# Instructor's Solutions Manual

*to accompany*

## Design of Concrete Structures, 14e

Nilson/Darwin/Dolan

## Chapters 1-4

The authors welcome feedback on the problem solutions and on the text in general. Please e-mail any comments to David Darwin at: **daved@ku.edu**

1.1 $A_s = 6.0\,in^2$

$f_y = 40{,}000\,psi$

$E_s = 29{,}000{,}000\,psi$

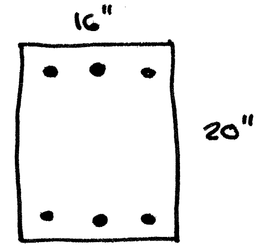
$n = \dfrac{29{,}000{,}000}{3{,}600{,}000} = 8$

$A_g = 16 \times 20 = 320\,in^2$

$A_c = 320 - 6 = 314\,in^2$

$f_c' = 4{,}000\,psi$

$E_c = 3{,}600{,}000$




a) $f_c = 1200\,psi$, $f_y = 40{,}000\,psi$

$P = 1200(314 + 8 \times 6)$
$= 434{,}000\,lbs$

$P_s = 1200(8 \times 6) = 57{,}600\,lbs$
$P_s = 13.3\%$ of $P$

$f_y = 60{,}000\,psi$

a) Same

b) $\epsilon_y = \dfrac{40{,}000}{29{,}000{,}000} = 0.00140$

for slow loading $f_c = 3000\,psi$

$P = 3000(314) + 40{,}000(6) =$
$= 1{,}182{,}000\,lbs$

$P_s = 40{,}000(6) = 240{,}000\,lb$
$= 20.3\%\,P$

$\epsilon_y = \dfrac{60{,}000}{29{,}000} = 0.00207$

$f_c = 3300\,psi$

$P = 3300(314) + 60{,}000(6)$
$= 1{,}396{,}000\,lbs$

$P_s = 60{,}000(6) = 360{,}000\,lb$
$= 25.6\%\,P$

c) $f_c = 3400\,psi$

$P_u = 3400(314) + 40{,}000(6)$
$= 1{,}308{,}000\,lb$

$P_s = 240{,}000\,lb$ $(18.3\%\,P_u)$

$P_u = 3400(314) + 60{,}000(6)$
$= 1{,}428{,}000\,lb$

$P_s = 360{,}000\,lb$ $(25.2\%\,P_u)$

Comments:

1. There is no difference in performance at $f_c = 1200\,psi$
2. As the strain increases, the steel with $f_y = 60{,}000\,psi$ contributes more to the total load and the column has a higher total load.
3. For the same cost, $f_y = 60{,}000\,psi$ provides a 9% increase in capacity.

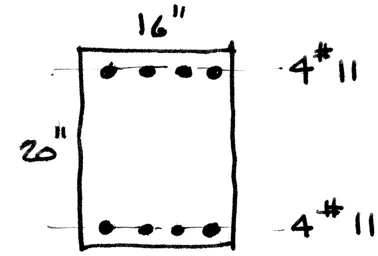
1.2

$A_s = 8(1.56) = 12.48\ in^2$

$A_g = 320\ in^2 \quad A_c = 307.5\ in^2$

$n = 8 \quad E_s = 29{,}000{,}000\ psi$

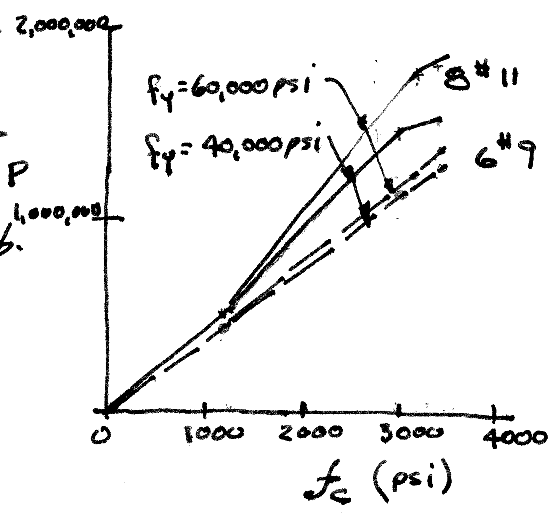
$E_c = 3{,}600{,}000\ psi$




| $f_y = 40{,}000\ psi$ | $f_y = 60{,}000\ psi$ |
|---|---|
| a) $P = 1200(307.5 + 8(12.48)) = 489{,}000\ lb$ | Same as $f_y = 40{,}000$ |
| $P_s = 1200(8)(12.48) = 120{,}000\ lb$ (24.5% P) | |
| b) $\epsilon_y = 0.00140$, $f_c = 3000\ psi$ | $\epsilon_y = 0.00207 \quad f_c = 3300\ psi$ |
| $P = 3000(307.5) + 40{,}000(12.48) = 1{,}424{,}000\ lb$ | $P = 3300(307.5) + 60{,}000(12.48) = 1{,}766{,}000\ lb$ |
| $P_s = 40{,}000(12.48) = 500{,}000\ lb$ (35.1% of P) | $P_s = 60{,}000(12.48) = 750{,}000$ (42.5% of P) |
| c) $f_c = 3400\ psi$ both cases | |
| $P_u = 3400(307.5) + 40{,}000(12.48) = 1{,}547{,}000\ lb$ | $P_u = 3400(307.5) + 60{,}000(12.48) = 1{,}797{,}000\ lb$ |
| $P_s = 500{,}000\ lb$ (32.3% of $P_u$) | $P_s = 750{,}000\ lb$ (41.7% of $P_u$) |

