Problems and Solutions Section 1.5 (1.82 through 1.93)

1.82 A bar of negligible mass fixed with a tip mass forms part of a machine used to punch holes in a sheet of metal as it passes past the fixture as illustrated in Figure P1.82. The impact to the mass and bar fixture causes the bar to vibrate and the speed of the process demands that frequency of vibration not interfere with the process. The static design yields a mass of 50 kg and that the bar be made of steel of length 0.25 m with a cross sectional area of 0.01 m^2. Compute the system's natural frequency.

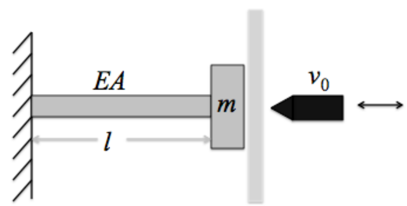


Figure P1.82 A bar model of a punch fixture.

Solution: From equation (1.63)

$$\omega_n = \sqrt{\frac{EA}{lm}} = \sqrt{\frac{(2.0\times 10^{11})(0.01)}{50(0.25)}\frac{(\text{N/m}^2)\text{m}^2}{\text{kg}\cdot\text{m}}} = \underline{1.26\times 10^4\,\text{rad/s}}$$

This is about 2000 Hz, which is likely too high to be a problem but could cause some undesirable noise.

1.83 Consider the punch fixture of Figure P1.82. If the system is giving an initial velocity of 10 m/s, what is the maximum displacement of the mass at the tip if the mass is 1000 kg and the bar is made of steel of length 0.25 m with a cross sectional area of 0.01 m^2?

Solution: First compute the frequency:

$$\omega_n = \sqrt{\frac{EA}{lm}} = \sqrt{\frac{(2.0\times 10^{11})(0.01)}{1000(0.25)}\frac{(\text{N/m}^2)\text{m}^2}{\text{kg}\cdot\text{m}}} = \underline{2.828\times 10^3\,\text{rad/s}}$$

From equation (1.9) the maximum amplitude is

$$A_{\max} = \frac{\sqrt{\omega_n^2 x_0^2 + v_0^2}}{\omega_n} = \frac{v_0}{\omega_n} = \frac{10}{2828}\frac{\text{m/s}}{1/\text{s}} = \underline{0.0035\text{ m}},$$

or about 0.35 mm, not much.

1.84 Consider the punch fixture of Figure P1.82. If the punch strikes the mass off center it is possible that the steel bar may vibrate in torsion. The mass is 1000 kg and the bar 0.25 m-long, with a square cross section of 0.1 m on a side. The mass polar moment of inertia of the tip mass is 10 kg/m^2. The polar moment of inertia for a square bar is $b^4/6$, where b is the length of the side of the square. Compute both the torsion and longitudinal frequencies. Which is larger?

Solution: First compute the longitudinal frequency of the bar:

$$\omega_n = \sqrt{\frac{EA}{lm}} = \sqrt{\frac{(2.0\times 10^{11})(0.01)}{1000(0.25)}\frac{(\text{N/m}^2)\text{m}^2}{\text{kg}\cdot\text{m}}} = \underline{2.828\times 10^3\,\text{rad/s}}$$

Next compute the torsional frequency of the bar (square cross section):

$$\omega_n = \sqrt{\frac{GJ_p}{lJ}} = \sqrt{\frac{8\times 10^8(0.1^4/6)}{0.25\times 10}} = \underline{73.03\text{ rad/s}}$$

In this case the torsional frequency is lower and should be considered in any design.

1.85 A helicopter landing gear consists of a metal framework rather than the coil spring based suspension system used in a fixed-wing aircraft. The vibration of the frame in the vertical direction can be modeled by a spring made of a slender bar as illustrated in Figure 1.23, where the helicopter is modeled as ground. Here l = 0.4 m, $E = 20\times 10^{10}$ N/m^2, and m = 100 kg. Calculate the cross-sectional area that should be used if the natural frequency is to be f_n = 500 Hz.

Solution: From equation (1.63)

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{EA}{lm}} \tag{1}$$

and

$$\omega_n = 500\text{ Hz}\left(\frac{2\pi\,\text{rad}}{1\text{ cycle}}\right) = 3142\text{ rad/s}$$

Solving (1) for A yields:

$$A = \frac{\omega_n^2 lm}{E} = \frac{(3142)^2(.4)(100)}{20\times 10^{10}} = 0.001974$$

$$A \approx 0.0020\text{ m}^2 = 20\text{cm}^2$$

1.86 The frequency of oscillation of a person on a diving board can be modeled as the transverse vibration of a beam as indicated in Figure 1.26. Let *m* be the mass of the diver ($m = 100$ kg) and $l = 1.5$ m. If the diver wishes to oscillate at 3 Hz, what value of *EI* should the diving board material have?

