INSTRUCTOR'S SOLUTIONS MANUAL

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COLLEGE ALGEBRA

SEVENTH EDITION

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TABLE OF CONTENTS for INSTRUCTOR SOLUTIONS

COLLEGE ALGEBRA 7E

Chapter P
Fundamental Concepts of Algebra

Section P.1

Check Point Exercises

1. $8+6(x-3)^2 = 8+6(13-3)^2$
$= 8+6(10)^2$
$= 8+6(100)$
$= 8+600$
$= 608$

2. **a.** Since 2014 is 14 years after 2000, substitute 14 for x.
$T = 4x^2 + 330x + 3310$
$= 4(14)^2 + 330(14) + 3310$
$= 8714$
The average cost of tuition and fees at public U.S. colleges for the school year ending in 2014 was \$8714.

b. The formula underestimates the actual answer by \$179.

3. The elements common to {3, 4, 5, 6, 7} and {3, 7, 8, 9} are 3 and 7.
$\{3,4,5,6,7\} \cap \{3,7,8,9\} = \{3,7\}$

4. The union is the set containing all the elements of either set.
$\{3,4,5,6,7\} \cup \{3,7,8,9\} = \{3,4,5,6,7,8,9\}$

5. $\left\{-9,\ -1.3,\ 0,\ 0.\overline{3},\ \frac{\pi}{2},\ \sqrt{9},\ \sqrt{10}\right\}$

a. Natural numbers: $\sqrt{9}$ because $\sqrt{9} = 3$

b. Whole numbers: 0, $\sqrt{9}$

c. Integers: –9, 0, $\sqrt{9}$

d. Rational numbers: –9, -1.3, 0, $0.\overline{3}$, $\sqrt{9}$

e. Irrational numbers: $\frac{\pi}{2}$, $\sqrt{10}$

f. Real numbers: –9, -1.3, 0, $0.\overline{3}$, $\frac{\pi}{2}$, $\sqrt{9}$, $\sqrt{10}$

6. **a.** $\left|1-\sqrt{2}\right|$
Because $\sqrt{2} \approx 1.4$, the number inside the absolute value bars is negative. The absolute value of x when $x < 0$ is $-x$. Thus,
$\left|1-\sqrt{2}\right| = -\left(1-\sqrt{2}\right) = \sqrt{2}-1$

b. $|\pi - 3|$
Because $\pi \approx 3.14$, the number inside the absolute value bars is positive. The absolute value of a positive number is the number itself. Thus,
$|\pi - 3| = \pi - 3.$

c. $\frac{|x|}{x}$
Because $x > 0$, $|x| = x$.
Thus, $\frac{|x|}{x} = \frac{x}{x} = 1$

7. $|-4-(5)| = |-9| = 9$
The distance between –4 and 5 is 9.

8. $7(4x^2+3x)+2(5x^2+x)$
$= 7(4x^2+3x)+2(5x^2+x)$
$= 28x^2+21x+10x^2+2x$
$= 38x^2+23x$

9. $6+4[7-(x-2)]$
$= 6+4[7-x+2)]$
$= 6+4[9-x]$
$= 6+36-4x$
$= 42-4x$

Concept and Vocabulary Check P.1

1. expression

2. b to the nth power; base; exponent

3. formula; modeling; models

4. intersection; $A \cap B$

5. union; $A \cup B$

6. natural

7. whole

8. integers

9. rational

10. irrational

11. rational; irrational

12. absolute value; x, $-x$

13. $b+a$; ba

14. $a+(b+c)$; $(ab)c$

15. $ab+ac$

16. 0; inverse; 0; identity

17. inverse; 1; identity

18. simplified

19. a

Exercise Set P.1

1. $7+5(10)=7+50=57$

2. $8+6(5)=8+30=38$

3. $6(3)-8=18-8=10$

4. $8(3)-4=24-4=20$

5. $8^2+3(8)=64+24=88$

6. $6^2+5(6)=36+30=66$

7. $7^2-6(7)+3=49-42+3=7+3=10$

8. $8^2-7(8)+4=64-56+4=8+4=12$

9. $$4+5(9-7)^3=4+5(2)^3$$
$$=4+5(8)=4+40=44$$

10. $$6+5(8-6)^3=6+5(2)^3$$
$$=6+5(8)$$
$$=6+40=46$$

11. $$8^2-3(8-2)=64-3(6)$$
$$=64-18=46$$

12. $8^2-4(8-3)=64-4(5)=64-20=44$

13. $$\frac{5(x+2)}{2x-14}=\frac{5(10+2)}{2(10)-14}$$
$$=\frac{5(12)}{6}$$
$$=5\cdot 2$$
$$=10$$

14. $$\frac{7(x-3)}{2x-16}=\frac{7(9-3)}{2(9)-16}=\frac{7(6)}{2}=7\cdot 3=21$$

15. $\frac{2x+3y}{x+1}$; $x=-2$, $y=4$
$$=\frac{2(-2)+3(4)}{-2+1}=\frac{-4+12}{-1}=\frac{8}{-1}=-8$$

16. $\frac{2x+y}{xy-2x}$; $x=-2$ and $y=4$
$$\frac{2(-2)+4}{(-2)(4)-2(-2)}=\frac{-4+4}{-8+4}=\frac{0}{4}=0$$

17. $C=\frac{5}{9}(50-32)=\frac{5}{9}(18)=10$
50°F is equivalent to 10°C.

18. $C=\frac{5}{9}(F-32)=\frac{5}{9}(86-32)=\frac{5}{9}(54)=30$
86°F is equivalent to 30°C.

19. $$h=4+60t-16t^2=4+60(2)-16(2)^2$$
$$=4+120-16(4)=4+120-64$$
$$=124-64=60$$
Two seconds after it is kicked, the ball's height is 60 feet.

20. $h = 4 + 60t - 16t^2$
$= 4 + 60(3) - 16(3)^2$
$= 4 + 180 - 16(9)$
$= 4 + 180 - 144$
$= 184 - 144 = 40$
Three seconds after it is kicked, the ball's height is 40 feet.

21. $\{1,2,3,4\} \cap \{2,4,5\} = \{2,4\}$

22. $\{1,3,7\} \cap \{2,3,8\} = \{3\}$

23. $\{s,e,t\} \cap \{t,e,s\} = \{s,e,t\}$

24. $\{r,e,a,l\} \cap \{l,e,a,r\} = \{r,e,a,l\}$

25. $\{1,3,5,7\} \cap \{2,4,6,8,10\} = \{\ \ \}$
The empty set is also denoted by $\varnothing$.

26. $\{1,3,5,7\} \cap \{-5,-3,-1\} = \{\ \ \}$ or $\varnothing$

27. $\{a,b,c,d\} \cap \varnothing = \varnothing$

28. $\{w,y,z\} \cap \varnothing = \varnothing$

29. $\{1,2,3,4\} \cup \{2,4,5\} = \{1,2,3,4,5\}$

30. $\{1,3,7,8\} \cup \{2,3,8\} = \{1,2,3,7,8\}$

31. $\{1,3,5,7\} \cup \{2,4,6,8,10\}$
$= \{1,2,3,4,5,6,7,8,10\}$

32. $\{0,1,3,5\} \cup \{2,4,6\} = \{0,1,2,3,4,5,6\}$

33. $\{a,e,i,o,u\} \cup \varnothing = \{a,e,i,o,u\}$

34. $\{e,m,p,t,y\} \cup \varnothing = \{e,m,p,t,y\}$

35. **a.** $\sqrt{100}$

b. $0, \sqrt{100}$

c. $-9, 0, \sqrt{100}$

d. $-9, -\frac{4}{5}, 0, 0.25, 9.2, \sqrt{100}$

e. $\sqrt{3}$

f. $-9, -\frac{4}{5}, 0, 0.25, \sqrt{3}, 9.2, \sqrt{100}$

36. **a.** $\sqrt{49}$

b. $0, \sqrt{49}$

c. $-7, 0, \sqrt{49}$

d. $-7, -0.\overline{6}, 0, \sqrt{49}$

e. $\sqrt{50}$

f. $-7, -0.\overline{6}, 0, \sqrt{49}, \sqrt{50}$

37. **a.** $\sqrt{64}$

b. $0, \sqrt{64}$

c. $-11, 0, \sqrt{64}$

d. $-11, -\frac{5}{6}, 0, 0.75, \sqrt{64}$

e. $\sqrt{5}, \pi$

f. $-11, -\frac{5}{6}, 0, 0.75, \sqrt{5}, \pi, \sqrt{64}$

38. **a.** $\sqrt{4}$

b. $0, \sqrt{4}$

c. $-5, 0, \sqrt{4}$

d. $-5, -0.\overline{3}, 0, \sqrt{4}$

e. $\sqrt{2}$

f. $-5, -0.\overline{3}, 0, \sqrt{2}, \sqrt{4}$

39. 0

40. Answers will vary. An example is $\frac{1}{2}$.

41. Answers will vary. An example is 2.

42. Answers will vary. An example is -2.

43. true; –13 is to the left of –2 on the number line.

44. false; –6 is to the left of 2 on the number line.

45. true; 4 is to the right of –7 on the number line.

46. true; –13 is to the left of –5 on the number line.

47. true; $-\pi = -\pi$

48. true; –3 is to the right of –13 on the number line.

49. true; 0 is to the right of –6 on the number line.

50. true; 0 is to the right of –13 on the number line.

51. $|300| = 300$

52. $|-203| = 203$

53. $|12-\pi| = 12-\pi$

54. $|7-\pi| = 7-\pi$

55. $\left|\sqrt{2}-5\right| = 5-\sqrt{2}$

56. $\left|\sqrt{5}-13\right| = 13-\sqrt{5}$

57. $\frac{-3}{|-3|} = \frac{-3}{3} = -1$

58. $\frac{-7}{|-7|} = \frac{-7}{7} = -1$

59. $\left||-3|-|-7|\right| = |3-7| = |-4| = 4$

60. $\left||-5|-|-13|\right| = |5-13| = |-8| = 8$

61. $|x+y| = |2+(-5)| = |-3| = 3$

62. $|x-y| = |2-(-5)| = |7| = 7$

63. $|x|+|y| = |2|+|-5| = 2+5 = 7$

64. $|x|-|y| = |2|-|-5| = 2-5 = -3$

65. $\frac{y}{|y|} = \frac{-5}{|-5|} = \frac{-5}{5} = -1$

66. $\frac{|x|}{x}+\frac{|y|}{y} = \frac{|2|}{2}+\frac{|-5|}{-5} = \frac{2}{2}+\frac{5}{-5} = 1+(-1) = 0$

67. The distance is $|2-17| = |-15| = 15$.

68. The distance is $|4-15| = |-11| = 11$.

69. The distance is $|-2-5| = |-7| = 7$.

70. The distance is $|-6-8| = |-14| = 14$.

71. The distance is $|-19-(-4)| = |-19+4| = |-15| = 15$.

72. The distance is $|-26-(-3)| = |-26+3| = |-23| = 23$.

73. The distance is
$|-3.6-(-1.4)| = |-3.6+1.4| = |-2.2| = 2.2$.

74. The distance is
$|-5.4-(-1.2)| = |-5.4+1.2| = |-4.2| = 4.2$.

75. 6 + (–4) = (–4) + 6;
commutative property of addition

76. $11\cdot(7+4) = 11\cdot 7+11\cdot 4$;
distributive property of multiplication over addition

77. 6 + (2 + 7) = (6 + 2) + 7;
associative property of addition

78. $6\cdot(2\cdot 3) = 6\cdot(3\cdot 2)$;
commutative property of multiplication

79. (2 + 3) + (4 + 5) = (4 + 5) + (2 + 3);
commutative property of addition

80. $7\cdot(11\cdot 8) = (11\cdot 8)\cdot 7$;
commutative property of multiplication

81. 2 (–8 + 6) = –16 + 12;
distributive property of multiplication over addition

82. $-8(3+11) = -24+(-88)$;
distributive property of multiplication over addition

83. $\frac{1}{x+3}(x+3) = 1; x \neq -3$,
inverse property of multiplication

84. $(x+4)+\left[-(x+4)\right] = 0$;
inverse property of addition

85. $$\begin{aligned} 5(3x+4)-4 &= 5\cdot 3x+5\cdot 4-4 \\ &= 15x+20-4 \\ &= 15x+16 \end{aligned}$$

86. $2(5x+4)-3 = 2\cdot 5x + 2\cdot 4 - 3$
$= 10x + 8 - 3$
$= 10x + 5$

87. $5(3x-2)+12x = 5\cdot 3x - 5\cdot 2 + 12x$
$= 15x - 10 + 12x$
$= 27x - 10$

88. $2(5x-1)+14x = 2\cdot 5x - 2\cdot 1 + 14x$
$= 10x - 2 + 14x$
$= 24x - 2$

89. $7(3y-5)+2(4y+3)$
$= 7\cdot 3y - 7\cdot 5 + 2\cdot 4y + 2\cdot 3$
$= 21y - 35 + 8y + 6$
$= 29y - 29$

90. $4(2y-6)+3(5y+10)$
$= 4\cdot 2y - 4\cdot 6 + 3\cdot 5y + 3\cdot 10$
$= 8y - 24 + 15y + 30$
$= 23y + 6$

91. $5(3y-2)-(7y+2) = 15y - 10 - 7y - 2$
$= 8y - 12$

92. $4(5y-3)-(6y+3) = 20y - 12 - 6y - 3$
$= 14y - 15$

93. $7 - 4\left[3 - (4y-5)\right] = 7 - 4[3 - 4y + 5]$
$= 7 - 4[8 - 4y]$
$= 7 - 32 + 16y$
$= 16y - 25$

94. $6 - 5[8 - (2y-4)] = 6 - 5[8 - 2y + 4]$
$= 6 - 5[12 - 2y]$
$= 6 - 60 + 10y$
$= 10y - 54$

95. $18x^2 + 4 - \left[6\left(x^2 - 2\right) + 5\right]$
$= 18x^2 + 4 - \left[6x^2 - 12 + 5\right]$
$= 18x^2 + 4 - \left[6x^2 - 7\right]$
$= 18x^2 + 4 - 6x^2 + 7$
$= 18x^2 - 6x^2 + 4 + 7$
$= (18-6)x^2 + 11 = 12x^2 + 11$

96. $14x^2 + 5 - \left[7\left(x^2 - 2\right) + 4\right]$
$= 14x^2 + 5 - \left[7x^2 - 14 + 4\right]$
$= 14x^2 + 5 - \left[7x^2 - 10\right]$
$= 14x^2 + 5 - 7x^2 + 10$
$= 14x^2 - 7x^2 + 5 + 10$
$= (14-7)x^2 + 15$
$= 7x^2 + 15$

97. $-(-14x) = 14x$

98. $-(-17y) = 17y$

99. $-(2x - 3y - 6) = -2x + 3y + 6$

100. $-(5x - 13y - 1) = -5x + 13y + 1$

101. $\frac{1}{3}(3x) + [(4y) + (-4y)] = x + 0 = x$

102. $\frac{1}{2}(2y) + [(-7x) + 7x] = y + 0 = y$

103. $|-6| \;\square\; |-3|$
$6 \;\square\; 3$
$6 > 3$
Since $6 > 3$, $|-6| > |-3|$.

104. $|-20| \;\square\; |-50|$
$20 \;\square\; 50$
$20 < 50$
Since $20 < 50$, $|-20| < |-50|$.

105. $\left|\frac{3}{5}\right| \;\square\; |-0.6|$
$|0.6| \;\square\; |-0.6|$
$0.6 \;\square\; 0.6$
$0.6 = 0.6$
Since $0.6 = 0.6$, $\left|\frac{3}{5}\right| = |-0.6|$.

106. $\left|\frac{5}{2}\right| \;\square\; |-2.5|$

$|2.5| \;\square\; |-2.5|$

$2.5 \;\square\; 2.5$

$2.5 = 2.5$

Since $2.5 = 2.5$, $\left|\frac{5}{2}\right| = |-2.5|$.

107. $\frac{30}{40} - \frac{3}{4} \;\square\; \frac{14}{15} \cdot \frac{15}{14}$

$\frac{30}{40} - \frac{30}{40} \;\square\; \frac{\cancel{14}}{\cancel{15}} \cdot \frac{\cancel{15}}{\cancel{14}}$

$0 \;\square\; 1$

$0 < 1$

Since $0 < 1$, $\frac{30}{40} - \frac{3}{4} < \frac{14}{15} \cdot \frac{15}{14}$.

108. $\frac{17}{18} \cdot \frac{18}{17} \;\square\; \frac{50}{60} - \frac{5}{6}$

$\frac{\cancel{17}}{\cancel{18}} \cdot \frac{\cancel{18}}{\cancel{17}} \;\square\; \frac{50}{60} - \frac{50}{60}$

$1 \;\square\; 0$

$1 > 0$

Since $1 > 0$, $\frac{17}{18} \cdot \frac{18}{17} > \frac{50}{60} - \frac{5}{6}$.

109. $\frac{8}{13} \div \frac{8}{13} \;\square\; |-1|$

$\frac{8}{13} \cdot \frac{13}{8} \;\square\; 1$

$1 \;\square\; 1$

$1 = 1$

Since $1 = 1$, $\frac{8}{13} \div \frac{8}{13} = |-1|$.

110. $|-2| \;\square\; \frac{4}{17} \div \frac{4}{17}$

$2 \;\square\; \frac{4}{17} \cdot \frac{17}{4}$

$2 \;\square\; 1$

$2 > 1$

Since $2 > 1$, $|-2| > \frac{4}{17} \div \frac{4}{17}$.

111.
$$\begin{aligned} 8^2 - 16 \div 2^2 \cdot 4 - 3 &= 64 - 16 \div 4 \cdot 4 - 3 \\ &= 64 - 4 \cdot 4 - 3 \\ &= 64 - 16 - 3 \\ &= 48 - 3 \\ &= 45 \end{aligned}$$

112.
$$\begin{aligned} 10^2 - 100 \div 5^2 \cdot 2 - 3 &= 100 - 100 \div 25 \cdot 2 - 3 \\ &= 100 - 4 \cdot 2 - 3 \\ &= 100 - 8 - 3 \\ &= 92 - 3 \\ &= 89 \end{aligned}$$

113.
$$\begin{aligned} \frac{5 \cdot 2 - 3^2}{[3^2 - (-2)]^2} &= \frac{5 \cdot 2 - 9}{[9 - (-2)]^2} \\ &= \frac{10 - 9}{[9 + 2]^2} \\ &= \frac{10 - 9}{11^2} \\ &= \frac{1}{121} \end{aligned}$$

114.
$$\begin{aligned} \frac{10 \div 2 + 3 \cdot 4}{(12 - 3 \cdot 2)^2} &= \frac{5 + 12}{(12 - 6)^2} \\ &= \frac{17}{6^2} \\ &= \frac{17}{36} \end{aligned}$$

115.
$$\begin{aligned} 8 - 3[-2(2 - 5) - 4(8 - 6)] &= 8 - 3[-2(-3) - 4(2)] \\ &= 8 - 3[6 - 8] \\ &= 8 - 3[-2] \\ &= 8 + 6 \\ &= 14 \end{aligned}$$

116.
$$\begin{aligned} 8 - 3[-2(5 - 7) - 5(4 - 2)] &= 8 - 3[-2(-2) - 5(2)] \\ &= 8 - 3[4 - 10] \\ &= 8 - 3[-6] \\ &= 8 + 18 \\ &= 26 \end{aligned}$$

117.
$$\begin{aligned} \frac{2(-2) - 4(-3)}{5 - 8} &= \frac{-4 + 12}{-3} \\ &= \frac{8}{-3} \\ &= -\frac{8}{3} \end{aligned}$$

118. $\dfrac{6(-4)-5(-3)}{9-10}=\dfrac{-24+15}{-1}$
$=\dfrac{-9}{-1}$
$=9$

119. $\dfrac{(5-6)^2-2|3-7|}{89-3\cdot 5^2}=\dfrac{(-1)^2-2|-4|}{89-3\cdot 25}$
$=\dfrac{1-2(4)}{89-75}$
$=\dfrac{1-8}{14}$
$=\dfrac{-7}{14}$
$=-\dfrac{1}{2}$

120. $\dfrac{12\div 3\cdot 5|2^2+3^2|}{7+3-6^2}=\dfrac{12\div 3\cdot 5|4+9|}{7+3-36}$
$=\dfrac{4\cdot 5|13|}{10-36}$
$=\dfrac{20(13)}{-26}$
$=\dfrac{260}{-26}$
$=-10$

121. $x-(x+4)=x-x-4=-4$

122. $x-(8-x)=x-8+x=2x-8$

123. $6(-5x)=-30x$

124. $10(-4x)=-40x$

125. $5x-2x=3x$

126. $6x-(-2x)=6x+2x=8x$

127. $8x-(3x+6)=8x-3x-6=5x-6$

128. $8-3(x+6)=8-3x-18=-3x-10$

129. a. $H=\dfrac{7}{10}(220-a)$
$H=\dfrac{7}{10}(220-20)$
$=\dfrac{7}{10}(200)$
$=140$
The lower limit of the heart rate for a 20-year-old with this exercise goal is 140 beats per minute.

b. $H=\dfrac{4}{5}(220-a)$
$H=\dfrac{4}{5}(220-20)$
$=\dfrac{4}{5}(200)$
$=160$
The upper limit of the heart rate for a 20-year-old with this exercise goal is 160 beats per minute.

130. a. $H=\dfrac{1}{2}(220-a)$
$H=\dfrac{1}{2}(220-30)$
$=\dfrac{1}{2}(190)$
$=95$
The lower limit of the heart rate for a 30-year-old with this exercise goal is 95 beats per minute.

b. $H=\dfrac{3}{5}(220-a)$
$H=\dfrac{3}{5}(220-30)$
$=\dfrac{3}{5}(190)$
$=114$
The upper limit of the heart rate for a 30-year-old with this exercise goal is 114 beats per minute.

131. a. $T=21x^2+862x+15,552$
$=21(14)^2+862(14)+15,552$
$=31,736$
The formula estimates the cost to have been \$31,736 in 2014.

b. This overestimates the value in the graph by \$35.

c. $T = 21x^2 + 862x + 15{,}552$
$= 21(20)^2 + 862(20) + 15{,}552$
$= 41{,}192$
The formula projects the cost to be \$41,192 in 2020.

132. a. $T = 21x^2 + 862x + 15{,}552$
$= 21(12)^2 + 862(12) + 15{,}552$
$= 28{,}920$
The formula estimates the cost to have been \$28,920 in 2012.

b. This underestimates the value in the graph by \$136.

c. $T = 21x^2 + 862x + 15{,}552$
$= 21(22)^2 + 862(22) + 15{,}552$
$= 44{,}680$
The formula projects the cost to be \$44,680 in 2022.

133. a. $0.05x + 0.12(10{,}000 - x)$
$= 0.05x + 1200 - 0.12x$
$= 1200 - 0.07x$

b. $1200 - 0.07x = 1200 - 0.07(6000)$
$= \$780$

134. a. $0.06t + 0.5(50 - t) = 0.06t + 25 - 0.5t$
$= 25 - 0.44t$

b. $0.06(20) + 0.5(50 - 20)$
$= 1.2 + 0.5(30)$
$= 1.2 + 15$
$= 16.2$ miles

135. – 143. Answers will vary.

144. does not make sense; Explanations will vary. Sample explanation: Models do not always accurately predict future values.

145. does not make sense; Explanations will vary. Sample explanation: To use the model, substitute 0 for x.

146. makes sense

147. does not make sense; Explanations will vary. Sample explanation: The commutative property changes order and the associative property changes groupings.

148. false; Changes to make the statement true will vary. A sample change is: Some rational numbers are not integers.

149. false; Changes to make the statement true will vary. A sample change is: All whole numbers are integers.

150. true

151. false; Changes to make the statement true will vary. A sample change is: Some irrational numbers are negative.

152. false; Changes to make the statement true will vary. A sample change is: The term x has a coefficient of 1.

153. false; Changes to make the statement true will vary. A sample change is:
$5 + 3(x - 4) = 5 + 3x - 12 = 3x - 7.$

154. false; Changes to make the statement true will vary. A sample change is: $-x - x = -2x.$

155. true

156. $\sqrt{2} \approx 1.4$
$1.4 < 1.5$
$\sqrt{2} < 1.5$

157. $-\pi > -3.5$

158. $-\dfrac{3.14}{2} = -1.57$
$-\dfrac{\pi}{2} \approx -1.571$
$-1.57 > -1.571$
$-\dfrac{3.14}{2} > -\dfrac{\pi}{2}$

159. a. $b^4 \cdot b^3 = (b \cdot b \cdot b \cdot b)(b \cdot b \cdot b) = b^7$

b. $b^5 \cdot b^5 = (b \cdot b \cdot b \cdot b \cdot b)(b \cdot b \cdot b \cdot b \cdot b) = b^{10}$

c. add the exponents

160. **a.** $\frac{b^7}{b^3} = \frac{b \cdot b \cdot b \cdot b \cdot b \cdot b \cdot b}{b \cdot b \cdot b} = b^4$

b. $\frac{b^8}{b^2} = \frac{b \cdot b \cdot b \cdot b \cdot b \cdot b \cdot b \cdot b}{b \cdot b} = b^6$

c. subtract the exponents

161. $6.2 \times 10^3 = 6.2 \times 10 \times 10 \times 10 = 6200$
It moves the decimal point 3 places to the right.

Section P.2

Check Point Exercises

1. **a.** $3^3 \cdot 3^2 = 3^{3+2} = 3^5$ or 243

b. $\left(4x^3y^4\right)\left(10x^2y^6\right) = 4 \cdot 10 \cdot x^3 \cdot x^2 \cdot y^4 \cdot y^6$
$= 40x^{3+2} \cdot y^{4+6}$
$= 40x^5y^{10}$

2. **a.** $\frac{(-3)^6}{(-3)^3} = (-3)^3 = -27$

b. $\frac{27x^{14}y^8}{3x^3y^5} = \frac{27}{3} \cdot \frac{x^{14}}{x^3} \cdot \frac{y^8}{y^5} = 9x^{14-3}y^{8-5} = 9x^{11}y^3$

3. **a.** $5^{-2} = \frac{1}{5^2} = \frac{1}{25}$

b. $(-3)^{-3} = \frac{1}{(-3)^3} = \frac{1}{-27} = -\frac{1}{27}$

c. $\frac{1}{4^{-2}} = \frac{1}{\frac{1}{4^2}} = 1 \cdot \frac{4^2}{1} = 4^2 = 16$

d. $3x^{-6}y^4 = 3 \cdot \frac{1}{x^6} \cdot y^4 = \frac{3y^4}{x^6}$

4. **a.** $\left(3^3\right)^2 = 3^{3 \cdot 2} = 3^6$ or 729

b. $\left(y^7\right)^{-2} = y^{7(-2)} = y^{-14} = \frac{1}{y^{14}}$

c. $\left(b^{-3}\right)^{-4} = b^{-3(-4)} = b^{12}$

5. $(-4x)^3 = (-4)^3(x)^3 = -64x^3$

6. **a.** $\left(-\frac{2}{y}\right)^5 = \frac{(-2)^5}{y^5} = \frac{-32}{y^5}$

b. $\left(\frac{x^5}{3}\right)^3 = \frac{\left(x^5\right)^3}{3^3} = \frac{x^{15}}{27}$

7. **a.** $\left(2x^3y^6\right)^4 = (2)^4\left(x^3\right)^4\left(y^6\right)^4 = 16x^{12}y^{24}$

b. $\left(-6x^2y^5\right)\left(3xy^3\right) = (-6) \cdot 3 \cdot x^2 \cdot x \cdot y^5 \cdot y^3$
$= -18x^3y^8$

c. $\frac{100x^{12}y^2}{20x^{16}y^{-4}} = \left(\frac{100}{20}\right)\left(\frac{x^{12}}{x^{16}}\right)\left(\frac{y^2}{y^{-4}}\right)$
$= 5x^{12-16}y^{2-(-4)}$
$= 5x^{-4}y^6$
$= \frac{5y^6}{x^4}$

d. $\left(\frac{5x}{y^4}\right)^{-2} = \frac{(5)^{-2}(x)^{-2}}{\left(y^4\right)^{-2}}$
$= \frac{(5)^{-2}(x)^{-2}}{\left(y^4\right)^{-2}}$
$= \frac{5^{-2}x^{-2}}{y^{-8}}$
$= \frac{y^8}{5^2x^2}$
$= \frac{y^8}{25x^2}$

8. **a.** $-2.6 \times 10^9 = -2{,}600{,}000{,}000$

b. $3.017 \times 10^{-6} = 0.000003017$

9. **a.** $5{,}210{,}000{,}000 = 5.21 \times 10^9$

b. $-0.00000006893 = -6.893 \times 10^{-8}$

10. $410\times10^7 = \left(4.1\times10^2\right)\times10^7$
$= 4.1\times\left(10^2\times10^7\right)$
$= 4.1\times10^9$

11 a. $\left(7.1\times10^5\right)\left(5\times10^{-7}\right)$
$= 7.1\cdot5\times10^5\cdot10^{-7}$
$= 35.5\times10^{-2}$
$= \left(3.55\times10^1\right)\times10^{-2}$
$= 3.55\times\left(10^1\times10^{-2}\right)$
$= 3.55\times10^{-1}$

b. $\frac{1.2\times10^6}{3\times10^{-3}} = \frac{1.2}{3}\cdot\frac{10^6}{10^{-3}}$
$= 0.4\times10^{6-(-3)}$
$= 0.4\times10^9$
$= 4\times10^8$

12. $\frac{4.08\times10^{10}}{680{,}000} = \frac{4.08\times10^{10}}{6.8\times10^5} = \frac{4.08}{6.8}\cdot\frac{10^{10}}{10^5}$
$= 0.6\times10^5$
$= 60{,}000$
The average salary was $60,000 per U.S. police officer.

Concept and Vocabulary Check P.2

1. b^{m+n}; add

2. b^{m-n}; subtract

3. 1

4. $\frac{1}{b^n}$

5. false

6. b^n

7. true

8. a number greater than or equal to 1 and less than 10; integer

9. true

10. false

Exercise Set P.2

1. $5^2\cdot2=(5\cdot5)\cdot2=25\cdot2=50$

2. $6^2\cdot2=(6\cdot6)\cdot2=36\cdot2=72$

3. $(-2)^6=(-2)(-2)(-2)(-2)(-2)(-2)=64$

4. $(-2)^4=(-2)(-2)(-2)(-2)=16$

5. $-2^6=-2\cdot2\cdot2\cdot2\cdot2\cdot2=-64$

6. $-2^4=-2\cdot2\cdot2\cdot2=-16$

7. $(-3)^0=1$

8. $(-9)^0=1$

9. $-3^0=-1$

10. $-9^0=-1$

11. $4^{-3}=\frac{1}{4^3}=\frac{1}{4\cdot4\cdot4}=\frac{1}{64}$

12. $2^{-6}=\frac{1}{2^6}=\frac{1}{2\cdot2\cdot2\cdot2\cdot2\cdot2}=\frac{1}{64}$

13. $2^2\cdot2^3=2^{2+3}=2^5=2\cdot2\cdot2\cdot2\cdot2=32$

14. $3^3\cdot3^2=3^{3+2}=3^5=3\cdot3\cdot3\cdot3\cdot3=243$

15. $(2^2)^3=2^{2\cdot3}=2^6=2\cdot2\cdot2\cdot2\cdot2\cdot2=64$

16. $(3^3)^2=3^{3\cdot2}=3^6=3\cdot3\cdot3\cdot3\cdot3\cdot3=729$

17. $\frac{2^8}{2^4}=2^{8-4}=2^4=2\cdot2\cdot2\cdot2=16$

18. $\frac{3^8}{3^4}=3^{8-4}=3^4=3\cdot3\cdot3\cdot3=81$

19. $3^{-3}\cdot3=3^{-3+1}=3^{-2}=\frac{1}{3^2}=\frac{1}{3\cdot3}=\frac{1}{9}$

20. $2^{-3}\cdot2=2^{-3+1}=2^{-2}=\frac{1}{2^2}=\frac{1}{2\cdot2}=\frac{1}{4}$

21. $\frac{2^3}{2^7}=2^{3-7}=2^{-4}=\frac{1}{2^4}=\frac{1}{2\cdot2\cdot2\cdot2}=\frac{1}{16}$

22. $\frac{3^4}{3^7}=3^{4-7}=3^{-3}=\frac{1}{3^3}=\frac{1}{3\cdot 3\cdot 3}=\frac{1}{27}$

23. $x^{-2}y=\frac{1}{x^2}\cdot y=\frac{y}{x^2}$

24. $xy^{-3}=x\cdot\frac{1}{y^3}=\frac{x}{y^3}$

25. $x^0y^5=1\cdot y^5=y^5$

26. $x^7\cdot y^0=x^7\cdot 1=x^7$

27. $x^3\cdot x^7=x^{3+7}=x^{10}$

28. $x^{11}\cdot x^5=x^{11+5}=x^{16}$

29. $x^{-5}\cdot x^{10}=x^{-5+10}=x^5$

30. $x^{-6}\cdot x^{12}=x^{-6+12}=x^6$

31. $(x^3)^7=x^{3\cdot 7}=x^{21}$

32. $(x^{11})^5=x^{11\cdot 5}=x^{55}$

33. $(x^{-5})^3=x^{-5\cdot 3}=x^{-15}=\frac{1}{x^{15}}$

34. $(x^{-6})^4=x^{-6\cdot 4}=x^{-24}=\frac{1}{x^{24}}$

35. $\frac{x^{14}}{x^7}=x^{14-7}=x^7$

36. $\frac{x^{30}}{x^{10}}=x^{30-10}=x^{20}$

37. $\frac{x^{14}}{x^{-7}}=x^{14-(-7)}=x^{14+7}=x^{21}$

38. $\frac{x^{30}}{x^{-10}}=x^{30-(-10)}=x^{30+10}=x^{40}$

39. $(8x^3)^2=8^2(x^3)^2=8^2x^{3\cdot 2}=64x^6$

40. $(6x^4)^2=(6)^2(x^4)^2=6^2x^{4\cdot 2}=36x^8$

41. $\left(-\frac{4}{x}\right)^3=\frac{(-4)^3}{x^3}=-\frac{64}{x^3}$

42. $\left(-\frac{6}{y}\right)^3=\frac{(-6)^3}{y^3}=-\frac{216}{y^3}$

43. $(-3x^2y^5)^2=(-3)^2(x^2)^2\cdot(y^5)^2$
$=9x^{2\cdot 2}y^{5\cdot 2}$
$=9x^4y^{10}$

44. $(-3x^4y^6)^3=(-3)^3(x^4)^3(y^6)^3$
$=-27x^{4\cdot 3}y^{6\cdot 3}$
$=-27x^{12}y^{18}$

45. $(3x^4)(2x^7)=3\cdot 2x^4\cdot x^7=6x^{4+7}=6x^{11}$

46. $(11x^5)(9x^{12})=11\cdot 9x^5x^{12}=99x^{5+12}=99x^{17}$

47. $(-9x^3y)(-2x^6y^4)=(-9)(-2)x^3x^6yy^4$
$=18x^{3+6}y^{1+4}$
$=18x^9y^5$

48. $(-5x^4y)(-6x^7y^{11})=(-5)(-6)x^4x^7yy^{11}$
$=30x^{4+7}y^{1+11}$
$=30x^{11}y^{12}$

49. $\frac{8x^{20}}{2x^4}=\left(\frac{8}{2}\right)\left(\frac{x^{20}}{x^4}\right)=4x^{20-4}=4x^{16}$

50. $\frac{20x^{24}}{10x^6}=\left(\frac{20}{10}\right)\left(\frac{x^{24}}{x^6}\right)=2x^{24-6}=2x^{18}$

51. $\frac{25a^{13}\cdot b^4}{-5a^2\cdot b^3}=\left(\frac{25}{-5}\right)\left(\frac{a^{13}}{a^2}\right)\left(\frac{b^4}{b^3}\right)$
$=-5a^{13-2}b^{4-3}$
$=-5a^{11}b$

52. $\frac{35a^{14}b^6}{-7a^7b^3}=\left(\frac{35}{-7}\right)\left(\frac{a^{14}}{a^7}\right)\left(\frac{b^6}{b^3}\right)$
$=-5a^{14-7}b^{6-3}$
$=-5a^7b^3$

53. $\frac{14b^7}{7b^{14}}=\left(\frac{14}{7}\right)\left(\frac{b^7}{b^{14}}\right)=2\cdot b^{7-14}=2b^{-7}=\frac{2}{b^7}$

54. $\frac{20b^{10}}{10b^{20}} = \left(\frac{20}{10}\right)\left(\frac{b^{10}}{b^{20}}\right)$
$= 2b^{10-20}$
$= 2b^{-10}$
$= \frac{2}{b^{10}}$

55. $(4x^3)^{-2} = (4^{-2})(x^3)^{-2}$
$= 4^{-2}x^{-6}$
$= \frac{1}{4^2x^6}$
$= \frac{1}{16x^6}$

56. $(10x^2)^{-3} = 10^{-3}x^{2\cdot(-3)}$
$= 10^{-3}x^{-6}$
$= \frac{1}{10^3x^6}$
$= \frac{1}{1000x^6}$

57. $\frac{24x^3 \cdot y^5}{32x^7y^{-9}} = \frac{3}{4}x^{3-7}y^{5-(-9)}$
$= \frac{3}{4}x^{-4}y^{14}$
$= \frac{3y^{14}}{4x^4}$

58. $\frac{10x^4y^9}{30x^{12}y^{-3}} = \frac{1}{3}x^{4-12}y^{9-(-3)}$
$= \frac{1}{3}x^{-8}y^{12}$
$= \frac{y^{12}}{3x^8}$

59. $\left(\frac{5x^3}{y}\right)^{-2} = \frac{5^{-2}x^{-6}}{y^{-2}} = \frac{y^2}{25x^6}$

60. $\left(\frac{3x^4}{y}\right)^{-3} = \left(\frac{y}{3x^4}\right)^3$
$= \frac{y^3}{3^3x^{4\cdot 3}}$
$= \frac{y^3}{27x^{12}}$

61. $\left(\frac{-15a^4b^2}{5a^{10}b^{-3}}\right)^3 = \left(\frac{-3b^{2-(-3)}}{a^{10-4}}\right)^3$
$= \left(\frac{-3b^5}{a^6}\right)^3$
$= \frac{-27b^{15}}{a^{18}}$

62. $\left(\frac{-30a^{14}b^8}{10a^{17}b^{-2}}\right)^3 = \left(\frac{-3b^{8-(-2)}}{a^{17-14}}\right)^3$
$= \left(\frac{-3b^{10}}{a^3}\right)^3$
$= \frac{-27b^{30}}{a^9}$

63. $\left(\frac{3a^{-5}b^2}{12a^3b^{-4}}\right)^0 = 1$

64. $\left(\frac{4a^{-5}b^3}{12a^3b^{-5}}\right)^0 = 1$

65. $3.8\times10^2 = 380$

66. $9.2\times10^2 = 920$

67. $6\times10^{-4} = 0.0006$

68. $7\times10^{-5} = 0.00007$

69. $-7.16\times10^6 = -7,160,000$

70. $-8.17\times10^6 = -8,170,000$

71. $7.9\times10^{-1} = 0.79$

72. $6.8\times10^{-1} = 0.68$

73. $-4.15\times10^{-3}=-0.00415$

74. $-3.14\times10^{-3}=-0.00314$

75. $-6.00001\times10^{10}=-60,000,100,000$

76. $-7.00001\times10^{10}=-70,000,100,000$

77. $32,000=3.2\times10^{4}$

78. $64,000=6.4\times10^{4}$

79. $638,000,000,000,000,000$
$=6.38\times10^{17}$

80. $579,000,000,000,000,000=5.79\times10^{17}$

81. $-5716=-5.716\times10^{3}$

82. $-3829=-3.829\times10^{3}$

83. $0.0027=2.7\times10^{-3}$

84. $0.0083=8.3\times10^{-3}$

85. $-0.00000000504=-5.04\times10^{-9}$

86. $-0.00000000405=-4.05\times10^{-9}$

87. $\left(3\times10^{4}\right)\left(2.1\times10^{3}\right)=(3\times2.1)\left(10^{4}\times10^{3}\right)$
$=6.3\times10^{4+3}=6.3\times10^{7}$

88. $\left(2\times10^{4}\right)\left(4.1\times10^{3}\right)=8.2\times10^{7}$

89. $\left(1.6\times10^{15}\right)\left(4\times10^{-11}\right)=(1.6\times4)\left(10^{15}\times10^{-11}\right)$
$=6.4\times10^{15+(-11)}$
$=6.4\times10^{4}$

90. $\left(1.4\times10^{15}\right)\left(3\times10^{-11}\right)=(1.4\times3)\left(10^{15}\times10^{-11}\right)$
$=4.2\times10^{15+(-11)}$
$=4.2\times10^{4}$

91. $\left(6.1\times10^{-8}\right)\left(2\times10^{-4}\right)=(6.1\times2)\left(10^{-8}\times10^{-4}\right)$
$=12.2\times10^{-8+(-4)}$
$=12.2\times10^{-12}$
$=1.22\times10^{-11}$

92. $\left(5.1\times10^{-8}\right)\left(3\times10^{-4}\right)=15.3\times10^{-12}$
$=1.53\times10^{-11}$

93. $\left(4.3\times10^{8}\right)\left(6.2\times10^{4}\right)$
$=(4.3\times6.2)\left(10^{8}\times10^{4}\right)$
$=26.66\times10^{8+4}$
$=26.66\times10^{12}$
$=2.666\times10^{13}\approx2.67\times10^{13}$

94. $\left(8.2\times10^{8}\right)\left(4.6\times10^{4}\right)$
$=37.72\times10^{8+4}=37.72\times10^{12}$
$=3.772\times10^{13}\approx3.77\times10^{13}$

95. $\dfrac{8.4\times10^{8}}{4\times10^{5}}=\dfrac{8.4}{4}\times\dfrac{10^{8}}{10^{5}}$
$=2.1\times10^{8-5}=2.1\times10^{3}$

96. $\dfrac{6.9\times10^{8}}{3\times10^{5}}=2.3\times10^{8-5}=2.3\times10^{3}$

97. $\dfrac{3.6\times10^{4}}{9\times10^{-2}}=\dfrac{3.6}{9}\times\dfrac{10^{4}}{10^{-2}}$
$=0.4\times10^{4-(-2)}$
$=0.4\times10^{6}=4\times10^{5}$

98. $\dfrac{1.2\times10^{4}}{2\times10^{-2}}=0.6\times10^{4-(-2)}=0.6\times10^{6}$
$=\left(6\times10^{-1}\right)\times10^{6}=6\times10^{5}$

99. $\dfrac{4.8\times10^{-2}}{2.4\times10^{6}}=\dfrac{4.8}{2.4}\times\dfrac{10^{-2}}{10^{6}}$
$=2\times10^{-2-6}=2\times10^{-8}$

100. $\dfrac{7.5\times10^{-2}}{2.5\times10^{6}}=3\times10^{-2-6}=3\times10^{-8}$

101. $\dfrac{2.4\times10^{-2}}{4.8\times10^{-6}}=\dfrac{2.4}{4.8}\times\dfrac{10^{-2}}{10^{-6}}$
$=0.5\times10^{-2-(-6)}$
$=0.5\times10^{4}=5\times10^{3}$

102. $\dfrac{1.5\times10^{-2}}{5\times10^{-6}} = 0.5\times10^{-2-(-6)}$
$= 0.5\times10^{4} = 5\times10^{3}$

103. $\dfrac{480,000,000,000}{0.00012} = \dfrac{4.8\times10^{11}}{1.2\times10^{-4}}$
$= \dfrac{4.8}{1.2}\times\dfrac{10^{11}}{10^{-4}}$
$= 4\times10^{11-(-4)}$
$= 4\times10^{15}$

104. $\dfrac{282,000,000,000}{0.00141} = \dfrac{2.82\times10^{11}}{1.41\times10^{-3}}$
$= 2\times10^{11-(-3)}$
$= 2\times10^{14}$

105. $\dfrac{0.00072\times0.003}{0.00024}$
$= \dfrac{\left(7.2\times10^{-4}\right)\left(3\times10^{-3}\right)}{2.4\times10^{-4}}$
$= \dfrac{7.2\times3}{2.4}\times\dfrac{10^{-4}\cdot10^{-3}}{10^{-4}} = 9\times10^{-3}$

106. $\dfrac{66000\times0.001}{0.003\times0.002} = \dfrac{\left(6.6\times10^{4}\right)\left(1\times10^{-3}\right)}{\left(3\times10^{-3}\right)\left(2\times10^{-3}\right)}$
$= \dfrac{6.6\times10^{1}}{6\times10^{-6}} = 1.1\times10^{1-(-6)}$
$= 1.1\times10^{7}$

107. $\dfrac{\left(x^{-2}y\right)^{-3}}{\left(x^{2}y^{-1}\right)^{3}} = \dfrac{x^{6}y^{-3}}{x^{6}y^{-3}}$
$= x^{6-6}y^{-3-(-3)} = x^{0}y^{0} = 1$

108. $\dfrac{\left(xy^{-2}\right)^{-2}}{\left(x^{-2}y\right)^{-3}} = \dfrac{x^{-2}y^{4}}{x^{6}y^{-3}}$
$= x^{-2-6}y^{4-(-3)} = x^{-8}y^{7} = \dfrac{y^{7}}{x^{8}}$

109. $\left(2x^{-3}yz^{-6}\right)\left(2x\right)^{-5} = 2x^{-3}yz^{-6}\cdot2^{-5}x^{-5}$
$= 2^{-4}x^{-8}yz^{-6} = \dfrac{y}{2^{4}x^{8}z^{6}} = \dfrac{y}{16x^{8}z^{6}}$

110. $\left(3x^{-4}yz^{-7}\right)\left(3x\right)^{-3} = 3x^{-4}yz^{-7}\cdot3^{-3}x^{-3}$
$= 3^{-2}x^{-7}yz^{-7} = \dfrac{y}{3^{2}x^{7}z^{7}} = \dfrac{y}{9x^{7}z^{7}}$

111. $\left(\dfrac{x^{3}y^{4}z^{5}}{x^{-3}y^{-4}z^{-5}}\right)^{-2} = \left(x^{6}y^{8}z^{10}\right)^{-2}$
$= x^{-12}y^{-16}z^{-20} = \dfrac{1}{x^{12}y^{16}z^{20}}$

112. $\left(\dfrac{x^{4}y^{5}z^{6}}{x^{-4}y^{-5}z^{-6}}\right)^{-4} = \left(x^{8}y^{10}z^{12}\right)^{-4}$
$= x^{-32}y^{-40}z^{-48} = \dfrac{1}{x^{32}y^{40}z^{48}}$

113. $\dfrac{\left(2^{-1}x^{-2}y^{-1}\right)^{-2}\left(2x^{-4}y^{3}\right)^{-2}\left(16x^{-3}y^{3}\right)^{0}}{\left(2x^{-3}y^{-5}\right)^{2}}$
$= \dfrac{\left(2^{2}x^{2}y^{2}\right)\left(2^{-2}x^{8}y^{-6}\right)(1)}{\left(2^{2}x^{-6}y^{-10}\right)}$
$= \dfrac{x^{18}y^{6}}{4}$

114. $\dfrac{\left(2^{-1}x^{-3}y^{-1}\right)^{-2}\left(2x^{-6}y^{4}\right)^{-2}\left(9x^{3}y^{-3}\right)^{0}}{\left(2x^{-4}y^{-6}\right)^{2}}$
$= \dfrac{\left(2^{2}x^{6}y^{2}\right)\left(2^{-2}x^{12}y^{-8}\right)(1)}{\left(2^{2}x^{-8}y^{-12}\right)}$
$= \dfrac{x^{26}y^{6}}{4}$

115. **a.** 3.18×10^{12}

b. 3.20×10^{8}

c. $\dfrac{3.18\times10^{12}}{3.20\times10^{8}} = \dfrac{3.18}{3.20}\times\dfrac{10^{12}}{10^{8}}$
$\approx 0.9938\times10^{4}$
≈ 9938
$9938 per American

116. **a.** 3.02×10^{12}

b. 3.19×10^{8}

c. $\frac{3.02\times10^{12}}{3.19\times10^{8}}=\frac{3.02}{3.19}\times\frac{10^{12}}{10^{8}}$
$\approx 0.9467\times10^{4}$
≈ 9467
$9467 per American

117. a. 1.89×10^{13}

b. 6×10^{4}

c. $\frac{1.89\times10^{13}}{6\times10^{4}}=\frac{1.89}{6}\times\frac{10^{13}}{10^{4}}$
$=0.315\times10^{9}$
$=3.15\times10^{8}$
$=315,000,000$
315,00,000 Americans

118. a. 1.89×10^{13}

b. 2.54×10^{11}

c. $\frac{1.89\times10^{13}}{2.54\times10^{11}}=\frac{1.89}{2.54}\times\frac{10^{13}}{10^{11}}$
$\approx 0.74\times10^{2}$
≈ 74
approximately 74 years

119. a. 1.09×10^{12}

b. 3.2×10^{7}

c. $\frac{1.09\times10^{12}}{3.2\times10^{7}}=\frac{1.09}{3.2}\times\frac{10^{12}}{10^{7}}$
$=0.340625\times10^{5}$
$=34,062.5$
34,062.5 years

120. – 128. Answers will vary.

129. does not make sense; Explanations will vary. Sample explanation: $36(x^3)^9=36x^{27}$ not $36x^{12}$.

130. makes sense

131. does not make sense; Explanations will vary. Sample explanation: 4.6×10^{12} represents over 4 trillion. The entire world population is measured in billions (10^9).

132. makes sense

133. false; Changes to make the statement true will vary. A sample change is: $4^{-2}>4^{-3}$.

134. true

135. false; Changes to make the statement true will vary. A sample change is: $(-2)^4\neq2^{-4}$ because $16\neq\frac{1}{16}$.

136. false; Changes to make the statement true will vary. A sample change is: $5^2\cdot5^{-2}=2^5\cdot2^{-5}$.

137. false; Changes to make the statement true will vary. A sample change is: $534.7\neq5347$.

138. false; Changes to make the statement true will vary. A sample change is:
$\frac{8\times10^{30}}{2\times10^{-5}}=4\times10^{30-(-5)}=4\times10^{35}$.

139. false; Changes to make the statement true will vary. A sample change is:
$(7\times10^5)+(2\times10^{-3})=700,000.002$.

140. true

141. The doctor has gathered:
$2^{-1}+2^{-2}=\frac{1}{2}+\frac{1}{2^2}=\frac{2}{4}+\frac{1}{4}=\frac{3}{4}$
So, $1-\frac{3}{4}=\frac{1}{4}$ is remaining.

142. $b^A=MN, b^C=M, b^D=N$
$b^A=b^Cb^D$
$A=C+D$

143. $\frac{70\text{ bts}}{\cancel{\text{min}}}\cdot\frac{60\,\cancel{\text{min}}}{\cancel{\text{hr}}}\cdot\frac{24\,\cancel{\text{hrs}}}{\cancel{\text{day}}}\cdot\frac{365\,\cancel{\text{days}}}{\cancel{\text{yr}}}\cdot80\,\cancel{\text{yrs}}$
$=70\cdot60\cdot24\cdot365\cdot80$ beats
$=2943360000$ beats
$=2.94336\times10^9$ beats
$\approx2.94\times10^9$ beats
The heartbeats approximately 2.94×10^9 times over a lifetime of 80 years.

144. Answers will vary.

145. **a.** $\sqrt{16}\cdot\sqrt{4}=4\cdot 2=8$

b. $\sqrt{16\cdot 4}=\sqrt{64}=8$

c. $\sqrt{16}\cdot\sqrt{4}=\sqrt{16\cdot 4}$

146. **a.** $\sqrt{300}\approx 17.32$

b. $10\sqrt{3}\approx 17.32$

c. $\sqrt{300}=10\sqrt{3}$

147. **a.** $21x+10x=31x$

b. $21\sqrt{2}+10\sqrt{2}=31\sqrt{2}$

Section P.3

Check Point Exercises

1. **a.** $\sqrt{81}=9$

b. $-\sqrt{9}=-3$

c. $\sqrt{\frac{1}{25}}=\frac{1}{5}$

d. $\sqrt{36+64}=\sqrt{100}=10$

e. $\sqrt{36}+\sqrt{64}=6+8=14$

2. **a.** $\sqrt{75}=\sqrt{25\cdot 3}=\sqrt{25}\sqrt{3}=5\sqrt{3}$

b. $\sqrt{5x}\cdot\sqrt{10x}=\sqrt{5x\cdot 10x}$
$=\sqrt{50x^2}$
$=\sqrt{25\cdot 2x^2}$
$=\sqrt{25x^2}\cdot\sqrt{2}$
$=5x\sqrt{2}$

3. **a.** $\sqrt{\frac{25}{16}}=\frac{\sqrt{25}}{\sqrt{16}}=\frac{5}{4}$

b. $\frac{\sqrt{150x^3}}{\sqrt{2x}}=\sqrt{\frac{150x^3}{2x}}$
$=\sqrt{75x^2}$
$=\sqrt{25x^2}\cdot\sqrt{3}$
$=5x\sqrt{3}$

4. **a.** $8\sqrt{13}+9\sqrt{13}=(8+9)\sqrt{3}$
$=17\sqrt{13}$

b. $\sqrt{17x}-20\sqrt{17x}$
$=1\sqrt{17x}-20\sqrt{17x}$
$=(1-20)\sqrt{17x}$
$=-19\sqrt{17x}$

5. **a.** $5\sqrt{27}+\sqrt{12}$
$=5\sqrt{9\cdot 3}+\sqrt{4\cdot 3}$
$=5\cdot 3\sqrt{3}+2\sqrt{3}$
$=15\sqrt{3}+2\sqrt{3}$
$=(15+2)\sqrt{3}$
$=17\sqrt{3}$

b. $6\sqrt{18x}-4\sqrt{8x}$
$=6\sqrt{9\cdot 2x}-4\sqrt{4\cdot 2x}$
$=6\cdot 3\sqrt{2x}-4\cdot 2\sqrt{2x}$
$=18\sqrt{2x}-8\sqrt{2x}$
$=(18-8)\sqrt{2x}$
$=10\sqrt{2x}$

6. **a.** If we multiply numerator and denominator by $\sqrt{3}$, the denominator becomes $\sqrt{3}\cdot\sqrt{3}=\sqrt{9}=3$. Therefore, multiply by 1, choosing $\frac{\sqrt{3}}{\sqrt{3}}$ for 1.

$$\frac{5}{\sqrt{3}}=\frac{5}{\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}}=\frac{5\sqrt{3}}{\sqrt{9}}=\frac{5\sqrt{3}}{3}$$

b. The *smallest* number that will produce a perfect square in the denominator of $\frac{6}{\sqrt{12}}$ is $\sqrt{3}$ because $\sqrt{12}\cdot\sqrt{3}=\sqrt{36}=6$. So multiply by 1, choosing $\frac{\sqrt{3}}{\sqrt{3}}$ for 1.

$$\frac{6}{\sqrt{12}}=\frac{6}{\sqrt{12}}\cdot\frac{\sqrt{3}}{\sqrt{3}}=\frac{6\sqrt{3}}{\sqrt{36}}=\frac{6\sqrt{3}}{6}=\sqrt{3}$$

7. Multiply by $\frac{4-\sqrt{5}}{4-\sqrt{5}}$.

$$\frac{8}{4+\sqrt{5}} = \frac{8}{4+\sqrt{5}} \cdot \frac{4-\sqrt{5}}{4-\sqrt{5}}$$
$$= \frac{8(4-\sqrt{5})}{4^2-(\sqrt{5})^2}$$
$$= \frac{8(4-\sqrt{5})}{16-5}$$
$$= \frac{8(4-\sqrt{5})}{11} \text{ or } \frac{32-8\sqrt{5}}{11}$$

8. a. $\sqrt[3]{40} = \sqrt[3]{8\cdot 5} = \sqrt[3]{8}\cdot\sqrt[3]{5} = 2\sqrt[3]{5}$

b. $\sqrt[5]{8}\cdot\sqrt[5]{8} = \sqrt[5]{64} = \sqrt[5]{32}\cdot\sqrt[5]{2} = 2\sqrt[5]{2}$

c. $\sqrt[3]{\frac{125}{27}} = \frac{\sqrt[3]{125}}{\sqrt[3]{27}} = \frac{5}{3}$

9. $3\sqrt[3]{81} - 4\sqrt[3]{3}$
$= 3\sqrt[3]{27\cdot 3} - 4\sqrt[3]{3}$
$= 3\cdot 3\sqrt[3]{3} - 4\sqrt[3]{3}$
$= 9\sqrt[3]{3} - 4\sqrt[3]{3}$
$= (9-4)\sqrt[3]{3}$
$= 5\sqrt[3]{3}$

10. a. $25^{\frac{1}{2}} = \sqrt{25} = 5$

b. $8^{\frac{1}{3}} = \sqrt[3]{8} = 2$

c. $-81^{\frac{1}{4}} = -\sqrt[4]{81} = -3$

d. $(-8)^{\frac{1}{3}} = \sqrt[3]{-8} = -2$

e. $27^{-\frac{1}{3}} = \frac{1}{27^{\frac{1}{3}}} = \frac{1}{\sqrt[3]{27}} = \frac{1}{3}$

11. a. $27^{\frac{4}{3}} = \left(\sqrt[3]{27}\right)^4 = (3)^4 = 81$

b. $4^{\frac{3}{2}} = \left(\sqrt[2]{4}\right)^3 = (2)^3 = 8$

c. $32^{-\frac{2}{5}} = \frac{1}{32^{\frac{2}{5}}} = \frac{1}{\left(\sqrt[5]{32}\right)^2} = \frac{1}{2^2} = \frac{1}{4}$

12. a. $\left(2x^{4/3}\right)\left(5x^{8/3}\right)$
$= 2\cdot 5x^{4/3}\cdot x^{8/3}$
$= 10x^{(4/3)+(8/3)}$
$= 10x^{12/3}$
$= 10x^4$

b. $\frac{20x^4}{5x^{3/2}} = \left(\frac{20}{5}\right)\left(\frac{x^4}{x^{3/2}}\right)$
$= 4x^{4-(3/2)}$
$= 4x^{(8/2)-(3/2)}$
$= 4x^{5/2}$

13. $\sqrt[6]{x^3} = x^{3/6} = x^{1/2} = \sqrt{x}$

Concept and Vocabulary Check P.3

1. principal
2. 8^2
3. $|a|$
4. $\sqrt{a}\cdot\sqrt{b}$
5. $\frac{\sqrt{a}}{\sqrt{b}}$
6. $18\sqrt{3}$
7. 5; $6\sqrt{3}$
8. $7-\sqrt{3}$
9. $\sqrt{10}+\sqrt{2}$
10. index; radicand
11. $(-2)^5$
12. a; $|a|$
13. $\sqrt[n]{a}$
14. 2; 8

Exercise Set P.3

1. $\sqrt{36} = \sqrt{6^2} = 6$

2. $\sqrt{25} = \sqrt{5^2} = 5$

3. $-\sqrt{36} = -\sqrt{6^2} = -6$

4. $-\sqrt{25} = -\sqrt{5^2} = -5$

5. $\sqrt{-36}$, The square root of a negative number is not real.

6. $\sqrt{-25}$, The square root of a negative number is not real.

7. $\sqrt{25-16} = \sqrt{9} = 3$

8. $\sqrt{144+25} = \sqrt{169} = 13$

9. $\sqrt{25} - \sqrt{16} = 5 - 4 = 1$

10. $\sqrt{144} + \sqrt{25} = 12 + 5 = 17$

11. $\sqrt{(-13)^2} = \sqrt{169} = 13$

12. $\sqrt{(-17)^2} = \sqrt{289} = 17$

13. $\sqrt{50} = \sqrt{25 \cdot 2} = \sqrt{25}\sqrt{2} = 5\sqrt{2}$

14. $\sqrt{27} = \sqrt{9 \cdot 3} = \sqrt{9}\sqrt{3} = 3\sqrt{3}$

15.
$$\begin{aligned} \sqrt{45x^2} &= \sqrt{9x^2 \cdot 5} \\ &= \sqrt{9x^2}\sqrt{5} \\ &= \sqrt{9}\sqrt{x^2}\sqrt{5} \\ &= 3|x|\sqrt{5} \end{aligned}$$

16.
$$\begin{aligned} \sqrt{125x^2} &= \sqrt{25x^2 \cdot 5} \\ &= \sqrt{25x^2}\sqrt{5} \\ &= \sqrt{25}\sqrt{x^2}\sqrt{5} \\ &= 5|x|\sqrt{5} \end{aligned}$$

17.
$$\begin{aligned} \sqrt{2x} \cdot \sqrt{6x} &= \sqrt{2x \cdot 6x} \\ &= \sqrt{12x^2} \\ &= \sqrt{4x^2} \cdot \sqrt{3} \\ &= 2x\sqrt{3} \end{aligned}$$

18.
$$\begin{aligned} \sqrt{10x} \cdot \sqrt{8x} &= \sqrt{10x \cdot 8x} \\ &= \sqrt{80x^2} \\ &= \sqrt{16x^2} \cdot \sqrt{5} \\ &= 4x\sqrt{5} \end{aligned}$$

19. $\sqrt{x^3} = \sqrt{x^2} \cdot \sqrt{x} = x\sqrt{x}$

20. $\sqrt{y^3} = \sqrt{y^2} \cdot \sqrt{y} = y\sqrt{y}$

21.
$$\begin{aligned} \sqrt{2x^2} \cdot \sqrt{6x} &= \sqrt{2x^2 \cdot 6x} \\ &= \sqrt{12x^3} \\ &= \sqrt{4x^2} \cdot \sqrt{3x} \\ &= 2x\sqrt{3x} \end{aligned}$$

22.
$$\begin{aligned} \sqrt{6x} \cdot \sqrt{3x^2} &= \sqrt{6x \cdot 3x^2} \\ &= \sqrt{18x^3} \\ &= \sqrt{9x^2} \cdot \sqrt{2x} \\ &= 3x\sqrt{2x} \end{aligned}$$

23. $\sqrt{\dfrac{1}{81}} = \dfrac{\sqrt{1}}{\sqrt{81}} = \dfrac{1}{9}$

24. $\sqrt{\dfrac{1}{49}} = \dfrac{\sqrt{1}}{\sqrt{49}} = \dfrac{1}{7}$

25. $\sqrt{\dfrac{49}{16}} = \dfrac{\sqrt{49}}{\sqrt{16}} = \dfrac{7}{4}$

26. $\sqrt{\dfrac{121}{9}} = \dfrac{\sqrt{121}}{\sqrt{9}} = \dfrac{11}{3}$

27. $\dfrac{\sqrt{48x^3}}{\sqrt{3x}} = \sqrt{\dfrac{48x^3}{3x}} = \sqrt{16x^2} = 4x$

28. $\dfrac{\sqrt{72x^3}}{\sqrt{8x}} = \sqrt{\dfrac{72x^3}{8x}} = \sqrt{9x^2} = 3x$

29. $\dfrac{\sqrt{150x^4}}{\sqrt{3x}} = \sqrt{\dfrac{150x^4}{3x}}$
$= \sqrt{50x^3}$
$= \sqrt{25x^2} \cdot \sqrt{2x}$
$= 5x\sqrt{2x}$

30. $\dfrac{\sqrt{24x^4}}{\sqrt{3x}} = \sqrt{\dfrac{24x^4}{3x}}$
$= \sqrt{8x^3}$
$= \sqrt{4x^2} \cdot \sqrt{2x}$
$= 2x\sqrt{2x}$

31. $\dfrac{\sqrt{200x^3}}{\sqrt{10x^{-1}}} = \sqrt{\dfrac{200x^3}{10x^{-1}}}$
$= \sqrt{20x^{3-(-1)}}$
$= \sqrt{20x^4}$
$= \sqrt{4 \cdot 5x^4}$
$= 2x^2\sqrt{5}$

32. $\dfrac{\sqrt{500x^3}}{\sqrt{10x^{-1}}} = \sqrt{\dfrac{500x^3}{10x^{-1}}} = \sqrt{50x^{3-(-1)}}$
$= \sqrt{50x^4} = \sqrt{25 \cdot 2x^4} = 5x^2\sqrt{2}$

33. $7\sqrt{3} + 6\sqrt{3} = (7+6)\sqrt{3} = 13\sqrt{3}$

34. $8\sqrt{5} + 11\sqrt{5} = (8+11)\sqrt{5} = 19\sqrt{5}$

35. $6\sqrt{17x} - 8\sqrt{17x} = (6-8)\sqrt{17x} = -2\sqrt{17x}$

36. $4\sqrt{13x} - 6\sqrt{13x} = (4-6)\sqrt{13x} = -2\sqrt{13x}$

37. $\sqrt{8} + 3\sqrt{2} = \sqrt{4 \cdot 2} + 3\sqrt{2}$
$= 2\sqrt{2} + 3\sqrt{2}$
$= (2+3)\sqrt{2}$
$= 5\sqrt{2}$

38. $\sqrt{20} + 6\sqrt{5} = \sqrt{4 \cdot 5} + 6\sqrt{5}$
$= 2\sqrt{5} + 6\sqrt{5}$
$= (2+6)\sqrt{5}$
$= 8\sqrt{5}$

39. $\sqrt{50x} - \sqrt{8x} = \sqrt{25 \cdot 2x} - \sqrt{4 \cdot 2x}$
$= 5\sqrt{2x} - 2\sqrt{2x}$
$= (5-2)\sqrt{2x}$
$= 3\sqrt{2x}$

40. $\sqrt{63x} - \sqrt{28x} = \sqrt{9 \cdot 7x} - \sqrt{4 \cdot 7x}$
$= 3\sqrt{7x} - 2\sqrt{7x}$
$= (3-2)\sqrt{7x}$
$= \sqrt{7x}$

41. $3\sqrt{18} + 5\sqrt{50} = 3\sqrt{9 \cdot 2} + 5\sqrt{25 \cdot 2}$
$= 3 \cdot 3\sqrt{2} + 5 \cdot 5\sqrt{2}$
$= 9\sqrt{2} + 25\sqrt{2}$
$= (9+25)\sqrt{2}$
$= 34\sqrt{2}$

42. $4\sqrt{12} - 2\sqrt{75} = 4\sqrt{4 \cdot 3} - 2\sqrt{25 \cdot 3}$
$= 4 \cdot 2\sqrt{3} - 2 \cdot 5\sqrt{3}$
$= 8\sqrt{3} - 10\sqrt{3}$
$= (8-10)\sqrt{3}$
$= -2\sqrt{3}$

43. $3\sqrt{8} - \sqrt{32} + 3\sqrt{72} - \sqrt{75}$
$= 3\sqrt{4 \cdot 2} - \sqrt{16 \cdot 2} + 3\sqrt{36 \cdot 2} - \sqrt{25 \cdot 3}$
$= 3 \cdot 2\sqrt{2} - 4\sqrt{2} + 3 \cdot 6\sqrt{2} - 5\sqrt{3}$
$= 6\sqrt{2} - 4\sqrt{2} + 18\sqrt{2} - 5\sqrt{3}$
$= 20\sqrt{2} - 5\sqrt{3}$

44. $3\sqrt{54} - 2\sqrt{24} - \sqrt{96} + 4\sqrt{63}$
$= 3\sqrt{9 \cdot 6} - 2\sqrt{4 \cdot 6} - \sqrt{16 \cdot 6} + 4\sqrt{9 \cdot 7}$
$= 3 \cdot 3\sqrt{6} - 2 \cdot 2\sqrt{6} - 4\sqrt{6} + 4 \cdot 3\sqrt{7}$
$= 9\sqrt{6} - 4\sqrt{6} - 4\sqrt{6} + 12\sqrt{7}$
$= \sqrt{6} + 12\sqrt{7}$

45. $\dfrac{1}{\sqrt{7}} = \dfrac{1}{\sqrt{7}} \cdot \dfrac{\sqrt{7}}{\sqrt{7}} = \dfrac{\sqrt{7}}{7}$

46. $\dfrac{2}{\sqrt{10}} = \dfrac{2}{\sqrt{10}} \cdot \dfrac{\sqrt{10}}{\sqrt{10}} = \dfrac{2\sqrt{10}}{10} = \dfrac{\sqrt{10}}{5}$

47. $\dfrac{\sqrt{2}}{\sqrt{5}} = \dfrac{\sqrt{2}}{\sqrt{5}} \cdot \dfrac{\sqrt{5}}{\sqrt{5}} = \dfrac{\sqrt{10}}{5}$

48. $\frac{\sqrt{7}}{\sqrt{3}}=\frac{\sqrt{7}}{\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}}=\frac{\sqrt{21}}{3}$

49. $\frac{13}{3+\sqrt{11}}=\frac{13}{3+\sqrt{11}}\cdot\frac{3-\sqrt{11}}{3-\sqrt{11}}$

$=\frac{13(3-\sqrt{11})}{3^2-(\sqrt{11})^2}$

$=\frac{13(3-\sqrt{11})}{9-11}$

$=\frac{13(3-\sqrt{11})}{-2}$

50. $\frac{3}{3+\sqrt{7}}=\frac{3}{3+\sqrt{7}}\cdot\frac{3-\sqrt{7}}{3-\sqrt{7}}$

$=\frac{3(3-\sqrt{7})}{3^2-(\sqrt{7})^2}$

$=\frac{3(3-\sqrt{7})}{9-7}$

$=\frac{3(3-\sqrt{7})}{2}$

51. $\frac{7}{\sqrt{5}-2}=\frac{7}{\sqrt{5}-2}\cdot\frac{\sqrt{5}+2}{\sqrt{5}+2}$

$=\frac{7(\sqrt{5}+2)}{(\sqrt{5})^2-2^2}$

$=\frac{7(\sqrt{5}+2)}{5-4}$

$=7(\sqrt{5}+2)$

52. $\frac{5}{\sqrt{3}-1}=\frac{5}{\sqrt{3}-1}\cdot\frac{\sqrt{3}+1}{\sqrt{3}+1}$

$=\frac{5(\sqrt{3}+1)}{(\sqrt{3})^2-1^2}$

$=\frac{5(\sqrt{3}+1)}{3-1}$

$=\frac{5(\sqrt{3}+1)}{2}$

53. $\frac{6}{\sqrt{5}+\sqrt{3}}=\frac{6}{\sqrt{5}+\sqrt{3}}\cdot\frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}-\sqrt{3}}$

$=\frac{6(\sqrt{5}-\sqrt{3})}{(\sqrt{5})^2-(\sqrt{3})^2}$

$=\frac{6(\sqrt{5}-\sqrt{3})}{5-3}$

$=\frac{6(\sqrt{5}-\sqrt{3})}{2}$

$=3(\sqrt{5}-\sqrt{3})$

54. $\frac{11}{\sqrt{7}-\sqrt{3}}=\frac{11}{\sqrt{7}-\sqrt{3}}\cdot\frac{\sqrt{7}+\sqrt{3}}{\sqrt{7}+\sqrt{3}}$

$=\frac{11(\sqrt{7}+\sqrt{3})}{(\sqrt{7})^2-(\sqrt{3})^2}$

$=\frac{11(\sqrt{7}+\sqrt{3})}{7-3}$

$=\frac{11(\sqrt{7}+\sqrt{3})}{4}$

55. $\sqrt[3]{125}=\sqrt[3]{5^3}=5$

56. $\sqrt[3]{8}=\sqrt[3]{2^3}=2$

57. $\sqrt[3]{-8}=\sqrt[3]{(-2)^3}=-2$

58. $\sqrt[3]{-125}=\sqrt[3]{(-5)^3}=-5$

59. $\sqrt[4]{-16}$ is not a real number.

60. $\sqrt[4]{-81}$ is not a real number.

61. $\sqrt[4]{(-3)^4}=|-3|=3$

62. $\sqrt[4]{(-2)^4}=|-2|=2$

63. $\sqrt[5]{(-3)^5}=-3$

64. $\sqrt[5]{(-2)^5}=-2$

65. $\sqrt[5]{-\frac{1}{32}}=\sqrt[5]{-\frac{1}{2^5}}=-\frac{1}{2}$

66. $\sqrt[6]{\frac{1}{64}} = \frac{\sqrt[6]{1}}{\sqrt[6]{2^6}} = \frac{1}{2}$

67. $\sqrt[3]{32} = \sqrt[3]{8 \cdot 4} = \sqrt[3]{8}\,\sqrt[3]{4} = 2 \cdot \sqrt[3]{4}$

68. $\sqrt[3]{150}$ cannot be simplified further.

69. $\sqrt[3]{x^4} = \sqrt[3]{x^3 \cdot x} = x \cdot \sqrt[3]{x}$

70. $\sqrt[3]{x^5} = \sqrt[3]{x^3 x^2} = x\sqrt[3]{x^2}$

71. $\sqrt[3]{9} \cdot \sqrt[3]{6} = \sqrt[3]{54} = \sqrt[3]{27 \cdot 2} = \sqrt[3]{27}\sqrt[3]{2} = 3\sqrt[3]{2}$

72. $\sqrt[3]{12} \cdot \sqrt[3]{4} = \sqrt[3]{48} = \sqrt[3]{8 \cdot 6} = 2\sqrt[3]{6}$

73. $\frac{\sqrt[5]{64x^6}}{\sqrt[5]{2x}} = \sqrt[5]{\frac{64x^6}{2x}} = \sqrt[5]{32x^5} = 2x$

74. $\frac{\sqrt[4]{162x^5}}{\sqrt[4]{2x}} = \sqrt[4]{\frac{162x^5}{2x}} = \sqrt[4]{81x^4} = 3x$

75. $4\sqrt[5]{2} + 3\sqrt[5]{2} = 7\sqrt[5]{2}$

76. $6\sqrt[5]{3} + 2\sqrt[5]{3} = 8\sqrt[5]{3}$

77. $5\sqrt[3]{16} + \sqrt[3]{54} = 5\sqrt[3]{8 \cdot 2} + \sqrt[3]{27 \cdot 2}$
$= 5 \cdot 2\sqrt[3]{2} + 3\sqrt[3]{2}$
$= 10\sqrt[3]{2} + 3\sqrt[3]{2}$
$= 13\sqrt[3]{2}$

78. $3\sqrt[3]{24} + \sqrt[3]{81} = \sqrt[3]{8 \cdot 3} + \sqrt[3]{27 \cdot 3}$
$= 3 \cdot 2\sqrt[3]{3} + 3\sqrt[3]{3}$
$= 6\sqrt[3]{3} + 3\sqrt[3]{3}$
$= 9\sqrt[3]{3}$

79. $\sqrt[3]{54xy^3} - y\sqrt[3]{128x}$
$= \sqrt[3]{27 \cdot 2xy^3} - y\sqrt[3]{64 \cdot 2x}$
$= 3y\sqrt[3]{2x} - 4y\sqrt[3]{2x}$
$= -y\sqrt[3]{2x}$

80. $\sqrt[3]{24xy^3} - y\sqrt[3]{81x}$
$= \sqrt[3]{8 \cdot 3xy^3} - y\sqrt[3]{27 \cdot 3x}$
$= 2y\sqrt[3]{3x} - 3y\sqrt[3]{3x}$
$= -y\sqrt[3]{3x}$

81. $\sqrt{2} + \sqrt[3]{8} = \sqrt{2} + 2$

82. $\sqrt{3} + \sqrt[3]{15}$ will not simplify.

83. $36^{1/2} = \sqrt{36} = 6$

84. $121^{1/2} = \sqrt{121} = 11$

85. $8^{1/3} = \sqrt[3]{8} = 2$

86. $27^{1/3} = \sqrt[3]{27} = 3$

87. $125^{2/3} = \left(\sqrt[3]{125}\right)^2 = 5^2 = 25$

88. $8^{2/3} = \left(\sqrt[3]{8}\right)^2 = 4$

89. $32^{-4/5} = \frac{1}{32^{4/5}} = \frac{1}{2^4} = \frac{1}{16}$

90. $16^{-5/2} = \frac{1}{16^{5/2}} = \frac{1}{(\sqrt{16})^5} = \frac{1}{4^5} = \frac{1}{1024}$

91. $\left(7x^{1/3}\right)\left(2x^{1/4}\right) = 7 \cdot 2x^{1/3} \cdot x^{1/4}$
$= 14 \cdot x^{1/3+1/4}$
$= 14x^{7/12}$

92. $(3x^{2/3})(4x^{3/4}) = 3 \cdot 4x^{2/3} \cdot x^{3/4}$
$= 12 \cdot x^{2/3+3/4}$
$= 12x^{17/12}$

93. $\frac{20x^{1/2}}{5x^{1/4}} = \left(\frac{20}{5}\right)\left(\frac{x^{1/2}}{x^{1/4}}\right)$
$= 4 \cdot x^{1/2-1/4}$
$= 4x^{1/4}$

94. $\frac{72x^{3/4}}{9x^{1/3}} = \left(\frac{72}{9}\right)\left(\frac{x^{3/4}}{x^{1/3}}\right) = 8 \cdot x^{3/4-1/3} = 8x^{5/12}$

95. $\left(x^{2/3}\right)^3 = x^{2/3\cdot 3} = x^2$

96. $(x^{4/5})^5 = x^{4/5\cdot 5} = x^4$

97. $(25x^4y^6)^{1/2} = 25^{1/2}x^{4\cdot 1/2}y^{6\cdot 1/2} = 5x^2|y|^3$

98. $(125x^9y^6)^{1/3} = 125^{1/3}x^{9/3}y^{6/3} = 5x^3y^2$

99. $\dfrac{\left(3y^{\frac{1}{4}}\right)^3}{y^{\frac{1}{12}}} = \dfrac{27y^{\frac{3}{4}}}{y^{\frac{1}{12}}} = 27y^{\frac{3}{4}-\frac{1}{12}}$

$= 27y^{\frac{8}{12}} = 27y^{\frac{2}{3}}$

100. $\dfrac{\left(2y^{1/5}\right)^4}{y^{3/10}} = \dfrac{2^4\left(y^{1/5}\right)^4}{y^{3/10}}$

$= \dfrac{16y^{4/5}}{y^{3/10}} = 16y^{4/5-3/10} = 16y^{1/2}$

101. $\sqrt[4]{5^2} = 5^{2/4} = 5^{1/2} = \sqrt{5}$

102. $\sqrt[4]{7^2} = 7^{2/4} = 7^{1/2} = \sqrt{7}$

103. $\sqrt[3]{x^6} = x^{6/3} = x^2$

104. $\sqrt[4]{x^{12}} = x^{12/4} = |x|^3$

105. $\sqrt[6]{x^4} = \sqrt[6/2]{x^{4/2}} = \sqrt[3]{x^2}$

106. $\sqrt[9]{x^6} = \sqrt[9/3]{x^{6/3}} = \sqrt[3]{x^2}$

107. $\sqrt[9]{x^6y^3} = x^{\frac{6}{9}}y^{\frac{3}{9}} = x^{\frac{2}{3}}y^{\frac{1}{3}} = \sqrt[3]{x^2y}$

108. $\sqrt[12]{x^4y^8} = |x|^{\frac{4}{12}}|y|^{\frac{8}{12}} = |x|^{\frac{1}{3}}|y|^{\frac{2}{3}} = \sqrt[3]{|x|y^2}$

109. $\sqrt[3]{\sqrt[4]{16} + \sqrt{625}} = \sqrt[3]{2+25} = \sqrt[3]{27} = 3$

110. $\sqrt[3]{\sqrt{\sqrt{169}+\sqrt{9}} + \sqrt{\sqrt[3]{1000}+\sqrt[3]{216}}}$

$= \sqrt[3]{\sqrt{13+3} + \sqrt{10+6}}$

$= \sqrt[3]{\sqrt{16} + \sqrt{16}}$

$= \sqrt[3]{4+4} = \sqrt[3]{8}$

$= 2$

111. $\left(49x^{-2}y^4\right)^{-1/2}\left(xy^{1/2}\right)$

$= (49)^{-1/2}\left(x^{-2}\right)^{-1/2}\left(y^4\right)^{-1/2}\left(xy^{1/2}\right)$

$= \dfrac{1}{49^{1/2}}x^{(-2)(-1/2)}y^{(4)(-1/2)}\left(xy^{1/2}\right)$

$= \dfrac{1}{7}x^1y^{-2}\cdot xy^{1/2} = \dfrac{1}{7}x^{1+1}y^{-2+(1/2)}$

$= \dfrac{1}{7}x^2y^{-3/2} = \dfrac{x^2}{7y^{3/2}}$

112. $\left(8x^{-6}y^3\right)^{1/3}\left(x^{5/6}y^{-1/3}\right)^6$

$= 8^{1/3}x^{(-6)(1/3)}y^{(3)(1/3)}x^{(5/6)(6)}y^{(-1/3)(6)}$

$= 2x^{-2}y^1x^5y^{-2} = 2x^{-2+5}y^{1+(-2)}$

$= 2x^3y^{-1} = \dfrac{2x^3}{y}$

113. $\left(\dfrac{x^{-5/4}y^{1/3}}{x^{-3/4}}\right)^{-6} = \left(x^{(-5/4)-(-3/4)}y^{1/3}\right)^{-6}$

$= \left(x^{-2/4}y^{1/3}\right)^{-6} = x^{(-2/4)(-6)}y^{(1/3)(-6)}$

$= x^3y^{-2} = \dfrac{x^3}{y^2}$

114. $\left(\dfrac{x^{1/2}y^{-7/4}}{y^{-5/4}}\right)^{-4} = \left(x^{1/2}y^{(-7/4)-(-5/4)}\right)^{-4}$

$= \left(x^{1/2}y^{-2/4}\right)^{-4} = x^{(1/2)(-4)}y^{(-2/4)(-4)}$

$= x^{-2}y^2 = \dfrac{y^2}{x^2}$

115. The message is "Paige Fox is bad at math."

116. a. For 2030: $E = 5.8\sqrt{x} + 56.4$

$= 5.8\sqrt{10} + 56.4$

For 2060: $E = 5.8\sqrt{x} + 56.4$

$= 5.8\sqrt{40} + 56.4$

$= 5.8 \cdot 2\sqrt{10} + 56.4$

$= 11.6\sqrt{10} + 56.4$

Difference:

$(11.6\sqrt{10} + 56.4) - (5.8\sqrt{10} + 56.4)$

$= 11.6\sqrt{10} + 56.4 - 5.8\sqrt{10} - 56.4$

$= 11.6\sqrt{10} - 5.8\sqrt{10} + 56.4 - 56.4$

$= 5.8\sqrt{10}$

The difference is $5.8\sqrt{10}$.

b. $5.8\sqrt{10} \approx 18.3$

This underestimates the difference projected by the graph of $98.2 - 74.1 = 24.1$ by 5.8. This represents a difference of 5.8 million people.

117. $\dfrac{2}{\sqrt{5}-1} \cdot \dfrac{\sqrt{5}+1}{\sqrt{5}+1} = \dfrac{2(\sqrt{5}+1)}{5-1}$

$= \dfrac{2(\sqrt{5}+1)}{4}$

$= \dfrac{\sqrt{5}+1}{2}$

≈ 1.62

About 1.62 to 1.

118. $R_a = R_f\sqrt{1-\left(\dfrac{v}{c}\right)^2}$

$= R_f\sqrt{1-\left(\dfrac{0.9c}{c}\right)^2}$

$= R_f\sqrt{1-(0.9)^2}$

$= R_f\sqrt{0.19}$

$\approx 0.44R_f$

$R_a = 0.44R_f$

$44 = 0.44R_f$

$\dfrac{44}{0.44} = \dfrac{0.44R_f}{0.44}$

$100 = R_f$

If you are gone for 44 weeks, then 100 weeks will have passed for your friend.

119. Perimeter:

$P = 2l + 2w$

$= 2 \cdot \sqrt{125} + 2 \cdot 2\sqrt{20}$

$= 2 \cdot \sqrt{25 \cdot 5} + 4\sqrt{4 \cdot 5}$

$= 2 \cdot 5\sqrt{5} + 4 \cdot 2\sqrt{5}$

$= 10\sqrt{5} + 8\sqrt{5}$

$= 18\sqrt{5}$ feet

Area:

$A = lw$

$= \sqrt{125} \cdot 2\sqrt{20}$

$= 2\sqrt{125 \cdot 20}$

$= 2\sqrt{2500}$

$= 2 \cdot 50$

$= 100$ square feet

120. Perimeter:

$P = 2l + 2w$

$= 2 \cdot 4\sqrt{20} + 2 \cdot \sqrt{80}$

$= 8\sqrt{4 \cdot 5} + 2\sqrt{16 \cdot 5}$

$= 8 \cdot 2\sqrt{5} + 2 \cdot 4\sqrt{5}$

$= 16\sqrt{5} + 8\sqrt{5}$

$= 24\sqrt{5}$ feet

Area:

$A = lw$

$= 4\sqrt{20} \cdot \sqrt{80}$

$= 4\sqrt{20 \cdot 80}$

$= 4\sqrt{1600}$

$= 4 \cdot 40$

$= 160$ square feet

121. – 128. Answers will vary.

129. does not make sense; Explanations will vary. Sample explanation: The denominator is rationalized correctly.

130. makes sense

131. does not make sense; Explanations will vary. Sample explanation: $2\sqrt{20} + 4\sqrt{75}$ simplifies to $4\sqrt{5} + 20\sqrt{3}$ and thus the radical terms are not common.

132. does not make sense; Explanations will vary. Sample explanation: Finding the *n*th root first often gives smaller numbers on the middle step.

133. false; Changes to make the statement true will vary. A sample change is: $7^{1/2} \cdot 7^{1/2} = 7^1 = 7$.

134. false; Changes to make the statement true will vary. A sample change is: $(8)^{-\frac{1}{3}} = \frac{1}{(8)^{\frac{1}{3}}} = \frac{1}{\sqrt[3]{8}} = \frac{1}{2}$.

135. false; Changes to make the statement true will vary. The cube root of –8 is the real number –2.

136. false; Changes to make the statement true will vary. A sample change is: $\frac{\sqrt{20}}{8} = \frac{\sqrt{5}}{4}$.

137. $\left(5+\sqrt{\boxed{3}}\right)\left(5-\sqrt{\boxed{3}}\right) = 22$

$$25 - \boxed{3} = 22$$
$$\boxed{3} = 3$$

138. $\sqrt{\boxed{25}x^{\boxed{14}}} = 5x^7$

139. $\sqrt{13+\sqrt{2}+\frac{7}{3+\sqrt{2}}}$

$$= \sqrt{13+\sqrt{2}+\frac{7}{3+\sqrt{2}}\cdot\frac{3-\sqrt{2}}{3-\sqrt{2}}}$$
$$= \sqrt{13+\sqrt{2}+\frac{21-7\sqrt{2}}{9-2}}$$
$$= \sqrt{13+\sqrt{2}+\frac{21-7\sqrt{2}}{7}}$$
$$= \sqrt{13+\sqrt{2}+3-\sqrt{2}}$$
$$= \sqrt{16}$$
$$= 4$$

140. a. $3^{\frac{1}{2}} \boxed{>} 3^{\frac{1}{3}}$

Calculator Check: $1.7321 > 1.4422$

b. $\sqrt{7}+\sqrt{18} \boxed{>} \sqrt{7+18}$

Calculator Check: $6.8884 > 5$

141. a. $\frac{ab}{a^2+ab+b^2} + \left(\frac{ac-ad-bc+bd}{ac-ad+bc-bd} \div \frac{a^3-b^3}{a^3+b^3}\right) = \frac{ab}{a^2+ab+b^2} + \left(\frac{a(c-d)-b(c-d)}{a(c-d)+b(c-d)} \cdot \frac{a^3+b^3}{a^3-b^3}\right)$

$$= \frac{ab}{a^2+ab+b^2} + \left(\frac{\cancel{(c-d)}\cancel{(a-b)}}{\cancel{(c-d)}\cancel{(a+b)}} \cdot \frac{\cancel{(a+b)}\left(a^2-ab+b^2\right)}{\cancel{(a-b)}\left(a^2+ab+b^2\right)}\right) = \frac{ab}{a^2+ab+b^2} + \frac{a^2-ab+b^2}{a^2+ab+b^2}$$

$$= \frac{ab+a^2-ab+b^2}{a^2+ab+b^2} = \frac{a^2+b^2}{a^2+ab+b^2}$$

Her son is 8 years old.

b. Son's portion:

$$\frac{8^{-\frac{4}{3}}+2^{-2}}{16^{-\frac{3}{4}}+2^{-1}}=\frac{\frac{1}{\left(\sqrt[3]{8}\right)^4}+\frac{1}{2^2}}{\frac{1}{\left(\sqrt[4]{16}\right)^3}+\frac{1}{2}}$$

$$=\frac{\frac{1}{2^4}+\frac{1}{4}}{\frac{1}{2^3}+\frac{1}{2}}$$

$$=\frac{\frac{1}{16}+\frac{1}{4}}{\frac{1}{8}+\frac{1}{2}}$$

$$=\frac{\frac{5}{16}}{\frac{5}{8}}$$

$$=\frac{8}{16}$$

$$=\frac{1}{2}$$

Mom's portion:

$$\frac{1}{2}\left(1-\frac{1}{2}\right)=\frac{1}{2}\left(\frac{1}{2}\right)=\frac{1}{4}$$

142. $(2x^3y^2)(5x^4y^7)=10x^7y^9$

143. $2x^4(8x^4+3x)=2x^4(8x^4)+2x^4(3x)$
$=16x^8+6x^5$

144. $2x(x^2+4x+5)+3(x^2+4x+5)$
$=2x^3+8x^2+10x+3x^2+12x+15$
$=2x^3+8x^2+3x^2+10x+12x+15$
$=2x^3+11x^2+22x+15$

Section P.4

Check Point Exercises

1. **a.** $(-17x^3+4x^2-11x-5)+(16x^3-3x^2+3x-15)$
$=(-17x^3+16x^3)+(4x^2-3x^2)+(-11x+3x)+(-5-15)$
$=-x^3+x^2-8x-20$

b. $(13x^2 - 9x^2 - 7x + 1) - (-7x^3 + 2x^2 - 5x + 9)$
$= (13x^3 - 9x^2 - 7x + 1) + (7x^3 - 2x^2 + 5x - 9)$
$= (13x^3 + 7x^3) + (-9x^2 - 2x^2) + (-7x + 5x) + (1 - 9)$
$= 20x^3 - 11x^2 - 2x - 8$

2. $(5x - 2)(3x^2 - 5x + 4)$
$= 5x(3x^2 - 5x + 4) - 2(3x^2 - 5x + 4)$
$= 5x \cdot 3x^2 - 5x \cdot 5x + 5x \cdot 4 - 2 \cdot 3x^2 + 2 \cdot 5x - 2 \cdot 4$
$= 15x^3 - 25x^2 + 20x - 6x^2 + 10x - 8$
$= 15x^3 - 31x^2 + 30x - 8$

3. $(7x - 5)(4x - 3) = 7x \cdot 4x + 7x(-3) + (-5)4x + (-5)(-3)$
$= 28x^2 - 21x - 20x + 15$
$= 28x^2 - 41x + 15$

4. **a.** Use the special-product formula shown.
$(A + B)(A - B) = A^2 - B^2$
$(7x + 8)(7x - 8) = (7x)^2 - (8)^2$
$= 49x^2 - 64$

b. Use the special-product formula shown.
$(A + B)(A - B) = A^2 - B^2$
$(2y^3 - 5)(2y^3 + 5) = (2y^3 + 5)(2y^3 - 5)$
$= (2y^3)^2 - (5)^2$
$= 4y^6 - 25$

5. **a.** Use the special-product formula shown.
$(A + B)^2 = A^2 + 2AB + B^2$
$(x + 10)^2 = x^2 + 2(x)(10) + 10^2$
$= x^2 + 20x + 100$

b. Use the special-product formula shown.
$(A + B)^2 = A^2 + 2AB + B^2$
$(5x + 4)^2 = (5x)^2 + 2(5x)(4) + 4^2$
$= 25x^2 + 40x + 16$

6. **a.** Use the special-product formula shown.
$(A - B)^2 = A^2 - 2AB + B^2$
$(x - 9)^2 = x^2 - 2(x)(9) + 9^2$
$= x^2 - 18x + 81$

b. Use the special-product formula shown.

$$(A-B)^2 = A^2 - 2AB + B^2$$
$$(7x-3)^2 = (7x)^2 - 2(7x)(3) + 3^2$$
$$= 49x^2 - 42x + 9$$

7. $(x^3 - 4x^2y + 5xy^2 - y^3) - (x^3 - 6x^2y + y^3)$
$= (x^3 - 4x^2y + 5xy^2 - y^3) + (-x^3 + 6x^2y - y^3)$
$= (x^3 - x^3) + (-4x^2y + 6x^2y) + (5xy^2) + (-y^3 - y^3)$
$= 2x^2y + 5xy^2 - 2y^3$

8. **a.** $(7x-6y)(3x-y) = (7x)(3x) + (7x)(-y) + (-6y)(3x) + (-6y)(-y)$
$= 21x^2 - 7xy - 18xy + 6y^2$
$= 21x^2 - 25xy + 6y^2$

b. $(2x+4y)^2 = (2x)^2 + 2(2x)(4y) + (4y)^2$
$= 4x^2 + 16xy + 16y^2$

Concept and Vocabulary Check P.4

1. whole

2. standard

3. monomial

4. binomial

5. trinomial

6. n

7. like;

8. distributive; $4x^3 - 8x^2 + 6$; $7x^3$

9. $5x$; 3; like

10. $3x^2$; $5x$; $21x$; 35

11. $A^2 - B^2$; minus

12. $A^2 + 2AB + B^2$; squared; product of the terms; squared

13. $A^2 - 2AB + B^2$; minus; product of the terms; plus

14. $n + m$

Exercise Set P.4

1. yes; $2x+3x^2-5=3x^2+2x-5$

2. no; The term $3x^{-1}$ does not have a whole number exponent.

3. no; The form of a polynomial involves addition and subtraction, not division.

4. yes; $x^2-x^3+x^4-5=x^4-x^3+x^2-5$

5. $3x^2$ has degree 2
$-5x$ has degree 1
4 has degree 0
$3x^2-5x+4$ has degree 2.

6. $-4x^3$ has degree 3
$7x^2$ has degree 2
-11 has degree 0
$-4x^3+7x^2-11$ has degree 3.

7. x^2 has degree 2
$-4x^3$ has degree 3
$9x$ has degree 1
$-12x^4$ has degree 4
63 has degree 0
$x^2-4x^3+9x-12x^4+63$ has degree 4.

8. x^2 has degree 2
$-8x^3$ has degree 3
$15x^4$ has degree 4
91 has degree 0
$x^2-8x^3+15x^4+91$ has degree 4.

9. $$\begin{aligned}(-6x^3+5x^2-8x+9)+(17x^3+2x^2-4x-13)&=(-6x^3+17x^3)+(5x^2+2x^2)+(-8x-4x)+(9-13)\\&=11x^3+7x^2-12x-4\end{aligned}$$
The degree is 3.

10. $$\begin{aligned}(-7x^3+6x^2-11x+13)+(19x^3-11x^2+7x-17)&=(-7x^3+19x^3)+(6x^2-11x^2)+(-11x+7x)+(13-17)\\&=12x^3-5x^2-4x-4\end{aligned}$$
The degree is 3.

11. $$\begin{aligned}(17x^3-5x^2+4x-3)-(5x^3-9x^2-8x+11)&=(17x^3-5x^2+4x-3)+(-5x^3+9x^2+8x-11)\\&=(17x^3-5x^3)+(-5x^2+9x^2)+(4x+8x)+(-3-11)\\&=12x^3+4x^2+12x-14\end{aligned}$$
The degree is 3.

12. $(18x^4 - 2x^3 - 7x + 8) - (9x^4 - 6x^3 - 5x + 7) = (18x^4 - 2x^3 - 7x + 8) + (-9x^4 + 6x^3 + 5x - 7)$
$= (18x^4 - 9x^4) + (-2x^3 + 6x^3) + (-7x + 5x) + (8 - 7)$
$= 9x^4 + 4x^3 - 2x + 1$

The degree is 4.

13. $(5x^2 - 7x - 8) + (2x^2 - 3x + 7) - (x^2 - 4x - 3) = (5x^2 - 7x - 8) + (2x^2 - 3x + 7) + (-x^2 + 4x + 3)$
$= (5x^2 + 2x^2 - x^2) + (-7x - 3x + 4x) + (-8 + 7 + 3)$
$= 6x^2 - 6x + 2$

The degree is 2.

14. $(8x^2 + 7x - 5) - (3x^2 - 4x) - (-6x^3 - 5x^2 + 3) = (8x^2 + 7x - 5) + (-3x^2 + 4x) + (6x^3 + 5x^2 - 3)$
$= 6x^3 + (8x^2 - 3x^2 + 5x^2) + (7x + 4x) + (-5 - 3)$
$= 6x^3 + 10x^2 + 11x - 8$

The degree is 3.

15. $(x+1)(x^2 - x + 1) = x(x^2) - x \cdot x + x \cdot 1 + 1(x^2) - 1 \cdot x + 1 \cdot 1$
$= x^3 - x^2 + x + x^2 - x + 1$
$= x^3 + 1$

16. $\left(x+5\right)\left(x^2 - 5x + 25\right) = x(x^2) - x(5x) + x(25) + 5(x^2) - 5(5x) + 5(25)$
$= x^3 - 5x^2 + 25x + 5x^2 - 25x + 125)$
$= x^3 + 125$

17. $(2x-3)(x^2 - 3x + 5) = (2x)(x^2) + (2x)(-3x) + (2x)(5) + (-3)(x^2) + (-3)(-3x) + (-3)(5)$
$= 2x^3 - 6x^2 + 10x - 3x^2 + 9x - 15$
$= 2x^3 - 9x^2 + 19x - 15$

18. $(2x-1)(x^2 - 4x + 3) = (2x)(x^2) + (2x)(-4x) + (2x)(3) + (-1)(x^2) + (-1)(-4x) + (-1)(3)$
$= 2x^3 - 8x^2 + 6x - x^2 + 4x - 3$
$= 2x^3 - 9x^2 + 10x - 3$

19. $(x+7)(x+3) = x^2 + 3x + 7x + 21 = x^2 + 10x + 21$

20. $(x+8)(x+5) = x^2 + 5x + 8x + 40 = x^2 + 13x + 40$

21. $(x-5)(x+3) = x^2 + 3x - 5x - 15 = x^2 - 2x - 15$

22. $(x-1)(x+2) = x^2 + 2x - x - 2 = x^2 + x - 2$

23. $(3x+5)(2x+1) = (3x)(2x) + 3x(1) + 5(2x) + 5 = 6x^2 + 3x + 10x + 5 = 6x^2 + 13x + 5$

24. $(7x+4)(3x+1) = (7x)(3x) + 7x(1) + 4(3x) + 4(1) = 21x^2 + 7x + 12x + 4 = 21x^2 + 19x + 4$

25. $(2x-3)(5x+3) = (2x)(5x) + (2x)(3) + (-3)(5x) + (-3)(3) = 10x^2 + 6x - 15x - 9 = 10x^2 - 9x - 9$

26. $(2x-5)(7x+2)=(2x)(7x)+(2x)(2)+(-5)(7x)+(-5)(2)=14x^2+4x-35x-10=14x^2-31x-10$

27. $(5x^2-4)(3x^2-7)=(5x^2)(3x^2)+(5x^2)(-7)+(-4)(3x^2)+(-4)(-7)=15x^4-35x^2-12x^2+28=15x^4-47x^2+28$

28. $(7x^2-2)(3x^2-5)=(7x^2)(3x^2)+(7x^2)(-5)+(-2)(3x^2)+(-2)(-5)=21x^4-35x^2-6x^2+10=21x^4-41x^2+10$

29. $\left(8x^3+3\right)\left(x^2-5\right)=\left(8x^3\right)\left(x^2\right)+\left(8x^3\right)\left(-5\right)+\left(3\right)\left(x^2\right)+\left(3\right)\left(-5\right)=8x^5-40x^3+3x^2-15$

30. $\left(7x^3+5\right)\left(x^2-2\right)=\left(7x^3\right)\left(x^2\right)+\left(7x^3\right)\left(-2\right)+\left(5\right)\left(x^2\right)+\left(5\right)\left(-2\right)=7x^5-14x^3+5x^2-10$

31. $(x+3)(x-3)=x^2-3^2=x^2-9$

32. $(x+5)(x-5)=x^2-5^2=x^2-25$

33. $(3x+2)(3x-2)=(3x)^2-2^2=9x^2-4$

34. $(2x+5)(2x-5)=(2x)^2-5^2=4x^2-25$

35. $(5-7x)(5+7x)=5^2-(7x)^2=25-49x^2$

36. $(4-3x)(4+3x)=4^2-(3x)^2=16-9x^2$

37. $(4x^2+5x)(4x^2-5x)=(4x^2)^2-(5x)^2=16x^4-25x^2$

38. $(3x^2+4x)(3x^2-4x)=(3x^2)^2-(4x)^2=9x^4-16x^2$

39. $\left(1-y^5\right)\left(1+y^5\right)=\left(1\right)^2-\left(y^5\right)^2=1-y^{10}$

40. $\left(2-y^5\right)\left(2+y^5\right)=\left(2\right)^2-\left(y^5\right)^2=4-y^{10}$

41. $(x+2)^2=x^2+2\cdot x\cdot 2+2^2=x^2+4x+4$

42. $(x+5)^2=x^2+2\cdot x\cdot 5+5^2=x^2+10x+25$

43. $(2x+3)^2=(2x)^2+2(2x)(3)+3^2=4x^2+12x+9$

44. $(3x+2)^2=(3x)^2+2(3x)(2)+2^2=9x^2+12x+4$

45. $(x-3)^2=x^2-2\cdot x\cdot 3+3^2=x^2-6x+9$

46. $(x-4)^2=x^2-2\cdot x\cdot 4+4^2=x^2-8x+16$

47. $(4x^2-1)^2=(4x^2)^2-2(4x^2)(1)+1^2=16x^4-8x^2+1$

48. $(5x^2-3)^2=(5x^2)^2-2(5x^2)(3)+3^2=25x^4-30x^2+9$

49. $(7-2x)^2 = 7^2 - 2(7)(2x) + (2x)^2 = 49 - 28x + 4x^2 = 4x^2 - 28x + 49$

50. $(9-5x)^2 = 9^2 - 2(9)(5x) + (5x)^2 = 81 - 90x + 25x^2$ or $25x^2 - 90x + 81$

51. $(x+1)^3 = x^3 + 3\cdot x^2 \cdot 1 + 3x\cdot 1^2 + 1^3 = x^3 + 3x^2 + 3x + 1$

52. $(x+2)^3 = x^3 + 3\cdot x^2 \cdot 2 + 3\cdot x \cdot 2^2 + 2^3 = x^3 + 6x^2 + 12x + 8$

53. $(2x+3)^3 = (2x)^3 + 3\cdot(2x)^2 \cdot 3 + 3(2x)\cdot 3^2 + 3^3 = 8x^3 + 36x^2 + 54x + 27$

54. $(3x+4)^3 = (3x)^3 + 3(3x)^2 \cdot 4 + 3(3x)\cdot 4^2 + 4^3 = 27x^3 + 108x^2 + 144x + 64$

55. $(x-3)^3 = x^3 - 3\cdot x^3 \cdot 3 + 3\cdot x \cdot 3^2 - 3^3 = x^3 - 9x^2 + 27x - 27$

56. $(x-1)^3 = x^3 - 3x^2 \cdot 1 + 3x \cdot 1^2 - 1^3 = x^3 - 3x^2 + 3x - 1$

57. $(3x-4)^3 = (3x)^3 - 3(3x)^2 \cdot 4 + 3(3x)\cdot 4^2 - 4^3 = 27x^3 - 108x^2 + 144x - 64$

58. $(2x-3)^3 = (2x)^3 - 3(2x)^2 \cdot 3 + 3(2x)\cdot 3^2 - 3^3 = 8x^3 - 36x^2 + 54x - 27$

59.
$$\begin{aligned}(5x^2y - 3xy) + (2x^2y - xy) &= (5x^2y + 2x^2y) + (-3xy - xy)\\ &= (5+2)x^2y + (-3-1)xy\\ &= 7x^2y - 4xy \text{ is of degree 3.}\end{aligned}$$

60.
$$\begin{aligned}(-2x^2y + xy) + (4x^2y + 7xy) &= (-2x^2y + 4x^2y) + (xy + 7xy)\\ &= (-2+4)x^2y + (1+7)xy\\ &= 2x^2y + 8xy \text{ is of degree 3.}\end{aligned}$$

61.
$$\begin{aligned}(4x^2y + 8xy + 11) + (-2x^2y + 5xy + 2) &= (4x^2y - 2x^2y) + (8xy + 5xy) + (11+2)\\ &= (4-2)x^2y + (8+5)xy + 13\\ &= 2x^2y + 13xy + 13 \text{ is of degree 3.}\end{aligned}$$

62.
$$\begin{aligned}(7x^4y^2 - 5x^2y^2 + 3xy) + (-18x^4y^2 - 6x^2y^2 - xy) &= (7x^4y^2 - 18x^4y^2) + (-5x^2y^2 - 6x^2y^2) + (3xy - xy)\\ &= (7-18)x^4y^2 + (-5-6)x^2y^2 + (3-1)xy\\ &= -11x^4y^2 - 11x^2y^2 + 2xy \text{ is of degree 6.}\end{aligned}$$

63.
$$\begin{aligned}(x^3 + 7xy - 5y^2) - (6x^3 - xy + 4y^2) &= (x^3 + 7xy - 5y^2)\\ &= (x^3 - 6x^3) + (7xy + xy) + (-5y^2 - 4y^2)\\ &= (1-6)x^3 + (7+1)xy + (-5-4)y^2\\ &= -5x^3 + 8xy - 9y^2 \text{ is of degree 3.}\end{aligned}$$

64. $(x^4-7xy-5y^3)-(6x^4-3xy+4y^3)=(x^4-7xy-5y^3)+(-6x^4+3xy-4y^3)$
$=(x^4-6x^4)+(-7xy+3xy)+(-5y^3-4y^3)$
$=(1-6)x^4+(-7+3)xy+(-5-4)y^3$
$=-5x^4-4xy-9y^3$ is of degree 4.

65. $(3x^4y^2+5x^3y-3y)-(2x^4y^2-3x^3y-4y+6x)=(3x^4y^2+5x^3y-3y)+(-2x^4y^2+3x^3y+4y-6x)$
$=(3x^4y^2-2x^4y^2)+(5x^3y+3x^3y)+(-3y+4y)-6x$
$=(3-2)x^4y^2+(5+3)x^3y+(-3+4)y-6x$
$=x^4y^2+8x^3y+y-6x$ is of degree 6.

66. $(5x^4y^2+6x^3y-7y)-(3x^4y^2-5x^3y-6y+8x)=(5x^4y^2+6x^3y-7y)+(-3x^4y^2+5x^3y+6y-8x)$
$=(5x^4y^2-3x^4y^2)+(6x^3y+5x^3y)+(-7y+6y)-8x$
$=(5-3)x^4y^2+(6+5)x^3y+(-7+6)y-8x$
$=2x^4y^2+11x^3y-y-8x$ is of degree 6.

67. $(x+5y)(7x+3y)=x(7x)+x(3y)+(5y)(7x)+(5y)(3y)$
$=7x^2+3xy+35xy+15y^2$
$=7x^2+38xy+15y^2$

68. $(x+9y)(6x+7y)=x(6x)+x(7y)+(9y)(6x)+(9y)(7y)$
$=6x^2+7xy+54xy+63y^2$
$=6x^2+61xy+63y^2$

69. $(x-3y)(2x+7y)=x(2x)+x(7y)+(-3y)(2x)+(-3y)(7y)$
$=2x^2+7xy-6xy-21y^2$
$=2x^2+xy-21y^2$

70. $(3x-y)(2x+5y)=(3x)(2x)+(3x)(5y)+(-y)(2x)+(-y)(5y)$
$=6x^2+15xy-2xy-5y^2$
$=6x^2+13xy-5y^2$

71. $(3xy-1)(5xy+2)=(3xy)(5xy)+(3xy)(2)+(-1)(5xy)+(-1)(2)$
$=15x^2y^2+6xy-5xy-2$
$=15x^2y^2+xy-2$

72. $(7x^2y+1)(2x^2y-3)=(7x^2y)(2x^2y)+(7x^2y)(-3)+(1)2x^2y+(1)(-3)$
$=14x^4y^2-21x^2y+2x^2y-3$
$=14x^4y^2-19x^2y-3$

73. $(7x+5y)^2=(7x)^2+2(7x)(5y)+(5y)^2=49x^2+70xy+25y^2$

74. $(9x+7y)^2=(9x)^2+2(9x)(7y)+(7y)^2=81x^2+126xy+49y^2$

75. $(x^2y^2-3)^2=(x^2y^2)^2-2(x^2y^2)(3)+3^2=x^4y^4-6x^2y^2+9$

76. $(x^2y^2-5)^2=(x^2y^2)^2-2(x^2y^2)(5)+5^2=x^4y^4-10x^2y^2+25$

77. $$\begin{aligned}(x-y)(x^2+xy+y^2)&=x(x^2)+x(xy)+x(y^2)+(-y)(x^2)+(-y)(xy)+(-y)(y^2)\\&=x^3+x^2y+xy^2-x^2y-xy^2-y^3\\&=x^3-y^3\end{aligned}$$

78. $$\begin{aligned}(x+y)(x^2-xy+y^2)&=x(x^2)+x(-xy)+x(y^2)+y(x^2)+y(-xy)+y(y^2)\\&=x^3-x^2y+xy^2+x^2y-xy^2+y^3\\&=x^3+y^3\end{aligned}$$

79. $(3x+5y)(3x-5y)=(3x)^2-(5y)^2=9x^2-25y^2$

80. $(7x+3y)(7x-3y)=(7x)^2-(3y)^2=49x^2-9y^2$

81. $\left(7xy^2-10y\right)\left(7xy^2+10y\right)=\left(7xy^2\right)^2-\left(10y\right)^2=49x^2y^4-100y^2$

82. $\left(3xy^2-4y\right)\left(3xy^2+4y\right)=\left(3xy^2\right)^2-\left(4y\right)^2=9x^2y^4-16y^2$

83. $$\begin{aligned}(3x+4y)^2-(3x-4y)^2&=\left[(3x)^2+2(3x)(4y)+(4y)^2\right]-\left[(3x)^2-2(3x)(4y)+(4y)^2\right]\\&=\left(9x^2+24xy+16y^2\right)-\left(9x^2-24xy+16y^2\right)\\&=9x^2+24xy+16y^2-9x^2+24xy-16y^2\\&=48xy\end{aligned}$$

84. $$\begin{aligned}(5x+2y)^2-(5x-2y)^2&=\left[(5x)^2+2(5x)(2y)+(2y)^2\right]-\left[(5x)^2-2(5x)(2y)+(2y)^2\right]\\&=\left(25x^2+20xy+4y^2\right)-\left(25x^2-20xy+4y^2\right)\\&=25x^2+20xy+4y^2-25x^2+20xy-4y^2\\&=40xy\end{aligned}$$

85. $$\begin{aligned}&(5x-7)(3x-2)-(4x-5)(6x-1)\\&=\left[15x^2-10x-21x+14\right]-\left[24x^2-4x-30x+5\right]\\&=\left(15x^2-31x+14\right)-\left(24x^2-34x+5\right)\\&=15x^2-31x+14-24x^2+34x-5\\&=-9x^2+3x+9\end{aligned}$$

86. $(3x+5)(2x-9)-(7x-2)(x-1)$
$=\left(6x^2-27x+10x-45\right)-\left(7x^2-7x-2x+2\right)$
$=\left(6x^2-17x-45\right)-\left(7x^2-9x+2\right)$
$=6x^2-17x-45-7x^2+9x-2$
$=-x^2-8x-47$

87. $(2x+5)(2x-5)\left(4x^2+25\right)$
$=\left[(2x)^2-5^2\right]\left(4x^2+25\right)$
$=\left(4x^2-25\right)\left(4x^2+25\right)$
$=\left(4x^2\right)^2-(25)^2$
$=16x^4-625$

88. $(3x+4)(3x-4)\left(9x^2+16\right)$
$=\left[(3x)^2-4^2\right]\left(9x^2+16\right)$
$=\left(9x^2-16\right)\left(9x^2+16\right)$
$=\left(9x^2\right)^2-(16)^2$
$=81x^4-256$

89. $\dfrac{(2x-7)^5}{(2x-7)^3}=(2x-7)^{5-3}$
$=(2x-7)^2$
$=(2x)^2-2(2x)(7)+(7)^2$
$=4x^2-28x+49$

90. $\dfrac{(5x-3)^6}{(5x-3)^4}=(5x-3)^{6-4}$
$=(5x-3)^2$
$=(5x)^2-2(5x)(3)+(3)^2$
$=25x^2-30x+9$

91. **a.** $S=0.2x^3-1.5x^2+3.4x+25+\left(0.1x^3-1.3x^2+3.3x+5\right)$
$S=0.2x^3-1.5x^2+3.4x+25+0.1x^3-1.3x^2+3.3x+5$
$S=0.3x^3-2.8x^2+6.7x+30$

b. $S = 0.3x^3 - 2.8x^2 + 6.7x + 30$

$S = 0.3(5)^3 - 2.8(5)^2 + 6.7(5) + 30$

$S = 31$

The model gives a score of 31 for the group in the 45-54 age range which is the same as the score displayed by the bar graph.

92. **a.** $S = -0.02x^3 + 0.4x^2 + 1.2x + 22 + \left(-0.01x^3 - 0.2x^2 + 1.1x + 2\right)$

$S = -0.02x^3 + 0.4x^2 + 1.2x + 22 - 0.01x^3 - 0.2x^2 + 1.1x + 2$

$S = -0.03x^3 + 0.2x^2 + 2.3x + 24$

b. $S = -0.03x^3 + 0.2x^2 + 2.3x + 24$

$S = -0.03(5)^3 + 0.2(5)^2 + 2.3(5) + 24$

$S = 36.75$

The model gives a score of 36.75 for the group of slightly conservative political identification group. This underestimates the score shown on the bar graph by 0.25.

93. $x(8-2x)(10-2x) = x\left(80 - 36x + 4x^2\right)$

$= 80x - 36x^2 + 4x^3$

$= 4x^3 - 36x^2 + 80x$

94. $x(8-2x)(5-2x) = x\left(40 - 26x + 4x^2\right)$

$= 40x - 26x^2 + 4x^3$

$= 4x^3 - 26x^2 + 40x$

95. $(x+9)(x+3) - (x+5)(x+1)$

$= x^2 + 12x + 27 - \left(x^2 + 6x + 5\right)$

$= x^2 + 12x + 27 - x^2 - 6x - 5$

$= 6x + 22$

96. $(x+4)(x+3) - (x+2)(x+1)$

$= x^2 + 7x + 12 - \left(x^2 + 3x + 2\right)$

$= x^2 + 7x + 12 - x^2 - 3x - 2$

$= 4x + 10$

97. – 102. Answers will vary.

103. makes sense

104. does not make sense; Explanations will vary. Sample explanation: FOIL is used to multiply two binomials.

105. makes sense

106. makes sense, although answers may vary

107. false; Changes to make the statement true will vary. A sample change is: $(3x^3 + 2)(3x^3 - 2) = 9x^6 - 4$

108. false; Changes to make the statement true will vary. A sample change is: $(x-5)^2 = x^2 - 10x + 25$

109. false; Changes to make the statement true will vary. A sample change is: $(x+1)^2 = x^2 + 2x + 1$

110. true

111. $$\begin{aligned}[(7x+5)+4y][(7x+5)-4y] &= (7x+5)^2 - 4y^2 \\ &= (7x)^2 + 2(7x)(5) + 5^2 - 16y^2 \\ &= 49x^2 + 70x + 25 - 16y^2\end{aligned}$$

112. $$\begin{aligned}&[(3x+y)+1]^2 \\ &= (3x+y)^2 + 2(3x+y)(1) + 1^2 \\ &= (3x)^2 + 2(3x)y + y^2 + 6x + 2y + 1 \\ &= 9x^2 + 6xy + y^2 + 6x + 2y + 1\end{aligned}$$

113. $$\begin{aligned}&(x^n+2)(x^n-2) - (x^n-3)^2 \\ &(x^n+2)(x^n-2) - (x^n-3)^2 \\ &= (x^{2n} - 4) - (x^{2n} - 6x^n + 9) \\ &= x^{2n} - 4 - x^{2n} + 6x^n - 9 \\ &= 6x^n - 13\end{aligned}$$

114. $$\begin{aligned}&(x+3)(x-1) + ((x+3)-x)(x-(x-1)) \\ &= (x+3)(x-1) + 3(x-x+1) \\ &= x^2 - x + 3x - 3 + 3 \\ &= x^2 + 2x\end{aligned}$$

115. $(x+3)(x+\boxed{4}) = x^2 + 7x + 12$

116. $(x-\boxed{2})(x-12) = x^2 - 14x + 24$

117. $(4x+1)(2x-\boxed{3}) = 8x^2 - 10x - 3$

Mid-Chapter P Check Point

1. $$\begin{aligned}(3x+5)(4x-7) &= (3x)(4x) + (3x)(-7) + (5)(4x) + (5)(-7) \\ &= 12x^2 - 21x + 20x - 35 \\ &= 12x^2 - x - 35\end{aligned}$$

2. $$\begin{aligned}(3x+5)-(4x-7) &= 3x + 5 - 4x + 7 \\ &= 3x - 4x + 5 + 7 \\ &= -x + 12\end{aligned}$$

3. $\sqrt{6} + 9\sqrt{6} = 10\sqrt{6}$

4. $3\sqrt{12} - \sqrt{27} = 3 \cdot 2\sqrt{3} - 3\sqrt{3} = 6\sqrt{3} - 3\sqrt{3} = 3\sqrt{3}$

5. $7x+3[9-(2x-6)]=7x+3[9-2x+6]=7x+3[15-2x]=7x+45-6x=x+45$

6. $(8x-3)^2=(8x)^2-2(8x)(3)+(3)^2=64x^2-48x+9$

7. $\left(x^{\frac{1}{3}}y^{-\frac{1}{2}}\right)^6=x^{\frac{1}{3}\cdot 6}y^{-\frac{1}{2}\cdot 6}=x^2y^{-3}=\frac{x^2}{y^3}$

8. $\left(\frac{2}{7}\right)^0-32^{-\frac{2}{5}}=1-\frac{1}{\left(\sqrt[5]{32}\right)^2}=1-\frac{1}{(2)^2}=1-\frac{1}{4}=\frac{3}{4}$

9. $(2x-5)-(x^2-3x+1)=2x-5-x^2+3x-1=-x^2+5x-6$

10. $$\begin{aligned}(2x-5)(x^2-3x+1)&=2x(x^2-3x+1)-5(x^2-3x+1)\\&=2x(x^2-3x+1)-5(x^2-3x+1)\\&=2x^3-6x^2+2x-5x^2+15x-5\\&=2x^3-6x^2-5x^2+2x+15x-5\\&=2x^3-11x^2+17x-5\end{aligned}$$

11. $x^3+x^3-x^3\cdot x^3=2x^3-x^6=-x^6+2x^3$

12. $$\begin{aligned}(9a-10b)(2a+b)&=(9a)(2a)+(9a)(b)+(-10b)(2a)+(-10b)(b)\\&=(9a)(2a)+(9a)(b)+(-10b)(2a)+(-10b)(b)\\&=18a^2+9ab-20ab-10b^2\\&=18a^2-11ab-10b^2\end{aligned}$$

13. $\{a,c,d,e\}\cup\{c,d,f,h\}=\{a,c,d,e,f,h\}$

14. $\{a,c,d,e\}\cap\{c,d,f,h\}=\{c,d\}$

15. $$\begin{aligned}\left(3x^2y^3-xy+4y^2\right)-\left(-2x^2y^3-3xy+5y^2\right)&=3x^2y^3-xy+4y^2+2x^2y^3+3xy-5y^2\\&=3x^2y^3-xy+4y^2+2x^2y^3+3xy-5y^2\\&=3x^2y^3+2x^2y^3-xy+3xy+4y^2-5y^2\\&=5x^2y^3+2xy-y^2\end{aligned}$$

16. $\frac{24x^2y^{13}}{-2x^5y^{-2}}=-12x^{2-5}y^{13-(-2)}=-12x^{-3}y^{15}=-\frac{12y^{15}}{x^3}$

17. $\left(\frac{1}{3}x^{-5}y^4\right)\left(18x^{-2}y^{-1}\right)=6x^{-5-2}y^{4-1}=\frac{6y^3}{x^7}$

18. $\sqrt[12]{x^4}=x^{\frac{4}{12}}=\left|x^{\frac{1}{3}}\right|=\left|\sqrt[3]{x}\right|$

19. $\dfrac{24\times10^3}{2\times10^6}=\dfrac{24}{2}\cdot\dfrac{10^3}{10^6}=12\times10^{-3}=\left(1.2\times10^1\right)\times10^{-3}=1.2\times\left(10^1\times10^{-3}\right)=1.2\times10^{-2}$

20. $\dfrac{\sqrt[3]{32}}{\sqrt[3]{2}}=\sqrt[3]{\dfrac{32}{2}}=\sqrt[3]{16}=\sqrt[3]{2^4}=2\sqrt[3]{2}$

21. $(x^3+2)(x^3-2)=x^6-4$

22. $(x^2+2)^2=(x^2)^2+2(x^2)(2)+(2)^2=x^4+4x^2+4$

23. $\sqrt{50}\cdot\sqrt{6}=5\sqrt{2}\cdot\sqrt{6}=5\sqrt{2\cdot6}=5\sqrt{12}=5\cdot2\sqrt{3}=10\sqrt{3}$

24. $\dfrac{11}{7-\sqrt{3}}=\dfrac{11}{7-\sqrt{3}}\cdot\dfrac{7+\sqrt{3}}{7+\sqrt{3}}=\dfrac{77+11\sqrt{3}}{49-3}=\dfrac{77+11\sqrt{3}}{46}$

25. $\dfrac{11}{\sqrt{3}}=\dfrac{11}{\sqrt{3}}\cdot\dfrac{\sqrt{3}}{\sqrt{3}}=\dfrac{11\sqrt{3}}{3}$

26. $\left\{-11,\ -\dfrac{3}{7},\ 0,\ 0.45,\ \sqrt{25}\right\}$

27. Since $2-\sqrt{13}<0$ then $\left|2-\sqrt{13}\right|=\sqrt{13}-2$

28. Since $x<0$ then $|x|=-x$. Thus $x^2|x|=-x^2x=-x^3$

29. $4.6\cdot3.0\times10^8=4.6\times10^8=13.8\times10^8=1.38\times10^9$
The U.S. produces 1.38×10^9 pounds of garbage per day.