Comments

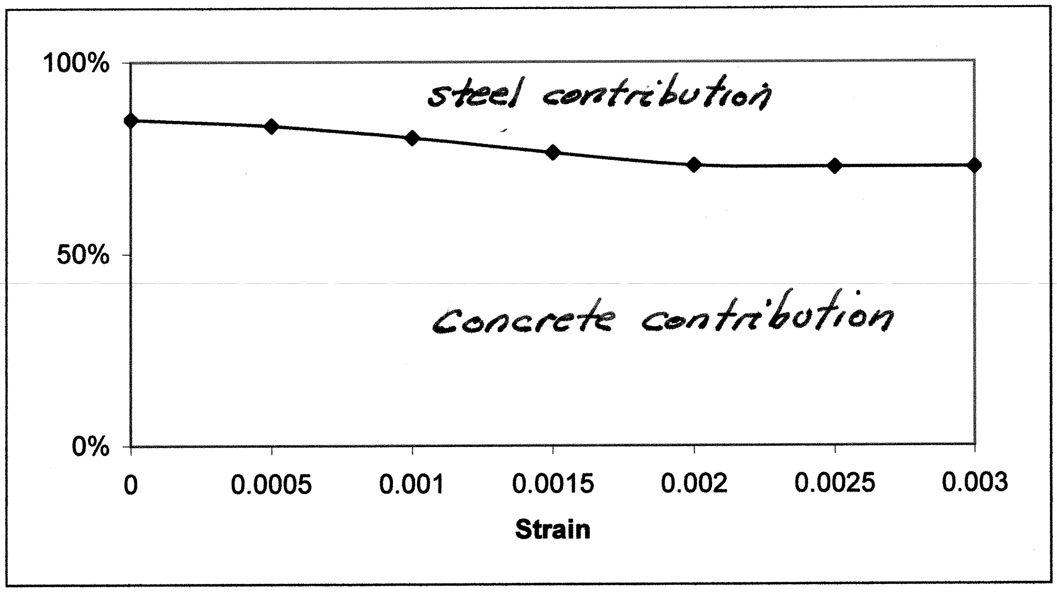
1. There is no strength difference at $f_c = 1200\ psi$
2. There is a 16% strength increase at ultimate using $f_y = 60{,}000\ psi$. This occurs at virtually no cost increase
3. The higher steel ratio produces a stronger column - compare to prob. 1.1



1.3

$A_s =$ 10.12 $in^2$
$A_c =$ 474 $in^2$
$f_y =$ 60000 psi
$f'_c =$ 4000 psi

| $\epsilon_c = \epsilon_s$ | $f_c$ (psi) | $P_c$ (kips) | $f_s$ (psi) | $P_s$ (kips) | $P_{total}$ (kips) | $P_c/P_{total}$ | $P_s/P_{total}$ |
|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 85.0% | 15.0% |
| 0.0005 | 1600 | 758.4 | 15000 | 151.8 | 910.2 | 83.3% | 16.7% |
| 0.001 | 2600 | 1232.4 | 30000 | 303.6 | 1536 | 80.2% | 19.8% |
| 0.0015 | 3100 | 1469.4 | 45000 | 455.4 | 1924.8 | 76.3% | 23.7% |
| 0.002 | 3300 | 1564.2 | 57000 | 576.84 | 2141.04 | 73.1% | 26.9% |
| 0.0025 | 3400 | 1611.6 | 60000 | 607.2 | 2218.8 | 72.6% | 27.4% |
| 0.003 | 3400 | 1611.6 | 60000 | 607.2 | 2218.8 | 72.6% | 27.4% |



1.4 A 20 x 24 in. column is made of the same concrete as Examples 1.1 and 1.2 but reinforced with six No. 11 (No. 36) bars with $f_y$ = 60 ksi. For this column section, determine (a) the axial load the section will carry at a concrete stress of 1400 psi, (b) the load on the section when the steel begins to yield, (c) the maximum load if the section is loaded slowly and (d) the maximum load if the section is loaded rapidly. The area of one No. 11 (No. 36) bar is 1.56 in$^2$. Determine the percent of the load carried by the steel and the concrete for each combination.

▶ Reinforcement Areas

Given Properties

$f'_c := 4000psi$ $f_y := 60000psi$ $f_c := 1400psi$ $n := 8$ $E_s := 29000000psi$

Column Properties

$b := 20in$ $h := 24in$ $A_{st} := 6 \cdot A_{s11} = 9.36\,in^2$ The total area of steel $A_{st}$ is six no. 11 bars

**Part (a)** Compute the axial capacity of the section loaded below the elastic limit.

**Solution:** The axial capacity is based on the gross area of the column plus the effective area of the steel. Since we count the holes where the steel is removed, the additional effective area of the steel is (n-1)$A_{st}$.

$A_g := b \cdot h$ $A_g = 480\,in^2$ $A_{st} = 9.36\,in^2$ Reinforcement ratio = $\frac{A_{st}}{A_g} = 0.0195$

$P := f_c \cdot \left[A_g + (n-1) \cdot A_{st}\right]$ $P = 764\,kip$

$P_c := f_c \cdot \left(A_g - A_{st}\right)$ $P_c = 659\,kip$

$P_s := f_c \cdot n \cdot A_{st}$ $P_s = 105\,kip$ $100 \cdot \frac{P_c}{P} = 86.3$ $100 \frac{P_s}{P} = 13.7$

**Part (b)**: Compute the capacity of the column when the steel begins to yield $\varepsilon := 0.002069$ or 2/10 of one percent

Examining Figure 1.16, we are **beyond the elastic portion of the concrete** stress strain curve, but we are at the elastic limit of the steel.

