Chapter 1

Functions, Graphs, and Limits

1.1 Functions

1. $f(x) = 3x + 5,$
$f(0) = 3(0) + 5 = 5,$
$f(-1) = 3(-1) + 5 = 2,$
$f(2) = 3(2) + 5 = 11.$

2. $f(x) = -7x + 1$
$f(0) = -7(0) + 1 = 1$
$f(1) = -7(1) + 1 = -6$
$f(-2) = -7(-2) + 1 = 15$

3. $f(x) = 3x^2 + 5x - 2,$
$f(0) = 3(0)^2 + 5(0) - 2 = -2,$
$f(-2) = 3(-2)^2 + 5(-2) - 2 = 0,$
$f(1) = 3(1)^2 + 5(1) - 2 = 6.$

4. $h(t) = (2t+1)^3 \qquad h(-1) = (-2+1)^3 = -1$
$h(0) = (0+1)^3 = 1 \qquad h(1) = (2+1)^3 = 27$

5. $g(x) = x + \frac{1}{x},$
$g(-1) = -1 + \frac{1}{-1} = -2,$
$g(1) = 1 + \frac{1}{1} = 2,$
$g(2) = 2 + \frac{1}{2} = \frac{5}{2}.$

6. $f(x) = \frac{x}{x^2+1}$
$f(0) = \frac{0}{0+1} = 0$
$f(-1) = \frac{-1}{(-1)^2+1} = -\frac{1}{2}$
$f(2) = \frac{2}{2^2+1} = \frac{2}{5}$

7. $h(t) = \sqrt{t^2 + 2t + 4},$
$h(2) = \sqrt{2^2 + 2(2) + 4} = 2\sqrt{3},$
$h(0) = \sqrt{0^2 + 2(0) + 4} = 2,$
$h(-4) = \sqrt{(-4)^2 + 2(-4) + 4} = 2\sqrt{3}.$

8. $g(u) = (u+1)^{3/2}$
$g(0) = (0+1)^{3/2} = 1$
$g(-1) = (-1+1)^{3/2} = 0$
$g(8) = (8+1)^{3/2} = \left(\sqrt{9}\right)^3 = 27$

9. $f(t) = (2t-1)^{-3/2} = \frac{1}{\left(\sqrt{2t-1}\right)^3},$
$f(1) = \frac{1}{\left[\sqrt{2(1)-1}\right]^3} = 1,$
$f(5) = \frac{1}{\left[\sqrt{2(5)-1}\right]^3} = \frac{1}{\left[\sqrt{9}\right]^3} = \frac{1}{27},$
$f(13) = \frac{1}{\left[\sqrt{2(13)-1}\right]^3} = \frac{1}{\left[\sqrt{25}\right]^3} = \frac{1}{125}.$

10. $f(t) = \frac{1}{\sqrt{3-2t}}$
$f(1) = \frac{1}{\sqrt{3-2(1)}} = 1$
$f(-3) = \frac{1}{\sqrt{3-2(-3)}} = \frac{1}{3}$
$f(0) = \frac{1}{\sqrt{3-2(0)}} = \frac{1}{\sqrt{3}}$

11. $f(x) = x - |x-2|$,
$f(1) = 1 - |1-2| = 1 - |-1| = 1 - 1 = 0,$
$f(2) = 2 - |2-2| = 2 - |0| = 2,$
$f(3) = 3 - |3-2| = 3 - |1| = 3 - 2 = 2.$

12. $g(x) = 4 + |x|$
$g(-2) = 4 + |-2| = 6$
$g(0) = 4 + |0| = 4$
$g(2) = 4 + |2| = 6$

13. $h(x) = \begin{cases} -2x+4 & \text{if} \quad x \le 1 \\ x^2+1 & \text{if} \quad x > 1 \end{cases}$
$h(3) = (3)^2 + 1 = 10$
$h(1) = -2(1) + 4 = 2$
$h(0) = -2(0) + 4 = 4$
$h(-3) = -2(-3) + 4 = 10$

14. $f(t) = \begin{cases} 3 & \text{if} \quad t < -5 \\ t+1 & \text{if} \quad -5 \le t \le 5 \\ \sqrt{t} & \text{if} \quad t > 5 \end{cases}$
$f(-6) = 3$
$f(-5) = -5 + 1 = -4$
$f(16) = \sqrt{16} = 4$

15. $g(x) = \dfrac{x}{1+x^2}$
Since $1 + x^2 \ne 0$ for any real number, the domain is the set of all real numbers.

16. Since $x^2 - 1 = 0$ for $x = \pm 1$, $f(x)$ is defined only for $x \ne \pm 1$ and the domain does not consist of the real numbers.

17. $f(t) = \sqrt{1-t}$
Since negative numbers do not have real square roots, the domain is all real numbers such that $1 - t \ge 0$, or $t \le 1$. Therefore, the domain is not the set of all real numbers.

18. The square root function only makes sense for non-negative numbers. Since $t^2 + 1 \ge 0$ for all real numbers t the domain of $h(t) = \sqrt{t^2+1}$ consists of all real numbers.

19. $g(x) = \dfrac{x^2+5}{x+2}$
Since the denominator cannot be 0, the domain consists of all real numbers such that $x \ne -2$.

20. $f(x) = x^3 - 3x^2 + 2x + 5$
The domain consists of all real numbers.

21. $f(x) = \sqrt{2x+6}$
Since negative numbers do not have real square roots, the domain is all real numbers such that $2x + 6 \ge 0$, or $x \ge -3$.

22. $f(t) = \dfrac{t+1}{t^2 - t - 2}$
$t^2 - t - 2 = (t-2)(t+1) \ne 0$
if $t \ne -1$ and $t \ne 2$.

23. $f(t) = \dfrac{t+2}{\sqrt{9-t^2}}$
Since negative numbers do not have real square roots and denominators cannot be zero, the domain is the set of all real numbers such that $9 - t^2 > 0$, namely $-3 < t < 3$.

24. $h(s) = \sqrt{s^2-4}$ is defined only if $s^2 - 4 \ge 0$ or equivalently $(s-2)(s+2) \ge 0$. This occurs when the factors $(s-2)$ and $(s+2)$ are zero or have the same sign. This happens when $s \ge 2$ or $s \le -2$ and these values of s form the domain of h.

25. $f(u) = 3u^2 + 2u - 6$ and $g(x) = x + 2$, so
$$\begin{aligned} f(g(x)) &= f(x+2) \\ &= 3(x+2)^2 + 2(x+2) - 6 \\ &= 3x^2 + 14x + 10. \end{aligned}$$

26. $f(u)=u^2+4$

$f(x-1)=(x-1)^2+4=x^2-2x+5$

27. $f(u)=(u-1)^3+2u^2$ and $g(x = x + 1$, so

$$f(g(x))=f(x+1)$$
$$=[(x+1)-1]^3+2(x+1)^2$$
$$=x^3+2x^2+4x+2.$$

28. $f(u)=(2u+10)^2$

$$f(x-5)=\left[2(x-5)+10\right]^2$$
$$=(2x-10+10)^2=4x^2$$

29. $f(u)=\dfrac{1}{u^2}$ and $g(x) = x - 1$, so

$$f(g(x))=f(x-1)=\frac{1}{(x-1)^2}.$$

30. $f(u)=\dfrac{1}{u}$

$$f(x^2+x-2)=\frac{1}{x^2+x-2}$$

31. $f(u)=\sqrt{u+1}$ and $g(x)=x^2-1$, so

$$f(g(x))=f(x^2-1)$$
$$=\sqrt{(x^2-1)+1}$$
$$=\sqrt{x^2}$$
$$=|x|.$$

32. $f(u)=u^2,\ f\left(\dfrac{1}{x-1}\right)=\dfrac{1}{(x-1)^2}$

33. $f(x) = 4 - 5x$

$$\frac{f(x+h)-f(x)}{h}=\frac{4-5(x+h)-(4-5x)}{h}$$
$$\frac{4-5x-5h-4+5x}{h}=\frac{-5h}{h}=-5$$

34. For $f(x)=2x+3$,

$$\frac{f(x+h)-f(x)}{h}=\frac{(2(x+h)+3)-(2x+3)}{h}$$
$$=\frac{2x+2h+3-2x-3}{h}$$
$$=\frac{2h}{h}$$
$$=2$$

35. $f(x)=4x-x^2$

$$\frac{f(x+h)-f(x)}{h}$$
$$=\frac{4(x+h)-(x+h)^2-(4x-x^2)}{h}$$
$$=\frac{4x+4h-(x^2+2xh+h^2)-4x+x^2}{h}$$
$$=\frac{4x+4h-x^2-2xh-h^2-4x+x^2}{h}$$
$$=\frac{4h-2xh-h^2}{h}$$
$$=\frac{h(4-2x-h)}{h}$$
$$=4-2x-h$$

36. $f(x)=x^2$

$$\frac{f(x+h)-f(x)}{h}=\frac{(x+h)^2-x^2}{h}$$
$$=\frac{x^2+2xh+h^2-x^2}{h}$$
$$=\frac{h(2x+h)}{h}$$
$$=2x+h$$

37. $f(x)=\dfrac{x}{x+1}$

$$\frac{f(x+h)-f(x)}{h}$$
$$=\frac{\frac{x+h}{(x+h)+1}-\frac{x}{x+1}}{h}$$
$$=\frac{\frac{x+h}{x+h+1}-\frac{x}{x+1}}{h}\cdot\frac{(x+h+1)(x+1)}{(x+h+1)(x+1)}$$
$$=\frac{(x+h)(x+1)-x(x+h+1)}{h(x+1)(x+h+1)}$$
$$=\frac{x^2+hx+x+h-x^2-xh-x}{h(x+1)(x+h+1)}$$
$$=\frac{h}{h(x+1)(x+h+1)}$$
$$=\frac{1}{(x+1)(x+h+1)}$$

38. $f(x)=\dfrac{1}{x}$

$$\frac{f(x+h)-f(x)}{h}=\frac{\frac{1}{x+h}-\frac{1}{x}}{h}\cdot\frac{x(x+h)}{x(x+h)}$$
$$=\frac{x-(x+h)}{hx(x+h)}$$
$$=\frac{x-x-h}{hx(x+h)}$$
$$=\frac{-h}{hx(x+h)}$$
$$=\frac{-1}{x(x+h)}$$

39. $f(g(x))=f(1-3x)=\sqrt{1-3x}$

$g(f(x))=g\left(\sqrt{x}\right)=1-3\sqrt{x}$

To solve $\sqrt{1-3x}=1-3\sqrt{x}$, square both sides, so

$$1-3x=1-6\sqrt{x}+9x$$
$$-3x=-6\sqrt{x}+9x$$
$$6\sqrt{x}=12x$$
$$\sqrt{x}=2x$$

Squaring both sides again,

$$x=4x^2$$
$$0=4x^2-x$$
$$0=x(4x-1)$$
$$x=0,\ x=\frac{1}{4}$$

Since squaring both sides can introduce extraneous solutions, one needs to check these values.

$$\sqrt{1-3(0)}\stackrel{?}{=}1-3\sqrt{0}$$
$$1=1$$
$$\sqrt{1-3\left(\frac{1}{4}\right)}\stackrel{?}{=}1-3\sqrt{\frac{1}{4}}$$
$$\frac{1}{2}\stackrel{?}{=}1-\frac{3}{2}$$
$$\frac{1}{2}\neq-\frac{1}{2}$$

Also check remaining value to see if it is in the domain of f and g functions. Since $f(0)$ and $g(0)$ are both defined, $f(g(x))=g(f(x))$ when $x=0$.

40. $f(x)=x^2+1,\ g(x)=1-x$

$$f(g(x))=(1-x)^2+1$$
$$=x^2-2x+2 \text{ and}$$
$$g(f(x))=1-(x^2+1)$$
$$=-x^2$$

So $f(g(x))=g(f(x))$ means

$$x^2-2x+2=-x^2$$
$$2x^2-2x+2=0$$
$$x^2-x+1=0$$

but, by the quadratic formula, this last equation has no solutions.

41. $f(g(x))=f\left(\dfrac{x+3}{x-2}\right)=\dfrac{2\left(\frac{x+3}{x-2}\right)+3}{\frac{x+3}{x-2}-1}=x$

$g(f(x))=g\left(\dfrac{2x+3}{x-1}\right)=\dfrac{\frac{2x+3}{x-1}+3}{\frac{2x+3}{x-1}-2}=x$

Answer will be all real numbers for which f and g are defined. So, $f(g(x))=g(f(x))$ for all real numbers except $x=1$ and $x=2$.

42. $f(g(x)) = \dfrac{1}{\left(\frac{4-x}{2+x}\right)} = \dfrac{2+x}{4-x}$,

$g(f(x)) = \dfrac{4-\frac{1}{x}}{2+\frac{1}{x}} = \dfrac{4x-1}{2x+1}$

so $f(g(x)) = g(f(x))$ means

$\dfrac{2+x}{4-x} = \dfrac{4x-1}{2x+1}$.

Clearing denominators, multiplying, and collecting terms gives

$$\begin{aligned}(2+x)(2x+1) &= (4x-1)(4-x)\\ 2x^2+5x+2 &= -4x^2+17x-4\\ 6x^2-12x+6 &= 6(x^2-2x+1)\\ &= 6(x-1)^2\\ &= 0\end{aligned}$$

The last equation has solution $x=1$ which is in the domains of $f(g(x))$ and $g(f(x))$. Thus $f(g(x)) = g(f(x))$ only for $x=1$.

43. $f(x) = 2x^2-3x+1,$

$$\begin{aligned}f(x-2) &= 2(x-2)^2-3(x-2)+1\\ &= 2x^2-11x+15.\end{aligned}$$

44. $f(x) = x^2+5$

$f(x+1) = (x+1)^2+5 = x^2+2x+6$

45. $f(x) = (x+1)^5-3x^2,$

$$\begin{aligned}f(x-1) &= [(x-1)+1]^5-3(x-1)^2\\ &= x^5-3x^2+6x-3.\end{aligned}$$

46. $f(x) = (2x-6)^2$

$$\begin{aligned}f(x+3) &= \left[2(x+3)-6\right]^2\\ &= (2x+6-6)^2\\ &= 4x^2\end{aligned}$$

47. $f(x) = \sqrt{x},$

$f(x^2+3x-1) = \sqrt{x^2+3x-1}.$

48. $f(x) = 3x+\dfrac{2}{x}$

$f\left(\dfrac{1}{x}\right) = (3)\left(\dfrac{1}{x}\right)+\dfrac{2}{\frac{1}{x}} = \dfrac{3}{x}+2x$

49. $f(x) = \dfrac{x-1}{x},$

$f(x+1) = \dfrac{(x+1)-1}{x+1} = \dfrac{x}{x+1}.$

50. $f(x) = 2x-20$

$$\begin{aligned}f(x^2-2x+9) &= 2(x^2-2x+9)-20\\ &= 2x^2-4x+18-20\\ &= 2x^2-4x-2\end{aligned}$$

51. $f(x) = (x-1)^2+2(x-1)+3$ can be rewritten as $g(h(x))$ with $g(u) = u^2+2u+3$ and $h(x) = x-1$.

52. $f(x) = (x^5-3x^2+12)^3$

$g(u) = u^3$

$h(x) = x^5-3x^2+12$

53. $f(x) = \dfrac{1}{x^2+1}$ can be rewritten as $g(h(x))$ with $g(u) = \dfrac{1}{u}$ and $h(x) = x^2+1$.

54. $f(x) = \sqrt{3x-5}$ can be written as $g\left[h(x)\right]$ with $g(u) = \sqrt{u}$ and $h(x) = 3x-5$.

55. $f(x) = \sqrt[3]{2-x}+\dfrac{4}{2-x}$ can be rewritten as $g(h(x))$ with $g(u) = \sqrt[3]{u}+\dfrac{4}{u}$ and $h(x) = 2-x$.

56. $f(x) = \sqrt{x+4}-\dfrac{1}{(x+4)^3}$. $g(u) = \sqrt{u}-\dfrac{1}{u^3}$

$h(x) = x+4$

57. $C(q) = 0.01q^2 + 0.9q + 2$

(a) Total cost of 10 units = $C(10)$

$C(10) = 0.01(10)^2 + 0.9(10) + 2 = 12$,
or \$12,000.
Average cost per unit when 10 units are manufactured is $\frac{C(10)}{10}$, or \$1,200 per unit.

(b) Cost of the 10th unit is

$C(10) - C(9)$

$= 12 - \left[0.01(9)^2 + 0.9(9) + 2\right]$

$= 12 - 10.91 = 1.09$ or \$1,090

58. **(a)** $C(q) = q^3 - 30q^2 + 400q + 500$ where q is the number of units.

Thus, $C(10) = (10)^3 - 30(10)^2 + 400(10) + 500 = 2{,}500$, or \$2.5 million

$AC(10) = \frac{C(10)}{10} = \frac{2{,}500}{10} = 250$, or \$250,000/unit

(b) The cost of producing the 10^{th} unit is

$C(10) - C(9) = 2{,}500 - \left[(9)^3 - 30(9)^2 + 400(9) + 500\right] = 2{,}500 - 2{,}399 = 101$, or \$101,000.

59. $D(x) = -0.02x + 29$; $C(x) = 1.43x^2 + 18.3x + 15.6$

(a) $R(x) = xD(x) = x(-0.02x + 29)$

$= -0.02x^2 + 29x$

$P(x) = R(x) - C(x)$

$= (-0.02x^2 + 29x)$

$- (1.43x^2 + 18.3x + 15.6)$

$= -1.45x^2 + 10.7x - 15.6$

(b) $P(x) > 0$ when $-1.45x^2 + 10.7x - 15.6 > 0$. Using the quadratic formula, the zeros of P are

$$x = \frac{-10.7 \pm \sqrt{(10.7)^2 - (4)(-1.45)(-15.6)}}{2(-1.45)}$$

$x = 2, 5.38$

so, $P(x) > 0$ when $2 < x < 5.38$.

60. **(a)** $R(x) = xD(x) = x(-0.37x + 47) = -0.37x^2 + 47x$

$P(x) = R(x) - C(x) = (-0.37x^2 + 47x) - (1.38x^2 + 15.15x + 115.5) = -1.75x^2 + 31.85x - 115.5$

(b) Since $P(x) = -1.75x^2 + 31.85x - 115.5$,
the quadratic formula tells us $P(x) = 0$ when $x = 5$ and $x = 13.2$. By evaluating $P(x)$ at a number of x values, or by graphing the function, it is easy to see that $P(x) > 0$, that is the commodity is profitable, for $5 < x < 13.2$.

61. $D(x) = -0.5x + 39$; $C(x) = 1.5x^2 + 9.2x + 67$

(a) $R(x) = xD(x) = x(-0.5x + 39)$
$= -0.5x^2 + 39x$
$P(x) = R(x) - C(x)$
$= (-0.5x^2 + 39x) - (1.5x^2 + 9.2x + 67)$
$= -2x^2 + 29.8x - 67$

(b) $P(x) > 0$ when $-2x^2 + 29.8x - 67 > 0$.
Using the quadratic formula, the zeros of P are

$$x = \frac{-29.8 \pm \sqrt{(29.8)^2 - (4)(-2)(-67)}}{2(-2)}$$

$x \approx 2.76, 12.14$
so, $P(x) > 0$ when $2.76 < x < 12.14$.

62. **(a)** $R(x) = xD(x) = x(-0.09x + 51) = -0.09x^2 + 51x$
$P(x) = R(x) - C(x) = -0.09x^2 + 51x - (1.32x^2 + 11.7x + 101.4) = -1.41x^2 + 39.3x - 101.4$

(b) Since $P(x) = -1.41x^2 + 39.3x - 101.4$, the quadratic formula tells us $P(x) = 0$ when $x \approx 2.8$ and $x \approx 25.0$. By evaluating $P(x)$ at a number of x values, it is easy to see that $P(x) > 0$, that is, the commodity is profitable for $2.8 < x < 25.0$.

63. $W(x) = \dfrac{600x}{300 - x}$

(a) $300 - x \neq 0$
$x \neq 300$
The domain is all real numbers except 300.

(b) Typically, the domain would be restricted to the first quadrant. That is, $x \geq 0$. However, since x is a percentage, the restriction should be $0 \leq x \leq 100$.

(c) When $x = 50$,

$$W(50) = \frac{600(50)}{300 - 50}$$

$= 120$ worker-hours.

(d) To distribute all of the households, $x = 100$ and

$$W(100) = \frac{600(100)}{300 - 100}$$

$= 300$ worker-hours.

(e) Need to find x when $W(x) = 150$.

$$150 = \frac{600x}{300 - x}$$

$(150)(300 - x) = (1)(600x)$
$300 - x = 4x$
$x = 60$
After 150 worker-hours, 60% of the households have received a new telephone book.

64. **(a)** $6 - t \geq 0$
$-t \geq -6$
$t \leq 6$
The domain is all $t \leq 6$, or $(-\infty, 6]$.

(b) In 2000, $t = 2000 - 2010 = -10$ and

$$R(-10) = 30\sqrt{6-(-10)} = 30\sqrt{16} = 120$$

There were 120 databases in 2000.

(c) In 2007, $t = 2007 - 2010 = -3$, and

$$R(-3) = 30\sqrt{6-(-3)} = 30\sqrt{9} = 90.$$

90 databases still needed to be transferred in 2007.

(d) In 2011, $t = 2011 - 2010 = 1$, and

$$R(1) = 30\sqrt{6-1} = 30\sqrt{5} \approx 67.1.$$

Since 120 – 67.1 = 52.9 ≈ 53, about 53 databases had been transferred as of 2011, with 67 still remaining.

(e) In 2015, $t = 2015 - 2010 = 5$, and

$$R(5) = 30\sqrt{6-5} = 30\sqrt{1} = 30.$$

No. In 2015, 30 databases will still remain to be transferred.

65. $S(t) = \begin{cases} 14.7 + 0.6t & \text{if } t \le 4 \\ 14.2t^2 - 128t + 304 & \text{if } t > 4 \end{cases}$

(a) In 1990, share price = $S(-10)$

$$S(-10) = 14.7 + 0.6(-10) = 8.7 \quad \text{or} \quad \$8.70$$

In 2006, share price = $S(6)$

$$S(6) = 14.2(6)^2 - 128(6) + 304 = 47.2 \quad \text{or} \quad \$47.20$$

(b) Share price = \$200 when

$$200 = 14.2t^2 - 128t + 304$$

$$0 = 14.2t^2 - 128t + 104$$

$$t = \frac{128 \pm \sqrt{(-128)^2 - 4(14.2)(104)}}{2(14.2)}$$

$$t \approx \frac{128 \pm 102.36}{28.4} \approx 0.90, 8.11$$

However, this branch of function applies when $t \ge 4$, so reject answer $t \approx 0.90$.

Share price = \$200 during the year 2008. (Note: $200 = 14.7 + 0.6t$ gives $t \approx 308.83$ but $t \le 4$ for this branch.)

(c) In the year 2012, $t = 12$

$$S(12) = 14.2(12)^2 - 128(12) + 304 = 812.8 \quad \text{or} \quad \$812.80$$

66. $f(x) = -x^3 + 6x^2 + 15x$

(a) $f(2) = -8 + 6(2)^2 + 15(2) = 46$

(b) $f(1) = -1 + 6 + 15 = 20$

$f(2) - f(1) = 46 - 20 = 26$

67. $Q(p) = \dfrac{4,374}{p^2}$ and

$p(t) = 0.04t^2 + 0.2t + 12$

(a) $Q(t) = \dfrac{4,374}{(0.04t^2 + 0.2t + 12)^2}$

(b) $$Q(10) = \frac{4,374}{(4 + 2 + 12)^2} = \frac{4,374}{324} = 13.5 \text{ kg/week}$$

(c) $$30.375 = \frac{4,374}{(0.04t^2 + 0.2t + 12)^2}$$

$$(0.04t^2 + 0.2t + 12)^2 = \frac{4,374}{30.375} = 144 = 12^2$$

So $0.04t^2 + 0.2t + 12 = \pm 12$.

The positive root leads to $t(0.04t + 0.2) = 0$ or $t = 0$. (Disregard $t < 0$.) The negative root produces imaginary numbers. $t = 0$ now.

68. **(a)** $C(q) = q^2 + q + 500$ and $q(t) = 25t$, Thus,

$$C[q(t)] = C(25t)$$
$$= (25t)^2 + 25t + 500$$
$$= 625t^2 + 25t + 500$$

(b) For $t = 3$,

$$C[q(3)] = 625(3)^2 + 25(3) + 500$$
$$= 6{,}200$$

or \$6,200.The average cost is

$$\frac{C[q(3)]}{3} = \frac{6{,}200}{3} \approx 2{,}066.67$$, or \$2,066.67/hour.

(c) $625t^2 + 25t + 500 = 11{,}000$

$625t^2 + 25t - 10{,}500 = 0$

Divide by 25 to get smaller numbers, then $25t^2 + t - 420 = 0$ or $t \approx 4.1$ hours. Disregard $t \approx -4.1$.

69. $C(x) = \dfrac{150x}{200 - x}$

(a) All real numbers except $x = 200$.

(b) All real numbers for which $0 \le x \le 100$. If $x < 0$ or $x > 200$ then $C(x) < 0$ but cost is non-negative. $x > 100$ means more than 100%.

(c) $C(50) = \dfrac{150(50)}{200 - 50} = 50$ million dollars.

(d) $C(100) = \dfrac{150(100)}{200 - 100} = 150$

$C(100) - C(50) = 100$ million dollars.

(e) $\dfrac{150x}{200 - x} = 37.5$

$187.5x = 37.5(200)$

$x = \dfrac{7{,}500}{187.5} = 40\%$

70. **(a)** $C(t) = -\dfrac{t^2}{6} + 4t + 10$ degrees Celsius, where t represents the number of hours after midnight.

Thus $t = 2$ at 2:00 A.M. and $C(2) = -\dfrac{(2)^2}{6} + 4(2) + 10 = 17\dfrac{1}{3}$.

(b) The difference in temperature between 6:00 P.M. $(t = 18)$ and 9:00 P.M. $(t = 21)$ is

$$C(21) - C(18) = \left[-\frac{(21)^2}{6} + 4(21) + 10\right] - \left[-\frac{(18)^2}{6} + 4(18) + 10\right] = 20\frac{1}{2} - 28 = -7\frac{1}{2}$$

71. $P(t) = 20 - \dfrac{6}{t+1}$

(a) $P(9) = 20 - \dfrac{6}{9+1}$ or 19,400 people.

(b) $P(8) = 20 - \dfrac{6}{8+1}$

$P(9) - P(8) = 20 - \dfrac{3}{5} - \left(20 - \dfrac{2}{3}\right) = \dfrac{1}{15}$

This accounts for about $\dfrac{1}{15}$ of 1,000 people, or 67 people.

(c) $P(t)$ approaches 20, or 20,000 people. Writing exercise—Answers will vary.

72. (a) $T(n) = 3 + \dfrac{12}{n}$

The domain consists of all real numbers $n \neq 0$ (because of the denominator).

(b) Since n represents the number of trials, n is a positive integer, like $n = 1, 2, 3, \cdots$

(c) For the third trial $n = 3$, thus

$T(3) = 3 + \dfrac{12}{3} = 7$ minutes.

(d)

$T(n) \leq 4,$ so $3 + \dfrac{12}{n} \leq 4,$ $\dfrac{12}{n} \leq 1$ or $n = 12$

(e) $\dfrac{12}{n}$ gets smaller and smaller as n increases. Thus $\dfrac{12}{n} \to 0$ as $n \to \infty$ and $T(n)$ gets closer and closer to 3. No, the rat will never be able to traverse the maze in less than 3 minutes.

73. $S(r) = C(R^2 - r^2)$

$= 1.76 \times 10^5 (1.2^2 \times 10^{-4} - r^2)$

(a) $S(0) = (1.76 \times 10^5)(1.44 \times 10^{-4})$

$= 25.344$ cm/sec

(b) $S(0.6 \times 10^{-2})$

$= 1.76 \times 10^5 (1.44 \times 10^{-4} - 0.6^2 \times 10^{-4})$

$= 1.76 \times 10^5 (1.08 \times 10^{-4})$

$= 19.008$ cm/sec

74. (a) $N = 91.2 / 300^{0.73} \approx 1.42$ elk per square kilometer.

(b) $N = 91.2 / m^{0.73} < 1$ when $m > 91.2^{1/0.73} = 484.088$. An animal of this species has an average mass of at least 48 kg.

(c) Let A denote the area, in square kilometers, of the reserve. If the second species has average mass m, then $\dfrac{91.2}{m^{0.73}} = \dfrac{100}{A}$ since $\dfrac{100}{A}$ is the number of animals per square kilometer. The first species has average mass $2m$ and so the estimated number per square kilometer is

$\dfrac{91.2}{(2m)^{0.73}} = \dfrac{91.2}{2^{0.73} m^{0.73}} = \dfrac{100}{2^{0.73} A} \approx \dfrac{60.29}{A}.$

Since there are A square kilometers on the reserve, there are approximately 60 of the heavier species.

75. $s(A) = 2.9\sqrt[3]{A}$

(a) $s(8) = 2.9\sqrt[3]{8} = 2.9 \times 2 = 5.8$

Since the number of species should be an integer, you would expect to find approximately 6 species.

