Preface

This manual contains detailed solutions to all of the exercises of the text *College Algebra*, twelfth edition, by R. David Gustafson and Jeff Hughes.

Many of the exercises in the text may be solved using more than one method, but it is not feasible to list all possible solutions in this manual. Also, some of the exercises may have been solved in this manual using a method that differs slightly from that presented in the text. There are a few exercises in the text whose solutions may vary from person to person. Some of these solutions may not have been included in this manual. For the solution to an exercise like this, the notation "answers may vary" has been included.

If you are a student using this manual, please remember that only reading a solution does not teach you how to solve a problem. To repeat a commonly used phrase, mathematics is not a spectator sport. You MUST make an honest attempt to solve each exercise in the text without using this manual first. This manual should be viewed more or less as a last resort. Above all, DO NOT simply copy the solution from this manual onto your own paper. Doing so will not help you learn how to do the exercise, nor will it help you to do better on quizzes or tests.

I would like to thank Paul McCombs from Rock Valley College and Samantha Lugtu of Cengage Learning for their help and support. This solutions manual was prepared using EXP 5.1.

This book is dedicated to John, who helps me to realize that mathematics cannot describe everything in life.

May your study of this material be successful and rewarding.

Michael G. Welden

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Exercises 0.1 (page 13)

1. set **2.** subset **3.** union **4.** intersection

5. decimal **6.** variable **7.** 2 **8.** even

9. composite **10.** rational **11.** decimals **12.** $\leq$

13. negative **14.** 0 **15.** $x + (y + z)$ **16.** yx

17. $5m + 5 \cdot 2$ **18.** Commutative, Multiplication **19.** interval **20.** no

21. two **22.** half-open **23.** positive **24.** distance

25. Every natural number is a whole number, so **N** $\subset$ **W**. **TRUE**

26. Every rational number is a real number, so **Q** $\subset$ **R**. **TRUE**

27. The rational number $\frac{1}{2}$ is **not** a natural number, so **Q** $\not\subset$ **N**. **FALSE**

28. Every integer is a rational number, so **Z** $\subset$ **Q**. **TRUE**

29. Every whole number is an integer, so **W** $\subset$ **Z**. **TRUE**

30. The real number $\sqrt{2}$ is **not** an integer, so **R** $\not\subset$ **Z**. **FALSE**

31. $A \cup B = \{\text{a, b, c, d, e, f, g}\}$

32. $A \cap B = \{\text{d, e}\}$

33. $A \cap C = \{\text{a, c, e}\}$

34. $B \cup C = \{\text{a, c, d, e, f, g}\}$

35. $\frac{9}{16} = 0.5625$; terminates

36. $\frac{3}{8} = 0.375$; terminates

37. $\frac{3}{11} = 0.272727...$; repeats

38. $\frac{5}{12} = 0.416666...$; repeats

39. natural: 1, 2, 6, 7

40. whole: 0, 1, 2, 6, 7

41. integers: $-5, -4$, 0, 1, 2, 6, 7

42. rational: $-5, -4, -\frac{2}{3}$, 0, 1, 2, 2.75, 6, 7

43. irrational: $\sqrt{2}$

44. prime: 2, 7

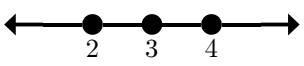
45. composite: 6

46. even: -4, 0, 2, 6

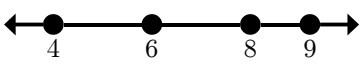
47. odd: -5, 1, 7

48. negative: $-5, -4, -\frac{2}{3}$

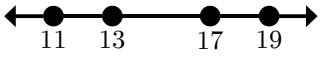
49.

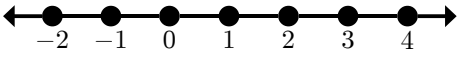


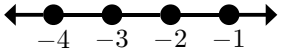
50.

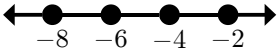


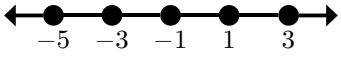
EXERCISES 0.1

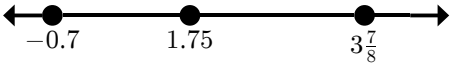
51. 

52. 

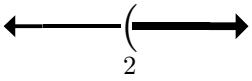
53. 

54. 

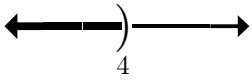
55. 

56. 

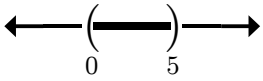
57. $x > 2 \rightarrow (2, \infty)$



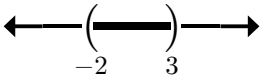
58. $x < 4 \rightarrow (-\infty, 4)$



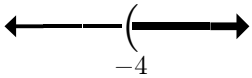
59. $0 < x < 5 \rightarrow (0, 5)$



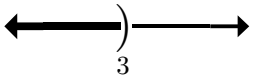
60. $-2 < x < 3 \rightarrow (-2, 3)$



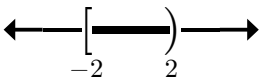
61. $x > -4 \rightarrow (-4, \infty)$



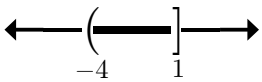
62. $x < 3 \rightarrow (-\infty, 3)$



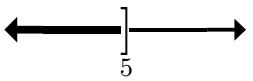
63. $-2 \leq x < 2 \rightarrow [-2, 2)$



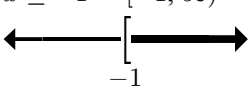
64. $-4 < x \leq 1 \rightarrow (-4, 1]$



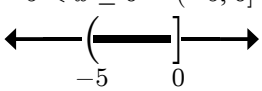
65. $x \leq 5 \rightarrow (-\infty, 5]$



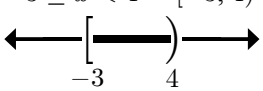
66. $x \geq -1 \rightarrow [-1, \infty)$



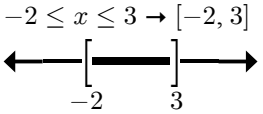
67. $-5 < x \leq 0 \rightarrow (-5, 0]$



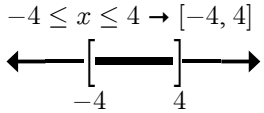
68. $-3 \leq x < 4 \rightarrow [-3, 4)$



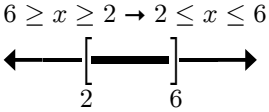
69. $-2 \leq x \leq 3 \rightarrow [-2, 3]$



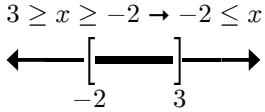
70. $-4 \leq x \leq 4 \rightarrow [-4, 4]$



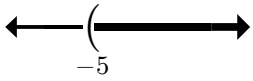
71. $6 \geq x \geq 2 \rightarrow 2 \leq x \leq 6 \rightarrow [2, 6]$

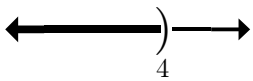


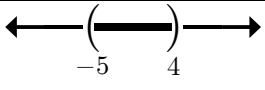
72. $3 \geq x \geq -2 \rightarrow -2 \leq x \leq 3 \rightarrow [-2, 3]$



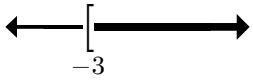
73. $x > -5$ and $x < 4 \rightarrow (-5, \infty) \cap (-\infty, 4)$

$(-5, \infty)$ 

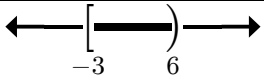
$(-\infty, 4)$ 

$(-5, \infty) \cap (-\infty, 4)$ 

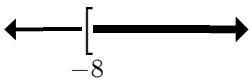
74. $x \geq -3$ and $x < 6 \rightarrow [-3, \infty) \cap (-\infty, 6)$

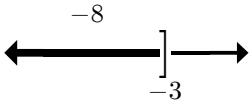
$[-3, \infty)$ 

$(-\infty, 6)$ 

$[-3, \infty) \cap (-\infty, 6)$ 

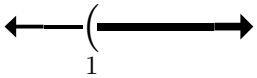
75. $x \geq -8$ and $x \leq -3 \rightarrow [-8, \infty) \cap (-\infty, -3]$

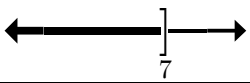
$[-8, \infty)$ 

$(-\infty, -3]$ 

$[-8, \infty) \cap (-\infty, -3]$ 

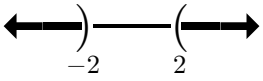
76. $x > 1$ and $x \leq 7 \rightarrow (1, \infty) \cap (-\infty, 7]$

$(1, \infty)$ 

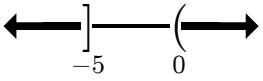
$(-\infty, 7]$ 

$(1, \infty) \cap (-\infty, 7]$ 

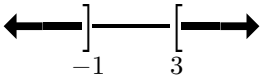
77. $x < -2$ or $x > 2 \rightarrow (-\infty, -2) \cup (2, \infty)$



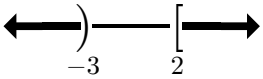
78. $x \leq -5$ or $x > 0 \rightarrow (-\infty, -5] \cup (0, \infty)$



79. $x \leq -1$ or $x \geq 3 \rightarrow (-\infty, -1] \cup [3, \infty)$



80. $x < -3$ or $x \geq 2 \rightarrow (-\infty, -3) \cup [2, \infty)$



81. Since $13 \geq 0$, $|13| = 13$.

82. Since $-17 < 0$, $|-17| = -(-17) = 17$.

83. Since $0 \geq 0$, $|0| = 0$.

84. Since $63 \geq 0$, $|63| = 63$.
$-|63| = -(63) = -63$

85. Since $-8 < 0$, $|-8| = -(-8) = 8$.
$-|-8| = -(8) = -8$

86. Since $-25 < 0$, $|-25| = -(-25) = 25$.

EXERCISES 0.1

87. Since $32 \geq 0$, $|32| = 32$.
$-|32| = -(32) = -32$

88. Since $-6 < 0$, $|-6| = -(-6) = 6$.
$-|-6| = -(6) = -6$

89. Since $\pi - 5 < 0$,
$|\pi - 5| = -(\pi - 5) = -\pi + 5 = 5 - \pi$.

90. Since $8 - \pi \geq 0$, $|8 - \pi| = 8 - \pi$.

91. $|\pi - \pi| = |0| = 0$

92. Since $2\pi \geq 0$, $|2\pi| = 2\pi$.

93. If $x \geq 2$, then $x + 1 \geq 0$. Then
$|x + 1| = x + 1$.

94. If $x \leq -2$, then $x + 1 < 0$. Then
$|x + 1| = -(x + 1)$.

95. If $x < 0$, then $x - 4 < 0$. Then
$|x - 4| = -(x - 4)$.

96. If $x > 10$, then $x - 7 \geq 0$. Then
$|x - 7| = x - 7$.

97. distance $= |8 - 3| = |5| = 5$

98. distance $= |12 - (-5)| = |17| = 17$

99. distance $= |-3 - (-8)| = |5| = 5$

100. distance $= |-20 - 6| = |-26| = 26$

101. Since population must be positive and never has a fractional part, the set of **natural numbers** should be used.

102. Since the subdivisions on a ruler are measured in fractions of an inch, the set of **rational numbers** should be used.

103. Since temperatures are usually reported without fractional parts and may be either positive or negative (or zero), the set of **integers** should be used.

104. Since the financial condition of a business is usually described in terms of dollars and cents (fractional parts of a dollar), the set of **rational numbers** should be used.

105. change $= |-7 - 15| = |-22| = 22$
The change is 22° F.

106. change $= |-20 - (-7)| = |-13| = 13$
The change is 13° C.

107. $-x$ will represent a positive number if x itself is negative. For instance, if $x = -3$, then $-x = -(-3) = 3$, which is a positive number.

108. Every integer is a rational number because every integer is equal to itself over 1.

109. The statement is always true.

110. The statement is always true.

111. The statement is not always true. (For example, let $a = 5$ and $b = -2$.)

112. The statement $a < b > c$ could be interpreted to mean that $a > c$, when this is not necessarily true.

113. False. There are 5 integers: -2, -1, 0, 1, and 2.

114. False. $\dfrac{725}{0}$ is not a rational number because the denominator cannot equal 0.

115. False. ∞ is not a number at all.

116. True.

117. True. (You cannot find an element in the 1st set that is not in the 2nd set.)

118. True. (You cannot find an element in the 1st set that is not in the 2nd set.)

119. False. There are eight subsets.

120. True.

Exercises 0.2 (page 25)

1. factor

2. natural

3. 3, $2x$

4. exponential

5. scientific, integer

6. **Answers may vary.**

7. $x^m x^n = x^{m+n}$

8. $(x^m)^n = x^{mn}$

9. $(xy)^n = x^n y^n$

10. $\dfrac{x^m}{x^n} = x^{m-n}$

11. $x^0 = 1$

12. $x^{-n} = \dfrac{1}{x^n}$

13. $13^2 = 13 \cdot 13 = 169$

14. $10^3 = 10 \cdot 10 \cdot 10 = 1{,}000$

15. $-5^2 = -1 \cdot 5 \cdot 5 = -25$

16. $(-5)^2 = (-5)(-5) = 25$

17. $4x^3 = 4 \cdot x \cdot x \cdot x$

18. $(4x)^3 = (4x)(4x)(4x)$

19. $(-5x)^4 = (-5x)(-5x)(-5x)(-5x)$

20. $-6x^2 = -6 \cdot x \cdot x$

21. $-8x^4 = -8 \cdot x \cdot x \cdot x \cdot x$

22. $(-8x)^4 = (-8x)(-8x)(-8x)(-8x)$

23. $7xxx = 7x^3$

24. $-8yyyy = -8y^4$

25. $(-x)(-x) = (-1)(-1)x^2 = x^2$

26. $(2a)(2a)(2a) = 2 \cdot 2 \cdot 2 \cdot a^3 = 8a^3$

27. $(3t)(3t)(-3t) = (3)(3)(-3)t^3 = -27t^3$

28. $-(2b)(2b)(2b)(2b) = -1 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot b^4 = -16b^4$

29. $xxxyy = x^3y^2$

30. $aaabbbb = a^3b^4$

31. $2.2^3 = 10.648$

32. $7.1^4 = 2541.1681$

33. $-0.5^4 = -0.0625$

34. $(-0.2)^4 = 0.0016$

35. $x^2x^3 = x^{2+3} = x^5$

36. $y^3y^4 = y^{3+4} = y^7$

37. $(z^2)^3 = z^{2 \cdot 3} = z^6$

38. $(t^6)^7 = t^{6 \cdot 7} = t^{42}$

39. $(y^5y^2)^3 = (y^7)^3 = y^{21}$

40. $(a^3a^6)a^4 = a^9a^4 = a^{13}$

41. $(z^2)^3(z^4)^5 = z^6z^{20} = z^{26}$

42. $(t^3)^4(t^5)^2 = t^{12}t^{10} = t^{22}$

EXERCISES 0.2

43. $(a^2)^3(a^4)^2 = a^6a^8 = a^{14}$

44. $(a^2)^4(a^3)^3 = a^8a^9 = a^{17}$

45. $(3x)^3 = 3^3x^3 = 27x^3$

46. $(-2y)^4 = (-2)^4y^4 = 16y^4$

47. $(x^2y)^3 = (x^2)^3y^3 = x^6y^3$

48. $(x^3z^4)^6 = (x^3)^6(z^4)^6 = x^{18}z^{24}$

49. $\left(\dfrac{a^2}{b}\right)^3 = \dfrac{(a^2)^3}{b^3} = \dfrac{a^6}{b^3}$

50. $\left(\dfrac{x}{y^3}\right)^4 = \dfrac{x^4}{(y^3)^4} = \dfrac{x^4}{y^{12}}$

51. $(-x)^0 = 1$

52. $4x^0 = 4 \cdot 1 = 4$

53. $(4x)^0 = 1$

54. $-2x^0 = -2 \cdot 1 = -2$

55. $z^{-4} = \dfrac{1}{z^4}$

56. $\dfrac{1}{t^{-2}} = t^2$

57. $y^{-2}y^{-3} = y^{-5} = \dfrac{1}{y^5}$

58. $-m^{-2}m^3 = -m^1 = -m$

59. $(x^3x^{-4})^{-2} = (x^{-1})^{-2} = x^2$

60. $(y^{-2}y^3)^{-4} = (y^1)^{-4} = y^{-4} = \dfrac{1}{y^4}$

61. $\dfrac{x^7}{x^3} = x^{7-3} = x^4$

62. $\dfrac{r^5}{r^2} = r^{5-2} = r^3$

63. $\dfrac{a^{21}}{a^{17}} = a^{21-17} = a^4$

64. $\dfrac{t^{13}}{t^4} = t^{13-4} = t^9$

65. $\dfrac{(x^2)^2}{x^2x} = \dfrac{x^4}{x^3} = x^{4-3} = x^1 = x$

66. $\dfrac{s^9s^3}{(s^2)^2} = \dfrac{s^{12}}{s^4} = s^{12-4} = s^8$

67. $\left(\dfrac{m^3}{n^2}\right)^3 = \dfrac{(m^3)^3}{(n^2)^3} = \dfrac{m^9}{n^6}$

68. $\left(\dfrac{t^4}{t^3}\right)^3 = \left(t^{4-3}\right)^3 = \left(t^1\right)^3 = t^3$

69. $\dfrac{(a^3)^{-2}}{aa^2} = \dfrac{a^{-6}}{a^3} = a^{-6-3} = a^{-9} = \dfrac{1}{a^9}$

70. $\dfrac{r^9r^{-3}}{(r^{-2})^3} = \dfrac{r^6}{r^{-6}} = r^{6-(-6)} = r^{12}$

71. $\left(\dfrac{a^{-3}}{b^{-1}}\right)^{-4} = \dfrac{(a^{-3})^{-4}}{(b^{-1})^{-4}} = \dfrac{a^{12}}{b^4}$

72. $\left(\dfrac{t^{-4}}{t^{-3}}\right)^{-2} = \dfrac{(t^{-4})^{-2}}{(t^{-3})^{-2}} = \dfrac{t^8}{t^6} = t^2$

73. $\left(\dfrac{r^4r^{-6}}{r^3r^{-3}}\right)^2 = \left(\dfrac{r^{-2}}{r^0}\right)^2 = \left(r^{-2}\right)^2 = r^{-4} = \dfrac{1}{r^4}$

74. $\dfrac{(x^{-3}x^2)^2}{(x^2x^{-5})^{-3}} = \dfrac{(x^{-1})^2}{(x^{-3})^{-3}} = \dfrac{x^{-2}}{x^9} = x^{-11} = \dfrac{1}{x^{11}}$

EXERCISES 0.2

75. $\left(\dfrac{x^5y^{-2}}{x^{-3}y^2}\right)^4 = \left(\dfrac{x^5x^3}{y^2y^2}\right)^4 = \left(\dfrac{x^8}{y^4}\right)^4 = \dfrac{x^{32}}{y^{16}}$

76. $\left(\dfrac{x^{-7}y^5}{x^7y^{-4}}\right)^3 = \left(\dfrac{y^5y^4}{x^7x^7}\right)^3 = \left(\dfrac{y^9}{x^{14}}\right)^3 = \dfrac{y^{27}}{x^{42}}$

77. $\left(\dfrac{5x^{-3}y^{-2}}{3x^2y^{-3}}\right)^{-2} = \left(\dfrac{3x^2y^{-3}}{5x^{-3}y^{-2}}\right)^2 = \left(\dfrac{3x^2x^3y^2}{5y^3}\right)^2 = \left(\dfrac{3x^5}{5y}\right)^2 = \dfrac{9x^{10}}{25y^2}$

78. $\left(\dfrac{3x^2y^{-5}}{2x^{-2}y^{-6}}\right)^{-3} = \left(\dfrac{2x^{-2}y^{-6}}{3x^2y^{-5}}\right)^3 = \left(\dfrac{2y^5}{3x^2x^2y^6}\right)^3 = \left(\dfrac{2}{3x^4y}\right)^3 = \dfrac{8}{27x^{12}y^3}$

79. $\left(\dfrac{3x^5y^{-3}}{6x^{-5}y^3}\right)^{-2} = \left(\dfrac{6x^{-5}y^3}{3x^5y^{-3}}\right)^2 = \left(\dfrac{2y^3y^3}{1x^5x^5}\right)^2 = \left(\dfrac{2y^6}{x^{10}}\right)^2 = \dfrac{4y^{12}}{x^{20}}$

80. $\left(\dfrac{12x^{-4}y^3z^{-5}}{4x^4y^{-3}z^5}\right)^3 = \left(\dfrac{3y^3y^3}{1x^4x^4z^5z^5}\right)^3 = \left(\dfrac{3y^6}{x^8z^{10}}\right)^3 = \dfrac{27y^{18}}{x^{24}z^{30}}$

81. $\dfrac{(8^{-2}z^{-3}y)^{-1}}{(5y^2z^{-2})^3(5yz^{-2})^{-1}} = \dfrac{8^2z^3y^{-1}}{5^3y^6z^{-6}\cdot 5^{-1}y^{-1}z^2} = \dfrac{64z^3y^{-1}}{5^2y^5z^{-4}} = \dfrac{64z^3z^4}{25y^5y^1} = \dfrac{64z^7}{25y^6}$

82. $\dfrac{(m^{-2}n^3p^4)^{-2}(mn^{-2}p^3)^4}{(mn^{-2}p^3)^{-4}(mn^2p)^{-1}} = \dfrac{m^4n^{-6}p^{-8}\cdot m^4n^{-8}p^{12}}{m^{-4}n^8p^{-12}\cdot m^{-1}n^{-2}p^{-1}} = \dfrac{m^8n^{-14}p^4}{m^{-5}n^6p^{-13}} = \dfrac{m^8m^5p^4p^{13}}{n^6n^{14}} = \dfrac{m^{13}p^{17}}{n^{20}}$

83. $-\dfrac{5[6^2+(9-5)]}{4(2-3)^2} = -\dfrac{5[36+4]}{4(-1)^2} = -\dfrac{5[40]}{4(1)} = -\dfrac{200}{4} = -50$

84. $\dfrac{6\left[3-(4-7)^2\right]}{-5(2-4^2)} = \dfrac{6\left[3-(-3)^2\right]}{-5(2-16)} = \dfrac{6[3-9]}{-5(-14)} = \dfrac{6[-6]}{70} = \dfrac{-36}{70} = -\dfrac{18}{35}$

85. $x^2 = (-2)^2 = 4$

86. $-x^2 = -(-2)^2 = -1\cdot 4 = -4$

87. $x^3 = (-2)^3 = -8$

88. $-x^3 = -(-2)^3 = -1\cdot(-8) = 8$

89. $(-xz)^3 = [-1\cdot(-2)\cdot 3]^3 = 6^3 = 216$

90. $-xz^3 = -1\cdot(-2)\cdot 3^3 = 2\cdot 27 = 54$

91. $\dfrac{-(x^2z^3)}{z^2-y^2} = \dfrac{-\left[(-2)^2\cdot 3^3\right]}{3^2-0^2} = \dfrac{-[4\cdot 27]}{9-0} = \dfrac{-108}{9} = -12$

92. $\dfrac{z^2(x^2-y^2)}{x^3z} = \dfrac{3^2\left[(-2)^2-0^2\right]}{(-2)^3(3)} = \dfrac{9(4-0)}{(-8)(3)} = \dfrac{9(4)}{-24} = \dfrac{36}{-24} = -\dfrac{3}{2}$

93. $5x^2-3y^3z = 5(-2)^2-3(0)^3(3) = 5(4)-3(0)(3) = 20-0 = 20$

94. $3(x-z)^2+2(y-z)^3 = 3(-2-3)^2+2(0-3)^3 = 3(-5)^2+2(-3)^3 = 3(25)+2(-27)$
$= 75+(-54) = 21$

EXERCISES 0.2

95. $\dfrac{-3x^{-3}z^{-2}}{6x^2z^{-3}} = \dfrac{-1z^3}{2x^2x^3z^2} = \dfrac{-z}{2x^5} = \dfrac{-3}{2(-2)^5} = \dfrac{-3}{2(-32)} = \dfrac{-3}{-64} = \dfrac{3}{64}$

96. $\dfrac{\left(-5x^2z^{-3}\right)^2}{5xz^{-2}} = \dfrac{25x^4z^{-6}}{5xz^{-2}} = \dfrac{5x^4}{xz^{-2}z^6} = \dfrac{5x^3}{z^4} = \dfrac{5(-2)^3}{3^4} = \dfrac{5(-8)}{81} = \dfrac{-40}{81} = -\dfrac{40}{81}$

97. $372{,}000 = 3.72 \times 10^5$

98. $89{,}500 = 8.95 \times 10^4$

99. $-177{,}000{,}000 = -1.77 \times 10^8$

100. $-23{,}470{,}000{,}000 = -2.347 \times 10^{10}$

101. $0.007 = 7 \times 10^{-3}$

102. $0.00052 = 5.2 \times 10^{-4}$

103. $-0.000000693 = -6.93 \times 10^{-7}$

104. $-0.000000089 = -8.9 \times 10^{-8}$

105. $1{,}000{,}000{,}000{,}000 = 1 \times 10^{12}$

106. $0.000001 = 1 \times 10^{-6}$

107. $9.37 \times 10^5 = 937{,}000$

108. $4.26 \times 10^9 = 4{,}260{,}000{,}000$

109. $2.21 \times 10^{-5} = 0.0000221$

110. $2.774 \times 10^{-2} = 0.02774$

111. $0.00032 \times 10^4 = 3.2$

112. $9{,}300.0 \times 10^{-4} = 0.93$

113. $-3.2 \times 10^{-3} = -0.0032$

114. $-7.25 \times 10^3 = -7{,}250$

115. $\dfrac{(65{,}000)(45{,}000)}{250{,}000} = \dfrac{(6.5 \times 10^4)(4.5 \times 10^4)}{2.5 \times 10^5} = \dfrac{(6.5)(4.5)}{2.5} \times 10^{4+4-5} = 11.7 \times 10^3$
$= 1.17 \times 10^1 \times 10^3$
$= 1.17 \times 10^4$

116. $\dfrac{(0.000000045)(0.00000012)}{45{,}000{,}000} = \dfrac{(4.5 \times 10^{-8})(1.2 \times 10^{-7})}{4.5 \times 10^7} = \dfrac{(4.5)(1.2)}{4.5} \times 10^{(-8)+(-7)-7}$
$= 1.2 \times 10^{-22}$

117. $\dfrac{(0.00000035)(170{,}000)}{0.00000085} = \dfrac{(3.5 \times 10^{-7})(1.7 \times 10^5)}{8.5 \times 10^{-7}} = \dfrac{(3.5)(1.7)}{8.5} \times 10^{(-7)+5-(-7)}$
$= 0.7 \times 10^5$
$= 7 \times 10^{-1} \times 10^5 = 7 \times 10^4$

118. $\dfrac{(0.0000000144)(12{,}000)}{600{,}000} = \dfrac{(1.44 \times 10^{-8})(1.2 \times 10^4)}{6 \times 10^5} = \dfrac{(1.44)(1.2)}{6} \times 10^{(-8)+4-5}$
$= 0.288 \times 10^{-9}$
$= 2.88 \times 10^{-1} \times 10^{-9} = 2.88 \times 10^{-10}$

EXERCISES 0.2

119. $\dfrac{(45{,}000{,}000{,}000)(212{,}000)}{0.00018} = \dfrac{(4.5\times10^{10})(2.12\times10^{5})}{1.8\times10^{-4}} = \dfrac{(4.5)(2.12)}{1.8}\times10^{10+5-(-4)}$
$= 5.3\times10^{19}$

120. $\dfrac{(0.00000000275)(4{,}750)}{500{,}000{,}000{,}000} = \dfrac{(2.75\times10^{-9})(4.75\times10^{3})}{5\times10^{11}} = \dfrac{(2.75)(4.75)}{5}\times10^{(-9)+3-11}$
$= 2.6125\times10^{-17}$

121. 3.31×10^{4} cm/sec $= \dfrac{3.31\times10^{4}\text{ cm}}{1\text{ sec}}\cdot\dfrac{1\text{ m}}{100\text{ cm}}\cdot\dfrac{60\text{ sec}}{1\text{ min}} = \dfrac{(3.31\times10^{4})(6\times10^{1})}{1\times10^{2}}$ m/min
$= \dfrac{(3.31)(6)}{1}\times10^{4+1-2}$ m/min
$= 19.86\times10^{3}$ m/min
$= 1.986\times10^{4}$ m/min

122. $V = lwh = (6{,}000\text{ mm})(9{,}700\text{ mm})(4{,}700\text{ mm}) = \left(6\times10^{3}\right)\left(9.7\times10^{3}\right)\left(4.7\times10^{3}\right)$ mm^3
$= (6)(9.7)(4.7)\times10^{3+3+3}$ mm^3
$= 273.54\times10^{9}$ mm^3
$= 2.7354\times10^{11}$ mm^3

123. mass $= 1{,}000{,}000{,}000(0.000000000000000000000000167248\text{ g})$
$= \left(1\times10^{9}\right)\left(1.67248\times10^{-24}\text{ g}\right) = 1.67248\times10^{-15}$ g

124. $30{,}000{,}000{,}000$ cm/sec $= \dfrac{30{,}000{,}000{,}000\text{ cm}}{1\text{ sec}}\cdot\dfrac{1\text{ mile}}{160{,}934.4\text{ cm}}\cdot\dfrac{60\text{ sec}}{1\text{ min}}\cdot\dfrac{60\text{ min}}{1\text{ hr}}$
$= \dfrac{(3\times10^{10})(6\times10^{1})(6\times10^{1})}{1.609344\times10^{5}}$ mile/hr
$= \dfrac{(3)(6)(6)}{1.609344}\times10^{10+1+1-5}$ mile/hr
$\approx 67.11\times10^{7}$ mile/hr $= 6.711\times10^{8}$ mile/hr

125. $10\cdot10\cdot10\cdot26\cdot26\cdot26 = 10^{3}\cdot26^{3}$; $10^{3}\cdot26^{3} = 17{,}576{,}000 = 1.7576\times10^{7}$

126.

