 | **F02** | 1 |
|  | **C03** | $29,500 | **C03** | $29,500 |
|  | **F03** | 1 | **F03** | 1 |
|  | **C04** | $27,300 | **C04** | $27,300 |
|  | **F04** | 1 | **F04** | 1 |
|  | I = 12% | | I = 12% | |
|  | NPV CPT | | NPV CPT | |
|  | $90,914.13 | | $12,914.13 | |

PI = $90,914.13 / $78,000 = 1.166

|  |  |  |
| --- | --- | --- |
| **18.** |  |  |
|  | **CF**o | $0 |
|  | **C01** | $23,200 |
|  | **F01** | 1 |
|  | **C02** | $5,600 |
|  | **F02** | 1 |
|  | **C03** | –$13,600 |
|  | **F03** | 1 |
|  | **C04** | –$26,200 |
|  | **F04** | 1 |
|  | CPT IRR | |
|  | 14.06% | |

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **20.** |  |  |  |  |
|  | **CF**o | –$745,382 | **CF**o | –$745,382 |
|  | **C01** | $265,381 | **C01** | $265,381 |
|  | **F01** | 1 | **F01** | 1 |
|  | **C02** | $304,172 | **C02** | $304,172 |
|  | **F02** | 1 | **F02** | 1 |
|  | **C03** | $225,153 | **C03** | $225,153 |
|  | **F03** | 1 | **F03** | 1 |
|  | **C04** | $208,614 | **C04** | $208,614 |
|  | **F04** | 1 | **F04** | 1 |
|  | I = 0 | | IRR CPT | |
|  | NPV CPT | | 13.79% | |
|  | $257,938 | |  | |

**21.** *b.*

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | *Project F* |  | *Project G* |  |
|  | **CF**o | –$180,000 | **CF**o | –$280,000 |
|  | **C01** | $93,600 | **C01** | $64,800 |
|  | **F01** | 1 | **F01** | 1 |
|  | **C02** | $64,800 | **C02** | $86,400 |
|  | **F02** | 1 | **F02** | 1 |
|  | **C03** | $81,600 | **C03** | $123,600 |
|  | **F03** | 1 | **F03** | 1 |
|  | **C04** | $72,000 | **C04** | $166,800 |
|  | **F04** | 1 | **F04** | 1 |
|  | **C05** | $64,800 | **C05** | $187,200 |
|  | **F05** | 1 | **F05** | 1 |
|  | I = 10% | | I = 10% | |
|  | NPV CPT | | NPV CPT | |
|  | $109,364.59 | | $173,339.68 | |

**24.**  Crossover rate:

|  |  |  |
| --- | --- | --- |
|  |  |  |
|  | **CF**o | $31,000 |
|  | **C01** | –$3,000 |
|  | **F01** | 1 |
|  | **C02** | –$1,000 |
|  | **F02** | 1 |
|  | **C03** | –$14,000 |
|  | **F03** | 1 |
|  | **C04** | –$21,000 |
|  | **F04** | 1 |
|  | **C05** | –$3,000 |
|  | **F05** | 1 |
|  | IRR CPT | |
|  | 9.24% | |

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | *Project R* |  | *Project S* |  |
|  | **CF**o | –$45,000 | **CF**o | –$76,000 |
|  | **C01** | $17,000 | **C01** | $20,000 |
|  | **F01** | 1 | **F01** | 1 |
|  | **C02** | $19,000 | **C02** | $20,000 |
|  | **F02** | 1 | **F02** | 1 |
|  | **C03** | $21,000 | **C03** | $35,000 |
|  | **F03** | 1 | **F03** | 1 |
|  | **C04** | $9,000 | **C04** | $30,000 |
|  | **F04** | 1 | **F04** | 1 |
|  | **C05** | $7,000 | **C05** | $10,000 |
|  | **F05** | 1 | **F05** |  |
|  | I = 9.24% | | I = 9.24% | |
|  | NPV CPT | | NPV CPT | |
|  | $13,414.39 | | $13,414.39 | |

|  |  |  |
| --- | --- | --- |
| **25.** |  |  |
|  | **CF**o | $91,000 |
|  | **C01** | –$55,000 |
|  | **F01** | 1 |
|  | **C02** | –$46,000 |
|  | **F02** | 1 |
|  | IRR CPT | |
|  | 7.47% | |

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  |  |  |  |  |  |  |
|  | **CF**o | $91,000 | **CF**o | $91,000 | **CF**o | $91,000 |
|  | **C01** | –$55,000 | **C01** | –$55,000 | **C01** | –$55,000 |
|  | **F01** | 1 | **F01** | 1 | **F01** | 1 |
|  | **C02** | –$46,000 | **C02** | –$46,000 | **C02** | –$46,000 |
|  | **F02** | 1 | **F02** | 1 | **F02** | 1 |
|  | I = 0% | | I = 10% | | I = 24% | |
|  | NPV CPT | | NPV CPT | | NPV CPT | |
|  | –$10,000 | | $2,983.47 | | $16,728.41 | |

**CHAPTER 9**

***MAKING CAPITAL INVESTMENT DECISIONS***

**Answers to Concepts Review and Critical Thinking Questions**

**1.** In this context, an opportunity cost refers to the value of an asset or other input that will be used in a project. The relevant cost is what the asset or input is actually worth today, not, for example, what it cost to acquire.

**2.** For tax purposes, a firm would choose MACRS because it provides for larger depreciation deductions earlier. These larger deductions reduce taxes, but have no other cash consequences. Notice that the choice between MACRS and straight-line is purely a time value issue; the total depreciation is the same, only the timing differs.

**3.** It’s probably only a mild over-simplification. Current liabilities will all be paid presumably. The cash portion of current assets will be retrieved. Some receivables won’t be collected, and some inventory will not be sold, of course. Counterbalancing these losses is the fact that inventory sold above cost (and not replaced at the end of the project’s life) acts to increase working capital. These effects tend to offset.

**4.** Management’s discretion to set the firm’s capital structure is applicable at the firm level. Since any one particular project could be financed entirely with equity, another project could be financed with debt, and the firm’s overall capital structure remain unchanged, financing costs are not relevant in the analysis of a project’s incremental cash flows according to the stand-alone principle.

**5.** Depreciation is a non-cash expense, but it is tax-deductible on the income statement. Thus depreciation causes taxes paid, an actual cash outflow, to be reduced by an amount equal to the depreciation tax shield TCD. A reduction in taxes that would otherwise be paid is the same thing as a cash inflow, so the effects of the depreciation tax shield must be added in to get the total incremental aftertax cash flows.

**6.** There are two particularly important considerations. The first is erosion. Will the essentialized book simply displace copies of the existing book that would have otherwise been sold? This is of special concern given the lower price. The second consideration is competition. Will other publishers step in and produce such a product? If so, then any erosion is much less relevant. A particular concern to book publishers (and producers of a variety of other product types) is that the publisher only makes money from the sale of new books. Thus, it is important to examine whether the new book would displace sales of used books (good from the publisher’s perspective) or new books (not good). The concern arises any time that there is an active market for used product.

**7.** Definitely. The damage to Porsche’s reputation is definitely a factor the company needed to consider. If the reputation was damaged, the company would have lost sales of its existing car lines.

**8.** One company may be able to produce at lower incremental cost or market better. Also, of course, one of the two may have made a mistake!

**9.** Porsche would recognize that the outsized profits would dwindle as more products come to market and competition becomes more intense.

**10.** With a sensitivity analysis, one variable is examined over a broad range of values. With a scenario analysis, all variables are examined for a limited range of values.

**11.** It is true that if average revenue is less than average cost, the firm is losing money. This much of the statement is therefore correct. At the margin, however, accepting a project with marginal revenue in excess of its marginal cost clearly acts to increase operating cash flow. At the margin, even if a firm is losing money, as long as marginal revenue exceeds marginal cost, the firm will lose less than it would without the new project.

**12.** The implication is that they will face hard capital rationing.

**13.** Forecasting risk is the risk that a poor decision is made because of errors in projected cash flows. The danger is greatest with a new project because the cash flows are usually harder to predict.

**14.** The option to abandon reflects our ability to reallocate assets if we find our initial estimates were too optimistic. The option to expand reflects our ability to increase cash flows from a project if we find our initial estimates were too pessimistic. Since the option to expand can increase cash flows and the option to abandon reduces losses, failing to consider these two options will generally lead us to underestimate a project’s NPV.

**Solutions to Questions and Problems**

*Basic*

*NOTE: All end-of-chapter problems were solved using a spreadsheet. Many problems require multiple steps. Due to space and readability constraints, when these intermediate steps are included in this solutions manual, rounding may appear to have occurred. However, the final answer for each problem is found without rounding during any step in the problem.*

**1.** The $5.3 million acquisition cost of the land six years ago is a sunk cost. The $7.4 million current aftertax value of the land is an opportunity cost if the land is used rather than sold off. The $26.5 million cash outlay and $1,320,000 grading expenses are the initial fixed asset investments needed to get the project going. Therefore, the proper year zero cash flow to use in evaluating this project is:

Cash flow = $7,400,000 + 26,500,000 + 1,320,000

Cash flow = $35,220,000

**2.** Sales due solely to the new product line are:

29,000($23,500) = $681,500,000

Increased sales of the motor home line occur because of the new product line introduction; thus:

2,500($77,000) = $192,500,000

in new sales is relevant. Erosion of luxury motor coach sales is also due to the new mid-size campers; thus:

750($120,000) = $90,000,000 loss in sales

is relevant. The net sales figure to use in evaluating the new line is thus:

Net sales = $681,500,000 + 192,500,000 – 90,000,000

Net sales = $784,000,000

**3.** We need to construct an income statement. The income statement is:

Sales $ 645,000

Variable costs 258,000

Fixed costs 168,000

Depreciation 83,000

EBIT $ 136,000

Taxes@35% 47,600

Net income $ 88,400

**4.** To find the OCF, we need to complete the income statement as follows:

Sales $ 558,400

Variable costs 346,800

Depreciation 94,500

EBIT $ 117,100

Taxes@35% 40,985

Net income $ 76,115

The OCF for the company is:

OCF = EBIT + Depreciation – Taxes

OCF = $117,100 + 94,500 – 40,985

OCF = $170,615

The depreciation tax shield is the depreciation times the tax rate, so:

Depreciation tax shield = Depreciation(*TC*)

Depreciation tax shield = .35($94,500)

Depreciation tax shield = $33,075

The depreciation tax shield shows us the increase in OCF by being able to expense depreciation.

**5.** The MACRS depreciation schedule is shown in Table 9.7. The ending book value for any year is the beginning book value minus the depreciation for the year. Remember, to find the amount of depreciation for any year, you multiply the purchase price of the asset times the MACRS percentage for the year. The depreciation schedule for this asset is:

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | Beginning  Year | Beginning  Book Value | Depreciation | Depreciation  Allowance | Ending  Book Value |
|  | 1 | $715,000.00 | 14.29% | $102,173.50 | $612,826.50 |
|  | 2 | 612,826.50 | 24.49% | 175,103.50 | 437,723.00 |
|  | 3 | 437,723.00 | 17.49% | 125,053.50 | 312,669.50 |
|  | 4 | 312,669.50 | 12.49% | 89,303.50 | 223,366.00 |
|  | 5 | 223,366.00 | 8.93% | 63,849.50 | 159,516.50 |
|  | 6 | 159,516.50 | 8.92% | 63,778.00 | 95,738.50 |
|  | 7 | 95,738.50 | 8.93% | 63,849.50 | 31,889.00 |
|  | 8 | 31,889.00 | 4.46% | 31,889.00 | 0 |

**6.** The asset has a useful life of 8 years and we want to find the book value of the asset after 5 years. With straight-line depreciation, the depreciation each year will be:

Annual depreciation = $545,000 / 8

Annual depreciation = $68,125

So, after five years, the accumulated depreciation will be:

Accumulated depreciation = 5($68,125)

Accumulated depreciation = $340,625

The book value at the end of year five is thus:

BV5 = $545,000 – 340,625

BV5 = $204,375

The asset is sold at a loss to book value, so the depreciation tax shield of the loss is recaptured.

Aftertax salvage value = $95,000 + ($204,375 – 95,000)(.35)

Aftertax salvage value = $133,281.25

To find the taxes on salvage value, remember to use the equation:

Taxes on salvage value = (BV – MV)*TC*

This equation will always give the correct sign for a tax inflow (refund) or outflow (payment).

**7.** To find the book value at the end of four years, we need to find the accumulated depreciation for the first four years. We could calculate a table as in Problem 5, but an easier way is to add the MACRS depreciation amounts for each of the first four years and multiply this percentage times the cost of the asset. We can then subtract this from the asset cost. Doing so, we get:

BV4 = $7,100,000 – 7,100,000(.2000 + .3200 + .1920 + .1152)

BV4 = $1,226,880

The asset is sold at a gain to book value, so this gain is taxable.

Aftertax salvage value = $1,460,000 + ($1,226,880 – 1,460,000)(.34)

Aftertax salvage value = $1,380,739.20

**8.** We need to calculate the OCF, so we need an income statement. The lost sales of the current sound board sold by the company will be a negative since it will lose the sales.

|  |  |  |
| --- | --- | --- |
|  | Sales of new | $40,950,000 |
|  | Lost sales of old | –8,217,000 |
|  | Variable costs | 18,003,150 |
|  | Fixed costs | 3,200,000 |
|  | Depreciation | 2,150,000 |
|  | EBT | $9,379,850 |
|  | Tax | 3,564,343 |
|  | Net income | $5,815,507 |

The OCF for the company is:

OCF = EBIT + Depreciation – Taxes

OCF = $9,379,850 + 2,150,000 – 3,564,343

OCF = $7,965,507

**9.** Using the tax shield approach to calculating OCF (Remember the approach is irrelevant; the final answer will be the same no matter which of the four methods you use.), we get:

OCF = (Sales – Costs)(1 – *TC*) + Depreciation(*TC*)

OCF = ($2,145,000 – 1,205,000)(1 – .35) + .35($1,950,000 / 3)

OCF = $838,500

**10.** Since we have the OCF, we can find the NPV as the initial cash outlay, plus the PV of the OCFs, which are an annuity, so the NPV is:

NPV = –$1,950,000 + $838,500(PVIFA14%,3)

NPV = –$3,311.55

**11.** The cash outflow at the beginning of the project will increase because of the spending on NWC. At the end of the project, the company will recover the NWC, so it will be a cash inflow. The sale of the equipment will result in a cash inflow, but we must also account for the taxes that will be paid on this sale. So, the cash flows for each year of the project will be:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | Year | Cash Flow |  |  |
|  | 0 | –$2,100,000 |  | = –$1,950,000 – 150,000 |
|  | 1 | 838,500 |  |  |
|  | 2 | 838,500 |  |  |
|  | 3 | 1,102,250 |  | = $838,500 + 150,000 + 175,000 + (0 – 175,000)(.35) |

And the NPV of the project is:

NPV = –$2,100,000 + $838,500(PVIFA14%,2) + ($1,102,250 / 1.143)

NPV = $24,712.19

**12.** First, we will calculate the annual depreciation for the equipment necessary for the project. The depreciation amount each year will be:

Year 1 depreciation = $1,950,000(.3333) = $649,935

Year 2 depreciation = $1,950,000(.4445) = $866,775

Year 3 depreciation = $1,950,000(.1481) = $288,795

So, the book value of the equipment at the end of three years, which will be the initial investment minus the accumulated depreciation, is:

Book value in 3 years = $1,950,000 – ($649,935 + 866,775 + 288,795)

Book value in 3 years = $144,495

The asset is sold at a gain to book value, so this gain is taxable.

Aftertax salvage value = $175,000 + ($144,495 – 175,000)(.35)

Aftertax salvage value = $164,323

To calculate the OCF, we will use the tax shield approach, so the cash flow each year is:

OCF = (Sales – Costs)(1 – *TC*) + Depreciation(*TC*)

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | Year | Cash Flow |  |  |
|  | 0 | –$2,100,000 |  | = –$1,950,000 – 150,000 |
|  | 1 | 838,477.25 |  | = ($940,000)(.65) + .35($649,935) |
|  | 2 | 914,371.25 |  | = ($940,000)(.65) + .35($866,775) |
|  | 3 | 1,026,401.50 |  | = ($940,000)(.65) + .35($288,795) + 164,323 + 150,000 |

Remember to include the NWC cost in Year 0, and the recovery of the NWC at the end of the project. The NPV of the project with these assumptions is:

NPV = –$2,100,000 + ($838,477.25 / 1.14) + ($914,371.25 / 1.142) + ($1,026,401.50 / 1.143)

NPV = $31,877.12

**13.** First, we will calculate the annual depreciation of the new equipment. It will be:

Annual depreciation = $655,000 / 5

Annual depreciation = $131,000

Now, we calculate the aftertax salvage value. The aftertax salvage value is the market price minus (or plus) the taxes on the sale of the equipment, so:

Aftertax salvage value = MV + (BV – MV)*TC*

Very often, the book value of the equipment is zero, as it is in this case. If the book value is zero, the equation for the aftertax salvage value becomes:

Aftertax salvage value = MV + (0 – MV)*TC*

Aftertax salvage value = MV(1 – *TC*)

We will use this equation to find the aftertax salvage value since we know the book value is zero. So, the aftertax salvage value is:

Aftertax salvage value = $85,000(1 – .34)

Aftertax salvage value = $56,100

Using the tax shield approach, we find the OCF for the project is:

OCF = $183,000(1 – .34) + .34($131,000)

OCF = $165,320

Now we can find the project NPV. Notice we include the NWC in the initial cash outlay. The recovery of the NWC occurs in Year 5, along with the aftertax salvage value.

NPV = –$655,000 – 35,000 + $165,320(PVIFA8%,5) + [($56,100 + 35,000) / 1.085]

NPV = $32,075.95

**14.** First, we will calculate the annual depreciation of the new equipment. It will be:

Annual depreciation charge = $410,000 / 5

Annual depreciation charge = $82,000

The aftertax salvage value of the equipment is:

Aftertax salvage value = $30,000(1 – .35)

Aftertax salvage value = $19,500

Using the tax shield approach, the OCF is:

OCF = $125,000(1 – .35) + .35($82,000)

OCF = $109,950

Now we can find the project IRR. There is an unusual feature that is a part of this project. Accepting this project means that we will reduce NWC. This reduction in NWC is a cash inflow at Year 0. This reduction in NWC implies that when the project ends, we will have to increase NWC. So, at the end of the project, we will have a cash outflow to restore the NWC to its level before the project. We must also include the aftertax salvage value at the end of the project. The IRR of the project is:

NPV = 0 = –$410,000 + 35,000 + $109,950(PVIFAIRR%,5) + [($19,500 – 35,000) / (1 + IRR)5]

IRR = 13.37%

**15.** To evaluate the project with a $145,000 cost savings, we need the OCF to compute the NPV. Using the tax shield approach, the OCF is:

OCF = $145,000(1 – .35) + .35($82,000)

OCF = $122,950

NPV = –$410,000 + 35,000 + $122,950(PVIFA10%,5) + [($19,500 – 35,000) / (1.10)5]

NPV = $81,452.95

The NPV with a $105,000 cost savings is:

OCF = $105,000(1 – .35) + .35($82,000)

OCF = $96,950

NPV = –$410,000 + 35,000 + $96,950(PVIFA10%,5) + [($19,500 – 35,000) / (1.10)5]

NPV = –$17,107.50

We would accept the project if the cost savings were $145,000, and reject the project if the cost savings were $105,000.

**16.** The base-case, best-case, and worst-case values are shown below. Remember that in the best-case, sales and price increase, while costs decrease. In the worst case, sales and price decrease, and costs increase.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | Scenario | Unit Sales | Unit Price | Unit Variable Cost | Fixed Costs |
|  | Base case | 55,000 | $960 | $350 | $3,600,000 |
|  | Best case | 63,250 | $1,104 | $298 | $3,060,000 |
|  | Worst case | 46,750 | $816 | $403 | $4,140,000 |

**17.** An estimate for the impact of changes in price on the profitability of the project can be found from the sensitivity of NPV with respect to price; ΔNPV/ΔP. This measure can be calculated by finding the NPV at any two different price levels and forming the ratio of the changes in these parameters. Whenever a sensitivity analysis is performed, all other variables are held constant at their base-case values.

**18.** *a.* We will use the tax shield approach to calculate the OCF. The OCF is:

OCFbase = [(P – v)Q – FC](1 – *TC*) + Depreciation(*TC*)

OCFbase = [($37.95 – 23.20)(91,000) – $815,000](.65) + .35($1,720,000 / 6)

OCFbase = $443,045.83

Now we can calculate the NPV using our base-case projections. There is no salvage value or NWC, so the NPV is:

NPVbase = –$1,720,000 + $443,045.83(PVIFA11%,6)

NPVbase = $154,322.17

To calculate the sensitivity of the NPV to changes in the quantity sold, we will calculate the NPV at a different quantity. We will use sales of 92,000 units. The NPV at this sales level is:

OCFnew = [($37.95 – 23.20)(92,000) – $815,000](.65) + .35($1,720,000 / 6)

OCFnew = $452,633.33

And the NPV is:

NPVnew = –$1,720,000 + $452,633.33(PVIFA11%,6)

NPVnew = $194,882.45

So, the change in NPV for every unit change in sales is:

ΔNPV / ΔS = [($154,322.17 – 194,882.45)] / (91,000 – 92,000)

ΔNPV / ΔS = +$40.56

If sales were to drop by 500 units, then NPV would drop by:

NPV drop = $40.56(500)

NPV drop = $20,280.14

You may wonder why we chose 92,000 units. Because it doesn’t matter! Whatever sales number we use, when we calculate the change in NPV per unit sold, the ratio will be the same.

*b.* To find out how sensitive OCF is to a change in variable costs, we will compute the OCF at a variable cost of $20. Again, the number we choose to use here is irrelevant: We will get the same ratio of OCF to a one dollar change in variable cost no matter what variable cost we use. So, using the tax shield approach, the OCF at a variable cost of $20 is:

OCFnew = [($37.95 – 20)(91,000) – $815,000](.65) + .35($1,720,000 / 6)

OCFnew = $632,325.83

So, the change in OCF for a $1 change in variable costs is:

ΔOCF / Δv = ($443,045.83 – 632,325.83) / ($23.20 – 20)

ΔOCF / Δv = –$59,150

If variable costs decrease by $1, then OCF would increase by $59,150.

**19.** We will use the tax shield approach to calculate the OCF for the best- and worst-case scenarios. For the best-case scenario, the price and quantity increase by 10 percent, so we will multiply the base case numbers by 1.1, a 10 percent increase. The variable and fixed costs both decrease by 10 percent, so we will multiply the base case numbers by .9, a 10 percent decrease. Doing so, we get:

OCFbest = {[$37.95(1.1) – $23.20(.9)](91,000)(1.1) – $815,000(.9)}(.65) + .35($1,720,000 / 6)

OCFbest = $981,139.56

The best-case NPV is:

NPVbest = –$1,720,000 + $981,139.56(PVIFA11%,6)

NPVbest = $2,430,748.04

For the worst-case scenario, the price and quantity decrease by 10 percent, so we will multiply the base case numbers by .9, a 10 percent decrease. The variable and fixed costs both increase by 10 percent, so we will multiply the base case numbers by 1.1, a 10 percent increase. Doing so, we get:

OCFworst = {[$37.95(.9) – $23.20(1.1)](91,000)(.9) – $815,000(1.1)}(.65) + .35($1,720,000 / 6)

OCFworst = –$22,707.44

The worst-case NPV is:

NPVworst = –$1,720,000 – $22,707.44(PVIFA11%,6)

NPVworst = –$1,816,064.69

**20.** First, we need to calculate the cash flows. The marketing study is a sunk cost and should be ignored. The net income each year will be:

|  |  |  |
| --- | --- | --- |
|  | Sales of new product | $815,000 |
|  | Variable costs | 163,000 |
|  | Fixed costs | 196,000 |
|  | Depreciation | 216,250 |
|  | EBT | $239,750 |
|  | Tax | 95,900 |
|  | Net income | $143,850 |

So, the OCF is:

OCF = EBIT + Depreciation – Taxes

OCF = $239,750 + 216,250 – 95,900

OCF = $360,100

The only initial cash flow is the cost of the equipment, so the payback period is:

Payback period = $865,000 / $360,100

Payback period = 2.40 years

The NPV is:

NPV = –$865,000 + $360,100(PVIFA13%,4)

NPV = $206,107.12

And the IRR is:

0 = –$865,000 + $360,100(PVIFAIRR%,4)

IRR = 24.05%

*Intermediate*

**21.** First, we will calculate the depreciation each year, which will be:

D1 = $375,000(.2000) = $75,000

D2 = $375,000(.3200) = $120,000

D3 = $375,000(.1920) = $72,000

D4 = $375,000(.1152) = $43,200

The book value of the equipment at the end of the project is:

BV4 = $375,000 – ($75,000 + 120,000 + 72,000 + 43,200)

BV4 = $64,800

The asset is sold at a loss to book value, so this creates a tax refund.

Aftertax salvage value = $45,000 + ($64,800 – 45,000)(.34)

Aftertax salvage value = $51,732

Using the depreciation tax shield approach, the OCF for each year will be:

OCF1 = $142,000(1 – .34) + .34($75,000) = $119,220

OCF2 = $142,000(1 – .34) + .34($120,000) = $134,520

OCF3 = $142,000(1 – .34) + .34($72,000) = $118,200

OCF4 = $142,000(1 – .34) + .34($43,200) = $108,408

Now, we have all the necessary information to calculate the project NPV. We need to be careful with the NWC in this project. Notice the project requires $15,000 of NWC at the beginning, and $2,000 more in NWC each successive year. We will subtract the $15,000 from the initial cash flow, and subtract $2,000 each year from the OCF to account for this spending. In Year 4, we will add back the total spent on NWC, which is $21,000. The $2,000 spent on NWC capital during Year 4 is irrelevant. Why? Well, during this year the project required an additional $2,000, but we would get the money back immediately. So, the net cash flow for additional NWC would be zero. With all this, the equation for the NPV of the project is:

NPV = – $375,000 – 15,000 + ($119,220 – 2,000) / 1.11 + ($134,520 – 2,000) / 1.112

+ ($118,200 – 2,000) / 1.113 + ($108,408 + 21,000 + 51,732) / 1.114

NPV = $27,446.78

**22.** Using the tax shield approach, the OCF at 90,000 units will be:

OCF = [(P – v)Q – FC](1 – *TC*) + Depreciation(*TC*)

OCF = [($38.70 – 29.65)(90,000) – 315,000](.66) + .34($645,000 / 5)

OCF = $373,530

We will calculate the OCF at 91,000 units. The choice of the second level of quantity sold is arbitrary and irrelevant. No matter what level of units sold we choose, we will still get the same sensitivity. So, the OCF at this level of sales is:

OCF = [($38.70 – 29.65)(91,000) – 315,000](.66) + .34($645,000 / 5)

OCF = $379,503

The sensitivity of the OCF to changes in the quantity sold is:

Sensitivity = ΔOCF / ΔQ = ($373,530 – 379,503) / (90,000 – 91,000)

ΔOCF / ΔQ = +$5.97

OCF will increase by $5.97 for every additional unit sold.