$f_s := \varepsilon \cdot E_s$ $f_s = 60001\,psi$

From Figure 1.16 $f_c := 3100psi$ for slow loading

Since the problem is nonlinear, we must break out the concrete and steel areas. We can no longer use the elastic equation from 1.1. $A_c := A_g - A_{st}$

$P := f_c \cdot A_c + f_s \cdot A_{st}$ $P = 2021\,kip$

$P_c := f_c \cdot A_c$ $P_c = 1459\,kip$

$P_s := f_s \cdot A_{st}$ $P_s = 562\,kip$ $100 \cdot \frac{P_c}{P} = 72.2$ $100 \frac{P_s}{P} = 27.8$

**Part (c)**: Compute the maximum load capacity of the section

Examining Figure 1.16, we are **beyond the elastic portion of the concrete** stress strain curve, but we are in the plastic range of the steel.

$f_s := f_y$ $f_s = 60000\,psi$

From Figure 1.16 $f_c := 3400psi$ for slow loading

Since the problem is nonlinear, we must break out the concrete and steel areas. We can no longer use the elastic equation from 1.1. $A_c := A_g - A_{st}$

$P := f_c \cdot A_c + f_s \cdot A_{st}$ $P = 2162\,kip$

$P_c := f_c \cdot A_c$ $P_c = 1600\,kip$

$P_s := f_s \cdot A_{st}$ $P_s = 562\,kip$ $100 \cdot \frac{P_c}{P} = 74.0$ $100 \frac{P_s}{P} = 26.0$

If we reexamine the problem with a fast loading as would occur in a building, then the concrete stress would be

$f_c := 4000psi$

$P := f_c \cdot A_c + f_s \cdot A_{st}$ $P = 2444\,kip$

$P_c := f_c \cdot A_c$ $P_c = 1883\,kip$ $100 \cdot \frac{P_c}{P} = 77.0$

$P_s := f_s \cdot A_{st}$ $P_s = 562\,kip$ $100 \frac{P_s}{P} = 23.0$

**Comments**

**1. As the concrete becomes non-linear, the steel picks up more load, but after the steel yields, the load goes to the conrete.**
**2. The slow loading is approximately 88% of the fast load scenario**

1.5 A 24 in. diameter column is made of the same concrete as Examples 1.1 and 1.2 The area of reinforcement equals 2.1 percent of the gross cross section (i.e., $A_s = 0.021A_g$) and $f_y$ = 60 ksi. For this column section, determine (a) the axial load the section will carry at a concrete stress of 1200 psi, (b) the load on the section when the steel begins to yield, (c) the maximum load if the section is loaded slowly, (d) the maximum load if the section is loaded rapidly and (e) the maximum load the reinforcement in the column is raised to 6.5 percent and the column is loaded slowly. Comment on your answer, especially the percent of the load carried by the steel and the concrete for each combination.

Given Properties

$f'_c := 4000\text{psi}$ $f_y := 60000\text{psi}$ $f_c := 1200\text{psi}$ $n := 8$ $E_s := 29000000\text{psi}$

Column Properties

$d := 24\text{in}$ $A_g := \pi \cdot \frac{d^2}{4}$ $\rho := 0.021$ ρ is the reinforcement ratio or the fraction of the section that is steel

$A_{st} := \rho \cdot A_g$ The total area of steel $A_{st}$ is $A_{st} = 9.5 \cdot \text{in}^2$

**Part (a)** Compute the axial capacity of the section loaded below the elastic limit.
**Solution:** The axial capacity is based on the gross area of the column plus the effective area of the steel. Since we count the holes where the steel is removed, the additional effective area of the steel is $(n-1)A_{st}$.

$A_c := A_g - A_{st}$ $A_g = 452 \cdot \text{in}^2$ $A_{st} = 9.50 \cdot \text{in}^2$ $A_c = 443 \cdot \text{in}^2$

$P := f_c \cdot \left[A_g + (n-1) \cdot A_{st}\right]$ $P = 623 \cdot \text{kip}$ Concrete and steel contribution

$P_c := f_c \cdot \left(A_g - A_{st}\right)$ $P_c = 531 \cdot \text{kip}$ $100 \cdot \frac{P_c}{P} = 85.4$

$P_s := f_c \cdot n \cdot A_{st}$ $P_s = 91 \cdot \text{kip}$ $100 \frac{P_s}{\Gamma} = 14.6$

**Part (b)**: Compute the capacity of the column when the steel begins to yield $\varepsilon_y := \frac{f_y}{E_s}$

$\varepsilon_y = 0.00207$ or 2/10 of one percent

Examining Figure 1.16, we are **beyond the elastic portion of the concrete** stress strain curve, but we are at the elastic limit of the steel.

$f_s := \varepsilon_y \cdot E_s$ $f_s = 60000 \cdot \text{psi}$

From Figure 1.16 $f_c := 3100\text{psi}$ for slow loading

Since the problem is nonlinear, we must break out the concrete and steel areas. We can no longer use the elastic equation from 1.1.

