Instructor's Manual

Digital Control System Analysis and Design

Fourth Edition

CHARLES L. PHILLIPS

H. TROY NAGLE

ARANYA CHAKRABORTTY

CONTENTS

CHAPTER 1

1.1-1. (a) Show that the transfer function of two systems in parallel, as shown in Fig. P1.1-l(a), is equal to the sum of the transfer functions.
(b) Show that the transfer function of two systems in series (cascade), as shown in Fig. Pl.1-l(b), is equal to the product of the transfer functions.

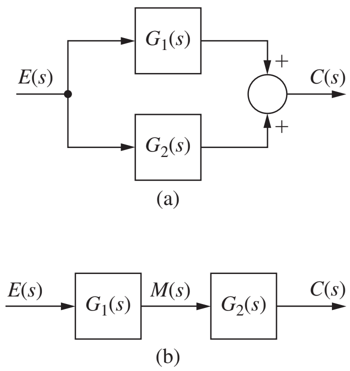


Fig. P1.1-1

Solution:

(a) $C(s) = G_1(s)E(s) + G_2(s)E(s) = [G_1(s) + G_2(s)]E(s)$

$$\therefore \frac{C(s)}{E(s)} = G_1(s) + G_2(s)$$

(b) $C(s) = G_2(s)M(s) = G_1(s)G_2(s)$

$$\therefore \frac{C(s)}{E(s)} = G_1(s)G_2(s)$$

1.1-2. By writing algebraic equations and eliminating variables, calculate the transfer function $C(s)/R(s)$ for the system of:
(a) Figure P1.1-2(a).
(b) Figure P1.1-2(b).
(c) Figure P1.1-2(c).

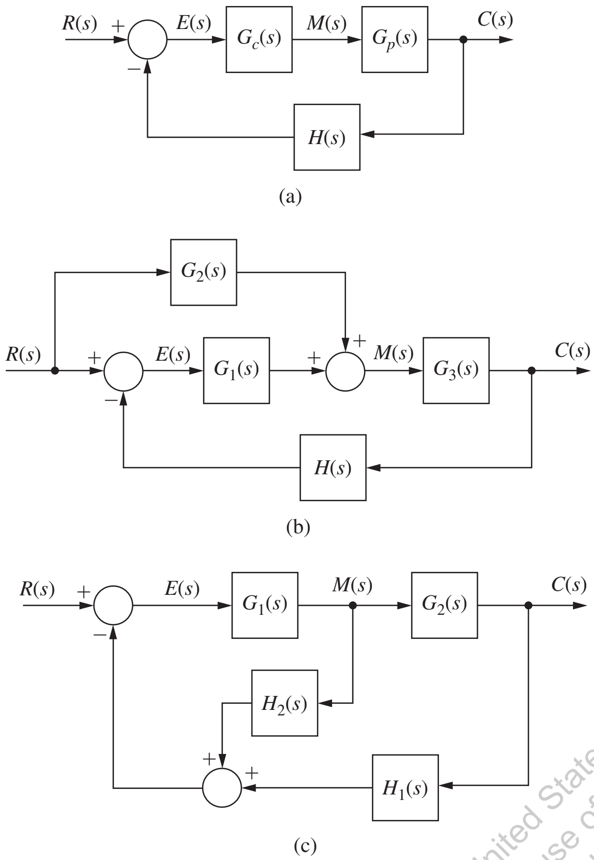


Fig. P1.1-2

Solution:

(a) $C(s) = G_p(s)M(s) = G_c(s)G_p(s)E(s) = G_c(s)G_p(s)[R(s) - H(s)C(s)]$

$[1 + G_c(s)G_p(s)H(s)]C(s) = G_c(s)G_p(s)R(s)$

$$\therefore \frac{C(s)}{R(s)} = \frac{G_c(s)G_p(s)}{1 + G_c(s)G_p(s)H(s)}$$

(b) $C(s) = G_3(s)M(s) = G_3(s)[G_1(s)E(s) + G_2(s)R(s)]$

$= G_3(s)G_1(s)[R(s) - H(s)C(s)] + G_2(s)G_3(s)R(s)$

$[1 + G_1(s)G_3(s)H(s)]C(s) = [G_1(s) + G_2(s)]G_3(s)R(s)$

$$\therefore \frac{C(s)}{R(s)} = \frac{[G_1(s) + G_2(s)]G_3(s)}{1 + G_1(s)G_3(s)H(s)}$$

(c) $C(s) = G_2(s)M(s) = G_1(s)G_2(s)E(s) = G_1(s)G_2(s)[R(s) - H_2(s)M(s) - H_1(s)C(s)]$

and $M(s) = C(s)/G_2(s)$

$$\therefore \left[1 + \frac{G_1G_2H_2}{G_2} + G_1G_2H_1\right]C(s) = G_1G_2R(s)$$

$$\therefore \frac{C(s)}{R(s)} = \frac{G_1(s)G_2(s)}{1 + G_1(s)H_2(s) + G_1(s)G_2(s)H_1(s)}$$

1.1-3. Use Mason's gain formula of Appendix II to verify the results of Problem 1.1-2 for the system of:

(a) Figure P1.1-2(a).
(b) Figure P1.1-2(b).
(c) Figure P1.1-2(c).