Earth: $n=3$	Mars: $n=4$
$d = 9{,}275{,}200\left[3\left(2^{n-2}\right)+4\right]$	$d = 9{,}275{,}200\left[3\left(2^{n-2}\right)+4\right]$
$= 9{,}275{,}200\left[3\left(2^{3-2}\right)+4\right]$	$= 9{,}275{,}200\left[3\left(2^{4-2}\right)+4\right]$
$= 9{,}275{,}200\left[3\left(2^{1}\right)+4\right]$	$= 9{,}275{,}200\left[3\left(2^{2}\right)+4\right]$
$= 9{,}275{,}200[10]$	$= 9{,}275{,}200[16]$
$= 92{,}752{,}000 \approx 93{,}000{,}000$	$= 148{,}403{,}200 \approx 148{,}000{,}000$
9.3×10^{7} mi	1.48×10^{8} mi

127. polar radius $= 6.356750\times10^{3}$ km
equatorial radius $= 6.378135\times10^{3}$ km

128. polar radius $= 3.941185\times10^{3}$ mi
equatorial radius $= 3.9544437\times10^{3}$ mi

129. $x^n x^2 = x^{n+2}$

130. $\dfrac{x^m}{x^3} = x^{m-3}$

131. $\dfrac{x^m x^2}{x^3} = \dfrac{x^{m+2}}{x^3} = x^{m+2-3} = x^{m-1}$

132. $\dfrac{x^{3m+5}}{x^2} = x^{3m+5-2} = x^{3m+3}$

133. $x^{m+1}x^3 = x^{m+1+3} = x^{m+4}$

134. $a^{n-3}a^3 = a^{n-3+3} = a^n$

135. In the expression $-x^4$, the base of the exponent is x, while in the expression $(-x)^4$, the base of the exponent is $-x$.

136. $(-x)^{55} = (-1)^{55} \cdot x^{55} = -x^{55}$

137. **Answers will vary.**

138. 32×10^2 is not in scientific notation because 32 is not a number between 1 and 10.

139. $x^{11} \cdot x^{11} = x^{11+11} = x^{22}$

140. $11^2 \cdot 11^3 = 11^{2+3} = 11^5$

141. $\dfrac{y^{50}}{y^{10}} = y^{50-10} = y^{40}$

142. $(6xyz)^6 = 6^6 x^6 y^6 z^6$

143. False. 0^0 is undefined.

144. True.

145. False. $x^{-n} = \dfrac{1}{x^n}$

146. False. $(x+y)^{-n} = \dfrac{1}{(x+y)^n}$

147. True. $\left(2^{-1} = \frac{1}{2},\ 2^{-2} = \frac{1}{4}\right)$

148. True. $\left[(-2)^{-1} = -\frac{1}{2},\ (-2)^{-2} = \frac{1}{4}\right]$

149.
$$\begin{aligned} 110 \times 365 \times 31{,}500{,}000 &= 1.1 \times 10^2 \times 3.65 \times 10^2 \times 3.15 \times 10^7 \\ &= 12.64725 \times 10^{11} \\ &= 1.264725 \times 10^1 \times 10^{11} = 1.264725 \times 10^{12} \end{aligned}$$

150.
$$\begin{aligned} 8 \times 365 \times 80 &= 8 \times 3.65 \times 10^2 \times 8 \times 10^1 \\ &= 233.6 \times 10^3 \\ &= 2.336 \times 10^2 \times 10^3 = 2.336 \times 10^5 \end{aligned}$$

Exercises 0.3 (page 39)

1. 0

2. positive

3. not

4. $(6^2)^{1/3}$, $\left(6^{1/3}\right)^2$

5. $a^{1/n}$

6. $|a|$

7. $\sqrt[n]{ab}$

8. $\dfrac{\sqrt[n]{a}}{\sqrt[n]{b}}$

9. $\neq$

10. $\sqrt[mn]{x}$

EXERCISES 0.3

11. $9^{1/2} = (3^2)^{1/2} = 3$

12. $8^{1/3} = (2^3)^{1/3} = 2$

13. $\left(\dfrac{1}{25}\right)^{1/2} = \left[\left(\dfrac{1}{5}\right)^2\right]^{1/2} = \dfrac{1}{5}$

14. $\left(\dfrac{16}{625}\right)^{1/4} = \left[\left(\dfrac{2}{5}\right)^4\right]^{1/4} = \dfrac{2}{5}$

15. $-81^{1/4} = -(3^4)^{1/4} = -3$

16. $-\left(\dfrac{8}{27}\right)^{1/3} = -\left[\left(\dfrac{2}{3}\right)^3\right]^{1/3} = -\dfrac{2}{3}$

17. $(10{,}000)^{1/4} = (10^4)^{1/4} = 10$

18. $(1{,}024)^{1/5} = (4^5)^{1/5} = 4$

19. $\left(-\dfrac{27}{8}\right)^{1/3} = \left[\left(-\dfrac{3}{2}\right)^3\right]^{1/3} = -\dfrac{3}{2}$

20. $-64^{1/3} = -\left(4^3\right)^{1/3} = -4$

21. $(-64)^{1/2} \Rightarrow$ not a real number

22. $(-125)^{1/3} = \left[(-5)^3\right]^{1/3} = -5$

23. $(16a^2)^{1/2} = \left[(4a)^2\right]^{1/2} = 4|a|$

24. $(25a^4)^{1/2} = \left[(5a^2)^2\right]^{1/2} = 5|a^2| = 5a^2$

25. $(16a^4)^{1/4} = \left[(2a)^4\right]^{1/4} = 2|a|$

26. $(-64a^3)^{1/3} = \left[(-4a)^3\right]^{1/3} = -4a$

27. $(-32a^5)^{1/5} = \left[(-2a)^5\right]^{1/5} = -2a$

28. $(64a^6)^{1/6} = \left[(2a)^6\right]^{1/6} = 2|a|$

29. $(-216b^6)^{1/3} = \left[(-6b^2)^3\right]^{1/3} = -6b^2$

30. $(256t^8)^{1/4} = \left[(4t^2)^4\right]^{1/4} = 4|t^2| = 4t^2$

31. $\left(\dfrac{16a^4}{25b^2}\right)^{1/2} = \left[\left(\dfrac{4a^2}{5b}\right)^2\right]^{1/2} = \left|\dfrac{4a^2}{5b}\right| = \dfrac{4a^2}{5|b|}$

32. $\left(-\dfrac{a^5}{32b^{10}}\right)^{1/5} = \left[\left(-\dfrac{a}{2b^2}\right)^5\right]^{1/5} = -\dfrac{a}{2b^2}$

33. $\left(-\dfrac{1000x^6}{27y^3}\right)^{1/3} = \left[\left(-\dfrac{10x^2}{3y}\right)^3\right]^{1/3} = -\dfrac{10x^2}{3y}$

34. $\left(\dfrac{49t^2}{100z^4}\right)^{1/2} = \left[\left(\dfrac{7t}{10z^2}\right)^2\right]^{1/2} = \left|\dfrac{7t}{10z^2}\right| = \dfrac{7|t|}{10z^2}$

35. $4^{3/2} = \left(4^{1/2}\right)^3 = 2^3 = 8$

36. $8^{2/3} = \left(8^{1/3}\right)^2 = 2^2 = 4$

37. $-16^{3/2} = -\left(16^{1/2}\right)^3 = -(4)^3 = -64$

38. $(-8)^{2/3} = \left[(-8)^{1/3}\right]^2 = (-2)^2 = 4$

EXERCISES 0.3

39. $-1000^{2/3} = -\left(1000^{1/3}\right)^2 = -(10)^2 = -100$

40. $100^{3/2} = \left(100^{1/2}\right)^3 = 10^3 = 1{,}000$

41. $64^{-1/2} = \dfrac{1}{64^{1/2}} = \dfrac{1}{8}$

42. $25^{-1/2} = \dfrac{1}{25^{1/2}} = \dfrac{1}{5}$

43. $64^{-3/2} = \dfrac{1}{64^{3/2}} = \dfrac{1}{\left(64^{1/2}\right)^3} = \dfrac{1}{8^3} = \dfrac{1}{512}$

44. $49^{-3/2} = \dfrac{1}{49^{3/2}} = \dfrac{1}{\left(49^{1/2}\right)^3} = \dfrac{1}{7^3} = \dfrac{1}{343}$

45. $-9^{-3/2} = -\dfrac{1}{9^{3/2}} = -\dfrac{1}{\left(9^{1/2}\right)^3} = -\dfrac{1}{3^3} = -\dfrac{1}{27}$

46. $(-27)^{-2/3} = \dfrac{1}{(-27)^{2/3}} = \dfrac{1}{\left[(-27)^{1/3}\right]^2} = \dfrac{1}{(-3)^2} = \dfrac{1}{9}$

47. $\left(\dfrac{4}{9}\right)^{5/2} = \left[\left(\dfrac{4}{9}\right)^{1/2}\right]^5 = \left(\dfrac{2}{3}\right)^5 = \dfrac{32}{243}$

48. $\left(\dfrac{25}{81}\right)^{3/2} = \left[\left(\dfrac{25}{81}\right)^{1/2}\right]^3 = \left(\dfrac{5}{9}\right)^3 = \dfrac{125}{729}$

49. $\left(-\dfrac{27}{64}\right)^{-2/3} = \left(-\dfrac{64}{27}\right)^{2/3} = \left[\left(-\dfrac{64}{27}\right)^{1/3}\right]^2 = \left(-\dfrac{4}{3}\right)^2 = \dfrac{16}{9}$

50. $\left(\dfrac{125}{8}\right)^{-4/3} = \left(\dfrac{8}{125}\right)^{4/3} = \left[\left(\dfrac{8}{125}\right)^{1/3}\right]^4 = \left(\dfrac{2}{5}\right)^4 = \dfrac{16}{625}$

51. $(100s^4)^{1/2} = 100^{1/2}(s^4)^{1/2} = 10s^2$

52. $(64u^6v^3)^{1/3} = 64^{1/3}(u^6)^{1/3}(v^3)^{1/3} = 4u^2v$

53. $(32y^{10}z^5)^{-1/5} = \dfrac{1}{(32y^{10}z^5)^{1/5}} = \dfrac{1}{32^{1/5}(y^{10})^{1/5}(z^5)^{1/5}} = \dfrac{1}{2y^2z}$

54. $(625a^4b^8)^{-1/4} = \dfrac{1}{(625a^4b^8)^{1/4}} = \dfrac{1}{625^{1/4}(a^4)^{1/4}(b^8)^{1/4}} = \dfrac{1}{5ab^2}$

55. $(x^{10}y^5)^{3/5} = x^{30/5}y^{15/5} = x^6y^3$

56. $(64a^6b^{12})^{5/6} = 64^{5/6}a^{30/6}b^{60/6} = \left(64^{1/6}\right)^5a^5b^{10} = 2^5a^5b^{10} = 32a^5b^{10}$

57. $\left(r^8s^{16}\right)^{-3/4} = r^{-24/4}s^{-48/4} = r^{-6}s^{-12} = \dfrac{1}{r^6s^{12}}$

58. $\left(-8x^9y^{12}\right)^{-2/3} = (-8)^{-2/3}x^{-18/3}y^{-24/3} = \dfrac{1}{(-8)^{2/3}}x^{-6}y^{-8} = \dfrac{1}{(-2)^2x^6y^8} = \dfrac{1}{4x^6y^8}$

EXERCISES 0.3

59. $\left(-\dfrac{8a^6}{125b^9}\right)^{2/3} = \dfrac{(-8)^{2/3}a^{12/3}}{125^{2/3}b^{18/3}} = \dfrac{(-2)^2a^4}{5^2b^6} = \dfrac{4a^4}{25b^6}$

60. $\left(\dfrac{16x^4}{625y^8}\right)^{3/4} = \dfrac{16^{3/4}x^{12/4}}{625^{3/4}y^{24/4}} = \dfrac{2^3x^3}{5^3y^6} = \dfrac{8x^3}{125y^6}$

61. $\left(\dfrac{27r^6}{1000s^{12}}\right)^{-2/3} = \left(\dfrac{1000s^{12}}{27r^6}\right)^{2/3} = \dfrac{1000^{2/3}s^{24/3}}{27^{2/3}r^{12/3}} = \dfrac{10^2s^8}{3^2r^4} = \dfrac{100s^8}{9r^4}$

62. $\left(-\dfrac{32m^{10}}{243n^{15}}\right)^{-2/5} = \left(\dfrac{-243n^{15}}{32m^{10}}\right)^{2/5} = \dfrac{(-243)^{2/5}n^{30/5}}{32^{2/5}m^{20/5}} = \dfrac{(-3)^2n^6}{2^2m^4} = \dfrac{9n^6}{4m^4}$

63. $\dfrac{a^{2/5}a^{4/5}}{a^{1/5}} = \dfrac{a^{6/5}}{a^{1/5}} = a^{5/5} = a$

64. $\dfrac{x^{6/7}x^{3/7}}{x^{2/7}x^{5/7}} = \dfrac{x^{9/7}}{x^{7/7}} = x^{2/7}$

65. $\sqrt{49} = \sqrt{7^2} = 7$

66. $\sqrt{81} = \sqrt{9^2} = 9$

67. $\sqrt[3]{125} = \sqrt[3]{5^3} = 5$

68. $\sqrt[3]{-64} = \sqrt[3]{(-4)^3} = -4$

69. $\sqrt[3]{-125} = \sqrt[3]{(-5)^3} = -5$

70. $\sqrt[5]{-243} = \sqrt[5]{(-3)^5} = -3$

71. $\sqrt[5]{-\dfrac{32}{100{,}000}} = \sqrt[5]{\left(-\dfrac{2}{10}\right)^5} = -\dfrac{2}{10} = -\dfrac{1}{5}$

72. $\sqrt[4]{\dfrac{256}{625}} = \sqrt[4]{\left(\dfrac{4}{5}\right)^4} = \dfrac{4}{5}$

73. $\sqrt{36x^2} = \sqrt{(6x)^2} = |6x| = 6|x|$

74. $-\sqrt{25y^2} = -\sqrt{(5y)^2} = -|5y| = -5|y|$

75. $\sqrt{9y^4} = \sqrt{(3y^2)^2} = |3y^2| = 3y^2$

76. $\sqrt{a^4b^8} = \sqrt{(a^2b^4)^2} = |a^2b^4| = a^2b^4$

77. $\sqrt[3]{8y^3} = \sqrt[3]{(2y)^3} = 2y$

78. $\sqrt[3]{-27z^9} = \sqrt[3]{(-3z^3)^3} = -3z^3$

79. $\sqrt[4]{\dfrac{x^4y^8}{z^{12}}} = \sqrt[4]{\left(\dfrac{xy^2}{z^3}\right)^4} = \left|\dfrac{xy^2}{z^3}\right| = \dfrac{|x|y^2}{|z^3|}$

80. $\sqrt[5]{\dfrac{a^{10}b^5}{c^{15}}} = \sqrt[5]{\left(\dfrac{a^2b}{c^3}\right)^5} = \dfrac{a^2b}{c^3}$

81. $\sqrt{8} - \sqrt{2} = \sqrt{4}\sqrt{2} - \sqrt{2} = 2\sqrt{2} - \sqrt{2} = \sqrt{2}$

82. $\sqrt{75} - 2\sqrt{27} = \sqrt{25}\sqrt{3} - 2\sqrt{9}\sqrt{3} = 5\sqrt{3} - 2(3)\sqrt{3} = 5\sqrt{3} - 6\sqrt{3} = -\sqrt{3}$

83. $\sqrt{200x^2} + \sqrt{98x^2} = \sqrt{100x^2}\sqrt{2} + \sqrt{49x^2}\sqrt{2} = 10x\sqrt{2} + 7x\sqrt{2} = 17x\sqrt{2}$

84. $\sqrt{128a^3} - a\sqrt{162a} = \sqrt{64a^2}\sqrt{2a} - a\sqrt{81}\sqrt{2a} = 8a\sqrt{2a} - 9a\sqrt{2a} = -a\sqrt{2a}$

EXERCISES 0.3

85. $2\sqrt{48y^5} - 3y\sqrt{12y^3} = 2\sqrt{16y^4}\sqrt{3y} - 3y\sqrt{4y^2}\sqrt{3y} = 2\left(4y^2\right)\sqrt{3y} - 3y(2y)\sqrt{3y}$
$= 8y^2\sqrt{3y} - 6y^2\sqrt{3y} = 2y^2\sqrt{3y}$

86. $y\sqrt{112y} + 4\sqrt{175y^3} = y\sqrt{16}\sqrt{7y} + 4\sqrt{25y^2}\sqrt{7y} = y(4)\sqrt{7y} + 4(5y)\sqrt{7y}$
$= 4y\sqrt{7y} + 20y\sqrt{7y} = 24y\sqrt{7y}$

87. $2\sqrt[3]{81} + 3\sqrt[3]{24} = 2\sqrt[3]{27}\sqrt[3]{3} + 3\sqrt[3]{8}\sqrt[3]{3} = 2(3)\sqrt[3]{3} + 3(2)\sqrt[3]{3} = 6\sqrt[3]{3} + 6\sqrt[3]{3} = 12\sqrt[3]{3}$

88. $3\sqrt[4]{32} - 2\sqrt[4]{162} = 3\sqrt[4]{16}\sqrt[4]{2} - 2\sqrt[4]{81}\sqrt[4]{2} = 3(2)\sqrt[4]{2} - 2(3)\sqrt[4]{2} = 6\sqrt[4]{2} - 6\sqrt[4]{2} = 0$

89. $\sqrt[4]{768z^5} + \sqrt[4]{48z^5} = \sqrt[4]{256z^4}\sqrt[4]{3z} + \sqrt[4]{16z^4}\sqrt[4]{3z} = 4z\sqrt[4]{3z} + 2z\sqrt[4]{3z} = 6z\sqrt[4]{3z}$

90. $-2\sqrt[5]{64y^2} + 3\sqrt[5]{486y^2} = -2\sqrt[5]{32}\sqrt[5]{2y^2} + 3\sqrt[5]{243}\sqrt[5]{2y^2} = -2(2)\sqrt[5]{2y^2} + 3(3)\sqrt[5]{2y^2}$
$= -4\sqrt[5]{2y^2} + 9\sqrt[5]{2y^2} = 5\sqrt[5]{2y^2}$

91. $\sqrt{8x^2y} - x\sqrt{2y} + \sqrt{50x^2y} = \sqrt{4x^2}\sqrt{2y} - x\sqrt{2y} + \sqrt{25x^2}\sqrt{2y}$
$= 2x\sqrt{2y} - x\sqrt{2y} + 5x\sqrt{2y} = 6x\sqrt{2y}$

92. $3x\sqrt{18x} + 2\sqrt{2x^3} - \sqrt{72x^3} = 3x\sqrt{9}\sqrt{2x} + 2\sqrt{x^2}\sqrt{2x} - \sqrt{36x^2}\sqrt{2x}$
$= 3x(3)\sqrt{2x} + 2x\sqrt{2x} - 6x\sqrt{2x}$
$= 9x\sqrt{2x} + 2x\sqrt{2x} - 6x\sqrt{2x} = 5x\sqrt{2x}$

93. $\sqrt[3]{16xy^4} + y\sqrt[3]{2xy} - \sqrt[3]{54xy^4} = \sqrt[3]{8y^3}\sqrt[3]{2xy} + y\sqrt[3]{2xy} - \sqrt[3]{27y^3}\sqrt[3]{2xy}$
$= 2y\sqrt[3]{2xy} + y\sqrt[3]{2xy} - 3y\sqrt[3]{2xy} = 0$

94. $\sqrt[4]{512x^5} - \sqrt[4]{32x^5} + \sqrt[4]{1{,}250x^5} = \sqrt[4]{256x^4}\sqrt[4]{2x} - \sqrt[4]{16x^4}\sqrt[4]{2x} + \sqrt[4]{625x^4}\sqrt[4]{2x}$
$= 4x\sqrt[4]{2x} - 2x\sqrt[4]{2x} + 5x\sqrt[4]{2x} = 7x\sqrt[4]{2x}$

95. $\dfrac{3}{\sqrt{3}} = \dfrac{3}{\sqrt{3}} \cdot \dfrac{\sqrt{3}}{\sqrt{3}} = \dfrac{3\sqrt{3}}{3} = \sqrt{3}$

96. $\dfrac{6}{\sqrt{5}} = \dfrac{6}{\sqrt{5}} \cdot \dfrac{\sqrt{5}}{\sqrt{5}} = \dfrac{6\sqrt{5}}{5}$

97. $\dfrac{2}{\sqrt{x}} = \dfrac{2}{\sqrt{x}} \cdot \dfrac{\sqrt{x}}{\sqrt{x}} = \dfrac{2\sqrt{x}}{x}$

98. $\dfrac{8}{\sqrt{y}} = \dfrac{8}{\sqrt{y}} \cdot \dfrac{\sqrt{y}}{\sqrt{y}} = \dfrac{8\sqrt{y}}{y}$

99. $\dfrac{2}{\sqrt[3]{2}} = \dfrac{2}{\sqrt[3]{2}} \cdot \dfrac{\sqrt[3]{4}}{\sqrt[3]{4}} = \dfrac{2\sqrt[3]{4}}{\sqrt[3]{8}} = \dfrac{2\sqrt[3]{4}}{2} = \sqrt[3]{4}$

100. $\dfrac{4d}{\sqrt[3]{9}} = \dfrac{4d}{\sqrt[3]{9}} \cdot \dfrac{\sqrt[3]{3}}{\sqrt[3]{3}} = \dfrac{4d\sqrt[3]{3}}{\sqrt[3]{27}} = \dfrac{4d\sqrt[3]{3}}{3}$

101. $\dfrac{5a}{\sqrt[3]{25a}} = \dfrac{5a}{\sqrt[3]{25a}} \cdot \dfrac{\sqrt[3]{5a^2}}{\sqrt[3]{5a^2}} = \dfrac{5a\sqrt[3]{5a^2}}{\sqrt[3]{125a^3}} = \dfrac{5a\sqrt[3]{5a^2}}{5a} = \sqrt[3]{5a^2}$

EXERCISES 0.3

102. $\dfrac{7}{\sqrt[3]{36c}} = \dfrac{7}{\sqrt[3]{36c}} \cdot \dfrac{\sqrt[3]{6c^2}}{\sqrt[3]{6c^2}} = \dfrac{7\sqrt[3]{6c^2}}{\sqrt[3]{216c^3}} = \dfrac{7\sqrt[3]{6c^2}}{6c}$

103. $\dfrac{2b}{\sqrt[4]{3a^2}} = \dfrac{2b}{\sqrt[4]{3a^2}} \cdot \dfrac{\sqrt[4]{27a^2}}{\sqrt[4]{27a^2}} = \dfrac{2b\sqrt[4]{27a^2}}{\sqrt[4]{81a^4}} = \dfrac{2b\sqrt[4]{27a^2}}{3a}$