(b) $s_1 = 2.9\sqrt[3]{A}$ and $s_2 = 2.9\sqrt[3]{2A}$

$s_2 = 2.9\sqrt[3]{2}\sqrt[3]{A} = \sqrt[3]{2}\left(2.9\sqrt[3]{A}\right) = \sqrt[3]{2} s_1.$

(c) $100 = 2.9\sqrt[3]{A}$

$\frac{100}{2.9} = \sqrt[3]{A}$

$\left(\frac{100}{2.9}\right)^3 = \left(\sqrt[3]{A}\right)^3$

$\left(\frac{100}{2.9}\right)^3 = A$

Need an area of approximately 41,000 square miles.

76. (a) $c(p) = 0.4p + 1$ and $p(t) = 8 + 0.2t^2$ thus

$c[p(t)] = 0.4(8 + 0.2t^2) + 1$

$= 4.2 + 0.08t^2$.

(b) $c[p(2)] = 4.2 + 0.08(2)^2 = 4.52$

It will be 4.52 parts per million (ppm).

(c) Solve $c[p(t)] = 6.2$.

$4.2 + 0.08t^2 = 6.2$

$0.08t^2 = 2$

$t^2 = 25$

$t = -5, 5$

Disregard $t = -5$. The level will reach 6.2 ppm when $t = 5$, or 5 years from now.

77. $H(t) = -16t^2 + 256$

(a) Height after 2 seconds = $H(2)$

$H(2) = -16(2)^2 + 256 = 192$ feet

(b) Distance traveled during 3rd second

$= H(3) - H(2)$

$H(3) - H(2)$

$= \left[-16(3)^2 + 256\right] - 192$

$= -80$

or 80 feet (negative denotes moving downward).

(c) Height of building = $H(0)$

$H(0) = -16(0)^2 + 256 = 256$ feet

(d) Hits ground when height = 0.

$0 = -16t^2 + 256$

$t = -4, 4$; reject $t = -4$

Hits ground after 4 seconds.

78. $y = \frac{7x^2 - 4}{x^3 - 2x + 4}$ is not defined when $x^3 - 2x + 4 = 0$.

Graphing this polynomial and looking for x-intercepts yields $x = -2$. This could also have been found by inspection. The domain is all numbers x such that $x \neq -2$.

79. To find the domain of $f(x) = \frac{4x^2 - 3}{2x^2 + x - 3}$, press [y =].

Enter (4*x* ^ 2 – 3) ÷ (2*x* ^ 2 + *x* – 3) for $y_1 =$

Press [graph].

For a better view of the vertical asymptotes, press [zoom] and enter Zoom In. Use arrow buttons to move cross-hair to the left-most vertical asymptote. When it appears cross-hair is on the line, zoom in again for a more accurate reading. Move cross-hair again to be on the line. It appears that $x = -1.5$ is not in the domain of *f*. Zoom out once to move cross-hair to the rightmost vertical asymptote and repeat the procedure of zoom in to find that $x = 1$ is not the domain of *f*.

The domain consists of all values except $x = -1.5$ and $x = 1$.

80. $f(x) = 2\sqrt{x-1}$, $g(x) = x^3 - 1.2$

$f(4.8) = 2\sqrt{3.8} \approx 3.90$

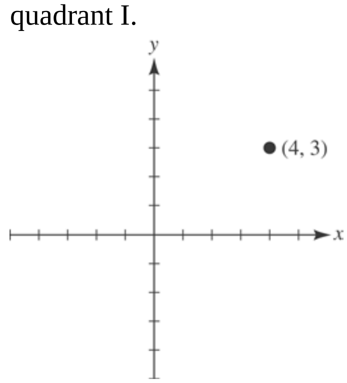
$g(f(4.8)) \approx g(3.90) = 3.90^3 - 1.3 = 58.02$

81. For $f(x) = 2\sqrt{x-1}$ and $g(x) = x^3 - 1.2$, to find $f(g(2.3))$, we must find $g(2.3)$ first and then input that answer into *f*.

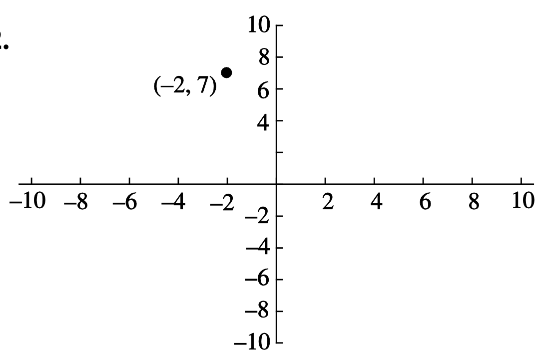
Press y =. Input $2\sqrt{(x-1)}$ for $y_1 =$ and press enter. Input $x \wedge 3 - 1.2$ for $y_2 =$.
Use window dimensions [–15, 15]1 by [–10, 10]1. Use the value function under the calc menu, input 2.3, and press enter.
Use ↑ and ↓ arrows to be sure that $y_2 = x \wedge 3 - 1.2$ is displayed in the upper left corner. The lower right corner display should read $y = 10.967$.
Use the value function again and input 10.967. Verify $y_1 = 2\sqrt{(x-1)}$ is displayed in the upper left corner.
The answer of $y = 6.31$ is displayed in lower right corner.

1.2 The Graph of a Function

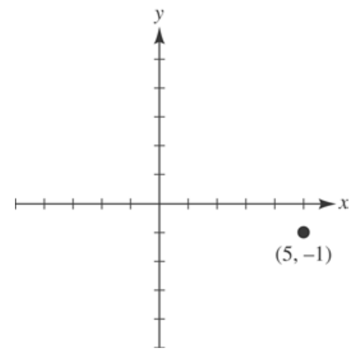
1. Since *x*-coordinate is positive and *y*-coordinate is positive, point is in quadrant I.



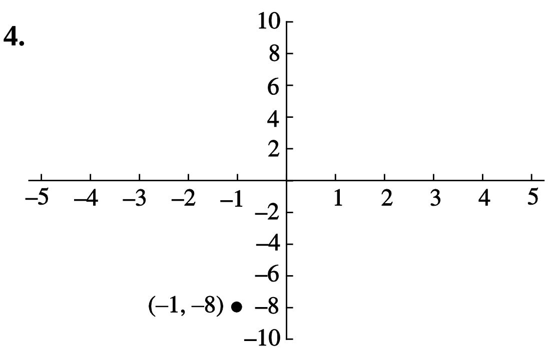
2.



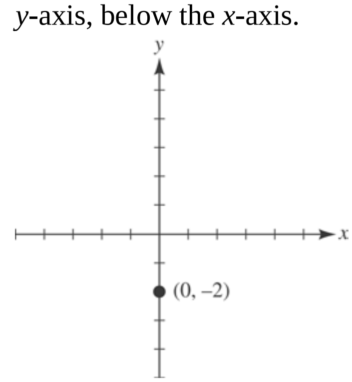
3. Since *x*-coordinate is positive and *y*-coordinate is negative, point is in quadrant IV.



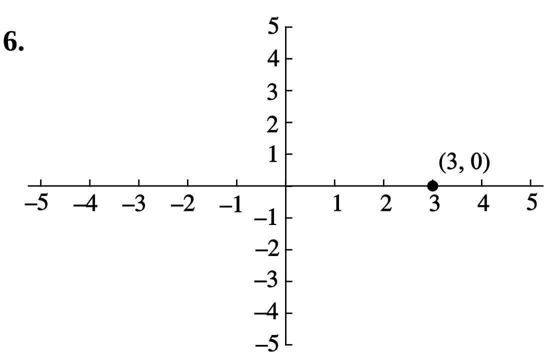
4.



5. Since *x*-coordinate is zero and *y*-coordinate is negative, point is on *y*-axis, below the *x*-axis.



6.



7. (3, –1), (7, 1)

$$D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{(1-(-1))^2 + (7-3)^2}$$
$$= \sqrt{4+16}$$
$$= \sqrt{20}$$
$$= \sqrt{4 \cdot 5}$$
$$= 2\sqrt{5}$$

8. $P(x_1, y_1) = (4, 5); Q(x_2, y_2) = (-2, -1)$

$$D = \sqrt{(-2-4)^2 + (-1-5)^2} = \sqrt{72} = 6\sqrt{2}$$

9. (7, –3), (5, 3)

$$D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{(3-(-3))^2 + (5-7)^2}$$
$$= \sqrt{36+4}$$
$$= \sqrt{40}$$
$$= \sqrt{4 \cdot 10}$$
$$= 2\sqrt{10}$$

10. $P(x_1, y_1) = \left(0, \frac{1}{2}\right); Q(x_2, y_2) = \left(-\frac{1}{5}, \frac{3}{8}\right)$

$$D = \sqrt{\left(-\frac{1}{5} - 0\right)^2 + \left(\frac{3}{8} - \frac{1}{2}\right)^2}$$
$$= \sqrt{\frac{1}{25} + \frac{1}{64}}$$
$$= \sqrt{\frac{89}{1,600}}$$
$$= \frac{\sqrt{89}}{40}$$

11. **(a)** Of the form x^n, where n is non-integer real number; is a power function.

(b) Of the form $a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$, where n is nonnegative integer; is a polynomial function.

(c) Can multiply out and simplify to form $a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$; is a polynomial function.

(d) Since is quotient of two polynomial functions is a rational function.

12. **(a)** polynomial

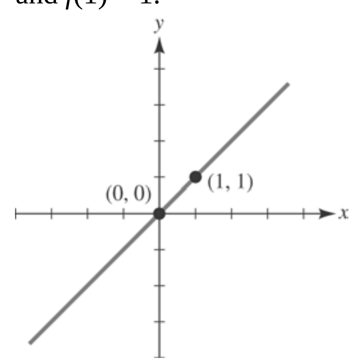
(b) different (since $\sqrt{x}$ is a non-integer power of x.)

(c) rational function (since $(x-3)(x+7) = x^2 + 4x - 21$ is a polynomial.)

(d) rational function (since
$$\left(\frac{2x+9}{x^2-3}\right)^3 = \frac{(2x+9)^3}{(x^2-3)^3}$$
and the numerator and denominator expand to polynomials.)

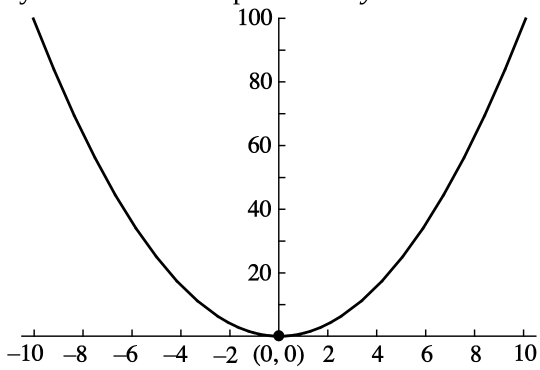
13. $f(x) = x$

A function of the form $y = f(x) = ax + b$ is a linear function, and its graph is a line. Two points are sufficient to draw that line. The x-intercept is 0, as is the y-intercept, and $f(1) = 1$.



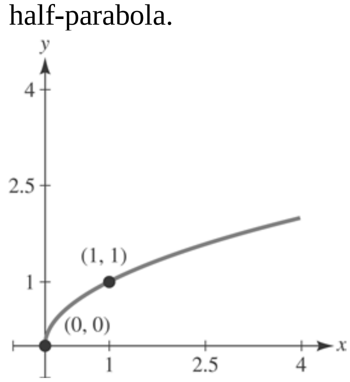
14. $f(x) = x^2$

$x = 0$ when $y = 0$, $y \geq 0$ and the curve is symmetric with respect to the y-axis.



15. $f(x)=\sqrt{x}$

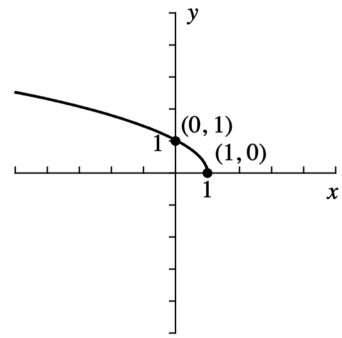
A function of the form $y=\sqrt{x}$ is the positive half of the function $y^2 = x$ (a parabola with vertex (0, 0), a horizontal axis and opening to the right). The x-intercept and y-intercept are the same, namely (0, 0). Choosing two more points on $y=\sqrt{x}$ (for example $P(1, 1)$ and $Q(2, 4)$), helps outline the shape of the half-parabola.



16. $f(x)=\sqrt{1-x}$

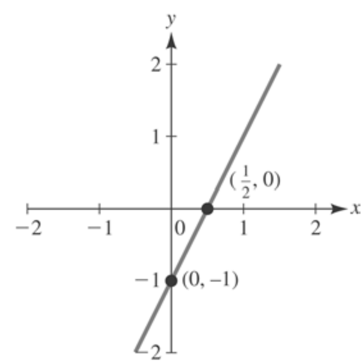
Note that the function is only defined for $x \le 1$.

$x = 0$ when $y = 1$ and $y = 0$ when $x = 1$.



17. $f(x) = 2x - 1$

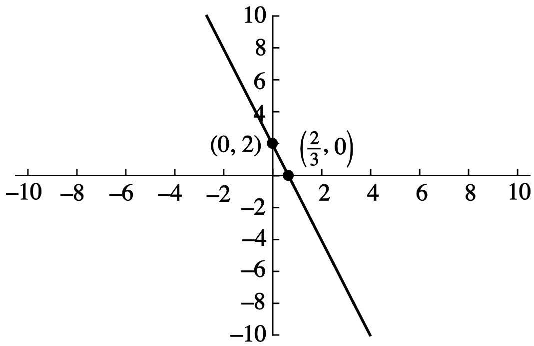
A function of the form $y = f(x) = ax + b$ is a linear function, and its graph is a line. Two points are sufficient to draw that line. The x-intercept is $\frac{1}{2}$ and the y-intercept is -1.



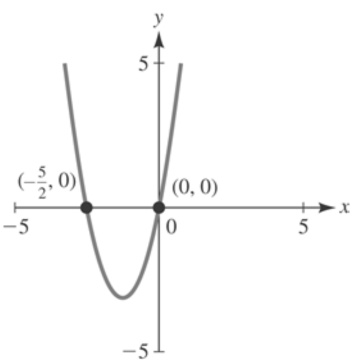
18. $f(x) = 2 - 3x$

Note that the graph is a straight line. The slope is –3. The curve falls.

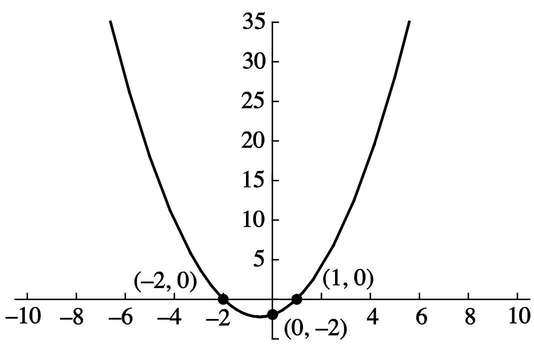
x	0	$\frac{2}{3}$	2
$f(x)$	2	0	–4



19. Since function is of the form $y = Ax^2 + Bx + C$ (where $C = 0$), the graph is a parabola; its vertex is $\left(-\frac{5}{4}, -\frac{25}{8}\right)$, it opens up ($A$ is positive), and its intercepts are (0, 0) and $\left(-\frac{5}{2}, 0\right)$.



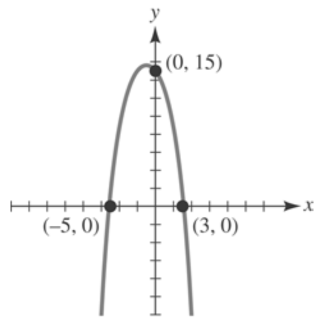
20. $f(x) = (x-1)(x+2)$
$x = 0$ when $y = -2$, and $y = 0$ when $x = 1$ or $x = -2$.



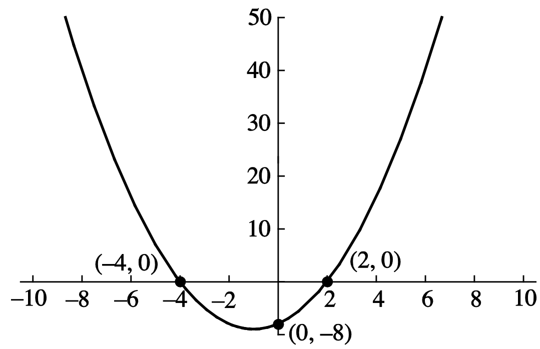
21. Since function is of the form $y = Ax^2 + Bx + C$, the graph is a parabola which opens down (A is negative) and its vertex is (–1, 16). Further,

$$\begin{aligned} f(x) &= -x^2 - 2x + 15 \\ &= -(x^2 + 2x - 15) \\ &= -(x+5)(x-3). \end{aligned}$$

So the x-intercepts are (–5, 0) and (3, 0), and the y-intercept is (0, 15).

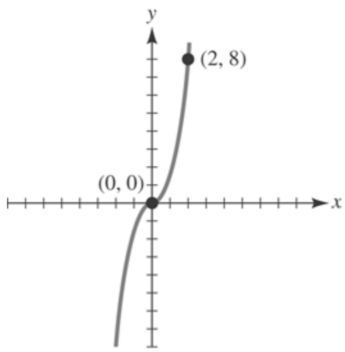


22. $f(x) = x^2 + 2x - 8 = (x+4)(x-2)$
When $x = 2$ or $x = -4$, $y = 0$.
When $x = 0$, $y = -8$.



23. $f(x) = x^3$
Note that if $x > 0$ then $f(x) > 0$ and if $x < 0$, then $f(x) < 0$. This means that the curve will only appear in the first and third quadrants. Since x^3 and $(-x)^3$ have the same absolute value, only their signs are opposites, the curve will be symmetric with respect to (wrt) the origin. The

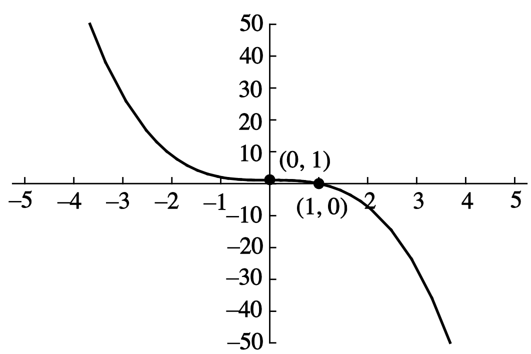
x-intercept is 0, as is the *y*-intercept.



24. $f(x) = -x^3 + 1$

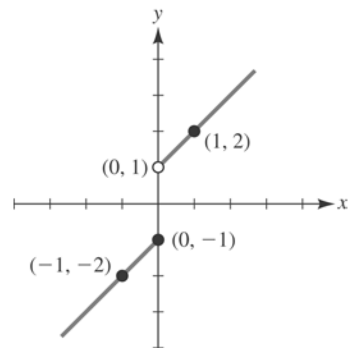
Note the similarities between this graph and the one in the previous exercise. The *y*-values here are the negatives of those and the curve is translated (moved up) by 1 unit.

x	0	1	2	3	5
$f(x)$	1	0	–7	–26	–124



25. $f(x) = \begin{cases} x-1 & \text{if } x \le 0 \\ x+1 & \text{if } x > 0 \end{cases}$

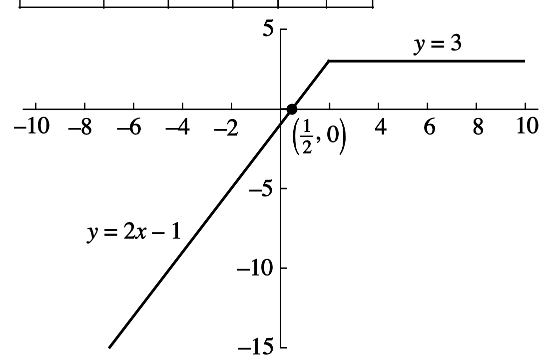
Note that the graph consists of two half lines on either side of *x* = 0. There is no *x*-intercept for either half line. The half line *y* = *x* – 1 has a *y*-intercept of –1, while the half line *y* = *x* + 1 has no *y*-intercept.



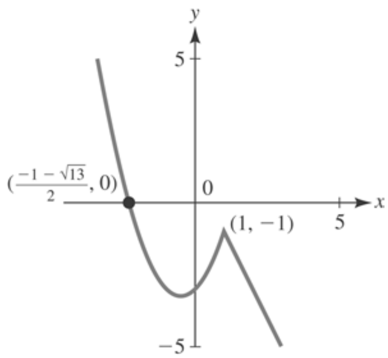
26. $f(x) = \begin{cases} 2x-1 & \text{if } x < 2 \\ 3 & \text{if } x \ge 2 \end{cases}$

The *x*-intercept is $x = \frac{1}{2}$. The *y*-intercept is $f(0) = -1$. Some points on the curve are:

x	–1	0	1	2	3
$f(x)$	–3	–1	1	3	3

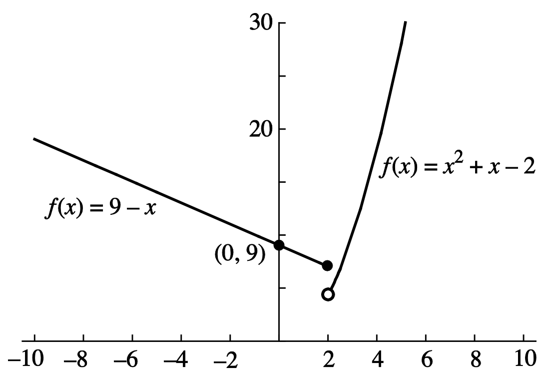


27. Graph consists of part of parabola $y = x^2 + x - 3$, namely the portion corresponding to *x* < 1, and a half line for *x* ≥ 1; for the parabola portion of the graph, the vertex is $\left(-\frac{1}{2}, -\frac{13}{4}\right)$, and the parabola opens up (*A* is positive); $\left(\frac{-1-\sqrt{13}}{2}, 0\right)$ and (0, –3) are its intercepts; the half line starts at (1, –1) and includes the point (2, –3).



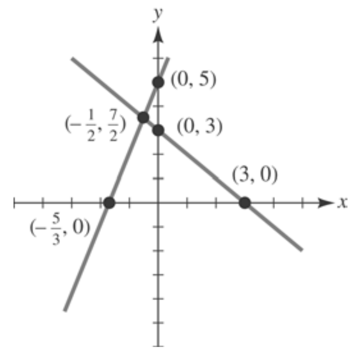
28. $f(x) = \begin{cases} 9 - x & \text{if } x \le 2 \\ x^2 + x - 2 & \text{if } x > 2 \end{cases}$

$x = 0$ when $y = 9$. There are no x intercepts.

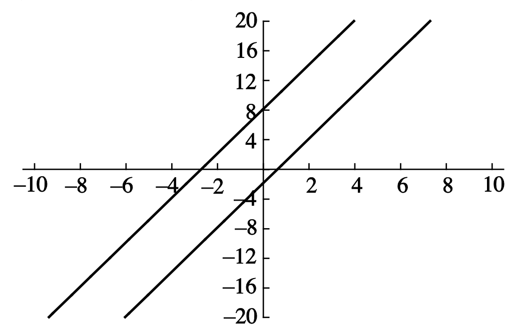


29. $y = 3x + 5$ and $y = –x + 3$

Add 3 times the second equation to the first. Then $4y = 14$ or $y = \frac{7}{2}$. Substitute in the first, then $x = 3 - y = -\frac{1}{2}$. The point of intersection is $P\left(-\frac{1}{2}, \frac{7}{2}\right)$.



30. $y = 3x + 8$ and $y = 3x - 2$



The slopes are the same, namely 3, so the lines are parallel, but the y intercepts differ. These lines do not intersect.

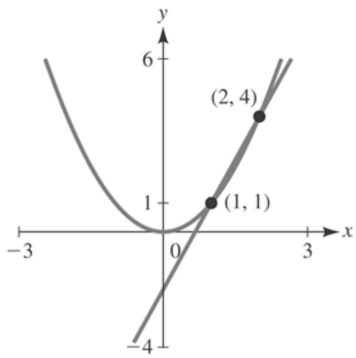
31. $y = x^2$ and $y = 3x - 2$

Setting the expressions equal to each other,

$$x^2 = 3x - 2$$
$$x^2 - 3x + 2 = 0$$
$$(x-1)(x-2) = 0$$
$$x = 1, 2$$

So points of intersection are $P_1(1, 1)$ and

$P_2(2, 4)$.

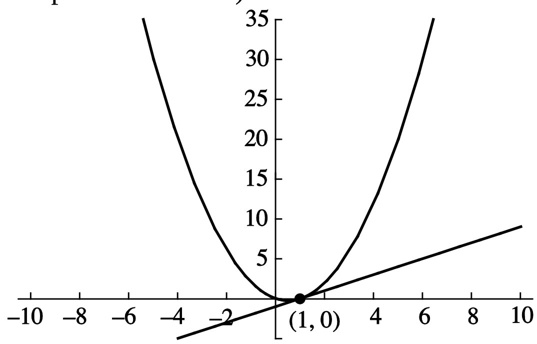


32. $y = x^2 - x$ and $y = x - 1$. Thus

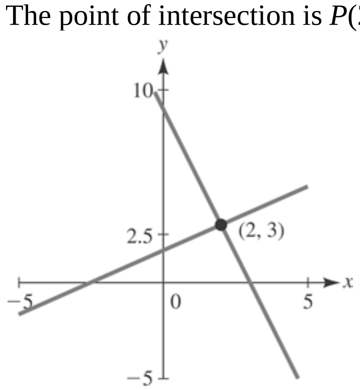
$$x^2 - x = x - 1$$
$$x(x-1) - (x-1) = 0$$
$$(x-1)(x-1) = 0$$

$x = 1$ and $y = 0$

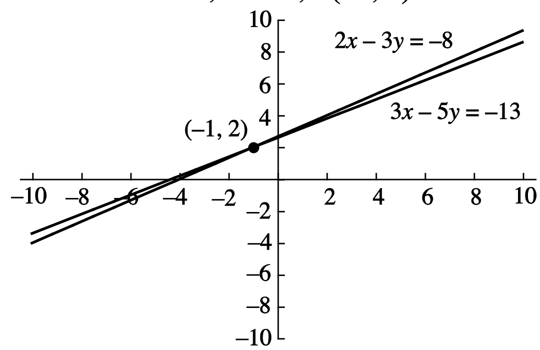
$P(1, 0)$ is the point of intersection (really the point of contact).



33. $3y - 2x = 5$ and $y + 3x = 9$.
Multiply the second equation by −3 and add it to the first one. Then,
$-2x - 9x = 5 - 27$,
$x = 2$, $y = 9 - 3(2) = 3$.
The point of intersection is $P(2, 3)$.



34. $2x - 3y = -8$ and $3x - 5y = -13$
Multiply the first equation by 3 and the second by -2, to get $y = 2$. Thus
$2x = -8 + 6 = -2$, $x = -1$, $P(-1, 2)$.



35. **(a)** Crosses y-axis at $y = -1$, y-intercept is $(0, -1)$.

(b) Crosses x-axis at $x = 1$, x-intercept is $(1, 0)$.

(c) Largest value of f is 3 and occurs at $x = 4$ (highest point on graph).

(d) Smallest value of f is −3 and occurs at $x = -2$ (lowest point on graph).

36. **(a)** $(0, 2)$

(b) $(1, 0)$ and $(3, 0)$

(c) Largest value of 3.5 at $x = -1$.

(d) Smallest value of −1 at $x = 2$.

37. **(a)** Crosses y-axis at $y = 2$, y-intercept is $(0, 2)$.

(b) Crosses x-axis at $x = -1$ and 3.5; x-intercepts are $(-1, 0)$ and $(3.5, 0)$.

(c) Largest value of f is 3 and occurs at $x = 2$ (highest point on graph).

(d) Smallest value of f is −3 and occurs at $x = 4$ (lowest point on graph).

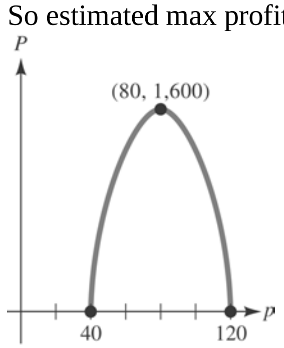
38. **(a)** $(0, 0)$

(b) $(-2, 0)$, $(0, 0)$ and $(3.5, 0)$

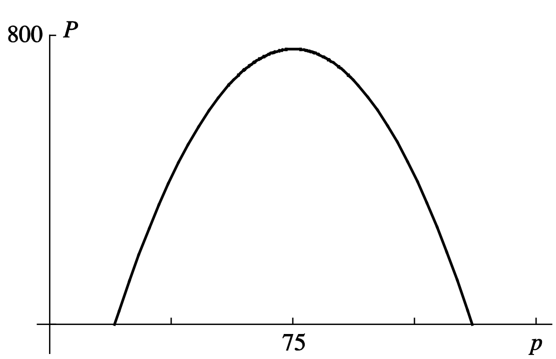
(c) Largest value of 2 at $x = -1$ and $x = 4$.

(d) Smallest value of –4 at $x = 2$.

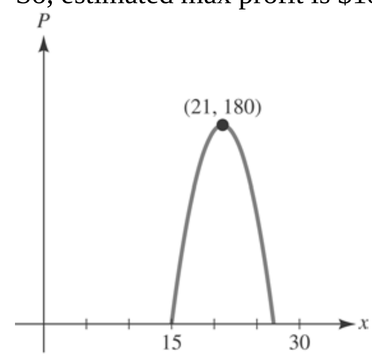
39. The monthly profit is
$P(p)$
= (number of recorders sold)(price – cost)
$= (120 - p)(p - 40)$
So, the intercepts are (40, 0), (120, 0), and (0, –4800). The graph suggests a maximum profit when $p \approx 80$, that is, when 80 recorders are sold.
$P(80) = (120 - 80)(80 - 40) = 1600$
So estimated max profit is \$1600.



