CHAPTER P
Prerequisites

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Section P.1 Real Numbers

1. rational

2. Irrational

3. prime

4. variables, constants

5. terms

6. Yes. $|5-2| = |2-5| \Rightarrow |3| = |-3| = 3$

7. (c) Commutative Property of Addition: $a+b=b+a$

8. (d) Associative Property of Multiplication: $(ab)c = a(bc)$

9. (e) Additive Inverse Property: $a+(-a)=0$

10. (b) Distributive Property: $a(b+c)=ab+ac$

11. (f) Associative Property of Addition: $(a+b)+c = a+(b+c)$

12. (a) Multiplicative Identity Property: $a \cdot 1 = a$

13. $\left\{-9, -\frac{7}{2}, 5, \frac{2}{3}, \sqrt{2}, 0, 1, -4, -1\right\}$
    (a) Natural numbers: 5, 1
    (b) Whole numbers: 5, 0, 1
    (c) Integers: $-9, 5, 0, 1, -4, -1$
    (d) Rational numbers: $-9, -\frac{7}{2}, 5, \frac{2}{3}, 0, 1, -4, -1$
    (e) Irrational number: $\sqrt{2}$

14. $\left\{\sqrt{5}, -7, -\frac{7}{3}, 0, 3.12, \frac{5}{4}, -2, -8, 3\right\}$
    (a) Natural number: 3
    (b) Whole numbers: 0, 3
    (c) Integers: $-7, 0, -2, -8, 3$
    (d) Rational numbers: $-7, -\frac{7}{3}, 0, 3.12, \frac{5}{4}, -2, -8, 3$
    (e) Irrational number: $\sqrt{5}$

15. $\{2.01, 0.666\ldots, -13, 0.010110111\ldots, 1, -10, 20\}$
    (a) Natural numbers: 1, 20
    (b) Whole numbers: 1, 20
    (c) Integers: $-13, 1, -10, 20$
    (d) Rational numbers: $2.01, 0.666\ldots, -13, 1, -10, 20$
    (e) Irrational number: $0.010110111\ldots$

16. $\{2.3030030003\ldots, 0.7575, -4.63, \sqrt{10}, -2, 0.3, 8\}$
    (a) Natural number: 8
    (b) Whole number: 8
    (c) Integers: $-2, 8$
    (d) Rational numbers: $0.7575, -4.63, -2, 0.3, 8$
    (e) Irrational numbers: $2.3030030003\ldots, \sqrt{10}$

17. $\left\{-\pi, -\frac{1}{3}, \frac{6}{3}, \frac{1}{2}\sqrt{2}, -7.5, -2, 3, -3\right\}$
    (a) Natural numbers: $\frac{6}{3}$ (since it equals 2), 3
    (b) Whole numbers: $\frac{6}{3}$, 3
    (c) Integers: $\frac{6}{3}, -2, 3, -3$
    (d) Rational numbers: $-\frac{1}{3}, \frac{6}{3}, -7.5, -2, 3, -3$
    (e) Irrational numbers: $-\pi, \frac{1}{2}\sqrt{2}$

18. $\left\{25, -17, -\frac{12}{5}, \sqrt{9}, 3.12, \frac{1}{2}\pi, 6, -4, 18\right\}$
    (a) Natural numbers: $25, \sqrt{9}=3, 6, 18$
    (b) Whole numbers: $25, \sqrt{9}=3, 6, 18$
    (c) Integers: $25, -17, \sqrt{9}, 6, -4, 18$
    (d) Rational numbers: $25, -17, -\frac{12}{5}, \sqrt{9}, 3.12, 6, -4, 18$
    (e) Irrational number: $\frac{1}{2}\pi$

19. $\frac{5}{16} = 0.3125$

20. $\frac{17}{4} = 4.25$

21. $\frac{41}{333} = 0.\overline{123}$

22. $\frac{3}{7} = 0.\overline{428571}$

23. $-\frac{100}{11} = -9.\overline{09}$

24. $-\frac{218}{33} = -6.\overline{60}$

25. $6.4 = 6\frac{4}{10} = \frac{64}{10} = \frac{32}{5}$

26. $-7.5 = -7\frac{5}{10} = -\frac{75}{10} = -\frac{15}{2}$

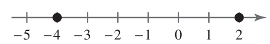
27. $-12.3 = -12\frac{3}{10} = -\frac{123}{10}$

28. $1.87 = 1\frac{87}{100} = \frac{187}{100}$

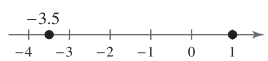
29. $-1 < 2.5$

30. $-6 < -2.5$

31. $-4 < 2$

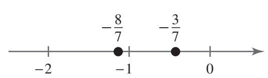


32. $-3.5 < 1$

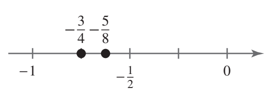


33. $\frac{3}{2} > -\frac{7}{2}$

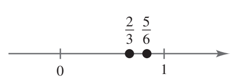
34. $-\frac{8}{7} < -\frac{3}{7}$



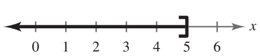
35. $-\frac{3}{4} < -\frac{5}{8}$



36. $\frac{5}{6} > \frac{2}{3}$



37. (a) The inequality $x \le 5$ is the set of all real numbers less than or equal to 5.

(b) 

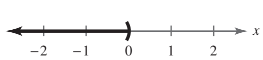
(c) The interval is unbounded.

38. (a) The inequality $x > -3$ is the set of all real numbers greater than -3.

(b) 

(c) The interval is unbounded.

39. (a) The inequality $x < 0$ is the set of all negative real numbers.

(b) 

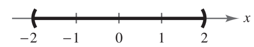
(c) The interval is unbounded.

40. (a) The inequality $x \ge 4$ is the set of all real numbers greater than or equal to 4.

(b) 

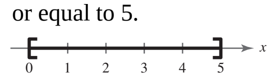
(c) The interval is unbounded.

41. (a) The inequality $-2 < x < 2$ is the set of all real numbers greater than -2 and less than 2.

(b) 

(c) The interval is bounded.

42. (a) The inequality $0 \le x \le 5$ is the set of all real numbers greater than or equal to zero and less than or equal to 5.

(b) 

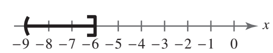
(c) The interval is bounded.

43. (a) The inequality $-1 \le x < 0$ is the set of all negative real numbers greater than or equal to -1.

(b) 

(c) The interval is bounded.

44. (a) The inequality $-9 < x \le -6$ is the set of all real numbers greater than -9 and then less than or equal to -6.

(b) 

(c) The interval is bounded.

45. $x < 0$; $(-\infty, 0)$

46. $y \ge 0$; $[0, \infty)$

47. $z \ge 10$, $[10, \infty)$

48. $y \le 25$, $(-\infty, 25]$

49. $9 \le t \le 24$; $[9, 24]$

50. $-1 \le k < 3$; $[-1, 3)$

51. $0 < m \le 5$ or $(0, 5]$

52. $2.5\% \le r \le 5\%$; $[0.025, 0.05]$

53. $-3 \le x < 8$ or $[-3, 8)$

54. $-4 < x \le 4$ or $(-4, 4]$

55. $[-a, a+4]$

56. $(-c+2, c+1)$

57. The interval $(-6, \infty)$ consists of all real numbers greater than -6.

58. The interval $(-\infty, 4]$ consists of all real numbers less than or equal to 4.

59. The interval $(-\infty, 2]$ consists of all real numbers less than or equal to 2.

60. The interval $(1, \infty)$ consists of all real numbers greater than 1.

61. $|-10| = -(-10) = 10$

62. $|0| = 0$

63. $-3 - |-3| = -3 - 3 = -6$

64. $|-1| - |-2| = (1) - (2) = -1$

65. $\dfrac{-5}{|-5|} = \dfrac{-5}{5} = -1$

66. $-3|-3| = -3(3) = -9$

67. (a) If $x > -1 \Rightarrow x + 1 > 0$, then $\dfrac{|x+1|}{x+1} = \dfrac{x+1}{x+1} = 1$.

(b) If $x < -1 \Rightarrow x + 1 < 0$, then $\dfrac{|x+1|}{x+1} = \dfrac{-(x+1)}{x+1} = -1$.

68. (a) If $x > 2 \Rightarrow x - 2 > 0$, then $\dfrac{|x-2|}{x-2} = \dfrac{x-2}{x-2} = 1$.

(b) If $x < 2 \Rightarrow x - 2 > 0$, then $\dfrac{|x-2|}{x-2} = \dfrac{-(x-2)}{x-2} = -1$.

69. $|y - 4x| = |-3 - 4(2)|$
$= |-3 - 8| = |-11| = 11$

70. $|x| - 2|y| = |-2| - 2|-1| = 2 - 2(1) = 0$

71. $\left|\dfrac{3x + 2y}{|x|}\right| = \left|\dfrac{3(4) + 2(1)}{|4|}\right|$
$= \left|\dfrac{12 + 2}{4}\right| = \dfrac{14}{4} = \dfrac{7}{2}$

72. $\left|\dfrac{3|x| - 2y}{2x + y}\right| = \left|\dfrac{3|-2| - 2(-4)}{2(-2) + (-4)}\right|$
$= \left|\dfrac{3(2) + 8}{(-4) + (-4)}\right| = \left|\dfrac{14}{-8}\right| = \dfrac{7}{4}$

73. $|-3| > -|-3|$ since $3 > -3$.

74. $|-4| = |4|$ since $|-4| = 4$ and $|4| = 4$.

75. $-5 = -|5|$ since $-5 = -5$.

76. $-|-6| < |-6|$ since $|-6| = 6$ and
$-|-6| = -(6) = -6$.

77. $-|-1| < -(-1)$ since $-1 < 1$.

78. $-(-2) > -|2|$ since $-(-2) = 2$.

79. $d(126, 75) = |75 - 126| = 51$

80. $d(-126, -75) = |-126 - (-75)|$
$= |-126 + 75|$
$= |-51| = 51$

81. $d\left(-\frac{5}{2}, \frac{9}{2}\right) = \left|\frac{9}{2} - \left(-\frac{5}{2}\right)\right| = \left|\frac{14}{2}\right| = |7| = 7$

82. $d\left(\frac{1}{4}, \frac{11}{4}\right) = \left|\frac{1}{4} - \frac{11}{4}\right|$
$= \left|-\frac{10}{4}\right| = \frac{10}{4} = \frac{5}{2}$

83. $d\left(\frac{16}{5}, \frac{112}{75}\right) = \left|\frac{112}{75} - \frac{16}{5}\right| = \frac{128}{75}$

84. $d\left(-\frac{15}{8}, \frac{7}{3}\right) = \left|\frac{7}{3} - \left(-\frac{15}{8}\right)\right| = \left|\frac{56+45}{24}\right| = \left|\frac{101}{24}\right| = \frac{101}{24}$

85. $d(x, 5) = |x - 5|$ and $d(x, 5) \leq 3$
Thus, $|x - 5| \leq 3$.

86. $d(x, -10) = |x - (-10)| = |x + 10|$, and
$d(x, -10) \geq 6$. So, $|x + 10| \geq 6$.

87. $d(y, 0) = |y - 0| = |y|$ and $d(y, 0) \geq 6$
Thus, $|y| \geq 6$.

88. $d(y, a) = |y - a|$ and $d(y, a) \leq 3$, So, $|y - a| \leq 3$.

		Receipts	*Expenditures*	$\lvert Receipts - Expenditures \rvert$
89.	1992	$1091.2 billion	$1381.5 billion	$290.3 billion
90.	1996	$1453.1 billion	$1560.5 billion	$107.4 billion
91.	2000	$2025.2 billion	$1789.0 billion	$236.2 billion
92.	2004	$1880.1 billion	$2292.8 billion	$412.7 billion
93.	2008	$2524.0 billion	$2982.5 billion	$458.5 billion
94.	2012	$2450.2 billion	$3537.1 billion	$1086.9 billion

95.

Budgeted Expense, b	*Actual Expense, a*	$\lvert a-b \rvert$	$0.05b$
$112,700	$113,356	$656	$5635

The actual expense difference is greater than $500 (but is less than 5% of the budget) so it does not pass the "budget variance test."

96.

Budgeted Expense, b	*Actual Expense, a*	$\lvert a-b \rvert$	$0.05b$
$9400	$9772	$372	0.05($9400) = $470

Because the difference between the actual expenses and the budget is less than $500 and less than 5% of the budgeted amount, there is compliance with the "budget variance test."

97.

Budgeted Expense, b	*Actual Expense, a*	$\lvert a-b \rvert$	$0.05b$
$37,600	$37,335	$265	$1880

Because the difference between the actual expenses and the budget is less than $500 and less than 5% of the budgeted amount, there is compliance with the "budget variance test."

98.

Budgeted Expense, b	*Actual Expense, a*	$\lvert a-b \rvert$	$0.05b$
$25,800	$25,263	$537	0.05(25,800) = $1290

The actual expense difference is greater than $500 (but is less than 5% of the budget) so it does not meet the "budget variance test."

99. $7x + 4$
Terms: $7x, 4$
Coefficient of $7x$: 7

100. $2x - 9$
Terms: $2x, -9$
Coefficient of $2x$: 2

101. $\sqrt{3}x^2 - 8x - 11$
Terms: $\sqrt{3}x^2, -8x, -11$
Coefficient of $\sqrt{3}x^2$: $\sqrt{3}$
Coefficient of $-8x$: -8

102. $7\sqrt{5}x^2 + 3$
Terms: $7\sqrt{5}x^2, 3$
Coefficient of $7\sqrt{5}x^2$: $7\sqrt{5}$

103. $4x^3 + \dfrac{x}{2} - 5$

Terms: $4x^3, \dfrac{x}{2}, -5$

Coefficient of $4x^3$: 4

Coefficient of $\dfrac{x}{2}$: $\dfrac{1}{2}$

104. $3x^4 + \dfrac{2x^3}{5}$

Terms: $3x^4, \dfrac{2}{5}x^3$

Coefficient of $3x^4$: 3

Coefficient of $\dfrac{2}{5}x^3$: $\dfrac{2}{5}$

105. $2x - 5$

(a) $2(4) - 5 = 8 - 5 = 3$

(b) $2\left(-\frac{1}{2}\right) - 5 = -1 - 5 = -6$

106. $4 - 3x$

(a) $4 - 3(2) = 4 - 6 = -2$

(b) $4 - 3\left(-\frac{5}{6}\right) = 4 + \frac{15}{6} = 4 + \frac{5}{2} = \frac{13}{2}$

107. $x^2 - 4$

(a) $(2)^2 - 4 = 4 - 4 = 0$

(b) $(-2)^2 - 4 = 4 - 4 = 0$

108. $\dfrac{x^2}{x+4}$

(a) $\dfrac{(1)^2}{1^2 + 4} = \dfrac{1}{1+4} = \dfrac{1}{5}$

(b) $\dfrac{(-4)^2}{-4+4} = \dfrac{16}{0}$, undefined

Division by zero is undefined.

109. $2(x+3) = 2x + 6$

Distributive Property

110. $(z-2) + 0 = z - 2$

Additive Identity Property

111. $x + 9 = 9 + x$

Commutative Property of Addition

112. $\dfrac{1}{(h+6)}(h+6) = 1,\ h \neq -6$

Multiplicative Inverse Property

113. $-y + (y + 10) = (-y + y) + 10 = 10$

Associative Property of Addition

114. $\frac{1}{7}(7 \cdot 12) = \left(\frac{1}{7} \cdot 7\right)12$ Associative Property of Multiplication

$= 1 \cdot 12$ Multiplicative Inverse Property

$= 12$ Multiplicative Identity Property

115. $\frac{3}{16} + \frac{5}{16} = \frac{8}{16} = \frac{1}{2}$

116. $\frac{6}{7} - \frac{5}{7} = \frac{1}{7}$

117. $\frac{5}{8} - \frac{5}{12} + \frac{1}{6} = \frac{15}{24} - \frac{10}{24} + \frac{4}{24} = \frac{9}{24} = \frac{3}{8}$

118. $\frac{10}{11} + \frac{6}{33} - \frac{13}{66} = \frac{60}{66} + \frac{12}{66} - \frac{13}{66} = \frac{59}{66}$

119. $\dfrac{x}{6} + \dfrac{4x}{12} = \dfrac{x}{6} + \dfrac{2x}{6} = \dfrac{3x}{6} = \dfrac{x}{2}$

120. $\dfrac{2x}{5} + \dfrac{x}{2} = \dfrac{4x}{10} + \dfrac{5x}{10} = \dfrac{9x}{10}$

121. $\dfrac{12}{x} \div \dfrac{1}{8} = \dfrac{12}{x} \cdot \dfrac{8}{1} = \dfrac{96}{x}$

122. $\dfrac{11}{x} \div \dfrac{3}{4} = \dfrac{11}{x} \cdot \dfrac{4}{3} = \dfrac{44}{3x}$

123. $\left(\frac{2}{5} \div 4\right) - \left(4 \cdot \frac{3}{8}\right) = \left(\frac{2}{5} \cdot \frac{1}{4}\right) - \frac{12}{8} = \frac{1}{10} - \frac{3}{2}$

$= \frac{1}{10} - \frac{15}{10} = -\frac{14}{10} = -\frac{7}{5}$

124. $\left(\frac{3}{5} \div 3\right) - \left(6 \cdot \frac{4}{8}\right) = \left(\frac{3}{5} \cdot \frac{1}{3}\right) - (3)$

$= \frac{1}{5} - 3 = \frac{1}{5} - \frac{15}{5} = -\frac{14}{5}$

125. $d(57,\ 236) = |236 - 57|$

$= 179$ miles

126. $|60° - 23°| = 37°$ change

127. False. The number 0 is nonnegative but positive.

128. False. If $a > 0$ and $b < 0$, then $ab < 0$.

129. False. For example, $3 > 2$, but $\frac{1}{3} < \frac{1}{2}$.

130. (a)

n	1	0.5	0.01	0.0001	0.000001
$5/n$	5	10	500	50,000	5,000,000

(b) As n approaches 0, $5/n$ approaches infinity (∞).
That is, $5/n$ increases without bound.

131. (a) $-A$ is negative, $-A < 0$, because $A > 0$.

(b) $-C$ is positive, $-C > 0$ because $C < 0$.

(c) $B - A$ is negative, $B - A < 0$, because $B < 0$ and $-A < 0$.

(d) $A - C$ is positive, $A - C > 0$ because $-C > 0$ and $A > 0$.

132. (a) Matches graph (ii).

(b) Matches graph (i).

A range of prices can only include zero and positive numbers with at most two decimal places. So, a range of prices can be represented by whole numbers and some noninteger positive fractions.

A range of lengths can only include positive numbers. So, a range of lengths can be represented by positive real numbers.

133. When u and v have the same sign, $|u+v| = |u| + |v|$. For example if $u = 2$ and $v = 1$, then $|2+1| = |2| + |1|$, or if $u = -2$ and $v = -1$, then $|-2+(-1)| = |-2| + |-1|$.

If u and v have different signs, then $|u+v| < |u| + |v|$.

For example if $u = 2$ and $v = -1$, $|2+(-1)| < |2| + |-1|$.

Finally, $|u+v| \ngtr |u| + |v|$, no matter the signs of u and v.

134. $a \le 0$; If the original value of a is negative, then $|a|$ results in a positive number. Because a is negative, the expression $|a| = a$ states that $|a|$ is equal to a negative number, which can never happen. So, if a is originally negative, $|a|$ must equal $-a$, which is a positive value.

Section P. 2 Exponents and Radicals

1. exponent, base

2. square root

3. principal *n*th root

4. index, radicand

5. rationalizing

6. power, index

7. The conjugate of $2 + 3\sqrt{5}$ is $2 - 3\sqrt{5}$.

8. An expression involving radicals is in simplest form when the following conditions are satisfied:

1. All possible factors have been removed from the radical.
2. All fractions have radical-free denominators.
3. The index of the radical is reduced.

9. No, -10.767×10^3 is not written in scientific notation. It should be -1.0767×10^4.

10. 64 is both a perfect square $\left(8^2 = 64\right)$ and a perfect cube $\left(4^3 = 64\right)$.

11. (a) $3 \cdot 3^3 = 3^4 = 81$

(b) $\dfrac{3^2}{3^4} = \dfrac{1}{3^2} = \dfrac{1}{9}$

12. (a) $\dfrac{5^3}{5^2} = 5^1 = 5$

(b) $4^2 \cdot 4^2 = 4^4 = 256$

13. (a) $\left(4^3\right)^0 = 4^0 = 1$

(b) $\dfrac{6}{6^{-3}} = 6^4 = 1296$

14. (a) $24(-2)^{-5} = \dfrac{24}{(-2)^5} = \dfrac{24}{-32} = -\dfrac{3}{4}$

(b) $-7^0 = -1$

15. (a) $(4 \cdot 3)^3 = 12^3 = 1728$

(b) $\left(-3^2\right)^3 = (-9)^3 = -729$

16. (a) $\left(2^3 \cdot 3^2\right)^2 = (8 \cdot 9)^2 = (72)^2 = 5184$

(b) $\left(\dfrac{5}{8}\right)^2 = \dfrac{5^2}{8^2} = \dfrac{25}{64}$

17. (a) $2^{-1} + 3^{-1} = \dfrac{1}{2} + \dfrac{1}{3} = \dfrac{3}{6} + \dfrac{2}{6} = \dfrac{5}{6}$

(b) $\left(3^{-1}\right)^{-3} = 3^3 = 27$

18. (a) $\dfrac{4 \cdot 3^{-2}}{2^{-2} \cdot 3^{-1}} = \dfrac{4\left(\frac{1}{9}\right)}{\left(\frac{1}{4}\right)\left(\frac{1}{3}\right)} = \dfrac{\frac{4}{9}}{\frac{1}{12}} = \dfrac{4}{9} \cdot \dfrac{12}{1} = \dfrac{48}{9} = \dfrac{16}{3}$

(b) $3^{-1} + 2^{-2} = \dfrac{1}{3} + \dfrac{1}{4} = \dfrac{4+3}{12} = \dfrac{7}{12}$

19. When $x = -3$, $2x^3 = 2(-3)^3 = 2(-27) = -54$.

20. When $x = 2$, $-3x^4 = -3(2)^4 = -3(16) = -48$.

21. When $x = 4$, $5(-x)^0 = 5(-4)^0 = 5(1) = 5$.

22. When $x = 7$,
$6x^0 - (6x)^0 = 6(7)^0 - (6 \cdot 7)^0 = 6(1) - 1 = 5.$

23. When $x = 2$, $7x^{-2} = 7(2^{-2}) = 7\left(\frac{1}{2^2}\right) = \frac{7}{4}.$

24. When $x = -5$, $20x^{-2} + x^{-1} = 20(-5)^{-2} + (-5)^{-1} = 20\left(\frac{1}{25}\right) + \frac{1}{-5} = \frac{4}{5} - \frac{1}{5} = \frac{3}{5}.$

25. (a) $x^3(x^2) = x^5$

(b) $x^4(3x^5) = 3x^9$

26. (a) $4z^4(-2z^3) = -8z^7$

(b) $-5x^3(4x^2) = -20x^5$

27. (a) $(3x)^2 = 3^2x^2 = 9x^2$

(b) $(4x^3)^0 = 1, x \neq 0$

28. (a) $6z^2(2z^5)^4 = 6z^2(16z^{20}) = 96z^{22}$

(b) $(3x^5)^3(2x^7)^2 = (27x^{15})(4x^{14}) = 108x^{29}$

29. (a) $\frac{7x^2}{x^3} = 7x^{2-3} = 7x^{-1} = \frac{7}{x}$

(b) $\frac{12(x+y)^3}{9(x+y)} = \frac{4}{3}(x+y)^{3-1}$

$= \frac{4}{3}(x+y)^2, \ x + y \neq 0$

30. (a) $\frac{r^5}{r^9} = \frac{1}{r^4}$

(b) $\frac{3x^2y^4}{15(xy)^2} = \frac{3x^2y^4}{15x^2y^2} = \frac{y^2}{5}, x \neq 0, y \neq 0$

31. (a) $\left[\left(x^2y^{-2}\right)^{-1}\right]^{-1} = x^2y^{-2} = \frac{x^2}{y^2}, x \neq 0$

(b) $\left(\frac{a^{-2}}{b^{-2}}\right)\left(\frac{b}{a}\right)^3 = \frac{b^2}{a^2} \cdot \frac{b^3}{a^3} = \frac{b^5}{a^5}, b \neq 0$

32. (a) $\left(\frac{4}{y}\right)^3\left(\frac{3}{y}\right)^4 = \left(\frac{64}{y^3}\right)\left(\frac{81}{y^4}\right) = \frac{5184}{y^7}$

(b) $(5x^2z^6)^3(5x^2z^6)^{-3} = 1, x \neq 0, z \neq 0$

33. $(-4)^3(5^2) = (-64)(25)$
$= -1600$

34. $(8^{-4})(10^3) \approx 0.244$

35. $\frac{3^6}{7^3} = \frac{729}{343} \approx 2.125$

36. $\frac{4^5}{9^3} = \frac{1024}{729} \approx 1.405$

37. $\frac{4^3 - 1}{3^{-4}} = 3^4(64 - 1) = (81)(63) = 5103$

38. $\frac{3^2 - 2}{4^2 - 3} = \frac{9 - 2}{16 - 3} = \frac{7}{13} \approx 0.538$

39. $973.50 = 9.735 \times 10^2$

40. $28{,}022.2 = 2.80222 \times 10^4$

41. $10{,}252.484 = 1.0252484 \times 10^4$

42. $525{,}252{,}118 = 5.25252118 \times 10^8$

43. $-1110.25 = -1.11025 \times 10^3$

44. $-5{,}222{,}145 = -5.222145 \times 10^6$

45. $0.0002485 = 2.485 \times 10^{-4}$

46. $0.0000025 = 2.5 \times 10^{-6}$

47. $-0.0000025 = -2.5 \times 10^{-6}$

48. $-0.000125005 = -1.25005 \times 10^{-4}$

49. $57{,}300{,}000 = 5.73 \times 10^7$ square miles

50. $9{,}460{,}000{,}000{,}000 = 9.46 \times 10^{12}$ kilometers

51. $0.0000899 = 8.99 \times 10^{-5}$ gram per cm^3

52. 0.000003281 foot $= 3.281 \times 10^{-6}$ foot

53. $1.08 \times 10^4 = 10{,}800$

54. $-4.816 \times 10^8 = -481{,}600{,}000$

55. $-7.65 \times 10^{-7} = -0.000000765$

56. $5.098 \times 10^{-10} = 0.0000000005098$

57. $5.14 \times 10^2 = 514$

58. $1.5 \times 10^7 = 15{,}000{,}000$ degrees Celsius

59. $9.0 \times 10^{-5} = 0.00009$ meter

60. 1.6022×10^{-19}
$= 0.00000000000000000016022$ coulomb

61. $(2.0\times10^7)(3.0\times10^{-3}) = 6.0\times10^4 = 60{,}000$

62. $(1.4\times10^5)(5.2\times10^{-2}) = (1.4)(5.2)\times10^3$
$= 7.28\times10^3 = 7280$

63. $\dfrac{7.0\times10^5}{4.0\times10^{-3}} = \dfrac{7.0}{4.0}\times10^8 = 1.75\times10^8 = 175{,}000{,}000$

64. $\dfrac{3.0\times10^{-3}}{6.0\times10^2} = \dfrac{3.0}{6.0}\times10^{-5} = 0.5\times10^{-5} = 5.0\times10^{-6}$
$= 0.000005$

65. $\sqrt{25\times10^8} = \sqrt{5^2\times(10^4)^2} = 5\times10^4 = 50{,}000$

66. $\sqrt[3]{8\times10^{15}} = \sqrt[3]{2^3\times(10^5)^3}$
$= 2\times10^5$
$= 200{,}000$

67. (a) $(9.3\times10^6)^3(6.1\times10^{-4}) \approx 4.907\times10^{17}$

(b) $\dfrac{(2.414\times10^4)^6}{(1.68\times10^5)^5} \approx 1.479$

68. (a) $750\left(1+\dfrac{0.11}{365}\right)^{800} \approx 954.448$

(b) $\dfrac{67{,}000{,}000+93{,}000{,}000}{0.0052}$
$\approx 30{,}769{,}230{,}769.2 \approx 3.077\times10^{10}$

69. (a) $\sqrt{4.5\times10^9} \approx 67{,}082.039$

(b) $(7.3\times10^4)^{1/5} \approx 9.390$

70. (a) $(2.65\times10^{-4})^{1/3} \approx 0.064$

(b) $\sqrt[4]{9.9\times10^6} \approx 56.093$

71. $\sqrt{121} = \sqrt{11^2} = 11$

72. $\sqrt{-49}$ is not possible. Not a real number.

73. $-\sqrt[3]{-64} = -\sqrt[3]{(-4)^3} = -(-4) = 4$

74. $\sqrt[3]{125} = 5$

75. $\sqrt[4]{-625}$ is not possible. Not a real number.

76. $-\sqrt[7]{-128} = -(-2) = 2$

77. $-\sqrt[6]{\dfrac{1}{729}} = -\dfrac{1}{3}$

78. $\dfrac{\sqrt[5]{-243}}{9} = \dfrac{-3}{9} = -\dfrac{1}{3}$

79. $\sqrt[3]{45^2} \approx 12.651$

80. $\sqrt[5]{-27^3} = (-27)^{3/5} \approx -7.225$

81. $(6.1)^{-2.9} \approx 0.005$

82. $(3.4)^{2.5} \approx 21.316$

83. $\sqrt[4]{90} - (4.1^3)\sqrt{17} \approx -281.088$

84. $(1.2^{-2})\sqrt{75} + 3\sqrt{8} \approx 14.499$

85. $\dfrac{-5+\sqrt{33}}{5} \approx 0.149$

86. $\dfrac{\sqrt[3]{-68}+4}{0.1} \approx -0.817$

87. $\dfrac{3.14}{\pi} + \sqrt[3]{5} \approx 2.709$

88. $\dfrac{\sqrt{10}}{2.5} - \pi^2 \approx -8.605$

89. $(2.8)^{-2} + 1.01\times10^6 \approx 1{,}010{,}000.128$

90. $2.12\times10^{-2} + \sqrt{15} \approx 3.894$

91. $(\sqrt[3]{20})^3 = 20$

92. $\sqrt[4]{(-3x)^4} = 3|x|$

93. $\sqrt{12}\cdot\sqrt{3} = \sqrt{36} = 6$

94. $\dfrac{\sqrt[3]{40x^5}}{\sqrt[3]{5x^2}} = \sqrt[3]{\dfrac{40x^5}{5x^2}} = \sqrt[3]{8x^3} = 2x$

95. (a) $\sqrt{45} = \sqrt{9\cdot5} = 3\sqrt{5}$

(b) $\sqrt[3]{\dfrac{32a^2}{b^2}} = \left(\dfrac{2^3\cdot2^2a^2}{b^2}\right)^{1/3} = 2\sqrt[3]{\dfrac{4a^2}{b^2}}$

96. (a) $\sqrt[3]{54} = \sqrt[3]{3^3\cdot2} = 3\sqrt[3]{2}$

(b) $\sqrt{32x^3y^4} = \sqrt{2^4\cdot2\cdot x\cdot x^2\cdot(y^2)^2} = 4xy^2\sqrt{2x}$

97. (a) $\sqrt[3]{16x^5} = \sqrt[3]{8 \cdot 2 \cdot x^3 \cdot x^2} = 2x^3\sqrt{2x^2}$

(b) $\sqrt{75x^2y^{-4}} = \sqrt{\dfrac{25 \cdot 3 \cdot x^2}{y^4}} = \dfrac{5|x|\sqrt{3}}{y^2}$

98. (a) $\sqrt[4]{3x^4y^2} = |x|\sqrt[4]{3y^2}$

(b) $\sqrt[5]{160x^8z^4} = \sqrt[5]{32 \cdot 5 \cdot x^5 \cdot x^3 \cdot z^4} = 2x\sqrt[5]{5x^3z^4}$

99. (a)
$$\begin{aligned} 2\sqrt{50}+12\sqrt{8} &= 2\sqrt{25\cdot 2}+12\sqrt{4\cdot 2} \\ &= 2\left(5\sqrt{2}\right)+12\left(2\sqrt{2}\right) \\ &= 10\sqrt{2}+24\sqrt{2} \\ &= 34\sqrt{2} \end{aligned}$$

(b)
$$\begin{aligned} 10\sqrt{32}-6\sqrt{18} &= 10\sqrt{16\cdot 2}-6\sqrt{9\cdot 2} \\ &= 10\left(4\sqrt{2}\right)-6\left(3\sqrt{2}\right) \\ &= 40\sqrt{2}-18\sqrt{2} \\ &= 22\sqrt{2} \end{aligned}$$

100. (a) $5\sqrt{x}-3\sqrt{x} = 2\sqrt{x}$

(b)
$$\begin{aligned} -2\sqrt{9y}+10\sqrt{y} &= -2\sqrt{3^2\cdot y}+10\sqrt{y} \\ &= -6\sqrt{y}+10\sqrt{y} = 4\sqrt{y} \end{aligned}$$

101. (a) $3\sqrt{x+1}+10\sqrt{x+1} = 13\sqrt{x+1}$

(b)
$$\begin{aligned} 7\sqrt{80x}-2\sqrt{125x} &= 7\sqrt{16\cdot 5x}-2\sqrt{25\cdot 5x} \\ &= 7(4)\sqrt{5x}-2(5)\sqrt{5x} \\ &= 28\sqrt{5x}-10\sqrt{5x} \\ &= 18\sqrt{5x} \end{aligned}$$

102. (a)
$$\begin{aligned} 5\sqrt{10x^2}-\sqrt{90x^2} &= 5\sqrt{10x^2}-\sqrt{3^2\cdot 10\cdot x^2} \\ &= 5\sqrt{10x^2}-3\sqrt{10x^2} \\ &= 2\sqrt{10x^2} = 2|x|\sqrt{10} \end{aligned}$$

(b)
$$\begin{aligned} 8\sqrt[3]{27x}-\tfrac{1}{2}\sqrt[3]{64x} &= 8\left(3^3\cdot x\right)^{1/3}-\tfrac{1}{2}\left(4^3\cdot x\right)^{1/3} \\ &= 24x^{1/3}-2x^{1/3} \\ &= 22x^{1/3} = 22\sqrt[3]{x} \end{aligned}$$

103. $\sqrt{\dfrac{3}{11}} = \dfrac{\sqrt{3}}{\sqrt{11}}$

104. $\sqrt{5}+\sqrt{3} \approx 3.968$ and $\sqrt{5+3} = \sqrt{8} \approx 2.828$

Thus, $\sqrt{5}+\sqrt{3} > \sqrt{5+3}$.

105. $\sqrt{3^2+2^2} = \sqrt{9+4} = \sqrt{13} \approx 3.606$

Thus, $5 > \sqrt{3^2+2^2}$.

106. $\sqrt{3^2+4^2} = \sqrt{9+16} = \sqrt{25} = 5$

So, $5 = \sqrt{3^2+4^2}$.

107. $\dfrac{1}{\sqrt{3}} = \dfrac{1}{\sqrt{3}}\cdot\dfrac{\sqrt{3}}{\sqrt{3}} = \dfrac{\sqrt{3}}{3}$

108. $\dfrac{8}{\sqrt[3]{2}} = \dfrac{8}{\sqrt[3]{2}}\cdot\dfrac{\sqrt[3]{4}}{\sqrt[3]{4}} = \dfrac{8\sqrt[3]{4}}{2} = 4\sqrt[3]{4}$

109.
$$\begin{aligned} \frac{5}{\sqrt{14}-2} &= \frac{5}{\sqrt{14}-2}\cdot\frac{\sqrt{14}+2}{\sqrt{14}+2} \\ &= \frac{5\left(\sqrt{14}+2\right)}{14-4} \\ &= \frac{5\left(\sqrt{14}+2\right)}{10} \\ &= \frac{\sqrt{14}+2}{2} \end{aligned}$$

110.
$$\begin{aligned} \frac{3}{\sqrt{5}+\sqrt{6}} &= \frac{3}{\sqrt{5}+\sqrt{6}}\cdot\frac{\sqrt{5}-\sqrt{6}}{\sqrt{5}-\sqrt{6}} \\ &= \frac{3\left(\sqrt{5}-\sqrt{6}\right)}{5-6} \\ &= 3\left(\sqrt{6}-\sqrt{5}\right) \end{aligned}$$

111. $\dfrac{\sqrt{12}}{2} = \dfrac{2\sqrt{3}}{2} = \sqrt{3} = \dfrac{\sqrt{3}\cdot\sqrt{3}}{1\cdot\sqrt{3}} = \dfrac{3}{\sqrt{3}}$

112. $\dfrac{\sqrt{3}}{3}\cdot\dfrac{\sqrt{3}}{\sqrt{3}} = \dfrac{3}{3\sqrt{3}} = \dfrac{1}{\sqrt{3}}$

113.
$$\begin{aligned} \frac{\sqrt{5}+\sqrt{3}}{3} &= \frac{\sqrt{5}+\sqrt{3}}{3}\cdot\frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}-\sqrt{3}} \\ &= \frac{5-3}{3\left(\sqrt{5}-\sqrt{3}\right)} \\ &= \frac{2}{3\left(\sqrt{5}-\sqrt{3}\right)} \end{aligned}$$

114.
$$\begin{aligned} \frac{\sqrt{7}-3}{4} &= \frac{\sqrt{7}-3}{4}\cdot\frac{\sqrt{7}+3}{\sqrt{7}+3} \\ &= \frac{7-9}{4\left(\sqrt{7}+3\right)} = \frac{-1}{2\left(\sqrt{7}+3\right)} \\ &= -\frac{1}{2\sqrt{7}+6} \end{aligned}$$

115. (a) $\sqrt[4]{3^2} = 3^{2/4} = 3^{1/2} = \sqrt{3}$

(b) $\sqrt[6]{(x+1)^4} = (x+1)^{4/6} = (x+1)^{2/3}$
$= \sqrt[3]{(x+1)^2}$

116. (a) $\sqrt[6]{x^3} = x^{3/6} = x^{1/2} = \sqrt{x}$

(b) $\sqrt[4]{(3x^2)^4} = 3x^2$

	Radical Form	*Rational Exponent Form*
117.	$\sqrt[3]{64}$ Given	$64^{1/3}$ Answer
118.	$-\sqrt{144}$ Answer	$-(144^{1/2})$ Given
119.	$\sqrt[5]{\frac{1}{32}}$ Answer	$(1/32)^{1/5}$ Given
120.	$\sqrt[3]{614.125}$ Given	$(614.125)^{1/3}$ Answer
121.	$\sqrt[5]{-243}$ Answer	$(-243)^{1/5}$ Given
122.	$\sqrt[3]{-216}$ Given	$(-216)^{1/3}$ Answer
123.	$\sqrt[4]{81^3}$ Given	$81^{3/4}$ Answer
124.	$\sqrt[4]{16^5}$ Answer	$16^{5/4}$ Given

125. $\dfrac{(2x^2)^{3/2}}{2^{1/2}x^4} = \dfrac{2^{3/2}(x^2)^{3/2}}{2^{1/2}x^4} = \dfrac{2^{3/2}x^3}{2^{1/2}x^4}$
$= 2^{3/2-1/2}x^{3-4} = 2^1x^{-1} = \dfrac{2}{|x|}$

126. $\dfrac{x^{4/3}y^{2/3}}{(xy)^{1/3}} = \dfrac{x^{4/3}y^{2/3}}{x^{1/3}y^{1/3}} = x^{(4/3)-(1/3)}y^{(2/3)-(1/3)}$
$= xy^{1/3},\ x \neq 0,\ y \neq 0$

127. $\dfrac{x^{-3}\cdot x^{1/2}}{x^{3/2}\cdot x^{-1}} = \dfrac{x^{1/2}\cdot x^1}{x^{3/2}\cdot x^3} = x^{(1/2)+1-(3/2)-3}$
$= x^{-3} = \dfrac{1}{x^3},\ x > 0$

128. $\dfrac{5^{-1/2}\cdot 5x^{5/2}}{(5x)^{3/2}} = \dfrac{5^{-1/2}\cdot 5x^{5/2}}{5^{3/2}x^{3/2}} = 5^{-1}x = \dfrac{x}{5},\ x > 0$

129. $32^{-3/5} = \dfrac{1}{32^{3/5}} = \dfrac{1}{(\sqrt[5]{32})^3} = \dfrac{1}{(2)^3} = \dfrac{1}{8}$

130. $\left(\dfrac{9}{4}\right)^{-1/2} = \left(\dfrac{4}{9}\right)^{1/2} = \dfrac{4^{1/2}}{9^{1/2}} = \dfrac{2}{3}$

131. $\left(-\dfrac{1}{27}\right)^{-1/3} = (-27)^{1/3} = \sqrt[3]{-27} = \sqrt[3]{(-3)^3} = -3$

132. $-\left(\frac{1}{125}\right)^{-4/3} = -(125)^{4/3}$
$= -(125^{1/3})^4 = -(5)^4 = -625$

133. $\sqrt{\sqrt{35}} = (35^{1/2})^{1/2} = 35^{1/4} = \sqrt[4]{35}$

134. $\sqrt{\sqrt[4]{2x}} = \left((2x)^{1/4}\right)^{1/2} = (2x)^{1/8} = \sqrt[8]{2x}$

135. $\sqrt{\sqrt{243(x+1)}} = \left(\left[243(x+1)\right]^{1/2}\right)^{1/2}$
$= (243(x+1))^{1/4}$
$= \sqrt[4]{243(x+1)}$
$= \sqrt[4]{3\cdot 3^4(x+1)}$
$= 3\sqrt[4]{3(x+1)}$

136. $\sqrt{\sqrt[3]{128a^7b}} = \left[(128a^7b)^{1/3}\right]^{1/2} = (128a^7b)^{1/6} = \sqrt[6]{128a^7b} = \sqrt[6]{64\cdot 2\cdot a^6\cdot a\cdot b} = 2a\sqrt[6]{2ab}$

137. Brazil: $\dfrac{2.19 \times 10^{12}}{2.01 \times 10^{8}} = 1.09 \times 10^{4}$

Canada: $\dfrac{1.83 \times 10^{12}}{3.46 \times 10^{7}} = 5.29 \times 10^{4}$

Germany: $\dfrac{3.59 \times 10^{12}}{8.11 \times 10^{7}} = 4.43 \times 10^{4}$

India: $\dfrac{1.76 \times 10^{12}}{1.22 \times 10^{9}} = 1.44 \times 10^{3}$

Iran: $\dfrac{4.12 \times 10^{11}}{7.99 \times 10^{7}} = 5.16 \times 10^{3}$

Ireland: $\dfrac{2.21 \times 10^{11}}{4.78 \times 10^{6}} = 4.62 \times 10^{4}$

Mexico: $\dfrac{1.33 \times 10^{12}}{1.19 \times 10^{8}} = 1.12 \times 10^{4}$

138. Paper: $0.274(2.51 \times 10^{8}) \approx 6.8774 \times 10^{7}$ tons

Metals: $0.089(2.51 \times 10^{8}) \approx 2.2339 \times 10^{7}$ tons

Glass: $0.046(2.51 \times 10^{8}) \approx 1.1546 \times 10^{7}$ tons

Plastics: $0.127(2.51 \times 10^{8}) \approx 3.1877 \times 10^{7}$ tons

Yard waste: $0.135(2.51 \times 10^{8}) \approx 3.3885 \times 10^{7}$ tons

Other: $0.329(2.51 \times 10^{8}) \approx 8.2579 \times 10^{7}$ tons

139. For $h = 7$,

$t = 0.03\left[12^{5/2} - (12 - 7)^{5/2}\right] = 0.03\left[12^{5/2} - 5^{5/2}\right]$

≈ 13.288 seconds.

140. True. For $x \neq 0$, $\dfrac{x^{k+1}}{x} = \dfrac{x^{k+1}}{x^1} = x^k$.

141. False. For example, let $a = 2$, $n = 3$ and $k = 2$. Then

$(a^n)^k = (2^3)^2 = 8^2 = 64$, whereas $a^{(n^k)} = 2^{(3^2)} = 2^9 = 512$.

142. False. For example, let $a = 5$ and $b = 4$.

$(a + b)^2 = (5 + 4)^2 = 9^2 = 81$, whereas

$(5)^2 + (4)^2 = 25 + 16 = 41$.

143. True.

$$x^{-1} + y^{-1} = \frac{1}{x} + \frac{1}{y} = \frac{y}{xy} + \frac{x}{xy} = \frac{y + x}{xy} = \frac{x + y}{xy}$$

144. The length of a side of package A is about 8 inches $(8^3 = 512)$, and the length of a side of package B is about 6.3 inches $(6.3^3 \approx 250)$. Twice the length of a side of package B is about $2(6.3) = 12.6$ inches, and $8 < 12.6$. So, the length x of a side of package A is less than twice the length of a side of package B.

145. For $a \neq 0$, $1 = \dfrac{a}{a} = \dfrac{a^1}{a^1} = a^{1-1} = a^0$.

Thus, $a^0 = 1$.

146. Consider $x^2 = n$, x a positive integer.

Unit digit of x	*Unit digit of* $n = x^2$
1	1
2	4
3	9
4	6
5	5
6	6
7	9
8	4
9	1
0	0

Therefore, the possible digits are 0, 1, 4, 5, 6, and 9 thus $\sqrt{5233}$ is *not* an integer because its unit digit is 3.

147. No. Rationalizing the denominator produces a number equivalent to the original fraction; squaring does not.

$$\left(\frac{5}{\sqrt{3}}\right)^2 = \frac{25}{3} \neq \frac{5}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{5\sqrt{3}}{3}$$

Section P.3 Polynomials and Factoring

1. n, a_n

2. monomial

3. First, Outer, Inner, Last

4. prime

5. A polynomial is completely factorial when each of its factors are prime.

6. Four guidelines for factoring polynomials are as follows:
(1) Factor out any common factors using the Distributive Property.
(2) Factor according to one of the special polynomial forms.
(3) Factor as $ax^2 + bx + c = (mx + r)(nx + s)$.
(4) Factor by grouping.

7. 7 is a polynomial of degree zero. Matches (d).

8. $-3x^5 + 2x^3 + x$ is a trinomial of degree 5. Matches (e).

9. $-4x^3 + 1$ is a binomial with leading coefficient -4. Matches (b).

10. $6x$ is a monomial of positive degree. Matches (a).

11. $\frac{3}{4}x^4 + x^2 + 14$ is a trinomial with leading coefficient $\frac{3}{4}$. Matches (f).

12. $x^3 + 2x^2 - 4x + 1$ is a third-degree polynomial with leading coefficient 1. Matches (c).

13. $-2x^3 + 4x$ is one possible answer.

14. $8x^5 + 14$ is one possible answer.

15. $-15x^4 + 2x$ is one possible answer.

16. $8x^3 + 3x + 14$ is one possible answer.

17. $3x + 4x^2 + 2 = 4x^2 + 3x + 2$ Standard form
Degree: 2
Leading coefficient: 4

18. $x^2 - 4 - 3x^4 = -3x^4 + x^2 - 4$ Standard form
Degree: 4
Leading coefficient: -3

19. $-8 + x^7 = x^7 - 8$ Standard form
Degree: 7
Leading coefficient: 1

20. $23 - x^3 = -x^3 + 23$ Standard form
Degree: 3
Leading coefficient: -1

21. $1 - x + 6x^4 - 2x^5 = -2x^5 + 6x^4 - x + 1$ Standard form
Degree: 5
Leading coefficient: -2

22. $-x^6 + 5 - 4x^5 + x^3 = -x^6 - 4x^5 + x^3 + 5$ Standard form
Degree: 6
Leading coefficient: -1

23. This is a polynomial: $-8y^2 + 2y$.

24. $5x^4 - 2x^2 + \dfrac{1}{x^2}$ is not a polynomial.

25. $\sqrt{x^2 - x^4}$ is not a polynomial.

26. $\dfrac{x^2 + 2x - 3}{6} = \dfrac{1}{6}x^2 + \dfrac{1}{3}x - \dfrac{1}{2}$ is a polynomial.

27. $(4x + 1) + (-x + 9) = 3x + 10$

28. $(t^2 - 3) + (6t^2 - 4t) = 7t^2 - 4t - 3$

29. $(8x + 5) - (6x - 12) = 8x + 5 - 6x + 12 = 2x + 17$

30. $(x^2 - 5) - (2x^2 - 3x) = x^2 - 5 - 2x^2 + 3x$
$= -x^2 + 3x - 5$

31. $(2x^3 - 9x^2 - 20) + (-2x^3 + 10x^2) = x^2 - 20$

32. $(y^3 - 6y + 3) + (5y^3 - 2y^2 + y - 10)$
$= 6y^3 - 2y^2 - 5y - 7$

33. $(15x^2 - 6) - (-8.1x^3 - 14.7x^2 - 17)$
$= 15x^2 - 6 + 8.1x^3 + 14.7x^2 + 17$
$= 8.1x^3 + 29.7x^2 + 11$

34. $(13.6w^4 - 14w - 17.4) - (16.9w^4 - 9.2w + 13)$
$= 13.6w^4 - 14w - 17.4 - 16.9w^4 + 9.2w - 13$
$= 3.3w^4 - 4.8w - 30.4$

35. $5z(z - 8) = 5z^2 - 40z$

36. $\left(\frac{1}{6}x + 1\right)(2x^2) = \frac{1}{3}x^3 + 2x^2$

37. $\left(5 - \dfrac{3}{2}y\right)(-4y) = (5)(-4y) - \left(\dfrac{3}{2}y\right)(-4y)$
$= 6y^2 - 20y$

38. $-7x(4 - x^3) = -28x + 7x^4$
$= 7x^4 - 28x$

39. $3x(x^2 - 2x + 1) = 3x(x^2) + 3x(-2x) + 3x(1)$
$= 3x^3 - 6x^2 + 3x$

40. $-y^2(4y^2 + 2y - 3) = -y^2(4y^2) - y^2(2y) - y^2(-3)$
$= -4y^4 - 2y^3 + 3y^2$

41. $(x + 3)(x + 4) = x^2 + 4x + 3x + 12$ FOIL
$= x^2 + 7x + 12$

42. $(x - 5)(x + 10) = x^2 + 10x - 5x - 50$ FOIL
$= x^2 + 5x - 50$

43. $(3x - 5)(2x + 1) = 6x^2 + 3x - 10x - 5$ FOIL
$= 6x^2 - 7x - 5$

44. $(7x - 2)(4x - 3) = 28x^2 - 21x - 8x + 6$ FOIL
$= 28x^2 - 29x + 6$

45. $(4y + 7)^2 = 16y^2 + 56y + 49$

46. $(3a - 3)^2 = 9a^2 - 18a + 9$

47. $(2 - 5x)^2 = 4 - 2(2)(5x) + 25x^2$
$= 4 - 20x + 25x^2$
$= 25x^2 - 20x + 4$

48. $(5x + 8y)^2 = 25x^2 + 2(5x)(8y) + 64y^2$
$= 25x^2 + 80xy + 64y^2$

49. $(x - 9)(x + 9) = x^2 - 9^2 = x^2 - 81$

50. $(5x + 6)(5x - 6) = (5x)^2 - 6^2 = 25x^2 - 36$

51. $(x + 2y)(x - 2y) = x^2 - (2y)^2 = x^2 - 4y^2$

52. $(2r^2 - 5)(2r^2 + 5) = (2r^2)^2 - 5^2 = 4r^4 - 25$

53. $(x + 1)^3 = x^3 + 3x^2(1) + 3x(1^2) + 1^3$
$= x^3 + 3x^2 + 3x + 1$

54. $(y - 4)^3 = y^3 - 3y^2(4) + 3y(4)^2 - 4^3$
$= y^3 - 12y^2 + 48y - 64$

55. $(2x - y)^3 = (2x)^3 - 3(2x)^2 y + 3(2x)y^2 - y^3$
$= 8x^3 - 12x^2y + 6xy^2 - y^3$

56. $(3x + 2y)^3 = (3x)^3 + 3(3x)^2(2y) + 3(3x)(2y)^2 + (2y)^3$
$= 27x^3 + 54x^2y + 36xy^2 + 8y^3$

57. $\left(\frac{1}{4}x - 3\right)\left(\frac{1}{4}x + 3\right) = \left(\frac{1}{4}x\right)^2 - 3^2 = \frac{1}{16}x^2 - 9$

58. $(1.5y + 0.6)(1.5y - 0.6) = 2.25y^2 - 0.36$

59. $\left(\frac{5}{2}x + 3\right)^2 = \frac{25}{4}x^2 + 15x + 9$

60. $(1.8y - 5)^2 = (1.8y)^2 + 2(1.8y)(-5) + (-5)^2$
$= 3.24y^2 - 18y + 25$

61.

$$\begin{array}{rrrrr} & & -x^2 & +\ x & -\ 5 \\ & & 3x^2 & +\ 4x & +\ 1 \\ \hline & & -x^2 & +\ x & -\ 5 \\ & -4x^3 & +\ 4x^2 & -\ 20x & \\ -3x^4 & +\ 3x^3 & -\ 15x^2 & & \\ \hline -3x^4 & -\ x^3 & -\ 12x^2 & -\ 19x & -\ 5 \end{array}$$

Answer: $-3x^4 - x^3 - 12x^2 - 19x - 5$

62.

$$\begin{array}{rrrrr} & & x^2 & +\ 3x & +\ 2 \\ & & 2x^2 & -\ x & +\ 4 \\ \hline & & 4x^2 & +\ 12x & +\ 8 \\ & -x^3 & -\ 3x^2 & -\ 2x & \\ 2x^4 & +\ 6x^3 & +\ 4x^2 & & \\ \hline 2x^4 & +\ 5x^3 & +\ 5x^2 & +\ 10x & +\ 8 \end{array}$$

63. $[(x + z) + 5][(x + z) - 5] = (x + z)^2 - 5^2$
$= x^2 + 2xz + z^2 - 25$

64. $[(x - 3y) + z][(x - 3y) - z]$
$= (x - 3y)^2 - z^2$
$= x^2 - 2x(3y) + (3y)^2 - z^2$
$= x^2 - 6xy + 9y^2 - z^2$

65. $[(x - 3) + y]^2 = (x - 3)^2 + 2y(x - 3) + y^2$
$= x^2 - 6x + 9 + 2xy - 6y + y^2$
$= x^2 + 2xy + y^2 - 6x - 6y + 9$

66. $[(x + 1) - y]^2 = (x + 1)^2 + 2(x + 1)(-y) + (-y)^2$
$= x^2 + 2x + 1 - 2xy - 2y + y^2$
$= x^2 - 2xy + y^2 + 2x - 2y + 1$

67. $5x - 40 = 5(x - 8)$

68. $4y + 20 = 4(y + 5)$

69. $2x^3 - 6x = 2x(x^2 - 3)$

70. $3z^4 - 6z^2 + 9z = 3z(z^3 - 2z + 3)$

71. $3x(x-5) + 8(x-5) = (3x+8)(x-5)$

72. $5(x+1) - x(x+1) = (x+1)(5-x)$
$= -(x+1)(x-5)$

73. $(5x-4)^2 + (5x-4) = (5x-4)[(5x-4)+1]$
$= (5x-4)(5x-3)$

74. $2x(x+3) - 3x(x+3)^2 = (x+3)[2x - 3x(x+3)]$
$= (x+3)(2x - 3x^2 - 9x)$
$= (x+3)(-3x^2 - 7x)$
$= -x(x+3)(3x+7)$

75. $x^2 - 36 = (x+6)(x-6)$

76. $x^2 - 81 = (x+9)(x-9)$

77. $48y^2 - 27 = 3(16y^2 - 9) = 3((4y)^2 - 3^2)$
$= 3(4y+3)(4y-3)$

78. $50 - 98z^2 = 2(25 - 49z^2)$
$= 2(5^2 - (7z)^2)$
$= 2(5+7z)(5-7z)$
$= -2(7z+5)(7z-5)$

79. $4x^2 - \frac{1}{9} = (2x)^2 - \left(\frac{1}{3}\right)^2 = \left(2x + \frac{1}{3}\right)\left(2x - \frac{1}{3}\right)$

80. $\frac{25}{36}y^2 - 49 = \left(\frac{5}{6}y\right)^2 - 7^2 = \left(\frac{5}{6}y + 7\right)\left(\frac{5}{6}y - 7\right)$

81. $(x-1)^2 - 4 = [(x-1)+2][(x-1)-2]$
$= (x+1)(x-3)$

82. $25 - (z+5)^2 = 5^2 - (z+5)^2$
$= (5 - (z+5))(5 + (z+5))$
$= (5 - z - 5)(5 + z + 5)$
$= -z(z+10)$

83. $x^2 - 4x + 4 = x^2 - 2(2)x + 2^2 = (x-2)^2$

84. $x^2 + 10x + 25 = x^2 + 2(5)(x) + 5^2 = (x+5)^2$

85. $x^2 + x + \frac{1}{4} = x^2 + 2\left(\frac{1}{2}\right)x + \left(\frac{1}{2}\right)^2 = \left(x + \frac{1}{2}\right)^2$

86. $x^2 - \frac{4}{3}x + \frac{4}{9} = x^2 - 2\left(\frac{2}{3}\right)x + \left(\frac{2}{3}\right)^2 = \left(x - \frac{2}{3}\right)^2$

87. $4x^2 - 12x + 9 = (2x)^2 - 2(2x)(3) + 3^2$
$= (2x-3)^2$

88. $25z^2 - 10z + 1 = (5z)^2 - 2(5z)(1) + 1$
$= (5z-1)^2$

89. $4x^2 - \frac{4}{3}x + \frac{1}{9} = (2x)^2 - 2(2x)\left(\frac{1}{3}\right) + \left(\frac{1}{3}\right)^2$
$= \left(2x - \frac{1}{3}\right)^2$

90. $9y^2 - \frac{3}{2}y + \frac{1}{16} = (3y)^2 - 2(3y)\left(\frac{1}{4}\right) + \left(\frac{1}{4}\right)^2$
$= \left(3y - \frac{1}{4}\right)^2$
$= \frac{(12y-1)^2}{16}$

91. $x^3 - 8 = (x)^3 - (2)^3$
$= (x-2)(x^2 + 2x + 4)$

92. $y^3 - 125 = (y)^3 - (5)^3$
$= (y-5)(y^2 + 5y + 25)$

93. $z^3 + 1 = (z)^3 + (1)^3$
$= (z+1)(z^2 - z + 1)$

94. $x^3 + 64 = (x)^3 + (4)^3$
$= (x+4)(x^2 - 4x + 16)$

95. $x^3 + \frac{1}{27} = (x)^3 + \left(\frac{1}{3}\right)^3$
$= \left(x + \frac{1}{3}\right)\left(x^2 - \frac{1}{3}x + \frac{1}{9}\right)$

96. $w^3 - \frac{27}{216} = w^3 - \frac{1}{8} = (w)^3 - \left(\frac{1}{2}\right)^3$
$= \left(w - \frac{1}{2}\right)\left(w^2 + \frac{1}{2}x + \frac{1}{4}\right)$

97. $125v^3 - 1 = (5v)^3 - (1)^3$
$= (5v - 1)(25v^2 + 5v + 1)$

98. $343a^3 + 8 = (7a)^3 + (2)^3$
$= (7a + 2)(49a^2 - 14a + 4)$

99. $(y + 1)^3 - x^3 = (y + 1)^3 - (x)^3$
$= [(y + 1) - x][(y + 1)^2 + x(y + 1) + x^2]$
$= (y - x + 1)(y^2 + 2y + 1 + xy + x + x^2)$
$= (-x + y + 1)(x^2 + y^2 + xy + x + 2y + 1)$

100. $(2x - z)^3 + 125y^3 = (2x - z)^3 + (5y)^3$
$= [(2x - z) + 5y][(2x - z)^2 - 5y(2x - z) + 25y^2]$
$= (2x + 5y - z)(4x^2 - 4xz + z^2 - 10xy + 5yz + 25y^2)$
$= (2x + 5y - z)(4x^2 + 25y^2 + z^2 - 10xy - 4xz + 5yz)$

101. $x^2 + x - 2 = (x + 2)(x - 1)$

102. $x^2 + 6x + 8 = (x + 4)(x + 2)$

103. $s^2 - 5s + 6 = (s - 3)(s - 2)$

104. $t^2 - t - 6 = t^2 + 2t - 3t - 6 = (t + 2)(t - 3)$

105. $20 - y - y^2 = (5 + y)(4 - y)$
$= -(y + 5)(y - 4)$

106. $24 + 5z - z^2 = 24 + 8z - 3z - z^2$
$= (8 - z)(3 + z)$

107. $3x^2 + 13x - 10 = (3x - 2)(x + 5)$

108. $2x^2 - x - 21 = (2x - 7)(x + 3)$

109. $5x^2 + 26x + 5 = (5x + 1)(x + 5)$

110. $8x^2 - 45x - 18 = (x - 6)(8x + 3)$

111. $-5u^2 - 13u + 6 = -(5u^2 + 13u - 6)$
$= -(5u - 2)(u + 3)$
$= (2 - 5u)(u + 3)$

112. $-6x^2 + 23x + 4 = -(6x^2 - 23x - 4)$
$= -(x - 4)(6x + 1)$
$= (4 - x)(6x + 1)$

113. $\frac{1}{8}x^2 - \frac{1}{96}x - \frac{1}{16} = \frac{1}{8}\left(x^2 - \frac{1}{12}x - \frac{1}{2}\right)$
$= \frac{1}{8}\left(x - \frac{3}{4}\right)\left(x + \frac{2}{3}\right)$
$= \frac{1}{96}(4x - 3)(3x + 2)$

114. $\frac{1}{81}x^2 + \frac{2}{9}x - 8 = \frac{1}{81}[x^2 + 18x - 648]$
$= \frac{1}{81}(x - 18)(x + 36)$
$= \left(\frac{1}{9}x - 2\right)\left(\frac{1}{9}x + 4\right)$

115. $x^3 - x^2 + 2x - 2 = x^2(x - 1) + 2(x - 1)$
$= (x - 1)(x^2 + 2)$

116. $x^3 + 5x^2 - 5x - 25 = x^2(x + 5) - 5(x + 5)$
$= (x + 5)(x^2 - 5)$

117. $x^3 - 5x^2 + x - 5 = x^2(x - 5) + (x - 5)$
$= (x^2 + 1)(x - 5)$

118. $x^3 - x^2 + 3x - 3 = x^2(x - 1) + 3(x - 1)$
$= (x^2 + 3)(x - 1)$

119. $x^2 + x - 20 = x^2 + 5x - 4x - 20$
$= x(x + 5) - 4(x + 5)$
$= (x + 5)(x - 4)$

120. $b^2 - 11b + 18 = b^2 - 9b - 2b + 18$
$= b(b - 9) - 2(b - 9)$
$= (b - 9)(b - 2)$

121. $6x^2 + x - 2$

$a = 6,\ c = -2,\ ac = -12 = 4(-3)$, and

$4 - 3 = 1 = b$.