104. $\sqrt{\dfrac{x}{2y}} = \dfrac{\sqrt{x}}{\sqrt{2y}} = \dfrac{\sqrt{x}}{\sqrt{2y}} \cdot \dfrac{\sqrt{2y}}{\sqrt{2y}} = \dfrac{\sqrt{2xy}}{2y}$

105. $\sqrt[3]{\dfrac{2u^4}{9v}} = \dfrac{\sqrt[3]{2u^4}}{\sqrt[3]{9v}} = \dfrac{\sqrt[3]{u^3}\sqrt[3]{2u}}{\sqrt[3]{9v}} \cdot \dfrac{\sqrt[3]{3v^2}}{\sqrt[3]{3v^2}} = \dfrac{u\sqrt[3]{6uv^2}}{\sqrt[3]{27v^3}} = \dfrac{u\sqrt[3]{6uv^2}}{3v}$

106. $\sqrt[3]{-\dfrac{3s^5}{4r^2}} = \dfrac{\sqrt[3]{-3s^5}}{\sqrt[3]{4r^2}} = \dfrac{\sqrt[3]{-s^3}\sqrt[3]{3s^2}}{\sqrt[3]{4r^2}} \cdot \dfrac{\sqrt[3]{2r}}{\sqrt[3]{2r}} = \dfrac{-s\sqrt[3]{6rs^2}}{\sqrt[3]{8r^3}} = -\dfrac{s\sqrt[3]{6rs^2}}{2r}$

107. $\dfrac{\sqrt{5}}{10} = \dfrac{\sqrt{5}}{10} \cdot \dfrac{\sqrt{5}}{\sqrt{5}} = \dfrac{5}{10\sqrt{5}} = \dfrac{1}{2\sqrt{5}}$

108. $\dfrac{\sqrt{y}}{3} = \dfrac{\sqrt{y}}{3} \cdot \dfrac{\sqrt{y}}{\sqrt{y}} = \dfrac{y}{3\sqrt{y}}$

109. $\dfrac{\sqrt[3]{9}}{3} = \dfrac{\sqrt[3]{9}}{3} \cdot \dfrac{\sqrt[3]{3}}{\sqrt[3]{3}} = \dfrac{\sqrt[3]{27}}{3\sqrt[3]{3}} = \dfrac{3}{3\sqrt[3]{3}} = \dfrac{1}{\sqrt[3]{3}}$

110. $\dfrac{\sqrt[3]{16b^2}}{16} = \dfrac{\sqrt[3]{16b^2}}{16} \cdot \dfrac{\sqrt[3]{4b}}{\sqrt[3]{4b}} = \dfrac{\sqrt[3]{64b^3}}{16\sqrt[3]{4b}} = \dfrac{4b}{16\sqrt[3]{4b}} = \dfrac{b}{4\sqrt[3]{4b}}$

111. $\dfrac{\sqrt[5]{16b^3}}{64a} = \dfrac{\sqrt[5]{16b^3}}{64a} \cdot \dfrac{\sqrt[5]{2b^2}}{\sqrt[5]{2b^2}} = \dfrac{\sqrt[5]{32b^5}}{64a\sqrt[5]{2b^2}} = \dfrac{2b}{64a\sqrt[5]{2b^2}} = \dfrac{b}{32a\sqrt[5]{2b^2}}$

112. $\sqrt{\dfrac{3x}{57}} = \dfrac{\sqrt{3x}}{\sqrt{57}} = \dfrac{\sqrt{3x}}{\sqrt{57}} \cdot \dfrac{\sqrt{3x}}{\sqrt{3x}} = \dfrac{3x}{\sqrt{171x}} = \dfrac{3x}{\sqrt{9}\sqrt{19x}} = \dfrac{3x}{3\sqrt{19x}} = \dfrac{x}{\sqrt{19x}}$

113. $\sqrt{\dfrac{1}{3}} - \sqrt{\dfrac{1}{27}} = \dfrac{\sqrt{1}}{\sqrt{3}} - \dfrac{\sqrt{1}}{\sqrt{27}} = \dfrac{1}{\sqrt{3}} \cdot \dfrac{\sqrt{3}}{\sqrt{3}} - \dfrac{1}{\sqrt{27}} \cdot \dfrac{\sqrt{3}}{\sqrt{3}} = \dfrac{\sqrt{3}}{3} - \dfrac{\sqrt{3}}{\sqrt{81}} = \dfrac{\sqrt{3}}{3} - \dfrac{\sqrt{3}}{9}$

$= \dfrac{3\sqrt{3}}{9} - \dfrac{\sqrt{3}}{9} = \dfrac{2\sqrt{3}}{9}$

114. $\sqrt[3]{\dfrac{1}{2}} + \sqrt[3]{\dfrac{1}{16}} = \dfrac{\sqrt[3]{1}}{\sqrt[3]{2}} + \dfrac{\sqrt[3]{1}}{\sqrt[3]{16}} = \dfrac{1}{\sqrt[3]{2}} \cdot \dfrac{\sqrt[3]{4}}{\sqrt[3]{4}} + \dfrac{1}{\sqrt[3]{16}} \cdot \dfrac{\sqrt[3]{4}}{\sqrt[3]{4}} = \dfrac{\sqrt[3]{4}}{\sqrt[3]{8}} + \dfrac{\sqrt[3]{4}}{\sqrt[3]{64}} = \dfrac{\sqrt[3]{4}}{2} + \dfrac{\sqrt[3]{4}}{4}$

$= \dfrac{2\sqrt[3]{4}}{4} + \dfrac{\sqrt[3]{4}}{4} = \dfrac{3\sqrt[3]{4}}{4}$

EXERCISES 0.3

115. $\sqrt{\frac{x}{8}} - \sqrt{\frac{x}{2}} + \sqrt{\frac{x}{32}} = \frac{\sqrt{x}}{\sqrt{8}} - \frac{\sqrt{x}}{\sqrt{2}} + \frac{\sqrt{x}}{\sqrt{32}} = \frac{\sqrt{x}}{\sqrt{8}} \cdot \frac{\sqrt{2}}{\sqrt{2}} - \frac{\sqrt{x}}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} + \frac{\sqrt{x}}{\sqrt{32}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$

$$= \frac{\sqrt{2x}}{\sqrt{16}} - \frac{\sqrt{2x}}{\sqrt{4}} + \frac{\sqrt{2x}}{\sqrt{64}}$$

$$= \frac{\sqrt{2x}}{4} - \frac{\sqrt{2x}}{2} + \frac{\sqrt{2x}}{8}$$

$$= \frac{2\sqrt{2x}}{8} - \frac{4\sqrt{2x}}{8} + \frac{\sqrt{2x}}{8} = -\frac{\sqrt{2x}}{8}$$

116. $\sqrt[3]{\frac{y}{4}} + \sqrt[3]{\frac{y}{32}} - \sqrt[3]{\frac{y}{500}} = \frac{\sqrt[3]{y}}{\sqrt[3]{4}} + \frac{\sqrt[3]{y}}{\sqrt[3]{32}} - \frac{\sqrt[3]{y}}{\sqrt[3]{500}} = \frac{\sqrt[3]{y}}{\sqrt[3]{4}} \cdot \frac{\sqrt[3]{2}}{\sqrt[3]{2}} + \frac{\sqrt[3]{y}}{\sqrt[3]{32}} \cdot \frac{\sqrt[3]{2}}{\sqrt[3]{2}} - \frac{\sqrt[3]{y}}{\sqrt[3]{500}} \cdot \frac{\sqrt[3]{2}}{\sqrt[3]{2}}$

$$= \frac{\sqrt[3]{2y}}{\sqrt[3]{8}} + \frac{\sqrt[3]{2y}}{\sqrt[3]{64}} - \frac{\sqrt[3]{2y}}{\sqrt[3]{1{,}000}}$$

$$= \frac{\sqrt[3]{2y}}{2} + \frac{\sqrt[3]{2y}}{4} - \frac{\sqrt[3]{2y}}{10}$$

$$= \frac{10\sqrt[3]{2y}}{20} + \frac{5\sqrt[3]{2y}}{20} - \frac{2\sqrt[3]{2y}}{20} = \frac{13\sqrt[3]{2y}}{20}$$

117. $\sqrt[4]{9} = 9^{1/4} = (3^2)^{1/4} = 3^{2/4} = 3^{1/2} = \sqrt{3}$

118. $\sqrt[6]{27} = 27^{1/6} = (3^3)^{1/6} = 3^{3/6} = 3^{1/2} = \sqrt{3}$

119. $\sqrt[10]{16x^6} = (16x^6)^{1/10} = (2^4x^6)^{1/10} = 2^{4/10}x^{6/10} = 2^{2/5}x^{3/5} = (2^2x^3)^{1/5} = \sqrt[5]{4x^3}$

120. $\sqrt[6]{27x^9} = (27x^9)^{1/6} = (3^3x^9)^{1/6} = 3^{3/6}x^{9/6} = 3^{1/2}x^{3/2} = (3x^3)^{1/2} = \sqrt{3x^3} = x\sqrt{3x}$

121. $s = \sqrt{120} = \sqrt{4}\sqrt{30} = 2\sqrt{30}$ inches

122. $s = \sqrt[3]{2000} = \sqrt[3]{1000}\sqrt[3]{2} = 10\sqrt[3]{2}$ inches

123. $\sqrt{2}\sqrt[3]{2} = 2^{1/2} \cdot 2^{1/3} = 2^{3/6} \cdot 2^{2/6} = \sqrt[6]{2^3}\sqrt[6]{2^2} = \sqrt[6]{8}\sqrt[6]{4} = \sqrt[6]{32}$

124. $\sqrt{3}\sqrt[3]{5} = 3^{1/2}5^{1/3} = 3^{3/6}5^{2/6} = \sqrt[6]{3^3}\sqrt[6]{5^2} = \sqrt[6]{27}\sqrt[6]{25} = \sqrt[6]{675}$

125. $\frac{\sqrt[4]{3}}{\sqrt{2}} = \frac{3^{1/4}}{2^{1/2}} = \frac{3^{1/4}}{2^{2/4}} = \frac{\sqrt[4]{3}}{\sqrt[4]{2^2}} = \frac{\sqrt[4]{3}}{\sqrt[4]{4}} = \frac{\sqrt[4]{3}}{\sqrt[4]{4}} \cdot \frac{\sqrt[4]{4}}{\sqrt[4]{4}} = \frac{\sqrt[4]{12}}{\sqrt[4]{16}} = \frac{\sqrt[4]{12}}{2}$

126. $\frac{\sqrt[3]{2}}{\sqrt{5}} = \frac{2^{1/3}}{5^{1/2}} = \frac{2^{2/6}}{5^{3/6}} = \frac{\sqrt[6]{2^2}}{\sqrt[6]{5^3}} = \frac{\sqrt[6]{4}}{\sqrt[6]{125}} = \frac{\sqrt[6]{4}}{\sqrt[6]{125}} \cdot \frac{\sqrt[6]{125}}{\sqrt[6]{125}} = \frac{\sqrt[6]{500}}{\sqrt[6]{15{,}625}} = \frac{\sqrt[6]{500}}{5}$

127. If $a^{1/n} = x$, then $x^n = a$. However, if n is even, x^n cannot be negative.

128. $\sqrt[4]{x^4} = |x|$. Since $|x| = x$ if $x \geq 0$, then $\sqrt[4]{x^4} = x$ if $x \geq 0$.

EXERCISES 0.3

129. To rationalize a denominator means to write an equivalent fraction with a denominator equal to a rational number.

130. $\sqrt[n]{\dfrac{x}{y}} = \left(\dfrac{x}{y}\right)^{1/n} = \dfrac{x^{1/n}}{y^{1/n}} = \dfrac{\sqrt[n]{x}}{\sqrt[n]{y}}$

131. $\left(\dfrac{x}{y}\right)^{-m/n} = \dfrac{x^{-m/n}}{y^{-m/n}} = \dfrac{x^{-m/n}}{y^{-m/n}} \cdot \dfrac{x^{m/n}y^{m/n}}{x^{m/n}y^{m/n}} = \dfrac{y^{m/n}}{x^{m/n}} = \dfrac{(y^m)^{1/n}}{(x^m)^{1/n}} = \left(\dfrac{y^m}{x^m}\right)^{1/n} = \sqrt[n]{\dfrac{y^m}{x^m}}$

132. Consider the case when n is even, m is odd and x is negative. Then $x^{m/n} = \left(x^{1/n}\right)^m = \left(\sqrt[n]{x}\right)^m$. Thus, $\sqrt[n]{x}$ must be a real number for the expression to be defined.

133. $(-16)^{1/4}$ is undefined. g

134. $(1024)^{1/10} = (2^{10})^{1/10} = 2$. c

135. $0^{-111/19} = 0$. d

136. $(-1)^{-12/19} = \left(\sqrt[19]{-1}\right)^{12} = 1$. f

137. $\sqrt[87]{-1} = -1$. h

138. $\sqrt[87]{x^{88}} = \sqrt[87]{x^{87}}\sqrt[87]{x} = x\sqrt[87]{x}$. b

139. $\dfrac{1}{\sqrt[87]{x}} = \dfrac{1}{\sqrt[87]{x}} \cdot \dfrac{\sqrt[87]{x^{86}}}{\sqrt[87]{x^{86}}} = \dfrac{\sqrt[87]{x^{86}}}{x}$. e

140. $\sqrt{\sqrt[3]{\sqrt[3]{-512}}} = \sqrt[3]{-8} = -2$. a

Exercises 0.4 (page 52)

1. monomial, variables

2. degree, variables

3. trinomial

4. binomial

5. one

6. zero

7. like

8. degree

9. coefficients, variables

10. $3\sqrt{x} - 2$

11. yes, trinomial, 2nd degree

12. yes, binomial, 3rd degree

13. no

14. no

15. yes, binomial, 3rd degree

16. yes, monomial, 5th degree

17. yes, monomial, 0th degree

18. no

19. yes, monomial, no degree

20. yes, none, 3rd degree

21. $(x^3 - 3x^2) + (5x^3 - 8x) = x^3 - 3x^2 + 5x^3 - 8x = x^3 + 5x^3 - 3x^2 - 8x = 6x^3 - 3x^2 - 8x$

22. $\begin{aligned}\left(2x^4 - 5x^3\right) + \left(7x^3 - x^4 + 2x\right) &= 2x^4 - 5x^3 + 7x^3 - x^4 + 2x \\ &= 2x^4 - x^4 - 5x^3 + 7x^3 + 2x = x^4 + 2x^3 + 2x\end{aligned}$

23. $\begin{aligned}\left(y^5 + 2y^3 + 7\right) - \left(y^5 - 2y^3 - 7\right) &= y^5 + 2y^3 + 7 - y^5 + 2y^3 + 7 \\ &= y^5 - y^5 + 2y^3 + 2y^3 + 7 + 7 = 4y^3 + 14\end{aligned}$

EXERCISES 0.4

24. $\big(3t^7 - 7t^3 + 3\big) - \big(7t^7 - 3t^3 + 7\big) = 3t^7 - 7t^3 + 3 - 7t^7 + 3t^3 - 7$
$= 3t^7 - 7t^7 - 7t^3 + 3t^3 + 3 - 7 = -4t^7 - 4t^3 - 4$

25. $2\big(x^2 + 3x - 1\big) - 3\big(x^2 + 2x - 4\big) + 4 = 2\big(x^2\big) + 2(3x) + 2(-1) - 3\big(x^2\big) - 3(2x) - 3(-4) + 4$
$= 2x^2 + 6x - 2 - 3x^2 - 6x + 12 + 4$
$= 2x^2 - 3x^2 + 6x - 6x - 2 + 12 + 4 = -x^2 + 14$

26. $5\big(x^3 - 8x + 3\big) + 2\big(3x^2 + 5x\big) - 7 = 5\big(x^3\big) + 5(-8x) + 5(3) + 2\big(3x^2\big) + 2(5x) - 7$
$= 5x^3 - 40x + 15 + 6x^2 + 10x - 7$
$= 5x^3 + 6x^2 - 40x + 10x + 15 - 7 = 5x^3 + 6x^2 - 30x + 8$

27. $8\big(t^2 - 2t + 5\big) + 4\big(t^2 - 3t + 2\big) - 6\big(2t^2 - 8\big)$
$= 8\big(t^2\big) + 8(-2t) + 8(5) + 4\big(t^2\big) + 4(-3t) + 4(2) - 6\big(2t^2\big) - 6(-8)$
$= 8t^2 - 16t + 40 + 4t^2 - 12t + 8 - 12t^2 + 48$
$= 8t^2 + 4t^2 - 12t^2 - 16t - 12t + 40 + 8 + 48 = -28t + 96$

28. $-3\big(x^3 - x\big) + 2\big(x^2 + x\big) + 3\big(x^3 - 2x\big) = -3\big(x^3\big) - 3(-x) + 2\big(x^2\big) + 2(x) + 3\big(x^3\big) + 3(-2x)$
$= -3x^3 + 3x + 2x^2 + 2x + 3x^3 - 6x$
$= -3x^3 + 3x^3 + 2x^2 + 3x + 2x - 6x = 2x^2 - x$

29. $y\big(y^2 - 1\big) - y^2(y + 2) - y(2y - 2) = y\big(y^2\big) + y(-1) - y^2(y) - y^2(2) - y(2y) - y(-2)$
$= y^3 - y - y^3 - 2y^2 - 2y^2 + 2y$
$= y^3 - y^3 - 2y^2 - 2y^2 - y + 2y = -4y^2 + y$

30. $-4a^2(a + 1) + 3a\big(a^2 - 4\big) - a^2(a + 2)$
$= -4a^2(a) - 4a^2(1) + 3a\big(a^2\big) + 3a(-4) - a^2(a) - a^2(2)$
$= -4a^3 - 4a^2 + 3a^3 - 12a - a^3 - 2a^2$
$= -4a^3 + 3a^3 - a^3 - 4a^2 - 2a^2 - 12a = -2a^3 - 6a^2 - 12a$

31. $xy(x - 4y) - y\big(x^2 + 3xy\big) + xy(2x + 3y)$
$= xy(x) + xy(-4y) - y\big(x^2\big) - y(3xy) + xy(2x) + xy(3y)$
$= x^2y - 4xy^2 - x^2y - 3xy^2 + 2x^2y + 3xy^2$
$= x^2y - x^2y + 2x^2y - 4xy^2 - 3xy^2 + 3xy^2 = 2x^2y - 4xy^2$

32. $3mn(m + 2n) - 6m(3mn + 1) - 2n(4mn - 1)$
$= 3mn(m) + 3mn(2n) - 6m(3mn) - 6m(1) - 2n(4mn) - 2n(-1)$
$= 3m^2n + 6mn^2 - 18m^2n - 6m - 8mn^2 + 2n$
$= 3m^2n - 18m^2n + 6mn^2 - 8mn^2 - 6m + 2n = -15m^2n - 2mn^2 - 6m + 2n$

33. $2x^2y^3(4xy^4) = 2(4)x^2xy^3y^4 = 8x^3y^7$

34. $-15a^3b\big(-2a^2b^3\big) = -15(-2)a^3a^2bb^3$
$= 30a^5b^4$

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35. $-3m^2n\left(2mn^2\right)\left(-\dfrac{mn}{12}\right) = (-3)(2)\left(-\dfrac{1}{12}\right)m^2mmnn^2n = \dfrac{6}{12}m^4n^4 = \dfrac{m^4n^4}{2}$

36. $-\dfrac{3r^2s^3}{5}\left(\dfrac{2r^2s}{3}\right)\left(\dfrac{15rs^2}{2}\right) = \left(-\dfrac{3}{5}\right)\left(\dfrac{2}{3}\right)\left(\dfrac{15}{2}\right)r^2r^2rs^3ss^2 = -3r^5s^6$

37. $-4rs(r^2+s^2) = -4rs(r^2) - 4rs(s^2) = -4r^3s - 4rs^3$

38. $6u^2v(2uv^2 - y) = 6u^2v(2uv^2) + 6u^2v(-y) = 12u^3v^3 - 6u^2vy$

39. $6ab^2c\left(2ac + 3bc^2 - 4ab^2c\right) = 6ab^2c(2ac) + 6ab^2c\left(3bc^2\right) + 6ab^2c\left(-4ab^2c\right)$
$= 12a^2b^2c^2 + 18ab^3c^3 - 24a^2b^4c^2$

40. $-\dfrac{mn^2}{2}\left(4mn - 6m^2 - 8\right) = -\dfrac{mn^2}{2}(4mn) - \dfrac{mn^2}{2}\left(-6m^2\right) - \dfrac{mn^2}{2}(-8)$
$= -2m^2n^3 + 3m^3n^2 + 4mn^2$

41. $(a+2)(a+2) = a^2 + 2a + 2a + 4$
$= a^2 + 4a + 4$

42. $(y-5)(y-5) = y^2 - 5y - 5y + 25$
$= y^2 - 10y + 25$

43. $(a-6)^2 = (a-6)(a-6)$
$= a^2 - 6a - 6a + 36$
$= a^2 - 12a + 36$

44. $(t+9)^2 = (t+9)(t+9)$
$= t^2 + 9t + 9t + 81$
$= t^2 + 18t + 81$

45. $(x+4)(x-4) = x^2 - 4x + 4x - 16$
$= x^2 - 16$

46. $(z+7)(z-7) = z^2 - 7z + 7z - 49$
$= z^2 - 49$

47. $(x-3)(x+5) = x^2 + 5x - 3x - 15$
$= x^2 + 2x - 15$

48. $(z+4)(z-6) = z^2 - 6z + 4z - 24$
$= z^2 - 2z - 24$

49. $(u+2)(3u-2) = 3u^2 - 2u + 6u - 4$
$= 3u^2 + 4u - 4$

50. $(4x+1)(2x-3) = 8x^2 - 12x + 2x - 3$
$= 8x^2 - 10x - 3$

51. $(5x-1)(2x+3) = 10x^2 + 15x - 2x - 3$
$= 10x^2 + 13x - 3$

52. $(4x-1)(2x-7) = 8x^2 - 28x - 2x + 7$
$= 8x^2 - 30x + 7$

53. $(3a-2b)^2 = (3a-2b)(3a-2b) = 9a^2 - 6ab - 6ab + 4b^2 = 9a^2 - 12ab + 4b^2$

54. $(4a+5b)(4a-5b) = 16a^2 - 20ab + 20ab - 25b^2 = 16a^2 - 25b^2$

55. $(3m+4n)(3m-4n) = 9m^2 - 12mn + 12mn - 16n^2 = 9m^2 - 16n^2$

56. $(4r+3s)^2 = (4r+3s)(4r+3s) = 16r^2 + 12rs + 12rs + 9s^2 = 16r^2 + 24rs + 9s^2$

57. $(2y-4x)(3y-2x) = 6y^2 - 4xy - 12xy + 8x^2 = 6y^2 - 16xy + 8x^2$

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58. $(-2x+3y)(3x+y) = -6x^2 - 2xy + 9xy + 3y^2 = -6x^2 + 7xy + 3y^2$

59. $(9x-y)(x^2-3y) = 9x^3 - 27xy - x^2y + 3y^2 = 9x^3 - x^2y - 27xy + 3y^2$

60. $(8a^2+b)(a+2b) = 8a^3 + 16a^2b + ab + 2b^2$

61. $(5z+2t)(z^2-t) = 5z^3 - 5tz + 2tz^2 - 2t^2 = 5z^3 + 2tz^2 - 5tz - 2t^2$

62. $(y-2x^2)(x^2+3y) = x^2y + 3y^2 - 2x^4 - 6x^2y = -2x^4 - 5x^2y + 3y^2$

63. $\left(\sqrt{5}+3x\right)\left(2-\sqrt{5}x\right) = 2\sqrt{5} - 5x + 6x - 3\sqrt{5}x^2 = -3\sqrt{5}\,x^2 + x + 2\sqrt{5}$

64. $\left(\sqrt{2}+x\right)\left(3+\sqrt{2}x\right) = 3\sqrt{2} + 2x + 3x + \sqrt{2}x^2 = \sqrt{2}\,x^2 + 5x + 3\sqrt{2}$

65.
$$\begin{aligned}(3x-1)^3 &= (3x-1)(3x-1)(3x-1)\\ &= \big(9x^2 - 3x - 3x + 1\big)(3x-1)\\ &= \big(9x^2 - 6x + 1\big)(3x-1)\\ &= 9x^2(3x) + 9x^2(-1) - 6x(3x) - 6x(-1) + 1(3x) + 1(-1)\\ &= 27x^3 - 9x^2 - 18x^2 + 6x + 3x - 1 = 27x^3 - 27x^2 + 9x - 1\end{aligned}$$

66.
$$\begin{aligned}(2x-3)^3 &= (2x-3)(2x-3)(2x-3)\\ &= \big(4x^2 - 6x - 6x + 9\big)(2x-3)\\ &= \big(4x^2 - 12x + 9\big)(2x-3)\\ &= 4x^2(2x) + 4x^2(-3) - 12x(2x) - 12x(-3) + 9(2x) + 9(-3)\\ &= 8x^3 - 12x^2 - 24x^2 + 36x + 18x - 27 = 8x^3 - 36x^2 + 54x - 27\end{aligned}$$

67.
$$\begin{aligned}(3x+1)\big(2x^2+4x-3\big) &= 3x\big(2x^2\big) + 3x(4x) + 3x(-3) + 1\big(2x^2\big) + 1(4x) + 1(-3)\\ &= 6x^3 + 12x^2 - 9x + 2x^2 + 4x - 3 = 6x^3 + 14x^2 - 5x - 3\end{aligned}$$

68.
$$\begin{aligned}(2x-5)\big(x^2-3x+2\big) &= 2x\big(x^2\big) + 2x(-3x) + 2x(2) - 5\big(x^2\big) - 5(-3x) - 5(2)\\ &= 2x^3 - 6x^2 + 4x - 5x^2 + 15x - 10 = 2x^3 - 11x^2 + 19x - 10\end{aligned}$$

69.
$$\begin{aligned}&(3x+2y)\big(2x^2 - 3xy + 4y^2\big)\\ &\quad= 3x\big(2x^2\big) + 3x(-3xy) + 3x\big(4y^2\big) + 2y\big(2x^2\big) + 2y(-3xy) + 2y\big(4y^2\big)\\ &\quad= 6x^3 - 9x^2y + 12xy^2 + 4x^2y - 6xy^2 + 8y^3 = 6x^3 - 5x^2y + 6xy^2 + 8y^3\end{aligned}$$