30. $\dfrac{3\times10^{10}}{7.5\times10^9}=\dfrac{3}{7.5}\cdot\dfrac{10^{10}}{10^9}=0.4\times10=4$
A human brain has 4 times as many neurons as a gorilla brain.

31. a. Model 1:
$D=1188x+16,218$
$D=1188(1)+16,218$
$D=17,406$
Model 2:
$D=46x^2+541x+17,650$
$D=46(1)^2+541(1)+17,650$
$D=18,237$

Model 1 best describes the data in 2001.

b. $D=46x^2+541x+17,650$
$D=46(13)^2+541(13)+17,650$
$D=32,457$
Model 2 underestimates the average student-loan debt in 2013 by $593.

Section P.5

Check Point Exercises

1. **a.** $10x^3 - 4x^2$
$= 2x^2(5x) - 2x^2(2)$
$= 2x^2(5x - 2)$

b. $2x(x-7) + 3(x-7)$
$= (x-7)(2x+3)$

2. $x^3 + 5x^2 - 2x - 10$
$= (x^3 + 5x^2) - (2x + 10)$
$= x^2(x+5) - 2(x+5)$
$= (x+5)(x^2 - 2)$

3. Find two numbers whose product is 40 and whose sum is 13. The required integers are 8 and 5. Thus,
$x^2 + 13x + 40 = (x+8)(x+5)$ or $(x+5)(x+8)$.

4. Find two numbers whose product is –14 and whose sum is –5. The required integers are –7 and 2. Thus,
$x^2 - 5x - 14 = (x-7)(x+2)$ or $(x+2)(x-7)$.

5. Find two First terms whose product is $6x^2$.
$6x^2 + 19x - 7 = (6x \quad)(x \quad)$
$6x^2 + 19x - 7 = (3x \quad)(2x \quad)$

Find two Last terms whose product is –7.
The possible factors are 1(–7) and –1(7).

Try various combinations of these factors to find the factorization in which the sum of the Outside and Inside products is 19*x*.

Possible Factors of $6x^2 + 19x - 7$	Sum of Outside and Inside Products (Should Equal 19*x*)
$(6x+1)(x-7)$	$-42x + x = -41x$
$(6x-7)(x+1)$	$6x - 7x = -x$
$(6x-1)(x+7)$	$42x - x = 41x$
$(6x+7)(x-1)$	$-6x + 7x = x$
$(3x+1)(2x-7)$	$-21x + 2x = -19x$
$(3x-7)(2x+1)$	$3x - 14x = -11x$
$(3x-1)(2x+7)$	$21x - 2x = 19x$
$(3x+7)(2x-1)$	$-3x + 14x = 11x$

Thus, $6x^2 + 19x - 7 = (3x-1)(2x+7)$ or $(2x+7)(3x-1)$.

6. Find two First terms whose product is $3x^2$.
$3x^2 - 13xy + 4y^2 = (3x \quad)(x \quad)$

Find two Last terms whose product is $4y^2$.
The possible factors are $(2y)(2y)$, $(-2y)(-2y)$, $(4y)(y)$, and $(-4y)(-y)$.

Try various combinations of these factors to find the factorization in which the sum of the Outside and Inside products is $-13xy$.

$3x^2 - 13xy + y^2 = (3x - y)(x - 4y)$ or $(x - 4y)(3x - y)$.

7. Express each term as the square of some monomial. Then use the formula for factoring $A^2 - B^2$.

a. $x^2 - 81 = x^2 - 9^2 = (x+9)(x-9)$

b. $36x^2 - 25 = (6x)^2 - 5^2 = (6x+5)(6x-5)$

8. Express $81x^4 - 16$ as the difference of two squares and use the formula for factoring $A^2 - B^2$.
$81x^4 - 16 = (9x^2)^2 - 4^2 = (9x^2+4)(9x^2-4)$

The factor $9x^2 - 4$ is the difference of two squares and can be factored. Express $9x^2 - 4$ as the difference of two squares and again use the formula for factoring $A^2 - B^2$.
$(9x^2+4)(9x^2-4) = (9x^2+4)\left[(3x)^2 - 2^2\right] = (9x^2+4)(3x+2)(3x-2)$

Thus, factored completely,
$81x^4 - 16 = (9x^2+4)(3x+2)(3x-2)$.

9. **a.** $x^2 + 14x + 49 = x^2 + 2 \cdot x \cdot 7 + 7^2 = (x+7)^2$

b. Since $16x^2 = (4x)^2$ and $49 = 7^2$, check to see if the middle term can be expressed as twice the product of $4x$ and 7. Since $2 \cdot 4x \cdot 7 = 56x$, $16x^2 - 56x + 49$ is a perfect square trinomial. Thus, $16x^2 - 56x + 49 = (4x)^2 - 2 \cdot 4x \cdot 7 + 7^2$
$= (4x-7)^2$

10. **a.**
$$\begin{aligned} x^3 + 1 &= x^3 + 1^3 \\ &= (x+1)(x^2 - x \cdot 1 + 1^2) \\ &= (x+1)(x^2 - x + 1) \end{aligned}$$

b.
$$\begin{aligned} 125x^3 - 8 &= (5x)^3 - 2^3 \\ &= (5x-2)\left[(5x)^2 + (5x)(2) + 2^2\right] \\ &= (5x-2)(25x^2 + 10x + 4) \end{aligned}$$

11. Factor out the greatest common factor.
$3x^3 - 30x^2 + 75x = 3x\left(x^2 - 10x + 25\right)$
Factor the perfect square trinomial.
$3x\left(x^2 - 10x + 25\right) = 3x\left(x-5\right)^2$

12. Reorder to write as a difference of squares.

$x^2 - 36a^2 + 20x + 100$
$= x^2 + 20x + 100 - 36a^2$
$= \left(x^2 + 20x + 100\right) - 36a^2$
$= \left(x + 10\right)^2 - 36a^2$
$= \left(x + 10 + 6a\right)\left(x + 10 - 6a\right)$

13. $x\left(x-1\right)^{-\frac{1}{2}} + \left(x-1\right)^{\frac{1}{2}}$
$= \left(x-1\right)^{-\frac{1}{2}}\left[x + \left(x-1\right)^{\frac{1}{2}-\left(-\frac{1}{2}\right)}\right]$
$= \left(x-1\right)^{-\frac{1}{2}}\left[x + \left(x-1\right)\right]$
$= \left(x-1\right)^{-\frac{1}{2}}\left(2x-1\right)$
$= \dfrac{2x-1}{\left(x-1\right)^{\frac{1}{2}}}$

Concept and Vocabulary Check P.5

1. d

2. g

3. b

4. c

5. c

6. a

7. f

8. $(x+1)^{\frac{1}{2}}$

Exercise Set P.5

1. $18x + 27 = 9 \cdot 2x + 9 \cdot 3 = 9(2x + 3)$

2. $16x - 24 = 8(2x) + 8(-3) = 8(2x - 3)$

3. $3x^2 + 6x = 3x \cdot x + 3x \cdot 2 = 3x(x + 2)$

4. $4x^2 - 8x = 4x(x) + 4x(-2) = 4x(x - 2)$

5. $9x^4 - 18x^3 + 27x^2$
$= 9x^2(x^2) + 9x^2(-2x) + 9x^2(3)$
$= 9x^2(x^2 - 2x + 3)$

6. $6x^4 - 18x^3 + 12x^2$
$= 6x^2(x^2) + 6x^2(-3x) + 6x^2(2)$
$= 6x^2\left(x^2 - 3x + 2\right)$

7. $x(x + 5) + 3(x + 5) = (x + 5)(x + 3)$

8. $x(2x + 1) + 4(2x + 1) = (2x + 1)(x + 4)$

9. $x^2(x-3) + 12(x-3) = (x-3)(x^2 + 12)$

10. $x^2\left(2x+5\right) + 17\left(2x+5\right) = \left(2x+5\right)\left(x^2 + 17\right)$

11. $x^3 - 2x^2 + 5x - 10 = x^2(x-2) + 5(x-2)$
$= (x^2 + 5)(x-2)$

12. $x^3 - 3x^2 + 4x - 12 = x^2\left(x-3\right) + 4\left(x-3\right)$
$= \left(x-3\right)\left(x^2 + 4\right)$

13. $x^3 - x^2 + 2x - 2 = x^2(x-1) + 2(x-1)$
$= (x-1)(x^2 + 2)$

14. $x^3 + 6x^2 - 2x - 12 = x^2\left(x+6\right) - 2\left(x+6\right)$
$= \left(x+6\right)\left(x^2 - 2\right)$

15. $3x^3 - 2x^2 - 6x + 4 = x^2(3x-2) - 2(3x-2)$
$= (3x-2)(x^2 - 2)$

16. $x^3 - x^2 - 5x + 5 = x^2\left(x-1\right) - 5\left(x-1\right)$
$= \left(x-1\right)\left(x^2 - 5\right)$

17. $x^2 + 5x + 6 = (x+2)(x+3)$

18. $x^2 + 8x + 15 = (x+3)(x+5)$

19. $x^2 - 2x - 15 = (x - 5)(x + 3)$

20. $x^2 - 4x - 5 = (x-5)(x+1)$

21. $x^2 - 8x + 15 = (x - 5)(x - 3)$

22. $x^2 - 14x + 45 = (x-5)(x-9)$

23. $3x^2 - x - 2 = (3x + 2)(x - 1)$

24. $2x^2+5x-3=(2x-1)(x+3)$

25. $3x^2-25x-28=(3x-28)(x+1)$

26. $3x^2-2x-5=(3x-5)(x+1)$

27. $6x^2-11x+4=(2x-1)(3x-4)$

28. $6x^2-17x+12=(2x-3)(3x-4)$

29. $4x^2+16x+15=(2x+3)(2x+5)$

30. $8x^2+33x+4=(8x+1)(x+4)$

31. $9x^2-9x+2=(3x-1)(3x-2)$

32. $9x^2+5x-4=(9x-4)(x+1)$

33. $20x^2+27x-8=(5x+8)(4x-1)$

34. $15x^2-19x+6=(3x-2)(5x-3)$

35. $2x^2+3xy+y^2=(2x+y)(x+y)$

36. $3x^2+4xy+y^2=(3x+y)(x+y)$

37. $6x^2-5xy-6y^2=(3x+2y)(2x-3y)$

38. $6x^2-7xy-5y^2=(3x-5y)(2x+y)$

39. $x^2-100=x^2-10^2=(x+10)(x-10)$

40. $x^2-144=x^2-12^2=(x+12)(x-12)$

41. $36x^2-49=(6x)^2-7^2=(6x+7)(6x-7)$

42. $64x^2-81=(8x)^2-9^2=(8x+9)(8x-9)$

43. $9x^2-25y^2=(3x)^2-(5y)^2$
$=(3x+5y)(3x-5y)$

44. $36x^2-49y^2=(6x)^2-(7y)^2$
$=(6x+7y)(6x-7y)$

45. $x^4-16=(x^2)^2-4^2$
$=(x^2+4)(x^2-4)$
$=(x^2+4)(x+2)(x-2)$

46. $x^4-1=(x^2)^2-1^2=(x^2+1)(x^2-1)$
$=(x^2+1)(x+1)(x-1)$

47. $16x^4-81=(4x^2)^2-9^2$
$=(4x^2+9)(4x^2-9)$
$=(4x^2+9)[(2x)^2-3^2]$
$=(4x^2+9)(2x+3)(2x-3)$

48. $81x^4-1=(9x^2)^2-1^2$
$=(9x^2+1)(9x^2-1)$
$=(9x^2+1)[(3x)^2-1^2]$
$=(9x^2+1)(3x+1)(3x-1)$

49. $x^2+2x+1=x^2+2\cdot x\cdot 1+1^2=(x+1)^2$

50. $x^2+4x+4=x^2+2\cdot x\cdot 2+2^2=(x+2)^2$

51. $x^2-14x+49=x^2-2\cdot x\cdot 7+7^2$
$=(x-7)^2$

52. $x^2-10x+25=x^2-2\cdot x\cdot 5+5^2=(x-5)^2$

53. $4x^2+4x+1=(2x)^2+2\cdot 2x\cdot 1+1^2$
$=(2x+1)^2$

54. $25x^2+10x+1=(5x)^2+2\cdot 5x\cdot 1+1^2=(5x+1)^2$

55. $9x^2-6x+1=(3x)^2-2\cdot 3x\cdot 1+1^2$
$=(3x-1)^2$

56. $64x^2-16x+1=(8x)^2-2\cdot 8x\cdot 1+1^2=(8x-1)^2$

57. $x^3+27=x^3+3^3$
$=(x+3)(x^2-x\cdot 3+3^2)$
$=(x+3)(x^2-3x+9)$

58. $x^3+64=x^3+4^3$
$=(x+4)(x^2-x\cdot 4+4^2)$
$=(x+4)(x^2-4x+16)$

59. $x^3-64=x^3-4^3$
$=(x-4)(x^2+x\cdot 4+4^2)$
$=(x-4)(x^2+4x+16)$

60. $x^3-27=x^3-3^3$
$=(x-3)(x^2+x\cdot 3+3^2)$
$=(x-3)(x^2+3x+9)$

61. $8x^3-1=(2x)^3-1^3$
$=(2x-1)[(2x)^2+(2x)(1)+1^2]$
$=(2x-1)(4x^2+2x+1)$

62. $27x^3-1=(3x)^3-1^3$
$=(3x-1)[(3x)^2+(3x)(1)+1^2]$
$=(3x-1)(9x^2+3x+1)$

63. $64x^3+27=(4x)^3+3^3$
$=(4x+3)[(4x)^2-(4x)(3)+3^2]$
$=(4x+3)(16x^2-12x+9)$

64. $8x^3+125=(2x)^3+5^3$
$=(2x+5)[(2x)^2-(2x)(5)+5^2]$
$=(2x+5)(4x^2-10x+25)$

65. $3x^3-3x=3x(x^2-1)=3x(x+1)(x-1)$

66. $5x^3-45x=5x(x^2-9)=5x(x+3)(x-3)$

67. $4x^2-4x-24=4(x^2-x-6)$
$=4(x+2)(x-3)$

68. $6x^2-18x-60=6(x^2-3x-10)$
$=6(x+2)(x-5)$

69. $2x^4-162=2(x^4-81)$
$=2[(x^2)^2-9^2]$
$=2(x^2+9)(x^2-9)$
$=2(x^2+9)(x^2-3^2)$
$=2(x^2+9)(x+3)(x-3)$

70. $7x^4-7=7(x^4-1)$
$=7[(x^2)^2-1^2]$
$=7(x^2+1)(x^2-1)$
$=7(x^2+1)(x+1)(x-1)$

71. $x^3+2x^2-9x-18=(x^3+2x^2)-(9x+18)$
$=x^2(x+2)-9(x+2)$
$=(x^2-9)(x+2)$
$=(x^2-3^2)(x+2)$
$=(x-3)(x+3)(x+2)$

72. $x^3+3x^2-25x-75=(x^3+3x^2)-(25x+75)$
$=x^2(x+3)-25(x+3)$
$=(x^2-25)(x+3)$
$=(x^2-5^2)(x+3)$
$=(x-5)(x+5)(x+3)$

73. $2x^2-2x-112=2(x^2-x-56)=2(x-8)(x+7)$

74. $6x^2-6x-12=6(x^2-x-2)$
$=6(x-2)(x+1)$

75. $x^3-4x=x(x^2-4)$
$=x(x^2-2^2)$
$=x(x-2)(x+2)$

76. $9x^3-9x=9x(x^2-1)=9x(x-1)(x+1)$

77. x^2+64 is prime.

78. x^2+36 is prime.

79. $x^3+2x^2-4x-8=(x^3+2x^2)+(-4x-8)$
$=x^2(x+2)-4(x+2)=(x^2-4)(x+2)=(x^2-2^2)(x+2)=(x-2)(x+2)(x+2)=(x-2)(x+2)^2$

80. x^3+2x^2-x-2
$=(x^3+2x^2)+(-x-2)=x^2(x+2)-1(x+2)=(x^2-1)(x+2)=(x^2-1^2)(x+2)=(x-1)(x+1)(x+2)$

81. y^5-81y
$=y(y^4-81)=y[(y^2)^2-9^2]=y(y^2+9)(y^2-9)=y(y^2+9)(y^2-3^2)=y(y^2+9)(y+3)(y-3)$

82. y^5-16y
$=y(y^4-16)=y[(y^2)^2-4^2]=y(y^2+4)(y^2-4)=y(y^2+4)(y^2-2^2)=y(y^2+4)(y+2)(y-2)$

83. $20y^4-45y^2=5y^2(4y^2-9)=5y^2[(2y)^2-3^2]=5y^2(2y+3)(2y-3)$

84. $48y^4-3y^2=3y^2(16y^2-1)=3y^2[(4y)^2-1^2]=3y^2(4y+1)(4y-1)$

85. $x^2-12x+36-49y^2=\left(x^2-12x+36\right)-49y^2=\left(x-6\right)^2-49y^2=(x-6+7y)\left(x-6-7y\right)$

86. $x^2-10x+25-36y^2=(x^2-10x+25)-36y^2=(x-5)^2-36y^2=\left(x-5+6y\right)\left(x-5-6y\right)$

87. $9b^2x-16y-16x+9b^2y$
$=\left(9b^2x+9b^2y\right)+\left(-16x-16y\right)=9b^2\left(x+y\right)-16\left(x+y\right)=\left(x+y\right)\left(9b^2-16\right)=\left(x+y\right)\left(3b+4\right)\left(3b-4\right)$

88. $16a^2x-25y-25x+16a^2y$
$=\left(16a^2x+16a^2y\right)+\left(-25y-25x\right)=16a^2\left(x+y\right)-25\left(x+y\right)=\left(x+y\right)\left(16a^2-25\right)=\left(x+y\right)\left(4a+5\right)\left(4a-5\right)$

89. $x^2y-16y+32-2x^2$
$=\left(x^2y-16y\right)+\left(-2x^2+32\right)=y\left(x^2-16\right)-2\left(x^2-16\right)=\left(x^2-16\right)\left(y-2\right)=\left(x+4\right)\left(x-4\right)\left(y-2\right)$

90. $12x^2y-27y-4x^2+9$
$=\left(12x^2y-27y\right)+\left(-4x^2+9\right)=3y\left(4x^2-9\right)-1\left(4x^2-9\right)=\left(4x^2-9\right)\left(3y-1\right)=\left(2x+3\right)\left(2x-3\right)\left(3y-1\right)$

91. $2x^3-8a^2x+24x^2+72x$
$=2x\left(x^2-4a^2+12x+36\right)=2x\left[\left(x^2+12x+36\right)-4a^2\right]=2x\left[\left(x+6\right)^2-4a^2\right]=2x\left(x+6-2a\right)\left(x+6+2a\right)$

92. $2x^3-98a^2x+28x^2+98x$
$=2x\left(x^2-49a^2+14x+49\right)=2x\left[\left(x^2+14x+49\right)-49a^2\right]=2x\left[\left(x+7\right)^2-49a^2\right]=2x\left(x+7-7a\right)\left(x+7+7a\right)$

93. $x^{\frac{3}{2}}-x^{\frac{1}{2}}=x^{\frac{1}{2}}\left(x^{\frac{3}{2}-\frac{1}{2}}\right)-1=x^{\frac{1}{2}}\left(x-1\right)$

94. $x^{\frac{3}{4}}-x^{\frac{1}{4}}=x^{\frac{1}{4}}\left(x^{\frac{3}{4}-\frac{1}{4}}-1\right)=x^{\frac{1}{4}}\left(x^{\frac{1}{2}}-1\right)$

95. $4x^{-\frac{2}{3}}+8x^{\frac{1}{3}}=4x^{-\frac{2}{3}}\left(1+2x^{\frac{1}{3}-\left(-\frac{2}{3}\right)}\right)=4x^{-\frac{2}{3}}(1+2x)=\dfrac{4(1+2x)}{x^{\frac{2}{3}}}$

96. $12x^{-\frac{3}{4}}+6x^{\frac{1}{4}}=6x^{-\frac{3}{4}}\left(2+x^{\frac{1}{4}-\left(-\frac{3}{4}\right)}\right)=6x^{-\frac{3}{4}}(2+x)=\dfrac{6(x+2)}{x^{\frac{3}{4}}}$

97. $(x+3)^{\frac{1}{2}}-(x+3)^{\frac{3}{2}}=(x+3)^{\frac{1}{2}}\left[1-(x+3)^{\frac{3}{2}-\frac{1}{2}}\right]=(x+3)^{\frac{1}{2}}\left[1-(x+3)\right]=(x+3)^{\frac{1}{2}}(-x-2)=-(x+3)^{\frac{1}{2}}(x+2)$

98. $\left(x^2+4\right)^{\frac{3}{2}}+\left(x^2+4\right)^{\frac{7}{2}}=\left(x^2+4\right)^{\frac{3}{2}}\left[1+\left(x^2+4\right)^{\frac{7}{2}-\frac{3}{2}}\right]=\left(x^2+4\right)^{\frac{3}{2}}\left[1+\left(x^2+4\right)^2\right]=\left(x^2+4\right)^{\frac{3}{2}}\left(x^4+8x^2+17\right)$

99. $(x+5)^{-\frac{1}{2}}-(x+5)^{-\frac{3}{2}}=(x+5)^{-\frac{3}{2}}\left[(x+5)^{-\frac{1}{2}-\left(-\frac{3}{2}\right)}-1\right]=(x+5)^{-\frac{3}{2}}\left[(x+5)-1\right]=(x+5)^{-\frac{3}{2}}(x+4)=\dfrac{x+4}{(x+5)^{\frac{3}{2}}}$

100. $\left(x^2+3\right)^{-\frac{2}{3}}+\left(x^2+3\right)^{-\frac{5}{3}}=\left(x^2+3\right)^{-\frac{5}{3}}\left[\left(x^2+3\right)^{-\frac{2}{3}-\left(-\frac{5}{3}\right)}+1\right]=\left(x^2+3\right)^{-\frac{5}{3}}\left[\left(x^2+3\right)+1\right]=\dfrac{x^2+4}{(x^2+3)^{5/3}}$

101. $(4x-1)^{\frac{1}{2}}-\frac{1}{3}(4x-1)^{\frac{3}{2}}$

$=(4x-1)^{\frac{1}{2}}\left[1-\frac{1}{3}(4x-1)^{\frac{3}{2}-\frac{1}{2}}\right]=(4x-1)^{\frac{1}{2}}\left[1-\frac{1}{3}(4x-1)\right]=(4x-1)^{\frac{1}{2}}\left[1-\frac{4}{3}x+\frac{1}{3}\right]$

$=(4x-1)^{\frac{1}{2}}\left(\frac{4}{3}-\frac{4}{3}x\right)=(4x-1)^{\frac{1}{2}}\frac{4}{3}(1-x)=\dfrac{-4(4x-1)^{\frac{1}{2}}(x-1)}{3}$

102. $-8(4x+3)^{-2}+10(5x+1)(4x+3)^{-1}=2(4x+3)^{-2}\left[-4+5(5x+1)(4x+3)\right]=\dfrac{2(100x^2+95x+11)}{(4x+3)^2}$

103. $10x^2(x+1)-7x(x+1)-6(x+1)=(x+1)\left(10x^2-7x-6\right)=(x+1)(5x-6)(2x+1)$

104. $12x^2(x-1)-4x(x-1)-5(x-1)=(x-1)\left(12x^2-4x-5\right)=(x-1)(6x-5)(2x+1)$

105. $6x^4+35x^2-6=\left(x^2+6\right)\left(6x^2-1\right)$

106. $7x^4+34x^2-5=\left(7x^2-1\right)\left(x^2+5\right)$

107. $y^7 + y = y\left(y^6+1\right) = y\left[\left(y^2\right)^3 + 1^3\right] = y\left(y^2+1\right)\left(y^4 - y^2 + 1\right)$

108. $(y+1)^3 + 1 = (y+1)^3 + 1^3 = \left[(y+1)+1\right]\left[(y+1)^2 - (y+1) + 1\right] = (y+2)\left[\left(y^2+2y+1\right) - y - 1 + 1\right]$
$= (y+2)\left(y^2+2y+1-y-1+1\right) = (y+2)\left(y^2+y+1\right)$

109. $x^4 - 5x^2y^2 + 4y^4 = \left(x^2 - 4y^2\right)\left(x^2 - y^2\right) = (x+2y)(x-2y)(x+y)(x-y)$

110. $x^4 - 10x^2y^2 + 9y^4 = \left(x^2 - 9y^2\right)\left(x^2 - y^2\right) = (x+3y)(x-3y)(x+y)(x-y)$

111. $(x-y)^4 - 4(x-y)^2$
$= (x-y)^2\left((x-y)^2 - 4\right) = (x-y)^2\left((x-y)+2\right)\left((x-y)-2\right) = (x-y)^2(x-y+2)(x-y-2)$

112. $(x+y)^4 - 100(x+y)^2 = (x+y)^2\left((x+y)^2 - 100\right) = (x+y)^2(x+y-10)(x+y+10)$

113. $2x^2 - 7xy^2 + 3y^4 = \left(2x - y^2\right)\left(x - 3y^2\right)$

114. $3x^2 + 5xy^2 + 2y^4 = \left(3x + 2y^2\right)\left(x + y^2\right)$

115. a. $(x-0.4x) - 0.4(x-0.4x) = (x-0.4x)(1-0.4) = (0.6x)(0.6) = 0.36x$

b. No, the computer is selling at 36% of its original price.

116. a. $(x-0.3x) - 0.3(x-0.3x) = (x-0.3x)(1-0.3) = (0.7x)(0.7) = 0.49x$

b. No, the computer is selling at 49% of its original price.

117. a. $(3x)^2 - 4\cdot 2^2 = 9x^2 - 16$

b. $9x^2 - 16 = (3x+4)(3x-4)$

118. a. $(7x)^2 - 4\cdot 3^2 = 49x^2 - 36$

b. $49x^2 - 36 = (7x+6)(7x-6)$

119. a. $x(x+y) - y(x+y)$

b. $x(x+y) - y(x+y) = (x+y)(x-y)$

120. a. $x^2 + xy + xy + y^2 = x^2 + 2xy + y^2$

b. $x^2 + 2xy + y^2 = (x+y)^2$

121. $V_{\text{shaded}} = V_{\text{outside}} - V_{\text{inside}}$

$$= a \cdot a \cdot 4a - b \cdot b \cdot 4a$$
$$= 4a^3 - 4ab^2$$
$$= 4a\left(a^2 - b^2\right)$$
$$= 4a\left(a+b\right)\left(a-b\right)$$

122. $V_{shaded} = V_{outside} - V_{inside}$

$$= a \cdot a \cdot 3a - b \cdot b \cdot 3a$$
$$= 3a^3 - 3ab^2$$
$$= 3a\left(a^2 - b^2\right)$$
$$= 3a\left(a+b\right)\left(a-b\right)$$

123. – 129. Answers will vary.

130. makes sense

131. makes sense

132. does not make sense; Explanations will vary. Sample explanation: $4x^2 - 100 = 4(x^2 - 25) = 4(x+5)(x-5)$

133. makes sense

134. false; Changes to make the statement true will vary. A sample change is:
$x^4 - 16 = (x^2 + 4)(x^2 - 4) = (x^2 + 4)(x+2)(x-2)$

135. true

136. false; Changes to make the statement true will vary. A sample change is: The binomial $x^2 + 36$ is prime.

137. false; Changes to make the statement true will vary. A sample change is: $x^3 - 64 = \left(x-4\right)\left(x+4x+16\right)$

138. $x^{2n} + 6x^n + 8 = \left(x^n + 4\right)\left(x^n + 2\right)$

139. $-x^2 - 4x + 5 = -1\left(x^2 + 4x - 5\right) = -1\left(x+5\right)\left(x-1\right) = -\left(x+5\right)(x-1)$

140. $x^4 - y^4 - 2x^3y + 2xy^3$

$$= \left(x^4 - y^4\right) + \left(-2x^3y + 2xy^3\right)$$
$$= \left(x^2 - y^2\right)\left(x^2 + y^2\right) - 2xy\left(x^2 - y^2\right)$$
$$= \left(x^2 - y^2\right)\left(x^2 + y^2 - 2xy\right)$$
$$= \left(x-y\right)\left(x+y\right)\left(x^2 - 2xy + y^2\right)$$
$$= \left(x-y\right)\left(x+y\right)\left(x-y\right)^2$$
$$= \left(x-y\right)^3\left(x+y\right)$$

141. $(x-5)^{-\frac{1}{2}}(x+5)^{-\frac{1}{2}}-(x+5)^{\frac{1}{2}}(x-5)^{-\frac{3}{2}}=(x-5)^{-\frac{3}{2}}(x+5)^{-\frac{1}{2}}\left[(x-5)^{-\frac{1}{2}-\left(-\frac{3}{2}\right)}-(x+5)^{\frac{1}{2}-\left(-\frac{1}{2}\right)}\right]$

$$=(x-5)^{-\frac{3}{2}}(x+5)^{-\frac{1}{2}}\left[(x-5)-(x+5)\right]$$

$$=(x-5)^{-\frac{3}{2}}(x+5)^{-\frac{1}{2}}(-10)=\frac{-10}{(x-5)^{3/2}(x+5)^{1/2}}$$

142. $x^2+bx+15$, $b=16$, -16, 8 or -8

143. $b = 0, 3, 4$, or $-c(c+4)$, where $c > 0$ is an integer.

144. $\frac{x^2+6x+5}{x^2-25}=\frac{(x+5)(x+1)}{(x+5)(x-5)}=\frac{x+1}{x-5}$

145. $\frac{5}{4}\cdot\frac{8}{15}=\frac{5}{4}\cdot\frac{4\cdot 2}{5\cdot 3}$

$$=\frac{1}{1}\cdot\frac{2}{3}$$

$$=\frac{2}{3}$$

146. $\frac{1}{2}+\frac{2}{3}=\frac{3}{6}+\frac{4}{6}$

$$=\frac{7}{6}$$

Section P.6

Check Point Exercises

1. **a.** The denominator would equal zero if $x=-5$, so –5 must be excluded from the domain.

b. $x^2-36=(x+6)(x-6)$
The denominator would equal zero if $x=-6$ or $x = 6$, so –6 and 6 both must be excluded from the domain.

c. $x^2-5x-14=(x+2)(x-7)$
The denominator would equal zero if $x=-2$ or $x = 7$, so –2 and 7 both must be excluded from the domain.

2. **a.** $\frac{x^3+3x^2}{x+3}=\frac{x^2(x+3)}{x+3}$

$$=\frac{x^2(x+3)}{x+3}$$

$$=x^2,\ x\neq -3$$

Because the denominator is $x + 3$, $x\neq -3$

b. $\frac{x^2-1}{x^2+2x+1}=\frac{(x-1)(x+1)}{(x+1)(x+1)}=\frac{x-1}{x+1}, x\neq -1$

Because the denominator is $(x+1)(x+1), x\neq -1$

3. $\frac{x+3}{x^2-4}\cdot\frac{x^2-x-6}{x^2+6x+9}$

$=\frac{x+3}{(x+2)(x-2)}\cdot\frac{(x-3)(x+2)}{(x+3)(x+3)}$

$=\frac{x+3}{(x+2)(x-2)}\cdot\frac{(x-3)(x+2)}{(x+3)(x+3)}$

$=\frac{x-3}{(x-2)(x+3)},\ x\neq-2,\ x\neq2,\ x\neq-3$

Because the denominator has factors of $x+2,\ x-2,$ and $x+3,\ x\neq-2,\ x\neq2,$ and $x\neq-3$.

4. $\frac{x^2-2x+1}{x^3+x}\div\frac{x^2+x-2}{3x^2+3}$

$=\frac{x^2-2x+1}{x^3+x}\cdot\frac{3x^2+3}{x^2+x-2}$

$=\frac{(x-1)(x-1)}{x(x^2+1)}\cdot\frac{3(x^2+1)}{(x+2)(x-1)}$

$=\frac{3(x-1)}{x(x+2)},\ x\neq1,\ x\neq0,\ x\neq-2$

5. $\frac{x}{x+1}-\frac{3x+2}{x+1}=\frac{x-3x-2}{x+1}$

$=\frac{-2x-2}{x+1}$

$=\frac{-2\left(x+1\right)}{x+1}$

$=-2, x\neq-1$

6. Factor each denominator completely.

$x+1=1(x+1)$

$x-1=1(x-1)$

List the factors of the first denominator.

$1,\ x+1$

Add any unlisted factors from the second denominator.

$1,\ x+1,\ x-1$

The least common denominator is the product of all factors in the final list.

$1(x+1)(x-1)$ or $(x+1)(x-1)$ is the least common denominator.

7. Factor each denominator completely.

$x^2-6x+9=(x-3)^2$

$x^2-9=(x+3)(x-3)$

List the factors of the first denominator.

$x-3,\ x-3$

Add any unlisted factors from the second denominator.

$x-3,\ x-3,\ x+3$

The least common denominator is the product of all factors in the final list.

$(x-3)(x-3)(x+3)$ or $(x-3)^2(x+3)$ is the least common denominator.

8. Find the least common denominator.

$x-3=1(x-3)$

$x+3=1(x+3)$

The least common denominator is $(x-3)(x+3)$.
Write all rational expressions in terms of the least common denominator.

$\frac{x}{x-3}+\frac{x-1}{x+3}$

$=\frac{x(x+3)}{(x-3)(x+3)}+\frac{(x-1)(x-3)}{(x-3)(x+3)}$

Add numerators, putting this sum over the least common denominator.

$=\frac{x(x+3)+(x-1)(x-3)}{(x-3)(x+3)}$

$=\frac{x^2+3x+\left(x^2-4x+3\right)}{(x-3)(x+3)}$

$=\frac{x^2+3x+x^2-4x+3}{(x-3)(x+3)}$

$=\frac{2x^2-x+3}{(x-3)(x+3)},\ x\neq3, x\neq-3$

9. Find the least common denominator.

$x^2-10x+25=(x-5)^2$

$2x-10=2(x-5)$

The least common denominator is $2(x-5)(x-5)$. Write all rational expressions in terms of the least common denominator.

$$\frac{x}{x^2-10x+25}-\frac{x-4}{2x-10}$$
$$=\frac{x}{(x-5)(x-5)}-\frac{x-4}{2(x-5)}$$
$$=\frac{2x}{2(x-5)(x-5)}-\frac{(x-4)(x-5)}{2(x-5)(x-5)}$$

Add numerators, putting this sum over the least common denominator.

$$=\frac{2x-(x-4)(x-5)}{2(x-5)(x-5)}$$
$$=\frac{2x-(x^2-5x-4x+20)}{2(x-5)(x-5)}$$
$$=\frac{2x-x^2+5x+4x-20}{2(x-5)(x-5)}$$
$$=\frac{2x-x^2+5x+4x-20}{2(x-5)(x-5)}$$
$$=\frac{-x^2+11x-20}{2(x-5)(x-5)}$$
$$=\frac{-x^2+11x-20}{2(x-5)^2},\ x\neq 5$$

10. $$\frac{\frac{1}{x}-\frac{3}{2}}{\frac{1}{x}+\frac{3}{4}}=\frac{\frac{2}{2x}-\frac{3x}{2x}}{\frac{4}{4x}+\frac{3x}{4x}},\ x\neq 0$$
$$=\frac{\frac{2-3x}{2x}}{\frac{4+3x}{4x}},\ x\neq\frac{-4}{3}$$
$$=\frac{2-3x}{2x}\div\frac{4+3x}{4x}$$
$$=\frac{2-3x}{2x}\cdot\frac{4x}{4+3x}$$
$$=\frac{2-3x}{4+3x}\cdot\frac{4}{2}$$
$$=\frac{2-3x}{4+3x}\cdot\frac{2}{1}$$
$$=\frac{2(2-3x)}{4+3x},\ x\neq 0,\ x\neq\frac{-4}{3}$$

11. Multiply each of the three terms, $\frac{1}{x+7}$, $\frac{1}{x}$, and 7 by the least common denominator of $x(x+7)$.

$$\frac{\frac{1}{x+7}-\frac{1}{x}}{7}=\frac{x(x+7)\left(\frac{1}{x+7}\right)-x(x+7)\left(\frac{1}{x}\right)}{7x(x+7)}$$
$$=\frac{x-(x+7)}{7x(x+7)}$$
$$=\frac{-7}{7x(x+7)}$$
$$=-\frac{1}{x(x+7)},\ x\neq 0,\ x\neq -7$$

Concept and Vocabulary Check P.6

1. polynomials

2. domain; 0

3. factoring; common factors

4. $\frac{x^2}{15}$

5. $\frac{3}{5}$

6. $\frac{x^2-x+4}{3}$

7. $x+3$ and $x-2$; $x+3$ and $x+1$; $(x+3)(x-2)(x+1)$

8. $3x+4$

9. complex; complex

10. x; $x+3$; -3; $\frac{1}{x(x+3)}$

Exercise Set P.6

1. $\frac{7}{x-3},\ x\neq 3$

2. $\frac{13}{x+9},\ x\neq -9$

3. $\frac{x+5}{x^2-25}=\frac{x+5}{(x+5)(x-5)}$, $x\neq 5, -5$

4. $\frac{x+7}{x^2-49}=\frac{x+7}{(x+7)(x-7)}$, $x\neq 7, -7$

5. $\frac{x-1}{x^2+11x+10}=\frac{x-1}{(x+1)(x+10)}$, $x\neq -1, -10$

6. $\frac{x-3}{x^2+4x-45}=\frac{x-3}{(x+9)(x-5)}$, $x\neq -9, 5$

7. $\frac{3x-9}{x^2-6x+9}=\frac{3(x-3)}{(x-3)(x-3)}$

$=\frac{3}{x-3}$, $x\neq 3$

8. $\frac{4x-8}{x^2-4x+4}=\frac{4(x-2)}{(x-2)(x-2)}=\frac{4}{x-2}$, $x\neq 2$

9. $\frac{x^2-12x+36}{4x-24}=\frac{(x-6)(x-6)}{4(x-6)}=\frac{x-6}{4}$.

$x\neq 6$

10. $\frac{x^2-8x+16}{3x-12}=\frac{(x-4)(x-4)}{3(x-4)}=\frac{x-4}{3}$, $x\neq 4$

11. $\frac{y^2+7y-18}{y^2-3y+2}=\frac{(y+9)(y-2)}{(y-2)(y-1)}=\frac{y+9}{y-1}$,

$y\neq 1, 2$

12. $\frac{y^2-4y-5}{y^2+5y+4}=\frac{(y-5)(y+1)}{(y+4)(y+1)}=\frac{y-5}{y+4}$, $y\neq -4, -1$

13. $\frac{x^2+12x+36}{x^2-36}=\frac{(x+6)^2}{(x+6)(x-6)}=\frac{x+6}{x-6}$,

$x\neq 6, -6$

14. $\frac{x^2-14x+49}{x^2-49}=\frac{(x-7)^2}{(x-7)(x+7)}$

$=\frac{x-7}{x+7}$,

$x\neq 7, -7$

15. $\frac{x-2}{3x+9}\cdot\frac{2x+6}{2x-4}=\frac{x-2}{3(x+3)}\cdot\frac{2(x+3)}{2(x-2)}$

$=\frac{2}{6}=\frac{1}{3}$, $x\neq 2, -3$

16. $\frac{6x+9}{3x-15}\cdot\frac{x-5}{4x+6}=\frac{3(2x+3)}{3(x-5)}\cdot\frac{x-5}{2(2x+3)}$

$=\frac{3}{6}$

$=\frac{1}{2}$,

$x\neq 5, -\frac{3}{2}$

17. $\frac{x^2-9}{x^2}\cdot\frac{x^2-3x}{x^2+x-12}$

$=\frac{(x-3)(x+3)}{x^2}\cdot\frac{x(x-3)}{(x+4)(x-3)}$

$=\frac{(x-3)(x+3)}{x(x+4)}$, $x\neq 0, -4, 3$

18. $\frac{x^2-4}{x^2-4x+4}\cdot\frac{2x-4}{x+2}=\frac{(x+2)(x-2)}{(x-2)^2}\cdot\frac{2(x-2)}{x+2}$

$=2$,

$x\neq 2, -2$

19. $\frac{x^2-5x+6}{x^2-2x-3}\cdot\frac{x^2-1}{x^2-4}$

$=\frac{(x-3)(x-2)}{(x-3)(x+1)}\cdot\frac{(x+1)(x-1)}{(x-2)(x+2)}$

$=\frac{x-1}{x+2}$, $x\neq -2, -1, 2, 3$

20. $\frac{x^2+5x+6}{x^2+x-6}\cdot\frac{x^2-9}{x^2-x-6}$

$=\frac{(x+3)(x+2)}{(x+3)(x-2)}\cdot\frac{(x-3)(x+3)}{(x-3)(x+2)}=\frac{x+3}{x-2}$,

$x\neq -3, -2, 2, 3$

21. $\frac{x^3-8}{x^2-4}\cdot\frac{x+2}{3x}=\frac{(x-2)(x^2+2x+4)}{(x-2)(x+2)}\cdot\frac{x+2}{3x}$

$=\frac{x^2+2x+4}{3x}$, $x\neq -2, 0, 2$

22. $\frac{x^2+6x+9}{x^3+27}\cdot\frac{1}{x+3}$

$=\frac{(x+3)(x+3)}{(x+3)(x^2-3x+9)}\cdot\frac{1}{x+3}=\frac{1}{x^2-3x+9}$,

$x\neq -3$

23. $\dfrac{x+1}{3}\div\dfrac{3x+3}{7}=\dfrac{x+1}{3}\div\dfrac{3(x+1)}{7}$

$=\dfrac{x+1}{3}\cdot\dfrac{7}{3(x+1)}$

$=\dfrac{7}{9},\ x\neq -1$

24. $\dfrac{x+5}{7}\div\dfrac{4x+20}{9}=\dfrac{x+5}{7}\div\dfrac{4(x+5)}{9}$

$=\dfrac{x+5}{7}\cdot\dfrac{9}{4(x+5)}$

$=\dfrac{9}{28},$

$x\neq -5$

25. $\dfrac{x^2-4}{x}\div\dfrac{x+2}{x-2}=\dfrac{(x-2)(x+2)}{x}\cdot\dfrac{x-2}{x+2}$

$=\dfrac{(x-2)^2}{x};\ x\neq 0,-2,2$

26. $\dfrac{x^2-4}{x-2}\div\dfrac{x+2}{4x-8}=\dfrac{(x-2)(x+2)}{x-2}\div\dfrac{x+2}{4(x-2)}$

$=\dfrac{(x-2)(x+2)}{x-2}\cdot\dfrac{4(x-2)}{x+2}$

$=4(x-2),$

$x\neq 2,-2$

27. $\dfrac{4x^2+10}{x-3}\div\dfrac{6x^2+15}{x^2-9}$

$=\dfrac{2(2x^2+5)}{x-3}\div\dfrac{3(2x^2+5)}{(x-3)(x+3)}$

$=\dfrac{2(2x^2+5)}{x-3}\cdot\dfrac{(x-3)(x+3)}{3(2x^2+5)}$

$=\dfrac{2(x+3)}{3},\ x\neq 3,-3$

28. $\dfrac{x^2+x}{x^2-4}\div\dfrac{x^2-1}{x^2+5x+6}$

$=\dfrac{x(x+1)}{(x-2)(x+2)}\div\dfrac{(x-1)(x+1)}{(x+2)(x+3)}$

$=\dfrac{x(x+1)}{(x-2)(x+2)}\cdot\dfrac{(x+2)(x+3)}{(x-1)(x+1)}$

$=\dfrac{x(x+3)}{(x-2)(x-1)},$

$x\neq 2,1,-1,-2,-3$

29. $\dfrac{x^2-25}{2x-2}\div\dfrac{x^2+10x+25}{x^2+4x-5}$

$=\dfrac{(x-5)(x+5)}{2(x-1)}\div\dfrac{(x+5)^2}{(x+5)(x-1)}$

$=\dfrac{(x-5)(x+5)}{2(x-1)}\cdot\dfrac{(x+5)(x-1)}{(x+5)^2}$

$=\dfrac{x-5}{2},\ x\neq 1,-5$

30. $\dfrac{x^2-4}{x^2+3x-10}\div\dfrac{x^2+5x+6}{x^2+8x+15}$

$=\dfrac{(x+2)(x-2)}{(x+5)(x-2)}\div\dfrac{(x+2)(x+3)}{(x+3)(x+5)}$

$=\dfrac{(x+2)(x-2)}{(x+5)(x-2)}\cdot\dfrac{(x+3)(x+5)}{(x+2)(x+3)}$

$=1$

$x\neq 2,-2,-3,-5$

31. $\dfrac{x^2+x-12}{x^2+x-30}\cdot\dfrac{x^2+5x+6}{x^2-2x-3}\div\dfrac{x+3}{x^2+7x+6}$

$=\dfrac{(x+4)(x-3)}{(x+6)(x-5)}\cdot\dfrac{(x+2)(x+3)}{(x+1)(x-3)}\cdot\dfrac{(x+6)(x+1)}{x+3}$

$=\dfrac{(x+4)(x+2)}{x-5}$

$x\neq -6,-3,-1,3,5$

32. $\dfrac{x^3-25x}{4x^2}\cdot\dfrac{2x^2-2}{x^2-6x+5}\div\dfrac{x^2+5x}{7x+7}$

$=\dfrac{x(x-5)(x+5)}{4x^2}\cdot\dfrac{2(x-1)(x+1)}{(x-1)(x-5)}\cdot\dfrac{7(x+1)}{x(x+5)}$

$=\dfrac{7(x+1)^2}{2x^2}$

$x\neq 0,1,-1,5,-5$

33. $\dfrac{4x+1}{6x+5}+\dfrac{8x+9}{6x+5}=\dfrac{4x+1+8x+9}{6x+5}$

$=\dfrac{12x+10}{6x+5}$

$=\dfrac{2(6x+5)}{6x+5}=2,\ x\neq -\dfrac{5}{6}$

34. $\frac{3x+2}{3x+4}+\frac{3x+6}{3x+4}=\frac{3x+2+3x+6}{3x+4}$

$=\frac{6x+8}{3x+4}$

$=\frac{2(3x+4)}{3x+4}$

$=2$

$x\neq -\frac{4}{3}$

35. $\frac{x^2-2x}{x^2+3x}+\frac{x^2+x}{x^2+3x}=\frac{x^2-2x+x^2+x}{x^2+3x}$

$=\frac{2x^2-x}{x^2+3x}$

$=\frac{x(2x-1)}{x(x+3)}$

$=\frac{2x-1}{x+3},\ x\neq 0,-3$

36. $\frac{x^2-4x}{x^2-x-6}+\frac{4x-4}{x^2-x-6}=\frac{x^2-4x+4x-4}{x^2-x-6}$

$=\frac{x^2-4}{(x-3)(x+2)}$

$=\frac{(x-2)(x+2)}{(x-3)(x+2)}$

$=\frac{x-2}{x-3},$

$x\neq -2,\ 3$

37. $\frac{4x-10}{x-2}-\frac{x-4}{x-2}=\frac{4x-10-(x-4)}{x-2}$

$=\frac{4x-10-x+4}{x-2}$

$=\frac{3x-6}{x-2}$

$=\frac{3(x-2)}{x-2}$

$=3,\ x\neq 2$

38. $\frac{2x+3}{3x-6}-\frac{3-x}{3x-6}=\frac{2x+3-(3-x)}{3x-6}$

$=\frac{2x+3-3+x}{3x-6}$

$=\frac{3x}{3(x-2)}$

$=\frac{x}{x-2},$

$x\neq 2$

39. $\frac{x^2+3x}{x^2+x-12}-\frac{x^2-12}{x^2+x-12}$

$=\frac{x^2+3x-(x^2-12)}{x^2+x-12}$

$=\frac{x^2+3x-x^2+12}{x^2+x-12}$

$=\frac{3x+12}{x^2+x-12}$

$=\frac{3(x+4)}{(x+4)(x-3)}$

$=\frac{3}{x-3},\ x\neq 3,-4$

40. $\frac{x^2-4x}{x^2-x-6}-\frac{x-6}{x^2-x-6}$

$=\frac{x^2-4x-(x-6)}{x^2-x-6}$

$=\frac{x^2-4x-x+6}{x^2-x-6}$

$=\frac{x^2-5x+6}{x^2-x-6}$

$=\frac{(x-2)(x-3)}{(x-3)(x+2)}$

$=\frac{x-2}{x+2},\ x\neq -2,\ 3$

41. $\frac{3}{x+4}+\frac{6}{x+5}=\frac{3(x+5)+6(x+4)}{(x+4)(x+5)}$

$=\frac{3x+15+6x+24}{(x+4)(x+5)}$

$=\frac{9x+39}{(x+4)(x+5)},\ x\neq -4,\ -5$

42. $\frac{8}{x-2}+\frac{2}{x-3}=\frac{8(x-3)+2(x-2)}{(x-2)(x-3)}$

$=\frac{8x-24+2x-4}{(x-2)(x-3)}$

$=\frac{10x-28}{(x-2)(x-3)},$

$x\neq 2,\ 3$

43. $\frac{3}{x+1}-\frac{3}{x}=\frac{3x-3(x+1)}{x(x+1)}$

$=\frac{3x-3x-3}{x(x+1)}=-\frac{3}{x(x+1)}$, $x \neq -1, 0$

44. $\frac{4}{x}-\frac{3}{x+3}=\frac{4(x+3)-3x}{x(x+3)}$

$=\frac{4x+12-3x}{x(x+3)}$

$=\frac{x+12}{x(x+3)}$

$x \neq -3, 0$

45. $\frac{2x}{x+2}+\frac{x+2}{x-2}=\frac{2x(x-2)+(x+2)(x+2)}{(x+2)(x-2)}$

$=\frac{2x^2-4x+x^2+4x+4}{(x+2)(x-2)}$

$=\frac{3x^2+4}{(x+2)(x-2)}$, $x \neq -2, 2$

46. $\frac{3x}{x-3}-\frac{x+4}{x+2}=\frac{3x(x+2)-(x+4)(x-3)}{(x-3)(x+2)}$

$=\frac{3x^2+6x-\left(x^2+x-12\right)}{(x-3)(x+2)}$

$=\frac{2x^2+5x+12}{(x-3)(x+2)}$,

$x \neq 3, -2$

47. $\frac{x+5}{x-5}+\frac{x-5}{x+5}$

$=\frac{(x+5)(x+5)+(x-5)(x-5)}{(x-5)(x+5)}$

$=\frac{x^2+10x+25+x^2-10x+25}{(x-5)(x+5)}$

$=\frac{2x^2+50}{(x-5)(x+5)}$, $x \neq -5, 5$

48. $\frac{x+3}{x-3}+\frac{x-3}{x+3}=\frac{(x+3)(x+3)+(x-3)(x-3)}{(x-3)(x+3)}$

$=\frac{x^2+6x+9+x^2-6x+9}{(x-3)(x+3)}$

$=\frac{2x^2+18}{(x-3)(x+3)}$,

$x \neq -3, 3$

49. $\frac{3}{2x+4}+\frac{2}{3x+6}=\frac{3}{2(x+2)}+\frac{2}{3(x+2)}$

$=\frac{9}{6(x+2)}+\frac{4}{6(x+2)}$

$=\frac{9+4}{6(x+2)}$

$=\frac{13}{6(x+2)}$

$x \neq -2$

50. $\frac{5}{2x+8}+\frac{7}{3x+12}=\frac{5}{2(x+4)}+\frac{7}{3(x+4)}$

$=\frac{15}{6(x+4)}+\frac{14}{6(x+4)}$

$=\frac{15+14}{6(x+4)}$

$=\frac{29}{6(x+4)}$

$x \neq -4$

51. $\frac{4}{x^2+6x+9}+\frac{4}{x+3}=\frac{4}{(x+3)^2}+\frac{4}{x+3}$

$=\frac{4+4(x+3)}{(x+3)^2}=\frac{4+4x+12}{(x+3)^2}=\frac{4x+16}{(x+3)^2}$,

$x \neq -3$

52. $\frac{3}{5x+2}+\frac{5x}{25x^2-4}=\frac{3}{5x+2}+\frac{5x}{(5x-2)(5x+2)}$

$=\frac{3(5x-2)+5x}{(5x-2)(5x+2)}$

$=\frac{15x-6+5x}{(5x-2)(5x+2)}$

$=\frac{20x-6}{(5x-2)(5x+2)}$,

$x \neq -\frac{2}{5}, \frac{2}{5}$

53. $\dfrac{3x}{x^2+3x-10}-\dfrac{2x}{x^2+x-6}$

$=\dfrac{3x}{(x+5)(x-2)}-\dfrac{2x}{(x+3)(x-2)}$

$=\dfrac{3x(x+3)-2x(x+5)}{(x+5)(x-2)(x+3)}$

$=\dfrac{3x^2+9x-2x^2-10x}{(x+5)(x-2)(x+3)}$

$=\dfrac{x^2-x}{(x+5)(x-2)(x+3)}$, $x \neq -5, 2, -3$

54. $\dfrac{x}{x^2-2x-24}-\dfrac{x}{x^2-7x+6}$

$=\dfrac{x}{(x-6)(x+4)}-\dfrac{x}{(x-6)(x-1)}$

$=\dfrac{x(x-1)-x(x+4)}{(x-6)(x+4)(x-1)}$

$=\dfrac{x^2-x-x^2-4x}{(x-6)(x+4)(x-1)}$

$=-\dfrac{5x}{(x-6)(x-1)(x+4)}$,

$x \neq 6, 1, -4$

55. $\dfrac{x+3}{x^2-1}-\dfrac{x+2}{x-1}$

$=\dfrac{x+3}{(x+1)(x-1)}-\dfrac{x+2}{x-1}$

$=\dfrac{x+3}{(x+1)(x-1)}-\dfrac{(x+1)(x+2)}{(x+1)(x-1)}$

$=\dfrac{x+3}{(x+1)(x-1)}-\dfrac{x^2+3x+2}{(x+1)(x-1)}$

$=\dfrac{x+3-x^2-3x-2}{(x+1)(x-1)}$

$=\dfrac{-x^2-2x+1}{(x+1)(x-1)}$

$x \neq 1, -1$

56. $\dfrac{x+5}{x^2-4}-\dfrac{x+1}{x-2}$

$=\dfrac{x+5}{(x+2)(x-2)}-\dfrac{x+1}{x-2}$

$=\dfrac{x+5}{(x+2)(x-2)}-\dfrac{(x+2)(x+1)}{(x+2)(x-2)}$

$=\dfrac{x+5}{(x+2)(x-2)}-\dfrac{x^2+3x+2}{(x+2)(x-2)}$

$=\dfrac{x+5-x^2-3x-2}{(x+2)(x-2)}$

$=\dfrac{-x^2-2x+3}{(x+2)(x-2)}$

$x \neq 2, -2$

57. $\dfrac{4x^2+x-6}{x^2+3x+2}-\dfrac{3x}{x+1}+\dfrac{5}{x+2}$

$=\dfrac{4x^2+x-6}{(x+1)(x+2)}+\dfrac{-3x}{x+1}+\dfrac{5}{x+2}$

$=\dfrac{4x^2+x-5}{(x+1)(x+2)}+\dfrac{-3x(x+2)}{(x+1)(x+2)}+\dfrac{5(x+1)}{(x+1)(x+2)}$

$=\dfrac{4x^2+x-6-3x^2-6x+5x+5}{(x+1)(x+2)}$

$=\dfrac{x^2-1}{(x+1)(x+2)}$

$=\dfrac{(x-1)(x+1)}{(x+1)(x+2)}$

$=\dfrac{x-1}{x+2}; x \neq -2, -1$

58. $\dfrac{6x^2+17x-40}{x^2+x-20}+\dfrac{3}{x-4}-\dfrac{5x}{x+5}$

$=\dfrac{6x^2+17x-40}{(x+5)(x-4)}+\dfrac{3}{x-4}-\dfrac{5x}{x+5}$

$=\dfrac{6x^2+17x-40+3(x+5)-5x(x-4)}{(x+5)(x-4)}$

$=\dfrac{6x^2+17x-40+3x+15-5x^2+20x}{(x+5)(x-4)}$

$=\dfrac{x^2+40x-25}{(x+5)(x-4)}; x \neq -5, 4$

59. $\dfrac{\frac{x}{3}-1}{x-3}=\dfrac{3\left[\frac{x}{3}-1\right]}{3[x-3]}=\dfrac{x-3}{3(x-3)}=\dfrac{1}{3},\ x\neq 3$

60. $\dfrac{\frac{x}{4}-1}{x-4}=\dfrac{4\left[\frac{x}{4}-1\right]}{4(x-4)}=\dfrac{x-4}{4(x-4)}=\dfrac{1}{4},\ x\neq 4$

61. $\dfrac{1+\frac{1}{x}}{3-\frac{1}{x}}=\dfrac{x\left[1+\frac{1}{x}\right]}{x\left[3-\frac{1}{x}\right]}=\dfrac{x+1}{3x-1},\ x\neq 0,\frac{1}{3}$

62. $\dfrac{8+\frac{1}{x}}{4-\frac{1}{x}}=\dfrac{x\left[8+\frac{1}{x}\right]}{x\left[4-\frac{1}{x}\right]}=\dfrac{8x+1}{4x-1},\ x\neq 0,\frac{1}{4}$

63. $\dfrac{\frac{1}{x}+\frac{1}{y}}{x+y}=\dfrac{xy\left[\frac{1}{x}+\frac{1}{y}\right]}{xy[x+y]}=\dfrac{y+x}{xy(x+y)}=\dfrac{1}{xy},$
$x\neq 0,\ y\neq 0,\ x\neq -y$

64. $\dfrac{1-\frac{1}{x}}{xy}=\dfrac{x\left[1-\frac{1}{x}\right]}{x(xy)}=\dfrac{x-1}{x^2y},\ x\neq 0,\ y\neq 0$

65. $\dfrac{x-\frac{x}{x+3}}{x+2}=\dfrac{(x+3)\left[x-\frac{x}{x+3}\right]}{(x+3)(x+2)}=\dfrac{x(x+3)-x}{(x+3)(x+2)}$
$=\dfrac{x^2+3x-x}{(x+3)(x+2)}=\dfrac{x^2+2x}{(x+3)(x+2)}$
$=\dfrac{x(x+2)}{(x+3)(x+2)}=\dfrac{x}{x+3},\ x\neq -2,-3$

66. $\dfrac{x-3}{x-\frac{3}{x-2}}=\dfrac{(x-2)[x-3]}{(x-2)\left[x-\frac{3}{x-2}\right]}=\dfrac{(x-2)(x-3)}{x(x-2)-3}$
$=\dfrac{(x-2)(x-3)}{x^2-2x-3}$
$=\dfrac{(x-2)(x-3)}{(x-3)(x+1)}=\dfrac{x-2}{x+1},\ x\neq 2,3,-1$

67. $\dfrac{\frac{3}{x-2}-\frac{4}{x+2}}{\frac{7}{x^2-4}}=\dfrac{\frac{3}{x-2}-\frac{4}{x+2}}{\frac{7}{(x-2)(x+2)}}$
$=\dfrac{\left[\frac{3}{x-2}-\frac{4}{x+2}\right](x-2)(x+2)}{\left[\frac{7}{(x-2)(x+2)}\right](x-2)(x+2)}$
$=\dfrac{3(x+2)-4(x-2)}{7}$
$=\dfrac{3x+6-4x+8}{7}=\dfrac{-x+14}{7}$
$=-\dfrac{x-14}{7}\quad x\neq -2,\ 2$

68. $\dfrac{\frac{x}{x-2}+1}{\frac{3}{x^2-4}+1}=\dfrac{\frac{x}{x-2}+1}{\frac{3}{(x-2)(x+2)}+1}$
$=\dfrac{\left[\frac{x}{x-2}+1\right](x-2)(x+2)}{\left[\frac{3}{(x-2)(x+2)}+1\right](x-2)(x+2}$
$=\dfrac{x(x+2)+(x-2)(x+2)}{3+(x-2)(x+2)}$
$=\dfrac{x^2+2x+x^2-4}{3+x^2-4}=\dfrac{2x^2+2x-4}{x^2-1}$
$=\dfrac{2(x^2+x-2)}{(x-1)(x+1)}$
$=\dfrac{2(x+2)(x-1)}{(x-1)(x+1)}=\dfrac{2(x+2)}{x+1},$
$x\neq 1,-1,\ 2,\ -2$

69. $\dfrac{\frac{1}{x+1}}{\frac{1}{x^2-2x-3}+\frac{1}{x-3}}=\dfrac{\frac{1}{x+1}}{\frac{1}{(x+1)(x-3)}+\frac{1}{x-3}}$
$=\dfrac{\frac{(x+1)(x-3)}{x+1}}{\frac{(x+1)(x-3)}{(x+1)(x-3)}+\frac{(x+1)(x-3)}{x-3}}$
$=\dfrac{x-3}{1+x+1}$
$=\dfrac{x-3}{x+2}\quad x\neq -2,-1,3$

70. $$\frac{\frac{6}{x^2+2x-15}-\frac{1}{x-3}}{\frac{1}{x+5}+1}=\frac{\frac{6}{(x+5)(x-3)}-\frac{1}{x-3}}{\frac{1}{x+5}+1}$$

$$=\frac{\frac{6(x+5)(x-3)}{(x+5)(x-3)}-\frac{(x+5)(x-3)}{x-3}}{\frac{(x+5)(x-3)}{x+5}+(x+5)(x-3)}$$

$$=\frac{6-(x+5)}{(x-3)+(x+5)(x-3)}$$

$$=\frac{6-x-5}{x-3+x^2+2x-15}$$

$$=\frac{1-x}{x^2+3x-18}$$

$$=\frac{1-x}{(x+6)(x-3)} \quad x\neq -6,-5,3$$

71. $$\frac{\frac{1}{(x+h)^2}-\frac{1}{x^2}}{h}=\frac{\frac{x^2(x+h)^2}{(x+h)^2}-\frac{x^2(x+h)^2}{x^2}}{hx^2(x+h)^2}$$

$$=\frac{x^2-(x+h)^2}{hx^2(x+h)^2}$$

$$=\frac{x^2-(x^2+2hx+h^2)}{hx^2(x+h)^2}$$

$$=\frac{x^2-x^2-2hx-h^2}{hx^2(x+h)^2}$$

$$=\frac{-2hx-h^2}{hx^2(x+h)^2}$$

$$=\frac{-h(2x+h)}{hx^2(x+h)^2}$$

$$=-\frac{(2x+h)}{x^2(x+h)^2}$$

72. $$\frac{\frac{x+h}{x+h+1}-\frac{x}{x+1}}{h}=\frac{\frac{(x+h)(x+h+1)(x+1)}{x+h+1}-\frac{x(x+h+1)(x+1)}{x+1}}{h(x+h+1)(x+1)}$$

$$=\frac{(x+h)(x+1)-x(x+h+1)}{h(x+h+1)(x+1)}$$

$$=\frac{x^2+x+hx+h-x^2-hx-x}{h(x+h+1)(x+1)}$$

$$=\frac{h}{h(x+h+1)(x+1)}$$

$$=\frac{1}{(x+h+1)(x+1)}$$

73. $\left(\frac{2x+3}{x+1}\cdot\frac{x^2+4x-5}{2x^2+x-3}\right)-\frac{2}{x+2}=\left(\frac{\cancel{(2x+3)}}{x+1}\cdot\frac{(x+5)\cancel{(x-1)}}{\cancel{(2x+3)}\cancel{(x-1)}}\right)-\frac{2}{x+2}=\frac{x+5}{x+1}-\frac{2}{x+2}$

$=\frac{(x+5)(x+2)}{(x+1)(x+2)}-\frac{2(x+1)}{(x+1)(x+2)}=\frac{(x+5)(x+2)-2(x+1)}{(x+1)(x+2)}=\frac{x^2+2x+5x+10-2x-2}{(x+1)(x+2)}=\frac{x^2+5x+8}{(x+1)(x+2)}$

74. $\frac{1}{x^2-2x-8}\cdot\left(\frac{1}{x-4}-\frac{1}{x+2}\right)=\frac{1}{(x-4)(x+2)}\div\left(\frac{(x+2)}{(x-4)(x+2)}-\frac{(x-4)}{(x-4)(x+2)}\right)$

$=\frac{1}{(x-4)(x+2)}\div\left(\frac{x+2-x+4}{(x-4)(x+2)}\right)=\frac{1}{(x-4)(x+2)}\div\left(\frac{6}{(x-4)(x+2)}\right)=\frac{1}{(x-4)(x+2)}\cdot\frac{(x-4)(x+2)}{6}=\frac{1}{6}$

75. $\left(2-\frac{6}{x+1}\right)\left(1+\frac{3}{x-2}\right)=\left(\frac{2(x+1)}{(x+1)}-\frac{6}{(x+1)}\right)\left(\frac{(x-2)}{(x-2)}+\frac{3}{(x-2)}\right)$

$=\left(\frac{2x+2-6}{x+1}\right)\left(\frac{x-2+3}{x-2}\right)=\left(\frac{2x-4}{x+1}\right)\left(\frac{x+1}{x-2}\right)=\frac{2\cancel{(x-2)}\cancel{(x+1)}}{\cancel{(x+1)}\cancel{(x-2)}}=2$

76. $\left(4-\frac{3}{x+2}\right)\left(1+\frac{5}{x-1}\right)=\left(\frac{4(x+2)}{x+2}-\frac{3}{x+2}\right)\left(\frac{(x-1)}{x-1}+\frac{5}{x-1}\right)$

$=\left(\frac{4x+8-3}{x+2}\right)\left(\frac{x-1+5}{x-1}\right)=\frac{4x+5}{x+2}\cdot\frac{x+4}{x-1}=\frac{(4x+5)(x+4)}{(x+2)(x-1)}$

77. $\frac{y^{-1}-(y+5)^{-1}}{5}=\frac{\frac{1}{y}-\frac{1}{y+5}}{5}$

LCD $= y(y+5)$

$\frac{\frac{1}{y}-\frac{1}{y+5}}{5}=\frac{y(y+5)\left(\frac{1}{y}-\frac{1}{y+5}\right)}{y(y+5)(5)}=\frac{y+5-y}{5y(y+5)}=\frac{5}{5y(y+5)}=\frac{1}{y(y+5)}$

78. $\frac{y^{-1}-(y+2)^{-1}}{2}=\frac{\frac{1}{y}-\frac{1}{y+2}}{2}$

LCD $= y(y+2)$

$\frac{\frac{1}{y}-\frac{1}{y+2}}{2}=\frac{y(y+2)\left(\frac{1}{y}-\frac{1}{y+2}\right)}{y(y+2)(2)}=\frac{y+2-y}{2y(y+2)}=\frac{2}{2y(y+2)}=\frac{1}{y(y+2)}$

79. $\left(\dfrac{1}{a^3-b^3}\cdot\dfrac{ac+ad-bc-bd}{1}\right)-\dfrac{c-d}{a^2+ab+b^2}=\left(\dfrac{1}{(a-b)\left(a^2+ab+b^2\right)}\cdot\dfrac{a(c+d)-b(c+d)}{1}\right)-\dfrac{c-d}{a^2+ab+b^2}$

$=\left(\dfrac{1}{\cancel{(a-b)}\left(a^2+ab+b^2\right)}\cdot\dfrac{(c+d)\cancel{(a-b)}}{1}\right)-\dfrac{c-d}{a^2+ab+b^2}=\dfrac{c+d}{a^2+ab+b^2}-\dfrac{c-d}{a^2+bd+b^2}$

$=\dfrac{c+d-c+d}{a^2+ab+b^2}=\dfrac{2d}{a^2+ab+b^2}$

80. $\dfrac{ab}{a^2+ab+b^2}+\left(\dfrac{ac-ad-bc+bd}{ac-ad+bc-bd}\div\dfrac{a^3-b^3}{a^3+b^3}\right)=\dfrac{ab}{a^2+ab+b^2}+\left(\dfrac{a(c-d)-b(c-d)}{a(c-d)+b(c-d)}\cdot\dfrac{a^3+b^3}{a^3-b^3}\right)$

$=\dfrac{ab}{a^2+ab+b^2}+\left(\dfrac{\cancel{(c-d)}\cancel{(a-b)}}{\cancel{(c-d)}\cancel{(a+b)}}\cdot\dfrac{\cancel{(a+b)}\left(a^2-ab+b^2\right)}{\cancel{(a-b)}\left(a^2+ab+b^2\right)}\right)=\dfrac{ab}{a^2+ab+b^2}+\dfrac{a^2-ab+b^2}{a^2+ab+b^2}$

$=\dfrac{ab+a^2-ab+b^2}{a^2+ab+b^2}=\dfrac{a^2+b^2}{a^2+ab+b^2}$

81. **a.** $\dfrac{130x}{100-x}$ is equal to

1. $\dfrac{130\cdot 40}{100-40}=\dfrac{130\cdot 40}{60}=86.67$,
   when $x = 40$
2. $\dfrac{130\cdot 80}{100-80}=\dfrac{130\cdot 80}{20}=520$,
   when $x = 80$
3. $\dfrac{130\cdot 90}{100-90}=\dfrac{130\cdot 90}{10}=1170$,
   when $x = 90$

It costs \$86,670,000 to inoculate 40% of the population against this strain of flu, and \$520,000,000 to inoculate 80% of the population, and \$1,170,000,000 to inoculate 90% of the population.

b. For $x = 100$, the function is not defined.

c. As x approaches 100, the value of the function increases rapidly. So it costs an astronomical amount of money to inoculate almost all of the people, and it is impossible to inoculate 100% of the population.

82. $\dfrac{2d}{\dfrac{d}{r_1}+\dfrac{d}{r_2}}$

LCD = r_1r_2

$$\frac{2d}{\frac{d}{r_1}+\frac{d}{r_2}} = \frac{r_1r_2(2d)}{r_1r_2\left(\frac{d}{r_1}+\frac{d}{r_2}\right)}$$
$$= \frac{2r_1r_2d}{r_2d+r_1d}$$
$$= \frac{2r_1r_2d}{d(r_2+r_1)} = \frac{2r_1r_2}{r_2+r_1}$$

If $r_1 = 40$ and $r_2 = 30$, the value of this expression will be

$$\frac{2\cdot 40\cdot 30}{30+40} = \frac{2400}{70}$$
$$= 34\frac{2}{7}.$$

Your average speed will be $34\frac{2}{7}$ miles per hour.

83. **a.** Substitute 4 for x in the model.

$W = -66x^2 + 526x + 1030$

$W = -66(4)^2 + 526(4) + 1030$

$W = 2078$

According to the model, women between the ages of 19 and 30 with this lifestyle need 2078 calories per day. This underestimates the actual value shown in the bar graph by 22 calories.

b. Substitute 4 for x in the model.

$M = -120x^2 + 998x + 590$

$M = -120(4)^2 + 998(4) + 590$

$M = 2662$

According to the model, men between the ages of 19 and 30 with this lifestyle need 2662 calories per day. This underestimates the actual value shown in the bar graph by 38 calories.

c. $$\frac{W}{M} = \frac{-66x^2+526x+1030}{-120x^2+998x+590}$$
$$= \frac{2(-33x^2+263x+515)}{2(-60x^2+499x+295)}$$
$$= \frac{-33x^2+263x+515}{-60x^2+499x+295}$$

84. $P = 2L + 2W$

$$= 2\left(\frac{x}{x+3}\right) + 2\left(\frac{x}{x-4}\right)$$

$$= \frac{2x}{x+3} + \frac{2x}{x+4}$$

$$= \frac{2x(x+4)}{(x+3)(x+4)} + \frac{2x(x+3)}{(x+3)(x+4)}$$

$$= \frac{2x^2 + 8x + 2x^2 + 6x}{(x+3)(x+4)}$$

$$= \frac{4x^2 + 14x}{(x+3)(x+4)}$$

85. $P = 2L + 2W$

$$= 2\left(\frac{x}{x+5}\right) + 2\left(\frac{x}{x+6}\right)$$

$$= \frac{2x}{x+5} + \frac{2x}{x+6}$$

$$= \frac{2x(x+6)}{(x+5)(x+6)} + \frac{2x(x+5)}{(x+5)(x+6)}$$

$$= \frac{2x^2 + 12x + 2x^2 + 10x}{(x+5)(x+6)}$$

$$= \frac{4x^2 + 22x}{(x+5)(x+6)}$$

86. – 97. Answers will vary.

98. does not make sense; Explanations will vary. Sample explanation: $\frac{3x-3}{4x(x-1)} = \frac{3(1)-3}{4(1)(1-1)} = \frac{0}{0}$ which is undefined.

99. does not make sense; Explanations will vary. Sample explanation: The numerator and denominator of $\frac{7}{14+x}$ do not share a common factor.

100. does not make sense; Explanations will vary. Sample explanation: The first step is to invert the second fraction.

101. makes sense

102. false; Changes to make the statement true will vary. A sample change is: $\frac{x^2-25}{x-5} = \frac{(x+5)(x-5)}{x-5} = x+5$

103. true

104. true

105. false; Changes to make the statement true will vary. A sample change is: $6 + \frac{1}{x} = \frac{6x}{x} + \frac{1}{x} = \frac{6x+1}{x}$

106. $\dfrac{1}{x^n-1}-\dfrac{1}{x^n+1}-\dfrac{1}{x^{2n}-1}$

$=\dfrac{x^n+1}{x^{2n}-1}-\dfrac{x^n-1}{x^{2n}-1}-\dfrac{1}{x^{2n}-1}$

$=\dfrac{x^n+1-x^n+1-1}{x^{2n}-1}$

$=\dfrac{1}{x^{2n}-1}$

107. $\left(1-\dfrac{1}{x}\right)\left(1-\dfrac{1}{x+1}\right)\left(1-\dfrac{1}{x+2}\right)\left(1-\dfrac{1}{x+3}\right)=\left(\dfrac{x}{x}-\dfrac{1}{x}\right)\left(\dfrac{x+1}{x+1}-\dfrac{1}{x+1}\right)\left(\dfrac{x+2}{x+2}-\dfrac{1}{x+2}\right)\left(\dfrac{x+3}{x+3}-\dfrac{1}{x+3}\right)$

$=\left(\dfrac{x-1}{x}\right)\left(\dfrac{(x+1)-1}{x+1}\right)\left(\dfrac{(x+2)-1}{x+2}\right)\left(\dfrac{(x+3)-1}{x+3}\right)$

$=\dfrac{x-1}{\cancel{x}}\cdot\dfrac{\cancel{x}}{\cancel{x+1}}\cdot\dfrac{\cancel{x+1}}{\cancel{x+2}}\cdot\dfrac{\cancel{x+2}}{x+3}=\dfrac{x-1}{x+3}$

108. $(x-y)^{-1}+(x-y)^{-2}=\dfrac{1}{(x-y)}+\dfrac{1}{(x-y)^2}=\dfrac{(x-y)}{(x-y)(x-y)}+\dfrac{1}{(x-y)^2}=\dfrac{x-y+1}{(x-y)^2}$

109. It cubes x.

$$\dfrac{\dfrac{1}{x}+\dfrac{1}{x^2}+\dfrac{1}{x^3}}{\dfrac{1}{x^4}+\dfrac{1}{x^5}+\dfrac{1}{x^6}}=\dfrac{\dfrac{x^6}{x}+\dfrac{x^6}{x^2}+\dfrac{x^6}{x^3}}{\dfrac{x^6}{x^4}+\dfrac{x^6}{x^5}+\dfrac{x^6}{x^6}}=\dfrac{x^5+x^4+x^3}{x^2+x+1}=\dfrac{x^3\left(x^2+x+1\right)}{x^2+x+1}=x^3$$

110. $y=4-x^2$

111. $y=1-x^2$

112. $y=|x+1|$

x	$y=\lvert x+1\rvert$
−4	$\lvert -4+1\rvert=3$
−3	$\lvert -3+1\rvert=2$
−2	$\lvert -2+1\rvert=1$
−1	$\lvert -1+1\rvert=0$
0	$\lvert 0+1\rvert=1$
1	$\lvert 1+1\rvert=2$
2	$\lvert 2+1\rvert=3$

Chapter P Review Exercises

1. $3+6(x-2)^3 = 3+6(4-2)^3$
$= 3+6(2)^3$
$= 3+6(8)$
$= 3+48$
$= 51$

2. $x^2-5(x-y) = 6^2-5(6-2)$
$= 36-5(4)$
$= 36-20$
$= 16$

3. $S = 0.015x^2+x+10$
$S = 0.015(60)^2+(60)+10$
$= 0.015(3600)+60+10$
$= 54+60+10$
$= 124$

4. $A = \{a,b,c\} \quad B = \{a,c,d,e\}$
$\{a,b,c\} \cap \{a,c,d,e\} = \{a,c\}$

5. $A = \{a,b,c\} \quad B = \{a,c,d,e\}$
$\{a,b,c\} \cup \{a,c,d,e\} = \{a,b,c,d,e\}$

6. $A = \{a,b,c\} \quad C = \{a,d,f,g\}$
$\{a,b,c\} \cup \{a,d,f,g\} = \{a,b,c,d,f,g\}$

7. $A = \{a,b,c\} \quad C = \{a,d,f,g\}$
$\{a,d,f,g\} \cap \{a,b,c\} = \{a\}$

8. **a.** $\sqrt{81}$

b. $0, \sqrt{81}$

c. $-17, 0, \sqrt{81}$

d. $-17, -\frac{9}{13}, 0, 0.75, \sqrt{81}$

e. $\sqrt{2}, \pi$

f. $-17, -\frac{9}{13}, 0, 0.75, \sqrt{2}, \pi, \sqrt{81}$

9. $|-103| = 103$

10. $\left|\sqrt{2}-1\right| = \sqrt{2}-1$

11. $\left|3-\sqrt{17}\right| = \sqrt{17}-3$ since $\sqrt{17}$ is greater than 3.

12. $|4-(-17)| = |4+17| = |21| = 21$

13. 3 + 17 = 17 + 3;
commutative property of addition.

14. $(6\cdot 3)\cdot 9 = 6\cdot(3\cdot 9)$;
associative property of multiplication.

15. $\sqrt{3}(\sqrt{5}+\sqrt{3}) = \sqrt{15}+3$;
distributive property of multiplication over addition.

16. $(6\cdot 9)\cdot 2 = 2\cdot(6\cdot 9)$;
commutative property of multiplication.

17. $\sqrt{3}(\sqrt{5}+\sqrt{3}) = (\sqrt{5}+\sqrt{3})\sqrt{3}$;
commutative property of multiplication.

18. $(3\cdot 7)+(4\cdot 7) = (4\cdot 7)+(3\cdot 7)$;
commutative property of addition.

19. $5(2x-3)+7x = 10x-15+7x = 17x-15$

20. $\frac{1}{5}(5x)+\left[(3y)+(-3y)\right]-(-x) = x+\left[0\right]+x = 2x$

21. $3(4y-5)-(7y+2) = 12y-15-7y-2 = 5y-17$

22. $8-2[3-(5x-1)] = 8-2[3-5x+1]$
$= 8-2[4-5x]$
$= 8-8+10x$
$= 10x$

23. $D = 0.005x^2+0.55x+34$
$D = 0.005(30)^2+0.55(30)+34$
$= 55$
The U.S. diversity index was 55% in 2010.
This is the same as the value displayed in the bar graph.

24. $(-3)^3(-2)^2 = (-27)\cdot(4) = -108$

25. $2^{-4}+4^{-1}=\frac{1}{2^4}+\frac{1}{4}$
$=\frac{1}{16}+\frac{1}{4}$
$=\frac{1}{16}+\frac{4}{16}$
$=\frac{5}{16}$

26. $5^{-3}\cdot 5=5^{-3}5^1=5^{-3+1}=5^{-2}=\frac{1}{5^2}=\frac{1}{25}$

27. $\frac{3^3}{3^6}=3^{3-6}=3^{-3}=\frac{1}{3^3}=\frac{1}{27}$

28. $(-2x^4y^3)^3=(-2)^3(x^4)^3(y^3)^3$
$=(-2)^3x^{4\cdot 3}y^{3\cdot 3}$
$=-8x^{12}y^9$

29. $(-5x^3y^2)(-2x^{-11}y^{-2})$
$=(-5)(-2)x^3x^{-11}y^2y^{-2}$
$=10\cdot x^{3-11}y^{2-2}$
$=10x^{-8}y^0$
$=\frac{10}{x^8}$

30. $(2x^3)^{-4}=(2)^{-4}(x^3)^{-4}$
$=2^{-4}x^{-12}$
$=\frac{1}{2^4x^{12}}$
$=\frac{1}{16x^{12}}$

31. $\frac{7x^5y^6}{28x^{15}y^{-2}}=\left(\frac{7}{28}\right)(x^{5-15})(y^{6-(-2)})$
$=\frac{1}{4}x^{-10}y^8$
$=\frac{y^8}{4x^{10}}$

32. $3.74\times 10^4=37,400$

33. $7.45\times 10^{-5}=0.0000745$

34. $3,590,000=3.59\times 10^6$

35. $0.00725=7.25\times 10^{-3}$

36. $(3\times 10^3)(1.3\times 10^2)=(3\times 1.3)\times(10^3\times 10^2)$
$=3.9\times 10^5$
$=390,000$

37. $\frac{6.9\times 10^3}{3\times 10^5}=\left(\frac{6.9}{3}\right)\times 10^{3-5}$
$=2.3\times 10^{-2}$
$=0.023$

38. 1.35×10^{12}

39. $32,000,000=3.2\times 10^7$

40. $\frac{1.35\times 10^{12}}{3.2\times 10^7}=\frac{1.35}{3.2}\cdot\frac{10^{12}}{10^7}\approx 0.42188\times 10^5=42,188$
1.35×10^{12} seconds is approximately 42,188 years.

41. $\sqrt{300}=\sqrt{100\cdot 3}=\sqrt{100}\cdot\sqrt{3}=10\sqrt{3}$

42. $\sqrt{12x^2}=\sqrt{4x^2\cdot 3}=\sqrt{4x^2}\cdot\sqrt{3}=2|x|\sqrt{3}$

43. $\sqrt{10x}\cdot\sqrt{2x}=\sqrt{20x^2}$
$=\sqrt{4x^2}\cdot\sqrt{5}$
$=2x\sqrt{5}$

44. $\sqrt{r^3}=\sqrt{r^2}\cdot\sqrt{r}=r\sqrt{r}$

45. $\sqrt{\frac{121}{4}}=\frac{\sqrt{121}}{\sqrt{4}}=\frac{11}{2}$

46. $\frac{\sqrt{96x^3}}{\sqrt{2x}}=\sqrt{\frac{96x^3}{2x}}$
$=\sqrt{48x^2}$
$=\sqrt{16x^2}\cdot\sqrt{3}$
$=4x\sqrt{3}$

47. $7\sqrt{5}+13\sqrt{5}=(7+13)\sqrt{5}=20\sqrt{5}$

48. $2\sqrt{50}+3\sqrt{8}=2\sqrt{25\cdot 2}+3\sqrt{4\cdot 2}$
$=2\cdot 5\sqrt{2}+3\cdot 2\sqrt{2}$
$=10\sqrt{2}+6\sqrt{2}$
$=16\sqrt{2}$

49. $4\sqrt{72}-2\sqrt{48}=4\sqrt{36\cdot 2}-2\sqrt{16\cdot 3}$
$=4\cdot 6\sqrt{2}-2\cdot 4\sqrt{3}$
$=24\sqrt{2}-8\sqrt{3}$

50. $\frac{30}{\sqrt{5}}=\frac{30}{\sqrt{5}}\cdot\frac{\sqrt{5}}{\sqrt{5}}=\frac{30\sqrt{5}}{5}=6\sqrt{5}$

51. $\frac{\sqrt{2}}{\sqrt{3}}=\frac{\sqrt{2}}{\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}}=\frac{\sqrt{6}}{3}$

52. $\frac{5}{6+\sqrt{3}}=\frac{5}{6+\sqrt{3}}\cdot\frac{6-\sqrt{3}}{6-\sqrt{3}}$
$=\frac{5(6-\sqrt{3})}{36-3}$
$=\frac{5(6-\sqrt{3})}{33}$

53. $\frac{14}{\sqrt{7}-\sqrt{5}}=\frac{14}{\sqrt{7}-\sqrt{5}}\cdot\frac{\sqrt{7}+\sqrt{5}}{\sqrt{7}+\sqrt{5}}$
$=\frac{14(\sqrt{7}+\sqrt{5})}{7-5}$
$=\frac{14(\sqrt{7}+\sqrt{5})}{2}$
$=7(\sqrt{7}+\sqrt{5})$

54. $\sqrt[3]{125}=5$

55. $\sqrt[5]{-32}=-2$

56. $\sqrt[4]{-125}$ is not a real number.

57. $\sqrt[4]{(-5)^4}=\sqrt[4]{625}=\sqrt[4]{5^4}=5$

58. $\sqrt[3]{81}=\sqrt[3]{27\cdot 3}=\sqrt[3]{27}\cdot\sqrt[3]{3}=3\sqrt[3]{3}$

59. $\sqrt[3]{y^5}=\sqrt[3]{y^3y^2}=y\sqrt[3]{y^2}$

60. $\sqrt[4]{8}\cdot\sqrt[4]{10}=\sqrt[4]{80}=\sqrt[4]{16\cdot 5}=\sqrt[4]{16}\cdot\sqrt[4]{5}=2\sqrt[4]{5}$

61. $4\sqrt[3]{16}+5\sqrt[3]{2}=4\sqrt[3]{8\cdot 2}+5\sqrt[3]{2}$
$=4\cdot 2\sqrt[3]{2}+5\sqrt[3]{2}$
$=8\sqrt[3]{2}+5\sqrt[3]{2}$
$=13\sqrt[3]{2}$

62. $\frac{\sqrt[4]{32x^5}}{\sqrt[4]{16x}}=\sqrt[4]{\frac{32x^5}{16x}}=\sqrt[4]{2x^4}=x\sqrt[4]{2}$

63. $16^{1/2}=\sqrt{16}=4$

64. $25^{-1/2}=\frac{1}{25^{1/2}}=\frac{1}{\sqrt{25}}=\frac{1}{5}$

65. $125^{1/3}=\sqrt[3]{125}=5$

66. $27^{-1/3}=\frac{1}{27^{1/3}}=\frac{1}{\sqrt[3]{27}}=\frac{1}{3}$

67. $64^{2/3}=(\sqrt[3]{64})^2=4^2=16$

68. $27^{-4/3}=\frac{1}{27^{4/3}}=\frac{1}{(\sqrt[3]{27})^4}=\frac{1}{3^4}=\frac{1}{81}$

69. $(5x^{2/3})(4x^{1/4})=5\cdot 4x^{2/3+1/4}=20x^{11/12}$

70. $\frac{15x^{3/4}}{5x^{1/2}}=\left(\frac{15}{5}\right)x^{3/4-1/2}=3x^{1/4}$

71. $(125\cdot x^6)^{2/3}=(\sqrt[3]{125x^6})^2$
$=(5x^2)^2$
$=25x^4$

72. $\sqrt[6]{y^3}=(y^3)^{1/6}=y^{3\cdot 1/6}=y^{1/2}=\sqrt{y}$

73. $(-6x^3+7x^2-9x+3)+(14x^3+3x^2-11x-7)=(-6x^3+14x^3)+(7x^2+3x^2)+(-9x-11x)+(3-7)$
$=8x^3+10x^2-20x-4$
The degree is 3.

74. $(13x^4-8x^3+2x^2)-(5x^4-3x^3+2x^2-6)=(13x^4-8x^3+2x^2)+(-5x^4+3x^3-2x^2+6)$
$=(13x^4-5x^4)+(-8x^3+3x^3)+(2x^2-2x^2)+6$
$=8x^4-5x^3+6$
The degree is 4.

75. $(3x-2)(4x^2+3x-5)=(3x)(4x^2)+(3x)(3x)+(3x)(-5)+(-2)(4x^2)+(-2)(3x)+(-2)(-5)$
$=12x^3+9x^2-15x-8x^2-6x+10$
$=12x^3+x^2-21x+10$

76. $(3x-5)(2x+1)=(3x)(2x)+(3x)(1)+(-5)(2x)+(-5)(1)$
$=6x^2+3x-10x-5$
$=6x^2-7x-5$

77. $(4x+5)(4x-5)=(4x^2)-5^2=16x^2-25$

78. $(2x+5)^2=(2x)^2+2(2x)\cdot 5+5^2=4x^2+20x+25$

79. $(3x-4)^2=(3x)^2-2(3x)\cdot 4+(-4)^2=9x^2-24x+16$

80. $(2x+1)^3=(2x)^3+3(2x)^2(1)+3(2x)(1)^2+1^3=8x^3+12x^2+6x+1$

81. $(5x-2)^3=(5x)^3-3(5x)^2(2)+3(5x)(2)^2-2^3=125x^3-150x^2+60x-8$

82. $(7x^2-8xy+y^2)+(-8x^2-9xy-4y^2)=(7x^2-8x^2)+(-8xy-9xy)+(y^2-4y^2)$
$=-x^2-17xy-3y^2$
The degree is 2.

83. $(13x^3y^2-5x^2y-9x^2)-(-11x^3y^2-6x^2y+3x^2-4)$
$=(13x^3y^2-5x^2y-9x^2)+(11x^3y^2+6x^2y-3x^2+4)$
$=(13x^3y^2+11x^3y^2)+(-5x^2y+6x^2y)+(-9x^2-3x^2)+4$
$=24x^3y^2+x^2y-12x^2+4$
The degree is 5.

84. $(x+7y)(3x-5y)=x(3x)+(x)(-5y)+(7y)(3x)+(7y)(-5y)$
$=3x^2-5xy+21xy-35y^2$
$=3x^2+16xy-35y^2$

85. $(3x-5y)^2=(3x)^2-2(3x)(5y)+(-5y)^2$
$=9x^2-30xy+25y^2$

86. $(3x^2+2y)^2 = (3x^2)^2 + 2(3x^2)(2y) + (2y)^2$
$= 9x^4 + 12x^2y + 4y^2$

87. $(7x+4y)(7x-4y) = (7x)^2 - (4y)^2$
$= 49x^2 - 16y^2$

88. $(a-b)(a^2+ab+b^2)$
$= a(a^2) + a(ab) + a(b^2) + (-b)(a^2)$
$+(-b)(ab) + (-b)(b^2)$
$= a^3 + a^2b + ab^2 - a^2b - ab^2 - b^3$
$= a^3 - b^3$

89. $15x^3 + 3x^2 = 3x^2 \cdot 5x + 3x^2 \cdot 1$
$= 3x^2(5x+1)$

90. $x^2 - 11x + 28 = (x-4)(x-7)$

91. $15x^2 - x - 2 = (3x+1)(5x-2)$

92. $64 - x^2 = 8^2 - x^2 = (8-x)(8+x)$

93. $x^2 + 16$ is prime.

94. $3x^4 - 9x^3 - 30x^2 = 3x^2(x^2 - 3x - 10)$
$= 3x^2(x-5)(x+2)$

95. $20x^7 - 36x^3 = 4x^3(5x^4 - 9)$

96. $x^3 - 3x^2 - 9x + 27 = x^2(x-3) - 9(x-3)$
$= (x^2-9)(x-3)$
$= (x+3)(x-3)(x-3)$
$= (x+3)(x-3)^2$

97. $16x^2 - 40x + 25 = (4x-5)(4x-5)$
$= (4x-5)^2$

98. $x^4 - 16 = (x^2)^2 - 4^2$
$= (x^2+4)(x^2-4)$
$= (x^2+4)(x+2)(x-2)$

99. $y^3 - 8 = y^3 - 2^3 = (y-2)(y^2+2y+4)$

100. $x^3 + 64 = x^3 + 4^3 = (x+4)(x^2-4x+16)$

101. $3x^4 - 12x^2 = 3x^2(x^2-4)$
$= 3x^2(x-2)(x+2)$

102. $27x^3 - 125 = (3x)^3 - 5^3$
$= (3x-5)[(3x)^2 + (3x)(5) + 5^2]$
$= (3x-5)(9x^2+15x+25)$

103. $x^5 - x = x(x^4-1)$
$= x(x^2-1)(x^2+1)$
$= x(x-1)(x+1)(x^2+1)$

104. $x^3 + 5x^2 - 2x - 10 = x^2(x+5) - 2(x+5)$
$= (x^2-2)(x+5)$

105. $x^2 + 18x + 81 - y^2 = \left(x^2+18x+81\right) - y^2$
$= \left(x+9\right)^2 - y^2$
$= \left(x+9-y\right)\left(x+9+y\right)$

106. $16x^{-3/4} + 32x^{1/4} = 16x^{-3/4}\left(1 + 2x^{1/4-(-3/4)}\right)$
$= 16x^{-3/4}\left(1+2x\right)$
$= \dfrac{16\left(1+2x\right)}{x^{3/4}}$

107. $\left(x^2-4\right)\left(x^2+3\right)^{\frac{1}{2}} - \left(x^2-4\right)^2\left(x^2+3\right)^{\frac{3}{2}}$
$= \left(x^2-4\right)\left(x^2+3\right)^{\frac{1}{2}}\left[1 - \left(x^2-4\right)\left(x^2+3\right)\right]$
$= \left(x-2\right)\left(x+2\right)\left(x^2+3\right)^{\frac{1}{2}}\left[1 - \left(x-2\right)\left(x+2\right)\left(x^2+3\right)\right]$
$= (x-2)(x+2)(x^2+3)^{\frac{1}{2}}(-x^4+x^2+13)$

108. $12x^{-\frac{1}{2}} + 6x^{-\frac{3}{2}} = 6x^{-\frac{3}{2}}\left(2x+1\right) = \dfrac{6(2x+1)}{x^{\frac{3}{2}}}$

109. $\dfrac{x^3+2x^2}{x+2} = \dfrac{x^2(x+2)}{x+2} = x^2$, $x \neq -2$

110. $\dfrac{x^2+3x-18}{x^2-36} = \dfrac{(x+6)(x-3)}{(x+6)(x-6)} = \dfrac{x-3}{x-6}$,
$x \neq -6, 6$

111. $\dfrac{x^2+2x}{x^2+4x+4} = \dfrac{x(x+2)}{(x+2)^2} = \dfrac{x}{x+2}$,
$x \neq -2$

112. $\frac{x^2+6x+9}{x^2-4}\cdot\frac{x+3}{x-2}=\frac{(x+3)^2}{(x-2)(x+2)}\cdot\frac{x+3}{x-2}$

$=\frac{(x+3)^3}{(x-2)^2(x+2)},$

$x \neq 2, -2$

113. $\frac{6x+2}{x^2-1}\div\frac{3x^2+x}{x-1}$

$=\frac{2(3x+1)}{(x-1)(x+1)}\div\frac{x(3x+1)}{x-1}$

$=\frac{2(3x+1)}{(x-1)(x+1)}\cdot\frac{x-1}{x(3x+1)}$

$=\frac{2}{x(x+1)},$

$x \neq 0, 1, -1, -\frac{1}{3}$

114. $\frac{x^2-5x-24}{x^2-x-12}\div\frac{x^2-10x+16}{x^2+x-6}$

$=\frac{(x-8)(x+3)}{(x-4)(x+3)}\div\frac{(x-2)(x-8)}{(x+3)(x-2)}$

$=\frac{x-8}{x-4}\cdot\frac{x+3}{x-8}$

$=\frac{x+3}{x-4},$

$x \neq -3, 4, 2, 8$

115. $\frac{2x-7}{x^2-9}-\frac{x-10}{x^2-9}=\frac{2x-7-(x-10)}{x^2-9}$

$=\frac{x+3}{(x+3)(x-3)}$

$=\frac{1}{x-3},$

$x \neq 3, -3$

116. $\frac{3x}{x+2}+\frac{x}{x-2}=\frac{3x}{x+2}\cdot\frac{x-2}{x-2}+\frac{x}{x-2}\cdot\frac{x+2}{x+2}$

$=\frac{3x^2-6x+x^2+2x}{(x+2)(x-2)}$

$=\frac{4x^2-4x}{(x+2)(x-2)}$

$=\frac{4x(x-1)}{(x+2)(x-2)},$

$x \neq 2, -2$

117. $\frac{x}{x^2-9}+\frac{x-1}{x^2-5x+6}$

$=\frac{x}{(x-3)(x+3)}+\frac{x-1}{(x-2)(x-3)}$

$=\frac{x}{(x-3)(x+3)}\cdot\frac{x-2}{x-2}+\frac{x-1}{(x-2)(x-3)}\cdot\frac{x+3}{x+3}$

$=\frac{x(x-2)+(x-1)(x+3)}{(x-3)(x+3)(x-2)}$

$=\frac{x^2-2x+x^2+2x-3}{(x-3)(x+3)(x-2)}$

$=\frac{2x^2-3}{(x-3)(x+3)(x-2)}$

$x \neq 3, -3, 2$

118. $\frac{4x-1}{2x^2+5x-3}-\frac{x+3}{6x^2+x-2}$

$=\frac{4x-1}{(2x-1)(x+3)}-\frac{x+3}{(2x-1)(3x+2)}$

$=\frac{4x-1}{(2x-1)(x+3)}\cdot\frac{3x+2}{3x+2}$

$-\frac{x+3}{(2x-1)(3x+2)}\cdot\frac{x+3}{x+3}$

$=\frac{12x^2+8x-3x-2-x^2-6x-9}{(2x-1)(x+3)(3x+2)}$

$=\frac{11x^2-x-11}{(2x-1)(x+3)(3x+2)},$

$x \neq \frac{1}{2}, -3, -\frac{2}{3}$

119. $\frac{\frac{1}{x}-\frac{1}{2}}{\frac{1}{3}-\frac{x}{6}}=\frac{\frac{1}{x}-\frac{1}{2}}{\frac{1}{3}-\frac{x}{6}}\cdot\frac{6x}{6x}$

$=\frac{6-3x}{2x-x^2}$

$=\frac{-3(x-2)}{-x(x-2)}$

$=\frac{3}{x},$

$x \neq 0, 2$

120. $\dfrac{3+\frac{12}{x}}{1-\frac{16}{x^2}}=\dfrac{3+\frac{12}{x}}{1-\frac{16}{x^2}}\cdot\dfrac{x^2}{x^2}$

$=\dfrac{3x^2+12x}{x^2-16}$

$=\dfrac{3x(x+4)}{(x+4)(x-4)}$

$=\dfrac{3x}{x-4},$

$x\neq 0, 4, -4$

121. $\dfrac{3-\frac{1}{x+3}}{3+\frac{1}{x+3}}=\dfrac{3-\frac{1}{x+3}}{3+\frac{1}{x+3}}\cdot\dfrac{x+3}{x+3}$

$=\dfrac{3(x+3)-1}{3(x+3)+1}$

$=\dfrac{3x+9-1}{3x+9+1}$

$=\dfrac{3x+8}{3x+10},$

$x\neq -3, -\dfrac{10}{3}$

Chapter P Test

1. $5(2x^2-6x)-(4x^2-3x)=10x^2-30x-4x^2+3x$

$=6x^2-27x$

2. $7+2[3(x+1)-2(3x-1)]$

$=7+2[3x+3-6x+2]$

$=7+2[-3x+5]$

$=7-6x+10$

$=-6x+17$

3. $\{1,2,5\}\cap\{5,a\}=\{5\}$

4. $\{1,2,5\}\cup\{5,a\}=\{1,2,5,a\}$

5. $(2x^2y^3-xy+y^2)-(-4x^2y^3-5xy-y^2)$

$=2x^2y^3-xy+y^2+4x^2y^3+5xy+y^2$

$=2x^2y^3+4x^2y^3-xy+5xy+y^2+y^2$

$=6x^2y^3+4xy+2y^2$

6. $\dfrac{30x^3y^4}{6x^9y^{-4}}=5x^{3-9}y^{4-(-4)}=5x^{-6}y^8=\dfrac{5y^8}{x^6}$

7. $\sqrt{6r}\cdot\sqrt{3r}=\sqrt{18r^2}=\sqrt{9r^2}\cdot\sqrt{2}=3r\sqrt{2}$

8. $4\sqrt{50}-3\sqrt{18}=4\sqrt{25\cdot 2}-3\sqrt{9\cdot 2}$

$=4\cdot 5\sqrt{2}-3\cdot 3\sqrt{2}$

$=20\sqrt{2}-9\sqrt{2}$

$=11\sqrt{2}$

9. $\dfrac{3}{5+\sqrt{2}}=\dfrac{3}{5+\sqrt{2}}\cdot\dfrac{5-\sqrt{2}}{5-\sqrt{2}}$

$=\dfrac{3(5-\sqrt{2})}{25-2}$

$=\dfrac{3(5-\sqrt{2})}{23}$

10. $\sqrt[3]{16x^4}=\sqrt[3]{8x^3\cdot 2x}$

$=\sqrt[3]{8x^3}\cdot\sqrt[3]{2x}$

$=2x\sqrt[3]{2x}$

11. $\dfrac{x^2+2x-3}{x^2-3x+2}=\dfrac{(x+3)(x-1)}{(x-2)(x-1)}=\dfrac{x+3}{x-2},$

$x\neq 2, 1$

12. $\dfrac{5\times 10^{-6}}{20\times 10^{-8}}=\dfrac{5}{20}\cdot\dfrac{10^{-6}}{10^{-8}}=0.25\times 10^2=2.5\times 10^1$

13. $(2x-5)(x^2-4x+3)$

$=2x^3-8x^2+6x-5x^2+20x-15$

$=2x^3-13x^2+26x-15$

14. $(5x+3y)^2=(5x)^2+2(5x)(3y)+(3y)^2$

$=25x^2+30xy+9y^2$

15. $\dfrac{2x+8}{x-3}\div\dfrac{x^2+5x+4}{x^2-9}$

$=\dfrac{2(x+4)}{x-3}\div\dfrac{(x+1)(x+4)}{(x-3)(x+3)}$

$=\dfrac{2(x+4)}{x-3}\cdot\dfrac{(x-3)(x+3)}{(x+1)(x+4)}$

$=\dfrac{2(x+3)}{x+1},$

$x\neq 3, -1, -4, -3$

16. $\frac{x}{x+3}+\frac{5}{x-3}$

$=\frac{x}{x+3}\cdot\frac{x-3}{x-3}+\frac{5}{x-3}\cdot\frac{x+3}{x+3}$

$=\frac{x(x-3)+5(x+3)}{(x+3)(x-3)}$

$=\frac{x^2-3x+5x+15}{(x+3)(x-3)}$

$=\frac{x^2+2x+15}{(x+3)(x-3)}, x\neq 3,-3$

17. $\frac{2x+3}{x^2-7x+12}-\frac{2}{x-3}$

$=\frac{2x+3}{(x-3)(x-4)}-\frac{2}{x-3}$

$=\frac{2x+3}{(x-3)(x-4)}-\frac{2}{x-3}\cdot\frac{x-4}{x-4}$

$=\frac{2x+3-2(x-4)}{(x-3)(x-4)}$

$=\frac{2x+3-2(x-4)}{(x-3)(x-4)}$

$=\frac{2x+3-2x+8}{(x-3)(x-4)}$

$=\frac{11}{(x-3)(x-4)},$

$x\neq 3, 4$

18. $\frac{\frac{1}{x}-\frac{1}{3}}{\frac{1}{x}}=\frac{\frac{1}{x}-\frac{1}{3}}{\frac{1}{x}}\cdot\frac{3x}{3x}=\frac{3-x}{3},$

$x\neq 0$

19. $x^2-9x+18=(x-3)(x-6)$

20. $x^3+2x^2+3x+6=x^2(x+2)+3(x+2)$

$=(x^2+3)(x+2)$

21. $25x^2-9=(5x)^2-3^2=(5x-3)(5x+3)$

22. $36x^2-84x+49=(6x)^2-2(6x)\cdot 7+7^2$

$=(6x-7)^2$

23. $y^3-125=y^3-5^3=(y-5)(y^2+5y+25)$

24. $(x^2+10x+25)-9y^2$

$=(x+5)^2-9y^2$

$=(x+5-3y)(x+5+3y)$

25. $x(x+3)^{-\frac{3}{5}}+(x+3)^{\frac{2}{5}}$

$=(x+3)^{-\frac{3}{5}}\left[x+(x+3)\right]$

$=(x+3)^{-\frac{3}{5}}(2x+3)=\frac{2x+3}{(x+3)^{\frac{3}{5}}}$

26. $-7,-\frac{4}{5},0,0.25,\sqrt{4},\frac{22}{7}$ are rational numbers.

27. 3(2 + 5) = 3(5 + 2);
commutative property of addition

28. $6(7+4)=6\cdot 7+6\cdot 4$
distributive property of multiplication over addition

29. $0.00076=7.6\times 10^{-4}$

30. $27^{-\frac{5}{3}}=\frac{1}{27^{\frac{5}{3}}}=\frac{1}{\left(\sqrt[3]{27}\right)^5}=\frac{1}{(3)^5}=\frac{1}{243}$

31. $2\left(6.6\times 10^9\right)=13.2\times 10^9=1.32\times 10^{10}$

32. a. 2003 is 14 years after 1989.

$M=-0.28n+47$

$M=-0.28(14)+47$

$=43.08$

In 2003, 43.08% of bachelor's degrees were awarded to men. This overestimates the actual percent shown by the bar graph by 0.08%.

b. $R=\frac{M}{W}=\frac{-0.28n+47}{0.28n+53}$

c. $R=\frac{-0.28n+47}{0.28n+53}$

$R=\frac{-0.28(25)+47}{0.28(25)+53}$

$=\frac{2}{3}$

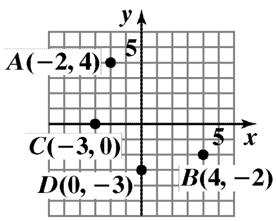
Three women received bachelor's degrees for every two men. This describes the data exactly.

Chapter 1
Equations and Inequalities

Section 1.1

Check Point Exercises

1. Plot points:



2. $x = -3,\ y = 7$

$x = -2,\ y = 6$

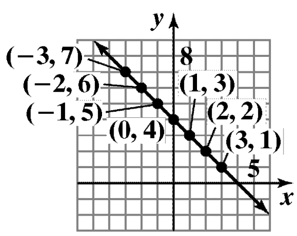
$x = -1,\ y = 5$

$x = 0,\ y = 4$

$x = 1,\ y = 3$

$x = 2,\ y = 2$

$x = 3,\ y = 1$



$y = 4 - x$

3. $x = -4,\ y = 3$

$x = -3,\ y = 2$

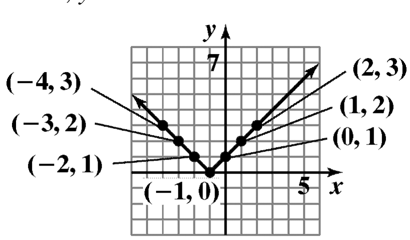
$x = -2,\ y = 1$

$x = -1,\ y = 0$

$x = 0,\ y = 1$

$x = 1,\ y = 2$

$x = 2,\ y = 3$



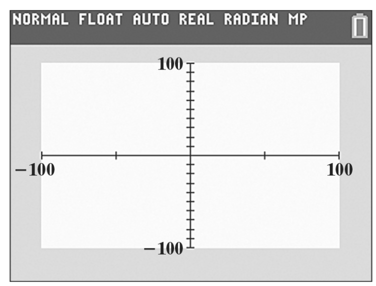
$y = |x + 1|$

4. The meaning of a $[-100,100,50]$ by $[-100,100,10]$ viewing rectangle is as follows:

$$[\overbrace{-100}^{\substack{\text{minimum}\\ x\text{-value}}},\ \overbrace{100}^{\substack{\text{maximum}\\ x\text{-value}}},\ \overbrace{50}^{\substack{\text{distance}\\ \text{between}\\ x\text{-axis}\\ \text{tick}\\ \text{marks}}}]$$

by

$$[\overbrace{-100}^{\substack{\text{minimum}\\ y\text{-value}}},\ \overbrace{100}^{\substack{\text{maximum}\\ y\text{-value}}},\ \overbrace{10}^{\substack{\text{distance}\\ \text{between}\\ y\text{-axis}\\ \text{tick}\\ \text{marks}}}]$$



5. a. The graph crosses the x-axis at (–3, 0).
Thus, the x-intercept is –3.
The graph crosses the y-axis at (0, 5).
Thus, the y-intercept is 5.

b. The graph does not cross the x-axis.
Thus, there is no x-intercept.
The graph crosses the y-axis at (0, 4).
Thus, the y-intercept is 4.

c. The graph crosses the x- and y-axes at the origin (0, 0).
Thus, the x-intercept is 0 and the y-intercept is 0.

d. The graph crosses the x-axis at (–1, 0) and (1, 0).
Thus, the x-intercepts are –1 and 1.
The graph crosses the y-axis at (0, 3).
Thus, the y-intercept is 3.

6. a. $d = 4n + 5$

$d = 4(15) + 5 = 65$

65% of marriages end in divorce after 15 years when the wife is under 18 at the time of marriage.

b. According to the line graph, 60% of marriages end in divorce after 15 years when the wife is under 18 at the time of marriage.

c. The mathematical model overestimates the actual percentage shown in the graph by 5%.

Concept and Vocabulary Check 1.1

1. x-axis

2. y-axis

3. origin

4. quadrants; four

5. x-coordinate; y-coordinate

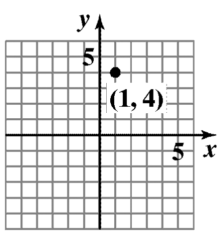
6. solution; satisfies

7. x-intercept; zero

8. y-intercept; zero

Exercise Set 1.1

1.



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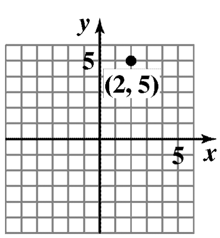

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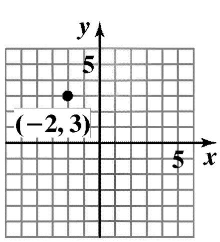

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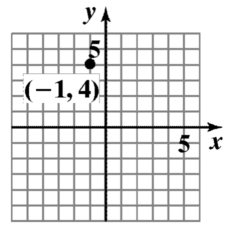

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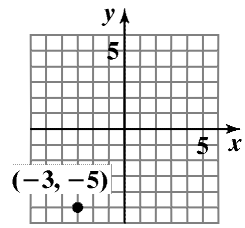

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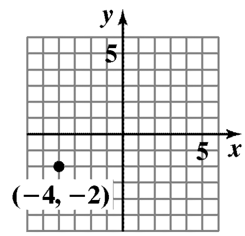

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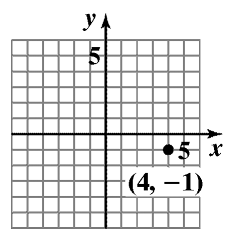

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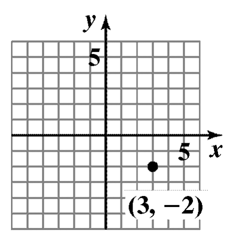

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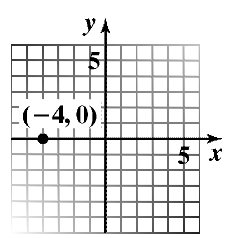
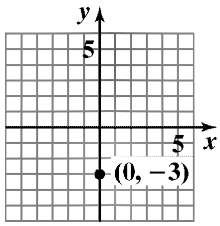
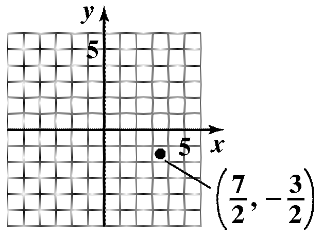


10.



11.



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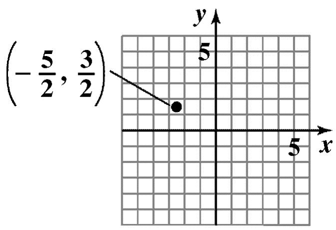

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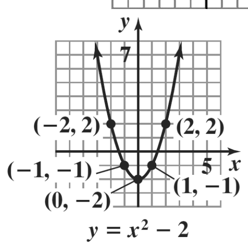


$x = -3,\ y = 7$

$x = -2,\ y = 2$

$x = -1,\ y = -1$

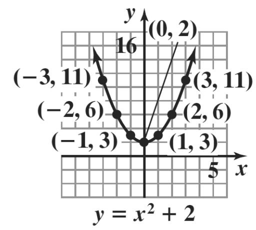
$x = 0,\ y = -2$

$x = 1,\ y = -1$

$x = 2,\ y = 2$

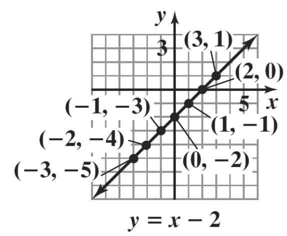
$x = 3,\ y = 7$

14.



$x = -3,\ y = 11$
$x = -2,\ y = 6$
$x = -1,\ y = 3$
$x = 0,\ y = 2$
$x = 1,\ y = 3$
$x = 2,\ y = 6$
$x = 3,\ y = 11$

15.



$x = -3,\ y = -5$

$x = -2,\ y = -4$

$x = -1,\ y = -3$

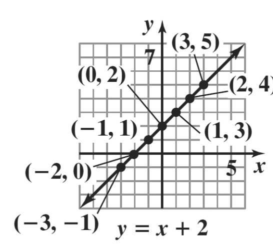
$x = 0,\ y = -2$

$x = 1,\ y = -1$

$x = 2,\ y = 0$

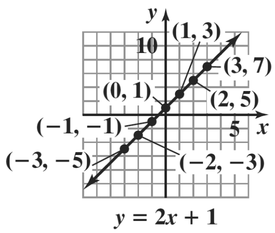
$x = 3,\ y = 1$

16.



$x = -3,\ y = -1$
$x = -2,\ y = 0$
$x = -1,\ y = 1$
$x = 0,\ y = 2$
$x = 1,\ y = 3$
$x = 2,\ y = 4$
$x = 3,\ y = 5$

17.



$x = -3,\ y = -5$

$x = -2,\ y = -3$

$x = -1,\ y = -1$

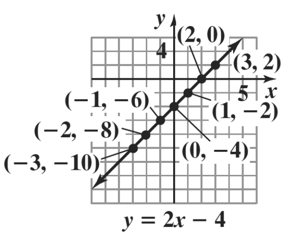
$x = 0,\ y = 1$

$x = 1,\ y = 3$

$x = 2,\ y = 5$

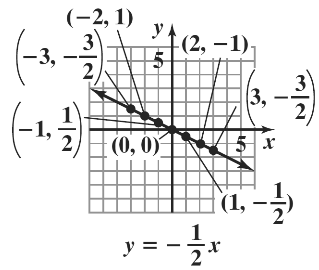
$x = 3,\ y = 7$

18.



$x = -3,\ y = -10$
$x = -2,\ y = -8$
$x = -1,\ y = -6$
$x = 0,\ y = -4$
$x = 1,\ y = -2$
$x = 2,\ y = 0$
$x = 3,\ y = 2$

19.



$x = -3,\ y = \dfrac{3}{2}$

$x = -2,\ y = 1$

$x = -1,\ y = \dfrac{1}{2}$

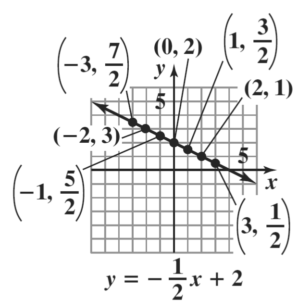
$x = 0,\ y = 0$

$x = 1,\ y = -\dfrac{1}{2}$

$x = 2,\ y = -1$

$x = 3,\ y = -\dfrac{3}{2}$

20.



$x = -3,\ y = \dfrac{7}{2}$

$x = -2,\ y = 3$

$x = -1,\ y = \dfrac{5}{2}$

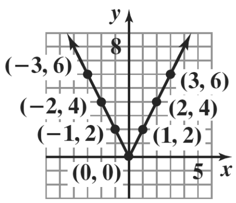
$x = 0,\ y = 2$

$x = 1,\ y = \dfrac{3}{2}$

$x = 2,\ y = 1$

$x = 3,\ y = \dfrac{1}{2}$

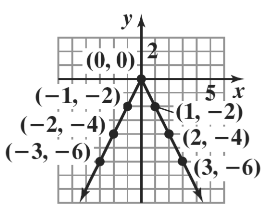
21.



$y = 2|x|$

$x = -3, y = 6$
$x = -2, y = 4$
$x = -1, y = 2$
$x = 0, y = 0$
$x = 1, y = 2$
$x = 2, y = 4$
$x = 3, y = 6$

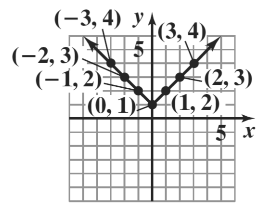
22.



$y = -2|x|$

$x = -3, y = -6$
$x = -2, y = -4$
$x = -1, y = -2$
$x = 0, y = 0$
$x = 1, y = -2$
$x = 2, y = -4$
$x = 3, y = -6$

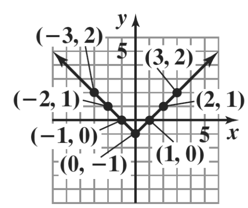
23.



$y = |x| + 1$

$x = -3, y = 4$
$x = -2, y = 3$
$x = -1, y = 2$
$x = 0, y = 1$
$x = 1, y = 2$
$x = 2, y = 3$
$x = 3, y = 4$

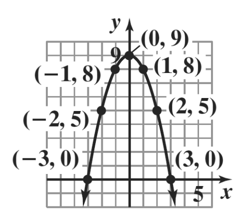
24.



$y = |x| - 1$

$x = -3, y = 2$
$x = -2, y = 1$
$x = -1, y = 0$
$x = 0, y = -1$
$x = 1, y = 0$
$x = 2, y = 1$
$x = 3, y = 2$

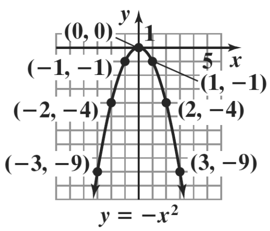
25.



$y = 9 - x^2$

$x = -3, y = 0$
$x = -2, y = 5$
$x = -1, y = 8$
$x = 0, y = 9$
$x = 1, y = 8$
$x = 2, y = 5$
$x = 3, y = 0$

26.



$x = -3, y = -9$

$x = -2, y = -4$

$x = -1, y = -1$

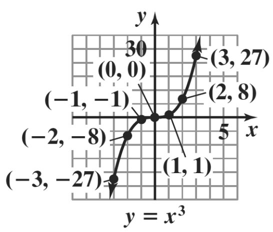
$x = 0, y = 0$

$x = 1, y = -1$

$x = 2, y = -4$

$x = 3, y = -9$

27.



$x = -3, y = -27$

$x = -2, y = -8$

$x = -1, y = 1$

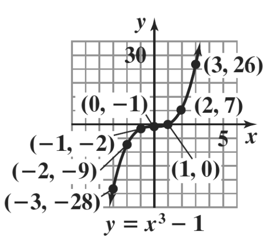
$x = 0, y = 0$

$x = 1, y = 1$

$x = 2, y = 8$

$x = 3, y = 27$

28.



$x = -3, y = -28$

$x = -2, y = -9$

$x = -1, y = -2$

$x = 0, y = -1$

$x = 1, y = 0$

$x = 2, y = 7$

$x = 3, y = 26$

29. (c) x-axis tick marks –5, –4, –3, –2, –1, 0, 1, 2, 3, 4, 5; y-axis tick marks are the same.

30. (d) x-axis tick marks –10, –8, –6, –4, –2, 0, 2, 4, 6, 8, 10; y-axis tick marks –4, –2, 0, 2, 4

31. (b); x-axis tick marks –20, –10, 0, 10, 20, 30, 40, 50, 60, 70, 80; y-axis tick marks –30, –20, –10, 0, 10, 20, 30, 40, 50, 60, 70

32. (a) x-axis tick marks –40, –20, 0, 20, 40; y-axis tick marks –1000, –900, –800, –700, . . . , 700, 800, 900, 1000

33. The equation that corresponds to Y_2 in the table is (c), $y_2 = 2 - x$. We can tell because all of the points $(-3,5)$, $(-2,4)$, $(-1,3)$, $(0,2)$, $(1,1)$, $(2,0)$, and $(3,-1)$ are on the line $y = 2 - x$, but all are not on any of the others.