Thus, $6x^2 + x - 2 = 6x^2 + 4x - 3x - 2$

$= 2x(3x + 2) - (3x + 2)$

$= (2x - 1)(3x + 2)$.

122. $a = 3,\ c = 8,\ ac = 24 = 6(4)$, and

$6 + 4 = 10 = b$.

Thus, $3x^2 + 10x + 8 = 3x^2 + 4x + 6x + 8$

$= x(3x + 4) + 2(3x + 4)$

$= (x + 2)(3x + 4)$.

123. $10x^2 - 40 = 10(x^2 - 4) = 10(x + 2)(x - 2)$

124. $7z^2 - 63 = 7(z^2 - 9) = 7(z + 3)(z - 3)$

125. $y^3 - y = y(y^2 - 1) = y(y + 1)(y - 1)$

126. $x^3 - 9x^2 = x^2(x - 9)$

127. $x^2 - 2x + 1 = (x - 1)^2$

128. $9x^2 - 6x + 1 = (3x - 1)^2$

129. $1 - 4x + 4x^2 = (1 - 2x)^2 = (2x - 1)^2$

130. $16 - 6x - x^2 = -(x^2 + 6x - 16)$

$= -(x + 8)(x - 2)$

$= (x + 8)(2 - x)$

131. $2x^2 + 6x - 2x^3 = -2x^3 + 2x^2 + 6x$

$= -2x(x^2 - x - 3)$

132. $7y^2 + 15y - 2y^3 = -y(2y^2 - 7y - 15)$

$= -y(2y + 3)(y - 5)$

133. $9x^2 + 10x + 1 = (9x + 1)(x + 1)$

134. $13x + 6 + 5x^2 = 5x^2 + 13x + 6$

$= 5x^2 + 10x + 3x + 6$

$= (5x + 3)(x + 2)$

135. $3x^3 + x^2 + 15x + 5 = x^2(3x + 1) + 5(3x + 1)$

$= (3x + 1)(x^2 + 5)$

136. $5 - x + 5x^2 - x^3 = 1(5 - x) + x^2(5 - x)$

$= (5 - x)(1 + x^2)$

137. $3u - 2u^2 + 6 - u^3 = -u^3 - 2u^2 + 3u + 6$

$= -u^2(u + 2) + 3(u + 2)$

$= (3 - u^2)(u + 2)$

138. $x^4 - 4x^3 + x^2 - 4x = x^3(x - 4) + x(x - 4)$

$= (x^3 + x)(x - 4)$

$= x(x^2 + 1)(x - 4)$

139. $2x^3 + x^2 - 8x - 4 = x^2(2x + 1) - 4(2x + 1)$

$= (x^2 - 4)(2x + 1)$

$= (x + 2)(x - 2)(2x + 1)$

140. $3x^3 + x^2 - 27x - 9 = x^2(3x + 1) - 9(3x + 1)$

$= (x^2 - 9)(3x + 1)$

$= (x + 3)(x - 3)(3x + 1)$

141. $(x^2 + 1)^2 - 4x^2 = [(x^2 + 1) + 2x][(x^2 + 1) - 2x]$

$= (x^2 + 2x + 1)(x^2 - 2x + 1)$

$= (x + 1)^2(x - 1)^2$

142. $(x^2 + 8)^2 - 36x^2 = (x^2 + 8)^2 - (6x)^2$

$= [(x^2 + 8) - 6x][(x^2 + 8) + 6x]$

$= (x^2 - 6x + 8)(x^2 + 6x + 8)$

$= (x - 4)(x - 2)(x + 4)(x + 2)$

143. $3t^3 + 24 = 3(t^3 + 8) = 3(t + 2)(t^2 - 2t + 4)$

144. $4x^3 - 32 = 4(x^3 - 8) = 4(x - 2)(x^2 + 2x + 4)$

145. $4x(2x - 1) + 2(2x - 1)^2 = 2(2x - 1)(2x + (2x - 1))$

$= 2(2x - 1)(4x - 1)$

146. $5(3 - 4x)^2 - 8(3 - 4x)(5x - 1)$

$= (3 - 4x)[5(3 - 4x) - 8(5x - 1)]$

$= (3 - 4x)[15 - 20x - 40x + 8]$

$= (3 - 4x)(23 - 60x)$

147. $2(x+1)(x-3)^2 - 3(x+1)^2(x-3)$
$= (x+1)(x-3)[2(x-3) - 3(x+1)]$
$= (x+1)(x-3)[2x-6-3x-3]$
$= (x+1)(x-3)(-x-9)$
$= -(x+1)(x-3)(x+9)$

148. $7(3x+2)^2(1-x)^2 + (3x+2)(1-x)^3$
$= (3x+2)(1-x)^2[7(3x+2)+(1-x)]$
$= (3x+2)(1-x)^2(21x+14+1-x)$
$= (3x+2)(1-x)^2(20x+15)$
$= 5(3x+2)(1-x)^2(4x+3)$

149. (a) $1000(1+r^2) = 1000(1+2r+r^2)$
$= 1000r^2 + 2000r + 1000$

(b)

r	1%	$1\frac{1}{2}$%	2%	$2\frac{1}{2}$%	3%
$1000(1+r)^2$	1020.10	1030.23	1040.40	1050.63	1060.90

(c) The amount increases as r increases.

150. $V = l \cdot w \cdot h = (26-2x)(18-2x)(x)$
$= 2(13-x)(2)(9-x)(x)$
$= 4x(-1)(x-13)(-1)(x-9)$
$= 4x(x-13)(x-9)$

When $x = 1$: $V = 4(1)(-12)(-8) = 384$ cubic inches.
When $x = 2$: $V = 4(2)(-11)(-7) = 616$ cubic inches.
When $x = 3$: $V = 4(3)(-10)(-6) = 720$ cubic inches.

151. (a) $T = R + B = 1.1x + (0.0475x^2 - 0.001x + 0.23)$
$= 0.0475x^2 + 1.099x + 0.23$

(b)

x mi/hr	30	40	55
T feet	75.95	120.19	204.36

(c) As the speed x increases, the total stopping distance increases.

152. (a) Estimates will vary. Actual safe loads for $x = 12$:

$S_6 = (0.06(12)^2 - 2.42(12) + 38.71)^2$
$= 335.2561$ (using a calculator)

$S_8 = (0.08(12)^2 - 3.30(12) + 51.93)^2$
$= 568.8225$ (using a calculator)

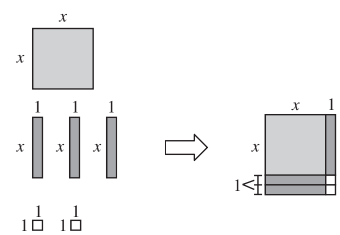
Difference in safe loads $= 568.8225 - 335.2561$
≈ 233.6 pounds

(b) The difference in safe loads decreases in magnitude as the span increases.

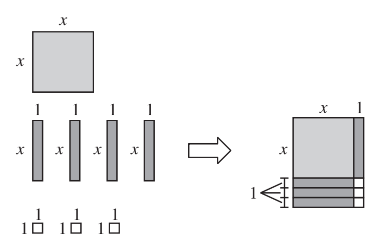
153. $a^2 - b^2 = (a+b)(a-b)$
Matches model (a).

154. $ab + a + b + 1 = (a+1)(b+1)$
Matches model (b).

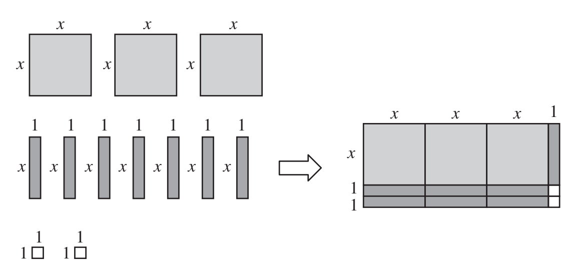
155. $x^2 + 3x + 2 = (x+2)(x+1)$



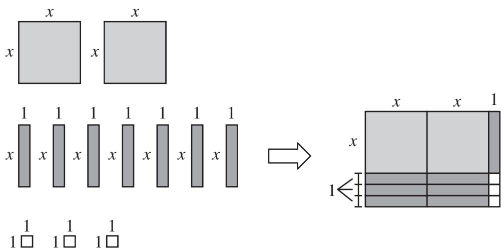
156. $x^2 + 4x + 3 = (x+3)(x+1)$



157. $3x^2 + 7x + 2 = (3x+1)(x+2)$



158. $2x^2 + 7x + 3 = (2x+1)(x+3)$



159. $A = \pi(r+2)^2 - \pi r^2 = \pi\left[(r+2)^2 - r^2\right]$

$= \pi\left[r^2 + 4r + 4 - r^2\right] = \pi(4r+4)$

$= 4\pi(r+1)$

160. Area $= \frac{1}{2}(x+3)\left(\frac{5}{4}\right)(x+3) - \frac{1}{2}(5)(4)$

$= \frac{5}{8}\left(x^2 + 6x + 9\right) - 10$

$= \frac{5}{8}\left(x^2 + 6x + 9 - \frac{80}{5}\right)$

$= \frac{5}{8}\left(x^2 + 6x + 9 - 16\right)$

$= \frac{5}{8}\left(x^2 + 6x - 7\right)$

$= \frac{5}{8}(x+7)(x-1)$

161. $x^4(4)(2x+1)^3(2x) + (2x+1)^4(4x^3)$

$= 4x^3(2x+1)^3\left[2x^2 + (2x+1)\right]$

$= 4x^3(2x+1)^3\left(2x^2 + 2x + 1\right)$

162. $x^3(3)\left(x^2+1\right)^2(2x) + \left(x^2+1\right)^3\left(3x^2\right)$

$= 3x^2\left(x^2+1\right)^2\left[2x^2 + \left(x^2+1\right)\right]$

$= 3x^2\left(x^2+1\right)^2\left(3x^2+1\right)$

163. $(2x-5)^4(3)(5x-4)^2(5) + (5x-4)^3(4)(2x-5)^3(2)$

$= (2x-5)^3(5x-4)^2\left[15(2x-5) + 8(5x-4)\right]$

$= (2x-5)^3(5x-4)^2\left[30x - 75 + 40x - 32\right]$

$= (2x-5)^3(5x-4)^2(70x - 107)$

164. $\left(x^2-5\right)^3(2)(4x+3)(4) + (4x+3)^2(3)\left(x^2-5\right)^2\left(x^2\right)$

$= \left(x^2-5\right)^2(4x+3)\left[8\left(x^2-5\right) + 3x^2(4x+3)\right]$

$= \left(x^2-5\right)^2(4x+3)\left(12x^3 + 17x^2 - 40\right)$

165. $\dfrac{4(2x+3)^2 - (4x-1)(2)(2x+3)(2)}{(2x+3)^4} = \dfrac{4(2x+3)\left[(2x+3) - (4x-1)\right]}{(2x+3)^4}$

$= \dfrac{4(2x+3)\left[-2x+4\right]}{(2x+3)^4}$

$= \dfrac{-8(2x+3)(x-2)}{(2x+3)^4}$

$= \dfrac{-8(x-2)}{(2x+3)^3}$

166. $\dfrac{3(5x-1)^3 - (3x+1)(3)(5x-1)^2(5)}{(5x-1)^6} = \dfrac{3(5x-1)^2\left[(5x-1) - (5)(3x+1)\right]}{(5x-1)^6}$

$= \dfrac{3(5x-1)^2\left[5x - 1 - 15x - 5\right]}{(5x-1)^6}$

$= \dfrac{3(5x-1)^2(-10x-6)}{(5x-1)^6}$

$= \dfrac{-6(5x-1)^2(5x+3)}{(5x-1)^6}$

$= -\dfrac{6(5x+3)}{(5x-1)^4}$

167. For $x^2 + bx - 15 = (x + m)(x + n)$ to be factorable, b must equal $m + n$ where $mn = -15$.

Factors of -15	Sum of factors
$(15)(-1)$	$15 + (-1) = 14$
$(-15)(1)$	$-15 + 1 = -14$
$(3)(-5)$	$3 + (-5) = -2$
$(-3)(5)$	$-3 + 5 = 2$

The possible b-value are $14,\ -14,\ -2,$ or $2.$

168. For $x^2 + bx - 12$ to be factorable, b must equal $m + n$ where $mn = -12$.

Factors of -12	Sum of factors
$(1)(-12)$	$1 - 12 = -11$
$(-1)(12)$	$-1 + 12 = 11$
$(2)(-6)$	$2 - 6 = -4$
$(-2)(6)$	$-2 + 6 = 4$
$(3)(-4)$	$3 - 4 = -1$
$(-3)(4)$	$-3 + 4 = 1$

The possible b-values are
$11,\ -11,\ 4,\ -4,\ 1,$ or $-1.$

169. For $x^2 + bx + 50 = (x + m)(x + n)$ to be factorable, b must equal $m + n$ where $mn = 50$.

Factors of 50	Sum of factors
$(50)(1)$	51
$(-50)(-1)$	–51
$(25)(2)$	27
$(-25)(-2)$	–27
$(10)(5)$	15
$(-10)(-5)$	–15

The possible b-values are
$51,\ -51,\ 27,\ -27,\ 15,$ or $-15.$

170. For $x^2 + bx + 24$ to be factorable, b must be equal to $m + n$ where $mn = 24$.

Factors of 24	Sum of factors
$(24)(1)$	25
$(-24)(-1)$	–25
$(12)(2)$	14
$(-12)(-2)$	–14
$(8)(3)$	11
$(-8)(-3)$	–11
$(6)(4)$	10
$(-6)(-4)$	–10

The possible b-values are
$25,\ -25,\ 14,\ -14,\ 11,\ -11,\ 10,$ or $-10.$

171. For $x^2 + x + c$ to be factorable, the factors of c must add up to 1.

Possible c-values	c	Factors of c that add up to 1
-2	-2	$(2)(-1) = -2$ and $2 + (-1) = 1$
-6	-6	$(3)(-2) = -6$ and $3 + (-2) = 1$
-20	-20	$(5)(-4) = -20$ and $5 + (-4) = 1$

These are a few possible c-values. There are many correct answers.

If $c = -2$: $x^2 + x - 2 = (x + 2)(x - 1)$

If $c = -6$: $x^2 + x - 6 = (x + 3)(x - 2)$

If $c = -20$: $x^2 + x - 20 = (x + 5)(x - 4)$

172. For $x^2 - 9x + c$ to be factorable, the factors of c must add up to -9.

Possible c-values	c	Factors of c that add up to -9
8	8	$(-1)(-8) = 8$ and $-1 + (-8) = -9$
14	14	$(-2)(-7) = 14$ and $-2 + (-7) = -9$
18	18	$(-3)(-6) = 18$ and $-3 + (-6) = -9$

These are a few possible c-values. There are many correct answers.

If $c = 8$: $x^2 - 9x + 8 = (x - 1)(x - 8)$

If $c = 14$: $x^2 - 9x + 14 = (x - 2)(x - 7)$

If $c = 18$: $x^2 - 9x + 18 = (x - 3)(x - 6)$

173. For $2x^2 + 5x + c$ to be factorable, the factors of $2c$ must add up to 5.

Possible c-values	2c	Factors of 2c that add up to 5
2	4	$(1)(4) = 4$ and $1 + 4 = 5$
3	6	$(2)(3) = 6$ and $2 + 3 = 5$
–3	–6	$(6)(-1) = -6$ and $6 + (-1) = 5$
–7	–14	$(7)(-2) = -14$ and $7 + (-2) = 5$
–12	–24	$(8)(-3) = -24$ and $8 + (-3) = 5$

These are a few possible c-values. There are many correct answers.

If $c = 2$: $2x^2 + 5x + 2 = (2x + 1)(x + 2)$

If $c = 3$: $2x^2 + 5x + 3 = (2x + 3)(x + 1)$

If $c = -3$: $2x^2 + 5x - 3 = (2x - 1)(x + 3)$

If $c = -7$: $2x^2 + 5x - 7 = (2x + 7)(x - 1)$

If $c = -12$: $2x^2 + 5x - 12 = (2x - 3)(x + 4)$

174. For $3x^2 - 10x + c$ to be factorable, of $3c$ must add up to -10.

Possible c-values	$3c$	Factors of $3c$ that must add up to -10
3	9	$(-9)(-1) = 9$ and $-9 - 1 = -10$
7	21	$(-3)(-7) = 21$ and $-3 - 7 = -10$
8	24	$(-4)(-6) = 24$ and $-4 - 6 = -10$
–8	–24	$(2)(-12) = -24$ and $-12 + 2 = -10$

Other c-values are possible. The above values yield the following factorizations.
There are many correct answers.
If $c = 3$: $3x^2 - 10x + 3 = (3x - 1)(x - 3)$
If $c = 7$: $3x^2 - 10x + 7 = (3x - 7)(x - 1)$
If $c = 8$: $3x^2 - 10x + 8 = (3x - 4)(x - 2)$
If $c = -8$: $3x^2 - 10x - 8 = (3x + 2)(x - 4)$

175. $V = \pi R^2 h - \pi r^2 h$

(a) $V = \pi h\left[R^2 - r^2\right] = \pi h(R - r)(R + r)$

(b) The average radius is $\dfrac{R + r}{2}$. The thickness of the shell is $R - r$. Therefore,

$$V = \pi h(R + r)(R - r) = 2\pi\left(\frac{R + r}{2}\right)(R - r)h$$
$$= 2\pi \text{ (average radius)(thickness)}h.$$

176. $kQx - kx^2 = kx(Q - x)$

177. False. The product of the two binomials is not always a second-degree polynomial. For instance, $(x^2 + 2)(x^2 - 3) = x^4 - x^2 - 6$ is a fourth-degree polynomial.

178. False. The product of the two binomials is not always a trinomial. For example, $(x + 2)(x - 2) = x^2 - 4$.

179. False. For example, $(x^2 - 3x + 1) + (-x^2 + x - 2) = -2x - 1$, which is a first-degree polynomial.

180. False. The sum of a third-degree polynomial and a fourth-degree polynomial will always be a fourth-degree polynomial.

181. False. $(3x - 6)(x + 1) = 3(x - 2)(x + 1)$

182. (a) The box could have been created by cutting squares of length x from the corners of the piece of cardboard. The original dimensions of the cardboard are 52 inches × 42 inches.

(b) The degree is 3 because the volume is length × width × height, and each dimension contains an x.

(c) $x(52 - 2x)(42 - 2x) = 4x(26 - x)(21 - x)$
The possible values of x are $0 < x < 21$.

183. If two polynomials have degree m and n, then their product is degree $m + n$.

184. $(x + y)^2 \neq x^2 + y^2$ because you cannot just distribute the squares. You have to use the FOIL Method.
$(x + y)^2 = x^2 + 2xy + y^2 \neq x^2 + y^2$

185. To cube a binomial difference, cube the first term. Next, subtract 3 times the square of the first term times the second term. Next, add 3 times the first term times the square of the second term. Finally, subtract the cube of the second term.
$(x - y)^3 = x^3 - 3x^2y + 3xy^2 - y^3$

186. A polynomial is in factored form when each of its factors is prime (it cannot be factored any further using integer coefficients).

187. $9x^2 - 9x - 54 = 9(x^2 - x - 6) = 9(x + 2)(x - 3)$

The error in the problem in the book was that 3 was factored out of the first binomial but not out of the second binomial.
$(3x+6)(3x-9) = 3(x+2)(3)(x-3) = 9(x+2)(x-3)$

188. Answers will vary. Sample answer: $x^2 - 3$

189. $x^{2n} - y^{2n} = (x^n + y^n)(x^n - y^n)$

190. $x^{3n} + y^{3n} = (x^n + y^n)(x^{2n} - x^n y^n + y^{2n})$

Section P.4 Rational Expressions

1. domain
2. rational expression
3. complex fractions
4. lesser
5. A rational expression is in simplest form when its numerator and denominator have no common factors aside from ± 1.
6. Values that make the denominator equal to zero are excluded from the domain of a rational expression.
7. The domain of the polynomial $x^2 + 7x - 3$ is the set of all real numbers.
8. The domain of the polynomial $6x^2 - x - 10$ is the set of all real numbers.
9. The domain of the polynomial $5x^2 + 1, x > 0,$ is the set of all positive real numbers.
10. The domain of the polynomial $9x - 4, x \le 0,$ is the set of all negative real numbers.
11. The domain of the expression $\dfrac{x}{x+2}$ is the set of all real numbers except $x = -2$, which would result in division by zero, which is undefined.
12. The domain of the expression $\dfrac{1-x}{4-x}$ is the set of all real numbers except $x = 4$, which would result in division by zero, which is undefined.
13. The domain of the expression $\dfrac{x^2 + 3x}{x^2 + 14x + 49} = \dfrac{x(x+3)}{(x+7)^2}$ is the set of all real numbers except $x = -7$, which would result in division by zero, which is undefined.
14. The domain of the expression $\dfrac{x^2 + 2x - 8}{x^2 - 4} = \dfrac{(x+4)(x-2)}{(x+2)(x-2)} = \dfrac{x+4}{x+2}, x \ne 2,$ is the set of all real numbers except $x = \pm 2$, which would result in division by zero, which is undefined.
15. The domain of the expression $\dfrac{x^2 - 2x - 3}{9x^2 - 1} = \dfrac{(x-3)(x+1)}{(3x+1)(3x-1)}$ is the set of all real numbers except $x = \pm\dfrac{1}{3}$, which would result in division by zero, which is undefined.
16. The domain of the expression $\dfrac{x^2 + 6x + 9}{x^2 - 10x + 25} = \dfrac{(x+3)^2}{(x-5)^2}$ is the set of all real numbers except $x = 5$, which would result in division by zero, which is undefined.
17. The domain of the radical expression $\sqrt{x+10}$ is the set of all real numbers greater than or equal to -10, because the square root of a negative number is not a real number.
18. The domain of the radical expression $\sqrt{x-7}$ is the set of all real numbers greater than or equal to 7 because the square root of a negative number is not a real number.
19. Because $12 - 3x \ge 0 \Rightarrow x \le 4$, the domain of the radical expression $\sqrt{12 - 3x}$ is the set of all real numbers less than or equal to 4.
20. Because $6 - 4x \ge 0 \Rightarrow x \le \dfrac{3}{2}$, the domain of the radical expression $\sqrt{6 - 4x}$ is the set of all real numbers less than or equal to $\dfrac{3}{2}$.
21. Because $x + 1 > 0 \Rightarrow x > -1$, the domain of the radical expression $\dfrac{1}{\sqrt{x+1}}$ is the set of all real numbers greater than -1.
22. Because $x - 5 > 0 \Rightarrow x > 5$, the domain of the radical expression $\dfrac{1}{\sqrt{x-5}}$ is the set of all real numbers greater than 5.

23. $\dfrac{5}{2x} = \dfrac{5(3x)}{(2x)(3x)} = \dfrac{5(3x)}{6x^2},\ x \neq 0$

Missing factor: $3x$

24. $\dfrac{2}{3x^2} = \dfrac{2(x^2)}{(3x^2)(x^2)} = \dfrac{2(x^2)}{3x^4},\ x \neq 0$

Missing factor: x^2

25. $\dfrac{3}{4} = \dfrac{3(x+1)}{4(x+1)},\ x \neq 1$

Missing factor: $(x+1)$

26. $\dfrac{2}{5} = \dfrac{2(x-3)}{5(x-3)},\ x \neq 3$

Missing factor: $x-3$

27. $\dfrac{x-1}{4(x+2)} = \dfrac{(x-1)(x+2)}{4(x+2)(x+2)}$

$= \dfrac{4(x-1)(x+2)}{4(x+2)^2},\ x \neq -2$

Missing factor: $x+2$

28. $\dfrac{x+3}{2(x-1)} = \dfrac{(x+3)(x-1)}{2(x-1)(x-1)}$

$= \dfrac{(x+3)(x-1)}{2(x-1)^2},\ x \neq 1$

Missing factor: $x-1$

29. $\dfrac{15x^2}{10x} = \dfrac{5x(3x)}{5x(2)} = \dfrac{3x}{2},\ x \neq 0$

30. $\dfrac{18y^2}{60y^5} = \dfrac{6y^2(3)}{6y^2(10y^3)} = \dfrac{3}{10y^3},\ y \neq 0$

31. $\dfrac{3xy}{x^2y + x^2} = \dfrac{3xy}{x^2(y+1)} = \dfrac{3y}{x(y+1)}$

32. $\dfrac{2x^2y}{xy-y} = \dfrac{y(2x^2)}{y(x-1)} = \dfrac{2x^2}{x-1},\ y \neq 0$

33. $\dfrac{4y-8y^2}{10y-5} = \dfrac{4y(1-2y)}{5(2y-1)}$

$= \dfrac{-4y(2y-1)}{5(2y-1)} = -\dfrac{4y}{5},\ y \neq \dfrac{1}{2}$

34. $\dfrac{9x^2+9x}{2x+2} = \dfrac{9x(x+1)}{2(x+1)} = \dfrac{9x}{2},\ x \neq -1$

35. $\dfrac{x-5}{10-2x} = \dfrac{x-5}{-2(x-5)} = -\dfrac{1}{2},\ x \neq 5$

36. $\dfrac{12-4x}{x-3} = \dfrac{-4(x-3)}{x-3} = -4,\ x \neq 3$

37. $\dfrac{y^2-16}{y+4} = \dfrac{(y+4)(y-4)}{y+4} = y-4,\ y \neq -4$

38. $\dfrac{x^2-25}{5-x} = \dfrac{(x+5)(x-5)}{-1(x-5)} = -(x+5),\ x \neq 5$

39. $\dfrac{x^3+5x^2+6x}{x^2-4} = \dfrac{x(x+2)(x+3)}{(x+2)(x-2)}$

$= \dfrac{x(x+3)}{x-2},\ x \neq -2$

40. $\dfrac{x^2+8x-20}{x^2+11x+10} = \dfrac{(x+10)(x-2)}{(x+10)(x+1)}$

$= \dfrac{x-2}{x+1},\ x \neq -10$

41. $\dfrac{y^2-7y+12}{y^2+3y-18} = \dfrac{(y-3)(y-4)}{(y+6)(y-3)} = \dfrac{y-4}{y+6},\ y \neq 3$

42. $\dfrac{-10-x}{x^2+11x+10} = \dfrac{-(x+10)}{(x+10)(x+1)}$

$= -\dfrac{1}{x+1},\ x \neq -10$

43. $\dfrac{2-x+2x^2-x^3}{x-2} = \dfrac{(2-x)+x^2(2-x)}{-(2-x)}$

$= \dfrac{(2-x)(1+x^2)}{-(2-x)}$

$= -(1+x^2),\ x \neq 2$

44. $\dfrac{x^2-9}{x^3+x^2-9x-9} = \dfrac{x^2-9}{(x^2-9)(x+1)}$

$= \dfrac{1}{x+1},\ x \neq \pm 3$

45. $\dfrac{z^3-8}{z^2+2z+4} = \dfrac{(z-2)(z^2+2z+4)}{z^2+2z+4} = z-2$

46. $\dfrac{y^3-2y^2-3y}{y^3+1} = \dfrac{y(y-3)(y+1)}{(y+1)(y^2-y+1)}$

$= \dfrac{y(y-3)}{y^2-y+1},\ y \neq -1$

47.

x	-4	-3	-2	-1	0	1	2
$\dfrac{x^2+2x-3}{x-1}$	-1	0	1	2	3	Undef.	5
$x+3$	-1	0	1	2	3	4	5

The expressions are equivalent except at $x = 1$. In fact, $\dfrac{x^2+2x-3}{x-1} = \dfrac{(x+3)(x-1)}{x-1} = x+3,\ x \neq 1$.

48.

x	0	1	2	3	4	5	6
$\dfrac{x-3}{x^2-x-6}$	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$	Undef.	$\frac{1}{6}$	$\frac{1}{7}$	$\frac{1}{8}$
$\dfrac{1}{x+2}$	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{5}$	$\frac{1}{6}$	$\frac{1}{7}$	$\frac{1}{8}$

The expressions are equivalent except at $x = 3$.

In fact, $\dfrac{x-3}{x^2-x-6} = \dfrac{x-3}{(x-3)(x+2)} = \dfrac{1}{x+2},\ x \neq 3$.

49. $\dfrac{5}{x-1}\cdot\dfrac{x-1}{25(x-2)} = \dfrac{1}{5(x-2)},\ x \neq 1$

50. $\dfrac{x+13}{x^3(3-x)}\cdot\dfrac{x(x-3)}{5} = \dfrac{x+13}{x^3(x-3)(-1)}\cdot\dfrac{x(x-3)}{5}$

$= \dfrac{x+13}{-5x^2} = -\dfrac{x+13}{5x^2},\ x \neq 3$

51. $\dfrac{r}{1-r} \div \dfrac{r^2}{r^2-1} = -\dfrac{r}{r-1}\cdot\dfrac{r^2-1}{r^2}$

$= -\dfrac{r}{r-1}\cdot\dfrac{(r+1)(r-1)}{r^2}$

$= -\dfrac{r+1}{r},\ r \neq -1, 1$

52. $\dfrac{4y-16}{5y+15} \div \dfrac{4-y}{2y+6} = \dfrac{4(y-4)}{5(y+3)}\cdot\dfrac{2(y+3)}{-(y-4)}$

$= \dfrac{8}{-5} = -\dfrac{8}{5},\ y \neq -3, 4$

53. $\dfrac{t^2-t-6}{t^2+6t+9}\cdot\dfrac{t+3}{t^2-4} = \dfrac{(t-3)(t+2)(t+3)}{(t+3)^2(t+2)(t-2)}$

$= \dfrac{t-3}{(t+3)(t-2)},\ t \neq -2$

54. $\dfrac{y^3-8}{2y^3}\cdot\dfrac{4y}{y^2-5y+6} = \dfrac{(y-2)(y^2+2y+4)}{2y^3}\cdot\dfrac{4y}{(y-2)(y-3)}$

$= \dfrac{2(y^2+2y+4)}{y^2(y-3)},\ y \neq 2$

55. $\dfrac{3(x+y)}{4} \div \dfrac{x+y}{2} = \dfrac{3(x+y)}{4}\cdot\dfrac{2}{x+y} = \dfrac{3}{2},\ x \neq -y$

56. $\dfrac{2x-y}{y-1} \div \dfrac{3y-6x}{y^2-6y+5} = \dfrac{2x-y}{y-1}\cdot\dfrac{y^2-6y+5}{-6x+3y}$

$= \dfrac{2x-y}{y-1}\cdot\dfrac{(y-5)(y-1)}{-3(2x-y)}$

$= -\dfrac{y-5}{3},\ y \neq 1, 5, 2x$

57. $\dfrac{\pi r^2}{(2r)^2} = \dfrac{\pi r^2}{4r^2} = \dfrac{\pi}{4}$

58. Area of shaded portion: $\left(\dfrac{x+5}{2}\right)^2 = \dfrac{(x+5)^2}{4}$

Area of total figure: $(2x+3)(x+5)$

Ratio: $\dfrac{(x+5)^2/4}{(2x+3)(x+5)} = \dfrac{(x+5)/4}{(2x+3)}$

$= \dfrac{x+5}{4(2x+3)},\ x \neq -5$

59. $\dfrac{5}{x-1} + \dfrac{x}{x-1} = \dfrac{5+x}{x-1} = \dfrac{x+5}{x-1}$

60. $\dfrac{2x-1}{x+3} - \dfrac{1-x}{x+3} = \dfrac{2x-1-1+x}{x+3} = \dfrac{3x-2}{x+3}$

61. $\dfrac{6}{2x+1} - \dfrac{x}{x+3} = \dfrac{6(x+3)-x(2x+1)}{(2x+1)(x+3)}$

$= \dfrac{6x+18-2x^2-x}{(2x+1)(x+3)}$

$= \dfrac{-2x^2+5x+18}{(2x+1)(x+3)}$

$= -\dfrac{2x^2-5x-18}{(2x+1)(x+3)}$

62. $\dfrac{3}{x-1}+\dfrac{5x}{3x+4}=\dfrac{3(3x+4)+5x(x-1)}{(x-1)(3x+4)}$

$=\dfrac{5x^2+4x+12}{(x-1)(3x+4)}$

63. $\dfrac{3}{x-2}+\dfrac{5}{2-x}=\dfrac{3}{x-2}-\dfrac{5}{x-2}=-\dfrac{2}{x-2}$

64. $\dfrac{2x}{x-5}-\dfrac{5}{5-x}=\dfrac{2x}{x-5}-\dfrac{5(-1)}{(-1)(5-x)}$

$=\dfrac{2x}{x-5}-\dfrac{-5}{x-5}=\dfrac{2x+5}{x-5}$

65. $\dfrac{1}{x^2-x-2}-\dfrac{x}{x^2-5x+6}=\dfrac{1}{(x-2)(x+1)}-\dfrac{x}{(x-2)(x-3)}$

$=\dfrac{(x-3)-x(x+1)}{(x+1)(x-2)(x-3)}$

$=\dfrac{-x^2-3}{(x+1)(x-2)(x-3)}$

$=-\dfrac{x^2+3}{(x+1)(x-2)(x-3)}$

66. $\dfrac{2}{x^2-x-2}+\dfrac{10}{x^2+2x-8}=\dfrac{2}{(x-2)(x+1)}+\dfrac{10}{(x+4)(x-2)}$

$=\dfrac{2(x+4)}{(x-2)(x+1)(x+4)}$

$+\dfrac{10(x+1)}{(x-2)(x+1)(x+4)}$

$=\dfrac{2x+8+10x+10}{(x-2)(x+1)(x+4)}$

$=\dfrac{12x+18}{(x-2)(x+1)(x+4)}$

$=\dfrac{6(2x+3)}{(x-2)(x+1)(x+4)}$

67. $\dfrac{2}{x+1}+\dfrac{2}{x-1}+\dfrac{1}{x^2-1}=\dfrac{2}{x+1}+\dfrac{2}{x-1}+\dfrac{1}{(x+1)(x-1)}$

$=\dfrac{2(x-1)}{(x+1)(x-1)}+\dfrac{2(x+1)}{(x+1)(x-1)}+\dfrac{1}{(x+1)(x-1)}$

$=\dfrac{2x-2+2x+2+1}{(x+1)(x-1)}$

$=\dfrac{4x+1}{(x+1)(x-1)}$

68. $-\dfrac{1}{x}+\dfrac{2}{x^2+1}-\dfrac{1}{x^3+x}=\dfrac{-(x^2+1)}{x(x^2+1)}+\dfrac{2x}{x(x^2+1)}-\dfrac{1}{x(x^2+1)}$

$=\dfrac{-x^2-1+2x-1}{x(x^2+1)}$

$=-\dfrac{x^2-2x+2}{x(x^2+1)}$

69. $\dfrac{1}{x^2+x}-\dfrac{6}{x^2}+\dfrac{5}{x+1}=\dfrac{1}{x(x+1)}-\dfrac{6}{x^2}+\dfrac{5}{x+1}$

$=\dfrac{x-6(x+1)+5x^2}{x^2(x+1)}$

$=\dfrac{x-6x-6+5x^2}{x^2(x+1)}$

$=\dfrac{5x^2-5x-6}{x^2(x+1)}$

70. $\dfrac{3}{x-3}-\dfrac{x}{x^2-9}-\dfrac{2}{x}=\dfrac{3}{x-3}-\dfrac{x}{(x-3)(x+3)}-\dfrac{2}{x}$

$=\dfrac{3x(x+3)-x(x)-2(x+3)(x-3)}{x(x-3)(x+3)}$

$=\dfrac{3x^2+9x-x^2-2x^2+18}{x(x-3)(x+3)}$

$=\dfrac{9x+18}{x(x-3)(x+3)}$

71. $\dfrac{\left(\dfrac{x}{2}-1\right)}{(x-2)} = \dfrac{\left(\dfrac{x}{2}-\dfrac{2}{2}\right)}{\left(\dfrac{x-2}{1}\right)}$

$= \dfrac{x-2}{2}\cdot\dfrac{1}{x-2} = \dfrac{1}{2},\ x \neq 2$

72. $\dfrac{(x-4)}{\left(\dfrac{x}{4}-\dfrac{4}{x}\right)} = \dfrac{\left(\dfrac{x-4}{1}\right)}{\left(\dfrac{x^2}{4x}-\dfrac{16}{4x}\right)} = \dfrac{\left(\dfrac{x-4}{1}\right)}{\left(\dfrac{x^2-16}{4x}\right)}$

$= \dfrac{x-4}{1}\cdot\dfrac{4x}{x^2-16}$

$= \dfrac{x-4}{1}\cdot\dfrac{4x}{(x+4)(x-4)}$

$= \dfrac{4x}{x+4},\ x \neq 0,\ 4$

73. $\dfrac{\left[\dfrac{x^2}{(x+1)^2}\right]}{\left[\dfrac{x}{(x+1)^3}\right]} = \dfrac{x^2}{(x+1)^2}\cdot\dfrac{(x+1)^3}{x}$

$= \dfrac{x(x+1)}{1}$

$= x^2 + x,\ x \neq 0,\ -1$

74. $\dfrac{\left(\dfrac{x^2-1}{x}\right)}{\left[\dfrac{(x-1)^2}{x}\right]} = \dfrac{x^2-1}{x}\cdot\dfrac{x}{(x-1)^2}$

$= \dfrac{(x+1)(x-1)}{(x-1)^2}$

$= \dfrac{x+1}{x-1},\ x \neq 0$

75. $\dfrac{\left[\dfrac{1}{(x+h)^2}-\dfrac{1}{x^2}\right]}{h} = \dfrac{\left[\dfrac{1}{(x+h)^2}-\dfrac{1}{x^2}\right]}{h}\cdot\dfrac{x^2(x+h)^2}{x^2(x+h)^2}$

$= \dfrac{x^2-(x+h)^2}{hx^2(x+h)^2}$

$= \dfrac{x^2-(x^2+2xh+h^2)}{hx^2(x+h)^2}$

$= \dfrac{-h(2x+h)}{hx^2(x+h)^2}$

$= -\dfrac{2x+h}{x^2(x+h)^2},\ h \neq 0$

76. $\dfrac{\left(\dfrac{x+h}{x+h+1}-\dfrac{x}{x+1}\right)}{h}$

$= \dfrac{\left(\dfrac{(x+h)(x+1)}{(x+h+1)(x+1)}-\dfrac{x(x+h+1)}{(x+h+1)(x+1)}\right)}{\dfrac{h}{1}}$

$= \left(\dfrac{(x+h)(x+1)}{(x+h+1)(x+1)}-\dfrac{x(x+h+1)}{(x+h+1)(x+1)}\right)\cdot\dfrac{1}{h}$

$= \left(\dfrac{x^2+x+hx+h-x^2-xh-x}{(x+h+1)(x+1)}\right)\cdot\dfrac{1}{h}$

$= \dfrac{h}{(x+h+1)(x+1)}\cdot\dfrac{1}{h} = \dfrac{1}{(x+h+1)(x+1)},\ h \neq 0$

77. $\dfrac{\left(\sqrt{x}-\dfrac{1}{2\sqrt{x}}\right)}{\sqrt{x}} = \dfrac{\left(\sqrt{x}-\dfrac{1}{2\sqrt{x}}\right)}{\sqrt{x}}\cdot\dfrac{2\sqrt{x}}{2\sqrt{x}}$

$= \dfrac{2x-1}{2x},\ x > 0$

78. $\dfrac{\left(\dfrac{t^2}{\sqrt{t^2-1}}-\sqrt{t^2-1}\right)}{t^2} = \dfrac{\left(\dfrac{t^2-(t^2-1)}{\sqrt{t^2-1}}\right)}{t^2}$

$= \dfrac{\dfrac{1}{\sqrt{t^2-1}}}{t^2}$

$= \dfrac{1}{\sqrt{t^2-1}}\cdot\dfrac{1}{t^2}$

$= \dfrac{1}{t^2\sqrt{t^2-1}}$

79. $x^5 - 2x^{-2} = x^{-2}(x^7-2) = \dfrac{x^7-2}{x^2}$

80. $x^5 - 5x^{-3} = x^{-3}(x^8-5) = \dfrac{x^8-5}{x^3}$

81. $x^2(x^2+1)^{-5} - (x^2+1)^{-4} = (x^2+1)^{-5}\left[x^2-(x^2+1)\right]$

$= -\dfrac{1}{(x^2+1)^5}$

82. $2x(x-5)^{-3} - 4x^2(x-5)^{-4} = 2x(x-5)^{-4}(x-5-2x)$

$= \dfrac{2x(-x-5)}{(x-5)^4} = \dfrac{-2x(x+5)}{(x-5)^4}$

83. $2x^2(x-1)^{1/2} - 5(x-1)^{-1/2} = (x-1)^{-1/2}\left(2x^2(x-1)-5\right)$

$= \dfrac{2x^3-2x^2-5}{(x-1)^{1/2}}$

84. $$4x^3(2x-1)^{3/2} - 2x(2x-1)^{-1/2} = 2x(2x-1)^{-1/2}\left(2x^2(2x-1)^2 - 1\right)$$
$$= 2x(2x-1)^{-1/2}\left(2x^2(4x^2-4x+1)-1\right)$$
$$= 2x(2x-1)^{-1/2}\left(8x^4 - 8x^3 + 2x^2 - 1\right)$$
$$= \frac{2x\left(8x^4 - 8x^3 + 2x^2 - 1\right)}{(2x-1)^{1/2}}$$

85. $$\frac{2x^{3/2} - x^{-1/2}}{x^2} = \frac{x^{-1/2}(2x^2-1)}{x^2} = \frac{2x^2-1}{x^{5/2}}$$

86. $$\frac{x^2(x^{-1/2}) - 3x^{1/2}(x^2)}{x^4} = \frac{x^{3/2} - 3x^{5/2}}{x^4}$$
$$= \frac{x^{3/2}(1-3x)}{x^4}$$
$$= -\frac{3x-1}{x^{5/2}}$$

87. $$\frac{-x^2(x^2+1)^{-1/2} + 2x(x^2+1)^{-3/2}}{x^3} = \frac{x(x^2+1)^{-3/2}\left[-x(x^2+1)+2\right]}{x^3}$$
$$= \frac{(x^2+1)^{-3/2}\left[-x^3-x+2\right]}{x^2} = \frac{-x^3-x+2}{x^2(x^2+1)^{3/2}}$$
$$= -\frac{(x-1)(x^2+x+2)}{x^2(x^2+1)^{3/2}}$$

88. $$\frac{x^3\left(4x^{-1/2}\right) - 3x^2\left(\frac{8}{3}x^{-3/2}\right)}{x^6} = \frac{4x^{5/2} - 8x^{1/2}}{x^6} = \frac{4x^{1/2}(x^2-2)}{x^6} = \frac{4(x^2-2)}{x^{11/2}}$$

89. $$\frac{(x^2+5)\left(\frac{1}{2}\right)(4x+3)^{-1/2}(4) - (4x+3)^{1/2}(2x)}{(x^2+5)^2} = \frac{2(4x+3)^{-1/2}\left[(x^2+5) - x(4x+3)\right]}{(x^2+5)^2}$$
$$= \frac{2\left(-3x^2-3x+5\right)}{(x^2+5)^2\sqrt{4x+3}}$$
$$= -\frac{2\left(3x^2+3x-5\right)}{(x^2+5)^2\sqrt{4x+3}}$$

90. $$\frac{(2x+1)^{1/2}3(x-5)^2 - (x-5)^3\left(\frac{1}{2}\right)(2x+1)^{-1/2}(2)}{2x+1} = \frac{(x-5)^2(2x+1)^{-1/2}\left[3(2x+1)-(x-5)\right]}{2x+1}$$
$$= \frac{(x-5)^2(6x+3-x+5)}{(2x+1)^{3/2}}$$
$$= \frac{(x-5)^2(5x+8)}{(2x+1)^{3/2}}$$

91. $$\frac{\sqrt{x+4}-\sqrt{x}}{4} = \frac{\sqrt{x+4}-\sqrt{x}}{4}\cdot\frac{\sqrt{x+4}+\sqrt{x}}{\sqrt{x+4}+\sqrt{x}} = \frac{(x+4)-(x)}{4\left(\sqrt{x+4}+\sqrt{x}\right)} = \frac{4}{4\left(\sqrt{x+4}+\sqrt{x}\right)} = \frac{1}{\sqrt{x+4}+\sqrt{x}}$$

92. $$\frac{\sqrt{z-3}-\sqrt{z}}{3} = \frac{\sqrt{z-3}-\sqrt{z}}{3}\cdot\frac{\sqrt{z-3}+\sqrt{z}}{\sqrt{z-3}+\sqrt{z}} = \frac{(z-3)-z}{3\left(\sqrt{z-3}+\sqrt{z}\right)} = \frac{-3}{3\left(\sqrt{z-3}+\sqrt{z}\right)} = \frac{-1}{\sqrt{z-3}+\sqrt{z}}$$

93. $$\frac{\sqrt{x+2}-\sqrt{2}}{x} = \frac{\sqrt{x+2}-\sqrt{2}}{x}\cdot\frac{\sqrt{x+2}+\sqrt{2}}{\sqrt{x+2}+\sqrt{2}} = \frac{(x+2)-2}{x\left(\sqrt{x+2}+\sqrt{2}\right)} = \frac{x}{x\left(\sqrt{x+2}+\sqrt{2}\right)} = \frac{1}{\sqrt{x+2}+\sqrt{2}},\ x\neq 0$$

94. $$\frac{\sqrt{x+5}-\sqrt{5}}{x} = \frac{\sqrt{x+5}-\sqrt{5}}{x}\cdot\frac{\sqrt{x+5}+\sqrt{5}}{\sqrt{x+5}+\sqrt{5}} = \frac{(x+5)-5}{x\left(\sqrt{x+5}+\sqrt{5}\right)} = \frac{x}{x\left(\sqrt{x+5}+\sqrt{5}\right)} = \frac{1}{\sqrt{x+5}+\sqrt{5}},\ x\neq 0$$

95. $$\frac{\sqrt{1-x}-1}{x} = \frac{\sqrt{1-x}-1}{x}\cdot\frac{\sqrt{1-x}+1}{\sqrt{1-x}+1} = \frac{(1-x)-(1)}{x\left(\sqrt{1-x}+1\right)} = \frac{-x}{x\left(\sqrt{1-x}+1\right)} = -\frac{1}{\sqrt{1-x}+1},\ x\neq 0$$

96. $$\frac{\sqrt{4+x}-2}{x} = \frac{\sqrt{4+x}-2}{x}\cdot\frac{\sqrt{4+x}+2}{\sqrt{4+x}+2} = \frac{(4+x)-(4)}{x\left(\sqrt{4+x}+2\right)} = \frac{x}{x\left(\sqrt{4+x}+2\right)} = \frac{1}{\sqrt{4+x}+2},\ x\neq 0$$

97. $$\text{Probability} = \frac{\text{Area shaded rectangle}}{\text{Area large rectangle}} = \frac{x(x/2)}{x(2x+1)} = \frac{x/2}{2x+1}\cdot\frac{2}{2} = \frac{x}{2(2x+1)}$$

98. $$\text{Probability} = \frac{\begin{array}{c}\text{Shaded area}\\ (\text{area of trapezoid})\end{array}}{\begin{array}{c}\text{Total area}\\ (\text{area of triangle})\end{array}} = \frac{\frac{1}{2}\cdot\frac{4}{x}(x+2)(x+x+4)}{\frac{1}{2}(x+4)\left[(x+2)+\frac{4}{x}(x+2)\right]}$$

$$= \frac{\frac{4(x+2)(2x+4)}{x}}{(x+4)(x+2)\left(1+\frac{4}{x}\right)} = \frac{\frac{4\cdot 2(x+2)^2}{x}}{(x+4)(x+2)\left(1+\frac{4}{x}\right)}$$

$$= \frac{8(x+2)^2}{x}\cdot\frac{1}{(x+4)(x+2)\left(1+\frac{4}{x}\right)}$$

$$= \frac{8(x+2)^2}{(x+4)(x+2)(x+4)} = \frac{8(x+2)}{(x+4)^2}$$

In Exercises 99 and 100, use the formula

$$r = \frac{\left[\frac{24(NM-P)}{N}\right]}{\left(P+\frac{NM}{12}\right)}.$$

99. (a) $N = (4)(12) = 48$

$M = \$475$

$P = \$20{,}000$

$$r = \frac{\left[\frac{24((48)(475)-20{,}000)}{48}\right]}{\left(20{,}000+\frac{(48)(475)}{12}\right)}$$

$r \approx 0.0639 \Rightarrow 6.39\%$

(b) $$r = \frac{\left[\frac{24(NM-P)}{N}\right]}{\left(P+\frac{NM}{12}\right)} = \frac{\frac{24(NM-P)}{N}}{\frac{12P+NM}{12}}$$

$$= \frac{24(NM-P)}{N}\cdot\frac{12}{12P+NM} = \frac{288(NM-P)}{N(12P+NM)}$$

$$r = \frac{288(48\cdot 475-20{,}000)}{48(12\cdot 20{,}000+48\cdot 475)} \approx 0.0639 \Rightarrow 6.39\%$$

100. (a) $N = (5)(12) = 60$

$M = \$525$

$P = \$28{,}000$

$$r = \frac{\left[\frac{24((60)(525)-28{,}000)}{60}\right]}{\left(28{,}000+\frac{(60)(525)}{12}\right)}$$

$r \approx 0.0457 \Rightarrow 4.57\%$

(b) $$r = \frac{\left[\frac{24(NM-P)}{N}\right]}{\left(P+\frac{NM}{12}\right)} = \frac{\frac{24(NM-P)}{N}}{\frac{12P+NM}{12}}$$

$$= \frac{24(NM-P)}{N}\cdot\frac{12}{12P+NM} = \frac{288(NM-P)}{N(12P+NM)}$$

$$r = \frac{288(60\cdot 525-28{,}000)}{60(12\cdot 28{,}000+60\cdot 525)} \approx 0.0457 \Rightarrow 4.57\%$$

101. $\text{Copy rate} = \dfrac{50 \text{ pages}}{1 \text{ minute}}$

(a) The time required to copy one page is $\frac{1}{50}$ minute.

(b) The time required to copy x pages is $x\left(\frac{1}{50}\right) = \frac{x}{50}$ minutes.

(c) The time required to copy 120 pages is $120\left(\frac{1}{50}\right) = \frac{12}{5}$ minutes or 2.4 minutes.

102. (a) $R_T = \dfrac{1}{\dfrac{1}{R_1} + \dfrac{1}{R_2} + \dfrac{1}{R_3}}$

$= \dfrac{1}{\dfrac{R_2R_3 + R_1R_3 + R_1R_2}{R_1R_2R_3}}$

$= \dfrac{R_1R_2R_3}{R_2R_3 + R_1R_3 + R_1R_2}$

(b) $R_T = \dfrac{(6)(4)(12)}{(4)(12) + (6)(12) + (6)(4)}$

$= \dfrac{288}{144} = 2$ ohms

103. (a)

Year	2005	2006	2007	2008
Births (in millions)	4.152	4.266	4.341	4.265
Population (in millions)	295.9	298.5	301.2	303.8

Year	2009	2010	2011	2012
Births (in millions)	4.125	4.027	3.971	3.939
Population (in millions)	306.5	309.1	311.7	314.4

(b) The models are close to the actual data.

(c) The ratio of the number of births B to the number of people P is given by

$$\frac{B}{P} = \frac{\dfrac{0.06815t^2 - 0.9865t + 3.948}{0.01753t^2 - 0.2530t + 1}}{2.64t + 282.7} = \frac{0.06815t^2 - 0.9865t + 3.948}{0.01753t^2 - 0.2530t + 1} \cdot \frac{1}{2.64t + 282.7}$$

$$= \frac{0.06815t^2 - 0.9865t + 3.948}{(0.01753t^2 - 0.2530t + 1)(2.64t + 282.7)}.$$

(d)

Year	2005	2006	2007	2008
Ratio	0.0140	0.0143	0.0144	0.0140

Year	2009	2010	2011	2012
Ratio	0.0135	0.0130	0.0127	0.0125

The ratio has remained fairly constant over time.

104. (a)

t	0	2	4	6	8	10
T	75	55.9	48.3	45	43.3	42.3

t	12	14	16	18	20	22
T	41.7	41.3	41.1	40.9	40.7	40.6

(b) $T = 10\left(\dfrac{4t^2 + 16t + 75}{t^2 + 4t + 10}\right)$ appears to be approaching 40.

105. $$\frac{(x+h)^2 - x^2}{h} = \frac{x^2 + 2xh + h^2 - x^2}{h} = \frac{h(2x+h)}{h} = 2x + h,\ h \neq 0$$

106. $$\frac{(x+h)^3 - x^3}{h} = \frac{x^3 + 3x^2h + 3xh^2 + h^3 - x^3}{h} = \frac{h(3x^2 + 3xh + h^2)}{h} = 3x^2 + 3xh + h^2,\ h \neq 0$$

107. $$\frac{\dfrac{1}{(x+h)^2} - \dfrac{1}{x^2}}{h} = \frac{x^2 - (x+h)^2}{hx^2(x+h)^2} = \frac{x^2 - (x^2 + 2xh + h^2)}{hx^2(x+h)^2} = \frac{-2xh - h^2}{hx^2(x+h)^2} = \frac{-2x - h}{x^2(x+h)^2},\ h \neq 0$$

108. $$\frac{\dfrac{1}{2(x+h)} - \dfrac{1}{2x}}{h} = \frac{2x - 2(x+h)}{h(2x)2(x+h)} = \frac{-2h}{h4x(x+h)} = \frac{-1}{2x(x+h)},\ h \neq 0$$

109. $$\frac{4}{n}\left(\frac{n(n+1)(2n+1)}{6}\right) + 2n\left(\frac{4}{n}\right) = \frac{2(n+1)(2n+1)}{3} + 8 = \frac{2(2n^2 + 3n + 1) + 24}{3} = \frac{4n^2 + 6n + 26}{3},\ n \neq 0$$

110. $$9\left(\frac{3}{n}\right)\left(\frac{n(n+1)(2n+1)}{6}\right) - n\left(\frac{3}{n}\right) = \frac{9(n+1)(2n+1)}{2} - 3 = \frac{9(2n^2 + 3n + 1) - 6}{2} = \frac{18n^2 + 27n + 3}{2} = \frac{3}{2}(6n^2 + 9n + 1),\ n \neq 0$$

111. False. For n odd, the domain of $(x^{2n} - 1)/(x^n - 1)$ is all $x \neq 1$, unlike the domain of the right-hand side.

112. False. The domain of the left-hand side is all $x \neq 1$, unlike the domain of the right-hand side, which is all real numbers x.

113. $$\frac{\sqrt{x+h} - \sqrt{x}}{h} = \frac{\sqrt{x+h} - \sqrt{x}}{h} \cdot \frac{\sqrt{x+h} + \sqrt{x}}{\sqrt{x+h} + \sqrt{x}} = \frac{x + h - x}{h\left(\sqrt{x+h} + \sqrt{x}\right)} = \frac{h}{h\left(\sqrt{x+h} + \sqrt{x}\right)} = \frac{1}{\sqrt{x+h} + \sqrt{x}}$$

However, the expression still has a radical in the denominator, so the expression is not in simplest form.

114. No. $\dfrac{ax - b}{b - ax}$ is undefined for values of a, b, and x such that $b = ax$.

115. $$\frac{5x^3}{2x^3 + 4} = \frac{5x^3}{2(x^3 + 2)}$$

There are no common factors, so this expression is in reduced form. In this case, factors of terms were incorrectly cancelled.

116. The negative sign in front of the second fraction was not distributed through the numerator before the fractions were added.

