Chapter 2 Functions and Relations

Section 2.1 The Rectangular Coordinate System and Graphing Utilities

1. origin

2. quadrants

3. $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

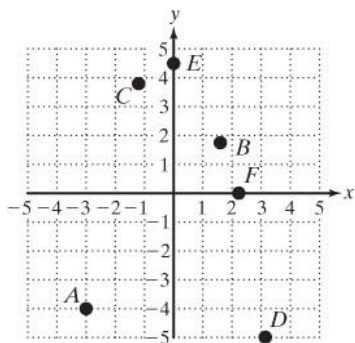**4.** $M = \left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)$
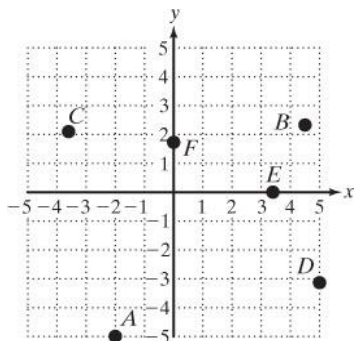
5. solution

6. 0

7. 0

8. 0; y

9.

10.

11. a. $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{\left[-4 - (-2)\right]^2 + (11 - 7)^2}$

$= \sqrt{(-2)^2 + (4)^2} = \sqrt{4 + 16}$

$= \sqrt{20} = 2\sqrt{5}$

b. $M = \left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)$

$= \left(\dfrac{-4 + (-2)}{2}, \dfrac{11 + 7}{2}\right)$

$= \left(\dfrac{-6}{2}, \dfrac{18}{2}\right) = (-3, 9)$

12. a. $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{\left[3 - (-1)\right]^2 + \left[-7 - (-3)\right]^2}$

$= \sqrt{(4)^2 + (-4)^2}$

$= \sqrt{16 + 16} = \sqrt{32} = 4\sqrt{2}$

b. $M = \left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)$

$= \left(\dfrac{3 + (-1)}{2}, \dfrac{-7 + (-3)}{2}\right)$

$= \left(\dfrac{2}{2}, \dfrac{-10}{2}\right) = (1, -5)$

13. a. $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{\left[2 - (-7)\right]^2 + \left[5 - (-4)\right]^2}$

$= \sqrt{(9)^2 + (9)^2}$

$= \sqrt{81 + 81} = \sqrt{162} = 9\sqrt{2}$

b. $M = \left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2} \right)$

$= \left(\dfrac{2 + (-7)}{2}, \dfrac{5 + (-4)}{2} \right)$

$= \left(\dfrac{-5}{2}, \dfrac{1}{2} \right) = \left(-\dfrac{5}{2}, \dfrac{1}{2} \right)$

14. a. $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(-4 - 3)^2 + (-1 - 6)^2}$

$= \sqrt{(-7)^2 + (7)^2}$

$= \sqrt{49 + 49} = \sqrt{98} = 7\sqrt{2}$

b. $M = \left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2} \right)$

$= \left(\dfrac{-4 + 3}{2}, \dfrac{-1 + 6}{2} \right)$

$= \left(\dfrac{-1}{2}, \dfrac{5}{2} \right) = \left(-\dfrac{1}{2}, \dfrac{5}{2} \right)$

15. a. $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(5.2 - 2.2)^2 + [-6.4 - (-2.4)]^2}$

$= \sqrt{(3)^2 + (4)^2} = \sqrt{9 + 16} = \sqrt{25} = 5$

b. $M = \left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2} \right)$

$= \left(\dfrac{5.2 + 2.2}{2}, \dfrac{-6.4 + (-2.4)}{2} \right)$

$= \left(\dfrac{7.4}{2}, \dfrac{-8.8}{2} \right) = (3.7, -4.4)$

16. a. $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(31.1 - 37.1)^2 + [-32.7 - (-24.7)]^2}$

$= \sqrt{(-6)^2 + (-8)^2}$

$= \sqrt{36 + 64} = \sqrt{100} = 10$

b. $M = \left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2} \right)$

$= \left(\dfrac{31.1 + 37.1}{2}, \dfrac{-32.7 + (-24.7)}{2} \right)$

$= \left(\dfrac{68.2}{2}, \dfrac{-57.4}{2} \right) = (34.1, -28.7)$

17. a. $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{\left(4\sqrt{5} - \sqrt{5}\right)^2 + \left[-7\sqrt{2} - \left(-\sqrt{2}\right)\right]^2}$

$= \sqrt{\left(3\sqrt{5}\right)^2 + \left(-6\sqrt{2}\right)^2}$

$= \sqrt{45 + 72} = \sqrt{117}$

b. $M = \left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2} \right)$

$= \left(\dfrac{4\sqrt{5} + \sqrt{5}}{2}, \dfrac{-7\sqrt{2} + \left(-\sqrt{2}\right)}{2} \right)$

$= \left(\dfrac{5\sqrt{5}}{2}, \dfrac{-8\sqrt{2}}{2} \right) = \left(\dfrac{5\sqrt{5}}{2}, -4\sqrt{2} \right)$

18. a. $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{\left(2\sqrt{7} - \sqrt{7}\right)^2 + \left[\sqrt{5} - \left(-3\sqrt{5}\right)\right]^2}$

$= \sqrt{\left(\sqrt{7}\right)^2 + \left(4\sqrt{5}\right)^2}$

$= \sqrt{7 + 80} = \sqrt{87}$

b. $M = \left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2} \right)$

$= \left(\dfrac{2\sqrt{7} + \sqrt{7}}{2}, \dfrac{\sqrt{5} + \left(-3\sqrt{5}\right)}{2} \right)$

$= \left(\dfrac{3\sqrt{7}}{2}, \dfrac{-2\sqrt{5}}{2} \right) = \left(\dfrac{3\sqrt{7}}{2}, -\sqrt{5} \right)$

19. $d_1 = \sqrt{(3 - 1)^2 + (1 - 3)^2}$

$= \sqrt{4 + 4} = 2\sqrt{2}$

$$d_2 = \sqrt{(0-3)^2 + (-2-1)^2}$$
$$= \sqrt{9+9} = 3\sqrt{2}$$
$$d_3 = \sqrt{(1-0)^2 + [3-(-2)]^2}$$
$$= \sqrt{1+25} = \sqrt{26}$$
$$d_1^2 + d_2^2 = d_3^2$$
$$\left(2\sqrt{2}\right)^2 + \left(3\sqrt{2}\right)^2 = \left(\sqrt{26}\right)^2$$
$$8+18 = 26$$
$$26 = 26 \checkmark \text{ True}$$
Yes

20. $d_1 = \sqrt{(3-1)^2 + (0-2)^2}$
$$= \sqrt{4+4} = 2\sqrt{2}$$
$$d_2 = \sqrt{(-3-3)^2 + (-2-0)^2}$$
$$= \sqrt{36+4} = 2\sqrt{10}$$
$$d_3 = \sqrt{[1-(-3)]^2 + [2-(-2)]^2}$$
$$= \sqrt{16+16} = 4\sqrt{2}$$
$$d_1^2 + d_3^2 = d_2^2$$
$$\left(2\sqrt{2}\right)^2 + \left(4\sqrt{2}\right)^2 = \left(2\sqrt{10}\right)^2$$
$$8+32 = 40$$
$$40 = 40 \checkmark \text{ True}$$
Yes

21. $d_1 = \sqrt{[5-(-2)]^2 + (0-4)^2}$
$$= \sqrt{49+16} = \sqrt{65}$$
$$d_2 = \sqrt{(-5-5)^2 + (1-0)^2}$$
$$= \sqrt{100+1} = \sqrt{101}$$
$$d_3 = \sqrt{[-2-(-5)]^2 + (4-1)^2}$$
$$= \sqrt{9+9} = 3\sqrt{2}$$
$$d_1^2 + d_3^2 = d_2^2$$
$$\left(\sqrt{65}\right)^2 + \left(3\sqrt{2}\right)^2 = \left(\sqrt{101}\right)^2$$
$$65+18 = 101$$
$$83 = 101 \text{ False}$$
No

22. $d_1 = \sqrt{(-6-3)^2 + (2-1)^2}$
$$= \sqrt{81+1} = \sqrt{82}$$
$$d_2 = \sqrt{(3-1)^2 + [1-(-2)]^2}$$
$$= \sqrt{4+9} = \sqrt{13}$$
$$d_3 = \sqrt{(-6-1)^2 + [2-(-2)]^2}$$
$$= \sqrt{49+16} = \sqrt{65}$$
$$d_2^2 + d_3^2 = d_1^2$$
$$\left(\sqrt{13}\right)^2 + \left(\sqrt{65}\right)^2 = \left(\sqrt{82}\right)^2$$
$$13+65 = 82$$
$$78 = 82 \text{ False}$$
No

23. a. $\quad x^2 + y = 1$
$$(-2)^2 + (-3) = 1$$
$$4-3 = 1$$
$$1 = 1 \checkmark$$
Yes

b. $\quad x^2 + y = 1$
$$(4)^2 + (-17) = 1$$
$$16-17 = 1$$
$$-1 = 1 \text{ False}$$
No

c. $\quad x^2 + y = 1$
$$\left(\frac{1}{2}\right)^2 + \left(\frac{3}{4}\right) = 1$$
$$\frac{1}{4} + \frac{3}{4} = 1$$
$$1 = 1 \checkmark$$
Yes

24. a. $\quad |x-3| - y = 4$
$$|(1)-3| - (-2) = 4$$
$$2+2 = 4$$
$$4 = 4 \checkmark$$
Yes

b. $\quad |x-3| - y = 4$

$\quad |(-2) - 3| - (-3) = 4$

$\qquad 5 + 3 = 4$

$\qquad\qquad 8 = 4 \text{ False}$

$\quad$ No

c. $|x-3| - y = 4$

$\left| \left(\dfrac{1}{10} \right) - 3 \right| - \left(-\dfrac{11}{10} \right) = 4$

$10 \left[\left| \left(\dfrac{1}{10} \right) - 3 \right| + \dfrac{11}{10} \right] = 10 \left[4 \right]$

$|1 - 30| + 11 = 40$

$29 + 11 = 40$

$40 = 40 \checkmark$

$\quad$ Yes

25. $x - 3 \neq 0$

$\quad x \neq 3$

$\quad \{ x \mid x \neq 3 \}$

26. $x + 7 \neq 0$

$\quad x \neq -7$

$\quad \{ x \mid x \neq -7 \}$

27. $x - 10 \geq 0$

$\quad x \geq 10$

$\quad \{ x \mid x \geq 10 \}$

28. $x + 11 \geq 0$

$\quad x \geq -11$

$\quad \{ x \mid x \geq -11 \}$

29. $1.5 - x \geq 0$

$\quad -x \geq -1.5$

$\quad x \leq 1.5$

$\quad \{ x \mid x \leq 1.5 \}$

30. $2.2 - x \geq 0$

$\quad -x \geq -2.2$

$\quad x \leq 2.2$

$\quad \{ x \mid x \leq 2.2 \}$

31. $y = x$

x	y	$y = x$	Ordered pair
–3	–3	$y = -3$	$(-3, -3)$
–2	–2	$y = -2$	$(-2, -2)$
–1	–1	$y = -1$	$(-1, -1)$
0	0	$y = 0$	$(0, 0)$
1	1	$y = 1$	$(1, 1)$
2	2	$y = 2$	$(2, 2)$
3	3	$y = 3$	$(3, 3)$
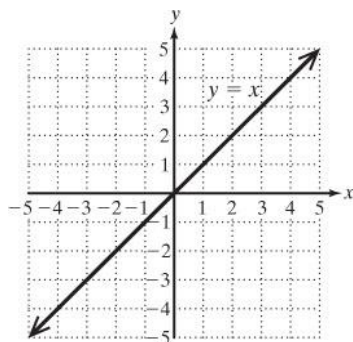
32. $y = x^2$

x	y	$y = x^2$	Ordered
–3	9	$y = (-3)^2 = 9$	$(-3, 9)$
–2	4	$y = (-2)^2 = 4$	$(-2, 4)$
–1	1	$y = (-1)^2 = 1$	$(-1, 1)$
0	0	$y = (0)^2 = 0$	$(0, 0)$
1	1	$y = (1)^2 = 1$	$(1, 1)$
2	4	$y = (2)^2 = 4$	$(2, 4)$
3	9	$y = (3)^2 = 9$	$(3, 9)$
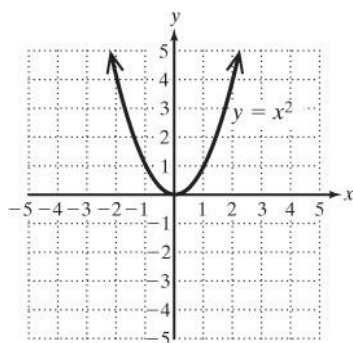
33. $y = \sqrt{x}$

x	y	$y = \sqrt{x}$	Ordered pair
0	0	$y = \sqrt{0} = 0$	$(0, 0)$
1	1	$y = \sqrt{1} = 1$	$(1, 1)$

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4	2	$y = \sqrt{4} = 2$	$\left(4, 2\right)$
9	3	$y = \sqrt{9} = 3$	$\left(9, 3\right)$

34. $y = \left|x\right|$

x	y	$y = \left|x\right|$	**Ordered pair**
–3	3	$y = \left|-3\right| = 3$	$\left(-3, 3\right)$
–2	2	$y = \left|-2\right| = 2$	$\left(-2, 2\right)$
–1	1	$y = \left|-1\right| = 1$	$\left(-1, 1\right)$
0	0	$y = \left|0\right| = 0$	$\left(0, 0\right)$
1	1	$y = \left|1\right| = 1$	$\left(1, 1\right)$
2	2	$y = \left|2\right| = 2$	$\left(2, 2\right)$
3	3	$y = \left|3\right| = 3$	$\left(3, 3\right)$
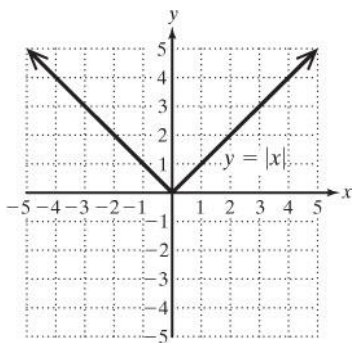
35. $y = x^3$

x	y	$y = x^3$	Ordered pair
–2	–8	$y = (-2)^3 = -8$	$(-2, -8)$
–1	–1	$y = (-1)^3 = -1$	$(-1, -1)$
0	0	$y = (0)^3 = 0$	$(0, 0)$
1	1	$y = (1)^3 = 1$	$(1, 1)$
2	8	$y = (2)^3 = 8$	$(2, 8)$
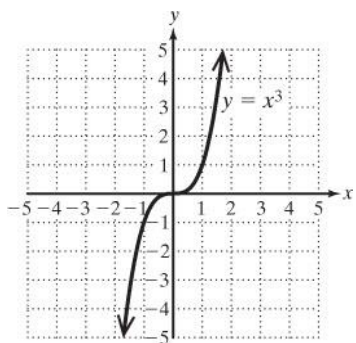
36. $y = \dfrac{1}{x}$

x	y	$y = \dfrac{1}{x}$	Ordered pair
–4	$-\dfrac{1}{4}$	$y = \dfrac{1}{(-4)} = -\dfrac{1}{4}$	$\left(-4, -\dfrac{1}{4}\right)$
–2	$-\dfrac{1}{2}$	$y = \dfrac{1}{(-2)} = -\dfrac{1}{2}$	$\left(-2, -\dfrac{1}{2}\right)$
–1	–1	$y = \dfrac{1}{(-1)} = -1$	$(-1, 1)$
$-\dfrac{1}{2}$	–2	$y = \dfrac{1}{\left(-\dfrac{1}{2}\right)} = -2$	$\left(-\dfrac{1}{2}, -2\right)$
$\dfrac{1}{2}$	2	$y = \dfrac{1}{\left(\dfrac{1}{2}\right)} = 2$	$\left(\dfrac{1}{2}, 2\right)$

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1	1	$y = \dfrac{1}{(1)} = 1$	$(1,1)$
2	$\dfrac{1}{2}$	$y = \dfrac{1}{(2)} = \dfrac{1}{2}$	$\left(2, \dfrac{1}{2}\right)$
4	$\dfrac{1}{4}$	$y = \dfrac{1}{(4)} = \dfrac{1}{4}$	$\left(4, \dfrac{1}{4}\right)$
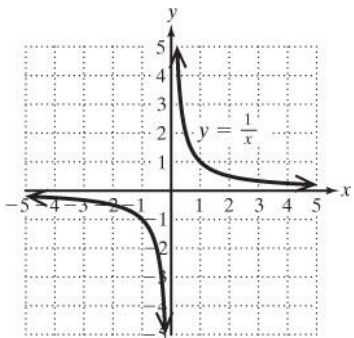
37. $y - |x| = 2$

$y = |x| + 2$

x	y	$y = |x| + 2$	**Ordered pair**
-3	5	$y = |-3| + 2 = 5$	$(-3, 5)$
-2	4	$y = |-2| + 2 = 4$	$(-2, 4)$
-1	3	$y = |-1| + 2 = 3$	$(-1, 3)$
0	2	$y = |0| + 2 = 2$	$(0, 2)$
1	3	$y = |1| + 2 = 3$	$(1, 3)$
2	4	$y = |2| + 2 = 4$	$(2, 4)$
3	5	$y = |3| + 2 = 5$	$(3, 5)$
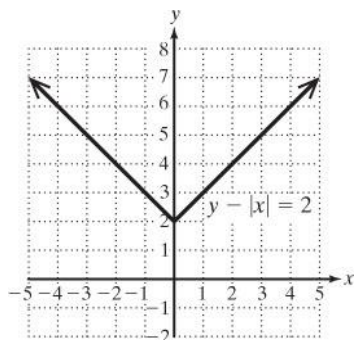
38. $|x| + y = 3$

$$y = 3 - |x|$$

x	y	$y = 3 - |x|$	Ordered pair
–3	0	$y = 3 - |-3| = 0$	$(-3, 0)$
–2	1	$y = 3 - |-2| = 1$	$(-2, 1)$
–1	2	$y = 3 - |-1| = 2$	$(-1, 2)$
0	3	$y = 3 - |0| = 3$	$(0, 3)$
1	2	$y = 3 - |1| = 2$	$(1, 2)$
2	1	$y = 3 - |2| = 1$	$(2, 1)$
3	0	$y = 3 - |3| = 0$	$(3, 0)$

39. $y^2 - x - 2 = 0$

$$y^2 = x + 2$$

$$y = \pm\sqrt{x + 2}$$

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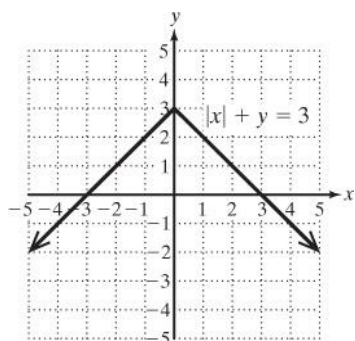
x	y	$y = \pm\sqrt{x+2}$	Ordered pairs
–2	1	$y = \pm\sqrt{(-2)+2} = 0$	$(-2, 0)$
–1	±1	$y = \pm\sqrt{(-1)+2} = \pm 1$	$(-1, 1), (-1, -1)$
2	±2	$y = \pm\sqrt{(2)+2} = \pm 2$	$(2, 2), (2, -2)$
7	±3	$y = \pm\sqrt{(7)+2} = \pm 3$	$(7, 3), (7, -3)$
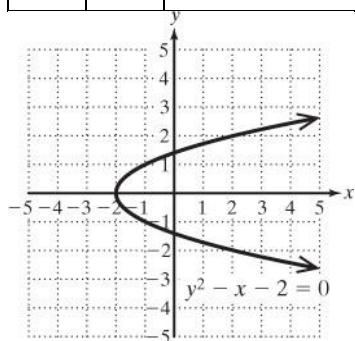
40. $y^2 - x + 1 = 0$

$y^2 = x - 1$

$y = \pm\sqrt{x-1}$

x	y	$y = \pm\sqrt{x-1}$	Ordered pairs
1	0	$y = \pm\sqrt{(1)-1} = 0$	$(1, 0)$
2	±1	$y = \pm\sqrt{(2)-1} = \pm 1$	$(2, 1), (2, -1)$
5	±2	$y = \pm\sqrt{(5)-1} = \pm 2$	$(5, 2), (5, -2)$
10	±3	$y = \pm\sqrt{(10)-1} = \pm 3$	$(10, 3), (10, -3)$
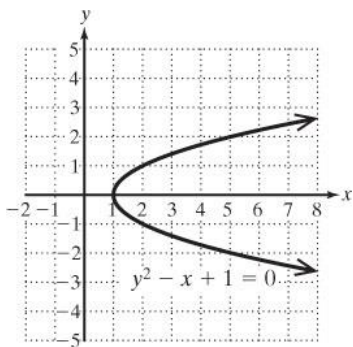
41. $x = |y| + 1$

$|y| = x - 1$

$y = \pm(x - 1)$

x	y	$y = \pm(x - 1)$	Ordered pairs
1	0	$y = \pm\left[(1) - 1\right] = 0$	$(1, 0)$
2	± 1	$y = \pm\left[(2) - 1\right] = \pm 1$	$(2, 1), (2, -1)$
3	± 2	$y = \pm\left[(3) - 1\right] = \pm 2$	$(3, 2), (3, -2)$
4	± 3	$y = \pm\left[(4) - 1\right] = \pm 3$	$(4, 3), (4, -3)$
5	± 4	$y = \pm\left[(5) - 1\right] = \pm 4$	$(5, 4), (5, -4)$
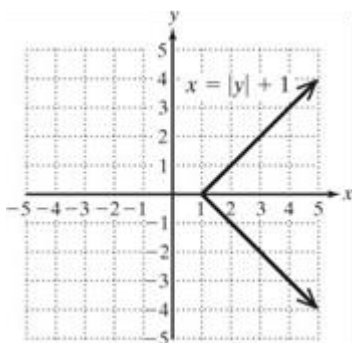
42. $x = |y| - 3$

$|y| = x + 3$

$y = \pm(x + 3)$

x	y	$y = \pm(x + 3)$	Ordered pairs
-3	0	$y = \pm\left[(-3) + 3\right] = 0$	$(-3, 0)$
-2	± 1	$y = \pm\left[(-2) + 3\right] = \pm 1$	$(-2, 1), (-2, -1)$

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-1	± 2	$y = \pm\left[(-1)+3\right] = \pm 2$	$(-1,2),(-1,-2)$
0	± 3	$y = \pm\left[(0)+3\right] = \pm 3$	$(0,3),(0,-3)$
1	± 4	$y = \pm\left[(1)+3\right] = \pm 4$	$(1,4),(1,-4)$
2	± 5	$y = \pm\left[(2)+3\right] = \pm 5$	$(2,5),(2,-5)$
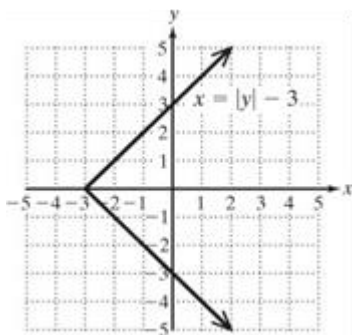
43. $y = |x+1|$

x	y	$y = |x+1|$	Ordered pair
-3	2	$y = |(-3)+1| = 2$	$(-3,2)$
-2	1	$y = |(-2)+1| = 1$	$(-2,1)$
-1	0	$y = |(-1)+1| = 0$	$(-1,0)$
0	1	$y = |(0)+1| = 1$	$(0,1)$
1	2	$y = |(1)+1| = 2$	$(1,2)$
2	3	$y = |(2)+1| = 3$	$(2,3)$
3	4	$y = |(3)+1| = 4$	$(3,4)$

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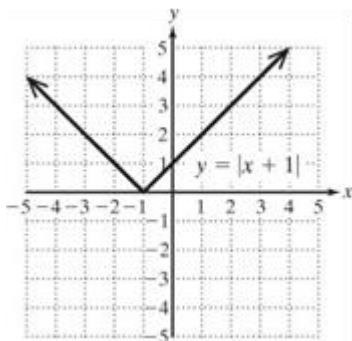
44. $y = |x - 2|$

x	y	$y = |x - 2|$	Ordered
-2	4	$y = |(-2) - 2| = 4$	$(-2, 4)$
-1	3	$y = |(-1) - 2| = 3$	$(-1, 3)$
0	2	$y = |(0) - 2| = 2$	$(0, 2)$
1	1	$y = |(1) - 2| = 1$	$(1, 1)$
2	0	$y = |(2) - 2| = 0$	$(2, 0)$
3	1	$y = |(3) - 2| = 1$	$(3, 1)$
4	2	$y = |(4) - 2| = 2$	$(4, 2)$
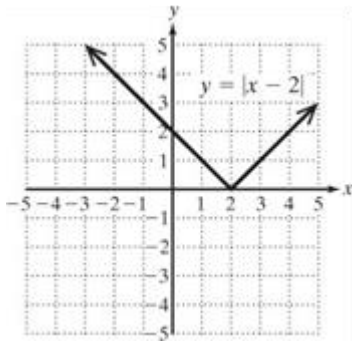
45. x-intercepts: $(-1, 0), (9, 0)$

y-intercepts: $(0, -3), (0, 3)$

46. x-intercepts: $(-16, 0), (4, 0)$

y-intercepts: $(0, -8), (0, 8)$

47. x-intercept: $(-2, 0)$; y-intercept: none

48. x-intercept: none; y-intercept: $(0, 1)$

49. x-intercept: $(0, 0)$; y-intercept: $(0, 0)$

50. x-intercept: $(0, 0), (6, 0)$; y-intercept: $(0, 0)$

51. Substitute 0 for y: Substitute 0 for x:

$$-2x + 4y = 12 \qquad -2x + 4y = 12$$
$$-2x + 4(0) = 12 \qquad -2(0) + 4y = 12$$
$$-2x = 12 \qquad\qquad 4y = 12$$
$$x = -6 \qquad\qquad y = 3$$

x-intercept: $(-6, 0)$; y-intercept: $(0, 3)$

52. Substitute 0 for y: Substitute 0 for x:

$$-3x - 5y = 60 \qquad -3x - 5y = 60$$
$$-3x - 5(0) = 60 \qquad -3(0) - 5y = 60$$
$$-3x = 60 \qquad\qquad -5y = 60$$
$$x = -20 \qquad\qquad y = -12$$

x-intercept: $(-20, 0)$; y-intercept:

$(0, -12)$

53. Substitute 0 for y: Substitute 0 for x:

$$x^2 + y = 9 \qquad x^2 + y = 9$$
$$x^2 + (0) = 9 \qquad (0)^2 + y = 9$$
$$x^2 = 9 \qquad\qquad y = 9$$
$$x = \pm 3$$

x-intercepts: $(-3, 0), (3, 0)$; y-intercept:

$(0, 9)$

54. Substitute 0 for y: Substitute 0 for x:

$$x^2 = -y + 16 \qquad x^2 = -y + 16$$
$$x^2 = -(0) + 16 \qquad (0)^2 = -y + 16$$
$$x^2 = 16 \qquad\qquad y = 16$$
$$x = \pm 4$$

x-intercepts: $(-4, 0), (4, 0)$; y-intercept:

$(0, 16)$

55. Substitute 0 for y:

$$y = |x - 5| - 2$$
$$(0) = |x - 5| - 2$$
$$|x - 5| = 2$$
$$x - 5 = -2 \quad \text{or} \quad x - 5 = 2$$
$$x = 3 \quad \text{or} \quad x = 7$$

Substitute 0 for x:

$$y = |x - 5| - 2$$
$$y = |(0) - 5| - 2 = 3$$

x-intercepts: $(3, 0), (7, 0)$; y-intercept:

$(0, 3)$

56. Substitute 0 for y:

$$y = |x + 4| - 3$$
$$(0) = |x + 4| - 3$$
$$|x + 4| = 3$$
$$x + 4 = -3 \quad \text{or} \quad x + 4 = 3$$
$$x = -7 \quad \text{or} \quad x = -1$$

Substitute 0 for x:

$$y = |x + 4| - 3$$
$$y = |(0) + 4| - 3 = 1$$

x-intercepts: $(-7, 0), (-1, 0)$; y-

intercept: $(0, 1)$

57. Substitute 0 for y: Substitute 0 for x:

$$x = y^2 - 1 \qquad x = y^2 - 1$$
$$x = (0)^2 - 1 = -1 \qquad (0) = y^2 - 1$$
$$1 = y^2$$
$$\pm 1 = y$$

x-intercept: $(-1, 0)$; y-intercepts:

$(0, -1), (0, 1)$

58. Substitute 0 for y: Substitute 0 for x:

$$x = y^2 - 4 \qquad x = y^2 - 4$$
$$x = (0)^2 - 4 \qquad (0) = y^2 - 4$$
$$x = -4 \qquad\qquad 4 = y^2$$
$$\pm 2 = y$$

x-intercept: $(-4, 0)$; y-intercepts:

$(0, -2), (0, 2)$

59. Substitute 0 for y: Substitute 0 for x:

$$|x| = |y| \qquad\qquad |x| = |y|$$
$$|x| = |(0)| \qquad\qquad |(0)| = |y|$$
$$x = 0 \qquad\qquad 0 = y$$

x-intercept: $(0, 0)$; y-intercepts: $(0, 0)$

60. Substitute 0 for y: Substitute 0 for x:

$$x = |5y| \qquad\qquad x = |5y|$$
$$|x| = |5(0)| \qquad\qquad (0) = |5y|$$
$$|x| = 0 \qquad\qquad 0 = y$$
$$x = 0$$

x-intercept: $(0, 0)$; y-intercepts: $(0, 0)$

61.

$$\frac{(x-3)^2}{4}+\frac{(y-4)^2}{9}=1$$

$$36\left[\frac{(x-3)^2}{4}+\frac{(y-4)^2}{9}\right]=36(1)$$

$$9(x-3)^2+4(y-4)^2=36$$

Substitute 0 for y:

$$9(x-3)^2+4(y-4)^2=36$$

$$9(x-3)^2+4[(0)-4]^2=36$$

$$9(x-3)^2+64=36$$

$$9(x-3)^2=-28$$

$$(x-3)^2=-\frac{28}{9}$$

$$x-3=\pm\sqrt{-\frac{28}{9}}$$

$$x=3\pm\sqrt{-\frac{28}{9}}$$

Not a real number.

Substitute 0 for x:

$$9(x-3)^2+4(y-4)^2=36$$

$$9[(0)-3]^2+4(y-4)^2=36$$

$$81+4(y-4)^2=36$$

$$4(y-4)^2=-45$$

$$(y-4)^2=-\frac{45}{4}$$

$$y-4=\pm\sqrt{-\frac{45}{4}}$$

$$y=4\pm\sqrt{-\frac{45}{4}}$$

Not a real number.

x-intercept: none; y-intercept: none

62.

$$\frac{(x+6)^2}{16}+\frac{(y+3)^2}{4}=1$$

$$16\left[\frac{(x+6)^2}{16}+\frac{(y+3)^2}{4}\right]=16(1)$$

$$(x+6)^2+4(y+3)^2=16$$

Substitute 0 for y:

$$(x+6)^2+4(y+3)^2=16$$

$$(x+6)^2+4[(0)+3]^2=16$$

$$(x+6)^2+36=16$$

$$(x+6)^2=-20$$

$$x+6=\pm\sqrt{-20}$$

$$x=-6\pm\sqrt{-20}$$

Not a real number.

Substitute 0 for x:

$$(x+6)^2+4(y+3)^2=16$$

$$[(0)+6]^2+4(y+3)^2=16$$

$$36+4(y+3)^2=16$$

$$4(y+3)^2=-20$$

$$(y+3)^2=-5$$

$$y+3=\pm\sqrt{-5}$$

$$y=-3\pm\sqrt{-5}$$

Not a real number.

x-intercept: none; y-intercept: none

63. $d_{AC}=\sqrt{[4-(-6)]^2+(8-10)^2}$

$$=\sqrt{100+4}=\sqrt{104}=2\sqrt{26}$$

$$d_{BC}=\sqrt{(4-6)^2+(8-0)^2}$$

$$=\sqrt{4+64}=\sqrt{68}=2\sqrt{17}$$

Observation tower B is closer.

64. a. $d=\sqrt{[1-(-2)]^2+(-3-3)^2}$

$$=\sqrt{9+36}=\sqrt{45}$$

$$=3\sqrt{5}\text{ mi}\approx6.7\text{ mi}$$

b. $M = \left(\dfrac{-2+1}{2}, \dfrac{3+(-3)}{2}\right)$

$= \left(\dfrac{-1}{2}, \dfrac{0}{2}\right) = \left(-\dfrac{1}{2}, 0\right)$

65. a. $d = \sqrt{(410-36)^2 + (53-315)^2}$

$= \sqrt{374^2 + (-262)^2}$

$= 456.64 \approx 457$ pixels

b. If both the player move directly towards each other at the same speed, then they will meet at the midpoint.

$\text{Midpoint} = \left(\dfrac{36+410}{2}, \dfrac{315+53}{2}\right)$

$= (223, 184)$

c. The midpoint between A and B is $(224, 184)$. If the A is 3 times faster than B, then A and B will meet at the midpoint of $(223, 184)$ and B.

$\left(\dfrac{223+410}{2}, \dfrac{184+53}{2}\right) = (316.5, 118.5)$

$\approx (317, 119)$

Therefore, A and B meet at $(317, 119)$

66. $d(A,B) = \sqrt{(80-460)^2 + (210-420)^2}$

$= 434.165 \approx 434$

$d(B,C) = \sqrt{(120-80)^2 + (60-210)^2}$

$= 10\sqrt{241} \approx 155$

The total distance between A to B to C is approximately 589 pixels.

At 120 pixels per second, the time required is only about 4.9 sec

Yes

67. $d(A,B) = \sqrt{(x-0)^2 + (0-0)^2}$

$= |x|$

$d(B,C) = \sqrt{\left(\dfrac{1}{2}x - x\right)^2 + \left(\dfrac{\sqrt{3}}{2}x - 0\right)^2}$

$= |x|$

$d(C,A) = \sqrt{\left(0 - \dfrac{1}{2}x\right)^2 + \left(0 - \dfrac{\sqrt{3}}{2}x\right)^2}$

$= |x|$

Therefore, the points A, B, and C make up the vertices of an equilateral triangle.

68. $d(A,B) = \sqrt{(x-0)^2 + (0-0)^2} = |x|$

$d(A,C) = \sqrt{(0-0)^2 + (x-0)^2} = |x|$

Therefore, the points A, B, and C make up the vertices of an isosceles triangle.

$d(B,C) = \sqrt{(0-x)^2 + (x-0)^2} = \sqrt{2}|x|$

$[d(B,C)]^2 = [d(A,B)]^2 + [d(A,C)]^2$

$\left(\sqrt{2}x\right)^2 = x^2 + x^2, \text{True}$

Therefore, the points A, B, and C make up the vertices of an isosceles right triangle.

69. a. $l = \sqrt{[1-(-2)]^2 + (-3-0)^2}$

$= \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$ ft

$w = \sqrt{(3-1)^2 + (1-3)^2}$

$= \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$ ft

b. $P = 2l + 2w$

$= 2\left(3\sqrt{2}\right) + 2\left(2\sqrt{2}\right)$

$= 2\left(5\sqrt{2}\right) = 10\sqrt{2}$ ft

$A = lw = \left(3\sqrt{2}\right)\left(2\sqrt{2}\right) = 6(2) = 12$ ft^2

70. a. $l = \sqrt{[5-(-1)]^2 + (-1-4)^2}$

$= \sqrt{36+25} = \sqrt{61}$ ft

$$w = \sqrt{\left[-1-(-2)\right]^2 + (4-3)^2}$$
$$= \sqrt{1+1} = \sqrt{2} \text{ ft}$$

b. $P = 2l + 2w$
$$= 2\left(\sqrt{61}\right) + 2\left(\sqrt{2}\right)$$
$$= 2\left(\sqrt{61} + \sqrt{2}\right) \text{ ft}$$

$$A = lw = \left(\sqrt{61}\right)\left(\sqrt{2}\right) = \sqrt{122} \text{ ft}^2$$

71. $C = \left(\dfrac{-2+4}{2}, \dfrac{1+3}{2}\right) = \left(\dfrac{2}{2}, \dfrac{4}{2}\right) = (1, 2)$

$$r = \frac{d}{2} = \frac{\sqrt{\left[4-(-2)\right]^2 + (3-1)^2}}{2}$$
$$= \frac{\sqrt{36+4}}{2} = \frac{\sqrt{40}}{2} = \frac{2\sqrt{10}}{2} = \sqrt{10}$$

Center: $(1, 2)$; Radius: $\sqrt{10}$

72. $C = \left(\dfrac{-5+2}{2}, \dfrac{3+(-1)}{2}\right)$
$$= \left(\frac{-3}{2}, \frac{2}{2}\right)$$
$$= \left(-\frac{3}{2}, 1\right)$$

$$r = \frac{d}{2} = \frac{\sqrt{\left[2-(-5)\right]^2 + (-1-3)^2}}{2}$$
$$= \frac{\sqrt{49+16}}{2} = \frac{\sqrt{65}}{2}$$

Center: $\left(-\dfrac{3}{2}, 1\right)$; Radius: $\dfrac{\sqrt{65}}{2}$

73. $M = \left(\dfrac{7+1}{2}, \dfrac{6+(-2)}{2}\right)$
$$= \left(\frac{8}{2}, \frac{4}{2}\right) = (4, 2)$$

$$h = \sqrt{(4-0)^2 + (2-5)^2}$$
$$= \sqrt{16+9}$$
$$= \sqrt{25} = 5$$

$$b = \sqrt{(7-1)^2 + \left[6-(-2)\right]^2}$$
$$= \sqrt{36+64} = \sqrt{100} = 10$$

$$\text{Area} = \frac{1}{2}bh = \frac{1}{2}(10)(5) = 25 \text{ m}^2$$

74. $M = \left(\dfrac{(-7)+(-4)}{2}, \dfrac{5+(-4)}{2}\right)$
$$= \left(-\frac{11}{2}, \frac{1}{2}\right)$$

$$h = \sqrt{\left[-1-\left(-\frac{11}{2}\right)\right]^2 + \left(2-\frac{1}{2}\right)^2}$$
$$= \sqrt{\frac{81}{4} + \frac{9}{4}}$$
$$= \sqrt{\frac{90}{4}} = \frac{3\sqrt{10}}{2}$$

$$b = \sqrt{\left[-7-(-4)\right]^2 + \left[5-(-4)\right]^2}$$
$$= \sqrt{9+81} = \sqrt{90} = 3\sqrt{10}$$

$$\text{Area} = \frac{1}{2}\left(\frac{3\sqrt{10}}{2}\right)\left(3\sqrt{10}\right) = \frac{90}{4} = 22.5 \text{ m}^2$$

75. $d_{AB} = \sqrt{(4-2)^2 + (3-2)^2}$
$$= \sqrt{4+1} = \sqrt{5}$$

$$d_{BC} = \sqrt{(8-4)^2 + (5-3)^2}$$
$$= \sqrt{16+4} = \sqrt{20} = 2\sqrt{5}$$

$$d_{AC} = \sqrt{(2-8)^2 + (2-5)^2}$$
$$= \sqrt{36+9} = \sqrt{45} = 3\sqrt{5}$$

$$d_{AB} + d_{BC} = d_{AC}$$
$$\sqrt{5} + 2\sqrt{5} = 3\sqrt{5}$$
$$3\sqrt{5} = 3\sqrt{5} \checkmark \text{ True}$$

Collinear

76. $d_{AB} = \sqrt{(4-2)^2 + (2-1.5)^2} = \sqrt{4+0.25}$
$$= \sqrt{4.25} = \sqrt{0.25 \cdot 17} = 0.5\sqrt{17}$$

$$d_{BC} = \sqrt{(8-4)^2 + (3-2)^2}$$
$$= \sqrt{16+1} = \sqrt{17}$$

$$d_{AC} = \sqrt{(2-8)^2 + (1.5-3)^2}$$
$$= \sqrt{36 + 2.25}$$
$$= \sqrt{38.25} = \sqrt{2.25 \cdot 17}$$
$$= 1.5\sqrt{17}$$
$$d_{AB} + d_{BC} = d_{AC}$$
$$0.5\sqrt{17} + \sqrt{17} = 1.5\sqrt{17}$$
$$1.5\sqrt{17} = 1.5\sqrt{17} \checkmark \text{ True}$$
Collinear

77. $d_{AB} = \sqrt{[1-(-2)]^2 + (2-8)^2}$
$$= \sqrt{9+36} = \sqrt{45} = 3\sqrt{5}$$
$$d_{BC} = \sqrt{(4-1)^2 + (-3-2)^2}$$
$$= \sqrt{9+25} = \sqrt{34}$$
$$d_{AC} = \sqrt{(-2-4)^2 + [8-(-3)]^2}$$
$$= \sqrt{36+121} = \sqrt{157}$$
$$d_{AB} + d_{BC} = d_{AC}$$
$$3\sqrt{5} + \sqrt{34} = \sqrt{157} \text{ False}$$
Not collinear

78. $d_{AB} = \sqrt{[0-(-1)]^2 + (3-5)^2}$
$$= \sqrt{1+4} = \sqrt{5}$$
$$d_{BC} = \sqrt{(5-0)^2 + (-13-3)^2}$$
$$= \sqrt{25+256} = \sqrt{281}$$
$$d_{AC} = \sqrt{(-1-5)^2 + [5-(-13)]^2}$$
$$= \sqrt{36+324} = \sqrt{360} = 6\sqrt{10}$$
$$d_{AB} + d_{BC} = d_{AC}$$
$$\sqrt{5} + \sqrt{281} = 6\sqrt{10} \text{ False}$$
Not collinear

79. The points (x_1, y_1) and (x_2, y_2) define the endpoints of the hypotenuse d of a right triangle. The lengths of the legs of the triangle are $|x_2 - x_1|$ and $|y_2 - y_1|$. Applying the Pythagorean theorem

produces $d^2 = |x_2 - x_1|^2 + |y_2 - y_1|^2$, or equivalently

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \text{ for } d \geq 0.$$

80. The midpoint formula results in an ordered pair. The x-coordinate of the midpoint is the average of the x-coordinates of the endpoints. The y-coordinate of the midpoint is the average of the y-coordinates of the endpoints.

81. To find the x-intercept(s), substitute 0 for y and solve for x. To find the y-intercept(s), substitute 0 for x and solve for y.

82. The graph of the equation represents the set of all solutions to the equation graphed in a rectangular coordinate system.

83. $d = \sqrt{(4-5)^2 + [6-(-3)]^2 + (-1-2)^2}$
$$= \sqrt{1+81+9} = \sqrt{91}$$

84. $d = \sqrt{(2-6)^2 + [3-(-4)]^2 + [1-(-1)]^2}$
$$= \sqrt{16+49+4} = \sqrt{69}$$

85. $d = \sqrt{(0-3)^2 + (-5-7)^2 + [1-(-2)]^2}$
$$= \sqrt{9+144+9} = \sqrt{162} = 9\sqrt{2}$$

86. $d = \sqrt{(2-9)^2 + [0-(-5)]^2 + [1-(-3)]^2}$
$$= \sqrt{49+25+16} = \sqrt{90} = 3\sqrt{10}$$

87. The viewing window is part of the Cartesian plane shown in the display screen of a calculator. The boundaries of the window are often denoted by [Xmin, Xmax, Xscl] by [Ymin, Ymax, Yscl].
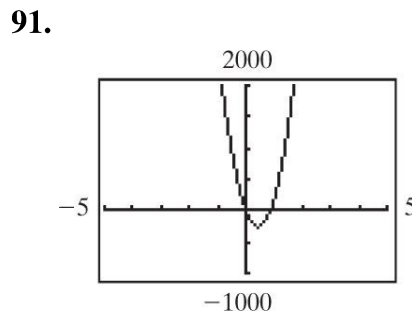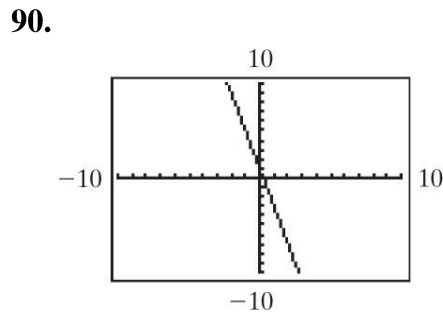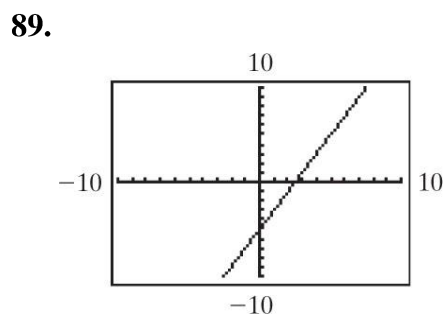
88. $780x - 42y = 5460$

$$42y = 780x - 5460$$

$$y = \frac{780x - 5460}{42}$$

$$y = \frac{130}{7}x - 130$$

```
Plot1 Plot2 Plot3
\Y1∎(130/7)X-130

\Y2=
\Y3=
\Y4=
\Y5=
\Y6=
```

```
WINDOW
 Xmin=-10
 Xmax=10
 Xscl=1
 Ymin=-150
 Ymax=10
 Yscl=10
↓Xres=1
```

Window d

89.

90.

91.

92.

93.

94.

