Chapter 2
The One-way Error Component Regression Model

2.2 (a) $Q = I_{NT} - P$ where $P = I_N \otimes \bar{J}_T$ is idempotent and is therefore its own generalized inverse. The variance-covariance matrix of the disturbance $\widetilde{\nu} = Q\nu$ in (2.6) is

$$E(\widetilde{\nu}\widetilde{\nu}') = E(Q\nu\nu'Q) = \sigma_\nu^2 Q$$

with generalized inverse Q/σ_ν^2. OLS on (2.6) yields

$$\widehat{\beta} = (X'QQX)^{-1}X'QQy = (X'QX)^{-1}XQy$$

which is $\widetilde{\beta}$ given in (2.7). Also, GLS on (2.6) using generalized inverse yields

$$\widehat{\widehat{\beta}} = (X'QQQX)^{-1}X'QQQy = (X'QX)^{-1}X'Qy = \widetilde{\beta}$$

(b) For the general linear model $y = X\beta + u$ with $E(uu') = \Omega$, a necessary and sufficient condition for OLS to be equivalent to GLS is given by $X'\Omega^{-1}\bar{P}_X = 0$, where $\bar{P}_X = I - P_X$ and $P_X = X(X'X)^{-1}X'$, see Baltagi (1989). For (2.6), this condition can be written as

$$(X'Q)\left(Q/\sigma_\nu^2\right)\bar{P}_{QX} = 0$$

using the fact that Q is idempotent, the left hand side can be written as $(X'Q)\bar{P}_{QX}/\sigma_\nu^2$ which is clearly 0, since $\bar{P}_{QX}$ is the orthogonal projection of QX.

2.5 $E(u'Pu) = E(\text{tr}(uu'P)) = \text{tr}(E(uu')P) = \text{tr}(\Omega P)$. From (2.18), $\Omega P = \sigma_1^2 P$ since $PQ = 0$. Hence, from (2.21), $E(\widehat{\sigma}_1^2) = \frac{E(u'Pu)}{\text{tr}(P)} = \frac{\text{tr}(\sigma_1^2 P)}{\text{tr}(P)} = \sigma_1^2$.

Similarly, $E(u'Qu) = \text{tr}(\Omega Q) = \text{tr}(\sigma_\nu^2 Q)$ where the last equality follows from (2.18) and the fact that $\Omega Q = \sigma_\nu^2 Q$ since $PQ = 0$. Hence from (2.22), $E(\widehat{\sigma}_\nu^2) = \frac{E(u'Qu)}{\text{tr}(Q)} = \frac{\text{tr}(\sigma_\nu^2 Q)}{\text{tr}(Q)} = \sigma_\nu^2$.

2.7 (b) GLS on (2.28) yields

$$\widehat{\delta}_{GLS} = \left[(Z'Q, Z'P) \begin{bmatrix} \sigma_\nu^2 Q & 0 \\ 0 & \sigma_1^2 P \end{bmatrix}^{-1} \begin{pmatrix} QZ \\ PZ \end{pmatrix} \right]^{-1}$$
$$\times (Z'Q, Z'P) \begin{bmatrix} \sigma_\nu^2 Q & 0 \\ 0 & \sigma_1^2 P \end{bmatrix}^{-1} \begin{pmatrix} Qy \\ Py \end{pmatrix}$$

Using generalized inverse

$$\begin{bmatrix} \sigma_\nu^2 Q & 0 \\ 0 & \sigma_1^2 P \end{bmatrix}^{-1} = \begin{bmatrix} Q/\sigma_\nu^2 & 0 \\ 0 & P/\sigma_1^2 \end{bmatrix}$$

one gets

$$\widehat{\delta}_{GLS} = \left[(Z'QZ)/\sigma_\nu^2 + (Z'PZ)/\sigma_1^2 \right]^{-1} \left[(Z'QZ)/\sigma_\nu^2 + (Z'Py)/\sigma_1^2 \right]$$
$$= (Z'\Omega^{-1}Z)^{-1} Z'\Omega^{-1} y$$

where Ω^{-1} is given by (2.19).
(a) OLS on (2.28) yields

$$\widehat{\delta}_{OLS} = \left[Z'Q, Z'P) \begin{pmatrix} QZ \\ PZ \end{pmatrix} \right]^{-1} (Z'Q, Z'P) \begin{pmatrix} Qy \\ Py \end{pmatrix}$$
$$= (Z'QZ + Z'PZ)^{-1}(Z'Qy + Z'Py)$$
$$= (Z'(Q+P)Z)^{-1} Z'(Q+P)y = (Z'Z)^{-1} Z'y$$

since $Q + P = I_{NT}$.

2.10 (a) From (2.2) and (2.38), $E(u_{i,T+S} u_{jt}) = \sigma_\mu^2$, for $i = j$ and zero otherwise. The only correlation over time occurs because of the presence of the same individual across the panel. The ν_{it}'s are not correlated for different time periods. In vector form

$$w = E(u_{i,T+S} u) = \sigma_\mu^2 (0, \ldots, 0, \ldots, 1, \ldots, 1, \ldots, 0, \ldots, 0)'$$

where there are T ones for the i-th individual. This can be rewritten as $w = \sigma_\mu^2 (\ell_i \otimes \iota_T)$ where ℓ_i is the i-th column of I_N, i.e., ℓ_i is a vector that has 1 in the i-th position and zero elsewhere. ι_T is a vector of ones of dimension T.
(b) $(\ell_i' \otimes \iota_T')P = (\ell_i' \otimes \iota_T')(I_N \otimes \frac{\iota_T \iota_T'}{T}) = (\ell_i' \otimes \iota_T')$. Therefore, in (2.39)

$$w'\Omega^{-1} = \sigma_\mu^2 (\ell_i' \otimes \iota_T') \left[\frac{1}{\sigma_1^2} P + \frac{1}{\sigma_\nu^2} Q \right] = \frac{\sigma_\mu^2}{\sigma_1^2} (\ell_i' \otimes \iota_T')$$

since $(\ell_i' \otimes \iota_T')Q = (\ell_i' \otimes \iota_T')(I_{NT} - P) = (\ell_i' \otimes \iota_T') - (\ell_i' \otimes \iota_T') = 0$.

