Chapter 1

Solving Linear Equations and Inequalities

Exercise Set 1.1

RC2. (f)

RC4. (c)

2.
$$\begin{array}{c|c} \multicolumn{2}{c}{47-x=23} \\ \hline 47-24 \ ? & 23 \\ 23 & \quad \text{TRUE} \end{array}$$

24 is a solution of the equation.

4.
$$\begin{array}{r|l} \multicolumn{2}{c}{3x+14=-27} \\ \hline 3(-10)+14 \ ? & -27 \\ -30+14 & \\ -16 & \quad \text{FALSE} \end{array}$$

-10 is not a solution of the equation.

6.
$$\begin{array}{r|l} \multicolumn{2}{c}{\dfrac{-x}{8}=-3} \\ \hline \dfrac{-32}{8} \ ? & -3 \\ -4 & \quad \text{FALSE} \end{array}$$

32 is not a solution of the equation.

8.
$$\begin{array}{r|l} \multicolumn{2}{c}{4-5x=59} \\ \hline 4-5(-11) \ ? & 59 \\ 4+55 & \\ 59 & \quad \text{TRUE} \end{array}$$

-11 is a solution of the equation.

10.
$$\begin{array}{r|l} \multicolumn{2}{c}{9y+5=86} \\ \hline 9\cdot 9+5 \ ? & 86 \\ 81+5 & \\ 86 & \quad \text{TRUE} \end{array}$$

9 is a solution of the equation.

12.
$$\begin{array}{r|l} \multicolumn{2}{c}{x+5=5+x} \\ \hline -13+5 \ ? & 5+(-13) \\ -8 & -8 \quad \text{TRUE} \end{array}$$

-13 is a solution of the equation.

14.
$$\begin{aligned} x+7&=14 \\ x+7-7&=14-7 \\ x+0&=7 \\ x&=7 \end{aligned}$$

16.
$$\begin{aligned} -27&=y-17 \\ -27+17&=y-17+17 \\ -10&=y+0 \\ -10&=y \end{aligned}$$

18.
$$\begin{aligned} -8+r&=17 \\ 8-8+r&=8+17 \\ 0+r&=25 \\ r&=25 \end{aligned}$$

20.
$$\begin{aligned} -37+x&=-89 \\ 37-37+x&=37-89 \\ 0+x&=-52 \\ x&=-52 \end{aligned}$$

22.
$$\begin{aligned} z-14.9&=-5.73 \\ z-14.9+14.9&=-5.73+14.9 \\ z+0&=9.17 \\ z&=9.17 \end{aligned}$$

24.
$$\begin{aligned} x+\frac{1}{12}&=-\frac{5}{6} \\ x+\frac{1}{12}-\frac{1}{12}&=-\frac{5}{6}-\frac{1}{12} \\ x+0&=-\frac{10}{12}-\frac{1}{12} \\ x&=-\frac{11}{12} \end{aligned}$$

26.
$$\begin{aligned} 5x&=30 \\ \frac{5x}{5}&=\frac{30}{5} \\ 1\cdot x&=\frac{30}{5} \\ x&=6 \end{aligned}$$

28.
$$\begin{aligned} -4x&=124 \\ \frac{-4x}{-4}&=\frac{124}{-4} \\ 1\cdot x&=\frac{124}{-4} \\ x&=-31 \end{aligned}$$

30.
$$\begin{aligned} -\frac{x}{3}&=-25 \\ -\frac{1}{3}x&=-25 \\ -3\left(-\frac{1}{3}\right)x&=-3(-25) \\ x&=75 \end{aligned}$$

32.
$$\begin{aligned} -120 &= -8y \\ \frac{-120}{-8} &= \frac{-8y}{-8} \\ \frac{-120}{-8} &= 1 \cdot y \\ 15 &= y \end{aligned}$$

34.
$$\begin{aligned} 0.39t &= -2.73 \\ \frac{0.39t}{0.39} &= \frac{-2.73}{0.39} \\ 1 \cdot t &= \frac{-2.73}{0.39} \\ t &= -7 \end{aligned}$$

36.
$$\begin{aligned} -\frac{7}{6}y &= -\frac{7}{8} \\ -\frac{6}{7}\Big(-\frac{7}{6}\Big)(y) &= -\frac{6}{7}\Big(-\frac{7}{8}\Big) \\ 1 \cdot y &= \frac{42}{56} \\ y &= \frac{3}{4} \end{aligned}$$

38.
$$\begin{aligned} 4x - 7 &= 81 \\ 4x &= 88 \\ x &= 22 \end{aligned}$$

40.
$$\begin{aligned} 6z - 7 &= 11 \\ 6z &= 18 \\ z &= 3 \end{aligned}$$

42.
$$\begin{aligned} 5x + 7 &= -108 \\ 5x &= -115 \\ x &= -23 \end{aligned}$$

44.
$$\begin{aligned} -\frac{9}{2}y + 4 &= -\frac{91}{2} \\ -9y + 8 &= -91 \\ -9y &= -99 \\ y &= 11 \end{aligned}$$

46.
$$\begin{aligned} \frac{9}{5}y + \frac{4}{10}y &= \frac{66}{10} \\ 18y + 4y &= 66 \quad \text{Multiplying by 10} \\ 22y &= 66 \\ y &= 3 \end{aligned}$$

48.
$$\begin{aligned} 0.8t - 0.3t &= 6.5 \\ 0.5t &= 6.5 \\ t &= 13 \end{aligned}$$

50.
$$\begin{aligned} 15x + 40 &= 8x - 9 \\ 15x &= 8x - 49 \\ 7x &= -49 \\ x &= -7 \end{aligned}$$

52.
$$\begin{aligned} 3x - 15 &= 15 + 3x \\ -15 &= 15 \quad \text{False equation} \end{aligned}$$
No solution

54.
$$\begin{aligned} 9t - 4 &= 14 + 15t \\ 9t - 18 &= 15t \\ -18 &= 6t \\ -3 &= t \end{aligned}$$

56.
$$\begin{aligned} 6 - 7x &= x - 14 \\ 20 - 7x &= x \\ 20 &= 8x \\ \frac{20}{8} &= x \\ \frac{5}{2} &= x \end{aligned}$$

58.
$$\begin{aligned} 5x - 8 &= -8 + 3x - x \\ 5x - 8 &= -8 + 2x \\ 3x &= 0 \\ x &= 0 \end{aligned}$$

60.
$$\begin{aligned} 6y + 20 &= 10 + 3y + y \\ 6y + 20 &= 10 + 4y \\ 2y &= -10 \\ y &= -5 \end{aligned}$$

62.
$$\begin{aligned} -3t + 4 &= 5 - 3t \\ 4 &= 5 \quad \text{False equation} \end{aligned}$$
No solution

64.
$$\begin{aligned} 5 - 2y &= -2y + 5 \\ 5 &= 5 \quad \text{True equation} \end{aligned}$$
All real numbers are solutions.

66.
$$\begin{aligned} 3(y + 6) &= 9y \\ 3y + 18 &= 9y \\ 18 &= 6y \\ 3 &= y \end{aligned}$$

68.
$$\begin{aligned} 27 &= 9(5y - 2) \\ 27 &= 45y - 18 \\ 45 &= 45y \\ 1 &= y \end{aligned}$$

70.
$$\begin{aligned} 210(x - 3) &= 840 \\ 210x - 630 &= 840 \\ 210x &= 1470 \\ x &= 7 \end{aligned}$$

72.
$$\begin{aligned} 8x - (3x - 5) &= 40 \\ 8x - 3x + 5 &= 40 \\ 5x &= 35 \\ x &= 7 \end{aligned}$$

74. $$\begin{aligned} 3(4-2x) &= 4-(6x-8) \\ 12-6x &= 4-6x+8 \\ 12-6x &= 12-6x \\ 12 &= 12 \quad \text{True equation} \end{aligned}$$
All real numbers are solutions.

76. $$\begin{aligned} -40x+45 &= 3[7-2(7x-4)] \\ -40x+45 &= 3[7-14x+8] \\ -40x+45 &= 3[-14x+15] \\ -40x+45 &= -42x+45 \\ 2x &= 0 \\ x &= 0 \end{aligned}$$

78. $$\begin{aligned} \frac{1}{6}(12t+48)-20 &= -\frac{1}{8}(24t-144) \\ 2t+8-20 &= -3t+18 \\ 5t &= 30 \\ t &= 6 \end{aligned}$$

80. $$\begin{aligned} 6[4(8-y)-5(9+3y)]-21 &= -7[3(7+4y)-4] \\ 6[32-4y-45-15y]-21 &= -7[21+12y-4] \\ 6[-13-19y]-21 &= -7[17+12y] \\ -78-114y-21 &= -119-84y \\ 20 &= 30y \\ \frac{2}{3} &= y \end{aligned}$$

82. $$\begin{aligned} \frac{3}{4}\left(3x-\frac{1}{2}\right)+\frac{2}{3} &= \frac{1}{3} \\ 54x-9+16 &= 8 \quad \text{Multiplying by 24} \\ 54x &= 1 \\ x &= \frac{1}{54} \end{aligned}$$

84. $$\begin{aligned} 9(4x+7)-3(5x-8) &= 6\left(\frac{2}{3}-x\right)-5\left(\frac{3}{5}+2x\right) \\ 36x+63-15x+24 &= 4-6x-3-10x \\ 21x+87 &= -16x+1 \\ 37x &= -86 \\ x &= -\frac{86}{37} \end{aligned}$$

86. $\dfrac{a^{-9}}{a^{23}} = a^{-32} = \dfrac{1}{a^{32}}$

88. $-2x^8y^3$

90. $-5+6x$

92. $-10x+35y-20$

94. $4(-x-6y)$, or $-4(x+6y)$

96. $5(-2x+7y-4)$, or $-5(2x-7y+4)$

98. $\{-8,-7,-6,-5,-4,-3,-2,-1\}$;
$\{x|x \text{ is a negative integer greater than } -9\}$

100. $$\begin{aligned} -0.00458y+1.7787 &= 13.002y-1.005 \\ -13.00658y &= -2.7837 \\ y &\approx 0.214 \end{aligned}$$

102. $$\begin{aligned} \frac{2x-5}{6}+\frac{4-7x}{8} &= \frac{10+6x}{3} \\ 4(2x-5)+3(4-7x) &= 8(10+6x) \\ &\quad \text{Multiplying by 24} \\ 8x-20+12-21x &= 80+48x \\ -88 &= 61x \\ -\frac{88}{61} &= x \end{aligned}$$

104. $$\begin{aligned} 23-2\{4+3(x-1)\}+5\{x-2(x+3)\} &= \\ 7\{x-2[5-(2x+3)]\} \\ 23-2\{4+3x-3\}+5\{x-2x-6\} &= \\ 7\{x-2[5-2x-3]\} \\ 23-2\{3x+1\}+5\{-x-6\} &= \\ 7\{x-2[-2x+2]\} \\ 23-6x-2-5x-30 &= \\ 7\{x+4x-4\} \\ -11x-9 &= \\ 7\{5x-4\} \\ -11x-9 &= \\ 35x-28 \\ 19 &= 46x \\ \frac{19}{46} &= x \end{aligned}$$

Exercise Set 1.2

RC2. $$\begin{aligned} qs+4r &= t \\ qs &= t-4r \\ q &= \frac{t-4r}{s} \end{aligned}$$
The answer is (b).

RC4. $$\begin{aligned} 4q &= 7r \\ q &= \frac{7r}{4}, \text{ or } \frac{7}{4}r \end{aligned}$$
The answer is (a).

RC6. $$\begin{aligned} 7r-t &= 4s \\ 7r &= 4s+t \\ r &= \frac{4s+t}{7} \end{aligned}$$
The answer is (e).

2. $$\begin{aligned} d &= rt \\ \frac{d}{r} &= t \end{aligned}$$

4. $$\begin{aligned} V &= \frac{4}{3}\pi r^3 \\ \frac{3V}{4\pi} &= r^3 \end{aligned}$$

6.
$$P = 2w + 2l$$
$$P - 2w = 2l$$
$$\frac{P-2w}{2} = l, \text{ or}$$
$$\frac{P}{2} - w = l$$

8.
$$A = \frac{1}{2}bh$$
$$2A = bh$$
$$\frac{2A}{b} = h$$

10.
$$A = \frac{a+b}{2}$$
$$2A = a + b$$
$$2A - a = b$$

12.
$$F = ma$$
$$\frac{F}{m} = a$$

14.
$$I = Prt$$
$$\frac{I}{rt} = P$$

16.
$$E = mc^2$$
$$\frac{E}{c^2} = m$$

18.
$$Q = \frac{p-q}{2}$$
$$2Q = p - q$$
$$q = p - 2Q$$

20.
$$Ax + By = c$$
$$Ax = c - By$$
$$x = \frac{c - By}{A}$$

22.
$$F = \frac{mv^2}{r}$$
$$\frac{Fr}{m} = v^2 \quad \text{Multiplying by } \frac{r}{m}$$

24.
$$N = \frac{1}{3}M(t+w)$$
$$3N = M(t+w)$$
$$\frac{3N}{M} = t + w$$
$$\frac{3N}{M} - t = w, \text{ or}$$
$$\frac{3N - Mt}{M} = w$$

26.
$$t = \frac{1}{6}(x - y + z)$$
$$6t = x - y + z$$
$$6t - x + y = z$$

28.
$$g = m + mnp$$
$$g = m(1 + np)$$
$$\frac{g}{1+np} = m$$

30.
$$Z = Q - Qab$$
$$Z = Q(1 - ab)$$
$$\frac{Z}{1-ab} = Q$$

32. a) 5 ft 6 in. = 5×12 in. + 6 in. = 66 in.
$R = 665 + 4.35(145) + 4.7(66) - 4.7(32) \approx$
1446 calories

b)
$$R = 655 + 4.35w + 4.7h - 4.7a$$
$$R - 655 - 4.35w + 4.7a = 4.7h$$
$$\frac{R - 655 - 4.35w + 4.7a}{4.7} = h$$

34. a) 6 ft 2 in. = 6×12 in. + 2 in. = 74 in.
$K = 102.3 + 9.66(210) + 19.69(74) - 10.54(34) \approx$
3230 calories

b)
$$K = 102.3 + 9.66w + 19.69h - 10.54a$$
$$K - 102.3 - 9.66w - 19.69h = -10.54a$$
$$\frac{K - 102.3 - 9.66w - 19.69h}{-10.54} = a, \text{ or}$$
$$\frac{102.3 + 9.66w + 19.69h - K}{10.54} = a$$

36. a)
$$P = 94.593c + 34.227a - 2134.616$$
$$P = 94.593(26.7) + 34.227(24.1) - 2134.616$$
$$P \approx 1216 \text{ g}$$

b)
$$P = 94.593c + 34.227a - 2134.616$$
$$P - 34.227a + 2134.616 = 94.593c$$
$$\frac{P - 34.227a + 2134.616}{94.593} = c$$

38. a)
$$F = \frac{n}{15}$$
$$F = \frac{42,690}{15}$$
$$F = 2846 \text{ students}$$

b)
$$F = \frac{n}{15}$$
$$15F = n$$

40. $-2000 \div (-8) = \frac{-2000}{-8} = 250$

42. $120 \div (-4.8) = \frac{120}{-4.8} = -25$

44. $\frac{-90}{-15} = 6$

46. $\frac{-80}{16} = -5$

48. Solve for a:
$$s = v_1 t + \frac{1}{2}at^2$$
$$s - v_1 t = \frac{1}{2}at^2$$
$$2(s - v_1 t) = at^2$$
$$\frac{2(s - v_1 t)}{t^2} = a$$

Solve for v_1:

$$s = v_1 t + \frac{1}{2}at^2$$
$$s - \frac{1}{2}at^2 = v_1 t$$
$$\frac{s - \frac{1}{2}at^2}{t} = v_1$$

50. Solve for T_2:

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$$
$$P_1V_1T_2 = P_2V_2T_1$$
$$T_2 = \frac{P_2V_2T_1}{P_1V_1}$$

Solve for P_1:

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$$
$$P_1 = \frac{P_2V_2T_1}{T_2V_1}$$

52. First find the length of $\overline{AB}$. This is the base of the shaded triangle. (Alternatively, we could consider $\overline{AB}$ to be the height of the triangle.)

$$A = \frac{1}{2}bh$$
$$20 = \frac{1}{2} \cdot b \cdot 8$$
$$20 = 4b$$
$$5 = b$$

The length of $\overline{AB}$ is 5 cm. This is one base of the trapezoid.

$$A = \frac{1}{2}h(b_1 + b_2)$$
$$A = \frac{1}{2} \cdot 8(5 + 13)$$
$$A = \frac{1}{2} \cdot 8(18)$$
$$A = 72 \text{ cm}^2$$

Exercise Set 1.3

RC2. Translate the problem to an equation.