34. The equation that corresponds to Y_1 in the table is (b), $y_1 = x^2$. We can tell because all of the points $(-3,9)$, $(-2,4)$, $(-1,1)$, $(0,0)$, $(1,1)$, $(2,4)$, and $(3,9)$ are on the graph $y = x^2$, but all are not on any of the others.

35. No. It passes through the point $(0,2)$.

36. Yes. It passes through the point $(0,0)$.

37. $(2,0)$

38. $(0,2)$

39. The graphs of Y_1 and Y_2 intersect at the points $(-2,4)$ and $(1,1)$.

40. The values of Y_1 and Y_2 are the same when $x = -2$ and $x = 1$.

41. **a.** 2; The graph intersects the x-axis at (2, 0).

b. –4; The graph intersects the y-axis at (0,–4).

42. **a.** 1; The graph intersects the x-axis at (1, 0).

b. 2; The graph intersects the y-axis at (0, 2).

43. **a.** 1, –2; The graph intersects the x-axis at (1, 0) and (–2, 0).

b. 2; The graph intersects the y-axis at (0, 2).

44. **a.** 1, –1; The graph intersects the x-axis at (1, 0) and (–1, 0).

b. 1; The graph intersect the y-axis at (0, 1).

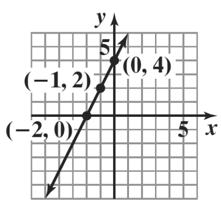
45. **a.** –1; The graph intersects the x-axis at (–1, 0).

b. none; The graph does not intersect the y-axis.

46. **a.** none; The graph does not intersect the x-axis.

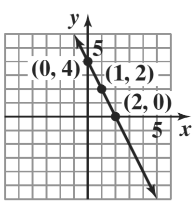
b. 2; The graph intersects the y-axis at (0, 2).

47.



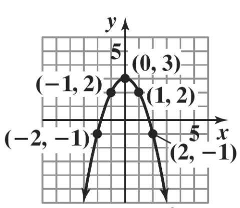
$y = 2x + 4$

48.



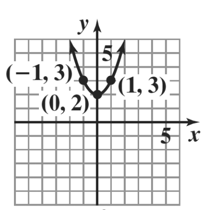
$y = 4 - 2x$

49.



$y = 3 - x^2$

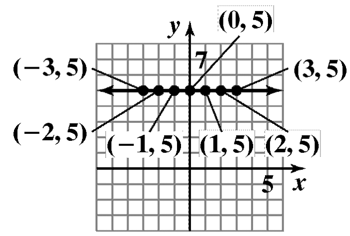
50.



$y = x^2 + 2$

51.

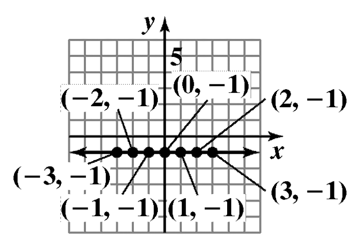
x	(x, y)
–3	(–3, 5)
–2	(–2, 5)
–1	(–1, 5)
0	(0, 5)
1	(1, 5)
2	(2, 5)
3	(3, 5)



$y = 5$

52.

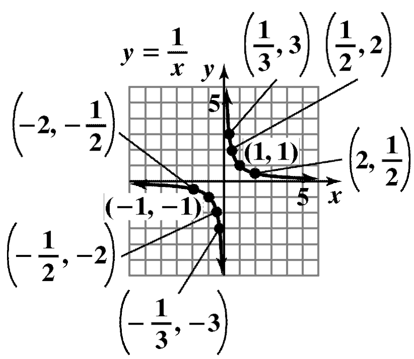
x	(x, y)
–3	(–3, –1)
–2	(–2, –1)
–1	(–1, –1)
0	(0, –1)
1	(1, –1)
2	(2, –1)
3	(3, –1)



$y = -1$

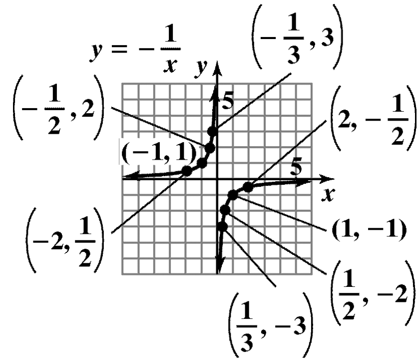
53.

x	(x, y)
-2	$\left(-2,-\frac{1}{2}\right)$
-1	$(-1,-1)$
$-\frac{1}{2}$	$\left(-\frac{1}{2},-2\right)$
$-\frac{1}{3}$	$\left(-\frac{1}{3},-3\right)$
$\frac{1}{3}$	$\left(\frac{1}{3},3\right)$
$\frac{1}{2}$	$\left(\frac{1}{2},2\right)$
1	$(1,1)$
2	$\left(2,\frac{1}{2}\right)$



54.

x	(x, y)
-2	$\left(-2,\frac{1}{2}\right)$
-1	$(-1,1)$
$-\frac{1}{2}$	$\left(-\frac{1}{2},2\right)$
$-\frac{1}{3}$	$\left(-\frac{1}{3},3\right)$
$\frac{1}{3}$	$\left(\frac{1}{3},-3\right)$
$\frac{1}{2}$	$\left(\frac{1}{2},-2\right)$
1	$(1,-1)$
2	$\left(2,-\frac{1}{2}\right)$



55. **a.** According to the line graph, about 44% of seniors used marijuana in 2010.

b. 2010 is 20 years after 1990.
$M = 0.1n + 43$
$M = 0.1(20) + 43 = 45$
According to formula, 45% of seniors used marijuana in 2010. It is greater than the estimate, although answers may vary.

c. According to the line graph, about 71% of seniors used alcohol in 2010.

d. 2010 is 20 years after 1990.
$A = -0.9n + 88$
$A = -0.9(20) + 88 = 70$
According to formula, 70% of seniors used alcohol in 2010. It is less than the estimate, although answers may vary.

e. The maximum for marijuana was reached in 2000.
According to the line graph, about 49% of seniors used marijuana in 1990.

56. **a.** According to the line graph, about 66% of seniors used alcohol in 2014.

b. 2014 is 24 years after 1990.
$A = -0.9n + 88$
$A = -0.9(24) + 88 = 66.4$
According to formula, 66.4% of seniors used alcohol in 2014. It is greater than the estimate, although answers may vary.

c. According to the line graph, about 44% of seniors used marijuana in 2014.

d. 2014 is 24 years after 1990.
$M = 0.1n + 43$
$M = 0.1(24) + 43 = 45.4$
According to formula, 45.4% of seniors used marijuana in 2014. It is greater than the estimate, although answers may vary.

e. The maximum for alcohol was reached in 1990. According to the line graph, about 90% of seniors used alcohol in 1990.

57. At age 8, women have the least number of awakenings, averaging about 1 awakening per night.

58. At age 65, men have the greatest number of awakenings, averaging about 8 awakenings per night.

59. The difference between the number of awakenings for 25-year-old men and women is about 1.9.

60. The difference between the number of awakenings for 18-year-old men and women is about 1.1.

61. – 66. Answers will vary.

67. makes sense

68. does not make sense; Explanations will vary. Sample explanation: Most graphing utilities do not display numbers on the axes.

69. does not make sense; Explanations will vary. Sample explanation: These three points are not collinear.

70. does not make sense; Explanations will vary. Sample explanation: As the time of day goes up, the total calories burned will also go up.

71. false; Changes to make the statement true will vary. A sample change is: The product of the coordinates of a point in quadrant III is also positive.

72. false; Changes to make the statement true will vary. A sample change is: A point on the x-axis will have $y = 0$.

73. true

74. false; Changes to make the statement true will vary. A sample change is: $3(5) - 2(2) \neq -4$.

75. I, III

76. II, IV

77. IV

78. II

79. (a)

80. (d)

81. (b)

82. (c)

83. (b)

84. (a)

85. (c)

86. (b)

87.
$$\begin{aligned} 2(x-3)-17 &= 13-3(x+2) \\ 2(6-3)-17 &= 13-3(6+2) \\ 2(3)-17 &= 13-3(8) \\ 6-17 &= 13-24 \\ -11 &= -11, \text{ true} \end{aligned}$$

88.
$$\begin{aligned} 12\left(\frac{x+2}{4}-\frac{x-1}{3}\right) &= 12\left(\frac{x+2}{4}\right)-12\left(\frac{x-1}{3}\right) \\ &= 3(x+2)-4(x-1) \\ &= 3x+6-4x+4 \\ &= -x+10 \end{aligned}$$

89.
$$\begin{aligned} (x-3)\ \frac{3}{x-3}+9 &= (x-3)\ \frac{3}{x-3}\ +(x-3)(9) \\ &= 3+9x-27 \\ &= 9x-24 \end{aligned}$$

Section 1.2

Check Point Exercises

1.
$$4x+5=29$$
$$4x+5-5=29-5$$
$$4x=24$$
$$\frac{4x}{4}=\frac{24}{4}$$
$$x=6$$
Check:
$$4x+5=29$$
$$4(6)+5=29$$
$$24+5=29$$
$$29=29 \text{ true}$$
The solution set is $\{6\}$.

2.
$$4(2x+1)=29+3(2x-5)$$
$$8x+4=29+6x-15$$
$$8x+4=6x+14$$
$$8x+4-6x=6x+14-6x$$
$$2x+4=14$$
$$2x+4-4=14-4$$
$$2x=10$$
$$\frac{2x}{2}=\frac{10}{2}$$
$$x=5$$
Check:
$$4(2x+1)=29+3(2x-5)$$
$$4[2(5)+1]=29+3[2(5)-5]$$
$$4[10+1]=29+3[10-5]$$
$$4[11]=29+3[5]$$
$$44=29+15$$
$$44=44 \text{ true}$$
The solution set is $\{5\}$.

3.
$$\frac{x-3}{4}=\frac{5}{14}-\frac{x+5}{7}$$
$$28\cdot\frac{x-3}{4}=28\left(\frac{5}{14}-\frac{x+5}{7}\right)$$
$$7(x-3)=2(5)-4(x+5)$$
$$7x-21=10-4x-20$$
$$7x-21=-4x-10$$
$$7x+4x=-10+21$$
$$11x=11$$
$$\frac{11x}{11}=\frac{11}{11}$$
$$x=1$$
Check:
$$\frac{x-3}{4}=\frac{5}{14}-\frac{x+5}{7}$$
$$\frac{1-3}{4}=\frac{5}{14}-\frac{1+5}{7}$$
$$\frac{-2}{4}=\frac{5}{14}-\frac{6}{7}$$
$$-\frac{1}{2}=-\frac{1}{2}$$
The solution set is $\{1\}$.

4.
$$\frac{5}{2x}=\frac{17}{18}-\frac{1}{3x},\ x\neq 0$$
$$18x\cdot\frac{5}{2x}=18x\left(\frac{17}{18}-\frac{1}{3x}\right)$$
$$18\cdot\frac{5}{2x}=18x\cdot\frac{17}{18}-18x\cdot\frac{1}{3x}$$
$$45=17x-6$$
$$45+6=17x-6+6$$
$$51=17x$$
$$\frac{51}{17}=\frac{17x}{17}$$
$$3=x$$
The solution set is $\{3\}$.

5. $$\frac{x}{x-2}=\frac{2}{x-2}-\frac{2}{3}, \quad x\neq 2$$
$$3(x-2)\cdot\frac{x}{x-2}=3(x-2)\left(\frac{2}{x-2}-\frac{2}{3}\right)$$
$$3(x-2)\cdot\frac{x}{x-2}=3(x-2)\cdot\frac{2}{x-2}-3(x-2)\cdot\frac{2}{3}$$
$$3x=6-(x-2)\cdot 2$$
$$3x=6-2(x-2)$$
$$3x=6-2x+4$$
$$3x=10-2x$$
$$3x+2x=10-2x+2x$$
$$5x=10$$
$$\frac{5x}{5}=\frac{10}{5}$$
$$x=2$$
The solution set is the empty set, $\varnothing$.

6. Set $y_1=y_2$.
$$\frac{1}{x+4}+\frac{1}{x-4}=\frac{22}{x^2-16}$$
$$\frac{1}{x+4}+\frac{1}{x-4}=\frac{22}{(x+4)(x-4)}$$
$$\frac{(x+4)(x-4)}{x+4}+\frac{(x+4)(x-4)}{x-4}=\frac{22(x+4)(x-4)}{(x+4)(x-4)}$$
$$(x-4)+(x+4)=22$$
$$x-4+x+4=22$$
$$2x=22$$
$$x=11$$
Check:
$$\frac{1}{x+4}+\frac{1}{x-4}=\frac{22}{x^2-16}$$
$$\frac{1}{11+4}+\frac{1}{11-4}=\frac{22}{11^2-16}$$
$$\frac{1}{15}+\frac{1}{7}=\frac{22}{105}$$
$$\frac{22}{105}=\frac{22}{105} \quad \text{true}$$

7. $$4x-7=4(x-1)+3$$
$$4x-7=4(x-1)+3$$
$$4x-7=4x-4+3$$
$$4x-7=4x-1$$
$$-7=-1$$
The original equation is equivalent to the statement $-7=-1$, which is false for every value of x.
The solution set is the empty set, $\varnothing$.
The equation is an inconsistent equation.

8. $$7x+9=9(x+1)-2x$$
$$7x+9=9(x+1)-2x$$
$$7x+9=9x+9-2x$$
$$7x+9=7x+9$$
$$9=9$$
The original equation is equivalent to the statement $9=9$, which is true for every value of x.
The equation is an identity, and all real numbers are solutions. The solution set $\{x|x \text{ is a real number}\}$.

9. $$D=\frac{10}{9}x+\frac{53}{9}$$
$$10=\frac{10}{9}x+\frac{53}{9}$$
$$9\cdot 10=9\left(\frac{10}{9}x+\frac{53}{9}\right)$$
$$90=10x+53$$
$$90-53=10x+53-53$$
$$37=10x$$
$$\frac{37}{10}=\frac{10x}{10}$$
$$3.7=x$$
$$x=3.7$$
The formula indicates that if the low-humor group averages a level of depression of 10 in response to a negative life event, the intensity of that event is 3.7. This is shown as the point whose corresponding value on the vertical axis is 10 and whose value on the horizontal axis is 3.7.

Concept and Vocabulary Check 1.2

1. linear

2. equivalent

3. apply the distributive property

4. least common denominator**;** 12

5. 0

6. $2x$

7. $(x+5)(x+1)$

8. $x\neq 2$ **;** $x\neq 4$

9. $5(x+3)+3(x+4)=12x+9$

10. identity

11. inconsistent

Exercise Set 1.2

1. $7x - 5 = 72$
$7x = 77$
$x = 11$
Check:
$7x-5=72$
$7(11)-5=72$
$77-5=72$
$72=72$
The solution set is $\{11\}$.

2. $6x - 3 = 63$
$6x = 66$
$x = 11$
The solution set is $\{11\}$.

Check:
$6x-3=63$
$6(11)-3=63$
$66-3=63$
$63=63$

3. $11x-(6x-5)=40$
$11x-6x+5=40$
$5x+5=40$
$5x=35$
$x=7$
The solution set is $\{7\}$.

Check:
$11x-(6x-5)=40$
$11(7)-[6(7)-5]=40$
$77-(42-5)=40$
$77-(37)=40$
$40=40$

4. $5x - (2x - 10) = 35$
$5x - 2x + 10 = 35$
$3x + 10 = 35$
$3x = 25$
$x = \frac{25}{3}$

The solution set is $\left\{\frac{25}{3}\right\}$.

Check:
$5x-(2x-10)=35$
$5\left(\frac{25}{3}\right)-\left[2\left(\frac{25}{3}\right)-10\right]=35$
$\frac{125}{3}-\left[\frac{50}{3}-10\right]=35$
$\frac{125}{3}-\frac{20}{3}=35$
$\frac{105}{3}=35$
$35=35$

5. $2x - 7 = 6 + x$
$x - 7 = 6$
$x = 13$
The solution set is $\{13\}$.

Check:
$2(13)-7=6+13$
$26-7=19$
$19=19$

6. $3x + 5 = 2x + 13$
$x + 5 = 13$
$x = 8$
The solution set is $\{8\}$.

Check:
$3x+5=2x+13$
$3(8)+5=2(8)+13$
$24+5=16+13$
$29=29$

7. $7x + 4 = x + 16$
$6x + 4 = 16$
$6x = 12$
$x = 2$
The solution set is $\{2\}$.

Check:
$7(2)+4=2+16$
$14+4=18$
$18=18$

8. $13x + 14 = 12x - 5$
$x + 14 = -5$
$x = -19$
The solution set is {–19}.

Check:
$$13x+14=12x-5$$
$$13(-19)+14=12(-19)-5$$
$$-247+14=-228-5$$
$$-233=-233$$

9. $3(x-2)+7=2(x+5)$
$3x-6+7=2x+10$
$3x+1=2x+10$
$x+1=10$
$x=9$
The solution set is {9}.

Check:
$$3(9-2)+7=2(9+5)$$
$$3(7)+7=2(14)$$
$$21+7=28$$
$$28=28$$

10. $2(x-1)+3=x-3(x+1)$
$2x-2+3=x-3x-3$
$2x+1=-2x-3$
$4x+1=-3$
$4x=-4$
$x=-1$
The solution set is {–1}.

Check:
$$2(x-1)+3=x-3(x+1)$$
$$2(-1-1)+3=-1-3(-1+1)$$
$$2(-2)+3=-1-3(0)$$
$$-4+3=-1+0$$
$$-1=-1$$

11. $3(x-4)-4(x-3)=x+3-(x-2)$
$3x-12-4x+12=x+3-x+2$
$-x=5$
$x=-5$
The solution set is {–5}.

Check:
$$3(-5-4)-4(-5-3)=-5+3-(-5-2)$$
$$3(-9)-4(-8)=-2-(-7)$$
$$-27+32=-2+7$$
$$5=5$$

12. $2-(7x+5)=13-3x$
$2-7x-5=13-3x$
$-7x-3=13-3x$
$-4x=16$
$x=-4$
The solution set is {–4}.

Check:
$$2-(7x+5)=13-3x$$
$$2-[7(-4)+5]=13-3(-4)$$
$$2-[-28+5]=13+12$$
$$2-[-23]=15$$
$$2+23=25$$
$$25=25$$

13. $16=3(x-1)-(x-7)$
$16=3x-3-x+7$
$16=2x+4$
$12=2x$
$6=x$
The solution set is {6}.

Check:
$$16=3(6-1)-(6-7)$$
$$16=3(5)-(-1)$$
$$16=15+1$$
$$16=16$$

14. $5x-(2x+2)=x+(3x-5)$
$5x-2x-2=x+3x-5$
$3x-2=4x-5$
$-x=-3$
$x=3$
The solution set is {3}.

Check:
$$5x-(2x+2)=x+(3x-5)$$
$$5(3)-[2(3)+2]=3+[3(3)-5]$$
$$15-[6+2]=3+[9-5]$$
$$15-8=3+4$$
$$7=7$$

15. $25-[2+5y-3(y+2)=-3(2y-5)-[5(y-1)-3y+3]$
$$25-[2+5y-3y-6]=-6y+15-[5y-5-3y+3]$$
$$25-[2y-4]=-6y+15-[2y-2]$$
$$25-2y+4=-6y+15-2y+2$$
$$-2y+29=-8y+17$$
$$6y=-12$$
$$y=-2$$
The solution set is {–2}.

Check:
$$25-[2+5y-3(y+2)=-3(2y-5)-[5(y-1)-3y+3]$$
$$25-[2+5(-2)-3(-2+2)=-3[2(-2)-5]-[5(-2-1)-3(-2)+3]$$
$$25-[2-10-3(0)]=-3[-4-5]-[5(-3)+6+3]$$
$$25-[-8]=-3(-9)-[-15+9]$$
$$25+8=27-(-6)$$
$$33=27+6$$
$$33=33$$

16. $45-[4-2y-4(y+7)]=-4(1+3y)-[4-3(y+2)-2(2y-5)]$
$$45-[4-2y-4y-28]=-4-12y-[4-3y-6-4y+10]$$
$$45-[-6y-24]=-4-12y-[-7y+8]$$
$$45+6y+24=-4-12y+7y-8$$
$$6y+69=-5y-12$$
$$11y=-81$$
$$y=-\frac{81}{11}$$
The solution set is $\left\{-\frac{81}{11}\right\}$.

17. $\frac{x}{3}=\frac{x}{2}-2$
$$6\left[\frac{x}{3}=\frac{x}{2}-2\right]$$
$$2x=3x-12$$
$$12=3x-2x$$
$$x=12$$
The solution set is {12}.

18.
$$\frac{x}{5}=\frac{x}{6}+1$$
$$30\left[\frac{x}{5}=\frac{x}{6}+1\right]$$
$$6x=5x+30$$
$$6x-5x=30$$
$$x=30$$
The solution set is {30}.

19. $20-\frac{x}{3}=\frac{x}{2}$

$6\left[20-\frac{x}{3}=\frac{x}{2}\right]$

$120-2x=3x$

$120=3x+2x$

$120=5x$

$x=\frac{120}{5}$

$x=24$

The solution set is $\{24\}$.

20. $\frac{x}{5}-\frac{1}{2}=\frac{x}{6}$

$30\left[\frac{x}{5}-\frac{1}{2}=\frac{x}{6}\right]$

$6x-15=5x$

$6x-5x=15$

$x=15$

The solution set is $\{15\}$.

21. $\frac{3x}{5}=\frac{2x}{3}+1$

$15\left[\frac{3x}{5}=\frac{2x}{3}+1\right]$

$9x=10x+15$

$9x-10x=15$

$-x=15$

$x=-15$

The solution set is $\{-15\}$.

22. $\frac{x}{2}=\frac{3x}{4}+5$

$4\left[\frac{x}{2}=\frac{3x}{4}+5\right]$

$2x=3x+20$

$2x-3x=20$

$-x=20$

$x=-20$

The solution set is $\{-20\}$.

23. $\frac{3x}{5}-x=\frac{x}{10}-\frac{5}{2}$

$10\left[\frac{3x}{5}-x=\frac{x}{10}-\frac{5}{2}\right]$

$6x-10x=x-25$

$-4x-x=-25$

$-5x=-25$

$x=5$

The solution set is $\{5\}$.

24. $2x-\frac{2x}{7}=\frac{x}{2}+\frac{17}{2}$

$14\left[2x-\frac{2x}{7}=\frac{x}{2}+\frac{17}{2}\right]$

$28x-4x=7x+119$

$24x-7x=119$

$17x=119$

$x=7$

The solution set is $\{7\}$.

25. $\frac{x+3}{6}=\frac{3}{8}+\frac{x-5}{4}$

$24\left[\frac{x+3}{6}=\frac{3}{8}+\frac{x-5}{4}\right]$

$4x+12=9+6x-30$

$4x-6x=-21-12$

$-2x=-33$

$x=\frac{33}{2}$

The solution set is $\left\{\frac{33}{2}\right\}$.

26. $\frac{x+1}{4}=\frac{1}{6}+\frac{2-x}{3}$

$12\left[\frac{x+1}{4}=\frac{1}{6}+\frac{2-x}{3}\right]$

$3x+3=2+8-4x$

$3x+4x=10-3$

$7x=7$

$x=1$

The solution set is $\{1\}$.

27. $\frac{x}{4}=2+\frac{x-3}{3}$

$12\left[\frac{x}{4}=2+\frac{x-3}{3}\right]$

$3x = 24 + 4x - 12$

$3x - 4x = 12$

$-x = 12$

$x = -12$

The solution set is $\{-12\}$.

28. $5+\frac{x-2}{3}=\frac{x+3}{8}$

$24\left[5+\frac{x-2}{3}=\frac{x+3}{8}\right]$

$120+8x-16=3x+9$

$8x-3x=9-104$

$5x=-95$

$x=-19$

The solution set is $\{-19\}$.

29. $\frac{x+1}{3}=5-\frac{x+2}{7}$

$21\left[\frac{x+1}{3}=5-\frac{x+2}{7}\right]$

$7x + 7 = 105 - 3x - 6$

$7x + 3x = 99 - 7$

$10x = 92$

$x=\frac{92}{10}$

$x=\frac{46}{5}$

The solution set is $\left\{\frac{46}{5}\right\}$.

30. $\frac{3x}{5}-\frac{x-3}{2}=\frac{x+2}{3}$

$30\left[\frac{3x}{5}-\frac{x-3}{2}=\frac{x+2}{3}\right]$

$18x-15x+45=10x+20$

$3x-10x=20-45$

$-7x=-25$

$x=\frac{25}{7}$

The solution set is $\left\{\frac{25}{7}\right\}$.

31. a. $\frac{4}{x}=\frac{5}{2x}+3\,(x\neq 0)$

b. $\frac{4}{x}=\frac{5}{2x}+3$

$8=5+6x$

$3=6x$

$\frac{1}{2}=x$

The solution set is $\left\{\frac{1}{2}\right\}$.

32. a. $\frac{5}{x}=\frac{10}{3x}+4\,(x\neq 0)$

b. $\frac{5}{x}=\frac{10}{3x}+4$

$15=10+12x$

$5=12x$

$x=\frac{5}{12}$

The solution set is $\left\{\frac{5}{12}\right\}$.

33. a. $\frac{2}{x}+3=\frac{5}{2x}+\frac{13}{4}\,(x\neq 0)$

b. $\frac{2}{x}+3=\frac{5}{2x}+\frac{13}{4}$

$8+12x=10+13x$

$-x=2$

$x=-2$

The solution set is $\{-2\}$.

34. a. $\frac{7}{2x}-\frac{5}{3x}=\frac{22}{3}\,(x\neq 0)$

b. $\frac{7}{2x}-\frac{5}{3x}=\frac{22}{3}$

$21-10=44x$

$11=44x$

$x=\frac{1}{4}$

The solution set is $\left\{\frac{1}{4}\right\}$.

35. a. $\frac{2}{3x}+\frac{1}{4}=\frac{11}{6x}-\frac{1}{3}\,(x\neq 0)$

b. $\frac{2}{3x}+\frac{1}{4}=\frac{11}{6x}-\frac{1}{3}$

$8+3x=22-4x$

$7x=14$

$x=2$

The solution set is $\{2\}$.

36. a. $\frac{5}{2x}-\frac{8}{9}=\frac{1}{18}-\frac{1}{3x}\,(x\neq 0)$

b. $\frac{5}{2x}-\frac{8}{9}=\frac{1}{18}-\frac{1}{3x}$

$45-16x=x-6$

$-17x=-51$

$x=3$

The solution set is $\{3\}$.

37. a. $\frac{x-2}{2x}+1=\frac{x+1}{x}\quad(x\neq 0)$

b. $\frac{x-2}{2x}+1=\frac{x+1}{x}$

$x-2+2x=2x+2$

$x-2=2$

$x=4$

The solution set is $\{4\}$.

38. a. $\frac{4}{x}=\frac{9}{5}-\frac{7x-4}{5x}\quad(x\neq 0)$

b. $\frac{4}{x}=\frac{9}{5}-\frac{7x-4}{5x}$

$20=9x-7x+4$

$16=2x$

$8=x$

The solution set is $\{8\}$.

39. a. $\frac{1}{x-1}+5=\frac{11}{x-1}\,(x\neq 1)$

b. $\frac{1}{x-1}+5=\frac{11}{x-1}$

$1+5(x-1)=11$

$1+5x-5=11$

$5x-4=11$

$5x=15$

$x=3$

The solution set is $\{3\}$.

40. a. $\frac{3}{x+4}-7=\frac{-4}{x+4}\,(x\neq -4)$

b. $\frac{3}{x+4}-7=\frac{-4}{x+4}$

$3-7(x+4)=-4$

$3-7x-28=-4$

$-7x=21$

$x=-3$

The solution set is $\{-3\}$.

41. a. $\frac{8x}{x+1}=4-\frac{8}{x+1}\,(x\neq -1)$

b. $\frac{8x}{x+1}=4-\frac{8}{x+1}$

$8x=4(x+1)-8$

$8x=4x+4-8$

$4x=-4$

$x=-1\Rightarrow$ no solution

The solution set is the empty set, $\varnothing$.

42. a. $\frac{2}{x-2}=\frac{x}{x-2}-2\,(x\neq 2)$

b. $\frac{2}{x-2}=\frac{x}{x-2}-2$

$2=x-2(x-2)$

$2=x-2x+4$

$x=2\Rightarrow$ no solution

The solution set is the empty set, $\varnothing$.

43. a. $\frac{3}{2x-2}+\frac{1}{2}=\frac{2}{x-1}\,(x\neq 1)$

b. $\frac{3}{2x-2}+\frac{1}{2}=\frac{2}{x-1}$

$\frac{3}{2(x-1)}+\frac{1}{2}=\frac{2}{x-1}$

$3+1(x-1)=4$

$3+x-1=4$

$x=2$

The solution set is $\{2\}$.

44. a. $\frac{3}{x+3}=\frac{5}{2x+6}+\frac{1}{x-2}\ (x\neq -3, x\neq 2)$

b. $\frac{3}{x+3}=\frac{5}{2(x+3)}+\frac{1}{x-2}$

$6(x-2)=5(x-2)+2(x+3)$

$6x-12=5x-10+2x+6$

$-x=8$

$x=-8$

The solution set is {–8}.

45. a. $\frac{3}{x+2}+\frac{2}{x-2}=\frac{8}{(x+2)(x-2)};(x\neq -2,2)$

b. $\frac{3}{x+2}+\frac{2}{x-2}=\frac{8}{(x+2)(x-2)}$

$(x\neq 2, x\neq -2)$

$3(x-2)+2(x+2)=8$

$3x-6+2x+4=8$

$5x=10$

$x=2\Rightarrow$ no solution

The solution set is the empty set, $\varnothing$.

46. a. $\frac{5}{x+2}+\frac{3}{x-2}=\frac{12}{(x+2)(x-2)}$

$(x\neq 2, x\neq -2)$

b. $\frac{5}{x+2}+\frac{3}{x-2}=\frac{12}{(x+2)(x-2)}$

$5(x-2)+3(x+2)=12$

$5x-10+3x+6=12$

$8x=16$

$x=2\Rightarrow$ no solution

The solution set is the empty set, $\varnothing$.

47. a. $\frac{2}{x+1}-\frac{1}{x-1}=\frac{2x}{x^2-1}\ (x\neq 1, x\neq -1)$

b.

$\frac{2}{x+1}-\frac{1}{x-1}=\frac{2x}{x^2-1}$

$\frac{2}{x+1}-\frac{1}{x-1}=\frac{2x}{(x+1)(x-1)}$

$2(x-1)-1(x+1)=2x$

$2x-2-x-1=2x$

$-x=3$

$x=-3$

The solution set is {–3}.

48. a. $\frac{4}{x+5}+\frac{2}{x-5}=\frac{32}{x^2-25};\ x\neq 5,-5$

b. $\frac{4}{x+5}+\frac{2}{x-5}=\frac{32}{(x+5)(x-5)}$

$(x\neq 5, x\neq -5)$

$4(x-5)+2(x+5)=32$

$4x-20+2x+10=32$

$6x=42$

$x=7$

The solution set is {7}.

49. a. $\frac{1}{x-4}-\frac{5}{x+2}=\frac{6}{(x-4)(x+2)};(x\neq -2,4)$

b. $\frac{1}{x-4}-\frac{5}{x+2}=\frac{6}{x^2-2x-8}$

$\frac{1}{x-4}-\frac{5}{x+2}=\frac{6}{(x-4)(x+2)}$

$(x\neq 4, x\neq -2)$

$1(x+2)-5(x-4)=6$

$x+2-5x+20=6$

$-4x=-16$

$x=4\Rightarrow$ no solution

The solution set is the empty set, $\varnothing$.

50. a. $\frac{6}{x+3}-\frac{5}{x-2}=\frac{-20}{x^2+x-6};\ x\neq -3,2$

b. $\frac{6}{x+3}-\frac{5}{x-2}=\frac{-20}{(x-2)(x+3)}$

$(x\neq -3, x\neq 2)$

$6(x-2)-5(x+3)=-20$

$6x-12-5x-15=-20$

$x=7$

The solution set is {7}.

51. Set $y_1=y_2$.

$5(2x-8)-2=5(x-3)+3$

$10x-40-2=5x-15+3$

$10x-42=5x-12$

$10x-5x=-12+42$

$5x=30$

$x=6$

The solution set is {6}.

52. Set $y_1 = y_2$.

$$\begin{aligned}7(3x-2)+5&=6(2x-1)+24\\21x-14+5&=12x-6+24\\21x-9&=12x+18\\21x-12x&=18+9\\9x&=27\\x&=3\end{aligned}$$

The solution set is $\{3\}$.

53. Set $y_1 - y_2 = 1$.

$$\begin{aligned}\frac{x-3}{5}-\frac{x-5}{4}&=1\\20\cdot\frac{x-3}{5}-20\cdot\frac{x-5}{4}&=20\cdot 1\\4(x-3)-5(x-5)&=20\\4x-12-5x+25&=20\\-x+13&=20\\-x&=7\\x&=-7\end{aligned}$$

The solution set is $\{-7\}$.

54. Set $y_1 - y_2 = -4$.

$$\begin{aligned}\frac{x+1}{4}-\frac{x-2}{3}&=-4\\12\cdot\frac{x+1}{4}-12\cdot\frac{x-2}{3}&=12(-4)\\3(x+1)-4(x-2)&=-48\\3x+3-4x+8&=-48\\-x+11&=-48\\-x&=-59\\x&=59\end{aligned}$$

The solution set is $\{59\}$.

55. Set $y_1 + y_2 = y_3$.

$$\begin{aligned}\frac{5}{x+4}+\frac{3}{x+3}&=\frac{12x+19}{x^2+7x+12}\\\frac{5}{x+4}+\frac{3}{x+3}&=\frac{12x+19}{(x+4)(x+3)}\\(x+4)(x+3)\left(\frac{5}{x+4}+\frac{3}{x+3}\right)&=(x+4)(x+3)\frac{12x+19}{(x+4)(x+3)}\\5(x+3)+3(x+4)&=12x+19\\5x+15+3x+12&=12x+19\\8x+27&=12x+19\\-4x&=-8\\x&=2\end{aligned}$$

The solution set is $\{2\}$.

56. Set $y_1 + y_2 = y_3$.

$$\frac{2x-1}{x^2+2x-8}+\frac{2}{x+4}=\frac{1}{x-2}$$
$$\frac{2x-1}{(x+4)(x-2)}+\frac{2}{x+4}=\frac{1}{x-2}$$
$$(x+4)(x-2)\left(\frac{2x-1}{(x+4)(x-2)}+\frac{2}{x+4}\right)=(x+4)(x-2)\frac{1}{x-2}$$
$$2x-1+2(x-2)=x+4$$
$$2x-1+2x-4=x+4$$
$$4x-5=x+4$$
$$3x=9$$
$$x=3$$

The solution set is $\{3\}$.

57.
$$0=4[x-(3-x)]-7(x+1)$$
$$0=4[x-3+x]-7x-7$$
$$0=4[2x-3]-7x-7$$
$$0=8x-12-7x-7$$
$$0=x-19$$
$$-x=-19$$
$$x=19$$

The solution set is $\{19\}$.

58.
$$0=2[3x-(4x-6)]-5(x-6)$$
$$0=2[3x-4x+6]-5x+30$$
$$0=2[-x+6]-5x+30$$
$$0=-2x+12-5x+30$$
$$0=-7x+42$$
$$7x=42$$
$$x=6$$

The solution set is $\{6\}$.

59.
$$0=\frac{x+6}{3x-12}-\frac{5}{x-4}-\frac{2}{3}$$
$$0=\frac{x+6}{3(x-4)}-\frac{5}{x-4}-\frac{2}{3}$$
$$3(x-4)\cdot 0=3(x-4)\left(\frac{x+6}{3(x-4)}-\frac{5}{x-4}-\frac{2}{3}\right)$$
$$0=\frac{3(x-4)(x+6)}{3(x-4)}-\frac{5\cdot 3(x-4)}{x-4}-\frac{2\cdot 3(x-4)}{3}$$
$$0=(x+6)-15-2(x-4)$$
$$0=x+6-15-2x+8$$
$$0=-x-1$$
$$x=-1$$

The solution set is $\{-1\}$.

60. $0 = \frac{1}{5x+5} - \frac{3}{x+1} + \frac{7}{5}$

$0 = \frac{1}{5(x+1)} - \frac{3}{x+1} + \frac{7}{5}$

$5(x+1) \cdot 0 = 5(x+1)\left(\frac{1}{5(x+1)} - \frac{3}{x+1} + \frac{7}{5}\right)$

$0 = \frac{1 \cdot 5(x+1)}{5(x+1)} - \frac{3 \cdot 5(x+1)}{x+1} + \frac{7 \cdot 5(x+1)}{5}$

$0 = 1 - 15 + 7(x+1)$

$0 = 1 - 15 + 7x + 7$

$0 = -7 + 7x$

$-7x = -7$

$x = 1$

The solution set is $\{1\}$.

61. $5x + 9 = 9(x+1) - 4x$

$5x + 9 = 9x + 9 - 4x$

$5x + 9 = 5x + 9$

$9 = 9$

The solution set $\{x | x \text{ is a real number}\}$.

The given equation is an identity.

62. $4x + 7 = 7(x+1) - 3x$

$4x + 7 = 7x + 7 - 3x$

$4x + 7 = 4x + 7$

$7 = 7$

The solution set $\{x | x \text{ is a real number}\}$.

The given equation is an identity.

63. $3(x+2) = 7 + 3x$

$3x + 6 = 7 + 3x$

$6 = 7$

The solution set $\varnothing$.

The given equation is an inconsistent equation.

64. $4(x+5) = 21 + 4x$

$4x + 20 = 21 + 4x$

$20 = 21$

The solution set $\varnothing$.

The given equation is an inconsistent equation.

65. $10x + 3 = 8x + 3$

$2x + 3 = 3$

$2x = 0$

$x = 0$

The solution set $\{0\}$.

The given equation is a conditional equation.

66. $5x + 7 = 2x + 7$

$3x + 7 = 7$

$3x = 0$

$x = 0$

The solution set $\{0\}$.

The given equation is a conditional equation.

67. $\frac{2x}{x-3} = \frac{6}{x-3} + 4$

$2x = 6 + 4(x-3)$

$2x = 6 + 4x - 12$

$-2x = -6$

$x = 3 \Rightarrow$ no solution

The given equation is an inconsistent equation.

68. $\frac{3}{x-3} = \frac{x}{x-3} + 3$

$3 = x + 3(x-3)$

$3 = x + 3x - 9$

$-4x = -12$

$x = 3 \Rightarrow$ no solution

The given equation is an inconsistent equation.

69. $\frac{x+5}{2} - 4 = \frac{2x-1}{3}$

$3(x+5) - 24 = 2(2x-1)$

$3x + 15 - 24 = 4x - 2$

$-x = 7$

$x = -7$

The solution set is $\{-7\}$.

The given equation is a conditional equation.

70. $\frac{x+2}{7} = 5 - \frac{x+1}{3}$

$3(x+2) = 105 - 7(x+1)$

$3x + 6 = 105 - 7x - 7$

$10x = 92$

$x = \frac{92}{10}$

$x = \frac{46}{5}$

The solution set is $\left\{\frac{46}{5}\right\}$.

The given equation is a conditional equation.

71. $\dfrac{2}{x-2}=3+\dfrac{x}{x-2}$

$2=3(x-2)+x$

$2=3x-6+x$

$-4x=-8$

$x=2\Rightarrow$ no solution

The solution set is the empty set, $\varnothing$.

The given equation is an inconsistent equation.

72. $\dfrac{6}{x+3}+2=\dfrac{-2x}{x+3}$

$6+2(x+3)=-2x$

$6+2x+6=-2x$

$4x=-12$

$x=-3\Rightarrow$ no solution

This equation is not true for any real numbers.

The given equation is an inconsistent equation.

73. $8x-(3x+2)+10=3x$

$8x-3x-2+10=3x$

$2x=-8$

$x=-4$

The solution set is $\{-4\}$.

The given equation is a conditional equation.

74. $2(x+2)+2x=4(x+1)$

$2x+4+2x=4x+4$

$0=0$

This equation is true for all real numbers.

The given equation is an identity.

75. $\dfrac{2}{x}+\dfrac{1}{2}=\dfrac{3}{4}$

$8+2x=3x$

$-x=-8$

$x=8$

The solution set is $\{8\}$.

The given equation is a conditional equation.

76. $\dfrac{3}{x}-\dfrac{1}{6}=\dfrac{1}{3}$

$18-x=2x$

$-3x=-18$

$x=6$

The solution set is $\{6\}$.

The given equation is a conditional equation.

77. $\dfrac{4}{x-2}+\dfrac{3}{x+5}=\dfrac{7}{(x+5)(x-2)}$

$4(x+5)+3(x-2)=7$

$4x+20+3x-6=7$

$7x=-7$

$x=-1$

The solution set is $\{-1\}$.

The given equation is a conditional equation.

78. $\dfrac{1}{x-1}=\dfrac{1}{(2x+3)(x-1)}+\dfrac{4}{2x+3}$

$1(2x+3)=1+4(x-1)$

$2x+3=1+4x-4$

$-2x=-6$

$x=3$

The solution set is $\{3\}$.

The given equation is a conditional equation.

79. $\dfrac{4x}{x+3}-\dfrac{12}{x-3}=\dfrac{4x^2+36}{x^2-9};x\neq 3,-3$

$4x(x-3)-12(x+3)=4x^2+36$

$4x^2-12x-12x-36=4x^2+36$

$4x^2-24x-36=4x^2+36$

$-24x-36=36$

$-24x=72$

$x=-3$ No solution

The solution set is $\{\ \}$.

The given equation is an inconsistent equation.

80. $\dfrac{4}{x^2+3x-10}-\dfrac{1}{x^2+x-6}=\dfrac{3}{x^2-x-12}$

$\dfrac{4}{(x+5)(x-2)}-\dfrac{1}{(x+3)(x-2)}=\dfrac{3}{(x+3)(x-4)}, x\neq -5, 2, -3, 4$

$4(x+3)(x-4)-1(x+5)(x-4)=3(x+5)(x-2)$

$4x^2-4x-48-x^2-x+20=3x^2+9x-30$

$3x^2-5x-28=3x^2+9x-30$

$2=14x$

$\dfrac{1}{7}=x$

The solution set is $\left\{\dfrac{1}{7}\right\}$.

The given equation is a conditional equation.

81. The equation is $3(x-4)=3(2-2x)$, and the solution is $x=2$.

82. The equation is $3(2x-5)=5x+2$, and the solution is $x=17$.

83. The equation is $-3(x-3)=5(2-x)$, and the solution is $x=0.5$.

84. The equation is $2x-5=4(3x+1)-2$, and the solution is $x=-0.7$.

85. Solve: $4(x-2)+2=4x-2(2-x)$

$4x-8+2=4x-4+2x$

$4x-6=6x-4$

$-2x-6=-4$

$-2x=2$

$x=-1$

Now, evaluate x^2-x for $x=-1$:

$x^2-x=(-1)^2-(-1)$

$=1-(-1)=1+1=2$

86. Solve: $2(x-6)=3x+2(2x-1)$

$2x-12=3x+4x-2$

$2x-12=7x-2$

$-5x-12=-2$

$-5x=10$

$x=-2$

Now, evaluate x^2-x for $x=-2$:

$x^2-x=(-2)^2-(-2)$

$=4-(-2)=4+2=6$

87. Solve for x: $\frac{3(x+3)}{5} = 2x+6$

$$3(x+3) = 5(2x+6)$$
$$3x+9 = 10x+30$$
$$-7x+9 = 30$$
$$-7x = 21$$
$$x = -3$$

Solve for y: $-2y-10 = 5y+18$

$$-7y-10 = 18$$
$$-7y = 28$$
$$y = -4$$

Now, evaluate $x^2-(xy-y)$ for $x=-3$ and $y=-4$:

$$x^2-(xy-y)$$
$$=(-3)^2-[-3(-4)-(-4)]$$
$$=(-3)^2-[12-(-4)]$$
$$=9-(12+4)=9-16=-7$$

88. Solve for x: $\frac{13x-6}{4} = 5x+2$

$$13x-6 = 4(5x+2)$$
$$13x-6 = 20x+8$$
$$-7x-6 = 8$$
$$-7x = 14$$
$$x = -2$$

Solve for y: $5-y = 7(y+4)+1$

$$5-y = 7y+28+1$$
$$5-y = 7y+29$$
$$5-8y = 29$$
$$-8y = 24$$
$$y = -3$$

Now, evaluate $x^2-(xy-y)$ for $x=-2$ and $y=-3$:

$$x^2-(xy-y)$$
$$=(-2)^2-[-2(-3)-(-3)]$$
$$=(-2)^2-[6-(-3)]$$
$$=4-(6+3)=4-9=-5$$

89. $\left[(3+6)^2 \div 3\right]\cdot 4 = -54x$

$$\left(9^2 \div 3\right)\cdot 4 = -54x$$
$$(81 \div 3)\cdot 4 = -54x$$
$$27\cdot 4 = -54x$$
$$108 = -54x$$
$$-2 = x$$

The solution set is $\{-2\}$.

90. $2^3-\left[4(5-3)^3\right] = -8x$

$$8-\left[4(2)^3\right] = -8x$$
$$8-4\cdot 8 = -8x$$
$$8-32 = -8x$$
$$-24 = -8x$$
$$3 = x$$

The solution set is $\{3\}$.

91. $5-12x = 8-7x-\left[6\div 3\left(2+5^3\right)+5x\right]$

$$5-12x = 8-7x-\left[6\div 3(2+125)+5x\right]$$
$$5-12x = 8-7x-[6\div 3\cdot 127+5x]$$
$$5-12x = 8-7x-[2\cdot 127+5x]$$
$$5-12x = 8-7x-[254+5x]$$
$$5-12x = 8-7x-254-5x$$
$$5-12x = -12x-246$$
$$5 = -246$$

The final statement is a contradiction, so the equation has no solution. The solution set is $\varnothing$.

92. $2(5x+58) = 10x+4(21\div 3.5-11)$

$$10x+116 = 10x+4(6-11)$$
$$10x+116 = 10x+4(-5)$$
$$10x+116 = 10x-20$$
$$116 = -20$$

The final statement is a contradiction, so the equation has no solution. The solution set is $\varnothing$.

93. $0.7x+0.4(20) = 0.5(x+20)$

$$0.7x+8 = 0.5x+10$$
$$0.2x+8 = 10$$
$$0.2x = 2$$
$$x = 10$$

The solution set is $\{10\}$.

94. $0.5(x+2)=0.1+3(0.1x+0.3)$
$0.5x+1=0.1+0.3x+0.9$
$0.5x+1=0.3x+1$
$0.2x+1=1$
$0.2x=0$
$x=0$
The solution set is $\{0\}$.

95. $4x+13-\{2x-[4(x-3)-5]\}=2(x-6)$
$4x+13-\{2x-[4x-12-5]\}=2x-12$
$4x+13-\{2x-[4x-17]\}=2x-12$
$4x+13-\{2x-4x+17\}=2x-12$
$4x+13-\{-2x+17\}=2x-12$
$4x+13+2x-17=2x-12$
$6x-4=2x-12$
$4x-4=-12$
$4x=-8$
$x=-2$
The solution set is $\{-2\}$.

96. $-2\{7-[4-2(1-x)+3]\}=10-[4x-2(x-3)]$
$-2\{7-[4-2+2x+3]\}=10-[4x-2x+6]$
$-2\{7-[2x+5]\}=10-[2x+6]$
$-2\{7-2x-5\}=10-2x-6$
$-2\{-2x+2\}=-2x+4$
$4x-4=-2x+4$
$6x-4=4$
$6x=8$
$x=\frac{8}{6}=\frac{4}{3}$
The solution set is $\left\{\frac{4}{3}\right\}$.

97. **a.** $p=\frac{4x}{5}+25$
$p=\frac{4(30)}{5}+25$
$p=24+25$
$p=49$
According to the model, 49% of U.S. college freshman had an average grade of A in high school in 2010. This overestimates the value shown in the bar graph by 1%.

b. $p=\frac{4x}{5}+25$
$57=\frac{4x}{5}+25$
$32=\frac{4x}{5}$
$160=4x$
$40=x$
According to the model, 57% of U.S. college freshman will have an average grade of A in high school 40 years after 1980, or 2020.

98. **a.** $p=\frac{4x}{5}+25$
$p=\frac{4(20)}{5}+25$
$p=16+25$
$p=41$
According to the model, 41% of U.S. college freshman had an average grade of A in high school in 2000. This underestimates the value shown in the bar graph by 2%.

b. $p=\frac{4x}{5}+25$
$65=\frac{4x}{5}+25$
$40=\frac{4x}{5}$
$200=4x$
$50=x$
According to the model, 65% of U.S. college freshman will have an average grade of A in high school 50 years after 1980, or 2030.

99. **a.** What cost \$10,000 in 1984 would cost about \$22,000 in 2010.

b. $C=442x+12{,}969$
$=442(20)+12{,}969$
$=\$21{,}809$
It describes the estimate from part (a) reasonably well.

c. $C=2x^2+390x+13{,}126$
$=2(20)^2+390(20)+13{,}126$
$=\$21{,}726$
It describes the estimate from part (a) reasonably well.

100. **a.** What cost \$10,000 in 1984 would cost about \$17,000 in 2000.

b. $C = 442x + 12,969$

$= 442(10) + 12,969$

$= \$17,389$

It describes the estimate from part (a) reasonably well.

c. $C = 2x^2 + 390x + 13,126$

$= 2(10)^2 + 390(10) + 13,126$

$= \$17,226$

It describes the estimate from part (a) reasonably well.

101.
$$\begin{aligned} C &= 442x + 12,969 \\ 26,229 &= 442x + 12,969 \\ 13,260 &= 442x \\ \frac{13,260}{442} &= \frac{442x}{442} \\ 30 &= x \end{aligned}$$
Model 1 predicts the cost will be \$26,229 30 years after 1990, or 2020.

102.
$$\begin{aligned} C &= 442x + 12,969 \\ 25,345 &= 442x + 12,969 \\ 12,376 &= 442x \\ \frac{12,376}{442} &= \frac{442x}{442} \\ 28 &= x \end{aligned}$$
Model 1 predicts the cost will be \$25,345 28 years after 1990, or 2018.

103. 11 learning trials; represented by the point $(11, 0.95)$ on the graph.

104. 1 learning trial; represented by the point $(1, 0.5)$ on the graph.

105.
$$\begin{aligned} C &= \frac{x + 0.1(500)}{x + 500} \\ 0.28 &= \frac{x + 0.1(500)}{x + 500} \\ 0.28(x + 500) &= x + 0.1(500) \\ 0.28x + 140 &= x + 50 \\ -0.72x &= -90 \\ \frac{-0.72x}{-0.72} &= \frac{-90}{-0.72} \\ x &= 125 \end{aligned}$$
125 liters of pure peroxide must be added.

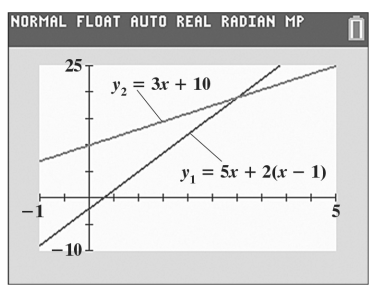
106. **a.** $C = \dfrac{x + 0.35(200)}{x + 200}$

b.
$$\begin{aligned} 0.74 &= \frac{x + 0.35(200)}{x + 200} \\ 0.74(x + 200) &= x + 0.35(200) \\ 0.74x + 148 &= x + 70 \\ -0.26x &= -78 \\ \frac{-0.26x}{-0.26} &= \frac{-78}{-0.26} \\ x &= 300 \end{aligned}$$
300 liters of pure acid must be added.

107. – 115. Answers will vary.

116. $5x + 2(x - 1) = 3x + 10$

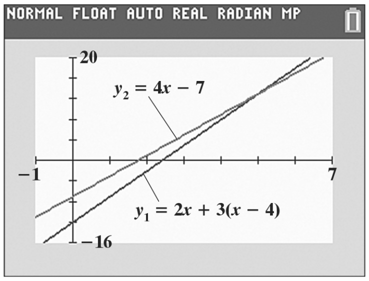
Let $y_1 = 5x + 2(x - 1)$ and let $y_2 = 3x + 10$.



The solution set is $\{3\}$.

117. $2x+3(x-4)=4x-7$

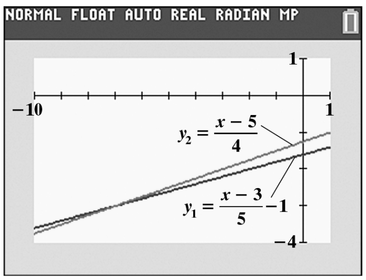
Let $y_1=2x+3(x-4)$ and let $y_2=4x-7$.



The solution set is {5}.

118. $\dfrac{x-3}{5}-1=\dfrac{x-5}{4}$

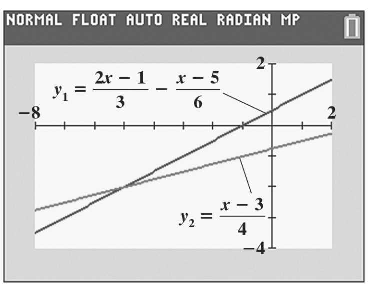
Let $y_1=\dfrac{x-3}{5}-1$ and let $y_2=\dfrac{x-5}{4}$.



The solution set is $\{-7\}$.

119. $\dfrac{2x-1}{3}-\dfrac{x-5}{6}=\dfrac{x-3}{4}$

Let $y_1=\dfrac{2x-1}{3}-\dfrac{x-5}{6}$ and let $y_2=\dfrac{x-3}{4}$.



The solution set is {−5}.

120. does not make sense; Explanations will vary. Sample explanation: Substitute $n=6$ into the equation to find P.

121. makes sense

122. makes sense

123. makes sense

124. false; Changes to make the statement true will vary. A sample change is: $x=0$ is a solution.

125. false; Changes to make the statement true will vary. A sample change is: In the first equation, $x\neq 4$.

126. true

127. false; Changes to make the statement true will vary. A sample change is: If $a = 0$, then $ax + b = 0$ is equivalent to $b = 0$, which either has no solution ($b \neq 0$) or infinitely many solutions ($b = 0$).

128. Answers will vary.

129.
$$\frac{7x+4}{b}+13=x$$
$$\frac{7(-6)+4}{b}+13=-6$$
$$\frac{-42+4}{b}+13=-6$$
$$\frac{-38}{b}+13=-6$$
$$\frac{-38}{b}=-19$$
$$-38=-19b$$
$$b=2$$

130. $\dfrac{4x-b}{x-5}=3$

$4x-b=3(x-5)$

The solution set will be $\varnothing$ if $x = 5$.

$4(5)-b=3(5-5)$

$20-b=0$

$20=b$

$b=20$

131. $x+150$

132. $20+0.05x$

133. $4x+400$

Section 1.3

Check Point Exercises

1. Let x = the average yearly salary, in thousands, of women with an associate's degree
Let $x + 14$ = the average yearly salary, in thousands, of women with a bachelor's degree
Let $x + 26$ = the average yearly salary, in thousands, of women with a master's degree

$$x+(x+14)+(x+26)=139$$
$$x+x+14+x+26=139$$
$$3x+40=139$$
$$3x=99$$
$$x=33$$

$x=33$, associate's degree: \$33,000
$x+14=47$, bachelor's degree: \$47,000
$x+26=59$, master's degree: \$59,000

2. Let x = the number of years after 1969.

$$85-0.9x=25$$
$$-0.9x=25-85$$
$$-0.9x=-60$$
$$x=\frac{-60}{-0.9}$$
$$x\approx 67$$

25% of freshmen will respond this way 67 years after 1969, or 2036.

3. Let x = the number of bridge crossings at which the costs of the two plans are the same.

$$\overbrace{5x}^{\text{No Pass}} = \overbrace{40+3x}^{\text{Discount Pass}}$$
$$5x-3x=40$$
$$2x=40$$
$$x=20$$

The two plans cost the same for 20 bridge crossings.

4. Let x = the computer's price before the reduction.

$$x-0.30x=840$$
$$0.70x=840$$
$$x=\frac{840}{0.70}$$
$$x=1200$$

Before the reduction the computer's price was \$1200.

5. Let x = the amount invested at 9%.
Let $5000 - x$ = the amount invested at 11%.

$$0.09x+0.11(5000-x)=487$$
$$0.09x+550-0.11x=487$$
$$-0.02x+550=487$$
$$-0.02x=-63$$
$$x=\frac{-63}{-0.02}$$
$$x=3150$$
$$5000-x=1850$$

\$3150 was invested at 9% and \$1850 was invested at 11%.

6. Let x = the width of the court.
Let $x + 44$ = the length of the court.

$$2l+2w=P$$
$$2(x+44)+2x=288$$
$$2x+88+2x=288$$
$$4x+88=288$$
$$4x=200$$
$$x=\frac{200}{4}$$
$$x=50$$
$$x+44=94$$

The dimensions of the court are 50 feet by 94 feet.

7.

$$2l+2w=P$$
$$2l+2w-2l=P-2l$$
$$2w=P-2l$$
$$\frac{2w}{2}=\frac{P-2l}{2}$$
$$w=\frac{P-2l}{2}$$

8.

$$P=C+MC$$
$$P=C(1+M)$$
$$\frac{P}{1+M}=\frac{C(1+M)}{1+M}$$
$$\frac{P}{1+M}=C$$
$$C=\frac{P}{1+M}$$

Concept and Vocabulary Check 1.3

1. $x + 658.6$

2. $31 + 2.4x$

3. $4 + 0.15x$

4. $x - 0.15x$ or $0.85x$

5. $0.12x + 0.09(30{,}000 - x)$

6. isolated on one side

7. factoring

Exercise Set 1.3

1. Let $x =$ the number of years spent watching TV.
Let $x + 19 =$ the number of years spent sleeping.

$$\begin{aligned} x + (x + 19) &= 37 \\ x + x + 19 &= 37 \\ 2x + 19 &= 37 \\ 2x &= 18 \\ x &= 9 \\ x + 19 &= 28 \end{aligned}$$

Americans will spend 9 years watching TV and 28 years sleeping.

2. Let $x =$ the number of years spent eating.
Let $x + 24 =$ the number of years spent sleeping.

$$\begin{aligned} x + (x + 24) &= 32 \\ x + x + 24 &= 32 \\ 2x + 24 &= 32 \\ 2x &= 8 \\ x &= 4 \\ x + 24 &= 28 \end{aligned}$$

Americans will spend 4 years eating and 28 years sleeping.

3. Let $x =$ the average salary, in thousands, for an American whose final degree is a bachelor's.
Let $2x - 70 =$ the average salary, in thousands, for an American whose final degree is a master's.

$$\begin{aligned} x + (2x - 70) &= 173 \\ x + 2x - 70 &= 173 \\ 3x - 70 &= 173 \\ 3x &= 243 \\ x &= 81 \\ 2x - 70 &= 92 \end{aligned}$$

The average salary for an American whose final degree is a bachelor's is $81 thousand and for an American whose final degree is a master's is $92 thousand.

4. Let $x =$ the average salary, in thousands, for an American whose final degree is a bachelor's.
Let $2x - 45 =$ the average salary, in thousands, for an American whose final degree is a doctorate.

$$\begin{aligned} x + (2x - 45) &= 198 \\ x + 2x - 45 &= 198 \\ 3x - 45 &= 198 \\ 3x &= 243 \\ x &= 81 \\ 2x - 45 &= 117 \end{aligned}$$

The average salary for an American whose final degree is a bachelor's is $81 thousand and for an American whose final degree is a doctorate is $117 thousand.

5. Let $x =$ the number of years after 2014.

$$\begin{aligned} 37{,}600 + 1250x &= 46{,}350 \\ 1250x &= 8750 \\ \frac{1250x}{1250} &= \frac{8750}{1250} \\ x &= 7 \end{aligned}$$

7 years after 2014, or in 2021, the average price of a new car will be $46,350.

6. Let $x =$ the number of years after 2014.

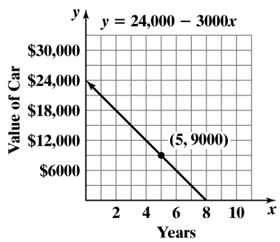
$$\begin{aligned} 11.3 + 0.2x &= 12.3 \\ 0.2x &= 1 \\ \frac{0.2x}{0.2} &= \frac{1}{0.2} \\ x &= 5 \end{aligned}$$

5 years after 2014, or in 2019, the average age of vehicles on U.S. roads will be 12.3 years.

7. **a.** $y = 24,000 - 3000x$

b.
$$y = 24,000 - 3000x$$
$$9000 = 24,000 - 3000x$$
$$9000 - 24,000 = -3000x$$
$$-15,000 = -3000x$$
$$x = \frac{-15,000}{-3000}$$
$$x = 5$$
The car's value will drop to $9000 after 5 years.

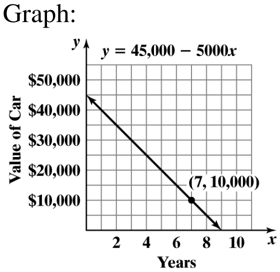
c. Graph:



8. **a.** $y = 45,000 - 5000x$

b.
$$y = 45,000 - 5000x$$
$$10,000 = 45,000 - 5000x$$
$$10,000 - 45,000 = -5000x$$
$$-35,000 = -5000x$$
$$x = \frac{-35,000}{-5000}$$
$$x = 7$$
The car's value will drop to $10,000 after 7 years.

c. Graph:



9. Let x = the number of months.
The cost for Club A: $25x + 40$
The cost for Club B: $30x + 15$
$$25x + 40 = 30x + 15$$
$$-5x + 40 = 15$$
$$-5x = -25$$
$$x = 5$$
The total cost for the clubs will be the same at 5 months. The cost will be
$25(5) + 40 = 30(5) + 15 = \$165$

10. Let g = the number of video games rented
$$9g = 4g + 50$$
$$5g = 50$$
$$g = 10$$
The total amount spent at each store will be the same after 10 rentals.
$9g = 9(10) = 90$
The total amount spent will be $90.

11. Let x = the number of uses.
Cost without discount pass: $1.25x$
Cost with discount pass: $15 + 0.75x$
$$1.25x = 15 + 0.75x$$
$$0.50x = 15$$
$$x = 30$$
The bus must be used 30 times in a month for the costs to be equal.

12. Cost per crossing: 5x$
Cost with discount pass: $30 + 3.50x$
$$5x = 30 + 3.50x$$
$$1.50x = 30$$
$$x = 20$$
The bridge must be used 20 times in a month for the costs to be equal.

13. **a.** Let x = the number of years (after 2010).
College A's enrollment: $13,300 + 1000x$

College B's enrollment: $26,800 - 500x$
$$13,300 + 1000x = 26,800 - 500x$$
$$13,300 + 1500x = 26,800$$
$$1500x = 13,500$$
$$x = 9$$
The two colleges will have the same enrollment 9 years after 2010, or 2019. That year the enrollment will be
$13,300 + 1000(9)$
$= 26,800 - 500(9)$
$= 22,300$ students

b. Check points to determine that $y_1 = 13,300 + 1000x$ and $y_2 = 26,800 - 500x$.

14. Let x = the number of years after 2000

$$10,600,000-28,000x=10,200,000-12,000x$$
$$-16,000x=-400,000$$
$$x=25$$

The countries will have the same population 25 years after the year 2000, or the year 2025.

$$10,200,000-12,000x=10,200,000-12,000(25)$$
$$=10,200,000-300,000$$
$$=9,900,000$$

The population in the year 2025 will be 9,900,000.

15. Let x = the cost of the television set.

$$x-0.20x=336$$
$$0.80x=336$$
$$x=420$$

The television set's price is \$420.

16. Let x = the cost of the dictionary

$$x-0.30x=30.80$$
$$0.70x=30.80$$
$$x=44$$

The dictionary's price before the reduction was \$44.

17. Let x = the nightly cost

$$x+0.08x=162$$
$$1.08x=162$$
$$x=150$$

The nightly cost is \$150.

18. Let x = the nightly cost

$$x+0.05x=252$$
$$1.05x=252$$
$$x=240$$

The nightly cost is \$240.

19. Let c = the dealer's cost

$$584=c+0.25c$$
$$584=1.25c$$
$$467.20=c$$

The dealer's cost is \$467.20.

20. Let c = the dealer's cost

$$15=c+0.25c$$
$$15=1.25c$$
$$12=c$$

The dealer's cost is \$12.

21. Let x = the amount invested at 6%.
Let $7000 - x$ = the amount invested at 8%.

$$0.06x+0.08(7000-x)=520$$
$$0.06x+560-0.08x=520$$
$$-0.02x+560=520$$
$$-0.02x=-40$$
$$x=\frac{-40}{-0.02}$$
$$x=2000$$
$$7000-x=5000$$

\$2000 was invested at 6% and \$5000 was invested at 8%.

22. Let x = the amount invested at 5%.
Let $11,000 - x$ = the amount invested at 8%.

$$0.05x+0.08(11,000-x)=730$$
$$0.05x+880-0.08x=730$$
$$-0.03x+880=730$$
$$-0.03x=-150$$
$$x=\frac{-150}{-0.03}$$
$$x=5000$$
$$11,000-x=6000$$

\$5000 was invested at 5% and \$6000 was invested at 8%.

23. Let x = amount invested at 12%
$8000 - x$ = amount invested at 5% loss

$$.12x-.05(8000-x)=620$$
$$.12x-400+.05x=620$$
$$.17x=1020$$
$$x=6000$$
$$8000-x=2000$$

\$6000 at 12%, \$2000 at 5% loss

24. Let x = amount at 14%
$12000 - x$ = amount at 6%

$$.14x-0.6\left(12000-x\right)=680$$
$$.14x-720+.06x=680$$
$$.2x=1400$$
$$x=7000$$
$$12000-7000=5000$$

\$7000 at 14%, \$5000 at 6% loss

25. Let w = the width of the field
Let $2w$ = the length of the field

$P = 2(\text{length}) + 2(\text{width})$
$300 = 2(2w) + 2(w)$
$300 = 4w + 2w$
$300 = 6w$
$50 = w$

If $w = 50$, then $2w = 100$. Thus, the dimensions are 50 yards by 100 yards.

26. Let w = the width of the swimming pool,
Let $3w$ = the length of the swimming pool

$P = 2(\text{length}) + 2(\text{width})$
$320 = 2(3w) + 2(w)$
$320 = 6w + 2w$
$320 = 8w$
$40 = w$

If w = 40, $3w$ = 3(40) = 120.
The dimensions are 40 feet by 120 feet.

27. Let w = the width of the field
Let $2w + 6$ = the length of the field

$228 = 6w + 12$
$216 = 6w$
$36 = w$

If $w = 36$, then $2w + 6 = 2(36) + 6 = 78$. Thus, the dimensions are 36 feet by 78 feet.

28. Let w = the width of the pool,
Let $2w – 6$ = the length of the pool

$P = 2(\text{length}) + 2(\text{width})$
$126 = 2(2w - 6) + 2(w)$
$126 = 4w - 12 + 2w$
$126 = 6w - 12$
$138 = 6w$
$23 = w$

Find the length.
$2w - 6 = 2(23) - 6 = 46 - 6 = 40$
The dimensions are 23 meters by 40 meters.

29. Let x = the width of the frame.
Total length: $16 + 2x$
Total width: $12 + 2x$

$P = 2(\text{length}) + 2(\text{width})$
$72 = 2(16 + 2x) + 2(12 + 2x)$
$72 = 32 + 4x + 24 + 4x$
$72 = 8x + 56$
$16 = 8x$
$2 = x$

The width of the frame is 2 inches.

30. Let w = the width of the path
Let 40 + $2w$ = the width of the pool and path
Let 60 + $2w$ = the length of the pool and path

$2(40 + 2w) + 2(60 + 2w) = 248$
$80 + 4w + 120 + 4w = 248$
$200 + 8w = 248$
$8w = 48$
$w = 6$

The width of the path is 6 feet.

31. Let x = number of hours
$35x$ = labor cost
$35x + 63 = 448$
$35x = 385$
$x = 11$
It took 11 hours.

32. Let x = number of hours
$63x$ = labor cost
$63x + 532 = 1603$
$63x = 1071$
$x = 17$
17 hours were required to repair the yacht.

33. Let x = inches over 5 feet
$100 + 5x = 135$
$5x = 35$
$x = 7$
A height of 5 feet 7 inches corresponds to 135 pounds.

34. Let g = the gross amount of the paycheck

$\text{Yearly Salary} = 2(12)g + 750$
$33150 = 24g + 750$
$32400 = 24g$
$1350 = g$

The gross amount of each paycheck is $1350.

35. Let x = the weight of unpeeled bananas.

$\frac{7}{8}x =$ weight of peeled bananas

$$x = \frac{7}{8}x + \frac{7}{8}$$
$$\frac{1}{8}x = \frac{7}{8}$$
$$x = 7$$

The banana with peel weighs 7 ounces.

36. Let x = the length of the call.

$$0.43 + 0.32(x-1) + 2.10 = 5.73$$
$$0.43 + 0.32x - 0.32 + 2.10 = 5.73$$
$$0.32x + 2.21 = 5.73$$
$$0.32x = 3.52$$
$$x = 11$$

The person talked for 11 minutes.

37. $A = lw$

$$w = \frac{A}{l}$$

area of rectangle

38. $D = RT$

$$R = \frac{D}{T}$$

distance, rate, time equation

39. $A = \frac{1}{2}bh$

$$2A = bh$$
$$b = \frac{2A}{h};$$

area of triangle

40. $V = \frac{1}{3}Bh$

$$3V = Bh$$
$$B = \frac{3V}{h}$$

volume of a cone

41. $I = Prt$

$$P = \frac{I}{rt};$$

interest

42. $C = 2\pi r$

$$r = \frac{C}{2\pi};$$

circumference of a circle

43. $E = mc^2$

$$m = \frac{E}{c^2};$$

Einstein's equation

44. $V = \pi r^2 h$

$$h = \frac{V}{\pi r^2};$$

volume of a cylinder

45. $T = D + pm$

$$T - D = pm$$
$$\frac{T-D}{m} = \frac{pm}{m}$$
$$\frac{T-D}{m} = p$$

total of payment

46. $P = C + MC$

$$P - C = MC$$
$$\frac{P-C}{C} = M$$

markup based on cost

47. $A = \frac{1}{2}h(a+b)$

$$2A = h(a+b)$$
$$\frac{2A}{h} = a+b$$
$$\frac{2A}{h} - b = a$$

area of trapezoid

48. $A = \frac{1}{2}h(a+b)$

$$2A = h(a+b)$$
$$\frac{2A}{h} = a+b$$
$$\frac{2A}{h} - a = b$$

area of trapezoid

49. $S = P + Prt$
$S - P = Prt$
$\frac{S-P}{Pt} = r;$
interest

50. $S = P + Prt$
$S - P = Prt$
$\frac{S-P}{Pr} = t;$
interest

51. $B = \frac{F}{S-V}$
$B(S-V) = F$
$S - V = \frac{F}{B}$
$S = \frac{F}{B} + V$

52. $S = \frac{C}{1-r}$
$S(1-r) = C$
$1 - r = \frac{C}{S}$
$-r = \frac{C}{S} - 1$
$r = -\frac{C}{S} + 1$
markup based on selling price

53. $IR + Ir = E$
$I(R+r) = E$
$I = \frac{E}{R+r}$
electric current

54. $A = 2lw + 2lh + 2wh$
$A - 2lw = h(2l + 2w)$
$\frac{A-2lw}{2l+2w} = h$
surface area

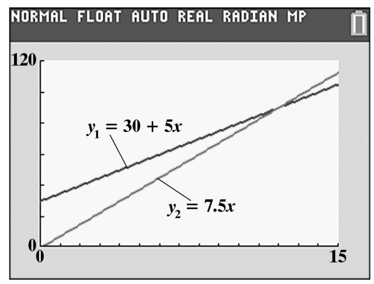
55. $\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$
$qf + pf = pq$
$f(q+p) = pq$
$f = \frac{pq}{p+q}$
thin lens equation

56. $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$
$R_1R_2 = RR_2 + RR_1$
$R_1R_2 - RR_1 = RR_2$
$R_1(R_2 - R) = RR_2$
$R_1 = \frac{RR_2}{R_2 - R}$
resistance

57. – 61. Answers will vary.

62. a. $F = 30 + 5x$
$F = 7.5x$

b.



c. Calculator shows the graphs to intersect at (12, 90); the two options both cost \$90 when 12 hours court time is used per month.

d. $30 + 5x = 7.5x$
$30 = 2.5x$
$x = 12$
Rent the court 12 hours per month.

63. does not make sense; Explanations will vary. Sample explanation: Though mathematical models can often provide excellent estimates about future attitudes, they cannot guaranty perfect precision.

64. makes sense

65. does not make sense; Explanations will vary. Sample explanation: Solving a formula for one of its variables does not produce a numerical value for the variable.

66. does not make sense; Explanations will vary. Sample explanation: The correct equation is $x - 0.35x = 780.$

67. $0.1x+.9(1000-x)=420$

$0.1+900-0.9x=420$

$-0.8x=-480$

$x=600$

600 students at the north campus, 400 students at south campus.

68. Let x = original price

$x - 0.4x = 0.6x$ = price after first reduction

$0.6x - 0.4(0.6x)$ = price after second reduction

$0.6x-0.24x=72$

$0.36x=72$

$x=200$

The original price was \$200.

69. Let x = woman's age

$3x$ = Coburn's age

$3x + 20 = 2(x + 20)$

$3x + 20 = 2x + 40$

$x + 20 = 40$

$x = 20$

Coburn is 60 years old the woman is 20 years old.

70. Let x = correct answers

$26 - x$ = incorrect answers

$8x-5(26-x)=0$

$8x-130+5x=0$

$13x-130=0$

$13x=130$

$x=10$

10 problems were solved correctly.

71. Let x = mother's amount

$2x$ = boy's amount

$\frac{x}{2}$ = girl's amount

$x+2x+\frac{x}{2}=14,000$

$\frac{7}{2}x=14,000$

$x=\$4,000$

The mother received \$4000, the boy received \$8000, and the girl received \$2000.

72. Let x = the number of plants originally stolen

After passing the first security guard, the thief has:

$$x-\left(\frac{1}{2}x+2\right)=x-\frac{1}{2}x-2=\frac{1}{2}x-2$$

After passing the second security guard, the thief has:

$$\frac{1}{2}x-2-\left(\frac{\frac{1}{2}x-2}{2}+2\right)=\frac{1}{4}x-3$$

After passing the third security guard, the thief has:

$$\frac{1}{4}x-3-\left(\frac{\frac{1}{4}x-3}{2}+2\right)=\frac{1}{8}x-\frac{7}{2}$$

Thus, $\frac{1}{8}x-\frac{7}{2}=1$

$x-28=8$

$x=36$

The thief stole 36 plants.

73. $V=C-\frac{C-S}{L}N$

$VL=CL-CN+SN$

$VL-SN=CL-CN$

$VL-SN=C(L-N)$

$\frac{VL-SN}{L-N}=C$

$C=\frac{VL-SN}{L-N}$

74. Answers will vary

75. $(7-3x)(-2-5x)=-14-35x+6x+15x^2$

$=-14-29x+15x^2$

or

$=15x^2-29x-14$

76. $\sqrt{18}-\sqrt{8}=\sqrt{9\cdot 2}-\sqrt{4\cdot 2}$

$=3\sqrt{2}-2\sqrt{2}$

$=\sqrt{2}$

77. $\frac{7+4\sqrt{2}}{2-5\sqrt{2}}\cdot\frac{2+5\sqrt{2}}{2+5\sqrt{2}}=\frac{14+35\sqrt{2}+8\sqrt{2}+40}{4+10\sqrt{2}-10\sqrt{2}-50}$

$=\frac{54+43\sqrt{2}}{-46}$

$=-\frac{54+43\sqrt{2}}{46}$

Section 1.4

Check Point Exercises

1. **a.** $(5-2i)+(3+3i)$
$=5-2i+3+3i$
$=(5+3)+(-2+3)i$
$=8+i$

b. $(2+6i)-(12-i)$
$=2+6i-12+i$
$=(2-12)+(6+1)i$
$=-10+7i$

2. **a.** $7i(2-9i)=7i(2)-7i(9i)$
$=14i-63i^2$
$=14i-63(-1)$
$=63+14i$

b. $(5+4i)(6-7i)=30-35i+24i-28i^2$
$=30-35i+24i-28(-1)$
$=30+28-35i+24i$
$=58-11i$

3. $\frac{5i}{7+i}=\frac{5i}{7+i}\cdot\frac{7-i}{7-i}$
$=\frac{35i-5i^2}{49+7i-7i-i^2}$
$=\frac{35i+5}{49+1}$
$=\frac{35i+5}{50}$
$=\frac{5}{50}+\frac{35}{50}i$
$=\frac{1}{10}+\frac{7}{10}i$

4. $\frac{5+4i}{4-i}=\frac{5+4i}{4-i}\cdot\frac{4+i}{4+i}$
$=\frac{20+5i+16i+4i^2}{16+4i-4i-i^2}$
$=\frac{20+21i-4}{16+1}$
$=\frac{16+21i}{17}$
$=\frac{16}{17}+\frac{21}{17}i$

5. **a.** $\sqrt{-27}+\sqrt{-48}=i\sqrt{27}+i\sqrt{48}$
$=i\sqrt{9\cdot 3}+i\sqrt{16\cdot 3}$
$=3i\sqrt{3}+4i\sqrt{3}$
$=7i\sqrt{3}$

b. $(-2+\sqrt{-3})^2=(-2+i\sqrt{3})^2$
$=(-2)^2+2(-2)(i\sqrt{3})+(i\sqrt{3})^2$
$=4-4i\sqrt{3}+3i^2$
$=4-4i\sqrt{3}+3(-1)$
$=1-4i\sqrt{3}$

c. $\frac{-14+\sqrt{-12}}{2}=\frac{-14+i\sqrt{12}}{2}$
$=\frac{-14+2i\sqrt{3}}{2}$
$=\frac{-14}{2}+\frac{2i\sqrt{3}}{2}$
$=-7+i\sqrt{3}$

Concept and Vocabulary Check 1.4

1. $\sqrt{-1}$; -1

2. complex; imaginary; real

3. $-6i$

4. $14i$

5. 18; $-15i$; $12i$; $-10i^2$; 10

6. $2+9i$

7. $2+5i$

8. i; $2i\sqrt{5}$

Exercise Set 1.4

1. $(7+2i)+(1-4i)=7+2i+1-4i$
$=7+1+2i-4i$
$=8-2i$

2. $(-2+6i)+(4-i)$
$=-2+6i+4-i$
$=-2+4+6i-i$
$=2+5i$

3. $(3 + 2i) - (5 - 7i) = 3 - 5 + 2i + 7i$
$= 3 + 2i - 5 + 7i$
$= -2 + 9i$

4. $(-7 + 5i) - (-9 - 11i) = -7 + 5i + 9 + 11i$
$= -7 + 9 + 5i + 11i$
$= 2 + 16i$

5. $6 - (-5 + 4i) - (-13 - i) = 6 + 5 - 4i + 13 + i$
$= 24 - 3i$

6. $7 - (-9 + 2i) - (-17 - i) = 7 + 9 - 2i + 17 + i$
$= 33 - i$

7. $8i - (14 - 9i) = 8i - 14 + 9i$
$= -14 + 8i + 9i$
$= -14 + 17i$

8. $15i - (12 - 11i) = 15i - 12 + 11i$
$= -12 + 15i + 11i$
$= -12 + 26i$

9. $-3i(7i - 5) = -21i^2 + 15i$
$= -21(-1) + 15i$
$= 21 + 15i$

10. $-8i(2i - 7) = -16i^2 + 56i = -16(-1) + 56i$
$= 9 - 25i^2 = 9 + 25 = 34 = 16 + 56i$

11. $(-5 + 4i)(3 + i) = -15 - 5i + 12i + 4i^2$
$= -15 + 7i - 4$
$= -19 + 7i$

12. $(-4 - 8i)(3 + i) = -12 - 4i - 24i - 8i^2$
$= -12 - 28i + 8$
$= -4 - 28i$

13. $(7 - 5i)(-2 - 3i) = -14 - 21i + 10i + 15i^2$
$= -14 - 15 - 11i$
$= -29 - 11i$

14. $(8 - 4i)(-3 + 9i) = -24 + 72i + 12i - 36i^2$
$= -24 + 36 + 84i$
$= 12 + 84i$

15. $(3 + 5i)(3 - 5i) = 9 - 15i + 15i - 25i^2$
$= 9 + 25$
$= 34$

16. $(2 + 7i)(2 - 7i) = 4 - 49i^2 = 4 + 49 = 53$

17. $(-5 + i)(-5 - i) = 25 + 5i - 5i - i^2$
$= 25 + 1$
$= 26$

18. $(-7 + i)(-7 - i) = 49 + 7i - 7i - i^2$
$= 49 + 1$
$= 50$

19. $(2 + 3i)^2 = 4 + 12i + 9i^2$
$= 4 + 12i - 9$
$= -5 + 12i$

20. $(5 - 2i)^2 = 25 - 20i + 4i^2$
$= 25 - 20i - 4$
$= 21 - 20i$

21. $\frac{2}{3-i} = \frac{2}{3-i} \cdot \frac{3+i}{3+i}$
$= \frac{2(3+i)}{9+1}$
$= \frac{2(3+i)}{10}$
$= \frac{3+i}{5}$
$= \frac{3}{5} + \frac{1}{5}i$

22. $\frac{3}{4+i} = \frac{3}{4+i} \cdot \frac{4-i}{4-i}$
$= \frac{3(4-i)}{16-i^2}$
$= \frac{3(4-i)}{17}$
$= \frac{12}{17} - \frac{3}{17}i$

23. $\frac{2i}{1+i} = \frac{2i}{1+i} \cdot \frac{1-i}{1-i} = \frac{2i-2i^2}{1+1} = \frac{2+2i}{2} = 1+i$

24. $\frac{5i}{2-i} = \frac{5i}{2-i} \cdot \frac{2+i}{2+i}$
$= \frac{10i+5i^2}{4+1}$
$= \frac{-5+10i}{5}$
$= -1+2i$

25. $\frac{8i}{4-3i}=\frac{8i}{4-3i}\cdot\frac{4+3i}{4+3i}$
$=\frac{32i+24i^2}{16+9}$
$=\frac{-24+32i}{25}$
$=-\frac{24}{25}+\frac{32}{25}i$

26. $\frac{-6i}{3+2i}=\frac{-6i}{3+2i}\cdot\frac{3-2i}{3-2i}=\frac{-18i+12i^2}{9+4}$
$=\frac{-12-18i}{13}=-\frac{12}{13}-\frac{18}{13}i$

27. $\frac{2+3i}{2+i}=\frac{2+3i}{2+i}\cdot\frac{2-i}{2-i}$
$=\frac{4+4i-3i^2}{4+1}$
$=\frac{7+4i}{5}$
$=\frac{7}{5}+\frac{4}{5}i$

28. $\frac{3-4i}{4+3i}=\frac{3-4i}{4+3i}\cdot\frac{4-3i}{4-3i}$
$=\frac{12-25i+12i^2}{16+9}$
$=\frac{-25i}{25}$
$=-i$

29. $\sqrt{-64}-\sqrt{-25}=i\sqrt{64}-i\sqrt{25}$
$=8i-5i=3i$

30. $\sqrt{-81}-\sqrt{-144}=i\sqrt{81}-i\sqrt{144}=9i-12i$
$=-3i$

31. $5\sqrt{-16}+3\sqrt{-81}=5(4i)+3(9i)$
$=20i+27i=47i$

32. $5\sqrt{-8}+3\sqrt{-18}$
$=5i\sqrt{8}+3i\sqrt{18}=5i\sqrt{4\cdot 2}+3i\sqrt{9\cdot 2}$
$=10i\sqrt{2}+9i\sqrt{2}$
$=19i\sqrt{2}$

33. $\left(-2+\sqrt{-4}\right)^2=\left(-2+2i\right)^2$
$=4-8i+4i^2$
$=4-8i-4$
$=-8i$

34. $\left(-5-\sqrt{-9}\right)^2=(-5-i\sqrt{9})^2=\left(-5-3i\right)^2$
$=25+30i+9i^2$
$=25+30i-9$
$=16+30i$

35. $\left(-3-\sqrt{-7}\right)^2=\left(-3-i\sqrt{7}\right)^2$
$=9+6i\sqrt{7}+i^2(7)$
$=9-7+6i\sqrt{7}$
$=2+6i\sqrt{7}$

36. $\left(-2+\sqrt{-11}\right)^2=\left(-2+i\sqrt{11}\right)^2$
$=4-4i\sqrt{11}+i^2(11)$
$=4-11-4i\sqrt{11}$
$=-7-4i\sqrt{11}$

37. $\frac{-8+\sqrt{-32}}{24}=\frac{-8+i\sqrt{32}}{24}$
$=\frac{-8+i\sqrt{16\cdot 2}}{24}$
$=\frac{-8+4i\sqrt{2}}{24}$
$=-\frac{1}{3}+\frac{\sqrt{2}}{6}i$

38. $\frac{-12+\sqrt{-28}}{32}=\frac{-12+i\sqrt{28}}{32}=\frac{-12+i\sqrt{4\cdot 7}}{32}$
$=\frac{-12+2i\sqrt{7}}{32}=-\frac{3}{8}+\frac{\sqrt{7}}{16}i$

39. $\frac{-6-\sqrt{-12}}{48}=\frac{-6-i\sqrt{12}}{48}$
$=\frac{-6-i\sqrt{4\cdot 3}}{48}$
$=\frac{-6-2i\sqrt{3}}{48}$
$=-\frac{1}{8}-\frac{\sqrt{3}}{24}i$

40. $\dfrac{-15-\sqrt{-18}}{33}=\dfrac{-15-i\sqrt{18}}{33}=\dfrac{-15-i\sqrt{9\cdot 2}}{33}$
$=\dfrac{-15-3i\sqrt{2}}{33}=-\dfrac{5}{11}-\dfrac{\sqrt{2}}{11}i$

41. $\sqrt{-8}\left(\sqrt{-3}-\sqrt{5}\right)=i\sqrt{8}(i\sqrt{3}-\sqrt{5})$
$=2i\sqrt{2}\left(i\sqrt{3}-\sqrt{5}\right)$
$=-2\sqrt{6}-2i\sqrt{10}$

42. $\sqrt{-12}\left(\sqrt{-4}-\sqrt{2}\right)=i\sqrt{12}(i\sqrt{4}-\sqrt{2})$
$=2i\sqrt{3}\left(2i-\sqrt{2}\right)$
$=4i^2\sqrt{3}-2i\sqrt{6}$
$=-4\sqrt{3}-2i\sqrt{6}$

43. $\left(3\sqrt{-5}\right)\left(-4\sqrt{-12}\right)=\left(3i\sqrt{5}\right)\left(-8i\sqrt{3}\right)$
$=-24i^2\sqrt{15}$
$=24\sqrt{15}$

44. $\left(3\sqrt{-7}\right)\left(2\sqrt{-8}\right)$
$=(3i\sqrt{7})(2i\sqrt{8})=(3i\sqrt{7})(2i\sqrt{4\cdot 2})$
$=\left(3i\sqrt{7}\right)\left(4i\sqrt{2}\right)=12i^2\sqrt{14}=-12\sqrt{14}$

45. $(2-3i)(1-i)-(3-i)(3+i)$
$=\left(2-2i-3i+3i^2\right)-\left(3^2-i^2\right)$
$=2-5i+3i^2-9+i^2$
$=-7-5i+4i^2$
$=-7-5i+4(-1)$
$=-11-5i$

46. $(8+9i)(2-i)-(1-i)(1+i)$
$=\left(16-8i+18i-9i^2\right)-\left(1^2-i^2\right)$
$=16+10i-9i^2-1+i^2$
$=15+10i-8i^2$
$=15+10i-8(-1)$
$=23+10i$

47. $(2+i)^2-(3-i)^2$
$=\left(4+4i+i^2\right)-\left(9-6i+i^2\right)$
$=4+4i+i^2-9+6i-i^2$
$=-5+10i$

48. $(4-i)^2-(1+2i)^2$
$=\left(16-8i+i^2\right)-\left(1+4i+4i^2\right)$
$=16-8i+i^2-1-4i-4i^2$
$=15-12i-3i^2$
$=15-12i-3(-1)$
$=18-12i$

49. $5\sqrt{-16}+3\sqrt{-81}$
$=5\sqrt{16}\sqrt{-1}+3\sqrt{81}\sqrt{-1}$
$=5\cdot 4i+3\cdot 9i$
$=20i+27i$
$=47i$ or $0+47i$

50. $5\sqrt{-8}+3\sqrt{-18}$
$=5\sqrt{4}\sqrt{2}\sqrt{-1}+3\sqrt{9}\sqrt{2}\sqrt{-1}$
$=5\cdot 2\sqrt{2}\,i+3\cdot 3\sqrt{2}\,i$
$=10i\sqrt{2}+9i\sqrt{2}$
$=(10+9)i\sqrt{2}$
$=19i\sqrt{2}$ or $0+19i\sqrt{2}$

51. $f(x)=x^2-2x+2$
$f(1+i)=(1+i)^2-2(1+i)+2$
$=1+2i+i^2-2-2i+2$
$=1+i^2$
$=1-1$
$=0$

52. $f(x)=x^2-2x+5$
$f(1-2i)=(1-2i)^2-2(1-2i)+5$
$=1-4i+4i^2-2+4i+5$
$=4+4i^2$
$=4-4$
$=0$

53. $f(x)=\dfrac{x^2+19}{2-x}$

$f(3i)=\dfrac{(3i)^2+19}{2-3i}$

$=\dfrac{9i^2+19}{2-3i}$

$=\dfrac{-9+19}{2-3i}$

$=\dfrac{10}{2-3i}$

$=\dfrac{10}{2-3i}\cdot\dfrac{2+3i}{2+3i}$

$=\dfrac{20+30i}{4-9i^2}$

$=\dfrac{20+30i}{4+9}$

$=\dfrac{20+30i}{13}$

$=\dfrac{20}{13}+\dfrac{30}{13}i$

54. $f(x)=\dfrac{x^2+11}{3-x}$

$f(4i)=\dfrac{(4i)^2+11}{3-4i}=\dfrac{16i^2+11}{3-4i}$

$=\dfrac{-16+11}{3-4i}$

$=\dfrac{-5}{3-4i}$

$=\dfrac{-5}{3-4i}\cdot\dfrac{3+4i}{3+4i}$

$=\dfrac{-15-20i}{9-16i^2}$

$=\dfrac{-15-20i}{9+16}$

$=\dfrac{-15-20i}{25}$

$=\dfrac{-15}{25}-\dfrac{20}{25}i$

$=-\dfrac{3}{5}-\dfrac{4}{5}i$

55. $E=IR=(4-5i)(3+7i)$

$=12+28i-15i-35i^2$

$=12+13i-35(-1)$

$=12+35+13i=47+13i$

The voltage of the circuit is $(47+13i)$ volts.

56. $E=IR=(2-3i)(3+5i)$

$=6+10i-9i-15i^2=6+i-15(-1)$

$=6+i+15=21+i$

The voltage of the circuit is $(21+i)$ volts.

57. Sum:

$\left(5+i\sqrt{15}\right)+\left(5-i\sqrt{15}\right)$

$=5+i\sqrt{15}+5-i\sqrt{15}$

$=5+5$

$=10$

Product:

$\left(5+i\sqrt{15}\right)\left(5-i\sqrt{15}\right)$

$=25-5i\sqrt{15}+5i\sqrt{15}-15i^2$

$=25+15$

$=40$

58. – 66. Answers will vary.

67. makes sense

68. does not make sense; Explanations will vary. Sample explanation: Imaginary numbers are not undefined.

69. does not make sense; Explanations will vary. Sample explanation: $i=\sqrt{-1}$; It is not a variable in this context.

70. makes sense

71. false; Changes to make the statement true will vary. A sample change is: All irrational numbers are complex numbers.

72. false; Changes to make the statement true will vary. A sample change is: $(3+7i)(3-7i)=9+49=58$ which is a real number.

73. false; Changes to make the statement true will vary. A sample change is:

$\dfrac{7+3i}{5+3i}=\dfrac{7+3i}{5+3i}\cdot\dfrac{5-3i}{5-3i}=\dfrac{44-6i}{34}=\dfrac{22}{17}-\dfrac{3}{17}i$

74. true

75.
$$\frac{4}{(2+i)(3-i)}=\frac{4}{6-2i+3i-i^2}$$
$$=\frac{4}{6+i+1}$$
$$=\frac{4}{7+i}$$
$$=\frac{4}{7+i}\cdot\frac{7-i}{7-i}$$
$$=\frac{28-4i}{49-i^2}$$
$$=\frac{28-4i}{49+1}$$
$$=\frac{28-4i}{50}$$
$$=\frac{28}{50}-\frac{4}{50}i$$
$$=\frac{14}{25}-\frac{2}{25}i$$

76.
$$\frac{1+i}{1+2i}+\frac{1-i}{1-2i}$$
$$=\frac{(1+i)(1-2i)}{(1+2i)(1-2i)}+\frac{(1-i)(1+2i)}{(1+2i)(1-2i)}$$
$$=\frac{(1+i)(1-2i)+(1-i)(1+2i)}{(1+2i)(1-2i)}$$
$$=\frac{1-2i+i-2i^2+1+2i-i-2i^2}{1-4i^2}$$
$$=\frac{1-2i+i+2+1+2i-i+2}{1+4}$$
$$=\frac{6}{5}$$
$$=\frac{6}{5}+0i$$

77.
$$\frac{8}{1+\frac{2}{i}}=\frac{8}{\frac{i}{i}+\frac{2}{i}}$$
$$=\frac{8}{\frac{2+i}{i}}$$
$$=\frac{8i}{2+i}$$
$$=\frac{8i}{2+i}\cdot\frac{2-i}{2-i}$$
$$=\frac{16i-8i^2}{4-i^2}$$
$$=\frac{16i+8}{4+1}$$
$$=\frac{8+16i}{5}$$
$$=\frac{8}{5}+\frac{16}{5}i$$

78. $2x^2+7x-4=(2x-1)(x+4)$

79. $x^2-6x+9=(x-3)(x-3)=(x-3)^2$

80.
$$\frac{-b-\sqrt{b^2-4ac}}{2a}=\frac{-(9)-\sqrt{(9)^2-4(2)(-5)}}{2(2)}$$
$$=\frac{-9-\sqrt{81+40}}{4}$$
$$=\frac{-9-\sqrt{121}}{4}$$
$$=\frac{-9-11}{4}$$
$$=-5$$

Section 1.5

Check Point Exercises

1. a. $3x^2 - 9x = 0$
$3x(x-3) = 0$
$3x = 0 \quad \text{or} \quad x - 3 = 0$
$x = 0 \qquad x = 3$
The solution set is $\{0, 3\}$.

b. $2x^2 + x = 1$
$2x^2 + x - 1 = 0$
$(2x-1)(x+1) = 0$
$2x - 1 = 0 \quad \text{or} \quad x + 1 = 0$
$2x = 1 \qquad x = -1$
$x = \frac{1}{2}$
The solution set is $\left\{-1, \frac{1}{2}\right\}$.

2. a. $3x^2 - 21 = 0$
$3x^2 = 21$
$\frac{3x^2}{3} = \frac{21}{3}$
$x^2 = 7$
$x = \pm\sqrt{7}$
The solution set is $\left\{-\sqrt{7}, \sqrt{7}\right\}$.

b. $5x^2 + 45 = 0$
$5x^2 = -45$
$x^2 = -9$
$x = \pm\sqrt{-9}$
$x = \pm 3i$
The solution set is $\{-3i, 3i\}$.

c. $(x+5)^2 = 11$
$x + 5 = \pm\sqrt{11}$
$x = -5 \pm \sqrt{11}$
The solution set is $\left\{-5+\sqrt{11}, -5-\sqrt{11}\right\}$.

3. a. The coefficient of the x-term is 6. Half of 6 is 3, and 3^2 is 9.
9 should be added to the binomial.
$x^2 + 6x + 9 = (x+3)^2$

b. The coefficient of the x-term is –5.
Half of –5 is $-\frac{5}{2}$, and $\left(-\frac{5}{2}\right)^2$ is $\frac{25}{4}$.
$\frac{25}{4}$ should be added to the binomial.
$x^2 - 5x + \frac{25}{4} = \left(x - \frac{5}{2}\right)^2$

c. The coefficient of the x-term is $\frac{2}{3}$.
Half of $\frac{2}{3}$ is $\frac{1}{3}$, and $\left(\frac{1}{3}\right)^2$ is $\frac{1}{9}$.
$\frac{1}{9}$ should be added to the binomial.
$x^2 + \frac{2}{3}x + \frac{1}{9} = \left(x + \frac{1}{3}\right)^2$

4. $x^2 + 4x - 1 = 0$
$x^2 + 4x = 1$
$x^2 + 4x + 4 = 1 + 4$
$(x+2)^2 = 5$
$x + 2 = \pm\sqrt{5}$
$x = -2 \pm \sqrt{5}$
The solution set is $\left\{-2 \pm \sqrt{5}\right\}$.

5. $2x^2 + 3x - 4 = 0$
$x^2 + \frac{3}{2}x - 2 = 0$
$x^2 + \frac{3}{2}x = 2$
$x^2 + \frac{3}{2}x + \frac{9}{16} = 2 + \frac{9}{16}$
$\left(x + \frac{3}{4}\right)^2 = \frac{41}{16}$
$x + \frac{3}{4} = \pm\sqrt{\frac{41}{16}}$
$x + \frac{3}{4} = \pm\frac{\sqrt{41}}{4}$
$x = -\frac{3}{4} \pm \frac{\sqrt{41}}{4}$
$x = \frac{-3 \pm \sqrt{41}}{4}$
The solution set is $\left\{\frac{-3 \pm \sqrt{41}}{4}\right\}$.

6. $2x^2 + 9x - 5 = 0$

$a = 2, b = 9, c = -5$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-9 \pm \sqrt{9^2 - 4(2)(-5)}}{2(2)}$$
$$= \frac{-9 \pm \sqrt{81+40}}{4}$$
$$= \frac{-9 \pm \sqrt{121}}{4}$$
$$= \frac{-9 \pm 11}{4}$$
$$x = \frac{-9+11}{4} \text{ or } x = \frac{-9-11}{4}$$
$$x = \frac{2}{4} = \frac{1}{2} \qquad x = \frac{-20}{4} = -5$$

The solution set is $\left\{-5, \frac{1}{2}\right\}$.

7. $2x^2 + 2x - 1 = 0$

$a = 2, b = 2, c = -1$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-2 \pm \sqrt{2^2 - 4(2)(-1)}}{2(2)}$$
$$= \frac{-2 \pm \sqrt{4+8}}{4}$$
$$= \frac{-2 \pm \sqrt{12}}{4}$$
$$= \frac{-2 \pm 2\sqrt{3}}{4}$$
$$= \frac{2(-1 \pm \sqrt{3})}{4}$$
$$= \frac{-1 \pm \sqrt{3}}{2}$$

The solution set is $\left\{\frac{-1 \pm \sqrt{3}}{2}\right\}$.

8. $x^2 - 2x + 2 = 0$

$a = 1, b = -2, c = 2$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(2)}}{2(1)}$$
$$x = \frac{2 \pm \sqrt{4-8}}{2}$$
$$x = \frac{2 \pm \sqrt{-4}}{2}$$
$$x = \frac{2 \pm 2i}{2}$$
$$x = 1 \pm i$$

The solution set is $\{1+i, 1-i\}$.

9. **a.** $a = 1, \quad b = 6, \quad c = 9$

$$b^2 - 4ac = (6)^2 - 4(1)(9)$$
$$= 36 - 36$$
$$= 0$$

Since $b^2 - 4ac = 0$, the equation has one real solution that is rational.

b. $a = 2, \quad b = -7, \quad c = -4$

$$b^2 - 4ac = (-7)^2 - 4(2)(-4)$$
$$= 49 + 32$$
$$= 81$$

Since $b^2 - 4ac > 0$, the equation has two real solutions. Since 81 is a perfect square, the two solutions are rational.

c. $a = 3, \quad b = -2, \quad c = 4$

$$b^2 - 4ac = (-2)^2 - 4(3)(4)$$
$$= 4 - 48$$
$$= -44$$

Since $b^2 - 4ac < 0$, the equation has two imaginary solutions that are complex conjugates.

10. $P = 0.01A^2 + 0.05A + 107$

$115 = 0.01A^2 + 0.05A + 107$

$0 = 0.01A^2 + 0.05A - 8$

$a = 0.01, \quad b = 0.05, \quad c = -8$

$A = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$A = \frac{-(0.05) \pm \sqrt{(0.05)^2 - 4(0.01)(-8)}}{2(0.01)}$

$A = \frac{-0.05 \pm \sqrt{0.3225}}{0.02}$

$A \approx \frac{-0.05 + \sqrt{0.3225}}{0.02} \qquad A \approx \frac{-0.05 - \sqrt{0.3225}}{0.02}$

$A \approx 26 \qquad A \approx -31$

Age cannot be negative, reject the negative answer. Thus, a woman whose normal systolic blood pressure is 115 mm Hg is approximately 26 years old.

11. Let c = the screen's diagonal.

$a^2 + b^2 = c^2$

$19.2^2 + 25.6^2 = c^2$

$368.64 + 655.36 = c^2$

$1024 = c^2$

$c = \sqrt{1024} \quad \text{or} \quad c = -\sqrt{1024}$

$c = 32 \qquad c = -32$

The dimension must be positive. Reject –32. The size of the screen is 32 inches.

Concept and Vocabulary Check 1.5

1. quadratic

2. $A = 0$ or $B = 0$

3. x-intercepts

4. $\pm\sqrt{d}$

5. $\pm\sqrt{7}$

6. $\frac{9}{4}$

7. $\frac{4}{25}$

8. 9

9. $\frac{1}{9}$

10. $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

11. 2; 9; –5

12. 1; –4; –1

13. $2 \pm \sqrt{2}$

14. $-1 \pm i\frac{\sqrt{6}}{2}$

15. $b^2 - 4ac$

16. no

17. two

18. the square root property

19. the quadratic formula

20. factoring and the zero-product principle

21. right; hypotenuse; legs

22. right; legs; the square of the length of the hypotenuse

Exercise Set 1.5

1. $x^2 - 3x - 10 = 0$

$(x + 2)(x - 5) = 0$

$x + 2 = 0 \quad \text{or} \quad x - 5 = 0$

$x = -2 \quad \text{or} \quad x = 5$

The solution set is {–2, 5}.

2. $x^2 - 13x + 36 = 0$

$(x - 4)(x - 9) = 0$

$x - 4 = 0 \quad \text{or} \quad x - 9 = 0$

$x = 4 \quad \text{or} \quad x = 9$

The solution set is {4, 9}.

3. $x^2 = 8x - 15$

$x^2 - 8x + 15 = 0$

$(x - 3)(x - 5) = 0$

$x - 3 = 0 \quad \text{or} \quad x - 5 = 0$

$x = 3 \quad \text{or} \quad x = 5$

The solution set is {3, 5}.

4. $x^2 = -11x - 10$
$x^2 + 11x + 10 = 0$
$(x + 10)(x + 1) = 0$
$x + 10 = 0$ or $x + 1 = 0$
$x = -10$ or $x = -1$
The solution set is $\{-10, -1\}$.

5. $6x^2 + 11x - 10 = 0$
$(2x + 5)(3x - 2) = 0$
$2x + 5 = 0$ or $3x - 2 = 0$
$2x = -5$ $\quad 3x = 2$
$x = -\frac{5}{2}$ or $x = \frac{2}{3}$
The solution set is $\left\{-\frac{5}{2}, \frac{2}{3}\right\}$.

6. $9x^2 + 9x + 2 = 0$
$(3x + 2)(3x + 1) = 0$
$3x + 2 = 0$ or $3x + 1 = 0$
$x = -\frac{2}{3}$ or $x = -\frac{1}{3}$
The solution set is $\left\{-\frac{2}{3}, -\frac{1}{3}\right\}$.

7. $3x^2 - 2x = 8$
$3x^2 - 2x - 8 = 0$
$(3x + 4)(x - 2) = 0$
$3x + 4 = 0$ or $x - 2 = 0$
$3x = -4$
$x = -\frac{4}{3}$ or $x = 2$
The solution set is $\left\{-\frac{4}{3}, 2\right\}$.

8. $4x^2 - 13x = -3$
$4x^2 - 13x + 3 = 0$
$(4x - 1)(x - 3) = 0$
$4x - 1 = 0$ or $x - 3 = 0$
$4x = 1$
$x = \frac{1}{4}$ or $x = 3$
The solution set is $\left\{\frac{1}{4}, 3\right\}$.

9. $3x^2 + 12x = 0$
$3x(x + 4) = 0$
$3x = 0$ or $x + 4 = 0$
$x = 0$ or $x = -4$
The solution set is $\{-4, 0\}$.

10. $5x^2 - 20x = 0$
$5x(x - 4) = 0$
$5x = 0$ or $x - 4 = 0$
$x = 0$ or $x = 4$
The solution set is $\{0, 4\}$.

11. $2x(x - 3) = 5x^2 - 7x$
$2x^2 - 6x - 5x^2 + 7x = 0$
$-3x^2 + x = 0$
$x(-3x + 1) = 0$
$x = 0$ or $-3x + 1 = 0$
$-3x = -1$
$x = \frac{1}{3}$
The solution set is $\left\{0, \frac{1}{3}\right\}$.

12. $16x(x - 2) = 8x - 25$
$16x^2 - 32x - 8x + 25 = 0$
$16x^2 - 40x + 25 = 0$
$(4x - 5)(4x - 5) = 0$
$4x - 5 = 0$
$4x = 5$
$x = \frac{5}{4}$
The solution set is $\left\{\frac{5}{4}\right\}$.

13. $7 - 7x = (3x + 2)(x - 1)$
$7 - 7x = 3x^2 - x - 2$
$7 - 7x - 3x^2 + x + 2 = 0$
$-3x^2 - 6x + 9 = 0$
$-3(x + 3)(x - 1) = 0$
$x + 3 = 0$ or $x - 1 = 0$
$x = -3$ or $x = 1$
The solution set is $\{-3, 1\}$.

14. $10x-1=(2x+1)^2$
$10x-1=4x^2+4x+1$
$10x-1-4x^2-4x-1=0$
$-4x^2+6x-2=0$
$-2(2x-1)(x-1)=0$
$2x-1=0$ or $x-1=0$
$2x=1$
$x=\frac{1}{2}$ or $x=1$
The solution set is $\left\{\frac{1}{2},1\right\}$.

15. $3x^2=27$
$x^2=9$
$x=\pm\sqrt{9}=\pm3$
The solution set is {–3, 3}.

16. $5x^2=45$
$x^2=9$
$x=\pm\sqrt{9}=\pm3$
The solution set is {–3, 3}.

17. $5x^2+1=51$
$5x^2=50$
$x^2=10$
$x=\pm\sqrt{10}$
The solution set is $\left\{-\sqrt{10},\sqrt{10}\right\}$.

18. $3x^2-1=47$
$3x^2=48$
$x^2=16$
$x=\pm\sqrt{16}=\pm4$
The solution set is {–4, 4}.

19. $2x^2-5=-55$
$2x^2=-50$
$x^2=-25$
$x=\pm\sqrt{-25}=\pm5i$
The solution set is $\{5i,-5i\}$.

20. $2x^2-7=-15$
$2x^2=-8$
$x^2=-4$
$x=\pm\sqrt{-4}=\pm2i$
The solution set is $\{2i,-2i\}$.

21. $(x+2)^2=25$
$x+2=\pm\sqrt{25}$
$x+2=\pm5$
$x=-2\pm5$
$x=-2+5$ or $x=-2-5$
$x=3$ $\quad$ $x=-7$
The solution set is $\{-7,3\}$.

22. $(x-3)^2=36$
$x-3=\pm\sqrt{36}$
$x-3=\pm6$
$x=3\pm6$
$x=3+6$ or $x=3-6$
$x=9$ $\quad$ $x=-3$
The solution set is $\{-3,9\}$.

23. $3(x-4)^2=15$
$(x-4)^2=5$
$x-4=\pm\sqrt{5}$
$x=4\pm\sqrt{5}$
The solution set is $\left\{4+\sqrt{5},4-\sqrt{5}\right\}$.

24. $3(x+4)^2=21$
$(x+4)^2=7$
$x+4=\pm\sqrt{7}$
$x=-4\pm\sqrt{7}$
The solution set is $\left\{-4+\sqrt{7},-4-\sqrt{7}\right\}$.

25. $(x+3)^2=-16$
$x+3=\pm\sqrt{-16}$
$x+3=\pm4i$
$x=-3\pm4i$
The solution set is $\{-3+4i,-3-4i\}$.

26. $(x-1)^2=-9$
$x-1=\pm\sqrt{-9}$
$x-1=\pm3i$
$x=1\pm3i$
The solution set is $\{1+3i,1-3i\}$.

27. $(x-3)^2 = -5$

$x-3 = \pm\sqrt{-5}$

$x-3 = \pm i\sqrt{5}$

$x = 3 \pm i\sqrt{5}$

The solution set is $\{3+i\sqrt{5}, 3-i\sqrt{5}\}$.

28. $(x+2)^2 = -7$

$x+2 = \pm\sqrt{-7}$

$x+2 = \pm i\sqrt{7}$

$x = -2 \pm i\sqrt{7}$

The solution set is $\{-2+i\sqrt{7}, -2-i\sqrt{7}\}$.

29. $(3x+2)^2 = 9$

$3x+2 = \pm\sqrt{9} = \pm 3$

$3x+2 = -3$ or $3x+2 = 3$

$3x = -5$ $\quad$ $3x = 1$

$x = -\frac{5}{3}$ or $x = \frac{1}{3}$

The solution set is $\left\{-\frac{5}{3}, \frac{1}{3}\right\}$.

30. $(4x-1)^2 = 16$

$4x-1 = \pm\sqrt{16} = \pm 4$

$4x-1 = -4$ or $4x-1 = 4$

$4x = -3$ $\quad$ $4x = 5$

$x = \frac{-3}{4}$ or $x = \frac{5}{4}$

The solution set is $\left\{-\frac{3}{4}, \frac{5}{4}\right\}$.

31. $(5x-1)^2 = 7$

$5x-1 = \pm\sqrt{7}$

$5x = 1 \pm \sqrt{7}$

$x = \frac{1 \pm \sqrt{7}}{5}$

The solution set is $\left\{\frac{1-\sqrt{7}}{5}, \frac{1+\sqrt{7}}{5}\right\}$.

32. $(8x-3)^2 = 5$

$8x-3 = \pm\sqrt{5}$

$8x = 3 \pm \sqrt{5}$

$x = \frac{3 \pm \sqrt{5}}{8}$

The solution set is $\left\{\frac{3-\sqrt{5}}{8}, \frac{3+\sqrt{5}}{8}\right\}$.

33. $(3x-4)^2 = 8$

$3x-4 = \pm\sqrt{8} = \pm 2\sqrt{2}$

$3x = 4 \pm 2\sqrt{2}$

$x = \frac{4 \pm 2\sqrt{2}}{3}$

The solution set is $\left\{\frac{4-2\sqrt{2}}{3}, \frac{4+2\sqrt{2}}{3}\right\}$.

34. $(2x+8)^2 = 27$

$2x+8 = \pm\sqrt{27} = \pm 3\sqrt{3}$

$2x = -8 + 3\sqrt{3}$

$x = \frac{-8 \pm 3\sqrt{3}}{2}$

The solution set is $\left\{\frac{-8-3\sqrt{3}}{2}, \frac{-8+3\sqrt{3}}{2}\right\}$.

35. $x^2 + 12x$

$\left(\frac{12}{2}\right)^2 = 6^2 = 36$

$x^2 + 12x + 36 = (x+6)^2$

36. $x^2 + 16x$

$\left(\frac{16}{2}\right)^2 = 8^2 = 64;$

$x^2 + 16x + 64 = (x+8)^2$

37. $x^2 - 10x$

$\left(\frac{10}{2}\right)^2 = 5^2 = 25$

$x^2 - 10x + 25 = (x-5)^2$

38. $x^2 - 14x$

$\left(\frac{-14}{2}\right)^2 = (-7)^2 = 49;$

$x^2 - 14x + 49 = (x-7)^2$

39. $x^2 + 3x$

$\left(\frac{3}{2}\right)^2 = \frac{9}{4}$

$x^2 + 3x + \frac{9}{4} = \left(x + \frac{3}{2}\right)^2$

40. $x^2 + 5x$

$\left(\frac{5}{2}\right)^2 = \frac{25}{4};$

$x^2 + 5x + \frac{25}{4} = \left(x + \frac{5}{2}\right)^2$

41. $x^2 - 7x$

$\left(\frac{7}{2}\right)^2 = \frac{49}{4}$

$x^2 - 7x + \frac{49}{4} = \left(x - \frac{7}{2}\right)^2$

42. $x^2 - 9x$

$\left(\frac{-9}{2}\right)^2 = \frac{81}{4};$

$x^2 - 9x + \frac{81}{4} = \left(x - \frac{9}{2}\right)^2$

43. $x^2 - \frac{2}{3}x$

$\left(\frac{\frac{2}{3}}{2}\right)^2 = \left(\frac{1}{3}\right)^2 = \frac{1}{9}$

$x^2 - \frac{2}{3}x + \frac{1}{9} = \left(x - \frac{1}{3}\right)^2$

44. $x^2 + \frac{4}{5}x$

$\left(\frac{\frac{4}{5}}{2}\right)^2 = \left(\frac{2}{5}\right)^2 = \frac{4}{25};$

$x^2 + \frac{4}{5}x + \frac{4}{25} = \left(x + \frac{2}{5}\right)^2$

45. $x^2 - \frac{1}{3}x$

$\left(\frac{\frac{1}{3}}{2}\right)^2 = \left(\frac{1}{6}\right)^2 = \frac{1}{36}$

$x^2 - \frac{1}{3}x + \frac{1}{36} = \left(x - \frac{1}{6}\right)^2$

46. $x^2 - \frac{1}{4}x$

$\left(\frac{\frac{-1}{4}}{2}\right)^2 = \left(\frac{-1}{8}\right)^2 = \frac{1}{64};$

$x^2 - \frac{1}{4}x + \frac{1}{64} = \left(x - \frac{1}{8}\right)^2$

47.

$$\begin{aligned} x^2 + 6x &= 7 \\ x^2 + 6x + 9 &= 7 + 9 \\ (x+3)^2 &= 16 \\ x + 3 &= \pm 4 \\ x &= -3 \pm 4 \end{aligned}$$

The solution set is {–7, 1}.

48.

$$\begin{aligned} x^2 + 6x &= -8 \\ x^2 + 6x + 9 &= -8 + 9 \\ (x+3)^2 &= 1 \\ x + 3 &= \pm 1 \\ x &= -3 \pm 1 \end{aligned}$$

The solution set is {–4, –2}.

49.

$$\begin{aligned} x^2 - 2x &= 2 \\ x^2 - 2x + 1 &= 2 + 1 \\ (x-1)^2 &= 3 \\ x - 1 &= \pm\sqrt{3} \\ x &= 1 \pm \sqrt{3} \end{aligned}$$

The solution set is $\left\{1+\sqrt{3}, 1-\sqrt{3}\right\}$.

50.

$$\begin{aligned} x^2 + 4x &= 12 \\ x^2 + 4x + 4 &= 12 + 4 \\ (x+2)^2 &= 16 \\ x + 2 &= \pm 4 \\ x &= -2 \pm 4 \end{aligned}$$

The solution set is {–6, 2}.

51.
$$x^2-6x-11=0$$
$$x^2-6x=11$$
$$x^2-6x+9=11+9$$
$$(x-3)^2=20$$
$$x-3=\pm\sqrt{20}$$
$$x=3\pm2\sqrt{5}$$
The solution set is $\left\{3+2\sqrt{5},3-2\sqrt{5}\right\}$.

52.
$$x^2-2x-5=0$$
$$x^2-2x=5$$
$$x^2-2x+1=5+1$$
$$(x-1)^2=6$$
$$x-1=\pm\sqrt{6}$$
$$x=1\pm\sqrt{6}$$
The solution set is $\left\{1+\sqrt{6},1-\sqrt{6}\right\}$.

53.
$$x^2+4x+1=0$$
$$x^2+4x=-1$$
$$x^2+4x+4=-1+4$$
$$(x+2)^2=3$$
$$x+2=\pm\sqrt{3}$$
$$x=-2\pm\sqrt{3}$$
The solution set is $\left\{-2+\sqrt{3},-2-\sqrt{3}\right\}$.

54.
$$x^2+6x-5=0$$
$$x^2+6x=5$$
$$x^2+6x+9=5+9$$
$$(x+3)^2=14$$
$$x+3=\pm\sqrt{14}$$
$$x=-3\pm\sqrt{14}$$
The solution set is $\left\{-3+\sqrt{14},-3-\sqrt{14}\right\}$.

55.
$$x^2-5x+6=0$$
$$x^2-5x=-6$$
$$x^2-5x+\frac{25}{4}=-6+\frac{25}{4}$$
$$\left(x-\frac{5}{2}\right)^2=\frac{1}{4}$$
$$x-\frac{5}{2}=\pm\sqrt{\frac{1}{4}}$$
$$x-\frac{5}{2}=\pm\frac{1}{2}$$
$$x=\frac{5}{2}\pm\frac{1}{2}$$
$$x=\frac{5}{2}+\frac{1}{2} \quad \text{or} \quad x=\frac{5}{2}-\frac{1}{2}$$
$$x=3 \qquad x=2$$
The solution set is {2, 3}.

56.
$$x^2+7x-8=0$$
$$x^2+7x=8$$
$$x^2+7x+\frac{49}{4}=8+\frac{49}{4}$$
$$\left(x+\frac{7}{2}\right)^2=\frac{81}{4}$$
$$x+\frac{7}{2}=\pm\sqrt{\frac{81}{4}}$$
$$x+\frac{7}{2}=\pm\frac{9}{2}$$
$$x=-\frac{7}{2}\pm\frac{9}{2}$$
$$x=-\frac{7}{2}+\frac{9}{2} \quad \text{or} \quad x=-\frac{7}{2}-\frac{9}{2}$$
$$x=1 \qquad x=-8$$
The solution set is {–8, 1}.

57.
$$x^2+3x-1=0$$
$$x^2+3x=1$$
$$x^2+3x+\frac{9}{4}=1+\frac{9}{4}$$
$$\left(x+\frac{3}{2}\right)^2=\frac{13}{4}$$
$$x+\frac{3}{2}=\pm\frac{\sqrt{13}}{2}$$
$$x=\frac{-3\pm\sqrt{13}}{2}$$
The solution set is $\left\{\frac{-3+\sqrt{13}}{2},\frac{-3-\sqrt{13}}{2}\right\}$.

58. $x^2-3x-5=0$

$$x^2-3x=5$$

$$x^2-3x+\frac{9}{4}=5+\frac{9}{4}$$

$$\left(x-\frac{3}{2}\right)^2=\frac{29}{4}$$

$$x-\frac{3}{2}=\frac{\pm\sqrt{29}}{2}$$

$$x=\frac{3\pm\sqrt{29}}{2}$$

The solution set is $\left\{\frac{3+\sqrt{29}}{2},\frac{3-\sqrt{29}}{2}\right\}$.

59. $2x^2-7x+3=0$

$$x^2-\frac{7}{2}x+\frac{3}{2}=0$$

$$x^2-\frac{7}{2}x=\frac{-3}{2}$$

$$x^2-\frac{7}{2}x+\frac{49}{16}=-\frac{3}{2}+\frac{49}{16}$$

$$\left(x-\frac{7}{4}\right)^2=\frac{25}{16}$$

$$x-\frac{7}{4}=\pm\frac{5}{4}$$

$$x=\frac{7}{4}\pm\frac{5}{4}$$

The solution set is $\left\{\frac{1}{2},3\right\}$.

60. $2x^2+5x-3=0$

$$x^2+\frac{5}{2}x-\frac{3}{2}=0$$

$$x^2+\frac{5}{2}x=\frac{3}{2}$$

$$x^2+\frac{5}{2}x+\frac{25}{16}=\frac{3}{2}+\frac{25}{16}$$

$$\left(x+\frac{5}{4}\right)^2=\frac{49}{16}$$

$$x+\frac{5}{4}=\pm\frac{7}{4}$$

$$x=-\frac{5}{4}\pm\frac{7}{4}$$

$$x=\frac{1}{2};-3$$

The solution set is $\left\{-3,\frac{1}{2}\right\}$.

61. $4x^2-4x-1=0$

$$4x^2-4x-1=0$$

$$x^2-x-\frac{1}{4}=0$$

$$x^2-x=\frac{1}{4}$$

$$x^2-x+\frac{1}{4}=\frac{1}{4}+\frac{1}{4}$$

$$\left(x-\frac{1}{2}\right)^2=\frac{2}{4}$$

$$x-\frac{1}{2}=\frac{\pm\sqrt{2}}{2}$$

$$x=\frac{1\pm\sqrt{2}}{2}$$

The solution set is $\left\{\frac{1+\sqrt{2}}{2},\frac{1-\sqrt{2}}{2}\right\}$.

62. $2x^2 - 4x - 1 = 0$

$$x^2 - 2x - \frac{1}{2} = 0$$

$$x^2 - 2x + 1 = \frac{1}{2} + 1$$

$$x^2 - 2x = \frac{1}{2}$$

$$(x-1)^2 = \frac{3}{2}$$

$$x - 1 = \pm\sqrt{\frac{3}{2}}$$

$$x = 1 \pm \frac{\sqrt{6}}{2}$$

$$x = \frac{2 \pm \sqrt{6}}{2}$$

The solution set is $\left\{\frac{2+\sqrt{6}}{2}, \frac{2-\sqrt{6}}{2}\right\}$.

