Complete Solutions Manual for Calculus of a Single Variable, Volume 1

Calculus

ELEVENTH EDITION

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Section P.1 Graphs and Models

1. To find the x-intercepts of the graph of an equation, let y be zero and solve the equation for x. To find the y-intercepts of the graph of an equation, let x be zero and solve the equation for y.

2. Substitute the x- and y-values of the ordered pair into both equations. If the ordered pair satisfies both equations, then the ordered pair is a point of intersection.

3. $y = -\frac{3}{2}x + 3$

x-intercept: $(2, 0)$

y-intercept: $(0, 3)$

Matches graph (b).

4. $y = \sqrt{9 - x^2}$

x-intercepts: $(-3, 0), (3, 0)$

y-intercept: $(0, 3)$

Matches graph (d).

5. $y = 3 - x^2$

x-intercepts: $(\sqrt{3}, 0), (-\sqrt{3}, 0)$

y-intercept: $(0, 3)$

Matches graph (a).

6. $y = x^3 - x$

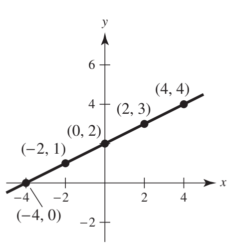
x-intercepts: $(0, 0), (-1, 0), (1, 0)$

y-intercept: $(0, 0)$

Matches graph (c).

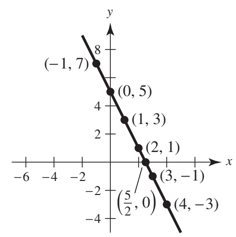
7. $y = \frac{1}{2}x + 2$

x	-4	-2	0	2	4
y	0	1	2	3	4



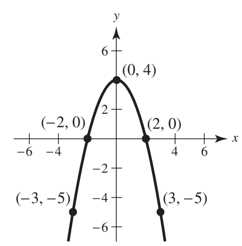
8. $y = 5 - 2x$

x	-1	0	1	2	$\frac{5}{2}$	3	4
y	7	5	3	1	0	-1	-3



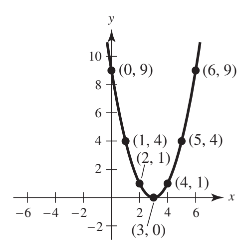
9. $y = 4 - x^2$

x	-3	-2	0	2	3
y	-5	0	4	0	-5



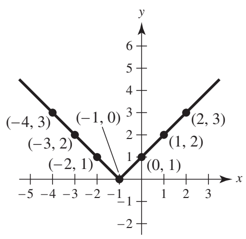
10. $y = (x - 3)^2$

x	0	1	2	3	4	5	6
y	9	4	1	0	1	4	9



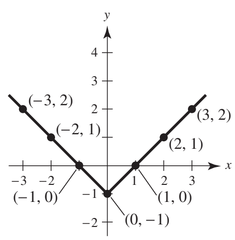
11. $y = |x + 1|$

x	-4	-3	-2	-1	0	1	2
y	3	2	1	0	1	2	3



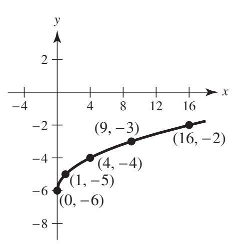
12. $y = |x| - 1$

x	-3	-2	-1	0	1	2	3
y	2	1	0	-1	0	1	2



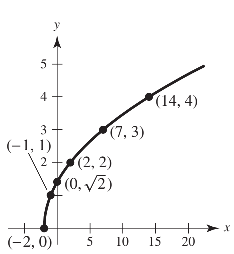
13. $y = \sqrt{x} - 6$

x	0	1	4	9	16
y	-6	-5	-4	-3	-2



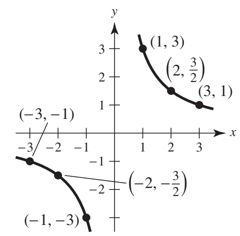
14. $y = \sqrt{x + 2}$

x	-2	-1	0	2	7	14
y	0	1	$\sqrt{2}$	2	3	4



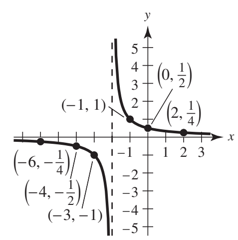
15. $y = \dfrac{3}{x}$

x	-3	-2	-1	0	1	2	3
y	-1	$-\frac{3}{2}$	-3	Undef.	3	$\frac{3}{2}$	1

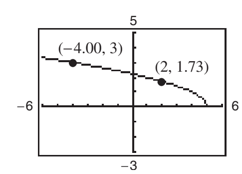


16. $y = \dfrac{1}{x + 2}$

x	-6	-4	-3	-2	-1	0	2
y	$-\frac{1}{4}$	$-\frac{1}{2}$	-1	Undef.	1	$\frac{1}{2}$	$\frac{1}{4}$



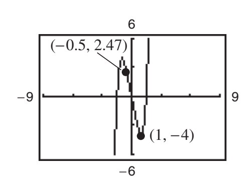
17. $y = \sqrt{5 - x}$



(a) $(2, y) = (2, 1.73) \quad \left(y = \sqrt{5 - 2} = \sqrt{3} \approx 1.73\right)$

(b) $(x, 3) = (-4, 3) \quad \left(3 = \sqrt{5 - (-4)}\right)$

18. $y = x^5 - 5x$



(a) $(-0.5, y) = (-0.5, 2.47)$

(b) $(x, -4) = (-1.65, -4)$ and $(x, -4) = (1, -4)$

19. $y = 2x - 5$

y-intercept: $y = 2(0) - 5 = -5$; $(0, -5)$

x-intercept: $0 = 2x - 5$

$5 = 2x$

$x = \frac{5}{2}$; $\left(\frac{5}{2}, 0\right)$

20. $y = 4x^2 + 3$

y-intercept: $y = 4(0)^2 + 3 = 3$; $(0, 3)$

x-intercept: $0 = 4x^2 + 3$

$-3 = 4x^2$

None. y cannot equal 0.

21. $y = x^2 + x - 2$

y-intercept: $y = 0^2 + 0 - 2$

$y = -2$; $(0, -2)$

x-intercepts: $0 = x^2 + x - 2$

$0 = (x + 2)(x - 1)$

$x = -2, 1$; $(-2, 0), (1, 0)$

22. $y^2 = x^3 - 4x$

y-intercept: $y^2 = 0^3 - 4(0)$

$y = 0$; $(0, 0)$

x-intercepts: $0 = x^3 - 4x$

$0 = x(x - 2)(x + 2)$

$x = 0, \pm 2$; $(0, 0), (\pm 2, 0)$

23. $y = x\sqrt{16 - x^2}$

y-intercept: $y = 0\sqrt{16 - 0^2} = 0$; $(0, 0)$

x-intercepts: $0 = x\sqrt{16 - x^2}$

$0 = x\sqrt{(4 - x)(4 + x)}$

$x = 0, 4, -4$; $(0, 0), (4, 0), (-4, 0)$

24. $y = (x - 1)\sqrt{x^2 + 1}$

y-intercept: $y = (0 - 1)\sqrt{0^2 + 1}$

$y = -1$; $(0, -1)$

x-intercept: $0 = (x - 1)\sqrt{x^2 + 1}$

$x = 1$; $(1, 0)$

25. $y = \dfrac{2 - \sqrt{x}}{5x + 1}$

y-intercept: $y = \dfrac{2 - \sqrt{0}}{5(0) + 1} = 2$; $(0, 2)$

x-intercept: $0 = \dfrac{2 - \sqrt{x}}{5x + 1}$

$0 = 2 - \sqrt{x}$

$x = 4$; $(4, 0)$

26. $y = \dfrac{x^2 + 3x}{(3x + 1)^2}$

y-intercept: $y = \dfrac{0^2 + 3(0)}{[3(0) + 1]^2}$

$y = 0$; $(0, 0)$

x-intercepts: $0 = \dfrac{x^2 + 3x}{(3x + 1)^2}$

$0 = \dfrac{x(x + 3)}{(3x + 1)^2}$

$x = 0, -3$; $(0, 0), (-3, 0)$

27. $x^2y - x^2 + 4y = 0$

y-intercept: $0^2(y) - 0^2 + 4y = 0$

$y = 0$; $(0, 0)$

x-intercept: $x^2(0) - x^2 + 4(0) = 0$

$x = 0$; $(0, 0)$

28. $y = 2x - \sqrt{x^2 + 1}$

y-intercept: $y = 2(0) - \sqrt{0^2 + 1}$

$y = -1$; $(0, -1)$

x-intercept: $0 = 2x - \sqrt{x^2 + 1}$

$2x = \sqrt{x^2 + 1}$

$4x^2 = x^2 + 1$

$3x^2 = 1$

$x^2 = \dfrac{1}{3}$

$x = \pm\dfrac{\sqrt{3}}{3}$

$x = \dfrac{\sqrt{3}}{3}$; $\left(\dfrac{\sqrt{3}}{3}, 0\right)$

Note: $x = -\sqrt{3}/3$ is an extraneous solution.

29. Symmetric with respect to the y-axis because
$y = (-x)^2 - 6 = x^2 - 6.$

30. $y = 9x - x^2$

No symmetry with respect to either axis or the origin.

31. Symmetric with respect to the x-axis because
$(-y)^2 = y^2 = x^3 - 8x.$

32. Symmetric with respect to the origin because

$$(-y) = (-x)^3 + (-x)$$
$$-y = -x^3 - x$$
$$y = x^3 + x.$$

33. Symmetric with respect to the origin because
$(-x)(-y) = xy = 4.$

34. Symmetric with respect to the x-axis because
$x(-y)^2 = xy^2 = -10.$

35. $y = 4 - \sqrt{x + 3}$

No symmetry with respect to either axis or the origin.

36. Symmetric with respect to the origin because

$$(-x)(-y) - \sqrt{4 - (-x)^2} = 0$$
$$xy - \sqrt{4 - x^2} = 0.$$

37. Symmetric with respect to the origin because

$$-y = \frac{-x}{(-x)^2 + 1}$$
$$y = \frac{x}{x^2 + 1}.$$

38. Symmetric with respect to the origin because

$$-y = \frac{(-x)^5}{4 - (-x)^2}$$
$$-y = \frac{-x^5}{4 - x^2}$$
$$y = \frac{x^5}{4 - x^2}.$$

39. $y = |x^3 + x|$ is symmetric with respect to the y-axis because $y = |(-x)^3 + (-x)| = |-(x^3 + x)| = |x^3 + x|.$

40. $|y| - x = 3$ is symmetric with respect to the x-axis because

$$|-y| - x = 3$$
$$|y| - x = 3.$$

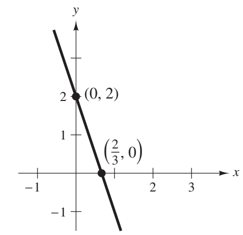
41. $y = 2 - 3x$

$y = 2 - 3(0) = 2,$ y-intercept

$0 = 2 - 3(x) \Rightarrow 3x = 2 \Rightarrow x = \frac{2}{3},$ x-intercept

Intercepts: $(0, 2), \left(\frac{2}{3}, 0\right)$

Symmetry: none



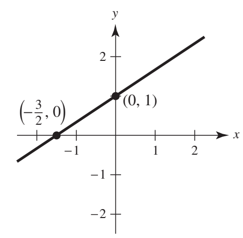
42. $y = \frac{2}{3}x + 1$

$y = \frac{2}{3}(0) + 1 = 1,$ y-intercept

$0 = \frac{2}{3}x + 1 \Rightarrow -\frac{2}{3}x = 1 \Rightarrow x = -\frac{3}{2},$ x-intercept

Intercepts: $(0, 1), \left(-\frac{3}{2}, 0\right)$

Symmetry: none



43. $y = 9 - x^2$

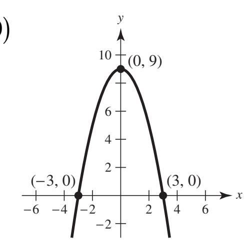
$y = 9 - (0)^2 = 9,$ y-intercept

$0 = 9 - x^2 \Rightarrow x^2 = 9 \Rightarrow x = \pm 3,$ x-intercepts

Intercepts: $(0, 9), (3, 0), (-3, 0)$

$y = 9 - (-x)^2 = 9 - x^2$

Symmetry: y-axis



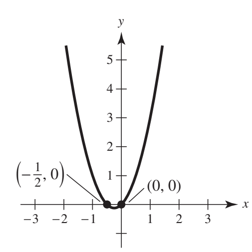
44. $y = 2x^2 + x = x(2x + 1)$

$y = 0(2(0) + 1) = 0,$ y-intercept

$0 = x(2x + 1) \Rightarrow x = 0, -\frac{1}{2},$ x-intercepts

Intercepts: $(0, 0), \left(-\frac{1}{2}, 0\right)$

Symmetry: none



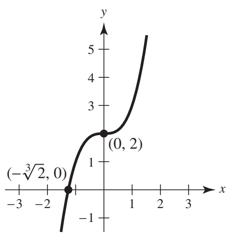
45. $y = x^3 + 2$

$y = 0^3 + 2 = 2$, y-intercept

$0 = x^3 + 2 \Rightarrow x^3 = -2 \Rightarrow x = -\sqrt[3]{2}$, x-intercept

Intercepts: $\left(-\sqrt[3]{2}, 0\right), (0, 2)$

Symmetry: none



46. $y = x^3 - 4x$

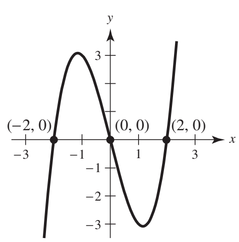
$y = 0^3 - 4(0) = 0$, y-intercept

$$x^3 - 4x = 0$$
$$x(x^2 - 4) = 0$$
$$x(x + 2)(x - 2) = 0$$
$$x = 0, \pm 2, \; x\text{-intercepts}$$

Intercepts: $(0, 0), (2, 0), (-2, 0)$

$y = (-x)^3 - 4(-x) = -x^3 + 4x = -(x^3 - 4x)$

Symmetry: origin



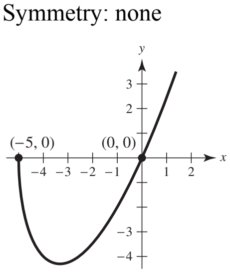
47. $y = x\sqrt{x + 5}$

$y = 0\sqrt{0 + 5} = 0$, y-intercept

$x\sqrt{x + 5} = 0 \Rightarrow x = 0, -5$, x-intercepts

Intercepts: $(0, 0), (-5, 0)$

Symmetry: none



48. $y = \sqrt{25 - x^2}$

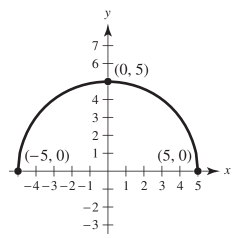
$y = \sqrt{25 - 0^2} = \sqrt{25} = 5$, y-intercept

$$\sqrt{25 - x^2} = 0$$
$$25 - x^2 = 0$$
$$(5 + x)(5 - x) = 0$$
$$x = \pm 5, \; x\text{-intercept}$$

Intercepts: $(0, 5), (5, 0), (-5, 0)$

$y = \sqrt{25 - (-x)^2} = \sqrt{25 - x^2}$

Symmetry: y-axis



49. $x = y^3$

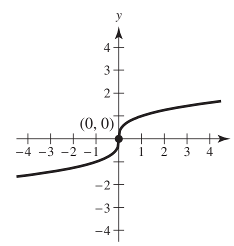
$y^3 = 0 \Rightarrow y = 0$, y-intercept

$x = 0$, x-intercept

Intercept: $(0, 0)$

$-x = (-y)^3 \Rightarrow -x = -y^3$

Symmetry: origin



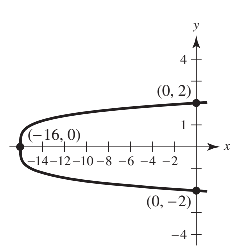
50. $x = y^4 - 16$

$$y^4 - 16 = 0$$
$$(y^2 - 4)(y^2 + 4) = 0$$
$$(y - 2)(y + 2)(y^2 + 4) = 0$$
$$y = \pm 2, \; y\text{-intercepts}$$

$x = 0^4 - 16 = -16$, x-intercept

Intercepts: $(0, 2), (0, -2), (-16, 0)$

Symmetry: x-axis because $x = (-y)^4 - 16 = y^4 - 16$



51. $y = \dfrac{8}{x}$

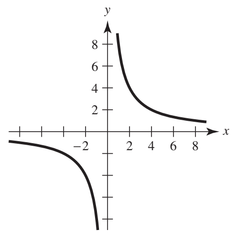
$y = \dfrac{8}{0} \Rightarrow$ Undefined $\Rightarrow$ no y-intercept

$\dfrac{8}{x} = 0 \Rightarrow$ No solution $\Rightarrow$ no x-intercept

Intercepts: none

$-y = \dfrac{8}{-x} \Rightarrow y = \dfrac{8}{x}$

Symmetry: origin



52. $y = \dfrac{10}{x^2 + 1}$

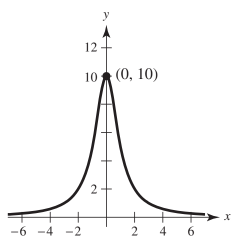
$y = \dfrac{10}{0^2 + 1} = 10$, y-intercept

$\dfrac{10}{x^2 + 1} = 0 \Rightarrow$ No solution $\Rightarrow$ no x-intercepts

Intercept: (0, 10)

$y = \dfrac{10}{(-x)^2 + 1} = \dfrac{10}{x^2 + 1}$

Symmetry: y-axis



53. $y = 6 - |x|$

$y = 6 - |0| = 6$, y-intercept

$6 - |x| = 0$

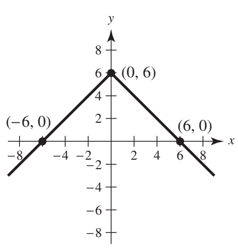
$6 = |x|$

$x = \pm 6$, x-intercepts

Intercepts: $(0, 6), (-6, 0), (6, 0)$

$y = 6 - |-x| = 6 - |x|$

Symmetry: y-axis



54. $y = |6 - x|$

$y = |6 - 0| = |6| = 6$, y-intercept

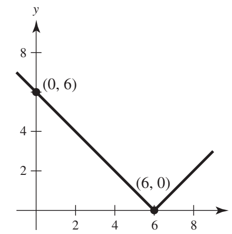
$|6 - x| = 0$

$6 - x = 0$

$6 = x$, x-intercept

Intercepts: (0, 6), (6, 0)

Symmetry: none



55. $3y^2 - x = 9$

$3y^2 = x + 9$

$y^2 = \frac{1}{3}x + 3$

$y = \pm\sqrt{\frac{1}{3}x + 3}$

$y = \pm\sqrt{0 + 3} = \pm\sqrt{3}$, y-intercepts

$\pm\sqrt{\frac{1}{3}x + 3} = 0$

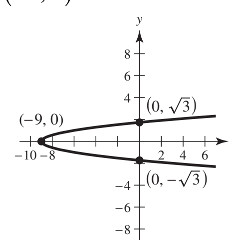
$\frac{1}{3}x + 3 = 0$

$x = -9$, x-intercept

Intercepts: $(0, \sqrt{3}), (0, -\sqrt{3}), (-9, 0)$

$3(-y)^2 - x = 3y^2 - x = 9$

Symmetry: x-axis



56. $x^2 + 4y^2 = 4 \Rightarrow y = \pm\dfrac{\sqrt{4 - x^2}}{2}$

$y = \pm\dfrac{\sqrt{4 - 0^2}}{2} = \pm\dfrac{\sqrt{4}}{2} = \pm 1$, y-intercepts

$x^2 + 4(0)^2 = 4$

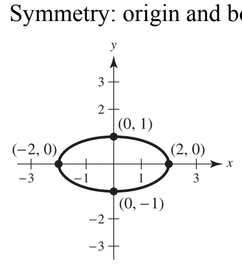
$x^2 = 4$

$x = \pm 2$, x-intercepts

Intercepts: $(-2, 0), (2, 0), (0, -1), (0, 1)$

$(-x)^2 + 4(-y)^2 = 4 \Rightarrow x^2 + 4y^2 = 4$

Symmetry: origin and both axes



57. $x + y = 8 \Rightarrow y = 8 - x$

$4x - y = 7 \Rightarrow y = 4x - 7$

$$8 - x = 4x - 7$$
$$15 = 5x$$
$$3 = x$$

The corresponding y-value is $y = 5$.

Point of intersection: (3, 5)

58. $3x - 2y = -4 \Rightarrow y = \dfrac{3x + 4}{2}$

$4x + 2y = -10 \Rightarrow y = \dfrac{-4x - 10}{2}$

$$\frac{3x + 4}{2} = \frac{-4x - 10}{2}$$
$$3x + 4 = -4x - 10$$
$$7x = -14$$
$$x = -2$$

The corresponding y-value is $y = -1$.

Point of intersection: $(-2, -1)$

59. $x^2 + y = 15 \Rightarrow y = -x^2 + 15$

$-3x + y = 11 \Rightarrow y = 3x + 11$

$$-x^2 + 15 = 3x + 11$$
$$0 = x^2 + 3x - 4$$
$$0 = (x + 4)(x - 1)$$
$$x = -4, 1$$

The corresponding y-values are $y = -1$ (for $x = -4$) and $y = 14$ (for $x = 1$).

Points of intersection: $(-4, -1)$, $(1, 14)$

60. $x = 3 - y^2 \Rightarrow y^2 = 3 - x$

$y = x - 1$

$$3 - x = (x - 1)^2$$
$$3 - x = x^2 - 2x + 1$$
$$0 = x^2 - x - 2 = (x + 1)(x - 2)$$
$$x = -1 \text{ or } x = 2$$

The corresponding y-values are $y = -2$ (for $x = -1$) and $y = 1$ (for $x = 2$).

Points of intersection: $(-1, -2)$, $(2, 1)$

61. $x^2 + y^2 = 5 \Rightarrow y^2 = 5 - x^2$

$x - y = 1 \Rightarrow y = x - 1$

$$5 - x^2 = (x - 1)^2$$
$$5 - x^2 = x^2 - 2x + 1$$
$$0 = 2x^2 - 2x - 4 = 2(x + 1)(x - 2)$$
$$x = -1 \text{ or } x = 2$$

The corresponding y-values are $y = -2$ (for $x = -1$) and $y = 1$ (for $x = 2$).

Points of intersection: $(-1, -2)$, $(2, 1)$

62. $x^2 + y^2 = 16$

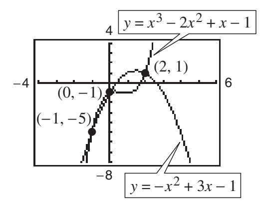
$x + 2y = 4 \Rightarrow x = 4 - 2y$

$$(4 - 2y)^2 + y^2 = 16$$
$$5y^2 - 16y + 16 = 16$$
$$y(5y - 16) = 0 \Rightarrow y = 0, \tfrac{16}{5}$$
$$x = 4 - 2(0) \Rightarrow x = 4$$
$$x = 4 - 2\left(\tfrac{16}{5}\right) \Rightarrow x = -\tfrac{12}{5}$$

Points of intersection: $(4, 0)$, $\left(-\tfrac{12}{5}, \tfrac{16}{5}\right)$

63. $y = x^3 - 2x^2 + x - 1$

$y = -x^2 + 3x - 1$



Points of intersection:

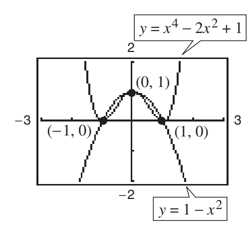
$(-1, -5)$, $(0, -1)$, $(2, 1)$

Analytically,

$$x^3 - 2x^2 + x - 1 = -x^2 + 3x - 1$$
$$x^3 - x^2 - 2x = 0$$
$$x(x - 2)(x + 1) = 0$$
$$x = -1, 0, 2.$$

64. $y = x^4 - 2x^2 + 1$

$y = 1 - x^2$



Points of intersection:

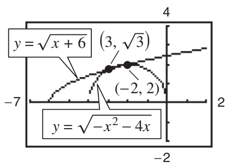
$(-1, 0)$, $(0, 1)$, $(1, 0)$

Analytically,

$$1 - x^2 = x^4 - 2x^2 + 1$$
$$0 = x^4 - x^2$$
$$0 = x^2(x + 1)(x - 1)$$
$$x = -1, 0, 1.$$

65. $y = \sqrt{x + 6}$

$y = \sqrt{-x^2 - 4x}$



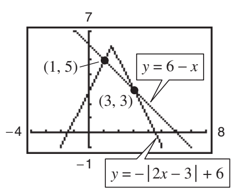
Points of intersection: $(-2, 2), (-3, \sqrt{3}) \approx (-3, 1.732)$

Analytically,

$$\sqrt{x + 6} = \sqrt{-x^2 - 4x}$$
$$x + 6 = -x^2 - 4x$$
$$x^2 + 5x + 6 = 0$$
$$(x + 3)(x + 2) = 0$$
$$x = -3, -2.$$

66. $y = -|2x - 3| + 6$

$y = 6 - x$



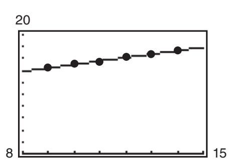
Points of intersection: (3, 3), (1, 5)

Analytically, $-|2x - 3| + 6 = 6 - x$

$$|2x - 3| = x$$
$$2x - 3 = x \text{ or } 2x - 3 = -x$$
$$x = 3 \text{ or } x = 1.$$

67. (a) Using a graphing utility, you obtain $y = 0.58t + 9.2$.

(b)



The model is a good fit for the data.

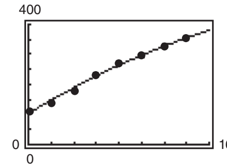
(c) For 2024, $t = 24$:

$$y = 0.58(24) + 9.2 \approx 23.1$$

The GDP in 2024 will be approximately \$23.1 trillion.

68. (a) Using a graphing utility, you obtain $y = -0.35t^2 + 22.9t + 103$.

(b)



The model is a good fit for the data.

(c) For 2024, $t = 24$:

$$y = -0.35(24)^2 + 22.9(24) + 103 \approx 451$$

There will be approximately 451 million cell phone subscribers in 2024.

69.

$$C = R$$
$$2.04x + 5600 = 3.29x$$
$$5600 = 3.29x - 2.04x$$
$$5600 = 1.25x$$
$$x = \frac{5600}{1.25} = 4480$$

To break even, 4480 units must be sold.

70. $y^2 = 4kx$

(a) $(1, 1)$: $1^2 = 4k(1)$

$1 = 4k$

$k = \frac{1}{4}$

(b) $(2, 4)$: $(4)^2 = 4k(2)$

$16 = 8k$

$k = 2$

(c) $(0, 0)$: $0^2 = 4k(0)$

k can be any real number.

(d) $(3, 3)$: $(3)^2 = 4k(3)$

$9 = 12k$

$k = \frac{9}{12} = \frac{3}{4}$

71. Answers may vary. *Sample answer*:

$y = \left(x + \frac{3}{2}\right)(x - 4)\left(x - \frac{5}{2}\right)$ has intercepts at

$x = -\frac{3}{2}$, $x = 4$, and $x = \frac{5}{2}$.

72. Yes. If (x, y) is on the graph, then so is $(-x, y)$ by y-axis symmetry. Because $(-x, y)$ is on the graph, then so is $(-x, -y)$ by x-axis symmetry. So, the graph is symmetric with respect to the origin. The converse is not true. For example, $y = x^3$ has origin symmetry but is not symmetric with respect to either the x-axis or the y-axis.

73. Yes. Assume that the graph has x-axis and origin symmetry. If (x, y) is on the graph, so is $(x, -y)$ by x-axis symmetry. Because $(x, -y)$ is on the graph, then so is $(-x, -(-y)) = (-x, y)$ by origin symmetry. Therefore, the graph is symmetric with respect to the y-axis. The argument is similar for y-axis and origin symmetry.

74. (a) Intercepts for $y = x^3 - x$:

y-intercept: $y = 0^3 - 0 = 0$; $(0, 0)$

x-intercepts: $0 = x^3 - x = x(x^2 - 1) = x(x - 1)(x + 1)$;

$(0, 0), (1, 0)\ (-1, 0)$

Intercepts for $y = x^2 + 2$:

y-intercept: $y = 0 + 2 = 2$; $(0, 2)$

x-intercepts: $0 = x^2 + 2$

None. y cannot equal 0.

(b) Symmetry with respect to the origin for $y = x^3 - x$ because $-y = (-x)^3 - (-x) = -x^3 + x.$

Symmetry with respect to the y-axis for $y = x^2 + 2$ because $y = (-x)^2 + 2 = x^2 + 2.$

(c)
$$x^3 - x = x^2 + 2$$
$$x^3 - x^2 - x - 2 = 0$$
$$(x - 2)(x^2 + x + 1) = 0$$
$$x = 2 \Rightarrow y = 6$$

Point of intersection : (2, 6)

Note: The polynomial $x^2 + x + 1$ has no real roots.

75. False. x-axis symmetry means that if $(-4, -5)$ is on the graph, then $(-4, 5)$ is also on the graph. For example, $(4, -5)$ is not on the graph of $x = y^2 - 29$, whereas $(-4, -5)$ is on the graph.

76. True. $f(4) = f(-4)$.

77. True. The x-intercepts are $\left(\dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}, 0\right)$.

78. True. The x-intercept is $\left(-\dfrac{b}{2a}, 0\right)$.

Section P.2 Linear Models and Rates of Change

1. In the form $y = mx + b$, m is the slope and b is the y-intercept.

2. No. Perpendicular lines have slopes that are negative reciprocals of each other. So, one line has a positive slope and the other line has a negative slope.

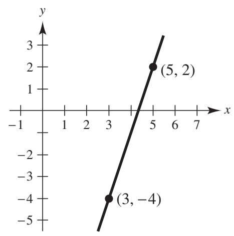
3. $m = 2$

4. $m = 0$

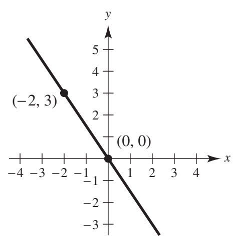
5. $m = -1$

6. $m = -12$

7. $m = \dfrac{2-(-4)}{5-3} = \dfrac{6}{2} = 3$

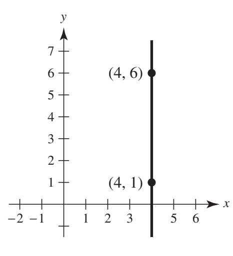


8. $m = \dfrac{3-0}{-2-0} = -\dfrac{3}{2}$



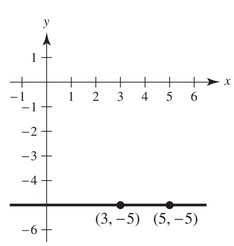
9. $m = \dfrac{1-6}{4-4} = \dfrac{-5}{0}$, undefined.

The line is vertical.

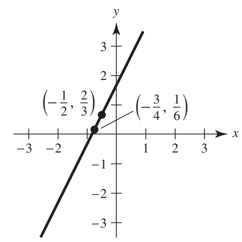


10. $m = \dfrac{-5-(-5)}{5-3} = \dfrac{0}{2} = 0$

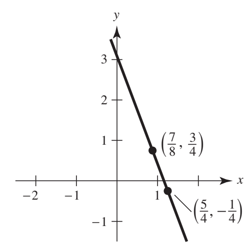
The line is horizontal.



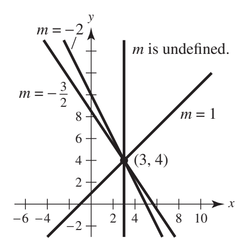
11. $m = \dfrac{\dfrac{2}{3}-\dfrac{1}{6}}{-\dfrac{1}{2}-\left(-\dfrac{3}{4}\right)} = \dfrac{\dfrac{1}{2}}{\dfrac{1}{4}} = 2$



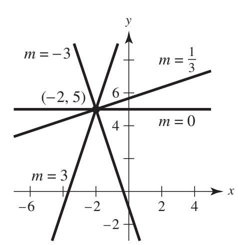
12. $m = \dfrac{\left(\dfrac{3}{4}\right)-\left(-\dfrac{1}{4}\right)}{\left(\dfrac{7}{8}\right)-\left(\dfrac{5}{4}\right)} = \dfrac{1}{-\dfrac{3}{8}} = -\dfrac{8}{3}$



13.



14.



15. Because the slope is 0, the line is horizontal and its equation is $y = 2$. Therefore, three additional points are $(0, 2), (1, 2), (5, 2)$.

16. Because the slope is undefined, the line is vertical and its equation is $x = -4$. Therefore, three additional points are $(-4, 0), (-4, 1), (-4, 2)$.

17. The equation of this line is

$$y - 7 = -3(x - 1)$$
$$y = -3x + 10.$$

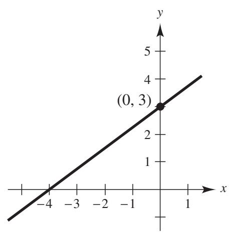
Therefore, three additional points are (0, 10), (2, 4), and (3, 1).

18. The equation of this line is

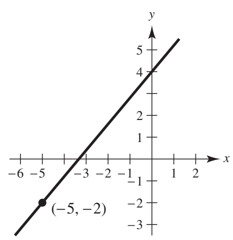
$$y + 2 = 2(x + 2)$$
$$y = 2x + 2.$$

Therefore, three additional points are $(-3, -4)$, $(-1, 0)$, and (0, 2).

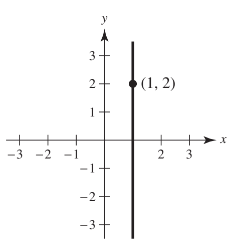
19.
$$y = \frac{3}{4}x + 3$$
$$4y = 3x + 12$$
$$0 = 3x - 4y + 12$$



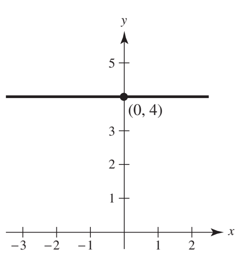
20.
$$y - (-2) = \frac{6}{5}[x - (-5)]$$
$$y + 2 = \frac{6}{5}(x + 5)$$
$$y + 2 = \frac{6}{5}x + 6$$
$$y = \frac{6}{5}x + 4$$
$$0 = 6x - 5y + 20$$



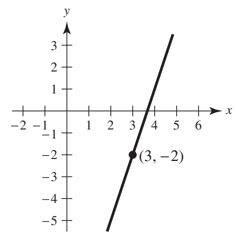
21. Because the slope is undefined, the line is vertical and its equation is $x = 1$.



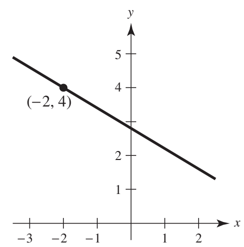
22.
$$y = 4$$
$$y - 4 = 0$$



23.
$$y + 2 = 3(x - 3)$$
$$y + 2 = 3x - 9$$
$$y = 3x - 11$$
$$0 = 3x - y - 11$$



24.
$$y - 4 = -\frac{3}{5}(x + 2)$$
$$5y - 20 = -3x - 6$$
$$3x + 5y - 14 = 0$$

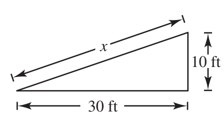


25.
$$\frac{6}{100} = \frac{x}{200}$$
$$100x = 1200$$
$$x = 12$$

Since the grade of the road is $\frac{6}{100}$, if you drive 200 feet, the vertical rise in the road will be 12 feet.

26. (a) Slope $= \frac{\Delta y}{\Delta x} = \frac{1}{3}$

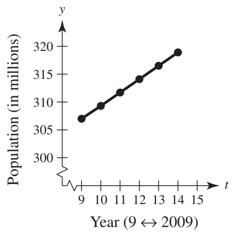
(b)



By the Pythagorean Theorem,

$$x^2 = 30^2 + 10^2 = 1000$$
$$x = 10\sqrt{10} \approx 31.623 \text{ feet.}$$

27. (a)



Slopes: $\dfrac{309.3 - 307.0}{10 - 9} = 2.3$

$\dfrac{311.7 - 309.3}{11 - 10} = 2.4$

$\dfrac{314.1 - 311.7}{12 - 11} = 2.4$

$\dfrac{316.5 - 314.1}{13 - 12} = 2.4$

$\dfrac{318.9 - 316.5}{14 - 13} = 2.4$

The population increased least rapidly from 2009 to 2010.

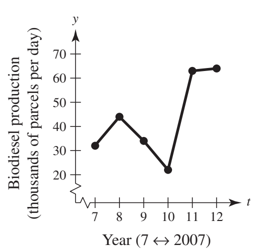
(b) $\dfrac{318.9 - 307.0}{14 - 9} = 2.38$ million people per year

(c) For 2025, $t = 25$:

$$\frac{P - 307.0}{25 - 9} = 2.38 \Rightarrow P = 2.38(16) + 307.0$$
$$\approx 345.1$$

The population of the United States in 2025 will be about 345.1 million people.

28. (a)



Slopes: $\dfrac{44 - 32}{8 - 7} = 12$

$\dfrac{34 - 44}{9 - 8} = -10$

$\dfrac{22 - 34}{10 - 9} = -12$

$\dfrac{63 - 22}{11 - 10} = 41$

$\dfrac{64 - 63}{12 - 11} = 1$

The population increased most rapidly from 2010 to 2011.

(b) $\dfrac{64 - 32}{12 - 7} = \dfrac{32}{5} = 6.4$ thousand barrels per day

(c) No. The production seems to randomly increase and decrease.

29. $y = 4x - 3$

The slope is $m = 4$ and the y-intercept is $(0, -3)$.

30. $-x + y = 1$

$y = x + 1$

The slope is $m = 1$ and the y-intercept is (0, 1).

31. $5x + y = 20$

$y = -5x + 20$

The slope is $m = -5$ and the y-intercept is (0, 20).

32. $6x - 5y = 15$

$y = \frac{6}{5}x - 3$

The slope is $m = \frac{6}{5}$ and the y-intercept is $(0, -3)$.

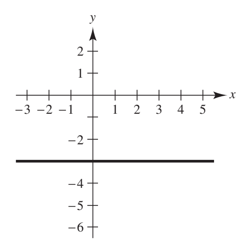
33. $x = 4$

The line is vertical. Therefore, the slope is undefined and there is no y-intercept.

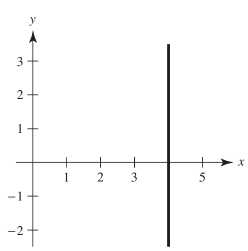
34. $y = -1$

The line is horizontal. Therefore, the slope is $m = 0$ and the y-intercept is $(0, -1)$.

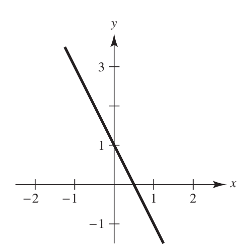
35. $y = -3$



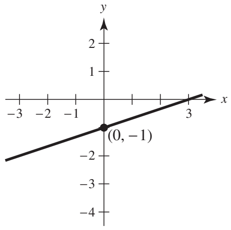
36. $x = 4$



37. $y = -2x + 1$

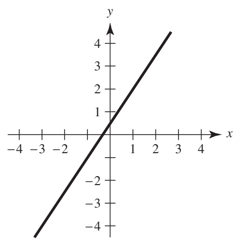


38. $y = \frac{1}{3}x - 1$



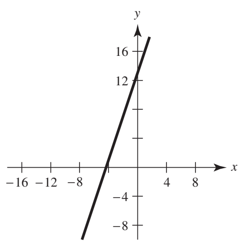
39. $y - 2 = \frac{3}{2}(x - 1)$

$y = \frac{3}{2}x + \frac{1}{2}$



40. $y - 1 = 3(x + 4)$

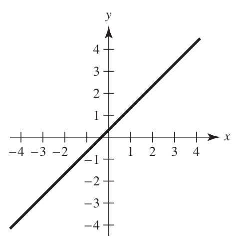
$y = 3x + 13$



41. $3x - 3y + 1 = 0$

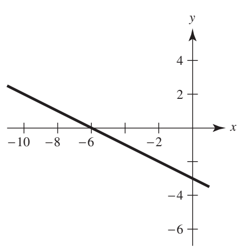
$3y = 3x + 1$

$y = x + \frac{1}{3}$



42. $x + 2y + 6 = 0$

$y = -\frac{1}{2}x - 3$

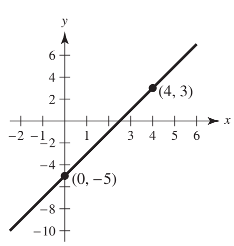


43. $m = \dfrac{-5 - 3}{0 - 4} = \dfrac{-8}{-4} = 2$

$y - (-5) = 2(x - 0)$

$y + 5 = 2x$

$0 = 2x - y - 5$



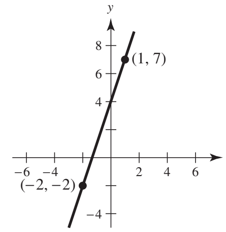
44. $m = \dfrac{7 - (-2)}{1 - (-2)} = \dfrac{9}{3} = 3$

$y - (-2) = 3(x - (-2))$

$y + 2 = 3(x + 2)$

$y = 3x + 4$

$0 = 3x - y + 4$

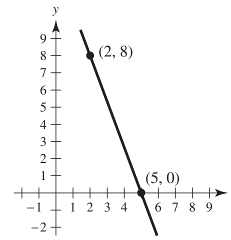


45. $m = \dfrac{8 - 0}{2 - 5} = -\dfrac{8}{3}$

$y - 0 = -\dfrac{8}{3}(x - 5)$

$y = -\dfrac{8}{3}x + \dfrac{40}{3}$

$8x + 3y - 40 = 0$

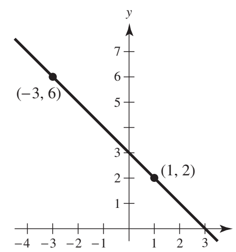


46. $m = \dfrac{6 - 2}{-3 - 1} = \dfrac{4}{-4} = -1$

$y - 2 = -1(x - 1)$

$y - 2 = -x + 1$

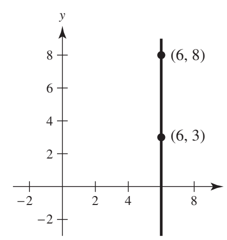
$x + y - 3 = 0$



47. $m = \dfrac{8 - 3}{6 - 6} = \dfrac{5}{0}$, undefined

The line is vertical.

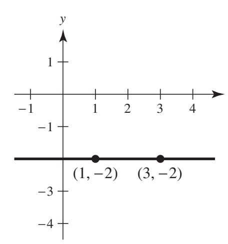
$x = 6$ or $x - 6 = 0$



48. $m = \dfrac{-2 - (-2)}{3 - 1} = \dfrac{0}{2} = 0$

$y = -2$

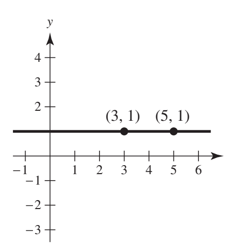
$y + 2 = 0$



49. $m = \dfrac{1 - 1}{5 - 3} = 0$

The line is horizontal.

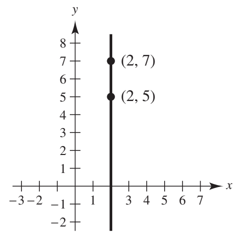
$y = 1$ or $y - 1 = 0$



50. $m = \dfrac{7-5}{2-2} = \dfrac{2}{0}$, undefined

The line is vertical.

$x = 2$ or $x - 2 = 0$



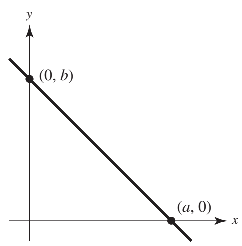
51. The slope is $\dfrac{1-b}{3-0} = \dfrac{1-b}{3}$.

The y-intercept is $(0, b)$. Hence,

$$y = mx + b = \left(\frac{1-b}{3}\right)x + b.$$

52. $m = -\dfrac{b}{a}$

$$y = \frac{-b}{a}x + b$$
$$\frac{b}{a}x + y = b$$
$$\frac{x}{a} + \frac{y}{b} = 1$$



53. $\dfrac{x}{2} + \dfrac{y}{3} = 1$

$$3x + 2y - 6 = 0$$

54. $\dfrac{x}{-\frac{2}{3}} + \dfrac{y}{-2} = 1$

$$\frac{-3x}{2} - \frac{y}{2} = 1$$
$$3x + y = -2$$
$$3x + y + 2 = 0$$

55. $\dfrac{x}{2a} + \dfrac{y}{a} = 1$

$$\frac{9}{2a} + \frac{-2}{a} = 1$$
$$\frac{9-4}{2a} = 1$$
$$5 = 2a$$
$$a = \frac{5}{2}$$
$$\frac{x}{2\left(\frac{5}{2}\right)} + \frac{y}{\left(\frac{5}{2}\right)} = 1$$
$$\frac{x}{5} + \frac{2y}{5} = 1$$
$$x + 2y = 5$$
$$x + 2y - 5 = 0$$

56. $\dfrac{x}{a} + \dfrac{y}{-a} = 1$

$$\frac{\left(-\frac{2}{3}\right)}{a} + \frac{(-2)}{-a} = 1$$
$$-\frac{2}{3} + 2 = a$$
$$a = \frac{4}{3}$$
$$\frac{x}{\left(\frac{4}{3}\right)} + \frac{y}{\left(-\frac{4}{3}\right)} = 1$$
$$x - y = \frac{4}{3}$$
$$3x - 3y - 4 = 0$$

57. The given line is vertical.

(a) $x = -7$, or $x + 7 = 0$

(b) $y = -2$, or $y + 2 = 0$

58. The given line is horizontal.

(a) $y = 0$

(b) $x = -1$, or $x + 1 = 0$

59. $x + y = 7$

$$y = -x + 7$$
$$m = -1$$

(a) $y - 2 = -1(x + 3)$

$$y - 2 = -x - 3$$
$$x + y + 1 = 0$$

(b) $y - 2 = 1(x + 3)$

$$y - 2 = x + 3$$
$$0 = x - y + 5$$

60. $x - y = -2$

$$y = x + 2$$
$$m = 1$$

(a) $y - 5 = 1(x - 2)$

$$y - 5 = x - 2$$
$$x - y + 3 = 0$$

(b) $y - 5 = -1(x - 2)$

$$y - 5 = -x + 2$$
$$x + y - 7 = 0$$

61. $5x - 3y = 0$

$$y = \frac{5}{3}x$$

$$m = \frac{5}{3}$$

(a) $y - \frac{7}{8} = \frac{5}{3}\left(x - \frac{3}{4}\right)$

$$24y - 21 = 40x - 30$$

$$0 = 40x - 24y - 9$$

(b) $y - \frac{7}{8} = -\frac{3}{5}\left(x - \frac{3}{4}\right)$

$$40y - 35 = -24x + 18$$

$$24x + 40y - 53 = 0$$

62. $7x + 4y = 8$

$$4y = -7x + 8$$

$$y = \frac{-7}{4}x + 2$$

$$m = -\frac{7}{4}$$

(a) $y + \frac{1}{2} = \frac{-7}{4}\left(x - \frac{5}{6}\right)$

$$y + \frac{1}{2} = \frac{-7}{4}x + \frac{35}{24}$$

$$24y + 12 = -42x + 35$$

$$42x + 24y - 23 = 0$$

(b) $y + \frac{1}{2} = \frac{4}{7}\left(x - \frac{5}{6}\right)$

$$42y + 21 = 24x - 20$$

$$24x - 42y - 41 = 0$$

63. The slope is 250.

$V = 1850$ when $t = 6$.

$$V = 250(t - 6) + 1850$$
$$= 250t + 250$$

64. The slope is -1600.

$V = 17{,}200$ when $t = 6$.

$$V = -1600(t - 6) + 17{,}200$$
$$= -1600t + 26{,}800$$

65. $m_1 = \frac{1 - 0}{-2 - (-1)} = -1$

$$m_2 = \frac{-2 - 0}{2 - (-1)} = -\frac{2}{3}$$

$$m_1 \neq m_2$$

The points are not collinear.