2.11 The following program, using the Grunfeld (1958) data on investment on the Wiley web site, gives the SAS program and output for the estimation of a "One-way Error Component Regression". The estimators are OLS, WITHIN, BETWEEN, WALHUS, AMEMIYA, SWAR, NERLOVE, AND IMLE. The results do not replicate exactly those in Table 2.1 because there is no correction for degrees of freedom in these programs. However, the output reported below using EViews should match the results in Table 2.1.

```
DATA GRUNFELD;
      INFILE 'A:\GRUNFELD.FIL';
      INPUT FIRM YEAR INVEST STOCK CAPITAL;

     PROC IML;
        USE GRUNFELD;  READ ALL INTO TEMP;
        N=10;    T=20;   NT=N*T;
        One=Repeat(1,NT,1);
   Y=Temp[,3];   X=Temp[,4:5];   Z=One||X;   K=NCOL(X);
                              /* I, J, P and Q Matrices */
   l_t=J(T,1,1); JT=(l_t*l_t'); Z_U=I(N)@l_t;
   P=I(N)@JT/T;    Q=I(NT)-P;
   JNT=Repeat(JT,N,N);          JNT_BAR=JNT/NT;

 *--------- OLS ESTIMATORS ------------*;

   OLS_BETA=INV(Z'*Z)*Z'*Y;
   OLS_RES=Y-Z*OLS_BETA;
   VAR_REG=SSQ(OLS_RES)/(NT-NCOL(Z));
   VAR_COV=VAR_REG*INV(Z'*Z);
   STD_OLS=SQRT(VECDIAG(VAR_COV));
   T_OLS=OLS_BETA/STD_OLS;

  LOOK1=OLS_BETA||STD_OLS||T_OLS;
  CTITLE={'PARAMETER' 'STANDARD ERROR' 'T-STATISTICS'};
  RTITLE={'INTERCEPT' 'FIRMV' 'CAPITALV'};
  PRINT 'RESULTS OF OLS',,
  LOOK1(|COLNAME=CTITLE ROWNAME=RTITLE FORMAT=8.5|);

 *--------- BETWEEN ESTIMATOR ------------------*;

   BW_BETA=INV(Z'*P*Z)*Z'*P*Y;
   BW_RES=P*Y-P*Z*BW_BETA;
   VAR_BW=SSQ(BW_RES)/(N-NCOL(Z));   /* Equation 2.27 */
   V_C_BW=VAR_BW*INV(Z'*P*Z);
   STD_BW=SQRT(VECDIAG(V_C_BW));
   T_BW=BW_BETA/STD_BW;

  LOOK1=BW_BETA||STD_BW||T_BW;
  CTITLE={'PARAMETER' 'STANDARD ERROR' 'T-STATISTICS'};
  RTITLE={'INTERCEPT' 'FIRMV' 'CAPITALV'};
  PRINT 'RESULTS OF BETWEEN',,
  LOOK1(|COLNAME=CTITLE ROWNAME=RTITLE FORMAT=8.5|);
```

```
*--------- WITHIN ESTIMATORS ------------*;

   WT_BETA=INV(X'*Q*X)*X'*Q*Y;                  /* Equation 2.7*/
   WT_RES=Q*Y-Q*X*WT_BETA;
   VAR_WT=SSQ(WT_RES)/(NT-N-NCOL(X));           /* Equation 2.24  */
   V_C_WT=VAR_WT*INV(X'*Q*X);
   STD_WT=SQRT(VECDIAG(V_C_WT));
   T_WT=WT_BETA/STD_WT;

    WH_V_1=(OLS_RES'*P*OLS_RES)/N;
               /* Equation 2.21 with U=OLS residuals */

    ******* Checking for negative VAR_MHU *******;

    WH_V_MHU=(WH_V_1-WH_V_V)/T;
    IF WH_V_MHU<0 THEN NEGA_WH=1; ELSE NEGA_WH=0;
    WH_V_MHU=WH_V_MHU # (WH_V_MHU>0);
    WH_V_1=(T*WH_V_MHU)+WH_V_V;
    WH_V_T=WH_V_V+WH_V_MHU;
    WH_RHO=WH_V_MHU/WH_V_T;

    *********************************************;

    OMEGA_WH=(Q/WH_V_V)+(P/WH_V_1);    /* Equation 2.19 */
    WH_BETA=INV(Z'*OMEGA_WH*Z)*Z'*OMEGA_WH*Y;
    THETA_WH=1-(SQRT(WH_V_V)/SQRT(WH_V_1));
    OMEGAWH=(Q/SQRT(WH_V_V))+(P/SQRT(WH_V_1));
    WH_RES=(OMEGAWH*Y)-(OMEGAWH*Z*WH_BETA);
    VAR_WH=SSQ(WH_RES)/(NT-NCOL(Z));
    V_C_WH=INV(Z'*OMEGA_WH*Z);
    STD_WH=SQRT(VECDIAG(V_C_WH));
    T_WH=WH_BETA/STD_WH;

   LOOK1=WH_BETA||STD_WH||T_WH;
   CTITLE={'PARAMETER' 'STANDARD ERROR' 'T-STATISTICS'};
   RTITLE={'INTERCEPT' 'FIRMV' 'CAPITALV'};
   PRINT 'RESULTS OF WALLACE-HUSSAIN',,
   LOOK1(|COLNAME=CTITLE ROWNAME=RTITLE FORMAT=8.5|);

    FREE OMEGA_WH OMEGAWH WH_RES;

 *---- AMEMIYA ESTIMATOR OF VARIANCE COMPONENTS ----*;

    Y_BAR=Y[:];   X_BAR=X[:,];
    ALPHA_WT=Y_BAR-X_BAR*WT_BETA;
    LSDV_RES=Y-ALPHA_WT*ONE-X*WT_BETA;
    AM_V_V=(LSDV_RES'*Q*LSDV_RES)/(NT-N);
                /* Equation 2.22 with U=LSDV residuals */
    AM_V_1=(LSDV_RES'*P*LSDV_RES)/N;
                /*Equation 2.21 with U=LSDV residuals */
```

```
  LOOK1=WT_BETA||STD_WT||T_WT;
  CTITLE={'PARAMETER' 'STANDARD ERROR' 'T-STATISTICS'};
  RTITLE={'FIRMV' 'CAPITALV'};
  PRINT 'RESULTS OF WITHIN',,
  LOOK1(|COLNAME=CTITLE ROWNAME=RTITLE FORMAT=8.5|);

*-------- WALLACE & HUSSAIN ESTIMATOR OF VARIANCE COMPONENTS ----------*;

   WH_V_V=(OLS_RES'*Q*OLS_RES)/(NT-N);
              /* Equation 2.22 with U=OLS residuals */

   THETA_AM=1-(SQRT(AM_V_V)/SQRT(AM_V_1));
   OMEGAAM=(Q/SQRT(AM_V_V))+(P/SQRT(AM_V_1));
   AM_RES=(OMEGAAM*Y)-(OMEGAAM*Z*AM_BETA);
   VAR_AM=SSQ(AM_RES)/(NT-NCOL(Z));
   V_C_AM=INV(Z'*OMEGA_AM*Z);
   STD_AM=SQRT(VECDIAG(V_C_AM));
   T_AM=AM_BETA/STD_AM;

  LOOK1=AM_BETA||STD_AM||T_AM;
  CTITLE={'PARAMETER' 'STANDARD ERROR' 'T-STATISTICS'};
  RTITLE={'INTERCEPT' 'FIRMV' 'CAPITALV'};
  PRINT 'RESULTS OF AMEMIYA',,
  LOOK1(|COLNAME=CTITLE ROWNAME=RTITLE FORMAT=8.5|);

   FREE OMEGA_AM OMEGAAM AM_RES;

*--------- SWAMY & ARORA ESTIMATOR OF VARIANCE COMPONENTS -------------*;
    ***** Checking for negative VAR_MHU *********;

    AM_V_MHU=(AM_V_1-AM_V_V)/T;
    IF AM_V_MHU<0 THEN NEGA_AM=1; ELSE NEGA_AM=0;
    AM_V_MHU=AM_V_MHU # (AM_V_MHU>0);
    AM_V_1=(T*AM_V_MHU)+AM_V_V;

    AM_V_T=AM_V_V+AM_V_MHU;
    AM_RHO=AM_V_MHU/AM_V_T;

    **********************************************;

    OMEGA_AM=(Q/AM_V_V)+(P/AM_V_1);     /*Equation 2.19 */
    AM_BETA=INV(Z'*OMEGA_AM*Z)*Z'*OMEGA_AM*Y;

    SA_V_V=(Y'*Q*Y-Y'*Q*X*INV(X'*Q*X)*X'*Q*Y)/(NT-N-K);
                                     /* Equation 2.24 */
    SA_V_1=(Y'*P*Y-Y'*P*Z*INV(Z'*P*Z)*Z'*P*Y)/(N-K-1);
                                     /* Equation 2.27 */

    ****** Checking for negative VAR_MHU ********;

    SA_V_MHU=(SA_V_1-SA_V_V)/T;
    IF SA_V_MHU<0 THEN NEGA_SA=1; ELSE NEGA_SA=0;
    SA_V_MHU=SA_V_MHU # (SA_V_MHU>0);
    SA_V_1=(T*SA_V_MHU)+SA_V_V;
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SA_V_T=SA_V_V+SA_V_MHU;
SA_RHO=SA_V_MHU/SA_V_T;

*********************************************;

OMEGA_SA=(Q/SA_V_V)+(P/SA_V_1);    /* Equation 2.19 */
SA_BETA=INV(Z'*OMEGA_SA*Z)*Z'*OMEGA_SA*Y;
THETA_SA=1-(SQRT(SA_V_V)/SQRT(SA_V_1));
OMEGASA=(Q/SQRT(SA_V_V))+(P/SQRT(SA_V_1));
SA_RES=(OMEGASA*Y)-(OMEGASA*Z*SA_BETA);
VAR_SA=SSQ(SA_RES)/(NT-NCOL(Z));
V_C_SA=INV(Z'*OMEGA_SA*Z);
STD_SA=SQRT(VECDIAG(V_C_SA));
T_SA=SA_BETA/STD_SA;

LOOK1=SA_BETA||STD_SA||T_SA;
CTITLE={'PARAMETER' 'STANDARD ERROR' 'T-STATISTICS'};
RTITLE={'INTERCEPT' 'FIRMV' 'CAPITALV'};
PRINT 'RESULTS OF SWAMY-ARORA',,
LOOK1(|COLNAME=CTITLE ROWNAME=RTITLE FORMAT=8.5|);

FREE OMEGA_SA OMEGASA SA_RES;

*--------- NERLOVE ESTIMATOR OF VARIANCE COMPONENTS AND BETA -----------*;

MHU=P*LSDV_RES;
MEAN_MHU=MHU[:];
DEV_MHU=MHU-(ONE*MEAN_MHU);
VAR_MHU=SSQ(DEV_MHU)/(T*(N-1));
NL_V_V=SSQ(WT_RES)/NT;
NL_V_1=T*VAR_MHU+NL_V_V;
OMEGA_NL=(Q/NL_V_V)+(P/NL_V_1);    /* Equation 2.19 */
NL_BETA=INV(Z'*OMEGA_NL*Z)*Z'*OMEGA_NL*Y;
THETA_NL=1-(SQRT(NL_V_V)/SQRT(NL_V_1));
OMEGANL=(Q/SQRT(NL_V_V))+(P/SQRT(NL_V_1));
NL_RES=(OMEGANL*Y)-(OMEGANL*Z*NL_BETA);
VAR_NL=SSQ(NL_RES)/(NT-NCOL(Z));
V_C_NL=INV(Z'*OMEGA_NL*Z);
STD_NL=SQRT(VECDIAG(V_C_NL));
T_NL=NL_BETA/STD_NL;

NL_V_T=NL_V_V+VAR_MHU;
NL_RHO=VAR_MHU/NL_V_T;

LOOK1=NL_BETA||STD_NL||T_NL;
CTITLE={'PARAMETER' 'STANDARD ERROR' 'T-STATISTICS'};
RTITLE={'INTERCEPT' 'FIRMV' 'CAPITALV'};
PRINT 'RESULTS OF NERLOVE',,
LOOK1(|COLNAME=CTITLE ROWNAME=RTITLE FORMAT=8.5|);

FREE OMEGA_NL OMEGANL NL_RES;

*---------- MAXIMUM LIKELIHOOD ESTIMATION ---------------------*;
/* START WITH WITHIN AND BETWEEN BETA SUGGESTED BY BREUSCH(1987) */;

CRITICAL=1;
BETA_W=WT_BETA;
BETA_B=BW_BETA[2:K+1,];
BETA_MLE=WT_BETA;
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```
    *OMEGA_ML=(Q+PHISQ_ML*P)/VAR_V_ML;
    *V_C_MLE=INV(X'*OMEGA_ML*X);
    *STD_MLE=SQRT(VECDIAG(V_C_MLE));
    VAR_1_ML=VAR_V_ML/PHISQ_ML;
    OMEGA_ML=(Q/VAR_V_ML)+(P/VAR_1_ML);

    ML_BETA=INV(Z'*OMEGA_ML*Z)*Z'*OMEGA_ML*Y;
    OMEGAML=(Q/SQRT(VAR_V_ML))+(P/SQRT(VAR_1_ML));
    ML_RES=(OMEGAML*Y)-(OMEGAML*Z*ML_BETA);
    VAR_ML=SSQ(ML_RES)/(NT-NCOL(Z));
    V_C_ML=INV(Z'*OMEGA_ML*Z);
    STD_ML=SQRT(VECDIAG(V_C_ML));
    T_ML=ML_BETA/STD_ML;

   LOOK1=ML_BETA||STD_ML||T_ML;
   CTITLE={'PARAMETER' 'STANDARD ERROR' 'T-STATISTICS'};
   RTITLE={'INTERCEPT' 'FIRMV' 'CAPITALV'};
   PRINT 'RESULTS OF MAXIMUM LIKELIHOOD',,
   LOOK1(|COLNAME=CTITLE ROWNAME=RTITLE FORMAT=8.5|);

    FREE OMEGA_ML;

*---------- PRINT AND OUTPUT INFORMATION ---------------------*;

    BETA=OLS_BETA'[,2:K+1]//BW_BETA'[,2:K+1]//WT_BETA'//
    WH_BETA'[,2:K+1]//AM_BETA'[,2:K+1]//
    SA_BETA'[,2:K+1]//NL_BETA'[,2:K+1]//ML_BETA'[,2:K+1];

    STD_ERR=STD_OLS'[,2:K+1]//STD_BW'[,2:K+1]//STD_WT'//
    STD_WH'[,2:K+1]//STD_AM'[,2:K+1]//STD_SA'[,2:K+1]//
    STD_NL'[,2:K+1]//STD_ML'[,2:K+1];

    THETAS={0, ., 1}//THETA_WH//THETA_AM//THETA_SA//THETA_NL//THETA_ML;

    NEGA_VAR={.,.,.}//NEGA_WH//NEGA_AM//NEGA_SA//{.,.};

    OUTPUT=BETA||STD_ERR||THETAS||NAGA_VAR;

 DO WHILE (CRITICAL>0.0001);
    WT_RES=Y-X*BETA_W;
    BW_RES=Y-X*BETA_B;
    PHISQ_W=(WT_RES'*Q*WT_RES)/((T-1)*(WT_RES'*(P-JNT_BAR)*WT_RES));
                                                      /* Equation 2.35 */
    PHISQ_B=(BW_RES'*Q*BW_RES)/((T-1)*(BW_RES'*(P-JNT_BAR)*BW_RES));
                                                      /* Equation 2.35 */
    CRITICAL=PHISQ_W-PHISQ_B;
    BETA_W=INV(X'*(Q+PHISQ_W*(P-JNT_BAR))*X)*X'*(Q+PHISQ_W*(P-JNT_BAR))*Y;
                                                        /* Equation 2.36 */
    BETA_B=INV(X'*(Q+PHISQ_B*(P-JNT_BAR))*X)*X'*(Q+PHISQ_B*(P-JNT_BAR))*Y;
                                                        /* Equation 2.36 */
    BETA_MLE=(BETA_W+BETA_B)/2;
  END;
      D_MLE=Y-X*BETA_MLE;
      PHISQ_ML=(D_MLE'*Q*D_MLE)/((T-1)*D_MLE'*(P-JNT_BAR)*D_MLE);
                                                     /* Equation 2.35 */
      THETA_ML=1-SQRT(PHISQ_ML);
      VAR_V_ML=D_MLE'*(Q+PHISQ_ML*(P-JNT_BAR))*D_MLE/NT;
```

```
C1={"BETA1" "BETA2" "STD_BETA1" "STD_BETA2" "THETA'};
C2={"BETA1"  "BETA2' 'BETA3' 'STD_BETA1"  "STD_BETA2'
    "STD_BETA3"  'THETA'};

R={"OLS ESTIMATOR'  "BETWEEN ESTIMATOR"  "WITHIN ESTIMATOR'
    "WALLACE & HUSSAIN ESTIMATOR"  "AMEMIYA ESTIMATOR"
    "SWAMY & ARORA ESTIMATOR"  'NERLOVE ESTIMATOR" "IMLE'};

PRINT 'ONE-WAY ERROR COMPONENT MODEL WITH GRUNFELD DATA:
       BETA, VARIANCES OF BETA, AND THETA'
      ,,OUTPUT (|ROWNAME=R COLNAME=C1  FORMAT=8.5|);

PRINT 'NEGATIVE VAR_MHU',,NEGA_VAR (|ROWNAME=R|);
WH_VAR=WH_V_V||WH_V_MHU||WH_V_1||WH_V_T||WH_RHO;
AM_VAR=AM_V_V||AM_V_MHU||AM_V_1||AM_V_T||AM_RHO;
SA_VAR=SA_V_V||SA_V_MHU||SA_V_1||SA_V_T||SA_RHO;
NL_VAR=NL_V_V||VAR_MHU||NL_V_1||NL_V_T||NL_RHO;

OUTPUT=WH_VAR//AM_VAR//SA_VAR//NL_VAR;

R={'WALLACE & HUSSAIN ESTIMATOR"  'AMEMIYA ESTIMATOR"
   'SWAMY & ARORA ESTIMATOR"  'NERLOVE ESTIMATOR"};

C1={"SIGMA-NU-SQ" "SIGMA-MU-SQ' 'SIGMA-1-SQ" "SIGMA-SQ" "RHO"};

PRINT 'ONE-WAY VARIANCE COMPONENTS FOR GRUNFELD DATA'
      ,,OUTPUT (|ROWNAME=R COLNAME=C1  FORMAT=12.3|);
```