Solution:

(a) $\dfrac{C(s)}{R(s)} = \dfrac{G_c(s)G_p(s)}{1+G_c(s)G_p(s)H(s)}$ (b) $\dfrac{C(s)}{R(s)} = \dfrac{G_1(s)G_3(s)+G_2(s)G_3(s)}{1+G_1(s)G_3(s)H(s)}$

(c) $\dfrac{C(s)}{R(s)} = \dfrac{G_1(s)G_2(s)}{1+G_1(s)H_2(s)+G_1(s)G_2(s)H_1(s)}$

1.1-4. A feedback control system is illustrated in Fig. P1.1-4. The plant transfer function is given by

$$G_p(s) = \frac{5}{0.2s+1}$$

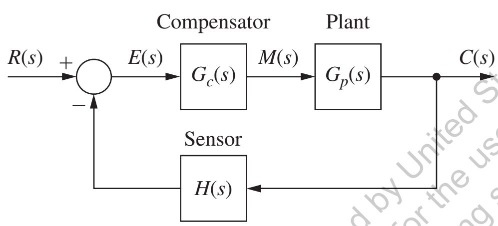


Fig. P1.1-4

(a) Write the differential equation of the plant. This equation relates $c(t)$ and $m(t)$.

(b) Modify the equation of part (a) to yield the system differential equation; this equation relates $c(t)$ and $r(t)$. The compensator and sensor transfer functions are given by

$$G_c(s) = 10, \qquad H(s) = 1$$

(c) Derive the system transfer function from the results of part (b).

(d) It is shown in Problem 1.1-2(a) that the closed-loop transfer function of the system of Fig. P1.1-4 is given by

$$\frac{C(s)}{R(s)} = \frac{G_c(s)G_p(s)}{1+G_c(s)G_P(s)H(s)}$$

Use this relationship to verify the results of part (c).

(e) Recall that the transfer-function pole term $(s+a)$ yields a time constant $\tau = 1/a$, where a is real. Find the time constants for both the open-loop and closed-loop systems.

Solution:

(a) $G_p(s) = \dfrac{C(s)}{M(s)} = \dfrac{5}{0.2s+1} = \dfrac{25}{s+5} \Rightarrow (s+5)C(s) = 25M(s)$

$$\therefore \frac{dc(t)}{dt} + 5c(t) = 25m(t)$$

(b) $m(t) = 10e(t) = 10[r(t) - c(t)]$

$$\therefore \dot{c}(t) + 5c(t) = 250[r(t) - c(t)]$$

$$\therefore \dot{c}(t) + 255c(t) = 250r(t)$$

(c) $5C(s) + 255C(s) = 250R(s)$

$$\therefore \frac{C(s)}{R(s)} = \frac{250}{s+255}$$

(d) $$\frac{C(s)}{R(s)} = \frac{(10)\frac{25}{s+5}}{1+(10)\frac{25}{s+5}(1)} = \frac{250}{s+255}$$

(e) open-loop: $\tau = 0.2s$

closed-loop: $\tau = 1/255 = 0.00392s$

1.1-5. Repeat Problem 1.1-4 with the transfer functions

$$G_c(s) = 2, \qquad G_p(s) = \frac{3s+8}{s^2+2s+2}, \qquad H(s) = 1$$

For part (e), recall that the transfer-function underdamped pole term $[(s+a)^2 + b^2]$ yields a time constant, $\tau = 1/a$.

Solution:

(a) $$G(s) = \frac{C(s)}{M(s)} = \frac{3s+8}{s^2+2s+2}$$

$$\therefore (s^2+2s+2)C(s) = (3s+8)M(s)$$
$$\ddot{c}(t) + 2\dot{c}(t) + 2c(t) = 3\dot{m}(t) + 8m(t)$$

(b) $m(t) = 2e(t) = 2[r(t) - c(t)]$

$$\therefore \ddot{c}(t) + 2\dot{c}(t) + 2c(t) = 6[\dot{r}(t) - \dot{c}(t)] + 16[r(t) - c(t)]$$

$$\therefore \ddot{c}(t) + 8\dot{c}(t) + 18c(t) = 6\dot{r}(t) + 16r(t)$$

(c) $(s^2 + 8s + 18)C(s) = (6s + 16)R(s)$

$$\therefore \frac{C(s)}{R(s)} = \frac{6s+16}{s^2+8s+18}$$

(d) $$\frac{C(s)}{R(s)} = \frac{(2)\frac{3s+8}{s^2+2s+2}}{1+(2)\frac{3s+8}{s^2+2s+2}(1)} = \frac{6s+16}{s^2+8s+18}$$

(e) open-loop: $s^2 + 2s + 2 \Rightarrow s = -1 \pm j$

$$\therefore \text{term} = A_1 \varepsilon^{-t} \cos(t + \theta_1);\ \tau = 1/1 = 1s$$

closed-loop: $s^2 + 8s + 18 \Rightarrow s = -4 \pm j\sqrt{2}$

$$\therefore \text{term} = A_2 \varepsilon^{-4t} \cos(\sqrt{2}t + \theta_2);\ \tau = 1/4 = 0.25s$$

1.1-6. Repeat Problem 1.1-4 with the transfer functions

$$G_c(s) = 2, \quad G_p(s) = \frac{5}{s^2 + 2s + 2}, \quad H(s) = 3s + 1$$

Solution:

(a) $G(s) = \dfrac{C(s)}{M(s)} = \dfrac{5}{s^2 + 2s + 2} \Rightarrow (s^2 + 2s + 2)C(s) = 5M(s)$

$$\therefore \ddot{c}(t) + 2\dot{c}(t) + 2c(t) = 5m(t)$$

(b) $E(s) = R(s) - (3s + 1)C(s)$

$\therefore e(t) = r(t) - 3\dot{c}(t) - c(t)$

$\therefore \ddot{c}(t) + 2\dot{c}(t) + 2c(t) = 5[2e(t)] = 10r(t) - 30\dot{c}(t) - 10c(t)$