70.
$$\begin{aligned}&(4r-3s)\big(2r^2 + 4rs - 2s^2\big)\\ &\quad= 4r\big(2r^2\big) + 4r(4rs) + 4r\big(-2s^2\big) - 3s\big(2r^2\big) - 3s(4rs) - 3s\big(-2s^2\big)\\ &\quad= 8r^3 + 16r^2s - 8rs^2 - 6r^2s - 12rs^2 + 6s^3 = 8r^3 + 10r^2s - 20rs^2 + 6s^3\end{aligned}$$

71. $2y^n(3y^n + y^{-n}) = 2y^n(3y^n) + 2y^n(y^{-n}) = 6y^{n+n} + 2y^{n+(-n)} = 6y^{2n} + 2y^0 = 6y^{2n} + 2$

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72. $3a^{-n}\left(2a^n+3a^{n-1}\right)=3a^{-n}(2a^n)+3a^{-n}\left(3a^{n-1}\right)=6a^{-n+n}+9a^{-n+n-1}=6a^0+9a^{-1}$
$=6+\dfrac{9}{a}$

73. $-5x^{2n}y^n\left(2x^{2n}y^{-n}+3x^{-2n}y^n\right)=-5x^{2n}y^n\left(2x^{2n}y^{-n}\right)-5x^{2n}y^n\left(3x^{-2n}y^n\right)$
$=-10x^{2n+2n}y^{n+(-n)}-15x^{2n+(-2n)}y^{n+n}$
$=-10x^{4n}y^0-15x^0y^{2n}=-10x^{4n}-15y^{2n}$

74. $-2a^{3n}b^{2n}\left(5a^{-3n}b-ab^{-2n}\right)=-2a^{3n}b^{2n}\left(5a^{-3n}b\right)-2a^{3n}b^{2n}\left(-ab^{-2n}\right)$
$=-10a^{3n+(-3n)}b^{2n+1}+2a^{3n+1}b^{2n+(-2n)}$
$=-10a^0b^{2n+1}+2a^{3n+1}b^0=-10b^{2n+1}+2a^{3n+1}$

75. $(x^n+3)(x^n-4)=x^nx^n-4x^n+3x^n-12=x^{2n}-x^n-12$

76. $(a^n-5)(a^n-3)=a^na^n-3a^n-5a^n+15=a^{2n}-8a^n+15$

77. $(2r^n-7)(3r^n-2)=2r^n(3r^n)-2r^n(2)-7(3r^n)+14$
$=6r^{2n}-4r^n-21r^n+14=6r^{2n}-25r^n+14$

78. $(4z^n+3)(3z^n+1)=4z^n(3z^n)+4z^n(1)+3(3z^n)+3$
$=12z^{2n}+4z^n+9z^n+3=12z^{2n}+13z^n+3$

79. $x^{1/2}\left(x^{1/2}y+xy^{1/2}\right)=x^{1/2}x^{1/2}y+x^{1/2}xy^{1/2}=x^{2/2}y+x^{3/2}y^{1/2}=xy+x^{3/2}y^{1/2}$

80. $ab^{1/2}\left(a^{1/2}b^{1/2}+b^{1/2}\right)=ab^{1/2}a^{1/2}b^{1/2}+ab^{1/2}b^{1/2}=a^{3/2}b^{2/2}+ab^{2/2}=a^{3/2}b+ab$

81. $\left(a^{1/2}+b^{1/2}\right)\left(a^{1/2}-b^{1/2}\right)=a^{1/2}a^{1/2}-a^{1/2}b^{1/2}+a^{1/2}b^{1/2}-b^{1/2}b^{1/2}$
$=a^{2/2}-b^{2/2}=a-b$

82. $\left(x^{3/2}+y^{1/2}\right)^2=\left(x^{3/2}+y^{1/2}\right)\left(x^{3/2}+y^{1/2}\right)=x^{3/2}x^{3/2}+x^{3/2}y^{1/2}+x^{3/2}y^{1/2}+y^{1/2}y^{1/2}$
$=x^{6/2}+2x^{3/2}y^{1/2}+y^{2/2}$
$=x^3+2x^{3/2}y^{1/2}+y$

83. $\dfrac{2}{\sqrt{3}-1}=\dfrac{2}{\sqrt{3}-1}\cdot\dfrac{\sqrt{3}+1}{\sqrt{3}+1}=\dfrac{2\left(\sqrt{3}+1\right)}{\left(\sqrt{3}\right)^2-1^2}=\dfrac{2\left(\sqrt{3}+1\right)}{3-1}=\dfrac{2\left(\sqrt{3}+1\right)}{2}=\sqrt{3}+1$

84. $\dfrac{1}{\sqrt{5}+2}=\dfrac{1}{\sqrt{5}+2}\cdot\dfrac{\sqrt{5}-2}{\sqrt{5}-2}=\dfrac{1\left(\sqrt{5}-2\right)}{\left(\sqrt{5}\right)^2-2^2}=\dfrac{\sqrt{5}-2}{5-4}=\dfrac{\sqrt{5}-2}{1}=\sqrt{5}-2$

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85. $\dfrac{3x}{\sqrt{7}+2} = \dfrac{3x}{\sqrt{7}+2}\cdot\dfrac{\sqrt{7}-2}{\sqrt{7}-2} = \dfrac{3x\left(\sqrt{7}-2\right)}{\left(\sqrt{7}\right)^2-2^2} = \dfrac{3x\left(\sqrt{7}-2\right)}{7-4} = \dfrac{3x\left(\sqrt{7}-2\right)}{3} = x\left(\sqrt{7}-2\right)$

86. $\dfrac{14y}{\sqrt{2}-3} = \dfrac{14y}{\sqrt{2}-3}\cdot\dfrac{\sqrt{2}+3}{\sqrt{2}+3} = \dfrac{14y\left(\sqrt{2}+3\right)}{\left(\sqrt{2}\right)^2-3^2} = \dfrac{14y\left(\sqrt{2}+3\right)}{2-9} = -2y\left(\sqrt{2}+3\right)$

87. $\dfrac{x}{x-\sqrt{3}} = \dfrac{x}{x-\sqrt{3}}\cdot\dfrac{x+\sqrt{3}}{x+\sqrt{3}} = \dfrac{x\left(x+\sqrt{3}\right)}{x^2-\left(\sqrt{3}\right)^2} = \dfrac{x\left(x+\sqrt{3}\right)}{x^2-3}$

88. $\dfrac{y}{2y+\sqrt{7}} = \dfrac{y}{2y+\sqrt{7}}\cdot\dfrac{2y-\sqrt{7}}{2y-\sqrt{7}} = \dfrac{y\left(2y-\sqrt{7}\right)}{(2y)^2-\left(\sqrt{7}\right)^2} = \dfrac{y\left(2y-\sqrt{7}\right)}{4y^2-7}$

89. $\dfrac{y+\sqrt{2}}{y-\sqrt{2}} = \dfrac{y+\sqrt{2}}{y-\sqrt{2}}\cdot\dfrac{y+\sqrt{2}}{y+\sqrt{2}} = \dfrac{\left(y+\sqrt{2}\right)\left(y+\sqrt{2}\right)}{y^2-\left(\sqrt{2}\right)^2} = \dfrac{y^2+2y\sqrt{2}+2}{y^2-2}$

90. $\dfrac{x-\sqrt{3}}{x+\sqrt{3}} = \dfrac{x-\sqrt{3}}{x+\sqrt{3}}\cdot\dfrac{x-\sqrt{3}}{x-\sqrt{3}} = \dfrac{\left(x-\sqrt{3}\right)\left(x-\sqrt{3}\right)}{x^2-\left(\sqrt{3}\right)^2} = \dfrac{x^2-2x\sqrt{3}+3}{x^2-3}$

91. $\dfrac{\sqrt{2}-\sqrt{3}}{1-\sqrt{3}} = \dfrac{\sqrt{2}-\sqrt{3}}{1-\sqrt{3}}\cdot\dfrac{1+\sqrt{3}}{1+\sqrt{3}} = \dfrac{\sqrt{2}+\sqrt{6}-\sqrt{3}-\left(\sqrt{3}\right)^2}{1^2-\left(\sqrt{3}\right)^2} = \dfrac{\sqrt{2}+\sqrt{6}-\sqrt{3}-3}{1-3}$

$$= \frac{\sqrt{2}+\sqrt{6}-\sqrt{3}-3}{-2}$$

$$= \frac{-\left(\sqrt{2}+\sqrt{6}-\sqrt{3}-3\right)}{2}$$

$$= \frac{\sqrt{3}+3-\sqrt{2}-\sqrt{6}}{2}$$

92. $\dfrac{\sqrt{3}-\sqrt{2}}{1+\sqrt{2}} = \dfrac{\sqrt{3}-\sqrt{2}}{1+\sqrt{2}}\cdot\dfrac{1-\sqrt{2}}{1-\sqrt{2}} = \dfrac{\sqrt{3}-\sqrt{6}-\sqrt{2}+\left(\sqrt{2}\right)^2}{1^2-\left(\sqrt{2}\right)^2} = \dfrac{\sqrt{3}-\sqrt{6}-\sqrt{2}+2}{1-2}$

$$= \frac{\sqrt{3}-\sqrt{6}-\sqrt{2}+2}{-1}$$

$$= -\left(\sqrt{3}-\sqrt{6}-\sqrt{2}+2\right)$$

$$= \sqrt{6}+\sqrt{2}-\sqrt{3}-2$$

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93. $\dfrac{\sqrt{x}-\sqrt{y}}{\sqrt{x}+\sqrt{y}} = \dfrac{\sqrt{x}-\sqrt{y}}{\sqrt{x}+\sqrt{y}} \cdot \dfrac{\sqrt{x}-\sqrt{y}}{\sqrt{x}-\sqrt{y}} = \dfrac{\sqrt{x^2}-\sqrt{xy}-\sqrt{xy}+\sqrt{y^2}}{\left(\sqrt{x}\right)^2-\left(\sqrt{y}\right)^2} = \dfrac{x-2\sqrt{xy}+y}{x-y}$

94. $\dfrac{\sqrt{2x}+y}{\sqrt{2x}-y} = \dfrac{\sqrt{2x}+y}{\sqrt{2x}-y} \cdot \dfrac{\sqrt{2x}+y}{\sqrt{2x}+y} = \dfrac{\sqrt{4x^2}+y\sqrt{2x}+y\sqrt{2x}+y^2}{\left(\sqrt{2x}\right)^2-y^2} = \dfrac{2x+2y\sqrt{2x}+y^2}{2x-y^2}$

95. $\dfrac{\sqrt{2}+1}{2} = \dfrac{\sqrt{2}+1}{2} \cdot \dfrac{\sqrt{2}-1}{\sqrt{2}-1} = \dfrac{\left(\sqrt{2}\right)^2-1^2}{2\left(\sqrt{2}-1\right)} = \dfrac{2-1}{2\left(\sqrt{2}-1\right)} = \dfrac{1}{2\left(\sqrt{2}-1\right)}$

96. $\dfrac{\sqrt{x}-3}{3} = \dfrac{\sqrt{x}-3}{3} \cdot \dfrac{\sqrt{x}+3}{\sqrt{x}+3} = \dfrac{\left(\sqrt{x}\right)^2-3^2}{3\left(\sqrt{x}+3\right)} = \dfrac{x-9}{3\left(\sqrt{x}+3\right)}$

97. $\dfrac{y-\sqrt{3}}{y+\sqrt{3}} = \dfrac{y-\sqrt{3}}{y+\sqrt{3}} \cdot \dfrac{y+\sqrt{3}}{y+\sqrt{3}} = \dfrac{y^2-\left(\sqrt{3}\right)^2}{y^2+y\sqrt{3}+y\sqrt{3}+\sqrt{9}} = \dfrac{y^2-3}{y^2+2y\sqrt{3}+3}$

98. $\dfrac{\sqrt{a}-\sqrt{b}}{\sqrt{a}+\sqrt{b}} = \dfrac{\sqrt{a}-\sqrt{b}}{\sqrt{a}+\sqrt{b}} \cdot \dfrac{\sqrt{a}+\sqrt{b}}{\sqrt{a}+\sqrt{b}} = \dfrac{\left(\sqrt{a}\right)^2-\left(\sqrt{b}\right)^2}{\sqrt{a^2}+\sqrt{ab}+\sqrt{ab}+\sqrt{b^2}} = \dfrac{a-b}{a+2\sqrt{ab}+b}$

99. $\dfrac{\sqrt{x+3}-\sqrt{x}}{3} = \dfrac{\sqrt{x+3}-\sqrt{x}}{3} \cdot \dfrac{\sqrt{x+3}+\sqrt{x}}{\sqrt{x+3}+\sqrt{x}} = \dfrac{\left(\sqrt{x+3}\right)^2-\left(\sqrt{x}\right)^2}{3\left(\sqrt{x+3}+\sqrt{x}\right)}$

$= \dfrac{x+3-x}{3\left(\sqrt{x+3}+\sqrt{x}\right)}$

$= \dfrac{3}{3\left(\sqrt{x+3}+\sqrt{x}\right)} = \dfrac{1}{\sqrt{x+3}+\sqrt{x}}$

100. $\dfrac{\sqrt{2+h}-\sqrt{2}}{h} = \dfrac{\sqrt{2+h}-\sqrt{2}}{h} \cdot \dfrac{\sqrt{2+h}+\sqrt{2}}{\sqrt{2+h}+\sqrt{2}} = \dfrac{\left(\sqrt{2+h}\right)^2-\left(\sqrt{2}\right)^2}{h\left(\sqrt{2+h}+\sqrt{2}\right)}$

$= \dfrac{2+h-2}{h\left(\sqrt{2+h}+\sqrt{2}\right)}$

$= \dfrac{h}{h\left(\sqrt{2+h}+\sqrt{2}\right)} = \dfrac{1}{\sqrt{2+h}+\sqrt{2}}$

101. $\dfrac{36a^2b^3}{18ab^6} = 2a^{2-1}b^{3-6} = 2a^1b^{-3} = \dfrac{2a}{b^3}$

102. $\dfrac{-45r^2s^5t^3}{27r^6s^2t^8} = -\dfrac{5}{3}r^{2-6}s^{5-2}t^{3-8} = -\dfrac{5}{3}r^{-4}s^3t^{-5} = -\dfrac{5s^3}{3r^4t^5}$

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103. $\dfrac{16x^6y^4z^9}{-24x^9y^6z^0} = -\dfrac{2}{3}x^{6-9}y^{4-6}z^{9-0} = -\dfrac{2}{3}x^{-3}y^{-2}z^9 = -\dfrac{2z^9}{3x^3y^2}$

104. $\dfrac{32m^6n^4p^2}{26m^6n^7p^2} = \dfrac{16}{13}m^{6-6}n^{4-7}p^{2-2} = \dfrac{16}{13}m^0n^{-3}p^0 = \dfrac{16}{13n^3}$

105. $\dfrac{5x^3y^2 + 15x^3y^4}{10x^2y^3} = \dfrac{5x^3y^2}{10x^2y^3} + \dfrac{15x^3y^4}{10x^2y^3}$
$= \dfrac{x}{2y} + \dfrac{3xy}{2}$

106. $\dfrac{9m^4n^9 - 6m^3n^4}{12m^3n^3} = \dfrac{9m^4n^9}{12m^3n^3} - \dfrac{6m^3n^4}{12m^3n^3}$
$= \dfrac{3mn^6}{4} - \dfrac{n}{2}$

107. $\dfrac{24x^5y^7 - 36x^2y^5 + 12xy}{60x^5y^4} = \dfrac{24x^5y^7}{60x^5y^4} - \dfrac{36x^2y^5}{60x^5y^4} + \dfrac{12xy}{60x^5y^4} = \dfrac{2y^3}{5} - \dfrac{3y}{5x^3} + \dfrac{1}{5x^4y^3}$

108. $\dfrac{9a^3b^4 + 27a^2b^4 - 18a^2b^3}{18a^2b^7} = \dfrac{9a^3b^4}{18a^2b^7} + \dfrac{27a^2b^4}{18a^2b^7} - \dfrac{18a^2b^3}{18a^2b^7} = \dfrac{a}{2b^3} + \dfrac{3}{2b^3} - \dfrac{1}{b^4}$

109.
$$\begin{array}{r|l} & 3x + 2 \\ \hline x+3 & 3x^2 + 11x + 6 \\ & \underline{3x^2 + 9x} \\ & \quad 2x + 6 \\ & \quad \underline{2x + 6} \\ & \quad\quad 0 \end{array}$$

110.
$$\begin{array}{r|l} & x + 3 \\ \hline 3x+2 & 3x^2 + 11x + 6 \\ & \underline{3x^2 + 2x} \\ & \quad 9x + 6 \\ & \quad \underline{9x + 6} \\ & \quad\quad 0 \end{array}$$

111.
$$\begin{array}{r|l} & x - 7 + \frac{2}{2x-5} \\ \hline 2x-5 & 2x^2 - 19x + 37 \\ & \underline{2x^2 - 5x} \\ & \quad -14x + 37 \\ & \quad \underline{-14x + 35} \\ & \quad\quad 2 \end{array}$$

112.
$$\begin{array}{r|l} & 2x - 5 \\ \hline x-7 & 2x^2 - 19x + 35 \\ & \underline{2x^2 - 14x} \\ & \quad -5x + 35 \\ & \quad \underline{-5x + 35} \\ & \quad\quad 0 \end{array}$$

113.
$$\begin{array}{r|l} & 2x^2 + 2x + 2 + \frac{3}{x-1} \\ \hline x-1 & 2x^3 + 0x^2 + 0x + 1 \\ & \underline{2x^3 - 2x^2} \\ & \quad 2x^2 + 0x + 1 \\ & \quad \underline{2x^2 - 2x} \\ & \quad\quad 2x + 1 \\ & \quad\quad \underline{2x - 2} \\ & \quad\quad\quad 3 \end{array}$$

114.
$$\begin{array}{r|l} & x^2 - x + 3 + \frac{1}{2x-7} \\ \hline 2x-7 & 2x^3 - 9x^2 + 13x - 20 \\ & \underline{2x^3 - 7x^2} \\ & \quad -2x^2 + 13x - 20 \\ & \quad \underline{-2x^2 + 7x} \\ & \quad\quad 6x - 20 \\ & \quad\quad \underline{6x - 21} \\ & \quad\quad\quad 1 \end{array}$$

115.
$$\begin{array}{r|l} & x - 3 \\ \hline x^2+x-1 & x^3 - 2x^2 - 4x + 3 \\ & \underline{x^3 + x^2 - x} \\ & \quad -3x^2 - 3x + 3 \\ & \quad \underline{-3x^2 - 3x + 3} \\ & \quad\quad 0 \end{array}$$

116.
$$\begin{array}{r|l} & x - 2 + \frac{-x-1}{x^2-3} \\ \hline x^2-3 & x^3 - 2x^2 - 4x + 5 \\ & \underline{x^3 \quad - 3x} \\ & \quad -2x^2 - x + 5 \\ & \quad \underline{-2x^2 \quad + 6} \\ & \quad\quad -x - 1 \end{array}$$

EXERCISES 0.4

117.
$$\begin{array}{r|l} x^3-2 & x^2-\quad 2+\frac{-x^2+5}{x^3-2} \\ & \underline{x^5+0x^4-2x^3-3x^2+0x+9} \\ & \underline{x^5\qquad\quad -2x^2} \\ & \quad -2x^3-\ x^2+0x+9 \\ & \quad \underline{-2x^3\qquad\quad +4} \\ & \qquad -\ x^2\qquad +5 \end{array}$$

118.
$$\begin{array}{r|l} x^3-3 & x^2-\quad 2+\frac{3}{x^3-3} \\ & \underline{x^5+0x^4-2x^3-3x^2+0x+9} \\ & \underline{x^5\qquad\quad -3x^2} \\ & \quad -2x^3\qquad +0x+9 \\ & \quad \underline{-2x^3\qquad\quad +6} \\ & \qquad\qquad 3 \end{array}$$

119.
$$\begin{array}{r|l} x-2 & x^4+2x^3+4x^2+\ 8x+\ 16 \\ & \underline{x^5+0x^4+0x^3+0x^2+\ 0x-32} \\ & \underline{x^5-2x^4} \\ & \quad 2x^4+0x^3 \\ & \quad \underline{2x^4-4x^3} \\ & \qquad 4x^3+0x^2 \\ & \qquad \underline{4x^3-8x^2} \\ & \qquad\quad 8x^2+\ 0x \\ & \qquad\quad \underline{8x^2-16x} \\ & \qquad\qquad 16x-32 \\ & \qquad\qquad \underline{16x-32} \\ & \qquad\qquad\quad 0 \end{array}$$

120.
$$\begin{array}{r|l} x+1 & x^3-\ x^2+\quad x-\ 1 \\ & \underline{x^4+0x^3+0x^2+0x-1} \\ & \underline{x^4+\ x^3} \\ & \quad -\ x^3+0x^2 \\ & \quad \underline{-\ x^3-\ x^2} \\ & \qquad\quad x^2+0x \\ & \qquad\quad \underline{x^2+\ x} \\ & \qquad\qquad -\ x-1 \\ & \qquad\qquad \underline{-\ x-1} \\ & \qquad\qquad\quad 0 \end{array}$$

121.
$$\begin{array}{r|l} 6x^2+11x-10 & 6x^2+\quad x-\quad 12 \\ & \underline{36x^4+72x^3-121x^2-142x+120} \\ & \underline{36x^4+66x^3-\ 60x^2} \\ & \qquad 6x^3-\ 61x^2-142x \\ & \qquad \underline{6x^3+\ 11x^2-\ 10x} \\ & \qquad\quad -\ 72x^2-132x+120 \\ & \qquad\quad \underline{-\ 72x^2-132x+120} \\ & \qquad\qquad\quad 0 \end{array}$$

122.
$$\begin{array}{r|l} 6x^2+x-12 & 6x^2+\ 11x-\quad 10 \\ & \underline{36x^4+72x^3-121x^2-142x+120} \\ & \underline{36x^4+\ 6x^3-\ 72x^2} \\ & \qquad 66x^3-\ 49x^2-142x \\ & \qquad \underline{66x^3+\ 11x^2-132x} \\ & \qquad\quad -\ 60x^2-\ 10x+120 \\ & \qquad\quad \underline{-\ 60x^2-\ 10x+120} \\ & \qquad\qquad\quad 0 \end{array}$$

123. Area = length · width = $(x+5)(x-2)$ ft^2 = $(x^2-2x+5x-10)$ ft^2 = $(x^2+3x-10)$ ft^2

124.
$$\begin{aligned} \text{Area} &= \frac{1}{2}\cdot \text{base} \cdot \text{height} \\ x^2+3x-40 &= \frac{1}{2}(x+8)\cdot \text{height} \\ 2\big(x^2+3x-40\big) &= (x+8)\cdot \text{height} \\ 2x^2+6x-80 &= (x+8)\cdot \text{height} \\ \frac{2x^2+6x-80}{x+8} &= \text{height} \end{aligned}$$

$$\begin{array}{r|l} x+8 & 2x-\ \ 10 \\ & \underline{2x^2+\ 6x-80} \\ & \underline{2x^2+16x} \\ & \quad -10x-80 \\ & \quad \underline{-10x-80} \\ & \qquad\quad 0 \end{array}$$

The height is $(2x-10)$ ft.