/*

SAS Output for Grunfeld Data

RESULTS OF OLS

LOOK1	PARAMETER	STANDARD ERROR	T-STATISTICS
INTERCEPT	-42.7144	9.51168	-4.49073
FIRMV	0.11556	0.00584	19.80259
CAPITALV	0.23068	0.02548	9.05481

RESULTS OF BETWEEN

LOOK1	PARAMETER	STANDARD ERROR	T-STATISTICS
INTERCEPT	-8.52711	47.51531	-0.17946
FIRMV	0.13465	0.02875	4.68408
CAPITALV	0.03203	0.19094	0.16776

RESULTS OF WITHIN

LOOK1	PARAMETER	STANDARD ERROR	T-STATISTICS
FIRMV	0.11012	0.01186	9.28790
CAPITALV	0.31007	0.01735	17.86656

RESULTS OF WALLACE-HUSSAIN

LOOK1	PARAMETER	STANDARD ERROR	T-STATISTICS
INTERCEPT	-57.5539	26.45011	-2.17594
FIRMV	0.10971	0.01063	10.32157
CAPITALV	0.30737	0.01803	17.04600

RESULTS OF AMEMIYA

LOOK1	PARAMETER	STANDARD ERROR	T-STATISTICS
INTERCEPT	-57.7711	27.75255	-2.08165
FIRMV	0.10976	0.01034	10.61206
CAPITALV	0.30795	0.01707	18.03867

RESULTS OF SWAMY-ARORA

LOOK1	PARAMETER	STANDARD ERROR	T-STATISTICS
INTERCEPT	-57.8344	28.88930	-2.00193
FIRMV	0.10978	0.01049	10.46615
CAPITALV	0.30811	0.01717	17.93989

RESULTS OF NERLOVE

LOOK1	PARAMETER	STANDARD ERROR	T-STATISTICS
INTERCEPT	-57.9074	29.24211	-1.98027
FIRMV	0.10980	0.01027	10.68948
CAPITALV	0.30829	0.01667	18.49906

RESULTS OF MAXIMUM LIKELIHOOD

LOOK1	PARAMETER	STANDARD ERROR	T-STATISTICS
INTERCEPT	-57.7632	27.64352	-2.08957
FIRMV	0.10976	0.01033	10.62083
CAPITALV	0.30793	0.01707	18.03485

ONE-WAY ERROR COMPONENT MODEL WITH GRUNFELD DATA:
BETA, VARIANCES OF BETA, AND THETA

OUTPUT	BETA1	BETA2	STD_BETA1	STD_BETA2	THETA
OLS ESTIMATOR	0.11556	0.23068	0.00584	0.02548	0.00000
BETWEEN ESTIMATOR	0.13465	0.03203	0.02875	0.19094	.
WITHIN ESTIMATOR	0.11012	0.31007	0.01186	0.01735	1.00000
WALLACE & HUSSAIN ESTIMATOR	0.10971	0.30737	0.01063	0.01803	0.83744
AMEMIYA ESTIMATOR	0.10976	0.30795	0.01034	0.01707	0.85569
SWAMY & ARORA ESTIMATOR	0.10978	0.30811	0.01049	0.01717	0.86122
NERLOVE ESTIMATOR	0.10980	0.30829	0.01027	0.01667	0.86774
IMLE	0.10976	0.30793	0.01033	0.01707	0.85501

```
                          NEGATIVE VAR_MHU

NEGA_VAR
OLS ESTIMATOR                         .
BETWEEN ESTIMATOR                     .
WITHIN ESTIMATOR                      .
WALLACE & HUSSAIN ESTIMATOR           0
AMEMIYA ESTIMATOR                     0
SWAMY & ARORA ESTIMATOR               0
NERLOVE ESTIMATOR                     .
IMLE                                  .

           ONE-WAY VARIANCE COMPONENTS FOR GRUNFELD DATA

OUTPUT             SIGMA-NU-SQ SIGMA-MU-SQ SIGMA-1-SQ SIGMA-SQ   RHO
WALLACE & HUSSAIN    3089.071    5690.182  116892.705 8779.252  0.648
   ESTIMATOR
AMEMIYA ESTIMATOR    2755.148    6477.298  132301.113 9232.446  0.702
SWAMY & ARORA        2784.458    7089.800  144580.460 9874.258  0.718
   ESTIMATOR
NERLOVE ESTIMATOR    2617.391    7350.062  149618.628 9967.453  0.737
**************************************************************************
```

The following output was produced using EViews 5.0. This should replicate Table 2.1 and differs from the above SAS output in that degrees of freedom adjustments are made when estimating the variance components.

For Pooled OLS:

Dependent Variable: I
Method: Panel Least Squares

Sample: 1935 1954
Cross-sections included: 10
Total panel (balanced) observations: 200

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	−42.71437	9.511676	−4.490730	0.0000
F	0.115562	0.005836	19.80259	0.0000
K	0.230678	0.025476	9.054808	0.0000

R-squared	0.812408	Mean dependent var	145.9582
Adjusted R-squared	0.810504	S.D. dependent var	216.8753
S.E. of regression	94.40840	Akaike info criterion	11.94802
Sum squared resid	1755850.	Schwarz criterion	11.99750
Log likelihood	−1191.802	F-statistic	426.5757
Durbin-Watson stat	0.219599	Prob(F-statistic)	0.000000

For the one-way fixed effects estimator with firm effects:

Dependent Variable: I
Method: Panel Least Squares

Sample: 1935 1954
Cross-sections included: 10
Total panel (balanced) observations: 200

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	−58.74394	12.45369	−4.716990	0.0000
F	0.110124	0.011857	9.287901	0.0000
K	0.310065	0.017355	17.86656	0.0000

Effects Specification

Cross-section fixed (dummy variables)

R-squared	0.944073	Mean dependent var	145.9582
Adjusted R-squared	0.940800	S.D. dependent var	216.8753
S.E. of regression	52.76797	Akaike info criterion	10.82781
Sum squared resid	523478.1	Schwarz criterion	11.02571
Log likelihood	−1070.781	F-statistic	288.4996
Durbin-Watson stat	0.716733	Prob(F-statistic)	0.000000

For the Wallace and Hussain random effects estimator reported in Table 2.2, we get:

Dependent Variable: I
Method: Panel EGLS (Cross-section random effects)