**23.** *a*. The base-case, best-case, and worst-case values are shown below. Remember that in the best-case, unit sales increase, while fixed and variable costs decrease. In the worst case, unit sales decrease, while fixed and variable costs increase.

Scenario Unit sales Variable cost Fixed costs

Base 180 $11,100 $535,000

Best 198 $9,990 $481,500

Worst 162 $12,120 $588,500

Using the tax shield approach, the OCF and NPV for the base case estimate is:

OCFbase = [($16,300 – 11,100)(180) – $535,000](.65) + .35($780,000 / 4)

OCFbase = $328,900

NPVbase = –$780,000 + $328,900(PVIFA11%,4)

NPVbase = $240,394.39

The OCF and NPV for the worst case estimate are:

OCFworst = [($16,300 – 12,210)(162) – $588,500](.65) + .35($780,000 / 4)

OCFworst = $116,402

NPVworst = –$780,000 + $116,402(PVIFA11%,4)

NPVworst = –$418,869.12

And the OCF and NPV for the best case estimate are:

OCFbest = [($16,300 – 9,990)(198) – $481,500](.65) + .35($780,000 / 4)

OCFbest = $567,372

NPVbest = –$780,000 + $567,372(PVIFA11%,4)

NPVbest = $980,240.82

*b*. To calculate the sensitivity of the NPV to changes in fixed costs, we choose another level of fixed costs. We will use fixed costs of $545,000. The OCF using this level of fixed costs and the other base-case values with the tax shield approach, we get:

OCF = [($16,300 – 11,100)(180) – $545,000](.65) + .35($780,000 / 4)

OCF = $322,400

And the NPV is:

NPV = –$780,000 + $322,400(PVIFA11%,4)

NPV = $220,228.49

The sensitivity of NPV to changes in fixed costs is:

ΔNPV/ΔFC = ($240,394.39 – 220,228.49)/($535,000 – 545,000)

ΔNPV/ΔFC = –$2.02

For every dollar FC increase, NPV falls by $2.02.

**24.** The marketing study and the research and development are both sunk costs and should be ignored. The initial cost is the equipment plus the net working capital, so:

Initial cost = $29,400,000 + 3,500,000

Initial cost = $32,900,000

Next, we will calculate the sales and variable costs. Since we will lose sales of the expensive clubs and gain sales of the cheap clubs, these must be accounted for as erosion. The total sales for the new project will be:

|  |  |  |
| --- | --- | --- |
|  | Sales |  |
|  | New clubs | $825 × 74,000 = 61,050,000 |
|  | Exp. clubs | $1,250 × (–8,900) = –11,125,000 |
|  | Cheap clubs | $375 × 11,000 = 4,125,000 |
|  |  | $54,050,000 |

For the variable costs, we must include the units gained or lost from the existing clubs. Note that the variable costs of the expensive clubs are an inflow. If we are not producing the sets anymore, we will save these variable costs, which is an inflow. So:

|  |  |  |
| --- | --- | --- |
|  | Var. costs |  |
|  | New clubs | $370 × 74,000 = $27,380,000 |
|  | Exp. clubs | $630 × (–8,900) = –5,607,000 |
|  | Cheap clubs | $140 × 11,000 = 1,540,000 |
|  |  | $23,313,000 |

The pro forma income statement will be:

|  |  |  |
| --- | --- | --- |
|  | Sales | $54,050,000 |
|  | Variable costs | 23,313,000 |
|  | Costs | 14,350,000 |
|  | Depreciation | 4,200,000 |
|  | EBT | $12,187,000 |
|  | Taxes | 4,874,800 |
|  | Net income | $7,312,200 |

Using the bottom up OCF calculation, we get:

OCF = NI + Depreciation

OCF = $7,312,200 + 4,200,000

OCF = $11,512,200

So, the payback period is:

Payback period = 2 + $9,875,600 / $11,512,200

Payback period = 2.86 years

The NPV is:

NPV = –$32,900,000 + $11,512,200(PVIFA14%,7) + $3,500,000 / 1.147

NPV = $17,866,553.60

And the IRR is:

0 = –$32,900,000 + $11,512,200(PVIFAIRR%,7) + $3,500,000 / IRR7

IRR = 29.89%

*Challenge*

**25.** This is an in-depth capital budgeting problem. We will use the bottom up approach, so we will need to construct an income statement for each year. Beginning with the initial cash flow at Time 0, the project will require an investment in equipment. The project will also require an initial investment in NWC of $1,500,000. So, the cash flow required for the project today will be:

|  |  |  |
| --- | --- | --- |
|  | Capital spending | –$18,500,000 |
|  | Change in NWC | –1,500,000 |
|  | Total cash flow | –$20,000,000 |

Now we can begin the remaining calculations. Sales figures are given for each year, along with the price per unit. The variable costs per unit are used to calculate total variable costs, and fixed costs are given at $1,950,000 per year. To calculate depreciation each year, we use the initial equipment cost of $18,500,000, times the appropriate MACRS depreciation each year. The remainder of each income statement is calculated below. Notice at the bottom of the income statement we added back depreciation to get the OCF for each year. The section labeled “Net cash flows” will be discussed below:

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  | Year | 1 | 2 | 3 | 4 | 5 |
|  | Ending book value | $15,856,350 | $11,325,700 | $8,090,050 | $5,779,400 | $4,127,350 |
|  |  |  |  |  |  |  |
|  | Sales | $23,962,500 | $29,784,500 | $35,038,500 | $30,530,000 | $25,560,000 |
|  | Variable costs | 15,525,000 | 19,297,000 | 22,701,000 | 19,780,000 | 16,560,000 |
|  | Fixed costs | 1,950,000 | 1,950,000 | 1,950,000 | 1,950,000 | 1,950,000 |
|  | Depreciation | 2,643,650 | 4,530,650 | 3,235,650 | 2,310,650 | 1,652,050 |
|  | EBIT | 3,843,850 | 4,006,850 | 7,151,850 | 6,489,350 | 5,397,950 |
|  | Taxes | 1,345,348 | 1,402,398 | 2,503,148 | 2,271,273 | 1,889,283 |
|  | Net income | 2,498,503 | 2,604,453 | 4,648,703 | 4,218,078 | 3,508,668 |
|  | Depreciation | 2,643,650 | 4,530,650 | 3,235,650 | 2,310,650 | 1,652,050 |
|  | Operating cash flow | $5,142,153 | $7,135,103 | $7,884,353 | $6,528,728 | $5,160,718 |
|  |  |  |  |  |  |  |
|  | *Net cash flows* |  |  |  |  |  |
|  | Operating cash flow | $5,142,153 | $7,135,103 | $7,884,353 | $6,528,728 | $5,160,718 |
|  | Change in NWC | –873,300 | –788,100 | 676,275 | 745,500 | 1,739,625 |
|  | Capital spending |  |  |  |  | 3,849,573 |
|  | Total cash flow | $4,268,853 | $6,347,003 | $8,560,628 | $7,274,228 | $10,749,915 |

After we calculate the OCF for each year, we need to account for any other cash flows. The other cash flows in this case are NWC cash flows and capital spending, which is the aftertax salvage of the equipment. The required NWC is 15 percent of the change in sales in the next year. We will work through the NWC cash flow for Year 1. The total NWC in Year 1 will be 15 percent of the sales increase from Year 1 to Year 2, or:

Change in NWC for Year 1 = .15($23,962,500 – 29,784,500)

Change in NWC for Year 1 = –$873,300

Notice that the NWC cash flow is negative. Since the sales are increasing, we will have to spend more money to increase NWC. In Year 3, the NWC cash flow is positive since sales are declining. And, in Year 5, the NWC cash flow is the recovery of all NWC the company still has in the project.

To calculate the aftertax salvage value, we first need the book value of the equipment. The book value at the end of the five years will be the purchase price, minus the total depreciation. So, the ending book value is:

Ending book value = $18,500,000 – ($2,643,650 + 4,530,650 + 3,235,650 + 2,310,650 + 1,652,050)

Ending book value = $4,127,350

The market value of the used equipment is 20 percent of the purchase price, or $3.7 million, so the aftertax salvage value will be:

Aftertax salvage value = $3,700,000 + ($4,127,350 – 3,700,000)(.35)

Aftertax salvage value = $3,849,573

The aftertax salvage value is included in the total cash flows as capital spending. Now we have all of the cash flows for the project. The NPV of the project is:

NPV = –$20,000,000 + $4,268,853 / 1.15 + $6,347,003 / 1.152 + $8,560,628 / 1.153

+ $7,274,228 / 1.154 + $10,749,915 / 1.155

NPV = $3,643,713.77

And the IRR is:

NPV = 0 = –$20,000,000 + $4,268,853 / (1 + IRR) + $6,347,003 / (1 + IRR)2

+ $8,560,628 / (1 + IRR)3 + $7,274,228 / (1 + IRR)4 + $10,749,915 / (1 + IRR)5

IRR = 21.47%

The company should accept the project.

**26.** To find the initial pretax cost savings necessary to buy the new machine, we should use the tax shield approach to find the OCF. We begin by calculating the depreciation each year using the MACRS depreciation schedule. The depreciation each year is:

D1 = $535,000(.3333) = $178,315.50

D2 = $535,000(.4445) = $237,807.50

D3 = $535,000(.1481) = $79,233.50

D4 = $535,000(.0741) = $39,643.50

Using the tax shield approach, the OCF each year is:

OCF1 = (S – C)(1 – .35) + .35($178,315.50)

OCF2 = (S – C)(1 – .35) + .35($237,807.50)

OCF3 = (S – C)(1 – .35) + .35($79,233.50)

OCF4 = (S – C)(1 – .35) + .35($39,643.50)

OCF5 = (S – C)(1 – .35)

Now we need the aftertax salvage value of the equipment. The aftertax salvage value is:

Aftertax salvage value = $50,000(1 – .35)

Aftertax salvage value = $32,500

To find the necessary cost reduction, we must realize that we can split the cash flows each year. The OCF in any given year is the cost reduction (S – C) times one minus the tax rate, which is an annuity for the project life, and the depreciation tax shield. To calculate the necessary cost reduction, we would require a zero NPV. The equation for the NPV of the project is:

NPV = 0 = –$535,000 – 38,000 + (S – C)(.65)(PVIFA12%,5) + .35($178,315.50 / 1.12

+ $237,807.50 / 1.122 + $79,233.50 / 1.123 + $39,643.50 / 1.124)

+ ($38,000 + 32,500) / 1.125

Solving this equation for the sales minus costs, we get:

(S – C)(.65)(PVIFA12%,5) = $382,363.42

(S – C) = $163,186.67

**CHAPTER 10**

***SOME LESSONS FROM CAPITAL MARKET HISTORY***

**Answers to Concepts Review and Critical Thinking Questions**

**1.** They all wish they had! Since they didn’t, it must have been the case that the stellar performance was not foreseeable, at least not by most.

**2.** As in the previous question, it’s easy to see after the fact that the investment was terrible, but it probably wasn’t so easy ahead of time.

**3.** No, stocks are riskier. Some investors are highly risk averse, and the extra possible return doesn’t attract them relative to the extra risk.

**4.** On average, the only return that is earned is the required return—investors buy assets with returns in excess of the required return (positive NPV), bidding up the price and thus causing the return to fall to the required return (zero NPV); investors sell assets with returns less than the required return (negative NPV), driving the price lower and thus the causing the return to rise to the required return (zero NPV).

**5.** The market is not weak form efficient.

**6.** Yes, historical information is also public information; weak form efficiency is a subset of semi-strong form efficiency.

**7.** Ignoring trading costs, on average, such investors merely earn what the market offers; the trades all have zero NPV. If trading costs exist, then these investors lose by the amount of the costs.

**8.** Unlike gambling, the stock market is a positive sum game; everybody can win. Also, speculators provide liquidity to markets and thus help to promote efficiency.

**9.** The EMH only says, that within the bounds of increasingly strong assumptions about the information processing of investors, that assets are fairly priced. An implication of this is that, on average, the typical market participant cannot earn excessive profits from a particular trading strategy. However, that does not mean that a few particular investors cannot outperform the market over a particular investment horizon. Certain investors who do well for a period of time get a lot of attention from the financial press, but the scores of investors who do not do well over the same period of time generally get considerably less attention.

**10.** *a.* If the market is not weak form efficient, then this information could be acted on and a profit earned from following the price trend. Under (2), (3), and (4), this information is fully impounded in the current price and no abnormal profit opportunity exists.

*b.* Under (2), if the market is not semi-strong form efficient, then this information could be used to buy the stock “cheap” before the rest of the market discovers the financial statement anomaly.

Since (2) is stronger than (1), both imply that a profit opportunity exists; under (3) and (4), this information is fully impounded in the current price and no profit opportunity exists.

*c.* Under (3), if the market is not strong form efficient, then this information could be used as a profitable trading strategy, by noting the buying activity of the insiders as a signal that the stock is underpriced or that good news is imminent. Since (1) and (2) are weaker than (3), all three imply that a profit opportunity exists. Note that this assumes the individual who sees the insider trading is the only one who sees the trading. If the information about the trades made by company management is public information, under (3) it will be discounted in the stock price and no profit opportunity exists. Under (4), this information does not signal any profit opportunity for traders; any pertinent information the manager-insiders may have is fully reflected in the current share price.

**Solutions to Questions and Problems**

*NOTE: All end-of-chapter problems were solved using a spreadsheet. Many problems require multiple steps. Due to space and readability constraints, when these intermediate steps are included in this solutions manual, rounding may appear to have occurred. However, the final answer for each problem is found without rounding during any step in the problem.*

*Basic*

**1.** The return of any asset is the increase in price, plus any dividends or cash flows, all divided by the initial price. The return of this stock is:

*R* = [($85 – 72) + 1.65] / $72

*R* = .2035, or 20.35%

The dividend yield is the dividend divided by price at the initial period price, so:

Dividend yield = $1.65 / $72

Dividend yield = .0229, or 2.29%

And the capital gains yield is the increase in price divided by the initial price, so:

Capital gains yield = ($85 – 72) / $72

Capital gains yield = .1806, or 18.06%

**2.** Using the equation for total return, we find:

*R* = [($62 – 72) + 1.65] / $72

*R* = –.1160, or –11.60%

And the dividend yield and capital gains yield are:

Dividend yield = $1.65 / $72

Dividend yield = .0229, or 2.29%

Capital gains yield = ($62 – 72) / $72

Capital gains yield = –.1389, or –13.89%

Here’s a question for you: Can the dividend yield ever be negative? No, that would mean you were paying the company for the privilege of owning the stock, however, it has happened on bonds.

**3.** To calculate the dollar return, we multiply the number of shares owned by the change in price per share and the dividend per share received. The total dollar return is:

Dollar return = 250($94.86 – 87.25 + 1.15)

Dollar return = $2,190

**4.** The total dollar return is the change in price plus the coupon payment, so:

Total dollar return = $1,020 – 1,032 + 65

Total dollar return = $53

The nominal percentage return of the bond is:

*R* = [($1,020 – 1,032) + 65] / $1,032

*R* = .0514, or 5.14%

Notice here that we could have simply used the total dollar return of $53 in the numerator of this equation.

Using the Fisher equation, the real return was:

(1 + *R*) = (1 + *r*)(1 + *h*)

*r* = (1.0514 / 1.03) – 1

*r* = .0207, or 2.07%

**5.** The nominal return is the stated return, which is 12.10 percent. Using the Fisher equation, the real return was:

(1 + *R*) = (1 + *r*)(1 + *h*)

*r* = (1.1210) / (1.030) – 1

*r* = .0883, or 8.83%

**6.** Using the Fisher equation, the real returns for government and corporate bonds were:

(1 + *R*) = (1 + *r*)(1 + *h*)

*r*G = 1.061 / 1.030 – 1

*r*G = .0301, or 3.01%

*r*C = 1.064 / 1.030 – 1

*r*C = .0330, or 3.30%

**7.** The average return is the sum of the returns, divided by the number of returns. The average return for each stock was:





Remembering back to “sadistics,” we calculate the variance of each stock as:



The standard deviation is the square root of the variance, so the standard deviation of each stock is:

*σX* = .024871/2

σ*X* = .1577, or 15.77%

*σY* = .062031/2

*σY* = .2491, or 24.91%

**8.** We will calculate the sum of the returns for each asset and the observed risk premium first. Doing so, we get:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | Year | Large co. stock return | T-bill return | Risk premium |
|  | 1973 | –14.69% | 7.29% | –21.98% |
|  | 1974 | –26.47% | 7.99% | –34.46% |
|  | 1975 | 37.23% | 5.87% | 31.36% |
|  | 1976 | 23.93% | 5.07% | 18.86% |
|  | 1977 | –7.16% | 5.45% | –12.61% |
|  | 1978 | 6.57% | 7.64% | –1.07% |
|  | Total | 19.41% | 39.31% | –19.90% |

*a*. The average return for large-company stocks over this period was:

Large-company stock average return = 19.41% / 6

Large-company stock average return = 3.24%

And the average return for T-bills over this period was:

1. T-bills average return = 39.31% / 6
2. T-bills average return = 6.55%

*b*. Using the equation for variance, we find the variance for large-company stocks over this period was:

Variance = 1/5[(–.1469 – .0324)2 + (–.2647 – .0324)2 + (.3723 – .0324)2 + (.2393 – .0324)2 +

(–.0716 – .0324)2 + (.0657 – .0324)2]

Variance = .058136

And the standard deviation for large company stocks over this period was:

Standard deviation = (.058136)1/2

Standard deviation = .2411, or 24.11%

Using the equation for variance, we find the variance for T-bills over this period was:

Variance = 1/5[(.0729 – .0655)2 + (.0799 – .0655)2 + (.0587 – .0655)2 + (.0507 – .0655)2 +

(.0545 – .0655)2 + (.0764 – .0655)2]

Variance = .000153

And the standard deviation for T-bills over this period was:

Standard deviation = .0001531/2

Standard deviation = .0124, or 1.24%

*c*. The average observed risk premium over this period was:

Average observed risk premium = –19.90% / 6

Average observed risk premium = –3.32%

The variance of the observed risk premium was:

Variance = 1/5[(–.2198 – (–.0332))2 + (–.3446 – (–.0332))2 + (.3136 – (–.0332))2 +

(.1886 – (–.0332))2 + (–.1261 – (–.0332))2 + (–.0107 – (–.0332))2]

Variance = .062078

And the standard deviation of the observed risk premium was:

Standard deviation = .0620781/2

Standard deviation = .2492, or 24.92%

*d.* Before the fact, for most assets the risk premium will be positive; investors demand compensation over and above the risk-free return to invest their money in the risky asset. After the fact, the observed risk premium can be negative if the asset’s nominal return is unexpectedly low, the risk-free return is unexpectedly high, or if some combination of these two events occurs.

**9.** *a*. To find the average return, we sum all the returns and divide by the number of returns, so:

Arithmetic average return = (–.12 + .23 + .18 + .07 + .13) / 5

Arithmetic average return = .0980, or 9.80%

*b*. Using the equation to calculate variance, we find:

Variance = 1/4[(–.12 – .098)2 + (.23 – .098)2 + (.18 – .098)2 + (.07 – .098)2 +

(.13 – .098)2]

Variance =.01837

So, the standard deviation is:

Standard deviation = .018371/2

Standard deviation = .1355, or 13.55%

**10.** *a*. To calculate the average real return, we can use the average return of the asset, and the average inflation rate in the Fisher equation. Doing so, we find:

(1 + *R*) = (1 + *r*)(1 + *h*)

 = (1.0980 / 1.032) – 1

 = .0640, or 6.40%

*b*. The average risk premium is simply the average return of the asset, minus the average risk-free rate, so, the average risk premium for this asset would be:

– 

= .0980 – .0430

= .0550, or 5.50%

**11.** We can find the average real risk-free rate using the Fisher equation. The average real risk-free rate was:

(1 + *R*) = (1 + *r*)(1 + *h*)

= (1.043 / 1.032) – 1

= .0107, or 1.07%

And to calculate the average real risk premium, we can subtract the average risk-free rate from the average real return. So, the average real risk premium was:

 – = 6.40% – 1.07%

= 5.33%

**12.** T-bill rates were highest in the early eighties. This was during a period of high inflation and is consistent with the Fisher effect.

**13.**  To find the return on the zero-coupon bond, we first need to find the price of the bond today. We need to remember that the price for zero-coupon bonds is calculated with semiannual periods. Since one year has elapsed, the bond now has 19 years to maturity, so the price today is:

P1 = $1,000 / 1.02738

P1 = $363.35

There are no intermediate cash flows on a zero-coupon bond, so the return is the capital gain, or:

*R* = ($363.35 – 352.81) / $352.81

*R* = .0299, or 2.99%

**14.**  The return of any asset is the increase in price, plus any dividends or cash flows, all divided by the initial price. Since preferred stock is assumed to have a par value of $100, the dividend was $5.50, so the return for the year was:

*R* = ($102.67 – 104.18 + 5.50) / $104.18

*R* = .0383, or 3.83%

**15.** The return of any asset is the increase in price, plus any dividends or cash flows, all divided by the initial price. This stock paid no dividend, so the return was:

*R* = ($45.38 – 42.67) / $42.67

*R* = .0635, or 6.35%

This is the return for three months, so the APR is:

APR = 4(6.35%)

APR = 25.40%

And the EAR is:

EAR = (1 + .0635)4 – 1

EAR = .2793, or 27.93%

**16.** To find the real return each year, we will use the Fisher equation, which is:

1 + *R* = (1 + *r*)(1 + h)

Using this relationship for each year, we find:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  |  | T-bills | Inflation | Real Return |
|  | 1926 | .0330 | –.0112 | .0447 |
|  | 1927 | .0315 | –.0226 | .0554 |
|  | 1928 | .0405 | –.0116 | .0527 |
|  | 1929 | .0447 | .0058 | .0387 |
|  | 1930 | .0227 | –.0640 | .0926 |
|  | 1931 | .0115 | –.0932 | .1155 |
|  | 1932 | .0088 | –.1027 | .1243 |

So, the average real return was:

Average = (.0447 + .0554 + .0527 + .0387 + .0926 + .1155 + .1243) / 7

Average = .0748, or 7.48%

Notice that the real return was higher than the nominal return during this period because of deflation, or negative inflation.

**17.** Looking at the long-term corporate bond return history in Figure 10.10, we see that the mean return was 6.4 percent, with a standard deviation of 8.4 percent. The range of returns you would expect to see 68 percent of the time is the mean plus or minus 1 standard deviation, or:

*R*∈ μ ± 1σ = 6.4% ± 8.4% = –2.00% to 14.80%

The range of returns you would expect to see 95 percent of the time is the mean plus or minus 2 standard deviations, or:

*R*∈ μ ± 2σ = 6.4% ± 2(8.4%) = –10.40% to 23.20%

**18.**  Looking at the large-company stock return history in Figure 10.10, we see that the mean return was 12.1 percent, with a standard deviation of 20.1 percent. The range of returns you would expect to see 68 percent of the time is the mean plus or minus 1 standard deviation, or:

*R*∈ μ ± 1σ = 12.1% ± 20.1% = –8.00% to 32.20%

The range of returns you would expect to see 95 percent of the time is the mean plus or minus 2 standard deviations, or:

*R*∈ μ ± 2σ = 12.1% ± 2(20.1%) = –28.10% to 52.30%

*Intermediate*

**19.** Here, we know the average stock return, and four of the five returns used to compute the average return. We can work the average return equation backward to find the missing return. The average return is calculated as:

.10 = (.17 – .13 + .26 + .08 + *R*) / 5

.50 = .17 – .13 + .26 + .08 + *R*

*R* = .12, or 12%

The missing return has to be 12 percent. Now, we can use the equation for the variance to find:

Variance = 1/4[(.17 – .10)2 + (–.13 – .10)2 + (.26 – .10)2 + (.08 – .10)2 + (.12 – .10)2]

Variance = .02105

And the standard deviation is:

Standard deviation = .021051/2

Standard deviation = .1451, or 14.51%

**20.** The arithmetic average return is the sum of the known returns divided by the number of returns, so:

Arithmetic average return = (–.23 + .09 + .37 –.08 + .28 +.19) / 6

Arithmetic average return = .1033, or 10.33%

Using the equation for the geometric return, we find:

Geometric average return = [(1 + *R*1) × (1 + *R*2) × … × (1 + *RT*)]1/*T* – 1

Geometric average return =[(1 – .23)(1 + .09)(1 + .37)(1 – .08)(1 + .28)(1 + .19)](1/6) – 1

Geometric average return = .0828, or 8.28%

Remember that the geometric average return will always be less than the arithmetic average return if the returns have any variation.