63. $3x^2 - 2x - 2 = 0$

$$x^2 - \frac{2}{3}x - \frac{2}{3} = 0$$

$$x^2 - \frac{2}{3}x = \frac{2}{3}$$

$$x^2 - \frac{2}{3}x + \frac{1}{9} = \frac{2}{3} + \frac{1}{9}$$

$$\left(x - \frac{1}{3}\right)^2 = \frac{7}{9}$$

$$x - \frac{1}{3} = \frac{\pm\sqrt{7}}{3}$$

$$x = \frac{1 \pm \sqrt{7}}{3}$$

The solution set is $\left\{\frac{1+\sqrt{7}}{3}, \frac{1-\sqrt{7}}{3}\right\}$.

64. $3x^2 - 5x - 10 = 0$

$$x^2 - \frac{5}{3}x - \frac{10}{3} = 0$$

$$x^2 - \frac{5}{3}x = \frac{10}{3}$$

$$x^2 - \frac{5}{3}x + \frac{25}{36} = \frac{10}{3} + \frac{25}{36}$$

$$\left(x - \frac{5}{6}\right)^2 = \frac{145}{36}$$

$$x - \frac{5}{6} = \frac{\pm\sqrt{145}}{6}$$

$$x = \frac{5 \pm \sqrt{145}}{6}$$

The solution set is $\left\{\frac{5\pm\sqrt{145}}{6}, \frac{5-\sqrt{145}}{6}\right\}$.

65. $x^2 + 8x + 15 = 0$

$$x = \frac{-8 \pm \sqrt{8^2 - 4(1)(15)}}{2(1)}$$

$$x = \frac{-8 \pm \sqrt{64 - 60}}{2}$$

$$x = \frac{-8 \pm \sqrt{4}}{2}$$

$$x = \frac{-8 \pm 2}{2}$$

The solution set is $\{-5, -3\}$.

66. $x^2 + 8x + 12 = 0$

$$x = \frac{-8 \pm \sqrt{8^2 - 4(1)(12)}}{2(1)}$$

$$x = \frac{-8 \pm \sqrt{64 - 48}}{2}$$

$$x = \frac{-8 \pm \sqrt{16}}{2}$$

$$x = \frac{-8 \pm 4}{2}$$

The solution set is $\{-6, -2\}$.

67. $x^2+5x+3=0$

$$x=\frac{-5\pm\sqrt{5^2-4(1)(3)}}{2(1)}$$
$$x=\frac{-5\pm\sqrt{25-12}}{2}$$
$$x=\frac{-5\pm\sqrt{13}}{2}$$

The solution set is $\left\{\frac{-5+\sqrt{13}}{2},\frac{-5-\sqrt{13}}{2}\right\}$.

68. $x^2+5x+2=0$

$$x=\frac{-5\pm\sqrt{5^2-4(1)(2)}}{2(1)}$$
$$x=\frac{-5\pm\sqrt{25-8}}{2}$$
$$x=\frac{-5\pm\sqrt{17}}{2}$$

The solution set is $\left\{\frac{-5+\sqrt{17}}{2},\frac{-5-\sqrt{17}}{2}\right\}$.

69. $3x^2-3x-4=0$

$$x=\frac{3\pm\sqrt{(-3)^2-4(3)(-4)}}{2(3)}$$
$$x=\frac{3\pm\sqrt{9+48}}{6}$$
$$x=\frac{3\pm\sqrt{57}}{6}$$

The solution set is $\left\{\frac{3+\sqrt{57}}{6},\frac{3-\sqrt{57}}{6}\right\}$.

70. $5x^2+x-2=0$

$$x=\frac{-1\pm\sqrt{1^2-4(5)(-2)}}{2(5)}$$
$$x=\frac{-1\pm\sqrt{1+40}}{10}$$
$$x=\frac{-1\pm\sqrt{41}}{10}$$

The solution set is $\left\{\frac{-1+\sqrt{41}}{10},\frac{-1-\sqrt{41}}{10}\right\}$.

71. $4x^2=2x+7$

$$4x^2-2x-7=0$$
$$x=\frac{2\pm\sqrt{(-2)^2-4(4)(-7)}}{2(4)}$$
$$x=\frac{2\pm\sqrt{4+112}}{8}$$
$$x=\frac{2\pm\sqrt{116}}{8}$$
$$x=\frac{2\pm2\sqrt{29}}{8}$$
$$x=\frac{1\pm\sqrt{29}}{4}$$

The solution set is $\left\{\frac{1+\sqrt{29}}{4},\frac{1-\sqrt{29}}{4}\right\}$.

72. $3x^2=6x-1$

$$3x^2-6x+1=0$$
$$x=\frac{6\pm\sqrt{(-6)^2-4(3)(1)}}{2(3)}$$
$$x=\frac{6\pm\sqrt{36-12}}{6}$$
$$x=\frac{6\pm\sqrt{24}}{6}$$
$$x=\frac{6\pm2\sqrt{6}}{6}$$
$$x=\frac{3\pm\sqrt{6}}{3}$$

The solution set is $\left\{\frac{3+\sqrt{6}}{3},\frac{3-\sqrt{6}}{3}\right\}$.

73. $x^2-6x+10=0$

$$x=\frac{6\pm\sqrt{(-6)^2-4(1)(10)}}{2(1)}$$
$$x=\frac{6\pm\sqrt{36-40}}{2}$$
$$x=\frac{6\pm\sqrt{-4}}{2}$$
$$x=\frac{6\pm2i}{2}$$
$$x=3\pm i$$

The solution set is $\{3+i,3-i\}$.

74. $x^2 - 2x + 17 = 0$

$x = \frac{2 \pm \sqrt{(-2)^2 - 4(1)(17)}}{2(1)}$

$x = \frac{2 \pm \sqrt{4-68}}{2}$

$x = \frac{2 \pm \sqrt{-64}}{2}$

$x = \frac{2 \pm 8i}{2}$

$x = 1 \pm 4i$

The solution set is $\{1+4i, 1-4i\}$.

75. $x^2 - 4x - 5 = 0$

$(-4)^2 - 4(1)(-5)$

= 16 + 20

= 36; 2 unequal real solutions

76. $4x^2 - 2x + 3 = 0$

$(-2)^2 - 4(4)(3)$

= 4 – 48

= –44; 2 complex imaginary solutions

77. $2x^2 - 11x + 3 = 0$

$(-11)^2 - 4(2)(3)$

= 121 – 24

= 97; 2 unequal real solutions

78. $2x^2 + 11x - 6 = 0$

$11^2 - 4(2)(-6)$

= 121 + 48

= 169; 2 unequal real solutions

79. $x^2 - 2x + 1 = 0$

$(-2)^2 - 4(1)(1)$

= 4 – 4

= 0; 1 real solution

80. $3x^2 = 2x - 1$

$3x^2 - 2x + 1 = 0$

$(-2)^2 - 4(3)(1)$

= 4 – 12

= –8; 2 complex imaginary solutions

81. $x^2 - 3x - 7 = 0$

$(-3)^2 - 4(1)(-7)$

= 9 + 28

= 37; 2 unequal real solutions

82. $3x^2 + 4x - 2 = 0$

$4^2 - 4(3)(-2)$

= 16 + 24

= 40; 2 unequal real solutions

83. $2x^2 - x = 1$

$2x^2 - x - 1 = 0$

$(2x+1)(x-1) = 0$

$2x + 1 = 0$ or $x - 1 = 0$

$2x = -1$

$x = -\frac{1}{2}$ or $x = 1$

The solution set is $\left\{-\frac{1}{2}, 1\right\}$.

84. $3x^2 - 4x = 4$

$3x^2 - 4x - 4 = 0$

$(3x+2)(x-2) = 0$

$3x + 2$ or $x - 2 = 0$

$3x = -2$

$x = -\frac{2}{3}$ or $x = -3$

The solution set is $\left\{-\frac{2}{3}, 2\right\}$.

85. $5x^2 + 2 = 11x$

$5x^2 - 11x + 2 = 0$

$(5x-1)(x-2) = 0$

$5x - 1 = 0$ or $x - 2 = 0$

$5x = 1$

$x = \frac{1}{5}$ or $x = 2$

The solution set is $\left\{\frac{1}{5}, 2\right\}$.

86. $5x^2 = 6 - 13x$

$5x^2 + 13x - 6 = 0$

$(5x-2)(x+3) = 0$

$5x - 2 = 0$ or $x + 3$

$5x = 2$

$x = \frac{2}{5}$ or $x = -3$

The solution set is $\left\{-3, \frac{2}{5}\right\}$.

87. $3x^2 = 60$
$x^2 = 20$
$x = \pm\sqrt{20}$
$x = \pm 2\sqrt{5}$
The solution set is $\{-2\sqrt{5}, 2\sqrt{5}\}$.

88. $2x^2 = 250$
$x^2 = 125$
$x = \pm\sqrt{125}$
$x = \pm 5\sqrt{5}$
The solution set is $\{-5\sqrt{5}, 5\sqrt{5}\}$.

89. $x^2 - 2x = 1$
$x^2 - 2x + 1 = 1 + 1$
$(x-1)^2 = 2$
$x - 1 = \pm\sqrt{2}$
$x = 1 \pm \sqrt{2}$
The solution set is $\{1+\sqrt{2}, 1-\sqrt{2}\}$.

90. $2x^2 + 3x = 1$
$2x^2 + 3x - 1 = 0$
$$x = \frac{-3 \pm \sqrt{3^2 - 4(2)(-1)}}{2(2)}$$
$$x = \frac{-3 \pm \sqrt{9+8}}{4}$$
$$x = \frac{-3 \pm \sqrt{17}}{4}$$
The solution set is $\left\{\frac{-3+\sqrt{17}}{4}, \frac{-3-\sqrt{17}}{4}\right\}$.

91. $(2x+3)(x+4) = 1$
$2x^2 + 8x + 3x + 12 = 1$
$2x^2 + 11x + 11 = 0$
$$x = \frac{-11 \pm \sqrt{11^2 - 4(2)(11)}}{2(2)}$$
$$x = \frac{-11 \pm \sqrt{121-88}}{4}$$
$$x = \frac{-11 \pm \sqrt{33}}{4}$$
The solution set is $\left\{\frac{-11+\sqrt{33}}{4}, \frac{-11-\sqrt{33}}{4}\right\}$.

92. $(2x-5)(x+1) = 2$
$2x^2 + 2x - 5x - 5 = 2$
$2x^2 - 3x - 7 = 0$
$$x = \frac{3 \pm \sqrt{(-3)^2 - 4(2)(-7)}}{2(2)}$$
$$x = \frac{3 \pm \sqrt{9+56}}{4}$$
$$x = \frac{3 \pm \sqrt{65}}{4}$$
The solution set is $\left\{\frac{3+\sqrt{65}}{4}, \frac{3-\sqrt{65}}{4}\right\}$.

93. $(3x-4)^2 = 16$
$3x - 4 = \pm\sqrt{16}$
$3x - 4 = \pm 4$
$3x = 4 \pm 4$
$3x = 8$ or $3x = 0$
$x = \frac{8}{3}$ or $x = 0$

The solution set is $\left\{0, \frac{8}{3}\right\}$.

94. $(2x+7)^2 = 25$
$2x + 7 = \pm 5$
$2x = -7 \pm 5$
$2x = -12$ or $2x = -2$
$x = 6$ or $x = -1$
The solution set is $\{-6, -1\}$.

95. $3x^2 - 12x + 12 = 0$
$x^2 - 4x + 4 = 0$
$(x-2)(x-2) = 0$
$x - 2 = 0$
$x = 2$
The solution set is $\{2\}$.

96. $9 - 6x + x^2 = 0$
$x^2 - 6x + 9 = 0$
$(x-3)(x-3) = 0$
$x - 3 = 0$
$x = 3$
The solution set is $\{3\}$.

97. $4x^2 - 16 = 0$
$4x^2 = 16$
$x^2 = 4$
$x = \pm 2$
The solution set is $\{-2, 2\}$.

98. $3x^2 - 27 = 0$
$3x^2 = 27$
$x^2 = 9$
$x = \pm 3$
The solution set is $\{-3, 3\}$.

99. $x^2 - 6x + 13 = 0$
$x^2 - 6x = -13$
$x^2 - 6x + 9 = -13 + 9$
$(x-3)^2 = -4$
$x - 3 = \pm 2i$
$x = 3 \pm 2i$
The solution set is $\{3 + 2i, 3 - 2i\}$.

100. $x^2 - 4x + 29 = 0$
$x^2 - 4x = -29$
$x^2 - 4x + 4 = -29 + 4$
$(x-2)^2 = -25$
$x - 2 = \pm 5i$
$x = 2 \pm 5i$
The solution set is $\{2 + 5i, 2 - 5i\}$.

101. $x^2 = 4x - 7$
$x^2 - 4x = -7$
$x^2 - 4x + 4 = -7 + 4$
$(x-2)^2 = -3$
$x - 2 = \pm i\sqrt{3}$
$x = 2 \pm i\sqrt{3}$
The solution set is $\{2 + i\sqrt{3}, 2 - i\sqrt{3}\}$.

102. $5x^2 = 2x - 3$
$5x^2 - 2x + 3 = 0$
$$x = \frac{2 \pm \sqrt{(-2)^2 - 4(5)(3)}}{2(5)}$$
$$x = \frac{2 \pm \sqrt{4 - 60}}{10}$$
$$x = \frac{2 \pm \sqrt{-56}}{10}$$
$$x = \frac{2 \pm 2i\sqrt{14}}{10}$$
$$x = \frac{1 \pm i\sqrt{14}}{5}$$
The solution set is $\left\{\frac{1 + i\sqrt{14}}{5}, \frac{1 - i\sqrt{14}}{5}\right\}$.

103. $2x^2 - 7x = 0$
$x(2x - 7) = 0$
$x = 0$ or $2x - 7 = 0$
$2x = 7$
$x = 0$ or $x = \frac{7}{2}$
The solution set is $\left\{0, \frac{7}{2}\right\}$.

104. $2x^2 + 5x = 3$
$2x^2 + 5x - 3 = 0$
$$x = \frac{-5 \pm \sqrt{5^2 - 4(2)(-3)}}{2(2)}$$
$$x = \frac{-5 \pm \sqrt{25 + 24}}{4}$$
$$x = \frac{-5 \pm \sqrt{49}}{4}$$
$$x = \frac{-5 \pm 7}{4}$$
$$x = -3, \frac{1}{2}$$
The solution set is $\left\{-3, \frac{1}{2}\right\}$.

105. $\frac{1}{x}+\frac{1}{x+2}=\frac{1}{3}; x\neq 0,-2$

$3x+6+3x=x^2+2x$

$0=x^2-4x-6$

$x=\frac{-(-4)\pm\sqrt{(-4)^2-4(1)(-6)}}{2(1)}$

$x=\frac{4\pm\sqrt{16+24}}{2}$

$x=\frac{4\pm\sqrt{40}}{2}$

$x=\frac{4\pm 2\sqrt{10}}{2}$

$x=2\pm\sqrt{10}$

The solution set is $\{2+\sqrt{10},\ 2-\sqrt{10}\}$.

106. $\frac{1}{x}+\frac{1}{x+3}=\frac{1}{4}; x\neq 0,-3$

$4x+12+4x=x^2+3x$

$0=x^2-5x-12$

$x=\frac{-(-5)\pm\sqrt{(-5)^2-4(1)(-12)}}{2(1)}$

$x=\frac{5\pm\sqrt{25+48}}{2}$

$x=\frac{5\pm\sqrt{73}}{2}$

The solution set is $\left\{\frac{5+\sqrt{73}}{2},\frac{5-\sqrt{73}}{2}\right\}$.

107. $\frac{2x}{x-3}+\frac{6}{x+3}=\frac{-28}{x^2-9}; x\neq 3,-3$

$2x(x+3)+6(x-3)=-28$

$2x^2+6x+6x-18=-28$

$2x^2+12x+10=0$

$x^2+6x+5=0$

$(x+1)(x+5)=0$

The solution set is $\{-5,\ -1\}$.

108. $\frac{3}{x-3}+\frac{5}{x-4}=\frac{x^2-20}{x^2-7x+12}; x\neq 3,4$

$3x-12+5x-15=x^2-20$

$0=x^2-8x+7$

$0=(x-7)(x-1)$

$x=7 \qquad x=1$

The solution set is $\{1, 7\}$.

109. $x^2-4x-5=0$

$(x+1)(x-5)=0$

$x+1=0$ or $x-5=0$

$x=-1 \qquad x=5$

This equation matches graph (d).

110. $x^2-6x+7=0$

$a=1,\quad b=-6,\quad c=7$

$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$x=\frac{-(-6)\pm\sqrt{(-6)^2-4(1)(7)}}{2(1)}$

$x=\frac{6\pm\sqrt{8}}{2}$

$x=3\pm\sqrt{2}$

$x\approx 1.6,\quad x\approx 4.4$

This equation matches graph (a).

111. $0=-(x+1)^2+4$

$(x+1)^2=4$

$x+1=\pm 2$

$x=-1\pm 2$

$x=-3,\quad x=1$

This equation matches graph (f).

112. $0=-(x+3)^2+1$

$(x+3)^2=1$

$x+3=\pm 1$

$x=-3\pm 1$

$x=-4,\quad x=-2$

This equation matches graph (e).

113. $x^2-2x+2=0$
$a=1,\quad b=-2,\quad c=2$
$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$
$$x=\frac{-(-2)\pm\sqrt{(-2)^2-4(1)(2)}}{2(1)}$$
$$x=\frac{2\pm\sqrt{-4}}{2}$$
$$x=\frac{2\pm 2i}{2}$$
$x=1\pm i$
This equation has no real roots. Thus, its equation has no x-intercepts. This equation matches graph (b).

114. $x^2+6x+9=0$
$(x+3)(x+3)=0$
$x+3=0$
$x=-3$
This equation matches graph (c).

115. $y=2x^2-3x$
$2=2x^2-3x$
$0=2x^2-3x-2$
$0=(2x+1)(x-2)$
$x=-\frac{1}{2},\quad x=2$

116. $y=5x^2+3x$
$2=5x^2+3x$
$0=5x^2+3x-2$
$0=(x+1)(5x-2)$
$x=-1,\quad x=\frac{2}{5}$

117. $y_1y_2=14$
$(x-1)(x+4)=14$
$x^2+3x-4=14$
$x^2+3x-18=0$
$(x+6)(x-3)=0$
$x=-6,\quad x=3$

118. $y_1y_2=-30$
$(x-3)(x+8)=-30$
$x^2+5x-24=-30$
$x^2+5x+6=0$
$(x+3)(x+2)=0$
$x=-3,\quad x=-2$

119. $y_1+y_2=1$
$$\frac{2x}{x+2}+\frac{3}{x+4}=1$$
$$(x+2)(x+4)\left(\frac{2x}{x+2}+\frac{3}{x+4}\right)=1(x+2)(x+4)$$
$$\frac{2x(x+2)(x+4)}{x+2}+\frac{3(x+2)(x+4)}{x+4}=(x+2)(x+4)$$
$$2x(x+4)+3(x+2)=(x+2)(x+4)$$
$$2x^2+8x+3x+6=x^2+6x+8$$
$$x^2+5x-2=0$$
$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$
$$x=\frac{-(5)\pm\sqrt{(5)^2-4(1)(-2)}}{2(1)}$$
$$x=\frac{-5\pm\sqrt{33}}{2}$$
The solution set is $\left\{\frac{-5+\sqrt{33}}{2},\frac{-5-\sqrt{33}}{2}\right\}$.

120. $y_1+y_2=3$
$$\frac{3}{x-1}+\frac{8}{x}=3$$
$$x(x-1)\left(\frac{3}{x-1}+\frac{8}{x}\right)=3(x)(x-1)$$
$$\frac{3x(x-1)}{x-1}+\frac{8x(x-1)}{x}=3x(x-1)$$
$$3x+8(x-1)=3x^2-3x$$
$$3x+8x-8=3x^2-3x$$
$$11x-8=3x^2-3x$$
$$0=3x^2-14x+8$$
$$0=(3x-2)(x-4)$$
$x=\frac{2}{3},\quad x=4$
The solution set is $\left\{\frac{2}{3},4\right\}$.

121. $y_1 - y_2 = 0$

$$\left(2x^2+5x-4\right)-\left(-x^2+15x-10\right)=0$$
$$2x^2+5x-4+x^2-15x+10=0$$
$$3x^2-10x+6=0$$
$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$
$$x=\frac{-(-10)\pm\sqrt{(-10)^2-4(3)(6)}}{2(3)}$$
$$x=\frac{10\pm\sqrt{28}}{6}$$
$$x=\frac{10\pm2\sqrt{7}}{6}$$
$$x=\frac{5\pm\sqrt{7}}{3}$$

The solution set is $\left\{\frac{5+\sqrt{7}}{3},\frac{5-\sqrt{7}}{3}\right\}$.

122. $y_1 - y_2 = 0$

$$\left(-x^2+4x-2\right)-\left(-3x^2+x-1\right)=0$$
$$-x^2+4x-2+3x^2-x+1=0$$
$$2x^2+3x-1=0$$
$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$
$$x=\frac{-(3)\pm\sqrt{(3)^2-4(2)(-1)}}{2(2)}$$
$$x=\frac{-3\pm\sqrt{17}}{4}$$

The solution set is $\left\{\frac{-3+\sqrt{17}}{4},\frac{-3-\sqrt{17}}{4}\right\}$.

123. Values that make the denominator zero must be excluded**.**

$$2x^2+4x-9=0$$
$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$
$$x=\frac{-(4)\pm\sqrt{(4)^2-4(2)(-9)}}{2(2)}$$
$$x=\frac{-4\pm\sqrt{88}}{4}$$
$$x=\frac{-4\pm2\sqrt{22}}{4}$$
$$x=\frac{-2\pm\sqrt{22}}{2}$$

124. Values that make the denominator zero must be excluded**.**

$$2x^2-8x+5=0$$
$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$
$$x=\frac{-(-8)\pm\sqrt{(-8)^2-4(2)(5)}}{2(2)}$$
$$x=\frac{8\pm\sqrt{24}}{4}$$
$$x=\frac{8\pm2\sqrt{6}}{4}$$
$$x=\frac{4\pm\sqrt{6}}{2}$$

125. $x^2-\left(6+2x\right)=0$

$$x^2-2x-6=0$$

Apply the quadratic formula.

$a=1 \quad b=-2 \quad c=-6$

$$x=\frac{-(-2)\pm\sqrt{(-2)^2-4(1)(-6)}}{2(1)}$$
$$=\frac{2\pm\sqrt{4-(-24)}}{2}$$
$$=\frac{2\pm\sqrt{28}}{2}$$
$$=\frac{2\pm\sqrt{4\cdot7}}{2}=\frac{2\pm2\sqrt{7}}{2}=1\pm\sqrt{7}$$

We disregard $1-\sqrt{7}$ because it is negative, and we are looking for a positive number.

Thus, the number is $1+\sqrt{7}$.

126. Let x = the number.

$$2x^2-(1+2x)=0$$
$$2x^2-2x-1=0$$

Apply the quadratic formula.

$a=2 \quad b=-2 \quad c=-1$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$
$$x=\frac{-(-2)\pm\sqrt{(-2)^2-4(2)(-1)}}{2(2)}$$
$$=\frac{2\pm\sqrt{4-(-8)}}{4}$$
$$=\frac{2\pm\sqrt{12}}{4}=\frac{2\pm\sqrt{4\cdot 3}}{4}=\frac{2\pm 2\sqrt{3}}{4}=\frac{1\pm\sqrt{3}}{2}$$

We disregard $\frac{1+\sqrt{3}}{2}$ because it is positive, and we are looking for a negative number. The number is $\frac{1-\sqrt{3}}{2}$.

127.
$$\frac{1}{x^2-3x+2}=\frac{1}{x+2}+\frac{5}{x^2-4}$$
$$\frac{1}{(x-1)(x-2)}=\frac{1}{x+2}+\frac{5}{(x+2)(x-2)}$$

Multiply both sides of the equation by the least common denominator, $(x-1)(x-2)(x+2)$. This results in the following:

$$x+2=(x-1)(x-2)+5(x-1)$$
$$x+2=x^2-2x-x+2+5x-5$$
$$x+2=x^2+2x-3$$
$$0=x^2+x-5$$

Apply the quadratic formula:

$a=1 \quad b=1 \quad c=-5$.

$$x=\frac{-1\pm\sqrt{1^2-4(1)(-5)}}{2(1)}=\frac{-1\pm\sqrt{1-(-20)}}{2}$$
$$=\frac{-1\pm\sqrt{21}}{2}$$

The solutions are $\frac{-1\pm\sqrt{21}}{2}$, and the solution set is $\left\{\frac{-1\pm\sqrt{21}}{2}\right\}$.

128.
$$\frac{x-1}{x-2}+\frac{x}{x-3}=\frac{1}{x^2-5x+6}$$
$$\frac{x-1}{x-2}+\frac{x}{x-3}=\frac{1}{(x-2)(x-3)}$$

Multiply both sides of the equation by the least common denominator, $(x-2)(x-3)$. This results in the following:

$$(x-3)(x-1)+x(x-2)=1$$
$$x^2-x-3x+3+x^2-2x=1$$
$$2x^2-6x+3=1$$
$$2x^2-6x+2=0$$

Apply the quadratic formula:

$a=2 \quad b=-6 \quad c=2$.

$$x=\frac{-(-6)\pm\sqrt{(-6)^2-4(2)(2)}}{2(2)}$$
$$=\frac{6\pm\sqrt{36-16}}{4}=\frac{6\pm\sqrt{20}}{4}$$
$$=\frac{6\pm\sqrt{4\cdot 5}}{4}=\frac{6\pm 2\sqrt{5}}{4}$$
$$=\frac{3\pm\sqrt{5}}{2}$$

The solutions are $\frac{3\pm\sqrt{5}}{2}$, and the solution set is $\left\{\frac{3\pm\sqrt{5}}{2}\right\}$.

129. $\sqrt{2}x^2 + 3x - 2\sqrt{2} = 0$

Apply the quadratic formula:

$a = \sqrt{2} \quad b = 3 \quad c = -2\sqrt{2}$

$$x = \frac{-3 \pm \sqrt{3^2 - 4\left(\sqrt{2}\right)\left(-2\sqrt{2}\right)}}{2\left(\sqrt{2}\right)}$$

$$= \frac{-3 \pm \sqrt{9-(-16)}}{2\sqrt{2}}$$

$$= \frac{-3 \pm \sqrt{25}}{2\sqrt{2}} = \frac{-3 \pm 5}{2\sqrt{2}}$$

Evaluate the expression to obtain two solutions.

$$x = \frac{-3-5}{2\sqrt{2}} \quad \text{or} \quad x = \frac{-3+5}{2\sqrt{2}}$$

$$= \frac{-8}{2\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} \qquad = \frac{2}{2\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$$

$$= \frac{-8\sqrt{2}}{4} \qquad = \frac{2\sqrt{2}}{4}$$

$$= -2\sqrt{2} \qquad = \frac{\sqrt{2}}{2}$$

The solutions are $-2\sqrt{2}$ and $\frac{\sqrt{2}}{2}$, and the solution set is $\left\{-2\sqrt{2}, \frac{\sqrt{2}}{2}\right\}$.

130. $\sqrt{3}x^2 + 6x + 7\sqrt{3} = 0$

Apply the quadratic formula:

$a = \sqrt{3} \quad b = 6 \quad c = 7\sqrt{3}$

$$x = \frac{-6 \pm \sqrt{6^2 - 4\left(\sqrt{3}\right)\left(7\sqrt{3}\right)}}{2\left(\sqrt{3}\right)}$$

$$= \frac{-6 \pm \sqrt{36-84}}{2\sqrt{3}}$$

$$= \frac{-6 \pm \sqrt{-48}}{2\sqrt{3}}$$

$$= \frac{-6 \pm \sqrt{16 \cdot 3 \cdot (-1)}}{2\sqrt{3}}$$

$$= \frac{-6 \pm 4\sqrt{3}i}{2\sqrt{3}}$$

$$= \frac{-6}{2\sqrt{3}} \pm \frac{4\sqrt{3}i}{2\sqrt{3}} = -\sqrt{3} \pm 2i$$

The solutions are $-\sqrt{3} \pm 2i$, and the solution set is $\left\{-\sqrt{3} \pm 2i\right\}$.

131. $N = \frac{x^2 - x}{2}$

$$21 = \frac{x^2 - x}{2}$$

$$42 = x^2 - x$$

$$0 = x^2 - x - 42$$

$$0 = (x+6)(x-7)$$

$$x + 6 = 0 \quad \text{or} \quad x - 7 = 0$$

$$x = -6 \qquad x = 7$$

Reject the negative value.
There were 7 players.

132. $N = \frac{x^2 - x}{2}$

$$36 = \frac{x^2 - x}{2}$$

$$72 = x^2 - x$$

$$0 = x^2 - x - 72$$

$$0 = (x+8)(x-9)$$

$$x + 8 = 0 \quad \text{or} \quad x - 9 = 0$$

$$x = -8 \qquad x = 9$$

Reject the negative value.
There were 9 players.

133. This is represented on the graph as point (7, 21).

134. This is represented on the graph as point (9, 36).

135. $f(x) = 0.013x^2 - 1.19x + 28.24$

$$3 = 0.013x^2 - 1.19x + 28.24$$

$$0 = 0.013x^2 - 1.19x + 25.24$$

Apply the quadratic formula.

$a = 0.013 \quad b = -1.19 \quad c = 25.24$

$$x = \frac{-(-1.19) \pm \sqrt{(-1.19)^2 - 4(0.013)(25.24)}}{2(0.013)}$$

$$= \frac{1.19 \pm \sqrt{1.4161 - 1.31248}}{0.026}$$

$$= \frac{1.19 \pm \sqrt{0.10362}}{0.026}$$

$$\approx \frac{1.19 \pm 0.32190}{0.026}$$

$$\approx 58.15 \text{ or } 33.39$$

The solutions are approximately 33.39 and 58.15.
Thus, 33 year olds and 58 year olds are expected to be in 3 fatal crashes per 100 million miles driven.
The function models the actual data well.

136. $f(x) = 0.013x^2 - 1.19x + 28.24$

$10 = 0.013x^2 - 1.19x + 28.24$

$0 = 0.013x^2 - 1.19x + 18.24$

$a = 0.013 \quad b = -1.19 \quad c = 18.24$

$$x = \frac{-(-1.19) \pm \sqrt{(-1.19)^2 - 4(0.013)(18.24)}}{2(0.013)}$$

$$= \frac{1.19 \pm \sqrt{1.4161 - 0.94848}}{0.026}$$

$$= \frac{1.19 \pm \sqrt{0.46762}}{0.026} \approx \frac{1.19 \pm 0.68383}{0.026}$$

Evaluate the expression to obtain two solutions.

$x = \frac{1.19 + 0.68383}{0.026}$ or $x = \frac{1.19 - 0.68383}{0.026}$

$x = \frac{1.87383}{0.026}$ $\quad$ $x = \frac{0.50617}{0.026}$

$x \approx 72.1$ $\quad$ $x \approx 19$

Drivers of approximately age 19 and age 72 are expected to be involved in 10 fatal crashes per 100 million miles driven. The model doesn't seem to predict the number of accidents very well. The model overestimates the number of fatal accidents.

137. Using the TRACE feature, we find that the height of the shot put is approximately 0 feet when the distance is 77.8 feet. Graph (b) shows the shot' path.

138. Using the ZERO feature, we find that the height of the shot put is approximately 0 feet when the distance is 55.3 feet. Graph (a) shows the shot's path.

139. $x^2 = 4^2 + 2^2$

$x^2 = 16 + 4$

$x^2 = 20$

$x = \pm\sqrt{20}$

$x = \pm 2\sqrt{5}$

We disregard $-2\sqrt{5}$ because we can't have a negative measurement. The path is $2\sqrt{5}$ miles, or approximately 4.5 miles.

140. $x^2 = 6^2 + 3^2$

$x^2 = 36 + 9$

$x^2 = 45$

$x = \pm\sqrt{45}$

$x = \pm 3\sqrt{5}$

We disregard $-3\sqrt{5}$ because we can't have a negative measurement. The path is $3\sqrt{5}$ miles, or approximately 6.7 miles.

141. $x^2 + 10^2 = 30^2$

$x^2 + 100 = 900$

$x^2 = 800$

Apply the square root property.

$x = \pm\sqrt{800} = \pm\sqrt{400 \cdot 2} = \pm 20\sqrt{2}$

We disregard $-20\sqrt{2}$ because we can't have a negative length measurement. The solution is $20\sqrt{2}$. We conclude that the ladder reaches $20\sqrt{2}$ feet, or approximately 28.3 feet, up the house.

142. $90^2 + 90^2 = x^2$

$8100 + 8100 = x^2$

$16200 = x^2$

$x \approx \pm 127.28$

The distance is 127.28 feet.

143. **a.** $x^2 + 120^2 = 122^2$

$x^2 + 14400 = 14884$

$x^2 = 484$

$x \approx \pm 22$

The ramp's vertical distance is 22 inches.

b. This ramp does not satisfy the requirement.

144. **a.** $h^2 = a^2 + a^2$

$h^2 = 2a^2$

$h = \sqrt{2a^2}$

$h = a\sqrt{2}$

b. The length of the hypotenuse of an isosceles right triangle is the length of the leg times $\sqrt{2}$.

145. Let w = the width
Let $w + 3$ = the length

Area $= lw$

$54 = (w+3)w$

$54 = w^2 + 3w$

$0 = w^2 + 3w - 54$

$0 = (w+9)(w-6)$

Apply the zero product principle.

$w + 9 = 0$ $\quad$ $w - 6 = 0$

$w = -9$ $\quad$ $w = 6$

The solution set is $\{-9, 6\}$. Disregard –9 because we can't have a negative length measurement. The width is 6 feet and the length is $6 + 3 = 9$ feet.

146. Let w = the width
Let w + 3 = the width
Area = lw

$$180=(w+3)w$$
$$180=w^2+3w$$
$$0=w^2+3w-180$$
$$0=(w+15)(w-12)$$
$$w+15=0 \qquad w-12=0$$
$$\cancel{w=-15} \qquad w=12$$

The width is 12 yards and the length is 12 yards + 3 yards = 15 yards.

147. Let x = the length of the side of the original square
Let x + 3 = the length of the side of the new, larger square

$$(x+3)^2=64$$
$$x^2+6x+9=64$$
$$x^2+6x-55=0$$
$$(x+11)(x-5)=0$$

Apply the zero product principle.

$$x+11=0 \qquad x-5=0$$
$$x=-11 \qquad x=5$$

The solution set is $\{-11,5\}$. Disregard –11 because we can't have a negative length measurement. This means that x, the length of the side of the original square, is 5 inches.

148. Let x = the side of the original square,
Let x + 2 = the side of the new, larger square

$$(x+2)^2=36$$
$$x^2+4x+4=36$$
$$x^2+4x-32=0$$
$$(x+8)(x-4)=0$$
$$x+8=0 \qquad x-4=0$$
$$\cancel{x=-8} \qquad x=4$$

The length of the side of the original square, is 4 inches.

149. Let x = the width of the path

$$(20+2x)(10+2x)=600$$
$$200+40x+20x+4x^2=600$$
$$200+60x+4x^2=600$$
$$4x^2+60x+200=600$$
$$4x^2+60x-400=0$$
$$4(x^2+15x-100)=0$$
$$4(x+20)(x-5)=0$$

Apply the zero product principle.

$$4(x+20)=0 \qquad x-5=0$$
$$x+20=0 \qquad x=5$$
$$x=-20$$

The solution set is $\{-20,5\}$. Disregard –20 because we can't have a negative width measurement. The width of the path is 5 meters.

150. Let x = the width of the path

$$(12+2x)(15+2x)=378$$
$$180+24x+30x+4x^2=378$$
$$4x^2+54x+180=378$$
$$4x^2+54x-198=0$$
$$2(2x^2+27x-99)=0$$
$$2(2x+33)(x-3)=0$$
$$2(2x+33)=0 \qquad x-3=0$$
$$2x+33=0 \qquad x=3$$
$$2x=-33$$
$$\cancel{x=-\frac{33}{2}}$$

The width of the path is 3 meters.

151. $x(x)(2)=200$

$$2x^2=200$$
$$x^2=100$$
$$x=\pm 10$$

The length and width are 10 inches.

152. $x(x)(3)=75$

$$3x^2=75$$
$$x^2=25$$
$$x=\pm 5$$

The length and width is 5 inches.

153. $x(20-2x)=13$

$20x-2x^2=13$

$0=2x^2-20x+13$

$x=\dfrac{-(-20)\pm\sqrt{(-20)^2-4(2)(13)}}{2(2)}$

$x=\dfrac{20\pm\sqrt{296}}{4}$

$x=\dfrac{10\pm17.2}{4}$

$x=9.3, 0.7$

9.3 in and 0.7 in

154. $\left(\dfrac{x}{4}\right)^2+\left(\dfrac{8-x}{4}\right)^2=2$

$\dfrac{x^2}{16}+\dfrac{64-16x+x^2}{16}=2$

$x^2+64-16x+x^2=32$

$2x^2-16x+32=0$

$x^2-8x+16=0$

$(x-4)(x-4)=0$

$x=4$ in

Both are 4 inches.

155. – 165. Answers will vary.

166. does not make sense; Explanations will vary. Sample explanation: The factoring method would be quicker.

167. does not make sense; Explanations will vary. Sample explanation: Higher degree polynomial equations can have only one *x*-intercept.

168. does not make sense; Explanations will vary. Sample explanation: The solutions are not irrational.

169. makes sense

170. false; Changes to make the statement true will vary. A sample change is: $(2x-3)^2=25$

$2x-3=\pm5$

171. true

172. false; Changes to make the statement true will vary. A sample change is: The quadratic formula is developed by completing the square.

173. false; Changes to make the statement true will vary. A sample change is: The first step is to collect all the terms on one side and have 0 on the other.

174. $(x+3)(x-5)=0$

$x^2-5x+3x-15=0$

$x^2-2x-15=0$

175. $s=-16t^2+v_0t$

$0=-16t^2+v_0t-s$

$a=-16,\ b=v_0,\ c=-s$

$t=\dfrac{-v_0\pm\sqrt{(v_0)^2-4(-16)(-s)}}{2(-16)}$

$t=\dfrac{-v_0\pm\sqrt{(v_0)^2-64s}}{-32}$

$t=\dfrac{v_0\pm\sqrt{{v_0}^2-64s}}{32}$

176. The dimensions of the pool are 12 meters by 8 meters. With the tile, the dimensions will be 12 + 2*x* meters by 8 + 2*x* meters. If we take the area of the pool with the tile and subtract the area of the pool without the tile, we are left with the area of the tile only.

$(12+2x)(8+2x)-12(8)=120$

$\cancel{96}+24x+16x+4x^2-\cancel{96}=120$

$4x^2+40x-120=0$

$x^2+10x-30=0$

$a=1 \quad b=10 \quad c=-30$

$x=\dfrac{-10\pm\sqrt{10^2-4(1)(-30)}}{2(1)}$

$=\dfrac{-10\pm\sqrt{100+120}}{2}$

$=\dfrac{-10\pm\sqrt{220}}{2}\approx\dfrac{-10\pm14.8}{2}$

Evaluate the expression to obtain two solutions.

$x=\dfrac{-10+14.8}{2}$ or $x=\dfrac{-10-14.8}{2}$

$x=\dfrac{4.8}{2}$ $\quad$ $x=\dfrac{-24.8}{2}$

$x=2.4$ $\quad$ $x=-12.4$

We disregard –12.4 because we can't have a negative width measurement. The solution is 2.4 and we conclude that the width of the uniform tile border is 2.4 meters. This is more than the 2-meter requirement, so the tile meets the zoning laws.

177. $x^3 + x^2 - 4x - 4 = x^2(x+1) - 4(x+1)$
$= (x+1)(x^2 - 4)$
$= (x+1)(x+2)(x-2)$

178. $(\sqrt{x+4} + 1)^2 = \sqrt{x+4}^2 + 2(\sqrt{x+4})(1) + 1^2$
$= x + 4 + 2\sqrt{x+4} + 1$
$= x + 5 + 2\sqrt{x+4}$

179. $5x^{2/3} + 11x^{1/3} + 2 = 0$
$5(-8)^{2/3} + 11(-8)^{1/3} + 2 = 0$
$5(-2)^2 + 11(-2)^1 + 2 = 0$
$5(4) + 11(-2) + 2 = 0$
$20 - 22 + 2 = 0$
$0 = 0$, true
The statement is true.

Mid-Chapter 1 Check Point

1. $-5 + 3(x+5) = 2(3x-4)$
$-5 + 3x + 15 = 6x - 8$
$3x + 10 = 6x - 8$
$-3x = -18$
$\frac{-3x}{-3} = \frac{-18}{-3}$
$x = 6$
The solution set is $\{6\}$.

2. $5x^2 - 2x = 7$
$5x^2 - 2x - 7 = 0$
$(5x-7)(x+1) = 0$
$5x - 7 = 0$ or $x + 1 = 0$
$5x = 7$ $\quad x = -1$
$x = \frac{7}{5}$
The solution set is $\left\{-1, \frac{7}{5}\right\}$.

3. $\frac{x-3}{5} - 1 = \frac{x-5}{4}$
$20\left(\frac{x-3}{5} - 1\right) = 20\left(\frac{x-5}{4}\right)$
$\frac{20(x-3)}{5} - 20(1) = \frac{20(x-5)}{4}$
$4(x-3) - 20 = 5(x-5)$
$4x - 12 - 20 = 5x - 25$
$4x - 32 = 5x - 25$
$-x = 7$
$x = -7$
The solution set is $\{-7\}$.

4. $3x^2 - 6x - 2 = 0$
$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(3)(-2)}}{2(3)}$
$x = \frac{6 \pm \sqrt{60}}{6}$
$x = \frac{6 \pm 2\sqrt{15}}{6}$
$x = \frac{3 \pm \sqrt{15}}{3}$
The solution set is $\left\{\frac{3+\sqrt{15}}{3}, \frac{3-\sqrt{15}}{3}\right\}$.

5. $4x - 2(1-x) = 3(2x+1) - 5$
$4x - 2(1-x) = 3(2x+1) - 5$
$4x - 2 + 2x = 6x + 3 - 5$
$6x - 2 = 6x - 2$
$0 = 0$
The equation is an identity.
The solution set is $\{x \mid x \text{ is a real number}\}$.

6. $5x^2+1=37$

$5x^2=36$

$\frac{5x^2}{5}=\frac{36}{5}$

$x^2=\frac{36}{5}$

$x=\pm\sqrt{\frac{36}{5}}$

$x=\pm\frac{6}{\sqrt{5}}$

$x=\pm\frac{6}{\sqrt{5}}\cdot\frac{\sqrt{5}}{\sqrt{5}}$

$x=\pm\frac{6\sqrt{5}}{5}$

The solution set is $\left\{-\frac{6\sqrt{5}}{5},\frac{6\sqrt{5}}{5}\right\}$.

7. $x(2x-3)=-4$

$2x^2-3x=-4$

$2x^2-3x+4=0$

$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$x=\frac{-(-3)\pm\sqrt{(-3)^2-4(2)(4)}}{2(2)}$

$x=\frac{3\pm\sqrt{-23}}{4}$

$x=\frac{3\pm i\sqrt{23}}{4}$

The solution set is $\left\{\frac{3+i\sqrt{23}}{4},\frac{3-i\sqrt{23}}{4}\right\}$.

8. $\frac{3x}{4}-\frac{x}{3}+1=\frac{4x}{5}-\frac{3}{20}$

$\frac{3x}{4}-\frac{x}{3}+1=\frac{4x}{5}-\frac{3}{20}$

$60\left(\frac{3x}{4}-\frac{x}{3}+1\right)=60\left(\frac{4x}{5}-\frac{3}{20}\right)$

$\frac{60(3x)}{4}-\frac{60x}{3}+60(1)=\frac{60(4x)}{5}-\frac{60(3)}{20}$

$45x-20x+60=48x-9$

$25x+60=48x-9$

$-23x=-69$

$\frac{-23x}{-23}=\frac{-69}{-23}$

$x=3$

The solution set is $\{3\}$.

9. $(x+3)^2=24$

$x+3=\pm\sqrt{24}$

$x=-3\pm 2\sqrt{6}$

The solution set is $\left\{-3+2\sqrt{6},-3-2\sqrt{6}\right\}$.

10. $\frac{1}{x^2}-\frac{4}{x}+1=0$

$x^2\left(\frac{1}{x^2}-\frac{4}{x}+1\right)=x^2(0)$

$\frac{x^2}{x^2}-\frac{4x^2}{x}+x^2=0$

$1-4x+x^2=0$

$x^2-4x+1=0$

$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$x=\frac{-(-4)\pm\sqrt{(-4)^2-4(1)(1)}}{2(1)}$

$x=\frac{4\pm\sqrt{12}}{2}$

$x=\frac{4\pm 2\sqrt{3}}{2}$

$x=2\pm\sqrt{3}$

The solution set is $\left\{2+\sqrt{3},2-\sqrt{3}\right\}$.

11. $3x+1-(x-5)=2x-4$

$2x+6=2x-4$

$6=-4$

The solution set is $\varnothing$.

12. $\frac{2x}{x^2+6x+8}=\frac{x}{x+4}-\frac{2}{x+2},\quad x\neq -2, x\neq -4$

$$\frac{2x}{(x+4)(x+2)}=\frac{x}{x+4}-\frac{2}{x+2}$$

$$\frac{2x(x+4)(x+2)}{(x+4)(x+2)}=(x+4)(x+2)\left(\frac{x}{x+4}-\frac{2}{x+2}\right)$$

$$2x=\frac{x(x+4)(x+2)}{x+4}-\frac{2(x+4)(x+2)}{x+2}$$

$$2x=x(x+2)-2(x+4)$$

$$2x=x^2+2x-2x-8$$

$$0=x^2-2x-8$$

$$0=(x+2)(x-4)$$

$x+2=0$ or $x-4=0$

$x=-2$ $\quad x=4$

–2 must be rejected.

The solution set is $\{4\}$.

13. Let $y=0$.

$$0=x^2+6x+2$$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-(6)\pm\sqrt{(6)^2-4(1)(2)}}{2(1)}$$

$$x=\frac{-6\pm\sqrt{28}}{2}$$

$$x=\frac{-6\pm 2\sqrt{7}}{2}$$

$$x=-3\pm\sqrt{7}$$

x-intercepts: $-3+\sqrt{7}$ and $-3-\sqrt{7}$.

14. Let $y=0$.

$$0=4(x+1)-3x-(6-x)$$

$$0=4x+4-3x-6+x$$

$$0=2x-2$$

$$-2x=-2$$

$$x=1$$

x-intercept: 1.

15. Let $y=0$.

$$0=2x^2+26$$

$$-2x^2=26$$

$$x^2=-13$$

$$x=\pm\sqrt{-13}$$

$$x=\pm i\sqrt{13}$$

There are no x-intercepts.

16. Let $y=0$.

$$0=\frac{x^2}{3}+\frac{x}{2}-\frac{2}{3}$$

$$6(0)=6\left(\frac{x^2}{3}+\frac{x}{2}-\frac{2}{3}\right)$$

$$0=\frac{6\cdot x^2}{3}+\frac{6\cdot x}{2}-\frac{6\cdot 2}{3}$$

$$0=2x^2+3x-4$$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-(3)\pm\sqrt{(3)^2-4(2)(-4)}}{2(2)}$$

$$x=\frac{-3\pm\sqrt{41}}{4}$$

x-intercepts: $\frac{-3+\sqrt{41}}{4}$ and $\frac{-3-\sqrt{41}}{4}$.

17. Let $y=0$.

$$0=x^2-5x+8$$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-(-5)\pm\sqrt{(-5)^2-4(1)(8)}}{2(1)}$$

$$x=\frac{5\pm\sqrt{-7}}{2}$$

$$x=\frac{5\pm i\sqrt{7}}{2}$$

There are no x-intercepts.

18. $y_1=y_2$

$$3(2x-5)-2(4x+1)=-5(x+3)-2$$

$$6x-15-8x-2=-5x-15-2$$

$$-2x-17=-5x-17$$

$$3x=0$$

$$x=0$$

The solution set is $\{0\}$.

19.
$$y_1 y_2 = 10$$
$$(2x+3)(x+2) = 10$$
$$2x^2 + 7x + 6 = 10$$
$$2x^2 + 7x - 4 = 0$$
$$(2x-1)(x+4) = 0$$
$$2x - 1 = 0 \quad \text{or} \quad x + 4 = 0$$
$$x = \frac{1}{2} \qquad x = -4$$
The solution set is $\left\{-4, \frac{1}{2}\right\}$.

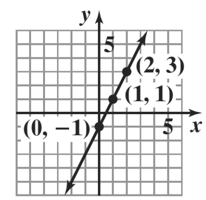
20.
$$x^2 + 10x - 3 = 0$$
$$x^2 + 10x \quad = 3$$
Since $b = 10$, we add $\left(\frac{10}{2}\right)^2 = 5^2 = 25$.
$$x^2 + 10x + 25 = 3 + 25$$
$$(x+5)^2 = 28$$
Apply the square root principle:
$$x + 5 = \pm\sqrt{28}$$
$$x + 5 = \pm\sqrt{4 \cdot 7} = \pm 2\sqrt{7}$$
$$x = -5 \pm 2\sqrt{7}$$
The solutions are $-5 \pm 2\sqrt{7}$, and the solution set is $\left\{-5 \pm 2\sqrt{7}\right\}$.

21.
$$2x^2 + 5x + 4 = 0$$
$$a = 2 \quad b = 5 \quad c = 4$$
$$b^2 - 4ac = 5^2 - 4(2)(4)$$
$$= 25 - 32 = -7$$
Since the discriminant is negative, there are no real solutions. There are two imaginary solutions that are complex conjugates.

22.
$$10x(x+4) = 15x - 15$$
$$10x^2 + 40x = 15x - 15$$
$$10x^2 - 25x + 15 = 0$$
$$a = 10 \quad b = -25 \quad c = 15$$
$$b^2 - 4ac = (-25)^2 - 4(10)(15)$$
$$= 625 - 600 = 25$$
Since the discriminant is positive and a perfect square, there are two rational solutions.

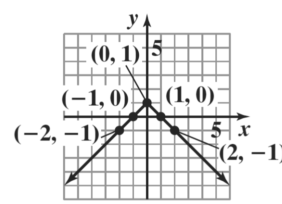
23.

x	(x, y)
–2	–5
–1	–3
0	–1
1	1
2	3



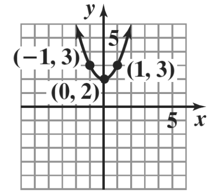
24.

x	(x, y)
–3	–2
–2	–1
–1	0
0	1
1	0
2	–1
3	–2



25.

x	(x, y)
–2	6
–1	3
0	2
1	3
2	6



26.
$$L = a + (n-1)d$$
$$L = a + dn - d$$
$$-dn = a - d - L$$
$$\frac{-dn}{-d} = \frac{a}{-d} - \frac{d}{-d} - \frac{L}{-d}$$
$$n = -\frac{a}{d} + 1 + \frac{L}{d}$$
$$n = \frac{L}{d} - \frac{a}{d} + 1$$
$$n = \frac{L-a}{d} + 1$$

27.
$$A = 2lw + 2lh + 2wh$$
$$-2lw - 2lh = 2wh - A$$
$$l\left(-2w - 2h\right) = 2wh - A$$
$$l = \frac{2wh - A}{-2w - 2h}$$
$$l = \frac{A - 2wh}{2w + 2h}$$

28.
$$f = \frac{f_1 f_2}{f_1 + f_2}$$
$$\left(f_1 + f_2\right)\left(f\right) = \left(f_1 + f_2\right)\frac{f_1 f_2}{f_1 + f_2}$$
$$f_1 f + f_2 f = f_1 f_2$$
$$f_1 f - f_1 f_2 = -f_2 f$$
$$f_1\left(f - f_2\right) = -f_2 f$$
$$f_1 = \frac{-f_2 f}{f - f_2}$$
$$f_1 = -\frac{ff_2}{f - f_2} \text{ or } f_1 = \frac{ff_2}{f_2 - f}$$

29. Let $x =$ the average yearly earnings, in thousands, of marketing majors.
Let $x + 19 =$ the average yearly earnings, in thousands, of engineering majors.
Let $x + 6 =$ the average yearly earnings, in thousands, of accounting majors.
$$x + (x+19) + (x+6) = 196$$
$$x + x + 19 + x + 6 = 196$$
$$3x + 25 = 196$$
$$3x = 171$$
$$x = 57$$
$$x + 19 = 76$$
$$x + 6 = 63$$
The average yearly earnings for marketing majors, engineering majors, and accounting majors were \$57 thousand, \$76 thousand, and \$63 thousand, respectively.

30. Let x = the number of years since 1960.
$$23 - 0.28x = 0$$
$$-0.28x = -23$$
$$\frac{-0.28x}{-0.28} = \frac{-23}{-0.28}$$
$$x \approx 82$$
If this trend continues, corporations will pay zero taxes 82 years after 1960, or 2042.

31. Let x = the amount invested at 8%.
Let 25,000 – x = the amount invested at 9%.
$$0.08x + 0.09(25{,}000 - x) = 2135$$
$$0.08x + 2250 - 0.09x = 2135$$
$$-0.01x + 2250 = 2135$$
$$-0.01x = -115$$
$$x = \frac{-115}{-0.01}$$
$$x = 11{,}500$$
$$25{,}000 - x = 13{,}500$$
\$11,500 was invested at 8% and \$13,500 was invested at 9%.

32. Let x = the number of bridge crossings.
Without discount pass: $8x$
With discount pass: $45 + 5x$
$$8x = 45 + 5x$$
$$3x = 45$$
$$x = 15$$
The cost will be the same for 15 bridge crossings.

33. Let x = the price before the reduction.
$$x - 0.40x = 468$$
$$0.60x = 468$$
$$\frac{0.60x}{0.60} = \frac{468}{0.60}$$
$$x = 780$$
The price before the reduction was \$780.

34. Let x = the amount invested at 4%.
Let $4000 - x$ = the amount invested that lost 3%.

$$0.04x - 0.03(4000 - x) = 55$$
$$0.04x - 120 + 0.03x = 55$$
$$0.07x - 120 = 55$$
$$0.07x = 175$$
$$x = \frac{175}{0.07}$$
$$x = 2500$$
$$4000 - x = 1500$$

\$2500 was invested at 4% and \$1500 lost 3%.

35. Let x = the width of the rectangle
Let $2x + 5$ = the length of the rectangle

$$2l + 2w = P$$
$$2(2x+5) + 2x = 46$$
$$4x + 10 + 2x = 46$$
$$6x + 10 = 46$$
$$6x = 36$$
$$\frac{6x}{6} = \frac{36}{6}$$
$$x = 6$$
$$2x + 5 = 17$$

The dimensions of the rectangle are 6 ft by 17 ft.

36. Let x = the width of the rectangle
Let $2x - 1$ = the length of the rectangle

$$lw = A$$
$$(2x-1)x = 28$$
$$2x^2 - x = 28$$
$$2x^2 - x - 28 = 0$$
$$(2x+7)(x-4) = 0$$
$$2x + 7 = 0 \quad \text{or} \quad x - 4 = 0$$
$$2x = -7 \qquad x = 4$$
$$x = -\frac{7}{2}$$

$-\frac{7}{2}$ must be rejected.

If $x = 4$, then $2x - 1 = 7$

The dimensions of the rectangle are 4 ft by 7 ft.

37. Let x = the height up the pole at which the wires are attached.

$$x^2 + 5^2 = 13^2$$
$$x^2 + 25 = 169$$
$$x^2 = 144$$
$$x = \pm 12$$

–12 must be rejected.
The wires are attached 12 yards up the pole.

38. **a.**

$$P = -10x^2 + 475x + 3500$$
$$5990 = -10x^2 + 475x + 3500$$
$$0 = -10x^2 + 475x - 2490$$
$$0 = 2x^2 - 95x + 498$$
$$0 = (x-6)(2x-83)$$
$$x - 6 = 0 \quad \text{or} \quad 2x - 83 = 0$$
$$x = 6 \qquad 2x = 83$$
$$x = 41.5$$

The population reached 5990 after 6 years.

b. This is represented by the point (6, 5990).

39.

$$p = 0.004x^2 - 0.35x + 13.9$$
$$19 = 0.004x^2 - 0.35x + 13.9$$
$$0 = 0.004x^2 - 0.35x - 5.1$$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{-(-0.35) \pm \sqrt{(-0.35)^2 - 4(0.004)(-5.1)}}{2(0.004)}$$
$$x = \frac{0.35 \pm \sqrt{0.1225 + 0.0816}}{0.008}$$
$$x \approx 100, \quad x \approx -13 \text{ (rejected)}$$

The percentage of foreign born Americans will be 19% about 100 years after 1920, or 2020.

40. $(6-2i) - (7-i) = 6 - 2i - 7 + i = -1 - i$

41. $3i(2+i) = 6i + 3i^2 = -3 + 6i$

42.
$$(1+i)(4-3i) = 4 - 3i + 4i - 3i^2$$
$$= 4 + i + 3 = 7 + i$$

43.
$$\frac{1+i}{1-i} = \frac{1+i}{1-i} \cdot \frac{1+i}{1+i} = \frac{1+i+i+i^2}{1-i^2}$$
$$= \frac{1+2i-1}{1+1}$$
$$= \frac{2i}{2}$$
$$= i$$

44. $\sqrt{-75} - \sqrt{-12} = 5i\sqrt{3} - 2i\sqrt{3} = 3i\sqrt{3}$

45.
$$\left(2 - \sqrt{-3}\right)^2 = \left(2 - i\sqrt{3}\right)^2$$
$$= 4 - 4i\sqrt{3} + 3i^2$$
$$= 4 - 4i\sqrt{3} - 3$$
$$= 1 - 4i\sqrt{3}$$

Section 1.6

Check Point Exercises

1. $4x^4 = 12x^2$

$4x^4 - 12x^2 = 0$

$4x^2(x^2 - 3) = 0$

$4x^2 = 0$ or $x^2 - 3 = 0$

$x^2 = 0 \qquad x^2 = 3$

$x = \pm\sqrt{0} \qquad x = \pm\sqrt{3}$

$x = 0 \qquad x = \pm\sqrt{3}$

The solution set is $\{-\sqrt{3}, 0, \sqrt{3}\}$.

2. $2x^3 + 3x^2 = 8x + 12$

$x^2(2x+3) - 4(2x+3) = 10$

$(2x+3)(x^2 - 4) = 0$

$2x + 3 = 0$ or $x^2 - 4 = 0$

$2x = -3 \qquad x^2 = 4$

$x = -\frac{3}{2} \qquad x = \pm 2$

The solution set is $\left\{-2, -\frac{3}{2}, 2\right\}$.

3. $\sqrt{x+3} + 3 = x$

$\sqrt{x+3} = x - 3$

$\left(\sqrt{x+3}\right)^2 = (x-3)^2$

$x + 3 = x^2 - 6x + 9$

$0 = x^2 - 7x + 6$

$0 = (x-6)(x-1)$

$x - 6 = 0$ or $x - 1 = 0$

$x = 6 \qquad x = 1$

1 does not check and must be rejected.
The solution set is $\{6\}$.

4. $\sqrt{x+5} - \sqrt{x-3} = 2$

$\sqrt{x+5} = 2 + \sqrt{x-3}$

$\left(\sqrt{x+5}\right)^2 = \left(2 + \sqrt{x-3}\right)^2$

$x + 5 = (2)^2 + 2(2)\left(\sqrt{x-3}\right) + \left(\sqrt{x-3}\right)^2$

$x + 5 = 4 + 4\sqrt{x-3} + x - 3$

$4 = 4\sqrt{x-3}$

$\frac{4}{4} = \frac{4\sqrt{x-3}}{4}$

$1 = \sqrt{x-3}$

$(1)^2 = \left(\sqrt{x-3}\right)^2$

$1 = x - 3$

$4 = x$

The check indicates that 4 is a solution.
The solution set is $\{4\}$.

5. a. $5x^{3/2} - 25 = 0$

$5x^{3/2} = 25$

$x^{3/2} = 5$

$\left(x^{3/2}\right)^{2/3} = (5)^{2/3}$

$x = 5^{2/3}$ or $\sqrt[3]{25}$

Check:

$5\left(5^{2/3}\right)^{3/2} - 25 = 0$

$5(5) - 25 = 0$

$25 - 25 = 0$

$0 = 0$

The solution set is $\{5^{2/3}\}$ or $\{\sqrt[3]{25}\}$.

b. $x^{\frac{2}{3}} - 8 = -4$

$x^{2/3} = 4$

$\left(x^{2/3}\right)^{3/2} = 4^{3/2}$ or

$x = \left(2^2\right)^{3/2} \qquad x = (-2)^3$

$x = 2^3 \qquad x = -8$

$x = 8$

The solution set is $\{-8, 8\}$.

6. $x^4 - 5x^2 + 6 = 0$

$\left(x^2\right)^2 - 5x^2 + 6 = 0$

Let $t = x^2$.

$t^2 - 5t + 6 = 0$

$(t-3)(t-2) = 0$

$t - 3 = 0$ or $t - 2 = 0$

$t = 3$ or $t = 2$

$x^2 = 3$ or $x^2 = 2$

$x = \pm\sqrt{3}$ or $x = \pm\sqrt{2}$

The solution set is $\left\{-\sqrt{3}, -\sqrt{2}, \sqrt{2}, \sqrt{3}\right\}$.

7. $3x^{2/3} - 11x^{1/3} - 4 = 0$

Let $t = x^{1/3}$.

$3t^2 - 11t - 4 = 0$

$(3t+1)(t-4) = 0$

$3t + 1 = 0$ or $t - 4 = 0$

$3t = -1$

$t = -\frac{1}{3}$ $\quad t = 4$

$x^{1/3} = -\frac{1}{3}$ $\quad x^{1/3} = 4$

$x = \left(-\frac{1}{3}\right)^3$ $\quad x = 4^3$

$x = -\frac{1}{27}$ $\quad x = 64$

The solution set is $\left\{-\frac{1}{27}, 64\right\}$.

8. $\left(x^2 - 4\right)^2 + \left(x^2 - 4\right) - 6 = 0$

Let $u = x^2 - 4$.

$u^2 + u - 6 = 0$

$(u+3)(u-2) = 0$

$u + 3 = 0$ or $u - 2 = 0$

$u = -3$ or $u = 2$

$x^2 - 4 = -3$ or $x^2 - 4 = 2$

$x = \pm 1$ or $x = \pm\sqrt{6}$

The solution set is $\left\{-\sqrt{6}, -1, 1, \sqrt{6}\right\}$.

9. $|2x - 1| = 5$

$2x - 1 = 5$ or $2x - 1 = -5$

$2x = 6$ $\quad 2x = -4$

$x = 3$ $\quad x = -2$

The solution set is {–2, 3}.

10. $4|1 - 2x| - 20 = 0$

$4|1 - 2x| = 20$

$|1 - 2x| = 5$

$1 - 2x = 5$ or $1 - 2x = -5$

$-2x = 4$ $\quad -2x = -6$

$x = -2$ $\quad x = 3$

The solution set is {–2, 3}.

11. $H = -2.3\sqrt{I} + 67.6$

$33.1 = -2.3\sqrt{I} + 67.6$

$-34.5 = -2.3\sqrt{I}$

$\frac{-34.5}{-2.3} = \frac{-2.3\sqrt{I}}{-2.3}$

$15 = \sqrt{I}$

$15^2 = \left(\sqrt{I}\right)^2$

$225 = I$

The model indicates that an annual income of 225 thousand dollars, or $225,000, corresponds to 33.1 hours per week watching TV.

Concept and Vocabulary Check 1.6

1. subtract 8*x* and subtract 12 from both sides

2. radical

3. extraneous

4. $2x + 1$; $x^2 + 14x + 49$

5. $x + 2$; $x + 8 - 6\sqrt{x - 1}$

6. $5^{\frac{4}{3}}$

7. $\pm 5^{\frac{3}{2}}$

8. x^2; $u^2 - 13u + 36 = 0$

9. $x^{\frac{1}{3}}$; $u^2+2u-3=0$

10. c; $-c$

11. $3x-1=7$; $3x-1=-7$

Exercise Set 1.6

1.
$$3x^4-48x^2=0$$
$$3x^2(x^2-16)=0$$
$$3x^2(x+4)(x-4)=0$$
$$3x^2=0 \quad x+4=0 \quad x-4=0$$
$$x^2=0 \quad x=-4 \quad x=4$$
$$x=0$$
The solution set is {–4, 0, 4}.

2.
$$5x^4-20x^2=0$$
$$5x^2(x^2-4)=0$$
$$5x^2(x+2)(x-2)=0$$
$$5x^2=0 \quad x+2=0 \quad x-2=0$$
$$x^2=0$$
$$x=0 \quad x=-2 \quad x=2$$
The solution set is {–2, 0, 2}.

3.
$$3x^3+2x^2=12x+8$$
$$3x^3+2x^2-12x-8=0$$
$$x^2(3x+2)-4(3x+2)=0$$
$$(3x+2)(x^2-4)=0$$
$$3x+2=0 \quad x^2-4=0$$
$$3x=-2 \quad x^2=4$$
$$x=-\frac{2}{3} \quad x=\pm 2$$
The solution set is $\left\{-2,-\frac{2}{3},2\right\}$.

4.
$$4x^3-12x^2=9x-27$$
$$4x^3-12x^2-9x+27=0$$
$$4x^2(x-3)-9(x-3)=0$$
$$(x-3)(4x^2-9)=0$$
$$x-3=0 \quad 4x^2-9=0$$
$$x=3 \quad 4x^2=9$$
$$x^2=\frac{9}{4}$$
$$x=\pm\frac{3}{2}$$
The solution set is $\left\{-\frac{3}{2},\frac{3}{2},3\right\}$.

5.
$$2x-3=8x^3-12x^2$$
$$8x^3-12x^2-2x+3=0$$
$$4x^2(2x-3)-(2x-3)=0$$
$$(2x-3)(4x^2-1)=0$$
$$2x-3=0 \quad 4x^2-1=0$$
$$2x=3 \quad 4x^2=1$$
$$x^2=\frac{1}{4}$$
$$x=\frac{3}{2} \quad x=\pm\frac{1}{2}$$
The solution set is $\left\{\frac{3}{2},\frac{1}{2},-\frac{1}{2}\right\}$.

6.
$$x+1=9x^3+9x^2$$
$$9x^3+9x^2-x-1=0$$
$$9x^2(x+1)-(x+1)=0$$
$$(x+1)(9x^2-1)=0$$
$$x+1=0 \quad 9x^2-1=1$$
$$x=-1 \quad 9x^2=1$$
$$x^2=\frac{1}{9}$$
$$x=\pm\frac{1}{3}$$
The solution set is $\left\{-1,-\frac{1}{3},\frac{1}{3}\right\}$.

7. $4y^3 - 2 = y - 8y^2$

$$4y^3 + 8y^2 - y - 2 = 0$$
$$4y^2(y+2) - (y+2) = 0$$
$$(y+2)(4y^2 - 1) = 0$$
$$y + 2 = 0 \qquad 4y^2 - 1 = 0$$
$$4y^2 = 1$$
$$y^2 = \frac{1}{4}$$
$$y = -2 \qquad y = \pm\frac{1}{2}$$

The solution set is $\left\{-2, \frac{1}{2}, -\frac{1}{2}\right\}$.

8. $9y^3 + 8 = 4y + 18y^2$

$$9y^3 - 18y^2 - 4y + 8 = 0$$
$$9y^2(y-2) - 4(y-2) = 0$$
$$(y-2)(9y^2 - 4) = 0$$
$$y - 2 = 0 \qquad 9y^2 - 4 = 0$$
$$y = 2 \qquad 9y^2 = 4$$
$$y^2 = \frac{4}{9}$$
$$y = \pm\frac{2}{3}$$

The solution set is $\left\{-\frac{2}{3}, \frac{2}{3}, 2\right\}$.

9. $2x^4 = 16x$

$$2x^4 - 16x = 0$$
$$2x\left(x^3 - 8\right) = 0$$
$$2x = 0 \qquad x^3 - 8 = 0$$
$$x = 0 \qquad (x-2)(x^2 + 2x + 2) = 0$$
$$x - 2 = 0 \qquad x^2 + 2x + 4 = 0$$
$$x = 2 \qquad x = \frac{-2 \pm \sqrt{2^2 - 4(1)(4)}}{2(1)}$$
$$x = \frac{-2 \pm \sqrt{-12}}{2}$$
$$x = \frac{-2 \pm 2i\sqrt{3}}{2}$$
$$x = -1 \pm i\sqrt{3}$$

The solution set is $\left\{0, 2, -1 \pm i\sqrt{3}\right\}$.

10. $3x^4 = 81x$

$$3x^4 - 81x = 0$$
$$3x(x^3 - 27) = 0$$
$$3x = 0 \qquad x^3 - 27 = 0$$
$$x = 0;$$
$$(x-3)(x^2 + 3x + 9) = 0$$
$$x - 3 = 0 \qquad x^2 + 3x + 9 = 0$$
$$x = 3 \qquad x = \frac{-3 \pm \sqrt{3^2 - 4(1)(9)}}{2(1)}$$
$$x = \frac{-3 \pm \sqrt{9 - 36}}{2}$$
$$x = \frac{-3 \pm \sqrt{-27}}{2}$$
$$x = \frac{-3 \pm 3i\sqrt{3}}{2}$$

The solution set is $\left\{0, 3, \frac{-3 \pm 3i\sqrt{3}}{2}\right\}$.

11. $\sqrt{3x + 18} = x$

$$3x + 18 = x^2$$
$$x^2 - 3x - 18 = 0$$
$$(x+3)(x-6) = 0$$
$$x + 3 = 0 \qquad x - 6 = 0$$
$$x = -3 \qquad x = 6$$
$$\sqrt{3(-3) + 18} = -3 \qquad \sqrt{3(6) + 18} = 6$$
$$\sqrt{-9 + 18} = -3 \qquad \sqrt{18 + 18} = 6$$
$$\sqrt{9} = -3 \quad \text{False} \qquad \sqrt{36} = 6$$

The solution set is $\{6\}$.

12. $\sqrt{20 - 8x} = x$

$$20 - 8x = x^2$$
$$x^2 + 8x - 20 = 0$$
$$(x+10)(x-2) = 0$$
$$x + 10 = 0 \qquad x - 2 = 0$$
$$x = -10 \qquad x = 2$$
$$\sqrt{20 - 8(-10)} = -10 \qquad \sqrt{20 - 8(2)} = 2$$
$$\sqrt{20 + 80} = -10 \qquad \sqrt{20 - 16} = 2$$
$$\sqrt{100} = -10 \quad \text{False} \qquad \sqrt{4} = 2$$

The solution set is $\{2\}$.

13. $\sqrt{x+3}=x-3$

$x+3=x^2-6x+9$

$x^2-7x+6=0$

$(x-1)(x-6)=0$

$x-1=0 \quad x-6=0$

$x=1 \qquad x=6$

$\sqrt{1+3}=1-3 \qquad \sqrt{6+3}=6-3$

$\sqrt{4}=-2$ False $\quad \sqrt{9}=3$

The solution set is $\{6\}$.

14. $\sqrt{x+10}=x-2$

$x+10=(x-2)^2$

$x+10=x^2-4x+4$

$x^2-5x-6=0$

$(x+1)(x-6)=0$

$x+1=0 \quad x-6=0$

$x=-1 \qquad x=6$

$\sqrt{-1+10}=-1-2 \qquad \sqrt{6+10}=6-2$

$\sqrt{9}=-3$ False $\quad \sqrt{16}=4$

The solution set is $\{6\}$.

15. $\sqrt{2x+13}=x+7$

$2x+13=(x+7)^2$

$2x+13=x^2+14x+49$

$x^2+12x+36=0$

$(x+6)^2=0$

$x+6=0$

$x=-6$

$\sqrt{2(-6)+13}=-6+7$

$\sqrt{-12+13}=1$

$\sqrt{1}=1$

The solution set is $\{-6\}$.

16. $\sqrt{6x+1}=x-1$

$6x+1=(x-1)^2$

$6x+1=x^2-2x+1$

$x^2-8x=0$

$x(x-8)=0$

$x-8=0 \qquad x=0$

$x=8$

$\sqrt{6(0)+1}=0-1 \qquad \sqrt{6(8)+1}=8-1$

$\sqrt{0+1}=-1 \qquad \sqrt{48+1}=7$

$\sqrt{1}=-1$ False $\quad \sqrt{49}=7$

The solution set is $\{8\}$.

17. $x-\sqrt{2x+5}=5$

$x-5=\sqrt{2x+5}$

$(x-5)^2=2x+5$

$x^2-10x+25=2x+5$

$x^2-12x+20=0$

$(x-2)(x-10)=0$

$x-2=0 \quad x-10=0$

$x=2 \qquad x=10$

$2-\sqrt{2(2)+5}=5 \qquad 10-\sqrt{2(10)+5}=5$

$2-\sqrt{9}=5 \qquad 10-\sqrt{25}=5$

$2-3=5$ False $\quad 10-5=5$

The solution set is $\{10\}$.

18. $x-\sqrt{x+11}=1$

$x-1=\sqrt{x+11}$

$(x-1)^2=x+11$

$x^2-2x+1=x+11$

$x^2-3x-10=0$

$(x+2)(x-5)=0$

$x+2=0 \quad x-5=0$

$x=-2 \qquad x=5$

$-2-\sqrt{-2+11}=1 \qquad 5-\sqrt{5+11}=1$

$-2-\sqrt{9}=1 \qquad 5-\sqrt{16}=1$

$-2-3=1$ False $\quad 5-4=1$

The solution set is $\{5\}$.

19. $\sqrt{2x+19}-8=x$

$\sqrt{2x+19}=x+8$

$\left(\sqrt{2x+19}\right)^2=(x+8)^2$

$2x+19=x^2+16x+64$

$0=x^2+14x+45$

$0=(x+9)(x+5)$

$x+9=0$ or $x+5=0$

$x=-9$ $\quad x=-5$

–9 does not check and must be rejected.

The solution set is $\{-5\}$.

20. $\sqrt{2x+15}-6=x$

$\sqrt{2x+15}=x+6$

$\left(\sqrt{2x+15}\right)^2=(x+6)^2$

$2x+15=x^2+12x+36$

$0=x^2+10x+21$

$0=(x+3)(x+7)$

$x+3=0$ or $x+7=0$

$x=-3$ $\quad x=-7$

–7 does not check and must be rejected.

The solution set is $\{-3\}$.

21. $\sqrt{3x}+10=x+4$

$\sqrt{3x}=x-6$

$3x=(x-6)^2$

$3x=x^2-12x+36$

$x^2-15x+36=0$

$(x-12)(x-3)=0$

$x-12=0 \quad x-3=0$

$x=12 \quad x=3$

$\sqrt{3(12)}+10=12+4 \quad \sqrt{3(3)}+10=3+4$

$\sqrt{36}+10=16 \quad \sqrt{9}+10=7$

$6+10=16 \quad 3+10=7$ False

The solution set is $\{12\}$.

22. $\sqrt{x}-3=x-9$

$\sqrt{x}=x-6$

$x=(x-6)^2$

$x=x^2-12x+36$

$x^2-13x+36=0$

$(x-9)(x-4)=0$

$x-9=0 \quad x-4=0$

$x=9 \quad x=4$

$\sqrt{9}-3=9-9 \quad \sqrt{4}-3=4-9$

$3-3=9-9 \quad 2-3=4-9$ False

The solution set is $\{9\}$.

23. $\sqrt{x+8}-\sqrt{x-4}=2$

$\sqrt{x+8}=\sqrt{x-4}+2$

$x+8=(\sqrt{x-4}+2)^2$

$x+8=x-4+4\sqrt{x-4}+4$

$x+8=x+4\sqrt{x-4}$

$8=4\sqrt{x-4}$

$2=\sqrt{x-4}$

$4=x-4$

$x=8$

$\sqrt{8+8}-\sqrt{8-4}=2$

$\sqrt{16}-\sqrt{4}=2$

$4-2=2$

The solution set is $\{8\}$.

24. $\sqrt{x+5}-\sqrt{x-3}=2$

$\sqrt{x+5}=\sqrt{x-3}+2$

$x+5=(\sqrt{x-3}+2)^2$

$x+5=x-3+4\sqrt{x-3}+4$

$x+5=x+1+4\sqrt{x-3}$

$5=1+4\sqrt{x-3}$

$4=4\sqrt{x-3}$

$1=\sqrt{x-3}$

$1=x-3$

$x=4$

$\sqrt{4+5}-\sqrt{4-3}=2$

$\sqrt{9}-\sqrt{1}=2$

$3-1=2$

The solution set is $\{4\}$.

25. $\sqrt{x-5}-\sqrt{x-8}=3$

$$\sqrt{x-5}=\sqrt{x-8}+3$$
$$x-5=(\sqrt{x-8}+3)^2$$
$$x-5=x-8+6\sqrt{x-8}+9$$
$$x-5=x+1+6\sqrt{x-8}$$
$$-6=6\sqrt{x-8}$$
$$-1=\sqrt{x-8}$$
$$1=x-8$$
$$x=9$$
$$\sqrt{9-5}-\sqrt{9-8}=3$$
$$\sqrt{4}-\sqrt{1}=3$$
$$2-1=3 \text{ False}$$

The solution set is the empty set, $\varnothing$.

26. $\sqrt{2x-3}-\sqrt{x-2}=1$

$$\sqrt{2x-3}=\sqrt{x-2}+1$$
$$2x-3=(\sqrt{x-2}+1)^2$$
$$2x-3=x-2+2\sqrt{x-2}+1$$
$$2x-3=x-1+2\sqrt{x-2}$$
$$x-2=2\sqrt{x-2}$$
$$\frac{x}{2}-1=\sqrt{x-2}$$
$$\left(\frac{x}{2}-1\right)^2=x-2$$
$$\frac{x^2}{4}-x+1=x-2$$
$$x^2-4x+4=4x-8$$
$$x^2-8x+12=0$$
$$(x-6)(x-2)=0$$
$$x-6=0 \quad x-2=0$$
$$x=6 \quad x=2$$
$$\sqrt{2(6)-3}-\sqrt{6-2}=1 \quad \sqrt{2(2)-3}-\sqrt{2-2}=1$$
$$\sqrt{12-3}-\sqrt{4}=1 \quad \sqrt{4-3}-\sqrt{0}=1$$
$$\sqrt{9}-\sqrt{4}=1 \quad \sqrt{1}-0=1$$
$$3-2=1 \quad 1-0=1$$

The solution set is {2, 6}.

27. $\sqrt{2x+3}+\sqrt{x-2}=2$

$$\sqrt{2x+3}=2-\sqrt{x-2}$$
$$2x+3=(2-\sqrt{x-2})^2$$
$$2x+3=4-4\sqrt{x-2}+x-2$$
$$x+1=-4\sqrt{x-2}$$
$$(x+1)^2=16(x-2)$$
$$x^2+2x+1=16x-32$$
$$x^2-14x+33=0$$
$$(x-11)(x-3)=0$$
$$x-11=0 \quad x-3=0$$
$$x=11 \quad x=3$$
$$\sqrt{2(11)+3}+\sqrt{11-2}=2$$
$$\sqrt{22+3}+\sqrt{9}=2$$
$$5+3=2 \text{ False}$$
$$\sqrt{2(3)+3}+\sqrt{3-2}=2$$
$$\sqrt{6+3}+\sqrt{1}=2$$
$$3+1=2 \text{ False}$$

The solution set is the empty set, $\varnothing$.

28. $\sqrt{x+2}+\sqrt{3x+7}=1$

$$\sqrt{x+2}=1-\sqrt{3x+7}$$
$$x+2=(1-\sqrt{3x+7})^2$$
$$x+2=1-2\sqrt{3x+7}+3x+7$$
$$-2x-6=-2\sqrt{3x+7}$$
$$x+3=\sqrt{3x+7}$$
$$(x+3)^2=3x+7$$
$$x^2+6x+9=3x+7$$
$$x^2+3x+2=0$$
$$(x+1)(x+2)=0$$
$$x+1=0 \quad x+2=0$$
$$x=-1 \quad x=-2$$
$$\sqrt{-1+2}+\sqrt{3(-1)+7}=1$$
$$\sqrt{1}+\sqrt{4}=1$$
$$1+2=1 \text{ False}$$
$$\sqrt{-2+2}+\sqrt{3(-2)+7}=1$$
$$\sqrt{0}+\sqrt{1}=1$$
$$0+1=1$$

The solution set is {–2}.

29. $\sqrt{3\sqrt{x+1}} = \sqrt{3x-5}$

$3\sqrt{x+1} = 3x-5$

$9(x+1) = 9x^2 - 30x + 25$

$9x^2 - 39x + 16 = 0$

$x = \dfrac{39 \pm \sqrt{945}}{18} = \dfrac{13 \pm \sqrt{105}}{6}$

Check proposed solutions.

The solution set is $\left\{\dfrac{13+\sqrt{105}}{6}\right\}$.

30. $\sqrt{1+4\sqrt{x}} = 1+\sqrt{x}$

$1+4\sqrt{x} = 1+2\sqrt{x}+x$

$2\sqrt{x} = x$

$4x = x^2$

$x^2 - 4x = 0$

$x(x-4) = 0$

$x = 0$ or $x = 4$

The solution set is $\{0, 4\}$.

31. $x^{3/2} = 8$

$(x^{3/2})^{2/3} = 8^{2/3}$

$x = \sqrt[3]{8}^2$

$x = 2^2$

$x = 4$

$4^{3/2} = 8$

$\sqrt{4}^3 = 8$

$2^3 = 8$

The solution set is $\{4\}$.

32. $x^{3/2} = 27$

$(x^{3/2})^{2/3} = 27^{2/3}$

$x = \sqrt[3]{27}^2$

$x = 3^2$

$x = 9$

$9^{3/2} = 27$

$\sqrt{9}^3 = 27$

$3^3 = 27$

The solution set is $\{9\}$.

33. $(x-4)^{3/2} = 27$

$((x-4)^{3/2})^{2/3} = 27^{2/3}$

$x-4 = \sqrt[3]{27}^2$

$x-4 = 3^2$

$x-4 = 9$

$x = 13$

$(13-4)^{3/2} = 27$

$9^{3/2} = 27$

$\sqrt{9}^3 = 27$

$3^3 = 27$

The solution set is $\{13\}$.

34. $(x+5)^{3/2} = 8$

$((x+5)^{3/2})^{2/3} = 8^{2/3}$

$x+5 = \sqrt[3]{8}^2$

$x+5 = 2^2$

$x+5 = 4$

$x = -1$

$(-1+5)^{3/2} = 8$

$4^{3/2} = 8$

$\sqrt{4}^3 = 8$

$2^3 = 8$

The solution set is $\{-1\}$.

35. $6x^{5/2} - 12 = 0$

$6x^{5/2} = 12$

$x^{5/2} = 2$

$(x^{5/2})^{2/5} = 2^{2/5}$

$x = \sqrt[5]{2^2}$

$x = \sqrt[5]{4}$

$6(\sqrt[5]{4})^{5/2} - 12 = 0$

$6(4^{1/5})^{5/2} - 12 = 0$

$6(4^{1/2}) - 12 = 0$

$6(2) - 12 = 0$

The solution set is $\left\{\sqrt[5]{4}\right\}$.

36. $8x^{5/3} - 24 = 0$

$$8x^{5/3} = 24$$
$$x^{5/3} = 3$$
$$(x^{5/3})^{3/5} = 3^{3/5}$$
$$x = \sqrt[5]{3^3}$$
$$x = \sqrt[5]{27}$$
$$8(\sqrt[5]{27})^{5/3} - 24 = 0$$
$$8(27^{1/5})^{5/3} - 24 = 0$$
$$8(27^{1/3}) - 24 = 0$$
$$8(3) - 24 = 0$$

The solution set is $\left\{\sqrt[5]{27}\right\}$.

37. $(x-4)^{2/3} = 16$

$$\left[(x-4)^{2/3}\right]^{3/2} = (16)^{3/2}$$
$$x - 4 = \left(2^4\right)^{3/2}$$
$$x - 4 = 4^3 \qquad x - 4 = (-4)^3$$
$$x - 4 = 64 \qquad x - 4 = -64$$
$$x = 68 \qquad x = -60$$

The solution set is {– 60, 68}.

38. $(x+5)^{\frac{2}{3}} = 4$

$$\left[(x+5)^{\frac{2}{3}}\right]^{\frac{3}{2}} = (4)^{\frac{3}{2}}$$
$$x + 5 = \left(2^2\right)^{\frac{3}{2}}$$
$$x + 5 = 2^3 \quad \text{or} \quad x + 5 = (-2)^3$$
$$x + 5 = 8 \qquad x + 5 = -8$$
$$x = 3 \qquad x = -13$$

The solution set is {–13, 3}.

39. $(x^2 - x - 4)^{3/4} - 2 = 6$

$$(x^2 - x - 4)^{3/4} = 8$$
$$((x^2 - x - 4)^{3/4})^{4/3} = 8^{4/3}$$
$$x^2 - x - 4 = \sqrt[3]{8}^4$$
$$x^2 - x - 4 = 2^4$$
$$x^2 - x - 4 = 16$$
$$x^2 - x - 20 = 0$$
$$(x-5)(x+4) = 0$$
$$x - 5 = 0 \qquad x + 4 = 0$$
$$x = 5 \qquad x = -4$$
$$(5^2 - 5 - 4)^{3/4} - 2 = 6$$
$$(25 - 9)^{3/4} - 2 = 6$$
$$16^{3/4} - 2 = 6$$
$$\sqrt[4]{16}^3 - 2 = 6$$
$$2^3 - 2 = 6$$
$$8 - 2 = 6$$
$$((-4)^2 - (-4) - 4)^{3/4} - 2 = 6$$
$$(16 + 4 - 4)^{3/4} - 2 = 6$$
$$16^{3/4} - 2 = 6$$
$$\sqrt[4]{16}^3 - 2 = 6$$
$$2^3 - 2 = 6$$
$$8 - 2 = 6$$

The solution set is {5, –4}.

40. $(x^2 - 3x + 3)^{3/2} - 1 = 0$

$$(x^2 - 3x + 3)^{3/2} = 1$$
$$x^2 - 3x + 3 = 1^{2/3}$$
$$x^2 - 3x + 3 = 1$$
$$x^2 - 3x + 2 = 0$$
$$(x-1)(x-2) = 0$$
$$x - 1 = 0 \qquad x - 2 = 0$$
$$x = 1 \qquad x = 2$$
$$(1^2 - 3(1) + 3)^{3/2} - 1 = 0$$
$$(1 - 3 + 3)^{3/2} - 1 = 0$$
$$1^{3/2} - 1 = 0$$
$$1 - 1 = 0$$
$$(2^2 - 3(2) + 3)^{3/2} - 1 = 0$$
$$(4 - 6 + 3)^{3/2} - 1 = 0$$
$$1^{3/2} - 1 = 0$$
$$1 - 1 = 0$$

The solution set is {1, 2}.

41. $x^4 - 5x^2 + 4 = 0$ let $t = x^2$

$t^2 - 5t + 4 = 0$

$(t-1)(t-4) = 0$

$t - 1 = 0 \quad t - 4 = 0$

$t = 1 \quad t = 4$

$x^2 = 1 \quad x^2 = 4$

$x = \pm 1 \quad x = \pm 2$

The solution set is {1, –1, 2, –2}.

42. $x^4 - 13x^2 + 36 = 0$ let $t = x^2$

$t^2 - 13t + 36 = 0$

$(t-4)(t-9) = 0$

$t - 4 = 0 \quad t - 9 = 0$

$t = 4 \quad t = 9$

$x^2 = 4 \quad x^2 = 9$

$x = \pm 2 \quad x = \pm 3$

The solution set is {–3, –2, 2, 3}.

43. $9x^4 = 25x^2 - 16$

$9x^4 - 25x^2 + 16 = 0$ let $t = x^2$

$9t^2 - 25t + 16 = 0$

$(9t - 16)(t - 1) = 0$

$9t - 16 = 0 \quad t - 1 = 0$

$9t = 16 \quad t = 1$

$t = \frac{16}{9} \quad x^2 = 1$

$x = \pm 1$

$x^2 = \frac{16}{9}$

$x = \pm\frac{4}{3}$

The solution set is $\left\{1, -1, \frac{4}{3}, -\frac{4}{3}\right\}$.

44. $4x^4 = 13x^2 - 9$

$4x^4 - 13x^2 + 9 = 0$ let $t = x^2$

$4t^2 - 13t + 9 = 0$

$(4t - 9)(t - 1) = 0$

$4t - 9 = 0 \quad t - 1 = 0$

$4t = 9 \quad t = 1$

$t = \frac{9}{4} \quad x^2 = 1$

$x^2 = \frac{9}{4} \quad x = \pm 1$

$x = \pm\frac{3}{2}$

The solution set is $\left\{-\frac{3}{2}, -1, 1, \frac{3}{2}\right\}$.

45. $x - 13\sqrt{x} + 40 = 0$ Let $t = \sqrt{x}$.

$t^2 - 13t + 40 = 0$

$(t-8)(t-5) = 0$

$t - 8 = 0 \quad t - 5 = 0$

$t = 8 \quad t = 5$

$\sqrt{x} = 8 \quad \sqrt{x} = 5$

$x = 64 \quad x = 25$

The solution set is {25, 64}.

46. $2x - 7\sqrt{x} - 30 = 0$ Let $t = \sqrt{x}$.

$2t^2 - 7t - 30 = 0$

$(2t + 5)(t - 6) = 0$

$2t + 5 = 0$

$t = \frac{5}{2} \quad t - 6 = 0$

$t = 6$

$\sqrt{x} = \frac{5}{2} \quad \sqrt{x} = 6$

$x = 36$

$x = \frac{25}{4}$

The solution set is {36} since 25/4 does not check in the original equation.

47. $x^{-2} - x^{-1} - 20 = 0$ Let $t = x^{-1}$

$t^2 - t - 20 = 0$

$(t-5)(t+4) = 0$

$t - 5 = 0 \quad t + 4 = 0$

$t = 5 \quad t = -4$

$x^{-1} = 5 \quad x^{-1} = -4$

$\frac{1}{x} = 5 \quad \frac{1}{x} = -4$

$1 = 5x \quad 1 = -4x$

$\frac{1}{5} = x \quad -\frac{1}{4} = x$

The solution set is $\left\{-\frac{1}{4}, \frac{1}{5}\right\}$.

48. $x^{-2} - x^{-1} - 6 = 0$ Let t = x^{-1}.

$t^2 - t - 6 = 0$

$(t-3)(t+2) = 0$

$t - 3 = 0 \quad t + 2 = 0$

$t = 3 \quad t = -2$

$x^{-1} = 3 \quad x^{-1} = -2$

$\frac{1}{x} = 3 \quad \frac{1}{x} = -2$

$1 = 3x \quad 1 = -2x$

$\frac{1}{3} = x \quad -\frac{1}{2} = x$

The solution set is $\left\{-\frac{1}{2}, \frac{1}{3}\right\}$.

49. $x^{2/3} - x^{1/3} - 6 = 0$ let $t = x^{1/3}$

$t^2 - t - 6 = 0$

$(t-3)(t+2) = 0$

$t - 3 = 0 \quad t + 2 = 0$

$t = 3 \quad t = -2$

$x^{1/3} = 3 \quad x^{1/3} = -2$

$x = 3^3 \quad x = (-2)^3$

$x = 27 \quad x = -8$

The solution set is {27, –8}.

50. $2x^{2/3} + 7x^{1/3} - 15 = 0$ let $t = x^{1/3}$

$2t^2 + 7t - 15 = 0$

$(2t-3)(t+5) = 0$

$2t - 3 = 0 \quad t + 5 = 0$

$2t = 3 \quad t = -5$

$t = \frac{3}{2} \quad x^{1/3} = -5$

$x^{1/3} = \frac{3}{2} \quad x = (-5)^2$

$x = \left(\frac{3}{2}\right)^3 \quad x = -125$

$x = \frac{27}{8}$

The solution set is $\left\{-125, \frac{27}{8}\right\}$.

51. $x^{3/2} - 2x^{3/4} + 1 = 0$ let $t = x^{3/4}$

$t^2 - 2t + 1 = 0$

$(t-1)(t-1) = 0$

$t - 1 = 0$

$t = 1$

$x^{3/4} = 1$

$x = 1^{4/3}$

$x = 1$

The solution set is {1}.

52. $x^{2/5} + x^{1/5} - 6 = 0$ let $t = x^{1/5}$

$t^2 + t - 6 = 0$

$(t+3)(t-2) = 0$

$t + 3 = 0 \quad t - 2 = 0$

$t = -3 \quad t = 2$

$x^{1/5} = -3 \quad x^{1/5} = 2$

$x = (-3)^5 \quad x = 2^5$

$x = -243 \quad x = 32$

The solution set is {–243, 32}.

53. $2x-3x^{1/2}+1=0$ let $t=x^{1/2}$

$2t^2-3t+1=0$

$(2t-1)(t-1)=0$

$2t-1=0 \qquad t-1=0$

$2t=1$

$t=\frac{1}{2} \qquad t=1$

$x^{1/2}=\frac{1}{2} \qquad x^{1/2}=1$

$x=\left(\frac{1}{2}\right)^2 \qquad x=1^2$

$x=\frac{1}{4} \qquad x=1$

The solution set is $\left\{\frac{1}{4},1\right\}$.

54. $x+3x^{1/2}-4=0$ let $t=x^{1/2}$

$t^2+3t-4=0$

$(t-1)(t+4)=0$

$t-1=0 \qquad t+4=0$

$t=1 \qquad t=-4$

$x^{1/2}=1 \qquad x^{1/2}=-4$

$x=1^2 \qquad x=(-4)^2$

$x=1 \qquad x=16$

The solution set is {1}.

55. $(x-5)^2-4(x-5)-21=0$ let $t=x-5$

$t^2-4t-21=0$

$(t+3)(t-7)=0$

$t+3=0 \qquad t-7=0$

$t=-3 \qquad t=7$

$x-5=-3 \qquad x-5=7$

$x=2 \qquad x=12$

The solution set is {2, 12}.

56. $(x+3)^2+7(x+3)-18=0$ let $t=x+3$

$t^2+7t-18=0$

$(t+9)(t-2)=0$

$t+9=0 \qquad t-2=0$

$t=-9 \qquad t=2$

$x+3=-9 \qquad x+3=2$

$x=-12 \qquad x=-1$

The solution set is {–12, –1}.

57. $\left(x^2-x\right)^2-14\left(x^2-x\right)+24=0$

Let $t=x^2-x$.

$t^2-14t+24=0$

$(t-2)(t-12)=0$

$t=2$ or $t=12$

$x^2-x=2 \qquad$ or $\qquad x^2-x=12$

$x^2-x-2=0 \qquad x^2-x-12=0$

$(x-2)(x+1)=0 \qquad (x-4)(x+3)=0$

The solution set is {–3, –1, 2, 4}.

58. $\left(x^2-2x\right)^2-11\left(x^2-2x\right)+24=0$

Let $t=x^2-2x$

$t^2-11t+24=0$

$(t-3)(t-8)=0$

$t=3$ or $t=8$

$x^2-2x=3 \qquad$ or $\qquad x^2-2x=8$

$x^2-2x-3=0 \qquad x^2-2x-8=0$

$(x-3)(x+1)=0 \qquad (x-4)(x+2)=0$

The solution set is {–2, –1, 3, 4}.

59. $\left(y-\frac{8}{y}\right)^2+5\left(y-\frac{8}{y}\right)-14=0$

Let $t=y-\frac{8}{y}$.

$t^2+5t-14=0$

$(t+7)(t-2)=0$

$t=-7$ or $t=2$

$y-\frac{8}{y}=-7 \qquad$ or $\qquad y-\frac{8}{y}=2$

$y^2+7y-8=0 \qquad y^2-2y-8=0$

$(y+8)(y-1)=0 \qquad (y-4)(y+2)=0$

The solution set is {–8, –2, 1, 4}.

60. $\left(y-\frac{10}{y}\right)^2+6\left(y-\frac{10}{y}\right)-27=0$

Let $t=y-\frac{10}{y}$.

$t^2+6t-27=0$

$(t+9)(t-3)=0$

$t=-9$ or $t=3$

$y-\frac{10}{y}=-9$ or $y-\frac{10}{y}=3$

$y^2+9y-10=0$ $\quad$ $y^2-3y-10=0$

$(y+10)(y-1)=0$ $\quad$ $(y-5)(y+2)=0$

The solution set is {–10, –2, 1, 5}.

61. $|x|=8$

$x=8, x=-8$

The solution set is {8, –8}.

62. $|x|=6$

$x=6, x=-6$

The solution set is {–6, 6}.

63. $|x-2|=7$

$x-2=7$ $\quad$ $x-2=-7$

$x=9$ $\quad$ $x=-5$

The solution set is {9, –5}.

64. $|x+1|=5$

$x+1=5$ $\quad$ $x+1=-5$

$x=4$ $\quad$ $x=-6$

The solution set is {–6, 4}.

65. $|2x-1|=5$

$2x-1=5$ $\quad$ $2x-1=-5$

$2x=6$ $\quad$ $2x=-4$

$x=3$ $\quad$ $x=-2$

The solution set is {3, –2}.

66. $|2x-3|=11$

$2x-3=11$ $\quad$ $2x-3=-11$

$2x=14$ $\quad$ $2x=-8$

$x=7$ $\quad$ $x=-4$

The solution set is {–4, 7}.

67. $2|3x-2|=14$

$|3x-2|=7$

$3x-2=7$ $\quad$ $3x-2=-7$

$3x=9$ $\quad$ $3x=-5$

$x=3$ $\quad$ $x=-5/3$

The solution set is {3, –5/3}.

68. $3|2x-1|=21$

$|2x-1|=7$

$2x-1=7$ or $2x-1=-7$

$2x=8$ $\quad$ $2x=-6$

$x=4$ $\quad$ $x=-3$

The solution set is {4, –3}.

69. $7|5x|+2=16$

$7|5x|=14$

$|5x|=2$

$5x=2$ $\quad$ $5x=-2$

$x=2/5$ $\quad$ $x=-2/5$

The solution set is $\left\{\frac{2}{5},-\frac{2}{5}\right\}$.

70. $7|3x|+2=16$

$7|3x|=14$

$|3x|=2$

$3x=2$ or $3x=-2$

$x=2/3$ $\quad$ $x=-2/3$

The solution set is {–2/3, 2/3}.

71. $2\left|4-\frac{5}{2}x\right|+6=18$

$2\left|4-\frac{5}{2}x\right|=12$

$\left|4-\frac{5}{2}x\right|=6$

$4-\frac{5}{2}x=6$ or $4-\frac{5}{2}x=-6$

$-\frac{5}{2}x=2$ $\quad$ $-\frac{5}{2}x=-10$

$-\frac{2}{5}\left(-\frac{5}{2}\right)x=-\frac{2}{5}(2)$ $\quad$ $-\frac{2}{5}\left(-\frac{5}{2}\right)x=-\frac{2}{5}(-10)$

$x=-\frac{4}{5}$ $\quad$ $x=4$

The solution set is $\left\{-\frac{4}{5},4\right\}$.

72. $4\left|1-\frac{3}{4}x\right|+7=10$

$4\left|1-\frac{3}{4}x\right|=3$

$\left|1-\frac{3}{4}x\right|=\frac{3}{4}$

$1-\frac{3}{4}x=\frac{3}{4}$ or $1-\frac{3}{4}x=-\frac{3}{4}$

$-\frac{3}{4}x=-\frac{1}{4}$ $\qquad$ $-\frac{3}{4}x=-\frac{7}{4}$

$-\frac{4}{3}\left(-\frac{3}{4}\right)x=-\frac{4}{3}\left(-\frac{1}{4}\right)$ $\qquad$ $-\frac{4}{3}\left(-\frac{3}{4}\right)x=-\frac{4}{3}\left(-\frac{7}{4}\right)$

$x=\frac{1}{3}$ $\qquad$ $x=\frac{7}{3}$

The solution set is $\left\{\frac{1}{3},\frac{7}{3}\right\}$.

73. $|x+1|+5=3$

$|x+1|=-2$

No solution

The solution set is { }.

74. $|x+1|+6=2$

$|x+1|=-4$

No solution

The solution set is { }.

75. $|2x-1|+3=3$

$|2x-1|=0$

$2x-1=0$

$2x=1$

$x=\frac{1}{2}$

The solution set is $\left\{\frac{1}{2}\right\}$.

76. $|3x-2|+4=4$

$|3x-2|=0$

$3x-2=0$

$3x=2$

$x=\frac{2}{3}$

The solution set is $\left\{\frac{2}{3}\right\}$.

77. $|3x-1|=|x+5|$

$3x-1=x+5$ $\qquad$ $3x-1=-x-5$

$2x-1=5$ $\qquad$ $4x-1=-5$

$2x=6$ $\qquad$ $4x=-4$

$x=3$ $\qquad$ $x=-1$

The solution set is {3, −1}.

78. $|2x-7|=|x+3|$

$2x-7=x+3$ or $2x-7=-(x+3)$

$x=10$ $\qquad$ $2x-7=-x-3$

$3x=4$

$x=\frac{4}{3}$

The solution set is $\left\{10,\frac{4}{3}\right\}$.

79. Set $y = 0$ to find the x–intercept(s).

$0=\sqrt{x+2}+\sqrt{x-1}-3$

$-\sqrt{x+2}=\sqrt{x-1}-3$

$\left(-\sqrt{x+2}\right)^2=\left(\sqrt{x-1}-3\right)^2$

$x+2=\left(\sqrt{x-1}\right)^2-2\left(\sqrt{x-1}\right)(3)+(3)^2$

$x+2=x-1-6\sqrt{x-1}+9$

$x+2=x-1-6\sqrt{x-1}+9$

$2=8-6\sqrt{x-1}$

$-6=-6\sqrt{x-1}$

$\frac{-6}{-6}=\frac{-6\sqrt{x-1}}{-6}$

$1=\sqrt{x-1}$

$(1)^2=\left(\sqrt{x-1}\right)^2$

$1=x-1$

$2=x$

The x-intercept is 2.

The corresponding graph is graph (c).

80. Set $y = 0$ to find the x–intercept(s).

$$0 = \sqrt{x-4} + \sqrt{x+4} - 4$$
$$-\sqrt{x-4} = \sqrt{x+4} - 4$$
$$\left(-\sqrt{x-4}\right)^2 = \left(\sqrt{x+4} - 4\right)^2$$
$$x - 4 = \left(\sqrt{x+4}\right)^2 - 2\left(\sqrt{x+4}\right)(4) + (4)^2$$
$$x - 4 = x + 4 - 8\sqrt{x+4} + 16$$
$$-4 = 20 - 8\sqrt{x+4}$$
$$-24 = -8\sqrt{x+4}$$
$$\frac{-24}{-8} = \frac{-8\sqrt{x+4}}{-8}$$
$$3 = \sqrt{x+4}$$
$$(3)^2 = \left(\sqrt{x+4}\right)^2$$
$$9 = x + 4$$
$$5 = x$$

The x-intercept is 5.
The corresponding graph is graph (a).

81. Set $y = 0$ to find the x–intercept(s).

$$0 = x^{\frac{1}{3}} + 2x^{\frac{1}{6}} - 3$$

Let $t = x^{\frac{1}{6}}$.

$$x^{\frac{1}{3}} + 2x^{\frac{1}{6}} - 3 = 0$$
$$\left(x^{\frac{1}{6}}\right)^2 + 2x^{\frac{1}{6}} - 3 = 0$$
$$t^2 + 2t - 3 = 0$$
$$(t+3)(t-1) = 0$$
$$t + 3 = 0 \quad \text{or} \quad t - 1 = 0$$
$$t = -3 \qquad\qquad t = 1$$

Substitute $x^{\frac{1}{6}}$ for t.

$$x^{\frac{1}{6}} = -3 \quad \text{or} \quad x^{\frac{1}{6}} = 1$$
$$\left(x^{\frac{1}{6}}\right)^6 = (-3)^6 \qquad \left(x^{\frac{1}{6}}\right)^6 = (1)^6$$
$$x = 729 \qquad\qquad x = 1$$

729 does not check and must be rejected.
The x-intercept is 1.
The corresponding graph is graph (e).

82. Set $y = 0$ to find the x–intercept(s).

$$0 = x^{-2} - x^{-1} - 6$$

Let $t = x^{-1}$.

$$x^{-2} - x^{-1} - 6 = 0$$
$$\left(x^{-1}\right)^2 - x^{-1} - 6 = 0$$
$$t^2 - t - 6 = 0$$
$$(t+2)(t-3) = 0$$
$$t + 2 = 0 \quad \text{or} \quad t - 3 = 0$$
$$t = -2 \qquad\qquad t = 3$$

Substitute x^{-1} for t.

$$x^{-1} = -2 \quad \text{or} \quad x^{-1} = 3$$
$$x = -\frac{1}{2} \qquad\qquad x = \frac{1}{3}$$

The x-intercepts are $-\frac{1}{2}$ and $\frac{1}{3}$.
The corresponding graph is graph (b).

83. Set $y = 0$ to find the x–intercept(s).

$$(x+2)^2 - 9(x+2) + 20 = 0$$

Let $t = x + 2$.

$$(x+2)^2 - 9(x+2) + 20 = 0$$
$$t^2 - 9t + 20 = 0$$
$$(t-5)(t-4) = 0$$
$$t - 5 = 0 \quad \text{or} \quad t - 4 = 0$$
$$t = 5 \qquad\qquad t = 4$$

Substitute $x + 2$ for t.

$$x + 2 = 5 \quad \text{or} \quad x + 2 = 4$$
$$x = 3 \qquad\qquad x = 2$$

The x-intercepts are 2 and 3.
The corresponding graph is graph (f).

84. Set $y = 0$ to find the x–intercept(s).

$0 = 2(x+2)^2 + 5(x+2) - 3$

Let $t = x+2$.

$$2(x+2)^2 + 5(x+2) - 3 = 0$$
$$2t^2 + 5t - 3 = 0$$
$$(2t-1)(t+3) = 0$$

$2t - 1 = 0$ or $t + 3 = 0$

$2t = 1$ $\quad t = -3$

$t = \frac{1}{2}$

Substitute $x+2$ for t.

$x + 2 = \frac{1}{2}$ or $x + 2 = -3$

$x = \frac{1}{2} - 2$ $\quad x = -5$

$x = -\frac{3}{2}$

The x-intercepts are -5 and $-\frac{3}{2}$.

The corresponding graph is graph (d).

85. $|5 - 4x| = 11$

$5 - 4x = 11$ $\quad 5 - 4x = -11$

$-4x = 6$ or $-4x = -16$

$x = -\frac{3}{2}$ $\quad x = 4$

The solution set is $\left\{-\frac{3}{2}, 4\right\}$.

86. $|2 - 3x| = 13$

$2 - 3x = 13$ $\quad 2 - 3x = -13$

$-3x = 11$ or $-3x = -15$

$x = -\frac{11}{3}$ $\quad x = 5$

The solution set is $\left\{-\frac{11}{3}, 5\right\}$.

87. $x + \sqrt{x+5} = 7$

$$\sqrt{x+5} = 7 - x$$
$$\left(\sqrt{x+5}\right)^2 = (7-x)^2$$
$$x + 5 = 49 - 14x + x^2$$
$$0 = x^2 - 15x + 44$$
$$0 = (x-4)(x-11)$$

$x - 4 = 0$ or $x - 11 = 0$

$x = 4$ $\quad x = 11$

11 does not check and must be rejected.

The solution set is $\{4\}$.

88. $x - \sqrt{x-2} = 4$

$$-\sqrt{x-2} = 4 - x$$
$$\left(-\sqrt{x-2}\right)^2 = (4-x)^2$$
$$x - 2 = 16 - 8x + x^2$$
$$0 = x^2 - 9x + 18$$
$$0 = (x-6)(x-3)$$

$x - 6 = 0$ or $x - 3 = 0$

$x = 6$ $\quad x = 3$

3 does not check and must be rejected.

The solution set is $\{6\}$.

89.

$$2x^3 + x^2 - 8x + 2 = 6$$
$$2x^3 + x^2 - 8x - 4 = 0$$
$$x^2(2x+1) - 4(2x+1) = 0$$
$$(2x+1)(x^2 - 4) = 0$$
$$(2x+1)(x+2)(x-2) = 0$$

$2x + 1 = 0$ or $x + 2 = 0$ or $x - 2 = 0$

$x = -\frac{1}{2}$ $\quad x = -2$ $\quad x = 2$

The solution set is $\left\{-\frac{1}{2}, -2, 2\right\}$.

90.

$$x^3 + 4x^2 - x + 6 = 10$$
$$x^3 + 4x^2 - x - 4 = 0$$
$$x^2(x+4) - 1(x+4) = 0$$
$$(x+4)(x^2 - 1) = 0$$
$$(x+4)(x+1)(x-1) = 0$$

$x + 4 = 0$ or $x + 1 = 0$ or $x - 1 = 0$

$x = -4$ $\quad x = -1$ $\quad x = 1$

The solution set is $\{-4, -1, 1\}$.

91. $(x+4)^{\frac{3}{2}} = 8$

$$\left((x+4)^{\frac{3}{2}}\right)^{\frac{2}{3}} = (8)^{\frac{2}{3}}$$

$$x+4 = \left(\sqrt[3]{8}\right)^2$$

$$x+4 = (2)^2$$

$$x+4 = 4$$

$$x = 0$$

The solution set is $\{0\}$.

92. $(x-5)^{\frac{3}{2}} = 125$

$$\left((x-5)^{\frac{3}{2}}\right)^{\frac{2}{3}} = (125)^{\frac{2}{3}}$$

$$x-5 = \left(\sqrt[3]{125}\right)^2$$

$$x-5 = (5)^2$$

$$x-5 = 25$$

$$x = 30$$

The solution set is $\{30\}$.

93. $y_1 = y_2 + 3$

$$\left(x^2-1\right)^2 = 2\left(x^2-1\right)+3$$

$$\left(x^2-1\right)^2 - 2\left(x^2-1\right) - 3 = 0$$

Let $t = x^2 - 1$ and substitute.

$$t^2 - 2t - 3 = 0$$

$$(t+1)(t-3) = 0$$

$t+1 = 0$ or $t-3 = 0$

$t = -1$ $\quad$ $t = 3$

Substitute $x^2 - 1$ for t.

$x^2 - 1 = -1$ or $x^2 - 1 = 3$

$x^2 = 0$ $\quad$ $x^2 = 4$

$x = 0$ $\quad$ $x = \pm 2$

The solution set is $\{-2, 0, 2\}$.

94. $y_1 = y_2 + 6$

$$6\left(\frac{2x}{x-3}\right)^2 = 5\left(\frac{2x}{x-3}\right)+6$$

$$6\left(\frac{2x}{x-3}\right)^2 - 5\left(\frac{2x}{x-3}\right) - 6 = 0$$

Let $t = \frac{2x}{x-3}$ and substitute.

$$6t^2 - 5t - 6 = 0$$

$$(3t+2)(2t-3) = 0$$

$3t+2 = 0$ or $2t-3 = 0$

$t = -\frac{2}{3}$ $\quad$ $t = \frac{3}{2}$

Substitute $\frac{2x}{x-3}$ for t.

$\frac{2x}{x-3} = -\frac{2}{3}$ or $\frac{2x}{x-3} = \frac{3}{2}$

First solve $\frac{2x}{x-3} = -\frac{2}{3}$

$$\frac{2x(3)(x-3)}{x-3} = -\frac{2(3)(x-3)}{3}$$

$$2x(3) = -2(x-3)$$

$$6x = -2x+6$$

$$8x = 6$$

$$x = \frac{3}{4}$$

Next solve $\frac{2x}{x-3} = \frac{3}{2}$

$$\frac{2x(2)(x-3)}{x-3} = \frac{3(2)(x-3)}{2}$$

$$2x(2) = 3(x-3)$$

$$4x = 3x - 9$$

$$x = -9$$

The solution set is $\left\{-9, \frac{3}{4}\right\}$.

95. $\left|x^2 + 2x - 36\right| = 12$

$x^2 + 2x - 36 = 12$ $\quad$ $x^2 + 2x - 36 = -12$

$x^2 + 2x - 48 = 0$ or $x^2 + 2x - 24 = 0$

$(x+8)(x-6) = 0$ $\quad$ $(x+6)(x-4) = 0$

Setting each of the factors above equal to zero gives $x = -8$, $x = 6$, $x = -6$, and $x = 4$.

The solution set is $\{-8, -6, 4, 6\}$.

96. $\left|x^2+6x+1\right|=8$

$x^2+6x+1=8$ or $x^2+6x+1=-8$

$x^2+6x-7=0$ $\quad$ $x^2+6x+9=0$

$(x+7)(x-1)=0$ $\quad$ $(x+3)(x+3)=0$

Setting each of the factors above equal to zero gives $x=-7$, $x=-3$, and $x=1$.

The solution set is $\{-7,-3,1\}$.

97. $x(x+1)^3-42(x+1)^2=0$

$(x+1)^2\left(x(x+1)-42\right)=0$

$(x+1)^2\left(x^2+x-42\right)=0$

$(x+1)^2(x+7)(x-6)=0$

Setting each of the factors above equal to zero gives $x=-7$, $x=-1$, and $x=6$.

The solution set is $\{-7,-1,6\}$.

98. $x(x-2)^3-35(x-2)^2=0$

$x(x-2)^3-35(x-2)^2=0$

$(x-2)^2\left(x(x-2)-35\right)=0$

$(x-2)^2\left(x^2-2x-35\right)=0$

$(x-2)^2(x+5)(x-7)=0$

Setting each of the factors above equal to zero gives $x=-5$, $x=2$, and $x=7$.

The solution set is $\{-5,2,7\}$.

99. Let x = the number.

$\sqrt{5x-4}=x-2$

$\left(\sqrt{5x-4}\right)^2=(x-2)^2$

$5x-4=x^2-4x+4$

$0=x^2-9x+8$

$0=(x-8)(x-1)$

$x-8=0$ or $x-1=0$

$x=8$ $\quad$ $x=1$

Check $x=8$: $\sqrt{5(8)-4}=8-2$

$\sqrt{40-4}=6$

$\sqrt{36}=6$

$6=6$

Check $x=1$: $\sqrt{5(1)-4}=1-2$

$\sqrt{5-4}=-1$

$\sqrt{-1}\neq-1$

Discard $x=1$. The number is 8.

100. Let x = the number.

$\sqrt{x-3}=x-5$

$\left(\sqrt{x-3}\right)^2=(x-5)^2$

$x-3=x^2-10x+25$

$0=x^2-11x+28$

$0=(x-7)(x-4)$

$x-7=0$ or $x-4=0$

$x=7$ $\quad$ $x=4$

Check $x=7$: $\sqrt{7-3}=7-5$

$\sqrt{4}=2$

$2=2$

Check $x=4$: $\sqrt{4-3}=4-5$

$\sqrt{1}=-1$

$1\neq-1$

Discard 4. The number is 7.

101.

$$r=\sqrt{\frac{3V}{\pi h}}$$

$$r^2=\left(\sqrt{\frac{3V}{\pi h}}\right)^2$$

$$r^2=\frac{3V}{\pi h}$$

$$\pi r^2h=3V$$

$$\frac{\pi r^2h}{3}=V$$

$$V=\frac{\pi r^2h}{3} \quad \text{or} \quad V=\frac{1}{3}\pi r^2h$$

102.

$$r=\sqrt{\frac{A}{4\pi}}$$

$$r^2=\left(\sqrt{\frac{A}{4\pi}}\right)^2$$

$$r^2=\frac{A}{4\pi}$$

$$4\pi r^2=A \quad \text{or} \quad A=4\pi r^2$$

103. Exclude any value that causes the denominator to equal zero.

$|x+2|-14=0$

$|x+2|=14$

$x+2=14$ or $x+2=-14$

$x=12$ $\quad x=-16$

–16 and 12 must be excluded from the domain.

104. Exclude any value that causes the denominator to equal zero.

$x^3+3x^2-x-3=0$

$x^2(x+3)-1(x+3)=0$

$(x+3)\left(x^2-1\right)=0$

$(x+3)(x+1)(x-1)=0$

Setting each of the factors above equal to zero gives $x=-3,\quad x=-1,\quad$ and $x=1$.

–3, –1, and 1 must be excluded from the domain.

105.

$t=\frac{\sqrt{d}}{2}$

$1.16=\frac{\sqrt{d}}{2}$

$2.32=\sqrt{d}$

$2.32^2=\left(\sqrt{d}\right)^2$

$d\approx 5.4$

The vertical distance was about 5.4 feet.

106.

$t=\frac{\sqrt{d}}{2}$

$0.85=\frac{\sqrt{d}}{2}$

$1.7=\sqrt{d}$

$1.7^2=\left(\sqrt{d}\right)^2$

$d\approx 2.9$

The vertical distance was about 2.9 feet.

107. It is represented by the point $(5.4, 1.16)$.

108. It is represented by the point $(2.9, 0.85)$.

109. a. According to the line graph, about 47% $\pm 1\%$ of U.S. women participated in the labor force in 2010.

b. $p=1.6\sqrt{t}+38$

$p=1.6\sqrt{40}+38\approx 48.1$

According to the formula, about 48.1% of U.S. women participated in the labor force in 2010.

c. $p=1.6\sqrt{t}+38$

$51=1.6\sqrt{t}+38$

$13=1.6\sqrt{t}$

$\frac{13}{1.6}=\frac{1.6\sqrt{t}}{1.6}$

$\frac{13}{1.6}=\sqrt{t}$

$\left(\frac{13}{1.6}\right)^2=\left(\sqrt{t}\right)^2$

$66\approx t$

According to the formula, 51% of U.S. women will participate in the labor force 66 years after 1970, or 2036.

110. a. According to the line graph, about 53% $\pm 1\%$ of U.S. men participated in the labor force in 2010.

b. $p=-1.6\sqrt{t}+62$

$p=-1.6\sqrt{40}+62\approx 51.9$

According to the formula, about 51.9% of U.S. men participated in the labor force in 2010.

c. $p=-1.6\sqrt{t}+62$

$49=-1.6\sqrt{t}+62$

$-13=-1.6\sqrt{t}$

$\frac{-13}{-1.6}=\frac{-1.6\sqrt{t}}{-1.6}$

$\frac{-13}{-1.6}=\sqrt{t}$

$\left(\frac{-13}{-1.6}\right)^2=\left(\sqrt{t}\right)^2$

$66\approx t$

According to the formula, 49% of U.S. men will participate in the labor force 66 years after 1970, or 2036.

111.

$$365 = 0.2x^{3/2}$$
$$\frac{365}{0.2} = \frac{0.2x^{3/2}}{0.2}$$
$$1825 = x^{3/2}$$
$$1825^2 = \left(x^{3/2}\right)^2$$
$$3,330,625 = x^3$$
$$\sqrt[3]{3,330,625} = \sqrt[3]{x^3}$$
$$149.34 \approx x$$

The average distance of the Earth from the sun is approximately 149 million kilometers.

112.

$$f(x) = 0.2x^{3/2}$$
$$88 = 0.2x^{3/2}$$
$$\frac{88}{0.2} = \frac{0.2x^{3/2}}{0.2}$$
$$440 = x^{3/2}$$
$$440^2 = \left(x^{3/2}\right)^2$$
$$193,600 = x^3$$
$$\sqrt[3]{193,600} = \sqrt[3]{x^3}$$
$$58 \approx x$$

The average distance of Mercury from the sun is approximately 58 million kilometers.

113. $\sqrt{6^2 + x^2} + \sqrt{8^2 + (10-x)^2} = 18$

$$\sqrt{36 + x^2} = 18 - \sqrt{64 + 100 - 20x + x^2}$$
$$36 + x^2 = 324 - 36\sqrt{x^2 - 20x + 164} + x^2 - 20x + 164$$
$$36\sqrt{x^2 - 20x + 164} = -20x + 452$$
$$9\sqrt{x^2 - 20x + 164} = -5x + 113$$
$$81(x^2 - 20x + 164) = 25x^2 - 1130x + 12769$$
$$81x^2 - 1620x + 13284 = 25x^2 - 1130x + 12769$$
$$56x^2 - 490x + 515 = 0$$
$$x = \frac{490 \pm \sqrt{(-490)^2 - 4(56)(515)}}{2(56)}$$
$$x = \frac{490 \pm 353.19}{112}$$
$$x \approx 1.2 \qquad x \approx 7.5$$

The point should be located approximately either 1.2 feet or 7.5 feet from the base of the 6-foot pole.

114. a. Distance from point $A = \sqrt{6^2 + x^2} + \sqrt{3^2 + (12-x)^2}$ or $A = \sqrt{x^2 + 36} + \sqrt{(12-x)^2 + 9}$.

b. Let the distance = 15.

$$\sqrt{6^2 + x^2} + \sqrt{3^2 + (12-x)^2} = 15$$
$$\sqrt{36 + x^2} = 15 - \sqrt{9 + 144 - 24x + x^2}$$
$$36 + x^2 = 225 - 30\sqrt{153 - 24x + x^2} + x^2 - 24x + 153$$
$$30\sqrt{x^2 - 24x + 153} = -24x + 342$$
$$5\sqrt{x^2 - 24x + 153} = -4x + 157$$
$$25(x^2 - 24x + 153) = 16x^2 - 456x + 3249$$
$$25x^2 - 600x + 3825 = 16x^2 - 456x + 3249$$
$$9x^2 - 144x + 576 = 0$$
$$x^2 - 16x + 64 = 0$$
$$(x-8)(x-8) = 0$$
$$x = 8$$

The distance is 8 miles.

