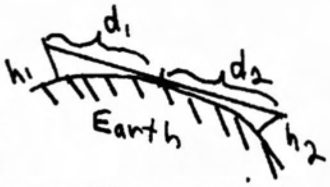
Chapter 1

1-1 $d_1 + d_2 = \sqrt{2(100)} + \sqrt{2(800)} = \underline{\underline{54.14 \text{ miles}}}$

1-2 Using (1-1),

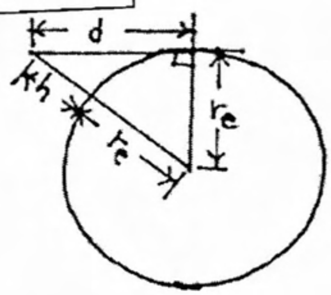


$d_1 = \sqrt{2h_1} = \sqrt{2(1000)}$, $d_2 = \sqrt{2h_2}$

$d_1 + d_2 = 55 \text{ miles} \Rightarrow \sqrt{2(1000)} + \sqrt{2h_2} = 55 \text{ miles}$

$\Rightarrow h_2 = \dfrac{\left(55 - \sqrt{2(1000)}\right)^2}{2} = \underline{\underline{52.8 \text{ ft}}}$

1-3



$(h + r_e)^2 = d^2 + r_e^2$

$\Rightarrow h^2 + 2hr_e = d^2$ where $h \ll r_e$

$\Rightarrow d^2 \approx 2hr_e$ and $r_e = \frac{4}{3}(3960 \text{ miles}) = 5280 \text{ miles}$

Let h = antenna height in feet, and d in miles.

$\Rightarrow d^2 (\text{miles})^2 = 2h\,\cancel{(\text{feet})}\left(\dfrac{1 \text{ mile}}{5280\,\cancel{\text{feet}}}\right)(5280 \text{ miles})$

$\Rightarrow d^2 = 2h$ or $\underline{\underline{d = \sqrt{2h}}}$

1-4

$2d = 30 \text{ miles} = 2\sqrt{2h} \Rightarrow 4(2h) = (30)^2$

$\Rightarrow h = (30)^2/8 = \underline{112.5 \text{ ft}}$

1-5 $d_1 = 2h = \sqrt{2(100)} = 14.14 \text{ m}$ $d_2 = 2h = \sqrt{2(4)} = 2.83 \text{ miles}$

Total radius of coverage $= d_1 + d_2 = 14.14 + 2.83 = \underline{\underline{16.97 \text{ miles}}}$

1-6 $\{m_i\} = \{-1.0, 0.0, 3.0, 4.0\}$ $i = 1, 4$; $P_1 = P_2 = 0.2$; $P_3 = P_4 = 0.3$

$\Rightarrow I_1 = I_2 = \dfrac{-\ln(0.2)}{\ln 2} = 2.322 \text{ bits}$, $I_3 = I_4 = \dfrac{-\ln(0.3)}{\ln 2} = 1.737 \text{ bits}$

$H = \sum_{j=1}^{M} P_j I_j = 2\left[(0.2)(2.322) + (0.3)(1.737)\right] = \underline{\underline{1.971 \text{ bits}}}$

1-7 $I = \log_2(x) \Rightarrow 2^I = x$ Thus, $\log_{10}(2^I) = \log_{10}(x)$

$\Rightarrow I \log_{10}(2) = \log_{10}(x) \Rightarrow I = \dfrac{\log_{10}(x)}{\log_{10}(2)} = \log_2(x)$

1-8

$$p_j = p = \frac{1}{M}\ ;\ j = 1, m$$

Eq. (1-3) $\Rightarrow$ $H = \sum_{j=1}^{M} P_j I_j = \sum_{j=1}^{M} P_j \log_2\left(\frac{1}{P_j}\right)$

or

$$H = \sum_{j=1}^{M} P \log_2\left(\frac{1}{P}\right) = M\left(\frac{1}{M}\right)\log_2\left(\frac{1}{P}\right) = \log_2\left(\frac{1}{P}\right)$$

1-9

Let p_1 = prob. of sending a binary 1

p_2 = prob. of sending a binary 0 $= 1 - p_1$

(a)

$$H = \sum_{i=1}^{2} P_i I_i = p_1 \log_2\left(\frac{1}{p_1}\right) + (1-p_1)\log_2\left(\frac{1}{1-p_1}\right)$$

$$H = \frac{1}{\ln 2}\left[-p_1 \ln(p_1) - (1-p_1)\ln(1-p_1)\right]$$

$$\frac{\partial H}{\partial p_1} = 0 \Rightarrow -(\ln p_1 + 1) - \left((-1)\ln(1-p_1) + \frac{1-p_1}{1-p_1}(-1)\right) = 0$$

$$\Rightarrow -\ln p_1 - \cancel{1} + \ln(1-p_1) + \cancel{1} = 0$$

or $\ln\left(\frac{1-p_1}{p_1}\right) = 0 = \ln 1$

thus $\frac{1}{p_1} - 1 = 1 \Rightarrow \underline{\underline{p_1 = \frac{1}{2} = p_2}}$