**21.** To calculate the arithmetic and geometric average returns, we must first calculate the return for each year. The return for each year is:

*R*1 = ($67.32 – 58.27 + 1.10) / $58.27 = .1742, or 17.42%

*R*2 = ($61.46 – 67.32 + 1.25) / $67.32 = –.0685, or –6.85%

*R*3 = ($69.32 – 61.46 + 1.45) / $61.46 = .1515, or 15.15%

*R*4 = ($75.15 – 69.32 + 1.60) / $69.32 = .1076, or 10.76%

*R*5 = ($84.32 – 75.18 + 1.75) / $75.18 = .1449, or 14.49%

The arithmetic average return was:

*R*A = (.1742 – .0685 + .1515 + .1076 + .1449) / 5

*R*A = .1019, or 10.19%

And the geometric average return was:

*R*G = [(1 + .1742)(1 – .0685)(1 + .1515)(1 + .1076)(1 + .1449)]1/5 – 1

*R*G = .0982, or 9.82%

**22.** To find the real return, we need to use the Fisher equation. Re-writing the Fisher equation to solve for the real return, we get:

*r* = [(1 + *R*) / (1 + *h*)] – 1

So, the real return each year was:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | Year | T-bill return | Inflation | Real return |
|  | 1973 | .0729 | .0871 | –.0131 |
|  | 1974 | .0799 | .1234 | –.0387 |
|  | 1975 | .0587 | .0694 | –.0100 |
|  | 1976 | .0507 | .0486 | .0020 |
|  | 1977 | .0545 | .0670 | –.0117 |
|  | 1978 | .0764 | .0902 | –.0127 |
|  | 1979 | .1056 | .1329 | –.0241 |
|  | 1980 | .1210 | .1252 | –.0037 |
|  |  | .6197 | .7438 | –.1120 |

*a.* The average return for T-bills over this period was:

Average return = .6197 / 8

Average return = .0775, or 7.75%

And the average inflation rate was:

1. Average inflation = .7438 / 8
2. Average inflation = .0930, or 9.30%

*b.* Using the equation for variance, we find the variance for T-bills over this period was:

Variance = 1/7[(.0729 – .0775)2 + (.0799 – .0775)2 + (.0587 – .0775)2 + (.0507 – .0775)2

+ (.0545 – .0775)2 + (.0764 – .0775)2 + (.1056 – .0775)2 + (.1210 − .0775)2]

Variance = .000616

And the standard deviation for T-bills was:

Standard deviation = .0006161/2

Standard deviation = .0248, or 2.48%

The variance of inflation over this period was:

Variance = 1/7[(.0871 – .0930)2 + (.1234 – .0930)2 + (.0694 – .0930)2 + (.0486 – .0930)2

+ (.0670 – .0930)2 + (.0902 – .0930)2 + (.1329 – .0930)2 + (.1252 − .0930)2]

Variance = .000971

And the standard deviation of inflation was:

Standard deviation = .0009711/2

Standard deviation = .0312, or 3.12%

*c*. The average observed real return over this period was:

Average observed real return = –.1122 / 8

Average observed real return = –.0140, or –1.40%

*d*. The statement that T-bills have no risk refers to the fact that there is only an extremely small chance of the government defaulting, so there is little default risk. Since T-bills are short term, there is also very limited interest rate risk. However, as this example shows, there is inflation risk, i.e. the purchasing power of the investment can actually decline over time even if the investor is earning a positive nominal return.

**23.** To find the return on the coupon bond, we first need to find the price of the bond today, so:

P1 = $65(PVIFA5.2%,9) + $1,000 / 1.0529

P1 = $1,091.58

You received the coupon payments on the bond, so the nominal return was:

*R* = ($1,091.58 – 1,032.15 + 65) / $1,032.15

*R* = .1206, or 12.06%

And using the Fisher equation to find the real return, we get:

*r* = (1.1206 / 1.035) – 1

*r* = .0827, or 8.27%

**24.** Looking at the long-term government bond return history in Figure 10.10, we see that the mean return was 6.1 percent, with a standard deviation of 10.0 percent. In the normal probability distribution, approximately 2/3 of the observations are within one standard deviation of the mean. This means that 1/3 of the observations are outside one standard deviation away from the mean. Or:

Pr(*R*< –3.9 or *R* > 16.1) ≈ 1/3

But we are only interested in one tail here, that is, returns less than –3.9 percent, so:

Pr(*R*< –3.9) ≈ 1/6

You can use the z-statistic and the cumulative normal distribution table to find the answer as well. Doing so, we find:

z = (X – µ) / σ

z = (–3.9% – 6.1) / 10.0% = –1.00

Looking at the z-table, this gives a probability of 15.87%, or:

Pr(*R*< –3.9) ≈ .1587, or 15.87%

The range of returns you would expect to see 95 percent of the time is the mean plus or minus 2 standard deviations, or:

95% level: *R*∈ μ ± 2σ = 6.1% ± 2(10.0%) = –13.90% to 26.10%

The range of returns you would expect to see 99 percent of the time is the mean plus or minus 3 standard deviations, or:

99% level: *R*∈ μ ± 3σ = 6.1% ± 3(10.0%) = –23.90% to 36.10%

**25.** The mean return for small company stocks was 16.7 percent, with a standard deviation of 32.1 percent. Doubling your money is a 100% return, so if the return distribution is normal, we can use the z-statistic. So:

z = (X – µ) / σ

z = (100% – 16.7) / 32.1% = 2.595 standard deviations above the mean

This corresponds to a probability of ≈ .473%, or less than once every 100 years. Tripling your money would be:

z = (200% – 16.7) / 32.1% = 5.710 standard deviations above the mean.

This corresponds to a probability of (much) less than .5%, or once every 200 years. The actual answer is ≈.00000056%, or less than once every 1 million years.

**26.** It is impossible to lose more than 100 percent of your investment. Therefore, return distributions are truncated on the lower tail at –100 percent, and cannot truly follow a normal distribution.

*Challenge*

**27.** Using the z-statistic, we find:

z = (X – µ) / σ

z = (0% – 12.1) / 20.1% = –.602

Pr(*R*0) ≈ 27.36%

**28.** For each of the questions asked here, we need to use the z-statistic, which is:

z = (X – µ) / σ

*a*. z1 = (10% – 6.4) / 8.4% = .4286

This z-statistic gives us the probability that the return is less than 10 percent, but we are looking for the probability the return is greater than 10 percent. Given the symmetry of the normal distribution, and the fact that the total probability is 100 percent (or 1), the probability of a return greater than 10 percent is 1 minus the probability of a return less than 10 percent. Using the NORMDIST function, we get:

Pr(*R*10%) = 1 – Pr(*R*10%) = 1 – .6659 ≈ 33.41%

For a return less than 0 percent:

Pr(*R*≤0%) ≈ 22.31%

*b.* The probability that T-bill returns will be greater than 10 percent is:

z3 = (10% – 3.50) / 3.1% = 2.0968

Pr(*R*10%) = 1 – Pr(*R*10%) = 1 – .9820 ≈ 1.80%

And the probability that T-bill returns will be less than 0 percent is:

z4 = (0% – 3.5) / 3.1% = –1.1290

Pr(*R*0) ≈ 12.94%

*c*. The probability that the return on long-term corporate bonds will be less than –4.18 percent is:

z5 = (–4.18% – 6.4) / 8.4% = –1.2595

Pr(*R*–4.18%) ≈ 10.39%

And the probability that T-bill returns will be greater than 10.56 percent is:

z6 = (10.56% – 3.5) / 3.1% = 2.2774

Pr(*R*10.56%) = 1 – Pr(*R*10.56%) = 1 – .9886 ≈ 1.14%

**CHAPTER 11**

***RISK AND RETURN***

**Answers to Concepts Review and Critical Thinking Questions**

**1.** Some of the risk in holding any asset is unique to the asset in question. By investing in a variety of assets, this unsystematic portion of the total risk can be eliminated at little cost. On the other hand, there are systematic risks that affect all investments. This portion of the total risk of an asset cannot be costlessly eliminated. In other words, systematic risk can be controlled, but only by a costly reduction in expected returns.

**2.** If the market expected the growth rate in the coming year to be 2 percent, then there would be no change in security prices if this expectation had been fully anticipated and priced. However, if the market had been expecting a growth rate different than 2 percent and the expectation was incorporated into security prices, then the government’s announcement would most likely cause security prices in general to change; prices would typically drop if the anticipated growth rate had been more than 2 percent, and prices would typically rise if the anticipated growth rate had been less than 2 percent.

**3.** *a.* systematic

*b.* unsystematic

*c.* both; probably mostly systematic

*d.* unsystematic

*e.* unsystematic

*f.* systematic

**4.** *a.* This is a systematic risk; market prices in general will most likely decline.

*b.* This is a firm specific risk; the company price will most likely stay constant.

*c.* This is a systematic risk; market prices in general will most likely stay constant.

*d.* This is a firm specific risk; the company price will most likely decline.

*e.* This is a systematic risk; market prices in general will most likely stay constant.

**5.** No to both questions. The portfolio expected return is a weighted average of the asset returns, so it must be less than the largest asset return and greater than the smallest asset return.

**6.** False. The variance of the individual assets is a measure of the total risk. The variance and expected return on a well-diversified portfolio are functions of systematic risk only.

**7.** Yes, the standard deviation can be less than that of every asset in the portfolio. However, β cannot be less than the smallest beta because βP is a weighted average of the individual asset betas.

**8.** Yes. It is possible, in theory, to construct a zero beta portfolio of risky assets whose return would be equal to the risk-free rate. It is also possible to have a negative beta; the return would be less than the risk-free rate. A negative beta asset would carry a negative risk premium because of its value as a diversification instrument.

**9.** Such layoffs generally occur in the context of corporate restructurings. To the extent that the market views a restructuring as value-creating, stock prices will rise. So, it’s not the layoffs per se that are being cheered on but the cost savings associated with the layoffs. Nonetheless, Wall Street does encourage corporations to take actions to create value, even if such actions involve layoffs.

**10.** Earnings contain information about recent sales and costs. This information is useful for projecting future growth rates and cash flows. Thus, unexpectedly low earnings often lead market participants to reduce estimates of future growth rates and cash flows; lower prices are the result. The reverse is often true for unexpectedly high earnings.

**Solutions to Questions and Problems**

*NOTE: All end-of-chapter problems were solved using a spreadsheet. Many problems require multiple steps. Due to space and readability constraints, when these intermediate steps are included in this solutions manual, rounding may appear to have occurred. However, the final answer for each problem is found without rounding during any step in the problem.*

*Basic*

**1.** The portfolio weight of an asset is total investment in that asset divided by the total portfolio value. First, we will find the portfolio value, which is:

Total value = 165($69) + 125($44)

Total value = $16,885

The portfolio weight for each stock is:

*x*A = 165($69) / $16,885

*x*A = .6743

*x*B = 125($44) / $16,885

*x*B = .3257

**2.** The expected return of a portfolio is the sum of the weight of each asset times the expected return of each asset. The total value of the portfolio is:

Total value = $2,750 + 3,900   
Total value = $6,650

So, the expected return of this portfolio is:

E(*R*p) = ($2,750 / $6,650)(.09) + ($3,900 / $6,650)(.14)

E(*R*p) = .1193, or 11.93%

**3.** The expected return of a portfolio is the sum of the weight of each asset times the expected return of each asset. So, the expected return of the portfolio is:

E(*R*p) = .15(.10) + .45(.13) + .45(.15)

E(*R*p) = .1345, or 13.45%

**4.** Here, we are given the expected return of the portfolio and the expected return of each asset in the portfolio, and are asked to find the weight of each asset. We can use the equation for the expected return of a portfolio to solve this problem. Since the total weight of a portfolio must equal 1 (100%), the weight of Stock Y must be one minus the weight of Stock X. Mathematically speaking, this means:

E(*R*p) = .122 = .13*x*X + .095(1 – *x*X)

We can now solve this equation for the weight of Stock X as:

.122 = .13*x*X + .095 – .095*x*X

.027 = .035*x*X

*x*X =.7714

So, the dollar amount invested in Stock X is the weight of Stock X times the total portfolio value, or:

Investment in X = .7714($10,000) = $7,714.29

And the dollar amount invested in Stock Y is:

Investment in Y = (1 – .7714)($10,000) = $2,285.71

**5.** The expected return of an asset is the sum of the probability of each state occurring times the rate of return if that state occurs. So, the expected return of each asset is:

E(*R*) = .30(–.11) + .70(.21)

E(*R*) = .1140, or 11.40%

**6.** The expected return of an asset is the sum of the probability of each state occurring times the rate of return if that state occurs. So, the expected return of each asset is:

E(*R*) = .15(–.09) + .60(.11) + .25(.30)

E(*R*) = .1275, or 12.75%

**7.** The expected return of an asset is the sum of the probability of each state occurring times the rate of return if that state occurs. So, the expected return of each asset is:

E(*R*A) = .10(.02) + .50(.10) + .40(.15)

E(*R*A) = .1120, or 11.20%

E(*R*B) = .10(–.30) + .50(.18) + .40(.31)

E(*R*B) = .1840, or 18.40%

To calculate the standard deviation, we first need to calculate the variance. To find the variance, we find the squared deviations from the expected return. We then multiply each possible squared deviation by its probability, and then sum. The result is the variance. So, the variance and standard deviation of each stock is:

σA2 = .10(.02 – .1120)2 + .50(.10 – .1120)2 + .40(.15 – .1120)2

σA2 = .00150

σA = .001501/2

σA = .0387, or 3.87%

σB2 = .10(–.30 – .1840)2 + .50(.18 – .1840)2 + .40(.31 – .1840)2

σB2 = .02978

σB = .029781/2

σB = .1726, or 17.26%

**8.** The expected return of an asset is the sum of the probability of each return occurring times the probability of that return occurring. So, the expected return of the asset is:

E(*R*) = .20(.092) + .35(.111) + .45(.138)

E(*R*) = .1194, or 11.94%

**9.** *a.* To find the expected return of the portfolio, we need to find the return of the portfolio in each state of the economy. This portfolio is a special case since all three assets have the same weight. To find the expected return in an equally weighted portfolio, we can sum the returns of each asset and divide by the number of assets, so the expected return of the portfolio in each state of the economy is:

Boom: *R*p = (.15 + .02 + .34) / 3

*R*p = .17, or 17%

Bust: *R*p = (.03 + .16 − .08) / 3

*R*p = .0367, or 3.67%

This is equivalent to multiplying the weight of each asset (1/3, or .3333) times its expected return and summing the results, which gives:

Boom: *R*p = 1/3(.15) + 1/3(.02) + 1/3(.34)

*R*p = .17, or 17%

Bust: *R*p = 1/3(.03) + 1/3(.16) + 1/3(–.08)

*R*p = .0367, or 3.67%

To find the expected return of the portfolio, we multiply the return in each state of the economy by the probability of that state occurring, and then sum. Doing this, we find:

E(*R*p) = .60(.17) + .40(.0367)

E(*R*p) = .1167, or 11.67%

*b.* This portfolio does not have an equal weight in each asset. We still need to find the return of the portfolio in each state of the economy. To do this, we will multiply the return of each asset by its portfolio weight and then sum the products to get the portfolio return in each state of the economy. Doing so, we get:

Boom: *R*p = .20(.15) +.20(.02) + .60(.40)

*R*p = .2380, or 23.80%

Bust: *R*p = .20(.03) +.20(.16) + .60(−.08)

*R*p = –.0100, or –1.00%

And the expected return of the portfolio is:

E(*R*p) = .60(.2380) + .40(–.0100)

E(*R*p) = .1388, or 13.88%

To find the variance, we find the squared deviations from the expected return. We then multiply each possible squared deviation by its probability, and then sum. The result is the variance. So, the variance of the portfolio is:

σp2 = .60(.2380 – .1428)2 + .40(–.0100 – .1388)2

σp2 = .01476

**10.** *a.* This portfolio does not have an equal weight in each asset. We first need to find the return of the portfolio in each state of the economy. To do this, we will multiply the return of each asset by its portfolio weight and then sum the products to get the portfolio return in each state of the economy. Doing so, we get:

Boom: *R*p = .25(.35) + .50(.45) + .25(.33)

*R*p = .3950, or 39.50%

Good: *R*p = .25(.12) + .50(.10) + .25(.17)

*R*p = .1225, or 12.25%

Poor: *R*p = .25(.01) + .50(.02) + .25(–.05)

*R*p = .0000, or 0.00%

Bust: *R*p = .25(–.11) + .50(–.25) + .25(–.09)

*R*p = –.1750, or –17.50%

And the expected return of the portfolio is:

E(*R*p) = .15(.3950) + .50(.1225) + .25(.0000) + .10(–.1750)

E(*R*p) = .1030, or 10.30%

*b.* To calculate the standard deviation, we first need to calculate the variance. To find the variance, we find the squared deviations from the expected return. We then multiply each possible squared deviation by its probability, and then sum. The result is the variance. So, the variance and standard deviation of the portfolio is:

σp2 = .15(.3950 – .1030)2 + .50(.1225 – .1030)2 + .25(.0000 – .1030)2

+ .10(–.1750 – .1030)2

σp2 = .02336

σp = .023361/2

σP = .1528, or 15.28%

**11.** The beta of a portfolio is the sum of the weight of each asset times the beta of each asset. So, the beta of the portfolio is:

βp = .15(.75) + .25(.87) + .40(1.26) + .20(1.76)

βp = 1.19

**12.** The beta of a portfolio is the sum of the weight of each asset times the beta of each asset. If the portfolio is as risky as the market, it must have the same beta as the market. Since the beta of the market is one, we know the beta of our portfolio is one. We also need to remember that the beta of the risk-free asset is zero. It has to be zero since the asset has no risk. Setting up the equation for the beta of our portfolio, we get:

βp = 1.0 = 1/3(0) + 1/3(1.63) + 1/3(βX)

Solving for the beta of Stock X, we get:

βX = 1.37

**13.** The CAPM states the relationship between the risk of an asset and its expected return. The CAPM is:

E(*R*i) = *R*f + [E(*R*M) – *R*f] × βi

Substituting the values we are given, we find:

E(*R*i) = .036 + (.1090 – .036)(1.23)

E(*R*i) = .1258, or 12.58%

**14.** We are given the values for the CAPM except for the β of the stock. We need to substitute these values into the CAPM, and solve for the β of the stock. One important thing we need to realize is that we are given the market risk premium. The market risk premium is the expected return of the market minus the risk-free rate. We must be careful not to use this value as the expected return of the market. Using the CAPM, we find:

E(*R*i) = *R*f + [E(*R*M) – *R*f] × βi

.114 = .037 + .071βi

βi = 1.08

**15.** Here, we need to find the expected return of the market, using the CAPM. Substituting the values given, and solving for the expected return of the market, we find:

E(*R*i) = *R*f + [E(*R*M) – *R*f] × βi

.109 = .028 + [E(*R*M) – .028](.85)

E(*R*M) = .1233, or 12.33%

**16.** Here, we need to find the risk-free rate, using the CAPM. Substituting the values given, and solving for the risk-free rate, we find:

E(*R*i) = *R*f + [E(*R*M) – *R*f] × βi

.1070 = *R*f + (.1150 – *R*f)(.91)

.1070 = *R*f + .10465 – .91*R*f

*R*f = .0261, or 2.61%

**17.** *a.* Again, we have a special case where the portfolio is equally weighted, so we can sum the returns of each asset and divide by the number of assets. The expected return of the portfolio is:

E(*R*p) = (.117 + .035) / 2

E(*R*p) = .0760, or 7.60%

*b.* We need to find the portfolio weights that result in a portfolio with a β of .7. We know the β of the risk-free asset is zero. We also know the weight of the risk-free asset is one minus the weight of the stock since the portfolio weights must sum to one, or 100 percent. So:

βp = .70 = *x*S(1.23) + (1 – *x*S)(0)

.70 = 1.23*x*S + 0 – 0*x*S

*x*S = .70 / 1.23

*x*S = .5691

And, the weight of the risk-free asset is:

*xR*f = 1 – .5691

*xR*f = .4309

*c.* We need to find the portfolio weights that result in a portfolio with an expected return of 9 percent. We also know the weight of the risk-free asset is one minus the weight of the stock since the portfolio weights must sum to one, or 100 percent. So:

E(*R*p) = .09 = .117*x*S + .035(1 – *x*S)

.09 = .117*x*S + .035 – .035*x*S

*x*S = .6707

So, the β of the portfolio will be:

βp = .6707(1.23) + (1 – .6707)(0)

βp = .825

*d.* Solving for the β of the portfolio as we did in part *a*, we find:

βp = 2.46 = *x*S(1.23) + (1 – *x*S)(0)

*x*S = 2.46 / 1.23

*x*S = 2

*xR*f = 1 – 2

*xR*f = –1

The portfolio is invested 200% in the stock and –100% in the risk-free asset. This represents borrowing at the risk-free rate to buy more of the stock.

**18.** First, we need to find the β of the portfolio. The β of the risk-free asset is zero, and the weight of the risk-free asset is one minus the weight of the stock, so the β of the portfolio is:

ßp = *x*W(1.18) + (1 – *x*W)(0) = 1.18*x*W

So, to find the β of the portfolio for any weight of the stock, we simply multiply the weight of the stock times its β.

Even though we are solving for the β and expected return of a portfolio of one stock and the risk-free asset for different portfolio weights, we are really solving for the SML. Any combination of this stock, and the risk-free asset will fall on the SML. For that matter, a portfolio of any stock and the risk-free asset, or any portfolio of stocks, will fall on the SML. We know the slope of the SML line is the market risk premium, so using the CAPM and the information concerning this stock, the market risk premium is:

E(*R*W) = .114 = .0315 + MRP(1.18)

MRP = .0825 / 1.18

MRP = .0699, or 6.99%

So, now we know the CAPM equation for any stock is:

E(*R*p) = .0315 + .0699βp

The slope of the SML is equal to the market risk premium, which is .0699. Using these equations to fill in the table, we get the following results:

*x*W E(*R*p) ßp

0% .0315 0

25 .0521 .295

50 .0728 .590

75 .0934 .885

100 .1140 1.180

125 .1346 1.475

150 .1553 1.770

**19.** There are two ways to correctly answer this question. We will work through both. First, we can use the CAPM. Substituting in the value we are given for each stock, we find:

E(*R*Y) = .041 + .07(1.20)

E(*R*Y) = .1250, or 12.50%

It is given in the problem that the expected return of Stock Y is 11.6 percent, but according to the CAPM, the return of the stock based on its level of risk, the expected return should be 12.5 percent. This means the stock return is too low, given its level of risk. Stock Y plots below the SML and is overvalued. In other words, its price must decrease to increase the expected return to 12.5 percent.

For Stock Z, we find:

E(*R*Z) = .041 + .07(.85)

E(*R*Z) = .1005, or 10.05%

The return given for Stock Z is 10.3 percent, but according to the CAPM the expected return of the stock should be 10.05 percent based on its level of risk. Stock Z plots above the SML and is undervalued. In other words, its price must increase to decrease the expected return to 10.3 percent.

We can also answer this question using the reward-to-risk ratio. All assets must have the same reward-to-risk ratio. The reward-to-risk ratio is the risk premium of the asset divided by its β. We are given the market risk premium, and we know the β of the market is one, so the reward-to-risk ratio for the market is .07, or 7 percent. Calculating the reward-to-risk ratio for Stock Y, we find:

Reward-to-risk ratio Y = (.116 – .041) / 1.20

Reward-to-risk ratio Y = .0625

The reward-to-risk ratio for Stock Y is too low, which means the stock plots below the SML, and the stock is overvalued. Its price must decrease until its reward-to-risk ratio is equal to the market reward-to-risk ratio. For Stock Z, we find:

Reward-to-risk ratio Z = (.103 – .041) / .85

Reward-to-risk ratio Z = .0729

The reward-to-risk ratio for Stock Z is too high, which means the stock plots above the SML, and the stock is undervalued. Its price must increase until its reward-to-risk ratio is equal to the market reward-to-risk ratio.

**20.** We need to set the reward-to-risk ratios of the two assets equal to each other, which is:

(.116 – *R*f) / 1.20 = (.103 – *R*f) / .85

We can cross multiply to get:

.85(.116 – *R*f) = 1.20(.103 – *R*f)

Solving for the risk-free rate, we find:

.0986 – .85*R*f = .1236 – 1.20*R*f

*R*f = .0714, or 7.14%

**21.** For a portfolio that is equally invested in large-company stocks and long-term corporate bonds:

*R* = (12.10% + 6.40%) / 2

*R* = 9.25%

For a portfolio that is equally invested in small stocks and Treasury bills:

*R* = (16.70% + 3.50%) / 2

*R* = 10.10%

**22.** Here, we are given the expected return of the portfolio and the expected return of the assets in the portfolio and are asked to calculate the dollar amount of each asset in the portfolio. So, we need to find the weight of each asset in the portfolio. Since we know the total weight of the assets in the portfolio must equal 1 (or 100%), we can find the weight of each asset as:

E(*R*p) = .111 = .129*x*H + .0980(1 – *x*H)

*x*H = .4194

*x*L = 1 – *x*H

*x*L = 1 – .4194

*x*L = .5806

So, the dollar investment in each asset is the weight of the asset times the value of the portfolio, so the dollar investment in each asset must be:

Investment in H = .4194($250,000)

Investment in H = $104,838.71

Investment in L = .5806($250,000)

Investment in L = $145,161.29

**23.** To find the expected return of the portfolio we first need to find the weight of each asset in the portfolio. The weights of the assets sum to 1 (or 100%), so we can solve for the weights, using the betas of the each asset and the beta of the portfolio. Doing so, we find:

ßp = 1 = *x*J(1.19) + (1 – *x*J)(.84)

*x*J = .4571

*x*K = 1 – .4571

*x*K = .5429

So, the expected return of the portfolio is:

E(*R*P) = .4571(.1295) + .5429(.1040)

E(*R*P) = .1157, or 11.57%

**24.** To find the expected return of the portfolio, we first need to find the weight of each asset in the portfolio. The weight of each asset is the dollar investment of that asset divided by the total dollar value of the portfolio, so:

Portfolio value = 645($43) + 830($29) + 475($94) + 765($51)

Portfolio value = $135,470

And the weight of each asset in the portfolio is:

*x*W = 645($43) / $135,470

*x*W = .2047

*x*X = 830($29) / $135,470

*x*X = .1777

*x*Y = 475($94) / $135,470

*x*Y = .3296

*x*Z = 765($51) / $135,470

*x*Z = .2880

With the weight of each asset, we can find the expected return of the portfolio, which is:

E(*R*P) = .2047(.10) + .1777(.15) + .3296(.11) + .2880(.14)

E(*R*P) = .1237, or 12.37%

*Intermediate*

**25.** *a.* We need to find the return of the portfolio in each state of the economy. To do this, we will multiply the return of each asset by its portfolio weight and then sum the products to get the portfolio return in each state of the economy. Doing so, we get:

Boom: *R*p = .40(.02) + .40(.32) + .20(.60)

*R*p = .2560, or 25.60%

Normal: *R*p = .40(.10) + .40(.12) + .20(.20)

*R*p = .1280, or 12.80%

Bust: *R*p = .40(.16) + .40(–.11) + .20(–.35)

*R*p = –.0500, or –5.00%

And the expected return of the portfolio is:

E(*R*p) = .15(.2560) + .60(.1280) + .25(–.0500)

E(*R*p) = .1027, or 10.27%

To calculate the standard deviation, we first need to calculate the variance. To find the variance, we find the squared deviations from the expected return. We then multiply each possible squared deviation by its probability, and then sum. The result is the variance. So, the variance and standard deviation of the portfolio are:

σ2p = .15(.2560 – .1027)2 + .60(.1280 – .1027)2 + .25(–.0500 – .1027)2

σ2p = .00974

σp = .009741/2

σp = .0987, or 9.87%

*b.* The risk premium is the return of a risky asset, minus the risk-free rate. T-bills are often used as the risk-free rate, so:

RPi = E(*R*p) – *R*f

RPi = .1027 – .0375

RPi = .0652, or 6.52%

**26.** We know that the reward-to-risk ratios for all assets must be equal. This can be expressed as:

[E(*R*A) – *R*f] / βA = [E(*R*B) – *R*f] / ßB

The numerator of each equation is the risk premium of the asset, so:

RPA / βA = RPB / βB

We can rearrange this equation to get:

βB / βA = RPB / RPA

If the reward-to-risk ratios are the same, the ratio of the betas of the assets is equal to the ratio of the risk premiums of the assets.