115. – 121. Answers will vary.

122. $x^3 + 3x^2 - x - 3 = 0$

The solution set is {–3, –1, 1}.

$$(-3)^3 + 3(-3)^2 - (-3) - 3 = 0$$
$$-27 + 27 + 3 - 3 = 0$$
$$(-1)^3 + 3(-1)^2 - (-1) - 3 = 0$$
$$-1 + 3 + 1 - 3 = 0$$
$$1^3 + 3(1)^2 - (1) - 3 = 0$$
$$1 + 3 - 1 - 3 = 0$$

123. $-x^4 + 4x^3 - 4x^2 = 0$

The solution set is {0, 2}.

$$-(0)^4 + 4(0)^3 - 4(0)^2 = 0$$
$$0 = 0$$
$$-(2)^4 + 4(2)^3 - 4(2)^2 = 0$$
$$-16 + 32 - 16 = 0$$
$$0 = 0$$

124. $\sqrt{2x + 13} - x - 5 = 0$

The solution set is {–2}.

$$\sqrt{2(-2) + 13} - (-2) - 5 = 0$$
$$\sqrt{-4 + 13} + 2 - 5 = 0$$
$$\sqrt{9} - 3 = 0$$
$$3 - 3 = 0$$

125. does not make sense; Explanations will vary. Sample explanation: You should substitute into the original equation.

126. makes sense

127. does not make sense; Explanations will vary. Sample explanation: Changing the order of the terms does not change the fact that this equation is quadratic in form.

128. makes sense

129. false; Changes to make the statement true will vary. A sample change is: Squaring $x+2$ results in $x^2+4x+4.$

130. false; Changes to make the statement true will vary. A sample change is: 21 satisfies the linear equation but not the radical equation.

131. false; Changes to make the statement true will vary. A sample change is: To solve the equation, let $u^2 = x.$

132. false; Changes to make the statement true will vary. A sample change is: Neither 6 nor –6 satisfies the absolute value equation.

133. $\sqrt{6x-2} = \sqrt{2x+3} - \sqrt{4x-1}$

$$6x-2 = 2x+3-2\sqrt{(2x+3)(4x-1)}+4x-1$$
$$-4 = -2\sqrt{(2x+3)(4x-1)}$$
$$2 = \sqrt{8x^2+10x-3}$$
$$4 = 8x^2+10x-3$$
$$8x^2+10x-7=0$$
$$x = \frac{-10\pm\sqrt{10^2-4(8)(-7)}}{2(8)}$$
$$x = \frac{-10\pm\sqrt{100+224}}{16}$$
$$x = \frac{-10\pm\sqrt{324}}{16}$$
$$x = \frac{-10\pm 18}{16}$$
$$x = \frac{-28}{26}, \frac{8}{16}$$
$$x = \frac{1}{2}$$

The solution set is $\left\{\frac{1}{2}\right\}$.

134. $5-\frac{2}{x} = \sqrt{5-\frac{2}{x}}$

or

$$5-\frac{2}{x}=0 \qquad 5-\frac{2}{x}=1$$
$$5=\frac{2}{x} \qquad -\frac{2}{x}=-4$$
$$5x=2 \qquad -4x=-2$$
$$x=\frac{2}{5} \qquad x=\frac{1}{2}$$

The solution set is $\left\{\frac{2}{5},\frac{1}{2}\right\}$.

135. $\sqrt[3]{x\sqrt{x}} = 9$

$$\sqrt[3]{x\sqrt{x}} = 9$$
$$\sqrt[3]{x^1x^{\frac{1}{2}}} = 9$$
$$\left(x^1x^{\frac{1}{2}}\right)^{\frac{1}{3}} = 9$$
$$\left(x^{\frac{3}{2}}\right)^{\frac{1}{3}} = 9$$
$$x^{\frac{1}{2}} = 9$$
$$\left(x^{\frac{1}{2}}\right)^2 = (9)^2$$
$$x = 81$$

The solution set is {81}.

136. $x^{5/6}+x^{2/3}-2x^{1/2}=0$

$$x^{1/2}(x^{2/6}+x^{1/6}-2)=0 \text{ let } t = x^{1/6}$$
$$x^{1/2}(t^2+t-2)=0$$
$$x^{1/2}=0 \quad t^2+t-2=0$$
$$(t-1)(t+2)=0$$
$$t-1=0 \qquad t+2=0$$
$$t=1 \qquad t=-2$$
$$x^{1/6}=1 \qquad x^{1/6}=-2$$
$$x=1^6 \qquad x=(-2)^6$$
$$x=0 \quad x=1 \qquad x=64$$

64 does not check and must be rejected.
The solution set is {0, 1}.

137. $3-2x \le 11$

$3-2(-1) \le 11$

$3+2 \le 11$

$5 \le 11$, true

Yes, –1 is a solution.

138. $-2x-4 = x+5$

$-2x-x = 5+4$

$-3x = 9$

$x = \frac{9}{-3}$

$x = -3$

The solution set is $\{-3\}$.

139. $\frac{x+3}{4} = \frac{x-2}{3}+\frac{1}{4}$

$12\left(\frac{x+3}{4}\right) = 12\left(\frac{x-2}{3}+\frac{1}{4}\right)$

$3(x+3) = 4(x-2)+3$

$3x+9 = 4x-8+3$

$3x+9 = 4x-5$

$3x-4x = -5-9$

$-x = -14$

$x = 14$

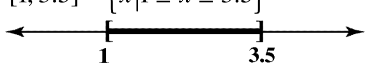
The solution set is $\{14\}$.

Section 1.7

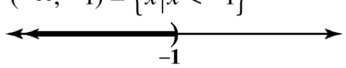
Check Point Exercises

1. **a.** $[-2,5) = \{x \mid -2 \le x < 5\}$

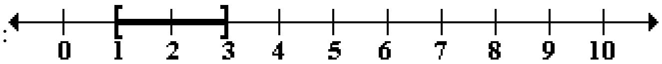
b. $[1,3.5] = \{x \mid 1 \le x \le 3.5\}$



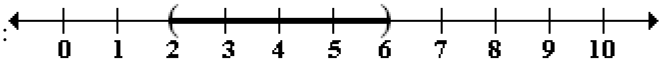
c. $(-\infty,-1) = \{x \mid x < -1\}$



2. a. Graph $[1,3]$:

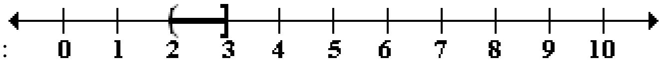


Graph $(2,6)$:



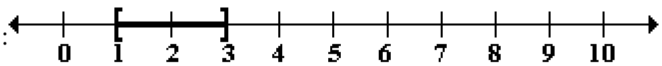
To find the intersection, take the portion of the number line that the two graphs have in common.

Numbers in both $[1,3]$ and $(2,6)$:

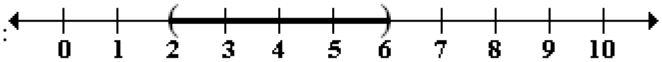


Thus, $[1,3] \cap (2,6) = (2,3]$.

b. Graph $[1,3]$:

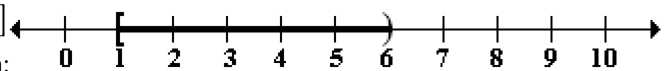


Graph $(2,6)$:



To find the union, take the portion of the number line representing the total collection of numbers in the two graphs.

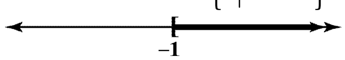
Numbers in either $[1,3]$ or $(2,6)$ or both:



Thus, $[1,3] \cup (2,6) = [1,6)$.

3.
$$2 - 3x \le 5$$
$$-3x \le 3$$
$$x \ge -1$$

The solution set is $\{x | x \ge -1\}$ or $[-1, \infty)$.

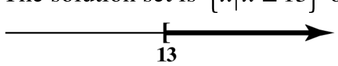


4.
$$3x + 1 > 7x - 15$$
$$-4x > -16$$
$$\frac{-4x}{-4} < \frac{-16}{-4}$$
$$x < 4$$

The solution set is $\{x | x < 4\}$ or $(-\infty, 4)$.

5.
$$\frac{x-4}{2} \ge \frac{x-2}{3} + \frac{5}{6}$$
$$6\left(\frac{x-4}{2}\right) \ge 6\left(\frac{x-2}{3} + \frac{5}{6}\right)$$
$$3(x-4) \ge 2(x-2) + 5$$
$$3x - 12 \ge 2x - 4 + 5$$
$$3x - 12 \ge 2x + 1$$
$$3x - 2x \ge 1 + 12$$
$$x \ge 13$$

The solution set is $\{x | x \ge 13\}$ or $[13, \infty)$.



6. **a.** $3(x+1) > 3x+2$

$3x+3 > 3x+2$

$3 > 2$

$3 > 2$ is true for all values of x.

The solution set is $\{x \mid x \text{ is a real number}\}$ or $\Re$ or $(-\infty, \infty)$.

b. $x+1 \le x-1$

$1 \le -1$

$1 \le -1$ is false for all values of x.

The solution set is $\varnothing$.

7. $1 \le 2x+3 < 11$

$-2 \le 2x < 8$

$-1 \le x < 4$

The solution set is $\{x \mid -1 \le x < 4\}$ or $[-1, 4)$.

8. $|x-2| < 5$

$-5 < x-2 < 5$

$-3 < x < 7$

The solution set is $\{x \mid -3 < x < 7\}$ or $(-3, 7)$.

9. $-3|5x-2| + 20 \ge -19$

$-3|5x-2| \ge -39$

$\frac{-3|5x-2|}{-3} \le \frac{-39}{-3}$

$|5x-2| \le 13$

$-13 \le 5x-2 \le 13$

$-11 \le 5x \le 15$

$\frac{-11}{5} \le \frac{5x}{5} \le \frac{15}{5}$

$-\frac{11}{5} \le x \le 3$

The solution set is $\left\{x \,\middle|\, -\frac{11}{5} \le x \le 3\right\}$ or $\left[-\frac{11}{5}, 3\right]$.

10. $18 < |6-3x|$

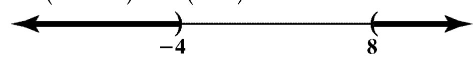
$6-3x < -18$ or $6-3x > 18$

$-3x < -24$ $\quad$ $-3x > 12$

$\frac{-3x}{-3} > \frac{-24}{-3}$ $\quad$ $\frac{-3x}{-3} < \frac{12}{-3}$

$x > 8$ $\quad$ $x < -4$

The solution set is $\{x \mid x < -4 \text{ or } x > 8\}$ or $(-\infty, -4) \cup (8, \infty)$.



11. Let x = the number of miles driven in a week.

$260 < 80 + 0.25x$

$180 < 0.25x$

$720 < x$

Driving more than 720 miles in a week makes Basic the better deal.

Concept and Vocabulary Check 1.7

1. 2; 5; 2; 5

2. greater than

3. less than or equal to

4. $(-\infty, 9)$; intersection

5. $(-\infty, 12)$; union

6. adding 4; dividing; -3; direction; $>$; $<$

7. $\varnothing$

8. $(-\infty, \infty)$

9. middle

10. $-c$; c

11. $-c$; c

12. $-2 < x-7 < 2$

13. $x-7 < -2$ or $-7 > 2$

Exercise Set 1.7

1. $1 < x \le 6$

2. $-2 < x \le 4$

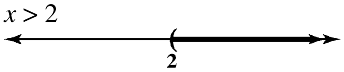
3. $-5 \le x < 2$

4. $-4 \le x < 3$

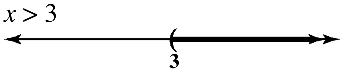
5. $-3 \le x \le 1$

6. $-2 \le x \le 5$

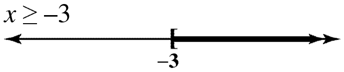
7. $x > 2$



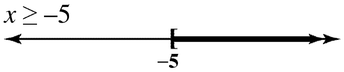
8. $x > 3$



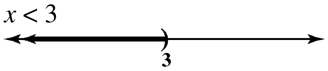
9. $x \ge -3$



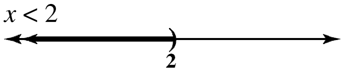
10. $x \ge -5$



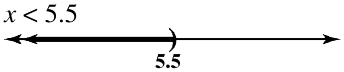
11. $x < 3$



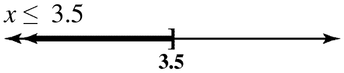
12. $x < 2$



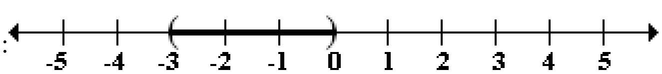
13. $x < 5.5$



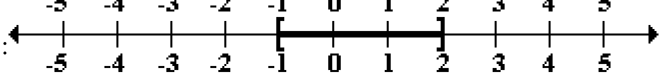
14. $x \le 3.5$



15. Graph $(-3, 0)$:

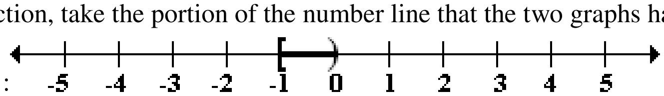


Graph $[-1, 2]$:



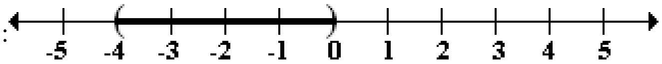
To find the intersection, take the portion of the number line that the two graphs have in common.

Numbers in both $(-3, 0)$ and $[-1, 2]$:

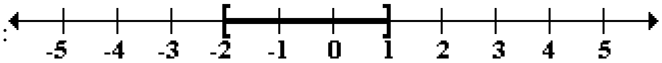


Thus, $(-3, 0) \cap [-1, 2] = [-1, 0)$.

16. Graph $(-4,0)$:

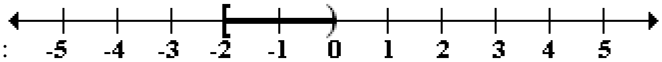


Graph $[-2,1]$:



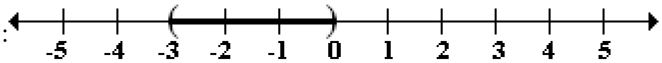
To find the intersection, take the portion of the number line that the two graphs have in common.

Numbers in both $(-4,0)$ and $[-2,1]$:

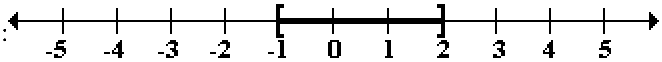


Thus, $(-4,0) \cap [-2,1] = [-2,0)$.

17. Graph $(-3,0)$:

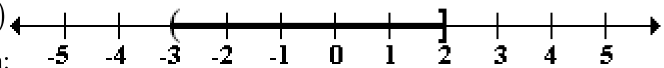


Graph $[-1,2]$:



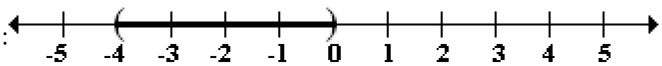
To find the union, take the portion of the number line representing the total collection of numbers in the two graphs.

Numbers in either $(-3,0)$ or $[-1,2]$ or both:

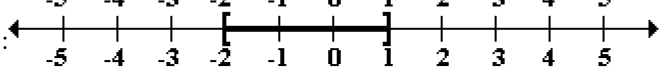


Thus, $(-3,0) \cup [-1,2] = (-3,2]$.

18. Graph $(-4,0)$:

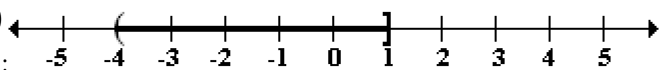


Graph $[-2,1]$:



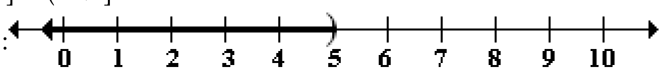
To find the union, take the portion of the number line representing the total collection of numbers in the two graphs.

Numbers in either $(-4,0)$ or $[-2,1]$ or both:

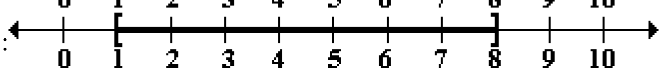


Thus, $(-4,0) \cup [-2,1] = (-4,1]$.

19. Graph $(-\infty,5)$:

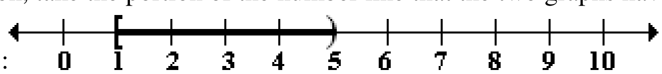


Graph $[1,8]$:



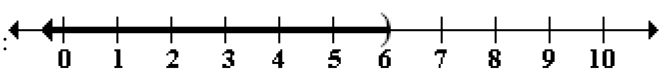
To find the intersection, take the portion of the number line that the two graphs have in common.

Numbers in both $(-\infty,5)$ and $[1,8]$:

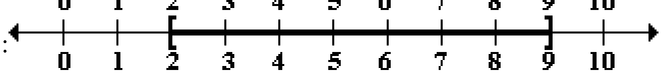


Thus, $(-\infty,5) \cap [1,8] = [1,5)$.

20. Graph $(-\infty,6)$:

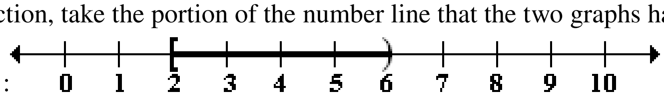


Graph $[2,9]$:



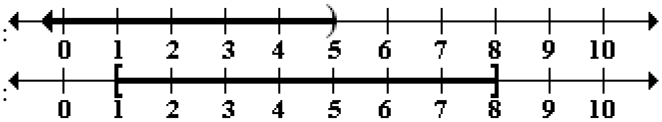
To find the intersection, take the portion of the number line that the two graphs have in common.

Numbers in both $(-\infty,6)$ and $[2,9]$:



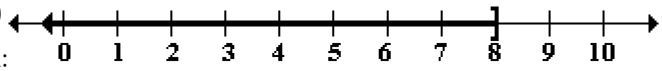
Thus, $(-\infty,6) \cap [2,9] = [2,6)$.

21. Graph $(-\infty,5)$: Graph $[1,8]$:



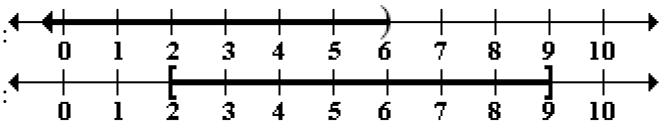
To find the union, take the portion of the number line representing the total collection of numbers in the two graphs.

Numbers in either $(-\infty,5)$ or $[1,8]$ or both:



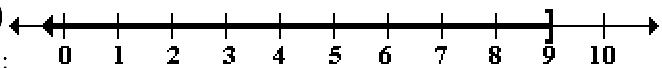
Thus, $(-\infty,5)\cup[1,8]=(-\infty,8]$.

22. Graph $(-\infty,6)$: Graph $[2,9]$:



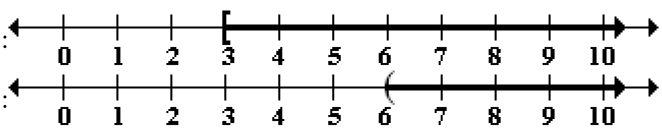
To find the union, take the portion of the number line representing the total collection of numbers in the two graphs.

Numbers in either $(-\infty,6)$ or $[2,9]$ or both:



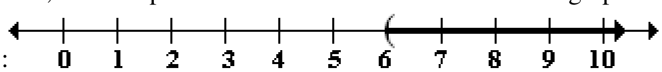
Thus, $(-\infty,6)\cup[2,9]=(-\infty,9]$.

23. Graph $[3,\infty)$: Graph $(6,\infty)$:



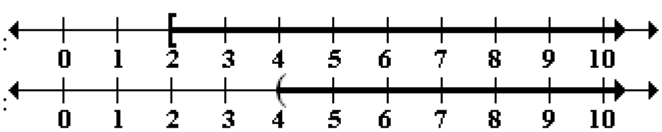
To find the intersection, take the portion of the number line that the two graphs have in common.

Numbers in both $[3,\infty)$ and $(6,\infty)$:



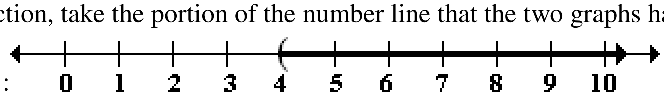
Thus, $[3,\infty)\cap(6,\infty)=(6,\infty)$.

24. Graph $[2,\infty)$: Graph $(4,\infty)$:



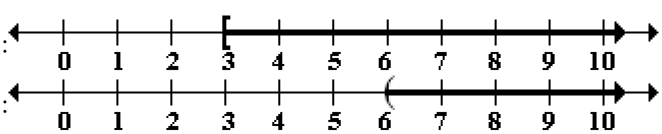
To find the intersection, take the portion of the number line that the two graphs have in common.

Numbers in both $[2,\infty)$ and $(4,\infty)$:



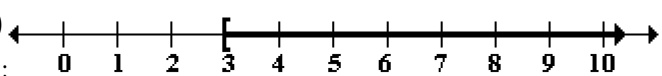
Thus, $[2,\infty)\cap(4,\infty)=(4,\infty)$.

25. Graph $[3,\infty)$: Graph $(6,\infty)$:



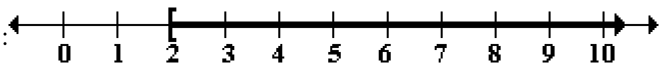
To find the union, take the portion of the number line representing the total collection of numbers in the two graphs.

Numbers in either $[3,\infty)$ or $(6,\infty)$ or both:

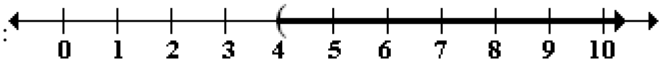


Thus, $[3,\infty)\cup(6,\infty)=[3,\infty)$.

26. Graph $[2,\infty)$:

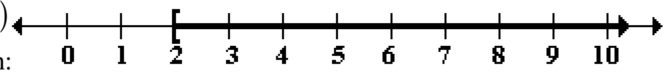


Graph $(4,\infty)$:



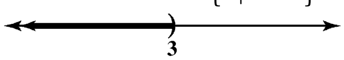
To find the union, take the portion of the number line representing the total collection of numbers in the two graphs.

Numbers in either $[2,\infty)$ or $(4,\infty)$ or both:

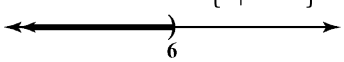


Thus, $[2,\infty) \cup (4,\infty) = [2,\infty)$.

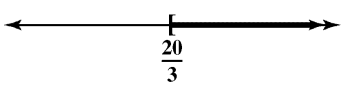
27. $5x + 11 < 26$
$5x < 15$
$x < 3$
The solution set is $\{x \mid x < 3\}$, or $(-\infty, 3)$.



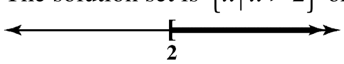
28. $2x + 5 < 17$
$2x < 12$
$x < 6$
The solution set is $\{x \mid x < 6\}$ or $(-\infty, 6)$.



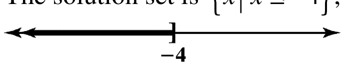
29. $3x - 7 \geq 13$
$3x \geq 20$
$x \geq \frac{20}{3}$
The solution set is $\left\{x \middle| x > \frac{20}{3}\right\}$, or $\left[\frac{20}{3},\infty\right)$.



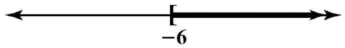
30. $8x - 2 \geq 14$
$8x \geq 16$
$x \geq 2$
The solution set is $\{x \mid x > 2\}$ or $[2,\infty)$.



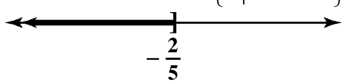
31. $-9x \geq 36$
$x \leq -4$
The solution set is $\{x \mid x \leq -4\}$, or $(-\infty,-4]$.



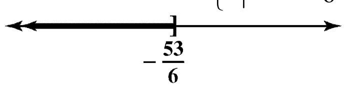
32. $-5x \le 30$
$x \ge -6$
The solution set is $\{x \mid x \ge -6\}$ or $[-6, \infty)$.



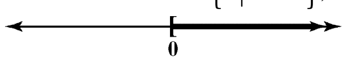
33. $8x - 11 \le 3x - 13$
$8x - 3x \le -13 + 11$
$5x \le -2$
$x \le -\frac{2}{5}$
The solution set is $\left\{x \,\middle|\, x \le -\frac{2}{5}\right\}$, or $\left(-\infty, -\frac{2}{5}\right]$.



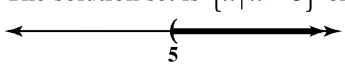
34. $18x + 45 \le 12x - 8$
$18x - 12x \le -8 - 45$
$6x \le -53$
$x \le -\frac{53}{6}$
The solution set is $\left\{x \,\middle|\, x \le -\frac{53}{6}\right\}$ or $\left(-\infty, -\frac{53}{6}\right]$.



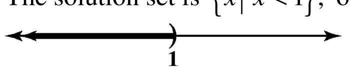
35. $4(x + 1) + 2 \ge 3x + 6$
$4x + 4 + 2 \ge 3x + 6$
$4x + 6 \ge 3x + 6$
$4x - 3x \ge 6 - 6$
$x \ge 0$
The solution set is $\{x \mid x > 0\}$, or $[0, \infty)$.



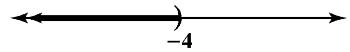
36. $8x + 3 > 3(2x + 1) + x + 5$
$8x + 3 > 6x + 3 + x + 5$
$8x + 3 > 7x + 8$
$8x - 7x > 8 - 3$
$x > 5$
The solution set is $\{x \mid x > 5\}$ or $(5, \infty)$.



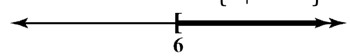
37. $2x - 11 < -3(x + 2)$
$2x - 11 < -3x - 6$
$5x < 5$
$x < 1$
The solution set is $\{x \mid x < 1\}$, or $(-\infty, 1)$.



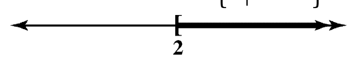
38. $-4(x + 2) > 3x + 20$
$-4x - 8 > 3x + 20$
$-7x > 28$
$x < -4$
The solution set is $\{x \mid x < -4\}$ or $(-\infty, -4)$.



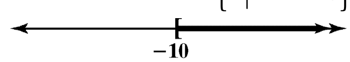
39. $1 - (x + 3) \ge 4 - 2x$
$1 - x - 3 \ge 4 - 2x$
$-x - 2 \ge 4 - 2x$
$x \ge 6$
The solution set is $\{x \mid x \ge 6\}$, or $[6, \infty)$.



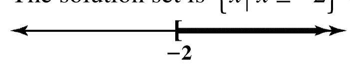
40. $5(3 - x) \le 3x - 1$
$15 - 5x \le 3x - 1$
$-8x \le -16$
$x \ge 2$
The solution set is $\{x \mid x \ge 2\}$ or $[2, \infty)$.



41. $\frac{x}{4} - \frac{3}{2} \le \frac{x}{2} + 1$
$\frac{4x}{4} - \frac{4 \cdot 3}{2} \le \frac{4 \cdot x}{2} + 4 \cdot 1$
$x - 6 \le 2x + 4$
$-x \le 10$
$x \ge -10$
The solution set is $\{x \mid x \ge -10\}$, or $[-10, \infty)$.



42. $\frac{3x}{10} + 1 \ge \frac{1}{5} - \frac{x}{10}$
$10\left(\frac{3x}{10} + 1\right) \ge 10\left(\frac{1}{5} - \frac{x}{10}\right)$
$3x + 10 \ge 2 - x$
$4x \ge -8$
$x \ge -2$
The solution set is $\{x \mid x \ge -2\}$ or $[-2, \infty)$.

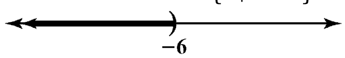


43. $1-\frac{x}{2}>4$

$-\frac{x}{2}>3$

$x<-6$

The solution set is $\{x \mid x, -6\}$, or $(-\infty, -6)$.

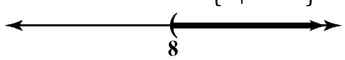


44. $7-\frac{4}{5}x<\frac{3}{5}$

$-\frac{4}{5}x<-\frac{32}{5}$

$x>8$

The solution set is $\{x \mid x>8\}$ or $(8, \infty)$.



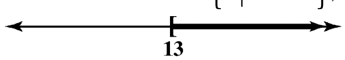
45. $\frac{x-4}{6}\geq\frac{x-2}{9}+\frac{5}{18}$

$3(x-4)\geq 2(x-2)+5$

$3x-12\geq 2x-4+5$

$x\geq 13$

The solution set is $\{x \mid x\geq 13\}$, or $[13, \infty)$.



46.

$\frac{4x-3}{6}+2\geq\frac{2x-1}{12}$

$2(4x-3)+24\geq 2x-1$

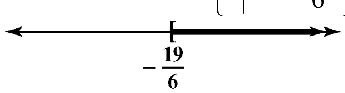
$8x-6+24\geq 2x-1$

$6x+18\geq -1$

$6x\geq -19$

$x\geq -\frac{19}{6}$

The solution set is $\left\{x \,\middle|\, x\geq\frac{-19}{6}\right\}$ or $\left[\frac{-19}{6}, \infty\right)$.



47. $4(3x-2)-3x<3(1+3x)-7$

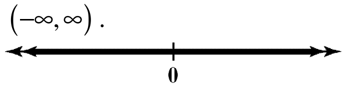
$12x-8-3x<3+9x-7$

$9x-8<-4+9x$

$-8<-4$

True for all x

The solution set is $\{x \mid x \text{ is any real number}\}$, or $(-\infty, \infty)$.



48. $3(x-8)-2(10-x)>5(x-1)$

$3x-24-20+2x>5x-5$

$5x-44>5x-5$

$-44>-5$

Not true for any x.

The solution set is the empty set, $\varnothing$.

49. $5(x-2)-3(x+4)\geq 2x-20$

$5x-10-3x-12\geq 2x-20$

$2x-22\geq 2x-20$

$-22\geq -20$

Not true for any x.

The solution set is the empty set, $\varnothing$.

50. $6(x-1)-(4-x)\geq 7x-8$

$6x-6-4+x\geq 7x-8$

$7x-10\geq 7x-8$

$-10\geq -8$

Not true for any x.

The solution set is the empty set, $\varnothing$.

51. $6<x+3<8$

$6-3<x+3-3<8-3$

$3<x<5$

The solution set is $\{x \mid 3<x<5\}$, or $(3, 5)$.

52. $7<x+5<11$

$7-5<x+5-5<11-5$

$2<x<6$

The solution set is $\{x \mid 2<x<6\}$ or $(2, 6)$.

53. $-3\leq x-2<1$

$-1\leq x<3$

The solution set is $\{x \mid -1\leq x<3\}$, or $[-1, 3)$.

54. $-6<x-4\leq 1$

$-2<x\leq 5$

The solution set is $\{x \mid -2<x\leq 5\}$ or $(-2, 5]$.

55. $-11<2x-1\leq -5$

$-10<2x\leq -4$

$-5<x\leq -2$

The solution set is $\{x \mid -5<x\leq -2\}$, or $(-5, -2]$.

56. $3 \le 4x - 3 < 19$

$6 \le 4x < 22$

$\frac{6}{4} \le x < \frac{22}{4}$

$\frac{3}{2} \le x < \frac{11}{2}$

The solution set is $\left\{x \middle| \frac{3}{2} \le x < \frac{11}{2}\right\}$ or $\left[\frac{3}{2}, \frac{11}{2}\right)$.

57. $-3 \le \frac{2}{3}x - 5 < -1$

$2 \le \frac{2}{3}x < 4$

$3 \le x < 6$

The solution set is $\{x \mid 3 \le x < 6\}$, or [3, 6).

58. $-6 \le \frac{1}{2}x - 4 < -3$

$-2 \le \frac{1}{2}x < 1$

$-4 \le x < 2$

The solution set is $\{x \mid -4 \ge x < 2\}$ or $[-4, 2)$.

59. $|x| < 3$

$-3 < x < 3$

The solution set is $\{x \mid -3 < x < 3\}$, or (–3, 3).

60. $|x| < 5$

$-5 < x < 5$

The solution set is $\{x \mid -5 < x < 5\}$ or (–5, 5).

61. $|x - 1| \le 2$

$-2 \le x - 1 \le 2$

$-1 \le x \le 3$

The solution set is $\{x \mid -1 \le x \le 3\}$, or [–1, 3].

62. $|x + 3| \le 4$

$-4 \le x + 3 \le 4$

$-7 \le x \le 1$

The solution set is $\{x \mid -7 \le x \le 1\}$ or [–7, 1].

63. $|2x - 6| < 8$

$-8 < 2x - 6 < 8$

$-2 < 2x < 14$

$-1 < x < 7$

The solution set is $\{x \mid -1 < x < 7\}$, or (–1, 7).

64. $|3x + 5| < 17$

$-17 < 3x + 5 < 17$

$-22 < 3x < 12$

The solution set is $\left\{x \middle| -\frac{22}{3} < x < 4\right\}$ or $\left(-\frac{22}{3}, 4\right)$.

65. $|2(x - 1) + 4| \le 8$

$-8 \le 2(x - 1) + 4 \le 8$

$-8 \le 2x - 2 + 4 \le 8$

$-8 \le 2x + 2 \le 8$

$-10 \le 2x \le 6$

$-5 \le x \le 3$

The solution set is $\{x \mid -5 \le x \le 3\}$, or [–5, 3].

66. $|3(x - 1) + 2| \le 20$

$-20 \le 3(x - 1) + 2 \le 20$

$-20 \le 3x - 1 \le 20$

$-19 \le 3x \le 21$

$-\frac{19}{3} \le x \le 7$

The solution set is $\left\{x \middle| -\frac{19}{3} \le x \le 7\right\}$ or $\left[-\frac{19}{3}, 7\right]$.

67. $\left|\frac{2y + 6}{3}\right| < 2$

$-2 < \frac{2y + 6}{3} < 2$

$-6 < 2y + 6 < 6$

$-12 < 2y < 0$

$-6 < y < 0$

The solution set is $\{x \mid -6 < y < 0\}$, or (–6, 0).

68. $\left|\frac{3(x - 1)}{4}\right| < 6$

$-6 < \frac{3(x - 1)}{4} < 6$

$-24 < 3x - 3 < 24$

$-21 < 3x < 27$

$-7 < x < 9$

The solution set is $\{x \mid -7 < x < 9\}$ or (–7, 9).

69. $|x| > 3$

$x > 3$ or $x < -3$

The solution set is $\{x \mid x > 3 \text{ or } x < -3\}$, that is, $(-\infty, -3)$ or $(3, \infty)$.

70. $|x| > 5$
$x > 5$ or $x < -5$
The solution set is $\{x \mid x < -5 \text{ or } x > 5\}$, that is, all x in $(-\infty, -5)$ or $(5, \infty)$.

71. $|x-1| \geq 2$
$x - 1 \geq 2$ or $x - 1 \leq -2$
$x \geq 3$ $\quad$ $x \leq -1$
The solution set is $\{x \mid x \leq -1 \text{ or } x \geq 3\}$, that is, $(-\infty, -1]$ or $[3, \infty)$.

72. $|x+3| \geq 4$
$x + 3 \geq 4$ or $x + 3 \leq -4$
$x \geq 1$ $\quad$ $x \leq -7$
The solution set is $\{x \mid x \leq -7 \text{ or } x \geq 1\}$ that is, $(-\infty, -7)$ or $(1, \infty)$.

73. $|3x-8| > 7$
$3x - 8 > 7$ or $3x - 8 < -7$
$3x > 15$ $\quad$ $3x < 1$
$x > 5$ $\quad$ $x < \frac{1}{3}$
The solution set is $\left\{x \,\middle|\, x < \frac{1}{3} \text{ or } x > 5\right\}$, that is, $\left(-\infty, \frac{1}{3}\right)$ or $(5, \infty)$.

74. $|5x-\ 2| > 13$
$5x - 2 > 13$ or $5x - 2 < -13$
$5x > 15$ $\quad$ $5x < -11$
$x > 3$ $\quad$ $x < -\frac{11}{5}$
The solution set is $\left\{x \,\middle|\, x < \frac{-11}{5} \text{ or } x > 3\right\}$,
that is, all x in $\left(-\infty, \frac{-11}{5}\right)$ or $(3, \infty)$

75. $\left|\frac{2x+2}{4}\right| \geq 2$
$\frac{2x+2}{4} \geq 2$ or $\frac{2x+2}{4} \leq -2$
$2x + 2 \geq 8$ $\quad$ $2x + 2 \leq -8$
$2x \geq 6$ $\quad$ $2x \leq -10$
$x \geq 3$ $\quad$ $x \leq -5$
The solution set is $\{x \mid x \leq -5 \text{ or } x \geq 3\}$, that is, $(-\infty, -5]$ or $[3, \infty)$.

76. $\left|\frac{3x-3}{9}\right| \geq 1$
$\frac{3x-3}{9} \geq 1$ or $\frac{3x-3}{9} \leq -1$
$3x - 3 \geq 9$ $\quad$ $3x - 3 \leq -9$
$3x \geq 12$ $\quad$ $3x \leq -6$
$x \geq 4$ $\quad$ $x \leq -2$
The solution set is $\{x \mid x \leq -2 \text{ or } x \geq 4\}$, or $(-\infty, -2]$ or $[4, \infty)$.

77. $\left|3 - \frac{2}{3}x\right| > 5$
$3 - \frac{2}{3}x > 5$ or $3 - \frac{2}{3}x < -5$
$-\frac{2}{3}x > 2$ $\quad$ $-\frac{2}{3}x < -8$
$x < -3$ $\quad$ $x > 12$
The solution set is $\{x \mid x < -3 \text{ or } x > 12\}$, that is, $(-\infty, -3)$ or $(12, \infty)$.

78. $\left|3 - \frac{3}{4}x\right| > 9$
$3 - \frac{3}{4}x > 9$ or $3 - \frac{3}{4}x < -9$
$-\frac{3}{4}x > 6$ $\quad$ $-\frac{3}{4}x < -12$
$x < -8$ $\quad$ $x > 16$
$\{x \mid x < -8 \text{ or } x > 16\}$, that is all x in $(-\infty, -8)$ or $(16, \infty)$.

79. $3|x-1| + 2 \geq 8$
$3|x-1| \geq 6$
$|x-1| \geq 2$
$x - 1 \geq 2$ or $x - 1 \leq -2$
$x \geq 3$ $\quad$ $x \leq -1$
The solution set is $\{x \mid x \leq 1 \text{ or } x \geq 3\}$, that is, $(-\infty, -1]$ or $[3, \infty)$.

80. $5|2x+1|-3 \ge 9$

$5|2x+1| \ge 12$

$|2x+1| \ge \frac{12}{5}$

$2x+1 \ge \frac{12}{5}$ or $2x+1 \le -\frac{12}{5}$

$2x \ge \frac{7}{5}$ or $2x \le -\frac{17}{5}$

$x \ge \frac{7}{10}$ or $x \le -\frac{17}{10}$

The solution set is $\left\{x \middle| x \le -\frac{17}{10} \text{ or } x \ge \frac{7}{10}\right\}$.

81. $-2|x-4| \ge -4$

$\frac{-2|x-4|}{-2} \le \frac{-4}{-2}$

$|x-4| \le 2$

$-2 \le x-4 \le 2$

$2 \le x \le 6$

The solution set is $\{x|2 \le x \le 6\}$.

82. $-3|x+7| \ge -27$

$\frac{-3|x+7|}{-3} \le \frac{-27}{-3}$

$|x+7| \le 9$

$-9 \le x+7 \le 9$

$-16 \le x \le 2$

The solution set is $\{x|-16 \le x \le 2\}$.

83. $-4|1-x| < -16$

$\frac{-4|1-x|}{-4} > \frac{-16}{-4}$

$|1-x| > 4$

$1-x > 4$ or $1-x < -4$

$-x > 3$ or $-x < -5$

$x < -3$ or $x > 5$

The solution set is $\{x|\ x < -3 \text{ or } x > 5\}$.

84. $-2|5-x| < -6$

$-2|5-x| < -6$

$\frac{-2|5-x|}{-2} > \frac{-6}{-2}$

$|5-x| > 3$

$5-x > 3$ or $5-x < -3$

$-x > -2$ or $-x < -8$

$x < 2$ or $x > 8$

The solution set is $\{x|\ x < 2 \text{ or } x > 8\}$.

85. $3 \le |2x-1|$

$2x-1 \ge 3$ or $2x-1 \le -3$

$2x \ge 4$ or $2x \le -2$

$x \ge 2$ or $x \le -1$

The solution set is $\{x|\ x \le -1 \text{ or } x \ge 2\}$.

86. $9 \le |4x+7|$

$4x+7 \ge 9$ or $4x+7 \le -9$

$4x \ge 2$ or $4x \le -16$

$x \ge \frac{2}{4}$ or $x \le -4$

$x \ge \frac{1}{2}$

The solution set is $\left\{x \middle| x \le -4 \text{ or } x \ge \frac{1}{2}\right\}$.

87. $5 > |4-x|$ is equivalent to $|4-x| < 5$.

$-5 < 4-x < 5$

$-9 < -x < 1$

$\frac{-9}{-1} > \frac{-x}{-1} > \frac{1}{-1}$

$9 > x > -1$

$-1 < x < 9$

The solution set is $\{x|-1 < x < 9\}$.

88. $2 > |11-x|$ is equivalent to $|11-x| < 2$.

$-2 < 11-x < 2$

$-13 < -x < -9$

$\frac{-13}{-1} > \frac{-x}{-1} > \frac{-9}{-1}$

$13 > x > 9$

$9 < x < 13$

The solution set is $\{x|\ 9 < x < 13\}$.

89. $1 < |2-3x|$ is equivalent to $|2-3x| > 1$.

$2-3x > 1$ or $2-3x < -1$

$-3x > -1$ $\quad$ $-3x < -3$

$\frac{-3x}{-3} < \frac{-1}{-3}$ $\quad$ $\frac{-3x}{-3} > \frac{-3}{-3}$

$x < \frac{1}{3}$ $\quad$ $x > 1$

The solution set is $\left\{x \,\middle|\, x < \frac{1}{3} \text{ or } x > 1\right\}$.

90. $4 < |2-x|$ is equivalent to $|2-x| > 4$.

$2-x > 4$ or $2-x < -4$

$-x > 2$ $\quad$ $-x < -6$

$\frac{-x}{-1} < \frac{2}{-1}$ $\quad$ $\frac{-x}{-1} > \frac{-6}{-1}$

$x < -2$ $\quad$ $x > 6$

The solution set is $\{x \mid x < -2 \text{ or } x > 6\}$.

91. $12 < \left|-2x + \frac{6}{7}\right| + \frac{3}{7}$

$\frac{81}{7} < \left|-2x + \frac{6}{7}\right|$

$-2x + \frac{6}{7} > \frac{81}{7}$ or $-2x + \frac{6}{7} < -\frac{81}{7}$

$-2x > \frac{75}{7}$ $\quad$ $-2x < -\frac{87}{7}$

$x < -\frac{75}{14}$ $\quad$ $x > \frac{87}{14}$

The solution set is $\left\{x \,\middle|\, x < -\frac{75}{14} \text{ or } x > \frac{87}{14}\right\}$, that is, $\left(-\infty, -\frac{75}{14}\right)$ or $\left(\frac{87}{14}, \infty\right)$.

92. $1 < \left|x - \frac{11}{3}\right| + \frac{7}{3}$

$-\frac{4}{3} < \left|x - \frac{11}{3}\right|$

Since $\left|x - \frac{11}{3}\right| > -\frac{4}{3}$ is true for all x, the solution set is $\{x \mid x \text{ is any real number}\}$ or $(-\infty, \infty)$.

93. $4 + \left|3 - \frac{x}{3}\right| \ge 9$

$\left|3 - \frac{x}{3}\right| \ge 5$

$3 - \frac{x}{3} \ge 5$ or $3 - \frac{x}{3} \le -5$

$-\frac{x}{3} \ge 2$ $\quad$ $-\frac{x}{3} \le -8$

$x \le -6$ $\quad$ $x \ge 24$

The solution set is $\{x \mid x \le -6 \text{ or } x \ge 24\}$, that is, $(-\infty, -6]$ or $[24, \infty)$.

94. $\left|2 - \frac{x}{2}\right| - 1 \le 1$

$\left|2 - \frac{x}{2}\right| \le 2$

$-2 \le 2 - \frac{x}{2} \le 2$

$-4 \le -\frac{x}{2} \le 0$

$8 \ge x \ge 0$

The solution set is $\{x \mid 0 \le x \le 8\}$ or $[0, 8]$.

95.

$$y_1 \le y_2$$

$$\frac{x}{2} + 3 \le \frac{x}{3} + \frac{5}{2}$$

$$6\left(\frac{x}{2} + 3\right) \le 6\left(\frac{x}{3} + \frac{5}{2}\right)$$

$$\frac{6x}{2} + 6(3) \le \frac{6x}{3} + \frac{6(5)}{2}$$

$$3x + 18 \le 2x + 15$$

$$x \le -3$$

The solution set is $(-\infty, -3]$.

96.

$$y_1 > y_2$$
$$\frac{2}{3}(6x-9)+4 > 5x+1$$
$$3\left(\frac{2}{3}(6x-9)+4\right) > 3(5x+1)$$
$$2(6x-9)+12 > 15x+3$$
$$12x-18+12 > 15x+3$$
$$12x-6 > 15x+3$$
$$-3x > 9$$
$$\frac{-3x}{-3} < \frac{9}{-3}$$
$$x < -3$$

The solution set is $(-\infty,-3)$.

97.

$$y \ge 4$$
$$1-(x+3)+2x \ge 4$$
$$1-x-3+2x \ge 4$$
$$x-2 \ge 4$$
$$x \ge 6$$

The solution set is $[6,\infty)$.

98.

$$y \le 0$$
$$2x-11+3(x+2) \le 0$$
$$2x-11+3x+6 \le 0$$
$$5x-5 \le 0$$
$$5x \le 5$$
$$x \le 1$$

The solution set is $(-\infty,1]$.

99.

$$y < 8$$
$$|3x-4|+2 < 8$$
$$|3x-4| < 6$$
$$-6 < 3x-4 < 6$$
$$-2 < 3x < 10$$
$$\frac{-2}{3} < \frac{3x}{3} < \frac{10}{3}$$
$$\frac{-2}{3} < x < \frac{10}{3}$$

The solution set is $\left(\frac{-2}{3},\frac{10}{3}\right)$.

100.

$$y > 9$$
$$|2x-5|+1 > 9$$
$$|2x-5| > 8$$
$$2x-5 < -8 \quad \text{or} \quad 2x-5 > 8$$
$$2x < -3 \qquad 2x > 13$$
$$x < -\frac{3}{2} \qquad x > \frac{13}{2}$$

The solution set is $\left(-\infty,-\frac{3}{2}\right)\cup\left(\frac{13}{2},\infty\right)$.

101.

$$y \le 4$$
$$7-\left|\frac{x}{2}+2\right| \le 4$$
$$-\left|\frac{x}{2}+2\right| \le -3$$
$$\left|\frac{x}{2}+2\right| \ge 3$$
$$\frac{x}{2}+2 \ge 3 \quad \text{or} \quad \frac{x}{2}+2 \le -3$$
$$x+4 \ge 6 \qquad x+4 \le -6$$
$$x \ge 2 \qquad x \le -10$$

The solution set is $(-\infty,-10]\cup[2,\infty)$.

102.

$$y \ge 6$$
$$8-|5x+3| \ge 6$$
$$-|5x+3| \ge -2$$
$$-\left(-|5x+3|\right) \le -(-2)$$
$$|5x+3| \le 2$$
$$-2 \le 5x+3 \le 2$$
$$-5 \le 5x \le -1$$
$$\frac{-5}{5} \le \frac{5x}{5} \le \frac{-1}{5}$$
$$-1 \le x \le -\frac{1}{5}$$

The solution set is $\left[-1,-\frac{1}{5}\right]$.

103. The graph's height is below 5 on the interval $(-1,9)$.

104. The graph's height is at or above 5 on the interval $(-\infty,-1]\cup[9,\infty)$.

105. The solution set is $\{x \mid -1 \le x < 2\}$ or $[-1,2)$.

106. The solution set is $\{x \mid 1 < x \le 4\}$ or $(1,4]$.

107. Let x be the number.

$|4-3x| \ge 5$ or $|3x-4| \ge 5$

$3x-4 \ge 5$ $\quad$ $3x-4 \le -5$

$3x \ge 9$ or $3x \le -1$

$x \ge 3$ $\quad$ $x \le -\frac{1}{3}$

The solution set is $\left\{x \mid x \le -\frac{1}{3} \text{ or } x \ge 3\right\}$ or $\left(-\infty, -\frac{1}{3}\right] \cup [3, \infty)$.

108. Let x be the number.

$|5-4x| \le 13$ or $|4x-5| \le 13$

$-13 \le 4x-5 \le 13$

$-8 \le 4x \le 18$

$-2 \le x \le \frac{9}{2}$

The solution set is $\left\{x \mid -2 \le x \le \frac{9}{2}\right\}$ or $\left[-2, -\frac{2}{9}\right]$.

109. $(0,4)$

110. $[0,5]$

111. passion $\le$ intimacy or intimacy $\ge$ passion

112. commitment $\ge$ intimacy or
intimacy $\le$ commitment

113. passion<commitment or
commitment > passion

114. commitment > passion or
passion < commitment

115. 9, after 3 years

116. after approximately $5\frac{1}{2}$ years

117. a. $I = \frac{1}{4}x + 26$

$\frac{1}{4}x + 26 > 33$

$\frac{1}{4}x > 7$

$x > 28$

More than 33% of U.S. households will have an interfaith marriage in years after 2016 (i.e. $1988+28$).

b. $N = \frac{1}{4}x + 6$

$\frac{1}{4}x + 6 > 14$

$\frac{1}{4}x > 8$

$x > 32$

More than 14% of U.S. households will have a person of faith married to someone with no religion in years after 2020 (i.e. $1988+32$).

c. More than 33% of U.S. households will have an interfaith marriage *and* more than 14% of U.S. households will have a person of faith married to someone with no religion in years after 2020.

d. More than 33% of U.S. households will have an interfaith marriage *or* more than 14% of U.S. households will have a person of faith married to someone with no religion in years after 2016.

118. a. $I = \frac{1}{4}x + 26$

$\frac{1}{4}x + 26 > 34$

$\frac{1}{4}x > 8$

$x > 32$

More than 34% of U.S. households will have an interfaith marriage in years after 2020 (i.e. $1988+32$).

b. $N = \frac{1}{4}x + 6$

$\frac{1}{4}x + 6 > 15$

$\frac{1}{4}x > 9$

$x > 36$

More than 15% of U.S. households will have a person of faith married to someone with no religion in years after 2024 (i.e. $1988+36$).

c. More than 34% of U.S. households will have an interfaith marriage *and* more than 15% of U.S. households will have a person of faith married to someone with no religion in years after 2024.

d. More than 34% of U.S. households will have an interfaith marriage *or* more than 15% of U.S. households will have a person of faith married to someone with no religion in years after 2020.

119. $15 \le \frac{5}{9}(F-32) \le 35$

$\frac{9}{5}(15) \le \frac{9}{5}\left(\frac{5}{9}(F-32)\right) \le \frac{9}{5}(35)$

$9(3) \le F-32 \le 9(7)$

$27 \le F-32 \le 63$

$59 \le F \le 95$

The range for Fahrenheit temperatures is $59°\text{F}$ to $95°\text{F}$, inclusive or $[59°\text{F}, 95°\text{F}]$.

120. $41 \le \frac{9}{5}C + 32 \le 50$

$41-32 \le \frac{9}{5}C + 32 - 32 \le 50-32$

$9 \le \frac{9}{5}C \le 18$

$\frac{5}{9}(9) \le \frac{5}{9}\left(\frac{9}{5}C\right) \le \frac{5}{9}(18)$

$5 \le C \le 10$

The range for Celsius temperatures is $5°\text{C}$ to $10°\text{C}$, inclusive or $[5°\text{C}, 10°\text{C}]$.

121. $\left|\frac{h-50}{5}\right| \ge 1.645$

$\frac{h-50}{5} \ge 1.645$ or $\frac{h-50}{5} \le -1.645$

$h-50 \ge 8.225$ $\quad$ $h-50 \le -8.225$

$h \ge 58.225$ $\quad$ $h \le 41.775$

The number of outcomes would be 59 or more, or 41 or less.

122. $50 + 0.20x < 20 + 0.50x$

$30 < 0.3x$

$100 < x$

Basic Rental is a better deal when driving more than 100 miles per day.

123. $15 + 0.08x < 3 + .12x$

$12 < 0.04x$

$300 < x$

Plan A is a better deal when texting more than 300 times per month.

124. $1800 + 0.03x < 200 + 0.08x$

$1600 < 0.05x$

$32000 < x$

A home assessment of greater than $32,000 would make the first bill a better deal.

125. $2 + 0.08x < 8 + 0.05x$

$0.03x < 6$

$x < 200$

The credit union is a better deal when writing less than 200 checks.

126. $2x > 10,000 + 0.40x$

$1.6x > 10,000$

$\frac{1.6x}{1.6} > \frac{10,000}{1.6}$

$x > 6250$

More than 6250 tapes need to be sold a week to make a profit.

127. $3000 + 3x < 5.5x$

$3000 < 2.5x$

$1200 < x$

More then 1200 packets of stationary need to be sold each week to make a profit.

128. $265 + 65x \le 2800$

$65x \le 2535$

$x \le 39$

39 bags or fewer can be lifted safely.

129. $245 + 95x \le 3000$

$95x \le 2755$

$x \le 29$

29 bags or less can be lifted safely.

130. Let $x =$ the grade on the final exam.

$\frac{86+88+92+84+x+x}{6} \ge 90$

$86+88+92+84+x+x \ge 540$

$2x + 350 \ge 540$

$2x \ge 190$

$x \ge 95$

You must receive at least a 95% to earn an A.

131. a. $\frac{86+88+x}{3} \ge 90$

$\frac{174+x}{3} \ge 90$

$174 + x \ge 270$

$x \ge 96$

You must get at least a 96.

b. $\dfrac{86+88+x}{3}<80$

$\dfrac{174+x}{3}<80$

$174+x<240$

$x<66$

This will happen if you get a grade less than 66.

132. Let x = the number of hours the mechanic works on the car.

$226 \le 175+34x \le 294$

$51 \le 34x \le 119$

$1.5 \le x \le 3.5$

The man will be working on the job at least 1.5 and at most 3.5 hours.

133. Let x = the number of times the bridge is crossed per three month period

The cost with the 3-month pass is $C_3 = 7.50+0.50x$.

The cost with the 6-month pass is $C_6 = 30$.

Because we need to buy two 3-month passes per 6-month pass, we multiply the cost with the 3-month pass by 2.

$2(7.50+0.50x)<30$

$15+x<30$

$x<15$

We also must consider the cost without purchasing a pass. We need this cost to be less than the cost with a 3-month pass.

$3x>7.50+0.50x$

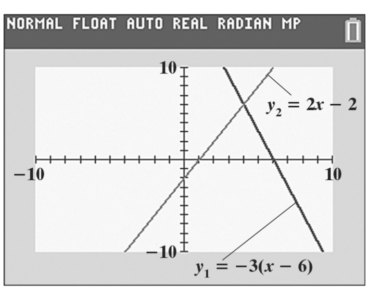
$2.50x>7.50$

$x>3$

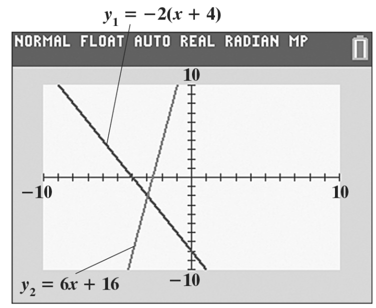
The 3-month pass is the best deal when making more than 3 but less than 15 crossings per 3-month period.

134. – 141. Answers will vary.

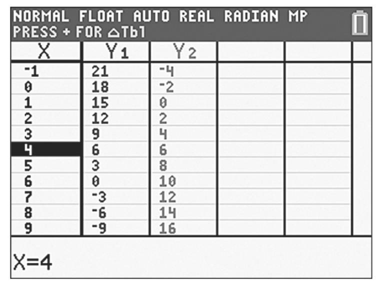
142. $x<4$



143. $x<-3$



144. Verify exercise 142.

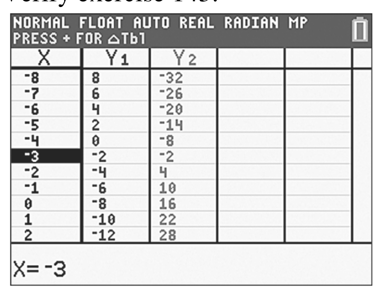


NORMAL FLOAT AUTO REAL RADIAN MP
PRESS + FOR △Tbl

X	Y1	Y2
-1	21	-4
0	18	-2
1	15	0
2	12	2
3	9	4
4	6	6
5	3	8
6	0	10
7	-3	12
8	-6	14
9	-9	16

X=4

Verify exercise 143.



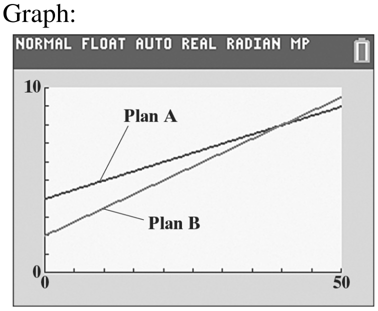
NORMAL FLOAT AUTO REAL RADIAN MP
PRESS + FOR △Tbl

X	Y1	Y2
-8	8	-32
-7	6	-26
-6	4	-20
-5	2	-14
-4	0	-8
-3	-2	-2
-2	-4	4
-1	-6	10
0	-8	16
1	-10	22
2	-12	28

X= -3

145. a. The cost of Plan A is $4 + 0.10x$;
The cost of Plan B is $2 + 0.15x$.

b. Graph:



c. 41 or more checks make Plan A better.

d. $4 + 0.10x < 2 + 0.15x$

$2 < 0.05x$

$x > 40$

The solution set is $\{x \mid x > 40\}$ or $(40, \infty)$.

146. makes sense

147. makes sense

148. makes sense

149. makes sense

150. true

151. false; Changes to make the statement true will vary. A sample change is: $(-\infty, 3) \cup (-\infty, -2) = (-\infty, 3)$

152. false; Changes to make the statement true will vary. A sample change is: $3x > 6$ is equivalent to $x > 2$.

153. true

154. Because $x > y$, $y - x$ represents a negative number. When both sides are multiplied by $(y - x)$ the inequality must be reversed.

155. a. $|x - 4| < 3$

b. $|x - 4| \geq 3$

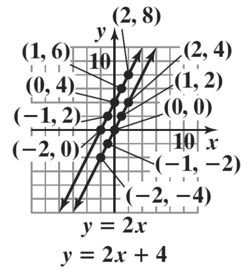
156. Answers will vary.

157. Set 1 has each x-coordinate paired with only one y-coordinate.

158.

x	$y = 2x$	(x, y)
–2	$y = 2(-2) = -4$	$(-2, -4)$
–1	$y = 2(-1) + 4 = 2$	$(-1, -2)$
0	$y = 2(0) = 0$	$(0, 0)$
1	$y = 2(1) = 2$	$(1, 2)$
2	$y = 2(2) = 4$	$(2, 4)$

x	$y = 2x + 4$	(x, y)
–2	$y = 2(-2) + 4 = 0$	$(-2, 0)$
–1	$y = 2(-1) + 4 = 2$	$(-1, 2)$
0	$y = 2(0) + 4 = 4$	$(0, 4)$
1	$y = 2(1) + 4 = 6$	$(1, 6)$
2	$y = 2(2) + 4 = 8$	$(2, 8)$



159. a. When the x-coordinate is 2, the y-coordinate is 3.

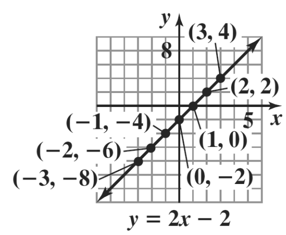
b. When the y-coordinate is 4, the x-coordinates are –3 and 3.

c. The x-coordinates are all real numbers.

d. The y-coordinates are all real numbers greater than or equal to 1.

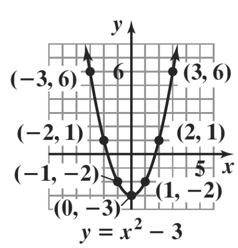
Chapter 1 Review Exercises

1.



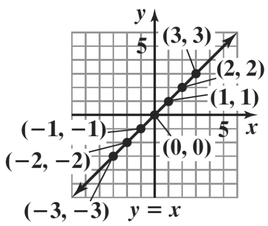
$x = -3, y = -8$
$x = -2, y = -6$
$x = -1, y = -4$
$x = 0, y = -2$
$x = 1, y = 0$
$x = 2, y = 2$
$x = 3, y = 4$

2.



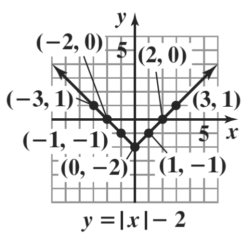
$x = -3, y = 6$
$x = -2, y = 1$
$x = -1, y = -2$
$x = 0, y = -3$
$x = 1, y = -2$
$x = 2, y = 1$
$x = 3, y = 6$

3.



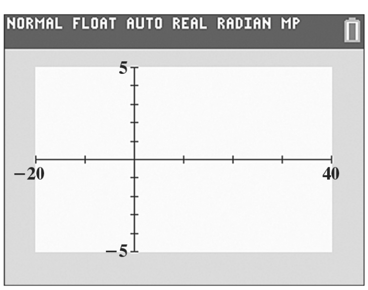
$x = -3, y = -3$
$x = -2, y = -2$
$x = -1, y = -1$
$x = 0, y = 0$
$x = 1, y = 1$
$x = 2, y = 2$
$x = 3, y = 3$

4.



$x = -3, y = 1$
$x = -2, y = 0$
$x = -1, y = -1$
$x = 0, y = -2$
$x = 1, y = -1$
$x = 2, y = 0$
$x = 3, y = 1$

5. A portion of Cartesian coordinate plane with minimum x-value equal to –20, maximum x-value equal to 40, x-scale equal to 10 and with minimum y-value equal to –5, maximum y-value equal to 5, and y-scale equal to 1.



6. x-intercept: –2; The graph intersects the x-axis at $(-2,\ 0)$.
y-intercept: 2; The graph intersects the y-axis at $(0,\ 2)$.

7. x-intercepts: 2, –2; The graph intersects the x-axis at $(-2,\ 0)$ and $(2,\ 0)$.
y-intercept: –4; The graph intercepts the y-axis at $(0,\ -4)$.

8. x-intercept: 5; The graph intersects the x-axis at $(5,\ 0)$.
y-intercept: None; The graph does not intersect the y-axis.

9. The coordinates are (20, 8). This means that 8% of college students anticipated a starting salary of $20 thousand.

10. The starting salary that was anticipated by the greatest percentage of college students was $30 thousand. 22% of students anticipated this salary.

11. The starting salary that was anticipated by the least percentage of college students was $70 thousand. 2% of students anticipated this salary.

12. Starting salaries of $25 thousand and $30 thousand were anticipated by more than 20% of college students

13. 14% of students anticipated a starting salary of $40 thousand.

14. $p = -0.01s^2 + 0.8s + 3.7$
$p = -0.01(40)^2 + 0.8(40) + 3.7$
$p = 19.7$
This is greater than the estimate of the previous question.

15. $2x - 5 = 7$
$2x = 12$
$x = 6$
The solution set is $\{6\}$.
This is a conditional equation.

16. $5x + 20 = 3x$
$2x = -20$
$x = -10$
The solution set is $\{-10\}$.
This is a conditional equation.

17. $7(x - 4) = x + 2$
$7x - 28 = x + 2$
$6x = 30$
$x = 5$
The solution set is $\{5\}$.
This is a conditional equation.

18. $1 - 2(6 - x) = 3x + 2$
$1 - 12 + 2x = 3x + 2$
$-11 - x = 2$
$-x = 13$
$x = -13$
The solution set is $\{-13\}$.
This is a conditional equation.

19. $2(x-4)+3(x+5) = 2x-2$
$2x-8+3x+15 = 2x-2$
$5x+7 = 2x-2$
$3x = -9$
$x = -3$
The solution set is $\{-3\}$.
This is a conditional equation.

20. $2x - 4(5x + 1) = 3x + 17$
$2x - 20x - 4 = 3x + 17$
$-18x - 4 = 3x + 17$
$-21x = 21$
$x = -1$
The solution set is $\{-1\}$.
This is a conditional equation.

21. $7x+5 = 5(x+3)+2x$
$7x+5 = 5x+15+2x$
$7x+5 = 7x+15$
$5 = 15$
The solution set is $\varnothing$.
This is an inconsistent equation.

22. $7x+13 = 2(2x-5)+3x+23$
$7x+13 = 2(2x-5)+3x+23$
$7x+13 = 4x-10+3x+23$
$7x+13 = 7x+13$
$13 = 13$
The solution set is all real numbers.
This is an identity.

23. $\frac{2x}{3} = \frac{x}{6}+1$
$2(2x) = x+6$
$4x = x+6$
$3x = 6$
$x = 2$
The solution set is $\{2\}$.
This is a conditional equation.

24. $\frac{x}{2}-\frac{1}{10} = \frac{x}{5}+\frac{1}{2}$
$5x-1 = 2x+5$
$3x = 6$
$x = 2$
The solution set is $\{2\}$.
This is a conditional equation.

25. $\frac{2x}{3} = 6-\frac{x}{4}$
$4(2x) = 12(6)-3x$
$8x = 72-3x$
$11x = 72$
$x = \frac{72}{11}$
The solution set is $\left\{\frac{72}{11}\right\}$.
This is a conditional equation.

26. $\frac{x}{4} = 2-\frac{x-3}{3}$
$\frac{12 \cdot x}{4} = 12(2)-\frac{12(x-3)}{3}$
$3x = 24-4x+12$
$7x = 36$
$x = \frac{36}{7}$
The solution set is $\left\{\frac{36}{7}\right\}$.
This is a conditional equation.

27. $\frac{3x+1}{3}-\frac{13}{2} = \frac{1-x}{4}$
$4(3x+1)-6(13) = 3(1-x)$
$12x+4-78 = 3-3x$
$12x-74 = 3-3x$
$15x = 77$
$x = \frac{77}{15}$
The solution set is $\left\{\frac{77}{15}\right\}$.
This is a conditional equation.

28. $\frac{9}{4}-\frac{1}{2x}=\frac{4}{x}$

$9x-2=16$

$9x=18$

$x=2$

The solution set is $\{2\}$.
This is a conditional equation.

29. $\frac{7}{x-5}+2=\frac{x+2}{x-5}$

$7+2(x-5)=x+2$

$7+2x-10=x+2$

$2x-3=x+2$

$x=5$

5 does not check and must be rejected.
The solution set is the empty set, $\varnothing$.
This is an inconsistent equation.

30. $\frac{1}{x-1}-\frac{1}{x+1}=\frac{2}{x^2-1}$

$\frac{1}{x-1}-\frac{1}{x+1}=\frac{2}{(x+1)(x-1)}$

$x+1-(x-1)=2$

$x+1-x+1=2$

$2=2$

The solution set is all real numbers except –1 and 1.
This is a conditional equation.

31. $\frac{5}{x+3}+\frac{1}{x-2}=\frac{8}{x^2+x-6}$

$\frac{5}{x+3}+\frac{1}{x-2}=\frac{8}{(x+3)(x-2)}$

$\frac{5(x+3)(x-2)}{x+3}+\frac{(x+3)(x-2)}{x-2}=\frac{8(x+3)(x-2)}{(x+3)(x-2)}$

$5(x-2)+1(x+3)=8$

$5x-10+x+3=8$

$6x-7=8$

$6x=15$

$x=\frac{15}{6}$

$x=\frac{5}{2}$

The solution set is $\left\{\frac{5}{2}\right\}$.

This is a conditional equation.

32. $\frac{1}{x+5}=0$

$(x+5)\frac{1}{x+5}=(x+5)(0)$

$1=0$

The solution set is the empty set, $\varnothing$.
This is an inconsistent equation.

33. $\frac{4}{x+2}+\frac{3}{x}=\frac{10}{x^2+2x}$

$\frac{4}{x+2}+\frac{3}{x}=\frac{10}{x(x+2)}$

$\frac{4\cdot x(x+2)}{x+2}+\frac{3\cdot x(x+2)}{x}=\frac{10\cdot x(x+2)}{x(x+2)}$

$4x+3(x+2)=10$

$4x+3x+6=10$

$7x+6=10$

$7x=4$

$x=\frac{4}{7}$

The solution set is $\left\{\frac{4}{7}\right\}$.

This is a conditional equation.

34. $3-5(2x+1)-2(x-4)=0$

$3-5(2x+1)-2(x-4)=0$

$3-10x-5-2x+8=0$

$-12x+6=0$

$-12x=-6$

$x=\frac{-6}{-12}$

$x=\frac{1}{2}$

The solution set is $\left\{\frac{1}{2}\right\}$.

This is a conditional equation.

35. $\frac{x+2}{x+3}+\frac{1}{x^2+2x-3}-1=0$

$\frac{x+2}{x+3}+\frac{1}{(x+3)(x-1)}-1=0$

$\frac{x+2}{x+3}+\frac{1}{(x+3)(x-1)}=1$

$\frac{(x+2)(x+3)(x-1)}{x+3}+1=(x+3)(x-1)$

$(x+2)(x-1)+1=(x+3)(x-1)$

$x^2+x-2+1=x^2+2x-3$

$x-1=2x-3$

$-x=-2$

$x=2$

The solution set is $\{2\}$.

This is a conditional equation.

36. Let $x=$ the number involving oversleeping.
Let $x+10=$ the number involving computer problems.
Let $x+80=$ the number involving illness.

$x+(x+10)+(x+80)=270$

$x+x+10+x+80=270$

$3x+90=270$

$3x=180$

$x=60$

$x+10=70$

$x+80=140$

The number involving oversleeping, computer problems, and illness, respectively, is 60, 70, and 140.

37. Let x = the number of years after 1980.

$2.69+0.17x=9.49$

$0.17x=6.8$

$x=40$

The average price of a movie ticket will be \$9.49 40 years after 1980, or 2020.

38. Let x = the number of GB used.
Plan A: $C=52+18x$
Plan B: $C=32+22x$
Set the costs equal to each other.

$52+18x=32+22x$

$52=32+4x$

$20=4x$

$5=x$

The cost will be the same for 5 GB.

39. Let x = the original price of the phone

$48=x-0.20x$

$48=0.80x$

$60=x$

The original price is \$60.

40. Let x = the amount sold to earn \$800 in one week

$800=300+0.05x$

$500=0.05x$

$10,000=x$

Sales must be \$10,000 in one week to earn \$800.

41. Let x = the amount invested at 4%
Let y = the amount invested at 7%

$x+y=9000$

$0.04x+0.07y=555$

Multiply the first equation by –0.04 and add.

$-0.04x-0.04y=-360$

$0.04x+0.07y=555$

$0.03y=195$

$y=6500$

Back-substitute 6500 for y in one of the original equations to find x.

$x+y=9000$

$x+6500=9000$

$x=2500$

There was \$2500 invested at 4% and \$6500 invested at 7%.

42. Let $x=$ the amount invested at 2%
Let $8000-x=$ the amount invested at 5%.

$0.05(8000-x)=0.02x+85$

$400-0.05x=0.02x+85$

$-0.05x-0.02x=85-400$

$-0.07x=-315$

$\frac{-0.07x}{-0.07}=\frac{-315}{-0.07}$

$x=4500$

$8000-x=3500$

\$4500 was invested at 2% and \$3500 was invested at 5%.

43. Let w = the width of the playing field,
Let $3w - 6$ = the length of the playing field

$$P = 2(\text{length}) + 2(\text{width})$$
$$340 = 2(3w - 6) + 2w$$
$$340 = 6w - 12 + 2w$$
$$340 = 8w - 12$$
$$352 = 8w$$
$$44 = w$$

The dimensions are 44 yards by 126 yards.

44. **a.** Let x = the number of years (after 2015).
College A's enrollment: $14,100 + 1500x$
College B's enrollment: $41,700 - 800x$
$14,100 + 1500x = 41,700 - 800x$

b. Check points to determine that $y_1 = 14,100 + 1500x$ and $y_2 = 41,700 - 800x$. Since $y_1 = y_2 = 32,100$ when $x = 12$, the two colleges will have the same enrollment in the year $2015 + 12 = 2027$. That year the enrollments will be 32,100 students.

45.
$$vt + gt^2 = s$$
$$gt^2 = s - vt$$
$$\frac{gt^2}{t^2} = \frac{s - vt}{t^2}$$
$$g = \frac{s - vt}{t^2}$$

46.
$$T = gr + gvt$$
$$T = g(r + vt)$$
$$\frac{T}{r + vt} = \frac{g(r + vt)}{r + vt}$$
$$\frac{T}{r + vt} = g$$
$$g = \frac{T}{r + vt}$$

47.
$$T = \frac{A - P}{\text{Pr}}$$
$$Pr(T) = Pr\frac{A - P}{\text{Pr}}$$
$$PrT = A - P$$
$$PrT + P = A$$
$$P(rT + 1) = A$$
$$P = \frac{A}{1 + rT}$$

48.
$$(8 - 3i) - (17 - 7i) = 8 - 3i - 17 + 7i$$
$$= -9 + 4i$$

49.
$$4i(3i - 2) = (4i)(3i) + (4i)(-2)$$
$$= 12i^2 - 8i$$
$$= -12 - 8i$$

50.
$$(7 - i)(2 + 3i)$$
$$= 7 \cdot 2 + 7(3i) + (-i)(2) + (-i)(3i)$$
$$= 14 + 21i - 2i + 3$$
$$= 17 + 19i$$

51.
$$(3 - 4i)^2 = 3^2 + 2 \cdot 3(-4i) + (-4i)^2$$
$$= 9 - 24i - 16$$
$$= -7 - 24i$$

52. $(7 + 8i)(7 - 8i) = 7^2 + 8^2 = 49 + 64 = 113$

53.
$$\frac{6}{5 + i} = \frac{6}{5 + i} \cdot \frac{5 - i}{5 - i}$$
$$= \frac{30 - 6i}{25 + 1}$$
$$= \frac{30 - 6i}{26}$$
$$= \frac{15 - 3i}{13}$$
$$= \frac{15}{13} - \frac{3}{13}i$$

54.
$$\frac{3 + 4i}{4 - 2i} = \frac{3 + 4i}{4 - 2i} \cdot \frac{4 + 2i}{4 + 2i}$$
$$= \frac{12 + 6i + 16i + 8i^2}{16 - 4i^2}$$
$$= \frac{12 + 22i - 8}{16 + 4}$$
$$= \frac{4 + 22i}{20}$$
$$= \frac{1}{5} + \frac{11}{10}i$$

55.
$$\sqrt{-32} - \sqrt{-18} = i\sqrt{32} - i\sqrt{18}$$
$$= i\sqrt{16 \cdot 2} - i\sqrt{9 \cdot 2}$$
$$= 4i\sqrt{2} - 3i\sqrt{2}$$
$$= (4i - 3i)\sqrt{2}$$
$$= i\sqrt{2}$$

56. $(-2+\sqrt{-100})^2 = (-2+i\sqrt{100})^2$
$= (-2+10i)^2$
$= 4-40i+(10i)^2$
$= 4-40i-100$
$= -96-40i$

57. $\frac{4+\sqrt{-8}}{2} = \frac{4+i\sqrt{8}}{2} = \frac{4+2i\sqrt{2}}{2} = 2+i\sqrt{2}$

58. $2x^2+15x = 8$
$2x^2+15x-8 = 0$
$(2x-1)(x+8) = 0$
$2x-1=0 \quad x+8=0$
$x=\frac{1}{2}$ or $x=-8$
The solution set is $\left\{\frac{1}{2}, -8\right\}$.

59. $5x^2+20x=0$
$5x(x+4)=0$
$5x=0 \quad x+4=0$
$x=0$ or $x=-4$
The solution set is {0, –4}.

60. $2x^2-3=125$
$2x^2=128$
$x^2=64$
$x=\pm 8$
The solution set is {8, –8}.

61. $\frac{x^2}{2}+5=-3$
$\frac{x^2}{2}=-8$
$x^2=-16$
$\sqrt{x^2}=\pm\sqrt{-16}$
$x=\pm 4i$

62. $(x+3)^2=-10$
$\sqrt{(x+3)^2}=\pm\sqrt{-10}$
$x+3=\pm i\sqrt{10}$
$x=-3\pm i\sqrt{10}$

63. $(3x-4)^2=18$
$\sqrt{(3x-4)^2}=\pm\sqrt{18}$
$3x-4=\pm 3\sqrt{2}$
$3x=4\pm 3\sqrt{2}$
$\frac{3x}{3}=\frac{4\pm 3\sqrt{2}}{3}$
$x=\frac{4\pm 3\sqrt{2}}{3}$

64. x^2+20x
$\left(\frac{20}{2}\right)^2=10^2=100$
$x^2+20x+100=(x+10)^2$

65. x^2-3x
$\left(\frac{3}{2}\right)^2=\frac{9}{4}$
$x^2-3x+\frac{9}{4}=\left(x-\frac{3}{2}\right)^2$

66. $x^2-12x=-27$
$x^2-12x+36=-27+36$
$(x-6)^2=9$
$x-6=\pm 3$
$x=6\pm 3$
$x=9, 3$
The solution set is {9, 3}.

67. $3x^2-12x+11=0$
$x^2-4x=-\frac{11}{3}$
$x^2-4x+4=-\frac{11}{3}+4$
$(x-2)^2=\frac{1}{3}$
$x-2=\pm\sqrt{\frac{1}{3}}$
$x=2\pm\frac{\sqrt{3}}{3}$
The solution set is $\left\{2+\frac{\sqrt{3}}{3}, 2-\frac{\sqrt{3}}{3}\right\}$.

68. $x^2 = 2x + 4$

$x^2 - 2x - 4 = 0$

$x = \dfrac{2 \pm \sqrt{(-2)^2 - 4(1)(-4)}}{2(1)}$

$x = \dfrac{2 \pm \sqrt{4+16}}{2}$

$x = \dfrac{2 \pm \sqrt{20}}{2}$

$x = \dfrac{2 \pm 2\sqrt{5}}{2}$

$x = 1 \pm \sqrt{5}$

The solution set is $\left\{1+\sqrt{5}, 1-\sqrt{5}\right\}$.

69. $x^2 - 2x + 19 = 0$

$x = \dfrac{2 \pm \sqrt{(-2)^2 - 4(1)(19)}}{2(1)}$

$x = \dfrac{2 \pm \sqrt{4-76}}{2}$

$x = \dfrac{2 \pm \sqrt{-72}}{2}$

$x = \dfrac{2 \pm 6i\sqrt{2}}{2}$

$x = 1 \pm 3i\sqrt{2}$

The solution set is $\left\{1+3i\sqrt{2}, 1-3i\sqrt{2}\right\}$.

70. $2x^2 = 3 - 4x$

$2x^2 + 4x - 3 = 0$

$x = \dfrac{-4 \pm \sqrt{4^2 - 4(2)(-3)}}{2(2)}$

$x = \dfrac{-4 \pm \sqrt{16+24}}{4}$

$x = \dfrac{-4 \pm \sqrt{40}}{4}$

$x = \dfrac{-4 \pm 2\sqrt{10}}{4}$

$x = \dfrac{-2 \pm \sqrt{10}}{2}$

The solution set is $\left\{\dfrac{-2+\sqrt{10}}{2}, \dfrac{-2-\sqrt{10}}{2}\right\}$.

71. $x^2 - 4x + 13 = 0$

$(-4)^2 - 4(1)(13)$

$= 16 - 52$

$= -36$; 2 complex imaginary solutions

72. $9x^2 = 2 - 3x$

$9x^2 + 3x - 2 = 0$

$3^2 - 4(9)(-2)$

$= 9 + 72$

$= 81$; 2 unequal real solutions

73. $2x^2 - 11x + 5 = 0$

$(2x - 1)(x - 5) = 0$

$2x - 1 = 0 \quad x - 5 = 0$

$x = \dfrac{1}{2}$ or $x = 5$

The solution set is $\left\{5, \dfrac{1}{2}\right\}$.

74. $(3x + 5)(x - 3) = 5$

$3x^2 + 5x - 9x - 15 = 5$

$3x^2 - 4x - 20 = 0$

$x = \dfrac{4 \pm \sqrt{(-4)^2 - 4(3)(-20)}}{2(3)}$

$x = \dfrac{4 \pm \sqrt{16+240}}{6}$

$x = \dfrac{4 \pm \sqrt{256}}{6}$

$x = \dfrac{4 \pm 16}{6}$

$x = \dfrac{20}{6}, \dfrac{-12}{6}$

$x = \dfrac{10}{3}, -2$

The solution set is $\left\{-2, \dfrac{10}{3}\right\}$.

75. $3x^2 - 7x + 1 = 0$

$x = \dfrac{7 \pm \sqrt{(-7)^2 - 4(3)(1)}}{2(3)}$

$x = \dfrac{7 \pm \sqrt{49 - 12}}{6}$

$x = \dfrac{7 \pm \sqrt{37}}{6}$

The solution set is $\left\{\dfrac{7+\sqrt{37}}{6}, \dfrac{7-\sqrt{37}}{6}\right\}$.

76. $x^2 - 9 = 0$

$x^2 = 9$

$x = \pm 3$

The solution set is {–3, 3}.

77. $(x-3)^2 - 25 = 0$

$(x-3)^2 = 25$

$x - 3 = \pm 5$

$x = 3 \pm 5$

$x = 8, -2$

The solution set is {8, –2}.

78. $3x^2 - x + 2 = 0$

$x = \dfrac{1 \pm \sqrt{(-1)^2 - 4(3)(2)}}{2(3)}$

$x = \dfrac{1 \pm \sqrt{1-24}}{6}$

$x = \dfrac{1 \pm \sqrt{-23}}{6}$

$x = \dfrac{1 \pm i\sqrt{23}}{6}$

The solution set is $\left\{\dfrac{1+i\sqrt{23}}{6}, \dfrac{1-i\sqrt{23}}{6}\right\}$.

79. $3x^2 - 10x = 8$

$3x^2 - 10x - 8 = 0$

$(3x+2)(x-4) = 0$

$3x + 2 = 0$ or $x - 4 = 0$

$3x = -2$ $\quad x = 4$

$x = -\dfrac{2}{3}$

The solution set is $\left\{-\dfrac{2}{3}, 4\right\}$.

80. $(x+2)^2 + 4 = 0$

$(x+2)^2 = -4$

$\sqrt{(x+2)^2} = \pm\sqrt{-4}$

$x + 2 = \pm 2i$

$x = -2 \pm 2i$

The solution set is $\{-2+2i, -2-2i\}$.

81. $\dfrac{5}{x+1} + \dfrac{x-1}{4} = 2$

$\dfrac{5 \cdot 4(x+1)}{x+1} + \dfrac{(x-1) \cdot 4(x+1)}{4} = 2 \cdot 4(x+1)$

$20 + (x-1)(x+1) = 8(x+1)$

$20 + x^2 - 1 = 8x + 8$

$x^2 - 8x - 11 = 0$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \dfrac{-(-8) \pm \sqrt{(-8)^2 - 4(1)(11)}}{2(1)}$

$x = \dfrac{8 \pm \sqrt{20}}{2}$

$x = \dfrac{8 \pm 2\sqrt{5}}{2}$

$x = 4 \pm \sqrt{5}$

The solution set is $\{4+\sqrt{5}, 4-\sqrt{5}\}$.

82. $W(t) = 3t^2$

$588 = 3t^2$

$196 = t^2$

Apply the square root property.

$t^2 = 196$

$t = \pm\sqrt{196}$

$t = \pm 14$

The solutions are –14 and 14. We disregard –14, because we cannot have a negative time measurement. The fetus will weigh 588 grams after 14 weeks.

83. **a.** $G = -82x^2 + 410x + 7079$

$G = -82(6)^2 + 410(6) + 7079$

$= 6587$

The model estimates the aid per student in 2011 was $6587. This underestimates the actual number shown in the bar graph by $13.

b.
$$G = -82x^2 + 410x + 7079$$
$$4127 = -82x^2 + 410x + 7079$$
$$82x^2 - 410x - 2952 = 0$$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{-(-410) \pm \sqrt{(-410)^2 - 4(82)(-2952)}}{2(82)}$$
$$x = \frac{410 \pm \sqrt{1136356}}{164}$$
$$x = \frac{410 \pm 1066}{164}$$
$$x = 9 \text{ or } -4$$

The model projects the government aid per student will be $4127 9 years after 2005, or 2014.

84.
$$A = lw$$
$$15 = l(2l - 7)$$
$$15 = 2l^2 - 7l$$
$$0 = 2l^2 - 7l - 15$$
$$0 = (2l + 3)(l - 5)$$
$$l = 5$$
$$2l - 7 = 3$$

The length is 5 yards, the width is 3 yards.

85. Let x = height of building
$2x$ = shadow height
$$x^2 + (2x)^2 = 300^2$$
$$x^2 + 4x^2 = 90{,}000$$
$$5x^2 = 90{,}000$$
$$x^2 = 18{,}000$$
$$x \approx \pm 134.164$$

Discard negative height.
The building is approximately 134 meters high.

86.
$$2x^4 = 50x^2$$
$$2x^4 - 50x^2 = 0$$
$$2x^2\left(x^2 - 25\right) = 0$$
$$x = 0$$
$$x = \pm 5$$

The solution set is {–5, 0, 5}.

87.
$$2x^3 - x^2 - 18x + 9 = 0$$
$$x^2(2x - 1) - 9(2x - 1) = 0$$
$$\left(x^2 - 9\right)(2x - 1) = 0$$
$$x = \pm 3, \; x = \frac{1}{2}$$

The solution set is $\left\{-3, \frac{1}{2}, 3\right\}$.

88.
$$\sqrt{2x - 3} + x = 3$$
$$\sqrt{2x - 3} = 3 - x$$
$$2x - 3 = 9 - 6x + x^2$$
$$x^2 - 8x + 12 = 0$$
$$x^2 - 8x = -12$$
$$x^2 - 8x + 16 = -12 + 16$$
$$(x - 4)^2 = 4$$
$$x - 4 = \pm 2$$
$$x = 4 + 2$$
$$x = 6, 2$$

The solution set is {2}.

89.
$$\sqrt{x - 4} + \sqrt{x + 1} = 5$$
$$\sqrt{x - 4} = 5 - \sqrt{x + 1}$$
$$x - 4 = 25 - 10\sqrt{x + 1} + (x + 1)$$
$$x - 4 = 26 + x - 10\sqrt{x + 1}$$
$$-30 = -10\sqrt{x + 1}$$
$$3 = \sqrt{x + 1}$$
$$9 = x + 1$$
$$x = 8$$

The solution set is {8}.

90.
$$3x^{\frac{3}{4}} - 24 = 0$$
$$3x^{\frac{3}{4}} = 24$$
$$x^{\frac{3}{4}} = 8$$
$$\left(x^{\frac{3}{4}}\right)^{\frac{4}{3}} = (8)^{\frac{4}{3}}$$
$$x = 16$$

The solution set is {16}.

91. $(x-7)^{\frac{2}{3}} = 25$

$\left[(x-7)^{\frac{2}{3}}\right]^{\frac{3}{2}} = 25^{\frac{3}{2}}$

$x-7 = \left(5^2\right)^{\frac{3}{2}}$

$x-7 = 5^3$

$x-7 = 125$

$x = 132$

The solution set is {132}.

92. $x^4 - 5x^2 + 4 = 0$

Let $t = x^2$

$t^2 - 5t + 4 = 0$

$t = 4$ or $t = 1$

$x^2 = 4$ $\quad$ $x^2 = 1$

$x = \pm 2$ $\quad$ $x = \pm 1$

The solution set is {–2, –1, 1, 2}.

93. $x^{1/2} + 3x^{1/4} - 10 = 0$

Let $t = x^{1/4}$

$t^2 + 3t - 10 = 0$

$(t+5)(t-2) = 0$

$t = -5$ or $t = 2$

$x^{\frac{1}{4}} = -5$ $\quad$ $x^{\frac{1}{4}} = 2$

$\left(x^{\frac{1}{4}}\right)^4 = (-5)^4$ $\quad$ $\left(x^{\frac{1}{4}}\right)^4 = (2)^4$

$x = 625$ $\quad$ $x = 16$

625 does not check and must be rejected.
The solution set is {16}.

94. $|2x+1| = 7$

$2x+1 = 7$ or $2x+1 = -7$

$2x = 6$ $\quad$ $2x = -8$

$x = 3$ $\quad$ $x = -8$

The solution set is {–4, 3}.

95. $2|x-3| - 6 = 10$

$2|x-3| = 16$

$|x-3| = 8$

$x-3 = 8$ or $x-3 = -8$

$x = 11$ $\quad$ $x = -5$

The solution set is {–5, 11}.

96. $3x^{4/3} - 5x^{2/3} + 2 = 0$

Let $t = x^{\frac{2}{3}}$.

$3t^2 - 5t + 2 = 0$

$(3t-2)(t-1) = 0$

$3t - 2 = 0$ or $t - 1 = 0$

$3t = 2$ $\quad$ $t = 1$

$t = \frac{2}{3}$ $\quad$ $x^{\frac{2}{3}} = 1$

$x^{\frac{2}{3}} = \frac{2}{3}$ $\quad$ $\left(x^{\frac{2}{3}}\right)^{\frac{3}{2}} = \pm(1)^{\frac{3}{2}}$

$\left(x^{\frac{2}{3}}\right)^{\frac{3}{2}} = \pm\left(\frac{2}{3}\right)^{\frac{3}{2}}$ $\quad$ $x = \pm 1$

$x = \pm\sqrt[2]{\left(\frac{2}{3}\right)^3}$

$x = \pm\frac{2}{3}\sqrt{\frac{2}{3}}$

$x = \pm\frac{2}{3}\cdot\frac{\sqrt{2}}{\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}}$

$x = \pm\frac{2\sqrt{6}}{9}$

The solution set is $\left\{-\frac{2\sqrt{6}}{9}, \frac{2\sqrt{6}}{9}, -1, 1\right\}$.

97. $2\sqrt{x-1} = x$

$4(x-1) = x^2$

$4x - 4 = x^2$

$x^2 - 4x + 4 = 0$

$(x-2)^2 = 0$

$x = 2$

The solution set is {2}.

98. $|2x-5| - 3 = 0$

$2x-5 = 3$ or $2x-5 = -3$

$2x = 8$ $\quad$ $2x = 2$

$x = 4$ $\quad$ $x = 1$

The solution set is {4, 1}.

99. $x^3+2x^2-9x-18=0$
$x^2(x+2)-9(x+2)=0$
$(x+2)(x^2-9)=0$
$(x+2)(x+3)(x-3)=0$
The solution set is $\{-3, -2, 3\}$.

100. $\sqrt{8-2x}-x=0$
$\sqrt{8-2x}=x$
$\left(\sqrt{8-2x}\right)^2=(x)^2$
$8-2x=x^2$
$0=x^2+2x-8$
$0=(x+4)(x-2)$
$x+4=0$ or $x-2=0$
$x=-4$ $\quad x=2$
–4 does not check.
The solution set is $\{2\}$.

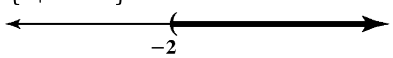
101. $x^3+3x^2-2x-6=0$
$x^2(x+3)-2(x+3)=0$
$(x+3)(x^2-2)=0$
$x+3=0$ or $x^2-2=0$
$x=-3$ $\quad x^2=2$
$x=\pm\sqrt{2}$
The solution set is $\{-3,-\sqrt{2},\sqrt{2}\}$.

102. $-4|x+1|+12=0$
$-4|x+1|=-12$
$|x+1|=3$
$x+1=3$ or $x+1=-3$
$x=2$ $\quad x=-4$
The solution set is $\{-4, 2\}$.

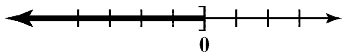
103. $p=-2.5\sqrt{t}+17$
$7=-2.5\sqrt{t}+17$
$-10=-2.5\sqrt{t}$
$4=\sqrt{t}$
$16=t$
The percentage dropped to 7% 16 years after 1993, or 2009.

104. $\{x|-3\le x<5\}$

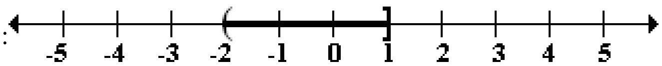
105. $\{x|x > -2\}$



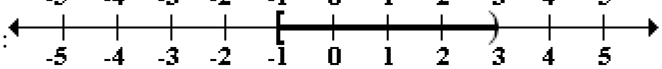
106. $\{x|x \le 0\}$



107. Graph $(-2,1]$:

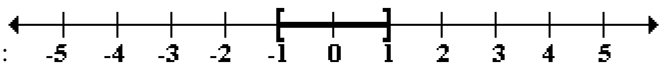


Graph $[-1,3)$:



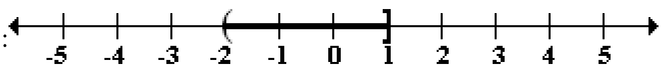
To find the intersection, take the portion of the number line that the two graphs have in common.

Numbers in both $(-2,1]$ and $[-1,3)$:

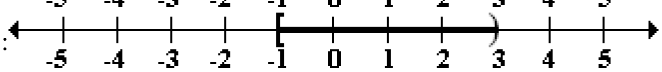


Thus, $(-2,1] \cap [-1,3) = [-1,1]$.

108. Graph $(-2,1]$:

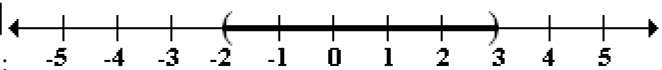


Graph $[-1,3)$:



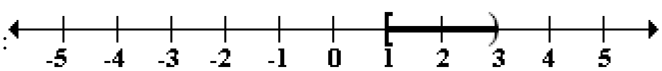
To find the union, take the portion of the number line representing the total collection of numbers in the two graphs.

Numbers in either $(-2,1]$ or $[-1,3)$ or both:

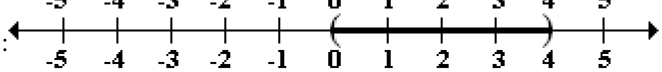


Thus, $(-2,1] \cup [-1,3) = (-2,3)$.

109. Graph $[1,3)$:

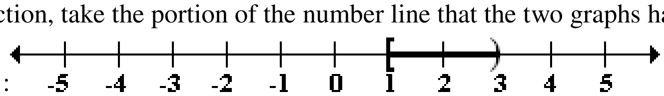


Graph $(0,4)$:



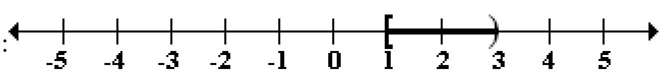
To find the intersection, take the portion of the number line that the two graphs have in common.

Numbers in both $[1,3)$ and $(0,4)$:

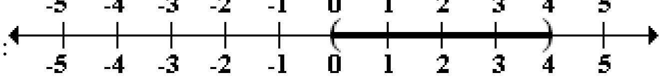


Thus, $[1,3) \cap (0,4) = [1,3)$.

110. Graph $[1,3)$:

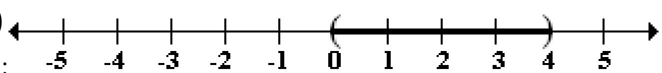


Graph $(0,4)$:



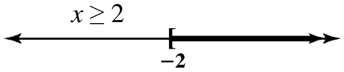
To find the union, take the portion of the number line representing the total collection of numbers in the two graphs.

Numbers in either $[1,3)$ or $(0,4)$ or both:



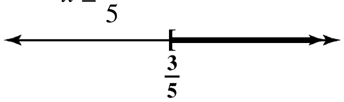
Thus, $[1,3) \cup (0,4) = (0,4)$.

111. $-6x + 3 \le 15$
$-6x \le 12$
$x \ge 2$



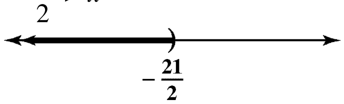
The solution set is $[-2, \infty)$.

112. $6x - 9 \ge -4x - 3$
$10x \ge 6$
$x \ge \frac{3}{5}$



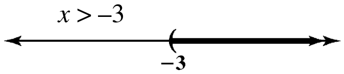
The solution set is $\left[\frac{3}{5}, \infty\right)$.

113. $\frac{x}{3} - \frac{3}{4} - 1 > \frac{x}{2}$
$12\left(\frac{x}{3} - \frac{3}{4} - 1\right) > 12\left(\frac{x}{2}\right)$
$4x - 9 - 12 > 6x$
$-21 > 2x$
$-\frac{21}{2} > x$



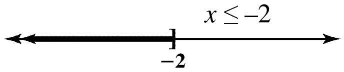
The solution set is $\left(-\infty, -\frac{21}{2}\right)$.

114. $6x + 5 > -2(x - 3) - 25$
$6x + 5 > -2x + 6 - 25$
$8x + 5 > -19$
$8x > -24$
$x > -3$



The solution set is $(-3, \infty)$.

115. $3(2x - 1) - 2(x - 4) \ge 7 + 2(3 + 4x)$
$6x - 3 - 2x + 8 \ge 7 + 6 + 8x$
$4x + 5 \ge 8x + 13$
$-4x \ge 8$
$x \le -2$



The solution set is $[-\infty, -2)$.

116. $5(x - 2) - 3(x + 4) \ge 2x - 20$
$5x - 10 - 3x - 12 \ge 2x - 20$
$2x - 22 \ge 2x - 20$
$-22 \ge -20$
The solution set is $\varnothing$.

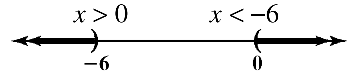
117. $7 < 2x + 3 \le 9$
$4 < 2x \le 6$
$2 < x \le 3$
(2, 3]

The solution set is $[2, 3)$.

118. $|2x + 3| \le 15$
$-15 \le 2x + 3 \le 15$
$-18 \le 2x \le 12$
$-9 \le x \le 6$

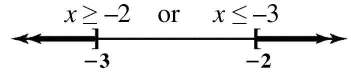
The solution set is $[-9, 6]$.

119. $\left|\frac{2x + 6}{3}\right| > 2$
$\frac{2x+6}{3} > 2 \quad \frac{2x+6}{3} < -2$
$2x + 6 > 6 \quad 2x + 6 < -6$
$2x > 0 \quad 2x < -12$
$x > 0 \quad x < -6$



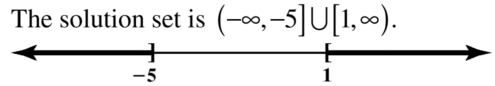
The solution set is $(-\infty, -6)$ or $(0, \infty)$.

120. $|2x + 5| - 7 \ge -6$
$|2x + 5| \ge 1$
$2x + 5 \ge 1$ or $2x + 5 \le -1$
$2x \ge -4 \quad 2x \le -6$
$x \ge -2$ or $x \le -3$



The solution set is $(-\infty, -3]$ or $[-2, \infty)$.

121. $-4|x + 2| + 5 \le -7$
$-4|x + 2| \le -12$
$|x + 2| \ge 3$
$x + 2 \ge 3$ or $x + 2 \le -3$
$x \ge 1 \quad x \le -5$
The solution set is $(-\infty, -5] \cup [1, \infty)$.



122.
$$\begin{aligned} y_1 &> y_2 \\ -10-3(2x+1) &> 8x+1 \\ -10-6x-3 &> 8x+1 \\ -6x-13 &> 8x+1 \\ -14x &> 14 \\ \frac{-14x}{-14} &< \frac{14}{-14} \\ x &< -1 \end{aligned}$$
The solution set is $(-\infty,-1)$.

123.
$$\begin{aligned} 3-|2x-5| &\ge -6 \\ -|2x-5| &\ge -9 \\ \frac{-|2x-5|}{-1} &\le \frac{-9}{-1} \\ |2x-5| &\le 9 \\ -9 \le 2x-5 &\le 9 \\ -4 \le 2x &\le 14 \\ -2 \le x &\le 7 \end{aligned}$$
The solution set is $[-2,7]$.

124.
$$\begin{aligned} 0.20x+24 &\le 40 \\ 0.20x &\le 16 \\ \frac{0.20x}{0.20} &\le \frac{16}{0.20} \\ x &\le 80 \end{aligned}$$
A customer can drive no more than 80 miles.

125.
$$\begin{aligned} 80 \le \frac{95+79+91+86+x}{5} &< 90 \\ 400 \le 95+79+91+86+x &< 450 \\ 400 \le 351+x &< 450 \\ 49 \le x &< 99 \end{aligned}$$
A grade of at least 49% but less than 99% will result in a B.

126.
$$\begin{aligned} 0.075x &\ge 9000 \\ \frac{0.075x}{0.075} &\ge \frac{9000}{0.075} \\ x &\ge 120{,}000 \end{aligned}$$
The investment must be at least $120,000.

Chapter 1 Test

1.
$$\begin{aligned} 7(x-2) &= 4(x+1)-21 \\ 7x-14 &= 4x+4-21 \\ 7x-14 &= 4x-17 \\ 3x &= -3 \\ x &= -1 \end{aligned}$$
The solution set is $\{-1\}$.

2.
$$\begin{aligned} -10-3(2x+1)-8x-1 &= 0 \\ -10-6x-3-8x-1 &= 0 \\ -14x-14 &= 0 \\ -14x &= 14 \\ x &= -1 \end{aligned}$$
The solution set is $\{-1\}$.

3.
$$\begin{aligned} \frac{2x-3}{4} &= \frac{x-4}{2}-\frac{x+1}{4} \\ 2x-3 &= 2(x-4)-(x+1) \\ 2x-3 &= 2x-8-x-1 \\ 2x-3 &= x-9 \\ x &= -6 \end{aligned}$$
The solution set is $\{-6\}$.

4.
$$\begin{aligned} \frac{2}{x-3}-\frac{4}{x+3} &= \frac{8}{(x-3)(x+3)} \\ 2(x+3)-4(x-3) &= 8 \\ 2x+6-4x+12 &= 8 \\ -2x+18 &= 8 \\ -2x &= -10 \\ x &= 5 \end{aligned}$$
The solution set is $\{5\}$.

5.
$$\begin{aligned} &2x^2-3x-2 = 0 \\ &(2x+1)(x-2) = 0 \\ &2x+1=0 \quad \text{or} \quad x-2=0 \\ &x=-\frac{1}{2} \quad \text{or} \quad x=2 \end{aligned}$$
The solution set is $\left\{-\frac{1}{2}, 2\right\}$.

6.
$$\begin{aligned} (3x-1)^2 &= 75 \\ 3x-1 &= \pm\sqrt{75} \\ 3x &= 1\pm 5\sqrt{3} \\ x &= \frac{1\pm 5\sqrt{3}}{3} \end{aligned}$$
The solution set is $\left\{\frac{1-5\sqrt{3}}{3}, \frac{1+5\sqrt{3}}{3}\right\}$.

7. $(x+3)^2+25=0$

$(x+3)^2=-25$

$x+3=\pm\sqrt{-25}$

$x=-3\pm 5i$

The solution set is $\{-3+5i, -3-5i\}$.

8. $x(x-2)=4$

$x^2-2x-4=0$

$x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

$x=\dfrac{2\pm\sqrt{(-2)^2-4(1)(-4)}}{2}$

$x=\dfrac{2\pm 2\sqrt{5}}{2}$

$x=1\pm\sqrt{5}$

The solution set is $\{1-\sqrt{5}, 1+\sqrt{5}\}$.

9. $4x^2=8x-5$

$4x^2-8x+5=0$

$x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

$x=\dfrac{8\pm\sqrt{(-8)^2-4(4)(5)}}{2(4)}$

$x=\dfrac{8\pm\sqrt{-16}}{8}$

$x=\dfrac{8\pm 4i}{8}$

$x=1\pm\dfrac{1}{2}i$

The solution set is $\left\{1+\dfrac{1}{2}i, 1-\dfrac{1}{2}i\right\}$.

10. $x^3-4x^2-x+4=0$

$x^2(x-4)-1(x-4)=0$

$(x^2-1)(x-4)=0$

$(x-1)(x+1)(x-4)=0$

$x=1$ or $x=-1$ or $x=4$

The solution set is $\{-1, 1, 4\}$.

11. $\sqrt{x-3}+5=x$

$\sqrt{x-3}=x-5$

$x-3=x^2-10x+25$

$x^2-11x+28=0$

$x=\dfrac{11\pm\sqrt{11^2-4(1)(28)}}{2(1)}$

$x=\dfrac{11\pm\sqrt{121-112}}{2}$

$x=\dfrac{11\pm\sqrt{9}}{2}$

$x=\dfrac{11\pm 3}{2}$

$x=7$ or $x=4$

4 does not check and must be rejected.

The solution set is $\{7\}$.

12. $\sqrt{8-2x}-x=0$

$\sqrt{8-2x}=x$

$\left(\sqrt{8-2x}\right)^2=(x)^2$

$8-2x=x^2$

$0=x^2+2x-8$

$0=(x+4)(x-2)$

$x+4=0$ or $x-2=0$

$x=-4$ $\quad$ $x=2$

–4 does not check and must be rejected.

The solution set is $\{2\}$.

13. $\sqrt{x+4}+\sqrt{x-1}=5$

$\sqrt{x+4}=5-\sqrt{x-1}$

$x+4=25-10\sqrt{x-1}+(x-1)$

$x+4=25-10\sqrt{x-1}+x-1$

$-20=-10\sqrt{x-1}$

$2=\sqrt{x-1}$

$4=x-1$

$x=5$

The solution set is $\{5\}$.

14. $5x^{3/2} - 10 = 0$

$5x^{3/2} = 10$

$x^{3/2} = 2$

$x = 2^{2/3}$

$x = \sqrt[3]{4}$

The solution set is $\left\{\sqrt[3]{4}\right\}$.

15. $x^{2/3} - 9x^{1/3} + 8 = 0$ let $t = x^{1/3}$

$t^2 - 9t + 8 = 0$

$(t-1)(t-8) = 0$

$t = 1 \qquad t = 8$

$x^{1/3} = 1 \quad x^{1/3} = 8$

$x = 1 \qquad x = 512$

The solution set is {1, 512}.

16. $\left|\frac{2}{3}x - 6\right| = 2$

$\frac{2}{3}x - 6 = 2 \qquad \frac{2}{3}x - 6 = -2$

$\frac{2}{3}x = 8 \qquad \frac{2}{3}x = 4$

$x = 12 \qquad x = 6$

The solution set is {6, 12}.

17. $-3|4x-7| + 15 = 0$

$-3|4x-7| = -15$

$|4x-7| = 5$

$4x - 7 = 5$ or $4x - 7 = -5$

$4x = 12 \qquad 4x = 2$

$x = 3 \qquad x = \frac{1}{2}$

The solution set is $\left\{\frac{1}{2}, 3\right\}$

18. $\frac{1}{x^2} - \frac{4}{x} + 1 = 0$

$\frac{x^2}{x^2} - \frac{4x^2}{x} + x^2 = 0$

$1 - 4x + x^2 = 0$

$x^2 - 4x + 1 = 0$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(1)}}{2(1)}$

$x = \frac{4 \pm \sqrt{12}}{2}$

$x = \frac{4 \pm 2\sqrt{3}}{2}$

$x = 2 \pm \sqrt{3}$

The solution set is $\left\{2 + \sqrt{3}, 2 - \sqrt{3}\right\}$.

19. $\frac{2x}{x^2 + 6x + 8} + \frac{2}{x+2} = \frac{x}{x+4}$

$\frac{2x}{(x+4)(x+2)} + \frac{2}{x+2} = \frac{x}{x+4}$

$\frac{2x(x+4)(x+2)}{(x+4)(x+2)} + \frac{2(x+4)(x+2)}{x+2} = \frac{x(x+4)(x+2)}{x+4}$

$2x + 2(x+4) = x(x+2)$

$2x + 2x + 8 = x^2 + 2x$

$2x + 8 = x^2$

$0 = x^2 - 2x - 8$

$0 = (x-4)(x+2)$

$x - 4 = 0$ or $x + 2 = 0$

$x = 4 \qquad x = -2$ (rejected)

The solution set is $\{4\}$.

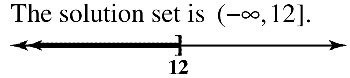
20. $3(x + 4) \geq 5x - 12$

$3x + 12 \geq 5x - 12$

$-2x \geq -24$

$x \leq 12$

The solution set is $(-\infty, 12]$.



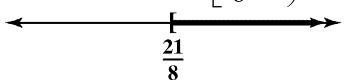
21. $\frac{x}{6}+\frac{1}{8}\le\frac{x}{2}-\frac{3}{4}$

$4x+3\le 12x-18$

$-8x\le -21$

$x\ge\frac{21}{8}$

The solution set is $\left[\frac{21}{8},\infty\right)$.



22. $-3\le\frac{2x+5}{3}<6$

$-9\le 2x+5<18$

$-14\le 2x<13$

$-7\le x<\frac{13}{2}$

The solution set is $\left[-7,\frac{13}{2}\right)$.

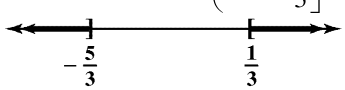
23. $|3x+2|\ge 3$

$3x+2\ge 3$ or $3x+2\le -3$

$3x\ge 1$ $\quad 3x\le -5$

$x\ge\frac{1}{3}$ $\quad x\le -\frac{5}{3}$

The solution set is $\left(-\infty,-\frac{5}{3}\right]\cup\left[\frac{1}{3},\infty\right)$.



24. $-3\le y\le 7$

$-3\le 2x-5\le 7$

$2\le 2x\le 12$

$1\le x\le 6$

The solution set is $[1,6]$.

25. $y\ge 1$

$\left|\frac{2-x}{4}\right|\ge 1$

$\frac{2-x}{4}\ge 1$ or $\frac{2-x}{4}\le -1$

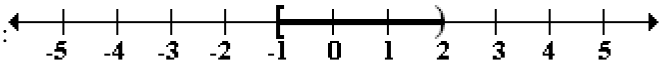
$2-x\ge 4$ $\quad 2-x\le -4$

$-x\ge 2$ $\quad -x\le -6$

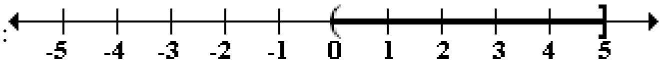
$x\le -2$ $\quad x\ge 6$

The solution set is $(-\infty,-2]\cup[6,\infty)$.

26. Graph $[-1,2)$:

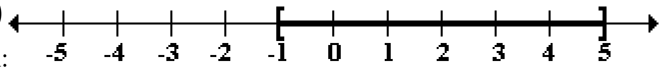


Graph $(0,5]$:



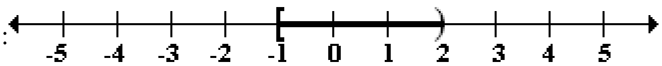
To find the union, take the portion of the number line representing the total collection of numbers in the two graphs.

Numbers in either $[-1,2)$ or $(0,5]$ or both:

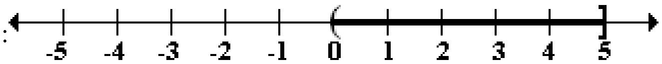


Thus,
$[-1,2) \cup (0,5] = [-1,5]$.

27. Graph $[-1,2)$:

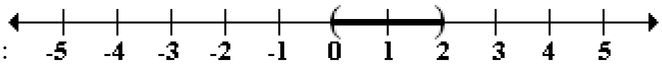


Graph $(0,5]$:



To find the intersection, take the portion of the number line that the two graphs have in common.

Numbers in both $[-1,2)$ and $(0,5]$:



Thus, $[-1,2) \cap (0,5] = (0,2)$.

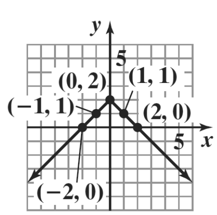
28.

$$V = \frac{1}{3}lwh$$
$$3V = lwh$$
$$\frac{3V}{lw} = \frac{lwh}{lw}$$
$$\frac{3V}{lw} = h$$
$$h = \frac{3V}{lw}$$

29.

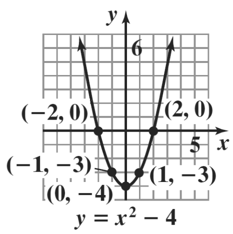
$$y - y_1 = m(x - x_1)$$
$$y - y_1 = mx - mx_1$$
$$-mx = y_1 - mx_1 - y$$
$$\frac{-mx}{-m} = \frac{y_1 - mx_1 - y}{-m}$$
$$x = \frac{y - y_1}{m} + x_1$$

30.



$y = 2 - |x|$

31.



32. $(6-7i)(2+5i) = 12+30i-14i-35i^2$
$= 12+16i+35$
$= 47+16i$

33. $\frac{5}{2-i} = \frac{5}{2-i}\cdot\frac{2+i}{2+i}$
$= \frac{5(2+i)}{4+1}$
$= \frac{5(2+i)}{5}$
$= 2+i$

34. $2\sqrt{-49}+3\sqrt{-64} = 2(7i)+3(8i)$
$= 14i+24i$
$= 38i$

35. $43x+575 = 1177$
$43x = 602$
$x = 14$
The system's income will be \$1177 billion 14 years after 2004, or 2018.

36. $B = 0.07x^2+47.4x+500$
$1177 = 0.07x^2+47.4x+500$
$0 = 0.07x^2+47.4x-677$
$0 = 0.07x^2+47.4x-677$
$x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$
$x = \frac{-(47.4)\pm\sqrt{(47.4)^2-4(0.07)(-677)}}{2(0.07)}$
$x \approx 14, \quad x \approx -691$ (rejected)
The system's income will be \$1177 billion 14 years after 2004, or 2018.

37. The formulas model the data quite well.

38. Let $x =$ the percentage of strikingly-attractive men.
Let $x+57 =$ the percentage of average-looking men.
Let $x+25 =$ the percentage of good-looking men.
$(x)+(x+57)+(x+25) = 88$
$x+x+57+x+25 = 88$
$3x+82 = 88$
$3x = 6$
$x = 2$
$x+57 = 59$
$x+25 = 27$
2% of men are strikingly-attractive.
59% of men are average-looking.
27% of men are good-looking.

39. $29700+150x = 5000+1100x$
$24700 = 950x$
$26 = x$
In 26 years, the cost will be \$33,600.

40. Let x = amount invested at 8%
$10000 - x$ = amount invested at 10%
$0.08x+0.1(10000-x) = 940$
$0.08x+1000-0.1x = 940$
$-0.02x = -60$
$x = 3000$
$10000-x = 7000$
\$3000 at 8%, \$7000 at 10%

41. $l = 2w+4$
$A = lw$
$48 = (2w+4)w$
$48 = 2w^2+4w$
$0 = 2w^2+4w-48$
$0 = w^2+2w-24$
$0 = (w+6)(w-4)$

$w+6 = 0 \qquad w-4 = 0$
$w = -6 \qquad w = 4$
$2w+4 = 2(4)+4 = 12$
width is 4 feet, length is 12 feet

42. $24^2+x^2 = 26^2$
$576+x^2 = 676$
$x^2 = 100$
$x = \pm 10$
The wire should be attached 10 feet up the pole.

43. Let x = the original selling price

$20 = x - 0.60x$

$20 = 0.40x$

$50 = x$

The original price is \$50.

44. Let x = the number text messages.
The monthly cost using Plan A is $C_A = 25$.
The monthly cost using Plan B is
$C_B = 13 + 0.06x$.
For Plan A to be better deal, it must cost less than Plan B.

$C_A < C_B$

$25 < 13 + 0.06x$

$12 < 0.06x$

$200 < x$

$x > 200$

Plan A is a better deal when more than 200 text messages per month are sent/received.

Chapter 2
Functions and Graphs

Section 2.1

Check Point Exercises

1. The domain is the set of all first components: {0, 10, 20, 30, 42}. The range is the set of all second components: {9.1, 6.7, 10.7, 13.2, 21.7}.

2. **a.** The relation is not a function since the two ordered pairs (5, 6) and (5, 8) have the same first component but different second components.

b. The relation is a function since no two ordered pairs have the same first component and different second components.

3. **a.** $2x + y = 6$

$y = 6 - 2x$

For each value of *x*, there is one and only one value for *y*, so the equation defines *y* as a function of *x*.

b. $x^2 + y^2 = 1$

$y^2 = 1 - x^2$

$y = \pm\sqrt{1 - x^2}$

Since there are values of *x* (all values between –1 and 1 exclusive) that give more than one value for *y* (for example, if *x* = 0, then $y = \pm\sqrt{1-0^2} = \pm 1$), the equation does not define *y* as a function of *x*.

4. **a.** $f(-5) = (-5)^2 - 2(-5) + 7$

$= 25 - (-10) + 7$

$= 42$

b. $f(x+4) = (x+4)^2 - 2(x+4) + 7$

$= x^2 + 8x + 16 - 2x - 8 + 7$

$= x^2 + 6x + 15$

c. $f(-x) = (-x)^2 - 2(-x) + 7$

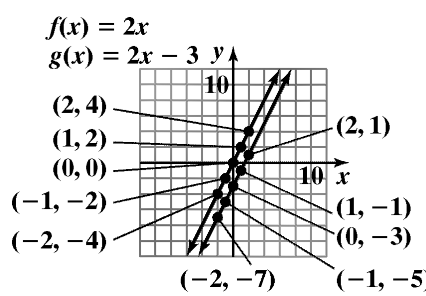
$= x^2 - (-2x) + 7$

$= x^2 + 2x + 7$

5.

x	$f(x) = 2x$	(x, y)
-2	–4	$(-2,-4)$
-1	–2	$(-1,-2)$
0	0	$(0,0)$
1	2	$(1,2)$
2	4	$(2,4)$

x	$g(x) = 2x - 3$	(x, y)
-2	$g(-2) = 2(-2) - 3 = -7$	$(-2,-7)$
-1	$g(-1) = 2(-1) - 3 = -5$	$(-1,-5)$
0	$g(0) = 2(0) - 3 = -3$	$(0,-3)$
1	$g(1) = 2(1) - 3 = -1$	$(1,-1)$
2	$g(2) = 2(2) - 3 = 1$	$(2,1)$



The graph of *g* is the graph of *f* shifted down 3 units.

6. The graph (a) passes the vertical line test and is therefore is a function.
The graph (b) fails the vertical line test and is therefore not a function.
The graph (c) passes the vertical line test and is therefore is a function.
The graph (d) fails the vertical line test and is therefore not a function.

7. **a.** $f(5) = 400$

b. $x = 9$, $f(9) = 100$

c. The minimum T cell count in the asymptomatic stage is approximately 425.

8. **a.** domain: $\{x|-2 \le x \le 1\}$ or $[-2,1]$.
range: $\{y|0 \le y \le 3\}$ or $[0,3]$.

b. domain: $\{x|-2 < x \le 1\}$ or $(-2,1]$.
range: $\{y|-1 \le y < 2\}$ or $[-1,2)$.

c. domain: $\{x|-3 \le x < 0\}$ or $[-3,0)$.
range: $\{y|y = -3,-2,-1\}$.

Concept and Vocabulary Check 2.1

1. relation; domain; range

2. function

3. f; x

4. true

5. false

6. x; $x+6$

7. ordered pairs

8. more than once; function

9. $[0,3)$; domain

10. $[1,\infty)$; range

11. 0; 0; zeros

12. false

Exercise Set 2.1

1. The relation is a function since no two ordered pairs have the same first component and different second components. The domain is {1, 3, 5} and the range is {2, 4, 5}.

2. The relation is a function because no two ordered pairs have the same first component and different second components The domain is {4, 6, 8} and the range is {5, 7, 8}.

3. The relation is not a function since the two ordered pairs (3, 4) and (3, 5) have the same first component but different second components (the same could be said for the ordered pairs (4, 4) and (4, 5)). The domain is {3, 4} and the range is {4, 5}.

4. The relation is not a function since the two ordered pairs (5, 6) and (5, 7) have the same first component but different second components (the same could be said for the ordered pairs (6, 6) and (6, 7)). The domain is {5, 6} and the range is {6, 7}.

5. The relation is a function because no two ordered pairs have the same first component and different second components The domain is {3, 4, 5, 7} and the range is {–2, 1, 9}.

6. The relation is a function because no two ordered pairs have the same first component and different second components The domain is {–2, –1, 5, 10} and the range is {1, 4, 6}.

7. The relation is a function since there are no same first components with different second components. The domain is {–3, –2, –1, 0} and the range is {–3, –2, –1, 0}.

8. The relation is a function since there are no ordered pairs that have the same first component but different second components. The domain is {–7, –5, –3, 0} and the range is {–7, –5, –3, 0}.

9. The relation is not a function since there are ordered pairs with the same first component and different second components. The domain is {1} and the range is {4, 5, 6}.

10. The relation is a function since there are no two ordered pairs that have the same first component and different second components. The domain is {4, 5, 6} and the range is {1}.