$P := f_c \cdot A_c + f_s \cdot A_{st}$ $P = 1943 \cdot \text{kip}$

$P_c := f_c \cdot A_c$ $P_c = 1373 \cdot \text{kip}$ $100 \cdot \frac{P_c}{P} = 27.4$

$P_s := f_s \cdot A_{st}$ $P_s = 570 \cdot \text{kip}$ $100 \frac{P_s}{P} = 29.3$

**Part (c)**: Compute the maximum load capacity of the section if loaded slowly

Examining Figure 1.16, we are **beyond the elastic portion of the concrete** stress strain curve and we are in the plastic range of the steel.

$f_s := f_y$ $f_s = 60000 \cdot psi$

From Figure 1.16 $f_c := 3400psi$ for slow loading

Since the problem is nonlinear, we must break out the concrete and steel areas. We can no longer use the elastic equation from 1.1:

$P := f_c \cdot A_c + f_s \cdot A_{st}$ $P = 2076 \cdot kip$

$P_c := f_c \cdot A_c$ $P_c = 1506 \cdot kip$ $100 \cdot \frac{P_c}{P} = 72.5$

$P_s := f_s \cdot A_{st}$ $P_s = 570 \cdot kip$ $100 \frac{P_s}{P} = 27.5$

**Part (d):** If we reexamine the problem with a fast loading as would occur in a building, then the concrete stress would be

$f_c := 4000psi$

$P := f_c \cdot A_c + f_s \cdot A_{st}$ $P = 2342 \cdot kip$

$P_c := f_c \cdot A_c$ $P_c = 1772 \cdot kip$ $100 \cdot \frac{P_c}{P} = 75.7$ $100 \frac{P_s}{P} = 24.3$

$P_s := f_s \cdot A_{st}$ $P_s = 570 \cdot kip$

**Part (e):** Determine the capacity for a slow loaded column with the steel changed to 6.5%

$A_{st} := 0.065 \cdot A_g$ $A_{st} = 29.4 \cdot in^2$

$f_s := f_y$ $f_s = 60000 \cdot psi$

From Figure 1.16 $f_c := 3400psi$ for slow loading

$P := f_c \cdot A_c + f_s \cdot A_{st}$ $P = 3270 \cdot kip$

$P_c := f_c \cdot A_c$ $P_c = 1506 \cdot kip$ $100 \cdot \frac{P_c}{P} = 46.0$

$P_s := f_s \cdot A_{st}$ $P_s = 1764 \cdot kip$ $100 \frac{P_s}{P} = 54.0$

**Comments**

**1. As the concrete becomes non-linear, the steel picks up more load, but after the steel yields, the load goes to the conrete.**
**2. The slow loading is approximately 88% of the fast load scenario - This is slightly higher than the 0.85 given in eq. 1.8.**

1.5 A 24 in. diameter column is made of the same concrete as Examples 1.1 and 1.2 The area of reinforcement equals 2.1 percent of the gross cross section (i.e., $A_s = 0.021 A_g$) and $f_y$ = 60 ksi. For this column section, determine (a) the axial load the section will carry at a concrete stress of 1200 psi, (b) the load on the section when the steel begins to yield, (c) the maximum load if the section is loaded slowly, (d) the maximum load if the section is loaded rapidly and (e) the maximum load the reinforcement in the column is raised to 6.5 percent and the column is loaded slowly. Comment on your answer, especially the percent of the load carried by the steel and the concrete for each combination.

Given Properties

$f'_c := 4000\text{psi}$ $\quad f_y := 60000\text{psi}$ $\quad f_c := 1200\text{psi}$ $\quad n := 8$ $\quad E_s := 29000000\text{psi}$

Column Properties

$d := 24\text{in}$ $\quad A_g := \pi \cdot \frac{d^2}{4}$ $\quad \rho := 0.021$ $\quad \rho$ is the reinforcement ratio or the fraction of the section that is steel

$A_{st} := \rho \cdot A_g$ $\quad$ The total area of steel $A_{st}$ is $\quad A_{st} = 9.5 \cdot \text{in}^2$

**Part (a)** Compute the axial capacity of the section loaded below the elastic limit.
**Solution:** The axial capacity is based on the gross area of the column plus the effective area of the steel. Since we count the holes where the steel is removed, the additional effective area of the steel is $(n-1)A_{st}$.

$A_c := A_g - A_{st}$ $\quad A_g = 452 \cdot \text{in}^2$ $\quad A_{st} = 9.50 \cdot \text{in}^2$ $\quad A_c = 443 \cdot \text{in}^2$

$P := f_c \cdot \left[A_g + (n-1) \cdot A_{st}\right]$ $\quad P = 623 \cdot \text{kip}$ $\quad$ Concrete and steel contribution

$P_c := f_c \cdot \left(A_g - A_{st}\right)$ $\quad P_c = 531 \cdot \text{kip}$ $\quad 100 \cdot \frac{P_c}{P} = 85.4$

$P_s := f_c \cdot n \cdot A_{st}$ $\quad P_s = 91 \cdot \text{kip}$ $\quad 100 \frac{P_s}{\Gamma} = 14.6$

**Part (b)**: Compute the capacity of the column when the steel begins to yield $\quad \varepsilon_y := \frac{f_y}{E_s}$

$\varepsilon_y = 0.00207$ $\quad$ or 2/10 of one percent

Examining Figure 1.16, we are **beyond the elastic portion of the concrete** stress strain curve, but we are at the elastic limit of the steel.