Sample: 1935 1954
Cross-sections included: 10
Total panel (balanced) observations: 200
Wallace and Hussain estimator of component variances

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	−57.86253	29.90492	−1.934883	0.0544
F	0.109789	0.010725	10.23698	0.0000
K	0.308183	0.017498	17.61207	0.0000

Effects Specification

	S.D.	Rho
Cross-section random S.D. / Rho	87.35803	0.7254
Idiosyncratic random S.D. / Rho	53.74518	0.2746

Weighted Statistics

R-squared	0.769410	Mean dependent var	19.89203
Adjusted R-squared	0.767069	S.D. dependent var	109.2808
S.E. of regression	52.74214	Sum squared resid	548001.4
F-statistic	328.6646	Durbin-Watson stat	0.683829
Prob(F-statistic)	0.000000		

Unweighted Statistics

R-squared	0.803285	Mean dependent var	145.9582
Sum squared resid	1841243.	Durbin-Watson stat	0.203525

For the Amemiya random effects estimator (which EViews names the Wansbeek-Kapteyn estimator) reported in Table 2.3, we get:

Dependent Variable: I
Method: Panel EGLS (Cross-section random effects)

Sample: 1935 1954
Cross-sections included: 10
Total panel (balanced) observations: 200
Wansbeek and Kapteyn estimator of component variances

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	−57.82187	28.68562	−2.015710	0.0452
F	0.109778	0.010471	10.48387	0.0000
K	0.308081	0.017172	17.94062	0.0000

Effects Specification

Cross-section random S.D. / Rho	83.52354	0.7147
Idiosyncratic random S.D. / Rho	52.76797	0.2853

Weighted Statistics

R-squared	0.769544	Mean dependent var	20.41664
Adjusted R-squared	0.767205	S.D. dependent var	109.4431
S.E. of regression	52.80503	Sum squared resid	549309.2
F-statistic	328.9141	Durbin-Watson stat	0.682171
Prob(F-statistic)	0.000000		

Unweighted Statistics

R-squared	0.803313	Mean dependent var	145.9582
Sum squared resid	1840981.	Durbin-Watson stat	0.203545

For the Swamy and Arora random effects estimator reported in Table 2.4, we get:

Dependent Variable: I
Method: Panel EGLS (Cross-section random effects)

Sample: 1935 1954
Cross-sections included: 10
Total panel (balanced) observations: 200
Swamy and Arora estimator of component variances

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	−57.83441	28.88930	−2.001932	0.0467
F	0.109781	0.010489	10.46615	0.0000
K	0.308113	0.017175	17.93989	0.0000

Effects Specification

Cross-section random S.D. / Rho	84.20095	0.7180
Idiosyncratic random S.D. / Rho	52.76797	0.2820

Weighted Statistics

R-squared	0.769503	Mean dependent var	20.25556
Adjusted R-squared	0.767163	S.D. dependent var	109.3928
S.E. of regression	52.78556	Sum squared resid	548904.1
F-statistic	328.8369	Durbin-Watson stat	0.682684
Prob(F-statistic)	0.000000		

Unweighted Statistics

R-squared	0.803304	Mean dependent var	145.9582
Sum squared resid	1841062.	Durbin-Watson stat	0.203539

Now we reproduce some of these estimates for the Grunfeld data set using Stata

```
. infile fn yr I F C using a:/grunfeld.fil
'FN' cannot be read as a number for fn[1]
'YR' cannot be read as a number for yr[1]
'I' cannot be read as a number for I[1]
'F' cannot be read as a number for F[1]
'C' cannot be read as a number for C[1]
(201 observations read)
. list

          fn        yr        I         F         C
  1.       .         .        .         .         .
  2.       1      1935    317.6    3078.5       2.8
  3.       1      1936    391.8    4661.7      52.6
  4.       1      1937    410.6    5387.1     156.9
  5.       1      1938    257.7    2792.2     209.2
  6.       1      1939    330.8    4313.2     203.4
  7.       1      1940    461.2    4643.9     207.2
  8.       1      1941      512    4551.2     255.2
  9.       1      1942      448    3244.1     303.7
```

```
. edit
- preserve
- drop in 1
- preserve
. sum

    Variable |       Obs        Mean    Std. Dev.       Min        Max
-------------+--------------------------------------------------------
          fn |       200         5.5    2.879489          1         10
          yr |       200      1944.5    5.780751       1935       1954
           I |       200    145.9583    216.8753        .93     1486.7
           F |       200    1081.681     1314.47      58.12     6241.7
           C |       200    276.0172    301.1039         .8     2226.3

. reg I F C

      Source |       SS       df       MS              Number of obs =     200
-------------+------------------------------           F(  2,   197) =  426.58
       Model |  7604093.48     2  3802046.74           Prob > F      =  0.0000
    Residual |  1755850.43   197  8912.94636           R-squared     =  0.8124
-------------+------------------------------           Adj R-squared =  0.8105
       Total |  9359943.92   199  47034.8941           Root MSE      =  94.408

------------------------------------------------------------------------------
           I |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
           F |   .1155622   .0058357    19.80   0.000     .1040537    .1270706
           C |   .2306785   .0254758     9.05   0.000     .1804382    .2809188
       _cons |  -42.71437   9.511676    -4.49   0.000    -61.47215   -23.95659
------------------------------------------------------------------------------
. iis fn
. xtreg I F C, re

Random-effects GLS regression                   Number of obs      =       200
Group variable (i) : fn                         Number of groups   =        10

R-sq:  within  = 0.7668                         Obs per group: min =        20
       between = 0.8196                                        avg =      20.0
       overall = 0.8061                                        max =        20

Random effects u_i ~ Gaussian                   Wald chi2(2)       =    657.67
corr(u_i, X)       = 0 (assumed)                Prob > chi2        =    0.0000

------------------------------------------------------------------------------
           I |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
           F |   .1097811   .0104927    10.46   0.000     .0892159    .1303464
           C |    .308113   .0171805    17.93   0.000     .2744399    .3417861
       _cons |  -57.83441   28.89893    -2.00   0.045    -114.4753   -1.193537
-------------+----------------------------------------------------------------
     sigma_u |   84.20095
     sigma_e |  52.767964
         rho |   .71800838   (fraction of variance due to u_i)
------------------------------------------------------------------------------

. xtreg I F C, fe

Fixed-effects (within) regression               Number of obs      =       200
Group variable (i) : fn                         Number of groups   =        10

R-sq:  within  = 0.7668                         Obs per group: min =        20
       between = 0.8194                                        avg =      20.0
       overall = 0.8060                                        max =        20
```

```
                                                F(2,188)           =    309.01
corr(u_i, Xb)  = -0.1517                        Prob > F           =    0.0000

------------------------------------------------------------------------------
           I |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
           F |   .1101238   .0118567     9.29   0.000     .0867345    .1335131
           C |   .3100653   .0173545    17.87   0.000     .2758308    .3442999
       _cons |  -58.74393   12.45369    -4.72   0.000    -83.31086     -34.177
-------------+----------------------------------------------------------------
     sigma_u |  85.732501
     sigma_e |  52.767964
         rho |  .72525012   (fraction of variance due to u_i)
------------------------------------------------------------------------------
F test that all u_i=0:     F(9, 188) =    49.18             Prob > F = 0.0000

. xtreg I F C, be

Between regression (regression on group means)  Number of obs      =       200
Group variable (i) : fn                         Number of groups   =        10

R-sq:  within  = 0.4778                         Obs per group: min =        20
       between = 0.8578                                        avg =      20.0
       overall = 0.7551                                        max =        20

                                                F(2,7)             =     21.11
sd(u_i + avg(e_i.))=  85.02366                  Prob > F           =    0.0011

------------------------------------------------------------------------------
           I |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
           F |   .1346461   .0287455     4.68   0.002     .0666739    .2026183
           C |   .0320315   .1909378     0.17   0.872    -.4194647    .4835276
       _cons |  -8.527114   47.51531    -0.18   0.863     -120.883    103.8287
------------------------------------------------------------------------------

. xtreg I F C, i(fn) re

Random-effects GLS regression                   Number of obs      =       200
Group variable (i) : fn                         Number of groups   =        10

R-sq:  within  = 0.7668                         Obs per group: min =        20
       between = 0.8196                                        avg =      20.0
       overall = 0.8061                                        max =        20

Random effects u_i ~ Gaussian                   Wald chi2(2)       =    657.67
corr(u_i, X)       = 0 (assumed)                Prob > chi2        =    0.0000

------------------------------------------------------------------------------
           I |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
           F |   .1097811   .0104927    10.46   0.000     .0892159    .1303464
           C |    .308113   .0171805    17.93   0.000     .2744399    .3417861
       _cons |  -57.83441   28.89893    -2.00   0.045    -114.4753   -1.193537
-------------+----------------------------------------------------------------
     sigma_u |   84.20095
     sigma_e |  52.767964
         rho |  .71800838   (fraction of variance due to u_i)
------------------------------------------------------------------------------
```

```
. xtreg I F C, mle

Fitting constant-only model:
Iteration 0:   log likelihood = -1387.6302
Iteration 1:   log likelihood = -1291.9897
Iteration 2:   log likelihood = -1254.2888
Iteration 3:   log likelihood = -1243.6309
Iteration 4:   log likelihood = -1242.0548
Iteration 5:   log likelihood = -1241.9709
Iteration 6:   log likelihood = -1241.9696
Iteration 7:   log likelihood = -1241.9696

Fitting full model:
Iteration 0:   log likelihood = -1105.6101
Iteration 1:   log likelihood = -1098.8418
Iteration 2:   log likelihood = -1095.4188
Iteration 3:   log likelihood = -1095.2576
Iteration 4:   log likelihood =  -1095.257

Random-effects ML regression                    Number of obs      =       200
Group variable (i) : fn                         Number of groups   =        10

Random effects u_i ~ Gaussian                   Obs per group: min =        20
                                                               avg =      20.0
                                                               max =        20

                                                LR chi2(2)         =    293.43
Log likelihood  =  -1095.257                    Prob > chi2        =    0.0000

------------------------------------------------------------------------------
           I |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
           F |   .1097626   .0103389    10.62   0.000     .0894988    .1300265
           C |    .307942   .0171006    18.01   0.000     .2744254    .3414585
       _cons |   -57.7672   27.70004    -2.09   0.037    -112.0583   -3.476114
-------------+----------------------------------------------------------------
    /sigma_u |   80.29729   18.37811     4.37   0.000     44.27685    116.3177
    /sigma_e |   52.49255    2.69306    19.49   0.000     47.21424    57.77085
-------------+----------------------------------------------------------------
         rho |   .7005943   .0985226                      .4881266    .8603709
------------------------------------------------------------------------------
Likelihood ratio test of sigma_u=0: chibar2(01)= 193.09 Prob>=chibar2 = 0.000

------------------------------------------------------------------------------
```

2.12 This program gives the SAS output for estimating "One-way Error Component Regression" for Gasoline. Estimators are OLS, WITHIN, BETWEEN, WALHUS, AMEMIYA, SWAR, NERLOVE, and IMLE. The results do not replicate exactly those in Table 2.5 because there is no correction for degrees of freedom in these programs. However, the output reported below using EViews should match the results in Table 2.5.

