CHAPTER 2

Problem 2.1

Given:

$$T_n = 2\pi\sqrt{\frac{m}{k}} = 0.5 \text{ sec} \qquad \text{(a)}$$

$$T_n' = 2\pi\sqrt{\frac{m + 50/\text{g}}{k}} = 0.75 \text{ sec} \qquad \text{(b)}$$

1. *Determine the weight of the table.*

Taking the ratio of Eq. (b) to Eq. (a) and squaring the result gives

$$\left(\frac{T_n'}{T_n}\right)^2 = \frac{m+50/\text{g}}{m} \qquad \Rightarrow \quad 1+\frac{50}{m\text{g}} = \left(\frac{0.75}{0.5}\right)^2 = 2.25$$

or

$$m\text{g} = \frac{50}{1.25} = 40 \text{ lbs}$$

2. *Determine the lateral stiffness of the table.*

Substitute for m in Eq. (a) and solve for k:

$$k = 16\pi^2 m = 16\pi^2\left(\frac{40}{386}\right) = 16.4 \text{lbs/in.}$$

Problem 2.2

1. *Determine the natural frequency.*

$$k = 100 \text{ lb/in.} \qquad m = \frac{400}{386} \text{ lb} - \text{sec}^2/\text{in.}$$

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{100}{400/386}} = 9.82 \text{ rads/sec}$$

2. *Determine initial deflection.*

Static deflection due to weight of the iron scrap

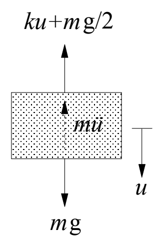
$$u(0) = \frac{200}{100} = 2 \text{ in.}$$

3. *Determine free vibration.*

$$u(t) = u(0) \cos \omega_n t = 2 \cos(9.82t)$$

Problem 2.3

1. *Set up equation of motion.*



$$m\ddot{u} + ku = \frac{mg}{2}$$

2. *Solve equation of motion.*

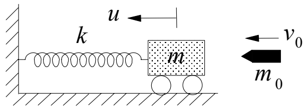
$$u(t) = A\cos\omega_n t + B\sin\omega_n t + \frac{mg}{2k}$$

At $t = 0$, $u(0) = 0$ and $\dot{u}(0) = 0$

$$\therefore\ A = -\frac{mg}{2k},\quad B = 0$$

$$u(t) = \frac{mg}{2k}(1 - \cos\omega_n t)$$

Problem 2.4



$$m = \frac{10}{386} = 0.0259 \text{ lb} - \text{sec}^2/\text{in.}$$

$$m_0 = \frac{0.5}{386} = 1.3 \times 10^{-3} \text{ lb} - \text{sec}^2/\text{in.}$$

$$k = 100 \text{ lb/in.}$$

Conservation of momentum implies

$$m_0 v_0 = (m + m_0)\,\dot{u}(0)$$

$$\dot{u}(0) = \frac{m_0 v_0}{m + m_0} = 2.857 \text{ ft/sec} = 34.29 \text{ in./sec}$$

After the impact the system properties and initial conditions are

Mass = $m + m_0 = 0.0272 \text{ lb} - \text{sec}^2/\text{in.}$

Stiffness = $k = 100$ lb/in.

Natural frequency:

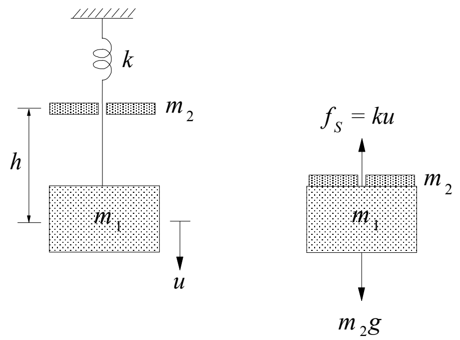
$$\omega_n = \sqrt{\frac{k}{m + m_0}} = 60.63 \text{ rads/sec}$$

Initial conditions: $u(0) = 0, \quad \dot{u}(0) = 34.29$ in./sec

The resulting motion is

$$u(t) = \frac{\dot{u}(0)}{\omega_n} \sin \omega_n t = 0.565 \sin(60.63t) \text{ in.}$$