11. $x + y = 16$

$y = 16 - x$

Since only one value of y can be obtained for each value of x, y is a function of x.

12. $x + y = 25$

$y = 25 - x$

Since only one value of y can be obtained for each value of x, y is a function of x.

13. $x^2 + y = 16$

$y = 16 - x^2$

Since only one value of y can be obtained for each value of x, y is a function of x.

14. $x^2 + y = 25$

$y = 25 - x^2$

Since only one value of y can be obtained for each value of x, y is a function of x.

15. $x^2 + y^2 = 16$

$y^2 = 16 - x^2$

$y = \pm\sqrt{16 - x^2}$

If $x = 0$, $y = \pm 4$.

Since two values, $y = 4$ and $y = -4$, can be obtained for one value of x, y is not a function of x.

16. $x^2 + y^2 = 25$

$y^2 = 25 - x^2$

$y = \pm\sqrt{25 - x^2}$

If $x = 0$, $y = \pm 5$.

Since two values, $y = 5$ and $y = -5$, can be obtained for one value of x, y is not a function of x.

17. $x = y^2$

$y = \pm\sqrt{x}$

If $x = 1$, $y = \pm 1$.

Since two values, $y = 1$ and $y = -1$, can be obtained for $x = 1$, y is not a function of x.

18. $4x = y2$

$y = \pm\sqrt{4x} = \pm 2\sqrt{x}$

If $x = 1$, then $y = \pm 2$.

Since two values, $y = 2$ and $y = -2$, can be obtained for $x = 1$, y is not a function of x.

19. $y = \sqrt{x + 4}$

Since only one value of y can be obtained for each value of x, y is a function of x.

20. $y = -\sqrt{x + 4}$

Since only one value of y can be obtained for each value of x, y is a function of x.

21. $x + y^3 = 8$

$y^3 = 8 - x$

$y = \sqrt[3]{8 - x}$

Since only one value of y can be obtained for each value of x, y is a function of x.

22. $x + y^3 = 27$

$y^3 = 27 - x$

$y = \sqrt[3]{27 - x}$

Since only one value of y can be obtained for each value of x, y is a function of x.

23. $xy + 2y = 1$

$y(x + 2) = 1$

$y = \dfrac{1}{x + 2}$

Since only one value of y can be obtained for each value of x, y is a function of x.

24. $xy - 5y = 1$

$y(x - 5) = 1$

$y = \dfrac{1}{x - 5}$

Since only one value of y can be obtained for each value of x, y is a function of x.

25. $|x| - y = 2$

$-y = -|x| + 2$

$y = |x| - 2$

Since only one value of y can be obtained for each value of x, y is a function of x.

26. $|x| - y = 5$

$-y = -|x| + 5$

$y = |x| - 5$

Since only one value of y can be obtained for each value of x, y is a function of x.

27. **a.** $f(6) = 4(6) + 5 = 29$

b. $f(x + 1) = 4(x + 1) + 5 = 4x + 9$

c. $f(-x) = 4(-x) + 5 = -4x + 5$

28. **a.** $f(4) = 3(4) + 7 = 19$

b. $f(x + 1) = 3(x + 1) + 7 = 3x + 10$

c. $f(-x) = 3(-x) + 7 = -3x + 7$

29. **a.** $$\begin{aligned} g(-1) &= (-1)^2 + 2(-1) + 3 \\ &= 1 - 2 + 3 \\ &= 2 \end{aligned}$$

b. $$\begin{aligned} g(x+5) &= (x+5)^2 + 2(x+5) + 3 \\ &= x^2 + 10x + 25 + 2x + 10 + 3 \\ &= x^2 + 12x + 38 \end{aligned}$$

c. $$\begin{aligned} g(-x) &= (-x)^2 + 2(-x) + 3 \\ &= x^2 - 2x + 3 \end{aligned}$$

30. **a.** $$\begin{aligned} g(-1) &= (-1)^2 - 10(-1) - 3 \\ &= 1 + 10 - 3 \\ &= 8 \end{aligned}$$

b. $$\begin{aligned} g(x+2) &= (x+2)^2 - 10(8+2) - 3 \\ &= x^2 + 4x + 4 - 10x - 20 - 3 \\ &= x^2 - 6x - 19 \end{aligned}$$

c. $$\begin{aligned} g(-x) &= (-x)^2 - 10(-x) - 3 \\ &= x^2 + 10x - 3 \end{aligned}$$

31. **a.** $$\begin{aligned} h(2) &= 2^4 - 2^2 + 1 \\ &= 16 - 4 + 1 \\ &= 13 \end{aligned}$$

b. $$\begin{aligned} h(-1) &= (-1)^4 - (-1)^2 + 1 \\ &= 1 - 1 + 1 \\ &= 1 \end{aligned}$$

c. $h(-x) = (-x)^4 - (-x)^2 + 1 = x^4 - x^2 + 1$

d. $$\begin{aligned} h(3a) &= (3a)^4 - (3a)^2 + 1 \\ &= 81a^4 - 9a^2 + 1 \end{aligned}$$

32. **a.** $h(3) = 3^3 - 3 + 1 = 25$

b. $$\begin{aligned} h(-2) &= (-2)^3 - (-2) + 1 \\ &= -8 + 2 + 1 \\ &= -5 \end{aligned}$$

c. $h(-x) = (-x)^3 - (-x) + 1 = -x^3 + x + 1$

d. $$\begin{aligned} h(3a) &= (3a)^3 - (3a) + 1 \\ &= 27a^3 - 3a + 1 \end{aligned}$$

33. **a.** $f(-6) = \sqrt{-6+6} + 3 = \sqrt{0} + 3 = 3$

b. $$\begin{aligned} f(10) &= \sqrt{10+6} + 3 \\ &= \sqrt{16} + 3 \\ &= 4 + 3 \\ &= 7 \end{aligned}$$

c. $f(x-6) = \sqrt{x-6+6} + 3 = \sqrt{x} + 3$

34. **a.** $f(16) = \sqrt{25-16} - 6 = \sqrt{9} - 6 = 3 - 6 = -3$

b. $$\begin{aligned} f(-24) &= \sqrt{25-(-24)} - 6 \\ &= \sqrt{49} - 6 \\ &= 7 - 6 = 1 \end{aligned}$$

c. $$\begin{aligned} f(25-2x) &= \sqrt{25-(25-2x)} - 6 \\ &= \sqrt{2x} - 6 \end{aligned}$$

35. **a.** $f(2) = \dfrac{4(2)^2 - 1}{2^2} = \dfrac{15}{4}$

b. $f(-2) = \dfrac{4(-2)^2 - 1}{(-2)^2} = \dfrac{15}{4}$

c. $f(-x) = \dfrac{4(-x)^2 - 1}{(-x)^2} = \dfrac{4x^2 - 1}{x^2}$

36. **a.** $f(2) = \dfrac{4(2)^3 + 1}{2^3} = \dfrac{33}{8}$

b. $f(-2) = \dfrac{4(-2)^3 + 1}{(-2)^3} = \dfrac{-31}{-8} = \dfrac{31}{8}$

c. $f(-x) = \dfrac{4(-x)^3 + 1}{(-x)^3} = \dfrac{-4x^3 + 1}{-x^3}$

or $\dfrac{4x^3 - 1}{x^3}$

37. **a.** $f(6) = \frac{6}{|6|} = 1$

b. $f(-6) = \frac{-6}{|-6|} = \frac{-6}{6} = -1$

c. $f(r^2) = \frac{r^2}{|r^2|} = \frac{r^2}{r^2} = 1$

38. **a.** $f(5) = \frac{|5+3|}{5+3} = \frac{|8|}{8} = 1$

b. $f(-5) = \frac{|-5+3|}{-5+3} = \frac{|-2|}{-2} = \frac{2}{-2} = -1$

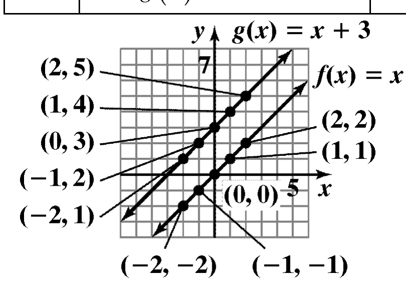
c. $f(-9-x) = \frac{|-9-x+3|}{-9-x+3}$

$= \frac{|-x-6|}{-x-6} = \begin{cases} 1, & \text{if } x < -6 \\ -1, & \text{if } x > -6 \end{cases}$

39.

x	$f(x) = x$	(x, y)
–2	$f(-2) = -2$	$(-2, -2)$
–1	$f(-1) = -1$	$(-1, -1)$
0	$f(0) = 0$	$(0, 0)$
1	$f(1) = 1$	$(1, 1)$
2	$f(2) = 2$	$(2, 2)$

x	$g(x) = x + 3$	(x, y)
–2	$g(-2) = -2 + 3 = 1$	$(-2, 1)$
–1	$g(-1) = -1 + 3 = 2$	$(-1, 2)$
0	$g(0) = 0 + 3 = 3$	$(0, 3)$
1	$g(1) = 1 + 3 = 4$	$(1, 4)$
2	$g(2) = 2 + 3 = 5$	$(2, 5)$

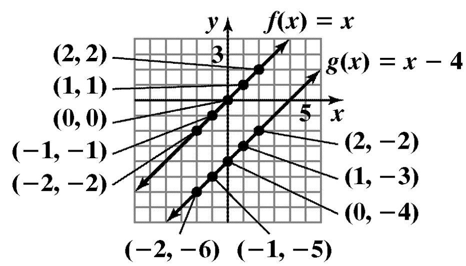


The graph of g is the graph of f shifted up 3 units.

40.

x	$f(x) = x$	(x, y)
–2	$f(-2) = -2$	$(-2, -2)$
–1	$f(-1) = -1$	$(-1, -1)$
0	$f(0) = 0$	$(0, 0)$
1	$f(1) = 1$	$(1, 1)$
2	$f(2) = 2$	$(2, 2)$

x	$g(x) = x - 4$	(x, y)
–2	$g(-2) = -2 - 4 = -6$	$(-2, -6)$
–1	$g(-1) = -1 - 4 = -5$	$(-1, -5)$
0	$g(0) = 0 - 4 = -4$	$(0, -4)$
1	$g(1) = 1 - 4 = -3$	$(1, -3)$
2	$g(2) = 2 - 4 = -2$	$(2, -2)$

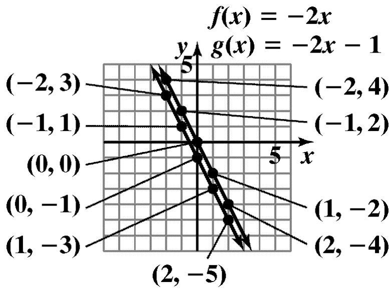


The graph of g is the graph of f shifted down 4 units.

41.

x	$f(x) = -2x$	(x, y)
–2	$f(-2) = -2(-2) = 4$	$(-2, 4)$
–1	$f(-1) = -2(-1) = 2$	$(-1, 2)$
0	$f(0) = -2(0) = 0$	$(0, 0)$
1	$f(1) = -2(1) = -2$	$(1, -2)$
2	$f(2) = -2(2) = -4$	$(2, -4)$

x	$g(x) = -2x - 1$	(x, y)
–2	$g(-2) = -2(-2) - 1 = 3$	$(-2, 3)$
–1	$g(-1) = -2(-1) - 1 = 1$	$(-1, 1)$
0	$g(0) = -2(0) - 1 = -1$	$(0, -1)$
1	$g(1) = -2(1) - 1 = -3$	$(1, -3)$
2	$g(2) = -2(2) - 1 = -5$	$(2, -5)$

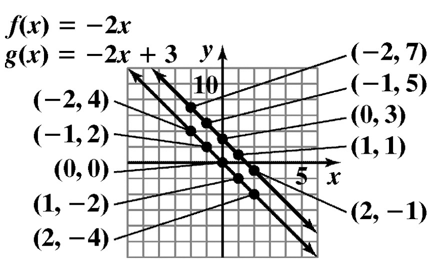


The graph of g is the graph of f shifted down 1 unit.

42.

x	$f(x)=-2x$	(x, y)
–2	$f(-2)=-2(-2)=4$	$(-2,4)$
–1	$f(-1)=-2(-1)=2$	$(-1,2)$
0	$f(0)=-2(0)=0$	$(0,0)$
1	$f(1)=-2(1)=-2$	$(1,-2)$
2	$f(2)=-2(2)=-4$	$(2,-4)$

x	$g(x)=-2x+3$	(x, y)
–2	$g(-2)=-2(-2)+3=7$	$(-2,7)$
–1	$g(-1)=-2(-1)+3=5$	$(-1,5)$
0	$g(0)=-2(0)+3=3$	$(0,3)$
1	$g(1)=-2(1)+3=1$	$(1,1)$
2	$g(2)=-2(2)+3=-1$	$(2,-1)$

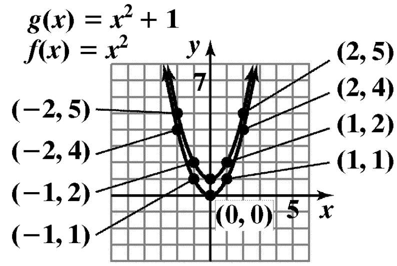


The graph of g is the graph of f shifted up 3 units.

43.

x	$f(x)=x^2$	(x, y)
–2	$f(-2)=(-2)^2=4$	$(-2,4)$
–1	$f(-1)=(-1)^2=1$	$(-1,1)$
0	$f(0)=(0)^2=0$	$(0,0)$
1	$f(1)=(1)^2=1$	$(1,1)$
2	$f(2)=(2)^2=4$	$(2,4)$

x	$g(x)=x^2+1$	(x, y)
–2	$g(-2)=(-2)^2+1=5$	$(-2,5)$
–1	$g(-1)=(-1)^2+1=2$	$(-1,2)$
0	$g(0)=(0)^2+1=1$	$(0,1)$
1	$g(1)=(1)^2+1=2$	$(1,2)$
2	$g(2)=(2)^2+1=5$	$(2,5)$

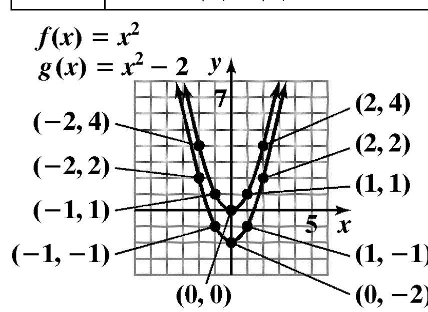


The graph of g is the graph of f shifted up 1 unit.

44.

x	$f(x)=x^2$	(x, y)
–2	$f(-2)=(-2)^2=4$	$(-2,4)$
–1	$f(-1)=(-1)^2=1$	$(-1,1)$
0	$f(0)=(0)^2=0$	$(0,0)$
1	$f(1)=(1)^2=1$	$(1,1)$
2	$f(2)=(2)^2=4$	$(2,4)$

x	$g(x)=x^2-2$	(x, y)
–2	$g(-2)=(-2)^2-2=2$	$(-2,2)$
–1	$g(-1)=(-1)^2-2=-1$	$(-1,-1)$
0	$g(0)=(0)^2-2=-2$	$(0,-2)$
1	$g(1)=(1)^2-2=-1$	$(1,-1)$
2	$g(2)=(2)^2-2=2$	$(2,2)$

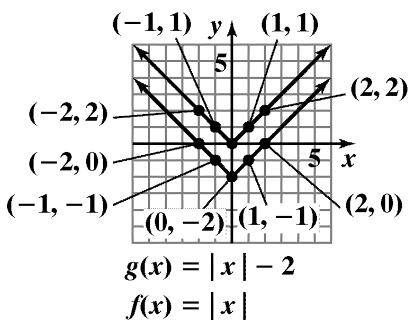


The graph of g is the graph of f shifted down 2 units.

45.

x	$f(x)=\lvert x\rvert$	(x,y)
-2	$f(-2)=\lvert -2\rvert=2$	$(-2,2)$
-1	$f(-1)=\lvert -1\rvert=1$	$(-1,1)$
0	$f(0)=\lvert 0\rvert=0$	$(0,0)$
1	$f(1)=\lvert 1\rvert=1$	$(1,1)$
2	$f(2)=\lvert 2\rvert=2$	$(2,2)$

x	$g(x)=\lvert x\rvert-2$	(x,y)
-2	$g(-2)=\lvert -2\rvert-2=0$	$(-2,0)$
-1	$g(-1)=\lvert -1\rvert-2=-1$	$(-1,-1)$
0	$g(0)=\lvert 0\rvert-2=-2$	$(0,-2)$
1	$g(1)=\lvert 1\rvert-2=-1$	$(1,-1)$
2	$g(2)=\lvert 2\rvert-2=0$	$(2,0)$

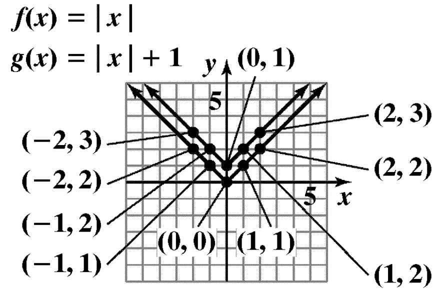


The graph of g is the graph of f shifted down 2 units.

46.

x	$f(x)=\lvert x\rvert$	(x,y)
-2	$f(-2)=\lvert -2\rvert=2$	$(-2,2)$
-1	$f(-1)=\lvert -1\rvert=1$	$(-1,1)$
0	$f(0)=\lvert 0\rvert=0$	$(0,0)$
1	$f(1)=\lvert 1\rvert=1$	$(1,1)$
2	$f(2)=\lvert 2\rvert=2$	$(2,2)$

x	$g(x)=\lvert x\rvert+1$	(x,y)
-2	$g(-2)=\lvert -2\rvert+1=3$	$(-2,3)$
-1	$g(-1)=\lvert -1\rvert+1=2$	$(-1,2)$
0	$g(0)=\lvert 0\rvert+1=1$	$(0,1)$
1	$g(1)=\lvert 1\rvert+1=2$	$(1,2)$
2	$g(2)=\lvert 2\rvert+1=3$	$(2,3)$

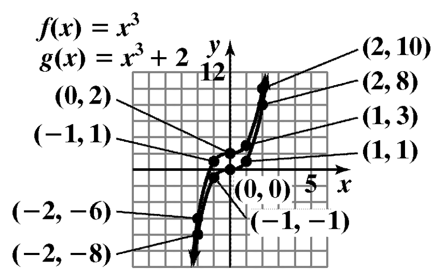


The graph of g is the graph of f shifted up 1 unit.

47.

x	$f(x)=x^3$	(x,y)
-2	$f(-2)=(-2)^3=-8$	$(-2,-8)$
-1	$f(-1)=(-1)^3=-1$	$(-1,-1)$
0	$f(0)=(0)^3=0$	$(0,0)$
1	$f(1)=(1)^3=1$	$(1,1)$
2	$f(2)=(2)^3=8$	$(2,8)$

x	$g(x)=x^3+2$	(x,y)
-2	$g(-2)=(-2)^3+2=-6$	$(-2,-6)$
-1	$g(-1)=(-1)^3+2=1$	$(-1,1)$
0	$g(0)=(0)^3+2=2$	$(0,2)$
1	$g(1)=(1)^3+2=3$	$(1,3)$
2	$g(2)=(2)^3+2=10$	$(2,10)$

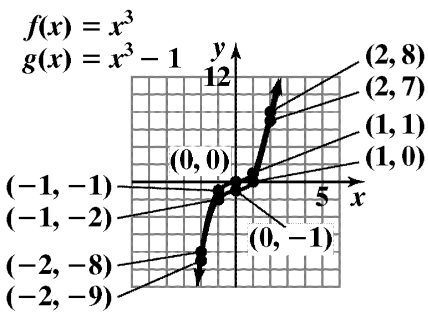


The graph of g is the graph of f shifted up 2 units.

48.

x	$f(x)=x^3$	(x,y)
–2	$f(-2)=(-2)^3=-8$	$(-2,-8)$
–1	$f(-1)=(-1)^3=-1$	$(-1,-1)$
0	$f(0)=(0)^3=0$	$(0,0)$
1	$f(1)=(1)^3=1$	$(1,1)$
2	$f(2)=(2)^3=8$	$(2,8)$

x	$g(x)=x^3-1$	(x,y)
–2	$g(-2)=(-2)^3-1=-9$	$(-2,-9)$
–1	$g(-1)=(-1)^3-1=-2$	$(-1,-2)$
0	$g(0)=(0)^3-1=-1$	$(0,-1)$
1	$g(1)=(1)^3-1=0$	$(1,0)$
2	$g(2)=(2)^3-1=7$	$(2,7)$

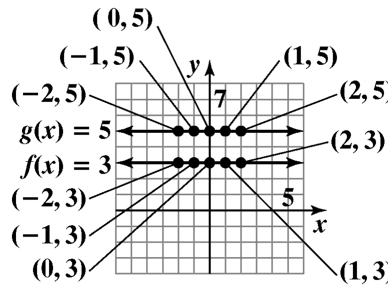


The graph of g is the graph of f shifted down 1 unit.

49.

x	$f(x)=3$	(x,y)
–2	$f(-2)=3$	$(-2,3)$
–1	$f(-1)=3$	$(-1,3)$
0	$f(0)=3$	$(0,3)$
1	$f(1)=3$	$(1,3)$
2	$f(2)=3$	$(2,3)$

x	$g(x)=5$	(x,y)
–2	$g(-2)=5$	$(-2,5)$
–1	$g(-1)=5$	$(-1,5)$
0	$g(0)=5$	$(0,5)$
1	$g(1)=5$	$(1,5)$
2	$g(2)=5$	$(2,5)$

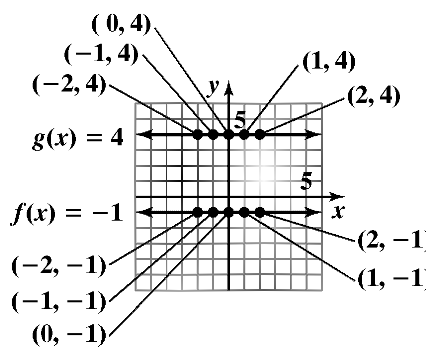


The graph of g is the graph of f shifted up 2 units.

50.

x	$f(x)=-1$	(x,y)
–2	$f(-2)=-1$	$(-2,-1)$
–1	$f(-1)=-1$	$(-1,-1)$
0	$f(0)=-1$	$(0,-1)$
1	$f(1)=-1$	$(1,-1)$
2	$f(2)=-1$	$(2,-1)$

x	$g(x)=4$	(x,y)
–2	$g(-2)=4$	$(-2,4)$
–1	$g(-1)=4$	$(-1,4)$
0	$g(0)=4$	$(0,4)$
1	$g(1)=4$	$(1,4)$
2	$g(2)=4$	$(2,4)$

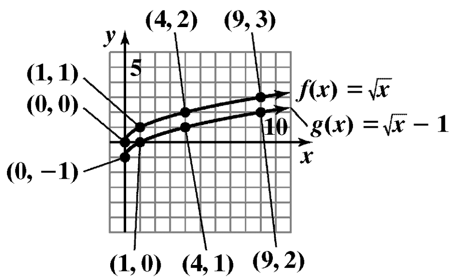


The graph of g is the graph of f shifted up 5 units.

51.

x	$f(x)=\sqrt{x}$	(x, y)
0	$f(0)=\sqrt{0}=0$	$(0,0)$
1	$f(1)=\sqrt{1}=1$	$(1,1)$
4	$f(4)=\sqrt{4}=2$	$(4,2)$
9	$f(9)=\sqrt{9}=3$	$(9,3)$

x	$g(x)=\sqrt{x}-1$	(x, y)
0	$g(0)=\sqrt{0}-1=-1$	$(0,-1)$
1	$g(1)=\sqrt{1}-1=0$	$(1,0)$
4	$g(4)=\sqrt{4}-1=1$	$(4,1)$
9	$g(9)=\sqrt{9}-1=2$	$(9,2)$

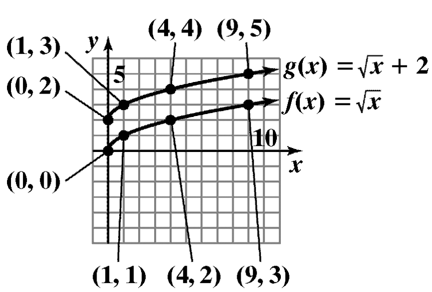


The graph of g is the graph of *f* shifted down 1 unit.

52.

x	$f(x)=\sqrt{x}$	(x, y)
0	$f(0)=\sqrt{0}=0$	$(0,0)$
1	$f(1)=\sqrt{1}=1$	$(1,1)$
4	$f(4)=\sqrt{4}=2$	$(4,2)$
9	$f(9)=\sqrt{9}=3$	$(9,3)$

x	$g(x)=\sqrt{x}+2$	(x, y)
0	$g(0)=\sqrt{0}+2=2$	$(0,2)$
1	$g(1)=\sqrt{1}+2=3$	$(1,3)$
4	$g(4)=\sqrt{4}+2=4$	$(4,4)$
9	$g(9)=\sqrt{9}+2=5$	$(9,5)$

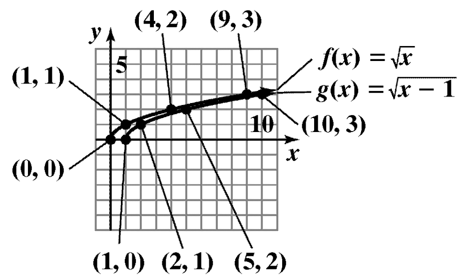


The graph of g is the graph of *f* shifted up 2 units.

53.

x	$f(x)=\sqrt{x}$	(x, y)
0	$f(0)=\sqrt{0}=0$	$(0,0)$
1	$f(1)=\sqrt{1}=1$	$(1,1)$
4	$f(4)=\sqrt{4}=2$	$(4,2)$
9	$f(9)=\sqrt{9}=3$	$(9,3)$

x	$g(x)=\sqrt{x-1}$	(x, y)
1	$g(1)=\sqrt{1-1}=0$	$(1,0)$
2	$g(2)=\sqrt{2-1}=1$	$(2,1)$
5	$g(5)=\sqrt{5-1}=2$	$(5,2)$
10	$g(10)=\sqrt{10-1}=3$	$(10,3)$

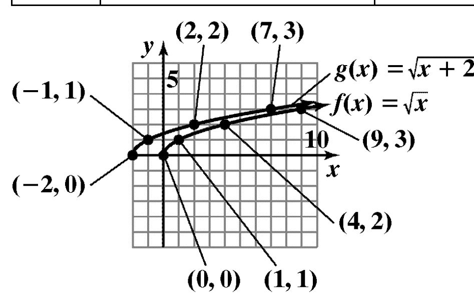


The graph of g is the graph of *f* shifted right 1 unit.

54.

x	$f(x)=\sqrt{x}$	(x, y)
0	$f(0)=\sqrt{0}=0$	$(0,0)$
1	$f(1)=\sqrt{1}=1$	$(1,1)$
4	$f(4)=\sqrt{4}=2$	$(4,2)$
9	$f(9)=\sqrt{9}=3$	$(9,3)$

x	$g(x)=\sqrt{x+2}$	(x, y)
–2	$g(-2)=\sqrt{-2+2}=0$	$(-2,0)$
–1	$g(-1)=\sqrt{-1+2}=1$	$(-1,1)$
2	$g(2)=\sqrt{2+2}=2$	$(2,2)$
7	$g(7)=\sqrt{7+2}=3$	$(7,3)$



The graph of g is the graph of *f* shifted left 2 units.

55. function

56. function

57. function

58. not a function

59. not a function

60. not a function

61. function

62. not a function

63. function

64. function

65. $f(-2) = -4$

66. $f(2) = -4$

67. $f(4) = 4$

68. $f(-4) = 4$

69. $f(-3) = 0$

70. $f(-1) = 0$

71. $g(-4) = 2$

72. $g(2) = -2$

73. $g(-10) = 2$

74. $g(10) = -2$

75. When $x = -2$, $g(x) = 1$.

76. When $x = 1$, $g(x) = -1$.

77. **a.** domain: $(-\infty, \infty)$

b. range: $[-4, \infty)$

c. x-intercepts: –3 and 1

d. y-intercept: –3

e. $f(-2) = -3$ and $f(2) = 5$

78. **a.** domain: $(-\infty, \infty)$

b. range: $(-\infty, 4]$

c. x-intercepts: –3 and 1

d. y-intercept: 3

e. $f(-2) = 3$ and $f(2) = -5$

79. **a.** domain: $(-\infty, \infty)$

b. range: $[1, \infty)$

c. x-intercept: none

d. y-intercept: 1

e. $f(-1) = 2$ and $f(3) = 4$

80. **a.** domain: $(-\infty, \infty)$

b. range: $[0, \infty)$

c. x-intercept: –1

d. y-intercept: 1

e. $f(-4) = 3$ and $f(3) = 4$

81. **a.** domain: $[0, 5)$

b. range: $[-1, 5)$

c. x-intercept: 2

d. y-intercept: –1

e. $f(3) = 1$

82. **a.** domain: $(-6, 0]$

b. range: $[-3, 4)$

c. x-intercept: –3.75

d. y-intercept: –3

e. $f(-5) = 2$

83. **a.** domain: $[0, \infty)$

b. range: $[1, \infty)$

c. x-intercept: none

d. y-intercept: 1

e. $f(4) = 3$

84. **a.** domain: $[-1, \infty)$

b. range: $[0, \infty)$

c. x-intercept: -1

d. y-intercept: 1

e. $f(3) = 2$

85. **a.** domain: $[-2, 6]$

b. range: $[-2, 6]$

c. x-intercept: 4

d. y-intercept: 4

e. $f(-1) = 5$

86. **a.** domain: $[-3, 2]$

b. range: $[-5, 5]$

c. x-intercept: $-\frac{1}{2}$

d. y-intercept: 1

e. $f(-2) = -3$

87. **a.** domain: $(-\infty, \infty)$

b. range: $(-\infty, -2]$

c. x-intercept: none

d. y-intercept: -2

e. $f(-4) = -5$ and $f(4) = -2$

88. **a.** domain: $(-\infty, \infty)$

b. range: $[0, \infty)$

c. x-intercept: $\{x \mid x \le 0\}$

d. y-intercept: 0

e. $f(-2) = 0$ and $f(2) = 4$

89. **a.** domain: $(-\infty, \infty)$

b. range: $(0, \infty)$

c. x-intercept: none

d. y-intercept: 1.5

e. $f(4) = 6$

90. **a.** domain: $(-\infty, 1) \cup (1, \infty)$

b. range: $(-\infty, 0) \cup (0, \infty)$

c. x-intercept: none

d. y-intercept: -1

e. $f(2) = 1$

91. **a.** domain: $\{-5, -2, 0, 1, 3\}$

b. range: $\{2\}$

c. x-intercept: none

d. y-intercept: 2

e. $f(-5) + f(3) = 2 + 2 = 4$

92. **a.** domain: $\{-5, -2, 0, 1, 4\}$

b. range: $\{-2\}$

c. x-intercept: none

d. y-intercept: -2

e. $f(-5) + f(4) = -2 + (-2) = -4$

93.
$$g(1) = 3(1) - 5 = 3 - 5 = -2$$
$$f(g(1)) = f(-2) = (-2)^2 - (-2) + 4$$
$$= 4 + 2 + 4 = 10$$

94.
$$g(-1) = 3(-1) - 5 = -3 - 5 = -8$$
$$f(g(-1)) = f(-8) = (-8)^2 - (-8) + 4$$
$$= 64 + 8 + 4 = 76$$

95.
$$\sqrt{3-(-1)} - (-6)^2 + 6 \div (-6) \cdot 4$$
$$= \sqrt{3+1} - 36 + 6 \div (-6) \cdot 4$$
$$= \sqrt{4} - 36 + -1 \cdot 4$$
$$= 2 - 36 + -4$$
$$= -34 + -4$$
$$= -38$$

96.
$$|-4-(-1)| - (-3)^2 + -3 \div 3 \cdot -6$$
$$= |-4+1| - 9 + -3 \div 3 \cdot -6$$
$$= |-3| - 9 + -1 \cdot -6$$
$$= 3 - 9 + 6 = -6 + 6 = 0$$

97.
$$f(-x) - f(x)$$
$$= (-x)^3 + (-x) - 5 - (x^3 + x - 5)$$
$$= -x^3 - x - 5 - x^3 - x + 5 = -2x^3 - 2x$$

98. $f(-x)-f(x)$
$=(-x)^2-3(-x)+7-(x^2-3x+7)$
$=x^2+3x+7-x^2+3x-7$
$=6x$

99. **a.** {(Iceland, 9.7), (Finland, 9.6), (New Zealand, 9.6), (Denmark, 9.5)}

b. Yes, the relation is a function because each country in the domain corresponds to exactly one corruption rating in the range.

c. {(9.7, Iceland), (9.6, Finland), (9.6, New Zealand), (9.5, Denmark)}

d. No, the relation is not a function because 9.6 in the domain corresponds to two countries in the range, Finland and New Zealand.

100. **a.** {(Bangladesh, 1.7), (Chad, 1.7), (Haiti, 1.8), (Myanmar, 1.8)}

b. Yes, the relation is a function because each country in the domain corresponds to exactly one corruption rating in the range.

c. {(1.7, Bangladesh), (1.7, Chad), (1.8, Haiti), (1.8, Myanmar)}

d. No, the relation is not a function because 1.7 in the domain corresponds to two countries in the range, Bangladesh and Chad.

101. **a.** $f(70)=83$ which means the chance that a 60-year old will survive to age 70 is 83%.

b. $g(70)=76$ which means the chance that a 60-year old will survive to age 70 is 76%.

c. Function *f* is the better model.

102. **a.** $f(90)=25$ which means the chance that a 60-year old will survive to age 90 is 25%.

b. $g(90)=10$ which means the chance that a 60-year old will survive to age 90 is 10%.

c. Function *f* is the better model.

103. **a.** $G(30)=-0.01(30)^2+(30)+60=81$
In 2010, the wage gap was 81%. This is represented as $(30, 81)$ on the graph.

b. $G(30)$ underestimates the actual data shown by the bar graph by 2%.

104. **a.** $G(10)=-0.01(10)^2+(10)+60=69$
In 1990, the wage gap was 69%. This is represented as $(10, 69)$ on the graph.

b. $G(10)$ underestimates the actual data shown by the bar graph by 2%.

105. $C(x)=100{,}000+100x$
$C(90)=100{,}000+100(90)=\$109{,}000$
It will cost $109,000 to produce 90 bicycles.

106. $V(x)=22{,}500-3200x$
$V(3)=22{,}500-3200(3)=\$12{,}900$
After 3 years, the car will be worth $12,900.

107. $T(x)=\frac{40}{x}+\frac{40}{x+30}$

$$T(30)=\frac{40}{30}+\frac{40}{30+30}$$
$$=\frac{80}{60}+\frac{40}{60}$$
$$=\frac{120}{60}$$
$$=2$$

If you travel 30 mph going and 60 mph returning, your total trip will take 2 hours.

108. $S(x)=0.10x+0.60(50-x)$
$S(30)=0.10(30)+0.60(50-30)=15$
When 30 mL of the 10% mixture is mixed with 20 mL of the 60% mixture, there will be 15 mL of sodium-iodine in the vaccine.

109. – 117. Answers will vary.

118. makes sense

119. does not make sense; Explanations will vary. Sample explanation: The parentheses used in function notation, such as $f(x)$, do not imply multiplication.

120. does not make sense; Explanations will vary. Sample explanation: The domain is the number of years worked for the company.

121. does not make sense; Explanations will vary. Sample explanation: This would not be a function because some elements in the domain would correspond to more than one age in the range.

122. false; Changes to make the statement true will vary. A sample change is: The domain is $[-4,4]$.

123. false; Changes to make the statement true will vary. A sample change is: The range is $[-2,2)$.

124. true

125. false; Changes to make the statement true will vary. A sample change is: $f(0)=0.8$

126. $f(a+h)=3(a+h)+7=3a+3h+7$

$f(a)=3a+7$

$$\frac{f(a+h)-f(a)}{h}$$
$$=\frac{(3a+3h+7)-(3a+7)}{h}$$
$$=\frac{3a+3h+7-3a-7}{h}=\frac{3h}{h}=3$$

127. Answers will vary.
An example is $\{(1,1),(2,1)\}$

128. It is given that $f(x+y)=f(x)+f(y)$ and $f(1)=3$.
To find $f(2)$, rewrite 2 as 1 + 1.

$f(2)=f(1+1)=f(1)+f(1)$
$=3+3=6$

Similarly:

$f(3)=f(2+1)=f(2)+f(1)$
$=6+3=9$

$f(4)=f(3+1)=f(3)+f(1)$
$=9+3=12$

While $f(x+y)=f(x)+f(y)$ is true for this function, it is not true for all functions. It is not true for $f(x)=x^2$, for example.

129. $-1+3(x-4)=2x$

$-1+3x-12=2x$

$3x-13=2x$

$-13=-x$

$13=x$

The solution set is $\{13\}$.

130. $\frac{x-3}{5}-\frac{x-4}{2}=5$

$10\left(\frac{x-3}{5}\right)-10\left(\frac{x-4}{2}\right)=10(5)$

$2x-6-5x+20=50$

$-3x+14=50$

$-3x=36$

$x=-12$

The solution set is $\{-12\}$.

131. Let x = the number of deaths by snakes, in thousands, in 2014
Let x + 661 = the number of deaths by mosquitoes, in thousands, in 2014
Let x + 106 = the number of deaths by snails, in thousands, in 2014

$x+(x+661)+(x+106)=1049$

$x+x+661+x+106=1049$

$3x+767=1049$

$3x=282$

$x=94$

$x=94$, thousand deaths by snakes

$x+661=755$, thousand deaths by mosquitoes

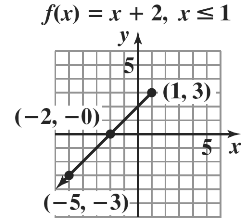
$x+106=200$, thousand deaths by snails

132. $C(t)=20+0.40(t-60)$

$C(100)=20+0.40(100-60)$
$=20+0.40(40)$
$=20+16$
$=36$

For 100 calling minutes, the monthly cost is \$36.

133. $f(x)=x+2,\ x\le 1$



134. $2(x+h)^2+3(x+h)+5-(2x^2+3x+5)$

$=2(x^2+2xh+h^2)+3x+3h+5-2x^2-3x-5$

$=2x^2+4xh+2h^2+3x+3h+5-2x^2-3x-5$

$=2x^2-2x^2+4xh+2h^2+3x-3x+3h+5-5$

$=4xh+2h^2+3h$

Section 2.2

Check Point Exercises

1. The function is increasing on the interval $(-\infty,-1)$, decreasing on the interval $(-1,1)$, and increasing on the interval $(1,\infty)$.

2. Test for symmetry with respect to the y-axis.

$y = x^2 - 1$

$y = (-x)^2 - 1$

$y = x^2 - 1$

The resulting equation is equivalent to the original. Thus, the graph is symmetric with respect to the y-axis.

Test for symmetry with respect to the x-axis.

$y = x^2 - 1$

$-y = x^2 - 1$

$y = -x^2 + 1$

The resulting equation is not equivalent to the original. Thus, the graph is not symmetric with respect to the x-axis.

Test for symmetry with respect to the origin.

$y = x^2 - 1$

$-y = (-x)^2 - 1$

$-y = x^2 - 1$

$y = -x^2 + 1$

The resulting equation is not equivalent to the original. Thus, the graph is not symmetric with respect to the origin.

3. Test for symmetry with respect to the y-axis.

$y^5 = x^3$

$y^5 = (-x)^3$

$y^5 = -x^3$

The resulting equation is not equivalent to the original. Thus, the graph is not symmetric with respect to the y-axis.

Test for symmetry with respect to the x-axis.

$y^5 = x^3$

$(-y)^5 = x^3$

$-y^5 = x^3$

$y^5 = -x^3$

The resulting equation is not equivalent to the original. Thus, the graph is not symmetric with respect to the x-axis.

Test for symmetry with respect to the origin.

$y^5 = x^3$

$(-y)^5 = (-x)^3$

$-y^5 = -x^3$

$y^5 = x^3$

The resulting equation is equivalent to the original. Thus, the graph is symmetric with respect to the origin.

4. **a.** The graph passes the vertical line test and is therefore the graph of a function. The graph is symmetric with respect to the y-axis. Therefore, the graph is that of an even function.

b. The graph passes the vertical line test and is therefore the graph of a function. The graph is neither symmetric with respect to the y-axis nor the origin. Therefore, the graph is that of a function which is neither even nor odd.

c. The graph passes the vertical line test and is therefore the graph of a function. The graph is symmetric with respect to the origin. Therefore, the graph is that of an odd function.

5. **a.** $f(-x) = (-x)^2 + 6 = x^2 + 6 = f(x)$

The function is even. The graph is symmetric with respect to the y-axis.

b. $g(-x) = 7(-x)^3 - (-x) = -7x^3 + x = -f(x)$

The function is odd. The graph is symmetric with respect to the origin.

c. $h(-x) = (-x)^5 + 1 = -x^5 + 1$

The function is neither even nor odd. The graph is neither symmetric to the y-axis nor the origin.

6. $C(t) = \begin{cases} 20 & \text{if } 0 \le t \le 60 \\ 20 + 0.40(t-60) & \text{if } t > 60 \end{cases}$

a. Since $0 \le 40 \le 60$, $C(40) = 20$

With 40 calling minutes, the cost is \$20.

This is represented by $(40, 20)$.

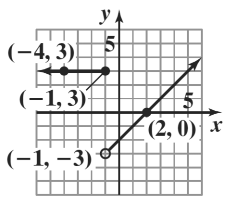
b. Since $80 > 60$,

$C(80) = 20 + 0.40(80 - 60) = 28$

With 80 calling minutes, the cost is \$28.

This is represented by $(80, 28)$.

7.



$$f(x)=\begin{cases}3 & \text{if } x\le -1\\ x-2 & \text{if } x>-1\end{cases}$$

8. a.

$$f(x)=-2x^2+x+5$$
$$f(x+h)=-2(x+h)^2+(x+h)+5$$
$$=-2(x^2+2xh+h^2)+x+h+5$$
$$=-2x^2-4xh-2h^2+x+h+5$$

b.

$$\frac{f(x+h)-f(x)}{h}$$
$$=\frac{-2x^2-4xh-2h^2+x+h+5-\left(-2x^2+x+5\right)}{h}$$
$$=\frac{-2x^2-4xh-2h^2+x+h+5+2x^2-x-5}{h}$$
$$=\frac{-4xh-2h^2+h}{h}$$
$$=\frac{h\left(-4x-2h+1\right)}{h}$$
$$=-4x-2h+1,\quad h\neq 0$$

Concept and Vocabulary Check 2.2

1. $< f(x_2)$; $> f(x_2)$; $= f(x_2)$

2. maximum; minimum

3. y-axis

4. x-axis

5. origin

6. $f(x)$; y-axis

7. $-f(x)$; origin

8. piecewise

9. less than or equal to x; 2; -3; 0

10. difference quotient; $x+h$; $f(x)$; h; h

11. false

12. false

Exercise Set 2.2

1. a. increasing: $(-1,\infty)$

b. decreasing: $(-\infty,-1)$

c. constant: none

2. a. increasing: $(-\infty,-1)$

b. decreasing: $(-1,\infty)$

c. constant: none

3. a. increasing: $(0,\infty)$

b. decreasing: none

c. constant: none

4. a. increasing: $(-1,\infty)$

b. decreasing: none

c. constant: none

5. a. increasing: none

b. decreasing: $(-2, 6)$

c. constant: none

6. a. increasing: $(-3, 2)$

b. decreasing: none

c. constant: none

7. a. increasing: $(-\infty,-1)$

b. decreasing: none

c. constant: $(-1,\infty)$

8. a. increasing: $(0,\infty)$

b. decreasing: none

c. constant: $(-\infty, 0)$

9. a. increasing: $(-\infty,0)$ or $(1.5,3)$

b. decreasing: $(0,1.5)$ or $(3,\infty)$

c. constant: none

10. a. increasing: $(-5,-4)$ or $(-2,0)$ or $(2,4)$

b. decreasing: $(-4,-2)$ or $(0,2)$ or $(4,5)$

c. constant: none

11. a. increasing: (–2, 4)

b. decreasing: none

c. constant: $(-\infty,-2)$ or $(4,\infty)$

12. a. increasing: none

b. decreasing: (–4, 2)

c. constant: $(-\infty,-4)$ or $(2,\infty)$

13. a. $x = 0$, relative maximum = 4

b. $x = -3, 3$, relative minimum = 0

14. a. $x = 0$, relative maximum = 2

b. $x = -3, 3$, relative minimum = –1

15. a. $x = -2$, relative maximum = 21

b. $x = 1$, relative minimum = –6

16. a. $x = 1$, relative maximum = 30

b. $x = 4$, relative minimum = 3

17. Test for symmetry with respect to the y-axis.

$$y = x^2 + 6$$
$$y = (-x)^2 + 6$$
$$y = x^2 + 6$$

The resulting equation is equivalent to the original. Thus, the graph is symmetric with respect to the y-axis.

Test for symmetry with respect to the x-axis.

$$y = x^2 + 6$$
$$-y = x^2 + 6$$
$$y = -x^2 - 6$$

The resulting equation is not equivalent to the original. Thus, the graph is not symmetric with respect to the x-axis.

Test for symmetry with respect to the origin.

$$y = x^2 + 6$$
$$-y = (-x)^2 + 6$$
$$-y = x^2 + 6$$
$$y = -x^2 - 6$$

The resulting equation is not equivalent to the original. Thus, the graph is not symmetric with respect to the origin.

18. Test for symmetry with respect to the y-axis.

$$y = x^2 - 2$$
$$y = (-x)^2 - 2$$
$$y = x^2 - 2$$

The resulting equation is equivalent to the original. Thus, the graph is symmetric with respect to the y-axis.

Test for symmetry with respect to the x-axis.

$$y = x^2 - 2$$
$$-y = x^2 - 2$$
$$y = -x^2 + 2$$

The resulting equation is not equivalent to the original. Thus, the graph is not symmetric with respect to the x-axis.

Test for symmetry with respect to the origin.

$$y = x^2 - 2$$
$$-y = (-x)^2 - 2$$
$$-y = x^2 - 2$$
$$y = -x^2 + 2$$

The resulting equation is not equivalent to the original. Thus, the graph is not symmetric with respect to the origin.

19. Test for symmetry with respect to the y-axis.

$$x = y^2 + 6$$
$$-x = y^2 + 6$$
$$x = -y^2 - 6$$

The resulting equation is not equivalent to the original. Thus, the graph is not symmetric with respect to the y-axis.

Test for symmetry with respect to the x-axis.

$$x = y^2 + 6$$
$$x = (-y)^2 + 6$$
$$x = y^2 + 6$$

The resulting equation is equivalent to the original. Thus, the graph is symmetric with respect to the x-axis.

Test for symmetry with respect to the origin.

$$x = y^2 + 6$$
$$-x = (-y)^2 + 6$$
$$-x = y^2 + 6$$
$$x = -y^2 - 6$$

The resulting equation is not equivalent to the original. Thus, the graph is not symmetric with respect to the origin.

20. Test for symmetry with respect to the y-axis.

$$x = y^2 - 2$$
$$-x = y^2 - 2$$
$$x = -y^2 + 2$$

The resulting equation is not equivalent to the original. Thus, the graph is not symmetric with respect to the y-axis.

Test for symmetry with respect to the x-axis.

$$x = y^2 - 2$$
$$x = (-y)^2 - 2$$
$$x = y^2 - 2$$

The resulting equation is equivalent to the original. Thus, the graph is symmetric with respect to the x-axis.

Test for symmetry with respect to the origin.

$$x = y^2 - 2$$
$$-x = (-y)^2 - 2$$
$$-x = y^2 - 2$$
$$x = -y^2 + 2$$

The resulting equation is not equivalent to the original. Thus, the graph is not symmetric with respect to the origin.

21. Test for symmetry with respect to the y-axis.

$$y^2 = x^2 + 6$$
$$y^2 = (-x)^2 + 6$$
$$y^2 = x^2 + 6$$

The resulting equation is equivalent to the original. Thus, the graph is symmetric with respect to the y-axis.

Test for symmetry with respect to the x-axis.

$$y^2 = x^2 + 6$$
$$(-y)^2 = x^2 + 6$$
$$y^2 = x^2 + 6$$

The resulting equation is equivalent to the original. Thus, the graph is symmetric with respect to the x-axis.

Test for symmetry with respect to the origin.

$$y^2 = x^2 + 6$$
$$(-y)^2 = (-x)^2 + 6$$
$$y = x^2 + 6$$

The resulting equation is equivalent to the original. Thus, the graph is symmetric with respect to the origin.

22. Test for symmetry with respect to the y-axis.

$$y^2 = x^2 - 2$$
$$y^2 = (-x)^2 - 2$$
$$y^2 = x^2 - 2$$

The resulting equation is equivalent to the original. Thus, the graph is symmetric with respect to the y-axis.

Test for symmetry with respect to the x-axis.

$$y^2 = x^2 - 2$$
$$(-y)^2 = x^2 - 2$$
$$y^2 = x^2 - 2$$

The resulting equation is equivalent to the original. Thus, the graph is symmetric with respect to the x-axis.

Test for symmetry with respect to the origin.

$$y^2 = x^2 - 2$$
$$(-y)^2 = (-x)^2 - 2$$
$$y = x^2 - 2$$

The resulting equation is equivalent to the original. Thus, the graph is symmetric with respect to the origin.

23. Test for symmetry with respect to the y-axis.

$$y = 2x + 3$$
$$y = 2(-x) + 3$$
$$y = -2x + 3$$

The resulting equation is not equivalent to the original. Thus, the graph is not symmetric with respect to the y-axis.

Test for symmetry with respect to the x-axis.

$$y = 2x + 3$$
$$-y = 2x + 3$$
$$y = -2x - 3$$

The resulting equation is not equivalent to the original. Thus, the graph is not symmetric with respect to the x-axis.

Test for symmetry with respect to the origin.

$$y = 2x + 3$$
$$-y = 2(-x) + 3$$
$$y = 2x - 3$$

The resulting equation is not equivalent to the original. Thus, the graph is not symmetric with respect to the origin.

24. Test for symmetry with respect to the *y*-axis.

$$y = 2x + 5$$
$$y = 2(-x) + 5$$
$$y = -2x + 5$$

The resulting equation is not equivalent to the original. Thus, the graph is not symmetric with respect to the *y*-axis.

Test for symmetry with respect to the *x*-axis.

$$y = 2x + 5$$
$$-y = 2x + 5$$
$$y = -2x - 5$$

The resulting equation is not equivalent to the original. Thus, the graph is not symmetric with respect to the *x*-axis.

Test for symmetry with respect to the origin.

$$y = 2x + 5$$
$$-y = 2(-x) + 5$$
$$y = 2x - 5$$

The resulting equation is not equivalent to the original. Thus, the graph is not symmetric with respect to the origin.

25. Test for symmetry with respect to the *y*-axis.

$$x^2 - y^3 = 2$$
$$(-x)^2 - y^3 = 2$$
$$x^2 - y^3 = 2$$

The resulting equation is equivalent to the original. Thus, the graph is symmetric with respect to the *y*-axis.

Test for symmetry with respect to the *x*-axis.

$$x^2 - y^3 = 2$$
$$x^2 - (-y)^3 = 2$$
$$x^2 + y^3 = 2$$

The resulting equation is not equivalent to the original. Thus, the graph is not symmetric with respect to the *x*-axis.

Test for symmetry with respect to the origin.

$$x^2 - y^3 = 2$$
$$(-x)^2 - (-y)^3 = 2$$
$$x^2 + y^3 = 2$$

The resulting equation is not equivalent to the original. Thus, the graph is not symmetric with respect to the origin.

26. Test for symmetry with respect to the *y*-axis.

$$x^3 - y^2 = 5$$
$$(-x)^3 - y^2 = 5$$
$$-x^3 - y^2 = 5$$

The resulting equation is not equivalent to the original. Thus, the graph is not symmetric with respect to the *y*-axis.

Test for symmetry with respect to the *x*-axis.

$$x^3 - y^2 = 5$$
$$x^3 - (-y)^2 = 5$$
$$x^3 - y^2 = 5$$

The resulting equation is equivalent to the original. Thus, the graph is symmetric with respect to the *x*-axis.

Test for symmetry with respect to the origin.

$$x^3 - y^2 = 5$$
$$(-x)^3 - (-y)^2 = 5$$
$$-x^3 - y^2 = 5$$

The resulting equation is not equivalent to the original. Thus, the graph is not symmetric with respect to the origin.

27. Test for symmetry with respect to the *y*-axis.

$$x^2 + y^2 = 100$$
$$(-x)^2 + y^2 = 100$$
$$x^2 + y^2 = 100$$

The resulting equation is equivalent to the original. Thus, the graph is symmetric with respect to the *y*-axis.

Test for symmetry with respect to the *x*-axis.

$$x^2 + y^2 = 100$$
$$x^2 + (-y)^2 = 100$$
$$x^2 + y^2 = 100$$

The resulting equation is equivalent to the original. Thus, the graph is symmetric with respect to the *x*-axis.

Test for symmetry with respect to the origin.

$$x^2 + y^2 = 100$$
$$(-x)^2 + (-y)^2 = 100$$
$$x^2 + y^2 = 100$$

The resulting equation is equivalent to the original. Thus, the graph is symmetric with respect to the origin.

28. Test for symmetry with respect to the *y*-axis.

$$x^2 + y^2 = 49$$
$$(-x)^2 + y^2 = 49$$
$$x^2 + y^2 = 49$$

The resulting equation is equivalent to the original. Thus, the graph is symmetric with respect to the *y*-axis.

Test for symmetry with respect to the *x*-axis.

$$x^2 + y^2 = 49$$
$$x^2 + (-y)^2 = 49$$
$$x^2 + y^2 = 49$$

The resulting equation is equivalent to the original. Thus, the graph is symmetric with respect to the *x*-axis.

Test for symmetry with respect to the origin.

$$x^2 + y^2 = 49$$
$$(-x)^2 + (-y)^2 = 49$$
$$x^2 + y^2 = 49$$

The resulting equation is equivalent to the original. Thus, the graph is symmetric with respect to the origin.

29. Test for symmetry with respect to the *y*-axis.

$$x^2y^2 + 3xy = 1$$
$$(-x)^2 y^2 + 3(-x)y = 1$$
$$x^2y^2 - 3xy = 1$$

The resulting equation is not equivalent to the original. Thus, the graph is not symmetric with respect to the *y*-axis.

Test for symmetry with respect to the *x*-axis.

$$x^2y^2 + 3xy = 1$$
$$x^2(-y)^2 + 3x(-y) = 1$$
$$x^2y^2 - 3xy = 1$$

The resulting equation is equivalent to the original. Thus, the graph is symmetric with respect to the *x*-axis.

Test for symmetry with respect to the origin.

$$x^2y^2 + 3xy = 1$$
$$(-x)^2(-y)^2 + 3(-x)(-y) = 1$$
$$x^2y^2 + 3xy = 1$$

The resulting equation is equivalent to the original. Thus, the graph is symmetric with respect to the origin.

30. Test for symmetry with respect to the *y*-axis.

$$x^2y^2 + 5xy = 2$$
$$(-x)^2 y^2 + 5(-x)y = 2$$
$$x^2y^2 - 5xy = 2$$

The resulting equation is not equivalent to the original. Thus, the graph is not symmetric with respect to the *y*-axis.

Test for symmetry with respect to the *x*-axis.

$$x^2y^2 + 5xy = 2$$
$$x^2(-y)^2 + 5x(-y) = 2$$
$$x^2y^2 - 5xy = 2$$

The resulting equation is equivalent to the original. Thus, the graph is symmetric with respect to the *x*-axis.

Test for symmetry with respect to the origin.

$$x^2y^2 + 5xy = 2$$
$$(-x)^2(-y)^2 + 5(-x)(-y) = 2$$
$$x^2y^2 + 5xy = 2$$

The resulting equation is equivalent to the original. Thus, the graph is symmetric with respect to the origin.

31. Test for symmetry with respect to the *y*-axis.

$$y^4 = x^3 + 6$$
$$y^4 = (-x)^3 + 6$$
$$y^4 = -x^3 + 6$$

The resulting equation is not equivalent to the original. Thus, the graph is not symmetric with respect to the *y*-axis.

Test for symmetry with respect to the *x*-axis.

$$y^4 = x^3 + 6$$
$$(-y)^4 = x^3 + 6$$
$$y^4 = x^3 + 6$$

The resulting equation is equivalent to the original. Thus, the graph is symmetric with respect to the *x*-axis.

Test for symmetry with respect to the origin.

$$y^4 = x^3 + 6$$
$$(-y)^4 = (-x)^3 + 6$$
$$y^4 = -x^3 + 6$$

The resulting equation is not equivalent to the original. Thus, the graph is not symmetric with respect to the origin.

32. Test for symmetry with respect to the *y*-axis.

$$y^5 = x^4 + 2$$
$$y^5 = (-x)^4 + 2$$
$$y^5 = x^4 + 2$$

The resulting equation is equivalent to the original. Thus, the graph is symmetric with respect to the *y*-axis.

Test for symmetry with respect to the *x*-axis.

$$y^5 = x^4 + 2$$
$$(-y)^5 = x^4 + 2$$
$$-y^5 = x^4 + 2$$
$$y^5 = -x^4 - 2$$

The resulting equation is not equivalent to the original. Thus, the graph is not symmetric with respect to the *x*-axis.

Test for symmetry with respect to the origin.

$$y^4 = x^3 + 6$$
$$(-y)^4 = (-x)^3 + 6$$
$$y^4 = -x^3 + 6$$

The resulting equation is not equivalent to the original. Thus, the graph is not symmetric with respect to the origin.

33. The graph is symmetric with respect to the *y*-axis. The function is even.

34. The graph is symmetric with respect to the origin. The function is odd.

35. The graph is symmetric with respect to the origin. The function is odd.

36. The graph is not symmetric with respect to the *y*-axis or the origin. The function is neither even nor odd.

37. $f(x) = x^3 + x$

$f(-x) = (-x)^3 + (-x)$

$f(-x) = -x^3 - x = -(x^3 + x)$

$f(-x) = -f(x)$, odd function

38. $f(x) = x^3 - x$

$f(-x) = (-x)^3 - (-x)$

$f(-x) = -x^3 + x = -(x^3 - x)$

$f(-x) = -f(x)$, odd function

39. $g(x) = x^2 + x$

$g(-x) = (-x)^2 + (-x)$

$g(-x) = x^2 - x$, neither

40. $g(x) = x^2 - x$

$g(-x) = (-x)^2 - (-x)$

$g(-x) = x^2 + x$, neither

41. $h(x) = x^2 - x^4$

$h(-x) = (-x)^2 - (-x)^4$

$h(-x) = x^2 - x^4$

$h(-x) = h(x)$, even function

42. $h(x) = 2x^2 + x^4$

$h(-x) = 2(-x)^2 + (-x)^4$

$h(-x) = 2x^2 + x^4$

$h(-x) = h(x)$, even function

43. $f(x) = x^2 - x^4 + 1$

$f(-x) = (-x)^2 - (-x)^4 + 1$

$f(-x) = x^2 - x^4 + 1$

$f(-x) = f(x)$, even function

44. $f(x) = 2x^2 + x^4 + 1$

$f(-x) = 2(-x)^2 + (-x)^4 + 1$

$f(-x) = 2x^2 + x^4 + 1$

$f(-x) = f(x)$, even function

45. $f(x) = \frac{1}{5}x^6 - 3x^2$

$f(-x) = \frac{1}{5}(-x)^6 - 3(-x)^2$

$f(-x) = \frac{1}{5}x^6 - 3x^2$

$f(-x) = f(x)$, even function

46. $f(x) = 2x^3 - 6x^5$
$f(-x) = 2(-x)^3 - 6(-x)^5$
$f(-x) = -2x^3 + 6x^5$
$f(-x) = -(2x^3 - 6x^5)$
$f(-x) = -f(x)$, odd function

47. $f(x) = x\sqrt{1-x^2}$
$f(-x) = -x\sqrt{1-(-x)^2}$
$f(-x) = -x\sqrt{1-x^2}$
$= -\left(x\sqrt{1-x^2}\right)$
$f(-x) = -f(x)$, odd function

48. $f(x) = x^2\sqrt{1-x^2}$
$f(-x) = (-x)^2\sqrt{1-(-x)^2}$
$f(-x) = x^2\sqrt{1-x^2}$
$f(-x) = f(x)$, even function

49. **a.** domain: $(-\infty, \infty)$

b. range: $[-4, \infty)$

c. x-intercepts: 1, 7

d. y-intercept: 4

e. $(4, \infty)$

f. $(0, 4)$

g. $(-\infty, 0)$

h. $x = 4$

i. $y = -4$

j. $f(-3) = 4$

k. $f(2) = -2$ and $f(6) = -2$

l. neither ; $f(-x) \neq x$, $f(-x) \neq -x$

50. **a.** domain: $(-\infty, \infty)$

b. range: $(-\infty, 4]$

c. x-intercepts: –4, 4

d. y-intercept: 1

e. $(-\infty, -2)$ or $(0, 3)$

f. $(-2, 0)$ or $(3, \infty)$

g. $(-\infty, -4]$ or $[4, \infty)$

h. $x = -2$ and $x = 3$

i. $f(-2) = 4$ and $f(3) = 2$

j. $f(-2) = 4$

k. $x = -4$ and $x = 4$

l. neither ; $f(-x) \neq x$, $f(-x) \neq -x$

51. **a.** domain: $(-\infty, 3]$

b. range: $(-\infty, 4]$

c. x-intercepts: –3, 3

d. $f(0) = 3$

e. $(-\infty, 1)$

f. $(1, 3)$

g. $(-\infty, -3]$

h. $f(1) = 4$

i. $x = 1$

j. positive; $f(-1) = +2$

52. **a.** domain: $(-\infty, 6]$

b. range: $(-\infty, 1]$

c. zeros of f: –3, 3

d. $f(0) = 1$

e. $(-\infty, -2)$

f. $(2, 6)$

g. $(-2, 2)$

h. $(-3,3)$

i. $x=-5$ and $x=5$

j. negative; $f(4)=-1$

k. neither

l. no; f(2) is not greater than the function values to the immediate left.

53. a. $f(-2) = 3(-2) + 5 = -1$

b. $f(0) = 4(0) + 7 = 7$

c. $f(3) = 4(3) + 7 = 19$

54. a. $f(-3) = 6(-3) - 1 = -19$

b. $f(0) = 7(0) + 3 = 3$

c. $f(4) = 7(4) + 3 = 31$

55. a. $g(0) = 0 + 3 = 3$

b. $g(-6) = -(-6 + 3) = -(-3) = 3$

c. $g(-3) = -3 + 3 = 0$

56. a. $g(0) = 0 + 5 = 5$

b. $g(-6) = -(-6 + 5) = -(-1) = 1$

c. $g(-5) = -5 + 5 = 0$

57. a. $h(5)=\dfrac{5^2-9}{5-3}=\dfrac{25-9}{2}=\dfrac{16}{2}=8$

b. $h(0)=\dfrac{0^2-9}{0-3}=\dfrac{-9}{-3}=3$

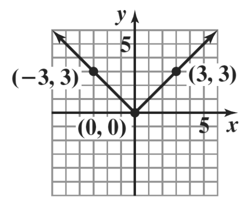
c. $h(3)=6$

58. a. $h(7)=\dfrac{7^2-25}{7-5}=\dfrac{49-25}{2}=\dfrac{24}{2}=12$

b. $h(0)=\dfrac{0^2-25}{0-5}=\dfrac{-25}{-5}=5$

c. $h(5)=10$

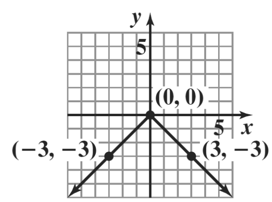
59. a.



$f(x)=\begin{cases}-x \text{ if } x<0\\ x \text{ if } x\ge 0\end{cases}$

b. range: $[0,\infty)$

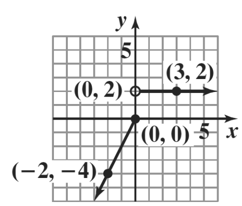
60. a.



$f(x)=\begin{cases}x \text{ if } x<0\\ -x \text{ if } x\ge 0\end{cases}$

b. range: $(-\infty,0]$

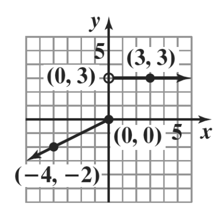
61. a.



$f(x)=\begin{cases}2x & \text{if } x\le 0\\ 2 & \text{if } x>0\end{cases}$

b. range: $(-\infty,0]\cup\{2\}$

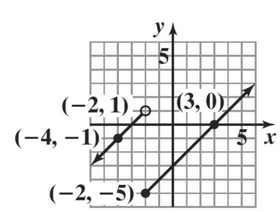
62. a.



$f(x)=\begin{cases}\frac{1}{2}x & \text{if } x\le 0\\ 3 & \text{if } x>0\end{cases}$

b. range: $(-\infty,0]\cup\{3\}$

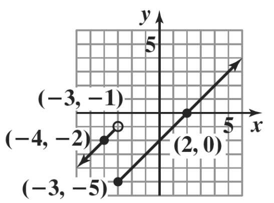
63. a.



$f(x)=\begin{cases}x+3 \text{ if } x<-2\\ x-3 \text{ if } x\ge -2\end{cases}$

b. range: $(-\infty,\infty)$

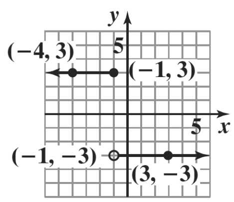
64. a.



$$f(x) = \begin{cases} x + 2 \text{ if } x < -3 \\ x - 2 \text{ if } x \geq -3 \end{cases}$$

b. range: $(-\infty, \infty)$

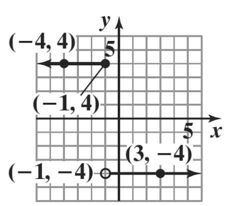
65. a.



$$f(x) = \begin{cases} 3 \text{ if } x \leq -1 \\ -3 \text{ if } x > -1 \end{cases}$$

b. range: $\{-3, 3\}$

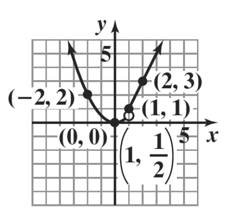
66. a.



$$f(x) = \begin{cases} 4 \text{ if } x \leq -1 \\ -4 \text{ if } x > -1 \end{cases}$$

b. range: $\{-4, 4\}$

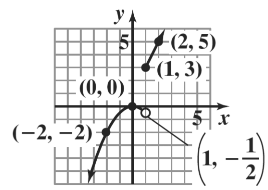
67. a.



$$f(x) = \begin{cases} \frac{1}{2}x^2 & \text{if } x < 1 \\ 2x - 1 & \text{if } x \geq 1 \end{cases}$$

b. range: $[0, \infty)$

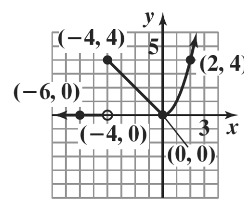
68. a.



$$f(x) = \begin{cases} -\frac{1}{2}x^2 & \text{if } x < 1 \\ 2x + 1 & \text{if } x \geq 1 \end{cases}$$

b. range: $(-\infty, 0] \cup [3, \infty)$

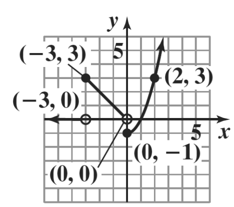
69. a.



$$f(x) = \begin{cases} 0 & \text{if } x < -4 \\ -x & \text{if } -4 \leq x < 0 \\ x^2 & \text{if } x \geq 0 \end{cases}$$

b. range: $[0, \infty)$

70. a.



$$f(x) = \begin{cases} 0 & \text{if } x < -3 \\ -x & \text{if } -3 \leq x < 0 \\ x^2 - 1 & \text{if } x \geq 0 \end{cases}$$

b. range: $[-1, \infty)$

71.
$$\frac{f(x+h) - f(x)}{h}$$
$$= \frac{4(x+h) - 4x}{h}$$
$$= \frac{4x + 4h - 4x}{h}$$
$$= \frac{4h}{h}$$
$$= 4$$

72. $\frac{f(x+h)-f(x)}{h}$
$=\frac{7(x+h)-7x}{h}$
$=\frac{7x+7h-7x}{h}$
$=\frac{7h}{h}$
$=7$

73. $\frac{f(x+h)-f(x)}{h}$
$=\frac{3(x+h)+7-(3x+7)}{h}$
$=\frac{3x+3h+7-3x-7}{h}$
$=\frac{3h}{h}$
$=3$

74. $\frac{f(x+h)-f(x)}{h}$
$=\frac{6(x+h)+1-(6x+1)}{h}$
$=\frac{6x+6h+1-6x-1}{h}$
$=\frac{6h}{h}$
$=6$

75. $\frac{f(x+h)-f(x)}{h}$
$=\frac{(x+h)^2-x^2}{h}$
$=\frac{x^2+2xh+h^2-x^2}{h}$
$=\frac{2xh+h^2}{h}$
$=\frac{h(2x+h)}{h}$
$=2x+h$

76. $\frac{f(x+h)-f(x)}{h}$
$=\frac{2(x+h)^2-2x^2}{h}$
$=\frac{2(x^2+2xh+h^2)-2x^2}{h}$
$=\frac{2x^2+4xh+2h^2-2x^2}{h}$
$=\frac{4xh+2h^2}{h}$
$=\frac{h(4x+2h)}{h}$
$=4x+2h$

77. $\frac{f(x+h)-f(x)}{h}$
$=\frac{(x+h)^2-4(x+h)+3-(x^2-4x+3)}{h}$
$=\frac{x^2+2xh+h^2-4x-4h+3-x^2+4x-3}{h}$
$=\frac{2xh+h^2-4h}{h}$
$=\frac{h(2x+h-4)}{h}$
$=2x+h-4$

78. $\frac{f(x+h)-f(x)}{h}$
$=\frac{(x+h)^2-5(x+h)+8-(x^2-5x+8)}{h}$
$=\frac{x^2+2xh+h^2-5x-5h+8-x^2+5x-8}{h}$
$=\frac{2xh+h^2-5h}{h}$
$=\frac{h(2x+h-5)}{h}$
$=2x+h-5$

79. $\frac{f(x+h)-f(x)}{h}$

$= \frac{2(x+h)^2+(x+h)-1-(2x^2+x-1)}{h}$

$= \frac{2x^2+4xh+2h^2+x+h-1-2x^2-x+1}{h}$

$= \frac{4xh+2h^2+h}{h}$

$= \frac{h(4x+2h+1)}{h}$

$= 4x+2h+1$

80. $\frac{f(x+h)-f(x)}{h}$

$= \frac{3(x+h)^2+(x+h)+5-(3x^2+x+5)}{h}$

$= \frac{3x^2+6xh+3h^2+x+h+5-3x^2-x-5}{h}$

$= \frac{6xh+3h^2+h}{h}$

$= \frac{h(6x+3h+1)}{h}$

$= 6x+3h+1$

81. $\frac{f(x+h)-f(x)}{h}$

$= \frac{-(x+h)^2+2(x+h)+4-(-x^2+2x+4)}{h}$

$= \frac{-x^2-2xh-h^2+2x+2h+4+x^2-2x-4}{h}$

$= \frac{-2xh-h^2+2h}{h}$

$= \frac{h(-2x-h+2)}{h}$

$= -2x-h+2$

82. $\frac{f(x+h)-f(x)}{h}$

$= \frac{-(x+h)^2-3(x+h)+1-(-x^2-3x+1)}{h}$

$= \frac{-x^2-2xh-h^2-3x-3h+1+x^2+3x-1}{h}$

$= \frac{-2xh-h^2-3h}{h}$

$= \frac{h(-2x-h-3)}{h}$

$= -2x-h-3$

83. $\frac{f(x+h)-f(x)}{h}$

$= \frac{-2(x+h)^2+5(x+h)+7-(-2x^2+5x+7)}{h}$

$= \frac{-2x^2-4xh-2h^2+5x+5h+7+2x^2-5x-7}{h}$

$= \frac{-4xh-2h^2+5h}{h}$

$= \frac{h(-4x-2h+5)}{h}$

$= -4x-2h+5$

84. $\frac{f(x+h)-f(x)}{h}$

$= \frac{-3(x+h)^2+2(x+h)-1-(-3x^2+2x-1)}{h}$

$= \frac{-3x^2-6xh-3h^2+2x+2h-1+3x^2-2x+1}{h}$

$= \frac{-6xh-3h^2+2h}{h}$

$= \frac{h(-6x-3h+2)}{h}$

$= -6x-3h+2$

85. $\frac{f(x+h)-f(x)}{h}$

$=\frac{-2(x+h)^2-(x+h)+3-(-2x^2-x+3)}{h}$

$=\frac{-2x^2-4xh-2h^2-x-h+3+2x^2+x-3}{h}$

$=\frac{-4xh-2h^2-h}{h}$

$=\frac{h(-4x-2h-1)}{h}$

$=-4x-2h-1$

86. $\frac{f(x+h)-f(x)}{h}$

$=\frac{-3(x+h)^2+(x+h)-1-(-3x^2+x-1)}{h}$

$=\frac{-3x^2-6xh-3h^2+x+h-1+3x^2-x+1}{h}$

$=\frac{-6xh-3h^2+h}{h}$

$=\frac{h(-6x-3h+1)}{h}$

$=-6x-3h+1$

87. $\frac{f(x+h)-f(x)}{h}=\frac{6-6}{h}=\frac{0}{h}=0$

88. $\frac{f(x+h)-f(x)}{h}=\frac{7-7}{h}=\frac{0}{h}=0$

89. $\frac{f(x+h)-f(x)}{h}$

$=\frac{\frac{1}{x+h}-\frac{1}{x}}{h}$

$=\frac{\frac{x}{x(x+h)}+\frac{-(x+h)}{x(x+h)}}{h}$

$=\frac{\frac{x-x-h}{x(x+h)}}{h}$

$=\frac{\frac{-h}{x(x+h)}}{h}$

$=\frac{-h}{x(x+h)}\cdot\frac{1}{h}$

$=\frac{-1}{x(x+h)}$

90. $\frac{f(x+h)-f(x)}{h}$

$=\frac{\frac{1}{2(x+h)}-\frac{1}{2x}}{h}$

$=\frac{\frac{x}{2x(x+h)}-\frac{x+h}{2x(x+h)}}{h}$

$=\frac{\frac{-h}{2x(x+h)}}{h}$

$=\frac{-h}{2x(x+h)}\cdot\frac{1}{h}$

$=\frac{-1}{2x(x+h)}$

91. $\frac{f(x+h)-f(x)}{h}$

$=\frac{\sqrt{x+h}-\sqrt{x}}{h}$

$=\frac{\sqrt{x+h}-\sqrt{x}}{h}\cdot\frac{\sqrt{x+h}+\sqrt{x}}{\sqrt{x+h}+\sqrt{x}}$

$=\frac{x+h-x}{h\left(\sqrt{x+h}+\sqrt{x}\right)}$

$=\frac{h}{h\left(\sqrt{x+h}+\sqrt{x}\right)}$

$=\frac{1}{\sqrt{x+h}+\sqrt{x}}$

92. $\frac{f(x+h)-f(x)}{h}$

$=\frac{\sqrt{x+h-1}-\sqrt{x-1}}{h}$

$=\frac{\sqrt{x+h-1}-\sqrt{x-1}}{h}\cdot\frac{\sqrt{x+h-1}+\sqrt{x-1}}{\sqrt{x+h-1}+\sqrt{x-1}}$

$=\frac{x+h-1-(x-1)}{h\left(\sqrt{x+h-1}+\sqrt{x-1}\right)}$

$=\frac{x+h-1-x+1}{h\left(\sqrt{x+h-1}+\sqrt{x-1}\right)}$

$=\frac{h}{h\left(\sqrt{x+h-1}+\sqrt{x-1}\right)}$

$=\frac{1}{\sqrt{x+h-1}+\sqrt{x-1}}$

93. $\sqrt{f(-1.5)+f(-0.9)}-\left[f(\pi)\right]^2+f(-3)\div f(1)\cdot f(-\pi)$

$=\sqrt{1+0}-[-4]^2+2\div(-2)\cdot 3$

$=\sqrt{1}-16+(-1)\cdot 3$

$=1-16-3$

$=-18$

94. $\sqrt{f(-2.5)-f(1.9)}-\left[f(-\pi)\right]^2+f(-3)\div f(1)\cdot f(\pi)$

$\sqrt{f(-2.5)-f(1.9)}-\left[f(-\pi)\right]^2+f(-3)\div f(1)\cdot f(\pi)$

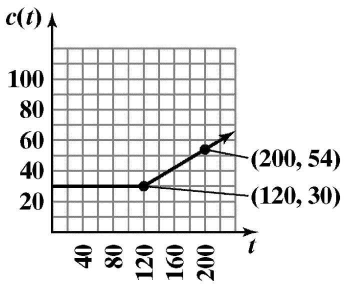
$=\sqrt{2-(-2)}-[3]^2+2\div(-2)\cdot(-4)$

$=\sqrt{4}-9+(-1)(-4)$

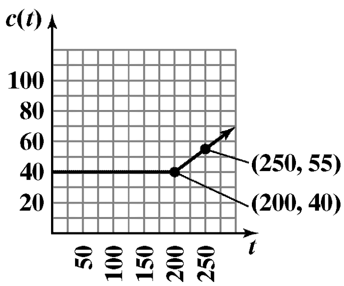
$=2-9+4$

$=-3$

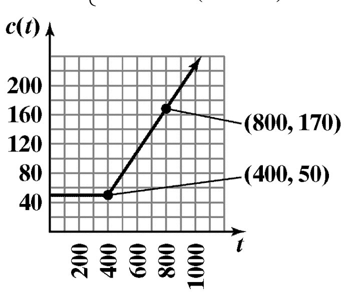
95. $30+0.30(t-120)=30+0.3t-36=0.3t-6$



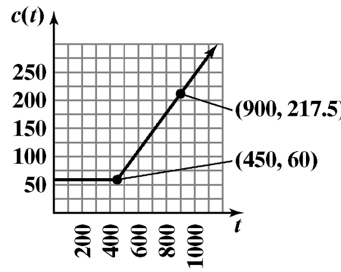
96. $40+0.30(t-200)=40+0.3t-60=0.3t-20$



97. $C(t)=\begin{cases}50 & \text{if } 0\le t\le 400\\ 50+0.30(t-400) & \text{if } t>400\end{cases}$



98. $C(t)=\begin{cases}60 & \text{if } 0\le t\le 450\\ 60+0.35(t-450) & \text{if } t>450\end{cases}$



99. increasing: (25, 55); decreasing: (55, 75)

100. increasing: (25, 65); decreasing: (65, 75)

101. The percent body fat in women reaches a maximum at age 55. This maximum is 38%.

102. The percent body fat in men reaches a maximum at age 65. This maximum is 26%.

103. domain: [25, 75]; range: [34, 38]

104. domain: [25, 75]; range: [23, 26]

105. This model describes percent body fat in men.

106. This model describes percent body fat in women.

107. $T(20{,}000)=850+0.15(20{,}000-8500)$

$=2575$

A single taxpayer with taxable income of $20,000 owes $2575.

108. $T(50{,}000)=4750+0.25(50{,}000-34{,}500)$

$=8625$

A single taxpayer with taxable income of $50,000 owes $8625.

109. $42{,}449+0.33(x-174{,}400)$

110. $110{,}016.50+0.35(x-(x-379{,}150)$

111. $f(3)=0.93$

The cost of mailing a first-class letter weighing 3 ounces is $0.93.

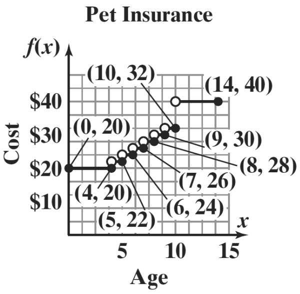
112. $f(3.5)=1.05$

The cost of mailing a first-class letter weighing 3.5 ounces is $1.05.

113. The cost to mail a letter weighing 1.5 ounces is $0.65.

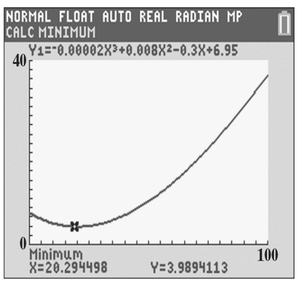
114. The cost to mail a letter weighing 1.8 ounces is \$0.65.

115.



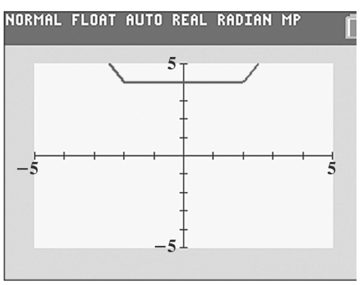
116. – 124. Answers will vary.

125.



The number of doctor visits decreases during childhood and then increases as you get older. The minimum is (20.29, 3.99), which means that the minimum number of doctor visits, about 4, occurs at around age 20.

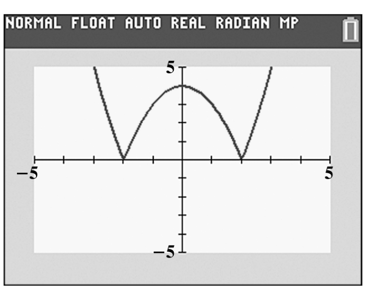
126.



Increasing: $(-\infty, 1)$ or $(3, \infty)$

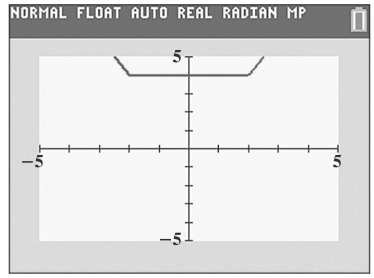
Decreasing: (1, 3)

127.



Increasing: (–2, 0) or (2, ∞)
Decreasing: (–∞, –2) or (0, 2)

128.

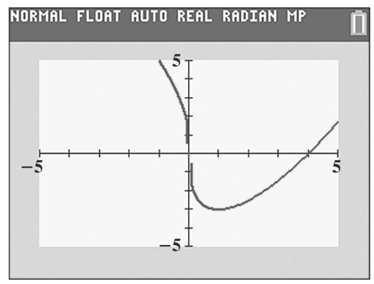


Increasing: $(2, \infty)$

Decreasing: $(-\infty, -2)$

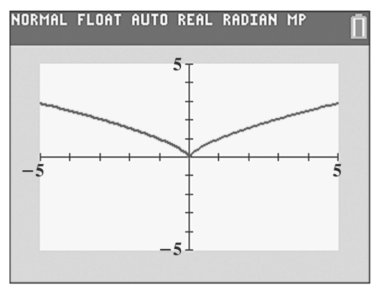
Constant: (–2, 2)

129.



Increasing: (1, ∞)
Decreasing: (–∞, 1)

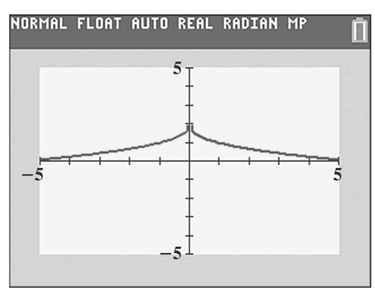
130.



Increasing: $(0, \infty)$

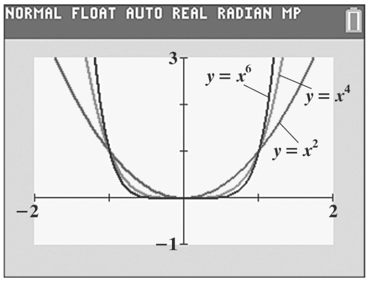
Decreasing: $(-\infty, 0)$

131.

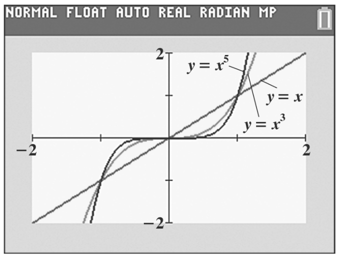


Increasing: (–∞, 0)
Decreasing: (0, ∞)

132. a.



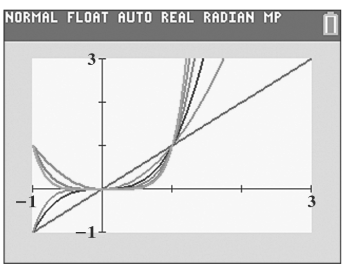
b.



c. Increasing: $(0, \infty)$
Decreasing: $(-\infty, 0)$

d. $f(x) = x^n$ is increasing from $(-\infty, \infty)$ when n is odd.

e.



133. does not make sense; Explanations will vary. Sample explanation: It's possible the graph is not defined at a.

134. makes sense

135. makes sense

136. makes sense

137. answers will vary

138. answers will vary

139. a. h is even if both f and g are even or if both f and g are odd.
f and g are both even:

$$h(-x) = \frac{f(-x)}{g(-x)} = \frac{f(x)}{g(x)} = h(x)$$

f and g are both odd:

$$h(-x) = \frac{f(-x)}{g(-x)} = \frac{-f(x)}{-g(x)} = \frac{f(x)}{g(x)} = h(x)$$

b. h is odd if f is odd and g is even or if f is even and g is odd.
f is odd and g is even:

$$h(-x) = \frac{f(-x)}{g(-x)} = \frac{-f(x)}{g(x)} = -\frac{f(x)}{g(x)} = -h(x)$$

f is even and g is odd:

$$h(-x) = \frac{f(-x)}{g(-x)} = \frac{f(x)}{-g(x)} = -\frac{f(x)}{g(x)} = -h(x)$$

140. Let x = the amount invested at 5%.
Let $80{,}000 - x$ = the amount invested at 7%.

$$0.05x + 0.07(80{,}000 - x) = 5200$$
$$0.05x + 5600 - 0.07x = 5200$$
$$-0.02x + 5600 = 5200$$
$$-0.02x = -400$$
$$x = 20{,}000$$
$$80{,}000 - x = 60{,}000$$

\$20,000 was invested at 5% and \$60,000 was invested at 7%.

141.

$$C = A + Ar$$
$$C = A + Ar$$
$$C = A(1 + r)$$
$$\frac{C}{1+r} = A$$

142. $5x^2 - 7x + 3 = 0$

$a = 5, b = -7, c = 3$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(5)(3)}}{2(5)}$$

$$x = \frac{7 \pm \sqrt{49 - 60}}{10}$$

$$x = \frac{7 \pm \sqrt{-11}}{10}$$

$$x = \frac{7 \pm i\sqrt{11}}{10}$$

$$x = \frac{7}{10} \pm i\frac{\sqrt{11}}{10}$$

The solution set is $\left\{\frac{7}{10} + i\frac{\sqrt{11}}{10}, \frac{7}{10} - \frac{\sqrt{11}}{10}\right\}$.

143. $\frac{y_2 - y_1}{x_2 - x_1} = \frac{4 - 1}{-2 - (-3)} = \frac{3}{1} = 3$

144. When $y = 0$:

$$4x - 3y - 6 = 0$$
$$4x - 3(0) - 6 = 0$$
$$4x - 6 = 0$$
$$4x = 6$$
$$x = \frac{3}{2}$$

The point is $\left(\frac{3}{2}, 0\right)$.

When $x = 0$:

$$4x - 3y - 6 = 0$$
$$4(0) - 3y - 6 = 0$$
$$-3y - 6 = 0$$
$$-3y = 6$$
$$x = -2$$

The point is $(0, -2)$.

145. $3x + 2y - 4 = 0$

$$2y = -3x + 4$$

$$y = \frac{-3x + 4}{2}$$

or

$$y = -\frac{3}{2}x + 2$$

Section 2.3

Check Point Exercises

1. **a.** $m = \frac{-2 - 4}{-4 - (-3)} = \frac{-6}{-1} = 6$

b. $m = \frac{5 - (-2)}{-1 - 4} = \frac{7}{-5} = -\frac{7}{5}$

2. Point-slope form:

$$y - y_1 = m(x - x_1)$$
$$y - (-5) = 6(x - 2)$$
$$y + 5 = 6(x - 2)$$

Slope-intercept form:

$$y + 5 = 6(x - 2)$$
$$y + 5 = 6x - 12$$
$$y = 6x - 17$$

3. $m = \frac{-6 - (-1)}{-1 - (-2)} = \frac{-5}{1} = -5$,

so the slope is –5.

Using the point (–2, –1), we get the following point-slope equation:

$$y - y_1 = m(x - x_1)$$
$$y - (-1) = -5[x - (-2)]$$
$$y + 1 = -5(x + 2)$$

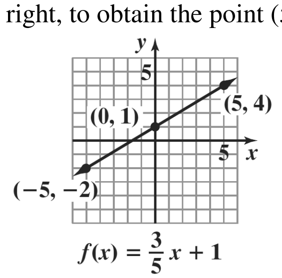
Using the point (–1, –6), we get the following point-slope equation:

$$y - y_1 = m(x - x_1)$$
$$y - (-6) = -5[x - (-1)]$$
$$y + 6 = -5(x + 1)$$

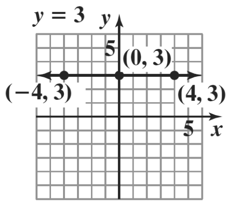
Solve the equation for y:

$$y + 1 = -5(x + 2)$$
$$y + 1 = -5x - 10$$
$$y = -5x - 11.$$

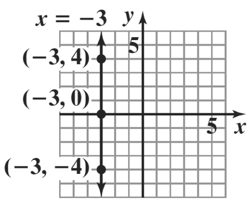
4. The slope m is $\frac{3}{5}$ and the y-intercept is 1, so one point on the line is (0, 1). We can find a second point on the line by using the slope $m = \frac{3}{5} = \frac{\text{Rise}}{\text{Run}}$: starting at the point (0, 1), move 3 units up and 5 units to the right, to obtain the point (5, 4).



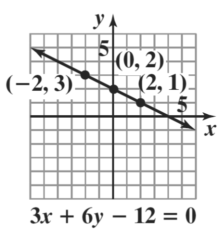
5. $y = 3$ is a horizontal line.



6. All ordered pairs that are solutions of $x = -3$ have a value of x that is always –3. Any value can be used for y.



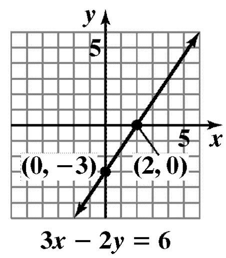
7.
$$3x + 6y - 12 = 0$$
$$6y = -3x + 12$$
$$y = \frac{-3}{6}x + \frac{12}{6}$$
$$y = -\frac{1}{2}x + 2$$



The slope is $-\frac{1}{2}$ and the y-intercept is 2.

8. Find the x-intercept:
$$3x - 2y - 6 = 0$$
$$3x - 2(0) - 6 = 0$$
$$3x - 6 = 0$$
$$3x = 6$$
$$x = 2$$

Find the y-intercept:
$$3x - 2y - 6 = 0$$
$$3(0) - 2y - 6 = 0$$
$$-2y - 6 = 0$$
$$-2y = 6$$
$$y = -3$$



9. First find the slope.
$$m = \frac{\text{Change in } y}{\text{Change in } x} = \frac{57.64 - 57.04}{354 - 317} = \frac{0.6}{37} \approx 0.016$$
Use the point-slope form and then find slope-intercept form.
$$y - y_1 = m(x - x_1)$$
$$y - 57.04 = 0.016(x - 317)$$
$$y - 57.04 = 0.016x - 5.072$$
$$y = 0.016x + 51.968$$
$$f(x) = 0.016x + 52.0$$
Find the temperature at a concentration of 600 parts per million.
$$f(x) = 0.016x + 52.0$$
$$f(600) = 0.016(600) + 52.0$$
$$= 61.6$$
The temperature at a concentration of 600 parts per million would be 61.6°F.

Concept and Vocabulary Check 2.3

1. scatter plot; regression

2. $\frac{y_2 - y_1}{x_2 - x_1}$

3. positive

4. negative

5. zero

6. undefined

7. $y - y_1 = m(x - x_1)$

8. $y = mx + b$; slope; y-intercept

9. $(0, 3)$; 2; 5

10. horizontal

11. vertical

12. general

Exercise Set 2.3

1. $m = \frac{10-7}{8-4} = \frac{3}{4}$; rises

2. $m = \frac{4-1}{3-2} = \frac{3}{1} = 3$; rises

3. $m = \frac{2-1}{2-(-2)} = \frac{1}{4}$; rises

4. $m = \frac{4-3}{2-(-1)} = \frac{1}{3}$; rises

5. $m = \frac{2-(-2)}{3-4} = \frac{0}{-1} = 0$; horizontal

6. $m = \frac{-1-(-1)}{3-4} = \frac{0}{-1} = 0$; horizontal

7. $m = \frac{-1-4}{-1-(-2)} = \frac{-5}{1} = -5$; falls

8. $m = \frac{-2-(-4)}{4-6} = \frac{2}{-2} = -1$; falls

9. $m = \frac{-2-3}{5-5} = \frac{-5}{0}$ undefined; vertical

10. $m = \frac{5-(-4)}{3-3} = \frac{9}{0}$ undefined; vertical

11. $m = 2,\ x_1 = 3,\ y_1 = 5$;
point-slope form: $y - 5 = 2(x - 3)$;
slope-intercept form: $y - 5 = 2x - 6$
$y = 2x - 1$

12. point-slope form: $y - 3 = 4(x - 1)$;
$m = 4, x_1 = 1, y_1 = 3$;
slope-intercept form: $y = 4x - 1$

13. $m = 6,\ x_1 = -2,\ y_1 = 5$;
point-slope form: $y - 5 = 6(x + 2)$;
slope-intercept form: $y - 5 = 6x + 12$
$y = 6x + 17$

14. point-slope form: $y + 1 = 8(x - 4)$;
$m = 8, x_1 = 4, y_1 = -1$;
slope-intercept form: $y = 8x - 33$

15. $m = -3,\ x_1 = -2,\ y_1 = -3$;
point-slope form: $y + 3 = -3(x + 2)$;
slope-intercept form: $y + 3 = -3x - 6$
$y = -3x - 9$

16. point-slope form: $y + 2 = -5(x + 4)$;
$m = -5, x_1 = -4, y_1 = -2$;
slope-intercept form: $y = -5x - 22$

17. $m = -4,\ x_1 = -4,\ y_1 = 0$;
point-slope form: $y - 0 = -4(x + 4)$;
slope-intercept form: $y = -4(x + 4)$
$y = -4x - 16$

18. point-slope form: $y + 3 = -2(x - 0)$
$m = -2, x_1 = 0, y_1 = -3$;
slope-intercept form: $y = -2x - 3$

19. $m = -1,\ x_1 = \frac{-1}{2},\ y_1 = -2$;
point-slope form: $y + 2 = -1\left(x + \frac{1}{2}\right)$;
slope-intercept form: $y + 2 = -x - \frac{1}{2}$
$y = -x - \frac{5}{2}$

20. point-slope form: $y + \frac{1}{4} = -1(x + 4)$;
$m = -1, x_1 = -4, y_1 = -\frac{1}{4}$;
slope-intercept form: $y = -x - \frac{17}{4}$

21. $m = \frac{1}{2},\ x_1 = 0,\ y_1 = 0$;
point-slope form: $y - 0 = \frac{1}{2}(x - 0)$;
slope-intercept form: $y = \frac{1}{2}x$

22. point-slope form: $y-0=\frac{1}{3}(x-0)$;

$m=\frac{1}{3}, x_1=0, y_1=0$;

slope-intercept form: $y=\frac{1}{3}x$

23. $m=-\frac{2}{3},\ x_1=6,\ y_1=-2$;

point-slope form: $y+2=-\frac{2}{3}(x-6)$;

slope-intercept form: $y+2=-\frac{2}{3}x+4$

$y=-\frac{2}{3}x+2$

24. point-slope form: $y+4=-\frac{3}{5}(x-10)$;

$m=-\frac{3}{5}, x_1=10, y_1=-4$;

slope-intercept form: $y=-\frac{3}{5}x+2$

25. $m=\frac{10-2}{5-1}=\frac{8}{4}=2$;
point-slope form: $y-2=2(x-1)$ using
$(x_1,\ y_1)=(1,\ 2)$, or $y-10=2(x-5)$ using
$(x_1,\ y_1)=(5,\ 10)$;
slope-intercept form: $y-2=2x-2$ or

$y-10=2x-10,$

$y=2x$

26. $m=\frac{15-5}{8-3}=\frac{10}{5}=2$;
point-slope form: $y-5=2(x-3)$ using
$(x_1,y_1)=(3,5)$, or $y-15=2(x-8)$ using
$(x_1,y_1)=(8,15)$;
slope-intercept form: $y=2x-1$

27. $m=\frac{3-0}{0-(-3)}=\frac{3}{3}=1$;
point-slope form: $y-0=1(x+3)$ using
$(x_1,\ y_1)=(-3,\ 0)$, or $y-3=1(x-0)$ using
$(x_1,\ y_1)=(0,\ 3)$; slope-intercept form: $y=x+3$

28. $m=\frac{2-0}{0-(-2)}=\frac{2}{2}=1$;
point-slope form: $y-0=1(x+2)$ using
$(x_1,y_1)=(-2,0)$, or $y-2=1(x-0)$ using
$(x_1,y_1)=(0,2)$;
slope-intercept form: $y=x+2$

29. $m=\frac{4-(-1)}{2-(-3)}=\frac{5}{5}=1$;
point-slope form: $y+1=1(x+3)$ using
$(x_1,\ y_1)=(-3,\ -1)$, or $y-4=1(x-2)$ using
$(x_1,\ y_1)=(2,\ 4)$; slope-intercept form:
$y+1=x+3$ or
$y-4=x-2$
$y=x+2$

30. $m=\frac{-1-(-4)}{1-(-2)}=\frac{3}{3}=1$;
point-slope form: $y+4=1(x+2)$ using
$(x_1,y_1)=(-2,-4)$, or $y+1=1(x-1)$ using
$(x_1,y_1)=(1,-1)$
slope-intercept form: $y=x-2$

31. $m=\frac{6-(-2)}{3-(-3)}=\frac{8}{6}=\frac{4}{3}$;

point-slope form: $y+2=\frac{4}{3}(x+3)$ using

$(x_1,\ y_1)=(-3,\ -2)$, or $y-6=\frac{4}{3}(x-3)$ using

$(x_1,\ y_1)=(3,\ 6)$;

slope-intercept form: $y+2=\frac{4}{3x}+4$ or

$y-6=\frac{4}{3}x-4,$

$y=\frac{4}{3}x+2$

32. $m=\frac{-2-6}{3-(-3)}=\frac{-8}{6}=-\frac{4}{3}$;

point-slope form: $y-6=-\frac{4}{3}(x+3)$ using

$(x_1,y_1)=(-3,6)$, or $y+2=-\frac{4}{3}(x-3)$ using

$(x_1,y_1)=(3,-2)$;

slope-intercept form: $y=-\frac{4}{3}x+2$

33. $m = \dfrac{-1-(-1)}{4-(-3)} = \dfrac{0}{7} = 0$;

point-slope form: $y + 1 = 0(x + 3)$ using $(x_1,\ y_1) = (-3, -1)$, or $y + 1 = 0(x - 4)$ using $(x_1,\ y_1) = (4, -1)$;

slope-intercept form: $y + 1 = 0$, so

$$y = -1$$

34. $m = \dfrac{-5-(-5)}{6-(-2)} = \dfrac{0}{8} = 0$;

point-slope form: $y + 5 = 0(x + 2)$ using $(x_1, y_1) = (-2, -5)$, or $y + 5 = 0(x - 6)$ using $(x_1, y_1) = (6, -5)$;

slope-intercept form: $y + 5 = 0$, so

$$y = -5$$

35. $m = \dfrac{0-4}{-2-2} = \dfrac{-4}{-4} = 1$;

point-slope form: $y - 4 = 1(x - 2)$ using $(x_1,\ y_1) = (2, 4)$, or $y - 0 = 1(x + 2)$ using $(x_1,\ y_1) = (-2, 0)$;

slope-intercept form: $y - 9 = x - 2$, or

$$y = x + 2$$

36. $m = \dfrac{0-(-3)}{-1-1} = \dfrac{3}{-2} = -\dfrac{3}{2}$

point-slope form: $y + 3 = -\dfrac{3}{2}(x - 1)$ using $(x_1, y_1) = (1, -3)$, or $y - 0 = -\dfrac{3}{2}(x + 1)$ using $(x_1, y_1) = (-1, 0)$;

slope-intercept form: $y + 3 = -\dfrac{3}{2}x + \dfrac{3}{2}$, or

$$y = -\frac{3}{2}x - \frac{3}{2}$$

37. $m = \dfrac{4-0}{0-\left(-\frac{1}{2}\right)} = \dfrac{4}{\frac{1}{2}} = 8$;

point-slope form: $y - 4 = 8(x - 0)$ using $(x_1,\ y_1) = (0, 4)$, or $y - 0 = 8\left(x + \frac{1}{2}\right)$ using $(x_1,\ y_1) = \left(-\frac{1}{2}, 0\right)$; or $y - 0 = 8\left(x + \frac{1}{2}\right)$

slope-intercept form: $y = 8x + 4$

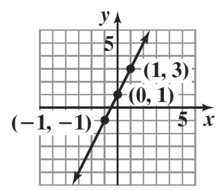
38. $m = \dfrac{-2-0}{0-4} = \dfrac{-2}{-4} = \dfrac{1}{2}$;

point-slope form: $y - 0 = \dfrac{1}{2}(x - 4)$ using $(x_1, y_1) = (4, 0)$,

or $y + 2 = \dfrac{1}{2}(x - 0)$ using $(x_1, y_1) = (0, -2)$;

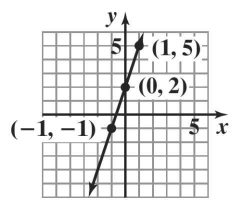
slope-intercept form: $y = \dfrac{1}{2}x - 2$

39. $m = 2$; $b = 1$



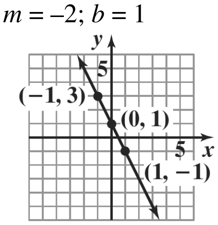
y = 2*x* + 1

40. $m = 3$; $b = 2$



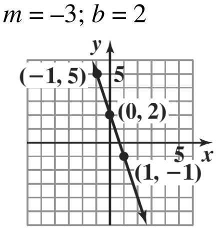
y = 3*x* + 2

41. $m = -2$; $b = 1$



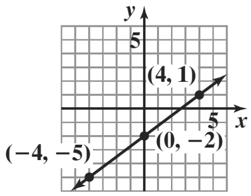
f(*x*) = −2*x* + 1

42. $m = -3$; $b = 2$



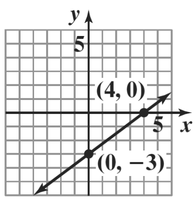
f(*x*) = −3*x* + 2

43. $m = \frac{3}{4}$; $b = -2$



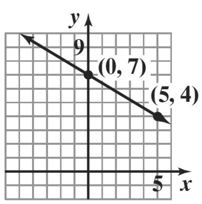
$f(x) = \frac{3}{4}x - 2$

44. $m = \frac{3}{4}$; $b = -3$



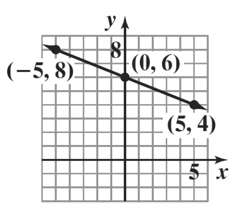
$f(x) = \frac{3}{4}x - 3$

45. $m = -\frac{3}{5}$; $b = 7$



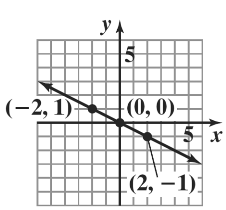
$y = -\frac{3}{5}x + 7$

46. $m = -\frac{2}{5}$; $b = 6$



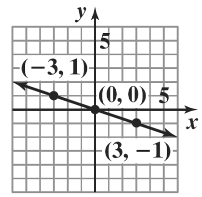
$y = -\frac{2}{5}x + 6$

47. $m = -\frac{1}{2}$; $b = 0$



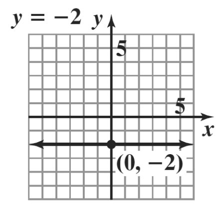
$g(x) = -\frac{1}{2}x$

48. $m = -\frac{1}{3}$; $b = 0$



$g(x) = -\frac{1}{3}x$

49.



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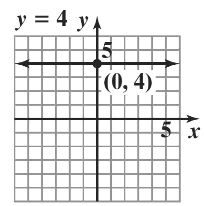

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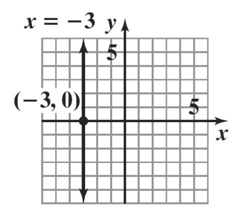

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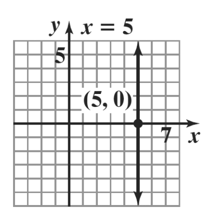

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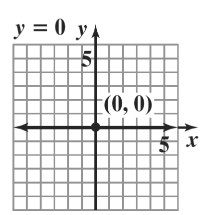

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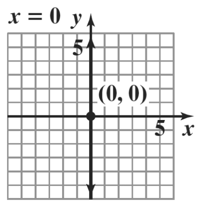

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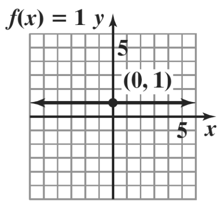

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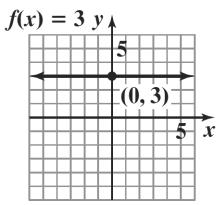
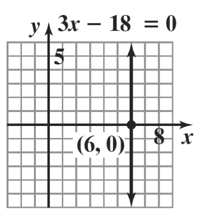


57. $3x-18=0$

$3x=18$

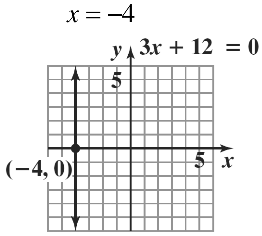
$x=6$



58. $3x+12=0$

$3x=-12$

$x=-4$



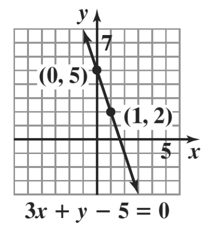
59. **a.** $3x+y-5=0$

$y-5=-3x$

$y=-3x+5$

b. $m=-3;\ b=5$

c.



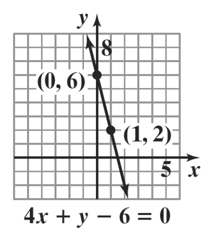
60. **a.** $4x+y-6=0$

$y-6=-4x$

$y=-4x+6$

b. $m=-4; b=6$

c.



61. **a.** $2x+3y-18=0$

$2x-18=-3y$

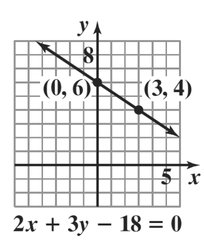
$-3y=2x-18$

$y=\frac{2}{-3}x-\frac{18}{-3}$

$y=-\frac{2}{3}x+6$

b. $m=-\frac{2}{3};\ b=6$

c.



62. **a.** $4x+6y+12=0$

$4x+12=-6y$

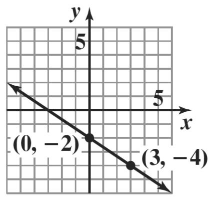
$-6y=4x+12$

$y=\frac{4}{-6}x+\frac{12}{-6}$

$y=-\frac{2}{3}x-2$

b. $m=-\frac{2}{3};\ b=-2$

c.



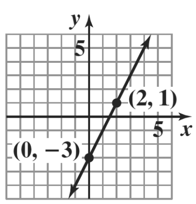
4x + 6y + 12 = 0

63. a. $8x-4y-12=0$

$$8x-12=4y$$
$$4y=8x-12$$
$$y=\frac{8}{4}x-\frac{12}{4}$$
$$y=2x-3$$

b. $m=2;\ b=-3$

c.



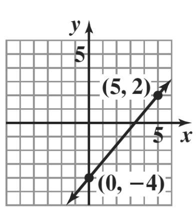
8x − 4y − 12 = 0

64. a. $6x-5y-20=0$

$$6x-20=5y$$
$$5y=6x-20$$
$$y=\frac{6}{5}x-\frac{20}{5}$$
$$y=\frac{6}{5}x-4$$

b. $m=\frac{6}{5};b=-4$

c.



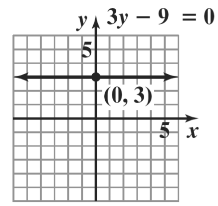
6x − 5y − 20 = 0

65. a. $3y-9=0$

$$3y=9$$
$$y=3$$

b. $m=0;\ b=3$

c.

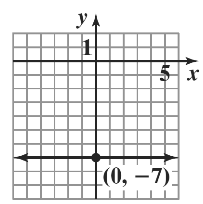


66. a. $4y+28=0$

$$4y=-28$$
$$y=-7$$

b. $m=0;\ b=-7$

c.



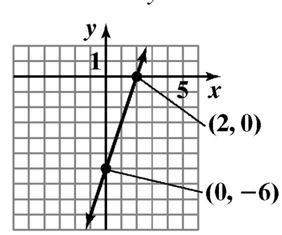
4y + 28 = 0

67. Find the x-intercept:

$$6x-2y-12=0$$
$$6x-2(0)-12=0$$
$$6x-12=0$$
$$6x=12$$
$$x=2$$

Find the y-intercept:

$$6x-2y-12=0$$
$$6(0)-2y-12=0$$
$$-2y-12=0$$
$$-2y=12$$
$$y=-6$$



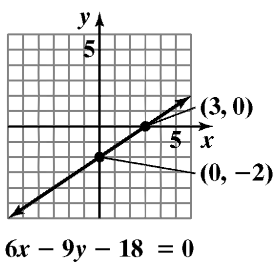
6x − 2y − 12 = 0

68. Find the x-intercept:

$$\begin{aligned}6x-9y-18&=0\\6x-9(0)-18&=0\\6x-18&=0\\6x&=18\\x&=3\end{aligned}$$

Find the y-intercept:

$$\begin{aligned}6x-9y-18&=0\\6(0)-9y-18&=0\\-9y-18&=0\\-9y&=18\\y&=-2\end{aligned}$$

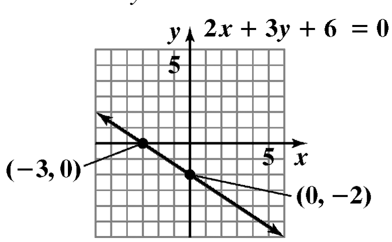


69. Find the x-intercept:

$$\begin{aligned}2x+3y+6&=0\\2x+3(0)+6&=0\\2x+6&=0\\2x&=-6\\x&=-3\end{aligned}$$

Find the y-intercept:

$$\begin{aligned}2x+3y+6&=0\\2(0)+3y+6&=0\\3y+6&=0\\3y&=-6\\y&=-2\end{aligned}$$

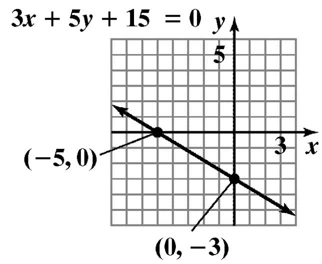


70. Find the x-intercept:

$$\begin{aligned}3x+5y+15&=0\\3x+5(0)+15&=0\\3x+15&=0\\3x&=-15\\x&=-5\end{aligned}$$

Find the y-intercept:

$$\begin{aligned}3x+5y+15&=0\\3(0)+5y+15&=0\\5y+15&=0\\5y&=-15\\y&=-3\end{aligned}$$

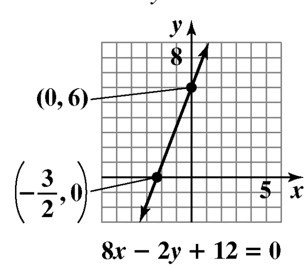


71. Find the x-intercept:

$$\begin{aligned}8x-2y+12&=0\\8x-2(0)+12&=0\\8x+12&=0\\8x&=-12\\\frac{8x}{8}&=\frac{-12}{8}\\x&=\frac{-3}{2}\end{aligned}$$

Find the y-intercept:

$$\begin{aligned}8x-2y+12&=0\\8(0)-2y+12&=0\\-2y+12&=0\\-2y&=-12\\y&=-6\end{aligned}$$

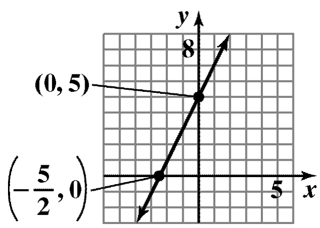


72. Find the x-intercept:

$$6x-3y+15=0$$
$$6x-3(0)+15=0$$
$$6x+15=0$$
$$6x=-15$$
$$\frac{6x}{6}=\frac{-15}{6}$$
$$x=-\frac{5}{2}$$

Find the y-intercept:

$$6x-3y+15=0$$
$$6(0)-3y+15=0$$
$$-3y+15=0$$
$$-3y=-15$$
$$y=5$$



6x + 3y + 15 = 0

73. $m=\frac{0-a}{b-0}=\frac{-a}{b}=-\frac{a}{b}$

Since a and b are both positive, $-\frac{a}{b}$ is negative. Therefore, the line falls.

74. $m=\frac{-b-0}{0-(-a)}=\frac{-b}{a}=-\frac{b}{a}$

Since a and b are both positive, $-\frac{b}{a}$ is negative. Therefore, the line falls.

75. $m=\frac{(b+c)-b}{a-a}=\frac{c}{0}$

The slope is undefined.
The line is vertical.

76. $m=\frac{(a+c)-c}{a-(a-b)}=\frac{a}{b}$

Since a and b are both positive, $\frac{a}{b}$ is positive. Therefore, the line rises.

77.

$$Ax+By=C$$
$$By=-Ax+C$$
$$y=-\frac{A}{B}x+\frac{C}{B}$$

The slope is $-\frac{A}{B}$ and the $y-$intercept is $\frac{C}{B}$.

78.

$$Ax=By-C$$
$$Ax+C=By$$
$$\frac{A}{B}x+\frac{C}{B}=y$$

The slope is $\frac{A}{B}$ and the $y-$intercept is $\frac{C}{B}$.

79.

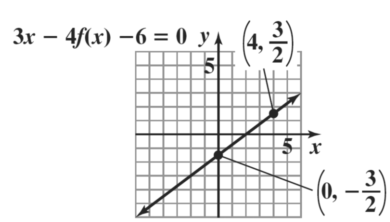
$$-3=\frac{4-y}{1-3}$$
$$-3=\frac{4-y}{-2}$$
$$6=4-y$$
$$2=-y$$
$$-2=y$$

80.

$$\frac{1}{3}=\frac{-4-y}{4-(-2)}$$
$$\frac{1}{3}=\frac{-4-y}{4+2}$$
$$\frac{1}{3}=\frac{-4-y}{6}$$
$$6=3(-4-y)$$
$$6=-12-3y$$
$$18=-3y$$
$$-6=y$$

81.

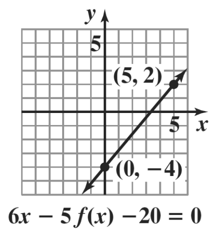
$$3x-4f(x)=6$$
$$-4f(x)=-3x+6$$
$$f(x)=\frac{3}{4}x-\frac{3}{2}$$



82. $6x-5f(x)=20$

$-5f(x)=-6x+20$

$f(x)=\frac{6}{5}x-4$



83. Using the slope-intercept form for the equation of a line:

$-1=-2(3)+b$

$-1=-6+b$

$5=b$

84. $-6=-\frac{3}{2}(2)+b$

$-6=-3+b$

$-3=b$

85. m_1, m_3, m_2, m_4

86. b_2, b_1, b_4, b_3

87. **a.** First, find the slope using $(20, 38.9)$ and $(30, 47.8)$.

$m=\frac{47.8-38.9}{30-20}=\frac{8.9}{10}=0.89$

Then use the slope and one of the points to write the equation in point-slope form.

$y-y_1=m(x-x_1)$

$y-47.8=0.89(x-30)$

or

$y-38.9=0.89(x-20)$

b. $y-47.8=0.89(x-30)$

$y-47.8=0.89x-26.7$

$y=0.89x+21.1$

$f(x)=0.89x+21.1$

c. $f(40)=0.89(40)+21.1=56.7$

The linear function predicts the percentage of never married American females, ages 25 – 29, to be 56.7% in 2020.

88. **a.** First, find the slope using $(20, 51.7)$ and $(30, 62.6)$.

$m=\frac{51.7-62.6}{20-30}=\frac{-10.9}{-10}=1.09$

Then use the slope and one of the points to write the equation in point-slope form.

$y-y_1=m(x-x_1)$

$y-62.6=1.09(x-30)$

or

$y-51.7=1.09(x-20)$

b. $y-62.6=1.09(x-30)$

$y-62.6=1.09x-32.7$

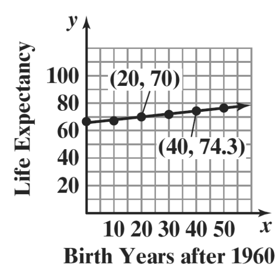
$y=1.09x+29.9$

$f(x)=1.09x+29.9$

c. $f(35)=1.09(35)+29.9=68.05$

The linear function predicts the percentage of never married American males, ages 25 – 29, to be 68.05% in 2015.

89. **a.**



b. $m=\frac{\text{Change in } y}{\text{Change in } x}=\frac{74.3-70.0}{40-20}=0.215$

$y-y_1=m(x-x_1)$

$y-70.0=0.215(x-20)$

$y-70.0=0.215x-4.3$

$y=0.215x+65.7$

$E(x)=0.215x+65.7$

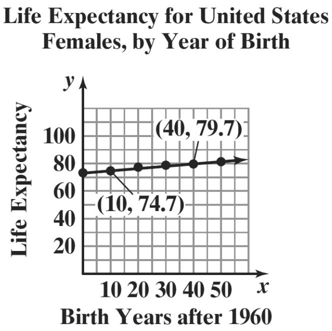
c. $E(x)=0.215x+65.7$

$E(60)=0.215(60)+65.7$

$=78.6$

The life expectancy of American men born in 2020 is expected to be 78.6.

90. a.



b. $m = \frac{\text{Change in } y}{\text{Change in } x} = \frac{79.7 - 74.7}{40 - 10} \approx 0.17$

$y - y_1 = m(x - x_1)$

$y - 74.7 = 0.17(x - 10)$

$y - 74.7 = 0.17x - 1.7$

$y = 0.17x + 73$

$E(x) = 0.17x + 73$

c. $E(x) = 0.17x + 73$

$E(60) = 0.17(60) + 73$

$= 83.2$

The life expectancy of American women born in 2020 is expected to be 83.2.

91. (10, 230) (60, 110) Points may vary.

$m = \frac{110 - 230}{60 - 10} = -\frac{120}{50} = -2.4$

$y - 230 = -2.4(x - 10)$

$y - 230 = -2.4x + 24$

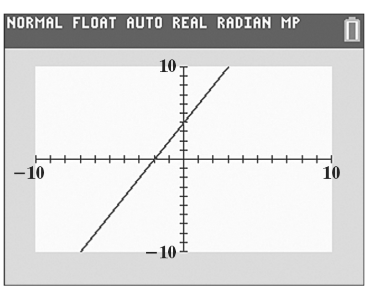
$y = -2.4x + 254$

Answers will vary for predictions.

92. – 99. Answers will vary.

100. Two points are (0,4) and (10,24).

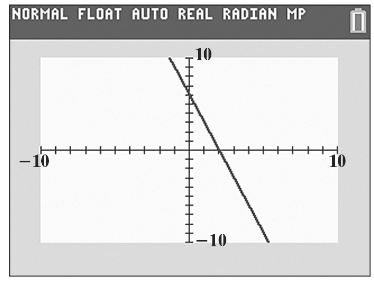
$m = \frac{24 - 4}{10 - 0} = \frac{20}{10} = 2.$



101. Two points are (0, 6) and (10, –24).

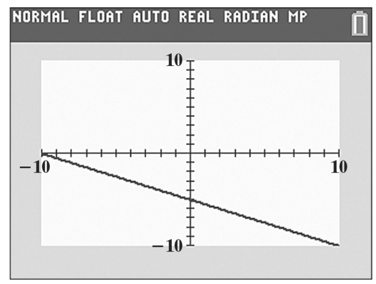
$m = \frac{-24 - 6}{10 - 0} = \frac{-30}{10} = -3.$

Check: $y = mx + b$: $y = -3x + 6$.



102. Two points are (0,–5) and (10,–10).

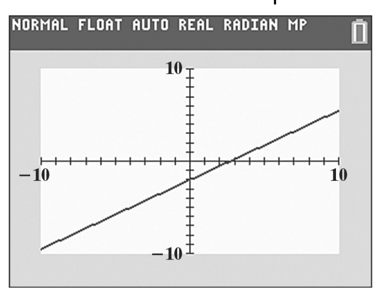
$m = \frac{-10 - (-5)}{10 - 0} = \frac{-5}{10} = -\frac{1}{2}.$



103. Two points are (0, –2) and (10, 5.5).

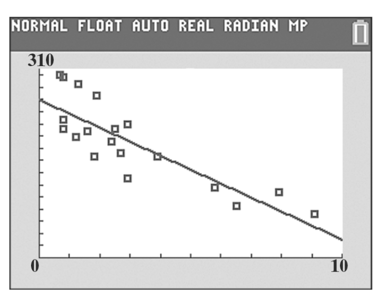
$m = \frac{5.5 - (-2)}{10 - 0} = \frac{7.5}{10} = 0.75 \text{ or } \frac{3}{4}.$

Check: $y = mx + b$: $y = \frac{3}{4}x - 2$.