RC4. Check the answer in the original problem.

2. Let m = the number of miles left to complete. Then $2m$ = the number of miles completed. The competition has a total of $2.4 + 112 + 26.2$, or 140.6 miles.

Solve: $2m + m = 140.6$

$m \approx 46.9$ mi

Then $2m = 2(46.9) \approx 94$ mi. This is the number of miles that have been completed.

4. Let x = the measure of the first angle. Then $3x$ = the measure of the second angle and $x + 25$ = the measure of the third angle.

Solve: $x + 3x + (x + 25) = 180$

$x = 31°$

The measures of the angles are 31°; $3 \cdot 31°$, or 93°; and $31° + 25°$, or 56°.

6. Let p = the number of passengers who cleared U.S. Customs in Nassau.

Solve: $4p + 103,264 = 4,808,984$

$p = 1,176,430$ passengers

8. Let p = the price of the headphone.

Solve: $p + 0.07p = 149.75$

$p \approx \$139.95$

10. Let l = the length. Then $l - 42$ = the width.

Solve: $2l + 2(l - 42) = 228$

$l = 78$

The length is 78 ft, and the width is $78 - 42$, or 36 ft.

12. Let s = the length of a side of the smaller square. Then $2s$ = the length of a side of the larger square.

Solve: $4s + 4 \cdot 2s = 100$

$s = \frac{25}{3}$

If $s = \frac{25}{3}$, then $4s = 4 \cdot \frac{25}{3} = \frac{100}{3}$, or $33\frac{1}{3}$;

$2s = 2 \cdot \frac{25}{3} = \frac{50}{3}$ and $4 \cdot \frac{50}{3} = \frac{200}{3}$, or $66\frac{2}{3}$.

The wire should be cut so that one piece is $33\frac{1}{3}$ cm long. Then the other piece will be $66\frac{2}{3}$ cm long.

14. Let p = the selling price of the house. Then $p - 100,000$ is the amount that exceeds \$100,000.

Solve:

$0.08(100,000) + 0.03(p - 100,000) = 9200$

$p = \$140,000$

16. Let x = the first even integer. Then $x + 2$ and $x + 4$ are the next two even integers.

Solve: $x + 5(x + 2) + 4(x + 4) = 1226$

$x = 120$

The numbers are 120; $120 + 2$, or 122; and $120 + 4$, or 124.

18. Let x = the first number. Then $x + 1$ = the second number.

Solve: $x + (x + 1) = 697$

$x = 348$

The numbers are 348 and $348 + 1$, or 349.

20. Let c = the number of square feet of carpet the customer had cleaned. Then the square footage that exceeds 200 sq ft is $c - 200$. The cost for cleaning the stairs is \$1.40(13), or \$18.20.

Solve: $75 + 0.25(c - 200) + 18.20 = 253.95$

$c = 843$ square feet

22. Let $s =$ the old salary.

Solve: $$\begin{aligned} s + 0.05s &= 40,530 \\ s &= \$38,600 \end{aligned}$$

24. Let $n =$ the number of people receiving assistance in 2008, in millions.

Solve: $$\begin{aligned} n + 0.684n &= 47.5 \\ n &\approx 28.2 \text{ million people} \end{aligned}$$

26. a) At age 50, $x = 50 - 40 = 10$.

$y = 2.06(10) + 10.08 = \$30.68$

b) Solve: $$\begin{aligned} 52 &= 2.06x + 10.08 \\ 20 &\approx x \end{aligned}$$

The monthly premium would be approximately \$52 at issue age $40 + 20$, or 60.

28. $d = 26,000 \text{ ft} - 11,000 \text{ ft} = 15,000$ ft. Let $t =$ the time required to reach the new altitude, in minutes.

Solve: $$\begin{aligned} 15,000 &= 2500t \\ t &= 6 \text{ min} \end{aligned}$$

30. Let $t =$ the time.

Solve: $$\begin{aligned} 725 &= (390 - 65)t \\ t &= \frac{29}{3}, \text{ or } 2\frac{3}{13}\text{hr} \end{aligned}$$

32. Let $t =$ the time, in hours, it took the *Delta Queen* to cruise 2 mi upstream. The speed of the boat traveling upstream was $7 - 3$, or 4 mph.

Solve: $$\begin{aligned} 2 &= 4t \\ \frac{1}{2}\text{ hr} &= t \end{aligned}$$

34. $$\begin{aligned} 16 \cdot 8 + 200 \div 25 \cdot 10 &= 128 + 8 \cdot 10 \\ &= 128 + 80 \\ &= 208 \end{aligned}$$

36. $$\begin{aligned} \frac{(9-4)^2 + (8-11)^2}{4^2 + 2^2} &= \frac{5^2 + (-3)^2}{16 + 4} \\ &= \frac{25 + 9}{20} \\ &= \frac{34}{20} \\ &= \frac{17}{10} \end{aligned}$$

38. Let S represent Christina's original salary and let x represent the number by which the reduced salary would have to be multiplied in order to return it to the original salary. Express $n\%$ in decimal notation as $0.01n$. The reduced salary is $S(1 - 0.01n)$ so we have $S(1 - 0.01n)(x) = S$.

$x = \dfrac{1}{1 - 0.01n}$, or $\dfrac{100}{100 - n}$, or $\dfrac{10,000}{100 - n}\%$.

40. Let $s =$ the number of seconds after which the watches will show the same time again. The difference in time between the two watches is

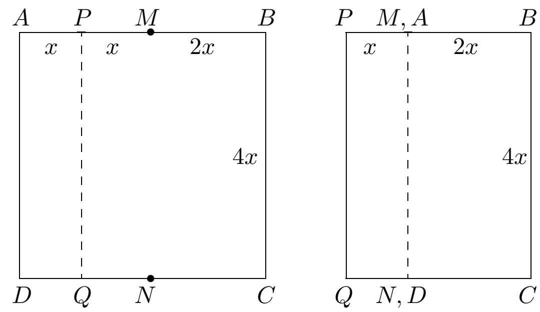
$2.5\dfrac{\text{sec}}{\text{hr}} = 2.5\dfrac{\text{sec}}{\cancel{\text{hr}}} \times \dfrac{1\ \cancel{\text{hr}}}{60\ \cancel{\text{min}}} \times \dfrac{1\ \cancel{\text{min}}}{60 \text{ sec}} = \dfrac{2.5 \text{ sec}}{3600 \text{ sec}}$.

The watches will show the same time again when the difference in time between them is

$12 \text{ hr} = 12\ \cancel{\text{hr}} \times \dfrac{60\ \cancel{\text{min}}}{1\ \cancel{\text{hr}}} \times \dfrac{60 \text{ sec}}{1\ \cancel{\text{min}}} = 43{,}200$ sec.

Solve: $$\begin{aligned} \frac{2.5}{3600}s &= 43,200 \\ s &= 62,208,000 \text{ sec} \end{aligned}$$

42. We add some labels to the figure in the text.



Before folding After folding

Let $x =$ the length of $\overline{AP}$. Then the length of a side of the square is $4x$. The smaller figure has sides of length $3x$ and $4x$.

Solve: $$\begin{aligned} 2 \cdot 3x + 2 \cdot 4x &= 25 \\ x &= \frac{25}{14} \end{aligned}$$

The square has sides of length $4 \cdot \dfrac{25}{14}$, or $\dfrac{50}{7}$ in. Its area is $\dfrac{50}{7} \text{ in.} \cdot \dfrac{50}{7} \text{ in.} = \dfrac{2500}{49}$ or $51\dfrac{1}{49}$ in^2.

Chapter 1 Mid-Chapter Review

1. The statement is true as shown by the following steps.

$$\begin{aligned} 2x + 3 &= 7 \\ 2x &= 4 \quad \text{Subtracting 3} \\ x &= 2 \quad \text{Dividing by 2} \end{aligned}$$

2. The statement is true. See Example 17 on page 9 in the text.

3. The statement is false. See Example 17 on page 9 in the text.

4. When we solve an applied problem, we check the possible solution in the *original problem.* The given statement is false.

5. $$\begin{array}{c|c} \multicolumn{2}{c}{x + 5 = 12} \\ \hline 7 + 5 \ ? & 12 \\ 12 & \text{TRUE} \end{array}$$

The number 7 is a solution of the equation.

6.
$$\begin{array}{r|l} 3x-4=5 & \\ \hline 3\cdot\frac{1}{3}-4 & ?\ 5 \\ 1-4 & \\ -3 & \qquad \text{FALSE} \end{array}$$
The number $\frac{1}{3}$ is not a solution of the equation.

7.
$$\begin{array}{r|l} \frac{-x}{8}=-3 & \\ \hline \frac{-(-24)}{8} & ?\ -3 \\ \frac{24}{8} & \\ 3 & \qquad \text{FALSE} \end{array}$$
The number -24 is not a solution of the equation.

8.
$$\begin{array}{r|l} 6(x-3)=36 & \\ \hline 6(9-3) & ?\ 36 \\ 6(6) & \\ 36 & \qquad \text{TRUE} \end{array}$$
The number 9 is a solution of the equation.

9.
$$\begin{aligned} x-7&=-10 \\ x-7+7&=-10+7 \\ x&=-3 \end{aligned}$$
The number -3 checks, so it is the solution.

10.
$$\begin{aligned} -7x&=56 \\ \frac{-7x}{-7}&=\frac{56}{-7} \\ x&=-8 \end{aligned}$$
The number -8 checks, so it is the solution.

11.
$$\begin{aligned} 8x-9&=23 \\ 8x&=32 && \text{Adding 9} \\ x&=4 && \text{Dividing by 8} \end{aligned}$$
The number 4 checks, so it is the solution.

12.
$$\begin{aligned} 1-x&=3x-7 \\ 1&=4x-7 && \text{Adding } x \\ 8&=4x && \text{Adding 7} \\ 2&=x && \text{Dividing by 4} \end{aligned}$$
The number 2 checks, so it is the solution.

13.
$$\begin{aligned} 2-4y&=-4y+2 \\ 2&=2 && \text{Adding } 4y \end{aligned}$$
We get an equation that is true for all real numbers, so all real numbers are solutions.

14.
$$\begin{aligned} \frac{3}{4}y+2&=\frac{7}{2} \\ \frac{3}{4}y&=\frac{3}{2} && \text{Subtracting 2} \\ \frac{4}{3}\cdot\frac{3}{4}y&=\frac{4}{3}\cdot\frac{3}{2} \\ y&=2 && \text{Simplifying} \end{aligned}$$
The number 2 checks, so it is the solution.

15.
$$\begin{aligned} 5t-9&=7t-4 \\ -9&=2t-4 && \text{Subtracting } 5t \\ -5&=2t && \text{Adding 4} \\ -\frac{5}{2}&=t && \text{Dividing by 2} \end{aligned}$$
The number $-\frac{5}{2}$ checks, so it is the solution.

16.
$$\begin{aligned} 4x-11&=11+4x \\ -11&=11 && \text{Subtracting } 4x \end{aligned}$$
We get a false equation. The equation has no solution.

17.
$$\begin{aligned} 2(y-4)&=8y \\ 2y-8&=8y \\ -8&=6y && \text{Subtracting } 2y \\ -\frac{4}{3}&=y && \text{Dividing by 6} \end{aligned}$$
The number $-\frac{4}{3}$ checks, so it is the solution.

18.
$$\begin{aligned} 4y-(y-1)&=16 \\ 4y-y+1&=16 \\ 3y+1&=16 && \text{Collecting like terms} \\ 3y&=15 && \text{Subtracting 1} \\ y&=5 && \text{Dividing by 3} \end{aligned}$$
The number 5 checks, so it is the solution.

19.
$$\begin{aligned} t-3(t-4)&=9 \\ t-3t+12&=9 \\ -2t+12&=9 && \text{Collecting like terms} \\ -2t&=-3 && \text{Subtracting 12} \\ t&=\frac{3}{2} && \text{Dividing by } -2 \end{aligned}$$
The number $\frac{3}{2}$ checks, so it is the solution.

20.
$$\begin{aligned} 6(2x+3)&=10-(4x-5) \\ 12x+18&=10-4x+5 \\ 12x+18&=15-4x && \text{Collecting like terms} \\ 16x+18&=15 && \text{Adding } 4x \\ 16x&=-3 && \text{Subtracting 18} \\ x&=-\frac{3}{16} && \text{Dividing by 16} \end{aligned}$$
The number $-\frac{3}{16}$ checks, so it is the solution.

21.
$$\begin{aligned} P&=mn \\ \frac{P}{m}&=n && \text{Dividing by } m \end{aligned}$$

22.
$$\begin{aligned} z&=3t+3w \\ z-3w&=3t && \text{Subtracting } 3w \\ \frac{z-3w}{3}&=t, \text{ or} && \text{Dividing by 3} \\ \frac{z}{3}-w&=t \end{aligned}$$

23.
$$N = \frac{r+s}{4}$$
$$4N = r + s \quad \text{Multiplying by 4}$$
$$4N - r = s \quad \text{Subtracting } r$$

24.
$$T = 1.5\frac{A}{B}$$
$$BT = 1.5A \quad \text{Multiplying by } B$$
$$B = \frac{1.5A}{T}, \text{ or } 1.5\frac{A}{T}$$

25.
$$H = \frac{2}{3}(t-5)$$
$$\frac{3}{2}H = t - 5 \quad \text{Multiplying by } \frac{3}{2}$$
$$\frac{3}{2}H + 5 = t, \text{ or} \quad \text{Adding 5}$$
$$\frac{3H+10}{2} = t$$

26.
$$f = g + ghm$$
$$f = g(1 + hm) \quad \text{Factoring}$$
$$\frac{f}{1+hm} = g \quad \text{Dividing by } 1 + hm$$

27. ***Familiarize***. Let f = number of female medical school graduates in 2002. Then an increase of 21.3% of this number is $f + 21.3\%f$, or $f + 0.213f$, or $1.213f$. This is the number of female medical school graduates in 2011.