$\therefore \ddot{c}(t) + 32\dot{c}(t) + 12c(t) = 10r(t)$

(c) $(s^2 + 32s + 12)C(s) = 10R(s)$

$\therefore \dfrac{C(s)}{R(s)} = \dfrac{10}{s^2 + 32s + 12}$

(d) $$\frac{C(s)}{R(s)} = \frac{(2)\dfrac{5}{s^2 + 2s + 2}}{1 + (2)\dfrac{5}{s^2 + 2s + 2}(3s + 1)} = \frac{10}{s^2 + 32s + 12}$$

(e) open-loop: $\tau = 1s$, from Problem 1.1-5(e)

closed-loop: poles $= -16 \pm \sqrt{244} = -31.62, -0.38$

$\therefore \tau_1 = 1/31.62 = 0.0316s;\ \tau_2 = 1/0.38 = 2.63s$

1.4-1. The satellite of Section 1.4 is connected in the closed-loop control system shown in Fig. P1.4-1. The torque is directly proportional to the error signal.

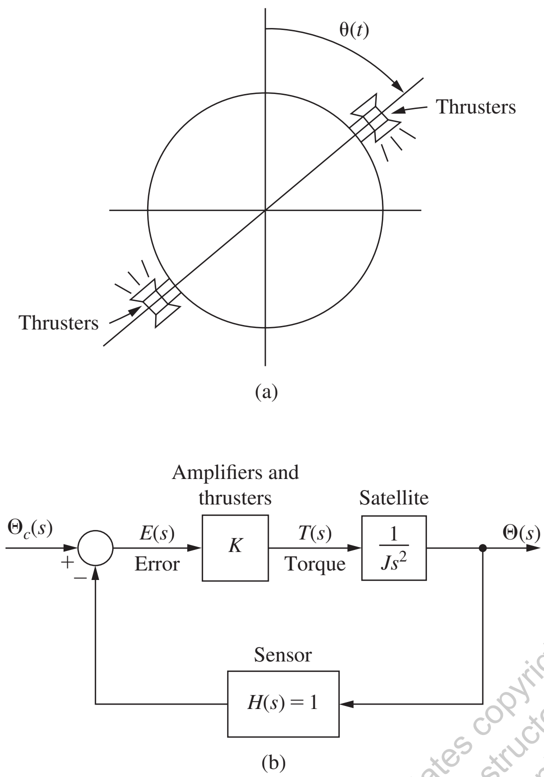


Fig. P1.4-1

(a) Derive the transfer function $\Theta(s)/\Theta_c(s)$, where $\theta(t) = \mathcal{L}^{-1}[\Theta(s)]$ is the commanded attitude angle.

(b) The state equations for the satellite are derived in Section 1.4. Modify these equations to model the closed-loop system of Fig. P1.4-1.

Solution:

(a) $$\Theta(s) = \frac{1}{Js^2}T(s) = \frac{K}{Js^2}E(s) = \frac{K}{Js^2}[\Theta_c(s) - \Theta(s)]$$

$$\therefore \left[1 + \frac{K}{Js^2}\right]\Theta(s) = \frac{K}{Js^2}\Theta_c(s)$$

$$\therefore \frac{\Theta(s)}{\Theta_c(s)} = \frac{K/Js^2}{1 + \dfrac{K}{Js^2}} = \frac{K/J}{s^2 + K/J}$$

(b) $$(1-4)\!: x_1 = 0;\ \ (1-5)\!: x_2 = \dot{\theta} = \dot{x}_1$$

$$\therefore \dot{x}_2(t) = \ddot{\theta}(t) = \frac{1}{J}\tau(t) = \frac{K}{J}e(t) = \frac{K}{J}[\theta_c(t) - \theta(t)]$$

$$= \frac{K}{J}[\theta_c(t) - x_1(t)]$$

$$\therefore \begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ K/J & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ K/J \end{bmatrix} \theta_c(t)$$

$$y(t) = [1 \quad 0]x(t)$$

1.4-2. (a) In the system of Problem 1.4-1, $J = 0.4$ and $K = 14.4$, in appropriate units. The attitude of the satellite is initially at 0°. At $t = 0$, the attitude is commanded to 20°; that is, a 20° step is applied at $t = 0$. Find the response $\theta(t)$.

(b) Repeat part (a), with the initial conditions $\theta(0) = 10°$ and $\dot{\theta}(0) = 30°/s$. Note that we have assumed that the units of time for the system is seconds.

(c) Verify the solution in part (b) by first checking the initial conditions and then substituting the solution into the system differential equation.

Solution:

(a) From Problem 1.4-1, $\dfrac{\Theta(s)}{\Theta_c(s)} = \dfrac{36}{s^2+36}$

$$\therefore \Theta(s) = \frac{36}{s^2+36} \times \frac{20}{s} = \frac{720}{s(s^2+36)} = \frac{20}{s} + \frac{as+b}{s^2+36}$$

$$= \frac{20s^2 + 720 + as^2 + bs}{s(s^2+36)}; \quad \therefore b = 0, \; a = -20$$

$$\therefore \Theta(s) = \frac{20}{s} + \frac{-20s}{s^2+36} \Rightarrow \Theta(t) = 20[1 - \cos 6t], \; t \geq 0$$