40. Number of tires sold:
$n(p) = 1{,}560 - 12p$ at p dollars apiece, and manufactured for \$20 each.
Revenue $R(p) = p(1{,}560 - 12p)$,
cost $C(p) = 20(1{,}560 - 12p)$,
profit $P(p) = R(p) - C(p)$
$= (p - 20)(1{,}560 - 12p)$
Relevant values are $20 \le p \le 130$. The maximum profit occurs at \$75 per tire.
$n(75) = 1{,}560 - 12(75) = 660$
660 tires will be sold at \$75 per tire.

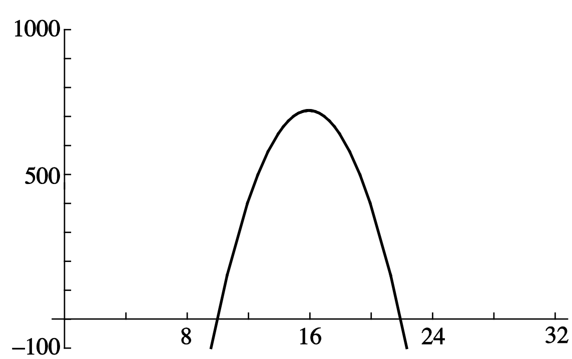


41. The weekly profit is
$P(x)$ = (number of sets sold) · (price-cost per set)
$= 5(27 - x)(x - 15)$
So, the intercepts are (27, 0), (15, 0) and (0, –2025). The graph suggests a maximum weekly profit when $x \approx 21$. That is, when the price per set is \$21.
$P(21) = 5(27 - 21)(21 - 15) = 180$
So, estimated max profit is \$180.

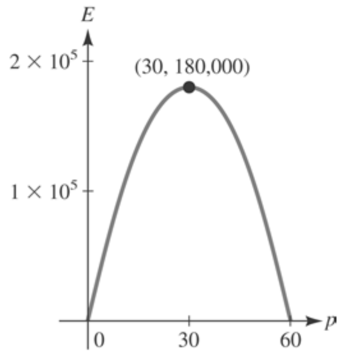


The number of sets corresponding to the max profit is $5(27 - 21) = 30$ sets.

42. Number of copies sold:
$n(x) = 20(22 - x)$ at x dollars each and bought for \$10 each.
Revenue $R(x) = 20x(22 - x)$,
cost $C(x) = 20 \times 10(22 - x)$, profit
$P(x) = R(x) - C(x) = 20(22 - x)(x - 10)$
Relevant values are $10 \le x \le 22$. The maximum profit occurs at \$16 per book.



43. (a) $E(p) = \text{(price per unit)(demand)}$
$= -200p(p-60)$

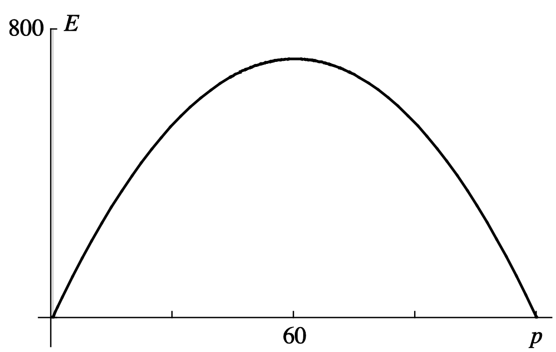


(b) The p intercepts represent prices at which consumers do not buy commodity.

(c) The graph suggests a maximum expenditure when $p \approx 30$.
$E(30) = -200(30)(30-60) = 180{,}000$
So estimated max expenditure is $180,000.

44. (a) $p = 5(24 - x)$, so $x = -\frac{1}{5}p + 24.$

$E(p) = px = p\left(-\frac{1}{5}p + 24\right)$ in thousands of dollars.

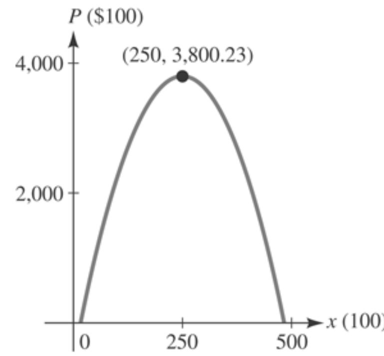


(b) The p intercepts represent prices at which consumers spend no money on the commodity.

(c) A price of $60 generates the greatest monthly expenditure. 12,000 units will be sold each month when the price is $60 per unit.

45. (a) profit = revenue − cost
= (# sold)(selling price) − cost

$P(x) = x(-0.05x + 38)$
$\quad - (0.02x^2 + 3x + 574.77)$
$= -0.07x^2 + 35x - 574.77$
hundred dollars



(b) Average profit $= \frac{P(x)}{x}$

$AP(x) = -0.07x + 35 - \frac{574.77}{x}$

when p = $37, $-0.05x + 38 = 37$ and $x = 20$.

$AP(20) = -0.07(20) + 35 - \frac{574.77}{20}$
$= \$4.86$ per unit

(c) The graph suggests a maximum profit when $x = 250$, that is, when 25,000 units are purchased. Note that the max profit is

$P(250) = -0.07(250)^2 + 35(20) - 574.77$
≈ 3800.23 hundred,

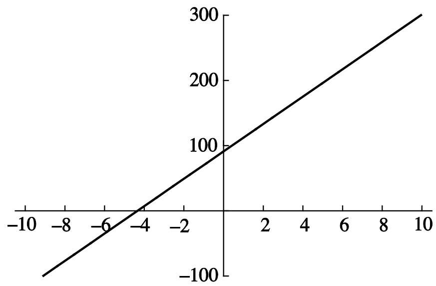
or $380,023. For the unit price, $p = -0.05(250) + 38 = \$25.50$.

46. (a)

x	2	5	7	10
$C(x)$	132	195	237	300

(b) $C(x) = 90 + 21x$

(c)

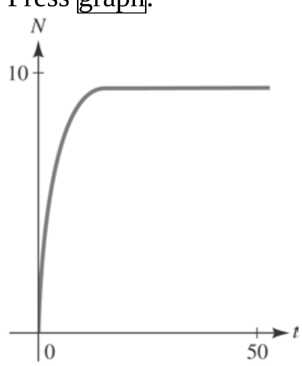


47. (a)

Days of Training	Mowers per Day
2	6
3	7.23
5	8.15
10	8.69
50	8.96

(b) The number of mowers per day approaches 9.

(c) To graph $N(t)=\dfrac{45t^2}{5t^2+t+8}$, press $\boxed{y=}$
Input (45*x* ^ 2) ÷ (5*x* ^ 2 + *x* + 8) for $Y_1 = .$
Use window dimensions [–10, 10]1 by [–10, 10]1 (z standard).
Press $\boxed{\text{graph}}$.



48. (a) $R(x) = xp$

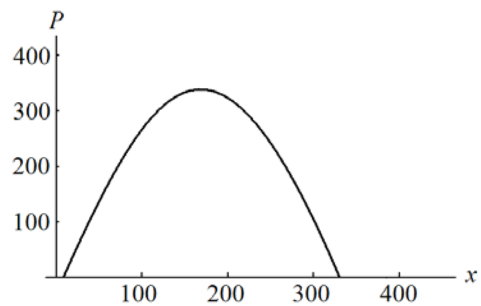
$$= x(4.2 - 0.01x)$$

$$= -0.01x^2 + 4.2x$$

$$P(x) = R(x) - C(x)$$

$$= -0.01x^2 + 4.2x - (0.002x^2 + 30)$$

$$= -0.012x^2 + 4.2x - 30$$

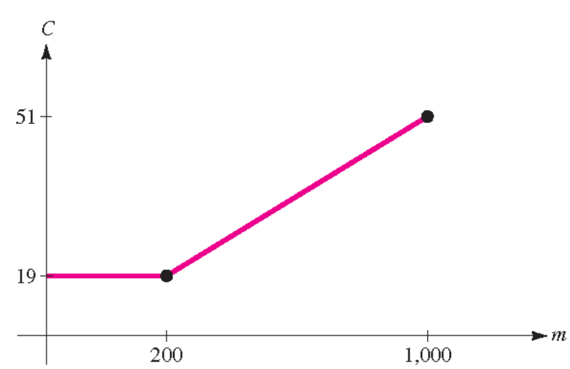


(b) The vertex occurs when *x* = 175, so Chuck should charge
$p = 4.2 - 0.01(175) = 2.45$,
or $2.45.

49. (a) When $m \le 200$, *C*(*m*) = 19 dollars. For each additional minute, that is $m-200$, the additional cost is $0.04 until a maximum of *m* = 1,000 is reached; so

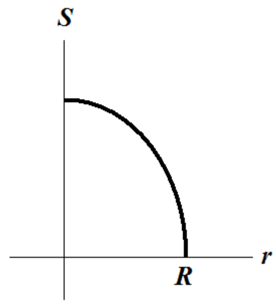
$$C(m) = \begin{cases} 19 & \text{if } m \le 200 \\ 19 + 0.04(m-200) & \text{if } 200 < m \le 1{,}000 \end{cases}$$

(b)



50. $S(r) = C(R^2 - r^2), 0 \le r \le R$

$S(0) = CR^2$ and $S(r) = 0$ when $r = R$



51. (a) revenue = (#apts)(rent per apt)

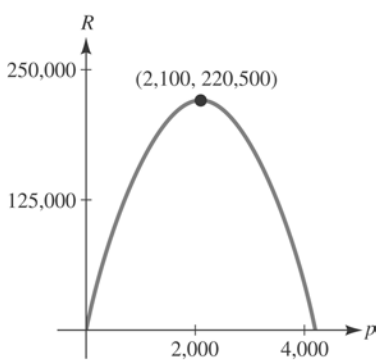
Since $\frac{p-1200}{100}$ represents the number of \$100 increases.

$$150 - 5\left(\frac{p-1200}{100}\right) = 210 - 0.05p$$

represents the number of apartments that will be leased. So,

$R(p) = 210p - 0.05p^2$.

(b)



(c) The graph suggests a maximum profit when $p = 2100$; that is, when the rental price is \$2,100. The max profit is

$$R(2100) = 210(2100) - 0.05(2100)^2 \approx \$220,500.$$

52. (a) Let x be the number of \$100 rent increases. Then the rental price of each apartment is $p = 1,200 + 100x$. The number of unrented apartments is $5x$, so the total number of apartments that are rented is $n(x) = 150 - 5x$ and the cost of maintaining and advertising the unrented apartments is

$c(x) = 500(5x) = 2,500x$.

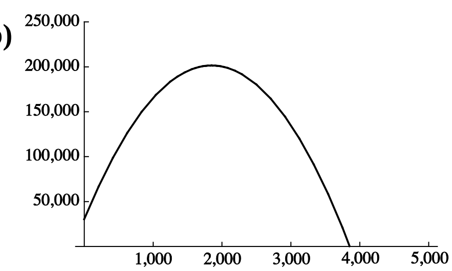
For the revenue as a function of p, use

$x = \frac{p}{100} - 12$. Then $n(p) = 210 - \frac{p}{20}$

and $c(p) = 25p - 30,000$.

$$R(p) = p \cdot n(p) - c(p) = p\left(210 - \frac{p}{210}\right) - (25p - 30,000) = -\frac{p^2}{20} + 185p + 30,000$$

(b)



(c) The vertex of the parabola is at

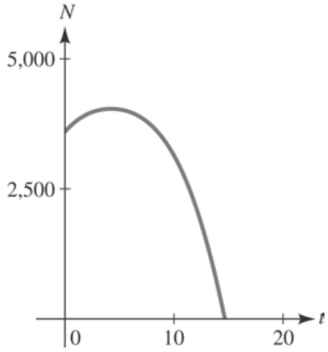
$$p = \frac{-B}{2A} = \frac{-185}{-\frac{2}{20}} = 1,850.$$

$$R(1850) = -\frac{1,850^2}{20} + 185(1,850) + 30,000 = 201,125$$

To maximize total revenue, the company should charge \$1,850. The maximum revenue is \$201,125.

53. $N(t) = -35t^2 + 299t + 3{,}347$

(a)



(b) Since the year 1995 is represented by $t = 5$, the amount predicted was

$$N(5) = -35(5)^2 + 299(5) + 3{,}347$$
$$= 3{,}967 \text{ thousand tons.}$$

(c) Based on the formula, the maximum lead emission would occur at the vertex, or when

$$t = -\frac{299}{2(-35)} \approx 4.27 \text{ years.}$$

This would be during March of the year 1994.

(d) No; from the graph, $N(t) < 0$ when $t \approx 15$, or during the year 2005.

54. (a) Let the origin be on the road at the middle of the base, so that the 6-meter width means that the parabola passes through (–3, 0) and (3, 0). The truck just fitting through means the parabola passes through (–2, 5) and (2, 5). This gives two equations in *a* and *b*.

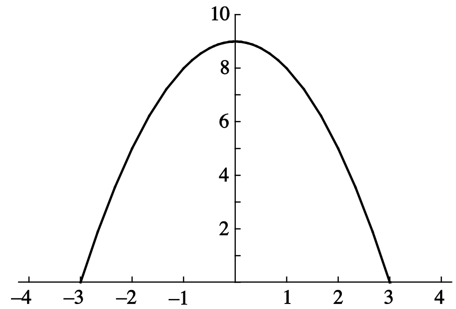
$0 = 9a + b$

$5 = 4a + b$

Subtracting gives $a = -1$, so $b = 9$.
The equation of the arch is

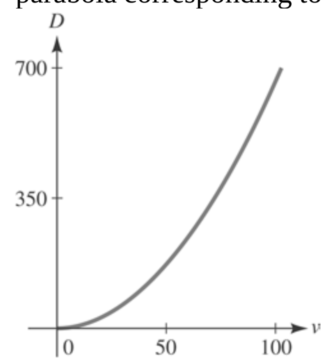
$$y = -x^2 + 9.$$

(b)



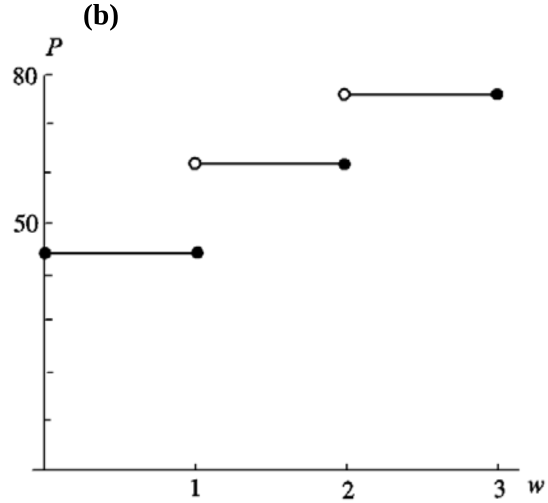
55. $D(v) = 0.065v^2 + 0.148v$

For practical domain, graph is part of parabola corresponding to $v \geq 0$.



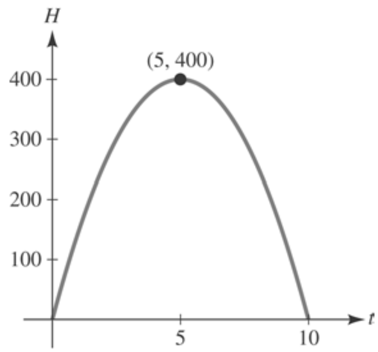
56. (a) $P(w) = \begin{cases} 44 & \text{if } 0 \leq w \leq 1 \\ 61 & \text{if } 1 < w \leq 2 \\ 78 & \text{if } 2 < w \leq 3 \end{cases}$

(b)



57. $H(t) = -16t^2 + 160t = -16t(t-10)$

(a) The intercepts of the graph are (0, 0) and (10, 0). Due to symmetry, the vertex is when $t = 5$ and
$y = H(5) = -16(5)^2 + 160(5) = 400.$

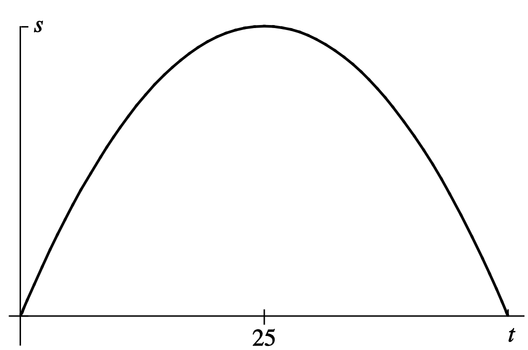


(b) Aside from when it is initially thrown, the height of the projectile is zero (ground level) when $t = 10$ seconds.

(c) The high point of the graph, which corresponds to the max height, is when *y*-coordinate is 400, or 400 feet.

58. (a) $s(0) = -15$
The bunker is 15 feet deep.

(b)



(c) The missile is at its highest point after 25 seconds, when its height is 9,985 feet.

59. The graph is a function because no vertical line intersects the graph more than once.

60. Since the *y* axis (a vertical line) intersects the curve more than once, the curve is not the graph of a function.

61. The graph is not a function because there are vertical lines intersecting the graph at more than one point; for example, the *y*-axis.

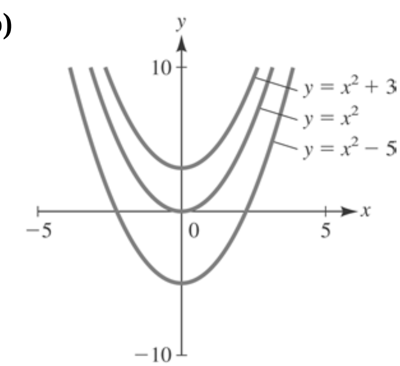
62. Since no vertical line will intersect the curve more than once, the curve is the graph of a function.

63. $f(x) = -9x^2 + 3600x - 358,200$
Answers will vary, but one viewing window has the following dimensions: [180, 200]10 by [–500, 1850]500.

64. The vertex of the parabola is the point (300, –5000) and this point should be included in the rectangle. One such viewing rectangle is
$[250, 350] \times [-8000, 5000]$.

65. (a) The graph of $y = x^2 + 3$ is graph of $y = x^2$ translated up 3 units.

(b)

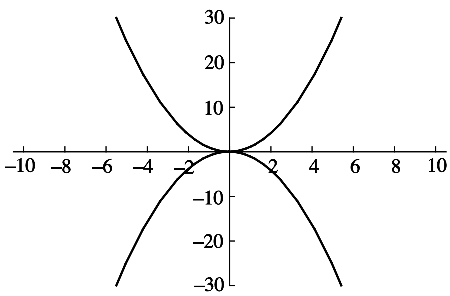


(c) When $c > 0$, the graph of *g* is the graph of *f* translated up *c* units. When $c < 0$, the graph is translated down $|c|$ units.

66. (a) Each *y* value for $y = -x^2$ is the negative of the corresponding *y* value of $y = x^2$. Hence the points on the graph of $y = -x^2$ are reflections across the x axis of the points of the graph of $y = x^2$.

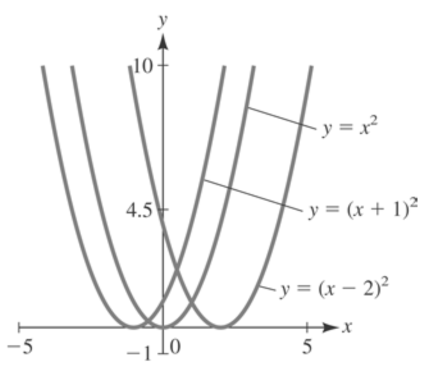
(b) If $g(x) = -f(x)$, the graph of $g(x)$ is the reflection across the x axis of the graph of $f(x)$.

x	–2	–1	0	1	2
$y = -x^2$	–4	–1	0	–1	–4
$y = x^2$	4	1	0	2	4



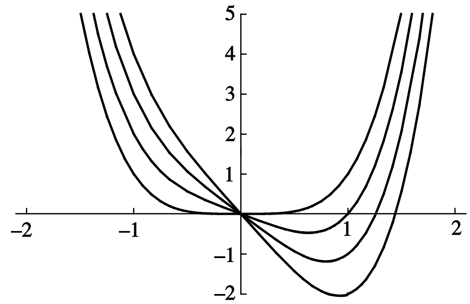
67. (a) The graph of $y = (x-2)^2$ is the graph of $y = x^2$ translated two units to the right.

(b)

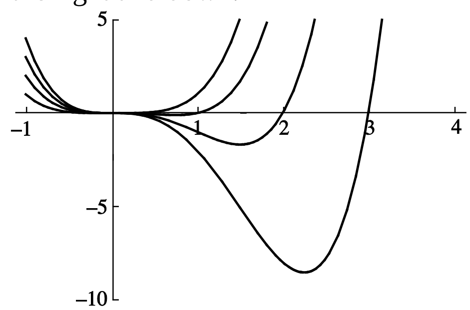


(c) When $c > 0$, the graph of g is the graph of f translated c units to the right. When $c < 0$, the graph is translated $|c|$ units to the left.

68.



The effect the additional term has is to pull the low point of the curve further to the right and down.



The extra terms create a valley further to the right and down from the low point of the original curve.

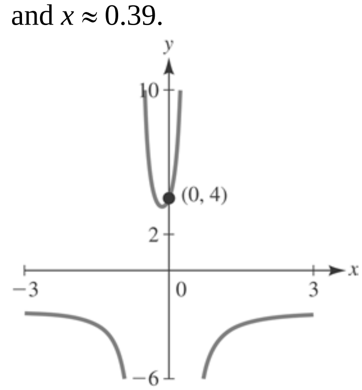
69. To graph $f(x) = \dfrac{-9x^2 - 3x - 4}{4x^2 + x - 1}$,

Press [y =]

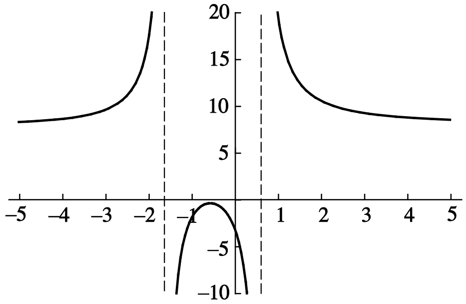
Input (–9x ^ 2 – 3x – 4) ÷ (4x ^ 2 + x – 1) for $y_1 =$

Press [graph]

Use the Zoom in function under the Zoom menu to find the vertical asymptotes to be $x_1 \approx -0.65$ and $x_2 \approx 0.39$. The function f is defined for all real x except $x_3 \approx -0.65$ and $x \approx 0.39$.

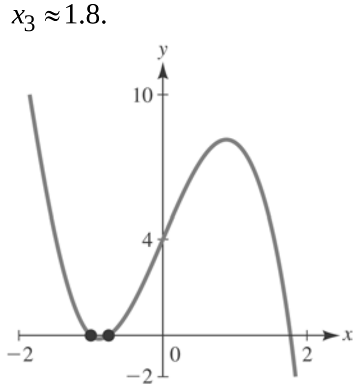


70.



The function is defined for all values x except for $x \approx 0.618$ and $x \approx -1.618$. These values can be determined with a graphing calculator or exactly by applying the quadratic formula to $x^2 + x - 1 = 0$ to find $x = \frac{-1 \pm \sqrt{5}}{2}$.

71. To graph $g(x) = -3x^3 + 7x + 4$ and find x-intercepts,
Press [y =]
Input –3*x* ^ 3 + 7*x* + 4 for $y_1 =$
Press [graph]
Press [trace]
Use left arrow to move cursor to the left most x-intercept. When the cursor appears to be at the x-intercept, use the Zoom In feature under the Zoom menu twice. It can be seen that there are two x-intercepts in close proximity to each other. These x-intercepts appear to be $x_1 \approx -1$ and $x_2 \approx -0.76$. To estimate the third x-intercept, use the z-standard function under the Zoom menu to view the original graph. Use right arrow and zoom in to estimate the third x-intercept to be $x_3 \approx 1.8$.



72. If (x, y) is a point on the circle, then its distance from the center of the circle is R. The distance between $P(a, b)$ and $Q(x, y)$ is $D = \sqrt{(x-a)^2 + (y-b)^2}$.
Thus, $\sqrt{(x-a)^2 + (y-b)^2} = R$ or $(x-a)^2 + (y-b)^2 = R^2$.

73. $(x-a)^2 + (y-b)^2 = R^2$

(a) Since the center of the circle is (2, –3), $a = 2$ and $b = -3$. Since its radius is 4, $R = 4$.

$$(x-2)^2 + (y-(-3))^2 = 4^2$$
$$(x-2)^2 + (y+3)^2 = 16$$

(b) $x^2 + y^2 - 4x + 6y = 11$

First, group the x terms together and the y terms together.

$$(x^2 - 4x) + (y^2 + 6y) = 11$$

Next, complete the square for each grouping

$$(x^2 - 4x + 4) - 4 + (y^2 + 6y + 9) - 9 = 11$$
$$(x-2)^2 + (y+3)^2 = 11 + 4 + 9$$
$$(x-2)^2 + (y+3)^2 = 24$$
$$(x-2)^2 + (y-(-3))^2 = \left(\sqrt{24}\right)^2$$

center: (2, –3)
radius: $\sqrt{24} = \sqrt{4 \cdot 6} = 2\sqrt{6}$

(c) Proceeding as in part (b),

$$(x^2 - 2x) + (y^2 + 4y) = -10$$
$$(x^2 - 2x + 1) - 1 + (y^2 + 4y + 4) - 4 = -10$$
$$(x-1)^2 + (y+2)^2 = -10 + 1 + 4$$
$$(x-1)^2 + (y+2)^2 = -5$$

Since the left-hand side is positive for all possible points (x, y) and the right-hand side is negative, the equality can never hold. That is, there are no points (x, y) that satisfy the equation.

74. First note that

$$A\left[\left(x+\frac{B}{2A}\right)^2+\left(\frac{C}{A}-\frac{B^2}{4A^2}\right)\right]$$
$$=A\left[x^2+2x\left(\frac{B}{2A}\right)+\left(\frac{B}{2A}\right)^2+\frac{C}{A}-\frac{B^2}{4A^2}\right]$$
$$=A\left[x^2+\left(\frac{B}{A}\right)x+\frac{C}{A}\right]=Ax^2+Bx+C$$

If $A>0$, the quantity $A\left[\left(x+\frac{B}{2A}\right)^2+\left(\frac{C}{A}-\frac{B^2}{4A^2}\right)\right]$ has no largest value and will be smallest when the squared term is as small as possible which is 0 when $x=\frac{-B}{2A}$.

Similarly, if $A<0$, there is no smallest value and the largest value occurs at $x=\frac{-B}{2A}$.

1.3 Lines and Linear Functions

1. For $P_1(2,-3)$ and $P_2(0,4)$ the slope is $m=\frac{4-(-3)}{0-2}=-\frac{7}{2}$.

2. $m=\frac{5-2}{2-(-1)}=\frac{3}{3}=1$

3. For $P_1(2,0)$ and $P_2(0,2)$ the slope is $m=\frac{2-0}{0-2}=-1$.

4. $m=\frac{-1-(-1)}{-2-5}=0$

5. For $P_1(2,6)$ and $P_2(2,-4)$ the slope is $m=\frac{6-(-4)}{2-2}$, which is undefined, since the denominator is 0. The line through the given points is vertical.

6. $$m=\frac{\frac{1}{8}-\left(-\frac{1}{5}\right)}{\left(-\frac{1}{7}\right)-\frac{2}{3}}$$
$$=\left(\frac{13}{40}\right)\left(-\frac{21}{17}\right)$$
$$=-\frac{273}{680}$$
$$\approx -0.4015$$

7. For $P_1\left(\frac{1}{7},5\right)$ and $P_2\left(-\frac{1}{11},5\right)$ the slope is $m=\frac{5-5}{-\frac{1}{11}-\frac{1}{7}}=\frac{0}{-\frac{18}{77}}=0.$

8. $m=\frac{-9-3.5}{-1.1-(-1.1)}=\frac{-12.5}{0}$

The slope is undefined.

9. The line has slope = 2 and an intercept of (0, 0). So, the equation of line is $y = 2x + 0$, or $y = 2x$.

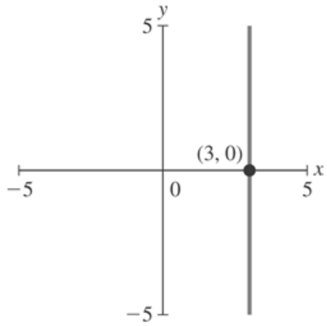
10. The line has slope $\frac{2\frac{1}{2}}{4}=\frac{\frac{5}{2}}{4}=\frac{5}{8}$.

The x intercept is (–4, 0) and the y intercept is $\left(0,\frac{5}{2}\right)$. The equation of the line is $y=\frac{5}{8}x+\frac{5}{2}$.

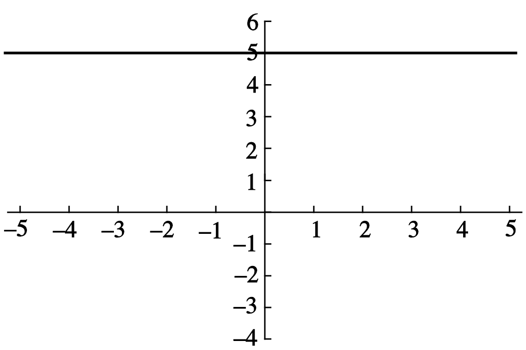
11. The slope of the line is $\frac{-5}{3}$. The x-intercept of the line is (3, 0) and the y-intercept is (0, 5). The equation of the line is $y=-\frac{5}{3}x+5$.