(b) $H_{max} = \frac{1}{2}\log_2 2 + \left(1 - \frac{1}{2}\right)\log_2 2 = \underline{\underline{1 \text{ bit}}}$

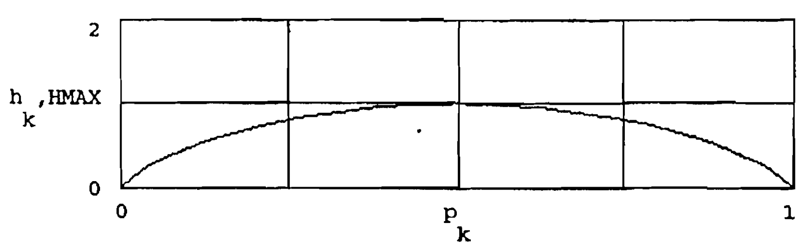
<u>Math CAD Solution</u>

LET p=The probability for sending a binary 1, then the probability for sending a binary 0 is (1-p). From the entropy formula for H(p), we can draw the figure of H(p) ,and from this figure , we can find the maximum entropy and the p.

H(p)=(p*ln(P)+(1-p)*ln(1-p))/(-ln(2))

$k \equiv 0 .. 50 \qquad p_k := \frac{k}{50} \qquad HMAX := 1$

$$h_k := \frac{-1}{\ln(2)}\left[p_k \ln\left[p_k\right] + \left[1 - p_k\right]\cdot \ln\left[1 - p_k\right]\right]$$



From the above figure, we know the maximum entropy is 1 where the probability for sending 1 or 0 is 0.5.

1-10 $P_1 = 0.25$; $P_2 = P_3 = 0.15$;

$P_4 = P_5 = P_6 = P_7 = P_8 = P_9 = 0.07$

$0.25 + 2(0.15) + 6(0.07) = 0.97$

$\therefore\ P_{10} = 0.03$ since $\sum_{j=1}^{10} P_j = 1.0$

$$H = \sum_{j=1}^{M} P_j \log_2\left(\frac{1}{P_j}\right) = \left[\frac{-1}{\ln 2}\right]\sum_{j=1}^{10} P_j \ln P_j$$

$$= \left[\frac{-1}{\ln 2}\right]\left[.25 \ln .25 + (2).15 \ln .15 + (6).07 \ln .07 + .03 \ln .03\right]$$

$$\underline{\underline{H = 3.084 \text{ bits}}}$$

1-11 (a.) $P_1 = 0.3$; $P_2 = 0.7$

$$H = \frac{-\sum_{j=1}^{2} P_j \ln P_j}{\ln 2} = \frac{0.3 \ln 0.3 + 0.7 \ln 0.7}{-\ln 2}$$

$$\underline{\underline{H = 0.881 \text{ bits}}}$$

(b.) H_{max} for $P_j = \frac{1}{M} = \frac{1}{2} = 0.5$ $\quad j = 1, M$

$$H_{max} = -\frac{2(0.5)\ln 0.5}{\ln 2} = \underline{\underline{1 \text{ bit} = H_{max}}}$$

1-12 $M = 10$ $\quad P_j = \frac{1}{10}$ $\quad j = 1, 10$ $\quad R = \frac{H}{T} = 3\,\frac{b}{s}$

$$H = \frac{-10(.1)\ln .1}{\ln 2} = 3.322 \text{ bits}$$

$$T = \frac{H}{R} = \frac{3.322 \text{ bits}}{3 \text{ bits/sec}} = \underline{\underline{1.11 \text{ sec.} = T}}$$

1-13

12 binary digits/word $\quad M=2^{12}\ ;\ P_j=\left(\frac{1}{2}\right)^{13},\ j=1,\frac{M}{2}\ ;\ P_j=3\left(\frac{1}{2}\right)^{13},\ j=\frac{M}{2},M$

$$H=\frac{\sum_{j=1}^{M} P_j \ln P_j}{-\ln 2}$$

$$=\frac{\frac{2^{12}}{2}\left[\left(\frac{1}{2}\right)^{13}\ln\left(\frac{1}{2}\right)^{13}+3\left(\frac{1}{2}\right)^{13}\ln 3\left(\frac{1}{2}\right)^{13}\right]}{-\ln 2}$$

$$\Rightarrow \underline{\underline{H=11.810 \text{ bits}}}$$

Check: $\sum_{j=1}^{M} P_j = 2^{11}\left[\left(\frac{1}{2}\right)^{13}+3\left(\frac{1}{2}\right)^{13}\right] = 2^{11}(4)\left(\frac{1}{2}\right)^{13} = 1.0$

1-14

$B = 300\text{ Hz} \qquad \left(\frac{S}{N}\right)_{dB} = 30\text{ dB}$

$$C = B\log_2\left[1+\left(\frac{S}{N}\right)\right] = \frac{B}{\ln 2}\ln\left[1+\frac{S}{N}\right] \quad \& \quad \left(\frac{S}{N}\right)_{dB} = 10\log_{10}\left(\frac{S}{N}\right)$$

$$\Rightarrow \frac{S}{N} = 10^{\frac{(S/N)_{dB}}{10}} = 10^3 = 1000$$

$$C = \frac{300}{\ln 2}\ln[1001] = \underline{\underline{2.99\times 10^3 \text{ bits/sec}}}$$