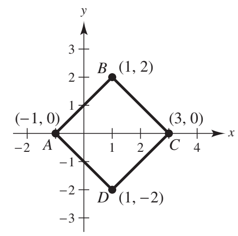
66. $m_1 = \frac{-6 - 4}{7 - 0} = -\frac{10}{7}$

$$m_2 = \frac{11 - 4}{-5 - 0} = -\frac{7}{5}$$

$$m_1 \neq m_2$$

The points are not collinear.

67.



The four sides are of equal length: $\sqrt{8} = 2\sqrt{2}$.

For example, the length of segment AB is

$$\sqrt{(1 - (-1))^2 + (2 - 0)^2} = \sqrt{4 + 4} = \sqrt{8} = 2\sqrt{2} \text{ units.}$$

Furthermore, the adjacent sides are perpendicular because the slope of $\overline{AB}$ is $\frac{2 - 0}{1 - (-1)} = \frac{2}{2} = 1$, whereas the slope of $\overline{BC}$ is $\frac{2 - 0}{1 - 3} = -1$.

68. $ax + by = 4$

(a) The line is parallel to the x-axis if $a = 0$ and $b \neq 0$.

(b) The line is parallel to the y-axis if $b = 0$ and $a \neq 0$.

(c) Answers will vary. *Sample answer*: $a = -5$ and $b = 8$.

$$-5x + 8y = 4$$

$$y = \frac{1}{8}(5x + 4) = \frac{5}{8}x + \frac{1}{2}$$

(d) The slope must be $-\frac{5}{2}$.

Answers will vary. *Sample answer*: $a = 5$ and $b = 2$.

$$5x + 2y = 4$$

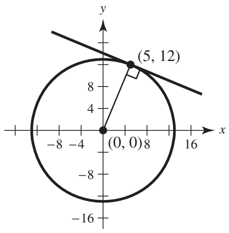
$$y = \frac{1}{2}(-5x + 4) = -\frac{5}{2}x + 2$$

(e) $a = \frac{5}{2}$ and $b = 3$.

$$\frac{5}{2}x + 3y = 4$$

$$5x + 6y = 8$$

69. The tangent line is perpendicular to the line joining the point (5, 12) and the center (0, 0).

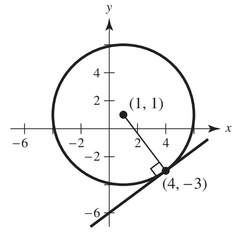


Slope of the line joining (5, 12) and (0, 0) is $\frac{12}{5}$.

The equation of the tangent line is

$$y - 12 = \frac{-5}{12}(x - 5)$$
$$y = \frac{-5}{12}x + \frac{169}{12}$$
$$5x + 12y - 169 = 0.$$

70. The tangent line is perpendicular to the line joining the point $(4, -3)$ and the center of the circle, (1, 1).



Slope of the line joining (1, 1) and $(4, -3)$ is

$\frac{1 + 3}{1 - 4} = \frac{-4}{3}$.

Tangent line:

$$y + 3 = \frac{3}{4}(x - 4)$$
$$y = \frac{3}{4}x - 6$$
$$0 = 3x - 4y - 24$$

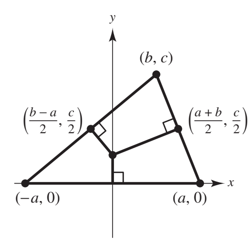
71. (a) The slope of the segment joining (b, c) and $(a, 0)$ is $\frac{c}{(b - a)}$. The slope of the perpendicular bisector of this segment is $\frac{(a - b)}{c}$. The midpoint of this segment is $\left(\frac{a + b}{2}, \frac{c}{2}\right)$.

So, the equation of the perpendicular bisector to this segment is

$$y - \frac{c}{2} = \frac{a - b}{c}\left(x - \frac{a + b}{2}\right).$$

Similarly, the equation of the perpendicular bisector of the segment joining $(-a, 0)$ and $(a, 0)$ is

$$y - \frac{c}{2} = \frac{a - b}{-c}\left(x - \frac{b - a}{2}\right).$$



Equating the right-hand sides of each equation, you obtain $x = 0$.

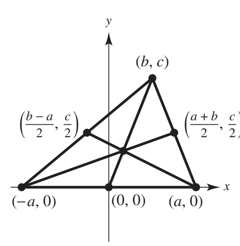
Letting $x = 0$ in either equation yields the point of intersection:

$$y = \frac{c}{2} + \frac{a - b}{c}\left(0 - \frac{a + b}{2}\right) = \frac{c^2}{2c} + \frac{b^2 - a^2}{2c} = \frac{c^2 + b^2 - a^2}{2c}.$$

The point of intersection is $\left(0, \frac{-a^2 + b^2 + c^2}{2c}\right)$.

(b) The equations of the medians are:

$$y = \frac{c}{b}x$$
$$y = \frac{c/2}{\left(\frac{b - a}{2}\right) - a}(x - a) = \frac{c}{b - 3a}(x - a)$$
$$y = \frac{c/2}{\left(\frac{a + b}{2} + a\right)}(x + a) = \frac{c}{3a + b}(x + a).$$



Solving these equation simultaneously for (x, y), you obtain the point of intersection $\left(\frac{b}{3}, \frac{c}{3}\right)$.

72. (a) Lines c, d, e and f have positive slopes.

(b) Lines a and b have negative slopes.

(c) Lines c and e appear parallel.
Lines d and f appear parallel.

(d) Lines b and f appear perpendicular.
Lines b and d appear perpendicular.

73. Find the equation of the line through the points (0, 32) and (100, 212).

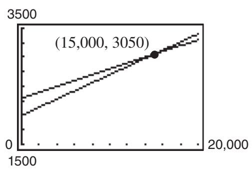
$$m = \frac{180}{100} = \frac{9}{5}$$
$$F - 32 = \frac{9}{5}(C - 0)$$
$$F = \frac{9}{5}C + 32$$

or

$$C = \frac{1}{9}(5F - 160)$$
$$5F - 9C - 160 = 0$$

For $F = 72°$, $C \approx 22.2°$.

74. (a) Current job: $W_1 = 0.07s + 2000$
New job offer: $W_2 = 0.05s + 2300$

(b)


Using a graphing utility, the point of intersection is (15,000, 3050).

Analytically, $W_1 = W_2$

$$0.07s + 2000 = 0.05s + 2300$$
$$0.02s = 300$$
$$s = 15{,}000$$

So, $W_1 = W_2 = 0.07(15{,}000) + 2000 = 3050.$

When sales exceed $15,000, the current job pays more.

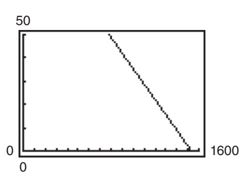
(c) No, if you can sell $20,000 worth of goods, then $W_1 > W_2$.
(**Note:** $W_1 = 3400$ and $W_2 = 3300$ when $s = 20{,}000$.)

75. (a) Two points are (50, 780) and (47, 825).
The slope is

$$m = \frac{825 - 780}{47 - 50} = \frac{45}{-3} = -15.$$
$$p - 780 = -15(x - 50)$$
$$p = -15x + 750 + 780 = -15x + 1530$$

or

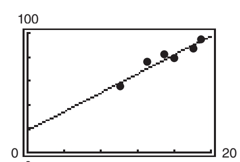
$$x = \frac{1}{15}(1530 - p)$$

(b)


If $p = 855$, then $x = 45$ units.

(c) If $p = 795$, then $x = \frac{1}{15}(1530 - 795) = 49$ units

76. (a) $y = 18.91 + 3.97x$
$(x = \text{quiz score},\ y = \text{test score})$

(b)


(c) If $x = 17$, $y = 18.91 + 3.97(17) = 86.4$.

(d) The slope shows the average increase in exam score for each unit increase in quiz score.

(e) The points would shift vertically upward 4 units. The new regression line would have a y-intercept 4 greater than before: $y = 22.91 + 3.97x$.

77. If $A = 0$, then $By + C = 0$ is the horizontal line $y = -C/B$. The distance to (x_1, y_1) is

$$d = \left|y_1 - \left(\frac{-C}{B}\right)\right| = \frac{|By_1 + C|}{|B|} = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}.$$

If $B = 0$, then $Ax + C = 0$ is the vertical line $x = -C/A$. The distance to (x_1, y_1) is

$$d = \left|x_1 - \left(\frac{-C}{A}\right)\right| = \frac{|Ax_1 + C|}{|A|} = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}.$$

(Note that A and B cannot both be zero.) The slope of the line $Ax + By + C = 0$ is $-A/B$.

The equation of the line through (x_1, y_1) perpendicular to $Ax + By + C = 0$ is:

$$y - y_1 = \frac{B}{A}(x - x_1)$$
$$Ay - Ay_1 = Bx - Bx_1$$
$$Bx_1 - Ay_1 = Bx - Ay$$

The point of intersection of these two lines is:

$$Ax + By = -C \quad \Rightarrow \quad A^2x + ABy = -AC \qquad (1)$$
$$Bx - Ay = Bx_1 - Ay_1 \Rightarrow \underline{B^2x - ABy = B^2x_1 - ABy_1} \qquad (2)$$
$$\left(A^2 + B^2\right)x = -AC + B^2x_1 - ABy_1 \quad \text{(By adding equations (1) and (2))}$$
$$x = \frac{-AC + B^2x_1 - ABy_1}{A^2 + B^2}$$

$$Ax + By = -C \quad \Rightarrow \quad ABx + B^2y = -BC \qquad (3)$$
$$Bx - Ay = Bx_1 - Ay_1 \Rightarrow \underline{-ABx + A^2y = -ABx_1 + A^2y_1} \qquad (4)$$
$$\left(A^2 + B^2\right)y = -BC - ABx_1 + A^2y_1 \quad \text{(By adding equations (3) and (4))}$$
$$y = \frac{-BC - ABx_1 + A^2y_1}{A^2 + B^2}$$

$$\left(\frac{-AC + B^2x_1 - ABy_1}{A^2 + B^2}, \frac{-BC - ABx_1 + A^2y_1}{A^2 + B^2}\right) \text{ point of intersection}$$

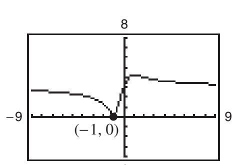
The distance between (x_1, y_1) and this point gives you the distance between (x_1, y_1) and the line $Ax + By + C = 0$.

$$d = \sqrt{\left[\frac{-AC + B^2x_1 - ABy_1}{A^2 + B^2} - x_1\right]^2 + \left[\frac{-BC - ABx_1 + A^2y_1}{A^2 + B^2} - y_1\right]^2}$$
$$= \sqrt{\left[\frac{-AC - ABy_1 - A^2x_1}{A^2 + B^2}\right]^2 + \left[\frac{-BC - ABx_1 - B^2y_1}{A^2 + B^2}\right]^2}$$
$$= \sqrt{\left[\frac{-A(C + By_1 + Ax_1)}{A^2 + B^2}\right]^2 + \left[\frac{-B(C + Ax_1 + By_1)}{A^2 + B^2}\right]^2} = \sqrt{\frac{\left(A^2 + B^2\right)\left(C + Ax_1 + By_1\right)^2}{\left(A^2 + B^2\right)^2}} = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}$$

78. $y = mx + 4 \Rightarrow mx + (-1)y + 4 = 0$

$$d = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}} = \frac{|m3 + (-1)(1) + 4|}{\sqrt{m^2 + (-1)^2}} = \frac{|3m + 3|}{\sqrt{m^2 + 1}}$$

The distance is 0 when $m = -1$. In this case, the line $y = -x + 4$ contains the point (3, 1).



79. $x - y - 2 = 0 \Rightarrow d = \dfrac{|1(-2) + (-1)(1) - 2|}{\sqrt{1^2 + 1^2}} = \dfrac{5}{\sqrt{2}} = \dfrac{5\sqrt{2}}{2}$

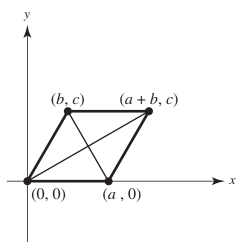
80. $4x + 3y - 10 = 0 \Rightarrow d = \dfrac{|4(2) + 3(3) - 10|}{\sqrt{4^2 + 3^2}} = \dfrac{7}{5}$

81. For simplicity, let the vertices of the rhombus be (0, 0), $(a, 0)$, (b, c), and $(a + b, c)$, as shown in the figure.

The slopes of the diagonals are then $m_1 = \dfrac{c}{a + b}$ and $m_2 = \dfrac{c}{b - a}$. Because the sides of the rhombus are equal, $a^2 = b^2 + c^2$, and you have

$$m_1 m_2 = \frac{c}{a + b} \cdot \frac{c}{b - a} = \frac{c^2}{b^2 - a^2} = \frac{c^2}{-c^2} = -1.$$

Therefore, the diagonals are perpendicular.



82. For simplicity, let the vertices of the quadrilateral be (0, 0), (a, 0), (b, c), and (d, e), as shown in the figure. The midpoints \of the sides are

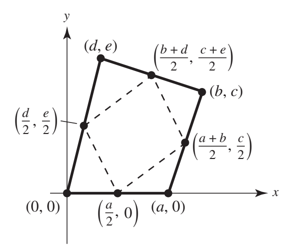
$$\left(\frac{a}{2}, 0\right), \left(\frac{a + b}{2}, \frac{c}{2}\right), \left(\frac{b + d}{2}, \frac{c + e}{2}\right), \text{ and } \left(\frac{d}{2}, \frac{e}{2}\right).$$

The slope of the opposite sides are equal:

$$\frac{\frac{c}{2} - 0}{\frac{a + b}{2} - \frac{a}{2}} = \frac{\frac{c + e}{2} - \frac{e}{2}}{\frac{b + d}{2} - \frac{d}{2}} = \frac{c}{b}$$

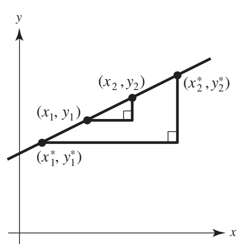
$$\frac{0 - \frac{e}{2}}{\frac{a}{2} - \frac{d}{2}} = \frac{\frac{c}{2} - \frac{c + e}{2}}{\frac{a + b}{2} - \frac{b + d}{2}} = -\frac{e}{a - d}$$

Therefore, the figure is a parallelogram.



83. Consider the figure below in which the four points are collinear. Because the triangles are similar, the result immediately follows.

$$\frac{y_2^* - y_1^*}{x_2^* - x_1^*} = \frac{y_2 - y_1}{x_2 - x_1}$$



84. If $m_1 = -1/m_2$, then $m_1 m_2 = -1$. Let L_3 be a line with slope m_3 that is perpendicular to L_1. Then $m_1 m_3 = -1$. So, $m_2 = m_3 \Rightarrow L_2$ and L_3 are parallel. Therefore, L_2 and L_1 are also perpendicular.

85. True.

$$ax + by = c_1 \Rightarrow y = -\frac{a}{b}x + \frac{c_1}{b} \Rightarrow m_1 = -\frac{a}{b}$$

$$bx - ay = c_2 \Rightarrow y = \frac{b}{a}x - \frac{c_2}{a} \Rightarrow m_2 = \frac{b}{a}$$

$$m_2 = -\frac{1}{m_1}$$

86. True. The slope must be positive.

Section P.3 Functions and Their Graphs

1. A relation between two sets X and Y is a set of ordered pairs of the form (x, y), where x is a member of X and y is at member of Y.

A function from X to Y is a relation between X and Y that has the property that any two ordered pairs with the same x-value also have the same y-value.

2. For a function f from X to Y, the domain is the set X. If y is the image of x, then the range is a subject of Y consisting of all images of numbers in X.

3. The three basic types are vertical shifts, horizontal shifts, and reflections.

4. Consider the nonconstant polynomial

$f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0.$

If n is even and $a_n > 0$, then the graph of f moves up to the left and up to the right.

If n is even and $a_n < 0$, then the graph of f moves down to the left and down to the right.

If n is odd and $a_n > 0$, then the graph of f moves down to the left and up to the right.

If n is odd and $a_n < 0$, then the graph of f moves up to the left and down to the right.

5. $f(x) = 3x - 2$

(a) $f(0) = 3(0) - 2 = -2$

(b) $f(5) = 3(5) - 2 = 13$

(c) $f(b) = 3(b) - 2 = 3b - 2$

(d) $f(x - 1) = 3(x - 1) - 2 = 3x - 5$

6. (a) $f(0) = 7(0) - 4 = -4$

(b) $f(-3) = 7(-3) - 4 = -25$

(c) $f(b) = 7(b) - 4 = 7b - 4$

(d) $f(x - 1) = 7(x - 1) - 4 = 7x - 11$

7. (a) $f(-2) = \sqrt{(-2)^2 + 4} = \sqrt{4 + 4} = \sqrt{8} = 2\sqrt{2}$

(b) $f(3) = \sqrt{3^2 + 4} = \sqrt{9 + 4} = \sqrt{13}$

(c) $f(2) = \sqrt{2^2 + 4} = \sqrt{4 + 4} = \sqrt{8} = 2\sqrt{2}$

(d) $f(x + bx) = \sqrt{(x + bx)^2 + 4}$

$= \sqrt{x^2 + 2bx^2 + b^2x^2 + 4}$

8. (a) $f(-4) = \sqrt{-4 + 5} = \sqrt{1} = 1$

(b) $f(11) = \sqrt{11 + 5} = \sqrt{16} = 4$

(c) $f(4) = \sqrt{4 + 5} = \sqrt{9} = 3$

(d) $f(x + \Delta x) = \sqrt{x + \Delta x + 5}$

9. (a) $g(0) = 5 - 0^2 = 5$

(b) $g(\sqrt{5}) = 5 - (\sqrt{5})^2 = 5 - 5 = 0$

(c) $g(-2) = 5 - (-2)^2 = 5 - 4 = 1$

(d) $g(t - 1) = 5 - (t - 1)^2 = 5 - (t^2 - 2t + 1)$

$= 4 + 2t - t^2$

10. (a) $g(4) = 4^2(4 - 4) = 0$

(b) $g\left(\frac{3}{2}\right) = \left(\frac{3}{2}\right)^2\left(\frac{3}{2} - 4\right) = \frac{9}{4}\left(-\frac{5}{2}\right) = -\frac{45}{8}$

(c) $g(c) = c^2(c - 4) = c^3 - 4c^2$

(d) $g(t + 4) = (t + 4)^2(t + 4 - 4)$

$= (t + 4)^2 t = t^3 + 8t^2 + 16t$

11. $$\frac{f(x + \Delta x) - f(x)}{\Delta x} = \frac{(x + \Delta x)^3 - x^3}{\Delta x} = \frac{x^3 + 3x^2\Delta x + 3x^2(\Delta x)^2 + (\Delta x)^3 - x^3}{\Delta x} = 3x^2 + 3x\Delta x + (\Delta x)^2,\ \Delta x \neq 0$$

12. $$\frac{f(x) - f(1)}{x - 1} = \frac{3x - 1 - (3 - 1)}{x - 1} = \frac{3(x - 1)}{x - 1} = 3,\ x \neq 1$$

13. $f(x) = 4x^2$

Domain: $(-\infty, \infty)$

Range: $[0, \infty)$

14. $g(x) = x^2 - 5$

Domain: $(-\infty, \infty)$

Range: $[-5, \infty)$

15. $f(x) = x^3$

Domain: $(-\infty, \infty)$

Range: $(-\infty, \infty)$

16. $h(x) = 4 - x^2$

Domain: $(-\infty, \infty)$

Range: $(-\infty, 4]$

17. $g(x) = \sqrt{6x}$

Domain: $6x \ge 0$

$x \ge 0 \Rightarrow [0, \infty)$

Range: $[0, \infty)$

18. $h(x) = -\sqrt{x + 3}$

Domain: $x + 3 \ge 0 \Rightarrow [-3, \infty)$

Range: $(-\infty, 0]$

19. $f(x) = \sqrt{16 - x^2}$

$16 - x^2 \ge 0 \Rightarrow x^2 \le 16$

Domain: $[-4, 4]$

Range: $[0, 4]$

Note: $y = \sqrt{16 - x^2}$ is a semicircle of radius 4.

20. $f(x) = |x - 3|$

Domain: $(-\infty, \infty)$

Range: $[0, \infty)$

21. $f(x) = \dfrac{3}{x}$

Domain: all $x \ne 0 \Rightarrow (-\infty, 0) \cup (0, \infty)$

Range: $(-\infty, 0) \cup (0, \infty)$

22. $f(x) = \dfrac{x - 2}{x + 4}$

Domain: all $x \ne -4$

Range: all $y \ne 1$

[**Note:** You can see that the range is all $y \ne 1$ by graphing *f*.]

23. $f(x) = \sqrt{x} + \sqrt{1 - x}$

$x \ge 0$ and $1 - x \ge 0$

$x \ge 0$ and $x \le 1$

Domain: $0 \le x \le 1 \Rightarrow [0, 1]$

24. $f(x) = \sqrt{x^2 - 3x + 2}$

$x^2 - 3x + 2 \ge 0$

$(x - 2)(x - 1) \ge 0$

Domain: $x \ge 2$ or $x \le 1$

Domain: $(-\infty, 1] \cup [2, \infty)$

25. $f(x) = \dfrac{1}{|x + 3|}$

$|x + 3| \ne 0$

$x + 3 \ne 0$

Domain: all $x \ne -3$

Domain: $(-\infty, -3) \cup (-3, \infty)$

26. $g(x) = \dfrac{1}{|x^2 - 4|}$

$|x^2 - 4| \ne 0$

$(x - 2)(x + 2) \ne 0$

Domain: all $x \ne \pm 2$

Domain: $(-\infty, -2) \cup (-2, 2) \cup (2, \infty)$

27. $f(x) = \begin{cases} 2x + 1, & x < 0 \\ 2x + 2, & x \ge 0 \end{cases}$

(a) $f(-1) = 2(-1) + 1 = -1$

(b) $f(0) = 2(0) + 2 = 2$

(c) $f(2) = 2(2) + 2 = 6$

(d) $f(t^2 + 1) = 2(t^2 + 1) + 2 = 2t^2 + 4$

(**Note:** $t^2 + 1 \ge 0$ for all t.)

Domain: $(-\infty, \infty)$

Range: $(-\infty, 1) \cup [2, \infty)$

28. $f(x) = \begin{cases} x^2 + 2, & x \le 1 \\ 2x^2 + 2, & x > 1 \end{cases}$

(a) $f(-2) = (-2)^2 + 2 = 6$

(b) $f(0) = 0^2 + 2 = 2$

(c) $f(1) = 1^2 + 2 = 3$

(d) $f(s^2 + 2) = 2(s^2 + 2)^2 + 2 = 2s^4 + 8s^2 + 10$

(**Note:** $s^2 + 2 > 1$ for all s.)

Domain: $(-\infty, \infty)$

Range: $[2, \infty)$

29. $f(x) = \begin{cases} |x| + 1, & x < 1 \\ -x + 1, & x \geq 1 \end{cases}$

(a) $f(-3) = |-3| + 1 = 4$

(b) $f(1) = -1 + 1 = 0$

(c) $f(3) = -3 + 1 = -2$

(d) $f(b^2 + 1) = -(b^2 + 1) + 1 = -b^2$

Domain: $(-\infty, \infty)$

Range: $(-\infty, 0] \cup [1, \infty)$

30. $f(x) = \begin{cases} \sqrt{x + 4}, & x \leq 5 \\ (x - 5)^2, & x > 5 \end{cases}$

(a) $f(-3) = \sqrt{-3 + 4} = \sqrt{1} = 1$

(b) $f(0) = \sqrt{0 + 4} = 2$

(c) $f(5) = \sqrt{5 + 4} = 3$

(d) $f(10) = (10 - 5)^2 = 25$

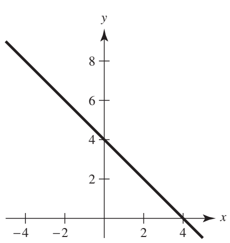
Domain: $[-4, \infty)$

Range: $[0, \infty)$

31. $f(x) = 4 - x$

Domain: $(-\infty, \infty)$

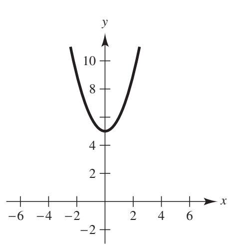
Range: $(-\infty, \infty)$



32. $f(x) = x^2 + 5$

Domain: $(-\infty, \infty)$

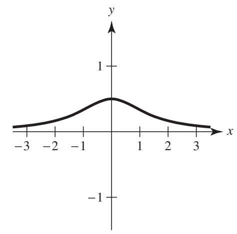
Range: $[5, \infty)$



33. $g(x) = \dfrac{1}{x^2 + 2}$

Domain: $(-\infty, \infty)$

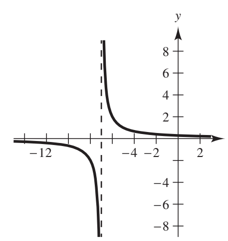
Range: $\left(0, \frac{1}{2}\right]$



34. $f(t) = \dfrac{2}{7 + t}$

Domain: $(-\infty, -7) \cup (-7, \infty)$

Range: $(-\infty, 0) \cup (0, \infty)$

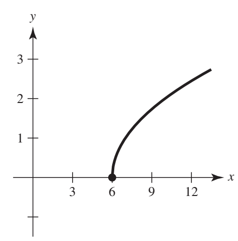


35. $h(x) = \sqrt{x - 6}$

Domain: $x - 6 \geq 0$

$x \geq 6 \Rightarrow [6, \infty)$

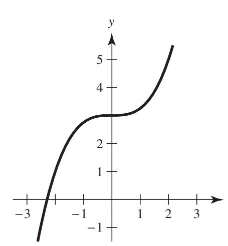
Range: $[0, \infty)$



36. $f(x) = \frac{1}{4}x^3 + 3$

Domain: $(-\infty, \infty)$

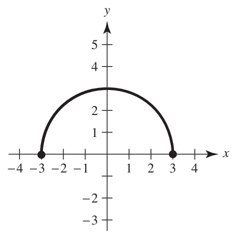
Range: $(-\infty, \infty)$



37. $f(x) = \sqrt{9 - x^2}$

Domain: $[-3, 3]$

Range: $[0, 3]$



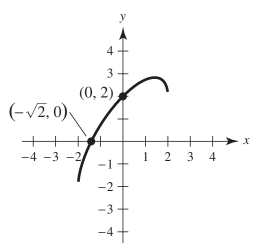
38. $f(x) = x + \sqrt{4 - x^2}$

Domain: $[-2, 2]$

Range: $\left[-2, 2\sqrt{2}\right] \approx [-2, 2.83]$

y-intercept: $(0, 2)$

x-intercept: $\left(-\sqrt{2}, 0\right)$



39. $x - y^2 = 0 \Rightarrow y = \pm\sqrt{x}$

y is not a function of x. Some vertical lines intersect the graph twice.

40. $\sqrt{x^2 - 4} - y = 0 \Rightarrow y = \sqrt{x^2 - 4}$

y is a function of x. Vertical lines intersect the graph at most once.

41. y is a function of x. Vertical lines intersect the graph at most once.

42. $x^2 + y^2 = 4$

$$y = \pm\sqrt{4 - x^2}$$

y is not a function of x. Some vertical lines intersect the graph twice.

43. $x^2 + y^2 = 16 \Rightarrow y = \pm\sqrt{16 - x^2}$

y is not a function of x because there are two values of y for some x.

44. $x^2 + y = 16 \Rightarrow y = 16 - x^2$

y is a function of x because there is one value of y for each x.

45. $y^2 = x^2 - 1 \Rightarrow y = \pm\sqrt{x^2 - 1}$

y is not a function of x because there are two values of y for some x.

46. $x^2y - x^2 + 4y = 0 \Rightarrow y = \dfrac{x^2}{x^2 + 4}$

y is a function of x because there is one value of y for each x.

47. The transformation is a horizontal shift two units to the right of the function $f(x) = \sqrt{x}$.

Shifted function: $y = \sqrt{x - 2}$

48. The transformation is a vertical shift 2 units upward of the function $f(x) = |x|$.

Shifted function: $y = |x| + 2$

49. The transformation is a horizontal shift 2 units to the right and a vertical shift 1 unit downward of the function $f(x) = x^2$.

Shifted function: $y = (x - 2)^2 - 1$

50. The transformation is a horizontal shift 1 unit to the left and a vertical shift 2 units upward of the function $f(x) = x^3$.

Shifted function: $y = (x + 1)^3 + 2$

51. $y = f(x + 5)$ is a horizontal shift 5 units to the left.

Matches d.

52. $y = f(x) - 5$ is a vertical shift 5 units downward.

Matches b.

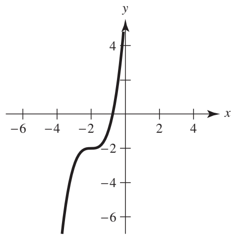
53. $y = -f(-x) - 2$ is a reflection in the y-axis, a reflection in the x-axis, and a vertical shift downward 2 units. Matches c.

54. $y = -f(x - 4)$ is a horizontal shift 4 units to the right, followed by a reflection in the x-axis. Matches a.

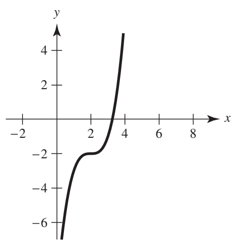
55. $y = f(x + 6) + 2$ is a horizontal shift to the left 6 units, and a vertical shift upward 2 units. Matches e.

56. $y = f(x - 1) + 3$ is a horizontal shift to the right 1 unit, and a vertical shift upward 3 units. Matches g.

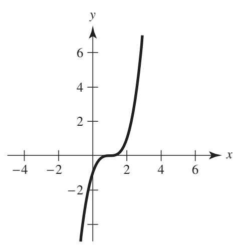
57. (a) The graph is shifted 3 units to the left.



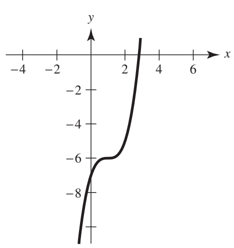
(b) The graph is shifted 1 unit to the right.



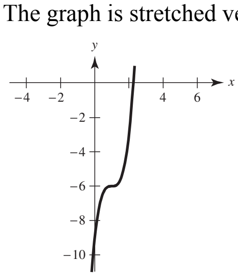
(c) The graph is shifted 2 units upward.



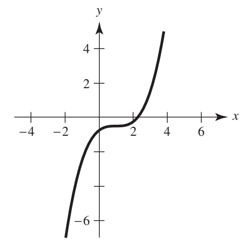
(d) The graph is shifted 4 units downward.



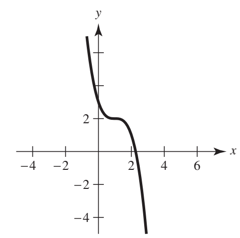
(e) The graph is stretched vertically by a factor of 3.



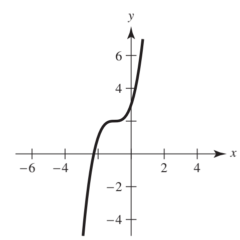
(f) The graph is stretched vertically by a factor of $\frac{1}{4}$.



(g) The graph is a reflection in the x-axis.



(h) The graph is a reflection about the origin.

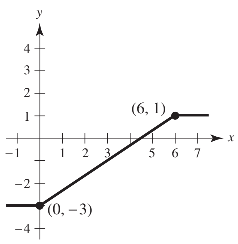


58. (a) $g(x) = f(x - 4)$

$g(6) = f(2) = 1$

$g(0) = f(-4) = -3$

The graph is shifted 4 units to the right.

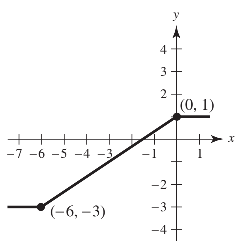


(b) $g(x) = f(x + 2)$

$g(0) = f(2) = 1$

$g(-6) = f(-4) = -3$

The graph is shifted 2 units to the left.

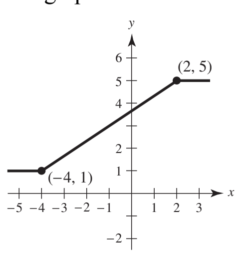


(c) $g(x) = f(x) + 4$

$g(2) = f(2) + 4 = 5$

$g(-4) = f(-4) + 4 = 1$

The graph is shifted 4 units upward.

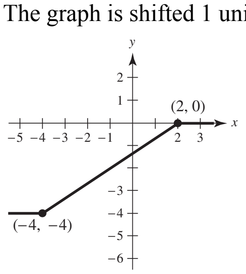


(d) $g(x) = f(x) - 1$

$g(2) = f(2) - 1 = 0$

$g(-4) = f(-4) - 1 = -4$

The graph is shifted 1 unit downward.

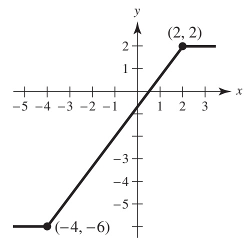


(e) $g(x) = 2f(x)$

$g(2) = 2f(2) = 2$

$g(-4) = 2f(-4) = -6$

The graph is stretched vertically by a factor of 2.

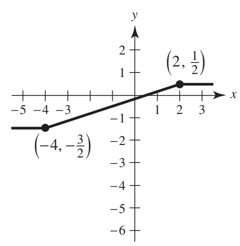


(f) $g(x) = \frac{1}{2}f(x)$

$g(2) = \frac{1}{2}f(2) = \frac{1}{2}$

$g(-4) = \frac{1}{2}f(-4) = -\frac{3}{2}$

The graph is stretched vertically by a factor of $\frac{1}{2}$.

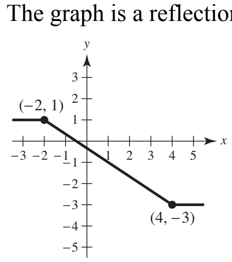


(g) $g(x) = f(-x)$

$g(-2) = f(2) = 1$

$g(4) = f(-4) = -3$

The graph is a reflection in the y-axis.

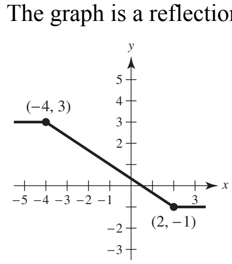


(h) $g(x) = -f(x)$

$g(2) = f(2) = -1$

$g(-4) = f(-4) = 3$

The graph is a reflection in the x-axis.



59. $f(x) = 2x - 5, g(x) = 4 - 3x$

(a) $f(x) + g(x) = (2x - 5) + (4 - 3x) = -x - 1$

(b) $f(x) - g(x) = (2x - 5) - (4 - 3x) = 5x - 9$

(c) $f(x) \cdot g(x) = (2x - 5)(4 - 3x) = -6x^2 + 8x + 15x - 20 = -6x^2 + 23x - 20$

(d) $\dfrac{f(x)}{g(x)} = \dfrac{2x - 5}{4 - 3x}$

60. $f(x) = x^2 + 5x + 4, \quad g(x) = x + 1$

(a) $f(x) + g(x) = (x^2 + 5x + 4) + (x + 1) = x^2 + 6x + 5$

(b) $f(x) - g(x) = (x^2 + 5x + 4) - (x + 1) = x^2 + 4x + 3$

(c) $f(x) \cdot g(x) = (x^2 + 5x + 4)(x + 1) = x^3 + 5x^2 + 4x + x^2 + 5x + 4 = x^3 + 6x^2 + 9x + 4$

(d) $f(x)/g(x) = \dfrac{x^2 + 5x + 4}{x + 1} = \dfrac{(x + 4)(x + 1)}{x + 1} = x + 4, x \neq -1$

61. (a) $f(g(1)) = f(0) = 0$

(b) $g(f(1)) = g(1) = 0$

(c) $g(f(0)) = g(0) = -1$

(d) $f(g(-4)) = f(15) = \sqrt{15}$

(e) $f(g(x)) = f(x^2 - 1) = \sqrt{x^2 - 1}$

(f) $g(f(x)) = g(\sqrt{x}) = (\sqrt{x})^2 - 1 = x - 1, \ (x \geq 0)$

62. $f(x) = 2x^3, g(x) = 4x + 3$

(a) $f(g(0)) = f(3) = 2(3)^3 = 54$

(b) $f\left(g\left(\tfrac{1}{2}\right)\right) = f\left(4\left(\tfrac{1}{2}\right) + 3\right) = f(5) = 2(5^3) = 250$

(c) $g(f(0)) = g(0) = 3$

(d) $g\left(f\left(-\tfrac{1}{4}\right)\right) = g\left(2\left(-\tfrac{1}{4}\right)^3\right) = g\left(-\tfrac{1}{32}\right) = 4\left(-\tfrac{1}{32}\right) + 3 = \tfrac{23}{8}$

(e) $f(g(x)) = f(4x + 3) = 2(4x + 3)^3 = 2(64x^3 + 144x^2 + 108x + 27) = 128x^3 + 288x^2 + 216x + 54$

(f) $g(f(x)) = g(2x^3) = 4(2x^3) + 3 = 8x^3 + 3$

63. $f(x) = x^2, \ g(x) = \sqrt{x}$

$(f \circ g)(x) = f(g(x)) = f(\sqrt{x}) = (\sqrt{x})^2 = x, \ x \geq 0$

Domain: $[0, \infty)$

$(g \circ f)(x) = g(f(x)) = g(x^2) = \sqrt{x^2} = |x|$

Domain: $(-\infty, \infty)$

No. Their domains are different. $(f \circ g) = (g \circ f)$ for $x \geq 0$.

64. $f(x) = x^2 - 1, g(x) = -x$

$(f \circ g)(x) = f(g(x)) = f(-x) = (-x)^2 - 1 = x^2 - 1$

Domain: $(-\infty, \infty)$

$(g \circ f)(x) = g(x^2 - 1) = -(x^2 - 1) = 1 - x^2$

Domain: $(-\infty, \infty)$

$f \circ g \neq g \circ f$

65. $f(x) = \dfrac{3}{x}, g(x) = x^2 - 1$

$(f \circ g)(x) = f(g(x)) = f(x^2 - 1) = \dfrac{3}{x^2 - 1}$

Domain: all $x \neq \pm 1 \Rightarrow (-\infty, -1) \cup (-1, 1) \cup (1, \infty)$

$(g \circ f)(x) = g(f(x)) = g\left(\dfrac{3}{x}\right) = \left(\dfrac{3}{x}\right)^2 - 1 = \dfrac{9}{x^2} - 1 = \dfrac{9 - x^2}{x^2}$

Domain: all $x \neq 0 \Rightarrow (-\infty, 0) \cup (0, \infty)$

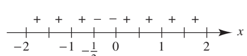
$f \circ g \neq g \circ f$

66. $(f \circ g)(x) = f\left(\sqrt{x + 2}\right) = \dfrac{1}{\sqrt{x + 2}}$

Domain: $(-2, \infty)$

$(g \circ f)(x) = g\left(\dfrac{1}{x}\right) = \sqrt{\dfrac{1}{x} + 2} = \sqrt{\dfrac{1 + 2x}{x}}$

You can find the domain of $g \circ f$ by determining the intervals where $(1 + 2x)$ and x are both positive, or both negative.



Domain: $\left(-\infty, -\frac{1}{2}\right], (0, \infty)$

67. (a) $(f \circ g)(3) = f(g(3)) = f(-1) = 4$

(b) $g(f(2)) = g(1) = -2$

(c) $g(f(5)) = g(-5)$, which is undefined

(d) $(f \circ g)(-3) = f(g(-3)) = f(-2) = 3$

(e) $(g \circ f)(-1) = g(f(-1)) = g(4) = 2$

(f) $f(g(-1)) = f(-4)$, which is undefined

68. $(A \circ r)(t) = A(r(t)) = A(0.6t) = \pi(0.6t)^2 = 0.36\pi t^2$

$(A \circ r)(t)$ represents the area of the circle at time t.

69. $F(x) = \sqrt{2x - 2}$

Let $h(x) = 2x$, $g(x) = x - 2$ and $f(x) = \sqrt{x}$.

Then $(f \circ g \circ h)(x) = f(g(2x)) = f((2x) - 2) = \sqrt{(2x) - 2} = \sqrt{2x - 2} = F(x)$.

(Other answers possible.)

70. $F(x) = \dfrac{1}{4x^6}$

Let $f(x) = \dfrac{1}{x}$, $g(x) = 4x$, and $h(x) = x^6$.

Then $(f \circ g \circ h)(x) = f(g(x^6)) = f(4x^6) = \dfrac{1}{4x^6}$.

(Other answers possible.)

71. (a) If f is even, then $\left(\frac{3}{2}, 4\right)$ is on the graph.

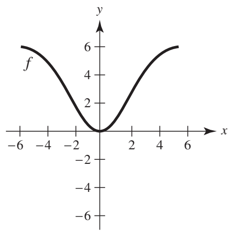
(b) If f is odd, then $\left(\frac{3}{2}, -4\right)$ is on the graph.

72. (a) If f is even, then $(-4, 9)$ is on the graph.

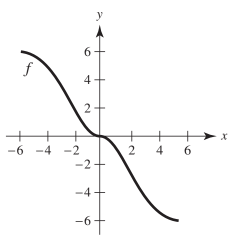
(b) If f is odd, then $(-4, -9)$ is on the graph.

73. f is even because the graph is symmetric about the y-axis. g is neither even nor odd. h is odd because the graph is symmetric about the origin.

74. (a) If f is even, then the graph is symmetric about the y-axis.



(b) If f is odd, then the graph is symmetric about the origin.



75. $f(x) = x^2(4 - x^2)$

$f(-x) = (-x)^2(4 - (-x)^2) = x^2(4 - x^2) = f(x)$

f is even.

$f(x) = x^2(4 - x^2) = 0$

$x^2(2 - x)(2 + x) = 0$

Zeros: $x = 0, -2, 2$

76. $f(x) = \sqrt[3]{x}$

$f(-x) = \sqrt[3]{(-x)} = -\sqrt[3]{x} = -f(x)$

f is odd.

$f(x) = \sqrt[3]{x} = 0 \Rightarrow x = 0$ is the zero.

77. $f(x) = 2\sqrt[6]{x}$

The domain of f is $x \geq 0$ and the range is $y \geq 0$. Hence, the function is neither even nor odd. The only zero is $x = 0$.

78. $f(x) = 4x^4 - 3x^2$

$f(-x) = 4(-x)^4 - 3(-x)^2 = 4x^4 - 3x^2 = f(x)$

f is even.

$4x^4 - 3x^2 = x^2(4x^2 - 3) = 0$

Zeros: $x = 0, \pm\dfrac{\sqrt{3}}{2}$

79. Slope $= \dfrac{4 - (-6)}{-2 - 0} = \dfrac{10}{-2} = -5$

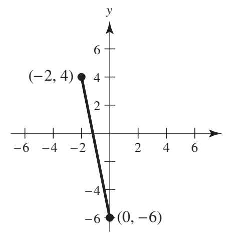
$y - 4 = -5(x - (-2))$

$y - 4 = -5x - 10$

$y = -5x - 6$

For the line segment, you must restrict the domain.

$f(x) = -5x - 6, -2 \leq x \leq 0$



80. Slope $= \dfrac{8 - 1}{5 - 3} = \dfrac{7}{2}$

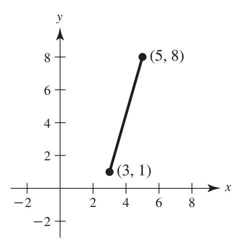
$y - 1 = \dfrac{7}{2}(x - 3)$

$y - 1 = \dfrac{7}{2}x - \dfrac{21}{2}$

$y = \dfrac{7}{2}x - \dfrac{19}{2}$

For the line segment, you must restrict the domain.

$f(x) = \dfrac{7}{2}x - \dfrac{19}{2}, 3 \leq x \leq 5$

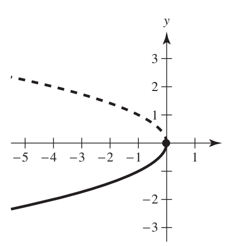


81. $x + y^2 = 0$

$y^2 = -x$

$y = -\sqrt{-x}$

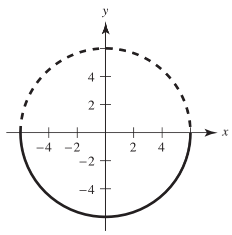
$f(x) = -\sqrt{-x}, x \leq 0$



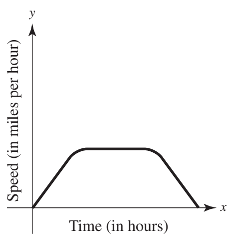
82. $x^2 + y^2 = 36$

$y^2 = 36 - x^2$

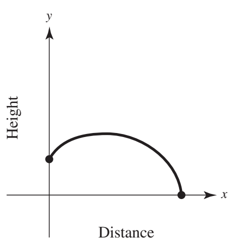
$y = -\sqrt{36 - x^2}, \ -6 \le x \le 6$



83. Answers will vary. *Sample answer*: Speed begins and ends at 0. The speed might be constant in the middle:

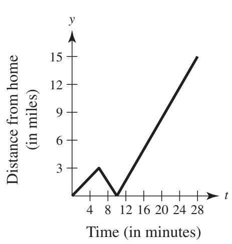


84. Answers will vary. *Sample answer*: Height begins a few feet above 0, and ends at 0.



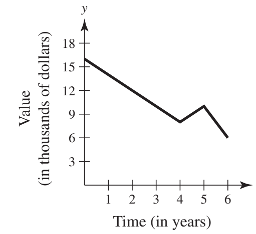
85. Answers will vary. *Sample answer:*

Distance begins at 0, then the graph has a sharp turn after a few minutes and goes back to 0. Then the graph goes back upward steeply.



86. Answers will vary. *Sample answer:*

The graph begins at time 0, then decreases until year 4. The graph then increases slightly for a few years and then decreases again.



87. $y = \sqrt{c - x^2}$

$y^2 = c - x^2$

$x^2 + y^2 = c$, a circle.

For the domain to be $[-5, 5]$, $c = 25$.

88. For the domain to be the set of all real numbers, you must require that $x^2 + 3cx + 6 \ne 0$. So, the discriminant must be less than zero:

$(3c)^2 - 4(6) < 0$

$9c^2 < 24$

$c^2 < \frac{8}{3}$

$-\sqrt{\frac{8}{3}} < c < \sqrt{\frac{8}{3}}$

$-\frac{2}{3}\sqrt{6} < c < \frac{2}{3}\sqrt{6}$

89. No. If a horizontal line intersects the graph more than once, then there is more than one x-value corresponding to the same y-value.

90. Answers will vary. *Sample answer:*

$f(x) = \frac{1}{x}$ and $g(x) = \frac{1}{x^2}$

$(f \circ g)(x) = f\left(\frac{1}{x^2}\right) = x^2$

$(g \circ f)(x) = g\left(\frac{1}{x}\right) = \frac{1}{\left(\frac{1}{x}\right)^2} = x^2$

91. No. For example, $y = x^3 + x + 2$ is not odd since $f(-x) \ne -f(x)$.

92. $f(x) = 0$ is even and odd.

$f(-x) = 0 = f(x) = -f(x)$

93. (a) $T(4) = 16°, T(15) \approx 23°$

(b) If $H(t) = T(t - 1)$, then the changes in temperature will occur 1 hour later.

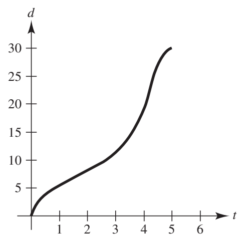
(c) If $H(t) = T(t) - 1$, then the overall temperature would be 1 degree lower.