12. The graph descends 3 units as you move 2.5 units to the right on the x axis. Thus $m=\frac{-3}{2.5}=\frac{-6}{5}$. The y intercept is (0,–3) so $b=-3$. The equation of the line is $y=\frac{-6}{5}x-3$.

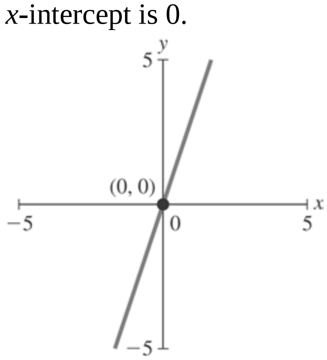
13. The line $x = 3$ is a vertical line that includes all points of the form $(3, y)$. Therefore, the x-intercept is $(3, 0)$ and there is no y-intercept. The slope of the line is undefined, since $x_2 - x_1 = 3 - 3 = 0$.



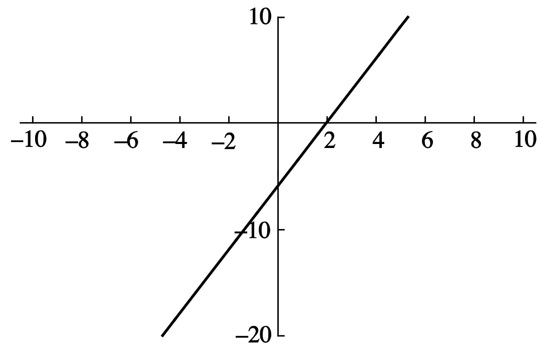
14. $y = 5$
Thus $m = 0$ and $b = 5$. There is no x intercept.



15. $y = 3x$
$m = 3$, y-intercept $b = 0$, and the x-intercept is 0.

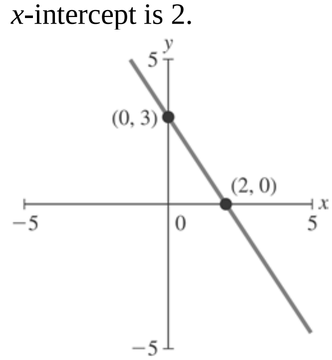


16. $y = 3x - 6$, $m = 3$
y-intercept $b = -6$.



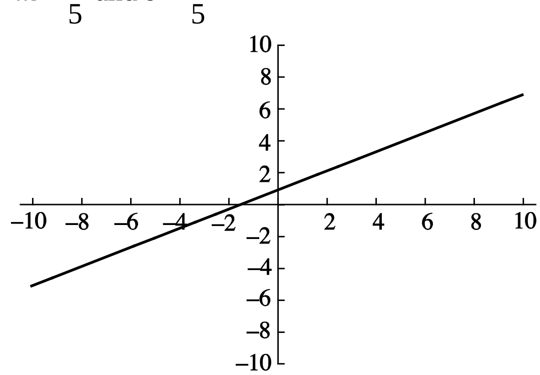
17. $3x + 2y = 6$ or $y = -\frac{3}{2}x + 3$

$m = -\frac{3}{2}$, y-intercept $b = 3$, and the x-intercept is 2.



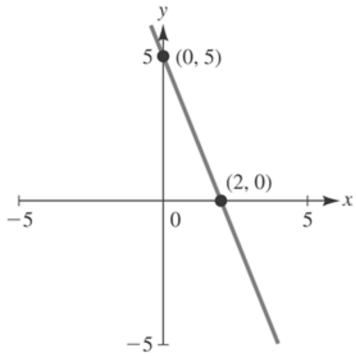
18. $5y - 3x = 4$ or $y = \frac{3}{5}x + \frac{4}{5}$

$m = \frac{3}{5}$ and $b = \frac{4}{5}$



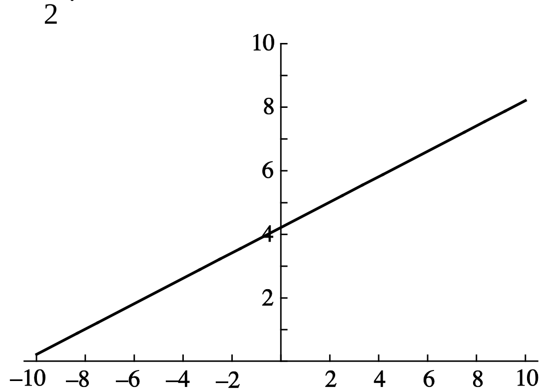
19. $\frac{x}{2}+\frac{y}{5}=1$ or $y=-\frac{5}{2}x+5$

$m=-\frac{5}{2}$, y-intercept b = 5, and the x-intercept is 2.



20.
$$\frac{x+3}{-5}+\frac{y-1}{2}=1$$
$$2(x+3)-5(y-1)=-10$$
$$2x+6-5y+5=-10$$
$$5y=2x+21$$
$$y=\frac{2}{5}x+\frac{21}{5}$$

Thus $m=\frac{2}{5}$ and $b=\frac{21}{5}$. The x intercept is $-\frac{21}{2}$.



21. m = 1 and $P(2, 0)$, so $y - 0 = (1)(x - 2)$, or $y = x - 2$.

22.
$$\frac{y-2}{x+1}=\frac{2}{3}$$
$$3y-6=2x+2$$
$$y=\frac{2}{3}x+\frac{8}{3}$$

23. $m=-\frac{1}{2}$ and $P(5, -2)$, so

$y-(-2)=-\frac{1}{2}(x-5)$, or $y=-\frac{1}{2}x+\frac{1}{2}$.

24. $\frac{y-0}{x-0}=5$, $y=5x$

25. Since the line is parallel to the x-axis, it is horizontal and its slope is 0. For $P(2, 5)$, the line is $y - 5 = 0(x - 2)$, or $y = 5$.

26. The slope is not defined and there is no y intercept. Thus $x=2$.

27. $m=\frac{1-0}{0-1}$ and for $P(1, 0)$ the equation of the line is $y - 0 = -1(x - 1)$ or $y = -x + 1$. The equation would be the same if the point (0, 1) had been used.

28.
$$\frac{y-5}{x-2}=\frac{5+2}{2-1}=7$$
$$y-5=7x-14$$
$$y=7x-9$$

29. $m=\frac{1-\left(\frac{1}{4}\right)}{-\left(\frac{1}{5}\right)-\left(\frac{2}{3}\right)}=-\frac{45}{52}$

For $P\left(-\frac{1}{5},1\right)$, the line is

$y-1=-\frac{45}{52}\left(x+\frac{1}{5}\right)$, or $y=-\frac{45}{52}x+\frac{43}{52}$.

30.
$$\frac{y-5}{x-0}=\frac{5-3}{0-(-2)}=1$$
$$y=x+5$$

31. The slope is 0 because the y-values are identical. So, $y = 5$.

32. The slope of the line through $(1,5)$ and $(1,-4)$ is not defined. We deal with a vertical line. Its equation is $x=1$.

33. The given line $2x + y = 3$, or $y = -2x + 3$, has a slope of -2. Since parallel lines have the same slope, $m = -2$ for the desired line. Given that the point (4, 1) is on the line, $y - 1 = -2(x - 4)$, or $y = -2x + 9$.

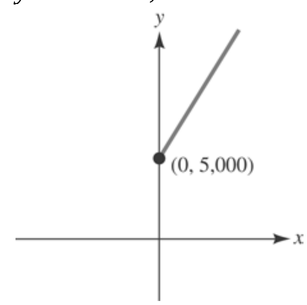
34. Writing $x+3y=5$ in slope-intercept form yields $y=-\frac{1}{3}x+\frac{5}{3}$. Therefore, the given line has slope $m=-\frac{1}{3}$. Using the point $(-2, 3)$ in the point-slope formula yields $y-3=-\frac{1}{3}(x+2)$.

35. The given line $x + y = 4$, or $y = -x + 4$, has a slope of -1. A perpendicular line has slope $m=-\frac{1}{-1}=1$. Given that the point (3, 5) is on the line, $y - 5 = 1(x - 3)$, or $y = x + 2$.

36. Writing $2x+5y=3$ in slope-intercept form yields $y=-\frac{2}{5}x+\frac{3}{5}$. The slope of a perpendicular line is $m=\frac{5}{2}$. Using the point $\left(-\frac{1}{2},1\right)$ in the point-slope formula yields $y-1=\frac{5}{2}\left(x+\frac{1}{2}\right)$.

37. **(a)** Let x be the number of units manufactured. Then $60x$ is the cost of producing x units, to which the fixed cost must be added.

$y = 60x + 5{,}000$

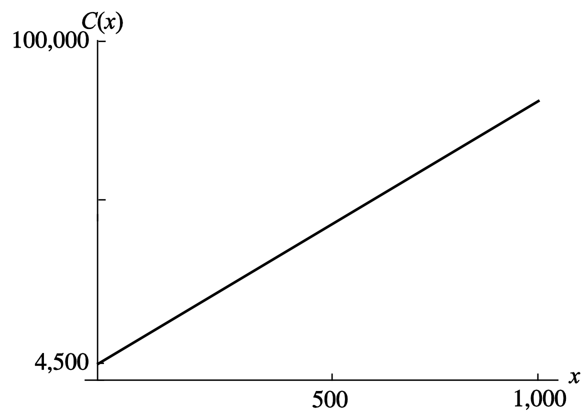


(b) Average cost $=\dfrac{y}{x}$

$$AC(x)=60+\frac{5{,}000}{x}$$

$$AC(20)=60+\frac{5{,}000}{20}=\$310 \text{ per unit}$$

38. **(a)** Producing x units costs $75x$ dollars. The total cost of producing x units is $C(x) = 75x + 4{,}500$.



(b)
$$AC(x)=\frac{C(x)}{x}=\frac{75x+4{,}500}{x}=\frac{4{,}500}{x}+75$$

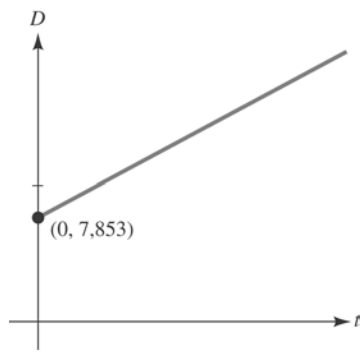
$AC(50)=\dfrac{4{,}500}{50}+75=165$, or \$165/unit.

39. **(a)** Since $t = 0$ in the year 2005, $t = 5$ is the year 2010. The given information translates to the points (0, 7853) and (5, 9127). The slope of a line through these points is

$$m = \frac{9127 - 7853}{5 - 0} = \frac{1274}{5} = 254.8.$$

So, the equation of the function is $D(t) = 254.8t + 7853$.

For practical purposes, the graph is limited to quadrant I.



(b) In the year 2015, $t = 10$ and the predicted debt is $D(10) = 254.8(10) + 7853 = 10{,}401$ or \$10,401.

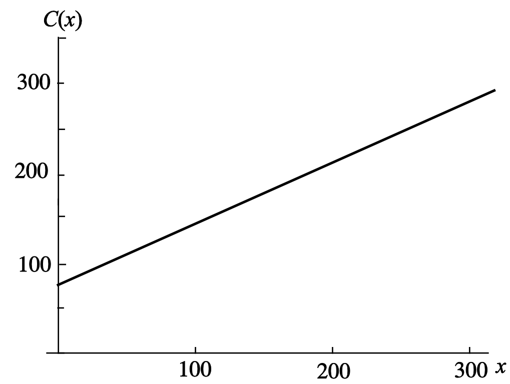
(c) Need to find t where $D(t) = 2(7{,}853) = 15{,}706$

$$254.8t + 7853 = 15706$$
$$254.8t = 7853$$
$$t \approx 30.8$$

Debt will be double the amount of 2005 during the year 2035.

40. **(a)** Let x denote the number of miles driven and $C(x)$ the corresponding cost (in dollars).

$C(x) = 0.70x + 75$



(b) The rental cost of a 50-mile trip is $C(50) = 0.75(50) + 75 = 110$.

(c) $125 = 0.7x + 75$, $0.7x = 50$, $x \approx 71.4$

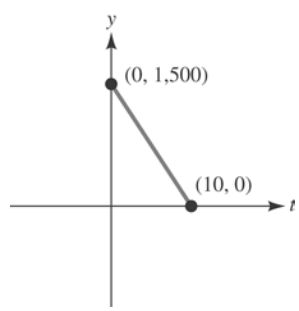
You must drive about 72 miles.

41. The slope is

$$m = \frac{1{,}500 - 0}{0 - 10} = -150$$

Originally (when time $x = 0$), the value of y of the books is 1,500 (this is the y-intercept.)

$y = -150x + 1{,}500$



42. **(a)** Let x denote the age in years of the machinery and V a linear function of x.

At the time of purchase, $x = 0$ and $V(0) = 20{,}000$.

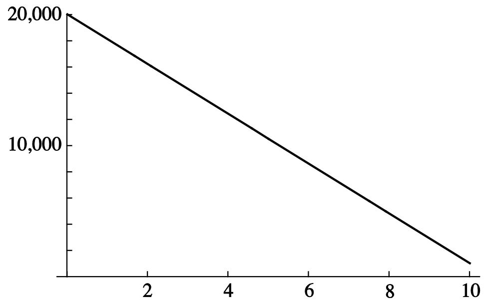
Ten years later, $x = 10$ and $V(10) = 1{,}000$.

The slope of the line through (0, 20,000) and (10, 1,000) is

$$m = \frac{1{,}000 - 20{,}000}{10 - 0} = -1{,}900.$$

Thus $V(x) = -1{,}900x + 20{,}000$.

$V(x)=0$ when $x=\dfrac{200}{19}$ and $V(x)$ is valid for $0 \le x \le \dfrac{200}{19}$.



(b) $V(4) = -1,900(4) + 20,000 = 12,400$

(c) $V(x) = 0$ when $x \approx$ 10.5, or after about 10.5 years.

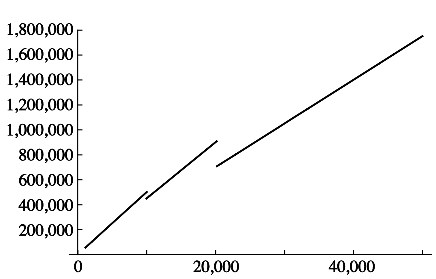
(d) Answers may vary.

43. (a) Using the points (0, *V*) and (*N*, *S*), the slope of the line is $\dfrac{S-V}{N}$. So, the value of an asset after *t* years is $B(t) = \dfrac{S-V}{N}t + V.$

(b) For this equipment, $B(T) = -6,400t + 50,000$. So, $B(3) = -6,400(3) + 50,000 = 30,800$. Value after three years is $30,800.

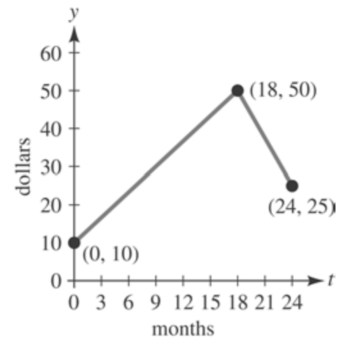
44. (a) $$F(N) = \begin{cases} 50N & \text{if } 1,000 \le N \le 10,000 \\ 40N & \text{if } 10,001 \le N \le 20,000 \\ 35N & \text{if } 20,001 \le N \le 50,000 \end{cases}$$

(b)

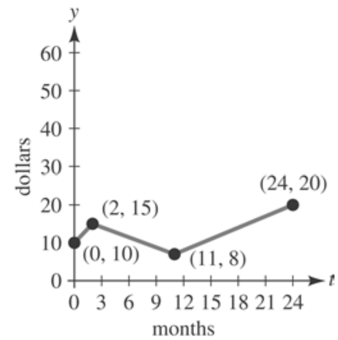


45. Let the *x*-axis represent time in months and the *y*-axis represent price per share.

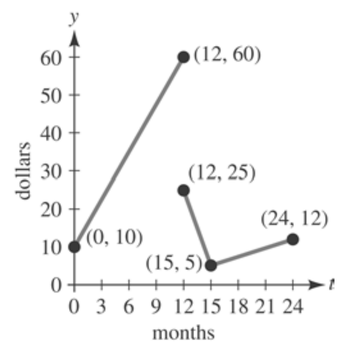
(a)



(b)



(c)



46. A rental company rents a piece of equipment for a $60.00 flat fee plus an hourly fee of $5.00 per hour.

(a) Let y = cost of renting the equipment and t = number of hours.

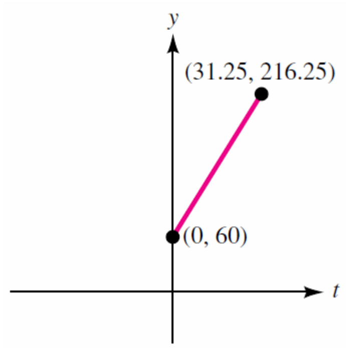
t	2	5	10	t
$y(t)$	70	85	110	$60+5t$

(b) $y(t) = 5t + 60, t \ge 0$

(c) Press $\boxed{y=}$

Input $5x+60$ for $y_1 =$.

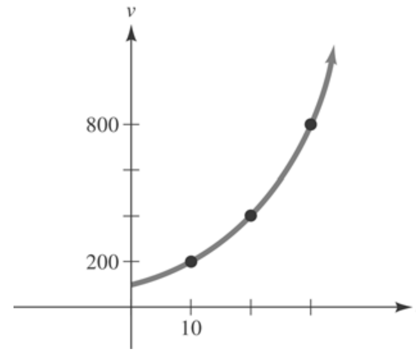
Use dimensions [–10, 10] 1 by [–10, 100] 10. Press $\boxed{\text{graph}}$.



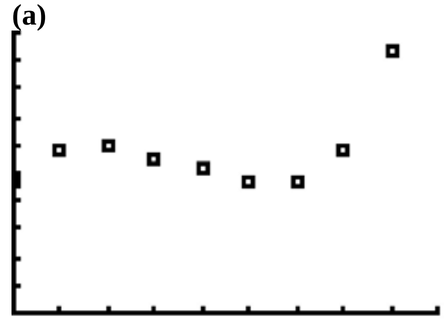
(d) To answer part (d), it may be easiest to use window dimensions [30, 33] 5 by [200, 230] 5. Press $\boxed{\text{graph}}$. Press $\boxed{\text{trace}}$ and move cross-hairs to be as close to y = 216.25 as possible. When y = 216.2234, the x-coordinate is 31.24. It takes approximately 31.24 hours for the rental charge to be \$216.25. Using algebra, we see it takes exactly 31.25 for the charge to be \$216.25.

47. **(a)** Since value doubles every 10 years,
in 1910, value is \$200
in 1920, value is \$400
in 1930, value is \$800
in 1940, value is \$1,600
in 1950, value is \$3,200
in 1960, value is \$6,400
in 1970, value is \$12,800
in 1980, value is \$25,600
in 1990, value is \$51,200
in 2000, value is \$102,400
in 2010, value is \$204,800
in 2020, value is \$409,600

(b) No, it is not linear.



48. **(a)**



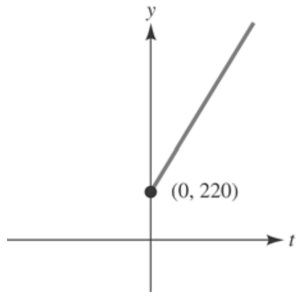
Use LinReg(ax+b) to obtain $y = 0.245x + 4.731$.

(b) According to the regression model, unemployment is changing by 0.245 percentage points per year.

(c) Answers will vary, but there are significant differences between the actual and predicted values.

49. **(a)** Let x be the number of hours spent registering students in person. During the first 4 hours (4)(35) = 140 students were registered. So, 360 – 140 = 220 students had pre-registered. Let y be the total number of students who register. Then,

$y = 35x + 220.$



(b) $y = (3)(35) + 220 = 325$

(c) From part **(a)**, we see that 220 students had pre-registered.

50. (a) From the data it is easy to see that every time the number of chirps increases by 5, there is a 1°F increase in temperature, Thus T is a linear function of C and the slope of this linear function is 1/5. Using this value, the point (0,38) and the point-slope formula

$$T - 38 = \frac{1}{5}(C - 0) \text{ or } T = \frac{1}{5}C + 38.$$

(b) Set $T = 75$ in the formula from part (a)

$75 = \frac{1}{5}C + 38$ and solve for

$C = 5(75 - 38) = 185$ chirps.

If 37 chirps are heard in 30 seconds then $C = 2(37) = 74$ chirps are heard per minute. Then

$T = \frac{1}{5}(37) + 38 = 45.4$ degrees F.

51. (a) $H(7) = 6.5(7) + 50 = 95.5$ cm tall

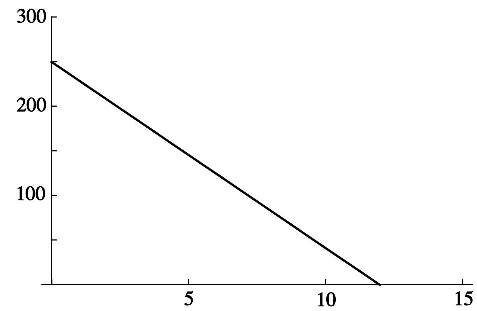
(b) $150 = 6.5A + 50$, $A = 15.4$ years old

(c) $H(0) = 6.5(0) + 50 = 50$ cm tall. This height (≈ 19.7 inches) seems reasonable.

(d) $H(20) = 6.5(20) + 50 = 180$ cm tall. This height (≈ 5.9 feet) seems reasonable.

52. (a) Since $\frac{250}{12}$ is the cost per week, then the value of the missed week is $\frac{250}{12}x$. Therefore the cost of the remaining weeks is

$$F(x) = -\frac{250}{12}x + 250$$



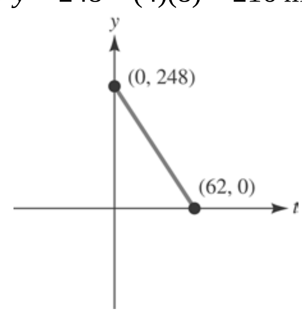
(b) The fee after 5 weeks is

$$F(5) = -\frac{250}{12}(5) + 250 = \$145.83$$

53. (a) Let x be the number of days. The slope is $m = \frac{200 - 164}{12 - 21} = -4.$

For $P(12, 200)$, $y - 200 = -4(x - 12)$, or $y = -4x + 248$.

(b) $y = 248 - (4)(8) = 216$ million gallons.



54. (a) Let x denote the number of days since the reduced rate went into effect and $N(x)$ the corresponding number of vehicles qualifying for the reduced rate. Since the number of qualifying vehicles is increasing at a constant rate, N is a linear function of x. Since $N(0) = 157$ (when the program began) and $N(30) = 247$ (30 days later), $m = \dfrac{247-157}{30-0} = 3.$

$N(x) = 3x + 157$ for $x \geq 0$

(b) In 14 days from now $N(44) = 3 \times 44 + 157 = 289$

55. (a) Let C be the temperature in degrees Celsius and F the temperature in degrees Fahrenheit. The slope is $m = \dfrac{212-32}{100-0} = \dfrac{9}{5}.$

So, $\dfrac{F-32}{C-0} = \dfrac{9}{5}$, or $F = \dfrac{9}{5}C + 32.$

(b) $F = \dfrac{9}{5}(15) + 32 = 59$ degrees

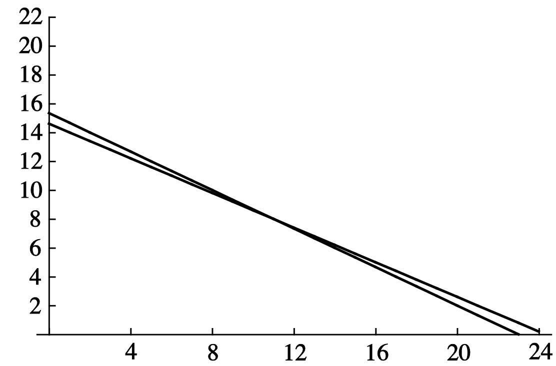
(c) $68 = \dfrac{9}{5}C + 32$

$36 = \dfrac{9}{5}C$

$C = 20$ degrees

(d) Solving $C = \dfrac{9}{5}C + 32$, $C = -40$. So, the temperature –40°C is also –40°F.

56. Let x be the number of ounces of Food I, and y be the number of ounces of Food II. Then $3x$ will be the number of grams of carbohydrate from the first food, and $5y$ the number of grams of carbohydrate from the second food.
Similarly $2x$ and $3y$ will be the number of grams of protein from the two foods. $3x + 5y$ is the total number of gm of carbohydrate, which must equal 73, while $2x + 3y$ is the total number of grams of protein, which must equal 46.



57. (a) Let t represent years after 2005. Using the points (0, 575) and (5, 545), the slope is $m = \dfrac{545-575}{5-0} = -6.$ If S represents the average SAT score, $S(t) = -6t + 575.$

(b) $S(10) = -6(10) + 575 = 515$

(c) $527 = -6t + 575$, $t = 8$, and the year would be 2013.

58. (a) If a liter of beer is 3% alcohol, then it contains $0.03 \times 1{,}000 = 30$ ml of alcohol. If alcohol is metabolized at 10 ml per hour, then 3 hours are required.

(b) $T = \dfrac{A}{10}$

(c) No one can have A ml of alcohol with fewer than T hours left in the party.

59. (a) Using the given points (100, 97) and (500, 110), slope is

$\dfrac{110-97}{500-100} = 0.0325$

$N - 97 = 0.0325(x - 100)$

$N(x) = 0.0325x + 93.75$

(b) When $x = 300$,

$N(300) = 0.0325(300) + 93.75$

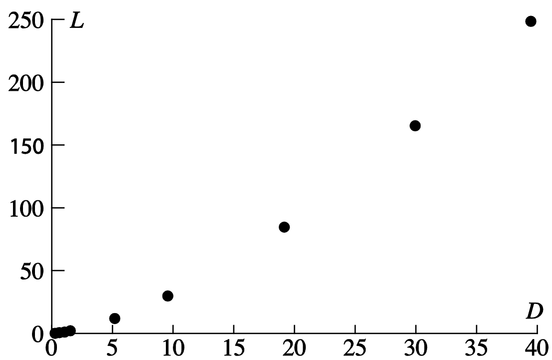
≈ 104 deaths

When there are 100 deaths,

$100 = 0.0325x + 93.75$
or $x \approx 192.3$ mg/m^3

(c) Writing exercise – Answers will vary.

60. **(a)** L and D are not linearly related.



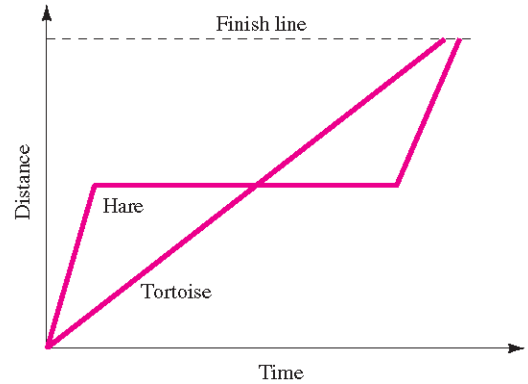
(b)

Planet	D	L	$\frac{L^2}{D^3}$
Mercury	0.388	0.241	0.994
Venus	0.722	0.615	1.005
Earth	1.000	1.000	1.000
Mars	1.523	1.881	1.002
Jupiter	5.203	11.862	0.999
Saturn	9.545	29.457	0.998
Uranus	19.189	84.013	0.999
Neptune	30.079	164.783	0.998
Pluto	39.463	248.420	1.004

$\frac{L^2}{D^3} \approx 1$ or $L = \sqrt[2]{D^3} = D^{3/2}$.

(c) Writing exercise, answers will vary.

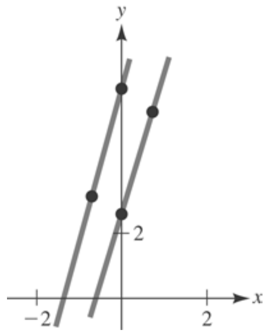
61. The segment with a constant, positive slope for all values of time represents the tortoise moving at a constant rate.
The segment with a positive slope, followed by a horizontal segment, and finished with another positive slope segment (slope of this segment matches slope of first segment) represents the hare's movement. Note that the horizontal segment represents the time during which the hare is napping.



62. $y = \frac{54}{270}x - \frac{63}{19}$ and $y = \frac{139}{695}x - \frac{346}{14}$ are parallel because $\frac{54}{270} = \frac{1}{5}$ and $\frac{139}{695} = \frac{1}{5}$.

63. To graph $y = \frac{25}{7}x + \frac{13}{2}$ and $y = \frac{144}{45}x + \frac{630}{229}$ on the same set of axes,
Press $y =$.
Input $\frac{(25x)}{7} + \frac{13}{2}$ for $y_1 =$ and press enter.
Input $\frac{(144x)}{45} + \frac{630}{229}$ for $y_2 =$.
Use the window dimensions [0, 4] 0.5 by [0, 14] 2. Press graph.
It does not appear that the lines are parallel. To verify this, press 2ND quit.
Input $\frac{25}{7} - \frac{144}{45}$ and enter.
If the lines were parallel the difference in their slopes would equal zero (the slopes would be the same). The difference of these slopes is 0.37 and therefore, the lines

are not parallel.



