Instructor's Solutions Manual

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Differential Equations With Boundary Value Problems

SECOND EDITION

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Chapter 1. Introduction

Section 1.1. Introduction to Differential Equations

1. Let $y(t)$ be the number of bacteria at time t. The rate of change of the number of bacteria is $y'(t)$. Since this rate of change is given to be proportional to $y(t)$, the resulting differential equation is $y'(t) = ky(t)$. Note that k is a positive constant since $y'(t)$ must be positive (i.e. the number of bacteria is growing).

2. Let $y(t)$ be the number of field mice at time t. The rate of change of the number of mice is $y'(t)$. Since this rate of change is given to be inversely proportional to the square root of $y(t)$, the resulting differential equation is $y'(t) = k/\sqrt{y(t)}$. Note that k is a positive constant since $y'(t)$ must be positive (i.e. the number of mice is growing).

3. Let $y(t)$ be the number of ferrets at time t. The rate of change of the number of ferrets is $y'(t)$. Since this rate of change is given to be proportional to the product of $y(t)$ and the difference between the maximum population and $y(t)$ (i.e. $100 - y(t)$), the resulting differential equation is $y'(t) = ky(t)(100 - y(t))$. Note that k is a positive constant since $y'(t)$ must be positive (i.e. the number of ferrets is growing provided $y(t) < 100$).

4. Let $y(t)$ be the quantity of radioactive substance at time t. The rate of change of the material is $y'(t)$. Since this rate of change (decay) is given to be proportional to $y(t)$, the resulting differential equation is $y'(t) = -ky(t)$. Note that k is a positive constant since $y'(t)$ must be negative (i.e. the quantity of radioactive material is decreasing).

5. Let $y(t)$ be the quantity of material at time t. The rate of change of the material is $y'(t)$. Since this rate of change (decay) is given to be inversely proportional to $y(t)$, the resulting differential equation is $y'(t) = -k/y(t)$. Note that k is a positive constant since $y'(t)$ must be negative (i.e. the quantity of material is decreasing).

6. Let $y(t)$ be the temperature of the potato at time t. The rate of change of the temperature is $y'(t)$. Since this rate of change is given to be proportional to the difference between the potato's temperature and that of the surrounding room (i.e. $y(t) - 65$), the resulting differential equation is $y'(t) = -k(y(t) - 65)$. Note that k is a positive constant since $y'(t)$ must be negative (i.e. the potato is cooling) and since $y(t) - 65 > 0$ (i.e. the potato is hotter than the surrounding room).

7. Let $y(t)$ be the temperature of the thermometer at time t. The rate of change of the temperature is $y'(t)$. Since this rate of change is given to be proportional to the difference between the thermometer's temperature and that of the surrounding room (i.e. $77 - y(t)$), the resulting differential equation is $y'(t) = k(77 - y(t))$. Note that k is a positive constant since $y'(t)$ must be positive (i.e. the thermometer is warming) and since $77 - y(t) > 0$ (i.e. the thermometer is cooler than the surrounding room).

8. Let $x(t)$ be the position (displacement) of the particle at time t. The force on the particle is given to be proportional to this displacement. Therefore, the force, F, is equal to $-kx(t)$ where k is a positive constant. The negative sign is present since the direction of F is opposite to that of $x(t)$. Newton's law states $F = ma$ where m is the mass of the object and $a = x''(t)$ is its acceleration. Therefore, $F = ma$ becomes $-kx(t) = mx''(t)$, which is the differential equation governing the motion of this particle.

9. Let $x(t)$ be the position (displacement) of the particle at time t. The force on the particle is given to be proportional to the square of the particle's velocity, i.e. $(x'(t))^2$. As a first guess, one might surmise that the force is given by $F = -k(x'(t))^2$, where k is a positive constant. However, closer inspection reveals that this will have the force pointing to

the left, regardless of whether the velocity is positive or negative. We can work around this difficulty by letting the force equal $F = -kx'(t)|x'(t)|$. The reader will recognize that the force is positive when $x'(t) < 0$, while the force is negative when $x'(t) > 0$, thus insuring that the force is always opposite the particle's motion. Newton's law states $F = ma$ where m is the mass of the object and $a = x''(t)$ is its acceleration. Therefore, $F = ma$ becomes $-k(x'(t))|x'(t)| = mx''(t)$, which is the differential equation governing the motion of this particle.

10. Let $x(t)$ be the position (displacement) of the particle at time t. The force on the particle is given to be inversely proportional to the square of this displacement. The direction of F is opposite to that of $x(t)$. Therefore, the force, F, is equal to $-k/[x(t)|x(t)|]$ where k is a positive constant. Note that we have written $x(t)|x(t)|$ instead of $x(t)^2$ since $-k/[x(t)|x(t)|]$ is negative when $x(t)$ is positive and $-k/[x(t)|x(t)|]$ is positive when $x(t)$ is negative. This agrees with the desired direction of F. Newton's law states $F = ma$ where m is the mass of the object and $a = x''(t)$ is its acceleration. Therefore, $F = ma$ becomes

$$\frac{-k}{x(t)|x(t)|} = mx''(t)$$

which is the differential equation governing the motion of this particle.