Translate.

$$\underbrace{\text{The number of female medical school graduates in 2011}}_{1.213f} \quad \underbrace{\text{was}}_{=} \quad \underbrace{8396.}_{8396}$$

Solve. We solve the equation.
$$1.213f = 8396$$
$$f \approx 6922$$

Check. 21.3% of 6922 is $0.213(6922) \approx 1474$ and $6922 + 1474 = 8396$. The answer checks.

State. There were 6922 female medical school graduates in 2002.

28. ***Familiarize***. Let c = the number of calories a 154-lb person would burn walking 3.5 mph for 30 min. Then 50 calories less than twice this number is $2c - 50$.

Translate. We know that $2c - 50$ represents 230 calories, so we have
$$2c - 50 = 230.$$

Solve. We solve the equation.
$$2c - 50 = 230$$
$$2c = 280$$
$$c = 140$$

Check. $2 \cdot 140 - 50 = 280 - 50 = 230$, so the answer checks.

State. A 154-lb person would burn 140 calories walking 3.5 mph for 30 min.

29. ***Familiarize***. Let l = the length of the carpet, in feet. Then $l - 2$ = the width.

Translate. We substitute in the formula for the perimeter of a rectangle, $P = 2l + 2w$.
$$24 = 2l + 2(l - 2)$$

Solve.
$$24 = 2l + 2l - 4$$
$$24 = 4l - 4$$
$$28 = 4l$$
$$7 = l$$

If $l = 7$, then $l - 2 = 7 - 2 = 5$.

Check. The width, 5 ft, is 2 ft less than the length, 7 ft. The perimeter is $2 \cdot 7$ ft $+ 2 \cdot 5$ ft, or 14 ft $+ 10$ ft, or 24 ft. The answer checks.

State. The length of the carpet is 7 ft, and the width is 5 ft.

30. First we will find how long it will take Frederick to travel 18 mi downstream.

Familiarize. Let t = the time, in hours, it will take Frederick to travel 18 mi downstream. The speed of the boat traveling downstream is $9 + 3$, or 12 mph.

Translate. We will substitute in the formula $d = rt$.
$$18 = 12t$$

Solve.
$$18 = 12t$$
$$\frac{18}{12} = t$$
$$1.5 = t \quad \text{Simplifying}$$

Check. At a speed of 12 mph, in 1.5 hr the boat travels 12(1.5), or 18 mi. The answer checks.

State. It will take Frederick 1.5 hr to travel 18 mi downstream.

Now we will find how long it will take Frederick to travel 18 mi upstream.

Familiarize. Let t = the time, in hours, it will take Frederick to travel 18 mi upstream. The speed of the boat traveling upstream is $9 - 3$, or 6 mph.

Translate. We will substitute in the formula $d = rt$.
$$18 = 6t$$

Solve.
$$18 = 6t$$
$$3 = t$$

Check. At a speed of 6 mph, in 3 hr the boat travels $6 \cdot 3$, or 18 mi. The answer checks.

State. It will take Frederick 3 hr to travel 18 mi upstream.

31. Equivalent expressions have the same value for all possible replacements. Any replacement that does not make any of the expressions undefined can be substituted for the variable. Equivalent equations have the same solution(s). True equations result only when a solution is substituted for the variable.

32. Answers may vary. A walker who knows how far and how long she walks each day wants to know her average speed each day.

33. Answers may vary. A decorator wants to have a carpet cut for a bedroom. The perimeter of the room is 54 ft and its length is 15 ft. How wide should the carpet be?

34. We can subtract by adding an opposite, so we can use the addition principle to subtract the same number on both sides of an equation. Similarly, we can divide by multiplying by a reciprocal, so we can use the multiplication principle to divide both sides of an equation by the same number.

35. The manner in which a guess or estimate is manipulated can give insight into the form of the equation to which the problem will be translated.

36. Labeling the variable clearly makes the Translate step more accurate. It also allows us to determine if the solution of the equation we translated to provides the information asked for in the original problem.

Exercise Set 1.4

RC2. (h)

RC4. (a)

RC6. (d)

2. $3x + 5 \le -10$

$-5:$ $3(-5) + 5 \le -10$, or $-10 \le -10$ is true.
-5 is a solution.

$-10:$ $3(-10) + 5 \le -10$, or $-25 \le -10$ is true.
-10 is a solution.

$0:$ $3 \cdot 0 + 5 \le -10$, or $5 \le -10$ is false.
0 is not a solution.

$27:$ $3 \cdot 27 + 5 \le -10$, or $86 \le -10$ is false.
27 is not a solution.

4. $5y - 7 < 8 - y$

$2:$ $5 \cdot 2 - 7 < 8 - 2$, or $3 < 6$ is true.
2 is a solution.

$-3:$ $5(-3) - 7 < 8 - (-3)$, or $-22 < 11$ is true.
-3 is a solution.

$0:$ $5 \cdot 0 - 7 < 8 - 0$, or $-7 < 8$ is true.
0 is a solution.

$3:$ $5 \cdot 3 - 7 < 8 - 3$, or $8 < 5$ is false.
3 is not a solution.

$\frac{2}{3}:$ $5 \cdot \frac{2}{3} - 7 < 8 - \frac{2}{3}$, or $-\frac{11}{3} < \frac{22}{3}$ is true.
$\frac{2}{3}$ is a solution.

6. $[-5, \infty)$

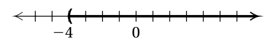
8. $(-10, 10]$

10. $\{x|13 > x \ge 5\} = \{x|5 \le x < 13\} = [5, 13)$

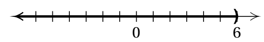
12. $[-20, 30)$

14. $(-\infty, 8]$

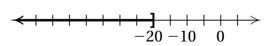
16. $x + 8 > 4$
$x > -4$
$\{x|x > -4\}$, or $(-4, \infty)$



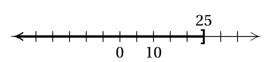
18. $y + 4 < 10$
$y < 6$
$\{y|y < 6\}$, or $(-\infty, 6)$



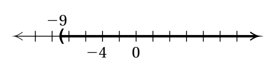
20. $a + 6 \le -14$
$a \le -20$
$\{a|a \le -20\}$, or $(-\infty, -20]$



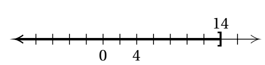
22. $x - 8 \le 17$
$x \le 25$
$\{x|x \le 25\}$, or $(-\infty, 25]$



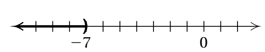
24. $y - 9 > -18$
$y > -9$
$\{y|y > -9\}$, or $(-9, \infty)$



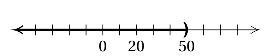
26. $y - 18 \le -4$
$y \le 14$
$\{y|y \le 14\}$, or $(-\infty, 14]$



28. $8t < -56$
$t < -7$
$\{t|t < -7\}$, or $(-\infty, -7)$



30. $0.6x < 30$
$x < 50$
$\{x|x < 50\}$, or $(-\infty, 50)$



32. $\frac{3}{5}x > -3$

$x > -5$

$\{x|x > -5\}$, or $(-5, \infty)$

34. $-5y \le 3.5$

$y \ge -0.7$

$\{y|y \ge -0.7\}$, or $[-0.7, \infty)$

36. $-\frac{1}{8}y \le -\frac{9}{8}$

$y \ge 9$

$\{y|y \ge 9\}$, or $[9, \infty)$

38. $5y + 13 > 28$

$5y > 15$

$y > 3$

$\{y|y > 3\}$, or $(3, \infty)$

40. $-9x + 3x \ge -24$

$-6x \ge -24$

$x \le 4$

$\{x|x \le 4\}$, or $(-\infty, 4]$

42. $8x - 9 < 3x - 11$

$5x < -2$

$x < -\frac{2}{5}$

$\left\{x\middle|x < -\frac{2}{5}\right\}$, or $\left(-\infty, -\frac{2}{5}\right)$

44. $0.2y + 1 > 2.4y - 10$

$-2.2y > -11$

$y < 5$

$\{y|y < 5\}$, or $(-\infty, 5)$

46. $2x - 3 < \frac{13}{4}x + 10 - 1.25x$

$8x - 12 < 13x + 40 - 5x$

$8x - 12 < 8x + 40$

$-12 < 40$ True for all real numbers

The solution set is all real numbers, or $(-\infty, \infty)$.

48. $2m + 5 \ge 16(m - 4)$

$2m + 5 \ge 16m - 64$

$69 \ge 14m$

$\frac{69}{14} \ge m$

$\left\{m\middle|m \le \frac{69}{14}\right\}$, or $\left(-\infty, \frac{69}{14}\right]$

50. $2(0.5 - 3y) + y > (4y - 0.2)8$

$1 - 6y + y > 32y - 1.6$

$1 - 5y > 32y - 1.6$

$-37y > -2.6$

$y < \frac{2.6}{37}$

$y < \frac{13}{185}$

$\left\{y\middle|y < \frac{13}{185}\right\}$, or $\left(-\infty, \frac{13}{185}\right)$

52. $[8x - 3(3x + 2)] - 5 \ge 3(x + 4) - 2x$

$[8x - 9x - 6] - 5 \ge 3x + 12 - 2x$

$-x - 11 \ge x + 12$

$-2x \ge 23$

$x \le -\frac{23}{2}$

$\left\{x\middle|x \le -\frac{23}{2}\right\}$, or $\left(-\infty, -\frac{23}{2}\right]$

54. $5(t + 3) + 9 < 3(t - 2) + 6$

$5t + 15 + 9 < 3t - 6 + 6$

$5t + 24 < 3t$

$2t < -24$

$t < -12$

$\{t|t < -12\}$, or $(-\infty, -12)$

56. $13 - (2c + 2) \ge 2(c + 2) + 3c$

$13 - 2c - 2 \ge 2c + 4 + 3c$

$11 - 2c \ge 5c + 4$

$-7c \ge -7$

$c \le 1$

$\{c|c \le 1\}$, or $(-\infty, 1]$

58. $\frac{1}{3}(6x + 24) - 20 > -\frac{1}{4}(12x - 72)$

$2x + 8 - 20 > -3x + 18$

$5x > 30$

$x > 6$

$\{x|x > 6\}$, or $(6, \infty)$

60. $5[3(7 - t) - 4(8 + 2t)] - 20 \le -6[2(6 + 3t) - 4]$

$5[21 - 3t - 32 - 8t] - 20 \le -6[12 + 6t - 4]$

$5[-11 - 11t] - 20 \le -6[8 + 6t]$

$-55 - 55t - 20 \le -48 - 36t$

$-19t \le 27$

$t \ge -\frac{27}{19}$

$\left\{t\middle|t \ge -\frac{27}{19}\right\}$, or $\left[-\frac{27}{19}, \infty\right)$

62. $\frac{2}{3}(4x-3) > 30$

$$4x-3 > 45 \quad \text{Multiplying by } \frac{3}{2}$$
$$4x > 48$$
$$x > 12$$

$\{x|x > 12\}$, or $(12, \infty)$

64. $\frac{7}{8}(5-4x) - 17 \ge 38$

$$7(5-4x) - 136 \ge 304$$
$$35 - 28x - 136 \ge 304$$
$$-28x \ge 405$$
$$x \le -\frac{405}{28}$$

$\left\{x\middle|x \le -\frac{405}{28}\right\}$, or $\left(-\infty, -\frac{405}{28}\right]$

66. $\frac{2}{3}\left(\frac{7}{8} - 4x\right) - \frac{5}{8} < \frac{3}{8}$

$$\frac{7}{12} - \frac{8x}{3} - \frac{5}{8} < \frac{3}{8}$$
$$14 - 64x - 15 < 9$$
$$-64x < 10$$
$$x > -\frac{10}{64}, \text{ or } -\frac{5}{32}$$

$\left\{x\middle|x > -\frac{5}{32}\right\}$, or $\left(-\frac{5}{32}, \infty\right)$

68. $0.9(2x+8) < 20 - (x+5)$

$$9(2x+8) < 200 - 10(x+5)$$
$$18x + 72 < 200 - 10x - 50$$
$$28x < 78$$
$$x < \frac{78}{28}, \text{ or } \frac{39}{14}$$

$\left\{x\middle|x < \frac{39}{14}\right\}$, or $\left(-\infty, \frac{39}{14}\right)$

70. $0.8 - 4(b-1) > 0.2 + 3(4-b)$

$$8 - 40(b-1) > 2 + 30(4-b)$$
$$8 - 40b + 40 > 2 + 120 - 30b$$
$$48 - 40b > 122 - 30b$$
$$-10b > 74$$
$$b < -\frac{74}{10}, \text{ or } -7.4$$

$\{b|b < -7.4\}$, or $(-\infty, -7.4)$

72. $\frac{703W}{77^2} < 25$

$$W < 210.8 \qquad \text{Rounding}$$

Weights of less than approximately 210.8 lb will keep Josiah's body mass index below 25. In terms of an inequality we write $\{W|W <$ (approximately) 210.8 lb$\}$.

74. Let x = the score on the fourth test. It is possible to score 100 on the fifth test, so we have the following:

$$94 + 90 + 89 + x + 100 \ge 450$$
$$x + 373 \ge 450$$
$$x \ge 77$$

Elizabeth must score 77 or better. In terms of an inequality we write $\{x|x \ge 77\}$.

76. Let m = the number of miles for which PDQ is less expensive. Solve:

$$25 + 0.75(m-10) < 15 + 1.25(m-10)$$
$$25 + 0.75m - 7.5 < 15 + 1.25m - 12.5$$
$$15 < 0.5m$$
$$30 < m$$

For deliveries of more than 30 mi, PDQ is less expensive. In terms of an inequality, we write $\{m|m > 30 \text{ mi}\}$.

78. $12.50n > 300 + 9n$

$$3.5n > 300$$
$$n > 85\frac{5}{7}$$

Plan B is better for values of n greater than $85\frac{5}{7}$ hr. In terms of an inequality we write $\left\{n\middle|n > 85\frac{5}{7} \text{ hr}\right\}$.

80. Let b = the amount of Giselle's medical bills.

$$250 + 0.1(b-250) < 50 + 0.2(b-50)$$
$$250 + 0.1b - 25 < 50 + 0.2b - 10$$
$$185 < 0.1b$$
$$1850 < b$$

Plan B will save Giselle money for medical bills greater than \$1850. In terms of an inequality we write $\{b|b > \$1850\}$.

82. Let x = the amount invested at 3%.

$$0.03x + 0.04(20{,}000 - x) \ge 650$$
$$0.03x + 800 - 0.04x \ge 650$$
$$-0.01x \ge -150$$
$$x \le 15{,}000$$

Matthew can invest at most \$15,000 at 3% and still be guaranteed at least \$650 in interest per year.

84. a) $\frac{5}{9}(F - 32) < 1063$

$$F - 32 < 1913.4 \quad \text{Multiplying by } \frac{9}{5}$$
$$F < 1945.4$$

Gold is solid at temperatures less than 1945.4°F.

In terms of an inequality we write

$\{F|F < 1945.4\}$.

b) $\frac{5}{9}(F-32) < 960.8$

$F - 32 < 1729.44$

$F < 1761.44$

Silver is solid at temperatures less than 1761.44°F. In terms of an inequality we write $\{F|F < 1761.44\}$.

86. Let d = the dewpoint spread. Then $\frac{d}{3}$ = the number of 3° blocks of dewpoint spread. Note that the number of thousands in 3500 is $\frac{3500}{1000}$, or 3.5.

$\frac{d}{3} > 3.5$

$d > 10.5$

Dewpoint spreads greater than 10.5° will allow the plane to fly.