64. Let's say that two lines being parallel means that they have no points in common. First, we will prove that if they are parallel, then they have the same slope.
Let's say the lines are $y = m_1 x + b$ and $y = m_2 x + c$ and $m_1 \neq m_2$. With some work, we find that both of these lines share the same point

$$\left(\frac{c-b}{m_1 - m_2}, \frac{m_1 c - m_2 b}{m_1 - m_2}\right)$$

Therefore they are not parallel.
This contradicts our assumption that they are parallel. Thus $m_1 = m_2$ and they must have the same slope.
Now we prove that if they have the same slope, they are parallel. Let's again suppose that they are not parallel, i.e., that they have a point in common, say (a,b) and let $(x, y_1), (x, y_2)$ be points on the two lines.
Calculating the slopes of these lines, we find $m_1 = \frac{y_1 - b}{x - a}$ and $m_1 = \frac{y_2 - b}{x - a}$.
But these slopes are different because $y_1 \neq y_2$. Therefore it must be that the lines do not have any points in common, i.e., they are parallel.

65. Lines L_1 and L_2 are given as perpendicular, and the slope of L_1 is $m_1 = \frac{b}{a}$ while that of L_2 is $m_2 = \frac{c}{a}$. In right triangle OAB, the hypotenuse has length $|AB| = |b - c|$ while the two legs have lengths $|OA| = \sqrt{a^2 + b^2}$ and $|OB| = \sqrt{a^2 + c^2}$. Thus, by the Pythagorean theorem

$$\begin{aligned}
(a^2 + b^2) + (a^2 + c^2) &= (b - c)^2 \\
a^2 + b^2 + a^2 + c^2 &= b^2 - 2bc + c^2 \\
2a^2 &= -2bc \\
\frac{bc}{a^2} &= -1
\end{aligned}$$

$$\left(\frac{b}{a}\right)\left(\frac{c}{a}\right) = -1$$

$$m_1 m_2 = -1$$

so $m_2 = \frac{-1}{m_1}$

1.4 Functional Models

1. Let x and y be the smaller and larger numbers, respectively. Then

$$xy = 318$$

$$y = \frac{318}{x}$$

The sum is $S = x + y = x + \frac{318}{x}$.

2. Let x and y be the smaller and larger numbers, respectively. Then $x + y = 18$ or $y = 18 - x$. The product is $P = xy = x(18 - x)$.

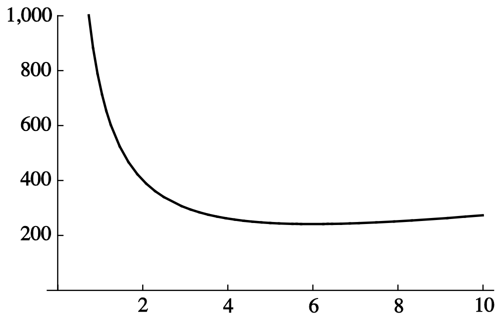
3. Let R denote the rate of population growth and p the population size. Since R is directly proportional to p, $R(p) = kp$, where k is the constant of proportionality.

4. Let x be the width, then $2x$ is the length. The area is $A = (2x)x = 2x^2$ square units.

5. This problem has two possible forms of the solution. Assume the stream is along the length, say l. Then w is the width and $l + 2w = 1{,}000$ or $l = 1{,}000 - 2w$.

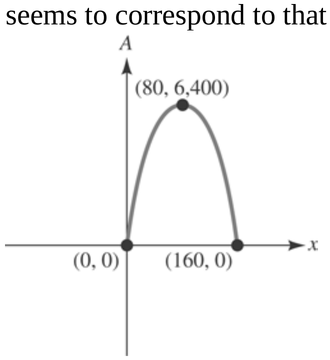
The area is $A = lw = 2w(500 - w)$ square feet.

6. Let x denote the length and y the width of the rectangular playground. Let P be the number of meters of fencing required to enclose the playground, then $P = 2x + 2y$. Since the area is $xy = 3,600$,
$y = \frac{3,600}{x}$ and $P(x) = 2x + \frac{7,200}{x}$.



The graph suggests that $P(x)$ is minimized near $x = 60$ meters. If so,
$y = \frac{3,600}{60} = 60$ meters and the playground is a square.

7. Let x be the length and y the width of the rectangle. Then $2x + 2y = 320$ or
$y = 160 - x$.
The area is (length)(width) or
$A(x) = x(160 - x)$.
The length is estimated to be 80 meters from the graph below, which also happens to be the width. So the maximum area seems to correspond to that of a square.



8. The rectangular box is closed. Let x be the length (and width) and y the depth. Since the volume, x^2y is 1,500 cu. in.,
$y = \frac{1,500}{x^2}$ and the surface area is

$$S(x) = 2x^2 + 4xy = 2x^2 + \frac{6,000}{x}.$$

9. Let x be the length of the square base and y the height of the box. The surface area is
$2x^2 + 4xy = 4,000$. So $y = \frac{2,000 - x^2}{2x}$
and the volume is

$$V = x^2 y = x\left(1,000 - \frac{x^2}{2}\right).$$

10. Let q be the amount of radium remaining and k a proportionality constant. Then
$R(q) = -kq$.
(Note $R(q) = -kq$ where $k > 0$ is usually used because decay means that less and less radium is left as time goes on.)

11. Let R denote the rate at which temperature changes, T_e the temperature of the medium, and T_0 the temperature of the object. Then $T_0 - T_e$ is the difference in the temperature between the object and the medium. Since the rate of change is directly proportional to the difference,
$R = k(T_0 - T_e)$, where k is the constant of proportionality.

12. Let q be the number of people who have caught the disease. Then $n - q$ is the number of people who have not yet caught the disease out of a total population of n people. If k is the proportionality constant then $R(q) = kq(n - q)$.

13. Let R denote the rate at which people are implicated, P the number of people implicated, and T the total number of people involved. Then $T - P$ is the number of people involved but not implicated. Since the rate of change is jointly proportional to those implicated and those not implicated, $R = kP(T - P)$, where k is the constant of proportionality.

14. Let x be the number of machines used. The setup cost is $k_1 x$ while the operating cost is $\frac{k_2}{x}$ where k_1, k_2 constants of proportionality. The total cost is
$C(x) = k_1 x + \frac{k_2}{x}$

15. Let R be the speed of the truck.
The cost due to wages is $\frac{k_1}{R}$, where k_1 is a constant of proportionality, and the cost due to gasoline is $k_2 R$, where k_2 is another constant of proportionality. If C is the total cost, $C = \frac{k_1}{R} + k_2 R.$

16. **(a)** $R(x) = 23x$
$C(x) = 14x + 1{,}200$
$$\begin{aligned} P(x) &= R(x) - C(x) \\ &= 23x - (14x + 1{,}200) \\ &= 9x - 1{,}200 \end{aligned}$$

(b) $P(2{,}000) = 9(2{,}000) - 1{,}200 = 16{,}800$
\$16,800 is generated when 2,000 units are produced.
$P(100) = 9(100) - 1{,}200 = -300$
There is a loss of \$300 when 100 units are produced.
For production to be profitable, solve
$$\begin{aligned} P(x) &> 0 \\ 9x - 1{,}200 &> 0 \\ x &> 133.\overline{3} \end{aligned}$$
At least 134 units must be produced.

(c) $AP(x) = \frac{P(x)}{x} = \frac{9x - 1{,}200}{x} = 9 - \frac{1{,}200}{x}$
$AP(2{,}500) = 9 - \frac{1{,}200}{2{,}500} = 8.52,$
or \$8.52/unit.

17. **(a)** Let p be the selling price of the commodity. Then
$$\text{Profit} = \text{Revenue} - \text{Costs}$$
$$\text{Revenue} = (\text{number sold}) \cdot (\text{selling price})$$
$$R(x) = xp$$
$$\text{Costs} = (\text{cost per unit}) \cdot (\text{number units}) + \text{fixed overhead}$$
$$C(x) = (p - 3)x + 17{,}000$$
$$C(x) = xp - 3x + 17{,}000$$
$$\begin{aligned} P(x) &= xp - (xp - 3x + 17{,}000) \\ &= 3x - 17{,}000 \end{aligned}$$

(b) $P(5{,}000) = 3(5{,}000) - 17{,}000$
$= -2{,}000$

or a loss of \$2,000

$P(20{,}000) = 3(20{,}000) - 17{,}000$
$= 43{,}000$

or a profit of \$43,000

Profit is zero (break-even point) when

$0 = 3x - 17{,}000$

$x \approx 5{,}666.67$ units are produced and sold so, when 5, 667 units are produced and sold, production becomes profitable.

(c) average profit $= \dfrac{P(x)}{x}$

$AP(x) = 3 - \dfrac{17{,}000}{x}$

when 10,000 units are produced,

$AP(10{,}000) = 3 - \dfrac{17{,}000}{10{,}000} = 1.3$

or \$1.30 per unit.

18. **(a)** Revenue = (number sold) (selling price)

$R(x) = xp = x(-6x + 100)$ thousand dollars

or, $R(x) = 1000x(-6x + 100)$ dollars

$R(15) = 1000(15)(-6(15) + 100)$
$= \$150{,}000$

(b) $AR(x) = \dfrac{R(x)}{x} = \dfrac{x(-6x + 100)}{x} = -6x + 100$

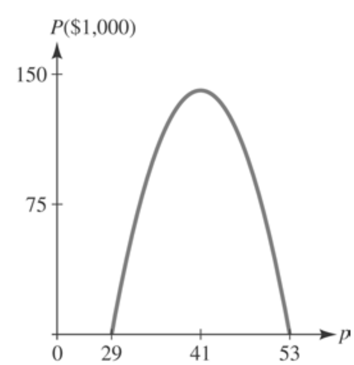
$AR(10) = -6(10) + 100 = 40$,

or \$40,000/unit.

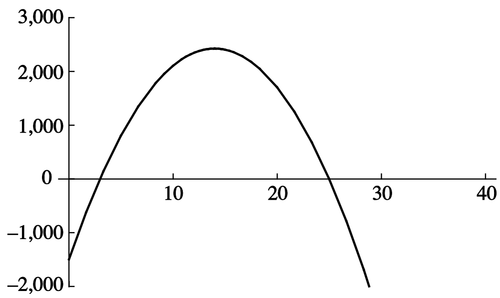
19. Let x be the sales price per lamp. Then, $x - 50$ will be the number of \$1.00 increases over the base price of \$50, and $1{,}000(x - 50)$ is the number of unsold lamps. Therefore the number of lamps sold is $3{,}000 - 1{,}000(x - 50)$. The profit is

$$P = [3{,}000 - 1{,}000(x - 50)]x - 29[3{,}000 - 1{,}000(x - 50)]$$
$$= [3{,}000 - 1{,}000(x - 50)](x - 29)$$
$$= (53{,}000 - 1{,}000x)(x - 29)$$

The optimal selling price is \$41.



20. Let x denote the selling price in dollars of the book and $P(x)$ the corresponding profit function.
If x is the price of the book then $(15-x)$ is the number of \$1 decreases in the price of the book from \$15. Since 20 more books (beyond 200) will be sold for each \$1 decrease, the total number of books sold at x dollars is $200+20(15-x)=500-20x$.
The bookstore's revenue function is then
$R(x)=x(500-20x)=500x-20x^2$
while the cost function is
$C(x)=3(500-20x)=1500-60x$.
Since profit is revenue minus cost
$$P(x)=R(x)-C(x)$$
$$=-20x^2+560x-1500.$$



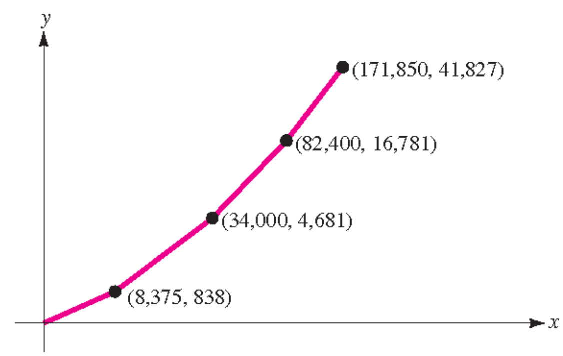
The graph of $P(x)$ suggests the profit is maximal when $x=14$, that is, when the books are sold for \$14 apiece. Note the vertex of this parabola is located at
$x=\dfrac{-560}{2(-20)}=14$ supporting the graphical estimate.

21. (a) For $0 \le x \le 8{,}375$, the tax is
$10\%x = 0.1x$.
For $8{,}375 < x \le 34{,}000$ the tax is
$837.5 + 15\%(x - 8{,}375)$
$= 837.5 + 0.15(x - 8{,}375)$
$= 837.5 + 0.15x - 1{,}256.25$
$= 0.15x - 418.75$.

For $34{,}000 < x \le 82{,}400$ the tax is
$4{,}681.25 + 25\%(x - 34{,}000)$
$= 4{,}681.25 + 0.25(x - 34{,}000)$
$= 4{,}681.25 + 0.25x - 8{,}500$
$= 0.25x - 3{,}818.75$

For $82{,}400 < x \le 171{,}850$ the tax is
$16{,}781.25 + 28\%(x - 82{,}400)$
$= 16{,}781.25 + 0.28(x - 82{,}400)$
$= 16{,}781.25 + 0.28x - 23{,}072$
$= 0.28x - 6{,}290.75$
So,
$$T(x)=\begin{cases} 0.1x & \text{if } 0 \le x \le 8{,}375 \\ 0.15x-418.75 & \text{if } 8{,}375 < x \le 34{,}000 \\ 0.25x-3{,}818.75 & \text{if } 34{,}000 < x \le 82{,}400 \\ 0.28x-6{,}290.75 & \text{if } 82{,}400 < x \le 171{,}850 \end{cases}$$

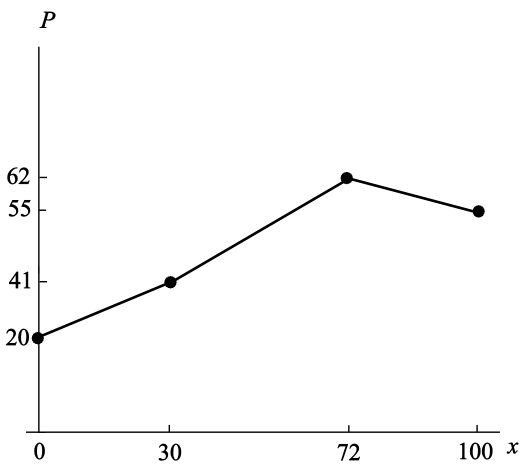
(b)



The slopes of the segments are 0.1, 0.15, 0.25 and 0.28, respectively. As taxable income increases, the slopes of the segments increase. So, as you earn more, you pay more on your earnings

22. (a)

$$P(x) = \begin{cases} 20 + 0.7x & \text{for} \quad 0 \le x < 30 \\ 26 + 0.5x & \text{for} \quad 30 \le x < 72 \\ 80 - 0.25x & \text{for} \quad 72 \le x \le 100 \end{cases}$$



(b) $P(50) = 26 + 0.5(50) = 51$
The profit is \$51,000.

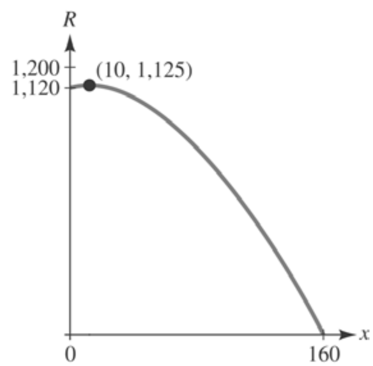
(c) In terms of x, $P(0) = 20$, $P(30) = 41$, $P(72) = 62$, and $P(100) = 55$.
Thus, in terms of y, $P(100) = 20$, $P(70) = 41$, $P(28) = 62$, and $P(0) = 55$.

$$P(y) = \begin{cases} 55 + 0.25y & \text{for} \quad 0 \le y \le 28 \\ 76 - 0.5y & \text{for} \quad 28 < y \le 70 \\ 90 - 0.7y & \text{for} \quad 70 < y \le 100 \end{cases}$$

23. Let x denote the number of days after July 1 and $R(x)$ the corresponding revenue (in dollars). Then
$R(x)$ = (number of bushels sold)
(price per bushel)
Since the crop increases at the rate of 1 bushel per day and 140 bushels were available on July 1, the number of bushels sold after x days is $140 + x$. Since the price per bushel decreases by 0.05 dollars per day and was \$8 on July 1, the price per bushel after x days is
$8 - 0.05x$ dollars. Putting it all together,

$$R(x) = (140 + x)(8 - 0.05x) = 0.05(160 - x)(140 + x)$$

The number of days to maximize revenue is approximately 10 days after July 1, or July 11. Note that
$R(10) = 0.05(160 - 10)(140 + 10) = 1{,}125$.
So, the estimated max revenue is \$1,125.

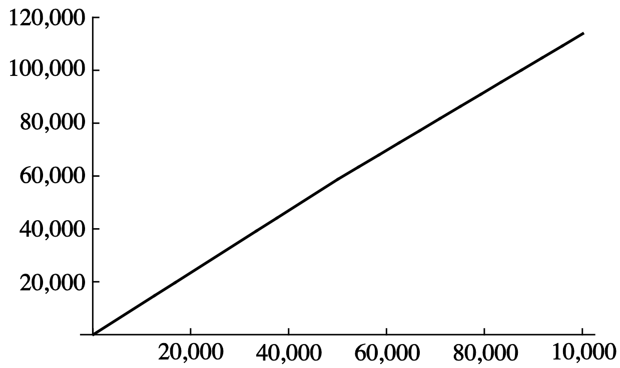


24. (a)

Winning bid	Total price
\$1,000	$\$1{,}000 + (0.175)(\$1{,}000) = \$1{,}175$
\$25,000	$\$25{,}000 + (0.175)(\$25{,}000) = \$29{,}375$
\$100,000	\$100,000 + (0.175)(\$50,000) + (0.10)(\$50,000) = \$113,750

(b) Let x denote the winning bid price. If x is less than or equal to 50,000, the total purchase price is $x + 0.175x = 1.175x$ dollars. If x exceeds 50,000, the first 50,000 carries a premium of $(0.175)(50{,}000) = 8{,}750$ dollars while the remaining $x - 50{,}000$ carries a 10% premium. The total price is then $x + 8{,}750 + (0.10)(x - 50{,}000) = 1.1x + 3{,}750$ dollars. Summarizing, the total price, $P(x)$, is given by the function

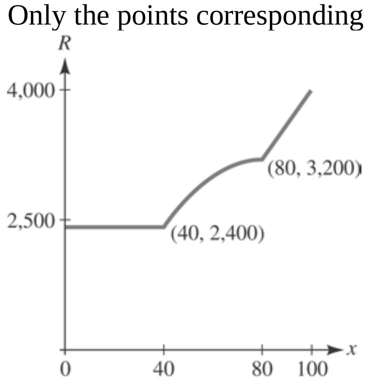
$$P(x) = \begin{cases} 1.175x & \text{if } x \le 50{,}000 \\ 1.1x + 3{,}750 & \text{if } x > 50{,}000 \end{cases}$$



25. Let x be the number of passengers. There will be $x - 40$ additional passengers between $40 < x \le 80$ (if the total number is below 80). The price for the second category is $60 - 0.5(x - 40) = 80 - 0.5x$. The revenue generated in this category is $80x - 0.5x^2$.

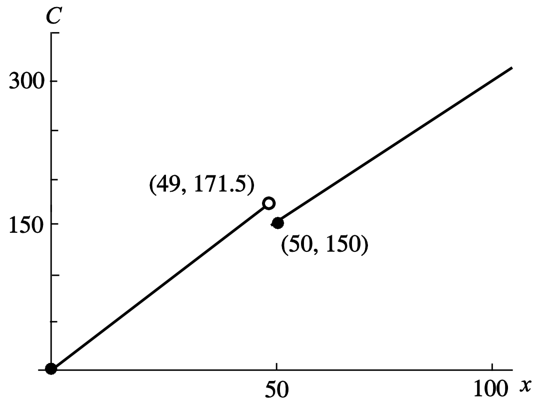
$$R(x) = \begin{cases} 2{,}400 & \text{if } 0 < x \le 40 \\ 80x - 0.5x^2 & \text{if } 40 < x < 80 \\ 40x & \text{if } x \ge 80 \end{cases}$$

Only the points corresponding to the integers $x = 0, 1, 2, \ldots$ are meaningful in the practical context.



26. (a) Let x denote the number of people in the group and $C(x)$ the corresponding total admission charge for the group. If $x \ge 50$, the group is charged $3x$ dollars and if $0 \le x < 50$, the group is charged $3.5x$ dollars. So $C(x) = \begin{cases} 3.5x & \text{if } 0 \le x \le 50 \\ 3.0x & \text{if } x \ge 50 \end{cases}$.

Only the points corresponding to the integers $x = 0, 1, 2, \ldots$ are meaningful in the practical context.



(b) $C(49) = 3.5(49) = 171.50$;
$C(50) = 150$. So, by recruiting one additional member, a group of 49 can save $171.50 - 150 = \$21.50$.

27. Royalties for publisher A are given by

$$R_A(N) = \begin{cases} 0.01(2)(N) & 0 < N \le 30,000 \\ 0.01(2)(30,000) + 0.035(2)(N - 30,000) & N > 30,000 \end{cases}$$

Royalties for publisher B are given by

$$R_B(N) = \begin{cases} 0 & N \le 4,000 \\ 0.02(3)(N - 4,000) & N > 4,000 \end{cases}$$

Clearly, for $N \le 4,000$, publisher A offers the better deal. When $N = 30,000$, publisher A pays \$600, but publisher B now pays more, paying \$1,560. Therefore, the plans pay the same amount for some value of $N < 30,000$. To find the value,

$$\begin{aligned} 0.01(2)(N) &= 0.02(3)(N - 4,000) \\ 0.02N &= 0.06N - 240 \\ 240 &= 0.04N \\ 6,000 &= N \end{aligned}$$

So, when $N < 6,000$, publisher A offers the better deal. When $N > 6,000$, publisher B initially offers the better deal. Then, the plans again pay the same amount when

$$\begin{aligned} 0.01(2)(30,000) + 0.035(2)(N - 30,000) &= 0.02(3)(N - 4,000) \\ 0.07N - 1,500 &= 0.06N - 240 \\ 0.01N &= 1,260 \\ N &= 126,000 \end{aligned}$$

So, when more than 126,000 copies are sold, plan A becomes the better plan.

28. Let x be the number of checks that clear the bank. Then the first bank charges $y = 0.10x + 12$ dollars while the second one charges $y = 0.14x + 10$ dollars.

Find the break even point by setting the two equal

$$\begin{aligned} 0.10x + 12 &= 0.14x + 10 \\ 0.04x &= 2 \\ x &= 50 \end{aligned}$$

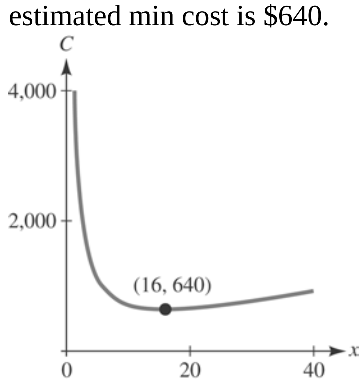
If fewer than 50 checks are written the second bank offers the better deal. If more than 50 checks will be written, the first bank is more economical.

29. Let x be the number of machines used and t the number of hours of production. The number of kickboards produced per machine per hour is $30x$. It costs $20x$ to set up all the machines. The cost of supervision is $19.20t$. The number of kickboards produced by x machines in t hours is $30xt$ which must account for all 8,000 kickboards. Solving $30xt = 8{,}000$ for t leads to $t = \frac{800}{3x}$.

Cost of supervision: $19.20\left(\frac{800}{3x}\right) = \frac{5{,}120}{x}$

Total cost: $C(x) = 20x + \frac{5{,}120}{x}$

The number of machines which minimize cost is approximately 16. Note that $C(16) = 20(16) + \frac{5{,}120}{16} = 640.$ So, the estimated min cost is \$640.

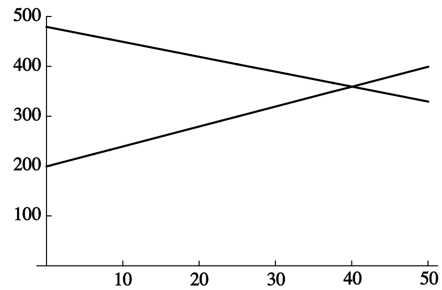


30. **(a)** Market equilibrium occurs when

$$S(x) = D(x)$$
$$4x + 200 = -3x + 480$$
$$7x = 280$$

so $x = \frac{280}{7}$ or $x_e = 40$.

(b)



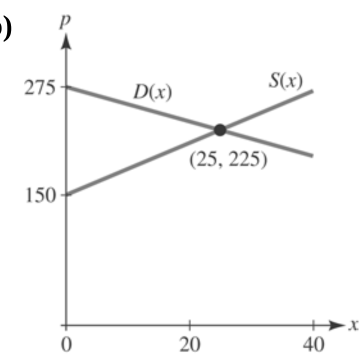
(c) A market shortage occurs when the graph of $S(x)$ is below the graph of $D(x)$ or when $x < 40$.
A market surplus occurs when the graph of $S(x)$ is above the graph of $D(x)$ or when $x > 40$.

31. **(a)** Equilibrium occurs when $S(x) = D(x)$, or $3x + 150 = -2x + 275$

$$5x = 125$$
$$x = 5$$

The corresponding equilibrium price is $p = S(x) = D(x)$ or $p = 3(25) + 150 = \$225$.

(b)



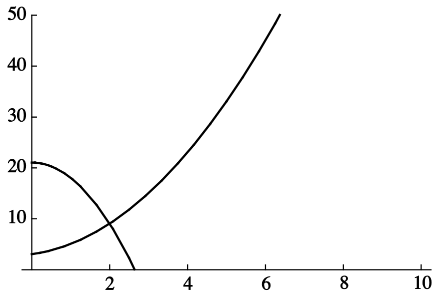
(c) There is a market shortage when demand exceeds supply. Here, a market shortage occurs when $0 < x < 25$. A market surplus occurs when supply exceeds demand. Here, a market surplus occurs when $x > 25$.

32. **(a)** Market equilibrium occurs when

$$S(x) = D(x)$$
$$x^2 + x + 3 = 21 - 3x^2$$
$$4x^2 + x - 18 = 0$$
$$(4x + 9)(x - 2) = 0$$

so $x = \frac{-9}{4}$ or $x = 2$. The negative value is not a valid level of production so $x_e = 2$ and $p_e = S(x_e) = D(x_e) = 9$.

(b)



(c) A market shortage occurs when the graph of $S(x)$ is below the graph of $D(x)$ or when $x < 2$.
A market surplus occurs when the graph of $S(x)$ is above the graph of $D(x)$ or when $x > 2$.

33. (a) Equilibrium occurs when $S(x) = D(x)$, or $2x + 7.43 = -0.21x^2 - 0.84x + 50$

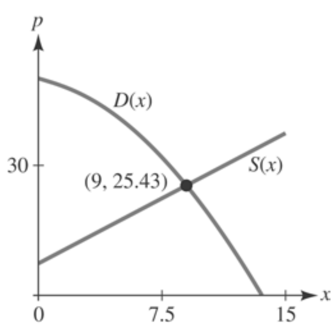
$$0.21x^2 + 2.84x - 42.57 = 0$$

Using the quadratic formula,

$$x = \frac{-2.84 \pm \sqrt{(2.84)^2 - 4(0.21)(-42.57)}}{2(0.21)}$$

so $x = 9$ (disregarding the negative root.) The corresponding equilibrium price is $p = S(x) = D(x)$, or $p = 2(9) + 7.43 = 25.43$.

(b)



(c) There is a market shortage when demand exceeds supply. Here, a market shortage occurs when $0 < x < 9$. A market surplus occurs when supply exceeds demand. Here, a market surplus occurs when $x > 9$.

34. The supply is $S(p) = \frac{p^2}{10}$ and the demand is $D(p) = 60 - p$. Supply will equal demand when $\frac{p^2}{10} = 60 - p$ or

$p^2 + 10p - 600 = 0$.

Solving this quadratic gives $p = 20, -30$. Disregarding the negative value, supply will equal demand when the blenders are priced at $20 apiece. At this price $S(20) = D(20) = 40$ blenders will be sold.

35. (a) Equilibrium occurs when $S(x) = D(x)$,

or $$2x + 15 = \frac{385}{x+1}$$

$$(2x + 15)(x + 1) = 385$$

$$2x^2 + 17x + 15 = 385$$

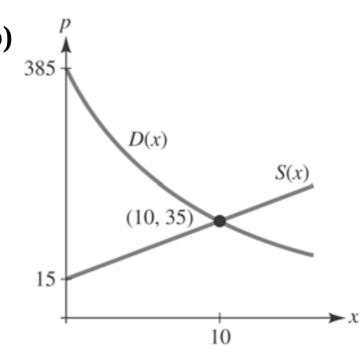
$$2x^2 + 17x - 370 = 0$$

Using the quadratic formula,

$$x = \frac{-17 \pm \sqrt{(17)^2 - 4(2)(-370)}}{2(2)}$$

so $x = 10$ (disregard the negative root). The corresponding equilibrium price is $p = S(x) = D(x)$, or $p = 2(10) + 15 = 35$.

(b)



(c) The supply curve intersects the y-axis at $S(0) = 15$. Since this is the price at which producers are willing to supply zero units, it corresponds to their overhead at the start of production.