11. Let $V(t)$ be the voltage drop across the inductor and $I(t)$ be the current at time t. The rate of change of the current is $I'(t)$. Since the voltage drop is proportional to the rate of change of I, we obtain the differential equation $V(t) = kI'(t)$, where k is a constant.

Section 1.2. The Derivative

1. $$\begin{aligned} D_x(3x-5) &= 3D_x x - D_x 5 \\ &= 3(1) - 0 \\ &= 3 \end{aligned}$$

2. $$\begin{aligned} D_x(5x^2-4x-8) &= 5D_x x^2 - 4D_x x - D_x 8 \\ &= 5(2x^2) - 4(1) - 0 \\ &= 10x^2 - 4 \end{aligned}$$

3. $$\begin{aligned} D_x(3\sin 5x) &= 3D_x \sin 5x \\ &= 3(\cos 5x)D_x(5x) \\ &= 15\cos 5x \end{aligned}$$

4. $$\begin{aligned} D_x(\cos 2\pi x) &= (-\sin 2\pi x)D_x(2\pi x) \\ &= -2\pi \sin 2\pi x \end{aligned}$$

5. $$\begin{aligned} D_x(e^{3x}) &= e^{3x}D_x(3x) \\ &= 3e^{3x} \end{aligned}$$

6. $$\begin{aligned} D_x(5e^{x^2}) &= 5D_x e^{x^2} \\ &= 5e^{x^2}D_x(x^2) \\ &= 5e^{x^2}(2x) \\ &= 10xe^{x^2} \end{aligned}$$

7. $$\begin{aligned} D_x \ln|5x| &= \tfrac{1}{5x}D_x(5x) \\ &= \tfrac{1}{x} \end{aligned}$$

8. $$\begin{aligned} D_x \ln(\cos 2x) &= \tfrac{1}{\cos 2x}D_x \cos 2x \\ &= \tfrac{1}{\cos 2x}(-\sin 2x)D_x(2x) \\ &= -2\tan 2x \end{aligned}$$

9. $$\begin{aligned} D_x x \ln x &= (D_x x)\ln x + xD_x \ln x \\ &= (1)\ln x + x\left(\tfrac{1}{x}\right) \\ &= 1 + \ln x \end{aligned}$$

10. $$\begin{aligned} D_x e^x \sin \pi x &= (D_x e^x)\sin \pi x + e^x D_x \sin \pi x \\ &= e^x \sin \pi x + e^x(-\cos \pi x)D_x(\pi x) \\ &= e^x(\sin \pi x - \pi \cos \pi x) \end{aligned}$$

11. $$\begin{aligned} D_x\left(\tfrac{x^2}{\ln x}\right) &= \tfrac{(D_x x^2)\ln x - x^2 D_x \ln x}{[\ln x]^2} \\ &= \tfrac{2x\ln x - x^2\left(\frac{1}{x}\right)}{[\ln x]^2} \\ &= \tfrac{2x\ln x - x}{[\ln x]^2} \end{aligned}$$

12. $D_x\left(\frac{x\ln x}{\cos x}\right) = \frac{D_x(x\ln x)\cos x - x\ln x D_x \cos x}{\cos^2 x}$
$= \frac{(1+\ln x)\cos x - x\ln x(-\sin x)}{\cos^2 x}$
$= \frac{(1+\ln x)\cos x + x\sin x\ln x}{\cos^2 x}$

13. If $L(x) = f(x_0) + f'(x_0)(x - x_0)$, then

$$\begin{aligned} R(x) &= f(x) - L(x) \\ &= f(x) - f(x_0) - f'(x_0)(x - x_0). \end{aligned}$$

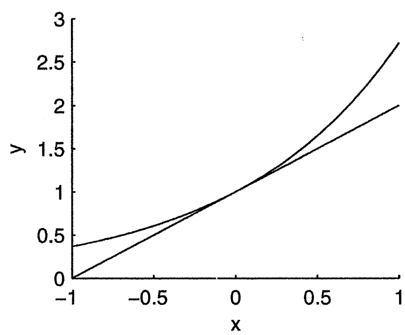
Thus,

$$\begin{aligned} \lim_{x\to x_0} \frac{R(x)}{x - x_0} &= \lim_{x\to x_0}\left[\frac{f(x) - f(x_0)}{x - x_0} - f'(x_0)\right] \\ &= f'(x_0) - f'(x_0) \\ &= 0. \end{aligned}$$

14. Given that $f(x) = e^x$, the derivative is $f'(x) = e^x$. At $x_0 = 0$, $f'(0) = 1$. Thus, the linearization is

$$L(x) = f(0) + f'(0)(x - 0) = 1 + x.$$

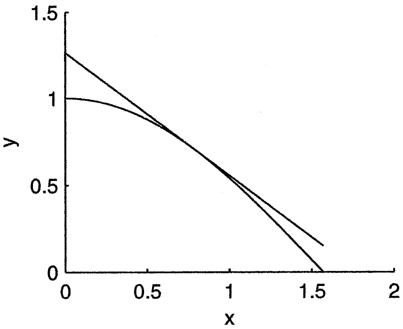
The graph of f, together with its linear approximation at $x_0 = 0$, is shown in the following figure.