$f_s := \varepsilon_y \cdot E_s$ $\quad f_s = 60000 \cdot \text{psi}$

From Figure 1.16 $\quad f_c := 3100\text{psi}$ for slow loading

Since the problem is nonlinear, we must break out the concrete and steel areas. We can no longer use the elastic equation from 1.1.

$P := f_c \cdot A_c + f_s \cdot A_{st}$ $\quad P = 1943 \cdot \text{kip}$

$P_c := f_c \cdot A_c$ $\quad P_c = 1373 \cdot \text{kip}$ $\quad 100 \cdot \frac{P_c}{P} = 27.4$

$P_s := f_s \cdot A_{st}$ $\quad P_s = 570 \cdot \text{kip}$ $\quad 100 \frac{P_s}{P} = 29.3$

**Part (c)**: Compute the maximum load capacity of the section if loaded slowly

Examining Figure 1.16, we are **beyond the elastic portion of the concrete** stress strain curve and we are in the plastic range of the steel.

$f_s := f_y$ $f_s = 60000 \cdot psi$

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Since the problem is nonlinear, we must break out the concrete and steel areas. We can no longer use the elastic equation from 1.1.

$P := f_c \cdot A_c + f_s \cdot A_{st}$ $P = 2076 \cdot kip$

$P_c := f_c \cdot A_c$ $P_c = 1506 \cdot kip$ $100 \cdot \frac{P_c}{P} = 72.5$

$P_s := f_s \cdot A_{st}$ $P_s = 570 \cdot kip$ $100 \frac{P_s}{P} = 27.5$

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$f_c := 4000psi$

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**Part (e):** Determine the capacity for a slow loaded column with the steel changed to 6.5%

$A_{st} := 0.065 \cdot A_g$ $A_{st} = 29.4 \cdot in^2$

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**Comments**

**1. As the concrete becomes non-linear, the steel picks up more load, but after the steel yields, the load goes to the conrete.**
**2. The slow loading is approximately 88% of the fast load scenario - This is slightly higher than the 0.85 given in eq. 1.8.**

2.1

$f'_c = 6000 psi$

(a) No prior results

$f'_{cr} = f'_c + 0.1 f'_c + 700 psi = 6000 + 0.1 \times 6000 + 700 = 7300 psi$

(b) 20 prior tests for concrete with $f'_c$ within 1000 psi of $f'_c$ for project. $s_s = 580 psi$ –

From Table 1.1, $1.08 \times 580 = 626 psi$

Because $f'_c > 5000 psi$, use Eqs. (2.1) + (2.2b)

$f'_{cr} = f'_c + 1.34 s_s = 6000 + 1.34 \times 626 = 6840 psi$

$f'_{cr} = 0.9 f'_c + 2.33 s_s = 0.9 \times 6000 + 2.33 \times 626 = 6860 psi$

Use $f'_{cr} = 6860 psi$

(c) 30 prior tests for concrete with $f'_c$ within 1000 psi of $f'_c$ for project. $s_s = 590 psi$ –

$f'_{cr} = f'_c + 1.34 s_s = 6000 + 1.34 \times 590 = 6790 psi$

$f'_{cr} = 0.9 f'_c + 2.33 s_s = 0.9 \times 6000 + 2.33 \times 590 = 6770 psi$

Use $f'_{cr} = 6790 psi$

## 2.2

(a) For $f'_c = 4000$ psi, the strength results indicate satisfactory concrete quality because (1) no individual test is below $f'_c - 500$ psi $= 3500$ psi, and (2) every arithimetic average of any three consecutive tests equals or exceeds $f'_c$.

(b) For $S_s = 510$ psi for 30 consecutive tests, Calculate $f'_{cr}$ using Eqs (2.1) and (2.2a)

$$f'_{cr} = f'_c + 1.34 S_s = 4000 + 1.34 \times 510 = 4680 \text{ psi}$$

$$f'_{cr} = f'_c + 2.33 S_s - 500 \text{ psi} = 4000 + 2.33 \times 510 - 500$$

$$= 4690 \text{ psi}$$

Use 4690 psi

$$(4590 + 4750 + 5280 + 4210 + 4460 + 4170 + 3750 + 5110 + 4640 + 4170)/10$$
$$= 4510 < f'_{cr}$$

Because the average compressive strength is less than $f'_{cr}$, the water-cement ratio must be decreased, either by adding cement or reducing water, to increase strength. If the water is reduced, a water reducer must be added or the quantity of water reducer must be increased to maintain concrete workability.

**Problem 3.1** A rectangular beam made using concrete with $f'_c$ = 6000 psi and steel with $f_y$ = 60,000 psi had a width $b$ = 20 in., and an effective depth of $d$ = 17.5 in and an $h$ =20 in. The Concrete modulus of rupture $f_r$ = 530 psi. The elastic modulus of the steel and concrete are, respectively $E_c$ = 4,030,000 psi and $E_s$ = 29,000,000 psi. The area of steel is four No. 11(No. 36) bars.

(a) Find the maximum service load that can be resisted without stressing the concrete above 0.45 $f'_c$ or the steel above 0.40 $f_y$.

(b) Determine if the beam will show cracking before reaching the service load

(c) Compute the nominal moment capacity of the beam

(d) Compute the ratio of the nominal capacity of the beam to the maximum service level capacity and compare your findings to the ACI load factors and strength reduction factor.