**27.** We know the total portfolio value and the investment of two stocks in the portfolio, so we can find the weight of these two stocks. The weights of Stock A and Stock B are:

*x*A = $105,000 / $500,000

*x*A = .21

*x*B = $155,000 / $500,000

*x*B = .31

Since the portfolio is as risky as the market, the β of the portfolio must be equal to one. We also know the β of the risk-free asset is zero. We can use the equation for the β of a portfolio to find the weight of the third stock. Doing so, we find:

βp = 1 = *x*A(.80) + *x*B(1.15) + *x*C(1.40) + *xR*f(0)

1 = .21(.80) + .31(1.15) + *x*C(1.40)

Solving for the weight of Stock C, we find:

*x*C = .339643

So, the dollar investment in Stock C must be:

Invest in Stock C = .339643($500,000)

Invest in Stock C = $169,821.43

We also know the total portfolio weight must be one, so the weight of the risk-free asset must be one minus the asset weights we know, or:

1 = *x*A + *x*B + *x*C + *xR*f = 1 – .21 – .31 – .339643 = *xR*f

*xR*f = .140357

So, the dollar investment in the risk-free asset must be:

Invest in risk-free asset = .140357($500,000)

Invest in risk-free asset = $70,178.57

**28.** We know the expected return of the portfolio and of each asset, but only one portfolio weight. We need to recognize that the weight of the risk-free asset is one minus the weight of the other two assets. Mathematically, the expected return of the portfolio is:

E(*R*p) = .099 = .50(.128) + *x*F(.093) + (1 – .50 – *x*F)(.038)

.099 = .50(.128) + *x*F(.093) + .038 – .019 – .038*x*F

*x*F = .2909

So, the weight of the risk-free asset is:

*xR*f = 1 – .50 – .2909

*xR*f = .2091

And the amount of Stock F to buy is:

Amount of stock F to buy = .2909($100,000)

Amount of stock F to buy = $29,090.91

**29.** *a.* The expected return of an asset is the sum of the probability of each state occurring times the rate of return if that state occurs. So, the expected return of each asset is:

E(*R*A) = .10(–.12) + .65(.09) + .25(.35) = .1340, or 13.40%

E(*R*B) = .10(–.05) + .65(.10) + .25(.21) = .1125, or 11.25%

*b.* We can use the expected returns we calculated to find the slope of the Security Market Line. We know that the beta of Stock Ais .25 greater than the beta of Stock B. Therefore, as beta increases by .25, the expected return on a security increases by .0215 (= .1340 – .1125). The slope of the security market line (SML) equals:

SlopeSML = Rise / Run

SlopeSML = Increase in expected return / Increase in beta

SlopeSML = (.1340 – .1125) / .25

SlopeSML = .0860, or 8.60%

Since the market’s beta is 1 and the risk-free rate has a beta of zero, the slope of the Security Market Line equals the expected market risk premium. So, the expected market risk premium must be 8.6 percent.

We could also solve this problem using CAPM. The equations for the expected returns of the two stocks are:

E(*R*A) = .1340 = *R*f + (βB + .25)(MRP)

E(*R*B) = .1125 = *R*f + βB(MRP)

We can rewrite the CAPM equation for Stock A as:

.1340 = *R*f + βB(MRP) + .25(MRP)

Subtracting the CAPM equation for Stock B from this equation yields:

.0215 = .25MRP

MRP = .086, or 8.6%

which is the same answer as our previous result.

**30.** The amount of systematic risk is measured by the β of an asset. Since we know the market risk premium and the risk-free rate, if we know the expected return of the asset, we can use the CAPM to solve for the β of the asset. The expected return of Stock I is:

E(*R*I) = .25(.02) + .60(.32) + .15(.18)

E(*R*I) = .2240, or 22.40%

Using the CAPM to find the β of Stock I, we find:

.2240 = .04 + .07βI

βI = 2.63

The total risk of the asset is measured by its standard deviation, so we need to calculate the standard deviation of Stock I. Beginning with the calculation of the stock’s variance, we find:

σI2 = .25(.02 – .2240)2 + .60(.32 – .2240)2 + .15(.18 – .2240)2

σI2 = .01622

σI = .016221/2

σI = .1274, or 12.74%

Using the same procedure for Stock II, we find the expected return to be:

E(*R*II) = .25(–.20) + .60(.12) + .15(.40)

E(*R*II) = .0820, or 8.20%

Using the CAPM to find the β of Stock II, we find:

.0820 = .04 + .07βII

βII = .60

And the standard deviation of Stock II is:

σII2 = .25(–.20 – .0820)2 + .60(.12 – .0820)2 + .15(.40 – .0820)2

σII2 = .03592

σII = .035921/2

σII = .1895, or 18.95%

Although Stock II has more total risk than I, it has much less systematic risk, since its beta is much smaller than I’s. Thus, I has more systematic risk, and II has more unsystematic and more total risk. Since unsystematic risk can be diversified away, I is actually the “riskier” stock despite the lack of volatility in its returns. Stock I will have a higher risk premium and a greater expected return.

**CHAPTER 12**

***COST OF CAPITAL***

**Answers to Concepts Review and Critical Thinking Questions**

**1.** It is the minimum rate of return the firm must earn overall on its existing assets. If it earns more than this, value is created.

**2.** Book values for debt are likely to be much closer to their market values than are book values for equity.

**3.** No. The cost of capital depends on the risk of the project, not the source of the money.

**4.** Interest expense is tax-deductible. There is no difference between pretax and aftertax equity costs.

**5.** The primary advantage of the DGM model is its simplicity. The method is disadvantaged in that (1) the model is applicable only to firms that actually pay dividends; many do not; (2) even if a firm does pay dividends, the DGM model requires a constant dividend growth rate forever; (3) the estimated cost of equity from this method is very sensitive to changes in g, which is a very uncertain parameter; and (4) the model does not explicitly consider risk, although risk is implicitly considered to the extent that the market has impounded the relevant risk of the stock into its market price. While the share price and most recent dividend can be observed in the market, the dividend growth rate must be estimated. Two common methods of estimating g are to use analysts’ earnings and payout forecasts, or determine some appropriate average historical g from the firm’s available data.

**6.** Two primary advantages of the SML approach are that the model explicitly incorporates the relevant risk of the stock, and the method is more widely applicable than is the DGM model, since the SML doesn’t make any assumptions about the firm’s dividends. The primary disadvantages of the SML method are (1) estimating three parameters: the risk-free rate, the expected return on the market, and beta, and (2) the method essentially uses historical information to estimate these parameters. The risk-free rate is usually estimated to be the yield on very short maturity T-bills and is hence observable; the market risk premium is usually estimated from historical risk premiums and is not directly observable. The stock beta, which is unobservable, is usually estimated either by determining some average historical beta from the firm and the market’s return data, or using beta estimates provided by analysts and investment firms.

**7.** The appropriate aftertax cost of debt to the company is the interest rate it would have to pay if it were to issue new debt today. Hence, if the YTM on outstanding bonds of the company is observed, the company has an accurate estimate of its cost of debt. If the debt is privately placed, the firm could still estimate its cost of debt by (1) looking at the cost of debt for similar firms in similar risk classes, (2) looking at the average debt cost for firms with the same credit rating (assuming the firm’s private debt is rated), or (3) consulting analysts and investment bankers. Even if the debt is publicly traded, an additional complication is when the firm has more than one issue outstanding; these issues rarely have the same yield because no two issues are ever completely homogeneous.

**8.** *a.* This only considers the dividend yield component of the required return on equity.

*b.* This is the current yield only, not the promised yield to maturity. In addition, it is based on the book value of the liability, and it ignores taxes.

*c.* Equity is inherently riskier than debt (except, perhaps, in the unusual case where a firm’s assets have a negative beta). For this reason, the cost of equity exceeds the cost of debt. If taxes are considered in this case, it can be seen that at reasonable tax rates, the cost of equity does exceed the cost of debt.

**9.** *R*Superior = .12 + .75(.08) = .18, or 18%

Both should proceed. The appropriate discount rate does not depend on which company is investing; it depends on the risk of the project. Since Superior is in the business, it is closer to a pure play. Therefore, its cost of capital should be used. With an 18% cost of capital, the project has an NPV of $1 million regardless of who takes it.

**10.** If the different operating divisions were in much different risk classes, then separate cost of capital figures should be used for the different divisions; the use of a single, overall cost of capital would be inappropriate. If the single hurdle rate were used, riskier divisions would tend to receive funds for investment projects, since their return would exceed the hurdle rate despite the fact that they may actually plot below the SML and hence be unprofitable projects on a risk-adjusted basis. The typical problem encountered in estimating the cost of capital for a division is that it rarely has its own securities traded on the market, so it is difficult to observe the market’s valuation of the risk of the division. Two typical ways around this are to use a pure play proxy for the division, or to use subjective adjustments of the overall firm hurdle rate based on the perceived risk of the division.

**Solutions to Questions and Problems**

*NOTE: All end-of-chapter problems were solved using a spreadsheet. Many problems require multiple steps. Due to space and readability constraints, when these intermediate steps are included in this solutions manual, rounding may appear to have occurred. However, the final answer for each problem is found without rounding during any step in the problem.*

*Basic*

**1.** With the information given, we can find the cost of equity using the dividend growth model. Using this model, the cost of equity is:

*RE* = [$2.55(1.05) / $43] + .05

*RE* = .1123, or 11.23%

**2.** Here we have information to calculate the cost of equity, using the CAPM. The cost of equity is:

*RE* = .036 + 1.15(.11 – .036)

*RE* = .1211, or 12.11%

**3.** We have the information available to calculate the cost of equity, using the CAPM and the dividend growth model. Using the CAPM, we find:

*RE* = .035 + 1.10(.078)

*RE* = .1208, or 12.08%

And using the dividend growth model, the cost of equity is

*RE* = [$2.35(1.05) / $45] + .05

*RE* = .1048, or 10.48%

Both estimates of the cost of equity seem reasonable. If we remember the historical return on large capitalization stocks, the estimate from the CAPM model is about the same as the historical average, and the estimate from the dividend growth model is about 1.5 percent lower than the historical average, so we cannot definitively say one of the estimates is incorrect. Given this, we will use the average of the two, so:

*RE* = (.1208 + .1048) / 2

*RE* = .1128, or 11.28%

**4.** To use the dividend growth model, we first need to find the growth rate in dividends. So, the increase in dividends each year was:

*g*1 = ($1.33 – 1.24) / $1.24

*g*1 = .0726, or 7.26%

*g*2 = ($1.44 – 1.33) / $1.33

*g*2 = .0827, or 8.27%

*g*3 = ($1.53 – 1.44) / $1.44

*g*3 = .0625, or 6.25%

*g*4 = ($1.65 – 1.53) / $1.53

*g*4 = .0784, or 7.84%

So, the average arithmetic growth rate in dividends was:

*g* = (.0726 + .0827 + .0625 + .0784) / 4

*g* = .0741, or 7.41%

Using this growth rate in the dividend growth model, we find the cost of equity is:

*RE* = [$1.65(1.0741) / $55] + .0741

*RE* = .1063, or 10.63%

Calculating the geometric growth rate in dividends, we find:

$1.65 = $1.24(1 + *g*)4

*g* = .0740, or 7.40%

The cost of equity using the geometric dividend growth rate is:

*RE* = [$1.65(1.0740) / $55] + .0740

*RE* = .1062, or 10.62%

**5.** The cost of preferred stock is the dividend payment divided by the price, so:

*RP* = $4.20 / $93

*RP* = .0452, or 4.52%

**6.** The pretax cost of debt is the YTM of the company’s bonds, so:

P0 = $960 = $24.50(PVIFA*R*%,14) + $1,000(PVIF*R*%,14)

*R* = 2.800%

YTM = 2 × 2.800%

YTM = 5.60%

And the aftertax cost of debt is:

*RD* = .0560(1 – .38)

*RD*= .0347, or 3.47%

**7.** *a.* The pretax cost of debt is the YTM of the company’s bonds, so:

*P0* = $1,070 = $31.50(PVIFA*R*%,44) + $1,000(PVIF*R*%,44)

*R* = 2.868%

YTM = 2 × 2.868%

YTM = 5.74%

*b.* The aftertax cost of debt is:

*RD* = .0574(1 – .35)

*RD* = .0373, or 3.73%

*c.* The aftertax rate is more relevant because that is the actual cost to the company.

**8.** The book value of debt is the total par value of all outstanding debt, so:

BVD = $145,000,000 + 75,000,000

BVD = $220,000,000

To find the market value of debt, we find the price of the bonds and multiply by the number of bonds. Alternatively, we can multiply the price quote of the bond times the par value of the bonds. Doing so, we find:

MVD = 1.07($145,000,000) + .674($75,000,000)

MVD = $205,700,000

The YTM of the zero coupon bonds is (Remember, even on zero coupon bonds, for consistency, the “payments” are assumed to be semiannual):

*PZ* = $674 = $1,000(PVIF*R*%,20)

*R* = .02216, or 2.216%

Which means the YTM is:

YTM = 2.216% × 2

YTM = 4.43%

So, the aftertax cost of the zero coupon bonds is:

*RZ* = .0443(1 – .35)

*RZ* = .0288, or 2.88%

The aftertax cost of debt for the company is the weighted average of the aftertax cost of debt for all outstanding bond issues. We need to use the market value weights of the bonds. The total aftertax cost of debt for the company is:

*RD* = .0373[(1.07)($145,000,000) / $205,700,000] + .0288[(.674)($75,000,000) / $205,700,000]

*RD* = .0352, or 3.52%

**9.** *a.* Using the equation to calculate the WACC, we find:

WACC = .75(.1090) + .05(.051) + .20(.058)(1 – .35)

WACC = .0918, or 9.18%

*b.* Since interest is tax deductible and dividends are not, we must look at the aftertax cost of debt, which is:

*RD* = .058(1 – .35)

*RD* = .0377, or 3.77%

Hence, on an aftertax basis, debt is cheaper than the preferred stock.

**10.** Here, we need to use the debt–equity ratio to calculate the WACC. A debt–equity ratio of .45 implies a weight of debt of .45/1.45 and an equity weight of 1/1.45. Using this relationship, we find:

WACC = .103(1/1.45) + .064(.45/1.45)(1 – .35)

WACC = .0839, or 8.39%

**11.** Here, we have the WACC and need to find the debt–equity ratio of the company. Setting up the WACC equation, we find:

WACC = .0865 = .104(*E*/*V*) + .053(*D*/*V*)(1 – .35)

Rearranging the equation, we find:

.0865(*V*/*E*) = .104 + .053(.65)(*D*/*E*)

Now we must realize that the *V*/*E* is just the equity multiplier, which is equal to:

*V*/*E* = 1 + *D*/*E*

.0865(*D*/*E* + 1) = .104 + .03445(*D*/*E*)

Now, we can solve for *D*/*E* as:

.05205(*D*/*E*) = .0175

*D*/*E* = .3362

**12.** *a.* The book value of equity is the book value per share times the number of shares, and the book value of debt is the face value of the company’s debt, so:

BVE = 3,900,000($11) = $42,900,000

BVD = $65,000,000 + 50,000,000 = $115,000,000

So, the total value of the company is:

*V* = $42,900,000 + 115,000,000 = $157,900,000

And the book value weights of equity and debt are:

*E/V* = $42,900,000 / $157,900,000 = .2717

*D*/*V* = 1 – *E*/*V* = .7283

*b.* The market value of equity is the share price times the number of shares, so:

MVE = 3,900,000($84) = $327,600,000

Using the relationship that the total market value of debt is the price quote times the par value of the bond, we find the market value of debt is:

MVD = .98($65,000,000) + .97($50,000,000) = $112,200,000

This makes the total market value of the company:

*V* = $327,600,000 + 112,200,000 = $439,800,000

And the market value weights of equity and debt are:

*E*/*V* = $327,600,000 / $439,800,000 = .7449

*D*/*V* = 1 – *E*/*V* = .2551

*c.* The market value weights are more relevant because they represent a more current valuation of the debt and equity.

**13.** First, we will find the cost of equity for the company. The information provided allows us to solve for the cost of equity using the dividend growth model, so:

*RE* = [$3.85(1.05) / $84] + .05

*RE* = .0981, or 9.81%

Next, we need to find the YTM on both bond issues. Doing so, we find:

*P1* = $980 = $31.50(PVIFA*R*%,40) + $1,000(PVIF*R*%,40)

*R* = 3.240%

YTM = 3.240% × 2

YTM = 6.48%

*P2* = $970 = $29(PVIFA*R*%,24) + $1,000(PVIF*R*%,24)

*R* = 3.079%

YTM = 3.079% × 2

YTM = 6.16%

To find the weighted average aftertax cost of debt, we need the weight of each bond as a percentage of the total debt. We find:

*x*D1 = .98($65,000,000) / $112,200,000

*x*D1 = .5677

*x*D2 = .97($50,000,000) / $112,200,000

*x*D2 = .4323

Now we can multiply the weighted average cost of debt times one minus the tax rate to find the weighted average aftertax cost of debt. This gives us:

*RD* = (1 – .35)[(.5677)(.0648) + (.4323)(.0616)]

*RD* = .0412, or 4.12%

Using the costs we have found and the weight of debt we calculated earlier, the WACC is:

WACC = .7449(.0981) + .2551(.0412)

WACC = .0836, or 8.36%

**14.** *a.* Using the equation to calculate WACC, we find:

WACC = .081 = (1/1.85)(.11) + (.85/1.85)(1 – .35)*RD*

*RD* = .0721, or 7.21%

*b.* Using the equation to calculate WACC, we find:

WACC = .081 = (1/1.85)*RE* + (.85/1.85)(.038)

*RE* = .1176, or 11.76%

**15.** We will begin by finding the market value of each type of financing. We find:

MVD = 15,500($1,000)(1.08) = $16,740,000

MVE = 495,000($81) = $40,095,000

MVP = 20,000($92) = $1,840,000

And the total market value of the firm is:

*V* = $16,740,000 + 40,095,000 + 1,840,000

*V* = $58,675,000

Now, we can find the cost of equity using the CAPM. The cost of equity is:

*RE* = .031 + 1.20(.07)

*RE* = .1150, or 11.50%

The cost of debt is the YTM of the bonds, so:

*P0* = $1,080 = $32(PVIFA*R*%,50) + $1,000(PVIF*R*%,50)

*R* = 2.895%

YTM = 2.895% × 2

YTM = 5.79%

And the aftertax cost of debt is:

*RD* = (1 – .35)(.0579)

*RD* = .0376, or 3.76%

The cost of preferred stock is:

*RP* = $4.20 / $92

*RP* = .0457, or 4.57%

Now we have all of the components to calculate the WACC. The WACC is:

WACC = .0376($16,740,000 / $58,675,000) + .1150($40,095,000 / $58,675,000)

+ .0457($1,840,000 / $58,675,000)

WACC = .0908, or 9.08%

Notice that we didn’t include the (1 – *TC*) term in the WACC equation. We used the aftertax cost of debt in the equation, so the term is not needed here.

**16.** *a.* We will begin by finding the market value of each type of financing. We find:

MVD = 125,000($1,000)(1.09) = $136,250,000

MVE = 5,400,000($64) = $345,600,000

MVP = 290,000($103) = $29,870,000

And the total market value of the firm is:

*V* = $136,250,000 + 345,600,000 + 29,870,000

*V* = $511,720,000

So, the market value weights of the company’s financing are:

*D*/*V* = $136,250,000 / $511,720,000 = .2663

*P*/*V* = $29,870,000 / $511,720,000 = .0584

*E*/*V* = $345,600,000 / $511,720,000 = .6754

*b.* For projects equally as risky as the firm itself, the WACC should be used as the discount rate.

First, we can find the cost of equity using the CAPM. The cost of equity is:

*RE* = .043 + 1.13(.068)

*RE* = .1198, or 11.98%

The cost of debt is the YTM of the bonds, so:

P0 = $1,090 = $33.50(PVIFA*R*%,40) + $1,000(PVIF*R*%,30)

*R* = 2.963%

YTM = 2.963% × 2

YTM = 5.93%

And the aftertax cost of debt is:

*RD* = (1 – .34)(.0593)

*RD* = .0391, or 3.91%

The cost of preferred stock is:

*RP* = $5.60 / $103

*RP* = .0544, or 5.44%

Now, we can calculate the WACC as:

WACC = .2663(.0391) + .0584(.0544) + .6754(.1198)

WACC = .0945, or 9.45%

**17.** *a.* Projects Y and Z.

*b.* Using the CAPM to consider the projects, we need to calculate the expected return of the project, given its level of risk. This expected return should then be compared to the expected return of the project. If the return calculated using the CAPM is lower than the project expected return, we should accept the project; if not, we reject the project. After considering risk via the CAPM:

*E*(*W*) = .04 + .80(.11 – .04) = .0960 > .0930, so reject W

*E*(*X*) = .04 + .90(.11 – .04) = .1030 < .1060, so accept X

*E*(*Y*) = .04 + 1.10(.11 – .04) = .1170 > .1140, so reject Y

*E*(*Z*) = .04 + 1.35(.11 – .04) = .1345 < .1410, so accept Z

1. Project X would be incorrectly rejected; Project Y would be incorrectly accepted.

**18.** We will begin by finding the market value of each type of financing. We find:

MVD = 13,000($1,000)(1.07) = $13,910,000

MVE = 345,000($73.50) = $25,357,500

MVP = 10,000($86) = $860,000

And the total market value of the firm is:

*V* = $13,910,000 + 25,357,500 + 860,000

*V* = $40,127,500

Now, we can find the cost of equity using the CAPM. The cost of equity is:

*RE1* = .035 + .90(.12 – .035)

*RE1* = .1115, or 11.15%

We can also find the cost of equity, using the dividend discount model. The cost of equity with the dividend discount model is:

*RE2* = ($3.35 / $73.50) + .05

*RE2* = .0956, or 9.56%

Both estimates for the cost of equity seem reasonable, so we will use the average of the two. The cost of equity estimate is:

*RE* = (.1115 + .0956) / 2

*RE* = .1035, or 10.35%

The cost of debt is the YTM of the bonds, so:

*P0* = $1,070 = $31(PVIFAR%,30) + $1,000(PVIFR%,30)

*R* = 2.754%

YTM = 2.754% × 2

YTM = 5.51%

And the aftertax cost of debt is:

*RD* = (1 – .35)(.0551)

*RD* = .0358, or 3.58%

The cost of preferred stock is:

*RP* = $4.10 / $86

*RP* = .0477, or 4.77%

Now, we have all of the components to calculate the WACC. The WACC is:

WACC = .0358($13,910,000 / $40,127,500) + .0477($860,000 / $40,127,500)

+ .1035($25,357,500 / $40,127,500)

WACC = .0789, or 7.89%

**19.** The bonds have 26 years to maturity so the price today is:

*P0* = $1,000 / (1 + .059 / 2)52

*P0* = $220.51

The market value of the debt is:

MVD = 175,000($220.51)

MVD = $38,589,379.83

So, the total value of the firm is:

*V* = $38,589,379.83 + 68,000,000

*V* = $106,589,379.83

This means the weight of debt in the capital structure is:

*D*/*V* = $38,589,379.83 / $106,589,379.83

*D*/*V* = .3620

**20.** To find the required return for the project, we need to adjust the company’s WACC for the level of risk in the project. A debt–equity ratio of .48 implies a weight of debt of .48/1.48 and a weight of equity of 1/1.48, so the company’s WACC is:

WACC = (.48/1.48)(.0560) + (1/1.48)(.1180)

WACC = .0979, or 9.79%

Adjusting for risk, the project discount rate is:

Project discount rate = .0979 + .03

Project discount rate = .1279, or 12.79%

*Intermediate*

**21.** First, we need to find the project discount rate. The project discount rate is the company’s cost of capital plus a risk adjustment factor. A debt–equity ratio of .25 implies a weight of debt of .25/1.25 and a weight of equity of 1/1.25, so the company’s WACC is:

WACC = (.25/1.25)(.032) + (1/1.25)(.104)

WACC = .0896, or 8.96%

Adjusting for risk, the project discount rate is:

Project discount rate = .0896 + .02

Project discount rate = .1096, or 10.96%

The company should only accept the project if the NPV is zero (hopefully greater than zero.) The cash flows are a growing annuity. The present value of a growing annuity can be found with the dividend discount equation. So, the present value of the savings is:

PV = [$4,200,000 / (.1096 – .019)]

PV = $46,357,615.89

The project should only be undertaken if its cost is less than $46,357,615.89.

**22.** To find the aftertax cost of equity for the company, we need to find the weighted average of the four debt issues. We will begin by calculating the market value of each debt issue, which is:

MV1 = 1.05($30,000,000)

MV1 = $31,500,000

MV2 = .954($50,000,000)

MV2 = $47,700,000

MV3 = 1.038($65,000,000)

MV3 = $67,470,000

MV4 = 1.057($85,000,000)

MV4 = $89,845,000

So, the total market value of the company’s debt is:

MVD = $31,500,000 + 47,700,000 + 67,470,000 + 89,845,000

MVD = $236,515,000

The weight of each debt issue is:

*x*1 = $31,500,000 / $236,515,000

*x*1 = .1332, or 13.32%

*x*2 = $47,700,000 / $236,515,000

*x*2 = .2017, or 20.17%

*x*3 = $67,470,000 / $236,515,000

*w*3 = .2853, or 28.53%

*x*4 = $89,845,000 / $236,515,000

*x*4 = .3799, or 37.99%

Next, we need to find the YTM for each bond issue. The YTM for each issue is:

*P1* = $1,050 = $32.50(PVIFAR%,10) + $1,000(PVIFR%,10)

*R1* = .02674, or 2.674%

YTM1 = 2.674% × 2

YTM1 = 5.35%

*P2* = $954 = $26.50(PVIFAR%,16) + $1,000(PVIFR%,16)

*R2* = .03017, or 3.017%

YTM2 = 3.017% × 2

YTM2 = 6.03%

*P3* = $1,038 = $34.50(PVIFAR%,31) + $1,000(PVIFR%,31)

*R3* = .03254, or 3.254%

YTM3 = 3.254% × 2

YTM3 = 6.51%

*P4* = $1,057 = $36.50(PVIFAR%,50) + $1,000(PVIFR%,50)

*R4* = .03411, or 3.411%

YTM4 = 3.411% × 2

YTM4 = 6.82%

The weighted average YTM of the company’s debt is thus:

YTM = .1332(.0535) + .2017(.0603) + .2853(.0651) + .3799(.0682)

YTM = .0638, or 6.38%

And the aftertax cost of debt is:

*RD* = .0638(1 – .34)

*RD* = .0421, or 4.21%

**23.** *a.* Using the dividend discount model, the cost of equity is:

*RE* = [(1.05)(1.04) / $83] + .04

*RE* = .0532, or 5.32%

*b.* Using the CAPM, the cost of equity is:

*RE* = .038 + 1.08(.109 – .038)

*RE* = .1147, or 11.47%

*c.* When using the dividend growth model or the CAPM, you must remember that both are estimates for the cost of equity. Additionally, and perhaps more importantly, each method of estimating the cost of equity depends upon different assumptions.