36. **(a)** If x is the number of tables produced then the manufacturer's cost function is $C(x) = 30,000 + 350x$ while the revenue function is $R(x) = 500x.$ The break even point is where $R(x) = C(x)$ or $500x = 350x + 30,000.$ Thus 150x = 30,000 and x = 200. The manufacturer must sell 200 tables to break even.

(b) Since profit is revenue minus cost, the profit function, $P(x)$, is

$$P(x) = 500x - (350x + 30,000) = 150x - 30,000.$$

For the profit to be \$6,000, x must satisfy

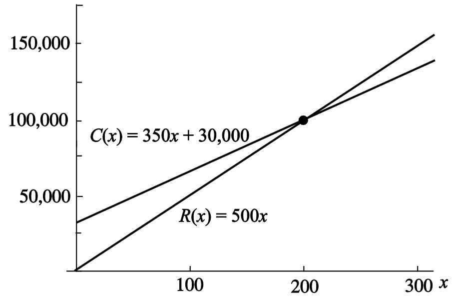
$$150x - 30,000 = 6,000$$
$$150x = 36,000$$
$$x = 240$$

Selling 240 tables yields a profit of \$6,000.

(c) Since $P(150) = 150(150) - 30,000 = 22,500 - 30,000 = -7,500$

there will be a loss of \$7,500 if only 150 tables are sold.

(d)



The overhead corresponds to the y intercept of the cost function.

37. **(a)** Total revenue = (selling price)(# units)
Let R represent the total revenue and x represent the number of units. Then,

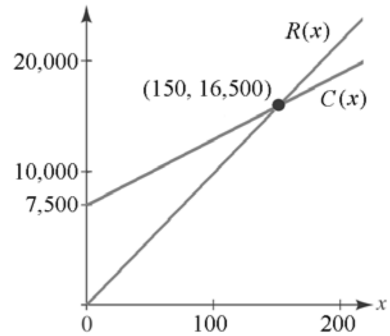
$$R(x) = 110x$$

Total cost = fixed overhead + (cost per unit)(#units)
Let C represent the total cost. Then,

$$C(x) = 7,500 + 60x$$

Total profit = total revenue – total cost
Let P represent the total profit. Then,

$$P(x) = 110x - (7,500 + 60x)$$
$$P(x) = 50x - 7,500$$



(b) To break even, need revenue = cost, or profit is zero.

$$0 = 50x - 7,500$$
$$x = 150 \text{ units}$$

(c) $P(100) = 50(100) - 7,500 = -2,500$
or a loss of \$2,500

(d) 1, 250 = 50x – 7,500
x = 175 units

38. **(a)**

$$R(x) = 2.75x$$
$$C(x) = 0.35x + 12,000$$
$$P(x) = R(x) - C(x)$$
$$= 2.75x - (0.35x + 12,000)$$
$$= 2.4x - 12,000$$

(b) To break even, solve

$$R(x) = C(x)$$
$$2.75x = 0.35x + 12,000$$
$$2.4x = 12,000$$
$$x = 5,000$$

They must sell at least 5,000 cards.

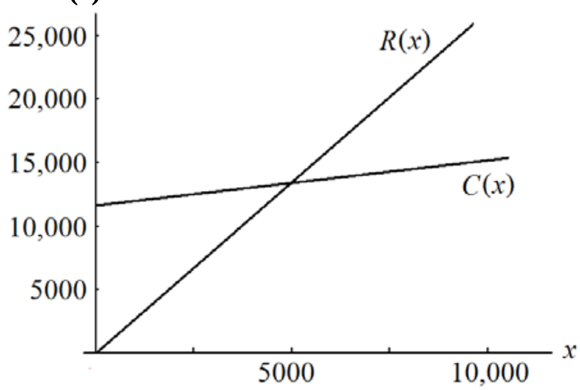
(c) They will break even. See part (b).

(d) To make a profit of $9,000, solve

$$P(x) = 9,000$$
$$2.4x - 12,000 = 9,000$$
$$2.4x = 21,000$$
$$x = 8,750$$

They must sell 8,750 cards.

(e)

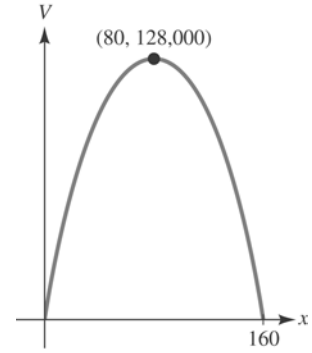


The graph of the cost function crosses the *y*-axis at (0, 12,000), which represents the fixed overhead costs.

39. **(a)** Volume = (length)(width)(height)
The height is given as 20 m and the perimeter is 320 m. So,

$$2(x+w) = 320$$
$$x + w = 160$$
$$w = 160 - x$$
$$V(x) = x(160 - x)(20) = 20x(160 - x)$$

(b) Since the high point of the graph occurs half-way between its intercepts, the max volume occurs when $x = 80$. The dimensions for the max volume are length = 80 m, width = 160 – 80 = 80 m and height = 20 m.



(c) Cost construction
= cost building + cost parking lot
Cost building = 75(80)(80)(20)
= $9,600,000
Cost parking lot
= cost top rectangle (across entire length) + cost right side rectangle (next to building)
Cost top rectangle
= 50(area rectangle)
= 50(length)(width)
= 50(length)(100 – width bldg)
= 50(120)(100 – 80)
= $120,000
Cost right rectangle
= 50(area rectangle)
= 50(length)(width)
= 50(120 – length bldg)(width)
= 50(120 – 80)(80)
= $160,000
Cost construction
= $9,600,000 + $120,000 + $160,000
= $9,880,000

40. **(a)** Let x = number of books and C = cost of producing x books.
$C(x) = 5.5x + 74{,}200$

x	2,000	4,000	6,000	8,000
$C(x)$	85,200	96,200	107,200	118,200

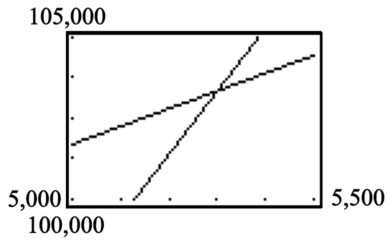
(b) Let x = number of books and R = revenue from the sale of x books.
$R(x) = 19.5x$

x	2,000	4,000	6,000	8,000
$R(x)$	39,000	78,000	117,000	156,000

(c) $y = 5.5x + 74{,}200$

(d) $y = 19.5x$

(e)



(f) To find where cost = revenue, press **TRACE**.
Use arrow buttons to move along one of the graphs to the apparent intersection. Use the Zoom in function under the calc menu. Repeat process using arrows buttons and zoom in for a more accurate reading. As an alternative, use the intersect function under the calc menu. Enter a value close to the point of intersection for y_1 and also for y_2. Finally, enter a guess. The coordinate (5,300, 103,350) appears to be the point at which cost equals revenue.

(g) Use arrow buttons to trace along the revenue graph. It appears that approximately 4,360 books must be sold for a revenue of \$85,000. The profit when 4,360 books are sold is –13,160, a loss of \$13,160.

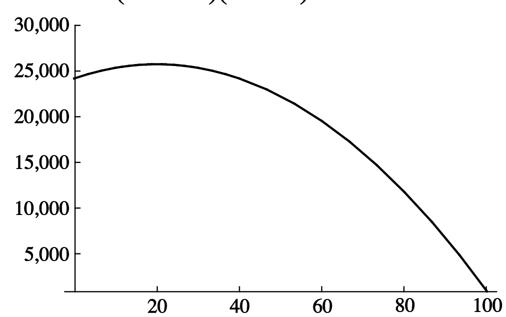
41. $S(q) = aq + b$
$D(q) = cq + d$

(a) The graph of S is rising, while the graph of D is falling. So, $a > 0$ and $c < 0$. Further, since both y-intercepts are positive, $b > 0$ and $d > 0$.

(b)
$$aq + b = cq + d$$
$$(a - c)q = d - b$$
$$q_e = \frac{d - b}{a - c}$$
$$p_e = aq_e + b$$
$$= a\left(\frac{d - b}{a - c}\right) + b$$
$$= \frac{ad - ab}{a - c} + b$$
$$= \frac{ad - ab + b(a - c)}{a - c}$$
$$= \frac{ad - bc}{a - c}$$

(c) As a increases, the denominator in the expression for q_e increases. This results in a decrease in q_e. As d increases, the numerator in the expression for q_e increases. This results in an increase in q_e.

42. Let x be the number of additional trees planted. Then the number of trees will be $60 + x$ and the average number of oranges per tree will be $400 - 4x$. The yield is then
$$y(x) = (400 - 4x)(60 + x)$$
$$= 4(100 - x)(60 + x).$$

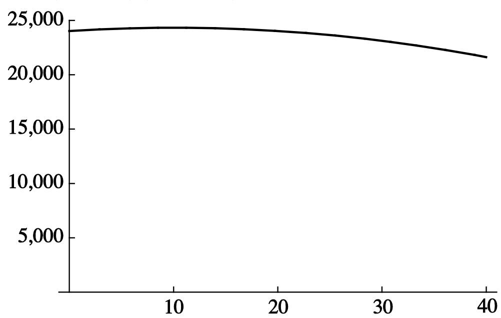


The number of additional trees for maximal yield appears to be 20 or 80 total trees.

43. Since I is proportional to the area, A, of the pupil, $I = kA$, where k is a constant of proportionality. Since the pupil of the eye is circular and the area of a circle is $A = \pi r^2$, $I = k\pi r^2$.

44. Let x be the number of additional days beyond 80 before the club takes all its glass to the recycling center.
The rate at which the club collects glass is $\frac{24,000}{80} = 300$ pounds per day. Thus after x additional days the club will have collected a total of $24,000 + 300x$ pounds.
The current price of 1 cent per pound will decrease by 1/100 cent for each day. Thus the clubs revenue, in cents, in x days will be

$$(24,000 + 300x)\left(1 - \frac{x}{100}\right)$$
$$= 3(80 + x)(100 - x).$$



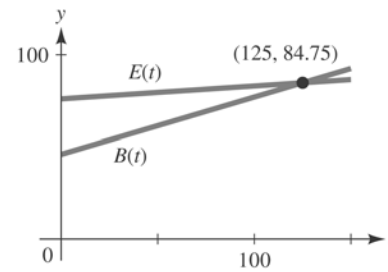
From the graph, it appears the club should collect glass for $x = 10$ additional days to maximize revenue.

45. **(a)** For a newborn child, the points (0, 46) and (100, 77) define the linear function. Its slope is
$m = \frac{77 - 46}{100 - 0} = 0.31$ and the function is
$B(t) = 0.31t + 46$.
For a 65-year-old, the points (0, 76) and (100, 83) define the linear function. Its slope is
$m = \frac{83 - 76}{100 - 0} = 0.07$ and the function is
$E(t) = 0.07t + 76$.

(b) Need to find $B(t) = E(t)$.

$$0.31t + 46 = 0.07t + 76$$
$$0.24t = 30$$
$$t = 125 \text{ years}$$

Note that this is where the graphs intersect.

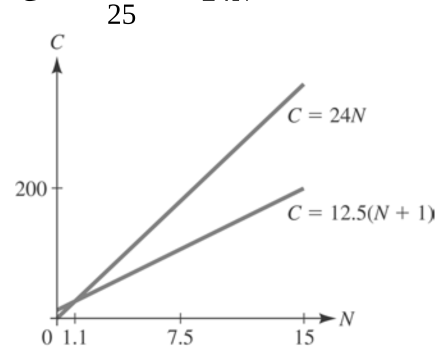


(c) Writing exercise—Answers will vary.

46. With $N = 11$ and $A = 300$, Cowling's rule suggests $C = \left(\frac{11+1}{24}\right)300 = 150$ mg while Friend's rule suggests
$C = \frac{2(11)(300)}{25} = 264$ mg.

47. $C = \left(\frac{N+1}{24}\right)(300) = \left(\frac{N+1}{2}\right)(25)$

$$C = \frac{2N \cdot 300}{25} = 24N$$



48. Setting the formulas for Cowling's and Friend's rules equal to each other gives
$\left(\frac{N+1}{24}\right)A = \frac{2NA}{25}$ or $\frac{N}{24} + \frac{1}{24} = \frac{2N}{25}$.

Solving for N gives $N = \frac{25}{23}$ or about 1 year, 1 month old. If N is smaller, Cowling's rule suggests the higher dosage. If N is larger, Friend's rule suggests the higher dosage.

49. **(a)** The estimated surface area of the child is:

$S = 0.0072(18)^{0.425}(91)^{0.725} \approx 0.6473$

so, $C = \dfrac{(0.6473)(250)}{1.7} \approx 95.2$ mg.

(b) Using $2H$ and $2W$ for the larger child,

$C = \dfrac{0.0072(2W)^{0.425}(2H)^{0.725}A}{1.7}$.

Comparing to drug dosage for the smaller child,

$$\frac{\dfrac{0.0072(2W)^{0.425}(2H)^{0.725}A}{1.7}}{\dfrac{0.0072W^{0.425}H^{0.725}A}{1.7}} = (2)^{0.425}(2)^{0.725}$$
$$\approx 2.22$$

So, the drug dosage for larger child is approximately 2.22 times the dosage for the smaller child.

50. (a) $V(0.73) = \dfrac{4}{3}\pi(0.73)^3 \approx 1.6295$

$V(0.95) = \dfrac{4}{3}\pi(0.95)^3 \approx 3.5914$

The volume increases by about 1.9619 cubic centimeters.

(b) If the radius at the beginning was r_1, then the radius at the end is $0.77r_1$ and the corresponding volume is

$V = \dfrac{4}{3}\pi(0.77r_1)^3 = 0.456533\left(\dfrac{4}{3}\pi r_1^3\right)$,

which is about 46% of the original volume. There is approximately a 54% decrease in the volume of the tumor.

51. $R = \dfrac{R_m[S]}{K_m + [S]}$

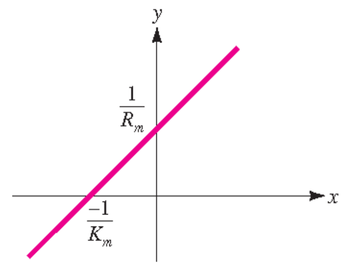
$y = \dfrac{1}{R} = \dfrac{K_m + [S]}{R_m[S]}$

Dividing numerator and denominator by [*S*] gives

$$y = \frac{K_m\left(\frac{1}{[S]}\right) + 1}{R_m} = \frac{K_m\left(\frac{1}{[S]}\right)}{R_m} + \frac{1}{R_m}$$
$$= \frac{K_m}{R_m}\left(\frac{1}{[S]}\right) + \frac{1}{R_m}$$

Letting $x = \dfrac{1}{[S]}$,

$y = \dfrac{K_m}{R_m}x + \dfrac{1}{R_m}$



52. Let r and h denote the radius and height of the can, respectively. The volume of the soda can is $V = \pi r^2 h = 6.89\pi$ and so $h = \dfrac{6.89}{r^2}$.

The surface area consists of the top and bottom circles, each of area πr^2, and the curved side. The curved side can be flattened out to a rectangle having width the height of the can *h*, and length the circumference of the circular top which is $2\pi r$. The total surface area of the can is then

$S = 2\pi r^2 + 2\pi rh = 2\pi r^2 + 2\pi r\dfrac{6.89}{r^2}$

$= 2\pi r^2 + \dfrac{13.78\pi}{r}$

53. Let r be the radius and h the height of the cylinder. The surface area of the closed cylinder is $S = 120\pi = 2\pi r^2 + 2\pi rh$ or $h = \dfrac{60 - r^2}{r}$. So

$V(r) = \pi r^2 h = \pi r(60 - r^2)$.

54. **(a)** Since $S = 2\pi rh + 2\pi r^2$, solving for h yields $h = \frac{S - 2\pi r^2}{2\pi r}$. Substituting into $V = \pi r^2 h$ yields

$$V = \pi r^2 \left(\frac{S - 2\pi r^2}{2\pi r}\right) = \frac{r}{2}(S - 2\pi r^2).$$

(b) Using $V = \pi r^2 h$, we can express h in terms of V and r, $h = \frac{V}{\pi r^2}$. Thus

$$S = 2\pi rh + 2\pi r^2$$
$$= 2\pi r\left(\frac{V}{\pi r^2}\right) + 2\pi r^2$$
$$= \frac{2V}{r} + 2\pi r^2$$

55. Let r be the radius and h the height of the cylinder. Since the volume is

$V = \pi r^2 h = 4\pi$, or $h = \frac{4}{r^2}$.

The cost of the top or bottom is $C_t = C_b = 2(0.02)\pi r^2$, while the cost of the side is $2\pi rh(0.02) = \frac{0.16\pi}{r}$.

The total cost is $C(x) = 0.08\pi r^2 + \frac{0.16\pi}{r}$

56. The surface area of the topless can is $S = \pi r^2 + 2\pi rh = 27\pi$. Solving for h yields

$$r^2 + 2rh = 27$$
$$h = \frac{27 - r^2}{2r}$$

The volume is

$$V = \pi r^2 \left(\frac{27\pi - r^2}{2r}\right) = \frac{r}{2}(27 - r^2).$$

57. Let x denote the width of the printed portion and y the length of the printed portion. Then $x + 4$ is the width of the poster and $y + 8$ is its length.
The area A of the poster is $A = (x + 4)(y + 8)$ which is a function of two variables.

$A = 25$ leads to $xy = 25$ or $y = \frac{25}{x}$.

So

$$A(x) = (x + 4)\left(\frac{25}{x} + 8\right) = 8x + 57 + \frac{100}{x}.$$

58. Let x denote the length of a side of the square base and y the height of the box.. The cost is given by C = (cost per m^2 of base top)(area of base and top) + (cost per m^2 of sides)(area of sides).

Thus $C = 2(2x^2) + 1(4xy) = 4x^2 + 4xy$.

Since the volume is 250, $x^2 y = 250$ or $y = \frac{250}{x^2}$. It follows that

$$C(x) = 4x^2 + \frac{1000}{x}.$$

59. Let x be the side of the square base and y the height of the open box. The area of the base is x^2 square meters and that of each side is xy square meters. The total cost is

$4x^2 + 3(4xy) = 48$.

Solving for y in terms of x,

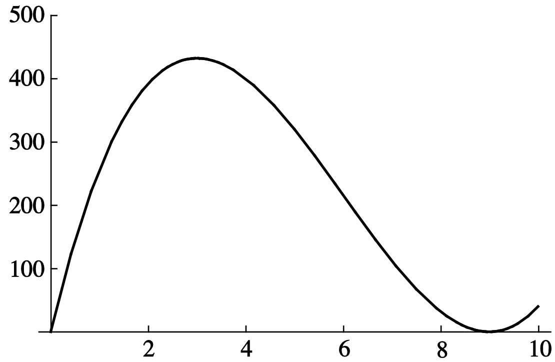
$$12xy = 48 - 4x^2$$
$$3xy = 12 - x^2$$
$$y = \frac{12 - x^2}{3x}$$

The volume of the box is

$$V = x^2 y$$
$$= \frac{x(12 - x^2)}{3}$$
$$= 4x - \frac{x^3}{3} \text{ cubic meters.}$$

60. Let x denote the length of the side of one of the removed squares and $V(x)$ the volume of the resulting box. Then

$$V(x) = \text{(area of base)(height)}$$
$$= (18 - 2x)(18 - 2x)x = 4x(9 - x)^2$$



From the graph, the value of x producing a box with greatest volume is estimated to be 3 in.

61. Let t be the number of hours the second plane has been flying. Since distance = (rate)(time), the equation for its distance is $d = 650t$.
The first plane has been flying for $t+\frac{1}{2}$ hours, so the equation for its distance is $d = 550\left(t+\frac{1}{2}\right)$.
The planes will meet when

$$650t = 550\left(t+\frac{1}{2}\right)$$
$$650t = 550t + 275$$
$$100t = 275$$
$$t = 2.75$$

Since three-quarters of an hour is 45 minutes, the second plane passes the first plane after it has been flying 2 hours and 45 minutes.

62. Let x denote the time in hours the spy has been traveling. Then $x-\frac{2}{3}$ is the time the smugglers have been traveling (since 40 minutes is $\frac{2}{3}$ of an hour.) The distance the spy travels is the $72x$ kilometers while the corresponding distance traveled by the smugglers is $168\left(x-\frac{2}{3}\right)$. The smugglers will overtake the spy when $72x = 168\left(x-\frac{2}{3}\right) = 168x - 112$. Solving for x yields $x = \frac{112}{96} = \frac{7}{6}$ hours. This corresponds to a distance of $72\left(\frac{7}{6}\right) = 84$ km which is beyond the 83.8 km to the border so the spy escapes pursuit.

1.5 Limits

1. $\lim_{x\to a} f(x) = b,$ even though $f(a)$ is not defined.

2. Yes the limit exists, because as $x \to a^+$ or $x \to a^-$, $y \to b,$ that is, $\lim_{x\to a} f(x) = b.$

3. $\lim_{x\to a} f(x) = b$ even though $f(a) = c$.

4. $\lim_{x\to a^-} f(x) = b$ but $\lim_{x\to a^+} f(x) = c$ and $b \neq c.$ The limit fails to exist.

5. $\lim_{x\to a} f(x)$ does not exist since as x approaches a from the left, the function becomes unbounded.

6. Yes the limit exists because $\lim_{x\to a^-} f(x) = \lim_{x\to a+} f(x) = b.$

7. $\lim_{x\to 2}(3x^2 - 5x + 2)$

$$= 3\lim_{x\to 2} x^2 - 5\lim_{x\to 2} x + \lim_{x\to 2} 2$$
$$= 3(2)^2 - 5(2) + 2$$
$$= 4$$

8. $\lim_{x\to -1}(x^3-2x^2+x-3)$
$= \lim_{x\to -1} x^3 - 2\lim_{x\to -1} x^2 + \lim_{x\to -1} x - 3$
$= -1-2\times 1+(-1)-3$
$= -7$

9. $\lim_{x\to 0}(x^5-6x^4+7)$
$= \lim_{x\to 0} x^5 - 6\lim_{x\to 0} x^4 + \lim_{x\to 0} 7$
$= 7$

10. $\lim_{x\to -1/2}(1-5x^3) = 1-5\lim_{x\to -1/2} x^3$
$= 1-5\left(\frac{-1}{2}\right)^3$
$= \frac{13}{8}$

11. $\lim_{x\to 3}(x-1)^2(x+1) = \lim_{x\to 3}(x-1)^2 \lim_{x\to 3}(x+1)$
$= (3-1)^2(3+1)$
$= 16$

12. $\lim_{x\to -1}(x^2+1)(1-2x)^2$
$= \left[\lim_{x\to -1}(x^2+1)\right]\left[\lim_{x\to -1}(1-2x)^2\right]$
$= \left[\lim_{x\to -1} x^2+1\right]\left[\lim_{x\to -1}(1-2x)\right]^2$
$= \left[(-1)^2+1\right]\left[1-2\lim_{x\to -1} x\right]^2$
$= 2[1-2(-1)]^2 = 2\times 3^2 = 18$

13. $\lim_{x\to 1/3}\frac{x+1}{x+2} = \frac{\lim_{x\to 1/3} x+1}{\lim_{x\to 1/3} x+2} = \frac{\frac{4}{3}}{\frac{7}{3}} = \frac{4}{7}$

14. $\lim_{x\to 1}\frac{2x+3}{x+1} = \frac{\lim_{x\to 1}(2x+3)}{\lim_{x\to 1}(x+1)}$
$= \frac{2\lim_{x\to 1} x+3}{\lim_{x\to 1} x+1}$
$= \frac{2+3}{1+1} = \frac{5}{2}$

15. $\lim_{x\to 5}\frac{x+3}{5-x}$ does not exist since the limit of the denominator is zero while the limit of the numerator is not zero.

16. $\lim_{x\to 3}\frac{2x+3}{x-3}$ is not defined because the denominator $\to 0$ (while the numerator does not.)

17. $\lim_{x\to 1}\frac{x^2-1}{x-1} = \lim_{x\to 1}\frac{(x+1)(x-1)}{x-1}$
$= \lim_{x\to 1}(x+1)$
$= 2$

18. $\lim_{x\to 3}\frac{9-x^2}{x-3} = -\lim_{x\to 3}\frac{(x+3)(x-3)}{x-3}$
$= -\lim_{x\to 3}(x+3)$
$= -6$

Note that $x \ne 3$ not matter how "close" x is to 3.

19. $\lim_{x\to 5}\frac{x^2-3x-10}{x-5} = \lim_{x\to 5}\frac{(x-5)(x+2)}{x-5}$
$= \lim_{x\to 5}(x+2)$
$= 7$

20. $\lim_{x\to 2}\frac{x^2+x-6}{x-2} = \lim_{x\to 2}\frac{(x+3)(x-2)}{x-2}$
$= \lim_{x\to 2}(x+3)$
$= 5$

21. $\lim_{x\to 4}\frac{(x+1)(x-4)}{(x-1)(x-4)} = \frac{\lim_{x\to 4}(x+1)}{\lim_{x\to 4}(x-1)} = \frac{5}{3}$

22. $\lim_{x\to 0} \frac{x(x^2-1)}{x^2} = \lim_{x\to 0} \frac{x^2-1}{x}$ is not defined because the denominator goes to 0 but the numerator does not.

23. $\lim_{x\to -2} \frac{x^2-x-6}{x^2+3x+2} = \lim_{x\to -2} \frac{(x-3)(x+2)}{(x+1)(x+2)}$

$$= \frac{\lim_{x\to -2}(x-3)}{\lim_{x\to -2}(x+1)}$$

$$= \frac{-5}{-1}$$

$$= 5$$

24. $\lim_{x\to 1} \frac{x^2+4x-5}{x^2-1} = \lim_{x\to 1} \frac{(x+5)(x-1)}{(x+1)(x-1)}$

$$= \lim_{x\to 1} \frac{x+5}{x+1}$$

$$= 3$$

25. $\lim_{x\to 4} \frac{\sqrt{x}-2}{x-4} = \lim_{x\to 4} \frac{\sqrt{x}-2}{x-4} \frac{\sqrt{x}+2}{\sqrt{x}+2}$

$$= \lim_{x\to 4} \frac{x-4}{(x-4)\left(\sqrt{x}+2\right)}$$

$$= \frac{1}{4}$$

26. $\lim_{x\to 9} \frac{\sqrt{x}-3}{x-9} = \lim_{x\to 9} \frac{\sqrt{x}-3}{(\sqrt{x}+3)(\sqrt{x}-3)}$

$$= \lim_{x\to 9} \frac{1}{\sqrt{x}+3}$$

$$= \frac{1}{\lim_{x\to 9} \sqrt{x}+3}$$

$$= \frac{1}{\sqrt{\lim_{x\to 9} x}+3}$$

$$= \frac{1}{6}$$

27. $f(x) = x^3 - 4x^2 - 4$

$$\lim_{x\to +\infty} f(x) = \lim_{x\to +\infty} x^3 = +\infty$$

$$\lim_{x\to -\infty} f(x) = \lim_{x\to -\infty} x^3 = -\infty$$

28. $f(x) = 1 - x + 2x^2 - 3x^3$

$$\lim_{x\to \infty} f(x) = \lim_{x\to \infty} (-3x^3) = -\infty$$

$$\lim_{x\to -\infty} f(x) = \lim_{x\to -\infty} (-3x^3) = \infty$$

29. $f(x) = (1-2x)(x+5) = -2x^2 - 9x + 5$

$$\lim_{x\to +\infty} f(x) = \lim_{x\to +\infty} -2x^2 = -\infty$$

$$\lim_{x\to -\infty} f(x) = \lim_{x\to -\infty} -2x^2 = -\infty$$

30. $f(x) = (1+x^2)^3$

$$\lim_{x\to \infty} f(x) = \lim_{x\to \infty} x^6 = \infty$$

$$\lim_{x\to -\infty} f(x) = \lim_{x\to -\infty} x^6 = \infty$$

31. $f(x) = \frac{x^2-2x+3}{2x^2+5x+1}$

$$\lim_{x\to +\infty} f(x) = \lim_{x\to +\infty} \frac{1-\frac{2}{x}+\frac{3}{x^2}}{2+\frac{5}{x}+\frac{1}{x^2}} = \frac{1}{2}$$

$$\lim_{x\to -\infty} f(x) = \lim_{x\to -\infty} \frac{1-\frac{2}{x}+\frac{3}{x^2}}{2+\frac{5}{x}+\frac{1}{x^2}} = \frac{1}{2}$$

32. $f(x) = \frac{1-3x^3}{2x^3-6x+2}$

$$\lim_{x\to \infty} f(x) = \lim_{x\to \infty} \frac{\frac{1}{x^3}-3}{2-\frac{6}{x^2}+\frac{2}{x^3}} = -\frac{3}{2}$$

$$\lim_{x\to -\infty} f(x) = \lim_{x\to -\infty} \frac{\frac{1}{x^3}-3}{2-\frac{6}{x^2}+\frac{2}{x^3}} = -\frac{3}{2}$$

33. $f(x)=\dfrac{2x+1}{3x^2+2x-7}$

$$\lim_{x\to+\infty} f(x)=\lim_{x\to+\infty}\frac{\frac{2}{x}+\frac{1}{x^2}}{3+\frac{2}{x}-\frac{7}{x^2}}=0$$

$$\lim_{x\to-\infty} f(x)=\lim_{x\to-\infty}\frac{\frac{2}{x}+\frac{1}{x^2}}{3+\frac{2}{x}-\frac{7}{x^2}}=0$$

34. $f(x)=\dfrac{x^2+x-5}{1-2x-x^3}$

$$\lim_{x\to\infty} f(x)=\lim_{x\to\infty}\frac{\frac{1}{x}+\frac{1}{x^2}-\frac{5}{x^3}}{\frac{1}{x^3}-\frac{2}{x^2}-1}=0$$

$$\lim_{x\to-\infty} f(x)=\lim_{x\to-\infty}\frac{\frac{1}{x}+\frac{1}{x^2}-\frac{5}{x^3}}{\frac{1}{x^3}-\frac{2}{x^2}-1}=0$$

35. $f(x)=\dfrac{3x^2-6x+2}{2x-9}$

$$\lim_{x\to+\infty} f(x)=\lim_{x\to+\infty}\frac{3x^2-6x+2}{2x-9}$$

$$=\lim_{x\to+\infty}\frac{3x-6+\frac{2}{2x}}{2-\frac{9}{x}}$$

$$\lim_{x\to+\infty} 3x-6+\frac{2}{x}=+\infty \text{ and } \lim_{x\to+\infty} 2-\frac{9}{x}=2$$

So, $\displaystyle\lim_{x\to+\infty}\frac{3x-6+\frac{2}{x}}{2-\frac{9}{x}}=+\infty$

$$\lim_{x\to-\infty} f(x)=\lim_{x\to-\infty}\frac{3x-6+\frac{2}{x}}{2-\frac{9}{x}}$$

$$\lim_{x\to-\infty} 3x-6+\frac{2}{x}=-\infty \text{ and } \lim_{x\to-\infty} 2-\frac{9}{x}=2$$

So, $\displaystyle\lim_{x\to-\infty}\frac{3x-6+\frac{2}{x}}{2-\frac{9}{x}}=-\infty$

36. $f(x)=\dfrac{1-2x^3}{x+1}$

$$\lim_{x\to\infty} f(x)=\lim_{x\to\infty}\frac{\frac{1}{x}-2x^2}{1+\frac{1}{x}}=-\infty$$

$$\lim_{x\to-\infty} f(x)=\lim_{x\to-\infty}\frac{\frac{1}{x}-2x^2}{1+\frac{1}{x}}=-\infty$$

37. $\displaystyle\lim_{x\to+\infty} f(x)=1$ and $\displaystyle\lim_{x\to-\infty} f(x)=-1$

38. As one moves to the right along the x axis, the graph approaches the horizontal line through 2. Similarly as one moves to the left through negative values, the graph approaches the horizontal line through –3. Thus $\displaystyle\lim_{x\to+\infty} f(x)=2$ and $\displaystyle\lim_{x\to-\infty} f(x)=-3$.