▶ Reinforcement sizes

Given data

*Note: for all MathCAD based solutions, the area and diameter of reinforcment bars is in a common database. Hence the notation $A_{s11}$ indicates the area of a single No. 11 (No. 36) bar.*

$A_s := 4 \cdot A_{s11}$ $\qquad A_s = 6.24 \cdot in^2$ $\qquad E_s := 29000000psi$

$b := 20in$ $\qquad d := 17.5in$ $\qquad h := 20in$

$f'_c := 6000psi$ $\qquad f_y := 60000psi$ $\qquad E_c := 57000\sqrt{f'_c \cdot psi}$ $\qquad E_c = 4415 \cdot ksi$

$f_r := 7.5\sqrt{f'_c \cdot psi}$ $\qquad f_r = 581\ psi$ $\qquad n := \frac{E_s}{E_c}$ $\qquad n = 6.6$

(a) Find the maximum service load that can be resisted without stressing the concrete above 0.45 *f'c* or the steel above 0.40 *fy*.

$f_c := 0.45f'_c$ $\qquad f_c = 2700\ psi$

$f_s := 0.60f_y$ $\qquad f_s = 36000\ psi$

$\rho := \frac{A_s}{b \cdot d}$ $\qquad \rho = 0.018$

$k := \sqrt{(\rho \cdot n)^2 + 2\rho \cdot n} - \rho \cdot n$ $\qquad k = 0.381$

$j := 1 - \frac{k}{3}$ $\qquad j = 0.873$

Moment due to concrete limits

$M_{sc} := \frac{1}{2} \cdot f_c \cdot b \cdot k \cdot d \cdot \left(d - \frac{k \cdot d}{3}\right)$ $\qquad M_{sc} = 229 \cdot ft \cdot kip$

Moment due to steel limit

$M_{ss} := A_s \cdot f_s \cdot j \cdot d$ $\qquad M_{ss} = 286 \cdot ft \cdot kip$

The maximum service moment is the minimum of the two values.

$M_s := \min(M_{ss}, M_{sc})$ $\qquad M_s = 229 \cdot ft \cdot kip$

*(b)* Determine if the beam will show cracking before reaching the service load

$$I_g := \frac{b \cdot h^3}{12} \qquad I_g = 13333 \cdot in^4$$

$$M_{cr1} := \frac{f_r \cdot I_g}{\frac{h}{2}} \qquad M_{cr1} = 64.5 \cdot ft \cdot kip$$

This is less than the service laod so the section cracks. To demonstrate that the transformed section does not affect this conclusion, the following checks the cracked transformed section.

$$\Delta y := \frac{n \cdot A_s \cdot \left(d - \frac{h}{2}\right)}{n \cdot A_s + b \cdot h} \qquad \Delta y = 0.697 \cdot in$$

$$I_{ut} := I_g + b \cdot d \cdot \Delta y^2 + n \cdot A_s \cdot \left(d - \frac{h}{2} - \Delta y\right)^2 \qquad I_{ut} = 15400 \cdot in^4$$

$$M_{cr2} := \frac{f_r \cdot I_{ut}}{\frac{h}{2} - \Delta y} \qquad M_{cr2} = 80.1 \cdot ft \cdot kip \qquad \frac{M_{cr2}}{M_{cr1}} = 1.242$$

*(c)* Determine the nominal moment capacity of the section.

$$a := \frac{A_s \cdot f_y}{0.85 f'_c \cdot b} \qquad a = 3.67 \cdot in$$

$$M_n := A_s \cdot f_y \cdot \left(d - \frac{a}{2}\right) \qquad M_n = 489 \cdot ft \cdot kip$$

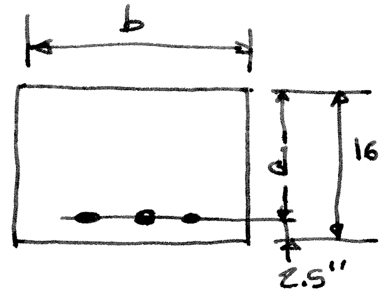
*(d)* Compute the ratio of the nominal capacity of the beam to the maximum service level capacity and compare your findings to the ACI load factors and strength reduction factor.

$$Ratio := \frac{M_n}{M_s} \qquad Ratio = 2.13$$

First, the extra computation of the uncracked transformed area gives only a 18% increase in the cracking moment. Comparing the cracking moment to the service moment shows that the service moment is almost 3 time the cracking moment. Therefore, unless the service moment is very close to the service moment, you can be assured that the section will crack based on the gross section calculation.

Second, the margin of safety between the service moment and the nominal capacity is 2.11. This is greater than the ultimate load factors and phi factors from ASCE-7 and ACI (1.8/0.9 =2.00 if the entire load is classified as live load) indicating that a service level design is more conservative than LRFD design.

