INSTRUCTOR'S RESOURCE MANUAL

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DISCRETE MATHEMATICAL STRUCTURES

Sixth Edition

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Prentice Hall
is an imprint of
PEARSON
Upper Saddle River, NJ 07458

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Prentice Hall is an imprint of PEARSON

Pearson Prentice Hall
Pearson Education, Inc.
Upper Saddle River, NJ 07458

Printed in the United States of America

ISBN-13: 978-0-13-229752-3
ISBN-10: 0-13-229752-3

Pearson Education Ltd., *London*
Pearson Education Singapore, Pte. Ltd.
Pearson Education Canada, Inc.
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Acknowledgements

I wish to thank Bernard Kolman and Robert C. Busby, Drexel University, who produced the Instructor's Manual for the first edition of *Discrete Mathematical Structures.* Also, special thanks go to my colleague, Linda Hawkins Boyd, Georgia Perimeter College, for her help in the production of the Instructor's Manual to accompany the third edition of *Discrete Mathematical Structures.* Thanks also go to Blaise de Sesa for checking exercise and test solutions in this and earlier editions. The artwork in this edition of the Instructor's Manual is a collaborative effort of Dennis Kletzing, Bill Ross the Younger, and the author.

Sharon Cutler Ross
May, 2008

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NOTES TO THE INSTRUCTOR

Our intent in *Discrete Mathematical Structures* is to introduce students to many of the basic mathematical concepts and ideas of discrete mathematics, focusing on those used throughout the computer science curriculum. The text can certainly be used for a general entry-level discrete mathematics course as well. We envision the text as serving as a reference book for many subsequent courses that cover these topics more formally or in greater depth. Relations are an organizing theme for pulling together nearly all the topics covered. Not only do relations underlie a broad spectrum from functions to relational data bases, they are also intuitive to students and can be represented in a variety of ways. The text uses the power of multiple representations to help students master the concepts presented.

A number of sections end with pseudocode versions of algorithms. These may be omitted without loss of continuity if they are not appropriate for the course. The details of the pseudocode used are given in Appendix A of the text. For courses where pseudocode is used or where students have programming experience, coding exercises for each chapter are given in Appendix C.

Each chapter begins with an overview of the chapter material and a brief historical feature, Looking Back. Chapters conclude with a discussion of proof related to the chapter (Tips for Proofs), a summary of Key Ideas, a Self-Test covering the chapter's material including conceptual questions, and a student Experiment. No experiment is included for the brief Chapter 11.

The writing of any text involves choices in topics, depth of coverage, and the sequencing of topics. Here for each chapter, we comment on reasons for some of our choices for this book. Included also are some thoughts from our experience in teaching the material.

Finally, the student experiments at the ends of chapters and in Appendix B of the text offer opportunities for exploring, conjecturing, and proving, either for individuals or for small groups. Each experiment requires students to engage with a problem for an extended period and to write about the mathematics. A further discussion of using the experiments and prerequisites for each is given after the chapter notes.

CHAPTER NOTES

CHAPTER 1 FUNDAMENTALS

Sets are fundamental to many later discussions and definitions so we have chosen to begin with them. Sequences, matrices, and properties of the integers also recur throughout the text in the development of examples as well as of basic ideas. Material on base n representations gives us a tool for beginning a discussion of cryptology. The last section pulls together some general ideas such as properties of operations and helps students see the similarities among the topics discussed earlier in the chapter. There is a danger that students will see discrete mathematics as a hodge-podge of unrelated ideas. By comparing and contrasting sets, integers, and matrices and operations on them, students begin the transition to a more global view – mathematical structures. These concepts are reinforced in later chapters. For many students the level of abstraction of many topics in discrete mathematics presents a problem. This may be the student's first mathematics course that is not skill-oriented and procedure-driven. The change from thinking about the distributive property to a distributive property does not come easily to most students. Working from familiar situations such as integer arithmetic and sets facilitates this change.

Although they are formally defined in Chapter 5, functions are used informally here in the discussion of computer representations of sets.

CHAPTER 2 LOGIC

While a number of discrete mathematics texts begin with logic, we chose to cover in Chapter 1 topics that are both more familiar and more concrete for students. This choice leads to some small inconsistencies in Chapter 1 where students are asked to prove some simple statements. However, the students have seen proofs before, so this is not a serious problem. Swapping the order of the two chapters is possible if one feels strongly about beginning with logic. In a course taught for computer science majors, it is easy to tie much of Chapter 2 to writing guards for loops and proving code. Students with programming experience will also have experience in determining the truth value of compound statements that can be utilized.

The idea of a mathematical structure is used to connect the system of logical statements and operations on them to ideas in Chapter 1.

While several types of proof are discussed in this chapter, the use of mathematical induction is emphasized, especially its role in proving code. We relate the structure of a proof based on mathematical induction to earlier work with implication. In particular, the induction step does not say "Assume that $P(k)$ is true." Students frequently stumble over the word "assume" and do not see why they can not just "assume $P(n)$" and be done. The strong form of mathematical induction is also included. Both forms of induction recur throughout the text both in examples and in exercises.

Two new sections provide more experiences with proofs and problem solving. In Section 2.5 we explore how mathematical statements are created and give more proof practice. By engaging students in the writing of mathematical statements, we demystify a common mathematical process and help students think mathematically. The last section uses sudoku puzzles as a hook to develop the problem-solving technique of modeling several problems in terms of an older problem.

CHAPTER 3 COUNTING

We use the notations ${}_nP_r$ and ${}_nC_r$ for the number of permutations and combinations, respectively, of n objects taken r at a time. These notations are the same as, or similar to, those used on many calculators and avoid the anomalous $\binom{n}{r}$ which is often a problem for students. Permutations and combinations with repeats are also discussed.

Counting techniques are necessary for algorithm analysis. Another tool for algorithm analysis is an understanding of recurrence relations. A section in this chapter covers the basic ideas of how to solve linear homogeneous recurrence relations. Recurrence relations are also one of a number of uses and examples of recursion throughout the text. We believe that students need to spend time with this concept and consequently we have woven it through the book.

A brief introduction to probability provides an application of the counting methods discussed as well as concepts needed for future discussions; e.g., probabilistic algorithms.

CHAPTER 4 RELATIONS AND DIGRAPHS

This chapter is the core of the text. Relations are used as the foundation of virtually all that follows. Properties and representations of relations are presented in detail. Verbal descriptions, sets of ordered pairs, incidence matrices, and digraphs are presented as representations together with discussion of how to determine, in that representation, whether a relation has a specified property. The use of multiple representations enables students to acquire the concept of a relation more easily and with greater understanding.

A taste of relational databases shows an application of ordered n-tuples before we restrict ourselves to ordered pairs.

The generation of new relations from given relations is covered for composition, union, intersection, complement, power, and closure. The inheritance of properties by the new relations is explored in examples and exercises.

The idea of an equivalence relation and its corresponding partition is a powerful one developed in this chapter and used in a number of later sections. Experiment 4 offers a strong reinforcement of this relationship.

The exercises in this chapter are particularly rich sources of opportunities for students to build their proof skills.

CHAPTER 5 FUNCTIONS

Functions are presented as a special type of relation. One-to-one correspondence is a key idea for later work and appears here as a function with certain characteristics. One-to-one functions are applied to the creation of substitution codes. The use of mod functions with check digits is also presented. Special emphasis is given to functions that are commonly used in computer science either in developing code or in analyzing algorithms. These include floor and ceiling functions, mod functions, and Boolean functions. Also, we extend the concept of a characteristic function to define a fuzzy set and have students revisit set operations and proofs using characteristic functions.

The chapter continues with a low-level introduction to the growth of functions including big-Oh notation and theta classes. This section also gives us a chance to return to some algorithms from earlier chapters.

Permutations are revisited from the point of view of functions in the final section. As one-to-one functions, they are used to develop some simple cryptography schemes.

CHAPTER 6 ORDER RELATIONS AND STRUCTURES

We return in this chapter to the theme of types of relations, namely partial orders (posets), lattices, and Boolean algebras. Section 7.1 contains the first appearance of isomorphism, a concept of great use in later sections and chapters. Lattices and Boolean algebras are also a way to return to the idea of a mathematical structure introduced in Chapter 1. The connections among Boolean algebras, Boolean functions, and circuit design are developed. Although circuit design and simplification are rarely done by hand any more, we think some experience with Karnaugh maps and basic ideas in this field is beneficial for students. A symbolic version of Karnaugh maps is included in the exercises.

