# CHAPTER 2

## Review of Basic Algebra

***Chapter Overview***

Chapter 2 reviews the basics of algebra, including simplifying algebraic expressions, evaluating algebraic expressions by substituting numbers into the variables, solving algebraic equations, and creating and solving word problems. Expressions involving exponents appear in the study of compound interest developed later in the text. Examples are presented showing how positive, negative, fractional, and zero exponents are defined. The study of terms involving positive, negative, and zero exponents serves as a prelude to the introduction of logarithms. Logarithms are useful in solving equations in which the unknown is an exponent. Such equations appear in the area of finance and will be useful in solving financial problems. A review of the steps involved in solving an algebraic equation is presented as well as a four-step procedure for reading and solving word problems.

***Learning Objectives***

After studying Chapter 2, your students will be able to:

1. Simplify algebraic expressions using fundamental operations and substitution.
2. Simplify and evaluate powers with positive exponents, negative exponents, and exponent zero.
3. Use an electronic calculator to compute the numerical value of arithmetic expressions involving fractional exponents.
4. Write exponential equations in logarithmic form and use an electronic calculator equipped with a natural logarithm function to determine the value of natural logarithms.
5. Solve basic equations using addition, subtraction, multiplication, and division.
6. Solve equations involving algebraic simplification and formula rearrangement.
7. Solve word problems by creating and solving equations.

***Suggested Priority of Chapter Topics***

Must Cover

- Simplification of algebraic expressions
- Evaluation of exponential expressions
- Logarithmic functions
- Use of the electronic calculator to evaluate exponential and logarithmic expressions
- Solving algebraic equations
- Translating word problems into algebraic equations

Optional

- Case Study on investing in a tax-free savings account

***<u>Chapter Outline</u>***

***Objective 1: Simplify algebraic expressions using fundamental operations and substitution.***

A. The simplification of algebraic expressions is introduced by noting that only like terms may be combined together when performing addition or subtraction. Like terms are said to have the same **literal coefficients.**

**Teaching Tip**

Review the terminology "coefficient" because use of the word "coefficient" is helpful in explaining operations with algebraic expressions. Use an example such as 3*xy*. The numerical coefficient of *xy* is 3, and the literal coefficient of 3 is *xy*.

Consider the expression: $3x^3 - 2x + 4x^3 + 5x$. Note that $4x^3$ and $3x^3$ can be combined because they have the same literal coefficient of $x^3$. Likewise, $-2x$ and $5x$ can be combined because they have the same literal coefficient of $x$. The original expression can be simplified to $7x^3 + 3x$.

B. Some algebraic expressions contain brackets, and the brackets must be removed in order to simplify the expression.

**Teaching Tip**

This is a good time to review the rules of signed numbers with the students. The four cases of addition, subtraction, multiplication, and division can be covered. Recall with multiplication and division that like signs produce a positive result, and unlike signs produce a negative result. Thus there is no change in sign if the brackets are preceded by a positive number and the signs are changed if the brackets are preceded by a negative number.

Present an example: $-(3x+4y)+(2x-3y) = -3x-4y+2x-3y = -x-7y$. Note the change of sign for each of the terms within the first set of brackets. It is important to point out to the student that <u>each term</u> within a set of brackets must be multiplied by the coefficient outside the brackets.

C. If you multiply two monomials together, multiply the numerical coefficients together, and then multiply the literal coefficients together. This can be expanded to the case of multiplying a polynomial by a monomial. In this case, each term in the polynomial is multiplied by the monomial, and any appropriate sign changes must be made.

D. Multiplying two polynomials together involves an extension of multiplying a polynomial by a monomial. Each term in the second polynomial is multiplied by each term in the first polynomial. Like terms are then collected. There is a lot of bookkeeping involved here because possible sign changes have to be taken into consideration as well.

Consider the example: $(2x+3y)(x-4y)$

Step 1: Multiply $x-4y$ by $2x$ to get $2x^2-8xy$.
Step 2: Multiply $x-4y$ by $3y$ to get $3xy-12y^2$.
Step 3: Add $2x^2-8xy+3xy-12y^2$ and simplify be combining like terms.
Step 4: The final result is $2x^2-5xy-12y^2$.

**Teaching Tip**

The **first, inner, outer, last** rule (FOIL) may help with expressions of the form $(ax+by)(cx+dy)$. The first terms *ax* and *cx* are multiplied together, the inner terms *by* and *cx* are multiplied together, the outer terms *ax* and *dy* are multiplied together, and the last terms *by* and *dy* are multiplied together. The expression is then simplified by combining like terms.

E. Monomials can be divided by finding the quotient of the numerical coefficients and finding the quotient of the literal coefficients. This can be expanded to the case of dividing a polynomial by a monomial, in which case each term of the polynomial is divided separately by the monomial.