**24.** First, we need to find the adjusted cash flow from assets (ACFA) for each year. We are given the projected EBIT, depreciation, increase in NWC, and capital spending. Each of these accounts increase at 18 percent per year. So, the ACFA for each of the next five years will be:

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  |  | *Year 1* | *Year 2* | *Year 3* | *Year 4* | *Year 5* |
|  | EBIT | $1,900,000 | $2,242,000 | $2,645,560 | $3,121,761 | $3,683,678 |
|  | Depreciation | 165,000 | 194,700 | 229,746 | 271,100 | 319,898 |
|  | Taxes\* | 665,000 | 784,700 | 925,946 | 1,092,616 | 1,289,287 |
|  | Capital spending | 115,000 | 135,700 | 160,126 | 188,949 | 222,959 |
|  | Change in NWC | 85,000 | 100,300 | 118,354 | 139,658 | 164,796 |
|  | ACFA | $1,200,000 | $1,416,000 | $1,670,880 | $1,971,638 | $2,326,533 |

The cash flows will grow at 3 percent in perpetuity, so the terminal value of the company in Year 5 will be:

Terminal value5 = ACFA6 / (WACC – *g*)

Terminal value5 = $2,326,533(1 + .03) / (.085 – .03)

Terminal value5 = $43,569,623.84

The value of the company today is the present value of the first five ACFAs, plus the value today of the terminal value, or:

Company value = $1,200,000 / 1.085 + $1,416,000 / 1.0852 + $1,670,880 / 1.0853

+ $1,971,638 / 1.0854 + ($2,326,533 + 43,569,623.84) / 1.0855

Company value = $35,562,677.78

To find the value of equity, we subtract the value of the debt from the total value of the company, which is:

Equity value = $35,562,677.78 – 13,000,000

Equity value = $22,562,677.78

Finally, the value per share is the total equity value divided by the shares outstanding, or:

Share price = $22,562,677.78 / 800,000

Share price = $28.20

**25.** The ACFA for each of the first five years will be the same as the previous problem. To calculate the terminal value, we can use the price–sales ratio, which will be:

Terminal value5 = 3.5($9,500,000)

Terminal value5 = $33,250,000

The value of the company today is the present value of the first five FCFs, plus the value today of the terminal value, or:

Company value = $1,200,000 / 1.085 + $1,416,000 / 1.0852 + $1,670,880 / 1.0853

+ $1,971,638 / 1.0854 + ($2,326,533 + 33,250,000) / 1.0855

Company value = $28,699,659.18

To find the value of equity, we subtract the value of the debt from the total value of the company, which is:

Equity value = $28,699,659.18 – 13,000,000

Equity value = $15,699,659.18

Finally, the value per share is the total equity value divided by the shares outstanding, or:

Share price = $15,699,659.18 / 800,000

Share price = $19.62

*Challenge*

**26.** First, we need to find the adjusted cash flow from assets (ACFA) for each year. At the growth rates given, the projected ACFA for each of the next five years will be:

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | *Year 1* | *Year 2* | *Year 3* | *Year 4* | *Year 5* |
| EBIT | $3,392,500 | $3,901,375 | $4,486,581 | $5,159,568 | $5,933,504 |
| Depreciation | 282,000 | 338,400 | 406,080 | 487,296 | 584,755 |
| Taxes\* | 1,187,375 | 1,365,481 | 1,570,303 | 1,805,849 | 2,076,726 |
| Capital spending | 570,000 | 684,000 | 820,800 | 984,960 | 1,181,952 |
| Change in NWC | 115,500 | 127,050 | 139,755 | 153,731 | 169,104 |
| ACFA | $1,801,625 | $2,063,244 | $2,361,803 | $2,702,325 | $3,090,477 |

The cash flows will grow at 3.5 percent in perpetuity, so the terminal value of the company in Year 5 will be:

Terminal value5 = ACFA6 / (WACC – *g*)

Terminal value5 = $3,090,477(1 + .035) / (.0925 – .035)

Terminal value5 = $55,628,587.03

The value of the company today is the present value of the first five ACFAs, plus the value today of the terminal value, or:

Company value = $1,801,625 / 1.0925 + $2,063,244 / 1.09252 + $2,361,803 / 1.09253

+ $2,702,325 / 1.09254 + ($3,090,477 + 55,628,587.03) / 1.09255

Company value = $44,814,627.66

To find the value of equity, we subtract the value of the debt from the total value of the company, which is:

Equity value = $44,814,627.66 – 19,500,000

Equity value = $25,314,627.66

Finally, the value per share is the total equity value divided by the shares outstanding, or:

Share price = $25,314,627.66 / 400,000

Share price = $63.29

**27.** We can use the debt–equity ratio to calculate the weights of equity and debt. The debt of the company has a weight for long-term debt and a weight for accounts payable. We can use the weight given for accounts payable to calculate the weight of accounts payable and the weight of long-term debt. The weight of each will be:

Accounts payable weight = .15/1.15 = .1304

Long-term debt weight = 1/1.15 = .8696

Since the accounts payable has the same cost as the overall WACC, we can write the equation for the WACC as:

WACC = (1/1.45)(.15) + (.45/1.45)[(.1304)WACC + (.8696)(.07)(1 – .35)]

Solving for WACC, we find:

WACC = .1034 + .3103[(.1304)WACC + .0396]

WACC = .1034 + (.0405)WACC + .0123

(.9595)WACC = .1157

WACC = .1206, or 12.06%

Since the cash flows go to perpetuity, we can calculate the present value using the equation for the PV of a perpetuity. The NPV is:

NPV = –$37,000,000 + ($5,100,000 / .1206)

NPV = $5,285,270.11

**28.** *a.* The $4.5 million cost of the land 3 years ago is a sunk cost and irrelevant; the $5 million appraised value of the land is an opportunity cost and is relevant. So, the total initial cash flow is:

CF0 = –$5,000,000 – 19,500,000 – 825,000

CF0 = –$25,325,000

*b.* To find the required return for the project, we need to adjust the company’s WACC for the level of risk in the project. We begin by calculating the market value of each type of financing, so:

MVD = 60,000($1,000)(.95) = $57,000,000

MVE = 1,250,000($97) = $121,250,000

MVP = 90,000($95) = $8,550,000

The total market value of the company is:

*V* = $57,000,000 + 121,250,000 + 8,550,000

*V* = $186,800,000

Next, we need to find the cost of funds. We have the information available to calculate the cost of equity using the CAPM, so:

*RE* = .0380 + 1.15(.07)

*RE* = .1185, or 11.85%

The cost of debt is the YTM of the company’s outstanding bonds, so:

*P0* = $950 = $31(PVIFA*R*%,50) + $1,000(PVIF*R*%,50)

*R* = .03306, or 3.306%

YTM = 3.306% × 2

YTM = 6.61%

And the aftertax cost of debt is:

*RD* = (1 – .34)(.0661)

*RD* = .0436, or 4.36%

The cost of preferred stock is:

*RP* = $5.80 / $95

*RP* = .0611, or 6.11%

So, the company’s WACC is:

WACC = .0436($57,000,000 / $186,800,000) + .0611($8,550,000 / $186,800,000)

+ .1185($121,250,000 / $186,800,000)

WACC = .0930, or 9.30%

The company wants to use the subjective approach to this project because it is located overseas. The adjustment factor is 2 percent, so the required return on this project is:

Project required return = .0930 + .02

Project required return = .1130, or 11.30%

*c.* The annual depreciation for the equipment will be:

$19,500,000 / 8 = $2,437,500

So, the book value of the equipment at the end of five years will be:

BV5 = $19,500,000 – 5($2,437,500)

BV5 = $7,312,500

So, the aftertax salvage value will be:

Aftertax salvage value = $2,100,000 + .34($7,312,500 – 2,100,000)

Aftertax salvage value = $3,872,250

*d.* Using the tax shield approach, the OCF for this project is:

OCF = [(P – v)Q – FC](1 – *TC*) + *TCD*

OCF = [($10,800 – 9,900)(13,000) – 3,500,000](1 – .34) + .34($19,500,000/8)

OCF = $6,240,750

*e.* We have calculated all cash flows of the project. We just need to make sure that in Year 5 we add back the aftertax salvage value, the recovery of the initial NWC, and the aftertax value of the land in five years since it will be an opportunity cost. So, the cash flows for the project are:

*Year*  *Flow Cash*

0 –$25,325,000

1 6,240,750

2 6,240,750

3 6,240,750

4 6,240,750

5 16,238,000

Using the required return of 11.30 percent, the NPV of the project is:

NPV = –$25,325,000 + $6,240,750(PVIFA11.30%,4) + $16,238,000 / 1.11305

NPV = $3,418,282.75

And the IRR is:

NPV = 0 = –$25,325,000 + $6,240,750(PVIFAIRR%,4) + $16,238,000 / (1 + IRR)5

IRR = 15.81%

**CHAPTER 13**

***LEVERAGE AND CAPITAL STRUCTURE***

**Answers to Concepts Review and Critical Thinking Questions**

**1.** Business risk is the equity risk arising from the nature of the firm’s operating activity, and is directly related to the systematic risk of the firm’s assets. Financial risk is the equity risk that is due entirely to the firm’s chosen capital structure. As financial leverage, or the use of debt financing, increases, so does financial risk and hence the overall risk of the equity. Thus, Firm B could have a higher cost of equity if it uses greater leverage.

**2.** No, it doesn’t follow. While it is true that the equity and debt costs are rising, the key thing to remember is that the cost of debt is still less than the cost of equity. Since we are using more and more debt, the WACC does not necessarily rise.

**3.** Because many relevant factors such as bankruptcy costs, tax asymmetries, and agency costs cannot easily be identified or quantified, it’s practically impossible to determine the precise debt–equity ratio that maximizes the value of the firm. However, if the firm’s cost of new debt suddenly becomes much more expensive, it’s probably true that the firm is too highly leveraged.

**4.** The more capital intensive industries, such as airlines, cable television, and electric utilities, tend to use greater financial leverage. Also, industries with less predictable future earnings, such as computers or drugs, tend to use less. Such industries also have a higher concentration of growth and startup firms. Overall, the general tendency is for firms with identifiable, tangible assets and relatively more predictable future earnings to use more debt financing. These are typically the firms with the greatest need for external financing and the greatest likelihood of benefiting from the interest tax shelter.

**5.** It’s called leverage (or “gearing” in the UK) because it magnifies gains or losses.

**6.** Homemade leverage refers to the use of borrowing on the personal level as opposed to the corporate level.

**7.** One answer is that the right to file for bankruptcy is a valuable asset, and the financial manager acts in shareholders’ best interest by managing this asset in ways that maximizes its value. To the extent that a bankruptcy filing prevents “a race to the courthouse steps,” it would seem to be a reasonable alternative to complicated and expensive litigation.

**8.** As in the previous question, it could be argued that using bankruptcy laws as a sword may simply be the best use of the asset. Creditors are aware at the time a loan is made of the possibility of bankruptcy, and the interest charged incorporates this possibility.

**9.** One side is that Continental was going to go bankrupt because its costs made it uncompetitive. The bankruptcy filing enabled Continental to restructure and keep flying. The other side is that Continental abused the bankruptcy code. Rather than renegotiate labor agreements, Continental simply abrogated them to the detriment of its employees. It is important thing to keep in mind that the bankruptcy code is a creation of law, not economics. A strong argument can always be made that making the best use of the bankruptcy code is no different from, for example, minimizing taxes by making best use of the tax code. Indeed, a strong case can be made that it is the financial manager’s duty to do so. As the case of Continental illustrates, the code can be changed if socially undesirable outcomes are a problem.

**10.** As with any management decision, the goal is to maximize the value of shareholder equity. To accomplish this with respect to the capital structure decision, management attempts to choose the capital structure with the lowest cost of capital.

**Solutions to Questions and Problems**

*NOTE: All end-of-chapter problems were solved using a spreadsheet. Many problems require multiple steps. Due to space and readability constraints, when these intermediate steps are included in this solutions manual, rounding may appear to have occurred. However, the final answer for each problem is found without rounding during any step in the problem.*

*Basic*

**1.** *a.* A table outlining the income statement for the three possible states of the economy is shown below. The EPS is the net income divided by the 7,350 shares outstanding. The last row shows the percentage change in EPS the company will experience in a recession or an expansion economy.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  |  |  | Recession | Normal | Expansion |
|  |  | EBIT | $8,970 | $13,800 | $16,560 |
|  |  | Interest | 0 | 0 | 0 |
|  |  | NI | $8,970 | $13,800 | $16,560 |
|  |  | EPS | $1.22 | $1.88 | $2.25 |
|  |  | %ΔEPS | –35% | – | +20% |

*b.* If the company undergoes the proposed recapitalization, it will repurchase:

Share price = Equity / Shares outstanding

Share price = $194,775 / 7,350

Share price = $26.50

Shares repurchased = Debt issued / Share price

Shares repurchased = $39,750 / $26.50

Shares repurchased = 1,500

The interest payment each year under all three scenarios will be:

Interest payment = $39,750(.06)

Interest payment = $2,385

The last row shows the percentage change in EPS the company will experience in a recession or an expansion economy under the proposed recapitalization.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  |  |  | Recession | Normal | Expansion |
|  |  | EBIT | $8,970 | $13,800 | $16,560 |
|  |  | Interest | 2,385 | 2,385 | 2,385 |
|  |  | NI | $6,585 | $11,415 | $14,175 |
|  |  | EPS | $1.13 | $1.95 | $2.42 |
|  |  | %ΔEPS | –42.31% | – | +24.18% |

**2.** *a.* A table outlining the income statement with taxes for the three possible states of the economy is shown below. The share price is still $26.50, and there are still 7,350 shares outstanding. The last row shows the percentage change in EPS the company will experience in a recession or an expansion economy.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  |  |  | Recession | Normal | Expansion |
|  |  | EBIT | $8,970 | $13,800 | $16,560 |
|  |  | Interest | 0 | 0 | 0 |
|  |  | Taxes | 3,140 | 4,830 | 5,796 |
|  |  | NI | $5,831 | $8,970 | $10,764 |
|  |  | EPS | $.79 | $1.22 | $1.46 |
|  |  | %ΔEPS | –35% | – | +20% |

*b.* A table outlining the income statement with taxes for the three possible states of the economy and assuming the company undertakes the proposed capitalization is shown below. The interest payment and shares repurchased are the same as in part *b* of Problem 1.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  |  |  | Recession | Normal | Expansion |
|  |  | EBIT | $8,970 | $13,800 | $16,560 |
|  |  | Interest | 2,385 | 2,385 | 2,385 |
|  |  | Taxes | 2,305 | 3,995 | 4,961 |
|  |  | NI | $4,280 | $7,420 | $9,214 |
|  |  | EPS | $.73 | $1.27 | $1.58 |
|  |  | %ΔEPS | –42.31% | – | +24.18 |

Notice that the percentage change in EPS is the same both with and without taxes

**3.** *a.* Since the company has a market-to-book ratio of 1.0, the total equity of the firm is equal to the market value of equity. Using the equation for ROE:

ROE = NI / $194,775

The ROE for each state of the economy under the current capital structure and no taxes is:

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  |  |  | Recession | Normal | Expansion |
|  |  | ROE | 4.61% | 7.09% | 8.50% |
|  |  | %ΔROE | –35% | – | +20% |

The second row shows the percentage change in ROE from the normal economy.

*b.* If the company undertakes the proposed recapitalization, the new equity value will be:

Equity = $194,775 – 39,750

Equity = $155,025

So, the ROE for each state of the economy is:

ROE = NI / $155,025

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  |  |  | Recession | Normal | Expansion |
|  |  | ROE | 4.25% | 7.36% | 9.14% |
|  |  | %ΔROE | –42.31% | – | +24.18% |

*c.* If there are corporate taxes and the company maintains its current capital structure, the ROE is:

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  |  | ROE | 2.99% | 4.61% | 5.53% |
|  |  | %ΔROE | –35% | – | +20% |

If the company undertakes the proposed recapitalization, and there are corporate taxes, the ROE for each state of the economy is:

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  |  | ROE | 2.76% | 4.79% | 5.94% |
|  |  | %ΔROE | –42.31% | – | +24.18% |

Notice that the percentage change in ROE is the same as the percentage change in EPS. The percentage change in ROE is also the same with or without taxes.

**4.** *a.* Under Plan I, the unlevered company, net income is the same as EBIT with no corporate tax. The EPS under this capitalization will be:

EPS = $600,000 / 300,000 shares

EPS = $2.00

Under Plan II, the levered company, net income will be reduced by the interest payment. The interest payment is the amount of debt times the interest rate, so:

NI = $600,000 – .10($2,367,000)

NI = $363,300

And the EPS will be:

EPS = $363,300 / 210,000 shares

EPS = $1.73

Plan I has the higher EPS when EBIT is $600,000.

*b.* Under Plan I, the net income is $900,000 and the EPS is:

EPS = $900,000 / 300,000 shares

EPS = $3.00

Under Plan II, the net income is:

NI = $900,000 – .10($2,367,000)

NI = $663,300

And the EPS is:

EPS = $663,300 / 210,000 shares

EPS = $3.16

Plan II has the higher EPS when EBIT is $900,000.

*c.* To find the breakeven EBIT for two different capital structures, we simply set the equations for EPS equal to each other and solve for EBIT. The breakeven EBIT is:

EBIT / 300,000 = [EBIT – .10($2,367,000)] / 210,000

EBIT = $789,000

**5.** We can find the price per share by dividing the amount of debt used to repurchase shares by the number of shares repurchased. Doing so, we find the share price is:

Share price = $2,367,000 / (300,000 – 210,000)

Share price = $26.30 per share

The value of the company under the all-equity plan is:

*V* = $26.30(300,000 shares)

*V* = $7,890,000

And the value of the company under the levered plan is:

*V* = $26.30(210,000 shares) + $2,367,000 debt

*V* = $7,890,000

**6.** *a.* The income statement for each capitalization plan is:

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  |  |  | *I* | *II* | *All-equity* |
|  |  | EBIT | $68,000 | $68,000 | $68,000 |
|  |  | Interest | 49,400 | 26,000 | \_\_ 0 |
|  |  | NI | $18,600 | $ 42,000 | $68,000 |
|  |  | EPS | $1.62 | $2.63 | $3.24 |

The all-equity plan has the highest EPS; Plan I has the lowest EPS.

*b.* The breakeven level of EBIT occurs when the capitalization plans result in the same EPS. The EPS is calculated as:

EPS = (EBIT – *RDD*) / Shares outstanding

This equation calculates the interest payment (*RDD*) and subtracts it from the EBIT, which results in the net income. Dividing by the shares outstanding gives us the EPS. For the all-equity capital structure, the interest term is zero. To find the breakeven EBIT for two different capital structures, we simply set the equations equal to each other and solve for EBIT. The breakeven EBIT between the all-equity capital structure and Plan I is:

EBIT / 21,000 = [EBIT – .10($494,000)] / 11,500

EBIT = $109,200

And the breakeven EBIT between the all-equity capital structure and Plan II is:

EBIT / 21,000 = [EBIT – .10($260,000)] / 16,000

EBIT = $109,200

The break-even levels of EBIT are the same because of M&M Proposition I.

*c.* Setting the equations for EPS from Plan I and Plan II equal to each other and solving for EBIT, we get:

[EBIT – .10($494,000)] / 11,500 = [EBIT – .10($260,000)] / 16,000

EBIT = $109,200

This break-even level of EBIT is the same as in part *b* again because of M&M Proposition I.

*d.* The income statement for each capitalization plan with corporate income taxes is:

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  |  |  | *I* | *II* | *All-equity* |
|  |  | EBIT | $68,000 | $68,000 | $68,000 |
|  |  | Interest | 49,400 | 26,000 | 0 |
|  |  | Taxes | 6,510 | 14,700 | 23,800 |
|  |  | NI | $12,090 | $27,300 | $44,200 |
|  |  | EPS | $1.05 | $1.71 | $2.10 |

The all-equity plan still has the highest EPS; Plan I still has the lowest EPS.

We can calculate the EPS as:

EPS = [(EBIT – *RDD*)(1 – *TC*)] / Shares outstanding

This is similar to the equation we used before, except now we need to account for taxes. Again, the interest expense term is zero in the all-equity capital structure. So, the breakeven EBIT between the all-equity plan and Plan I is:

EBIT(1 – .35) / 21,000 = [EBIT – .10($494,000)](1 – .35) / 11,500

EBIT = $109,200

The breakeven EBIT between the all-equity plan and Plan II is:

EBIT(1 – .35) / 21,000 = [EBIT – .10($260,000)](1 – .35) / 16,000

EBIT = $109,200

And the breakeven between Plan I and Plan II is:

[EBIT – .10($494,000)](1 – .35) / 11,500 = [EBIT – .10($260,000)](1 – .35) / 16,000

EBIT = $109,200

The break-even levels of EBIT do not change because the addition of taxes reduces the income of all three plans by the same percentage; therefore, they do not change relative to one another.

**7.** To find the value per share of the stock under each capitalization plan, we can calculate the price as the value of shares repurchased divided by the number of shares repurchased. So, under Plan I, the value per share is:

*P* = $494,000 / (21,000 shares – 11,500 shares)

*P* = $52 per share

And under Plan II, the value per share is:

*P* = $260,000 / (21,000 shares – 16,000 shares)

*P* = $52 per share

This shows that when there are no corporate taxes, the stockholder does not care about the capital structure decision of the firm. This is M&M Proposition I without taxes.

**8.** *a.* The earnings per share are:

EPS = $19,300 / 6,400 shares

EPS = $3.02

Since all earnings are paid as dividends, the cash flow for the investors will be:

Cash flow = $3.02(100 shares)

Cash flow = $301.56

*b.* To determine the cash flow to the shareholder, we need to determine the EPS of the firm under the proposed capital structure. The market value of the firm is:

*V* = $55(6,400)

*V* = $352,000

Under the proposed capital structure, the firm will raise new debt in the amount of:

*D* = .30($352,000)

*D* = $105,600

This means the number of shares repurchased will be:

Shares repurchased = $105,600 / $55

Shares repurchased = 1,920

Under the new capital structure, the company will have to make an interest payment on the new debt. The net income with the interest payment will be:

NI = $19,300 – .08($105,600)

NI = $10,852

This means the EPS under the new capital structure will be:

EPS = $10,852 / (6,400 – 1,920)

EPS = $2.42

Since all earnings are paid as dividends, the shareholder will receive:

Shareholder cash flow = $2.42(100 shares)

Shareholder cash flow = $242.23

*c.* To replicate the original capital structure, the shareholder should sell 30 percent of her shares, or 30 shares, and lend the proceeds at 8 percent. The shareholder will have an interest cash flow of:

Interest cash flow = 30($55)(.08)

Interest cash flow = $132.00

The shareholder will receive dividend payments on the remaining 70 shares, so the dividends received will be:

Dividends received = $2.42(70 shares)

Dividends received = $169.56

The total cash flow for the shareholder under these assumptions will be:

Total cash flow = $132 + 169.56

Total cash flow = $301.56

This is the same cash flow we calculated in part *a*.

*d.* The capital structure is irrelevant because shareholders can create their own leverage or unlever the stock to create the payoff they desire, regardless of the capital structure the firm actually chooses.

**9.** *a.* The total value of the company is the share price times the number of shares, so:

*V* = $60(6,400)

*V* = $384,000

The investor will receive dividends in proportion to the percentage of the company’s shares they own. The total dividends received by the shareholder will be:

Dividends received = $47,000($18,000 / $384,000)

Dividends received = $2,203.13

*b.* Under the proposed capital structure, the firm will raise new debt in the amount of:

*D* = .25($384,000)

*D* = $96,000

This means the number of shares repurchased will be:

Shares repurchased = $96,000 / $60

Shares repurchased = 1,600

Under the new capital structure, the company will have to make an interest payment on the new debt. The net income with the interest payment will be:

NI = $47,000 – .07($96,000)

NI = $40,280

This means the EPS under the new capital structure will be:

EPS = $40,280 / (6,400 – 1,600)

EPS = $8.39

The number of shares owned by the shareholder is the dollar amount invested divided by the share price, so:

Shares owned = $18,000 / $60

Shares owned = 300

Since all earnings are paid as dividends, the shareholder will receive:

Shareholder cash flow = $8.39(300 shares)

Shareholder cash flow = $2,517.50

*c.* To replicate the proposed capital structure, the shareholder should sell 25 percent of their shares, or 75 shares, and lend the proceeds at 7 percent. The shareholder will have an interest cash flow of:

Interest cash flow = 75($60)(.07)

Interest cash flow = $315

The shareholder will receive dividend payments on the remaining 225 shares, so the dividends received will be:

Dividends received = $8.39(225 shares)

Dividends received = $1,888.13

The total cash flow for the shareholder under these assumptions will be:

Total cash flow = $315 + 1,888.13

Total cash flow = $2,203.13

This is the same cash flow we calculated in part *a*.

*d.* The capital structure is irrelevant because shareholders can create their own leverage or unlever the stock to create the payoff they desire, regardless of the capital structure the firm actually chooses.