15. Given that $f(x) = \cos x$, the derivative is $f'(x) = -\sin x$. At $x_0 = \pi/4$, $f'(\pi/4) = -\sqrt{2}/2$. Thus, the linearization is

$$\begin{aligned} L(x) &= f(\pi/4) + f'(\pi/4)(x - \pi/4) \\ &= \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}\left(x - \frac{\pi}{4}\right). \end{aligned}$$

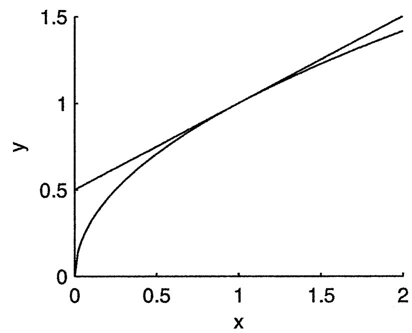
The graph of f, together with its linear approximation at $x_0 = \pi/4$, is shown in the following figure.



16. Given that $f(x) = \sqrt{x}$, the derivative is $f'(x) = 1/(2\sqrt{x})$. At $x_0 = 1$, $f'(1) = 1/2$. Thus, the linearization is

$$L(x) = f(1) + f'(1)(x - 1) = 1 + \frac{1}{2}(x - 1).$$

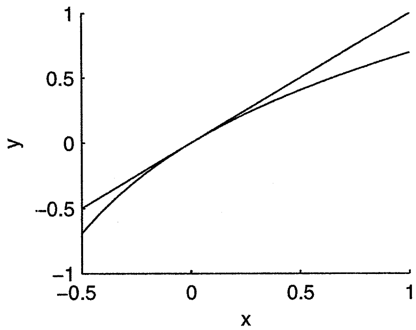
The graph of f, together with its linear approximation at $x_0 = 1$, is shown in the following figure.



17. Given that $f(x) = \ln(1 + x)$, the derivative is $f'(x) = 1/(1 + x)$. At $x_0 = 0$, $f'(0) = 1$. Thus, the linearization is

$$L(x) = f(0) + f'(0)(x - 0) = x.$$

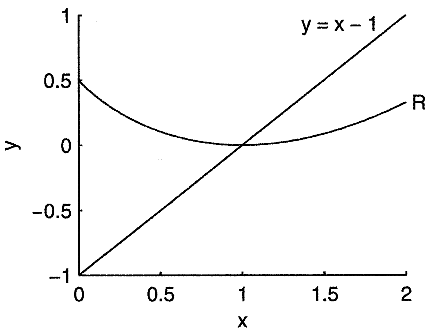
The graph of f, together with its linear approximation at $x_0 = 0$, is shown in the following figure.



18. Given that $f(x) = x^{3/2}$, the derivative is $f'(x) = (3/2)x^{1/2}$. At $x_0 = 1$, $f'(1) = 3/2$. Thus, the linearization is

$$L(x) = f(1) + f'(1)(x-1) = 1 + \frac{3}{2}(x-1).$$

The graph of $y = x - 1$, together with the graph of the remainder $R(x) = f(x) - L(x)$, is shown in the following figure.

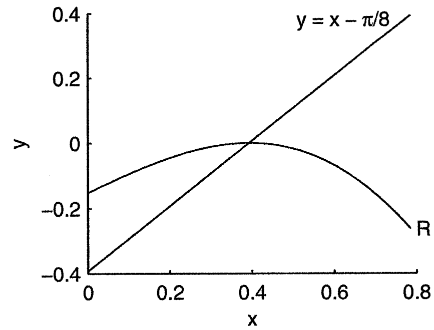


Note that both graphs approach zero as $x \to 1$, but the graph of R approaches zero at a more rapid rate.

19. Given that $f(x) = \sin 2x$, the derivative is $f'(x) = 2\cos 2x$. At $x_0 = \pi/8$, $f'(\pi/8) = \sqrt{2}$. Thus, the linearization is

$$\begin{aligned} L(x) &= f(\pi/8) + f'(\pi/8)(x - \pi/8) \\ &= \frac{\sqrt{2}}{2} + \sqrt{2}\left(x - \frac{\pi}{8}\right). \end{aligned}$$

The graph of $y = x - \pi/8$, together with the graph of the remainder $R(x) = f(x) - L(x)$, is shown in the following figure.

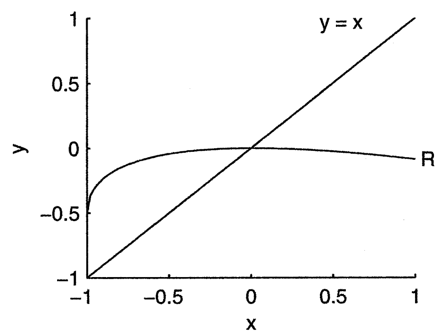


Note that both graphs approach zero as $x \to \pi/8$, but the graph of R approaches zero at a more rapid rate.

20. Given that $f(x) = \sqrt{x+1}$, the derivative is $f'(x) = 1/(2\sqrt{x+1})$. At $x_0 = 0$, $f'(0) = 1/2$. Thus, the linearization is

$$L(x) = f(0) + f'(0)(x - 0) = 1 + \frac{1}{2}x.$$

The graph of $y = x$, together with the graph of the remainder $R(x) = f(x) - L(x)$, is shown in the following figure.