39. The corresponding table values are:

$f(1.9)=(1.9)^2-1.9=1.71$

$f(1.99)=(1.99)^2-1.99=1.9701$

$f(1.999)=(1.999)^2-1.999=1.997001$

$f(2.001)=(2.001)^2-2.001=2.003001$

$f(2.01)=(2.01)^2-2.01=2.0301$

$f(2.1)=(2.1)^2-2.1=2.31$

$\displaystyle\lim_{x\to 2} f(x)=2$

40. $f(x)=x-\dfrac{1}{x}$

x	–0.09	–0.009	0	0.0009	0.009	0.09
$f(x)$	11.02111	111.10211	╳	–1,111.11021	–111.10211	–11.02111

For $x < 0$, the table suggests that the values of $f(x)$ increase without bound, while for $x > 0$, the values decrease without bound, so $\lim_{x\to 0} f(x)$ does not exist.

41. The corresponding table values are

$$f(0.9) = \frac{(0.9)^3+1}{0.9-1} = -17.29$$

$$f(0.99) = \frac{(0.99)^3+1}{0.99-1} = -197.0299$$

$$f(0.999) = \frac{(0.999)^3+1}{0.999-1} = -1{,}997.002999$$

$$f(1.001) = \frac{(1.001)^3+1}{1.001-1} = 2{,}003.003001$$

$$f(1.01) = \frac{(1.01)^3+1}{1.01-1} = 203.0301$$

$$f(1.1) = \frac{(1.1)^3+1}{1.1-1} = 23.31$$

$\lim_{x\to 1} f(x)$ does not exist.

42. $f(x) = \dfrac{x^3+1}{x+1}$

x	–1.1	–1.01	–1.001	–1	–0.999	–0.99	–0.9
$f(x)$	3.31	3.0301	3.003001	✕	2.997001	2.9701	2.71

The table indicates that $\lim_{x\to -1} f(x) = 3$.

43.
$$\begin{aligned}&\lim_{x\to c}[2f(x)-3g(x)]\\&= \lim_{x\to c} 2f(x) - \lim_{x\to c} 3g(x)\\&= 2\lim_{x\to c} f(x) - 3\lim_{x\to c} g(x)\\&= 2(5)-3(-2)\\&= 16\end{aligned}$$

44.
$$\begin{aligned}\lim_{x\to c} f(x)g(x) &= \left(\lim_{x\to c} f(x)\right)\left(\lim_{x\to c} g(x)\right)\\&= 5(-2)\\&= -10\end{aligned}$$

45. $\lim_{x\to c}\sqrt{f(x)+g(x)}$
$= \lim_{x\to c}[f(x)+g(x)]^{1/2}$
$= \left[\lim_{x\to c}(f(x)+g(x))\right]^{1/2}$
$= \left[\lim_{x\to c} f(x)+\lim_{x\to c} g(x)\right]^{1/2}$
$= [5+(-2)]^{1/2}$
$= \sqrt{3}$

46. $\lim_{x\to c} f(x)[g(x)-3] = \left(\lim_{x\to c} f(x)\right)\left(\lim_{x\to c}[g(x)-3]\right)$
$= \left(\lim_{x\to c} f(x)\right)\left[\lim_{x\to c} g(x)-3\right]$
$= 5[-2-3]$
$= -25$

47. $\lim_{x\to c}\dfrac{f(x)}{g(x)} = \dfrac{\lim_{x\to c} f(x)}{\lim_{x\to c} g(x)} = \dfrac{5}{-2} = -\dfrac{5}{2}$

48. Check the denominator.
$\lim_{x\to c}[5g(x)+2f(x)] = \lim_{x\to c} 5g(x)+\lim_{x\to c} 2f(x)$
$= 5\left(\lim_{x\to c} g(x)\right)+2\left(\lim_{x\to c} f(x)\right)$
$= 5(-2)+2(5)$
$= 0$
Since the limit of the denominator is 0, the limit of the quotient does not exist. Note that $\lim_{x\to c}[2f(x)-g(x)] = 12 \neq 0$.

49. $\lim\limits_{x\to\infty} \dfrac{2f(x)+g(x)}{x+f(x)}$

$= \lim\limits_{x\to\infty} \dfrac{\frac{1}{x}\cdot 2f(x)+\frac{1}{x}\cdot g(x)}{1+\frac{1}{x}\cdot f(x)}$

$= \dfrac{\lim\limits_{x\to\infty}\left[\frac{1}{x}\cdot 2f(x)+\frac{1}{x}\cdot g(x)\right]}{\lim\limits_{x\to\infty}\left[1+\frac{1}{x}\cdot f(x)\right]}$

$= \dfrac{\lim\limits_{x\to\infty}\frac{1}{x}\cdot 2f(x)+\lim\limits_{x\to\infty}\frac{1}{x}\cdot g(x)}{\lim\limits_{x\to\infty}1+\lim\limits_{x\to\infty}\frac{1}{x}\cdot f(x)}$

$= \dfrac{\lim\limits_{x\to\infty}\frac{1}{x}\cdot 2\lim\limits_{x\to\infty} f(x)+\lim\limits_{x\to\infty}\frac{1}{x}\cdot \lim\limits_{x\to\infty} g(x)}{\lim\limits_{x\to\infty}1+\lim\limits_{x\to\infty}\frac{1}{x}\cdot \lim\limits_{x\to\infty} f(x)}$

Since $\lim\limits_{x\to\infty}\dfrac{1}{x}=0$ and $\lim\limits_{x\to\infty}1=1$,

$= \dfrac{0+0}{1+0}$

$= 0$

50. $\lim\limits_{x\to\infty}\sqrt{g(x)} = \sqrt{\lim\limits_{x\to\infty} g(x)} = \sqrt{4} = 2$

51. $p = 0.2t + 1{,}500$; $E(t)=\sqrt{9t^2+0.5t+179}$

(a) Since the units of p are thousands and the units of E are millions, the units of E/p will be thousands.

$P(t)=\dfrac{\sqrt{9t^2+0.5t+179}}{0.2t+1500}$ thousand dollars per person

(b) Dividing each term by t (note that each term under the square root will be divided by t^2 since $\left(\sqrt{t^2}=t\right)$,

$\lim\limits_{t\to\infty} P(t) = \lim\limits_{t\to\infty}\dfrac{\sqrt{9+\frac{0.5}{t}+\frac{179}{t^2}}}{0.2+\frac{1500}{t}}$

$= \dfrac{\sqrt{\lim\limits_{t\to\infty}\left(9+\frac{0.5}{t}+\frac{179}{t^2}\right)}}{\lim\limits_{t\to\infty}\left(0.2+\frac{1500}{t}\right)}$

$= \dfrac{\sqrt{9}}{0.2}$

$= 15$

52. $\lim\limits_{t\to\infty} P(t) = \lim\limits_{t\to\infty}\dfrac{6t^2+5t}{(t+1)^2}$

$= \lim\limits_{t\to\infty}\dfrac{\frac{6t^2}{t^2}+\frac{5t}{t^2}}{\frac{t^2}{t^2}+\frac{2t}{t^2}+\frac{1}{t^2}}$

$= \lim\limits_{t\to\infty}\dfrac{6+\frac{5}{t}}{1+\frac{2}{t}+\frac{1}{t^2}}$

$= \dfrac{\lim\limits_{t\to\infty}6+\lim\limits_{t\to\infty}\frac{5}{t}}{\lim\limits_{t\to\infty}1+\lim\limits_{t\to\infty}\frac{2}{t}+\lim\limits_{t\to\infty}\frac{1}{t^2}}$

$= \dfrac{6+0}{1+0+0}$

$= 6$

In the long run, production approaches 6,000 units.

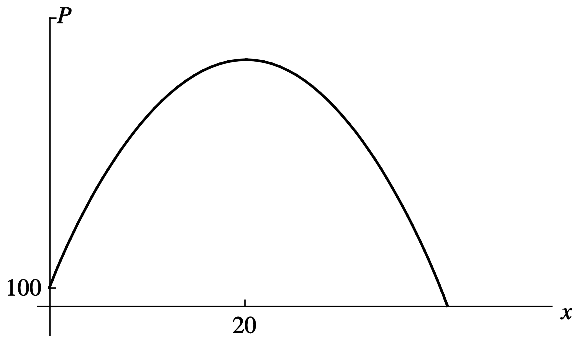
53. $\lim\limits_{x\to+\infty}\dfrac{7.5x+120{,}000}{x}$

$= \lim\limits_{x\to+\infty} 7.5+\dfrac{120{,}000}{x}$

$= 7.5$

As the number of units produced increases indefinitely, the average cost per unit decreases, approaching a minimum of \$7.50. The average cost cannot decrease further, as the expense of materials cannot be eliminated completely.

54. (a) $P(x)$
$= R(x) - C(x)$
$= (400 + 120x - x^2) - (2x^2 + 300)$
$= 100 + 120x - 3x^2$



(b) The maximum profit occurs when $x = 20$, so the event should be announced 20 days in advance. The maximum profit is $1,300,000.

(c) $Q(x) = \frac{400 + 120x - x^2}{2x^2 + 300}$

$Q(20) = \frac{24}{11} \approx 2.18$

$$\lim_{x \to 0} Q(x) = \lim_{x \to 0} \frac{400 + 120x - x^2}{2x^2 + 300} = \frac{\lim_{x \to 0}(400 + 120x - x^2)}{\lim_{x \to 0}(2x^2 + 300)} = \frac{400}{300} \approx 1.33$$

At the optimal announcement time, the revenue is more than double the advertising cost. As the announcement date gets closer to the event, the revenue gets closer to $\frac{4}{3}$ of the advertising cost.

55. $C(x) = \frac{8x^2 - 636x - 320}{x^2 - 68x - 960}$

Need to find $\lim_{x \to 80} C(x)$. Substituting x = 80 into C yields $\frac{0}{0}$, so need to try to factor and cancel.

$$\lim_{x \to 80} \frac{4(2x+1)(x-80)}{(x+12)(x-80)} = \lim_{x \to 80} \frac{4(2x+1)}{x+12} = \frac{4[2(80)+1]}{80+12} = 7$$

So, the operating cost will be $700.

56. (a) $A(t) = \text{rate} \times \text{number}$
$= 0.20\left(70 - \frac{150}{t+4}\right)$

(b) $\lim_{t \to \infty} A(t) = \lim_{t \to \infty} 0.20\left(70 - \frac{150}{t+4}\right)$
$= 0.20(70 - 0) = 14$

An employee should expect to earn $14/hour in the long run.

57.

x	1	0.1	0.01	0.001	0.0001
$1{,}000(1 + 0.05x)^{1/x}$	1,050.00	1,051.14	1,051.26	1,051.27	1,051.27

Limit appears to be $1,051.27.

58. (a) $P(0) = \frac{40(0)}{0^2 + 10} - \frac{50}{0+1} + 70 = 20$

The current population is 20,000.

(b) $P(2)=\frac{40(2)}{2^2+10}-\frac{50}{2+1}+70\approx 59.048$

$P(3)=\frac{40(3)}{3^2+10}-\frac{50}{3+1}+70\approx 63.816$

$P(3)-P(2)\approx 4.768$

The population increased by 4,768 during the third year.

(c) $\lim_{t\to\infty} P(t)$

$=\lim_{t\to\infty}\left(\frac{40t}{t^2+10}-\frac{50}{t+1}+70\right)$

$=\lim_{t\to\infty}\frac{40t}{t^2+10}-\lim_{t\to\infty}\frac{50}{t+1}+70$

$=\lim_{t\to\infty}\frac{\frac{40}{t}}{1+\frac{10}{t^2}}-\lim_{t\to\infty}\frac{\frac{50}{t}}{1+\frac{1}{t}}+70$

$=0-0+70$

$=70$

In the long run, the population approaches 70,000.

59. $C(t)=\frac{0.4}{t^{1.2}+1}+0.013$

(a) $C(0)=\frac{0.4}{0^{1.2}+1}+0.013$

$=0.413$ mg/ml

(b) Need to find

$C(5)-C(4)$

$=\left[\frac{0.4}{5^{1.2}+1}+0.013\right]-\left[\frac{0.4}{4^{1.2}+1}+0.013\right]$

$=\frac{0.4}{5^{1.2}+1}-\frac{0.4}{4^{1.2}+1}$

$\approx 0.0506-0.0637$

$=-0.0131$

So, the concentration decreases approximately 0.013 mg/ml during this hour.

(c) $\lim_{t\to+\infty} C(t)$

$=\lim_{t\to+\infty}\left[\frac{0.4}{t^{1.2}+1}+0.013\right]$

$=\lim_{t\to+\infty}\frac{0.4}{t^{1.2}+1}+\lim_{t\to+\infty}0.013$

$=0+0.013$

$=0.013$ mg/ml

60. $\lim_{n\to+\infty}\frac{5n+17}{n}=\lim_{n\to+\infty}\left(5+\frac{17}{n}\right)$

$=5+0$

$=5$

The limit tells us that as more trials are conducted, the rat's traversal time will approach a minimum time of 5 minutes.

61. $P(t)=\frac{30}{3+t},\ Q(t)=\frac{64}{4-t}$

(a) $P(0)=\frac{30}{3}=10$ thousand, or 10,000

$Q(0)=\frac{64}{4}=16$ thousand, or 16,000

(b) Since the function P accepts all $t\ge 0$, the function values decrease as t increases. Further,

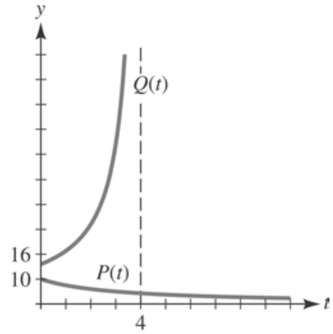
$\lim_{t\to+\infty}P(t)=\lim_{t\to+\infty}\frac{30}{3+t}=0.$

So, in the long run, P tends to zero. The Q function, however, only accepts values of t such that $0\le t\le 4$. The function values increase as t increases.

Further, $\lim_{t\to 4^-}\frac{64}{4-5}=+\infty$

(a t/Q table is an easy way to see this). So, Q increases without bound.

(c)



(d) Writing exercise—Answers will vary.

62. **(a)** The growth rate doubles from about 0.5 generation/hr to 1.0 generations/hr between about 10°C and 15°C. It also doubles from about 0.75 generation/hr to 1.5 generations/hr between about 12°C and 20°C.

(b) The growth rate is constant for $25 < T < 45$.

(c) The growth rate begins to decrease at about 45°C, then drops rapidly. It appears that $\lim_{T\to 50} R(T) = 0$.

(d) Writing exercise; answers will vary.

63. **(a)** $\lim_{S\to\infty} \frac{aS}{S+c} = \lim_{S\to\infty} \frac{a}{1+\frac{c}{S}} = a$

As bite size increases indefinitely, intake approaches a limit of a. This signifies that the animal has a limit of how much it can consume, no matter how large its bites become.

(b) Writing exercise—answers will vary.

64. Answers will vary. The answer corresponding to each problem should include a sequence of numbers approaching the limiting value of x from the right and left, along with the corresponding values of $f(x)$.

65. $$\lim_{x\to+\infty} = \frac{a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0}{b_m x^m + b_{m-1}x^{m-1} + \cdots + b_1 x + b_0}$$

(a) When $n < m$,

$$= \lim_{x\to+\infty} \frac{a_n + \frac{a_{n-1}}{x} + \cdots + \frac{a_1}{x^{n-1}} + \frac{a_0}{x^n}}{b_m \frac{x^m}{x^n} + b_{m-1}\frac{x^{m-1}}{x^n} + \cdots + b_1 \frac{x}{x^n} + b_0 \frac{1}{x^n}}$$

Since $\lim_{x\to+\infty} \frac{x^m}{x^n} = +\infty$, $\lim_{x\to+\infty} f(x) = 0$

(b) When $n = m$, $\frac{x^m}{x^n} = 1$ and

$$\lim_{x\to+\infty} f(x) = \frac{a_n}{b_m}$$

(c) When $n > m$,

$$= \lim_{x\to+\infty} \frac{a_n \frac{x^n}{x^m} + a_{n-1}\frac{x^{n-1}}{x^m} + \cdots + a_1 \frac{x}{x^m} + a_0 \frac{1}{x^m}}{b_m + \frac{b_{m-1}}{x} + \cdots + \frac{b_1}{x^{m-1}} + \frac{b_0}{x^m}}$$

Now,

$$\lim_{x\to+\infty} a_n \frac{x^n}{x^m} + a_{n-1}\frac{x^{n-1}}{x^m} + \cdots + a_1 \frac{x}{x^m} + a_0 \frac{1}{x^m}$$
$$= \pm\infty,$$

depending on the sign of a_n. Also

$$\lim_{x\to+\infty} b_m + \frac{b_{m-1}}{x} + \cdots + \frac{b_1}{x^{m-1}} + \frac{b_0}{x^m}$$
$$= b_m$$

So, $$\lim_{x\to+\infty} \frac{a_n \frac{x^n}{x^m} + a_{n-1}\frac{x^{n-1}}{x^m} + \cdots}{b_m + \frac{b_{m-1}}{x} + \cdots} = \pm\infty,$$

depending on the signs of a_n and b_m. When a_n and b_m have the same sign, the limit is $+\infty$; when they have opposite signs, the limit is $-\infty$.

66. $\lim_{x\to 0} f(x)$ does not exist because $f(x)$ oscillates infinitely many times between −1 and 1, regardless of how close *x* gets to 0.

67. As the weight approaches 18 lb, displacement approaches a limit of 1.8 inches.

1.6 One-Sided Limits and Continuity

1. $\lim_{x\to 2^-} f(x) = -2;\ \lim_{x\to 2^+} f(x) = 1$
Since $-2 \neq 1$, $\lim_{x\to 2} f(x)$ does not exist.

2. As *x* approaches 2 from the left, the curve approaches the point (2, 4) so $\lim_{x\to 2^-} f(x) = 4$. From the right the curve approaches the point (2, 2) so $\lim_{x\to 2^+} f(x) = 2$. Since the one-sided limits at $x = 2$ are not equal, $\lim_{x\to 2} f(x)$ does not exist.

3. $\lim_{x\to 2^-} f(x) = 2;\ \lim_{x\to 2^+} f(x) = 2$
Since limits are the same, $\lim_{x\to 2} f(x) = 2$.

4. As *x* approaches 2 from the left, the curve approaches the point (2, 2) so $\lim_{x\to 2^-} f(x) = 2$. From the right the curve assumes larger and larger values as it nears 2 so $\lim_{x\to 2^+} f(x) = +\infty$. Since the one-sided limits at $x = 2$ are not equal, $\lim_{x\to 2} f(x)$ does not exist.

5.
$$\begin{aligned}\lim_{x\to 4^+} (3x^2 - 9) &= \lim_{x\to 4^+} 3x^2 - \lim_{x\to 4^+} 9\\ &= 3(4)^2 - 9\\ &= 39\end{aligned}$$

6. $\lim_{x\to 1^-} x(2-x) = 1(2-1) = 1$

7. $\lim_{x\to 3^+} \sqrt{3x-9} = \sqrt{3(3)-9} = 0$

8. $\lim_{x\to 2^-} \sqrt{4-2x} = \sqrt{\lim_{x\to 2^-} (4-2x)} = \sqrt{0} = 0$

9.
$$\begin{aligned}\lim_{x\to 2^-} \frac{x+3}{x+2} &= \frac{\lim_{x\to 2^-} (x+3)}{\lim_{x\to 2^-} (x+2)}\\ &= \frac{2+3}{2+2}\\ &= \frac{2+3}{2+2}\\ &= \frac{5}{4}\end{aligned}$$

10. The rational function $\frac{x^2+4}{x-2}$ is not continuous at $x = 2$ since the denominator is 0 there. As *x* approaches 2 from the left, the numerator approaches $2^2 + 4 = 8$ while the denominator approaches 0 through negative values. Thus
$$\lim_{x\to 2^-} \frac{x^2+4}{x-2} = -\infty.$$

11. $\lim_{x\to 0^+} \left(x - \sqrt{x}\right) = 0 - 0 = 0$

12. $\lim\limits_{x\to 1^-} \dfrac{x-\sqrt{x}}{x-1}$
$= \lim\limits_{x\to 1^-} \dfrac{x-\sqrt{x}}{x-1} \cdot \dfrac{x+\sqrt{x}}{x+\sqrt{x}}$
$= \lim\limits_{x\to 1^-} \dfrac{x^2-\sqrt{x}^2}{(x-1)(x+\sqrt{x})}$
$= \lim\limits_{x\to 1^-} \dfrac{x^2-x}{(x-1)(x+\sqrt{x})}$
$= \lim\limits_{x\to 1^-} \dfrac{x(x-1)}{(x-1)(x+\sqrt{x})}$
$= \lim\limits_{x\to 1^-} \dfrac{x}{x+\sqrt{x}}$
$= \dfrac{1}{1+1}$
$= \dfrac{1}{2}$

13. $\lim\limits_{x\to 3^+} \dfrac{\sqrt{x+1}-2}{x-3}$
$= \lim\limits_{x\to 3^+} \dfrac{\sqrt{x+1}-2}{x-3} \cdot \dfrac{\sqrt{x+1}+2}{\sqrt{x+1}+2}$
$= \lim\limits_{x\to 3^+} \dfrac{x+1-4}{(x-3)\left(\sqrt{x+1}+2\right)}$
$= \dfrac{1}{4}$

14. $\lim\limits_{x\to 5^-} \dfrac{\sqrt{2x-1}-3}{x-5}$
$= \lim\limits_{x\to 5^-} \dfrac{\sqrt{2x-1}-3}{x-5} \dfrac{\sqrt{2x-1}+3}{\sqrt{2x-1}+3}$
$= \lim\limits_{x\to 5^-} \dfrac{2}{\sqrt{2x-1}+3}$
$= \dfrac{1}{3}$

15. $\lim\limits_{x\to 3^-} f(x) = \lim\limits_{x\to 3^-} (2x^2-x)$
$= 2(3)^2-3$
$= 15$
$\lim\limits_{x\to 3^+} f(x) = \lim\limits_{x\to 3^+} (3-x) = 3-3 = 0$

16. $f(x) = \begin{cases} \dfrac{1}{x-1} & \text{if } x<-1 \\ x^2+2x & \text{if } -1\le x \end{cases}$
$\lim\limits_{x\to -1^-} f(x) = \lim\limits_{x\to -1^-} \dfrac{1}{x-1} = -\dfrac{1}{2}$
$\lim\limits_{x\to -1^+} f(x) = \lim\limits_{x\to -1^+} (x^2+2x) = -1$

17. If $f(x)=5x^2-6x+1,$ then *f*(2) = 9 and $\lim\limits_{x\to 2} f(x)=9,$ so *f* is continuous at *x* = 2.

18. $\lim\limits_{x\to 0} f(x) = 0^3-2\times 0^2+0-5 = f(0)$
thus $f(x)$ is continuous at $x=0.$ Note that all polynomials are continuous.

19. If $f(x)=\dfrac{x+2}{x+1},$ then $f(1)=\dfrac{3}{2}$ and
$\lim\limits_{x\to 1} f(x) = \lim\limits_{x\to 1} \dfrac{x+2}{x+1} = \dfrac{\lim\limits_{x\to 1}(x+2)}{\lim\limits_{x\to 1}(x+1)} = \dfrac{3}{2}.$
So, *f* is continuous at *x* = 1.

20. $\lim\limits_{x\to 2} f(x) = \dfrac{2\lim\limits_{x\to 2} x-4}{3\lim\limits_{x\to 2} x-2}$
$= \dfrac{0}{6-2}$
$= 0$
$= f(2)$
thus $f(x)$ is continuous at $x=2.$

21. If $f(x)=\dfrac{x+1}{x-1},$ *f*(1) is undefined since the denominator is zero, and so *f* is not continuous at *x* = 1.

22. $\lim\limits_{x\to 2} f(x) = \dfrac{2\lim\limits_{x\to 2} x+1}{3\lim\limits_{x\to 2} x-6} = \dfrac{5}{0}$ which is not defined, thus $f(x)$ is not continuous at $x=2.$

23. If $f(x)=\frac{\sqrt{x}-2}{x-4}$, $f(4)$ is undefined since the denominator is zero, and so f is not continuous at $x = 4$.

24. $\lim_{x\to 2} f(x)=\frac{\lim_{x\to 2}\sqrt{x}-2}{\lim_{x\to 2} x-4}$
$=\frac{\sqrt{2}-2}{2-4}$
$=f(2)$
thus $f(x)$ is continuous at $x=2$.

25. If $f(x)=\begin{cases} x+1 & \text{if } x\le 2 \\ 2 & \text{if } x>2 \end{cases}$, then $f(2) = 3$ and $\lim_{x\to 2} f(x)$ must be determined.
As x approaches 2 from the left,
$\lim_{x\to 2^-} f(x)=\lim_{x\to 2^-}(x+1)=3$ and as x approaches 2 from the right,
$\lim_{x\to 2^+} f(x)=\lim_{x\to 2^+} 2=2.$
So the limit does not exist (since different limits are obtained from the left and the right), and f is not continuous at $x = 2$.

26. $f(x)=\begin{cases} x+1 & \text{if } x<0 \\ x-1 & \text{if } 0\le x \end{cases}$
$f(0)=-1$ and $\lim_{x\to 0} f(x)$ must be determined. As x approaches 0 from the right, $\lim_{x\to 0^+} f(x)=\lim_{x\to 0^+}(x-1)=-1$
and as x approaches 0 from the left,
$\lim_{x\to 0^-} f(x)=\lim_{x\to 0^-}(x+1)=1$
Hence the limit does not exist (since different limits are obtained form the left and right), and so f is not continuous at $x=0$.

27. If $f(x)=\begin{cases} x^2+1 & \text{if } x\le 3 \\ 2x+4 & \text{if } x>3 \end{cases}$, then
$f(3)=(3)^2+1=10$ and $\lim_{x\to 3} f(x)$ must be determined. As x approaches 3 from the left,
$\lim_{x\to 3^-} f(x)=\lim_{x\to 3^-}(x^2+1)=(3)^2+1=10$
and as x approaches 3 from the right,
$\lim_{x\to 3^+} f(x)=\lim_{x\to 3^+}(2x+4)=2(3)+4=10.$
So $\lim_{x\to 3} f(x)=10.$ Since $f(x)=\lim_{x\to 3} f(x),$ f is continuous at $x = 3$.

28. $f(x)=\begin{cases} \frac{x^2-1}{x+1} & \text{if } x<-1 \\ x^2-3 & \text{if } -1\le x \end{cases}$
then $f(-1)=-2$ and $\lim_{x\to -1} f(x)$ must be determined. As x approaches –1 from the right, $\lim_{x\to -1^+} f(x)=\lim_{x\to -1^+}(x^2-3)=-2$
and as x approaches –1 from the left,
$$\lim_{x\to -1^-} f(x)=\lim_{x\to -1^-}\frac{x^2-1}{x+1}$$
$$=\lim_{x\to -1^-}\frac{(x-1)(x+1)}{x+1}$$
$$=\lim_{x\to -1^-}(x-1)$$
$$=-2$$
Hence the limit exists and is equal to $f(-1)$, and so f is continuous at $x=-1$.