117. Answers will vary. For example, let $x = y = 1$:

$$\frac{1}{1+1} = \frac{1}{2} \neq \frac{1}{1} + \frac{1}{1} = 2$$

118. Answers will vary. Sample answer: When $t = 0$, the percent is 100%. After one week, the percent drops by one-half and then starts to increase as the time increases, slowly approaching 100% again.

119. $\dfrac{2x^{-2} - x^{-4}}{1} = \dfrac{x^{-4}\left(2x^2 - 1\right)}{1}$

$= \dfrac{2x^2 - 1}{x^4}$

or

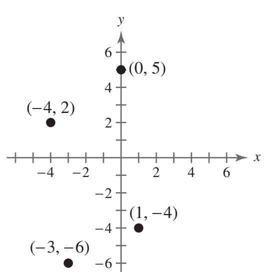
$\dfrac{2x^{-2} - x^{-4}}{1} \cdot \dfrac{x^4}{x^4} = \dfrac{2x^2 - 1}{x^4}$

Answers will vary.

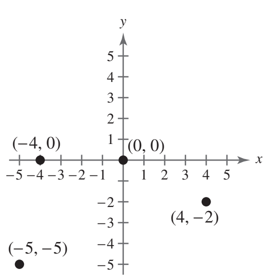
Section P.5 The Cartesian Plane

1. Cartesian
2. Distance Formula
3. Midpoint Formula
4. $(x - h)^2 + (y - k)^2 = r^2$, center, radius
5. The x-axis is the horizontal real number line.
   Matches (c).
6. The y-axis is the vertical real number line.
   Matches (f).
7. The origin is the point of intersection of the vertical and horizontal axes.
   Matches (a).
8. The quadrants are four regions of the coordinate plane.
   Matches (d).
9. An x-coordinate is the directed distance from the y-axis.
   Matches (e).
10. A y-coordinate is the directed distance from the x-axis.
    Matches (b).
11. $A: (2, 6)$, $B: (-6, -2)$, $C: (4, -4)$, $D: (-3, 2)$
12. $A: \left(\frac{3}{2}, -4\right)$; $B: (0, -2)$; $C: \left(-3, \frac{5}{2}\right)$; $D: (-6, 0)$

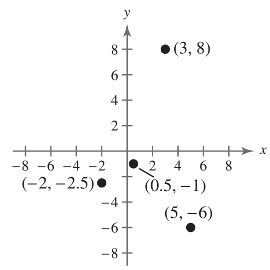
13.



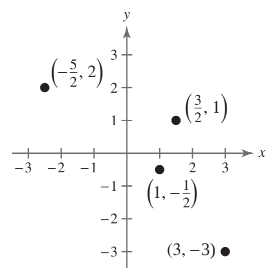
14.



15.



16.



17. $(-5, 4)$
18. $(2, -3)$
19. $(0, -6)$
20. $(-11, 0)$
21. $x > 0 \Rightarrow$ The point lies in Quadrant I or in Quadrant IV.
    $y < 0 \Rightarrow$ The point lies in Quadrant III or in Quadrant IV.
    $x > 0$ and $y < 0 \Rightarrow (x, y)$ lies in Quadrant IV.
22. If $x < 0$ and $y < 0$ then (x, y) is in Quadrant III.
23. $x = -4 \Rightarrow x$ is negative $\Rightarrow$ The point lies in Quadrant II or in Quadrant III.
    $y > 0 \Rightarrow$ The point lies in Quadrant I or Quadrant II.
    $x = -4$ and $y > 0 \Rightarrow (x, y)$ lies in Quadrant II.
24. If $x > 2$ and $y = 3$ then $(x, 3)$ is in Quadrant I.
25. $y < -5 \Rightarrow y$ is negative $\Rightarrow$ The point lies in either Quadrant III or Quadrant IV.

26. If $x > 4$ then (x, y) is in Quadrant I or IV.

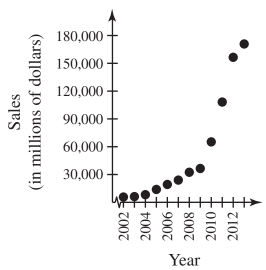
27. If $-y > 0$, then $y < 0$.
$x < 0 \Rightarrow$ The point lies in Quadrant II or in Quadrant III.
$y < 0 \Rightarrow$ The point lies in Quadrant III or in Quadrant IV.
$x < 0$ and $y < 0 \Rightarrow (x, y)$ lies in Quadrant III.

28. If $(-x, y)$ is in Quadrant IV, then (x, y) must be in Quadrant III.

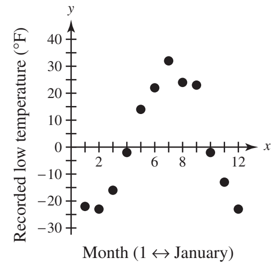
29. If $xy > 0$, then either x and y are both positive, or both negative. Hence, (x, y) lies in either Quadrant I or Quadrant III.

30. If $xy < 0$, then x and y have opposite signs. This happens in Quadrants II and IV.

31.



32.



33. $(6, -3), (6, 5)$

$d = \sqrt{(6-6)^2 + (5-(-3))^2} = \sqrt{0^2 + 8^2} = \sqrt{64} = 8$

34. $(-11, 4), (-1, 4)$

$d = \sqrt{(-1-(-11))^2 + (4-4)^2} = \sqrt{10^2 + 0^2}$
$= \sqrt{100} = 10$

35. $(-2, 6), (3, -6)$

$d = \sqrt{(3-(-2))^2 + (-6-6)^2} = \sqrt{5^2 + (-12)^2}$
$= \sqrt{25+144} = \sqrt{169} = 13$

36. $(8, 5), (0, 20)$

$d = \sqrt{(0-8)^2 + (20-5)^2} = \sqrt{8^2 + 15^2}$
$= \sqrt{64+225} = \sqrt{289} = 17$

37. $(2, 6), (-5, 5)$

$d = \sqrt{(-5-2)^2 + (5-6)^2} = \sqrt{(-7)^2 + (-1)^2} = \sqrt{50} = 5\sqrt{2}$

38. $(-3, -7), (1, -15)$

$d = \sqrt{[1-(-3)]^2 + [-15-(-7)]^2} = \sqrt{(4)^2 + (-8)^2} = \sqrt{80} = 4\sqrt{5}$

39. $\left(\frac{1}{2}, \frac{4}{3}\right), (2, -1)$

$d = \sqrt{\left(\frac{1}{2}-2\right)^2 + \left(\frac{4}{3}+1\right)^2}$
$= \sqrt{\frac{9}{4} + \frac{49}{9}}$
$= \sqrt{\frac{277}{36}} = \frac{\sqrt{277}}{6} \approx 2.77$

40. $\left(-\frac{2}{3}, 3\right), \left(-1, \frac{5}{4}\right)$

$d = \sqrt{\left(-\frac{2}{3}+1\right)^2 + \left(3-\frac{5}{4}\right)^2}$
$= \sqrt{\frac{1}{9} + \frac{49}{16}}$
$= \sqrt{\frac{457}{144}} = \frac{\sqrt{457}}{12} \approx 1.78$

41. $(-4.2, 3.1), (-12.5, 4.8)$

$d = \sqrt{(-4.2+12.5)^2 + (3.1-4.8)^2}$
$= \sqrt{68.89 + 2.89}$
$= \sqrt{71.78} \approx 8.47$

42. $(9.5, -2.6), (-3.9, 8.2)$

$d = \sqrt{(9.5+3.9)^2 + (-2.6-8.2)^2}$
$= \sqrt{179.56 + 116.64}$
$= \sqrt{296.2} \approx 17.21$

43. (a) $(1, 1), (4, 5)$

$d = \sqrt{(4-1)^2 + (5-1)^2}$
$= \sqrt{3^2 + 4^2}$
$= \sqrt{25} = 5$

$(4, 5), (4, 1)$

$d = |1-5| = |-4| = 4$

$(4, 1), (1, 1)$

$d = |1-4| = |-3| = 3$

(b) $3^2 + 4^2 = 9 + 16 = 25 = 5^2$

44. (a) $(1, 0), (13, 5)$

$d = \sqrt{(13-1)^2 + (5-0)^2}$

$= \sqrt{12^2 + 5^2}$

$= \sqrt{169} = 13$

$(13, 5), (13, 0)$

$d = |5-0| = |5| = 5$

$(1, 0), (13, 0)$

$d = |1-13| = |-12| = 12$

(b) $5^2 + 12^2 = 25 + 144 = 169 = 13^2$

45. (a) $(-1, 1), (9, 4)$

$d = \sqrt{(9-(-1))^2 + (4-1)^2}$

$= \sqrt{10^2 + 3^2}$

$= \sqrt{109}$

$(9, 4), (9, 1)$

$d = |1-4| = |-3| = 3$

$(9, 1), (-1, 1)$

$d = |-1-9| = |-10| = 10$

(b) $10^2 + 3^2 = 100 + 9 = 109 = \left(\sqrt{109}\right)^2$

46. (a) $(1, 5), (5, -2)$

$d = \sqrt{(1-5)^2 + (5-(-2))^2}$

$= \sqrt{(-4)^2 + (7)^2}$

$= \sqrt{16+49} = \sqrt{65}$

$(1, 5), (1, -2)$

$d = |5-(-2)| = |5+2| = |7| = 7$

$(1, -2), (5, -2)$

$d = |1-5| = |-4| = 4$

(b) $4^2 + 7^2 = 16 + 49 = 65 = \left(\sqrt{65}\right)^2$

47. Find the distances between pairs of points.

$d_1 = \sqrt{(4-2)^2 + (0-1)^2} = \sqrt{4+1} = \sqrt{5}$

$d_2 = \sqrt{(4+1)^2 + (0+5)^2} = \sqrt{25+25} = \sqrt{50}$

$d_3 = \sqrt{(2+1)^2 + (1+5)^2} = \sqrt{9+36} = \sqrt{45}$

$\left(\sqrt{5}\right)^2 + \left(\sqrt{45}\right)^2 = \left(\sqrt{50}\right)^2$

Because $d_1^2 + d_3^2 = d_2^2$, the triangle is a right triangle.

48. Find the distances between pairs of points.

$d_1 = \sqrt{(3-(-1))^2 + (5-3)^2}$

$= \sqrt{16+4} = \sqrt{20}$

$d_2 = \sqrt{(5-3)^2 + (1-5)^2}$

$= \sqrt{4+16} = \sqrt{20}$

$d_3 = \sqrt{(5-(-1))^2 + (1-3^2)}$

$= \sqrt{36+4} = \sqrt{40}$

$\left(\sqrt{20}\right)^2 + \left(\sqrt{20}\right)^2 = \left(\sqrt{40}\right)^2$

Because $d_1^2 + d_2^2 = d_3^2$, the triangle is a right triangle.

49. Find the distances between pairs of points.

$d_1 = \sqrt{(1-3)^2 + (-3-2)^2} = \sqrt{4+25} = \sqrt{29}$

$d_2 = \sqrt{(3+2)^2 + (2-4)^2} = \sqrt{25+4} = \sqrt{29}$

$d_3 = \sqrt{(1+2)^2 + (-3-4)^2} = \sqrt{9+49} = \sqrt{58}$

Because $d_1 = d_2$, the triangle is isosceles.

50. Find the distances between the pairs of points.

$d_1 = \sqrt{(4-2)^2 + (9-3)^2}$

$= \sqrt{4+36} = \sqrt{40} = 2\sqrt{10}$

$d_2 = \sqrt{(-2-4)^2 + (7-9)^2}$

$= \sqrt{36+4} = \sqrt{40} = 2\sqrt{10}$

$d_3 = \sqrt{(-2-2)^2 + (7-3)^2}$

$= \sqrt{16+16} = \sqrt{32} = 4\sqrt{2}$

Because $d_1 = d_2$, the triangle is isosceles.

51. Find the distances between pairs of points.

$d_1 = \sqrt{(0-2)^2 + (9-5)^2} = \sqrt{4+16} = \sqrt{20} = 2\sqrt{5}$

$d_2 = \sqrt{(-2-0)^2 + (0-9)^2} = \sqrt{4+81} = \sqrt{85}$

$d_3 = \sqrt{(0-(-2))^2 + (-4-0)^2} = \sqrt{4+16} = \sqrt{20} = 2\sqrt{5}$

$d_4 = \sqrt{(0-2)^2 + (-4-5)^2} = \sqrt{4+81} = \sqrt{85}$

Opposite sides have equal lengths of $2\sqrt{5}$ and $\sqrt{85}$, so the figure is a parallelogram.

52. Find the distances between pairs of points

$d_1 = \sqrt{(0-3)^2 + (1-7)^2} = \sqrt{9+36} = \sqrt{45} = 3\sqrt{5}$

$d_2 = \sqrt{(3-4)^2 + (7-4)^2} = \sqrt{1+9} = \sqrt{10}$

$d_3 = \sqrt{(4-1)^2 + (4+2)^2} = \sqrt{9+36} = \sqrt{45} = 3\sqrt{5}$

$d_4 = \sqrt{(0-1)^2 + (1+2)^2} = \sqrt{1+9} = \sqrt{10}$

Opposite sides have equal lengths of $3\sqrt{5}$ and $\sqrt{10}$. The figure is a parallelogram.

53. First show that the diagonals are equal in length.

$$d_1 = \sqrt{\left(0-(-3)\right)^2 + (8-1)^2} = \sqrt{9+49} = \sqrt{58}$$

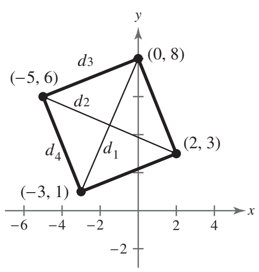
$$d_2 = \sqrt{\left(2-(-5)\right)^2 + (3-6)^2} = \sqrt{49+9} = \sqrt{58}$$

Now use the Pythagorean Theorem to verify that at least one angle is 90° (and hence, they are all right angles).

$$d_3 = \sqrt{\left(0-(-5)\right)^2 + (8-6)^2} = \sqrt{25+4} = \sqrt{29}$$

$$d_4 = \sqrt{\left(-3-(-5)\right)^2 + (1-6)^2} = \sqrt{4+25} = \sqrt{29}$$

Thus, $d_3^{\,2} + d_4^{\,2} = d_1^{\,2}$.



54. First show that the diagonals are equal in length.

$$d_1 = \sqrt{(3-2)^2 + (1-4)^2} = \sqrt{1+9} = \sqrt{10}$$

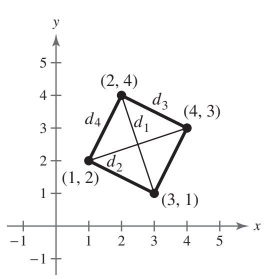
$$d_2 = \sqrt{(4-1)^2 + (3-2)^2} = \sqrt{9+1} = \sqrt{10}$$

Now use the Pythagorean Theorem to verify that at least one angle is 90°.

$$d_3 = \sqrt{(4-2)^2 + (3-4)^2} = \sqrt{4+1} = \sqrt{5}$$

$$d_4 = \sqrt{(2-1)^2 + (4-2)^2} = \sqrt{1+4} = \sqrt{5}$$

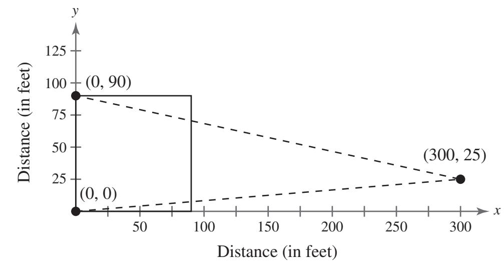
Thus, $d_3^{\,2} + d_4^{\,2} = d_1^{\,2}$.



55. $d = \sqrt{(45-10)^2 + (40-15)^2} = \sqrt{35^2 + 25^2}$

$= \sqrt{1850} = 5\sqrt{74} \approx 43$ yards

56.



Distance from $(300, 25)$ to home plate:

$$d_1 = \sqrt{(300-0)^2 + (25-0)^2}$$

$$= \sqrt{90{,}625} \approx 301.0 \text{ feet}$$

Distance from $(300, 25)$ to third base:

$$d_2 = \sqrt{(300-0)^2 + (25-90)^2}$$

$$= \sqrt{94{,}225} \approx 307.0 \text{ feet}$$

57. Let $(0, 0)$ represent the point of departure, Naples, and $(120, 150)$ represent the destination, Rome.

$$d = \sqrt{(120-0)^2 + (150-0)^2}$$

$$= \sqrt{36{,}900} \approx 192.1 \text{ km}$$

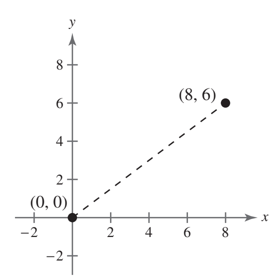
58. Let $(0, 0)$ represent the location of Oklahoma City, $(0, -360)$ represent the location of Austin, and $(-514, 0)$ represent the location of Albuquerque.

$$d = \sqrt{(-514-0)^2 + \left[0-(-360)\right]^2}$$

$$= \sqrt{(-514)^2 + 360}$$

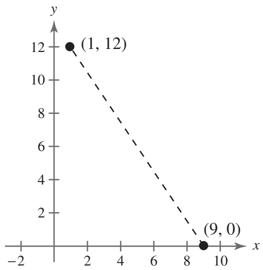
$$= \sqrt{393{,}796} \approx 627.53 \text{ mi}$$

59. (a)



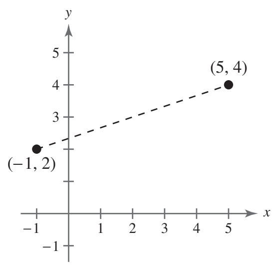
(b) $\left(\dfrac{8+0}{2}, \dfrac{6+0}{2}\right) = \left(\dfrac{8}{2}, \dfrac{6}{2}\right) = (4, 3)$

60. (a)



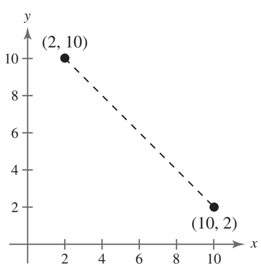
(b) $\left(\dfrac{1+9}{2}, \dfrac{12+0}{2}\right) = \left(\dfrac{10}{2}, \dfrac{12}{2}\right) = (5, 6)$

61. (a)



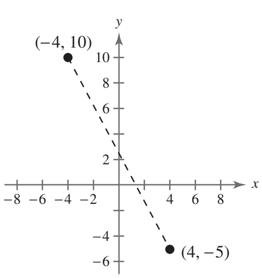
(b) $\left(\dfrac{-1+5}{2}, \dfrac{2+4}{2}\right) = \left(\dfrac{4}{2}, \dfrac{6}{2}\right) = (2, 3)$

62. (a)



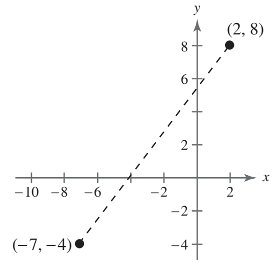
(b) $\left(\dfrac{2+10}{2}, \dfrac{10+2}{2}\right) = \left(\dfrac{12}{2}, \dfrac{12}{2}\right) = (6, 6)$

63. (a)



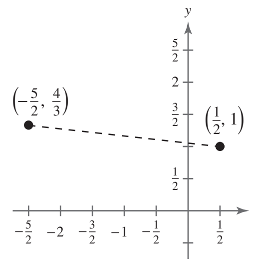
(b) $\left(\dfrac{4-4}{2}, \dfrac{-5+10}{2}\right) = \left(\dfrac{0}{2}, \dfrac{5}{2}\right) = \left(0, \dfrac{5}{2}\right)$

64. (a)



(b) $\left(\dfrac{-7+2}{2}, \dfrac{-4+8}{2}\right) = \left(\dfrac{-5}{2}, \dfrac{4}{2}\right) = \left(-\dfrac{5}{2}, 2\right)$

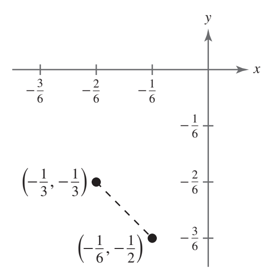
65. (a)



(b) $\left(\dfrac{-\frac{5}{2}+\frac{1}{2}}{2}, \dfrac{\frac{4}{3}+1}{2}\right) = \left(\dfrac{-4/2}{2}, \dfrac{7/3}{2}\right)$

$= \left(-1, \dfrac{7}{6}\right)$

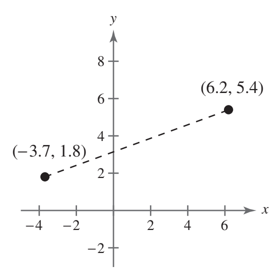
66. (a)



(b) $\left(\dfrac{(-1/3)-(1/6)}{2}, \dfrac{(-1/3)-(1/2)}{2}\right)$

$= \left(\dfrac{-1/2}{2}, \dfrac{-5/6}{2}\right) = \left(-\dfrac{1}{4}, -\dfrac{5}{12}\right)$

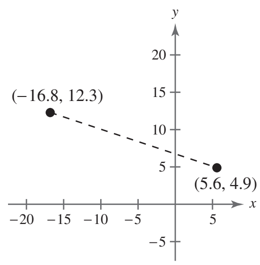
67. (a)



(b) $\left(\dfrac{6.2-3.7}{2}, \dfrac{5.4+1.8}{2}\right) = \left(\dfrac{2.5}{2}, \dfrac{7.2}{2}\right)$

$= (1.25, 3.6)$

68. (a)



(b) $\left(\frac{-16.8+5.6}{2}, \frac{12.3+4.9}{2}\right)$

$=\left(\frac{-11.2}{2}, \frac{17.2}{2}\right)=(-5.6, 8.6)$

69. Calculate the midpoint.

$\left(\frac{2009 + 2013}{2}, \frac{924 + 1423}{2}\right) = (2011, 1182.5)$

The revenue for Texas Roadhouse was \$1182.5 million in 2011.

70. Calculate the midpoint.

$\left(\frac{2009 + 2013}{2}, \frac{1106 + 1439}{2}\right) = (2011, 1272.5)$

The revenue for Papa John's Intl. was \$1272.5 million in 2011.

71. $(x-0)^2+(y-0)^2=5^2$

$x^2+y^2=25$

72. $(x-0)^2+(y-0)^2=6^2$

$x^2+y^2=36$

73. $(x-2)^2+(y+1)^2=4^2$

$(x-2)^2+(y+1)^2=16$

74. $(x+5)^2+(y-3)^2=2^2$

$(x+5)^2+(y-3)^2=4$

75. $(x+1)^2+(y-2)^2=r^2$

$(0+1)^2+(0-2)^2=r^2 \Rightarrow r^2=5$

$(x+1)^2+(y-2)^2=5$

76. $r=\sqrt{(3-(-1))^2+(-2-1)^2}=\sqrt{16+9}=5$

$(x-3)^2+(y+2)^2=5^2=25$

77. Center: $\left(\frac{-4+4}{2}, \frac{-1+1}{2}\right)=(0, 0)$

$r=\sqrt{(4-0)^2+(1-0)^2}=\sqrt{17}$

$x^2+y^2=17$

78. $r=\frac{1}{2}\sqrt{(6-0)^2+(8-0)^2}=\frac{1}{2}\sqrt{100}=5$

Center: $\left(\frac{0+6}{2}, \frac{0+8}{2}\right)=(3, 4)$

$(x-3)^2+(y-4)^2=25$

79. Because the circle is tangent to the x-axis, the radius is 1.

$(x+2)^2+(y-1)^2=1$

80. Because the circle is tangent to the y-axis, the radius is 3.

$(x-3)^2+(y+2)^2=9$

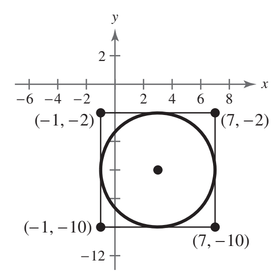
81. The center is the midpoint of one of the diagonals of the square.

Center: $\left(\frac{7+(-1)}{2}, \frac{-2+(-10)}{2}\right)=(3, -6)$

The radius is one half the length of a side of the square.

Radius: $\frac{1}{2}(7-(-1))=4$

Circle: $(x-3)^2+(y+6)^2=16$



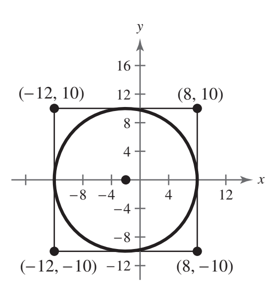
82. The center is the midpoint of one of the diagonals of the square.

Center: $\left(\frac{8+(-12)}{2}, \frac{10+(-10)}{2}\right)=(-2, 0)$

The radius is one half the length of a side of the square.

Radius: $\frac{1}{2}(8-(-12))=10$

Circle: $(x+2)^2+y^2=100$



83. From the graph, you can estimate the center to be $(2, -1)$ and the radius to be 4.

$(x-2)^2 + (y+1)^2 = 16$

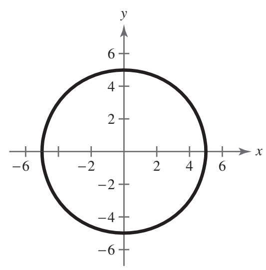
84. From the graph, you can estimate the center to be $(-3, 1)$ and the radius to be 5.

$(x+3)^2 + (y-1)^2 = 25$

85. $x^2 + y^2 = 25$

Center: $(0, 0)$

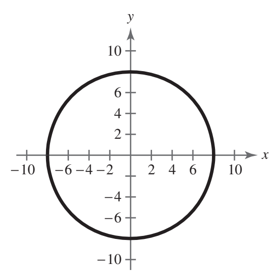
Radius: 5



86. $x^2 + y^2 = 64$

Center: $(0, 0)$

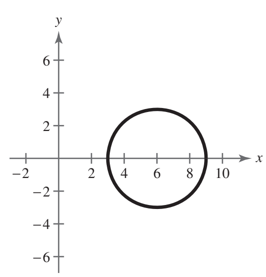
Radius: 8



87. $(x - 6)^2 + y^2 = 9$

Center: $(6, 0)$

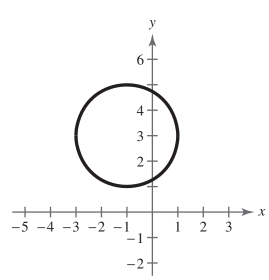
Radius: 3



88. $(x + 1)^2 + (y - 3)^2 = 4$

Center: $(-1, 3)$

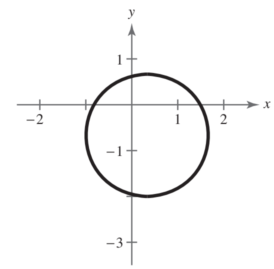
Radius: 2



89. $\left(x - \frac{1}{3}\right)^2 + \left(y + \frac{2}{3}\right)^2 = \frac{16}{9}$

Center: $\left(\frac{1}{2}, -\frac{2}{3}\right)$

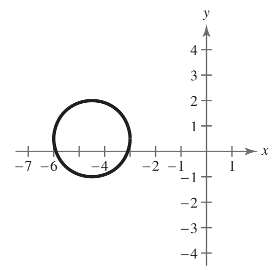
Radius: $\frac{4}{3}$



90. $(x + 4.5)^2 + (y - 0.5)^2 = 2.25$

Center: $(-4.5, 0.5)$

Radius: 1.5



91. The x-coordinates are increased by 2, and the y-coordinates are increased by 5.

Original vertex	*Shifted vertex*
$(-1, -1)$	$(-1+2, -1+5) = (1, 4)$
$(-2, -4)$	$(-2+2, -4+5) = (0, 1)$
$(2, -3)$	$(2+2, -3+5) = (4, 2)$

92. The x-coordinates are increased by 6, and the y-coordinates are decreased by 3.

Original vertex	*Shifted vertex*
$(-5, 3)$	$(-5+6, 3-3) = (1, 0)$
$(-3, 6)$	$(-3+6, 6-3) = (3, 3)$
$(-1, 3)$	$(-1+6, 3-3) = (5, 0)$
$(-3, 0)$	$(-3+6, 0-3) = (3, -3)$

93. The x-coordinates are decreased by 1, and the y-coordinates are increased by 3.

Original vertex	*Shifted vertex*
$(0, 2)$	$(0-1, 2+3) = (-1, 5)$
$(-3, 5)$	$(-3-1, 5+3) = (-4, 8)$
$(-5, 2)$	$(-5-1, 2+3) = (-6, 5)$
$(-2, -1)$	$(-2-1, -1+3) = (-3, 2)$

94. The x-coordinates are decreased by 3, and the y-coordinates are decreased by 2.

Original vertex	*Shifted vertex*
$(1, -1)$	$(1-3, -1-2) = (-2, -3)$
$(3, 2)$	$(3-3, 2-2) = (0, 0)$
$(1, -2)$	$(1-3, -2-2) = (-2, -4)$

95. (a) The point $(65, 83)$ represents a final exam score of 83, given an entrance score of 65.

(b) No. There are many variables that will affect the final exam score.

96. In the last two years, six performers have been elected to the Rock and Rock Hall of Fame. If this pattern continues, then there will be six performers elected in 2015.

97. True. The side joining $(-8, 4)$ and $(2, 11)$ has length $\sqrt{(-8-2)^2 + (4-11)^2} = \sqrt{149}$.

The side joining $(2, 11)$ and $(-5, 1)$ has length $\sqrt{(2+5)^2 + (11-1)^2} = \sqrt{149}$.

98. False. The polygon could be a rhombus. For example, consider the points $(4, 0)$, $(0, 6)$, $(-4, 0)$, and $(0, -6)$.

99. The y-coordinate of a point on the x-axis is 0. The x-coordinate of a point on the y-axis is 0.

100. Since $x_m = \dfrac{x_1 + x_2}{2}$ and $y_m = \dfrac{y_1 + y_2}{2}$ we have:

$$2x_m = x_1 + x_2 \qquad 2y_m = y_1 + y_2$$
$$2x_m - x_1 = x_2 \qquad 2y_m - y_1 = y_2$$

So, $(x_2, y_2) = (2x_m - x_1, 2y_m - y_1)$.

(a) $(x_2, y_2) = (2x_m - x_1, 2y_m - y_1)$
$$= [2(4) - 1, 2(-1) - (-2)] = (7, 0)$$

(b) $(x_2, y_2) = (2x_m - x_1, 2y_m - y_1)$
$$= [2(2) - (-5), 2(4) - 11] = (9, -3)$$

101. Midpoint of segment: $\left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)$

Midpoint between (x_1, y_1) and $\left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)$:

$$\left(\frac{x_1 + \frac{x_1 + x_2}{2}}{2}, \frac{y_1 + \frac{y_1 + y_2}{2}}{2}\right) = \left(\frac{\frac{2x_1 + x_1 + x_2}{2}}{2}, \frac{\frac{2y_1 + y_1 + y_2}{2}}{2}\right)$$
$$= \left(\frac{3x_1 + x_2}{4}, \frac{3y_1 + y_2}{4}\right)$$

Midpoint between $\left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)$ and (x_2, y_2):

$$\left(\frac{\frac{x_1 + x_2}{2} + x_2}{2}, \frac{\frac{y_1 + y_2}{2} + y_2}{2}\right) = \left(\frac{\frac{x_1 + x_2 + 2x_2}{2}}{2}, \frac{\frac{y_1 + y_2 + 2y_2}{2}}{2}\right)$$
$$= \left(\frac{x_1 + 3x_2}{4}, \frac{y_1 + 3y_2}{4}\right)$$

(a) $$\left(\frac{3x_1 + x_2}{4}, \frac{3y_1 + y_2}{4}\right) = \left(\frac{3(1) + 4}{4}, \frac{3(-2) - 1}{4}\right) = \left(\frac{7}{4}, -\frac{7}{4}\right)$$
$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{1 + 4}{2}, \frac{-2 - 1}{2}\right) = \left(\frac{5}{2}, -\frac{3}{2}\right)$$
$$\left(\frac{x_1 + 3x_2}{4}, \frac{y_1 + 3y_2}{4}\right) = \left(\frac{1 + 3(4)}{4}, \frac{-2 + 3(-1)}{4}\right) = \left(\frac{13}{4}, -\frac{5}{4}\right)$$

(b) $$\left(\frac{3x_1 + x_2}{4}, \frac{3y_1 + y_2}{4}\right) = \left(\frac{3(-2) + 0}{4}, \frac{3(-3) + 0}{4}\right) = \left(-\frac{3}{2}, -\frac{9}{4}\right)$$
$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{-2 + 0}{2}, \frac{-3 + 0}{2}\right) = \left(-1, -\frac{3}{2}\right)$$
$$\left(\frac{x_1 + 3x_2}{4}, \frac{y_1 + 3y_2}{4}\right) = \left(\frac{-2 + 0}{4}, \frac{-3 + 0}{4}\right) = \left(-\frac{1}{2}, -\frac{3}{4}\right)$$

102. $x_0 < 0,\ y_0 > 0$

(a) $x_0 < 0,\ -y_0 > 0$

So, $(x_0,\ -y_0)$ is in Quadrant III. Matches (ii).

(b) $-2x_0 > 0,\ y_0 > 0$

So, $(-2x_0,\ y_0)$ is in Quadrant I. Matches (iii).

(c) $x_0 < 0,\ \frac{1}{2}y_0 > 0$

So, $\left(x_0,\ \frac{1}{2}y_0\right)$ is in Quadrant II. Matches (iv).

(d) $-x_0 > 0,\ -y_0 < 0$

So, $(-x_0,\ -y_0)$ is in Quadrant IV. Matches (i).

103. Midpoint between $(0,\ 0)$ and $(a+b,\ c)$:

$$\left(\frac{0+a+b}{2},\ \frac{0+c}{2}\right) = \left(\frac{a+b}{2},\ \frac{c}{2}\right)$$

Midpoint between $(a,\ 0)$ and $(b,\ c)$:

$$\left(\frac{a+b}{2},\ \frac{0+c}{2}\right) = \left(\frac{a+b}{2},\ \frac{c}{2}\right)$$

Therefore, the diagonals of the parallelogram intersect at their midpoints.

Section P.6 Representing Data Graphically

1. Line plots

2. Line graphs

3. Line plot matches (c).

4. Bar graph matches (d).

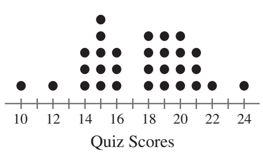
5. Histogram matches (a).

6. Line graph matches (b).

7. (a) The price \$3.52 occurred with the greatest frequency.
   (b) The range of prices is $\$3.98 - \$3.42 = \$0.56$.

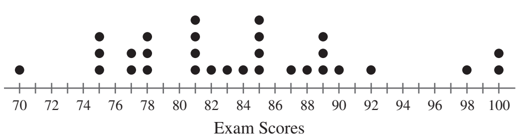
8. (a) The weight of 900 pounds occurred with the greatest frequency (9).
   (b) The weights range from 600 to 1300 pounds. The range of weights is $1300 - 600 = 700$ pounds.

9.



The score of 15 occurred with the greatest frequency.

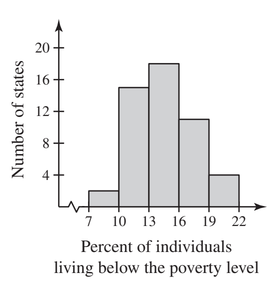
10.



The scores of 81 and 85 occurred with the greatest frequency.

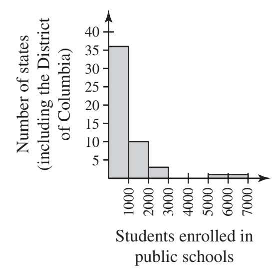
11.

Interval	Tally
[7, 10)	||
[10, 13)	𝍸 𝍸 𝍸
[13, 16)	𝍸 𝍸 𝍸 |||
[16, 19)	𝍸 𝍸 |
[19, 22)	||||

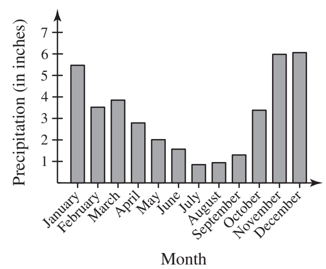


12.

Interval	Tally
[0, 1000)	𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 |
[1000, 2000)	𝍸 𝍸
[2000, 3000)	|||
[3000, 4000)	
[4000, 5000)	
[5000, 6000)	|
[6000, 7000)	|

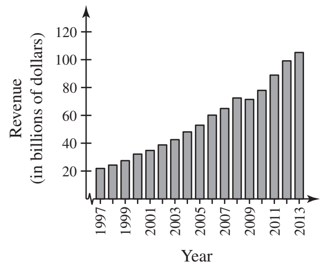


13.



Answers will vary. Sample answer: The amount of precipitation decreases at a fairly constant rate from January to July, and then it starts to increase at a fairly constant rate until December.

14.



Answers will vary. Sample answer: As time progresses from 1997 to 2013, the revenue of Costco Wholesale increases at a fairly constant rate.

15.

Year	2008	2009	2010
Differences in tuition charges (in dollars)	16,681	17,058	16,693

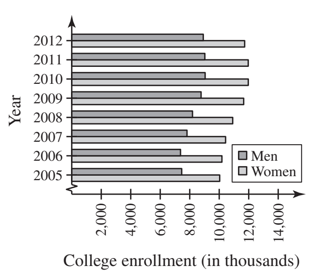
Year	2011	2012	2013
Differences in tuition charges (in dollars)	17,110	17,291	18,044

16.

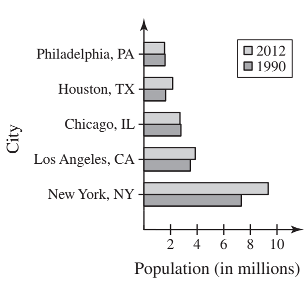
	2008–2009	2009–2010	2010-2011
Public	221	239	325
Private	598	–126	742

	2011–2012	2012–2013
Public	483	340
Private	664	1093

17.



18.



19. The price increased at a fairly constant rate from 2005 to 2008.

20. The price decreased dramatically from 2008 to 2009.

21. In 2008, the price was about \$3.50 and dropped to about \$3.05 in 2010, which is a decrease of

$$\frac{\$3.50 - \$3.05}{\$3.50} = \frac{\$0.45}{\$3.50} \approx 0.129 = 12.9\%.$$

22. In 2009, the price was about \$2.60 and rose to about \$3.90 in 2013, which is an increase of

$$\frac{\$3.90 - \$2.60}{\$2.60} = \frac{\$1.30}{\$2.60} = 0.50 = 50\%.$$

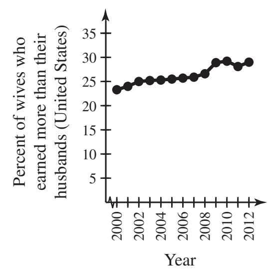
23. In December, the price paid for one dozen Grade A large eggs was approximately \$2.03.

24. The highest price was \$2.03 and the lowest price was \$1.83. The difference is $\$2.03 - \$1.83 = \$0.20$.

25. Because $\$2.03 - \$1.92 = \$0.11$ is the greatest difference between months, the greatest rate of increase occurred from November to December.

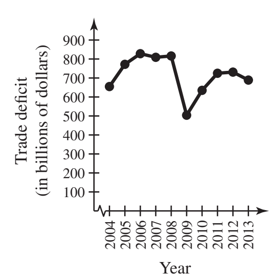
26. Answers will vary. Sample answer: The highest prices seemed to occur in the winter months, while the lowest prices seemed to occur in the summer months. According to the data, a price of about \$2.10 per dozen seems reasonable. If the trend continues, the price should be within \$0.10 of the actual price in February 2014.

27.



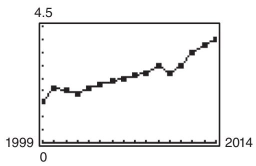
Answers will vary. Sample answer: From 2000 to 2012, the percent of wives who earned more income than their husbands increases at a fairly constant rate.

28.

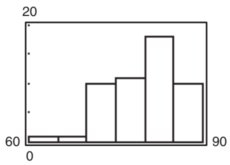


Answers will vary. Sample answer: From 2004 to 2008, the trade deficit increased at a fairly constant rate, then dropped significantly in 2009, and then increased at a fairly constant rate until 2013, when it dropped slightly.

29.

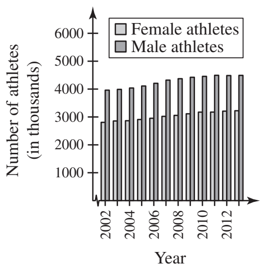


30.



Answers will vary. Sample answer: A histogram is best because the data are percents within a year that do not relate to increasing or decreasing behavior.

31.



Answers will vary. Sample answer: A double bar graph is best because there are two different sets of data within the same time interval that do not deal primarily with increasing or decreasing behavior.

32. A histogram has a portion of the real number line as its horizontal axis, and the bars are not separated by spaces. A bar graph can be either horizontal or vertical. The labels are not necessarily numbers, and the bars are usually separated by spaces.

33. A line plot and a histogram both use a portion of the real number line to order numbers. A line plot is especially useful for ordering small sets of data, and recording the frequency of each value; while a histogram is more useful to organize large sets of data and then grouping the data into intervals and plotting the frequency of the data in each interval.

34. The second graph is misleading because the vertical scale is too small which makes small changes look large. Answers will vary.

35. Answers will vary. Line plots are useful for ordering small sets of data. Histograms or bar graphs can be used to organize larger sets. Line graphs are used to show trends over time.

Chapter P Review

1. $\left\{11, -14, -\frac{8}{9}, \frac{5}{2}, \sqrt{6}, 0.4\right\}$

(a) Natural number: 11
(b) Whole number: 11
(c) Integers: $11, -14$
(d) Rational numbers: $11, -14, -\frac{8}{9}, \frac{5}{2}, 0.4$
(e) Irrational number: $\sqrt{6}$

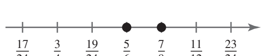
2. $\left\{\sqrt{15}, -22, -\frac{10}{3}, 0, 5.2, \frac{3}{7}\right\}$

(a) Natural numbers: none
(b) Whole number: 0
(c) Integers: $-22, 0$
(d) Rational numbers: $-22, -\frac{10}{3}, 0, 5.2, \frac{3}{7}$
(e) Irrational number: $\sqrt{15}$

3. (a) $\frac{5}{6} = 0.8\overline{3}$

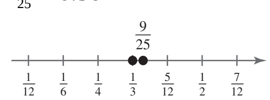
(b) $\frac{7}{8} = 0.875$

$\frac{5}{6} < \frac{7}{8}$



4. (a) $\frac{1}{3} = 0.\overline{3}$

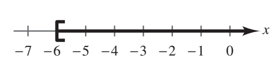
(b) $\frac{9}{25} = 0.36$



$0.\overline{3} = \frac{1}{3} < \frac{9}{25} = 0.36$

5. (a) The inequality $x \geq -6$ is the set of all real numbers greater than or equal to -6.

(b)



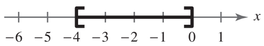
(c) The interval is unbounded.

6. (a) The inequality $x < 1$ is the set of all real numbers less than 1.

(b)

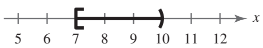
(c) The interval is unbounded.

7. (a) The inequality $-4 \le x \le 0$ is the set of all real numbers greater than or equal to -4 and less than or equal to 0.

(b) 

(c) The interval is bounded.

8. (a) The inequality $7 \le x < 10$ is the set of all real numbers greater than or equal to 7and less than 10.

(b) 

(c) The interval is bounded.

9. $d(a, b) = |48 - (-74)|$
$= |48 + 74| = 122$

10. $d(-123, -9) = |-9 - (-123)|$
$= |114| = 114$

11. $|x - 7| \ge 6$

12. $d(y, -30) = |y - (-30)| = |y + 30|$ and $d(y, -30) < 5$, so $|y + 30| < 5$.

13. $9x - 2$

(a) $x = -1: \ 9(-1) - 2 = -9 - 2 = -11$

(b) $x = 3: \ 9(3) - 2 = 27 - 2 = 25$

14. $x^2 - 11x + 24$

(a) $x = -2:$
$(-2)^2 - 11(-2) + 24 = 4 + 22 + 24 = 50$

(b) $x = 2: \ 2^2 - 11(2) + 24 = 4 - 22 + 24 = 6$

15. $\dfrac{-2x + 3}{x}$

(a) $x = 0$: not defined

(b) $x = 6: \ \dfrac{-2(6) + 3}{6} = \dfrac{-12 + 3}{6} = \dfrac{-9}{6} = -\dfrac{3}{2}$

16. $\dfrac{4x}{x - 1}$

(a) $x = -1: \ \dfrac{4(-1)}{(-1) - 1} = \dfrac{-4}{-2} = 2$

(b) $x = 1$: not defined

17. $2x + (3x - 10) = (2x + 3x) - 10$

Associative Property of Addition

18. $4(t + 2) = 4 \cdot t + 4 \cdot 2$

Distributive Property

19. $0 + (a - 5) = a - 5$

Additive Identity Property

20. $(t^2 + 1) + 3 = 3 + (t^2 + 1)$

Commutative Property of Addition

21. $\dfrac{2}{y + 4} \cdot \dfrac{y + 4}{2} = 1, \ y \ne -4$

Multiplicative Inverse Property

22. $1 \cdot (3x + 4) = 3x + 4$

Multiplicative Identity Property

23. (a) $(-2z)^3 = (-2)^3 z^3 = -8z^3$

(b) $(a^2b^4)(3ab^{-2}) = 3a^{2+1}b^{4-2} = 3a^3b^2$

24. (a) $\dfrac{(4y^2)^3}{y^2} = \dfrac{4^3 y^6}{y^2} = 64y^4, \ y \ne 0$

(b) $\dfrac{40(b - 3)^3}{75(b - 3)^5} = \dfrac{8}{15(b - 3)^2}$

25. (a) $\dfrac{36u^0v^{-3}}{12u^{-2}v} = \dfrac{36u^{0-(-2)}v^{-3-1}}{12} = 3u^2v^{-4} = \dfrac{3u^2}{v^4}$

(b) $\dfrac{3^{-4}m^{-1}n^{-3}}{9^{-2}mn^{-3}} = \dfrac{9^2n^3}{3^4mmn^3} = \dfrac{81}{81m^2} = \dfrac{1}{m^2}$

26. (a) $(a^4b^{-3}c^0)^{-1}a^2 = \dfrac{a^2}{(a^4b^{-3}c^0)} = \dfrac{b^3}{a^2}$

(b) $\left(\dfrac{y^{-2}}{x}\right)^{-1}\left(\dfrac{x^2}{y^{-2}}\right) = \left(\dfrac{1}{xy^2}\right)^{-1}\left(\dfrac{x^2y^2}{1}\right)$
$= \left(\dfrac{xy^2}{1}\right)\left(\dfrac{x^2y^2}{1}\right)$
$= x^3y^4, \ x \ne 0, \ y \ne 0$

27. $2,585,000,000 = 2.585 \times 10^9$

28. $-3,250,000 = -3.25 \times 10^6$

29. $-0.000000125 = -1.25 \times 10^{-7}$

30. $0.00000008064 = 8.064 \times 10^{-8}$

31. $1.28 \times 10^5 = 128,000$

32. $-4.002 \times 10^2 = -400.2$

33. $1.80 \times 10^{-5} = 0.000018$

34. $-4.02 \times 10^{-2} = -0.0402$

35. $\left(\sqrt[4]{78}\right)^4 = \left(78^{1/4}\right)^4 = 78^1 = 78$

36. $\sqrt{(5x)^2} = 5|x|$

37. $\sqrt[5]{8} \cdot \sqrt[5]{4} = \sqrt[5]{2^3} \cdot \sqrt[5]{2^2} = 2^{3/5} \cdot 2^{2/5} = 2^{5/5} = 2$

38. $\sqrt[3]{\sqrt{xy}} = \left[(xy)^{1/2}\right]^{1/3} = (xy)^{1/6} = \sqrt[6]{xy}$

39. $\sqrt{25a^3} = \sqrt{5^2 \cdot a^2 \cdot a} = 5a\sqrt{a}$

40. $\sqrt[5]{64x^6} = \sqrt[5]{(2x)(2x)^5} = 2x\sqrt[5]{2x}$

41. $\sqrt{\dfrac{81}{144}} = \sqrt{\dfrac{9 \cdot 9}{12 \cdot 12}} = \dfrac{9}{12} = \dfrac{3}{4}$

42. $\sqrt[3]{\dfrac{125}{216}} = \sqrt[3]{\dfrac{5^3}{6^3}} = \dfrac{5}{6}$

43. $\sqrt{\dfrac{75x^2}{y^4}} = \sqrt{\dfrac{3 \cdot 5^2 \cdot x^2}{y^4}} = \dfrac{5|x|}{y^2}\sqrt{3}$

44. $\sqrt[3]{\dfrac{2x^3}{27}} = \sqrt[3]{\dfrac{2x^3}{3^3}} = \dfrac{x}{3}\sqrt[3]{2}$

45. $\sqrt{48} - \sqrt{27} = \sqrt{3 \cdot 4^2} - \sqrt{3 \cdot 3^2}$
$= 4\sqrt{3} - 3\sqrt{3}$
$= \sqrt{3}$

46. $3\sqrt{32} + 4\sqrt{98} = 3\sqrt{2^5} + 4\sqrt{2 \cdot 7^2}$
$= 3 \cdot 4 \cdot \sqrt{2} + 4 \cdot 7 \cdot \sqrt{2}$
$= 40\sqrt{2}$

47. $8\sqrt{3x} - 5\sqrt{3x} = 3\sqrt{3x}$

48. $-11\sqrt{36y} - 6\sqrt{y} = -11(6)\sqrt{y} - 6\sqrt{y}$
$= -72\sqrt{y}$

49. $\sqrt{8x^3} + \sqrt{2x} = \sqrt{2 \cdot 2^2 \cdot x \cdot x^2} + \sqrt{2x}$
$= 2x\sqrt{2x} + \sqrt{2x}$
$= (2x+1)\sqrt{2x}$

50. $3\sqrt{14x^2} - \sqrt{56x^2} = 3|x|\sqrt{14} - 2|x|\sqrt{14}$
$= |x|\sqrt{14}$

51. $\dfrac{1}{3-\sqrt{5}} = \dfrac{1}{3-\sqrt{5}} \cdot \dfrac{3+\sqrt{5}}{3+\sqrt{5}}$
$= \dfrac{3+\sqrt{5}}{9-5} = \dfrac{3+\sqrt{5}}{4}$

52. $\dfrac{1}{\sqrt{x}-1} = \dfrac{1}{\sqrt{x}-1} \cdot \dfrac{\sqrt{x}+1}{\sqrt{x}+1} = \dfrac{\sqrt{x}+1}{x-1}$

53. $\dfrac{\sqrt{20}}{4} = \dfrac{2\sqrt{5}}{4} = \dfrac{\sqrt{5}}{2} \cdot \dfrac{\sqrt{5}}{\sqrt{5}} = \dfrac{5}{2\sqrt{5}}$

54. $\dfrac{\sqrt{2}-\sqrt{11}}{3} = \dfrac{\sqrt{2}-\sqrt{11}}{3} \cdot \dfrac{\sqrt{2}+\sqrt{11}}{\sqrt{2}+\sqrt{11}}$
$= \dfrac{(2-11)}{3\left(\sqrt{2}+\sqrt{11}\right)} = \dfrac{-9}{3\left(\sqrt{2}+\sqrt{11}\right)}$
$= \dfrac{-3}{\sqrt{2}+\sqrt{11}}$

55. $64^{5/2} = \left(\sqrt{64}\right)^5 = 8^5 = 32{,}768$

56. $64^{-2/3} = \dfrac{1}{64^{2/3}} = \dfrac{1}{\left(64^{1/3}\right)^2} = \dfrac{1}{4^2} = \dfrac{1}{16}$

57. $\left(-3x^{-1/6}\right)\left(-2x^{1/2}\right) = 6x^{-1/6}x^{1/2}$
$= 6x^{-1/6+3/6}$
$= 6x^{2/6}$
$= 6x^{1/3},\ x \neq 0$

58. $(x-1)^{1/3}(x-1)^{-1/4} = (x-1)^{1/3-1/4}$
$= (x-1)^{1/12},\ x \neq 1$

59. $15x^2 - 2x^5 + 3x^3 + 5 - x^4$
$= -2x^5 - x^4 + 3x^3 + 15x^2 + 5$ Standard form
Degree: 5
Leading coefficient: -2

60. $-4x^4 + x^2 - 10 - x + x^3$
$= -4x^4 + x^3 + x^2 - x - 10$ Standard form
Degree: 4
Leading coefficient: -4

61. $(3x^2 + 2x) - (1 - 5x) = 3x^2 + 2x - 1 + 5x$
$= 3x^2 + 7x - 1$

62. $(8y^2 + 2y) + (3y - 8) = 8y^2 + 5y - 8$

63. $(2x^3 - 5x^2 + 10x - 7) + (4x^2 - 7x - 2)$
$= 2x^3 - 5x^2 + 4x^2 + 10x - 7x - 7 - 2$
$= 2x^3 - x^2 + 3x - 9$

64. $(6x^4 - 4x^3 - x + 3 - 20x^2) - (16 + 9x^4 - 11x^2)$
$= 6x^4 - 4x^3 - x + 3 - 20x^2 - 16 - 9x^4 + 11x^2$
$= -3x^4 - 4x^3 - 9x^2 - x - 13$

65. $-2a(a^2 + a - 3) = -2a^3 - 2a^2 + 6a$

66. $(y^2 - 4y)(y^3) = y^5 - 4y^4$

67. $(x+4)(x+9) = x^2 + 9x + 4x + 36$
$= x^2 + 13x + 36$

68. $(z+1)(5z-6) = 5z^2 - 6z + 5z - 6$
$= 5z^2 - z - 6$

69. $(x+8)(x-8) = x^2 - 8^2 = x^2 - 64$

70. $(7x - 4)^2 = (7x)^2 - 2(7x)(4) + (4)^2$
$= 49x^2 - 56x + 16$

71. $(x-4)^3 = x^3 - 3x^2(4) + 3x(4)^2 - 4^3$
$= x^3 - 12x^2 + 48x - 64$

72. $(2x + 1)^3 = (2x)^3 + 3(2x)^2(1) + 3(2x)(1)^2 + (1)^3$
$= 8x^3 + 12x^2 + 6x + 1$

73. $[(m-7)+n][(m-7)-n] = (m-7)^2 - n^2$
$= m^2 - 14m - n^2 + 49$

74. $[(x-y)-4][(x-y)+4] = (x-y)^2 + 4^2$
$= x^2 - 2xy + y^2 - 16$

75. $(x+3)(x+5) = x(x+5) + 3(x+5)$
Distributive Property

76. $2500(1+r^2) = 2500(1 + 2r + r^2)$
$= 2500r^2 + 5000r + 2500$

77. $7x + 35 = 7(x+5)$

78. $4b - 12 = 4(b-3)$

79. $2x^3 + 18x^2 - 4x = 2x(x^2 + 9x - 2)$

80. $-6x^4 - 3x^3 + 12x = -3x(2x^3 + x^2 - 4)$

81. $x(x-3) + 4(x-3) = (x-3)(x+4)$

82. $8(2 - y) - (2 - y)^2 = (2 - y)[8 - (2 - y)]$
$= (2 - y)(8 - 2 + y)$
$= (2 - y)(6 + y)$
$= -(y - 2)(y + 6)$

83. $x^2 - 169 = (x+13)(x-13)$

84. $9x^2 - \frac{1}{25} = (3x - \frac{1}{5})(3x + \frac{1}{5})$

85. $x^2 + 6x + 9 = (x+3)(x+3) = (x+3)^2$

86. $4x^2 - 4x + 1 = (2x-1)(2x-1) = (2x-1)^2$

87. $x^3 + 216 = x^3 + 6^3 = (x+6)(x^2 - 6x + 36)$

88. $64x^3 - 27 = (4x)^3 - 3^3$
$= (4x-3)(16x^2 + 12x + 9)$

89. $x^2 - 6x - 27 = (x-9)(x+3)$

90. $x^2 - 9x + 14 = (x-2)(x-7)$

91. $2x^2 + 21x + 10 = (2x+1)(x+10)$

92. $3x^2 + 14x + 8 = (3x+2)(x+4)$

93. $x^3 - 4x^2 - 3x + 12 = x^2(x-4) - 3(x-4)$
$= (x-4)(x^2 - 3)$

94. $x^3 - 6x^2 - x + 6 = x^2(x-6) - (x-6)$
$= (x-6)(x^2 - 1)$
$= (x-6)(x-1)(x+1)$

95. $2x^2 - x - 15$
$a = 2,\ c = -15,\ ac = -30 = (-6)5$ and
$-6 + 5 = -1 = b$
So, $2x^2 - x - 15 = 2x^2 - 6x + 5x - 15$
$= 2x(x-3) + 5(x-3)$
$= (2x+5)(x-3).$

96. $6x^2 + x - 12$
$a = 6,\ c = -12,\ ac = -72 = (-8)9$ and
$-8 + 9 = 1 = b$
Thus, $6x^2 + x - 12 = 6x^2 - 8x + 9x - 12$
$= 2x(3x-4) + 3(3x-4)$
$= (2x+3)(3x-4).$

97. Domain: all real numbers x

98. Domain: all real numbers $x < 0$

99. Domain: all real numbers except $x = \frac{3}{2}$

100. Domain: all real numbers $x \geq -12$

101. $\dfrac{4x^2}{4x^3 + 28x} = \dfrac{4x^2}{4x(x^2 + 7)} = \dfrac{x}{x^2 + 7},\ x \neq 0$

102. $\dfrac{6xy}{xy + 2x} = \dfrac{6xy}{x(y + 2)} = \dfrac{6y}{y + 2},\ x \neq 0$

103. $\dfrac{x^2 - x - 30}{x^2 - 25} = \dfrac{(x - 6)(x + 5)}{(x + 5)(x - 5)} = \dfrac{x - 6}{x - 5},\ x \neq -5$

104. $\dfrac{x^2 - 9x + 18}{8x - 48} = \dfrac{(x - 6)(x - 3)}{8(x - 6)} = \dfrac{x - 3}{8},\ x \neq 6$

105. $\dfrac{x^2 - 4}{x^4 - 2x^2 - 8} \cdot \dfrac{x^2 + 2}{x^2} = \dfrac{(x - 2)(x + 2)}{(x^2 - 4)(x^2 + 2)} \cdot \dfrac{x^2 + 2}{x^2}$

$= \dfrac{(x - 2)(x + 2)}{(x - 2)(x + 2)} \cdot \dfrac{1}{x^2}$

$= \dfrac{1}{x^2},\ x \neq \pm 2$

106. $\dfrac{2x - 1}{x + 1} \cdot \dfrac{x^2 - 1}{2x^2 - 7x + 3} = \dfrac{2x - 1}{x + 1} \cdot \dfrac{(x + 1)(x - 1)}{(2x - 1)(x - 3)}$

$= \dfrac{x - 1}{x - 3},\ x \neq \dfrac{1}{2},\ -1$

107. $\dfrac{x^2(5x - 6)}{2x + 3} \div \dfrac{5x}{2x + 3} = \dfrac{x^2(5x - 6)}{2x + 3} \cdot \dfrac{2x + 3}{5x}$

$= \dfrac{x(5x - 6)}{5},\ x \neq 0,\ -\dfrac{3}{2}$

108. $\dfrac{4x - 6}{(x - 1)^2} \div \dfrac{2x^2 - 3x}{x^2 + 2x - 3} = \dfrac{4x - 6}{(x - 1)^2} \cdot \dfrac{x^2 + 2x - 3}{2x^2 - 3x}$

$= \dfrac{2(2x - 3)}{(x - 1)^2} \cdot \dfrac{(x + 3)(x - 1)}{x(2x - 3)}$

$= \dfrac{2(x + 3)}{x(x - 1)},\ x \neq -3,\ \dfrac{3}{2}$

109. $x - 1 + \dfrac{1}{x + 2} + \dfrac{1}{x - 1} = \dfrac{(x - 1)^2(x + 2) + (x - 1) + (x + 2)}{(x + 2)(x - 1)}$

$= \dfrac{(x^2 - 2x + 1)(x + 2) + 2x + 1}{(x + 2)(x - 1)}$

$= \dfrac{x^3 - 2x^2 + x + 2x^2 - 4x + 2 + (2x + 1)}{(x + 2)(x - 1)}$

$= \dfrac{x^3 - x + 3}{(x + 2)(x - 1)}$

110. $2x + \dfrac{3}{2(x - 4)} - \dfrac{1}{2(x + 2)}$

$= \dfrac{2x(2)(x - 4)(x + 2) + 3(x + 2) - (x - 4)}{2(x - 4)(x + 2)}$

$= \dfrac{4x(x^2 - 2x - 8) + 3x + 6 - x + 4}{2(x - 4)(x + 2)}$

$= \dfrac{4x^3 - 8x^2 - 32x + 2x + 10}{2(x - 4)(x + 2)}$

$= \dfrac{4x^3 - 8x^2 - 30x + 10}{2(x - 4)(x + 2)}$

$= \dfrac{2x^3 - 4x^2 - 15x + 5}{(x - 4)(x + 2)}$

111. $\dfrac{1}{x} - \dfrac{x - 1}{x^2 + 1} = \dfrac{1(x^2 + 1) - x(x - 1)}{x(x^2 + 1)} = \dfrac{x^2 + 1 - x^2 + x}{x(x^2 + 1)} = \dfrac{x + 1}{x(x^2 + 1)}$

112. $$\frac{1}{x-1}+\frac{1-x}{x^2+x+1}=\frac{x^2+x+1+(1-x)(x-1)}{(x-1)(x^2+x+1)}$$
$$=\frac{x^2+x+1+x-1-x^2+x}{(x-1)(x^2+x+1)}=\frac{3x}{(x-1)(x^2+x+1)}$$

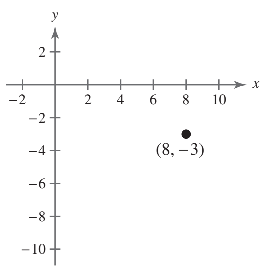
113. $$\frac{\frac{1}{x}-\frac{1}{y}}{(x^2-y^2)}=\frac{y-x}{xy}\cdot\frac{1}{(x-y)(x+y)}=\frac{-1}{xy(x+y)},\ x\neq y$$

114. $$\frac{\frac{1}{2x-3}-\frac{1}{2x+3}}{\frac{1}{2x}-\frac{1}{2x+3}}=\frac{\frac{(2x+3)-(2x-3)}{(2x-3)(2x+3)}}{\frac{(2x+3)-2x}{2x(2x+3)}}$$
$$=\frac{\frac{6}{(2x-3)(2x+3)}}{\frac{3}{2x(2x+3)}}$$
$$=\frac{6}{(2x-3)(2x+3)}\cdot\frac{2x(2x+3)}{3}$$
$$=\frac{4x}{2x-3},\ x\neq-\frac{3}{2},\ 0$$

115. $$x^3(2x^2+1)^{-4}+x(2x^2+1)^{-3}=x(2x^2+1)^{-4}\left[x^2+(2x^2+1)\right]$$
$$=x(2x^2+1)^{-4}(3x^2+1)$$
$$=\frac{x(3x^2+1)}{(2x^2+1)^4}$$

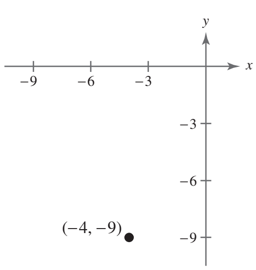
116. $$\frac{x^2(5x^{-3/2})-2x^3(x^{-1/2})}{x^3}=\frac{5x^{1/2}-2x^{5/2}}{x^3}$$
$$=\frac{x^{1/2}(5-2x^2)}{x^3}$$
$$=\frac{5-2x^2}{x^{5/2}}$$

117.