EXERCISES 0.4

125. Volume $= l \cdot w \cdot h$
$= (12-2x)(12-2x)x$ in.^{3}
$= \left(144-48x+4x^2\right)x$ in.^{3}
$= \left(144x-48x^2+4x^3\right)$ in.^{3}
$= \left(4x^3-48x^2+144x\right)$ in.^{3}

126. $t = \dfrac{d}{r} = \dfrac{3x^2+19x+20}{3x+4}$

$$\begin{array}{r} x+\ \ 5 \\ 3x+4\,\overline{)\,3x^2+19x+20} \\ \underline{3x^2+\ \ 4x} \\ 15x+20 \\ \underline{15x+20} \\ 0 \end{array}$$

$t = x+5$

127. $(a+b+c)^2 = (a+b+c)(a+b+c) = a(a+b+c)+b(a+b+c)+c(a+b+c)$
$= a^2+ab+ac+ab+b^2+bc+ac+bc+c^2$
$= a^2+b^2+c^2+2ab+2bc+2ac$

128. $(a+b+c+d)^2 = (a+b+c+d)(a+b+c+d)$
$= a(a+b+c+d)+b(a+b+c+d)+c(a+b+c+d)+d(a+b+c+d)$
$= a^2+ab+ac+ad+ab+b^2+bc+bd+ac+bc+c^2+cd+ad+bd+cd+d^2$
$= a^2+b^2+c^2+d^2+2ab+2ac+2ad+2bc+2bd+2cd$

129. Answers may vary.

130. Multiply the numerator and denominator by the conjugate of the numerator $\left(\sqrt{x}-2\right)$.

131. Check the formula with $a=1$ and $b=2$.

132. Check the formula with $a=3$ and $b=4$.

133. False. Some polynomials are trinomials.

134. True.

135. True. $(12x-5y)^2 = (12x-5y)(12x-5y) = 144x^2-60xy-60xy+25y^2$
$= 144x^2-120xy+25y^2$

136. False. $(6x+y)^2 = (6x+y)(6x+y) = 36x^2+6xy+6xy+y^2 = 36x^2+12xy+y^2$

137. False. $\left(x^{1/3}-6\right)\left(4x^{1/3}+7\right) = 4x^{2/3}+7x^{1/3}-24x^{1/3}-42 = 4x^{2/3}-17x^{1/3}-42$

138. False. $(x^{-3}+5)(x^{-3}-5) = x^{-6}-5x^{-3}+5x^{-3}-25 = x^{-6}-25 = \frac{1}{x^6}-25$

139. Profit $=$ Revenue $-$ Cost $= \left(x^2+200x\right)-(-200x+500) = x^2+200x+200x-500$
$= x^2+400x-500$

140. Use the answer to #139.
$x^2+400x-500 = (100)^2+400(100)-500 = 49{,}500 \Rightarrow$ False.

Exercises 0.5 (page 63)

1. factor

2. integer, prime

3. $ax + bx = x(a + b)$

4. $x^2 - y^2 = (x + y)(x - y)$

5. $x^2 + 2xy + y^2 = (x + y)(x + y) = (x + y)^2$

6. $x^2 - 2xy + y^2 = (x - y)(x - y) = (x - y)^2$

7. $x^3 + y^3 = (x + y)(x^2 - xy + y^2)$

8. $x^3 - y^3 = (x - y)(x^2 + xy + y^2)$

9. $3x - 6 = 3(x - 2)$

10. $5y - 15 = 5(y - 3)$

11. $8x^2 + 4x^3 = 4x^2(2 + x)$

12. $9y^3 + 6y^2 = 3y^2(3y + 2)$

13. $7x^2y^2 + 14x^3y^2 = 7x^2y^2(1 + 2x)$

14. $25y^2z - 15yz^2 = 5yz(5y - 3z)$

15. $a(x + y) + b(x + y) = (x + y)(a + b)$

16. $b(x - y) + a(x - y) = (x - y)(b + a)$

17. $4a + b - 12a^2 - 3ab = 4a + b - 3a(4a + b) = 1(4a + b) - 3a(4a + b) = (4a + b)(1 - 3a)$

18. $x^2 + 4x + xy + 4y = x(x + 4) + y(x + 4) = (x + 4)(x + y)$

19. $4x^2 - 9 = (2x)^2 - 3^2 = (2x + 3)(2x - 3)$

20. $36z^2 - 49 = (6z)^2 - 7^2 = (6z + 7)(6z - 7)$

21. $4 - 9r^2 = 2^2 - (3r)^2 = (2 + 3r)(2 - 3r)$

22. $16 - 49x^2 = 4^2 - (7x)^2 = (4 + 7x)(4 - 7x)$

23. $81x^4 - 1 = (9x^2)^2 - 1^2 = (9x^2 + 1)(9x^2 - 1) = (9x^2 + 1)(3x + 1)(3x - 1)$

24. $81 - x^4 = 9^2 - (x^2)^2 = (9 + x^2)(9 - x^2) = (9 + x^2)(3 + x)(3 - x)$

25. $(x + z)^2 - 25 = (x + z)^2 - 5^2 = (x + z + 5)(x + z - 5)$

26. $(x - y)^2 - 9 = (x - y)^2 - 3^2 = (x - y + 3)(x - y - 3)$

27. $x^2 + 8x + 16 = (x + 4)(x + 4) = (x + 4)^2$

28. $a^2 - 12a + 36 = (a - 6)(a - 6) = (a - 6)^2$

29. $b^2 - 10b + 25 = (b - 5)(b - 5) = (b - 5)^2$

30. $y^2 + 14y + 49 = (y + 7)(y + 7) = (y + 7)^2$

31. $m^2 + 4mn + 4n^2 = (m + 2n)(m + 2n) = (m + 2n)^2$

32. $r^2 - 8rs + 16s^2 = (r - 4s)(r - 4s) = (r - 4s)^2$

33. $12x^2 - xy - 6y^2 = (4x - 3y)(3x + 2y)$

34. $8x^2 - 10xy - 3y^2 = (4x + y)(2x - 3y)$

EXERCISES 0.5

35. $x^2 + 10x + 21$: $a = 1, b = 10, c = 21$
key number $= ac = 1(21) = 21$
$x^2 + 10x + 21 = x^2 + 7x + 3x + 21$
$= x(x + 7) + 3(x + 7)$
$= (x + 7)(x + 3)$

36. $x^2 + 7x + 10$: $a = 1, b = 7, c = 10$
key number $= ac = 1(10) = 10$
$x^2 + 7x + 10 = x^2 + 5x + 2x + 10$
$= x(x + 5) + 2(x + 5)$
$= (x + 5)(x + 2)$

37. $x^2 - 4x - 12$: $a = 1, b = -4, c = -12$
key number $= ac = 1(-12) = -12$
$x^2 - 4x - 12 = x^2 - 6x + 2x - 12$
$= x(x - 6) + 2(x - 6)$
$= (x - 6)(x + 2)$

38. $x^2 - 2x - 63$: $a = 1, b = -2, c = -63$
key number $= ac = 1(-63) = -63$
$x^2 - 2x - 63 = x^2 - 9x + 7x - 63$
$= x(x - 9) + 7(x - 9)$
$= (x - 9)(x + 7)$

39. $6p^2 + 7p - 3$: $a = 6, b = 7, c = -3$
key number $= ac = 6(-3) = -18$
$6p^2 + 7p - 3 = 6p^2 + 9p - 2p - 3$
$= 3p(2p + 3) - (2p + 3)$
$= (2p + 3)(3p - 1)$

40. $4q^2 - 19q + 12$: $a = 4, b = -19, c = 12$
key number $= ac = 4(12) = 48$
$4q^2 - 19q + 12 = 4q^2 - 3q - 16q + 12$
$= q(4q - 3) - 4(4q - 3)$
$= (4q - 3)(q - 4)$

41. $t^3 + 343 = t^3 + 7^3 = (t + 7)[t^2 - (t)(7) + 7^2] = (t + 7)(t^2 - 7t + 49)$

42. $r^3 + 8s^3 = r^3 + (2s)^3 = (r + 2s)\big[r^2 - (r)(2s) + (2s)^2\big] = (r + 2s)(r^2 - 2rs + 4s^2)$

43. $125y^3 + 216z^3 = (5y)^3 + (6z)^3 = (5y + 6z)\big[(5y)^2 - (5y)(6z) + (6z)^2\big]$
$= (5y + 6z)\big(25y^2 - 30yz + 36z^2\big)$

44. $27y^3 + 1000z^3 = (3y)^3 + (10z)^3 = (3y + 10z)\big[(3y)^2 - (3y)(10z) + (10z)^2\big]$
$= (3y + 10z)\big(9y^2 - 30yz + 100z^2\big)$

45. $8z^3 - 27 = (2z)^3 - 3^3 = (2z - 3)\big[(2z)^2 + (2z)(3) + 3^2\big] = (2z - 3)(4z^2 + 6z + 9)$

46. $125a^3 - 64 = (5a)^3 - 4^3 = (5a - 4)\big[(5a)^2 + (5a)(4) + 4^2\big] = (5a - 4)(25a^2 + 20a + 16)$

47. $343y^3 - z^3 = (7y)^3 - z^3 = (7y - z)\big[(7y)^2 + (7y)(z) + z^2\big] = (7y - z)(49y^2 + 7yz + z^2)$

48. $27y^3 - 512z^3 = (3y)^3 - (8z)^3 = (3y - 8z)\big[(3y)^2 + (3y)(8z) + (8z)^2\big]$
$= (3y - 8z)\big(9y^2 + 24yz + 64z^2\big)$

49. $3a^2bc + 6ab^2c + 9abc^2 = 3abc(a + 2b + 3c)$

50. $5x^3y^3z^3 + 25x^2y^2z^2 - 125xyz = 5xyz(x^2y^2z^2 + 5xyz - 25)$

51. $3x^3 + 3x^2 - x - 1 = 3x^2(x + 1) - 1(x + 1) = (x + 1)(3x^2 - 1)$

52. $4x + 6xy - 9y - 6 = 2x(2 + 3y) - 3(3y + 2) = (3y + 2)(2x - 3)$

EXERCISES 0.5

53. $2txy + 2ctx - 3ty - 3ct = t(2xy + 2cx - 3y - 3c) = t[2x(y + c) - 3(y + c)] = t(y + c)(2x - 3)$

54. $2ax + 4ay - bx - 2by = 2a(x + 2y) - b(x + 2y) = (x + 2y)(2a - b)$

55. $ax + bx + ay + by + az + bz = x(a + b) + y(a + b) + z(a + b) = (a + b)(x + y + z)$

56. $6x^2y^3 + 18xy + 3x^2y^2 + 9x = 3x\big(2xy^3 + 6y + xy^2 + 3\big) = 3x\big[2y\big(xy^2 + 3\big) + 1\big(xy^2 + 3\big)\big]$
$= 3x\big(xy^2 + 3\big)(2y + 1)$

57. $x^2 - (y - z)^2 = [x + (y - z)][x - (y - z)]$
$= (x + y - z)(x - y + z)$

58. $z^2 - (y + 3)^2 = [z + (y + 3)][z - (y + 3)]$
$= (z + y + 3)(z - y - 3)$

59. $(x - y)^2 - (x + y)^2 = [(x - y) + (x + y)][(x - y) - (x + y)] = (x - y + x + y)(x - y - x - y)$
$= (2x)(-2y) = -4xy$

60. $(2a + 3)^2 - (2a - 3)^2 = [(2a + 3) + (2a - 3)][(2a + 3) - (2a - 3)]$
$= (2a + 3 + 2a - 3)(2a + 3 - 2a + 3) = (4a)(6) = 24a$

61. $x^4 - y^4 = (x^2)^2 - (y^2)^2 = (x^2 + y^2)(x^2 - y^2) = (x^2 + y^2)(x + y)(x - y)$

62. $z^4 - 81 = (z^2)^2 - 9^2 = (z^2 + 9)(z^2 - 9) = (z^2 + 9)(z^2 - 3^2) = (z^2 + 9)(z + 3)(z - 3)$

63. $3x^2 - 12 = 3(x^2 - 4) = 3(x + 2)(x - 2)$

64. $3x^3y - 3xy = 3xy\big(x^2 - 1\big)$
$= 3xy(x + 1)(x - 1)$

65. $18xy^2 - 8x = 2x\big(9y^2 - 4\big)$
$= 2x(3y + 2)(3y - 2)$

66. $27x^2 - 12 = 3\big(9x^2 - 4\big)$
$= 3(3x + 2)(3x - 2)$

67. $x^2 - 2x + 15 \Rightarrow$ prime

68. $x^2 + x + 2 \Rightarrow$ prime

69. $-15 + 2a + 24a^2 = 24a^2 + 2a - 15$
$= (6a + 5)(4a - 3)$

70. $-32 - 68x + 9x^2 = 9x^2 - 68x - 32$
$= (9x + 4)(x - 8)$

71. $6x^2 + 29xy + 35y^2 = (3x + 7y)(2x + 5y)$

72. $10x^2 - 17xy + 6y^2 = (5x - 6y)(2x - y)$

73. $12p^2 - 58pq - 70q^2 = 2(6p^2 - 29pq - 35q^2) = 2(6p - 35q)(p + q)$

74. $3x^2 - 6xy - 9y^2 = 3(x^2 - 2xy - 3y^2) = 3(x - 3y)(x + y)$

75. $-6m^2 + 47mn - 35n^2 = -(6m^2 - 47mn + 35n^2) = -(6m - 5n)(m - 7n)$

76. $-14r^2 - 11rs + 15s^2 = -(14r^2 + 11rs - 15s^2) = -(7r - 5s)(2r + 3s)$

77. $-6x^3 + 23x^2 + 35x = -x(6x^2 - 23x - 35) = -x(6x + 7)(x - 5)$

EXERCISES 0.5

78. $-y^3 - y^2 + 90y = -y(y^2 + y - 90) = -y(y + 10)(y - 9)$

79. $6x^4 - 11x^3 - 35x^2 = x^2(6x^2 - 11x - 35) = x^2(2x - 7)(3x + 5)$

80. $12x + 17x^2 - 7x^3 = -7x^3 + 17x^2 + 12x = -x(7x^2 - 17x - 12) = -x(x - 3)(7x + 4)$

81. $x^4 + 2x^2 - 15 = (x^2 + 5)(x^2 - 3)$

82. $x^4 - x^2 - 6 = (x^2 - 3)(x^2 + 2)$

83. $a^{2n} - 2a^n - 3 = (a^n - 3)(a^n + 1)$

84. $a^{2n} + 6a^n + 8 = (a^n + 4)(a^n + 2)$

85. $6x^{2n} - 7x^n + 2 = (3x^n - 2)(2x^n - 1)$

86. $9x^{2n} + 9x^n + 2 = (3x^n + 2)(3x^n + 1)$

87. $\begin{aligned} 4x^{2n} - 9y^{2n} &= (2x^n)^2 - (3y^n)^2 \\ &= (2x^n + 3y^n)(2x^n - 3y^n) \end{aligned}$

88. $8x^{2n} - 2x^n - 3 = (4x^n - 3)(2x^n + 1)$

89. $10y^{2n} - 11y^n - 6 = (5y^n + 2)(2y^n - 3)$

90. $\begin{aligned} 16y^{4n} - 25y^{2n} &= y^{2n}\big(16y^{2n} - 25\big) \\ &= y^{2n}\big[(4y^n)^2 - 5^2\big] \\ &= y^{2n}(4y^n + 5)(4y^n - 5) \end{aligned}$

91. $2x^3 + 2000 = 2(x^3 + 1000) = 2(x^3 + 10^3) = 2(x + 10)(x^2 - 10x + 100)$

92. $3y^3 + 648 = 3(y^3 + 216) = 3(y^3 + 6^3) = 3(y + 6)(y^2 - 6y + 36)$

93. $\begin{aligned} (x + y)^3 - 64 = (x + y)^3 - 4^3 &= [(x + y) - 4]\big[(x + y)^2 + 4(x + y) + 4^2\big] \\ &= (x + y - 4)\big(x^2 + 2xy + y^2 + 4x + 4y + 16\big) \end{aligned}$

94. $\begin{aligned} (x - y)^3 + 27 = (x - y)^3 + 3^3 &= [(x - y) + 3]\big[(x - y)^2 - 3(x - y) + 3^2\big] \\ &= (x - y + 3)\big(x^2 - 2xy + y^2 - 3x + 3y + 9\big) \end{aligned}$

95. $\begin{aligned} 64a^6 - y^6 = \big(8a^3\big)^2 - \big(y^3\big)^2 &= \big(8a^3 + y^3\big)\big(8a^3 - y^3\big) \\ &= (2a + y)\big(4a^2 - 2ay + y^2\big)(2a - y)\big(4a^2 + 2ay + y^2\big) \\ &= (2a + y)(2a - y)\big(4a^2 - 2ay + y^2\big)\big(4a^2 + 2ay + y^2\big) \end{aligned}$

96. $a^6 + b^6 = (a^2)^3 + (b^2)^3 = (a^2 + b^2)\Big((a^2)^2 - a^2b^2 + (b^2)^2\Big) = (a^2 + b^2)(a^4 - a^2b^2 + b^4)$

97. $a^3 - b^3 + a - b = (a - b)(a^2 + ab + b^2) + (a - b)1 = (a - b)(a^2 + ab + b^2 + 1)$

98. $(a^2 - y^2) - 5(a + y) = (a + y)(a - y) - 5(a + y) = (a + y)(a - y - 5)$

99. $\begin{aligned} 64x^6 + y^6 = \big(4x^2\big)^3 + \big(y^2\big)^3 &= \big(4x^2 + y^2\big)\Big(\big(4x^2\big)^2 - 4x^2y^2 + \big(y^2\big)^2\Big) \\ &= \big(4x^2 + y^2\big)\big(16x^4 - 4x^2y^2 + y^4\big) \end{aligned}$

EXERCISES 0.5

100. $z^2+6z+9-225y^2=(z+3)(z+3)-225y^2=(z+3)^2-(15y)^2$
$=(z+3+15y)(z+3-15y)$

101. $x^2-6x+9-144y^2=(x-3)(x-3)-144y^2=(x-3)^2-(12y)^2$
$=(x-3+12y)(x-3-12y)$

102. $x^2+2x-9y^2+1=x^2+2x+1-9y^2=(x+1)(x+1)-9y^2$
$=(x+1)^2-(3y)^2=(x+1+3y)(x+1-3y)$

103. $(a+b)^2-3(a+b)-10=[(a+b)-5][(a+b)+2]=(a+b-5)(a+b+2)$

104. $2(a+b)^2-5(a+b)-3=[2(a+b)+1][(a+b)-3]=(2a+2b+1)(a+b-3)$

105. $x^6+7x^3-8=(x^3+8)(x^3-1)=(x+2)(x^2-2x+4)(x-1)(x^2+x+1)$

106. $x^6-13x^4+36x^2=x^2(x^4-13x^2+36)=x^2(x^2-9)(x^2-4)=x^2(x+3)(x-3)(x+2)(x-2)$

107. $x^4+x^2+1=x^4+2x^2+1-x^2$
$=\big(x^2+1\big)\big(x^2+1\big)-x^2$
$=\big(x^2+1\big)^2-x^2$
$=\big(x^2+1+x\big)\big(x^2+1-x\big)$
$=\big(x^2+x+1\big)\big(x^2-x+1\big)$

108. $x^4+3x^2+4=x^4+4x^2+4-x^2$
$=\big(x^2+2\big)\big(x^2+2\big)-x^2$
$=\big(x^2+2\big)^2-x^2$
$=\big(x^2+2+x\big)\big(x^2+2-x\big)$
$=\big(x^2+x+2\big)\big(x^2-x+2\big)$

109. $x^4+7x^2+16=x^4+8x^2+16-x^2$
$=\big(x^2+4\big)\big(x^2+4\big)-x^2$
$=\big(x^2+4\big)^2-x^2$
$=\big(x^2+4+x\big)\big(x^2+4-x\big)$
$=\big(x^2+x+4\big)\big(x^2-x+4\big)$

110. $y^4+2y^2+9=y^4+6y^2+9-4y^2$
$=\big(y^2+3\big)\big(y^2+3\big)-4y^2$
$=\big(y^2+3\big)^2-(2y)^2$
$=\big(y^2+3+2y\big)\big(y^2+3-2y\big)$
$=\big(y^2+2y+3\big)\big(y^2-2y+3\big)$

111. $4a^4+1+3a^2=4a^4+4a^2+1-a^2=\big(2a^2+1\big)\big(2a^2+1\big)-a^2=\big(2a^2+1\big)^2-a^2$
$=\big(2a^2+1+a\big)\big(2a^2+1-a\big)$
$=\big(2a^2+a+1\big)\big(2a^2-a+1\big)$

112. $x^4+25+6x^2=x^4+10x^2+25-4x^2=\big(x^2+5\big)\big(x^2+5\big)-4x^2=\big(x^2+5\big)^2-(2x)^2$
$=\big(x^2+5+2x\big)\big(x^2+5-2x\big)$
$=\big(x^2+2x+5\big)\big(x^2-2x+2\big)$

113. $V=\dfrac{4}{3}\pi r_1^3-\dfrac{4}{3}\pi r_2^3$
$=\frac{4}{3}\pi(r_1^3-r_2^3)$
$=\frac{4}{3}\pi(r_1-r_2)(r_1^2+r_1r_2+r_2^2)$

114. $f=144-16t^2$
$=16\big(9-t^2\big)$
$=16(3+t)(3-t)$

EXERCISES 0.5

115-118. Answers may vary.

119. $3x + 2 = 2\left(\frac{3x}{2} + \frac{2}{2}\right) = 2\left(\frac{3}{2}x + 1\right)$

120. $5x - 3 = 5\left(\frac{5x}{5} - \frac{3}{5}\right) = 5\left(x - \frac{3}{5}\right)$

121. $x^2 + 2x + 4 = 2\left(\frac{x^2}{2} + \frac{2x}{2} + \frac{4}{2}\right)$
$= 2\left(\frac{1}{2}x^2 + x + 2\right)$

122. $3x^2 - 2x - 5 = 3\left(\frac{3x^2}{3} - \frac{2x}{3} - \frac{5}{3}\right)$
$= 3\left(x^2 - \frac{2}{3}x - \frac{5}{3}\right)$

123. $a + b = a\left(\frac{a}{a} + \frac{b}{a}\right) = a\left(1 + \frac{b}{a}\right)$

124. $a - b = b\left(\frac{a}{b} - \frac{b}{b}\right) = b\left(\frac{a}{b} - 1\right)$

125. $x + x^{1/2} = x^{1/2}\left(x^{1-1/2} + x^{1/2-1/2}\right)$
$= x^{1/2}\left(x^{1/2} + 1\right)$

126. $x^{3/2} - x^{1/2} = x^{1/2}\left(x^{3/2-1/2} - x^{1/2-1/2}\right)$
$= x^{1/2}(x - 1)$

127. $2x + \sqrt{2}y = \sqrt{2}\left(\dfrac{2x}{\sqrt{2}} + \dfrac{\sqrt{2}y}{\sqrt{2}}\right)$
$= \sqrt{2}\left(\sqrt{2}x + y\right)$

128. $\sqrt{3}a - 3b = \sqrt{3}\left(\dfrac{\sqrt{3}a}{\sqrt{3}} - \dfrac{3b}{\sqrt{3}}\right)$
$= \sqrt{3}\left(a - \sqrt{3}b\right)$

129. $ab^{3/2} - a^{3/2}b = ab\left(\dfrac{ab^{3/2}}{ab} - \dfrac{a^{3/2}b}{ab}\right)$
$= ab\left(b^{1/2} - a^{1/2}\right)$

130. $ab^2 + b = b^{-1}\left(\dfrac{ab^2}{b^{-1}} + \dfrac{b}{b^{-1}}\right)$
$= b^{-1}\left(ab^3 + b^2\right)$

131. $x^2 + x - 6 + xy - 2y = (x + 3)(x - 2) + y(x - 2) = (x - 2)(x + 3 + y)$

132. $2x^2 + 5x + 2 - xy - 2y = (2x + 1)(x + 2) - y(x + 2) = (x + 2)(2x + 1 - y)$

133. $a^4 + 2a^3 + a^2 + a + 1 = a^2\left(a^2 + 2a + 1\right) + a + 1 = a^2(a + 1)(a + 1) + 1(a + 1)$
$= (a + 1)\left[a^2(a + 1) + 1\right]$
$= (a + 1)\left(a^3 + a^2 + 1\right)$

134. $a^4 + a^3 - 2a^2 + a - 1 = a^2\left(a^2 + a - 2\right) + a - 1 = a^2(a + 2)(a - 1) + 1(a - 1)$
$= (a - 1)\left[a^2(a + 2) + 1\right]$
$= (a - 1)\left(a^3 + 2a^2 + 1\right)$

135. True.

136. False. $25x^{200}z^{200} + 36$ is the sum of two squares and is thus prime.

137. False. $p^3q^3r^3 + 64 = (pqr)^3 + 4^3 = (pqr + 4)(p^2q^2r^2 - 4pqr + 16)$

138. True. $27x^3 - 125y^3 = (3x)^3 - (5y)^3 = (3x - 5y)(9x^2 + 15xy + 25y^2)$

139. True.

140. False. It can be factored for these values of k: 7, 8, 12, -7, -8, -12.

Exercises 0.6 (page 73)

1. numerator **2.** denominator **3.** $ad = bc$ **4.** zero

5. $\frac{ac}{bd}$ **6.** $\frac{ad}{bc}$ **7.** $\frac{a+c}{b}$ **8.** $\frac{a-c}{b}$

9.
$$\frac{8x}{3y} \stackrel{?}{=} \frac{16x}{6y}$$
$$8x \cdot 6y \stackrel{?}{=} 3y \cdot 16x$$
$$48xy = 48xy$$
EQUAL

10.
$$\frac{3x^2}{4y^2} \stackrel{?}{=} \frac{12y^2}{16x^2}$$
$$3x^2 \cdot 16x^2 \stackrel{?}{=} 4y^2 \cdot 12y^2$$
$$48x^4 \neq 48y^4$$
NOT EQUAL

11.
$$\frac{25xyz}{12ab^2c} \stackrel{?}{=} \frac{50a^2bc}{24xyz}$$
$$25xyz \cdot 24xyz \stackrel{?}{=} 12ab^2c \cdot 50a^2bc$$
$$600x^2y^2z^2 \neq 600a^3b^3c^2$$
NOT EQUAL

12.
$$\frac{15rs^2}{4rs^2} \stackrel{?}{=} \frac{37.5a^3}{10a^3}$$
$$15rs^2 \cdot 10a^3 \stackrel{?}{=} 4rs^2 \cdot 37.5a^3$$
$$150rs^2a^3 = 150rs^2a^3$$
EQUAL

13. $\frac{7a^2b}{21ab^2} = \frac{a \cdot 7ab}{3b \cdot 7ab} = \frac{a}{3b} \cdot \frac{7ab}{7ab} = \frac{a}{3b}$

14. $\frac{35p^3q^2}{49p^4q} = \frac{5q \cdot 7p^3q}{7p \cdot 7p^3q} = \frac{5q}{7p} \cdot \frac{7p^3q}{7p^3q} = \frac{5q}{7p}$

15. $\frac{4x}{7} \cdot \frac{2}{5a} = \frac{4x \cdot 2}{7 \cdot 5a} = \frac{8x}{35a}$

16. $\frac{-5y}{2z} \cdot \frac{4}{y^2} = \frac{-5y \cdot 4}{2z \cdot y^2} = \frac{-20y}{2y^2z} = -\frac{10}{yz}$

17. $\frac{8m}{5n} \div \frac{3m}{10n} = \frac{8m}{5n} \cdot \frac{10n}{3m} = \frac{80mn}{15mn} = \frac{16}{3}$

18. $\frac{15p}{8q} \div \frac{-5p}{16q^2} = \frac{15p}{8q} \cdot \frac{16q^2}{-5p} = \frac{240pq^2}{-40pq} = -6q$

19. $\frac{3z}{5c} + \frac{2z}{5c} = \frac{3z + 2z}{5c} = \frac{5z}{5c} = \frac{z}{c}$

20. $\frac{7a}{4b} - \frac{3a}{4b} = \frac{7a - 3a}{4b} = \frac{4a}{4b} = \frac{a}{b}$

21. $\frac{15x^2y}{7a^2b^3} - \frac{x^2y}{7a^2b^3} = \frac{14x^2y}{7a^2b^3} = \frac{2x^2y}{a^2b^3}$

22. $\frac{8rst^2}{15m^4t^2} + \frac{7rst^2}{15m^4t^2} = \frac{15rst^2}{15m^4t^2} = \frac{rs}{m^4}$

23. $\frac{2x-4}{x^2-4} = \frac{2(x-2)}{(x+2)(x-2)} = \frac{2}{x+2}$

24. $\frac{x^2-16}{x^2-8x+16} = \frac{(x+4)(x-4)}{(x-4)(x-4)} = \frac{x+4}{x-4}$

25. $\frac{4-x^2}{x^2-5x+6} = \frac{(2+x)(2-x)}{(x-3)(x-2)} = -\frac{x+2}{x-3}$

26. $\frac{25-x^2}{x^2+10x+25} = \frac{(5+x)(5-x)}{(x+5)(x+5)} = -\frac{x-5}{x+5}$