CHAPTER 7 TREES

Trees are presented as yet another type of relation with (naturally) most emphasis on their graphical representation. The groundwork is laid in this chapter for the use of a tree as an abstract data type. Huffman code trees introduce coding for efficiency. This relates to earlier (and later) discussions of coding for security and coding for error detection and correction. The two versions of Prim's algorithm are synthesized in a matrix-based method for weighted graphs.

Recursive procedures for traversing a tree bring the student to the idea of recursion again. Spanning trees and minimal spanning trees illustrate for the student ways in which trees are natural models for real-world situations. By this time students have seen a number of algorithms and have developed some intuition about recursion, backtracking, greedy algorithms, and other standard programming tools.

CHAPTER 8 TOPICS IN GRAPH THEORY

Graphs, in particular digraphs, are used to represent relations in Chapter 4, but students are likely to see graphs used in a number of other contexts. This chapter looks at graphs as entities in their own right. As with other topics, our aim is to present a solid foundation, but to leave deep results to later courses.

Euler and Hamiltonian paths and circuits are topics that arise naturally in many applications of graph theory. They are also ideas that students find accessible, and the fundamental theorems are nice examples of existence theorems.

Sections on transport networks and matching problems offer another context for the application of graph theory. The section on coloring graphs is an opportunity for students to work with counting techniques, functions, graphs, and relations. We also point out another case of using one model to solve a variety of problems.

CHAPTER 9 SEMIGROUPS AND GROUPS

This chapter is another extension of the ideas in Section 1.6 on mathematical structures. We believe that students should learn to deal with situations like the change of definition of binary operation in Section 9.1. Certainly we could have defined operation in the same way in Sections 1.6 and 9.1, but asking students to deal with a different version is useful. The main structures in this chapter are semigroups and groups. Many ideas and examples defined earlier are pulled together here to give the students lots of examples of operations, properties, semigroups, and groups. The results of Experiment A (Appendix B) can be discussed again if it has been assigned. Equivalence relations reappear as congruence relations and isomorphism is redefined to preserve operations. A final section gives a brief introduction to rings and fields. In particular, we develop $\mathbb{Z}_p$ for use in Chapter 11.

CHAPTER 10 LANGUAGES AND FINITE STATE MACHINES

Compiler and theory of languages courses for computer science majors generally expect students to absorb very quickly formal presentations of languages and topics such as parsing and derivation. The intent of this chapter is to lay some groundwork for these courses, and others. Approaching a phrase structure grammar as one more mathematical structure lets students use experience gained earlier in the course to aid in acquiring these concepts. Sections 10.1 and 10.2 go very smoothly for nearly all students.

Finite-state machines as language recognizers complete the chapter. The ideas are developed through a nice blend of earlier work – semigroups, graphs, functions, and equivalence relations.

CHAPTER 11 GROUPS AND CODING

This is an authors' choice chapter. There are a number of topics appropriate for courses using this text that would have allowed us to apply ideas from earlier chapters. In earlier editions we focused on codes for error detection and correction. Now we also present the culmination of the cryptology (codes for security) thread that begins in Chapter 1. These ideas use many topics from earlier chapters and are inherently interesting to students.

STUDENT EXPERIMENTS

Extended-time assignments in which students, individually or collaboratively, investigate, conjecture, and justify are an excellent way for students to take ownership of the mathematics they study. Each of the experiments is designed for a 10 – 14 day time frame. Some experiments extend concepts in the text; others preview topics. All require students to write about the mathematics they have done. A typical grading rubric is the following.

This assignment is worth a maximum of 35 points of the 100 point experiment grade. Be sure to answer each question using standard written English. Points will be distributed as follows:

10 points	clarity and completeness of presentation	
15 points	correctness of computations and mathematics	
10 points	style and neatness	*You are encouraged to use word-processing.*

Reading through all the reports before grading seems to speed up the process.

Encourage students to talk with you if they are stuck, but it works best to confine yourself to lots of questions and only tiny hints. What students may need most is to hear (repeatedly perhaps) that these are doable assignments.

<u>Experiment 1</u> weighted voting systems
- when: after Section 1.1
- goals: present a novel application of power sets; begin the course with something fresh

<u>Experiment 2</u> finite geometries
- when: after Chapter 2
- goal: provide practice in logical problem solving and proving conjectures

<u>Experiment 3</u> Markov chains
- when: after Section 3.4
- goals: introduce the concept of a Markov chain; apply probability ideas

<u>Experiment 4</u> compatibility relations
- when: after Section 4.5
- goals: extend idea of equivalence relation; reinforce connection between equivalence relation and partition; build parallel structure for compatibility relation and covering; investigate inheritance of properties

<u>Experiment 5</u> simple algorithm analysis
- when: after Section 5.3; also needs Appendix A and Section 3.5
- goals: perform simple algorithm analysis; introduce two simple search algorithms; utilize recurrence relations

<u>Experiment 6</u> Petri nets
- when: after Section 6.6
- goals: adapt digraphs to a new use; introduce the basic concepts of operating systems

Experiment 7	B-trees
when	after Section 7.3
goals	introduce concept of a B-tree
	investigate characteristics of B-trees

Experiment 8	cliques
when	after Section 8.6, but if question 7 is omitted, it may be assigned after Section 4.4
goal	provide practice with relations, digraphs, and graphs
	provide another method for determining $\chi(G)$

Experiment 9	subgroups
when	after Section 9.4
goals	introduce order of an element
	investigate the relationship between an element's order and the group's order
	investigate the relationship between the orders of subgroups and groups

Experiment 10	pushdown automata
when	after Section 10.3
goals	preview concept of a pushdown automaton
	use stack operations of push and pop

Experiment A	modular arithmetic
when	after Section 1.6, but before Chapter 9; needs Example 4, Section 4.5
goals	provide experience with modular arithmetic
	develop criteria to determine if $\mathbb{Z}_n$ has unique solutions to certain equations
	build examples of groups for later use

Experiment B	Towers of Hanoi
when	after Section 3.5, but if question 4 is restated, students need not have seen recurrence relations
goals	give experience with recursion
	practice mathematical induction proofs

Experiment C	Catalan numbers
when	after Section 5.2
goals	create models for the Catalan numbers
	demonstrate the power of modeling as a problem-solving tool
	practice proof techniques

Sample Test Items

Chapter 1 Test Items

1. Let $A = \{x \mid x = 3m,\ m \in \mathbb{Z}\}$, $B = \{0, 2, 4, 6, \dots\}$, $C = \{y \mid y^2 \leq 150\}$, and $D = \{y \mid y^2 + 4 = 0\}$.
   (a) Tell if each of the following is true or false.
   $(i)\ \{6, 12\} \subseteq A$ $(ii)\ \{6, 12\} \subseteq C$ $(iii)\ B \subseteq A$ $(iv)\ D \subseteq \emptyset$
   (b) Tell if each of the following is true or false.
   $(i)\ D \cup A \subseteq C$ $(ii)\ A \cap D = D$ $(iii)\ B \cap A \subseteq C$ $(iv)\ C \subseteq A \cup B$
2. Let $A = \{1, 2, 3, 4\}$, $B = \{x \mid x \in \mathbb{Z}^+ \text{ and } x^2 \leq 16\}$, $C = \{1, 2, 3\}$, $D = \{x \mid x \in \mathbb{Z} \text{ and } x^2 = 4\}$, and E $= \{x \mid x \in \mathbb{Z} \text{ and } x^2 = -4\}$.
   Tell if each of the following is true or false.
   (a) $A = C$ (b) $A \subseteq B$ (c) $C \subseteq B$
   (d) $D \subseteq E$ (e) $E \subseteq A$ (f) $C \subseteq (A \cap D)$
3. Draw a Venn diagram that shows the relationships among the five sets given in Question 2.
4. Let $S = \{\emptyset, a, \{b\}\}$.
   (a) Give P(S).
   (b) What is |P(S)|?
   (c) Tell if each of the following is true or false.
   $(i)\ \emptyset \in S$ $(ii)\ \{a, b\} \subseteq S$ $(iii)\ \{b\} \subseteq S$ $(iv)\ \emptyset \subseteq S$
   $(v)\ \{b\} \in S$ $(vi)\ \{\,\} \cap S \neq \{\,\}$ $(vii)\ \{\emptyset\} \in S$ $(viii)\ \{a\} \cup S = S$
5. Let $A = \{x \mid x = 3n,\ n \in \mathbb{Z}^+\}$, $B = \{x \mid x = 4m,\ m \in \mathbb{Z}^+\}$, $C = \{t \mid t = 2n - 1,\ n \in \mathbb{Z}^+\}$, and $D = \{s \mid s = 2k,\ k \in \mathbb{Z}\}$. Use $\mathbb{Z}$ as the universal set and compute:
   (a) $A \cap B$ (b) $A \oplus C$ (c) $\overline{B}$ (d) $D - C$ (e) $B \cup D$
6. Let $J = \{$M, A, N$\}$, $K = \{$F, A, U, S, T$\}$, $L = \{$S, I, A, M$\}$, and $U = \{$M, A, T, H, I, S, F, U, N$\}$.
   Compute:
   (a) $(J \cup \overline{K}) \cap L$ (b) $(J \oplus L) \cap (L - J)$
7. Prove that if $A \cap B = A \cup B$, then $A = B$.
8. The records of 200 students at Verysmall College show the following courses taken:
   104 students took Latin 103 students took Greek 35 students took Sanskrit
   46 students took Latin and Greek 24 students took Greek and Sanskrit
   9 students took all three languages 28 students have taken none of these languages
   (a) How many students took only Greek?
   (b) How many students took only Latin and Sanskrit?
9. Prove that $A \cup (A \cap B) = A$.
10. Write a formula for the nth term of the sequence 2, 5, 8, 11, 14, … .
11. Define a sequence as follows: $a_0 = 2$, $a_1 = 3$, $a_n = 2a_{n-1} - 3a_{n-2}$. Give the first six terms of this sequence.
12. Let $U = \{m, n, o, p, q, r, s, t, u, v\}$, $A = \{o, u\}$, $B = \{m, o, s, t\}$, $C = \{n, p, r\}$, and $D = \{n, o, r, t, u, v\}$.
   Represent each of the following sets by an array of zeros and ones.
   (a) $A \cup C$ (b) $B \cap D$ (c) $C \cup (A \cap D)$ (d) $(\overline{A} \cap B) \cup C$
13. Let $I = \{0, 1\}$. Describe the regular subsets of I^* corresponding to the following regular expressions.
   (a) $(01)^*10^*1$ (b) 110^*10^* (c) $(1 \vee 0)^*011$ (d) $0^*10^*(10^*10^*)^*$
14. Let $I = \{a, b, c\}$. In each part below is listed a string in I^* and a regular expression over I. For each, state whether the string belongs to the regular set corresponding to the expression.
   (a) abc abc^* (b) abc $(a^*b \vee c)^*$
   (c) abc $(a \vee b)^*c$ (d) abc $(ab)^* \vee c$
15. Compute GCD(1350, 297) and write it as $s(1350) + t(297)$.
16. Use the Euclidean algorithm to compute GCD(4389, 7293).
17. Compute LCM(180, 294).
18. Let g be the mod-13 function. Compute each of the following.
   (a) $g(92)$ (b) $g(8)$ (c) $g(182)$ (d) $g(115)$