(b) From (a), $\ddot{\theta}(t) + 36\,\theta(t) = 36\,\theta_c(t)$

$$\therefore (s^2+36)\Theta(s) - s\theta(0) - \dot{\theta}(0) = 36\Theta_c(s)$$

$$\therefore \Theta(s) = \frac{36}{s^2+36}\Theta_c(s) + \frac{10s}{s^2+36} + \frac{30}{s^2+36}$$

$$= \frac{20}{s} - \frac{20s}{s^2+36} + \frac{10s}{s^2+36} + \frac{30}{s^2+36}$$

$$= \frac{20}{s} - \frac{10s}{s^2+36} + \frac{30}{s^2+36}$$

$$\therefore \theta(t) = 20 - 10\cos 6t + 5\sin 6t, \; t \geq 0$$

(c) $\theta(0) = 20 - 10 = 10°$

$$\dot{\theta}(t) = 60\sin 6t + 30\cos 6t \Rightarrow \dot{\theta}(0) = 30°/s$$

From (b), $\ddot{\theta} + 36\theta = 720$

$$\therefore 360\cos 6t - 180\sin 6t + 720 - 360\cos 6t + 180\sin 6t = 720$$

$$\therefore \; 720 = 720$$

1.4-3. The input to the satellite system of Fig. P1.4-1 is a step function $\theta_c(t) = 5u(t)$ in degrees. As a result, the satellite angle $\theta(t)$ varies sinusoidally at a frequency of 10 cycles per minute. Find the amplifier gain *K* and the moment of inertia *J* for the system, assuming that the units of time in the system differential equation are seconds.

Solution:

From Problem 1.4-1,

$$\Theta(s) = \frac{K/J}{s^2 + K/J} \times \frac{5}{s} = \frac{a^2 5}{(s^2 + a^2)s}, \quad a^2 = K/J$$

$$\therefore \Theta(s) = \frac{5}{s} - \frac{5s}{s^2 + a^2}$$

$$\therefore \theta(t) = 5 - 5\cos at, \; t \geq 0$$

$$10 \text{ cy/min} \times \frac{1 \text{min}}{60 s} = \frac{1}{6} \text{cy/s} \Rightarrow 2\pi\left(\frac{1}{6}\right) \text{rad}/\text{s} = a$$

$\therefore a = \pi/3 = \sqrt{K/J}$, $\therefore$ *K* and *J* cannot be determined without additional data.

1.4-4. The satellite control system of Fig. P1.4-1 is not usable, since the response to any excitation includes an undamped sinusoid. The usual compensation for this system involves measuring the angular velocity $d\theta(t)/dt$. The feedback signal is then a linear sum of the position signal $\theta(t)$ and the velocity signal $d\theta(t)/dt$. This system is depicted in Fig. P1.4-4, and is said to have *rate feedback*.

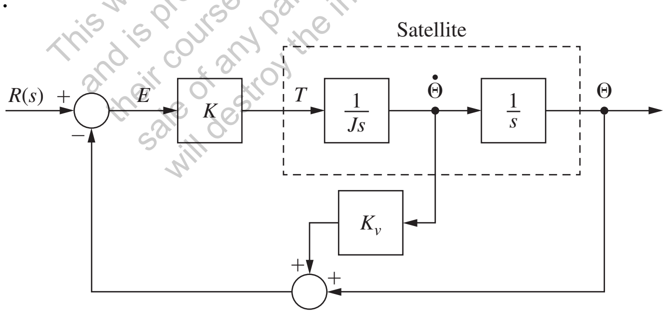


Fig. P1.4-4

(a) Derive the transfer function $\Theta(s) / \Theta_c(s)$ for this system.

(b) The state equations for the satellite are derived in Section 1.4. Modify these equations to model the closed-loop system of Fig. P1.4-4.

(c) The state equations in part (b) can be expressed as

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\theta_c(t)$$

The system characteristic equation is

$$|s\mathbf{I} - \mathbf{A}| = 0$$

Show that $|s\mathbf{I} - \mathbf{A}|$ in part (b) is equal to the transfer function denominator in part (a).

Solution:

(a) $\Theta(s) = \frac{1}{s}\dot{\Theta}(s) = \frac{1}{Js^2}T(s) = \frac{K}{Ts^2}[\Theta_c(s) - K_v\dot{\Theta}(s) - \Theta(s)]$

$\dot{\Theta}(s) = s\Theta(s)$

$\therefore \Theta(s) = \frac{K}{Js^2}[\Theta_c(s) - (K_v s + 1)\Theta(s)]$

$\therefore \frac{\Theta(s)}{\Theta_c(s)} = \frac{K}{Js^2 + KK_v s + K} = \frac{K/J}{s^2 + \frac{KK_v}{J}s + \frac{K}{J}}$

(b) $x_1(t) = \theta(t);\ \ x_2(t) = \dot{x}_1(t) = \theta(t)$

$\therefore \dot{x}_2(t) = \ddot{\theta}(t) = \frac{1}{J}\tau(t) = \frac{K}{J}[\theta_c(t) - K_v x_2(t) - x_1(t)]$

$\therefore \dot{\mathbf{x}}(t) = \begin{bmatrix} 0 & 1 \\ -K/J & -KK_v/J \end{bmatrix}\mathbf{x}(t) + \begin{bmatrix} 0 \\ K/J \end{bmatrix}\theta_c(t)$

$\theta(t) = \begin{bmatrix} 1 & 0 \end{bmatrix}x(t)$

(c) $|s\mathbf{I} - \mathbf{A}| = \begin{vmatrix} s & -1 \\ K/J & S + KK_v/J \end{vmatrix} = s^2 + \frac{KK_v}{J}s + \frac{K}{J}$