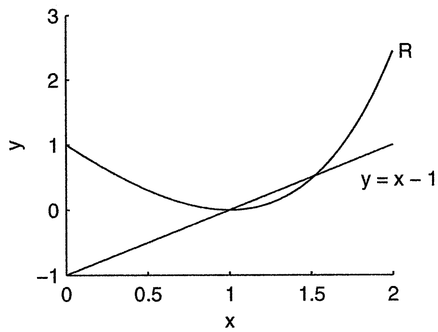


Note that both graphs approach zero as $x \to 0$, but the graph of R approaches zero at a more rapid rate.

21. Given that $f(x) = xe^{x-1}$, the derivative is $f'(x) = (x+1)e^{x-1}$. At $x_0 = 1$, $f'(1) = 2$. Thus, the linearization is

$$L(x) = f(1) + f'(1)(x-1) = 1 + 2(x-1).$$

The graph of $y = x - 1$, together with the graph of the remainder $R(x) = f(x) - L(x)$, is shown in the following figure.



Note that both graphs approach zero as $x \to 1$, but the graph of R approaches zero at a more rapid rate.

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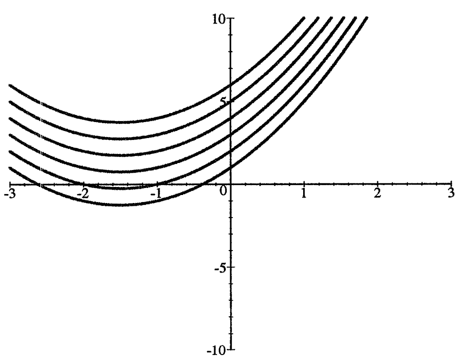
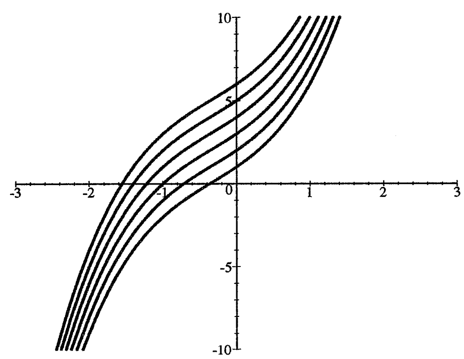
Section 1.3. Integration

1. $y' = 2t + 3$. Integrate to obtain $y = t^2 + 3t + C$.



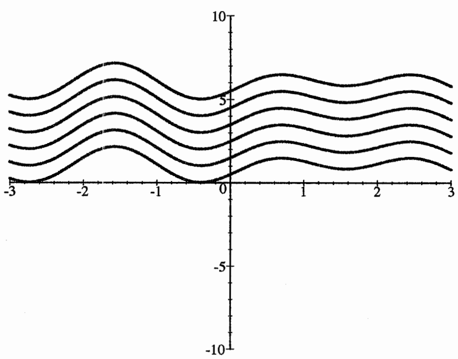
2. $y' = 3t^2 + 2t + 3$. Integrate to obtain $y =$

$t^3 + t^2 + 3t + C$.

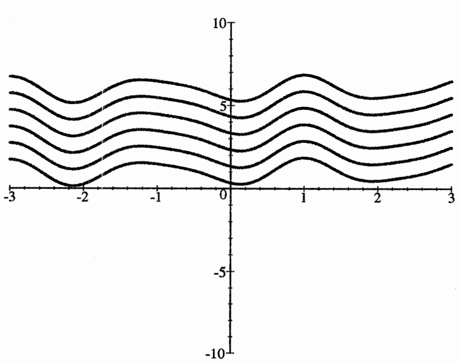


3. $y' = \sin 2t + 2\cos 3t$. Integrate to obtain $y =$

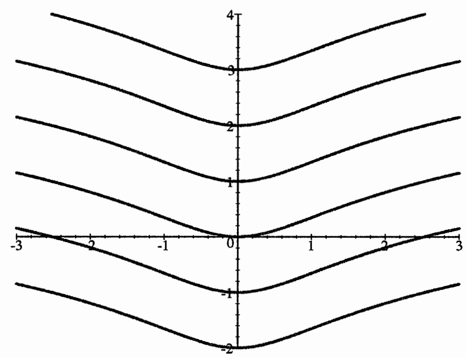
$(-1/2)\cos 2t + (2/3)\sin 3t + C$.



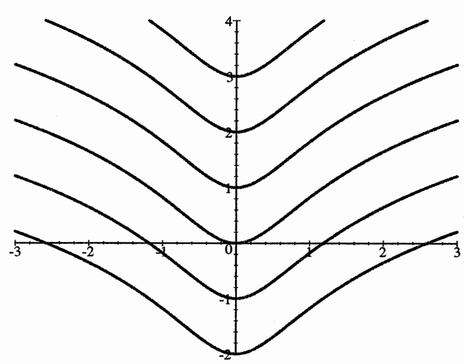
4. $y' = 2\sin 3t - \cos 5t$. Integrate to obtain $y = (-2/3)\cos 3t - (1/5)\sin 5t + C$.