19. Write the base 8 expansion of each of the following numbers.
(a) 12 (b) 139 (c) 486
20. Write the base 10 expansion of each of the following numbers.
(a) $(123)_5$ (b) $(71)_8$ (c) $(1011011)_2$
21. Let $\mathbf{A} = \begin{bmatrix} 4 & 5 \\ -3 & 0 \end{bmatrix}$ and $\mathbf{B} = \begin{bmatrix} 6 & 1 & 6 \\ -2 & 4 & 3 \end{bmatrix}$. Compute, if possible, each of the following.
(a) $\mathbf{AB}$ (b) $\mathbf{BA}$ (c) $\mathbf{A}^T$
22. Let $\mathbf{A} = \begin{bmatrix} 4 & 5 \\ -3 & 0 \end{bmatrix}$ and $\mathbf{B} = \begin{bmatrix} 6 & 1 & 6 \\ -2 & 4 & 3 \end{bmatrix}$. Compute, if possible, each of the following.
(a) $\mathbf{A} + \mathbf{B}$ (b) $\mathbf{A}^T\mathbf{B}$ (c) $\mathbf{A}^{-1}$
23. Describe all 2×2 matrices $\mathbf{A}$ such that $\mathbf{A}^2$ is symmetric.
24. Let $\mathbf{C} = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ and $\mathbf{D} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$. Compute each of the following.
(a) $\mathbf{C} \odot \mathbf{D}$ (b) $\mathbf{C} \vee \mathbf{D}$ (c) $\mathbf{C} \wedge \mathbf{D}$
25. Define $a \,\square\, b = \frac{ab}{2}$ and $a \nabla b = a - b$. Let $R =$ (even integers, $\square$, ∇). Show that R is closed with respect to $\square$.
26. Let $R =$ (even integers, $\square$, ∇) as defined in Question 25. Give the identity element, if one exists, for:
(a) $\square$ (b) ∇
27. Let R be the mathematical structure in Question 25. Show that R has a distributive property.
28. Show that $A \subseteq A \cup B$.
29 Show that $A \cap B \subseteq A$.
30. Show that $A - A = \{\,\}$.
31. Let a and b be integers. Show that if p is a prime such that $p \mid ab$, then $p \mid a$ or $p \mid b$.
32. Let $\mathbf{A}$ be a Boolean matrix. (a) Show that $\mathbf{A} \vee \mathbf{A} = \mathbf{A}$. (b) Show that $\mathbf{A} \wedge \mathbf{A} = \mathbf{A}$.

Chapter 2 Test Items

1. Let p: Jack is nimble, q: Jill is quick, and r: Jose jumped over the candlestick.
(a) Write each of the following in terms of p, q, r, and logical connectives.
(*i*) Jack is nimble or Jill is not quick.
(*ii*) Jose jumped over the candlestick and Jack is nimble.
(*iii*) Jack is nimble and Jill is quick, or Jose did not jump over the candlestick.
(b) Write an English sentence that corresponds to each of the following.
(*i*) $\sim p \wedge \sim r$ (*ii*) $q \vee (p \wedge r)$ (*iii*) $\sim (p \vee q)$
2. Determine the truth value for each of the following statements where x and y are integers.
(a) $\forall x \;\exists y \;\; x + y$ is even.
(b) $\exists x \;\exists y \;\; x + y$ is even.
3. Make a truth table for the statement $\sim (p \wedge \sim (p \vee q))$.
4. Make a truth table for the statement $p \vee (\sim p \vee q)$.
5. Make a truth table for the statement $(\sim p \vee \sim q) \wedge (p \vee \sim r)$.
6. Make a truth table for the statement $((q \wedge r) \wedge p) \vee (\sim (p \wedge r) \vee q)$.
7. Let p: Jack is nimble, q: Jill is quick, and r: Jose jumped over the candlestick.
(a) Write the negation of each of the following sentences.
(*i*) If Jack is nimble, then Jill is quick.
(*ii*) Jose jumps over the candlestick and Jill is not quick.

(b) Write an English sentence that corresponds to each of the following.

(i) $(\sim q) \Rightarrow p$

(ii) $(p \wedge q) \Rightarrow \sim r$

(iii) $(r \vee q) \Leftrightarrow p$

8. If $p \Rightarrow q$ is true, what is the truth value of $\sim(p \vee q) \Rightarrow \sim q$? Explain your reasoning.
9. Make a truth table for the statement $(\sim p) \wedge (p \Rightarrow q)$.
10. Let p: If you do your homework and you ask questions, then you will succeed in discrete mathematics.
    (a) Define simple statements and use them to write p in symbolic form.
    (b) Give the converse of p in English and in symbolic form.
    (c) Give the contrapositive of p in English and in symbolic form.
11. Translate into symbolic form and test the validity of the argument.
    (a) If I do not practice, then I will not win the match.
    I practice.
    ∴ I will win the match.
    (b) If I work, then I cannot study.
    Either I study or I pass my mathematics class.
    I work.
    ∴ I pass my mathematics class.
12. Determine whether the given argument is valid or not. Explain your reasoning.
    (a) $p \Rightarrow (q \vee r)$
    $(\sim q) \wedge p$
    $\therefore\ r$
    (b) $(\sim p) \Rightarrow (\sim q)$
    q
    $\therefore\ p$
13. Prove or disprove by giving a counterexample that the sum of any three consecutive even integers is divisible by 6.
14. Prove or disprove by giving a counterexample that the sum of any three consecutive odd integers is divisible by 3.
15. Prove that the following statement is true by using mathematical induction.
    $\frac{1}{1\cdot3} + \frac{1}{3\cdot5} + \cdots + \frac{1}{(2n-1)(2n+1)} = \frac{n}{2n+1}$
16. Prove that $1^3 + 2^3 + 3^3 + \cdots + n^3 = \frac{n^2(n+1)^2}{4}$.
17. Prove that $1 + 5 + 9 + \cdots + (4n-3) = n(2n-1)$.
18. Prove that $5^n - 3$ is divisible by 2.
19. Prove that $1 + 2 + 3 + \cdots + n \leq \frac{(n+1)^2}{2}$.
20. Prove that $n^5 - n$ is divisible by 5 for $n \geq 1$.
21. Prove that $n! \geq 2^{n-1}$.
22. Prove that every integer greater than 24 can be written as the sum of a multiple of 5 and a multiple of 7.
23. Consider the following function where the input X is a positive integer.