3.2 $w_d = 500\text{plf}$ $\quad f_c' = 3000\text{psi}$
$w_L = 1200\text{plf}$ $\quad f_y = 60{,}000\text{psi}$
$L = 22'$ $\quad \rho = 0.6\rho_{max}$




Assume $w_o = w_d = 500\text{plf}$ and check assumption at conclusion of the design

$$w_u = 1.2(500+500) + 1.6(1200) = 3120\text{plf}$$

$$M_u = \frac{wL^2}{8} = \frac{3.12(22)^2}{8} = 189 \text{ ft-kip}$$

$$\rho_{max} = 0.85\beta_1 \frac{f_c'}{f_y}\left(\frac{\epsilon_u}{\epsilon_u + 0.004}\right) = 0.85(.85)\frac{3}{60}\left(\frac{0.003}{0.003+0.004}\right)$$

$= 0.0155$ (or from Table A.4)

$$\rho = 0.6\rho_{max} = 0.6(0.155) = 0.0093$$

$$R = \rho f_y\left(1 - 0.588\frac{\rho f_y}{f_c'}\right) = .0093(60000)\left(1 - 0.588\,\frac{.0093(60)}{3}\right)$$

$R = 497$ (or by interpolation from Table A.5b)

Since $\rho = 0.6\rho_b$ assume $\phi = 0.9$ (Confirmed by Table A.4)

$$bd^2 = \frac{M_u}{\phi R} = \frac{189(12000)}{0.9(497)} = 5070$$

$d = h - 2.5 = 16 - 2.5 = 13.5$

Solve for b

$$b = \frac{5070}{13.5^2} = 27.8$$

Use $b = 28\text{in}$

Check weight assumption $w_o = \frac{16 \times 28}{144}(150) = 467\text{plf}$ which is less than assumed.

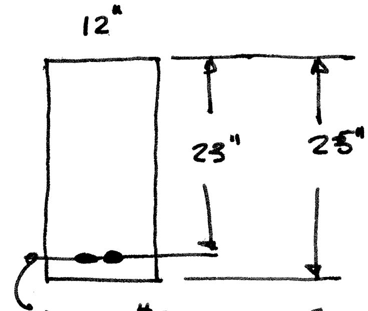
$$A_s = \rho bd = 0.0093(27.8)(13.5) = 3.49\text{ in}^2$$

$3$ #10 (#36) $= 3.81\text{in}^2$

3.3

$L = 20'$
$w_s = 2450$ plf

$w_o = \frac{12 \times 25}{144}(150) = 313$ plf




$A_s = 3\ \#9 = 3.0\,in^2$
$f_y = 60{,}000$ psi
$f_c' = 4000$ psi

$$M_s = \frac{wL^2}{8} = \frac{(2450+313)(20)^2}{8}\frac{1}{1000}$$
$$= 138 \text{ ft kip}$$

a)

$$a = \frac{A_s f_y}{0.85 f_c' b} = \frac{3.0(60)}{.85(4)(12)} = 4.41 in$$

$$M_u = A_s f_y (d - a/2) = 3.0(60)(23 - \tfrac{4.41}{2})\tfrac{1}{12} = 311 \text{ ft kips}$$

$$FS = \frac{M_u}{M_s} = \frac{311}{138} = 2.26$$

This exceeds the target value of 1.85

b)

$n = 8$ from $E_c = 57{,}000\sqrt{f_c'} = 3{,}600{,}000$

$\rho = A_s/bd = \frac{3}{12 \times 23} = 0.0109$

From Table A.6 $k = 0.339$, $j = 0.887$ by interpolation

or $k = \sqrt{(8 \times .0109)^2 + 2 \cdot 8 \cdot 0.0109} - 8(0.0109) = 0.339$

$$f_s = \frac{M_s}{A_s jd} = \frac{138(12000)}{3(.887)23} = 27{,}600 \text{ psi}$$

$$f_c = \frac{M_s}{kj\,bd^2} = \frac{138(12000)}{.339(.887)12(23)^2} = 868 \text{ psi}$$

c) $f_r = 7.5\sqrt{f_c'} = 7.5\sqrt{4000} = 474$ psi

$I_g = bh^3/12 = 12(25)^3/12 = 15{,}625\ in^4$

$$M_{cr} = \frac{f_r I}{y} = \frac{474(15{,}625)}{25/2}\frac{1}{12000} = 49.4 \text{ ft k}$$

$M_{cr} \ll M_s$ ∴ Beam will crack

**Problem 3.4** A rectangular reinforced concrete section has dimension $b$=14 in., $d$=25 in, and h = 28 in., and is reinforced with 3 No. 10 (No. 32) bars. The material strengths are $f'_c$ = 5000 psi, $fy$ = 60,000 psi.

- *(a)* Find the moment that will produce first cracking at the bottom surface of the section basing your calculations on $I_g$, the moment of inertial of the gross section.
- *(b)* Repeat the calculation using $I_{ut}$, the uncracked transformed moment of inertia.
- *(c)* Determine the maximum moment that can be carried without the concrete stress exceeding 0.45 $f'_c$ or the steel stress exceeding 0.60 $f_y$.
- *(d)* Determine the nominal moment capacity of the section.
- *(e)* Compute the ratio of nominal moment capacity from part *(d)* to the service level moment from part *(c)*
- *(f)* Comment on your results with particular attention to comparing parts *(a)* and *(b)* and comparing part *(e)* to established load factors.