F. Since simple interest will be formally covered in future chapters, expressions dealing with simple interest can be used to explain how to substitute numerical values into literal equations. Consider the expression $S=P(1+rt)$.
Let $P=5000$, $r=0.03$, and $t=30/365$. The order of operations can be recalled. The expression within the brackets is evaluated first. Next, *r* is multiplied by *t* and the number one is added to this result to give 1.002465753, which is then multiplied by 5000 to give a final answer of 5012.33 rounded.

***Objective 2: Simplify and evaluate powers with positive exponents, negative exponents, and exponent zero.***

A. Review base and exponent terminology. Convey to the student the various rules for interpreting integral, negative, positive, fractional, and zero exponents.

Related Exponent Rules:
Note: $a \neq 0$

$$a^n = (a)(a)(a)\ldots(a) \quad \text{to } n \text{ factors of } a$$
$$a^{-n} = \frac{1}{a^n}$$
$$a^0 = 1$$
$$a^{1/n} = \sqrt[n]{a}$$
$$a^{m/n} = \sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m$$

B. Numerical examples can be presented for each case.

$$3^4 = 81$$
$$3^{-4} = \frac{1}{81}$$
$$3^0 = 1$$
$$81^{-1/4} = \frac{1}{81^{1/4}} = \frac{1}{\sqrt[4]{81}} = \frac{1}{3}$$
$$81^{3/4} = \left(\sqrt[4]{81}\right)^3 = 3^3 = 27$$

**Teaching Tip**

Distinguish between expressions like $-3^4 = -81$ and $(-3)^4 = 81$.

C. After discussing how to interpret exponential notation, the rules for operations with powers can be introduced.

Operations with Powers:

$$a^m \times a^n = a^{m+n}$$
$$a^m \div a^n = a^{m-n}$$
$$\left(a^m\right)^n = a^{mn}$$

D. Numerical examples can be presented for each of the above rules.

$$3^2 \times 3^4 = 3^{2+4} = 3^6$$
$$3^8 \div 3^6 = 3^{8-6} = 3^2$$
$$\left(3^6\right)^2 = 3^{6\times 2} = 3^{12}$$

**Teaching Tip**

Distinguish between expressions like $3^2 \times 3^4$ and $\left(3^2\right)^4$. These two expressions look alike but are evaluated differently.

E. The definition of the zero exponent can be motivated through examples such as $\frac{2^5}{2^5} = 1$.

F. The definition of the negative exponent can be motivated through examples such as $\frac{6^3}{6^5} = \frac{1}{6^2} = 6^{-2}$.

***Objective 3: Use an electronic calculator to compute the numerical value of arithmetic expressions involving fractional exponents.***

A. Calculators evaluate exponents using keys that may include $x^y$, $y^x$, or ^.

Consider $(1.03)^{(1/4)}$.
On a calculator like the TI83 or TI84, we enter everything in the order it appears:
$1.03\wedge(1\div 4)$ ENTER
Calculators like the Sharp EL series follow a similar set of steps: $1.03\,x^y\,(1\div 4) =$
Finally, the BA II Plus, used in many financial texts, follows: $1.03\,y^x\,(1\div 4) =$
In each case, the result is 1.0074.

B. Calculators can be used to evaluate various expressions as illustrated below from the mathematics of finance. The student will be encountering such expressions in the chapters on compound interest and annuities.

a) $\frac{1.04^{20} - 1}{0.04} = 29.778$

b) $(1.05)^3 = 1.158$

c) $(1.085)^{-10} = 0.442$

***Objective 4: Write exponential equations in logarithmic form and use an electronic calculator equipped with a natural logarithm function to determine the value of natural logarithms.***

A. Motivate the study of logarithms by pointing out that some expressions from the mathematics of finance involve solving an equation for the value of an unknown exponent. Logarithms will enable you to do that.

B. Emphasize that the exponential and logarithmic functions are related. The concept of a logarithm can be introduced by starting with an exponential expression such as:

$$10^3 = 1000$$

This expression can be written in logarithmic form as $\log_{10} 1000 = 3$. Note by definition that the logarithm is defined as the exponent to which a base must be raised to give a certain number. By comparing the logarithmic form with the exponential form, it can be easily seen that the logarithm is an exponent. Note that the base is the same in the exponential and logarithmic forms.

**Teaching Tip**

Give the students an exercise in which it is required to write exponential expressions in logarithmic form and logarithmic expressions in exponential form. See exercises 2.4 A and B on page 61 of the text for such problems.