241
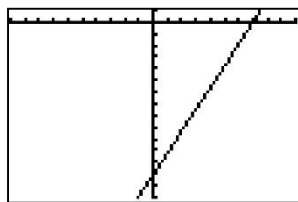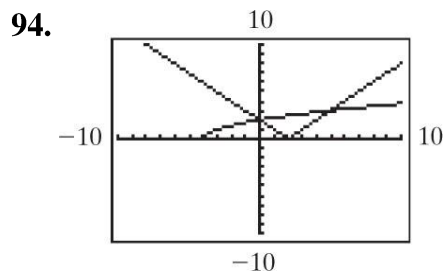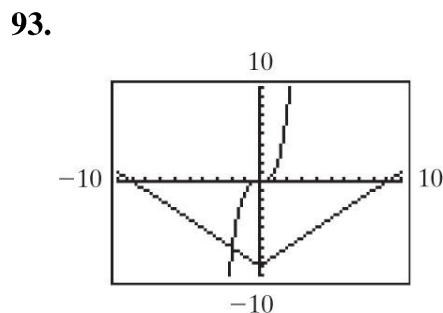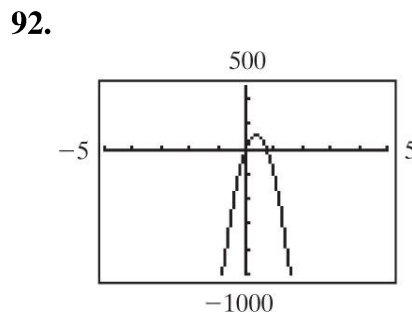
Section 2.2 Circles

1. circle; center

2. radius

3. $(x-h)^2 + (y-k)^2 = r^2$

4. general

5. $(x-2)^2 + (y-7)^2 = 4$
$(2-2)^2 + (7-7)^2 = 4$
$0 + 0 = 4$
$0 = 4$ False
No

6. $(x-3)^2 + (y-5)^2 = 36$
$(3-3)^2 + (5-5)^2 = 36$
$0 + 0 = 36$
$0 = 36$ False
No

7. $(x+1)^2 + (y-3)^2 = 25$
$(-4+1)^2 + (7-3)^2 = 25$
$9 + 16 = 25$
$25 = 25$ ✓ True
Yes

8. $(x+6)^2 + (y+1)^2 = 100$
$(2+6)^2 + (-7+1)^2 = 100$
$64 + 36 = 100$
$100 = 100$ ✓ True
Yes

9. $r^2 = 81$
$r = \sqrt{81} = 9$
Center: $(4, -2)$; Radius: 9

10. $r^2 = 16$
$r = \sqrt{16} = 4$
Center: $(-3, 1)$; Radius: 4

11. $r^2 = 6.25$
$r = \sqrt{6.25} = 2.5$
Center: $(0, 2.5)$; Radius: 2.5

12. $r^2 = 2.25$
$r = \sqrt{2.25} = 1.5$
Center: $(1.5, 0)$; Radius: 1.5

13. $r^2 = 20$
$r = \sqrt{20} = 2\sqrt{5}$
Center: $(0, 0)$; Radius: $2\sqrt{5}$

14. $r^2 = 28$
$r = \sqrt{28} = 2\sqrt{7}$
Center: $(0, 0)$; Radius: $2\sqrt{7}$

15. $r^2 = \dfrac{81}{49}$
$r = \sqrt{\dfrac{81}{49}} = \dfrac{9}{7}$
Center: $\left(\dfrac{3}{2}, -\dfrac{3}{4}\right)$; Radius: $\dfrac{9}{7}$

16. $r^2 = \dfrac{25}{9}$
$r = \sqrt{\dfrac{25}{9}} = \dfrac{5}{3}$
Center: $\left(-\dfrac{1}{7}, \dfrac{3}{5}\right)$; Radius: $\dfrac{5}{3}$

17. a. $(x-h)^2 + (y-k)^2 = r^2$
$[x-(-2)]^2 + (y-5)^2 = (1)^2$
$(x+2)^2 + (y-5)^2 = 1$

b.
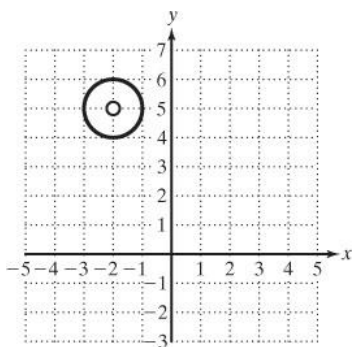
18. a. $(x-h)^2+(y-k)^2=r^2$

$[x-(-3)]^2+(y-2)^2=(4)^2$

$(x+3)^2+(y-2)^2=16$

b.
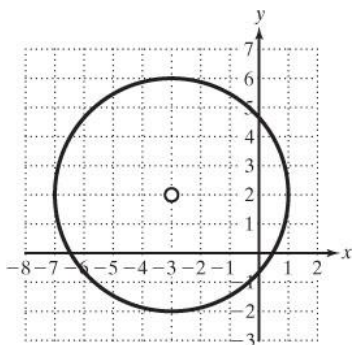
19. a. $(x-h)^2+(y-k)^2=r^2$

$[x-(-4)]^2+(y-1)^2=(3)^2$

$(x+4)^2+(y-1)^2=9$

b.
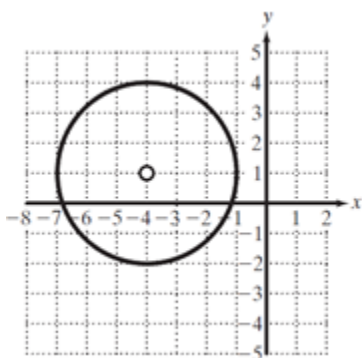
20. a. $(x-h)^2+(y-k)^2=r^2$

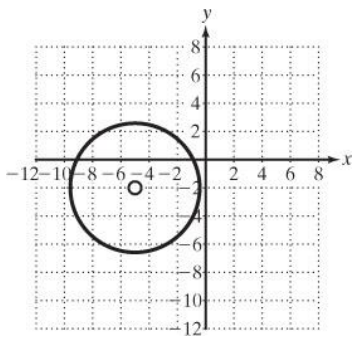$(x-6)^2+[y-(-2)]^2=(6)^2$

$(x-6)^2+(y+2)^2=36$

b.
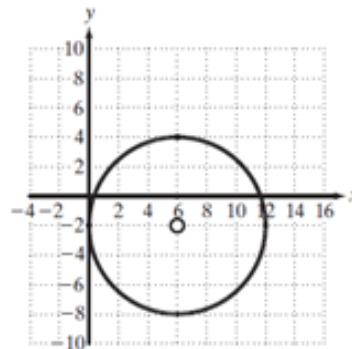
21. a. $(x-h)^2+(y-k)^2=r^2$

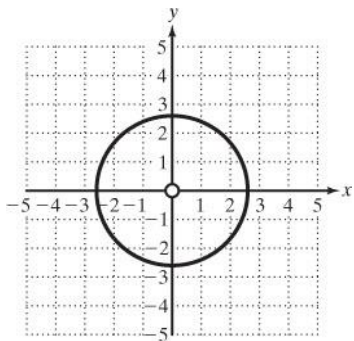$[x-(-4)]^2+[y-(-3)]^2=(\sqrt{11})^2$

$(x+4)^2+(y+3)^2=11$

b.
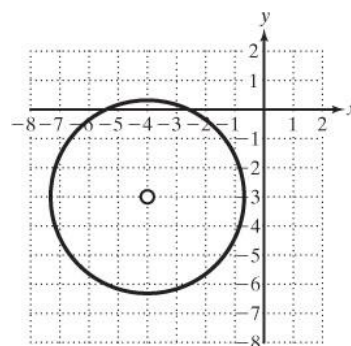
22. a. $(x-h)^2+(y-k)^2=r^2$

$[x-(-5)]^2+[y-(-2)]^2=(\sqrt{21})^2$

$(x+5)^2+(y+2)^2=21$

243

b.

23. a. $(x-h)^2 + (y-k)^2 = r^2$

$(x-0)^2 + (y-0)^2 = (2.6)^2$

$x^2 + y^2 = 6.76$

b.

24. a. $(x-h)^2 + (y-k)^2 = r^2$

$(x-0)^2 + (y-0)^2 = (4.2)^2$

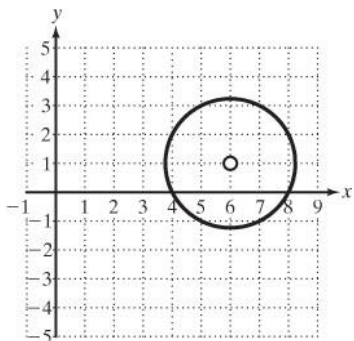$x^2 + y^2 = 17.64$

b.
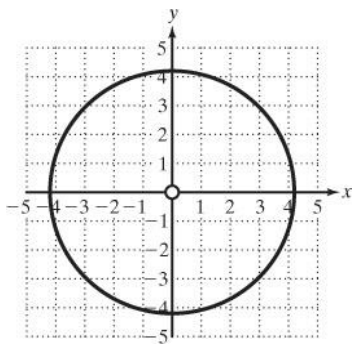
25. a. $C = \left(\dfrac{x_1+x_2}{2}, \dfrac{y_1+y_2}{2} \right)$

$= \left(\dfrac{-2+6}{2}, \dfrac{4+(-2)}{2} \right)$

$= \left(\dfrac{4}{2}, \dfrac{2}{2} \right) = (2,1)$

$r = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$
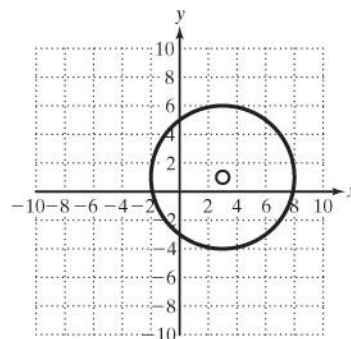
$= \sqrt{(-2-2)^2 + (4-1)^2}$

$= \sqrt{16+9} = \sqrt{25} = 5$

$(x-h)^2 + (y-k)^2 = r^2$

$(x-2)^2 + (y-1)^2 = (5)^2$

$(x-2)^2 + (y-1)^2 = 25$

b.
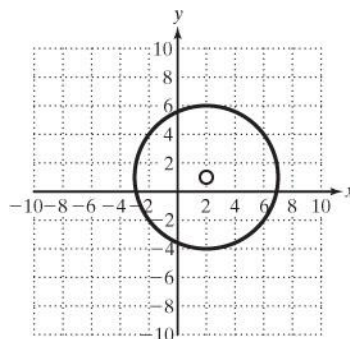
26. a. $C = \left(\dfrac{x_1+x_2}{2}, \dfrac{y_1+y_2}{2} \right)$

$= \left(\dfrac{7+5}{2}, \dfrac{3+(-1)}{2} \right)$

$= \left(\dfrac{12}{2}, \dfrac{2}{2} \right) = (6,1)$

$r = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$

$= \sqrt{(7-6)^2 + (3-1)^2}$

$= \sqrt{1+4} = \sqrt{5}$

$$(x-h)^2 + (y-k)^2 = r^2$$
$$(x-6)^2 + (y-1)^2 = (\sqrt{5})^2$$
$$(x-6)^2 + (y-1)^2 = 5$$

b.

$$(x-h)^2 + (y-k)^2 = r^2$$
$$(x-3)^2 + (y-1)^2 = (5)^2$$
$$(x-3)^2 + (y-1)^2 = 25$$

b.

27. a. $r = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$$= \sqrt{(-2-6)^2 + (-1-5)^2}$$
$$= \sqrt{64 + 36} = \sqrt{100} = 10$$
$$(x-h)^2 + (y-k)^2 = r^2$$
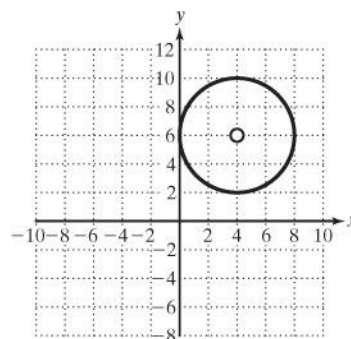$$[x-(-2)]^2 + [y-(-1)]^2 = (10)^2$$
$$(x+2)^2 + (y+1)^2 = 100$$

b.
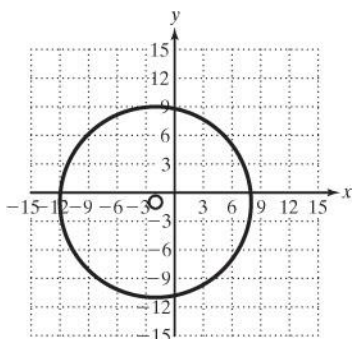
28. a. $r = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
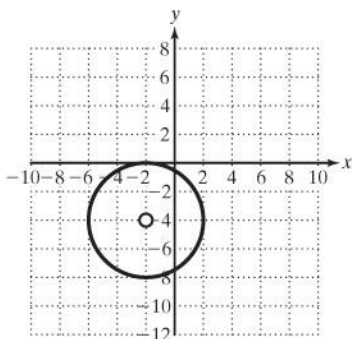
$$= \sqrt{(3-6)^2 + (1-5)^2}$$
$$= \sqrt{9 + 16} = \sqrt{25} = 5$$

29. a. The circle must touch the y-axis at $(0, 6)$, at one side of a diameter.

$$r = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{(4-0)^2 + (6-6)^2}$$
$$= \sqrt{16} = 4$$
$$(x-h)^2 + (y-k)^2 = r^2$$
$$(x-4)^2 + (y-6)^2 = (4)^2$$
$$(x-4)^2 + (y-6)^2 = 16$$

b.

30. a. The circle must touch the x-axis at $(-2, 0)$, at one side of a diameter.

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$$r = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{[-2-(-2)]^2 + (-4-0)^2}$$
$$= \sqrt{16} = 4$$
$$(x-h)^2 + (y-k)^2 = r^2$$
$$[x-(-2)]^2 + [y-(-4)]^2 = (4)^2$$
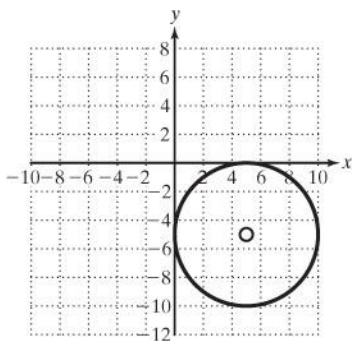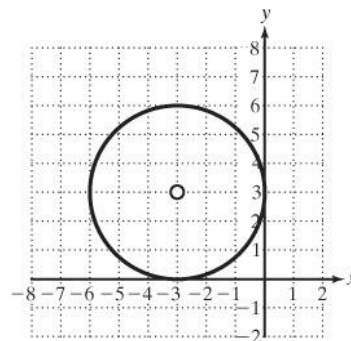$$(x+2)^2 + (y+4)^2 = 16$$

b.

31. a. The circle must touch the *x*-axis and the *y*-axis at a distance *r* from the center. Since the center is in quadrant IV, the center must be $(5, -5)$.

$$(x-h)^2 + (y-k)^2 = r^2$$
$$(x-5)^2 + [y-(-5)]^2 = (5)^2$$
$$(x-5)^2 + (y+5)^2 = 25$$

b.

32. a. The circle must touch the *x*-axis and the *y*-axis at a distance *r* from the center. Since the center is in quadrant II, the center must be $(-3, 3)$.

$$(x-h)^2 + (y-k)^2 = r^2$$
$$[x-(-3)]^2 + (y-3)^2 = (3)^2$$
$$(x+3)^2 + (y-3)^2 = 9$$

b.

33.
$$(x-h)^2 + (y-k)^2 = r^2$$
$$(x-8)^2 + [y-(-11)]^2 = (5)^2$$
$$(x-8)^2 + (y+11)^2 = 25$$

34.
$$(x-h)^2 + (y-k)^2 = r^2$$
$$[x-(-4)]^2 + (y-16)^2 = (9)^2$$
$$(x+4)^2 + (y-16)^2 = 81$$

35.
$$(x-h)^2 + (y-k)^2 = r^2$$
$$\left(x-\sqrt{7}\right)^2 + \left(y-\sqrt{7}\right)^2 = \left(\sqrt{7}\right)^2$$
$$\left(x-\sqrt{7}\right)^2 + \left(y-\sqrt{7}\right)^2 = 7$$

36.
$$(x-h)^2 + (y-k)^2 = r^2$$
$$\left[\left[x-\left(-\sqrt{11}\right)\right]^2 + \left[y-\left(-\sqrt{11}\right)\right]^2 \right] = \left(\sqrt{11}\right)^2$$
$$\left(x+\sqrt{11}\right)^2 + \left(y+\sqrt{11}\right)^2 = 11$$

37. $(x+1)^2 + (y-5)^2 = 0$

The sum of two squares will equal zero only if each individual term is zero. Therefore, $x = -1$ and $y = 5$.

$\{(-1, 5)\}$

38. $(x-3)^2 + (y+12)^2 = 0$

The sum of two squares will equal zero only if each individual term is zero. Therefore, $x = 3$ and $y = -12$.

$\{(3, -12)\}$

39. $(x-17)^2 + (y+1)^2 = -9$

The sum of two squares cannot be negative, so there is no solution.

$\{\ \}$

40. $(x+15)^2 + (y-3)^2 = -25$

The sum of two squares cannot be negative, so there is no solution.

$\{\ \}$

41. $\quad x^2 + y^2 + 6x - 2y + 6 = 0$

$(x^2 + 6x \quad) + (y^2 - 2y \quad) = -6$

$$\boxed{\left[\frac{1}{2}(6)\right]^2 = 9 \quad \left[\frac{1}{2}(-2)\right]^2 = 1}$$

$\left[\begin{array}{l}(x^2 + 6x + 9) \\ + (y^2 - 2y + 1)\end{array}\right] = -6 + 9 + 1$

$(x+3)^2 + (y-1)^2 = 4$

Center: $(-3, 1)$; Radius: $\sqrt{4} = 2$

42. $\quad x^2 + y^2 + 12x - 14y + 84 = 0$

$(x^2 + 12x \quad) + (y^2 - 14y \quad) = -84$

$$\boxed{\left[\frac{1}{2}(12)\right]^2 = 36 \quad \left[\frac{1}{2}(-14)\right]^2 = 49}$$

$\left[\begin{array}{l}(x^2 + 12x + 36) \\ + (y^2 - 14y + 49)\end{array}\right] = -84 + 36 + 49$

$(x+6)^2 + (y-7)^2 = 1$

Center: $(-6, 7)$; Radius: $\sqrt{1} = 1$

43. $\quad x^2 + y^2 - 22x + 6y + 129 = 0$

$(x^2 - 22x \quad) + (y^2 + 6y \quad) = -129$

$$\boxed{\left[\frac{1}{2}(-22)\right]^2 = 121 \quad \left[\frac{1}{2}(6)\right]^2 = 9}$$

$\left[\begin{array}{l}(x^2 - 22x + 121) \\ + (y^2 + 6y + 9)\end{array}\right] = -129 + 121 + 9$

$(x-11)^2 + (y+3)^2 = 1$

Center: $(11, -3)$; Radius: $\sqrt{1} = 1$

44. $\quad x^2 + y^2 - 10x + 4y - 20 = 0$

$(x^2 - 10x \quad) + (y^2 + 4y \quad) = 20$

$$\boxed{\left[\frac{1}{2}(-10)\right]^2 = 25 \quad \left[\frac{1}{2}(4)\right]^2 = 4}$$

$\left[\begin{array}{l}(x^2 - 10x + 25) \\ + (y^2 + 4y + 4)\end{array}\right] = 20 + 25 + 4$

$(x-5)^2 + (y+2)^2 = 49$

Center: $(5, -2)$; Radius: $\sqrt{49} = 7$

45. $\quad x^2 + y^2 - 20y - 4 = 0$

$x^2 + (y^2 - 20y \quad) = 4$

$$\boxed{\left[\frac{1}{2}(-20)\right]^2 = 100}$$

$x^2 + (y^2 - 20y + 100) = 4 + 100$

$x^2 + (y - 10)^2 = 104$

Center: $(0, 10)$; Radius: $\sqrt{104} = 2\sqrt{26}$

46. $x^2 + y^2 + 22x - 4 = 0$

$\left(x^2 + 22x \quad\right) + y^2 = 4$

$\boxed{\left[\dfrac{1}{2}(22)\right]^2 = 121}$

$\left(x^2 + 22x + 121\right) + y^2 = 4 + 121$

$\left(x + 11\right)^2 + y^2 = 125$

Center: $\left(-11, 0\right)$; Radius: $\sqrt{125} = 5\sqrt{5}$

47. $10x^2 + 10y^2 - 80x + 200y + 920 = 0$

$x^2 + y^2 - 8x + 20y + 92 = 0$

$\left(x^2 - 8x \quad\right) + \left(y^2 + 20y \quad\right) = -92$

$\boxed{\left[\dfrac{1}{2}(-8)\right]^2 = 16 \qquad \left[\dfrac{1}{2}(20)\right]^2 = 100}$

$\left[\begin{array}{l}\left(x^2 - 8x + 16\right) \\ + \left(y^2 + 20y + 100\right)\end{array}\right] = -92 + 16 + 100$

$\left(x - 4\right)^2 + \left(y + 10\right)^2 = 24$

Center: $\left(4, -10\right)$; Radius: $\sqrt{24} = 2\sqrt{6}$

48. $2x^2 + 2y^2 - 32x + 12y + 90 = 0$

$x^2 + y^2 - 16x + 6y + 45 = 0$

$\left(x^2 - 16x \quad\right) + \left(y^2 + 6y \quad\right) = -45$

$\boxed{\left[\dfrac{1}{2}(-16)\right]^2 = 64 \qquad \left[\dfrac{1}{2}(6)\right]^2 = 9}$

$\left[\begin{array}{l}\left(x^2 - 16x + 64\right) \\ + \left(y^2 + 6y + 9\right)\end{array}\right] = -45 + 64 + 9$

$\left(x - 8\right)^2 + \left(y + 3\right)^2 = 28$

Center: $\left(8, -3\right)$; Radius: $\sqrt{28} = 2\sqrt{7}$

49. $x^2 + y^2 - 4x - 18y + 89 = 0$

$\left(x^2 - 4x \quad\right) + \left(y^2 - 18y \quad\right) = -89$

$\boxed{\left[\dfrac{1}{2}(-4)\right]^2 = 4 \qquad \left[\dfrac{1}{2}(-18)\right]^2 = 81}$

$\left[\begin{array}{l}\left(x^2 - 4x + 4\right) \\ + \left(y^2 - 18y + 81\right)\end{array}\right] = -89 + 4 + 81$

$\left(x - 2\right)^2 + \left(y - 9\right)^2 = -4$

Degenerate case: $\{\ \}$

50. $x^2 + y^2 - 10x - 22y + 155 = 0$

$\left(x^2 - 10x \quad\right) + \left(y^2 - 22y \quad\right) = -155$

$\boxed{\left[\dfrac{1}{2}(-10)\right]^2 = 25 \qquad \left[\dfrac{1}{2}(-22)\right]^2 = 121}$

$\left[\begin{array}{l}\left(x^2 - 10x + 25\right) \\ + \left(y^2 - 22y + 121\right)\end{array}\right] = -155 + 25 + 121$

$\left(x - 5\right)^2 + \left(y - 11\right)^2 = -9$

Degenerate case: $\{\ \}$

51. $4x^2 + 4y^2 - 20y + 25 = 0$

$x^2 + y^2 - 5y + \dfrac{25}{4} = 0$

$x^2 + \left(y^2 - 5y \quad\right) = -\dfrac{25}{4}$

$\boxed{\left[\dfrac{1}{2}(-5)\right]^2 = \dfrac{25}{4}}$

$x^2 + \left(y^2 - 5y + \dfrac{25}{4}\right) = -\dfrac{25}{4} + \dfrac{25}{4}$

$x^2 + \left(y - \dfrac{5}{2}\right)^2 = 0$

Degenerate case (single point):

$\left\{\left(0, \dfrac{5}{2}\right)\right\}$

52. $4x^2 + 4y^2 - 12x + 9 = 0$

$x^2 + y^2 - 3x + \dfrac{9}{4} = 0$

$\left(x - 3x \quad\right)^2 + y^2 = -\dfrac{9}{4}$

$\boxed{\left[\dfrac{1}{2}(-3)\right]^2 = \dfrac{9}{4}}$

$$\left(x - 3x + \frac{9}{4}\right)^2 + y^2 = -\frac{9}{4} + \frac{9}{4}$$

$$\left(x - \frac{3}{2}\right)^2 + y^2 = 0$$

Degenerate case (single point):

$$\left\{\left(\frac{3}{2}, 0\right)\right\}$$

53. $\qquad x^2 + y^2 - x - \frac{3}{2}y - \frac{3}{4} = 0$

$$\left(x^2 - x \quad \right) + \left(y^2 - \frac{3}{2}y \quad \right) = \frac{3}{4}$$

$$\boxed{\left[\frac{1}{2}(-1)\right]^2 = \frac{1}{4} \quad \left[\frac{1}{2}\left(-\frac{3}{2}\right)\right]^2 = \frac{9}{16}}$$

$$\left[\begin{array}{l}\left(x^2 - x + \frac{1}{4}\right) \\ + \left(y^2 - \frac{3}{2}y + \frac{9}{16}\right)\end{array}\right] = \frac{3}{4} + \frac{1}{4} + \frac{9}{16}$$

$$\left(x - \frac{1}{2}\right)^2 + \left(y - \frac{3}{4}\right)^2 = \frac{25}{16}$$

Center: $\left(\frac{1}{2}, \frac{3}{4}\right)$; Radius: $\sqrt{\frac{25}{16}} = \frac{5}{4}$

54. $\qquad x^2 + y^2 - \frac{2}{3}x - \frac{5}{3}y - \frac{5}{9} = 0$

$$\left(x^2 - \frac{2}{3}x \quad \right) + \left(y^2 - \frac{5}{3}y \quad \right) = \frac{5}{9}$$

$$\boxed{\left[\frac{1}{2}\left(-\frac{2}{3}\right)\right]^2 = \frac{1}{9} \quad \left[\frac{1}{2}\left(-\frac{5}{3}\right)\right]^2 = \frac{25}{36}}$$

$$\left[\begin{array}{l}\left(x^2 - \frac{2}{3}x + \frac{1}{9}\right) \\ + \left(y^2 - \frac{5}{3}y + \frac{25}{36}\right)\end{array}\right] = \frac{5}{9} + \frac{1}{9} + \frac{25}{36}$$

$$\left(x - \frac{1}{3}\right)^2 + \left(y - \frac{5}{6}\right)^2 = \frac{49}{36}$$

Center: $\left(\frac{1}{3}, \frac{5}{6}\right)$; Radius: $\sqrt{\frac{49}{36}} = \frac{7}{6}$

55. $(x - 4)^2 + (y - 6)^2 = (1.5)^2$

$\qquad (x - 4)^2 + (y - 6)^2 = 2.25$

56. $\left[x - (-32)\right]^2 + (y - 40)^2 = (20)^2$

$\qquad (x + 32)^2 + (y - 40)^2 = 400$

57.
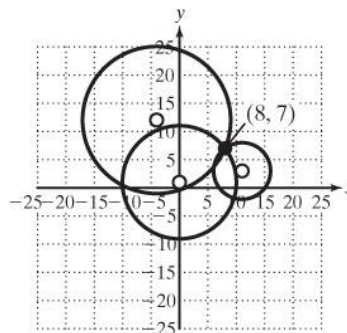
The approximate location of the earthquake is $(8, 7)$.

58.
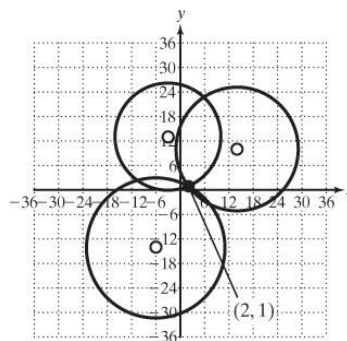
The fire is located at approximately coordinate $(2, 1)$.

59. A circle is the set of all points in a plane that are equidistant from a fixed point called the center.

60. The center and radius can easily be identified from an equation of a circle written in standard form.

61. $\sqrt{(-2-4)^2+(6-y)^2}=10$

$\sqrt{36+36-12y+y^2}=10$

$\sqrt{72-12y+y^2}=10$

$72-12y+y^2=100$

$y^2-12y-28=0$

$(y+2)(y-14)=0$

$y=-2 \quad$ or $\quad y=14$

$y=-2$ and $y=14$

62. $\sqrt{(4-x)^2+[2-(-1)]^2}=5$

$\sqrt{16-8x+x^2+9}=5$

$\sqrt{25-8x+x^2}=5$

$25-8x+x^2=25$

$x^2-8x=0$

$x(x-8)=0$

$x=0 \quad$ or $\quad x=8$

$x=0$ and $x=8$

63. $\sqrt{(2-x)^2+(4-x)^2}=6$

$\sqrt{4-4x+x^2+16-8x+x^2}=6$

$\sqrt{20-12x+2x^2}=6$

$20-12x+2x^2=36$

$2x^2-12x-16=0$

$x^2-6x-8=0$

$x=\dfrac{-(-6)\pm\sqrt{(-6)^2-4(1)(-8)}}{2(1)}$

$=\dfrac{6\pm\sqrt{68}}{2}=\dfrac{6\pm2\sqrt{17}}{2}=3\pm\sqrt{17}$

$\left(3+\sqrt{17},3+\sqrt{17}\right)$ and

$\left(3-\sqrt{17},3-\sqrt{17}\right)$

64. $\sqrt{(-4-x)^2+[6-(-x)]^2}=4$

$\sqrt{(-4-x)^2+(6+x)^2}=4$

$\sqrt{16+8x+x^2+36+12x+x^2}=4$

$\sqrt{52+20x+2x^2}=4$

$52+20x+2x^2=16$

$2x^2+20x+36=0$

$x^2+10x+18=0$

$x=\dfrac{-(10)\pm\sqrt{(10)^2-4(1)(18)}}{2(1)}$

$=\dfrac{-10\pm\sqrt{28}}{2}=\dfrac{-10\pm2\sqrt{7}}{2}$

$=-5\pm\sqrt{7}$

$\left(-5+\sqrt{7},5-\sqrt{7}\right)$ and

$\left(-5-\sqrt{7},5+\sqrt{7}\right)$

65. a.

b.

c.

c.

d.

d.

66. a.

67. a.

b.

b.

c.

d.

68. a.

b.

c.

d.
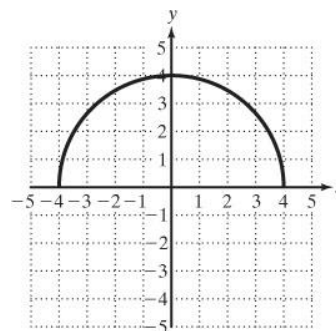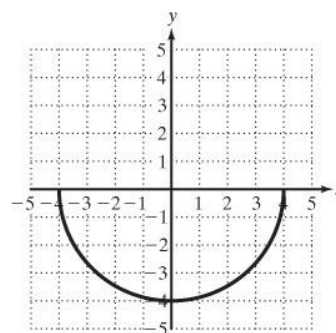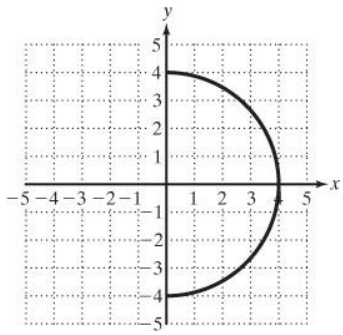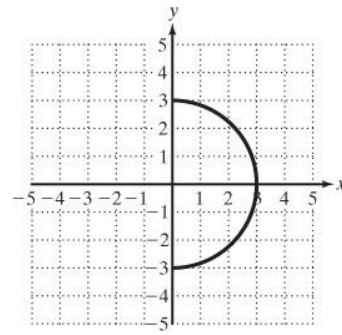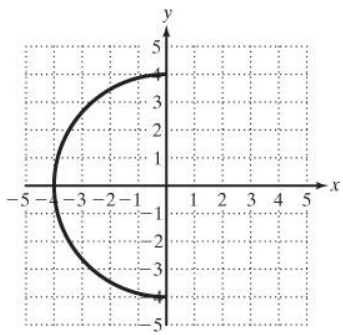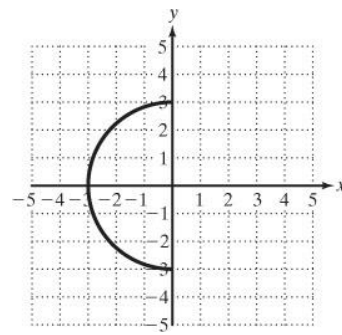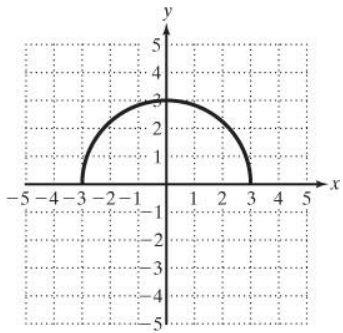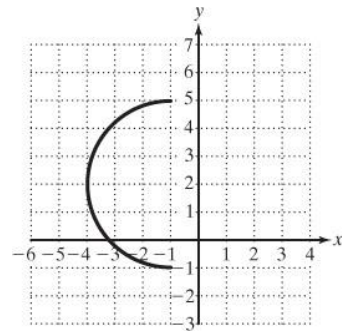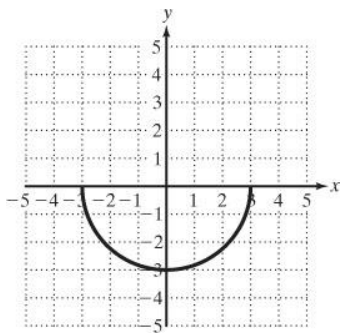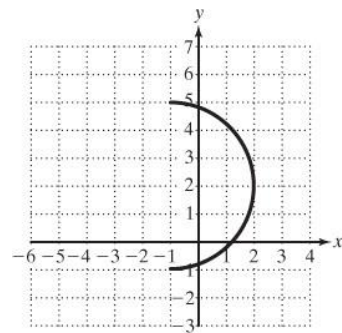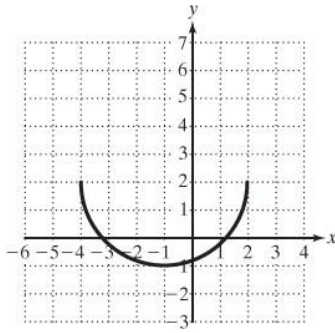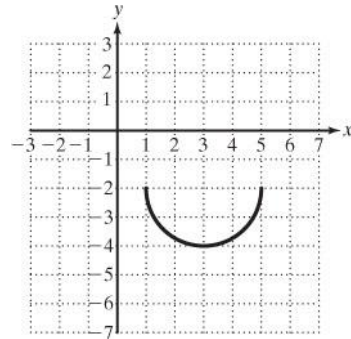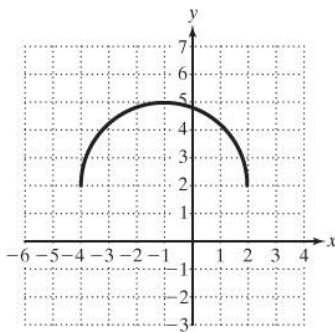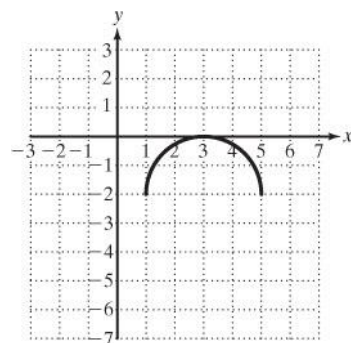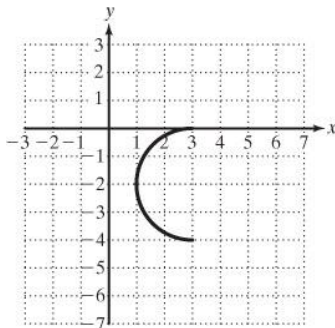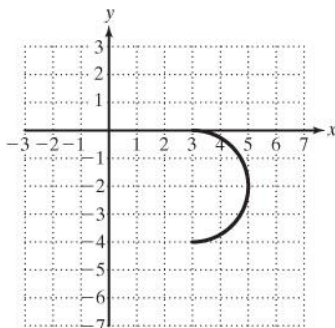
69.

$$x^2 + y^2 - 6x - 12y + 41 = 0$$

$$\left(x^2 - 6x \quad\right) + \left(y^2 - 12y \quad\right) = -41$$

$$\left[\frac{1}{2}(-6)\right]^2 = 9 \qquad \left[\frac{1}{2}(-12)\right]^2 = 36$$

$$\left[\begin{array}{l}\left(x^2 - 6x + 9\right)\\ + \left(y^2 - 12y + 36\right)\end{array}\right] = -41 + 9 + 36$$

$$\left(x - 3\right)^2 + \left(y - 6\right)^2 = 4$$

Center: $\left(3, 6\right)$; Radius: $\sqrt{4} = 2$

The distance between the origin $(0, 0)$
and the center $(3, 6)$ is $\sqrt{3^2 + 6^2} = 3\sqrt{5}$.
Therefore, the distance between the
origin and the point on the circle is
$3\sqrt{5} - 2$.

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70.
$$x^2 + y^2 + 4x - 12y + 31 = 0$$
$$\left(x^2 + 4x \quad\right) + \left(y^2 - 12y \quad\right) = -31$$

$$\left[\frac{1}{2}(4)\right]^2 = 4 \qquad \left[\frac{1}{2}(-12)\right]^2 = 36$$

$$\begin{bmatrix} \left(x^2 + 4x + 4\right) \\ + \left(y^2 - 12y + 36\right) \end{bmatrix} = -31 + 4 + 36$$
$$\left(x + 2\right)^2 + \left(y - 6\right)^2 = 9$$

Center: $\left(-2, 6\right)$; Radius: $\sqrt{9} = 3$

The distance between the origin (0, 0) and the center (3, 6) is

$$\sqrt{2^2 + 6^2} = 2\sqrt{10} \,.$$

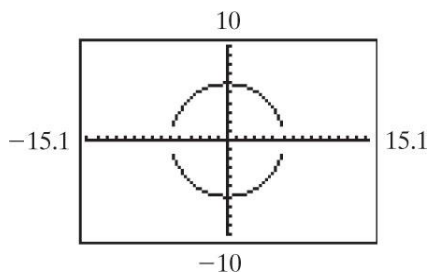Therefore, the distance between the origin and the point on the circle is

$2\sqrt{10} - 3$.

71. $\left[-15.1, 15.1, 1\right]$ by $\left[-10, 10, 1\right]$

72. $\left[-15.1, 15.1, 1\right]$ by $\left[-10, 10, 1\right]$
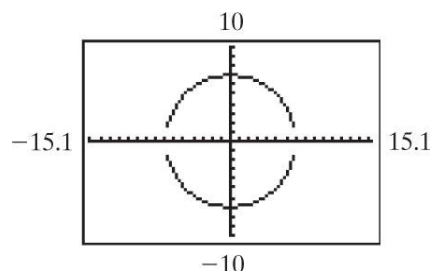
73. $\left[-14, 39, 5\right]$ by $\left[-30, 5, 5\right]$
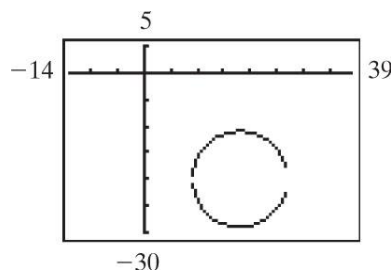
74. $\left[-0.28, 0.18, 0.02\right]$ by $\left[-0.1, 0.2, 0.02\right]$
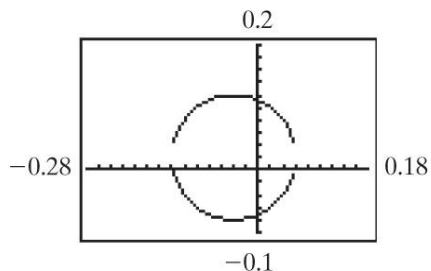
Section 2.3 Functions and Relations

1. relation; domain; y

2. $(2, 4)$

3. y

4. $f(x)$

5. –5

6. 5

7. Yes. For a given time after tree is planted, there cannot be two or more different heights.
That is, the height is unique at any given time.

8. No. There are numerous times during the year when the temperature in Fort Collins, is 70°.
Therefore, given an input value of 70°, there is more than one output value (time).

9. a. {(Tom Hanks, 5), (Jack Nicholson, 12), (Sean Penn, 5), (Dustin Hoffman, 7)}

  b. {Tom Hanks, Jack Nicholson, Sean Penn, Dustin Hoffman}

  c. {5, 12, 7}

  d. Yes

10. a. {(Albany, 285), (Denver, 5883), (Miami, 11), (San Francisco, 11)}

  b. {Albany, Denver, Miami, San Francisco}

  c. {285, 5883, 11}

  d. Yes

11. a. $\left\{ \begin{array}{l} (-4, 3), (-2, -3), (1, 4), \\ (3, -2), (3, 1) \end{array} \right\}$

  b. $\left\{ -4, -2, 1, 3 \right\}$

  c. $\left\{ 3, -3, 4, -2, 1 \right\}$

  d. No

12. a. $\left\{ \begin{array}{l} (-2, 2), (-1, 0), (0, -2), \\ (1, 0), (1, 2) \end{array} \right\}$

  b. $\left\{ -2, -1, 0, 1 \right\}$

  c. $\left\{ 2, 0, -2 \right\}$

  d. No

13. False

14. True

15. No vertical line intersects the graph in more than one point. This relation is a function.

16. No vertical line intersects the graph in more than one point. This relation is a function.

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17. There is at least one vertical line that intersects the graph in more than one point.
This relation is not a function.

18. No vertical line intersects the graph in more than one point. This relation is a function.

19. No vertical line intersects the graph in more than one point. This relation is a function.

20. There is at least one vertical line that intersects the graph in more than one point.
This relation is not a function.

21. No vertical line intersects the graph in more than one point. This relation is a function.

22. There is at least one vertical line that intersects the graph in more than one point.
This relation is not a function.

23. This mapping defines the set of ordered pairs: $\left\{\left(-8, 3\right), \left(-8, 4\right), \left(-8, 5\right), \left(-8, 6\right)\right\}$.
All four ordered pairs have the same x value but different y values. This relation is not a function.

24. This mapping defines the set of ordered pairs: $\left\{\left(-3, 11\right), \left(-6, 13\right), \left(-6, 9\right)\right\}$.
Two ordered pairs have the same x value but different y values. This relation is not a function.

25. This mapping defines the set of ordered pairs: $\left\{\left(1, 4\right), \left(2, 5\right), \left(3, 5\right)\right\}$.

No two ordered pairs have the same x value but different y values. This relation is a function.

26. This mapping defines the set of ordered pairs:
$\left\{\left(5, -4\right), \left(6, -4\right), \left(7, -4\right), \left(8, -4\right)\right\}$.
No two ordered pairs have the same x value but different y values. This relation is a function.

27. $\left(x + 1\right)^{2} + \left(y + 5\right)^{2} = 25$
This equation represents the graph of a circle with center $\left(-1, -5\right)$ and radius 5. This relation is not a function because it fails the vertical line test.

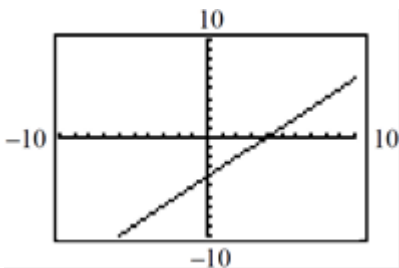**28.** $\left(x + 3\right)^{2} + \left(y + 4\right)^{2} = 1$
This equation represents the graph of a circle with center $\left(-3, -4\right)$ and radius 1. This relation is not a function because it fails the vertical line test.
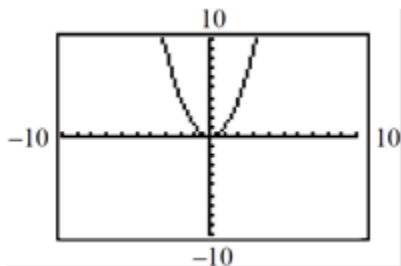
29. $y = x + 3$
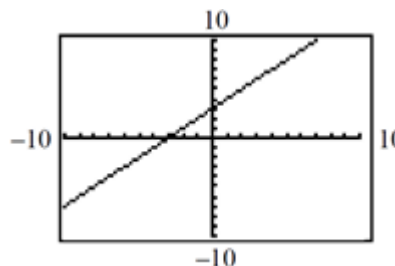
No vertical line intersects the graph in more than one point. This relation is a function.

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30. $y = x - 4$

No vertical line intersects the graph in more than one point. This relation is a function.

31. a. $y = x^2$

No vertical line intersects the graph in more than one point. This relation is a function.
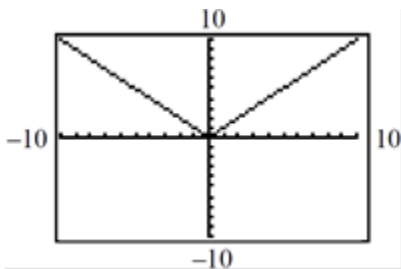
b. $x = y^2$

$y^2 = x$

$y = \pm\sqrt{x}$

x	y	$y = \pm\sqrt{x}$	Ordered
0	0	$y = \pm\sqrt{0} = 0$	$(0, 0)$
1	± 1	$y = \pm\sqrt{1} = \pm 1$	$(1, \pm 1)$
4	± 2	$y = \pm\sqrt{4} = \pm 2$	$(4, \pm 2)$
9	± 3	$y = \pm\sqrt{9} = \pm 3$	$(9, \pm 3)$

Two or more ordered pairs have the same x value but different y values. This relation is not a function.

32. a. $y = |x|$

$y = \pm x$

No vertical line intersects the graph in more than one point. This relation is a function.

b. $x = |y|$

$|y| = x$

$y = \pm x$

x	y	$y = \pm x$	Ordered
0	0	$y = \pm 0$	$(0, 0)$
1	± 1	$y = \pm 1$	$(1, \pm 1)$
2	± 2	$y = \pm 2$	$(2, \pm 2)$
3	± 3	$y = \pm 3$	$(3, \pm 3)$

Two or more ordered pairs have the same x value but different y values. This relation is not a function.

33. $(4, 1)$

34. $(7, -5)$

35. $f(x) = x^2 + 3x$

   a. $f(-2) = (-2)^2 + 3(-2) = 4 - 6 = -2$

   b. $f(-1) = (-1)^2 + 3(-1) = 1 - 3 = -2$

   c. $f(0) = (0)^2 + 3(0) = 0 + 0 = 0$

   d. $f(1) = (1)^2 + 3(1) = 1 + 3 = 4$

   e. $f(2) = (2)^2 + 3(2) = 4 + 6 = 10$

36. $g(x) = \dfrac{1}{x}$

   a. $g(-2) = \dfrac{1}{(-2)} = -\dfrac{1}{2}$

b. $g(-1) = \dfrac{1}{(-1)} = -1$

c. $g\left(-\dfrac{1}{2}\right) = \dfrac{1}{\left(-\dfrac{1}{2}\right)} = -2$

d. $g\left(\dfrac{1}{2}\right) = \dfrac{1}{\left(\dfrac{1}{2}\right)} = 2$

e. $g(2) = \dfrac{1}{(2)} = \dfrac{1}{2}$

37. $h(x) = 5$

   a. $h(-2) = 5$

   b. $h(-1) = 5$

   c. $h(0) = 5$

   d. $h(1) = 5$

   e. $h(2) = 5$

38. $k(x) = \sqrt{x + 1}$

   a. $k(-2) = \sqrt{(-2) + 1} = \sqrt{-1}$

     Undefined

   b. $k(-1) = \sqrt{(-1) + 1} = \sqrt{0} = 0$

   c. $k(0) = \sqrt{(0) + 1} = \sqrt{1} = 1$

   d. $k(1) = \sqrt{(1) + 1} = \sqrt{2}$

   e. $k(3) = \sqrt{(3) + 1} = \sqrt{4} = 2$

39. $g(3) = \dfrac{1}{(3)} = \dfrac{1}{3}$

40. $h(-7) = 5$

41. $g\left(\dfrac{1}{3}\right) = \dfrac{1}{\left(\dfrac{1}{3}\right)} = 3$

42. $h(7) = 5$

43. $k(-5) = \sqrt{(-5) + 1} = \sqrt{-4}$

   Undefined

44. $f(5) = (5)^2 + 3(5) = 25 + 15 = 40$

45. $k(8) = \sqrt{(8)+1} = \sqrt{9} = 3$

46. $f(-5) = (-5)^2 + 3(-5) = 25 - 15 = 10$

47. $g(t) = \dfrac{1}{(t)} = \dfrac{1}{t}$

48. $f(a) = (a)^2 + 3(a) = a^2 + 3a$

49. $k(x+h) = \sqrt{(x+h)+1} = \sqrt{x+h+1}$

50. $h(x+h) = 5$

51. $f(a+4) = (a+4)^2 + 3(a+4)$
$\qquad = a^2 + 8a + 16 + 3a + 12$
$\qquad = a^2 + 11a + 28$

52. $f(t-3) = (t-3)^2 + 3(t-3)$
$\qquad = t^2 - 6t + 9 + 3t - 9 = t^2 - 3t$

53. $g(0) = \dfrac{1}{0}$
Undefined

54. $k(-10) = \sqrt{(-10)+1} = \sqrt{-9}$
Undefined

55. $f(x+h) = (x+h)^2 + 3(x+h)$
$\qquad = x^2 + 2xh + h^2 + 3x + 3h$

56. $g(x+h) = \dfrac{1}{x+h}$

57. $f(x+h) = -4(x+h)^2 - 5(x+h) + 2$
$\qquad = -4(x^2 + 2xh + h^2) - 5x - 5h + 2$
$\qquad = -4x^2 - 8xh - 4h^2 - 5x - 5h + 2$

58. $f(x+h) = -2(x+h)^2 + 6(x+h) - 3$
$\qquad = -2(x^2 + 2xh + h^2) + 6x + 6h - 3$
$\qquad = -2x^2 - 4xh - 2h^2 + 6x + 6h - 3$

59. $f(x+h) = 7 - 3(x+h)^2$
$\qquad = 7 - 3(x^2 + 2xh + h^2)$
$\qquad = -3x^2 - 6xh - 3h^2 + 7$

60. $f(x+h) = 11 - 5(x+h)^2$
$\qquad = 11 - 5(x^2 + 2xh + h^2)$
$\qquad = -5x^2 - 10xh - 5h^2 + 11$

61. $f(x+h) = (x+h)^3 + 2(x+h) - 5$
$\qquad = (x^3 + h^3 + 3x^2h + 3xh^2)$
$\qquad\quad + 2x + 2h - 5$
$\qquad = x^3 + h^3 + 3x^2h + 3xh^2$
$\qquad\quad + 2x + 2h - 5$

62. $f(x+h) = (x+h)^3 - 4(x+h) + 2$
$\qquad = (x^3 + h^3 + 3x^2h + 3xh^2)$
$\qquad\quad - 4x - 4h + 2$
$\qquad = x^3 + h^3 + 3x^2h + 3xh^2$
$\qquad\quad - 4x - 4h + 2$

63. $f(9) = 7$

64. $f(-1) = 6$

65. $f(3) = 4$

66. $f(2) = 3$

67. $f(x) = 6$ when $x = -1$

68. $f(x) = 7$ when $x = 9$

69. $f(x) = 3$ when $x = 2$

70. $f(x) = 4$ when $x = 3$

71. a. $d(t) = 18t$
$\qquad d(2) = 18(2) = 36$
$\qquad$ Joe rides 36 mi in 2 hr.