▶ Reinforcement sizes

Given data

$A_s := 3 \cdot A_{s10}$ $\quad A_s = 3.81\,in^2$ $\quad E_s := 29000000psi$

$b := 14in$ $\quad d := 25in$ $\quad h := 28in$

$f'_c := 5000psi$ $\quad f_y := 60000psi$ $\quad E_c := 57000\sqrt{f'_c \cdot psi}$ $\quad E_c = 4031\,ksi$

$f_r := 7.5\sqrt{f'_c \cdot psi}$ $\quad f_r = 530\,psi$ $\quad n := \frac{E_s}{E_c}$ $\quad n = 7.2$

*(a)* Find the moment that will produce first cracking at the bottom surface of the section basing your calculations on $I_g$, the moment of inertial of the gross section.

$$I_g := \frac{b \cdot h^3}{12} \quad I_g = 25611\,in^4 \quad M_{cr1} := \frac{f_r \cdot I_g}{\frac{h}{2}} \quad M_{cr1} = 80.8\,ft \cdot kip$$

*(b)* Repeat the calculation using $I_{ut}$, the uncracked transformed moment of inertia.

$$\Delta y := \frac{n \cdot A_s \cdot \left(d - \frac{h}{2}\right)}{n \cdot A_s + b \cdot h} \quad \Delta y = 0.719\,in$$

$$I_{ut} := I_g + b \cdot d \cdot \Delta y^2 + n \cdot A_s \cdot \left(d - \frac{h}{2} - \Delta y\right)^2 \quad I_{ut} = 28689\,in^4$$

$$M_{cr2} := \frac{f_r \cdot I_{ut}}{\frac{h}{2} - \Delta y} \quad M_{cr2} = 95.5\,ft \cdot kip \quad \frac{M_{cr2}}{M_{cr1}} = 1.181$$

*(c)* Determine the maximum moment that can be carried without the concrete stress exceeding 0.45 $f'_c$ or the steel stress exceeding 0.60 $f_y$.

$f_c := 0.45f'_c$ $\quad f_c = 2250\,psi$

$f_s := 0.60 f_y$ $f_s = 36000\,\text{psi}$

$\rho := \dfrac{A_s}{b \cdot d}$ $\rho = 0.011$ $k := \sqrt{(\rho \cdot n)^2 + 2\rho \cdot n} - \rho \cdot n$ $k = 0.325$ $j := 1 - \dfrac{k}{3}$ $j = 0.892$

Moment due to concrete limits

Moment due to steel limit

$M_{sc} := \dfrac{1}{2} \cdot f_c \cdot b \cdot k \cdot d \cdot \left(d - \dfrac{k \cdot d}{3}\right)$ $M_{sc} = 238\,\text{ft} \cdot \text{kip}$ $M_{ss} := A_s \cdot f_s \cdot j \cdot d$ $M_{ss} = 255\,\text{ft} \cdot \text{kip}$

The maximum service moment is the minimum of the two values.

$M_s := \min(M_{ss}, M_{sc})$ $M_s = 238\,\text{ft} \cdot \text{kip}$

*(d)* Determine the nominal moment capacity of the section.

$a := \dfrac{A_s \cdot f_y}{0.85 f'_c \cdot b}$ $a = 3.84\,\text{in}$ $M_n := A_s \cdot f_y \cdot \left(d - \dfrac{a}{2}\right)$ $M_n = 440\,\text{ft} \cdot \text{kip}$

*(e)* Compute the ratio of nominal moment capacity from part *(d)* to the service level moment from part *(c)*

$\text{Ratio} := \dfrac{M_n}{M_s}$ $\text{Ratio} = 1.85$ $0.9 \cdot \text{Ratio} = 1.66$

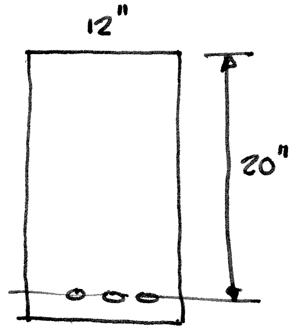
$\text{Ratio1} := \dfrac{M_s}{M_{cr1}}$ $\text{Ratio1} = 2.942$

*(f)* Comment on your results with particular attention to comparing parts *(a)* and *(b)* and comparing part *(e)* to established load factors.

First, the extra computation of the uncracked transformed area gives an 18% increase in the cracking moment. Comparing the cracking moment to the service moment, Ratio1, shows that the service moment is almost 3 time the cracking moment. Therefore, unless the service moment is very close to the service moment, you can be assured that the section will crack.

Second, the margin of safety between the service moment and the nominal capacity is 1.8, 1.6 if a $\phi$ factor is included. This is greater than the ultimate load factors from ASCE-7 indicating that a service level design is far more conservative than LRFD design.

3.5 $f_y = 60{,}000$ psi $f'_c = 5000$ psi




a) $A_s = 2\#8 = 2(0.79) = 1.58 \text{ in}^2$

$$a = \frac{1.58(60)}{.85(5)\,12} = 1.86 \text{ in}$$

$$M_n = A_s f_y (d - a/2) = 1.58(60)\left(20 - \frac{1.86}{2}\right)\frac{1}{12}$$

$= 151$ ft kips

b) $A_s = 2\#10 = 2(1.27) = 2.54 \text{ in}^2$

$$a = \frac{A_s f_y}{.85 f'_c b} = \frac{2.54(60)}{.85(5)\,12} = 2.99 \text{ in}$$

$$M_n = 2.54(60)\left(20 - \frac{2.99}{2}\right)\frac{1}{12}$$

$= 235$ ft kips

c) $A_s = 3\#10 = 3(1.27) = 3.81 \text{ in}^2$

$$a = \frac{3.81(60)}{.85(5)(12)} = 4.48 \text{ in}$$

$$M_n = 3.81(60)\left(20 - \frac{4.48}{2}\right)\frac{1}{12}$$

$= 338$ ft kips

a check of net tensile strain shows $\epsilon_t = 0.0077$ so tensile steel has yielded.