FUNCTION F(X)
1. $Z \leftarrow 1$
2. $C \leftarrow 1$
3. **WHILE** $(C < X)$
    a. $Z \leftarrow Z + 2C + 1$
    b. $C \leftarrow C + 1$
4. **RETURN** Z

END OF FUNCTION F

(a) Prove that $Z = C^2$ is a loop invariant and give the value of Z when the loop terminates.
(b) Describe what the function F produces.

Chapter 3 Test Items

1. Compute the number of
   (a) arrangements of the letters of FINAL (b) baseball lineups for a 15-member team
   (c) distinct arrangements of the letters of CLEVELAND
2. How many three-letter "words" can be formed from the letters of COMPUTER if
   (a) no letter may be repeated? (b) letters may be repeated?
3. A palindrome is a string that reads the same forwards as backwards like POP. How many seven-letter palindromes can be made using the English alphabet?
4. A fair six-sided die is rolled 4 times. If the results of each roll are recorded, how many
   (a) record sequences are possible? (b) record sequences end 5, 6?
5. Doughnut Whirl has 16 different types of doughnuts this morning. If you allow repeats, how many different ways can a six-doughnut box be filled?
6. The Acian parliament needs to form a joint committee of six members from the lower house and seven members from the upper house. If there are 29 eligible members of the lower house and 18 eligible members of the upper house, how many different committees are possible?
7. Prove that ${}_nP_r = r \cdot {}_{n-1}P_{r-1} + {}_{n-1}P_r$.
8. Prove that ${}_{m+n}C_r = \sum_{i=1}^{r} {}_mC_i \cdot {}_nC_{r-i}$.
9. How many choices does a student have if she must answer
   (a) eight out of ten questions on a quiz?
   (b) eight out of ten questions on a quiz and must answer the first three questions?
10. Complete and prove the following statement. At least _____ people in a city of 2,384 people have the same pair of first and last initials.
11. Suppose your history book has 648 pages and 14 chapters. Prove that at least one chapter has 45 pages.
12. A baker has decorated the top of a cake with 52 small hard candies. If the cake is cut into 12 equal-size pieces, at least one piece must have _____ candies on it. Justify your answer.
13. What is the probability that exactly 3 sixes will be recorded when 5 fair dice are tossed?
14. Chris has five different pairs of gloves in a drawer. If two gloves are chosen at random, what is the probability that the gloves will be a matched pair?
15. A basket contains three apples, five bananas, four oranges, and six pears. A piece of fruit is chosen at random from the basket. Compute the probability that
    (a) an apple or a pear is chosen. (b) the fruit chosen is not an orange.
16. In the state tournament, the probability of the Marmots winning is 0.40. The other two teams, the Toads and the Emus, are equally likely to win. What is the probability that
    (a) the Marmots will not win? (b) either the Toads or the Marmots will win?
17. Suppose your company work group has 10 members. What is the probability that you will be chosen to represent the group if
    (a) one representative is chosen? (b) two representatives are chosen?
18. Let $p(A) = 0.24$, $p(B) = 0.68$, and $p(A \cap B) = 0.15$.
    (a) What is $p(A \cup B)$? (b) Are A and B mutually exclusive events? Justify your answer.
19. Find an explicit formula for the sequence defined by $a_1 = 3,\ a_n = 3a_{n-1} + 1$.
20. Solve the recurrence relation $b_n = 8b_{n-1} - 15b_{n-2}$, $b_1 = 2$, $b_2 = 16$.
21. Solve the recurrence relation $c_n = -8c_{n-1} - 16c_{n-2}$, $c_1 = -1$, $c_2 = 8$.
22. Develop a formula for the solution of a recurrence relation of the form $a_n = a_{n-1} + m$, $a_1 = m$.

Chapter 4 Test Items

1. Let A be the set of positive divisors of 15 and $B = \{a, b, c\}$.
   (a) What is $|A \times B|$? (b) List $A \times B$.
2. Prove that if $A \subseteq B$, then $A \times C \subseteq B \times C$.
3. Give all two-element partitions of $\{a, b, c, d\}$.

4. Often email records show the date received, the sender, the subject, message size, and the status of the message (read, deleted, et al.). Use the operations select, project, and standard set operations to describe answers to the following queries. Assume the database is named EMAIL.
(a) How many messages with the subject "Saturday night" have been received?
(b) Who sent messages to you on January 11, 2008?

5. Let $S = \{a, b\}$. How many relations are there on $P(S)$?

6. Let $A = \{a, b, c, d\}$ and R be a relation on A whose matrix is $\mathbf{M}_R = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$.
(a) Give R as a set.
(b) Draw the digraph of R.

7. Let $B = \{1, 2, 3, 4, 5\}$ and R be the relation on A whose digraph is given in Figure 1.
(a) Give R as a set. (b) Give $\mathbf{M}_R$.

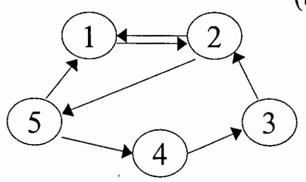


Figure 1

8. Let $C = \{3, 7, 11\}$. Define a relation on C by xRy if and only if $x - y < 4$.
(a) Draw the digraph of R. (b) Give $\mathbf{M}_R$.

9. Let $A = \{1, 2, 3, 4, 5\}$ and R be the relation on A whose digraph is given in Figure 2.
(a) List all paths of length 3. (b) List all cycles.

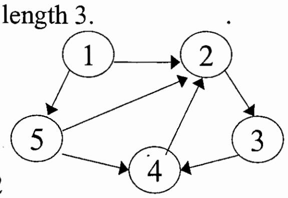


Figure 2

10. Using the relation R whose digraph is given in Figure 2,
(a) Give the matrix of R^2. (b) List the elements of R^4.
(c) Draw the digraph of R^3.

11. Let $B = \{1, 2, 3, 4\}$ and $R = \{(2, 4), (4, 3), (3, 2), (2, 1), (1, 1)\}$.
(a) Draw the digraphs of R and R^2. (b) Give $\mathbf{M}_R$ and $\mathbf{M}_{R^2}$. (c) Give $\mathbf{M}_{R^\infty}$.

12. Let R be a relation defined by $\mathbf{M}_R = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}$.
(a) Give $\mathbf{M}_{R^2}$. (b) Give $\mathbf{M}_{R^\infty}$.

13. Determine whether the relation R on the set A is reflexive, irreflexive, symmetric, asymmetric, antisymmetric, or transitive, where
$A = \{1, 2, 3, 4\}$ and $R = \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 1), (1, 4), (3, 2), (3, 1), (3, 4), (4, 2), (4, 1)\}$.

14. Let $D = \{1, 2, 3, 4, 5\}$ and R be the relation on D whose matrix is

$$\mathbf{M}_R = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 \end{bmatrix}.$$

Determine whether R is reflexive, irreflexive, symmetric, asymmetric, antisymmetric, or transitive.

15. Let $B = \{1, 2, 3, 4, 5\}$ and R be a relation on B whose digraph is given in Figure 3. Determine whether R is reflexive, irreflexive, symmetric, asymmetric, antisymmetric, or transitive.

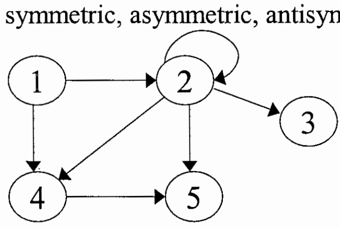


Figure 3

16. Explain why we can define the symmetric closure of a relation, but not the asymmetric closure of a relation.

17. Let t be a fixed real number. Define a relation on $\mathbb{R}$ as follows: xRy if and only if there exists an integer m such that $x - y = mt$. (Note that m will depend on x and y.) Prove that R is an equivalence relation.

18. Let $A = \mathbb{Z}^+ \times \mathbb{Z}^+$, and define R on A as follows: $(x, y)R(u, v)$ if and only if $2 \mid (x - u)$ and $3 \mid (y - v)$.
(a) Prove that R is an equivalence relation.
(b) Compute $[(5, 4)]$, the equivalence class of $(5, 4)$.

19. Let $C = \{1, 2, 3, 4\}$, $A = C \times C$, and define an equivalence relation R on A by $(u, v)R(x, y)$ if and only if $uv = xy$. Compute the corresponding partition of A.

20. Let $A = \{a, b, c, d, e, f\}$ and R be the relation on A defined by

$$\mathbf{M}_R = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}.$$

(a) Prove R is an equivalence relation.
(b) Give the partition of A corresponding to R.