$\quad$ **b.** 40 min is $\dfrac{40}{60} = \dfrac{2}{3}$ hr.
$\qquad d(t) = 18t$
$\qquad d\left(\dfrac{2}{3}\right) = 18\left(\dfrac{2}{3}\right) = 12$
$\quad$ 12 mi

72. a. $r(x) = 250 - x$
$\qquad r(50) = 250 - (50) = 200$

258

After having driven 50 mi, Frank still has 200 mi remaining.

b. $r(x) = 250 - x$

$r(122) = 250 - (122) = 128$

128 mi

73. $C(x) = x + 0.06x + 0.18x$

$C(225) = (225) + 0.06(225) + 0.18(225)$

$= 225 + 13.5 + 40.5 = 279$

If the cost of the food is $225, then the total bill including tax and tip is $279.

74. $P(x) = 1.075(x + 0.40x)$

$P(60) = 1.075\big[(60) + 0.40(60)\big]$

$= 1.075(60 + 24) = 1.075(84)$

$= 90.30$

If the cost of the book from the publisher is $60, the cost to the student after the bookstore markup and sales tax is $90.30.

75. Solve $f(x) = 0$:   Evaluate $f(0)$:

$f(x) = 2x - 4$ $f(x) = 2x - 4$

$0 = 2x - 4$ $f(0) = 2(0) - 4$

$2x = 4$ $= -4$

$x = 2$

x-intercept: $(2, 0)$; y-intercept: $(0, -4)$

76. Solve $g(x) = 0$:   Evaluate $g(0)$:

$g(x) = 3x - 12$ $g(x) = 3x - 12$

$0 = 3x - 12$ $g(0) = 3(0) - 12$

$3x = 12$ $= -12$

$x = 4$

x-intercept: $(4, 0)$; y-intercept: $(0, -12)$

77. Solve $h(x) = 0$:   Evaluate $h(0)$:

$h(x) = |x| - 8$ $h(x) = |x| - 8$

$0 = |x| - 8$ $h(0) = |(0)| - 8$

$|x| = 8$ $= -8$

$x = \pm 8$

x-intercepts: $(8, 0), (-8, 0)$;

y-intercept: $(0, -8)$

78. Solve $k(x) = 0$:   Evaluate $k(0)$:

$k(x) = -|x| + 2$ $k(x) = -|x| + 2$

$0 = -|x| + 2$ $k(0) = -|(0)| + 2$

$|x| = 2$ $= 2$

$x = \pm 2$

x-intercepts: $(2, 0), (-2, 0)$;

y-intercept: $(0, 2)$

79. Solve $p(x) = 0$:   Evaluate $p(0)$:

$p(x) = -x^2 + 12$

$0 = -x^2 + 12$

$x^2 = 12$

$x = \pm\sqrt{12} = \pm 2\sqrt{3}$

$p(x) = -x^2 + 12$

$p(0) = -(0)^2 + 12$

$= 12$

x-intercepts: $(2\sqrt{3}, 0), (-2\sqrt{3}, 0)$;

y-intercept: $(0, 12)$

80. Solve $q(x) = 0$:   Evaluate $q(0)$:

$q(x) = x^2 - 8$ $q(x) = x^2 - 8$

$0 = x^2 - 8$ $q(0) = (0)^2 - 8$

$x^2 = 8$ $= -8$

$x = \pm\sqrt{8} = \pm 2\sqrt{2}$

x-intercepts: $(2\sqrt{2}, 0), (-2\sqrt{2}, 0)$;

y-intercept: $(0, -8)$

81. Solve $r(x) = 0$:   Evaluate $r(0)$:

$$r(x) = |x - 8| \qquad r(x) = |x - 8|$$
$$0 = |x - 8| \qquad r(0) = |(0) - 8|$$
$$x - 8 = 0 \qquad\qquad = 8$$
$$x = 8$$

x-intercept: $(8, 0)$; y-intercept: $(0, 8)$

82. Solve $s(x) = 0$: Evaluate $s(0)$:

$$s(x) = |x + 3| \qquad s(x) = |x + 3|$$
$$0 = |x + 3| \qquad s(0) = |(0) + 3|$$
$$x + 3 = 0 \qquad\qquad = 3$$
$$x = -3$$

x-intercept: $(-3, 0)$; y-intercept: $(0, 3)$

83. Solve $f(x) = 0$: Evaluate $f(0)$:

$$f(x) = \sqrt{x} - 2 \qquad f(x) = \sqrt{x} - 2$$
$$0 = \sqrt{x} - 2 \qquad f(0) = \sqrt{(0)} - 2$$
$$\sqrt{x} = 2 \qquad\qquad = -2$$
$$x = 4$$

x-intercept: $(4, 0)$; y-intercept: $(0, -2)$

84. Solve $g(x) = 0$: Evaluate $g(0)$:

$$g(x) = -\sqrt{x} + 3 \quad g(x) = -\sqrt{x} + 3$$
$$0 = -\sqrt{x} + 3 \quad g(0) = -\sqrt{(0)} + 3$$
$$\sqrt{x} = 3 \qquad\qquad = 3$$
$$x = 9$$

x-intercept: $(9, 0)$; y-intercept: $(0, 3)$

85. Evaluate $A(0)$:

$$A(t) = 14,280 - 247t$$
$$A(0) = 14,820 - 0 = 14,280$$

The y-intercept is $(0, 14,280)$ and means that the amount owed after the initial down payment is \$14,280. The t-intercept is $(60, 0)$ and means that after 60 months, the amount owed is \$0.

86. Evaluate $f(0)$:

$$f(x) = 9.4x + 35.7$$
$$f(0) = 0 + 35.7 = 35.7$$

$(0,\ 35.7)$; The y-intercept means that for $x = 0$ (the year 2006), the average amount spent on video games per person in the United States was \$35.70.

87. Domain: $\{-3, -2, -1, 2, 3\}$;

Range: $\{-4, -3, 3, 4, 5\}$

88. Domain: $\{-4, -2, 0, 2, 4\}$;

Range: $\{-2, 0, 2, 4, 6\}$

89. Domain: $(-3, \infty)$; Range: $[1, \infty)$

90. Domain: $(-\infty, 4)$; Range: $(-\infty, 4]$

91. Domain: $(-\infty, \infty)$; Range: $(-\infty, \infty)$

92. Domain: $(-\infty, \infty)$; Range: $(-\infty, \infty)$

93. Domain: $(-\infty, \infty)$; Range: $[-3, \infty)$

94. Domain: $(-\infty, \infty)$; Range: $(-\infty, 2]$

95. Domain: $(-5, 1]$; Range: $\{-1, 1, 3\}$

96. Domain: $[-4, 5)$; Range: $\{-2, 1, 4\}$

97. a. $x - 4 \neq 0$

$$x \neq 4$$
$$(-\infty, 4) \cup (4, \infty)$$

b. $(x + 2) \neq 0 \qquad (x - 2) \neq 0$

$$x \neq -2 \qquad\qquad x \neq 2$$
$$(-\infty, -2) \cup (-2, 2) \cup (2, \infty)$$

c. The denominator $x^2 + 4$ is always positive.

$$(-\infty, \infty)$$

98. a. $x - 2 \neq 0$

$$x \neq 2$$
$$(-\infty, 2) \cup (2, \infty)$$

b. The denominator $x^2 + 2$ is always positive.

$$\left(-\infty, \infty\right)$$

c. $\left(x + \sqrt{2}\right) \neq 0 \qquad \left(x - \sqrt{2}\right) \neq 0$

$$x \neq -\sqrt{2} \qquad\qquad x \neq \sqrt{2}$$

$$\left(-\infty, -\sqrt{2}\right) \cup \left(-\sqrt{2}, \sqrt{2}\right) \cup \left(\sqrt{2}, \infty\right)$$

99. a. $x + 9 \geq 0$

$$x \geq -9$$

$$[-9, \infty)$$

b. $9 - x \geq 0$

$$x \leq 9$$

$$(-\infty, 9]$$

c. $x + 9 > 0$

$$x > -9$$

$$\left(-9, \infty\right)$$

100. a. $16 - t \geq 0$

$$t \leq 16$$

$$(-\infty, 16]$$

b. $t - 16 \geq 0$

$$t \geq 16$$

$$[16, \infty)$$

c. $16 - t > 0$

$$t < 16$$

$$(-\infty, 16)$$

101. a. There is no restriction on t.

$$\left(-\infty, \infty\right)$$

b. There is no restriction on t.

$$\left(-\infty, \infty\right)$$

c. $t - 5 > 0$

$$t > 5$$

$$\left(-\infty, 5\right) \cup \left(5, \infty\right)$$

102. a. There is no restriction on x.

$$\left(-\infty, \infty\right)$$

b. There is no restriction on x.

$$\left(-\infty, \infty\right)$$

c. $x - 3 > 0$

$$x > 3$$

$$\left(-\infty, 3\right) \cup \left(3, \infty\right)$$

103. a. There is no restriction on x.

$$\left(-\infty, \infty\right)$$

b. $x^2 - 3x - 28 \neq 0$

$$\left(x - 7\right)\left(x + 4\right) \neq 0$$

$$x \neq 7 \text{ and } x \neq -4$$

$$\left(-\infty, -4\right) \cup \left(-4, 7\right) \cup \left(7, \infty\right)$$

c. $x + 2 \neq 0$

$$x \neq -2$$

$$\left(-\infty, -2\right) \cup \left(-2, \infty\right)$$

104. a. There is no restriction on x.

$$\left(-\infty, \infty\right)$$

b. $x + 1 \neq 0$

$$x \neq -1$$

$$\left(-\infty, -1\right) \cup \left(-1, \infty\right)$$

c. $x^2 - 4x - 12 \neq 0$

$$\left(x - 6\right)\left(x + 2\right) \neq 0$$

$$x \neq 6 \text{ and } x \neq -2$$

$$\left(-\infty, -2\right) \cup \left(-2, 6\right) \cup \left(6, \infty\right)$$

105. a. $|x + 1| + 4$ is always positive.

$$\left(-\infty, \infty\right)$$

b. The denominator $|x + 1| + 4$ is always positive.

$$\left(-\infty, \infty\right)$$

c. $|x + 1| - 4 \neq 0$

$$\left(x + 1\right) - 4 \neq 0 \qquad -\left(x + 1\right) - 4 \neq 0$$

$$x \neq 3 \qquad\qquad x \neq -5$$

$$\left(-\infty, -5\right) \cup \left(-5, 3\right) \cup \left(3, \infty\right)$$

106. a. There is no restriction on a.

$$\left(-\infty, \infty\right)$$

b. $8 - |a - 2| \neq 0$

$8 - (a - 2) \neq 0 \qquad 8 + (a - 2) \neq 0$

$\qquad a \neq 10 \qquad\qquad a \neq -6$

$(-\infty, -6) \cup (-6, 10) \cup (10, \infty)$

c. The denominator $8 + |a - 2|$ is always positive.

$(-\infty, \infty)$

107. a. $x + 15 \geq 0$

$\qquad x \geq -15$

$\qquad [-15, \infty)$

b. $x + 15 \geq 0$

$\qquad x \geq -15$

$\qquad [-15, \infty)$

c. $x + 15 > 0 \qquad \sqrt{x + 15} - 2 \neq 0$

$\qquad x > -15 \qquad\quad x + 15 \neq 4$

$\qquad\qquad\qquad\qquad x \neq -11$

$[-15, -11] \cup (-11, \infty)$

108. a. $c + 20 \geq 0$

$\qquad c \geq -20$

$\qquad [-20, \infty)$

b. $c + 20 \geq 0$

$\qquad c \geq -20$

$\qquad [-20, \infty)$

c. $c + 20 > 0 \qquad \sqrt{c + 20} - 1 \neq 0$

$\qquad c > -20 \qquad\quad c + 20 \neq 1$

$\qquad\qquad\qquad\qquad c \neq -19$

$[-20, -19] \cup (-19, \infty)$

109. a. There is no restriction on x.

$\qquad (-\infty, \infty)$

b. $[0, \infty)$

c. $[0, 7)$

110. a. There is no restriction on x.

$\qquad (-\infty, \infty)$

b. $(-\infty, 0)$

c. $(-2, 2)$

111. a. $f(-2) = -4$

b. $f(3) = 2$

c. $f(x) = -1$ for $x = -3$, $x = -1$, $x = 1$

d. $f(x) = -4$ for $x = -2$, $x = 2$

e. x-intercepts: $(0, 0)$ and $\left(-\dfrac{10}{3}, 0\right)$

f. y-intercept: $(0, 0)$

g. Domain: $(-\infty, \infty)$

h. Range: $[-4, \infty)$

112. a. $f(-2) = 1$

b. $f(3) = 3$

c. There are no x-values where $f(x) = -1$.

d. $f(x) = -4$ for all x on the interval $[0, 2)$

e. x-intercept: $(-1, 0)$

f. y-intercept: $(0, -4)$

g. Domain: $(-\infty, \infty)$

h. Range: $\{-4\} \cup [0, \infty)$

113. a. $f(-2) = 0$

b. $f(3) = 5$

c. $f(x) = -1$ for $x = -3$, $x = -1$, $x = 1$

d. $f(x) = -4$ for $x = -4$, $x = 0$

e. x-intercepts: $(-2, 0)$ and $\left(\dfrac{4}{3}, 0\right)$

f. y-intercept: $(0, -4)$

g. Domain: $[-4, \infty)$

h. Range: $[-4, 5]$

114. a. $f(-2) = -3$

b. $f(3) = -2$

c. $f(x) = -1$ for $x = 0$, $x = 2$

d. $f(x) = -4$ for $x = -3$, $x = 5$

e. x-intercept: $(1, 0)$

f. y-intercept: $(0, -1)$

g. Domain: $(-\infty, 5]$

h. Range: $[-4, 0] \cup \{3\}$

115. $r(x) + x = 400$

$r(x) = 400 - x$

116. $w(x) + x = 200$

$w(x) = 200 - x$

117. $P(x) = x + x + x$

$P(x) = 3x$

118. $2A(x) + x = 180$

$2A(x) = 180 - x$

$A(x) = \dfrac{180 - x}{2}$

119. $C(x) + x = 90$

$C(x) = 90 - x$

120. $S(x) + x = 180$

$S(x) = 180 - x$

121. $f(x) = 3x^2 - 2$

122. $f(x) = \sqrt{x} + 3$

123. If two points in a set of ordered pairs are aligned vertically in a graph, then they have the same x-coordinate but different y-coordinates. This contradicts the definition of a function. Therefore, the points do not define y as a function of x.

124. To find the x-intercept(s), find the real solutions to the equation $f(x) = 0$. To find the y-intercept, evaluate $f(0)$.

125. a. $P(s) = 4s$

b. $A(s) = s^2$

c. $A(P) = \left(\dfrac{P}{4}\right)^2$ or $A(P) = \dfrac{P^2}{16}$

d. $P(A) = 4\sqrt{A}$

e. $\left[d(s)\right]^2 = s^2 + s^2 = 2s^2$

$d(s) = \sqrt{2s^2}$

$d(s) = s\sqrt{2}$

f. $\left[s(d)\right]^2 + \left[s(d)\right]^2 = d^2$

$2\left[s(d)\right]^2 = d^2$

$\left[s(d)\right]^2 = \dfrac{d^2}{2}$

$s(d) = \sqrt{\dfrac{d^2}{2}}$

$s(d) = \dfrac{d}{\sqrt{2}}$

or $s(d) = \dfrac{d\sqrt{2}}{2}$

g. $P(d) = 4\left(\dfrac{d\sqrt{2}}{2}\right)$

$P(d) = 2\sqrt{2}d$

h. $A(d) = \left(\dfrac{d\sqrt{2}}{2}\right)^2$

$A(d) = \dfrac{2d^2}{4}$

$A(d) = \dfrac{d^2}{2}$

126. a. $C(r) = 2\pi r$

b. $A(r) = \pi r^2$

c. $r(d) = \dfrac{d}{2}$

d. $d(r) = 2r$

e. $C(d) = \pi d$

f. $A(d) = \pi\left(\dfrac{d}{2}\right)^2$

$A(d) = \dfrac{\pi d^2}{4}$

g. $A(C) = \pi\left(\dfrac{C}{2\pi}\right)^2$

$A(C) = \dfrac{\pi C^2}{4\pi^2}$

$A(C) = \dfrac{C^2}{4\pi}$

h. $C(A) = 2\pi\sqrt{\dfrac{A}{\pi}}$

$C(A) = 2\sqrt{\dfrac{\pi^2 A}{\pi}}$

$C(A) = 2\sqrt{\pi A}$

Section 2.4 Linear Equations in Two Variables and Linear Functions

1. linear

2. vertical

3. horizontal

4. $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

5. zero; undefined

6. $mx + b$

7. $m = \dfrac{f(x_2) - f(x_1)}{x_2 - x_1}$

8. horizontal

9. $-3x + 4y = 12$

$\qquad 4y = 3x + 12$

$\qquad y = \dfrac{3}{4}x + 3$

x	y
-4	0
-2	$\dfrac{3}{2}$
0	3
2	$\dfrac{9}{2}$
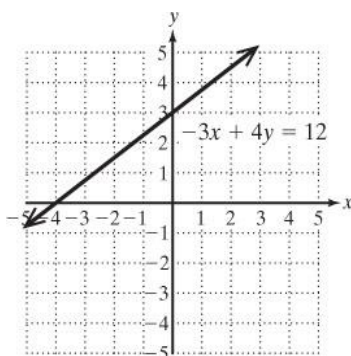
x-intercept: $(-4, 0)$; y-intercept: $(0, 3)$

10. $-2x + y = 4$

$y = 2x + 4$

x	y
-3	-2
-2	0
-1	2
0	4
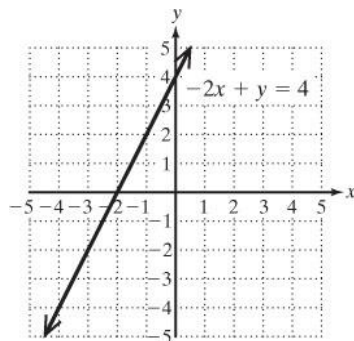
x-intercept: $\left(-2, 0\right)$; y-intercept: $\left(0, 4\right)$

11. $2y = -5x + 2$

$y = -\dfrac{5}{2}x + 1$

x	y
-1	$\dfrac{7}{2}$
0	1
$\dfrac{2}{5}$	0
2	-4
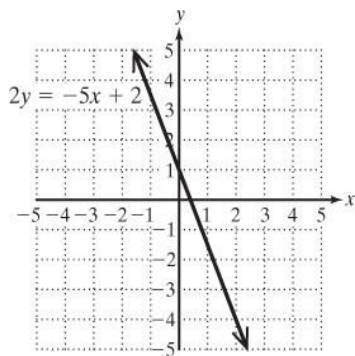
x-intercept: $\left(\dfrac{2}{5}, 0\right)$; y-intercept: $\left(0, 1\right)$

12. $3y = -4x + 6$

$y = -\dfrac{4}{3}x + 2$

x	y
-1	$\dfrac{10}{3}$
0	2
$\dfrac{3}{2}$	0
3	-2

265
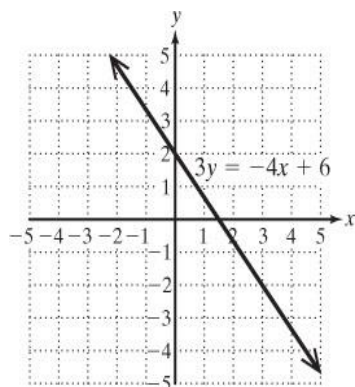
x-intercept: $\left(\dfrac{3}{2}, 0\right)$; y-intercept: $\left(0, 2\right)$

13. $x = -6$

x	y
-6	-2
-6	0
-6	2
-6	4
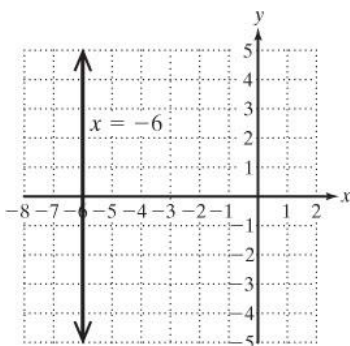
x-intercept: $\left(-6, 0\right)$; y-intercept: None

14. $y = 4$

x	y
-2	4
0	4
2	4
4	4
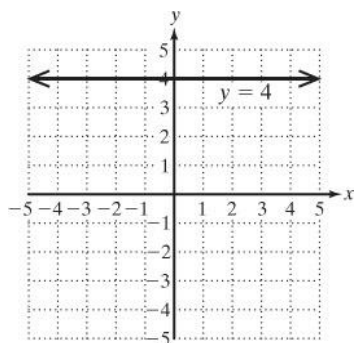
x-intercept: None; y-intercept: $\left(0, 4\right)$

15. $5y + 1 = 11$

$\quad\quad 5y = 10$

$\quad\quad\quad y = 2$

x	y
-2	2
0	2
2	2
4	2

x-intercept: None; y-intercept: $\left(0, 2\right)$

266
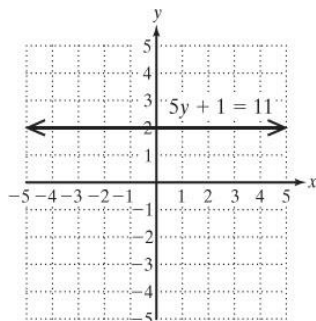
16. $3x - 2 = 4$

$\qquad 3x = 6$

$\qquad x = 2$

x	*y*
2	−2
2	0
2	2
2	4
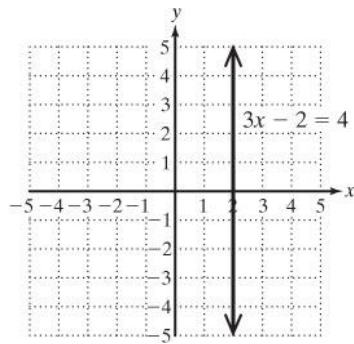
x-intercept: $(2, 0)$; y-intercept: None

17. $0.02x + 0.05y = 0.1$

$\qquad 2x + 5y = 10$

$\qquad 5y = -2x + 10$

$\qquad y = -\dfrac{2}{5}x + 2$

x	*y*
−5	4
0	2
1	$\dfrac{8}{5}$
5	0
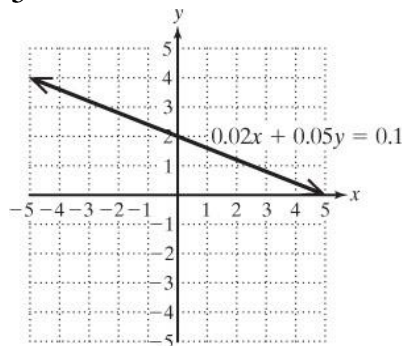
x-intercept: $(5, 0)$; y-intercept: $(0, 2)$

18. $0.03x + 0.07y = 0.21$

$\qquad 3x + 7y = 21$

$\qquad 7y = -3x + 21$

$\qquad y = -\dfrac{3}{7}x + 3$

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x	y
-1	$\dfrac{24}{7}$
0	3
1	$\dfrac{18}{7}$
7	0

$0.03x + 0.07y = 0.21$
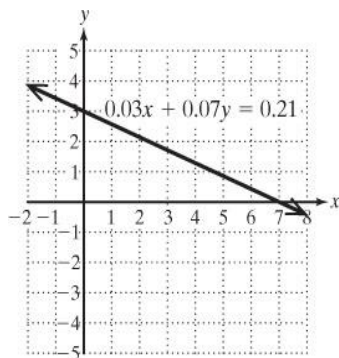
x-intercept: $\left(7, 0\right)$; *y*-intercept: $\left(0, 3\right)$

19. $2x = 3y$

$y = \dfrac{2}{3}x$

x	y
-3	-2
0	0
3	2
6	4

$2x = 3y$
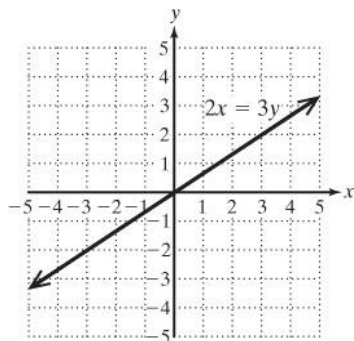
x-intercept: $\left(0, 0\right)$; *y*-intercept: $\left(0, 0\right)$

20. $2x = -5y$

$y = -\dfrac{2}{5}x$

x	y
-5	2
0	0
1	$-\dfrac{2}{5}$
5	-2

$2x = -5y$
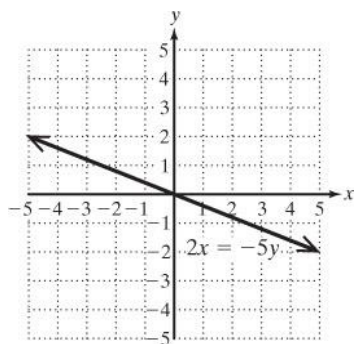
x-intercept: $\left(0, 0\right)$; *y*-intercept: $\left(0, 0\right)$

21. $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

$= \dfrac{300}{1000} = \dfrac{3}{10}$

22. $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

$= \dfrac{5}{80} = \dfrac{1}{16}$

23. $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

$= \dfrac{2.5}{100} = \dfrac{1}{40}$

24. $\text{pitch} = \dfrac{\text{rafter rise}}{\text{rafter run}} = \dfrac{7}{12}$

25. $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{-1 - (-7)}{2 - 4}$

$= \dfrac{6}{-2} = -3$

26. $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{6 - (-8)}{4 - (-3)}$

$= \dfrac{14}{7} = 2$

27. $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{-6 - 9}{42 - 17}$

$= \dfrac{-15}{25} = -\dfrac{3}{5}$

28. $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{-6 - 4}{-1 - (-9)}$

$= \dfrac{-10}{8} = -\dfrac{5}{4}$

29. $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{-39 - (-52)}{-22 - 30}$

$= \dfrac{13}{-52} = -\dfrac{1}{4}$

30. $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{30 - (-16)}{84 - (-100)}$

$= \dfrac{46}{184} = \dfrac{1}{4}$

31. $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{-3.7 - 4.1}{9.5 - 2.6}$

$= \dfrac{-7.8}{6.9} = -\dfrac{26}{23}$

32. $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{7.9 - 6.2}{-5.1 - 8.5}$

$= \dfrac{1.7}{-13.6} = -\dfrac{1}{8}$

33. $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{1 - 6}{\dfrac{5}{2} - \dfrac{3}{4}}$

$= \dfrac{-5}{\dfrac{10}{4} - \dfrac{3}{4}} = \dfrac{-5}{\dfrac{7}{4}} = -\dfrac{20}{7}$

34. $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{\dfrac{3}{10} - \dfrac{2}{5}}{4 - (-3)}$

$= \dfrac{\dfrac{3}{10} - \dfrac{4}{10}}{7} = \dfrac{-\dfrac{1}{10}}{7} = -\dfrac{1}{70}$

35. $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{\sqrt{5} - 2\sqrt{5}}{\sqrt{6} - 3\sqrt{6}}$

$= \dfrac{-\sqrt{5}}{-2\sqrt{6}} = \dfrac{\sqrt{5} \cdot \sqrt{6}}{2\sqrt{6} \cdot \sqrt{6}} = \dfrac{\sqrt{30}}{12}$

36. $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{-5\sqrt{3} - (-3\sqrt{3})}{\sqrt{11} - 2\sqrt{11}}$

$= \dfrac{-2\sqrt{3}}{-\sqrt{11}} = \dfrac{2\sqrt{3} \cdot \sqrt{11}}{\sqrt{11} \cdot \sqrt{11}} = \dfrac{2\sqrt{33}}{11}$

37. Use points $(-1, 1)$ and $(0, 4)$:

$m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{4 - 1}{0 - (-1)} = \dfrac{3}{1} = 3$

38. Use points $(-2, 0)$ and $(0, 1)$:

$m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{1 - 0}{0 - (-2)} = \dfrac{1}{2}$

39. Use points $(0, 2)$ and $(3, 1)$:

$m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{1 - 2}{3 - 0} = \dfrac{-1}{3} = -\dfrac{1}{3}$

40. Use points $(0, 3)$ and $(1, -1)$:

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1 - 3}{1 - 0} = \frac{-4}{1} = -4$$

41. Use points $(0, -4)$ and $(2, -4)$:

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-4 - (-4)}{2 - 0} = \frac{0}{2} = 0$$

42. Use points $\left(\frac{3}{2}, 1\right)$ and $\left(\frac{3}{2}, 3\right)$:

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3 - 1}{\frac{3}{2} - \frac{3}{2}} = \frac{2}{0} \text{ (undefined)}$$

43. Undefined

44. 0

45. 0

46. Undefined

47.
$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$\frac{4}{5} = \frac{y_2 - y_1}{52}$$
$$5(y_2 - y_1) = 208$$
$$y_2 - y_1 = 41.6 \text{ ft}$$

48.
$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$\frac{5}{8} = \frac{216}{x_2 - x_1}$$
$$5(x_2 - x_1) = 1728$$
$$x_2 - x_1 = 345.6 \text{ m}$$

49. Change in population over change in time.

50. Change in distance over change in time which is speed.

51. a. $2x - 4y = 8$
$$-4y = -2x + 8$$
$$y = \frac{1}{2}x - 2$$

$m = \frac{1}{2}$; y-intercept: $(0, -2)$

b.
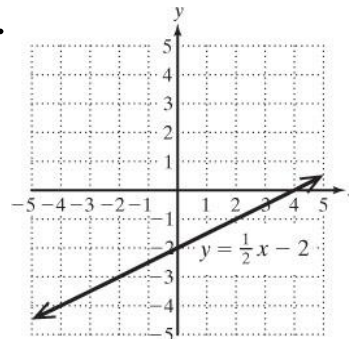
52. a. $3x - y = 6$
$$y = 3x - 6$$

$m = 3$; y-intercept: $(0, -6)$

b.
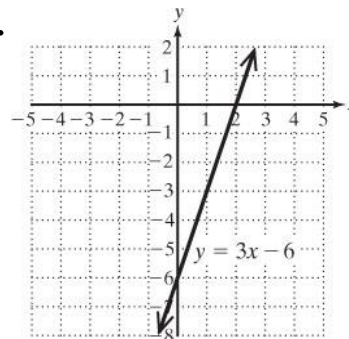
53. a. $3x = 2y - 4$
$$2y = 3x + 4$$
$$y = \frac{3}{2}x + 2$$

$m = \frac{3}{2}$; y-intercept: $(0, 2)$

b.
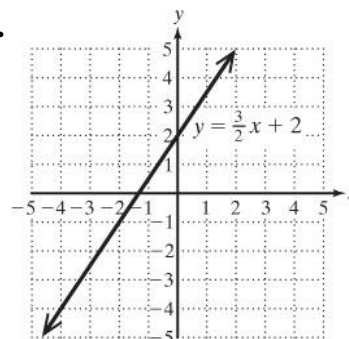
54. a. $5x = 3y - 6$

$3y = 5x + 6$

$y = \dfrac{5}{3}x + 2$

$m = \dfrac{5}{3}$; y-intercept: $\left(0, 2\right)$

b.

b.

55. a. $3x = 4y$

$y = \dfrac{3}{4}x$
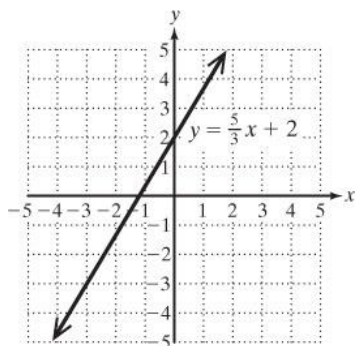
$m = \dfrac{3}{4}$; y-intercept: $\left(0, 0\right)$

b.
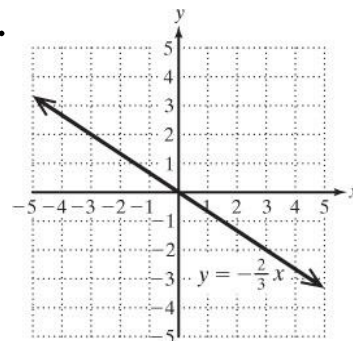
57. a. $2y - 6 = 8$

$2y = 14$

$y = 7$

$m = 0$; y-intercept: $\left(0, 7\right)$

b.

56. a. $-2x = 3y$

$y = -\dfrac{2}{3}x$

$m = -\dfrac{2}{3}$; y-intercept: $\left(0, 0\right)$

58. a. $3y + 9 = 6$

$3y = -3$

$y = -1$

$m = 0$; y-intercept: $\left(0, -1\right)$

b.

59. a. $0.02x + 0.06y = 0.06$

$$2x + 6y = 6$$
$$6y = -2x + 6$$
$$y = -\frac{1}{3}x + 1$$

$m = -\frac{1}{3}$; y-intercept: $(0, 1)$

b.

$y = -\frac{1}{3}x + 1$
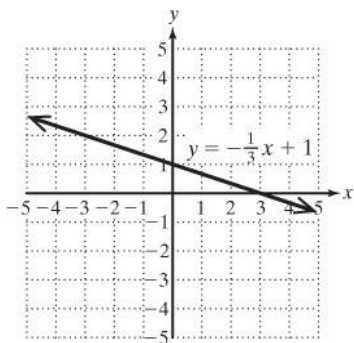
60. a. $0.03x + 0.04y = 0.12$

$$3x + 4y = 12$$
$$4y = -3x + 12$$
$$y = -\frac{3}{4}x + 3$$

$m = -\frac{3}{4}$; y-intercept: $(0, 3)$

b.

$y = -\frac{3}{4}x + 3$
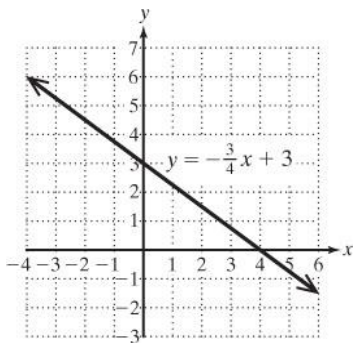
61. a. $\dfrac{x}{4} + \dfrac{y}{7} = 1$

$$28\left(\frac{x}{4} + \frac{y}{7}\right) = 28(1)$$
$$7x + 4y = 28$$
$$4y = -7x + 28$$
$$y = -\frac{7}{4}x + 7$$

$m = -\frac{7}{4}$; y-intercept: $(0, 7)$

b.

$y = -\frac{7}{4}x + 7$
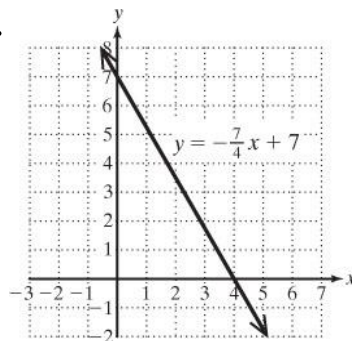
62. a. $\dfrac{x}{3} + \dfrac{y}{4} = 1$

$$12\left(\frac{x}{3} + \frac{y}{4}\right) = 12(1)$$
$$4x + 3y = 12$$
$$3y = -4x + 12$$
$$y = -\frac{4}{3}x + 4$$

$m = -\frac{4}{3}$; y-intercept: $(0, 4)$

b.

$y = -\frac{4}{3}x + 4$
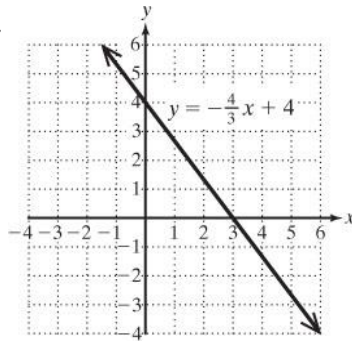
63. a. Linear

   b. Linear

   c. Neither

   d. Constant

64. a. Linear

   b. Neither

   c. Constant

   d. Linear

65. a. $y = mx + b$

$$y = \frac{1}{2}x + b$$
$$9 = \frac{1}{2}(0) + b$$
$$9 = b$$
$$y = \frac{1}{2}x + 9$$

   b. $f(x) = \frac{1}{2}x + 9$

66. a. $y = mx + b$

$$y = \frac{1}{3}x + b$$
$$-4 = \frac{1}{3}(0) + b$$
$$-4 = b$$
$$y = \frac{1}{3}x - 4$$

   b. $f(x) = \frac{1}{3}x - 4$

67. a. $y = mx + b$

$$y = -3x + b$$
$$-6 = -3(1) + b$$
$$-6 = -3 + b$$
$$-3 = b$$
$$y = -3x - 3$$

   b. $f(x) = -3x - 3$

68. a. $y = mx + b$

$$y = -5x + b$$
$$-8 = -5(2) + b$$
$$-8 = -10 + b$$
$$2 = b$$
$$y = -5x + 2$$

   b. $f(x) = -5x + 2$

69. a. $y = mx + b$

$$y = \frac{2}{3}x + b$$
$$-3 = \frac{2}{3}(-5) + b$$
$$-3 = -\frac{10}{3} + b$$
$$\frac{1}{3} = b$$
$$y = \frac{2}{3}x + \frac{1}{3}$$

   b. $f(x) = \frac{2}{3}x + \frac{1}{3}$

70. a. $y = mx + b$

$$y = \frac{3}{2}x + b$$
$$-2 = \frac{3}{2}(-4) + b$$
$$-2 = -6 + b$$
$$4 = b$$
$$y = \frac{3}{2}x + 4$$

   b. $f(x) = \frac{3}{2}x + 4$

71. a. $y = mx + b$

$$y = 0x + b$$
$$y = b$$
$$5 = b$$
$$y = 5$$

   b. $f(x) = 5$

72. a. $y = mx + b$

$y = 0x + b$

$y = b$

$-3 = b$

$y = -3$

b. $f(x) = -3$

73. a. $y = mx + b$

$y = 1.2x + b$

$5.1 = 1.2(3.6) + b$

$5.1 = 4.32 + b$

$0.78 = b$

$y = 1.2x + 0.78$

b. $f(x) = 1.2x + 0.78$

74. a. $y = mx + b$

$y = 2.4x + b$

$2.8 = 2.4(1.2) + b$

$2.8 = 2.88 + b$

$-0.08 = b$

$y = 2.4x - 0.08$

b. $f(x) = 2.4x - 0.08$

75. a. $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{-6 - 2}{0 - 4} = \dfrac{-8}{-4} = 2$

$y = mx + b$

$y = 2x + b$

$-6 = 2(0) + b$

$-6 = b$

$y = 2x - 6$

b. $f(x) = 2x - 6$

76. a. $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{-3 - 1}{0 - (-8)} = \dfrac{-4}{8} = -\dfrac{1}{2}$

$y = mx + b$

$y = -\dfrac{1}{2}x + b$

$-3 = -\dfrac{1}{2}(0) + b$

$-3 = b$

$y = -\dfrac{1}{2}x - 3$

b. $f(x) = -\dfrac{1}{2}x - 3$

77. a. $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{1 - (-3)}{4 - 7} = \dfrac{4}{-3} = -\dfrac{4}{3}$

$y = mx + b$

$y = -\dfrac{4}{3}x + b$

$1 = -\dfrac{4}{3}(4) + b$

$\dfrac{19}{3} = b$

$y = -\dfrac{4}{3}x + \dfrac{19}{3}$

b. $f(x) = -\dfrac{4}{3}x + \dfrac{19}{3}$

78. a. $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{3 - (-4)}{-1 - 2} = \dfrac{7}{-3} = -\dfrac{7}{3}$

$y = mx + b$

$y = -\dfrac{7}{3}x + b$

$3 = -\dfrac{7}{3}(-1) + b$

$\dfrac{2}{3} = b$

$y = -\dfrac{7}{3}x + \dfrac{2}{3}$

b. $f(x) = -\dfrac{7}{3}x + \dfrac{2}{3}$

79. $m = \dfrac{f(x_2) - f(x_1)}{x_2 - x_1} = \dfrac{4 - 1}{3 - 1} = \dfrac{3}{2}$

80. $m = \dfrac{f(x_2) - f(x_1)}{x_2 - x_1} = \dfrac{2 - 6}{5 - 1} = \dfrac{-4}{4} = -1$

81. a. Average rate of change

$= \dfrac{f(x_2) - f(x_1)}{x_2 - x_1}$

$= \dfrac{8244 - 6420}{10 - 5}$

$= \dfrac{1824}{5} = \$364.80/\text{yr}$

b. Average rate of change

$$= \frac{f(x_2)-f(x_1)}{x_2-x_1}$$

$$= \frac{17{,}452-13{,}591}{25-20}$$

$$= \frac{3861}{5} = \$772.20/\text{yr}$$

c. Increasing

82. a. Average rate of change

$$= \frac{f(x_2)-f(x_1)}{x_2-x_1}$$

$$= \frac{75-64}{5-3}$$

$$= \frac{11}{2} = 5.5°\text{F/month}$$

b. Average rate of change

$$= \frac{f(x_2)-f(x_1)}{x_2-x_1}$$

$$= \frac{64-79}{11-9}$$

$$= \frac{-15}{2} = -7.5°\text{F/month}$$

c. A positive rate of change means that average monthly temperature is increasing. A negative rate of change means that average monthly temperature is decreasing.

83. a. Average rate of change

$$= \frac{N(t_2)-N(t_1)}{t_2-t_1}$$

$$= \frac{4475.125-5000}{2-0}$$

$$= -262.437 \approx 262/\text{month}$$

b. Between months 4 and 6:
Average rate of change

$$= \frac{N(t_2)-N(t_1)}{t_2-t_1}$$

$$= \frac{1842.837-3208.565}{6-4}$$

$$= -682.864 \approx 683/\text{month}$$

Between months 10 and 12:

Average rate of change

$$= \frac{N(t_2)-N(t_1)}{t_2-t_1}$$

$$= \frac{92.265-312.5}{12-10}$$

$$= -110.118 \approx 110/\text{month}$$

c.
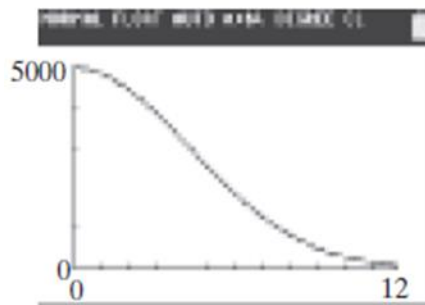
The number of new flu cases dropped slowly during the first two months. Then the rate of new cases dropped more rapidly between months 4 and 6 (perhaps as health department officials managed the outbreak). Finally, the rate of new cases dropped more slowly toward the end of the outbreak.

84. a. Average rate of change

$$= \frac{v(L_2)-v(L_1)}{L_2-L_1}$$

$$= \frac{2.4-1.2}{4-1} = 0.4\,\text{m/sec}$$

b. Average rate of change

275

$$= \frac{v(L_2) - v(L_1)}{L_2 - L_1}$$
$$= \frac{3.6 - 2.4}{9 - 4} = 0.24 \text{ m/sec}$$

c.

NORMAL FLOAT AUTO a+bi DEGREE CL
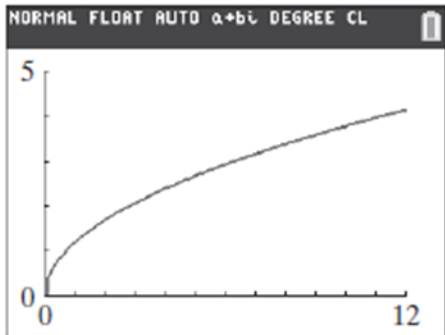
From the graph, the longer the wavelength, the faster the wave. However, the *rate* at which a wave gains speed decreases as wavelength increases.

