

INSTRUCTOR'S SOLUTIONS MANUAL

DISCRETE MATHEMATICS

FIFTH EDITION

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Publishing as Pearson Addison-Wesley, 75 Arlington Street, Boston, MA 02116.

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ISBN 0-321-30516-7

1 2 3 4 5 6 XXX 08 07 06 05



Contents

1	An Introduction to Combinatorial Problems and Techniques	1
1.1	The Time to Complete a Project	1
1.2	A Matching Problem	4
1.3	A Knapsack Problem	4
1.4	Algorithms and Their Efficiency	4
	Supplementary Exercises	5
2	Sets, Relations, and Functions	6
2.1	Set Operations	6
2.2	Equivalence Relations	7
2.3	Partial Ordering Relations	8
2.4	Functions	10
2.5	Mathematical Induction	11
2.6	Applications	11
	Supplementary Exercises	12
3	Coding Theory	14
3.1	Congruence	14
3.2	The Euclidean Algorithm	15
3.3	The RSA Method	17
3.4	Error-Detecting and Error-Correcting Codes	17
3.5	Matrix Codes	18
3.6	Matrix Codes That Correct All Single-Digit Errors	19
	Supplementary Exercises	20

Table of Contents

4	Graphs	22
4.1	Graphs and Their Representations	22
4.2	Paths and Circuits	24
4.3	Shortest Paths and Distance	27
4.4	Coloring a Graph	28
4.5	Directed Graphs and Multigraphs	30
	Supplementary Exercises	34
5	Trees	36
5.1	Properties of Trees	36
5.2	Spanning Trees	38
5.3	Depth-First Search	40
5.4	Rooted Trees	42
5.5	Binary Trees and Traversals	46
5.6	Optimal Binary Trees and Binary Search Trees	51
	Supplementary Exercises	60
6	Matching	63
6.1	Systems of Distinct Representatives	63
6.2	Matchings in Graphs	63
6.3	A Matching Algorithm	66
6.4	Applications of the Algorithm	66
6.5	The Hungarian Method	67
	Supplementary Exercises	67
7	Network Flows	68
7.1	Flows and Cuts	68
7.2	A Flow Augmentation Algorithm	70
7.3	The Max-Flow Min-Cut Theorem	73
7.4	Flows and Matchings	74
	Supplementary Exercises	75

8	Counting Techniques	78
8.1	Pascal's Triangle and the Binomial Theorem	78
8.2	Three Fundamental Principles	78
8.3	Permutations and Combinations	79
8.4	Arrangements and Selections with Repetitions	79
8.5	Probability	80
8.6	The Principle of Inclusion-Exclusion	81
8.7	Generating Permutations and r -Combinations	82
	Supplementary Exercises	82
9	Recurrence Relations and Generating Functions	84
9.1	Recurrence Relations	84
9.2	The Method of Iteration	88
9.3	Linear Difference Equations with Constant Coefficients	91
9.4*	Analyzing the Efficiency of Algorithms with Recurrence Relations	94
9.5	Counting with Generating Functions	99
9.6	The Algebra of Generating Functions	100
	Supplementary Exercises	102
10	Combinatorial Circuits and Finite State Machines	105
10.1	Logical Gates	105
10.2	Creating Combinatorial Circuits	107
10.3	Karnaugh Maps	109
10.4	Finite State Machines	111
	Supplementary Exercises	115

Table of Contents

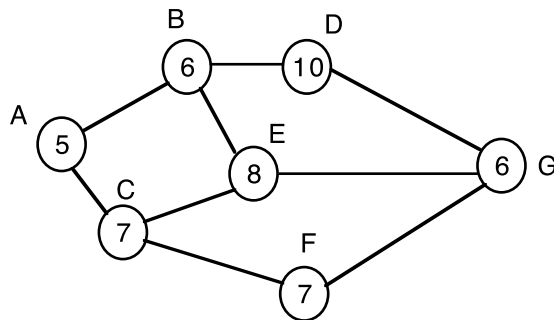
Appendices	120
A An Introduction to Logic and Proof	120
A.1 Statements and Connectives	120
A.2 Logical Equivalence	121
A.3 Methods of Proof	125
Supplementary Exercises	127
B Matrices	130

