

SOLUTIONS MANUAL FOR

Computational Fluid
Mechanics and
Heat Transfer,
Third Edition

by

Richard H. Pletcher, John Tannehill
and Dale A. Anderson

SOLUTIONS MANUAL FOR

Computational Fluid Mechanics and Heat Transfer, Third Edition

_____ by _____

Richard H. Pletcher, John Tannehill
and Dale A. Anderson



CRC Press

Taylor & Francis Group

Boca Raton London New York

CRC Press is an imprint of the
Taylor & Francis Group, an informa business

Taylor & Francis
Taylor & Francis Group
6000 Broken Sound Parkway NW, Suite 300
Boca Raton, FL 33487-2742

© 2012 by Taylor & Francis Group, LLC
Taylor & Francis is an Informa business

No claim to original U.S. Government works

Printed on acid-free paper
Version Date: 20131018

International Standard Book Number-13: 978-1-4398-7256-7 (Ancillary)

This book contains information obtained from authentic and highly regarded sources. Reasonable efforts have been made to publish reliable data and information, but the author and publisher cannot assume responsibility for the validity of all materials or the consequences of their use. The authors and publishers have attempted to trace the copyright holders of all material reproduced in this publication and apologize to copyright holders if permission to publish in this form has not been obtained. If any copyright material has not been acknowledged please write and let us know so we may rectify in any future reprint.

Except as permitted under U.S. Copyright Law, no part of this book may be reprinted, reproduced, transmitted, or utilized in any form by any electronic, mechanical, or other means, now known or hereafter invented, including photocopying, microfilming, and recording, or in any information storage or retrieval system, without written permission from the publishers.

For permission to photocopy or use material electronically from this work, please access www.copyright.com (<http://www.copyright.com/>) or contact the Copyright Clearance Center, Inc. (CCC), 222 Rosewood Drive, Danvers, MA 01923, 978-750-8400. CCC is a not-for-profit organization that provides licenses and registration for a variety of users. For organizations that have been granted a photocopy license by the CCC, a separate system of payment has been arranged.

Trademark Notice: Product or corporate names may be trademarks or registered trademarks, and are used only for identification and explanation without intent to infringe.

Visit the Taylor & Francis Web site at
<http://www.taylorandfrancis.com>

and the CRC Press Web site at
<http://www.crcpress.com>

Computational Fluid Mechanics and Heat Transfer

Solutions Manual

Chapter 2

2.1

The solution of Laplace's equation is

$$T(x, y) = \sum_{n=1}^{\infty} A_n \sin(n\pi x) \sinh[n\pi(y-1)]$$

To verify that the coefficient A_n given in Example 2.1 is correct, we can first use the boundary condition $T(x, 0) = T_0$. Multiply this equation by $\sin(n\pi x)$, and integrate from 0 to 1:

$$\int_0^1 T_0 \sin(n\pi x) dx = \frac{T_0 [1 - (-1)^n]}{n\pi} = A_n \sinh(-n\pi) \frac{1}{2}$$

Using the trigonometry identity $\sinh(-x) = -\sinh(x)$ the coefficient becomes

$$A_n = \frac{2T_0 [(-1)^n - 1]}{n\pi \sinh(n\pi)}$$

2.2

For this problem, $F(r, \theta) = r - r_b = 0$, thus $\nabla F = \mathbf{i}_r$ and the boundary condition is

$u_r = \mathbf{V} \cdot \mathbf{i}_r = \nabla \phi \cdot \mathbf{i}_r = \frac{\partial \phi}{\partial r} = 0$. Since $\phi = V_\infty r \cos \theta + K \cos \theta / r$, we have $u_r = \cos \theta \left(V_\infty - \frac{K}{r^2} \right)$. The

quantity in parenthesis must vanish on the cylinder ($r = r_b$) so $K = r_b^2 V_\infty$ and the required velocity boundary condition is satisfied.

2.3

Classical separation of variable provides the general term $X(x)T(t)$. Substituting into the wave equation $y_{tt} = a^2 y_{xx}$ yields the following set of differential equations:

$$X'' + \alpha^2 X = 0 \quad T'' + \alpha^2 a^2 T = 0$$

The boundary and initial conditions are

$$X(0) = X(l) = 0 \quad T(0) = \sin\left(\frac{\pi x}{l}\right) \quad T'(t) = 0$$

This leads to a solution

$$y(x,t) = \sum A_n \sin\left(\frac{an\pi t}{l}\right) \cos\left(\frac{n\pi x}{l}\right)$$

In this case, only one term of the expansion is necessary to satisfy the specified initial displacement. Applying the boundary conditions eliminates all but the first term in the series.

2.5

Applying the transformation to Equation 2.18a for the hyperbolic case results in the equation

$$-\frac{b^2 - 4ac}{a} \phi_{\xi\eta} + (e - d\lambda_1) \phi_{\xi} + (e - d\lambda_2) \phi_{\eta} + f \phi_{\eta} = \phi = g(\xi, \eta)$$

2.6

Let $\lambda_2 = \frac{b}{2a}$ and $\lambda_1 = c$. These selections provide transformed coordinates that are linearly

independent. The coefficient of the $\phi_{\xi\eta}$ term is

$$a\lambda_1^2 - b\lambda_2 + c = -b^2 + 4ac = 0$$

and the cross derivative coefficient is

$$2a(\lambda_1\lambda_2) - b(\lambda_1 + \lambda_2) + 2c = -b^2 + 4ac = 0$$

and the correct form is obtained.