94. (a) For each time t, there corresponds a depth d.

(b) Domain: $0 \le t \le 5$

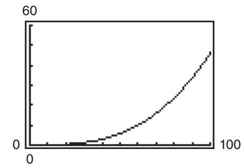
Range: $0 \le d \le 30$

(c)



(d) $d(4) \approx 18$. At time 4 seconds, the depth is approximately 18 cm.

95. (a)



(b) $H\left(\dfrac{x}{1.6}\right) = 0.00004636\left(\dfrac{x}{1.6}\right)^3 \approx 0.00001132x^3$

96. $p_1(x) = x^3 - x + 1$ has one zero. $p_2(x) = x^3 - x$ has three zeros. Every cubic polynomial has at least one zero. Given $p(x) = Ax^3 + Bx^2 + Cx + D$, you have $p \to -\infty$ as $x \to -\infty$ and $p \to \infty$ as $x \to \infty$ if $A > 0$. Furthermore, $p \to \infty$ as $x \to -\infty$ and $p \to -\infty$ as $x \to \infty$ if $A < 0$. Because the graph has no breaks, the graph must cross the x-axis at least one time.

97.
$$\begin{aligned} f(-x) &= a_{2n+1}(-x)^{2n+1} + \cdots + a_3(-x)^3 + a_1(-x) \\ &= -\left[a_{2n+1}x^{2n+1} + \cdots + a_3x^3 + a_1x\right] \\ &= -f(x) \end{aligned}$$

Odd

98.
$$\begin{aligned} f(-x) &= a_{2n}(-x)^{2n} + a_{2n-2}(-x)^{2n-2} + \cdots + a_2(-x)^2 + a_0 \\ &= a_{2n}x^{2n} + a_{2n-2}x^{2n-2} + \cdots + a_2x^2 + a_0 \\ &= f(x) \end{aligned}$$

Even

99. Let $F(x) = f(x)g(x)$ where f and g are even. Then $F(-x) = f(-x)g(-x) = f(x)g(x) = F(x)$.

So, $F(x)$ is even. Let $F(x) = f(x)g(x)$ where f and g are odd. Then

$F(-x) = f(-x)g(-x) = \left[-f(x)\right]\left[-g(x)\right] = f(x)g(x) = F(x)$.

So, $F(x)$ is even.

100. Let $F(x) = f(x)g(x)$ where f is even and g is odd. Then

$F(-x) = f(-x)g(-x) = f(x)\left[-g(x)\right] = -f(x)g(x) = -F(x)$.

So, $F(x)$ is odd.

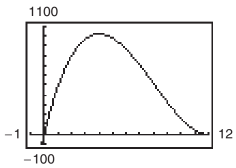
101. By equating slopes,
$$\begin{aligned} \frac{y-2}{0-3} &= \frac{0-2}{x-3} \\ y - 2 &= \frac{6}{x-3} \\ y &= \frac{6}{x-3} + 2 = \frac{2x}{x-3}, \end{aligned}$$

$$L = \sqrt{x^2 + y^2} = \sqrt{x^2 + \left(\frac{2x}{x-3}\right)^2}.$$

102. (a) $V = x(24 - 2x)^2$

Domain: $0 < x < 12$

(b)



Maximum volume occurs at $x = 4$. So, the dimensions of the box would be $4 \times 16 \times 16$ cm.

103. False. If $f(x) = x^2$, then $f(-3) = f(3) = 9$, but $-3 \neq 3$.

104. True

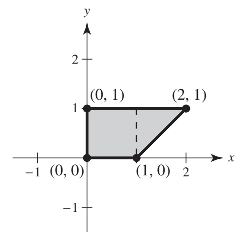
105. True. The function is even.

106. False. If $f(x) = x^2$ then, $f(3x) = (3x)^2 = 9x^2$ and $3f(x) = 3x^2$. So, $3f(x) \neq f(3x)$.

107. False. The constant function $f(x) = 0$ has symmetry with respect to the x-axis.

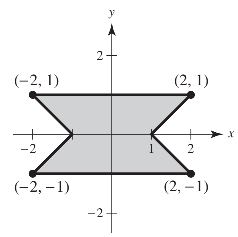
108. True. If the domain is $\{a\}$, then the range is $\{f(a)\}$.

109. First consider the portion of R in the first quadrant: $x \geq 0,\ 0 \leq y \leq 1$ and $x - y \leq 1$; shown below.



The area of this region is $1 + \frac{1}{2} = \frac{3}{2}$.

By symmetry, you obtain the entire region R:



The area of R is $4\left(\frac{3}{2}\right) = 6$.

110. Let $g(x) = c$ be constant polynomial.

Then $f(g(x)) = f(c)$ and $g(f(x)) = c$.

So, $f(c) = c$. Because this is true for all real numbers c, f is the identity function: $f(x) = x$.

Section P.4 Review of Trigonometric Functions

1. In general, if θ is any angle measured in degrees, then the angle $\theta + n(360°)$, n a nonzero integer, is coterminal with θ.

2. If the degree measure of angle θ is x, then the radian measure of θ is $x\left(\dfrac{\pi}{180°}\right)$.

3. $\sin\theta = \dfrac{\text{opp}}{\text{hyp}} = \dfrac{7}{25}$

$\cos\theta = \dfrac{\text{adj}}{\text{hyp}} = \dfrac{24}{25}$

$\tan\theta = \dfrac{\text{opp}}{\text{adj}} = \dfrac{7}{24}$

4. The amplitude of the graph of $y = a \sin bx$ or $y = a \cos bx$ is $|a|$. This value is the maximum value of the function, and $-|a|$ is the minimum value.

A function is periodic if there exists a positive real number p such that $f(x + p) = f(x)$ for all x in the domain of f. The period of f is the least positive value of p.

5. (a) $\theta + 360° = 36° + 360° = 396°$

$\theta - 360° = 36° - 360° = -324°$

(b) $\theta + 360° = -120° + 360° = 240°$

$\theta - 360° = -120° - 360° = -480°$

6. (a) $\theta + 360° = 300° + 360° = 660°$

$\theta - 360° = 300° - 360° = -60°$

(b) $\theta + 2(360°) = -420° + 720° = 300°$

$\theta + 360° = -420° + 360° = -60°$

7. (a) $\theta + 2\pi = \dfrac{\pi}{9} + 2\pi = \dfrac{19\pi}{9}$

$\theta - 2\pi = \dfrac{\pi}{9} - 2\pi = -\dfrac{17\pi}{9}$

(b) $\theta + 2\pi = \dfrac{4\pi}{3} + 2\pi = \dfrac{10\pi}{3}$

$\theta - 2\pi = \dfrac{4\pi}{3} - 2\pi = -\dfrac{2\pi}{3}$

8. (a) $\theta + 2\pi = -\dfrac{9\pi}{4} + 2\pi = -\dfrac{\pi}{4}$

$\theta + 4\pi = -\dfrac{9\pi}{4} + 4\pi = \dfrac{7\pi}{4}$

(b) $\theta + 2\pi = \dfrac{8\pi}{9} + 2\pi = \dfrac{26\pi}{9}$

$\theta - 2\pi = \dfrac{8\pi}{9} - 2\pi = -\dfrac{10\pi}{9}$

9. (a) $30°\left(\dfrac{\pi}{180°}\right) = \dfrac{\pi}{6} \approx 0.524$

(b) $150°\left(\dfrac{\pi}{180°}\right) = \dfrac{5\pi}{6} \approx 2.618$

(c) $315°\left(\dfrac{\pi}{180°}\right) = \dfrac{7\pi}{4} \approx 5.498$

(d) $120°\left(\dfrac{\pi}{180°}\right) = \dfrac{2\pi}{3} \approx 2.094$

10. (a) $-20°\left(\dfrac{\pi}{180°}\right) = -\dfrac{\pi}{9} \approx -0.349$

(b) $-240°\left(\dfrac{\pi}{180°}\right) = -\dfrac{4\pi}{3} \approx -4.189$

(c) $-270°\left(\dfrac{\pi}{180°}\right) = -\dfrac{3\pi}{2} \approx -4.712$

(d) $144°\left(\dfrac{\pi}{180°}\right) = -\dfrac{4\pi}{5} \approx 2.513$

11. (a) $\dfrac{3\pi}{2}\left(\dfrac{180°}{\pi}\right) = 270°$

(b) $\dfrac{7\pi}{6}\left(\dfrac{180°}{\pi}\right) = 210°$

(c) $-\dfrac{7\pi}{12}\left(\dfrac{180°}{\pi}\right) = -105°$

(d) $-2.367\left(\dfrac{180°}{\pi}\right) \approx -135.62°$

12. (a) $\dfrac{7\pi}{3}\left(\dfrac{180°}{\pi}\right) = 420°$

(b) $-\dfrac{11\pi}{30}\left(\dfrac{180°}{\pi}\right) = -66°$

(c) $\dfrac{11\pi}{6}\left(\dfrac{180°}{\pi}\right) = 330°$

(d) $0.438\left(\dfrac{180°}{\pi}\right) \approx 25.1°$

13.

r	8 ft	15 in.	85 cm	24 in.	$\dfrac{12{,}963}{\pi}$ mi
s	12 ft	24 in.	63.75π cm	96 in.	8642 mi
θ	1.5	1.6	$\dfrac{3\pi}{4}$	4	$\dfrac{2\pi}{3}$

14. (a) $50 \text{ mi/h} = \dfrac{50(5280)}{60} = 4400 \text{ ft/min}$

Circumference of tire: $C = 2.5\pi$ feet

Revolutions per minute: $\dfrac{4400}{2.5\pi} \approx 560.2$

(b) $\theta = \dfrac{4400}{2.5\pi}(2\pi) = 3520$ radians

Angular speed: $\dfrac{\theta}{t} = \dfrac{3520 \text{ radians}}{1 \text{ minute}} = 3520 \text{ rad/min}$

15. (a) $x = 3, y = 4, r = 5$

$\sin\theta = \frac{4}{5}$ $\csc\theta = \frac{5}{4}$

$\cos\theta = \frac{3}{5}$ $\sec\theta = \frac{5}{3}$

$\tan\theta = \frac{4}{3}$ $\cot\theta = \frac{3}{4}$

(b) $x = -12, y = -5, r = 13$

$\sin\theta = -\frac{5}{13}$ $\csc\theta = -\frac{13}{5}$

$\cos\theta = -\frac{12}{13}$ $\sec\theta = -\frac{13}{12}$

$\tan\theta = \frac{5}{12}$ $\cot\theta = \frac{12}{5}$

16. (a) $x = 8, y = -15, r = 17$

$\sin\theta = -\frac{15}{17}$ $\quad\csc\theta = -\frac{17}{15}$

$\cos\theta = \frac{8}{17}$ $\quad\sec\theta = \frac{17}{8}$

$\tan\theta = -\frac{15}{8}$ $\quad\cot\theta = -\frac{8}{15}$

(b) $x = 1, y = -1, r = \sqrt{2}$

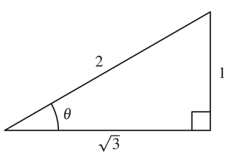
$\sin\theta = -\frac{\sqrt{2}}{2}$ $\quad\csc\theta = -\sqrt{2}$

$\cos\theta = \frac{\sqrt{2}}{2}$ $\quad\sec\theta = \sqrt{2}$

$\tan\theta = -1$ $\quad\cot\theta = -1$

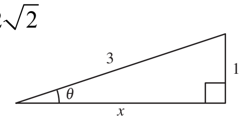
17. $x^2 + 1^2 = 2^2 \Rightarrow x = \sqrt{3}$

$\cos\theta = \frac{x}{2} = \frac{\sqrt{3}}{2}$



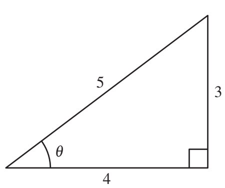
18. $x^2 + 1^2 = 3^2 \Rightarrow x = \sqrt{8} = 2\sqrt{2}$

$\tan\theta = \frac{1}{x} = \frac{1}{2\sqrt{2}} = \frac{\sqrt{2}}{4}$



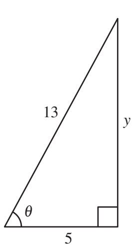
19. $4^2 + y^2 = 5^2 \Rightarrow y = 3$

$\cot\theta = \frac{4}{y} = \frac{4}{3}$



20. $5^2 + y^2 = 13^2 \Rightarrow y = 12$

$\csc\theta = \frac{13}{y} = \frac{13}{12}$



21. (a) $\sin 60° = \frac{\sqrt{3}}{2}$

$\cos 60° = \frac{1}{2}$

$\tan 60° = \sqrt{3}$

(b) $\sin 120° = \sin 60° = \frac{\sqrt{3}}{2}$

$\cos 120° = -\cos 60° = -\frac{1}{2}$

$\tan 120° = -\tan 60° = -\sqrt{3}$

(c) $\sin\frac{\pi}{4} = \frac{\sqrt{2}}{2}$

$\cos\frac{\pi}{4} = \frac{\sqrt{2}}{2}$

$\tan\frac{\pi}{4} = 1$

(d) $\sin\frac{5\pi}{4} - \sin\frac{\pi}{4} = -\frac{\sqrt{2}}{2}$

$\cos\frac{5\pi}{4} = \cos\frac{\pi}{4} = -\frac{\sqrt{2}}{2}$

$\tan\frac{5\pi}{4} = \tan\frac{\pi}{4} = 1$

22. (a) $\sin(-30°) = -\sin 30° = -\frac{1}{2}$

$\cos(-30°) = \cos 30° = \frac{\sqrt{3}}{2}$

$\tan(-30°) = -\tan 30° = -\frac{\sqrt{3}}{3}$

(b) $\sin 150° = \sin 30° = \frac{1}{2}$

$\cos 150° = -\cos 30° = -\frac{\sqrt{3}}{2}$

$\tan 150° = -\tan 30° = -\frac{\sqrt{3}}{3}$

(c) $\sin\left(-\frac{\pi}{6}\right) = -\sin\frac{\pi}{6} = -\frac{1}{2}$

$\cos\left(-\frac{\pi}{6}\right) = \cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$

$\tan\left(-\frac{\pi}{6}\right) = -\tan\frac{\pi}{6} = -\frac{\sqrt{3}}{3}$

(d) $\sin\frac{\pi}{2} = 1$

$\cos\frac{\pi}{2} = 0$

$\tan\frac{\pi}{2}$ is undefined.

23. (a) $\sin 225° = -\sin 45° = -\dfrac{\sqrt{2}}{2}$

$\cos 225° = -\cos 45° = -\dfrac{\sqrt{2}}{2}$

$\tan 225° = \tan 45° = 1$

(b) $\sin(-225°) = \sin 45° = \dfrac{\sqrt{2}}{2}$

$\cos(-225°) = -\cos 45° = -\dfrac{\sqrt{2}}{2}$

$\tan(-225°) = -\tan 45° = -1$

(c) $\sin\dfrac{5\pi}{3} = -\sin\dfrac{\pi}{3} = -\dfrac{\sqrt{3}}{2}$

$\cos\dfrac{5\pi}{3} = \cos\dfrac{\pi}{3} = \dfrac{1}{2}$

$\tan\dfrac{5\pi}{3} = -\tan\dfrac{\pi}{3} = -\sqrt{3}$

(d) $\sin\dfrac{11\pi}{6} = -\sin\dfrac{\pi}{6} = -\dfrac{1}{2}$

$\cos\dfrac{11\pi}{6} = \cos\dfrac{\pi}{6} = \dfrac{\sqrt{3}}{2}$

$\tan\dfrac{11\pi}{6} = -\tan\dfrac{\pi}{6} = -\dfrac{\sqrt{3}}{3}$

24. (a) $\sin 750° = \sin 30° = \dfrac{1}{2}$

$\cos 750° = \cos 30° = \dfrac{\sqrt{3}}{2}$

$\tan 750° = \tan 30° = \dfrac{\sqrt{3}}{3}$

(b) $\sin 510° = \sin 30° = \dfrac{1}{2}$

$\cos 510° = -\cos 30° = -\dfrac{\sqrt{3}}{2}$

$\tan 510° = -\tan 30° = -\dfrac{\sqrt{3}}{3}$

(c) $\sin\dfrac{10\pi}{3} = -\sin\dfrac{\pi}{3} = -\dfrac{\sqrt{3}}{2}$

$\cos\dfrac{10\pi}{3} = -\cos\dfrac{\pi}{3} = -\dfrac{1}{2}$

$\tan\dfrac{10\pi}{3} = \tan\dfrac{\pi}{3} = \sqrt{3}$

(d) $\sin\dfrac{17\pi}{3} = -\sin\dfrac{\pi}{3} = -\dfrac{\sqrt{3}}{2}$

$\cos\dfrac{17\pi}{3} = \cos\dfrac{\pi}{3} = \dfrac{1}{2}$

$\tan\dfrac{17\pi}{3} = -\tan\dfrac{\pi}{3} = \sqrt{3}$

25. (a) $\sin 10° \approx 0.1736$

(b) $\csc 10° \approx 5.759$

26. (a) $\sec 225° \approx -1.414$

(b) $\sec 135° \approx -1.414$

27. (a) $\tan\dfrac{\pi}{9} \approx 0.3640$

(b) $\tan\dfrac{10\pi}{9} \approx 0.3640$

28. (a) $\cot 1.35 \approx 0.2245$

(b) $\tan 1.35 \approx 4.455$

29. (a) $\sin\theta < 0 \Rightarrow \theta$ is in Quadrant III or IV.

$\cos\theta < 0 \Rightarrow \theta$ is in Quadrant II or III.

$\sin\theta < 0$ **and** $\cos\theta < 0 \Rightarrow \theta$ is in Quadrant III.

(b) $\sec\theta > 0 \Rightarrow \theta$ is in Quadrant I or IV.

$\cot\theta < 0 \Rightarrow \theta$ is in Quadrant II or IV.

$\sec\theta > 0$ **and** $\cot\theta < 0 \Rightarrow \theta$ is in Quadrant IV.

30. (a) $\sin\theta > 0 \Rightarrow \theta$ is in Quadrant I or II.

$\cos\theta < 0 \Rightarrow \theta$ is in Quadrant II or III.

$\sin\theta > 0$ **and** $\cos\theta < 0 \Rightarrow \theta$ is in Quadrant II.

(b) $\csc\theta < 0 \Rightarrow \theta$ is in Quadrant III or IV.

$\tan\theta > 0 \Rightarrow \theta$ is in Quadrant I or III.

$\csc\theta < 0$ **and** $\tan\theta > 0 \Rightarrow \theta$ is in Quadrant III.

31. (a) $\cos\theta = \dfrac{\sqrt{2}}{2}$

$\theta = \dfrac{\pi}{4}, \dfrac{7\pi}{4}$

(b) $\cos\theta = -\dfrac{\sqrt{2}}{2}$

$\theta = \dfrac{3\pi}{4}, \dfrac{5\pi}{4}$

32. (a) $\sec\theta = 2$

$\theta = \dfrac{\pi}{3}, \dfrac{5\pi}{3}$

(b) $\sec\theta = -2$

$\theta = \dfrac{2\pi}{3}, \dfrac{4\pi}{3}$

33. (a) $\tan\theta = 1$

$\theta = \dfrac{\pi}{4}, \dfrac{5\pi}{4}$

(b) $\cot\theta = -\sqrt{3}$

$\theta = \dfrac{5\pi}{6}, \dfrac{11\pi}{6}$

34. (a) $\sin\theta = \dfrac{\sqrt{3}}{2}$

$\theta = \dfrac{\pi}{3}, \dfrac{2\pi}{3}$

(b) $\sin\theta = -\dfrac{\sqrt{3}}{2}$

$\theta = \dfrac{4\pi}{3}, \dfrac{5\pi}{3}$

35. $2\sin^2\theta = 1$

$\sin\theta = \pm\dfrac{\sqrt{2}}{2}$

$\theta = \dfrac{\pi}{4}, \dfrac{3\pi}{4}, \dfrac{5\pi}{4}, \dfrac{7\pi}{4}$

36. $\tan^2\theta = 3$

$\tan\theta = \pm\sqrt{3}$

$\theta = \dfrac{\pi}{3}, \dfrac{2\pi}{3}, \dfrac{4\pi}{3}, \dfrac{5\pi}{3}$

37. $\tan^2\theta - \tan\theta = 0$

$\tan\theta(\tan\theta - 1) = 0$

$\tan\theta = 0 \qquad \tan\theta = 1$

$\theta = 0, \pi, 2\pi \qquad \theta = \dfrac{\pi}{4}, \dfrac{5\pi}{4}$

38. $2\cos^2\theta - \cos\theta - 1 = 0$

$(2\cos\theta + 1)(\cos\theta - 1) = 0$

$\cos\theta = -\dfrac{1}{2} \qquad \cos\theta = 1$

$\theta = \dfrac{2\pi}{3}, \dfrac{4\pi}{3} \qquad \theta = 0, 2\pi$

39. $\sec\theta\csc\theta - 2\csc\theta = 0$

$\csc\theta(\sec\theta - 2) = 0$

$(\csc\theta \neq 0$ for any value of $\theta)$

$\sec\theta = 2$

$\theta = \dfrac{\pi}{3}, \dfrac{5\pi}{3}$

40. $\sin\theta = \cos\theta$

$\tan\theta = 1$

$\theta = \dfrac{\pi}{4}, \dfrac{5\pi}{4}$

41. $\cos^2\theta + \sin\theta = 1$

$1 - \sin^2\theta + \sin\theta = 1$

$\sin^2\theta - \sin\theta = 0$

$\sin\theta(\sin\theta - 1) = 0$

$\sin\theta = 0 \qquad \sin\theta = 1$

$\theta = 0, \pi, 2\pi \qquad \theta = \dfrac{\pi}{2}$

42. $\cos\left(\dfrac{\theta}{2}\right) - \cos\theta = 1$

$\cos\left(\dfrac{\theta}{2}\right) = \cos\theta + 1$

$\sqrt{\left(\dfrac{1}{2}\right)(1 + \cos\theta)} = \cos\theta + 1$

$\left(\dfrac{1}{2}\right)(1 + \cos\theta) = \cos^2\theta + 2\cos\theta + 1$

$0 = \cos^2\theta + \left(\dfrac{3}{2}\right)\cos\theta + \left(\dfrac{1}{2}\right)$

$0 = \left(\dfrac{1}{2}\right)(2\cos^2\theta + 3\cos\theta + 1)$

$0 = \left(\dfrac{1}{2}\right)(2\cos\theta + 1)(\cos\theta + 1)$

$\cos\theta = -\dfrac{1}{2} \qquad \cos\theta = -1$

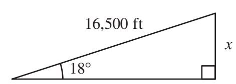
$\theta = \dfrac{2\pi}{3} \qquad \theta = \pi$

$(0 = 4\pi/3$ is extraneous$)$

43. $(275 \text{ ft/sec})(60 \text{ sec}) = 16{,}500$ feet

$\sin 18^\circ = \dfrac{a}{16{,}500}$

$a = 16{,}500 \sin 18^\circ \approx 5099$ feet



44. $\tan 3.5^\circ = \dfrac{h}{13 + x}$ and $\tan 9^\circ = \dfrac{h}{x}$

$(13 + x)\tan 3.5^\circ = h \qquad x\tan 9^\circ = h$

$13\tan 3.5^\circ + x\tan 3.5^\circ = x\tan 9^\circ$

$13\tan 3.5^\circ = x(\tan 9^\circ - \tan 3.5^\circ)$

$\dfrac{13\tan 3.5^\circ}{\tan 9^\circ - \tan 3.5^\circ} = x$

$h = x\tan 9^\circ = \dfrac{13\tan 3.5^\circ\tan 9^\circ}{\tan 9^\circ - \tan 3.5^\circ}$

≈ 1.295 mi or 6839.307 ft

45. $y = 2 \sin 2x$

Period $= \dfrac{2\pi}{2} = \pi$

Amplitude $= |2| = 2$

46. $y = \dfrac{3}{2} \cos \dfrac{x}{2}$

Period $= \dfrac{2\pi}{(1/2)} = 4\pi$

Amplitude $= \left|\dfrac{3}{2}\right| = \dfrac{3}{2}$

47. $y = -3 \sin 4\pi x$

Period $= \dfrac{2\pi}{4\pi} = \dfrac{1}{2}$

Amplitude $= |-3| = 3$

48. $y = \dfrac{2}{3} \cos \dfrac{\pi x}{10}$

Period $= \dfrac{2\pi}{(\pi/10)} = 20$

Amplitude $= \left|\dfrac{2}{3}\right| = \dfrac{2}{3}$

49. $y = 5 \tan 2x$

Period $= \dfrac{\pi}{2}$

50. $y = 7 \tan 2\pi x$

Period $= \dfrac{\pi}{2\pi} = \dfrac{1}{2}$

51. $y = \sec 5x$

Period $= \dfrac{2\pi}{5}$

52. $y = \csc 4x$

Period $= \dfrac{2\pi}{4} = \dfrac{\pi}{2}$

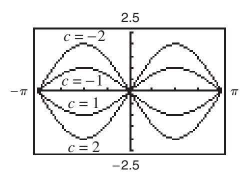
53. (a) $f(x) = c \sin x$; changing c changes the amplitude.

When $c = -2$: $f(x) = -2 \sin x$.

When $c = -1$: $f(x) = -\sin x$.

When $c = 1$: $f(x) = \sin x$.

When $c = 2$: $f(x) = 2 \sin x$.



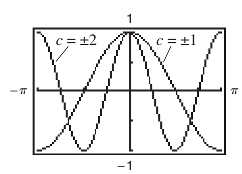
(b) $f(x) = \cos(cx)$; changing c changes the period.

When $c = -2$: $f(x) = \cos(-2x) = \cos 2x$.

When $c = -1$: $f(x) = \cos(-x) = \cos x$.

When $c = 1$: $f(x) = \cos x$.

When $c = 2$: $f(x) = \cos 2x$.



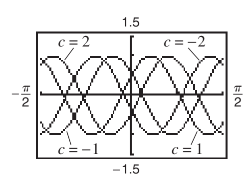
(c) $f(x) = \cos(\pi x - c)$; changing c causes a horizontal shift.

When $c = -2$: $f(x) = \cos(\pi x + 2)$.

When $c = -1$: $f(x) = \cos(\pi x + 1)$.

When $c = 1$: $f(x) = \cos(\pi x - 1)$.

When $c = 2$: $f(x) = \cos(\pi x - 2)$.



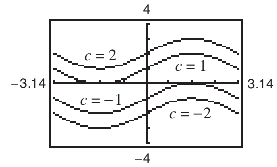
54. (a) $f(x) = \sin x + c$; changing c causes a vertical shift.

When $c = -2$: $f(x) = \sin x - 2$.

When $c = -1$: $f(x) = \sin x - 1$.

When $c = 1$: $f(x) = \sin x + 1$.

When $c = 2$: $f(x) = \sin x + 2$.



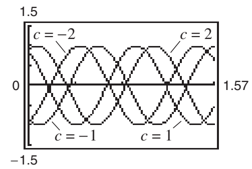
(b) $f(x) = -\sin(2\pi x - c)$; changing c causes a horizontal shift.

When $c = -2$: $f(x) = -\sin(2\pi x + 2)$.

When $c = -1$: $f(x) = \sin(2\pi x + 1)$.

When $c = 1$: $f(x) = \sin(2\pi x - 1)$.

When $c = 2$: $f(x) = -\sin(2\pi x - 2)$.



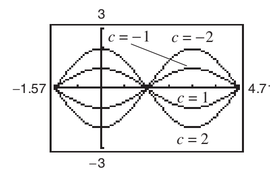
(c) $f(x) = c\cos x$; changing c changes the amplitude.

When $c = -2$: $f(x) = -2\cos x$.

When $c = -1$: $f(x) = -\cos x$.

When $c = 1$: $f(x) = \cos x$.

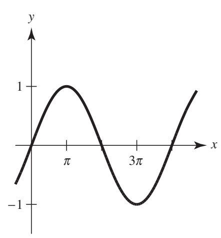
When $c = 2$: $f(x) = 2\cos x$.



55. $y = \sin \dfrac{x}{2}$

Period: 4π

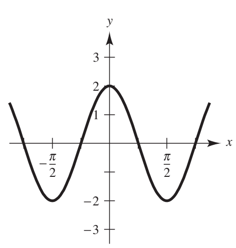
Amplitude: 1



56. $y = 2\cos 2x$

Period: π

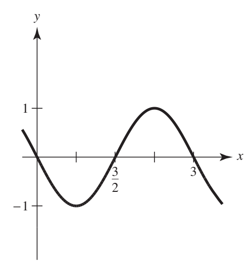
Amplitude: 2



57. $y = -\sin \dfrac{2\pi x}{3}$

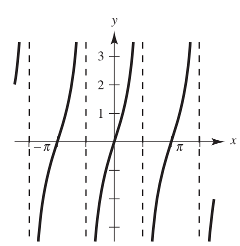
Period: 3

Amplitude: 1



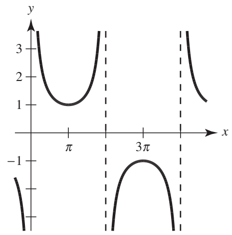
58. $y = 2\tan x$

Period: π



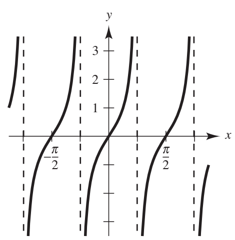
59. $y = \csc\frac{x}{2}$

Period: 4π



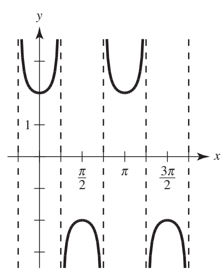
60. $y = \tan 2x$

Period: $\frac{\pi}{2}$



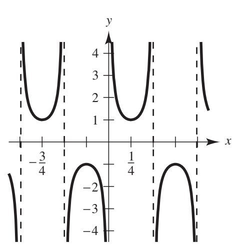
61. $y = 2 \sec 2x$

Period: π



62. $y = \csc 2\pi x$

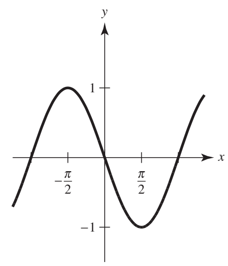
Period: 1



63. $y = \sin(x + \pi)$

Period: 2π

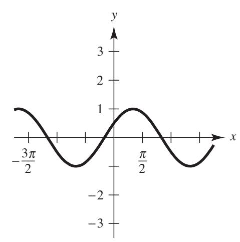
Amplitude: 1



64. $y = \cos\left(x - \frac{\pi}{3}\right)$

Period: 2π

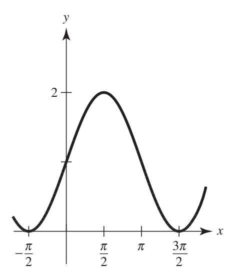
Amplitude: 1



65. $y = 1 + \cos\left(x - \frac{\pi}{2}\right)$

Period: 2π

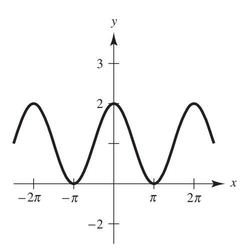
Amplitude: 1



66. $y = 1 + \sin\left(x + \frac{\pi}{2}\right)$

Period: 2π

Amplitude: 1



67. $y = a\cos(bx - c)$

From the graph, we see that the amplitude is 3, the period is 4π, and the horizontal shift is π. Thus,

$$a = 3$$

$$\frac{2\pi}{b} = 4\pi \Rightarrow b = \frac{1}{2}$$

$$\frac{c}{d} = \pi \Rightarrow c = \frac{\pi}{2}.$$

Therefore, $y = 3\cos\left[(1/2)x - (\pi/2)\right]$.

68. $y = a\sin(bx - c)$

From the graph, we see that the amplitude is $\frac{1}{2}$, the period is π, and the horizontal shift is 0. Also, the graph is reflected about the x-axis. Thus,

$$a = -\frac{1}{2}$$

$$\frac{2\pi}{b} = \pi \Rightarrow b = 2$$

$$\frac{c}{b} = 0 \Rightarrow c = 0.$$

Therefore, $y = -\frac{1}{2}\sin 2x$.

69. Yes. Use the right-triangle definitions of the trigonometric functions.

70. The sine function is one-to-one on the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$. Other intervals are possible.

71. The range of the cosine function is $[-1, 1]$. The range of the secant function is $(-\infty, -1] \cup [1, \infty)$.

72. As θ increases from $0°$ to $90°$ with $r = 12$ centimeters,

x decreases from 12 to 0 centimeters,

y increases from 0 to 12 centimeters,

$\sin\theta$ increases from 0 to 1,

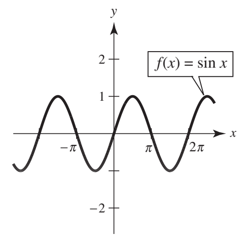
$\cos\theta$ decreases from 1 to 0, and

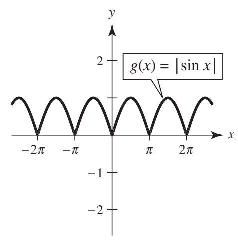
$\tan\theta$ increases from 0 to (positive) infinity.

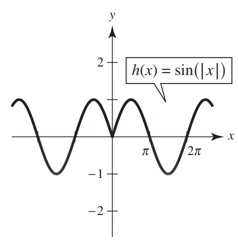
73. $f(x) = \sin x$

$g(x) = |\sin x|$

$h(x) = \sin|x|$





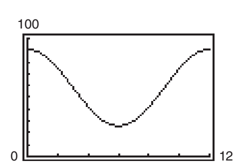


The graph of $|f(x)|$ will reflect any parts of the graph of $f(x)$ below the x-axis about the y-axis.

The graph of $f(|x|)$ will reflect the part of the graph of $f(x)$ to the right of the y-axis about the y-axis.

74. If $h = 51 + 50\sin\left(8\pi t - \frac{\pi}{2}\right)$, then $h = 1$ when $t = 0$.

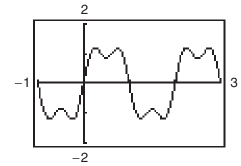
75. $S = 58.3 + 32.5\cos\frac{\pi t}{6}$



Sales exceed 75,000 during the months of January, November, and December.

76. $f(x) = \frac{4}{\pi}\left(\sin \pi x + \frac{1}{3}\sin 3\pi x\right)$

$g(x) = \frac{4}{\pi}\left(\sin \pi x + \frac{1}{3}\sin 3\pi x + \frac{1}{5}\sin 5\pi x\right)$



Pattern: $f(x) = \frac{4}{\pi}\left(\sin \pi x + \frac{1}{3}\sin 3\pi x + \frac{1}{5}\sin 5\pi x + \cdots + \frac{1}{2n-1}\sin(2n-1)\pi x\right), \ n = 1, 2, 3\ldots$

77. False. 4π radians (not 4 radians) corresponds to two complete revolutions from the initial side to the terminal side of an angle.

78. True

79. False. The amplitude of the function $y = \frac{1}{2}\sin 2x$ is one-half the amplitude of the function $y = \sin x$.

80. True

Review Exercises for Chapter P

1. $y = 5x - 8$

$x = 0: y = 5(0) - 8 = -8 \Rightarrow (0, -8)$, y-intercept

$y = 0: 0 = 5x - 8 \Rightarrow x = \frac{8}{5} \Rightarrow \left(\frac{8}{5}, 0\right)$, x-intercept

2. $y = x^2 - 8x + 12$

$x = 0: y = (0)^2 - 8(0) + 12 = 12 \Rightarrow (0, 12)$, y-intercept

$y = 0: x^2 - 8x + 12 = (x - 6)(x - 2) = 0 \Rightarrow x = 2, 6 \Rightarrow (2, 0), (6, 0)$, x-intercepts

3. $y = \frac{x - 3}{x - 4}$

$x = 0: y = \frac{0 - 3}{0 - 4} = \frac{3}{4} \Rightarrow \left(0, \frac{3}{4}\right)$, y-intercept

$y = 0: 0 = \frac{x - 3}{x - 4} \Rightarrow x = 3 \Rightarrow (3, 0)$, x-intercept

4. $y = (x - 3)\sqrt{x + 4}$

$x = 0: y = (0 - 3)\sqrt{0 + 4} = -3\sqrt{4} = -3(2) = -6 \Rightarrow (0, -6)$, y-intercept

$y = 0: (x - 3)\sqrt{x + 4} = 0 \Rightarrow x = 3, -4 \Rightarrow (3, 0), (-4, 0)$, x-intercepts

5. $y = x^2 + 4x$ does not have symmetry with respect to either axis or the origin.

6. Symmetric with respect to y-axis because

$y = (-x)^4 - (-x)^2 + 3$

$y = x^4 - x^2 + 3.$

7. Symmetric with respect to both axes and the origin because:

$y^2 = (-x^2) - 5$	$(-y)^2 = x^2 - 5$	$(-y)^2 = (-x)^2 - 5$
$y^2 = x^2 - 5$	$y^2 = x^2 - 5$	$y^2 = x^2 - 5$

8. Symmetric with respect to the origin because:

$(-x)(-y) = -2$

$xy = -2.$

9. $y = -\frac{1}{2}x + 3$

y-intercept: $y = -\frac{1}{2}(0) + 3 = 3$

$(0, 3)$

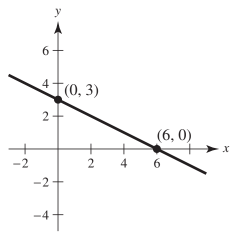
x-intercept: $-\frac{1}{2}x + 3 = 0$

$-\frac{1}{2}x = -3$

$x = 6$

$(6, 0)$

Symmetry: none



10. $y = -x^2 + 4$

y-intercept: $y = -(0)^2 + 4 = 4$

$(0, 4)$

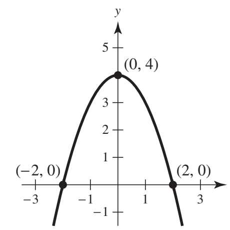
x-intercepts: $-x^2 + 4 = 0$

$(2 - x)(2 + x) = 0$

$x = \pm 2$

$(2, 0), (-2, 0)$

Symmetric with respect to the y-axis because $-(-x)^2 + 4 = -x^2 + 4$.



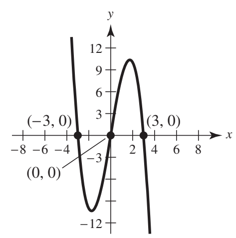
11. $y = 9x - x^3$

$9x - x^2 = x(9 - x^2) = x(3 - x)(3 + x) = 0 \Rightarrow x = 0, 3, -3$

Intercepts: $(0, 0), (3, 0), (-3, 0)$

Symmetric with respect to the origin because

$f(-x) = 9(-x) - (-x)^3 = -9x + x^3 = -(9x - x^3) = -f(x)$.



12. $y^2 = 9 - x$

$y^2 + x - 9 = 0$

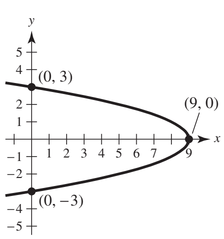
y-intercept: $y^2 = 9 - 0 = 9 \Rightarrow y = \pm 3$

$(0, 3), (0, -3)$

x-intercept: $0^2 = 9 - x \Rightarrow x = 9$

$(9, 0)$

Symmetric with respect to the x-axis because $(-y)^2 + x - 9 = y^2 + x - 9 = 0$.



13. $y = 2\sqrt{4 - x}$

y-intercept: $y = 2\sqrt{4 - 0} = 2\sqrt{4} = 4$

$(0, 4)$

x-intercept: $2\sqrt{4 - x} = 0$

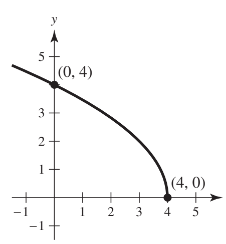
$\sqrt{4 - x} = 0$

$4 - x = 0$

$x = 4$

$(4, 0)$

Symmetry: none



14. $y = |x - 4| - 4$

y-intercept: $y = |0 - 4| - 4 = |-4| - 4 = 4 - 4 = 0$

$(0, 0)$

x-intercepts: $|x - 4| - 4 = 0$

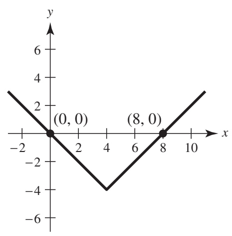
$|x - 4| = 4$

$x - 4 = 4$ or $x - 4 = -4$

$x = 8 \qquad x = 0$

$(0, 0), (8, 0)$

Symmetry: none



15. $5x + 3y = -1 \Rightarrow y = \frac{1}{3}(-5x - 1)$

$x - y = -5 \Rightarrow y = x + 5$

$\frac{1}{3}(-5x - 1) = x + 5$

$-5x - 1 = 3x + 15$

$-16 = 8x$

$-2 = x$

For $x = -2$, $y = x + 5 = -2 + 5 = 3$.

Point of intersection is: $(-2, 3)$

16. $2x + 4y = 9 \Rightarrow y = \dfrac{-2x + 9}{4}$

$6x - 4y = 7 \Rightarrow y = \dfrac{6x - 7}{4}$

$\dfrac{-2x + 9}{4} = \dfrac{6x - 7}{4}$

$-2x + 9 = 6x - 7$

$-8x = -16$

$x = 2$

For $x = 2$, $y = \dfrac{6(2) - 7}{4} = \dfrac{5}{4}$

Point of intersection: $\left(2, \dfrac{5}{4}\right)$

17. $x - y = -5 \Rightarrow y = x + 5$

$x^2 - y = 1 \Rightarrow y = x^2 - 1$

$x + 5 = x^2 - 1$

$0 = x^2 - x - 6$

$0 = (x - 3)(x + 2)$

$x = 3$ or $x = -2$

For $x = 3$, $y = 3 + 5 = 8$.

For $x = -2$, $y = -2 + 5 = 3$.

Points of intersection: $(3, 8)$, $(-2, 3)$

18. $x^2 + y^2 = 1 \Rightarrow y^2 = 1 - x^2$

$-x + y = 1 \Rightarrow y = x + 1$

$1 - x^2 = (x + 1)^2$

$1 - x^2 = x^2 + 2x + 1$

$0 = 2x^2 + 2x$

$0 = 2x(x + 1)$

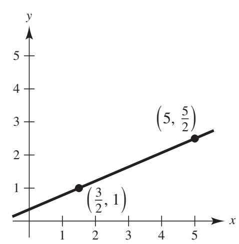
$x = 0$ or $x = -1$

For $x = 0$, $y = 0 + 1 = 1$.

For $x = -1$, $y = -1 + 1 = 0$.

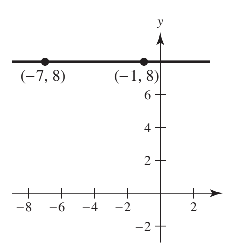
Points of intersection: $(0, 1)$, $(-1, 0)$

19.



$$\text{Slope} = \frac{\left(\frac{5}{2}\right) - 1}{5 - \left(\frac{3}{2}\right)} = \frac{\frac{3}{2}}{\frac{7}{2}} = \frac{3}{7}$$

20. The line is horizontal and has slope 0.

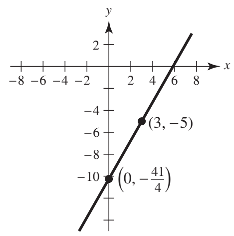


21. $y - (-5) = \frac{7}{4}(x - 3)$

$y + 5 = \frac{7}{4}x - \frac{21}{4}$

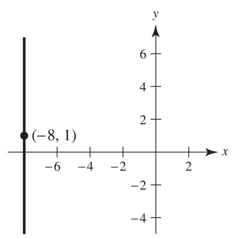
$4y + 20 = 7x - 21$

$0 = 7x - 4y - 41$



22. Because m is undefined the line is vertical.

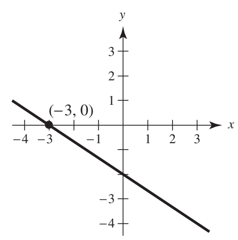
$x = -8$ or $x + 8 = 0$



23. $y - 0 = -\frac{2}{3}(x - (-3))$

$y = -\frac{2}{3}x - 2$

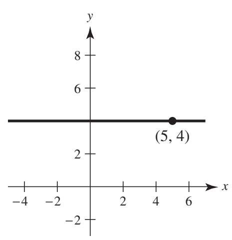
$2x + 3y + 6 = 0$



24. Because $m = 0$, the line is horizontal.

$y - 4 = 0(x - 5)$

$y = 4$ or $y - 4 = 0$



25. $y - 3x = 5$

$y = 3x + 5$

Slope: $m = 3$

y-intercept: $(0, 5)$

26. $9 - y = x$

$y = -x + 9$

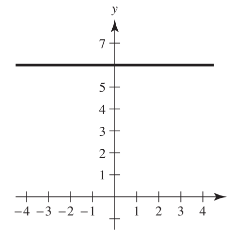
Slope: $m = -1$

y-intercept: $(0, 9)$

27. $y = 6$

Slope: 0

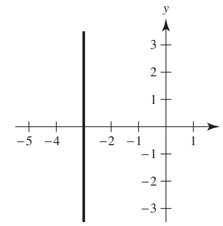
y-intercept: $(0, 6)$



28. $x = -3$

Slope: undefined

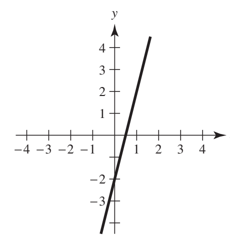
Line is vertical.