1.5-1. The antenna positioning system described in Section 1.5 is shown in Fig. P1.5–1. In this problem we consider the yaw angle control system, where $\theta(t)$ is the yaw angle. Suppose that the gain of the power amplifier is 10 V/V, and that the gear ratio and the angle sensor (the shaft encoder and the data hold) are such that

$$v_o(t) = 0.04\theta(t)$$

where the units of $v_o(t)$ are volts and of $\theta(t)$ are degrees. Let $e(t)$ be the input voltage to the motor; the transfer function of the motor pedestal is given as

$$\frac{\Theta(s)}{E(s)} = \frac{20}{s(s+6)}$$

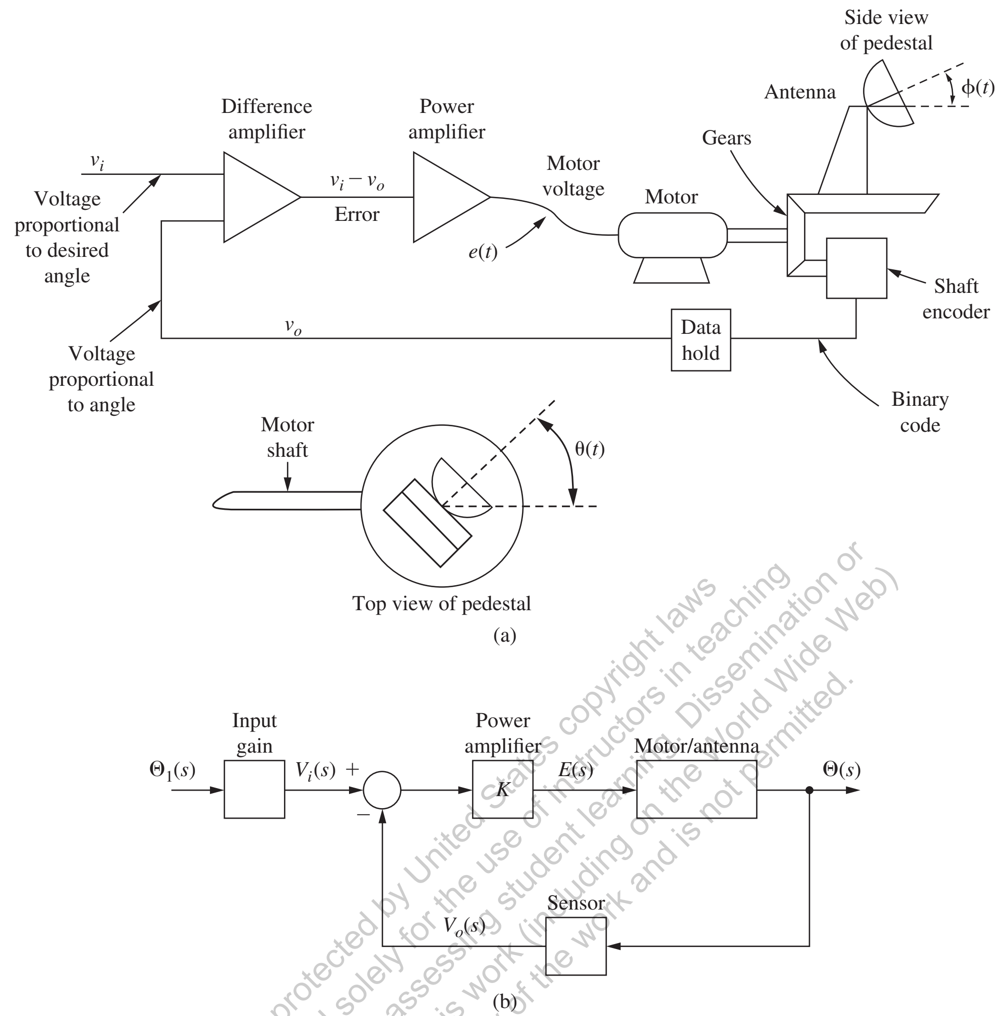


Fig. P1.5-1

(a) With the system open loop [$v_o(t)$ is always zero], a unit step function of voltage is applied to the motor $[E(s) = 1/s]$. Consider only the *steady-state response*. Find the output angle $\theta(t)$ in degrees, and the angular velocity of the antenna pedestal, $\theta(t)$, in both degrees per second and rpm.

(b) The system block diagram is given in Fig. P1.5-1(b), with the angle signals shown in degrees and the voltages in volts. Add the required gains and the transfer functions to this block diagram.

(c) Make the changes necessary in the gains in part (b) such that the units of $\theta(t)$ are radians.

(d) A step input of $\theta_i(t) = 10°$ is applied at the system input at $t = 0$. Find the response $\theta(t)$.

(e) The response in part (d) reaches steady state in approximately how many seconds?