88. $2(x-y)+10(3x-7y)$

$= 2x-2y+30x-70y$

$= 32x-72y$

90. $-3(2a-3b)+8b$

$= -6a+9b+8b$

$= -6a+17b$

92. $-12a+30ab = -6a(2-5b)$

94. $10n-45mn+100m = 5(2n-9mn+20m)$

96. $-2.3+8.9 = 6.6$

98. $-2.3-(-8.9) = -2.3+8.9 = 6.6$

100. False; $-3 < -2$, but $(-3)^2 > (-2)^2$.

102. No. Let $x = 2$. Then $x < 3$ is true, but $0 \cdot x < 0 \cdot 3$, or $0 < 0$, is false.

104. $x+8 < 3+x$

$8 < 3$ Subtracting x

We get a false inequality. Thus, the original inequality has no solution.

Exercise Set 1.5

RC2. False

RC4. True

2. $\{1,5,10,15\} \cap \{5,15,20\} = \{5,15\}$

4. {m, n, o, p} $\cap$ {m, o, p} = {m, o, p}

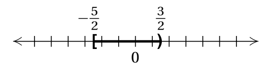
6. $\{1,5,10,15\} \cup \{5,15,20\} = \{1,5,10,15,20\}$

8. {m, n, o, p} $\cup$ {m, o, p} = {m, n, o, p}

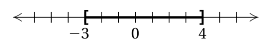
10. {a, e, i, o, u} $\cap$ {m, q, w, s, t} = $\emptyset$

12. $\{3,5,7\} \cap \emptyset = \emptyset$

14. Interval notation for $-\frac{5}{2} \le m$ *and* $m < \frac{3}{2}$ is $\left[-\frac{5}{2}, \frac{3}{2}\right)$.



16. Interval notation for $-3 \le y \le 4$ is $[-3,4]$.

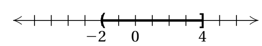


18. $-11 < 4x-3$ *and* $4x-3 \le 13$

$-8 < 4x$ *and* $4x \le 16$

$-2 < x$ *and* $x \le 4$

$\{x|-2 < x \le 4\}$, or $(-2,4]$

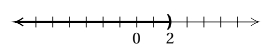


20. $4x-7 < 1$ *and* $7-3x > -8$

$4x < 8$ *and* $-3x > -15$

$x < 2$ *and* $x < 5$

$\{x|x < 2\}$, or $(-\infty,2)$



22. $5-7x > 19$ *and* $2-3x < -4$

$-7x > 14$ *and* $-3x < -6$

$x < -2$ *and* $x > 2$

$\emptyset$

24. $-6 < x+6 \le 8$

$-12 < x \le 2$

$\{x|-12 < x \le 2\}$, or $(-12,2]$

26. $3 > -x \ge -5$

$-3 < x \le 5$ Multiplying by -1

$\{x|-3 < x \le 5\}$, or $(-3,5]$

28. $-6 \le x+1 < 9$

$-7 \le x < 8$

$\{x|-7 \le x < 8\}$, or $[-7,8)$

30. $5 \le 8x+5 \le 21$

$0 \le 8x \le 16$

$0 \le x \le 2$

$\{x|0 \le x \le 2\}$, or $[0,2]$

32. $-6 \le 2x - 3 < 6$

$-6+3 \le 2x-3+3 < 6+3$

$-3 \le 2x < 9$

$\dfrac{-3}{2} \le \dfrac{2x}{2} < \dfrac{9}{2}$

$-\dfrac{3}{2} \le x < \dfrac{9}{2}$

The solution set is $\left\{x\middle| -\dfrac{3}{2} \le x < \dfrac{9}{2}\right\}$, or $\left[-\dfrac{3}{2}, \dfrac{9}{2}\right)$.

34. $4 > -3m - 7 \ge 2$

$11 > -3m \ge 9$

$-\dfrac{11}{3} < m \le -3$

$\left\{m\middle| -\dfrac{11}{3} < m \le -3\right\}$, or $\left(-\dfrac{11}{3}, -3\right]$

36. $-\dfrac{2}{3} \le 4 - \dfrac{1}{4}x < \dfrac{2}{3}$

$-\dfrac{14}{3} \le -\dfrac{1}{4}x < -\dfrac{10}{3}$

$\dfrac{56}{3} \ge x > \dfrac{40}{3}$

$\left\{x\middle|\dfrac{40}{3} < x \le \dfrac{56}{3}\right\}$, or $\left(\dfrac{40}{3}, \dfrac{56}{3}\right]$

38. $-3 < \dfrac{2x-5}{4} < 8$

$4(-3) < 4\left(\dfrac{2x-5}{4}\right) < 4\cdot 8$

$-12 < 2x - 5 < 32$

$-12+5 < 2x-5+5 < 32+5$

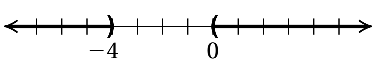
$-7 < 2x < 37$

$\dfrac{-7}{2} < \dfrac{2x}{2} < \dfrac{37}{2}$

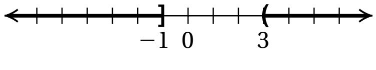
$-\dfrac{7}{2} < x < \dfrac{37}{2}$

$\left\{x\middle| -\dfrac{7}{2} < x < \dfrac{37}{2}\right\}$, or $\left(-\dfrac{7}{2}, \dfrac{37}{2}\right)$.

40. $x < -4$ *or* $x > 0$ can be written in interval notation as $(-\infty, -4) \cup (0, \infty)$.



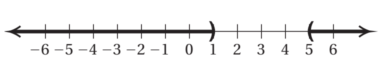
42. $x \le -1$ *or* $x > 3$ can be written in interval notation as $(-\infty, -1] \cup (3, \infty)$.



44. $x - 2 < -1$ *or* $x - 2 > 3$

$x < 1$ *or* $x > 5$

$\{x|x < 1 \text{ or } x > 5\}$, or $(-\infty, 1) \cup (5, \infty)$

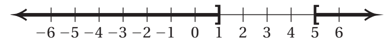


46. $x - 5 \le -4$ *or* $2x - 7 \ge 3$

$x \le 1$ *or* $2x \ge 10$

$x \le 1$ *or* $x \ge 5$

$\{x|x \le 1 \text{ or } x \ge 5\}$, or $(-\infty, 1] \cup [5, \infty)$

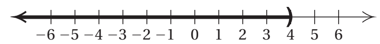


48. $4x - 4 < -8$ *or* $4x - 4 < 12$

$4x < -4$ *or* $4x < 16$

$x < -1$ *or* $x < 4$

$\{x|x < 4\}$, or $(-\infty, 4)$



50. $6 > 2x - 1$ *or* $-4 \le 2x - 1$

$6 + 1 > 2x - 1 + 1$ *or* $-4 + 1 \le 2x - 1 + 1$

$7 > 2x$ *or* $-3 \le 2x$

$\dfrac{1}{2}\cdot 7 > \dfrac{1}{2}\cdot 2x$ *or* $\dfrac{1}{2}(-3) \le \dfrac{1}{2}\cdot 2x$

$\dfrac{7}{2} > x$ *or* $-\dfrac{3}{2} \le x$

All real numbers, or $(-\infty, \infty)$

52. $3x + 2 < 2$ *or* $4 - 2x < 14$

$3x < 0$ *or* $-2x < 10$

$x < 0$ *or* $x > -5$

All real numbers, or $(-\infty, \infty)$

54. $-3m - 7 < -5$ *or* $-3m - 7 > 5$

$-3m < 2$ *or* $-3m > 12$

$m > -\dfrac{2}{3}$ *or* $m < -4$

$\left\{m\middle|m < -4 \text{ or } m > -\dfrac{2}{3}\right\}$, or

$(-\infty, -4) \cup \left(-\dfrac{2}{3}, \infty\right)$

56. $\dfrac{1}{4} - 3x \le -3.7$ *or* $\dfrac{1}{4} - 5x \ge 4.8$

$40\left(\dfrac{1}{4} - 3x\right) \le 40(-3.7)$ *or* $40\left(\dfrac{1}{4} - 5x\right) \ge 40(4.8)$

$10 - 120x \le -148$ *or* $10 - 200x \ge 192$

$-120x \le -158$ *or* $-200x \ge 182$

$x \ge \dfrac{79}{60}$ *or* $x \le -\dfrac{91}{100}$

$\left\{x\middle|x \le -\dfrac{91}{100} \text{ or } x \ge \dfrac{79}{60}\right\}$, or

$\left(-\infty, -\dfrac{91}{100}\right] \cup \left[\dfrac{79}{60}, \infty\right)$

58. $\dfrac{7-3x}{5} < -4 \quad or \quad \dfrac{7-3x}{5} > 4$

$7-3x < -20 \quad or \quad 7-3x > 20$

$-3x < -27 \quad or \quad -3x > 13$

$x > 9 \quad or \quad x < -\dfrac{13}{3}$

$\left\{x \middle| x < -\dfrac{13}{3} \text{ or } x > 9\right\}$, or $\left(-\infty, -\dfrac{13}{3}\right) \cup (9, \infty)$

60. a) Solve: $1063° \le \dfrac{5}{9}(F-32) < 2660°$

$1945.4° \le F < 4820°$

b) Solve: $960.8° \le \dfrac{5}{9}(F-32) < 2180°$

$1761.44° \le F < 3956°$

62. Let c = the number of crossings per year. Then at the rate of \$6 per crossing, the total cost of c crossings is \$6c. Two six-month passes cost $2 \cdot \$50$, or \$100. The additional toll of \$2 per crossing brings the total cost of c crossings to \$100 + \$2c. A one-year pass costs \$400 regardless of the number of crossings.

Solve: $100 + 2c < 6c$ *and* $100 + 2c < 400$

We get $c > 25$ *and* $c < 150$, or $25 < c < 150$, so for more than 25 crossings but fewer than 150 crossings per year the six-month passes are the most economical choice. The solution set is $\{c|25 < c < 150\}$.

64. Solve: $18.5 < \dfrac{703W}{77^2} < 24.9$

$156.0 < W < 210.0$

The solution set is $\{W|156.0 \text{ lb} < W < 210.0 \text{ lb}\}$.

66. Solve: $50 < \dfrac{5d}{5+12} < 100$

$170 < d < 340$

The solution set is $\{d|170 \text{ mg} < d < 340 \text{ mg}\}$.

68. $-\dfrac{1}{2}t + 5 = -\dfrac{7}{2}t$

$5 = -3t$

$-\dfrac{5}{3} = t$

70. $3x - (x-1) = 19$

$2x + 1 = 19$

$2x = 18$

$x = 9$

72. $6(x-5) = 2(x+3)$

$6x - 30 = 2x + 6$

$4x = 36$

$x = 9$

74. $4m - 8 > 6m \quad or \quad 5m - 8 < -2$

$-8 > 2m \quad or \quad 5m < 6$

$-4 > m \quad or \quad m < \dfrac{6}{5}$

$\left\{m \middle| m < \dfrac{6}{5}\right\}$, or $\left(-\infty, \dfrac{6}{5}\right)$

76. $2[5(3-y) - 2(y-2)] > y + 4$

$2[15 - 5y - 2y + 4] > y + 4$

$2[19 - 7y] > y + 4$

$38 - 14y > y + 4$

$-15y > -34$

$y < \dfrac{34}{15}$

$\left\{y \middle| y < \dfrac{34}{15}\right\}$, or $\left(-\infty, \dfrac{34}{15}\right)$

78. $2x - \dfrac{3}{4} < -\dfrac{1}{10} \quad or \quad 2x - \dfrac{3}{4} > \dfrac{1}{10}$

$2x < \dfrac{13}{20} \quad or \quad 2x > \dfrac{17}{20}$

$x < \dfrac{13}{40} \quad or \quad x > \dfrac{17}{40}$

$\left\{x \middle| x < \dfrac{13}{40} \text{ or } x > \dfrac{17}{40}\right\}$, or $\left(-\infty, \dfrac{13}{40}\right) \cup \left(\dfrac{17}{40}, \infty\right)$

80. $2x + 3 \le x - 6 \quad or \quad 3x - 2 \le 4x + 5$

$x \le -9 \quad or \quad -7 \le x$

$\{x|x \le -9 \text{ or } x \ge -7\}$, or $(-\infty, -9] \cup [-7, \infty)$

82. We can write $a \le c$ and $c \le b$ as $a \le c \le b$. Then $a \le b$, or $b \ge a$. The statement is true.

84. If $-a < c$, then $-1(-a) > -1 \cdot c$, or $a > -c$. Then if $a > -c$ and $-c > b$, we have $a > -c > b$, so $a > b$ and the given statement is true.

Exercise Set 1.6

RC2. $|x| \ge 3$

$x \le -3 \quad or \quad x \ge 3$

The answer is (b).

RC4. $|x| = 3$

$x = -3 \quad or \quad x = 3$

The answer is (c).

RC6. $|x| > -3$

Since $|x|$ is always nonnegative, the solution is all real numbers, or $(-\infty, \infty)$. The answer is (d).

2. $|26x| = |26| \cdot |x| = 26|x|$

4. $|8x^2| = |8| \cdot |x^2| = 8x^2$

6. $|-20x^2| = |-20| \cdot |x^2| = 20x^2$

8. $|-17y| = |-17| \cdot |y| = 17|y|$

10. $\left|\dfrac{y}{3}\right| = \dfrac{|y|}{|3|} = \dfrac{|y|}{3}$

12. $\left|\dfrac{x^4}{-y}\right| = \dfrac{|x^4|}{|-y|} = \dfrac{x^4}{|y|}$

14. $\left|\dfrac{-9y^2}{3y}\right| = |-3y| = |-3|\cdot|y| = 3|y|$

16. $\left|\dfrac{5x^3}{-25x}\right| = \left|\dfrac{x^2}{-5}\right| = \dfrac{|x^2|}{|-5|} = \dfrac{x^2}{5}$

18. $|-7-(-32)| = |25| = 25$

20. $|52-18| = |34| = 34$

22. $|-1.8-(-3.7)| = |1.9| = 1.9$

24. $\left|\dfrac{2}{3}-\left(-\dfrac{5}{6}\right)\right| = \left|\dfrac{4}{6}+\dfrac{5}{6}\right| = \left|\dfrac{9}{6}\right| = \dfrac{3}{2}$

26. $|x| = 5$

$x = -5$ *or* $x = 5$

$\{-5, 5\}$

28. $|x| = -9$

The absolute value of a number is always nonnegative. The solution set is $\emptyset$.