Chapter 1

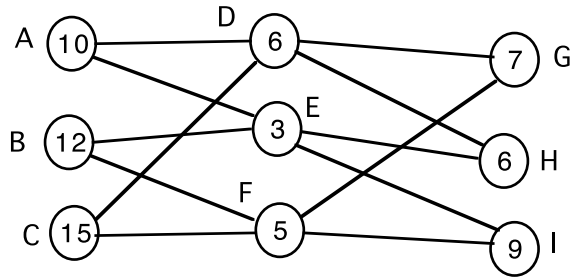
An Introduction to Combinatorial Problems and Techniques

1.1 THE TIME TO COMPLETE A PROJECT

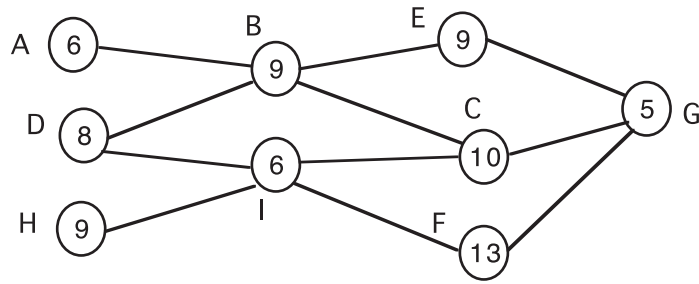
- 2. 31; A-B-E-G
- 4. 39; A-C-G-H
- 6. 16; B-D-F-H
- 8. 27; A-D-E-H
- 10. 27; A-B-D-G



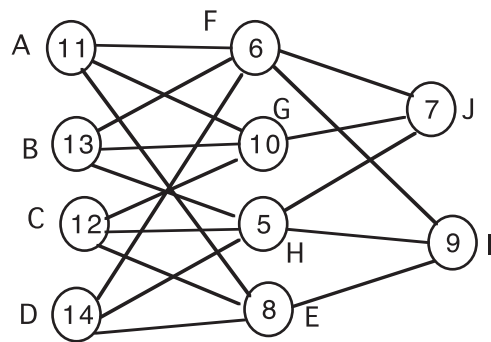
12. 29; C-F-I



14. 33; H-I-F-G



16. 31; D-E-I



18. 20 minutes

1.2 A MATCHING PROBLEM

2. 720 4. 210 6. 84 8. 1680
10. 19,958,400 12. $\frac{1}{8}$ 14. 5040 16. 126
18. 210 20. 119 22. 1320 24. 5040
26. 240 28. 1200

1.3 A KNAPSACK PROBLEM

2. T 4. F 6. F 8. F
10. T 12. T 14. T 16. no
18. yes; 32
20. \emptyset , {1}, {2}, {3}, {4}, {1, 2}, {1, 3}, {1, 4}, {2, 3}, {2, 4}, {3, 4}, {1, 2, 3}, {1, 2, 4}, {1, 3, 4}, {2, 3, 4}, {1, 2, 3, 4}; 16
22. 128 24. 1024 26. 256 28. 26
30. {1, 4, 6, 7, 8, 9, 10, 11, 12}

1.4 ALGORITHMS AND THEIR EFFICIENCY

2. yes; 0 4. no
6. no 8. -1, 9, 84; 3, 17, 84
10. -4, -4, 41, 95; 2, 11, 33, 95 12. 111000
14. 001010

16.

k	j	a_1	a_2	a_3
3		1	1	1
2		1	1	1
1		1	1	1
0		1	1	1

18.

k	j	a_1	a_2	a_3	a_4
4		1	1	1	0
		1	1	1	1

20. The circled numbers in the table below indicated the items being compared.

a_1	a_2	a_3	a_4	j	k
23	5	17	12	1	3
23	5	12	17		2
23	5	12	17		1
5	23	12	17	2	3
5	23	12	17		2
5	12	23	17	3	3
5	12	17	23		