Solution: From equation (1.67),

$$\omega_n^2 = \frac{3EI}{ml^3}$$

and

$$\omega_n = 3Hz\left(\frac{2\pi \text{ rad}}{1 \text{ cycle}}\right) = 6\pi \text{ rad/s}$$

Solving for *EI*

$$EI = \frac{\omega_n^2 ml^3}{3} = \frac{(6\pi)^2 (100)(1.5)^3}{3} = \underline{3.997 \times 10^4 \text{ Nm}^2}$$

1.87 Consider the spring system of Figure 1.32. Let $k_1 = k_5 = k_2 = 100$ N/m, $k_3 = 50$ N/m, and $k_4 = 1$ N/m. What is the equivalent stiffness?

Solution: Given: $k_1 = k_2 = k_5 = 100 \text{ N/m}, k_3 = 50 \text{ N/m}, \ k_4 = 1 \text{ N/m}$

From Example 1.5.4

$$k_{eq} = k_1 + k_2 + k_5 + \frac{k_3 k_4}{k_3 + k_4}$$

$$\Rightarrow \underline{k_{eq} = 300.98 \text{ N/m}}$$

1.88 Springs are available in stiffness values of 10, 100, and 1000 N/m. Design a spring system using these values only, so that a 100-kg mass is connected to ground with frequency of about 1.5 rad/s.

Solution: Using the definition of natural frequency:

$$\omega_n = \sqrt{\frac{k_{eq}}{m}}$$

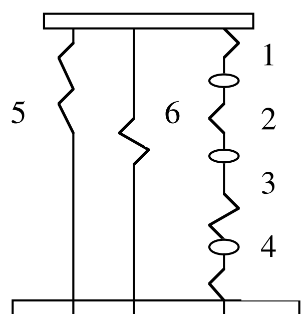
With m = 100 kg and ω_n = 1.5 rad/s the equivalent stiffness must be:

$$k_{eq} = m\omega_n^2 = (100)(1.5)^2 = 225 \text{ N/m}$$

There are many configurations of the springs given and no clear way to determine one configuration over another. Here is one possible solution. Choose two 100 N/m springs in parallel to get 200 N/m, then use four 100 N/m springs in series to get an equivalent spring of 25 N/m to put in parallel with the other 3 springs since

$$k_{eq} = \frac{1}{\dfrac{1}{k_1} + \dfrac{1}{k_2} + \dfrac{1}{k_3} + \dfrac{1}{k_4}} = \frac{1}{4/100} = 25$$

Thus using six 100 N/m springs in the following arrangement will produce an equivalent stiffness of 225 N/m



1.89 Calculate the natural frequency of the system in Figure 1.32(a) if $k_1 = k_2 = 0$. Choose m and nonzero values of k_3, k_4, and k_5 so that the natural frequency is 100 Hz.

Solution: Given: $k_1 = k_2 = 0$ and $\omega_n = 2\pi(100) = 628.3$ rad/s

From Figure 1.29, the natural frequency is

$$\omega_n = \sqrt{\frac{k_5 k_3 + k_5 k_4 + k_3 k_4}{m(k_3 + k_4)}} \quad \text{and} \quad k_{eq} = \left(k_5 + \frac{k_3 k_4}{k_3 + k_4}\right)$$

Equating the given value of frequency to the analytical value yields:

$$\omega_n^2 = (628.3)^2 = \frac{k_5 k_3 + k_5 k_4 + k_3 k_4}{m(k_3 + k_4)}$$

Any values of k_3, k_4, k_5, and m that satisfy the above equation will do. Again, the answer is *not unique*. One solution is

$$k_3 = 1\text{ N/m}, k_4 = 1\text{ N/m}, k_5 = 50{,}000\text{ N/m}, \text{and } m = 0.127\text{ kg}$$

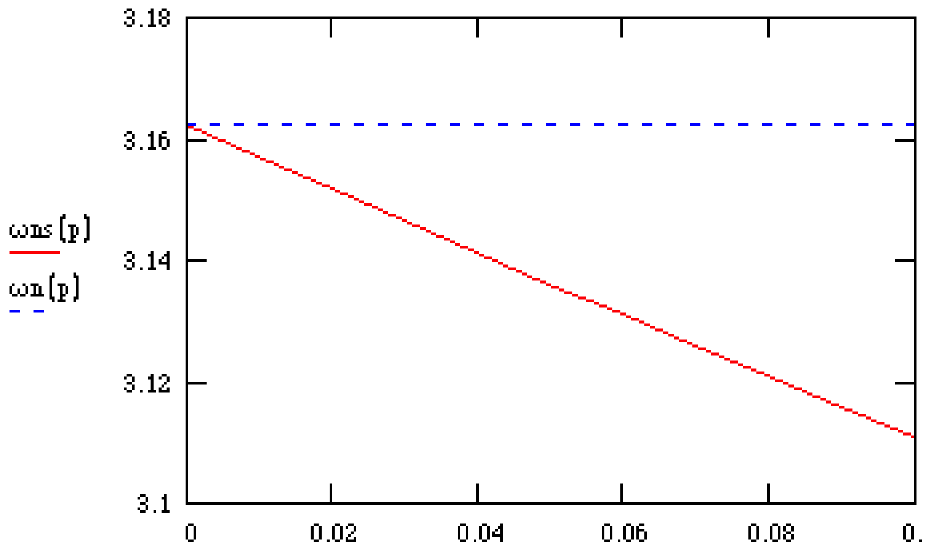
1.90* Example 1.4.4 examines the effect of the mass of a spring on the natural frequency of a simple spring-mass system. Use the relationship derived there and plot the natural frequency (normalized by the natural frequency, ω_n, for a massless spring) versus the percent that the spring mass is of the oscillating mass. Determine from the plot (or by algebra) the percentage where the natural frequency changes by 1% and therefore the situation when the mass of the spring should not be neglected.