21. Let $B = \{1, 2, 3, 4, 5\}$ and R be a relation on B whose digraph is given in Figure 3. Define arrays VERT, TAIL, HEAD, and NEXT, describing R as a linked list.

22. Give a linked list representation of the relation given by the matrix in Question 20.

23. The following arrays describe a relation R on the set $A = \{1, 2, 3, 4, 5\}$. Draw the digraph of R.
VERT $= [4, 2, 7, 1, 9]$
TAIL $= [4, 2, 2, 1, 1, 1, 3, 3, 5, 5, 1]$
HEAD $= [2, 3, 5, 2, 1, 5, 2, 4, 5, 4, 4]$
NEXT $= [0, 3, 0, 5, 6, 11, 8, 0, 10, 0, 0]$

24. Let $A = \{a, b, c, d\}$, $B = \{1, 2, 3\}$, and $C = \{\square, \triangle, ▱\}$. Let R and S be the following relations from A to B and from B to C, respectively.
$R = \{(a, 1), (a, 2), (b, 2), (b, 3), (c, 1), (d, 3), (d, 2)\}$
$S = \{(1, \square), (2, \triangle), (3, \triangle), (1, ▱)\}$.
(a) Is $(b, \triangle) \in S \circ R$? (b) Is $(c, \triangle) \in S \circ R$? (c) Compute $S \circ R$.

25. Let $A = B =$ the set of real numbers. Let R be the relation $<$ and S be the relation $>$.
Describe (a) $R \cap S$; (b) $R \cup S$.

26. Let $A =$ the set of all people in the Social Security database. Let aRb if and only if a and b receive the same benefits; let aSb if and only if a and b have the same last name. Describe $R \cap S$.

27. Let R be a relation from A to B. prove that
(a) $\text{Dom}(R^{-1}) = \text{Ran}(R)$. (b) $\text{Ran}(R^{-1}) = \text{Dom}(R)$.

28. Let R be a relation defined by $\mathbf{M}_R = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}$.
(a) Give $\mathbf{M}_{R^{-1}}$. (b) Give $\mathbf{M}_{\overline{R}}$.

29. Let R be a relation on a set A.
(a) Prove that if R is transitive, then R^{-1} is transitive.
(b) Prove that if $\text{Dom}(R) = A$, then $R \circ R^{-1}$ is reflexive.

30. Let R and S be relations defined by the matrices

$$\mathbf{M}_R = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \end{bmatrix} \qquad \mathbf{M}_S = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix}$$

(a) Give $\mathbf{M}_{R \cup \overline{S}}$. (b) Give $\mathbf{M}_{R^{-1} \cap S}$.
(c) Give $\mathbf{M}_{R \circ S}$. (d) Give $\mathbf{M}_{S^2}$ and $\mathbf{M}_{S^\infty}$.

31. Using the digraph in Figure 4, give
(a) the associated matrix and (b) the matrix of the transitive closure of the relation.

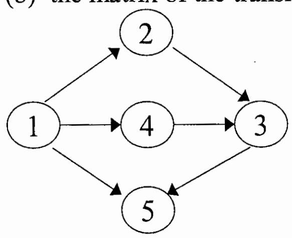


Figure 4

32. Let R be a relation defined by the digraph in Figure 5. Use Warshall's algorithm to find the matrix of the connectivity relation based on R.

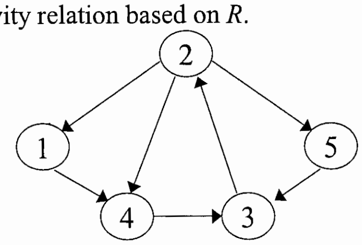


Figure 5

33. Let R be a relation defined by the matrix $\mathbf{M}_R = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix}$.

Using Warshall's algorithm, compute the matrix of the transitive closure of R.

34. Let R and S be relations on a set A described by the matrices

$$\mathbf{M}_R = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 \end{bmatrix} \quad \text{and} \quad \mathbf{M}_S = \begin{bmatrix} 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Use Warshall's algorithm to compute the transitive closure of $R \cup S$.

Chapter 5 Test Items

1. Let $A = \{a, b, c, d\}$, $B = \{1, 2, 3\}$, and $S = \{(a, 1), (b, 1), (c, 2), (b, 3)\}$.
Is S a function? Is S^{-1} a function? Explain your answers.
2. Let $A = B = \mathbb{R}$. Let $f\colon A \to B$ be the function defined by $f(x) = 3x - 4$.
(a) Show that f is one to one and onto. (b) Find f^{-1}.
3. Prove that if $f\colon A \to B$ and $g\colon B \to C$ are one-to-one functions, then $g \circ f$ is one to one.
4. Prove that if $f\colon A \to B$ and $g\colon B \to C$ are onto functions, then $g \circ f$ is onto.

5. For each of the following relations from A to B
(a) Determine if the relation is a function.
(b) For each function, determine whether it is one to one or onto.
(c) Find the inverse for each invertible function.
(*i*) $A = B = \{1, 2, 3\};\ R = \{(1, 1), (2, 3), (3, 1), (2, 1)\}$
(*ii*) $A = B = \mathbb{Z};\ R = \{(a, b) \mid b = a - 4\}$
(*iii*) $A = B = \mathbb{R};\ R = \{(a, b) \mid a \geq 5, b = a + 3\} \cup \{(a, b) \mid a \leq 5, b = a)\}$
(*iv*) $A = B = \mathbb{Z};\ R = \{(a, b) \mid b = a + 7\}$
(*v*) $A = \mathbb{Z}^+, \mathrm{B} = \{0, 1\};\ R = \{(a, b) \mid a \text{ even}, b = 0\} \cup \{(a, b) \mid a \text{ odd}, b = 1\}$

6. Let $A = \{1, 2, 3\}$, $B = \{p, q, r\}$, $f: A \to B$ be defined by $f = \{(1, q), (2, r), (3, p)\}$, and $g: B \to A$ be defined by $g = \{(p, 2), (q, 3), (r, 1)\}$. Prove that $(f \circ g)^{-1} = g^{-1} \circ f^{-1}$.

7. Compute (a) $\lfloor\sqrt{19}\rfloor$ (b) $\lfloor 5.87 \rfloor$ (c) $\lceil\sqrt{19}\rceil$ (d) $\lceil 5.87 \rceil$

8. Compute (a) $lg(32)$ (b) $lg(1/2)$

9. (a) Let P be the propositional function defined by $P(x, y)$: $(x \wedge \sim y) \vee y$. Evaluate $P(\text{true, true})$ and $P(\text{false, true})$.
(b) Let Q be the propositional function defined by $Q(x)$: $\forall y \ \ xy = y$.
Evaluate $Q\left(\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\right)$ and $Q\left(\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right)$.

10. Assume that 8,000 account records need to be stored using the hashing function h which takes the first three digits of the account number as one number and the last three digits as another number, adds them, and then applies the mod-73 function.
(a) How many linked lists will be needed?
(b) If an approximately even distribution of records is achieved, roughly how many records will be stored each linked list?
(c) Compute $h(473810)$, $h(125332)$, and $h(308691)$.

11. Let $A = \{1, 2, 3, 4, 5, 6\}$ and let $p_1 = (4, 2, 5)$ and $p_2 = (2, 6, 1)$ be permutations of A.
(a) Compute $p_1 \circ p_2$ and write the result as a product of cycles. (b) Compute $p_1^{-1} \circ p_2^{-1}$.

12. (a) Write the permutation $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 7 & 8 & 6 & 1 & 2 & 3 & 4 & 5 \end{pmatrix}$ as a product of disjoint cycles.
(b) Let $A = \{1, 2, 3, 4\}$, $p_1 = (1, 2, 4)$, and $p_2 = (1, 4, 3)$. Compute $p_2^2 \circ p_1^{-1}$.

13. Let $p_1 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 5 & 1 & 7 & 6 & 2 & 4 & 3 \end{pmatrix}$ and $p_2 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 3 & 5 & 7 & 1 & 6 & 2 & 4 \end{pmatrix}$.
(a) Compute $p_1 \circ p_2$. (b) Compute p_1^{-1}.
(c) Write p_2 as a product of disjoint cycles and as a product of transpositions.

14. Define a simple substitution code using the keyword JOURNALISM and use this to encode the message SEND MORE MONEY.

15. Define a simple substitution code using the keyword JOURNALISM and use this to decode the message QINKSUNSPFEASKN

16. Let $A = \{1, 2, 3, 4, 5, 6\}$. Let $p_1 = (1, 2, 3, 4)$ and $p_2 = (2, 3, 5, 6)$ be cycles on A.
(a) Write $p_1 \circ p_2$ as a product of disjoint cycles. (b) Is $p_1 \circ p_2$ even or odd? Justify your answer.