27. $\frac{6x^3+x^2-12x}{4x^3+4x^2-3x} = \frac{x(6x^2+x-12)}{x(4x^2+4x-3)} = \frac{x(2x+3)(3x-4)}{x(2x+3)(2x-1)} = \frac{3x-4}{2x-1}$

28. $\frac{6x^4-5x^3-6x^2}{2x^3-7x^2-15x} = \frac{x^2(6x^2-5x-6)}{x(2x^2-7x-15)} = \frac{x^2(2x-3)(3x+2)}{x(2x+3)(x-5)} = \frac{x(2x-3)(3x+2)}{(2x+3)(x-5)}$

EXERCISES 0.6

29. $$\frac{x^3-8}{x^2+ax-2x-2a}=\frac{x^3-2^3}{x(x+a)-2(x+a)}=\frac{(x-2)(x^2+2x+4)}{(x+a)(x-2)}=\frac{x^2+2x+4}{x+a}$$

30. $$\frac{xy+2x+3y+6}{x^3+27}=\frac{x(y+2)+3(y+2)}{x^3+3^3}=\frac{(y+2)(x+3)}{(x+3)(x^2-3x+9)}=\frac{y+2}{x^2-3x+9}$$

31. $$\frac{x^2-1}{x}\cdot\frac{x^2}{x^2+2x+1}=\frac{(x+1)(x-1)}{x}\cdot\frac{x^2}{(x+1)(x+1)}=\frac{x(x-1)}{x+1}$$

32. $$\frac{y^2-2y+1}{y}\cdot\frac{y+2}{y^2+y-2}=\frac{(y-1)(y-1)}{y}\cdot\frac{y+2}{(y+2)(y-1)}=\frac{y-1}{y}$$

33. $$\frac{3x^2+7x+2}{x^2+2x}\cdot\frac{x^2-x}{3x^2+x}=\frac{(3x+1)(x+2)}{x(x+2)}\cdot\frac{x(x-1)}{x(3x+1)}=\frac{x-1}{x}$$

34. $$\frac{x^2+x}{2x^2+3x}\cdot\frac{2x^2+x-3}{x^2-1}=\frac{x(x+1)}{x(2x+3)}\cdot\frac{(2x+3)(x-1)}{(x+1)(x-1)}=1$$

35. $$\frac{x^2+x}{x-1}\cdot\frac{x^2-1}{x+2}=\frac{x(x+1)}{x-1}\cdot\frac{(x+1)(x-1)}{x+2}=\frac{x(x+1)^2}{x+2}$$

36. $$\frac{x^2+5x+6}{x^2+6x+9}\cdot\frac{x+2}{x^2-4}=\frac{(x+2)(x+3)}{(x+3)(x+3)}\cdot\frac{x+2}{(x+2)(x-2)}=\frac{x+2}{(x+3)(x-2)}$$

37. $$\frac{2x^2+32}{8}\div\frac{x^2+16}{2}=\frac{2x^2+32}{8}\cdot\frac{2}{x^2+16}=\frac{2(x^2+16)}{8}\cdot\frac{2}{x^2+16}=\frac{1}{2}$$

38. $$\frac{x^2+x-6}{x^2-6x+9}\div\frac{x^2-4}{x^2-9}=\frac{x^2+x-6}{x^2-6x+9}\cdot\frac{x^2-9}{x^2-4}=\frac{(x+3)(x-2)}{(x-3)(x-3)}\cdot\frac{(x+3)(x-3)}{(x+2)(x-2)}$$
$$=\frac{(x+3)^2}{(x-3)(x+2)}$$

39. $$\frac{z^2+z-20}{z^2-4}\div\frac{z^2-25}{z-5}=\frac{z^2+z-20}{z^2-4}\cdot\frac{z-5}{z^2-25}=\frac{(z+5)(z-4)}{(z+2)(z-2)}\cdot\frac{z-5}{(z+5)(z-5)}$$
$$=\frac{z-4}{(z+2)(z-2)}$$

40. $$\frac{ax+bx+a+b}{a^2+2ab+b^2}\div\frac{x^2-1}{x^2-2x+1}=\frac{ax+bx+a+b}{a^2+2ab+b^2}\cdot\frac{x^2-2x+1}{x^2-1}$$
$$=\frac{x(a+b)+1(a+b)}{(a+b)(a+b)}\cdot\frac{(x-1)(x-1)}{(x+1)(x-1)}$$
$$=\frac{(a+b)(x+1)}{(a+b)(a+b)}\cdot\frac{(x-1)(x-1)}{(x+1)(x-1)}=\frac{x-1}{a+b}$$

EXERCISES 0.6

41. $$\frac{3x^2+5x-2}{x^3+2x^2} \div \frac{6x^2+13x-5}{2x^3+5x^2} = \frac{3x^2+5x-2}{x^3+2x^2} \cdot \frac{2x^3+5x^2}{6x^2+13x-5}$$
$$= \frac{(3x-1)(x+2)}{x^2(x+2)} \cdot \frac{x^2(2x+5)}{(3x-1)(2x+5)} = 1$$

42. $$\frac{x^2+13x+12}{8x^2-6x-5} \div \frac{2x^2-x-3}{8x^2-14x+5} = \frac{x^2+13x+12}{8x^2-6x-5} \cdot \frac{8x^2-14x+5}{2x^2-x-3}$$
$$= \frac{(x+12)(x+1)}{(4x-5)(2x+1)} \cdot \frac{(4x-5)(2x-1)}{(2x-3)(x+1)} = \frac{(x+12)(2x-1)}{(2x+1)(2x-3)}$$

43. $$\frac{x^2+7x+12}{x^3-x^2-6x} \cdot \frac{x^2-3x-10}{x^2+2x-3} \cdot \frac{x^3-4x^2+3x}{x^2-x-20}$$
$$= \frac{(x+3)(x+4)}{x(x-3)(x+2)} \cdot \frac{(x-5)(x+2)}{(x+3)(x-1)} \cdot \frac{x(x-3)(x-1)}{(x-5)(x+4)} = 1$$

44. $$\frac{x(x-2)-3}{x(x+7)-3(x-1)} \cdot \frac{x(x+1)-2}{x(x-7)+3(x+1)} = \frac{x^2-2x-3}{x^2+7x-3x+3} \cdot \frac{x^2+x-2}{x^2-7x+3x+3}$$
$$= \frac{x^2-2x-3}{x^2+4x+3} \cdot \frac{x^2+x-2}{x^2-4x+3}$$
$$= \frac{(x-3)(x+1)}{(x+3)(x+1)} \cdot \frac{(x+2)(x-1)}{(x-3)(x-1)} = \frac{x+2}{x+3}$$

45. $$\frac{x^2-2x-3}{21x^2-50x-16} \cdot \frac{3x-8}{x-3} \div \frac{x^2+6x+5}{7x^2-33x-10} = \frac{x^2-2x-3}{21x^2-50x-16} \cdot \frac{3x-8}{x-3} \cdot \frac{7x^2-33x-10}{x^2+6x+5}$$
$$= \frac{(x-3)(x+1)}{(7x+2)(3x-8)} \cdot \frac{3x-8}{x-3} \cdot \frac{(7x+2)(x-5)}{(x+5)(x+1)}$$
$$= \frac{x-5}{x+5}$$

46. $$\frac{x^3+27}{x^2-4} \div \left(\frac{x^2+4x+3}{x^2+2x} \div \frac{x^2+x-6}{x^2-3x+9} \right) = \frac{x^3+27}{x^2-4} \div \left(\frac{x^2+4x+3}{x^2+2x} \cdot \frac{x^2-3x+9}{x^2+x-6} \right)$$
$$= \frac{x^3+27}{x^2-4} \div \left(\frac{(x+3)(x+1)}{x(x+2)} \cdot \frac{x^2-3x+9}{(x+3)(x-2)} \right)$$
$$= \frac{x^3+27}{x^2-4} \div \frac{(x+1)(x^2-3x+9)}{x(x+2)(x-2)}$$
$$= \frac{(x+3)(x^2-3x+9)}{(x+2)(x-2)} \cdot \frac{x(x+2)(x-2)}{(x+1)(x^2-3x+9)}$$
$$= \frac{x(x+3)}{x+1}$$

47. $$\frac{3}{x+3} + \frac{x+2}{x+3} = \frac{3+x+2}{x+3} = \frac{x+5}{x+3}$$

48. $$\frac{3}{x+1} + \frac{x+2}{x+1} = \frac{3+x+2}{x+1} = \frac{x+5}{x+1}$$

49. $$\frac{4x}{x-1} - \frac{4}{x-1} = \frac{4x-4}{x-1} = \frac{4(x-1)}{x-1} = 4$$

50. $$\frac{6x}{x-2} - \frac{3}{x-2} = \frac{6x-3}{x-2} = \frac{3(2x-1)}{x-2}$$

EXERCISES 0.6

51. $\dfrac{2}{5-x}+\dfrac{1}{x-5}=\dfrac{-2}{x-5}+\dfrac{1}{x-5}=\dfrac{-1}{x-5}$ **52.** $\dfrac{3}{x-6}-\dfrac{2}{6-x}=\dfrac{3}{x-6}-\dfrac{-2}{x-6}=\dfrac{5}{x-6}$

53. $\dfrac{3}{x+1}+\dfrac{2}{x-1}=\dfrac{3(x-1)}{(x+1)(x-1)}+\dfrac{2(x+1)}{(x-1)(x+1)}=\dfrac{3x-3}{(x+1)(x-1)}+\dfrac{2x+2}{(x-1)(x+1)}$
$=\dfrac{5x-1}{(x+1)(x-1)}$

54. $\dfrac{3}{x+4}+\dfrac{x}{x-4}=\dfrac{3(x-4)}{(x+4)(x-4)}+\dfrac{x(x+4)}{(x-4)(x+4)}=\dfrac{3x-12}{(x+4)(x-4)}+\dfrac{x^2+4x}{(x-4)(x+4)}$
$=\dfrac{x^2+7x-12}{(x+4)(x-4)}$

55. $\dfrac{a+3}{a^2+7a+12}+\dfrac{a}{a^2-16}=\dfrac{a+3}{(a+3)(a+4)}+\dfrac{a}{(a+4)(a-4)}$
$=\dfrac{1}{a+4}+\dfrac{a}{(a+4)(a-4)}$
$=\dfrac{1(a-4)}{(a+4)(a-4)}+\dfrac{a}{(a+4)(a-4)}$
$=\dfrac{a-4}{(a+4)(a-4)}+\dfrac{a}{(a+4)(a-4)}$
$=\dfrac{2a-4}{(a+4)(a-4)}=\dfrac{2(a-2)}{(a+4)(a-4)}$

56. $\dfrac{a}{a^2+a-2}+\dfrac{2}{a^2-5a+4}=\dfrac{a}{(a+2)(a-1)}+\dfrac{2}{(a-4)(a-1)}$
$=\dfrac{a(a-4)}{(a+2)(a-1)(a-4)}+\dfrac{2(a+2)}{(a-4)(a-1)(a+2)}$
$=\dfrac{a^2-4a}{(a+2)(a-1)(a-4)}+\dfrac{2a+4}{(a+2)(a-1)(a-4)}$
$=\dfrac{a^2-2a+4}{(a+2)(a-1)(a-4)}$

57. $\dfrac{x}{x^2-4}-\dfrac{1}{x+2}=\dfrac{x}{(x+2)(x-2)}-\dfrac{1}{x+2}=\dfrac{x}{(x+2)(x-2)}-\dfrac{1(x-2)}{(x+2)(x-2)}$
$=\dfrac{x}{(x+2)(x-2)}-\dfrac{x-2}{(x+2)(x-2)}$
$=\dfrac{2}{(x+2)(x-2)}$

EXERCISES 0.6

58. $$\frac{b^2}{b^2-4}-\frac{4}{b^2+2b}=\frac{b^2}{(b+2)(b-2)}-\frac{4}{b(b+2)}=\frac{b^2(b)}{b(b+2)(b-2)}-\frac{4(b-2)}{b(b+2)(b-2)}$$
$$=\frac{b^3}{b(b+2)(b-2)}-\frac{4b-8}{b(b+2)(b-2)}$$
$$=\frac{b^3-4b+8}{b(b+2)(b-2)}$$

59. $$\frac{3x-2}{x^2+2x+1}-\frac{x}{x^2-1}=\frac{3x-2}{(x+1)(x+1)}-\frac{x}{(x+1)(x-1)}$$
$$=\frac{(3x-2)(x-1)}{(x+1)(x+1)(x-1)}-\frac{x(x+1)}{(x+1)(x-1)(x+1)}$$
$$=\frac{3x^2-5x+2}{(x+1)(x+1)(x-1)}-\frac{x^2+x}{(x+1)(x+1)(x-1)}$$
$$=\frac{2x^2-6x+2}{(x+1)(x+1)(x-1)}=\frac{2(x^2-3x+1)}{(x+1)^2(x-1)}$$

60. $$\frac{2t}{t^2-25}-\frac{t+1}{t^2+5t}=\frac{2t}{(t+5)(t-5)}-\frac{t+1}{t(t+5)}=\frac{2t(t)}{t(t+5)(t-5)}-\frac{(t+1)(t-5)}{t(t+5)(t-5)}$$
$$=\frac{2t^2}{t(t+5)(t-5)}-\frac{t^2-4t-5}{t(t+5)(t-5)}$$
$$=\frac{t^2+4t+5}{t(t+5)(t-5)}$$

61. $$\frac{2}{y^2-1}+3+\frac{1}{y+1}=\frac{2}{(y+1)(y-1)}+\frac{3}{1}+\frac{1}{y+1}$$
$$=\frac{2}{(y+1)(y-1)}+\frac{3(y+1)(y-1)}{1(y+1)(y-1)}+\frac{1(y-1)}{(y+1)(y-1)}$$
$$=\frac{2}{(y+1)(y-1)}+\frac{3y^2-3}{(y+1)(y-1)}+\frac{y-1}{(y+1)(y-1)}$$
$$=\frac{3y^2+y-2}{(y+1)(y-1)}=\frac{(3y-2)(y+1)}{(y+1)(y-1)}=\frac{3y-2}{y-1}$$

62. $$2+\frac{4}{t^2-4}-\frac{1}{t-2}=\frac{2}{1}+\frac{4}{(t+2)(t-2)}-\frac{1}{t-2}$$
$$=\frac{2(t+2)(t-2)}{1(t+2)(t-2)}+\frac{4}{(t+2)(t-2)}-\frac{1(t+2)}{(t+2)(t-2)}$$
$$=\frac{2t^2-8}{(t+2)(t-2)}+\frac{4}{(t+2)(t-2)}-\frac{t+2}{(t+2)(t-2)}$$
$$=\frac{2t^2-t-6}{(t+2)(t-2)}=\frac{(2t+3)(t-2)}{(t+2)(t-2)}=\frac{2t+3}{t+2}$$

EXERCISES 0.6

63. $\dfrac{1}{x-2}+\dfrac{3}{x+2}-\dfrac{3x-2}{x^2-4}=\dfrac{1}{x-2}+\dfrac{3}{x+2}-\dfrac{3x-2}{(x+2)(x-2)}$

$$=\frac{1(x+2)}{(x-2)(x+2)}+\frac{3(x-2)}{(x+2)(x-2)}-\frac{3x-2}{(x+2)(x-2)}$$

$$=\frac{x+2}{(x+2)(x-2)}+\frac{3x-6}{(x+2)(x-2)}-\frac{3x-2}{(x+2)(x-2)}$$

$$=\frac{x-2}{(x+2)(x-2)}=\frac{1}{x+2}$$

64. $\dfrac{x}{x-3}-\dfrac{5}{x+3}+\dfrac{3(3x-1)}{x^2-9}=\dfrac{x}{x-3}-\dfrac{5}{x+3}+\dfrac{9x-3}{(x+3)(x-3)}$

$$=\frac{x(x+3)}{(x-3)(x+3)}-\frac{5(x-3)}{(x+3)(x-3)}+\frac{9x-3}{(x+3)(x-3)}$$

$$=\frac{x^2+3x}{(x+3)(x-3)}-\frac{5x-15}{(x+3)(x-3)}+\frac{9x-3}{(x+3)(x-3)}$$

$$=\frac{x^2+7x+12}{(x+3)(x-3)}=\frac{(x+3)(x+4)}{(x+3)(x-3)}=\frac{x+4}{x-3}$$

65. $\left(\dfrac{1}{x-2}+\dfrac{1}{x-3}\right)\cdot\dfrac{x-3}{2x}=\left(\dfrac{1(x-3)}{(x-2)(x-3)}+\dfrac{1(x-2)}{(x-3)(x-2)}\right)\cdot\dfrac{x-3}{2x}$

$$=\left(\frac{x-3}{(x-2)(x-3)}+\frac{x-2}{(x-2)(x-3)}\right)\cdot\frac{x-3}{2x}$$

$$=\frac{2x-5}{(x-2)(x-3)}\cdot\frac{x-3}{2x}=\frac{2x-5}{2x(x-2)}$$

66. $\left(\dfrac{1}{x+1}-\dfrac{1}{x-2}\right)\div\dfrac{1}{x-2}=\left(\dfrac{1(x-2)}{(x+1)(x-2)}-\dfrac{1(x+1)}{(x-2)(x+1)}\right)\cdot\dfrac{x-2}{1}$

$$=\left(\frac{x-2}{(x+1)(x-2)}-\frac{x+1}{(x+1)(x-2)}\right)\cdot\frac{x-2}{1}$$

$$=\frac{-3}{(x+1)(x-2)}\cdot\frac{x-2}{1}=\frac{-3}{x+1}$$

67. $\dfrac{3x}{x-4}-\dfrac{x}{x+4}-\dfrac{3x+1}{16-x^2}=\dfrac{3x}{x-4}-\dfrac{x}{x+4}-\dfrac{3x+1}{(4+x)(4-x)}$

$$=\frac{3x}{x-4}-\frac{x}{x+4}+\frac{3x+1}{(x+4)(x-4)}$$

$$=\frac{3x(x+4)}{(x-4)(x+4)}-\frac{x(x-4)}{(x+4)(x-4)}+\frac{3x+1}{(x+4)(x-4)}$$

$$=\frac{3x^2+12x}{(x+4)(x-4)}-\frac{x^2-4x}{(x+4)(x-4)}+\frac{3x+1}{(x+4)(x-4)}$$

$$=\frac{2x^2+19x+1}{(x+4)(x-4)}$$

EXERCISES 0.6

68. $$\begin{aligned}\frac{7x}{x-5}+\frac{3x}{5-x}+\frac{3x-1}{x^2-25} &= \frac{7x}{x-5}+\frac{-3x}{x-5}+\frac{3x-1}{(x+5)(x-5)}\\ &= \frac{4x}{x-5}+\frac{3x-1}{(x+5)(x-5)}\\ &= \frac{4x(x+5)}{(x-5)(x+5)}+\frac{3x-1}{(x+5)(x-5)}\\ &= \frac{4x^2+20x}{(x+5)(x-5)}+\frac{3x-1}{(x+5)(x-5)} = \frac{4x^2+23x-1}{(x+5)(x-5)}\end{aligned}$$

69. $$\begin{aligned}&\frac{1}{x^2+3x+2}-\frac{2}{x^2+4x+3}+\frac{1}{x^2+5x+6}\\ &= \frac{1}{(x+2)(x+1)}-\frac{2}{(x+3)(x+1)}+\frac{1}{(x+2)(x+3)}\\ &= \frac{1(x+3)}{(x+2)(x+1)(x+3)}-\frac{2(x+2)}{(x+3)(x+1)(x+2)}+\frac{1(x+1)}{(x+2)(x+3)(x+1)}\\ &= \frac{x+3}{(x+2)(x+1)(x+3)}-\frac{2x+4}{(x+2)(x+1)(x+3)}+\frac{x+1}{(x+2)(x+1)(x+3)}\\ &= \frac{x+3-2x-4+x+1}{(x+2)(x+1)(x+3)} = \frac{0}{(x+2)(x+1)(x+3)} = 0\end{aligned}$$

70. $$\begin{aligned}\frac{-2}{x-y}+\frac{2}{x-z}-\frac{2z-2y}{(y-x)(z-x)} &= \frac{2}{y-x}+\frac{-2}{z-x}-\frac{2z-2y}{(y-x)(z-x)}\\ &= \frac{2(z-x)}{(y-x)(z-x)}+\frac{-2(y-x)}{(z-x)(y-x)}-\frac{2z-2y}{(y-x)(z-x)}\\ &= \frac{2z-2x}{(y-x)(z-x)}+\frac{-2y+2x}{(y-x)(z-x)}-\frac{2z-2y}{(y-x)(z-x)}\\ &= \frac{2z-2x-2y+2x-2z+2y}{(y-x)(z-x)} = \frac{0}{(y-x)(z-x)} = 0\end{aligned}$$

71. $$\begin{aligned}&\frac{3x-2}{x^2+x-20}-\frac{4x^2+2}{x^2-25}+\frac{3x^2-25}{x^2-16}\\ &= \frac{3x-2}{(x+5)(x-4)}-\frac{4x^2+2}{(x+5)(x-5)}+\frac{3x^2-25}{(x+4)(x-4)}\\ &= \frac{(3x-2)(x-5)(x+4)}{(x+5)(x-4)(x-5)(x+4)}-\frac{(4x^2+2)(x-4)(x+4)}{(x+5)(x-5)(x-4)(x+4)}\ \cdots\\ &\qquad\qquad +\frac{(3x^2-25)(x+5)(x-5)}{(x+4)(x-4)(x+5)(x-5)}\\ &= \frac{3x^3-5x^2-58x+40}{(x+5)(x-4)(x-5)(x+4)}-\frac{4x^4-62x^2-32}{(x+5)(x-4)(x-5)(x+4)}\ \cdots\\ &\qquad\qquad +\frac{3x^4-100x^2+625}{(x+5)(x-4)(x-5)(x+4)}\end{aligned}$$

continued on next page...

71. **continued**

$$= \frac{3x^3 - 5x^2 - 58x + 40}{(x+5)(x-4)(x-5)(x+4)} - \frac{4x^4 - 62x^2 - 32}{(x+5)(x-4)(x-5)(x+4)} \cdots + \frac{3x^4 - 100x^2 + 625}{(x+5)(x-4)(x-5)(x+4)}$$

$$= \frac{3x^3 - 5x^2 - 58x + 40 - 4x^4 + 62x^2 + 32 + 3x^4 - 100x^2 + 625}{(x+5)(x-4)(x-5)(x+4)}$$

$$= \frac{-x^4 + 3x^3 - 43x^2 - 58x + 697}{(x+5)(x-4)(x-5)(x+4)}$$

72. $\dfrac{3x+2}{8x^2-10x-3} + \dfrac{x+4}{6x^2-11x+3} - \dfrac{1}{4x+1}$

$$= \frac{3x+2}{(4x+1)(2x-3)} + \frac{x+4}{(3x-1)(2x-3)} - \frac{1}{4x+1}$$

$$= \frac{(3x+2)(3x-1)}{(4x+1)(2x-3)(3x-1)} + \frac{(x+4)(4x+1)}{(3x-1)(2x-3)(4x+1)} - \frac{1(2x-3)(3x-1)}{(4x+1)(2x-3)(3x-1)}$$

$$= \frac{9x^2+3x-2}{(4x+1)(2x-3)(3x-1)} + \frac{4x^2+17x+4}{(4x+1)(2x-3)(3x-1)} - \frac{6x^2-11x+3}{(4x+1)(2x-3)(3x-1)}$$

$$= \frac{9x^2+3x-2+4x^2+17x+4-6x^2+11x-3}{(4x+1)(2x-3)(3x-1)} = \frac{7x^2+31x-1}{(4x+1)(2x-3)(3x-1)}$$

73. $\dfrac{\frac{3a}{b}}{\frac{6ac}{b^2}} = \dfrac{3a}{b} \div \dfrac{6ac}{b^2} = \dfrac{3a}{b} \cdot \dfrac{b^2}{6ac} = \dfrac{b}{2c}$

74. $\dfrac{\frac{3t^2}{9x}}{\frac{t}{18x}} = \dfrac{3t^2}{9x} \div \dfrac{t}{18x} = \dfrac{3t^2}{9x} \cdot \dfrac{18x}{t} = 6t$

75. $\dfrac{3a^2b}{\frac{ab}{27}} = \dfrac{3a^2b}{1} \div \dfrac{ab}{27} = \dfrac{3a^2b}{1} \cdot \dfrac{27}{ab} = 81a$

76. $\dfrac{\frac{3u^2v}{4t}}{3uv} = \dfrac{3u^2v}{4t} \div \dfrac{3uv}{1} = \dfrac{3u^2v}{4t} \cdot \dfrac{1}{3uv} = \dfrac{u}{4t}$

77. $\dfrac{\frac{x-y}{ab}}{\frac{y-x}{ab}} = \dfrac{x-y}{ab} \div \dfrac{y-x}{ab} = \dfrac{x-y}{ab} \cdot \dfrac{ab}{y-x} = -1$

78. $\dfrac{\frac{x^2-5x+6}{2x^2y}}{\frac{x^2-9}{2x^2y}} = \dfrac{x^2-5x+6}{2x^2y} \div \dfrac{x^2-9}{2x^2y} = \dfrac{x^2-5x+6}{2x^2y} \cdot \dfrac{2x^2y}{x^2-9}$

$$= \frac{(x-3)(x-2)}{2x^2y} \cdot \frac{2x^2y}{(x+3)(x-3)} = \frac{x-2}{x+3}$$

79. $\dfrac{\frac{1}{x}+\frac{1}{y}}{xy} = \dfrac{xy\left(\frac{1}{x}+\frac{1}{y}\right)}{xy(xy)} = \dfrac{xy\left(\frac{1}{x}\right)+xy\left(\frac{1}{y}\right)}{x^2y^2} = \dfrac{y+x}{x^2y^2}$

80. $\dfrac{xy}{\frac{11}{x}+\frac{11}{y}} = \dfrac{xy(xy)}{xy\left(\frac{11}{x}+\frac{11}{y}\right)} = \dfrac{x^2y^2}{xy\left(\frac{11}{x}\right)+xy\left(\frac{11}{y}\right)} = \dfrac{x^2y^2}{11y+11x} = \dfrac{x^2y^2}{11(y+x)}$