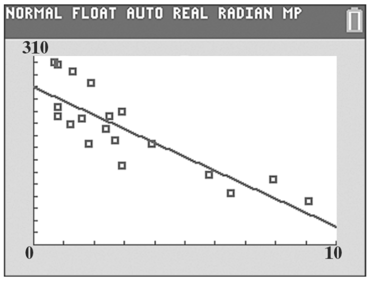
104. a. Enter data from table.

b.



c. $a = -22.96876741$
$b = 260.5633751$
$r = -0.8428126855$

d.



105. does not make sense; Explanations will vary. Sample explanation: Linear functions never change from increasing to decreasing.

106. does not make sense; Explanations will vary. Sample explanation: Since college cost are going up, this function has a positive slope.

107. does not make sense; Explanations will vary. Sample explanation: The slope of line's whose equations are in this form can be determined in several ways. One such way is to rewrite the equation in slope-intercept form.

108. makes sense

109. false; Changes to make the statement true will vary. A sample change is: It is possible for m to equal b.

110. false; Changes to make the statement true will vary. A sample change is: Slope-intercept form is $y = mx + b$. Vertical lines have equations of the form $x = a$. Equations of this form have undefined slope and cannot be written in slope-intercept form.

111. true

112. false; Changes to make the statement true will vary. A sample change is: The graph of $x = 7$ is a vertical line through the point (7, 0).

113. We are given that the $x-$intercept is -2 and the $y-$intercept is 4. We can use the points $(-2,0)$ and $(0,4)$ to find the slope.

$$m = \frac{4-0}{0-(-2)} = \frac{4}{0+2} = \frac{4}{2} = 2$$

Using the slope and one of the intercepts, we can write the line in point-slope form.

$$y - y_1 = m(x - x_1)$$
$$y - 0 = 2(x - (-2))$$
$$y = 2(x+2)$$
$$y = 2x + 4$$
$$-2x + y = 4$$

Find the x– and y–coefficients for the equation of the line with right-hand-side equal to 12. Multiply both sides of $-2x + y = 4$ by 3 to obtain 12 on the right-hand-side.

$$-2x + y = 4$$
$$3(-2x + y) = 3(4)$$
$$-6x + 3y = 12$$

Therefore, the coefficient of x is –6 and the coefficient of y is 3.

114. We are given that the $y-$intercept is -6 and the slope is $\frac{1}{2}$.

So the equation of the line is $y = \frac{1}{2}x - 6$.

We can put this equation in the form $ax + by = c$ to find the missing coefficients.

$$y = \frac{1}{2}x - 6$$
$$y - \frac{1}{2}x = -6$$
$$2\left(y - \frac{1}{2}x\right) = 2(-6)$$
$$2y - x = -12$$
$$x - 2y = 12$$

Therefore, the coefficient of x is 1 and the coefficient of y is –2.

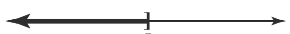
115. Answers will vary.

116. Let (25, 40) and (125, 280) be ordered pairs (M, E) where M is degrees Madonna and E is degrees Elvis. Then $m = \frac{280-40}{125-25} = \frac{240}{100} = 2.4$. Using $(x_1, y_1) = (25, 40)$, point-slope form tells us that
$E - 40 = 2.4\ (M - 25)$ or
$E = 2.4\ M - 20$.

117. Answers will vary.

118. Let x = the number of years after 1994.
$$714 - 17x = 289$$
$$-17x = -425$$
$$x = 25$$
Violent crime incidents will decrease to 289 per 100,000 people 25 years after 1994, or 2019.

119.
$$\frac{x+3}{4} \geq \frac{x-2}{3} + 1$$
$$12\left(\frac{x+3}{4}\right) \geq 12\left(\frac{x-2}{3} + 1\right)$$
$$3(x+3) \geq 4(x-2) + 12$$
$$3x + 9 \geq 4x - 8 + 12$$
$$3x + 9 \geq 4x + 4$$
$$5 \geq x$$
$$x \leq 5$$



The solution set is $\{x | x \leq 5\}$ or $(-\infty, 5]$.

120. $3|2x+6| - 9 < 15$
$$3|2x+6| < 24$$
$$\frac{3|2x+6|}{3} < \frac{24}{3}$$
$$|2x+6| < 8$$
$$-8 < 2x + 6 < 8$$
$$-14 < 2x < 2$$
$$-7 < x < 1$$

The solution set is $\{x | -7 < x < 1\}$ or $(-7, 1)$.

121. Since the slope is the same as the slope of $y = 2x + 1$, then $m = 2$.
$$y - y_1 = m(x - x_1)$$
$$y - 1 = 2(x - (-3))$$
$$y - 1 = 2(x + 3)$$
$$y - 1 = 2x + 6$$
$$y = 2x + 7$$

122. Since the slope is the negative reciprocal of $-\frac{1}{4}$, then $m = 4$.
$$y - y_1 = m(x - x_1)$$
$$y - (-5) = 4(x - 3)$$
$$y + 5 = 4x - 12$$
$$-4x + y + 17 = 0$$
$$4x - y - 17 = 0$$

123.
$$\frac{f(x_2) - f(x_1)}{x_2 - x_1} = \frac{f(4) - f(1)}{4 - 1}$$
$$= \frac{4^2 - 1^2}{4 - 1}$$
$$= \frac{15}{3}$$
$$= 5$$

Section 2.4

Check Point Exercises

1. The slope of the line $y = 3x + 1$ is 3.
$$y - y_1 = m(x - x_1)$$
$$y - 5 = 3(x - (-2))$$
$$y - 5 = 3(x + 2) \quad \text{point-slope}$$
$$y - 5 = 3x + 6$$
$$y = 3x + 11 \quad \text{slope-intercept}$$

2. **a.** Write the equation in slope-intercept form:
$$x + 3y - 12 = 0$$
$$3y = -x + 12$$
$$y = -\frac{1}{3}x + 4$$
The slope of this line is $-\frac{1}{3}$ thus the slope of any line perpendicular to this line is 3.

b. Use $m = 3$ and the point (–2, –6) to write the equation.
$$y - y_1 = m(x - x_1)$$
$$y - (-6) = 3(x - (-2))$$
$$y + 6 = 3(x + 2)$$
$$y + 6 = 3x + 6$$
$$-3x + y = 0$$
$$3x - y = 0 \quad \text{general form}$$

3. $m = \frac{\text{Change in } y}{\text{Change in } x} = \frac{15-11.2}{2013-2000} = \frac{3.8}{13} \approx 0.29$

The slope indicates that the number of U.S. men living alone increased at a rate of 0.29 million each year.
The rate of change is 0.29 million men per year.

4. **a.** $\frac{f(x_2)-f(x_1)}{x_2-x_1} = \frac{1^3-0^3}{1-0} = 1$

b. $\frac{f(x_2)-f(x_1)}{x_2-x_1} = \frac{2^3-1^3}{2-1} = \frac{8-1}{1} = 7$

c. $\frac{f(x_2)-f(x_1)}{x_2-x_1} = \frac{0^3-(-2)^3}{0-(-2)} = \frac{8}{2} = 4$

5. $\frac{f(x_2)-f(x_1)}{x_2-x_1} = \frac{f(3)-f(1)}{3-1}$

$= \frac{0.05-0.03}{3-1}$

$= 0.01$

The average rate of change in the drug's concentration between 1 hour and 3 hours is 0.01 mg per 100 mL per hour.

Concept and Vocabulary Check 2.4

1. the same

2. -1

3. $-\frac{1}{3}$; 3

4. -2 ; $\frac{1}{2}$

5. y; x

6. $\frac{f(x_2)-f(x_1)}{x_2-x_1}$

Exercise Set 2.4

1. Since L is parallel to $y = 2x$, we know it will have slope $m = 2$. We are given that it passes through $(4, 2)$. We use the slope and point to write the equation in point-slope form.

$y - y_1 = m(x - x_1)$

$y - 2 = 2(x - 4)$

Solve for y to obtain slope-intercept form.

$y - 2 = 2(x - 4)$

$y - 2 = 2x - 8$

$y = 2x - 6$

In function notation, the equation of the line is $f(x) = 2x - 6.$

2. L will have slope $m = -2$. Using the point and the slope, we have $y - 4 = -2(x - 3)$. Solve for y to obtain slope-intercept form.

$y - 4 = -2x + 6$

$y = -2x + 10$

$f(x) = -2x + 10$

3. Since L is perpendicular to $y = 2x$, we know it will have slope $m = -\frac{1}{2}$. We are given that it passes through $(2, 4)$. We use the slope and point to write the equation in point-slope form.

$y - y_1 = m(x - x_1)$

$y - 4 = -\frac{1}{2}(x - 2)$

Solve for y to obtain slope-intercept form.

$y - 4 = -\frac{1}{2}(x - 2)$

$y - 4 = -\frac{1}{2}x + 1$

$y = -\frac{1}{2}x + 5$

In function notation, the equation of the line is $f(x) = -\frac{1}{2}x + 5.$

4. L will have slope $m=\frac{1}{2}$. The line passes through (–1, 2). Use the slope and point to write the equation in point-slope form.

$y-2=\frac{1}{2}(x-(-1))$

$y-2=\frac{1}{2}(x+1)$

Solve for y to obtain slope-intercept form.

$y-2=\frac{1}{2}x+\frac{1}{2}$

$y=\frac{1}{2}x+\frac{1}{2}+2$

$y=\frac{1}{2}x+\frac{5}{2}$

$f(x)=\frac{1}{2}x+\frac{5}{2}$

5. $m = –4$ since the line is parallel to $y=-4x+3$; $x_1=-8$, $y_1=-10$;
point-slope form: $y + 10 = –4(x + 8)$
slope-intercept form: $y + 10 = –4x – 32$
$y = –4x – 42$

6. $m = –5$ since the line is parallel to $y=-5x+4$;
$x_1=-2, y_1=-7$;
point-slope form: $y + 7 = –5(x + 2)$
slope-intercept form: $y+7=-5x-10$
$y=-5x-17$

7. $m = –5$ since the line is perpendicular to
$y=\frac{1}{5}x+6$; $x_1=2$, $y_1=-3$;
point-slope form: $y + 3 = –5(x – 2)$
slope-intercept form: $y+3=-5x+10$
$y=-5x+7$

8. $m=-3$ since the line is perpendicular to $y=\frac{1}{3}x+7$;
$x_1=-4, y_1=2$;
point-slope form: $y-2=-3(x+4)$
slope-intercept form: $y-2=-3x-12$
$y=-3x-10$

9. $2x-3y-7=0$

$-3y=-2x+7$

$y=\frac{2}{3}x-\frac{7}{3}$

The slope of the given line is $\frac{2}{3}$, so $m=\frac{2}{3}$ since the lines are parallel.

point-slope form: $y-2=\frac{2}{3}(x+2)$

general form: $2x-3y+10=0$

10. $3x-2y-=0$

$-2y=-3x+5$

$y=\frac{3}{2}x-\frac{5}{2}$

The slope of the given line is $\frac{3}{2}$, so $m=\frac{3}{2}$ since the lines are parallel.

point-slope form: $y-3=\frac{3}{2}(x+1)$

general form: $3x-2y+9=0$

11. $x-2y-3=0$

$-2y=-x+3$

$y=\frac{1}{2}x-\frac{3}{2}$

The slope of the given line is $\frac{1}{2}$, so $m = –2$ since the lines are perpendicular.

point-slope form: $y+7=-2(x-4)$

general form: $2x+y-1=0$

12. $x+7y-12=0$

$7y=-x+12$

$y=\frac{-1}{7}x+\frac{12}{7}$

The slope of the given line is $-\frac{1}{7}$, so $m = 7$ since the lines are perpendicular.

point-slope form: $y + 9 = 7(x – 5)$

general form: $7x-y-44=0$

13. $\frac{15-0}{5-0}=\frac{15}{5}=3$

14. $\frac{24-0}{4-0}=\frac{24}{4}=6$

15. $\dfrac{5^2+2\cdot5-(3^2+2\cdot3)}{5-3}=\dfrac{25+10-(9+6)}{2}$
$=\dfrac{20}{2}$
$=10$

16. $\dfrac{6^2-2(6)-(3^2-2\cdot3)}{6-3}=\dfrac{36-12-(9-6)}{3}=\dfrac{21}{3}=7$

17. $\dfrac{\sqrt{9}-\sqrt{4}}{9-4}=\dfrac{3-2}{5}=\dfrac{1}{5}$

18. $\dfrac{\sqrt{16}-\sqrt{9}}{16-9}=\dfrac{4-3}{7}=\dfrac{1}{7}$

19. Since the line is perpendicular to $x=6$ which is a vertical line, we know the graph of f is a horizontal line with 0 slope. The graph of f passes through $(-1,5)$, so the equation of f is $f(x)=5$.

20. Since the line is perpendicular to $x=-4$ which is a vertical line, we know the graph of f is a horizontal line with 0 slope. The graph of f passes through $(-2,6)$, so the equation of f is $f(x)=6$.

21. First we need to find the equation of the line with $x-$intercept of 2 and $y-$intercept of -4. This line will pass through $(2,0)$ and $(0,-4)$. We use these points to find the slope.
$m=\dfrac{-4-0}{0-2}=\dfrac{-4}{-2}=2$
Since the graph of f is perpendicular to this line, it will have slope $m=-\dfrac{1}{2}$.
Use the point $(-6,4)$ and the slope $-\dfrac{1}{2}$ to find the equation of the line.
$y-y_1=m(x-x_1)$
$y-4=-\dfrac{1}{2}(x-(-6))$
$y-4=-\dfrac{1}{2}(x+6)$
$y-4=-\dfrac{1}{2}x-3$
$y=-\dfrac{1}{2}x+1$
$f(x)=-\dfrac{1}{2}x+1$

22. First we need to find the equation of the line with $x-$intercept of 3 and $y-$intercept of -9. This line will pass through $(3,0)$ and $(0,-9)$. We use these points to find the slope.
$m=\dfrac{-9-0}{0-3}=\dfrac{-9}{-3}=3$
Since the graph of f is perpendicular to this line, it will have slope $m=-\dfrac{1}{3}$.

Use the point $(-5,6)$ and the slope $-\dfrac{1}{3}$ to find the equation of the line.
$y-y_1=m(x-x_1)$
$y-6=-\dfrac{1}{3}(x-(-5))$
$y-6=-\dfrac{1}{3}(x+5)$
$y-6=-\dfrac{1}{3}x-\dfrac{5}{3}$
$y=-\dfrac{1}{3}x+\dfrac{13}{3}$
$f(x)=-\dfrac{1}{3}x+\dfrac{13}{3}$

23. First put the equation $3x-2y-4=0$ in slope-intercept form.

$$3x-2y-4=0$$
$$-2y=-3x+4$$
$$y=\frac{3}{2}x-2$$

The equation of f will have slope $-\frac{2}{3}$ since it is perpendicular to the line above and the same $y-$intercept -2.

So the equation of f is $f(x)=-\frac{2}{3}x-2$.

24. First put the equation $4x-y-6=0$ in slope-intercept form.

$$4x-y-6=0$$
$$-y=-4x+6$$
$$y=4x-6$$

The equation of f will have slope $-\frac{1}{4}$ since it is perpendicular to the line above and the same $y-$intercept -6.

So the equation of f is $f(x)=-\frac{1}{4}x-6$.

25. $p(x)=-0.25x+22$

26. $p(x)=0.22x+3$

27. $m=\frac{1163-617}{1998-1994}=\frac{546}{4}\approx 137$

There was an average increase of approximately 137 discharges per year.

28. $m=\frac{623-1273}{2006-2001}=\frac{-650}{5}\approx -130$

There was an average decrease of approximately 130 discharges per year.

29. **a.** $f(x)=1.1x^3-35x^2+264x+557$

$f(0)=1.1(0)^3-35(0)^2+264(0)+557=557$

$f(4)=1.1(4)^3-35(4)^2+264(4)+557=1123.4$

$$m=\frac{1123.4-557}{4-0}\approx 142$$

b. This overestimates by 5 discharges per year.

30. **a.** $f(x)=1.1x^3-35x^2+264x+557$

$f(0)=1.1(7)^3-35(7)^2+264(7)+557=1067.3$

$f(12)=1.1(12)^3-35(12)^2+264(12)+557=585.8$

$$m=\frac{585.8-1067.3}{12-7}\approx -96$$

b. This underestimates the decrease by 34 discharges per year.

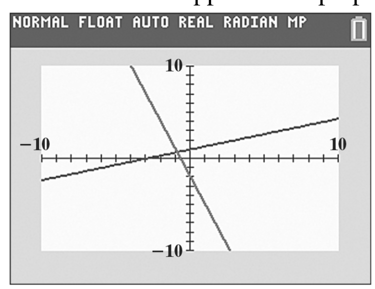
31. – 36. Answers will vary.

37. $y=\frac{1}{3}x+1$

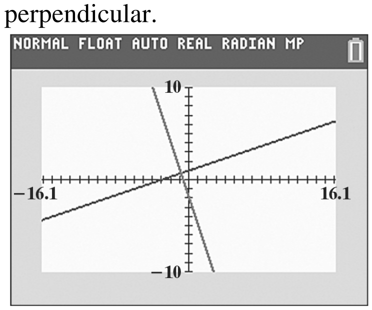
$y=-3x-2$

a. The lines are perpendicular because their slopes are negative reciprocals of each other. This is verified because product of their slopes is –1.

b. The lines do not appear to be perpendicular.



c. The lines appear to be perpendicular. The calculator screen is rectangular and does not have the same width and height. This causes the scale of the *x*–axis to differ from the scale on the *y*–axis despite using the same scale in the window settings. In part (b), this causes the lines not to appear perpendicular when indeed they are. The zoom square feature compensates for this and in part (c), the lines appear to be perpendicular.



38. does not make sense; Explanations will vary. Sample explanation: Perpendicular lines have slopes with opposite signs.

39. makes sense

40. does not make sense; Explanations will vary. Sample explanation: Slopes can be used for segments of the graph.

41. makes sense

42. Write $Ax+By+C=0$ in slope-intercept form.

$$Ax+By+C=0$$
$$By=-Ax-C$$
$$\frac{By}{B}=\frac{-Ax}{B}-\frac{C}{B}$$
$$y=-\frac{A}{B}x-\frac{C}{B}$$

The slope of the given line is $-\frac{A}{B}$.

The slope of any line perpendicular to $Ax+By+C=0$ is $\frac{B}{A}$.

43. The slope of the line containing $(1,-3)$ and $(-2,4)$ has slope $m=\frac{4-(-3)}{-2-1}=\frac{4+3}{-3}=\frac{7}{-3}=-\frac{7}{3}$

Solve $Ax+y-2=0$ for y to obtain slope-intercept form.

$$Ax+y-2=0$$
$$y=-Ax+2$$

So the slope of this line is $-A$.

This line is perpendicular to the line above so its slope is $\frac{3}{7}$. Therefore, $-A=\frac{3}{7}$ so $A=-\frac{3}{7}$.

44.
$$24+3(x+2)=5(x-12)$$
$$24+3x+6=5x-60$$
$$3x+30=5x-60$$
$$90=2x$$
$$45=x$$

The solution set is $\{45\}$.

45. Let x = the television's price before the reduction.

$$x-0.30x=980$$
$$0.70x=980$$
$$x=\frac{980}{0.70}$$
$$x=1400$$

Before the reduction the television's price was \$1400.

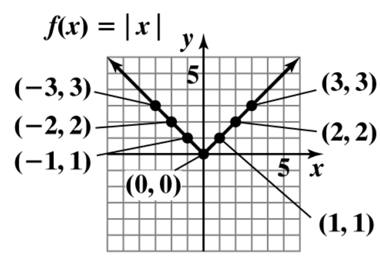
46. $2x^{2/3}-5x^{1/3}-3=0$

Let $t=x^{1/3}$.

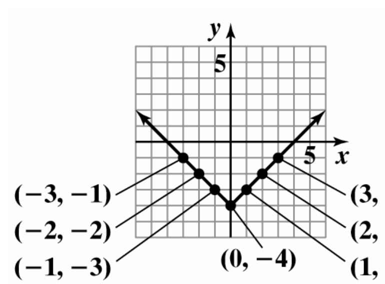
$$2t^2-5t-3=0$$
$$(2t+1)(t-3)=0$$
$$2t+1=0 \quad \text{or} \quad t-3=0$$
$$2t=-1$$
$$t=-\frac{1}{2} \qquad t=3$$
$$x^{1/3}=-\frac{1}{2} \qquad x^{1/3}=3$$
$$x=\left(-\frac{1}{2}\right)^3 \qquad x=3^3$$
$$x=-\frac{1}{8} \qquad x=27$$

The solution set is $\left\{-\frac{1}{8},27\right\}$.

47. **a.**

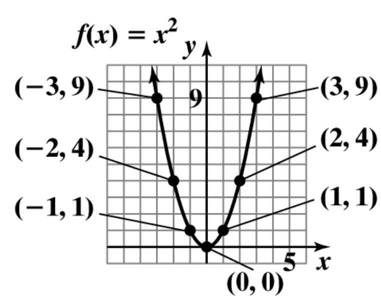


b.

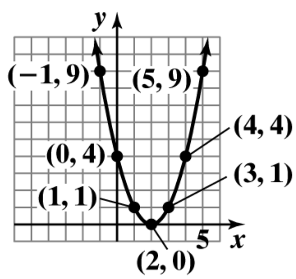


c. The graph in part (b) is the graph in part (a) shifted down 4 units.

48. **a.**

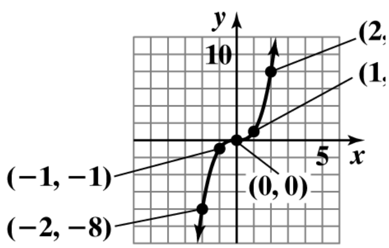


b.

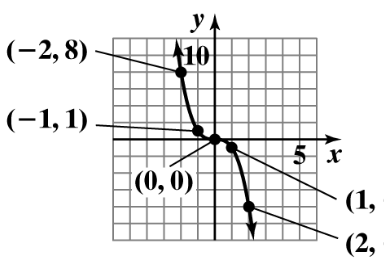


c. The graph in part (b) is the graph in part (a) shifted to the right 2 units.

49. a.



b.



c. The graph in part (b) is the graph in part (a) reflected across the y-axis.

Mid-Chapter 2 Check Point

1. The relation is not a function.
The domain is $\{1, 2\}$.
The range is $\{-6, 4, 6\}$.

2. The relation is a function.
The domain is $\{0, 2, 3\}$.
The range is $\{1, 4\}$.

3. The relation is a function.
The domain is $\{x \mid -2 \le x < 2\}$.
The range is $\{y \mid 0 \le y \le 3\}$.

4. The relation is not a function.
The domain is $\{x \mid -3 < x \le 4\}$.
The range is $\{y \mid -1 \le y \le 2\}$.

5. The relation is not a function.
The domain is $\{-2, -1, 0, 1, 2\}$.
The range is $\{-2, -1, 1, 3\}$.

6. The relation is a function.
The domain is $\{x \mid x \le 1\}$.
The range is $\{y \mid y \ge -1\}$.

7. $x^2 + y = 5$

$$y = -x^2 + 5$$

For each value of x, there is one and only one value for y, so the equation defines y as a function of x.

8. $x + y^2 = 5$

$$y^2 = 5 - x$$
$$y = \pm\sqrt{5 - x}$$

Since there are values of x that give more than one value for y (for example, if $x = 4$, then $y = \pm\sqrt{5-4} = \pm 1$), the equation does not define y as a function of x.

9. No vertical line intersects the graph in more than one point. Each value of x corresponds to exactly one value of y.

10. Domain: $(-\infty, \infty)$

11. Range: $(-\infty, 4]$

12. x-intercepts: –6 and 2

13. y-intercept: 3

14. increasing: $(-\infty, -2)$

15. decreasing: $(-2, \infty)$

16. $x = -2$

17. $f(-2) = 4$

18. $f(-4) = 3$

19. $f(-7) = -2$ and $f(3) = -2$

20. $f(-6) = 0$ and $f(2) = 0$

21. $(-6, 2)$

22. $f(100)$ is negative.

23. neither; $f(-x) \neq x$ and $f(-x) \neq -x$

24. $\dfrac{f(x_2)-f(x_1)}{x_2-x_1} = \dfrac{f(4)-f(-4)}{4-(-4)} = \dfrac{-5-3}{4+4} = -1$

25. Test for symmetry with respect to the *y*-axis.

$$x = y^2 + 1$$
$$-x = y^2 + 1$$
$$x = -y^2 - 1$$

The resulting equation is not equivalent to the original. Thus, the graph is not symmetric with respect to the *y*-axis.

Test for symmetry with respect to the *x*-axis.

$$x = y^2 + 1$$
$$x = (-y)^2 + 1$$
$$x = y^2 + 1$$

The resulting equation is equivalent to the original. Thus, the graph is symmetric with respect to the *x*-axis.

Test for symmetry with respect to the origin.

$$x = y^2 + 1$$
$$-x = (-y)^2 + 1$$
$$-x = y^2 + 1$$
$$x = -y^2 - 1$$

The resulting equation is not equivalent to the original. Thus, the graph is not symmetric with respect to the origin.

26. Test for symmetry with respect to the *y*-axis.

$$y = x^3 - 1$$
$$y = (-x)^3 - 1$$
$$y = -x^3 - 1$$

The resulting equation is not equivalent to the original. Thus, the graph is not symmetric with respect to the *y*-axis.

Test for symmetry with respect to the *x*-axis.

$$y = x^3 - 1$$
$$-y = x^3 - 1$$
$$y = -x^3 + 1$$

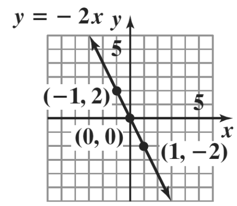
The resulting equation is not equivalent to the original. Thus, the graph is not symmetric with respect to the *x*-axis.

Test for symmetry with respect to the origin.

$$y = x^3 - 1$$
$$-y = (-x)^3 - 1$$
$$-y = -x^3 - 1$$
$$y = x^3 + 1$$

The resulting equation is not equivalent to the original. Thus, the graph is not symmetric with respect to the origin.

27.



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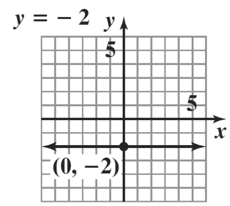

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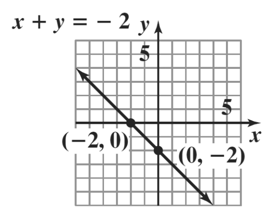

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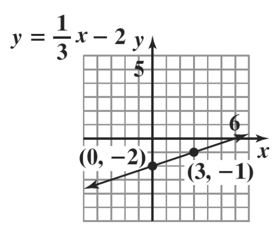
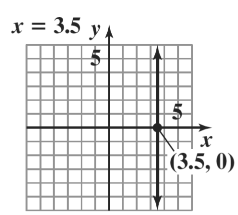
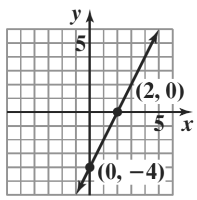


31.

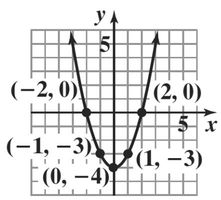


32.



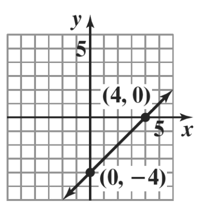
$4x - 2y = 8$

33.



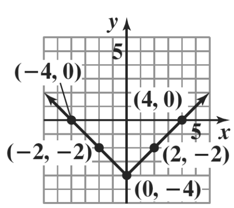
$f(x) = x^2 - 4$

34.



$f(x) = x - 4$

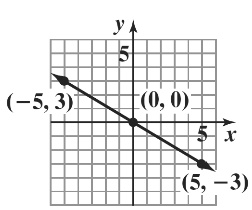
35.



$f(x) = |x| - 4$

36. $5y = -3x$

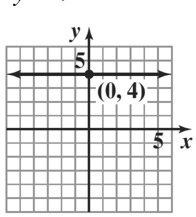
$y = -\dfrac{3}{5}x$



$5y = -3x$

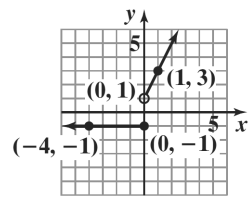
37. $5y = 20$

$y = 4$



$5y = 20$

38.



$f(x) = \begin{cases} -1 \text{ if } x \le 0 \\ 2x + 1 \text{ if } x > 0 \end{cases}$

39. **a.** $f(-x) = -2(-x)^2 - x - 5$

$= -2x^2 - x - 5$

neither; $f(-x) \neq x$ and $f(-x) \neq -x$

b. $\dfrac{f(x+h) - f(x)}{h}$

$= \dfrac{-2(x+h)^2 + (x+h) - 5 - (-2x^2 + x - 5)}{h}$

$= \dfrac{-2x^2 - 4xh - 2h^2 + x + h - 5 + 2x^2 - x + 5}{h}$

$= \dfrac{-4xh - 2h^2 + h}{h}$

$= \dfrac{h(-4x - 2h + 1)}{h}$

$= -4x - 2h + 1$

40. $C(x) = \begin{cases} 30 & \text{if } \quad 0 \le t \le 200 \\ 30 + 0.40(t - 200) & \text{if } \quad t > 200 \end{cases}$

a. $C(150) = 30$

b. $C(250) = 30 + 0.40(250 - 200) = 50$

41. $y - y_1 = m(x - x_1)$

$y - 3 = -2(x - (-4))$

$y - 3 = -2(x + 4)$

$y - 3 = -2x - 8$

$y = -2x - 5$

$f(x) = -2x - 5$

42. $m = \dfrac{\text{Change in } y}{\text{Change in } x} = \dfrac{1 - (-5)}{2 - (-1)} = \dfrac{6}{3} = 2$

$y - y_1 = m(x - x_1)$

$y - 1 = 2(x - 2)$

$y - 1 = 2x - 4$

$y = 2x - 3$

$f(x) = 2x - 3$

43. $3x - y - 5 = 0$

$$-y = -3x + 5$$
$$y = 3x - 5$$

The slope of the given line is 3, and the lines are parallel, so $m = 3$.

$$y - y_1 = m(x - x_1)$$
$$y - (-4) = 3(x - 3)$$
$$y + 4 = 3x - 9$$
$$y = 3x - 13$$
$$f(x) = 3x - 13$$

44. $2x - 5y - 10 = 0$

$$-5y = -2x + 10$$
$$\frac{-5y}{-5} = \frac{-2x}{-5} + \frac{10}{-5}$$
$$y = \frac{2}{5}x - 2$$

The slope of the given line is $\frac{2}{5}$, and the lines are perpendicular, so $m = -\frac{5}{2}$.

$$y - y_1 = m(x - x_1)$$
$$y - (-3) = -\frac{5}{2}\left(x - (-4)\right)$$
$$y + 3 = -\frac{5}{2}x - 10$$
$$y = -\frac{5}{2}x - 13$$
$$f(x) = -\frac{5}{2}x - 13$$

45. $m_1 = \dfrac{\text{Change in } y}{\text{Change in } x} = \dfrac{0 - (-4)}{7 - 2} = \dfrac{4}{5}$

$m_2 = \dfrac{\text{Change in } y}{\text{Change in } x} = \dfrac{6 - 2}{1 - (-4)} = \dfrac{4}{5}$

The slope of the lines are equal thus the lines are parallel.

46. **a.** $m = \dfrac{\text{Change in } y}{\text{Change in } x} = \dfrac{42 - 26}{180 - 80} = \dfrac{16}{100} = 0.16$

b. For each minute of brisk walking, the percentage of patients with depression in remission increased by <u>0.16</u>%. The rate of change is <u>0.16</u>% per <u>minute of brisk walking</u>.

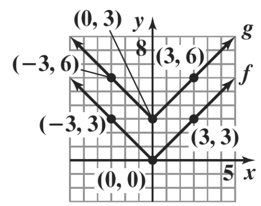
47. $\dfrac{f(x_2) - f(x_1)}{x_2 - x_1} = \dfrac{f(2) - f(-1)}{2 - (-1)}$

$$= \frac{\left(3(2)^2 - 2\right) - \left(3(-1)^2 - (-1)\right)}{2 + 1}$$
$$= 2$$

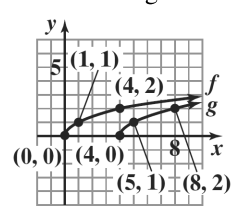
Section 2.5

Check Point Exercises

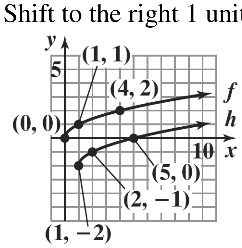
1. Shift up vertically 3 units.



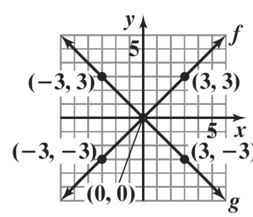
2. Shift to the right 4 units.



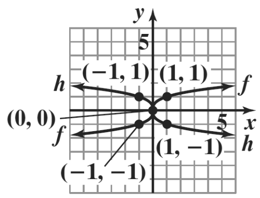
3. Shift to the right 1 unit and down 2 units.



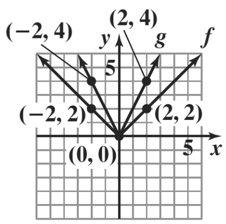
4. Reflect about the x-axis.



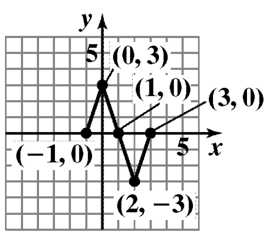
5. Reflect about the y-axis.



6. Vertically stretch the graph of $f(x)=|x|$.

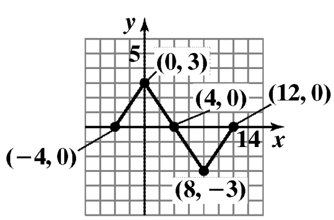


7. **a.** Horizontally shrink the graph of $y=f(x)$.



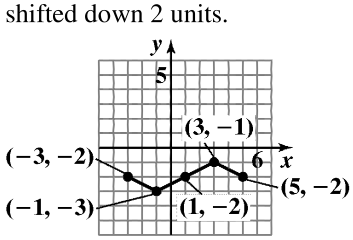
$g(x)=f(2x)$

b. Horizontally stretch the graph of $y=f(x)$.



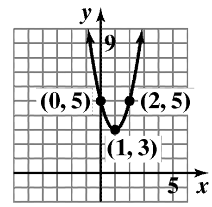
$h(x)=f\left(\frac{1}{2}x\right)$

8. The graph of $y=f(x)$ is shifted 1 unit left, shrunk by a factor of $\frac{1}{3}$, reflected about the x-axis, then shifted down 2 units.



$y=-\frac{1}{3}f(x+1)-2$

9. The graph of $f(x)=x^2$ is shifted 1 unit right, stretched by a factor of 2, then shifted up 3 units.



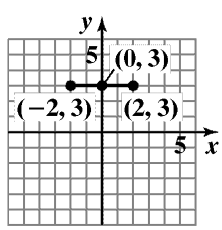
$g(x)=2(x-1)^2+3$

Concept and Vocabulary Check 2.5

1. vertical; down
2. horizontal; to the right
3. x-axis
4. y-axis
5. vertical; y
6. horizontal; x
7. false

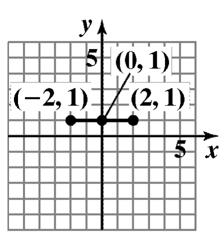
Exercise Set 2.5

1.



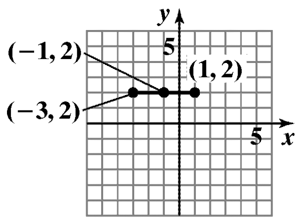
$g(x)=f(x)+1$

2.



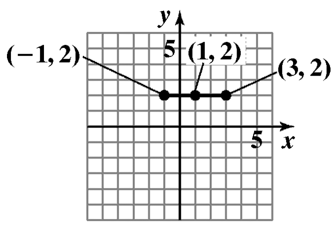
$g(x)=f(x)-1$

3.



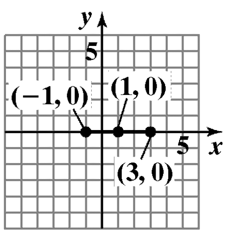
$g(x) = f(x + 1)$

4.



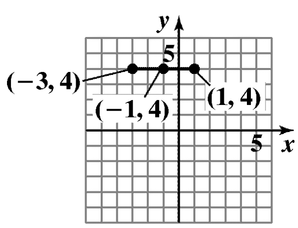
$g(x) = f(x - 1)$

5.



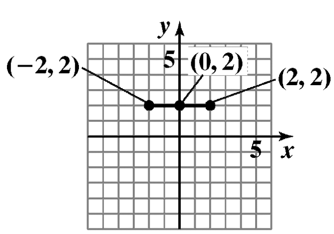
$g(x) = f(x - 1) - 2$

6.



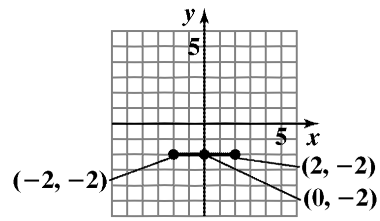
$g(x) = f(x + 1) + 2$

7.



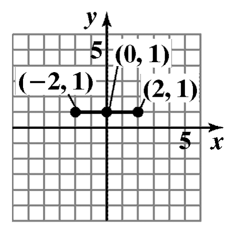
$g(x) = -f(x)$

8.



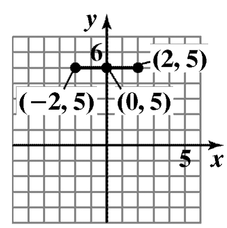
$g(x) = -f(x)$

9.



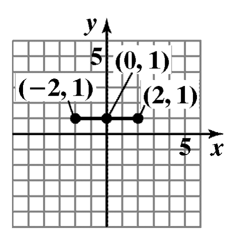
$g(x) = -f(x) + 3$

10.



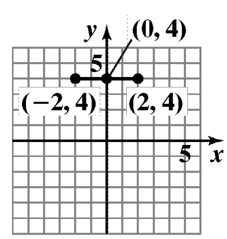
$g(x) = f(-x) + 3$

11.



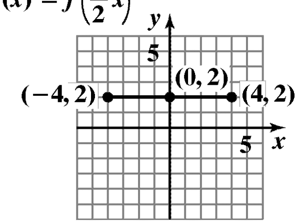
$g(x) = \frac{1}{2}f(x)$

12.

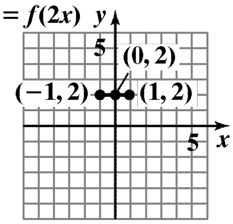


$g(x) = 2f(x)$

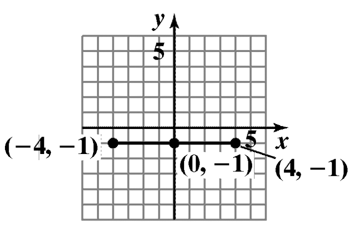
13. $g(x) = f\left(\frac{1}{2}x\right)$



14. $g(x) = f(2x)$

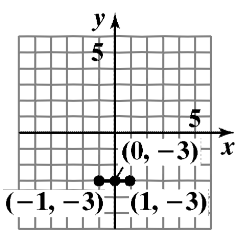


15.



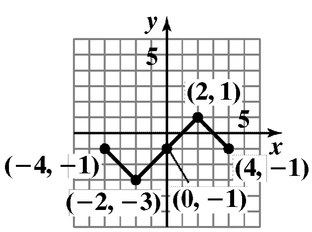
$g(x) = -f\left(\frac{1}{2}x\right) + 1$

16.



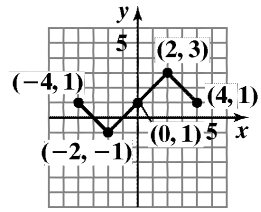
$g(x) = -f(2x) - 1$

17.



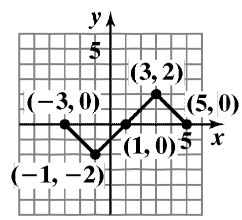
$g(x) = f(x) - 1$

18.



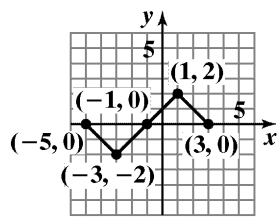
$g(x) = f(x) + 1$

19.



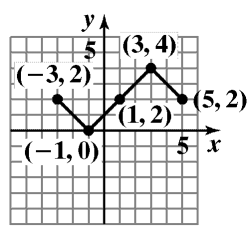
$g(x) = f(x - 1)$

20.



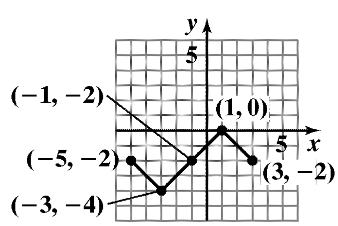
$g(x) = f(x + 1)$

21.



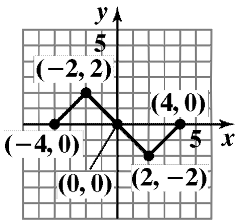
$g(x) = f(x - 1) + 2$

22.



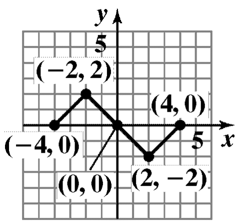
$g(x) = f(x + 1) - 2$

23.



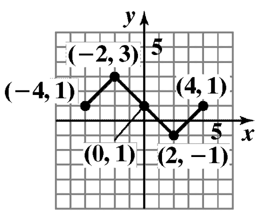
$g(x) = -f(x)$

24.



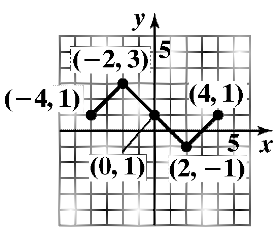
$g(x) = f(-x)$

25.



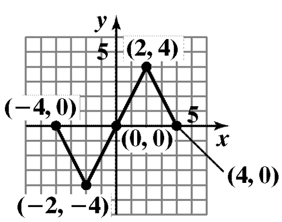
$g(x) = f(-x) + 1$

26.



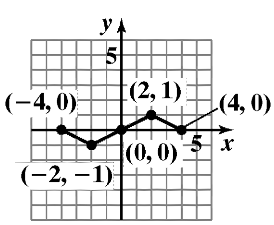
$g(x) = -f(x) + 1$

27.



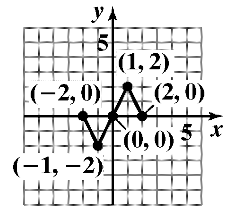
$g(x) = 2f(x)$

28.



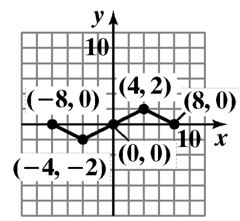
$g(x) = \frac{1}{2}f(x)$

29.



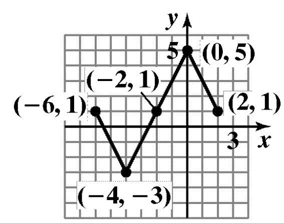
$g(x) = f(2x)$

30.



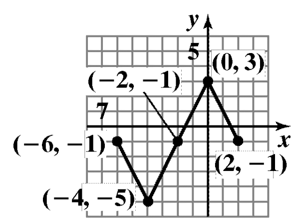
$g(x) = f\left(\frac{1}{2}x\right)$

31.



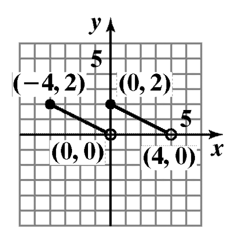
$g(x) = 2f(x + 2) + 1$

32.



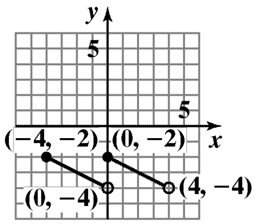
$g(x) = 2f(x + 2) - 1$

33.



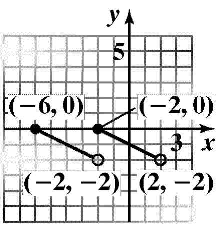
$g(x) = f(x) + 2$

34.



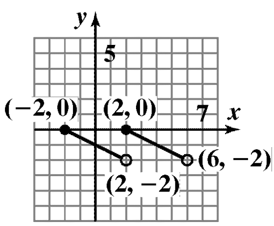
$g(x) = f(x) - 2$

35.



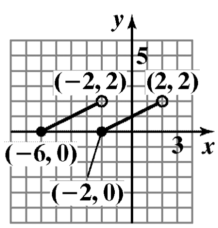
$g(x) = f(x + 2)$

36.



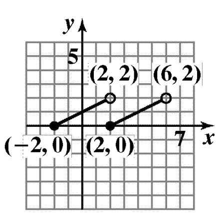
$g(x) = f(x - 2)$

37.



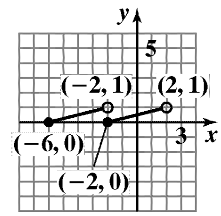
$g(x) = -f(x + 2)$

38.



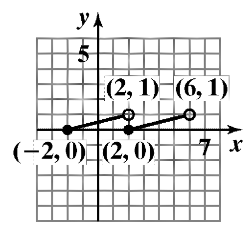
$g(x) = -f(x - 2)$

39.



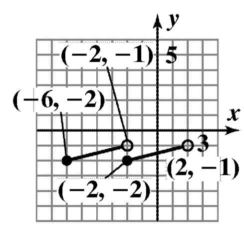
$g(x) = -\frac{1}{2}f(x + 2)$

40.



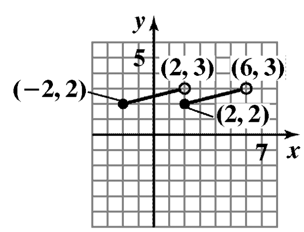
$g(x) = -\frac{1}{2}f(x - 2)$

41.



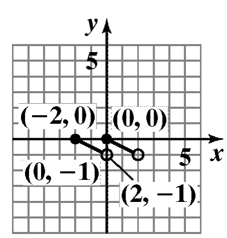
$g(x) = -\frac{1}{2}f(x + 2) - 2$

42.



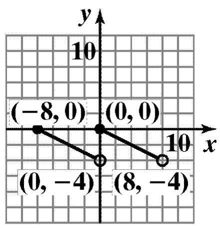
$g(x) = -\frac{1}{2}f(x - 2) + 2$

43.



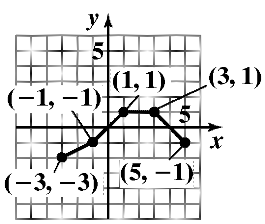
$g(x) = \frac{1}{2}f(2x)$

44.



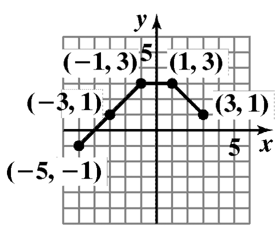
$g(x) = 2f\left(\frac{1}{2}x\right)$

45.



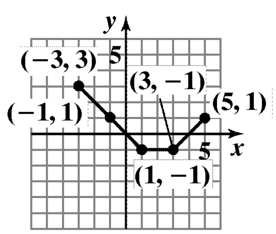
$g(x) = f(x - 1) - 1$

46.



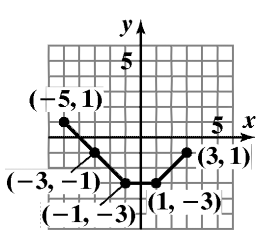
$g(x) = f(x + 1) + 1$

47.



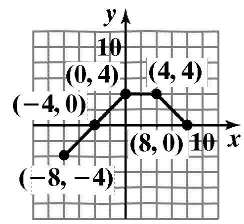
$g(x) = -f(x - 1) + 1$

48.



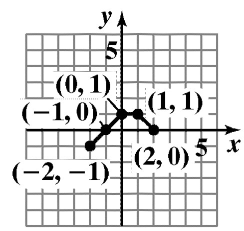
$g(x) = -f(x + 1) - 1$

49.



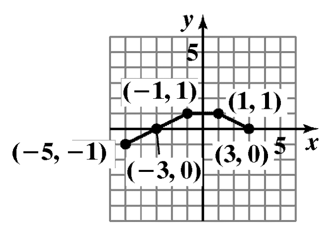
$g(x) = 2f\left(\frac{1}{2}x\right)$

50.



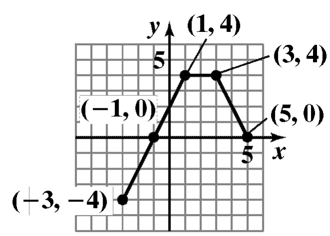
$g(x) = \frac{1}{2}f(2x)$

51.



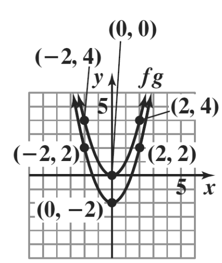
$g(x) = \frac{1}{2}f(x + 1)$

52.



$g(x) = 2f(x - 1)$

53.



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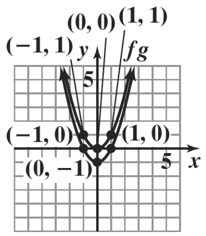

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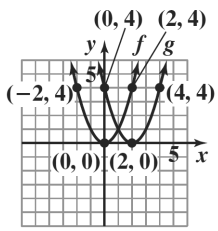

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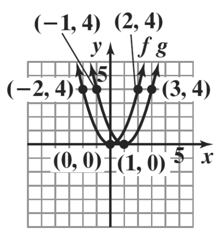

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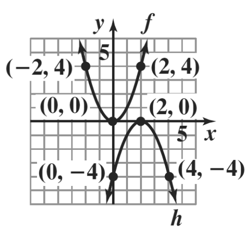

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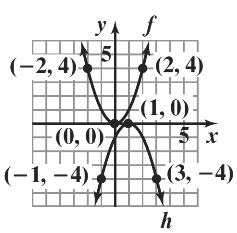

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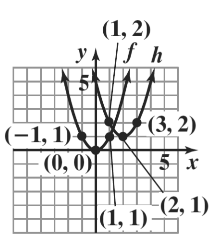

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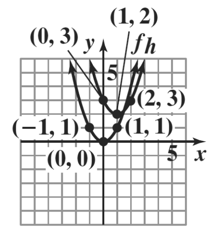

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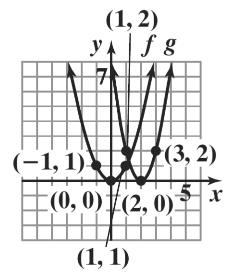

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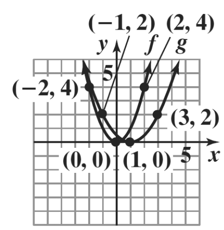

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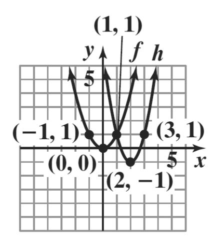

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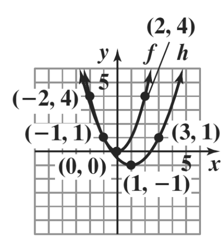

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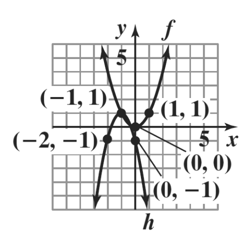

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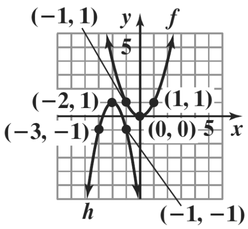

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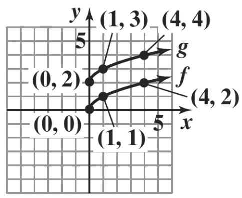

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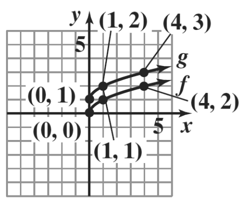

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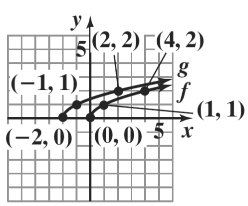

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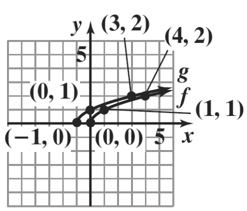

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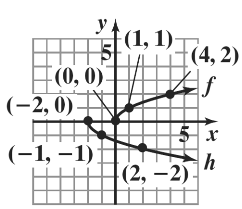

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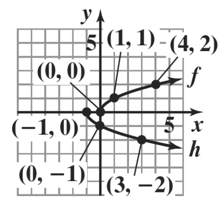

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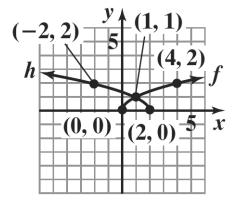

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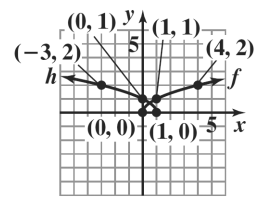

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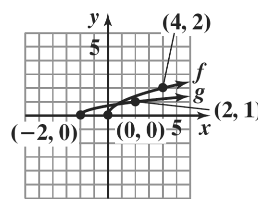

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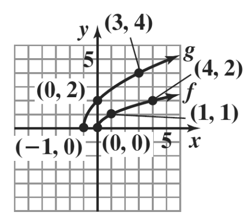

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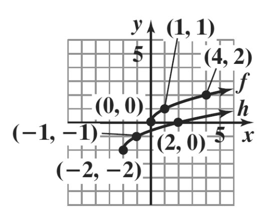

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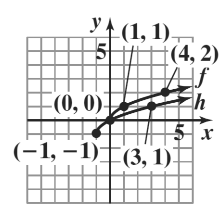

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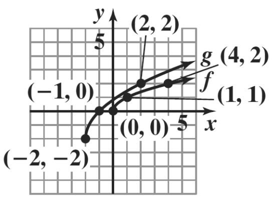

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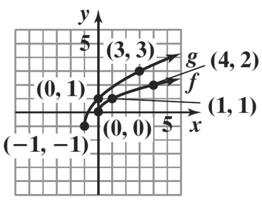

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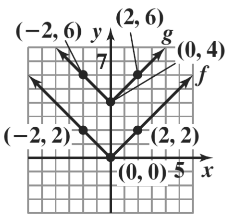

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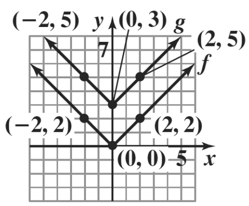

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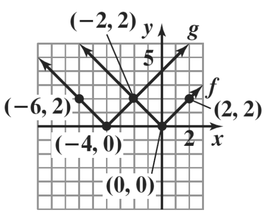

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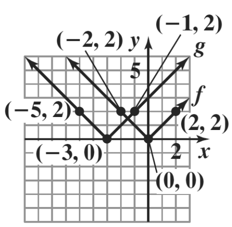

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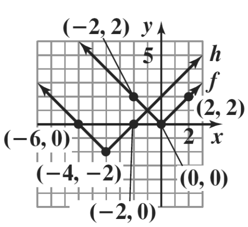

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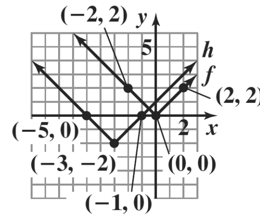

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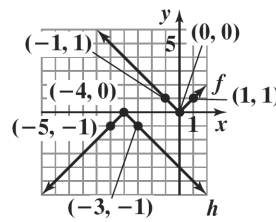

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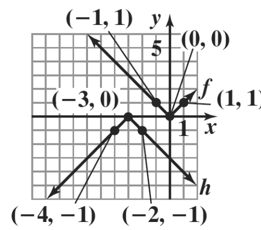

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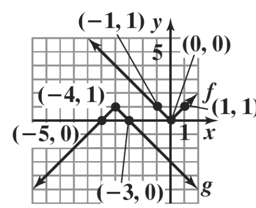

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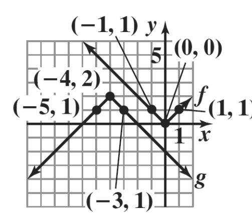

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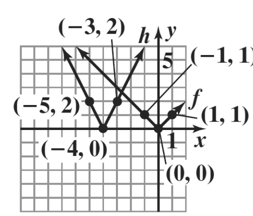

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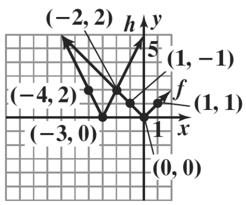

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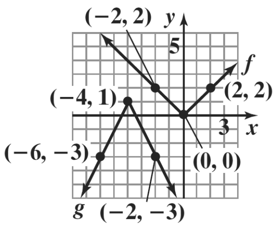

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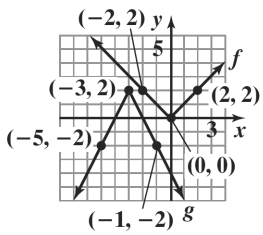

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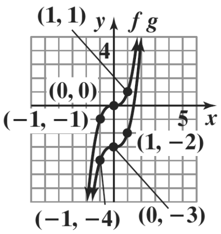

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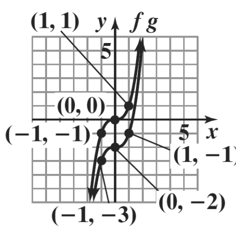

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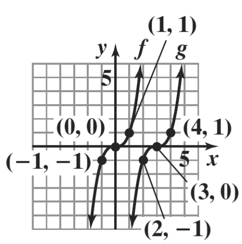

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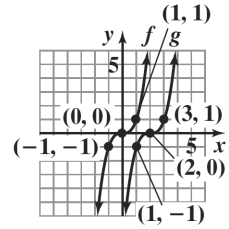

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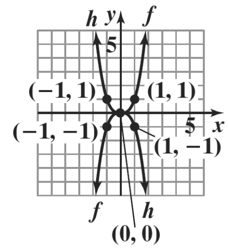

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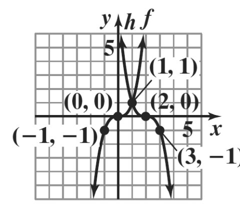

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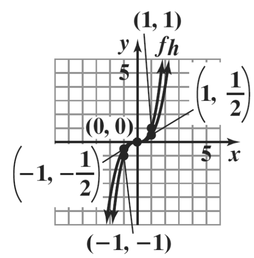

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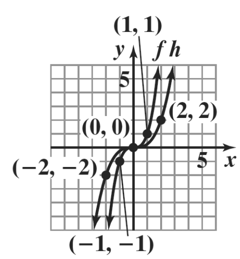

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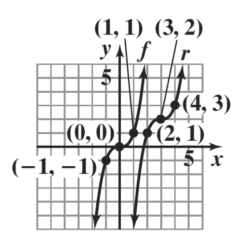

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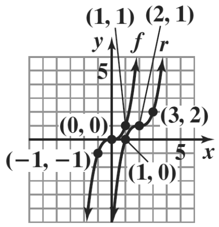

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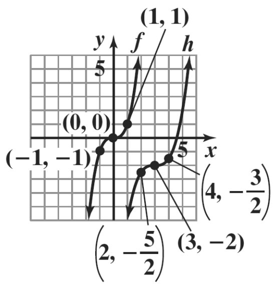

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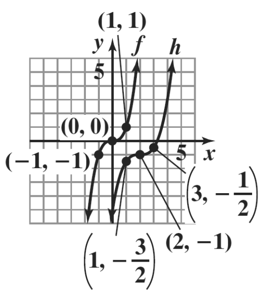

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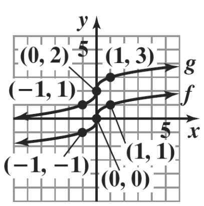

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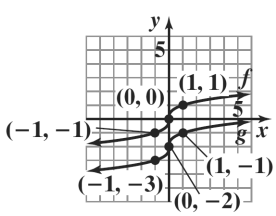

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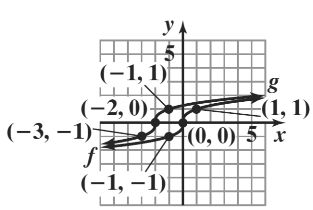

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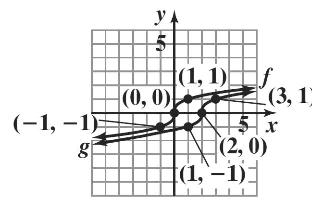

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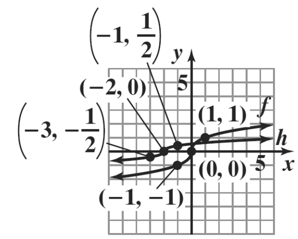

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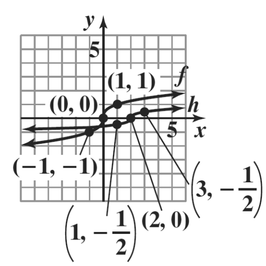

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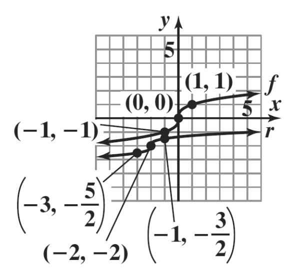

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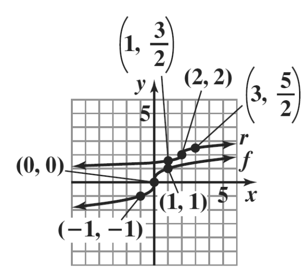

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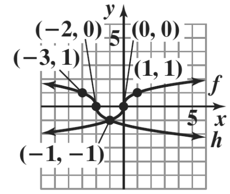

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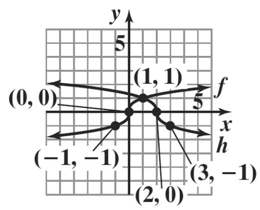

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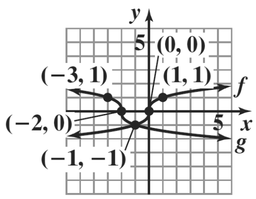

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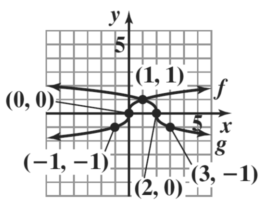

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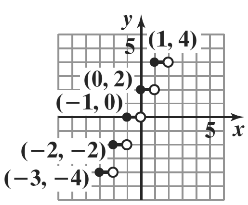
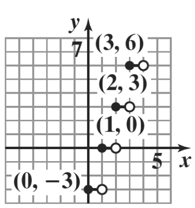


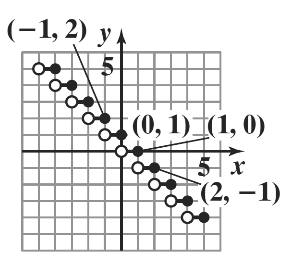
$g(x) = 2 \text{ int } (x + 1)$

120.



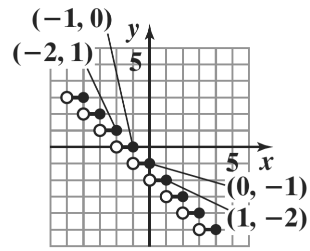
$g(x) = 3 \text{ int } (x - 1)$

121.



$h(x) = \text{int } (-x) + 1$

122.



$h(x) = \text{int } (-x) - 1$

123. $y = \sqrt{x-2}$

124. $y = -x^3 + 2$

125. $y = (x+1)^2 - 4$

126. $y = \sqrt{x-2} + 1$

127. a. First, vertically stretch the graph of $f(x) = \sqrt{x}$ by the factor 2.9; then shift the result up 20.1 units.

b. $f(x) = 2.9\sqrt{x} + 20.1$

$f(48) = 2.9\sqrt{48} + 20.1 \approx 40.2$

The model describes the actual data very well.

c.
$$\frac{f(x_2) - f(x_1)}{x_2 - x_1}$$
$$= \frac{f(10) - f(0)}{10 - 0}$$
$$= \frac{\left(2.9\sqrt{10} + 20.1\right) - \left(2.9\sqrt{0} + 20.1\right)}{10 - 0}$$
$$= \frac{29.27 - 20.1}{10}$$
$$\approx 0.9$$

0.9 inches per month

d.
$$\frac{f(x_2) - f(x_1)}{x_2 - x_1}$$
$$= \frac{f(60) - f(50)}{60 - 50}$$
$$= \frac{\left(2.9\sqrt{60} + 20.1\right) - \left(2.9\sqrt{50} + 20.1\right)}{60 - 50}$$
$$= \frac{42.5633 - 40.6061}{10}$$
$$\approx 0.2$$

This rate of change is lower than the rate of change in part (c). The relative leveling off of the curve shows this difference.

128. a. First, vertically stretch the graph of $f(x) = \sqrt{x}$ by the factor 3.1; then shift the result up 19 units.

b. $f(x) = 3.1\sqrt{x} + 19$

$f(48) = 3.1\sqrt{48} + 19 \approx 40.5$

The model describes the actual data very well.

c. $\dfrac{f(x_2) - f(x_1)}{x_2 - x_1}$

$= \dfrac{f(10) - f(0)}{10 - 0}$

$= \dfrac{\left(3.1\sqrt{10} + 19\right) - \left(3.1\sqrt{0} + 19\right)}{10 - 0}$

$= \dfrac{28.8031 - 19}{10}$

≈ 1.0

1.0 inches per month

d. $\dfrac{f(x_2) - f(x_1)}{x_2 - x_1}$

$= \dfrac{f(60) - f(50)}{60 - 50}$

$= \dfrac{\left(3.1\sqrt{60} + 19\right) - \left(3.1\sqrt{50} + 19\right)}{60 - 50}$

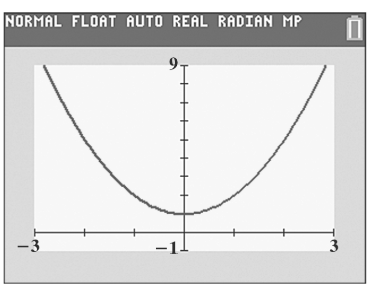
$= \dfrac{43.0125 - 40.9203}{10}$

≈ 0.2

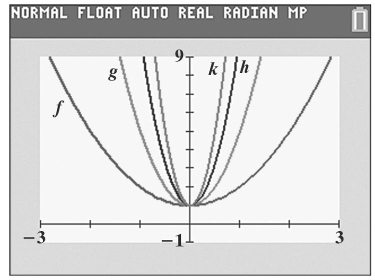
This rate of change is lower than the rate of change in part (c). The relative leveling off of the curve shows this difference.

129. – 134. Answers will vary.

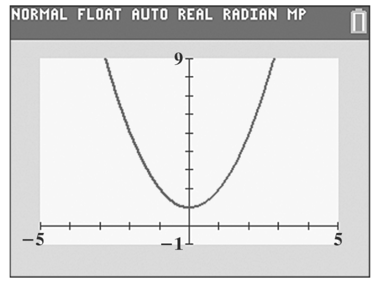
135. a.



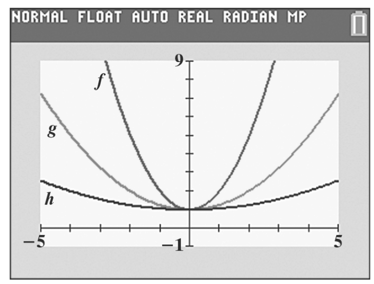
b.



136. a.



b.



137. makes sense

138. makes sense

139. does not make sense; Explanations will vary. Sample explanation: The reprogram should be $y = f(t+1)$.

140. does not make sense; Explanations will vary. Sample explanation: The reprogram should be $y = f(t-1)$.

141. false; Changes to make the statement true will vary. A sample change is: The graph of g is a translation of f three units to the <u>left</u> and three units upward.

142. false; Changes to make the statement true will vary. A sample change is: The graph of f is a reflection of the graph of $y = \sqrt{x}$ in the x-axis, while the graph of g is a reflection of the graph of $y = \sqrt{x}$ in the y-axis.

143. false; Changes to make the statement true will vary. A sample change is: The stretch will be 5 units and the downward shift will be 10 units.

144. true

145. $g(x) = -(x+4)^2$

146. $g(x) = -|x-5|+1$

147. $g(x) = -\sqrt{x-2}+2$

148. $g(x) = -\frac{1}{4}\sqrt{16-x^2}-1$

149. $(-a, b)$

150. $(a, 2b)$

151. $(a+3, b)$

152. $(a, b-3)$

153. Let x = the width of the rectangle.
Let $x + 13$ = the length of the rectangle.

$$\begin{aligned} 2l + 2w &= P \\ 2(x+13)+2x &= 82 \\ 2x+26+2x &= 82 \\ 4x+26 &= 82 \\ 4x &= 56 \\ x &= \frac{56}{4} \\ x &= 14 \\ x+13 &= 27 \end{aligned}$$

The dimensions of the rectangle are 14 yards by 27 yards.

154.
$$\begin{aligned} \sqrt{x+10}-4 &= x \\ \sqrt{x+10} &= x+4 \\ \left(\sqrt{x+10}\right)^2 &= (x+4)^2 \\ x+10 &= x^2+8x+16 \\ 0 &= x^2+7x+6 \\ 0 &= (x+6)(x+1) \end{aligned}$$

$x+6=0$ or $x+1=0$

$x=-6$ $\quad$ $x=-1$

–6 does not check and must be rejected.
The solution set is $\{-1\}$.

155.
$$\begin{aligned} (3-7i)(5+2i) &= 15+6i-35i-14i^2 \\ &= 15+6i-35i-14(-1) \\ &= 15+6i-35i+14 \\ &= 29-29i \end{aligned}$$

156.
$$\begin{aligned} (2x-1)(x^2+x-2) &= 2x(x^2+x-2)-1(x^2+x-2) \\ &= 2x^3+2x^2-4x-x^2-x+2 \\ &= 2x^3+2x^2-x^2-4x-x+2 \\ &= 2x^3+x^2-5x+2 \end{aligned}$$

157.
$$\begin{aligned} \left(f(x)\right)^2-2f(x)+6 &= (3x-4)^2-2(3x-4)+6 \\ &= 9x^2-24x+16-6x+8+6 \\ &= 9x^2-24x-6x+16+8+6 \\ &= 9x^2-30x+30 \end{aligned}$$

158. $\dfrac{2}{\frac{3}{x}-1} = \dfrac{2x}{\frac{3x}{x}-x} = \dfrac{2x}{3-x}$

Section 2.6

Check Point Exercises

1. **a.** The function $f(x) = x^2+3x-17$ contains neither division nor an even root. The domain of f is the set of all real numbers or $(-\infty, \infty)$.

b. The denominator equals zero when x = 7 or x = –7. These values must be excluded from the domain.
domain of g = $(-\infty,-7)\cup(-7,7)\cup(7,\infty)$.

c. Since $h(x) = \sqrt{9x-27}$ contains an even root; the quantity under the radical must be greater than or equal to 0.

$$\begin{aligned} 9x-27 &\ge 0 \\ 9x &\ge 27 \\ x &\ge 3 \end{aligned}$$

Thus, the domain of h is $\{x|x \ge 3\}$, or the interval $[3, \infty)$.

d. Since the denominator of $j(x)$ contains an even root; the quantity under the radical must be greater than or equal to 0. But that quantity must also not be 0 (because we cannot have division by 0). Thus, $24-3x$ must be strictly greater than 0.

$$24-3x>0$$
$$-3x>-24$$
$$x<8$$

Thus, the domain of *j* is $\{x|x<8\}$, or the interval $(-\infty,8)$.

2. a. $(f+g)(x)=f(x)+g(x)$
$$=x-5+(x^2-1)$$
$$=x-5+x^2-1$$
$$=-x^2+x-6$$
domain: $(-\infty,\infty)$

b. $(f-g)(x)=f(x)-g(x)$
$$=x-5-(x^2-1)$$
$$=x-5-x^2+1$$
$$=-x^2+x-4$$
domain: $(-\infty,\infty)$

c. $(fg)(x)=(x-5)(x^2-1)$
$$=x(x^2-1)-5(x^2-1)$$
$$=x^3-x-5x^2+5$$
$$=x^3-5x^2-x+5$$
domain: $(-\infty,\infty)$

d. $\left(\frac{f}{g}\right)(x)=\frac{f(x)}{g(x)}$
$$=\frac{x-5}{x^2-1},\ x\neq\pm1$$
domain: $(-\infty,-1)\cup(-1,1)\cup(1,\infty)$

3. a. $(f+g)(x)=f(x)+g(x)$
$$=\sqrt{x-3}+\sqrt{x+1}$$

b. domain of *f*: $x-3\geq0$
$$x\geq3$$
$$[3,\ \infty)$$
domain of g: $x+1\geq0$
$$x\geq-1$$
$$[-1,\ \infty)$$
The domain of *f* + *g* is the set of all real numbers that are common to the domain of *f* and the domain of *g*. Thus, the domain of *f* + *g* is [3, ∞).

4. a. $(B+D)(x)$
$$=B(x)+D(x)$$
$$=(-2.6x^2+49x+3994)+(-0.6x^2+7x+2412)$$
$$=-2.6x^2+49x+3994-0.6x^2+7x+2412$$
$$=-3.2x^2+56x+6406$$

b. $(B+D)(x)=-3.2x^2+56x+6406$
$$(B+D)(5)=-3.2(3)^2+56(3)+6406$$
$$=6545.2$$
The number of births and deaths in the U.S. in 2003 was 6545.2 thousand.

c. $(B+D)(x)$ overestimates the actual number of births and deaths in 2003 by 7.2 thousand.

5. a. $(f\circ g)(x)=f(g(x))$
$$=5(2x^2-x-1)+6$$
$$=10x^2-5x-5+6$$
$$=10x^2-5x+1$$

b. $(g\circ f)(x)=g(f(x))$
$$=2(5x+6)^2-(5x+6)-1$$
$$=2(25x^2+60x+36)-5x-6-1$$
$$=50x^2+120x+72-5x-6-1$$
$$=50x^2+115x+65$$

c. $(f\circ g)(x)=10x^2-5x+1$
$$(f\circ g)(-1)=10(-1)^2-5(-1)+1$$
$$=10+5+1$$
$$=16$$

6. a. $(f\circ g)(x)=\frac{4}{\frac{1}{x}+2}=\frac{4x}{1+2x}$

b. domain: $\left\{x\middle|x\neq0,\ x\neq-\frac{1}{2}\right\}$

or $\left(-\infty,-\frac{1}{2}\right)\cup\left(-\frac{1}{2},0\right)\cup(0,\infty)$

7. $h(x)=f\circ g$ where $f(x)=\sqrt{x}$; $g(x)=x^2+5$

Concept and Vocabulary Check 2.6

1. zero

2. negative

3. $f(x)+g(x)$

4. $f(x)-g(x)$

5. $f(x)\cdot g(x)$

6. $\dfrac{f(x)}{g(x)}$; $g(x)$

7. $(-\infty,\infty)$

8. $(2,\infty)$

9. $(0,3)$; $(3,\infty)$

10. composition; $f(g(x))$

11. f; $g(x)$

12. composition; $g(f(x))$

13. g; $f(x)$

14. false

15. false

16. 2

Exercise Set 2.6

1. The function contains neither division nor an even root. The domain $=(-\infty,\infty)$

2. The function contains neither division nor an even root. The domain $=(-\infty,\infty)$

3. The denominator equals zero when $x=4$. This value must be excluded from the domain.
domain: $(-\infty,4)\cup(4,\infty)$.

4. The denominator equals zero when $x=-5$. This value must be excluded from the domain.
domain: $(-\infty,-5)\cup(-5,\infty)$.

5. The function contains neither division nor an even root. The domain $=(-\infty,\infty)$

6. The function contains neither division nor an even root. The domain $=(-\infty,\infty)$

7. The values that make the denominator equal zero must be excluded from the domain.
domain: $(-\infty,-3)\cup(-3,5)\cup(5,\infty)$

8. The values that make the denominator equal zero must be excluded from the domain.
domain: $(-\infty,-4)\cup(-4,3)\cup(3,\infty)$

9. The values that make the denominators equal zero must be excluded from the domain.
domain: $(-\infty,-7)\cup(-7,9)\cup(9,\infty)$

10. The values that make the denominators equal zero must be excluded from the domain.
domain: $(-\infty,-8)\cup(-8,10)\cup(10,\infty)$

11. The first denominator cannot equal zero. The values that make the second denominator equal zero must be excluded from the domain.
domain: $(-\infty,-1)\cup(-1,1)\cup(1,\infty)$

12. The first denominator cannot equal zero. The values that make the second denominator equal zero must be excluded from the domain.
domain: $(-\infty,-2)\cup(-2,2)\cup(2,\infty)$

13. Exclude x for $x=0$.
Exclude x for $\frac{3}{x}-1=0$.

$$\frac{3}{x}-1=0$$
$$x\left(\frac{3}{x}-1\right)=x(0)$$
$$3-x=0$$
$$-x=-3$$
$$x=3$$

domain: $(-\infty,0)\cup(0,3)\cup(3,\infty)$

14. Exclude x for $x = 0$.

Exclude x for $\frac{4}{x} - 1 = 0$.

$$\frac{4}{x} - 1 = 0$$
$$x\left(\frac{4}{x} - 1\right) = x(0)$$
$$4 - x = 0$$
$$-x = -4$$
$$x = 4$$

domain: $(-\infty, 0) \cup (0, 4) \cup (4, \infty)$

15. Exclude x for $x - 1 = 0$.

$$x - 1 = 0$$
$$x = 1$$

Exclude x for $\frac{4}{x-1} - 2 = 0$.

$$\frac{4}{x-1} - 2 = 0$$
$$(x-1)\left(\frac{4}{x-1} - 2\right) = (x-1)(0)$$
$$4 - 2(x-1) = 0$$
$$4 - 2x + 2 = 0$$
$$-2x + 6 = 0$$
$$-2x = -6$$
$$x = 3$$

domain: $(-\infty, 1) \cup (1, 3) \cup (3, \infty)$

16. Exclude x for $x - 2 = 0$.

$$x - 2 = 0$$
$$x = 2$$

Exclude x for $\frac{4}{x-2} - 3 = 0$.

$$\frac{4}{x-2} - 3 = 0$$
$$(x-2)\left(\frac{4}{x-2} - 3\right) = (x-2)(0)$$
$$4 - 3(x-2) = 0$$
$$4 - 3x + 6 = 0$$
$$-3x + 10 = 0$$
$$-3x = -10$$
$$x = \frac{10}{3}$$

domain: $(-\infty, 2) \cup \left(2, \frac{10}{3}\right) \cup \left(\frac{10}{3}, \infty\right)$

17. The expression under the radical must not be negative.

$$x - 3 \ge 0$$
$$x \ge 3$$

domain: $[3, \infty)$

18. The expression under the radical must not be negative.

$$x + 2 \ge 0$$
$$x \ge -2$$

domain: $[-2, \infty)$

19. The expression under the radical must be positive.

$$x - 3 > 0$$
$$x > 3$$

domain: $(3, \infty)$

20. The expression under the radical must be positive.

$$x + 2 > 0$$
$$x > -2$$

domain: $(-2, \infty)$

21. The expression under the radical must not be negative.

$$5x + 35 \ge 0$$
$$5x \ge -35$$
$$x \ge -7$$

domain: $[-7, \infty)$

22. The expression under the radical must not be negative.

$$7x - 70 \ge 0$$
$$7x \ge 70$$
$$x \ge 10$$

domain: $[10, \infty)$

23. The expression under the radical must not be negative.

$$24 - 2x \ge 0$$
$$-2x \ge -24$$
$$\frac{-2x}{-2} \le \frac{-24}{-2}$$
$$x \le 12$$

domain: $(-\infty, 12]$

24. The expression under the radical must not be negative.

$$84-6x \ge 0$$
$$-6x \ge -84$$
$$\frac{-6x}{-6} \le \frac{-84}{-6}$$
$$x \le 14$$

domain: $(-\infty, 14]$

25. The expressions under the radicals must not be negative.

$x-2 \ge 0$ and $x+3 \ge 0$

$x \ge 2$ $\quad$ $x \ge -3$

To make both inequalities true, $x \ge 2$.

domain: $[2, \infty)$

26. The expressions under the radicals must not be negative.

$x-3 \ge 0$ and $x+4 \ge 0$

$x \ge 3$ $\quad$ $x \ge -4$

To make both inequalities true, $x \ge 3$.

domain: $[3, \infty)$

27. The expression under the radical must not be negative.

$$x-2 \ge 0$$
$$x \ge 2$$

The denominator equals zero when $x = 5$.

domain: $[2,5) \cup (5, \infty)$.

28. The expression under the radical must not be negative.

$$x-3 \ge 0$$
$$x \ge 3$$

The denominator equals zero when $x = 6$.

domain: $[3,6) \cup (6, \infty)$.

29. Find the values that make the denominator equal zero and must be excluded from the domain.

$$x^3 - 5x^2 - 4x + 20$$
$$= x^2(x-5) - 4(x-5)$$
$$= (x-5)(x^2-4)$$
$$= (x-5)(x+2)(x-2)$$

–2, 2, and 5 must be excluded.

domain: $(-\infty,-2) \cup (-2,2) \cup (2,5) \cup (5,\infty)$

30. Find the values that make the denominator equal zero and must be excluded from the domain.

$$x^3 - 2x^2 - 9x + 18$$
$$= x^2(x-2) - 9(x-2)$$
$$= (x-2)(x^2-9)$$
$$= (x-2)(x+3)(x-3)$$

–3, 2, and 3 must be excluded.

domain: $(-\infty,-3) \cup (-3,2) \cup (2,3) \cup (3,\infty)$

31. $(f+g)(x) = 3x + 2$

domain: $(-\infty, \infty)$

$$(f-g)(x) = f(x) - g(x)$$
$$= (2x + 3) - (x - 1)$$
$$= x + 4$$

domain: $(-\infty, \infty)$

$$(fg)(x) = f(x) \cdot g(x)$$
$$= (2x+3) \cdot (x-1)$$
$$= 2x^2 + x - 3$$

domain: $(-\infty, \infty)$

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{2x+3}{x-1}$$

domain: $(-\infty, 1) \cup (1, \infty)$

32. $(f+g)(x) = 4x - 2$

domain: $(-\infty, \infty)$

$(f-g)(x) = (3x - 4) - (x + 2) = 2x - 6$

domain: $(-\infty, \infty)$

$(fg)(x) = (3x - 4)(x + 2) = 3x^2 + 2x - 8$

domain: $(-\infty, \infty)$

$$\left(\frac{f}{g}\right)(x) = \frac{3x-4}{x+2}$$

domain: $(-\infty, -2) \cup (-2, \infty)$

33. $(f+g)(x) = 3x^2 + x - 5$

domain: $(-\infty, \infty)$

$(f-g)(x) = -3x^2 + x - 5$

domain: $(-\infty, \infty)$

$(fg)(x) = (x-5)(3x^2) = 3x^3 - 15x^2$

domain: $(-\infty, \infty)$

$$\left(\frac{f}{g}\right)(x) = \frac{x-5}{3x^2}$$

domain: $(-\infty, 0) \cup (0, \infty)$

34. $(f+g)(x)=5x^2+x-6$
domain: $(-\infty, \infty)$
$(f-g)(x)=-5x^2+x-6$
domain: $(-\infty, \infty)$
$(fg)(x)=(x-6)(5x^2)=5x^3-30x^2$
domain: $(-\infty, \infty)$
$\left(\frac{f}{g}\right)(x)=\frac{x-6}{5x^2}$
domain: $(-\infty,0)\cup(0,\infty)$

35. $(f+g)(x)=2x^2-2$
domain: $(-\infty, \infty)$
$(f-g)(x)=2x^2-2x-4$
domain: $(-\infty, \infty)$
$(fg)(x)=(2x^2-x-3)(x+1)$
$=2x^3+x^2-4x-3$
domain: $(-\infty, \infty)$
$\left(\frac{f}{g}\right)(x)=\frac{2x^2-x-3}{x+1}$
$=\frac{(2x-3)(x+1)}{(x+1)}=2x-3$
domain: $(-\infty,-1)\cup(-1,\infty)$

36. $(f+g)(x)=6x^2-2$
domain: $(-\infty, \infty)$
$(f-g)(x)=6x^2-2x$
domain: $(-\infty, \infty)$
$(fg)(x)=(6x^2-x-1)(x-1)=6x^3-7x^2+1$
domain: $(-\infty, \infty)$
$\left(\frac{f}{g}\right)(x)=\frac{6x^2-x-1}{x-1}$
domain: $(-\infty,1)\cup(1,\infty)$

37. $(f+g)(x)=(3-x^2)+(x^2+2x-15)$
$=2x-12$
domain: $(-\infty, \infty)$
$(f-g)(x)=(3-x^2)-(x^2+2x-15)$
$=-2x^2-2x+18$
domain: $(-\infty, \infty)$
$(fg)(x)=(3-x^2)(x^2+2x-15)$
$=-x^4-2x^3+18x^2+6x-45$
domain: $(-\infty, \infty)$
$\left(\frac{f}{g}\right)(x)=\frac{3-x^2}{x^2+2x-15}$
domain: $(-\infty,-5)\cup(-5,3)\cup(3,\infty)$

38. $(f+g)(x)=(5-x^2)+(x^2+4x-12)$
$=4x-7$
domain: $(-\infty, \infty)$
$(f-g)(x)=(5-x^2)-(x^2+4x-12)$
$=-2x^2-4x+17$
domain: $(-\infty, \infty)$
$(fg)(x)=(5-x^2)(x^2+4x-12)$
$=-x^4-4x^3+17x^2+20x-60$
domain: $(-\infty, \infty)$
$\left(\frac{f}{g}\right)(x)=\frac{5-x^2}{x^2+4x-12}$
domain: $(-\infty,-6)\cup(-6,2)\cup(2,\infty)$

39. $(f+g)(x)=\sqrt{x}+x-4$
domain: $[0,\infty)$
$(f-g)(x)=\sqrt{x}-x+4$
domain: $[0,\infty)$
$(fg)(x)=\sqrt{x}(x-4)$
domain: $[0,\infty)$
$\left(\frac{f}{g}\right)(x)=\frac{\sqrt{x}}{x-4}$
domain: $[0,4)\cup(4,\infty)$

40. $(f+g)(x)=\sqrt{x}+x-5$
domain: $[0,\infty)$
$(f-g)(x)=\sqrt{x}-x+5$
domain: $[0,\infty)$
$(fg)(x)=\sqrt{x}(x-5)$
domain: $[0,\infty)$
$\left(\frac{f}{g}\right)(x)=\frac{\sqrt{x}}{x-5}$
domain: $[0,5)\cup(5,\infty)$

41. $(f+g)(x) = 2+\frac{1}{x}+\frac{1}{x} = 2+\frac{2}{x} = \frac{2x+2}{x}$

domain: $(-\infty,0)\cup(0,\infty)$

$(f-g)(x) = 2+\frac{1}{x}-\frac{1}{x} = 2$

domain: $(-\infty,0)\cup(0,\infty)$

$(fg)(x) = \left(2+\frac{1}{x}\right)\cdot\frac{1}{x} = \frac{2}{x}+\frac{1}{x^2} = \frac{2x+1}{x^2}$

domain: $(-\infty,0)\cup(0,\infty)$

$\left(\frac{f}{g}\right)(x) = \frac{2+\frac{1}{x}}{\frac{1}{x}} = \left(2+\frac{1}{x}\right)\cdot x = 2x+1$

domain: $(-\infty,0)\cup(0,\infty)$

42. $(f+g)(x) = 6-\frac{1}{x}+\frac{1}{x} = 6$

domain: $(-\infty,0)\cup(0,\infty)$

$(f-g)(x) = 6-\frac{1}{x}-\frac{1}{x} = 6-\frac{2}{x} = \frac{6x-2}{x}$

domain: $(-\infty,0)\cup(0,\infty)$

$(fg)(x) = \left(6-\frac{1}{x}\right)\cdot\frac{1}{x} = \frac{6}{x}-\frac{1}{x^2} = \frac{6x-1}{x^2}$

domain: $(-\infty,0)\cup(0,\infty)$

$\left(\frac{f}{g}\right)(x) = \frac{6-\frac{1}{x}}{\frac{1}{x}} = \left(6-\frac{1}{x}\right)\cdot x = 6x-1$

domain: $(-\infty,0)\cup(0,\infty)$

43. $(f+g)(x) = f(x)+g(x)$

$$= \frac{5x+1}{x^2-9}+\frac{4x-2}{x^2-9}$$

$$= \frac{9x-1}{x^2-9}$$

domain: $(-\infty,-3)\cup(-3,3)\cup(3,\infty)$

$(f-g)(x) = f(x)-g(x)$

$$= \frac{5x+1}{x^2-9}-\frac{4x-2}{x^2-9}$$

$$= \frac{x+3}{x^2-9}$$

$$= \frac{1}{x-3}$$

domain: $(-\infty,-3)\cup(-3,3)\cup(3,\infty)$

$(fg)(x) = f(x)\cdot g(x)$

$$= \frac{5x+1}{x^2-9}\cdot\frac{4x-2}{x^2-9}$$

$$= \frac{(5x+1)(4x-2)}{\left(x^2-9\right)^2}$$

domain: $(-\infty,-3)\cup(-3,3)\cup(3,\infty)$

$$\left(\frac{f}{g}\right)(x) = \frac{\frac{5x+1}{x^2-9}}{\frac{4x-2}{x^2-9}}$$

$$= \frac{5x+1}{x^2-9}\cdot\frac{x^2-9}{4x-2}$$

$$= \frac{5x+1}{4x-2}$$

The domain must exclude –3, 3, and any values that make $4x-2=0$.

$4x-2=0$

$4x=2$

$x=\frac{1}{2}$

domain: $(-\infty,-3)\cup\left(-3,\frac{1}{2}\right)\cup\left(\frac{1}{2},3\right)\cup(3,\infty)$

44. $(f+g)(x) = f(x)+g(x)$

$$= \frac{3x+1}{x^2-25} + \frac{2x-4}{x^2-25}$$
$$= \frac{5x-3}{x^2-25}$$

domain: $(-\infty,-5)\cup(-5,5)\cup(5,\infty)$

$(f-g)(x) = f(x)-g(x)$

$$= \frac{3x+1}{x^2-25} - \frac{2x-4}{x^2-25}$$
$$= \frac{x+5}{x^2-25}$$
$$= \frac{1}{x-5}$$

domain: $(-\infty,-5)\cup(-5,5)\cup(5,\infty)$

$(fg)(x) = f(x)\cdot g(x)$

$$= \frac{3x+1}{x^2-25} \cdot \frac{2x-4}{x^2-25}$$
$$= \frac{(3x+1)(2x-4)}{\left(x^2-25\right)^2}$$

domain: $(-\infty,-5)\cup(-5,5)\cup(5,\infty)$

$$\left(\frac{f}{g}\right)(x) = \frac{\frac{3x+1}{x^2-25}}{\frac{2x-4}{x^2-25}}$$
$$= \frac{3x+1}{x^2-25} \cdot \frac{x^2-25}{2x-4}$$
$$= \frac{3x+1}{2x-4}$$

The domain must exclude –5, 5, and any values that make $2x-4=0$.

$2x-4=0$

$2x=4$

$x=2$

domain: $(-\infty,-5)\cup(-5,2)\cup(2,5)\cup(5,\infty)$

45. $(f+g)(x) = f(x)+g(x)$

$$= \frac{8x}{x-2} + \frac{6}{x+3}$$
$$= \frac{8x(x+3)}{(x-2)(x+3)} + \frac{6(x-2)}{(x-2)(x+3)}$$
$$= \frac{8x^2+24x}{(x-2)(x+3)} + \frac{6x-12}{(x-2)(x+3)}$$
$$= \frac{8x^2+30x-12}{(x-2)(x+3)}$$

domain: $(-\infty,-3)\cup(-3,2)\cup(2,\infty)$

$(f+g)(x) = f(x)-g(x)$

$$= \frac{8x}{x-2} - \frac{6}{x+3}$$
$$= \frac{8x(x+3)}{(x-2)(x+3)} - \frac{6(x-2)}{(x-2)(x+3)}$$
$$= \frac{8x^2+24x}{(x-2)(x+3)} - \frac{6x-12}{(x-2)(x+3)}$$
$$= \frac{8x^2+18x+12}{(x-2)(x+3)}$$

domain: $(-\infty,-3)\cup(-3,2)\cup(2,\infty)$

$(fg)(x) = f(x)\cdot g(x)$

$$= \frac{8x}{x-2} \cdot \frac{6}{x+3}$$
$$= \frac{48x}{(x-2)(x+3)}$$

domain: $(-\infty,-3)\cup(-3,2)\cup(2,\infty)$

$$\left(\frac{f}{g}\right)(x) = \frac{\frac{8x}{x-2}}{\frac{6}{x+3}}$$
$$= \frac{8x}{x-2} \cdot \frac{x+3}{6}$$
$$= \frac{4x(x+3)}{3(x-2)}$$

The domain must exclude –3, 2, and any values that make $3(x-2)=0$.

$3(x-2)=0$

$3x-6=0$

$3x=6$

$x=2$

domain: $(-\infty,-3)\cup(-3,2)\cup(2,\infty)$

46. $(f+g)(x) = f(x)+g(x)$

$$= \frac{9x}{x-4} + \frac{7}{x+8}$$
$$= \frac{9x(x+8)}{(x-4)(x+8)} + \frac{7(x-4)}{(x-4)(x+8)}$$
$$= \frac{9x^2+72x}{(x-4)(x+8)} + \frac{7x-28}{(x-4)(x+8)}$$
$$= \frac{9x^2+79x-28}{(x-4)(x+8)}$$

domain: $(-\infty,-8)\cup(-8,4)\cup(4,\infty)$

$(f+g)(x) = f(x)-g(x)$

$$= \frac{9x}{x-4} - \frac{7}{x+8}$$
$$= \frac{9x(x+8)}{(x-4)(x+8)} - \frac{7(x-4)}{(x-4)(x+8)}$$
$$= \frac{9x^2+72x}{(x-4)(x+8)} - \frac{7x-28}{(x-4)(x+8)}$$
$$= \frac{9x^2+65x+28}{(x-4)(x+8)}$$

domain: $(-\infty,-8)\cup(-8,4)\cup(4,\infty)$

$(fg)(x) = f(x)\cdot g(x)$

$$= \frac{9x}{x-4}\cdot\frac{7}{x+8}$$
$$= \frac{63x}{(x-4)(x+8)}$$

domain: $(-\infty,-8)\cup(-8,4)\cup(4,\infty)$

$$\left(\frac{f}{g}\right)(x) = \frac{\frac{9x}{x-4}}{\frac{7}{x+8}}$$
$$= \frac{9x}{x-4}\cdot\frac{x+8}{7}$$
$$= \frac{9x(x+8)}{7(x-4)}$$

The domain must exclude –8, 4, and any values that make $7(x-4)=0$.

$7(x-4)=0$

$7x-28=0$

$7x=28$

$x=4$

domain: $(-\infty,-8)\cup(-8,4)\cup(4,\infty)$

47. $(f+g)(x) = \sqrt{x+4}+\sqrt{x-1}$

domain: $[1,\infty)$

$(f-g)(x) = \sqrt{x+4}-\sqrt{x-1}$

domain: $[1,\infty)$

$(fg)(x) = \sqrt{x+4}\cdot\sqrt{x-1} = \sqrt{x^2+3x-4}$

domain: $[1,\infty)$

$\left(\frac{f}{g}\right)(x) = \frac{\sqrt{x+4}}{\sqrt{x-1}}$

domain: $(1,\infty)$

48. $(f+g)(x) = \sqrt{x+6}+\sqrt{x-3}$

domain: $[3,\infty)$

$(f-g)(x) = \sqrt{x+6}-\sqrt{x-3}$

domain: $[3,\infty)$

$(fg)(x) = \sqrt{x+6}\cdot\sqrt{x-3} = \sqrt{x^2+3x-18}$

domain: $[3,\infty)$

$\left(\frac{f}{g}\right)(x) = \frac{\sqrt{x+6}}{\sqrt{x-3}}$

domain: $(3,\infty)$

49. $(f+g)(x) = \sqrt{x-2}+\sqrt{2-x}$

domain: $\{2\}$

$(f-g)(x) = \sqrt{x-2}-\sqrt{2-x}$

domain: $\{2\}$

$(fg)(x) = \sqrt{x-2}\cdot\sqrt{2-x} = \sqrt{-x^2+4x-4}$

domain: $\{2\}$

$\left(\frac{f}{g}\right)(x) = \frac{\sqrt{x-2}}{\sqrt{2-x}}$

domain: $\varnothing$

50. $(f+g)(x) = \sqrt{x-5}+\sqrt{5-x}$

domain: $\{5\}$

$(f-g)(x) = \sqrt{x-5}-\sqrt{5-x}$

domain: $\{5\}$

$(fg)(x) = \sqrt{x-5}\cdot\sqrt{5-x} = \sqrt{-x^2+10x-25}$

domain: $\{5\}$

$\left(\frac{f}{g}\right)(x) = \frac{\sqrt{x-5}}{\sqrt{5-x}}$

domain: $\varnothing$

51. $f(x) = 2x;\ g(x) = x + 7$

a. $(f \circ g)(x) = 2(x+7) = 2x+14$

b. $(g \circ f)(x) = 2x+7$

c. $(f \circ g)(2) = 2(2)+14 = 18$

52. $f(x) = 3x;\ g(x) = x - 5$

a. $(f \circ g)(x) = 3(x-5) = 3x-15$

b. $(g \circ f)(x) = 3x-5$

c. $(f \circ g)(2) = 3(2)-15 = -9$

53. $f(x) = x + 4;\ g(x) = 2x + 1$

a. $(f \circ g)(x) = (2x+1)+4 = 2x+5$

b. $(g \circ f)(x) = 2(x+4)+1 = 2x+9$

c. $(f \circ g)(2) = 2(2)+5 = 9$

54. $f(x) = 5x + 2\ ;\ \ g(x) = 3x - 4$

a. $(f \circ g)(x) = 5(3x-4)+2 = 15x-18$

b. $(g \circ f)(x) = 3(5x+2)-4 = 15x+2$

c. $(f \circ g)(2) = 15(2)-18 = 12$

55. $f(x) = 4x - 3;\ \ g(x) = 5x^2 - 2$

a. $(f \circ g)(x) = 4(5x^2-2)-3$
$= 20x^2 - 11$

b. $(g \circ f)(x) = 5(4x-3)^2 - 2$
$= 5(16x^2 - 24x + 9) - 2$
$= 80x^2 - 120x + 43$

c. $(f \circ g)(2) = 20(2)^2 - 11 = 69$

56. $f(x) = 7x+1;\ g(x) = 2x^2 - 9$

a. $(f \circ g)(x) = 7(2x^2-9)+1 = 14x^2 - 62$

b. $(g \circ f)(x) = 2(7x+1)^2 - 9$
$= 2(49x^2 + 14x + 1) - 9$
$= 98x^2 + 28x - 7$

c. $(f \circ g)(2) = 14(2)^2 - 62 = -6$

57. $f(x) = x^2 + 2;\ g(x) = x^2 - 2$

a. $(f \circ g)(x) = (x^2-2)^2 + 2$
$= x^4 - 4x^2 + 4 + 2$
$= x^4 - 4x^2 + 6$

b. $(g \circ f)(x) = (x^2+2)^2 - 2$
$= x^4 + 4x^2 + 4 - 2$
$= x^4 + 4x^2 + 2$

c. $(f \circ g)(2) = 2^4 - 4(2)^2 + 6 = 6$

58. $f(x) = x^2 + 1;\ g(x) = x^2 - 3$

a. $(f \circ g)(x) = (x^2-3)^2 + 1$
$= x^4 - 6x^2 + 9 + 1$
$= x^4 - 6x^2 + 10$

b. $(g \circ f)(x) = (x^2+1)^2 - 3$
$= x^4 + 2x^2 + 1 - 3$
$= x^4 + 2x^2 - 2$

c. $(f \circ g)(2) = 2^4 - 6(2)^2 + 10 = 2$

59. $f(x) = 4 - x;\quad g(x) = 2x^2 + x + 5$

a. $(f \circ g)(x) = 4 - \left(2x^2 + x + 5\right)$
$= 4 - 2x^2 - x - 5$
$= -2x^2 - x - 1$

b. $(g \circ f)(x) = 2\left(4-x\right)^2 + \left(4-x\right) + 5$
$= 2(16 - 8x + x^2) + 4 - x + 5$
$= 32 - 16x + 2x^2 + 4 - x + 5$
$= 2x^2 - 17x + 41$

c. $(f \circ g)(2) = -2(2)^2 - 2 - 1 = -11$

60. $f(x)=5x-2;\quad g(x)=-x^2+4x-1$

a. $$\begin{aligned}(f\circ g)(x)&=5\left(-x^2+4x-1\right)-2\\&=-5x^2+20x-5-2\\&=-5x^2+20x-7\end{aligned}$$

b. $$\begin{aligned}(g\circ f)(x)&=-\left(5x-2\right)^2+4\left(5x-2\right)-1\\&=-(25x^2-20x+4)+20x-8-1\\&=-25x^2+20x-4+20x-8-1\\&=-25x^2+40x-13\end{aligned}$$

c. $(f\circ g)(2)=-5(2)^2+20(2)-7=13$

61. $f(x)=\sqrt{x};\ g(x)=x-1$

a. $(f\circ g)(x)=\sqrt{x-1}$

b. $(g\circ f)(x)=\sqrt{x}-1$

c. $(f\circ g)(2)=\sqrt{2-1}=\sqrt{1}=1$

62. $f(x)=\sqrt{x}\,;g(x)=x+2$

a. $(f\circ g)(x)=\sqrt{x+2}$

b. $(g\circ f)(x)=\sqrt{x}+2$

c. $(f\circ g)(2)=\sqrt{2+2}=\sqrt{4}=2$

63. $f(x)=2x-3;\ g(x)=\dfrac{x+3}{2}$

a. $$\begin{aligned}(f\circ g)(x)&=2\left(\frac{x+3}{2}\right)-3\\&=x+3-3\\&=x\end{aligned}$$

b. $(g\circ f)(x)=\dfrac{(2x-3)+3}{2}=\dfrac{2x}{2}=x$

c. $(f\circ g)(2)=2$

64. $f(x)=6x-3;g(x)=\dfrac{x+3}{6}$

a. $(f\circ g)(x)=6\left(\dfrac{x+3}{6}\right)-3=x+3-3=x$

b. $(g\circ f)(x)=\dfrac{6x-3+3}{6}=\dfrac{6x}{6}=x$

c. $(f\circ g)(2)=2$

65. $f(x)=\dfrac{1}{x};\quad g(x)=\dfrac{1}{x}$

a. $(f\circ g)(x)=\dfrac{1}{\frac{1}{x}}=x$

b. $(g\circ f)(x)=\dfrac{1}{\frac{1}{x}}=x$

c. $(f\circ g)(2)=2$

66. $f(x)=\dfrac{2}{x};\quad g(x)=\dfrac{2}{x}$

a. $(f\circ g)(x)=\dfrac{2}{\frac{2}{x}}=x$

b. $(g\circ f)(x)=\dfrac{2}{\frac{2}{x}}=x$

c. $(f\circ g)(2)=2$

67. **a.** $$\begin{aligned}(f\circ g)(x)&=f\left(\frac{1}{x}\right)=\frac{2}{\frac{1}{x}+3},x\neq 0\\&=\frac{2(x)}{\left(\frac{1}{x}+3\right)(x)}\\&=\frac{2x}{1+3x}\end{aligned}$$

b. We must exclude 0 because it is excluded from g.

We must exclude $-\dfrac{1}{3}$ because it causes the denominator of $f\circ g$ to be 0.

domain: $\left(-\infty,-\dfrac{1}{3}\right)\cup\left(-\dfrac{1}{3},0\right)\cup(0,\infty).$

68. a. $f \circ g(x) = f\left(\frac{1}{x}\right) = \frac{5}{\frac{1}{x}+4} = \frac{5x}{1+4x}$

b. We must exclude 0 because it is excluded from *g*.
We must exclude $-\frac{1}{4}$ because it causes the denominator of $f \circ g$ to be 0.
domain: $\left(-\infty,-\frac{1}{4}\right)\cup\left(-\frac{1}{4},0\right)\cup(0,\infty)$.

69. a. $(f \circ g)(x) = f\left(\frac{4}{x}\right) = \frac{\frac{4}{x}}{\frac{4}{x}+1}$

$= \frac{\left(\frac{4}{x}\right)(x)}{\left(\frac{4}{x}+1\right)(x)}$

$= \frac{4}{4+x}, x \neq -4$

b. We must exclude 0 because it is excluded from *g*.
We must exclude -4 because it causes the denominator of $f \circ g$ to be 0.
domain: $(-\infty,-4)\cup(-4,0)\cup(0,\infty)$.

70. a. $f \circ g(x) = f\left(\frac{6}{x}\right) = \frac{\frac{6}{x}}{\frac{6}{x}+5} = \frac{6}{6+5x}$

b. We must exclude 0 because it is excluded from *g*.
We must exclude $-\frac{6}{5}$ because it causes the denominator of $f \circ g$ to be 0.
domain: $\left(-\infty,-\frac{6}{5}\right)\cup\left(-\frac{6}{5},0\right)\cup(0,\infty)$.

71. a. $f \circ g(x) = f(x-2) = \sqrt{x-2}$

b. The expression under the radical in $f \circ g$ must not be negative.
$x-2 \geq 0$
$x \geq 2$
domain: $[2,\infty)$.

72. a. $f \circ g(x) = f(x-3) = \sqrt{x-3}$

b. The expression under the radical in $f \circ g$ must not be negative.
$x-3 \geq 0$
$x \geq 3$
domain: $[3,\infty)$.

73. a. $(f \circ g)(x) = f(\sqrt{1-x})$

$= \left(\sqrt{1-x}\right)^2 + 4$

$= 1-x+4$

$= 5-x$

b. The domain of $f \circ g$ must exclude any values that are excluded from *g*.
$1-x \geq 0$
$-x \geq -1$
$x \leq 1$
domain: (–•, 1].

74. a. $(f \circ g)(x) = f(\sqrt{2-x})$

$= \left(\sqrt{2-x}\right)^2 + 1$

$= 2-x+1$

$= 3-x$

b. The domain of $f \circ g$ must exclude any values that are excluded from *g*.
$2-x \geq 0$
$-x \geq -2$
$x \leq 2$
domain: (–•, 2].

75. $f(x) = x^4 \quad g(x) = 3x-1$

76. $f(x) = x^3; g(x) = 2x-5$

77. $f(x) = \sqrt[3]{x} \quad g(x) = x^2-9$

78. $f(x) = \sqrt{x}; g(x) = 5x^2+3$

79. $f(x) = |x| \quad g(x) = 2x-5$

80. $f(x) = |x|; \; g(x) = 3x-4$

81. $f(x) = \frac{1}{x} \quad g(x) = 2x-3$

82. $f(x) = \frac{1}{x}; g(x) = 4x+5$

83. $(f+g)(-3) = f(-3) + g(-3) = 4 + 1 = 5$

84. $(g-f)(-2) = g(-2) - f(-2) = 2 - 3 = -1$

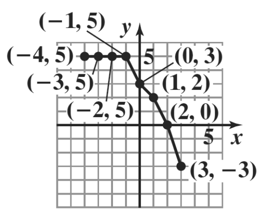
85. $(fg)(2) = f(2)g(2) = (-1)(1) = -1$

86. $\left(\frac{g}{f}\right)(3) = \frac{g(3)}{f(3)} = \frac{0}{-3} = 0$

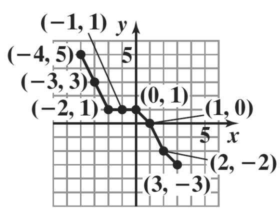
87. The domain of $f+g$ is $[-4,3]$.

88. The domain of $\frac{f}{g}$ is $(-4,3)$.

89. The graph of $f+g$



90. The graph of $f-g$



91. $(f \circ g)(-1) = f(g(-1)) = f(-3) = 1$

92. $(f \circ g)(1) = f(g(1)) = f(-5) = 3$

93. $(g \circ f)(0) = g(f(0)) = g(2) = -6$

94. $(g \circ f)(-1) = g(f(-1)) = g(1) = -5$

95.
$$\begin{aligned}
(f \circ g)(x) &= 7\\
2(x^2 - 3x + 8) - 5 &= 7\\
2x^2 - 6x + 16 - 5 &= 7\\
2x^2 - 6x + 11 &= 7\\
2x^2 - 6x + 4 &= 0\\
x^2 - 3x + 2 &= 0\\
(x-1)(x-2) &= 0\\
x - 1 = 0 \quad &\text{or} \quad x - 2 = 0\\
x = 1 \quad & \quad\quad x = 2
\end{aligned}$$

96.
$$\begin{aligned}
(f \circ g)(x) &= -5\\
1 - 2(3x^2 + x - 1) &= -5\\
1 - 6x^2 - 2x + 2 &= -5\\
-6x^2 - 2x + 3 &= -5\\
-6x^2 - 2x + 8 &= 0\\
3x^2 + x - 4 &= 0\\
(3x+4)(x-1) &= 0\\
3x + 4 = 0 \quad &\text{or} \quad x - 1 = 0\\
3x = -4 \quad & \quad\quad x = 1\\
x = -\frac{4}{3} &
\end{aligned}$$

97. **a.**
$$\begin{aligned}
(M+F)(x) &= M(x) + F(x)\\
&= (1.48x + 115.1) + (1.44x + 120.9)\\
&= 2.92x + 236
\end{aligned}$$

b.
$$\begin{aligned}
(M+F)(x) &= 2.92x + 236\\
(M+F)(25) &= 2.92(25) + 236\\
&= 309
\end{aligned}$$
The total U.S. population in 2010 was 309 million.

c. It is the same.

98. **a.**
$$\begin{aligned}
(F-M)(x) &= F(x) - M(x)\\
&= (1.44x + 120.9) - (1.48x + 115.1)\\
&= -0.04x + 5.8
\end{aligned}$$

b.
$$\begin{aligned}
(F-M)(x) &= -0.04x + 5.8\\
(F-M)(25) &= -0.04(25) + 5.8\\
&= 4.8
\end{aligned}$$
In 2010 there were 4.8 million more women than men.

c. The result in part (b) underestimates the actual difference by 0.2 million.

99. $(R-C)(20,000)$
$= 65(20,000) - (600,000 + 45(20,000))$
$= -200,000$
The company lost $200,000 since costs exceeded revenues.

$(R - C)(30,000)$
$= 65(30,000) - (600,000 + 45(30,000))$
$= 0$
The company broke even.

$(R - C)(40,000)$
$= 65(40,000) - (600,000 + 45(40,000))$
$= 200,000$
The company gained $200,000 since revenues exceeded costs.

100. a. The slope for *f* is -0.44 This is the decrease in profits for the first store for each year after 2012.

b. The slope of *g* is 0.51 This is the increase in profits for the second store for each year after 2012.

c. $f + g = -.044x + 13.62 + 0.51x + 11.14$
$= 0.07x + 24.76$
The slope for $f + g$ is 0.07 This is the profit for the two stores combined for each year after 2012.

101. a. *f* gives the price of the computer after a $400 discount. *g* gives the price of the computer after a 25% discount.

b. $(f \circ g)(x) = 0.75x - 400$
This models the price of a computer after first a 25% discount and then a $400 discount.

c. $(g \circ f)(x) = 0.75(x - 400)$
This models the price of a computer after first a $400 discount and then a 25% discount.

d. The function $f \circ g$ models the greater discount, since the 25% discount is taken on the regular price first.

102. a. *f* gives the cost of a pair of jeans for which a $5 rebate is offered.
g gives the cost of a pair of jeans that has been discounted 40%.

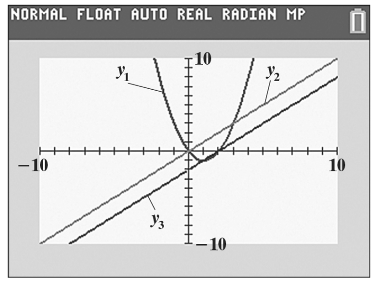
b. $(f \circ g)(x) = 0.6x - 5$
The cost of a pair of jeans is 60% of the regular price minus a $5 rebate.

c. $(g \circ f)(x) = 0.6(x - 5)$
$= 0.6x - 3$
The cost of a pair of jeans is 60% of the regular price minus a $3 rebate.

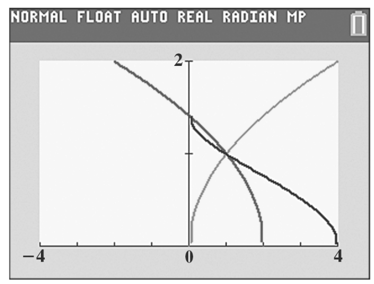
d. $f \circ g$ because of a $5 rebate.

103. – 107. Answers will vary.

108. When your trace reaches $x = 0$, the *y* value disappears because the function is not defined at $x = 0$.



109.



$(f \circ g)(x) = \sqrt{2 - \sqrt{x}}$
The domain of g is $[0, \infty)$.
The expression under the radical in $f \circ g$ must not be negative.
$2 - \sqrt{x} \geq 0$
$-\sqrt{x} \geq -2$
$\sqrt{x} \leq 2$
$x \leq 4$
domain: $[0, 4]$

110. makes sense

111. makes sense

112. does not make sense; Explanations will vary. Sample explanation: It is common that $f \circ g$ and $g \circ f$ are not the same.

113. does not make sense; Explanations will vary. Sample explanation: The diagram illustrates $g(f(x)) = x^2 + 4.$

114. false; Changes to make the statement true will vary. A sample change is:

$$\begin{aligned}(f \circ g)(x) &= f\left(\sqrt{x^2 - 4}\right) \\ &= \left(\sqrt{x^2-4}\right)^2 - 4 \\ &= x^2 - 4 - 4 \\ &= x^2 - 8\end{aligned}$$

115. false; Changes to make the statement true will vary. A sample change is:

$f(x) = 2x; g(x) = 3x$

$(f \circ g)(x) = f(g(x)) = f(3x) = 2(3x) = 6x$

$(g \circ f)(x) = g(f(x)) = g(f(x)) = 3(2x) = 6x$

116. false; Changes to make the statement true will vary. A sample change is:

$(f \circ g)(4) = f(g(4)) = f(7) = 5$

117. true

118. $(f \circ g)(x) = (f \circ g)(-x)$

$f(g(x)) = f(g(-x))$ since g is even

$f(g(x)) = f(g(x))$ so $f \circ g$ is even

119. Answers will vary.

120.

$$\begin{aligned}\frac{x-1}{5} - \frac{x+3}{2} &= 1 - \frac{x}{4} \\ 20\left(\frac{x-1}{5} - \frac{x+3}{2}\right) &= 20\left(1 - \frac{x}{4}\right) \\ 4(x-1) - 10(x+3) &= 20 - 5x \\ 4x - 4 - 10x - 30 &= 20 - 5x \\ -6x - 34 &= 20 - 5x \\ -6x + 5x &= 20 + 34 \\ -1x &= 54 \\ x &= -54\end{aligned}$$

The solution set is {-54}.

121. Let x = the number of bridge crossings at which the costs of the two plans are the same.

$$\underbrace{6x}_{\text{No Pass}} = \underbrace{30 + 4x}_{\text{Discount Pass}}$$

$$\begin{aligned}6x - 4x &= 30 \\ 2x &= 30 \\ x &= 15\end{aligned}$$

The two plans cost the same for 15 bridge crossings. The monthly cost is $\$6(15) = \$90.$

122.

$$\begin{aligned}Ax + By &= Cy + D \\ By - Cy &= D - Ax \\ y(B - C) &= D - Ax \\ y &= \frac{D - Ax}{B - C}\end{aligned}$$

123. $\{(4,-2),(1,-1),(1,1),(4,2)\}$

The element 1 in the domain corresponds to two elements in the range.
Thus, the relation is not a function.

124.

$$\begin{aligned}x &= \frac{5}{y} + 4 \\ y(x) &= y\left(\frac{5}{y} + 4\right) \\ xy &= 5 + 4y \\ xy - 4y &= 5 \\ y(x-4) &= 5 \\ y &= \frac{5}{x-4}\end{aligned}$$

125.

$$\begin{aligned}x &= y^2 - 1 \\ x + 1 &= y^2 \\ \sqrt{x+1} &= \sqrt{y^2} \\ \sqrt{x+1} &= y \\ y &= \sqrt{x+1}\end{aligned}$$

Section 2.7

Check Point Exercises

1. $f\left(g(x)\right)=4\left(\frac{x+7}{4}\right)-7$

$=x+7-7$

$=x$

$g\left(f(x)\right)=\frac{(4x-7)+7}{4}$

$=\frac{4x-7+7}{4}$

$=\frac{4x}{4}$

$=x$

$f\left(g(x)\right)=g\left(f(x)\right)=x$

2. $f(x)=2x+7$

Replace $f(x)$ with *y*:

$y=2x+7$

Interchange *x* and *y*:

$x=2y+7$

Solve for *y*:

$x=2y+7$

$x-7=2y$

$\frac{x-7}{2}=y$

Replace y with $f^{-1}(x)$:

$f^{-1}(x)=\frac{x-7}{2}$

3. $f(x)=4x^3-1$

Replace $f(x)$ with *y*:

$y=4x^3-1$

Interchange *x* and *y*:

$x=4y^3-1$

Solve for *y*:

$x=4y^3-1$

$x+1=4y^3$

$\frac{x+1}{4}=y^3$

$\sqrt[3]{\frac{x+1}{4}}=y$

Replace *y* with $f^{-1}(x)$:

$f^{-1}(x)=\sqrt[3]{\frac{x+1}{4}}$

Alternative form for answer:

$f(x)^{-1}=\sqrt[3]{\frac{x+1}{4}}=\frac{\sqrt[3]{x+1}}{\sqrt[3]{4}}$

$=\frac{\sqrt[3]{x+1}}{\sqrt[3]{4}}\cdot\frac{\sqrt[3]{2}}{\sqrt[3]{2}}=\frac{\sqrt[3]{2x+2}}{\sqrt[3]{8}}$

$=\frac{\sqrt[3]{2x+2}}{2}$

4. $f(x)=\frac{x+1}{x-5}, x\neq 5$

Replace $f(x)$ with *y*:

$y=\frac{x+1}{x-5}$

Interchange *x* and *y*:

$x=\frac{y+1}{y-5}$

Solve for *y*:

$x=\frac{y+1}{y-5}$

$x\left(y-5\right)=y+1$

$xy-5x=y+1$

$xy-y=5x+1$

$y(x-1)=5x+1$

$y=\frac{5x+1}{x-1}$

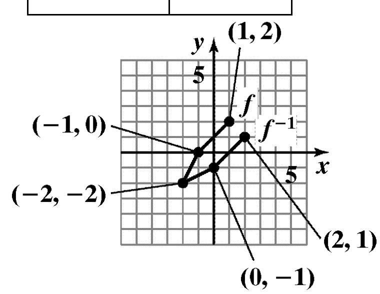
Replace *y* with $f^{-1}(x)$:

$f^{-1}(x)=\frac{5x+1}{x-1}$

5. The graphs of (b) and (c) pass the horizontal line test and thus have an inverse.

6. Find points of f^{-1}.

$f(x)$	$f^{-1}(x)$
$(-2,-2)$	$(-2,-2)$
$(-1,0)$	$(0,-1)$
$(1,2)$	$(2,1)$



7. $f(x) = x^2 + 1$

Replace $f(x)$ with y:

$y = x^2 + 1$

Interchange x and y:

$x = y^2 + 1$

Solve for y:

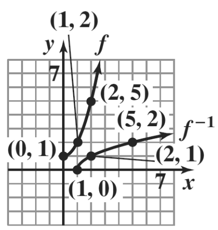
$x = y^2 + 1$

$x - 1 = y^2$

$\sqrt{x-1} = y$

Replace y with $f^{-1}(x)$:

$f^{-1}(x) = \sqrt{x-1}$



Concept and Vocabulary Check 2.7

1. inverse

2. x; x

3. horizontal; one-to-one

4. $y = x$

Exercise Set 2.7

1. $f(x) = 4x;\ g(x) = \frac{x}{4}$

$f(g(x)) = 4\left(\frac{x}{4}\right) = x$

$g(f(x)) = \frac{4x}{4} = x$

f and g are inverses.

2. $f(x) = 6x;\ g(x) = \frac{x}{6}$

$f\left(g(x)\right) = 6\left(\frac{x}{6}\right) = x$

$g\left(f(x)\right) = \frac{6x}{6} = x$

f and g are inverses.

3. $f(x) = 3x + 8;\ g(x) = \frac{x-8}{3}$

$f(g(x)) = 3\left(\frac{x-8}{3}\right) + 8 = x - 8 + 8 = x$

$g(f(x)) = \frac{(3x+8)-8}{3} = \frac{3x}{3} = x$

f and g are inverses.