**10.** *a.* With the information provided, we can use the equation for calculating WACC to find the cost of equity. The equation for WACC (assuming no taxes) is:

WACC = (*E*/*V*)*RE* + (*D*/*V*)*RD*

The company has a debt–equity ratio of 1.5, which implies the weight of debt is 1.5/2.5, and the weight of equity is 1/2.5, so:

WACC = .091 = (1/2.5)*RE* + (1.5/2.5)(.055)

*RE* = .1450, or 14.50%

*b.* To find the cost of equity under different capital structures, we can again use the WACC equation. With a debt–equity ratio of 2, the cost of equity is:

.091 = (1/3)*RE* + (2/3)(.055)

*RE* = .1630, or 16.30%

With a debt–equity ratio of .5, the cost of equity is:

.091 = (1/1.5)*RE* + (.5/1.5)(.055)

RE = .1090, or 10.90%

And with a debt–equity ratio of 0, the cost of equity is:

.091 = (1)*RE* + (0)(.055)

*RE* = WACC = .0910, or 9.10%

**11.** *a*. For an all-equity financed company:

WACC = *RE* = .0960, or 9.60%

*b.* To find the cost of equity for the company with leverage, we need to use M&M Proposition I with no taxes, so:

*RE* = *RA* + (*RA* – *RD*)(*D*/*E*)

*RE* = .0960 + (.0960 – .059)(.30/.70)

*RE* = .1119, or 11.19%

*c.* Using M&M Proposition I with no taxes again, we get:

*RE* = *RA* + (*RA* – *RD*)(*D*/*E*)

*RE* = .0960 + (.0960 – .059)(.60/.40)

*RE* = .1515, or 15.15%

*d.* The WACC with 30 percent debt is:

WACC = (*E*/*V*)*RE* + (*D*/*V*)*RD*

WACC = .70(.1119) + .30(.059)

WACC = .0960, or 9.60%

And the WACC with 60 percent debt is:

WACC = (*E*/*V*)*RE* + (*D*/*V*)*RD*

WACC = .40(.1515) + .60(.059)

WACC = .0960, or 9.60%

**12.** Using M&M Proposition I with taxes, the value of the levered firm is:

*VL* = *VU* + *TCD*

*VL* = $595,000 + .35($310,000)

*VL* = $703,500

**13.** The interest tax shield is the total interest paid times the tax rate, so:

Interest tax shield = Interest paid(*TC*)

Interest tax shield = $13,800,000(.38)

Interest tax shield = $5,244,000

The interest tax shield represents the tax savings in current income due to the deductibility of a firm’s qualified debt expenses.

*Intermediate*

**14.** M&M Proposition I with no taxes states the value of the levered firm is equal to the value of the unlevered firm. So, with no taxes, the value of the firm if it issues debt is:

*VU* = *VL* = $53,000,000

With corporate taxes, we need to use M&M Proposition I with taxes, so the value of the firm is:

*VL* = *VU* + *TCD*

*VL* = $53,000,000 + .40($19,400,000)

*VL* = $60,760,000

**15.** The value of the firm is the value of the debt plus the value of the equity. We can use this relationship to find the value of equity in each case. So, the debt–equity ratio with no taxes is:

*V* = *E* + *D*

$33,600,000 = *E* + 19,400,000

*E* = $29,000,000

So, the debt–equity ratio is:

Debt–equity ratio = $19,400,000 / $33,600,000

Debt–equity ratio = .58

With taxes, the debt–equity ratio becomes:

*V* = *E* + *D*

$60,760,000 = *E* + 19,400,000

*E* = $41,360,000

So, the debt–equity ratio is:

Debt–equity ratio = $19,400,000 / $41,360,000

Debt–equity ratio = .47

**16.** When the company is all-equity financed, the cost of equity is:

WACC = *RE* = .0940, or 9.40%

Using M&M Proposition I with no taxes, the cost of equity will be:

*RE* = *RA* + (*RA* – *RD*)(*D*/*E*)

*RE* = .094 + (.094 – .061)(1)

*RE* = .1270, or 12.70%

And the new WACC will be:

WACC = (*E*/*V*)*RE* + (*D*/*V*)*RD*

WACC = (.50).1270 + (.50).0610

WACC = .0940, or 9.40%

*Challenge*

**17.** *a.* With no debt, we are finding the value of an unlevered firm, so:

*VU* = EBIT(1 – *TC*) / *R*A

*VU* = $22,300(1 – .35) / .15

*VU* = $96,633.33

*b.* The general expression for the value of a leveraged firm is:

*VL*= *VU* + *TCD*

If debt is 50 percent of *VU*, then *D* = (.50)*VU*, and we have:

*VL*= *VU* + *TC*[(.50)*VU*]

*VL* = $96,633.33 + .35(.50)($96,633.33)

*VL* = $113,544.17

And if debt is 100 percent of *VU*, then *D* = (1)*VU*, and we have:

*VL*= *VU*  + *TC*[(1)*VU*]

*VL* = $96,633.33 + .35(1.0)($96,633.33)

*VL* = $130,455.00

*c.* According to M&M Proposition I with taxes:

*VL*= *VU*+ *TCD*

With debt being 50 percent of the value of the levered firm, *D* must equal (.50)*VL*, so:

*VL* = *VU* + *TC*[(.50)*VL*]

*VL* = $96,633.33 + .35(.50)(*VL*)

*VL* = $117,131.31

If the debt is 100 percent of the levered value, *D* must equal *VL*, so:

*VL* = *VU + TC*[(1)(*VL*]

*VL =* $96,633.33 + .35(1.0)(*VL*)

*VL* = $148,666.67

**18.** The return on equity is net income divided by equity. Net income can be expressed as:

NI = (EBIT – *RDD*)(1 – *TC*)

So, ROE is:

*RE* = (EBIT – *RDD*)(1 – *TC*)/E

Now we can rearrange and substitute as follows to arrive at M&M Proposition II with taxes:

*RE* = [EBIT(1 – *TC*)/*E*] – [*RD*(*D*/*E*)(1 – *TC*)]

*RE* = *RAVU*/*E* – [*RD*(*D*/*E*)(1 – *TC*)]

*RE* = *RA*(*VL* – *TCD*)/*E* – [*RD*(*D*/*E*)(1 – *TC*)]

*RE* = *RA*(*E* + *D* – *TCD*)/*E* – [*RD*(*D*/*E*)(1 – *TC*)]

*RE* = *RA* + (*RA* – *RD*)(*D*/*E*)(1 – *TC*)

And the equation for WACC is:

WACC = (*E*/*V*)*RE* + (*D*/*V*)*RD*(1 – *TC*)

Substituting the M&M Proposition II equation into the equation for WACC, we get:

WACC = (*E*/*V*)[*RA* + (*RA* – *RD*)(*D*/*E*)(1 – *TC*)] + (*D*/*V*)*RD*(1 – *TC*)

Rearranging and reducing the equation, we get:

WACC = *RA*[(*E*/*V*) + (*E*/*V*)(*D*/*E*)(1 – *TC*)] + *RD*(1 – *TC*)[(*D*/*V*) – (*E*/*V*)(*D*/*E*)]

WACC = *RA*[(*E*/*V*) + (*D*/*V*)(1 – *TC*)]

WACC = *RA*[{(*E* + *D*)/*V*} – *TC*(*D*/*V*)]

WACC = *RA*[1 – *TC*(*D*/*V*)]

Thus, with a debt-to-value ratio of 1, the WACC is *RA*(1 – *TC*). This result makes intuitive sense. It says that the cost of capital is the same as it would be for an all equity firm, but with the tax benefit of debt.

To find the value of the firm, M&M Proposition I with taxes states:

*VL* = *VU* + *TCD*

Since the firm is entirely financed by debt, the value of the firm must be equal to the amount of debt, so:

*VL* = *D*

Substituting, we get:

*D* = *VU* + *TCD*

*D* – *TCD* = *VU*

*D*(1 – *TC*) = *VU*

*D* = *VU* / (1 – *TC*)

*VL* = *VU* / (1 – *TC*)

Again, this result makes intuitive sense. The value of the firm is equal to its all-equity value grossed up by the tax benefit of debt.

**19.** The return on equity is net income divided by equity. Net income can be expressed as:

NI = (EBIT – *RDD*)(1 – *TC*)

So, ROE is:

*RE* = (EBIT – *RDD*)(1 – *TC*)/*E*

Now we can rearrange and substitute as follows:

*RE* = [EBIT(1 – *TC*)/*E*] – [*RD*(*D*/*E*)(1 – *TC*)]

If we assume that EBIT and *TC* are constant, then we can treat the unlevered firm as a perpetuity whose per-period cash flows are EBIT(1 – *TC*). Then:

VU = EBIT(1 – *TC*)/*RA*

Substituting, we get:

*RE* = *RAVU*/*E* – [*RD*(*D*/*E*)(1 – *TC*)]

Note that we’ve implicitly assumed, with M&M, that *RA* is independent of capital structure, so that *RA* of the unlevered firm can be treated as the required return on assets of the leveraged firm.

Recalling, from M&M Proposition I with taxes that *VL* = *VU* + *TCD*, rearranging, and substituting, we get:

*RE* = *RA*(*VL* – *TCD*)/*E* – [*RD*(*D*/*E*)(1 – *TC*)]

*RE* = *RA*(*E* + *D* – *TCD*)/*E* – [*RD*(*D*/*E*)(1 – *TC*)]

*RE* = *RA* + (*RA* – *RD*)(*D*/*E*)(1 – *TC*)

This result is known as M&M Proposition II with taxes.

**20.** M&M Proposition II, with no taxes is:

*RE* = *RA* + (*RA* – *Rf*)(*D*/*E*)

Note that we assumed the return on debt was the risk-free rate. This is an important assumption of M&M Proposition II. The CAPM to calculate the cost of equity is expressed as:

*RE* = βE(*RM* – *Rf*) + *Rf*

We can rewrite the CAPM to express the return on an unlevered company as:

*RA* = βA(*RM* – *Rf*) + *Rf*

We can now substitute the CAPM for an unlevered company into M&M Proposition II. Doing so and rearranging the terms we get:

*RE* = βA(*RM* – *Rf*) + *Rf* + [βA(*RM* – *Rf*) + *Rf* – *Rf*](*D*/*E*)

*RE* = βA(*RM* – *Rf*) + *Rf* + [βA(*RM* – *Rf*)](*D*/*E*)

*RE* = (1 + *D*/*E*)βA(*RM* – *Rf*) + *Rf*

Now we set this equation equal to the CAPM equation to calculate the cost of equity and reduce:

βE(*RM* – *Rf*) + *Rf* = (1 + *D*/*E*)βA(*RM* – *Rf*) + *Rf*

βE(*RM* – *Rf*) = (1 + D/E)βA(*RM* – *Rf*)

βE = βA(1 + *D*/*E*)

**CHAPTER 14**

***DIVIDENDS AND DIVIDEND POLICY***

**Answers to Concepts Review and Critical Thinking Questions**

**1.** Dividend policy deals with the timing of dividend payments, not the amounts ultimately paid. Dividend policy is irrelevant when the timing of dividend payments doesn’t affect the present value of all future dividends.

**2.** A stock repurchase reduces equity while leaving debt unchanged. The debt ratio rises. A firm could, if desired, use excess cash to reduce debt instead. This is a capital structure decision.

**3.** First, relatively young and less profitable firms generally should not make cash distributions. They need the cash to fund investments (and flotation costs discourage the raising of outside cash). However, as a firm matures, it begins to generate free cash flow (which, you will recall, is internally generated cash flow beyond that needed to fund profitable investment activities). Significant free cash flow can lead to agency problems if it is not distributed. Managers may become tempted to pursue empire building or otherwise spend the excess cash in ways not in the shareholders’ best interests. Thus, firms come under pressure to make distributions rather than hoard cash. And, consistent with what we observe, we expect large firms with a history of profitability to make large distributions. Thus, the life cycle theory says that firms trade off the agency costs of excess cash retention against the potential future costs of external equity financing. A firm should begin making distributions when it generates sufficient internal cash flow to fund its investment needs now and into the foreseeable future.

**4.** Friday, December 29 is the ex-dividend day. Remember not to count January 1 because it is a holiday, and the exchanges are closed. Anyone who buys the stock before December 29 is entitled to the dividend, assuming they do not sell it again before December 29.

**5.** No, because the money could be better invested in stocks that pay dividends in cash that will benefit the fundholders directly.

**6.** The change in price is due to the change in dividends, not to the change in dividend *policy*. Dividend policy can still be irrelevant without a contradiction.

**7.** The stock price dropped because of an expected drop in future dividends. Since the stock price is the present value of all future dividend payments, if the expected future dividend payments decrease, then the stock price will decline.

**8.**  The plan will probably have little effect on shareholder wealth. The shareholders can reinvest on their own, and the shareholders must pay the taxes on the dividends either way. However, the shareholders who take the option may benefit at the expense of the ones who don’t (because of the discount). Also as a result of the plan, the firm will be able to raise equity by paying a 10% flotation cost (the discount), which may be a smaller discount than the market flotation costs of a new issue.

**9.** If these firms just went public, they probably did so because they were growing and needed the additional capital. Growth firms typically pay very small cash dividends, if they pay a dividend at all. This is because they have numerous projects, and therefore reinvest the earnings in the firm instead of paying cash dividends.

**10.** It would not be irrational to find low-dividend, high-growth stocks. The trust should be indifferent between receiving dividends or capital gains since it does not pay taxes on either one (ignoring possible restrictions on invasion of principal, etc.). It would be irrational, however, to hold municipal bonds. Since the trust does not pay taxes on the interest income it receives, it does not need the tax break associated with the municipal bonds. Therefore, it should prefer to hold higher yielding, taxable bonds.

**Solutions to Questions and Problems**

*Basic*

*NOTE: All end-of-chapter problems were solved using a spreadsheet. Many problems require multiple steps. Due to space and readability constraints, when these intermediate steps are included in this solutions manual, rounding may appear to have occurred. However, the final answer for each problem is found without rounding during any step in the problem.*

**1.** With no taxes we would expect the stock price to drop by exactly the amount of the dividend, so the new stock price will be:

New stock price = $87 – 1.32

New stock price = $85.68

Your total stock investment will be worth:

Stock value = 150 × $85.68

Stock value = $12,852

**2.** Your total portfolio value will be the total stock value plus the dividends received, so:

Portfolio value = $12,852 + (150 × $1.32)

Portfolio value = $13,050

Technically speaking, the dividends will not arrive in your portfolio until the payment date.

**3.** The aftertax dividend is the pretax dividend times one minus the tax rate, so:

Aftertax dividend = $7.40(1 – .15)

Aftertax dividend = $6.29

The stock price should drop by the aftertax dividend amount, or:

Ex-dividend price = $92 – 6.29

Ex-dividend price = $85.71

**4.** *a.* The shares outstanding increases by 10 percent, so:

New shares outstanding = 35,000(1.10)

New shares outstanding = 38,500

New shares issued = 3,500

Since the par value of the new shares is $1, the capital surplus per share is $41. The additional capital surplus is therefore:

Capital surplus on new shares = 3,500($41)

Capital surplus on new shares = $143,500

The new equity account balances will be:

|  |  |  |
| --- | --- | --- |
|  | Common stock ($1 par value) | $ 38,500 |
|  | Capital surplus | 292,500 |
|  | Retained earnings | 418,000 |
|  |  | $749,000 |

*b.* The shares outstanding increases by 25 percent, so:

New shares outstanding = 35,000(1.25)

New shares outstanding = 43,750

New shares issued = 8,750

Since the par value of the new shares is $1, the capital surplus per share is $41. The additional capital surplus is therefore:

Capital surplus on new shares = 8,750($41)

Capital surplus on new shares = $358,750

The new equity account balances will be:

|  |  |  |
| --- | --- | --- |
|  | Common stock ($1 par value) | $ 43,750 |
|  | Capital surplus | 507,750 |
|  | Retained earnings | 197,500 |
|  |  | $749,000 |

**5.** *a.* To find the new shares outstanding, we multiply the current shares outstanding times the ratio of old shares to new shares, so:

New shares outstanding = 35,000(2/1)

New shares outstanding = 70,000

The equity accounts are unchanged except that the par value of the stock is changed by the ratio of new shares to old shares, so the new par value is:

New par value = $1(1/2)

New par value = $.50 per share

*b.* To find the new shares outstanding, we multiply the current shares outstanding times the ratio of new shares to old shares, so:

New shares outstanding = 35,000(1/5)

New shares outstanding = 7,000

The equity accounts are unchanged except the par value of the stock is changed by the ratio of new shares to old shares, so the new par value is:

New par value = $1(5/1)

New par value = $5.00 per share

**6.** To find the new stock price, we multiply the current stock price by the ratio of old shares to new shares, so:

*a.* $86(3/5) = $51.60

*b.* $86(1/1.15) = $74.78

*c.* $86(1/1.425) = $60.35

*d.* $86(7/4) = $150.50

*e.* To find the new shares outstanding, we multiply the current shares outstanding times the ratio of new shares to old shares, so:

*a:* 465,000(5/3) = 775,000

*b:* 465,000(1.15) = 534,750

*c:* 465,000(1.425) = 662,625

*d:* 465,000(4/7) = 265,714

**7.** The stock price is the total market value of equity divided by the shares outstanding, so:

*P0* = $724,500 equity / 23,000 shares

*P0* = $31.50 per share

Ignoring tax effects, the stock price will drop by the amount of the dividend, so:

*PX* = $31.50 – 1.35

*PX* = $30.15

The total dividends paid will be:

$1.35 per share(23,000 shares) = $31,050

The equity and cash accounts will both decline by $31,050. The new balance sheet will be:

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | *Market Value Balance Sheet* | | | | |
|  | Cash | $128,950 |  | Equity | $693,450 |
|  | Fixed assets | 564,500 |  |  |  |
|  | Total | $693,450 |  | Total | $693,450 |

**8.** Repurchasing the shares will reduce shareholders’ equity by $31,050. The shares repurchased will be the total purchase amount divided by the stock price, so:

Shares bought = $31,050 / $31.50

Shares bought = 986

And the new shares outstanding will be:

New shares outstanding = 23,000 – 986

New shares outstanding = 22,014

After repurchase, the new stock price is:

Share price = ($724,500 – 31,050) / 22,014 shares

Share price = $31.50

The repurchase is effectively the same as the cash dividend because you either hold a share worth $31.50, or a share worth $30.15 and $1.35 in cash. Therefore, if you participate in the repurchase according to the dividend payout percentage, you are unaffected.

**9.** The stock price is the total market value of equity divided by the shares outstanding, so:

*P0* = $650,000 equity / 24,000 shares

*P0* = $27.08 per share

The shares outstanding will increase by 20 percent, so:

New shares outstanding = 24,000(1.20)

New shares outstanding = 28,800

The new stock price is the market value of equity divided by the new shares outstanding, so:

*PX* = $650,000 / 28,800 shares

*PX* = $22.57

**10.** With a stock dividend, the shares outstanding will increase by one plus the dividend percentage, so:

New shares outstanding = 245,000(1.10)

New shares outstanding = 269,500

The capital surplus is the capital paid in excess of par value, which is $1, so:

Capital surplus for new shares = 24,500($52)

Capital surplus for new shares = $1,274,000

The new capital surplus will be the old capital surplus plus the additional capital surplus for the new shares, so:

Capital surplus = $1,280,000 + 1,274,000

Capital surplus = $2,554,000

The new equity portion of the balance sheet will look like this:

|  |  |  |
| --- | --- | --- |
|  | Common stock ($1 par value) | $ 269,500 |
|  | Capital surplus | 2,554,000 |
|  | Retained earnings | 1,663,500 |
|  |  | $4,457,000 |

**11.** The only equity account that will be affected is the par value of the stock. The par value will change by the ratio of old shares to new shares, so:

New par value = $1(1/2)

New par value = $.50 per share

The total dividends paid this year will be the dividend amount times the number of shares outstanding. The company had 245,000 shares outstanding before the split. We must remember to adjust the shares outstanding for the stock split, so:

Total dividends paid this year = $.73(245,000 shares)(2/1 split)

Total dividends paid this year = $357,700

The dividends increased by 10 percent, so the total dividends paid last year were:

Last year’s dividends = $357,700 / 1.10

Last year’s dividends = $325,181.82

And to find the dividends per share, we simply divide this amount by the shares outstanding last year. Doing so, we get:

Dividends per share last year = $325,181.82 / 245,000 shares

Dividends per share last year = $1.33

*Intermediate*

**12.** *a.* If the company makes a dividend payment, we can calculate the wealth of a shareholder as:

Dividend per share = $9,065 / 4,900 shares

Dividend per share = $1.85

The stock price after the dividend payment will be:

*PX* = $59 – 1.85

*PX* = $57.15 per share

The shareholder will have a stock worth $57.15 and a $1.85 dividend for a total wealth of $59. If the company makes a repurchase, the company will repurchase:

Shares repurchased = $9,065 / $59

Shares repurchased = 153.64 shares

If the shareholder lets their shares be repurchased, they will have $59 in cash. If the shareholder keeps their shares, they’re still worth $59.

*b.* If the company pays dividends, the current EPS is $2.65, and the PE ratio is:

PE = $57.15 / $2.65

PE = 21.57

If the company repurchases stock, the number of shares will decrease. The total net income is the EPS times the current number of shares outstanding. Dividing net income by the new number of shares outstanding, we find the EPS under the repurchase is:

EPS = $2.65(4,900) / (4,900 − 153.64)

EPS = $2.74

The stock price will remain at $59 per share, so the PE ratio is:

PE = $59 / $2.74

PE = 21.57

1. A share repurchase would seem to be the preferred course of action. Only those shareholders who wish to sell will do so, giving the shareholder a tax timing option that he or she doesn’t get with a dividend payment.

**13.** *a.* The price of a share of stock is the present value of the dividends. The dividend per share each year will be:

Year 1 dividend per share = $550,000 / 20,000 = $27.50

Year 2 dividend per share = $840,000 / 20,000 = $42

The required return is 12 percent, so the stock price today is:

*P*0 = $27.50 / 1.12 + $42 / 1.122 = $58.04

*b.* The dividend increase for existing shareholders for Year 1 will be the new dividend amount minus by the current dividend amount, which is:

Dividend increase = $650,000 – 550,000 = $100,000

In order to pay this increase in dividends, the firm must sell new shares. The number of new shares that must be sold is the dividend increase divided by the stock price, or:

New shares to sell = $100,000 / $58.04 = 1,723.08

The new shareholders invested a total of $100,000 in the company at Year 1. At Year 2, the total dividends that must be paid to these new shareholders is the investment amount of $100,000 times one plus the required return of 12 percent, or:

Dividends to new shareholders at Year 2 = $100,000(1.12) = $112,000

Since this amount must be paid to the new shareholders at Year 2, it will reduce the amount available to current shareholders. The amount available to current shareholders at Year 2 will be:

Total dividends to current shareholders at Year 2 = $840,000 – 112,000 = $728,000

So, the dividend per share to the current shareholders will now be:

Year 1 dividend per share = $650,000 / 20,000 = $32.50

Year 2 dividend per share = $728,000 / 20,000 = $36.40

The new stock price today is:

*P*0 = $32.50 / 1.12 + $36.40 / 1.122 = $58.04

This is the same price as we found with the original dividend policy.

*Challenge*

**14.** Assuming no capital gains tax, the aftertax return for the Gordon Company is the capital gains growth rate, plus the dividend yield times one minus the tax rate. Using the constant growth dividend model, we get:

Aftertax return = *g* + *D*(1 – *T*) = .12

Solving for *g*, we get:

.12 = *g* + .035(1 – .35)

*g* = .0973, or 9.73%

The equivalent pretax return for the Gordon Company, which pays a dividend, is:

Pretax return = *g* + *D* = .0973 + .035

Pretax return = .1323, or 13.23%

**15.**  Using the equation for the decline in the stock price ex-dividend for each of the tax rate policies, we get:

(*P0* – *PX*) / *D* = (1 – *TP*) / (1 – *TG*)

*a.* *P0* – *PX* = *D*(1 – 0) / (1 – 0)

*P0* – *PX* = *D*

*b.* *P0* – *PX* = *D*(1 – .15) / (1 – 0)

*P0* – *PX* = .85*D*

*c.* *P0* – *PX* = *D*(1 – .15) / (1 – .30)

*P0* – *PX* = 1.2143*D*

*d.* With this tax policy, we simply need to multiply the corporate tax rate times one minus the dividend exemption percentage, so:

*P0* – *PX* = *D*[1 – (.35)(.30)] / (1 – .35)

*P0* – *PX* = 1.377*D*

*e.* Since different investors have widely varying tax rates on ordinary income and capital gains, dividend payments have different after-tax implications for different investors. This differential taxation among investors is one aspect of what we have called the clientele effect.

**CHAPTER 15**

***RASING CAPITAL***

**Answers to Concepts Review and Critical Thinking Questions**

**1.** A company’s internally generated cash flow provides a source of equity financing. For a profitable company, outside equity may never be needed. Debt issues are larger because large companies have the greatest access to public debt markets (small companies tend to borrow more from private lenders). Equity issuers are frequently small companies going public; such issues are often quite small.

**2.** From the previous question, economies of scale are part of the answer. Beyond this, debt issues are simply easier and less risky to sell from an investment bank’s perspective. The two main reasons are that very large amounts of debt securities can be sold to a relatively small number of buyers, particularly large institutional buyers, such as pension funds and insurance companies, and debt securities are much easier to price.

**3.** They are riskier and harder to market from an investment bank’s perspective.

**4.** Yields on comparable bonds can usually be readily observed, so pricing a bond issue accurately is much less difficult.

**5.** It is clear that the stock was sold too cheaply, so Zipcar had reason to be unhappy.

**6.** No, but, in fairness, pricing the stock in such a situation is extremely difficult.

**7.** It’s an important factor. Only 9.68 million of the shares were underpriced. The other 30 million were, in effect, priced completely correctly.

**8.** He could have done worse since his access to the oversubscribed and, presumably, underpriced issues was restricted, while the bulk of his funds were allocated in stocks from the undersubscribed and, quite possibly, overpriced issues.

**9.** Going back to the chapter on risk and return, the greater the risk of an investment, the greater the required return. Venture capital is expensive simply because it is extremely risk. Although all companies are begun with an expectation of future cash flows, the reality is that many, if not the majority, will fail without ever have provided a positive return to investors.

**10.** *a*. The price will probably go up because IPOs are generally underpriced. This is especially true for smaller issues, such as this one.

*b.* It is probably safe to assume that they are having trouble moving the issue, and it is likely that the issue is not substantially underpriced.

**Solutions to Questions and Problems**

*Basic*

*NOTE: All end-of-chapter problems were solved using a spreadsheet. Many problems require multiple steps. Due to space and readability constraints, when these intermediate steps are included in this solutions manual, rounding may appear to have occurred. However, the final answer for each problem is found without rounding during any step in the problem.*

**1.** If you receive 1,000 shares of each, the profit is:

Profit = 1,000($9.50) – 1,000($5.25)

Profit = $4,250

Since you will only receive one-half of the shares of the oversubscribed issue, your profit will be:

Expected profit = 500($9.50) – 1,000($5.25)

Expected profit = –$500

This is an example of the winner’s curse.