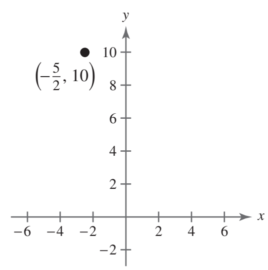
Quadrant IV

118.



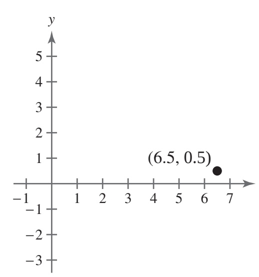
Quadrant III

119.



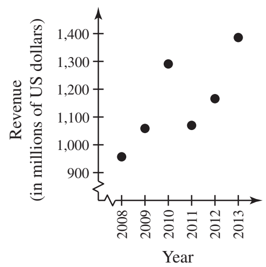
Quadrant II

120.



Quadrant I

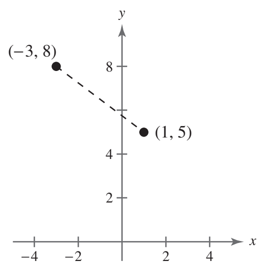
121.



122. The revenues increased from 2008 to 2013.

123. $(-3, 8), (1, 5)$

(a)



(b) $d = \sqrt{(1-(-3))^2 + (5-8)^2}$

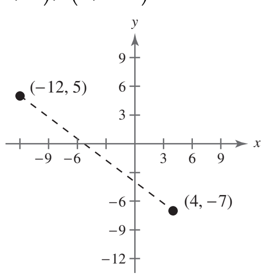
$= \sqrt{4^2 + (-3)^2} = \sqrt{16+9}$

$= \sqrt{25} = 5$

(c) Midpoint: $\left(\dfrac{-3+1}{2}, \dfrac{8+5}{2}\right) = \left(-1, \dfrac{13}{2}\right)$

124. $(-12, 5), (4, -7)$

(a)



(b) $d = \sqrt{(4-(-12))^2 + (-7-5)^2}$

$= \sqrt{16^2 + (-12)^2}$

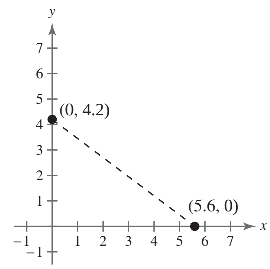
$= \sqrt{256+144}$

$= \sqrt{400} = 20$

(c) Midpoint: $\left(\dfrac{-12+4}{2}, \dfrac{5+(-7)}{2}\right) = (-4, -1)$

125. $(5.6, 0), (0, 4.2)$

(a)



(b) $d = \sqrt{(0-5.6)^2 + (4.2-0)^2}$

$= \sqrt{(-5.6)^2 + (4.2)^2}$

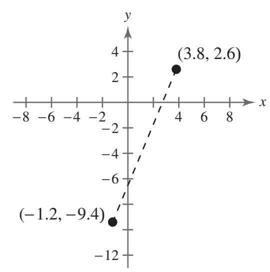
$= \sqrt{31.36+17.64}$

$= \sqrt{49} = 7$

(c) Midpoint: $\left(\dfrac{5.6+0}{2}, \dfrac{0+4.2}{2}\right) = (2.8, 2.1)$

126. $(3.8, 2.6), (-1.2, -9.4)$

(a)



(b) $d = \sqrt{(-1.2-3.8)^2 + (-9.4-2.6)^2}$

$= \sqrt{(-5)^2 + (-12)^2}$

$= \sqrt{25+144} = \sqrt{169} = 13$

(c) Midpoint: $\left(\dfrac{3.8+(-1.2)}{2}, \dfrac{2.6+(-9.4)}{2}\right)$

$= \left(\dfrac{2.6}{2}, \dfrac{-6.8}{2}\right) = (1.3, -3.4)$

127. Radius:

$\sqrt{(3-(-5))^2 + (-1-1)^2} = \sqrt{64+4} = \sqrt{68}$

Circle: $(x-3)^2 + (y+1)^2 = 68$

128. Center: $\left(\dfrac{-4+10}{2}, \dfrac{6-2}{2}\right) = (3, 2)$

Radius:

$\dfrac{1}{2}\sqrt{(10+4)^2 + (-2-6)^2} = \dfrac{1}{2}\sqrt{14^2 + (-8)^2} = \sqrt{65}$

Circle: $(x-3)^2 + (y-2)^2 = 65$

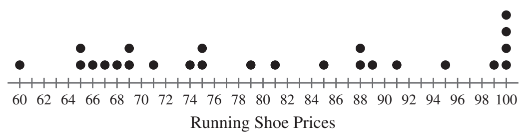
129. The x-coordinates are decreased by 2, and the y-coordinates are decreased by 3.

Original vertices	*Shifted vertices*
$(4, 8)$	$(4-2, 8-3) = (2, 5)$
$(6, 8)$	$(6-2, 8-3) = (4, 5)$
$(4, 3)$	$(4-2, 3-3) = (2, 0)$
$(6, 3)$	$(6-2, 3-3) = (4, 0)$

130. The x-coordinates are increased by 4, and the y-coordinates are increased by 5.

Original vertices	*Shifted vertices*
$(0, 1)$	$(0+4, 1+5) = (4, 6)$
$(3, 3)$	$(3+4, 3+5) = (7, 8)$
$(0, 5)$	$(0+4, 5+5) = (4, 10)$
$(-3, 3)$	$(-3+4, 3+5) = (1, 8)$

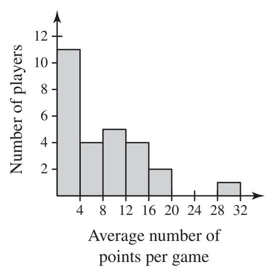
131.



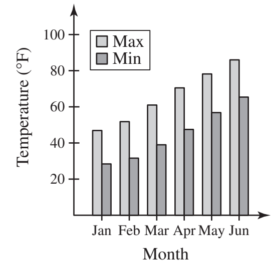
The price of \$100 occurs with the greatest frequency (4).

132.

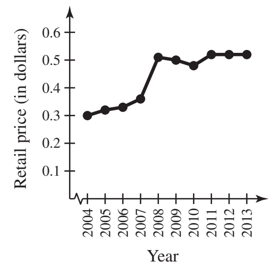
Interval	Tally
$[0, 4)$	卌 卌 |
$[4, 8)$	||||
$[8, 12)$	卌
$[12, 16)$	||||
$[16, 20)$	||
$[20, 24)$	
$[24, 28)$	
$[28, 32)$	|



133.



134.



The price increased from 2004 to 2008, dropped slightly in 2009 and 2010, then stayed constant from 2011 to 2013.

Chapter P Test

1. $-\frac{10}{3} \approx -3.3$ and $-|-4| = -4$, hence $-\frac{10}{3} > -|-4|$.

2. $d = |-16 - 38| = |-54| = 54$

3. $5 \cdot (1 - x) \cdot 2 = 5 \cdot 2 \cdot (1 - x)$

Commutative Property of Multiplication

4. (a) $\left(\frac{3^2}{2}\right)^{-3} = \left(\frac{2}{3^2}\right)^3 = \frac{2^3}{3^6} = \frac{8}{729}$

(b) $\sqrt{5} \cdot \sqrt{125} = \sqrt{5} \cdot 5\sqrt{5} = 25$

(c) $\frac{5.4 \times 10^8}{3 \times 10^3} = \frac{5.4}{3} \times 10^5 = 1.8 \times 10^5$

(d) $(3 \times 10^4)^3 = 3^3 \times (10^4)^3$

$= 27 \times 10^{12} = 2.7 \times 10^{13}$

5. (a) $3z^2(2z^3)^2 = 3z^2 4 \cdot z^6 = 12z^8$

(b) $(u-2)^{-4}(u-2)^{-3} = (u-2)^{-7} = \dfrac{1}{(u-2)^7}$

(c) $\left(\dfrac{x^{-2}y^2}{3}\right)^{-1} = \dfrac{3}{x^{-2}y^2} = \dfrac{3x^2}{y^2}$

6. (a) $9z\sqrt{8z} - 3\sqrt{2z^3} = 9z \cdot 2\sqrt{2z} - 3z\sqrt{2z}$
$= 15z\sqrt{2z}$

(b) $-5\sqrt{16y} + 10\sqrt{y} = -5 \cdot 4\sqrt{y} + 10\sqrt{y}$
$= -10\sqrt{y}$

(c) $\sqrt[3]{\dfrac{16}{v^5}} = \sqrt[3]{\dfrac{2^3 \cdot 2}{v^3v^2}} = \dfrac{2}{v}\sqrt[3]{\dfrac{2}{v^2}}$

7. $3 - 2x^5 + 3x^3 - x^4$
$= -2x^5 - x^4 + 3x^3 + 3$ Standard form

Degree: 5
Leading coefficient: -2

8. $(x^2+3) - \left[3x + (8 - x^2)\right] = x^2 + 3 - 3x - 8 + x^2$
$= 2x^2 - 3x - 5$

9. $(2x-5)(4x^2+6) = 8x^3 + 12x - 20x^2 - 30$
$= 8x^3 - 20x^2 + 12x - 30$

10. $\dfrac{8x}{x-3} + \dfrac{24}{3-x} = \dfrac{8x}{x-3} - \dfrac{24}{x-3}$
$= \dfrac{8x-24}{x-3} = \dfrac{8(x-3)}{x-3}$
$= 8,\ x \neq 3$

11. $\left(\dfrac{2}{x} - \dfrac{2}{x+1}\right) \div \dfrac{4}{x^2-1}$
$= \dfrac{2(x+1) - 2x}{x(x+1)} \cdot \dfrac{(x-1)(x+1)}{4}$
$= \dfrac{2}{x} \cdot \dfrac{x-1}{4} = \dfrac{x-1}{2x},\ x \neq \pm 1$

12. $(x+\sqrt{5})(x-\sqrt{5}) = x^2 - (\sqrt{5})^2 = x^2 - 5$

13. $(x-2)^3 = x^3 - 3x^2(2) + 3x(2^2) - 2^3$
$= x^3 - 6x^2 + 12x - 8$

14. $\left[(x+y) - z\right]\left[(x+y) + z\right] = (x+y)^2 - z^2$
$= x^2 + 2xy + y^2 - z^2$

15. $2x^4 - 3x^3 - 2x^2 = x^2(2x^2 - 3x - 2)$
$= x^2(2x+1)(x-2)$

16. $x^3 + 2x^2 - 4x - 8 = x^2(x+2) - 4(x+2)$
$= (x+2)(x^2-4)$
$= (x+2)(x+2)(x-2)$
$= (x+2)^2(x-2)$

17. $8x^3 - 64 = 8(x^3 - 8)$
$= 8\left[x^3 - (2)^3\right]$
$= 8(x-2)(x^2 + 2x + 4)$

18. (a) The domain of the rational expression $\dfrac{x+3}{x^2-16} = \dfrac{x+3}{(x+4)(x-4)}$ is all real numbers x except $x = \pm 4$, because division by zero is undefined.

(b) The domain of the radical expression $\sqrt{7-x}$ is all real numbers x such that $x \leq 7$, because the square root of a negative number is not a real number.

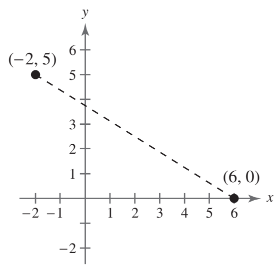
19. (a) $\dfrac{16}{\sqrt[3]{16}} = \dfrac{16}{\sqrt[3]{8 \cdot 2}} = \dfrac{16}{2}\dfrac{1}{2^{1/3}} \cdot \dfrac{2^{2/3}}{2^{2/3}}$
$= 4 \cdot 2^{2/3} = 4\sqrt[3]{4}$

(b) $\dfrac{6}{1-\sqrt{3}} = \dfrac{6}{1-\sqrt{3}} \cdot \dfrac{1+\sqrt{3}}{1+\sqrt{3}}$
$= \dfrac{6(1+\sqrt{3})}{1-3} = -3(1+\sqrt{3})$

(c) $\dfrac{1}{\sqrt{x+2}-\sqrt{2}} \cdot \dfrac{\sqrt{x+2}+\sqrt{2}}{\sqrt{x+2}+\sqrt{2}} = \dfrac{\sqrt{x+2}+\sqrt{2}}{(x+2)-2}$
$= \dfrac{\sqrt{x+2}+\sqrt{2}}{x}$

20. Shaded region $=$ (area big triangle)
$-$ (area small triangle)
$= \frac{1}{2}(3x)(\sqrt{3}x) - \frac{1}{2}(2x)\left(\frac{2}{3}\sqrt{3}x\right)$
$= \frac{1}{2}3\sqrt{3}x^2 - \frac{2}{3}\sqrt{3}x^2$
$= \left(\frac{3}{2} - \frac{2}{3}\right)\sqrt{3}x^2 = \frac{5}{6}\sqrt{3}x^2$

21.



Midpoint: $\left(\dfrac{-2+6}{2}, \dfrac{5+0}{2}\right) = \left(2, \dfrac{5}{2}\right)$

$$d = \sqrt{(-2-6)^2 + (5-0)^2}$$
$$= \sqrt{64+25} = \sqrt{89} \approx 9.43$$

22. (a) The endpoints of a diameter $(-3, 4)$ and $(1, -8)$ shifted five units to the left are $(-3 - 5, 4) = (-8, 4)$ and $(1 - 5, -8) = (-4, -8)$.

(b) Use the midpoint of the diameter to find the center.

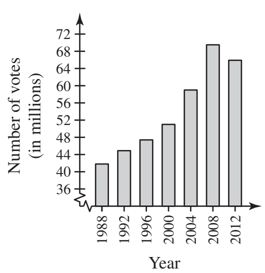
$$(h, k) = \left(\frac{-8 + (-4)}{2}, \frac{4 + (-8)}{2}\right) = (-6, -2)$$

Use the distance from the center to an endpoint of a diameter to find the radius.

$$d = \sqrt{\left[-6 - (-8)\right]^2 + (-2 - 4)^2} = \sqrt{(2)^2 + (-6)^2} = \sqrt{40}$$

So, the equation of the circle is $(x + 6)^2 + (y + 2)^2 = 40$.

23.



CHAPTER 1
Functions and Their Graphs

CHAPTER 1
Functions and Their Graphs

Section 1.1 Graphs of Equations

1. solution point

2. graph

3. Three common approaches that can be used to solve problems mathematically are algebraic, graphical, and numerical.

4. Steps sketching the graph of an equation by point-plotting are:

1. If possible, rewrite the equation so that one of the variables is isolated on one side of the equation.
2. Make a table of values showing several solution points.
3. Plot these points on a rectangular coordinate system.
4. Connect the points with a smooth curve or line.

5. $y = \sqrt{x+4}$

(a) $(0, 2): 2 \stackrel{?}{=} \sqrt{0+4}$

$2 \stackrel{?}{=} \sqrt{4}$

$2 = 2$

Yes, the point is on the graph.

(b) $(12, 4): 4 \stackrel{?}{=} \sqrt{12+4}$

$4 \stackrel{?}{=} \sqrt{16}$

$4 = 4$

Yes, the point is on the graph.

6. $y = \sqrt{5-x}$

(a) $(1,\ 2):\ 2 \stackrel{?}{=} \sqrt{5-(1)}$

$2 \stackrel{?}{=} \sqrt{4}$

$2 = 2$

Yes, the point is on the graph.

(b) $(5,\ 0):\ 0 \stackrel{?}{=} \sqrt{5-(5)}$

$0 \stackrel{?}{=} \sqrt{0}$

$0 = 0$

Yes, the point is on the graph.

7. $y = 4 - |x-2|$

(a) $(1, 5): 5 \stackrel{?}{=} 4 - |1-2|$

$5 \neq 4 - 1$

No, the point is not on the graph.

(b) $(1.2, 3.2): 3.2 \stackrel{?}{=} 4 - |1.2-2|$

$3.2 \stackrel{?}{=} 4 - |-0.8|$

$3.2 \stackrel{?}{=} 4 - 0.8$

$3.2 \stackrel{?}{=} 3.2$

Yes, the point is on the graph

8. $y = |x-1| + 2$

(a) $(2, 1): 1 \stackrel{?}{=} |(2) - 1| + 2$

$1 \stackrel{?}{=} 1 + 2$

$1 \neq 3$

No, the point is not on the graph.

(b) $(3.2, 4.2): 4.2 \stackrel{?}{=} |(3.2) - 1| + 2$

$4.2 \stackrel{?}{=} 2.2 + 2$

$4.2 = 4.2$

Yes, the point is on the graph.

9. $2x - y - 3 = 0$

(a) $(1,\ 2): 2(1) - 2 - 3 \stackrel{?}{=} 0$

$-3 \neq 0$

No, the point is not on the graph.

(b) $(1,\ -1): 2(1) - (-1) - 3 \stackrel{?}{=} 0$

$0 = 0$

Yes, the point is on the graph.

10. $x^2 + y^2 = 20$

(a) $(3,\ -2): 3^2 + (-2)^2 \stackrel{?}{=} 20$

$9 + 4 \stackrel{?}{=} 20$

$13 \neq 20$

No, the point is not on the graph.

(b) $(-4,\ 2): (-4)^2 + 2^2 \stackrel{?}{=} 20$

$16 + 4 \stackrel{?}{=} 20$

$20 = 20$

Yes, the point is on the graph.

11. $y = x^2 - 3x + 2$

(a) $\left(\frac{5}{2}, \frac{3}{4}\right): \frac{3}{4} \stackrel{?}{=} \left(\frac{5}{2}\right)^2 - 3\left(\frac{5}{2}\right) + 2$

$$\frac{3}{4} \stackrel{?}{=} \frac{25}{4} - \frac{15}{2} + 2$$

$$\frac{3}{4} = \frac{3}{4}$$

Yes, the point is on the graph.

(b) $(-2, 8): 8 \stackrel{?}{=} (-2)^2 - 3(-2) + 2$

$$8 \stackrel{?}{=} 4 + 6 + 2$$

$$8 \neq 12$$

No, the point is not on the graph.

12. $y = \frac{1}{3}x^3 - 2x^2$

(a) $\left(2, -\frac{16}{3}\right): \frac{1}{3}(2)^3 - 2(2)^2 \stackrel{?}{=} -\frac{16}{3}$

$$\frac{1}{3} \cdot 8 - 2 \cdot 4 \stackrel{?}{=} -\frac{16}{3}$$

$$\frac{8}{3} - 8 \stackrel{?}{=} -\frac{16}{3}$$

$$\frac{8}{3} - \frac{24}{3} \stackrel{?}{=} -\frac{16}{3}$$

$$-\frac{16}{3} = -\frac{16}{3}$$

Yes, the point is on the graph.

(b) $(-3, 9): \frac{1}{3}(-3)^3 - 2(-3)^2 \stackrel{?}{=} 9$

$$\frac{1}{3}(-27) - 2(9) \stackrel{?}{=} 9$$

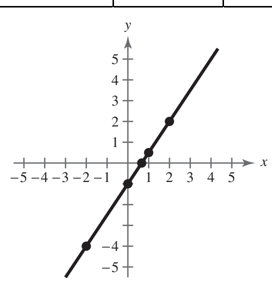
$$-9 - 18 \stackrel{?}{=} 9$$

$$-27 \neq 9$$

No, the point is not on the graph.

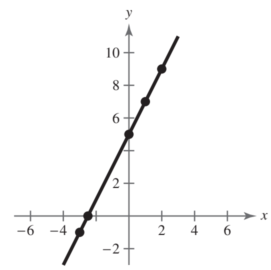
13. $3x - 2y = 2 \Rightarrow y = \frac{3}{2}x - 1$

x	-2	0	$\frac{2}{3}$	1	2
y	-4	-1	0	$\frac{1}{2}$	2
Solution point	$(-2, -4)$	$(0, -1)$	$\left(\frac{2}{3}, 0\right)$	$\left(1, \frac{1}{2}\right)$	$(2, 2)$



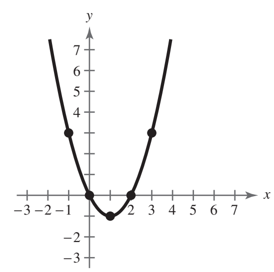
14. $-4x + 2y = 10 \Rightarrow y = 2x + 5$

x	-3	$-\frac{5}{2}$	0	1	2
y	-1	0	5	7	9
Solution point	$(-3, -1)$	$\left(-\frac{5}{2}, 0\right)$	$(0, 5)$	$(1, 7)$	$(2, 9)$



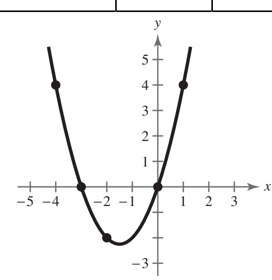
15. $2x + y = x^2 \Rightarrow y = x^2 - 2x$

x	-1	0	1	2	3
y	3	0	-1	0	3
Solution point	$(-1, 3)$	$(0, 0)$	$(1, -1)$	$(2, 0)$	$(3, 3)$



16. $6x - 2y = -2x^2 \Rightarrow y = x^2 + 3x$

x	-4	-3	-2	0	1
y	4	0	-2	0	4
Solution point	$(-4, 4)$	$(-3, 0)$	$(-2, -2)$	$(0, 0)$	$(1, 4)$



17. $y = 2\sqrt{x}$ has one intercept $(0, 0)$.

Matches graph (b).

18. $y = 4 - x^2$ has intercepts $(0, 4)$, $(2, 0)$, and $(-2, 0)$.

Matches graph (d).

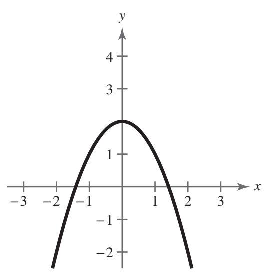
19. $y = \sqrt{9 - x^2}$ has intercepts $(0, 3)$, $(-3, 0)$, and $(3, 0)$.
Matches graph (c).

20. $y = |x| - 3$ has intercepts $(0, -3)$, $(3, 0)$, and $(-3, 0)$.
Matches graph (a).

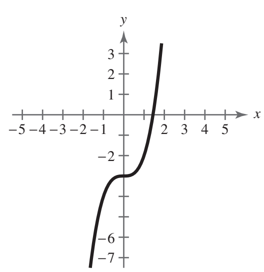
21. $y = x^3 - 3x$ has intercepts $(-\sqrt{3}, 0)$, $(0, 0)$, and $(\sqrt{3}, 0)$.
Matches graph (e).

22. $x = 5 - y^2$ has intercepts $(5, 0)$, $(0, \sqrt{5})$, and $(0, -\sqrt{5})$.
Matches graph (f).

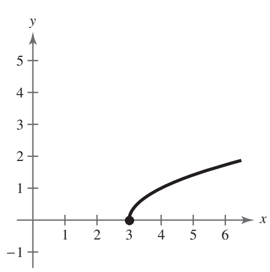
23. $y = 2 - x^2$



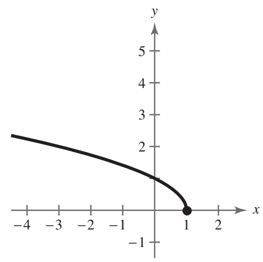
24. $y = x^3 - 3$



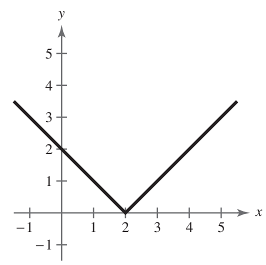
25. $y = \sqrt{x - 3}$



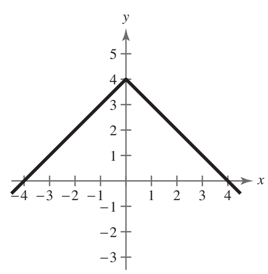
26. $y = \sqrt{1 - x}$



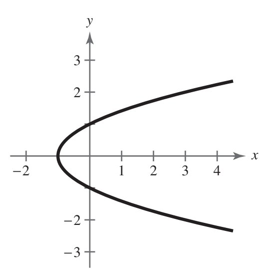
27. $y = |x - 2|$



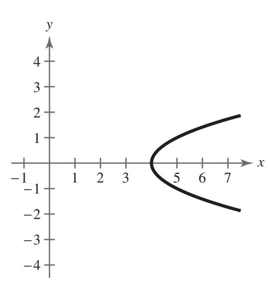
28. $y = 4 - |x|$



29. $x = y^2 - 1$

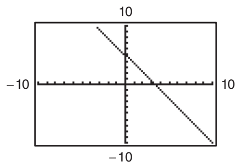


30. $x = y^2 + 4$



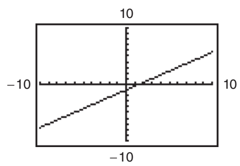
31. $y = 5 - \frac{3}{2}x$

Intercepts: $\left(\frac{10}{3}, 0\right), (0, 5)$



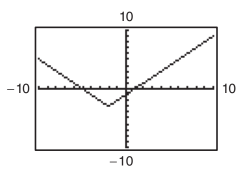
32. $y = \dfrac{2}{3}x - 1$

Intercepts: $(0, -1), \left(\dfrac{3}{2}, 0\right)$



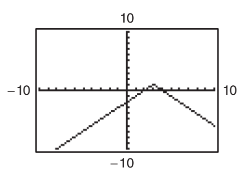
33. $y = |x + 2| - 3$

Intercepts: $(-5, 0), (1, 0), (0, -1)$



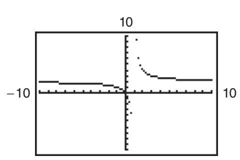
34. $y = -|x - 3| + 1$

Intercepts: $(2, 0), (4, 0), (0, -2)$



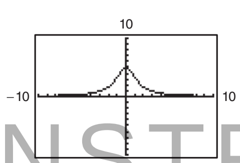
35. $y = \dfrac{2x}{x - 1}$

Intercept: $(0, 0)$



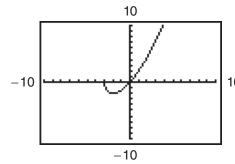
36. $y = \dfrac{10}{x^2 + 2}$

Intercept: $(0, 5)$



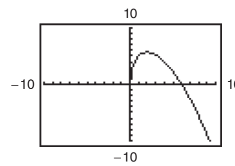
37. $y = x\sqrt{x + 3}$

Intercepts: $(0, 0), (-3, 0)$



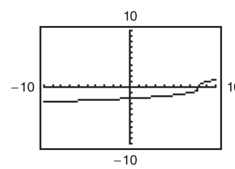
38. $y = (6 - x)\sqrt{x}$

Intercepts: $(0, 0), (6, 0)$



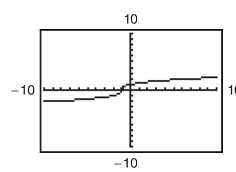
39. $y = \sqrt[3]{x - 8}$

Intercepts: $(8, 0), (0, -2)$



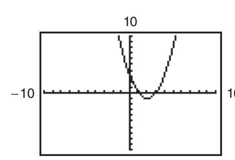
40. $y = \sqrt[3]{x + 1}$

Intercepts: $(-1, 0), (0, 1)$



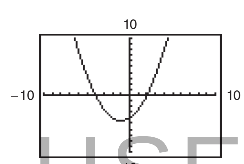
41. $y = x^2 - 4x + 3$

Intercepts: $(3, 0), (1, 0), (0, 3)$



42. $y = \dfrac{x^2 + 2x - 8}{2}$

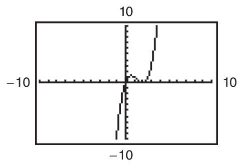
Intercepts: $(2, 0), (-4, 0), (0, -4)$



43. $y = x^2(x-4) + 4x$

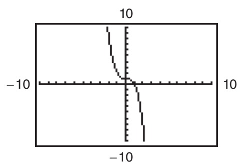
$= x^3 - 4x^2 + 4x$

Intercepts: $(0, 0)$, $(2, 0)$



44. $y = 1 - x^3$

Intercepts: $(0, 1)$, $(1, 0)$



45. $y = -10x + 50$

Range/Window

Xmin = −10
Xmax = 10
Xscl = 2
Ymin = −50
Ymax = 100
Yscl = 25

46. $y = \sqrt{x+2} - 1$

Range/Window

Xmin = −5
Xmax = 1
Xscl = 1
Ymin = −3
Ymax = 1
Yscl = 1

47. $y_1 = \frac{1}{4}(x^2 - 8)$

$y_2 = \frac{1}{4}x^2 - 2$

Graphing these equations with a graphing utility shows that the graphs are identical. The Distributive Property is illustrated.

48. $y_1 = \frac{1}{2}x + (x+1)$

$y_2 = \frac{3}{2}x + 1$

Graphing these equations with a graphing utility shows that their graphs are identical. The Associative Property of Addition is illustrated.

49. $y_1 = \frac{1}{5}\left[10(x^2 - 1)\right]$

$y_2 = 2(x^2 - 1)$

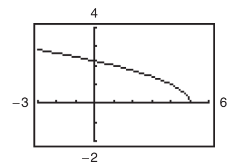
Graphing these equations with a graphing utility shows that the graphs are identical. The Associative Property of Multiplication is illustrated.

50. $y_1 = (x^2 + 3) \cdot \dfrac{1}{x^2 + 3}$

$y_2 = 1$

Graphing these equations with a graphing utility shows that their graphs are identical. The Multiplicative Inverse Property is illustrated.

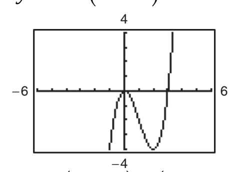
51. $y = \sqrt{5 - x}$



(a) $(3, y) \approx (3, 1.41)$

(b) $(x, 3) = (-4, 3)$

52. $y = x^2(x - 3)$



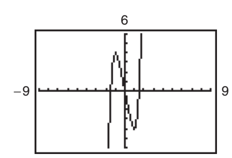
(a) $(-1, y) = (-1, -4)$

(b) $(x, 6) \approx (3.49, 6)$

53. $y = x^5 - 5x$

(a) $(-0.5, y) \approx (-0.5, 2.47)$

(b) $(x, -2) \approx (-1.58, -2), (0.40, -2), (1.37, -2)$

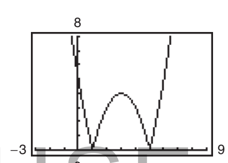


54. $y = \left|x^2 - 6x + 5\right|$

(a) $(2, y) \approx (2, 3)$

(b) $(x, 1.5) \approx (0.65, 1.5)$, $(1.42, 1.5)$

$(4.58, 1.5)$, $(5.35, 1.5)$

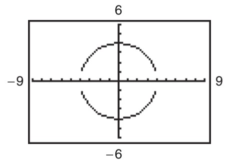


55. $x^2 + y^2 = 16$

$y^2 = 16 - x^2$

$y = \pm\sqrt{16 - x^2}$

Use $y_1 = \sqrt{16 - x^2}$ and $y_2 = -\sqrt{16 - x^2}$.

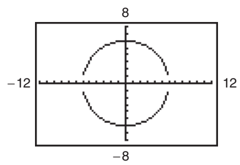


56. $x^2 + y^2 = 36$

$y^2 = 36 - x^2$

$y = \pm\sqrt{36 - x^2}$

Use $y_1 = \sqrt{36 - x^2}$ and $y_2 = -\sqrt{36 - x^2}$.



57. $(x-1)^2 + (y-2)^2 = 49$

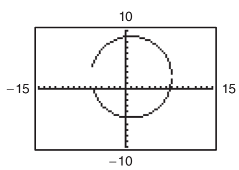
$(y-2)^2 = 49 - (x-1)^2$

$y - 2 = \pm\sqrt{49 - (x-1)^2}$

$y = 2 \pm \sqrt{49 - (x-1)^2}$

Use $y_1 = 2 + \sqrt{49 - (x-1)^2}$ and

$y_2 = 2 - \sqrt{49 - (x-1)^2}$.



58. $(x-3)^2 + (y-1)^2 = 25$

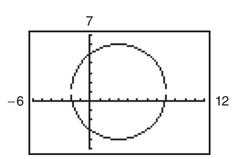
$(y-1)^2 = 25 - (x-3)^2$

$y - 1 = \pm\sqrt{25 - (x-3)^2}$

$y = 1 \pm \sqrt{25 - (x-3)^2}$

Use $y_1 = 1 + \sqrt{25 - (x-3)^2}$ and

$y_2 = 1 - \sqrt{25 - (x-3)^2}$.



59. $(x-1)^2 + (y-2)^2 = 25$

(a) $(1, 3): (1-1)^2 + (3-2)^2 \stackrel{?}{=} 25$

$1 \neq 25$

No

(b) $(-2, 6): (-2-1)^2 + (6-2)^2 \stackrel{?}{=} 25$

$(-3)^2 + (4)^2 \stackrel{?}{=} 25$

$25 = 25$

Yes

(c) $(5, -1): (5-1)^2 + (-1-2)^2 \stackrel{?}{=} 25$

$(4)^2 + (-3)^2 \stackrel{?}{=} 25$

$25 = 25$

Yes

(d) $\left(0, 2+2\sqrt{6}\right): (0-1)^2 + \left(2+2\sqrt{6}-2\right)^2 \stackrel{?}{=} 25$

$(-1)^2 + \left(2\sqrt{6}\right)^2 \stackrel{?}{=} 25$

$25 = 25$

Yes

60. $(x+2)^2 + (y-3)^2 = 25$

(a) $(-2, 3): (-2+2)^2 + (3-3)^2 = 0 \neq 25$ No

(b) $(0, 0): (0+2)^2 + (0-3)^2 = 4 + 9$

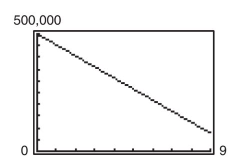
$= 13 \neq 25$ No

(c) $(1, -1): (1+2)^2 + (-1-3)^2 = 9 + 16 = 25$ Yes

(d) $\left(-1, 3-2\sqrt{6}\right): (-1+2)^2 + \left(3-2\sqrt{6}-3\right)^2$

$= 1 + 24 = 25$ Yes

61. (a)



(b) Using the *value* feature, when $t = 5.8$, $y = 227{,}400$.

Algebraically, $y = 500{,}000 - 47{,}000t$

$= 500{,}000 - 47{,}000(5.8)$

$= 227{,}400.$

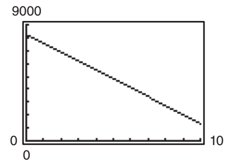
(c) Using the *zoom* and *trace* features, when $y = 156{,}000$, $t \approx 7.3$.

Algebraically, $y = 500{,}000 - 47{,}000t$

$156{,}000 = 500{,}000 - 47{,}000t$

$7.3 \approx t.$

62. (a)



(b) Using the *zoom* and *trace* features, when $y = 5545.25$, $t \approx 3.93$.

Algebraically,
$$y = 8250 - 689t$$
$$5545.25 = 8250 - 689t$$
$$3.93 \approx t.$$

(c) Using the *value* feature, when $t = 5.5$, $y = 4460.50$.

Algebraically,
$$y = 8250 - 689t$$
$$= 8250 - 689(5.5)$$
$$= 4460.50.$$

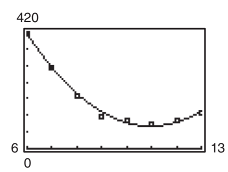
63. (a)

Year	2006	2007	2008	2009
New houses (in thousands)	410.5	290.9	198.1	132.1

Year	2010	2011	2012	2013
New houses (in thousands)	93.0	80.7	95.3	136.7

The model fits the data well.

(b)



The model fits the data well.

(c) In 2015, $t = 15$.

$$y = 13.42(15)^2 - 294.1(15) + 1692$$
$$= 30 \Rightarrow \$300{,}000$$

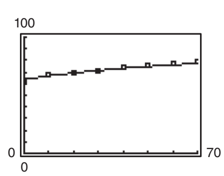
In 2017, $t = 17$.

$$y = 13.42(17)^2 - 294.1(17) + 1692$$
$$= 570.68 \Rightarrow \$570{,}680$$

Yes, the answers seem reasonable. Answers will vary.

(d) Using the *zoom* and *trace* features, there were 100,000 new houses during the years 2009 and 2012.

64. (a)



The model fits the data well.

(b) When $t = 0$, $y = \dfrac{63.6 + 0.97(0)}{1 + 0.01(0)} = 63.6$.

The y-intercept is 63.6, which represents the life expectancy in 1940.

(c) Using the *zoom* and *trace* features, when $y = 70.1$, $t = 24.2$. Algebraically,

$$y = \frac{63.6 + 0.97t}{1 + 0.01t}$$
$$70.1 = \frac{63.6 + 0.97t}{1 + 0.01t}$$
$$70.1 + 0.701t = 63.6 + 0.97t$$
$$6.5 = 0.269t$$
$$24.2 \approx t$$

So, in the year 1964, the life expectancy was 70.1.

(d) Graphically, when $t = 38$, $y = 72.8$. Algebraically,

$$y = \frac{63.6 + 0.97t}{1 + 0.01t}$$
$$= \frac{63.6 + 0.97(38)}{1 + 0.01(38)}$$
$$= 72.8.$$

So, in the year 1978, the life expectancy was 72.8.

65. False. $y = x^2 - 1$ has two x-intercepts, $(1, 0)$ and $(-1, 0)$. Also, $y = x^2 + 1$ has no x-intercepts.

66. False. The line $y = 0$ has an infinite number of x-intercepts.

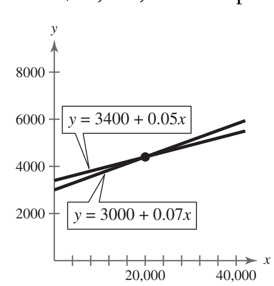
67. Option 1: $w_1 = 3000 + 0.07x$

Option 2: $w_2 = 3400 + 0.05x$

(x is amount of sales)

$$w_1 = w_2$$
$$3000 + 0.07x = 3400 + 0.05x$$
$$0.02x = 400$$
$$x = 20{,}000$$

If sales equal \$20,000, the options are equivalent. For sales less than \$20,000, choose option 2. For sales greater than \$20,000, choose option 1.



68. (a)

Xmin = −9
Xmax = 9
Xscl = 1
Ymin = −6
Ymax = 6
Yscl = 1

(b) x-intercepts: $(-1, 0), (3, 0)$

y-intercept: $(0, -3)$

(c) $(-4, 1)$: $1 \stackrel{?}{=} [(-4) - 1]^2 - 4$

$1 \stackrel{?}{=} 25 - 4$

$1 \neq 21$

No, the point is not on the graph.

$(2, -3)$: $-3 \stackrel{?}{=} [(2) - 1]^2 - 4$

$-3 \stackrel{?}{=} 1 - 4$

$-3 = -3$

Yes, the point is on the graph.

69. $(9x - 4) + (2x^2 - x + 15) = 2x^2 + 8x + 11$

70. $(3x^2 - 5)(-x^2 + 1) = -3x^4 + 5x^2 + 3x^2 - 5$

$= -3x^4 + 8x^2 - 5$

Section 1.2 Lines in the Plane

1. (a) iii (b) i (c) v (d) ii (e) iv

2. slope

3. parallel

4. They are perpendicular to each other.

5. Since $x = 3$ is a vertical line, all horizontal lines are perpendicular and have slope $m = 0$.

6. Since the line $y - (-1) = \frac{1}{4}(x - 8)$ is in point-slope form, the point $(8, -1)$ lies on the line.

7. (a) $m = \frac{2}{3}$. Since the slope is positive, the line rises. Matches L_2.

(b) m is undefined. The line is vertical. Matches L_3.

(c) $m = -2$. The line falls. Matches L_1.

8. (a) $m = 0$. The line is horizontal. Matches L_2.

(b) $m = -\frac{3}{4}$. Because the slope is negative, the line falls. Matches L_1.

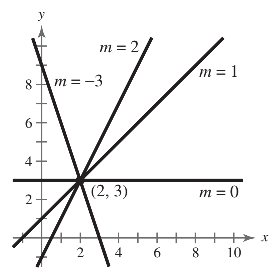
(c) $m = 1$. Because the slope is positive, the line rises. Matches L_3.

9. $\text{Slope} = \frac{\text{rise}}{\text{run}} = \frac{3}{2}$

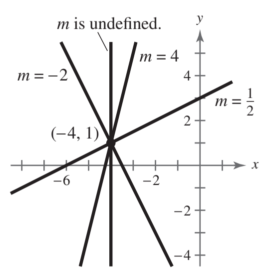
10. The line appears to go through $(0, 8)$ and $(2, 0)$.

$\text{Slope} = \frac{8 - 0}{0 - 2} = -4$

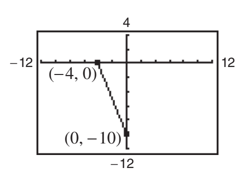
11.



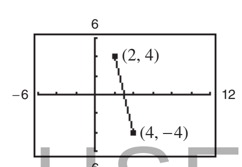
12.



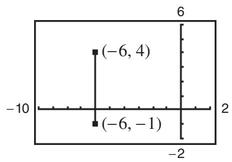
13. $\text{Slope} = \frac{0 - (-10)}{-4 - 0} = \frac{10}{-4} = -\frac{5}{2}$



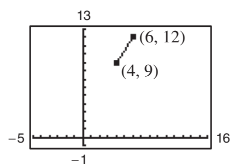
14. $\text{Slope} = \frac{-4 - 4}{4 - 2} = -4$



15. Slope $= \dfrac{4 - 1}{-6 - (-6)} = \dfrac{3}{0}$; slope is undefined.



16. Slope $= \dfrac{12 - 9}{6 - 4} = \dfrac{3}{2}$



17. Since $m = 0$, y does not change. Three additional points are $(0, 1)$, $(3, 1)$, and $(-1, 1)$.

18. Since $m = 0$, y does not change. Three additional points are $(0, -2)$, $(1, -2)$, and $(4, -2)$.

19. Since m is undefined, x does not change and the line is vertical. Three additional points are $(1, 1)$, $(1, 2)$, and $(1, 3)$.

20. Because m is undefined, x does not change. Three additional points are $(-4, 0)$, $(-4, 3)$, and $(-4, 5)$.

21. Since $m = -2$, y decreases 2 for every unit increase in x. Three additional points are $(1, -11)$, $(2, -13)$, and $(3, -15)$.

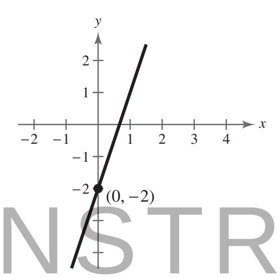
22. Since $m = 4$, y increases 4 for every unit increase in x. Three additional points are $(-4, 8)$, $(-3, 12)$, and $(-2, 16)$.

23. Since $m = \dfrac{1}{2}$, y increases 1 for every increase of 2 units in x. Three additional points are $(9, -1)$, $(11, 0)$, and $(13, 1)$.

24. Since $m = -\dfrac{1}{3}$, y decreases 1 for every increase of 3 units in x. Three additional points are $(2, -7)$, $(5, -8)$, and $(8, -9)$.

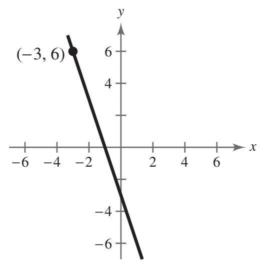
25. $m = 3$, $(0, -2)$

$$y + 2 = 3(x - 0)$$
$$y = 3x - 2 \Rightarrow 3x - y - 2 = 0$$



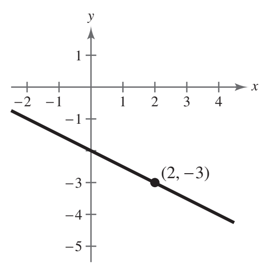
26. $m = -3$, $(-3, 6)$

$$y - 6 = -3(x + 3)$$
$$y - 6 = -3x - 9$$
$$y = -3x - 3$$



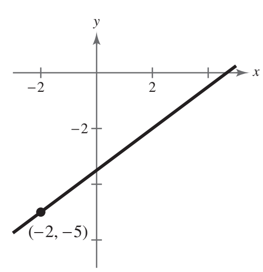
27. $m = -\dfrac{1}{2}$, $(2, -3)$

$$y - (-3) = -\frac{1}{2}(x - 2)$$
$$y + 3 = -\frac{1}{2}x + 1$$
$$y = -\frac{1}{2}x - 2$$



28. $m = \dfrac{3}{4}$, $(-2, -5)$

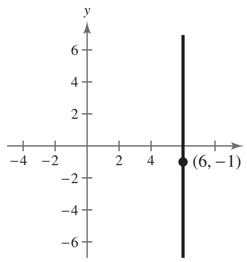
$$y + 5 = \frac{3}{4}(x + 2)$$
$$y - 5 = \frac{3}{4}x + \frac{3}{2}$$
$$y = \frac{3}{4}x - \frac{7}{2}$$



29. m is undefined, $(6, -1)$

$x = 6$

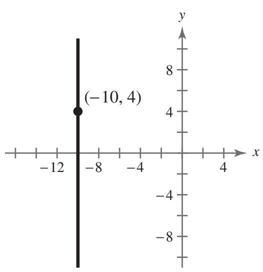
vertical line



30. m is undefined, $(-10, 4)$

$x = 10$

vertical line

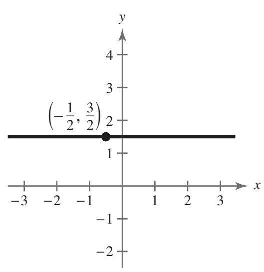


31. $m = 0, \left(-\frac{1}{2}, \frac{3}{2}\right)$

$y - \frac{3}{2} = 0\left(x + \frac{1}{2}\right)$

$y - \frac{3}{2} = 0$

$y = \frac{3}{2}$ horizontal line

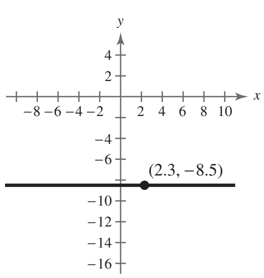


32. $m = 0, (2.3, -8.5)$

$y - (-8.5) = 0(x - 2.3)$

$y + 8.5 = 0$

$y = -8.5$ horizontal line



33. Begin by letting $x = 7$ correspond to 2007. Then using the points $(7, 1.5)$ and $(13, 1.7)$, you have

$$m = \frac{1.7 - 1.5}{13 - 7} = \frac{0.2}{6} = \frac{1}{30}$$

$$y - 1.5 = \frac{1}{30}(x - 7)$$

$$y - 1.5 = \frac{1}{30}x - \frac{7}{30}$$

$$y = \frac{1}{30}x + \frac{19}{15}$$

When $x = 19$: $y = \frac{1}{30}(19) + \frac{19}{15} = \$1.9$ million

34. Begin by letting $x = 4$ correspond to 2004. Then using the points $(4, 348{,}000)$ and $(13, 555{,}000)$, you have

$$m = \frac{555{,}000 - 348{,}000}{13 - 4} = \frac{207{,}000}{9} = 23{,}000$$

$$y - 348{,}000 = 23{,}000(x - 4)$$

$$y - 348{,}000 = 23{,}000x - 92{,}000$$

$$y = 23{,}000x + 256{,}000$$

When $x = 19$:

$$y = 23{,}000(19) + 256{,}000 = \$693{,}000$$

35. $2x - 3y = 9$

$-3y = -2x + 9$

$y = \frac{2}{3}x - 3$

Slope: $\frac{2}{3}$

y-intercept: $(0, -3)$

The line passes through $(0, -3)$ and rises 2 units for each horizontal increase of 3 units.

36. $3x + 4y = 1$

$4y = -3x + 1$

$y = \frac{-3}{4}x + \frac{1}{4}$

Slope: $-\frac{3}{4}$

y-intercept: $\left(0, \frac{1}{4}\right)$

The line passes through $\left(0, \frac{1}{4}\right)$ and falls 3 units for each horizontal increase of 4 units.

37. $2x - 5y + 10 = 0$

$$-5y = -2x - 10$$
$$y = \frac{2}{5}x + 2$$

Slope: $\frac{2}{5}$

y-intercept: $(0, 2)$

The line passes through $(0, 2)$ and rises 2 units for each horizontal increase of 5 units.

38. $4x - 3y - 9 = 0$

$$-3y = -4x + 9$$
$$y = \frac{4}{3}x - 3$$

Slope: $\frac{4}{3}$

y-intercept: $(0, -3)$

The line passes through $(0, -3)$ and rises 4 units for each horizontal increase of 3 units.

39. $x = -6$

Slope is undefined; no y-intercept.

The line is vertical and passes through $(-6, 0)$.

40. $y = 12$

Slope: 0

y-intercept: $(0, 12)$

The line is horizontal and passes through $(0, 12)$.

41. $3y + 2 = 0$

$$3y = -2$$
$$y = -\frac{2}{3}$$

Slope: 0

y-intercept: $\left(0, -\frac{2}{3}\right)$

The line is horizontal and passes through $\left(0, -\frac{2}{3}\right)$.

42. $2x - 5 = 0$

$$2x = 5$$
$$x = \frac{5}{2}$$

Slope is undefined; no y-intercept.

The line is vertical and passes through $\left(\frac{5}{2}, 0\right)$.

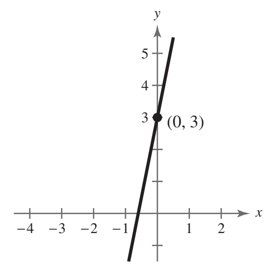
43. $5x - y + 3 = 0$

$$y = 5x + 3$$

(a) Slope: $m = 5$

y-intercept: $(0, 3)$

(b)



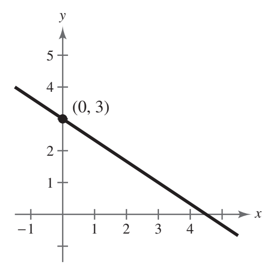
44. $2x + 3y - 9 = 0$

$$3y = -2x + 9$$
$$y = -\frac{2}{3}x + 3$$

(a) Slope: $m = -\frac{2}{3}$

y-intercept: $(0, 3)$

(b)



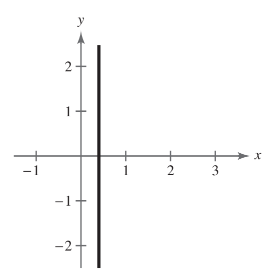
45. $5x - 2 = 0$

$$x = \frac{2}{5}$$

(a) Slope: undefined

No y-intercept

(b)



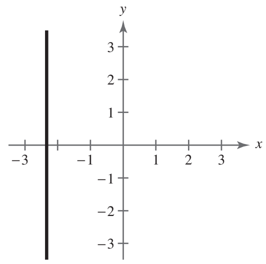
46. $3x + 7 = 0$

$$x = -\frac{7}{3}$$

(a) Slope: undefined

No y-intercept

(b)



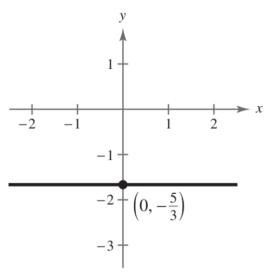
47. $3y + 5 = 0$

$$y = -\frac{5}{3}$$

(a) Slope: $m = 0$

y-intercept: $\left(0, -\frac{5}{3}\right)$

(b)



48. $-11 - 4y = 0$

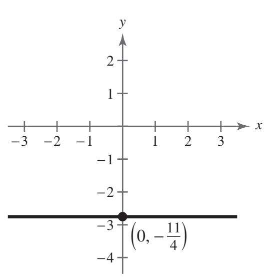
$$-4y = 11$$

$$y = -\frac{11}{4}$$

(a) Slope: $m = 0$

y-intercept: $\left(0, -\frac{11}{4}\right)$

(b)



49. The slope is $\dfrac{-3 - (-7)}{1 - (-1)} = \dfrac{4}{2} = 2.$

$$y - (-3) = 2(x - 1)$$

$$y + 3 = 2x - 2$$

$$y = 2x - 5$$

50. The slope is $\dfrac{-1 - \frac{3}{2}}{4 - (-1)} = \dfrac{-\frac{5}{2}}{5} = -\dfrac{1}{2}.$

$$y - (-1) = -\frac{1}{2}(x - 4)$$

$$y + 1 = -\frac{1}{2}x + 2$$

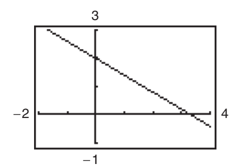
$$y = -\frac{1}{2}x + 1$$

51. $(5, -1), (-5, 5)$

$$y + 1 = \frac{5 + 1}{-5 - 5}(x - 5)$$

$$y = -\frac{3}{5}(x - 5) - 1$$

$$y = -\frac{3}{5}x + 2$$

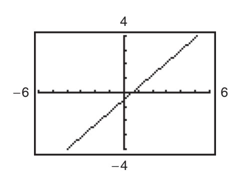


52. $(4, 3), (-4, -4)$

$$y - 3 = \frac{-4 - 3}{-4 - 4}(x - 4)$$

$$y - 3 = \frac{7}{8}(x - 4)$$

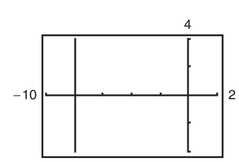
$$y = \frac{7}{8}x - \frac{1}{2}$$



53. $(-8, 1), (-8, 7)$

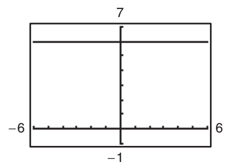
Since both points have an x-coordinate of -8, the slope is undefined and the line is vertical.

$$x + 8 = 0$$



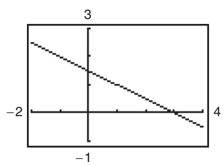
54. $(-1, 6), (5, 6)$

$$y - 6 = \frac{6 - 6}{5 - (-1)}(x - (-1))$$
$$y - 6 = 0(x + 1)$$
$$y - 6 = 0$$
$$y = 6$$



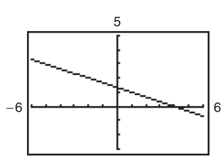
55. $\left(2, \frac{1}{2}\right), \left(\frac{1}{2}, \frac{5}{4}\right)$

$$y - \frac{1}{2} = \frac{\frac{5}{4} - \frac{1}{2}}{\frac{1}{2} - 2}(x - 2)$$
$$y = -\frac{1}{2}(x - 2) + \frac{1}{2}$$
$$y = -\frac{1}{2}x + \frac{3}{2}$$



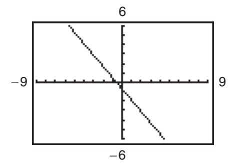
56. $(1, 1), \left(6, -\frac{2}{3}\right)$

$$y - 1 = \frac{-\frac{2}{3} - 1}{6 - 1}(x - 1)$$
$$y - 1 = -\frac{1}{3}(x - 1)$$
$$y - 1 = -\frac{1}{3}x + \frac{1}{3}$$
$$y = -\frac{1}{3}x + \frac{4}{3}$$



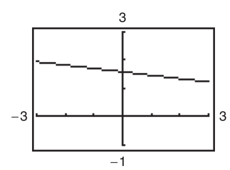
57. $\left(-\frac{1}{10}, -\frac{3}{5}\right), \left(\frac{9}{10}, -\frac{9}{5}\right)$

$$y + \frac{3}{5} = \frac{-\frac{9}{5} + \frac{3}{5}}{\frac{9}{10} + \frac{1}{10}}\left(x + \frac{1}{10}\right)$$
$$y + \frac{3}{5} = -\frac{6}{5}\left(x + \frac{1}{10}\right)$$
$$y = -\frac{6}{5}x - \frac{18}{25}$$



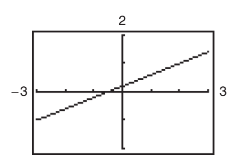
58. $\left(\frac{3}{4}, \frac{3}{2}\right), \left(-\frac{4}{3}, \frac{7}{4}\right)$

$$y - \frac{3}{2} = \frac{\frac{7}{4} - \frac{3}{2}}{-\frac{4}{3} - \frac{3}{4}}\left(x - \frac{3}{4}\right)$$
$$y - \frac{3}{2} = -\frac{3}{25}\left(x - \frac{3}{4}\right)$$
$$y - \frac{3}{2} = -\frac{3}{25}x + \frac{9}{100}$$
$$y = -\frac{3}{25}x + \frac{159}{100}$$



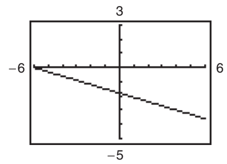
59. $(1, 0.6), (-2, -0.6)$

$$y - 0.6 = \frac{-0.6 - 0.6}{-2 - 1}(x - 1)$$
$$y = 0.4(x - 1) + 0.6$$
$$y = 0.4x + 0.2$$

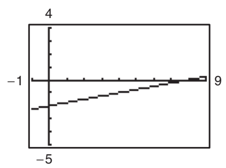


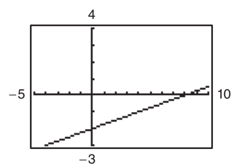
60. $(-8, 0.6),\ (2, -2.4)$

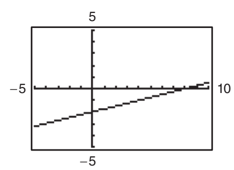
$$y - 0.6 = \frac{-2.4 - 0.6}{2 - (-8)}(x + 8)$$
$$y - 0.6 = -0.3(x + 8)$$
$$y = -0.3x - 1.8$$



61.