30. $|y| = 7.4$

$y = -7.4$ *or* $y = 7.4$

$\{-7.4, 7.4\}$

32. $|3x-2| = 6$

$$\begin{aligned} 3x-2 &= -6 \quad or \quad 3x-2 = 6 \\ 3x &= -4 \quad or \quad 3x = 8 \\ x &= -\frac{4}{3} \quad or \quad x = \frac{8}{3} \end{aligned}$$

$\left\{-\dfrac{4}{3}, \dfrac{8}{3}\right\}$

34. $|5x+2| = 3$

$$\begin{aligned} 5x+2 &= -3 \quad or \quad 5x+2 = 3 \\ 5x &= -5 \quad or \quad 5x = 1 \\ x &= -1 \quad or \quad x = \frac{1}{5} \end{aligned}$$

$\left\{-1, \dfrac{1}{5}\right\}$

36. $|9y-2| = 17$

$$\begin{aligned} 9y-2 &= -17 \quad or \quad 9y-2 = 17 \\ 9y &= -15 \quad or \quad 9y = 19 \\ y &= -\frac{5}{3} \quad or \quad y = \frac{19}{9} \end{aligned}$$

$\left\{-\dfrac{5}{3}, \dfrac{19}{9}\right\}$

38. $|x| - 2 = 6.3$

$|x| = 8.3$

$x = -8.3$ *or* $x = 8.3$

$\{-8.3, 8.3\}$

40. $-562 = -2000 + |x|$

$1438 = |x|$

$x = -1438$ *or* $x = 1438$

$\{-1438, 1438\}$

42. $|2y| = 18$

$$\begin{aligned} 2y &= -18 \quad or \quad 2y = 18 \\ y &= -9 \quad or \quad y = 9 \end{aligned}$$

$\{-9, 9\}$

44. $|6x| + 8 = 32$

$|6x| = 24$

$$\begin{aligned} 6x &= -24 \quad or \quad 6x = 24 \\ x &= -4 \quad or \quad x = 4 \end{aligned}$$

$\{-4, 4\}$

46. $5|x| + 10 = 26$

$5|x| = 16$

$|x| = \dfrac{16}{5}$

$x = -\dfrac{16}{5}$ *or* $x = \dfrac{16}{5}$

$\left\{-\dfrac{16}{5}, \dfrac{16}{5}\right\}$

48. $\left|\dfrac{4-5x}{6}\right| = 7$

$$\begin{aligned} \frac{4-5x}{6} &= -7 \quad or \quad \frac{4-5x}{6} = 7 \\ 4-5x &= -42 \quad or \quad 4-5x = 42 \\ -5x &= -46 \quad or \quad -5x = 38 \\ x &= \frac{46}{5} \quad or \quad x = -\frac{38}{5} \end{aligned}$$

$\left\{-\dfrac{38}{5}, \dfrac{46}{5}\right\}$

50. $|t-7| - 5 = 4$

$|t-7| = 9$

$$\begin{aligned} t-7 &= -9 \quad or \quad t-7 = 9 \\ t &= -2 \quad or \quad t = 16 \end{aligned}$$

$\{-2, 16\}$

52. $2|2x-7| + 11 = 25$

$2|2x-7| = 14$

$|2x-7| = 7$

$$\begin{aligned} 2x-7 &= -7 \quad or \quad 2x-7 = 7 \\ 2x &= 0 \quad or \quad 2x = 14 \\ x &= 0 \quad or \quad x = 7 \end{aligned}$$

$\{0, 7\}$

54. $|x-6| = -8$

The absolute value of a number is always nonnegative. The solution set is $\emptyset$.

56. $\left|\frac{2}{3}-4x\right|=\frac{4}{5}$

$$\begin{aligned} \frac{2}{3}-4x &= -\frac{4}{5} \quad \text{or} \quad \frac{2}{3}-4x = \frac{4}{5} \\ -4x &= -\frac{22}{15} \quad \text{or} \quad -4x = \frac{2}{15} \\ x &= \frac{11}{30} \quad \text{or} \quad x = -\frac{1}{30} \end{aligned}$$

$\left\{-\frac{1}{30}, \frac{11}{30}\right\}$

58. $|2x-8|=|x+3|$

$$\begin{aligned} 2x-8 = x+3 &\quad \text{or} \quad 2x-8 = -(x+3) \\ x = 11 &\quad \text{or} \quad 2x-8 = -x-3 \\ x = 11 &\quad \text{or} \quad 3x = 5 \\ x = 11 &\quad \text{or} \quad x = \frac{5}{3} \end{aligned}$$

$\left\{11, \frac{5}{3}\right\}$

60. $|x-15|=|x+8|$

$$\begin{aligned} x-15 = x+8 &\quad \text{or} \quad x-15 = -(x+8) \\ -15 = 8 &\quad \text{or} \quad x-15 = -x-8 \\ -15 = 8 &\quad \text{or} \quad 2x = 7 \\ -15 = 8 &\quad \text{or} \quad x = \frac{7}{2} \end{aligned}$$

The first equation has no solution. The solution set is $\left\{\frac{7}{2}\right\}$.

62. $|5p+7|=|4p+3|$

$$\begin{aligned} 5p+7 = 4p+3 &\quad \text{or} \quad 5p+7 = -(4p+3) \\ p = -4 &\quad \text{or} \quad 5p+7 = -4p-3 \\ p = -4 &\quad \text{or} \quad 9p = -10 \\ p = -4 &\quad \text{or} \quad p = -\frac{10}{9} \end{aligned}$$

$\left\{-4, -\frac{10}{9}\right\}$

64. $|m-7|=|7-m|$

$$\begin{aligned} m-7 = 7-m &\quad \text{or} \quad m-7 = -(7-m) \\ 2m = 14 &\quad \text{or} \quad m-7 = -7+m \\ m = 7 &\quad \text{or} \quad 0 = 0 \end{aligned}$$

All real numbers are solutions.

66. $|8-q|=|q+19|$

$$\begin{aligned} 8-q = q+19 &\quad \text{or} \quad 8-q = -(q+19) \\ -2q = 11 &\quad \text{or} \quad 8-q = -q-19 \\ q = -\frac{11}{2} &\quad \text{or} \quad 8 = -19 \end{aligned}$$

The second equation has no solution. The solution set is $\left\{-\frac{11}{2}\right\}$.

68. $\left|\frac{6-8x}{5}\right|=\left|\frac{7+3x}{2}\right|$

$$\begin{aligned} \frac{6-8x}{5} = \frac{7+3x}{2} &\quad \text{or} \quad \frac{6-8x}{5} = -\left(\frac{7+3x}{2}\right) \\ 12-16x = 35+15x &\quad \text{or} \quad 12-16x = -35-15x \\ -31x = 23 &\quad \text{or} \quad -x = -47 \\ x = -\frac{23}{31} &\quad \text{or} \quad x = 47 \end{aligned}$$

$\left\{-\frac{23}{31}, 47\right\}$

70. $\left|2-\frac{2}{3}x\right|=\left|4+\frac{7}{8}x\right|$

$$\begin{aligned} 2-\frac{2}{3}x = 4+\frac{7}{8}x &\quad \text{or} \quad 2-\frac{2}{3}x = -\left(4+\frac{7}{8}x\right) \\ -\frac{37}{24}x = 2 &\quad \text{or} \quad 2-\frac{2}{3}x = -4-\frac{7}{8}x \\ x = -\frac{48}{37} &\quad \text{or} \quad \frac{5}{24}x = -6 \\ x = -\frac{48}{37} &\quad \text{or} \quad x = -\frac{144}{5} \end{aligned}$$

$\left\{-\frac{48}{37}, -\frac{144}{5}\right\}$

72. $|x| \le 5$

$-5 \le x \le 5$

$\{x| -5 \le x \le 5\}$, or $[-5, 5]$

74. $|y| > 12$

$y < -12$ *or* $y > 12$

$\{y|y < -12 \text{ or } y > 12\}$, or $(-\infty, -12) \cup (12, \infty)$

76. $|x+4| \le 9$

$-9 \le x+4 \le 9$

$-13 \le x \le 5$

$\{x| -13 \le x \le 5\}$, or $[-13, 5]$

78. $2|x-2| > 6$

$|x-2| > 3$

$x-2 < -3$ *or* $x-2 > 3$

$x < -1$ *or* $x > 5$

$\{x|x < -1 \text{ or } x > 5\}$, or $(-\infty, -1) \cup (5, \infty)$

80. $|5x+2| \le 3$

$-3 \le 5x+2 \le 3$

$-5 \le 5x \le 1$

$-1 \le x \le \frac{1}{5}$

$\left\{x\middle| -1 \le x \le \frac{1}{5}\right\}$, or $\left[-1, \frac{1}{5}\right]$

82. $|3y-4| > 8$

$$\begin{aligned} 3y-4 < -8 &\quad \text{or} \quad 3y-4 > 8 \\ 3y < -4 &\quad \text{or} \quad 3y > 12 \\ y < -\frac{4}{3} &\quad \text{or} \quad y > 4 \end{aligned}$$

$\left\{y\middle|y < -\frac{4}{3} \text{ or } y > 4\right\}$, or $\left(-\infty, -\frac{4}{3}\right) \cup (4, \infty)$

84. $|9y-2| \geq 17$

$9y-2 \leq -17 \quad or \quad 9y-2 \geq 17$

$9y \leq -15 \quad or \quad 9y \geq 19$

$y \leq -\frac{5}{3} \quad or \quad y \geq \frac{19}{9}$

$\left\{y\middle|y \leq -\frac{5}{3} \ \ or \ \ y \geq \frac{19}{9}\right\}$, or

$\left(-\infty, -\frac{5}{3}\right] \cup \left[\frac{19}{9}, \infty\right)$

86. $|p-2| < 6$

$-6 < p-2 < 6$

$-4 < p < 8$

$\{p|-4 < p < 8\}$, or $(-4, 8)$

88. $|5x+2| \leq 13$

$-13 \leq 5x+2 \leq 13$

$-15 \leq 5x \leq 11$

$-3 \leq x \leq \frac{11}{5}$

$\left\{x\middle|-3 \leq x \leq \frac{11}{5}\right\}$, or $\left[-3, \frac{11}{5}\right]$

90. $|7-2y| > 5$

$7-2y < -5 \quad or \quad 7-2y > 5$

$-2y < -12 \quad or \quad -2y > -2$

$y > 6 \quad or \quad y < 1$

$\{y|y < 1 \ \ or \ \ y > 6\}$, or $(-\infty, 1) \cup (6, \infty)$

92. $|2-9p| \geq 17$

$2-9p \leq -17 \quad or \quad 2-9p \geq 17$

$-9p \leq -19 \quad or \quad -9p \geq 15$

$p \geq \frac{19}{9} \quad or \quad p \leq -\frac{5}{3}$

$\left\{p\middle|p \leq -\frac{5}{3} \ \ or \ \ p \geq \frac{19}{9}\right\}$, or

$\left(-\infty, -\frac{5}{3}\right] \cup \left[\frac{19}{9}, \infty\right)$

94. $|-5-7x| \leq 30$

$-30 \leq -5-7x \leq 30$

$-25 \leq -7x \leq 35$

$\frac{25}{7} \geq x \geq -5$

$\left\{x\middle|-5 \leq x \leq \frac{25}{7}\right\}$, or $\left[-5, \frac{25}{7}\right]$

96. $\left|\frac{1}{4}y-6\right| > 24$

$\frac{1}{4}y-6 < -24 \quad or \quad \frac{1}{4}y-6 > 24$

$\frac{1}{4}y < -18 \quad or \quad \frac{1}{4}y > 30$

$y < -72 \quad or \quad y > 120$

$\{y|y < -72 \ \ or \ \ y > 120\}$, or $(-\infty, -72) \cup (120, \infty)$

98. $\left|\frac{x+5}{4}\right| \leq 2$

$-2 \leq \frac{x+5}{4} \leq 2$

$-8 \leq x+5 \leq 8$

$-13 \leq x \leq 3$

$\{x|-13 \leq x \leq 3\}$, or $[-13, 3]$

100. $\left|\frac{1+3x}{5}\right| > \frac{7}{8}$

$\frac{1+3x}{5} < -\frac{7}{8} \quad or \quad \frac{1+3x}{5} > \frac{7}{8}$

$1+3x < -\frac{35}{8} \quad or \quad 1+3x > \frac{35}{8}$

$3x < -\frac{43}{8} \quad or \quad 3x > \frac{27}{8}$

$x < -\frac{43}{24} \quad or \quad x > \frac{9}{8}$

$\left\{x\middle|x < -\frac{43}{24} \ \ or \ \ x > \frac{9}{8}\right\}$, or

$\left(-\infty, -\frac{43}{24}\right) \cup \left(\frac{9}{8}, \infty\right)$

102. $|t-7|+3 \geq 4$

$|t-7| \geq 1$

$t-7 \leq -1 \quad or \quad t-7 \geq 1$

$t \leq 6 \quad or \quad t \geq 8$

$\{t|t \leq 6 \ \ or \ \ t \geq 8\}$, or $(-\infty, 6] \cup [8, \infty)$

104. $16 \leq |2x-3|+9$

$7 \leq |2x-3|$

$2x-3 \leq -7 \quad or \quad 2x-3 \geq 7$

$2x \leq -4 \quad or \quad 2x \geq 10$

$x \leq -2 \quad or \quad x \geq 5$

$\{x|x \leq -2 \ \ or \ \ x \geq 5\}$, or $(-\infty, -2] \cup [5, \infty)$

106. $\left|\frac{3x-2}{5}\right| \geq 1$

$\frac{3x-2}{5} \leq -1 \quad or \quad \frac{3x-2}{5} \geq 1$

$3x-2 \leq -5 \quad or \quad 3x-2 \geq 5$

$3x \leq -3 \quad or \quad 3x \geq 7$

$x \leq -1 \quad or \quad x \geq \frac{7}{3}$

$\left\{x\middle|x \leq -1 \ \ or \ \ x \geq \frac{7}{3}\right\}$, or $(-\infty, -1] \cup \left[\frac{7}{3}, \infty\right)$

108. $\frac{7}{9}y < -\frac{7}{10}$

$y < -\frac{9}{10}$

$\left\{y|y < -\frac{9}{10}\right\}$, or $\left(-\infty, -\frac{9}{10}\right)$

110. $8 > -x \geq 4$

$-8 < x \leq -4$

$\{x|-8 < x \leq -4\}$, or $(-8, -4]$

112. $-2 \le 6x - 4 < 20$

$2 \le 6x < 24$

$\frac{1}{3} \le x < 4$

$\left\{x \middle| \frac{1}{3} \le x < 4\right\}$, or $\left[\frac{1}{3}, 4\right)$

114. $l \ge w + 3,$

$2l + 2w \le 24$

The width must be more than 0 in. The maximum value of w occurs when $l = w + 3$. Then

$$\begin{aligned} 2l + 2w &\le 24 \\ 2(w+3) + 2w &\le 24 \\ 2w + 6 + 2w &\le 24 \\ 4w + 6 &\le 24 \\ 4w &\le 18 \\ w &\le 4.5 \end{aligned}$$

Thus, the solution set is $\{w | 0 \text{ in.} < w \le 4.5 \text{ in.}\}$.

116. $1 - \left|\frac{1}{4}x + 8\right| = \frac{3}{4}$

$-\left|\frac{1}{4}x + 8\right| = -\frac{1}{4}$

$\left|\frac{1}{4}x + 8\right| = \frac{1}{4}$

$\frac{1}{4}x + 8 = \frac{1}{4}$ *or* $\frac{1}{4}x + 8 = -\frac{1}{4}$

$\frac{1}{4}x = -\frac{31}{4}$ *or* $\frac{1}{4}x = -\frac{33}{4}$

$x = -31$ *or* $x = -33$

$\{-31, -33\}$

118. $|x-1| = x-1$ only when $x - 1 \ge 0$, or $x \ge 1$. The solution set is $\{x | x \ge 1\}$, or $[1, \infty)$.

120. $|3x - 4| > -2$

From the definition of absolute value we know that $|3x - 4| \ge 0$. Thus, $|3x - 4| > -2$ is true for all x. The solution set is the set of all real numbers.