17. Show that $f(n) = 5n^2 + 3n - 1$ is $O(n^2)$.

18. Determine the Θ-class of each of the following.
(a) $f(n) = lg(n) + n^2 + 2^n$ (b) $g(n) = n^2(n + 3)(n - 1)$
(c) $h(n) = n \cdot lg(n) + n^2$

19. Show that $f(n) = 18n^4$ is of lower order than $g(n) = n^5$.

20. Consider the following pseudocode.
1. $X \leftarrow 5$
2. $I \leftarrow 0$
3. **UNTIL** $(I > N)$
    a. $X \leftarrow X + 2I$
    b. $I \leftarrow I + 1$

Write a function of N that describes the number of steps required and give the Θ-class of the function.

Chapter 6 Test Items

1. Determine whether the given relation is a partial order. Explain your answer.
   (a) $A = \{1, 2, 3\}$; $R = \{(1, 1), (2, 2), (3, 1), (1, 3)\}$.
   (b) $A = \mathbb{Z}^+$; zRt if and only if $z \cdot t$ is an even integer.
2. Let $B = \{1, 2, 3, 6, 12, 18\}$ and R be defined by xRy if and only if $x \mid y$.
   (a) Draw the digraph and the Hasse diagram of R.
   (b) Give a subset of B that is linearly ordered.
3. Let A be $\mathbb{Z}^+ \times \mathbb{Z}^+$ and define R on A by $(x, y)R(u, v)$ if and only if $x \mid u$ and $y \mid v$. Prove that R is a partial order.
4. A relation R on A is defined by the digraph in Figure 1.
   (a) Confirm that R is a partial order on A.
   (b) Draw the Hasse diagram of R.

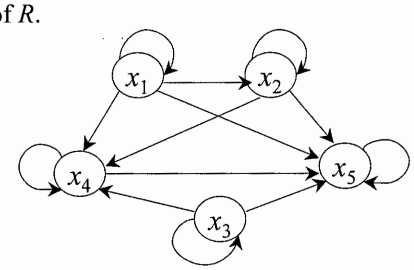


Figure 1

5. Let B and R be as defined in Question 2.
   (a) Determine all minimal and all maximal elements of the poset.
   (b) Find all least and greatest elements of the poset.
   (c) Give all upper bounds and the LUB of $A = \{2, 3, 6\}$.
   (d) Give all lower bounds and the GLB of $A = \{2, 3, 6\}$.
6. Let $A = P(\{a, b, c\})$ and $\leq$ denote the partial order of containment. Let $B = P(\{a, b\})$. Find, if they exist, (a) all upper bounds of B, (b) all lower bounds of B, (c) the least upper bound of B, and (d) the greatest lower bound of B.
7. Let $A = \{2, 3, 4, 6, 8, 12, 24, 48\}$ and $\leq$ denote the partial order of divisibility. Let $B = \{4, 6, 12\}$. Find, if they exist, (a) all upper bounds of B, (b) all lower bounds of B, (c) the least upper bound of B, and (d) the greatest lower bound of B.
8. Let (A, R) be the poset given in Figure 1.
   (a) What is the least upper bound of x_1, x_3? Why? (b) Is R a linear order on A? Explain.
9. Let $(A, \leq)$ be the poset whose Hasse diagram is given in Figure 2.
   (a) Find all minimal and maximal elements of A. (b) Find the least and greatest elements of A.

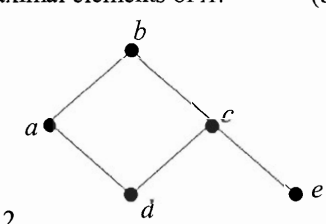


Figure 2

10. Let $(A, \leq)$ be the poset whose Hasse diagram is given in Figure 3.
    (a) Find all minimal and maximal elements of A. (b) Find the least and greatest elements of A.

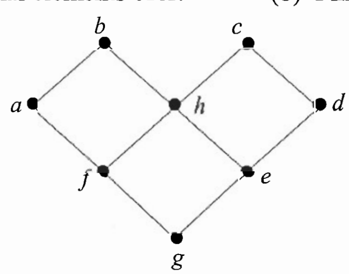
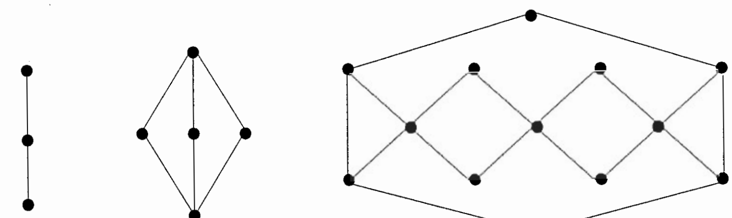


Figure 3

11. Let $(D_{42}, \leq)$ be the lattice of all positive divisors of 42 and $x \leq y$ mean $x \mid y$.
 (a) Draw the Hasse diagram of the lattice.
 (b) Give the GLB of $\{2, 7, 14\}$ and the LUB of $\{3, 6, 7\}$.

12. Let $A = \{a, b, c, d\}$ and R be a relation on A whose matrix is $\mathbf{M}_R = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$.
 (a) Compute LUB($\{a, c\}$).
 (b) Compute GLB($\{b, d\}$).
 (c) Is the poset (A, R) a lattice? Explain your answer.

13. Consider the Hasse diagrams given in Figure 4. State whether the diagram represents a lattice.



(a) (b) (c)

Figure 4

14. Let R and S be defined by the Hasse diagrams in Figure 5.

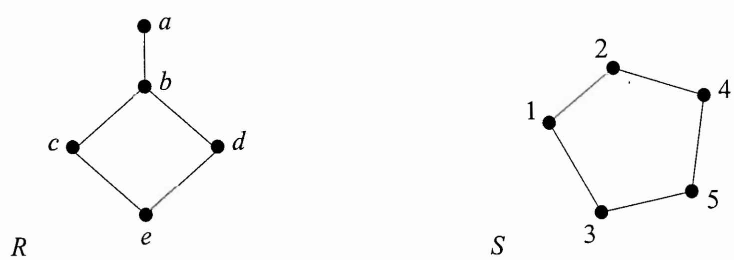


Figure 5

 (a) Complete the following tables for R.

GLB	a	b	c	d	e
a					
b					
c					
d					
e					

LUB	a	b	c	d	e
a					
b					
c					
d					
e					

 (b) Is R a lattice? Explain.
 (c) Is S distributive? Explain.
 (d) Is S complemented? Explain.
 (e) Give the matrix of R.

15. Prove that the lattice $\mathbb{Z}^+$ under the usual partial order $\leq$ is distributive.
16. Prove that a linearly ordered poset is a distributive lattice.
17. Is the dual of a distributive lattice also distributive? Justify your conclusion.
18. Prove that if $a \leq (b \wedge c)$ for some a, b, and c in a poset L, then the distributive properties of a lattice are satisfied by a, b, and c.
19. Find the complement of each element in D_{105}.
20. Let $S = \{a, b, c\}$ and $L = P(S)$. Prove that $(L, \subseteq)$ is isomorphic to D_{42}.
21. Let $(D_{72}, \leq)$ be the lattice of all positive divisors of 72 and $x \leq y$ mean $x \mid y$.
 (a) Draw the Hasse diagram of the lattice.
 (b) Give the GLB of $\{4, 6\}$ and the LUB of $\{4, 6\}$.
 (c) Is this lattice a Boolean algebra? Explain.
22. Let a, b, c be elements in a lattice L. Prove that $a \wedge b \leq a \wedge (b \vee c)$.

23. Consider the Boolean polynomial $p(x, y, z) = (x \wedge y) \vee (x' \wedge z)$.
(a) If $B = \{0, 1\}$, compute the truth table of the function $f: B_3 \to B$ defined by p.
(b) Construct a logic diagram implementing the function f.

24. Consider the Boolean polynomial $p(x, y, z) = (x \wedge y) \wedge (x' \wedge (y \wedge z'))$.
(a) If $B = \{0, 1\}$, compute the truth table of the function $f: B_3 \to B$ defined by p.
(b) Construct a logic diagram implementing the function f.

25 Apply the rules of Boolean arithmetic to show the given Boolean polynomials are equivalent.
$((x \vee y) \wedge y')' \wedge (x \wedge y')'$ and $x' \vee y$

26. (a) Write the Boolean expression represented by the logic diagram in Figure 6.