5. $y' = t/(1+t^2)$. Use $u = 1+t^2$, $du = 2t\,dt$ and get $dy = (1/2)du/u$. Integrate to obtain $y = (1/2)\ln u + C = (1/2)\ln(1+t^2) + C$.



6. $y' = 3t/(1+2t^2)$. Let $u = 1+2t^2$, $du = 4t\,dt$ and get $dy = (3/4)du/u$. Integrate to obtain $y = (3/4)\ln u + C = (3/4)\ln(1+2t^2) + C$.

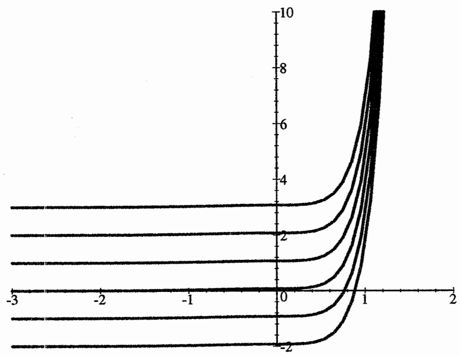


7. $y' = t^2e^{3t}$. Integrate by parts with $u = t^2$ and $dv = e^{3t}$ to obtain

$$y = \frac{t^2e^{3t}}{3} - \int \frac{e^{3t}2t}{3}dt$$

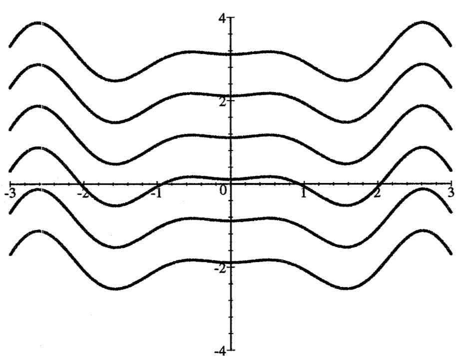
Integrate by parts once more and obtain

$$y = \frac{t^2e^{3t}}{3} - \frac{2te^{3t}}{9} + \frac{2e^{3t}}{27} + C$$



8. $y' = t\cos 3t$. Integrate by parts with $u = t$, $dv = \cos 3t$ and obtain

$$y = \frac{t\sin 3t}{3} + \frac{\cos 3t}{9} + C$$



9. $y' = e^{-2\omega}\sin\omega$. Integrate by parts with $u = e^{-2\omega}$ and $dv = \sin\omega$ to obtain

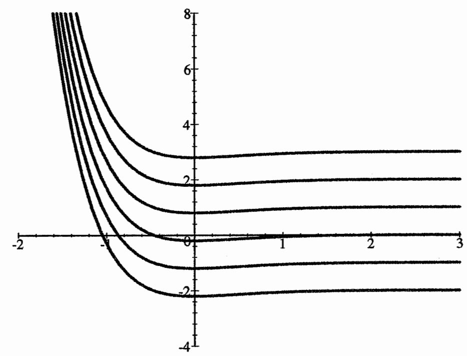
$$\int e^{-2\omega}\sin\omega\, d\omega = -e^{-2\omega}\cos\omega - 2\int \cos\omega e^{-2\omega}\, d\omega.$$

Integrate by parts again with $u = e^{-2\omega}$ and $dv = \cos\omega$, to obtain

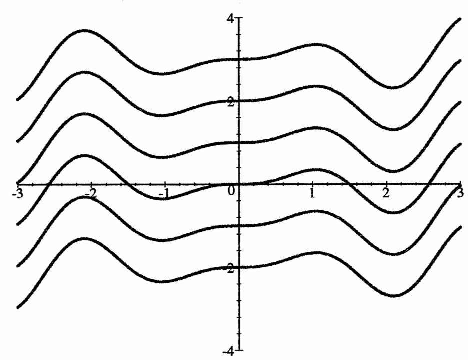
$$\int e^{-2\omega}\sin\omega\, d\omega = -e^{-2\omega}\cos\omega - 2\sin\omega e^{-2\omega} - 4\int \sin\omega e^{-2\omega}\, d\omega$$

Then add the integral on the right to the integral on the left, which then becomes $5\int \sin\omega e^{-2\omega}\, d\omega$; divide by the 5 and obtain the answer:

$$y = \left(-e^{-2\omega}\cos\omega - 2\sin\omega e^{-2\omega}\right)/5 + C$$



10. $y' = x\sin 3x$. Integrate by parts with $u = x$ and $dv = \sin 3x$ to obtain $y = (-x/3)\cos 3x + (1/9)\sin 3x + C$.

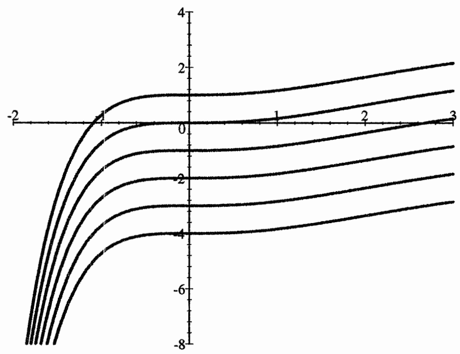


11. $x' = s^2 e^{-s}$. Integrate by parts with $u = s^2$ and $dv = e^{-s}$ and obtain

$$x = -s^2 e^{-s} + 2\int e^{-s} s\, ds.$$

Integrate by parts again with $u = s$ and $dv = e^{-s}$ to obtain the answer:

$$x = -s^2 e^{-s} - 2se^{-s} - 2e^{-s} + C.$$



12.