C. The general interrelationship can be expressed as follows:

$$N = b^y$$
$$y = \log_b N$$

D. Describe the difference between the natural and common logarithmic systems. The common log has a base of 10 and is often designated as log N, whereas the natural log has a base of *e* and is often designated as ln N. It might be pointed out that the symbol *e* represents a special number in mathematics. It is an irrational number with a non-terminating decimal representation and is approximately equal to 2.718. I often liken *e* to π for the purpose of explanation as many students have seen π before. The log key on the calculator can be used to find the common logarithm of a number, and the ln key can be used to find the natural logarithm of a number.

E. The following points related to the ln function should be noted. The student can check these points on the calculator.

$$\ln e = 1$$
$$\ln 1 = 0$$
If $a > 1$, $\ln a$ is a positive number.

If $a < 1$, $\ln a$ is a negative number.

$\ln 0$ is not defined.

F. The logarithmic function has special properties that offer tools for evaluating expressions. It is important to note that these properties work for both log and ln.

$$\ln(ab) = \ln a + \ln b$$
$$\ln(abc) = \ln a + \ln b + \ln c$$
$$\ln\left(\frac{a}{b}\right) = \ln a - \ln b$$
$$\ln\left(a^k\right) = k \ln a$$

**Teaching Tip**

The property that $\ln\left(a^k\right) = k \ln a$ is useful when solving financial equations of the form $600(1.09)^n = 1000$. The unknown in this equation is an exponent. In this case we can simplify the equation before using the properties of logarithms:

$$(1.09)^n = 1.6667$$

Now we apply logarithms:

$$n \ln(1.09) = \ln(1.6667)$$

This equation can be solved for n giving a result of $n = 5.93$. (You will not get an exact answer of 1000 upon substituting the result back into the original equation because of rounding, but it is very close.)

***Objective 5: Solve basic equations using addition, subtraction, multiplication, and division.***

A. State the formal definition of a linear equation as one in which the unknown appears to the power of 1.

B. Initial examples can involve solving equations that require only one step. You can illustrate during the solutions that addition and subtraction are inverse operations, and that multiplication and division are inverse operations.

| Examples | To Isolate for $x$ |
|---|---|
| $x+4=12$ | Subtract 4 from each side – the inverse of addition |
| $x-3=9$ | Add 3 to both sides – the inverse of subtraction |
| $2x=10$ | Divide both sides by 2 – the inverse of multiplication |
| $\frac{x}{3}=12$ | Multiply both sides by 3 = the inverse of division |

***Objective 6: Solve equations involving algebraic simplification and formula rearrangement.***

A. If a linear equation contains common fractions, both sides of the equation can be multiplied by a common denominator to eliminate the fractions. It is important to follow through with such an operation on both sides of the equation.

B. An equation may involve brackets. Appropriate changes, including sign changes, must be made to each term within the brackets when the brackets are removed.

C. The term containing the unknown can be set up on the left hand side of the equation with all constant terms on the right hand side of the equation. This involves formula rearrangement. Recall how to “undo” operations by the use of inverse operations. If an operation is performed on one side of the equation, it must be performed on the other side of the equation as well to maintain the equality.

***Objective 7: Solve word problems by creating and solving equations.***

A. The text provides a four-step procedure for solving word problems.

To Start: Read the problem

1. Introduce the unknown variable
2. Translate the problem into an algebraic expression
3. Set up the equation
4. Solve the equation

To finish: Check you answer

The text has provided word problems in Exercise 2.7 to give students practice in applying the four steps. Select a few problems and go through the four steps for each case.

Students tend to struggle with word problems. It is often helpful to discuss some of the key words used to set up the equation, as in Pointers and Pitfalls on page 75.

***Assignment Grid***

| **Assignment** | **Topic(s)** | **Learning Objective** | **Estimated Time in Minutes** | **Level of Difficulty** |
|---|---|---|---|---|
| Ex. 2.1 | Simplifying and evaluating algebraic expressions | 1 | 20-30 | Easy |
| Ex. 2.2 | Evaluating powers | 2 | 20-30 | Easy |
| Ex. 2.3B, 1-18 | Evaluating expressions with exponents | 2, 3 | 25-35 | Medium |
| Ex. 2.3B, 19-22 | Solving for an unknown in an exponential expression | 2,3 | 20-30 | Difficult |
| Ex. 2.4 | Writing expressions in logarithmic form and evaluating logarithmic expressions | 4 | 20-30 | Easy |
| Ex. 2.5 | Solving algebraic equations using addition, subtraction, multiplication, and division | 5 | 20-30 | Easy |
| Ex. 2.6 | Solving equations using formula rearrangement | 6 | 30-40 | Medium |
| Ex. 2.7 (odd numbers) | Solving word problems | 7 | 40-50 | Medium/Difficult |
| Review Exercise | Preparation for exam | 1-7 | 60-75 | Medium |