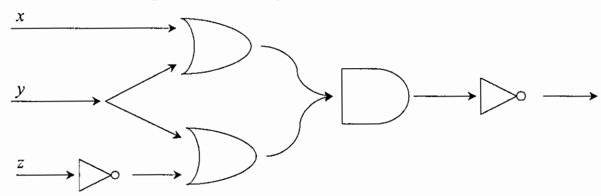


Figure 6

(b) Use the rules of Boolean arithmetic to find an expression that is equivalent to the expression found in part (a) and that uses as few symbols as possible.
(c) Draw a logic diagram for the expression found in part (b).

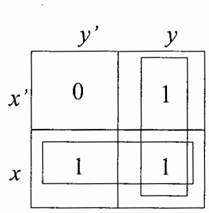
27. Construct the Karnaugh map for the function f whose truth table is

x	y	z	$f(x, y, z)$
0	0	0	1
0	0	1	0
0	1	0	1
0	1	1	0
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	0

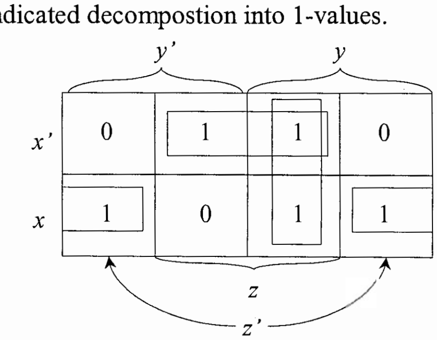
28. Construct the Karnaugh map for the function f whose truth table is

x	y	z	$f(x, y, z)$
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	1

29. Write a Boolean expression for the function whose Karnaugh map is given using the indicated decomposition into 1-values.



30. Write a Boolean expression for the function whose Karnaugh map is given using the indicated decompostion into 1-values.



Chapter 7 Test Items

1. Determine if the relation $R = \{(7, 5), (7, 6), (4, 2), (3, 4), (4, 1), (4, 7)\}$ is a tree on the set $A = \{1, 2, 3, 4, 5, 6, 7\}$. If it is a tree, what is the root?
2. Consider T, the tree whose digraph is given in Figure 1.
   (a) How many proper subtrees does T have, that is, subtrees that are not all of T.
   (b) How many subtrees of T contain v_3?
   (c) How many subtrees of T have height 2?

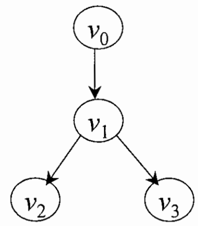


Figure 1

3. Draw all possible binary trees with three vertices.
4. Construct the labeled tree representing the algebraic expression
   $((x - 2) + 3) \div ((2 - (3 + y)) \times (w - 8))$.
5. Construct the labeled tree representing the algebraic expression
   $((a - (b \div c)) \times (((a \times c) \times d) \div b))$.
6. Represent the labeled binary, positional tree whose digraph is given in Figure 2 as a doubly-linked list. Use the arrays LEFT, DATA, and RIGHT.

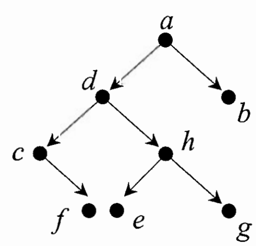


Figure 2

7. The arrays LEFT, DATA, RIGHT give a doubly-linked list representation of a labeled binary, positional tree. Construct the digraph of this tree.

INDEX	LEFT	DATA	RIGHT
1	3		0
2	6	D	0
3	5	C	2
4	0	F	0
5	0	A	7
6	4	E	9
7	0	B	0
8	0	G	0
9	8	H	10
10	0	I	0

8. Use the Huffman code tree in Figure 3 to find the string that represents the given word.

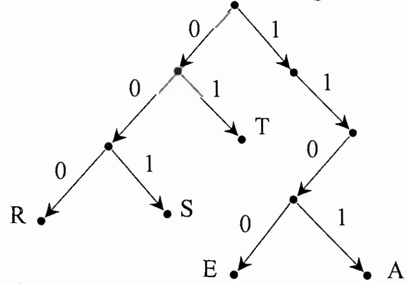


Figure 3

(a) SEAT (b) TARE

9. Use the Huffman code tree in Figure 3 to decode each of the following messages.
(a) 0010111011100 (b) 000110000101
10. Here to visit a node means to print the contents of the node.
(a) Show the results of performing a preorder search on the tree constructed in Question 4
(b) Show the results of performing a postorder search on the tree constructed in Question 5.
11. Assume that visiting a node means to print the contents of the node. For the tree shown in Figure 4, show the result of performing
(a) a preorder search. (b) an inorder search. (c) a postorder search.

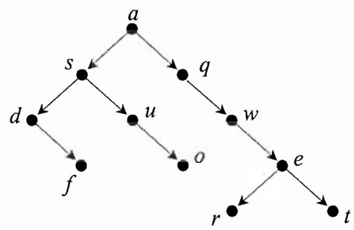


Figure 4

12. The digraph of a labeled, ordered tree T is shown in Figure 5.
Construct the digraph of the corresponding labeled binary, positional tree $B(T)$.
13. (a) Give the results of performing a preorder search on $B(T)$ as constructed in Question 12.
(b) Give the results of performing a postorder search on $B(T)$ as constructed in Question 12.

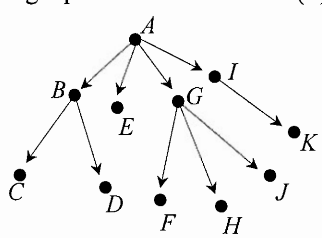


Figure 5

14. The digraph of a labeled, binary positional tree T is shown in Figure 6. Construct the digraph of the labeled ordered tree T' such that $T = B(T')$.

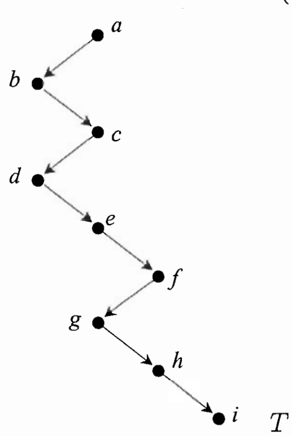


Figure 6

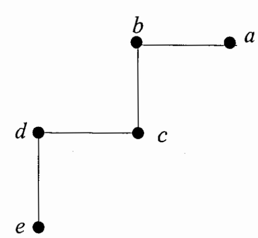


Figure 7

15. Give two different trees whose symmetric closure is given by the digraph in Figure 7.
16. Find an undirected spanning tree for the connected relation whose graph is given in Figure 8.

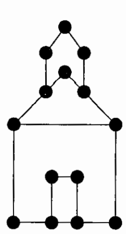


Figure 8

17. Use Prim's algorithm to construct a spanning tree for the connected graph in Figure 9. Use vertex E as the root and draw the digraph of the spanning tree.

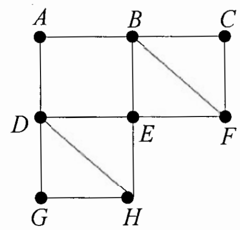


Figure 9

18. For the undirected tree shown in Figure 10, show the digraphs of all spanning trees.

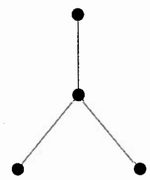


Figure 10

Use the weighted graphs in Figures 11 and 12 for problems 19 through 22.

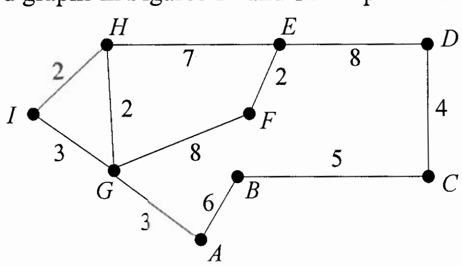


Figure 11

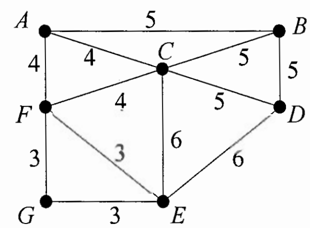


Figure 12

19. Use Prim's greedy algorithm to find a minimal spanning tree for the graph in Figure 11. Use vertex E as the initial vertex and list the edges in the order in which they are chosen.
20. Use Prim's greedy algorithm to find two different minimal spanning trees for the graph in Figure 12.
21. Use Kruskal's algorithm to find a minimal spanning tree for the graph in Figure 11. List the edges in the order in which they are chosen.
22. Use Kruskal's algorithm to find two different minimal spanning trees for the graph in Figure 12. List the edges in the order in which they are chosen.