Solution:

(a) $$\Theta(s) = \frac{20}{s^2(s+6)} = \frac{3.33}{s^2} + \frac{k}{s} + \frac{5/9}{s+6}$$

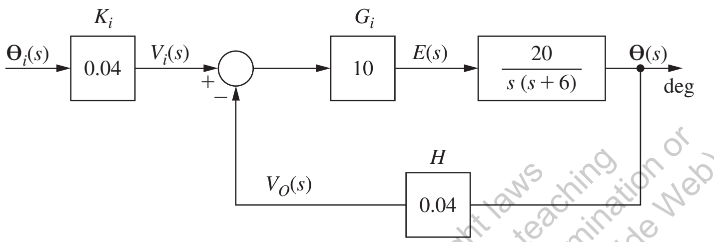
$$k = \frac{d}{ds}\left[\frac{20}{s+6}\right]_{s=0} = \left.\frac{-20}{(s+6)^2}\right|_{s=0} = -5/9$$

$$\therefore \theta_{ss}(t) = \left(3.33t - \frac{5}{9}\right) \text{ in degrees}$$

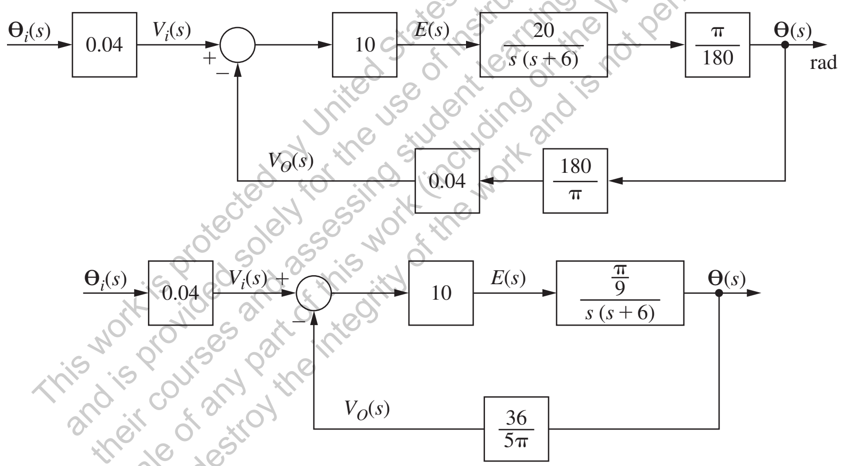
$$\dot{\theta}_{ss}(t) = 3.33 \text{ deg/s}$$

$$\therefore 3.33\frac{d}{s} \times \frac{60s}{1 \text{ min}} \times \frac{1\text{rev}}{360°} = \frac{5}{9}\text{rpm}$$

(b)



(c) $1 \text{ degree} = \frac{\pi}{180}\text{rad}$



(d) $$\frac{\Theta(s)}{\Theta_i(s)} = \frac{(0.04)(10)\frac{20}{s(s+6)}}{1+(0.04)(10)\frac{20}{s(s+6)}} = \frac{8}{s^2+6s+8}$$

$$\Theta(s) = \frac{8}{(s+2)(s+4)} \times \frac{10}{s} = \frac{10}{s} + \frac{-20}{s+2} + \frac{10}{s+4}$$

$$\therefore \theta(t) = 10 - 20\varepsilon^{-2t} + 10e^{-4t},\ t \ge 0$$

(e) $\varepsilon^{-t/\tau} \Rightarrow \frac{1}{\tau_1} = 2,\ \tau_1 = 0.5,\ \tau_2 = 1/4 = 0.25,\ \therefore t_{ss} \approx 4(0.5) = 2s$

1.5-2. The state-variable model of a servomotor is given in Section 1.5. Expand these state equations to model the antenna pointing system of Problem 1.5-1(b).

Solution:

From Eqn. (1-15)

$$Gp(s) = \frac{K_T / JR_a}{s\left(s + \dfrac{BR_a + K_T K_b}{JR_a}\right)} = \frac{20}{s(s+6)},\ \therefore \frac{K_T}{JR_a} = 20;\ \frac{BR_a + K_T K_b}{JR_a} = 6$$

From Eqn (1−16), $x_1 = \theta;\ x_2 = \dot{\theta} = \dot{x}_1$

From Eqn (1−17), $\dot{x}_2(t) = \ddot{\theta}(t) = -6x_2(t) + 20e(t)$

$$= -6x_2(t) + 20(10[0.04\theta_i(t) - 0.04\theta(t)])$$

$$= -6x_2(t) + 8\theta_i(t) - 8x_1(t)$$

$$\therefore \dot{\mathbf{x}}(t) = \begin{bmatrix} 0 & 1 \\ -8 & -6 \end{bmatrix}\mathbf{x}(t) + \begin{bmatrix} 0 \\ 8 \end{bmatrix}\theta_i(t)$$

$$\theta(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} x(t)$$

1.5-3. (a) Find the transfer function $\Theta(s)/\Theta_i(s)$ for the antenna pointing system of Problem 1.5-1(b). This transfer function yields the angle $\theta(t)$ in degrees.

(b) Modify the transfer function in part (a) such that use of the modified transfer function yields $\theta(t)$ in radians.

(c) Verify the results of part (b) using the block diagram of Problem 1.5-1(b).

Solution:

(a) From Problem 1.5-1(d), $\dfrac{\Theta(s)}{\Theta_i(s)} = \dfrac{8}{s^2 + 6s + 8}$

(b) 1 deg = $(\pi/180)$ rad

$$\therefore \frac{\Theta(s)}{\Theta_i(s)} = \frac{8}{s^2 + 6s + 8} \times \frac{\pi}{180} = \frac{2\pi/45}{s^2 + 6s + 8}$$

(c) $$\frac{\Theta(s)}{\Theta_i(s)} = \frac{K_i G_1 G_2}{1 + G_1 G_2 H} = \frac{\left(\dfrac{1}{25}\right)(10)\left(\dfrac{\pi/9}{s(s+6)}\right)}{1 + (10)\dfrac{(\pi/9)}{s(s+6)}\left(\dfrac{36}{5\pi}\right)} = \frac{2\pi/45}{s^2 + 6s + 8}$$

1.5-4. Shown in Fig. P1.5-4 is the block diagram of one joint of a robot arm. This system is described in Section 1.5. The input $M(s)$ is the controlling signal, $E_a(s)$ is the servomotor input voltage, $\Theta_m(s)$ is the motor shaft angle, and the output $\Theta_a(s)$ is the angle of the arm. The inductance of the armature of the servomotor has been neglected such that the servomotor transfer function is second order. The servomotor transfer function includes the inertia of both the gears and the robot arm. Derive the transfer functions $\Theta_a(s) / M(s)$ and $\Theta_a(s) / E_a(s)$.