Problem 2.5



With u measured from the static equilibrium position of m_1 and k, the equation of motion after impact is

$$(m_1 + m_2)\ddot{u} + ku = m_2 g \qquad \text{(a)}$$

The general solution is

$$u(t) = A\cos\omega_n t + B\sin\omega_n t + \frac{m_2 g}{k} \qquad \text{(b)}$$

$$\omega_n = \sqrt{\frac{k}{m_1 + m_2}} \qquad \text{(c)}$$

The initial conditions are

$$u(0) = 0 \qquad \dot{u}(0) = \frac{m_2}{m_1 + m_2}\sqrt{2gh} \qquad \text{(d)}$$

The initial velocity in Eq. (d) was determined by conservation of momentum during impact:

$$m_2\dot{u}_2 = (m_1 + m_2)\dot{u}(0)$$

where

$$\dot{u}_2 = \sqrt{2gh}$$

Impose initial conditions to determine A and B:

$$u(0) = 0 \Rightarrow A = -\frac{m_2 g}{k} \qquad \text{(e)}$$

$$\dot{u}(0) = \omega_n B \Rightarrow B = \frac{m_2}{m_1 + m_2}\frac{\sqrt{2gh}}{\omega_n} \qquad \text{(f)}$$

Substituting Eqs. (e) and (f) in Eq. (b) gives

$$u(t) = \frac{m_2 g}{k}(1 - \cos\omega_n t) + \frac{\sqrt{2gh}}{\omega_n}\frac{m_2}{m_1 + m_2}\sin\omega_n t$$

Problem 2.6

1. *Determine deformation and velocity at impact.*

$$u(0) = \frac{mg}{k} = \frac{10}{50} = 0.2 \text{ in.}$$

$$\dot{u}(0) = -\sqrt{2gh} = -\sqrt{2(386)(36)} = -166.7 \text{ in./sec}$$

2. *Determine the natural frequency.*

$$\omega_n = \sqrt{\frac{kg}{w}} = \sqrt{\frac{(50)(386)}{10}} = 43.93 \text{ rad/sec}$$

3. *Compute the maximum deformation.*

$$u(t) = u(0)\cos\omega_n t + \frac{\dot{u}(0)}{\omega_n}\sin\omega_n t$$

$$= (0.2)\cos 316.8t - \left(\frac{166.7}{43.93}\right)\sin 316.8t$$

$$u_o = \sqrt{[u(0)]^2 + \left[\frac{\dot{u}(0)}{\omega_n}\right]^2}$$

$$= \sqrt{0.2^2 + (-3.795)^2} = 3.8 \text{ in.}$$

4. *Compute the maximum acceleration.*

$$\ddot{u}_o = \omega_n{}^2 u_o = (43.93)^2(3.8)$$

$$= 7334 \text{ in./sec}^2 = 18.98g$$

Problem 2.7

Given:

$$m = \frac{200}{32.2} = 6.211 \text{ lb} - \text{sec}^2/\text{ft}$$

$$f_n = 2 \text{ Hz}$$

Determine EI:

$$k = \frac{3EI}{L^3} = \frac{3EI}{3^3} = \frac{EI}{9} \text{ lb/ft}$$

$$f_n = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \Rightarrow 2 = \frac{1}{2\pi}\sqrt{\frac{EI}{55.90}} \Rightarrow$$

$$EI = (4\pi)^2 \, 55.90 = 8827 \text{ lb} - \text{ft}^2$$

Problem 2.8

Equation of motion:

$$m\ddot{u} + c\dot{u} + ku = 0 \qquad \text{(a)}$$

Dividing Eq. (a) through by *m* gives

$$\ddot{u} + 2\zeta\omega_n\dot{u} + \omega_n^2 u = 0 \qquad \text{(b)}$$

where $\zeta = 1$.

