Chapter 2

Chapter overview

This chapter introduces linear functions, with applications. The straight line is one of the simplest mathematical functions, yet students are surprisingly vague about its properties, its equation and how to derive its equation. In spite of its simplicity, the line serves as a model for numerous economic/business situations (at least at an introductory level): demand, supply, cost, revenue, consumption, savings. Therefore, having studied a simple mathematical function, the student is in a position to:

- set up economic models, such as demand functions, to describe economic behaviour.
- use the model to analyse economic behaviour.

4th edition. All sections have been revised and updated. A new section: 2.3.4 introduces profit as the difference between total revenue and total cost.

4th edition: new on-line teaching and learning material.

An animated Worked Example (Worked Example 2.7 Analysis of a linear supply function) is available on on-line. The narrative is designed to give the student a broader and more in-depth understanding of supply. It describes the quantity supplied as a response to the price per unit as well as giving a verbal description of slope and intercept.

An on-line question bank is available in both WileyPlus and MapleTA.
The question types provided for this chapter are the following.

- Questions that require a single answer
- Matching questions. For example, given
  - (i) 6 graphs, the reader is asked to match each plot with its equation
  - (ii) graphs of TR, TC, FC and Profit, the reader is asked to match each function with its equation.
- Questions that require several inputs: this type of question is designed to help the student work through standard methods, to give intermediate results and give answers that demonstrate their understanding. This type of question is given in two formats
  - (i) Questions that submitted after all inputs are entered: the question is then graded and feedback given.
  - (ii) Questions which are graded at intermediate stages: feedback may be allowed at each stage so that students can progress to the next stage.

  To demonstrate that even very basic linear functions are used in everyday situations, questions are given that require the student to write and equation/formula to model situations from the verbal description.

  All questions are algorithmic, that is, numbers are randomised so that it is unlikely that two students will be given identical questions.

For the student: to help students to work independently, immediate feedback is given to most questions. The algorithmic nature of questions provide practise for student on topics of difficulty.

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For the instructor: The virtual learning environments (VLEs) allow the instructor to manage the course material and students' progress. For further details consult the website at www.wileyplus.com

A problem in Context This problem is presented from an economist's perspective. The problem discusses and calculates income elasticity of demand and cross price elasticity of demand.

The following sections are available on the Website

www.wiley.com/college/bradley.

2.5 Translations of linear functions. Additional Worked Examples on applications of translations have been included.

2.6.2 Further material on elasticity, including income elasticity of demand.

2.7 Budget and cost constraints

Topics covered in this chapter

The Mathematics covered in this chapter is summarised as follows.

Describe the slope and intercept of a line verbally: Illustrate these ideas graphically.
Explain the equation of a line (or any curve!) as the formula that describes the relationship between the *x* and *y* co-ordinate at every point on the line.
Explain how the intercept and slope of a line are used to give in the formula/equation of a line
Deduce the equation of a line when given
- (a) slope and intercept
- (b) slope and one point
- (c) two points.

Write down the slope and intercept of a line from its equation.
Plot a line when given
- (a) slope and intercept
- (b) its equation in any format

The Applications of Linear functions covered in this chapter are the following**.**

Demand;

Supply; price
Cost functions
Total revenue functions
Profit
Price elasticity of demand and supply.
Applications on the Website: income elasticity; budget and cost constraints.

Note:
Due to space constraints, other applications modelled by linear functions, such as consumption: $C = C_0 + bY$ and savings: $S = Y - C = Y - (C_0 + bY)$
could not be included in the text but are given on the web. However, having completed this chapter, the student should have no difficulty using linear functions to model these and other applications.

PowerPoint presentations

The presentations include the following

- Describe and illustrate graphically the properties of linear functions
- The concept of a linear equation, illustrate graphically
- How go about plotting graphs, defining a sensible scale for both the horizontal and vertical axis, calculating a table of points, then plotting the graph.
- Plot a linear function by calculating, then joining the horizontal and vertical intercepts. This is particularly useful in when plotting demand, supply, cost revenue functions which are only economically meaningful for positive values of the variables.
- Plotting and describing demand, supply, cost, revenue and profit functions.
- Plotting and describing budget and cost constraints. Describing the effect of changes in prices and budget limits and illustrating these graphically.

Software

Excel
Useful for calculating tables and graph plotting. Details are given in section 2.8, chapter 2 in the text. Exercises in Excel are given in Progress Exercises 2.9.

Excel exercises and solutions are available on-line at www.wiley.com/college/bradley

Areas perceived as difficult.

In my experience, the following are just some of the areas where students experience difficulties

Graph Plotting
Plotting straight lines is a very simple process. In economic applications graphs are normally required in the first quadrant only. Students need some guidelines
(Note: several examples are also given on the PowerPoint presentations)

Suggestion

1. Decide which variable should be plotted on the horizontal and vertical axis.
2. Determine the appropriate interval over which to plot the graph.
3. Calculate at least two points on the line from the equation of the line.
4. Plot the graph by drawing a line through the points calculated above.

For example, since most economic/business applications require the graph in the first quadrant only, then
(a) find the points of intersection with the axis, join the points to draw the line
or
(b) calculate a table of points and draw the line as outlined in the guidelines above.

Note
(b) it might seem a bit superfluous for linear functions, since any two points, such as those calculated in (a) are sufficient. However, some students feel more confident using a table of points. The practise of calculating a table of points is a useful exercise and is necessary later on when plotting non-linear functions.