122. $|y| \le 5$

124. $-5 < x < 1$

$-3 < x + 2 < 3$ Adding 2

$|x + 2| < 3$

Chapter 1 Vocabulary Reinforcement

1. An <u>inequality</u> is a sentence containing $<, \le, >, \ge$, or $\neq$.
2. Using <u>set-builder</u> notation, we write the solution set for $x < 7$ as $\{x | x < 7\}$.
3. Using <u>interval</u> notation, we write the solution set of $-5 \le y < 16$ as $[-5, 16)$.
4. The <u>intersection</u> of two sets A and B is the set of all members that are common to A and B.
5. When two or more sentences are joined by the word *and* to make a compound sentence, the new sentence is called a <u>conjunction</u> of the sentences.
6. When two sets have no elements in common, the intersection of the two sets is the <u>empty set</u>.
7. Two sets with an empty intersection are said to be <u>disjoint sets</u>.
8. The <u>union</u> of two sets A and B is the collection of elements belonging to A and/or B.
9. When two or more sentences are joined by the word *or* to make a compound sentence, the new sentence is called a <u>disjunction</u> of the sentences.
10. The <u>addition principle</u> for equations states that for any real numbers a, b, and c, $a = b$ is equivalent to $a+c = b+c$.
11. The <u>multiplication principle</u> for equations states that for any real numbers a, b, and c, $a = b$ is equivalent to $a \cdot c = b \cdot c$.
12. For any real numbers a and b, the <u>distance</u> between them is $|a - b|$.

Chapter 1 Concept Reinforcement

1. True; see page 5 in the text.
2. False; the variable t appears on both sides of the formula $t = \frac{3B - mt}{n}$, so the original formula has not been solved for t.
3. False; see page 38 in the text.
4. False; numbers in the interval $(1, 2)$ are solutions of $x < 2$, but they are not solutions of $x \le 1$.
5. True; see page 57 in the text.
6. False; $|0| = 0$.
7. True; we have $|a - b| = |-1 \cdot (-a + b)| = |-1| \cdot |-a + b| = 1 \cdot |-a + b| = |-a + b|$, or $|b - a|$.

Chapter 1 Study Guide

1.

$$\begin{array}{r|l} \multicolumn{2}{c}{28 - 7x = 7} \\ \hline 28 - 7(-3) & ?\ 7 \\ 28 + 21 & \\ 49 & \quad \text{FALSE} \end{array}$$

The number -3 is not a solution of the equation.

2. $$\begin{aligned} 2(x+2) &= 5(x-4) \\ 2x+4 &= 5x-20 \\ 4 &= 3x-20 \\ 24 &= 3x \\ 8 &= x \end{aligned}$$

The solution is 8.

3. $$\begin{aligned} F &= \frac{1}{4}gh \\ 4F &= gh \\ \frac{4F}{g} &= h \end{aligned}$$

4. $8 - 3x \leq 3x + 6$

-2 : We substitute and get $8 - 3(-2) \leq 3(-2) + 6$, or $8 + 6 \leq -6 + 6$, or $14 \leq 0$, a false sentence. Therefore, -2 is not a solution.

5 : We substitute and get $8 - 3 \cdot 5 \leq 3 \cdot 5 + 6$, or $8 - 15 \leq 15 + 6$, or $-7 \leq 21$, a true sentence. Therefore, 5 is a solution.

5. a) Interval notation for $\{t|t < -8\}$ is $(-\infty, -8)$.

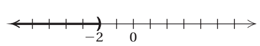
b) Interval notation for $\{x| -7 \leq x < 10\}$ is $[-7, 10)$.

c) Interval notation for $\{b|b \geq 3\}$ is $[3, \infty)$.

6. $$\begin{aligned} 5y + 5 &< 2y - 1 \\ 3y + 5 &< -1 \\ 3y &< -6 \\ y &< -2 \end{aligned}$$

The solution set is $\{y|y < -2\}$, or $(-\infty, -2)$.

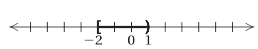
The graph of the solution set is shown below.



7. $$\begin{aligned} -4 \leq 5x &+ 6 < 11 \\ -10 \leq 5x &< 5 \\ -2 \leq x &< 1 \end{aligned}$$

The solution set is $\{x| -2 \leq x < 1\}$, or $[-2, 1)$.

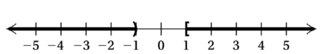
The graph of the solution set is shown below.



8. $$\begin{aligned} z + 4 < 3 \quad &\textit{or} \quad 4z + 1 \geq 5 \\ z < -1 \quad &\textit{or} \quad 4z \geq 4 \\ z < -1 \quad &\textit{or} \quad z \geq 1 \end{aligned}$$

The solution set is $\{z|z < -1 \textit{ or } z \geq 1\}$, or $(-\infty, -1) \cup [1, \infty)$.

The graph of the solution set is shown below.



9. $$\begin{aligned} |8y^2| &= |8| \cdot |y^2| \\ &= 8y^2 \qquad \text{Since } y^2 \text{ is never negative} \end{aligned}$$

10. $|8 - (-20)| = |8 + 20| = |28| = 28$

11. $|5x - 1| = 9$

$$\begin{aligned} 5x - 1 = -9 \quad &\textit{or} \quad 5x - 1 = 9 \\ 5x = -8 \quad &\textit{or} \quad 5x = 10 \\ x = -\frac{8}{5} \quad &\textit{or} \quad x = 2 \end{aligned}$$

The solution set is $\left\{-\frac{8}{5}, 2\right\}$.

12. $|z + 4| = |3z - 2|$

$$\begin{aligned} z + 4 = 3z - 2 \quad &\textit{or} \quad z + 4 = -(3z - 2) \\ -2z + 4 = -2 \quad &\textit{or} \quad z + 4 = -3z + 2 \\ -2z = -6 \quad &\textit{or} \quad 4z + 4 = 2 \\ z = 3 \quad &\textit{or} \quad 4z = -2 \\ z = 3 \quad &\textit{or} \quad z = -\frac{1}{2} \end{aligned}$$

The solution set is $\left\{3, -\frac{1}{2}\right\}$.

13. a) $$\begin{aligned} |2x + 3| &< 5 \\ -5 < 2x &+ 3 < 5 \\ -8 < 2x &< 2 \\ -4 < x &< 1 \end{aligned}$$

The solution set is $\{x| -4 < x < 1\}$, or $(-4, 1)$.

b) $|3x + 2| \geq 8$

$$\begin{aligned} 3x + 2 \leq -8 \quad &\textit{or} \quad 3x + 2 \geq 8 \\ 3x \leq -10 \quad &\textit{or} \quad 3x \geq 6 \\ x \leq -\frac{10}{3} \quad &\textit{or} \quad x \geq 2 \end{aligned}$$

The solution set is $\left\{x \middle| x \leq -\frac{10}{3} \textit{ or } x \geq 2\right\}$, or $\left(-\infty, -\frac{10}{3}\right] \cup [2, \infty)$.

Chapter 1 Review Exercises

1. $$\begin{aligned} -11 + y &= -3 \\ -11 + y + 11 &= -3 + 11 \\ y &= 8 \end{aligned}$$

The number 8 checks, so it is the solution.

2. $$\begin{aligned} -7x &= -3 \\ \frac{-7x}{-7} &= \frac{-3}{-7} \\ x &= \frac{3}{7} \end{aligned}$$

The number $\frac{3}{7}$ checks, so it is the solution.

3.
$$\begin{aligned} -\frac{5}{3}x+\frac{7}{3} &= -5 \\ 3\left(-\frac{5}{3}x+\frac{7}{3}\right) &= 3(-5) \quad \text{Clearing fractions} \\ -5x+7 &= -15 \\ -5x &= -22 \\ x &= \frac{22}{5} \end{aligned}$$

The number $\frac{22}{5}$ checks, so it is the solution.

4.
$$\begin{aligned} 6(2x-1) &= 3-(x+10) \\ 12x-6 &= 3-x-10 \\ 12x-6 &= -7-x \\ 13x-6 &= -7 \\ 13x &= -1 \\ x &= -\frac{1}{13} \end{aligned}$$

The number $-\frac{1}{13}$ checks, so it is the solution.

5.
$$\begin{aligned} 2.4x+1.5 &= 1.02 \\ 100(2.4x+1.5) &= 100(1.02) \quad \text{Clearing decimals} \\ 240x+150 &= 102 \\ 240x &= -48 \\ x &= -0.2 \end{aligned}$$

The number -0.2 checks, so it is the solution.

6.
$$\begin{aligned} 2(3-x)-4(x+1) &= 7(1-x) \\ 6-2x-4x-4 &= 7-7x \\ 2-6x &= 7-7x \\ 2+x &= 7 \\ x &= 5 \end{aligned}$$

The number 5 checks, so it is the solution.

7.
$$\begin{aligned} C &= \frac{4}{11}d+3 \\ C-3 &= \frac{4}{11}d \qquad \text{Subtracting 3} \\ \frac{11}{4}(C-3) &= d \qquad \text{Multiplying by } \frac{11}{4} \end{aligned}$$

8.
$$\begin{aligned} A &= 2a-3b \\ A-2a &= -3b \\ \frac{A-2a}{-3} &= b, \text{ or} \\ \frac{2a-A}{3} &= b \end{aligned}$$

9. ***Familiarize***. Let $x =$ the smaller number. Then $x+1 =$ the larger number.

Translate.

$$\underbrace{\text{Smaller number}}_{x} \ \underbrace{\text{plus}}_{+} \ \underbrace{\text{larger number}}_{(x+1)} \ \underbrace{\text{is}}_{=} \ \underbrace{371.}_{371}$$

Solve. We solve the equation.

$$\begin{aligned} x+(x+1) &= 371 \\ 2x+1 &= 371 \\ 2x &= 370 \\ x &= 185 \end{aligned}$$

If $x = 185$, then $x+1 = 185+1 = 186$.

Check. 185 and 186 are consecutive integers and $185+186 = 371$. The answer checks.

State. The numbers on the markers are 185 and 186.

10. ***Familiarize***. Let $x =$ the length of the longer piece of rope, in meters. Then $\frac{4}{5}x =$ the length of the shorter piece.

Translate.

$$\underbrace{\text{Length of longer piece}}_{x} \ \underbrace{\text{plus}}_{+} \ \underbrace{\text{Length of shorter piece}}_{\frac{4}{5}x} \ \underbrace{\text{is}}_{=} \ \underbrace{27\text{ m}}_{27}.$$

Solve. We solve the equation.

$$\begin{aligned} x+\frac{4}{5}x &= 27 \\ \frac{9}{5}x &= 27 \\ x &= \frac{5}{9}\cdot 27 \\ x &= 15 \end{aligned}$$

If $x = 15$, then $\frac{4}{5}x = \frac{4}{5}\cdot 15 = 12$.

Check. 12 m is $\frac{4}{5}$ of 15 m and 12 m + 15 m = 27, so the answer checks.

State. The lengths of the pieces are 15 m and 12 m.

11. ***Familiarize***. Let $p =$ the former population.

Translate.

$$\underbrace{\text{Former population}}_{p} \ \underbrace{\text{plus}}_{+} \ \underbrace{12\%}_{12\%} \ \underbrace{\text{of}}_{\cdot} \ \underbrace{\text{former population}}_{p} \ \underbrace{\text{is}}_{=} \ \underbrace{179,200}_{179,200}$$

Solve. We solve the equation.

$$\begin{aligned} p+12\%\cdot p &= 179,200 \\ p+0.12p &= 179,200 \\ 1.12p &= 179,200 \\ p &= 160,000 \end{aligned}$$

Check. 12% of 160,000 is $0.12(160,000) = 19,200$ and $160,000+19,200 = 179,200$. The answer checks.

State. The former population is 160,000.

12. ***Familiarize***. We will use the formula $d = rt$. Arnie's speed on the walkway is $3+6 = 9$ ft/sec.

Translate.

$$\begin{aligned} d &= rt \\ 360 &= 9t \end{aligned}$$

Solve. We solve the equation.

$360 = 9t$

$40 = t$

Check. If Arnie travels at a speed of 9 ft/sec for 40 sec, he travels $9 \cdot 40 = 360$ ft. The answer checks.

State. It will take Arnie 40 sec to walk the length of the walkway.

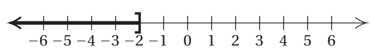
13. Interval is $[-8, 9)$.

14. Interval notation is $(-\infty, 40]$.

15. $x - 2 \le -4$

$x \le -2$

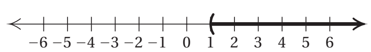
The solution set is $(-\infty, -2]$.



16. $x + 5 > 6$

$x > 1$

The solution set is $(1, \infty)$.



17. $a + 7 \le -14$

$a \le -21$

The solution set is $\{a|a \le -21\}$, or $(-\infty, -21]$.

18. $y - 5 \ge -12$

$y \ge -7$

The solution set is $\{y|y \ge -7\}$, or $[-7, \infty)$.

19. $4y > -16$

$y > -4$

The solution set is $\{y|y > -4\}$, or $(-4, \infty)$.

20. $-0.3y < 9$

$y > -30$ Reversing the inequality symbol

The solution set is $\{y|y > -30\}$, or $(-30, \infty)$.

21. $-6x - 5 < 13$

$-6x < 18$

$x > -3$ Reversing the inequality symbol

The solution set is $\{x|x > -3\}$, or $(-3, \infty)$.

22. $4y + 3 \le -6y - 9$

$10y + 3 \le -9$

$10y \le -12$

$y \le -\dfrac{6}{5}$

The solution set is $\left\{y\middle|y \le -\dfrac{6}{5}\right\}$, or $\left(-\infty, -\dfrac{6}{5}\right]$.

23. $-\dfrac{1}{2}x - \dfrac{1}{4} > \dfrac{1}{2} - \dfrac{1}{4}x$

$-\dfrac{1}{4}x - \dfrac{1}{4} > \dfrac{1}{2}$

$-\dfrac{1}{4}x > \dfrac{3}{4}$

$x < -3$ Reversing the inequality symbol

The solution set is $\{x|x < -3\}$, or $(-\infty, -3)$.

24. $0.3y - 8 < 2.6y + 15$

$-2.3y - 8 < 15$

$-2.3y < 23$

$y > -10$ Reversing the inequality symbol

The solution set is $\{y|y > -10\}$, or $(-10, \infty)$.

25. $-2(x - 5) \ge 6(x + 7) - 12$

$-2x + 10 \ge 6x + 42 - 12$

$-2x + 10 \ge 6x + 30$

$-8x + 10 \ge 30$

$-8x \ge 20$

$x \le -\dfrac{5}{2}$ Reversing the inequality symbol

The solution set is $\left\{x\middle|x \le -\dfrac{5}{2}\right\}$, or $\left(-\infty, -\dfrac{5}{2}\right]$.

26. ***Familiarize***. Let t = the length of time of the move, in hours. Then Metro Movers charges $85+40t$ and Champion Moving charges $60t$.

Translate.

$$\underbrace{\text{Cost of Champion Moving}}_{60t} \quad \underbrace{\text{is more than}}_{>} \quad \underbrace{\text{Cost of Metro Movers}}_{85 + 40t}$$

Solve. We solve the inequality.

$60t > 85 + 40t$

$20t > 85$

$t > \dfrac{17}{4}$, or $4\dfrac{1}{4}$

Check. When $t = \dfrac{17}{4}$ hr, Champion Moving charges $60 \cdot \dfrac{17}{4}$, or \$255, and Metro Movers charges $85 + 40 \cdot \dfrac{17}{4} = 85 + 170 = \$255$. For a value of t greater than $4\dfrac{1}{4}$, say 5, Champion Moving charges $60 \cdot 5 = \$300$, and Metro Movers charges $85 + 40 \cdot 5 = 85 + 200 = \$285$. This partial check tells us that the answer is probably correct.