Equation (b) thus reads

$$\ddot{u} + 2\omega_n\dot{u} + \omega_n^2 u = 0 \qquad \text{(c)}$$

Assume a solution of the form $u(t) = e^{st}$. Substituting this solution into Eq. (c) yields

$$(s^2 + 2\omega_n s + \omega_n^2)\, e^{st} = 0$$

Because e^{st} is never zero, the quantity within parentheses must be zero:

$$s^2 + 2\omega_n s + \omega_n^2 = 0$$

or

$$s = \frac{-2\omega_n \pm \sqrt{(2\omega_n)^2 - 4\omega_n^2}}{2} = -\omega_n$$

(double root)

The general solution has the following form:

$$u(t) = A_1\, e^{-\omega_n t} + A_2\, t\, e^{-\omega_n t} \qquad \text{(d)}$$

where the constants A_1 and A_2 are to be determined from the initial conditions: $u(0)$ and $\dot{u}(0)$.

Evaluate Eq. (d) at $t = 0$:

$$u(0) = A_1 \Rightarrow A_1 = u(0) \qquad \text{(e)}$$

Differentiating Eq. (d) with respect to *t* gives

$$\dot{u}(t) = -\omega_n A_1\, e^{-\omega_n t} + A_2(1 - \omega_n t)\, e^{-\omega_n t} \qquad \text{(f)}$$

Evaluate Eq. (f) at $t = 0$:

$$\dot{u}(0) = -\omega_n A_1 + A_2(1 - 0)$$

$$\therefore\; A_2 = \dot{u}(0) + \omega_n A_1 = \dot{u}(0) + \omega_n u(0) \qquad \text{(g)}$$

Substituting Eqs. (e) and (g) for A_1 and A_2 in Eq. (d) gives

$$u(t) = \left\{u(0) + \left[\dot{u}(0) + \omega_n u(0)\right]t\right\}e^{-\omega_n t} \qquad \text{(h)}$$

Problem 2.9

Equation of motion:

$$m\ddot{u} + c\dot{u} + ku = 0 \tag{a}$$

Dividing Eq. (a) through by m gives

$$\ddot{u} + 2\zeta\omega_n\dot{u} + \omega_n^2 u = 0 \tag{b}$$

where $\zeta > 1$.

Assume a solution of the form $u(t) = e^{st}$. Substituting this solution into Eq. (b) yields

$$(s^2 + 2\zeta\omega_n s + \omega_n^2)\, e^{st} = 0$$

Because e^{st} is never zero, the quantity within parentheses must be zero:

$$s^2 + 2\zeta\omega_n s + \omega_n^2 = 0$$

or

$$s = \frac{-2\zeta\omega_n \pm \sqrt{(2\zeta\omega_n)^2 - 4\omega_n^2}}{2}$$

$$= \left(-\zeta \pm \sqrt{\zeta^2 - 1}\right)\omega_n$$

The general solution has the following form:

$$u(t) = A_1 \exp\left[\left(-\zeta - \sqrt{\zeta^2-1}\right)\omega_n t\right] + A_2 \exp\left[\left(-\zeta + \sqrt{\zeta^2-1}\right)\omega_n t\right] \tag{c}$$

where the constants A_1 and A_2 are to be determined from the initial conditions: $u(0)$ and $\dot{u}(0)$.

Evaluate Eq. (c) at $t = 0$:

$$u(0) = A_1 + A_2 \Rightarrow A_1 + A_2 = u(0) \tag{d}$$

Differentiating Eq. (c) with respect to t gives

$$\dot{u}(t) = A_1\left(-\zeta - \sqrt{\zeta^2-1}\right)\omega_n \exp\left[\left(-\zeta - \sqrt{\zeta^2-1}\right)\omega_n t\right] + A_2\left(-\zeta + \sqrt{\zeta^2-1}\right)\omega_n \exp\left[\left(-\zeta + \sqrt{\zeta^2-1}\right)\omega_n t\right] \tag{e}$$