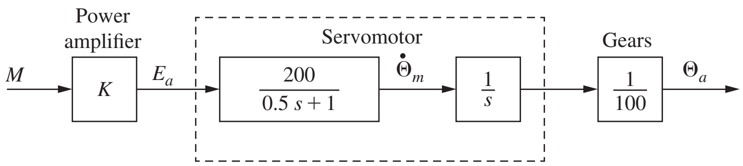


Fig. P1.5-4

Solution:

$$\Theta_a(s) = \frac{1}{100}\Theta_m(s) \quad \frac{1}{100s}\dot{\Theta}_m(s) = \frac{2}{s(0.5s+1)}E_a(s) = \frac{2K}{s(0.5s+1)}M(s)$$

$$\therefore \frac{\Theta_a(s)}{M(s)} = \frac{2K}{s(0.5s+1)}; \ \frac{\Theta_a(s)}{E_a(s)} = \frac{2}{s(0.5s+1)}$$

1.5-5. Consider the robot arm depicted in Fig. P1.5-4.

(a) Suppose that the units of $e_a(t)$ are volts, that the units of both $\theta_m(t)$ and $\theta_a(t)$ are degrees, and that the units of time are seconds. If the servomotor is rated at 24 V [the voltage $e_a(t)$ should be less than or equal to 24 V], find the rated rpm of the motor (the motor rpm, in steady state, with 24 V applied).

(b) Find the maximum rate of movement of the robot arm, in degrees per second, with a step voltage of $e_a(t) = 24u(t)$ volts applied.

(c) Assume that $e_a(t)$ is a step function of 24 V. Give the time required for the arm to be moving at 99 percent of the maximum rate of movement found in part (b).

(d) Suppose that the input $m(t)$ is constrained by system hardware to be less than or equal to 10 V in magnitude. What value would you choose for the gain *K*. Why?

Solution:

(a) $$\dot{\Theta}_a(s) = \frac{2}{0.5s+1}\times\frac{24}{s} = \frac{48}{s(0.5s+2)} = \frac{96}{s(s+2)} = \frac{k_1}{s} + \frac{k_2}{s+2}$$

$$= \frac{48}{s} - \frac{48}{s+2} \Rightarrow \dot{\theta}_a(t) = 48 - 48\varepsilon^{-2t}, \ t \ge 0$$

$$\therefore \dot{\theta}_{ass}(t) = 48°/s \times \frac{60s}{1\,\text{min}} \times \frac{1\,\text{rev}}{360°} = 8\,\text{rpm}$$

$\therefore \dot{\theta}_{mss}(t) = 100\dot{\theta}_a(t) = 800\,\text{rpm}$

(b) From (a), $\dot{\theta}_a(t) = 48°/s$

(c) From (a), $\dot{\theta}_a(t) = 48(1-\varepsilon^{-2t}),\ t \ge 0$

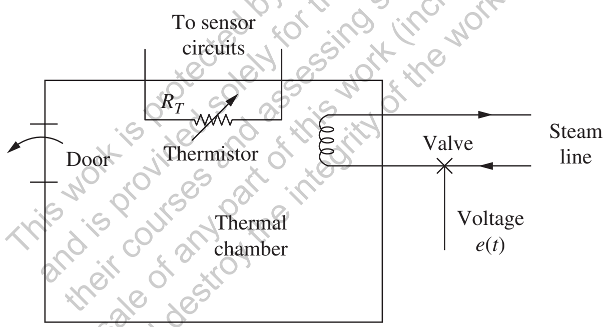
$\therefore \tau = 0.5s$

$\varepsilon^{-2t_1} = 0.01 \Rightarrow 2t_1 = 4.60 \Rightarrow t_1 = 2.30s$

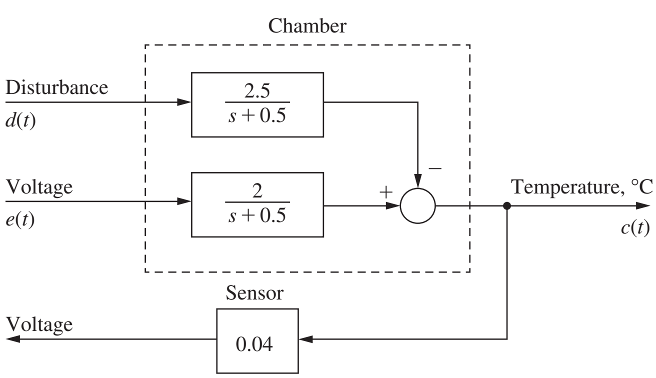
(d) $K = 2.4 \Rightarrow e_a(t) = 24V,$ rated voltage

1.6-1. A thermal test chamber is illustrated in Fig. P1.6-1(a). This chamber, which is a large room, is used to test large devices under various thermal stresses. The chamber is heated with steam, which is controlled by an electrically activated valve. The temperature of the chamber is measured by a sensor based on a thermistor, which is a semiconductor resistor whose resistance varies with temperature. Opening the door into the chamber affects the chamber temperature and thus must be considered as a disturbance.