85. $f(x) = x^2 - 3$

$$f(0) = (0)^2 - 3 = -3$$
$$f(1) = (1)^2 - 3 = 1 - 3 = -2$$
$$f(3) = (3)^2 - 3 = 9 - 3 = 6$$
$$f(-2) = (-2)^2 - 3 = 4 - 3 = 1$$

a. Average rate of change

$$= \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$
$$= \frac{f(1) - f(0)}{1 - 0}$$
$$= \frac{-2 - (-3)}{1} = \frac{1}{1} = 1$$

b. Average rate of change

$$= \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$
$$= \frac{f(3) - f(1)}{3 - 1}$$
$$= \frac{6 - (-2)}{2} = \frac{8}{2} = 4$$

c. Average rate of change

$$= \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$
$$= \frac{f(0) - f(-2)}{0 - (-2)}$$
$$= \frac{-3 - 1}{2} = \frac{-4}{2} = -2$$

86. $g(x) = 2x^2 + 2$

$$g(0) = 2(0)^2 + 2 = 2$$
$$g(1) = 2(1)^2 + 2 = 2 + 2 = 4$$
$$g(3) = 2(3)^2 + 2 = 18 + 2 = 20$$
$$g(-2) = 2(-2)^2 + 2 = 8 + 2 = 10$$

a. Average rate of change

$$= \frac{g(x_2) - g(x_1)}{x_2 - x_1}$$
$$= \frac{g(1) - g(0)}{1 - 0}$$
$$= \frac{4 - 2}{1} = \frac{2}{1} = 2$$

b. Average rate of change

$$= \frac{g(x_2) - g(x_1)}{x_2 - x_1}$$
$$= \frac{g(3) - g(1)}{3 - 1}$$
$$= \frac{20 - 4}{2} = \frac{16}{2} = 8$$

c. Average rate of change

$$= \frac{g(x_2) - g(x_1)}{x_2 - x_1}$$

$$= \frac{g(0) - g(-2)}{0 - (-2)}$$

$$= \frac{2 - 10}{2} = \frac{-8}{2} = -4$$

87. $h(x) = x^3$

$$h(-1) = (-1)^3 = -1$$

$$h(0) = (0)^3 = 0$$

$$h(1) = (1)^3 = 1$$

$$h(2) = (2)^3 = 8$$

a. Average rate of change

$$= \frac{h(x_2) - h(x_1)}{x_2 - x_1}$$

$$= \frac{h(0) - h(-1)}{0 - (-1)}$$

$$= \frac{0 - (-1)}{1} = \frac{1}{1} = 1$$

b. Average rate of change

$$= \frac{h(x_2) - h(x_1)}{x_2 - x_1}$$

$$= \frac{h(1) - h(0)}{1 - 0}$$

$$= \frac{1 - 0}{1} = \frac{1}{1} = 1$$

c. Average rate of change

$$= \frac{h(x_2) - h(x_1)}{x_2 - x_1}$$

$$= \frac{h(2) - h(1)}{2 - 1}$$

$$= \frac{8 - 1}{1} = \frac{7}{1} = 7$$

88. $k(x) = x^3 - 2$

$$k(-1) = (-1)^3 - 2 = -1 - 2 = -3$$

$$k(0) = (0)^3 - 2 = 0 - 2 = -2$$

$$k(1) = (1)^3 - 2 = 1 - 2 = -1$$

$$k(2) = (2)^3 - 2 = 8 - 2 = 6$$

a. Average rate of change

$$= \frac{k(x_2) - k(x_1)}{x_2 - x_1}$$

$$= \frac{k(0) - k(-1)}{0 - (-1)}$$

$$= \frac{-2 - (-3)}{1} = \frac{1}{1} = 1$$

b. Average rate of change

$$= \frac{k(x_2) - k(x_1)}{x_2 - x_1}$$

$$= \frac{k(1) - k(0)}{1 - 0}$$

$$= \frac{-1 - (-2)}{1} = \frac{1}{1} = 1$$

c. Average rate of change

$$= \frac{k(x_2) - k(x_1)}{x_2 - x_1}$$

$$= \frac{k(2) - k(1)}{2 - 1}$$

$$= \frac{6 - (-1)}{1} = \frac{7}{1} = 7$$

89. $m(x) = \sqrt{x}$

$$m(0) = \sqrt{0} = 0$$

$$m(1) = \sqrt{1} = 1$$

$$m(4) = \sqrt{4} = 2$$

$$m(9) = \sqrt{9} = 3$$

a. Average rate of change

$$= \frac{m(x_2) - m(x_1)}{x_2 - x_1}$$

$$= \frac{m(1) - m(0)}{1 - 0}$$

$$= \frac{1 - 0}{1} = \frac{1}{1} = 1$$

b. Average rate of change

$$= \frac{m(x_2) - m(x_1)}{x_2 - x_1}$$

$$= \frac{m(4) - m(1)}{4 - 1}$$

$$= \frac{2 - 1}{3} = \frac{1}{3}$$

c. Average rate of change

$$= \frac{m(x_2) - m(x_1)}{x_2 - x_1}$$

$$= \frac{m(9) - m(4)}{9 - 4}$$

$$= \frac{3 - 2}{5} = \frac{1}{5}$$

90. $n(x) = \sqrt{x - 1}$

$n(1) = \sqrt{1 - 1} = \sqrt{0} = 0$

$n(2) = \sqrt{2 - 1} = \sqrt{1} = 1$

$n(5) = \sqrt{5 - 1} = \sqrt{4} = 2$

$n(10) = \sqrt{10 - 1} = \sqrt{9} = 3$

a. Average rate of change

$$= \frac{n(x_2) - n(x_1)}{x_2 - x_1}$$

$$= \frac{n(2) - n(1)}{2 - 1}$$

$$= \frac{1 - 0}{1} = \frac{1}{1} = 1$$

b. Average rate of change

$$= \frac{n(x_2) - n(x_1)}{x_2 - x_1}$$

$$= \frac{n(5) - n(2)}{5 - 2}$$

$$= \frac{2 - 1}{3} = \frac{1}{3}$$

c. Average rate of change

$$= \frac{n(x_2) - n(x_1)}{x_2 - x_1}$$

$$= \frac{n(10) - n(5)}{10 - 5}$$

$$= \frac{3 - 2}{5} = \frac{1}{5}$$

91. a. $\{-1\}$

    b. $(-\infty, -1)$

    c. $[-1, \infty)$

92. a. $\{1\}$

    b. $(-\infty, 1)$

    c. $[1, \infty)$

93. a. $\{2\}$

    b. $(-\infty, 2)$

    c. $[2, \infty)$

94. a. $\{1\}$

    b. $[1, \infty)$

    c. $(-\infty, 1)$

95. a. $\{-5\}$

    b. $[-5, \infty)$

    c. $(-\infty, -5]$

96. a. $\{-9.5\}$

    b. $(-\infty, -9.5]$

    c. $[-9.5, \infty)$

97. a. $\{14\}$

b. $\left(14, \infty\right)$

c. $\left(-\infty, 14\right)$

98. a. $\{1.5\}$

b. $\left(-\infty, 1.5\right)$

c. $\left(1.5, \infty\right)$

99. The line will be slanted if both *A* and *B* are nonzero. If *A* is zero and *B* is not zero, then the equation can be written in the form $y = k$ and the graph is a horizontal line. If *B* is zero and *A* is not zero, then the equation can be written in the form $x = k$, and the graph is a vertical line.

100. If *C* is zero, then the line passes through the origin.

101. The slope and *y*-intercept are easily determined by inspection of the equation.

102. The average rate of change of *f* on the interval $\left[x_1, x_2\right]$ is the slope of the secant line passing through the points $\left(x_1, f\left(x_1\right)\right)$ and $\left(x_2, f\left(x_2\right)\right)$.

103.
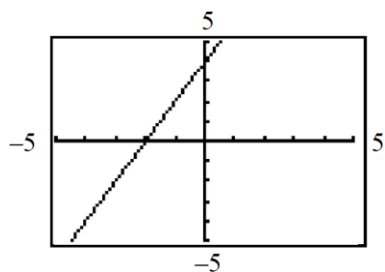
The *x*-intercept of the line is $\left(-2, 0\right)$. The base of the triangle is 2 units. The *y*-intercept of the line is $\left(0, 4\right)$. The height of the triangle is 4.

$$A = \frac{1}{2}bh = \frac{1}{2}(2)(4) = 4 \text{ units}^2$$

104.
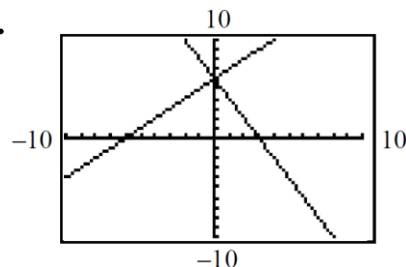
The *x*-intercepts of the two lines are $\left(-6, 0\right)$ and $\left(3, 0\right)$. The base of the triangle is $3 - \left(-6\right) = 9$ units. The *y*-intercept of the two lines is $\left(0, 6\right)$. The height of the triangle is 6.

$$A = \frac{1}{2}bh = \frac{1}{2}(9)(6) = 27 \text{ units}^2$$

105.
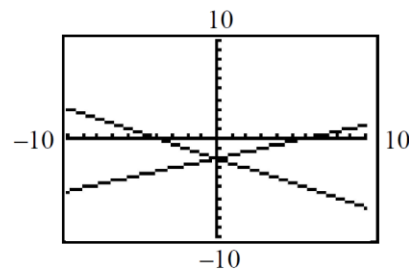
The *x*-intercepts of the two lines are $\left(-4, 0\right)$ and $\left(6, 0\right)$. The base of the triangle is $6 - \left(-4\right) = 10$ units. The *y*-intercept of the two lines is $\left(0, -2\right)$. The height of the triangle is 2.

$$A = \frac{1}{2}bh = \frac{1}{2}(10)(2) = 10 \text{ units}^2$$

106.
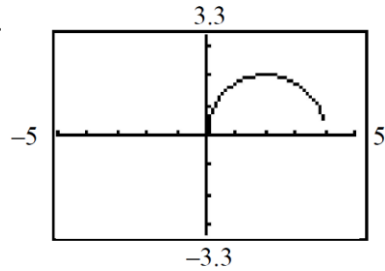
The *x*-intercepts of the line are $(0, 0)$ and $(4, 0)$. The diameter of the half circle is 4 units, so the radius is 2 units.

$$A = \frac{1}{2}\pi r^2 = \frac{1}{2}\pi(2)^2 = 2\pi \text{ units}^2$$

107. a. $Ax + By = C$

$$By = -Ax + C$$

$$y = -\frac{A}{B}x + \frac{C}{B}$$

b. $m = -\dfrac{A}{B}$

c. $\left(0, \dfrac{C}{B}\right)$

108. a. $m = -\dfrac{A}{B} = -\dfrac{5}{-9} = \dfrac{5}{9}$

b. $\left(0, \dfrac{C}{B}\right) = \left(0, \dfrac{6}{-9}\right) = \left(0, -\dfrac{2}{3}\right)$

109. a. $3.1 - 2.2(t+1) = 6.3 + 1.4t$

$$3.1 - 2.2t - 2.2 = 6.3 + 1.4t$$

$$-2.2t + 0.9 = 1.4t + 6.3$$

$$-3.6t = 5.4$$

$$t = -1.5$$

$$\{-1.5\}$$

b. $\left(-\infty, -1.5\right)$

c. $\left(-1.5, \infty\right)$

110. a. $\left[\begin{matrix} -11.2 - 4.6(c-3) \\ +1.8c \end{matrix}\right] = 0.4(c+2)$

$$\left[\begin{matrix} -11.2 - 4.6c + 13.8 \\ +1.8c \end{matrix}\right] = 0.4c + 0.8$$
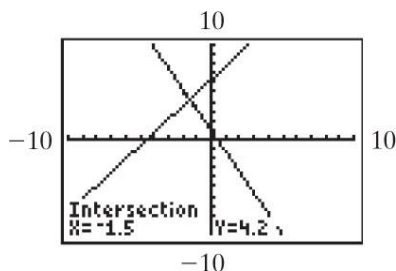
$$-2.8c + 2.6 = 0.4c + 0.8$$

$$-3.2c = -1.8$$

$$c = 0.5625$$

$$\{0.5625\}$$
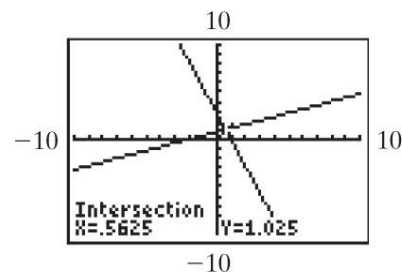
b. $\left(-\infty, 0.5625\right)$

c. $\left(0.5625, \infty\right)$

111. a. $|2x - 3.8| - 4.6 = 7.2$

$$|2x - 3.8| = 11.8$$

$$2x - 3.8 = 11.8 \text{ or } 2x - 3.8 = -11.8$$

$$2x = 15.6 \text{ or } \qquad 2x = -8$$

$$x = 7.8 \text{ or } \qquad x = -4$$

$$\{-4, 7.8\}$$

280
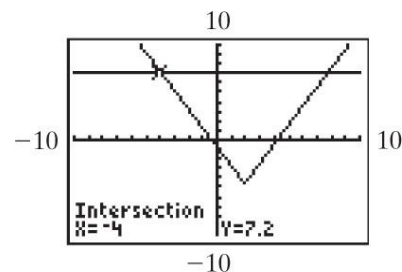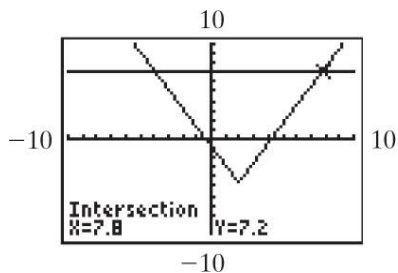
b. $\left(-\infty, -4\right] \cup \left[7.8, \infty\right)$

c. $\left[-4, 7.8\right]$

112. a. $|x - 1.7| + 4.95 = 11.15$

$|x - 1.7| = 6.2$

$x - 1.7 = 6.2 \ \text{ or } \ x - 1.7 = -6.2$

$x = 7.9 \ \text{ or } \qquad x = -4.5$

$\left\{-4.5, 7.9\right\}$
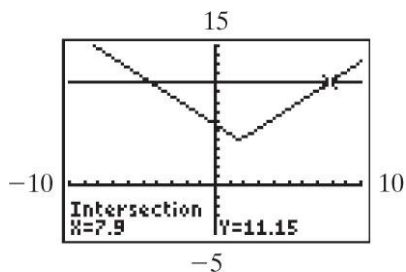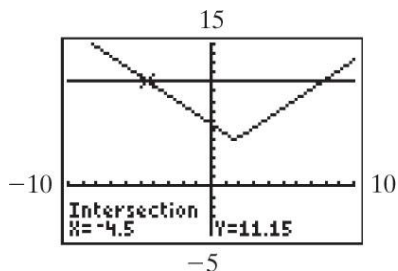
b. $\left(-\infty, -4.5\right] \cup \left[7.9, \infty\right)$

c. $\left[-4.5, 7.9\right]$

113.
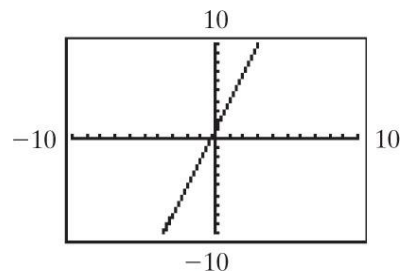
The lines are not exactly the same. The slopes are different.

114.
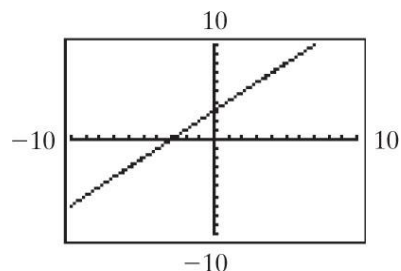
The lines are not exactly the same. The y-intercepts are different.

Section 2.5 Applications of Linear Equations and Modeling

1. $y - y_1 = m\left(x - x_1\right)$

2. parallel

3. -1

4. $R\left(x\right) - C\left(x\right)$

5. $y - y_1 = m(x - x_1)$

$\quad y - 5 = -2\big[x - (-3)\big]$

$\quad y - 5 = -2(x + 3)$

$\quad y - 5 = -2x - 6$

$\quad\quad y = -2x - 1$

6. $\quad y - y_1 = m(x - x_1)$

$\quad y - (-6) = 3(x - 4)$

$\quad\quad y + 6 = 3x - 12$

$\quad\quad\quad y = 3x - 18$

7. $y - y_1 = m(x - x_1)$

$\quad y - 0 = \dfrac{2}{3}\big[x - (-1)\big]$

$\quad\quad y = \dfrac{2}{3}(x + 1)$

$\quad\quad y = \dfrac{2}{3}x + \dfrac{2}{3}$

8. $y - y_1 = m(x - x_1)$

$\quad y - 0 = \dfrac{3}{5}\big[x - (-4)\big]$

$\quad\quad y = \dfrac{3}{5}(x + 4)$

$\quad\quad y = \dfrac{3}{5}x + \dfrac{12}{5}$

9. $\quad y - y_1 = m(x - x_1)$

$\quad y - 2.6 = 1.2(x - 3.4)$

$\quad y - 2.6 = 1.2x - 4.08$

$\quad\quad\quad y = 1.2x - 1.48$

10. $\quad y - y_1 = m(x - x_1)$

$\quad y - 4.1 = 2.4(x - 2.2)$

$\quad y - 4.1 = 2.4x - 5.28$

$\quad\quad\quad y = 2.2x - 1.18$

11. $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{1 - 2}{-3 - 6} = \dfrac{-1}{-9} = \dfrac{1}{9}$

$y - y_1 = m(x - x_1)$

$\quad y - 2 = \dfrac{1}{9}(x - 6)$

$\quad y - 2 = \dfrac{1}{9}x - \dfrac{2}{3}$

$\quad\quad y = \dfrac{1}{9}x + \dfrac{4}{3}$

12. $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{-3 - 8}{-7 - (-4)} = \dfrac{-11}{-3} = \dfrac{11}{3}$

$y - y_1 = m(x - x_1)$

$\quad y - 8 = \dfrac{11}{3}\big[x - (-4)\big]$

$\quad y - 8 = \dfrac{11}{3}(x + 4)$

$\quad y - 8 = \dfrac{11}{3}x + \dfrac{44}{3}$

$\quad\quad y = \dfrac{11}{3}x + \dfrac{68}{3}$

13. $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{0 - 8}{5 - 0} = \dfrac{-8}{5} = -\dfrac{8}{5}$

$y - y_1 = m(x - x_1)$

$\quad y - 0 = -\dfrac{8}{5}(x - 5)$

$\quad\quad y = -\dfrac{8}{5}x + 8$

14. $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{0 - (-6)}{11 - 0} = \dfrac{6}{11}$

$y - y_1 = m(x - x_1)$

$\quad y - 0 = \dfrac{6}{11}(x - 11)$

$\quad\quad y = \dfrac{6}{11}x - 6$

15. $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{3.7 - 5.1}{1.9 - 2.3} = \dfrac{-1.4}{-0.4} = 3.5$

$y - y_1 = m(x - x_1)$

$\quad y - 5.1 = 3.5(x - 2.3)$

$\quad y - 5.1 = 3.5x - 8.05$

$\quad\quad\quad y = 3.5x - 2.95$

16. $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{6 - 4.8}{0.8 - 1.6} = \dfrac{1.2}{-0.8} = -1.5$

$\quad y - y_1 = m(x - x_1)$

$\quad\quad y - 6 = -1.5(x - 0.8)$

$\quad\quad y - 6 = -1.5x + 1.2$

$\quad\quad\quad y = -1.5x + 7.2$

17. $\quad y - y_1 = m(x - x_1)$

$\quad y - (-4) = 0(x - 3)$

$\quad\quad y + 4 = 0$

$\quad\quad\quad y = -4$

18. $y - y_1 = m(x - x_1)$

$\quad y - 1 = 0\big[x - (-5)\big]$

$\quad y - 1 = 0$

$\quad\quad y = 1$

19. A line with an undefined slope is a vertical line. Every coordinate has the same x-value, so the equation is $x = \dfrac{2}{3}$.

20. A line with an undefined slope is a vertical line. Every coordinate has the same x-value, so the equation is $x = -\dfrac{4}{7}$.

21. Undefined

22. 0

23. a. $m = \dfrac{3}{11}$

b. $m = -\dfrac{1}{\frac{3}{11}} = -\dfrac{11}{3}$

24. a. $m = \dfrac{6}{7}$

b. $m = -\dfrac{1}{\frac{6}{7}} = -\dfrac{7}{6}$

25. a. $m = -6$

b. $m = -\dfrac{1}{-6} = \dfrac{1}{6}$

26. a. $m = -10$

b. $m = -\dfrac{1}{-10} = -\dfrac{1}{10}$

27. a. $m = 1$

b. $m = -\dfrac{1}{1} = -1$

28. a. m is undefined

b. $m = 0$

29. $m_1 = -\dfrac{1}{m_2}$

$\quad 2 = -\dfrac{1}{-\frac{1}{2}}$

$\quad 2 = -2$ ✓ True

Perpendicular

30. $m_1 = -\dfrac{1}{m_2}$

$\quad \dfrac{4}{3} = -\dfrac{1}{-\frac{3}{4}}$

$\quad \dfrac{4}{3} = \dfrac{4}{3}$ ✓ True

Perpendicular

31. $8x - 5y = 3 \qquad\qquad 2x = \dfrac{5}{4}y + 1$

$\quad -5y = -8x + 3 \qquad \dfrac{5}{4}y = 2x - 1$

$\quad\quad y = \dfrac{8}{5}x - \dfrac{3}{5} \qquad\quad y = \dfrac{8}{5}x - \dfrac{4}{5}$

$m_1 = m_2$; Parallel

32. $2x + 3y = 7 \qquad\qquad 4x = -6y + 2$

$\quad 3y = -2x + 7 \qquad 6y = -4x + 2$

$\quad y = -\dfrac{2}{3}x + \dfrac{7}{3} \qquad y = -\dfrac{2}{3}x + \dfrac{1}{3}$

$m_1 = m_2$; Parallel

33. $2x = 6$ $\qquad$ $5 = y$

$\qquad x = 3$ $\qquad\qquad y = 5$

$\qquad m = \text{undefined} \qquad m = 0$

$\qquad$ Perpendicular

34. $3y = 5$ $\qquad\quad x = 1$

$\qquad y = \dfrac{5}{3}$ $\qquad m = \text{undefined}$

$\qquad m = 0$

$\qquad$ Perpendicular

35. $6x = 7y$ $\qquad \dfrac{7}{2}x - 3y = 0$

$\qquad y = \dfrac{6}{7}x$ $\qquad -3y = -\dfrac{7}{2}x$

$\qquad\qquad\qquad\qquad y = \dfrac{7}{6}x$

$\qquad m_1 \neq m_2$

$\qquad m_1 = -\dfrac{1}{m_2}$

$\qquad \dfrac{6}{7} = -\dfrac{1}{\dfrac{7}{6}}$

$\qquad \dfrac{6}{7} = -\dfrac{6}{7}$ False

$\qquad$ Neither

36. $5y = 2x$ $\qquad \dfrac{5}{2}x - y = 0$

$\qquad y = \dfrac{2}{5}x$ $\qquad -y = -\dfrac{5}{2}x$

$\qquad\qquad\qquad\qquad y = \dfrac{5}{2}x$

$\qquad m_1 \neq m_2$

$\qquad m_1 = -\dfrac{1}{m_2}$

$\qquad \dfrac{2}{5} = -\dfrac{1}{\dfrac{5}{2}}$

$\qquad \dfrac{2}{5} = -\dfrac{2}{5}$ False

$\qquad$ Neither

37. $2x + y = 6$

$\qquad y = -2x + 6;\ m = -2$

$\qquad y - y_1 = m(x - x_1)$

$\qquad y - 5 = -2(x - 2)$

$\qquad y - 5 = -2x + 4$

$\qquad y = -2x + 9 \text{ or } 2x + y = 9$

38. $-3x + y = 4$

$\qquad y = 3x + 4;\ m = 3$

$\qquad y - y_1 = m(x - x_1)$

$\qquad y - (-1) = 3(x - 3)$

$\qquad y + 1 = 3x - 9$

$\qquad y = 3x - 10 \text{ or } 3x - y = 10$

39. $x - 5y = 1$

$\qquad -5y = -x + 1$

$\qquad y = \dfrac{1}{5}x - \dfrac{1}{5};\ m = \dfrac{1}{5}$

$\qquad -\dfrac{1}{m} = -\dfrac{1}{\dfrac{1}{5}} = -5$

$\qquad y - y_1 = m(x - x_1)$

$\qquad y - (-4) = -5(x - 6)$

$\qquad y + 4 = -5x + 30$

$\qquad y = -5x + 26 \text{ or } 5x + y = 26$

40. $x - 2y = 7$

$\qquad -2y = -x + 7$

$\qquad y = \dfrac{1}{2}x - \dfrac{7}{2};\ m = \dfrac{1}{2}$

$\qquad -\dfrac{1}{m} = -\dfrac{1}{\dfrac{1}{2}} = -2$

$\qquad y - y_1 = m(x - x_1)$

$\qquad y - 4 = -2(x - 5)$

$\qquad y - 4 = -2x + 10$

$\qquad y = -2x + 14 \text{ or } 2x + y = 14$

41. $3x = 7y + 5$

$\qquad 7y = 3x - 5$

$\qquad y = \dfrac{3}{7}x - \dfrac{5}{7};\ m = \dfrac{3}{7}$

$$y - y_1 = m(x - x_1)$$
$$y - 8 = \frac{3}{7}(x - 6)$$
$$y - 8 = \frac{3}{7}x - \frac{18}{7}$$
$$y = \frac{3}{7}x + \frac{38}{7} \text{ or } 7y = 3x + 38$$
$$3x - 7y = -38$$

42. $2x = 5y - 4$
$$5y = 2x + 4$$
$$y = \frac{2}{5}x + \frac{4}{5}; \ m = \frac{2}{5}$$
$$y - y_1 = m(x - x_1)$$
$$y - (-6) = \frac{2}{5}(x - 7)$$
$$y + 6 = \frac{2}{5}x - \frac{14}{5}$$
$$y = \frac{2}{5}x - \frac{44}{5} \text{ or } 5y = 2x - 44$$
$$2x - 5y = 44$$

43. $2x = 4 - y$
$$y = -2x + 4; \ m = -2$$
$$-\frac{1}{m} = -\frac{1}{-2} = \frac{1}{2} = 0.5$$
$$y - y_1 = m(x - x_1)$$
$$y - 6.4 = 0.5(x - 2.2)$$
$$y - 6.4 = 0.5x - 1.1$$
$$y = 0.5x + 5.3 \text{ or } 10y = 5x + 53$$
$$5x - 10y = -53$$

44. $4x = 9 - y$
$$y = -4x + 9; \ m = -4$$
$$-\frac{1}{m} = -\frac{1}{-4} = \frac{1}{4} = 0.25$$
$$y - y_1 = m(x - x_1)$$
$$y - 1.2 = 0.25(x - 3.6)$$
$$y - 1.2 = 0.25x - 0.9$$
$$y = 0.25x + 0.3 \text{ or } 20y = 5x + 6$$
$$5x - 20y = -6$$

45. A line that is parallel to the *x*-axis is a horizontal line with slope 0. Every coordinate has the same *y*-value, so the equation is $y = 6$.

46. A line that is parallel to the *y*-axis is a vertical line with undefined slope. Every coordinate has the same *x*-value, so the equation is $x = -11$.

47. A line that is perpendicular to the *y*-axis is a horizontal line with slope 0. Every coordinate has the same *y*-value, so the equation is $y = -\frac{3}{4}$.

48. A line that is perpendicular to the *x*-axis is a vertical line with undefined slope. Every coordinate has the same *x*-value, so the equation is $x = -\frac{7}{9}$.

49. A line that is parallel to a vertical line is also a vertical line with undefined slope. Every coordinate has the same *x*-value, so the equation is $x = -61.5$.

50. A line that is parallel to a horizontal line is also a horizontal line with slope 0. Every coordinate has the same *y*-value, so the equation is $y = 0.009$.

51. a. $S(x) = 0.12x + 400$ for $x \geq 0$

b. $S(x) = 0.12(8000) + 400$
$$= 960 + 400 = 1360$$
$S(8000) = 1360$ means that the sales person will make \$1360 if \$8000 in merchandise is sold for the week.

52. a. $P(t) = 1.5t + 2$ for $t > 0$

b. $P(1.6) = 1.5(1.6) + 2$
$$= 2.4 + 2 = 4.4$$

$P(1.6) = 4.4$ means that it costs $4.40 to park for 1.6 hr (1 hr 36 min).

53. a. $T(x) = 0.019x + 172$ for $x > 0$

  b. $T(80,000) = 0.019(80,000) + 172$
  $= 1520 + 172 = 1692$

  $T(80,000) = 1692$ means that the property tax is $1692 for a home with a taxable value of 80,000.

54. a. $W(t) = -0.25t + 6.8$ for $0 \le t \le 27.2$

  b. $W(20) = -0.25(20) + 6.8$
  $= -5 + 6.8 = 1.8$

  $W(20) = 1.8$ means that after 20 days of drought, the water level will be 1.8 ft.

55. a. $C(x) = 34.5x + 2275$

  b. $R(x) = 80x$

  c. $P(x) = R(x) - C(x)$
  $= 80x - (34.5x + 2275)$
  $= 45.5x - 2275$

  d. $\qquad P(x) = 0$
  $45.5x - 2275 = 0$
  $45.5x = 2275$
  $x = 50$ items

56. a. $C(x) = 0.4x + 5625$

  b. $R(x) = 1.3x$

  c. $P(x) = R(x) - C(x)$
  $= 1.3x - (0.4x + 5625)$
  $= 0.9x - 5625$

  d. $\qquad P(x) = 0$
  $0.9x - 5625 = 0$
  $0.9x = 5625$
  $x = 6250$ items

57. a. $\{730\}$

  b. $[0, 730)$

c. $(730, \infty)$

58. a. $\{25,000\}$

  b. $[0, 25,000)$

  c. $(25,000, \infty)$

59. a. $C(x) = 0.24(12)x + 790$
  $= 2.88x + 790$

  b. $R(x) = 6x$

  c. $P(x) = R(x) - C(x)$
  $= 6x - (2.88x + 790)$
  $= 3.12x - 790$

  d. $\qquad P(x) > 0$
  $3.12x - 790 > 0$
  $3.12x > 790$
  $x > 253.2$

  The business will make a profit if it produces and sells 254 dozen or more cookies.

  e. $\quad P(x) = 3.12x - 790$
  $P(150) = 3.12(150) - 790$
  $= 468 - 790 = -322$

  The business will lose $322.

60. a. $C(x) = 36x + 680$

  b. $R(x) = 60x$

  c. $P(x) = R(x) - C(x)$
  $= 60x - (36x + 680)$
  $= 24x - 680$

  d. $\qquad P(x) > 0$
  $24x - 680 > 0$
  $24x > 680$
  $x > 28.\overline{3}$

  The business will make a profit if 29 or more maintenance calls are made per month.

e. $P(x) = 24x - 680$

$P(42) = 24(42) - 680$

$= 1008 - 680 = 328$

The business will make \$328.

61. a. $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{50 - 110}{1000 - 950} = -1.2$

$y - y_1 = m(x - x_1)$

$y - 110 = -1.2(x - 950)$

$y - 110 = -1.2x + 1140$

$y = -1.2x + 1250$

b. $m = -1.2$ mph/mb means that for an increase of 1 mb in pressure, the wind speed decreases by 1.2 mph.

c. $y = (-1.2 \times 900) + 1250$

$= 170$ mph/mb

d. No. There is no guarantee that the linear trend continues outside the interval of the observed data points.

62. a. $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{22 - 11}{40 - 0} = 0.275$

$y - y_1 = m(x - x_1)$

$y - 11 = 0.275(x - 0)$

$y = 0.275x + 11$

b. $m = 0.275$ means that Dodger's weight increased by 0.275 lb per day during this period.

c. The y-intercept is (0, 11) and indicates that Dodger's weight the day he was adopted was 11 lb.

d. $\left(70 \times \dfrac{90}{100}\right) = 0.275x + 11$

$x = \dfrac{\left(70 \times \dfrac{90}{100}\right) - 11}{0.275}$

$= 189.090 \approx 189$ days

e. No. Dodger will eventually stop growing (or at least Caroline hopes so).

63. a. $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{46 - 35}{6 - 2} = 2.75$

$y - y_1 = m(x - x_1)$

$y - 35 = 2.75(x - 2)$

$y - 35 = 2.75 - 5.5$

$y = 2.75x + 29.5$

b. $m = 2.75$ means that the average height of girls increased by 2.75 in. per year during this time period.

c. $y = (2.75 \times 11) + 2.95 = 59.75$ in

d. Average height of adult women

$= \left(59.75 \times \dfrac{100}{90}\right)$

$= 66.38 \approx 66.4$ in

64. a. $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{13.0 - 11.2}{14 - 4}$

$= \dfrac{1.8}{10} = 0.18$

$y - y_1 = m(x - x_1)$

$y - 11.2 = 0.18(x - 4)$

$y - 11.2 = 0.18x - 0.72$

$y = 0.18x + 10.48$

b. $m = 0.18$ means that enrollment in public colleges increased at an average rate of 0.18 million per yr (180,000 per yr).

c. The y-intercept is (0, 10.48) and means that in the year 1990, there were approximately 10,480,000 students enrolled in public colleges.

d. $y = 0.18x + 10.48$

$\qquad y = 0.18(25) + 10.48$

$\qquad = 4.5 + 10.48 = 14.98$

Approximately 14.98 million

65. a.

Amount of Cholesterol vs. Number of Calories for Selected Hamburgers
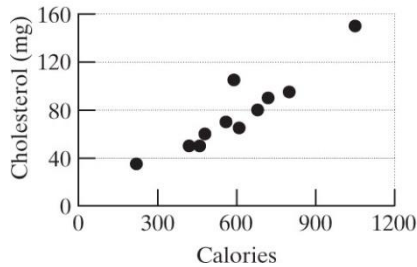
b. $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{90 - 60}{720 - 480}$

$\qquad = \dfrac{30}{240} = 0.125$

$\qquad y - y_1 = m(x - x_1)$

$\qquad y - 60 = 0.125(x - 480)$

$\qquad y - 60 = 0.125x - 60$

$\qquad\qquad y = 0.125x$

$\qquad c(x) = 0.125x$

c. $m = 0.125$ means that the amount of cholesterol increases at an average rate of 0.125 mg per calorie of hamburger.

d. $\quad c(x) = 0.125x$

$\qquad c(650) = 0.125(650) = 81.25$ mg

66. a.

Longevity of Selected Animals vs. Gestation/Incubation Period
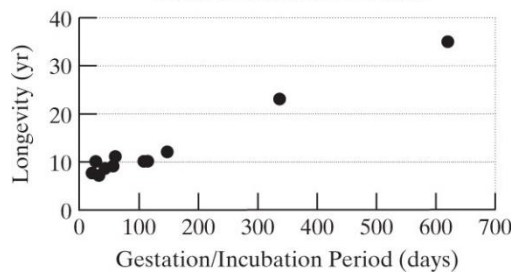
b. $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{35 - 8.5}{620 - 44}$

$\qquad = \dfrac{26.5}{576} \approx 0.046$

$\qquad y - y_1 = m(x - x_1)$

$\qquad y - 8.5 = 0.046(x - 44)$

$\qquad y - 8.5 = 0.046x - 2.024$

$\qquad\qquad y = 0.046x + 6.476$

$\qquad L(x) = 0.0.046x + 6.48$

c. $m = 0.046$ means that longevity increases at an average rate of 0.046 yr per 1 day increase in the gestation or incubation period.

d. $\quad L(x) = 0.0.046x + 6.48$

$\qquad L(80) = 0.0.046(80) + 6.48$

$\qquad\qquad = 3.68 + 6.48 \approx 10$

Approximately 10 yr.

67. Yes. From the graph, the data appear to follow a linear trend.

68. Yes. From the graph, the data appear to follow a linear trend.

69. No. From the graph, the data do not appear to follow a linear trend.

70. No. From the graph, the data do not appear to follow a linear trend.

71. a. $y = -1.22x + 1273$

b.
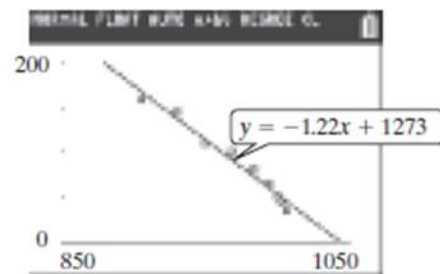
c. $y = -(1.22 \times 900) + 1273 = 175$ mph

d. From Exercise 61(c), $y = 170$ mph.
Result of part (c) differ 5 mph from the result of 61(c).

72. a. $y = 0.27x + 11.1$

b.
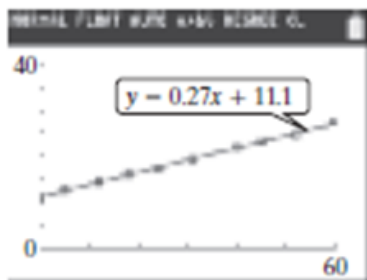
c. $\left(70 \times \dfrac{90}{100}\right) = 0.27x + 11.1$

$x = \dfrac{\left(70 \times \dfrac{90}{100}\right) - 11.1}{0.27}$

$= 192.22 \approx 192$ days

d. From Exercise 62(d), $y = 189$ days.
Result of part (c) differ 3 days from the result of 62(d).

73. a. $y = 2.48x + 31.0$

b.
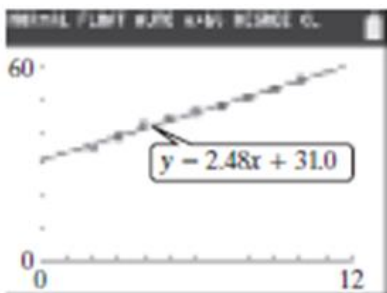
c. $y = (2.48 \times 11) + 31.0 = 58.28$ in

d. Average height of adult women

$= \left(58.28 \times \dfrac{100}{90}\right)$

$= 64.75 \approx 64.8$ in

e. From Exercise 63(d), $y = 66.4$ in.
Result of part (d) differ 1.6 in from the result of 63(d).

74. a. $y = 0.18x + 10.4$

b.
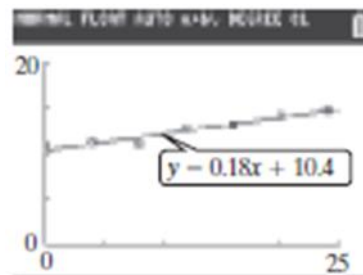
c. For the year 2020, $x = 30$.
$y = (0.18 \times 30) + 10.4 = 15.8$ million

d. From Exercise 64(d),
$y = 15.88$ million.
Result of part (c) differ 0.08 million (or 80,000) from the result of 64(d).

75. a. $y = 0.118x + 4.97$

b.
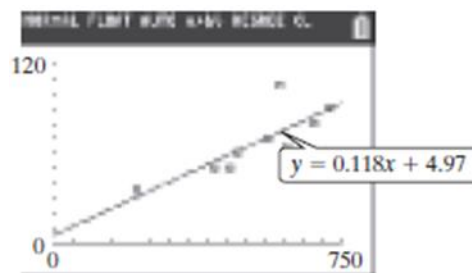
c. $y = (0.118 \times 650) + 4.97$
$= 81.67 \approx 82$ mg

76. a. $y = 0.046x + 6.28$

b.
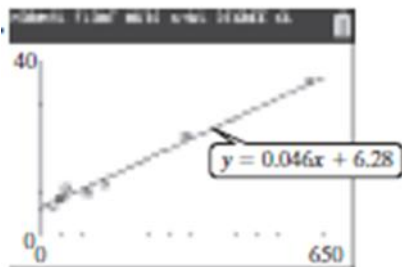
c. $y = (0.046 \times 80) + 6.28 = 9.96 \approx 10$ yr

77. $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{-1 - (-6)}{2 - 4} = \dfrac{5}{-2} = -\dfrac{5}{2}$

$y - y_1 = m(x - x_1)$

$y - (-6) = -\dfrac{5}{2}(x - 4)$

$y + 6 = -\dfrac{5}{2}x + 10$

$y = -\dfrac{5}{2}x + 4$

To find the *x*-intercept, solve $y = 0$.

$0 = -\dfrac{5}{2}x + 4$

$\dfrac{5}{2}x = 4$

$x = \dfrac{8}{5} \qquad \left(\dfrac{8}{5}, 0\right)$

78. $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{7 - (-5)}{-4 - 2} = \dfrac{12}{-6} = -2$

$y - y_1 = m(x - x_1)$

$y - (-5) = -2(x - 2)$

$y + 5 = -2x + 4$

$y = -2x - 1$

To find the *x*-intercept, solve $y = 0$.

$0 = -2x - 1$

$2x = -1$

$x = -\dfrac{1}{2} \qquad \left(-\dfrac{1}{2}, 0\right)$

79. Use the points $(0, 4)$ and $(3, 11)$.

$m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{11 - 4}{3 - 0} = \dfrac{7}{3}$

$y - y_1 = m(x - x_1)$

$y - 4 = \dfrac{7}{3}(x - 0)$

$y - 4 = \dfrac{7}{3}x$

$y = \dfrac{7}{3}x + 4 \text{ or } f(x) = \dfrac{7}{3}x + 4$

80. Use the points $(0, 7)$ and $(-2, 4)$.

$m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{4 - 7}{-2 - 0} = \dfrac{-3}{-2} = \dfrac{3}{2}$

$y - y_1 = m(x - x_1)$

$y - 7 = \dfrac{3}{2}(x - 0)$

$y - 7 = \dfrac{3}{2}x$

$y = \dfrac{3}{2}x + 7 \text{ or } g(x) = \dfrac{3}{2}x + 7$

81. Use the points $(1, 6)$ and $(-3, 2)$.

$m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{2 - 6}{-3 - 1} = \dfrac{-4}{-4} = 1$

$y - y_1 = m(x - x_1)$

$y - 6 = 1(x - 1)$

$y - 6 = x - 1$

$y = x + 5 \text{ or } h(x) = x + 5$

82. Use the points $(-2, 10)$ and $(5, -18)$.

$m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{-18 - 10}{5 - (-2)} = \dfrac{-28}{7} = -4$

$y - y_1 = m(x - x_1)$

$y - 10 = -4\left[x - (-2)\right]$

$y - 10 = -4(x + 2)$

$y - 10 = -4x - 8$

$y = -4x + 2 \text{ or } h(x) = -4x + 2$

83. If the slopes of the two lines are the same and the y-intercepts are different, then the lines are parallel. If the slope of one line is the opposite of the reciprocal of the slope of the other line, then the lines are perpendicular.

84. The point-slope formula is used to construct an equation of a line if a point on the line and the slope of the line are known.

85. Profit is equal to revenue minus cost.

86. The break-even point is found by determining where revenue equals cost or where profit equals zero.

87. $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{3 - (-1)}{1 - (-3)} = \dfrac{4}{4} = 1$

$-\dfrac{1}{m} = -\dfrac{1}{1} = -1$

$y - y_1 = m(x - x_1)$

$y - 3 = -1(x - 1)$

$y - 3 = -x + 1$

$y = -x + 4$

88. $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{3 - 0}{4 - 0} = \dfrac{3}{4}$

$-\dfrac{1}{m} = -\dfrac{1}{\frac{3}{4}} = -\dfrac{4}{3}$

$y - y_1 = m(x - x_1)$

$y - 3 = -\dfrac{4}{3}(x - 4)$

$y - 3 = -\dfrac{4}{3}x + \dfrac{16}{3}$

$y = -\dfrac{4}{3}x + \dfrac{25}{3}$

89. Let $c = -2$.

$\left(c, c^3 + 1\right) = \left(-2, (-2)^3 + 1\right) = \left(-2, -7\right)$

$m = 3c^2 = 3(-2)^2 = 3(4) = 12$

$y - y_1 = m(x - x_1)$

$y - (-7) = 12\left[x - (-2)\right]$

$y + 7 = 12(x + 2)$

$y + 7 = 12x + 24$

$y = 12x + 17$

90. Let $c = 2$.

$\left(c, \dfrac{1}{c}\right) = \left(2, \dfrac{1}{2}\right)$

$m = -\dfrac{1}{c^2} = -\dfrac{1}{(2)^2} = -\dfrac{1}{4}$

$y - y_1 = m(x - x_1)$

$y - \dfrac{1}{2} = -\dfrac{1}{4}(x - 2)$

$y - \dfrac{1}{2} = -\dfrac{1}{4}x + \dfrac{1}{2}$

$y = -\dfrac{1}{4}x + 1$

91. $M = \left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)$

$= \left(\dfrac{-2 + 4}{2}, \dfrac{9 + 7}{2}\right) = \left(\dfrac{2}{2}, \dfrac{16}{2}\right) = (1, 8)$

$m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{-2 - 8}{5 - 1} = \dfrac{-10}{4} = -\dfrac{5}{2}$

$y - y_1 = m(x - x_1)$

$y - 8 = -\dfrac{5}{2}(x - 1)$

$y - 8 = -\dfrac{5}{2}x + \dfrac{5}{2}$

$y = -\dfrac{5}{2}x + \dfrac{21}{2}$ for $1 \leq x \leq 5$

92. $M = \left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)$

$= \left(\dfrac{-4 + 12}{2}, \dfrac{1 + 3}{2}\right) = \left(\dfrac{8}{2}, \dfrac{4}{2}\right) = (4, 2)$

$m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{-5 - 2}{6 - 4} = \dfrac{-7}{2} = -\dfrac{7}{2}$

$$y - y_1 = m\left(x - x_1\right)$$

$$y - 2 = -\frac{7}{2}\left(x - 4\right)$$

$$y - 2 = -\frac{7}{2}x + 14$$

$$y = -\frac{7}{2}x + 16 \text{ for } 4 \le x \le 6$$

Problem Recognition Exercises: Comparing Graphs of Equations

1.

2.

3.

4.

5.

6.

7.
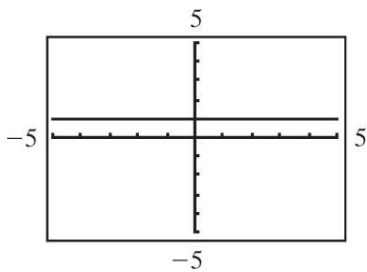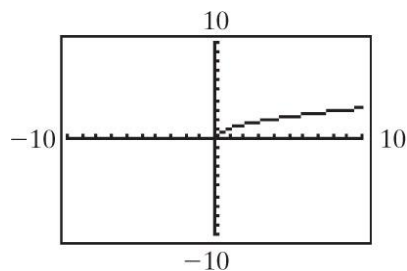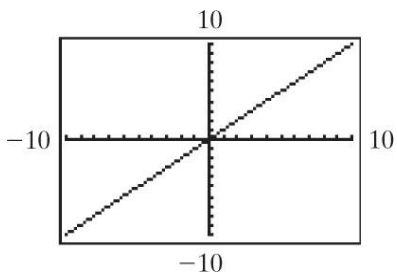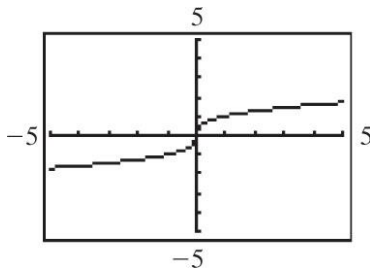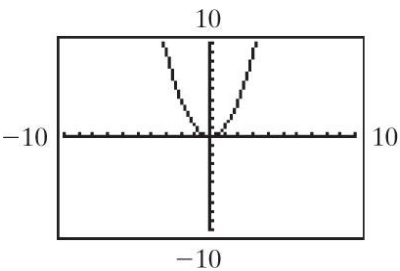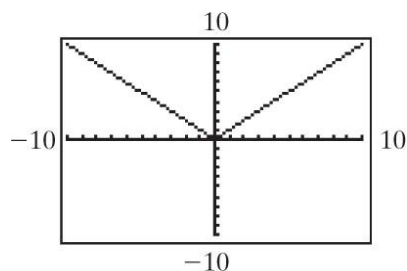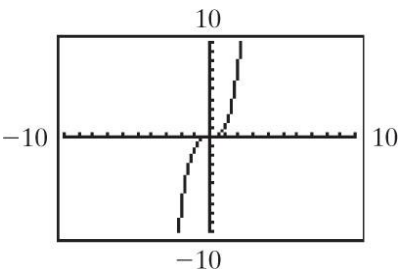
8.
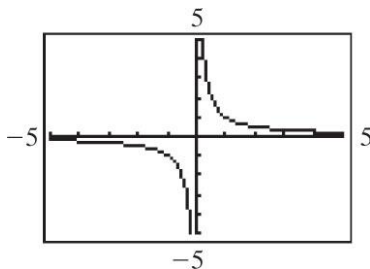
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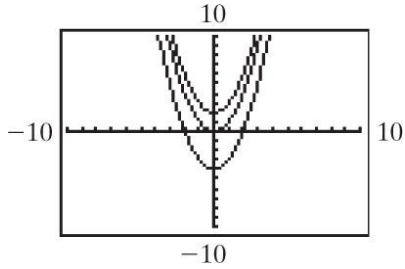
The graphs have the shape of $y = x^2$ with a vertical shift.

10.
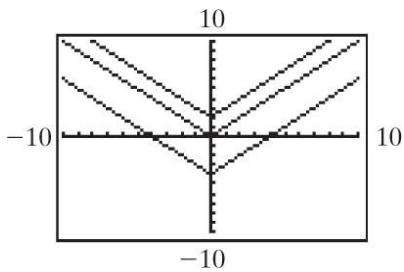
The graphs have the shape of $y = |x|$ with a vertical shift.