**2.** Using X to stand for the required sale proceeds, the equation to calculate the total sale proceeds, including flotation costs, is:

X(1 – .07) = $87,000,000

X = $93,548,387 required total proceeds from sale.

So the number of shares offered is the total amount raised divided by the offer price, which is:

Number of shares offered = $93,548,387 / $48

Number of shares offered = 1,948,925

**3.** This is basically the same as the previous problem, except that we need to include the $1,425,000 of expenses in the amount the company needs to raise, so:

X(1 – .07) = $88,425,000

X = $95,080,645 required total proceeds from sale

Number of shares offered = $95,080,645 / $48

Number of shares offered = 1,980,847

**4.** We need to calculate the net amount raised and the costs associated with the offer. The net amount raised is the number of shares offered times the price received by the company, minus the costs associated with the offer, so:

Net amount raised = (9,500,000 shares)($22.32) – 1,475,000 – 350,000

Net amount raised = $210,215,000

The company received $210,215,000 from the stock offering. Now, we can calculate the direct costs. Part of the direct costs are given in the problem, but the company also had to pay the underwriters. The stock was offered at $24 per share, and the company received $22.32 per share. The difference, which is the underwriters spread, is also a direct cost. The total direct costs were:

Total direct costs = $1,475,000 + ($24 – 22.32)(9,500,000 shares)

Total direct costs = $17,435,000

We are given part of the indirect costs in the problem. Another indirect cost is the immediate price appreciation. The total indirect costs were:

Total indirect costs = $350,000 + ($27.50 – 24)(9,500,000 shares)

Total indirect costs = $33,600,000

This makes the total costs:

Total costs = $17,435,000 + 33,600,000

Total costs = $51,035,000

The flotation costs as a percentage of the amount raised is the total cost divided by the amount raised, so:

Flotation cost percentage = $51,035,000 / $210,215,000

Flotation cost percentage = .2428, or 24.28%

**5.** Using X to stand for the required sale proceeds, the equation to calculate the total sale proceeds, including flotation costs, is:

X(1 – .075) = $52,500,000

X = $56,756,757 required total proceeds from sale.

So the number of shares offered is the total amount raised divided by the offer price, which is:

Number of shares offered = $56,756,757 / $21

Number of shares offered = 2,702,703

**6.** This is basically the same as the previous problem, except that we need to include the $1,450,000 of expenses in the amount the company needs to raise, so:

X(1 – .075) = $53,950,000

X = $58,324,324 required total proceeds from sale

Number of shares offered = $58,324,324 / $21

Number of shares offered = 2,777,349

**7.** We need to calculate the net amount raised and the costs associated with the offer. The net amount raised is the number of shares offered times the price received by the company, minus the costs associated with the offer, so:

Net amount raised = (7,750,000 shares)($31.75) – 1,350,000 – 210,000

Net amount raised = $244,502,500

The company received $244,502,500 from the stock offering. Now, we can calculate the direct costs. Part of the direct costs are given in the problem, but the company also had to pay the underwriters. The stock was offered at $34 per share, and the company received $31.75 per share. The difference, which is the underwriters spread, is also a direct cost. The total direct costs were:

Total direct costs = $1,350,000 + ($34 – 31.75)(7,750,000 shares)

Total direct costs = $18,787,500

We are given part of the indirect costs in the problem. Another indirect cost is the immediate price appreciation. The total indirect costs were:

Total indirect costs = $210,000 + ($43.85 – 34)(7,750,000 shares)

Total indirect costs = $76,547,500

This makes the total costs:

Total costs = $18,787,500 + 76,547,500

Total costs = $95,335,000

The flotation cost as a percentage of the amount raised is the total cost divided by the amount raised, so:

Flotation cost percentage = $95,335,000 / $244,502,500

Flotation cost percentage = .3899, or 38.99%

**CHAPTER 16**

***SHORT-TERM FINANCIAL PLANNING***

**Answers to Concepts Review and Critical Thinking Questions**

**1.** These are firms with relatively long inventory periods and/or relatively long receivables periods. Thus, such firms tend to keep inventory on hand, and they allow customers to purchase on credit and take a relatively long time to pay.

**2.** These are firms that have a relatively long time between the time purchased inventory is paid for and the time that inventory is sold and payment received. Thus, these are firms that have relatively short payables periods and/or relatively long receivable cycles.

**3.** *a.* Use: The cash balance declined by $200 to pay the dividend.

*b.* Source: The cash balance increased by $500 assuming the goods bought on payables credit were sold for cash.

*c.* Use: The cash balance declined by $900 to pay for the fixed assets.

*d.* Use: The cash balance declined by $625 to pay for the higher level of inventory.

*e.* Use: The cash balance declined by $1,200 to pay for the redemption of debt.

**4.** Carrying costs will decrease because they are not holding goods in inventory. Shortage costs will probably increase, depending on how close the suppliers are and how well they can estimate need. The operating cycle will decrease because the inventory period is decreased.

**5.** Since the cash cycle equals the operating cycle minus the accounts payable period, it is not possible for the cash cycle to be longer than the operating cycle if the accounts payable period is positive. Moreover, it is unlikely that the accounts payable period would ever be negative since that implies the firm pays its bills before they are incurred.

**6.** It lengthened its payables period, thereby shortening its cash cycle. There was no effect on the operating cycle.

**7.** Their receivables period increased, thereby increasing their operating and cash cycles.

**8.** It is sometimes argued that large firms “take advantage of” smaller firms by threatening to take their business elsewhere. However, considering a move to another supplier to get better terms is the nature of competitive free enterprise.

**9.** They would like to! The payables period is a subject of much negotiation, and it is one aspect of the price a firm pays its suppliers. A firm will generally negotiate the best possible combination of payables period and price. Typically, suppliers provide strong financial incentives for rapid payment. This issue is discussed in detail in a later chapter on credit policy.

**10.** The company will need less financing because it is essentially borrowing more from its suppliers. Among other things, the company will likely need less short-term borrowing from other sources, so it will save on interest expense.

**Solutions to Questions and Problems**

*NOTE: All end-of-chapter problems were solved using a spreadsheet. Many problems require multiple steps. Due to space and readability constraints, when these intermediate steps are included in this solutions manual, rounding may appear to have occurred. However, the final answer for each problem is found without rounding during any step in the problem.*

*Basic*

**1.** *a.* No change. A dividend paid for by the sale of debt will not change cash since the cash raised from the debt offer goes immediately to shareholders.

*b.* No change. The real estate is paid for by the cash raised from the debt, so this will not change the cash balance.

*c.* No change. Inventory and accounts payable will increase, but neither will impact the cash account.

*d.* Decrease. The short-term bank loan is repaid with cash, which will reduce the cash balance.

*e*. Decrease. The payment of taxes is a cash transaction.

*f*. Decrease. The preferred stock will be repurchased with cash.

*g.* No change. Accounts receivable will increase, but cash will not increase until the sales are paid off.

*h*. Decrease. The interest is paid with cash, which will reduce the cash balance.

*i*. Increase. When payments for previous sales, or accounts receivable, are paid off, the cash balance increases since the payment must be made in cash.

*j.* Decrease. The accounts payable are reduced through cash payments to suppliers.

*k.* Decrease. Here, the dividend payments are made with cash, which is generally the case. This is different from part *a*, where debt was raised to make the dividend payment.

*l.* No change. The supplies are paid by the cash raised by the short-term note, so the cash balance will not change.

*m.* Decrease. The utility bills must be paid in cash.

*n.* Decrease. A cash payment will reduce cash.

*o.* Decrease. If marketable securities are purchased, the company pays cash to the seller.

**2.** The total liabilities and equity of the company are the book value of equity, plus current liabilities and long-term debt, so:

Total liabilities and equity = $13,465 + 1,630 + 8,200

Total liabilities and equity = $23,295

We have NWC other than cash. Since NWC is current assets minus current liabilities, NWC other than cash is:

NWC other than cash = Accounts receivable + Inventory – Current liabilities

$2,275 = Accounts receivable + Inventory – $1,630

Accounts receivable + Inventory = $2,275 + 1,630

Accounts receivable + Inventory = $3,905

Since total assets must equal total liabilities and equity, we can solve for cash as:

Cash = Total assets – Fixed assets – (Accounts receivable + Inventory)

Cash = $23,295 – 18,380 – 3,905

Cash = $1,010

So, the current assets are:

Current assets = Cash + (Accounts receivable + Inventory)

Current assets = $1,010 + 3,905

Current assets = $4,915

**3.** *a.* Increase. If receivables go up, the time to collect the receivables would increase, which increases the operating cycle.

*b.* Increase. If credit repayment times are increased, customers will take longer to pay their bills, which will lead to an increase in the operating cycle.

*c.* Decrease. If the inventory turnover increases, the inventory period decreases.

*d.* No change. The accounts payable period is part of the cash cycle, not the operating cycle.

*e.* Decrease. If the receivables turnover increases, the receivables period decreases.

*f.* No change. Payments to suppliers affect the accounts payable period, which is part of the cash cycle, not the operating cycle.

**4.** *a.* Increase; Increase. If the terms of the cash discount are made less favorable to customers, the accounts receivable period will lengthen. This will increase both the cash cycle and the operating cycle.

*b.* Increase; No change. This will shorten the accounts payable period, which will increase the cash cycle. It will have no effect on the operating cycle since the length of the operating cycle is not affected by the payables period.

*c.* Decrease; Decrease. If more customers pay in cash, the accounts receivable period will decrease. This will decrease both the cash cycle and the operating cycle.

*d.* Decrease; Decrease. Fewer raw materials purchased will reduce the inventory period, which will decrease both the cash cycle and the operating cycle.

*e.* Decrease; No change. If more raw materials are purchased on credit, the accounts payable period will tend to increase, which would decrease the cash cycle. We should say that this may not be the case. The accounts payable period is a decision made by the company’s management. The company could increase the accounts payable account and still make the payments in the same number of days. This would leave the accounts payable period unchanged, which would leave the cash cycle unchanged. The change in purchases made on credit will not affect the inventory period or the accounts receivable period, so the operating cycle will not change.

*f.* Increase; Increase. If more goods are produced for inventory, the inventory period will increase. This will increase both the cash cycle and operating cycle.

**5.** *a.* 45-day collection period implies all receivables outstanding from previous quarter are collected in the current quarter, and (90 – 45) / 90 = 1/2 of current sales are collected.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  |  | *Q1* | *Q2* | *Q3* | *Q4* |
|  | Beginning receivables | $330 | $293 | $338 | $315 |
|  | Sales | 585 | 675 | 630 | 895 |
|  | Cash collections | 623 | 630 | 653 | 763 |
|  | Ending receivables | $293 | $338 | $315 | $448 |

*b.* 60-day collection period implies all receivables outstanding from previous quarter are collected in the current quarter, and (90 – 60) / 90 = 1/3 of current sales are collected.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  |  | *Q1* | *Q2* | *Q3* | *Q4* |
|  | Beginning receivables | $330 | $390 | $450 | $420 |
|  | Sales | 585 | 675 | 630 | 895 |
|  | Cash collections | 525 | 615 | 660 | 718 |
|  | Ending receivables | $390 | $450 | $420 | $597 |

*c.* 30-day collection period implies all receivables outstanding from previous quarter are collected in the current quarter, and (90 – 30) / 90 = 2/3 of current sales are collected.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  |  | *Q1* | *Q2* | *Q3* | *Q4* |
|  | Beginning receivables | $330 | $195 | $225 | $210 |
|  | Sales | 585 | 675 | 630 | 895 |
|  | Cash collections | 720 | 645 | 645 | 807 |
|  | Ending receivables | $195 | $225 | $210 | $298 |

**6.** The operating cycle is the inventory period plus the receivables period. The inventory turnover and inventory period are:

Inventory turnover = COGS / Average inventory

Inventory turnover = $77,681 / [($9,605 + 11,302) / 2]

Inventory turnover = 7.4311 times

Inventory period = 365 days / Inventory turnover

Inventory period = 365 days / 7.4311

Inventory period = 49.12 days

And the receivables turnover and receivables period are:

Receivables turnover = Credit sales / Average receivables

Receivables turnover = $124,652 / [($4,093 + 4,994) / 2]

Receivables turnover = 27.4352 times

Receivables period = 365 days / Receivables turnover

Receivables period = 365 days / 27.4352

Receivables period = 13.30 days

So, the operating cycle is:

Operating cycle = 49.12 days + 13.30 days

Operating cycle = 62.42 days

The cash cycle is the operating cycle minus the payables period. The payables turnover and payables period are:

Payables turnover = COGS / Average payables

Payables turnover = $77,681 / [($4,822 + 5,749) / 2]

Payables turnover = 14.6970 times

Payables period = 365 days / Payables turnover

Payables period = 365 days / 14.6970

Payables period = 24.83 days

So, the cash cycle is:

Cash cycle = 62.42 days – 24.83 days

Cash cycle = 37.59 days

The firm is receiving cash on average 37.59 days after it pays its bills.

**7.** If we factor immediately, we receive cash on an average of 38 days sooner. The number of periods in a year is:

Number of periods = 365 / 38

Number of periods = 9.6053

The EAR of this arrangement is:

EAR = (1 + Periodic rate)*m* – 1

EAR = (1 + 2/98)9.6053 – 1

EAR = .2142, or 21.42%

**8.** *a.* The payables period is zero since the company pays immediately. The payment in each period is 30 percent of next period’s sales, so:

*Q1 Q2 Q3 Q4*

Payment of accounts $229.50 $274.50 $252.00 $234.60

*b.* Since the payables period is 90 days, the payment in each period is 30 percent of the current period sales, so:

*Q1 Q2 Q3 Q4*

Payment of accounts $204.00 $229.50 $274.50 $252.00

*c.* Since the payables period is 60 days, the payment in each period is 2/3 of last quarter’s orders, plus 1/3 of this quarter’s orders, or:

Quarterly payments = 2/3(.30)(Current sales) + 1/3(.30)(Next period sales)

*Q1 Q2 Q3 Q4*

Payment of accounts $212.50 $244.50 $267.00 $246.20

**9.** Since the payables period is 60 days, payables in each period = 2/3 of last quarter’s orders, and 1/3 of this quarter’s orders, or 2/3(.75)(Current sales) + 1/3(.75)(Next period sales)

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  |  | *Q1* | *Q2* | *Q3* | *Q4* |
|  | Payment of accounts | $1,445.00 | $1,680.00 | $1,883.75 | $1,865.00 |
|  | Wages, taxes, other expenses | 547.50 | 639.00 | 738.00 | 784.50 |
|  | Long-term financing expenses | 120.00 | 120.00 | 120.00 | 120.00 |
|  | Total | $2,112.50 | $2,439.00 | $2,741.75 | $2,769.50 |

**10.** *a.* The November sales must have been the total uncollected sales minus the uncollected sales from December, divided by the collection rate two months after the sale, so:

November sales = ($97,000 – 71,000) / .15

November sales = $173,333.33

*b.* The December sales are the uncollected sales from December divided by the sum of the collection rates from the previous two months’ sales, so:

December sales = $71,000 / .35

December sales = $202,857.14

*c.* The collections each month for this company are:

Collections = .15(Sales from 2 months ago) + .20(Last month’s sales) + .65(Current sales)

January collections = .15($173,333.33) + .20($202,857.14) + .65($153,000)

January collections = $166,021.43

February collections = .15($202,857.14) + .20($153,000) + .65($169,000)

February collections = $170,878.57

March collections = .15($153,000) + .20($169,000) + .65($181,000)

March collections = $174,400.00

**11.** The sales collections each month will be:

Sales collections = .35(Current month sales) + .60(Previous month sales)

Given this collection, the cash budget will be:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  |  | April | May | June |
|  | Beginning cash balance | $152,000 | $221,100 | $361,850 |
|  | Cash receipts |  |  |  |
|  | Cash collections from credit sales | 481,700 | 519,050 | 518,500 |
|  | Total cash available | $633,700 | $740,150 | $880,350 |
|  | Cash disbursements |  |  |  |
|  | Purchases | $241,000 | $252,000 | $235,000 |
|  | Wages, taxes, and expenses | 114,000 | 107,800 | 149,000 |
|  | Interest | 13,000 | 13,000 | 13,000 |
|  | Equipment purchases | 44,600 | 5,500 | 207,000 |
|  | Total cash disbursements | $412,600 | $378,300 | $604,000 |
|  | Ending cash balance | $221,100 | $361,850 | $276,350 |

**12.** *a.* 45-day collection period implies all receivables outstanding from previous quarter are collected in the current quarter, and (90 – 45) / 90 = 1/2 of current sales are collected.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  |  | *Q1* | *Q2* | *Q3* | *Q4* |
|  | Beginning receivables | $1,900 | $2,150 | $2,600 | $2,400 |
|  | Sales | 4,300 | 5,200 | 4,800 | 4,000 |
|  | Cash collections | 4,050 | 4,750 | 5,000 | 4,400 |
|  | Ending receivables | $2,150 | $2,600 | $2,400 | $2,000 |

*b.* 60-day collection period implies all receivables outstanding from previous quarter are collected in the current quarter, and (90 – 60) / 90 = 1/3 of current sales are collected.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  |  | *Q1* | *Q2* | *Q3* | *Q4* |
|  | Beginning receivables | $1,900 | $2,867 | $3,467 | $3,200 |
|  | Sales | 4,300 | 5,200 | 4,800 | 4,000 |
|  | Cash collections | 3,333 | 4,600 | 5,067 | 4,533 |
|  | Ending receivables | $2,867 | $3,467 | $3,200 | $2,667 |

*c.* 30-day collection period implies all receivables outstanding from previous quarter are collected in the current quarter, and (90 – 30) / 90 = 2/3 of current sales are collected.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  |  | *Q1* | *Q2* | *Q3* | *Q4* |
|  | Beginning receivables | $1,900 | $1,433 | $1,733 | $1,600 |
|  | Sales | 4,300 | 5,200 | 4,800 | 4,000 |
|  | Cash collections | 4,767 | 4,900 | 4,933 | 4,267 |
|  | Ending receivables | $1,433 | $1,733 | $1,600 | $1,333 |

*Intermediate*

**13.** *a.* The EAR of the loan without the compensating balance is:

EAR = (1 + .00485)12 – 1

EAR = .0598, or 5.98%

However, with the compensating balance, you will only get the use of part of the amount, so the EAR with the compensating balance will be:

EAR = .0598 / (1 – .05)

EAR = .0629, or 6.29%

*b.* To end up with $15,000,000, you must borrow:

Amount to borrow = $15,000,000 / (1 – .05)

Amount to borrow = $15,789,473.68

The total interest you will pay on the loan is:

Total interest paid = $15,789,473.68(1.00485)6 – 15,789,473.68

Total interest paid = $465,080.96

**14.** *a.* The EAR of your investment account is:

EAR = 1.00494 – 1

EAR = .0197, or 1.97%

*b.* To calculate the EAR of the loan, we can divide the interest on the loan by the amount of the loan. The interest on the loan includes the opportunity cost of the compensating balance. The opportunity cost is the amount of the compensating balance times the potential interest rate you could have earned. The compensating balance is only on the unused portion of the credit line, so:

Opportunity cost = .04($75,000,000 – 40,000,000)(1.0049)4 – .04($75,000,000 – 40,000,000)

Opportunity cost = $27,642.34

And the interest you will pay to the bank on the loan is:

Interest cost = $40,000,000(1.0165)4 – 40,000,000

Interest cost = $2,706,061.70

So, the EAR of the loan in the amount of $40,000,000 is:

EAR = ($27,642.34 + 2,706,061.70) / $40,000,000

EAR = .0683, or 6.83%

*c.* The compensating balance is only applied to the unused portion of the credit line, so the EAR of a loan on the full credit line is:

EAR = 1.01654 – 1

EAR = .0677, or 6.77%

**15.** Here, we need to use the cash cycle and operating cycles to calculate the average accounts payable and average accounts receivable. We are given the cash cycle and the operating cycle, so the payables period is:

Cash cycle = Operating cycle – Payables period

36.50 days = 59.40 days – Accounts payable period

Accounts payable period = 22.90 days

Now, we can use the payables period equation to find the accounts payable turnover, which is:

Payables period = 365 / Payables turnover

22.90 = 365 / Payables turnover

Payables turnover = 15.94 times

Now, we can solve the payables turnover equation to find the average payables:

Payables turnover = Cost of goods sold / Average payables

15.94 = $445,000 / Average payables

Average payables = $27,919.18

Next, we can find the average accounts receivable. Using the equation for the operating cycle, the accounts receivable period is:

Operating cycle = Inventory period + Receivables period

59.40 days = 23.20 days + Receivables period

Receivables period = 36.20 days

Using the receivables period equation, we find:

Receivables period = 365 / Receivables turnover

36.20 = 365 / Receivables turnover

Receivables turnover = 10.08 times

Finally, the average accounts receivable balance is:

Receivables turnover = Credit sales / Average accounts receivable

10.08 = $724,000 / Average accounts receivable

Average accounts receivable = $71,804.93

**16.** Since the company has a 32-day collection period, only those sales made in the first 58 days of the quarter will be collected in that quarter. Total cash collections in the first quarter will be:

Q1 cash collections = Beginning receivables + (58/90)(Quarter 1 sales)

Q1 cash collections = $218 + (58/90)($378)

Q1 cash collections = $462

And cash collection in the second quarter will be sales made in the first 58 days of the quarter plus sales made in the last 32 days of the first quarter, so:

Q2 cash collections = (58/90)(Quarter 2 sales) + (32/90)(Quarter 1 sales)

Q2 cash collections = (58/90)($468) + (32/90)($378)

Q2 cash collections = $436

Quarter 3 and Quarter 4 collections will be:

Q3 cash collections = (58/90)(Quarter 3 sales) + (32/90)(Quarter 2 sales)

Q3 cash collections = (58/90)($570) + (32/90)($468)

Q3 cash collections = $534

Q4 cash collections = (58/90)(Quarter 4 sales) + (32/90)(Quarter 3 sales)

Q4 cash collections = (58/90)($522) + (32/90)($570)

Q4 cash collections = $539

The beginning receivables in each quarter will be the sales made in the last 32 days of the previous quarter, so:

Q2 beginning receivables = (32/90)(Quarter 1 sales)

Q2 beginning receivables = (32/90)($378)

Q2 beginning receivables = $134

Q3 beginning receivables = (32/90)(Quarter 2 sales)

Q3 beginning receivables = (32/90)($468)

Q3 beginning receivables = $166

Q4 beginning receivables = (32/90)(Quarter 3 sales)

Q4 beginning receivables = (32/90)($570)

Q4 beginning receivables = $203

The cash budget (in millions) for the company is:

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  |  | *Q1* | *Q2* | *Q3* | *Q4* |
|  | Beginning receivables | $218 | $134 | $166 | $203 |
|  | Sales | 378 | 468 | 570 | 522 |
|  | Cash collections | 462 | 436 | 534 | 539 |
|  | Ending receivables | $134 | $166 | $203 | $186 |
|  | Total cash collections | $462 | $436 | $534 | $539 |
|  | Total cash disbursements | 318 | 408 | 726 | 450 |
|  | Net cash inflow | $144 | $28 | –$192 | $89 |
|  | Beginning cash balance | $20 | $164 | $192 | –$1 |
|  | Net cash inflow | 144 | 28 | –192 | 89 |
|  | Ending cash balance | $164 | $192 | –$1 | $88 |
|  | Minimum cash balance | –20 | –20 | –20 | –20 |
|  | Cumulative surplus (deficit) | $144 | $172 | –$21 | $68 |

The company has a cash surplus for much of the year, but in Quarter 3, it will need to raise $21 million to cover its cash flows.

*Challenge*

**17.** *a.* For every dollar borrowed, you pay quarterly interest of:

Interest = $1(.0173) = $.0173

You also must maintain a compensating balance of 5 percent of the funds borrowed, so for each dollar borrowed, you will only receive:

Amount received = $1(1 – .05) = $.95

We can adjust the EAR equation we have been using to account for the compensating balance by dividing the EAR by one minus the compensating balance, so:

EAR = [(1.0173)4 – 1] / (1 – .05)

EAR = .0748, or 7.48%

Another way to calculate the EAR is using the FVIF (or PVIF). For each dollar borrowed, we must repay:

Amount owed = $1(1.0173)4

Amount owed = $1.07102

At the end of the year the compensating will be returned, so your net cash flow at the end of the year will be:

End of year cash flow = $1.07102 – .05

End of year cash flow = $1.02102

The present value of the end of year cash flow is the amount you receive at the beginning of the year, so the EAR is:

FV = PV(1 + *R*)

$1.02102 = $.95(1 + *R*)

*R* = $1.02102 / $.95 – 1

EAR = .0748, or 7.48%

*b.* The EAR is the amount of interest paid on the loan divided by the amount received when the loan is originated. The amount of interest you will pay on the loan is the amount of the loan times the effective annual interest rate, so:

Interest = $210,000,000[(1.0173)4 – 1]

Interest = $14,913,473.49

For whatever loan amount you take, you will only receive 95 percent of that amount since you must maintain a 5 percent compensating balance on the portion of the credit line used. The credit line also has a fee of .25 percent, so you will only get to use:

Amount received = .95($210,000,000) – .0025($400,000,000)

Amount received = $198,500,000

At the end of the loan, you will not repay the amount received since the commitment fee is a one-time fee paid up front. The amount you will repay (excluding interest) at the end of the loan will be the amount received plus the commitment, or:

Amount repaid (excluding interest) = .95($210,000,000)

Amount repaid (excluding interest) = $199,500,000

So, the total amount repaid will be:

Total repaid = $199,500,000 + 14,913,473.49

Total repaid = $214,413,473.49

So, the EAR of the loan is:

FV = PV(1 + *R*)

$214,413,473.49 = $198,500,000(1 + *R*)

*R* = .0802, or 8.02%

**18.** You will pay interest of:

Interest = $20,000,000(.065) = $1,300,000

Additionally, the compensating balance on the loan is:

Compensating balance = $20,000,000(.04) = $800,000

Since this is a discount loan, you will receive the loan amount minus the interest payment. You will also not get to use the compensating balance. So, the amount of money you will actually receive on a $20 million loan is:

Cash received = $20,000,000 – 1,300,000 – 800,000

Cash received = $17,900,000

The EAR is the interest amount divided by the loan amount, so:

EAR = $1,300,000 / $17,900,000

EAR = .0726, or 7.26%

We can also use the FVIF (or PVIF) here to calculate the EAR. Your cash flow at the beginning of the year is $17,900,000. At the end of the year, your cash flow is the loan repayment, but you will also receive your compensating balance back, so:

End of year cash flow = $20,000,000 – 800,000

End of year cash flow = $19,200,000

So, using the time value of money, the EAR is:

$19,200,000 = $17,900,000(1 + *R*)

*R* = $19,200,000 / $17,900,000 – 1

EAR = .0726, or 7.26%

**CHAPTER 17**

***WORKING CAPITAL MANAGEMENT***

**Answers to Concepts Review and Critical Thinking Questions**

**1.** Yes. Once a firm has more cash than it needs for operations and planned expenditures, the excess cash has an opportunity cost. It could be invested (by shareholders) in potentially more profitable ways. Question 9 discusses another reason.