A simplified model of the test chamber is shown in Fig. P1.6-1(b), with the units of time in minutes. The control input is the voltage $e(t)$, which controls the valve in the steam line, as shown. For the disturbance $d(t)$, a unit step function is used to model the opening of the door. With the door closed, $d(t) = 0$.



(a)



(b)

Fig. P1.6-1

(a) Find the time constant of the chamber.

(b) With the controlling voltage $e(t) = 5u(t)$ and the chamber door closed, find and plot the chamber temperature $c(t)$. In addition, give the steady-state temperature.

(c) A tacit assumption in part (a) is an initial chamber temperature of zero degrees Celsius. Repeat part (b), assuming that the initial chamber temperature is $c(0) = 25°\text{C}$.

(d) Two minutes after the application of the voltage in part (c), the door is opened, and it remains open. Add the effects of this disturbance to the plot of part (c).

(e) The door in part (d) remains open for 12 min. and is then closed. Add the effects of this disturbance to the plot of part (d).

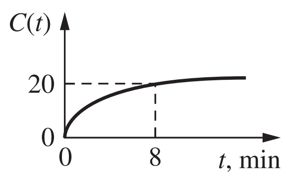
Solution:

(a) $G(s) = \dfrac{2}{s+0.5} = \dfrac{K}{\tau s+1} = \dfrac{K/\tau}{s+1/\tau};\ \therefore \tau = 2 \text{ min}$

(b) $C(s) = \dfrac{2}{s+0.5} \times \dfrac{5}{s} = \dfrac{20}{s} + \dfrac{-20}{s+0.5}$

$\therefore c(t) = 20(1-\varepsilon^{-0.5t}),\ t \geq 0$

$c_{ss}(t) = 20°\text{C}$



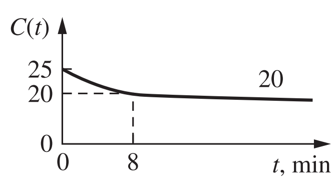
(c) $\dfrac{C(s)}{E(s)} = \dfrac{2}{s+0.5}$

$\therefore \dot{c}(t) + 0.5c(t) = 2e(t)$

$sC(s) - c(0) + 0.5C(s) = 2E(s)$

$\therefore C(s) = \dfrac{2E(s)}{s+0.5} + \dfrac{c(0)}{s+0.5} \Rightarrow c(t) = \overbrace{20(1-\varepsilon^{-0.5t})}^{\text{from (b)}} + 25e^{-0.5t}$

$\therefore c(t) = 20 + 5\varepsilon^{-0.5t},\ t \geq 0$



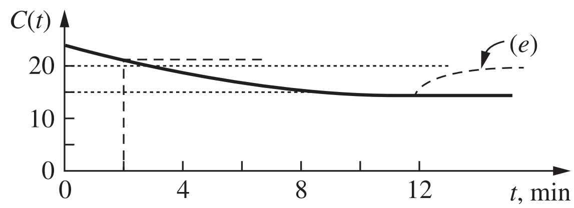
(d) Disturbance response:

$C_d(s) = \dfrac{-2.5}{s+0.5} \cdot \dfrac{1}{s} = \dfrac{-5}{s} + \dfrac{5}{s+0.5}$

$\therefore c_d(t) = -5\left(1 - \varepsilon^{-0.5(t-2)}\right)u(t-2)$, since door opened at $t_0 = 2$.

$\therefore$ from (c) and (d),

$$c(t) = \left(20 + 5\varepsilon^{-0.5t}\right)u(t) - 5\left(1 - \varepsilon^{-0.5(t-2)}\right)u(t-2)$$



(e) $c(12) = 20 + 5e^{-6} - 5 + 5e^{-5} \approx 15$

From (c), with c(0) = 15

$$c_1(t) = 20\left(1 - \varepsilon^{-0.5t}\right) + 15\varepsilon^{-0.5t} = 20 - 5e^{-0.5t}$$

$$\therefore c(t) = \left[20 - 5\varepsilon^{-0.5(t-12)}\right]u(t-12),\ t \geq 12$$

1.6-2. The thermal chamber transfer function $C(s)/E(s) = 2/(s+0.5)$ of Problem 1.6-1 is based on the units of time being minutes.

(a) Modify this transfer function to yield the chamber temperature $c(t)$ based on seconds.

(b) Verify the result in part (a) by solving for $c(t)$ with the door closed and the input $e(t) = 5u(t)$ volts, (i) using the chamber transfer function found in part (a), and (ii) using the transfer function of Fig. P1.6-1. Show that (i) and (ii) yield the same temperature at $t = 1$ min.

Solution:

(a) $\tau = 2\,\text{min} \rightarrow 120s$

$$\therefore \frac{C(s)}{E(s)} = \frac{K}{\tau s + 1} = \frac{4}{2s+1} \Rightarrow \frac{4}{120s+1} = \frac{0.0333}{s + 0.00833}$$

(b) (i) From Problem 1.1-4(b), $c(t) = 20\left(1 - \varepsilon^{-0.5t}\right),\ t \geq 0$

$$\therefore c(1) = 20\left(1 - \varepsilon^{-0.5}\right)$$

(ii) $C(s) = \dfrac{0.0333}{s + 0.00833} \times \dfrac{5}{s} = \dfrac{20}{s} + \dfrac{-20}{s + 0.00833}$

$$\therefore c(t) = 20\left(1 - \varepsilon^{-0.00833t}\right),\ t \geq 0$$

$$c(1) = 20\left(1 - \varepsilon^{-0.00833(60)}\right) = 20\left(1 - \varepsilon^{-0.5}\right)$$