22. The circled numbers in the table below indicated the items being compared.

a_1	a_2	a_3	a_4	a_5	j	k
88	2	75	10	48	1	4
88	2	75	10	48		3
88	2	10	75	48		2
88	2	10	75	48		1
2	88	10	75	48	2	4
2	88	10	48	75		3
2	88	10	48	75		2
2	10	88	48	75	3	4
2	10	88	48	75		3
2	10	48	88	75	4	4
2	10	48	75	88		

24. 6.5 years, 2.7 seconds

26. 2.3×10^{10} years, 12.5 seconds

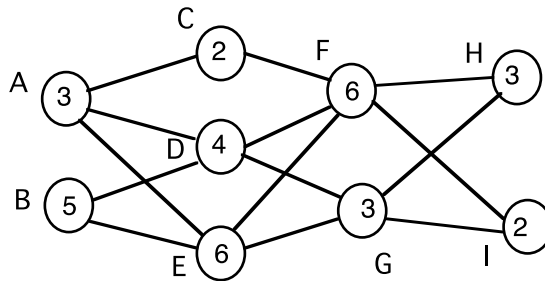
28. $4n - 3$

30. $3n - 2$

32. $-4, -4, 41, 95$

SUPPLEMENTARY EXERCISES

2. 20; B-E-F-H



4. 336

6. 40

8. 14040

10. T

12. F

14. T

16. T

18. 16

20. no

22. yes; 0

24. $-5, 7, 7, 88$

26. $\emptyset, \{4\}, \{3\}, \{3, 4\}, \{2\}, \{2, 4\}, \{2, 3\}, \{2, 3, 4\}, \{1\}, \{1, 4\}, \{1, 3\}, \{1, 3, 4\}, \{1, 2\}, \{1, 2, 4\}, \{1, 2, 3\}, \{1, 2, 3, 4\}$

28. 4.92×10^8 years

30. 4

32. $4r - 3$

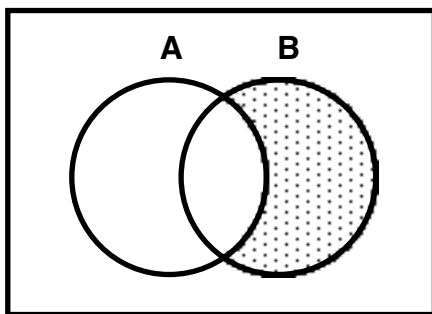
Chapter 2

Sets, Relations, and Functions

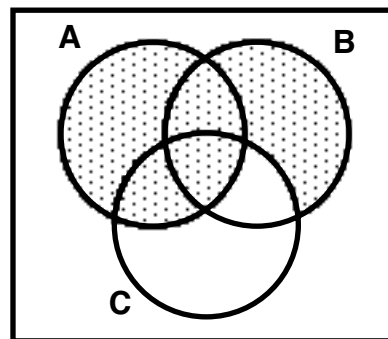
2.1 SET OPERATIONS

2. $A \cup B = \{1, 2, 4, 5, 6, 7, 9\}$, $A \cap B = \{1, 4, 6, 9\}$, $A - B = \emptyset$, $\bar{A} = \{2, 3, 5, 7, 8\}$, and $\bar{B} = \{3, 8\}$
4. $A \cup B = \{2, 3, 4, 5, 6, 7, 8, 9\}$, $A \cap B = \{7, 9\}$, $A - B = \{3, 4, 6, 8\}$, $\bar{A} = \{1, 2, 5\}$, and $\bar{B} = \{1, 3, 4, 6, 8\}$
6. $\{(3, 1), (3, 2), (3, 3), (4, 1), (4, 2), (4, 3), (5, 1), (5, 2), (5, 3)\}$
8. $\{(p, a), (p, c), (p, e), (q, a), (q, c), (q, e), (r, a), (r, c), (r, e), (s, a), (s, c), (s, e)\}$

10.



12.