**2.** If it has too much cash, it can simply pay a dividend, or, more likely in the current financial environment, buy back stock. It can also reduce debt. If it has insufficient cash, then it must either borrow, sell stock, or improve profitability.

**3.** Probably not. Creditors would probably want substantially more.

**4.** Auto manufacturers often argue that due to the cyclical nature of their business, cash reserves are a good way to deal with future economic downturns. This is debatable, but it is true that auto manufacturers’ operating cash flows are very sensitive to the business cycle, and enormous losses have occurred during recent downturns.

**5.** Such instruments go by a variety of names, but the key feature is that the dividend adjusts, keeping the price relatively stable. This price stability, along with the dividend tax exemption, makes so-called adjustable rate preferred stock very attractive relative to interest-bearing instruments.

**6.** Net disbursement float is more desirable because the bank thinks the firm has more money than it actually does, and the firm is therefore receiving interest on funds it has already spent.

**7.** The firm has a net disbursement float of $500,000. If this is an ongoing situation, the firm may be tempted to write checks for more than it actually has in its account.

**8.** *a.* About the only disadvantage to holding T-bills are the generally lower yields compared to alternative money market investments.

*b.* Some ordinary preferred stock issues pose both credit and price risks that are not consistent with most short-term cash management plans.

*c.* The primary disadvantage of NCDs is the normally large transactions sizes, which may not be feasible for the short-term investment plans of many smaller to medium-sized corporations.

*d.* The primary disadvantages of the commercial paper market are the higher default risk characteristics of the security, and the lack of an active secondary market which may excessively restrict the flexibility of corporations to meet their liquidity adjustment needs.

**9.** The concern is that excess cash on hand can lead to poorly thought-out investments. The thought is that keeping cash levels relatively low forces management to pay careful attention to cash flow and capital spending.

**10.** A potential advantage is that the quicker payment often means a better price. The disadvantage is that doing so increases the firm’s cash cycle.

**11.** This is really a capital structure decision. If the firm has an optimal capital structure, paying off debt moves it to an under-leveraged position. However, a combination of debt reduction and stock buybacks could be structured to leave capital structure unchanged.

**12.** It is unethical because you have essentially tricked the grocery store into making you an interest-free loan, and the grocery store is harmed because it could have earned interest on the money instead of loaning it to you.

**13.** *a.* A sight draft is a commercial draft that is payable immediately.

*b.* A time draft is a commercial draft that does not require immediate payment.

*c.* A banker’s acceptance is when a bank guarantees the future payment of a commercial draft.

*d.* A promissory note is an IOU that the customer signs.

*e.* A trade acceptance is when the buyer accepts the commercial draft and promises to pay it in the future.

**14.** Trade credit is usually granted on open account. The invoice is the credit instrument.

**15.** The costs of granting credit are carrying costs, namely, the cost of debt, possibility of default, and the cash discount. The major cost of not granting credit is the opportunity cost of lost sales. The sum of these costs for different levels of receivables creates the total credit cost curve.

**16.** *1.* Character: determines if a customer is willing to pay his or her debts.

*2.* Capacity: determines if a customer is able to pay debts out of operating cash flow.

*3.* Capital: determines the customer’s financial reserves in case problems occur with operating cash flow.

*4.* Collateral: assets that can be liquidated to pay off the loan in case of default.

*5.* Conditions: customer’s ability to weather an economic downturn and whether such a downturn is likely.

**17.** *1.* Perishability and collateral value

*2.* Consumer demand

*3.* Cost, profitability, and standardization

*4.* Credit risk

*5.* The size of the account

*6.* Competition

*7.* Customer type

If the credit period exceeds a customer’s operating cycle, then the firm is financing the receivables and other aspects of the customer’s business that go beyond the purchase of the selling firm’s merchandise.

**18.** *a.* B: A is likely to sell for cash only, unless the product really works. If it does, then they might grant longer credit periods to entice buyers.

*b.* A: Landlords have significantly greater collateral, and that collateral is not mobile.

*c.* A: Since A’s customers turn over inventory less frequently, they have a longer inventory period and thus will most likely have a longer credit period as well.

*d.* B: Since A’s merchandise is perishable and B’s is not, B will probably have a longer credit period.

*e.* A: Rugs are fairly standardized and they are transportable, while carpets are custom fit and are not particularly transportable.

**19.** The three main categories of inventory are: raw material (initial inputs to the firm’s production process), work-in-progress (partially completed products), and finished goods (products ready for sale). From the firm’s perspective, the demand for finished goods is independent of the demand for the other types of inventory. The demand for raw material and work-in-progress is derived from, or dependent on, the firm’s needs for these inventory types in order to achieve the desired levels of finished goods.

**20.** JIT systems reduce inventory amounts. Assuming no adverse effects on sales, inventory turnover will increase. Since assets will decrease, total asset turnover will also increase. Recalling the Du Pont equation, an increase in total asset turnover, all else being equal, has a positive effect on ROE.

**Solutions to Questions and Problems**

*NOTE: All end-of-chapter problems were solved using a spreadsheet. Many problems require multiple steps. Due to space and readability constraints, when these intermediate steps are included in this solutions manual, rounding may appear to have occurred. However, the final answer for each problem is found without rounding during any step in the problem.*

*Basic*

**1.** The available balance is the amount you have on deposit, or $95,000. By writing a check, you now have a disbursement float. Your book balance is the amount on deposit minus the amount of the check, or:

Book balance = $95,000 – 24,300

Book balance = $70,700

**2.** The available balance is the amount you have on deposit, or $12,700. This is a collection float since you are waiting for the deposited check to clear into your account. The book balance is the amount on deposit plus the collection float, or:

Book balance = $12,700 + 2,400

Book balance = $15,100

**3.** Your disbursement float is the amount of the check you wrote, or $4,200. The collection float is the amount of the check deposited, or –$5,100. The net float is the sum of the disbursement float and collection float, or:

Net float = $4,200 – 5,100

Net float = –$900

**4.** *a.* There are 30 days until account is overdue. If you take the full period, you must remit:

Remittance = 540($62)

Remittance = $33,480

*b.* There is a 1 percent discount offered, with a 10 day discount period. If you take the discount, you will only have to remit:

Remittance = (1 – .01)($33,480)

Remittance = $33,145

*c.* The implicit interest is the difference between the two remittance amounts, or:

Implicit interest = $33,480 – 33,145

Implicit interest = $335

The number of days’ credit offered is:

Days’ credit = 30 – 10

Days’ credit = 20 days

**5.** The average daily receipts are the total amount of checks received divided by the number of days in a month. Assuming 30 days in a month, the average daily float is:

Average daily receipts = $59,200 / 30

Average daily receipts = $1,973.33

This is the average amount of checks received per day. The average check takes three days to clear, so the average daily float is:

Average daily float = 3($1,973.33)

Average daily float = $5,920

**6.** *a.* The disbursement float is the average monthly checks written times the average number of days for the checks to clear, so:

Disbursement float = 4($23,400)

Disbursement float = $93,600

The collection float is the average monthly checks received times the average number of days for the checks to clear, so:

Collection float = 2(–$38,100)

Collection float = –$76,200

The net float is the disbursement float plus the collection float, so:

Net float = $93,600 – 76,200

Net float = $17,400

*b.* The new collection float will be:

Collection float = 1(–$38,100)

Collection float = –$38,100

And the new net float will be:

Net float = $93,600 – 38,100

Net float = $55,500

**7.** The total sales of the firm are equal to the total credit sales since all sales are on credit, so:

Total credit sales = 7,400($235)

Total credit sales = $1,739,000

The average collection period is the percentage of accounts taking the discount times the discount period, plus the percentage of accounts not taking the discount times the days’ until full payment is required, so:

Average collection period = .40(10) + .60(30)

Average collection period = 22 days

The receivables turnover is 365 divided by the average collection period, so:

Receivables turnover = 365 / 22

Receivables turnover = 16.591 times

And the average receivables are the credit sales divided by the receivables turnover so:

Average receivables = $1,739,000 / 16.591

Average receivables = $104,816.44

If the firm increases the cash discount, more people will pay sooner, thus lowering the average collection period. If the ACP declines, the receivables turnover increases, which will lead to a decrease in the average receivables.

**8.** The receivables turnover is 365 divided by the average collection period, so:

Receivables turnover = 365 / 33

Receivables turnover = 11.061 times

And the average receivables are the credit sales divided by the receivables turnover, so:

Average receivables = $31,200,000 / 11.061

Average receivables = $2,820,821.92

**9.** *a.* The average collection period is the percentage of accounts taking the discount times the discount period, plus the percentage of accounts not taking the discount times the days’ until full payment is required, so:

Average collection period = .55(10 days) + .45(30 days)

Average collection period = 19 days

*b.* And the average daily balance is:

Average balance = 1,250($2,300)(19)(12/365)

Average balance = $1,795,890.41

**10.** The daily sales are:

Daily sales = $31,800 / 7

Daily sales = $4,542.86

Since the average collection period is 36 days, the average accounts receivable is:

Average accounts receivable = $4,542.86(36)

Average accounts receivable = $163,542.86

**11.** The interest rate for the term of the discount is:

Interest rate = .01/.99

Interest rate = .0101, or 1.01%

And the interest is for:

30 – 10 = 20 days

So, using the EAR equation, the effective annual interest rate is:

EAR = (1 + Periodic rate)m – 1

EAR = (1.0101)365/20 – 1

EAR = .2013, or 20.13%

*a.* The periodic interest rate is:

Interest rate = .02/.98

Interest rate = .0204, or 2.04%

And the EAR is:

EAR = (1.0204)365/20 – 1

EAR = .4459, or 44.59%

*b.* The EAR is:

EAR = (1.0101)365/30 – 1

EAR = .1301, or 13.01%

*c.* The EAR is:

EAR = (1.0101)365/10 – 1

EAR = .4432, or 44.32%

**12.** The receivables turnover is:

Receivables turnover = 365 / Average collection period

Receivables turnover = 365 / 33

Receivables turnover = 11.061 times

And the annual credit sales are:

Annual credit sales = Receivables turnover × Average daily receivables

Annual credit sales = 11.061($87,600)

Annual credit sales = $968,909.09

**13.** The carrying costs are the average inventory times the cost of carrying an individual unit, so:

Carrying costs = (1,150 / 2)($6.75)

Carrying costs = $3,881.25

The order costs are the number of orders times the cost of an order, so:

Restocking costs = 52($425)

Restocking costs = $22,100

The economic order quantity is:

EOQ = [(2T × F)/CC]1/2

EOQ = [2(52)(1,150)($425) / $6.75]1/2

EOQ = 2,744.15

The number of orders per year will be the total units sold per year divided by the EOQ, so:

Number of orders per year = 52(1,150) / 2,744.15

Number of orders per year = 21.79

The firm’s policy is not optimal, since the carrying costs and the order costs are not equal. The company should increase the order size and decrease the number of orders.

**14.** The carrying costs are the average inventory times the cost of carrying an individual unit, so:

Carrying costs = (765 / 2)($27)

Carrying costs = $10,327.50

The order costs are the number of orders times the cost of an order, so:

Restocking costs = 12($385)

Restocking costs = $4,620

The economic order quantity is:

EOQ = [(2T × F) / CC]1/2

EOQ = [2(12)(765)($385) / $27]1/2

EOQ = 511.66

The number of orders per year will be the total units sold per year divided by the EOQ, so:

Number of orders per year = 12(765) / 511.66

Number of orders per year = 17.94

The firm’s policy is not optimal, since the carrying costs and the order costs are not equal. The company should decrease the order size and increase the number of orders.

*Intermediate*

**15.** The total carrying costs are:

Carrying costs = (Q/2) × CC

where CC is the carrying cost per unit. The restocking costs are:

Restocking costs = F × (T/Q)

So, the total cost is:

Total cost = Carrying cost + Restocking costs

Total cost = (Q/2) × CC + F × (T/Q)

Using calculus to find the minimum point of the curve, we take the derivative and set it equal to zero. Doing so, we find:

∂/∂Q = 0 = (CC/2) + (F × T × –Q–2)

–Q–2 = –CC/ (2 × F × T)

Q–2 = CC/ (2 × F × T)

Q2 = (2 × F × T) / CC

Q = [(2 × F × T) / CC]1/2

To prove this point is a minimum, we can find the second derivative, which is:

∂/∂Q[(CC/2) + F × T × –Q–2) ] = F × T × 2Q–3 = (2 × F × T) / Q3

Since the second derivative is greater than zero so long as F, T, and Q are all positive, the first derivative is at a minimum.

*Challenge*

**16.** Since the company sells 700 suits per week, and there are 52 weeks per year, the total number of suits sold is:

Total suits sold = 700 × 52 = 36,400

And, the EOQ is 500 suits, so the number of orders per year is:

Orders per year = 36,400 / 500 = 72.80

To determine the day when the next order is placed, we need to determine when the last order was placed. Since the suits arrived on Monday and there is a 3-day delay from the time the order was placed until the suits arrive, the last order was placed Friday. Since there are five days between the orders, the next order will be placed on Wednesday

Alternatively, we could consider that the store sells 100 suits per day (700 per week / 7 days). This implies that the store will be at the safety stock of 100 suits on Saturday when it opens. Since the suits must arrive before the store opens on Saturday, they should be ordered 3 days prior to account for the delivery time, which again means the suits should be ordered on Wednesday.

**CHAPTER 18**

***INTERNATIONAL ASPECTS OF FINANCIAL MANAGEMENT***

**Answers to Concepts Review and Critical Thinking Questions**

**1.**  *a.* The dollar is selling at a premium, because it is more expensive in the forward market than in the spot market (SF 1.13 versus SF 1.10).

*b.* The franc is expected to depreciate relative to the dollar, because it will take more francs to buy one dollar in the future than it does today.

*c.* Inflation in Switzerland is higher than in the United States, as are interest rates.

**2.** The exchange rate will increase, as it will take progressively more rubles to purchase a dollar as the higher inflation in Russia will devalue the ruble. This is the relative PPP relationship.

**3.** *a.* The Australian dollar is expected to weaken relative to the dollar, because it will take more A$ in the future to buy one dollar than it does today.

*b.* The inflation rate in Australia should be higher.

*c.* Nominal interest rates in Australia should be higher; relative real rates in the two countries should be the same.

**4.** A Yankee bond is most accurately described by *d*.

**5.** Either. For example, if a country’s currency strengthens, imports become cheaper (good), but its exports become more expensive for others to buy (bad). The reverse is true for a currency depreciation.

**6.** The main advantage is the avoidance of the tariff. Additional advantages include being closer to the final consumer and, thereby, saving on transportation, significantly lower wages, and less exposure to exchange rate risk. Disadvantages include political risk and costs of supervising distant operations.

**7.** One key thing to remember is that dividend payments are made in the home currency. More generally, it may be that the owners of the multinational are primarily domestic and are ultimately concerned about their wealth denominated in their home currency because, unlike a multinational, they are not internationally diversified.

**8.** *a.* False. If prices are rising faster in Great Britain, it will take more pounds to buy the same amount of goods that one dollar can buy; the pound will depreciate relative to the dollar.

*b.* False. The forward market would already reflect the projected deterioration of the euro relative to the dollar. Only if you feel that there might be additional, unanticipated weakening of the euro that isn’t reflected in forward rates today will the forward hedge protect you against additional declines.

*c.* True. The market would only be correct on average, while you would be correct all the time.

**9.** *a.* American exporters: their situation in general improves because a sale of the exported goods for a fixed number of pesos will be worth more dollars.

American importers: their situation in general worsens because the purchase of the imported goods for a fixed number of pesos will cost more in dollars.

*b.* American exporters: they would generally be better off if the British government’s intentions result in a strengthened pound.

American importers: they would generally be worse off if the pound strengthens.

*c.* American exporters: would generally be much worse off, because an extreme case of fiscal expansion like this one will make American goods prohibitively expensive to buy, or else Brazilian sales, if fixed in reais, would become worth an unacceptably low number of dollars.

American importers: would generally be much better off, because Brazilian goods will become much cheaper to purchase in dollars.

**10.** False. If the financial markets are perfectly competitive, the difference between the Eurodollar rate and the U.S. rate will be due to differences in risk and government regulation. Therefore, speculating in those markets will not be beneficial.

**Solutions to Questions and Problems**

*NOTE: All end-of-chapter problems were solved using a spreadsheet. Many problems require multiple steps. Due to space and readability constraints, when these intermediate steps are included in this solutions manual, rounding may appear to have occurred. However, the final answer for each problem is found without rounding during any step in the problem.*

*Basic*

**1.** Using the quotes from the table, we get:

*a.* $100(Z3.3598/$1) = Z335.98

*b.* $1.2452

*c.* €5,000,000($1.2452/€) = $6,226,000

Alternatively the question can be answered as:

€5,000,000 / (€.8031/$) = $6,225,875

The difference is due to rounding in the exchange rate quote.

*d.* New Zealand dollar

*e.* Mexican peso

*f.* (SF.9655/$)($1.2452/€) = 1.2022 SF/€

This is a cross rate.

*g.* Most valuable: Kuwaiti dinar = $3.429

Least valuable: Vietnamese dong = $.00005

**2.** *a.* You would prefer £100, since:

(£100)($1.5649/£1) = $156.49

*b.* You would still prefer £100. Using the $/£ exchange rate and the C$/$ exchange rate to find the amount of Canadian dollars £100 will buy, we get:

(£100)($1.5649/£1)(C$1.1417/$1) = C$178.66

*c.* Using the quotes in the book to find the C$/£ cross rate, we find:

(C$1.1417/$1)($1.5649/£1) = C$1.7866/£1

The £/C$ exchange rate is the inverse of the C$/£ exchange rate, so:

£1/C$1.7866 = £.5597/C$

**3.** *a.* *F*180 = ¥118.37(per $). The yen is selling at a premium because it is more expensive in the forward market than in the spot market ($.008430 versus $.008448).

*b.* *F*90 = $.8452/A$. The Australian dollar is selling at a discount because it is less expensive in the forward market than in the spot market ($.8507 versus $.8452).

*c.* The value of the dollar will fall relative to the yen, since it takes more dollars to buy one yen in the future than it does today. The value of the dollar will rise relative to the Australian dollar, because it will take fewer US dollars to buy the Australian dollar in the future than it does today.

**4.** *a.* The U.S. dollar, since one Canadian dollar will buy:

(Can$1)/(Can$1.09/$1) = $.9174

*b.* The cost in U.S. dollars is:

(Can$2.49)/(Can$1.09/$1) = $2.28

Among the reasons that absolute PPP doesn’t hold are tariffs and other barriers to trade, transactions costs, taxes, and different tastes.

*c.* The U.S. dollar is selling at a premium because it is more expensive in the forward market than in the spot market (Can$1.09 versus Can$1.14).

*d.* The U.S. dollar is expected to appreciate in value relative to the Canadian dollar, because it takes more Canadian dollars to buy one U.S. dollar in the future than it does today.

*e.* Interest rates in the United States are probably higher than they are in Canada.

**5.** *a.* The cross rate in ¥/£ terms is:

(¥121/$1)($1.53/£1) = ¥185.13/£1

*b.* The yen is quoted high relative to the pound. Take out a loan for $1 and buy £.6536. Use the pounds to purchase yen at the cross-rate, which will give you:

£.6536(£1/¥189) = ¥123.529

Use the yen to buy back dollars and repay the loan. The cost to repay the loan will be:

¥123.529($1/¥121) = $1.0209

Your arbitrage profit is $.0209 per dollar used.

**6.** We can rearrange the approximate interest rate parity condition to answer this question. The equation we will use is:

*RFC* = (*Ft* – *S0*) / *S0* + *RUS*

Using this relationship, we find:

Australia: *RFC* = (A$1.1909 – A$1.1755) / A$1.1755 + .012 = .0251, or 2.51%

Japan: *RFC* = (¥118.37 – ¥118.62) / ¥118.62 + .012 = .0099, or .99%

Great Britain: *RFC* = (£.6400 – £.6390) / £.6390 + .012 = .0136, or 1.36%

**7.** If we invest in the U.S. for the next three months, we will have:

$30,000,000(1.0019)3 = $30,171,325.11

If we invest in Great Britain, we must exchange the dollars today for pounds, and exchange the pounds for dollars in three months. After making these transactions, the dollar amount we would have in three months would be:

($30,000,000)(£.63/$1)(1.0022)3/(£.64/$1) = $29,726,585.36

We should invest in the U.S.

**8.** Using the relative purchasing power parity equation:

*Ft* = *S0* × [1 + (*hFC* – *hUS*)]t

We find:

R51.80 = R49.35[1 + (*hFC* – *hUS*)]3

*hFC* – *hUS* = (R51.80 / R49.35)1/3 – 1

*hFC* – *hUS* = .0163, or 1.63%

Inflation in Russia is expected to exceed that in the U.S. by 1.63% per year over this period.

**9.** The profit will be the quantity sold, times the sales price minus the cost of production. The production cost is in Singapore dollars, so we must convert this to U.S. dollars. Doing so, we find that if the exchange rate stays the same, the profit will be:

Profit = 30,000[$150 – {(S$185.50)/(S$1.3043/$1)}]

Profit = $233,343.56

If the exchange rate rises, we must adjust the cost by the increased exchange rate, so:

Profit = 30,000[$150 – {(S$185.50)/1.1(S$1.3043/$1)}]

Profit = $621,221.41

If the exchange rate falls, we must adjust the cost by the decreased exchange rate, so:

Profit = 30,000[$150 – {(S$185.50)/.9(S$1.3043/$1)}]

Profit = –$240,729.38

To calculate the breakeven change in the exchange rate, we need to find the exchange rate that makes the cost in Singapore dollars equal to the selling price in U.S. dollars, so:

$150 = S$185.50/*ST*

*ST* = S$1.2367/$1

This is a decline of:

Decline = (S$1.2367 – 1.3043) / S$1.3043

Decline = –.0519, or –5.19%

**10.** *a.* If IRP holds, then:

*F*180 = (W1,108.27)[1 + (.031 – .025)]1/2

*F*180 = W1,111.5898

Since given F180 is W1,110.32, an arbitrage opportunity exists; the forward premium is too low. Borrow $1 today at 2.5% interest. Agree to a 180-day forward contract at W1110.32. Convert the loan proceeds into won:

$1(W1,108.27/$1) = W1,108.27

Invest these won at 3.1%, ending up with W1125.08. Convert the won back into dollars as

W1,125.08($1/W1,110.32) = $1.01330

Repay the $1 loan, ending with a profit of:

$1.01330 – 1.01225 = $.00104

*b.* To find the forward rate that eliminates arbitrage, we use the interest rate parity condition, so:

*F*180 = (W1,108.27)[1 + (.031 – .025)]1/2

*F*180 = W1,111.5898

*Intermediate*

**11.** *a.* The yen is expected to get stronger, since it will take fewer yen to buy one dollar in the future than it does today.

*b.* *hUS* – *hJAP* ≈ (¥112.63 – ¥113.21) / ¥113.21

*hUS* – *hJAP* = – .0051, or –.51%

(1 – .0051)4 – 1 = –.0203, or –2.03%

The approximate inflation differential between the U.S. and Japan is –2.03% annually.

**12.** We need to find the change in the exchange rate over time, so we need to use the interest rate parity relationship:

*Ft* = *S0* × [1 + (*RFC* – *RUS*)]*t*

Using this relationship, we find the exchange rate in one year should be:

*F1* = 249[1 + (.048 – .027)]1

*F1* = HUF 254.23

The exchange rate in two years should be:

*F2* = 249[1 + (.048 – .027)]2

*F2* = HUF 259.57

And the exchange rate in five years should be:

*F5* = 249[1 + (.048 – .027)]5

*F5* = HUF 276.27

**13.** Pounds are cheaper in New York, so we start there. Buy:

$10,000(£/$1.5769) = £6,341.56

in New York. Sell the £6,341.56 in London for

£6,341.56($1.5872/£) = $10,065.32

Your profit is $10,065.32 – 10,000 = $65.32

for each $10,000 transaction.

**14.** If purchasing power parity holds, the exchange rate will be:

Krona135.34 / $4.79 = Krona28.2547/$

**15.** *a.* To construct the balance sheet in dollars, we need to convert the account balances to dollars. At the current exchange rate, we get:

Assets = solaris 40,000 × ($/solaris 1.20) = $33,333.33

Debt = solaris 12,500 × ($/solaris 1.20) = $10,416.67

Equity = solaris 27,500 × ($/solaris 1.20) = $22,916.67

*b.* In one year, if the exchange rate is solaris 1.30/$, the accounts will be:

Assets = solaris 40,000 × ($/solaris 1.30) = $30,769.23

Debt = solaris 12,500 × ($/solaris 1.30) = $9,615.38

Equity = solaris 27,500 × ($/solaris 1.30) = $21,153.85

*c.* If the exchange rate is solaris 1.08/$, the accounts will be:

Assets = solaris 40,000 × ($/solaris 1.08) = $37,037.04

Debt = solaris 12,500 × ($/solaris 1.08) = $11,574.07

Equity = solaris 27,500 × ($/solaris 1.08) = $25,462.96

*Challenge*

**16.** First, we need to construct the end of year balance sheet in solaris. Since the company has retained earnings, the equity account will increase, which necessarily implies the assets will also increase by the same amount. So, the balance sheet at the end of the year in solaris will be:

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  |  | Balance Sheet (solaris) | | |  |
|  |  |  |  | Liabilities | 12,500.00 |
|  |  |  |  | Equity | 29,000.00 |
|  | Assets | 41,500.00 |  | Total liabilities & equity | 41,500.00 |

Now we need to convert the balance sheet accounts to dollars, which gives us:

Assets = solaris 41,500 × ($/solaris 1.24) = $33,467.74

Debt = solaris 12,500 × ($/solaris 1.24) = $10,080.65

Equity = solaris 29,000 × ($/solaris 1.24) = $23,387.10