SAS Output for Gasoline Data

RESULTS OF OLS

LOOK1	PARAMETER	STANDARD ERROR	T-STATISTICS
INTERCEPT	2.39133	0.11693	20.45017
INCOME	0.88996	0.03581	24.85523
PRICE	-0.89180	0.03031	-29.4180
CAR	-0.76337	0.01861	-41.0232

RESULTS OF BETWEEN

LOOK1	PARAMETER	STANDARD ERROR	T-STATISTICS
INTERCEPT	2.54163	0.52678	4.82480
INCOME	0.96758	0.15567	6.21571
PRICE	-0.96355	0.13292	-7.24902
CAR	-0.79530	0.08247	-9.64300

RESULTS OF WITHIN

LOOK1	PARAMETER	STANDARD ERROR	T-STATISTICS
INCOME	0.66225	0.07339	9.02419
PRICE	-0.32170	0.04410	-7.29496
CAR	-0.64048	0.02968	-21.5804

RESULTS OF WALLACE-HUSSAIN

LOOK1	PARAMETER	STANDARD ERROR	T-STATISTICS
INTERCEPT	1.90580	0.19403	9.82195
INCOME	0.54346	0.06353	8.55377
PRICE	-0.47111	0.04550	-10.3546
CAR	-0.60613	0.02840	-21.3425

RESULTS OF AMEMIYA

LOOK1	PARAMETER	STANDARD ERROR	T-STATISTICS
INTERCEPT	2.18445	0.21453	10.18228
INCOME	0.60093	0.06542	9.18559
PRICE	-0.36639	0.04138	-8.85497
CAR	-0.62039	0.02718	-22.8227

RESULTS OF SWAMY-ARORA

LOOK1	PARAMETER	STANDARD ERROR	T-STATISTICS
INTERCEPT	1.99670	0.17824	11.20260
INCOME	0.55499	0.05717	9.70689
PRICE	-0.42039	0.03866	-10.8748
CAR	-0.60684	0.02467	-24.5964

RESULTS OF NERLOVE

LOOK1	PARAMETER	STANDARD ERROR	T-STATISTICS
INTERCEPT	2.20177	0.21252	10.36040
INCOME	0.60561	0.06432	9.41526
PRICE	-0.36243	0.04049	-8.95152
CAR	-0.62189	0.02666	-23.3289

RESULTS OF MAXIMUM LIKELIHOOD

LOOK1	PARAMETER	STANDARD ERROR	T-STATISTICS
INTERCEPT	2.10623	0.20096	10.48088
INCOME	0.58044	0.06286	9.23388
PRICE	-0.38582	0.04072	-9.47515
CAR	-0.61401	0.02647	-23.2001

ONE-WAY ERROR COMPONENT MODEL WITH GASOLINE DATA:
BETA, VARIANCES OF BETA, AND THETA

OUTPUT	INCOME	PRICE	CAR	STD_INC	STD_PRICE	STD_CAR	THETA
OLS ESTIMATOR	0.88996	-0.89180	-0.76337	0.03581	0.03031	0.01861	0.00000
BETWEEN ESTIMATOR	0.96758	-0.96355	-0.79530	0.15567	0.13292	0.08247	.
WITHIN ESTIMATOR	0.66225	-0.32170	-0.64048	0.07339	0.04410	0.02968	1.00000
WALLACE & HUSSAIN ESTIMATOR	0.54346	-0.47111	-0.60613	0.06353	0.04550	0.02840	0.84802
AMEMIYA ESTIMATOR	0.60093	-0.36639	-0.62039	0.06542	0.04138	0.02718	0.93773
SWAMY & ARORA ESTIMATOR	0.55499	-0.42039	-0.60684	0.05717	0.03866	0.02467	0.89231
NERLOVE ESTIMATOR	0.60561	-0.36243	-0.62189	0.06432	0.04049	0.02666	0.94120
IMLE	0.58044	-0.38582	-0.61401	0.06286	0.04072	0.02647	0.92126

NEGATIVE VAR_MHU

NEGA_VAR	COL1
OLS ESTIMATOR	.
BETWEEN ESTIMATOR	.
WITHIN ESTIMATOR	.
WALLACE & HUSSAIN ESTIMATOR	0
AMEMIYA ESTIMATOR	0
SWAMY & ARORA ESTIMATOR	0
NERLOVE ESTIMATOR	.
IMLE	.

ONE-WAY VARIANCE COMPONENTS FOR GASOLINE DATA

OUTPUT	SIGMA-NU-SQ	SIGMA-MU-SQ	SIGMA-1-SQ	SIGMA-SQ	RHO
WALLACE & HUSSAIN ESTIMATOR	0.01351	0.03007	0.58487	0.04358	0.69003
AMEMIYA ESTIMATOR	0.00845	0.11420	2.17830	0.12265	0.93114
SWAMY & ARORA ESTIMATOR	0.00852	0.03824	0.73504	0.04676	0.81770
NERLOVE ESTIMATOR	0.00800	0.12139	2.31444	0.12939	0.93816

The following output was produced using EViews 5.0. This should replicate Table 2.5 and differs from the above SAS output in that degrees of freedom adjustments are made when estimating the variance components.

For Pooled OLS:

Dependent Variable: GAS
Method: Panel Least Squares

Sample: 1960 1978
Cross-sections included: 18
Total panel (balanced) observations: 342

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	2.391326	0.116934	20.45017	0.0000
INC	0.889962	0.035806	24.85523	0.0000
PMG	−0.891798	0.030315	−29.41796	0.0000
CAR	−0.763373	0.018608	−41.02325	0.0000

R-squared	0.854935	Mean dependent var	4.296242
Adjusted R-squared	0.853648	S.D. dependent var	0.548907
S.E. of regression	0.209990	Akaike info criterion	−0.271888
Sum squared resid	14.90436	Schwarz criterion	−0.227037
Log likelihood	50.49289	F-statistic	663.9993
Durbin-Watson stat	0.137461	Prob(F-statistic)	0.000000

For the **one-way fixed effects estimator** with country effects:

Dependent Variable: GAS
Method: Panel Least Squares

Sample: 1960 1978
Cross-sections included: 18
Total panel (balanced) observations: 342

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	2.402670	0.225309	10.66387	0.0000
INC	0.662250	0.073386	9.024191	0.0000
PMG	−0.321702	0.044099	−7.294964	0.0000
CAR	−0.640483	0.029679	−21.58045	0.0000

Effects Specification

Cross-section fixed (dummy variables)

R-squared	0.973366	Mean dependent var	4.296242
Adjusted R-squared	0.971706	S.D. dependent var	0.548907
S.E. of regression	0.092330	Akaike info criterion	−1.867450
Sum squared resid	2.736491	Schwarz criterion	−1.631979
Log likelihood	340.3340	F-statistic	586.5556
Durbin-Watson stat	0.326578	Prob(F-statistic)	0.000000

For the **Wallace and Hussain random effects estimator**, we get:

Dependent Variable: GAS
Method: Panel EGLS (Cross-section random effects)

Sample: 1960 1978
Cross-sections included: 18
Total panel (balanced) observations: 342
Wallace and Hussain estimator of component variances

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	1.938318	0.201817	9.604333	0.0000
INC	0.545202	0.065555	8.316682	0.0000
PMG	−0.447490	0.045763	−9.778438	0.0000
CAR	−0.605086	0.028838	−20.98191	0.0000

Effects Specification

Cross-section random S.D. / Rho	0.196715	0.7508
Idiosyncratic random S.D. / Rho	0.113320	0.2492

Weighted Statistics

R-squared	0.826568	Mean dependent var	0.562884
Adjusted R-squared	0.825029	S.D. dependent var	0.233119
S.E. of regression	0.097513	Sum squared resid	3.213953
F-statistic	536.9632	Durbin-Watson stat	0.299781
Prob(F-statistic)	0.000000		

Unweighted Statistics

R-squared	0.750484	Mean dependent var	4.296242
Sum squared resid	25.63603	Durbin-Watson stat	0.037583

For the **Amemiya random effects estimator** (which EViews names the Wansbeek-Kapteyn estimator), we get:

Dependent Variable: GAS
Method: Panel EGLS (Cross-section random effects)

Sample: 1960 1978
Cross-sections included: 18
Total panel (balanced) observations: 342
Wansbeek and Kapteyn estimator of component variances

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	2.188322	0.216372	10.11372	0.0000
INC	0.601969	0.065876	9.137941	0.0000
PMG	−0.365500	0.041620	−8.781832	0.0000
CAR	−0.620725	0.027356	−22.69053	0.0000

Effects Specification

Cross-section random S.D. / Rho	0.343826	0.9327
Idiosyncratic random S.D. / Rho	0.092330	0.0673

Weighted Statistics

R-squared	0.835065	Mean dependent var	0.264177
Adjusted R-squared	0.833602	S.D. dependent var	0.225791
S.E. of regression	0.092104	Sum squared resid	2.867329
F-statistic	570.4327	Durbin-Watson stat	0.315210
Prob(F-statistic)	0.000000		

Unweighted Statistics

R-squared	0.670228	Mean dependent var	4.296242
Sum squared resid	33.88179	Durbin-Watson stat	0.026675

For the **Swamy and Arora random effects estimator**, we get:

Dependent Variable: GAS
Method: Panel EGLS (Cross-section random effects)

Sample: 1960 1978
Cross-sections included: 18
Total panel (balanced) observations: 342
Swamy and Arora estimator of component variances

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	1.996698	0.178235	11.20260	0.0000
INC	0.554986	0.057174	9.706890	0.0000
PMG	−0.420389	0.038657	−10.87482	0.0000
CAR	−0.606840	0.024672	−24.59636	0.0000