State. Champion Moving is more expensive for moves taking more than $4\dfrac{1}{4}$ hr. The solution set is $\left\{t\middle|t > 4\dfrac{1}{4}\text{ hr}\right\}$.

27. ***Familiarize***. Let x = the amount invested at 3%. Then $30,000 - x$ = the amount invested at 4%. The interest earned on the 3% investment is $3\%x$, or $0.03x$, and the interest earned on the 4% investment is $4\%(30,000 - x)$, or $0.04(30,000 - x)$.

Translate.

$$\begin{array}{ccccc} \underbrace{\text{Interest on 3\% investment}} & \text{plus} & \underbrace{\text{Interest on 4\% investment}} & \underbrace{\text{is at least}} & \$1100 \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ 0.03x & + & 0.04(30,000-x) & \geq & 1100 \end{array}$$

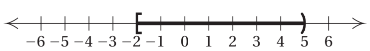
Solve. We solve the inequality.

$$\begin{aligned} 0.03x + 0.04(30,000 - x) &\geq 1100 \\ 0.03x + 1200 - 0.04x &\geq 1100 \\ -0.01x + 1200 &\geq 1100 \\ -0.01x &\geq -100 \\ x &\leq 10,000 \end{aligned}$$

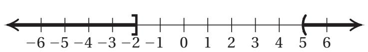
Check. If \$10,000 is invested at 3%, then the amount invested at 4% is $\$30,000-\$10,000$, or \$20,000. The interest earned is $0.03(\$10,000) + 0.04(\$20,000)$, or $\$300+\$800$, or \$1100. Then if less than \$10,000 is invested at 3%, the interest earned will be more than \$1100. This partial check shows that the answer is probably correct.

State. At most \$10,000 can be invested at 3% interest.

28. Interval notation for $-2 \leq x < 5$ is $[-2, 5)$.



29. Interval notation for $x \leq -2$ *or* $x > 5$ is $(-\infty, -2] \cup (5, \infty)$.



30. $\{1, 2, 5, 6, 9\} \cap \{1, 3, 5, 9\} = \{1, 5, 9\}$

31. $\{1, 2, 5, 6, 9\} \cup \{1, 3, 5, 9\} = \{1, 2, 3, 5, 6, 9\}$

32.
$$\begin{aligned} 2x - 5 < -7 \quad &\textit{and} \quad 3x + 8 \geq 14 \\ 2x < -2 \quad &\textit{and} \quad 3x \geq 6 \\ x < -1 \quad &\textit{and} \quad x \geq 2 \end{aligned}$$

The intersection of $\{x|x < -1\}$ *and* $\{x \geq 2\}$ is $\emptyset$, so the solution set is $\emptyset$.

33.
$$\begin{aligned} -4 < x + 3 \leq 5 & \\ -7 < x \leq 2 & \quad \text{Subtracting 3} \end{aligned}$$

The solution set is $\{x| -7 < x \leq 2\}$, or $(-7, 2]$.

34.
$$\begin{aligned} -15 < -4x - 5 < 0 & \\ -10 < -4x < 5 & \quad \text{Adding 5} \\ \frac{5}{2} > x > -\frac{5}{4} & \quad \text{Dividing by } -4 \text{ and reversing the inequality symbol} \end{aligned}$$

The solution set is $\left\{x \middle| \frac{5}{2} > x > -\frac{5}{4}\right\}$, or $\left\{x \middle| -\frac{5}{4} < x < \frac{5}{2}\right\}$, or $\left(-\frac{5}{4}, \frac{5}{2}\right)$.

35.
$$\begin{aligned} 3x < -9 \quad &\textit{or} \quad -5x < -5 \\ x < -3 \quad &\textit{or} \quad x > 1 \end{aligned}$$

The solution set is $\{x|x < -3 \textit{ or } x > 1\}$, or $(-\infty, -3) \cup (1, \infty)$.

36.
$$\begin{aligned} 2x + 5 < -17 \quad &\textit{or} \quad -4x + 10 \leq 34 \\ 2x < -22 \quad &\textit{or} \quad -4x \leq 24 \\ x < -11 \quad &\textit{or} \quad x \geq -6 \end{aligned}$$

The solution set is $\{x|x < -11 \textit{ or } x \geq -6\}$, or $(-\infty, -11) \cup [-6, \infty)$.

37.
$$\begin{aligned} 2x + 7 \leq -5 \quad &\textit{or} \quad x + 7 \geq 15 \\ 2x \leq -12 \quad &\textit{or} \quad x \geq 8 \\ x \leq -6 \quad &\textit{or} \quad x \geq 8 \end{aligned}$$

The solution set is $\{x|x \leq -6 \textit{ or } x \geq 8\}$, or $(-\infty, -6] \cup [8, \infty)$.

38. $\left|-\frac{3}{x}\right| = \left|\frac{-3}{x}\right| = \frac{|-3|}{|x|} = \frac{3}{|x|}$

39. $\left|\frac{2x}{y^2}\right| = \frac{|2x|}{|y^2|} = \frac{|2| \cdot |x|}{y^2} = \frac{2|x|}{y^2}$

40. $\left|\frac{12y}{-3y^2}\right| = \left|\frac{-4}{y}\right| = \frac{|-4|}{|y|} = \frac{4}{|y|}$

41. $|-23 - 39| = |-62| = 62$, or

$|39 - (-23)| = |39 + 23| = |62| = 62$

42. $|x| = 6$

$x = -6$ *or* $x = 6$ Absolute-value principle

The solution set is $\{-6, 6\}$.

43. $|x - 2| = 7$

$$\begin{aligned} x - 2 = -7 \quad &\textit{or} \quad x - 2 = 7 \\ x = -5 \quad &\textit{or} \quad x = 9 \end{aligned}$$

The solution set is $\{-5, 9\}$.

44. $|2x + 5| = |x - 9|$

$$\begin{aligned} 2x + 5 = x - 9 \quad &\textit{or} \quad 2x + 5 = -(x - 9) \\ x + 5 = -9 \quad &\textit{or} \quad 2x + 5 = -x + 9 \\ x = -14 \quad &\textit{or} \quad 3x + 5 = 9 \\ x = -14 \quad &\textit{or} \quad 3x = 4 \\ x = -14 \quad &\textit{or} \quad x = \frac{4}{3} \end{aligned}$$

The solution set is $\left\{-14, \frac{4}{3}\right\}$.

45. $|5x + 6| = -8$

The absolute value of a number is always nonnegative. Thus, the solution set is $\emptyset$.

46. $|2x + 5| < 12$

$$\begin{aligned} -12 < 2x + 5 < 12 \\ -17 < 2x < 7 \\ -\frac{17}{2} < x < \frac{7}{2} \end{aligned}$$

The solution set is $\left\{x \middle| -\frac{17}{2} < x < \frac{7}{2}\right\}$, or $\left(-\frac{17}{2}, \frac{7}{2}\right)$.

47. $|x| \geq 3.5$

$x \leq -3.5$ *or* $x \geq 3.5$

The solution set is $\{x|x \leq -3.5 \text{ or } x \geq 3.5\}$, or $(-\infty, -3.5] \cup [3.5, \infty)$.

48. $|3x - 4| \geq 15$

$$\begin{aligned} 3x - 4 \leq -15 \quad &or \quad 3x - 4 \geq 15 \\ 3x \leq -11 \quad &or \quad 3x \geq 19 \\ x \leq -\frac{11}{3} \quad &or \quad x \geq \frac{19}{3} \end{aligned}$$

The solution set is $\left\{x\middle|x \leq -\frac{11}{3} \text{ or } x \geq \frac{19}{3}\right\}$, or $\left(-\infty, -\frac{11}{3}\right] \cup \left[\frac{19}{3}, \infty\right)$.

49. $|x| < 0$

The absolute value of a number is always greater than or equal to 0, so the solution set is $\emptyset$.

50. In 2010, $t = 2010 - 1980 = 30$.

$G = 0.506t + 18.3$

$G = 0.506(30) + 18.3 = 15.18 + 18.3 = 33.48$

We estimate carbon dioxide emissions to be 33.48 billion metric tons in 2010. Answer B is correct.

51. We want to find the value of t for which $35 < G < 40$. We have

$$\begin{aligned} 35 &< 0.506t + 18.3 < 40 \\ 16.7 &< 0.506t < 21.7 \\ 33 &< t < 43. \quad \text{Rounding} \end{aligned}$$

Thus, for years between 33 yr after 1980 and 43 yr after 1980, global carbon dioxide emissions are predicted to be between 35 and 40 billion metric tons. These are the years between 2013 and 2023. Answer A is correct.

52. $|2x + 5| \leq |x + 3|$

$|2x + 5| \leq x + 3$ *or* $|2x + 5| \leq -(x + 3)$

First we solve $|2x + 5| \leq x + 3$.

$$\begin{aligned} -(x + 3) \leq 2x + 5 \quad &and \quad 2x + 5 \leq x + 3 \\ -x - 3 \leq 2x + 5 \quad &and \quad x \leq -2 \\ -8 \leq 3x \quad &and \quad x \leq -2 \\ -\frac{8}{3} \leq x \quad &and \quad x \leq -2 \end{aligned}$$

The solution set for this portion of the inequality is $\left\{x\middle| -\frac{8}{3} \leq x \leq -2\right\}$.

Now we solve $|2x + 5| \leq -(x + 3)$.

$$\begin{aligned} -[-(x + 3)] \leq 2x + 5 \quad &and \quad 2x + 5 \leq -(x + 3) \\ x + 3 \leq 2x + 5 \quad &and \quad 2x + 5 \leq -x - 3 \\ -2 \leq x \quad &and \quad 3x \leq -8 \\ -2 \leq x \quad &and \quad x \leq -\frac{8}{3} \end{aligned}$$

The solution set for this portion of the inequality is $\emptyset$.

Then the solution set for the original inequality is $\left\{x\middle| -\frac{8}{3} \leq x \leq -2\right\} \cup \emptyset$, or $\left\{x\middle| -\frac{8}{3} \leq x \leq -2\right\}$. This is expressed in interval notation as $\left[-\frac{8}{3}, -2\right]$.

53. When the signs of the quantities on either side of the inequality symbol are changed, their relative positions on the number line are reversed.

54. The distance between x and -5 is $|x - (-5)|$, or $|x + 5|$. Then the solutions of the inequality $|x + 5| \leq 2$ can be interpreted as "all those numbers x whose distance from -5 is at most 2 units."

55. When $b \geq c$, then $[a, b] \cup [c, d] = [a, d]$.

56. The solutions of $|x| \geq 6$ are those numbers whose distance from zero is greater than or equal to 6. In addition to the numbers in $[6, \infty)$, the distance of the numbers in $(-\infty, -6]$ from 0 is also greater than or equal to 6. Thus, $[6, \infty)$ is only part of the solution of the inequality.

57. (1) $-9(x + 2) = -9x - 18$, not $-9x + 2$. (2) This would be correct if (1) were correct except that the inequality symbol should not have been reversed. (3) If (2) were correct, the right-hand side would be -5, not 8. (4) The inequality symbol should be reversed. The correct solution is

$$\begin{aligned} 7 - 9x + 6x &< -9(x + 2) + 10x \\ 7 - 9x + 6x &< -9x - 18 + 10x \\ 7 - 3x &< x - 18 \\ -4x &< -25 \\ x &> \frac{25}{4}. \end{aligned}$$

58. By definition, the notation $3 < x < 5$ indicates that $3 < x$ *and* $x < 5$. A solution of the disjunction $3 < x$ *or* $x < 5$ must be in at least one of these sets but not necessarily in both, so the disjunction cannot be written as $3 < x < 5$.

Chapter 1 Test

1.
$$\begin{aligned} x + 7 &= 5 \\ x + 7 - 7 &= 5 - 7 \\ x &= -2 \end{aligned}$$

The number -2 checks, so it is the solution.

2.
$$\begin{aligned} -12x &= -8 \\ \frac{-12x}{-12} &= \frac{-8}{-12} \\ x &= \frac{2}{3} \end{aligned}$$

The number $\frac{2}{3}$ checks, so it is the solution.

3.
$$\begin{aligned} x - \frac{3}{5} &= \frac{2}{3} \\ x - \frac{3}{5} + \frac{3}{5} &= \frac{2}{3} + \frac{3}{5} \\ x &= \frac{10}{15} + \frac{9}{15} \\ x &= \frac{19}{15} \end{aligned}$$
The number $\frac{19}{15}$ checks, so it is the solution.

4.
$$\begin{aligned} 3y - 4 &= 8 \\ 3y &= 12 \quad \text{Adding 4} \\ y &= 4 \quad \text{Dividing by 3} \end{aligned}$$
The number 4 checks, so it is the solution.

5.
$$\begin{aligned} 1.7y - 0.1 &= 2.1 - 0.3y \\ 2y - 0.1 &= 2.1 \quad \text{Adding } 0.3y \\ 2y &= 2.2 \quad \text{Adding 0.1} \\ y &= 1.1 \quad \text{Dividing by 2} \end{aligned}$$
The number 1.1 checks, so it is the solution.

6.
$$\begin{aligned} 5(3x + 6) &= 6 - (x + 8) \\ 15x + 30 &= 6 - x - 8 \\ 15x + 30 &= -2 - x \\ 16x + 30 &= -2 \\ 16x &= -32 \\ x &= -2 \end{aligned}$$
The number -2 checks, so it is the solution.

7.
$$\begin{aligned} A &= 3B - C \\ A + C &= 3B \quad \text{Adding } C \\ \frac{A + C}{3} &= B \quad \text{Dividing by 3} \end{aligned}$$

8.
$$\begin{aligned} m &= n - nt \\ m &= n(1 - t) \quad \text{Factoring out } n \\ \frac{m}{1 - t} &= n \quad \text{Dividing by } 1 - t \end{aligned}$$

9. ***Familiarize***. Let l = the length of the room, in feet. Then $\frac{2}{3}l$ = the width. Recall that the formula for the perimeter P of a rectangle with length l and width w is $P = 2l + 2w$.

Translate. We substitute in the formula.
$$\begin{aligned} P &= 2l + 2w \\ 48 &= 2l + 2 \cdot \frac{2}{3}l \end{aligned}$$

Solve. We solve the equation.
$$\begin{aligned} 48 &= 2l + 2 \cdot \frac{2}{3}l \\ 48 &= 2l + \frac{4}{3}l \\ 48 &= \frac{10}{3}l \\ \frac{3}{10} \cdot 48 &= l \\ \frac{72}{5} &= l, \text{ or} \\ 14\frac{2}{5} &= l \end{aligned}$$
If $l = \frac{72}{5}$, then $\frac{2}{3}l = \frac{2}{3} \cdot \frac{72}{5} = \frac{48}{5}$, or $9\frac{3}{5}$.

Check. $9\frac{3}{5}$ ft is two-thirds of $14\frac{2}{5}$ ft and $2 \cdot 14\frac{2}{5} + 2 \cdot 9\frac{3}{5} = 2 \cdot \frac{72}{5} + 2 \cdot \frac{48}{5} = \frac{144}{5} + \frac{96}{5} = \frac{240}{5} = 48$. The answer checks.

State. The length of the room is $14\frac{2}{5}$ ft and the width is $9\frac{3}{5}$ ft.

10. ***Familiarize***. Let c = the number of copies the firm can make. The rental cost for 3 months is $3 \cdot \$240$, or \$720, and the cost of the copies is $1.5¢ \cdot c$, or $\$0.015c$.