Note
Plotting lines from the points of intersection with the axis is a useful practise to encourage. It will be used elsewhere in the text, for example, when plotting isocost lines, budget lines, inequality constraints in linear programming, demand and supply functions when finding equilibrium, consumer and producer surplus etc.

If Excel™ is available, students should be encouraged to use it for calculating tables and plotting graphs. See section 2.8, page 92 in the text

Elasticity
Students often find this concept difficult.

Suggestion

It may help to calculate elasticity from the definition directly,

$$\varepsilon = \frac{\%\text{ change in Quantity}}{\%\text{ change in Price}} = \frac{\left(\frac{\Delta Q}{Q}\right)\times 100}{\left(\frac{\Delta P}{P}\right)\times 100},$$

as well as calculating it from one of the any of the standard formulae such as

$$\varepsilon = \frac{\Delta Q}{\Delta P}\frac{P}{Q}, \text{ where } \varepsilon = -\frac{1}{b}\frac{P}{Q} \text{ or } \varepsilon = \frac{P}{P-a}, \text{ for the demand function } P = a - bQ$$

After calculating elasticity directly from the definition, the student should appreciate the convenience formulae.

The signs associated with elasticity. This should be explained.

Suggestion

- Elasticity of demand negative (in the first quadrant) since
demand normally drops as price increases
hence the slope of the demand function is negative.

Consider the demand function $P = a - bQ$, slope = $\frac{\Delta P}{\Delta Q} = -b$

But by definition, $\varepsilon = \frac{\Delta Q}{\Delta P}\frac{P}{Q} = \frac{1}{\frac{\Delta P}{\Delta Q}}\frac{P}{Q} = -\frac{1}{b}\frac{P}{Q}$ where $b > 0$, $P > 0$ and $Q > 0$

So ε is negative for demand functions, $P > 0$ and $Q > 0$.

- Elasticity of supply positive (in the first quadrant) because
the quantity supplied normally increases as the market price increases
hence the slope of the supply function is positive**.**

Consider the supply function $P = c + dQ$, slope = $\frac{\Delta Q}{\Delta P} = d$

By definition, $\varepsilon = \frac{\Delta Q}{\Delta P}\frac{P}{Q} = \frac{1}{\frac{\Delta Q}{\Delta P}}\frac{P}{Q} = \frac{1}{d}\frac{P}{Q}$ where $d > 0$, $P > 0$ and $Q > 0$

So ε is positive for supply functions, $P > 0$ and $Q > 0$

- $|\,\varepsilon\,|$ may also be confusing at first and again, some explanation should be given.

Suggestion

In some applications, we are interested only in whether the percentage change in demand (quantity) or supply in greater or less than the percentage change in price.

Elastic Demand: the percentage change in demand (quantity) is greater than the percentage change in price, then the value of the fraction: $\varepsilon = \frac{\text{\% change in Quantity}}{\text{\% change in Price}} = \frac{\left(\frac{\Delta Q}{Q}\right)\times 100}{\left(\frac{\Delta P}{P}\right)\times 100}$, assumes values such as – 1.2, – 1.5, – 2, – 2.2 etc. (since the numerator is greater than the denominator)

Elastic Supply: the percentage change in supply is greater than the percentage change in price, then the value of the fraction: $\varepsilon = \frac{\text{\% change in Quantity}}{\text{\% change in Price}} = \frac{\left(\frac{\Delta Q}{Q}\right)\times 100}{\left(\frac{\Delta P}{P}\right)\times 100}$, assumes values such as 1.2, 1.5, 2, 2.2 etc.

In each case the % change quantity is greater than the percentage change in price. So we refer to the magnitude of elasticity , $|\varepsilon|$, hence

$|\varepsilon| > 1$ when demand or supply is elastic
$|\varepsilon| < 1$ when demand or supply is inelastic etc.

A sketch of the number line, (see Figures 2.33, 2.34, 2.35) indicating the intervals in which demand or supply is described as elastic or inelastic is helpful:

Applications
As stated in the introduction, some students may find it difficult to think of applications having just mastered the mathematics. Sometimes even changing variables from *x* and *y* to *Q* and *P* is confusing, but this it is necessary for the development and analysis of general economic models.

Suggestion
Introduce a specific, tangible example first, such as a cost function. Such an example is given Worked Example 2.9, illustrated in Table 2.5 and Figure 2.25 (also included in Powerpoint);

To make the example tangible, consider an entrepreneur who is about to supply wooden vegetable crates to vegetable growers. His fixed costs could consist of basic tools such as a stapler, saw etc. Let this fixed cost be £100. To make each crate he requires some light wood and staples. This costs £12 per crate. Therefore, if he produces *Q* crates his variable costs are £12 × *Q* = £ 12*Q*.

Q (quantity)	$FC = 100$	$VC =$ cost per unit × number of units	$TC = FC + VC$
0	$FC = 100$	$12 \times 0 = 0$	$TC = 100 + 0Q = 100$
1	$FC = 100$	$12 \times 1 = 12$	$TC = 100 + 12$

2	$FC = 100$	$12 \times 2 = 24$	$TC = 100 + 24$
3	$FC = 100$	$12 \times 3 = 36$	$TC = 100 + 36$
:	:	:	:
in general ...			
Q	$FC = 100$	$12 \times Q = 12Q$	$TC = 100 + 12Q$

His overall costs (total costs) are the sum of his fixed costs and his variable costs
$TC = 100 + 12Q$

Another tangible example is the modelling and graphing of total revenue as illustrated in Worked Example 2.10 (in text and also on PowerPoint).

Note: In the text, demand and supply are introduced first. However, Worked Examples in the text may be used in any sensible order.