11.
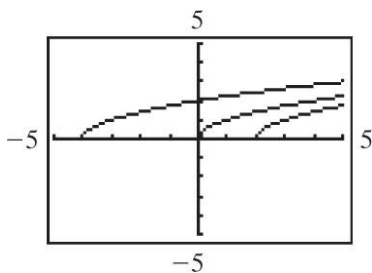
The graphs have the shape of $y = \sqrt{x}$ with a horizontal shift.

12.
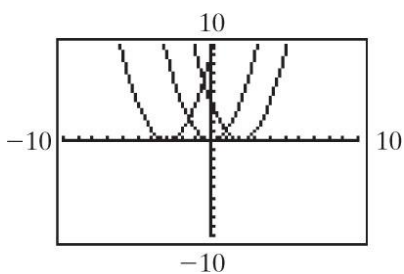
The graphs have the shape of $y = x^2$ with a horizontal shift.

13.
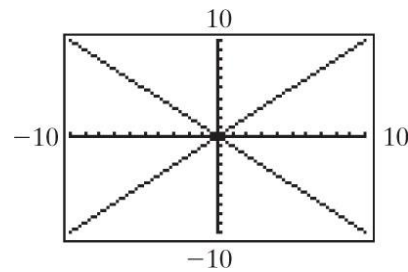
The graph of $g(x) = -|x|$ has the shape of the graph of $y = |x|$ but is reflected across the x-axis.

14.
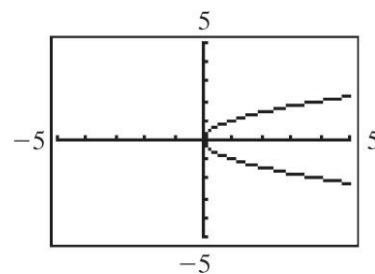
The graph of $g(x) = -\sqrt{x}$ has the shape of the graph of $y = \sqrt{x}$ but is reflected across the x-axis.

15.
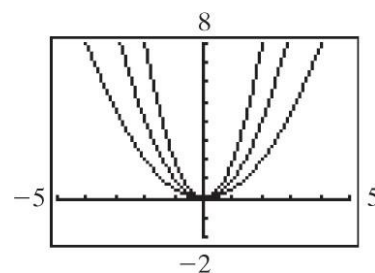
The graphs have the shape of $y = x^2$ but show a vertical shrink or stretch.

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16.
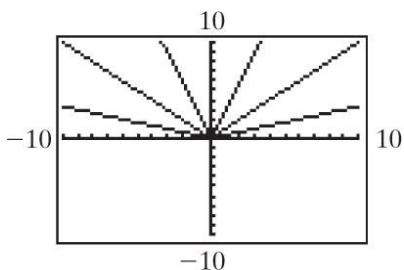
The graphs have the shape of $y = |x|$ but show a vertical shrink or stretch.

17.
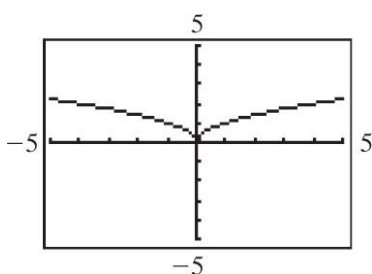
The graph of $g(x) = \sqrt{-x}$ has the shape of the graph of $y = \sqrt{x}$ but is reflected across the y-axis.

18.
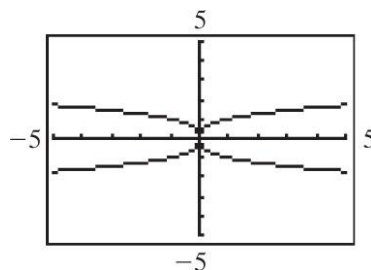
The graph of $g(x) = \sqrt[3]{-x}$ has the shape of the graph of $y = \sqrt[3]{x}$ but is reflected across the y-axis.

Section 2.6 Transformations of Graphs

1. left

2. right

3. down

4. vertical stretch

5. horizontal shrink

6. horizontal stretch

7. vertical shrink

8. x

9. e

10. f

11. b

12. c

13. a

14. d

15. The graph of f is the graph of $f(x) = |x|$ shifted upward 1 unit.
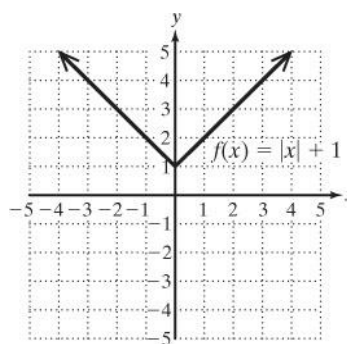
16. The graph of g is the graph of $f(x) = \sqrt{x}$ shifted upward 2 units.
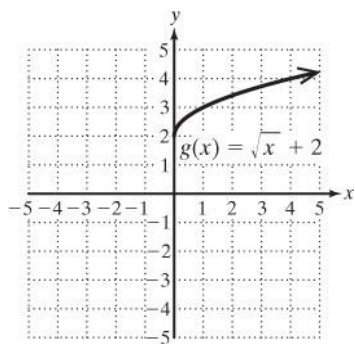
17. The graph of k is the graph of $f(x) = x^3$ shifted downward 2 units.
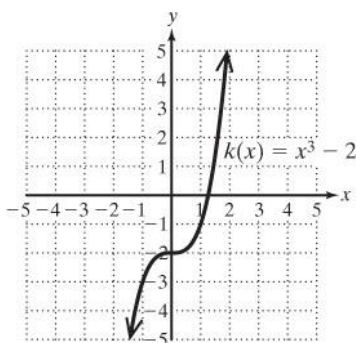
18. The graph of h is the graph of $f(x) = \dfrac{1}{x}$ shifted downward 2 units.
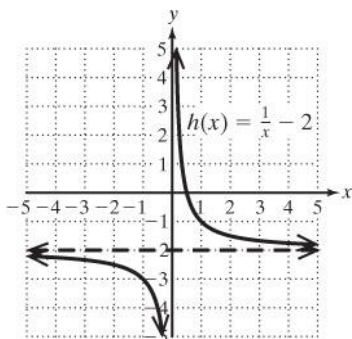
19. The graph of g is the graph of $f(x) = \sqrt{x}$ shifted to the left 5 units.
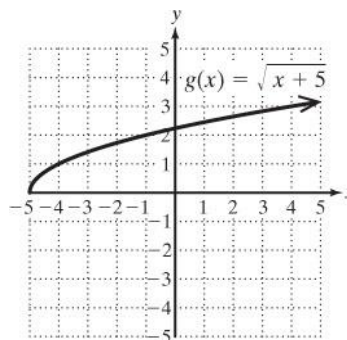
20. The graph of m is the graph of $f(x) = |x|$ shifted to the left 1 unit.
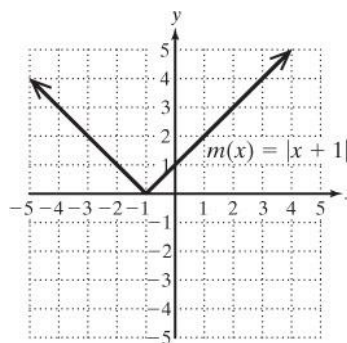
21. The graph of r is the graph of $f(x) = x^2$ shifted to the right 4 units.
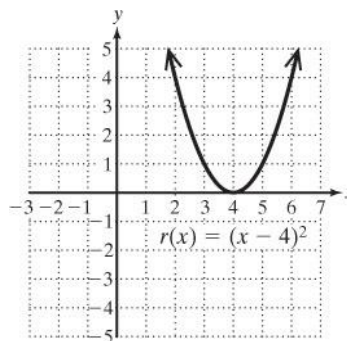
22. The graph of t is the graph of $f(x) = \sqrt[3]{x}$ shifted to the right 2 units.

$t(x) = \sqrt[3]{x} - 2$

$c(x) = \frac{1}{x-3} + 1$
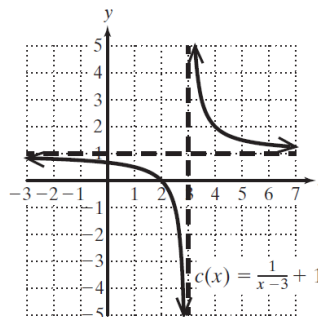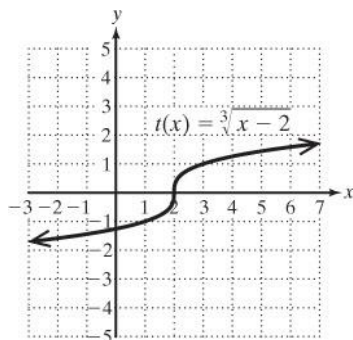
23. The graph of a is the graph of $f(x) = \sqrt{x}$ shifted to the left 1 unit and downward 3 units.
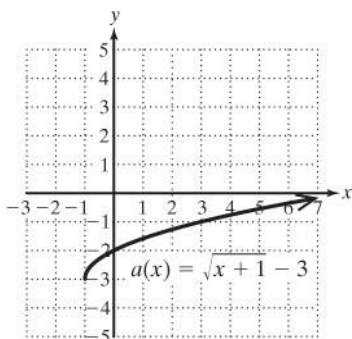
26. The graph of d is the graph of $f(x) = \frac{1}{x}$ shifted to the left 4 units and downward 1 unit.

$a(x) = \sqrt{x+1} - 3$

$d(x) = \frac{1}{x+4} - 1$
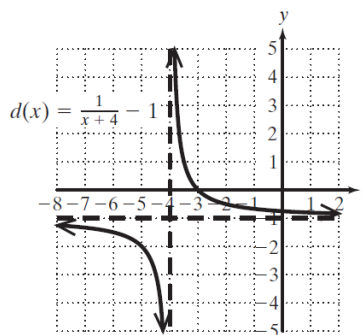
24. The graph of b is the graph of $f(x) = |x|$ shifted to the right 2 units and upward 4 units.
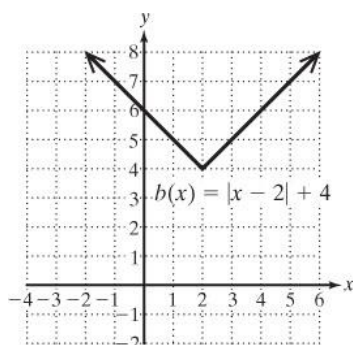
27. The graph of m is the graph of $f(x) = \sqrt[3]{x}$ stretched vertically by a factor of 4.

$b(x) = |x - 2| + 4$

$m(x) = 4\sqrt[3]{x}$
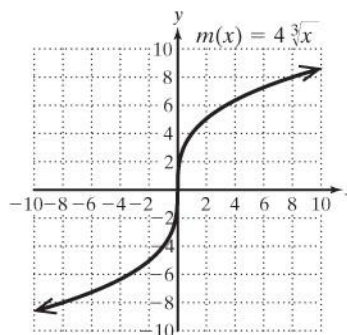
25. The graph of c is the graph of $f(x) = \frac{1}{x}$ shifted to the right 3 units and upward 1 unit.

28. The graph of n is the graph of $f(x) = |x|$ stretched vertically by a factor of 3.

The graph of p is the graph of $f(x) = |x|$ stretched vertically by a factor of 2.
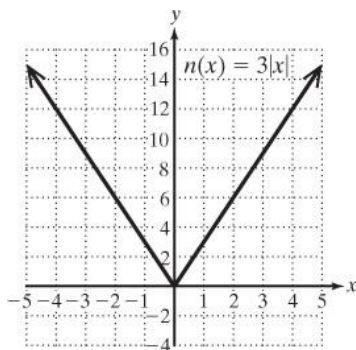
29. The graph of r is the graph of $f(x) = x^2$ shrunk vertically by a factor of $\dfrac{1}{2}$.
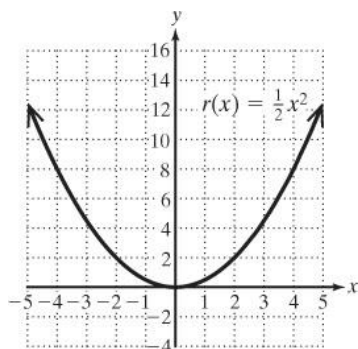
30. The graph of t is the graph of $f(x) = |x|$ shrunk vertically by a factor of $\dfrac{1}{3}$.
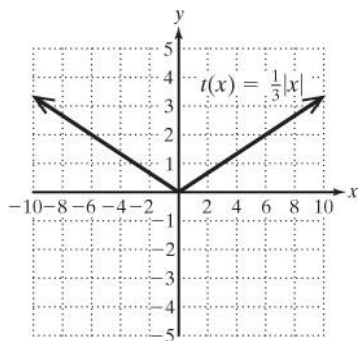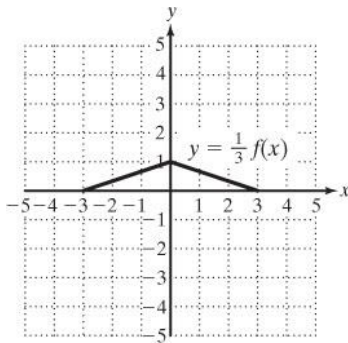

31. $p(x) = |2x| = 2|x|$
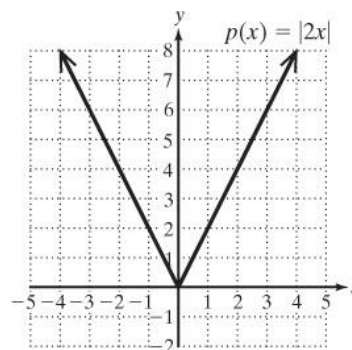
32. $q(x) = \sqrt{2x} = \sqrt{2} \cdot \sqrt{x}$

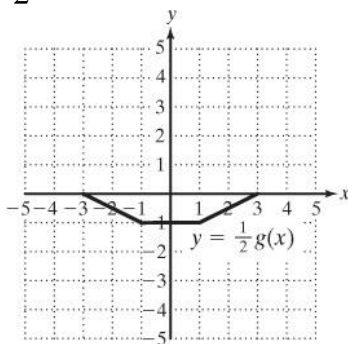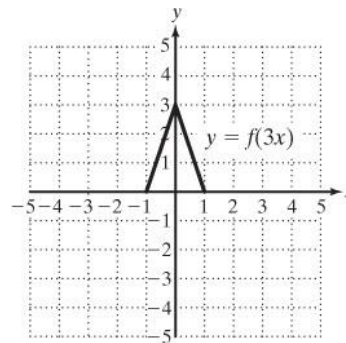The graph of q is the graph of $f(x) = \sqrt{x}$ stretched vertically by a factor of $\sqrt{2}$.
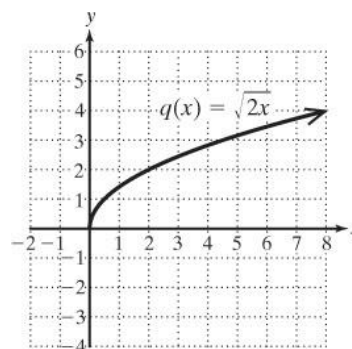
33. The graph of the function is the graph of $f(x)$ shrunk vertically by a factor of $\dfrac{1}{3}$.

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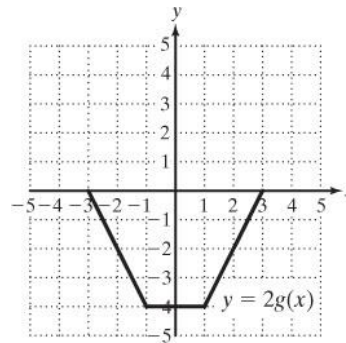
34. The graph of the function is the graph of $g(x)$ shrunk vertically by a factor of $\dfrac{1}{2}$.
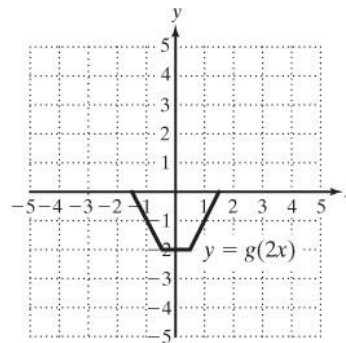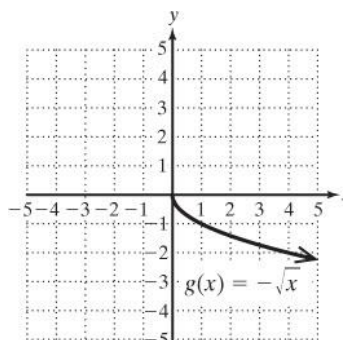

35. The graph of the function is the graph of $f(x)$ stretched vertically by a factor of 3.

36. The graph of the function is the graph of $g(x)$ stretched vertically by a factor of 2.

37. The graph of the function is the graph of $f(x)$ shrunk horizontally by a factor of 3.
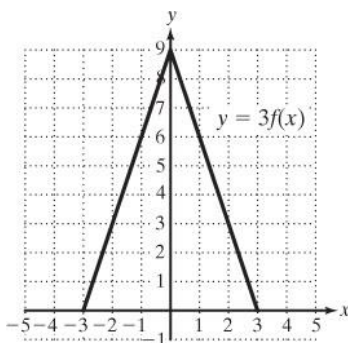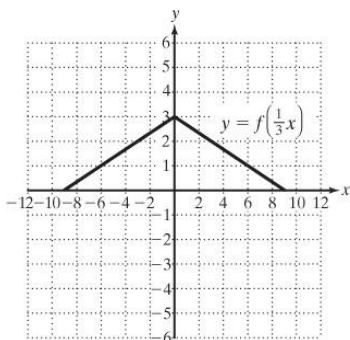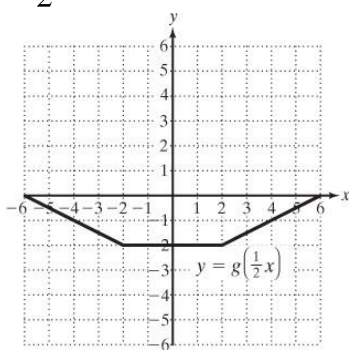

38. The graph of the function is the graph of $g(x)$ shrunk horizontally by a factor of 2.
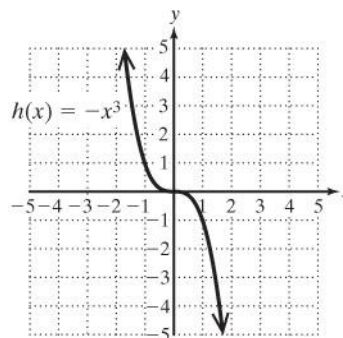

39. The graph of the function is the graph of $f(x)$ stretched horizontally by a factor of $\dfrac{1}{3}$.
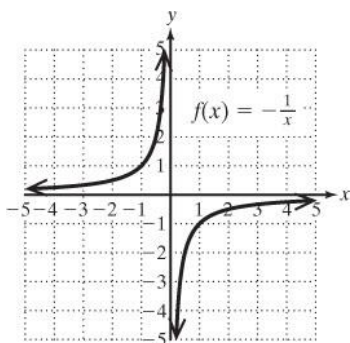
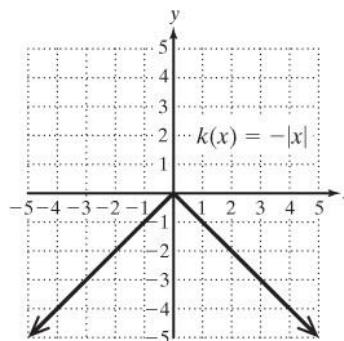**40.** The graph of the function is the graph of $g(x)$ stretched horizontally by a factor of $\dfrac{1}{2}$.

43. The graph of h is the graph of $f(x) = x^3$ reflected across the x-axis.
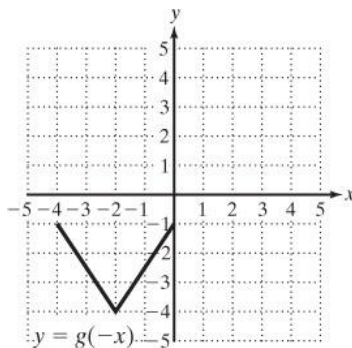

44. The graph of k is the graph of $f(x) = |x|$ reflected across the x-axis.

41. The graph of f is the graph of $f(x) = \dfrac{1}{x}$ reflected across the x-axis.
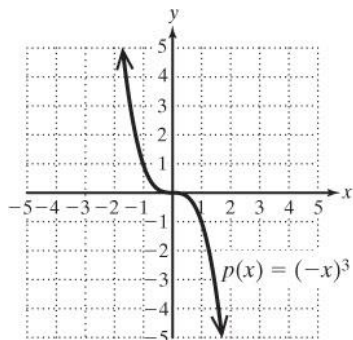

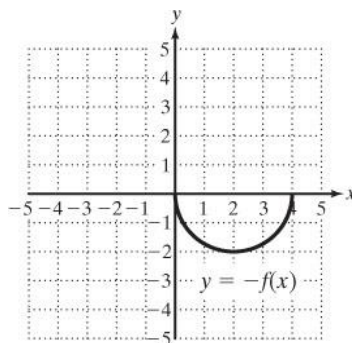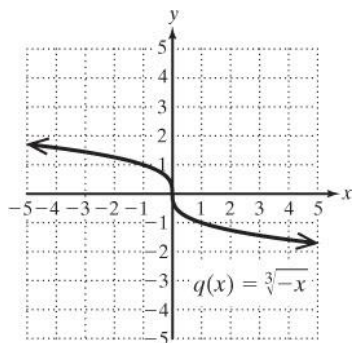**45.** The graph of p is the graph of $f(x) = x^3$ reflected across the y-axis.

42. The graph of g is the graph of $f(x) = \sqrt{x}$ reflected across the x-axis.

46. The graph of q is the graph of

$f(x) = \sqrt[3]{x}$ reflected across the y-axis.

47. The graph of the function is the graph of

$f(x)$ reflected across the y-axis.

48. The graph of the function is the graph of

$g(x)$ reflected across the y-axis.

49. The graph of the function is the graph of

$f(x)$ reflected across the x-axis.

50. The graph of the function is the graph of

$g(x)$ reflected across the x-axis.

51. The graph of the function is the graph of

$f(x)$ reflected across the y-axis.

52. The graph of the function is the graph of $g(x)$ reflected across the *y*-axis.
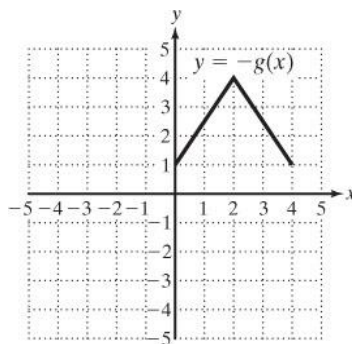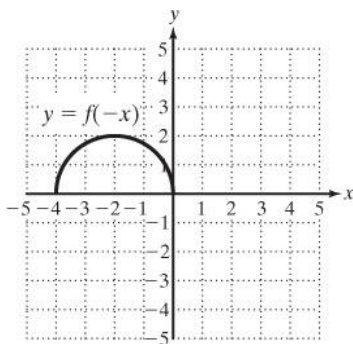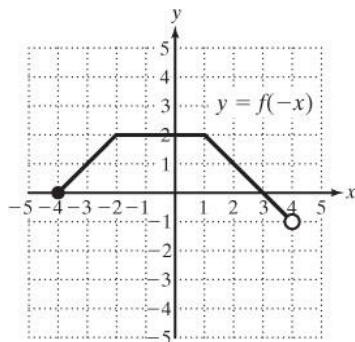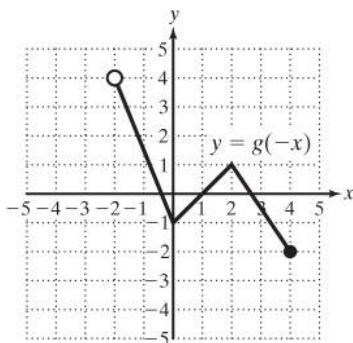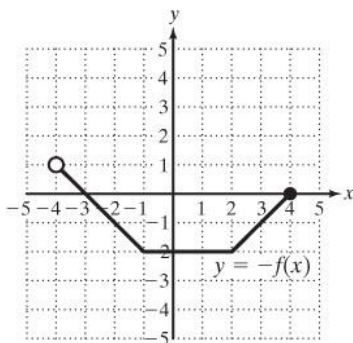

53. The graph of the function is the graph of $f(x)$ reflected across the *x*-axis.

54. The graph of the function is the graph of $g(x)$ reflected across the *x*-axis.
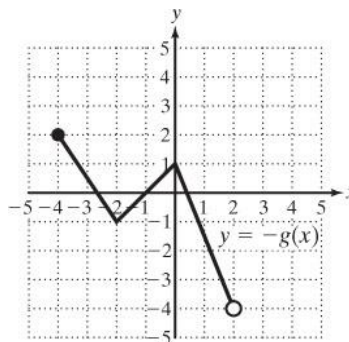
55. Parent function: $f(x) = \dfrac{1}{x}$; Shift the graph of *f* to the left 1 unit, stretch the graph vertically by a factor of 3, and shift the graph downward by 2 units.

56. Parent function: $f(x) = \dfrac{1}{x}$; Shift the graph of *f* to the right 4 unit, stretch the graph vertically by a factor of 5, and shift the graph upward by 1 units.

57. Parent function: $f(x) = x^2$; Shift the graph of *f* to the right 2.1 units, shrink the graph vertically by a factor of $\dfrac{1}{3}$, and shift the graph upward by 7.9 units.

58. Parent function: $f(x) = \sqrt{x}$; Shift the graph of *f* to the left 4.3 unit, shrink the graph vertically by a factor of $\dfrac{1}{2}$, and shift the graph downward by 8.4 units.

59. Parent function: $f(x) = \sqrt{x}$; Shift the graph of *f* to the left 5 unit, shrink the graph horizontally by a factor of $\dfrac{1}{2}$, stretch the graph vertically by a factor of 2, and reflect the graph across the *y*-axis.

60. Parent function: $f(x) = |x|$; Shift the graph of *f* to the right 4 unit, stretch the

graph horizontally by a factor of 2, stretch the graph vertically by a factor of 3, and reflect the graph across the *y*-axis.

61. Parent function: $f(x) = \sqrt{x}$; Stretch the graph *f* horizontally by a factor of 3, reflect the graph across the *x*-axis, and shift the graph downward by 6 units.

62. Parent function: $f(x) = |x|$; Shrink the graph *f* horizontally by a factor of $\dfrac{1}{2}$, reflect the graph across the *x*-axis, and shift the graph upward by 8 units.

63. The graph of *v* is the graph of $f(x) = x^2$ shifted to the left 2 units, reflected across the *x*-axis, and shifted upward 1 unit.
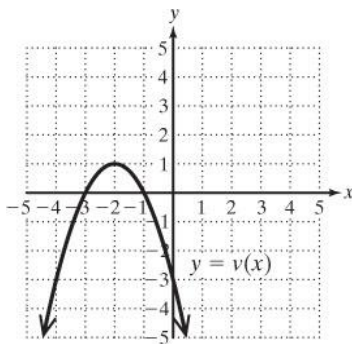
64. The graph of *u* is the graph of $f(x) = x^2$ shifted to the right 1 unit, reflected across the *x*-axis, and shifted downward 2 units.
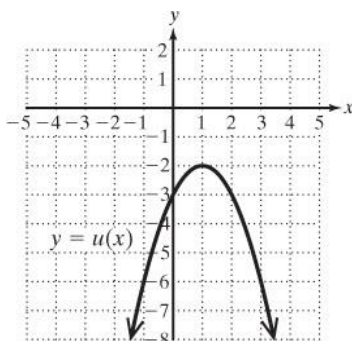
65. The graph of *f* is the graph of $f(x) = \sqrt{x}$ shifted to the left 3 units, stretched vertically by a factor of 2, and shifted downward 1 unit.
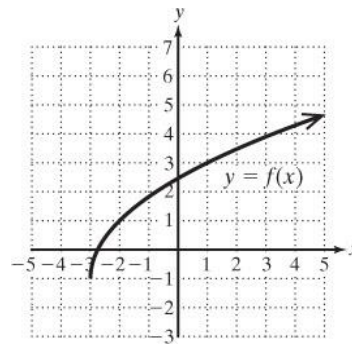
66. The graph of *g* is the graph of $f(x) = \sqrt{x}$ shifted to the right 1 unit, stretched vertically by a factor of 2, and shifted upward 3 units.
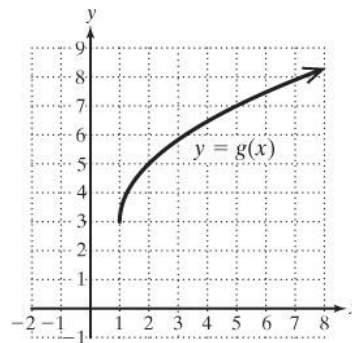
67. The graph of *p* is the graph of $f(x) = |x|$ shifted to the right 1 unit, shrunk vertically by a factor of $\dfrac{1}{2}$, and shifted downward 2 units.

$y = p(x)$
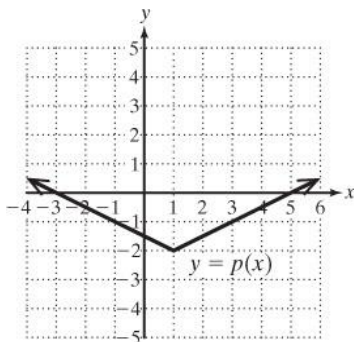
68. The graph of q is the graph of $f(x) = |x|$ shifted to the left 2 units, shrunk vertically by a factor of $\dfrac{1}{3}$, and shifted downward 1 unit.

$y = q(x)$
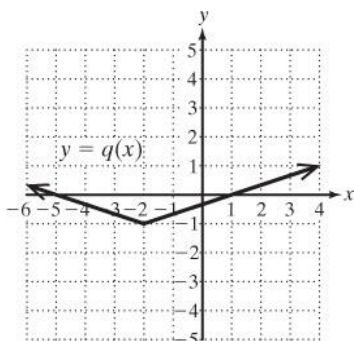
69. The graph of r is the graph of $f(x) = \sqrt{x}$ reflected across the y-axis, reflected across the x-axis, and shifted upward 1 unit.

$y = r(x)$
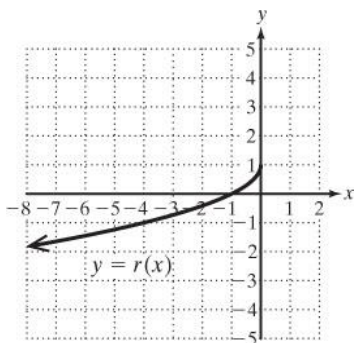
70. The graph of s is the graph of $f(x) = \sqrt{x}$ reflected across the y-axis, reflected across the x-axis, and shifted downward 2 units.

$y = s(x)$
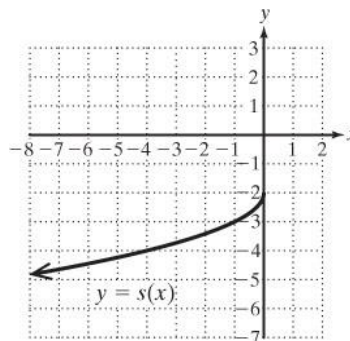
71. $f(x) = \sqrt{-x+3} = \sqrt{-(x-3)}$

The graph of f is the graph of $f(x) = \sqrt{x}$ reflected across the y-axis and shifted right 3 units.

$y = f(x)$
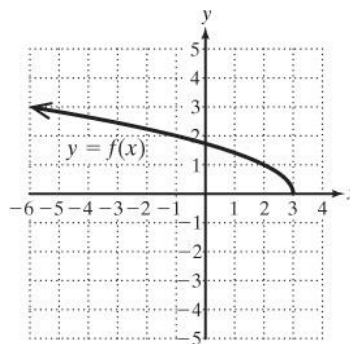
72. $g(x) = \sqrt{-x-4} = \sqrt{-(x+4)}$

The graph of g is the graph of $f(x) = \sqrt{x}$ reflected across the y-axis and shifted to the left 4 units.
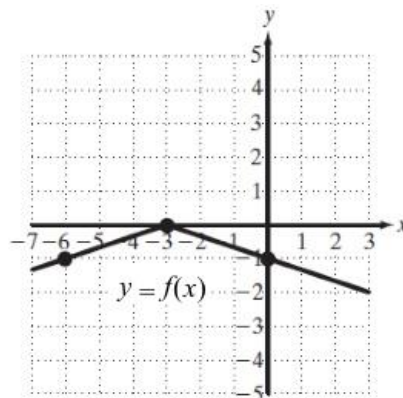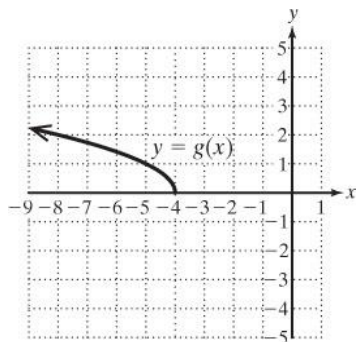
73. The graph of *n* is the graph of $f(x) = |x|$ stretch horizontally by a factor of 2, shifted to the right 3 units, and reflected across *x*-axis.

74. The graph of *m* is the graph of $f(x) = |x|$ stretch horizontally by a factor of 3, shifted to the left 1 unit, and reflected across *x*-axis.

75. The graph of *f* is the graph of $f(x) = x^2$ shifted to the right 3 units, shrunk vertically by a factor of $\frac{1}{2}$, reflected across *x*-axis, and shifted upward by 8 units.

76. The graph of *g* is the graph of $f(x) = x^2$ shifted to the left 2 units, shrunk vertically by a factor of $\frac{1}{3}$, reflected across *x*-axis, and shifted upward by 3 units.
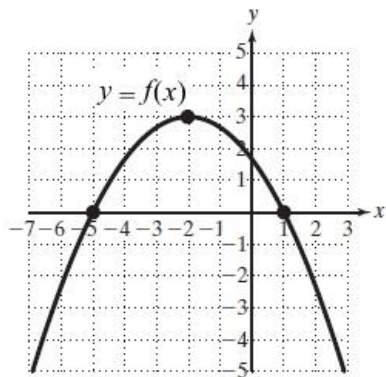
77. The graph p is the graph of $f(x) = |x|$ shifted to the left 2 units, stretch vertically by a factor of 4, reflected across x-axis, and shifted downward by 1 unit.
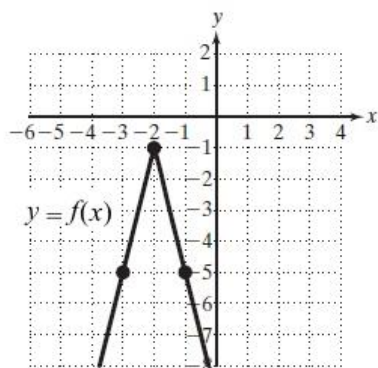
78. The graph q is the graph of $f(x) = |x|$ shifted to the right 1 unit, stretch vertically by a factor of 2, reflected across x-axis, and shifted upward by 4 units.
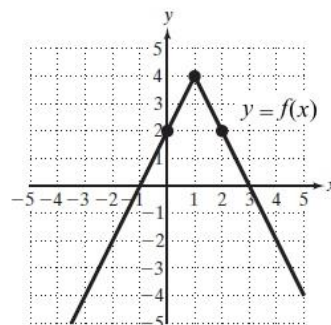
79. The graph of the function is the graph of $f(x)$ shifted to the right 1 unit, reflected across the x-axis, and shifted upward 2 units.
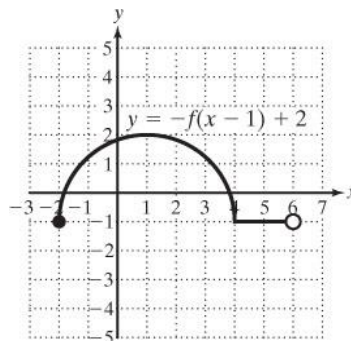
80. The graph of the function is the graph of $f(x)$ shifted to the left 1 unit, reflected across the x-axis, and shifted downward 2 units.
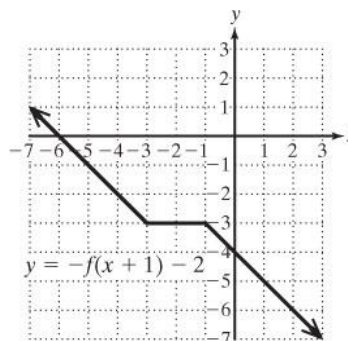
81. The graph of the function is the graph of $f(x)$ shifted to the right 2 units, stretched vertically by a factor of 2, and shifted downward 3 units.

$y = 2f(x - 2) - 3$
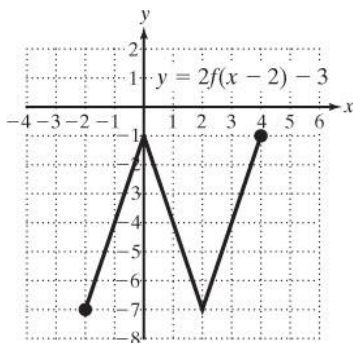
82. The graph of the function is the graph of $f(x)$ shifted to the left 2 units, stretched vertically by a factor of 2, and shifted downward 4 units.

$y = 2f(x + 2) - 4$
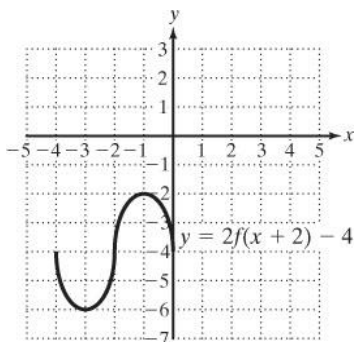
83. The graph of the function is the graph of $f(x)$ shrunk horizontally by a factor of 2, stretched vertically by a factor of 3, and reflected across the x-axis.

$y = -3f(2x)$
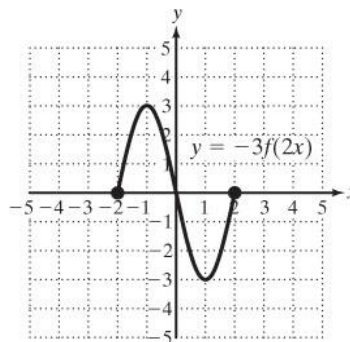
84. The graph of the function is the graph of $f(x)$ stretched horizontally by a factor of $\dfrac{1}{2}$, shrunk vertically by a factor of $\dfrac{1}{2}$, and reflected across the x-axis.

$y = -\frac{1}{2}f\left(\frac{1}{2}x\right)$
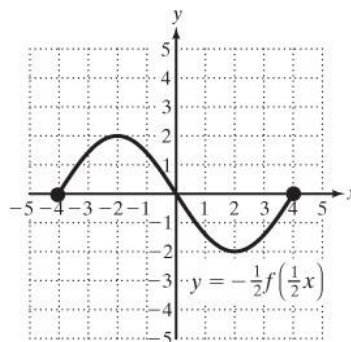
85. The graph of the function is the graph of $f(x)$ reflected across the y-axis and shifted downward 2 units.

$y = f(-x) - 2$
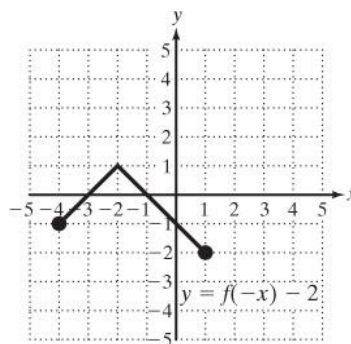
86. The graph of the function is the graph of $f(x)$ reflected across the *y*-axis and shifted upward 3 units.
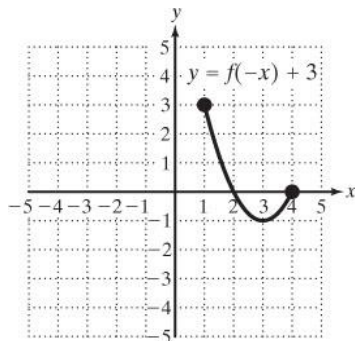
87. $y = (-x + 4.5)^3 + 2.1$

88. $y = -\sqrt[3]{\dfrac{1}{4}x + 1}$

89. $y = -\dfrac{2}{x} - 3$

90. $y = |-3x - 3.7|$

91. As written, $g(x) = |2x|$ is in the form $g(x) = f(ax)$ with $a > 1$. This indicates a horizontal shrink. However, $g(x)$ can also be written as $g(x) = |2| \cdot |x| = 2|x|$. This is written in the form $g(x) = af(x)$ with $a > 1$. This represents a vertical stretch.

92. As written, $h(x) = \sqrt{\dfrac{1}{2}x}$ is in the form $h(x) = f(ax)$ with $0 < a < 1$. This indicates a horizontal stretch. However, $h(x)$ can also be written as $h(x) = \sqrt{\dfrac{1}{2}} \cdot \sqrt{x}$. This is written in the form $h(x) = af(x)$ with $0 < a < 1$. This represents a vertical shrink.

93. The graph of *f* is the same as the graph of y $y = x$ with a horizontal shift to the right 2 units and a vertical shift downward 3 units. By contrast, the graph of *g* is the graph of $y = |x|$ with a horizontal shift to the right 3 units and a vertical shift downward 2 units.

94. The equation $g(x) = \dfrac{1}{-x + 1}$ can also be written as $g(x) = \dfrac{1}{-(x - 1)} = -\dfrac{1}{x - 1}$. Written in this alternate form, we see that shifting *f* one unit to the right and reflecting over the *x*-axis produces the same graph.

95. The graph is $f(x) = x^2$ shifted to the right 2 units and downward 3 units.

$f(x) = (x - 2)^2 - 3$

96. The graph is $f(x) = |x|$ shifted to the left 3 units and downward 4 units.

$f(x) = |x + 3| - 4$

97. The graph is $f(x) = \dfrac{1}{x}$ shifted to the left 3 units.

$f(x) = \dfrac{1}{x + 3}$

98. The graph is $f(x) = \sqrt{x}$ shifted downward 2 units.

$f(x) = \sqrt{x} - 2$

99. The graph is $f(x) = x^3$ reflected across the *x*-axis and shifted upward 1 unit.

$f(x) = -x^3 + 1$

100. The graph is $f(x) = x^2$ shifted left 2 units and reflected across the *x*-axis.

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$$f(x) = -(x+2)^2$$

101. $y = 2t^2 + 1$

102. $y = 3\sqrt{t} + 6$

103. a.

b.
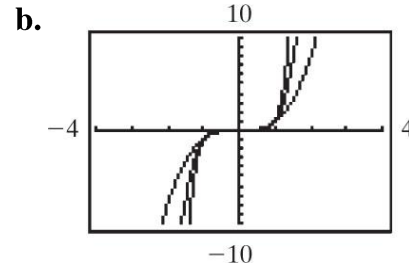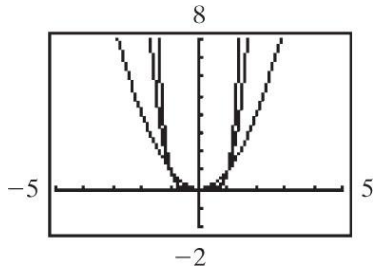
c. The general shape of $y = x^n$ is similar to the graph of $y = x^2$ for even values of n greater than 1.

d. The general shape of $y = x^n$ is similar to the graph of $y = x^3$ for odd values of n greater than 1.

Section 2.7 Analyzing Graphs of Functions and Piecewise-Defined Functions

1. y

2. x

3. origin

4. y-axis

5. origin

6. $[\![x]\!]$ or int(x) or floor(x)

7. $y = x^2 + 3$

Replace x by $-x$.

$$y = (-x)^2 + 3$$
$$y = x^2 + 3$$

This equation *is* equivalent to the original equation, so the graph is symmetric with respect to the y-axis.

Replace y by $-y$.

$$-y = x^2 + 3$$
$$y = -x^2 - 3$$

This equation is *not* equivalent to the original equation, so the graph is not symmetric with respect to the x-axis.

Replace x by $-x$ and y by $-y$.

$$-y = (-x)^2 + 3$$
$$-y = x^2 - 3$$
$$y = -x^2 - 3$$

This equation is *not* equivalent to the original equation, so the graph is not symmetric with respect to the origin.

8. $y = -|x| - 4$

Replace x by $-x$.

$$y = -|-x| - 4$$
$$y = -|x| - 4$$

This equation *is* equivalent to the original equation, so the graph is symmetric with

respect to the *y*-axis.

Replace *y* by $-y$.

$-y = -|x| - 4$

$y = |x| + 4$

This equation is *not* equivalent to the original equation, so the graph is not symmetric with respect to the *x*-axis.

Replace *x* by $-x$ and *y* by $-y$.

$-y = -|-x| - 4$

$-y = -|x| - 4$

$y = |x| + 4$

This equation is *not* equivalent to the original equation, so the graph is not symmetric with respect to the origin.

9. $x = -|y| - 4$

Replace *x* by $-x$.

$-x = -|y| - 4$

$x = |y| + 4$

This equation is *not* equivalent to the original equation, so the graph is not symmetric with respect to the *y*-axis.

Replace *y* by $-y$.

$x = -|-y| - 4$

$x = -|y| - 4$

This equation *is* equivalent to the original equation, so the graph is symmetric with respect to the *x*-axis.

Replace *x* by $-x$ and *y* by $-y$.

$x = -|y| - 4$

$-x = -|-y| - 4$

$x = |y| + 4$

This equation is *not* equivalent to the original equation, so the graph is not symmetric with respect to the origin.

10. $x = y^2 + 3$

Replace *x* by $-x$.

$-x = y^2 + 3$

$x = -y^2 - 3$

This equation is *not* equivalent to the original equation, so the graph is not symmetric with respect to the *y*-axis.

Replace *y* by $-y$.

$x = (-y)^2 + 3$

$x = y^2 + 3$

This equation *is* equivalent to the original equation, so the graph is symmetric with respect to the *x*-axis.

Replace *x* by $-x$ and *y* by $-y$.

$-x = (-y)^2 + 3$

$-x = y^2 + 3$

$x = -y^2 - 3$

This equation is *not* equivalent to the original equation, so the graph is not symmetric with respect to the origin.

11. $x^2 + y^2 = 3$

Replace *x* by $-x$.

$(-x)^2 + y^2 = 3$

$x^2 + y^2 = 3$

This equation *is* equivalent to the original equation, so the graph is symmetric with respect to the *y*-axis.

Replace *y* by $-y$.

$x^2 + (-y)^2 = 3$

$x^2 + y^2 = 3$

This equation *is* equivalent to the original equation, so the graph is symmetric with respect to the *x*-axis.

Replace *x* by $-x$ and *y* by $-y$.

$$\left(-x\right)^2 + \left(-y\right)^2 = 3$$
$$x^2 + y^2 = 3$$

This equation *is* equivalent to the original equation, so the graph is symmetric with respect to the origin.

12. $\left|x\right| + \left|y\right| = 4$

Replace x by $-x$.

$$\left|-x\right| + \left|y\right| = 4$$
$$\left|x\right| + \left|y\right| = 4$$

This equation *is* equivalent to the original equation, so the graph is symmetric with respect to the y-axis.

Replace y by $-y$.

$$\left|x\right| + \left|-y\right| = 4$$
$$\left|x\right| + \left|y\right| = 4$$

This equation *is* equivalent to the original equation, so the graph is symmetric with respect to the x-axis.

Replace x by $-x$ and y by $-y$.

$$\left|-x\right| + \left|-y\right| = 4$$
$$\left|x\right| + \left|y\right| = 4$$

This equation *is* equivalent to the original equation, so the graph is symmetric with respect to the origin.

13. $y = \left|x\right| + 2x + 7$

Replace x by $-x$.

$$y = \left|-x\right| + 2\left(-x\right) + 7$$
$$y = \left|x\right| - 2x + 7$$

This equation is *not* equivalent to the original equation, so the graph is not symmetric with respect to the y-axis.

Replace y by $-y$.

$$-y = \left|x\right| + 2x + 7$$
$$y = -\left|x\right| - 2x - 7$$

This equation is *not* equivalent to the original equation, so the graph is not symmetric with respect to the x-axis.