Evaluate Eq. (e) at $t = 0$:

$$\dot{u}(0) = A_1\left(-\zeta - \sqrt{\zeta^2-1}\right)\omega_n + A_2\left(-\zeta + \sqrt{\zeta^2-1}\right)\omega_n$$

$$= [u(0) - A_2]\left(-\zeta - \sqrt{\zeta^2-1}\right)\omega_n + A_2\left(-\zeta + \sqrt{\zeta^2-1}\right)\omega_n$$

or

$$A_2\,\omega_n\left[-\zeta + \sqrt{\zeta^2-1} + \zeta + \sqrt{\zeta^2-1}\right] = \dot{u}(0) + \left(\zeta + \sqrt{\zeta^2-1}\right)\omega_n u(0)$$

or

$$A_2 = \frac{\dot{u}(0) + \left(\zeta + \sqrt{\zeta^2-1}\right)\omega_n u(0)}{2\sqrt{\zeta^2-1}\;\omega_n} \tag{f}$$

Substituting Eq. (f) in Eq. (d) gives

$$A_1 = u(0) - \frac{\dot{u}(0) + \left(\zeta + \sqrt{\zeta^2-1}\right)\omega_n u(0)}{2\sqrt{\zeta^2-1}\,\omega_n}$$

$$= \frac{2\sqrt{\zeta^2-1}\,\omega_n u(0) - \dot{u}(0) - \left(\zeta + \sqrt{\zeta^2-1}\right)\omega_n u(0)}{2\sqrt{\zeta^2-1}\,\omega_n}$$

$$= \frac{-\dot{u}(0) + \left(-\zeta + \sqrt{\zeta^2-1}\right)\omega_n u(0)}{2\sqrt{\zeta^2-1}\,\omega_n} \tag{g}$$

The solution, Eq. (c), now reads:

$$u(t) = e^{-\zeta\omega_n t}\left(A_1 e^{-\omega_D' t} + A_2 e^{\omega_D' t}\right)$$

where

$$\omega_D' = \sqrt{\zeta^2 - 1}\,\omega_n$$

$$A_1 = \frac{-\dot{u}(0) + \left(-\zeta + \sqrt{\zeta^2-1}\right)\omega_n u(0)}{2\omega_D'}$$

$$A_2 = \frac{\dot{u}(0) + \left(\zeta + \sqrt{\zeta^2-1}\right)\omega_n u(0)}{2\omega_D'}$$

Problem 2.10

Equation of motion:

$$\ddot{u} + 2\zeta\omega_n\dot{u} + \omega_n^2 u = 0 \qquad \text{(a)}$$

Assume a solution of the form

$$u(t) = e^{st}$$

Substituting this solution into Eq. (a) yields:

$$\left(s^2 + 2\zeta\omega_n s + \omega_n^2\right)e^{st} = 0$$

Because e^{st} is never zero

$$s^2 + 2\zeta\omega_n s + \omega_n^2 = 0 \qquad \text{(b)}$$

The roots of this characteristic equation depend on ζ.

(a) Underdamped Systems, ζ<1

The two roots of Eq. (b) are

$$s_{1,2} = \omega_n\left(-\zeta \pm i\sqrt{1-\zeta^2}\right) \qquad \text{(c)}$$

Hence the general solution is

$$u(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$$

which after substituting in Eq. (c) becomes

$$u(t) = e^{-\zeta\omega_n t}\left(A_1 e^{i\omega_D t} + A_2 e^{-i\omega_D t}\right) \qquad \text{(d)}$$

where

$$\omega_D = \omega_n\sqrt{1-\zeta^2} \qquad \text{(e)}$$

Rewrite Eq. (d) in terms of trigonometric functions:

$$u(t) = e^{-\zeta\omega_n t}\left(A\cos\omega_D t + B\sin\omega_D t\right) \qquad \text{(f)}$$

Determine A and B from initial conditions $u(0) = 0$ and $\dot{u}(0)$:

$$A = 0 \qquad B = \frac{\dot{u}(0)}{\omega_D}$$

Substituting A and B into Eq. (f) gives

$$u(t) = \frac{\dot{u}(0)}{\omega_n\sqrt{1-\zeta^2}}e^{-\zeta\omega_n t}\sin\left(\omega_n\sqrt{1-\zeta^2}\right)t \qquad \text{(g)}$$