Translate.
$$\underbrace{\text{Rental cost}}_{720} \quad \underbrace{\text{plus}}_{+} \quad \underbrace{\text{copy cost}}_{0.015c} \quad \underbrace{\text{is no more than}}_{\leq} \quad \underbrace{\$1500}_{1500}$$

Solve. We solve the inequality.
$$\begin{aligned} 720 + 0.015c &\leq 1500 \\ 0.015c &\leq 780 \\ c &\leq 52,000 \end{aligned}$$

Check. If 52,000 copies are made, the total cost is $\$720 + \$0.015(52,000) = \$1500$. For more than 52,000 copies, say 52,001, the total cost is $\$720 + \$0.015(52,001) \approx \$1500.02$. The answer checks.

State. The law firm can make at most 52,000 copies.

11. ***Familiarize***. Let p = the former population.

Translate.
$$\underbrace{\text{Former population}}_{p} \quad \underbrace{\text{minus}}_{-} \quad \underbrace{12\%}_{12\%} \quad \underbrace{\text{of}}_{\cdot} \quad \underbrace{\text{Former population}}_{p} \quad \underbrace{\text{is}}_{=} \quad \underbrace{158,400.}_{158,400}$$

Solve. We solve the equation.
$$\begin{aligned} p - 12\% \cdot p &= 158,400 \\ p - 0.12p &= 158,400 \\ 0.88p &= 158,400 \\ p &= 180,000 \end{aligned}$$

Check. 12% of 180,000 is $0.12(180,000) = 21,600$ and $180,000 - 21,600 = 158,400$ so the answer checks.

State. The former population of Baytown was 180,000.

12. ***Familiarize***. Let $x =$ the measure of the smallest angle. Then $x+1$ and $x+2$ represent the measures of the other two angles. Recall that the sum of the measures of the angles in a triangle is $180°$.

Translate.

$$\begin{array}{ccc} \underbrace{\text{The sum of the measures}} & \text{is} & 180° \\ \downarrow & \downarrow & \downarrow \\ x+(x+1)+(x+2) & = & 180 \end{array}$$

Solve. We solve the equation.

$$\begin{aligned} x+(x+1)+(x+2) &= 180 \\ 3x+3 &= 180 \\ 3x &= 177 \\ x &= 59 \end{aligned}$$

If $x = 59$, then $x+1 = 59+1 = 60$ and $x+2 = 59+2 = 61$.

Check. The numbers 59, 60, and 61 are consecutive integers and $59° + 60° + 61° = 180°$. The answer checks.

State. The measures of the angles are $59°$, $60°$, and $61°$.

13. First we will find how long it takes the boat to travel 36 mi downstream.

Familiarize. We will use the formula $d = rt$. Let $t =$ the time, in hours, it will take the boat to travel 36 mi downstream. The speed of the boat traveling downstream is $12+3$, or 15 mph.

Translate.

$$\begin{aligned} d &= rt \\ 36 &= 15t \end{aligned}$$

Solve. We solve the equation.

$$\begin{aligned} 36 &= 15t \\ \frac{12}{5} &= t, \text{ or} \\ 2\frac{2}{5} &= t \end{aligned}$$

Check. If the boat travels at 15 mph for $\frac{12}{5}$ hr, it travels $15 \cdot \frac{12}{5}$, or 36 mi. The answer checks.

State. It will take the boat $2\frac{2}{5}$ hr to travel 36 mi downstream.

Now we find how long it will take the boat to travel 36 mi upstream.

Familiarize. We will use the formula $d = rt$. Let $t =$ the time, in hours, it will take the boat to travel 36 mi upstream. The speed of the boat traveling upstream is $12-3$, or 9 mph.

Translate.

$$\begin{aligned} d &= rt \\ 36 &= 9t \end{aligned}$$

Solve. We solve the equation.

$$\begin{aligned} 36 &= 9t \\ 4 &= t \end{aligned}$$

Check. If the boat travels at 9 mph for 4 hr, it travels $9 \cdot 4$, or 36 mi. The answer checks.

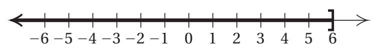
State. It will take the boat 4 hr to travel 36 mi upstream.

14. Interval notation for $\{x| -3 < x \le 2\}$ is $(-3, 2]$.

15. Interval notation is $(-4, \infty)$.

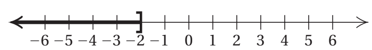
16. $$\begin{aligned} x-2 &\le 4 \\ x &\le 6 \quad \text{Adding 2} \end{aligned}$$

The solution set is $\{x|x \le 6\}$, or $(-\infty, 6]$.



17. $$\begin{aligned} -4y-3 &\ge 5 \\ -4y &\ge 8 \\ y &\le -2 \quad \text{Reversing the inequality symbol} \end{aligned}$$

The solution set is $\{y|y \le -2\}$, or $(-\infty, -2]$.



18. $$\begin{aligned} x-4 &\ge 6 \\ x &\ge 10 \quad \text{Adding 4} \end{aligned}$$

The solution set is $\{x|x \ge 10\}$, or $[10, \infty)$.

19. $$\begin{aligned} -0.6y &< 30 \\ y &> -50 \quad \text{Reversing the inequality symbol} \end{aligned}$$

The solution set is $\{y|y > -50\}$, or $(-50, \infty)$.

20. $$\begin{aligned} 3a-5 &\le -2a+6 \\ 5a-5 &\le 6 \\ 5a &\le 11 \\ a &\le \frac{11}{5} \end{aligned}$$

The solution set is $\left\{a\middle|a \le \frac{11}{5}\right\}$, or $\left(-\infty, \frac{11}{5}\right]$.

21. $$\begin{aligned} -5y-1 &> -9y+3 \\ 4y-1 &> 3 \\ 4y &> 4 \\ y &> 1 \end{aligned}$$

The solution set is $\{y|y > 1\}$, or $(1, \infty)$.

22. $$\begin{aligned} 4(5-x) &< 2x+5 \\ 20-4x &< 2x+5 \\ 20-6x &< 5 \\ -6x &< -15 \\ x &> \frac{5}{2} \end{aligned}$$

The solution set is $\left\{x\middle|x > \frac{5}{2}\right\}$, or $\left(\frac{5}{2}, \infty\right)$.

23. $$\begin{aligned} -8(2x+3)+6(4-5x) &\ge 2(1-7x)-4(4+6x) \\ -16x-24+24-30x &\ge 2-14x-16-24x \\ -46x &\ge -14-38x \\ -8x &\ge -14 \\ x &\le \frac{7}{4} \end{aligned}$$

The solution set is $\left\{x\middle|x \le \frac{7}{4}\right\}$, or $\left(-\infty, \frac{7}{4}\right]$.

24. ***Familiarize***. Let t = the length of time of the move, in hours. Then Motivated Movers charges $105 + 30t$ and Quick-Pak Moving charges $80t$.

Translate.

$$\underbrace{\text{Cost of Quick-Pak}}_{\displaystyle\downarrow\atop\displaystyle 80t}\quad\underbrace{\text{is more than}}_{\displaystyle\downarrow\atop\displaystyle >}\quad\underbrace{\text{Cost of Motivated Movers}}_{\displaystyle\downarrow\atop\displaystyle 105+30t}$$

Solve. We solve the inequality.

$$80t > 105 + 30t$$
$$50t > 105$$
$$t > \frac{21}{10},\ \text{or } 2\frac{1}{10}$$

Check. When $t = \frac{21}{10}$ hr, Motivated Movers charges $105 + 30 \cdot \frac{21}{10}$, or \$168, and Quick-Pak charges $80 \cdot \frac{21}{10}$, or \$168. For a value of t greater than $2\frac{1}{10}$, say 3, Motivated Movers charges $105+30\cdot3$, or \$195, and Quick-Pak charges $80\cdot3$, or \$240, so Quick-Pak is more expensive. This partial check tells us that the answer is probably correct.

State. Quick-Pak is more expensive for moves more than $2\frac{1}{10}$ hr. The solution set is $\left\{t\middle|t > 2\frac{1}{10}\text{ hr}\right\}$.

25. ***Familiarize***. We will use the formula $P = 1 + \frac{d}{33}$.

Translate. We want to find those values of P for which

$$2 \le P \le 8$$

or

$$2 \le 1 + \frac{d}{33} \le 8.$$

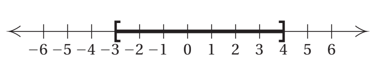
Solve. We solve the inequality.

$$2 \le 1 + \frac{d}{33} \le 8$$
$$1 \le \frac{d}{33} \le 7$$
$$33 \le d \le 231$$

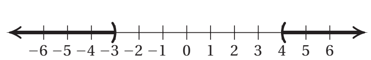
Check. We could do a partial check by substituting some values for d in the formula. The result checks.

State. The pressure is at least 2 atm and at most 8 atm for depths d in the set $\{d|33\text{ ft} \le d \le 231\text{ ft}\}$.

26. Interval notation for $-3 \le x \le 4$ is $[-3, 4]$.



27. Interval notation for $x < -3$ *or* $x > 4$ is $(-\infty, -3)\cup(4, \infty)$.



28.
$$5 - 2x \le 1 \quad \textit{and} \quad 3x + 2 \ge 14$$
$$-2x \le -4 \quad \textit{and} \quad 3x \ge 12$$
$$x \ge 2 \quad \textit{and} \quad x \ge 4$$

The intersection of $\{x|x \ge 2\}$ *and* $\{x|x \ge 4\}$, is $\{x|x \ge 4\}$, or $[4, \infty)$.

29.
$$-3 < x - 2 < 4$$
$$-1 < x < 6 \qquad \text{Adding 2}$$

The solution set is $\{x| - 1 < x < 6\}$, or $(-1, 6)$.

30.
$$-11 \le -5x - 2 < 0$$
$$-9 \le -5x < 2$$
$$\frac{9}{5} \ge x > -\frac{2}{5}$$

The solution set is $\left\{x\middle|\frac{9}{5} \ge x > -\frac{2}{5}\right\}$, or $\left\{x\middle| -\frac{2}{5} < x \le \frac{9}{5}\right\}$, or $\left(-\frac{2}{5}, \frac{9}{5}\right]$.

31.
$$-3x > 12 \quad \textit{or} \quad 4x > -10$$
$$x < -4 \quad \textit{or} \quad x > -\frac{5}{2}$$

The solution set is $\left\{x\middle|x < -4 \textit{ or } x > -\frac{5}{2}\right\}$, or $(-\infty, -4) \cup \left(-\frac{5}{2}, \infty\right)$.

32.
$$x - 7 \le -5 \quad \textit{or} \quad x - 7 \ge -10$$
$$x \le 2 \quad \textit{or} \quad x \ge -3$$

The union of $(-\infty, 2]$ and $[-3, \infty)$ is the set of all real numbers, or $(-\infty, \infty)$.

33.
$$3x - 2 < 7 \quad \textit{or} \quad x - 2 > 4$$
$$3x < 9 \quad \textit{or} \quad x > 6$$
$$x < 3 \quad \textit{or} \quad x > 6$$

The solution set is $\{x|x < 3 \textit{ or } x > 6\}$, or $(-\infty, 3)\cup(6, \infty)$.

34. $\left|\frac{7}{x}\right| = \frac{|7|}{|x|} = \frac{7}{|x|}$

35. $\left|\frac{-6x^2}{3x}\right| = | - 2x| = | - 2| \cdot |x| = 2|x|$

36. $|4.8 - (-3.6)| = |4.8 + 3.6| = |8.4| = 8.4$, or
$| - 3.6 - 4.8| = | - 8.4| = 8.4$

37. $\{1, 3, 5, 7, 9\} \cap \{3, 5, 11, 13\} = \{3, 5\}$

38. $\{1, 3, 5, 7, 9\} \cup \{3, 5, 11, 13\} = \{1, 3, 5, 7, 9, 11, 13\}$

39. $|x| = 9$

$x = -9$ *or* $x = 9$ Absolute-value principle

The solution set is $\{-9, 9\}$.

40. $|x - 3| = 9$
$$x - 3 = -9 \quad \textit{or} \quad x - 3 = 9$$
$$x = -6 \quad \textit{or} \quad x = 12$$

The solution set is $\{-6, 12\}$.

41. $|x + 10| = |x - 12|$
$$x + 10 = x - 12 \quad \textit{or} \quad x + 10 = -(x - 12)$$
$$10 = -12 \quad \textit{or} \quad x + 10 = -x + 12$$
$$10 = -12 \quad \textit{or} \quad 2x = 2$$
$$10 = -12 \quad \textit{or} \quad x = 1$$

The first equation has no solution. The solution of the second equation is 1, so the solution set is $\{1\}$.

42. $|2-5x| = -10$

The absolute value of a number is always nonnegative. Thus, the solution set is $\emptyset$.

43. $|4x-1| < 4.5$

$$-4.5 < 4x - 1 < 4.5$$
$$-3.5 < 4x < 5.5$$
$$-0.875 < x < 1.375$$

The solution set is $\{x|-0.875 < x < 1.375\}$, or $(-0.875, 1.375)$. This could also be expressed as $\left\{x\middle|-\frac{7}{8} < x < \frac{11}{8}\right\}$, or $\left(-\frac{7}{8}, \frac{11}{8}\right)$.

44. $|x| > 3$

$x < -3$ *or* $x > 3$

The solution set is $\{x|x < -3 \text{ or } x > 3\}$, or $(-\infty, -3) \cup (3, \infty)$.

45.

$$\left|\frac{6-x}{7}\right| \le 15$$
$$-15 \le \frac{6-x}{7} \le 15$$
$$-105 \le 6 - x \le 105 \quad \text{Multiplying by 7}$$
$$-111 \le -x \le 99$$
$$111 \ge x \ge -99$$

The solution set is $\{x|111 \ge x \ge -99\}$, or $\{x|-99 \le x \le 111\}$, or $[-99, 111]$.

46. $|-5x-3| \ge 10$

$$-5x - 3 \le -10 \quad or \quad -5x - 3 \ge 10$$
$$-5x \le -7 \quad or \quad -5x \ge 13$$
$$x \ge \frac{7}{5} \quad or \quad x \le -\frac{13}{5}$$

The solution set is $\left\{x\middle|x \le -\frac{13}{5} \text{ or } x \ge \frac{7}{5}\right\}$, or $\left(-\infty, -\frac{13}{5}\right] \cup \left[\frac{7}{5}, \infty\right)$.

47.

$$2(3x-6) + 5 = 1 - (x-6)$$
$$6x - 12 + 5 = 1 - x + 6$$
$$6x - 7 = 7 - x$$
$$7x - 7 = 7$$
$$7x = 14$$
$$x = 2$$

The number 2 checks, so it is the solution. The solution is between 1 and 3, so answer C is correct.

48. $|3x-4| \le -3$

The absolute value of a number is always nonnegative, so $|3x-4|$ cannot be less than -3. Thus, the solution set is $\emptyset$.

49. $7x < 8 - 3x < 6 + 7x$

$$7x < 8 - 3x \quad and \quad 8 - 3x < 6 + 7x$$
$$10x < 8 \quad and \quad -10x < -2$$
$$x < \frac{4}{5} \quad and \quad x > \frac{1}{5}$$

The intersection of $\left\{x\middle|x < \frac{4}{5}\right\}$ *and* $\left\{x\middle|x > \frac{1}{5}\right\}$ is $\left\{x\middle|\frac{1}{5} < x < \frac{4}{5}\right\}$, or $\left(\frac{1}{5}, \frac{4}{5}\right)$.