Solution: The solution here depends on the value of the stiffness and mass ratio and hence the frequency. Almost any logical discussion is acceptable as long as the solution indicates that for smaller values of m_s, the approximation produces a reasonable frequency. Here is one possible answer. For

$k := 1000 \qquad m := 100$

$p := 0, 0.01 .. 0.1$

$$\omega ns(p) := \sqrt{\frac{k}{m + \frac{p \cdot m}{3}}} \qquad \omega n(p) := \sqrt{\frac{k}{m}}$$



From this plot, for these values of *m* and *k*, ***a 10 % spring mass causes less then a 1 % error in the frequency.***

1.91 Calculate the natural frequency and damping ratio for the system in Figure P1.91 given the values m = 10 kg, c = 100 kg/s, k_1 = 4000 N/m, k_2 = 200 N/m and k_3 = 1000 N/m. Assume that no friction acts on the rollers. Is the system overdamped, critically damped or underdamped?

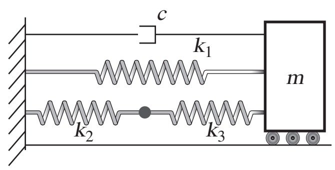


Figure P1.91

Solution: Following the procedure of Example 1.5.4, the equivalent spring constant is:

$$k_{eq} = k_1 + \frac{k_2 k_3}{k_2 + k_3} = 4000 + \frac{(200)(1000)}{1200} = 4167 \text{ N/m}$$

Then using the standard formulas for frequency and damping ratio:

$$\omega_n = \sqrt{\frac{k_{eq}}{m}} = \sqrt{\frac{4167}{10}} = 20.412 \text{ rad/s}$$

$$\zeta = \frac{c}{2m\omega_n} = \frac{100}{2(10)(20.412)} = 0.245$$

Thus the system is underdamped.

1.92 Calculate the natural frequency and damping ratio for the system in Figure P1.92. Assume that no friction acts on the rollers. Is the system overdamped, critically damped or underdamped?.

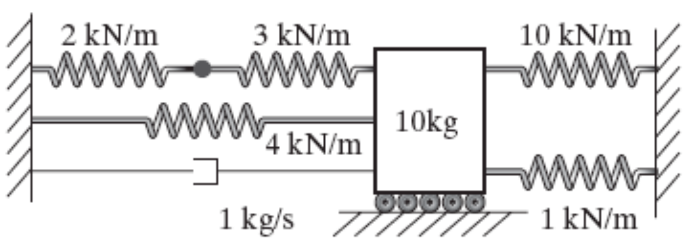


Figure P1.92

Solution: Again using the procedure of Example 1.5.4, the equivalent spring constant is:

$$k_{eq} = k_1 + k_2 + k_3 + \frac{k_4 k_5}{k_4 + k_5} = (10 + 1 + 4 + \frac{2 \times 3}{2+3})\text{kN/m} = 16.2 \text{ kN/m}$$

Then using the standard formulas for frequency and damping ratio:

$$\omega_n = \sqrt{\frac{k_{eq}}{m}} = \sqrt{\frac{16.2\times 10^3}{10}} = 40.25 \text{ rad/s}$$

$$\zeta = \frac{c}{2m\omega_n} = \frac{1}{2(10)(40.25)} = 0.001242 \approx \underline{0.001}$$

Thus the system is underdamped, in fact very lightly damped.

1.93 A manufacturer makes a cantilevered leaf spring from steel (E = 2 x 10^{11} N/m^2) and sizes the spring so that the device has a specific frequency. Later, to save weight, the spring is made of aluminum (E = 7.1 x 10^{10} N/m^2). Assuming that the mass of the spring is much smaller than that of the device the spring is attached to, determine if the frequency increases or decreases and by how much.

Solution: Use equation (1.67) to write the expression for the frequency twice:

$$\omega_{\text{al}} = \sqrt{\frac{3E_{\text{al}}}{m\ell^3}} \text{ and } \omega_{\text{steel}} = \sqrt{\frac{3E_{\text{steel}}}{m\ell^3}} \text{ rad/s}$$

Dividing yields:

$$\frac{\omega_{\text{al}}}{\omega_{\text{steel}}} = \frac{\sqrt{\frac{3E_{\text{al}}}{m\ell^3}}}{\sqrt{\frac{3E_{\text{steel}}}{m\ell^3}}} = \sqrt{\frac{7.1\times 10^{10}}{2\times 10^{11}}} = 0.596$$

Thus the ***frequency is decreased by about 40% by using aluminum***.