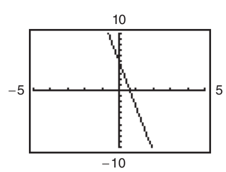


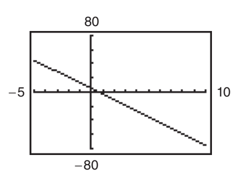


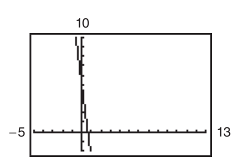


The first graph does not show both intercepts. The third graph is best because it shows both intercepts and gives the most accurate view of the slope by using a square setting.

62.







The second graph does not give a good view of the intercepts. The third graph is best because it gives the most accurate view of the slope by using a square setting.

63. L_1: $(0, -1),\ (5, 9)$

$$m_1 = \frac{9 + 1}{5 - 0} = 2$$

L_2: $(0, 3),\ (4, 1)$

$$m_2 = \frac{1 - 3}{4 - 0} = -\frac{1}{2} = -\frac{1}{m_1}$$

L_1 and L_2 are perpendicular.

64. L_1: $(-2, -1),\ (1, 5)$

$$m_1 = \frac{5 - (-1)}{1 - (-2)} = \frac{6}{3} = 2$$

L_2: $(1, 3),\ (5, -5)$

$$m_2 = \frac{-5 - 3}{5 - 1} = \frac{-8}{4} = -2$$

The lines are neither parallel nor perpendicular.

65. L_1: $(3, 6),\ (-6, 0)$

$$m_1 = \frac{0 - 6}{-6 - 3} = \frac{2}{3}$$

L_2: $(0, -1),\ \left(5, \frac{7}{3}\right)$

$$m_2 = \frac{\frac{7}{3} + 1}{5 - 0} = \frac{2}{3} = m_1$$

L_1 and L_2 are parallel.

66. L_1: $(4, 8),\ (-4, 2)$

$$m_1 = \frac{2 - 8}{-4 - 4} = \frac{-6}{-8} = \frac{3}{4}$$

L_2: $(3, -5),\ \left(-1, \frac{1}{3}\right)$

$$m_2\text{: } \frac{(1/3) - (-5)}{-1 - 3} = \frac{16/3}{-4} = -\frac{4}{3}$$

The lines are perpendicular.

67. $4x - 2y = 3$

$$y = 2x - \frac{3}{2}$$

Slope: $m = 2$

(a) Parallel slope: $m = 2$

$$y - 1 = 2(x - 2)$$
$$y = 2x - 3$$

(b) Perpendicular slope: $m = -\frac{1}{2}$

$$y - 1 = -\frac{1}{2}(x - 2)$$
$$y = -\frac{1}{2}x + 2$$

68. $x + y = 7$

$y = -x + 7$

Slope: $m = -1$

(a) Parallel slope: $m = -1$

$y - 2 = -1(x + 3)$

$y = -x - 1$

(b) Perpendicular slope: $m = 1$

$y - 2 = 1(x + 3)$

$y = x + 5$

69. $3x + 4y = 7$

$y = -\frac{3}{4}x + \frac{7}{4}$

Slope: $m = -\frac{3}{4}$

(a) Parallel slope: $m = -\frac{3}{4}$

$y - \frac{7}{8} = -\frac{3}{4}\left(x + \frac{2}{3}\right)$

$y = -\frac{3}{4}x + \frac{3}{8}$

(b) Perpendicular slope: $m = \frac{4}{3}$

$y - \frac{7}{8} = \frac{4}{3}\left(x + \frac{2}{3}\right)$

$y = \frac{4}{3}x + \frac{127}{72}$

70. $3x - 2y = 6$

$y = \frac{3}{2}x - 6$

Slope: $m = \frac{3}{2}$

(a) Parallel slope: $m = \frac{3}{2}$

$y + 1 = \frac{3}{2}\left(x - \frac{2}{5}\right)$

$y = \frac{3}{2}x - \frac{8}{5}$

(b) Perpendicular slope: $m = -\frac{2}{3}$

$y + 1 = -\frac{2}{3}\left(x - \frac{2}{5}\right)$

$y = -\frac{2}{3}x - \frac{11}{15}$

71. $6x + 5y = 9$

$5y = -6x + 9$

$y = -\frac{6}{5}x + \frac{9}{5}$

Slope: $m = -\frac{6}{5}$

(a) Parallel slope: $m = -\frac{6}{5}$

$y + 1.4 = -\frac{6}{5}(x + 3.9)$

$y + 1.4 = -\frac{6}{5}x - 4.68$

$y = -\frac{6}{5}x - 6.08$

(b) Perpendicular slope: $m = \frac{5}{6}$

$y + 1.4 = \frac{5}{6}(x + 3.9)$

$y + 1.4 = \frac{5}{6}x + 3.25$

$y = \frac{5}{6}x + 1.85$

72. $5x + 4y = 1$

$y = -\frac{5}{4}x + \frac{1}{4}$

Slope: $m = -\frac{5}{4} = -1.25$

(a) Parallel slope: $m = -\frac{5}{4}$

$y - 2.4 = -\frac{5}{4}(x + 1.2)$

$y = -1.25x + 0.9$

(b) Perpendicular slope: $m = 0.8$

$y - 2.4 = 0.8(x + 1.2)$

$y = 0.8x + 3.36$

73. $x - 4 = 0$ vertical line

Slope is undefined.

(a) $x - 3 = 0$ passes through $(3, -2)$ and is vertical.

(b) $y = -2$ passes through $(3, -2)$ and is horizontal.

74. $y - 2 = 0$

$y = 2$ horizontal line

Slope: $m = 0$

(a) $y = -1$ passes through $(3, -1)$ and is horizontal.

(b) $x - 3 = 0$ passes through $(3, -1)$ and is vertical.

75. $y + 2 = 0$

$y = -2$ horizontal line

Slope: $m = 0$

(a) $y = 1$ passes through $(-5, 1)$ and is horizontal.

(b) $x + 5 = 0$ passes through $(-5, 1)$ and is vertical.

76. $x + 5 = 0$ vertical line

Slope is undefined.

(a) $x + 2 = 0$ passes through $(-2, 4)$ and is vertical.

(b) $y = 4$ passes through $(-2, 4)$ and is horizontal.

77. The slope is 2 and $(-1, -1)$ lies on the line. Hence,

$$y - (-1) = 2(x - (-1))$$
$$y + 1 = 2(x + 1)$$
$$y = 2x + 1.$$

78. The slope is -2 and $(-1, 1)$ lies on the line. Hence,

$$y - 1 = -2(x - (-1))$$
$$y - 1 = -2(x + 1)$$
$$y = -2x - 1.$$

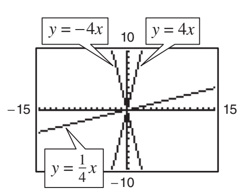
79. The slope of the given line is 2. Then y_2 has slope $-\frac{1}{2}$. Hence,

$$y - 2 = -\frac{1}{2}(x - (-2))$$
$$y - 2 = -\frac{1}{2}(x + 2)$$
$$y = -\frac{1}{2}x + 1.$$

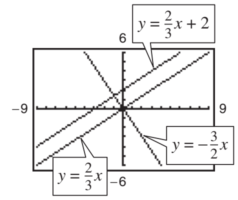
80. The slope of the given line is 3. Then y_2 has slope $-\frac{1}{3}$. Hence,

$$y - 5 = -\frac{1}{3}(x - (-3))$$
$$y - 5 = -\frac{1}{3}(x + 3)$$
$$y = -\frac{1}{3}x + 4.$$

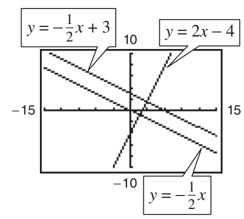
81. The lines $y = -4x$ and $y = \frac{1}{4}x$ are perpendicular.



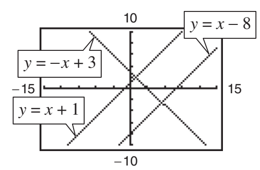
82. The lines $y = \frac{2}{3}x$ and $y = \frac{2}{3}x + 2$ are parallel. Both are perpendicular to $y = -\frac{3}{2}x$.



83. The lines $y = -\frac{1}{2}x$ and $y = -\frac{1}{2}x + 3$ are parallel. Both are perpendicular to $y = 2x - 4$.



84. The lines $y = x - 8$ and $y = x + 1$ are parallel. Both are perpendicular to $y = -x + 3$.



85. $\frac{\text{rise}}{\text{run}} = \frac{3}{4} = \frac{x}{\frac{1}{2}(32)}$

$$\frac{3}{4} = \frac{x}{16}$$
$$4x = 48$$
$$x = 12$$

The maximum height in the attic is 12 feet.

86. $\text{Slope} = \frac{\text{rise}}{\text{run}}$

$$\frac{-12}{100} = \frac{-2000}{x}$$
$$-12x = (-2000)(100)$$
$$x = 16{,}666\frac{2}{3}\text{ ft} \approx 3.16 \text{ miles}$$

87. (a)

Years	Slope
2005–2006	$24.088 - 23.104 = 0.984$
2006–2007	$28.857 - 24.088 = 4.769$
2007–2008	$31.944 - 28.857 = 3.087$
2008–2009	$30.990 - 31.944 = -0.954$
2009–2010	$35.123 - 30.990 = 4.133$
2010–2011	$46.554 - 35.123 = 11.431$
2011–2012	$48.017 - 46.554 = 1.463$

The greatest increase was $11.431 billion from 2010 to 2011.
The greatest decrease was $954 million from 2008 to 2009.

(b) Using the points $(5, 23.104)$ and $(12, 48.017)$, the slope is $m = \dfrac{48.017 - 23.104}{12 - 5} = 3.559$.

Then $y - 23.104 = 3.559(x - 5)$

$y - 23.104 = 3.559x - 17.795$

$y = 3.559x + 5.309$

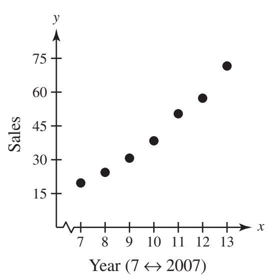
(c) There was an average increase in sales of about $3.559 billion per year from 2005 to 2012.

(d) When $x = 17$: $y = 3.559(17) + 5.309$

$y = \$65.812$ billion

Answers will vary.

88. (a)



(b)

Years	Slope
2007–2008	$24.4 - 19.7 = 4.7$
2008–2009	$30.7 - 24.4 = 6.3$
2009–2010	$38.4 - 30.7 = 7.7$
2010–2011	$50.4 - 38.4 = 12.0$
2011–2012	$57.3 - 50.4 = 6.9$
2012–2013	$71.6 - 57.3 = 14.3$

The greatest increase was $14.3 million from 2012 to 2013.
The least increase was $4.7 million from 2007 to 2008.

(c) Using the points $(7, 19.7)$ and $(13, 71.6)$, the slope is

$m = \dfrac{71.6 - 19.7}{13 - 7} = 8.65.$

Then $y - 19.7 = 8.65(x - 7)$

$y - 19.7 = 8.65x - 60.55$

$y = 8.65x - 40.85$

(d) There was an average increase in profit of approximately $8.65 million per year from 2007 to 2013.

(e) When $x = 17$, $y = 8.65(17) - 40.85 = \$106.2$ million.

Answers will vary.

For Exercises 89–92, $t = 15$ corresponds to 2015.

89. $(15, 2540)$, $m = 125$

$V - 2540 = 125(t - 15)$

$V - 2540 = 125t - 1875$

$V = 125t + 665$

90. $(15, 156)$, $m = 5.50$

$V - 156 = 5.50(t - 15)$

$V - 156 = 5.5t - 82.5$

$V = 5.5t + 73.5$

91. $(15, 20{,}400)$, $m = -2000$

$V - 20{,}400 = -2000(t - 15)$

$V - 20{,}400 = -2000t + 30{,}000$

$V = -2000t + 50{,}400$

92. $(15, 245{,}000)$, $m = -5600$

$V - 245{,}000 = -5600(t - 15)$

$V - 245{,}000 = -5600t + 84{,}000$

$V = -5600t + 329{,}000$

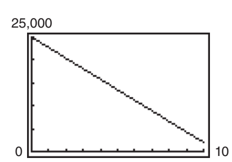
93. (a) $(0, 25{,}000)$, $(10, 2000)$

$V - 25{,}000 = \dfrac{2000 - 25{,}000}{10 - 0}(t - 0)$

$V - 25{,}000 = -2300t$

$V = -2300t + 25{,}000$

(b)



t	0	1	2	3	4	5
V	25,000	22,700	20,400	18,100	15,800	13,500

t	6	7	8	9	10
V	11,200	8900	6600	4300	2000

(c) $t = 0$: $V = -2300(0) + 25{,}000 = 25{,}000$

$t = 1$: $V = -2300(1) + 25{,}000 = 22{,}700$

etc.

94. (a) Using the points $(0, 32)$ and $(100, 212)$, you have

$$m = \frac{212-32}{100-0} = \frac{180}{100} = \frac{9}{5}$$
$$F - 32 = \frac{9}{5}(C - 0)$$
$$F = \frac{9}{5}C + 32.$$

(b) $F = \frac{9}{5}C + 32$

$F = 0°$: $0 = \frac{9}{5}C + 32$, $-32 = \frac{9}{5}C$, $-17.8 \approx C$

$C = 10°$: $F = \frac{9}{5}(10) + 32$, $F = 18 + 32$, $F = 50$

$F = 90°$: $90 = \frac{9}{5}C + 32$, $58 = \frac{9}{5}C$, $32.2 \approx C$

$C = -10°$: $F = \frac{9}{5}(-10) + 32$, $F = -18 + 32$, $F = 14$

$F = 68°$: $68 = \frac{9}{5}C + 32$, $36 = \frac{9}{5}C$, $20 = C$

$C = 177°$: $F = \frac{9}{5}(177) + 32$, $F = 318.6 + 32$, $F = 350.6$

C	−17.8°	−10°	10°	20°	32.2°	177°
F	0°	14°	50°	68°	90°	350.6°

95. (a) $C = 36{,}500 + 11.25t + 19.50t$
$C = 36{,}500 + 30.75t$

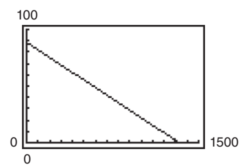
(b) $R = tp$ (t hours at \$$p$ per hour)
$R = t(80)$
$R = 80t$

(c) $P = R - C$
$P = 80t - (36{,}500 + 30.75t)$
$P = 49.25t - 36{,}500$

(d) $P = 0$:
$49.25t - 36{,}500 = 0$
$49.25t = 36{,}500$
$t \approx 741.1$ h

96. (a) $(580, 50)$, $(625, 47)$

$$x - 50 = \frac{47-50}{625-580}(p - 580)$$
$$x - 50 = \frac{-1}{15}(p - 580)$$
$$x = -\frac{1}{15}p + \frac{266}{3}$$

(b) 

If $p = 655$, $x = 45$ units.

Algebraically, $x = -\frac{1}{15}(655) + \frac{266}{3} = 45.$

(c) If $p = 595$, $x = 49$ units.

Algebraically, $x = -\frac{1}{15}(595) + \frac{266}{3} = 49.$

97. (a) Using the points $(1994, 73{,}500)$ and $(2013, 98{,}097)$, the slope is

$$m = \frac{98{,}097 - 73{,}500}{2013 - 1994} = \frac{24{,}597}{19} \approx 1295.$$

The average annual increase in enrollment was about 1295 students per year.

(b) 1996: $73{,}500 + 2(1{,}295) = 76{,}090$ students
2006: $73{,}500 + 12(1{,}295) = 89{,}040$ students
2011: $73{,}500 + 17(1{,}295) = 95{,}515$ students

(c) Using $m = 1295$ and letting $x = 4$ correspond to 1994, $y - 73{,}500 = 1295(x - 4)$
$y - 73{,}500 = 1295x - 5180$
$y = 1295x + 68{,}320$

The slope is 1295 and it determines the average increase in enrollment per year from 1994 to 2013.

98. Answers will vary. Sample answer: Slope is the rate of change over an interval; average rate of change is the slope of the line passing through the first and last points of a plot.

99. False. The slopes are different:

$$\frac{4-2}{-1+8} = \frac{2}{7}$$

$$\frac{7+4}{-7-0} = -\frac{11}{7}$$

100. False.

The equation of the line joining $(10, -3)$ and $(2, -9)$ is

$$y+3 = \frac{-9+3}{2-10}(x-10)$$

$$y+3 = \frac{3}{4}(x-10)$$

$$y = \frac{3}{4}x - \frac{21}{2}.$$

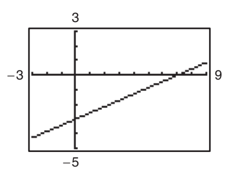
For $x = -12$, $y = \frac{3}{4}(-12) - \frac{21}{2}$

$$= -19.5$$

$$\neq \frac{-37}{2}$$

$$= -18.5$$

101.

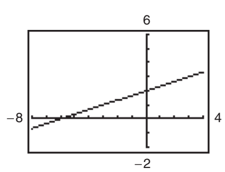


$$\frac{x}{7} + \frac{y}{-3} = 1$$

$$-3x + 7y + 21 = 0$$

a and *b* are the *x*- and *y*-intercepts.

102.



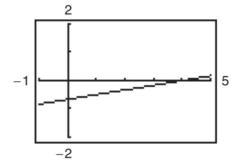
$$\frac{x}{-6} + \frac{y}{2} = 1$$

$$y = 2\left(1 + \frac{x}{6}\right)$$

$$y = \frac{x}{3} + 2$$

a and *b* are the *x*- and *y*-intercepts.

103.



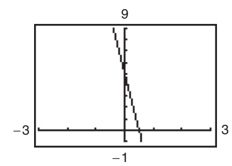
$$\frac{x}{4} + \frac{y}{-\frac{2}{3}} = 1$$

$$-\frac{2}{3}x + 4y = \frac{-8}{3}$$

$$-2x + 12y = -8$$

a and *b* are the *x*- and *y*-intercepts.

104.



$$\frac{x}{\frac{1}{2}} + \frac{y}{5} = 1$$

$$5x + \frac{1}{2}y = \frac{5}{2}$$

$$10x + y = 5$$

a and *b* are the *x*- and *y*-intercepts.

105.

$$\frac{x}{2} + \frac{y}{9} = 1$$

$$9x + 2y - 18 = 0$$

106.

$$\frac{x}{a} + \frac{y}{b} = 1$$

$$\frac{x}{-5} + \frac{y}{-4} = 1$$

$$4x + 5y + 20 = 0$$

107.

$$\frac{x}{-1/6} + \frac{y}{-2/3} = 1$$

$$-6x - \frac{3}{2}y = 1$$

$$12x + 3y + 2 = 0$$

108.

$$\frac{x}{a} + \frac{y}{b} = 1$$

$$\frac{x}{3/4} + \frac{y}{4/5} = 1$$

$$\frac{4}{5}x + \frac{3}{4}y = \frac{3}{5}$$

$$16x + 15y - 12 = 0$$

109. The slope is positive and the y-intercept is positive. Matches (a).

110. The slope is negative and the y-intercept is negative. Matches (b).

111. Both lines have positive slope, but their y-intercepts differ in sign. Matches (c).

112. The lines intersect in the first quadrant at a point (x, y) where $x < y$. Matches (a).

113. No. The line $y = 2$ does not have an x-intercept.

114. No. $x = 1$ cannot be written in slope-intercept form because the slope is undefined.

115. Yes. Once a parallel line is established to the given line, there are an infinite number of distances away from that line, and thus an infinite number of parallel lines.

116. (a) The slope is $m = -10$. This represents the decrease in the amount of the loan each week. Matches graph (ii).

(b) The y-intercept is 13.5 and the slope is $m = 2$, which represents the increase in hourly wage per unit produced. Matches graph (iii).

(c) The slope is $m = 0.5$. This represents the increase in travel cost for each mile driven. Matches graph (i).

(d) The y-intercept is 600 and the slope is $m = -100$, which represents the decrease in the value of the computer each year. Matches graph (iv).

117. Yes. $x + 20$

118. Yes. $3x - 10x^2 + 1 = -10x^2 + 3x + 1$

119. No. The term $x^{-1} = \dfrac{1}{x}$ causes the expression to not be a polynomial.

120. Yes. $2x^2 - 2x^4 - x^3 + \sqrt{2} = -2x^4 - x^3 + 2x^2 + \sqrt{2}$

121. No. This expression is not defined for $x = \pm 3$.

122. No.

123. $x^2 - 6x - 27 = (x - 9)(x + 3)$

124. $x^2 + 11x + 28 = (x + 4)(x + 7)$

125. $2x^2 + 11x - 40 = (2x - 5)(x + 8)$

126. $3x^2 - 16x + 5 = (3x - 1)(x - 5)$

127. Answers will vary.

Section 1.3 Functions

1. domain, range, function

2. independent, dependent

3. No. The input element $x = 3$ cannot be assigned to more than exactly one output element.

4. To find $g(x+1)$ for $g(x) = 3x - 2$, substitute x with the quantity $x + 1$.

$$\begin{aligned} g(x+1) &= 3(x+1) - 2 \\ &= 3x + 3 - 2 \\ &= 3x + 1 \end{aligned}$$

5. No. The domain of the function $f(x) = \sqrt{1+x}$ is $[-1, \infty)$ which does not include $x = -2$.

6. The domain of a piece-wise function must be explicitly described, so that it can determine which equation is used to evaluate the function.

7. Yes. Each domain value is matched with only one range value.

8. No. The domain value of -1 is matched with two output values.

9. No. The National Football Conference, an element in the domain, is assigned to three elements in the range, the Giants, the Saints, and the Seahawks; The American Football Conference, an element in the domain, is also assigned to three elements in the range, the Patriots, the Ravens, and the Steelers.

10. Yes. Each element, or state, in the domain is assigned to exactly one element, or electoral votes, in the range.

11. Yes, the table represents y as a function of x. Each domain value is matched with only one range value.

12. No, the table does not represent a function. The input values of 0 and 1 are each matched with two different output values.

13. No, the graph does not represent a function. The input values 1, 2, and 3 are each matched with two outputs.

14. Yes, the graph represents a function. Each input value is matched with one output value.

15. (a) Each element of A is matched with exactly one element of B, so it does represent a function.

(b) The element 1 in A is matched with two elements, -2 and 1 of B, so it does not represent a function.

(c) Each element of A is matched with exactly one element of B, so it does represent a function.

16. (a) The element c in A is matched with two elements, 2 and 3 of B, so it is not a function.
(b) Each element of A is matched with exactly one element of B, so it does represent a function.
(c) This is not a function from A to B (it represents a function from B to A instead).

17. Both are functions. For each year there is exactly one and only one average price of a name brand prescription and average price of a generic prescription.

18. Since $b(t)$ represents the average price of a name brand prescription, $b(2009) \approx \$151$. Since $g(t)$ represents the average price of a generic prescription, $g(2006) \approx \$31$.

19. $x^2 + y^2 = 4 \Rightarrow y = \pm\sqrt{4 - x^2}$

Thus, y *is not* a function of x. For instance, the values $y = 2$ and $y = -2$ both correspond to $x = 0$.

20. $x = y^2 + 1$

$y = \pm\sqrt{x - 1}$

This *is not* a function of x. For example, the values $y = 2$ and $y = -2$ both correspond to $x = 5$.

21. $y = \sqrt{x^2 - 1}$

This *is* a function of x.

22. $y = \sqrt{x + 5}$

This *is* a function of x.

23. $2x + 3y = 4 \Rightarrow y = \frac{1}{3}(4 - 2x)$

Thus, y *is* a function of x.

24. $x = -y + 5 \Rightarrow y = -x + 5$

This *is* a function of x.

25. $y^2 = x^2 - 1 \Rightarrow y = \pm\sqrt{x^2 - 1}$

Thus, y *is not* a function of x. For instance, the values $y = \sqrt{3}$ and $y = -\sqrt{3}$ both correspond to $x = 2$.

26. $x + y^2 = 3 \Rightarrow y = \pm\sqrt{3 - x}$

Thus, y *is not* a function of x.

27. $y = |4 - x|$

This *is* a function of x.

28. $|y| = 3 - 2x \Rightarrow y = 3 - 2x$ or $y = -(3 - 2x)$

Thus, y *is not* a function of x.

29. $x = -7$ *does not* represent y as a function of x. All values of y correspond to $x = -7$.

30. $y = 8$ *is* a function of x, a constant function.

31. $f(t) = 3t + 1$

(a) $f(2) = 3(2) + 1 = 7$
(b) $f(-4) = 3(-4) + 1 = -11$
(c) $f(t + 2) = 3(t + 2) + 1 = 3t + 7$

32. $g(y) = 7 - 3y$

(a) $g(0) = 7 - 3(0) = 7$

(b) $g\left(\frac{7}{3}\right) = 7 - 3\left(\frac{7}{3}\right) = 0$

(c) $g(s + 5) = 7 - 3(s + 5)$
$= 7 - 3s - 15 = -3s - 8$

33. $h(t) = t^2 - 2t$

(a) $h(2) = 2^2 - 2(2) = 0$

(b) $h(1.5) = (1.5)^2 - 2(1.5) = -0.75$

(c) $h(x - 4) = (x - 4)^2 - 2(x - 4)$
$= x^2 - 8x + 16 - 2x + 8$
$= x^2 - 10x + 24$

34. $V(r) = \frac{4}{3}\pi r^3$

(a) $V(3) = \frac{4}{3}\pi(3)^3 = 36\pi$

(b) $V\left(\frac{3}{2}\right) = \frac{4}{3}\pi\left(\frac{3}{2}\right)^3 = \frac{4}{3}\cdot\frac{27}{8}\pi = \frac{9\pi}{2}$

(c) $V(2r) = \frac{4}{3}\pi(2r)^3 = \frac{32\pi r^3}{3}$

35. $f(y) = 3 - \sqrt{y}$

(a) $f(4) = 3 - \sqrt{4} = 1$
(b) $f(0.25) = 3 - \sqrt{0.25} = 2.5$
(c) $f(4x^2) = 3 - \sqrt{4x^2} = 3 - 2|x|$

36. $f(x) = \sqrt{x + 8} + 2$

(a) $f(-4) = \sqrt{-4 + 8} + 2 = 4$

(b) $f(8) = \sqrt{8 + 8} + 2 = 6$

(c) $f(x - 8) = \sqrt{x - 8 + 8} + 2 = \sqrt{x} + 2$

37. $q(x) = \frac{1}{x^2 - 9}$

(a) $q(-3) = \frac{1}{(-3)^2 - 9} = \frac{1}{9 - 9} = \frac{1}{0}$ undefined

(b) $q(2) = \frac{1}{(2)^2 - 9} = \frac{1}{4 - 9} = -\frac{1}{5}$

(c) $q(y + 3) = \frac{1}{(y + 3)^2 - 9} = \frac{1}{y^2 + 6y + 9 - 9} = \frac{1}{y^2 + 6y}$

38. $q(t) = \dfrac{2t^2 + 3}{t^2}$

(a) $q(2) = \dfrac{2(2)^2 + 3}{(2)^2} = \dfrac{8+3}{4} = \dfrac{11}{4}$

(b) $q(0) = \dfrac{2(0)^2 + 3}{(0)^2}$ Division by zero is undefined.

(c) $q(-x) = \dfrac{2(-x)^2 + 3}{(-x)^2} = \dfrac{2x^2 + 3}{x^2}$

39. $f(x) = \dfrac{|x|}{x}$

(a) $f(9) = \dfrac{|9|}{9} = 1$

(b) $f(-9) = \dfrac{|-9|}{-9} = -1$

(c) $f(t) = \dfrac{|t|}{t} = \begin{cases} 1, & t > 0 \\ -1, & t < 0 \end{cases}$

$f(0)$ is undefined.

40. $f(x) = |x| + 4$

(a) $f(5) = |5| + 4 = 9$

(b) $f(-5) = |-5| + 4 = 9$

(c) $f(t) = |t| + 4$

41. $f(x) = \begin{cases} 2x + 1, & x < 0 \\ 2x + 2, & x \ge 0 \end{cases}$

(a) $f(-1) = 2(-1) + 1 = -1$

(b) $f(0) = 2(0) + 2 = 2$

(c) $f(2) = 2(2) + 2 = 6$

42. $f(x) = \begin{cases} 2x + 5, & x \le 0 \\ 2 - x, & x > 0 \end{cases}$

(a) $f(-2) = 2(-2) + 5 = 1$

(b) $f(0) = 2(0) + 5 = 5$

(c) $f(1) = 2 - 1 = 1$

43. $f(x) = \begin{cases} x^2 + 2, & x \le 1 \\ 2x^2 + 2, & x > 1 \end{cases}$

(a) $f(-2) = (-2)^2 + 2 = 6$

(b) $f(1) = (1)^2 + 2 = 3$

(c) $f(2) = 2(2)^2 + 2 = 10$

44. $f(x) = \begin{cases} x^2 - 4, & x \le 0 \\ 1 - 2x^2, & x > 0 \end{cases}$

(a) $f(-2) = (-2)^2 - 4 = 4 - 4 = 0$

(b) $f(0) = 0^2 - 4 = -4$

(c) $f(1) = 1 - 2(1^2) = 1 - 2 = -1$

45. $f(x) = \begin{cases} x + 2, & x < 0 \\ 4, & 0 \le x < 2 \\ x^2 + 1, & x \ge 2 \end{cases}$

(a) $f(-2) = (-2) + 2 = 0$

(b) $f(0) = 4$

(c) $f(2) = (2)^2 + 1 = 5$

46. $f(x) = \begin{cases} 5 - 2x, & x < 0 \\ 5, & 0 \le x < 1 \\ 4x + 1, & x \ge 1 \end{cases}$

(a) $f(-4) = 5 - 2(-4) = 13$

(b) $f(0) = 5$

(c) $f(1) = 4(1) + 1 = 5$

47. $f(x) = (x - 1)^2$

$\{(-2, 9), (-1, 4), (0, 1), (1, 0), (2, 1)\}$

48. $f(x) = x^2 - 3$

$\{(-2, 1), (-1, -2), (0, -3), (1, -2), (2, 1)\}$

49. $f(x) = |x| + 2$

$\{(-2, 4), (-1, 3), (0, 2), (1, 3), (2, 4)\}$

50. $f(x) = |x + 1|$

$\{(-2, 1), (-1, 0), (0, 1), (1, 2), (2, 3)\}$

51. $h(t) = \frac{1}{2}|t+3|$

$h(-5) = \frac{1}{2}|-5+3| = \frac{1}{2}|-2| = \frac{1}{2}(2) = 1$

$h(-4) = \frac{1}{2}|-4+3| = \frac{1}{2}|-1| = \frac{1}{2}(1) = \frac{1}{2}$

$h(-3) = \frac{1}{2}|-3+3| = \frac{1}{2}|0| = 0$

$h(-2) = \frac{1}{2}|-2+3| = \frac{1}{2}|1| = \frac{1}{2}(1) = \frac{1}{2}$

$h(-1) = \frac{1}{2}|-1+3| = \frac{1}{2}|2| = \frac{1}{2}(2) = 1$

t	–5	–4	–3	–2	–1
$h(t)$	1	$\frac{1}{2}$	0	$\frac{1}{2}$	1

52. $f(s) = \frac{|s-2|}{s-2}$

$f(0) = \frac{|0-2|}{0-2} = \frac{2}{-2} = -1$

$f(1) = \frac{|1-2|}{1-2} = \frac{1}{-1} = -1$

$f\left(\frac{3}{2}\right) = \frac{\left|\frac{3}{2}-2\right|}{\frac{3}{2}-2} = \frac{\frac{1}{2}}{-\frac{1}{2}} = -1$

$f\left(\frac{5}{2}\right) = \frac{\left|\frac{5}{2}-2\right|}{\frac{5}{2}-2} = \frac{\frac{1}{2}}{\frac{1}{2}} = 1$

$f(4) = \frac{|4-2|}{4-2} = \frac{2}{2} = 1$

s	0	1	$\frac{3}{2}$	$\frac{5}{2}$	4
$f(s)$	–1	–1	–1	1	1

53. $f(x) = 15 - 3x = 0$

$3x = 15$

$x = 5$

54. $f(x) = 5x + 1 = 0$

$5x = -1$

$x = -\frac{1}{5}$

55. $f(x) = \frac{9x-4}{5} = 0$

$9x - 4 = 0$

$9x = 4$

$x = \frac{4}{9}$

56. $f(x) = \frac{2x-3}{7} = 0$

$2x - 3 = 0$

$2x = 3$

$x = \frac{3}{2}$

57. $f(x) = 5x^2 + 2x - 1$

Since $f(x)$ is a polynomial, the domain is all real numbers x.

58. $g(x) = 1 - 2x^2$

Because $g(x)$ is a polynomial, the domain is all real numbers x.

59. $h(t) = \frac{4}{t}$

Domain: all real numbers except $t = 0$

60. $s(y) = \frac{3y}{y+5}$

$y + 5 \neq 0$

$y \neq -5$

Domain: all real numbers except $y = -5$

61. $f(x) = \sqrt[3]{x-4}$

Domain: all real numbers x

62. $f(x) = \sqrt[4]{x^2+3x}$

$x^2 + 3x = x(x+3) \geq 0$

Domain: $x \leq -3$ or $x \geq 0$

63. $g(x) = \frac{1}{x} - \frac{3}{x+2}$

Domain: all real numbers except $x = 0, x = -2$

64. $h(x) = \frac{10}{x^2 - 2x}$

$x^2 - 2x \neq 0$

$x(x-2) \neq 0$

Domain: all real numbers except $x = 0, x = 2$

65. $g(y) = \dfrac{y+2}{\sqrt{y-10}}$

$$y - 10 > 0$$
$$y > 10$$

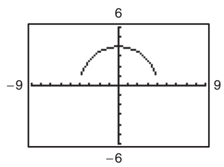
Domain: all $y > 10$

66. $f(x) = \dfrac{\sqrt{x+6}}{6+x}$

$x + 6 \ge 0$ for numerator and $x \ne -6$ for denominator.

Domain: all $x > -6$

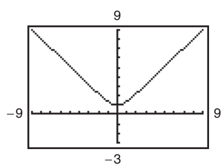
67. $f(x) = \sqrt{16 - x^2}$



Domain: $[-4, 4]$

Range: $[0, 4]$

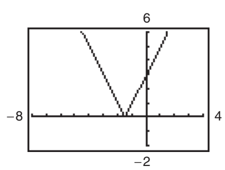
68. $f(x) = \sqrt{x^2 + 1}$



Domain: all real numbers

Range: $1 \le y$

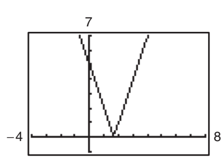
69. $g(x) = |2x + 3|$



Domain: $(-\infty, \infty)$

Range: $[0, \infty)$

70. $g(x) = |3x - 5|$



Domain: all real numbers

Range: $y \ge 0$

71. $A = \pi r^2,\ C = 2\pi r$

$$r = \frac{C}{2\pi}$$

$$A = \pi\left(\frac{C}{2\pi}\right)^2 = \frac{C^2}{4\pi}$$

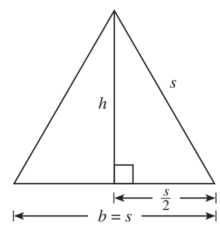
72. $A = \dfrac{1}{2}bh,$ in an equilateral triangle $b = s$ and:

$$s^2 = h^2 + \left(\frac{s}{2}\right)^2$$

$$h = \sqrt{s^2 - \left(\frac{s}{2}\right)^2}$$

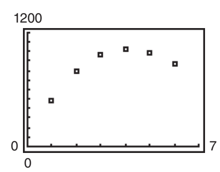
$$h = \sqrt{\frac{4s^2}{4} - \frac{s^2}{4}} = \frac{\sqrt{3}s}{2}$$

$$A = \frac{1}{2}s \cdot \frac{\sqrt{3}s}{2} = \frac{\sqrt{3}s^2}{4}$$



73. (a) From the table, the maximum volume seems to be 1024 cm^3, corresponding to $x = 4$.

(b)



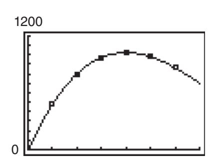
Yes, V is a function of x.

(c) $V = \text{length} \times \text{width} \times \text{height}$

$$= (24 - 2x)(24 - 2x)x$$
$$= x(24 - 2x)^2 = 4x(12 - x)^2$$

Domain: $0 < x < 12$

(d)



The function is a good fit. Answers will vary.

74. $A = \frac{1}{2}(\text{base})(\text{height}) = \frac{1}{2}xy.$

Since $(0, y)$, $(2, 1)$, and $(x, 0)$ all lie on the same line, the slopes between any pair of points are equal.

$$\frac{1-y}{2-0} = \frac{1-0}{2-x}$$
$$1 - y = \frac{2}{2-x}$$
$$y = 1 - \frac{2}{2-x} = \frac{x}{x-2}$$

Therefore, $A = \frac{1}{2}xy = \frac{1}{2}x\left(\frac{x}{x-2}\right) = \frac{x^2}{2x-4}.$

The domain is $x > 2$, since $A > 0$.

75. $A = l \cdot w = (2x)y = 2xy$

But $y = \sqrt{36 - x^2}$, so $A = 2x\sqrt{36 - x^2}$, $0 < x < 6$.

76. (a) $V = (\text{length})(\text{width})(\text{height}) = yx^2$

But, $y + 4x = 108$, or $y = 108 - 4x$.

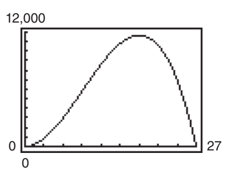
Thus, $V = (108 - 4x)x^2$.

Since $y = 108 - 4x > 0$

$$4x < 108$$
$$x < 27.$$

Domain: $0 < x < 27$

(b)



(c) The highest point on the graph occurs at $x = 18$. The dimensions that maximize the volume are $18 \times 18 \times 36$ inches.

77. (a) Total cost = Variable costs + Fixed costs

$$C = 68.75x + 248{,}000$$

(b) Revenue = Selling price × Units sold

$$R = 99.99x$$

(c) Since $P = R - C$

$$P = 99.99x - (68.75x + 248{,}000)$$
$$P = 31.24x - 248{,}000.$$

78. (a) The independent variable is x and represents the month. The dependent variable is y and represents the monthly revenue.

(b) $f(x) = \begin{cases} -1.97x + 26.3, & 7 \le x \le 12 \\ 0.505x^2 - 1.47x + 6.3, & 1 \le x \le 6 \end{cases}$

Answers will vary.

(c) $f(5) = 11.575$, and represents the revenue in May: \$11,575.

(d) $f(11) = 4.63$, and represents the revenue in November: \$4630.

(e) The values obtained from the model are close approximations to the actual data.

79. (a) The independent variable is t and represents the year. The dependent variable is n and represents the numbers of miles traveled.

(b)

t	0	1	2	3	4	5
$n(t)$	3.95	3.96	3.98	3.99	4.00	4.02

t	6	7	8	9	10	11
$n(t)$	4.03	4.04	4.05	4.07	4.08	4.09

(c) The model fits the data well.

(d) Sample answer: No. The function may not accurately model other years

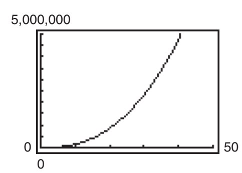
80. (a) $F(y) = 149.76\sqrt{10}y^{5/2}$

y	5	10	20	30	40
$F(y)$	26,474	149,760	847,170	2,334,527	4,792,320

(Answers will vary.)

F increases very rapidly as y increases.

(b)



$X_{min} = 0$
$X_{max} = 50$
$X_{scl} = 10$
$Y_{min} = 0$
$Y_{max} = 5{,}000{,}000$
$Y_{scl} = 500{,}000$

(c) From the table, $y \approx 22$ ft (slightly above 20). You could obtain a better approximation by completing the table for values of y between 20 and 30.

(d) By graphing $F(y)$ together with the horizontal line $y_2 = 1{,}000{,}000$, you obtain $y \approx 21.37$ feet.

81. Yes. If $x = 30$, $y = -0.01(30)^2 + 3(30) + 6$

$$y = 6 \text{ feet}$$

Since the child trying to catch the throw is holding the glove at a height of 5 feet, the ball will fly over the glove.

82. (a) $\dfrac{f(2013) - f(2005)}{2013 - 2005} \approx$ \$525 million/year

This represents the increase in sales per year from 2005 to 2013.

(b)

t	5	6	7	8	9
S(t)	217.3	136.9	237.4	518.8	981.1

t	10	11	12	13
S(t)	1624.2	2448.2	3453.1	4638.9

The model approximates the data well.

83. $f(x) = 2x$

$$\frac{f(x+c) - f(x)}{c} = \frac{2(x+c) - 2x}{c} = \frac{2c}{c} = 2,\ c \neq 0$$

84. $g(x) = 3x - 1$

$$g(x+h) = 3(x+h) - 1 = 3x + 3h - 1$$
$$g(x+h) - g(x) = (3x + 3h - 1) - (3x - 1) = 3h$$
$$\frac{g(x+h) - g(x)}{h} = \frac{3h}{h} = 3,\ h \neq 0$$

85. $f(x) = x^2 - x + 1,\ f(2) = 3$

$$\frac{f(2+h) - f(2)}{h} = \frac{(2+h)^2 - (2+h) + 1 - 3}{h} = \frac{4 + 4h + h^2 - 2 - h + 1 - 3}{h} = \frac{h^2 + 3h}{h} = h + 3,\ h \neq 0$$

86. $f(x) = x^3 + x$

$$f(x+h) = (x+h)^3 + (x+h) = x^3 + 3x^2h + 3xh^2 + h^3 + x + h$$
$$f(x+h) - f(x) = (x^3 + 3x^2h + 3xh^2 + h^3 + x + h) - (x^3 + x) = 3x^2h + 3xh^2 + h^3 + h = h(3x^2 + 3xh + h^2 + 1)$$
$$\frac{f(x+h) - f(x)}{h} = \frac{h(3x^2 + 3xh + h^2 + 1)}{h} = 3x^2 + 3xh + h^2 + 1,\ h \neq 0$$

87. False. The range of $f(x)$ is $(-1, \infty)$.

88. True. The first number in each ordered pair corresponds to exactly one second number.

89. $f(x) = \sqrt{x} + 2$

Domain: $[0, \infty)$ or $x \geq 0$

Range: $[2, \infty)$ or $y \geq 2$

90. $f(x) = \sqrt{x+3}$

Domain: $[-3, \infty)$ or $x \geq -3$

Range: $[0, \infty)$ or $y \geq 0$

91. No. f is not the independent variable. Because the value of f depends on the value of x, x is the independent variable and f is the dependent variable.

92. (a) The height h is a function of t because for each value of t there is exactly one corresponding value of h for $0 \leq t \leq 2.6$.

(b) The height after 0.5 second is about 20 feet. The height after 1.25 seconds is about 28 feet.

(c) From the graph, the domain is $0 \leq t \leq 2.6$.

(d) The time t is not a function of h because some values of h correspond to more than one value of t.

93. $12 - \dfrac{4}{x+2} = \dfrac{12(x+2) - 4}{x+2} = \dfrac{12x + 20}{x+2}$

94.

$$\frac{3}{x^2 + x - 20} + \frac{2x}{x^2 + 4x - 5} = \frac{3}{(x+5)(x-4)} + \frac{2x}{(x+5)(x-1)}$$
$$= \frac{3(x-1)}{(x+5)(x-4)(x-1)} + \frac{2x(x-4)}{(x+5)(x-1)(x-4)}$$
$$= \frac{3x - 3 + 2x^2 - 8x}{(x+5)(x-4)(x-1)} = \frac{2x^2 - 5x - 3}{(x+5)(x-4)(x-1)}$$

95.

$$\frac{2x^3 + 11x^2 - 6x}{5x} \cdot \frac{x+10}{2x^2 + 5x - 3} = \frac{x(2x^2 + 11x - 6)(x+10)}{5x(2x-1)(x+3)} = \frac{(2x-1)(x+6)(x+10)}{5(2x-1)(x+3)} = \frac{(x+6)(x+10)}{5(x+3)},\ x \neq 0,\ \frac{1}{2}$$

96. $\dfrac{x+7}{2(x-9)} \div \dfrac{x-7}{2(x-9)} = \dfrac{x+7}{2(x-9)} \cdot \dfrac{2(x-9)}{x-7} = \dfrac{x+7}{x-7},\ x \neq 9$

Section 1.4 Graphs of Functions

1. decreasing

2. even

3. Domain: $1 \le x \le 4$ or $[1, 4]$

4. No. If a vertical line intersects the graph more than once, then it does not represent y as a function of x.

5. If $f(2) \ge f(2)$ for all x in $(0, 3)$, then $(2, f(2))$ is a relative maximum of f.

6. Since $f(x) = [\![x]\!] = n$, where n is an integer and $n \le x$, the input value of x needs to be greater than or equal to 5 but less than 6 in order to produce an output value of 5. So the interval $[5, 6)$ would yield a function value of 5.

7. Domain: all real numbers, $(-\infty, \infty)$

Range: $(-\infty, 1]$

$f(0) = 1$

8. Domain: all real numbers, $(-\infty, \infty)$

Range: all real numbers, $(-\infty, \infty)$

$f(0) = 2$

9. Domain: $[-4, 4]$

Range: $[0, 4]$

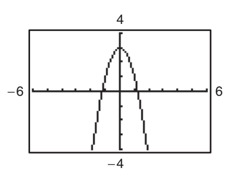
$f(0) = 4$

10. Domain: all real numbers, $(-\infty, \infty)$

Range: $[-3, \infty)$

$f(0) = -3$

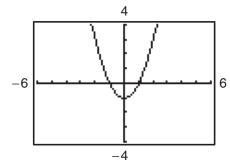
11. $f(x) = -2x^2 + 3$



Domain: $(-\infty, \infty)$

Range: $(-\infty, 3]$

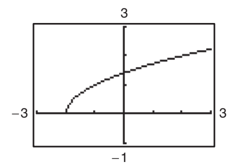
12. $f(x) = x^2 - 1$



Domain: $(-\infty, \infty)$

Range: $[-1, \infty)$

13. $f(x) = \sqrt{x + 2}$



$x + 2 \ge 0$

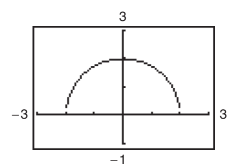
$x \ge -2$

Domain: $[-2, \infty)$

Range: $[0, \infty)$

14. $h(t) = \sqrt{4 - t^2}$

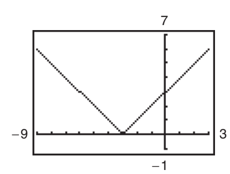
$4 - t^2 \ge 0 \Rightarrow t^2 \le 4$



Domain: $[-2, 2]$

Range: $[0, 2]$

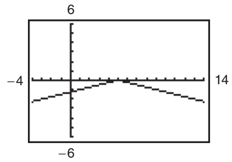
15. $f(x) = |x + 3|$



Domain: $(-\infty, \infty)$

Range: $[0, \infty)$

16. $f(x) = -\frac{1}{4}|x - 5|$



Domain: $(-\infty, \infty)$

Range: $(-\infty, 0]$

17. (a) Domain: $(-\infty, \infty)$

(b) Range: $[-2, \infty)$

(c) $f(x) = 0$ at $x = -1$ and $x = 3$.

(d) The values of $x = -1$ and $x = 3$ are the x-intercepts of the graph of f.

(e) $f(0) = -1$

(f) The value of $y = -1$ is the y-intercept of the graph of f.

(g) The value of f at $x = 1$ is $f(1) = -2$.

The coordinates of the point are $(1, -2)$.

(h) The value of f at $x = -1$ is $f(-1) = 0$.

The coordinates of the point are $(-1, 0)$.

(i) The coordinates of the point are $(-3, f(-3))$ or $(-3, 2)$.

18. (a) Domain: $(-\infty, \infty)$

(b) Range: $(-\infty, 4]$

(c) $f(x) = 0$ at $x = -4$ and $x = 2$.

(d) The values of $x = -4$ and $x = 2$ are the x-intercepts of the graph of f.

(e) $f(0) = 4$

(f) The value of $y = 4$ is the y-intercept of the graph of f.

(g) The value of f at $x = 1$ is $f(1) = 3$.

The coordinates of the point are $(1, 3)$.

(h) The value of f at $x = -1$ is $f(-1) = 3$.

The coordinates of the point are $(-1, 3)$.

(i) The coordinates of the point are $(-3, f(-3))$ or $(-3, 1)$.

19. $y = \frac{1}{2}x^2$

A vertical line intersects the graph just once, so y is a function of x. Graph $y_1 = \frac{1}{2}x^2$.

20. $x - y^2 = 1 \Rightarrow y = \pm\sqrt{x - 1}$

y is not a function of x. The vertical line $x = 2$ intersects the graph twice. Graph $y_1 = \sqrt{x - 1}$ and $y_2 = -\sqrt{x - 1}$.

21. $0.25x^2 + y^2 = 25$

$\frac{1}{4}x^2 + y^2 = 1$

A vertical line intersects the graph more than once, so y is not a function of x. Graph the circle as

$y_1 = \frac{1}{2}\sqrt{4 - x^2}$ and $y_2 = -\frac{1}{2}\sqrt{25 - x^2}$.

22. $x^2 = 2xy - 1$

A vertical line intersects the graph just once, so y is a function of x. Solve for y and graph $y_1 = \frac{x^2 + 1}{2x}$.

23. $f(x) = \frac{3}{2}x$

f is increasing on $(-\infty, \infty)$.

24. $f(x) = x^2 - 4x$

The graph is decreasing on $(-\infty, 2)$ and increasing on $(2, \infty)$.

25. $f(x) = x^3 - 3x^2 + 2$

f is increasing on $(-\infty, 0)$ and $(2, \infty)$.

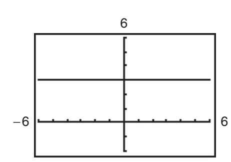
f is decreasing on $(0, 2)$.

26. $f(x) = \sqrt{x^2 - 1}$

The graph is decreasing on $(-\infty, -1)$ and increasing on $(1, \infty)$.

27. $f(x) = 3$

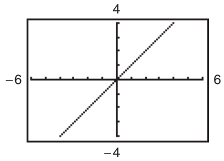
(a)



(b) f is constant on $(-\infty, \infty)$.

28. $f(x) = x$

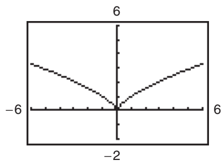
(a)



(b) Increasing on $(-\infty, \infty)$

29. $f(x) = x^{2/3}$

(a)

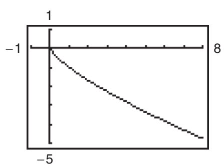


(b) Increasing on $(0, \infty)$

Decreasing on $(-\infty, 0)$

30. $f(x) = -x^{3/4}$

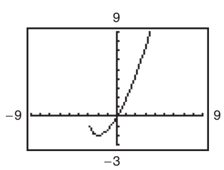
(a)



(b) Decreasing on $(0, \infty)$

31. $f(x) = x\sqrt{x+3}$

(a)

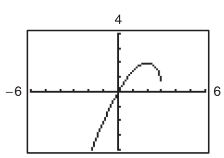


(b) Increasing on $(-2, \infty)$

Decreasing on $(-3, -2)$

32. $f(x) = x\sqrt{3 - x}$

(a)

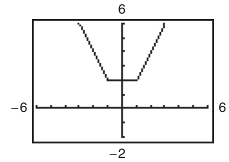


(b) Increasing on $(-\infty, 2)$

Decreasing on $(2, 3)$

33. $f(x) = |x+1| + |x-1|$

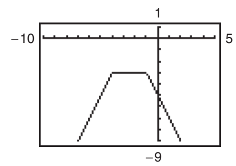
(a)



(b) Increasing on $(1, \infty)$, constant on $(-1, 1)$, decreasing on $(-\infty, -1)$

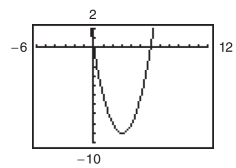
34. $f(x) = -|x+4| - |x+1|$

(a)



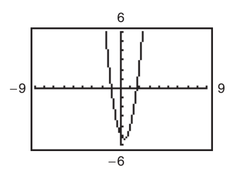
(b) Increasing on $(-\infty, -4)$, constant on $(-4, -1)$, decreasing on $(-1, \infty)$

35. $f(x) = x^2 - 6x$



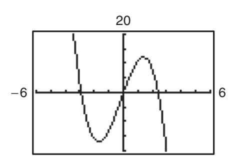
Relative minimum: $(3, -9)$

36. $f(x) = 3x^2 - 2x - 5$



Relative minimum: $(0.33, -5.33)$

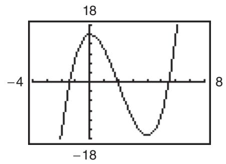
37. $y = -2x^3 - x^2 + 14x$



Relative minimum: $(-1.70, -16.86)$

Relative maximum: $(1.37, 12.16)$

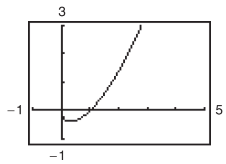
38. $y = x^3 - 6x^2 + 15$



Relative minimum: $(4, -17)$

Relative maximum: $(0, 15)$

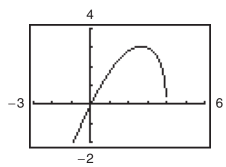
39. $h(x) = (x - 1)\sqrt{x}$



Relative minimum: $(0.33, -0.38)$

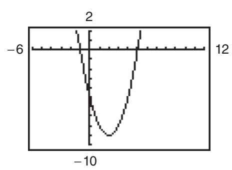
$(0, 0)$ is not a relative maximum because it occurs at the endpoint of the domain $[0, \infty)$.