Effects Specification

Cross-section random S.D. / Rho	0.195545	0.8177
Idiosyncratic random S.D. / Rho	0.092330	0.1823

Weighted Statistics

R-squared	0.829310	Mean dependent var	0.462676
Adjusted R-squared	0.827795	S.D. dependent var	0.230099
S.E. of regression	0.095485	Sum squared resid	3.081707
F-statistic	547.3996	Durbin-Watson stat	0.304481
Prob(F-statistic)	0.000000		

Unweighted Statistics

R-squared	0.730918	Mean dependent var	4.296242
Sum squared resid	27.64625	Durbin-Watson stat	0.033940

Now we reproduce some of these estimates for the Gasoline data set using Stata

```
 infile str8 co year c y p car using a:\gasoline.dat
'YR' cannot be read as a number for year[1]
'LN(Gas/Car)' cannot be read as a number for c[1]
'LN(Y/N)' cannot be read as a number for y[1]
'LN(Pmg/Pgdp)' cannot be read as a number for p[1]
'LN(Car/N)' cannot be read as a number for car[1]
(343 observations read)

. sum

    Variable |     Obs        Mean    Std. Dev.       Min        Max
-------------+-----------------------------------------------------
          co |       0
        year |     342        1969    5.485251       1960       1978
           c |     342    4.296242    .5489071   3.380209   6.156644
           y |     342   -6.139425    .6345925  -8.072523  -5.221232
           p |     342   -.5231032    .6782225  -2.896497   1.125311
         car |     342   -9.041805    1.218896  -13.47518  -7.536176

 . edit
 - preserve
 - drop in 1
 - preserve

 . sum

    Variable |     Obs        Mean    Std. Dev.       Min        Max
-------------+-----------------------------------------------------
          co |       0
        year |     342        1969    5.485251       1960       1978
           c |     342    4.296242    .5489071   3.380209   6.156644
           y |     342   -6.139425    .6345925  -8.072523  -5.221232
           p |     342   -.5231032    .6782225  -2.896497   1.125311
         car |     342   -9.041805    1.218896  -13.47518  -7.536176


 . gen coun=int([_n-1]/19)+1

 . edit
 - preserve
 . reg c y p car

      Source |       SS       df       MS              Number of obs =     342
-------------+------------------------------           F(  3,   338) =  664.00
       Model |  87.8386024     3  29.2795341           Prob > F      =  0.0000
    Residual |  14.9043581   338  .044095734           R-squared     =  0.8549
-------------+------------------------------           Adj R-squared =  0.8536
       Total |  102.742961   341  .301299005           Root MSE      =  .20999

------------------------------------------------------------------------------
           c |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
           y |   .8899616   .0358058    24.86   0.000     .8195313    .9603919
           p |  -.8917979   .0303147   -29.42   0.000    -.9514272   -.8321685
         car |  -.7633727   .0186083   -41.02   0.000    -.7999754   -.7267701
       _cons |   2.391326   .1169343    20.45   0.000     2.161315    2.621336
------------------------------------------------------------------------------
```

```
. iis coun

. xtreg c y p car, re theta

Random-effects GLS regression                   Number of obs      =       342
Group variable (i) : coun                       Number of groups   =        18

R-sq:  within  = 0.8363                         Obs per group: min =        19


     between = 0.7099                                          avg =      19.0
     overall = 0.7309                                          max =        19

Random effects u_i ~ Gaussian                   Wald chi2(3)       =   1642.20
corr(u_i, X)       = 0 (assumed)                Prob > chi2        =    0.0000
theta              = .89230675

------------------------------------------------------------------------------
           c |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
           y |   .5549858   .0591282     9.39   0.000     .4390967    .6708749
           p |  -.4203893   .0399781   -10.52   0.000     -.498745   -.3420336
         car |  -.6068402    .025515   -23.78   0.000    -.6568487   -.5568316
       _cons |   1.996699    .184326    10.83   0.000     1.635427    2.357971
-------------+----------------------------------------------------------------
     sigma_u |  .19554468
     sigma_e |  .09233034
         rho |  .81769856   (fraction of variance due to u_i)
------------------------------------------------------------------------------

. xtreg c y p car , fe

Fixed-effects (within) regression               Number of obs      =       342
Group variable (i) : coun                       Number of groups   =        18

R-sq:  within  = 0.8396                         Obs per group: min =        19
       between = 0.5755                                        avg =      19.0
       overall = 0.6150                                        max =        19

                                                F(3,321)           =    560.09
corr(u_i, Xb)  = -0.2468                        Prob > F           =    0.0000

------------------------------------------------------------------------------
           c |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
           y |   .6622498    .073386     9.02   0.000     .5178715    .8066282
           p |  -.3217025   .0440992    -7.29   0.000    -.4084626   -.2349425
         car |  -.6404829   .0296788   -21.58   0.000    -.6988725   -.5820933
       _cons |    2.40267   .2253094    10.66   0.000     1.959401     2.84594
-------------+----------------------------------------------------------------
     sigma_u |  .34841289
     sigma_e |  .09233034
         rho |  .93438173   (fraction of variance due to u_i)
------------------------------------------------------------------------------
F test that all u_i=0:     F(17, 321) =    83.96            Prob > F = 0.0000

. xtreg c y p car, be

Between regression (regression on group means)  Number of obs      =       342
Group variable (i) : coun                       Number of groups   =        18
```

```
R-sq:  within  = 0.7337                         Obs per group: min =        19
       between = 0.8799                                        avg =      19.0
       overall = 0.8529                                        max =        19

                                                F(3,14)            =     34.19

sd(u_i + avg(e_i.))=  .1966886                  Prob > F           =    0.0000

------------------------------------------------------------------------------
           c |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
           y |   .9675763   .1556662     6.22   0.000     .6337055    1.301447
           p |  -.9635503   .1329214    -7.25   0.000    -1.248638   -.6784622
         car |   -.795299   .0824742    -9.64   0.000    -.9721887   -.6184094
       _cons |    2.54163   .5267845     4.82   0.000     1.411789     3.67147
------------------------------------------------------------------------------

. xtreg c y p car, mle

Fitting constant-only model:
Iteration 0:   log likelihood = -298.79473
Iteration 1:   log likelihood =  -120.7272
Iteration 2:   log likelihood = -48.602465
Iteration 3:   log likelihood = -26.799172
Iteration 4:   log likelihood = -22.837386
Iteration 5:   log likelihood = -22.420508
Iteration 6:   log likelihood = -22.396956
Iteration 7:   log likelihood = -22.396801

Fitting full model:
Iteration 0:   log likelihood =  216.74308
Iteration 1:   log likelihood =  230.51837
Iteration 2:   log likelihood =   273.0581
Iteration 3:   log likelihood =  281.79287
Iteration 4:   log likelihood =  282.47033
Iteration 5:   log likelihood =  282.47697
Iteration 6:   log likelihood =  282.47697

Random-effects ML regression                    Number of obs      =       342
Group variable (i) : coun                       Number of groups   =        18

Random effects u_i ~ Gaussian                   Obs per group: min =        19
                                                               avg =      19.0
                                                               max =        19

                                                LR chi2(3)         =    609.75
Log likelihood  =  282.47697                    Prob > chi2        =    0.0000

------------------------------------------------------------------------------
           c |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
           y |   .5881334   .0659581     8.92   0.000     .4588578     .717409
           p |  -.3780466   .0440663    -8.58   0.000     -.464415   -.2916782
         car |  -.6163722   .0272054   -22.66   0.000    -.6696938   -.5630506
       _cons |   2.136168   .2156039     9.91   0.000     1.713593    2.558744
-------------+----------------------------------------------------------------
    /sigma_u |   .2922939   .0545496     5.36   0.000     .1853786    .3992092
    /sigma_e |   .0922537   .0036482    25.29   0.000     .0851033     .099404
-------------+----------------------------------------------------------------
         rho |   .9094086   .0317608                      .8303747    .9571561
------------------------------------------------------------------------------
Likelihood ratio test of sigma_u=0: chibar2(01)= 463.97 Prob>=chibar2 = 0.000
```

2.14 (a) This solution is based on Baltagi and Krämer (1994). From (2.3), one gets $\widehat{\delta}_{OLS} = (Z'Z)^{-1}Z'y$ and $\widehat{u}_{OLS} = y - Z\widehat{\delta}_{OLS} = \bar{P}_Z u$ where $\bar{P}_Z = I_{NT} - P_Z$ with $P_Z = Z(Z'Z)^{-1}Z'$.

Also, $E(s^2) = E[\widehat{u}'\widehat{u}/(NT - K')] = E[u'\bar{P}_Z u/(NT - K')] = \text{tr}(\Omega\bar{P}_Z)/(NT - K')$ which from (2.17) reduces to

$$E(s^2) = \sigma_\nu^2 + \sigma_\mu^2(NT - \text{tr}(I_N \otimes J_T)P_Z)/(NT - K')$$

since $\text{tr}(I_{NT}) = \text{tr}(I_N \otimes J_T) = NT$ and $\text{tr}(P_Z) = K'$. By adding and subtracting σ_μ^2, one gets

$$E(s^2) = \sigma^2 + \sigma_\mu^2[K' - \text{tr}(I_N \otimes J_T)P_Z]/(NT - K')$$

where $\sigma^2 = E(u_{it}^2) = \sigma_\mu^2 + \sigma_\nu^2$ for all i and t.

(b) Nerlove (1971) derived the characteristic roots and vectors of Ω given in (2.17). These characteristic roots turn out to be σ_ν^2 with multiplicity $N(T-1)$ and $(T\sigma_\mu^2 + \sigma_\nu^2)$ with multiplicity N. Therefore, the smallest $(n - K')$ characteristic roots are made up of the $(n - N)$ σ_ν^2's and $(N - K')$ of the $(T\sigma_\mu^2 + \sigma_\nu^2)$'s. This implies that the mean of the $(n - K')$ smallest characteristic roots of $\Omega = [(n - N)\sigma_\nu^2 + (N - K')(T\sigma_\mu^2 + \sigma_\nu^2)]/(n - K')$. Similarly, the largest $(n - K')$ characteristic roots are made up of the $N(T\sigma_\mu^2 + \sigma_\nu^2)$'s and $(n - N - K')$ of the σ_ν^2's. This implies that the mean of the $(n - K')$ largest characteristic roots of $\Omega = [N(T\sigma_\mu^2 + \sigma_\nu^2) + (n - N - K')\sigma_\nu^2]/(n - K')$. Using the Kiviet and Krämer (1992) inequalities, one gets

$$0 \leqslant \sigma_\nu^2 + (n - TK')\sigma_\mu^2/(n - K') \leqslant E(s^2)$$
$$\leqslant \sigma_\nu^2 + n\sigma_\mu^2/(n - K') \leqslant n\sigma^2/(n - K')$$

As $n \to \infty$, both bounds tend to σ^2, and s^2 is asymptotically unbiased, irrespective of the particular evolution of X.