29. $y = 4x - 2$

Slope: 4

y-intercept: $(0, -2)$



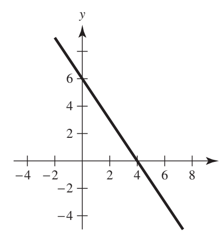
30. $3x + 2y = 12$

$2y = -3x + 12$

$y = \frac{-3}{2}x + 6$

Slope : $-\frac{3}{2}$

y-intercept: $(0, 6)$

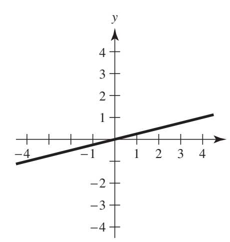


31. $m = \frac{2 - 0}{8 - 0} = \frac{1}{4}$

$y - 0 = \frac{1}{4}(x - 0)$

$y = \frac{1}{4}x$

$4y - x = 0$

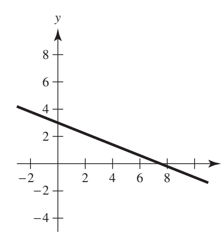


32. $m = \frac{-1 - 5}{10 - (-5)} = \frac{-6}{15} = -\frac{2}{5}$

$y - 5 = \frac{-2}{5}(x - (-5))$

$5y - 25 = -2x - 10$

$5y + 2x - 15 = 0$



33. (a)
$$y - 5 = \frac{7}{16}(x + 3)$$
$$16y - 80 = 7x + 21$$
$$0 = 7x - 16y + 101$$

(b) $5x - 3y = 3$ has slope $\frac{5}{3}$.
$$y - 5 = \frac{5}{3}(x + 3)$$
$$3y - 15 = 5x + 15$$
$$0 = 5x - 3y + 30$$

(c) $3x + 4y = 8$
$$4y = -3x + 8$$
$$y = \frac{-3}{4}x + 2$$

Perpendicular line has slope $\frac{4}{3}$.
$$y - 5 = \frac{4}{3}(x - (-3))$$
$$3y - 15 = 4x + 12$$
$$4x - 3y + 27 = 0 \quad \text{or} \quad y = \frac{4}{3}x + 9$$

(d) Slope is undefined so the line is vertical.
$$x = -3$$
$$x + 3 = 0$$

34. (a)
$$y - 4 = -\frac{2}{3}(x - 2)$$
$$3y - 12 = -2x + 4$$
$$2x + 3y - 16 = 0$$

(b) $x + y = 0$ has slope -1. Slope of the perpendicular line is 1.
$$y - 4 = 1(x - 2)$$
$$y = x + 2$$
$$0 = x - y + 2$$

(c) The slope of the line $3x - y = 0$ is 3. The parallel line through $(2, 4)$ is
$$y - 4 = 3(x - 2)$$
$$y - 4 = 3x - 6$$
$$y - 3x + 2 = 0.$$

(d) Because the line is horizontal the slope is 0.
$$y = 4$$
$$y - 4 = 0$$

35. The slope is -850.
$$V = -850t + 12{,}500.$$
$$V(3) = -850(3) + 12{,}500 = \$9950$$

36. (a) $C = 9.25t + 13.50t + 36{,}500 = 22.75t + 36{,}500$

(b) $R = 30t$

(c)
$$30t = 22.75t + 36{,}500$$
$$7.25t = 36{,}500$$
$$t \approx 5034.48 \text{ hours to break even}$$

37. $f(x) = 5x + 4$

(a) $f(0) = 5(0) + 4 = 4$

(b) $f(5) = 5(5) + 4 = 29$

(c) $f(-3) = 5(-3) + 4 = -11$

(d) $f(t + 1) = 5(t + 1) + 4 = 5t + 9$

38. $f(x) = x^3 - 2x$

(a) $f(-3) = (-3)^3 - 2(-3) = -27 + 6 = -21$

(b) $f(2) = 2^3 - 2(2) = 8 - 4 = 4$

(c) $f(-1) = (-1)^3 - 2(-1) = -1 + 2 = 1$

(d)
$$f(c - 1) = (c - 1)^3 - 2(c - 1)$$
$$= c^3 - 3c^2 + 3c - 1 - 2c + 2$$
$$= c^3 - 3c^2 + c + 1$$

39. $f(x) = 4x^2$
$$\frac{f(x + \Delta x) - f(x)}{\Delta x} = \frac{4(x + \Delta x)^2 - 4x^2}{\Delta x}$$
$$= \frac{4\left(x^2 + 2x\Delta x + (\Delta x)^2\right) - 4x^2}{\Delta x}$$
$$= \frac{4x^2 + 8x\Delta x + 4(\Delta x)^2 - 4x^2}{\Delta x}$$
$$= \frac{8x\Delta x + 4(\Delta x)^2}{\Delta x}$$
$$= 8x + 4\Delta x, \quad \Delta x \neq 0$$

40. $f(x) = 2x - 6$
$$f(1) = 2(1) - 6 = -4$$
$$\frac{f(x) - f(1)}{x - 1} = \frac{(2x - 6) - (-4)}{x - 1}$$
$$= \frac{2x - 6 + 4}{x - 1}$$
$$= \frac{2x - 2}{x - 1}$$
$$= \frac{2(x - 1)}{x - 1}$$
$$= 2, \quad x \neq 1$$

41. $f(x) = x^2 + 3$

Domain: $(-\infty, \infty)$

Range: $[3, \infty)$

42. $g(x) = \sqrt{6 - x}$

Domain: $6 - x \geq 0$

$6 \geq x$

$(-\infty, 6]$

Range: $[0, \infty)$

43. $f(x) = -|x + 1|$

Domain: $(-\infty, \infty)$

Range: $(-\infty, 0]$

44. $h(x) = \dfrac{2}{x + 1}$

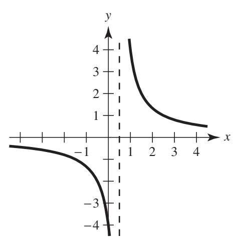
Domain: all $x \neq -1$; $(-\infty, -1) \cup (-1, \infty)$

Range: all $y \neq 0$; $(-\infty, 0) \cup (0, \infty)$

45. $f(x) = \dfrac{4}{2x - 1}$

Domain: $\left(-\infty, \frac{1}{2}\right) \cup \left(\frac{1}{2}, \infty\right)$

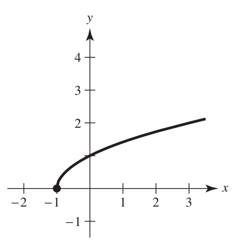
Range: $(-\infty, 0) \cup (0, \infty)$



46. $g(x) = \sqrt{x + 1}$

Domain: $[-1, \infty)$

Range: $[0, \infty)$



47. $x + y^2 = 2 \Rightarrow y = \pm\sqrt{2 - x}$

y is not a function of x.

Some vertical lines intersect the graph more than once.

48. $x^2 - y = 0 \Rightarrow y = x^2$

y is a function of x.

A vertical line intersects the graph exactly once.

49. $xy + x^3 - 2y = 0$

$(x - 2)y = -x^3$

$y = \dfrac{-x^3}{x - 2}$

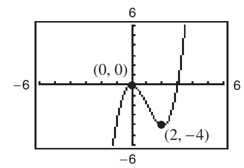
y is a function of x.

50. $x = 9 - y^2 \Rightarrow y = \pm\sqrt{9 - x}$

y is not a function of x.

Some y-values correspond to the same x-value.

51. $f(x) = x^3 - 3x^2$



(a) The graph of g is obtained from f by a vertical shift down 1 unit, followed by a reflection in the x-axis:

$g(x) = -[f(x) - 1] = -x^3 + 3x^2 + 1$

(b) The graph of g is obtained from f by a vertical shift upwards of 1 and a horizontal shift of 2 to the right.

$g(x) = f(x - 2) + 1 = (x - 2)^3 - 3(x - 2)^2 + 1$

52. (a) 3 (cubic), negative leading coefficient

(b) 4 (quartic), positive leading coefficient

(c) 2 (quadratic), negative leading coefficient

(d) 5, positive leading coefficient

53. $f(x) = 3x + 1$, $g(x) = -x$

$(f \circ g)(x) = f(g(x)) = f(-x) = -3x + 1$

Domain: $(-\infty, \infty)$

$$\begin{aligned}(g \circ f)(x) &= g(f(x)) \\ &= g(3x + 1) \\ &= -(3x + 1) \\ &= -3x - 1\end{aligned}$$

Domain: $(-\infty, \infty)$

$f \circ g \neq g \circ f$

54. $f(x) = \sqrt{x - 2}$, $g(x) = x^2$

$(f \circ g)(x) = f(g(x)) = f(x^2) = \sqrt{x^2 - 2}$

Domain: $\left(-\infty, -\sqrt{2}\right] \cup \left[\sqrt{2}, \infty\right)$

$$\begin{aligned}(g \circ f)(x) &= g(f(x)) \\ &= g\left(\sqrt{x - 2}\right) \\ &= \left(\sqrt{x - 2}\right)^2 \\ &= x - 2\end{aligned}$$

$f \circ g \neq g \circ f$

55. $f(x) = x^4 - x^2$

$f(-x) = (-x)^4 - (-x)^2 = x^4 - x^2 = f(x)$

f is even.

$$f(x) = x^4 - x^2 = 0$$
$$x^2(x^2 - 1) = 0$$
$$x^2(x + 1)(x - 1) = 0$$

Zeros: $x = 0, -1, 1$

56. $f(x) = \sqrt{x^3 + 1}$

$f(-x) = \sqrt{(-x)^3 + 1} = \sqrt{-x^3 + 1}$

f is neither even nor odd.

$$f(x) = \sqrt{x^3 + 1} = 0$$
$$x^3 + 1 = 0$$
$$x^3 = -1$$

Zero: $x = -1$

57. $340°\left(\dfrac{\pi}{180°}\right) = \dfrac{17\pi}{9} \approx 5.934$

58. $300°\left(\dfrac{\pi}{180°}\right) = \dfrac{5\pi}{3} \approx 5.236$

59. $-480°\left(\dfrac{\pi}{180°}\right) = -\dfrac{8\pi}{3} \approx -8.378$

60. $-900°\left(\dfrac{\pi}{180°}\right) = -5\pi \approx -15.708$

61. $\dfrac{\pi}{6}\left(\dfrac{180°}{\pi}\right) = 30°$

62. $\dfrac{11\pi}{4}\left(\dfrac{180°}{\pi}\right) = 495°$

63. $-\dfrac{2\pi}{3}\left(\dfrac{180°}{\pi}\right) = -120°$

64. $-\dfrac{13\pi}{6}\left(\dfrac{180°}{\pi}\right) = -390°$

65. $\sin(-45°) = -\sin 45° = -\dfrac{\sqrt{2}}{2}$

$\cos(-45°) = \cos 45° = \dfrac{\sqrt{2}}{2}$

$\tan(-45°) = -\tan 45° = -1$

66. $\sin 240° = -\sin 60° = -\dfrac{\sqrt{3}}{2}$

$\cos 240° = -\cos 60° = -\dfrac{1}{2}$

$\tan 240° = \tan 60° = \sqrt{3}$

67. $\sin \dfrac{13\pi}{6} = \sin \dfrac{\pi}{6} = \dfrac{1}{2}$

$\cos \dfrac{13\pi}{6} = \cos \dfrac{\pi}{6} = \dfrac{\sqrt{3}}{2}$

$\tan \dfrac{13\pi}{6} = \tan \dfrac{\pi}{6} = \dfrac{\sqrt{3}}{3}$

68. $\sin\left(-\dfrac{4\pi}{3}\right) = \sin \dfrac{\pi}{3} = \dfrac{\sqrt{3}}{2}$

$\cos\left(-\dfrac{4\pi}{3}\right) = -\cos \dfrac{\pi}{3} = -\dfrac{1}{2}$

$\tan\left(-\dfrac{4\pi}{3}\right) = -\tan \dfrac{\pi}{3} = -\sqrt{3}$

69. $\sin 405° = \sin 45° = \dfrac{\sqrt{2}}{2}$

$\cos 405° = \cos 45° = \dfrac{\sqrt{2}}{2}$

$\tan 405° = \tan 45° = 1$

70. $\sin 180° = 0$

$\cos 180° = -1$

$\tan 180° = 0$

71. $\tan 33° \approx 0.6494$

72. $\cot 401° = \dfrac{1}{\tan 401°} \approx 1.1504$

73. $\sec \dfrac{12\pi}{5} = \dfrac{1}{\cos(12\pi/5)} \approx 3.2361$

74. $\csc \dfrac{2\pi}{9} = \dfrac{1}{\sin(2\pi/9)} \approx 1.5557$

75. $\sin\left(-\dfrac{\pi}{9}\right) \approx -0.3420$

76. $\cos\left(-\dfrac{3\pi}{7}\right) \approx 0.2225$

77. $2\cos\theta + 1 = 0$

$$\cos\theta = -\frac{1}{2}$$

$$\theta = \frac{2\pi}{3}, \frac{4\pi}{3}$$

78. $2\cos^2\theta = 1$

$$\cos^2\theta = \frac{1}{2}$$

$$\cos\theta = \pm\frac{\sqrt{2}}{2}$$

$$\theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$$

79. $2\sin^2\theta + 3\sin\theta + 1 = 0$

$$(2\sin\theta - 1)(\sin\theta + 1) = 0$$

$$\sin\theta = -\frac{1}{2} \text{ or } \sin\theta = -1$$

$$\theta = \frac{7\pi}{6}, \frac{11\pi}{6} \text{ or } \theta = \frac{3\pi}{2}$$

80. $\cos^3\theta = \cos\theta$

$$\cos\theta\left(\cos^2\theta - 1\right) = 0$$

$$\cos\theta\left(-\sin^2\theta\right) = 0$$

$$\cos\theta = 0 \text{ or } \sin\theta = 0$$

$$\theta = \frac{\pi}{2}, \frac{3\pi}{2} \text{ or}$$

$$\theta = 0, \pi, 2\pi$$

81. $\sec^2\theta - \sec\theta - 2 = 0$

$$(\sec\theta - 2)(\sec\theta + 1) = 0$$

$$\sec\theta = 2 \text{ or } \sec\theta = -1$$

$$\cos\theta = \frac{1}{2} \text{ or } \cos\theta = -1$$

$$\theta = \frac{\pi}{3}, \frac{5\pi}{3} \text{ or } \theta = \pi$$

82. $2\sec^2\theta + \tan^2\theta - 5 = 0$

$$2\sec^2\theta + \left(\sec^2\theta - 1\right) - 5 = 0$$

$$3\sec^2\theta = 6$$

$$\sec^2\theta = 2$$

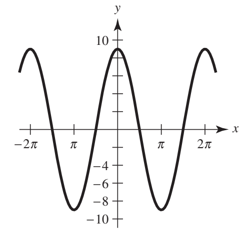
$$\sec\theta = \pm\sqrt{2}$$

$$\theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$$

83. $y = 9\cos x$

Period: 2π

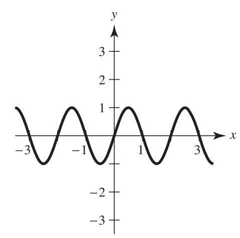
Amplitude: 9



84. $y = \sin \pi x$

Period: 2

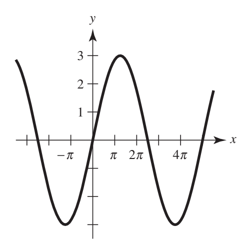
Amplitude: 1



85. $y = 3\sin\dfrac{2x}{5}$

Period: 5π

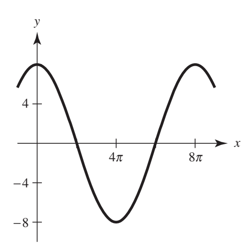
Amplitude: 3



86. $y = 8\cos\dfrac{x}{4}$

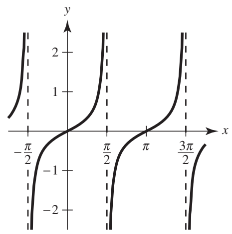
Period: 8π

Amplitude: 8



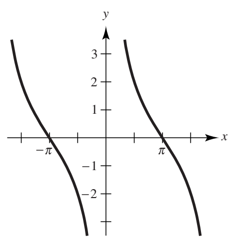
87. $y = \frac{1}{3}\tan x$

Period: π



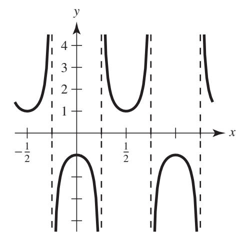
88. $y = \cot \dfrac{x}{2}$

Period: 2π



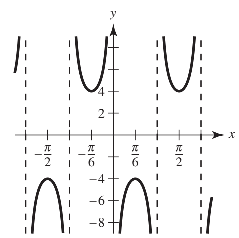
89. $y = -\sec 2\pi x$

Period: 1



90. $y = -4 \csc 3x$

Period: $\dfrac{2\pi}{3}$



Problem Solving for Chapter P

1. (a)

$$x^2 - 6x + y^2 - 8y = 0$$
$$(x^2 - 6x + 9) + (y^2 - 8y + 16) = 9 + 16$$
$$(x - 3)^2 + (y - 4)^2 = 25$$

Center: (3, 4); Radius: 5

(b) Slope of line from (0, 0) to (3, 4) is $\dfrac{4}{3}$.

Slope of tangent line is $-\dfrac{3}{4}$. So, $y - 0 = -\dfrac{3}{4}(x - 0) \Rightarrow y = -\dfrac{3}{4}x$, Tangent line

(c) Slope of line from (6, 0) to (3, 4) is $\dfrac{4 - 0}{3 - 6} = -\dfrac{4}{3}$.

Slope of tangent line is $\dfrac{3}{4}$. So, $y - 0 = \dfrac{3}{4}(x - 6) \Rightarrow y = \dfrac{3}{4}x - \dfrac{9}{2}$, Tangent line

(d)

$$-\frac{3}{4}x = \frac{3}{4}x - \frac{9}{2}$$
$$\frac{3}{2}x = \frac{9}{2}$$
$$x = 3$$

Intersection: $\left(3, -\dfrac{9}{4}\right)$

2. Let $y = mx + 1$ be a tangent line to the circle from the point (0, 1). Because the center of the circle is at $(0, -1)$ and the radius is 1 you have the following.

$$x^2 + (y + 1)^2 = 1$$

$$x^2 + (mx + 1 + 1)^2 = 1$$

$$(m^2 + 1)x^2 + 4mx + 3 = 0$$

Setting the discriminant $b^2 - 4ac$ equal to zero,

$$16m^2 - 4(m^2 + 1)(3) = 0$$

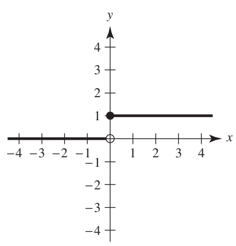
$$16m^2 - 12m^2 = 12$$

$$4m^2 = 12$$

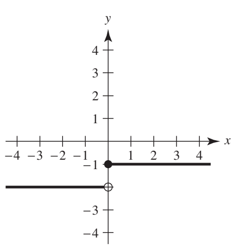
$$m = \pm\sqrt{3}$$

Tangent lines: $y = \sqrt{3}x + 1$ and $y = -\sqrt{3}x + 1$.

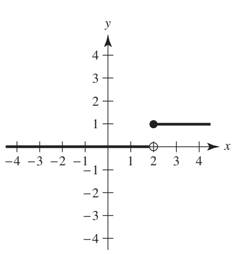
3. $H(x) = \begin{cases} 1, & x \ge 0 \\ 0, & x < 0 \end{cases}$



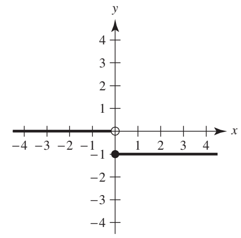
(a) $H(x) - 2 = \begin{cases} -1, & x \ge 0 \\ -2, & x < 0 \end{cases}$



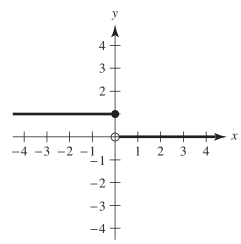
(b) $H(x - 2) = \begin{cases} 1, & x \ge 2 \\ 0, & x < 2 \end{cases}$



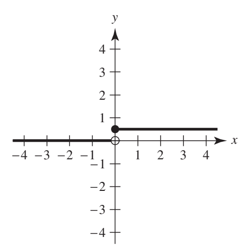
(c) $-H(x) = \begin{cases} -1, & x \ge 0 \\ 0, & x < 0 \end{cases}$



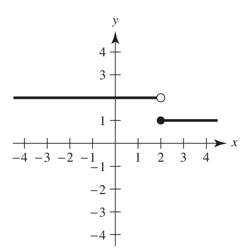
(d) $H(-x) = \begin{cases} 1, & x \le 0 \\ 0, & x > 0 \end{cases}$



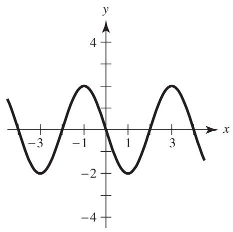
(e) $\frac{1}{2}H(x) = \begin{cases} \frac{1}{2}, & x \ge 0 \\ 0, & x < 0 \end{cases}$



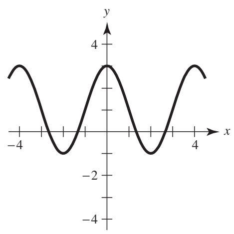
(f) $-H(x - 2) + 2 = \begin{cases} 1, & x \ge 2 \\ 2, & x < 2 \end{cases}$



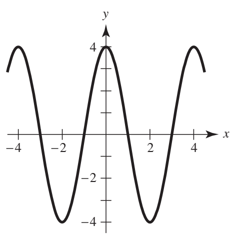
4. (a) $f(x+1)$



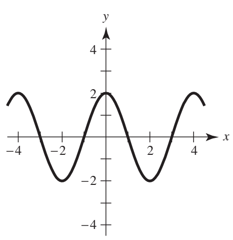
(b) $f(x) + 1$



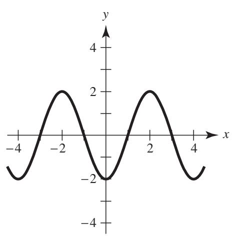
(c) $2f(x)$



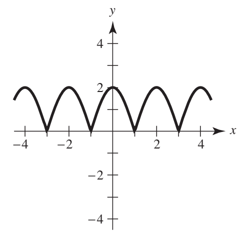
(d) $f(-x)$



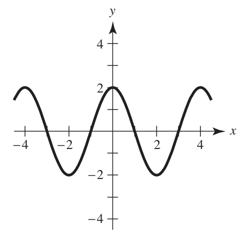
(e) $-f(x)$



(f) $|f(x)|$



(g) $f(|x|)$

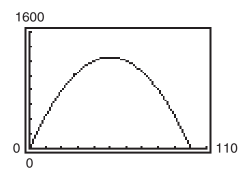


5. (a) $x + 2y = 100 \Rightarrow y = \dfrac{100 - x}{2}$

$A(x) = xy = x\left(\dfrac{100 - x}{2}\right) = -\dfrac{x^2}{2} + 50x$

Domain: $0 < x < 100$ or $(0, 100)$

(b)



Maximum of 1250 m^2 at $x = 50$ m, $y = 25$ m.

(c) $A(x) = -\frac{1}{2}(x^2 - 100x)$

$= -\frac{1}{2}(x^2 - 100x + 2500) + 1250$

$= -\frac{1}{2}(x - 50)^2 + 1250$

$A(50) = 1250$ m^2 is the maximum.

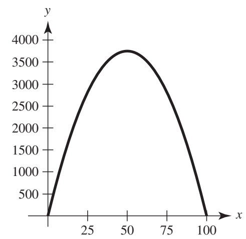
$x = 50$ m, $y = 25$ m

6. (a) $4y + 3x = 300 \Rightarrow y = \dfrac{300 - 3x}{4}$

$A(x) = x(2y) = x\left(\dfrac{300 - 3x}{2}\right) = \dfrac{-3x^2 + 300x}{2}$

Domain: $0 < x < 100$

(b)



Maximum of 3750 ft^2 at $x = 50$ ft, $y = 37.5$ ft.

(c) $A(x) = -\frac{3}{2}(x^2 - 100x)$

$= -\frac{3}{2}(x^2 - 100x + 2500) + 3750$

$= -\frac{3}{2}(x - 50)^2 + 3750$

$A(50) = 3750$ square feet is the maximum area, where $x = 50$ ft and $y = 37.5$ ft.

7. The length of the trip in the water is $\sqrt{2^2 + x^2}$, and the length of the trip over land is $\sqrt{1 + (3 - x)^2}$. So, the total time is $T = \dfrac{\sqrt{4 + x^2}}{2} + \dfrac{\sqrt{1 + (3 - x)^2}}{4}$ hours.

8. Let d be the distance from the starting point to the beach.

$$\text{Average speed} = \frac{\text{distance}}{\text{time}} = \frac{2d}{\dfrac{d}{120} + \dfrac{d}{60}} = \frac{2}{\dfrac{1}{120} + \dfrac{1}{60}} = 80 \text{ km/h}$$

9. (a) Slope $= \dfrac{9 - 4}{3 - 2} = 5$. Slope of tangent line is less than 5.

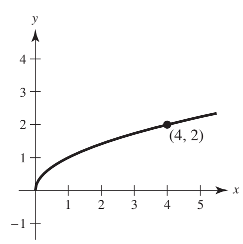
(b) Slope $= \dfrac{4 - 1}{2 - 1} = 3$. Slope of tangent line is greater than 3.

(c) Slope $= \dfrac{4.41 - 4}{2.1 - 2} = 4.1$. Slope of tangent line is less than 4.1.

(d) Slope $= \dfrac{f(2 + h) - f(2)}{(2 + h) - 2} = \dfrac{(2 + h)^2 - 4}{h} = \dfrac{4h + h^2}{h} = 4 + h,\ h \neq 0$

(e) Letting h get closer and closer to 0, the slope approaches 4. So, the slope at (2, 4) is 4.

10.



(a) Slope $= \dfrac{3 - 2}{9 - 4} = \dfrac{1}{5}$. Slope of tangent line is greater than $\dfrac{1}{5}$.

(b) Slope $= \dfrac{2 - 1}{4 - 1} = \dfrac{1}{3}$. Slope of tangent line is less than $\dfrac{1}{3}$.

(c) Slope $= \dfrac{2.1 - 2}{4.41 - 4} = \dfrac{10}{41}$. Slope of tangent line is greater than $\dfrac{10}{41}$.

(d) Slope $= \dfrac{f(4 + h) - f(4)}{(4 + h) - 4} = \dfrac{\sqrt{4 + h} - 2}{h}$

(e) $\dfrac{\sqrt{4 + h} - 2}{h} = \dfrac{\sqrt{4 + h} - 2}{h} \cdot \dfrac{\sqrt{4 + h} + 2}{\sqrt{4 + h} + 2} = \dfrac{(4 + h) - 4}{h\left(\sqrt{4 + h} + 2\right)} = \dfrac{1}{\sqrt{4 + h} + 2},\ h \neq 0$

As h gets closer to 0, the slope gets closer to $\dfrac{1}{4}$. The slope is $\dfrac{1}{4}$ at the point (4, 2).

11. $f(x) = y = \dfrac{1}{1-x}$

(a) Domain: all $x \neq 1$ or $(-\infty, 1) \cup (1, \infty)$

Range: all $y \neq 0$ or $(-\infty, 0) \cup (0, \infty)$

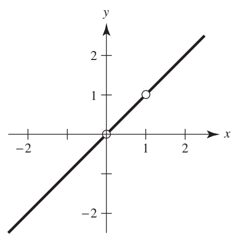
(b) $f(f(x)) = f\left(\dfrac{1}{1-x}\right) = \dfrac{1}{1-\left(\dfrac{1}{1-x}\right)} = \dfrac{1}{\dfrac{1-x-1}{1-x}} = \dfrac{1-x}{-x} = \dfrac{x-1}{x}$

Domain: all $x \neq 0, 1$ or $(-\infty, 0) \cup (0, 1) \cup (1, \infty)$

(c) $f(f(f(x))) = f\left(\dfrac{x-1}{x}\right) = \dfrac{1}{1-\left(\dfrac{x-1}{x}\right)} = \dfrac{1}{\dfrac{1}{x}} = x$

Domain: all $x \neq 0, 1$ or $(-\infty, 0) \cup (0, 1) \cup (1, \infty)$

(d) The graph is not a line. It has holes at (0, 0) and (1, 1).



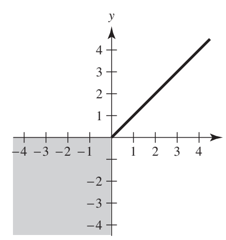
12. Using the definition of absolute value, you can rewrite the equation.

$$y + |y| = x + |x|$$

$$\begin{cases} 2y, & y > 0 \\ 0, & y \le 0 \end{cases} = \begin{cases} 2x, & x > 0 \\ 0, & x \le 0 \end{cases}.$$

For $x > 0$ and $y > 0$, you have $2y = 2x \Rightarrow y = x$.

For any $x \le 0$, y is any $y \le 0$. So, the graph of $y + |y| = x + |x|$ is as follows.

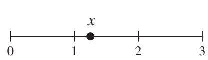


13. (a) $\dfrac{I}{x^2} = \dfrac{2I}{(x-3)^2}$

$x^2 - 6x + 9 = 2x^2$

$x^2 + 6x - 9 = 0$

$$x = \frac{-6 \pm \sqrt{36 + 36}}{2} = -3 \pm \sqrt{18} \approx 1.2426, -7.2426$$



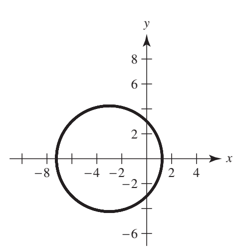
(b) $\dfrac{I}{x^2 + y^2} = \dfrac{2I}{(x-3)^2 + y^2}$

$(x-3)^2 + y^2 = 2(x^2 + y^2)$

$x^2 - 6x + 9 + y^2 = 2x^2 + 2y^2$

$x^2 + y^2 + 6x - 9 = 0$

$(x+3)^2 + y^2 = 18$



Circle of radius $\sqrt{18}$ and center $(-3, 0)$.

14. (a) $\dfrac{I}{x^2 + y^2} = \dfrac{kI}{(x-4)^2 + y^2}$

$$(x-4)^2 + y^2 = k(x^2 + y^2)$$

$$(k-1)x^2 + 8x + (k-1)y^2 = 16$$

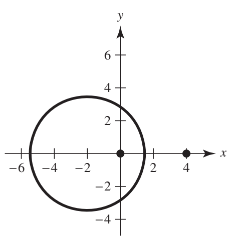
If $k = 1$, then $x = 2$ is a vertical line. Assume $k \neq 1$.

$$x^2 + \frac{8x}{k-1} + y^2 = \frac{16}{k-1}$$

$$x^2 + \frac{8x}{k-1} + \frac{16}{(k-1)^2} + y^2 = \frac{16}{k-1} + \frac{16}{(k-1)^2}$$

$$\left(x + \frac{4}{k-1}\right)^2 + y^2 = \frac{16k}{(k-1)^2}, \text{ Circle}$$

(b) If $k = 3$, $(x+2)^2 + y^2 = 12$



(c) As k becomes very large, $\dfrac{4}{k-1} \to 0$ and $\dfrac{16k}{(k-1)^2} \to 0$.

The center of the circle gets closer to (0, 0), and its radius approaches 0.

15.

$$d_1 d_2 = 1$$

$$\left[(x+1)^2 + y^2\right]\left[(x-1)^2 + y^2\right] = 1$$

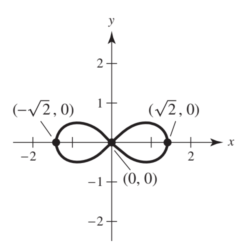
$$(x+1)^2(x-1)^2 + y^2\left[(x+1)^2 + (x-1)^2\right] + y^4 = 1$$

$$(x^2-1)^2 + y^2\left[2x^2 + 2\right] + y^4 = 1$$

$$x^4 - 2x^2 + 1 + 2x^2y^2 + 2y^2 + y^4 = 1$$

$$(x^4 + 2x^2y^2 + y^4) - 2x^2 + 2y^2 = 0$$

$$(x^2 + y^2)^2 = 2(x^2 - y^2)$$



Let $y = 0$. Then $x^4 = 2x^2 \Rightarrow x = 0$ or $x^2 = 2$.

So, $(0, 0)$, $(\sqrt{2}, 0)$ and $(-\sqrt{2}, 0)$ are on the curve.

CHAPTER 1
Limits and Their Properties

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Limits and Their Properties

Section 1.1 A Preview of Calculus

1. Calculus is the mathematics of change. Precalculus is more static. Answers will vary. *Sample answer:*

Precalculus: Area of a rectangle
Calculus: Area under a curve

Precalculus: Work done by a constant force
Calculus: Work done by a variable force

Precalculus: Center of a rectangle
Calculus: Centroid of a region

2. A secant line through a point P is a line joining P and another point Q on the graph.

The slope of the tangent line P is the limit of the slopes of the secant lines joining P and Q, as Q approaches P.

3. Precalculus: $(20 \text{ ft/sec})(15 \text{ sec}) = 300 \text{ ft}$

4. Calculus required: Velocity is not constant.

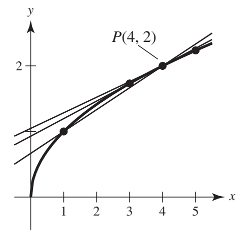
Distance $\approx (20 \text{ ft/sec})(15 \text{ sec}) = 300 \text{ ft}$

5. Calculus required: Slope of the tangent line at $x = 2$ is the rate of change, and equals about 0.16.

6. Precalculus: rate of change $=$ slope $= 0.08$

7. $f(x) = \sqrt{x}$

(a)



(b) $$\text{slope} = m = \frac{\sqrt{x} - 2}{x - 4} = \frac{\sqrt{x} - 2}{(\sqrt{x} + 2)(\sqrt{x} - 2)} = \frac{1}{\sqrt{x} + 2}, \ x \neq 4$$

$$x = 1: m = \frac{1}{\sqrt{1} + 2} = \frac{1}{3}$$

$$x = 3: m = \frac{1}{\sqrt{3} + 2} \approx 0.2679$$

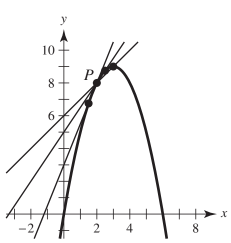
$$x = 5: m = \frac{1}{\sqrt{5} + 2} \approx 0.2361$$

(c) At $P(4, 2)$ the slope is $\dfrac{1}{\sqrt{4} + 2} = \dfrac{1}{4} = 0.25.$

You can improve your approximation of the slope at $x = 4$ by considering x-values very close to 4.

8. $f(x) = 6x - x^2$

(a)



(b) $$\text{slope} = m = \frac{(6x - x^2) - 8}{x - 2} = \frac{(x - 2)(4 - x)}{x - 2} = (4 - x), \ x \neq 2$$

For $x = 3, m = 4 - 3 = 1$

For $x = 2.5, m = 4 - 2.5 = 1.5 = \dfrac{3}{2}$

For $x = 1.5, m = 4 - 1.5 = 2.5 = \dfrac{5}{2}$

(c) At $P(2, 8)$, the slope is 2. You can improve your approximation by considering values of x close to 2.

9. (a) Area $\approx 5 + \frac{5}{2} + \frac{5}{3} + \frac{5}{4} \approx 10.417$

Area $\approx \frac{1}{2}\left(5 + \frac{5}{1.5} + \frac{5}{2} + \frac{5}{2.5} + \frac{5}{3} + \frac{5}{3.5} + \frac{5}{4} + \frac{5}{4.5}\right) \approx 9.145$

(b) You could improve the approximation by using more rectangles.

10. Answers will vary. *Sample answer*:

The instantaneous rate of change of an automobile's position is the velocity of the automobile, and can be determined by the speedometer.

11. (a) $D_1 = \sqrt{(5-1)^2 + (1-5)^2} = \sqrt{16+16} \approx 5.66$

(b) $D_2 = \sqrt{1 + \left(\frac{5}{2}\right)^2} + \sqrt{1 + \left(\frac{5}{2} - \frac{5}{3}\right)^2} + \sqrt{1 + \left(\frac{5}{3} - \frac{5}{4}\right)^2} + \sqrt{1 + \left(\frac{5}{4} - 1\right)^2}$

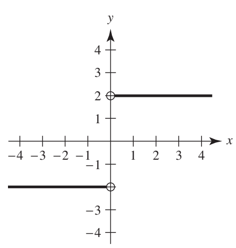
$\approx 2.693 + 1.302 + 1.083 + 1.031 \approx 6.11$

(c) Increase the number of line segments.

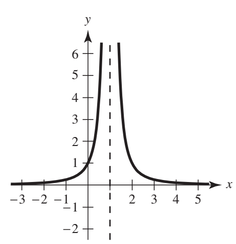
Section 1.2 Finding Limits Graphically and Numerically

1. As the graph of the function approaches 8 on the horizontal axis, the graph approaches 25 on the vertical axis.

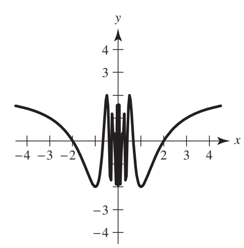
2. (i) The values of f approach different numbers as x approaches c from different sides of c:



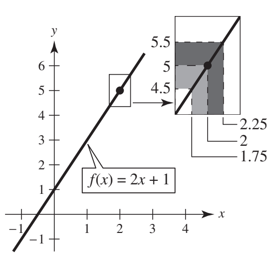
(ii) The values of f increase without bound as x approaches c:



(iii) The values of f oscillate between two fixed numbers as x approaches c:



3.



4. No. For example, consider Example 2 from this section.

$$f(x) = \begin{cases} 1, & x \neq 2 \\ 0, & x = 2 \end{cases}$$

$\lim_{x \to 2} f(x) = 1$, but $f(2) = 0$

5.

x	3.9	3.99	3.999	4	4.001	4.01	4.1
$f(x)$	0.3448	0.3344	0.3334	?	0.3332	0.3322	0.3226

$\lim_{x \to 4} \frac{x-4}{x^2 - 5x - 4} \approx 0.3333$ $\left(\text{Actual limit is } \frac{1}{3}.\right)$

6.

x	2.9	2.99	2.999	3	3.001	3.01	3.1
$f(x)$	0.1695	0.1669	0.1667	?	0.1666	0.1664	0.1639

$\lim_{x \to 3} \frac{x-3}{x^2 - 9} \approx 0.1667$ $\left(\text{Actual limit is } \frac{1}{6}.\right)$

7.

x	–0.1	–0.01	–0.001	0	0.001	0.01	0.1
$f(x)$	0.5132	0.5013	0.5001	?	0.4999	0.4988	0.4881

$\lim_{x\to 0} \frac{\sqrt{x+1}-1}{x} \approx 0.5000$ $\left(\text{Actual limit is } \frac{1}{2}.\right)$

8.

x	2.9	2.99	2.999	3.001	3.01	3.1
$f(x)$	–0.0641	–0.0627	–0.0625	–0.0625	–0.0623	–0.0610

$\lim_{x\to 3} \frac{[1/(x+1)]-(1/4)}{x-3} \approx -0.0625$ $\left(\text{Actual limit is } -\frac{1}{16}.\right)$

9.

x	–0.1	–0.01	–0.001	0.001	0.01	0.1
$f(x)$	0.9983	0.99998	1.0000	1.0000	0.99998	0.9983

$\lim_{x\to 0} \frac{\sin x}{x} \approx 1.0000$ (Actual limit is 1.) (Make sure you use radian mode.)

10.

x	–0.1	–0.01	–0.001	0.001	0.01	0.1
$f(x)$	0.0500	0.0050	0.0005	–0.0005	–0.0050	–0.0500

$\lim_{x\to 0} \frac{\cos x - 1}{x} \approx 0.0000$ (Actual limit is 0.) (Make sure you use radian mode.)

11.

x	0.9	0.99	0.999	1.001	1.01	1.1
$f(x)$	0.2564	0.2506	0.2501	0.2499	0.2494	0.2439

$\lim_{x\to 1} \frac{x-2}{x^2+x-6} \approx 0.2500$ $\left(\text{Actual limit is } \frac{1}{4}.\right)$

12.

x	–4.1	–4.01	–4.001	–4	–3.999	–3.99	–3.9
$f(x)$	1.1111	1.0101	1.0010	?	0.9990	0.9901	0.9091

$\lim_{x\to -4} \frac{x+4}{x^2+9x+20} \approx 1.0000$ (Actual limit is 1.)

13.

x	0.9	0.99	0.999	1.001	1.01	1.1
$f(x)$	0.7340	0.6733	0.6673	0.6660	0.6600	0.6015

$\lim_{x\to 1} \frac{x^4-1}{x^6-1} \approx 0.6666$ $\left(\text{Actual limit is } \frac{2}{3}.\right)$

14.

x	–3.1	–3.01	–3.001	–3	–2.999	–2.99	–2.9
$f(x)$	27.91	27.0901	27.0090	?	26.9910	26.9101	26.11

$\lim_{x\to -3} \frac{x^3+27}{x+3} \approx 27.0000$ (Actual limit is 27.)

15.

x	–6.1	–6.01	–6.001	–6	–5.999	–5.99	–5.9
$f(x)$	–0.1248	–0.1250	–0.1250	?	–0.1250	–0.1250	–0.1252

$\lim_{x\to -6} \frac{\sqrt{10-x}-4}{x+6} \approx -0.1250 \qquad \left(\text{Actual limit is } -\frac{1}{8}.\right)$

16.

x	1.9	1.99	1.999	2	2.001	2.01	2.1
$f(x)$	0.1149	0.115	0.1111	?	0.1111	0.1107	0.1075

$\lim_{x\to 2} \frac{x/(x+1)-2/3}{x-2} \approx 0.1111 \qquad \left(\text{Actual limit is } \frac{1}{9}.\right)$

17.

x	–0.1	–0.01	–0.001	0.001	0.01	0.1
$f(x)$	1.9867	1.9999	2.0000	2.0000	1.9999	1.9867

$\lim_{x\to 0} \frac{\sin 2x}{x} \approx 2.0000$ (Actual limit is 2.) (Make sure you use radian mode.)

18.

x	–0.1	–0.01	–0.001	0.001	0.01	0.1
$f(x)$	0.4950	0.5000	0.5000	0.5000	0.5000	0.4950

$\lim_{x\to 0} \frac{\tan x}{\tan 2x} \approx 0.5000 \qquad \left(\text{Actual limit is } \frac{1}{2}.\right)$

19. $f(x) = \frac{2}{x^3}$

x	–0.1	–0.01	–0.001	0	0.001	0.01	0.1
$f(x)$	–2000	-2×10^6	-2×10^9	?	2×10^9	2×10^6	2000

As x approaches 0 from the left, the function decreases without bound. As x approaches 0 from the right, the function increases without bound.

20. $f(x) = \frac{3|x|}{x^2}$

x	–0.1	–0.01	–0.001	0	0.001	0.01	0.1
$f(x)$	30	300	3000	?	3000	300	30

As x approaches 0 from either side, the function increases without bound.

21. $\lim_{x\to 3} (4 - x) = 1$

22. $\lim_{x\to 0} \sec x = 1$

23. $\lim_{x\to 2} f(x) = \lim_{x\to 2} (4 - x) = 2$

24. $\lim_{x\to 1} f(x) = \lim_{x\to 1} (x^2 + 3) = 4$

25. $\lim_{x\to 2} \frac{|x-2|}{x-2}$ does not exist.

For values of x to the left of 2, $\frac{|x-2|}{(x-2)} = -1$, whereas for values of x to the right of 2, $\frac{|x-2|}{(x-2)} = 1$.

26. $\lim_{x\to 5} \frac{2}{x-5}$ does not exist because the function increases and decreases without bound as x approaches 5.

27. $\lim_{x\to 0} \cos(1/x)$ does not exist because the function oscillates between –1 and 1 as x approaches 0.

28. $\lim_{x\to \pi/2} \tan x$ does not exist because the function increases without bound as x approaches $\frac{\pi}{2}$ from the left and decreases without bound as x approaches $\frac{\pi}{2}$ from the right.

29. (a) $f(1)$ exists. The black dot at (1, 2) indicates that $f(1) = 2$.

(b) $\lim_{x\to 1} f(x)$ does not exist. As x approaches 1 from the left, $f(x)$ approaches 3.5, whereas as x approaches 1 from the right, $f(x)$ approaches 1.

(c) $f(4)$ does not exist. The hollow circle at $(4, 2)$ indicates that f is not defined at 4.

(d) $\lim_{x\to 4} f(x)$ exists. As x approaches 4, $f(x)$ approaches 2: $\lim_{x\to 4} f(x) = 2$.

30. (a) $f(-2)$ does not exist. The vertical dotted line indicates that f is not defined at –2.

(b) $\lim_{x\to -2} f(x)$ does not exist. As x approaches –2, the values of $f(x)$ do not approach a specific number.

(c) $f(0)$ exists. The black dot at $(0, 4)$ indicates that $f(0) = 4$.

(d) $\lim_{x\to 0} f(x)$ does not exist. As x approaches 0 from the left, $f(x)$ approaches $\frac{1}{2}$, whereas as x approaches 0 from the right, $f(x)$ approaches 4.

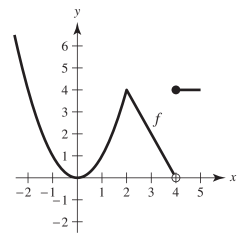
(e) $f(2)$ does not exist. The hollow circle at $\left(2, \frac{1}{2}\right)$ indicates that $f(2)$ is not defined.

(f) $\lim_{x\to 2} f(x)$ exists. As x approaches 2, $f(x)$ approaches $\frac{1}{2}$: $\lim_{x\to 2} f(x) = \frac{1}{2}$.

(g) $f(4)$ exists. The black dot at $(4, 2)$ indicates that $f(4) = 2$.

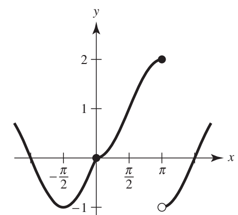
(h) $\lim_{x\to 4} f(x)$ does not exist. As x approaches 4, the values of $f(x)$ do not approach a specific number.

31.



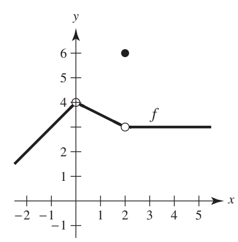
$\lim_{x\to c} f(x)$ exists for all values of $c \neq 4$.

32.

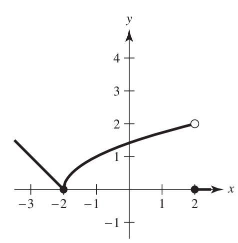


$\lim_{x\to c} f(x)$ exists for all values of $c \neq \pi$.

33. One possible answer is



34. One possible answer is



35. You need $|f(x) - 3| = |(x + 1) - 3| = |x - 2| < 0.4$.
So, take $\delta = 0.4$. If $0 < |x - 2| < 0.4$,
then $|x - 2| = |(x + 1) - 3| = |f(x) - 3| < 0.4$,
as desired.

36. You need $\left|f(x) - 1\right| = \left|\dfrac{1}{x-1} - 1\right| = \left|\dfrac{2-x}{x-1}\right| < 0.01$. Let $\delta = \dfrac{1}{101}$. If $0 < \left|x - 2\right| < \dfrac{1}{101}$, then

$$-\frac{1}{101} < x - 2 < \frac{1}{101} \Rightarrow 1 - \frac{1}{101} < x - 1 < 1 + \frac{1}{101}$$
$$\Rightarrow \frac{100}{101} < x - 1 < \frac{102}{101}$$
$$\Rightarrow \left|x - 1\right| > \frac{100}{101}$$

and you have

$$\left|f(x) - 1\right| = \left|\frac{1}{x-1} - 1\right| = \left|\frac{2-x}{x-1}\right| < \frac{1/101}{100/101} = \frac{1}{100} = 0.01.$$

37. You need to find δ such that $0 < \left|x - 1\right| < \delta$ implies

$\left|f(x) - 1\right| = \left|\dfrac{1}{x} - 1\right| < 0.1$. That is,

$$-0.1 < \frac{1}{x} - 1 < 0.1$$
$$1 - 0.1 < \frac{1}{x} < 1 + 0.1$$
$$\frac{9}{10} < \frac{1}{x} < \frac{11}{10}$$
$$\frac{10}{9} > x > \frac{10}{11}$$
$$\frac{10}{9} - 1 > x - 1 > \frac{10}{11} - 1$$
$$\frac{1}{9} > x - 1 > -\frac{1}{11}.$$

So take $\delta = \dfrac{1}{11}$. Then $0 < \left|x - 1\right| < \delta$ implies

$$-\frac{1}{11} < x - 1 < \frac{1}{11}$$
$$-\frac{1}{11} < x - 1 < \frac{1}{9}.$$

Using the first series of equivalent inequalities, you obtain

$$\left|f(x) - 1\right| = \left|\frac{1}{x} - 1\right| < 0.1.$$

38. You need to find δ such that $0 < \left|x - 1\right| < \delta$ implies

$\left|f(x) - 1\right| = \left|2 - \dfrac{1}{x} - 1\right| = \left|1 - \dfrac{1}{x}\right| < \varepsilon$.

$$-\varepsilon < \frac{1}{x} - 1 < \varepsilon$$
$$1 - \varepsilon < \frac{1}{x} < 1 + \varepsilon$$
$$\frac{1}{1-\varepsilon} > x > \frac{1}{1+\varepsilon}$$
$$\frac{1}{1-\varepsilon} - 1 > x - 1 > \frac{1}{1+\varepsilon} - 1$$
$$\frac{\varepsilon}{1-\varepsilon} > x - 1 > \frac{-\varepsilon}{1+\varepsilon}$$

For $\varepsilon = 0.05$, take $\delta = \dfrac{0.05}{1 - 0.05} \approx 0.05$.

For $\varepsilon = 0.01$, take $\delta = \dfrac{0.01}{1 - 0.01} \approx 0.01$.

For $\varepsilon = 0.005$, take $\delta = \dfrac{0.005}{1 - 0.005} \approx 0.005$.