40. $g(x) = x\sqrt{4 - x}$



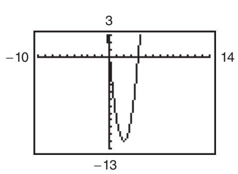
Relative maximum: $(2.67, 3.08)$

41. $f(x) = x^2 - 4x - 5$



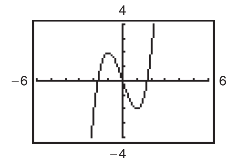
Relative minimum: $(2, -9)$

42. $f(x) = 3x^2 - 12x$



Relative minimum: $(2, -12)$

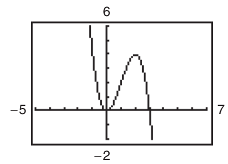
43. $f(x) = x^3 - 3x$



Relative minimum: $(1, -2)$

Relative maximum: $(-1, 2)$

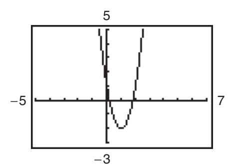
44. $f(x) = -x^3 + 3x^2$



Relative minimum: $(0, 0)$

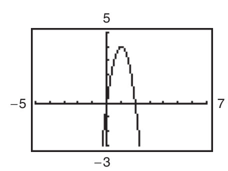
Relative maximum: $(2, 4)$

45. $f(x) = 3x^2 - 6x + 1$



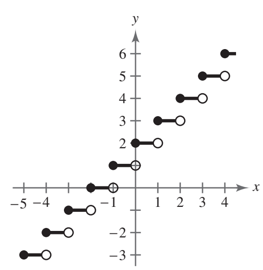
Relative minimum: $(1, -2)$

46. $f(x) = 8x - 4x^2$

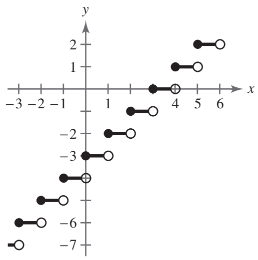


Relative maximum: $(1, 4)$

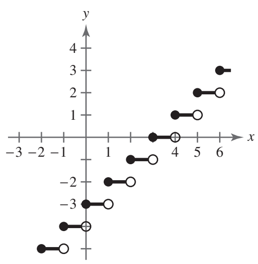
47. $f(x) = [\![x]\!] + 2$



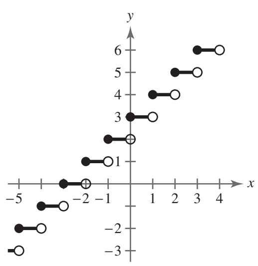
48. $f(x) = [\![x]\!] - 3$



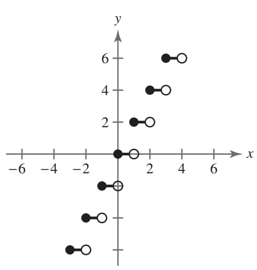
49. $f(x) = [\![x - 1]\!] - 2$



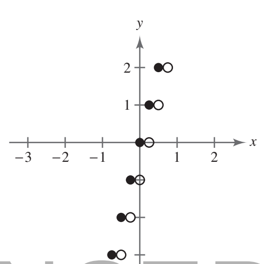
50. $f(x) = [\![x + 2]\!] + 1$



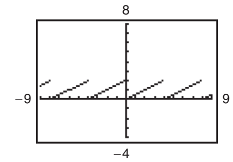
51. $f(x) = 2[\![x]\!]$



52. $f(x) = [\![4x]\!]$



53. $s(x) = 2\left(\frac{1}{4}x - [\![\frac{1}{4}x]\!]\right)$

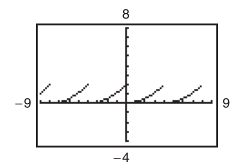


Domain: $(-\infty, \infty)$

Range: $[0, 2)$

Sawtooth pattern

54. $g(x) = 2\left(\frac{1}{4}x - [\![\frac{1}{4}x]\!]\right)^2$

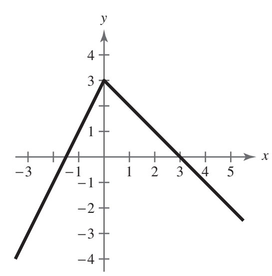


Domain: $(-\infty, \infty)$

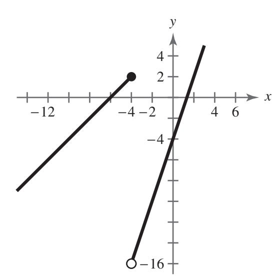
Range: $[0, 2)$

Sawtooth pattern

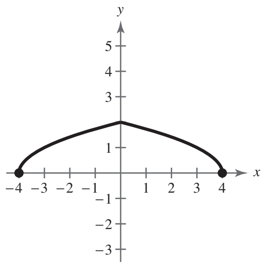
55. $f(x) = \begin{cases} 2x + 3, & x < 0 \\ 3 - x, & x \ge 0 \end{cases}$



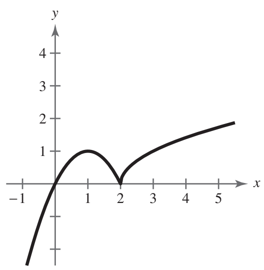
56. $f(x) = \begin{cases} x + 6, & x \le -4 \\ 3x - 4, & x > -4 \end{cases}$



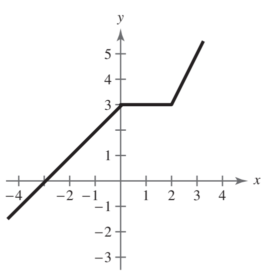
57. $f(x) = \begin{cases} \sqrt{x+4}, & x < 0 \\ \sqrt{4-x}, & x \ge 0 \end{cases}$



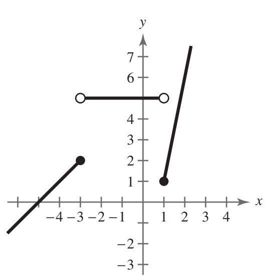
58. $f(x) = \begin{cases} 1-(x-1)^2, & x \le 2 \\ \sqrt{x-2}, & x > 2 \end{cases}$



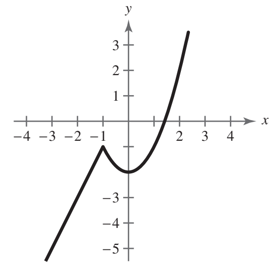
59. $f(x) = \begin{cases} x+3, & x \le 0 \\ 3, & 0 < x \le 2 \\ 2x-1, & x > 2 \end{cases}$



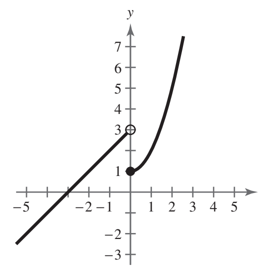
60. $g(x) = \begin{cases} x + 5, & x \le -3 \\ 5, & -3 < x < 1 \\ 5x - 4, & x \ge 1 \end{cases}$



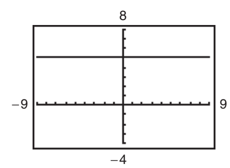
61. $f(x) = \begin{cases} 2x+1, & x \le -1 \\ x^2-2, & x > -1 \end{cases}$



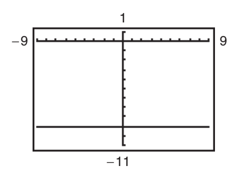
62. $h(x) = \begin{cases} 3+x, & x < 0 \\ x^2+1, & x \ge 0 \end{cases}$



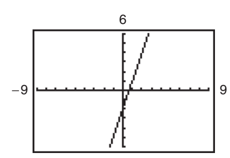
63. $f(x) = 5$ is even.



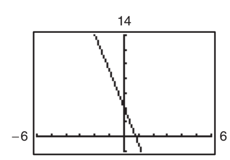
64. $f(x) = -9$ is even.



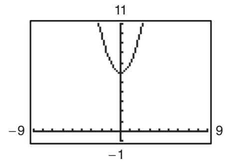
65. $f(x) = 3x - 2$ is neither even nor odd.



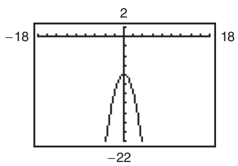
66. $f(x) = 4 - 5x$ is neither even nor odd.



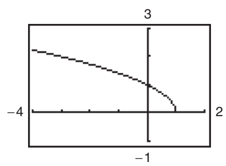
67. $h(x) = x^2 + 6$ is even.



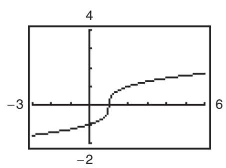
68. $f(x) = -x^2 - 8$ is even.



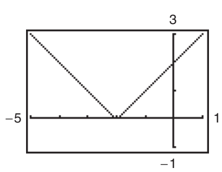
69. $f(x) = \sqrt{1-x}$ is neither even nor odd.



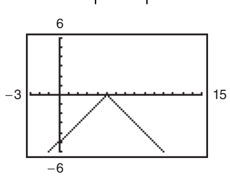
70. $g(t) = \sqrt[3]{t-1}$ is neither even nor odd.



71. $f(x) = |x+2|$ is neither even nor odd.



72. $f(x) = -|x-5|$ is neither even nor odd.



73. $\left(\frac{3}{2}, 4\right)$

(a) If f is even, another point is $\left(-\frac{3}{2}, 4\right)$.

(b) If f is odd, another point is $\left(-\frac{3}{2}, -4\right)$.

74. $\left(-\frac{5}{3}, -7\right)$

(a) If f is even, another point is $\left(\frac{5}{3}, -7\right)$.

(b) If f is odd, another point is $\left(\frac{5}{3}, 7\right)$.

75. $(-2, -9)$

(a) If f is even, another point is $(2, -9)$.

(b) If f is odd, another point is $(2, 9)$.

76. $(5, -1)$

(a) If f is even, another point is $(-5, -1)$.

(b) If f is odd, another point is $(-5, 1)$.

77. $(x, -y)$

(a) If f is even, another point is $(-x, -y)$.

(b) If f is odd, another point is $(-x, y)$.

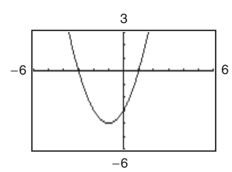
78. $(2a, 2c)$

(a) If f is even, another point is $(-2a, 2c)$.

(b) If f is odd, another point is $(-2a, -2c)$.

79. (a)
$$\begin{aligned} f(-t) &= (-t)^2 + 2(-t) - 3 \\ &= t^2 - 2t - 3 \\ &\neq f(t) \neq -f(t) \end{aligned}$$
f is neither even nor odd.

(b)

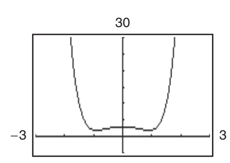


The graph is neither symmetric with respect to the origin nor with respect to the y-axis. So, f is neither even nor odd.

(c) Tables will vary. f is neither even nor odd.

80. (a)
$$\begin{aligned} f(-x) &= (-x)^6 - 2(-x)^2 + 3 \\ &= x^6 - 2x^2 + 3 \\ &= f(x) \end{aligned}$$
f is even.

(b)

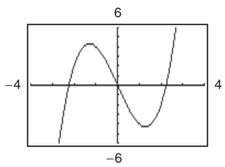


The graph is symmetric with respect to the y-axis. So, f is even.

(c) Tables will vary. f is even.

81. (a) $g(-x) = (-x)^3 - 5(-x)$
$= -x^3 + 5x$
$= -(x^3 - 5x)$
$= -g(x)$

g is odd.

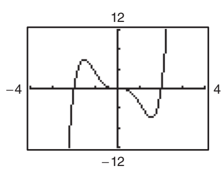
(b) 

The graph is symmetric with respect to the origin. So, g is odd.

(c) Tables will vary. g is odd.

82. (a) $h(-x) = (-x)^5 - 4(-x)^3$
$= -x^5 + 4x^3$
$= -(x^5 - 4x^3)$
$= -h(x)$

h is odd.

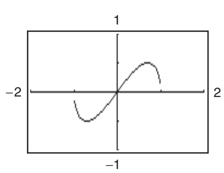
(b) 

The graph is symmetric with respect to the origin. So, h is odd.

(c) Tables will vary. h is odd.

83. (a) $f(-x) = (-x)\sqrt{1-(-x)^2}$
$= -x\sqrt{1-x^2}$
$= -f(x)$

f is odd.

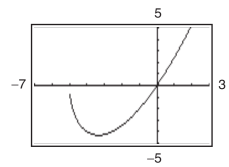
(b) 

The graph is symmetric with respect to the origin. So, f is odd.

(c) Tables will vary. f is odd.

84. (a) $f(-x) = (-x)\sqrt{(-x)+5}$
$= -x\sqrt{-x+5}$
$\neq f(x) \neq -f(x)$

f is neither even nor odd.

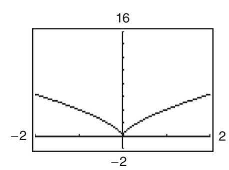
(b) 

The graph is neither symmetric with respect to the origin nor with respect to the y-axis. So, f is neither even nor odd.

(c) Tables will vary. f is neither even nor odd.

85. (a) $g(-s) = 4(-s)^{2/3}$
$= 4\left(\sqrt[3]{-s}\right)^2$
$= 4s^{2/3}$
$= g(s)$

g is even.

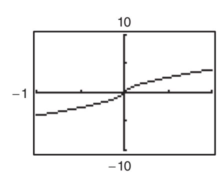
(b) 

The graph is symmetric with respect to the y-axis. So, g is even.

(c) Tables will vary. g is even.

86. (a) $f(-s) = 4(-s)^{3/5}$
$= -4s^{3/5}$
$= -f(s)$

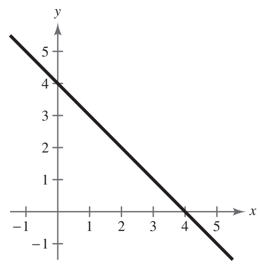
f is odd.

(b) 

The graph is symmetric with respect to the origin. So, f is odd.

(c) Tables will vary. f is odd.

87. $f(x) = 4 - x \geq 0$



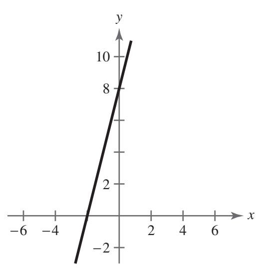
$f(x) \geq 0$

$4 - x \geq 0$

$4 \geq x$

$(-\infty, 4]$

88. $f(x) = 4x + 8$



$f(x) \geq 0$

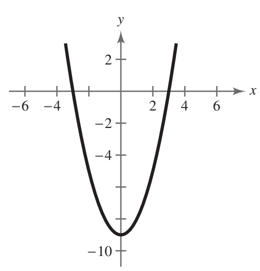
$4x + 8 \geq 0$

$4x \geq -8$

$x \geq -2$

$[-2, \infty)$

89. $f(x) = x^2 - 9 \geq 0$



$f(x) \geq 0$

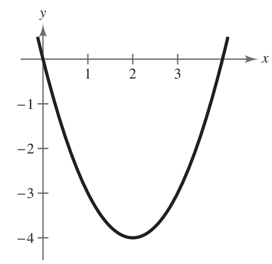
$x^2 - 9 \geq 0$

$x^2 \geq 9$

$x \geq 3$ or $x \leq -3$

$(-\infty, -3], [3, \infty)$

90. $f(x) = x^2 - 4x$



$f(x) \geq 0$

$x^2 - 4x \geq 0$

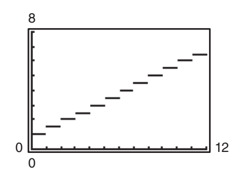
$x(x - 4) \geq 0$

$(-\infty, 0], [4, \infty)$

91. (a) C_2 is the appropriate model.

The cost of the first hour is \$1.00 and the cost increases \$0.50 when the next hour begins, and so on.

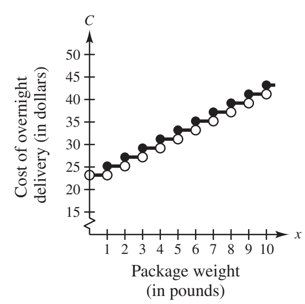
(b)



$$C_2\left(7 + \tfrac{1}{6}\right) = C_2(7.167)$$
$$= 1.00 - 0.50[\![-(7.167 - 1)]\!]$$
$$= 4.50$$

The cost for parking for 7 hours and 10 minutes is \$4.50.

92. $C = 23.20 + 2[\![x]\!], x > 0$



93. $h = \text{top} - \text{bottom}$

$= (-x^2 + 4x - 1) - 2$

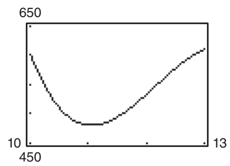
$= -x^2 + 4x - 3, \; 1 \leq x \leq 3$

94. $h = \text{top} - \text{bottom}$

$= 3 - (4x - x^2)$

$= 3 - 4x + x^2, 0 \leq x \leq 1$

95. (a)



(b) The number of cooperative homes and condos was decreasing from 2010 to early 2011 and increasing from early 2011 to 2013.

(c) The minimum number of cooperative homes and condos was about 480.8 thousand or 480,800 in early 2011.

96.

Interval	Intake Pipe	Drain Pipe 1	Drain Pipe 2
[0, 5]	Open	Closed	Closed
[5, 10]	Open	Open	Closed
[10, 20]	Closed	Closed	Closed
[20, 30]	Closed	Closed	Open
[30, 40]	Open	Open	Open
[40, 45]	Open	Closed	Open
[45, 50]	Open	Open	Open
[50, 60]	Open	Open	Closed

97. False. The domain of $f(x) = \sqrt{x^2}$ is the set of all real numbers.

98. False. The domain must be symmetric about the *y*-axis.

99. c

100. d

101. b

102. e

103. a

104. f

105. No. Each *y*-value corresponds to two distinct *x*-values when $y > 0$.

106. No. Each *y*-value corresponds to two distinct *x*-values when $-5 < y < 5$.

107. Yes, $f(x) = \begin{cases} -1, & -1 \le x < 0 \\ 0, & 0 \le x < 1 \\ 1, & 1 \le x < 2 \end{cases}$

108. The graph of the greatest integer function is a series of steps with a closed circle at the left end and an open circle at the right end. The graph of a line with a slope of zero is one continuous horizontal line with no steps.

109. f is an even function.

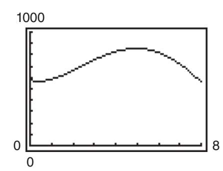
(a) $g(x) = -f(x)$ is even because
$g(-x) = -f(-x) = -f(x) = g(x)$.

(b) $g(x) = f(-x)$ is even because
$g(-x) = f(-(-x)) = f(x) = f(-x) = g(x)$.

(c) $g(x) = f(x) - 2$ is even because
$g(-x) = f(-x) - 2 = f(x) - 2 = g(x)$.

(d) $g(x) = -f(x + 3)$ is neither even nor odd because
$$g(-x) = -f(-x + 3) = -f(-(x - 3))$$
$$= -f(x - 3) \ne g(x) \text{ nor } -g(x).$$

110. (a)



(b) Using the graph from part (a), the domain is $(-\infty, \infty)$ and the range is $(-\infty, \infty)$.

(c) Using the graph from part (a), you can see that the graph is increasing on $-1 < x < 1$, and the graph is decreasing on $x < -1$ and $x > 1$.

(d) Using the graph from part (a), there is a relative minimum at $(-1, -2)$ and a relative maximum at $(1, 2)$.

111. $f(x) = a_{2n+1}x^{2n+1} + a_{2n-1}x^{2n-1} + \cdots + a_3x^3 + a_1x$
$$f(-x) = a_{2n+1}(-x)^{2n+1} + a_{2n-1}(-x)^{2n-1} + \cdots + a_3(-x)^3 + a_1(-x)$$
$$= -a_{2n+1}x^{2n+1} - a_{2n-1}x^{2n-1} - \cdots - a_3x^3 - a_1x = -f(x)$$
Therefore, $f(x)$ is odd.

112. $f(x) = a_{2n}x^{2n} + a_{2n-2}x^{2n-2} + \cdots + a_2x^2 + a_0$
$$f(-x) = a_{2n}(-x)^{2n} + a_{2n-2}(-x)^{2n-2} + \cdots + a_2(-x)^2 + a_0$$
$$= a_{2n}x^{2n} + a_{2n-2}x^{2n-2} + \cdots + a_2x^2 + a_0 = f(x)$$
$f(-x) = f(x)$; thus, $f(x)$ is even.

113. $-2x^2 + 11x + 3$

Terms: $-2x^2, 11x, 3$

Coefficients: $-2, 11$

114. $10 + 3x$

Terms: $3x, 10$

Coefficient: 3

115. $\dfrac{x}{3} - 5x^2 + x^3$

Terms: $\dfrac{x}{3}, -5x^2, x^3$

Coefficients: $\dfrac{1}{3}, -5, 1$

116. $7x^4 + \sqrt{2}x^2 - x$

Terms: $7x^4, \sqrt{2}x^2, -x$

Coefficients: $7, \sqrt{2}, -1$

117. $f(x) = -x^2 - x + 3$

(a) $f(4) = -(4)^2 - 4 + 3 = -17$

(b) $f(-5) = -(-5)^2 - (-5) + 3 = 12$

(c) $f(x-2) = -(x-2)^2 - (x-2) + 3$

$= -(x^2 - 4x + 4) - x + 2 + 3$

$= -x^2 + 3x + 1$

118. $f(x) = x\sqrt{x-3}$

(a) $f(3) = 3\sqrt{3-3} = 0$

(b) $f(12) = 12\sqrt{12-3}$

$= 12\sqrt{9} = 12(3) = 36$

(c) $f(6) = 6\sqrt{6-3} = 6\sqrt{3}$

119. $f(x) = x^2 - 2x + 9$

$f(3+h) = (3+h)^2 - 2(3+h) + 9 = 9 + 6h + h^2 - 6 - 2h + 9$

$= h^2 + 4h + 12$

$f(3) = 3^2 - 2(3) + 9 = 12$

$\dfrac{f(3+h) - f(3)}{h} = \dfrac{(h^2 + 4h + 12) - 12}{h} = \dfrac{h(h+4)}{h}$

$= h + 4, h \neq 0$

120. $f(x) = 5 + 6x - x^2$

$f(6+h) = 5 + 6(6+h) - (6+h)^2$

$= 5 + 36 + 6h - (36 + 12h + h^2)$

$= -h^2 - 6h + 5$

$f(6) = 5 + 6(6) - 6^2 = 5$

$\dfrac{f(h+6) - f(6)}{h} = \dfrac{(-h^2 - 6h + 5) - 5}{h} = \dfrac{h(-h-6)}{h}$

$= -h - 6, h \neq 0$

Section 1.5 Shifting, Reflecting, and Stretching Graphs

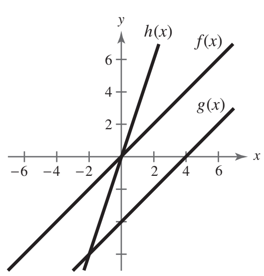
1. Horizontal shifts, vertical shifts, and reflections are rigid transformations.

2. (a) ii
(b) iv
(c) iii
(d) i

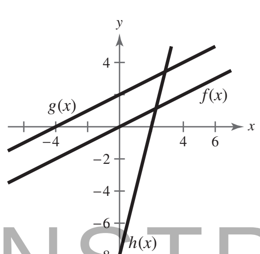
3. $-f(x), f(-x)$

4. $c > 1, 0 < c < 1$

5.



6.



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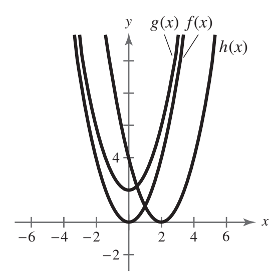

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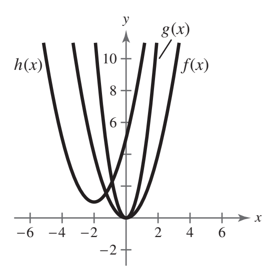

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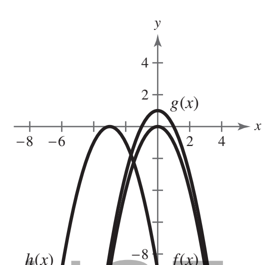

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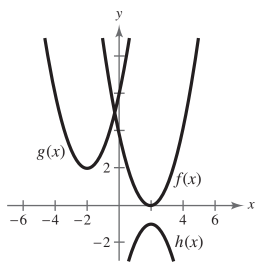
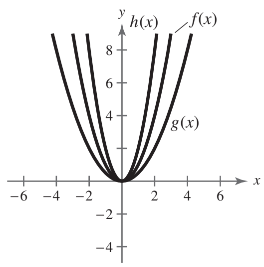
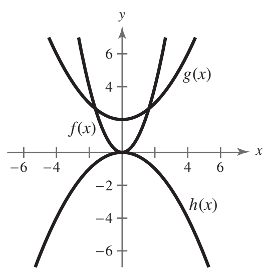


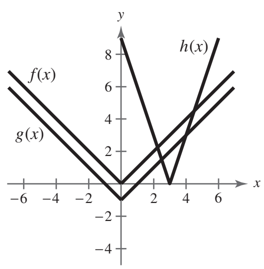
11.



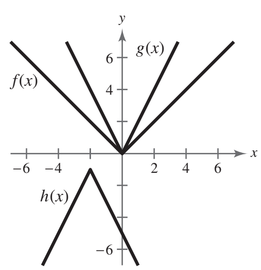
12.



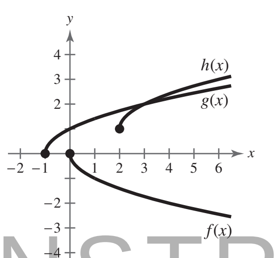
13.



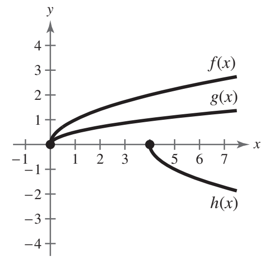
14.



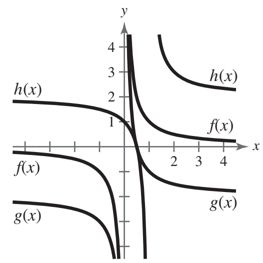
15.



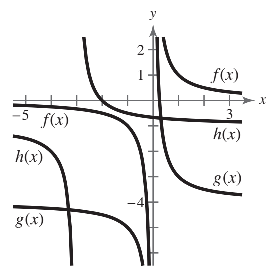
16.



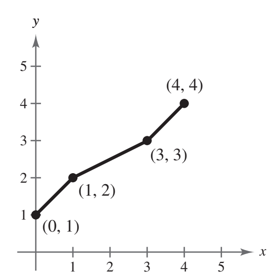
17.



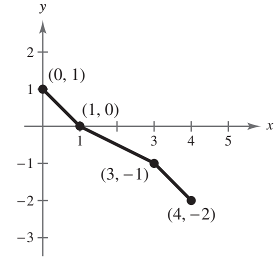
18.



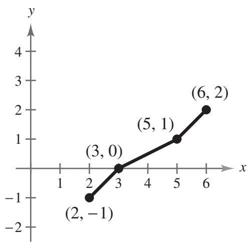
19. (a) $y = f(x) + 2$



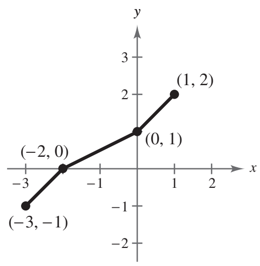
(b) $y = -f(x)$



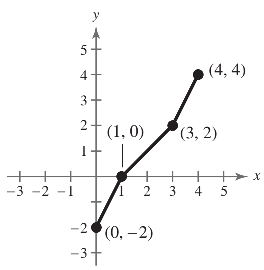
(c) $y = f(x-2)$



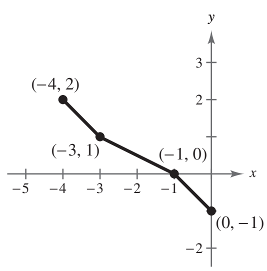
(d) $y = f(x+3)$



(e) $y = 2f(x)$



(f) $y = f(-x)$



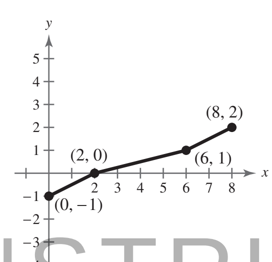
(g) Let $g(x) = f\left(\frac{1}{2}x\right)$. Then from the graph,

$$g(0) = f\left(\tfrac{1}{2}(0)\right) = f(0) = -1$$

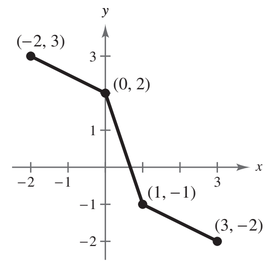
$$g(2) = f\left(\tfrac{1}{2}(2)\right) = f(1) = 0$$

$$g(6) = f\left(\tfrac{1}{2}(6)\right) = f(3) = 1$$

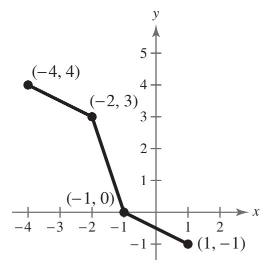
$$g(8) = f\left(\tfrac{1}{2}(8)\right) = f(4) = 2.$$



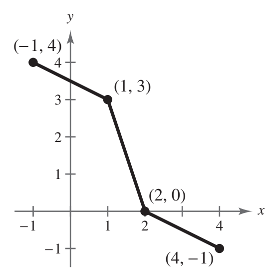
20. (a) $y = f(x) - 1$



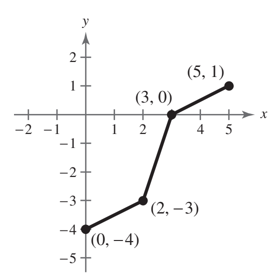
(b) $y = f(x+2)$



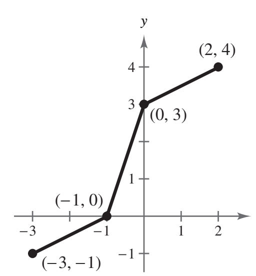
(c) $y = f(x-1)$



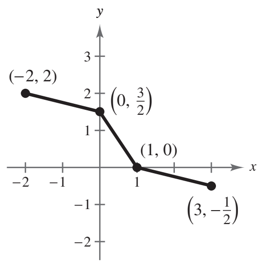
(d)



(e)



(f)



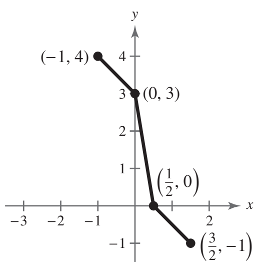
(g) Let $g(x) = f(2x)$. Then from the graph,

$$g(-1) = f(2(-1)) = f(-2) = 4$$

$$g(0) = f(2(0)) = f(0) = 3$$

$$g\left(\tfrac{1}{2}\right) = f\left(2\left(\tfrac{1}{2}\right)\right) = f(1) = 0$$

$$g\left(\tfrac{3}{2}\right) = f\left(2\left(\tfrac{3}{2}\right)\right) = f(3) = -1.$$



21. The graph of $f(x) = x^2$ should have been shifted one unit to the left instead of one unit to the right.

22. The graph of $f(x) = x^2$ should have been shifted one unit to the right instead of one unit downward.

23. $y = \sqrt{x} + 2$ is $f(x) = \sqrt{x}$ shifted vertically upward two units.

24. $y = \dfrac{1}{x} - 5$ is $f(x) = \dfrac{1}{x}$ shifted vertically five units downward.

25. $y = (x-4)^3$ is $f(x) = x^3$ shifted horizontally four units to the right.

26. $y = |x+5|$ is $f(x) = |x|$ shifted horizontally five units to the left.

27. $y = x^2 - 2$ is $f(x) = x^2$ shifted vertically two units downward.

28. $y = \sqrt{x-2}$ is $f(x) = \sqrt{x}$ shifted horizontally two units to the right.

29. Horizontal shift three units to left of $y = x$: $y = x + 3$ (or vertical shift three units upward)

30. Horizontal shift two units to the left of $y = \dfrac{1}{x} : y = \dfrac{1}{x+2}$

31. Vertical shift three units downward of $y = x^2$:
$y = x^2 - 3$

32. Horizontal shift two units to the left of $y = |x|$:
$y = |x+2|$

33. Reflection in the x-axis and a vertical shift one unit upward of $y = \sqrt{x}$: $y = 1 - \sqrt{x}$

34. Reflection in the x-axis and a vertical shift one unit upward of $y = x^3$: $y = 1 - x^3$

35. $y = -x$ is $f(x)$ reflected in the x-axis.

36. $y = |-x|$ is a reflection in the y-axis. In fact $y = |-x| = |x|$, therefore $y = |-x|$ is identical to $y = |x|$.

37. $y = (-x)^2$ is a reflection in the y-axis. In fact, $y = (-x)^2 = x^2$, therefore $y = (-x)^2$ is identical to $y = x^2$.

38. $y = -x^3$ is a reflection of $f(x) = x^3$ in the x-axis. However, since $y = -x^3 = (-x)^3$, either a reflection in the x-axis or a reflection in the y-axis produces the same graph.

39. $y = \dfrac{1}{-x}$ is a reflection of $f(x) = \dfrac{1}{x}$ in the y-axis.

However, since $y = \dfrac{1}{-x} = -\dfrac{1}{x}$, either a reflection in the y-axis or a reflection in the x-axis produces the same graph.

40. $y = -\dfrac{1}{x}$ is a reflection of $f(x) = \dfrac{1}{x}$ in the x-axis.

However, since $y = -\dfrac{1}{x} = \dfrac{1}{-x}$, either a reflection in the x-axis or a reflection in the y-axis produces the same graph.

41. $y = 4|x|$ is a vertical stretch of $f(x) = |x|$.

42. $p(x) = \dfrac{1}{2}x^2$ is a vertical shrink of $f(x) = \dfrac{x-1}{4}$.

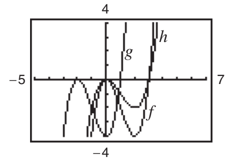
43. $g(x) = \dfrac{1}{4}x^3$ is a vertical shrink of $f(x) = x^3$.

44. $y = 2\sqrt{x}$ is a vertical stretch of $f(x) = \sqrt{x}$.

45. $f(x) = \sqrt{4x}$ is a horizontal shrink of $f(x) = \sqrt{x}$. However, since $f(x) = \sqrt{4x} = 2\sqrt{x}$, it also can be described as a vertical stretch of $f(x) = \sqrt{x}$.

46. $y = \left|\dfrac{1}{2}x\right| = \dfrac{1}{2}|x|$ is a vertical shrink of $f(x) = |x|$.

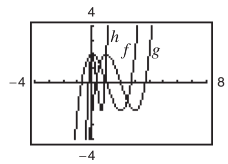
47. $f(x) = x^3 - 3x^2$



$g(x) = f(x+2) = (x+2)^3 - 3(x+2)^2$ is a horizontal shift two units to the left.

$h(x) = \frac{1}{2}f(x) = \frac{1}{2}(x^3 - 3x^2)$ is a vertical shrink.

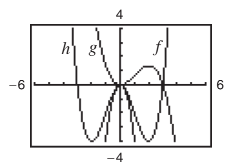
48. $f(x) = x^3 - 3x^2 + 2$



$g(x) = f(x-1) = (x-1)^3 - 3(x-1)^2 + 2$ is a horizontal shift one unit to the right.

$h(x) = f(3x) = (3x)^3 - 3(3x)^2 + 2$ is a horizontal shrink.

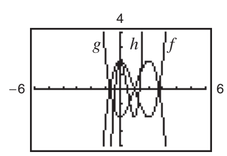
49. $f(x) = x^3 - 3x^2$



$g(x) = -\frac{1}{3}f(x) = -\frac{1}{3}(x^3 - 3x^2)$ is a reflection in the x-axis and a vertical shrink.

$h(x) = f(-x) = (-x)^3 - 3(-x)^2$ is a reflection in the y-axis.

50. $f(x) = x^3 - 3x^2 + 2$



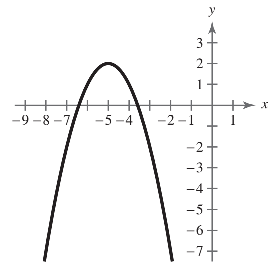
$g(x) = -f(x) = -(x^3 - 3x^2 + 2)$ is a reflection in the x-axis.

$h(x) = f(2x) = (2x)^3 - 3(2x)^2 + 2$ is a horizontal shrink.

51. (a) $f(x) = x^2$

(b) $g(x) = 2 - (x+5)^2$ is obtained from f by a horizontal shift to the left five units, a reflection in the x-axis, and a vertical shift upward two units.

(c)

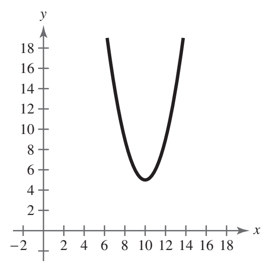


(d) $g(x) = 2 - f(x+5)$

52. (a) $f(x) = x^2$

(b) $g(x) = (x - 10)^2 + 5$ is obtained from f by a horizontal shift 10 units to the right and a vertical shift 5 units upward.

(c)

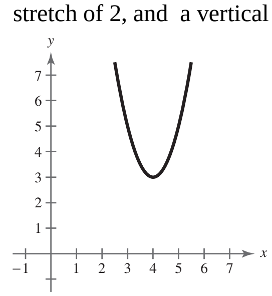


(d) $g(x) = f(x - 10) + 5$

53. (a) $f(x) = x^2$

(b) $g(x) = 3 + 2(x-4)^2$ is obtained from f by a horizontal shift four units to the right, a vertical stretch of 2, and a vertical shift upward three units.

(c)

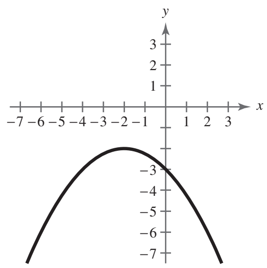


(d) $g(x) = 3 + 2f(x-4)$

54. (a) $f(x) = x^2$

(b) $g(x) = -\frac{1}{4}(x+2)^2 - 2$ is obtained from f by a horizontal shift two units to the left, a vertical shrink of $\frac{1}{4}$, a reflection in the x-axis, and a vertical shift two units downward.

(c)

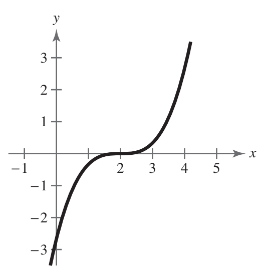


(d) $g(x) = -\frac{1}{4}f(x+2) - 2$

55. (a) $f(x) = x^3$

(b) $g(x) = \frac{1}{3}(x-2)^3$ is obtained from f by a horizontal shift two units to the right followed by a vertical shrink of $\frac{1}{3}$.

(c)

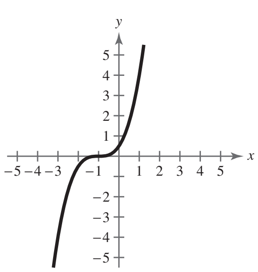


(d) $g(x) = \frac{1}{3}f(x-2)$

56. (a) $f(x) = x^3$

(b) $g(x) = \frac{1}{2}(x+1)^3$ is obtained from f by a horizontal shift one unit to the left and a vertical shrink of $\frac{1}{2}$.

(c)

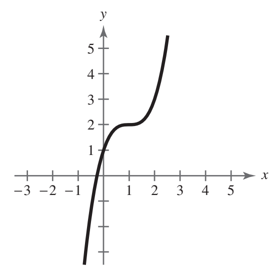


(d) $g(x) = \frac{1}{2}f(x+1)$

57. (a) $f(x) = x^3$

(b) $g(x) = (x-1)^3 + 2$ is obtained from f by a horizontal shift one unit to the right and a vertical shift upward two units.

(c)

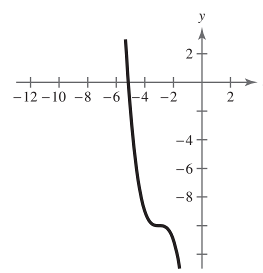


(d) $g(x) = f(x-1) + 2$

58. (a) $f(x) = x^3$

(b) $g(x) = -(x+3)^3 - 10$ is obtained from f by a horizontal shift 3 units to the left, a reflection in the x-axis, and a vertical shift 10 units downward.

(c)

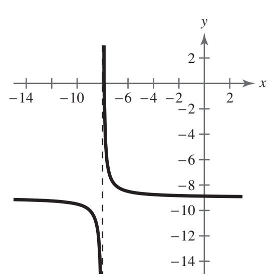


(d) $g(x) = -f(x+3) - 10$

59. (a) $f(x) = \frac{1}{x}$

(b) $g(x) = \frac{1}{x+8} - 9$ is obtained from f by a horizontal shift eight units to the left and a vertical shift nine units downward.

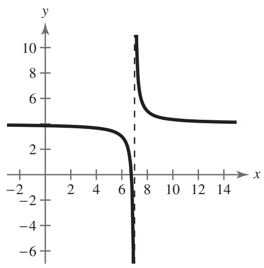
(c)



(d) $g(x) = f(x+8) - 9$

60. (a) $f(x) = \dfrac{1}{x}$

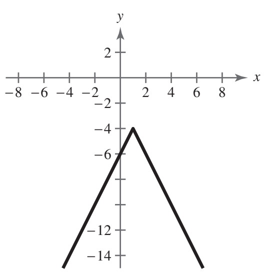
(b) $g(x) = \dfrac{1}{x-7} + 4$ is obtained from f by a horizontal shift seven units to the right and a vertical shift four units upward.

(c)


(d) $g(x) = f(x-7) + 4$

61. (a) $f(x) = |x|$

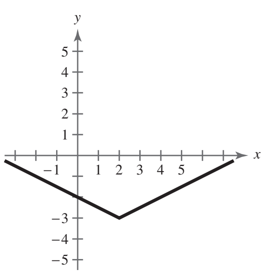
(b) $g(x) = -2|x-1| - 4$ is obtained from f by a horizontal shift one unit to the right, a vertical stretch of 2, a reflection in the x-axis, and a vertical shift downward four units.

(c)


(d) $g(x) = -2f(x-1) - 4$

62. (a) $f(x) = |x|$

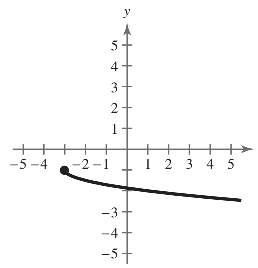
(b) $g(x) = \dfrac{1}{2}|x-2| - 3$ is obtained from f by a horizontal shift two units to the right, a vertical shrink, and a vertical shift three units downward.

(c)


(d) $g(x) = -\dfrac{1}{2}f(x-2) - 3$

63. (a) $f(x) = \sqrt{x}$

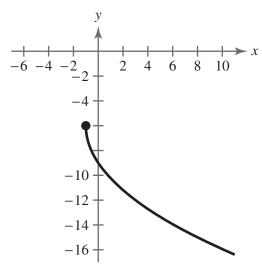
(b) $g(x) = -\dfrac{1}{2}\sqrt{x+3} - 1$ is obtained from f by a horizontal shift three units to the left, a vertical shrink, a reflection in the x-axis, and a vertical shift one unit downward.

(c)


(d) $g(x) = -\dfrac{1}{2}f(x+3) - 1$

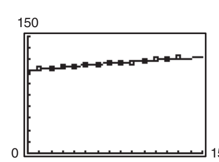
64. (a) $f(x) = \sqrt{x}$

(b) $g(x) = -3\sqrt{x+1} - 6$ is obtained from f by a horizontal shift one unit to the left, a reflection in the x-axis, a vertical stretch of 3, and a vertical shift six units downward.

(c)


(d) $g(x) = -3f(x+1) - 6$

65. (a) $N(t)$ is a horizontal shift of 26.17 units to the right, a vertical shift of 126.5 units upward, a reflection in the t-axis (horizontal axis), and a vertical shrink of 0.03.

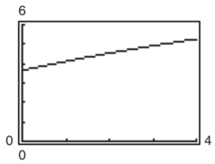
(b)


(c)
$$\begin{aligned} G(t) &= N(t+9) \\ &= -0.03[(t+9) - 26.17]^2 + 126.5 \\ &= -0.03(t-17.17)^2 + 126.5 \end{aligned}$$

To make a horizontal shift 9 years backward (9 units left), add 9 to t.

66. (a) $D(t)$ is a horizontal shift of 3.7 units to the left and a vertical stretch of 1.9.

(b)



(c) Let $D(t) = 6$ and solve for t.

$$19\sqrt{t + 3.7} = 60$$
$$\sqrt{t + 3.7} = \frac{60}{19}$$
$$t + 3.7 = \frac{3600}{361}$$
$$t = \frac{3600}{361} - 3.7$$
$$t \approx 6.27 \quad (\text{or year } 2015)$$

In the year 2015, the depreciation of assets will be approximately $6 million.

(d) $G(t) = D(t + 2) = 1.9\sqrt{(t + 2) + 3.7}$
$= 1.9\sqrt{t + 5.7}$

To make a horizontal shift 2 years backward (2 units left), add 2 to t.

67. False. $y = f(-x)$ is a reflection in the y-axis.

68. True. $y = |x| + 6$ and $y = |-x| + 6$ are identical because a reflection in the y-axis of $y = |x| + 6$ will be identical to itself. Additionally it is an even function.

69. $y = f(-x)$ is a reflection in the y-axis, so the x-intercepts are $x = -2$ and $x = 3$.

70. $y = 2f(x)$ is a vertical stretch, so the x-intercepts are the same: $x = 2, \ -3$.

71. $y = f(x) + 2$ is a vertical shift, so you cannot determine the x-intercepts.

72. $y = f(x - 3)$ is a horizontal shift 3 units to the right, so the x-intercepts are $x = 5$ and $x = 0$.

73. The vertex is approximately at $(2, 1)$ and the graph opens upward. Matches (c).

74. The domain is $[0, \ -\infty)$ and $(0, \ -4)$ is approximately on the graph, and $f(x) < 0$. Matches (a) and (b).

75. The vertex is approximately $(2, \ -4)$ and the graph opens upward. Matches (c).

76. The graph of f is $y = x^3$ shifted to the left approximately four units, reflected in the x-axis, and shifted upward approximately two units. Matches (b) and (d).

77. Answers will vary.

78. Since $y = f(x + 2) - 1$ is a horizontal shift of two units to the left and a vertical shift one unit downward, the point $(0, 1)$ will shift to $(-2, 0)$, $(1, 2)$ will shift to $(-1, 1)$, and $(2, 3)$ will shift to $(0, 2)$.

79. (a) Since $0 < a < 1$, $g(x) = ax^2$ will be a vertical shrink of $f(x) = x^2$.

(b) Since $a > 1$, $g(x) = ax^2$ will be a vertical stretch of $f(x) = x^2$.

80. (a) Increasing on the interval $(-2, 1)$ and decreasing on the intervals $(-\infty, -2)$ and $(1, \infty)$

(b) Increasing on the intervals $(-\infty, -2)$ and $(1, \infty)$, and decreasing on the interval $(-2, 1)$

(c) Increasing on the intervals $(-\infty, -1)$ and $(2, \infty)$, and decreasing on the interval $(-1, 2)$

81. Slope $L_1 : \dfrac{10 + 2}{2 + 2} = 3$

Slope $L_2 : \dfrac{9 - 3}{3 + 1} = \dfrac{3}{2}$

Neither parallel nor perpendicular

82. Slope $L_1 : \dfrac{3 - (-7)}{4 - (-1)} = \dfrac{10}{5} = 2$

Slope $L_2 : \dfrac{-7 - 5}{-2 - 1} = \dfrac{-12}{-3} = 4$

Neither parallel nor perpendicular

83. Domain: All $x \neq 9$

84. $f(x) = \dfrac{\sqrt{x - 5}}{x - 7}$

Domain: $x \geq 5$ and $x \neq 7$

85. Domain:
$100 - x^2 \geq 0 \Rightarrow x^2 \leq 100 \Rightarrow -10 \leq x \leq 10$

86. $f(x) = \sqrt[3]{16 - x^2}$

Domain: all real numbers

Section 1.6 Combinations of Functions

1. addition, subtraction, multiplication, division

2. composition

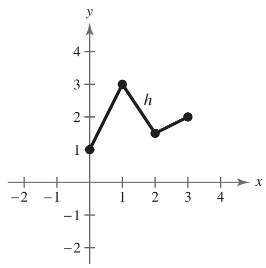
3. $g(x)$

4. inner, outer

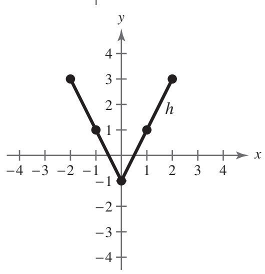
5. Since $(fg)(x) = 2x(x^2 + 1)$ and $f(x) = x^2 + 1$, $g(x) = 2x$, and $(fg)(x) = (gf)(x) = (2x)f(x)$.

6. Since $(f \circ g)(x) = f(g(x))$ and $(f \circ g)(x) = f(x^2 + 1)$, $g(x)$ must equal $x^2 + 1$.

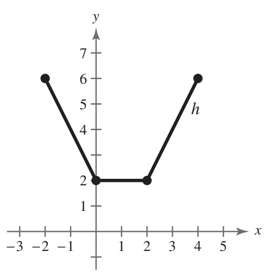
7.



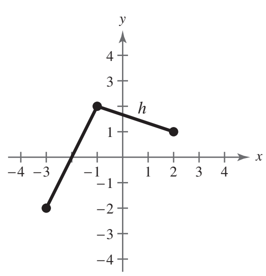
8.



9.



10.



11. $f(x) = x + 3,\ g(x) = x - 3$

(a) $(f + g)(x) = f(x) + g(x) = (x + 3) + (x - 3) = 2x$

(b) $(f - g)(x) = f(x) - g(x) = (x + 3) - (x - 3) = 6$

(c) $(fg)(x) = f(x)g(x) = (x + 3)(x - 3) = x^2 - 9$

(d) $\left(\dfrac{f}{g}\right)(x) = \dfrac{f(x)}{g(x)} = \dfrac{x + 3}{x - 3}$

Domain: all $x \neq 3$

12. $f(x) = 2x - 5,\ g(x) = 1 - x$

(a) $(f + g)(x) = 2x - 5 + 1 - x = x - 4$

(b) $(f - g)(x) = 2x - 5 - (1 - x)$
$= 2x - 5 - 1 + x$
$= 3x - 6$

(c) $(fg)(x) = (2x - 5)(1 - x)$
$= 2x - 2x^2 - 5 + 5x$
$= -2x^2 + 7x - 5$

(d) $\left(\dfrac{f}{g}\right)(x) = \dfrac{2x - 5}{1 - x}$

Domain: $1 - x \neq 0$
$x \neq 1$

13. $f(x) = 3x^2,\ g(x) = 6 - 5x$

(a) $(f + g)(x) = f(x) + g(x)$
$= 3x^2 + (6 - 5x)$
$= 3x^2 - 5x + 6$

(b) $(f - g)(x) = f(x) - g(x)$
$= 3x^2 - (6 - 5x)$
$= 3x^2 + 5x - 6$

(c) $(fg)(x) = f(x) \cdot g(x)$
$= 3x^2(6 - 5x)$
$= 18x^2 - 15x^3$
$= -15x^3 + 18x^2$

(d) $\left(\dfrac{f}{g}\right)(x) = \dfrac{f(x)}{g(x)} = \dfrac{3x^2}{6 - 5x}$

Domain: all $x \neq \dfrac{6}{5}$

14. $f(x) = 2x + 5,\ g(x) = x^2 - 9$

(a) $(f + g)(x) = (2x + 5) + (x^2 - 9)$
$= x^2 + 2x - 4$

(b) $(f - g)(x) = (2x + 5) - (x^2 - 9)$
$= -x^2 + 2x + 14$

(c) $(fg)(x) = (2x + 5)(x^2 - 9)$
$= 2x^3 + 5x^2 - 18x - 45$

(d) $\left(\dfrac{f}{g}\right)(x) = \dfrac{2x + 5}{x^2 - 9}$

Domain: all $x \neq \pm 3$

15. $f(x) = x^2 + 5,\ g(x) = \sqrt{1-x}$

(a) $(f+g)(x) = x^2 + 5 + \sqrt{1-x}$

(b) $(f-g)(x) = x^2 + 5 - \sqrt{1-x}$

(c) $(fg)(x) = (x^2+5)\sqrt{1-x}$

(d) $\left(\dfrac{f}{g}\right)(x) = \dfrac{x^2+5}{\sqrt{1-x}}$

Domain: $x < 1$

16. $f(x) = \sqrt{x^2-4},\ g(x) = \dfrac{x^2}{x^2+1}$

(a) $(f+g)(x) = \sqrt{x^2-4} + \dfrac{x^2}{x^2+1}$

(b) $(f-g)(x) = \sqrt{x^2-4} - \dfrac{x^2}{x^2+1}$

(c) $(fg)(x) = \left(\sqrt{x^2-4}\right)\left(\dfrac{x^2}{x^2+1}\right) = \dfrac{x^2\sqrt{x^2-4}}{x^2+1}$

(d) $\left(\dfrac{f}{g}\right)(x) = \sqrt{x^2-4} \div \dfrac{x^2}{x^2+1}$

$$= \frac{(x^2+1)\sqrt{x^2-4}}{x^2}$$

Domain: $x^2 - 4 \ge 0$ and $x \ne 0$

$x \ge 2$ or $x \le -2$

17. $f(x) = \dfrac{1}{x},\ g(x) = \dfrac{1}{x^2}$

(a) $(f+g)(x) = \dfrac{1}{x} + \dfrac{1}{x^2} = \dfrac{x+1}{x^2}$

(b) $(f-g)(x) = \dfrac{1}{x} - \dfrac{1}{x^2} = \dfrac{x-1}{x^2}$

(c) $(fg)(x) = \dfrac{1}{x}\cdot\dfrac{1}{x^2} = \dfrac{1}{x^3}$

(d) $\left(\dfrac{f}{g}\right)(x) = \dfrac{\frac{1}{x}}{\frac{1}{x^2}} = x,\ x \ne 0$

Domain: $x \ne 0$

18. $f(x) = \dfrac{x}{x+1},\ g(x) = \dfrac{1}{x^3}$

(a) $(f+g)(x) = \dfrac{x}{x+1} + \dfrac{1}{x^3} = \dfrac{x^4+x+1}{x^3(x+1)}$

(b) $(f-g)(x) = \dfrac{x}{x+1} - \dfrac{1}{x^3} = \dfrac{x^4-x-1}{x^3(x+1)}$

(c) $(fg)(x) = \dfrac{x}{x+1}\cdot\dfrac{1}{x^3} = \dfrac{1}{x^2(x+1)}$

(d) $\left(\dfrac{f}{g}\right)(x) = \dfrac{x}{x+1} \div \dfrac{1}{x^3}$

$$= \frac{x}{x+1}\cdot\frac{x^3}{1}$$

$$= \frac{x^4}{x+1}$$

Domain: $x \ne 0,\ x \ne -1$

19. $(f+g)(3) = f(3) + g(3)$

$= (3^2-1) + (3-2)$

$= 8 + 1 = 9$

20. $(f-g)(-2) = f(-2) - g(-2)$

$= ((-2)^2 - 1) - (-2-2)$

$= 3 - (-4) = 7$

21. $(f-g)(0) = f(0) - g(0)$

$= (0-1) - (0-2)$

$= 1$

22. $(f+g)(1) = f(1) + g(1)$

$= (1^2-1) + (1-2)$

$= 0 + (-1)$

$= -1$

23. $(fg)(-6) = f(-6)g(-6)$

$= \left((-6)^2 - 1\right)\left((-6) - 2\right)$

$= (35)(-8)$

$= -280$

24. $(fg)(4) = f(4)g(4)$

$= \left((4)^2 - 1\right)(4-2)$

$= (15)(2)$

$= 30$

25. $\left(\dfrac{f}{g}\right)(-5) = \dfrac{f(-5)}{g(-5)}$

$= \dfrac{(-5)^2 - 1}{-5 - 2}$

$= \dfrac{24}{-7}$

$= -\dfrac{24}{7}$

26. $\left(\dfrac{f}{g}\right)(0) = \dfrac{f(0)}{g(0)}$

$= \dfrac{0 - 1}{0 - 2}$

$= \dfrac{1}{2}$

27. $(f - g)(t + 1) = f(t + 1) - g(t + 1)$

$= \left((t + 1)^2 - 1\right) - \left((t + 1) - 2\right)$

$= \left(t^2 + 2t + 1 - 1\right) - (t - 1)$

$= t^2 + t + 1$

28. $(f + g)(t - 3) = f(t - 3) + g(t - 3)$

$= \left((t - 3)^2 - 1\right) + (t - 3 - 2)$

$= \left(t^2 - 6t + 9 - 1\right) + (t - 5)$

$= t^2 - 5t + 3$

29. $(fg)(-5t) = f(-5t)g(-5t)$

$= ((-5t)^2 - 1)(-5t - 2)$

$= (25t^2 - 1)(-5t - 2)$

$= -125t^3 - 50t^2 + 5t + 2$

30. $(fg)(3t^2) = f(3t^2)g(3t^2)$

$= ((3t^2)^2 - 1)(3t^2 - 2)$

$= (9t^4 - 1)(3t^2 - 2)$

$= 27t^6 - 18t^4 - 3t^2 + 2$

31. $\left(\dfrac{f}{g}\right)(t - 4) = \dfrac{f(t - 4)}{g(t - 4)}$

$= \dfrac{(t - 4)^2 - 1}{(t - 4) - 2}$

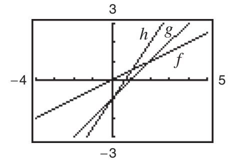
$= \dfrac{t^2 - 8t + 15}{t - 6}$

32. $\left(\dfrac{f}{g}\right)(t + 2) = \dfrac{f(t + 2)}{g(t + 2)}$

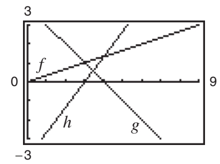
$= \dfrac{(t + 2)^2 - 1}{(t + 2) - 2}$

$= \dfrac{t^2 + 4t + 3}{t},\ t \neq 0$

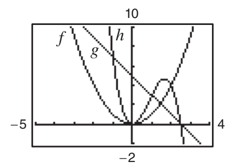
33.



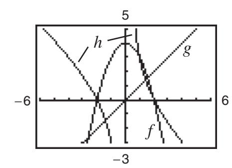
34.



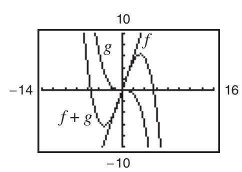
35.



36.



37. $f(x) = 3x,\ g(x) = -\dfrac{x^3}{10},\ (f + g)(x) = 3x - \dfrac{x^3}{10}$

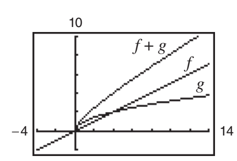


For $0 \le x \le 2$, $f(x)$ contributes more to the magnitude.

For $x > 6$, $g(x)$ contributes more to the magnitude.

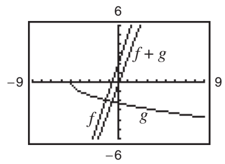
38. $f(x) = \dfrac{x}{2},\ g(x) = \sqrt{x},$

$(f + g)(x) = \dfrac{x}{2} + \sqrt{x}$



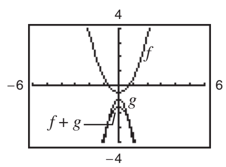
$g(x)$ contributes more to the magnitude of the sum for $0 \le x \le 2$. $f(x)$ contributes more to the magnitude of the sum for $x > 6$.

39. $f(x) = 3x + 2,\ g(x) = -\sqrt{x+5},$
$(f+g)(x) = 3x + 2 - \sqrt{x+5}$



$f(x)$ contributes more to the magnitude in both intervals.

40. $f(x) = x^2 - \frac{1}{2},\ g(x) = -3x^2 - 1,$
$(f+g)(x) = \left(x^2 - \frac{1}{2}\right) + (-3x^2 - 1) = -2x^2 - \frac{3}{2}$



$g(x)$ contributes more to the magnitude on both intervals.

41. $f(x) = 2x^2,\ g(x) = x + 4$

(a) $(f \circ g)(x) = f(g(x))$
$= f(x + 4)$
$= 2(x + 4)^2$
$= 2x^2 + 16x + 32$

(b) $(g \circ f)(x) = g(f(x))$
$= g(2x^2)$
$= (2x^2) + 4$
$= 2x^2 + 4$

(c) $(f \circ g)(0) = 2(0 + 4)^2 = 32$

42. $f(x) = \sqrt[3]{x-1},\ g(x) = x^3 + 1$

(a) $(f \circ g)(x) = f(g(x))$
$= f(x^3 + 1)$
$= \sqrt[3]{(x^3+1) - 1}$
$= \sqrt[3]{x^3} = x$

(b) $(g \circ f)(x) = g(f(x))$
$= g\left(\sqrt[3]{x-1}\right)$
$= \left(\sqrt[3]{x-1}\right)^3 + 1$
$= (x-1) + 1 = x$

(c) $(f \circ g)(0) = 0$

43. $f(x) = 3x + 5,\ g(x) = 5 - x$

(a) $(f \circ g)(x) = f(g(x)) = f(5 - x) = 3(5 - x) + 5 = 20 - 3x$

(b) $(g \circ f)(x) = g(f(x)) = g(3x + 5) = 5 - (3x + 5) = -3x$

(c) $(f \circ g)(0) = 20$

44. $f(x) = x^3,\ g(x) = \frac{1}{x}$

(a) $(f \circ g)(x) = f(g(x)) = f\left(\frac{1}{x}\right) = \left(\frac{1}{x}\right)^3 = \frac{1}{x^3}$

(b) $(g \circ f)(x) = g(f(x)) = g(x^3) = \frac{1}{x^3}$

(c) $(f \circ g)(0)$ is not defined.