Replace x by $-x$ and y by $-y$.

$$-y = \left|-x\right| + 2\left(-x\right) + 7$$
$$-y = \left|x\right| - 2x + 7$$
$$y = -\left|x\right| + 2x - 7$$

This equation is *not* equivalent to the original equation, so the graph is not symmetric with respect to the origin.

14. $y = x^2 + 6x + 1$

Replace x by $-x$.

$$y = \left(-x\right)^2 + 6\left(-x\right) + 1$$
$$y = x^2 - 6x + 1$$

This equation is *not* equivalent to the original equation, so the graph is not symmetric with respect to the y-axis.

Replace y by $-y$.

$$-y = x^2 + 6x + 1$$
$$y = -x^2 - 6x - 1$$

This equation is *not* equivalent to the original equation, so the graph is not symmetric with respect to the x-axis.

Replace x by $-x$ and y by $-y$.

$$-y = \left(-x\right)^2 + 6\left(-x\right) + 1$$
$$-y = x^2 - 6x + 1$$
$$y = -x^2 + 6x - 1$$

This equation is *not* equivalent to the original equation, so the graph is not symmetric with respect to the origin.

15. $x^2 = 5 + y^2$

Replace x by $-x$.

$$\left(-x\right)^2 = 5 + y^2$$
$$x^2 = 5 + y^2$$

This equation *is* equivalent to the original equation, so the graph is symmetric with respect to the *y*-axis. Replace *y* by $-y$.

$$x^2 = 5 + (-y)^2$$
$$x^2 = 5 + y^2$$

This equation *is* equivalent to the original equation, so the graph is symmetric with respect to the *x*-axis. Replace *x* by $-x$ and *y* by $-y$.

$$(-x)^2 = 5 + (-y)^2$$
$$x^2 = 5 + y^2$$

This equation *is* equivalent to the original equation, so the graph is symmetric with respect to the origin.

16. $y^4 = 2 + x^2$

Replace *x* by $-x$.

$$y^4 = 2 + (-x)^2$$
$$y^4 = 2 + x^2$$

This equation *is* equivalent to the original equation, so the graph is symmetric with respect to the *y*-axis. Replace *y* by $-y$.

$$(-y)^4 = 2 + x^2$$
$$y^4 = 2 + x^2$$

This equation *is* equivalent to the original equation, so the graph is symmetric with respect to the *x*-axis. Replace *x* by $-x$ and *y* by $-y$.

$$(-y)^4 = 2 + (-x)^2$$
$$y^4 = 2 + x^2$$

This equation *is* equivalent to the original equation, so the graph is symmetric with respect to the origin.

17. $y = \dfrac{1}{2}x - 3$

Replace *x* by $-x$.

$$y = \frac{1}{2}(-x) - 3$$
$$y = -\frac{1}{2}x - 3$$

This equation is *not* equivalent to the original equation, so the graph is not symmetric with respect to the *y*-axis. Replace *y* by $-y$.

$$-y = \frac{1}{2}x - 3$$
$$y = -\frac{1}{2}x + 3$$

This equation is *not* equivalent to the original equation, so the graph is not symmetric with respect to the *x*-axis. Replace *x* by $-x$ and *y* by $-y$.

$$-y = \frac{1}{2}(-x) - 3$$
$$-y = -\frac{1}{2}x - 3$$
$$y = \frac{1}{2}x + 3$$

This equation is *not* equivalent to the original equation, so the graph is not symmetric with respect to the origin.

18. $y = \dfrac{2}{5}x + 1$

Replace *x* by $-x$.

$$y = \frac{2}{5}(-x) + 1$$
$$y = -\frac{2}{5}x + 1$$

This equation is *not* equivalent to the original equation, so the graph is not symmetric with respect to the *y*-axis. Replace *y* by $-y$.

$$-y = \frac{2}{5}x + 1$$

$$y = -\frac{2}{5}x - 1$$

This equation is *not* equivalent to the original equation, so the graph is not symmetric with respect to the *x*-axis. Replace *x* by $-x$ and *y* by $-y$.

$$-y = \frac{2}{5}(-x) + 1$$

$$-y = -\frac{2}{5}x + 1$$

$$y = \frac{2}{5}x - 1$$

This equation is *not* equivalent to the original equation, so the graph is not symmetric with respect to the origin.

19. *y*-axis symmetry

20. Origin symmetry

21. The function is symmetric to the origin. Therefore, the function is an odd function.

22. The function is symmetric to the origin. Therefore, the function is an odd function.

23. The function is symmetric to the *y*-axis. Therefore, the function is an even function.

24. The function is symmetric to the *y*-axis. Therefore, the function is an even function.

25. The function is not symmetric with respect to either the *y*-axis or the origin. Therefore, the function is neither even nor odd.

26. The function is not symmetric with respect to either the *y*-axis or the origin. Therefore, the function is neither even nor odd.

27. a. $f(x) = 4x^2 - 3|x|$

$f(-x) = 4(-x)^2 - 3|-x| = 4x^2 - 3|x|$

b. Yes

c. Even

28. a. $g(x) = -x^8 + |3x|$

$g(-x) = -(-x)^8 + |3(-x)| = -x^8 + |3x|$

b. Yes

c. Odd

29. a. $h(x) = 4x^3 - 2x$

$h(-x) = 4(-x)^3 - 2(-x) = -4x^3 + 2x$

b. $-h(x) = -(4x^3 - 2x) = -4x^3 + 2x$

c. Yes

d. Odd

30. a. $k(x) = -8x^5 - 6x^3$

$k(-x) = -8(-x)^5 - 6(-x)^3$

$= 8x^5 + 6x^3$

b. $-k(x) = -(-8x^5 - 6x^3) = 8x^5 + 6x^3$

c. Yes

d. Odd

31. a. $m(x) = 4x^2 + 2x - 3$

$m(-x) = 4(-x)^2 + 2(-x) - 3$

$= 4x^2 - 2x - 3$

b. $-m(x) = -(4x^2 + 2x - 3)$

$= -4x^2 - 2x + 3$

c. No

d. No

e. Neither

32. a. $n(x) = 7|x| + 3x - 1$

$n(-x) = 7|-x| + 3(-x) - 1$

$\qquad = 7|x| - 3x - 1$

b. $-n(x) = -\left(7|x| + 3x - 1\right)$

$\qquad = -7|x| - 3x + 1$

c. No

d. No

e. Neither

33. $f(x) = 3x^6 + 2x^2 + |x|$

$f(-x) = 3(-x)^6 + 2(-x) + |-x|$

$f(-x) = 3x^6 + 2x^2 + |x|$

$f(-x) = f(x)$

The function is even.

34. $p(x) = -|x| + 12x^{10} + 5$

$p(-x) = -|-x| + 12(-x)^{10} + 5$

$p(-x) = -|x| + 12x^{10} + 5$

$p(-x) = p(x)$

The function is even.

35. $k(x) = 13x^3 + 12x$

$k(-x) = 13(-x)^3 + 12(-x)$

$k(-x) = -13x^3 - 12x$

$-k(x) = -\left(13x^3 + 12x\right) = -13x^3 - 12x$

$k(-x) = -k(x)$

The function is odd.

36. $m(x) = -4x^5 + 2x^3 + x$

$m(-x) = -4(-x)^5 + 2(-x)^3 + (-x)$

$m(-x) = 4x^5 - 2x^3 - x$

$-m(x) = -\left(-4x^5 + 2x^3 + x\right)$

$\qquad = 4x^5 - 2x^3 - x$

$m(-x) = -m(x)$

The function is odd.

37. $n(x) = \sqrt{16 - (x-3)^2}$

$n(-x) = \sqrt{16 - (-x-3)^2}$

$-n(x) = -\sqrt{16 - (x-3)^2}$

$n(-x) \neq n(x)$

$n(-x) \neq -n(x)$

The function is neither even nor odd.

38. $r(x) = \sqrt{81 - (x+2)^2}$

$r(-x) = \sqrt{81 - (-x+2)^2}$

$-r(x) = -\sqrt{81 - (x+2)^2}$

$r(-x) \neq r(x)$

$r(-x) \neq -r(x)$

The function is neither even nor odd.

39. $q(x) = \sqrt{16 + x^2}$

$q(-x) = \sqrt{16 + (-x)^2} = \sqrt{16 + x^2}$

$q(-x) = q(x)$

The function is even.

40. $z(x) = \sqrt{49 + x^2}$

$z(-x) = \sqrt{49 + (-x)^2} = \sqrt{49 + x^2}$

$z(-x) = z(x)$

The function is even.

41. $h(x) = 5x$

$h(-x) = 5(-x) = -5x$

$-h(x) = -(5x) = -5x$

$h(-x) = -h(x)$

The function is odd.

42. $g(x) = -x$

$g(-x) = -(-x) = x$

$-g(x) = -(-x) = x$

$g(-x) = -g(x)$

The function is odd.

43. $f(x) = \dfrac{x^2}{3(x-4)^2}$

$f(-x) = \dfrac{(-x)^2}{3(-x-4)^2}$

$f(-x) = \dfrac{x^2}{3(x+4)^2}$

$f(x) \neq f(-x)$

The function is neither even nor odd.

44. $g(x) = \dfrac{x^3}{2(x-1)^3}$

$g(-x) = \dfrac{(-x)^3}{2(-x-1)^3}$

$g(-x) = \dfrac{x^3}{2(x+1)^3}$

$g(x) \neq g(-x)$

The function is neither even nor odd.

45. $v(x) = \dfrac{-x^5}{|x|+2}$

$v(-x) = \dfrac{-(-x)^5}{|x|+2}$

$v(-x) = \dfrac{x^5}{|x|+2}$

$v(-x) = -v(x)$

The function is odd.

46. $w(x) = \dfrac{-\sqrt[3]{x}}{x^2+1}$

$w(-x) = \dfrac{-\sqrt[3]{-x}}{(-x)^2+1}$

$w(-x) = \dfrac{\sqrt[3]{x}}{x^2+1}$

$w(-x) = -w(x)$

The function is odd.

47. a. Use the second rule.

$f(3) = (3)^2 + 3 = 9 + 3 = 12$

b. Use the first rule.

$f(-2) = -3(-2) + 7 = 6 + 7 = 13$

c. Use the second rule.

$f(-1) = (-1)^2 + 3 = 1 + 3 = 4$

d. Use the third rule. $f(4) = 5$

e. Use the third rule. $f(5) = 5$

48. a. Use the first rule.

$g(-3) = -2|-3| - 3 = -6 - 3 = -9$

b. Use the third rule. $g(3) = 4$

c. Use the first rule.

$g(-2) = -2|-2| - 3 = -4 - 3 = -7$

d. Use the second rule.

$g(0) = 5(0) + 6 = 0 + 6 = 6$

e. Use the third rule. $g(4) = 4$

49. a. Use the second rule. $h(-1.7) = 1$

b. Use the first rule. $h(-2.5) = 2$

c. Use the second rule. $h(0.05) = -1$

d. Use the fourth rule. $h(-2) = 1$

e. Use the second rule. $h(0) = -1$

50. a. Use the second rule.

$t(1.99) = 2(1.99) = 3.98$

b. Use the first rule. $t(0.4) = 0.4$

c. Use the third rule. $t(3) = 3(3) = 9$

d. Use the first rule. $t(1) = 1$

e. Use the fourth rule.

$t(3.001) = 4(3.001) = 12.004$

51. Use the first rule to find the speed of the sled after 10 sec:

$s(10) = 1.5 \times 10 = 15$ ft/sec

Use the first rule to find the speed of the sled after 20 sec:

$s(20) = 1.5 \times 20 = 30$ ft/sec

Use the second rule to find the speed of the sled after 30 sec:

$$s(30) = \frac{30}{30-19} = 2.7 \text{ ft/sec}$$

Use the second rule to find the speed of the sled after 40 sec:

$$s(40) = \frac{30}{40-19} = 1.4 \text{ ft/sec}$$

52. Use the first rule to find the speed of the car 6 sec after the car begins motion:

$$s(6) = \frac{5}{12}6^2 = 15 \text{ mph}$$

Use the first rule to find the speed of the car 12 sec after the car begins motion:

$$s(12) = \frac{5}{12}12^2 = 60 \text{ mph}$$

Use the second rule to find the speed of the car 45 sec after the car begins motion:

$$s(45) = 60 \text{ mph}$$

Use the third rule to find the speed of the car 80 sec after the car begins motion:

$$s(80) = \frac{3}{20}(92-80)^2 = 21.6 \text{ mph}$$

53. c

54. a

55. d

56. b

57. a.

b.

c.

58. a.

b.

c.

60. a.

59. a.

b.

b.

c.

c.

61.

$y = f(x)$
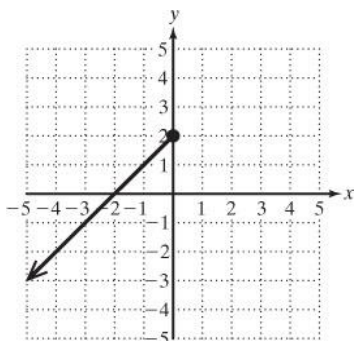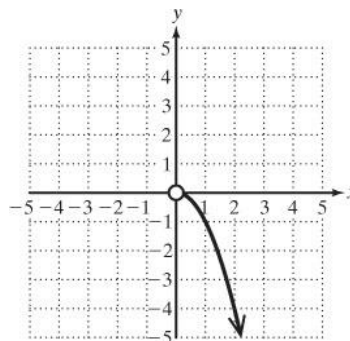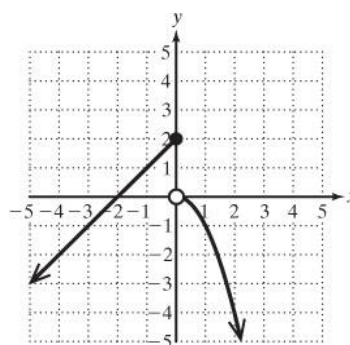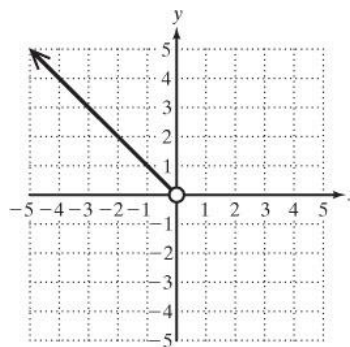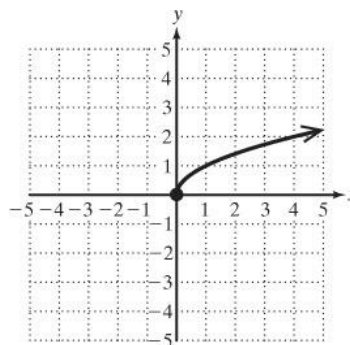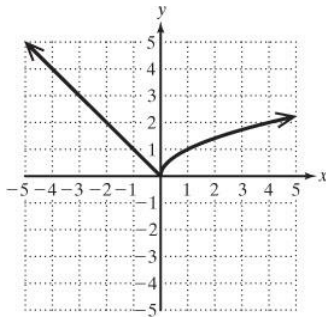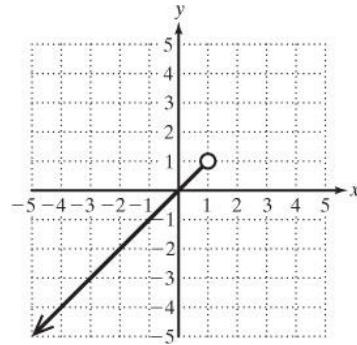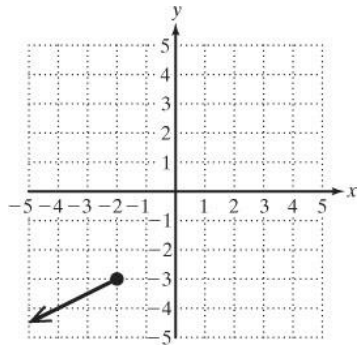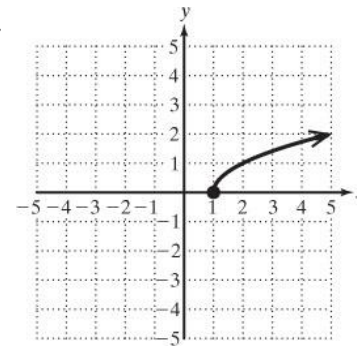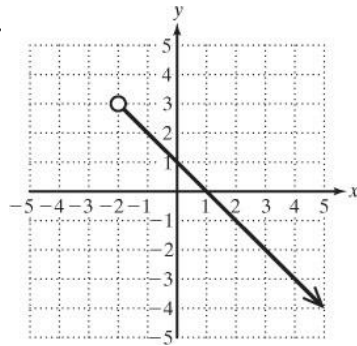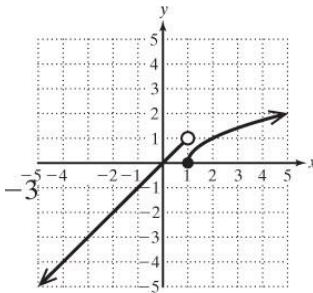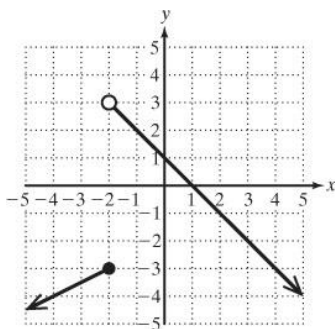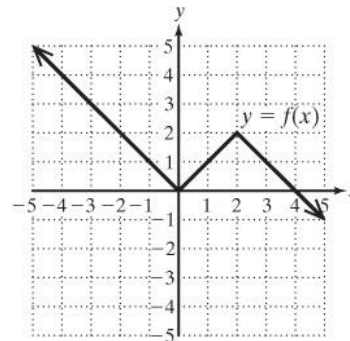
62.

63.

64.

65.

66.

67.

68.

69.

317
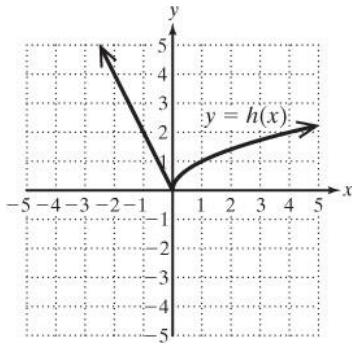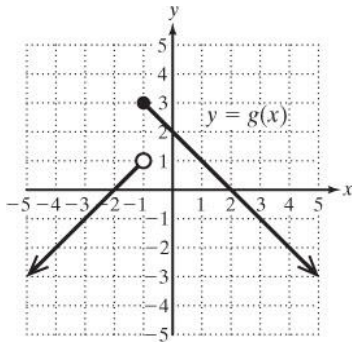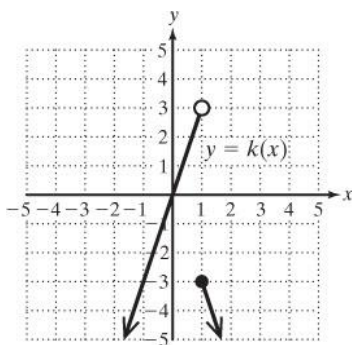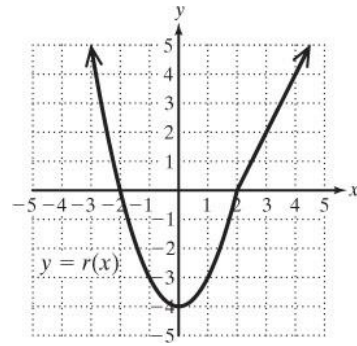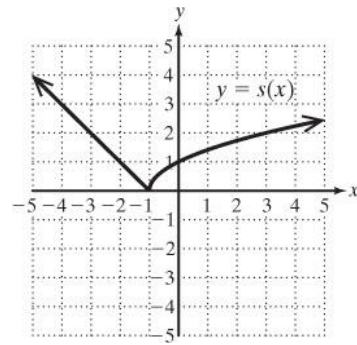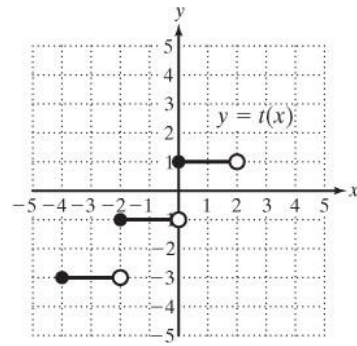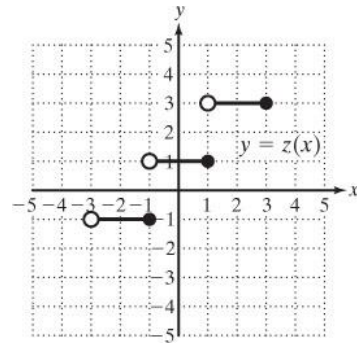
69.
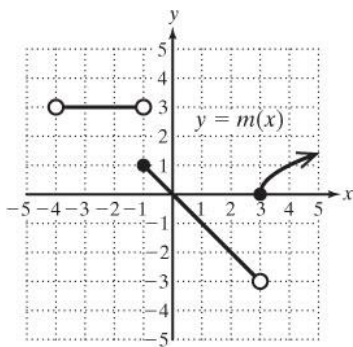
70.
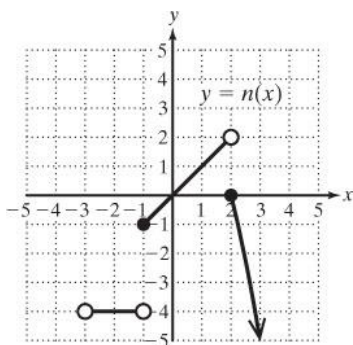
71. a.
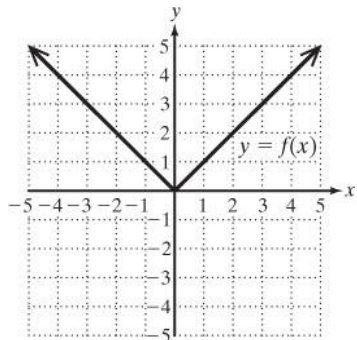
b. $y = |x|$

72. $f(x) = [\![x]\!]$
$f(-3.7) = [\![-3.7]\!] = -4$

73. $f(x) = [\![x]\!]$
$f(-4.2) = [\![-4.2]\!] = -5$

74. $f(x) = [\![x]\!]$
$f(-0.5) = [\![-0.5]\!] = -1$

75. $f(x) = [\![x]\!]$
$f(-0.09) = [\![-0.09]\!] = -1$

76. $f(x) = [\![x]\!]$
$f(0.5) = [\![0.5]\!] = 0$

77. $f(x) = [\![x]\!]$
$f(0.09) = [\![0.09]\!] = 0$

78. $f(x) = [\![x]\!]$
$f(6) = [\![6]\!] = 6$

79. $f(x) = [\![x]\!]$
$f(-9) = [\![-9]\!] = -9$

80. $f(x) = [\![x]\!]$
$f(-5) = [\![-5]\!] = -5$

81.

82.

83.

84.

$y = h(x)$
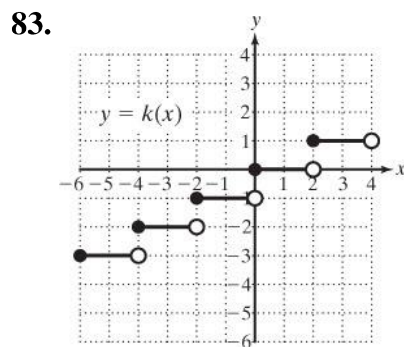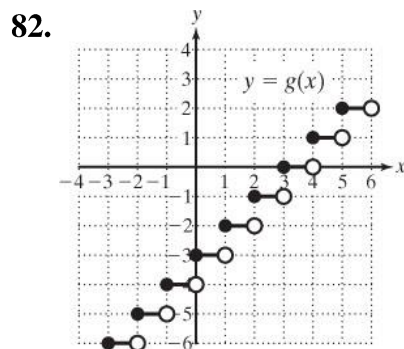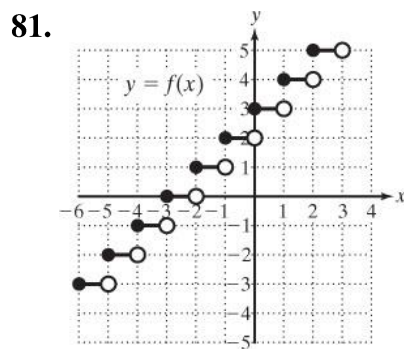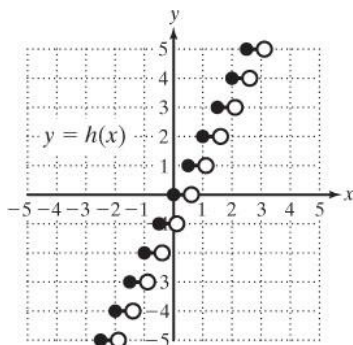
85. $C(x) = \begin{cases} 0.44 & \text{for } 0 < x \le 1 \\ 0.61 & \text{for } 1 < x \le 2 \\ 0.78 & \text{for } 2 < x \le 3 \\ 0.95 & \text{for } 3 < x \le 3.5 \end{cases}$

86. $L(x) = \begin{cases} 60 - 2x & \text{for } 0 \le x \le 14 \\ 32 + 2.5(x - 14) & \text{for } 14 < x \le 19 \end{cases}$

87. $S(x) = \begin{cases} 2000 & \text{for } 2 < x \le 3 \\ 2000 + 0.05(x - 40{,}000) & \text{for } 3 < x \le 3.5 \end{cases}$

88. $C(x) = \begin{cases} 63.97 & \text{for } 0 \le x \le 600 \\ 63.97 + 0.40(x - 600) & \text{for } x > 600 \end{cases}$

89. a. $(2, \infty)$

  b. $(-3, -2)$

  c. $(-2, 2)$

90. a. $(-\infty, 0)$

  b. $(0, 1)$

  c. $(1, 4)$

91. a. $(-\infty, \infty)$

  b. Never decreasing

  c. Never constant

92. a. $(-\infty, \infty)$

  b. Never decreasing

  c. Never constant

93. a. $(1, \infty)$

  b. $(-\infty, 1)$

  c. Never constant

94. a. $(-\infty, -1)$

  b. $(-1, \infty)$

  c. Never constant

95. a. $(-\infty, -2) \cup (2, \infty)$

  b. Never decreasing

  c. $(-2, 2)$

96. a. Never increasing

  b. $(-\infty, 0) \cup (2, \infty)$

  c. $(0, 2)$

97. At $x = 1$, the function has a relative minimum of -3.

98. At $x = -1$, the function has a relative maximum of 2.

99. At $x = -2$, the function has a relative minimum of 0. At $x = 0$, the function has a relative maximum of 2. At $x = 2$, the function has a relative minimum of 0.

100. At $x = -2$, the function has a relative maximum of 3. At $x = 0$, the function

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has a relative minimum of 1. At $x=2$, the function has a relative maximum of 3.

101. At $x=-2$, the function has a relative minimum of -4. At $x=0$, the function has a relative maximum of 0. At $x=2$, the function has a relative minimum of -4.

102. At $x=-3$, the function has a relative maximum of 5. At $x=0$, the function has a relative minimum of 0. At $x=3$, the function has a relative maximum of 5.

103. a. $(8,12)$ and $(18,20)$

  b. $(0,8)$ and $(12,18)$

  c. The function has relative minima of 3 ft and 3.5 ft at approximately 8 days and 18 days after recording began. The function has a relative maximum of 4.5 ft at a time 12 days after recording began.

  d. The weather was dry on the intervals of decreasing depth and water from the pond evaporated. The weather was rainy during intervals of increasing depth.

104. a. $(0,20)$ and $(30,50)$

  b. $(20,30)$ and $(50,70)$

  c. The function has relative maxima of 50 m and 90 m at approximately 20 sec and 50 sec into the ride. The function has a relative minimum of 40 m at a time 30 sec into the ride.

105. $f(x) = \begin{cases} -2 & \text{for } x < 1 \\ 3 & \text{for } x \geq 1 \end{cases}$

106. $f(x) = \begin{cases} 4 & \text{for } x \leq -2 \\ 1 & \text{for } x > -2 \end{cases}$

107. $f(x) = \begin{cases} -|x| & \text{for } x < 2 \\ -2 & \text{for } x \geq 2 \end{cases}$

108. $f(x) = \begin{cases} x^2 & \text{for } x < 2 \\ 4 & \text{for } x \geq 2 \end{cases}$

109. $f(x) = \begin{cases} \dfrac{1}{x} & \text{for } x < 0 \\ x & \text{for } x > 0 \end{cases}$

110. $f(x) = \begin{cases} -x & \text{for } x < 0 \\ \sqrt{x}+1 & \text{for } x \geq 0 \end{cases}$

111. a.
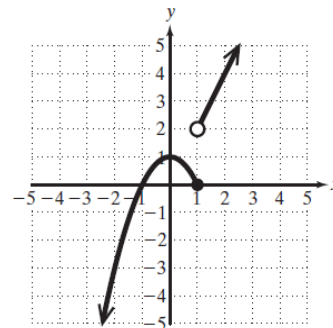
  b. $(-\infty, \infty)$

  c. $(-\infty, 1) \cup (2, \infty)$

  d. $f(-1) = -(-1)^2 + 1 = 0$

   $f(1) = -(1)^2 + 1 = 0$

   $f(2) = 2(2) = 4$

  e. $f(x) = 6$

   $2x = 6$

   $x = 3$

  f. $f(x) = -3$

   $-x^2 + 1 = -3$

   $x^2 = 4$

   $x = -2$

  g. Increasing: $(-\infty, 0) \cup (1, \infty)$;

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Decreasing: $(0,1)$;

Never constant

112. a.
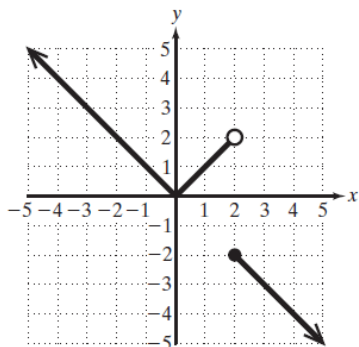
b. $(-\infty, \infty)$

c. $(-\infty, -2] \cup [0, \infty)$

d. $f(-1) = |-1| = 1$

$f(1) = |1| = 1$

$f(2) = -2$

e. $f(x) = 6$

$|x| = 6$

$x = -6$

f. $f(x) = -3$

$-x = -3$

$x = 3$

g. Increasing: $(0,2)$;

Decreasing: $(-\infty, 0) \cup (2, \infty)$

Never constant

113. a. 2

b. -4

c. 4

d. 3

e. -3

f. 4

114. a. 5

b. -1

c. -2

d. 6

e. 0

f. -2

115. If replacing y by $-y$ in the equation results in an equivalent equation, then the graph is symmetric with respect to the x-axis. If replacing x by $-x$ in the equation results in an equivalent equation, then the graph is symmetric with respect to the y-axis. If replacing both x by $-x$ and y by $-y$ results in an equivalent equation, then the graph is symmetric with respect to the origin.

116. If the graph is symmetric with respect to the y-axis, then the function is even. If the graph is symmetric with respect to the origin, then the function is odd.

117. At $x = 1$, there are two different y values. The relation contains the ordered pairs (1, 2) and (1, 3).

118. The two rules defining $f(x)$ are stated for $x < 1$ and for $x > 1$. The function is not defined at $x = 1$. Therefore, there is a "hole" in the function at the point $(1, 3)$.

119. A relative maximum of a function is the greatest function value relative to other points on the function nearby.

120. A function is increasing on an interval I if for all $x_1 < x_2$ on I, $f(x_1) < f(x_2)$. In other words, the function "rises" from left to right over the interval.

121. If the average rate of change is negative and then positive, the function is

decreasing and then increasing. Therefore it must have a relative minimum at a.

122. If the average rate of change is positive and then negative, the function is increasing and then decreasing. Therefore it must have a relative maximum at a.

123. a. Concave down

   b. Decreasing

124. a. Concave up

   b. Increasing

125. a. Concave up

   b. Decreasing

126. a. Concave down

   b. Increasing

127. $f(x) = \begin{cases} 0.1x & \text{if } 1 < x \le 8925 \\ 892.50 + 0.15(x - 8925) & \text{if } 8925 < x \le 36{,}250 \\ 4991.25 + 0.25(x - 36{,}250) & \text{if } 36{,}250 < x \le 87{,}850 \end{cases}$

Or

$f(x) = \begin{cases} 0.1x & \text{if } 1 < x \le 8925 \\ 0.15x - 446.25 & \text{if } 8925 < x \le 36{,}250 \\ 0.25x\, 4071.25 & \text{if } 36{,}250 < x \le 87{,}850 \end{cases}$

128.
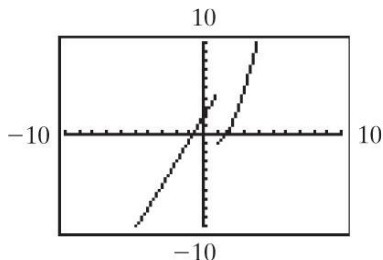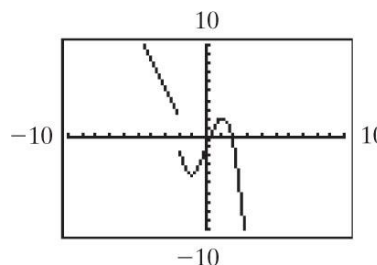
129.

130.
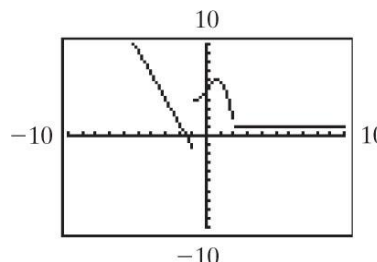
131.


```
Plot1  Plot2  Plot3
\Y1=(2.5X+8)/(X<
-2)
\Y2=(-2X²+X+4)/(
X≥-2)/(X<2)
\Y3=(-2)/(X≥2)
\Y4=
\Y5=
```
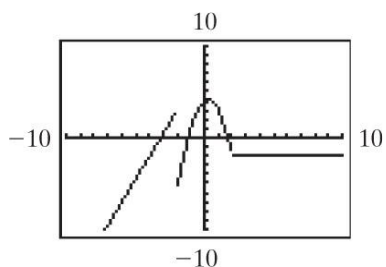

132. a. Relative maximum of 4.667 at

$x = 1.667$
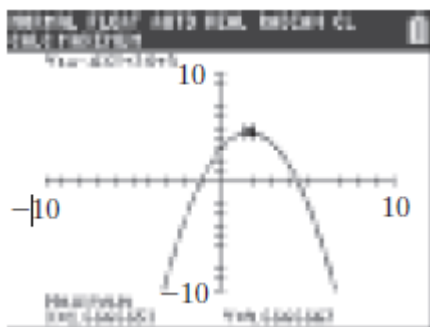
b. Increasing on $(-\infty, 1.667)$;

Decreasing on $(1.667, \infty)$

133. a. Relative maximum of -7.825 at

$x = 3.750$
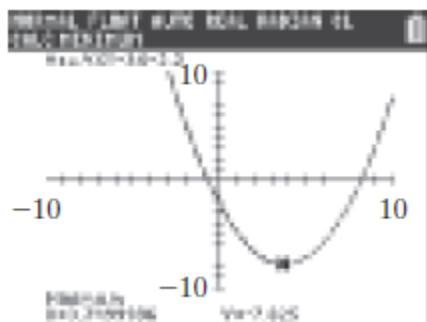
b. Increasing on $(3.750, \infty)$;

Decreasing on $(-\infty, 3.750)$

134. a. Relative maximum of 7.824 at

$x = 23.390$;

Relative minimum of -7.936 at

$x = 0.590$

323
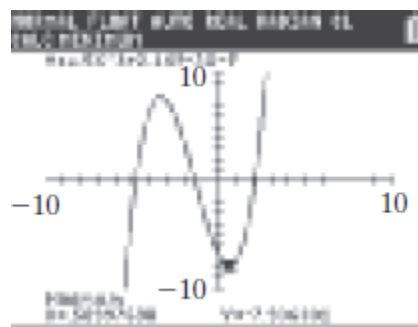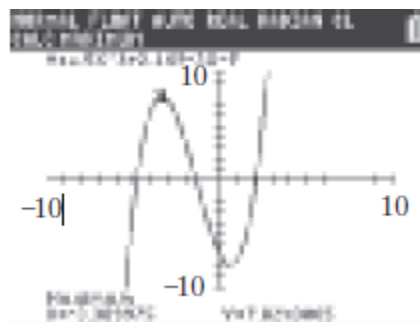
b. Increasing on

$$\left(-\infty,-3.390\right)\cup\left(0.590,\infty\right)$$

Decreasing on $\left(-3.390, 0.590\right)$

135. a. Relative maximum of 3.726 at

$x = 0.667$;

Relative minimum of -2.625 at

$x = -2.500$
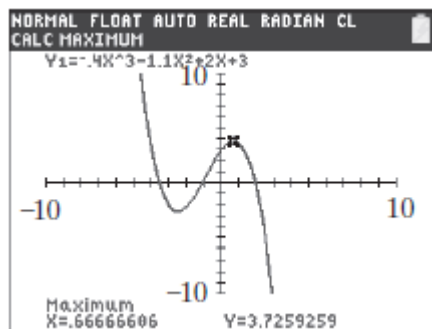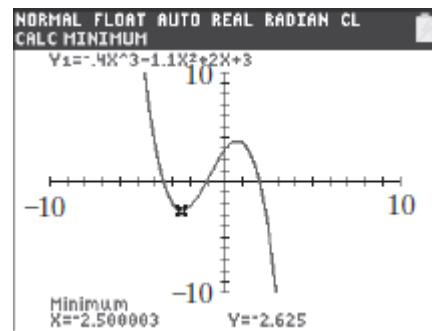
b. Increasing on $\left(-2.500, 0.667\right)$

Decreasing on

$$\left(-\infty, 2.500\right)\cup\left(0.667,\infty\right)$$

Section 2.8 Algebra of Functions and Function Composition

1. $f\left(x\right)$; $g\left(x\right)$

2. $\dfrac{f\left(x\right)}{g\left(x\right)}$; $g\left(x\right)$

3. $\dfrac{f\left(x+h\right)-f\left(x\right)}{h}$

4. $f\left(g\left(x\right)\right)$

5. $\left(f+g\right)\left(x\right)=\left|x\right|+3$; Graph d

6. $\left(f+g\right)\left(x\right)=\left|x\right|+\left(-4\right)=\left|x\right|-4$; Graph b

7. $\left(f+g\right)\left(x\right)=x^{2}+\left(-4\right)=x^{2}-4$; Graph a

8. $\left(f+g\right)\left(x\right)=x^{2}+3$; Graph c

9. $\left(f-g\right)\left(3\right)=f\left(3\right)-g\left(3\right)$

$$=-2\left(3\right)-\left|3+4\right|$$

$$=-6-7=-13$$

10. $\left(g-h\right)\left(2\right)=g\left(2\right)-h\left(2\right)$

$$=\left|2+4\right|-\dfrac{1}{2-3}$$

$$=6-\dfrac{1}{-1}=7$$

11. $\left(f\cdot g\right)\left(-1\right)=f\left(-1\right)\cdot g\left(-1\right)$

$$=-2\left(-1\right)\cdot\left|-1+4\right|$$

$$=2\cdot3=6$$

12. $\left(h\cdot g\right)\left(4\right)=h\left(4\right)\cdot g\left(4\right)$

$$=\dfrac{1}{4-3}\cdot\left|4+4\right|$$

$$=\dfrac{1}{1}\cdot8=8$$

13. $\left(g+h\right)\left(0\right)=g\left(0\right)+h\left(0\right)$

$$=\left|0+4\right|+\dfrac{1}{0-3}$$

$$=4-\dfrac{1}{3}=\dfrac{11}{3}$$

14. $(f+h)(5)=f(5)+h(5)$

$$=-2(5)+\frac{1}{5-3}$$

$$=-10+\frac{1}{2}=-\frac{19}{2}$$

15. $\left(\dfrac{f}{g}\right)(8)=\dfrac{f(8)}{g(8)}=\dfrac{-2(8)}{|8+4|}=\dfrac{-16}{12}=-\dfrac{4}{3}$

16. $\left(\dfrac{h}{f}\right)(7)=\dfrac{h(7)}{f(7)}=\dfrac{\frac{1}{7-3}}{-2(7)}=\dfrac{\frac{1}{4}}{-14}=-\dfrac{1}{56}$

17. $\left(\dfrac{g}{f}\right)(0)=\dfrac{g(0)}{f(0)}$

$$=\frac{|0+4|}{-2(0)}=\frac{4}{0}\quad\text{Undefined}$$

18. $\left(\dfrac{h}{g}\right)(-4)=\dfrac{h(-4)}{g(-4)}=\dfrac{\frac{1}{-4-3}}{|-4+4|}=\dfrac{\frac{1}{-7}}{0}$

Undefined

19. $(r-p)(x)=r(x)-p(x)$

$$=-3x-\left(x^2+3x\right)$$

$$=3x-x^2-3x=-x^2-6x$$

The domain of r is $\left(-\infty,\infty\right)$.

The domain of p is $\left(-\infty,\infty\right)$.

The intersection of their domains is

$\left(-\infty,\infty\right)$.

20. $(p-r)(x)=p(x)-r(x)$

$$=\left(x^2+3x\right)-\left(-3x\right)$$

$$=x^2+3x+3x=x^2+6x$$

The domain of p is $\left(-\infty,\infty\right)$.

The domain of r is $\left(-\infty,\infty\right)$.

The intersection of their domains is

$\left(-\infty,\infty\right)$.

21. $(p\cdot q)(x)=p(x)\cdot q(x)$

$$=\left(x^2+3x\right)\cdot\sqrt{1-x}$$

$1-x\geq0$

$\quad-x\geq-1$

$\quad\ x\leq1$

The domain of p is $\left(-\infty,\infty\right)$.

The domain of q is $\left(-\infty,1\right]$.

The intersection of their domains is

$\left(-\infty,1\right]$.

22. $(r\cdot q)(x)=r(x)\cdot q(x)=-3x\sqrt{1-x}$

The domain of r is $\left(-\infty,\infty\right)$.

The domain of q is $\left(-\infty,1\right]$.

The intersection of their domains is

$\left(-\infty,1\right]$.

23. $\left(\dfrac{q}{p}\right)(x)=\dfrac{q(x)}{p(x)}=\dfrac{\sqrt{1-x}}{x^2+3x}$

The domain of q is $\left(-\infty,1\right]$.

The domain of p is $\left(-\infty,\infty\right)$.

The intersection of their domains is

$\left(-\infty,1\right]$.

$x^2+3x=0$

$x\left(x+3\right)=0$

$\quad\ x=0\ \text{ or }\ x=-3$

Also exclude the values $x=0$ and

$x=-3$, which make the denominator

zero.

The domain is

$\left(-\infty,-3\right)\cup\left(-3,0\right)\cup\left(0,1\right]$.

24. $\left(\dfrac{q}{r}\right)(x)=\dfrac{q(x)}{r(x)}=\dfrac{\sqrt{1-x}}{-3x}$

The domain of q is $\left(-\infty,1\right]$.

The domain of r is $\left(-\infty,\infty\right)$.

The intersection of their domains is
$\left(-\infty, 1\right]$.

$-3x = 0$

$x = 0$

Also exclude the value $x = 0$, which makes the denominator zero.

The domain is $\left(-\infty, 0\right) \cup \left(0, 1\right]$.

25. $\left(\dfrac{p}{q}\right)(x) = \dfrac{p(x)}{q(x)} = \dfrac{x^2 + 3x}{\sqrt{1-x}}$

The domain of p is $\left(-\infty, \infty\right)$.

The domain of q is $\left(-\infty, 1\right]$.

The intersection of their domains is
$\left(-\infty, 1\right]$.

Also exclude the value $x = 1$, which makes the denominator zero.

The domain is $\left(-\infty, 1\right)$.

26. $\left(\dfrac{r}{q}\right)(x) = \dfrac{r(x)}{q(x)} = \dfrac{-3x}{\sqrt{1-x}}$

The domain of r is $\left(-\infty, \infty\right)$.

The domain of q is $\left(-\infty, 1\right]$.

The intersection of their domains is
$\left(-\infty, 1\right]$.

Also exclude the value $x = 1$, which makes the denominator zero.

The domain is $\left(-\infty, 1\right)$.

27. $(s \cdot t)(x) = s(x) \cdot t(x)$

$= \dfrac{x-2}{x^2-9} \cdot \dfrac{x-3}{2-x}$

$= \dfrac{-1}{x+3}$

The domain of s is
$\left(-\infty, -3\right) \cup \left(-3, 3\right) \cup \left(3, \infty\right)$.

The domain of t is $\left(-\infty, 2\right) \cup \left(2, \infty\right)$.

The intersection of their domain is
$\left(-\infty, -3\right) \cup \left(-3, 2\right) \cup \left(2, 3\right) \cup \left(3, \infty\right)$

28. $\left(\dfrac{s}{t}\right)(x) = \dfrac{s(x)}{t(x)} = -\dfrac{(x-2)^2}{(x-3)^2(x+3)}$

The domain of s is
$\left(-\infty, -3\right) \cup \left(-3, 3\right) \cup \left(3, \infty\right)$.

The domain of t is $\left(-\infty, 2\right) \cup \left(2, \infty\right)$.

The intersection of their domain is
$\left(-\infty, -3\right) \cup \left(-3, 2\right) \cup \left(2, 3\right) \cup \left(3, \infty\right)$.

29. $(s+t)(x) = s(x) + t(x)$

$= -\dfrac{x^3 - 4x^2 - 5x + 23}{(x+3)(x-3)(x-2)}$

The domain of s is
$\left(-\infty, -3\right) \cup \left(-3, 3\right) \cup \left(3, \infty\right)$.

The domain of t is $\left(-\infty, 2\right) \cup \left(2, \infty\right)$.

The intersection of their domain is
$\left(-\infty, -3\right) \cup \left(-3, 2\right) \cup \left(2, 3\right) \cup \left(3, \infty\right)$.

30. $(s-t)(x) = s(x) - t(x)$

$= \dfrac{x^3 - 2x^2 - 13x + 31}{(x+3)(x-3)(x-2)}$

The domain of s is
$\left(-\infty, -3\right) \cup \left(-3, 3\right) \cup \left(3, \infty\right)$.

The domain of t is $\left(-\infty, 2\right) \cup \left(2, \infty\right)$.

The intersection of their domain is
$\left(-\infty, -3\right) \cup \left(-3, 2\right) \cup \left(2, 3\right) \cup \left(3, \infty\right)$.

31. $(s \cdot v)(x) = s(x) \cdot v(x) = \dfrac{(x-2)\sqrt{x+3}}{(x-3)(x+3)}$

The domain of s is
$\left(-\infty, -3\right) \cup \left(-3, 3\right) \cup \left(3, \infty\right)$.

The domain of v is $[-3, \infty)$.

The intersection of their domain is
$\left(-3, 3\right) \cup \left(3, \infty\right)$.

32. $\left(\dfrac{v}{s}\right)(x)=\dfrac{v(x)}{s(x)}=\dfrac{\sqrt{x+3}\,(x+3)(x-3)}{x-2}$

The domain of s is

$(-\infty,-3)\cup(-3,3)\cup(3,\infty)$.

The domain of v is $[-3,\infty)$.

Also exclude the value $x=2$, which makes the denominator zero.

The intersection of their domain is

$(-3,2)\cup(2,3)\cup(3,\infty)$.