As ε decreases, so does ε.

39. $\lim_{x\to 2}(3x+2) = 3(2)+2 = 8 = L$

(a) $\left|(3x+2)-8\right| < 0.01$

$$\left|3x-6\right| < 0.01$$
$$3\left|x-2\right| < 0.01$$
$$0 < \left|x-2\right| < \tfrac{0.01}{3} \approx 0.0033 = \delta$$

So, if $0 < \left|x-2\right| < \delta = \frac{0.01}{3}$, you have

$$3\left|x-2\right| < 0.01$$
$$\left|3x-6\right| < 0.01$$
$$\left|(3x+2)-8\right| < 0.01$$
$$\left|f(x)-L\right| < 0.01.$$

(b) $\left|(3x+2)-8\right| < 0.005$

$$\left|3x-6\right| < 0.005$$
$$3\left|x-2\right| < 0.005$$
$$0 < \left|x-2\right| < \frac{0.005}{3} \approx 0.00167 = \delta$$

Finally, as in part (a), if $0 < \left|x-2\right| < \dfrac{0.005}{3}$,

you have $\left|(3x+2)-8\right| < 0.005$.

40. $\lim_{x\to 6}\left(6-\frac{x}{3}\right) = 6-\frac{6}{3} = 4 = L$

(a) $\left|\left(6-\frac{x}{3}\right)-4\right| < 0.01$

$$\left|2-\frac{x}{3}\right| < 0.01$$
$$\left|-\tfrac{1}{3}(x-6)\right| < 0.01$$
$$\left|x-6\right| < 0.03$$
$$0 < \left|x-6\right| < 0.03 = \delta$$

So, if $0 < \left|x-6\right| < \delta = 0.03$, you have

$$\left|-\tfrac{1}{3}(x-6)\right| < 0.01$$
$$\left|2-\frac{x}{3}\right| < 0.01$$
$$\left|\left(6-\frac{x}{3}\right)-4\right| < 0.01$$
$$\left|f(x)-L\right| < 0.01.$$

(b) $\left|\left(6-\frac{x}{3}\right)-4\right| < 0.005$

$$\left|2-\frac{x}{3}\right| < 0.005$$
$$\left|-\frac{1}{3}(x-6)\right| < 0.005$$
$$\left|x-6\right| < 0.015$$
$$0 < \left|x-6\right| < 0.015 = \delta$$

As in part (a), if $0 < \left|x-6\right| < 0.015$, you have

$\left|\left(6-\frac{x}{3}\right)-4\right| < 0.005.$

41. $\lim_{x\to 2}\left(x^2-3\right) = 2^2-3 = 1 = L$

(a) $\left|\left(x^2-3\right)-1\right| < 0.01$

$$\left|x^2-4\right| < 0.01$$
$$\left|(x+2)(x-2)\right| < 0.01$$
$$\left|x+2\right|\left|x-2\right| < 0.01$$
$$\left|x-2\right| < \frac{0.01}{\left|x+2\right|}$$

If you assume $1 < x < 3$, then
$\delta \approx 0.01/5 = 0.002$.

So, if $0 < \left|x-2\right| < \delta \approx 0.002$, you have

$$\left|x-2\right| < 0.002 = \frac{1}{5}(0.01) < \frac{1}{\left|x+2\right|}(0.01)$$
$$\left|x+2\right|\left|x-2\right| < 0.01$$
$$\left|x^2-4\right| < 0.01$$
$$\left|\left(x^2-3\right)-1\right| < 0.01$$
$$\left|f(x)-L\right| < 0.01.$$

(b) $\left|\left(x^2-3\right)-1\right| < 0.005$

$$\left|x^2-4\right| < 0.005$$
$$\left|(x+2)(x-2)\right| < 0.005$$
$$\left|x+2\right|\left|x-2\right| < 0.005$$
$$\left|x-2\right| < \frac{0.005}{\left|x+2\right|}$$

If you assume $1 < x < 3$, then
$\delta = \dfrac{0.005}{5} = 0.001$.

Finally, as in part (a), if $0 < \left|x-2\right| < 0.001$,

you have $\left|\left(x^2-3\right)-1\right| < 0.005$.

42. $\lim_{x \to 4} (x^2 + 6) = 4^2 + 6 = 22 = L$

(a) $|(x^2 + 6) - 22| < 0.01$

$$|x^2 - 16| < 0.01$$
$$|(x + 4)(x - 4)| < 0.01$$
$$|x + 4||x - 4| < 0.01$$
$$|x - 4| < \frac{0.01}{|x + 4|}$$

If you assume $3 < x < 5$, then

$\delta = \dfrac{0.01}{9} \approx 0.00111.$

So, if $0 < |x - 4| < \delta \approx \dfrac{0.01}{9}$, you have

$$|x - 4| < \frac{0.01}{9} < \frac{0.01}{|x + 4|}$$
$$|(x + 4)(x - 4)| < 0.01$$
$$|x^2 - 16| < 0.01$$
$$|(x^2 + 6) - 22| < 0.01$$
$$|f(x) - L| < 0.01.$$

(b) $|(x^2 + 6) - 22| < 0.005$

$$|x^2 - 16| < 0.005$$
$$|(x - 4)(x + 4)| < 0.005$$
$$|x - 4||x + 4| < 0.005$$
$$|x - 4| < \frac{0.05}{|x + 4|}$$

If you assume $3 < x < 5$, then

$\delta = \dfrac{0.005}{9} \approx 0.00056.$

Finally, as in part (a), if $0 < |x - 4| < \dfrac{0.005}{9}$,

you have $|(x^2 + 6) - 22| < 0.005.$

43. $\lim_{x \to 4} (x^2 - x) = 16 - 4 = 12 = L$

(a) $|(x^2 - x) - 12| < 0.01$

$$|(x - 4)(x + 3)| < 0.01$$
$$|x - 4||x + 3| < 0.01$$
$$|x - 4| < \frac{0.01}{|x + 3|}$$

If you assume $3 < x < 5$, then

$\delta = \dfrac{0.01}{8} = 0.00125.$

So, if $0 < |x - 4| < \dfrac{0.01}{8}$, you have

$$|x - 4| < \frac{0.01}{|x + 3|}$$
$$|x - 4||x + 3| < 0.01$$
$$|x^2 - x - 12| < 0.01$$
$$|(x^2 - x) - 12| < 0.01$$
$$|f(x) - L| < 0.01$$

(b) $|(x^2 - x) - 12| < 0.005$

$$|(x - 4)(x + 3)| < 0.005$$
$$|x - 4||x + 3| < 0.005$$
$$|x - 4| < \frac{0.005}{|x + 3|}$$

If you assume $3 < x < 5$, then

$\delta = \dfrac{0.005}{8} = 0.000625.$

Finally, as in part (a), if $0 < |x - 4| < \dfrac{0.005}{8}$,

you have $|(x^2 - x) - 12| < 0.005.$

44. $\lim_{x \to 3} x^2 = 3^2 = 9 = L$

(a) $|x^2 - 9| < 0.01$

$|(x - 3)(x + 3)| < 0.01$

$|x - 3||x + 3| < 0.01$

$|x - 3| < \dfrac{0.01}{|x + 3|}$

If you assume $2 < x < 4$, then

$\delta = \dfrac{0.01}{7} \approx 0.0014.$

So, if $0 < |x - 3| < \dfrac{0.01}{7}$, you have

$|x - 3| < \dfrac{0.01}{|x + 3|}$

$|x - 3||x + 3| < 0.01$

$|x^2 - 9| < 0.01$

$|f(x) - L| < 0.01$

(b) $|x^2 - 9| < 0.005$

$|(x - 3)(x + 3)| < 0.005$

$|x - 3||x + 3| < 0.005$

$|x - 3| < \dfrac{0.005}{|x + 3|}$

If you assume $2 < x < 4$, then

$\delta = \dfrac{0.005}{7} \approx 0.00071.$

Finally, as in part (a), if $0 < |x - 3| < \dfrac{0.005}{7}$, you have $|x^2 - 9| < 0.005$.

45. $\lim_{x \to 4} (x + 2) = 4 + 2 = 6$

Given $\varepsilon > 0$:

$|(x + 2) - 6| < \varepsilon$

$|x - 4| < \varepsilon = \delta$

So, let $\delta = \varepsilon$. So, if $0 < |x - 4| < \delta = \varepsilon$, you have

$|x - 4| < \varepsilon$

$|(x + 2) - 6| < \varepsilon$

$|f(x) - L| < \varepsilon.$

46. $\lim_{x \to -2} (4x + 5) = 4(-2) + 5 = -3$

Given $\varepsilon > 0$:

$|(4x + 5) - (-3)| < \varepsilon$

$|4x + 8| < \varepsilon$

$4|x + 2| < \varepsilon$

$|x + 2| < \dfrac{\varepsilon}{4} = \delta$

So, let $\delta = \dfrac{\varepsilon}{4}.$

So, if $0 < |x + 2| < \delta = \dfrac{\varepsilon}{4}$, you have

$|x + 2| < \dfrac{\varepsilon}{4}$

$|4x + 8| < \varepsilon$

$|(4x + 5) - (-3)| < \varepsilon$

$|f(x) - L| < \varepsilon.$

47. $\lim_{x \to -4} \left(\frac{1}{2}x - 1\right) = \frac{1}{2}(-4) - 1 = -3$

Given $\varepsilon > 0$:

$\left|\left(\frac{1}{2}x - 1\right) - (-3)\right| < \varepsilon$

$\left|\frac{1}{2}x + 2\right| < \varepsilon$

$\frac{1}{2}|x - (-4)| < \varepsilon$

$|x - (-4)| < 2\varepsilon$

So, let $\delta = 2\varepsilon.$

So, if $0 < |x - (-4)| < \delta = 2\varepsilon$, you have

$|x - (-4)| < 2\varepsilon$

$\left|\frac{1}{2}x + 2\right| < \varepsilon$

$\left|\left(\frac{1}{2}x - 1\right) + 3\right| < \varepsilon$

$|f(x) - L| < \varepsilon.$

48. $\lim_{x \to 3} \left(\frac{3}{4}x + 1\right) = \frac{3}{4}(3) + 1 = \frac{13}{4}$

Given $\varepsilon > 0$:

$$\left|\left(\frac{3}{4}x + 1\right) - \frac{13}{4}\right| < \varepsilon$$
$$\left|\frac{3}{4}x - \frac{9}{4}\right| < \varepsilon$$
$$\frac{3}{4}|x - 3| < \varepsilon$$
$$|x - 3| < \frac{4}{3}\varepsilon$$

So, let $\delta = \frac{4}{3}\varepsilon$.

So, if $0 < |x - 3| < \delta = \frac{4}{3}\varepsilon$, you have

$$|x - 3| < \frac{4}{3}\varepsilon$$
$$\frac{3}{4}|x - 3| < \varepsilon$$
$$\left|\frac{3}{4}x - \frac{9}{4}\right| < \varepsilon$$
$$\left|\left(\frac{3}{4}x + 1\right) - \frac{13}{4}\right| < \varepsilon$$
$$|f(x) - L| < \varepsilon.$$

49. $\lim_{x \to 6} 3 = 3$

Given $\varepsilon > 0$:

$$|3 - 3| < \varepsilon$$
$$0 < \varepsilon$$

So, any $\delta > 0$ will work.

So, for any $\delta > 0$, you have

$$|3 - 3| < \varepsilon$$
$$|f(x) - L| < \varepsilon.$$

50. $\lim_{x \to 2} (-1) = -1$

Given $\varepsilon > 0$: $|-1 - (-1)| < \varepsilon$

$$0 < \varepsilon$$

So, any $\delta > 0$ will work.

So, for any $\delta > 0$, you have

$$|(-1) - (-1)| < \varepsilon$$
$$|f(x) - L| < \varepsilon.$$

51. $\lim_{x \to 0} \sqrt[3]{x} = 0$

Given $\varepsilon > 0$: $\left|\sqrt[3]{x} - 0\right| < \varepsilon$

$$\left|\sqrt[3]{x}\right| < \varepsilon$$
$$|x| < \varepsilon^3 = \delta$$

So, let $\delta = \varepsilon^3$.

So, for $0|x - 0|\delta = \varepsilon^3$, you have

$$|x| < \varepsilon^3$$
$$\left|\sqrt[3]{x}\right| < \varepsilon$$
$$\left|\sqrt[3]{x} - 0\right| < \varepsilon$$
$$|f(x) - L| < \varepsilon.$$

52. $\lim_{x \to 4} \sqrt{x} = \sqrt{4} = 2$

Given $\varepsilon > 0$: $\left|\sqrt{x} - 2\right| < \varepsilon$

$$\left|\sqrt{x} - 2\right|\left|\sqrt{x} + 2\right| < \varepsilon\left|\sqrt{x} + 2\right|$$
$$|x - 4| < \varepsilon\left|\sqrt{x} + 2\right|$$

Assuming $1 < x < 9$, you can choose $\delta = 3\varepsilon$. Then,

$$0 < |x - 4| < \delta = 3\varepsilon \Rightarrow |x - 4| < \varepsilon\left|\sqrt{x} + 2\right|$$
$$\Rightarrow \left|\sqrt{x} - 2\right| < \varepsilon.$$

53. $\lim_{x \to -5} |x - 5| = |(-5) - 5| = |-10| = 10$

Given $\varepsilon > 0$: $||x - 5| - 10| < \varepsilon$

$$|-(x - 5) - 10| < \varepsilon \quad (x - 5 < 0)$$
$$|-x - 5| < \varepsilon$$
$$|x - (-5)| < \varepsilon$$

So, let $\delta = \varepsilon$.

So for $|x - (-5)| < \delta = \varepsilon$, you have

$$|-(x + 5)| < \varepsilon$$
$$|-(x - 5) - 10| < \varepsilon$$
$$||x - 5| - 10| < \varepsilon \quad (\text{because } x - 5 < 0)$$
$$|f(x) - L| < \varepsilon.$$

54. $\lim_{x \to 3} |x - 3| = |3 - 3| = 0$

Given $\varepsilon > 0$: $||x - 3| - 0| < \varepsilon$

$|x - 3| < \varepsilon$

So, let $\delta = \varepsilon$.

So, for $0 < |x - 3| < \delta = \varepsilon$, you have

$|x - 3| < \varepsilon$

$||x - 3| - 0| < \varepsilon$

$|f(x) - L| < \varepsilon$.

55. $\lim_{x \to 1} (x^2 + 1) = 1^2 + 1 = 2$

Given $\varepsilon > 0$: $|(x^2 + 1) - 2| < \varepsilon$

$|x^2 - 1| < \varepsilon$

$|(x + 1)(x - 1)| < \varepsilon$

$|x - 1| < \dfrac{\varepsilon}{|x + 1|}$

If you assume $0 < x < 2$, then $\delta = \varepsilon/3$.

So for $0 < |x - 1| < \delta = \dfrac{\varepsilon}{3}$, you have

$|x - 1| < \dfrac{1}{3}\varepsilon < \dfrac{1}{|x + 1|}\varepsilon$

$|x^2 - 1| < \varepsilon$

$|(x^2 + 1) - 2| < \varepsilon$

$|f(x) - 2| < \varepsilon$.

56. $\lim_{x \to -4} (x^2 + 4x) = (-4)^2 + 4(-4) = 0$

Given $\varepsilon > 0$: $|(x^2 + 4x) - 0| < \varepsilon$

$|x(x + 4)| < \varepsilon$

$|x + 4| < \dfrac{\varepsilon}{|x|}$

If you assume $-5 < x < -3$, then $\delta = \dfrac{\varepsilon}{5}$.

So for $0 < |x - (-4)| < \delta = \dfrac{\varepsilon}{5}$, you have

$|x + 4| < \dfrac{\varepsilon}{5} < \dfrac{1}{|x|}\varepsilon$

$|x(x + 4)| < \varepsilon$

$|(x^2 + 4x) - 0| < \varepsilon$

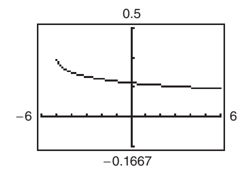
$|f(x) - L| < \varepsilon$.

57. $\lim_{x \to \pi} f(x) = \lim_{x \to \pi} 4 = 4$

58. $\lim_{x \to \pi} f(x) = \lim_{x \to \pi} x = \pi$

59. $f(x) = \dfrac{\sqrt{x + 5} - 3}{x - 4}$

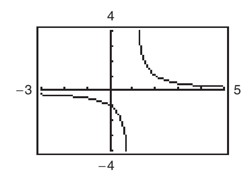
$\lim_{x \to 4} f(x) = \dfrac{1}{6}$



The domain is $[-5, 4) \cup (4, \infty)$. The graphing utility does not show the hole at $\left(4, \dfrac{1}{6}\right)$.

60. $f(x) = \dfrac{x - 3}{x^2 - 4x + 3}$

$\lim_{x \to 3} f(x) = \dfrac{1}{2}$



The domain is all $x \neq 1, 3$. The graphing utility does not show the hole at $\left(3, \dfrac{1}{2}\right)$.

61. $C(t) = 9.99 - 0.79[\![1 - t]\!], t > 0$

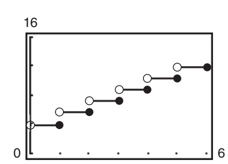
(a) $C(10.75) = 9.99 - 0.79[\![1 - 10.75]\!]$

$= 9.99 - 0.79(-10)$

$= \$17.89$

$C(10.75)$ represents the cost of a 10-minute, 45-second call.

(b)

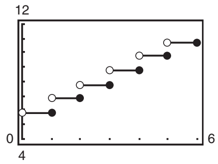


(c) The limit does not exist because the limits from the left and right are not equal.

62. $C(t) = 5.79 - 0.99[\![1 - t]\!], t > 0$

(a) $C(10.75) = 5.79 - 0.99[\![1 - 10.75]\!]$
$= 5.79 - 0.99(-10)$
$= \$15.69$

$C(10.75)$ represents the cost of a 10-minute, 45-second call.

(b) 

(c) The limit does not exist because the limits from the left and right are not equal.

63. Choosing a smaller positive value of δ will still satisfy the inequality $|f(x) - L| < \varepsilon$.

64. In the definition of $\lim_{x \to c} f(x)$, f must be defined on both sides of c, but does not have to be defined at c itself. The value of f at c has no bearing on the limit as x approaches c.

65. No. The fact that $f(2) = 4$ has no bearing on the existence of the limit of $f(x)$ as x approaches 2.

66. No. The fact that $\lim_{x \to 2} f(x) = 4$ has no bearing on the value of f at 2.

67. (a) $C = 2\pi r$

$$r = \frac{C}{2\pi} = \frac{6}{2\pi} = \frac{3}{\pi} \approx 0.9549 \text{ cm}$$

(b) When $C = 5.5$: $r = \dfrac{5.5}{2\pi} \approx 0.87535$ cm

When $C = 6.5$: $r = \dfrac{6.5}{2\pi} \approx 1.03451$ cm

So $0.87535 < r < 1.03451$.

(c) $\lim_{x \to 3/\pi} (2\pi r) = 6; \varepsilon = 0.5; \delta \approx 0.0796$

68. $V = \frac{4}{3}\pi r^3, V = 2.48$

(a) $2.48 = \frac{4}{3}\pi r^3$

$r^3 = \dfrac{1.86}{\pi}$

$r \approx 0.8397$ in.

(b) $2.45 \le V \le 2.51$

$2.45 \le \frac{4}{3}\pi r^3 \le 2.51$

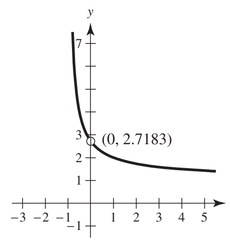
$0.5849 \le r^3 \le 0.5992$

$0.8363 \le r \le 0.8431$

(c) For $\varepsilon = 2.51 - 2.48 = 0.03, \delta \approx 0.003$

69. $f(x) = (1 + x)^{1/x}$

$\lim_{x \to 0} (1 + x)^{1/x} = e \approx 2.71828$



x	$f(x)$
−0.1	2.867972
−0.01	2.731999
−0.001	2.719642
−0.0001	2.718418
−0.00001	2.718295
−0.000001	2.718283

x	$f(x)$
0.1	2.593742
0.01	2.704814
0.001	2.716942
0.0001	2.718146
0.00001	2.718268
0.000001	2.718280

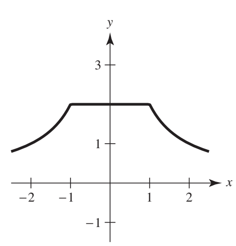
70. $f(x) = \dfrac{|x + 1| - |x - 1|}{x}$

x	−1	−0.5	−0.1	0	0.1	0.5	1.0
$f(x)$	2	2	2	Undef.	2	2	2

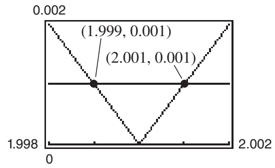
$\lim_{x \to 0} f(x) = 2$

Note that for

$-1 < x < 1, x \ne 0, f(x) = \dfrac{(x + 1) + (x - 1)}{x} = 2.$



71.



Using the zoom and trace feature, $\delta = 0.001$. So $(2 - \delta, 2 + \delta) = (1.999, 2.001)$.

Note: $\dfrac{x^2 - 4}{x - 2} = x + 2$ for $x \neq 2$.

72. (a) $\lim_{x \to c} f(x)$ exists for all $c \neq -3$.

(b) $\lim_{x \to c} f(x)$ exists for all $c \neq -2, 0$.

73. False. The existence or nonexistence of $f(x)$ at $x = c$ has no bearing on the existence of the limit of $f(x)$ as $x \to c$.

74. True

75. False. Let

$$f(x) = \begin{cases} x - 4, & x \neq 2 \\ 0, & x = 2 \end{cases}$$

$f(2) = 0$

$\lim_{x \to 2} f(x) = \lim_{x \to 2} (x - 4) = 2 \neq 0$

76. False. Let

$$f(x) = \begin{cases} x - 4, & x \neq 2 \\ 0, & x = 2 \end{cases}$$

$\lim_{x \to 2} f(x) = \lim_{x \to 2} (x - 4) = 2$ and $f(2) = 0 \neq 2$

77. $f(x) = \sqrt{x}$

$\lim_{x \to 0.25} \sqrt{x} = 0.5$ is true.

As x approaches $0.25 = \frac{1}{4}$ from either side, $f(x) = \sqrt{x}$ approaches $\frac{1}{2} = 0.5$.

78. $f(x) = \sqrt{x}$

$\lim_{x \to 0} \sqrt{x} = 0$ is false.

$f(x) = \sqrt{x}$ is not defined on an open interval containing 0 because the domain of f is $x \geq 0$.

79. Using a graphing utility, you see that

$\lim_{x \to 0} \dfrac{\sin x}{x} = 1$

$\lim_{x \to 0} \dfrac{\sin 2x}{x} = 2$, etc.

So, $\lim_{x \to 0} \dfrac{\sin nx}{x} = n$.

80. Using a graphing utility, you see that

$\lim_{x \to 0} \dfrac{\tan x}{x} = 1$

$\lim_{x \to 0} \dfrac{\tan 2x}{x} = 2$, etc.

So, $\lim_{x \to 0} \dfrac{\tan(nx)}{x} = n$.

81. If $\lim_{x \to c} f(x) = L_1$ and $\lim_{x \to c} f(x) = L_2$, then for every $\varepsilon > 0$, there exists $\delta_1 > 0$ and $\delta_2 > 0$ such that $|x - c| < \delta_1 \Rightarrow |f(x) - L_1| < \varepsilon$ and $|x - c| < \delta_2 \Rightarrow |f(x) - L_2| < \varepsilon$. Let δ equal the smaller of δ_1 and δ_2. Then for $|x - c| < \delta$, you have $|L_1 - L_2| = |L_1 - f(x) + f(x) - L_2| \leq |L_1 - f(x)| + |f(x) - L_2| < \varepsilon + \varepsilon$. Therefore, $|L_1 - L_2| < 2\varepsilon$. Since $\varepsilon > 0$ is arbitrary, it follows that $L_1 = L_2$.

82. $f(x) = mx + b, m \neq 0$. Let $\varepsilon > 0$ be given.

Take $\delta = \dfrac{\varepsilon}{|m|}$.

If $0 < |x - c| < \delta = \dfrac{\varepsilon}{|m|}$, then

$$|m||x - c| < \varepsilon$$

$$|mx - mc| < \varepsilon$$

$$|(mx + b) - (mc + b)| < \varepsilon$$

which shows that $\lim_{x \to c} (mx + b) = mc + b$.

83. $\lim_{x \to c} [f(x) - L] = 0$ means that for every $\varepsilon > 0$ there exists $\delta > 0$ such that if $0 < |x - c| < \delta$, then

$|(f(x) - L) - 0| < \varepsilon$.

This means the same as $|f(x) - L| < \varepsilon$ when $0 < |x - c| < \delta$.

So, $\lim_{x \to c} f(x) = L$.

84. (a) $(3x+1)(3x-1)x^2 + 0.01 = (9x^2-1)x^2 + \frac{1}{100}$

$= 9x^4 - x^2 + \frac{1}{100}$

$= \frac{1}{100}(10x^2-1)(90x^2-1)$

So, $(3x+1)(3x-1)x^2 + 0.01 > 0$ if

$10x^2 - 1 < 0$ and $90x^2 - 1 < 0$.

Let $(a, b) = \left(-\frac{1}{\sqrt{90}}, \frac{1}{\sqrt{90}}\right)$.

For all $x \neq 0$ in (a, b), the graph is positive.

You can verify this with a graphing utility.

(b) You are given $\lim_{x\to c} g(x) = L > 0$. Let $\varepsilon = \frac{1}{2}L$.

There exists $\delta > 0$ such that $0 < |x - c| < \delta$ implies that $|g(x) - L| < \varepsilon = \frac{L}{2}$. That is,

$$-\frac{L}{2} < g(x) - L < \frac{L}{2}$$
$$\frac{L}{2} < g(x) < \frac{3L}{2}$$

For x in the interval $(c-\delta, c+\delta)$, $x \neq c$, you have $g(x) > \frac{L}{2} > 0$, as desired.

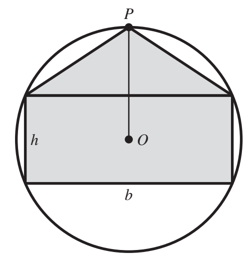
85. The radius OP has a length equal to the altitude z of the triangle plus $\frac{h}{2}$. So, $z = 1 - \frac{h}{2}$.

Area triangle $= \frac{1}{2}b\left(1 - \frac{h}{2}\right)$

Area rectangle $= bh$

Because these are equal,

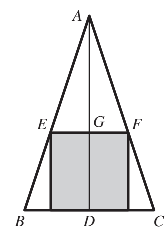
$$\frac{1}{2}b\left(1-\frac{h}{2}\right) = bh$$
$$1 - \frac{h}{2} = 2h$$
$$\frac{5}{2}h = 1$$
$$h = \frac{2}{5}.$$



86. Consider a cross section of the cone, where EF is a diagonal of the inscribed cube. $AD = 3$, $BC = 2$. Let x be the length of a side of the cube. Then $EF = x\sqrt{2}$.

By similar triangles,

$$\frac{EF}{BC} = \frac{AG}{AD}$$
$$\frac{x\sqrt{2}}{2} = \frac{3-x}{3}$$



Solving for x,

$$3\sqrt{2}x = 6 - 2x$$
$$(3\sqrt{2}+2)x = 6$$
$$x = \frac{6}{3\sqrt{2}+2} = \frac{9\sqrt{2}-6}{7} \approx 0.96.$$

Section 1.3 Evaluating Limits Analytically

1. For polynomial functions $p(x)$, substitute c for x, and simplify.

2. An indeterminant form is obtained when evaluating a limit using direct substitution produces a meaningless fractional expression such as $0/0$. That is,

$$\lim_{x\to c}\frac{f(x)}{g(x)}$$

for which $\lim_{x\to c} f(x) = \lim_{x\to c} g(x) = 0$

3. If a function f is squeezed between two functions h and g, $h(x) \le f(x) \le g(x)$, and h and g have the same limit L as $x \to c$, then $\lim_{x\to c} f(x)$ exists and equals L

4. $\lim_{x\to 0}\frac{\sin x}{x} = 1$

$\lim_{x\to 0}\frac{1-\cos x}{x} = 0$

5. $\lim_{x\to 2} x^3 = 2^3 = 8$

6. $\lim_{x\to -3} x^4 = (-3)^4 = 81$

7. $\lim_{x\to -3}(2x+5) = 2(-3) + 5 = -1$

8. $\lim_{x\to 9}(4x-1) = 4(9) - 1 = 36 - 1 = 35$

9. $\lim_{x\to -3}(x^2+3x) = (-3)^2 + 3(-3) = 9 - 9 = 0$

10. $\lim_{x\to 2}\left(-x^3+1\right) = \left(-2\right)^3 + 1 = -8 + 1 = -7$

11. $\lim_{x\to -3}\left(2x^2+4x+1\right) = 2\left(-3\right)^2 + 4\left(-3\right) + 1$

$= 18 - 12 + 1 = 7$

12. $\lim_{x\to 1}\left(2x^3-6x+5\right) = 2\left(1\right)^3 - 6\left(1\right) + 5$

$= 2 - 6 + 5 = 1$

13. $\lim_{x\to 3}\sqrt{x+8} = \sqrt{3+8} = \sqrt{11}$

14. $\lim_{x\to 2}\sqrt[3]{12x+3} = \sqrt[3]{12(2)+3}$

$= \sqrt[3]{24+3} = \sqrt[3]{27} = 3$

15. $\lim_{x\to -4}\left(1-x\right)^3 = \left[1-\left(-4\right)\right]^3 = 5^3 = 125$

16. $\lim_{x\to 0}\left(3x-2\right)^4 = \left(3(0)-2\right)^4 = \left(-2\right)^4 = 16$

17. $\lim_{x\to 2}\dfrac{3}{2x+1} = \dfrac{3}{2\left(2\right)+1} = \dfrac{3}{5}$

18. $\lim_{x\to -5}\dfrac{5}{x+3} = \dfrac{5}{-5+3} = -\dfrac{5}{2}$

19. $\lim_{x\to 1}\dfrac{x}{x^2+4} = \dfrac{1}{1^2+4} = \dfrac{1}{5}$

20. $\lim_{x\to 1}\dfrac{3x+5}{x+1} = \dfrac{3(1)+5}{1+1} = \dfrac{3+5}{2} = \dfrac{8}{2} = 4$

21. $\lim_{x\to 7}\dfrac{3x}{\sqrt{x+2}} = \dfrac{3(7)}{\sqrt{7+2}} = \dfrac{21}{3} = 7$

22. $\lim_{x\to 3}\dfrac{\sqrt{x+6}}{x+2} = \dfrac{\sqrt{3+6}}{3+2} = \dfrac{\sqrt{9}}{5} = \dfrac{3}{5}$

23. (a) $\lim_{x\to 1} f\left(x\right) = 5 - 1 = 4$

(b) $\lim_{x\to 4} g\left(x\right) = 4^3 = 64$

(c) $\lim_{x\to 1} g\left(f\left(x\right)\right) = g\left(f\left(1\right)\right) = g\left(4\right) = 64$

24. (a) $\lim_{x\to -3} f\left(x\right) = \left(-3\right) + 7 = 4$

(b) $\lim_{x\to 4} g\left(x\right) = 4^2 = 16$

(c) $\lim_{x\to -3} g\left(f\left(x\right)\right) = g\left(4\right) = 16$

25. (a) $\lim_{x\to 1} f\left(x\right) = 4 - 1 = 3$

(b) $\lim_{x\to 3} g\left(x\right) = \sqrt{3+1} = 2$

(c) $\lim_{x\to 1} g\left(f\left(x\right)\right) = g\left(3\right) = 2$

26. (a) $\lim_{x\to 4} f\left(x\right) = 2\left(4^2\right) - 3\left(4\right) + 1 = 21$

(b) $\lim_{x\to 21} g\left(x\right) = \sqrt[3]{21+6} = 3$

(c) $\lim_{x\to 4} g\left(f\left(x\right)\right) = g\left(21\right) = 3$

27. $\lim_{x\to \pi/2}\sin x = \sin\dfrac{\pi}{2} = 1$

28. $\lim_{x\to \pi}\tan x = \tan \pi = 0$

29. $\lim_{x\to 1}\cos\dfrac{\pi x}{3} = \cos\dfrac{\pi}{3} = \dfrac{1}{2}$

30. $\lim_{x\to 2}\sin\dfrac{\pi x}{12} = \sin\dfrac{\pi\left(2\right)}{12} = \sin\dfrac{\pi}{6} = \dfrac{1}{2}$

31. $\lim_{x\to 0}\sec 2x = \sec 0 = 1$

32. $\lim_{x\to \pi}\cos 3x = \cos 3\pi = -1$

33. $\lim_{x\to 5\pi/6}\sin x = \sin\dfrac{5\pi}{6} = \dfrac{1}{2}$

34. $\lim_{x\to 5\pi/3}\cos x = \cos\dfrac{5\pi}{3} = \dfrac{1}{2}$

35. $\lim_{x\to 3}\tan\left(\dfrac{\pi x}{4}\right) = \tan\dfrac{3\pi}{4} = -1$

36. $\lim_{x\to 7}\sec\left(\dfrac{\pi x}{6}\right) = \sec\dfrac{7\pi}{6} = \dfrac{-2\sqrt{3}}{3}$

37. $\lim_{x\to c} f\left(x\right) = \dfrac{2}{5}, \lim_{x\to c} g\left(x\right) = 2$

(a) $\lim_{x\to c}\left[5g\left(x\right)\right] = 5\lim_{x\to c} g\left(x\right) = 5\left(2\right) = 10$

(b) $\lim_{x\to c}\left[f\left(x\right) + g\left(x\right)\right] = \lim_{x\to c} f\left(x\right) + \lim_{x\to c} g\left(x\right) = \dfrac{2}{5} + 2 = \dfrac{12}{5}$

(c) $\lim_{x\to c}\left[f\left(x\right) + g\left(x\right)\right] = \left[\lim_{x\to c} f\left(x\right)\right] + \left[\lim_{x\to c} g\left(x\right)\right] = \dfrac{2}{5}\left(2\right) = \dfrac{4}{5}$

(d) $\lim_{x\to c}\dfrac{f\left(x\right)}{g\left(x\right)} = \dfrac{\lim_{x\to c} f\left(x\right)}{\lim_{x\to c} g\left(x\right)} = \dfrac{2/5}{2} = \dfrac{1}{5}$

38. $\lim_{x \to c} f(x) = 2, \lim_{x \to c} g(x) = \frac{3}{4}$

(a) $\lim_{x \to c} [4f(x)] = 4 \lim_{x \to c} f(x) = 4(2) = 8$

(b) $\lim_{x \to c} [f(x) + g(x)] = \lim_{x \to c} f(x) + \lim_{x \to c} g(x) = 2 + \frac{3}{4} = \frac{11}{4}$

(c) $\lim_{x \to c} [f(x)g(x)] = \left[\lim_{x \to c} f(x)\right]\left[\lim_{x \to c} g(x)\right] = 2\left(\frac{3}{4}\right) = \frac{3}{2}$

(d) $\lim_{x \to c} \frac{f(x)}{g(x)} = \frac{\lim_{x \to c} f(x)}{\lim_{x \to c} g(x)} = \frac{2}{(3/4)} = \frac{8}{3}$

39. $\lim_{x \to c} f(x) = 16$

(a) $\lim_{x \to c} [f(x)]^2 = \left[\lim_{x \to c} f(x)\right]^2 = (16)^2 = 256$

(b) $\lim_{x \to c} \sqrt{f(x)} = \sqrt{\lim_{x \to c} f(x)} = \sqrt{16} = 4$

(c) $\lim_{x \to c} [3f(x)] = 3\left[\lim_{x \to c} f(x)\right] = 3(16) = 48$

(d) $\lim_{x \to c} [f(x)]^{3/2} = \left[\lim_{x \to c} f(x)\right]^{3/2} = (16)^{3/2} = 64$

40. $\lim_{x \to c} f(x) = 27$

(a) $\lim_{x \to c} \sqrt[3]{f(x)} = \sqrt[3]{\lim_{x \to c} f(x)} = \sqrt[3]{27} = 3$

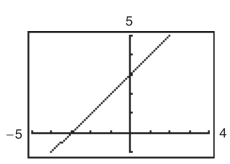
(b) $\lim_{x \to c} \frac{f(x)}{18} = \frac{\lim_{x \to c} f(x)}{\lim_{x \to c} 18} = \frac{27}{18} = \frac{3}{2}$

(c) $\lim_{x \to c} [f(x)]^2 = \left[\lim_{x \to c} f(x)\right]^2 = (27)^2 = 729$

(d) $\lim_{x \to c} [f(x)]^{2/3} = \left[\lim_{x \to c} f(x)\right]^{2/3} = (27)^{2/3} = 9$

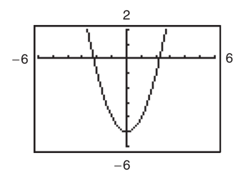
41. $f(x) = \frac{x^2 + 3x}{x} = \frac{x(x + 3)}{x}$ and $g(x) = x + 3$ agree except at $x = 0$.

$\lim_{x \to 0} f(x) = \lim_{x \to 0} g(x) = \lim_{x \to 0} (x + 3) = 0 + 3 = 3$



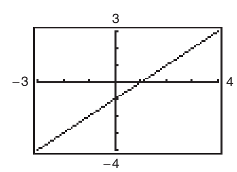
42. $f(x) = \frac{x^4 - 5x^2}{x^2} = \frac{x^2(x^2 - 5)}{x^2}$ and $g(x) = x^2 - 5$ agree except at $x = 0$.

$\lim_{x \to 0} f(x) = \lim_{x \to 0} g(x) = \lim_{x \to 0} (x^2 - 5) = 0^2 - 5 = -5$



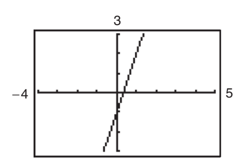
43. $f(x) = \frac{x^2 - 1}{x + 1} = \frac{(x + 1)(x - 1)}{x + 1}$ and $g(x) = x - 1$ agree except at $x = -1$.

$\lim_{x \to -1} f(x) = \lim_{x \to -1} g(x) = \lim_{x \to -1} (x - 1) = -1 - 1 = -2$



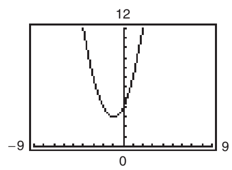
44. $f(x) = \frac{3x^2 + 5x - 2}{x + 2} = \frac{(x + 2)(3x - 1)}{x + 2}$ and $g(x) = 3x - 1$ agree except at $x = -2$.

$\lim_{x \to -2} f(x) = \lim_{x \to -2} g(x) = \lim_{x \to -2} (3x - 1)$
$= 3(-2) - 1 = -7$



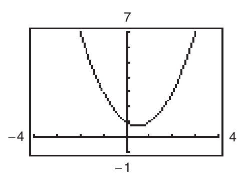
45. $f(x) = \dfrac{x^3 - 8}{x - 2}$ and $g(x) = x^2 + 2x + 4$ agree except at $x = 2$.

$$\lim_{x\to 2} f(x) = \lim_{x\to 2} g(x) = \lim_{x\to 2}\left(x^2 + 2x + 4\right) = 2^2 + 2(2) + 4 = 12$$



46. $f(x) = \dfrac{x^3 + 1}{x + 1}$ and $g(x) = x^2 - x + 1$ agree except at $x = -1$.

$$\lim_{x\to -1} f(x) = \lim_{x\to -1} g(x) = \lim_{x\to -1}\left(x^2 - x + 1\right) = (-1)^2 - (-1) + 1 = 3$$



47. $\displaystyle\lim_{x\to 0}\frac{x}{x^2 - x} = \lim_{x\to 0}\frac{x}{x(x-1)} = \lim_{x\to 0}\frac{1}{x-1} = \frac{1}{0-1} = -1$

48. $\displaystyle\lim_{x\to 0}\frac{7x^3 - x^2}{x} = \lim_{x\to 0}\left(7x^2 - x\right) = 0 - 0 = 0$

49. $\displaystyle\lim_{x\to 4}\frac{x-4}{x^2-16} = \lim_{x\to 4}\frac{x-4}{(x+4)(x-4)} = \lim_{x\to 4}\frac{1}{x+4} = \frac{1}{4+4} = \frac{1}{8}$

50. $\displaystyle\lim_{x\to 5}\frac{5-x}{x^2-25} = \lim_{x\to 5}\frac{-(x-5)}{(x-5)(x+5)} = \lim_{x\to 5}\frac{-1}{x+5} = \frac{-1}{5+5} = -\frac{1}{10}$

51. $\displaystyle\lim_{x\to -3}\frac{x^2+x-6}{x^2-9} = \lim_{x\to -3}\frac{(x+3)(x-2)}{(x+3)(x-3)} = \lim_{x\to -3}\frac{x-2}{x-3} = \frac{-3-2}{-3-3} = \frac{-5}{-6} = \frac{5}{6}$

52. $\displaystyle\lim_{x\to 2}\frac{x^2+2x-8}{x^2-x-2} = \lim_{x\to 2}\frac{(x-2)(x+4)}{(x-2)(x+1)} = \lim_{x\to 2}\frac{x+4}{x+1} = \frac{2+4}{2+1} = \frac{6}{3} = 2$

53. $\displaystyle\lim_{x\to 4}\frac{\sqrt{x+5}-3}{x-4} = \lim_{x\to 4}\frac{\sqrt{x+5}-3}{x-4}\cdot\frac{\sqrt{x+5}+3}{\sqrt{x+5}+3}$

$\displaystyle= \lim_{x\to 4}\frac{(x+5)-9}{(x-4)\left(\sqrt{x+5}+3\right)} = \lim_{x\to 4}\frac{1}{\sqrt{x+5}+3} = \frac{1}{\sqrt{9}+3} = \frac{1}{6}$

54. $\displaystyle\lim_{x\to 3}\frac{\sqrt{x+1}-2}{x-3} = \lim_{x\to 3}\frac{\sqrt{x+1}-2}{x-3}\cdot\frac{\sqrt{x+1}+2}{\sqrt{x+1}+2} = \lim_{x\to 3}\frac{x-3}{(x-3)\left[\sqrt{x+1}+2\right]}$

$\displaystyle= \lim_{x\to 3}\frac{1}{\sqrt{x+1}+2} = \frac{1}{\sqrt{4}+2} = \frac{1}{4}$

55. $\displaystyle\lim_{x\to 0}\frac{\sqrt{x+5}-\sqrt{5}}{x} = \lim_{x\to 0}\frac{\sqrt{x+5}-\sqrt{5}}{x}\cdot\frac{\sqrt{x+5}+\sqrt{5}}{\sqrt{x+5}+\sqrt{5}}$

$\displaystyle= \lim_{x\to 0}\frac{(x+5)-5}{x\left(\sqrt{x+5}+\sqrt{5}\right)} = \lim_{x\to 0}\frac{1}{\sqrt{x+5}+\sqrt{5}} = \frac{1}{\sqrt{5}+\sqrt{5}} = \frac{1}{2\sqrt{5}} = \frac{\sqrt{5}}{10}$

56. $\displaystyle\lim_{x\to 0}\frac{\sqrt{2+x}-\sqrt{2}}{x} = \lim_{x\to 0}\frac{\sqrt{2+x}-\sqrt{2}}{x}\cdot\frac{\sqrt{2+x}+\sqrt{2}}{\sqrt{2+x}+\sqrt{2}}$

$\displaystyle= \lim_{x\to 0}\frac{2+x-2}{\left(\sqrt{2+x}+\sqrt{2}\right)x} = \lim_{x\to 0}\frac{1}{\sqrt{2+x}+\sqrt{2}} = \frac{1}{\sqrt{2}+\sqrt{2}} = \frac{1}{2\sqrt{2}} = \frac{\sqrt{2}}{4}$

57. $\lim_{x\to 0} \dfrac{\dfrac{1}{3+x} - \dfrac{1}{3}}{x} = \lim_{x\to 0} \dfrac{3-(3+x)}{(3+x)3(x)} = \lim_{x\to 0} \dfrac{-x}{(3+x)(3)(x)} = \lim_{x\to 0} \dfrac{-1}{(3+x)3} = \dfrac{-1}{(3)3} = -\dfrac{1}{9}$

58. $\lim_{x\to 0} \dfrac{\dfrac{1}{x+4} - \dfrac{1}{4}}{x} = \lim_{x\to 0} \dfrac{\dfrac{4-(x+4)}{4(x+4)}}{x}$

$= \lim_{x\to 0} \dfrac{-1}{4(x+4)} = \dfrac{-1}{4(4)} = -\dfrac{1}{16}$

59. $\lim_{\Delta x\to 0} \dfrac{2(x+\Delta x) - 2x}{\Delta x} = \lim_{\Delta x\to 0} \dfrac{2x + 2\Delta x - 2x}{\Delta x} = \lim_{\Delta x\to 0} \dfrac{2\Delta x}{\Delta x} = \lim_{\Delta x\to 0} 2 = 2$

60. $\lim_{\Delta x\to 0} \dfrac{(x+\Delta x)^2 - x^2}{\Delta x} = \lim_{\Delta x\to 0} \dfrac{x^2 + 2x\Delta x + (\Delta x)^2 - x^2}{\Delta x} = \lim_{\Delta x\to 0} \dfrac{\Delta x(2x+\Delta x)}{\Delta x} = \lim_{\Delta x\to 0} (2x+\Delta x) = 2x$

61. $\lim_{\Delta x\to 0} \dfrac{(x+\Delta x)^2 - 2(x+\Delta x) + 1 - (x^2 - 2x + 1)}{\Delta x} = \lim_{\Delta x\to 0} \dfrac{x^2 + 2x\Delta x + (\Delta x)^2 - 2x - 2\Delta x + 1 - x^2 + 2x - 1}{\Delta x}$

$= \lim_{\Delta x\to 0} (2x + \Delta x - 2) = 2x - 2$

62. $\lim_{\Delta x\to 0} \dfrac{(x+\Delta x)^3 - x^3}{\Delta x} = \lim_{\Delta x\to 0} \dfrac{x^3 + 3x^2\Delta x + 3x(\Delta x)^2 + (\Delta x)^3 - x^3}{\Delta x}$

$= \lim_{\Delta x\to 0} \dfrac{\Delta x\left(3x^2 + 3x\Delta x + (\Delta x)^2\right)}{\Delta x} = \lim_{\Delta x\to 0} \left(3x^2 + 3x\Delta x + (\Delta x)^2\right) = 3x^2$

63. $\lim_{x\to 0} \dfrac{\sin x}{5x} = \lim_{x\to 0} \left[\left(\dfrac{\sin x}{x}\right)\left(\dfrac{1}{5}\right)\right] = (1)\left(\dfrac{1}{5}\right) = \dfrac{1}{5}$

64. $\lim_{x\to 0} \dfrac{3(1-\cos x)}{x} = \lim_{x\to 0} \left[3\left(\dfrac{(1-\cos x)}{x}\right)\right] = (3)(0) = 0$

65. $\lim_{x\to 0} \dfrac{(\sin x)(1-\cos x)}{x^2} = \lim_{x\to 0} \left[\dfrac{\sin x}{x} \cdot \dfrac{1-\cos x}{x}\right]$

$= (1)(0) = 0$

66. $\lim_{\theta\to 0} \dfrac{\cos\theta \tan\theta}{\theta} = \lim_{\theta\to 0} \dfrac{\sin\theta}{\theta} = 1$

67. $\lim_{x\to 0} \dfrac{\sin^2 x}{x} = \lim_{x\to 0} \left[\dfrac{\sin x}{x} \sin x\right] = (1)\sin 0 = 0$

68. $\lim_{x\to 0} \dfrac{\tan^2 x}{x} = \lim_{x\to 0} \dfrac{\sin^2 x}{x\cos^2 x} = \lim_{x\to 0} \left[\dfrac{\sin x}{x} \cdot \dfrac{\sin x}{\cos^2 x}\right]$

$= (1)(0) = 0$

69. $\lim_{h\to 0} \dfrac{(1-\cos h)^2}{h} = \lim_{h\to 0} \left[\dfrac{1-\cos h}{h}(1-\cos h)\right]$

$= (0)(0) = 0$

70. $\lim_{\phi\to\pi} \phi \sec\phi = \pi(-1) = -\pi$

71. $\lim_{x\to 0} \dfrac{6 - 6\cos x}{3} = \dfrac{6 - 6\cos 0}{3} = \dfrac{6-6}{3} = 0$

72. $\lim_{x\to 0} \dfrac{\cos x - \sin x - 1}{2x} = \lim_{x\to 0} \dfrac{-\sin x}{2x} + \lim_{x\to 0} \dfrac{\cos x - 1}{2x}$

$= -\dfrac{1}{2} \lim_{x\to 0} \dfrac{\sin x}{x} - \dfrac{1}{2} \lim_{x\to 0} \dfrac{1-\cos x}{x}$

$= -\dfrac{1}{2}(1) - \dfrac{1}{2}(0) = -\dfrac{1}{2}$

73. $\lim_{t\to 0}\frac{\sin 3t}{2t} = \lim_{t\to 0}\left(\frac{\sin 3t}{3t}\right)\left(\frac{3}{2}\right) = (1)\left(\frac{3}{2}\right) = \frac{3}{2}$

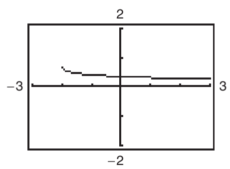
74. $\lim_{x\to 0}\frac{\sin 2x}{\sin 3x} = \lim_{x\to 0}\left[2\left(\frac{\sin 2x}{2x}\right)\left(\frac{1}{3}\right)\left(\frac{3x}{\sin 3x}\right)\right]$

$= 2(1)\left(\frac{1}{3}\right)(1) = \frac{2}{3}$

75. $f(x) = \frac{\sqrt{x+2}-\sqrt{2}}{x}$

x	–0.1	–0.01	–0.001	0	0.001	0.01	0.1
$f(x)$	0.358	0.354	0.354	?	0.354	0.353	0.349

It appears that the limit is 0.354.