(b) Critically Damped Systems, $\zeta = 1$

The roots of the characteristic equation [Eq. (b)] are:

$$s_1 = -\omega_n \qquad s_2 = -\omega_n \qquad \text{(h)}$$

The general solution is

$$u(t) = A_1 e^{-\omega_n t} + A_2\, t\, e^{-\omega_n t} \qquad \text{(i)}$$

Determined from the initial conditions $u(0) = 0$ and $\dot{u}(0)$:

$$A_1 = 0 \qquad A_2 = \dot{u}(0) \qquad \text{(j)}$$

Substituting in Eq. (i) gives

$$u(t) = \dot{u}(0)\, t\, e^{-\omega_n t} \qquad \text{(k)}$$

(c) Overdamped Systems, ζ>1

The roots of the characteristic equation [Eq. (b)] are:

$$s_{1,2} = \omega_n\left(-\zeta \pm \sqrt{\zeta^2 - 1}\right) \qquad \text{(l)}$$

The general solution is:

$$u(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t} \qquad \text{(m)}$$

which after substituting Eq. (l) becomes

$$u(t) = A_1 e^{\left(-\zeta+\sqrt{\zeta^2-1}\right)\omega_n t} + A_2 e^{\left(-\zeta-\sqrt{\zeta^2-1}\right)\omega_n t} \qquad \text{(n)}$$

Determined from the initial conditions $u(0) = 0$ and $\dot{u}(0)$:

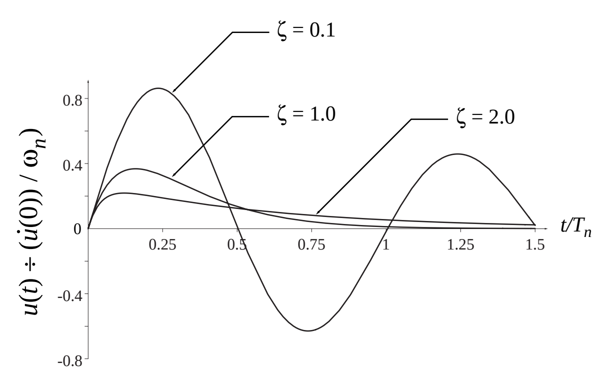
$$-A_1 = A_2 = \frac{\dot{u}(0)}{2\omega_n\sqrt{\zeta^2 - 1}} \qquad \text{(o)}$$

Substituting in Eq. (n) gives

$$u(t) = \frac{\dot{u}(0)\, e^{-\zeta\omega_n t}}{2\omega_n\sqrt{\zeta^2-1}}\left(e^{\omega_n t\sqrt{\zeta^2-1}} - e^{-\omega_n t\sqrt{\zeta^2-1}}\right) \qquad \text{(p)}$$

(d) Response Plots

Plot Eq. (g) with ζ = 0.1; Eq. (k), which is for ζ = 1; and Eq. (p) with ζ = 2.



Problem 2.11

$$\frac{1}{j}\ln\left(\frac{u_1}{u_{j+1}}\right) \approx 2\pi\zeta \Rightarrow \frac{1}{j_{10\%}}\ln\left(\frac{1}{0.1}\right) \approx 2\pi\zeta$$

$$\therefore\ j_{10\%} \approx \ln(10)/2\pi\zeta \approx 0.366/\zeta$$

Problem 2.12

$$\frac{u_i}{u_{i+1}} = \exp\left(\frac{2\pi\zeta}{\sqrt{1-\zeta^2}}\right)$$

(a) $\zeta = 0.01: \quad \frac{u_i}{u_{i+1}} = 1.065$

(b) $\zeta = 0.05: \quad \frac{u_i}{u_{i+1}} = 1.37$

(c) $\zeta = 0.25: \quad \frac{u_i}{u_{i+1}} = 5.06$

Problem 2.13

Given:

$w = 20.03$ kips (empty); $m = 0.0519$ kip-sec²/in.

$k = 2\ (8.2) = 16.4$ kips/in.

$c = 0.0359$ kip-sec/in.