Name_____________________________Date______________________Section_____

# CHAPTER 2
# TEN-MINUTE QUIZ

**Circle the letter of the best response.**

1. Consider the exponential expression: $2^6 = 64$. The logarithmic form of this expression is:
   a. $\log_2 6 = 64$
   b. $\log_6 2 = 64$
   c. $\log_2 64 = 6$
   d. $\log_{64} 6 = 2$

2. Simplify the following algebraic expression: $3x^2 + 4y - \left(x^3 + 7x^2 + 3y\right)$.
   a) $-4x^2 + y - x^3$
   b) $10x^2 + y - x^3$
   c) $-4x^2 + 7y - x^3$
   d) $-4x^2 - y + x^3$

3. Evaluate the following expression: $\dfrac{S}{1+rt}$.

   Note: $S = 1800,\ r = 0.125,\ t = 120/365$

   a) 1728.95
   b) 1615.89
   c) 4674.59
   d) 1699.50

4. Find the value of the following expression: $\ln\left[4000\left(1.0325^{-1}\right)\right]$
   a) 8.326
   b) 8.262
   c) 8.29
   d) 8.12

5. A logarithm is best defined by which of the following words:
   a) base
   b) radical
   c) radicand
   d) exponent

6. Consider the following algebraic equation and solve for *x*.

$$\frac{2(x-4)}{3} = (3x+6)$$

   a) -7.50
   b) -7.00
   c) -3.71
   d) 4.28

7. The base of the natural logarithm system is:
   a) 10
   b) e
   c) π
   d) 2

8. Jim, Sue, and Marion have set up a partnership. The total amount contributed to the partnership is $315 000. Sue's contribution is one-half of Jim's contribution, and Marion's contribution is two times Jim's contribution. Find the amount of Jim's contribution to the partnership.
   a) $90 000
   b) $45 000
   c) $180 000
   d) $60 000

9. Evaluate the following exponential expression: $1.05^{-3/2}$
   a) 1.076
   b) 1.575
   c) 0.929
   d) 0.968

10. Solve the following equation for $k$. (Hint: Note that $k$ is an exponent.)

$$(1.06)^k = 2000$$

a) 130.45
b) 186.66
c) 7.17
d) 100.55

**Answers:**

| | | | | |
|---|---|---|---|---|
| **1. c** | **2. a** | **3. a** | **4. b** | **5. d** |
| **6. c** | **7. b** | **8. a** | **9. c** | **10. a** |

Additional Questions:

1. Evaluate: $PMT\left[\frac{(1+i)^n - 1}{i}\right]$ for $PMT = 200,\ i = 0.02,\ n = 18$.
   a. 9714.35
   b. 14 232.46
   c. 4282.46
   d. 14 082.46

2. Simplify: $-(2a+4b-c)-(a+b+c)$.
   a. $-3a-3b+2c$
   b. $-3a+5b$
   c. $-3a-3b$
   d. $-3a-5b$

3. Evaluate using a calculator: $\frac{\ln(5.6)}{\ln(1.015)}$.
   a. 1.723
   b. 115.710
   c. 0.015
   d. 1.708

4. Solve: $x+1.3x=21.5$.
   a. 9.348
   b. 16.538
   c. 8.269
   d. 10.1

5. Simplify: $\left(16a^3-24a\right)\div 4a$.
   a. $16a^3-6$
   b. $-2a$
   c. $4a^2-6$
   d. $4a^2-6a$

6. Compute: $\ln\left[4e^{-0.5}\right]$.
   a. 0.8863
   b. 4
   c. -0.5
   d. 0.8408

7. Carla is framing in a square sand box in her back yard. She has set aside an area of 24 square feet for the sandbox. The sandbox is to be 1.5 times longer than it is wide. What is the length of the longer side for the sandbox?
   a. 16 ft
   b. 6 ft
   c. 4 ft
   d. 4.7 ft

8. Solve: $x+\frac{1}{2}x+\frac{3}{5}x=27$.
   a. 47.25
   b. 12.86
   c. 15
   d. 19.29

9. Given: $S=P\left(1+rt\right)$ solve for *t*.
   a. $t=\frac{S-P}{1+r}$
   b. $t=\frac{S}{\Pr}-1$
   c. $t=\frac{S-P}{r}$
   d. $t=\frac{S-P}{\Pr}$

10. Express in logarithmic form: $3^5 = 243$.
    a. $5 = \log_3 243$
    b. $5 = 3\log 243$
    c. $3 = \log_5 243$
    d. $3 = 5\log 243$

**Answers:**

| | | | | |
|---|---|---|---|---|
| **1. c** | **2. d** | **3. b** | **4. a** | **5. c** |
| **6. a** | **7. b** | **8. b** | **9. d** | **10. a** |