The graph has a hole at $x = 0$.

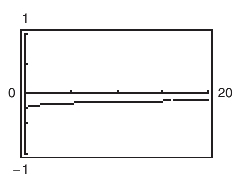
Analytically, $\lim_{x\to 0}\frac{\sqrt{x+2}-\sqrt{2}}{x} = \lim_{x\to 0}\frac{\sqrt{x+2}-\sqrt{2}}{x}\cdot\frac{\sqrt{x+2}+\sqrt{2}}{\sqrt{x+2}+\sqrt{2}}$

$= \lim_{x\to 0}\frac{x+2-2}{x\left(\sqrt{x+2}+\sqrt{2}\right)} = \lim_{x\to 0}\frac{1}{\sqrt{x+2}+\sqrt{2}} = \frac{1}{2\sqrt{2}} = \frac{\sqrt{2}}{4} \approx 0.354.$

76. $f(x) = \frac{4-\sqrt{x}}{x-16}$

x	15.9	15.99	15.999	16	16.001	16.01	16.1
$f(x)$	–0.1252	–0.125	–0.125	?	–0.125	–0.125	–0.1248

It appears that the limit is –0.125.



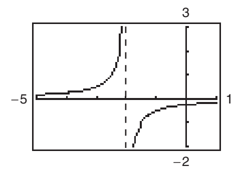
The graph has a hole at $x = 16$.

Analytically, $\lim_{x\to 16}\frac{4-\sqrt{x}}{x-16} = \lim_{x\to 16}\frac{\left(4-\sqrt{x}\right)}{\left(\sqrt{x}+4\right)\left(\sqrt{x}-4\right)} = \lim_{x\to 16}\frac{-1}{\sqrt{x}+4} = -\frac{1}{8}.$

77. $f(x) = \dfrac{\dfrac{1}{2+x} - \dfrac{1}{2}}{x}$

x	–0.1	–0.01	–0.001	0	0.001	0.01	0.1
$f(x)$	–0.263	–0.251	–0.250	?	–0.250	–0.249	–0.238

It appears that the limit is –0.250.



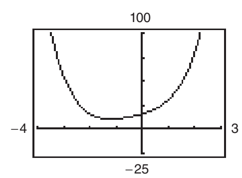
The graph has a hole at $x = 0$.

Analytically, $\displaystyle\lim_{x\to 0} \frac{\dfrac{1}{2+x} - \dfrac{1}{2}}{x} = \lim_{x\to 0} \frac{2-(2+x)}{2(2+x)} \cdot \frac{1}{x} = \lim_{x\to 0} \frac{-x}{2(2+x)} \cdot \frac{1}{x} = \lim_{x\to 0} \frac{-1}{2(2+x)} = -\frac{1}{4}.$

78. $f(x) = \dfrac{x^5 - 32}{x - 2}$

x	1.9	1.99	1.999	1.9999	2.0	2.0001	2.001	2.01	2.1
$f(x)$	72.39	79.20	79.92	79.99	?	80.01	80.08	80.80	88.41

It appears that the limit is 80.



The graph has a hole at $x = 2$.

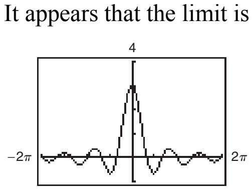
Analytically, $\displaystyle\lim_{x\to 2} \frac{x^5 - 32}{x - 2} = \lim_{x\to 2} \frac{(x-2)(x^4 + 2x^3 + 4x^2 + 8x + 16)}{x - 2} = \lim_{x\to 2}(x^4 + 2x^3 + 4x^2 + 8x + 16) = 80.$

(*Hint:* Use long division to factor $x^5 - 32$.)

79. $f(t) = \dfrac{\sin 3t}{t}$

t	–0.1	–0.01	–0.001	0	0.001	0.01	0.1
$f(t)$	2.96	2.9996	3	?	3	2.9996	2.96

It appears that the limit is 3.



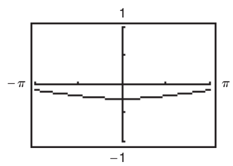
The graph has a hole at $t = 0$.

Analytically, $\displaystyle\lim_{t\to 0} \frac{\sin 3t}{t} = \lim_{t\to 0} 3\left(\frac{\sin 3t}{3t}\right) = 3(1) = 3.$

80. $f(x) = \dfrac{\cos x - 1}{2x^2}$

x	–1	–0.1	–0.01	0.01	0.1	1
$f(x)$	–0.2298	–0.2498	–0.25	–0.25	–0.2498	–0.2298

It appears that the limit is –0.25.



The graph has a hole at $x = 0$.

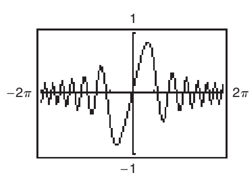
Analytically, $\dfrac{\cos x - 1}{2x^2} \cdot \dfrac{\cos x + 1}{\cos x + 1} = \dfrac{\cos^2 x - 1}{2x^2(\cos x + 1)} = \dfrac{-\sin^2 x}{2x^2(\cos x + 1)} = \dfrac{\sin^2 x}{x^2} \cdot \dfrac{-1}{2(\cos x + 1)}$

$$\lim_{x \to 0}\left[\frac{\sin^2 x}{x^2} \cdot \frac{-1}{2(\cos x + 1)}\right] = 1\left(\frac{-1}{4}\right) = -\frac{1}{4} = -0.25$$

81. $f(x) = \dfrac{\sin x^2}{x}$

x	–0.1	–0.01	–0.001	0	0.001	0.01	0.1
$f(x)$	–0.099998	–0.01	–0.001	?	0.001	0.01	0.099998

It appears that the limit is 0.



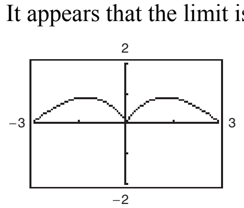
The graph has a hole at $x = 0$.

Analytically, $\displaystyle\lim_{x \to 0} \frac{\sin x^2}{x} = \lim_{x \to 0} x\left(\frac{\sin x^2}{x^2}\right) = 0(1) = 0.$

82. $f(x) = \dfrac{\sin x}{\sqrt[3]{x}}$

x	–0.1	–0.01	–0.001	0	0.001	0.01	0.1
$f(x)$	0.215	0.0464	0.01	?	0.01	0.0464	0.215

It appears that the limit is 0.



The graph has a hole at $x = 0$.

Analytically, $\displaystyle\lim_{x \to 0} \frac{\sin x}{\sqrt[3]{x}} = \lim_{x \to 0} \sqrt[3]{x^2}\left(\frac{\sin x}{x}\right) = (0)(1) = 0.$

83. $f(x) = 3x - 2$

$$\lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} = \lim_{\Delta x \to 0} \frac{3(x + \Delta x) - 2 - (3x - 2)}{\Delta x} = \lim_{\Delta x \to 0} \frac{3x + 3\Delta x - 2 - 3x + 2}{\Delta x} = \lim_{\Delta x \to 0} \frac{3\Delta x}{\Delta x} = 3$$

84. $f(x) = -6x + 3$

$$\lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} = \lim_{\Delta x \to 0} \frac{[-6(x + \Delta x) + 3] - [-6x + 3]}{\Delta x} = \lim_{\Delta x \to 0} \frac{-6x - 6\Delta x + 3 + 6x - 3}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{-6\Delta x}{\Delta x} = \lim_{\Delta x \to 0} (-6) = -6$$

85. $f(x) = x^2 - 4x$

$$\lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} = \lim_{\Delta x \to 0} \frac{(x + \Delta x)^2 - 4(x + \Delta x) - (x^2 - 4x)}{\Delta x} = \lim_{\Delta x \to 0} \frac{x^2 + 2x\Delta x + \Delta x^2 - 4x - 4\Delta x - x^2 + 4x}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{\Delta x(2x + \Delta x - 4)}{\Delta x} = \lim_{\Delta x \to 0} (2x + \Delta x - 4) = 2x - 4$$

86. $f(x) = 3x^2 + 1$

$$\lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} = \lim_{\Delta x \to 0} \frac{\left[3(x + \Delta x)^2 + 1\right] - \left[3x^2 + 1\right]}{\Delta x} = \lim_{\Delta x \to 0} \frac{(3x^2 + 6x\Delta x + 1) - (3x^2 + 1)}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{6x\Delta x}{\Delta x} = \lim_{\Delta x \to 0} 6x = 6x$$

87. $f(x) = 2\sqrt{x}$

$$\lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} = \lim_{\Delta x \to 0} \frac{2\sqrt{x + \Delta x} - 2\sqrt{x}}{\Delta x} = \lim_{\Delta x \to 0} \frac{2\left(\sqrt{x + \Delta x} - \sqrt{x}\right)}{\Delta x} \cdot \frac{\sqrt{x + \Delta x} + \sqrt{x}}{\sqrt{x + \Delta x} + \sqrt{x}}$$
$$= \lim_{\Delta x \to 0} \frac{2(x + \Delta x - x)}{\Delta x\left(\sqrt{x + \Delta x} + \sqrt{x}\right)} = \lim_{\Delta x \to 0} \frac{2\Delta x}{\Delta x\left(\sqrt{x + \Delta x} + \sqrt{x}\right)}$$
$$= \lim_{\Delta x \to 0} \frac{2}{\sqrt{x + \Delta x} + \sqrt{x}} = \frac{2}{2\sqrt{x}} = \frac{1}{\sqrt{x}} = x^{-1/2}$$

88. $f(x) = \sqrt{x} - 5$

$$\lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} = \lim_{\Delta x \to 0} \frac{\left(\sqrt{x + \Delta x} - 5\right) - \left(\sqrt{x} - 5\right)}{\Delta x} = \lim_{\Delta x \to 0} \frac{\sqrt{x + \Delta x} - \sqrt{x}}{\Delta x} \cdot \frac{\sqrt{x + \Delta x} + \sqrt{x}}{\sqrt{x + \Delta x} + \sqrt{x}}$$
$$= \lim_{\Delta x \to 0} \frac{(x + \Delta x) - x}{\Delta x\left(\sqrt{x + \Delta x} + \sqrt{x}\right)} = \lim_{\Delta x \to 0} \frac{\Delta x}{\Delta x\left(\sqrt{x + \Delta x} + \sqrt{x}\right)}$$
$$= \lim_{\Delta x \to 0} \frac{1}{\sqrt{x + \Delta x} + \sqrt{x}} = \frac{1}{2\sqrt{x}}$$

89. $f(x) = \dfrac{1}{x + 3}$

$$\lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} = \lim_{\Delta x \to 0} \frac{\dfrac{1}{x + \Delta x + 3} - \dfrac{1}{x + 3}}{\Delta x} = \lim_{\Delta x \to 0} \frac{x + 3 - (x + \Delta x + 3)}{(x + \Delta x + 3)(x + 3)} \cdot \frac{1}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{-\Delta x}{(x + \Delta x + 3)(x + 3)\Delta x} = \lim_{\Delta x \to 0} \frac{-1}{(x + \Delta x + 3)(x + 3)} = \frac{-1}{(x + 3)^2}$$

90. $f(x) = \dfrac{1}{x^2}$

$$\lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} = \lim_{\Delta x \to 0} \frac{\dfrac{1}{(x + \Delta x)^2} - \dfrac{1}{x^2}}{\Delta x} = \lim_{\Delta x \to 0} \frac{x^2 - (x + \Delta x)^2}{x^2(x + \Delta x)^2 \Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{x^2 - \left[x^2 + 2x\Delta x + (\Delta x)^2\right]}{x^2(x + \Delta x)^2 \Delta x} = \lim_{\Delta x \to 0} \frac{-2x\Delta x - (\Delta x)^2}{x^2(x + \Delta x)^2 \Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{-2x - \Delta x}{x^2(x + \Delta x)^2} = \frac{-2x}{x^4} = -\frac{2}{x^3}$$

91. $\lim_{x \to 0} \left(4 - x^2\right) \le \lim_{x \to 0} f(x) \le \lim_{x \to 0} \left(3 + x^2\right)$

$$4 \le \lim_{x \to 0} f(x) \le 4$$

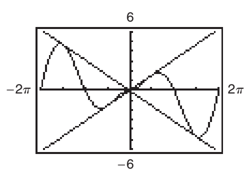
Therefore, $\lim_{x \to 0} f(x) = 4$.

92. $\lim_{x \to a} \left[b - |x - a|\right] \le \lim_{x \to a} f(x) \le \lim_{x \to a} \left[b + |x - a|\right]$

$$b \le \lim_{x \to a} f(x) \le b$$

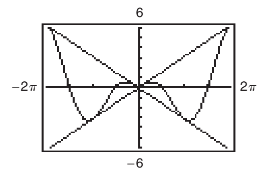
Therefore, $\lim_{x \to a} f(x) = b$.

93. $f(x) = |x| \sin x$



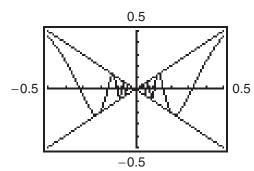
$\lim_{x \to 0} |x| \sin x = 0$

94. $f(x) = |x| \cos x$



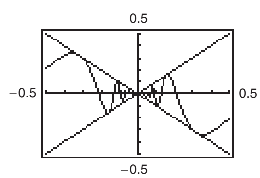
$\lim_{x \to 0} |x| \cos x = 0$

95. $f(x) = x \sin \dfrac{1}{x}$



$\lim_{x \to 0} \left(x \sin \dfrac{1}{x}\right) = 0$

96. $f(x) = x \cos \dfrac{1}{x}$



$\lim_{x \to 0} \left(x \cos \dfrac{1}{x}\right) = 0$

97. (a) Two functions f and g agree at all but one point (on an open interval) if $f(x) = g(x)$ for all x in the interval except for $x = c$, where c is in the interval.

(b) $f(x) = \dfrac{x^2 - 1}{x - 1} = \dfrac{(x + 1)(x - 1)}{x - 1}$ and $g(x) = x + 1$ agree at all points except $x = 1$.

(Other answers possible.)

98. Answers will vary. *Sample answers:*

(a) linear: $f(x) = \frac{1}{2}x$; $\lim_{x \to 8} \frac{1}{2}x = \frac{1}{2}(8) = 4$

(b) polynomial of degree 2: $f(x) = x^2 - 60$; $\lim_{x \to 8} \left(x^2 - 60\right) = 8^2 - 60 = 4$

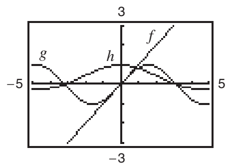
(c) rational: $f(x) = \dfrac{x}{2x - 14}$; $\lim_{x \to 8} \dfrac{x}{2x - 14} = \dfrac{8}{2(8) - 14} = \dfrac{8}{2} = 4$

(d) radical: $f(x) = \sqrt{x + 8}$; $\lim_{x \to 8} \sqrt{x + 8} = \sqrt{8 + 8} = \sqrt{16} = 4$

(e) cosine: $f(x) = 4 \cos(\pi x)$; $\lim_{x \to 8} 4 \cos(\pi x) = 4 \cos 8\pi = 4(1) = 4$

(f) sine: $f(x) = 4 \sin\left(\dfrac{\pi}{16}x\right)$; $\lim_{x \to 8} 4 \sin\left(\dfrac{\pi}{16}x\right) = 4 \sin \dfrac{\pi}{2} = 4(1) = 4$

99. $f(x) = x, g(x) = \sin x, h(x) = \dfrac{\sin x}{x}$



When the x-values are "close to" 0 the magnitude of f is approximately equal to the magnitude of g. So, $|g|/|f| \approx 1$ when x is "close to" 0.

100. (a) Use the dividing out technique because the numerator and denominator have a common factor.

(b) Use the rationalizing technique because the numerator involves a radical expression.

101. $s(t) = -16t^2 + 500$

$$\lim_{t\to 2}\frac{s(2)-s(t)}{2-t} = \lim_{t\to 2}\frac{-16(2)^2 + 500 - (-16t^2 + 500)}{2-t}$$
$$= \lim_{t\to 2}\frac{436 + 16t^2 - 500}{2-t}$$
$$= \lim_{t\to 2}\frac{16(t^2-4)}{2-t}$$
$$= \lim_{t\to 2}\frac{16(t-2)(t+2)}{2-t}$$
$$= \lim_{t\to 2} -16(t+2) = -64 \text{ ft/sec}$$

The paint can is falling at about 64 feet/second.

102. $s(t) = -16t^2 + 500 = 0$ when $t = \sqrt{\dfrac{500}{16}} = \dfrac{5\sqrt{5}}{2}$ sec. The velocity at time $a = \dfrac{5\sqrt{5}}{2}$ is

$$\lim_{t\to\left(\frac{5\sqrt{5}}{2}\right)}\frac{s\left(\frac{5\sqrt{5}}{2}\right) - s(t)}{\frac{5\sqrt{5}}{2} - t} = \lim_{t\to\left(\frac{5\sqrt{5}}{2}\right)}\frac{0 - (-16t^2 + 500)}{\frac{5\sqrt{5}}{2} - t}$$
$$= \lim_{t\to\left(\frac{5\sqrt{5}}{2}\right)}\frac{16\left(t^2 - \frac{125}{4}\right)}{\frac{5\sqrt{5}}{2} - t}$$
$$= \lim_{t\to\left(\frac{5\sqrt{5}}{2}\right)}\frac{16\left(t + \frac{5\sqrt{5}}{2}\right)\left(t - \frac{5\sqrt{5}}{2}\right)}{\frac{5\sqrt{5}}{2} - t}$$
$$= \lim_{t\to\frac{5\sqrt{5}}{2}}\left[-16\left(t + \frac{5\sqrt{5}}{2}\right)\right] = -80\sqrt{5} \text{ ft/sec}$$
$$\approx -178.9 \text{ ft/sec.}$$

The velocity of the paint can when it hits the ground is about 178.9 ft/sec.

103. $s(t) = -4.9t^2 + 200$

$$\lim_{t\to 3}\frac{s(3)-s(t)}{3-t} = \lim_{t\to 3}\frac{-4.9(3)^2 + 200 - (-4.9t^2 + 200)}{3-t}$$
$$= \lim_{t\to 3}\frac{4.9(t^2-9)}{3-t}$$
$$= \lim_{t\to 3}\frac{4.9(t-3)(t+3)}{3-t}$$
$$= \lim_{t\to 3}\left[-4.9(t+3)\right]$$
$$= -29.4 \text{ m/sec}$$

The object is falling about 29.4 m/sec.

104. $-4.9t^2 + 200 = 0$ when $t = \sqrt{\frac{200}{4.9}} = \frac{20\sqrt{5}}{7}$ sec. The velocity at time $a = \frac{20\sqrt{5}}{7}$ is

$$\lim_{t\to a}\frac{s(a) - s(t)}{a - t} = \lim_{t\to a}\frac{0 - \left[-4.9t^2 + 200\right]}{a - t}$$
$$= \lim_{t\to a}\frac{4.9(t + a)(t - a)}{a - t}$$
$$= \lim_{t\to\frac{20\sqrt{5}}{7}}\left[-4.9\left(t + \frac{20\sqrt{5}}{7}\right)\right] = -28\sqrt{5}\text{ m/sec}$$
$$\approx -62.6\text{ m/sec.}$$

The velocity of the object when it hits the ground is about 62.6 m/sec.

105. Let $f(x) = 1/x$ and $g(x) = -1/x$. $\lim_{x\to 0} f(x)$ and $\lim_{x\to 0} g(x)$ do not exist. However,

$\lim_{x\to 0}\left[f(x) + g(x)\right] = \lim_{x\to 0}\left[\frac{1}{x} + \left(-\frac{1}{x}\right)\right] = \lim_{x\to 0}[0] = 0$ and therefore does not exist.

106. Suppose, on the contrary, that $\lim_{x\to c} g(x)$ exists. Then, because $\lim_{x\to c} f(x)$ exists, so would $\lim_{x\to c}\left[f(x) + g(x)\right]$, which is a contradiction. So, $\lim_{x\to c} g(x)$ does not exist.

107. Given $f(x) = b$, show that for every $\varepsilon > 0$ there exists a $\delta > 0$ such that $\left|f(x) - b\right| < \varepsilon$ whenever $\left|x - c\right| < \delta$. Because $\left|f(x) - b\right| = \left|b - b\right| = 0 < \varepsilon$ for every $\varepsilon > 0$, any value of $\delta > 0$ will work.

108. Given $f(x) = x^n$, n is a positive integer, then

$$\lim_{x\to c} x^n = \lim_{x\to c}\left(xx^{n-1}\right)$$
$$= \left[\lim_{x\to c} x\right]\left[\lim_{x\to c} x^{n-1}\right] = c\left[\lim_{x\to c}\left(xx^{n-2}\right)\right]$$
$$= c\left[\lim_{x\to c} x\right]\left[\lim_{x\to c} x^{n-2}\right] = c(c)\lim_{x\to c}\left(xx^{n-3}\right)$$
$$= \cdots = c^n.$$

109. If $b = 0$, the property is true because both sides are equal to 0. If $b \neq 0$, let $\varepsilon > 0$ be given. Because $\lim_{x\to c} f(x) = L$, there exists $\delta > 0$ such that $\left|f(x) - L\right| < \varepsilon/\left|b\right|$ whenever $0 < \left|x - c\right| < \delta$. So, whenever $0 < \left|x - c\right| < \delta$, we have

$$\left|b\right|\left|f(x) - L\right| < \varepsilon \quad\text{or}\quad \left|bf(x) - bL\right| < \varepsilon$$

which implies that $\lim_{x\to c}\left[bf(x)\right] = bL$.

110. Given $\lim_{x\to c} f(x) = 0$:

For every $\varepsilon > 0$, there exists $\delta > 0$ such that $\left|f(x) - 0\right| < \varepsilon$ whenever $0 < \left|x - c\right| < \delta$.

Now $\left|f(x) - 0\right| = \left|f(x)\right| = \left|\left|f(x)\right| - 0\right| < \varepsilon$ for $\left|x - c\right| < \delta$. Therefore, $\lim_{x\to c}\left|f(x)\right| = 0$.

111.

$$-M\left|f(x)\right| \leq f(x)g(x) \leq M\left|f(x)\right|$$
$$\lim_{x\to c}\left(-M\left|f(x)\right|\right) \leq \lim_{x\to c}\left[f(x)g(x)\right] \leq \lim_{x\to c}\left(M\left|f(x)\right|\right)$$
$$-M(0) \leq \lim_{x\to c}\left[f(x)g(x)\right] \leq M(0)$$
$$0 \leq \lim_{x\to c}\left[f(x)g(x)\right] \leq 0$$

Therefore, $\lim_{x\to c}\left[f(x)g(x)\right] = 0$.

112. (a) If $\lim_{x\to c}\left|f(x)\right| = 0$, then $\lim_{x\to c}\left[-\left|f(x)\right|\right] = 0$.

$$-\left|f(x)\right| \leq f(x) \leq \left|f(x)\right|$$
$$\lim_{x\to c}\left[-\left|f(x)\right|\right] \leq \lim_{x\to c} f(x) \leq \lim_{x\to c}\left|f(x)\right|$$
$$0 \leq \lim_{x\to c} f(x) \leq 0$$

Therefore, $\lim_{x\to c} f(x) = 0$.

(b) Given $\lim_{x\to c} f(x) = L$:

For every $\varepsilon > 0$, there exists $\delta > 0$ such that $\left|f(x) - L\right| < \varepsilon$ whenever $0 < \left|x - c\right| < \delta$. Since $\left|\left|f(x)\right| - \left|L\right|\right| \leq \left|f(x) - L\right| < \varepsilon$ for $\left|x - c\right| < \delta$, then $\lim_{x\to c}\left|f(x)\right| = \left|L\right|$.

113. Let

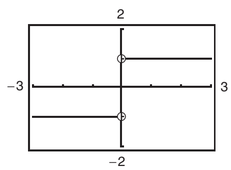
$$f(x) = \begin{cases} 4, & \text{if } x \geq 0 \\ -4, & \text{if } x < 0 \end{cases}$$

$\lim_{x \to 0} |f(x)| = \lim_{x \to 0} 4 = 4.$

$\lim_{x \to 0} f(x)$ does not exist because for $x < 0, f(x) = -4$ and for $x \geq 0, f(x) = 4$.

114. The graphing utility was set in degree mode, instead of *radian* mode.

115. The limit does not exist because the function approaches 1 from the right side of 0 and approaches -1 from the left side of 0.



116. False. $\lim_{x \to \pi} \frac{\sin x}{x} = \frac{0}{\pi} = 0$

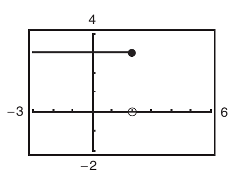
117. True.

118. False. Let

$$f(x) = \begin{cases} x & x \neq 1 \\ 3 & x = 1 \end{cases}, \quad c = 1.$$

Then $\lim_{x \to 1} f(x) = 1$ but $f(1) \neq 1$.

119. False. The limit does not exist because $f(x)$ approaches 3 from the left side of 2 and approaches 0 from the right side of 2.



120. False. Let $f(x) = \frac{1}{2}x^2$ and $g(x) = x^2$.

Then $f(x) < g(x)$ for all $x \neq 0$. But

$\lim_{x \to 0} f(x) = \lim_{x \to 0} g(x) = 0.$

121.
$$\begin{aligned} \lim_{x \to 0} \frac{1 - \cos x}{x} &= \lim_{x \to 0} \frac{1 - \cos x}{x} \cdot \frac{1 + \cos x}{1 + \cos x} \\ &= \lim_{x \to 0} \frac{1 - \cos^2 x}{x(1 + \cos x)} = \lim_{x \to 0} \frac{\sin^2 x}{x(1 + \cos x)} \\ &= \lim_{x \to 0} \frac{\sin x}{x} \cdot \frac{\sin x}{1 + \cos x} \\ &= \left[\lim_{x \to 0} \frac{\sin x}{x}\right]\left[\lim_{x \to 0} \frac{\sin x}{1 + \cos x}\right] \\ &= (1)(0) = 0 \end{aligned}$$

122.
$$f(x) = \begin{cases} 0, & \text{if } x \text{ is rational} \\ 1, & \text{if } x \text{ is irrational} \end{cases}$$

$$g(x) = \begin{cases} 0, & \text{if } x \text{ is rational} \\ x, & \text{if } x \text{ is irrational} \end{cases}$$

$\lim_{x \to 0} f(x)$ does not exist.

No matter how "close to" 0 x is, there are still an infinite number of rational and irrational numbers so that $\lim_{x \to 0} f(x)$ does not exist.

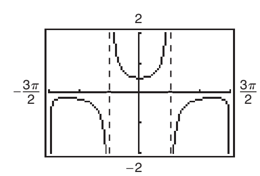
$\lim_{x \to 0} g(x) = 0$

when x is "close to" 0, both parts of the function are "close to" 0.

123. $f(x) = \frac{\sec x - 1}{x^2}$

(a) The domain of f is all $x \neq 0, \pi/2 + n\pi$.

(b)



The domain is not obvious. The hole at $x = 0$ is not apparent.

(c) $\lim_{x \to 0} f(x) = \frac{1}{2}$

(d)
$$\begin{aligned} \frac{\sec x - 1}{x^2} &= \frac{\sec x - 1}{x^2} \cdot \frac{\sec x + 1}{\sec x + 1} = \frac{\sec^2 x - 1}{x^2(\sec x + 1)} \\ &= \frac{\tan^2 x}{x^2(\sec x + 1)} = \frac{1}{\cos^2 x}\left(\frac{\sin^2 x}{x^2}\right)\frac{1}{\sec x + 1} \end{aligned}$$

So,
$$\begin{aligned} \lim_{x \to 0} \frac{\sec x - 1}{x^2} &= \lim_{x \to 0} \frac{1}{\cos^2 x}\left(\frac{\sin^2 x}{x^2}\right)\frac{1}{\sec x + 1} \\ &= 1(1)\left(\frac{1}{2}\right) = \frac{1}{2}. \end{aligned}$$

124. (a) $\lim_{x \to 0} \frac{1 - \cos x}{x^2} = \lim_{x \to 0} \frac{1 - \cos x}{x^2} \cdot \frac{1 + \cos x}{1 + \cos x}$

$= \lim_{x \to 0} \frac{1 - \cos^2 x}{x^2(1 + \cos x)}$

$= \lim_{x \to 0} \frac{\sin^2 x}{x^2} \cdot \frac{1}{1 + \cos x}$

$= (1)\left(\frac{1}{2}\right) = \frac{1}{2}$

(b) From part (a),

$\frac{1 - \cos x}{x^2} \approx \frac{1}{2} \Rightarrow 1 - \cos x$

$\approx \frac{1}{2}x^2 \Rightarrow \cos x$

$\approx 1 - \frac{1}{2}x^2$ for x

≈ 0.

(c) $\cos(0.1) \approx 1 - \frac{1}{2}(0.1)^2 = 0.995$

(d) $\cos(0.1) \approx 0.9950$, which agrees with part (c).

Section 1.4 Continuity and One-Sided Limits

1. A function f is continuous at a point c if there is no interruption of the graph at c.

2. $c = -1$ because $\lim_{x \to -1^+} 2\sqrt{x + 1} = 2\sqrt{-1 + 1} = 0$

3. The limit exists because the limit from the left and the limit from the right and equivalent.

4. If f is continuous on a close interval $[a, b]$ and $f(a) \neq f(b)$, then f takes on all values between $f(a)$ and $f(b)$.

5. (a) $\lim_{x \to 4^+} f(x) = 3$

(b) $\lim_{x \to 4^-} f(x) = 3$

(c) $\lim_{x \to 4} f(x) = 3$

The function is continuous at $x = 4$ and is continuous on $(-\infty, \infty)$.

6. (a) $\lim_{x \to -2^+} f(x) = -2$

(b) $\lim_{x \to -2^-} f(x) = -2$

(c) $\lim_{x \to -2} f(x) = -2$

The function is continuous at $x = -2$.

7. (a) $\lim_{x \to 3^+} f(x) = 0$

(b) $\lim_{x \to 3^-} f(x) = 0$

(c) $\lim_{x \to 3} f(x) = 0$

The function is NOT continuous at $x = 3$.

8. (a) $\lim_{x \to -3^+} f(x) = 3$

(b) $\lim_{x \to -3^-} f(x) = 3$

(c) $\lim_{x \to -3} f(x) = 3$

The function is NOT continuous at $x = -3$ because $f(-3) = 4 \neq \lim_{x \to -3} f(x)$.

9. (a) $\lim_{x \to 2^+} f(x) = -3$

(b) $\lim_{x \to 2^-} f(x) = 3$

(c) $\lim_{x \to 2} f(x)$ does not exist

The function is NOT continuous at $x = 2$.

10. (a) $\lim_{x \to -1^+} f(x) = 0$

(b) $\lim_{x \to -1^-} f(x) = 2$

(c) $\lim_{x \to -1} f(x)$ does not exist.

The function is NOT continuous at $x = -1$.

11. $\lim_{x \to 8^+} \frac{1}{x + 8} = \frac{1}{8 + 8} = \frac{1}{16}$

12. $\lim_{x \to 3^+} \frac{2}{x + 3} = \frac{2}{3 + 3} = \frac{1}{3}$

13. $\lim_{x \to 5^+} \frac{x - 5}{x^2 - 25} = \lim_{x \to 5^+} \frac{x - 5}{(x + 5)(x - 5)}$

$= \lim_{x \to 5^+} \frac{1}{x + 5} = \frac{1}{10}$

14. $\lim_{x\to 4^+} \dfrac{4-x}{x^2-16} = \lim_{x\to 4^+} \dfrac{-(x-4)}{(x+4)(x-4)} = \lim_{x\to 4^+} \dfrac{-1}{x+4}$
$= \dfrac{-1}{4+4} = -\dfrac{1}{8}$

15. $\lim_{x\to -3^-} \dfrac{x}{\sqrt{x^2-9}}$ does not exist because $\dfrac{x}{\sqrt{x^2-9}}$ decreases without bound as $x \to -3^-$.

16. $\lim_{x\to 4^-} \dfrac{\sqrt{x}-2}{x-4} = \lim_{x\to 4^-} \dfrac{\sqrt{x}-2}{x-4} \cdot \dfrac{\sqrt{x}+2}{\sqrt{x}+2}$
$= \lim_{x\to 4^-} \dfrac{x-4}{(x-4)(\sqrt{x}+2)}$
$= \lim_{x\to 4^-} \dfrac{1}{\sqrt{x}+2} = \dfrac{1}{\sqrt{4}+2} = \dfrac{1}{4}$

17. $\lim_{x\to 0^-} \dfrac{|x|}{x} = \lim_{x\to 0^-} \dfrac{-x}{x} = -1$

18. $\lim_{x\to 10^+} \dfrac{|x-10|}{x-10} = \lim_{x\to 10^+} \dfrac{x-10}{x-10} = 1$

19. $\lim_{\Delta x\to 0^-} \dfrac{\dfrac{1}{x+\Delta x} - \dfrac{1}{x}}{\Delta x} = \lim_{\Delta x\to 0^-} \dfrac{x-(x+\Delta x)}{x(x+\Delta x)} \cdot \dfrac{1}{\Delta x} = \lim_{\Delta x\to 0^-} \dfrac{-\Delta x}{x(x+\Delta x)} \cdot \dfrac{1}{\Delta x}$
$= \lim_{\Delta x\to 0^-} \dfrac{-1}{x(x+\Delta x)}$
$= \dfrac{-1}{x(x+0)} = -\dfrac{1}{x^2}$

20. $\lim_{\Delta x\to 0^+} \dfrac{(x+\Delta x)^2 + (x+\Delta x) - (x^2+x)}{\Delta x} = \lim_{\Delta x\to 0^+} \dfrac{x^2 + 2x(\Delta x) + (\Delta x)^2 + x + \Delta x - x^2 - x}{\Delta x}$
$= \lim_{\Delta x\to 0^+} \dfrac{2x(\Delta x) + (\Delta x)^2 + \Delta x}{\Delta x}$
$= \lim_{\Delta x\to 0^+} (2x + \Delta x + 1)$
$= 2x + 0 + 1 = 2x + 1$

21. $\lim_{x\to 3^-} f(x) = \lim_{x\to 3^-} \dfrac{x+2}{2} = \dfrac{5}{2}$

22. $\lim_{x\to 3^-} f(x) = \lim_{x\to 3^-} (x^2 - 4x + 6) = 9 - 12 + 6 = 3$
$\lim_{x\to 3^+} f(x) = \lim_{x\to 3^+} (-x^2 + 4x - 2) = -9 + 12 - 2 = 1$
Since these one-sided limits disagree, $\lim_{x\to 3} f(x)$ does not exist.

23. $\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} (x+1) = 2$
$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} (x^3+1) = 2$
$\lim_{x\to 1} f(x) = 2$

24. $\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} (1-x) = 0$

25. $\lim_{x\to \pi} \cot x$ does not exist because $\lim_{x\to \pi^+} \cot x$ and $\lim_{x\to \pi^-} \cot x$ do not exist.

26. $\lim_{x\to \pi/2} \sec x$ does not exist because $\lim_{x\to (\pi/2)^+} \sec x$ and $\lim_{x\to (\pi/2)^-} \sec x$ do not exist.

27. $\lim_{x\to 4^-} (5[\![x]\!] - 7) = 5(3) - 7 = 8$
$([\![x]\!] = 3 \text{ for } 3 \le x < 4)$

28. $\lim_{x\to 2^+} (2x - [\![x]\!]) = 2(2) - 2 = 2$

29. $\lim_{x\to -1} \left(\left[\!\left[\dfrac{x}{3}\right]\!\right] + 3\right) = \left[\!\left[-\dfrac{1}{3}\right]\!\right] + 3 = -1 + 3 = 2$

30. $\lim_{x\to 1} \left(1 - \left[\!\left[-\dfrac{x}{2}\right]\!\right]\right) = 1 - (-1) = 2$

31. $f(x) = \dfrac{1}{x^2-4}$
has discontinuities at $x = -2$ and $x = 2$ because $f(-2)$ and $f(2)$ are not defined.

32. $f(x) = \dfrac{x^2 - 1}{x + 1}$

has a discontinuity at $x = -1$ because $f(-1)$ is not defined.

33. $f(x) = \dfrac{[\![x]\!]}{2} + x$

has discontinuities at each integer k because $\lim_{x \to k^-} f(x) \neq \lim_{x \to k^+} f(x)$.

34. $f(x) = \begin{cases} x, & x < 1 \\ 2, & x = 1 \\ 2x - 1, & x > 1 \end{cases}$ has a discontinuity at $x = 1$

because $f(1) = 2 \neq \lim_{x \to 1} f(x) = 1$.

35. $g(x) = \sqrt{49 - x^2}$ is continuous on $[-7, 7]$.

36. $f(t) = 3 - \sqrt{9 - t^2}$ is continuous on $[-3, 3]$.

37. $\lim_{x \to 0^-} f(x) = 3 = \lim_{x \to 0^+} f(x)$. f is continuous on $[-1, 4]$.

38. $g(2)$ is not defined. g is continuous on $[-1, 2)$.

39. $f(x) = \dfrac{6}{x}$ has a nonremovable discontinuity at $x = 0$ because $\lim_{x \to 0} f(x)$ does not exist.

40. $f(x) = \dfrac{4}{x - 6}$ has a nonremovable discontinuity at $x = 6$ because $\lim_{x \to 6} f(x)$ does not exist.

41. $f(x) = \dfrac{1}{4 - x^2} = \dfrac{1}{(2 - x)(2 + x)}$ has nonremovable discontinuities at $x = \pm 2$ because $\lim_{x \to 2} f(x)$ and $\lim_{x \to -2} f(x)$ do not exist.

42. $f(x) = \dfrac{1}{x^2 + 1}$ is continuous for all real x.

43. $f(x) = 3x - \cos x$ is continuous for all real x.

44. $f(x) = \sin x - 8x$ is continuous for all real x.

45. $f(x) = \dfrac{x}{x^2 - x}$ is not continuous at $x = 0, 1$.

Because $\dfrac{x}{x^2 - x} = \dfrac{1}{x - 1}$ for $x \neq 0$, $x = 0$ is a removable discontinuity, whereas $x = 1$ is a nonremovable discontinuity.

46. $f(x) = \dfrac{x}{x^2 - 4}$ has nonremovable discontinuities at $x = 2$ and $x = -2$ because $\lim_{x \to 2} f(x)$ and $\lim_{x \to -2} f(x)$ do not exist.

47. $f(x) = \dfrac{x + 2}{x^2 - 3x - 10} = \dfrac{x + 2}{(x + 2)(x - 5)}$

has a nonremovable discontinuity at $x = 5$ because $\lim_{x \to 5} f(x)$ does not exist, and has a removable discontinuity at $x = -2$ because

$$\lim_{x \to -2} f(x) = \lim_{x \to -2} \frac{1}{x - 5} = -\frac{1}{7}.$$

48. $f(x) = \dfrac{x + 2}{x^2 - x - 6} = \dfrac{x + 2}{(x - 3)(x + 2)}$

has a nonremovable discontinuity at $x = 3$ because $\lim_{x \to 3} f(x)$ does not exist, and has a removable discontinuity at $x = -2$ because

$$\lim_{x \to -2} f(x) = \lim_{x \to -2} \frac{1}{x - 3} = -\frac{1}{5}.$$

49. $f(x) = \dfrac{|x + 7|}{x + 7}$

has a nonremovable discontinuity at $x = -7$ because $\lim_{x \to -7} f(x)$ does not exist.

50. $f(x) = \dfrac{2|x - 3|}{x - 3}$ has a nonremovable discontinuity at $x = 3$ because $\lim_{x \to 3} f(x)$ does not exist.

51. $f(x) = \begin{cases} \dfrac{x}{2} + 1, & x \le 2 \\ 3 - x, & x > 2 \end{cases}$

has a **possible** discontinuity at $x = 2$.

1. $f(2) = \dfrac{2}{2} + 1 = 2$

2. $\left.\begin{array}{l} \lim\limits_{x \to 2^-} f(x) = \lim\limits_{x \to 2^-} \left(\dfrac{x}{2} + 1\right) = 2 \\ \lim\limits_{x \to 2^+} f(x) = \lim\limits_{x \to 2^+} (3 - x) = 1 \end{array}\right\} \lim\limits_{x \to 2} f(x)$ does not exist.

Therefore, f has a nonremovable discontinuity at $x = 2$.

52. $f(x) = \begin{cases} -2x, & x \le 2 \\ x^2 - 4x + 1, & x > 2 \end{cases}$

has a **possible** discontinuity at $x = 2$.

1. $f(2) = -2(2) = -4$

2. $\left.\begin{array}{l} \lim\limits_{x \to 2^-} f(x) = \lim\limits_{x \to 2^-} (-2x) = -4 \\ \lim\limits_{x \to 2^+} f(x) = \lim\limits_{x \to 2^+} \left(x^2 - 4x + 1\right) = -3 \end{array}\right\} \lim\limits_{x \to 2} f(x)$ does not exist.

Therefore, f has a nonremovable discontinuity at $x = 2$.

53. $f(x) = \begin{cases} \tan \dfrac{\pi x}{4}, & |x| < 1 \\ x, & |x| \ge 1 \end{cases}$

$= \begin{cases} \tan \dfrac{\pi x}{4}, & -1 < x < 1 \\ x, & x \le -1 \text{ or } x \ge 1 \end{cases}$

has **possible** discontinuities at $x = -1, x = 1$.

1. $f(-1) = -1$ $\quad f(1) = 1$

2. $\lim\limits_{x \to -1} f(x) = -1$ $\quad \lim\limits_{x \to 1} f(x) = 1$

3. $f(-1) = \lim\limits_{x \to -1} f(x)$ $\quad f(1) = \lim\limits_{x \to 1} f(x)$

f is continuous at $x = \pm 1$, therefore, f is continuous for all real x.

54. $f(x) = \begin{cases} \csc \dfrac{\pi x}{6}, & |x - 3| \le 2 \\ 2, & |x - 3| > 2 \end{cases}$

$= \begin{cases} \csc \dfrac{\pi x}{6}, & 1 \le x \le 5 \\ 2, & x < 1 \text{ or } x > 5 \end{cases}$

has **possible** discontinuities at $x = 1, x = 5$.

1. $f(1) = \csc \dfrac{\pi}{6} = 2$ $\quad f(5) = \csc \dfrac{5\pi}{6} = 2$

2. $\lim\limits_{x \to 1} f(x) = 2$ $\quad \lim\limits_{x \to 5} f(x) = 2$

3. $f(1) = \lim\limits_{x \to 1} f(x)$ $\quad f(5) = \lim\limits_{x \to 5} f(x)$

f is continuous at $x = 1$ and $x = 5$, therefore, f is continuous for all real x.

55. $f(x) = \csc 2x$ has nonremovable discontinuities at integer multiples of $\pi/2$.

56. $f(x) = \tan \dfrac{\pi x}{2}$ has nonremovable discontinuities at each $2k + 1$, k is an integer.

57. $f(x) = [\![x - 8]\!]$ has nonremovable discontinuities at each integer k.

58. $f(x) = 5 - [\![x]\!]$ has nonremovable discontinuities at each integer k.

59. $f(1) = 3$

Find a so that $\lim_{x\to 1^-} (ax - 4) = 3$

$$a(1) - 4 = 3$$
$$a = 7.$$

60. $f(1) = 3$

Find a so that $\lim_{x\to 1^+} (ax + 5) = 3$

$$a(1) + 5 = 3$$
$$a = -2.$$

61. $f(2) = 8$

Find a so that $\lim_{x\to 2^+} ax^2 = 8 \Rightarrow a = \frac{8}{2^2} = 2.$

62. $\lim_{x\to 0^-} g(x) = \lim_{x\to 0^-} \dfrac{4 \sin x}{x} = 4$

$\lim_{x\to 0^+} g(x) = \lim_{x\to 0^+} (a - 2x) = a$

Let $a = 4$.

63. Find a and b such that $\lim_{x\to -1^+} (ax + b) = -a + b = 2$ and $\lim_{x\to 3^-} (ax + b) = 3a + b = -2$.

$$\begin{aligned} a - b &= -2 \\ (+)3a + b &= -2 \\ \hline 4a &= -4 \\ a &= -1 \\ b &= 2 + (-1) = 1 \end{aligned}$$

$$f(x) = \begin{cases} 2, & x \le -1 \\ -x + 1, & -1 < x < 3 \\ -2, & x \ge 3 \end{cases}$$

64. $\lim_{x\to a} g(x) = \lim_{x\to a} \dfrac{x^2 - a^2}{x - a}$

$= \lim_{x\to a} (x + a) = 2a$

Find a such $2a = 8 \Rightarrow a = 4.$

65. $f(g(x)) = (x - 1)^2$

Continuous for all real x

66. $f(g(x)) = 5(x^3) + 1 = 5x^3 + 1$

Continuous for all real x

67. $f(g(x)) = \dfrac{1}{(x^2 + 5) - 6} = \dfrac{1}{x^2 - 1}$

Nonremovable discontinuities at $x = \pm 1$

68. $f(g(x)) = \dfrac{1}{\sqrt{x - 1}}$

Nonremovable discontinuity at $x = 1$; continuous for all $x > 1$

69. $f(g(x)) = \tan\dfrac{x}{2}$

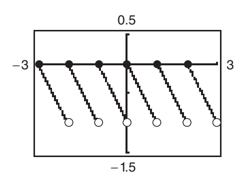
Not continuous at $x = \pm\pi, \pm 3\pi, \pm 5\pi, \ldots$ Continuous on the open intervals $\ldots, (-3\pi, -\pi), (-\pi, \pi), (\pi, 3\pi), \ldots$

70. $f(g(x)) = \sin x^2$

Continuous for all real x

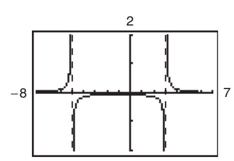
71. $y = [\![x]\!] - x$

Nonremovable discontinuity at each integer



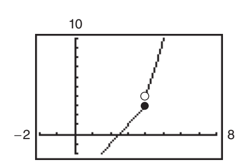
72. $h(x) = \dfrac{1}{x^2 + 2x - 15} = \dfrac{1}{(x + 5)(x - 3)}$

Nonremovable discontinuities at $x = -5$ and $x = 3$



73. $g(x) = \begin{cases} x^2 - 3x, & x > 4 \\ 2x - 5, & x \le 4 \end{cases}$

Nonremovable discontinuity at $x = 4$



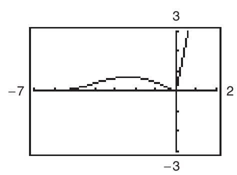
74. $f(x) = \begin{cases} \dfrac{\cos x - 1}{x}, & x < 0 \\ 5x, & x \ge 0 \end{cases}$

$f(0) = 5(0) = 0$

$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} \dfrac{(\cos x - 1)}{x} = 0$

$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} (5x) = 0$

Therefore, $\lim_{x\to 0} f(x) = 0 = f(0)$ and f is continuous on the entire real line.
$(x = 0$ was the only possible discontinuity.)