1-15

(a) chars := 110 Number of characters available

$b := \text{ceil}\left[\frac{\log(\text{chars})}{\log(2)}\right]$ Number of bits required to represent a character

$\Longrightarrow$ b = 7 bits

(b) B := 3200 Hz Channel bandwidth
SNRdB := 20 dB Signal to noise ratio

$\text{SNR} := 10^{\frac{\text{SNRdB}}{10}} \Longrightarrow$ SNR = 100 (Absolute power ratio)

$C := B\cdot\left[\frac{\log(1+\text{SNR})}{\log(2)}\right] \Longrightarrow C = 2.131\cdot 10^4$ Channel capacity (bits/sec)

$c := \frac{C}{b} \Longrightarrow c = 3.044\ 10^3$ Channel capacity (chars/sec)

(c) Assuming equally likely characters, information content of each character is:

$P := \frac{1}{\text{chars}}$ Probability of each character

$I := \frac{\log\left[\frac{1}{P}\right]}{\log(2)} \Longrightarrow I = 6.781$ bits

1-16

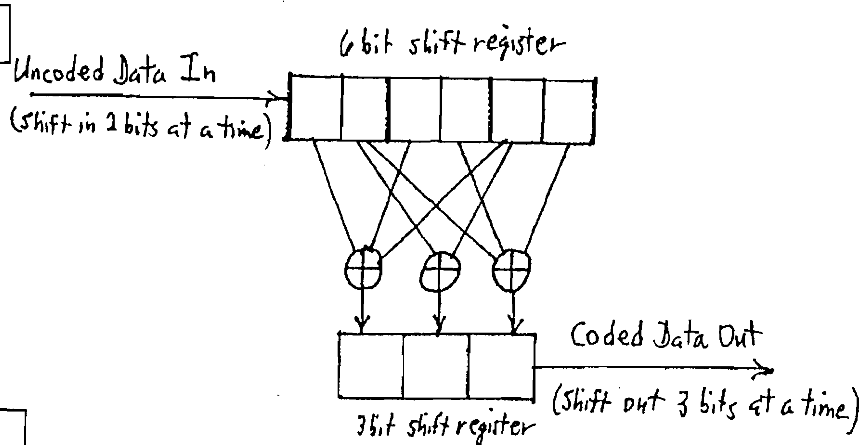
$$R = B\log_2\left(1+\frac{S}{N}\right) = B\,\frac{\log_{10}\left(1+10^{45/10}\right)}{\log_{10}(2)} = B(14.949)$$

(a) $B = 1200-300 = 900 \Rightarrow R = (900)(14.949) = \underline{\underline{13.45\ \text{kbits/s}}}$

(b) $B = 3200-1500 = 1700 \Rightarrow R = (1700)(14.949) = \underline{\underline{25.41\ \text{kbits/s}}}$

(C) $B = 3200-300 = 2900 \Rightarrow R = (2900)(14.949) = \underline{\underline{43.35\ \text{kbits/s}}}$

1-17



1-18

$x_0 := 1 \quad x_1 := 0 \quad x_2 := 1 \quad x_3 := 1 \quad x_4 := 1$ Input vector

$ga_0 := 1 \quad ga_1 := 0 \quad ga_2 := 0 \quad ga_3 := 1$ Gain vector, mod2 adder

$gb_0 := 1 \quad gb_1 := 1 \quad gb_2 := 1 \quad gb_3 := 1$ Gain vector, mod2 adder

$k := 0\,..\,\text{length}(ga) - 2$

$v_k := 0 \qquad k := 0\,..\,\text{length}(x) - 1 \qquad v_{k+\text{length}(ga)-1} := x_k$

$k := \text{length}(x) + \text{length}(ga) - 1\,..\,\text{length}(x) + 2\,\text{length}(ga) - 3$

$v_k := 0 \qquad i := 0\,..\,\text{length}(v) - \text{length}(ga)$

$\qquad\qquad j := 0\,..\,\text{length}(ga) - 1$

$$sa_i := \sum_j \left[ga_{\text{length}(ga)-j-1}\, v_{j+i}\right]$$

$$sa_i := \text{mod}\left[sa_i, 2\right]$$

$$sb_i := \sum_j \left[gb_{\text{length}(gb)-j-1}\, v_{j+i}\right]$$

$sb_i := \text{mod}\left[sb_i, 2\right] \qquad s_{2\,i} := sa_i \qquad s_{2\,i+1} := sb_i$

$i := 0\,..\,2\,\text{length}(x) - 1$

$out_i := s_i$

For $x^T = (1 \;\; 0 \;\; 1 \;\; 1 \;\; 1)$

$\Longrightarrow out^T = (1 \;\; 1 \;\; 0 \;\; 1 \;\; 1 \;\; 0 \;\; 0 \;\; 1 \;\; 1 \;\; 1)$