EXERCISES 0.6

81. $\dfrac{\frac{1}{x}+\frac{1}{y}}{\frac{1}{x}-\frac{1}{y}} = \dfrac{xy\left(\frac{1}{x}+\frac{1}{y}\right)}{xy\left(\frac{1}{x}-\frac{1}{y}\right)} = \dfrac{xy\left(\frac{1}{x}\right)+xy\left(\frac{1}{y}\right)}{xy\left(\frac{1}{x}\right)-xy\left(\frac{1}{y}\right)} = \dfrac{y+x}{y-x}$

82. $\dfrac{\frac{1}{x}-\frac{1}{y}}{\frac{1}{x}+\frac{1}{y}} = \dfrac{xy\left(\frac{1}{x}-\frac{1}{y}\right)}{xy\left(\frac{1}{x}+\frac{1}{y}\right)} = \dfrac{xy\left(\frac{1}{x}\right)-xy\left(\frac{1}{y}\right)}{xy\left(\frac{1}{x}\right)+xy\left(\frac{1}{y}\right)} = \dfrac{y-x}{y+x}$

83. $\dfrac{\frac{3a}{b}-\frac{4a^2}{x}}{\frac{1}{b}+\frac{1}{ax}} = \dfrac{abx\left(\frac{3a}{b}-\frac{4a^2}{x}\right)}{abx\left(\frac{1}{b}+\frac{1}{ax}\right)} = \dfrac{abx\left(\frac{3a}{b}\right)-abx\left(\frac{4a^2}{x}\right)}{abx\left(\frac{1}{b}\right)+abx\left(\frac{1}{ax}\right)} = \dfrac{3a^2x-4a^3b}{ax+b} = \dfrac{a^2(3x-4ab)}{ax+b}$

84. $\dfrac{1-\frac{x}{y}}{\frac{x^2}{y^2}-1} = \dfrac{y^2\left(1-\frac{x}{y}\right)}{y^2\left(\frac{x^2}{y^2}-1\right)} = \dfrac{y^2(1)-y^2\left(\frac{x}{y}\right)}{y^2\left(\frac{x^2}{y^2}\right)-y^2(1)} = \dfrac{y^2-xy}{x^2-y^2} = \dfrac{y(y-x)}{(x+y)(x-y)} = \dfrac{-y}{x+y}$

85. $\dfrac{x+1-\frac{6}{x}}{x+5+\frac{6}{x}} = \dfrac{x\left(x+1-\frac{6}{x}\right)}{x\left(x+5+\frac{6}{x}\right)} = \dfrac{x(x)+x(1)-x\left(\frac{6}{x}\right)}{x(x)+x(5)+x\left(\frac{6}{x}\right)} = \dfrac{x^2+x-6}{x^2+5x+6} = \dfrac{(x+3)(x-2)}{(x+2)(x+3)} = \dfrac{x-2}{x+2}$

86. $\dfrac{2z}{1-\frac{3}{z}} = \dfrac{z(2z)}{z\left(1-\frac{3}{z}\right)} = \dfrac{2z^2}{z(1)-z\left(\frac{3}{z}\right)} = \dfrac{2z^2}{z-3}$

87. $\dfrac{3xy}{1-\frac{1}{xy}} = \dfrac{xy(3xy)}{xy\left(1-\frac{1}{xy}\right)} = \dfrac{3x^2y^2}{xy(1)-xy\left(\frac{1}{xy}\right)} = \dfrac{3x^2y^2}{xy-1}$

88. $\dfrac{x-3+\frac{1}{x}}{-\frac{1}{x}-x+3} = \dfrac{x\left(x-3+\frac{1}{x}\right)}{x\left(-\frac{1}{x}-x+3\right)} = \dfrac{x^2-3x+1}{-1-x^2+3x} = \dfrac{x^2-3x+1}{-(x^2-3x+1)} = -1$

89. $\dfrac{3x}{x+\frac{1}{x}} = \dfrac{x(3x)}{x\left(x+\frac{1}{x}\right)} = \dfrac{3x^2}{x^2+1}$

90. $\dfrac{2x^2+4}{2+\frac{4x}{5}} = \dfrac{5(2x^2+4)}{5\left(2+\frac{4x}{5}\right)} = \dfrac{10x^2+20}{10+4x} = \dfrac{2(5x^2+10)}{2(5+2x)} = \dfrac{5x^2+10}{5+2x}$

91. $\dfrac{\frac{x}{x+2}-\frac{2}{x-1}}{\frac{3}{x+2}+\frac{x}{x-1}} = \dfrac{(x+2)(x-1)\left(\frac{x}{x+2}-\frac{2}{x-1}\right)}{(x+2)(x-1)\left(\frac{3}{x+2}+\frac{x}{x-1}\right)} = \dfrac{(x+2)(x-1)\left(\frac{x}{x+2}\right)-(x+2)(x-1)\left(\frac{2}{x-1}\right)}{(x+2)(x-1)\left(\frac{3}{x+2}\right)+(x+2)(x-1)\left(\frac{x}{x-1}\right)}$

$= \dfrac{(x-1)(x)-(x+2)(2)}{(x-1)(3)+(x+2)(x)}$

$= \dfrac{x^2-x-2x-4}{3x-3+x^2+2x} = \dfrac{x^2-3x-4}{x^2+5x-3}$

92. $\dfrac{\frac{2x}{x-3}+\frac{1}{x-2}}{\frac{3}{x-3}-\frac{x}{x-2}} = \dfrac{(x-3)(x-2)\left(\frac{2x}{x-3}+\frac{1}{x-2}\right)}{(x-3)(x-2)\left(\frac{3}{x-3}-\frac{x}{x-2}\right)} = \dfrac{(x-3)(x-2)\left(\frac{2x}{x-3}\right)+(x-3)(x-2)\left(\frac{1}{x-2}\right)}{(x-3)(x-2)\left(\frac{3}{x-3}\right)-(x-3)(x-2)\left(\frac{x}{x-2}\right)}$

$= \dfrac{(x-2)(2x)+(x-3)(1)}{(x-2)(3)-(x-3)(x)}$

$= \dfrac{2x^2-4x+x-3}{3x-6-x^2+3x}$

$= \dfrac{2x^2-3x-3}{-x^2+6x-6} = -\dfrac{2x^2-3x-3}{x^2-6x+6}$

93. $\dfrac{1}{1+x^{-1}} = \dfrac{1}{1+\frac{1}{x}} = \dfrac{x(1)}{x\left(1+\frac{1}{x}\right)} = \dfrac{x}{x+1}$

94. $\dfrac{y^{-1}}{x^{-1}+y^{-1}} = \dfrac{\frac{1}{y}}{\frac{1}{x}+\frac{1}{y}} = \dfrac{xy\left(\frac{1}{y}\right)}{xy\left(\frac{1}{x}+\frac{1}{y}\right)} = \dfrac{x}{y+x}$

95. $\dfrac{3(x+2)^{-1}+2(x-1)^{-1}}{(x+2)^{-1}} = \dfrac{\frac{3}{x+2}+\frac{2}{x-1}}{\frac{1}{x+2}} = \dfrac{(x+2)(x-1)\left(\frac{3}{x+2}+\frac{2}{x-1}\right)}{(x+2)(x-1)\left(\frac{1}{x+2}\right)}$

$= \dfrac{(x+2)(x-1)\left(\frac{3}{x+2}\right)+(x+2)(x-1)\left(\frac{2}{x-1}\right)}{x-1}$

$= \dfrac{(x-1)(3)+(x+2)(2)}{x-1}$

$= \dfrac{3x-3+2x+4}{x-1} = \dfrac{5x+1}{x-1}$

96. $\dfrac{2x(x-3)^{-1}-3(x+2)^{-1}}{(x-3)^{-1}(x+2)^{-1}} = \dfrac{\frac{2x}{x-3}-\frac{3}{x+2}}{\frac{1}{(x-3)(x+2)}} = \dfrac{(x-3)(x+2)\left(\frac{2x}{x-3}-\frac{3}{x+2}\right)}{(x-3)(x+2)\left(\frac{1}{(x-3)(x+2)}\right)}$

$= \dfrac{(x+2)(2x)-(x-3)(3)}{1}$

$= 2x^2+4x-3x+9 = 2x^2+x+9$

97. $\dfrac{1}{\frac{1}{k_1}+\frac{1}{k_2}} = \dfrac{k_1k_2(1)}{k_1k_2\left(\frac{1}{k_1}+\frac{1}{k_2}\right)} = \dfrac{k_1k_2}{k_1k_2\left(\frac{1}{k_1}\right)+k_1k_2\left(\frac{1}{k_2}\right)} = \dfrac{k_1k_2}{k_2+k_1}$

98. $\dfrac{1}{\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}} = \dfrac{R_1R_2R_3(1)}{R_1R_2R_3\left(\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}\right)} = \dfrac{R_1R_2R_3}{R_1R_2R_3\left(\frac{1}{R_1}\right)+R_1R_2R_3\left(\frac{1}{R_2}\right)+R_1R_2R_3\left(\frac{1}{R_3}\right)}$

$= \dfrac{R_1R_2R_3}{R_2R_3+R_1R_3+R_1R_2}$

99-104. Answers may vary.

105. $\frac{a}{b}+\frac{c}{d} = \frac{a}{b}\cdot\frac{d}{d}+\frac{c}{d}\cdot\frac{b}{b} = \frac{ad}{bd}+\frac{bc}{bd} = \frac{ad+bc}{bd}$.

106. Let $x = \frac{a}{b}\div\frac{c}{d}$. Then $x\cdot\frac{c}{d} = \frac{a}{b}$, and $x\cdot\frac{c}{d}\cdot\frac{d}{c} = \frac{a}{b}\cdot\frac{d}{c}$. Thus, $\frac{a}{b}\div\frac{c}{d} = x = \frac{a}{b}\cdot\frac{d}{c}$.

107. $\dfrac{x}{1+\frac{1}{3x^{-1}}} = \dfrac{x}{1+\frac{1}{\frac{3}{x}}} = \dfrac{x}{1+\frac{x(1)}{x\left(\frac{3}{x}\right)}} = \dfrac{x}{1+\frac{x}{3}} = \dfrac{3x}{3\left(1+\frac{x}{3}\right)} = \dfrac{3x}{3+x}$

108. $\dfrac{ab}{2+\frac{3}{2a^{-1}}} = \dfrac{ab}{2+\frac{3}{\frac{2}{a}}} = \dfrac{ab}{2+\frac{a(3)}{a\left(\frac{2}{a}\right)}} = \dfrac{ab}{2+\frac{3a}{2}} = \dfrac{2ab}{2\left(2+\frac{3a}{2}\right)} = \dfrac{2ab}{4+3a}$

109. $\dfrac{1}{1+\frac{1}{1+\frac{1}{x}}} = \dfrac{1}{1+\frac{x(1)}{x\left(1+\frac{1}{x}\right)}} = \dfrac{1}{1+\frac{x}{x+1}} = \dfrac{(x+1)1}{(x+1)\left(1+\frac{x}{x+1}\right)} = \dfrac{x+1}{x+1+x} = \dfrac{x+1}{2x+1}$

110. $\dfrac{y}{2+\frac{2}{2+\frac{2}{y}}} = \dfrac{y}{2+\frac{y(2)}{y\left(2+\frac{2}{y}\right)}} = \dfrac{y}{2+\frac{2y}{2y+2}} = \dfrac{(2y+2)(y)}{(2y+2)\left(2+\frac{2y}{2y+2}\right)} = \dfrac{2y^2+2y}{(2y+2)(2)+2y}$

$$= \frac{2y(y+1)}{4y+4+2y}$$

$$= \frac{2y(y+1)}{2(3y+2)} = \frac{y(y+1)}{3y+2}$$

111. False. The denominator can never equal 0.

112. True.

113. False. $\frac{x+7}{x+7} = 1$ for all values of x except $x = -7$.

114. False. $\frac{x-7}{7-x} = -1$ for all values of x except $x = 7$.

115. $-\dfrac{(x-y)^3}{(y-x)^3} = -\dfrac{(x-y)^3}{[-(x-y)]^3} = -\dfrac{(x-y)^3}{(-1)^3(x-y)^3} = -\dfrac{1}{-1} = 1$. True.

116. False. $\dfrac{25+x}{25} = \dfrac{25}{25} + \dfrac{x}{25} = 1 + \dfrac{x}{25}$.

117. False. $\dfrac{5}{x} + \dfrac{5}{y} = \dfrac{5}{x}\cdot\dfrac{y}{y} + \dfrac{5}{y}\cdot\dfrac{x}{x} = \dfrac{5y+5x}{xy}$.

118. False. $10 - \dfrac{1}{x} = \dfrac{10}{1}\cdot\dfrac{x}{x} - \dfrac{1}{x} = \dfrac{10x-1}{x}$.

119. The domain is the set of all real numbers except $x = 6$.

120. The domain is the set of all real numbers.

Chapter 0 Review (page 76)

1. natural: 3, 6, 8

2. whole: 0, 3, 6, 8

3. integers: -6, -3, 0, 3, 6, 8

4. rational: -6, -3, 0, $\frac{1}{2}$, 3, 6, 8

5. irrational: π, $\sqrt{5}$

6. real: -6, -3, 0, $\frac{1}{2}$, 3, π, $\sqrt{5}$, 6, 8

7. prime: 3

8. composite: 6, 8

9. even integers: $-6, 0, 6, 8$

10. odd integers: $-3, 3$

11. Associative Property of Addition

12. Commutative Property of Addition

13. Associative Property of Multiplication

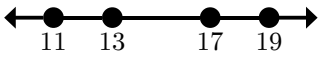
14. Distributive Property

15. Commutative Property of Multiplication

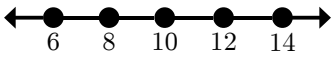
16. Commutative Property of Addition

17. Double Negative Rule

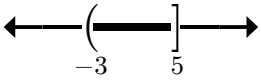
18.



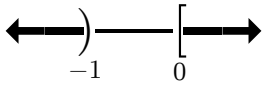
19.



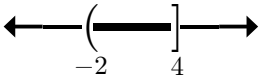
20. $-3 < x \leq 5$



21. $x \geq 0$ or $x < -1$

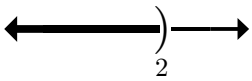


22. $(-2, 4]$

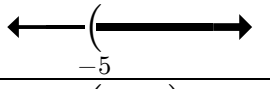


23. $(-\infty, 2) \cap (-5, \infty)$

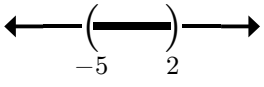
$(-\infty, 2)$



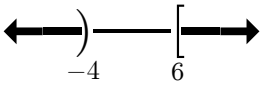
$(-5, \infty)$



$(-\infty, 2) \cap (-5, \infty)$



24. $(-\infty, -4) \cup [6, \infty)$



25. Since $6 \geq 0$, $|6| = 6$.

26. Since $-25 < 0$, $|-25| = -(-25) = 25$.

27. Since $1 - \sqrt{2} < 0$,
$\left|1 - \sqrt{2}\right| = -\left(1 - \sqrt{2}\right) = \sqrt{2} - 1.$

28. Since $\sqrt{3} - 1 \geq 0$,
$\left|\sqrt{3} - 1\right| = \sqrt{3} - 1.$

29. distance $= |7 - (-5)| = |12| = 12$

30. $-5a^3 = -5aaa$

31. $(-5a)^2 = (-5a)(-5a)$

32. $3ttt = 3t^3$

33. $(-2b)(3b) = (-2)(3)bb = -6b^2$

34. $n^2n^4 = n^{2+4} = n^6$

35. $(p^3)^2 = p^{3\cdot 2} = p^6$

36. $(x^3y^2)^4 = (x^3)^4(y^2)^4 = x^{12}y^8$

37. $\left(\dfrac{a^4}{b^2}\right)^3 = \dfrac{(a^4)^3}{(b^2)^3} = \dfrac{a^{12}}{b^6}$

38. $\left(m^{-3}n^0\right)^2 = \left(m^{-3}\cdot 1\right)^2 = m^{-6} = \dfrac{1}{m^6}$

39. $\left(\dfrac{p^{-2}q^2}{2}\right)^3 = \left(\dfrac{q^2}{2p^2}\right)^3 = \dfrac{(q^2)^3}{(2p^2)^3} = \dfrac{q^6}{8p^6}$

40. $\dfrac{a^5}{a^8} = a^{5-8} = a^{-3} = \dfrac{1}{a^3}$

41. $\left(\dfrac{a^2}{b^3}\right)^{-2} = \left(\dfrac{b^3}{a^2}\right)^2 = \dfrac{b^6}{a^4}$

42. $\left(\dfrac{3x^2y^{-2}}{x^2y^2}\right)^{-2} = \left(\dfrac{x^2y^2}{3x^2y^{-2}}\right)^2 = \left(\dfrac{x^2y^2y^2}{3x^2}\right)^2 = \left(\dfrac{y^4}{3}\right)^2 = \dfrac{y^8}{9}$

43. $\left(\dfrac{a^{-3}b^2}{ab^{-3}}\right)^{-2} = \left(\dfrac{ab^{-3}}{a^{-3}b^2}\right)^2 = \left(\dfrac{aa^3}{b^2b^3}\right)^2 = \left(\dfrac{a^4}{b^5}\right)^2 = \dfrac{a^8}{b^{10}}$

44. $\left(\dfrac{-3x^3y}{xy^3}\right)^{-2} = \left(\dfrac{xy^3}{-3x^3y}\right)^2 = \left(\dfrac{y^2}{-3x^2}\right)^2 = \dfrac{y^4}{9x^4}$

45. $\left(-\dfrac{2m^{-2}n^0}{4m^2n^{-1}}\right)^{-3} = \left(-\dfrac{4m^2n^{-1}}{2m^{-2}n^0}\right)^3 = \left(-\dfrac{2m^2m^2}{n^1n^0}\right)^3 = \left(-\dfrac{2m^4}{n}\right)^3 = -\dfrac{8m^{12}}{n^3}$

46. $-x^2 - xy^2 = -(-3)^2 - (-3)(3)^2 = -(+9) - (-3)(9) = -9 - (-27) = -9 + 27 = 18$

47. $6750 = 6.750 \times 10^3$

48. $0.00023 = 2.3 \times 10^{-4}$

49. $4.8 \times 10^2 = 480$

50. $0.25 \times 10^{-3} = 0.00025$

51. $\dfrac{(45{,}000)(350{,}000)}{0.000105} = \dfrac{(4.5\times 10^4)(3.5\times 10^5)}{1.05\times 10^{-4}} = \dfrac{4.5\times 3.5\times 10^4\times 10^5}{1.05\times 10^{-4}} = \dfrac{15.75\times 10^9}{1.05\times 10^{-4}}$
$= 15\times 10^{13}$
$= 1.5\times 10^{14}$

52. $121^{1/2} = (11^2)^{1/2} = 11$

53. $\left(\dfrac{27}{125}\right)^{1/3} = \left[\left(\dfrac{3}{5}\right)^3\right]^{1/3} = \dfrac{3}{5}$

54. $(32x^5)^{1/5} = 32^{1/5}(x^5)^{1/5} = 2x$

55. $(81a^4)^{1/4} = 81^{1/4}(a^4)^{1/4} = 3|a|$

56. $\left(-1000x^6\right)^{1/3} = (-1000)^{1/3}\left(x^6\right)^{1/3}$
$= -10x^2$

57. $\left(-25x^2\right)^{1/2} = (-25)^{1/2}\left(x^2\right)^{1/2}$
$\Rightarrow$ not a real number

58. $(x^{12}y^2)^{1/2} = (x^{12})^{1/2}(y^2)^{1/2} = x^6|y|$

59. $\left(\dfrac{x^{12}}{y^4}\right)^{-1/2} = \left(\dfrac{y^4}{x^{12}}\right)^{1/2} = \dfrac{y^2}{x^6}$

60. $\left(\dfrac{-c^{2/3}c^{5/3}}{c^{-2/3}}\right)^{1/3} = \left(\dfrac{-c^{7/3}}{c^{-2/3}}\right)^{1/3} = \left(-c^{9/3}\right)^{1/3} = \left(-c^3\right)^{1/3} = -c$

61. $\left(\dfrac{a^{-1/4}a^{3/4}}{a^{9/2}}\right)^{-1/2} = \left(\dfrac{a^{9/2}}{a^{-1/4}a^{3/4}}\right)^{1/2} = \left(\dfrac{a^{9/2}}{a^{2/4}}\right)^{1/2} = \left(\dfrac{a^{9/2}}{a^{1/2}}\right)^{1/2} = \left(a^{8/2}\right)^{1/2} = \left(a^4\right)^{1/2} = a^2$

62. $64^{2/3} = \left(64^{1/3}\right)^2 = 4^2 = 16$

63. $32^{-3/5} = \dfrac{1}{32^{3/5}} = \dfrac{1}{\left(32^{1/5}\right)^3} = \dfrac{1}{2^3} = \dfrac{1}{8}$

64. $\left(\dfrac{16}{81}\right)^{3/4} = \dfrac{16^{3/4}}{81^{3/4}} = \dfrac{\left(16^{1/4}\right)^3}{\left(81^{1/4}\right)^3} = \dfrac{2^3}{3^3} = \dfrac{8}{27}$

65. $\left(\dfrac{32}{243}\right)^{2/5} = \dfrac{32^{2/5}}{243^{2/5}} = \dfrac{\left(32^{1/5}\right)^2}{\left(243^{1/5}\right)^2} = \dfrac{2^2}{3^2} = \dfrac{4}{9}$

66. $\left(\dfrac{8}{27}\right)^{-2/3} = \left(\dfrac{27}{8}\right)^{2/3} = \dfrac{27^{2/3}}{8^{2/3}} = \dfrac{\left(27^{1/3}\right)^2}{\left(8^{1/3}\right)^2} = \dfrac{3^2}{2^2} = \dfrac{9}{4}$

67. $\left(\dfrac{16}{625}\right)^{-3/4} = \left(\dfrac{625}{16}\right)^{3/4} = \dfrac{625^{3/4}}{16^{3/4}} = \dfrac{\left(625^{1/4}\right)^3}{\left(16^{1/4}\right)^3} = \dfrac{5^3}{2^3} = \dfrac{125}{8}$

68. $(-216x^3)^{2/3} = (-216)^{2/3}(x^3)^{2/3} = 36x^2$

69. $\dfrac{p^{a/2}p^{a/3}}{p^{a/6}} = \dfrac{p^{3a/6}p^{2a/6}}{p^{a/6}} = \dfrac{p^{5a/6}}{p^{a/6}} = p^{4a/6} = p^{2a/3}$

70. $\sqrt{36} = 6$

71. $-\sqrt{49} = -7$

72. $\sqrt{\dfrac{9}{25}} = \dfrac{\sqrt{9}}{\sqrt{25}} = \dfrac{3}{5}$

73. $\sqrt[3]{\dfrac{27}{125}} = \dfrac{\sqrt[3]{27}}{\sqrt[3]{125}} = \dfrac{3}{5}$

74. $\sqrt{x^2y^4} = \sqrt{x^2}\sqrt{y^4} = |x|y^2$

75. $\sqrt[3]{x^3} = x$

76. $\sqrt[4]{\dfrac{m^8n^4}{p^{16}}} = \dfrac{\sqrt[4]{m^8}\sqrt[4]{n^4}}{\sqrt[4]{p^{16}}} = \dfrac{m^2|n|}{p^4}$

77. $\sqrt[5]{\dfrac{a^{15}b^{10}}{c^5}} = \dfrac{\sqrt[5]{a^{15}}\sqrt[5]{b^{10}}}{\sqrt[5]{c^5}} = \dfrac{a^3b^2}{c}$

78. $\sqrt{50} + \sqrt{8} = \sqrt{25}\sqrt{2} + \sqrt{4}\sqrt{2} = 5\sqrt{2} + 2\sqrt{2} = 7\sqrt{2}$

79. $\sqrt{12} + \sqrt{3} - \sqrt{27} = \sqrt{4}\sqrt{3} + \sqrt{3} - \sqrt{9}\sqrt{3} = 2\sqrt{3} + \sqrt{3} - 3\sqrt{3} = 3\sqrt{3} - 3\sqrt{3} = 0$

80. $\sqrt[3]{24x^4} - \sqrt[3]{3x^4} = \sqrt[3]{8x^3}\sqrt[3]{3x} - \sqrt[3]{x^3}\sqrt[3]{3x} = 2x\sqrt[3]{3x} - x\sqrt[3]{3x} = x\sqrt[3]{3x}$

81. $\dfrac{\sqrt{7}}{\sqrt{5}} = \dfrac{\sqrt{7}}{\sqrt{5}} \cdot \dfrac{\sqrt{5}}{\sqrt{5}} = \dfrac{\sqrt{35}}{5}$

82. $\dfrac{8}{\sqrt{8}} = \dfrac{8}{\sqrt{8}} \cdot \dfrac{\sqrt{2}}{\sqrt{2}} = \dfrac{8\sqrt{2}}{\sqrt{16}} = \dfrac{8\sqrt{2}}{4} = 2\sqrt{2}$

83. $\dfrac{1}{\sqrt[3]{2}} = \dfrac{1}{\sqrt[3]{2}} \cdot \dfrac{\sqrt[3]{4}}{\sqrt[3]{4}} = \dfrac{\sqrt[3]{4}}{\sqrt[3]{8}} = \dfrac{\sqrt[3]{4}}{2}$

84. $\dfrac{2}{\sqrt[3]{25}} = \dfrac{2}{\sqrt[3]{25}} \cdot \dfrac{\sqrt[3]{5}}{\sqrt[3]{5}} = \dfrac{2\sqrt[3]{5}}{\sqrt[3]{125}} = \dfrac{2\sqrt[3]{5}}{5}$

85. $\dfrac{\sqrt{2}}{5} = \dfrac{\sqrt{2}}{5} \cdot \dfrac{\sqrt{2}}{\sqrt{2}} = \dfrac{2}{5\sqrt{2}}$

86. $\dfrac{\sqrt{5}}{5} = \dfrac{\sqrt{5}}{5} \cdot \dfrac{\sqrt{5}}{\sqrt{5}} = \dfrac{5}{5\sqrt{5}} = \dfrac{1}{\sqrt{5}}$

87. $\dfrac{\sqrt{2x}}{3} = \dfrac{\sqrt{2x}}{3} \cdot \dfrac{\sqrt{2x}}{\sqrt{2x}} = \dfrac{2x}{3\sqrt{2x}}$