75. $f(x) = \dfrac{x}{x^2 + x + 2}$

Continuous on $(-\infty, \infty)$

76. $f(x) = \dfrac{x + 1}{\sqrt{x}}$

Continuous on $(0, \infty)$

77. $f(x) = 3 - \sqrt{x}$

Continuous on $[0, \infty)$

78. $f(x) = x\sqrt{x + 3}$

Continuous on $[-3, \infty)$

79. $f(x) = \sec\dfrac{\pi x}{4}$

Continuous on:
$\dots, (-6, -2), (-2, 2), (2, 6), (6, 10), \dots$

80. $f(x) = \cos\dfrac{1}{x}$

Continuous on $(-\infty, 0)$ and $(0, \infty)$

81. $f(x) = \begin{cases} \dfrac{x^2 - 1}{x - 1}, & x \ne 1 \\ 2, & x = 1 \end{cases}$

Since $\lim_{x\to 1} f(x) = \lim_{x\to 1} \dfrac{x^2 - 1}{x - 1} = \lim_{x\to 1} \dfrac{(x - 1)(x + 1)}{x - 1}$
$= \lim_{x\to 1} (x + 1) = 2,$

f is continuous on $(-\infty, \infty)$.

82. $f(x) = \begin{cases} 2x - 4, & x \ne 3 \\ 1, & x = 3 \end{cases}$

Since $\lim_{x\to 3} f(x) = \lim_{x\to 3} (2x - 4) = 2 \ne 1,$

f is continuous on $(-\infty, 3)$ and $(3, \infty)$.

83. $f(x) = \frac{1}{12}x^4 - x^3 + 4$ is continuous on the interval $[1, 2]$. $f(1) = \frac{37}{12}$ and $f(2) = -\frac{8}{3}$. By the Intermediate Value Theorem, there exists a number c in $[1, 2]$ such that $f(c) = 0$.

84. $f(x) = x^3 + 5x - 3$ is continuous on the interval $[0, 1]$. $f(0) = -3$ and $f(1) = 3$. By the Intermediate Value Theorem, there exists a number c in $[0, 1]$ such that $f(c) = 0$.

85. $f(x) = x^2 - 2 - \cos x$ is continuous on $[0, \pi]$. $f(0) = -3$ and $f(\pi) = \pi^2 - 1 \approx 8.87 > 0$. By the Intermediate Value Theorem, $f(c) = 0$ for at least one value of c between 0 and π.

86. $f(x) = -\dfrac{5}{x} + \tan\left(\dfrac{\pi x}{10}\right)$ is continuous on the interval $[1, 4]$.

$f(1) = -5 + \tan\left(\dfrac{\pi}{10}\right) \approx -4.7$ and

$f(4) = -\dfrac{5}{4} + \tan\left(\dfrac{2\pi}{5}\right) \approx 1.8$. By the Intermediate Value Theorem, there exists a number c in $[1, 4]$ such that $f(c) = 0$.

87. Consider the intervals $[1, 3]$ and $[3. 5]$ for $f(x) = (x - 3)^2 = 2$.

$f(1) = 2 > 0$ and $f(3) = -2 < 0$, so f has at least one zero in $[1, 3]$.

$f(3) = -2 < 0$ and $f(5) = 2 > 0$, so f has at least one zero in $[3, 5]$.

So, f has at least two zeros in $[1, 5]$.

88. Consider the intervals $[1, 3]$ and $[3, 5]$ for $f(x) = 2\cos x$.

$f(1) = 2\cos 1 \approx 1.08 > 0$ and $f(3) = 2\cos 3 \approx -1.98 < 0$, so f has at least one zero in $[1, 3]$.

$f(3) = 2\cos 3 \approx 1.98 < 0$ and $f(5) = 2\cos 5 \approx 0.57 > 0$, so f has at least one zero in $[3, 5]$.

So, f has at least two zeros in $[1, 5]$.

89. $f(x) = x^3 + x - 1$

$f(x)$ is continuous on $[0, 1]$.

$f(0) = -1$ and $f(1) = 1$

By the Intermediate Value Theorem, $f(c) = 0$ for at least one value of c between 0 and 1. Using a graphing utility to zoom in on the graph of $f(x)$, you find that $x \approx 0.68$. Using the *root* feature, you find that $x \approx 0.6823$.

90. $f(x) = x^4 - x^2 + 3x - 1$

$f(x)$ is continuous on $[0, 1]$.

$f(0) = -1$ and $f(1) = 2$

By the Intermediate Value Theorem, $f(c) = 0$ for at least one value of c between 0 and 1. Using a graphing utility to zoom in on the graph of $f(x)$, you find that $x \approx 0.37$. Using the *root* feature, you find that $x \approx 0.3733$.

91. $f(x) = \sqrt{x^2 + 17x + 19} - 6$

f is continuous on $[0, 1]$.

$f(0) = \sqrt{19} - 6 \approx -1.64 < 0$

$f(1) = \sqrt{37} - 6 \approx 0.08 > 0$

By the Intermediate Value Theorem, $f(c) = 0$ for at least one value of c between 0 and 1. Using a graphing utility to zoom in on the graph of $f(x)$, you find that $x \approx 0.95$. Using the *root* feature, you find that $x \approx 0.9472$.

92. $f(x) = \sqrt{x^4 + 39x + 13} - 4$

f is continuous on $[0, 1]$.

$f(0) = \sqrt{13} - 4 \approx -0.39 < 0$

$f(1) = \sqrt{53} - 4 \approx 3.28 > 0$

By the Intermediate Value Theorem, $f(c) = 0$ for at least one value of c between 0 and 1. Using a graphing utility to zoom in on the graph of $f(x)$, you find that $x \approx 0.08$. Using the *root* feature, you find that $x \approx 0.0769$.

93. $g(t) = 2\cos t - 3t$

g is continuous on $[0, 1]$.

$g(0) = 2 > 0$ and $g(1) \approx -1.9 < 0$.

By the Intermediate Value Theorem, $g(c) = 0$ for at least one value of c between 0 and 1. Using a graphing utility to zoom in on the graph of $g(t)$, you find that $t \approx 0.56$. Using the *root* feature, you find that $t \approx 0.5636$.

94. $h(\theta) = \tan\theta + 3\theta - 4$ is continuous on $[0, 1]$.

$h(0) = -4$ and $h(1) = \tan(1) - 1 \approx 0.557$.

By the Intermediate Value Theorem, $h(c) = 0$ for at least one value of c between 0 and 1. Using a graphing utility to zoom in on the graph of $h(\theta)$, you find that $\theta \approx 0.91$. Using the *root* feature, you obtain $\theta \approx 0.9071$.

95. $f(x) = x^2 + x - 1$

f is continuous on $[0, 5]$.

$f(0) = -1$ and $f(5) = 29$

$-1 < 11 < 29$

The Intermediate Value Theorem applies.

$$x^2 + x - 1 = 11$$
$$x^2 + x - 12 = 0$$
$$(x + 4)(x - 3) = 0$$
$$x = -4 \text{ or } x = 3$$

$c = 3$ ($x = -4$ is not in the interval.)

So, $f(3) = 11$.

96. $f(x) = x^2 - 6x + 8$

f is continuous on $[0, 3]$.

$f(0) = 8$ and $f(3) = -1$

$-1 < 0 < 8$

The Intermediate Value Theorem applies.

$$x^2 - 6x + 8 = 0$$
$$(x - 2)(x - 4) = 0$$
$$x = 2 \text{ or } x = 4$$

$c = 2$ ($x = 4$ is not in the interval.)

So, $f(2) = 0$.

97. $f(x) = \sqrt{x + 7} - 2$

f is continuous on $[0, 5]$.

$f(0) = \sqrt{7} - 2 \approx 0.6458 < 1$

$f(5) = \sqrt{12} - 2 \approx 1.4641 > 1$

The Intermediate Value Theorem applies.

$$\begin{aligned} \sqrt{x + 7} - 2 &= 1 \\ \sqrt{x + 7} &= 3 \\ x + 7 &= 9 \\ x &= 2 \\ c &= 2 \end{aligned}$$

So, $f(2) = 1$.

98. $f(x) = \sqrt[3]{x} + 8$

f is continuous on $[-9, -6]$.

$f(-9) = (-9)^{1/3} + 8 \approx 5.9199 < 6$

$f(-6) = (-6)^{1/3} + 8 \approx 6.1829 > 6$

The Intermediate Value Theorem applies.

$$\begin{aligned} \sqrt[3]{x} + 8 &= 6 \\ \sqrt[3]{x} &= -2 \\ x &= (-2)^3 = -8 \\ c &= -8 \end{aligned}$$

So, $f(-8) = 6$.

99. $f(x) = \dfrac{x - x^3}{x - 4}$

f is continuous on $[1, 3]$. The nonremovable discontinuity, $x = 4$, lies outside the interval.

$f(1) = \dfrac{1 - 1}{1 - 4} = 0 < 3$

$f(3) = 24 > 3$

The Intermediate Value Theorem applies.

$$\begin{aligned} \frac{x - x^3}{x - 4} &= 3 \\ x - x^3 &= 3x - 12 \\ x^3 + 2^x - 12 &= 0 \\ (x - 2)(x^2 + 2x + 6) &= 0 \\ x &= 2 \end{aligned}$$

$(x^2 + 2x + 6$ has no real solution.$)$

$$c = 2$$

So, $f(2) = 3$.

100. $f(x) = \dfrac{x^2 + x}{x - 1}$

f is continuous on $\left[\frac{5}{2}, 4\right]$. The nonremovable discontinuity, $x = 1$, lies outside the interval.

$f\left(\frac{5}{2}\right) = \frac{35}{6}$ and $f(4) = \frac{20}{3}$

$\frac{35}{6} < 6 < \frac{20}{3}$

The Intermediate Value Theorem applies.

$$\begin{aligned} \frac{x^2 + x}{x - 1} &= 6 \\ x^2 + x &= 6x - 6 \\ x^2 - 5x + 6 &= 0 \\ (x - 2)(x - 3) &= 0 \\ x = 2 \text{ or } x &= 3 \end{aligned}$$

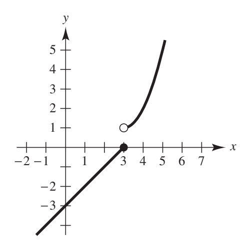
$c = 3$ ($x = 2$ is not in the interval.)

So, $f(3) = 6$.

101. Answers will vary. *Sample answer:*

$f(x) = \dfrac{1}{(x - a)(x - b)}$

102. Answers will vary. *Sample answer*:



The function is not continuous at $x = 3$ because $\lim_{x \to 3^+} f(x) = 1 \neq 0 = \lim_{x \to 3^-} f(x)$.

103. If f and g are continuous for all real x, then so is $f + g$ (Theorem 1.11, part 2). However, f/g might not be continuous if $g(x) = 0$. For example, let $f(x) = x$ and $g(x) = x^2 - 1$. Then f and g are continuous for all real x, but f/g is not continuous at $x = \pm 1$.

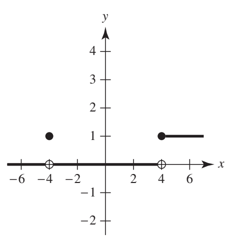
104. A discontinuity at c is removable if the function f can be made continuous at c by appropriately defining (or redefining) $f(c)$. Otherwise, the discontinuity is nonremovable.

(a) $f(x) = \dfrac{|x - 4|}{x - 4}$

(b) $f(x) = \dfrac{\sin(x + 4)}{x + 4}$

(c) $f(x) = \begin{cases} 1, & x \geq 4 \\ 0, & -4 < x < 4 \\ 1, & x = -4 \\ 0, & x < -4 \end{cases}$

$x = 4$ is nonremovable, $x = -4$ is removable



105. True

1. $f(c) = L$ is defined.

2. $\lim_{x \to c} f(x) = L$ exists.

3. $f(c) = \lim_{x \to c} f(x)$

All of the conditions for continuity are met.

106. True. If $f(x) = g(x), x \neq c$, then $\lim_{x \to c} f(x) = \lim_{x \to c} g(x)$ (if they exist) and at least one of these limits then does not equal the corresponding function value at $x = c$.

107. False. $f(x) = \cos x$ has two zeros in $[0, 2\pi]$. However, $f(0)$ and $f(2\pi)$ have the same sign.

108. True. For $x \in (-1, 0)$, $[\![x]\!] = -1$, which implies that $\lim_{x \to 0^-} [\![x]\!] = -1$.

109. False. A rational function can be written as $P(x)/Q(x)$ where P and Q are polynomials of degree m and n, respectively. It can have, at most, n discontinuities.

110. False. $f(1)$ is not defined and $\lim_{x \to 1} f(x)$ does not exist.

111. The functions agree for integer values of x:

$$\left.\begin{aligned} g(x) &= 3 - [\![-x]\!] = 3 - (-x) = 3 + x \\ f(x) &= 3 + [\![x]\!] = 3 + x \end{aligned}\right\} \text{for } x \text{ an integer}$$

However, for non-integer values of x, the functions differ by 1.

$f(x) = 3 + [\![x]\!] = g(x) - 1 = 2 - [\![-x]\!]$.

For example,

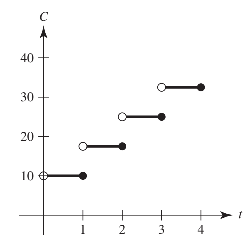
$f\left(\frac{1}{2}\right) = 3 + 0 = 3, g\left(\frac{1}{2}\right) = 3 - (-1) = 4.$

112. $\lim_{t \to 4^-} f(t) \approx 28$

$\lim_{t \to 4^+} f(t) \approx 56$

At the end of day 3, the amount of chlorine in the pool has decreased to about 28 ounces. At the beginning of day 4, more chlorine was added, and the amount is now about 56 ounces.

113. $C(t) = 10 - 7.5[\![1 - t]\!], t > 0$

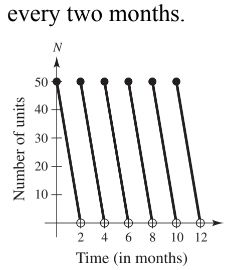


There is a nonremovable discontinuity at every integer value of t, or gigabyte.

114. $N(t) = 25\left(2\left[\!\!\left[\dfrac{t + 2}{2}\right]\!\!\right] - t\right)$

t	0	1	1.8	2	3	3.8
$N(t)$	50	25	5	50	25	5

There is a nonremovable discontinuity at every positive even integer. The company replenishes its inventory every two months.



115. Let $s(t)$ be the position function for the run up to the campsite. $s(0) = 0$ ($t = 0$ corresponds to 8:00 A.M., $s(20) = k$ (distance to campsite)). Let $r(t)$ be the position function for the run back down the mountain: $r(0) = k$, $r(10) = 0$. Let $f(t) = s(t) - r(t)$.

When $t = 0$ (8:00 A.M.), $f(0) = s(0) - r(0) = 0 - k < 0$.

When $t = 10$ (8:00 A.M.), $f(10) = s(10) - r(10) > 0$.

Because $f(0) < 0$ and $f(10) > 0$, then there must be a value t in the interval $[0, 10]$ such that $f(t) = 0$. If $f(t) = 0$, then $s(t) - r(t) = 0$, which gives us $s(t) = r(t)$. Therefore, at some time t, where $0 \le t \le 10$, the position functions for the run up and the run down are equal.

116. Let $V = \frac{4}{3}\pi r^3$ be the volume of a sphere with radius r.

V is continuous on $[5, 8]$. $V(5) = \frac{500\pi}{3} \approx 523.6$ and $V(8) = \frac{2048\pi}{3} \approx 2144.7$. Because $523.6 < 1500 < 2144.7$, the Intermediate Value Theorem guarantees that there is at least one value r between 5 and 8 such that $V(r) = 1500$. (In fact, $r \approx 7.1012$.)

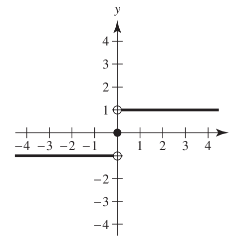
117. Suppose there exists x_1 in $[a, b]$ such that $f(x_1) > 0$ and there exists x_2 in $[a, b]$ such that $f(x_2) < 0$. Then by the Intermediate Value Theorem, $f(x)$ must equal zero for some value of x in $[x_1, x_2]$ (or $[x_2, x_1]$ if $x_2 < x_1$). So, f would have a zero in $[a, b]$, which is a contradiction. Therefore, $f(x) > 0$ for all x in $[a, b]$ or $f(x) < 0$ for all x in $[a, b]$.

118. Let c be any real number. Then $\lim_{x\to c} f(x)$ does not exist because there are both rational and irrational numbers arbitrarily close to c. Therefore, f is not continuous at c.

119. If $x = 0$, then $f(0) = 0$ and $\lim_{x\to 0} f(x) = 0$. So, f is continuous at $x = 0$.

If $x \ne 0$, then $\lim_{t\to x} f(t) = 0$ for x rational, whereas $\lim_{t\to x} f(t) = \lim_{t\to x} kt = kx \ne 0$ for x irrational. So, f is not continuous for all $x \ne 0$.

120. $\operatorname{sgn}(x) = \begin{cases} -1, & \text{if } x < 0 \\ 0, & \text{if } x = 0 \\ 1, & \text{if } x > 0 \end{cases}$

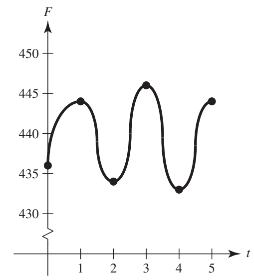


(a) $\lim_{x\to 0^-} \operatorname{sgn}(x) = -1$

(b) $\lim_{x\to 0^+} \operatorname{sgn}(x) = 1$

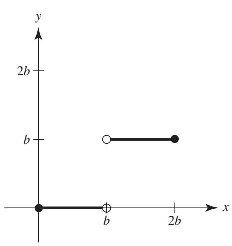
(c) $\lim_{x\to 0} \operatorname{sgn}(x)$ does not exist.

121. (a)



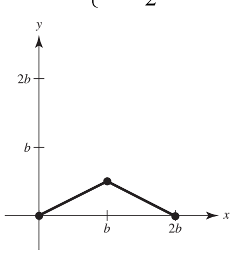
(b) No. The frequency is oscillating.

122. (a) $f(x) = \begin{cases} 0, & 0 \le x < b \\ b, & b < x \le 2b \end{cases}$



NOT continuous at $x = b$.

(b) $g(x) = \begin{cases} \frac{x}{2}, & 0 \le x \le b \\ b - \frac{x}{2}, & b < x \le 2b \end{cases}$



Continuous on $[0, 2b]$.

123. $f(x) = \begin{cases} 1 - x^2, & x \le c \\ x, & x > c \end{cases}$

f is continuous for $x < c$ and for $x > c$. At $x = c$, you need $1 - c^2 = c$. Solving $c^2 + c - 1$, you obtain

$$c = \frac{-1 \pm \sqrt{1+4}}{2} = \frac{-1 \pm \sqrt{5}}{2}.$$

124. Let y be a real number. If $y = 0$, then $x = 0$. If $y > 0$, then let $0 < x_0 < \pi/2$ such that $M = \tan x_0 > y$ (this is possible since the tangent function increases without bound on $[0, \pi/2)$). By the Intermediate Value Theorem, $f(x) = \tan x$ is continuous on $[0, x_0]$ and $0 < y < M$, which implies that there exists x between 0 and x_0 such that $\tan x = y$. The argument is similar if $y < 0$.

125. $f(x) = \dfrac{\sqrt{x + c^2} - c}{x}, c > 0$

Domain: $x + c^2 \ge 0 \Rightarrow x \ge -c^2$ and $x \ne 0, \left[-c^2, 0\right) \cup (0, \infty)$

$$\lim_{x\to 0} \frac{\sqrt{x+c^2} - c}{x} = \lim_{x\to 0} \frac{\sqrt{x+c^2} - c}{x} \cdot \frac{\sqrt{x+c^2} + c}{\sqrt{x+c^2} + c} = \lim_{x\to 0} \frac{\left(x + c^2\right) - c^2}{x\left[\sqrt{x+c^2} + c\right]} = \lim_{x\to 0} \frac{1}{\sqrt{x+c^2} + c} = \frac{1}{2c}$$

Define $f(0) = 1/(2c)$ to make f continuous at $x = 0$.

126. **1.** $f(c)$ is defined.

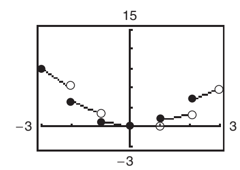
2. $\lim_{x\to c} f(x) = \lim_{\Delta x \to 0} f(c + \Delta x) = f(c)$ exists.

[Let $x = c + \Delta x$. As $x \to c, \Delta x \to 0$]

3. $\lim_{x\to c} f(x) = f(c)$.

Therefore, f is continuous at $x = c$.

127. $h(x) = x[\![x]\!]$



h has nonremovable discontinuities at $x = \pm 1, \pm 2, \pm 3, \ldots$.

128. (a) Define $f(x) = f_2(x) - f_1(x)$. Because f_1 and f_2 are continuous on $[a, b]$, so is f.

$f(a) = f_2(a) - f_1(a) > 0$ and $f(b) = f_2(b) - f_1(b) < 0$

By the Intermediate Value Theorem, there exists c in $[a, b]$ such that $f(c) = 0$.

$f(c) = f_2(c) - f_1(c) = 0 \Rightarrow f_1(c) = f_2(c)$

(b) Let $f_1(x) = x$ and $f_2(x) = \cos x$, continuous on $[0, \pi/2]$, $f_1(0) < f_2(0)$ and $f_1(\pi/2) > f_2(\pi/2)$.

So by part (a), there exists c in $[0, \pi/2]$ such that $c = \cos(c)$.

Using a graphing utility, $c \approx 0.739$.

129. The statement is true.

If $y \ge 0$ and $y \le 1$, then $y(y - 1) \le 0 \le x^2$, as desired. So assume $y > 1$. There are now two cases.

Case 1:

If $x \le y - \frac{1}{2}$, then $2x + 1 \le 2y$ and

$$\begin{aligned} y(y-1) &= y(y+1) - 2y \\ &\le (x+1)^2 - 2y \\ &= x^2 + 2x + 1 - 2y \\ &\le x^2 + 2y - 2y \\ &= x^2 \end{aligned}$$

Case 2:

If $x \ge y - \frac{1}{2}$

$$\begin{aligned} x^2 &\ge \left(y - \tfrac{1}{2}\right)^2 \\ &= y^2 - y + \tfrac{1}{4} \\ &> y^2 - y \\ &= y(y-1) \end{aligned}$$

In both cases, $y(y - 1) \le x^2$.

130. $P(1) = P(0^2 + 1) = P(0)^2 + 1 = 1$

$P(2) = P(1^2 + 1) = P(1)^2 + 1 = 2$

$P(5) = P(2^2 + 1) = P(2)^2 + 1 = 5$

Continuing this pattern, you see that $P(x) = x$ for infinitely many values of x.

So, the finite degree polynomial must be constant: $P(x) = x$ for all x.

Section 1.5 Infinite Limits

1. A limit in which $f(x)$ increases or decreases without bound as x approaches c is called an infinite limit. ∞ is not a number. Rather, the symbol

$\lim_{x \to c} f(x) = \infty$

Says how the limit fails to exist.

2. The line $x = c$ is a vertical asymptote if the graph of f approaches $\pm\infty$ as x approaches c.

3. $\lim_{x \to -2^+} 2\left|\dfrac{x}{x^2 - 4}\right| = \infty$

$\lim_{x \to -2^-} 2\left|\dfrac{x}{x^2 - 4}\right| = \infty$

4. $\lim_{x \to -2^+} \dfrac{1}{x + 2} = \infty$

$\lim_{x \to -2^-} \dfrac{1}{x + 2} = -\infty$

5. $\lim_{x \to -2^+} \tan \dfrac{\pi x}{4} = -\infty$

$\lim_{x \to -2^-} \tan \dfrac{\pi x}{4} = \infty$

6. $\lim_{x \to -2^+} \sec \dfrac{\pi x}{4} = \infty$

$\lim_{x \to -2^-} \sec \dfrac{\pi x}{4} = -\infty$

7. $f(x) = \dfrac{1}{x - 4}$

As x approaches 4 from the left, $x - 4$ is a small negative number. So,

$\lim_{x \to 4^-} f(x) = -\infty$

As x approaches 4 from the right, $x - 4$ is a small positive number. So,

$\lim_{x \to 4^+} f(x) = \infty$

8. $f(x) = \dfrac{-1}{x - 4}$

As x approaches 4 from the left, $x - 4$ is a small negative number. So,

$\lim_{x \to 4^-} f(x) = \infty.$

As x approaches 4 from the right, $x - 4$ is a small positive number. So,

$\lim_{x \to 4^+} f(x) = -\infty.$

9. $f(x) = \dfrac{1}{(x - 4)^2}$

As x approaches 4 from the left or right, $(x - 4)^2$ is a small positive number. So,

$\lim_{x \to 4^+} f(x) = \lim_{x \to 4^-} f(x) = \infty.$

10. $f(x) = \dfrac{-1}{(x - 4)^2}$

As x approaches 4 from the left or right, $(x - 4)^2$ is a small positive number. So,

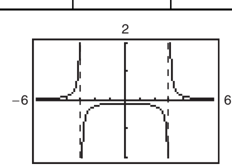
$\lim_{x \to 4^-} f(x) = \lim_{x \to 4^+} f(x) = -\infty.$

11. $f(x) = \dfrac{1}{x^2 - 9}$

x	–3.5	–3.1	–3.01	–3.001	–2.999	–2.99	–2.9	–2.5
$f(x)$	0.308	1.639	16.64	166.6	–166.7	–16.69	–1.695	–0.364

$\lim_{x \to -3^-} f(x) = \infty$

$\lim_{x \to -3^+} f(x) = -\infty$

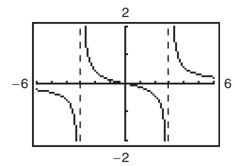


12. $f(x) = \dfrac{x}{x^2 - 9}$

x	–3.5	–3.1	–3.01	–3.001	–2.999	–2.99	–2.9	–2.5
$f(x)$	–1.077	–5.082	–50.08	–500.1	499.9	49.92	4.915	0.9091

$\lim_{x \to -3^-} f(x) = -\infty$

$\lim_{x \to -3^+} f(x) = \infty$

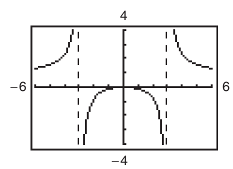


13. $f(x) = \dfrac{x^2}{x^2 - 9}$

x	–3.5	–3.1	–3.01	–3.001	–2.999	–2.99	–2.9	–2.5
$f(x)$	3.769	15.75	150.8	1501	–1499	–149.3	–14.25	–2.273

$\lim_{x \to -3^-} f(x) = \infty$

$\lim_{x \to -3^+} f(x) = -\infty$

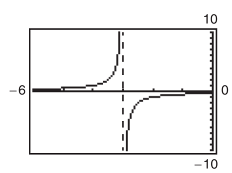


14. $f(x) = -\dfrac{1}{3 + x}$

x	–3.5	–3.1	–3.01	–3.001	–2.999	–2.99	–2.9	–2.5
$f(x)$	2	10	100	1000	–1000	–100	–10	–2

$\lim_{x \to -3^-} f(x) = \infty$

$\lim_{x \to -3^+} f(x) = -\infty$

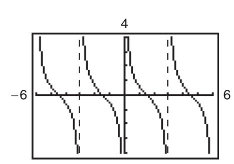


15. $f(x) = \cot \dfrac{\pi x}{3}$

x	–3.5	–3.1	–3.01	–3.001	–2.999	–2.99	–2.9	–2.5
$f(x)$	–1.7321	–9.514	–95.49	–954.9	954.9	95.49	9.514	1.7321

$\lim_{x \to -3^-} f(x) = -\infty$

$\lim_{x \to -3^+} f(x) = \infty$

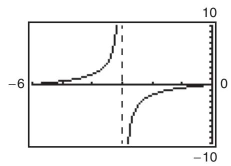


16. $f(x) = \tan \dfrac{\pi x}{6}$

x	–3.5	–3.1	–3.01	–3.001	–2.999	–2.99	–2.9	–2.5
$f(x)$	3.73	19.08	190.98	1909.9	–11909.9	–190.98	–19.08	–3.73

$\lim_{x \to -3^-} f(x) = \infty$

$\lim_{x \to -3^+} f(x) = -\infty$



17. $f(x) = \dfrac{1}{x^2}$

$\lim_{x \to 0^+} \dfrac{1}{x^2} = \infty = \lim_{x \to 0^-} \dfrac{1}{x^2}$

Therefore, $x = 0$ is a vertical asymptote.

18. $f(x) = \dfrac{2}{(x - 3)^3}$

$\lim_{x \to 3^-} \dfrac{2}{(x - 3)^3} = -\infty$

$\lim_{x \to 3^+} \dfrac{2}{(x - 3)^3} = \infty$

Therefore, $x = 3$ is a vertical asymptote.

19. $f(x) = \dfrac{x^2}{x^2 - 4} = \dfrac{x^2}{(x + 2)(x - 2)}$

$\lim_{x \to -2^-} \dfrac{x^2}{x^2 - 4} = \infty$ and $\lim_{x \to -2^+} \dfrac{x^2}{x^2 - 4} = -\infty$

Therefore, $x = -2$ is a vertical asymptote.

$\lim_{x \to 2^-} \dfrac{x^2}{x^2 - 4} = -\infty$ and $\lim_{x \to 2^+} \dfrac{x^2}{x^2 - 4} = \infty$

Therefore, $x = 2$ is a vertical asymptote.

20. $f(x) = \dfrac{3x}{x^2 + 9}$

No vertical asymptotes because the denominator is never zero.

21. $g(t) = \dfrac{t - 1}{t^2 + 1}$

No vertical asymptotes because the denominator is never zero.

22. $h(s) = \dfrac{3s + 4}{s^2 - 16} = \dfrac{3s + 4}{(s - 4)(s + 4)}$

$\lim_{s \to 4^-} \dfrac{3s + 4}{s^2 - 16} = -\infty$ and $\lim_{s \to 4^+} \dfrac{3s + 4}{s^2 - 16} = \infty$

Therefore, $s = 4$ is a vertical asymptote.

$\lim_{s \to -4^-} \dfrac{3s + 4}{s^2 - 16} = -\infty$ and $\lim_{s \to -4^+} \dfrac{3s + 4}{s^2 - 16} = \infty$

Therefore, $s = -4$ is a vertical asymptote.

23. $f(x) = \dfrac{3}{x^2 + x - 2} = \dfrac{3}{(x + 2)(x - 1)}$

$\lim_{x \to -2^-} \dfrac{3}{x^2 + x - 2} = \infty$ and $\lim_{x \to -2^+} \dfrac{3}{x^2 + x - 2} = -\infty$

Therefore, $x = -2$ is a vertical asymptote.

$\lim_{x \to 1^-} \dfrac{3}{x^2 + x - 2} = -\infty$ and $\lim_{x \to 1^+} \dfrac{3}{x^2 + x - 2} = \infty$

Therefore, $x = 1$ is a vertical asymptote.

24. $g(x) = \dfrac{x^2 - 5x + 25}{x^3 + 125}$

$= \dfrac{x^2 - 5x + 25}{(x + 5)(x^2 - 5x + 25)}$

$= \dfrac{1}{x + 5}$

$\lim_{x \to -5^-} \dfrac{1}{x + 5} = -\infty$ and $\lim_{x \to -5^+} \dfrac{1}{x + 5} = \infty$

Therefore, $x = -5$ is a vertical asymptote.

25. $f(x) = \dfrac{4(x^2 + x - 6)}{x(x^3 - 2x^2 - 9x + 18)}$

$= \dfrac{4(x+3)(x-2)}{x(x-2)(x^2-9)}$

$= \dfrac{4}{x(x-3)}, x \neq -3, 2$

$\lim_{x\to 0^-} f(x) = \infty$ and $\lim_{x\to 0^+} f(x) = -\infty$

Therefore, $x = 0$ is a vertical asymptote.

$\lim_{x\to 3^-} f(x) = -\infty$ and $\lim_{x\to 3^+} f(x) = \infty$

Therefore, $x = 3$ is a vertical asymptote.

$\lim_{x\to 2} f(x) = \dfrac{4}{2(2-3)}$

$= -2$

and

$\lim_{x\to -3} f(x) = \dfrac{4}{-3(-3-3)} = \dfrac{2}{9}$

Therefore, the graph has holes at $x = 2$ and $x = -3$.

26. $h(x) = \dfrac{x^2 - 9}{x^3 + 3x^2 - x - 3}$

$= \dfrac{(x-3)(x+3)}{(x-1)(x+1)(x+3)}$

$= \dfrac{x-3}{(x+1)(x-1)}, x \neq -3$

$\lim_{x\to -1^-} h(x) = -\infty$ and $\lim_{x\to -1^+} h(x) = \infty$

Therefore, $x = -1$ is a vertical asymptote.

$\lim_{x\to 1^-} h(x) = \infty$ and $\lim_{x\to 1^+} h(x) = -\infty$

Therefore, $x = 1$ is a vertical asymptote.

$\lim_{x\to -3} h(x) = \dfrac{-3-3}{(-3+1)(-3-1)} = -\dfrac{3}{4}$

Therefore, the graph has a hole at $x = -3$.

27. $f(x) = \dfrac{x^2 - 2x - 15}{x^3 - 5x^2 + x - 5}$

$= \dfrac{(x-5)(x+3)}{(x-5)(x^2+1)}$

$= \dfrac{x+3}{x^2+1}, x \neq 5$

$\lim_{x\to 5} f(x) = \dfrac{5+3}{5^2+1} = \dfrac{15}{26}$

There are no vertical asymptotes. The graph has a hole at $x = 5$.

28. $h(t) = \dfrac{t^2 - 2t}{t^4 - 16} = \dfrac{t(t-2)}{(t-2)(t+2)(t^2+4)}$

$= \dfrac{t}{(t+2)(t^2+4)}, t \neq 2$

$\lim_{t\to -2^-} h(t) = \infty$ and $\lim_{t\to -2^+} h(t) = -\infty$

Therefore, $t = -2$ is a vertical asymptote.

$\lim_{t\to 2} h(t) = \dfrac{2}{(2+2)(2^2+4)} = \dfrac{1}{16}$

Therefore, the graph has a hole at $t = 2$.

29. $f(x) = \csc \pi x = \dfrac{1}{\sin \pi x}$

Let n be any integer.

$\lim_{x\to n} f(x) = -\infty$ or ∞

Therefore, the graph has vertical asymptotes at $x = n$.

30. $f(x) = \tan \pi x = \dfrac{\sin \pi x}{\cos \pi x}$

$\cos \pi x = 0$ for $x = \dfrac{2n+1}{2}$, where n is an integer.

$\lim_{x\to \frac{2n+1}{2}} f(x) = \infty$ or $-\infty$

Therefore, the graph has vertical asymptotes at $x = \dfrac{2n+1}{2}$.

31. $s(t) = \dfrac{t}{\sin t}$

$\sin t = 0$ for $t = n\pi$, where n is an integer.

$\lim_{t\to n\pi} s(t) = \infty$ or $-\infty$ (for $n \neq 0$)

Therefore, the graph has vertical asymptotes at $t = n\pi$, for $n \neq 0$.

$\lim_{t\to 0} s(t) = 1$

Therefore, the graph has a hole at $t = 0$.

32. $g(\theta) = \dfrac{\tan \theta}{\theta} = \dfrac{\sin \theta}{\theta \cos \theta}$

$\cos \theta = 0$ for $\theta = \dfrac{\pi}{2} + n\pi$, where n is an integer.

$\lim_{\theta\to \frac{\pi}{2}+n\pi} g(\theta) = \infty$ or $-\infty$

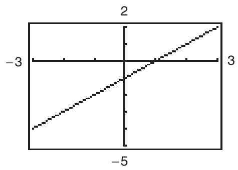
Therefore, the graph has vertical asymptotes at $\theta = \dfrac{\pi}{2} + n\pi$.

$\lim_{\theta\to 0} g(\theta) = 1$

Therefore, the graph has a hole at $\theta = 0$.

33. $\lim_{x \to -1} \frac{x^2 - 1}{x + 1} = \lim_{x \to -1} (x - 1) = -2$

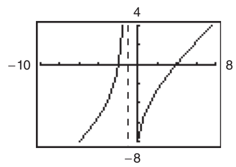
Removable discontinuity at $x = -1$



34. $\lim_{x \to -1^-} \frac{x^2 - 2x - 8}{x + 1} = \infty$

$\lim_{x \to -1^+} \frac{x^2 - 2x - 8}{x + 1} = -\infty$

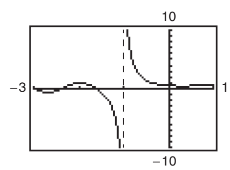
Vertical asymptote at $x = -1$



35. $\lim_{x \to -1^-} \frac{\cos(x^2 - 1)}{x + 1} = -\infty$

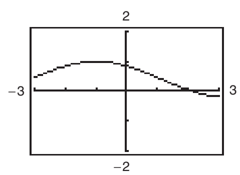
$\lim_{x \to -1^+} \frac{\cos(x^2 - 1)}{x + 1} = \infty$

Vertical asymptote at $x = 1$



36. $\lim_{x \to -1} \frac{\sin(x + 1)}{x + 1} = 1$

Removable discontinuity at $x = -1$



37. $\lim_{x \to 2^+} \frac{x}{x - 2} = \infty$

38. $\lim_{x \to 2^-} \frac{x^2}{x^2 + 4} = \frac{4}{4 + 4} = \frac{1}{2}$

39. $\lim_{x \to -3^-} \frac{x + 3}{(x^2 + x - 6)} = \lim_{x \to -3^-} \frac{x + 3}{(x + 3)(x - 2)}$

$= \lim_{x \to -3^-} \frac{1}{x - 2} = -\frac{1}{5}$

40. $\lim_{x \to -(1/2)^+} \frac{6x^2 + x - 1}{4x^2 - 4x - 3} = \lim_{x \to -(1/2)^+} \frac{(3x - 1)(2x + 1)}{(2x - 3)(2x + 1)}$

$= \lim_{x \to -(1/2)^+} \frac{3x - 1}{2x - 3} = \frac{5}{8}$

41. $\lim_{x \to 0^-} \left(1 + \frac{1}{x}\right) = -\infty$

42. $\lim_{x \to 0^+} \left(6 - \frac{1}{x^3}\right) = -\infty$

43. $\lim_{x \to -4^-} \left(x^2 + \frac{2}{x + 4}\right) = -\infty$

44. $\lim_{x \to 0^+} \left(x - \frac{1}{x} + 3\right) = -\infty$

45. $\lim_{x \to 0^+} \left(\sin x + \frac{1}{x}\right) = \infty$

46. $\lim_{x \to (\pi/2)^+} \frac{-2}{\cos x} = \infty$

47. $\lim_{x \to \pi^+} \frac{\sqrt{x}}{\csc x} = \lim_{x \to \pi^+} \left(\sqrt{x} \sin x\right) = 0$

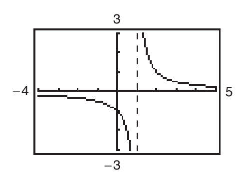
48. $\lim_{x \to 0^-} \frac{x + 2}{\cot x} = \lim_{x \to 0^-} \left[(x + 2) \tan x\right] = 0$

49. $\lim_{x \to (1/2)^-} x \sec \pi x = \lim_{x \to (1/2)^-} \frac{x}{\cos \pi x} = \infty$

50. $\lim_{x \to (1/2)^+} x^2 \tan \pi x = -\infty$

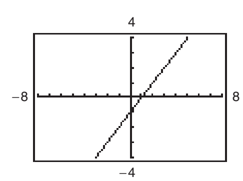
51. $f(x) = \frac{x^2 + x + 1}{x^3 - 1} = \frac{x^2 + x + 1}{(x - 1)(x^2 + x + 1)}$

$\lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} \frac{1}{x - 1} = \infty$



52. $f(x) = \frac{x^3 - 1}{x^2 + x + 1} = \frac{(x - 1)(x^2 + x + 1)}{x^2 + x + 1}$

$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} (x - 1) = 0$



53. $\lim_{x \to c} f(x) = \infty$ and $\lim_{x \to c} g(x) = -2$

(a) $\lim_{x \to c} \left[f(x) + g(x)\right] = \infty - 2 = \infty$

(b) $\lim_{x \to c} \left[f(x)g(x)\right] = \infty(-2) = -\infty$

(c) $\lim_{x \to c} \frac{g(x)}{f(x)} = \frac{-2}{\infty} = 0$

54. $\lim_{x\to c} f(x) = -\infty$ and $\lim_{x\to c} g(x) = 3$

(a) $\lim_{x\to c}\left[f(x) + g(x)\right] = -\infty + 3 = -\infty$

(b) $\lim_{x\to c}\left[f(x)g(x)\right] = (-\infty)(3) = -\infty$

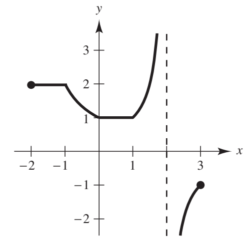
(c) $\lim_{x\to c}\dfrac{g(x)}{f(x)} = \dfrac{3}{-\infty} = 0$

55. One answer is

$$f(x) = \frac{x-3}{(x-6)(x+2)} = \frac{x-3}{x^2-4x-12}.$$

56. No. For example, $f(x) = \dfrac{1}{x^2+1}$ has no vertical asymptote.

57.

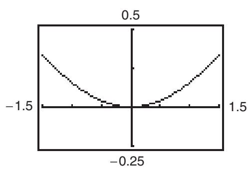


58. $m = \dfrac{m_0}{\sqrt{1-(v^2/c^2)}}$

$$\lim_{v\to c^-} m = \lim_{v\to c^-}\frac{m_0}{\sqrt{1-(v^2/c^2)}} = \infty$$

59. (a)

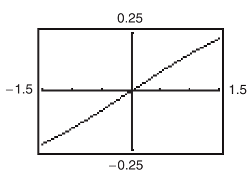
x	1	0.5	0.2	0.1	0.01	0.001	0.0001
$f(x)$	0.1585	0.0411	0.0067	0.0017	≈ 0	≈ 0	≈ 0



$$\lim_{x\to 0^+}\frac{x-\sin x}{x} = 0$$

(b)

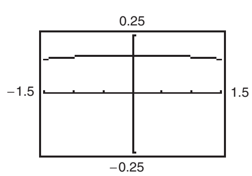
x	1	0.5	0.2	0.1	0.01	0.001	0.0001
$f(x)$	0.1585	0.0823	0.0333	0.0167	0.0017	≈ 0	≈ 0



$$\lim_{x\to 0^+}\frac{x-\sin x}{x^2} = 0$$

(c)

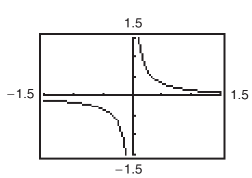
x	1	0.5	0.2	0.1	0.01	0.001	0.0001
$f(x)$	0.1585	0.1646	0.1663	0.1666	0.1667	0.1667	0.1667



$$\lim_{x\to 0^+}\frac{x-\sin x}{x^3} = 0.1667\ (1/6)$$

(d)

x	1	0.5	0.2	0.1	0.01	0.001	0.0001
$f(x)$	0.1585	0.3292	0.8317	1.6658	16.67	166.7	1667.0



$$\lim_{x\to 0^+}\frac{x-\sin x}{x^4} = \infty \text{ or } n > 3,\ \lim_{x\to 0^+}\frac{x-\sin x}{x^n} = \infty.$$

60. $\lim_{V \to 0^+} P = \infty$

As the volume of the gas decreases, the pressure increases.

61. (a) $r = \dfrac{2(7)}{\sqrt{625 - 49}} = \dfrac{7}{12}$ ft/sec

(b) $r = \dfrac{2(15)}{\sqrt{625 - 225}} = \dfrac{3}{2}$ ft/sec

(c) $\lim_{x \to 25^-} \dfrac{2x}{\sqrt{625 - x^2}} = \infty$

62. (a) $\text{Average speed} = \dfrac{\text{Total distance}}{\text{Total time}}$

$$50 = \frac{2d}{(d/x) + (d/y)}$$

$$50 = \frac{2xy}{y + x}$$

$$50y + 50x = 2xy$$

$$50x = 2xy - 50y$$

$$50x = 2y(x - 25)$$

$$\frac{25x}{x - 25} = y$$

Domain: $x > 25$

(b)

x	30	40	50	60
y	150	66.667	50	42.857

(c) $\lim_{x \to 25^+} \dfrac{25x}{\sqrt{x - 25}} = \infty$

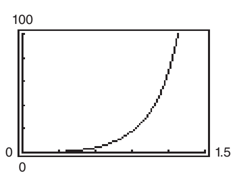
As x gets close to 25 miles per hour, y becomes larger and larger.

63. (a) $A = \dfrac{1}{2}bh - \dfrac{1}{2}r^2\theta = \dfrac{1}{2}(10)(10 \tan \theta) - \dfrac{1}{2}(10)^2\theta = 50 \tan \theta - 50\theta$

Domain: $\left(0, \dfrac{\pi}{2}\right)$

(b)

θ	0.3	0.6	0.9	1.2	1.5
$f(\theta)$	0.47	4.21	18.0	68.6	630.1



(c) $\lim_{\theta \to \pi/2^-} A = \infty$

64. (a) Because the circumference of the motor is half that of the saw arbor, the saw makes $1700/2 = 850$ revolutions per minute.

(b) The direction of rotation is reversed.

(c) $2(20 \cot \phi) + 2(10 \cot \phi)$: straight sections. The angle subtended in each circle is $2\pi - \left(2\left(\dfrac{\pi}{2} - \phi\right)\right) = \pi + 2\phi$.

So, the length of the belt around the pulleys is $20(\pi + 2\phi) + 10(\pi + 2\phi) = 30(\pi + 2\phi)$.

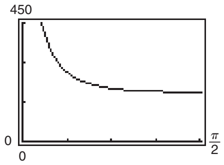
$\text{Total length} = 60 \cot \phi + 30(\pi + 2\phi)$

Domain: $\left(0, \dfrac{\pi}{2}\right)$

(d)

ϕ	0.3	0.6	0.9	1.2	1.5
L	306.2	217.9	195.9	189.6	188.5

(e)



(f) $\lim_{\phi \to (\pi/2)^-} L = 60\pi \approx 188.5$

(All the belts are around pulleys.)