Chapter 8 Test Items

1. Draw a picture of the graph $G = (V, E, \gamma)$ where $V = \{t, u, v, w, z\}$, $E = \{e_1, e_2, e_3, e_4, e_5\}$, and $\gamma(e_1) = \{v, w\}$, $\gamma(e_2) = \{t, u\}$, $\gamma(e_3) = \{u, v\}$, $\gamma(e_4) = \{v, w\}$, and $\gamma(e_5) = \{t, v\}$.
2. Using the graph in Figure 1,
   (a) List all paths of length 3 that begin at a.
   (b) Give the degree of each vertex.

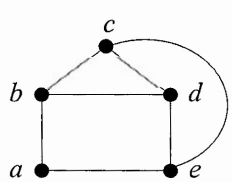


Figure 1

3. If R is the equivalence relation defined by the partition $\{\{v_3, v_4, v_5\}, \{v_1, v_2\}, \{v_6\}\}$, find the quotient graph G^R of the graph G represented by Figure 2.

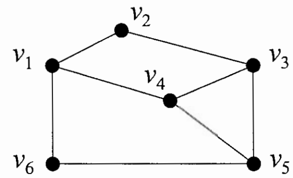


Figure 2

4. Given the graph G represented by Figure 3, draw both G_e and G^e when
(a) $e = \{y, z\}$ (b) $e = \{v, w\}$.

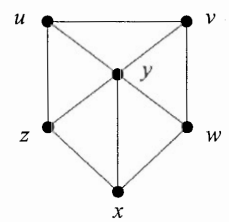


Figure 3

Use Figures 4 and 5 for Questions 5 and 9.

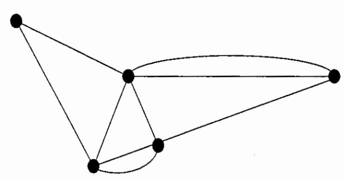


Figure 4

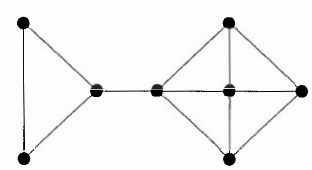


Figure 5

5. Tell whether the graph in the specified figure has an Euler circuit, an Euler path but no Euler circuit, or neither. Give reasons for your decision.
(a) Figure 4 (b) Figure 5

6. Give an Euler circuit, by numbering edges, for the graph in Figure 6. Begin the circuit at A.

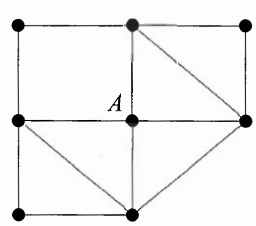


Figure 6

7. The graph represented by Figure 7 does not have an Euler circuit. Mark the minimal number of edges that would have to be traveled twice in order to travel every edge and return to the starting vertex.

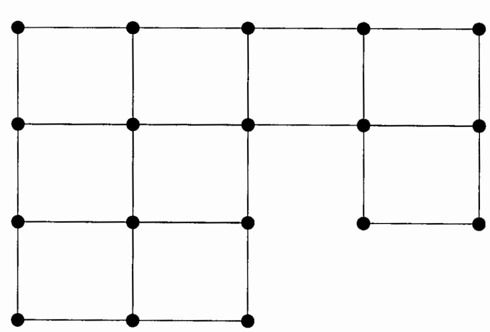


Figure 7

8. For each edge you marked in Question 7, add an edge between the same vertices. Use Fleury's algorithm to find an Euler circuit for the modified version of Figure 7.
9. Tell whether the graph in the specified figure has a Hamiltonian circuit, a Hamiltonian path but no Hamiltonian circuit, or neither. Give reasons for your decision.
(a) Figure 4 (b) Figure 5
10. Give a Hamiltonian circuit, by numbering edges, for the graph represented by Figure 8.

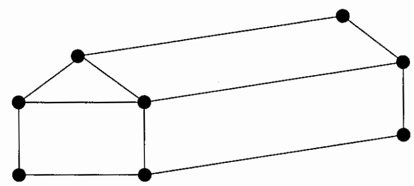
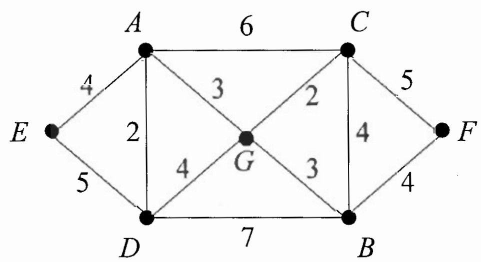


Figure 8

11. Find a Hamiltonian circuit of minimal weight for the graph represented by Figure 9.

Figure 9



12. Find a Hamiltonian circuit of maximal weight for the graph represented by Figure 9.
13. Label the network given in Figure 10 with a flow that conserves flow at each node, except the source and the sink. Each edge is labeled with its maximum capacity.

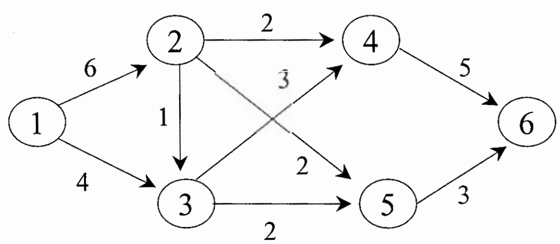


Figure 10

14. Find a maximum flow for the network in Figure 10 by using the labeling algorithm.
15. Find the minimum cut that corresponds to the maximum flow for the network in Figure 10.
16. The matrix $\mathbf{M}_R$ for a relation from A to B is given. Find a maximal matching for A, B, and R.

$$\mathbf{M}_R = \begin{bmatrix} 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 \end{bmatrix}$$

17. The matrix $\mathbf{M}_R$ for a relation from A to B is given. Find a maximal matching for A, B, and R.

$$\mathbf{M}_R = \begin{bmatrix} 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 \end{bmatrix}$$

18. Determine the number of colors needed for a proper coloring of the graph represented by Figure 1.
19. Determine the number of colors needed for a proper coloring of the graph represented by Figure 5.
20. (a) If $P_G(x) = x^2(x-1)(x-3)$, what is $\chi(G)$? (b) If $P_G(x) = (x-2)^2(x-1)^3$, what is $\chi(G)$?

21. (a) Construct a graph for the map given in Figure 11.
(b) Determine the number of colors required for a proper coloring of the map.

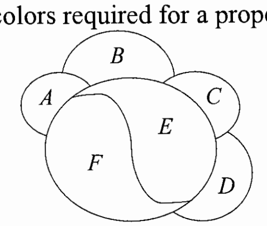


Figure 11

22. Compute the chromatic polynomial for the graph constructed in Question 21(a) and use it to prove the result of Question 21(b).
23. Find P_G and $\chi(G)$ for the graph G given in Figure 12. Confirm your results by giving a proper coloring of G.

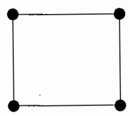


Figure 12

Chapter 9 Test Items

1. For each of the following, determine whether the description of $*$ is a valid definition of a binary operation on the given set.
(a) On $\mathbb{R}$, where $a * b = a + b$
(b) On $\mathbb{Z}$, where $a * b = ab$
(c) On the set of odd integers, where $a * b = a + b$
(d) On $\mathbb{R}^+$, the positive real numbers, where $a * b = a + b$
(e) On $\mathbb{Z}^+$, where $a * b = a^b$
2. Complete the operation table so that $*$ is a commutative binary operation.

$*$	a	b	c
a			a
b	c		
c			b

3. Consider the binary operation $*$ defined by the following table.

$*$	a	b	c	d
a	d	d	a	a
b	d	c	b	a
c	a	b	c	d
d	a	a	d	d

(a) Compute $b * c$ and $a * (c * b)$.
(b) Is $*$ commutative? Explain your answer.
(c) Is $*$ associative? Explain your answer.
4. Give an example of a binary operation on $A = \{x, y, z\}$ that satisfies the idempotent and commutative properties.
5. Let L be a lattice with greatest element I and $a \wedge b = \text{GLB}(a, b)$. Show that L with the operation $\wedge$ is a monoid.
6. Determine whether the set together with the operation is a semigroup, a monoid, or neither for each of the pairs given in Question 1.
7. Let $(G, *)$ be a commutative semigroup. Define $f: G \to G$ by $f(a) = a * a$. Show that f is a homomorphism.
8. If S_1 and S_2 are semigroups, show that S_1 is a homomorphic image of $S_1 \times S_2$.
9. Let S be the semigroup whose operation table is given below. Let R be the equivalence relation on S defined by the partition $\{\{x, y\}, \{z, w\}\}$. Show that R is a congruence relation on S.

$\times$	x	y	z	w
x	x	y	z	w
y	y	x	w	z
z	z	z	z	z
w	w	w	w	w

10. Using the semigroup S and the relation R in Question 9, construct the operation table for S/R.