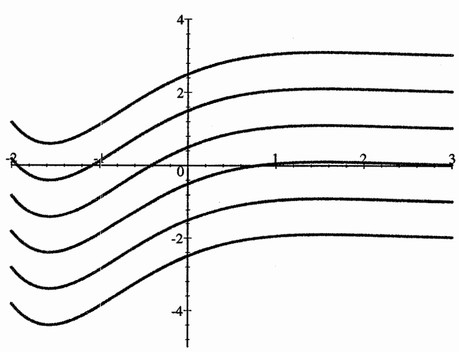
$$\int e^{-u}\cos u\,du = e^{-u}\sin u + \int \sin u e^{-u}\,du.$$

Integrate by parts again, with $U = e^{-u}$ and $dV = \sin u$ and obtain

$$\int e^{-u}\cos u\,du = e^{-u}\sin u - e^{-u}\cos u - \int \cos u e^{-u}\,du.$$

Add the integral on the right to the left side; then divide by 2 and obtain the answer:

$$y = \left(e^{-u}\sin u - e^{-u}\cos u\right)/2 + C.$$

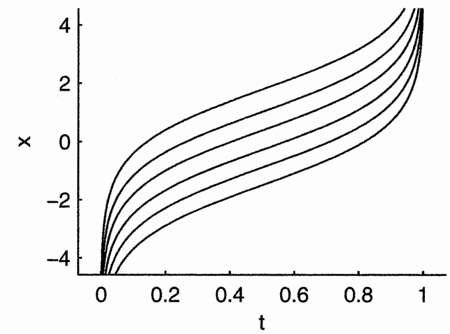


13. Use partial fractions to write

$$r' = \left[\frac{1}{u} + \frac{1}{1-u}\right].$$

Then integrate to obtain

$$r = \ln u - \ln(1-u) = \ln\left(\frac{u}{1-u}\right).$$

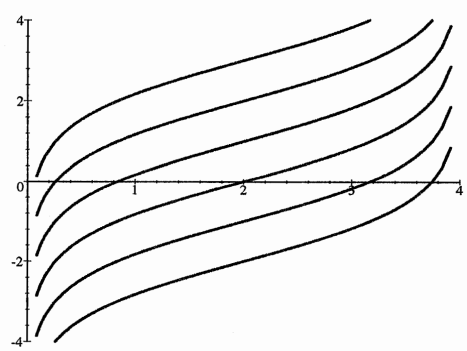


14. Use partial fractions to write

$$y' = \frac{3}{4}\left[\frac{1}{x} + \frac{1}{4-x}\right].$$

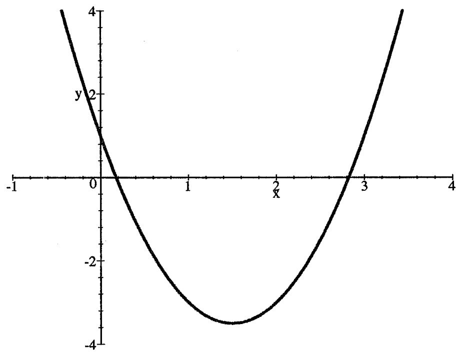
Then integrate to obtain

$$y = (3/4)(\ln x - \ln(4-x)) = \ln\left(\frac{x}{4-x}\right)^{3/4}.$$

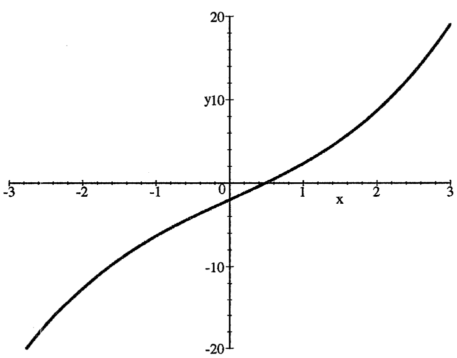


15. $y' = 4t - 6$. Integrate y' to obtain $y = 2t^2 - 6t + C$; the initial condition $y(0) = 1$ gives $1 = C$; so

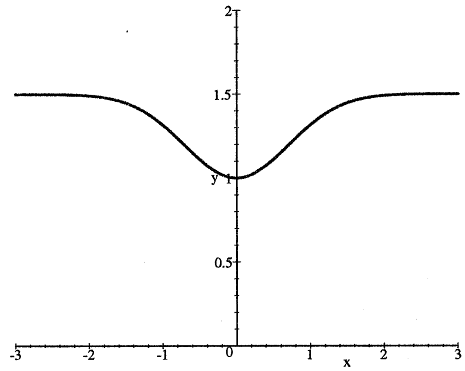
$y(t) = 2t^2 - 6t + 1.$



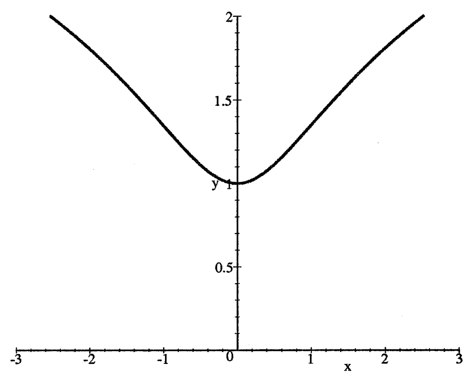
16. $y' = x^2 + 4$. Integrate to obtain $y = x^3/3 + 4x + C$; the initial condition $y(0) = -2$ gives $-2 = C$; so $y(t) = x^3/3 + 4x - 2$.