2.15 This gives the SAS output for the estimation of "One-way Error Component Regression" for Public Capital data. Estimators are OLS, WITHIN, BETWEEN, WALHUS, AMEMIYA, SWAR, NERLOVE, and IMLE. The results do not replicate exactly those in Table 2.6 because there is no correction for degrees of freedom in these programs. However, the output reported below using EViews should match the results in Table 2.6.

SAS Output Public Capital Data

RESULTS OF OLS

LOOK1	PARAMETER	STANDARD ERROR	T-STATISTICS
INTERCEPT	1.64330	0.05759	28.53587
PUBLIC_K	0.15501	0.01715	9.03632
PRIVATE_K	0.30919	0.01027	30.10033
LABOR	0.59393	0.01375	43.20324
UNEMP	-0.00673	0.00142	-4.75366

RESULTS OF BETWEEN

LOOK1	PARAMETER	STANDARD ERROR	T-STATISTICS
INTERCEPT	1.58944	0.23298	6.82225
PUBLIC_K	0.17937	0.07197	2.49215
PRIVATE_K	0.30195	0.04182	7.22007
LABOR	0.57613	0.05637	10.21963
UNEMP	-0.00389	0.00991	-0.39263

RESULTS OF WITHIN

LOOK1	PARAMETER	STANDARD ERROR	T-STATISTICS
PUBLIC_K	-0.02615	0.02900	-0.90166
PRIVATE_K	0.29201	0.02512	11.62463
LABOR	0.76816	0.03009	25.52725
UNEMP	-0.00530	0.00099	-5.35815

RESULTS OF WALLACE-HUSSAIN

LOOK1	PARAMETER	STANDARD ERROR	T-STATISTICS
INTERCEPT	2.10882	0.13314	15.83867
PUBLIC_K	0.00860	0.02369	0.36325
PRIVATE_K	0.31283	0.01986	15.75035
LABOR	0.72435	0.02510	28.86296
UNEMP	-0.00628	0.00093	-6.75795

RESULTS OF AMEMIYA

LOOK1	PARAMETER	STANDARD ERROR	T-STATISTICS
INTERCEPT	2.15640	0.13661	15.78461
PUBLIC_K	0.00125	0.02373	0.05268
PRIVATE_K	0.30871	0.02019	15.29068
LABOR	0.73378	0.02532	28.98434
UNEMP	-0.00609	0.00091	-6.68722

RESULTS OF SWAMY-ARORA

LOOK1	PARAMETER	STANDARD ERROR	T-STATISTICS
INTERCEPT	2.13541	0.13299	16.05651
PUBLIC_K	0.00444	0.02334	0.19021
PRIVATE_K	0.31055	0.01974	15.73569
LABOR	0.72967	0.02483	29.38330
UNEMP	-0.00617	0.00090	-6.82720

RESULTS OF NERLOVE

LOOK1	PARAMETER	STANDARD ERROR	T-STATISTICS
INTERCEPT	2.16831	0.13473	16.09397
PUBLIC_K	-0.00052	0.02328	-0.02253

PRIVATE_K	0.30766	0.01986	15.48794
LABOR	0.73608	0.02486	29.61146
UNEMP	-0.00604	0.00089	-6.80225

RESULTS OF MAXIMUM LIKELIHOOD

LOOK1	PARAMETER	STANDARD ERROR	T-STATISTICS
INTERCEPT	2.14292	0.13426	15.96155
PUBLIC_K	0.00329	0.02347	0.14010
PRIVATE_K	0.30989	0.01989	15.57784
LABOR	0.73115	0.02500	29.24478
UNEMP	-0.00614	0.00091	-6.77855

ONE-WAY ERROR COMPONENT MODEL WITH PUBLIC CAPITAL DATA (A. MUNNELL): BETA, VARIANCES OF BETA, AND THETA

OUTPUT	PUBLIC_K	PRIV_K	LABOR	UNEMPLOY	STD-PUBK	STD-PRIVK	STD-LAB	STD-UNEMP	THETA
OLS ESTIMATOR	0.15501	0.30919	0.59393	-0.00673	0.01715	0.01027	0.01375	0.00142	0.00000
BETWEEN ESTIMATOR	0.17937	0.30195	0.57613	-0.00389	0.07197	0.04182	0.05637	0.00991	.
WITHIN ESTIMATOR	-0.02615	0.29201	0.76816	-0.00530	0.02900	0.02512	0.03009	0.00099	1.00000
WALLACE & HUSSAIN ESTIMATOR	0.00860	0.31283	0.72435	-0.00628	0.02369	0.01986	0.02510	0.00093	0.87809
AMEMIYA ESTIMATOR	0.00125	0.30871	0.73378	-0.00609	0.02373	0.02019	0.02532	0.00091	0.89707
SWAMY & ARORA ESTIMATOR	0.00444	0.31055	0.72967	-0.00617	0.02334	0.01974	0.02483	0.00090	0.88884
NERLOVE ESTIMATOR	-0.00052	0.30766	0.73608	-0.00604	0.02328	0.01986	0.02486	0.00089	0.90166
IMLE	0.00329	0.30989	0.73115	-0.00614	0.02347	0.01989	0.02500	0.00091	0.89180

NEGATIVE VAR_MHU

NEGA_VAR	COL1
OLS ESTIMATOR	.
BETWEEN ESTIMATOR	.
WITHIN ESTIMATOR	.
WALLACE & HUSSAIN ESTIMATOR	0
AMEMIYA ESTIMATOR	0
SWAMY & ARORA ESTIMATOR	0
NERLOVE ESTIMATOR	.
IMLE	.

ONE-WAY VARIANCE COMPONENTS FOR PUBLIC CAPITAL DATA

OUTPUT	SIGMA-NU-SQ	SIGMA-MU-SQ	SIGMA-1-SQ	SIGMA-SQ	RHO
WALLACE & HUSSAIN ESTIMATOR	0.00157	0.00614	0.10594	0.00771	0.79588
AMEMIYA ESTIMATOR	0.00145	0.00795	0.13655	0.00939	0.84599
SWAMY & ARORA ESTIMATOR	0.00145	0.00684	0.11770	0.00829	0.82460
NERLOVE ESTIMATOR	0.00136	0.00820	0.14082	0.00957	0.85764

****

The following output was produced using EViews 5.0. This should replicate Table 2.6 and differs from the above SAS output in that degrees of freedom adjustments are made when estimating the variance components.

For Pooled OLS:

Dependent Variable: LNY
Method: Panel Least Squares

Sample: 1970 1986
Cross-sections included: 48
Total panel (balanced) observations: 816

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	1.643302	0.057587	28.53588	0.0000
LNK1	0.155007	0.017154	9.036311	0.0000
LNK2	0.309190	0.010272	30.10036	0.0000
LNL	0.593935	0.013747	43.20329	0.0000
U	−0.006733	0.001416	−4.753682	0.0000

R-squared	0.992593	Mean dependent var	10.50885
Adjusted R-squared	0.992557	S.D. dependent var	1.021132
S.E. of regression	0.088096	Akaike info criterion	−2.014663
Sum squared resid	6.294143	Schwarz criterion	−1.985837
Log likelihood	826.9824	F-statistic	27171.71
Durbin-Watson stat	0.079269	Prob(F-statistic)	0.000000

For the one-way fixed effects estimator with state effects:

Dependent Variable: LNY
Method: Panel Least Squares

Sample: 1970 1986
Cross-sections included: 48
Total panel (balanced) observations: 816

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	2.352878	0.174813	13.45938	0.0000
LNK1	−0.026146	0.029002	−0.901545	0.3676
LNK2	0.292008	0.025120	11.62467	0.0000
LNL	0.768156	0.030092	25.52708	0.0000
U	−0.005298	0.000989	−5.358296	0.0000

Effects Specification

Cross-section fixed (dummy variables)

R-squared	0.998692	Mean dependent var	10.50885
Adjusted R-squared	0.998605	S.D. dependent var	1.021132
S.E. of regression	0.038137	Akaike info criterion	−3.633657
Sum squared resid	1.111188	Schwarz criterion	−3.333866
Log likelihood	1534.532	F-statistic	11441.65
Durbin-Watson stat	0.413532	Prob(F-statistic)	0.000000

For the Wallace and Hussain random effects estimator, we get:

Dependent Variable: LNY
Method: Panel EGLS (Cross-section random effects)

Sample: 1970 1986
Cross-sections included: 48
Total panel (balanced) observations: 816
Wallace and Hussain estimator of component variances

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	2.128128	0.134569	15.81442	0.0000
LNK1	0.005565	0.023698	0.234824	0.8144
LNK2	0.311180	0.019998	15.56080	0.0000
LNL	0.728225	0.025192	28.90685	0.0000
U	–0.006202	0.000921	–6.731730	0.0000

Effects Specification

Cross-section random S.D. / Rho	0.082369	0.8169
Idiosyncratic random S.D. / Rho	0.038992	0.1831

Weighted Statistics

R-squared	0.959943	Mean dependent var	1.198662
Adjusted R-squared	0.959745	S.D. dependent var	0.191062
S.E. of regression	0.038334	Sum squared resid	1.191751
F-statistic	4858.753	Durbin-Watson stat	0.393903
Prob(F-statistic)	0.000000		

Unweighted Statistics

R-squared	0.991686	Mean dependent var	10.50885
Sum squared resid	7.065010	Durbin-Watson stat	0.066445

For the Amemiya random effects estimator (which EViews names the Wansbeek-Kapteyn estimator), we get:

Dependent Variable: LNY
Method: Panel EGLS (Cross-section random effects)

Sample: 1970 1986
Cross-sections included: 48
Total panel (balanced) observations: 816
Wansbeek and Kapteyn estimator of component variances

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	2.153284	0.136382	15.78857	0.0000
LNK1	0.001719	0.023725	0.072440	0.9423
LNK2	0.308985	0.020167	15.32094	0.0000
LNL	0.733179	0.025302	28.97685	0.0000
U	−0.006100	0.000911	−6.692963	0.0000