45. (a) Domain of *f*: $x - 7 \geq 0$ or $x \geq 7$

(b) Domain of *g*: all real numbers

(c) $(f \circ g)(x) = f(g(x)) = f(4x^2) = \sqrt{4x^2 - 7}$

Domain: all real $x \leq -\frac{\sqrt{7}}{2} \cup x \geq \frac{\sqrt{7}}{2}$

46. (a) Domain of f: $x + 3 \geq 0 \Rightarrow x \geq -3$

(b) Domain of g : all real numbers

(c) $(f \circ g)(x) = f\left(\frac{x}{2}\right) = \sqrt{\frac{x}{2} + 3}$

Domain: $\frac{x}{2} + 3 \geq 0 \Rightarrow x \geq -6$

47. (a) Domain of *f*: all real numbers
(b) Domain of *g*: all $x \geq 0$
(c) $(f \circ g)(x) = f(g(x)) = f\left(\sqrt{x}\right)$
$= \left(\sqrt{x}\right)^2 + 1 = x + 1,\ x \geq 0$

Domain: $x \geq 0$

48. (a) Domain of f: $x \geq 0$
(b) Domain of *g*: all real numbers
(c) $(f \circ g)(x) = f(g(x)) = f(x^4) = (x^4)^{1/4} = x$

Domain: all real numbers

49. (a) Domain of *f*: all $x \neq 0$

(b) Domain of *g*: all $x \neq 3$

(c) $(f \circ g)(x) = f\left(\frac{1}{x+3}\right) = \frac{1}{\frac{1}{x+3}} = x + 3$

Domain: all $x \neq -3$

50. (a) Domain of f: all $x \neq 0$

(b) Domain of g: all $x \neq 0$

(c) $(f \circ g)(x) = f\left(\dfrac{1}{2x}\right) = 2x,\ x \neq 0$

Domain: all $x \neq 0$

51. (a) Domain of f: all real numbers

(b) Domain of g: all real numbers

(c) $(f \circ g)(x) = f(g(x)) = f(3 - x)$

$= |(3 - x) - 4| = |-x - 1| = |x + 1|$

Domain: all real numbers

52. (a) Domain of f: all $x \neq 0$

(b) Domain of g: all real numbers

(c) $(f \circ g)(x) = f(g(x)) = f(x - 5) = \dfrac{2}{|x - 5|}$

Domain: all $x \neq 5$

53. (a) Domain of f: all real numbers

(b) Domain of g: all $x \neq \pm 2$

(c) $(f \circ g)(x) = f(g(x)) = f\left(\dfrac{1}{x^2 - 4}\right) = \dfrac{1}{x^2 - 4} + 2$

Domain: $x \neq \pm 2$

54. (a) Domain of f: all $x \neq \pm 1$

(b) Domain of g: all real numbers

(c) $(f \circ g)(x) = f(x + 1) = \dfrac{3}{(x + 1)^2 - 1}$

$= \dfrac{3}{x^2 + 2x} = \dfrac{3}{x(x + 2)}$

Domain: all $x \neq 0, -2$

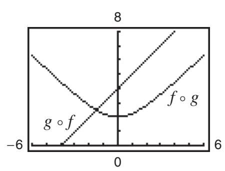
55. (a) $(f \circ g)(x) = f(g(x)) = f(x^2) = \sqrt{x^2 + 4}$

Domain: all real numbers

$(g \circ f)(x) = g(f(x)) = g\left(\sqrt{x + 4}\right) = \left(\sqrt{x + 4}\right)^2$

$= x + 4,\ x \geq -4$

(b)



They are not equal.

56. (a) $(f \circ g)(x) = f(g(x)) = f(x^3 - 1)$

$= \sqrt[3]{(x^3 - 1) + 1} = \sqrt[3]{x^3} = x$

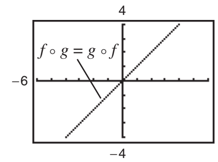
Domain: all real numbers

$(g \circ f)(x) = g(f(x)) = g\left(\sqrt[3]{x + 1}\right)$

$= \left[\sqrt[3]{x + 1}\right]^3 - 1$

$= (x + 1) - 1 = x$

(b)



They are equal.

57. (a) $(f \circ g)(x) = f(g(x)) = f(3x + 9)$

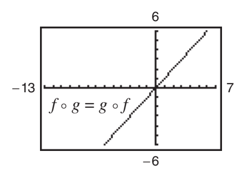
$= \dfrac{1}{3}(3x + 9) - 3 = x$

Domain: all real numbers

$(g \circ f)(x) = g(f(x)) = g\left(\dfrac{1}{3}x - 3\right)$

$= 3\left(\dfrac{1}{3}x - 3\right) + 9 = x$

(b)

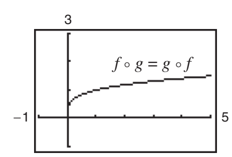


They are equal.

58. (a) $(f \circ g)(x) = (g \circ f)(x) = \sqrt{\sqrt{x}} = x^{1/4}$

Domain: all $x \geq 0$

(b)



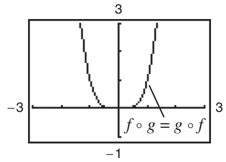
They are equal.

59. (a) $(f \circ g)(x) = f(g(x)) = f(x^6) = (x^6)^{2/3} = x^4$

Domain: all real numbers

$(g \circ f)(x) = g(f(x)) = g(x^{2/3}) = (x^{2/3})^6 = x^4$

(b)



They are equal.

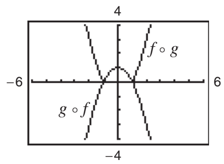
60. (a) $(f \circ g)(x) = f(g(x)) = f(-x^2 + 1) = \left|-x^2 + 1\right|$

Domain: all real numbers

$(g \circ f)(x) = g(f(x)) = g(|x|) = -|x|^2 + 1$

$= 1 - x^2$

(b)



They are not equal.

61. (a) $(f \circ g)(x) = f(g(x))$

$= f\left(\frac{1}{5}(x - 4)\right)$

$= 5\left(\frac{1}{5}(x - 4)\right) + 4$

$= x - 4 + 4$

$= x$

$(g \circ f)(x) = g(f(x))$

$= g(5x + 4)$

$= \frac{1}{5}((5x + 4) - 4)$

$= \frac{1}{5}(5x)$

$= x$

(b) They are equal because $x = x$.

(c)

x	0	1	2	3
$g(x)$	$-\frac{4}{5}$	$-\frac{3}{5}$	$-\frac{2}{5}$	$-\frac{1}{5}$
$(f \circ g)(x)$	0	1	2	3

x	0	1	2	3
$f(x)$	4	9	14	19
$(g \circ f)(x)$	0	1	2	3

62. (a) $(f \circ g)(x) = f(4x+1) = \frac{1}{4}[(4x+1) - 1] = \frac{1}{4}[4x] = x$

$(g \circ f)(x) = g\left(\frac{1}{4}(x-1)\right) = 4\left[\frac{1}{4}(x-1)\right] + 1 = (x-1) + 1 = x$

(b) They are equal because $x = x$.

(c)

x	$f(g(x))$	$g(f(x))$
–1	–1	–1
0	0	0
1	1	1
2	2	2
3	3	3

63. (a)

$(f \circ g)(x) = f(g(x)) = f(x^2 - 5) = \sqrt{(x^2 - 5) + 6} = \sqrt{x^2 + 1}$

$(g \circ f)(x) = g(f(x)) = g\left(\sqrt{x+6}\right) = \left(\sqrt{x+6}\right)^2 - 5$

$= (x+6) - 5 = x + 1,\ x \ge -6$

(b) They are not equal because $\sqrt{x^2 + 1} \ne x + 1$.

(c)

x	$f(g(x))$	$g(f(x))$
0	1	1
–2	$\sqrt{5}$	–1
3	$\sqrt{10}$	4

64. (a) $(f \circ g)(x) = f\left(\sqrt[3]{x+10}\right) = \left[\sqrt[3]{x+10}\right]^3 - 4$

$= (x+10) - 4 = x + 6$

$(g \circ f)(x) = g(x^3 - 4) = \sqrt[3]{(x^3 - 4) + 10} = \sqrt[3]{x^3 + 6}$

(b) They are not equal because $x + 6 \ne \sqrt[3]{x^3 + 6}$.

(c)

x	$f(g(x))$	$g(f(x))$
–2	4	$\sqrt[3]{-2}$
0	6	$\sqrt[3]{6}$
1	7	$\sqrt[3]{7}$
2	8	$\sqrt[3]{14}$
3	9	$\sqrt[3]{33}$

65. (a) $(f\circ g)(x) = f(g(x)) = f(2x^3) = \left|2x^3\right|$

$(g\circ f)(x) = g(f(x)) = g(\left|x\right|) = 2\left|x\right|^3$

(b) They are equal because $\left|2x^3\right| = 2\left|x\right|^3$.

(c)

x	$f(g(x))$	$g(f(x))$
–1	2	2
0	0	0
1	2	2
2	16	16

66. (a)

$(f\circ g)(x) = f(g(x)) = f(-x) = \dfrac{6}{3(-x)-5} = \dfrac{6}{-3x-5}$

$(g\circ f)(x) = g\left(\dfrac{6}{3x-5}\right) = -\left(\dfrac{6}{3x-5}\right) = \dfrac{-6}{3x-5}$

(b) They are not equal because $\dfrac{6}{-3x-5} \neq \dfrac{-6}{3x-5}$.

(c)

x	$f(g(x))$	$g(f(x))$
0	$-\frac{6}{5}$	$\frac{6}{5}$
1	$-\frac{3}{4}$	3
2	$-\frac{6}{11}$	–6
3	$-\frac{3}{7}$	$-\frac{3}{2}$

67. (a) $(f+g)(3) = f(3) + g(3) = 2 + 1 = 3$

(b) $\left(\dfrac{f}{g}\right)(2) = \dfrac{f(2)}{g(2)} = \dfrac{0}{2} = 0$

68. (a) $(f-g)(1) = f(1) - g(1) = 2 - 3 = -1$

(b) $(fg)(4) = f(4)\cdot g(4) = 4\cdot 0 = 0$

69. (a) $(f\circ g)(3) = f(g(3)) = f(1) = 2$

(b) $(g\circ f)(2) = g(f(2)) = g(0) = 4$

70. (a) $(f\circ g)(1) = f(g(1)) = f(3) = 2$

(b) $(g\circ f)(3) = g(f(3)) = g(2) = 2$

71. Let $f(x) = x^2$ and $g(x) = 2x + 1$,
then $(f\circ g)(x) = h(x)$. This is not a unique solution.
Another possibility is $f(x) = (x+1)^2$ and $g(x) = 2x$.

72. Let $g(x) = 1 - x$ and $f(x) = x^3$, then $(f\circ g)(x) = h(x)$.
This answer is not unique. Another possibility is
$f(x) = (x+1)^3$ and $g(x) = -x$.

73. Let $f(x) = \sqrt[3]{x}$ and $g(x) = x^2 - 4$,
then $(f\circ g)(x) = h(x)$. This answer is not unique.
Other possibilities are

$f(x) = \sqrt[3]{x-4}$ and $g(x) = x^2$ or
$f(x) = \sqrt[3]{-x}$ and $g(x) = 4 - x^2$ or
$f(x) = \sqrt[9]{x}$ and $g(x) = (x^2-4)^3$.

74. Let $g(x) = 9 - x$ and $f(x) = \sqrt{x}$,
then $(f\circ g)(x) = h(x)$. This answer is not unique.
Another possibility is $f(x) = \sqrt{9+x}$ and $g(x) = -x$.

75. Let $f(x) = \dfrac{1}{x}$ and $g(x) = x + 2$,
then $(f\circ g)(x) = h(x)$. This is not a unique solution.
Other possibilities are

$f(x) = \dfrac{1}{x+2}$ and $g(x) = x$ or $f(x) = \dfrac{1}{x+1}$ and
$g(x) = x + 1$.

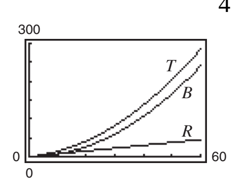
76. Let $g(x) = 5x + 2$ and $f(x) = \dfrac{4}{x^2}$, then $(f\circ g)(x) = h(x)$.
This answer is not unique. Another possibility is
$f(x) = \dfrac{4}{x}$ and $g(x) = (5x+2)^2$.

77. Let $f(x) = x^2 + 2x$ and $g(x) = x + 4$, then $(f\circ g)(x) = h(x)$.
This answer is not unique. Another possibility is $f(x) = x$ and $g(x) = (x+4)^2 + 2(x+4)$.

78. Let $g(x) = x + 3$ and $f(x) = x^{3/2} + 4x^{1/2}$,
then $(f\circ g)(x) = h(x)$. This answer is not unique.
Another possibility is $f(x) = (x+1)^{3/2} + 4(x+1)^{1/2}$ and
$g(x) = x + 2$.

79. (a) $T(x) = R(x) + B(x) = \dfrac{3}{4}x + \dfrac{1}{15}x^2$

(b)



(c) $B(x)$ contributes more to $T(x)$ at higher speeds.

80. (a)

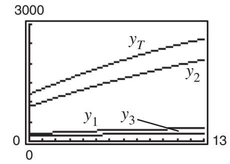
Year	2002	2003	2004	2005
y_1	222.92	238.52	252.88	266
y_2	1115.36	1222.46	1325.84	1425.5
y_3	144.12	151.82	159.28	166.5

Year	2006	2007	2008	2009
y_1	277.88	288.52	297.92	306.08
y_2	1521.44	1613.66	1702.16	1786.94
y_3	173.48	180.22	186.72	192.98

Year	2010	2011	2012
y_1	313	318.68	323.12
y_2	1868	1945.34	2018.96
y_3	199	204.78	210.32

The models are a good fit for the data. The variation of the model from the actual data is small in comparison to the sizes of the numbers.

(b)



y_T represents the total out-of-pocket payments, insurance premiums, and other types of payments in billions of dollars spent on health consumption expenditures in the United States and Puerto Rico for each year t.

81. (a) $r(x) = \dfrac{x}{2}$

(b) $A(r) = \pi r^2$

(c) $(A \circ r)(x) = A(r(x))$

$$= A\left(\frac{x}{2}\right) = \pi\left(\frac{x}{2}\right)^2 = \frac{1}{4}\pi x^2$$

$A \circ r$ represents the area of the circular base of the tank with radius $\dfrac{x}{2}$.

82. $(A \circ r)(t) = A(r(t))$

$$= A(0.6t)$$

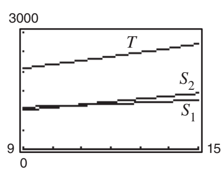
$$= \pi(0.6t)^2 = 0.36\pi t^2$$

$(A \circ r)(t)$ gives the area of the circle as a function of time.

83. (a) Since $T = S_1 + S_2$, $T = \left(973 + 1.3t^2\right) + \left(349 + 72.4t\right)$

$T = 1.3t^2 + 72.4t + 1322$

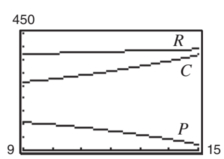
(b)



84. (a) Since $P = R - C$, $P = \left(341 + 3.2t\right) - \left(254 - 9t + 1.1t^2\right) - \left(341 + 3.2t\right)$

$P = -1.1t^2 + 12.2t + 87$

(b)



85. (a) $(N \circ T)(t) = N(T(t))$
$= N(2t + 1)$
$= 10(2t + 1)^2 - 20(2t + 1) + 600$
$= 40t^2 + 590$

$N \circ T$ represents the number of bacteria after t hours outside the refrigerator.

(b) $(N \circ T)(12) = 10(25^2) - 20(25) + 600 = 6350$

There are 6350 bacteria in a refrigerated food product after 12 hours outside the refrigerator.

(c) $40t^2 + 590 = 1200$
$40t^2 = 610$
$t^2 = 15.25$
$t \approx \pm 3.9$

$N = 1200$ when $t \approx 3.9$ hours.

86. (a) Area $= \pi r^2$, $r(t) = 5.25\sqrt{t}$. Hence

$(A \circ r)(t) = \pi\left[5.25\sqrt{t}\right]^2 = 27.5625\pi t, \quad t \geq 0$

(b) $(A \circ r)(36) = 27.5625\pi(36) = 992.25\pi$
≈ 3117 square meters

(c) $A = 6250 = 27.5625\pi t \Rightarrow t \approx 72.2$ hours

87. First, write the distance each plane is from point P. The plane that is 200 miles from point P is traveling at 450 miles per hour. Its distance is $200 - 450t$. Similarly, the other plane is $150 - 450t$ from point P.

So, the distance between the planes $s(t)$ can be found using the distance formula (or the Pythagorean Theorem):
$s(t) = \sqrt{(200 - 450t)^2 + (150 - 450t)^2}$
$s(t) = 50\sqrt{162t^2 - 126t + 25}$

88. (a) $R(p) = p - 2000$

(b) $S(p) = 0.91p$

(c) $(R \circ S)(p) = 0.91p - 2000$

This is the cost if the discount is taken before the rebate.

$(S \circ R)(p) = 0.91(p - 2000)$

This is the cost if the rebate is taken before the discount.

(d) $(R \circ S)(24{,}795) = \$20{,}563.45$
$(S \circ R)(24{,}795) = \$20{,}743.45$

The discount first yields a lower cost because the discount is applied to the full amount and then the rebate is taken.

89. False. $g(x) = x - 3$

90. True. $(f \circ g)(x) = f(g(x))$ is only defined when $g(x)$ is in the domain of *f*.

91. (a) If $f(x) = x^2$ and $g(x) = \dfrac{1}{x-2}$, then

$f(g(x)) = \left(\dfrac{1}{x-2}\right)^2 = \dfrac{1^2}{(x-2)^2} = \dfrac{1}{(x-2)^2} = h(x).$

(b) If $f(x) = \dfrac{1}{x-2}$ and $g(x) = x^2$, then

$f(g(x)) = \dfrac{1}{x^2 - 2} \neq h(x).$

(c) If $f(x) = \dfrac{1}{x}$ and $g(x) = (x-2)^2$, then

$f(g(x)) = \dfrac{1}{(x-2)^2} = h(x).$

92. Let $f(x)$ and $g(x)$ be odd functions, and define $h(x) = f(x)g(x)$. Then
$h(-x) = f(-x)g(-x)$
$= [-f(x)][-g(x)]$ since f and g are both odd
$= f(x)g(x) = h(x).$

Thus, *h* is even.

Let $f(x)$ and $g(x)$ be even functions, and define $h(x) = f(x)g(x)$. Then
$h(-x) = f(-x)g(-x)$
$= f(x)g(x)$ since f and g are both even
$= h(x).$

Thus, *h* is even.

93. The product of an odd function and an even function is odd. Let *f* be odd and *g* even.
Then $(fg)(-x) = f(-x)g(-x) = -f(x)g(x) = -(fg)(x).$
Thus, *fg* is odd.

94. Let *A*, *B*, and *C* be the three siblings, in decreasing age.
Then $A = 2B$ and $B = \dfrac{1}{2}C + 6.$

(a) $A = 2B = 2\left(\dfrac{1}{2}C + 6\right) = C + 12$

(b) If $A = 16$, then $B = 8$ and $C = 4$.

95. From Exercise 94, $A = 2B$ and $B = \dfrac{1}{2}C + 6.$

(a) $2(B - 6) = C$ and $B = \dfrac{1}{2}A$. Hence,

$C = 2\left(\dfrac{1}{2}A - 6\right) = A - 12.$

(b) If $C = 2$, then $B = 7$ and $A = 14$.

96. (a) Matches L_2 because each S-value is half the p-value.

(b) Matches L_1 because each S-value is 5 less than the p-value.

(c) Matches L_4 because $(g \circ f)(p)$ represents subtracting \$5 after applying a 50% discount.

(d) Matches L_3 because $(f \circ g)(p)$ represents subtracting \$5 from the price and then taking a 50% discount.

97. Three points on the graph of $y = -x^2 + x - 5$ are $(0, -5)$, $(1, -5)$, and $(2, -7)$.

98. Three points on the graph of $y = \frac{1}{5}x^3 - 4x^2 + 1$ are $(0, 1)$, $(1, -2.8)$, and $(-1, -3.2)$.

99. Three points on the graph of $x^2 + y^2 = 49$ are $(7, 0)$, $(-7, 0)$, and $(0, 7)$.

100. Three points on the graph of $y = \dfrac{x}{x^2 - 5}$ are $(0, 0)$, $\left(1, -\frac{1}{4}\right)$, and $\left(-1, \frac{1}{4}\right)$.

Section 1.7 Inverse Functions

1. inverse, f^{-1}

2. range, domain

3. $y = x$

4. one-to-one

5. If a function is one-to-one, no two x-values in the domain can correspond to the same y-value in the range. Therefore, a horizontal line can intersect the graph at most once.

6. No. If both the points $(1, 4)$ and $(2, 4)$ lie on the graph of a function, then the function is not one-to-one; it would not pass the Horizontal Line Test.

7.
$$f(x) = 6x$$
$$f^{-1}(x) = \frac{1}{6}x$$
$$f\left(f^{-1}(x)\right) = f\left(\frac{1}{6}x\right) = 6\left(\frac{1}{6}x\right) = x$$
$$f^{-1}\left(f(x)\right) = f^{-1}(6x) = \frac{1}{6}(6x) = x$$

8.
$$f(x) = \frac{1}{3}x$$
$$f^{-1}(x) = 3x$$
$$f\left(f^{-1}(x)\right) = f(3x) = \frac{1}{3}(3x) = x$$
$$f^{-1}\left(f(x)\right) = f^{-1}\left(\frac{1}{3}x\right) = 3\left(\frac{1}{3}x\right) = x$$

9.
$$f(x) = x + 11$$
$$f^{-1}(x) = x - 11$$
$$f\left(f^{-1}(x)\right) = f(x - 11) = (x - 11) + 11 = x$$
$$f^{-1}\left(f(x)\right) = f^{-1}(x + 11) = (x + 11) - 11 = x$$

10.
$$f(x) = x + 3$$
$$f^{-1}(x) = x - 3$$
$$f\left(f^{-1}(x)\right) = f(x - 3) = (x - 3) + 3 = x$$
$$f^{-1}\left(f(x)\right) = f^{-1}(x + 3) = (x + 3) - 3 = x$$

11.
$$f(x) = \frac{x - 1}{2}$$
$$f^{-1}(x) = 2x + 1$$
$$f\left(f^{-1}(x)\right) = f(2x + 1) = \frac{2x + 1 - 1}{2} = \frac{2x}{2} = x$$
$$f^{-1}\left(f(x)\right) = f^{-1}\left(\frac{x - 1}{2}\right) = 2\left(\frac{x - 1}{2}\right) + 1$$
$$= x - 1 + 1 = x$$

12.
$$f(x) = 4(x - 1)$$
$$f^{-1}(x) = \tfrac{1}{4}x + 1$$
$$f\left(f^{-1}(x)\right) = f\left(\tfrac{1}{4}x + 1\right) = 4\left(\left(\tfrac{1}{4}x + 1\right) - 1\right)$$
$$= 4\left(\tfrac{1}{4}x\right) = x$$
$$f^{-1}\left(f(x)\right) = f^{-1}\left(4(x - 1)\right) = \tfrac{1}{4}\left(4(x - 1)\right) + 1$$
$$= x - 1 + 1 = x$$

13.
$$f(x) = \sqrt[3]{x}$$
$$f^{-1}(x) = x^3$$
$$f\left(f^{-1}(x)\right) = f\left(x^3\right) = \sqrt[3]{x^3} = x$$
$$f^{-1}\left(f(x)\right) = f^{-1}\left(\sqrt[3]{x}\right) = \left(\sqrt[3]{x}\right)^3 = x$$

14. $f(x) = x^7$

$f^{-1}(x) = \sqrt[7]{x}$

$f\left(f^{-1}(x)\right) = f\left(\sqrt[7]{x}\right) = \left(\sqrt[7]{x}\right)^7 = x$

$f^{-1}\left(f(x)\right) = f^{-1}\left(x^7\right) = \sqrt[7]{x^7} = x$

15. The inverse is a line through $(-1, 0)$. Matches graph (c).

16. The inverse is a line through $(0, 6)$ and $(6, 0)$. Matches graph (b).

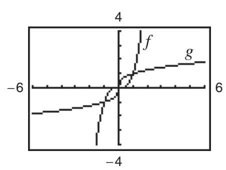
17. The inverse is half a parabola starting at $(1, 0)$. Matches graph (a).

18. The inverse is a reflection in $y = x$ of a third-degree equation through $(0, 0)$. Matches graph (d).

19. $f(x) = x^3,\ g(x) = \sqrt[3]{x}$

$f\left(g(x)\right) = f\left(\sqrt[3]{x}\right) = \left(\sqrt[3]{x}\right)^3 = x$

$g\left(f(x)\right) = g\left(x^3\right) = \sqrt[3]{x^3} = x$

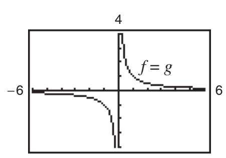


Reflections in the line $y = x$

20. $f(x) = \dfrac{1}{x},\ g(x) = \dfrac{1}{x}$

$f\left(g(x)\right) = f\left(\dfrac{1}{x}\right) = \dfrac{1}{1/x} = 1 \div \dfrac{1}{x} = 1 \cdot \dfrac{x}{1} = x$

$g\left(f(x)\right) = g\left(\dfrac{1}{x}\right) = \dfrac{1}{1/x} = 1 \div \dfrac{1}{x} = 1 \cdot \dfrac{x}{1} = x$



The graphs are the same.

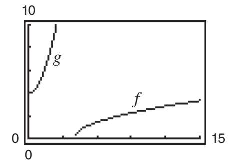
21. $f(x) = \sqrt{x-4};\ g(x) = x^2 + 4,\ x \ge 0$

$f\left(g(x)\right) = f\left(x^2 + 4\right),\ x \ge 0$

$= \sqrt{\left(x^2 + 4\right) - 4} = x$

$g\left(f(x)\right) = g\left(\sqrt{x-4}\right)$

$= \left(\sqrt{x-4}\right)^2 + 4 = x$



Reflections in the line $y = x$

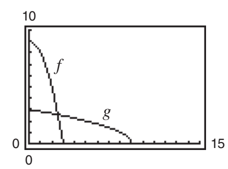
22. $f(x) = 9 - x^2,\ x \ge 0$

$g(x) = \sqrt{9-x},\ x \le 9$

$f\left(g(x)\right) = f\left(\sqrt{9-x}\right) = 9 - \left(\sqrt{9-x}\right)^2$

$= 9 - (9 - x) = x$

$g\left(f(x)\right) = g\left(9 - x^2\right) = \sqrt{9 - \left(9 - x^2\right)} = \sqrt{x^2} = x$

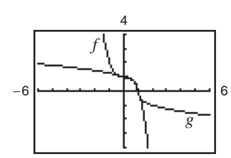


Reflections in the line $y = x$

23. $f(x) = 1 - x^3,\ g(x) = \sqrt[3]{1-x}$

$f\left(g(x)\right) = f\left(\sqrt[3]{1-x}\right) = 1 - \left(\sqrt[3]{1-x}\right)^3 = 1 - (1 - x) = x$

$g\left(f(x)\right) = g\left(1 - x^3\right) = \sqrt[3]{1 - \left(1 - x^3\right)} = \sqrt[3]{x^3} = x$

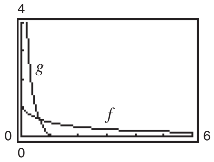


Reflections in the line $y = x$

24. $f(x) = \dfrac{1}{1+x},\ x \geq 0;\ g(x) = \dfrac{1-x}{x},\ 0 < x \leq 1$

$$f(g(x)) = f\left(\frac{1-x}{x}\right) = \frac{1}{1+\left(\frac{1-x}{x}\right)} = \frac{1}{\frac{x}{x}+\frac{1-x}{x}} = \frac{1}{\frac{1}{x}} = x$$

$$g(f(x)) = g\left(\frac{1}{1+x}\right) = \frac{1-\left(\frac{1}{1+x}\right)}{\left(\frac{1}{1+x}\right)} = \frac{\frac{1+x}{1+x}-\frac{1}{1+x}}{\frac{1}{1+x}} = \frac{\frac{x}{1+x}}{\frac{1}{1+x}} = \frac{x}{1+x}\cdot\frac{x+1}{1} = x$$



Reflections in the line $y = x$

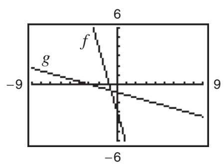
25. (a) $f(g(x)) = f\left(-\dfrac{2x+6}{7}\right)$

$$= -\frac{7}{2}\left(-\frac{2x+6}{7}\right) - 3 = x$$

$$g(f(x)) = g\left(-\frac{7}{2}x - 3\right)$$

$$= -\frac{2\left(-\frac{7}{2}x-3\right)+6}{7} = x$$

(b)



(c) $Y_1 = -\dfrac{7}{2}X - 3$

$Y_2 = -\dfrac{2X+6}{7}$

$Y_3 = Y_1(Y_2)$

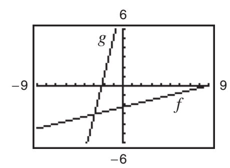
$Y_4 = Y_2(Y_1)$

X	Y_3	Y_4
–4	–4	–4
–2	–2	–2
0	0	0
2	2	2
4	4	4

26. (a) $f(g(x)) = f(4x+9) = \dfrac{(4x+9)-9}{4} = x$

$g(f(x)) = g\left(\dfrac{x-9}{4}\right) = 4\left(\dfrac{x-9}{4}\right) + 9 = x$

(b)



(c) $Y_1 = \dfrac{X-9}{4}$

$Y_2 = 4X + 9$

$Y_3 = Y_1(Y_2)$

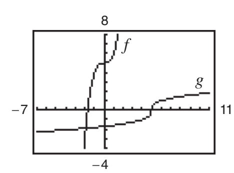
$Y_4 = Y_2(Y_1)$

X	Y_3	Y_4
–3	–3	–3
1	1	1
5	5	5
9	9	9

27. (a) $f(g(x)) = f\left(\sqrt[3]{x-5}\right) = \left(\sqrt[3]{x-5}\right)^3 + 5 = x$

$g(f(x)) = g(x^3+5) = \sqrt[3]{(x^3+5)-5} = x$

(b)



(c) $Y_1 = X^3 + 5$

$Y_2 = \sqrt[3]{X-5}$

$Y_3 = Y_1(Y_2)$

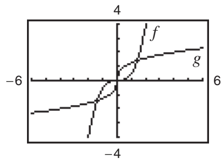
$Y_4 = Y_2(Y_1)$

X	Y_3	Y_4
–2	–2	–2
–1	–1	–1
0	0	0
1	1	1
4	4	4

28. (a) $f(g(x)) = f\left(\sqrt[3]{2x}\right) = \dfrac{\left(\sqrt[3]{2x}\right)^3}{2} = x$

$g(f(x)) = g\left(\dfrac{x^3}{2}\right) = \sqrt[3]{2\left(\dfrac{x^3}{2}\right)} = x$

(b)



(c) $Y_1 = \dfrac{X^3}{2}$

$Y_2 = \sqrt[3]{2X}$

$Y_3 = Y_1(Y_2)$

$Y_4 = Y_2(Y_1)$

X	Y_3	Y_4
–2	–2	–2
0	0	0
2	2	2
4	4	4
6	6	6

29. (a) $f(g(x)) = f(8 + x^2)$

$= -\sqrt{(8 + x^2) - 8}$

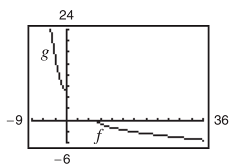
$= -\sqrt{x^2} = -(-x) = x,\ x \le 0$

$g(f(x)) = g(-\sqrt{x - 8})$

$= 8 + (-\sqrt{x - 8})^2$

$= 8 + (x - 8) = x,\ x \ge 8$

(b)



(c) $Y_1 = -\sqrt{x - 8}$

$Y_2 = 8 + x^2,\ x = 0$

$Y_3 = Y_1(Y_2)$

$Y_4 = Y_2(Y_1)$

X	Y_3	Y_4
8	8	8
9	9	9
12	12	12
15	15	15
20	20	20

30. (a) $f(g(x)) = f\left(\dfrac{x^4 + 10}{3}\right)$

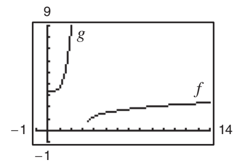
$= \sqrt[4]{3\left(\dfrac{x^4 + 10}{3}\right) - 10} = x,\ x \ge 0$

$g(f(x)) = g\left(\sqrt[4]{3x - 10}\right)$

$= \dfrac{\left(\sqrt[4]{3x - 10}\right)^4 + 10}{3} = \dfrac{3x - 10 + 10}{3}$

$= x,\ x \ge \dfrac{10}{3}$

(b)



(c) $Y_1 = \sqrt[4]{3x - 10}$

$Y_2 = \dfrac{x^4 + 10}{3},\ x \ge 0$

$Y_3 = Y_1(Y_2)$

$Y_4 = Y_2(Y_1)$

X	$\frac{10}{3}$	4	5	7	10
Y_3	0	$\sqrt[4]{2}$	$\sqrt[4]{5}$	$\sqrt[4]{11}$	$\sqrt[4]{20}$

X	0	$\sqrt[4]{2}$	$\sqrt[4]{5}$	$\sqrt[4]{11}$	$\sqrt[4]{20}$
Y_4	$\frac{10}{3}$	4	5	7	10

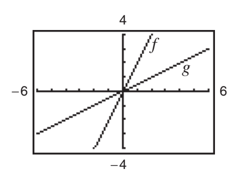
31. (a) $f(g(x)) = f\left(\dfrac{x}{2}\right)$

$= 2\left(\dfrac{x}{2}\right) = x$

$g(f(x)) = g(2x)$

$= \dfrac{2x}{2} = x$

(b)



(c) $Y_1 = 2x$

$Y_2 = \dfrac{x}{2}$

$Y_3 = Y_1(Y_2)$

$Y_4 = Y_2(Y_1)$

X	Y_3	Y_4
–4	–4	–4
–2	–2	–2
0	0	0
2	2	2
4	4	4

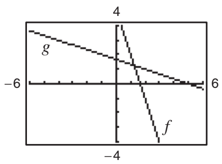
32. (a) $f(g(x)) = f\left(-\frac{x-5}{3}\right) = -3\left(-\frac{x-5}{3}\right) + 5$

$= x - 5 + 5 = x$

$g(f(x)) = g(-3x - 5) = -\frac{(-3x + 5) - 5}{3}$

$= \frac{3x}{3} = x$

(b)



(c) $Y_1 = -3x + 5$

$Y_2 = -\frac{x-5}{3}$

$Y_3 = Y_1(Y_2)$

$Y_4 = Y_2(Y_1)$

X	–5	–3	0	4	10
Y_3	20	14	5	–7	–25

X	20	14	5	–7	–25
Y_4	–5	–3	0	4	10

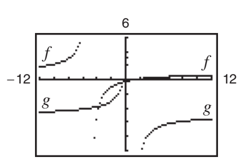
33. (a) $f(g(x)) = f\left(-\frac{5x+1}{x-1}\right)$

$= \frac{\left(-\frac{5x+1}{x-1}\right)-1}{\left(-\frac{5x+1}{x-1}\right)+5} = \frac{\frac{6x}{x-1}}{\frac{6}{x-1}} = x,\ x \neq 1$

$g(f(x)) = g\left(\frac{x-1}{x+5}\right)$

$= -\frac{5\left(\frac{x-1}{x+5}\right)+1}{\left(\frac{x-1}{x+5}\right)-1} = \frac{\frac{6x}{x+5}}{\frac{6}{x+5}} = x,\ x \neq -5$

(b)



(c) $Y_1 = \frac{x-1}{x+5}$

$Y_2 = -\frac{5x+1}{x-1}$

$Y_3 = Y_1(Y_2)$

$Y_4 = Y_2(Y_1)$

X	Y_3	Y_4
–2	–2	–2
–1	–1	–1
0	0	0
1	1	1
2	2	2

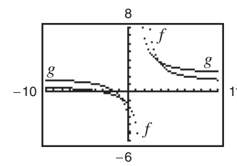
34. (a) $f(g(x)) = f\left(\frac{2x+3}{x-1}\right)$

$= \frac{\left(\frac{2x+3}{x-1}\right)+3}{\left(\frac{2x+3}{x-1}\right)-2} = \frac{\frac{5x}{x-1}}{\frac{5}{x-1}} = x,\ x \neq 1$

$g(f(x)) = g\left(\frac{x+3}{x-2}\right)$

$= \frac{2\left(\frac{x+3}{x-2}\right)+3}{\left(\frac{x+3}{x-2}\right)-1} = \frac{\frac{5x}{x-2}}{\frac{5}{x-2}} = x,\ x \neq 2$

(b)



(c) $Y_1 = \frac{x+3}{x-2}$

$Y_2 = \frac{2x+3}{x-1}$

$Y_3 = Y_1(Y_2)$

$Y_4 = Y_2(Y_1)$

X	Y_3	Y_4
–2	–2	–2
–1	–1	–1
0	0	0
1	1	1
3	3	3

35. Yes. No two elements, number of cans, in the domain correspond to the same element, price, in the range.

36. No. Both elements, ½ hour and 1 hour, in the domain correspond to the same element, \$40, in the range.

37. No. Both x-values, –3 and 0, in the domain correspond to the y-value 6 in the range.

38. Yes. No two x-values in the domain correspond to the same y-value in the range.

39. Not a function

40. It is the graph of a function, but not one-to-one.

41. It is the graph of a one-to-one function.

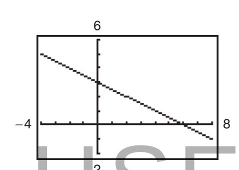
42. It is the graph of a one-to-one function.

43. It is the graph of a one-to-one function.

44. Not a function

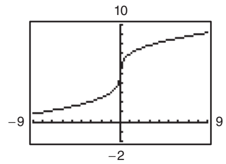
45. $f(x) = 3 - \frac{1}{2}x$

f is one-to-one because a horizontal line will intersect the graph at most once.



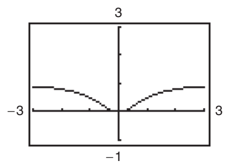
46. $f(x) = 2x^{1/3} + 5$

f does pass the Horizontal Line Test, so f is one-to-one.



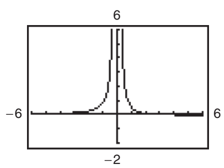
47. $h(x) = \dfrac{x^2}{x^2 + 1}$

h is not one-to-one because some horizontal lines intersect the graph twice.



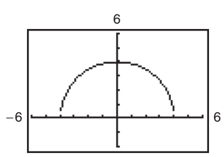
48. $g(x) = \dfrac{4 - x}{6x^2}$

g does not pass the Horizontal Line Test, so *g* is not one-to-one.



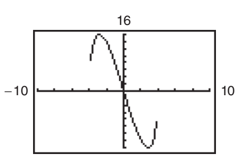
49. $h(x) = \sqrt{16 - x^2}$

h is not one-to-one because some horizontal lines intersect the graph twice.



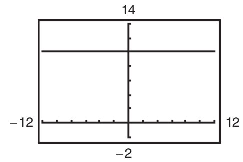
50. $f(x) = -2x\sqrt{16 - x^2}$

f is not one-to-one because it does not pass the Horizontal Line Test.



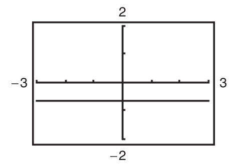
51. $f(x) = 10$

f is not one-to-one because the horizontal line $y = 10$ intersects the graph at every point on the graph.



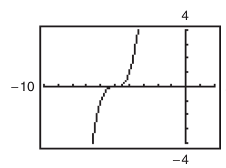
52. $f(x) = -0.65$

f is not one-to-one because it does not pass the Horizontal Line Test.



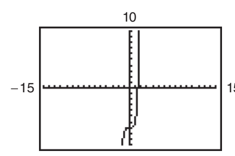
53. $g(x) = (x + 5)^3$

g is one-to-one because a horizontal line will intersect the graph at most once.



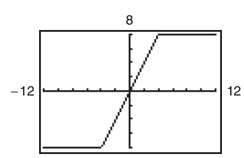
54. $f(x) = x^5 - 7$

f is one-to-one because it passes the Horizontal Line Test.



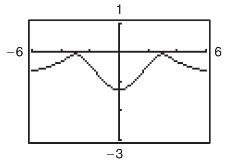
55. $h(x) = |x| - |x - 4|$

h is not one-to-one because some horizontal lines intersect the graph more than once.

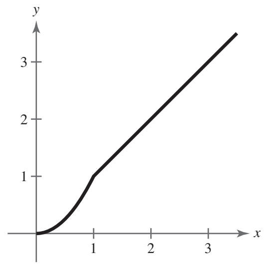


56. $f(x) = -\dfrac{|x-9|}{|x+7|}$

f is not one-to-one because it does not pass the Horizontal Line Test.

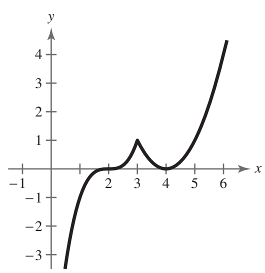


57.



The graph of the function passes the Horizontal Line Test and does have an inverse function.

58.

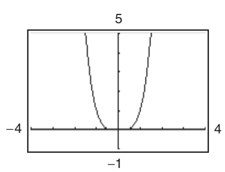


The graph of the function does not pass the Horizontal Line Test and does not have an inverse function.

59. $f(x) = x^4$

f is not one-to-one.

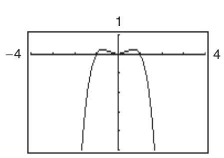
For example, $f(2) = f(-2) = 16$.



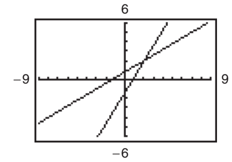
60. $g(x) = x^2 - x^4$

g is not one-to-one.

For example, $g(1) = g(-1) = 0$.

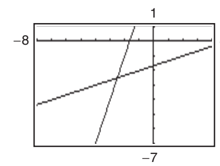


61.
$$f(x) = \frac{3x+4}{5}$$
$$y = \frac{3x+4}{5}$$
$$x = \frac{3y+4}{5}$$
$$5x = 3y+4$$
$$5x-4 = 3y$$
$$\frac{5x-4}{3} = y$$
$$f^{-1}(x) = \frac{5x-4}{3}$$



f is one-to-one and has an inverse function.

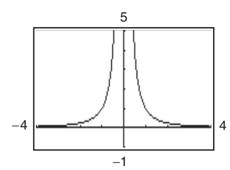
62.
$$f(x) = 3x+5$$
$$y = 3x+5$$
$$x = 3y+5$$
$$x-5 = 3y$$
$$\frac{x-5}{3} = y$$
$$f^{-1}(x) = \frac{x-5}{3}$$



f is one-to-one and has an inverse function.

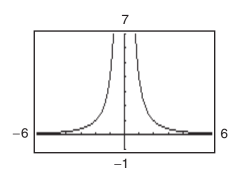
63. $f(x) = \dfrac{1}{x^2}$ is not one-to-one.

For example, $f(1) = f(-1) = 1$.

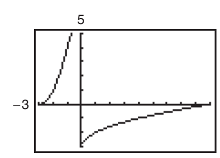


64. $h(x) = \dfrac{4}{x^2}$ is not one-to-one.

For example, $h(1) = h(-1) = 4$.



65.
$$f(x) = (x+3)^2,\ x \ge -3,\ y \ge 0$$
$$y = (x+3)^2,\ x \ge -3,\ y \ge 0$$
$$x = (y+3)^2,\ y \ge -3,\ x \ge 0$$
$$\sqrt{x} = y+3,\ y \ge -3,\ x \ge 0$$
$$y = \sqrt{x} - 3,\ x \ge 0,\ y \ge -3$$



f is one-to-one and has an inverse function.

66. $q(x) = (x-5)^2$

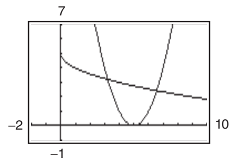
$y = (x-5)^2,\ x \le 5$

$x = (y-5)^2,\ y \le 5$

$-\sqrt{x} = y - 5,\ y \le 5$

$y = -\sqrt{x} + 5$

q is one-to-one and has an inverse function.



67. $f(x) = \sqrt{2x+3} \Rightarrow x \ge -\dfrac{3}{2},\ y \ge 0$

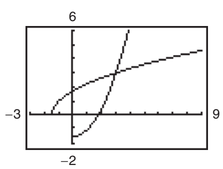
$y = \sqrt{2x+3},\ x \ge -\dfrac{3}{2},\ y \ge 0$

$x = \sqrt{2y+3},\ y \ge -\dfrac{3}{2},\ x \ge 0$

$x^2 = 2y+3,\ x \ge 0,\ y \ge -\dfrac{3}{2}$

$y = \dfrac{x^2-3}{2},\ x \ge 0,\ y \ge -\dfrac{3}{2}$

f is one-to-one and has an inverse function.



68. $f(x) = \sqrt{x-2} \Rightarrow x \ge 2,\ y \ge 0$

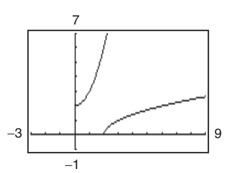
$y = \sqrt{x-2},\ x \ge 2,\ y \ge 0$

$x = \sqrt{y-2},\ y \ge 2,\ x \ge 0$

$x^2 = y-2,\ x \ge 0,\ y \ge 2$

$x^2 + 2 = y,\ x \ge 0,\ y \ge 2$

f is one-to-one and has an inverse function.



69. $f(x) = |x-2|,\ x \le 2,\ y \ge 0$

$y = |x-2|$

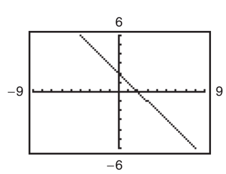
$x = |y-2|,\ y \le 2,\ x \ge 0$

$x = -(y-2)$ since $y - 2 \le 0.$

$x = -y + 2$

$y = -x + 2,\ x \ge 0,\ y \le 2$

f is one-to-one and has an inverse function.



70. $f(x) = \dfrac{x^2}{x^2+1},\ x \ge 0$ is one-to-one.

$f(x) = \dfrac{x^2}{x^2+1},\ x \ge 0,\ 0 \le y < 1$

$y = \dfrac{x^2}{x^2+1}$

$x = \dfrac{y^2}{y^2+1}$

$x(y^2+1) = y^2$

$xy^2 + x = y^2$

$xy^2 - y^2 = -x$

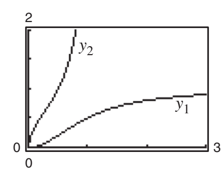
$y^2(x-1) = -x$

$y^2 = \dfrac{x}{1-x}$

$y = \sqrt{\dfrac{x}{1-x}},\ 0 \le x < 1,\ y \ge 0$

$f^{-1}(x) = \sqrt{\dfrac{x}{1-x}},\ 0 \le x < 1,\ y \ge 0$

f does have an inverse.



71. $f(x) = 4x - 9$

$y = 4x - 9$

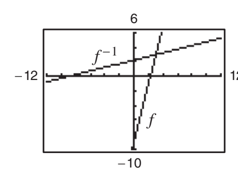
$x = 4y - 9$

$x + 9 = 4y$

$\dfrac{x+9}{4} = y$

$f^{-1}(x) = \dfrac{x+9}{4}$

Reflections in the line $y = x$



72. $f(x) = 3x$

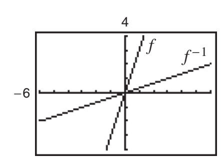
$y = 3x$

$x = 3y$

$\dfrac{x}{3} = y$

$f^{-1}(x) = \dfrac{x}{3}$

Reflections in the line $y = x$



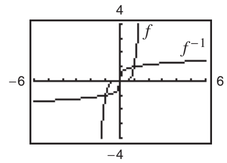
73. $f(x) = x^5$

$y = x^5$

$x = y^5$

$y = \sqrt[5]{x}$

$f^{-1}(x) = \sqrt[5]{x}$



Reflections in the line $y = x$

74. $f(x) = x^3 + 1$

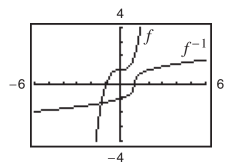
$y = x^3 + 1$

$x = y^3 + 1$

$x - 1 = y^3$

$\sqrt[3]{x-1} = y$

$f^{-1}(x) = \sqrt[3]{x-1}$



Reflections in the line $y = x$

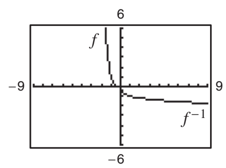
75. $f(x) = x^4, x \le 0, y \ge 0$

$y = x^4$

$x = y^4$

$-\sqrt[4]{x} = y$

$f^{-1}(x) = -\sqrt[4]{x}, x \ge 0, y \le 0$



Reflections in the line $y = x$

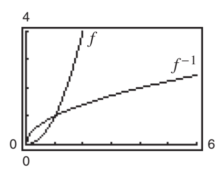
76. $f(x) = x^2,\ x \ge 0$

$y = x^2$

$x = y^2$

$\sqrt{x} = y$

$f^{-1}(x) = \sqrt{x}$



Reflections in the line $y = x$

77. $f(x) = \sqrt{4 - x^2},\ 0 \le x \le 2$

$y = \sqrt{4 - x^2}$

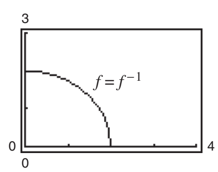
$x = \sqrt{4 - y^2}$

$x^2 = 4 - y^2$

$y^2 = 4 - x^2$

$y = \sqrt{4 - x^2}$

$f^{-1}(x) = \sqrt{4 - x^2},\ 0 \le x \le 2$



The graphs are the same.

78. $f(x) = \sqrt{16 - x^2},\ -4 \le x \le 0$

$y = \sqrt{16 - x^2}$

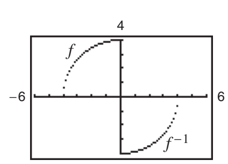
$x = \sqrt{16 - y^2},\ -4 \le y \le 0$

$x^2 = 16 - y^2$

$y^2 = 16 - x^2$

$y = -\sqrt{16 - x^2}$

$f^{-1}(x) = -\sqrt{16 - x^2},\ 0 \le x \le 4$



Reflections in the line $y = x$

79. $f(x) = \dfrac{4}{x^3}, x \ne 0$

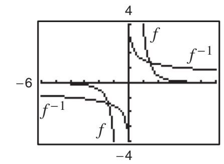
$y = \dfrac{4}{x^3}$

$x = \dfrac{4}{y^3}$

$y^3 = \dfrac{4}{x}$

$y = \sqrt[3]{\dfrac{4}{x}}$

$f^{-1}(x) = \sqrt[3]{\dfrac{4}{x}}, x \ne 0$



Reflections in the line $y = x$

80. $f(x) = \dfrac{6}{\sqrt{x}}$

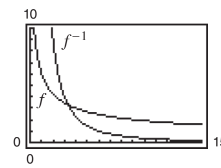
$y = \dfrac{6}{\sqrt{x}}$

$x = \dfrac{6}{\sqrt{y}}$

$x^2 = \dfrac{36}{y}$

$y = \dfrac{36}{x^2}, x > 0$

$f^{-1}(x) = \dfrac{36}{x^2}, x > 0$



Reflections in the line $y = x$

81. If you let $f(x) = (x-2)^2,\ x \ge 2,$ then f has an inverse function. [**Note:** You could also let $x \le 2$.]

$y = (x-2)^2$

$x = (y-2)^2$

$\sqrt{x} = y - 2$

$\sqrt{x} + 2 = y$

$f^{-1}(x) = \sqrt{x} + 2$

Domain of f: $x \ge 2$ Range of f: $y \ge 0$

Domain of f^{-1}: $x \ge 0$ Range of f^{-1}: $y \ge 2$

82. If you let $f(x) = x^4 + 1, x \geq 0$, then f has an inverse function. [**Note:** You could also let $x \leq 0$.]

$$y = x^4 + 1$$
$$x = y^4 + 1$$
$$x - 1 = y^4$$
$$\sqrt[4]{x - 1} = y$$
$$f^{-1}(x) = \sqrt[4]{x - 1}$$

Domain of $f: x \geq 0$ Range of $f: y \geq 1$
Domain of $f^{-1}: x \geq 1$ Range of $f^{-1}: y \geq 0$

83. If you let $f(x) = |x + 2|$, $x \geq -2$, then f has an inverse function. [**Note:** You could also let $x \leq -2$.]

$$y = x + 2$$
$$x = y + 2$$
$$x - 2 = y$$
$$f^{-1}(x) = x - 2$$

Domain of f: $x \geq -2$

Domain of f^{-1}: $x \geq 0$

Range of f: $y \geq 0$

Range of f^{-1}: $y \geq -2$

84. If you let $f(x) = |x - 2|$, $x \geq 2$, then f has an inverse function. [**Note:** You could also let $x \leq 2$.]

$$y = x - 2$$
$$x = y - 2$$
$$x + 2 = y$$
$$f^{-1}(x) = x + 2$$

Domain of f: $x \geq -2$ Range of f: $y \geq 0$

Domain of f^{-1}: $x \geq 0$ Range of f^{-1}: $y \geq -2$

85. If you let $f(x) = (x + 3)^2$, $x \geq -3$ then f has an inverse function. [**Note:** You could also let $x \leq -3$.]

$$y = (x + 3)^2$$
$$x = (y + 3)^2$$
$$\sqrt{x} = y + 3$$
$$y = \sqrt{x} - 3$$
$$f^{-1}(x) = \sqrt{x} - 3$$

Domain of $f: x \geq -3$ Range of $f: y \geq 0$
Domain of $f^{-1}: x \geq 0$ Range of $f^{-1}: y \geq -3$

86. If you let $f(x) = (x - 4)^2$, $x \geq 4$, then f has an inverse function. [**Note:** You could also let $x \leq 4$.]

$$y = (x - 4)^2$$
$$x = (y - 4)^2$$
$$\sqrt{x} = y - 4$$
$$y = \sqrt{x} + 4$$
$$f^{-1}(x) = \sqrt{x} + 4$$

Domain of $f: x \geq 4$ Range of $f: y \geq 0$
Domain of $f^{-1}: x \geq 0$ Range of $f^{-1}: y \geq 4$

87. If you let $f(x) = -2x^2 - 5, x \geq 0$, then f has an inverse function. [**Note:** You could also let $x \leq 0$.]

$$y = -2x^2 - 5$$
$$x = -2y^2 - 5$$
$$x + 5 = -2y^2$$
$$-\frac{x + 5}{2} = y^2$$
$$\sqrt{-\frac{x + 5}{2}} = y$$
$$f^{-1}(x) = \sqrt{-\frac{x + 5}{2}}$$

Domain of $f: x \geq 0$ Range of $f: y \leq -5$
Domain of $f^{-1}: x \leq -5$ Range of $f^{-1}: y \geq 0$

88. If you let $f(x) = \frac{1}{2}x^2 + 1, x \geq 0$, then f has an inverse function. [**Note:** You could also let $x \leq 0$.]

$$y = \frac{1}{2}x^2 + 1$$
$$x = \frac{1}{2}y^2 + 1$$
$$x - 1 = \frac{1}{2}y^2$$
$$2(x - 1) = y^2$$
$$\sqrt{2(x - 1)} = y$$
$$f^{-1}(x) = \sqrt{2(x - 1)}$$

or

$$f^{-1}(x) = \sqrt{2x - 2}$$

Domain of $f: x \geq 0$ Range of $f: y \geq 1$
Domain of $f^{-1}: x \geq 1$ Range of $f^{-1}: y \geq 0$

89. If you let $f(x) = |x-4|+1,\ x \ge 4,$ then f has an inverse function. [**Note:** You could also let $x \le 4$.]

$$y = |x-4|+1$$
$$y = x-3 \text{ because } x \ge 4$$
$$x = y-3$$
$$y = x+3$$
$$f^{-1}(x) = x+3$$

Domain of $f: x \ge 4$ Range of $f: y \ge 1$

Domain of $f^{-1}: x \ge 1$ Range of $f^{-1}: y \ge 4$

90. If you let $f(x) = -|x-1|-2,\ x \ge 1,$ then f has an inverse function. [**Note:** You could also let $x \le 1$.]

$$y = -|x-1|-2 = -(x-1)-2 \text{ because } x \ge 1$$
$$y = -x-1$$
$$x = -y-1$$
$$x+1 = -y$$
$$f^{-1}(x) = -x-1$$

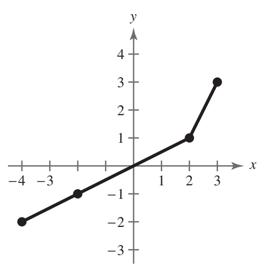
Domain of $f: x \ge 1$ Range of $f: y \le -2$

Domain of $f^{-1}: x \le -2$ Range of $f^{-1}: y \ge 1$

91.

x	$f(x)$
-2	-4
-1	-2
1	2
3	3

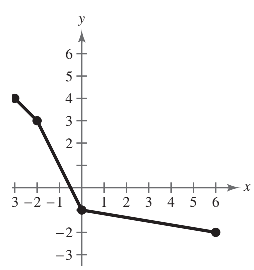
x	$f^{-1}(x)$
-4	-2
-2	-1
2	1
3	3



92.

x	$f(x)$
4	-3
3	-2
-1	0
-2	6

x	$f^{-1}(x)$
-3	4
-2	3
0	-1
6	-2



93. $f^{-1}(-4) = 4$ because $f(4) = -4$.

94. $g^{-1}(0) = -2$ because $g(-2) = 0$.

95. $(f \circ g)(2) = f(3) = -2$

96. $g(f(-4)) = g(4) = 6$

97. $f^{-1}(g(0)) = f^{-1}(2) = 0$

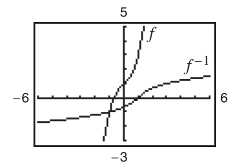
98. $(g^{-1} \circ f)(3) = g^{-1}(-2) = -3$

99. $(g \circ f^{-1})(2) = g(0) = 2$

100. $(f^{-1} \circ g^{-1})(6) = f^{-1}(g^{-1}(6)) = f^{-1}(4) = -4$

101. $f(x) = x^3 + x + 1$

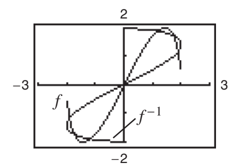
(a) and (b)



(c) The graph of the inverse relation is an inverse function since it satisfies the Vertical Line Test.

102. $f(x) = x\sqrt{4-x^2}$

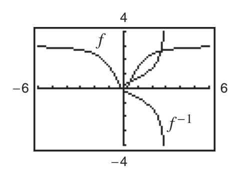
(a) and (b)



(c) Not an inverse function since it does not satisfy the Vertical Line Test.