(a) $T_n = 2\pi\sqrt{\dfrac{m}{k}} = 2\pi\sqrt{\dfrac{0.0519}{16.4}} = 0.353 \text{ sec}$

(b) $\zeta = \dfrac{c}{2\sqrt{k\,m}} = \dfrac{0.0359}{2\sqrt{(16.4)\,(0.0519)}} = 0.0194$

$= 1.94\%$

Problem 2.14

(a) The stiffness coefficient is

$$k = \frac{3000}{2} = 1500 \quad \text{lb/in.}$$

The damping coefficient is

$$c = c_{cr} = 2\sqrt{km}$$
$$c = 2\sqrt{1500\frac{3000}{386}} = 215.9 \quad \text{lb - sec / in.}$$

(b) With passengers the weight is w = 3640 lb. The damping ratio is

$$\zeta = \frac{c}{2\sqrt{km}} = \frac{215.9}{2\sqrt{1500\frac{3640}{386}}} = 0.908$$

(c) The natural vibration frequency for case (b) is

$$\begin{aligned}
\omega_D &= \omega_n\sqrt{1-\zeta^2} \\
&= \sqrt{\frac{1500}{3640/386}}\sqrt{1-(0.908)^2} \\
&= 12.61 \times 0.419 \\
&= 5.28 \text{ rads / sec}
\end{aligned}$$

Problem 2.15

1. *Determine* ζ *and* ω_n.

$$\zeta \approx \frac{1}{2\pi j} \ln\left(\frac{u_1}{u_{j+1}}\right) = \frac{1}{2\pi(20)} \ln\left(\frac{1}{0.2}\right) = 0.0128 = 1.28\%$$

Therefore the assumption of small damping implicit in the above equation is valid.

$$T_D = \frac{3}{20} = 0.15 \text{ sec}; \; T_n \approx T_D = 0.15 \text{ sec};$$

$$\omega_n = \frac{2\pi}{0.15} = 41.89 \text{ rads/sec}$$

2. *Determine stiffness coefficient.*

$$k = \omega_n^2 m = (41.89)^2 \, 0.1 = 175.5 \text{ lbs/in.}$$

3. *Determine damping coefficient.*

$$c_{cr} = 2m\omega_n = 2\,(0.1)\,(41.89) = 8.377 \text{ lb} - \text{sec/in.}$$

$$c = \zeta c_{cr} = 0.0128\,(8.377) = 0.107 \text{ lb} - \text{sec/in.}$$

Problem 2.16

(a) $k = \dfrac{250}{0.8} = 312.5 \text{ lbs/in.}$

$$m = \frac{w}{g} = \frac{250}{386} = 0.647 \text{ lb} - \text{sec}^2/\text{in.}$$

$$\omega_n = \sqrt{\frac{k}{m}} = 21.98 \text{ rads/sec}$$

(b) Assuming small damping,

$$\ln\left(\frac{u_1}{u_{j+1}}\right) \approx 2 j \pi \zeta \Rightarrow$$

$$\ln\left(\frac{u_0}{u_0/8}\right) = \ln(8) \approx 2(2)\pi\zeta \Rightarrow \zeta = 0.165$$

This value of ζ may be too large for small damping assumption; therefore we use the exact equation:

$$\ln\left(\frac{u_1}{u_{j+1}}\right) = \frac{2 j \pi \zeta}{\sqrt{1 - \zeta^2}}$$

or,

$$\ln(8) = \frac{2(2)\pi\zeta}{\sqrt{1 - \zeta^2}} \Rightarrow \frac{\zeta}{\sqrt{1 - \zeta^2}} = 0.165 \Rightarrow$$

$$\zeta^2 = 0.027(1 - \zeta^2) \Rightarrow$$

$$\zeta = \sqrt{0.0267} = 0.163$$

(c) $\omega_D = \omega_n \sqrt{1 - \zeta^2} = 21.69 \text{ rads/sec}$

Damping decreases the natural frequency.