Effects Specification

Cross-section random S.D./Rho	0.088336	0.8429
Idiosyncratic random S.D./Rho	0.038137	0.1571

Weighted Statistics

R-squared	0.957828	Mean dependent var	1.094387
Adjusted R-squared	0.957620	S.D. dependent var	0.185195
S.E. of regression	0.038125	Sum squared resid	1.178804
F-statistic	4604.932	Durbin-Watson stat	0.397124
Prob(F-statistic)	0.000000		

Unweighted Statistics

R-squared	0.991617	Mean dependent var	10.50885
Sum squared resid	7.123861	Durbin-Watson stat	0.065713

For the Swamy and Arora random effects estimator, we get:

Dependent Variable: LNY
Method: Panel EGLS (Cross-section random effects)

Sample: 1970 1986
Cross-sections included: 48
Total panel (balanced) observations: 816
Swamy and Arora estimator of component variances

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	2.135397	0.132994	16.05640	0.0000
LNK1	0.004441	0.023335	0.190316	0.8491
LNK2	0.310549	0.019735	15.73573	0.0000
LNL	0.729668	0.024833	29.38316	0.0000
U	−0.006173	0.000904	−6.827327	0.0000

Effects Specification

Cross-section random S.D./Rho	0.082690	0.8246
Idiosyncratic random S.D./Rho	0.038137	0.1754

Weighted Statistics

R-squared	0.959332	Mean dependent var	1.168214
Adjusted R-squared	0.959132	S.D. dependent var	0.189313
S.E. of regression	0.038271	Sum squared resid	1.187862
F-statistic	4782.777	Durbin-Watson stat	0.394873
Prob(F-statistic)	0.000000		

Unweighted Statistics

R-squared	0.991667	Mean dependent var	10.50885
Sum squared resid	7.081797	Durbin-Watson stat	0.066234

Now we reproduce some of these estimates for the Public Capital Productivity data set using Stata

```
. infile str12 st str2 abb yr pubk hwy wat util privk gsp emp u using a:\produc.prn
'YR' cannot be read as a number for yr[1]
'P-CAP' cannot be read as a number for pubk[1]
'HWY' cannot be read as a number for hwy[1]
'WATER' cannot be read as a number for wat[1]
'UTIL' cannot be read as a number for util[1]
'PC' cannot be read as a number for privk[1]
'GSP' cannot be read as a number for gsp[1]
'EMP' cannot be read as a number for emp[1]
'UNEMP' cannot be read as a number for u[1]
'-' cannot be read as a number for yr[818]
(eof not at end of obs)
(818 observations read)

. edit
- preserve
- drop in 1
- drop in 817
- preserve

. sum

    Variable |     Obs        Mean    Std. Dev.        Min        Max
-------------+-----------------------------------------------------
          st |       0
         abb |       0
          yr |     816        1978    4.901984        1970       1986
        pubk |     816    25036.66     27780.4     2627.12   140217.3
         hwy |     816    10218.42    9253.597     1827.14   47699.42
         wat |     816    3618.784    4311.742      228.46   24592.33
        util |     816    11199.45    14768.87      538.49   80728.14
       privk |     816    58188.29    59770.78     4052.71   375341.6
         gsp |     816    61014.32     69973.9        4354     464550
         emp |     816    1747.101    1855.988       108.3      11258
           u |     816    6.602206    2.233217         2.8         18
```

```
. gen lny=log(gsp)

. gen lnk1=log(pubk)

. gen lnk2=log(privk)

. gen lnl=log(emp)

. reg lny lnk1 lnk2 lnl u

      Source |       SS       df       MS              Number of obs =     816
-------------+------------------------------           F(  4,   811) =27171.66
       Model |  843.514726     4  210.878682           Prob > F      =  0.0000
    Residual |  6.29415387   811  .007760979           R-squared     =  0.9926
-------------+------------------------------           Adj R-squared =  0.9926
       Total |   849.80888   815  1.04271028           Root MSE      =   .0881

------------------------------------------------------------------------------
         lny |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
        lnk1 |    .155007   .0171538     9.04   0.000      .121336    .1886781
        lnk2 |   .3091902    .010272    30.10   0.000     .2890273     .329353
         lnl |   .5939349   .0137475    43.20   0.000     .5669501    .6209197
           u |   -.006733   .0014164    -4.75   0.000    -.0095132   -.0039528
       _cons |   1.643302   .0575873    28.54   0.000     1.530265     1.75634
------------------------------------------------------------------------------


. gen stid=int([_n-1]/17)+1

. edit
- preserve

. iis stid

. xtreg lny lnk1 lnk2 lnl u, re theta

Random-effects GLS regression                   Number of obs      =       816
Group variable (i) : stid                       Number of groups   =        48

R-sq:  within  = 0.9412                         Obs per group: min =        17
       between = 0.9928                                        avg =      17.0
       overall = 0.9917                                        max =        17

Random effects u_i ~ Gaussian                   Wald chi2(4)       =  19131.09
corr(u_i, X)       = 0 (assumed)                Prob > chi2        =    0.0000
theta              = .8888353

------------------------------------------------------------------------------
         lny |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
        lnk1 |   .0044388   .0234173     0.19   0.850    -.0414583    .0503359
        lnk2 |   .3105483   .0198047    15.68   0.000     .2717317    .3493649
         lnl |   .7296705   .0249202    29.28   0.000     .6808278    .7785132
           u |  -.0061725   .0009073    -6.80   0.000    -.0079507   -.0043942
       _cons |   2.135411   .1334615    16.00   0.000     1.873831     2.39699
-------------+----------------------------------------------------------------
     sigma_u |   .0826905
     sigma_e |  .03813705
         rho |  .82460109   (fraction of variance due to u_i)
------------------------------------------------------------------------------
```

```
. xtreg lny lnk1 lnk2 lnl u, fe

Fixed-effects (within) regression               Number of obs      =       816
Group variable (i) : stid                       Number of groups   =        48

R-sq:  within  = 0.9413                         Obs per group: min =        17
       between = 0.9921                                        avg =      17.0
       overall = 0.9910                                        max =        17

                                                F(4,764)           =   3064.81
corr(u_i, Xb)  = 0.0608                         Prob > F           =    0.0000

------------------------------------------------------------------------------
         lny |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
        lnk1 |  -.0261493   .0290016    -0.90   0.368    -.0830815    .0307829
        lnk2 |   .2920067   .0251197    11.62   0.000     .2426949    .3413185
         lnl |   .7681595   .0300917    25.53   0.000     .7090872    .8272318
           u |  -.0052977   .0009887    -5.36   0.000    -.0072387   -.0033568
       _cons |   2.352898   .1748131    13.46   0.000     2.009727    2.696069
-------------+----------------------------------------------------------------
     sigma_u |  .09057293

     sigma_e |  .03813705
         rho |   .8494045   (fraction of variance due to u_i)
------------------------------------------------------------------------------
F test that all u_i=0:     F(47, 764) =    75.82           Prob > F = 0.0000

. xtreg lny lnk1 lnk2 lnl u, be

Between regression (regression on group means) Number of obs      =       816
Group variable (i) : stid                      Number of groups   =        48

R-sq:  within  = 0.9330                        Obs per group: min =        17
       between = 0.9939                                       avg =      17.0
       overall = 0.9925                                       max =        17

                                               F(4,43)            =   1754.11
sd(u_i + avg(e_i.))=  .0832062                 Prob > F           =    0.0000

------------------------------------------------------------------------------
         lny |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
        lnk1 |   .1793651   .0719719     2.49   0.017     .0342199    .3245104
        lnk2 |   .3019542   .0418215     7.22   0.000     .2176132    .3862953
         lnl |   .5761274   .0563746    10.22   0.000     .4624372    .6898176
           u |  -.0038903   .0099084    -0.39   0.697    -.0238724    .0160918
       _cons |   1.589444   .2329796     6.82   0.000     1.119596    2.059292
------------------------------------------------------------------------------
```

```
. xtreg lny lnk1 lnk2 lnl u, mle

Fitting constant-only model:
Iteration 0:   log likelihood = -7979.4497
Iteration 1:   log likelihood = -4019.3964
Iteration 2:   log likelihood = -1886.2544
Iteration 3:   log likelihood = -774.44513
Iteration 4:   log likelihood = -226.38653
Iteration 5:   log likelihood =  21.808149
Iteration 6:   log likelihood =  124.88827
Iteration 7:   log likelihood =  168.03251
Iteration 8:   log likelihood =  186.72907
Iteration 9:   log likelihood =  193.60615
Iteration 10:  log likelihood =  195.27915
Iteration 11:  log likelihood =  195.44879
Iteration 12:  log likelihood =  195.45155
Iteration 13:  log likelihood =  195.45155

Fitting full model:
Iteration 0:   log likelihood =  1374.1026
Iteration 1:   log likelihood =  1398.8952
Iteration 2:   log likelihood =  1401.8628
Iteration 3:   log likelihood =  1401.9041
Iteration 4:   log likelihood =  1401.9041

Random-effects ML regression                    Number of obs      =        81
Group variable (i) : stid                       Number of groups   =         4

Random effects u_i ~ Gaussian                   Obs per group: min =        17
                                                               avg =      17.0
                                                               max =        17

                                                LR chi2(4)         =   2412.91
Log likelihood  =  1401.9041                    Prob > chi2        =    0.0000

------------------------------------------------------------------------------
         lny |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
        lnk1 |   .0031446   .0239185     0.13   0.895    -.0437348     .050024
        lnk2 |    .309811    .020081    15.43   0.000      .270453     .349169
         lnl |   .7313372   .0256936    28.46   0.000     .6809787    .7816957
           u |  -.0061382   .0009143    -6.71   0.000    -.0079302   -.0043462
       _cons |   2.143865   .1376582    15.57   0.000      1.87406    2.413671
-------------+----------------------------------------------------------------
    /sigma_u |    .085162   .0090452     9.42   0.000     .0674337    .1028903
    /sigma_e |   .0380836   .0009735    39.12   0.000     .0361756    .0399916
-------------+----------------------------------------------------------------
         rho |   .8333481   .0304597                      .7668537    .8861754
------------------------------------------------------------------------------
Likelihood ratio test of sigma_u=0: chibar2(01)= 1149.84 Prob>=chibar2 = 0.000

------------------------------------------------------------------------------
```