2.7

The divergence theorem is $\iint_D \nabla^2 u dA = \int_B \frac{\partial u}{\partial n} dl$. Since the original equation is Laplace's equation on the domain D , the integral must vanish and substituting $r = 1$ on the boundary yields

$$\int_B f(\theta) d\theta = \int_B \frac{\partial u}{\partial n}(1) d\theta = 0$$

2.8

(a) For the equation $y^2 u_{xx} - x^2 u_{yy} = 0$ we have $a = y^2$, $b = 0$, $c = -x^2$, $b^2 - 4ac = 4x^2 y^2$.

The discriminant is positive so the equation is always hyperbolic except when $x = 0$ and $y = 0$. For this isolated case, the equation is parabolic.

(b) Let $\xi = x^2 + y^2$ and $\eta = x^2 - y^2$. The equation is transformed to

$$2(\xi^2 - \eta^2)u_{\xi\eta} - \eta u_\xi + \xi u_\eta = 0$$

2.9

(a) $2u_{xx} - 4u_{xy} + 2u_{yy} + 3u = 0$ The discriminant is zero so the equation is parabolic.

(b) $\xi = y + kx$, $\eta = y - x$ assuming the second characteristic is a constant $k \neq 1$.

(c) $2v_x - 4w_x + 2w_y + 3u = 0$

$$w_x - v_y = 0$$

Letting $\mathbf{Z} = (v, w)$

$$[A]\mathbf{Z}_x + [C]\mathbf{Z}_y = [F]$$

where $[A] = \begin{bmatrix} 2 & -4 \\ 0 & 1 \end{bmatrix}$, $[C] = \begin{bmatrix} 0 & 2 \\ -1 & 0 \end{bmatrix}$ and $[F] = \begin{bmatrix} -3u \\ 0 \end{bmatrix}$.

(d) $D = (4)^2 - 4(2)(2) = 0$ Therefore, the system of equations is parabolic.

2.10

Classify the system of equations:

$$\frac{\partial u}{\partial t} + 8 \frac{\partial v}{\partial x} = 0$$

$$\frac{\partial v}{\partial t} + 2 \frac{\partial u}{\partial x} = 0$$

$$a_1 = 1, \quad b_1 = 0, \quad c_1 = 0, \quad d_1 = 8$$

$$a_2 = 0, \quad b_2 = 1, \quad c_2 = 2, \quad d_2 = 0$$

Since $D > 0$, the system of equations is hyperbolic.

2.11

Answer: The equation is elliptic for all values of $a \neq 0$.

2.12

Answer: hyperbolic

2.13

Answer: elliptic, hyperbolic

2.18

Answer: elliptic, hyperbolic

2.19

(a) $f(x) = \sin x, 0 \leq x \leq \pi$

Cosine series is $f(x) = \frac{a_0}{2} + \sum A_n \cos(n\pi x)$ where $n = 1 \rightarrow \infty$. In this series, all basis functions $[\cos(n\pi x)]$ are orthogonal to the function that is to be expanded. Thus, all of the Fourier coefficients vanish except $a_0 = \frac{2}{\pi}$. For the prescribed function, this is the best that can be done with the cosine series.

(b) In this case $f(x) = \cos x$ is itself the cosine series and only one term of the Fourier series survives.

2.20

(a) $a = 1, b = 3, c = 2$

$$\frac{dy}{dx} = 1, 2$$

(b) $a = 1, b = 12, c = 2$

$$\frac{dy}{dx} = -1$$

This is a parabolic equation and the other characteristic may be chosen with the restriction that the two are linearly independent.

2.21

(a) Hyperbolic $\lambda_1 = 2, \lambda_2 = 1$. Let $\xi = y - x, \eta = y + 2x$. This transforms to $u_{\xi\eta} = 0$.

(b) Parabolic $\lambda_1 = -1$. Let $\lambda_2 = 1, \xi = y - x, \eta = y + x$. This transforms to $u_{\xi\xi} = 0$.

2.22

(a) Answer: $u_{\xi\xi} + u_{\eta\eta} = 0$

(b) Answer: $u_{\xi\xi} + u_{\eta\eta} + \frac{u_{\xi}}{4} = 0$

2.23

(a) $\lambda_1 = -3$. Let $\lambda_2 = 1$, $\xi = y - x$, $\eta = y + 3x$. After transforming we have

$$16u_{\xi\xi} - u_{\xi} + 3u_{\eta} - e^{xy} = 0 \text{ where } x = \frac{\eta - \xi}{4}, y = \frac{\eta + 3\xi}{4}.$$

2.24

Answer: $u(x, y) = -\sinh(y - \pi) \frac{\sin x}{\sinh \pi} - 2 \sinh(y - \pi) \frac{\sin 2x}{\sinh 2\pi}$

2.25

Answer: $u(x, y) = -\frac{\sin x}{\sinh \pi} - \frac{2 \sin 2x}{\sinh 2\pi}$

2.26

Answer: $u(x, y) = \sum_{n=1}^{\infty} A_n \sinh[n(y - \pi)] \sin(nx)$

where $A_n = -\frac{2}{n \sinh(n\pi)} \left[2 \left(\pi^2 - \frac{5}{n^2} - \frac{12}{n^2 n^4} \right) + \frac{2 \cos(n\pi)}{n} \left(\frac{10\pi}{n^2} - \frac{24}{\pi n^4} \right) \right]$

2.27

Answer: $T(x, y) = e^{-4\pi^2 t} \sin(2\pi x)$