Problem 2.17

Reading values directly from Fig. 1.1.4b:

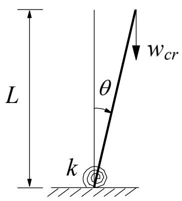
Peak	Time, t_i (sec)	Peak, $\ddot{u}_i$ (g)
1	0.80	0.78
31	7.84	0.50

$$T_D = \frac{7.84 - 0.80}{30} = 0.235 \text{ sec}$$

$$\zeta = \frac{1}{2\pi(30)} \ln\left(\frac{0.78g}{0.50g}\right) = 0.00236 = 0.236\%$$

Problem 2.18

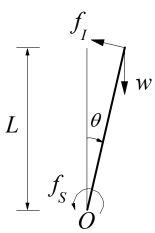
1. *Determine buckling load.*



$$w_{cr}\,(L\,\theta) = k\,\theta$$

$$w_{cr} = \frac{k}{L}$$

2. *Draw free-body diagram and set up equilibrium equation.*



$$\sum M_O = 0 \Rightarrow f_I\,L + f_S = w\,L\theta \qquad \text{(a)}$$

where

$$f_I = \frac{w}{g}L^2\,\ddot{\theta} \qquad f_S = k\,\theta \qquad \text{(b)}$$

Substituting Eq. (b) in Eq. (a) gives

$$\frac{w}{g}L^2\,\ddot{\theta} + (k - w\,L)\,\theta = 0 \qquad \text{(c)}$$

3. *Compute natural frequency.*

$$\omega_n' = \sqrt{\frac{k - wL}{(w/g)\,L^2}} = \sqrt{\frac{k}{(w/g)\,L^2}\left(1 - \frac{wL}{k}\right)}$$

or

$$\omega_n' = \omega_n\sqrt{1 - \frac{w}{w_{cr}}} \qquad \text{(d)}$$

Problem 2.19

For motion of the building from left to right, the governing equation is

$$m\ddot{u} + ku = -F \qquad \text{(a)}$$

for which the solution is

$$u(t) = A_2 \cos \omega_n t + B_2 \sin \omega_n t - u_F \qquad \text{(b)}$$

With initial velocity of $\dot{u}(0)$ and initial displacement $u(0) = 0$, the solution of Eq. (b) is

$$u(t) = \frac{\dot{u}(0)}{\omega_n} \sin \omega_n t + u_F (\cos \omega_n t - 1) \qquad \text{(c)}$$

$$\dot{u}(t) = \dot{u}(0) \cos \omega_n t - u_F \omega_n \sin \omega_n t \qquad \text{(d)}$$

At the extreme right, $\dot{u}(t) = 0$; hence from Eq. (d)

$$\tan \omega_n t = \frac{\dot{u}(0)}{\omega_n} \frac{1}{u_F} \qquad \text{(e)}$$

Substituting $\omega_n = 4\pi$, $u_F = 0.15$ in. and $\dot{u}(0) = 20$ in./sec in Eq. (e) gives

$$\tan \omega_n t = \frac{20}{4\pi} \frac{1}{0.15} = 10.61$$

or

$$\sin \omega_n t = 0.9956; \quad \cos \omega_n t = 0.0938$$

Substituting in Eq. (c) gives the displacement to the right:

$$u = \frac{20}{4\pi}(0.9956) + 0.15(0.0938 - 1) = 1.449 \text{ in.}$$

After half a cycle of motion the amplitude decreases by

$$2u_F = 2 \times 0.15 = 0.3 \text{ in.}$$

Maximum displacement on the return swing is

$$u = 1.449 - 0.3 = 1.149 \text{ in.}$$

Problem 2.20

Given:

$$F = 0.1w, \; T_n = 0.25 \text{ sec}$$

$$u_F = \frac{F}{k} = \frac{0.1w}{k} = \frac{0.1mg}{k} = \frac{0.1\text{g}}{\omega_n^2} = \frac{0.1\text{g}}{(2\pi/T_n)^2}$$

$$= \frac{0.1\text{g}}{(8\pi)^2} = 0.061 \text{ in.}$$

The reduction in displacement amplitude per cycle is

$$4u_F = 0.244 \text{ in.}$$

The displacement amplitude after 6 cycles is

$$2.0 - 6(0.244) = 2.0 - 1.464 = 0.536 \text{ in.}$$

Motion stops at the end of the half cycle for which the displacement amplitude is less than u_F. Displacement amplitude at the end of the 7th cycle is 0.536 – 0.244 = 0.292 in.; at the end of the 8th cycle it is 0.292 – 0.244 = 0.048 in.; which is less than u_F. Therefore, the motion stops after 8 cycles.