4. $f(x) = 4x + 9;\ g(x) = \frac{x-9}{4}$

$f\left(g(x)\right) = 4\left(\frac{x-9}{4}\right) + 9 = x - 9 + 9 = x$

$g\left(f(x)\right) = \frac{(4x+9)-9}{4} = \frac{4x}{4} = x$

f and g are inverses.

5. $f(x) = 5x - 9;\ g(x) = \frac{x+5}{9}$

$f(g(x)) = 5\left(\frac{x+5}{9}\right) - 9$

$= \frac{5x+25}{9} - 9$

$= \frac{5x-56}{9}$

$g(f(x)) = \frac{5x-9+5}{9} = \frac{5x-4}{9}$

f and g are not inverses.

6. $f(x) = 3x - 7;\ g(x) = \frac{x+3}{7}$

$f\left(g(x)\right) = 3\left(\frac{x+3}{7}\right) - 7 = \frac{3x+9}{7} - 7 = \frac{3x-40}{7}$

$g\left(f(x)\right) = \frac{3x-7+3}{7} = \frac{3x-4}{7}$

f and g are not inverses.

7. $f(x) = \frac{3}{x-4};\ g(x) = \frac{3}{x} + 4$

$f(g(x)) = \frac{3}{\frac{3}{x}+4-4} = \frac{3}{\frac{3}{x}} = x$

$g(f(x)) = \frac{3}{\frac{3}{x-4}} + 4$

$= 3 \cdot \left(\frac{x-4}{3}\right) + 4$

$= x - 4 + 4$

$= x$

f and g are inverses.

8. $f(x)=\dfrac{2}{x-5};\ g(x)=\dfrac{2}{x}+5$

$f(g(x))=\dfrac{2}{\left(\frac{2}{x}+5\right)-5}=\dfrac{2x}{2}=x$

$g\left(f(x)\right)=\dfrac{2}{\frac{2}{x-5}}+5=2\left(\dfrac{x-5}{2}\right)+5=x-5+5=x$

f and *g* are inverses.

9. $f(x)=-x;\ g(x)=-x$

$f(g(x))=-(-x)=x$

$g(f(x))=-(-x)=x$

f and *g* are inverses.

10. $f(x)=\sqrt[3]{x-4};\ g(x)=x^3+4$

$f\left(g(x)\right)=\sqrt[3]{x^3+4-4}=\sqrt[3]{x^3}=x$

$g\left(f(x)\right)=\left(\sqrt[3]{x-4}\right)^3+4=x-4+4=x$

f and *g* are inverses.

11. **a.** $f(x)=x+3$

$y=x+3$

$x=y+3$

$y=x-3$

$f^{-1}(x)=x-3$

b. $f(f^{-1}(x))=x-3+3=x$

$f^{-1}(f(x))=x+3-3=x$

12. **a.** $f(x)=x+5$

$y=x+5$

$x=y+5$

$y=x-5$

$f^{-1}(x)=x-5$

b. $f\left(f^{-1}(x)\right)=x-5+5=x$

$f^{-1}\left(f(x)\right)=x+5-5=x$

13. **a.** $f(x)=2x$

$y=2x$

$x=2y$

$y=\dfrac{x}{2}$

$f^{-1}(x)=\dfrac{x}{2}$

b. $f(f^{-1}(x))=2\left(\dfrac{x}{2}\right)=x$

$f^{-1}(f(x))=\dfrac{2x}{2}=x$

14. **a.** $f(x)=4x$

$y=4x$

$x=4y$

$y=\dfrac{x}{4}$

$f^{-1}(x)=\dfrac{x}{4}$

b. $f\left(f^{-1}(x)\right)=4\left(\dfrac{x}{4}\right)=x$

$f^{-1}\left(f(x)\right)=\dfrac{4x}{4}=x$

15. **a.** $f(x)=2x+3$

$y=2x+3$

$x=2y+3$

$x-3=2y$

$y=\dfrac{x-3}{2}$

$f^{-1}(x)=\dfrac{x-3}{2}$

b. $f(f^{-1}(x))=2\left(\dfrac{x-3}{2}\right)+3$

$=x-3+3$

$=x$

$f^{-1}(f(x))=\dfrac{2x+3-3}{2}=\dfrac{2x}{2}=x$

16. **a.** $f(x)=3x-1$

$y=3x-1$

$x=3y-1$

$x+1=3y$

$y=\dfrac{x+1}{3}$

$f^{-1}(x)=\dfrac{x+1}{3}$

b. $f\left(f^{-1}(x)\right)=3\left(\dfrac{x+1}{3}\right)-1=x+1-1=x$

$f^{-1}\left(f(x)\right)=\dfrac{3x-1+1}{3}=\dfrac{3x}{3}=x$

17. **a.**
$$f(x) = x^3 + 2$$
$$y = x^3 + 2$$
$$x = y^3 + 2$$
$$x - 2 = y^3$$
$$y = \sqrt[3]{x-2}$$
$$f^{-1}(x) = \sqrt[3]{x-2}$$

b.
$$f(f^{-1}(x)) = \left(\sqrt[3]{x-2}\right)^3 + 2$$
$$= x - 2 + 2$$
$$= x$$
$$f^{-1}(f(x)) = \sqrt[3]{x^3 + 2 - 2} = \sqrt[3]{x^3} = x$$

18. **a.**
$$f(x) = x^3 - 1$$
$$y = x^3 - 1$$
$$x = y^3 - 1$$
$$x + 1 = y^3$$
$$y = \sqrt[3]{x+1}$$
$$f^{-1}(x) = \sqrt[3]{x+1}$$

b.
$$f(f^{-1}(x)) = \left(\sqrt[3]{x+1}\right)^3 - 1$$
$$= x + 1 - 1$$
$$= x$$
$$f^{-1}(f(x)) = \sqrt[3]{x^3 - 1 + 1} = \sqrt[3]{x^3} = x$$

19. **a.**
$$f(x) = (x+2)^3$$
$$y = (x+2)^3$$
$$x = (y+2)^3$$
$$\sqrt[3]{x} = y + 2$$
$$y = \sqrt[3]{x} - 2$$
$$f^{-1}(x) = \sqrt[3]{x} - 2$$

b.
$$f(f^{-1}(x)) = \left(\sqrt[3]{x} - 2 + 2\right)^3 = \left(\sqrt[3]{x}\right)^3 = x$$
$$f^{-1}(f(x)) = \sqrt[3]{(x+2)^3} - 2$$
$$= x + 2 - 2$$
$$= x$$

20. **a.**
$$f(x) = (x-1)^3$$
$$y = (x-1)^3$$
$$x = (y-1)^3$$
$$\sqrt[3]{x} = y - 1$$
$$y = \sqrt[3]{x} + 1$$

b.
$$f\left(f^{-1}(x)\right) = \left(\sqrt[3]{x} + 1 - 1\right)^3 = \left(\sqrt[3]{x}\right)^3 = x$$
$$f^{-1}\left(f(x)\right) = \sqrt[3]{(x-1^3} + 1 = x - 1 + 1 = x$$

21. **a.**
$$f(x) = \frac{1}{x}$$
$$y = \frac{1}{x}$$
$$x = \frac{1}{y}$$
$$xy = 1$$
$$y = \frac{1}{x}$$
$$f^{-1}(x) = \frac{1}{x}$$

b.
$$f(f^{-1}(x)) = \frac{1}{\frac{1}{x}} = x$$
$$f^{-1}(f(x)) = \frac{1}{\frac{1}{x}} = x$$

22. **a.**
$$f(x) = \frac{2}{x}$$
$$y = \frac{2}{x}$$
$$x = \frac{2}{y}$$
$$xy = 2$$
$$y = \frac{2}{x}$$
$$f^{-1}(x) = \frac{2}{x}$$

b.
$$f(f^{-1}(x)) = \frac{2}{\frac{2}{x}} = 2 \cdot \frac{x}{2} = x$$
$$f^{-1}(f(x)) = \frac{2}{\frac{2}{x}} = 2 \cdot \frac{x}{2} = x$$

23. **a.**

$$f(x)=\sqrt{x}$$
$$y=\sqrt{x}$$
$$x=\sqrt{y}$$
$$y=x^2$$
$$f^{-1}(x)=x^2,\ x\ge 0$$

b. $f(f^{-1}(x))=\sqrt{x^2}=|x|=x$ for $x\ge 0$.

$$f^{-1}(f(x))=(\sqrt{x})^2=x$$

24. **a.**

$$f(x)=\sqrt[3]{x}$$
$$y=\sqrt[3]{x}$$
$$x=\sqrt[3]{y}$$
$$y=x^3$$
$$f^{-1}(x)=x^3$$

b. $f\left(f^{-1}(x)\right)=\sqrt[3]{x^3}=x$

$$f^{-1}\left(f(x)\right)=\left(\sqrt[3]{x}\right)^3=x$$

25. **a.**

$$f(x)=\frac{x+4}{x-2}$$
$$y=\frac{x+4}{x-2}$$
$$x=\frac{y+4}{y-2}$$
$$xy-2x=y+4$$
$$xy-y=2x+4$$
$$y(x-1)=2x+4$$
$$y=\frac{2x+4}{x-1}$$
$$f^{-1}(x)=\frac{2x+4}{x-1},\ x\ne 1$$

b.

$$f\left(f^{-1}(x)\right)=\frac{\frac{2x+4}{x-1}+4}{\frac{2x+4}{x-1}-2}=\frac{2x+4+4(x-1)}{2x+4-2(x-1)}=\frac{6x}{6}=x$$

$$f^{-1}\left(f(x)\right)=\frac{2\left(\frac{x+4}{x-2}\right)+4}{\frac{x+4}{x-2}-1}=\frac{2x+8+4(x-2)}{x+4-(x-2)}=\frac{6x}{6}=x$$

26. **a.**

$$f(x)=\frac{x+5}{x-6}$$
$$y=\frac{x+5}{x-6}$$
$$x=\frac{y+5}{y-6}$$
$$xy-6x=y+5$$
$$xy-y=6x+5$$
$$y(x-1)=6x+5$$
$$y=\frac{6x+5}{x-1}$$
$$f^{-1}(x)=\frac{6x+5}{x-1},\ x\ne 1$$

b. $f\left(f^{-1}(x)\right)=\dfrac{\dfrac{6x+5}{x-1}+5}{\dfrac{6x+5}{x-1}-6}$

$=\dfrac{6x+5+5(x-1)}{6x+5-6(x-1)}$

$=\dfrac{11x}{11}$

$=x$

$f^{-1}\left(f(x)\right)=\dfrac{6\left(\dfrac{x+5}{x-6}\right)+5}{\dfrac{x+5}{x-6}-1}$

$=\dfrac{6x+30+5(x-6)}{x+5-(x-6)}$

$=\dfrac{11x}{11}$

$=x$

27. a. $f(x)=\dfrac{2x+1}{x-3}$

$y=\dfrac{2x+1}{x-3}$

$x=\dfrac{2y+1}{y-3}$

$x(y-3)=2y+1$

$xy-3x=2y+1$

$xy-2y=3x+1$

$y(x-2)=3x+1$

$y=\dfrac{3x+1}{x-2}$

$f^{-1}(x)=\dfrac{3x+1}{x-2}$

b. $f(f^{-1}(x))=\dfrac{2\left(\dfrac{3x+1}{x-2}\right)+1}{\dfrac{3x+1}{x-2}-3}$

$=\dfrac{2(3x+1)+x-2}{3x+1-3(x-2)}=\dfrac{6x+2+x-2}{3x+1-3x+6}$

$=\dfrac{7x}{7}=x$

$f^{-1}(f(x))=\dfrac{3\left(\dfrac{2x+1}{x-3}\right)+1}{\dfrac{2x+1}{x-3}-2}$

$=\dfrac{3(2x+1)+x-3}{2x+1-2(x-3)}$

$=\dfrac{6x+3+x-3}{2x+1-2x+6}=\dfrac{7x}{7}=x$

28. a. $f(x)=\dfrac{2x-3}{x+1}$

$y=\dfrac{2x-3}{x+1}$

$x=\dfrac{2y-3}{y+1}$

$xy+x=2y-3$

$y(x-2)=-x-3$

$y=\dfrac{-x-3}{x-2}$

$f^{-1}(x)=\dfrac{-x-3}{x-2},\quad x\neq 2$

b. $f\left(f^{-1}(x)\right)=\dfrac{2\left(\dfrac{-x-3}{x-2}\right)-3}{\dfrac{-x-3}{x-2}+1}$

$=\dfrac{-2x-6-3x+6}{-x-3+x-2}=\dfrac{-5x}{-5}=x$

$f^{-1}(f(x))=\dfrac{-\left(\dfrac{2x-3}{x+1}\right)-3}{\dfrac{2x-3}{x+1}-2}$

$=\dfrac{-2x+3-3x-3}{2x-3-2x-2}=\dfrac{-5x}{-5}=x$

29. The function fails the horizontal line test, so it does not have an inverse function.

30. The function passes the horizontal line test, so it does have an inverse function.

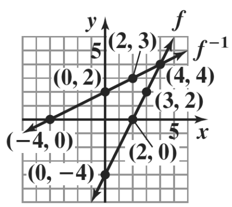
31. The function fails the horizontal line test, so it does not have an inverse function.

32. The function fails the horizontal line test, so it does not have an inverse function.

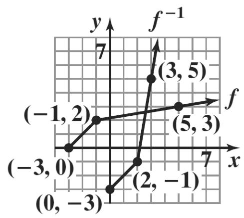
33. The function passes the horizontal line test, so it does have an inverse function.

34. The function passes the horizontal line test, so it does have an inverse function.

35.



36.



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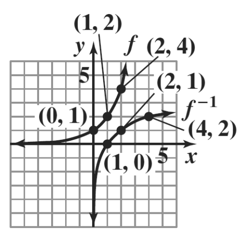

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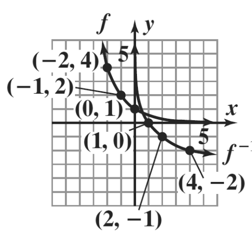
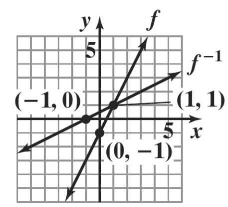


39. **a.**

$$f(x) = 2x - 1$$
$$y = 2x - 1$$
$$x = 2y - 1$$
$$x + 1 = 2y$$
$$\frac{x+1}{2} = y$$
$$f^{-1}(x) = \frac{x+1}{2}$$

b.



c. domain of f : $(-\infty, \infty)$

range of f : $(-\infty, \infty)$

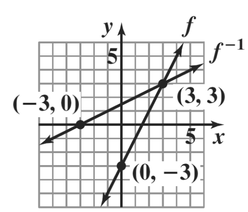
domain of f^{-1}: $(-\infty, \infty)$

range of f^{-1}: $(-\infty, \infty)$

40. **a.**

$$f(x) = 2x - 3$$
$$y = 2x - 3$$
$$x = 2y - 3$$
$$x + 3 = 2y$$
$$\frac{x+3}{2} = y$$
$$f^{-1}(x) = \frac{x+3}{2}$$

b.



c. domain of f : $(-\infty, \infty)$

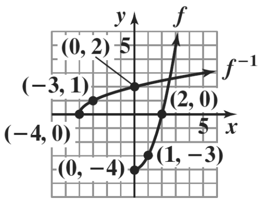
range of f : $(-\infty, \infty)$

domain of f^{-1}: $(-\infty, \infty)$

range of f^{-1}: $(-\infty, \infty)$

41. **a.**
$$f(x) = x^2 - 4$$
$$y = x^2 - 4$$
$$x = y^2 - 4$$
$$x + 4 = y^2$$
$$\sqrt{x+4} = y$$
$$f^{-1}(x) = \sqrt{x+4}$$

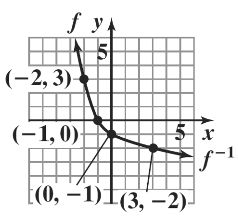
b.



c. domain of f : $[0, \infty)$
range of f : $[-4, \infty)$
domain of f^{-1}: $[-4, \infty)$
range of f^{-1}: $[0, \infty)$

42. **a.**
$$f(x) = x^2 - 1$$
$$y = x^2 - 1$$
$$x = y^2 - 1$$
$$x + 1 = y^2$$
$$-\sqrt{x+1} = y$$
$$f^{-1}(x) = -\sqrt{x+1}$$

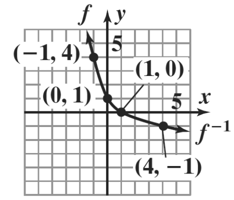
b.



c. domain of f : $(-\infty, 0]$
range of f : $[-1, \infty)$
domain of f^{-1}: $[-1, \infty)$
range of f^{-1}: $(-\infty, 0]$

43. **a.**
$$f(x) = (x-1)^2$$
$$y = (x-1)^2$$
$$x = (y-1)^2$$
$$-\sqrt{x} = y - 1$$
$$-\sqrt{x} + 1 = y$$
$$f^{-1}(x) = 1 - \sqrt{x}$$

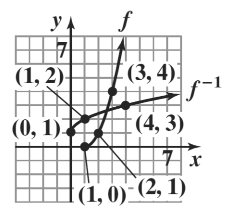
b.



c. domain of f : $(-\infty, 1]$
range of f : $[0, \infty)$
domain of f^{-1}: $[0, \infty)$
range of f^{-1}: $(-\infty, 1]$

44. **a.**
$$f(x) = (x-1)^2$$
$$y = (x-1)^2$$
$$x = (y-1)^2$$
$$\sqrt{x} = y - 1$$
$$\sqrt{x} + 1 = y$$
$$f^{-1}(x) = 1 + \sqrt{x}$$

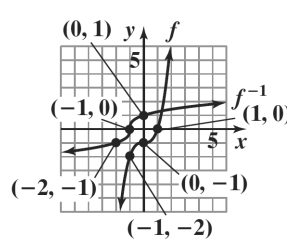
b.



c. domain of f : $[1, \infty)$
range of f : $[0, \infty)$
domain of f^{-1}: $[0, \infty)$
range of f^{-1}: $[1, \infty)$

45. **a.**
$$f(x) = x^3 - 1$$
$$y = x^3 - 1$$
$$x = y^3 - 1$$
$$x + 1 = y^3$$
$$\sqrt[3]{x+1} = y$$
$$f^{-1}(x) = \sqrt[3]{x+1}$$

b.



c. domain of f : $(-\infty,\infty)$

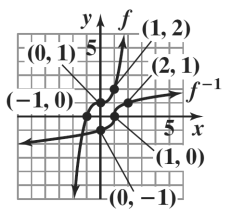
range of f : $(-\infty,\infty)$

domain of f^{-1}: $(-\infty,\infty)$

range of f^{-1}: $(-\infty,\infty)$

46. a.
$$f(x)=x^3+1$$
$$y=x^3+1$$
$$x=y^3+1$$
$$x-1=y^3$$
$$\sqrt[3]{x-1}=y$$
$$f^{-1}(x)=\sqrt[3]{x-1}$$

b.



c. domain of f : $(-\infty,\infty)$

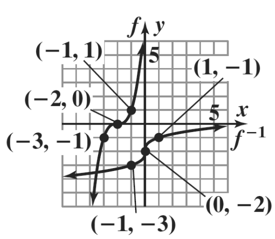
range of f : $(-\infty,\infty)$

domain of f^{-1}: $(-\infty,\infty)$

range of f^{-1}: $(-\infty,\infty)$

47. a.
$$f(x)=(x+2)^3$$
$$y=(x+2)^3$$
$$x=(y+2)^3$$
$$\sqrt[3]{x}=y+2$$
$$\sqrt[3]{x}-2=y$$
$$f^{-1}(x)=\sqrt[3]{x}-2$$

b.



c. domain of f : $(-\infty,\infty)$

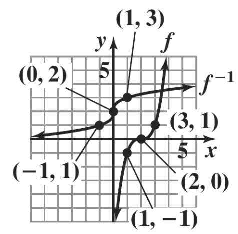
range of f : $(-\infty,\infty)$

domain of f^{-1}: $(-\infty,\infty)$

range of f^{-1}: $(-\infty,\infty)$

48. a.
$$f(x)=(x-2)^3$$
$$y=(x-2)^3$$
$$x=(y-2)^3$$
$$\sqrt[3]{x}=y-2$$
$$\sqrt[3]{x}+2=y$$
$$f^{-1}(x)=\sqrt[3]{x}+2$$

b.



c. domain of f : $(-\infty,\infty)$

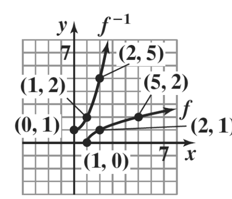
range of f : $(-\infty,\infty)$

domain of f^{-1}: $(-\infty,\infty)$

range of f^{-1}: $(-\infty,\infty)$

49. a.
$$f(x)=\sqrt{x-1}$$
$$y=\sqrt{x-1}$$
$$x=\sqrt{y-1}$$
$$x^2=y-1$$
$$x^2+1=y$$
$$f^{-1}(x)=x^2+1$$

b.



c. domain of f : $[1,\infty)$

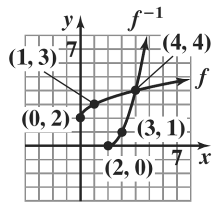
range of f : $[0,\infty)$

domain of f^{-1}: $[0,\infty)$

range of f^{-1}: $[1,\infty)$

50. a.
$$f(x)=\sqrt{x}+2$$
$$y=\sqrt{x}+2$$
$$x=\sqrt{y}+2$$
$$x-2=\sqrt{y}$$
$$(x-2)^2=y$$
$$f^{-1}(x)=(x-2)^2$$

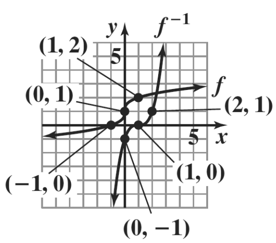
b.



c. domain of f : $[0,\infty)$
range of f : $[2,\infty)$
domain of f^{-1}: $[2,\infty)$
range of f^{-1}: $[0,\infty)$

51. a.
$$f(x)=\sqrt[3]{x}+1$$
$$y=\sqrt[3]{x}+1$$
$$x=\sqrt[3]{y}+1$$
$$x-1=\sqrt[3]{y}$$
$$(x-1)^3=y$$
$$f^{-1}(x)=(x-1)^3$$

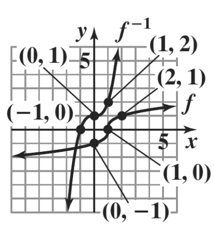
b.



c. domain of f : $(-\infty,\infty)$
range of f : $(-\infty,\infty)$
domain of f^{-1}: $(-\infty,\infty)$
range of f^{-1}: $(-\infty,\infty)$

52. a.
$$f(x)=\sqrt[3]{x-1}$$
$$y=\sqrt[3]{x-1}$$
$$x=\sqrt[3]{y-1}$$
$$x^3=y-1$$
$$x^3+1=y$$
$$f^{-1}(x)=x^3+1$$

b.



c. domain of f : $(-\infty,\infty)$
range of f : $(-\infty,\infty)$
domain of f^{-1}: $(-\infty,\infty)$
range of f^{-1}: $(-\infty,\infty)$

53. $f(g(1))=f(1)=5$

54. $f(g(4))=f(2)=-1$

55. $(g\circ f)(-1)=g(f(-1))=g(1)=1$

56. $(g\circ f)(0)=g(f(0))=g(4)=2$

57. $f^{-1}(g(10))=f^{-1}(-1)=2$, since $f(2)=-1$.

58. $f^{-1}(g(1))=f^{-1}(1)=-1$, since $f(-1)=1$.

59.
$$\begin{aligned}(f\circ g)(0)&=f(g(0))\\&=f(4\cdot 0-1)\\&=f(-1)=2(-1)-5=-7\end{aligned}$$

60.
$$\begin{aligned}(g\circ f)(0)&=g(f(0))\\&=g(2\cdot 0-5)\\&=g(-5)=4(-5)-1=-21\end{aligned}$$

61. Let $f^{-1}(1)=x$. Then
$$f(x)=1$$
$$2x-5=1$$
$$2x=6$$
$$x=3$$
Thus, $f^{-1}(1)=3$

62. Let $g^{-1}(7)=x$. Then
$$g(x)=7$$
$$4x-1=7$$
$$4x=8$$
$$x=2$$
Thus, $g^{-1}(7)=2$

63. $g\left(f\left[h(1)\right]\right) = g\left(f\left[1^2+1+2\right]\right)$
$= g\left(f(4)\right)$
$= g\left(2\cdot 4-5\right)$
$= g\left(3\right)$
$= 4\cdot 3-1=11$

64. $f\left(g\left[h(1)\right]\right) = f\left(g\left[1^2+1+2\right]\right)$
$= f\left(g(4)\right)$
$= f\left(4\cdot 4-1\right)$
$= f\left(15\right)$
$= 2\cdot 15-5=25$

65. **a.** {(Zambia, 4.2), (Colombia, 4.5), (Poland, 3.3), (Italy, 3.3), (United States, 2.5)}

b. {(4.2, Zambia), (4.5 , Colombia), (3.3 , Poland), (3.3, Italy), (2.5, United States)}
f is not a one-to-one function because the inverse of *f* is not a function.

66. **a.** {(Zambia,- 7.3), (Colombia, - 4.5), (Poland, - 2.8), (Italy, - 2.8), (United States, - 1.9)}

b. { (- 7.3, Zambia), (- 4.5, Colombia), (- 2.8, Po land), (- 2.8, Italy), (- 1.9, United States)}
g is not a one-to-one function because the inverse of *g* is not a function.

67. **a.** It passes the horizontal line test and is one-to-one.

b. $f^{-1}(0.25) = 15$ If there are 15 people in the room, the probability that 2 of them have the same birthday is 0.25.
$f^{-1}(0.5) = 21$ If there are 21 people in the room, the probability that 2 of them have the same birthday is 0.5.
$f^{-1}(0.7) = 30$ If there are 30 people in the room, the probability that 2 of them have the same birthday is 0.7.

68. **a.** This function fails the horizontal line test. Thus, this function does not have an inverse.

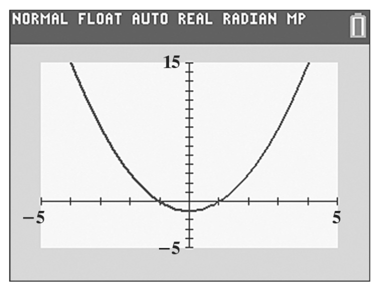
b. The average happiness level is 3 at 12 noon and at 7 p.m. These values can be represented as $(12,3)$ and $(19,3)$.

c. The graph does not represent a one-to-one function. $(12,3)$ and $(19,3)$ are an example of two *x*-values that correspond to the same *y*-value.

69. $f(g(x)) = \frac{9}{5}\left[\frac{5}{9}(x-32)\right]+32$
$= x-32+32$
$= x$
$g(f(x)) = \frac{5}{9}\left[\left(\frac{9}{5}x+32\right)-32\right]$
$= x+32-32$
$= x$
f and *g* are inverses.

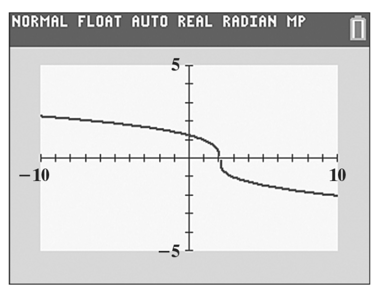
70. – 75. Answers will vary.

76.



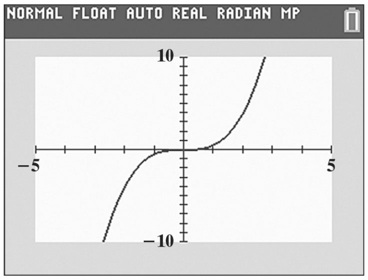
not one-to-one

77.



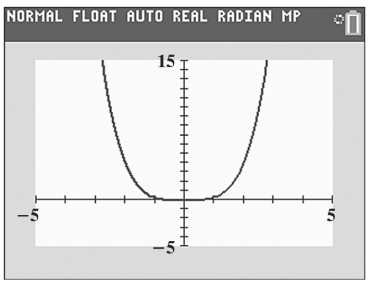
one-to-one

78.



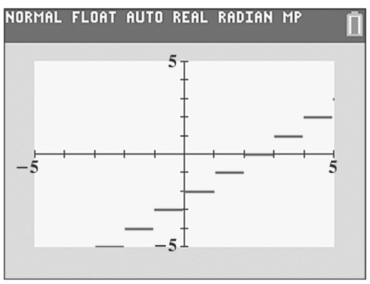
one-to-one

79.



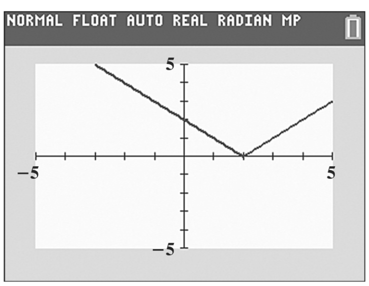
not one-to-one

80.



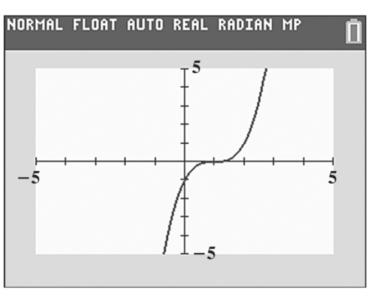
not one-to-one

81.



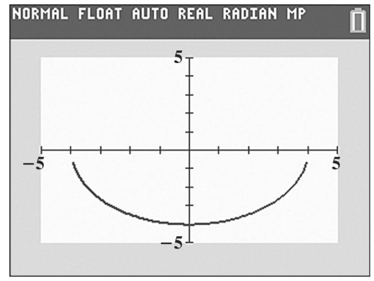
not one-to-one

82.



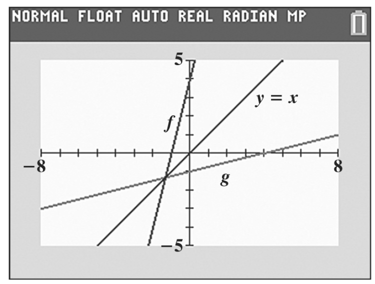
one-to-one

83.



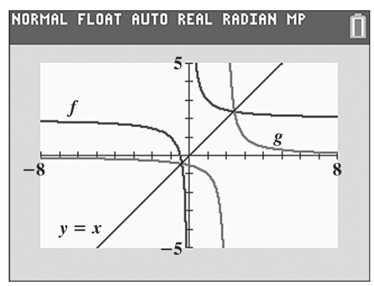
not one-to-one

84.



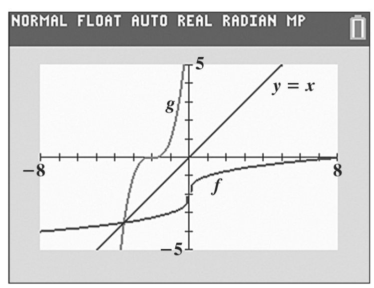
f and *g* are inverses

85.



f and *g* are inverses

86.



f and *g* are inverses

87. makes sense

88. makes sense

89. makes sense

90. does not make sense; Explanations will vary. Sample explanation: The vertical line test is used to determine if a relation is a function, but does not tell us if a function is one-to-one.

91. false; Changes to make the statement true will vary. A sample change is: The inverse is $\{(4,1), (7,2)\}$.

92. false; Changes to make the statement true will vary. A sample change is: $f(x) = 5$ is a horizontal line, so it does not pass the horizontal line test.

93. false; Changes to make the statement true will vary. A sample change is: $f^{-1}(x) = \frac{x}{3}$.

94. true

95. $(f \circ g)(x) = 3(x+5) = 3x+15.$

$$y = 3x+15$$
$$x = 3y+15$$
$$y = \frac{x-15}{3}$$
$$(f \circ g)^{-1}(x) = \frac{x-15}{3}$$
$$g(x) = x+5$$
$$y = x+5$$
$$x = y+5$$
$$y = x-5$$
$$g^{-1}(x) = x-5$$
$$f(x) = 3x$$
$$y = 3x$$
$$x = 3y$$
$$y = \frac{x}{3}$$
$$f^{-1}(x) = \frac{x}{3}$$
$$\left(g^{-1} \circ f^{-1}\right)(x) = \frac{x}{3} - 5 = \frac{x-15}{3}$$

96.
$$f(x) = \frac{3x-2}{5x-3}$$
$$y = \frac{3x-2}{5x-3}$$
$$x = \frac{3y-2}{5y-3}$$
$$x(5y-3) = 3y-2$$
$$5xy-3x = 3y-2$$
$$5xy-3y = 3x-2$$
$$y(5x-3) = 3x-2$$
$$y = \frac{3x-2}{5x-3}$$
$$f^{-1}(x) = \frac{3x-2}{5x-3}$$

Note: An alternative approach is to show that $(f \circ f)(x) = x$.

97. No, there will be 2 times when the spacecraft is at the same height, when it is going up and when it is coming down.

98.
$$8 + f^{-1}(x-1) = 10$$
$$f^{-1}(x-1) = 2$$
$$f(2) = x-1$$
$$6 = x-1$$
$$7 = x$$
$$x = 7$$

99. Answers will vary.

100. $2x^2 - 5x + 1 = 0$

$$x^2 - \frac{5}{2}x + \frac{1}{2} = 0$$
$$x^2 - \frac{5}{2}x \quad = -\frac{1}{2}$$
$$x^2 - \frac{5}{2}x + \frac{25}{16} = -\frac{1}{2} + \frac{25}{16}$$
$$\left(x - \frac{5}{4}\right)^2 = \frac{17}{16}$$
$$x - \frac{5}{4} = \pm\sqrt{\frac{17}{16}}$$
$$x - \frac{5}{4} = \pm\frac{\sqrt{17}}{4}$$
$$x = \frac{5}{4} \pm \frac{\sqrt{17}}{4}$$
$$x = \frac{5 \pm \sqrt{17}}{4}$$

The solution set is $\left\{\frac{5 \pm \sqrt{17}}{4}\right\}$.

101. $5x^{3/4} - 15 = 0$

$$5x^{3/2} = 15$$
$$x^{3/4} = 3$$
$$\left(x^{3/4}\right)^{4/3} = (3)^{4/3}$$
$$x = 3^{4/3}$$

The solution set is $\left\{3^{4/3}\right\}$.

102. $3|2x - 1| \geq 21$

$$|2x - 1| \geq 7$$

$2x - 1 \leq -7$ or $2x - 1 \geq 7$

$2x \leq -6 \qquad 2x \geq 8$

$\frac{2x}{2} \leq \frac{-6}{2} \qquad \frac{2x}{2} \geq \frac{8}{2}$

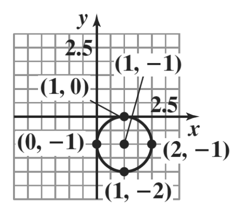
$x \leq -3 \qquad x \geq 4$

The solution set is $\{x \mid x \leq -3 \text{ or } x \geq 4\}$ or $(-\infty, -3] \cup [4, \infty)$.

103. $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(1-7)^2 + (-1-2)^2}$

$$= \sqrt{(-6)^2 + (-3)^2}$$
$$= \sqrt{36 + 9}$$
$$= \sqrt{45}$$
$$= 3\sqrt{5}$$

104.



105. $y^2 - 6y - 4 = 0$

$$y^2 - 6y \quad = 4$$
$$y^2 - 6y + 9 = 4 + 9$$
$$(y - 3)^2 = 13$$
$$y - 3 = \pm\sqrt{13}$$
$$y = 3 \pm \sqrt{13}$$

Solution set: $\left\{3 \pm \sqrt{13}\right\}$

Section 2.8

Check Point Exercises

1. $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$d = \sqrt{(2-(-1))^2 + (3-(-3))^2}$

$= \sqrt{3^2 + 6^2}$

$= \sqrt{9+36}$

$= \sqrt{45}$

$= 3\sqrt{5}$

≈ 6.71

2. $\left(\frac{1+7}{2}, \frac{2+(-3)}{2}\right) = \left(\frac{8}{2}, \frac{-1}{2}\right) = \left(4, -\frac{1}{2}\right)$

3. $h = 0,\ k = 0,\ r = 4;$

$(x-0)^2 + (y-0)^2 = 4^2$

$x^2 + y^2 = 16$

4. $h = 0,\ k = -6,\ r = 10;$

$(x-0)^2 + [y-(-6)]^2 = 10^2$

$(x-0)^2 + (y+6)^2 = 100$

$x^2 + (y+6)^2 = 100$

5. **a.** $(x+3)^2 + (y-1)^2 = 4$

$[x-(-3)]^2 + (y-1)^2 = 2^2$

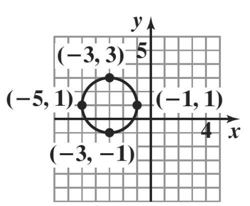
So in the standard form of the circle's equation $(x-h)^2 + (y-k)^2 = r^2$,

we have $h = -3,\ k = 1,\ r = 2.$

center: $(h, k) = (-3, 1)$

radius: $r = 2$

b.



$(x + 3)^2 + (y - 1)^2 = 4$

c. domain: $[-5, -1]$

range: $[-1, 3]$

6. $x^2 + y^2 + 4x - 4y - 1 = 0$

$x^2 + y^2 + 4x - 4y - 1 = 0$

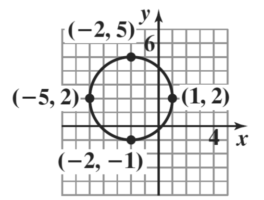
$(x^2 + 4x \quad) + (y^2 - 4y \quad) = 0$

$(x^2 + 4x + 4) + (y^2 + 4y + 4) = 1 + 4 + 4$

$(x+2)^2 + (y-2)^2 = 9$

$[x-(-x)]^2 + (y-2)^2 = 3^2$

So in the standard form of the circle's equation $(x-h)^2 + (y-k)^2 = r^2$, we have $h = -2,\ k = 2,\ r = 3$.



$x^2 + y^2 + 4x - 4y - 1 = 0$

Concept and Vocabulary Check 2.8

1. $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

2. $\frac{x_1 + x_2}{2}$; $\frac{y_1 + y_2}{2}$

3. circle; center; radius

4. $(x-h)^2 + (y-k)^2 = r^2$

5. general

6. 4; 16

Exercise Set 2.8

1. $d = \sqrt{(14-2)^2 + (8-3)^2}$

$= \sqrt{12^2 + 5^2}$

$= \sqrt{144 + 25}$

$= \sqrt{169}$

$= 13$

2. $d = \sqrt{(8-5)^2 + (5-1)^2}$
$= \sqrt{3^2 + 4^2}$
$= \sqrt{9+16}$
$= \sqrt{25}$
$= 5$

3. $d = \sqrt{(-6-4)^2 + (3-(-1))^2}$
$= \sqrt{(-10)^2 + (4)^2}$
$= \sqrt{100+16}$
$= \sqrt{116}$
$= 2\sqrt{29}$
≈ 10.77

4. $d = \sqrt{(-1-2)^2 + (5-(-3))^2}$
$= \sqrt{(-3)^2 + (8)^2}$
$= \sqrt{9+64}$
$= \sqrt{73}$
≈ 8.54

5. $d = \sqrt{(-3-0)^2 + (4-0)^2}$
$= \sqrt{3^2 + 4^2}$
$= \sqrt{9+16}$
$= \sqrt{25}$
$= 5$

6. $d = \sqrt{(3-0)^2 + (-4-0)^2}$
$= \sqrt{3^2 + (-4)^2}$
$= \sqrt{9+16}$
$= \sqrt{25}$
$= 5$

7. $d = \sqrt{[3-(-2)]^2 + [-4-(-6)]^2}$
$= \sqrt{5^2 + 2^2}$
$= \sqrt{25+4}$
$= \sqrt{29}$
≈ 5.39

8. $d = \sqrt{[2-(-4)]^2 + [-3-(-1)]^2}$
$= \sqrt{6^2 + (-2)^2}$
$= \sqrt{36+4}$
$= \sqrt{40}$
$= 2\sqrt{10}$
≈ 6.32

9. $d = \sqrt{(4-0)^2 + [1-(-3)]^2}$
$= \sqrt{4^2 + 4^2}$
$= \sqrt{16+16}$
$= \sqrt{32}$
$= 4\sqrt{2}$
≈ 5.66

10. $d = \sqrt{(4-0)^2 + [3-(-2)]^2}$
$= \sqrt{4^2 + [3+2]^2}$
$= \sqrt{16+5^2}$
$= \sqrt{16+25}$
$= \sqrt{41}$
≈ 6.40

11. $d = \sqrt{(-.5-3.5)^2 + (6.2-8.2)^2}$
$= \sqrt{(-4)^2 + (-2)^2}$
$= \sqrt{16+4}$
$= \sqrt{20}$
$= 2\sqrt{5}$
≈ 4.47

12. $d = \sqrt{(1.6-2.6)^2 + (-5.7-1.3)^2}$
$= \sqrt{(-1)^2 + (-7)^2}$
$= \sqrt{1+49}$
$= \sqrt{50}$
$= 5\sqrt{2}$
≈ 7.07

13. $$\begin{aligned} d &= \sqrt{(\sqrt{5}-0)^2+[0-(-\sqrt{3})]^2} \\ &= \sqrt{(\sqrt{5})^2+(\sqrt{3})^2} \\ &= \sqrt{5+3} \\ &= \sqrt{8} \\ &= 2\sqrt{2} \\ &\approx 2.83 \end{aligned}$$

14. $$\begin{aligned} d &= \sqrt{\left(\sqrt{7}-0\right)^2+\left[0-\left(-\sqrt{2}\right)\right]^2} \\ &= \sqrt{\left(\sqrt{7}\right)^2+\left[-\sqrt{2}\right]^2} \\ &= \sqrt{7+2} \\ &= \sqrt{9} \\ &= 3 \end{aligned}$$

15. $$\begin{aligned} d &= \sqrt{(-\sqrt{3}-3\sqrt{3})^2+(4\sqrt{5}-\sqrt{5})^2} \\ &= \sqrt{(-4\sqrt{3})^2+(3\sqrt{5})^2} \\ &= \sqrt{16(3)+9(5)} \\ &= \sqrt{48+45} \\ &= \sqrt{93} \\ &\approx 9.64 \end{aligned}$$

16. $$\begin{aligned} d &= \sqrt{\left(-\sqrt{3}-2\sqrt{3}\right)^2+\left(5\sqrt{6}-\sqrt{6}\right)^2} \\ &= \sqrt{\left(-3\sqrt{3}\right)^2+\left(4\sqrt{6}\right)^2} \\ &= \sqrt{9\cdot 3+16\cdot 6} \\ &= \sqrt{27+96} \\ &= \sqrt{123} \\ &\approx 11.09 \end{aligned}$$

17. $$\begin{aligned} d &= \sqrt{\left(\frac{1}{3}-\frac{7}{3}\right)^2+\left(\frac{6}{5}-\frac{1}{5}\right)^2} \\ &= \sqrt{(-2)^2+1^2} \\ &= \sqrt{4+1} \\ &= \sqrt{5} \\ &\approx 2.24 \end{aligned}$$

18. $$\begin{aligned} d &= \sqrt{\left[\frac{3}{4}-\left(-\frac{1}{4}\right)\right]^2+\left[\frac{6}{7}-\left(-\frac{1}{7}\right)\right]^2} \\ &= \sqrt{\left(\frac{3}{4}+\frac{1}{4}\right)^2+\left[\frac{6}{7}+\frac{1}{7}\right]^2} \\ &= \sqrt{1^2+1^2} \\ &= \sqrt{2} \\ &\approx 1.41 \end{aligned}$$

19. $\left(\frac{6+2}{2}, \frac{8+4}{2}\right) = \left(\frac{8}{2}, \frac{12}{2}\right) = (4, 6)$

20. $\left(\frac{10+2}{2}, \frac{4+6}{2}\right) = \left(\frac{12}{2}, \frac{10}{2}\right) = (6, 5)$

21. $$\begin{aligned} &\left(\frac{-2+(-6)}{2}, \frac{-8+(-2)}{2}\right) \\ &= \left(\frac{-8}{2}, \frac{-10}{2}\right) = (-4, -5) \end{aligned}$$

22. $$\begin{aligned} &\left(\frac{-4+(-1)}{2}, \frac{-7+(-3)}{2}\right) = \left(\frac{-5}{2}, \frac{-10}{2}\right) \\ &= \left(\frac{-5}{2}, -5\right) \end{aligned}$$

23. $$\begin{aligned} &\left(\frac{-3+6}{2}, \frac{-4+(-8)}{2}\right) \\ &= \left(\frac{3}{2}, \frac{-12}{2}\right) = \left(\frac{3}{2}, -6\right) \end{aligned}$$

24. $\left(\frac{-2+(-8)}{2} \frac{-1+6}{2}\right) = \left(\frac{-10}{2}, \frac{5}{2}\right) = \left(-5, \frac{5}{2}\right)$

25. $$\begin{aligned} &\left(\frac{\frac{-7}{2}+\left(-\frac{5}{2}\right)}{2}, \frac{\frac{3}{2}+\left(-\frac{11}{2}\right)}{2}\right) \\ &= \left(\frac{\frac{-12}{2}}{2}, \frac{\frac{-8}{2}}{2}\right) = \left(-\frac{6}{2}, \frac{-4}{2}\right) = (-3, -2) \end{aligned}$$

26. $\left(\dfrac{-\frac{2}{5}+\left(-\frac{2}{5}\right)}{2},\dfrac{\frac{7}{15}+\left(-\frac{4}{15}\right)}{2}\right)=\left(\dfrac{-\frac{4}{5}}{2},\dfrac{\frac{3}{15}}{2}\right)$

$=\left(-\frac{4}{5}\cdot\frac{1}{2},\frac{3}{15}\cdot\frac{1}{2}\right)=\left(-\frac{2}{5},\frac{1}{10}\right)$

27. $\left(\dfrac{8+(-6)}{2},\dfrac{3\sqrt{5}+7\sqrt{5}}{2}\right)$

$=\left(\dfrac{2}{2},\dfrac{10\sqrt{5}}{2}\right)=\left(1,5\sqrt{5}\right)$

28. $\left(\dfrac{7\sqrt{3}+3\sqrt{3}}{2},\dfrac{-6+(-2)}{2}\right)=\left(\dfrac{10\sqrt{3}}{2},\dfrac{-8}{2}\right)$

$=\left(5\sqrt{3},-4\right)$

29. $\left(\dfrac{\sqrt{18}+\sqrt{2}}{2},\dfrac{-4+4}{2}\right)$

$=\left(\dfrac{3\sqrt{2}+\sqrt{2}}{2},\dfrac{0}{2}\right)=\left(\dfrac{4\sqrt{2}}{2},0\right)=(2\sqrt{2},0)$

30. $\left(\dfrac{\sqrt{50}+\sqrt{2}}{2},\dfrac{-6+6}{2}\right)=\left(\dfrac{5\sqrt{2}+\sqrt{2}}{2},\dfrac{0}{2}\right)$

$=\left(\dfrac{6\sqrt{2}}{2},0\right)=\left(3\sqrt{2},0\right)$

31. $(x-0)^2+(y-0)^2=7^2$

$x^2+y^2=49$

32. $(x-0)^2+(y-0)^2=8^2$

$x^2+y^2=64$

33. $(x-3)^2+(y-2)^2=5^2$

$(x-3)^2+(y-2)^2=25$

34. $(x-2)^2+[y-(-1)]^2=4^2$

$(x-2)^2+(y+1)^2=16$

35. $[x-(-1)]^2+(y-4)^2=2^2$

$(x+1)^2+(y-4)^2=4$

36. $[x-(-3)]^2+(y-5)^2=3^2$

$(x+3)^2+(y-5)^2=9$

37. $[x-(-3)]^2+[y-(-1)]^2=\left(\sqrt{3}\right)^2$

$(x+3)^2+(y+1)^2=3$

38. $[x-(-5)]^2+[y-(-3)]^2=\left(\sqrt{5}\right)^2$

$(x+5)^2+(y+3)^2=5$

39. $[x-(-4)]^2+(y-0)^2=10^2$

$(x+4)^2+(y-0)^2=100$

40. $[x-(-2)]^2+(y-0)^2=6^2$

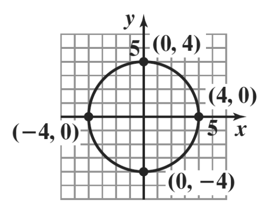
$(x+2)^2+y^2=36$

41. $x^2+y^2=16$

$(x-0)^2+(y-0)^2=y^2$

$h=0,\ k=0,\ r=4;$

center = (0, 0); radius = 4



$x^2+y^2=16$

domain: $[-4,4]$

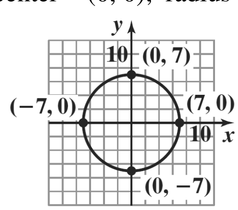
range: $[-4,4]$

42. $x^2+y^2=49$

$(x-0)^2+(y-0)^2=7^2$

$h=0, k=0, r=7;$

center = (0, 0); radius = 7



$x^2+y^2=49$

domain: $[-7,7]$

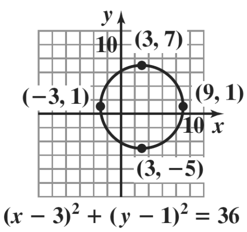
range: $[-7,7]$

43. $(x-3)^2+(y-1)^2=36$

$(x-3)^2+(y-1)^2=6^2$

$h=3,\ k=1,\ r=6;$

center = (3, 1); radius = 6



domain: $[-3,9]$

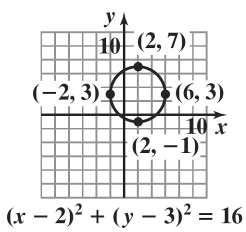
range: $[-5,7]$

44. $(x-2)^2+(y-3)^2=16$

$(x-2)^2+(y-3)^2=4^2$

$h=2, k=3, r=4;$

center = (2, 3); radius = 4



domain: $[-2,6]$

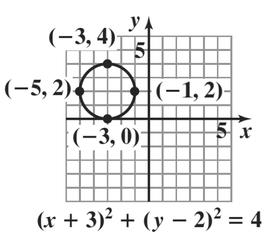
range: $[-1,7]$

45. $(x+3)^2+(y-2)^2=4$

$[x-(-3)]^2+(y-2)^2=2^2$

$h=-3,\ k=2,\ r=2$

center = (–3, 2); radius = 2



domain: $[-5,-1]$

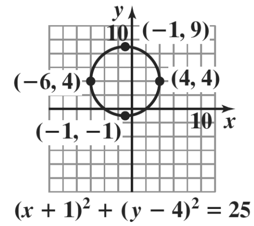
range: $[0,4]$

46. $(x+1)^2+(y-4)^2=25$

$[x-(-1)]^2+(y-4)^2=5^2$

$h=-1, k=4, r=5;$

center = (–1, 4); radius = 5



domain: $[-6,4]$

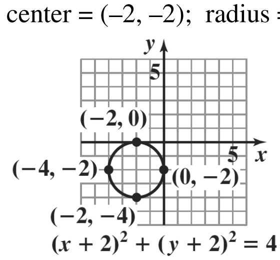
range: $[-1,9]$

47. $(x+2)^2+(y+2)^2=4$

$[x-(-2)]^2+[y-(-2)]^2=2^2$

$h=-2,\ k=-2,\ r=2$

center = (–2, –2); radius = 2



domain: $[-4,0]$

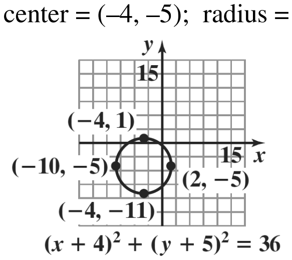
range: $[-4,0]$

48. $(x+4)^2+(y+5)^2=36$

$[x-(-4)]^2+[y-(-5)]^2=6^2$

$h=-4, k=-5, r=6;$

center = (–4, –5); radius = 6



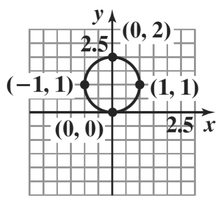
domain: $[-10,2]$

range: $[-11,1]$

49. $x^2+(y-1)^2=1$

$h=0, k=1, r=1;$

center = (0, 1); radius = 1



$x^2+(y-1)^2=1$

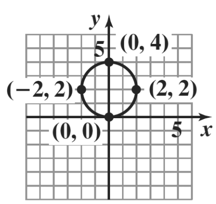
domain: $[-1,1]$

range: $[0,2]$

50. $x^2+(y-2)^2=4$

$h=0, k=2, r=2;$

center = (0,2); radius = 2



$x^2+(y-2)^2=4$

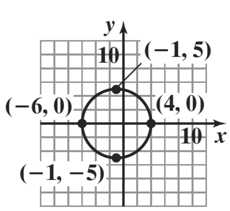
domain: $[-2,2]$

range: $[0,4]$

51. $(x+1)^2+y^2=25$

$h=-1, k=0, r=5;$

center = (–1,0); radius = 5



$(x+1)^2+y^2=25$

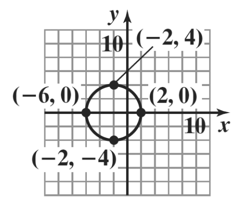
domain: $[-6,4]$

range: $[-5,5]$

52. $(x+2)^2+y^2=16$

$h=-2, k=0, r=4;$

center = (–2,0); radius = 4



$(x+2)^2+y^2=16$

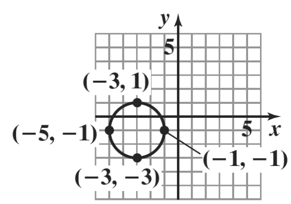
domain: $[-6,2]$

range: $[-4,4]$

53.

$$x^2+y^2+6x+2y+6=0$$
$$(x^2+6x)+(y^2+2y)=-6$$
$$(x^2+6x+9)+(y^2+2y+1)=9+1-6$$
$$(x+3)^2+(y+1)^2=4$$
$$[x-(-3)]^2+[9-(-1)]^2=2^2$$

center = (–3, –1); radius = 2

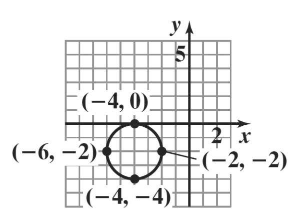


$x^2+y^2+6x+2y+6=0$

54. $x^2+y^2+8x+4y+16=0$

$$(x^2+8x)+(y^2+4y)=-16$$
$$(x^2+8x+16)+(y^2+4y+4)=20-16$$
$$(x+4)^2+(y+2)^2=4$$
$$[x-(-4)]^2+[y-(-2)]^2=2^2$$

center = (–4, –2); radius = 2



$x^2+y^2+8x+4y+16=0$

55. $x^2 + y^2 - 10x - 6y - 30 = 0$

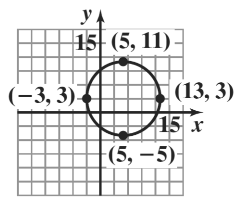
$(x^2 - 10x) + (y^2 - 6y) = 30$

$(x^2 - 10x + 25) + (y^2 - 6y + 9) = 25 + 9 + 30$

$(x-5)^2 + (y-3)^2 = 64$

$(x-5)^2 + (y-3)^2 = 8^2$

center = (5, 3); radius = 8



$x^2 + y^2 - 10x - 6y - 30 = 0$

56. $x^2 + y^2 - 4x - 12y - 9 = 0$

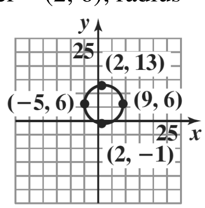
$(x^2 - 4x) + (y^2 - 12y) = 9$

$(x^2 - 4x + 4) + (y^2 - 12y + 36) = 4 + 36 + 9$

$(x-2)^2 + (y-6)^2 = 49$

$(x-2)^2 + (y-6)^2 = 7^2$

center = (2, 6); radius = 7



$x^2 + y^2 - 4x - 12y - 9 = 0$

57. $x^2 + y^2 + 8x - 2y - 8 = 0$

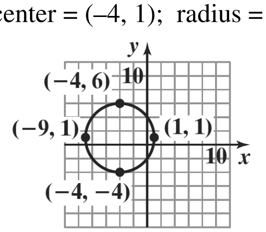
$(x^2 + 8x) + (y^2 - 2y) = 8$

$(x^2 + 8x + 16) + (y^2 - 2y + 1) = 16 + 1 + 8$

$(x+4)^2 + (y-1)^2 = 25$

$[x-(-4)]^2 + (y-1)^2 = 5^2$

center = (–4, 1); radius = 5



$x^2 + y^2 + 8x - 2y - 8 = 0$

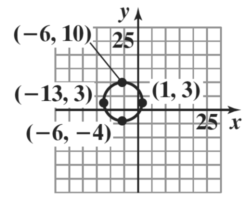
58. $x^2 + y^2 + 12x - 6y - 4 = 0$

$(x^2 + 12x) + (y^2 - 6y) = 4$

$(x^2 + 12x + 36) + (y^2 - 6y + 9) = 36 + 9 + 4$

$[x-(-6)]^2 + (y-3)^2 = 7^2$

center = (–6, 3); radius = 7



$x^2 + y^2 + 12x - 6y - 4 = 0$

59. $x^2 - 2x + y^2 - 15 = 0$

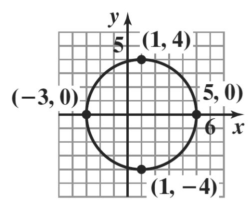
$(x^2 - 2x) + y^2 = 15$

$(x^2 - 2x + 1) + (y-0)^2 = 1 + 0 + 15$

$(x-1)^2 + (y-0)^2 = 16$

$(x-1)^2 + (y-0)^2 = 4^2$

center = (1, 0); radius = 4



$x^2 - 2x + y^2 - 15 = 0$

60. $x^2 + y^2 - 6y - 7 = 0$

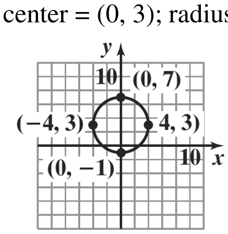
$x^2 + (y^2 - 6y) = 7$

$(x-0)^2 = (y^2 - 6y + 9) = 0 + 9 + 7$

$(x-0)^2 + (y-3)^2 = 16$

$(x-0)^2 + (y-3)^2 = 4^2$

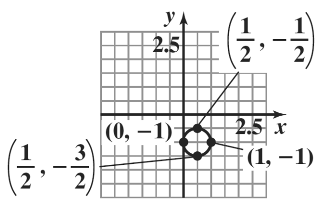
center = (0, 3); radius = 4



$x^2 + y^2 - 6y - 7 = 0$

61.
$$\begin{aligned} x^2+y^2-x+2y+1&=0\\ x^2-x\quad+y^2+2y\quad&=-1\\ x^2-x+\frac{1}{4}+y^2+2y+1&=-1+\frac{1}{4}+1\\ \left(x-\frac{1}{2}\right)^2+(y+1)^2&=\frac{1}{4}\end{aligned}$$

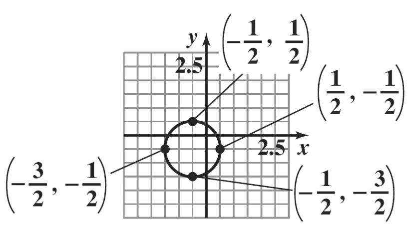
center = $\left(\frac{1}{2},-1\right)$; radius = $\frac{1}{2}$



$x^2 + y^2 - x + 2y + 1 = 0$

62.
$$\begin{aligned} x^2+y^2+x+y-\frac{1}{2}&=0\\ x^2+x\quad+y^2+y\quad&=\frac{1}{2}\\ x^2+x+\frac{1}{4}+y^2+y+\frac{1}{4}&=\frac{1}{2}+\frac{1}{4}+\frac{1}{4}\\ \left(x-\frac{1}{2}\right)^2+\left(y-\frac{1}{2}\right)^2&=1\end{aligned}$$

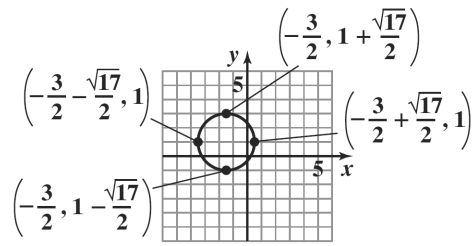
center = $\left(\frac{1}{2},\frac{1}{2}\right)$; radius = 1



$x^2 + y^2 + x + y - \frac{1}{2} = 0$

63.
$$\begin{aligned} x^2+y^2+3x-2y-1&=0\\ x^2+3x\quad+y^2-2y\quad&=1\\ x^2+3x+\frac{9}{4}+y^2-2y+1&=1+\frac{9}{4}+1\\ \left(x+\frac{3}{2}\right)^2+(y-1)^2&=\frac{17}{4}\end{aligned}$$

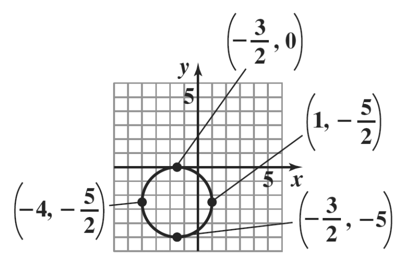
center = $\left(-\frac{3}{2},1\right)$; radius = $\frac{\sqrt{17}}{2}$



$x^2 + y^2 + 3x - 2y - 1 = 0$

64.
$$\begin{aligned} x^2+y^2+3x+5y+\frac{9}{4}&=0\\ x^2+3x\quad+y^2+5y\quad&=-\frac{9}{4}\\ x^2+3x+\frac{9}{4}+y^2+5y+\frac{25}{4}&=-\frac{9}{4}+\frac{9}{4}+\frac{25}{4}\\ \left(x+\frac{3}{2}\right)^2+\left(y+\frac{5}{2}\right)^2&=\frac{25}{4}\end{aligned}$$

center = $\left(-\frac{3}{2},-\frac{5}{2}\right)$; radius = $\frac{5}{2}$



$x^2 + y^2 + 3x + 5y + \frac{9}{4} = 0$

65. a. Since the line segment passes through the center, the center is the midpoint of the segment.

$$\begin{aligned} M&=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)\\ &=\left(\frac{3+7}{2},\frac{9+11}{2}\right)=\left(\frac{10}{2},\frac{20}{2}\right)\\ &=(5,10)\end{aligned}$$

The center is $(5,10)$.

b. The radius is the distance from the center to one of the points on the circle. Using the point $(3,9)$, we get:

$$d=\sqrt{(5-3)^2+(10-9)^2}$$
$$=\sqrt{2^2+1^2}=\sqrt{4+1}$$
$$=\sqrt{5}$$

The radius is $\sqrt{5}$ units.

c. $(x-5)^2+(y-10)^2=\left(\sqrt{5}\right)^2$

$(x-5)^2+(y-10)^2=5$

66. a. Since the line segment passes through the center, the center is the midpoint of the segment.

$$M=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$$
$$=\left(\frac{3+5}{2},\frac{6+4}{2}\right)=\left(\frac{8}{2},\frac{10}{2}\right)$$
$$=(4,5)$$

The center is $(4,5)$.

b. The radius is the distance from the center to one of the points on the circle. Using the point $(3,6)$, we get:

$$d=\sqrt{(4-3)^2+(5-6)^2}$$
$$=\sqrt{1^2+(-1)^2}=\sqrt{1+1}$$
$$=\sqrt{2}$$

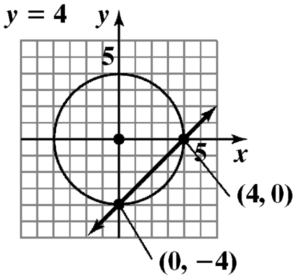
The radius is $\sqrt{2}$ units.

c. $(x-4)^2+(y-5)^2=\left(\sqrt{2}\right)^2$

$(x-4)^2+(y-5)^2=2$

67. $\boldsymbol{x^2+y^2=16}$

$\boldsymbol{x-y=4}$



Intersection points: $(0,-4)$ and $(4,0)$

Check $(0,-4)$:

$0^2+(-4)^2=16 \qquad 0-(-4)=4$

$16=16$ true $\qquad 4=4$ true

Check $(4,0)$:

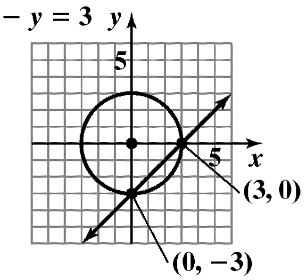
$4^2+0^2=16 \qquad 4-0=4$

$16=16$ true $\qquad 4=4$ true

The solution set is $\{(0,-4),(4,0)\}$.

68. $\boldsymbol{x^2+y^2=9}$

$\boldsymbol{x-y=3}$



Intersection points: $(0,-3)$ and $(3,0)$

Check $(0,-3)$:

$0^2+(-3)^2=9 \qquad 0-(-3)=3$

$9=9$ true $\qquad 3=3$ true

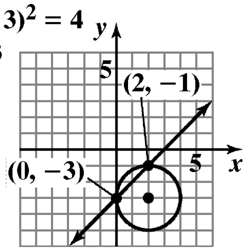
Check $(3,0)$:

$3^2+0^2=9 \qquad 3-0=3$

$9=9$ true $\qquad 3=3$ true

The solution set is $\{(0,-3),(3,0)\}$.

69. $(x-2)^2+(y+3)^2=4$
$y=x-3$



Intersection points: $(0,-3)$ and $(2,-1)$

Check $(0,-3)$:

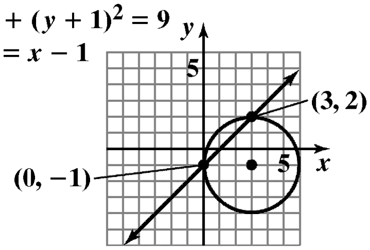
$(0-2)^2+(-3+3)^2=9 \qquad -3=0-3$
$(-2)^2+0^2=4 \qquad -3=-3$ true
$4=4$
true

Check $(2,-1)$:

$(2-2)^2+(-1+3)^2=4 \qquad -1=2-3$
$0^2+2^2=4 \qquad -1=-1$ true
$4=4$
true

The solution set is $\{(0,-3),(2,-1)\}$.

70. $(x-3)^2+(y+1)^2=9$
$y=x-1$



Intersection points: $(0,-1)$ and $(3,2)$

Check $(0,-1)$:

$(0-3)^2+(-1+1)^2=9 \qquad -1=0-1$
$(-3)^2+0^2=9 \qquad -1=-1$ true
$9=9$
true

Check $(3,2)$:

$(3-3)^2+(2+1)^2=9 \qquad 2=3-1$
$0^2+3^2=9 \qquad 2=2$ true
$9=9$
true

The solution set is $\{(0,-1),(3,2)\}$.

71. $d=\sqrt{(8495-4422)^2+(8720-1241)^2}\cdot\sqrt{0.1}$
$d=\sqrt{72{,}524{,}770}\cdot\sqrt{0.1}$
$d\approx 2693$
The distance between Boston and San Francisco is about 2693 miles.

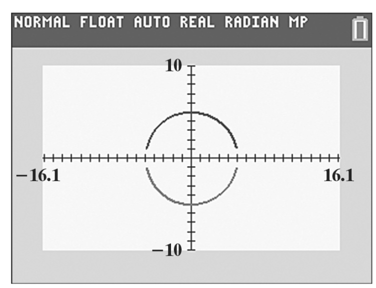
72. $d=\sqrt{(8936-8448)^2+(3542-2625)^2}\cdot\sqrt{0.1}$
$d=\sqrt{1{,}079{,}033}\cdot\sqrt{0.1}$
$d\approx 328$
The distance between New Orleans and Houston is about 328 miles.

73. If we place L.A. at the origin, then we want the equation of a circle with center at $(-2.4,-2.7)$ and radius 30.
$(x-(-2.4))^2+(y-(-2.7))^2=30^2$
$(x+2.4)^2+(y+2.7)^2=900$

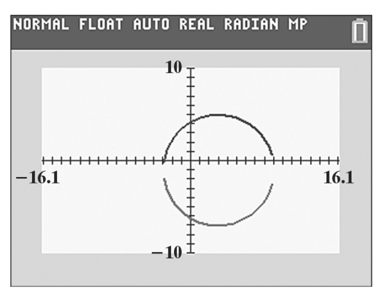
74. $C(0, 68+14)=(0, 82)$
$(x-0)^2+(y-82)^2=68^2$
$x^2+(y-82)^2=4624$

75. – 82. Answers will vary.

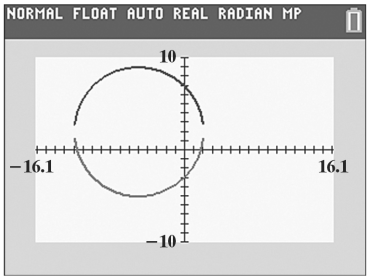
83.



84.



85.



86. makes sense

87. makes sense

88. does not make sense; Explanations will vary. Sample explanation: Since $r^2 = -4$ this is not the equation of a circle.

89. makes sense

90. false; Changes to make the statement true will vary. A sample change is: The equation would be $x^2 + y^2 = 256$.

91. false; Changes to make the statement true will vary. A sample change is: The center is at (3, –5).

92. false; Changes to make the statement true will vary. A sample change is: This is not an equation for a circle.

93. false; Changes to make the statement true will vary. A sample change is: Since $r^2 = -36$ this is not the equation of a circle.

94. The distance for A to B:

$$\overline{AB} = \sqrt{(3-1)^2 + [3+d-(1+d)]^2}$$
$$= \sqrt{2^2 + 2^2}$$
$$= \sqrt{4+4}$$
$$= \sqrt{8}$$
$$= 2\sqrt{2}$$

The distance from B to C:

$$\overline{BC} = \sqrt{(6-3)^2 + [3+d-(6+d)]^2}$$
$$= \sqrt{3^2 + (-3)^2}$$
$$= \sqrt{9+9}$$
$$= \sqrt{18}$$
$$= 3\sqrt{2}$$

The distance for A to C:

$$\overline{AC} = \sqrt{(6-1)^2 + [6+d-(1+d)]^2}$$
$$= \sqrt{5^2 + 5^2}$$
$$= \sqrt{25+25}$$
$$= \sqrt{50}$$
$$= 5\sqrt{2}$$

$$\overline{AB} + \overline{BC} = \overline{AC}$$
$$2\sqrt{2} + 3\sqrt{2} = 5\sqrt{2}$$
$$5\sqrt{2} = 5\sqrt{2}$$

95. **a.** d_1 is distance from (x_1, x_2) to midpoint

$$d_1 = \sqrt{\left(\frac{x_1+x_2}{2} - x_1\right)^2 + \left(\frac{y_1+y_2}{2} - y_1\right)^2}$$
$$d_1 = \sqrt{\left(\frac{x_1+x_2-2x_1}{2}\right)^2 + \left(\frac{y_1+y_2-2y_1}{2}\right)^2}$$
$$d_1 = \sqrt{\left(\frac{x_2-x_1}{2}\right)^2 + \left(\frac{y_2-y_1}{2}\right)^2}$$
$$d_1 = \sqrt{\frac{x_2-2x_1x_2+x_1^2}{4} + \frac{y_2^2-2y_2y_1+y_1^2}{4}}$$
$$d_1 = \sqrt{\frac{1}{4}\left(x_2-2x_1x_2+x_1+y_2^2-2y_2y_1+y_1^2\right)}$$
$$d_1 = \frac{1}{2}\sqrt{x_2-2x_1x_2+x_1+y_2^2-2y_2y_1+y_1^2}$$

d_2 is distance from midpoint to (x_2, y_2)

$$d_2 = \sqrt{\left(\frac{x_1+x_2}{2} - x_2\right)^2 + \left(\frac{y_1+y_2}{2} - y_2\right)^2}$$
$$d_2 = \sqrt{\left(\frac{x_1+x_2-2x_2}{2}\right)^2 + \left(\frac{y_1+y_2-2y_2}{2}\right)^2}$$
$$d_2 = \sqrt{\left(\frac{x_1-x_2}{2}\right)^2 + \left(\frac{y_1-y_2}{2}\right)^2}$$
$$d_2 = \sqrt{\frac{x_1^2-2x_1x_2+x_2^2}{4} + \frac{y_1^2-2y_2y_1+y_2^2}{4}}$$
$$d_2 = \sqrt{\frac{1}{4}\left(x_1^2-2x_1x_2+x_2^2+y_1^2-2y_2y_1+y_2^2\right)}$$
$$d_2 = \frac{1}{2}\sqrt{x_1^2-2x_1x_2+x_2^2+y_1^2-2y_2y_1+y_2^2}$$
$$d_1 = d_2$$

b. d_3 is the distance from (x_1, y_1) to $(x_2 y_2)$

$d_3 = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$d_3 = \sqrt{x_2^2 - 2x_1x_2 + x_1^2 + y_2^2 - 2y_2y_1 + y_1^2}$

$d_1 + d_2 = d_3$ because $\frac{1}{2}\sqrt{a} + \frac{1}{2}\sqrt{a} = \sqrt{a}$

96. Both circles have center (2, –3). The smaller circle has radius 5 and the larger circle has radius 6. The smaller circle is inside of the larger circle. The area between them is given by

$$\begin{aligned}\pi(6)^2 - \pi(5)^2 &= 36\pi - 25\pi \\ &= 11\pi \\ &\approx 34.56 \text{ square units.}\end{aligned}$$

97. The circle is centered at (0,0). The slope of the radius with endpoints (0,0) and (3,–4) is

$m = -\frac{-4-0}{3-0} = -\frac{4}{3}$. The line perpendicular to the radius has slope $\frac{3}{4}$. The tangent line has slope $\frac{3}{4}$ and passes through (3,–4), so its equation is:

$y + 4 = \frac{3}{4}(x - 3)$.

98.

$$\begin{aligned}7(x-2)+5 &= 7x-9 \\ 7x-14+5 &= 7x-9 \\ 7x-9 &= 7x-9 \\ -9 &= -9\end{aligned}$$

The original equation is equivalent to the statement $-9 = -9$, which is true for every value of x.

The equation is an identity, and all real numbers are solutions. The solution set $\{x \mid x \text{ is a real number}\}$.

99.

$$\begin{aligned}\frac{4i+7}{5-2i} &= \frac{4i+7}{5-2i}\cdot\frac{5+2i}{5+2i} \\ &= \frac{20i+8i^2+35+14i}{25+10i-10i-4i^2} \\ &= \frac{34i-8+35}{25+4} \\ &= \frac{34i+27}{29} \\ &= \frac{27}{29}+\frac{34}{29}i\end{aligned}$$

100.

$$\begin{aligned}-9 \le 4x-1 &< 15 \\ -8 \le 4x &< 16 \\ -2 \le x &< 4\end{aligned}$$

The solution set is $\{x \mid -2 \le x < 4\}$ or $[-2, 4)$.

101.

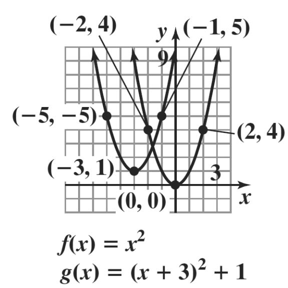
$$\begin{aligned}0 &= -2(x-3)^2 + 8 \\ 2(x-3)^2 &= 8 \\ (x-3)^2 &= 4 \\ x - 3 &= \pm\sqrt{4} \\ x &= 3 \pm 2 \\ x &= 1, 5\end{aligned}$$

102.

$$\begin{aligned}-x^2 - 2x + 1 &= 0 \\ x^2 + 2x - 1 &= 0 \\ x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ x &= \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-1)}}{2(1)} \\ &= \frac{2 \pm \sqrt{8}}{2} \\ &= \frac{2 \pm 2\sqrt{2}}{2} \\ &= 1 \pm \sqrt{2}\end{aligned}$$

The solution set is $\{1 \pm \sqrt{2}\}$.

103. The graph of g is the graph of f shifted 1 unit up and 3 units to the left.



Chapter 2 Review Exercises

1. function
domain: {2, 3, 5}
range: {7}

2. function
domain: {1, 2, 13}
range: {10, 500, π}

3. not a function
domain: {12, 14}
range: {13, 15, 19}

4. $2x + y = 8$
$y = -2x + 8$
Since only one value of y can be obtained for each value of x, y is a function of x.

5. $3x^2 + y = 14$
$y = -3x^2 + 14$
Since only one value of y can be obtained for each value of x, y is a function of x.

6. $2x + y^2 = 6$
$y^2 = -2x + 6$
$y = \pm\sqrt{-2x+6}$
Since more than one value of y can be obtained from some values of x, y is not a function of x.

7. $f(x) = 5 - 7x$

a. $f(4) = 5 - 7(4) = -23$

b.
$$\begin{aligned} f(x+3) &= 5 - 7(x+3) \\ &= 5 - 7x - 21 \\ &= -7x - 16 \end{aligned}$$

c. $f(-x) = 5 - 7(-x) = 5 + 7x$

8. $g(x) = 3x^2 - 5x + 2$

a. $g(0) = 3(0)^2 - 5(0) + 2 = 2$

b.
$$\begin{aligned} g(-2) &= 3(-2)^2 - 5(-2) + 2 \\ &= 12 + 10 + 2 \\ &= 24 \end{aligned}$$

c.
$$\begin{aligned} g(x-1) &= 3(x-1)^2 - 5(x-1) + 2 \\ &= 3(x^2 - 2x + 1) - 5x + 5 + 2 \\ &= 3x^2 - 11x + 10 \end{aligned}$$

d.
$$\begin{aligned} g(-x) &= 3(-x)^2 - 5(-x) + 2 \\ &= 3x^2 + 5x + 2 \end{aligned}$$

9. a. $g(13) = \sqrt{13-4} = \sqrt{9} = 3$

b. $g(0) = 4 - 0 = 4$

c. $g(-3) = 4 - (-3) = 7$

10. a. $f(-2) = \dfrac{(-2)^2 - 1}{-2-1} = \dfrac{3}{-3} = -1$

b. $f(1) = 12$

c. $f(2) = \dfrac{2^2 - 1}{2-1} = \dfrac{3}{1} = 3$

11. The vertical line test shows that this is not the graph of a function.

12. The vertical line test shows that this is the graph of a function.

13. The vertical line test shows that this is the graph of a function.

14. The vertical line test shows that this is not the graph of a function.

15. The vertical line test shows that this is not the graph of a function.

16. The vertical line test shows that this is the graph of a function.

17. a. domain: $[-3, 5)$

b. range: $[-5, 0]$

c. x-intercept: -3

d. y-intercept: -2

e. increasing: $(-2, 0)$ or $(3, 5)$
decreasing: $(-3, -2)$ or $(0, 3)$

f. $f(-2) = -3$ and $f(3) = -5$

18. **a.** domain: $(-\infty, \infty)$

b. range: $(-\infty, 3]$

c. x-intercepts: –2 and 3

d. y-intercept: 3

e. increasing: $(-\infty, 0)$
decreasing: $(0, \infty)$

f. $f(-2) = 0$ and $f(6) = -3$

19. **a.** domain: $(-\infty, \infty)$

b. range: [–2, 2]

c. x-intercept: 0

d. y-intercept: 0

e. increasing: (–2, 2)
constant: $(-\infty, -2)$ or $(2, \infty)$

f. $f(-9) = -2$ and $f(14) = 2$

20. **a.** 0, relative maximum −2

b. −2, 3, relative minimum −3, –5

21. **a.** 0, relative maximum 3

b. none

22. Test for symmetry with respect to the y-axis.

$$y = x^2 + 8$$
$$y = (-x)^2 + 8$$
$$y = x^2 + 8$$

The resulting equation is equivalent to the original. Thus, the graph is symmetric with respect to the y-axis.

Test for symmetry with respect to the x-axis.

$$y = x^2 + 8$$
$$-y = x^2 + 8$$
$$y = -x^2 - 8$$

The resulting equation is not equivalent to the original. Thus, the graph is not symmetric with respect to the x-axis.

Test for symmetry with respect to the origin.

$$y = x^2 + 8$$
$$-y = (-x)^2 + 8$$
$$-y = x^2 + 8$$
$$y = -x^2 - 2$$

The resulting equation is not equivalent to the original. Thus, the graph is not symmetric with respect to the origin.

23. Test for symmetry with respect to the y-axis.

$$x^2 + y^2 = 17$$
$$(-x)^2 + y^2 = 17$$
$$x^2 + y^2 = 17$$

The resulting equation is equivalent to the original. Thus, the graph is symmetric with respect to the y-axis.

Test for symmetry with respect to the x-axis.

$$x^2 + y^2 = 17$$
$$x^2 + (-y)^2 = 17$$
$$x^2 + y^2 = 17$$

The resulting equation is equivalent to the original. Thus, the graph is symmetric with respect to the x-axis.

Test for symmetry with respect to the origin.

$$x^2 + y^2 = 17$$
$$(-x)^2 + (-y)^2 = 17$$
$$x^2 + y^2 = 17$$

The resulting equation is equivalent to the original. Thus, the graph is symmetric with respect to the origin.

24. Test for symmetry with respect to the y-axis.

$$x^3 - y^2 = 5$$
$$(-x)^3 - y^2 = 5$$
$$-x^3 - y^2 = 5$$

The resulting equation is not equivalent to the original. Thus, the graph is not symmetric with respect to the y-axis.

Test for symmetry with respect to the x-axis.

$$x^3 - y^2 = 5$$
$$x^3 - (-y)^2 = 5$$
$$x^3 - y^2 = 5$$

The resulting equation is equivalent to the original. Thus, the graph is symmetric with respect to the x-axis.

Test for symmetry with respect to the origin.

$$x^3 - y^2 = 5$$
$$(-x)^3 - (-y)^2 = 5$$
$$-x^3 - y^2 = 5$$

The resulting equation is not equivalent to the original. Thus, the graph is not symmetric with respect to the origin.

25. The graph is symmetric with respect to the origin. The function is odd.

26. The graph is not symmetric with respect to the y-axis or the origin. The function is neither even nor odd.

27. The graph is symmetric with respect to the y-axis. The function is even.

28.
$$f(x) = x^3 - 5x$$
$$f(-x) = (-x)^3 - 5(-x)$$
$$= -x^3 + 5x$$
$$= -f(x)$$

The function is odd. The function is symmetric with respect to the origin.

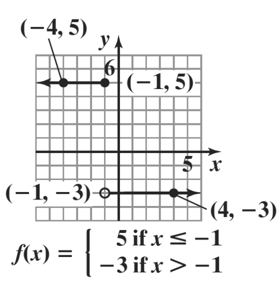
29.
$$f(x) = x^4 - 2x^2 + 1$$
$$f(-x) = (-x)^4 - 2(-x)^2 + 1$$
$$= x^4 - 2x^2 + 1$$
$$= f(x)$$

The function is even. The function is symmetric with respect to the y-axis.

30.
$$f(x) = 2x\sqrt{1-x^2}$$
$$f(-x) = 2(-x)\sqrt{1-(-x)^2}$$
$$= -2x\sqrt{1-x^2}$$
$$= -f(x)$$

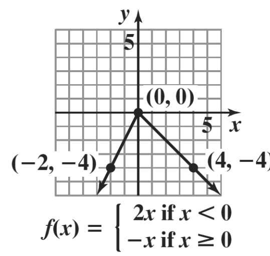
The function is odd. The function is symmetric with respect to the origin.

31. **a.**



b. range: {–3, 5}

32. **a.**



b. range: $\{y \mid y \le 0\}$

33.
$$\frac{8(x+h)-11-(8x-11)}{h}$$
$$= \frac{8x+8h-11-8x+11}{h}$$
$$= \frac{8h}{8}$$
$$= 8$$

34.
$$\frac{-2(x+h)^2+(x+h)+10-\left(-2x^2+x+10\right)}{h}$$
$$= \frac{-2\left(x^2+2xh+h^2\right)+x+h+10+2x^2-x-10}{h}$$
$$= \frac{-2x^2-4xh-2h^2+x+h+10+2x^2-x-10}{h}$$
$$= \frac{-4xh-2h^2+h}{h}$$
$$= \frac{h(-4x-2h+1)}{h}$$
$$-4x-2h+1$$

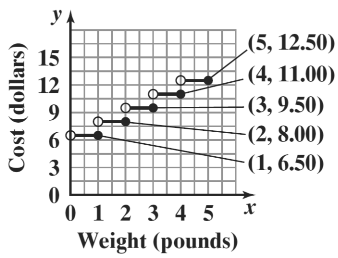
35. **a.** Yes, the eagle's height is a function of time since the graph passes the vertical line test.

b. Decreasing: (3, 12)
The eagle descended.

c. Constant: (0, 3) or (12, 17)
The eagle's height held steady during the first 3 seconds and the eagle was on the ground for 5 seconds.

d. Increasing: (17, 30)
The eagle was ascending.

36.



37. $m = \frac{1-2}{5-3} = \frac{-1}{2} = -\frac{1}{2}$; falls

38. $m = \frac{-4-(-2)}{-3-(-1)} = \frac{-2}{-2} = 1$; rises

39. $m = \frac{\frac{1}{4}-\frac{1}{4}}{6-(-3)} = \frac{0}{9} = 0$; horizontal

40. $m = \frac{10-5}{-2-(-2)} = \frac{5}{0}$ undefined; vertical

41. point-slope form: $y - 2 = -6(x + 3)$
slope-intercept form: $y = -6x - 16$

42. $m = \frac{2-6}{-1-1} = \frac{-4}{-2} = 2$
point-slope form: $y - 6 = 2(x - 1)$
or $y - 2 = 2(x + 1)$
slope-intercept form: $y = 2x + 4$

43. $3x + y - 9 = 0$
$y = -3x + 9$
$m = -3$
point-slope form:
$y + 7 = -3(x - 4)$
slope-intercept form:
$y = -3x + 12 - 7$
$y = -3x + 5$

44. perpendicular to $y = \frac{1}{3}x + 4$
$m = -3$
point-slope form:
$y - 6 = -3(x + 3)$
slope-intercept form:
$y = -3x - 9 + 6$
$y = -3x - 3$

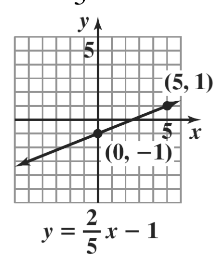
45. Write $6x - y - 4 = 0$ in slope intercept form.

$$6x - y - 4 = 0$$
$$-y = -6x + 4$$
$$y = 6x - 4$$

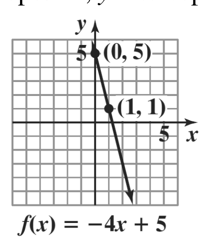
The slope of the perpendicular line is 6, thus the slope of the desired line is $m = -\frac{1}{6}$.

$$y - y_1 = m(x - x_1)$$
$$y - (-1) = -\tfrac{1}{6}\left(x - (-12)\right)$$
$$y + 1 = -\tfrac{1}{6}(x + 12)$$
$$y + 1 = -\tfrac{1}{6}x - 2$$
$$6y + 6 = -x - 12$$
$$x + 6y + 18 = 0$$

46. slope: $\frac{2}{5}$; y-intercept: -1



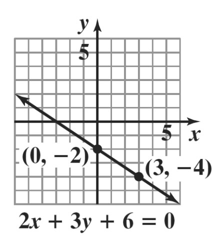
47. slope: -4; y-intercept: 5



48.

$$2x + 3y + 6 = 0$$
$$3y = -2x - 6$$
$$y = -\frac{2}{3}x - 2$$

slope: $-\frac{2}{3}$; y-intercept: -2

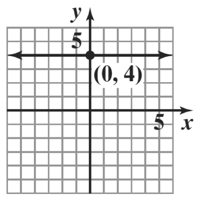


49. $2y - 8 = 0$

$2y = 8$

$y = 4$

slope: 0; y-intercept: 4



$2y - 8 = 0$

50. $2x - 5y - 10 = 0$

Find x-intercept:

$2x - 5(0) - 10 = 0$

$2x - 10 = 0$

$2x = 10$

$x = 5$

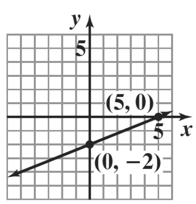
Find y-intercept:

$2(0) - 5y - 10 = 0$

$-5y - 10 = 0$

$-5y = 10$

$y = -2$



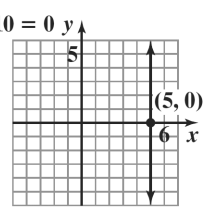
$2x - 5y - 10 = 0$

51. $2x - 10 = 0$

$2x = 10$

$x = 5$

$2x - 10 = 0$



52. a. First, find the slope using the points $(2, 28.2)$ and $(4, 28.6)$.

$$m = \frac{28.6 - 28.2}{4 - 2} = \frac{0.4}{2} = 0.2$$

Then use the slope and one of the points to write the equation in point-slope form.

$$y - y_1 = m(x - x_1)$$
$$y - 28.2 = 0.2(x - 2)$$

or

$$y - 28.6 = 0.2(x - 4)$$

b. Solve for y to obtain slope-intercept form.

$$y - 28.2 = 0.2(x - 2)$$
$$y - 28.2 = 0.2x - 0.4$$
$$y = 0.2x + 27.8$$
$$f(x) = 0.2x + 27.8$$

c. $f(x) = 0.2x + 27.8$

$f(7) = 0.2(12) + 27.8$

$= 30.2$

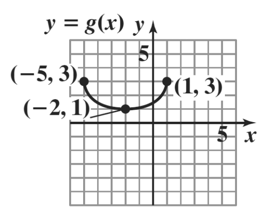
The linear function predicts men's average age of first marriage will be 30.2 years in 2020.

53. a. $m = \dfrac{27 - 21}{2010 - 1980} = \dfrac{6}{30} = 0.2$

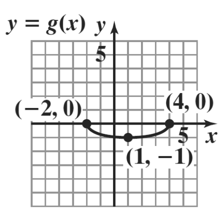
b. For the period shown, the number of the percentage of liberal college freshman increased each year by approximately 0.2. The rate of change was 0.2% per year.

54. $\dfrac{f(x_2) - f(x_1)}{x_2 - x_1} = \dfrac{[9^2 - 4(9)] - [4^2 - 4 \cdot 5]}{9 - 5} = 10$

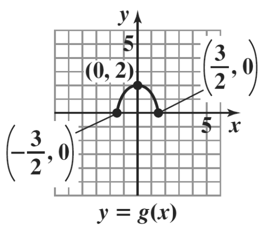
55.



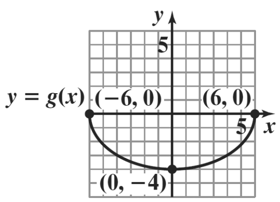
56.



57.



58.



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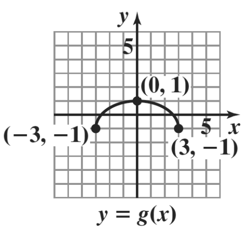

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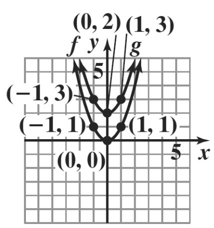

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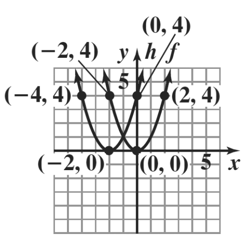

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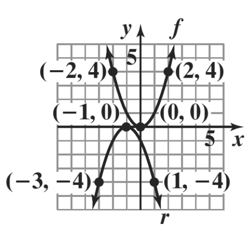

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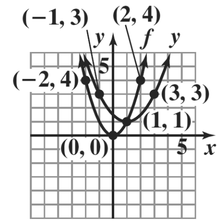

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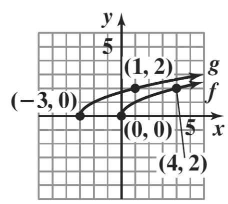

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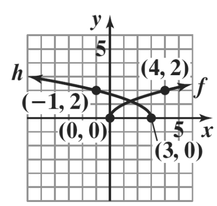

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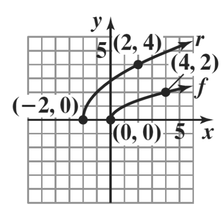

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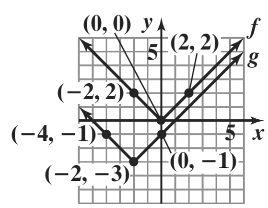

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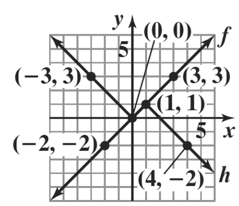

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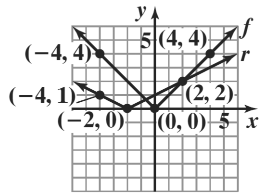

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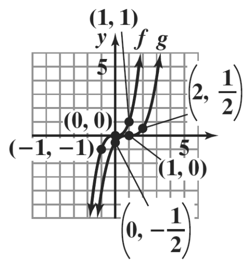

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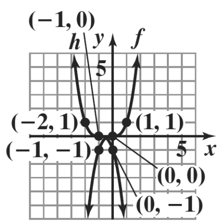

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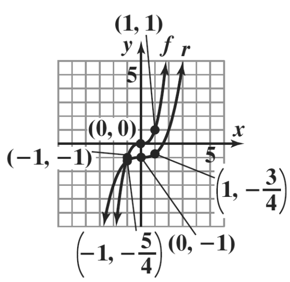

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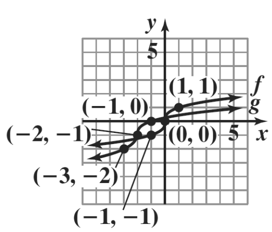

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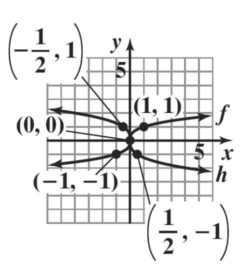

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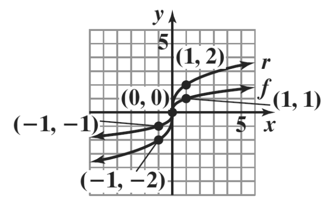


76. domain: $(-\infty, \infty)$

77. The denominator is zero when $x = 7$. The domain is $(-\infty, 7) \cup (7, \infty)$.

78. The expressions under each radical must not be negative.
$8 - 2x \geq 0$
$-2x \geq -8$
$x \leq 4$
domain: $(-\infty, 4]$.

79. The denominator is zero when $x = -7$ or $x = 3$.
domain: $(-\infty, -7) \cup (-7, 3) \cup (3, \infty)$

80. The expressions under each radical must not be negative. The denominator is zero when $x = 5$.
$x - 2 \geq 0$
$x \geq 2$
domain: $[2, 5) \cup (5, \infty)$

81. The expressions under each radical must not be negative.
$x - 1 \geq 0$ and $x + 5 \geq 0$
$x \geq 1$ $\quad$ $x \geq -5$
domain: $[1, \infty)$

82. $f(x) = 3x - 1$; $g(x) = x - 5$
$(f + g)(x) = 4x - 6$
domain: $(-\infty, \infty)$
$(f - g)(x) = (3x - 1) - (x - 5) = 2x + 4$
domain: $(-\infty, \infty)$
$(fg)(x) = (3x - 1)(x - 5) = 3x^2 - 16x + 5$
domain: $(-\infty, \infty)$
$\left(\frac{f}{g}\right)(x) = \frac{3x - 1}{x - 5}$
domain: $(-\infty, 5) \cup (5, \infty)$

83. $f(x)=x^2+x+1;\ g(x)=x^2-1$

$(f+g)(x)=2x^2+x$

domain: $(-\infty,\infty)$

$(f-g)(x)=(x^2+x+1)-(x^2-1)=x+2$

domain: $(-\infty,\infty)$

$(fg)(x)=(x^2+x+1)(x^2-1)$

$=x^4+x^3-x-1$

$\left(\frac{f}{g}\right)(x)=\frac{x^2+x+1}{x^2-1}$

domain: $(-\infty,-1)\cup(-1,1)\cup(1,\infty)$

84. $f(x)=\sqrt{x+7};\ g(x)=\sqrt{x-2}$

$(f+g)(x)=\sqrt{x+7}+\sqrt{x-2}$

domain: $[2,\infty)$

$(f-g)(x)=\sqrt{x+7}-\sqrt{x-2}$

domain: $[2,\infty)$

$(fg)(x)=\sqrt{x+7}\cdot\sqrt{x-2}$

$=\sqrt{x^2+5x-14}$

domain: $[2,\infty)$

$\left(\frac{f}{g}\right)(x)=\frac{\sqrt{x+7}}{\sqrt{x-2}}$

domain: $(2,\infty)$

85. $f(x)=x^2+3;\ g(x)=4x-1$

a. $(f\circ g)(x)=(4x-1)^2+3$

$=16x^2-8x+4$

b. $(g\circ f)(x)=4(x^2+3)-1$

$=4x^2+11$

c. $(f\circ g)(3)=16(3)^2-8(3)+4=124$

86. $f(x)=\sqrt{x};\quad g(x)=x+1$

a. $(f\circ g)(x)=\sqrt{x+1}$

b. $(g\circ f)(x)=\sqrt{x}+1$

c. $(f\circ g)(3)=\sqrt{3+1}=\sqrt{4}=2$

87. **a.** $(f\circ g)(x)=f\left(\frac{1}{x}\right)$

$$=\frac{\frac{1}{x}+1}{\frac{1}{x}-2}=\frac{\left(\frac{1}{x}+1\right)x}{\left(\frac{1}{x}-2\right)x}=\frac{1+x}{1-2x}$$

b. $x\neq 0 \qquad 1-2x\neq 0$

$x\neq\frac{1}{2}$

$(-\infty,0)\cup\left(0,\frac{1}{2}\right)\cup\left(\frac{1}{2},\infty\right)$

88. **a.** $(f\circ g)(x)=f(x+3)=\sqrt{x+3-1}=\sqrt{x+2}$

b. $x+2\geq 0$

$x\geq -2 \qquad [-2,\infty)$

89. $f(x)=x^4 \qquad g(x)=x^2+2x-1$

90. $f(x)=\sqrt[3]{x} \qquad g(x)=7x+4$

91. $f(x)=\frac{3}{5}x+\frac{1}{2};\ g(x)=\frac{5}{3}x-2$

$f(g(x))=\frac{3}{5}\left(\frac{5}{3}x-2\right)+\frac{1}{2}$

$=x-\frac{6}{5}+\frac{1}{2}$

$=x-\frac{7}{10}$

$g(f(x))=\frac{5}{3}\left(\frac{3}{5}x+\frac{1}{2}\right)-2$

$=x+\frac{5}{6}-2$

$=x-\frac{7}{6}$

f and *g* are not inverses of each other.

92. $f(x) = 2 - 5x;\ g(x) = \dfrac{2-x}{5}$

$$f(g(x)) = 2 - 5\left(\frac{2-x}{5}\right)$$
$$= 2 - (2 - x)$$
$$= x$$
$$g(f(x)) = \frac{2-(2-5x)}{5} = \frac{5x}{5} = x$$

f and *g* are inverses of each other.

93. **a.**
$$f(x) = 4x - 3$$
$$y = 4x - 3$$
$$x = 4y - 3$$
$$y = \frac{x+3}{4}$$
$$f^{-1}(x) = \frac{x+3}{4}$$

b.
$$f(f^{-1}(x)) = 4\left(\frac{x+3}{4}\right) - 3$$
$$= x + 3 - 3$$
$$= x$$
$$f^{-1}(f(x)) = \frac{(4x-3)+3}{4} = \frac{4x}{4} = x$$

94. **a.**
$$f(x) = 8x^3 + 1$$
$$y = 8x^3 + 1$$
$$x = 8y^3 + 1$$
$$x - 1 = 8y^3$$
$$\frac{x-1}{8} = y^3$$
$$\sqrt[3]{\frac{x-1}{8}} = y$$
$$\frac{\sqrt[3]{x-1}}{2} = y$$
$$f^{-1}(x) = \frac{\sqrt[3]{x-1}}{2}$$

b.
$$f\left(f^{-1}(x)\right) = 8\left(\frac{\sqrt[3]{x-1}}{2}\right)^3 + 1$$
$$= 8\left(\frac{x-1}{8}\right) + 1$$
$$= x - 1 + 1$$
$$= x$$
$$f^{-1}\left(f(x)\right) = \frac{\sqrt[3]{\left(8x^3+1\right)-1}}{2}$$
$$= \frac{\sqrt[3]{8x^3}}{2}$$
$$= \frac{2x}{2}$$
$$= x$$

95. **a.**
$$f(x) = \frac{x-7}{x+2}$$
$$y = \frac{x-7}{x+2}$$
$$x = \frac{y-7}{y+2}$$
$$xy + 2x = y - 7$$
$$xy - y = -2x - 7$$
$$y\left(x-1\right) = -2x - 7$$
$$y = \frac{-2x-7}{x-1}$$
$$f^{-1}(x) = \frac{-2x-7}{x-1},\ x \neq 1$$

b. $$f\left(f^{-1}(x)\right)=\frac{\frac{-2x-7}{x-1}-7}{\frac{-2x-7}{x-1}+2}$$
$$=\frac{-2x-7-7(x-1)}{-2x-7+2(x-1)}$$
$$=\frac{-9x}{-9}$$
$$=x$$

$$f^{-1}\left(f(x)\right)=\frac{-2\left(\frac{x-7}{x+2}\right)-7}{\frac{x-7}{x+2}-1}$$
$$=\frac{-2x+14-7(x+2)}{x-7-(x+2)}$$
$$=\frac{-9x}{-9}$$
$$=x$$

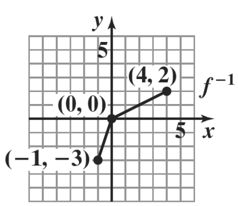
96. The inverse function exists.

97. The inverse function does not exist since it does not pass the horizontal line test.

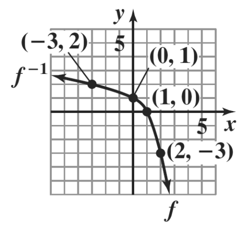
98. The inverse function exists.

99. The inverse function does not exist since it does not pass the horizontal line test.

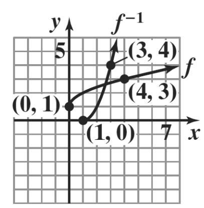
100.



101. $f(x)=1-x^2$
$$y=1-x^2$$
$$x=1-y^2$$
$$y^2=1-x$$
$$y=\sqrt{1-x}$$
$$f^{-1}(x)=\sqrt{1-x}$$



102. $f(x)=\sqrt{x}+1$
$$y=\sqrt{x}+1$$
$$x=\sqrt{y}+1$$
$$x-1=\sqrt{y}$$
$$(x-1)^2=y$$
$$f^{-1}(x)=(x-1)^2,\quad x\ge 1$$



103. $d=\sqrt{[3-(-2)]^2+[9-(-3)]^2}$
$$=\sqrt{5^2+12^2}$$
$$=\sqrt{25+144}$$
$$=\sqrt{169}$$
$$=13$$

104. $d=\sqrt{[-2-(-4)]^2+(5-3)^2}$
$$=\sqrt{2^2+2^2}$$
$$=\sqrt{4+4}$$
$$=\sqrt{8}$$
$$=2\sqrt{2}$$
$$\approx 2.83$$

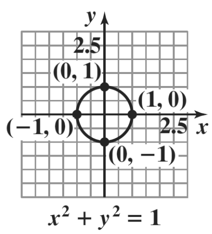
105. $\left(\frac{2+(-12)}{2},\frac{6+4}{2}\right)=\left(\frac{-10}{2},\frac{10}{2}\right)=(-5,5)$

106. $\left(\frac{4+(-15)}{2},\frac{-6+2}{2}\right)=\left(\frac{-11}{2},\frac{-4}{2}\right)=\left(\frac{-11}{2},-2\right)$

107. $x^2+y^2=3^2$
$x^2+y^2=9$

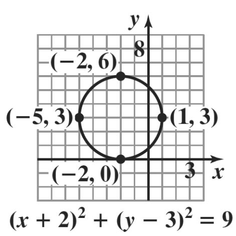
108. $(x-(-2))^2+(y-4)^2=6^2$
$(x+2)^2+(y-4)^2=36$

109. center: (0, 0); radius: 1



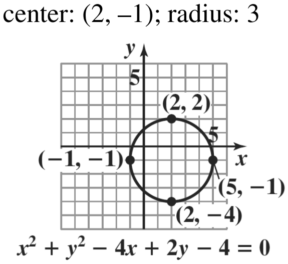
domain: $[-1,1]$
range: $[-1,1]$

110. center: (–2, 3); radius: 3



domain: $[-5,1]$
range: $[0,6]$

111. $x^2+y^2-4x+2y-4=0$
$x^2-4x\quad+y^2+2y\quad=4$
$x^2-4x+4+y^2+2y+1=4+4+1$
$(x-2)^2+(y+1)^2=9$
center: (2, –1); radius: 3



domain: $[-1,5]$
range: $[-4,2]$

Chapter 2 Test

1. (b), (c), and (d) are not functions.

2. **a.** $f(4)-f(-3)=3-(-2)=5$

b. domain: (–5, 6]

c. range: [–4, 5]

d. increasing: (–1, 2)

e. decreasing: $(-5,-1)$ or $(2,6)$

f. 2, $f(2)=5$

g. (–1, –4)

h. x-intercepts: –4, 1, and 5.

i. y-intercept: –3

3. **a.** –2, 2

b. –1, 1

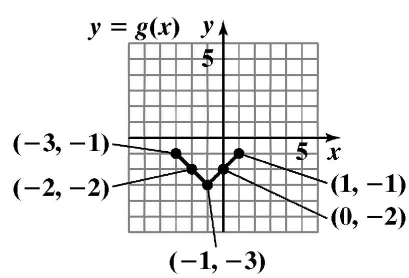
c. 0

d. even; $f(-x)=f(x)$

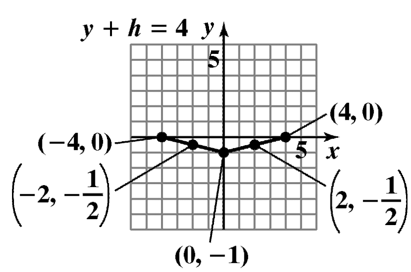
e. no; f fails the horizontal line test

f. $f(0)$ is a relative minimum.

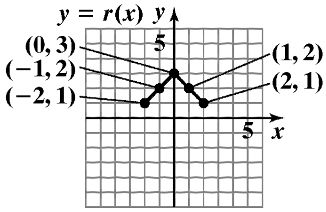
g.



h.

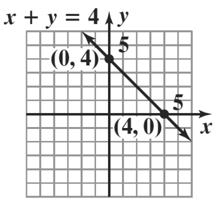


i.



j. $\frac{f(x_2)-f(x_1)}{x_2-x_1}=\frac{-1-0}{1-(-2)}=-\frac{1}{3}$

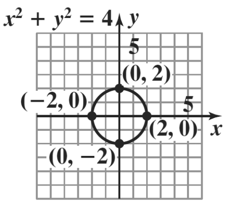
4.



domain: $(-\infty,\infty)$

range: $(-\infty,\infty)$

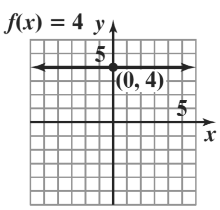
5.



domain: $[-2,2]$

range: $[-2,2]$

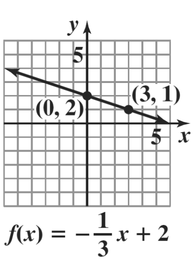
6.



domain: $(-\infty,\infty)$

range: $\{4\}$

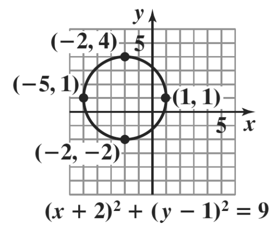
7.



domain: $(-\infty,\infty)$

range: $(-\infty,\infty)$

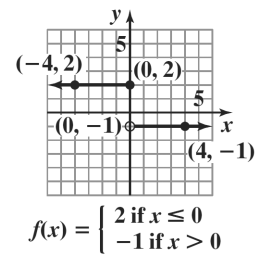
8.



domain: $[-5,1]$

range: $[-2,4]$

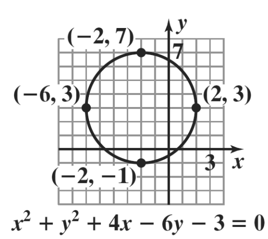
9.



domain: $(-\infty,\infty)$

range: $\{-1,2\}$

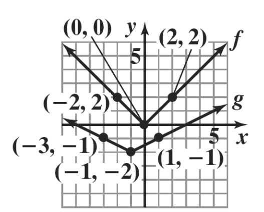
10.



domain: $[-6,2]$

range: $[-1,7]$

11.



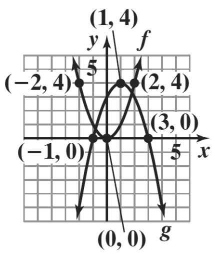
domain of *f*: $(-\infty,\infty)$

range of *f*: $[0,\infty)$

domain of *g*: $(-\infty,\infty)$

range of *g*: $[-2,\infty)$

12.



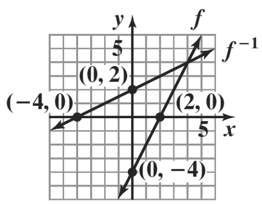
domain of *f*: $(-\infty, \infty)$

range of *f*: $[0, \infty)$

domain of *g*: $(-\infty, \infty)$

range of *g*: $(-\infty, 4]$

13.



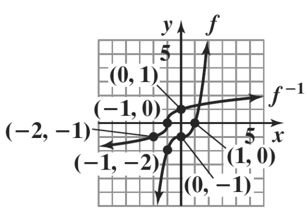
domain of *f*: $(-\infty, \infty)$

range of *f*: $(-\infty, \infty)$

domain of f^{-1}: $(-\infty, \infty)$

range of f^{-1}: $(-\infty, \infty)$

14.



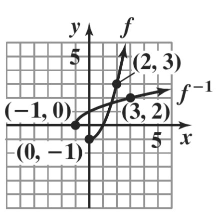
domain of *f*: $(-\infty, \infty)$

range of *f*: $(-\infty, \infty)$

domain of f^{-1}: $(-\infty, \infty)$

range of f^{-1}: $(-\infty, \infty)$

15.



domain of *f*: $[0, \infty)$

range of *f*: $[-1, \infty)$

domain of f^{-1}: $[-1, \infty)$

range of f^{-1}: $[0, \infty)$

16.
$$\begin{aligned} f(x) &= x^2 - x - 4 \\ f(x-1) &= (x-1)^2 - (x-1) - 4 \\ &= x^2 - 2x + 1 - x + 1 - 4 \\ &= x^2 - 3x - 2 \end{aligned}$$

17.
$$\begin{aligned} &\frac{f(x+h) - f(x)}{h} \\ &= \frac{(x+h)^2 - (x+h) - 4 - \left(x^2 - x - 4\right)}{h} \\ &= \frac{x^2 + 2xh + h^2 - x - h - 4 - x^2 + x + 4}{h} \\ &= \frac{2xh + h^2 - h}{h} \\ &= \frac{h\left(2x + h - 1\right)}{h} \\ &= 2x + h - 1 \end{aligned}$$

18.
$$\begin{aligned} (g - f)(x) &= 2x - 6 - \left(x^2 - x - 4\right) \\ &= 2x - 6 - x^2 + x + 4 \\ &= -x^2 + 3x - 2 \end{aligned}$$

19.
$$\left(\frac{f}{g}\right)(x) = \frac{x^2 - x - 4}{2x - 6}$$

domain: $(-\infty, 3) \cup (3, \infty)$

20.
$$\begin{aligned} (f \circ g)(x) &= f\left(g(x)\right) \\ &= (2x-6)^2 - (2x-6) - 4 \\ &= 4x^2 - 24x + 36 - 2x + 6 - 4 \\ &= 4x^2 - 26x + 38 \end{aligned}$$

21.
$$\begin{aligned} (g \circ f)(x) &= g\left(f(x)\right) \\ &= 2\left(x^2 - x - 4\right) - 6 \\ &= 2x^2 - 2x - 8 - 6 \\ &= 2x^2 - 2x - 14 \end{aligned}$$

22.
$$\begin{aligned} g\left(f(-1)\right) &= 2\left((-1)^2 - (-1) - 4\right) - 6 \\ &= 2\left(1 + 1 - 4\right) - 6 \\ &= 2\left(-2\right) - 6 \\ &= -4 - 6 \\ &= -10 \end{aligned}$$

23. $f(x)=x^2-x-4$
$$f(-x)=(-x)^2-(-x)-4$$
$$=x^2+x-4$$
f is neither even nor odd.

24. Test for symmetry with respect to the y-axis.
$$x^2+y^3=7$$
$$(-x)^2+y^3=7$$
$$x^2+y^3=7$$
The resulting equation is equivalent to the original. Thus, the graph is symmetric with respect to the y-axis.

Test for symmetry with respect to the x-axis.
$$x^2+y^3=7$$
$$x^2+(-y)^3=7$$
$$x^2-y^3=7$$
The resulting equation is not equivalent to the original. Thus, the graph is not symmetric with respect to the x-axis.

Test for symmetry with respect to the origin.
$$x^2+y^3=7$$
$$(-x)^2+(-y)^3=7$$
$$x^2-y^3=7$$
The resulting equation is not equivalent to the original. Thus, the graph is not symmetric with respect to the origin.

25. $m=\dfrac{-8-1}{-1-2}=\dfrac{-9}{-3}=3$
point-slope form: $y-1=3(x-2)$
or $y+8=3(x+1)$
slope-intercept form: $y=3x-5$

26. $y=-\dfrac{1}{4}x+5$ so $m=4$
point-slope form: $y-6=4(x+4)$
slope-intercept form: $y=4x+22$

27. Write $4x+2y-5=0$ in slope intercept form.
$$4x+2y-5=0$$
$$2y=-4x+5$$
$$y=-2x+\frac{5}{2}$$
The slope of the parallel line is –2, thus the slope of the desired line is $m=-2$.
$$y-y_1=m(x-x_1)$$
$$y-(-10)=-2\big(x-(-7)\big)$$
$$y+10=-2(x+7)$$
$$y+10=-2x-14$$
$$2x+y+24=0$$

28. **a.** Find slope: $m=\dfrac{25.8-24.6}{20-10}=\dfrac{1.2}{10}=0.12$
point-slope form:
$$y-y_1=m(x-x_1)$$
$$y-24.6=0.12(x-10)$$

b. slope-intercept form:
$$y-24.6=0.12(x-10)$$
$$y-24.6=0.12x-1.2$$
$$y=0.12x+23.4$$
$$f(x)=0.12x+23.4$$

c. $f(x)=0.12x+23.4$
$$=0.12(40)+23.4$$
$$=28.2$$
According to the model, 28.2% of U.S. households will be one-person households in 2020.

29. $\dfrac{3(10)^2-5-[3(6)^2-5]}{10-6}$
$$=\frac{205-103}{4}$$
$$=\frac{192}{4}$$
$$=48$$

30. $g(-1)=3-(-1)=4$
$g(7)=\sqrt{7-3}=\sqrt{4}=2$

31. The denominator is zero when $x=1$ or $x=-5$.
domain: $(-\infty,-5)\cup(-5,1)\cup(1,\infty)$

32. The expressions under each radical must not be negative.
$x+5\ge 0$ and $x-1\ge 0$
$x\ge -5$ $\quad$ $x\ge 1$
domain: $[1,\infty)$

33. $(f \circ g)(x) = \dfrac{7}{\frac{2}{x} - 4} = \dfrac{7x}{2-4x}$

$x \neq 0, \quad 2 - 4x \neq 0$

$x \neq \dfrac{1}{2}$

domain: $(-\infty, 0) \cup \left(0, \dfrac{1}{2}\right) \cup \left(\dfrac{1}{2}, \infty\right)$

34. $f(x) = x^7 \qquad g(x) = 2x + 3$

35. $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$$\begin{aligned} d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(5-2)^2 + (2-(-2))^2} \\ &= \sqrt{3^2 + 4^2} \\ &= \sqrt{9+16} \\ &= \sqrt{25} \\ &= 5 \end{aligned}$$

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{2+5}{2}, \frac{-2+2}{2}\right) = \left(\frac{7}{2}, 0\right)$$

The length is 5 and the midpoint is $\left(\dfrac{7}{2}, 0\right)$ or $(3.5, 0)$.

Cumulative Review Exercises (Chapters 1–2)

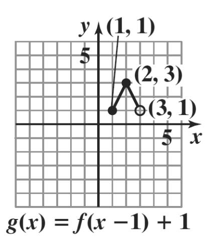
1. domain: $[0, 2)$

range: $[0, 2]$

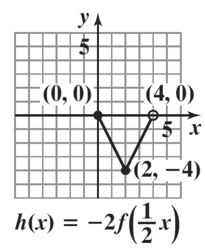
2. $f(x) = 1$ at $\dfrac{1}{2}$ and $\dfrac{3}{2}$.

3. relative maximum: 2

4.



5.



6. $(x+3)(x-4) = 8$

$x^2 - x - 12 = 8$

$x^2 - x - 20 = 0$

$(x+4)(x-5) = 0$

$x + 4 = 0$ or $x - 5 = 0$

$x = -4$ or $x = 5$

7. $3(4x-1) = 4 - 6(x-3)$

$12x - 3 = 4 - 6x + 18$

$18x = 25$

$x = \dfrac{25}{18}$

8. $\sqrt{x} + 2 = x$

$\sqrt{x} = x - 2$

$(\sqrt{x})^2 = (x-2)^2$

$x = x^2 - 4x + 4$

$0 = x^2 - 5x + 4$

$0 = (x-1)(x-4)$

$x - 1 = 0$ or $x - 4 = 0$

$x = 1$ or $x = 4$

A check of the solutions shows that $x = 1$ is an extraneous solution.
The solution set is $\{4\}$.

9. $x^{2/3} - x^{1/3} - 6 = 0$

Let $u = x^{1/3}$. Then $u^2 = x^{2/3}$.

$u^2 - u - 6 = 0$

$(u+2)(u-3) = 0$

$u = -2$ or $u = 3$

$x^{1/3} = -2$ or $x^{1/3} = 3$

$x = (-2)^3$ or $x = 3^3$

$x = -8$ or $x = 27$

10. $\frac{x}{2} - 3 \le \frac{x}{4} + 2$

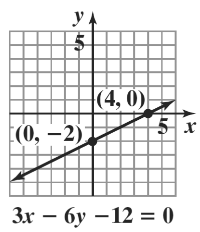
$4\left(\frac{x}{2} - 3\right) \le 4\left(\frac{x}{4} + 2\right)$

$2x - 12 \le x + 8$

$x \le 20$

The solution set is $(-\infty, 20]$.

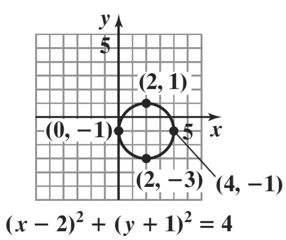
11.



domain: $(-\infty, \infty)$

range: $(-\infty, \infty)$

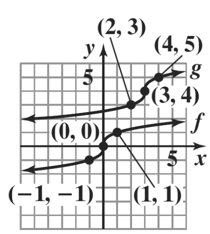
12.



domain: $[0, 4]$

range: $[-3, 1]$

13.



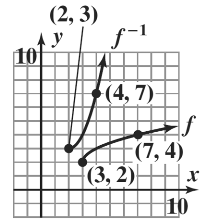
domain of *f*: $(-\infty, \infty)$

range of *f*: $(-\infty, \infty)$

domain of *g*: $(-\infty, \infty)$

range of *g*: $(-\infty, \infty)$

14.



domain of *f*: $[3, \infty)$

range of *f*: $[2, \infty)$

domain of f^{-1}: $[2, \infty)$

range of f^{-1}: $[3, \infty)$

15. $\frac{f(x+h) - f(x)}{h}$

$= \frac{\left(4 - (x+h)^2\right) - \left(4 - x^2\right)}{h}$

$= \frac{4 - (x^2 + 2xh + h^2) - \left(4 - x^2\right)}{h}$

$= \frac{4 - x^2 - 2xh - h^2 - 4 + x^2}{h}$

$= \frac{-2xh - h^2}{h}$

$= \frac{h\left(-2x - h\right)}{h}$

$= -2x - h$

16. $(f \circ g)(x) = f\left(g(x)\right)$

$(f \circ g)(x) = f\left(x + 5\right)$

$0 = 4 - \left(x + 5\right)^2$

$0 = 4 - (x^2 + 10x + 25)$

$0 = 4 - x^2 - 10x - 25$

$0 = -x^2 - 10x - 21$

$0 = x^2 + 10x + 21$

$0 = (x + 7)(x + 3)$

The value of $(f \circ g)(x)$ will be 0 when $x = -3$ or $x = -7$.

17. $y = -\frac{1}{4}x + \frac{1}{3}$, so $m = 4$.

point-slope form: $y - 5 = 4(x + 2)$

slope-intercept form: $y = 4x + 13$

general form: $4x - y + 13 = 0$

18. $0.07x + 0.09(6000 - x) = 510$
$$0.07x + 540 - 0.09x = 510$$
$$-0.02x = -30$$
$$x = 1500$$
$$6000 - x = 4500$$
\$1500 was invested at 7% and \$4500 was invested at 9%.

19. $200 + 0.05x = .15x$
$$200 = 0.10x$$
$$2000 = x$$

For \$2000 in sales, the earnings will be the same.

20. width = w
length = $2w + 2$
$$2(2w + 2) + 2w = 22$$
$$4w + 4 + 2w = 22$$
$$6w = 18$$
$$w = 3$$
$$2w + 2 = 8$$
The garden is 3 feet by 8 feet.