14. $A = \{1, 2\}$, $B = \{1, 3\}$, and $C = \{1\}$
16. $A = \{1, 2\}$, $B = \{1, 3\}$, and $C = \{1\}$

18. A 20. \emptyset
 22. $A \cap B$ 24. $A \cup \overline{B}$
 26. The equality $A - B = B - A$ holds if and only if $A = B$.
 28. The equality $A \cap B = A$ holds if and only if $A \subseteq B$.

2.2 EQUIVALENCE RELATIONS

2. reflexive and symmetric 4. reflexive, symmetric, and transitive
 6. reflexive and symmetric 8. none
 10. reflexive and symmetric 12. reflexive and transitive
 14. The equivalence class of R containing ABCD consists of the string ABC and the strings of 4 letters having A as their first letter and C as their third letter. There are $26^2 = 676$ distinct equivalence classes of R .
 16. The equivalence class of R containing $\{1, 2, 3\}$ is the set containing the following four elements of S : $\{1, 3\}$, $\{1, 2, 3\}$, $\{1, 2, 3, 4\}$, and $\{1, 3, 4\}$. There are 8 different equivalence classes of R , namely the sets consisting of the elements S , $S \cup \{2\}$, $S \cup \{4\}$, and $S \cup \{2, 4\}$ for every $S \subseteq \{1, 3, 5\}$.
 18. The equivalence class of R containing $(5, 8)$ is the set

$$\{(x_1, x_2): \text{each } x_i \text{ is an integer and } x_1 - x_2 = 5 - 8\}.$$

There are infinitely many distinct equivalence classes of R , namely, the sets of the form

$$\{(x_1, x_2): \text{each } x_i \text{ is an integer and } x_1 - x_2 = k\},$$

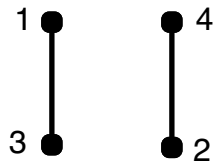
where $k = 0, \pm 1, \pm 2, \pm 3, \dots$

20. $\{(1, 1), (1, 3), (1, 6), (3, 1), (3, 3), (3, 6), (6, 1), (6, 3), (6, 6), (2, 2), (2, 5), (5, 2), (5, 5), (4, 4)\}$
 24. The equivalence classes have the form $E_1 \times E_2$, where E_i is an equivalence class of R_i .
 28. There are 5 partitions of a set with three elements.
 32. Let S be a nonempty set and R an equivalence relation on S . Then there is a function f with domain S such that $s_1 R s_2$ if and only if $f(s_1) = f(s_2)$.

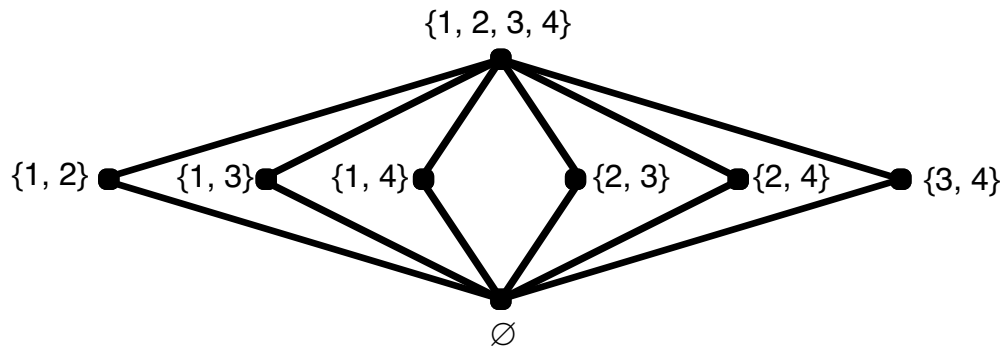
2.3 PARTIAL ORDERING RELATIONS

- 2. not antisymmetric
- 4. partial ordering
- 6. not antisymmetric
- 8. not antisymmetric

10.

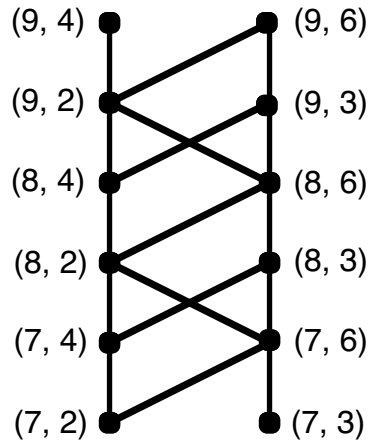


12.