33. a. $f(x+h)=5(x+h)+9=5x+5h+9$

b. $\dfrac{f(x+h)-f(x)}{h}$

$=\dfrac{5x+5h+9-(5x+9)}{h}$

$=\dfrac{5x+5h+9-5x-9}{h}=\dfrac{5h}{h}=5$

34. a. $f(x+h)=8(x+h)+4=8x+8h+4$

b. $\dfrac{f(x+h)-f(x)}{h}$

$=\dfrac{8x+8h+4-(8x+4)}{h}$

$=\dfrac{8x+8h+4-8x-4}{h}=\dfrac{8h}{h}=8$

35. a. $f(x+h)=(x+h)^2+4(x+h)$

$=x^2+2xh+h^2+4x+4h$

b. $\dfrac{f(x+h)-f(x)}{h}$

$=\dfrac{x^2+2xh+h^2+4x+4h-\left(x^2+4x\right)}{h}$

$=\dfrac{x^2+2xh+h^2+4x+4h-x^2-4x}{h}$

$=\dfrac{2xh+h^2+4h}{h}=2x+h+4$

36. a. $f(x+h)=(x+h)^2-3(x+h)$

$=x^2+2xh+h^2-3x-3h$

b. $\dfrac{f(x+h)-f(x)}{h}$

$=\dfrac{x^2+2xh+h^2-3x-3h-\left(x^2-3x\right)}{h}$

$=\dfrac{x^2+2xh+h^2-3x-3h-x^2+3x}{h}$

$=\dfrac{2xh+h^2-3h}{h}=2x+h-3$

37. $\dfrac{f(x+h)-f(x)}{h}$

$=\dfrac{-2(x+h)+5-(-2x+5)}{h}$

$=\dfrac{-2x-2h+5+2x-5}{h}=\dfrac{-2h}{h}=-2$

38. $\dfrac{f(x+h)-f(x)}{h}$

$=\dfrac{-3(x+h)+8-(-3x+8)}{h}$

$=\dfrac{-3x-3h+8+3x-8}{h}=\dfrac{-3h}{h}=-3$

39. $\dfrac{f(x+h)-f(x)}{h}=\dfrac{-5(x+h)^2-4(x+h)+2-\left(-5x^2-4x+2\right)}{h}$

$=\dfrac{-5\left(x^2+2xh+h^2\right)-4x-4h+2+5x^2+4x-2}{h}$

$=\dfrac{-5x^2-10xh-5h^2-4x-4h+2+5x^2+4x-2}{h}$

$=\dfrac{-10xh-5h^2-4h}{h}$

$=-10x-5h-4$

40. $\dfrac{f(x+h)-f(x)}{h} = \dfrac{-4(x+h)^2 - 2(x+h)+6-(-4x^2-2x+6)}{h}$

$$= \dfrac{-4(x^2+2xh+h^2)-2x-2h+6+4x^2+2x-6}{h}$$

$$= \dfrac{-4x^2-8xh-4h^2-2x-2h+6+4x^2+2x-6}{h}$$

$$= \dfrac{-8xh-4h^2-2h}{h} = -8x-4h-2$$

41. $\dfrac{f(x+h)-f(x)}{h}$

$$= \dfrac{(x+h)^3+5-(x^3+5)}{h}$$

$$= \dfrac{x^3+3x^2h+3xh^2+h^3-x^3}{h}$$

$$= 3x^2+3xh+h^2$$

42. $\dfrac{f(x+h)-f(x)}{h}$

$$= \dfrac{(x+h)^3-2-(x^3-2)}{h}$$

$$= \dfrac{x^3+3x^2h+3xh^2+h^3-x^3}{h}$$

$$= 3x^2+3xh+h^2$$

43. $\dfrac{f(x+h)-f(x)}{h} = \dfrac{\dfrac{1}{x+h}-\dfrac{1}{x}}{h}$

$$= \dfrac{\dfrac{x}{x(x+h)}-\dfrac{x+h}{x(x+h)}}{h}$$

$$= \dfrac{\dfrac{x-(x+h)}{x(x+h)}}{h}$$

$$= \dfrac{-\dfrac{h}{x(x+h)}}{h}$$

$$= -\dfrac{1}{x(x+h)}$$

44. $\dfrac{f(x+h)-f(x)}{h}$

$$= \dfrac{\dfrac{1}{x+h+2}-\dfrac{1}{x+2}}{h}$$

$$= \dfrac{\dfrac{x+2}{(x+2)(x+h+2)}-\dfrac{x+h+2}{(x+2)(x+h+2)}}{h}$$

$$= \dfrac{\dfrac{x+2-x-h-2}{(x+2)(x+h+2)}}{h}$$

$$= \dfrac{\dfrac{-h}{(x+2)(x+h+2)}}{h}$$

$$= -\dfrac{1}{(x+2)(x+h+2)}$$

45. a. $\dfrac{f(x+h)-f(x)}{h} = \dfrac{4\sqrt{x+h}-4\sqrt{x}}{h}$

b. $\dfrac{4\sqrt{1+1}-4\sqrt{1}}{1} = 4\sqrt{2}-4$

$$\approx 1.6569$$

$$\dfrac{4\sqrt{1+0.1}-4\sqrt{1}}{0.1} = \dfrac{4\sqrt{1.1}-4}{0.1}$$

$$\approx 1.9524$$

$$\dfrac{4\sqrt{1+0.01}-4\sqrt{1}}{0.01} = \dfrac{4\sqrt{1.01}-4}{0.01}$$

$$\approx 1.9950$$

$$\dfrac{4\sqrt{1+0.001}-4\sqrt{1}}{0.001} = \dfrac{4\sqrt{1.001}-4}{0.001}$$

$$\approx 1.9995$$

c. 2

46. a. $\dfrac{f(x+h)-f(x)}{h} = \dfrac{\dfrac{12}{x+h}-\dfrac{12}{x}}{h}$

b.
$$\frac{\dfrac{12}{2+0.1}-\dfrac{12}{2}}{0.1}=\frac{\dfrac{12}{2.1}-6}{0.1}$$
$$\approx -2.8571$$

$$\frac{\dfrac{12}{2+0.01}-\dfrac{12}{2}}{0.01}=\frac{\dfrac{12}{2.01}-6}{0.01}$$
$$\approx -2.9851$$

$$\frac{\dfrac{12}{2+0.001}-\dfrac{12}{2}}{0.001}=\frac{\dfrac{12}{2.001}-6}{0.001}$$
$$\approx -2.9985$$

$$\frac{\dfrac{12}{2+0.0001}-\dfrac{12}{2}}{0.0001}=\frac{\dfrac{12}{2.0001}-6}{0.0001}$$
$$\approx -2.9999$$

c. -3

47. $f\left(g(8)\right)=f\left(\sqrt{2(8)}\right)=f\left(\sqrt{16}\right)=f(4)$
$$=(4)^3-4(4)=64-16=48$$

48. $h\left(g(2)\right)=h\left(\sqrt{2(2)}\right)=h(2)$
$$=2(2)+3=4+3=7$$

49. $h\left(f(1)\right)=h\left(1^3-4(1)\right)=h(-3)$
$$=2(-3)+3=-6+3=-3$$

50. $g\left(f(3)\right)=g\left((3)^3-4(3)\right)=g(27-12)$
$$=g(15)=\sqrt{2(15)}=\sqrt{30}$$

51. $(f\circ g)(18)=f\left(g(18)\right)$
$$=f\left(\sqrt{2(18)}\right)$$
$$=f\left(\sqrt{36}\right)=f(6)$$
$$=(6)^3-4(6)$$
$$=216-24=192$$

52. $(f\circ h)(-1)=f\left(h(-1)\right)$
$$=f\left(2(-1)+3\right)$$
$$=f(1)=(1)^3-4(1)$$
$$=1-4=-3$$

53. $(g\circ f)(5)=g\left(f(5)\right)$
$$=g\left((5)^3-4(5)\right)$$
$$=g(125-20)=g(105)$$
$$=\sqrt{2(105)}=\sqrt{210}$$

54. $(h\circ f)(-2)=h\left(f(-2)\right)$
$$=h\left((-2)^3-4(-2)\right)$$
$$=h(-8+8)=h(0)$$
$$=2(0)+3=3$$

55. $(h\circ f)(-3)=h\left(f(-3)\right)$
$$=h\left((-3)^3-4(-3)\right)$$
$$=h(-27+12)=h(-15)$$
$$=2(-15)+3$$
$$=-30+3=-27$$

56. $(h\circ g)(72)=h\left(g(72)\right)$
$$=h\left(\sqrt{2(72)}\right)$$
$$=h\left(\sqrt{144}\right)=h(12)$$
$$=2(12)+3$$
$$=24+3=27$$

57. $(g\circ f)(1)=g\left(f(1)\right)=g\left((1)^3-4(1)\right)$
$$=g(1-4)=g(-3)$$
$$=\sqrt{2(-3)}=\sqrt{-6}$$
Undefined

58. $(g\circ f)(-4)=g\left(f(-4)\right)$
$$=g\left((-4)^3-4(-4)\right)$$
$$=g(-64+16)=g(-48)$$
$$=\sqrt{2(-48)}=\sqrt{-96}$$
Undefined

59. $(f\circ f)(3)=f\left(f(3)\right)=f\left((3)^3-4(3)\right)$
$$=f(27-12)=f(15)$$
$$=(15)^3-4(15)=3315$$

60. $(h \circ h)(-4) = h(h(-4)) = h(2(-4)+3)$
$$= h(-8+3) = h(-5)$$
$$= (2(-5)+3) = -7$$

61. $(f \circ h \circ g)(2) = f\left(h(g(2))\right)$
$$= f\left(h\left(\sqrt{2(2)}\right)\right)$$
$$= f\left(h(2)\right) = f(2(2)+3)$$
$$= f(7) = (7^3 - 4(7))$$
$$= 315$$

62. $(f \circ h \circ g)(8) = f\left(h(g(8))\right)$
$$= f\left(h\left(\sqrt{2(8)}\right)\right)$$
$$= f\left(h(4)\right) = f(2(4)+3)$$
$$= f(11) = (11^3 - 4(11))$$
$$= 1287$$

63. a. $(f \circ g)(x) = f(g(x))$
$$= 2(g(x))^2 + 4$$
$$= 2x^2 + 4$$

b. $(g \circ f)(x) = g(f(x)) = (f(x))^2$
$$= (2x+4)^2$$
$$= 4x^2 + 16x + 16$$

c. No

64. a. $(k \circ m)(x) = k(m(x))$
$$= 3(m(x)) + 1$$
$$= -3\left(\frac{1}{x}\right) + 1$$
$$= -\frac{3}{x} + 1$$

b. $(m \circ k)(x) = m(k(x))$
$$= \frac{1}{k(x)} = \frac{1}{-3x+1}$$

c. No

65. $(n \circ p)(x) = n(p(x))$
$$= p(x) - 5 = x^2 - 9x - 5$$

Domain: $(-\infty, \infty)$

66. $(p \circ n)(x) = p(n(x))$
$$= [n(x)]^2 - 9(n(x))$$
$$= (x-5)^2 - 9(x-5)$$
$$= x^2 - 10x + 25 - 9x + 45$$
$$= x^2 - 19x + 70$$

Domain: $(-\infty, \infty)$

67. $(m \circ n)(x) = m(n(x)) = \sqrt{n(x)+8}$
$$= \sqrt{x-5+8} = \sqrt{x+3}$$
$$x + 3 \geq 0$$
$$x \geq -3$$

Domain: $[-3, \infty)$

68. $(n \circ m)(x) = n(m(x)) = m(x) - 5$
$$= \sqrt{x+8} - 5$$
$$x + 8 \geq 0$$
$$x \geq -8$$

Domain: $[-8, \infty)$

69. $(q \circ n)(x) = q(n(x)) = \dfrac{1}{n(x)-10}$
$$= \frac{1}{x-5-10} = \frac{1}{x-15}$$
$$x - 15 \neq 0$$
$$x \neq 15$$

Domain: $(-\infty, 15) \cup (15, \infty)$

70. $(q \circ p)(x) = q(p(x))$
$$= \frac{1}{p(x)-10} = \frac{1}{x^2 - 9x - 10}$$
$$x^2 - 9x - 10 \neq 0$$
$$(x-10)(x+1) \neq 0$$
$$x \neq 10 \text{ or } x \neq -1$$

Domain: $(-\infty, -1) \cup (-1, 10) \cup (10, \infty)$

71. $(q \circ r)(x) = q(r(x))$
$$= \frac{1}{r(x)-10} = \frac{1}{|2x+3|-10}$$

$$|2x+3|-10 \neq 0$$
$$|2x+3| \neq 10$$
$$2x+3 \neq 10 \quad \text{or} \quad 2x+3 \neq -10$$
$$2x \neq 7 \quad \text{or} \quad 2x \neq -13$$
$$x \neq \frac{7}{2} \quad \text{or} \quad x \neq -\frac{13}{2}$$

Domain:
$$\left(-\infty, -\frac{13}{2}\right) \cup \left(-\frac{13}{2}, \frac{7}{2}\right) \cup \left(\frac{7}{2}, \infty\right)$$

72. $(q \circ m)(x) = q(m(x))$
$$= \frac{1}{m(x)-10} = \frac{1}{\sqrt{x+8}-10}$$
$$\sqrt{x+8}-10 \neq 0$$
$$\sqrt{x+8} \neq 10$$
$$x+8 \neq 100$$
$$x \neq 92$$
$$x+8 \geq 0$$
$$x \geq -8$$
Domain: $\left[-8, 92\right) \cup \left(92, \infty\right)$

73. $(n \circ r)(x) = n(r(x))$
$$= r(x)-5$$
$$= |2x+3|-5$$
Domain: $(-\infty, \infty)$

74. $(r \circ n)(x) = r(n(x))$
$$= |2(n(x))+3| = |2(x-5)+3|$$
$$= |2x-10+3| = |2x-7|$$
Domain: $(-\infty, \infty)$

75. $(n \circ n)(x) = n(n(x)) = n(x)-5$
$$= x-5-5 = x-10$$
Domain: $(-\infty, \infty)$

76. $(p \circ p)(x) = p(p(x))$

$$= [p(x)]^2 - 9(p(x))$$
$$= (x^2-9x)^2 - 9(x^2-9x)$$
$$= x^4 - 18x^3 + 81x^2 - 9x^2 + 81x$$
$$= x^4 - 18x^3 + 72x^2 + 81x$$
Domain: $(-\infty, \infty)$

77. $(f \circ g)(x) = f(g(x)) = f\left(\sqrt{2-x}\right)$
$$= \frac{3}{\left(\sqrt{2-x}\right)^2 - 16} = -\frac{3}{x+14}$$
Domain: $(-\infty, -14) \cup (-14, 2)$

78. $(f \circ g)(x) = f(g(x)) = f\left(\sqrt{3-x}\right)$
$$= \frac{4}{\left(\sqrt{3-x}\right)^2 - 9} = -\frac{4}{x+6}$$
Domain: $(-\infty, -6) \cup (-6, 3]$

79. $(f \circ g)(x) = f(g(x)) = f\left(\frac{9}{x^2-16}\right)$
$$= \frac{\left(\frac{9}{x^2-16}\right)}{\left(\frac{9}{x^2-16}\right)-1} = \frac{9}{25-x^2}$$
Domain: $\begin{bmatrix} (-\infty, -5) \cup (-5, -4) \cup (-4, 4) \\ \cup (4, 5) \cup (5, \infty) \end{bmatrix}$

80. $(f \circ g)(x) = f(g(x)) = f\left(\frac{3}{x^2-1}\right)$
$$= \frac{\left(\frac{3}{x^2-1}\right)}{\left(\frac{3}{x^2-1}\right)+4} = \frac{3}{4x^2-1}$$
Domain:
$$\begin{bmatrix} (-\infty, -1) \cup \left(-1, -\frac{1}{2}\right) \cup \left(-\frac{1}{2}, \frac{1}{2}\right) \\ \cup \left(\frac{1}{2}, 1\right) \cup (1, \infty) \end{bmatrix}$$

81. $(f \circ f)(x) = f(f(x)) = f\left(\dfrac{1}{x-2}\right)$

$$= \dfrac{1}{\left(\dfrac{1}{x-2}\right)-2} = \dfrac{x-2}{-2x+5}$$

Domain: $(-\infty, 2) \cup \left(2, \dfrac{5}{2}\right) \cup \left(\dfrac{5}{2}, \infty\right)$

82. $(g \circ g)(x) = g(g(x)) = g\left(\sqrt{x-3}\right)$

$$= \sqrt{\left(\sqrt{x-3}\right)-3}$$

Domain: $[12, \infty)$

83. $(f \circ g \circ h)(x) = f\left(g(h(x))\right)$

$$= f\left(g\left(\sqrt[3]{x}\right)\right)$$

$$= f\left(\left(\sqrt[3]{x}\right)^2\right)$$

$$= 2\left(\left(\sqrt[3]{x}\right)^2\right)+1$$

84. $(g \circ f \circ h)(x) = g\left(f(h(x))\right)$

$$= g\left(f\left(\sqrt[3]{x}\right)\right)$$

$$= g\left(2\left(\sqrt[3]{x}\right)+1\right)$$

$$= \left(2\left(\sqrt[3]{x}\right)+1\right)^2$$

85. $(h \circ g \circ f)(x) = h\left(g(f(x))\right)$

$$= h\left(g(2x+1)\right)$$

$$= h\left((2x+1)^2\right)$$

$$= \sqrt[3]{(2x+1)^2}$$

86. $(g \circ h \circ f)(x) = g\left(h(f(x))\right)$

$$= g\left(h(2x+1)\right)$$

$$= g\left(\sqrt[3]{2x+1}\right)$$

$$= \left(\sqrt[3]{2x+1}\right)^2$$

87. a. $C(x) = 21.95x$

b. $T(a) = a + 0.06a + 10.99$

$$= 1.06a + 10.99$$

c. $(T \circ C)(x) = T(C(x))$

$$= 1.06(C(x)) + 10.99$$

$$= 1.06(21.95x) + 10.99$$

$$= 23.267x + 10.99$$

d. $(T \circ C)(4) = 23.267(4) + 10.99$

$$= 104.058$$

The total cost to purchase 4 boxes of stationery is $104.06.

88. a. $C(x) = 60x$

b. $T(a) = a + 0.055a + 8 = 1.055a + 8$

c. $(T \circ C)(x) = T(C(x))$

$$= 1.055(C(x)) + 8$$

$$= 1.055(60x) + 8$$

$$= 63.3x + 8$$

d. $(T \circ C)(6) = 63.3(6) + 8 = 387.8$

The total cost to purchase 6 tickets is $387.80.

89. a. $r(t) = 80t$

b. $d(r) = 7.2r$

c. $(d \circ r)(t) = d(r(t)) = 7.2(r(t))$

$$= 7.2(80t) = 576t$$

This function represents the distance traveled (in ft) in t minutes.

d. $(d \circ r)(30) = 576(30) = 17{,}280$ means that the bicycle will travel 17,280 ft (approximately 3.27 mi) in 30 min.

90. a. $C(x) = 2.4x$

b. $D(C) = \dfrac{C}{0.8}$

c. $(D \circ C)(x) = D(C(x))$

$$= \frac{C(x)}{0.8}$$

$$= \frac{2.4x}{0.8} = 3x$$

d. $(D \circ C)(12) = 3(12) = 36$ means that 12 croissants cost \$36.

91. $f(x) = x^2$ and $g(x) = x + 7$

92. $f(x) = x^2$ and $g(x) = x - 8$

93. $f(x) = \sqrt[3]{x}$ and $g(x) = 2x + 1$

94. $f(x) = \sqrt[4]{x}$ and $g(x) = 9x - 5$

95. $f(x) = |x|$ and $g(x) = 2x^2 - 3$

96. $f(x) = |x|$ and $g(x) = 4 - x^2$

97. $f(x) = \dfrac{5}{x}$ and $g(x) = x + 4$

98. $f(x) = \dfrac{11}{x}$ and $g(x) = x - 3$

99. a. $(f + g)(0) = f(0) + g(0)$

$$= 3 + (-2) = 1$$

b. $(g - f)(2) = g(2) - f(2)$

$$= 0 - 1 = -1$$

c. $(g \cdot f)(-1) = g(-1) \cdot f(-1)$

$$= -2 \cdot 3 = -6$$

d. $\left(\dfrac{g}{f}\right)(1) = \dfrac{g(1)}{f(1)} = \dfrac{-1}{2} = -\dfrac{1}{2}$

e. $(f \circ g)(4) = f(g(4)) = f(2) = 1$

f. $(g \circ f)(0) = g(f(0)) = g(3) = 1$

g. $g(f(4)) = g(-1) = -2$

100. a. $(f + g)(0) = f(0) + g(0)$

$$= 1 + (-3) = -2$$

b. $(g - f)(1) = g(1) - f(1) = -2 - 1 = -3$

c. $(g \cdot f)(2) = g(2) \cdot f(2) = -1 \cdot 2 = -2$

d. $\left(\dfrac{f}{g}\right)(-3) = \dfrac{f(-3)}{g(-3)} = \dfrac{-2}{-1} = 2$

e. $(f \circ g)(3) = f(g(3)) = f(0) = 1$

f. $(g \circ f)(0) = g(f(0)) = g(1) = -2$

g. $g(f(-4)) = g(-3) = -1$

101. a. $(h + k)(-1) = h(-1) + k(-1)$

$$= 2 + (-3) = -1$$

b. $(h \cdot k)(4) = h(4) \cdot k(4) = -1 \cdot 0 = 0$

c. $\left(\dfrac{k}{h}\right)(-3) = \dfrac{k(-3)}{h(-3)} = \dfrac{-3}{0}$ Undefined

d. $(k - h)(1) = k(1) - h(1)$

$$= -1 - 2 = -3$$

e. $(k \circ h)(4) = k(h(4)) = k(-1) = -3$

f. $(h \circ k)(-2) = h(k(-2)) = h(-3) = 0$

g. $h(k(3)) = h(-1) = 2$

102. a. $(m + p)(1) = m(1) + p(1)$

$$= 4 + 2 = 6$$

b. $(p - m)(-4) = p(-4) - m(-4)$

$$= 4 - (-2) = 6$$

c. $\left(\dfrac{m}{p}\right)(3) = \dfrac{m(3)}{p(3)} = \dfrac{2}{0}$ Undefined

d. $(m \cdot p)(3) = m(3) \cdot p(3) = 2 \cdot 0 = 0$

e. $(m \circ p)(0) = m(p(0)) = m(2) = 3$

f. $(p \circ m)(0) = p(m(0))$

$$= p(4) \text{ Undefined}$$

g. $p(m(-4)) = p(-2) = 4$

103. $(f + g)(4) = f(4) + g(4) = -2 + 3 = 1$

104. $(g \cdot f)(0) = g(0) \cdot f(0) = 6 \cdot 3 = 18$

105. $(g \circ f)(2) = g(f(2)) = g(4) = 3$

106. $(f \circ g)(0) = f(g(0)) = f(6) = -1$

107. $(g \circ g)(6) = g(g(6)) = g(0) = 6$

108. $(f \circ f)(-1) = f(f(-1)) = f(6) = -1$

109. $(f \circ g)(5) = f(g(5))$
$\qquad = f(7) \text{ Undefined}$

110. $(g \circ f)(0) = g(f(0))$
$\qquad = g(3) \text{ Undefined}$

111. a. $d(r) = 2r$

b. $r(d) = \dfrac{d}{2}$

c. $(V \circ r)(d) = V(r(d)) = V\left(\dfrac{d}{2}\right)$

$\qquad = \dfrac{4}{3}\pi\left(\dfrac{d}{2}\right)^3 = \dfrac{1}{6}\pi d^3$

This is a volume of a cone as a function of its diameter.

112. a. $d(r) = 2r$

b. $r(d) = \dfrac{d}{2}$

c. $(V \circ r)(d) = V(r(d)) = V\left(\dfrac{d}{2}\right)$

$\qquad = \dfrac{1}{3}\pi\left(\dfrac{d}{2}\right)^2 (6) = \dfrac{1}{2}\pi d^2$

This is a volume of a cone as a function of its diameter.

113. $(A \circ A)(x) = (1.045)^2\, x$ represents the amount of money in the account after 2 years compounded annually.

114. $(P \circ P)(x) = (0.98)^2\, x$ represents the population 2 years later.

115. $\left(\dfrac{H+L}{2}\right)(x)$ represents the average of the high and low temperatures for day x.

116. a. $A_1(x) = \pi(x+5)^2$ represents the area of the outer circle in terms of the radius of the inner circle, x.

b. $A_2(x) = \pi x^2$ represents the area of the inner circle based on its radius, x.

c. $(A_1 - A_2)(x) = A_1(x) - A_2(x)$

$\qquad = \pi(x+5)^2 - \pi x^2$
$\qquad = \pi(x^2 + 10x + 25) - \pi x^2$
$\qquad = \pi x^2 + 10\pi x + 25\pi - \pi x^2$
$\qquad = 10\pi x + 25\pi$

This function represents the area of the region outside the inner circle and inside the outer circle.

117. a. $S_1(x) = x(x+4) = x^2 + 4x$

b. $S_2(x) = \dfrac{1}{2}\pi r^2 = \dfrac{1}{2}\pi\left(\dfrac{x}{2}\right)^2$

$\qquad = \dfrac{1}{2}\pi\,\dfrac{x^2}{4} = \dfrac{1}{8}\pi x^2$

c. $(S_1 - S_2)(x) = A_1(x) - A_2(x)$

$\qquad = x^2 + 4x - \dfrac{1}{8}\pi x^2$

This function represents the area of the region outside the semicircle, but inside the rectangle.

118. The domain of $\left(\dfrac{f}{g}\right)(x)$ is the intersection of the domains of f and g with the further restriction to exclude values of x for which $g(x) = 0$.

119. The domain of $(f \circ g)(x)$ is the set of real numbers x in the domain of g such that $g(x)$ is in the domain of f.

120. For a positive real number h, the difference quotient represents the average rate of change of a function f between points $(x, f(x))$ and $(x+h, f(x+h))$. This is also

interpreted as the slope of the secant line between the points $\left(x, f\left(x\right)\right)$ and $\left(x+h, f\left(x+h\right)\right)$.

121. a. $\dfrac{f\left(x+h\right)-f\left(x\right)}{h}=\dfrac{\sqrt{x+h+3}-\sqrt{x+3}}{h}$

b. $\dfrac{\sqrt{x+h+3}-\sqrt{x+3}}{h}=\dfrac{\sqrt{x+h+3}-\sqrt{x+3}}{h}\cdot\dfrac{\sqrt{x+h+3}+\sqrt{x+3}}{\sqrt{x+h+3}+\sqrt{x+3}}$

$$=\dfrac{x+h+3-\left(x+3\right)}{h\left(\sqrt{x+h+3}+\sqrt{x+3}\right)}$$

$$=\dfrac{h}{h\left(\sqrt{x+h+3}+\sqrt{x+3}\right)}=\dfrac{1}{\sqrt{x+h+3}+\sqrt{x+3}}$$

c. $\dfrac{1}{\sqrt{x+0+3}+\sqrt{x+3}}=\dfrac{1}{2\sqrt{x+3}}$

122. a. $\dfrac{f\left(x+h\right)-f\left(x\right)}{h}=\dfrac{\sqrt{x+h-4}-\sqrt{x-4}}{h}$

b. $\dfrac{\sqrt{x+h-4}-\sqrt{x-4}}{h}=\dfrac{\sqrt{x+h-4}-\sqrt{x-4}}{h}\cdot\dfrac{\sqrt{x+h-4}+\sqrt{x-4}}{\sqrt{x+h-4}+\sqrt{x-4}}$

$$=\dfrac{x+h-4-\left(x-4\right)}{h\left(\sqrt{x+h-4}+\sqrt{x-4}\right)}$$

$$=\dfrac{h}{h\left(\sqrt{x+h-4}+\sqrt{x-4}\right)}=\dfrac{1}{\sqrt{x+h-4}+\sqrt{x-4}}$$

c. $\dfrac{1}{\sqrt{x+0-4}+\sqrt{x-4}}=\dfrac{1}{2\sqrt{x-4}}$

123. a. $\dfrac{d\left(t+h\right)-d\left(t\right)}{h}=\dfrac{-4.84\left(t+h\right)^{2}+88\left(t+h\right)-\left(-4.84t^{2}+88t\right)}{h}$

$$=\dfrac{-4.84\left(t^{2}+2th+h^{2}\right)+88t+88h+4.84t^{2}-88t}{h}$$

$$=\dfrac{-4.84t^{2}-9.68th-4.84h^{2}+88h+4.84t^{2}}{h}$$

$$=\dfrac{-9.68th-4.84h^{2}+88h}{h}=-9.68t-4.84h+88$$

b. $-9.68t-4.84h+88=-9.68\left(0\right)-4.84\left(2\right)+88=-9.68+88=78.32$ ft/sec

c. $-9.68t-4.84h+88=-9.68\left(2\right)-4.84\left(2\right)+88=-19.36-9.68+88=58.96$ ft/sec

d. $-9.68t-4.84h+88=-9.68\left(4\right)-4.84\left(2\right)+88=-38.72-9.68+88=39.6$ ft/sec

e. $-9.68t - 4.84h + 88 = -9.68(6) - 4.84(2) + 88 = -58.08 - 9.68 + 88 = 20.24$ ft/sec

124. a. $\dfrac{d(t+h) - d(t)}{h} = \dfrac{5(t+h)^2 - 5t^2}{h}$

$$= \dfrac{5(t^2 + 2th + h^2) - 5t^2}{h}$$

$$= \dfrac{5t^2 + 10th + 5h^2 - 5t^2}{h}$$

$$= \dfrac{10th + 5h^2}{h}$$

$$= 10t + 5h$$

b. $10t + 5h = 10(0) + 5(2) = 10$ ft/sec

c. $10t + 5h = 10(2) + 5(2) = 20 + 10 = 30$ ft/sec

d. $10t + 5h = 10(4) + 5(2) = 40 + 10 = 50$ ft/sec

e. $10t + 5h = 10(6) + 5(2) = 60 + 10 = 70$ ft/sec

125. $a = b + 8$ and $c = a^2$

$c(b) = (b+8)^2$

126. $q = r - 7$ and $s = \sqrt{q}$

$s(r) = \sqrt{r-7}$

127. $x = 2y$ and $z = x - 4$

$z(y) = 2y - 4$

128. $m = \dfrac{1}{3}n$ and $p = m - 2$

$p(n) = \dfrac{1}{3}n - 2$

129. $m(x) = \sqrt[3]{x}$

$n(x) = x + 1$

$h(x) = 4x$

$k(x) = x^2$

130. $m(x) = |x|$

$n(x) = x - 4$

$h(x) = -2x$

$k(x) = x^3$

Chapter 2 Review Exercises

1. a. $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{[4 - (-1)]^2 + (-2 - 8)^2}$

$= \sqrt{25 + 100} = \sqrt{125} = 5\sqrt{5}$

b. $M = \left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)$

$= \left(\dfrac{-1+4}{2}, \dfrac{8 + (-2)}{2}\right)$

$= \left(\dfrac{3}{2}, \dfrac{6}{2}\right) = \left(\dfrac{3}{2}, 3\right)$

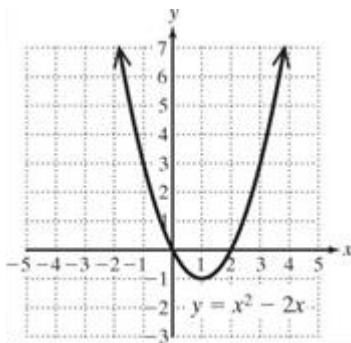**2. a.** $d = \sqrt{\left(x_2 - x_1\right)^2 + \left(y_2 - y_1\right)^2}$

$= \sqrt{\left(3\sqrt{3} - \sqrt{3}\right)^2 + \left[4\sqrt{6} - \left(-\sqrt{6}\right)\right]^2}$

$= \sqrt{12 + 150} = \sqrt{162} = 8\sqrt{2}$

b. $M = \left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)$

$= \left(\dfrac{\sqrt{3} + 3\sqrt{3}}{2}, \dfrac{-\sqrt{6} + 4\sqrt{6}}{2}\right)$

$= \left(\dfrac{4\sqrt{3}}{2}, \dfrac{3\sqrt{6}}{2}\right) = \left(2\sqrt{3}, \dfrac{3\sqrt{6}}{2}\right)$

3. a. $\qquad 4|x - 1| + y = 18$

$4|(-3) - 1| + (2) = 18$

$4(4) + 2 = 18$

$18 = 18 \checkmark$ True

Yes

b. $\qquad 4|x - 1| + y = 18$

$4|(5) - 1| + (-2) = 18$

$4(4) - 2 = 18$

$14 = 18$ False

No

4. x-intercept: $\left(\dfrac{3}{2}, 0\right)$; y-intercept: $\left(0, -2\right)$

5. x-intercept: $\left(4, 0\right)$; y-intercepts:

$\left(0, -4\right), \left(0, -10\right)$

6. x-intercepts: $\left(-1, 0\right), \left(-7, 0\right)$;

y-intercept: none

7. $y = x^2 - 2x$

x	y	$y = x^2 - 2x$	**Ordered pair**
-2	8	$y = (-2)^2 - 2(-2) = 8$	$(-2, 8)$
-1	3	$y = (-1)^2 - 2(-1) = 3$	$(-1, 3)$
0	0	$y = (0)^2 - 2(0) = 0$	$(0, 0)$
1	-1	$y = (1)^2 - 2(1) = -1$	$(1, -1)$
2	0	$y = (2)^2 - 2(2) = 0$	$(2, 0)$
3	3	$y = (3)^2 - 2(3) = 3$	$(3, 3)$
4	8	$y = (4)^2 - 2(4) = 8$	$(4, 8)$

$y = x^2 - 2x$

8. $d = \sqrt{\left[5-(-3)\right]^2 + \left[2-(-2)\right]^2}$

$= \sqrt{64+16}$

$= \sqrt{80} = 4\sqrt{5}$ units

9. $r^2 = 4$

$r = \sqrt{4} = 2$

Center: $(4, -3)$; Radius: 2

10. $r^2 = 17$

$r = \sqrt{17}$

Center: $\left(0, \dfrac{3}{2}\right)$; Radius: $\sqrt{17}$

11. a. $(x-h)^2 + (y-k)^2 = r^2$

$\left[x-(-3)\right]^2 + (y-1)^2 = \left(\sqrt{11}\right)^2$

$(x+3)^2 + (y-1)^2 = 11$

b.
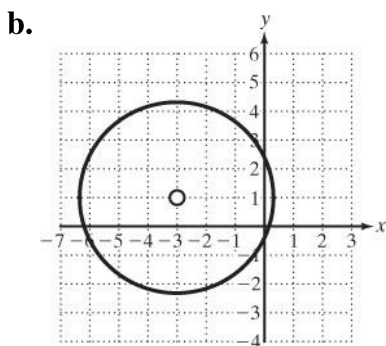
12. a. $(x-h)^2 + (y-k)^2 = r^2$

$(x-0)^2 + (y-0)^2 = (3.2)^2$

$x^2 + y^2 = 10.24$

b.
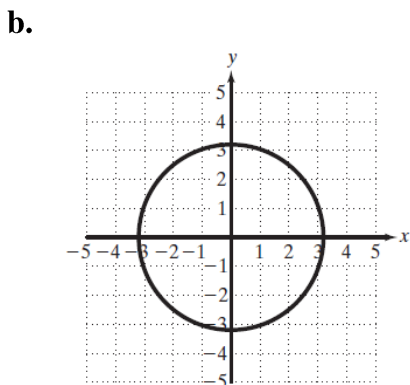
13. a. $C = \left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)$

$= \left(\dfrac{7+1}{2}, \dfrac{5+(-3)}{2}\right)$

$= \left(\dfrac{8}{2}, \dfrac{2}{2}\right) = (4, 1)$

$r = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(7-4)^2 + (5-1)^2}$

$= \sqrt{9+16} = \sqrt{25} = 5$

$(x-h)^2 + (y-k)^2 = r^2$

$(x-4)^2 + (y-1)^2 = (5)^2$

$(x-4)^2 + (y-1)^2 = 25$

b.
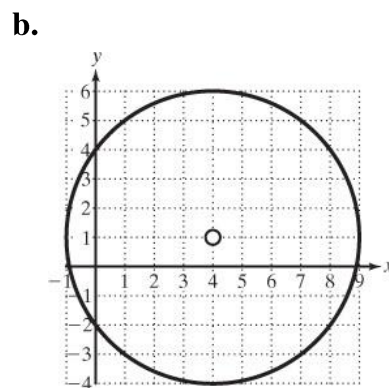
14. a. The circle must touch the *x*-axis and the *y*-axis at a distance *r* from the center. Since the center is in quadrant IV, the center must be $(4, -4)$.

$(x-h)^2 + (y-k)^2 = r^2$

$(x-4)^2 + \left[y-(-4)\right]^2 = (4)^2$

$(x-4)^2 + (y+4)^2 = 16$

b.
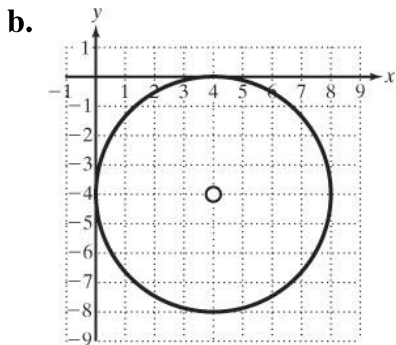
15. a. $x^2 + y^2 + 10x - 2y + 17 = 0$

$\left(x^2 + 10x \quad\right) + \left(y^2 - 2y \quad\right) = -17$

$\boxed{\left[\frac{1}{2}(10)\right]^2 = 25 \quad \left[\frac{1}{2}(-2)\right]^2 = 1}$

$\left[\begin{array}{l}\left(x^2 + 10x + 25\right) \\ +\left(y^2 - 2y + 1\right)\end{array}\right] = -17 + 25 + 1$

$(x+5)^2 + (y-1)^2 = 9$

b. Center: $(-5, 1)$; Radius: $\sqrt{9} = 3$

16. a. $x^2 + y^2 - 8y + 3 = 0$

$x^2 + \left(y^2 - 8y \quad\right) = -3$

$\boxed{\left[\frac{1}{2}(-8)\right]^2 = 16}$

$x^2 + \left(y^2 - 8y + 16\right) = -3 + 16$

$x^2 + (y-4)^2 = 13$

b. Center: $(0, 4)$; Radius: $\sqrt{13}$

17. $(x+3)^2 + (y-5)^2 = 0$

The sum of two squares will equal zero only if each individual term is zero. Therefore, $x = -3$ and $y = 5$.

$\{(-3, 5)\}$

18. $x^2 + y^2 + 6x - 4y + 15 = 0$

$\left(x^2 + 6x \quad\right) + \left(y^2 - 4y \quad\right) = -15$

$\boxed{\left[\frac{1}{2}(6)\right]^2 = 9 \quad \left[\frac{1}{2}(-4)\right]^2 = 4}$

$\left[\begin{array}{l}\left(x^2 + 6x + 9\right) \\ +\left(y^2 - 4y + 4\right)\end{array}\right] = -15 + 9 + 4$

$(x+3)^2 + (y-2)^2 = -2$

The sum of two squares cannot be negative, so there is no solution.

$\{\ \}$

19. a. {(Dara Torres, 12), (Carl Lewis, 10), (Bonnie Blair, 6), (Michael Phelps, 16)}

b. {Dara Torres, Carl Lewis, Bonnie Blair, Michael Phelps}

c. {12, 10, 6, 16}

d. Yes

20. No vertical line intersects the graph in more than one point. This relation is a function.

21. There is at least one vertical line that intersects the graph in more than one point. This relation is not a function.
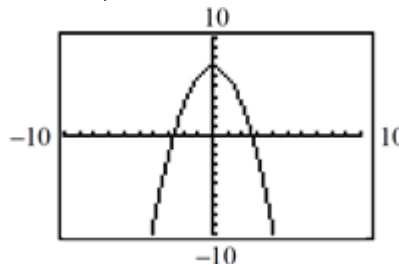
22. $x^2 + (y-3)^2 = 4$

This equation represents the graph of a circle with center $(0, 3)$ and radius 2.

This relation is not a function because it fails the vertical line test.

23. $x^2 + y - 3 = 4$

$y = -x^2 + 7$

No vertical line intersects the graph in more than one point. This relation is a function.

24. $f(x) = -2x^2 + 4x$

    a. $f(0) = -2(0)^2 + 4(0) = 0$

    b. $f(-1) = -2(-1)^2 + 4(-1)$
          $= -2 - 4 = -6$

    c. $f(3) = -2(3)^2 + 4(3)$
          $= -18 + 12 = -6$

    d. $f(t) = -2(t)^2 + 4(t) = -2t^2 + 4t$

    e. $f(x+4) = -2(x+4)^2 + 4(x+4)$
             $= -2(x^2 + 8x + 16) + 4x + 16$
             $= -2x^2 - 16x - 32 + 4x + 16$
             $= -2x^2 - 12x - 16$

25. **a.** $f(1) = 5$

    b. $f(0) = 4$

    c. $f(x) = -1$ for $x = 3$

26. $P(x) = 1.065(x + 0.32x)$
    $P(189) = 1.065\big[(189) + 0.32(189)\big]$
          $= 1.065(189 + 60.48)$
          $= 1.065(248.48)$
          ≈ 265.70

    $P(189) = 265.70$ means that a drill that costs \$189 from the manufacturer will cost the customer \$265.70 after the department store markup and sales tax.

27. Solve $p(x) = 0$:
    $p(x) = |x - 3| - 1$
       $0 = |x - 3| - 1$
    $|x - 3| = 1$
    $x - 3 = 1$ or $x - 3 = -1$
      $x = 4$ or $x = 2$
    Evaluate $p(0)$:

$p(x) = |x - 3| - 1$
$p(0) = |(0) - 3| - 1 = 3 - 1 = 2$

x-intercepts: $(4, 0), (2, 0)$; y-intercept: $(0, 2)$

28. Solve $q(x) = 0$:
    $q(x) = -\sqrt{x} + 2$
       $0 = -\sqrt{x} + 2$
     $\sqrt{x} = 2$
       $x = 4$

    Evaluate $q(0)$:
    $q(x) = -\sqrt{x} + 2$
    $q(0) = -\sqrt{(0)} + 2 = 2$

    x-intercept: $(4, 0)$; y-intercept: $(0, 2)$

29. Domain: $\{-4, -2, 0, 2, 3, 5\}$;

    Range: $\{-3, 0, 2, 1\}$

30. Domain: $(-\infty, 4)$; Range: $(-\infty, 3]$

31. $x - 5 \neq 0$
     $x \neq 5$
    $(-\infty, 5) \cup (5, \infty)$

32. $|x| - 3 \neq 0$
      $|x| \neq 3$
     $x \neq -3$ and $x \neq 3$
    $(-\infty, -3) \cup (-3, 3) \cup (3, \infty)$

33. There is no restriction on x.

    $(-\infty, \infty)$

34. $2 - x > 0$
      $-x > -2$
       $x < 2$
    $(-\infty, 2)$

35. **a.** $f(-2) = -2$

    b. $f(3) = -1$

c. $f(x) = -1$ for $x = -1$, $x = 3$

d. $f(x) = -4$ for $x = -4$

e. x-intercepts: $(0,0)$ and $(2,0)$

f. y-intercept: $(0,0)$

g. Domain: $(-\infty, \infty)$
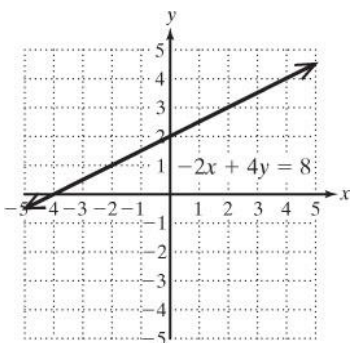
h. Range: $(-\infty, 1]$

36. $f(x) = 2x^2 - 4$

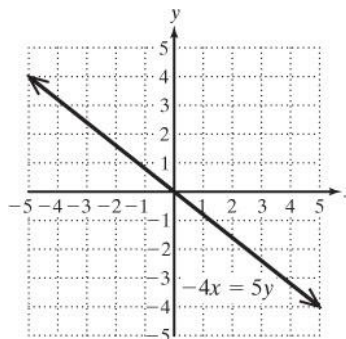**37.** $-2x + 4y = 8$

$$4y = 2x + 8$$

$$y = \frac{1}{2}x + 2$$

x	y
-4	0
-2	1
0	2
2	3

x-intercept: $(-4, 0)$; y-intercept: $(0, 2)$

38. $-4x = 5y$

$$y = -\frac{4}{5}x$$

x	y
-5	4
0	0
1	$-\dfrac{4}{5}$
5	-4

x-intercept: $(0,0)$; y-intercept: $(0,0)$

39. $y = 2$

x	y
-2	2
0	2
2	2
4	2
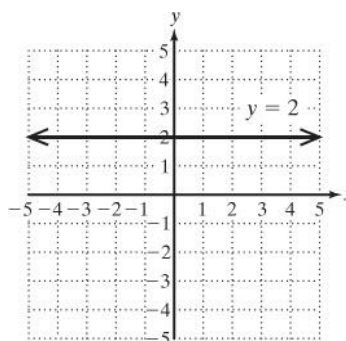
x-intercept: none; y-intercept: $(0, 2)$

40. $3x = 5$

$$x = \frac{5}{3}$$

x	y
$\dfrac{5}{3}$	-2
$\dfrac{5}{3}$	0
$\dfrac{5}{3}$	2
$\dfrac{5}{3}$	4
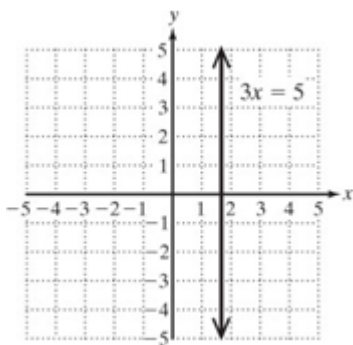
x-intercept: $\left(\dfrac{5}{3}, 0\right)$; y-intercept: none

41. $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{-4 - (-2)}{-12 - 4} = \dfrac{-2}{-16} = \dfrac{1}{8}$

42. $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{-\dfrac{4}{3} - \dfrac{2}{3}}{1 - (-3)} = \dfrac{-2}{4} = -\dfrac{1}{2}$

43. $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{f(b) - f(a)}{b - a}$

44. 0

45. Undefined

46. Undefined

47. $\dfrac{\Delta C}{\Delta t}$ represents the change in cost per change in time.