17. $x'(t) = te^{-t^2}$. Integrate to obtain $x(t) = (-1/2)e^{-t^2} + C$; the initial condition $x(0) = 1$ gives $1 = (-1/2) + C$; so $C = 3/2$ and $x(t) = (-1/2)e^{-t^2} + (3/2)$.



18. $r'(t) = t/(1 + t^2)$. Integrate to obtain $r(t) = (1/2)\ln(1 + t^2) + C$; the initial condition, $r(0) = 1$ gives $1 = (1/2)\ln 1 + C$ or $C = 1$; so $r(t) = (1/2)\ln(1 + t^2) + 1$.

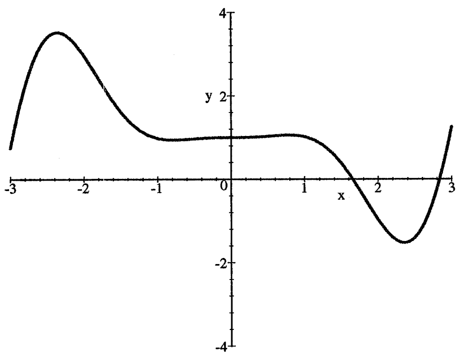


19. $s'(r) = r^2 \cos 2r$. Integrate by parts twice with dv being the trig - term (cos $2r$ and then sin $2r$ to obtain

$$s(r) = \frac{r^2 \sin 2r}{2} + \frac{r \cos 2r}{2} - \frac{\sin 2r}{4} + C.$$

The initial condition, $s(0) = 1$ gives $1 = C$ so

$$s(r) = \frac{r^2 \sin 2r}{2} + \frac{r \cos 2r}{2} - \frac{\sin 2r}{4} + 1$$

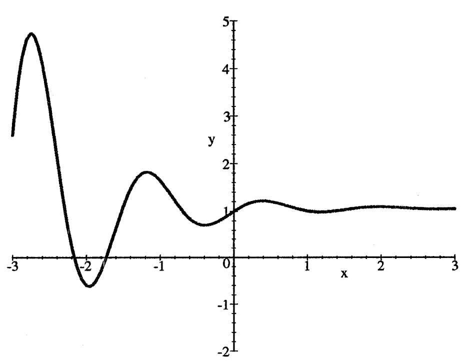


20. $P'(t) = e^{-t}\cos 4t$. Integrate by parts twice as done in the solution to Exercise 12 above to obtain

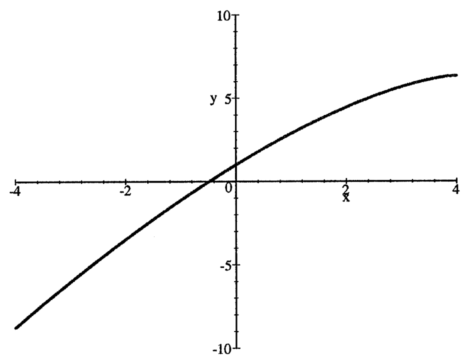
$$P(t) = \frac{4}{17}\left(e^{-t}\sin 4t - (1/4)e^{-t}\cos 4t\right) + C$$

The initial condition, $P(0) = 1$ gives $1 = -1/17 + C$ or $C = 18/17$; so

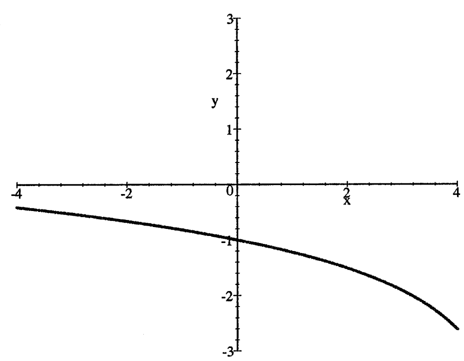
$$P(t) = \frac{4}{17}\left(e^{-t}\sin 4t - (1/4)e^{-t}\cos 4t\right) + \frac{18}{17}$$



21. $x'(t) = \sqrt{4-t}$. Integrate to obtain $x(t) = (-2/3)(4-t)^{3/2} + C$. The initial condition, $x(0) = 1$ gives $1 = (-2/3)(4)^{3/2} + C = -16/3 + C$ or $C = 19/3$. So $x(t) = (-2/3)(4-t)^{3/2} + 19/3$.



22. $u'(x) = 1/(x-5)$. Integrate to obtain $u(x) = \ln|x-5| + C$. The initial condition, $u(0) = -1$ gives $-1 = \ln 5 + C$ or $C = -1 - \ln 5$; so $u(x) = \ln|x-5| - 1 - \ln 5$.