103. $f(x) = \dfrac{3x^2}{x^2+1}$

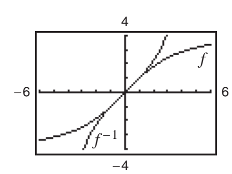
(a) and (b)



The graph of the inverse relation is not an inverse function since it does not satisfy the Vertical Line Test.

104. $f(x) = \dfrac{4x}{\sqrt{x^2+15}}$

(a) and (b)



(c) Inverse function since it satisfies the Vertical Line Test.

In Exercises 105 – 110, $f(x) = \frac{1}{8}x - 3$, $f^{-1}(x) = 8(x+3)$, $g(x) = x^3$, $g^{-1}(x) = \sqrt[3]{x}$.

105. $(f^{-1} \circ g^{-1})(1) = f^{-1}(g^{-1}(1)) = f^{-1}\left(\sqrt[3]{1}\right)$

$$= 8\left(\sqrt[3]{1}+3\right) = 8(1+3) = 32$$

106. $(g^{-1} \circ f^{-1})(-3) = g^{-1}(f^{-1}(-3)) = g^{-1}(8(-3+3))$
$= g^{-1}(0) = \sqrt[3]{0} = 0$

107. $(f^{-1} \circ f^{-1})(-6) = f^{-1}(f^{-1}(-6))$
$= f^{-1}(8[-6 + 3])$
$= f^{-1}(-24) = 8(-24 + 3) = -168$

108. $(g^{-1} \circ g^{-1})(4) = g^{-1}(g^{-1}(4))$
$= g^{-1}(\sqrt[3]{4})$
$= \sqrt[3]{\sqrt[3]{4}} = \sqrt[9]{4}$

109. $f^{-1}(x) = 8(x + 3)$ and $g^{-1}(x) = \sqrt[3]{x}$.

$(f^{-1} \circ g^{-1})(x) = f^{-1}(g^{-1}(x))$
$= f^{-1}(\sqrt[3]{x})$
$= 8(\sqrt[3]{x} + 3)$
$= 8\sqrt[3]{x} + 24$

110. $g^{-1}(x) = \sqrt[3]{x}$ and $f^{-1}(x) = 8(x + 3)$.

$(g^{-1} \circ f^{-1})(x) = g^{-1}(f^{-1}(x))$
$= g^{-1}(8(x + 3))$
$= \sqrt[3]{8(x + 3)}$
$= 2\sqrt[3]{x + 3}$

In Exercises 111–114, $f(x) = x + 4$, $f^{-1}(x) = x - 4$, $g(x) = 2x - 5$, $g^{-1}(x) = \dfrac{x+5}{2}$.

111. $(g^{-1} \circ f^{-1})(x) = g^{-1}(f^{-1}(x))$
$= g^{-1}(x-4)$
$= \dfrac{(x-4)+5}{2}$
$= \dfrac{x+1}{2}$

112. $(f^{-1} \circ g^{-1})(x) = f^{-1}(g^{-1}(x))$
$= f^{-1}\left(\dfrac{x+5}{2}\right)$
$= \dfrac{x+5}{2} - 4$
$= \dfrac{x+5-8}{2}$
$= \dfrac{x-3}{2}$

113. $(f \circ g)(x) = f(g(x)) = f(2x-5) = (2x-5) + 4 = 2x - 1.$

Now find the inverse function of $(f \circ g)(x) = 2x - 1$.

$y = 2x - 1$
$x = 2y - 1$
$x + 1 = 2y$
$y = \dfrac{x+1}{2}$
$(f \circ g)^{-1}(x) = \dfrac{x+1}{2}$

Note that $(f \circ g)^{-1}(x) = (g^{-1} \circ f^{-1})(x)$; see Exercise 111.

114. $(g \circ f)(x) = g(f(x)) = g(x+4) = 2(x+4) - 5$
$= 2x + 8 - 5 = 2x + 3.$

Now find the inverse function of $(g \circ f)(x) = 2x + 3$.

$y = 2x + 3$
$x = 2y + 3$
$x - 3 = 2y$
$\dfrac{x-3}{2} = y$
$(g \circ f)^{-1}(x) = \dfrac{x-3}{2}$

Note that $(g \circ f)^{-1} = f^{-1} \circ g^{-1}$.

115. (a) Yes, *f* is one-to-one. For each European shoe size, there is exactly one U.S. shoe size.

(b) $f(11) = 44$

(c) $f^{-1}(43) = 10$ because $f(10) = 43$.

(d) $f(f^{-1}(41)) = f(8) = 41$

(e) $f^{-1}(f(12)) = f^{-1}(45) = 12$

116. If two functions are inverse functions of each other, then $g(g^{-1}(x)) = g^{-1}(g(x)) = x$, so $g^{-1}(g(6)) = 6$.

117. (a) CALL ME LATER corresponds to numerical values: 3 1 12 12 0 13 5 0 12 1 20 5 18. Using f to encode,

$$f(3)=19$$
$$f(1)=9$$
$$f(12)=64$$
$$f(12)=64$$
$$f(0)=4$$
$$f(13)=69$$
$$f(5)=29$$
$$f(0)=4$$
$$f(12)=64$$
$$f(1)=9$$
$$f(20)=104$$
$$f(5)=29$$
$$f(18)=94$$

(b) For $f(x)=5x+4,\quad f^{-1}(x)=\dfrac{x-4}{5}.$

Using f^{-1} to decode, $f^{-1}(119)=23$

$$f^{-1}(44)=8$$
$$f^{-1}(9)=1$$
$$f^{-1}(104)=20$$
$$f^{-1}(4)=0$$
$$f^{-1}(104)=20$$
$$f^{-1}(49)=9$$
$$f^{-1}(69)=13$$
$$f^{-1}(29)=5$$

Converting from numerical values to letters, the message is WHAT TIME.

118. (a) $y = 12 + 0.55x$

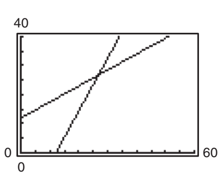
$x = 12 + 0.55y$

$x - 12 = 0.55y$

$\dfrac{x-12}{0.55} = y$

So, $y^{-1} = \dfrac{x-12}{0.55}$, where y^{-1} is the number of units produced while x is the hourly wage.

(b)



(c) When $y^{-1} = 9$, $x = \$16.95$.

(d) When $x = \$21.35$, $y^{-1} = 17$ units.

119. False. $f(x)=x^2$ is even, but f^{-1} does not exist.

120. True. If $(0, b)$ is the y-intercept of f, then $(b, 0)$ is the x-intercept of f^{-1}.

121. Yes. The inverse would give the time it took to complete n miles.

122. This situation could not be represented by a one-to-one function. Since the population was the greatest in 2012, it would have increased sometime from 2005 to 2012, reached a maximum in 2012, and would have decreased sometime from 2012 to 2015. Therefore, the graph would not pass the Horizontal Line Test, and not be one-to-one.

123. No. The function oscillates.

124. This situation could not be represented by a one-to-one function because height remains constant after a certain age.

125. The graph of f^{-1} is a reflection of the graph of f in the line $y = x$.

126. If the domain of f is $[0, 9]$ and the range is $[-3, 3]$, and since the graphs of f and f^{-1} can be described as if the point (a, b) lies on the graph of f, then the point (b, a) lies on the graph of f^{-1}, then the domain of f^{-1} is $[-3, 3]$ and the range is $[0, 9]$.

127. (a) The function f will have an inverse function because no two temperatures in degrees Celsius will correspond to the same temperature in degrees Fahrenheit.

(b) $f^{-1}(50)$ would represent the temperature in degrees Celsius for a temperature of $50°$ Fahrenheit.

128. Yes. The function would pass the Horizontal Line Test and therefore have an inverse function.

129. The constant function $f(x)=c$, whose graph is a horizontal line, would never have an inverse function.

130. (a) No, the graphs are not reflections of each other in the line $y = x$.

(b) Yes, the graphs are reflections of each other in the line $y = x$.

(c) Yes, the graphs are reflections of each other in the line $y = x$.

(d) Yes, the graphs are reflections of each other in the line $y = x$.

131. We will show that $(f \circ g)^{-1}(x) = (g^{-1} \circ f^{-1})(x)$ for all x in their domains.

Let $y = (f \circ g)^{-1}(x) \Rightarrow (f \circ g)(y) = x$ then

$f(g(y)) = x \Rightarrow f^{-1}(x) = g(y)$.

Hence,

$(g^{-1} \circ f^{-1})(x) = g^{-1}(f^{-1}(x)) = g^{-1}(g(y)) = y = (f \circ g)^{-1}(x)$.

Thus, $g^{-1} \circ f^{-1} = (f \circ g)^{-1}$.

132. If f is one-to-one, then f^{-1} exists. If f is odd, then $f(-x) = -f(x)$. Consider $f(x) = y \leftrightarrow f^{-1}(y) = x$. Then $f^{-1}(-y) = f^{-1}(-f(x)) = f^{-1}(f(-x)) = -x = -f^{-1}(y)$. Thus, f^{-1} is odd.

133. $\dfrac{27x^3}{3x^2} = 9x,\ x \neq 0$

134. $\dfrac{5x^2y^2 + 25x^2y}{xy + 5x} = \dfrac{5x^2y(y+5)}{x(y+5)} = 5xy,\ x(y+5) \neq 0$

135. $\dfrac{x^2 - 36}{6 - x} = \dfrac{(x-6)(x+6)}{-(x-6)} = \dfrac{x+6}{-1} = -x - 6,\ x \neq 6$

136. $\dfrac{x^2 + 3x - 40}{x^2 - 3x - 10} = \dfrac{(x-5)(x+8)}{(x-5)(x+2)} = \dfrac{x+8}{x+2},\ x \neq 5$

137. $x = 5$. No, it does not pass the Vertical Line Test.

138. $y - 7 = -3$

$y = 4$

Yes, y is a function of x.

139. $x^2 + y = 5$

$y = -x^2 + 5$

Yes, y is a function of x.

140. $x - y^2 = 0$

$y^2 = x$

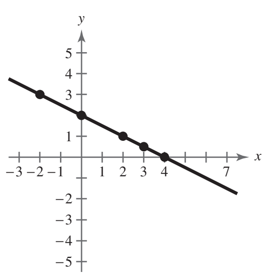
$y = \pm\sqrt{x}$

No, y is not a function of x.

Chapter 1 Review

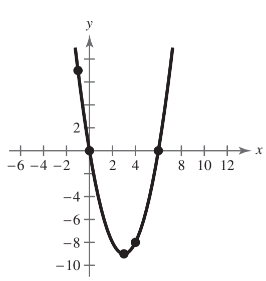
1. $y = -\frac{1}{2}x + 2$

x	-2	0	2	3	4
y	3	2	1	$\frac{1}{2}$	0
Solution point	$(-2, 3)$	$(0, 2)$	$(2, 1)$	$(3, \frac{1}{2})$	$(4, 0)$

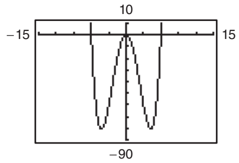


2. $y = x^2 - 6x$

x	-1	0	3	4	6
y	7	0	-9	-8	0
Solution point	$(-1, 7)$	$(0, 0)$	$(3, -9)$	$(4, -8)$	$(6, 0)$

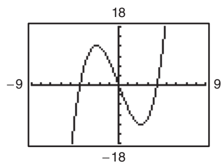


3. $y = \frac{1}{4}x^4 - 9x^2$



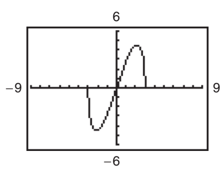
Intercepts: $(0, 0), (\pm 6, 0)$

4. $y = \frac{1}{2}x^3 - 8x$



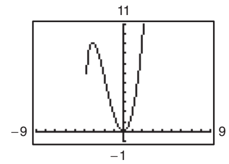
Intercepts: $(0, 0), (\pm 4, 0)$

5. $y = x\sqrt{9 - x^2}$



Intercepts: $(0, 0), (\pm 3, 0)$

6. $y = x^2\sqrt{x + 4}$



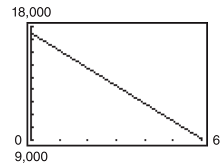
Intercept: $(0, 0)$

7. $y = 17{,}500 - 1400t,\ 0 \le t \le 6$

(a)

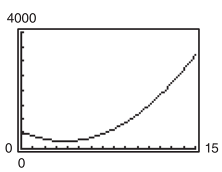
Xmin = 0
Xmax = 6
Xscl = 1
Ymin = 9000
Ymax = 18,000
Yscl = 1000

(b)



(c) When $y = \$11{,}900,\ t = 4$.

8. (a)

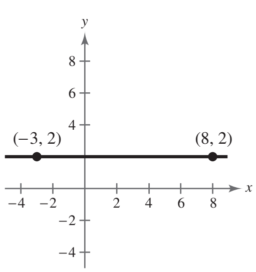


(b)

Year	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013
Revenue	261.6	295.9	377.3	505.7	681.3	903.9	1173.6	1490.4	1854.2	2265.2

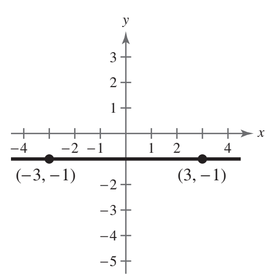
(c) When $R = 500,\ t \approx 6.96$. So, 2007 is the first year that the revenues exceed $5 billion.

9.



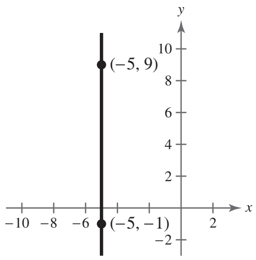
$$m = \frac{2 - 2}{8 - (-3)} = \frac{0}{11} = 0$$

10.



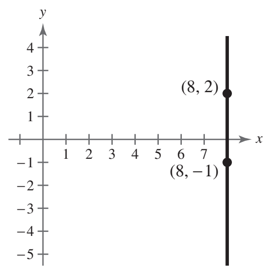
$$m = \frac{-1 - (-1)}{-3 - 3} = \frac{0}{-6} = 0$$

11.



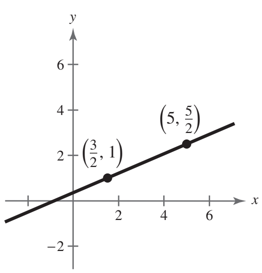
$$m = \frac{9 - (-1)}{-5 - (-5)} = \frac{10}{0}; \ m \text{ is undefined.}$$

12.



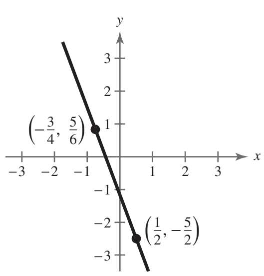
$$m = \frac{2 - (-1)}{8 - 8} = \frac{3}{0}; \ m \text{ is undefined.}$$

13.



$$m = \frac{(5/2) - 1}{5 - (3/2)} = \frac{3/2}{7/2} = \frac{3}{7}$$

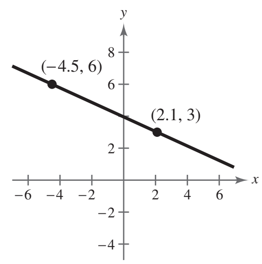
14.



$$m = \frac{\frac{5}{6} - \left(-\frac{5}{2}\right)}{-\frac{3}{4} - \frac{1}{2}} = \frac{\frac{5}{6} + \frac{15}{6}}{-\frac{3}{4} - \frac{2}{4}} = \frac{\frac{10}{3}}{-\frac{5}{4}}$$

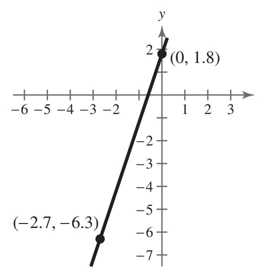
$$= -\frac{10}{3} \cdot \frac{4}{5} = -\frac{8}{3}$$

15.



$$m = \frac{3 - 6}{2.1 - (-4.5)} = \frac{-3}{6.6} = -\frac{30}{66} = -\frac{5}{11}$$

16.



$$m = \frac{1.8 - (-6.3)}{0 - (-2.7)} = \frac{8.1}{2.7} = 3$$

17. (a)

$$y + 1 = \frac{1}{4}(x - 2)$$
$$4y + 4 = x - 2$$
$$-x + 4y + 6 = 0$$

(b) Three additional points:

$(2 + 4, -1 + 1) = (6, 0)$

$(6 + 4, 0 + 1) = (10, 1)$

$(10 + 4, 1 + 1) = (14, 2)$

(Other answers possible.)

18. (a)

$$y - 5 = -\frac{3}{2}(x + 3)$$
$$2y - 10 = -3x - 9$$
$$3x + 2y - 1 = 0$$

(b) Three additional points:

$(-3 + 2,\ 5 - 3) = (-1,\ 2)$

$(-1 + 2,\ 2 - 3) = (1,\ -1)$

$(1 + 2,\ -1 - 3) = (3,\ -4)$

(Other answers possible.)

19. (a)

$$y + 5 = -\frac{3}{2}(x - 0)$$

$$y + 5 = -\frac{3}{2}x$$

$$y = -\frac{3}{2}x - 5$$

$$3x + 2y + 10 = 0$$

(b) Three additional points:

$(0 + 2, -5 - 3) = (2, -8)$

$(2 + 2, -8 - 3) = (4, -11)$

$(4 + 2, -11 - 3) = (6, -14)$

(Other answers possible.)

20. (a)

$$y - 1 = \frac{4}{5}(x - 0)$$

$$y - 1 = \frac{4}{5}x$$

$$-\frac{4}{5}x + y - 1 = 0$$

$$4x - 5y + 5 = 0$$

(b) Three additional points:

$(0 - 5, 1 - 4) = (-5, -3)$

$(0 + 5, 1 + 4) = (5, 5)$

$(5 + 5, 5 + 4) = (10, 9)$

(Other answers possible.)

21. (a) $y - 6 = 0(x + 2) = 0$

$y = 6$ (horizontal line)

$y - 6 = 0$

(b) Three additional points:

$(0, 6)$, $(1, 6)$, $(-1, 6)$

(Other answers possible.)

22. (a) $y - 8 = 0(x + 8) = 0$

$y = 8$ (horizontal line)

$y - 8 = 0$

(b) Three additional points: $(0, 8)$, $(1, 8)$, $(2, 8)$

(Other answers possible.)

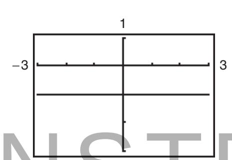
23. $(2, -1)$, $(4, -1)$

$$m = \frac{-1 - (-1)}{4 - 2} = \frac{0}{2} = 0$$

The line is horizontal.

$y - (-1) = 0(x - 2)$

$y = -1$

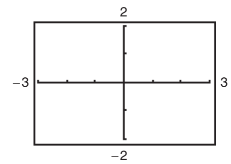


24. $(0, 0)$, $(0, 10)$

$$m = \frac{10 - 0}{0 - 0} = \frac{10}{0}$$

The slope is undefined and the line is vertical.

$x = 0$

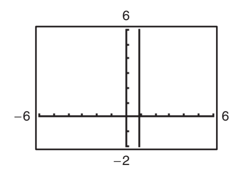


25. $\left(\frac{5}{6}, -1\right), \left(\frac{5}{6}, 3\right)$

$$m = \frac{3 - (-1)}{\frac{5}{6} - \frac{5}{6}} = \frac{4}{0}$$

The slope is undefined and the line is vertical.

$$x = \frac{5}{6}$$



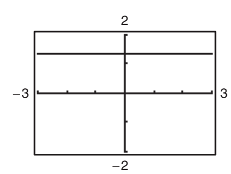
26. $\left(7, \frac{4}{3}\right), \left(9, \frac{4}{3}\right)$

$$m = \frac{\frac{4}{3} - \frac{4}{3}}{9 - 7} = \frac{0}{2} = 0$$

The line is horizontal.

$$y - \frac{4}{3} = 0(x - 7)$$

$$y = \frac{4}{3}$$

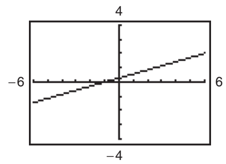


27. $(-1, 0), (6, 2)$

$$m = \frac{2-0}{6-(-1)} = \frac{2}{7}$$

$$y - 0 = \frac{2}{7}(x+1)$$

$$y = \frac{2}{7}x + \frac{2}{7}$$

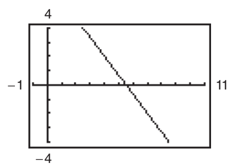


28. $(1, 6), (4, 2)$

$$m = \frac{2-6}{4-1} = \frac{-4}{3}$$

$$y - 6 = -\frac{4}{3}(x-1)$$

$$y = -\frac{4}{3}x + \frac{22}{3}$$

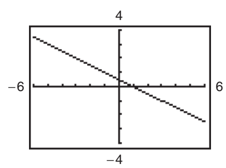


29. $(3, -1), (-3, 2)$

$$m = \frac{2-(-1)}{-3-3} = \frac{3}{-6} = -\frac{1}{2}$$

$$y - (-1) = -\frac{1}{2}(x-3)$$

$$y = -\frac{1}{2}x + \frac{1}{2}$$

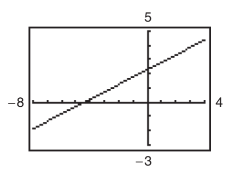


30. $\left(-\frac{5}{2}, 1\right), \left(-4, \frac{2}{9}\right)$

$$m = \frac{\frac{2}{9} - 1}{-4 - \left(-\frac{5}{2}\right)} = \frac{-\frac{7}{9}}{-\frac{3}{2}} = \frac{14}{27}$$

$$y - 1 = \frac{14}{27}\left(x - \left(-\frac{5}{2}\right)\right)$$

$$y = \frac{14}{27}x + \frac{62}{27}$$



31. $5x - 4y = 8 \Rightarrow y = \frac{5}{4}x - 2$ and $m = \frac{5}{4}$

(a) Parallel slope: $m = \frac{5}{4}$

$$y - (-2) = \frac{5}{4}(x-3)$$

$$4y + 8 = 5x - 15$$

$$0 = 5x - 4y - 23$$

$$y = \frac{5}{4}x - \frac{23}{4}$$

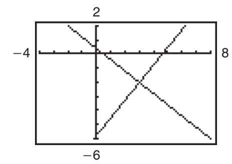
(b) Perpendicular slope: $m = -\frac{4}{5}$

$$y - (-2) = -\frac{4}{5}(x-3)$$

$$5y + 10 = -4x + 12$$

$$4x + 5y - 2 = 0$$

$$y = -\frac{4}{5}x + \frac{2}{5}$$



32. $2x + 3y = 5 \Rightarrow y = -\frac{2}{3}x + \frac{5}{3}$ and $m = -\frac{2}{3}$

(a) Parallel slope: $m = -\frac{2}{3}$

$$y - 3 = -\frac{2}{3}(x+8)$$

$$3y - 9 = -2x - 16$$

$$2x + 3y + 7 = 0$$

$$y = -\frac{2}{3}x - \frac{7}{3}$$

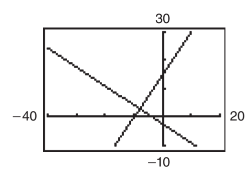
(b) Perpendicular slope: $m = \frac{3}{2}$

$$y - 3 = \frac{3}{2}(x+8)$$

$$2y - 6 = 3x + 24$$

$$3x - 2y + 30 = 0$$

$$y = \frac{3}{2}x + 15$$



33. (a) Not a function. 20 is assigned two different values.
(b) Function

34. (a) Function
(b) Not a function. w is assigned two different values and u is unassigned.

35. $16x^2 - y^2 = 0 \Rightarrow y = \pm 4x$

No, y is not a function of x. Some x-values correspond to two y-values. For example, $x = 1$ corresponds to $y = 4$ and $y = -4$.

36. $x^3 + y^2 = 64 \Rightarrow y = \pm\sqrt{64 - x^3}$

No, y is not a function of x. Some x-values correspond to two y-values. For example, $x = 0$ corresponds to $y = 8$ and $y = -8$.

37. $y = 2x - 3$
This is a function of x.

38. $y = -2x + 10$
This is a function of x.

39. $y = \sqrt{1 - x}$
This is a function of x.

40. $y = \sqrt{x^2 + 4}$
This is a function of x.

41. $f(x) = x^2 + 1$

(a) $f(1) = 1^2 + 1 = 2$

(b) $f(-3) = (-3)^2 + 1 = 10$

(c) $f(b^3) = (b^3)^2 + 1 = b^6 + 1$

(d) $f(x-1) = (x-1)^2 + 1 = x^2 - 2x + 2$

42. $g(x) = \sqrt{x^2 + 1}$

(a) $g(-1) = \sqrt{(-1)^2 + 1} = \sqrt{1+1} = \sqrt{2}$

(b) $g(3) = \sqrt{3^2 + 1} = \sqrt{9+1} = \sqrt{10}$

(c) $g(3x) = \sqrt{(3x)^2 + 1} = \sqrt{9x^2 + 1}$

(d) $g(x+2) = \sqrt{(x+2)^2 + 1} = \sqrt{x^2 + 4x + 4 + 1}$
$= \sqrt{x^2 + 4x + 5}$

43. The domain of $f(x) = \dfrac{x-1}{x+2}$ is all real numbers $x \neq -2$.

44. The domain of $f(x) = \dfrac{x^2}{x^2+1}$ is the set of all real numbers.

45. $f(x) = \sqrt{25 - x^2}$
$25 - x^2 \geq 0$
$(5+x)(5-x) \geq 0$
The domain is $[-5, 5]$.

46. $f(x) = \sqrt{x^2 - 16}$
$x^2 - 16 \geq 0$
$x^2 \geq 16$
The domain is $(-\infty, -4] \cup [4, \infty)$.

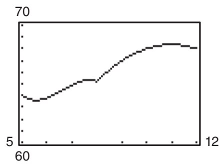
47. (a) $C(x) = 17{,}500 + 5.25x$

(b) $P(x) = R(x) - C(x)$
$P = 8.43x - (5.25x - 17{,}500)$
$P = 3.18x - 17{,}500$

48. (a)

t	5	6	7	8
$n(t)$	64	63.8	64.9	65.2

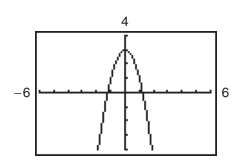
t	9	10	11	12
$n(t)$	66.95	68	68.35	68

(b) 

2013: 66.95 million; 2014: 65.2 million;
2015: 62.75 million; 2016: 59.6 million;
2017: 55.75 million; Answers will vary.

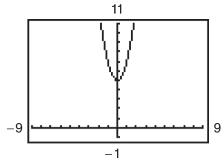
49.
$$f(x) = 2x^2 + 3x - 1$$
$$f(x+h) = 2(x+h)^2 + 3(x+h) - 1$$
$$= 2x^2 + 4xh + 2h^2 + 3x + 3h - 1$$
$$\frac{f(x+h) - f(x)}{h} =$$
$$\frac{(2x^2 + 4xh + 2h^2 + 3x + 3h - 1) - (2x^2 + 3x - 1)}{h}$$
$$= \frac{4xh + 2h^2 + 3h}{h}$$
$$= 4x + 2h + 3,\ h \neq 0$$

50.
$$f(x) = x^2 - 3x + 5$$
$$f(x+h) = (x+h)^2 - 3(x+h) + 5$$
$$= x^2 + 2xh + h^2 - 3x - 3h + 5$$
$$\frac{f(x+h) - f(x)}{h} = \frac{x^2 + 2xh + h^2 - 3x - 3h + 5 - (x^2 - 3x + 5)}{h}$$
$$= \frac{2xh + h^2 - 3h}{h}$$
$$= \frac{h(2x + h - 3)}{h}$$
$$= 2x + h - 3,\ h \neq 0$$

51. 

Domain: all real numbers x
Range: $y \leq 3$

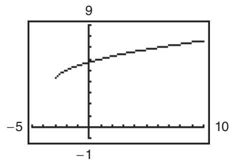
52.



Domain: all real numbers x

Range: $[5, \infty)$

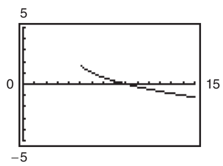
53.



Domain: $[-3, \infty)$

Range: $[4, \infty)$

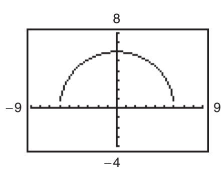
54.



Domain: $[5, \infty)$

Range: $(-\infty, 2]$

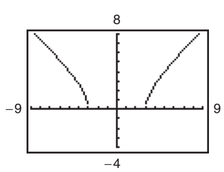
55.



Domain: $36 - x^2 \ge 0 \Rightarrow x^2 \le 36 \Rightarrow -6 \le x \le 6$

Range: $0 \le y \le 6$

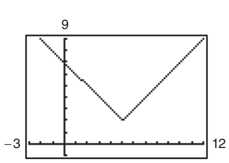
56.



Domain: $(-\infty, -3], [3, \infty)$

Range: $[0, \infty)$

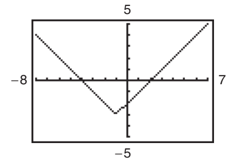
57.



Domain: all real numbers x

Range: $[2, \infty)$

58.



Domain: all real numbers x

Range: $[-3, \infty)$

59. $y - 4x = x^2$

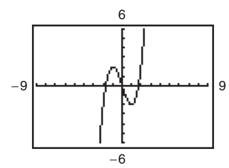
A vertical line intersects the graph just once, so y is a function of x. Solve for y and graph $y_1 = x^2 + 4x$.

60. $3x + y^2 - 2 = 0$

A vertical line intersects the graph more than once, so y is not a function of x. Solve for y and graph $y_1 = \sqrt{-3x + 2}$ and $y_2 = -\sqrt{-3x + 2}$.

61. $f(x) = x^3 - 3x$

(a)

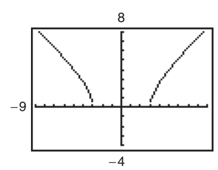


(b) Increasing on $(-\infty, -1)$ and $(1, \infty)$

Decreasing on $(-1, 1)$

62. $f(x) = \sqrt{x^2 - 9}$

(a)

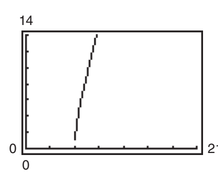


(b) Increasing on $(3, \infty)$

Decreasing on $(-\infty, -3)$

63. $f(x) = x\sqrt{x - 6}$

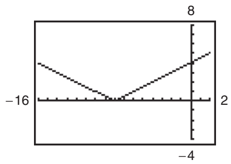
(a)



(b) Increasing on $(6, \infty)$

64. $f(x) = \dfrac{|x+8|}{2}$

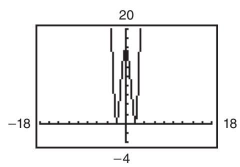
(a)



(b) Increasing on $(-8, \infty)$

Decreasing on $(-\infty, -8)$

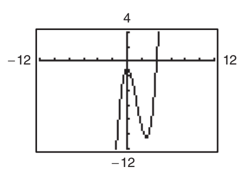
65. $f(x) = (x^2 - 4)^2$



Relative minima: $(-2, 0)$ and $(2, 0)$

Relative maximum: $(0, 16)$

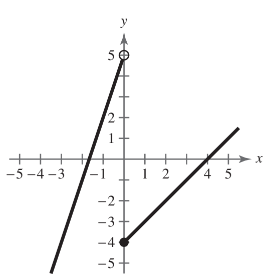
66. $f(x) = x^3 - 4x^2 - 1$



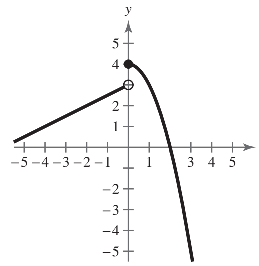
Relative maximum: $(0, -1)$

Relative minimum: $(2.67, -10.48)$

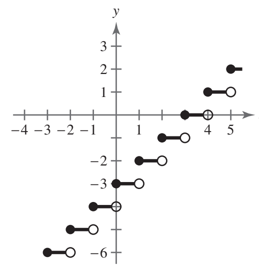
67. $f(x) = \begin{cases} 3x + 5, & x < 0 \\ x - 4, & x \geq 0 \end{cases}$



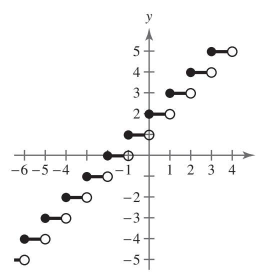
68. $f(x) = \begin{cases} \frac{1}{2}x + 3, & x < 0 \\ 4 - x^2, & x \geq 0 \end{cases}$



69. $f(x) = [\![x]\!] - 3$



70. $f(x) = [\![x + 2]\!]$

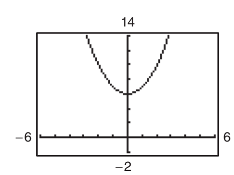


71. $f(-x) = (-x)^2 + 6$

$= x^2 + 6$

$= f(x)$

f is even.



The graph is symmetric with respect to the y-axis. So, f is even.

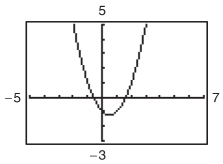
72. $f(-x) = (-x)^2 - (-x) - 1$

$= x^2 + x - 1$

$\neq f(x)$

and $f(-x) \neq -f(x)$

f is neither even nor odd.



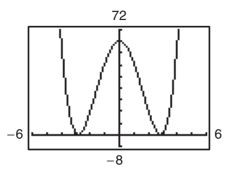
The graph is neither symmetric with respect to the origin nor with respect to the *y*-axis. So, *f* is neither even nor odd.

73. $f(-x) = \left((-x)^2 - 8\right)^2$

$= \left(x^2 - 8\right)^2$

$= f(x)$

f is even.



The graph is symmetric with respect to the *y*-axis. So, *f* is even.

74. $f(-x) = 2(-x)^3 - (-x)$

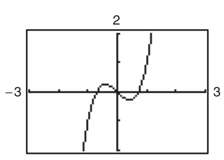
$= -2x^3 + x$

$= -\left(2x^3 - x\right)$

$= -f(x)$

and $f(-x) \neq -f(x)$

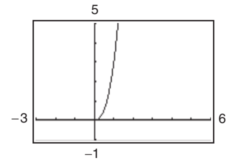
f is odd.



The graph is symmetric with respect to the origin. So, *f* is odd.

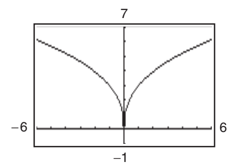
75. $f(-x) = 3(-x)^{5/2} \neq f(x)$ and $f(-x) \neq -f(x)$

f is neither even nor odd.
(Note that the domain of *f* is $x \geq 0$.)



The graph is neither symmetric with respect to the origin nor with respect to the *y*-axis. So, *f* is neither even nor odd.

76. $f(-x) = 3(-x)^{2/5} = 3x^{2/5} = f(x)$
f is even.



The graph is symmetric with respect to the *y*-axis. So, *f* is even.

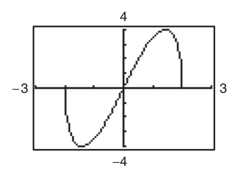
77. $f(-x) = 2(-x)\sqrt{4 - (-x)^2}$

$= -2x\sqrt{4 - x^2}$

$= -\left(2x\sqrt{4 - x^2}\right)$

$= -f(x)$

f is odd.

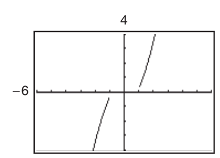


The graph is symmetric with respect to the origin. So, *f* is odd.

78. $f(-x) = (-x)\sqrt{(-x)^2 - 1}$

$= -x\sqrt{x^2 - 1}$

$= -f(x)$



f is odd.
The graph is symmetric with respect to the origin. So, *f* is odd.

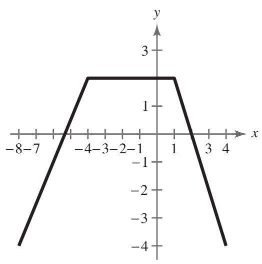
79. Horizontal shift three units to the right of $f(x) = \frac{1}{x}$: $y = \frac{1}{x-3}$

80. Horizontal shift two units to the right, followed by a vertical shift one unit upward of $f(x) = x^2$: $y = (x-2)^2 + 1$

81. Vertical shift three units upward of $f(x) = |x|$: $y = |x| + 3$

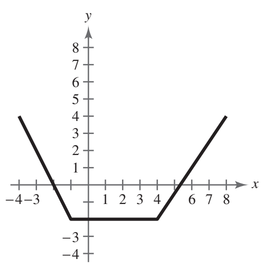
82. Horizontal shift three units to the right, followed by a reflection in the x-axis of $f(x) = \sqrt{x}$: $y = -\sqrt{x-3}$

83.



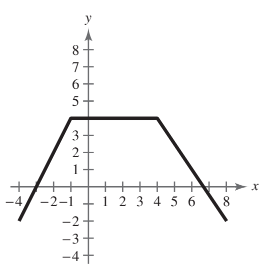
$y = f(-x)$ is a reflection in the y-axis.

84.



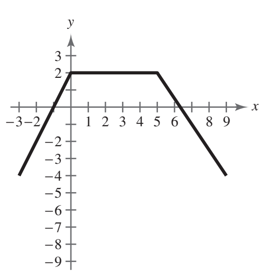
$y = -f(x)$ is a reflection in the x-axis.

85.



$y = f(x) + 2$ is a vertical shift two units upward.

86.



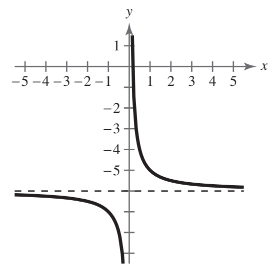
$y = f(x-1)$ is a horizontal shift one unit to the right.

87. $h(x) = \frac{1}{x} - 6$

(a) $f(x) = \frac{1}{x}$

(b) The graph of h is a vertical shift six units downward of f.

(c)



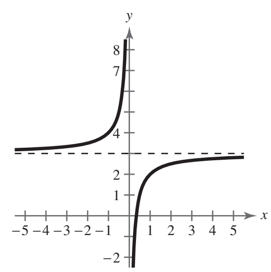
(d) $h(x) = f(x) - 6$

88. $h(x) = -\frac{1}{x} + 3$

(a) $f(x) = \frac{1}{x}$

(b) The graph of h is a reflection in the x-axis and a vertical shift three units upward of f.

(c)



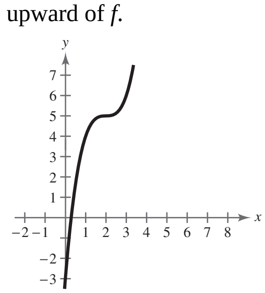
(d) $h(x) = -f(x) + 3$

89. $h(x) = (x-2)^3 + 5$

(a) $f(x) = x^3$

(b) The graph of h is a horizontal shift of f two units to the right, followed by a vertical shift five units upward of f.

(c)



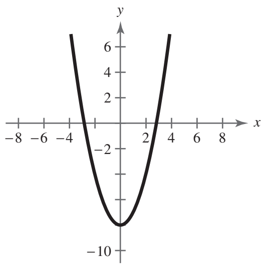
(d) $h(x) = (x-2)^3 + 5 = f(x-2) + 5$

90. $h(x) = (-x)^2 - 8$

(a) $f(x) = x^2$

(b) The graph of h is a reflection in the y-axis, followed by a vertical shift eight units downward of f.

(c)



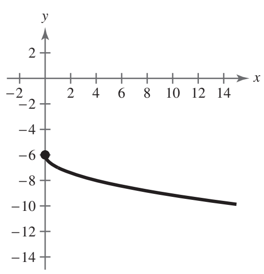
(d) $h(x) = f(-x) - 8$

91. $h(x) = -\sqrt{x} - 6$

(a) $f(x) = \sqrt{x}$

(b) The graph of h is a reflection in the x-axis and a vertical shift six units downward of f.

(c)



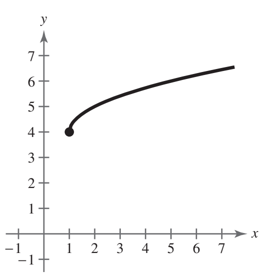
(d) $h(x) = -f(x) - 6$

92. $h(x) = \sqrt{x-1} + 4$

(a) $f(x) = \sqrt{x}$

(b) The graph of h is a horizontal shift one unit to the right and a vertical shift four units upward of f.

(c)



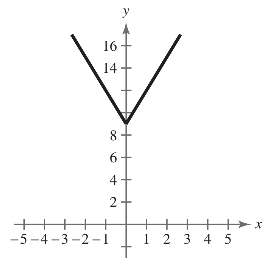
(d) $h(x) = f(x-1) + 4$

93. $h(x) = |3x| + 9$

(a) $f(x) = |x|$

(b) The graph of h is a vertical shift nine units upward, and a horizontal shrink of f.

(c)



(d) $h(x) = f(3x) + 9$

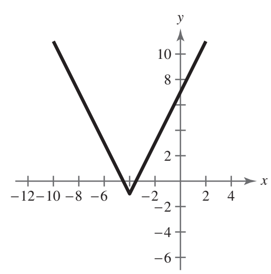
94. $h(x) = |2x + 8| - 1$

$h(x) = |2(x + 4)| - 1$

(a) $f(x) = |x|$

(b) h is a horizontal shift four units to the left, followed by a vertical shift one unit downward, and a horizontal shrink of f.

(c)



(d) $h(x) = f(2(x + 4)) - 1$

95. $(f - g)(4) = f(4) - g(4)$

$= [3 - 2(4)] - \sqrt{4}$

$= -5 - 2$

$= -7$

96. $(f + h)(5) = f(5) + h(5)$

$= -7 + 77$

$= 70$

97. $(f + g)(25) = f(25) + g(25)$

$= -47 + 5$

$= -42$

98. $(g - h)(1) = g(1) - h(1) = 1 - 5 = -4$

99. $(fh)(1) = f(1)h(1) = (3 - 2(1))(3(1)^2 + 2)$

$= (1)(5) = 5$

100. $\left(\dfrac{g}{h}\right)(1) = \dfrac{g(1)}{h(1)} = \dfrac{1}{5}$

101. $(h \circ g)(5) = h(g(5))$
$$= h\left(\sqrt{5}\right)$$
$$= 3\left(\sqrt{5}\right)^2 + 2 = 17$$

102. $(g \circ f)(-3) = g(f(-3))$
$$= g(9)$$
$$= \sqrt{9} = 3$$

103. $f(x) = x^2,\ g(x) = x + 3$
$$(f \circ g)(x) = f(x+3)$$
$$= (x+3)^2 = h(x)$$

104. $f(x) = x^3,\ g(x) = 1 - 2x$
$$(f \circ g)(x) = f(1-2x) = (1-2x)^3 = h(x)$$

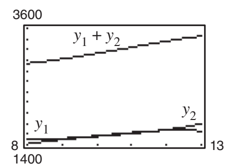
105. $f(x) = \sqrt{x},\ g(x) = 4x + 2$
$$(f \circ g)(x) = f(4x+2) = \sqrt{4x+2} = h(x)$$

106. $f(x) = \sqrt[3]{x},\ g(x) = (x+2)^2$
$$(f \circ g)(x) = f\left((x+2)^2\right) = \sqrt[3]{(x+2)^2} = h(x)$$

107. $f(x) = \dfrac{3}{x},\ g(x) = x + 2$
$$(f \circ g)(x) = f(x+2) = \frac{4}{x+2} = h(x)$$

108. $f(x) = \dfrac{6}{x^3},\ g(x) = 3x + 1$
$$(f \circ g)(x) = f(3x+1) = \frac{6}{(3x+1)^3} = h(x)$$

109.



110. For 2017, $t = 17$.
$$y_1 + y_2 = \left[-5.824(17)^3 + 182.05(17)^2 - 1832.8(17) + 7515\right] + \left[3.398(17)^3 - 103.63(17)^2 + 1106.5(17) - 2543\right]$$
$$= 3{,}369.342 \text{ thousand students or } 3{,}369{,}342 \text{ students}$$

111. $f(x) = 6x$
$$f^{-1}(x) = \frac{1}{6}x$$
$$f(f^{-1}(x)) = f\left(\frac{1}{6}x\right) = 6\left(\frac{1}{6}x\right) = x$$
$$f^{-1}(f(x)) = f^{-1}(6x) = \frac{1}{6}(6x) = x$$

112. $f(x) = x + 5$
$$f^{-1}(x) = x - 5$$
$$f(f^{-1}(x)) = f(x-5) = (x-5) + 5 = x$$
$$f^{-1}(f(x)) = f^{-1}(x+5) = (x+5) - 5 = x$$

113. $f(x) = \dfrac{1}{2}x + 3$
$$f^{-1}(x) = 2(x-3) = 2x - 6$$
$$f(f^{-1}(x)) = f\left(2(x-3)\right)$$
$$= \frac{1}{2}\left(2(x-3)\right) + 3 = x - 3 + 3 = x$$
$$f^{-1}(f(x)) = f^{-1}\left(\frac{1}{2}x + 3\right)$$
$$= 2\left(\frac{1}{2}x + 3 - 3\right) = 2\left(\frac{1}{2}x\right) = x$$

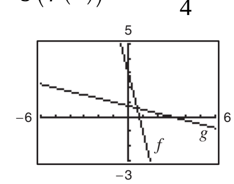
114. $f(x) = \dfrac{x-4}{5}$
$$f^{-1}(x) = 5x + 4$$
$$f(f^{-1}(x)) = f(5x+4) = \frac{5x+4-4}{5} = \frac{5x}{5} = x$$
$$f^{-1}(f(x)) = f^{-1}\left(\frac{x-4}{5}\right)$$
$$= 5\left(\frac{x-4}{5}\right) + 4 = x - 4 + 4 = x$$

115. $f(x) = 3 - 4x,\ g(x) = \dfrac{3-x}{4}$

(a) $f\left(g(x)\right) = 3 - 4\left(\dfrac{3-x}{4}\right) = 3 - (3-x) = x$

$g\left(f(x)\right) = \dfrac{3-(3-4x)}{4} = \dfrac{4x}{4} = x$

(b)



(c) $Y_1 = 3 - 4X$

$Y_2 = \dfrac{3-X}{4}$

$Y_3 = Y_1(Y_2)$

$Y_4 = Y_2(Y_1)$

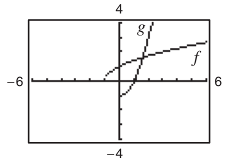
X	Y_3	Y_4
−2	−2	−2
−1	−1	−1
0	0	0
1	1	1
2	2	2

116. $f(x) = \sqrt{x+1},\ g(x) = x^2 - 1,\ x \ge 0$

(a) $f(g(x)) = \sqrt{(x^2 - 1) + 1} = \sqrt{x^2} = x$

$g(f(x)) = \left(\sqrt{x+1}\right)^2 - 1 = x + 1 - 1 = x$

(b)



(c) $Y_1 = \sqrt{x} + 1$

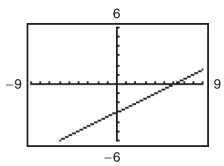
$Y_2 = X^2 - 1,\ X \ge 0$

$Y_3 = Y_1(Y_2)$

$Y_4 = Y_2(Y_1)$

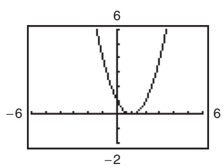
X	Y_3	Y_4
0	0	0
1	1	1
2	2	2
3	3	3
4	4	4

117.



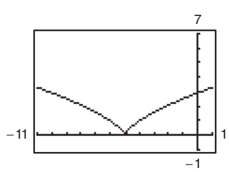
$f(x) = \frac{1}{2}x - 3$ passes the Horizontal Line Test, and hence is one-to-one and has an inverse function.

118.



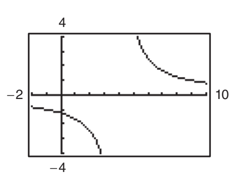
$f(x) = (x-1)^2$ does not pass the Horizontal Line Test, so f is not one-to-one and does not have an inverse function.

119.



$h(t) = (t + 5)^{2/3}$ does not pass the Horizontal Line Test, and hence is not one-to-one and does not have an inverse function.

120.



$g(x) = \dfrac{5}{x - 4}$ passes the Horizontal Line Test, and hence is one-to-one and has an inverse function.

121.

$$y = \frac{1}{2}x - 5$$
$$x = \frac{1}{2}y - 5$$
$$x + 5 = \frac{1}{2}y$$
$$y = 2(x + 5)$$
$$f^{-1}(x) = 2x + 10$$

122.

$$f(x) = \frac{7x+3}{8}$$
$$y = \frac{1}{8}(7x + 3)$$
$$x = \frac{1}{8}(7y + 3)$$
$$8x = 7y + 3$$
$$8x - 3 = 7y$$
$$f^{-1}(x) = \frac{1}{7}(8x - 3)$$

123.

$$f(x) = -5x^3 - 3$$
$$y = -5x^3 - 3$$
$$x = -5y^3 - 3$$
$$x + 3 = -5y^3$$
$$\frac{x+3}{-5} = y^3$$
$$f^{-1}(x) = \sqrt[3]{\frac{x+3}{-5}} = -\sqrt[3]{\frac{x+3}{5}}$$

124.

$$y = 5x^3 + 2$$
$$x = 5y^3 + 2$$
$$x - 2 = 5y^3$$
$$\frac{x-2}{5} = y^3$$
$$f^{-1}(x) = \sqrt[3]{\frac{x-2}{5}}$$

125.

$$f(x) = \sqrt{x + 10}$$
$$y = \sqrt{x+10},\ x \ge -10,\ y \ge 0$$
$$x = \sqrt{y+10},\ y \ge -10,\ x \ge 0$$
$$x^2 = y + 10$$
$$x^2 - 10 = y$$
$$f^{-1}(x) = x^2 - 10,\ x \ge 0$$

126. $f(x) = 4\sqrt{6-x},\ x \le 6,\ y \ge 0$

$$y = 4\sqrt{6-x}$$
$$x = 4\sqrt{6-y},\ y \le 6,\ x \ge 0$$
$$x^2 = 16(6-y) = 96 - 16y$$
$$16y = 96 - x^2$$
$$y = \frac{96-x^2}{16}$$
$$f^{-1}(x) = \frac{96-x^2}{16},\ x \ge 0$$

127. $f(x) = \frac{1}{4}x^2 + 1,\ x \ge 0$

$$y = \frac{1}{4}x^2 + 1$$
$$x = \frac{1}{4}y^2 + 1$$
$$x - 1 = \frac{1}{4}y^2$$
$$4(x-1) = y^2$$
$$f^{-1}(x) = \sqrt{4(x-1)} = 2\sqrt{x-1}$$

The positive square root is chosen as f^{-1} since the domain of f is $[0, \infty)$.

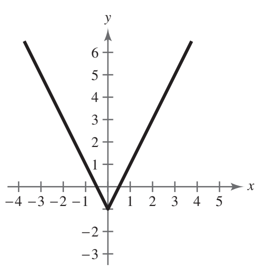
128. $f(x) = 5 - \frac{1}{9}x^2,\ x \le 0$

$$y = 5 - \frac{1}{9}x^2$$
$$x = 5 - \frac{1}{9}y^2$$
$$x - 5 = -\frac{1}{9}y^2$$
$$-9(x-5) = y^2$$
$$f^{-1}(x) = -\sqrt{-9(x-5)} = -3\sqrt{5-x}$$

The negative square root is chosen as f^{-1} since the domain of f is $(-\infty, 0]$.

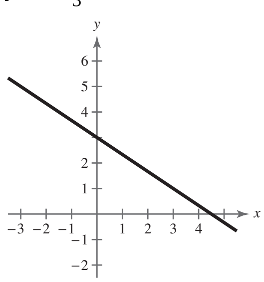
Chapter 1 Test

1. $y = 2|x| - 1$



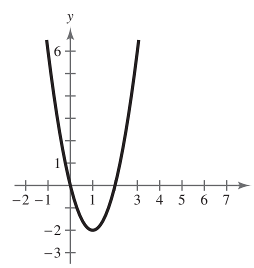
Intercepts: $(0, 1), \left(-\frac{1}{2}, 0\right), \left(\frac{1}{2}, 0\right)$

2. $y = -\frac{2}{3}x + 3$



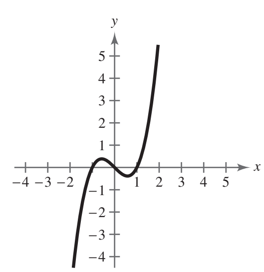
Intercepts: $(0, 3), \left(\frac{9}{2}, 0\right)$

3. $y = 2x^2 - 4x$



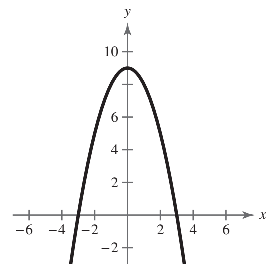
Intercepts: $(0, 0), (2, 0)$

4. $y = x^3 - x$



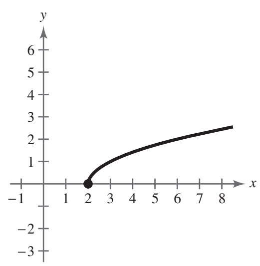
Intercepts: $(-1, 0), (0, 0), (1, 0)$

5. $y = -x^2 + 9$



Intercepts: $(-3, 0), (3, 0), (0, 9)$

6. $y = \sqrt{x - 2}$



Intercept: $(2, 0)$

7. $5x + 2y = 3$

$$2y = -5x + 3$$

$$y = -\frac{5}{2}x + \frac{3}{2}$$

$$\text{Slope} = -\frac{5}{2}$$

(a) Parallel line slope: $-\frac{5}{2}$

$$y - 4 = -\frac{5}{2}(x - 0)$$

$$y = -\frac{5}{2}x + 4$$

$$5x + 2y - 8 = 0$$

(b) Perpendicular line slope: $\frac{2}{5}$

$$y - 4 = \frac{2}{5}(x - 0)$$

$$y = \frac{2}{5}x + 4$$

$$2x - 5y + 20 = 0$$

8. $\text{Slope} = \frac{4 - (-1)}{-3 - 2} = \frac{5}{-5} = -1$

$$y + 1 = -1(x - 2)$$

$$y = -x + 1$$

9. No. For some x there corresponds more than one value of y. For instance, if $x = 1$, $y = \pm\frac{1}{\sqrt{3}}$.

10. $f(x) = |x + 2| - 15$

(a) $f(-8) = |-8 + 2| - 15 = 6 - 15 = -9$

(b) $f(14) = |14 + 2| - 15 = 16 - 15 = 1$

(c) $f(t - 6) = |t - 6 + 2| - 15 = |t - 4| - 15$

11. $3 - x \geq 0 \Rightarrow$ domain is all $x \leq 3$.

12. Total cost = Variable costs + Fixed costs

$$C = 24.60x + 25{,}000$$

Revenue = Price per unit × Number of units

$$R = 101.50x$$

Profit = Revenue − Cost

$$P = 101.50x - (24.60x + 25{,}000)$$

$$P = 76.90x - 25{,}000$$

13. $f(-x) = 2(-x)^3 - 3(-x)$

$$= -2x^3 + 3x = -f(x)$$

Odd

14. $f(-x) = 3(-x)^4 + 5(-x)^2$

$$= 3x^4 + 5x^2 = f(x)$$

Even

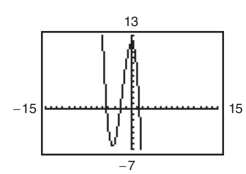
15. $h(x) = \frac{1}{4}x^4 - 2x^2 = \frac{1}{4}x^2(x^2 - 8)$

By graphing h, you see that the graph is increasing on $(-2, 0)$ and $(2, \infty)$ and decreasing on $(-\infty, -2)$ and $(0, 2)$.

16. $g(t) = |t + 2| - |t - 2|$

By graphing g, you see that the graph is increasing on $(-2, 2)$, and constant on $(-\infty, -2)$ and $(2, \infty)$.

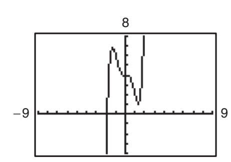
17.



Relative minimum: $(-3.33, -6.52)$

Relative maximum: $(0, 12)$

18.



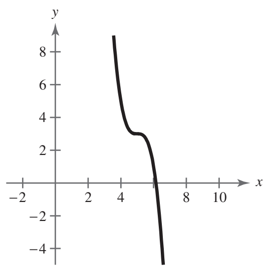
Relative minimum: $(1.34, 1.10)$

Relative maximum: $(-1.34, 6.90)$

19. (a) $f(x) = x^3$

(b) The graph of g is a horizontal shift five units to the right, a vertical stretch of 2, a reflection in the x-axis, and a vertical shift three units upward of f.

(c)



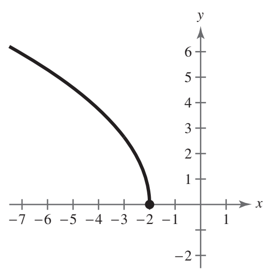
20. $g(x) = \sqrt{-7x - 14}$

$= \sqrt{-7(x + 2)}$

(a) $f(x) = \sqrt{x}$

(b) The graph of g is a reflection in the y-axis, a horizontal shrink, and a horizontal shift two units to the left.

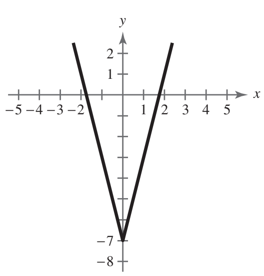
(c)



21. (a) $f(x) = |x|$

(b) g is obtained from f by a vertical stretch of 4 followed by a vertical shift seven units downward.

(c)



22. (a) $(f - g)(x) = x^2 - \sqrt{2 - x}$

Domain: $x \le 2$

(b) $\left(\dfrac{f}{g}\right)(x) = \dfrac{x^2}{\sqrt{2 - x}}$

Domain: $x < 2$

(c) $(f \circ g)(x) = f\left(\sqrt{2 - x}\right) = 2 - x$

Domain: $x \le 2$

(d) $(g \circ f)(x) = g\left(x^2\right) = \sqrt{2 - x^2}$

Domain: $-\sqrt{2} \le x \le \sqrt{2}$

23. $f(x) = x^3 + 8$

Yes, f is one-to-one and has an inverse function.

$$y = x^3 + 8$$
$$x = y^3 + 8$$
$$x - 8 = y^3$$
$$\sqrt[3]{x - 8} = y$$
$$f^{-1}(x) = \sqrt[3]{x - 8}$$

24. $f(x) = x^2 + 6$

No, f is not one-to-one, and does not have an inverse function.

25. $f(x) = \dfrac{3x\sqrt{x}}{8}$

Yes, f is one-to-one and has an inverse function.

$$y = \frac{3}{8}x^{3/2},\ x \ge 0,\ y \ge 0$$
$$x = \frac{3}{8}y^{3/2},\ y \ge 0,\ x \ge 0$$
$$\frac{8}{3}x = y^{3/2}$$
$$\left(\frac{8}{3}x\right)^{2/3} = y$$
$$f^{-1}(x) = \left(\frac{8}{3}x\right)^{2/3},\ x \ge 0$$