48. a. Linear

   b. Neither

   c. Constant

49. $y = mx + b$

$y = -\dfrac{2}{3}x + b$

$-5 = -\dfrac{2}{3}(1) + b$

$-5 = -\dfrac{2}{3} + b$

$-\dfrac{13}{3} = b$

$y = -\dfrac{2}{3}x - \dfrac{13}{3}; \ f(x) = -\dfrac{2}{3}x - \dfrac{13}{3}$

50. $y = mx + b$

$y = 0x + b$

$y = b$

$\dfrac{1}{4} = b$

$y = \dfrac{1}{4}; \ f(x) = \dfrac{1}{4}$

51. $m = \dfrac{f(x_2) - f(x_1)}{x_2 - x_1} = \dfrac{3 - 0}{4 - (-3)} = \dfrac{3}{7}$

52. a. Average rate of change

$= \dfrac{f(x_2) - f(x_1)}{x_2 - x_1}$

$= \dfrac{14{,}577 - 10{,}799}{10 - 5}$

$= \dfrac{3778}{5} = \$755.60/\text{yr}$

   b. Average rate of change

$= \dfrac{f(x_2) - f(x_1)}{x_2 - x_1}$

$= \dfrac{35{,}854 - 26{,}561}{25 - 20}$

$= \dfrac{9293}{5} = \$1858.60/\text{yr}$

   c. Increasing

53. $f(x) = -x^3 + 4$

$f(0) = -(0)^3 + 4 = 4$

$f(2) = -(2)^3 + 4 = -8 + 4 = -4$

$f(4) = -(4)^3 + 4 = -64 + 4 = -60$

a. Average rate of change

$$= \frac{f(x_2) - h(x_1)}{x_2 - x_1}$$

$$= \frac{f(2) - f(0)}{2 - 0}$$

$$= \frac{-4 - 4}{2} = \frac{-8}{2} = -4$$

b. Average rate of change

$$= \frac{f(x_2) - h(x_1)}{x_2 - x_1}$$

$$= \frac{f(4) - f(2)}{4 - 2}$$

$$= \frac{-60 - (-4)}{2}$$

$$= \frac{-56}{2} = -28$$

54. a. $\{1\}$

b. $(-\infty, 1)$

c. $[1, \infty)$

55. a. $\dfrac{2}{3}$

b. $-\dfrac{1}{m} = -\dfrac{1}{\dfrac{2}{3}} = -\dfrac{3}{2}$

56. $2x - 4y = 3$

$-4y = -2x + 3$

$y = \dfrac{1}{2}x - \dfrac{3}{4};\ m_1 = \dfrac{1}{2}$

a. $12x + 6y = 6$

$6y = -12x + 6$

$y = -2x + 1;\ m_2 = -2$

$m_1 = -\dfrac{1}{m_2}$

$\dfrac{1}{2} = -\dfrac{1}{-2}$

$\dfrac{1}{2} = \dfrac{1}{2}$ ✓ True

Perpendicular

b. $3y = 1.5x - 5$

$3y = \dfrac{3}{2}x - 5$

$y = \dfrac{1}{2}x - \dfrac{5}{3};\ m_2 = \dfrac{1}{2}$

$m_1 = m_2$; Parallel

c. $4x + 8y = 8$

$8y = -4x + 8$

$y = -\dfrac{1}{2}x + 1;\ m_2 = -\dfrac{1}{2}$

$m_1 \neq m_2$

$m_1 = -\dfrac{1}{m_2}$

$\dfrac{1}{2} = -\dfrac{1}{-\dfrac{1}{2}}$

$\dfrac{1}{2} = 2$ False

Neither

57. $y - y_1 = m(x - x_1)$

$y - (-7) = 3[x - (-2)]$

$y + 7 = 3(x + 2)$

$y + 7 = 3x + 6$

$y = 3x - 1$

58. $y - y_1 = m(x - x_1)$

$y - 5 = -\dfrac{2}{5}(x - 0)$

$y - 5 = -\dfrac{2}{5}x$

$y = -\dfrac{2}{5}x + 5$

59. $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{7.1 - 5.3}{-0.9 - 1.1} = \dfrac{1.8}{-2} = -0.9$

$y - y_1 = m(x - x_1)$

$y - 5.3 = -0.9(x - 1.1)$

$y - 5.3 = -0.9x + 0.99$

$y = -0.9x + 6.29$

60. A line with an undefined slope is a vertical line. Every coordinate has the same x-value, so the equation is $x = 5$.

61. $2x - y = 4$

$-y = -2x + 6$

$y = 2x - 6; \ m = 2$

$y - y_1 = m(x - x_1)$

$y - (-6) = 2(x - 2)$

$y + 6 = 2x - 4$

$y = 2x - 10$

62. $5y = 2x$

$y = \dfrac{2}{5}x; \ m = \dfrac{2}{5}$

$-\dfrac{1}{m} = -\dfrac{1}{\frac{2}{5}} = -\dfrac{5}{2}$

$y - y_1 = m(x - x_1)$

$y - 3 = -\dfrac{5}{2}\left[x - (-2)\right]$

$y - 3 = -\dfrac{5}{2}(x + 2)$

$y - 3 = -\dfrac{5}{2}x - 5$

$y = -\dfrac{5}{2}x - 2$

63. A line that is perpendicular to the y-axis is a horizontal line with slope 0. Every coordinate has the same y-value, so the equation is $y = 7$.

64. a. $G(t) = 15 - t \ \text{hr} \cdot \dfrac{60 \ \text{mi}}{\text{hr}} \cdot \dfrac{1 \ \text{gal}}{30 \ \text{mi}}$

$G(t) = 15 - 2t$

b. $G(4.5) = 15 - 2(4.5) = 15 - 9 = 6$

$G(4.5) = 6$ means that after 4.5 hr of driving, the tank has 6 gal left.

65. a. $C(x) = 1500 + 35x$

b. $R(x) = 60x$

c. $P(x) = R(x) - C(x)$

$= 60x - (1500 + 35x)$

$= 25x - 1500$

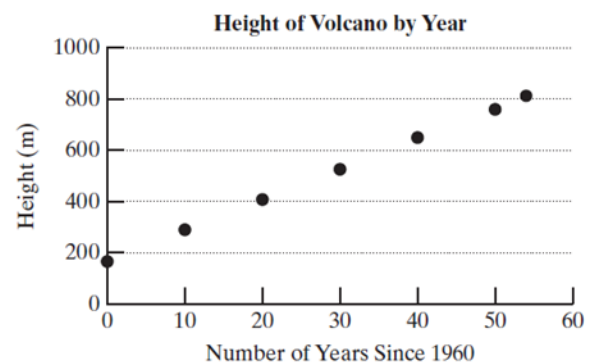**d.** $\quad P(x) > 0$

$25x - 1500 > 0$

$25x > 1500$

$x > 60$

The studio needs more than 60 private lessons per month to make a profit.

e. $P(x) = 25x - 1500$

$P(82) = 25(82) - 1500$

$= 2050 - 1500 = 550$

The studio will make \$550.

66. a.

Height of Volcano by Year

b. $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

$= \dfrac{650 - 166}{40 - 0} = \dfrac{484}{40} = 12.1$
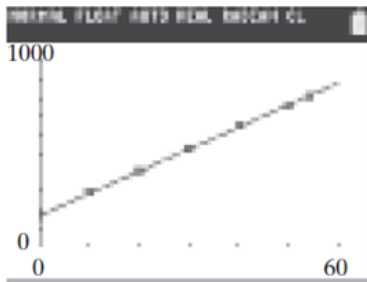
$$y - y_1 = m(x - x_1)$$
$$y - 166 = 12.1(x - 0)$$
$$y = 12.1x + 166$$

c. The slope is 12.1 and means that the volcano's height increases by 12.1 m per year.

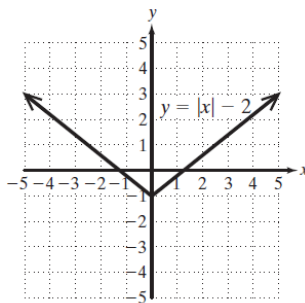**d.** For 2030, $x = 2030 - 1960 = 70$
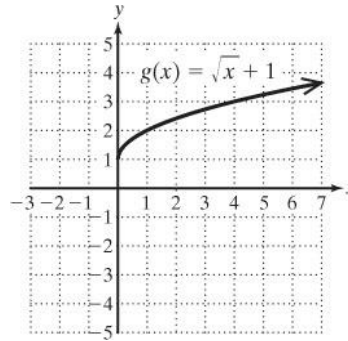
$$y = 12.1(70) + 166 = 1013 \text{ m}$$

67. a. $y = 11.9x + 169$

b.

c. For 2030, $x = 2030 - 1960 = 30$

$$y = 11.9(70) + 169 = 1002 \text{ m}$$
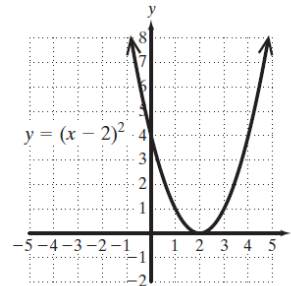
68. $y = (-x + 5)^2 - 2$

69. The graph of f is the graph of $f(x) = |x|$ shifted downward 2 unit.
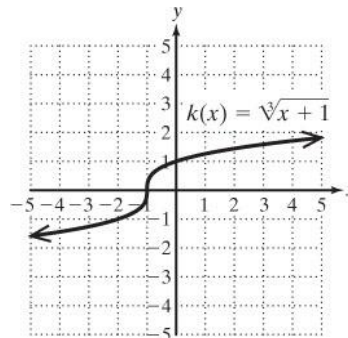

70. The graph of g is the graph of $f(x) = \sqrt{x}$ shifted upward 1 unit.
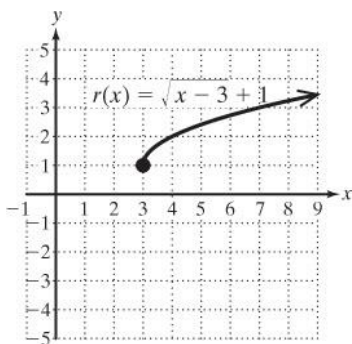

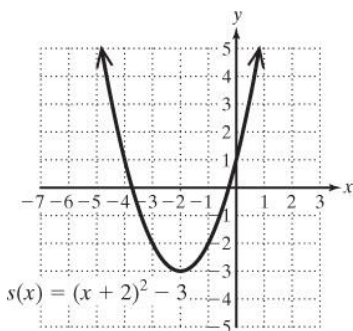**71.** The graph of h is the graph of $f(x) = x^2$ shifted to the right 2 units.

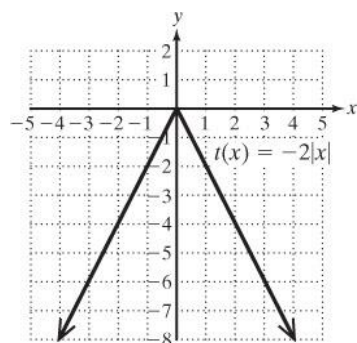**72.** The graph of k is the graph of $f(x) = \sqrt[3]{x}$ shifted to the left 1 unit.

73. The graph of r is the graph of $f(x) = \sqrt{x}$ shifted to the right 3 units and upward 1 unit.

$r(x) = \sqrt{x - 3} + 1$

74. The graph of s is the graph of $f(x) = x^2$ shifted to the left 2 units and downward 3 units.

$s(x) = (x + 2)^2 - 3$

75. The graph of t is the graph of $f(x) = |x|$ reflected across the x-axis and stretched vertically by a factor of 2.

$t(x) = -2|x|$

76. The graph of v is the graph of $f(x) = |x|$ reflected across the x-axis and shrunk vertically by a factor of $\dfrac{1}{2}$.

$v(x) = -\tfrac{1}{2}|x|$
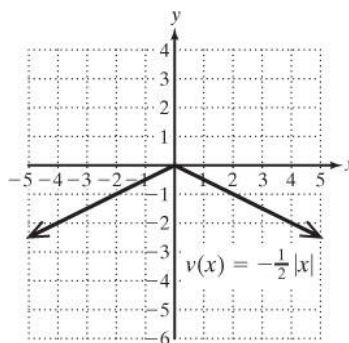
77. $m(x) = \sqrt{-x + 5} = \sqrt{-(x - 5)}$

The graph of m is the graph of $f(x) = \sqrt{x}$ reflected across the y-axis and shifted right 5 units.

$m(x) = \sqrt{-x + 5}$
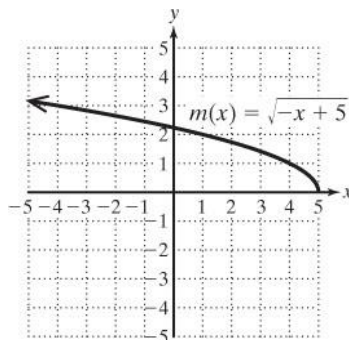
78. $n(x) = \sqrt{-x - 1} = \sqrt{-(x + 1)}$

The graph of n is the graph of $f(x) = \sqrt{x}$ reflected across the y-axis and shifted to the left 4 units.

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$$y = \sqrt{-x} - 1$$

79. The graph of the function is the graph of $f(x)$ shrunk horizontally by a factor of 2.
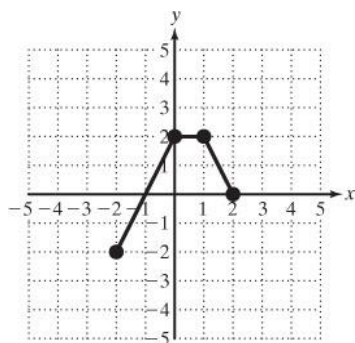
80. The graph of the function is the graph of $f(x)$ stretched horizontally by a factor of 2.
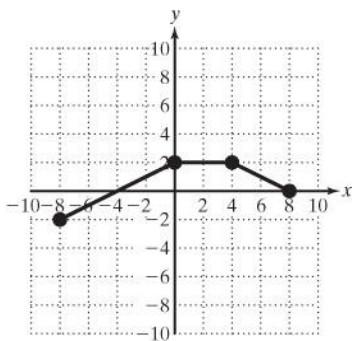
81. The graph of the function is the graph of $f(x)$ shifted to the left 1 unit, reflected

across the *x*-axis, and shifted downward 3 units.
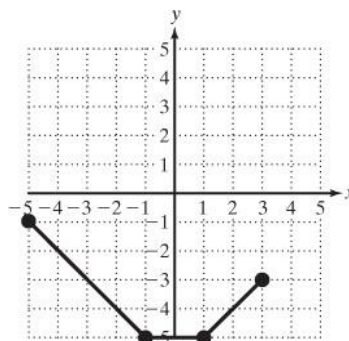
82. The graph of the function is the graph of $f(x)$ shifted to the right 4 units, reflected across the *x*-axis, and shifted downward 1 unit.

83. The graph of the function is the graph of $f(x)$ shifted to the right 3 units, stretched vertically by a factor of 2, and shifted upward 1 unit.
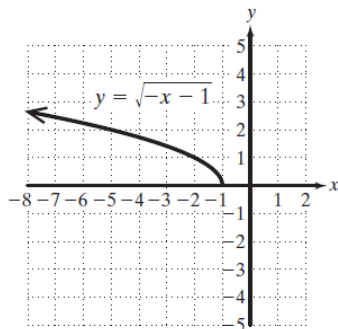
84. The graph of the function is the graph of $f(x)$ shifted to the left 2 units, shrunk vertically by a factor of $\dfrac{1}{2}$, and shifted downward 3 units.

85. $y = x^4 - 3$

Replace x by $-x$.

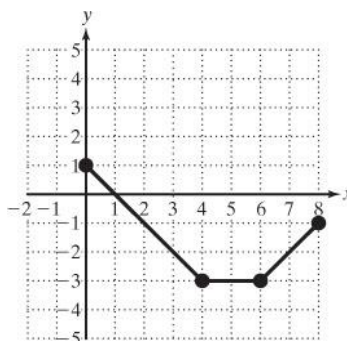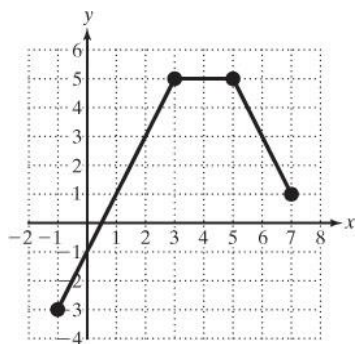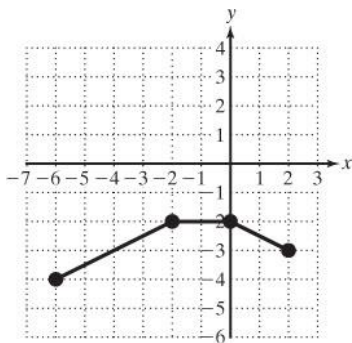$y = (-x)^4 - 3$

$y = x^4 - 3$

This equation *is* equivalent to the original equation, so the graph is symmetric with respect to the *y*-axis.
Replace y by $-y$.

$-y = x^4 - 3$

$y = -x^4 + 3$

This equation is *not* equivalent to the original equation, so the graph is not symmetric with respect to the *x*-axis.
Replace x by $-x$ and y by $-y$.

$-y = (-x)^4 - 3$

$-y = x^4 - 3$

$y = -x^4 + 3$

This equation is *not* equivalent to the original equation, so the graph is not symmetric with respect to the origin.

86. $x = |y| + y^2$

Replace x by $-x$.

$-x = |y| + y^2$

$x = -|y| - y^2$

This equation is *not* equivalent to the original equation, so the graph is not symmetric with respect to the *y*-axis.
Replace y by $-y$.

$x = |-y| + (-y)^2$

$x = |y| + y^2$

This equation *is* equivalent to the original equation, so the graph is symmetric with respect to the *x*-axis.
Replace x by $-x$ and y by $-y$.

$-x = |-y| + (-y)^2$

$-x = |y| + y^2$

$x = -|y| - y^2$

This equation is *not* equivalent to the original equation, so the graph is not symmetric with respect to the origin.

87. $y = \dfrac{1}{3}x - 1$

Replace x by $-x$.

$y = \dfrac{1}{3}(-x) - 1$

$y = -\dfrac{1}{3}x - 1$

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This equation is *not* equivalent to the original equation, so the graph is not symmetric with respect to the *y*-axis.
Replace *y* by $-y$.

$$-y = \frac{1}{3}x - 1$$
$$y = -\frac{1}{3}x + 1$$

This equation is *not* equivalent to the original equation, so the graph is not symmetric with respect to the *x*-axis.
Replace *x* by $-x$ and *y* by $-y$.

$$-y = \frac{1}{3}(-x) - 1$$
$$-y = -\frac{1}{3}x - 1$$
$$y = \frac{1}{3}x + 1$$

This equation is *not* equivalent to the original equation, so the graph is not symmetric with respect to the origin.

88. $x^2 = y^2 + 1$

Replace *x* by $-x$.

$$(-x)^2 = y^2 + 1$$
$$x^2 = y^2 + 1$$

This equation *is* equivalent to the original equation, so the graph is symmetric with respect to the *y*-axis.
Replace *y* by $-y$.

$$x^2 = (-y)^2 + 1$$
$$x^2 = y^2 + 1$$

This equation *is* equivalent to the original equation, so the graph is symmetric with respect to the *x*-axis.

Replace *x* by $-x$ and *y* by $-y$.

$$(-x)^2 = (-y)^2 + 1$$
$$x^2 = y^2 + 1$$

This equation *is* equivalent to the original equation, so the graph is symmetric with respect to the origin.

89. $f(x) = -4x^3 + x$

$$f(-x) = -4(-x)^3 + (-x)$$
$$f(-x) = 4x^3 - x$$
$$-f(x) = -(-4x^3 + x) = 4x^3 - x$$
$$f(-x) = -f(x)$$

The function is odd.

90. $g(x) = \sqrt[3]{x}$

$$g(-x) = \sqrt[3]{-x} = -x$$
$$-g(x) = -\left(\sqrt[3]{x}\right) = -x$$
$$g(-x) = -g(x)$$

The function is odd.

91. $p(x) = \sqrt{4 - x^2}$

$$p(-x) = \sqrt{4 - (-x)^2} = \sqrt{4 - x^2}$$
$$p(-x) = p(x)$$

The function is even.

92. $q(x) = -|x|$

$$q(-x) = -|-x| = -|x|$$
$$q(-x) = q(x)$$

The function is even.

93. $k(x) = (x - 3)^2$

$$k(-x) = (-x - 3)^2$$
$$-k(x) = -(x - 3)^2$$
$$k(-x) \neq k(x)$$
$$k(-x) \neq -k(x)$$

The function is neither even nor odd.

94. $m(x) = |x + 2|$

$m(-x) = |-x + 2|$

$-m(x) = -|x + 2|$

$m(-x) \neq m(x)$

$m(-x) \neq -m(x)$

The function is neither even nor odd.

95. a. Use the first rule.

$f(-4) = -4(-4) + 2$

$\qquad = 16 + 2 = 18$

b. Use the second rule.

$f(-1) = (-1)^2 = 1$

c. Use the third rule. $f(3) = 5$

d. Use the second rule.

$f(2) = (2)^2 = 4$

96.
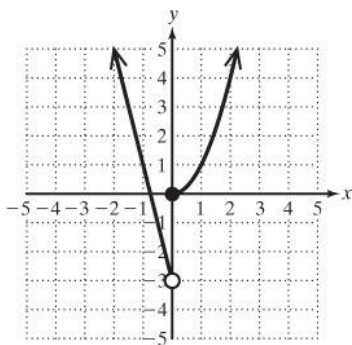
97.
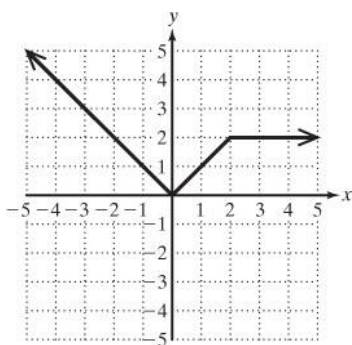
98.
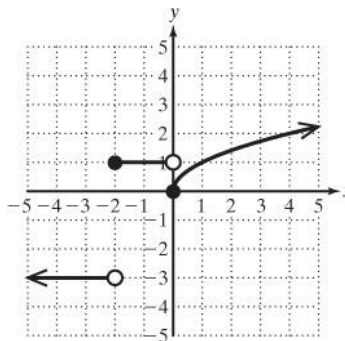
99. a. $f(x) = [\![x - 1]\!]$

$f(-1.5) = [\![-1.5 - 1]\!] = [\![-2.5]\!] = -3$

b. $f(x) = [\![x - 1]\!]$

$f(-2) = [\![-2 - 1]\!] = [\![-3]\!] = -3$

c. $f(x) = [\![x - 1]\!]$

$f(0.1) = [\![0.1 - 1]\!] = [\![-0.9]\!] = -1$

d. $f(x) = [\![x - 1]\!]$

$f(6.3) = [\![6.3 - 1]\!] = [\![5.3]\!] = 5$

100. a. $(-\infty, -3) \cup (-2, 0)$

b. $(-3, -2) \cup (0, 3)$

c. $(-3, \infty)$

101. a. $(2, \infty)$

b. $(-\infty, 2)$

c. Never constant

102. At $x = -2$, the function has a relative minimum of -2. At $x = 4$, the function has a relative minimum of -1. At $x = 2$, the function has a relative maximum of 1.

103. At $x = -2$, the function has a relative maximum of 4.

104. $f(x) = \begin{cases} -|x| + 4 & \text{for } x \leq 3 \\ 1 & \text{for } x > 3 \end{cases}$

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105. $(f-h)(2)=f(2)-h(2)$

$$=-3(2)-\frac{1}{2+1}$$

$$=-6-\frac{1}{3}=-\frac{19}{3}$$

106. $(g\cdot h)(3)=g(3)\cdot h(3)$

$$=|3-2|\cdot\frac{1}{3+1}$$

$$=1\cdot\frac{1}{4}=\frac{1}{4}$$

107. $\left(\dfrac{g}{h}\right)(-5)=\dfrac{g(-5)}{h(-5)}$

$$=\frac{|-5-2|}{\dfrac{1}{-5+1}}$$

$$=\frac{7}{\dfrac{1}{-4}}=-28$$

108. $(f\circ g)(5)=f\big(g(5)\big)=f\big(|5-2|\big)$

$$=f(3)=-3(3)=-9$$

109. $(g\circ f)(5)=g\big(f(5)\big)=g\big(-3(5)\big)$

$$=g(-15)=|-15-2|=17$$

110. a. $(f+g)(2)=f(2)+g(2)$

$$=3+(-2)=1$$

b. $(g\cdot f)(-4)=g(-4)\cdot f(-4)$

$$=2\cdot-1=-2$$

c. $\left(\dfrac{g}{f}\right)(-3)=\dfrac{g(-3)}{f(-3)}=\dfrac{1}{0}$ Undefined

d. $f\big(g(-4)\big)=f(2)=3$

e. $(g\circ f)(-4)=g\big(f(-4)\big)=g(-1)=1$

f. $(g\circ f)(5)=g\big(f(5)\big)$ Undefined

111. $(n-m)(x)=n(x)-m(x)$

$$=x^2-4x-(-4x)=x^2$$

Domain: $(-\infty,\infty)$

112. $\left(\dfrac{p}{n}\right)(x)=\dfrac{p(x)}{n(x)}=\dfrac{\sqrt{x-2}}{x^2-4x}$

$x-2\geq 0$

$x\geq 2$

$x^2-4x\neq 0$

$x(x-4)\neq 0$

$x\neq 0$ and $x\neq 4$

Domain: $[2,4)\cup(4,\infty)$

113. $\left(\dfrac{n}{p}\right)(x)=\dfrac{n(x)}{p(x)}=\dfrac{x^2-4x}{\sqrt{x-2}}$

$x-2>0$

$x>2$

Domain: $(2,\infty)$

114. $(m\cdot p)(x)=m(x)\cdot p(x)=-4x\sqrt{x-2}$

$x-2\geq 0$

$x\geq 2$

Domain: $[2,\infty)$

115. $(q\circ n)(x)=q\big(n(x)\big)$

$$=\frac{1}{n(x)-5}$$

$$=\frac{1}{x^2-4x-5}$$

$x^2-4x-5\neq 0$

$(x-5)(x+1)\neq 0$

$x\neq 5$ and $x\neq -1$

Domain: $(-\infty,-1)\cup(-1,5)\cup(5,\infty)$

116. $(q\circ p)(x)=q\big(p(x)\big)$

$$=\frac{1}{p(x)-5}=\frac{1}{\sqrt{x-2}-5}$$

$x-2\geq 0$

$x\geq 2$

$\sqrt{x-2}-5\neq 0$

$\sqrt{x-2}\neq 5$

$x-2\neq 25$

$x\neq 27$

Domain: $\left[2, 27\right) \cup \left(27, \infty\right)$

117. $\dfrac{f\left(x+h\right)+f\left(x\right)}{h} = \dfrac{-6\left(x+h\right)-5-\left(-6x-5\right)}{h}$

$= \dfrac{-6x-6h-5+6x+5}{h}$

$= \dfrac{-6h}{h} = -6$

118. $\dfrac{f\left(x+h\right)-f\left(x\right)}{h} = \dfrac{3\left(x+h\right)^2 - 4\left(x+h\right)+9-\left(3x^2-4x+9\right)}{h}$

$= \dfrac{3\left(x^2+2xh+h^2\right)-4x-4h+9-3x^2+4x-9}{h}$

$= \dfrac{3x^2+6xh+3h^2-4h-3x^2}{h} = \dfrac{6xh+3h^2-4h}{h}$

$= 6x+3h-4$

119. $f\left(x\right)=x^2$ and $g\left(x\right)=x-4$

120. $f\left(x\right)=\dfrac{12}{x}$ and $g\left(x\right)=x+5$

121. a. $d\left(t\right)=60t$

    b. $n\left(d\right)=\dfrac{d}{28}$

c. $\left(n \circ d\right)\left(t\right)=n\left(d\left(t\right)\right)=\dfrac{d\left(t\right)}{28}=\dfrac{60t}{28}$

This function represents the number of gallons of gasoline used in t hours.

d. $\left(n \circ d\right)\left(7\right)=\dfrac{60\left(7\right)}{28}=15$ means that 15 gal of gasoline is used in 7 hr.

Chapter 2 Test

1. a. $C = \left(\dfrac{x_1+x_2}{2}, \dfrac{y_1+y_2}{2}\right)$

$= \left(\dfrac{8+\left(-2\right)}{2}, \dfrac{-5+3}{2}\right)$

$= \left(\dfrac{6}{2}, \dfrac{-2}{2}\right) = \left(3, -1\right)$

b. $r = \sqrt{\left(x_2-x_1\right)^2 + \left(y_2-y_1\right)^2}$

$= \sqrt{\left(-2-3\right)^2 + \left[3-\left(-1\right)\right]^2}$

$= \sqrt{25+16} = \sqrt{41}$

c. $\left(x-h\right)^2 + \left(y-k\right)^2 = r^2$

$\left(x-3\right)^2 + \left[y-\left(-1\right)\right]^2 = \left(\sqrt{41}\right)^2$

$\left(x-3\right)^2 + \left(y+1\right)^2 = 41$

2. a. Substitute 0 for y: Substitute 0 for x:

$x = \left|y\right|-4$ $x = \left|y\right|-4$

$x = \left|\left(0\right)\right|-4$ $\left(0\right) = \left|y\right|-4$

$x = -4$ $\left|y\right| = 4$

                    $y = \pm 4$

x-intercept: $\left(-4, 0\right)$; y-intercepts:

$\left(0, -4\right), \left(0, 4\right)$

b. No

3. a.
$$x^2 + y^2 + 14x - 10y + 70 = 0$$

$$\left(x^2 + 14x \quad\right) + \left(y^2 - 10y \quad\right) = -70$$

$$\boxed{\left[\frac{1}{2}(14)\right]^2 = 49 \quad \left[\frac{1}{2}(10)\right]^2 = 25}$$

$$\left[\begin{array}{c}\left(x^2 + 14x + 49\right) \\ +\left(y^2 - 10y + 25\right)\end{array}\right] = -70 + 49 + 25$$

$$\left(x + 7\right)^2 + \left(y - 5\right)^2 = 4$$

b. Center: $\left(-7, 5\right)$; Radius: $\sqrt{4} = 2$

4. This mapping defines the set of ordered pairs: $\left\{\left(2, 5\right), \left(3, 5\right), \left(4, 5\right)\right\}$. No two ordered pairs have the same x value but different y values. This relation is a function.

5. There is at least one vertical line that intersects the graph in more than one point. This relation is not a function.

6. a. $f\left(-1\right) = -2\left(-1\right)^2 + 7\left(-1\right) - 3$

$$= -2 - 7 - 3$$

$$= -12$$

b. $f\left(x + h\right) = -2\left(x + h\right)^2 + 7\left(x + h\right) - 3$

$$= -2\left(x^2 + 2xh + h^2\right) + 7x + 7h - 3$$

$$= -2x^2 - 4xh - 2h^2 + 7x + 7h - 3$$

c. $\dfrac{f\left(x + h\right) - f\left(x\right)}{h} = \dfrac{-2x^2 - 4xh - 2h^2 + 7x + 7h - 3 - \left(-2x^2 + 7x - 3\right)}{h}$

$$= \dfrac{-2x^2 - 4xh - 2h^2 + 7x + 7h - 3 + 2x^2 - 7x + 3}{h}$$

$$= \dfrac{-4xh - 2h^2 + 7h}{h} = -4x - 2h + 7$$

d. Substitute 0 for $f\left(x\right)$:

$$0 = -2x^2 + 7x - 3$$

$$0 = 2x^2 - 7x + 3$$

$$0 = \left(2x - 1\right)\left(x - 3\right)$$

$$2x - 1 = 0 \quad \text{or} \quad x - 3 = 0$$

$$2x = 1 \quad \text{or} \quad x = 3$$

$$x = \frac{1}{2}$$

$$\left(\frac{1}{2}, 0\right) \text{ and } \left(3, 0\right)$$

e. Substitute 0 for x:

$$f\left(x\right) = -2\left(0\right)^2 + 7\left(0\right) - 3 = -3$$

$$\left(0, -3\right)$$

f. $-4x - 2h + 7 = -4\left(1\right) - 2\left(2\right) + 7$

$$= -4 - 4 + 7 = -1$$

7. a. $f\left(0\right) = -2$

b. $f\left(-4\right) = 0$

c. $f\left(x\right) = 2$ for $x = -2$ and $x = 2$

d. $\left(-\infty, -2\right) \cup \left(0, 2\right)$

e. $\left(-2, 0\right) \cup \left(2, \infty\right)$

f. At $x = 0$, the function has a relative minimum of -2.

g. At $x = -2$, the function has a relative minimum of 2.

At $x = 2$, the function has a relative minimum of 2.

h. $\left(-\infty, \infty\right)$

i. $\left(-\infty, 2\right]$

j. $f\left(x\right) = f\left(-x\right)$, so the function is even.

8. $3w + 7 \neq 0$

$3w \neq -7$

$w \neq -\dfrac{7}{3}$

$\left(-\infty, -\dfrac{7}{3}\right) \cup \left(-\dfrac{7}{3}, \infty\right)$

9. $4 - c \geq 0$

$-c \geq -4$

$c \leq 4$

$\left(-\infty, 4\right]$
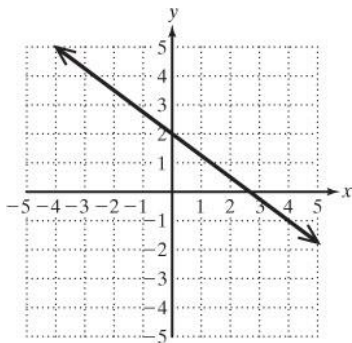
10. $3x = -4y + 8$

$4y = -3x + 8$

$y = -\dfrac{3}{4}x + 2$

a. $m = -\dfrac{3}{4}$

b. $\left(0, 2\right)$

c.

d. $m = -\dfrac{1}{\left(-\dfrac{3}{4}\right)} = \dfrac{4}{3}$

e. $m = -\dfrac{3}{4}$

11. $x + 3y = 4$

$3y = -x + 4$

$y = -\dfrac{1}{3}x + \dfrac{4}{3}; \ m = -\dfrac{1}{3}$

$-\dfrac{1}{m} = -\dfrac{1}{-\dfrac{1}{3}} = 3$

$y - y_1 = m\left(x - x_1\right)$

$y - 6 = 3\left[x - \left(-2\right)\right]$

$y - 6 = 3\left(x + 2\right)$

$y - 6 = 3x + 6$

$y = 3x + 12$

12. a. $\left\{-2\right\}$

b. $\left(-\infty, -2\right)$

c. $\left(-2, \infty\right)$
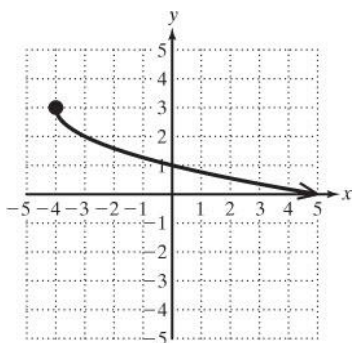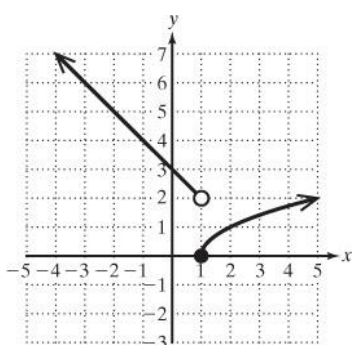
13.

14.
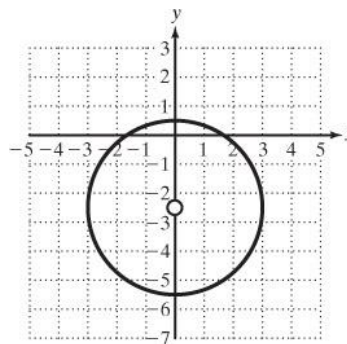
354
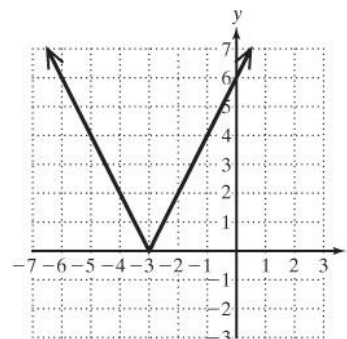
15.

16.

17. $x^2 + |y| = 8$

Replace x by $-x$.

$(-x)^2 + |y| = 8$

$x^2 + |y| = 8$

This equation *is* equivalent to the original equation, so the graph is symmetric with respect to the y-axis.

Replace y by $-y$.

$x^2 + |-y| = 8$

$x^2 + |y| = 8$

This equation *is* equivalent to the original equation, so the graph is symmetric with respect to the x-axis.

Replace x by $-x$ and y by $-y$.

$(-x)^2 + |-y| = 8$

$x^2 + |y| = 8$

This equation *is* equivalent to the original equation, so the graph is symmetric with respect to the origin.

18. $f(x) = x^3 - x$

$f(-x) = (-x)^3 - (-x)$

$f(-x) = -x^3 + x$

$-f(x) = -(x^3 - x) = -x^3 + x$

$f(-x) = -f(x)$

The function is odd.

19. $g(x) = x^4 + x^3 + x$

$g(-x) = (-x)^4 + (-x)^3 + (-x)$

$\quad = x^4 - x^3 - x$

$-g(x) = -(x^4 + x^3 + x)$

$\quad = -x^4 - x^3 - x$

$g(-x) \neq g(x)$

$g(-x) \neq -g(x)$

The function is neither even nor odd.

20. a. $f(x) = [\![x]\!]$

$f(4.27) = [\![4.27]\!] = 4$

b. $f(x) = [\![x]\!]$

$f(-4.27) = [\![-4.27]\!] = -5$

21. $(f - h)(6) = f(6) - h(6)$

$= 6 - 4 - \sqrt{6 - 5}$

$= 2 - 1$

$= 1$

22. $(g \cdot h)(5) = g(5) \cdot h(5)$

$= \dfrac{1}{5 - 3} \cdot \sqrt{5 - 5}$

$= \dfrac{1}{2} \cdot 0$

$= 0$

23. $(h \circ f)(1) = h(f(1))$
$= h(1-4)$
$= h(3)$
$= \sqrt{3-5}$
$= \sqrt{-2}$
Undefined

24. $(f \cdot g)(x) = f(x) \cdot g(x)$
$= (x-4) \cdot \left(\dfrac{1}{x-3} \right)$
$= \dfrac{x-4}{x-3}$
$x - 3 \neq 0$
$x \neq 3$
Domain: $(-\infty, 3) \cup (3, \infty)$

25. $\left(\dfrac{g}{f} \right)(x) = \dfrac{g(x)}{f(x)} = \dfrac{\frac{1}{x-3}}{x-4}$
$= \dfrac{1}{(x-3)(x-4)}$
$(x-3)(x-4) \neq 0$
$x \neq 3$ and $x \neq 4$
Domain: $(-\infty, 3) \cup (3, 4) \cup (4, \infty)$

26. $(g \circ h)(x) = g(h(x)) = \dfrac{1}{h(x)-3}$
$= \dfrac{1}{\sqrt{x-5}-3}$
$x - 5 \geq 0$
$x \geq 5$
$\sqrt{x-5} - 3 \neq 0$
$\sqrt{x-5} \neq 3$
$x - 5 \neq 9$
$x \neq 14$
Domain: $[5, 14) \cup (14, \infty)$

27. $f(x) = \sqrt[3]{x}$ and $g(x)x - 7$

28. a. $(f + g)(3) = f(3) + g(3)$
$= 1 + (-2) = -1$

b. $(f \cdot g)(0) = f(0) \cdot g(0)$
$= 1 \cdot (-1) = -1$

c. $g(f(3)) = g(1) = -2$

d. $(f \circ g)(2) = f(g(2))$
$= f(-3)$ Undefined

e. $(-3, 0)$

f. $(-\infty, 2)$

29. a.

Number of Attendees by Week
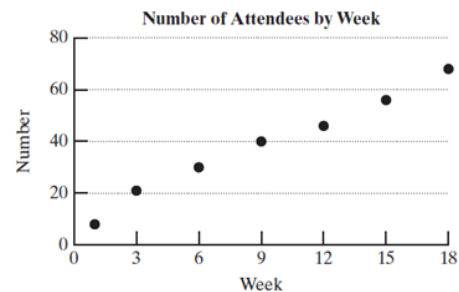
b. $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{40 - 8}{9 - 1} = 4$
$(y - y_1) = m(x - x_1)$
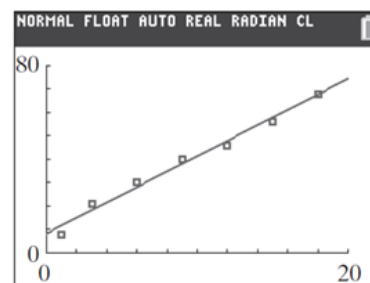$(y - 8) = 4(x - 1)$
$y = 4x + 4$

c. The slope is 4 and means that the number of attendees has increased by approximately 4 people per week.

d. $y = 4(24) + 4 = 100$ people

30. a. $y = 3.3x + 8.5$

b.

356

c. $y = 3.3(24) + 8.5 = 87.7 \approx 88$ people

Chapter 2 Cumulative Review Exercises

1. a. $f(2) = -1$

    b. $f(x) = 0$ for $x = 0$ and $x = 3$

    c. $(-4, \infty)$

    d. $[-2, \infty)$

    e. $(1, \infty)$

    f. $(-1, 1)$

    g. $(-4, -1)$

    h. $(f \circ f)(-1) = f(f(-1)) = f(2) = -1$

2. a. $x^2 + y^2 + 12x - 4y + 31 = 0$

$$(x^2 + 12x) + (y^2 - 4y) = -31$$

$$\boxed{\left[\frac{1}{2}(12)\right]^2 = 36 \qquad \left[\frac{1}{2}(-4)\right]^2 = 4}$$

$$\left[\begin{array}{c}(x^2 + 12x + 36) \\ +(y^2 - 4y + 4)\end{array}\right] = -31 + 36 + 4$$

$$(x + 6)^2 + (y - 2)^2 = 9$$

    b. Center: $(-6, 2)$; Radius: $\sqrt{9} = 3$

3. $(g \circ f)(x) = g(f(x)) = \dfrac{1}{f(x)} = \dfrac{1}{-x^2 + 3x}$

$$-x^2 + 3x \neq 0$$

$$x^2 - 3x \neq 0$$

$$x(x - 3) \neq 0$$

$$x \neq 0 \text{ and } x \neq 3$$

Domain: $(-\infty, 0) \cup (0, 3) \cup (3, \infty)$

4. $(g \cdot h)(x) = g(x) \cdot h(x)$

$$= \frac{1}{x} \cdot \sqrt{x + 2} = \frac{\sqrt{x + 2}}{x}$$

$$x \neq 0$$

$$x + 2 \geq 0$$

$$x \geq -2$$

Domain: $[-2, 0) \cup (0, \infty)$

5. $\dfrac{f(x + h) - f(x)}{h}$

$$= \frac{-(x + h)^2 + 3(x + h) - (-x^2 + 3x)}{h}$$

$$= \frac{-(x^2 + 2xh + h^2) + 3x + 3h + x^2 - 3x}{h}$$

$$= \frac{-x^2 - 2xh - h^2 + 3h + x^2}{h}$$

$$= \frac{-2xh - h^2 + 3h}{h}$$

$$= -2x - h + 3$$

6. $-2x - h + 3 = -2(0) - (3) + 3$

$$= 0 - 3 + 3 = 0$$

7. Substitute 0 for $f(x)$:

$$f(x) = -x^2 + 3x$$

$$0 = -x^2 + 3x$$

$$x^2 - 3x = 0$$

$$x(x - 3) = 0$$
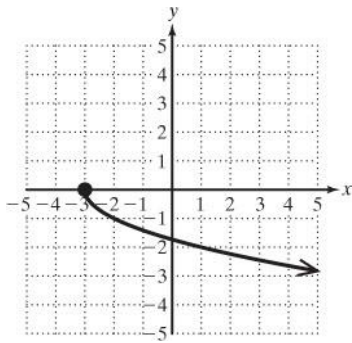
$$x = 0 \text{ or } x = 3$$

Substitute 0 for x:

$$f(x) = -x^2 + 3x$$
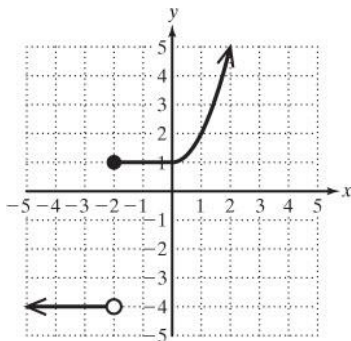
$$f(0) = -(0)^2 + 3(0) = 0$$

x-intercepts: $(0, 0)$ and $(3, 0)$;

y-intercept: $(0, 0)$

8.

9.

10. $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{1-(-3)}{-2-8} = \dfrac{4}{-10} = -\dfrac{2}{5}$

$y = mx + b$

$y = -\dfrac{2}{5}x + b$

$1 = -\dfrac{2}{5}(-2) + b$

$1 = \dfrac{4}{5} + b$

$\dfrac{1}{5} = b$

$y = -\dfrac{2}{5}x + \dfrac{1}{5}$

11. $|x - 7|$ or $|7 - x|$

12. $2x^3 - 128 = 2\left(x^3 - 64\right)$

$= 2\left[(x)^3 - (4)^3\right]$

$= 2(x - 4)(x^2 + 4x + 16)$

13. $-3t(t - 1) = 2t + 6$

$-3t^2 + 3t = 2t + 6$

$-3t^2 + t - 6 = 0$

$3t^2 - t + 6 = 0$

$t = \dfrac{-(-1) \pm \sqrt{(-1)^2 - 4(3)(6)}}{2(3)}$

$= \dfrac{1 \pm \sqrt{1 - 72}}{6} = \dfrac{1 \pm \sqrt{-71}}{6} = \dfrac{1}{6} \pm \dfrac{\sqrt{71}}{6}i$

$\left\{\dfrac{1}{6} \pm \dfrac{\sqrt{71}}{6}i\right\}$

14. $7 = |4x - 2| + 5$

$2 = |4x - 2|$

$4x - 2 = 2 \quad$ or $\quad 4x - 2 = -2$

$4x = 4 \quad$ or $\qquad 4x = 0$

$x = 1 \quad$ or $\qquad x = 0$

$\{0, 1\}$

15. Let $u = x^{1/5}$.

$x^{2/5} - 3x^{1/5} + 2 = 0$

$u^2 - 3u + 2 = 0$

$(u - 1)(u - 2) = 0$

$\qquad\qquad u = 1 \quad$ or $\quad u = 2$

$\qquad\qquad x^{1/5} = 1 \qquad x^{1/5} = 2$

$\qquad\qquad\quad x = 1 \qquad\quad x = 32$

$\{1, 32\}$

16. $|3a + 1| - 2 \le 9$

$|3a + 1| \le 11$

$-11 \le 3a + 1 \le 11$

$-12 \le 3a \le 10$

$-4 \le a \le \dfrac{10}{3}$

$\left[-4, \dfrac{10}{3}\right]$

17. $3 \le -2x + 1 < 7$

$2 \le -2x < 6$

$-1 \ge x > -3$ or $-3 < x \le -1$

$(-3, -1]$

18. $\dfrac{6}{\sqrt{15}+\sqrt{11}} = \dfrac{6}{\sqrt{15}+\sqrt{11}} \cdot \dfrac{\sqrt{15}-\sqrt{11}}{\sqrt{15}-\sqrt{11}}$

$\qquad\qquad = \dfrac{6\left(\sqrt{15}-\sqrt{11}\right)}{15-11}$

$\qquad\qquad = \dfrac{6\sqrt{15}-6\sqrt{11}}{4}$

$\qquad\qquad = \dfrac{3\sqrt{15}-3\sqrt{11}}{2}$

19. $3c\sqrt{8c^2 d^3} + c^2\sqrt{50d^3} - 2d\sqrt{2c^4 d} = 3c\sqrt{2^2 c^2 d^2 \cdot 2d} + c^2\sqrt{5^2 d^2 \cdot 2d} - 2d\sqrt{\left(c^2\right)^2 \cdot 2d}$

$\qquad\qquad\qquad\qquad\qquad\qquad = 6c^2 d\sqrt{2d} + 5c^2 d\sqrt{2d} - 2c^2 d\sqrt{2d}$

$\qquad\qquad\qquad\qquad\qquad\qquad = 9c^2 d\sqrt{2d}$

20. $\dfrac{2u^{-1}-w^{-1}}{4u^{-2}-w^{-2}} = \dfrac{\dfrac{2}{u}-\dfrac{1}{w}}{\dfrac{4}{u^2}-\dfrac{1}{w^2}} \cdot \dfrac{u^2 w^2}{u^2 w^2}$

$\qquad\qquad = \dfrac{2uw^2 - u^2 w}{4w^2 - u^2}$

$\qquad\qquad = \dfrac{uw\left(2w-u\right)}{\left(2w+u\right)\left(2w-u\right)} = \dfrac{uw}{2w+u}$