88. $\dfrac{3\sqrt[3]{7x}}{2} = \dfrac{3\sqrt[3]{7x}}{2} \cdot \dfrac{\sqrt[3]{49x^2}}{\sqrt[3]{49x^2}} = \dfrac{3\sqrt[3]{343x^3}}{2\sqrt[3]{49x^2}} = \dfrac{21x}{2\sqrt[3]{49x^2}}$

89. 3rd degree, binomial

90. 2nd degree, trinomial

91. 2nd degree, monomial

92. 4th degree, trinomial

93. $2(x+3)+3(x-4) = 2x+6+3x-12 = 5x-6$

94. $3x^2(x-1)-2x(x+3)-x^2(x+2) = 3x^3-3x^2-2x^2-6x-x^3-2x^2 = 2x^3-7x^2-6x$

95. $(3x+2)(3x+2) = 9x^2+6x+6x+4 = 9x^2+12x+4$

96. $(3x+y)(2x-3y) = 6x^2-9xy+2xy-3y^2 = 6x^2-7xy-3y^2$

97. $(4a+2b)(2a-3b) = 8a^2-12ab+4ab-6b^2 = 8a^2-8ab-6b^2$

98. $(z+3)(3z^2+z-1) = 3z^3+z^2-z+9z^2+3z-3 = 3z^3+10z^2+2z-3$

99. $(a^n+2)(a^n-1) = a^{2n}-a^n+2a^n-2 = a^{2n}+a^n-2$

100. $\left(\sqrt{2}+x\right)^2 = \left(\sqrt{2}+x\right)\left(\sqrt{2}+x\right) = \left(\sqrt{2}\right)^2 + x\sqrt{2}+x\sqrt{2}+x^2 = 2+2x\sqrt{2}+x^2$

101. $\left(\sqrt{2}+1\right)\left(\sqrt{3}+1\right) = \sqrt{6}+\sqrt{2}+\sqrt{3}+1$

102. $\left(\sqrt[3]{3}-2\right)\left(\sqrt[3]{9}+2\sqrt[3]{3}+4\right) = \sqrt[3]{27}+2\sqrt[3]{9}+4\sqrt[3]{3}-2\sqrt[3]{9}-4\sqrt[3]{3}-8 = 3-8 = -5$

103. $\dfrac{2}{\sqrt{3}-1} = \dfrac{2}{\sqrt{3}-1} \cdot \dfrac{\sqrt{3}+1}{\sqrt{3}+1} = \dfrac{2\left(\sqrt{3}+1\right)}{\left(\sqrt{3}\right)^2-1^2} = \dfrac{2\left(\sqrt{3}+1\right)}{3-1} = \dfrac{2\left(\sqrt{3}+1\right)}{2} = \sqrt{3}+1$

104. $$\frac{-2}{\sqrt{3}-\sqrt{2}} = \frac{-2}{\sqrt{3}-\sqrt{2}} \cdot \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}} = \frac{-2\left(\sqrt{3}+\sqrt{2}\right)}{\left(\sqrt{3}\right)^2-\left(\sqrt{2}\right)^2} = \frac{-2\left(\sqrt{3}+\sqrt{2}\right)}{3-2}$$
$$= \frac{-2\left(\sqrt{3}+\sqrt{2}\right)}{1}$$
$$= -2\left(\sqrt{3}+\sqrt{2}\right)$$

105. $$\frac{2x}{\sqrt{x}-2} = \frac{2x}{\sqrt{x}-2} \cdot \frac{\sqrt{x}+2}{\sqrt{x}+2} = \frac{2x\left(\sqrt{x}+2\right)}{\left(\sqrt{x}\right)^2-2^2} = \frac{2x\left(\sqrt{x}+2\right)}{x-4}$$

106. $$\frac{\sqrt{x}-\sqrt{y}}{\sqrt{x}+\sqrt{y}} = \frac{\sqrt{x}-\sqrt{y}}{\sqrt{x}+\sqrt{y}} \cdot \frac{\sqrt{x}-\sqrt{y}}{\sqrt{x}-\sqrt{y}} = \frac{\sqrt{x^2}-\sqrt{xy}-\sqrt{xy}+y}{\left(\sqrt{x}\right)^2-\left(\sqrt{y}\right)^2} = \frac{x-2\sqrt{xy}+y}{x-y}$$

107. $$\frac{\sqrt{x}+2}{5} = \frac{\sqrt{x}+2}{5} \cdot \frac{\sqrt{x}-2}{\sqrt{x}-2} = \frac{\left(\sqrt{x}\right)^2-2^2}{5\left(\sqrt{x}-2\right)} = \frac{x-4}{5\left(\sqrt{x}-2\right)}$$

108. $$\frac{1-\sqrt{a}}{a} = \frac{1-\sqrt{a}}{a} \cdot \frac{1+\sqrt{a}}{1+\sqrt{a}} = \frac{1^2-\left(\sqrt{a}\right)^2}{a\left(1+\sqrt{a}\right)} = \frac{1-a}{a\left(1+\sqrt{a}\right)}$$

109. $$\frac{3x^2y^2}{6x^3y} = \frac{y}{2x}$$

110. $$\frac{4a^2b^3+6ab^4}{2b^2} = \frac{4a^2b^3}{2b^2} + \frac{6ab^4}{2b^2}$$
$$= 2a^2b + 3ab^2$$

111.
$$\begin{array}{r} x^2+\ 2x+\ 1 \\ 2x+3\ \overline{)\ 2x^3+7x^2+8x+3} \\ \underline{2x^3+3x^2} \\ 4x^2+8x \\ \underline{4x^2+6x} \\ 2x+3 \\ \underline{2x+3} \\ 0 \end{array}$$

112.
$$\begin{array}{r} x^3+\ 2x-\ \ 3+\ \frac{-6}{x^2-1} \\ x^2-1\ \overline{)\ x^5+0x^4+\ x^3-3x^2-2x-3} \\ \underline{x^5\ \ \ \ -\ x^3} \\ 2x^3-3x^2-2x \\ \underline{2x^3\ \ \ \ -2x} \\ -3x^2\ \ -3 \\ \underline{-3x^2\ \ +3} \\ -6 \end{array}$$

113. $3t^3 - 3t = 3t(t^2-1) = 3t(t+1)(t-1)$

114. $5r^3 - 5 = 5(r^3-1) = 5(r^3-1^3) = 5(r-1)(r^2+r+1)$

115. $6x^2 + 7x - 24 = (3x+8)(2x-3)$

116. $3a^2 + ax - 3a - x = a(3a+x) - 1(3a+x) = (3a+x)(a-1)$

117. $8x^3 - 125 = (2x)^3 - 5^3 = (2x-5)\left[(2x)^2 + (2x)(5) + 5^2\right] = (2x-5)(4x^2+10x+25)$

118. $6x^2 - 20x - 16 = 2(3x^2-10x-8) = 2(3x+2)(x-4)$

119. $x^2+6x+9-t^2=(x+3)(x+3)-t^2=(x+3)^2-t^2=(x+3+t)(x+3-t)$

120. $3x^2-1+5x=3x^2+5x-1 \Rightarrow$ prime

121. $8z^3+343=(2z)^3+7^3=(2z+7)\left[(2z)^2-(2z)(7)+7^2\right]=(2z+7)(4z^2-14z+49)$

122. $1+14b+49b^2=49b^2+14b+1=(7b+1)(7b+1)=(7b+1)^2$

123. $121z^2+4-44z=121z^2-44z+4=(11z-2)(11z-2)=(11z-2)^2$

124. $64y^3-1000=8(8y^3-125)=8\left[(2y)^3-5^3\right]=8(2y-5)(4y^2+10y+25)$

125. $2xy-4zx-wy+2zw=2x(y-2z)-w(y-2z)=(y-2z)(2x-w)$

126.
$$\begin{aligned} x^8+x^4+1=x^8+2x^4+1-x^4&=\left(x^4+1\right)\left(x^4+1\right)-x^4\\ &=\left(x^4+1\right)^2-\left(x^2\right)^2\\ &=\left(x^4+1+x^2\right)\left(x^4+1-x^2\right)\\ &=\left(x^4+2x^2+1-x^2\right)\left(x^4-x^2+1\right)\\ &=\left[\left(x^2+1\right)\left(x^2+1\right)-x^2\right]\left(x^4-x^2+1\right)\\ &=\left(x^2+1+x\right)\left(x^2+1-x\right)\left(x^4-x^2+1\right)\end{aligned}$$

127. $\dfrac{2-x}{x^2-4x+4}=\dfrac{-(x-2)}{(x-2)(x-2)}=\dfrac{-1}{x-2}$

128. $\dfrac{a^2-9}{a^2-6a+9}=\dfrac{(a+3)(a-3)}{(a-3)(a-3)}=\dfrac{a+3}{a-3}$

129. $\dfrac{x^2-4x+4}{x+2}\cdot\dfrac{x^2+5x+6}{x-2}=\dfrac{(x-2)(x-2)}{x+2}\cdot\dfrac{(x+2)(x+3)}{x-2}=(x-2)(x+3)$

130. $\dfrac{2y^2-11y+15}{y^2-6y+8}\cdot\dfrac{y^2-2y-8}{y^2-y-6}=\dfrac{(2y-5)(y-3)}{(y-4)(y-2)}\cdot\dfrac{(y-4)(y+2)}{(y-3)(y+2)}=\dfrac{2y-5}{y-2}$

131.
$$\begin{aligned}\frac{2t^2+t-3}{3t^2-7t+4}\div\frac{10t+15}{3t^2-t-4}&=\frac{2t^2+t-3}{3t^2-7t+4}\cdot\frac{3t^2-t-4}{10t+15}\\ &=\frac{(2t+3)(t-1)}{(3t-4)(t-1)}\cdot\frac{(3t-4)(t+1)}{5(2t+3)}=\frac{t+1}{5}\end{aligned}$$

132.
$$\begin{aligned}\frac{p^2+7p+12}{p^3+8p^2+4p}\div\frac{p^2-9}{p^2}=\frac{p^2+7p+12}{p^3+8p^2+4p}\cdot\frac{p^2}{p^2-9}&=\frac{(p+3)(p+4)}{p(p^2+8p+4)}\cdot\frac{p^2}{(p+3)(p-3)}\\ &=\frac{p(p+4)}{(p^2+8p+4)(p-3)}\end{aligned}$$

133. $\dfrac{x^2+x-6}{x^2-x-6}\cdot\dfrac{x^2-x-6}{x^2+x-2}\div\dfrac{x^2-4}{x^2-5x+6}$

$$=\frac{x^2+x-6}{x^2-x-6}\cdot\frac{x^2-x-6}{x^2+x-2}\cdot\frac{x^2-5x+6}{x^2-4}$$

$$=\frac{(x+3)(x-2)}{(x-3)(x+2)}\cdot\frac{(x-3)(x+2)}{(x+2)(x-1)}\cdot\frac{(x-2)(x-3)}{(x+2)(x-2)}=\frac{(x+3)(x-2)(x-3)}{(x+2)^2(x-1)}$$

134. $\left(\dfrac{2x+6}{x+5}\div\dfrac{2x^2-2x-4}{x^2-25}\right)\dfrac{x^2-x-2}{x^2-2x-15}=\dfrac{2x+6}{x+5}\cdot\dfrac{x^2-25}{2(x^2-x-2)}\cdot\dfrac{x^2-x-2}{x^2-2x-15}$

$$=\frac{2(x+3)}{x+5}\cdot\frac{(x+5)(x-5)}{2(x-2)(x+1)}\cdot\frac{(x-2)(x+1)}{(x-5)(x+3)}=1$$

135. $\dfrac{2}{x-4}+\dfrac{3x}{x+5}=\dfrac{2(x+5)}{(x-4)(x+5)}+\dfrac{3x(x-4)}{(x+5)(x-4)}=\dfrac{2x+10}{(x-4)(x+5)}+\dfrac{3x^2-12x}{(x-4)(x+5)}$

$$=\frac{3x^2-10x+10}{(x-4)(x+5)}$$

136. $\dfrac{5x}{x-2}-\dfrac{3x+7}{x+2}+\dfrac{2x+1}{x+2}=\dfrac{5x}{x-2}+\dfrac{-x-6}{x+2}=\dfrac{5x(x+2)}{(x-2)(x+2)}+\dfrac{(-x-6)(x-2)}{(x+2)(x-2)}$

$$=\frac{5x^2+10x}{(x-2)(x+2)}+\frac{-x^2-4x+12}{(x-2)(x+2)}$$

$$=\frac{4x^2+6x+12}{(x-2)(x+2)}=\frac{2(2x^2+3x+6)}{(x-2)(x+2)}$$

137. $\dfrac{x}{x-1}+\dfrac{x}{x-2}+\dfrac{x}{x-3}$

$$=\frac{x(x-2)(x-3)}{(x-1)(x-2)(x-3)}+\frac{x(x-1)(x-3)}{(x-2)(x-1)(x-3)}+\frac{x(x-1)(x-2)}{(x-3)(x-1)(x-2)}$$

$$=\frac{x^3-5x^2+6x}{(x-1)(x-2)(x-3)}+\frac{x^3-4x^2+3x}{(x-1)(x-2)(x-3)}+\frac{x^3-3x^2+2x}{(x-1)(x-2)(x-3)}$$

$$=\frac{3x^3-12x^2+11x}{(x-1)(x-2)(x-3)}=\frac{x(3x^2-12x+11)}{(x-1)(x-2)(x-3)}$$

138. $\dfrac{x}{x+1}-\dfrac{3x+7}{x+2}+\dfrac{2x+1}{x+2}=\dfrac{x}{x+1}+\dfrac{-3x-7}{x+2}+\dfrac{2x+1}{x+2}$

$$=\frac{x}{x+1}+\frac{-x-6}{x+2}$$

$$=\frac{x(x+2)}{(x+1)(x+2)}+\frac{(-x-6)(x+1)}{(x+2)(x+1)}$$

$$=\frac{x^2+2x}{(x+1)(x+2)}+\frac{-x^2-7x-6}{(x+1)(x+2)}=\frac{-5x-6}{(x+1)(x+2)}$$

139. $\dfrac{3(x+1)}{x} - \dfrac{5(x^2+3)}{x^2} + \dfrac{x}{x+1} = \dfrac{3x(x+1)(x+1)}{x^2(x+1)} - \dfrac{5(x^2+3)(x+1)}{x^2(x+1)} + \dfrac{x(x^2)}{x^2(x+1)}$

$= \dfrac{3x^3+6x^2+3x}{x^2(x+1)} - \dfrac{5x^3+5x^2+15x+15}{x^2(x+1)} + \dfrac{x^3}{x^2(x+1)}$

$= \dfrac{-x^3+x^2-12x-15}{x^2(x+1)}$

140. $\dfrac{3x}{x+1} + \dfrac{x^2+4x+3}{x^2+3x+2} - \dfrac{x^2+x-6}{x^2-4} = \dfrac{3x}{x+1} + \dfrac{(x+3)(x+1)}{(x+1)(x+2)} - \dfrac{(x+3)(x-2)}{(x+2)(x-2)}$

$= \dfrac{3x}{x+1} + \dfrac{x+3}{x+2} - \dfrac{x+3}{x+2} = \dfrac{3x}{x+1}$

141. $\dfrac{\frac{5x}{2}}{\frac{3x^2}{8}} = \dfrac{5x}{2} \div \dfrac{3x^2}{8} = \dfrac{5x}{2} \cdot \dfrac{8}{3x^2} = \dfrac{20}{3x}$

142. $\dfrac{\frac{3x}{y}}{\frac{6x}{y^2}} = \dfrac{3x}{y} \div \dfrac{6x}{y^2} = \dfrac{3x}{y} \cdot \dfrac{y^2}{6x} = \dfrac{y}{2}$

143. $\dfrac{\frac{1}{x}+\frac{1}{y}}{x-y} = \dfrac{xy\left(\frac{1}{x}+\frac{1}{y}\right)}{xy(x-y)} = \dfrac{xy\left(\frac{1}{x}\right)+xy\left(\frac{1}{y}\right)}{xy(x-y)} = \dfrac{y+x}{xy(x-y)}$

144. $\dfrac{x^{-1}+y^{-1}}{y^{-1}-x^{-1}} = \dfrac{\frac{1}{x}+\frac{1}{y}}{\frac{1}{y}-\frac{1}{x}} = \dfrac{xy\left(\frac{1}{x}+\frac{1}{y}\right)}{xy\left(\frac{1}{y}-\frac{1}{x}\right)} = \dfrac{xy\left(\frac{1}{x}\right)+xy\left(\frac{1}{y}\right)}{xy\left(\frac{1}{y}\right)-xy\left(\frac{1}{x}\right)} = \dfrac{y+x}{x-y}$

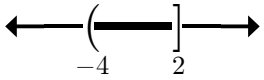
Chapter 0 Test (page 84)

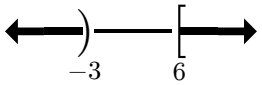
1. odd integers: $-7, 1, 3$

2. prime numbers: 3

3. Commutative Property of Addition

4. Distributive Property

5. $-4 < x \leq 2 \Rightarrow$ 

6. $(-\infty, -3) \cup [6, \infty) \Rightarrow$ 

7. Since $-17 < 0$, $|-17| = -(-17) = 17$

8. If $x < 0$, then $x - 7 < 0$. Then $|x-7| = -(x-7)$.

9. distance $= |12-(-4)| = |16| = 16$

10. distance $= |-12-(-20)| = |8| = 8$

11. $x^4x^5x^2 = x^{4+5+2} = x^{11}$

12. $\dfrac{r^2r^3s}{r^4s^2} = \dfrac{r^5s}{r^4s^2} = \dfrac{r}{s}$

13. $\dfrac{(a^{-1}a^2)^{-2}}{a^{-3}} = \dfrac{(a^1)^{-2}}{a^{-3}} = \dfrac{a^{-2}}{a^{-3}} = a$

14. $\left(\dfrac{x^0x^2}{x^{-2}}\right)^6 = \left(\dfrac{x^2}{x^{-2}}\right)^6 = \left(x^4\right)^6 = x^{24}$

15. $450{,}000 = 4.5 \times 10^5$

16. $0.000345 = 3.45 \times 10^{-4}$

17. $3.7 \times 10^3 = 3{,}700$

18. $1.2 \times 10^{-3} = 0.0012$

19. $(25a^4)^{1/2} = 25^{1/2}(a^4)^{1/2} = 5a^2$

20. $\left(\dfrac{36}{81}\right)^{3/2} = \dfrac{36^{3/2}}{81^{3/2}} = \dfrac{\left(36^{1/2}\right)^3}{\left(81^{1/2}\right)^3} = \dfrac{216}{729} = \dfrac{8}{27}$

21. $\left(\dfrac{8t^6}{27s^9}\right)^{-2/3} = \left(\dfrac{27s^9}{8t^6}\right)^{2/3} = \dfrac{27^{2/3}(s^9)^{2/3}}{8^{2/3}(t^6)^{2/3}} = \dfrac{\left(27^{1/3}\right)^2 s^6}{\left(8^{1/3}\right)^2 t^4} = \dfrac{9s^6}{4t^4}$

22. $\sqrt[3]{27a^6} = \sqrt[3]{27}\sqrt[3]{a^6} = 3a^2$

23. $\sqrt{12} + \sqrt{27} = \sqrt{4}\sqrt{3} + \sqrt{9}\sqrt{3}$
$= 2\sqrt{3} + 3\sqrt{3} = 5\sqrt{3}$

24. $2\sqrt[3]{3x^4} - 3x\sqrt[3]{24x} = 2\sqrt[3]{x^3}\sqrt[3]{3x} - 3x\sqrt[3]{8}\sqrt[3]{3x} = 2x\sqrt[3]{3x} - 3x(2)\sqrt[3]{3x} = 2x\sqrt[3]{3x} - 6x\sqrt[3]{3x}$
$= -4x\sqrt[3]{3x}$

25. $\dfrac{x}{\sqrt{x}-2} = \dfrac{x}{\sqrt{x}-2} \cdot \dfrac{\sqrt{x}+2}{\sqrt{x}+2} = \dfrac{x\left(\sqrt{x}+2\right)}{\left(\sqrt{x}\right)^2 - 2^2} = \dfrac{x\left(\sqrt{x}+2\right)}{x-4}$

26. $\dfrac{\sqrt{x}-\sqrt{y}}{\sqrt{x}+\sqrt{y}} = \dfrac{\sqrt{x}-\sqrt{y}}{\sqrt{x}+\sqrt{y}} \cdot \dfrac{\sqrt{x}+\sqrt{y}}{\sqrt{x}+\sqrt{y}} = \dfrac{\left(\sqrt{x}\right)^2 - \left(\sqrt{y}\right)^2}{\sqrt{x^2}+\sqrt{xy}+\sqrt{xy}+\sqrt{y^2}} = \dfrac{x-y}{x+2\sqrt{xy}+y}$

27. $\left(a^2+3\right) - \left(2a^2-4\right) = a^2 + 3 - 2a^2 + 4$
$= -a^2 + 7$

28. $(3a^3b^2)(-2a^3b^4) = -6a^6b^6$

29. $(3x-4)(2x+7) = 6x^2 + 21x - 8x - 28$
$= 6x^2 + 13x - 28$

30. $(a^n+2)(a^n-3) = a^{2n} - 3a^n + 2a^n - 6$
$= a^{2n} - a^n - 6$

31. $(x^2+4)(x^2-4) = x^4 - 4x^2 + 4x^2 - 16 = x^4 - 16$

32. $(x^2 - x + 2)(2x-3) = 2x^3 - 3x^2 - 2x^2 + 3x + 4x - 6 = 2x^3 - 5x^2 + 7x - 6$

33.
$$\begin{array}{r} 6x + 19 + \frac{34}{x-3} \\ x-3 \overline{)\,6x^2 + x - 23} \\ \underline{6x^2 - 18x} \\ 19x - 23 \\ \underline{19x - 57} \\ 34 \end{array}$$

34.
$$\begin{array}{r} x^2 + 2x + 1 \\ 2x-1 \overline{)\,2x^3 + 3x^2 + 0x - 1} \\ \underline{2x^3 - x^2} \\ 4x^2 + 0x \\ \underline{4x^2 - 2x} \\ 2x - 1 \\ \underline{2x - 1} \\ 0 \end{array}$$

35. $3x + 6y = 3(x + 2y)$

36. $x^2 - 100 = x^2 - 10^2 = (x+10)(x-10)$

37. $45x^2 - 20y^2 = 5\left(9x^2 - 4y^2\right)$
$= 5(3x+2y)(3x-2y)$

38. $10t^2 - 19tw + 6w^2 = (5t - 2w)(2t - 3w)$

39. $64m^3 + 125n^3 = (4m + 5n)(16m^2 - 20mn + 25n^2)$

40. $3a^3 - 648 = 3(a^3 - 216) = 3(a - 6)(a^2 + 6a + 36)$

41. $x^4 - x^2 - 12 = \big(x^2 - 4\big)\big(x^2 + 3\big)$
$= (x + 2)(x - 2)\big(x^2 + 3\big)$

42. $6x^4 + 11x^2 - 10 = (3x^2 - 2)(2x^2 + 5)$

43. $\dfrac{44p^3q^6}{33p^4q^2} = \dfrac{4q^4}{3p}$

44. $\dfrac{49 - x^2}{x^2 + 14x + 49} = \dfrac{-(x + 7)(x - 7)}{(x + 7)(x + 7)} = -\dfrac{x - 7}{x + 7}$

45. $\dfrac{x}{x + 2} + \dfrac{2}{x + 2} = \dfrac{x + 2}{x + 2} = 1$

46. $\dfrac{x}{x + 1} - \dfrac{x}{x - 1} = \dfrac{x(x - 1)}{(x + 1)(x - 1)} - \dfrac{x(x + 1)}{(x + 1)(x - 1)} = \dfrac{x^2 - x - x^2 - x}{(x + 1)(x - 1)} = \dfrac{-2x}{(x + 1)(x - 1)}$

47. $\dfrac{x^2 + x - 20}{x^2 - 16} \cdot \dfrac{x^2 - 25}{x - 5} = \dfrac{(x + 5)(x - 4)}{(x + 4)(x - 4)} \cdot \dfrac{(x + 5)(x - 5)}{x - 5} = \dfrac{(x + 5)^2}{x + 4}$

48. $\dfrac{x + 2}{x^2 + 2x + 1} \div \dfrac{x^2 - 4}{x + 1} = \dfrac{x + 2}{(x + 1)(x + 1)} \cdot \dfrac{x + 1}{(x + 2)(x - 2)} = \dfrac{1}{(x + 1)(x - 2)}$

49. $\dfrac{\frac{1}{a} + \frac{1}{b}}{\frac{1}{b}} = \dfrac{ab\big(\frac{1}{a} + \frac{1}{b}\big)}{ab\big(\frac{1}{b}\big)} = \dfrac{ab\big(\frac{1}{a}\big) + ab\big(\frac{1}{b}\big)}{a} = \dfrac{b + a}{a}$

50. $\dfrac{x^{-1}}{x^{-1} + y^{-1}} = \dfrac{\frac{1}{x}}{\frac{1}{x} + \frac{1}{y}} = \dfrac{xy\big(\frac{1}{x}\big)}{xy\Big(\frac{1}{x} + \frac{1}{y}\Big)} = \dfrac{y}{xy\big(\frac{1}{x}\big) + xy\Big(\frac{1}{y}\Big)} = \dfrac{y}{y + x}$

Exercises 1.1 (page 95)

1. root, solution

2. identity

3. no

4. conditional

5. linear

6. rational

7. one

8. denominator

9. $2x + 5 = -17$
no restrictions

10. $\dfrac{1}{2}x - 7 = 14$
no restrictions

11. $\dfrac{1}{x} = 12$
$x \neq 0$

12. $\dfrac{3}{x - 2} = 9x$
$x \neq 2$

13. $\dfrac{8}{x - 6} = \dfrac{5}{x + 2}$
$x - 6 \neq 0 \quad x + 2 \neq 0$
$x \neq 6 \qquad x \neq -2$
$x \neq 6, x \neq -2$

14. $\dfrac{x}{x - 3} = -\dfrac{4}{x + 4}$
$x - 3 \neq 0 \quad x + 4 \neq 0$
$x \neq 3 \qquad x \neq -4$
$x \neq 3, x \neq -4$