- 14. $R = \{(a, a), (b, b), (b, a), (c, c), (c, b), (c, a), (d, d), (d, a)\}$
- 16. $R = \{(1, 1), (2, 2), (2, 1), (2, 4), (4, 4), (8, 8), (8, 4)\}$
- 18. The maximal elements are $\{1\}$, $\{2\}$, and $\{3\}$; the only minimal element is $\{1, 2, 3\}$.
- 20. The only minimal element is 0; there are no maximal elements.
- 22. One possible sequence is 1, 3, 2, 4.
- 24. One possible sequence is 1, 3, 2, 6, 4, 12.
- 26. Let S denote the set of residents of Illinois and R be defined so that $x R y$ means that x is a sister of y .
- 28. Every prime integer is a minimal element; there are no maximal elements.
- 30. $(1, 1), (1, 2), (1, 3), (1, 4), (2, 1), (2, 2), (2, 3), (2, 4), (3, 1), (3, 2), (3, 3), (3, 4)$

32.



36. (a) Suppose that y is element of S such that $x R y$ is false. If there is no element y_1 in S such that $y_1 R y$, then y is a minimal element of S , contradicting that x is the unique minimal element of S . Thus there must be such an element y_1 . If there is no element y_2 in S such that $y_2 R y_1$, then y_1 is a minimal element of S , another contradiction. So there must be such an element y_2 . Because S is finite, continuing in this manner must produce a minimal element y_k of S different from x . Because x is the unique minimal element of S , the assumption that there is an element y of S such that $x R y$ is false must be incorrect. Thus $x R s$ is true for every $s \in S$.
- (b) Let Z denote the set of integers and $S = Z \cup \{\emptyset\}$. Let R be the relation defined on S by $x R y$ if and only if one of the following holds: (i) $x, y \in Z$ and $x \leq y$, or (ii) $x = y = \emptyset$. Then \emptyset is the unique minimal element in S , but $\emptyset R z$ is false for every $z \in Z$.

40. 2^n 42. $2^n \cdot 3^{n(n-1)/2}$

2.4 FUNCTIONS

2. not a function with domain X 4. a function with domain X
6. a function with domain X 8. a function with domain X
10. a function with domain X 12. not a function with domain X
14. 4 16. 13 18. 8 20. 7
22. -1 24. 6 26. -2 28. 10
30. 0.78 32. 6.64 34. 3.22 36. -2.56
38. $gf(x) = g(f(x)) = \sqrt{x^2 + 1}$ and $fg(x) = f(g(x)) = x + 1$
40. $gf(x) = \frac{1}{3x}$ and $fg(x) = \frac{3}{x}$
42. $gf(x) = 5(2^x) - 2^{2x}$ and $fg(x) = 2^{5x-x^2}$
44. $gf(x) = \frac{4x-1}{2-x}$ and $fg(x) = \frac{3x-2}{3-x}$
46. one-to-one and onto 48. neither one-to-one nor onto
50. one-to-one but not onto 52. onto but not one-to-one
54. $f^{-1}(x) = \frac{x+2}{3}$ 56. $f^{-1}(x)$ does not exist.
58. $f^{-1}(x)$ does not exist. 60. $f^{-1}(x) = \sqrt[3]{x+1}$
62. For $Y = \{x \in X: x \neq 0\}$, we have $g^{-1}(x) = \frac{-1}{x}$.
64. If $n < m$, there are no one-to-one functions; and if $m \leq n$, there are
- $$P(n, m) = n(n-1)(n-2) \cdots (n-m+1)$$
- one-to-one functions.
68. Let $X = \{1\}$, $Y = \{2, 3\}$, and $Z = 4$. Define $f: X \rightarrow Y$ by $f(x) = 2$ for all $x \in X$ and $g: Y \rightarrow Z$ by $g(y) = 4$ for all $y \in Y$.