(g) $\lim_{\phi \to 0^+} L = \infty$

65. True. The function is undefined at a vertical asymptote.

66. True

67. False. The graphs of $y = \tan x$, $y = \cot x$, $y = \sec x$ and $y = \csc x$ have vertical asymptotes.

68. False. Let

$$f(x) = \begin{cases} \frac{1}{x}, & x \neq 0 \\ 3, & x = 0. \end{cases}$$

The graph of f has a vertical asymptote at $x = 0$, but $f(0) = 3$.

69. Let $f(x) = \frac{1}{x^2}$ and $g(x) = \frac{1}{x^4}$, and $c = 0$.

$\lim_{x \to 0} \frac{1}{x^2} = \infty$ and $\lim_{x \to 0} \frac{1}{x^4} = \infty$, but $\lim_{x \to 0} \left(\frac{1}{x^2} - \frac{1}{x^4}\right) = \lim_{x \to 0} \left(\frac{x^2 - 1}{x^4}\right) = -\infty \neq 0$.

70. Given $\lim_{x \to c} f(x) = \infty$ and $\lim_{x \to c} g(x) = L$:

(1) Difference:

Let $h(x) = -g(x)$. Then $\lim_{x \to c} h(x) = -L$, and $\lim_{x \to c} [f(x) - g(x)] = \lim_{x \to c} [f(x) + h(x)] = \infty$, by the Sum Property.

(2) Product:

If $L > 0$, then for $\varepsilon = L/2 > 0$ there exists $\delta_1 > 0$ such that $|g(x) - L| < L/2$ whenever $0 < |x - c| < \delta_1$. So, $L/2 < g(x) < 3L/2$. Because $\lim_{x \to c} f(x) = \infty$ then for $M > 0$, there exists $\delta_2 > 0$ such that $f(x) > M(2/L)$ whenever $|x - c| < \delta_2$. Let δ be the smaller of δ_1 and δ_2. Then for $0 < |x - c| < \delta$, you have $f(x)g(x) > M(2/L)(L/2) = M$. Therefore $\lim_{x \to c} f(x)g(x) = \infty$. The proof is similar for $L < 0$.

(3) Quotient: Let $\varepsilon > 0$ be given.

There exists $\delta_1 > 0$ such that $f(x) > 3L/2\varepsilon$ whenever $0 < |x - c| < \delta_1$ and there exists $\delta_2 > 0$ such that $|g(x) - L| < L/2$ whenever $0 < |x - c| < \delta_2$. This inequality gives us $L/2 < g(x) < 3L/2$. Let δ be the smaller of δ_1 and δ_2. Then for $0 < |x - c| < \delta$, you have

$$\left|\frac{g(x)}{f(x)}\right| < \frac{3L/2}{3L/2\varepsilon} = \varepsilon.$$

Therefore, $\lim_{x \to c} \frac{g(x)}{f(x)} = 0$.

71. Given $\lim_{x \to c} f(x) = \infty$, let $g(x) = 1$. Then $\lim_{x \to c} \frac{g(x)}{f(x)} = 0$ by Theorem 1.15.

72. Given $\lim_{x\to c}\frac{1}{f(x)} = 0$. Suppose $\lim_{x\to c} f(x)$ exists and equals L.

Then, $\lim_{x\to c}\frac{1}{f(x)} = \frac{\lim_{x\to c} 1}{\lim_{x\to c} f(x)} = \frac{1}{L} = 0.$

This is not possible. So, $\lim_{x\to c} f(x)$ does not exist.

73. $f(x) = \frac{1}{x-3}$ is defined for all $x > 3$. Let $M > 0$ be given. You need $\delta > 0$ such that $f(x) = \frac{1}{x-3} > M$ whenever $3 < x < 3 + \delta$. Equivalently, $x - 3 < \frac{1}{M}$ whenever $|x-3| < \delta, x > 3$. So take $\delta = \frac{1}{M}$. Then for $x > 3$ and $|x-3| < \delta, \frac{1}{x-3} > \frac{1}{8} = M$ and so $f(x) > M$. Thus, $\lim_{x\to 3^+}\frac{1}{x-3} = \infty$.

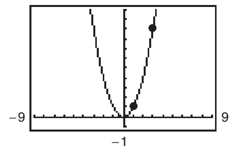
74. $f(x) = \frac{1}{x-5}$ is defined for all $x < 5$. Let $N < 0$ be given. You need $\delta > 0$ such that $f(x) = \frac{1}{x-5} < N$ whenever $5 - \delta < x < 5$. Equivalently, $x - 5 > \frac{1}{N}$ whenever $|x-5| < \delta, x < 5$. Equivalently, $\frac{1}{|x-5|} < -\frac{1}{N}$ whenever $|x-5| < \delta, x < 5$. So take $\delta = -\frac{1}{N}$. Note that $\delta > 0$ because $N < 0$. For $|x-5| < \delta$ and $x < 5, \frac{1}{|x-5|} > \frac{1}{\delta} = -N$, and $\frac{1}{x-5} = -\frac{1}{|x-5|} < N$. Thus, $\lim_{x\to 5^-}\frac{1}{x-5} = -\infty$.

75. $f(x) = \frac{3}{8-x}$ is defined for all $x > 8$. Let $N < 0$ be given. You need $\delta > 0$ such that $f(x) = \frac{3}{8-x} < N$ whenever $8 < x < 8 + \delta$. Equivalently, $\frac{8-x}{3} > \frac{1}{N}$ whenever $|x-8| < \delta, x > 8$. Equivalently, $|8-x| < \frac{-3}{N}$ whenever $|x-8| < \delta, x > 8$. So, let $\delta = \frac{-3}{N}$. Note that $\delta > 0$ because $N < 0$. Finally, for $|x-8| < \delta$ and $x > 8$, $\frac{1}{|x-8|} > \frac{1}{\delta} = \frac{N}{-3}, \frac{-3}{|x-8|} < N$, and $\frac{3}{8-x} < N$. Thus, $\lim_{x\to 8^+} f(x) = -\infty$.

76. $f(x) = \frac{6}{9-x}$ is defined for all $x < 9$. Let $M > 0$ be given. You need $\delta > 0$ such that $f(x) = \frac{6}{9-x} > M$ whenever $9 - \delta < x < 9$. Equivalently, $9 - x < \frac{6}{M}$ whenever $|x-9| < \delta, x < 9$. So, let $\delta = \frac{6}{M}$. Finally, for $|x-9| < \delta$ and $x < 9$, $|x-9| < \frac{6}{M}, \frac{1}{|x-9|} > \frac{M}{6}$, and $\frac{6}{9-x} > M$. Thus, $\lim_{x\to 9^-} f(x) = \delta$.

Review Exercises for Chapter 1

1. Calculus required. Using a graphing utility, you can estimate the length to be 8.3. Or, the length is slightly longer than the distance between the two points, approximately 8.25.

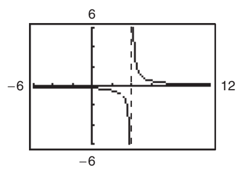


2. Precalculus. $L = \sqrt{(9-1)^2 + (3-1)^2} \approx 8.25$

3. $f(x) = \dfrac{x - 3}{x^2 - 7x + 12}$

x	2.9	2.99	2.999	3	3.001	3.01	3.1
$f(x)$	–0.9091	–0.9901	–0.9990	?	–1.0010	–1.0101	–1.1111

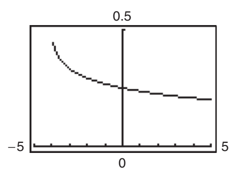
$\lim_{x\to 3} f(x) \approx -1.0000$ (Actual limit is -1.)



4. $f(x) = \dfrac{\sqrt{x + 4} - 2}{x}$

x	–0.1	–0.01	–0.001	0	0.001	0.01	0.1
$f(x)$	0.2516	0.2502	0.2500	?	0.2500	0.2498	0.2485

$\lim_{x\to 0} f(x) \approx 0.2500$ $\left(\text{Actual limit is } \frac{1}{4}.\right)$



5. $h(x) = \left[\!\left[-\dfrac{x}{2}\right]\!\right] + x^2$

(a) The limit does not exist at $x = 2$. The function approaches 3 from the left side of 2, but it approaches 2 from the right side of 2.

(b) $\lim_{x\to 1} h(x) = \left[\!\left[-\dfrac{1}{2}\right]\!\right] + x^2 = -1 + 1 = 0$

6. $g(x) = \dfrac{-2x}{x - 3}$

(a) $\lim_{x\to 3} g(x)$ does not exist because the function increases and decreases without bound as x approaches 3.

(b) $\lim_{x\to 0} g(x) = \dfrac{-2(0)}{0 - 3} = 0$

7. $\lim_{x\to 1} (x + 4) = 1 + 4 = 5$

Let $\varepsilon > 0$ be given. Choose $\delta = \varepsilon$. Then for $0 < |x - 1| < \delta = \varepsilon$, you have

$$|x - 1| < \varepsilon$$
$$|(x + 4) - 5| < \varepsilon$$
$$|f(x) - L| < \varepsilon.$$

8. $\lim_{x\to 9} \sqrt{x} = \sqrt{9} = 3$

Let $\varepsilon > 0$ be given. You need

$\left|\sqrt{x} - 3\right| < \varepsilon \Rightarrow \left|\sqrt{x} + 3\right|\left|\sqrt{x} - 3\right| < \varepsilon\left|\sqrt{x} + 3\right| \Rightarrow |x - 9| < \varepsilon\left|\sqrt{x} + 3\right|.$

Assuming $4 < x < 16$, you can choose $\delta = 5\varepsilon$.

So, for $0 < |x - 9| < \delta = 5\varepsilon$, you have

$$|x - 9| < 5\varepsilon < \left|\sqrt{x} + 3\right|\varepsilon$$
$$\left|\sqrt{x} - 3\right| < \varepsilon$$
$$|f(x) - L| < \varepsilon.$$

9. $\lim_{x\to 2} \left(1 - x^2\right) = 1 - 2^2 = -3$

Let $\varepsilon > 0$ be given. You need

$\left|1 - x^2 - (-3)\right| < \varepsilon \Rightarrow \left|x^2 - 4\right| = |x - 2||x + 2| < \varepsilon \Rightarrow |x - 2| < \dfrac{1}{|x + 2|}\varepsilon$

Assuming $1 < x < 3$, you can choose $\delta = \dfrac{\varepsilon}{5}$.

So, for $0 < |x - 2| < \delta = \dfrac{\varepsilon}{5}$, you have

$$|x - 2| < \frac{\varepsilon}{5} < \frac{\varepsilon}{|x + 2|}$$
$$|x - 2||x + 2| < \varepsilon$$
$$\left|x^2 - 4\right| < \varepsilon$$
$$\left|4 - x^2\right| < \varepsilon$$
$$\left|\left(1 - x^2\right) - (-3)\right| < \varepsilon$$
$$|f(x) - L| < \varepsilon.$$

10. $\lim_{x\to 5} 9 = 9$. Let $\varepsilon > 0$ be given. δ can be any positive number. So, for $0 < |x - 5| < \delta$, you have

$$|9 - 9| < \varepsilon$$
$$|f(x) - L| < \varepsilon.$$

11. $\lim_{x\to -6} x^2 = (-6)^2 = 36$

12. $\lim_{x\to 0} (5x - 3) = 5(0) - 3 = -3$

13. $\lim_{t\to 4} \sqrt{t + 2} = \sqrt{4 + 2} = \sqrt{6} \approx 2.45$

14. $\lim_{x\to 2} \sqrt{x^3 + 1} = \sqrt{2^3 + 1} = \sqrt{8 + 1} = \sqrt{9} = 3$

15. $\lim_{x\to 27} \left(\sqrt[3]{x} - 1\right)^4 = \left(\sqrt[3]{27} - 1\right)^4 = (3 - 1)^4 = 2^4 = 16$

16. $\lim_{x\to 7} (x - 4)^3 = (7 - 4)^3 = 3^3 = 27$

17. $\lim_{x\to 4} \dfrac{4}{x - 1} = \dfrac{4}{4 - 1} = \dfrac{4}{3}$

18. $\lim_{x\to 2} \dfrac{x}{x^2 + 1} = \dfrac{2}{2^2 + 1} = \dfrac{2}{4 + 1} = \dfrac{2}{5}$

19.
$$\lim_{x\to -3} \frac{2x^2 + 11x + 15}{x + 3} = \lim_{x\to -3} \frac{(2x + 5)(x + 3)}{x + 3}$$
$$= \lim_{x\to -3} (2x + 5)$$
$$= 2(-3) + 5$$
$$= -1$$

20.
$$\lim_{t\to 4} \frac{t^2 - 16}{t - 4} = \lim_{t\to 4} \frac{(t - 4)(t + 4)}{t - 4}$$
$$= \lim_{t\to 4} (t + 4) = 4 + 4 = 8$$

21. $\lim_{x\to 4}\frac{\sqrt{x-3}-1}{x-4} = \lim_{x\to 4}\frac{\sqrt{x-3}-1}{x-4}\cdot\frac{\sqrt{x-3}+1}{\sqrt{x-3}+1}$

$= \lim_{x\to 4}\frac{(x-3)-1}{(x-4)\left(\sqrt{x-3}+1\right)}$

$= \lim_{x\to 4}\frac{1}{\sqrt{x-3}+1} = \frac{1}{2}$

22. $\lim_{x\to 0}\frac{\sqrt{4+x}-2}{x} = \lim_{x\to 0}\frac{\sqrt{4+x}-2}{x}\cdot\frac{\sqrt{4+x}+2}{\sqrt{4+x}+2}$

$= \lim_{x\to 0}\frac{1}{\sqrt{4+x}+2} = \frac{1}{4}$

23. $\lim_{x\to 0}\frac{\left[1/(x+1)\right]-1}{x} = \lim_{x\to 0}\frac{1-(x+1)}{x(x+1)}$

$= \lim_{x\to 0}\frac{-1}{x+1} = -1$

24. $\lim_{s\to 0}\frac{\left(1/\sqrt{1+s}\right)-1}{s} = \lim_{s\to 0}\left[\frac{\left(1/\sqrt{1+s}\right)-1}{s}\cdot\frac{\left(1/\sqrt{1+s}\right)+1}{\left(1/\sqrt{1+s}\right)+1}\right]$

$= \lim_{s\to 0}\frac{\left[1/(1+s)\right]-1}{s\left[\left(1/\sqrt{1+s}\right)+1\right]} = \lim_{s\to 0}\frac{-1}{(1+s)\left[\left(1/\sqrt{1+s}\right)+1\right]} = -\frac{1}{2}$

25. $\lim_{x\to 0}\frac{1-\cos x}{\sin x} = \lim_{x\to 0}\left(\frac{x}{\sin x}\right)\left(\frac{1-\cos x}{x}\right) = (1)(0) = 0$

26. $\lim_{x\to(\pi/4)}\frac{4x}{\tan x} = \frac{4(\pi/4)}{1} = \pi$

27. $\lim_{\Delta x\to 0}\frac{\sin\left[(\pi/6)+\Delta x\right]-(1/2)}{\Delta x} = \lim_{\Delta x\to 0}\frac{\sin(\pi/6)\cos\Delta x+\cos(\pi/6)\sin\Delta x-(1/2)}{\Delta x}$

$= \lim_{\Delta x\to 0}\frac{1}{2}\cdot\frac{(\cos\Delta x-1)}{\Delta x} + \lim_{\Delta x\to 0}\frac{\sqrt{3}}{2}\cdot\frac{\sin\Delta x}{\Delta x} = 0 + \frac{\sqrt{3}}{2}(1) = \frac{\sqrt{3}}{2}$

28. $\lim_{\Delta x\to 0}\frac{\cos(\pi+\Delta x)+1}{\Delta x} = \lim_{\Delta x\to 0}\frac{\cos\pi\cos\Delta x-\sin\pi\sin\Delta x+1}{\Delta x}$

$= \lim_{\Delta x\to 0}\left[-\frac{(\cos\Delta x-1)}{\Delta x}\right] - \lim_{\Delta x\to 0}\left[\sin\pi\frac{\sin\Delta x}{\Delta x}\right]$

$= -0-(0)(1) = 0$

29. $\lim_{x\to c}\left[f(x)g(x)\right] = \left[\lim_{x\to c}f(x)\right]\left[\lim_{x\to c}g(x)\right]$

$= (-6)\left(\frac{1}{2}\right) = -3$

30. $\lim_{x\to c}\frac{f(x)}{g(x)} = \frac{\lim_{x\to c}f(x)}{\lim_{x\to c}g(x)} = \frac{-6}{\left(\frac{1}{2}\right)} = -12$

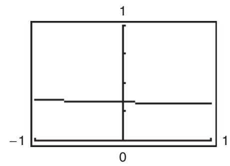
31. $\lim_{x\to c}\left[f(x)+2g(x)\right] = \lim_{x\to c}f(x) + 2\lim_{x\to c}g(x)$

$= -6+2\left(\frac{1}{2}\right) = -5$

32. $\lim_{x\to c}\left[f(x)\right]^2 = \left[\lim_{x\to c}f(x)\right]^2$

$= (-6)^2 = 36$

33. $f(x) = \dfrac{\sqrt{2x+9}-3}{x}$



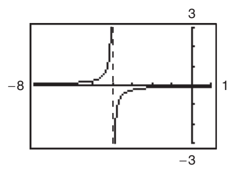
The limit appears to be $\frac{1}{3}$.

x	–0.01	–0.001	0	0.001	0.01
$f(x)$	0.3335	0.3333	?	0.3333	0.331

$\lim_{x\to 0} f(x) \approx 0.3333$

$$\lim_{x\to 0} \frac{\sqrt{2x+9}-3}{x} \cdot \frac{\sqrt{2x+9}+3}{\sqrt{2x+9}+3} = \lim_{x\to 0} \frac{(2x+9)-9}{x\left[\sqrt{2x+9}+3\right]} = \lim_{x\to 0} \frac{2}{\sqrt{2x+9}+3} = \frac{2}{\sqrt{9}+3} = \frac{1}{3}$$

34. $f(x) = \dfrac{\left[1/(x+4)\right]-(1/4)}{x}$



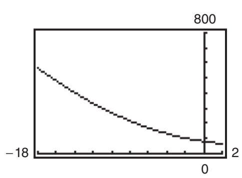
The limit appears to be $-\frac{1}{16}$

x	–0.01	–0.001	0	0.001	0.01
$f(x)$	–0.0627	–0.0625	?	–0.0625	–0.0623

$\lim_{x\to 0} f(x) \approx -0.0625 = -\frac{1}{16}$

$$\lim_{x\to 0} \frac{\frac{1}{x+4}-\frac{1}{4}}{x} = \lim_{x\to 0} \frac{4-(x+4)}{(x+4)4(x)} = \lim_{x\to 0} \frac{-1}{(x+4)4} = -\frac{1}{16}$$

35. $f(x) = \dfrac{x^3+729}{x+9}$



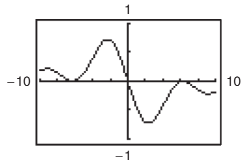
The limit appears to be 243.

x	–9.1	–9.01	–9.001	–9	–8.999	–8.99	–8.9
$f(x)$	245.7100	243.2701	243.0270	?	242.9730	242.7301	24.3100

$\lim_{x\to -9} \dfrac{x^3+729}{x+9} \approx 243.00$

$$\lim_{x\to -9} \frac{x^3+729}{x+9} = \lim_{x\to -9} \frac{(x+9)(x^2-9x+81)}{x+9} = \lim_{x\to -9} (x^2-9x+81) = 81+81+81 = 243$$

36. $f(x) = \dfrac{\cos x - 1}{x}$



The limit appears to be 0.

x	−0.01	−0.001	0	0.001	0.01
$f(x)$	0.005	0.0005	0	−0.0005	−0.005

$\lim_{x\to 0} f(x) \approx 0.000$

$$\lim_{x\to 0}\frac{\cos x - 1}{x} = \lim_{x\to 0}\frac{\cos x - 1}{x}\cdot\frac{\cos x + 1}{\cos x + 1}$$
$$= \lim_{x\to 0}\frac{\cos^2 x - 1}{x(\cos x + 1)}$$
$$= \lim_{x\to 0}\frac{-\sin^2 x}{x(\cos x + 1)}$$
$$= \lim_{x\to 0}\left(\frac{\sin x}{x}\right)\left(\frac{-\sin x}{\cos x + 1}\right)$$
$$= (1)\left(\frac{0}{2}\right)$$
$$= 0$$

37. $$v = \lim_{t\to 4}\frac{s(4) - s(t)}{4 - t}$$
$$= \lim_{t\to 4}\frac{\left[-4.9(16) + 250\right] - \left[-4.9t^2 + 250\right]}{4 - t}$$
$$= \lim_{t\to 4}\frac{4.9\left(t^2 - 16\right)}{4 - t}$$
$$= \lim_{t\to 4}\frac{4.9(t - 4)(t + 4)}{4 - t}$$
$$= \lim_{t\to 4}\left[-4.9(t + 4)\right] = -39.2 \text{ m/sec}$$

The object is falling at about 39.2 m/sec.

38. $-4.9t^2 + 250 = 0 \Rightarrow t = \dfrac{50}{7} \approx 7.143$

The object will hit the ground after about 7.1 seconds.

When $a = \dfrac{50}{7}$, the velocity is

$$\lim_{t\to a}\frac{s(a) - s(t)}{a - t} = \lim_{t\to a}\frac{\left[-4.9a^2 + 250\right] - \left[-4.9t^2 + 250\right]}{a - t}$$
$$= \lim_{t\to a}\frac{4.9\left(t^2 - a^2\right)}{a - t}$$
$$= \lim_{t\to a}\frac{4.9(t - a)(t + a)}{a - t}$$
$$= \lim_{t\to a}\left[-4.9(t + a)\right]$$
$$= -4.9(2a) \qquad \left(a = \frac{50}{7}\right)$$
$$= -70 \text{ m/sec.}$$

39. $\displaystyle\lim_{x\to 3^+}\frac{1}{x + 3} = \frac{1}{3 + 3} = \frac{1}{6}$

40. $$\lim_{x\to 6^-}\frac{x - 6}{x^2 - 36} = \lim_{x\to 6^-}\frac{x - 6}{(x - 6)(x + 6)}$$
$$= \lim_{x\to 6^-}\frac{1}{x + 6} = \frac{1}{12}$$

41. $$\lim_{x\to 25^+}\frac{\sqrt{x} - 5}{x - 25} = \lim_{x\to 25^+}\frac{\sqrt{x} - 5}{\left(\sqrt{x} + 5\right)\left(\sqrt{x} - 5\right)}$$
$$= \lim_{x\to 25^+}\frac{1}{\sqrt{x} + 5}$$
$$= \frac{1}{\sqrt{25} + 5} = \frac{1}{5 + 5} = \frac{1}{10}$$

42. $\displaystyle\lim_{x\to 3^-}\frac{|x - 3|}{x - 3} = \lim_{x\to 3^-}\frac{-(x - 3)}{x - 3} = -1$

43. $\lim_{x\to 2} f(x) = 0$

44. $\lim_{x\to 1^+} g(x) = 1 + 1 = 2$

45. $\lim_{t\to 1} h(t)$ does not exist because $\lim_{t\to 1^-} h(t) = 1 + 1 = 2$ and $\lim_{t\to 1^+} h(t) = \frac{1}{2}(1 + 1) = 1$.

46. $\lim_{s\to -2} f(s) = 2$

47. $\lim_{x\to 2^-}\left(2[\![x]\!] + 1\right) = 2(1) + 1 = 3$

48. $\lim_{x\to 4}[\![x - 1]\!]$ does not exist. There is a break in the graph at $x = 4$.

49. $$\lim_{x\to 2^-}\frac{x^2 - 4}{|x - 2|} = \lim_{x\to 2^-}\frac{(x - 2)(x + 2)}{2 - x}$$
$$= \lim_{x\to 2^-} -(x + 2) = -(2 + 2) = -4$$

50. $\lim_{x\to 1^+}\sqrt{x(x - 1)} = \sqrt{1(1 - 1)} = 0$

51. The function $g(x) = \sqrt{8 - x^3}$ is continuous on $[-2, 2]$ because $8 - x^3 \ge 0$ on $[-2, 2]$.

52. The function $h(x) = \dfrac{3}{5 - x}$ is not continuous on $[0, 5]$ because $h(5)$ is not defined.

53. $f(x) = x^4 - 81x$ is continuous for all real x.

54. $f(x) = x^2 - x + 20$ is continuous for all real x.

55. $f(x) = \dfrac{4}{x-5}$ has a nonremovable discontinuity at $x = 5$ because $\lim_{x\to 5} f(x)$ does not exist.

56. $f(x) = \dfrac{1}{x^2 - 9} = \dfrac{1}{(x-3)(x+3)}$

has nonremovable discontinuities at $x = \pm 3$ because $\lim_{x\to 3} f(x)$ and $\lim_{x\to -3} f(x)$ do not exist.

57. $f(x) = \dfrac{x}{x^3 - x} = \dfrac{x}{x(x^2-1)} = \dfrac{1}{(x-1)(x+1)},\ x \neq 0$

has nonremovable discontinuities at $x = \pm 1$ because $\lim_{x\to -1} f(x)$ and $\lim_{x\to 1} f(x)$ do not exist, and has a removable discontinuity at $x = 0$ because

$$\lim_{x\to 0} f(x) = \lim_{x\to 0} \frac{1}{(x-1)(x+1)} = -1.$$

58. $f(x) = \dfrac{x+3}{x^2 - 3x - 18}$

$= \dfrac{x+3}{(x+3)(x-6)}$

$= \dfrac{1}{x-6},\ x \neq -3$

has a nonremovable discontinuity at $x = 6$ because $\lim_{x\to 6} f(x)$ does not exist, and has a removable discontinuity at $x = -3$ because

$$\lim_{x\to -3} f(x) = \lim_{x\to -3} \frac{1}{x-6} = -\frac{1}{9}.$$

59. $f(2) = 5$

Find c so that $\lim_{x\to 2^+} (cx + 6) = 5$.

$c(2) + 6 = 5$

$2c = -1$

$c = -\dfrac{1}{2}$

60. $\lim_{x\to 1^+} (x+1) = 2$

$\lim_{x\to 3^-} (x+1) = 4$

Find b and c so that $\lim_{x\to 1^-} (x^2 + bx + c) = 2$ and $\lim_{x\to 3^+} (x^2 + bx + c) = 4$.

Consequently you get $1 + b + c = 2$ and $9 + 3b + c = 4$.

Solving simultaneously, $b = -3$ and $c = 4$.

61. $f(x) = -3x^2 + 7$

Continuous on $(-\infty, \infty)$

62. $f(x) = \dfrac{4x^2 + 7x - 2}{x+2} = \dfrac{(4x-1)(x+2)}{x+2}$

Continuous on $(-\infty, -2) \cup (-2, \infty)$. There is a removable discontinuity at $x = -2$.

63. $f(x) = \sqrt{x} + \cos x$ is continuous on $[0, \infty)$.

64. $f(x) = [\![x + 3]\!]$

$\lim_{x\to k^+} [\![x+3]\!] = k + 3$ where k is an integer.

$\lim_{x\to k^-} [\![x+3]\!] = k + 2$ where k is an integer.

Nonremovable discontinuity at each integer k

Continuous on $(k, k+1)$ for all integers k

65. $f(x) = \dfrac{3x^2 - x - 2}{x-1} = \dfrac{(3x+2)(x-1)}{x-1}$

$\lim_{x\to 1} f(x) = \lim_{x\to 1} (3x + 2) = 5$

Removable discontinuity at $x = 1$

Continuous on $(-\infty, 1) \cup (1, \infty)$

66. $f(x) = \begin{cases} 5 - x, & x \le 2 \\ 2x - 3, & x > 2 \end{cases}$

$\lim_{x\to 2^-} (5 - x) = 3$

$\lim_{x\to 2^+} (2x - 3) = 1$

Nonremovable discontinuity at $x = 2$

Continuous on $(-\infty, 2) \cup (2, \infty)$

67. $f(x) = 2x^3 - 3$

f is continuous on $[1, 2]$. $f(1) = -1 < 0$ and $f(2) = 13 > 0$. Therefore by the Intermediate Value Theorem, there is at least one value c in $(1, 2)$ such that $2c^3 - 3 = 0$.

68. $f(x) = x^2 + x - 2$

Consider the intervals $[-3, 0]$ and $[0, 3]$.

$f(-3) = (-3)^2 - 3 - 2 = 4 > 0$

$f(0) = -2 < 0$

By the Intermediate Value Theorem, there is at least one zero in $[-3, 0]$.

$f(0) = -2 < 0$

$f(3) = (3)^2 + 3 - 2 = 10 > 0$

Again, there is at least one zero in $[0, 3]$.

So, there are at least two zeros in $[-3, 3]$.

69. $f(x) = x^2 + 5x - 4$

f is continuous on $[-1, 2]$.

$f(-1) = (-1)^2 + 5(-1) - 4 = -8 < 2$

$f(2) = 2^2 + 5(2) - 4 = 10 > 2$

The Intermediate Value Theorem applies.

$$\begin{aligned} x^2 + 5x - 4 &= 2 \\ x^2 + 5x - 6 &= 0 \\ (x + 6)(x - 1) &= 0 \\ x &= 1 \ (x = -6 \text{ lies outside the interval.}) \\ c &= 1 \end{aligned}$$

So, $f(1) = 2$.

70. $f(x) = (x - 6)^3 + 4$

f is continuous on $[4, 7]$.

$f(4) = (4 - 6)^3 + 4 = -8 + 4 = -4 < 3$

$f(7) = (7 - 6)^3 + 4 = 1 + 4 = 5 > 3$

The Intermediate Value Theorem applies.

$$\begin{aligned} (x - 6)^3 + 4 &= 3 \\ (x - 6)^3 &= -1 \\ x - 6 &= -1 \\ x &= -1 \\ c &= 5 \end{aligned}$$

So, $f(5) = 3$.

71. $\lim_{x\to 6^-} \dfrac{1}{x - 6} = -\infty$

$\lim_{x\to 6^+} \dfrac{1}{x - 6} = \infty$

72. $\lim_{x\to 6^-} \dfrac{-1}{(x - 6)^2} = -\infty$

$\lim_{x\to 6^+} \dfrac{-1}{(x - 6)^2} = -\infty$

73. $f(x) = \dfrac{3}{x}$

$\lim_{x\to 0^-} \dfrac{3}{x} = -\infty$

$\lim_{x\to 0^+} \dfrac{3}{x} = \infty$

Therefore, $x = 0$ is a vertical asymptote.

74. $f(x) = \dfrac{5}{(x - 2)^4}$

$\lim_{x\to 2^-} \dfrac{5}{(x - 2)^4} = \infty = \lim_{x\to 2^+} \dfrac{5}{(x - 2)^4}$

Therefore, $x = 2$ is a vertical asymptote.

75. $f(x) = \dfrac{x^3}{x^2 - 9} = \dfrac{x^3}{(x + 3)(x - 3)}$

$\lim_{x\to -3^-} \dfrac{x^3}{x^2 - 9} = -\infty$ and $\lim_{x\to -3^+} \dfrac{x^3}{x^2 - 9} = \infty$

Therefore, $x = -3$ is a vertical asymptote.

$\lim_{x\to -3^-} \dfrac{x^3}{x^2 - 9} = -\infty$ and $\lim_{x\to 3^+} \dfrac{x^3}{x^2 - 9} = \infty$

Therefore, $x = 3$ is a vertical asymptote.

76. $f(x) = \dfrac{6x}{36 - x^2} = -\dfrac{6x}{(x + 6)(x - 6)}$

$\lim_{x\to -6^-} \dfrac{6x}{36 - x^2} = \infty$ and $\lim_{x\to -6^+} \dfrac{6x}{36 - x^2} = -\infty$

Therefore, $x = -6$ is a vertical asymptote.

$\lim_{x\to 6^-} \dfrac{6x}{36 - x^2} = \infty$ and $\lim_{x\to 6^+} \dfrac{6x}{36 - x^2} = -\infty$

Therefore, $x = 6$ is a vertical asymptote.

77. $f(x) = \sec \dfrac{\pi x}{2} = \dfrac{1}{\cos \dfrac{\pi x}{2}}$

$\cos \dfrac{\pi x}{2} = 0$ when $x = \pm 1, \pm 3, \ldots$

Therefore, the graph has vertical asymptotes at $x = 2n + 1$, where n is an integer.

78. $f(x) = \csc \pi x = \dfrac{1}{\sin \pi x}$

$\sin \pi x = 0$ for $x = n$, where n is an integer.

$\lim_{x \to n} f(x) = \infty$ or $-\infty$

Therefore, the graph has vertical asymptotes at $x = n$.

79. $\lim_{x \to 1^-} \dfrac{x^2 + 2x + 1}{x - 1} = -\infty$

80. $\lim_{x \to (1/2)^+} \dfrac{x}{2x - 1} = \infty$

81. $\lim_{x \to -1^+} \dfrac{x + 1}{x^3 + 1} = \lim_{x \to -1^+} \dfrac{1}{x^2 - x + 1} = \dfrac{1}{3}$

82. $\lim_{x \to -1^-} \dfrac{x + 1}{x^4 - 1} = \lim_{x \to -1^-} \dfrac{1}{(x^2 + 1)(x - 1)} = -\dfrac{1}{4}$

83. $\lim_{x \to 0^+} \left(x - \dfrac{1}{x^3}\right) = -\infty$

84. $\lim_{x \to 2^-} \dfrac{1}{\sqrt[3]{x^2 - 4}} = -\infty$

85. $\lim_{x \to 0^+} \dfrac{\sin 4x}{5x} = \lim_{x \to 0^+} \left[\dfrac{4}{5}\left(\dfrac{\sin 4x}{4x}\right)\right] = \dfrac{4}{5}$

86. $\lim_{x \to 0^-} \dfrac{\sec x^3}{2x} = -\infty$

(**Note:** $\sec x^3 \approx 1$ for x near 0.)

87. $\lim_{x \to 0^+} \dfrac{\csc 2x}{x} = \lim_{x \to 0^+} \dfrac{1}{x \sin 2x} = \infty$

88. $\lim_{x \to 0^-} \dfrac{\cos^2 x}{x} = -\infty$

89. $C = \dfrac{80{,}000p}{100 - p}, \ 0 \le p < 0$

(a) $C(50) = \dfrac{80{,}000(50)}{100 - 50} = \$80{,}000$

(b) $C(90) = \dfrac{80{,}000(90)}{100 - 90} = \$720{,}000$

(c) $\lim_{p \to 100^-} C(p) = \infty$

It would be financially impossible to remove 100% of the pollutants.

Problem Solving for Chapter 1

1. (a) Perimeter $\Delta PAO = \sqrt{x^2 + (y - 1)^2} + \sqrt{x^2 + y^2} + 1$

$= \sqrt{x^2 + (x^2 - 1)^2} + \sqrt{x^2 + x^4} + 1$

Perimeter $\Delta PBO = \sqrt{(x - 1)^2 + y^2} + \sqrt{x^2 + y^2} + 1$

$= \sqrt{(x - 1)^2 + x^4} + \sqrt{x^2 + x^4} + 1$

(b) $r(x) = \dfrac{\sqrt{x^2 + (x^2 - 1)^2} + \sqrt{x^2 + x^4} + 1}{\sqrt{(x - 1)^2 + x^4} + \sqrt{x^2 + x^4} + 1}$

x	4	2	1	0.1	0.01
Perimeter ΔPAO	33.02	9.08	3.41	2.10	2.01
Perimeter ΔPBO	33.77	9.60	3.41	2.00	2.00
$r(x)$	0.98	0.95	1	1.05	1.005

(c) $\lim_{x \to 0^+} r(x) = \dfrac{1 + 0 + 1}{1 + 0 + 1} = \dfrac{2}{2} = 1$

2. (a) Area $\Delta PAO = \frac{1}{2}bh = \frac{1}{2}(1)(x) = \frac{x}{2}$

Area $\Delta PBO = \frac{1}{2}bh = \frac{1}{2}(1)(y) = \frac{y}{2} = \frac{x^2}{2}$

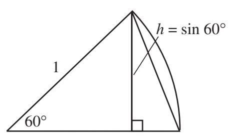
(b) $a(x) = \frac{\text{Area } \Delta PBO}{\text{Area } \Delta PAO} = \frac{x^2/2}{x/2} = x$

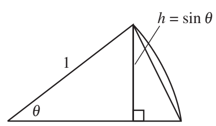
x	4	2	1	0.1	0.01
Area ΔPAO	2	1	1/2	1/20	1/200
Area ΔPBO	8	2	1/2	1/200	1/20,000
$a(x)$	4	2	1	1/10	1/100

(c) $\lim_{x \to 0^+} a(x) = \lim_{x \to 0^+} x = 0$

3. (a) There are 6 triangles, each with a central angle of $60° = \pi/3$. So,

Area hexagon $= 6\left[\frac{1}{2}bh\right] = 6\left[\frac{1}{2}(1)\sin\frac{\pi}{3}\right] = \frac{3\sqrt{3}}{2} \approx 2.598.$





Error = Area (Circle) − Area (Hexagon) $= \pi - \frac{3\sqrt{3}}{2} \approx 0.5435$

(b) There are n triangles, each with central angle of $\theta = 2\pi/n$. So,

$A_n = n\left[\frac{1}{2}bh\right] = n\left[\frac{1}{2}(1)\sin\frac{2\pi}{n}\right] = \frac{n\sin(2\pi/n)}{2}.$

(c)

n	6	12	24	48	96
A_n	2.598	3	3.106	3.133	3.139

As n gets larger and larger, $2\pi/n$ approaches 0. Letting $x = 2\pi/n$, $A_n = \frac{\sin(2\pi/n)}{2/n} = \frac{\sin(2\pi/n)}{(2\pi/n)}\pi = \frac{\sin x}{x}\pi$

which approaches $(1)\pi = \pi$, which is the area of the circle.

4. (a) Slope $= \frac{4-0}{3-0} = \frac{4}{3}$

(b) Slope $= -\frac{3}{4}$

Tangent line: $y - 4 = -\frac{3}{4}(x - 3)$

$y = -\frac{3}{4}x + \frac{25}{4}$

(c) Let $Q = (x, y) = \left(x, \sqrt{25 - x^2}\right)$

$m_x = \frac{\sqrt{25 - x^2} - 4}{x - 3}$

(d) $\lim_{x \to 3} m_x = \lim_{x \to 3} \frac{\sqrt{25 - x^2} - 4}{x - 3} \cdot \frac{\sqrt{25 - x^2} + 4}{\sqrt{25 - x^2} + 4}$

$= \lim_{x \to 3} \frac{25 - x^2 - 16}{(x - 3)\left(\sqrt{25 - x^2} + 4\right)}$

$= \lim_{x \to 3} \frac{(3 - x)(3 + x)}{(x - 3)\left(\sqrt{25 - x^2} + 4\right)}$

$= \lim_{x \to 3} \frac{-(3 + x)}{\sqrt{25 - x^2} + 4} = \frac{-6}{4 + 4} = -\frac{3}{4}$

This is the slope of the tangent line at P.

5. (a) Slope $= -\dfrac{12}{5}$

(b) Slope of tangent line is $\dfrac{5}{12}$.

$$y + 12 = \frac{5}{12}(x - 5)$$

$$y = \frac{5}{12}x - \frac{169}{12} \quad \text{Tangent line}$$

(c) $Q = (x, y) = \left(x, -\sqrt{169 - x^2}\right)$

$$m_x = \frac{-\sqrt{169 - x^2} + 12}{x - 5}$$

(d) $$\lim_{x\to5} m_x = \lim_{x\to5} \frac{12 - \sqrt{169 - x^2}}{x - 5} \cdot \frac{12 + \sqrt{169 - x^2}}{12 + \sqrt{169 - x^2}}$$

$$= \lim_{x\to5} \frac{144 - (169 - x^2)}{(x - 5)\left(12 + \sqrt{169 - x^2}\right)}$$

$$= \lim_{x\to5} \frac{x^2 - 25}{(x - 5)\left(12 + \sqrt{169 - x^2}\right)}$$

$$= \lim_{x\to5} \frac{(x + 5)}{12 + \sqrt{169 - x^2}} = \frac{10}{12 + 12} = \frac{5}{12}$$

This is the same slope as part (b).

6. $$\frac{\sqrt{a + bx} - \sqrt{3}}{x} = \frac{\sqrt{a + bx} - \sqrt{3}}{x} \cdot \frac{\sqrt{a + bx} + \sqrt{3}}{\sqrt{a + bx} + \sqrt{3}} = \frac{(a + bx) - 3}{x\left(\sqrt{a + bx} + \sqrt{3}\right)}$$

Letting $a = 3$ simplifies the numerator.

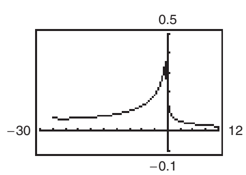
So, $$\lim_{x\to0} \frac{\sqrt{3 + bx} - \sqrt{3}}{x} = \lim_{x\to0} \frac{bx}{x\left(\sqrt{3 + bx} + \sqrt{3}\right)} = \lim_{x\to0} \frac{b}{\sqrt{3 + bx} + \sqrt{3}}$$

Setting $\dfrac{b}{\sqrt{3} + \sqrt{3}} = \sqrt{3}$, you obtain $b = 6$. So, $a = 3$ and $b = 6$.

7. (a) $$3 + x^{1/3} \ge 0$$
$$x^{1/3} \ge -3$$
$$x \ge -27$$

Domain: $x \ge -27, x \ne 1$ or $[-27, 1) \cup (1, \infty)$

(b)



(c) $$\lim_{x\to-27^+} f(x) = \frac{\sqrt{3 + (-27)^{1/3}} - 2}{-27 - 1} = \frac{-2}{-28} = \frac{1}{14} \approx 0.0714$$

(d) $$\lim_{x\to1} f(x) = \lim_{x\to1} \frac{\sqrt{3 + x^{1/3}} - 2}{x - 1} \cdot \frac{\sqrt{3 + x^{1/3}} + 2}{\sqrt{3 + x^{1/3}} + 2} = \lim_{x\to1} \frac{3 + x^{1/3} - 4}{(x - 1)\left(\sqrt{3 + x^{1/3}} + 2\right)}$$

$$= \lim_{x\to1} \frac{x^{1/3} - 1}{\left(x^{1/3} - 1\right)\left(x^{2/3} + x^{1/3} + 1\right)\left(\sqrt{3 + x^{1/3}} + 2\right)} = \lim_{x\to1} \frac{1}{\left(x^{2/3} + x^{1/3} + 1\right)\left(\sqrt{3 + x^{1/3}} + 2\right)}$$

$$= \frac{1}{(1 + 1 + 1)(2 + 2)} = \frac{1}{12}$$

8. $$\lim_{x\to0^-} f(x) = \lim_{x\to0^-} \left(a^2 - 2\right) = a^2 - 2$$

$$\lim_{x\to0^+} f(x) = \lim_{x\to0^+} \frac{ax}{\tan x} = a\left(\text{because } \lim_{x\to0} \frac{\tan x}{x} = 1\right)$$

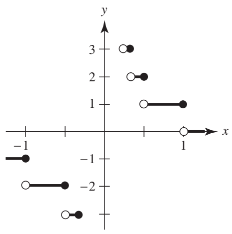
Thus, $$a^2 - 2 = a$$
$$a^2 - a - 2 = 0$$
$$(a - 2)(a + 1) = 0$$
$$a = -1, 2$$

9. (a) $\lim_{x\to2} f(x) = 3$: g_1, g_4

(b) f continuous at 2: g_1

(c) $\lim_{x\to2^-} f(x) = 3$: g_1, g_3, g_4

10.



(a) $f\left(\frac{1}{4}\right) = [\![4]\!] = 4$

$f(3) = \left[\!\left[\frac{1}{3}\right]\!\right] = 0$

$f(1) = [\![1]\!] = 1$

(b) $\lim_{x\to 1^-} f(x) = 1$

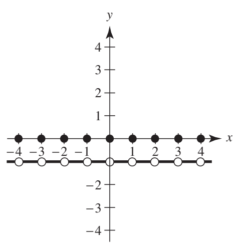
$\lim_{x\to 1^+} f(x) = 0$

$\lim_{x\to 0^-} f(x) = -\infty$

$\lim_{x\to 0^+} f(x) = \infty$

(c) f is continuous for all real numbers except $x = 0, \pm 1, \pm\frac{1}{2}, \pm\frac{1}{3}, \ldots$

11.



(a) $f(1) = [\![1]\!] + [\![-1]\!] = 1 + (-1) = 0$

$f(0) = 0$

$f\left(\frac{1}{2}\right) = 0 + (-1) = -1$

$f(-2.7) = -3 + 2 = -1$

(b) $\lim_{x\to 1^-} f(x) = -1$

$\lim_{x\to 1^+} f(x) = -1$

$\lim_{x\to 1/2} f(x) = -1$

(c) f is continuous for all real numbers except $x = 0, \pm 1, \pm 2, \pm 3, \ldots$

12. (a)

$$v^2 = \frac{192{,}000}{r} + v_0^2 - 48$$

$$\frac{192{,}000}{r} = v^2 - v_0^2 + 48$$

$$r = \frac{192{,}000}{v^2 - v_0^2 + 48}$$

$$\lim_{v\to 0} r = \frac{192{,}000}{48 - v_0^2}$$

Let $v_0 = \sqrt{48} = 4\sqrt{3}$ mi/sec.

(b)

$$v^2 = \frac{1920}{r} + v_0^2 - 2.17$$

$$\frac{1920}{r} = v^2 - v_0^2 + 2.17$$

$$r = \frac{1920}{v^2 - v_0^2 + 2.17}$$

$$\lim_{v\to 0} r = \frac{1920}{2.17 - v_0^2}$$

Let $v_0 = \sqrt{2.17}$ mi/sec $(\approx 1.47$ mi/sec$)$.

(c)

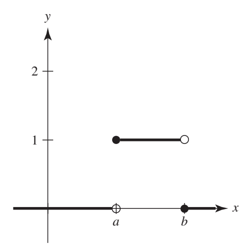
$$r = \frac{10{,}600}{v^2 - v_0^2 + 6.99}$$

$$\lim_{v\to 0} r = \frac{10{,}600}{6.99 - v_0^2}$$

Let $v_0 = \sqrt{6.99} \approx 2.64$ mi/sec.

Because this is smaller than the escape velocity for Earth, the mass is less.

13. (a)



(b) (i) $\lim_{x\to a^+} P_{a,b}(x) = 1$

(ii) $\lim_{x\to a^-} P_{a,b}(x) = 0$

(iii) $\lim_{x\to b^+} P_{a,b}(x) = 0$

(iv) $\lim_{x\to b^-} P_{a,b}(x) = 1$

(c) $P_{a,b}$ is continuous for all positive real numbers except $x = a, b$.

(d) The area under the graph of U, and above the x-axis, is 1.

14. Let $a \neq 0$ and let $\varepsilon > 0$ be given. There exists $\delta_1 > 0$ such that if $0 < |x - 0| < \delta_1$ then $|f(x) - L| < \varepsilon$. Let $\delta = \delta_1/|a|$. Then for $0 < |x - 0| < \delta = \delta_1/|a|$, you have

$$|x| < \frac{\delta_1}{|a|}$$

$$|ax| < \delta_1$$

$$|f(ax) - L| < \varepsilon.$$

As a counterexample, let $a = 0$ and

$$f(x) = \begin{cases} 1, & x \neq 0 \\ 2, & x = 0 \end{cases}.$$

Then $\lim_{x\to 0} f(x) = 1 = L$, but

$$\lim_{x\to 0} f(ax) = \lim_{x\to 0} f(0) = \lim_{x\to 0} 2 = 2.$$

C H A P T E R 2
Differentiation