23. $y'(t) = \frac{t+1}{t(t+4)}$. Partial fractions gives

$$y'(t) = \frac{1/4}{t} + \frac{3/4}{t+4}$$

Integrating, we obtain

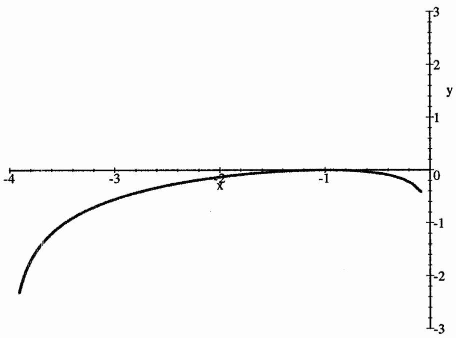
$$y(t) = (1/4)\ln|t| + (3/4)\ln|t+4| + C$$

The initial condition, $y(-1) = 0$ gives

$$0 = (1/4)\ln 1 + (3/4)\ln 3 + C = (3/4)\ln 3 + C$$

or $C = -(3/4)\ln 3$. So

$$y(t) = (1/4)\ln|t| + (3/4)\ln|t+4| - (3/4)\ln 3$$



24. $v'(r) = \frac{r^2}{r+1}$. By long division, we obtain

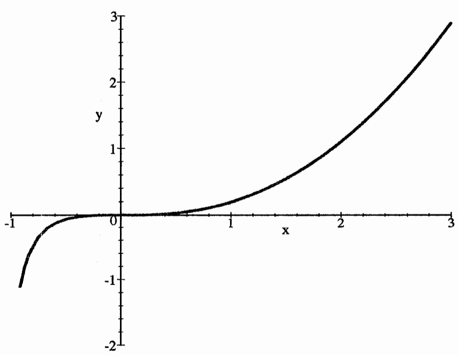
$$v'(r) = r - 1 + \frac{1}{r+1}$$

Integrating, we obtain

$$v(r) = \frac{r^2}{2} - r + \ln|r+1| + C$$

The initial condition, $v(0) = 0$ gives $0 = \ln 1 + C = C$; so

$$v(r) = \frac{r^2}{2} - r + \ln|r+1|$$



25. Let $s(t)$ be the height of the ball at time t seconds. If $g = -9.8$ is the gravitational constant, then $s''(t) = g$. Integrating we obtain, $s' = gt + v_0$, where v_0 is a constant. The initial condition $s'(0) = 50$ gives $v_0 = 50$; so $s'(t) = gt + 50$. The velocity at $t = 3$ seconds is $s'(3) = 3g + 50 = 20.6$ meters/second. Integrating s', gives $s(t) = gt^2/2 + 50t + s_0$. The initial condition, $s(0) = 3$ gives $s_0 = 3$, so $s(t) = gt^2/2 + 50t + 3$. The height at $t = 3$ seconds is $s(3) = (9/2)g + 153 = 108.9$ meters.

26. Let $s(t)$ be the height of the ball at time t seconds. If $g = -9.8$ is the gravitational constant, then $s''(t) = g$. Integrating we obtain, $s' = gt + v_0$, where v_0 is a constant. The initial condition $s'(0) = 0$ (dropped from rest) gives $v_0 = 0$; so $s'(t) = gt$. The velocity at $t = 3$ is $s'(3) = 3g = -29.4$ meters/sec. Integrating s', gives $s(t) = gt^2/2 + s_0$. The initial condition $s(0) = 200$ gives $s_0 = 200$, so $s(t) = gt^2/2 + 200$; the height at $t = 3$ seconds is $s(3) = (9/2)g + 200 = 155.9$ meters.

27. Let $s(t)$ be the height of the ball at time t seconds. If $g = -9.8$ is the gravitational constant, then $s''(t) = g$. Integrating we obtain, $s' = gt + v_0$, where v_0 is a constant. The initial condition $s'(0) = 120$ gives $v_0 = 120$, so $s'(t) = gt + 120$. The maximum height occurs when the velocity reaches zero, i.e. when $gt + 120 = 0$, or $t = -120/g = 12.24$ seconds. Integrating s' gives $s(t) = gt^2/2 + 120t + s_0$. The initial condition $s(0) = 6$ gives $s_0 = 6$, so $s(t) = gt^2/2 + 120t + 6$. When $t = 12.24$, the maximum height is $s(12.24) = 740.69$ meters.

28. Let $s(t)$ be the height of the ball at time t seconds. If $g = -9.8$ is the gravitational constant, then $s''(t) = g$. Integrating we obtain, $s' = gt + v_0$, where v_0 is a constant. The initial condition $s'(0) = -25$ gives $v_0 = -25$, so $s'(t) = gt - 25$. Integrating again gives $s(t) = gt^2/2 - 25t + s_0$. The initial condition $s(0) = 1000$ gives $s_0 = 1000$ and so $s(t) = gt^2/2 - 25t + 1000 = -4.9t^2 - 25t + 1000$. The ball hits the ground when $s(t) = 0$ which occurs at approximately $t = 11.96$ seconds.