2.5 MATHEMATICAL INDUCTION

2. 7, 9, 13, 21, 37, 69

$$4. x_n = \begin{cases} x & \text{if } n = 1 \\ x \cdot x^{n-1} & \text{if } n \geq 2 \end{cases}$$

6. Let x_n denote the n th odd positive integer. Then $x_n = \begin{cases} 1 & \text{if } n = 1 \\ x_{n-1} + 2 & \text{if } n \geq 2. \end{cases}$

8. If $k = 1$, the sets $\{x_1, x_2, \dots, x_k\}$ and $\{x_2, x_3, \dots, x_{k+1}\}$ are disjoint.

10. If $k = 0$, the induction hypothesis cannot be applied to a^{k-1} .

$$28. s_0 + s_1 + \dots + s_n = (n + 1) \left(s_0 + \frac{nd}{2} \right)$$

2.6 APPLICATIONS

2. 56

4. 924

6. 120

8. 715

10. n

12. $r!$

14. 31

16. 4096

18. 128

20. 15

22. 36

24. 286

26. 126

28. 64

44. There are $\frac{n^2 + n + 2}{2}$ regions.

SUPPLEMENTARY EXERCISES

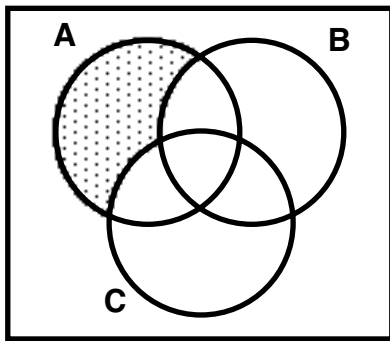
2. $\{1, 2, 3, 4, 5\}$

4. $\{1, 2, 4\}$.

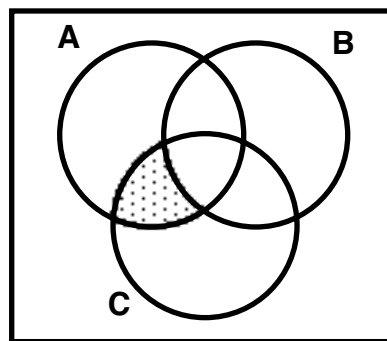
6. $\{1, 2, 4, 5, 6\}$

8. $\{2, 3\}$

10.



12.



14. $gf(x) = 2(x^3 + 1) - 5 = 2x^3 - 3$ and $fg(x) = (2x - 5)^3 + 1 = 8x^3 - 60x^2 + 150x - 124$

16. not a function with domain X

18. not a function with domain X

20. neither one-to-one nor onto

22. one-to-one and onto

24. f^{-1} does not exist.

26. $f^{-1} = \sqrt[3]{x-5}$

28. 84

30. 210

32. $\{1, 7\}, \{2, 6\}, \{3, 5\}, \{4\}, \{8\}$

34. the sets of positive and negative real numbers

36. 5

38. 6

40. (b) $a = \pm 1$

42. reflexive only

48. $x \vee x = x$ for all $x \in S$, $1 \vee y = y \vee 1 = y$ for all $y \in S$, $2 \vee 3 = 3 \vee 2 = 6$, $2 \vee 4 = 4 \vee 2 = 4$, $2 \vee 6 = 6 \vee 2 = 6$, $3 \vee 6 = 6 \vee 3 = 6$, $x \vee y$ is undefined in all other cases

52. Let S denote the set of all subsets of $\{1, 2, 3\}$ that contain at most two elements, and let R be the relation “is a subset of.” If $A = \{1\}$, $B = \{2\}$, and $C = \{3\}$, then $A \vee B = \{1, 2\}$, $B \vee C = \{2, 3\}$, and $A \vee C = \{1, 3\}$, but $(A \vee B) \vee C$ does not exist.
54. $[n] = \{-n, n\}$
56. $f(x) = \begin{cases} x & \text{if } x \equiv 0 \pmod{3} \\ x - 1 & \text{if } x \equiv 1 \pmod{3} \\ x - 2 & \text{if } x \equiv 2 \pmod{3} \end{cases}$