29. $f(a)=3a^2-6a+9$ so f is defined for all real numbers. $\lim_{x\to a} f(a)=3(a)^2-6a+9,$ so the limit of f exists for all real numbers. Since $f(a)=\lim_{x\to a} f(a),$ there are no values for which f is not continuous.

30. $f(x)=x^5-x^3$ is continuous for all values of x. Polynomials are continuous everywhere.

31. $f(x)=\frac{x+1}{x-2}$ is not defined at $x = 2$, so f is *not* continuous at $x = 2$.

32. $f(x) = \dfrac{3x-1}{2x-6}$ is not continuous at $x = 3$, where the denominator is zero.

33. $f(x) = \dfrac{3x+3}{x+1}$ is not defined at $x = -1$, so f is *not* continuous at $x = -1$.

34. $f(x) = \dfrac{x^2-1}{x+1}$ is not continuous at $x = -1$, where the denominator is zero.

35. $f(x) = \dfrac{3x-2}{(x+3)(x-6)}$ is not defined at $x = -3$ and $x = 6$, so f is *not* continuous at $x = -3$ and $x = 6$.

36. $f(x) = \dfrac{x}{(x+5)(x-1)}$ is not continuous at $x = -5$ or $x = 1$, where the denominator is zero.

37. $f(x) = \dfrac{x}{x^2-x}$ is not defined at $x = 0$ and $x = 1$, so f is *not* continuous at $x = 0$ and $x = 1$.

38. $f(x) = \dfrac{x^2-2x+1}{x^2-x-2} = \dfrac{(x-1)^2}{(x-2)(x+1)}$ is not continuous at $x = -1$ or $x = 2$, where the denominator is zero.

39. f is defined for all real numbers. Further,

$$\begin{aligned}\lim_{x\to 1^-} f(x) &= 2+3 \\ &= 5 \\ &= \lim_{x\to 1^+} f(x) \\ &= 6-1 \\ &= f(1),\end{aligned}$$

so there are no values for which f is not continuous.

40. $f(x) = \begin{cases} x^2 & \text{if } x \le 2 \\ 9 & \text{if } 2 < x \end{cases}$

is possibly not continuous at $x = 2$. As x approaches 2 from the right,

$\lim\limits_{x\to 2^+} f(x) = \lim\limits_{x\to 2^+} 9 = 9$

and as x approaches 2 form the left,

$\lim\limits_{x\to 2^-} f(x) = \lim\limits_{x\to 2^-} x^2 = 4.$

Hence the limit does not exist and so f is continuous at $x = 2$.

41. f is defined for all real numbers. However,

$\lim\limits_{x\to 0^-} f(x) = \lim\limits_{x\to 0^-} 3x-2 = 3(0)-2 = 2$

$\lim\limits_{x\to 0^+} f(x) = \lim\limits_{x\to 0^+} x^2+x = 0+0 = 0$

So $\lim\limits_{x\to 0} f(x)$ does not exist and therefore f is *not* continuous at $x = 0$.

42. $f(x) = \begin{cases} 2-3x & \text{if } x \le -1 \\ x^2-x+3 & \text{if } x > -1 \end{cases}$

is possibly not continuous only at $x = -1$. As x approaches –1 from the left,

$\lim\limits_{x\to -1^-} f(x) = \lim\limits_{x\to -1^-} (2-3x) = 5$

and as x approaches –1 from the right,

$\lim\limits_{x\to -1^+} f(x) = \lim\limits_{x\to -1^+} (x^2-x+3) = 5$.

Hence the limit as x approaches –1 exists and is equal to $f(-1)$. Thus there are no values of x at which $f(x)$ is not continuous.

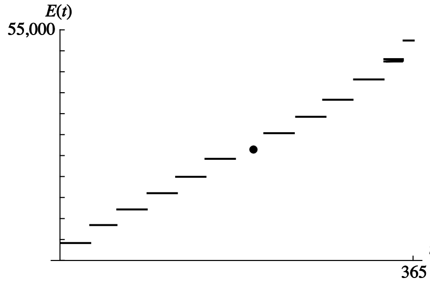
43. $C(x) = \dfrac{8x^2-636x-320}{x^2-68x-960}$

(a) $C(0) = \dfrac{-320}{-960} = \dfrac{1}{3} \approx 0.333$

$$\begin{aligned}C(100) &= \frac{8(100)^2-636(100)-320}{(100)^2-68(100)-960} \\ &\approx 7.179\end{aligned}$$

(b) Since the denominator factors as $(x+12)(x-80)$, the function has a vertical asymptote when $x = 80$. This means that C is not continuous on the interval $0 \le x \le 100$, and the intermediate value theorem cannot be used.

44. **(a)**



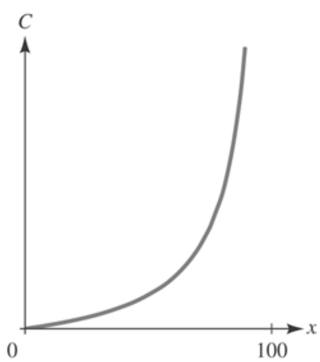
(b) The graph is discontinuous at $t = 31$ (Jan. 31), $t = 59$ (Feb. 28), $t = 90$ (March 31), $t = 120$ (April 30), $t = 151$ (May 31), $t = 181$ (June 30), $t = 182$ (July 1 commission), $t = 212$ (July 31), $t = 243$ (Aug. 31), $t = 273$ (Sept. 30), $t = 304$ (Oct. 31), $t = 334$ (Nov. 30), $t = 355$ (Dec. 21), and $t = 365$ (Dec. 31)

45. $C(x) = \dfrac{12x}{100 - x}$

(a) $C(25) = \dfrac{12(25)}{100-25} = 4,$ or \$4000

$C(50) = \dfrac{12(50)}{100-50} = 12,$ or \$12,000

(b)



(c) From the graph, $\lim\limits_{x \to 100^-} C(x) = \infty.$

So, it is not possible to remove all of the pollution.

46. The graph is discontinuous at $x = 6$ and $x = 12$. What happened to cause these jumps is a writing exercise; answers will vary.

47. The graph is discontinuous at $x = 10$ and $x = 25$. Sue is probably at the gas station replenishing fuel.

48. Since the thickness is assumed to be a continuous function of x, we would expect the thickness at the source to be

$$\lim_{x \to 0^+} \frac{0.5(x^2 + 3x)}{x^3 + x^2 + 4x} = \lim_{x \to 0^+} \frac{0.5x(x+3)}{x(x^2 + x + 4)}$$
$$= \lim_{x \to 0^+} \frac{0.5(x+3)}{x^2 + x + 4}$$
$$= \frac{3}{8} \text{ meters.}$$

49. $p(t) = 20 - \dfrac{7}{t+2};\ c(p) = 0.4\sqrt{p^2 + p + 21}$

$$\lim_{t \to \infty} p(t) = \lim_{t \to \infty} 20 - \frac{7}{t+2} = 20$$
$$\lim_{t \to \infty} c(p) = \lim_{t \to 20} 0.4\sqrt{p^2 + p + 21}$$
$$= 0.4\sqrt{(20)^2 + 20 + 21}$$
$$= 0.4(21) = 8.4$$

In the long run, the level of pollution will be 8.4 parts per million.

50. **(a)** When $v = 20$, the middle expression is used to find $W(v)$.

$$W(20) = 1.25(20) - 18.67\sqrt{20} + 62.3$$
$$\approx 3.75°\text{F}$$

For $v = 50$, the bottom expression is used to find $W(v)$, so $W(50) = -7°$F.

(b) If $0 \le v \le 4, W(v) = 30°\text{F}$, so v cannot be between 0 and 4 (inclusive). If $v \ge 45,\ W(v) = -7,$ so v cannot be 45 or more. If $4 < v < 45,$

$$W(v) = 1.25v - 18.67\sqrt{v} + 62.3$$

If $W(v) = 0$, then

$$0 = 1.25v - 18.67\sqrt{v} + 62.3$$

Using the quadratic formula, $v = 25$ mph.

(c) When rounded to the nearest degree, for practical purposes,

$$\lim_{v\to 4^-} W(v) = \lim_{v\to 4^-} 30 = 30$$
$$\lim_{v\to 4^+} W(v) = \lim_{v\to 4^+} \left(1.25v - 18.67\sqrt{v} + 62.3\right)$$
$$= 1.25(4) - 18.67\sqrt{4} + 62.3 = 30$$

So, W is continuous at $v = 4$.
Similarly, for $v = 45$,

$$\lim_{v\to 45^-} W(v) = \lim_{v\to 45^-} \left(1.25v - 18.67\sqrt{v} + 62.3\right)$$
$$= 1.25(45) - 18.67\sqrt{45} + 62.3$$
$$\approx -7$$
$$= \lim_{v\to 45^+} W(v)$$
$$= \lim_{v\to 45^+} -7 = -7$$

So, W is continuous at $v = 45$.

51. (a) $I(30) = 80°$ F, using the top branch of function

$I(90) = 0.005(90)^2 - 0.65(90) + 104$
$= 86°$ F, using the bottom branch of the function

(b) First, whenever $0 \le h \le 40$, the heat index is 80° F.
Using the middle branch of the function,

$83 = 80 + 0.1(h - 40)$

$30 = h - 40;\ h = 70$

A relative humidity of 70% produces a heat index of 83° F.
Using the bottom branch of the function,

$$83 = 0.005h^2 - 0.65h + 104$$
$$0 = 0.005h^2 - 0.65h + 21$$
$$h = \frac{0.65 \pm \sqrt{(-0.65) - 4(21)(0.005)}}{2(0.005)}$$
$$h = \frac{0.65 \pm \sqrt{-1.07}}{2(0.005)}$$

for which there is no solution.

(c)
$$\lim_{h\to 40^-} I(h) = \lim_{h\to 40^-} 80 = 80$$
$$\lim_{h\to 40^+} I(h) = \lim_{h\to 40^+} 80 + 0.1(h - 40)$$
$$= 80 + 0.1(40 - 40) = 80$$

$I(40) = 80$
so, continuous at $h = 40$

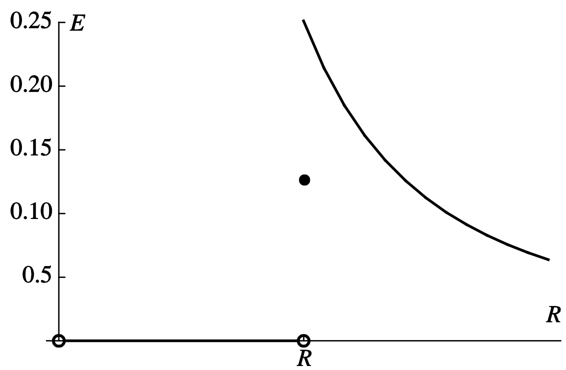
$$\lim_{h\to 80^-} I(h) = \lim_{h\to 80^-} 80 + 0.1(h - 40)$$
$$= 80 + 0.1(80 - 40) = 84$$
$$\lim_{h\to 80^+} I(h) = \lim_{h\to 80^+} 0.005h^2 - 0.65h + 104$$
$$= 0.005(80)^2 - 0.65(80) + 104$$
$$= 84$$

$I(80) = 84$
so, continuous at $h = 80$

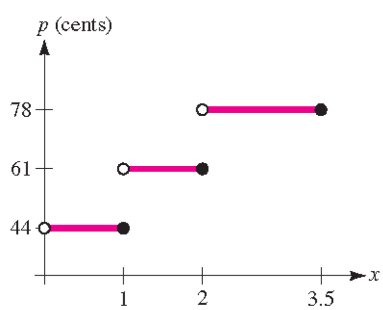
52. $$E(x) = \begin{cases} 0 & \text{if } 0 < x < R \\ \dfrac{1}{2x^2} & \text{if } x = R \\ \dfrac{1}{x^2} & \text{if } R < x \end{cases}$$

$E(x)$ is not continuous at $x = R$ since

$$\lim_{x\to R^+} \frac{1}{x^2} = \frac{1}{R^2} \ne \frac{1}{2R^2}.$$



53. The graph will consist of horizontal line segments, with the left endpoints open and the right endpoints closed (from the inequalities). The height of each segment corresponds to the respective costs of postage.



The function p is discontinuous where the price jumps or when $x = 1$ and $x = 2$.

54. $f(x) = x^2 - 3x$ is a polynomial in the open interval, and thus $f(x)$ is continuous for all x in the open interval. But at $x = 2$

$\lim_{x \to 2^-} f(x) = 2^2 - 3 \times 2 = -2$

$f(2) = 4 + 2 \times 2 = 8$, thus

$f(2) \neq \lim_{x \to 2^-} f(x)$. $f(x)$ is not continuous on the closed interval.

55. On the open interval $0 < x < 1$, since $x \neq 0$,

$f(x) = x\left(1 + \frac{1}{x}\right) = x + 1$. So, $f(x)$, a polynomial on $0 < x < 1$, is continuous. On the closed interval $0 \le x \le 1$, the endpoints must now be considered.

$f(x) = x\left(x + \frac{1}{x}\right)$ is not continuous at $x = 0$ since $f(0)$ is not defined. However, f is continuous at $x = 1$ since

$f(1) = 1\left(1 + \frac{1}{1}\right) = 2$ and as x approaches 1 from the left,

$$\lim_{x \to 1^-} x\left(x + \frac{1}{x}\right) = \lim_{x \to 1^-} x \cdot \lim_{x \to 1^-}\left(x + \frac{1}{x}\right)$$
$$= 1\left(1 + \frac{1}{1}\right)$$
$$= 2.$$

56. $\lim_{x \to 4^-} f(x) = 1 - 3 \times 4 = -11$

$f(4) = 16A + 8 - 3 = 16A + 5$

To be continuous at $x = 4$ we need $16A + 5 = -11$ or $A = -1$.

57. $f(x) = \begin{cases} Ax - 3 & \text{if } x < 2 \\ 3 - x + 2x^2 & \text{if } 2 \le x \end{cases}$

f is continuous everywhere except possibly at $x = 2$, since $Ax - 3$ and $3 - x + 2x^2$ are polynomials. Since $f(2) = 3 - 2 + 2(2)^2 = 9$, in order that f be continuous at $x = 2$, A must be chosen so that $\lim_{x \to 2} f(x) = 9$.

As x approaches 2 from the right,

$$\lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} (3 - x + 2x^2)$$
$$= \lim_{x \to 2^+} 3 - \lim_{x \to 2^+} x + 2 \lim_{x \to 2^+} x^2$$
$$= 3 - 2 + 2(2)^2$$
$$= 9$$

and as x approaches from the left,

$$\lim_{x \to 2^-} f(x) = \lim_{x \to 2^-} (Ax - 3)$$
$$= A \lim_{x \to 2^-} x - \lim_{x \to 2^-} 3$$
$$= 2A - 3.$$

For $\lim_{x \to 2} f(x) = 9$, $2A - 3$ must equal 9, or $A = 6$. f is continuous at $x = 2$ only when $A = 6$.

58. Define $F(x) = x^2 - x - 1 - \frac{1}{x+1}$. This function is continuous except at $x = -1$.

(a) The equation

$$x^2 - x - 1 = \frac{1}{x+1}$$

has a solution wherever $F(x) = 0$. Since $F(1) = -\frac{3}{2}$ and $F(2) = \frac{2}{3}$, and is continuous between 1 and 2, it follows from the intermediate value property that $F(x) = 0$ for some x between 1 and 2.

(b) Since $F(1.5) = -\frac{13}{20}$ and $F(1.6) = -\frac{138}{325}$, the test is inconclusive. (There might be two solutions in the interval $1.5 < x < 1.6$, for example.) Since $F(1.7) = -\frac{487}{2{,}700}$ and $F(1.8) = \frac{29}{350}$, there is a solution between 1.7 and 1.8.

59. Rewrite as $\sqrt[3]{x-8} + 9x^{2/3} - 29$ and notice that at $x = 0$ this expression is negative and at $x = 8$ it is positive. Therefore, by the intermediate value property, there must be a value of x between 0 and 8 such that this expression is 0 or $\sqrt[3]{x-8} + 9x^{2/3} = 29$.

60. Let $f(x) = \sqrt[3]{x} - (x^2 + 2x - 1)$

$f(x)$ is continuous at all x and $f(0) = 1$, $f(1) = 1 - (1 + 2 - 1) = -1$.

By the root location property, there is at least one number $0 < c \le 1$ such that $f(c) = 0$, and $x = c$ is a solution.

61. To investigate the behavior of

$$f(x) = \frac{2x^2 - 5x + 2}{x^2 - 4}:$$

Press [y=]. Input (2*x* ^ 2 – 5*x* + 2)/(*x* ^ 2 – 4) for y_1 = Press [graph].

(a) Press [trace]. Using arrows to move cursor to be near $x = 2$ we see that (1.9, 0.72) and (2.1, 0.79) are two points on the graph. By zooming in, we find (1.97, 0.74) and (2.02, 0.76) to be two points on the graph. The $\lim_{x \to 2} f(x) = \frac{3}{4}$, however, the function is not continuous at $x = 2$ since $f(2)$ is undefined. To show this, use the value function under the calc menu and enter $x = 2$. There is no y-value displayed, which indicates the function is undefined for $x = 2$.

(b) Use the *z* standard function under the Zoom menu to return to the original graph. We see from the graph that there is a vertical asymptote at $x = -2$. The $\lim_{x \to 2^-} f(x) = \infty$ and $\lim_{x \to 2^+} f(x) = -\infty$ and therefore $\lim_{x \to -2} f(x)$ does not exist. So f is not continuous at $x = -2$.

62. Your weight in pounds increased (and/or decreased) continuously from your minimum weight at birth to your present weight, like from 7 pounds to 150 pounds. Your (present) height in inches is some number, like 65. By the intermediate value property, your weight must have been 65 pounds at least once in your lifetime.

63. Let's assume the hands of a clock move in a continuous fashion. During each hour the minute hand moves from being behind the hour to being ahead of the hour. Therefore, at some time, the hands must be in the same place.

64. Let's use some numbers for the purpose of the illustration. Suppose Michaela is 60 inches tall at age 15 (say in 1980) when

Juan is 30 inches tall. Assume Michaela is 70 inches tall at age 31 (in 1996) when Juan is 76 inches tall. Draw a continuous curve (it could be a straight line) from $(1980, 60)$ to $(1996, 70)$. This represents Michaela's growth curve. Now draw a continuous curve from $(1980, 30)$ to (1996, 76). This represents Juan's growth curve. The two curves cross at one point, say in 1992 when they are both 66 inches tall. By the intermediate value property, 66 inches lies between 60 and 70 as well as between 30 and 76.

Checkup for Chapter 1

1. Since negative numbers do not have square roots and denominators cannot be zero, the domain of the function $f(x) = \frac{2x-1}{\sqrt{4-x^2}}$ is all real numbers such that $4-x^2 > 0$ or $(2 + x)(2 - x) > 0$, namely $-2 < x < 2$.

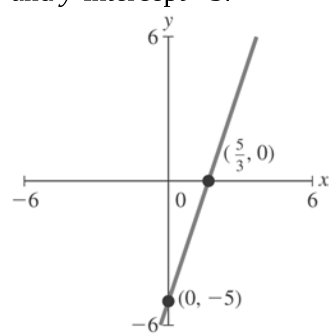
2.
$$g(h(x)) = g\left(\frac{x+2}{2x+1}\right) = \frac{1}{2\left(\frac{x+2}{2x+1}\right)+1} = \frac{1}{\frac{2x+4}{2x+1}+1} = \frac{1}{\frac{2x+4+2x+1}{2x+1}} = \frac{2x+1}{4x+5},\ x \neq -\frac{1}{2}$$

3. (a) Since $m = -\frac{1}{2}$ and the point $(-1, 2)$ is on the line, the equation of the line is

$$y-2 = -\frac{1}{2}(x-(-1))$$
$$y-2 = -\frac{1}{2}(x+1)$$
$$y-2 = -\frac{1}{2}x - \frac{1}{2}$$
$$y = -\frac{1}{2}x - \frac{1}{2} + 2$$
$$y = -\frac{1}{2}x + \frac{3}{2}$$

(b) Since $m = 2$ and $b = -3$, the equation of the line is $y = 2x - 3$.

4. (a) The graph is a line with x-intercept $\frac{5}{3}$ and y-intercept -5.

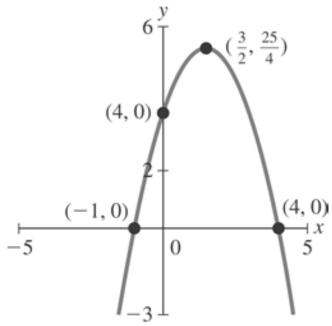


(b) The graph is a parabola which opens down (since $A < 0$). The vertex is $\left(-\frac{b}{2a},\ f\left(-\frac{b}{2a}\right)\right)$, or $\left(\frac{3}{2}, \frac{25}{4}\right)$.

The x-intercepts are

$$0 = -x^2 + 3x + 4$$
$$0 = x^2 - 3x - 4$$
$$0 = (x-4)(x+1)$$
$$x = 4, -1$$

The y-intercept is 4.



5. (a) $\lim\limits_{x\to -1} \dfrac{x^2+2x-3}{x-1} = \dfrac{(-1)^2+2(-1)-3}{-1-1}$
$= \dfrac{1-2-3}{-2}$
$= 2$

(b) $\lim\limits_{x\to 1} \dfrac{x^2+2x-3}{x-1} = \lim\limits_{x\to 1} \dfrac{(x+3)(x-1)}{x-1}$
$= \lim\limits_{x\to 1} (x+3)$
$= 4$

(c) $\lim\limits_{x\to 1} \dfrac{x^2-x-1}{x-2} = \dfrac{(1)^2-1-1}{1-2} = \dfrac{-1}{-1} = 1$

(d) $\lim\limits_{x\to +\infty} \dfrac{2x^3+3x-5}{-x^2+2x+7} = \lim\limits_{x\to +\infty} \dfrac{2x+3-\frac{5}{x^2}}{-1+\frac{2}{x}+\frac{7}{x^2}}$

Since $\lim\limits_{x\to +\infty} 2x+3-\dfrac{5}{x^2} = +\infty$ and

$\lim\limits_{x\to +\infty} -1+\dfrac{2}{x}+\dfrac{7}{x^2} = -1,$

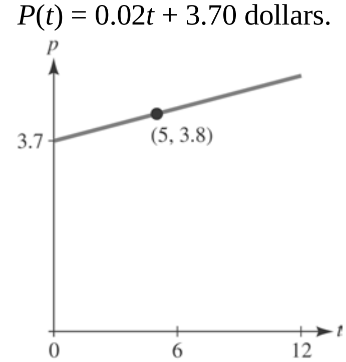
$\lim\limits_{x\to +\infty} \dfrac{2x^3+3x-5}{-x^2+2x+7} = -\infty.$

6. The function is defined at $x = 1$, and $f(1) = 2(1) + 1 = 3$. If $\lim\limits_{x\to 1} f(x) = 3,$ the function will be continuous at $x = 1$. From the left of $x = 1$,
$\lim\limits_{x\to 1^-} f(x) = \lim\limits_{x\to 1^-} 2x+1 = 2(1)+1 = 3.$
From the right of $x = 1$,

$$\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} \frac{x^2+2x-3}{x-1}$$
$$= \lim_{x\to 1^+} \frac{(x+3)(x-1)}{x-1}$$
$$= \lim_{x\to 1^+} (x+3)$$
$$= 1+3$$
$$= 4.$$

Since $\lim\limits_{x\to 1^-} f(x) \neq \lim\limits_{x\to 1^+} f(x),$ the limit does not exist and the function is not continuous at $x = 1$.

7. (a) Let t denote the time in months since the beginning of the year and $P(t)$ the corresponding price (in cents) of gasoline. Since the price increases at a constant rate of 2 cents per gallon per month, P is a linear function of t with slope $m = 2$. Since the price on June first (when $t = 5$) is 380 cents, the graph passes through (5, 380). The equation is therefore
$P - 380 = 2(t - 5)$ or
$P(t) = 2t + 370$ cents,
$P(t) = 0.02t + 3.70$ dollars.



(b) When $t = 0$,
$P(0) = 0.02(0) + 3.70 = 3.70$
The price was \$3.70.

(c) On October 1st, $t = 9$ and
$P(9) = 0.02(9) + 3.70 = 3.88$
The price will be \$3.88.

8. Let t be the time, in hours, that has passed since the truck was 300 miles due east of the car. The distance the truck is from the car's original location is $300 - 30t$. The car's distance from its original location is $60t$ (due north). These two distances form the legs of a right triangle, where the distance between the car and the truck is its hypotenuse. So,

$$D(t) = \sqrt{(60t)^2 + (300 - 30t)^2}$$
$$= 30\sqrt{5t^2 - 20t + 100}$$

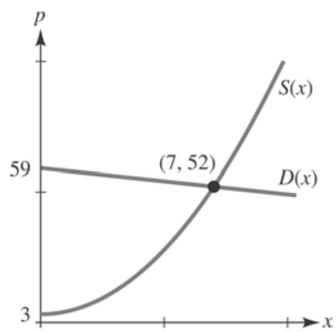
9. $S(x) = x^2 + A$; $D(x) = Bx + 59$

(a) Since no units are supplied until the selling price is greater than \$3 (assuming continuity), $3 = 0 + A$, or $A = 3$.
Equilibrium occurs when

$$S(7) = D(7)$$
$$(7)^2 + 3 = B(7) + 59$$
$$-1 = B$$

The equilibrium price is
$S(7) = (7)^2 + 3 = \$52.$

(b)



(c) When 5 units are produced the supply price is $S(5) = (5)^2 + 3 = \$28$ and the demand price is
$D(5) = -(5) + 59 = \$54.$ When 10 units are produced, the supply price is $S(10) = (10)^2 + 3 = \$103,$ and the demand price is
$D(10) = -(10) + 59 = \$49$. The difference is $\$103 - \$49 = \$54$. (Note that for 5 units, the demand price is higher than the supply price. However, for 10 units, the opposite is true.)

10. **(a)** The population is positive and increasing for $0 \le t < 5$. However, for $t \ge 5$, the population decreases. Therefore, the colony dies out when $-8t + 72 = 0$, or $t = 9$.

(b) $f(1) = 8$ and $f(7) = -56 + 72 = 16$.
Since $f(5) = \lim_{x \to 5} f(x) = 32,$ f is continuous. Since $8 < 10 < 16$, by the intermediate value property there exists a value $1 < c < 7$ such that $f(c) = 10$.

11. Since M is a linear function of D, $M = aD + b$, for some constants a and b. Using $M = 7.7$ when $D = 3$, and $M = 12.7$ when $D = 5$, solve the system

$$a \cdot 3 + b = 7.7$$
$$a \cdot 5 + b = 12.7$$

So, $a = 2.5$ and $b = 0.2$. Thus, $M = 2.5D + 0.2$. When $D = 0$, $M = 0.2$, so 0.2% will mutate when no radiation is used.

Review Exercises

1. **(a)** The domain of the quadratic function

$$f(x) = x^2 - 2x + 6$$

is all real numbers x.

(b) Since denominators cannot be zero, the domain of the rational function

$$f(x) = \frac{x-3}{x^2 + x - 2} = \frac{x-3}{(x+2)(x-1)}$$

is all real numbers x except $x = -2$ and $x = 1$.

(c) Since negative numbers do not have square roots, the domain of the function

$$f(x) = \sqrt{x^2 - 9} = \sqrt{(x+3)(x-3)}$$

is all real numbers x such that $(x + 3)(x - 3) \ge 0$, that is for $x \le -3$, or $x \ge 3$, or $|x| \ge 3$.

2. **(a)** $f(x)=4-(3-x)^2$ is a polynomial function, so it has domain all real numbers.

(b) $f(x)=\dfrac{x-1}{x^2-2x+1}$ is defined for $x^2-2x+1\neq 0$.
$x^2-2x+1=(x-1)^2=0$ when $x = 1$, so the domain is $x \neq 1$.

(c) $f(x)=\dfrac{1}{\sqrt{4-3x}}$ is defined for $4 - 3x > 0$, so the domain is $x<\dfrac{4}{3}$.

3. **(a)** If $g(u)=u^2+2u+1$ and $h(x) = 1 - x$ then

$$g(h(x))=g(1-x)$$
$$=(1-x)^2+2(1-x)+1$$
$$=x^2-4x+4.$$

(b) If $g(u)=\dfrac{1}{2u+1}$ and $h(x) = x + 2$, then

$$g(h(x))=g(x+2)$$
$$=\frac{1}{2(x+2)+1}$$
$$=\frac{1}{2x+5}.$$

4. **(a)** $g(h(x))=g\left(\sqrt{x+1}\right)=\left(1-2\sqrt{x+1}\right)^2$

(b)

$$g(h(x))=g(2x+4)$$
$$=\sqrt{1-(2x+4)}$$
$$=\sqrt{-3-2x}$$

5. **(a)**

$$f(3-x)=4-(3-x)-(3-x)^2$$
$$=4-3+x-(9-6x+x^2)$$
$$=1+x-9+6x-x^2$$
$$=-x^2+7x-8$$

(b) $f(x^2-3)=(x^2-3)-1=x^2-4$

(c)

$$f(x+1)-f(x)=\frac{1}{(x+1)-1}-\frac{1}{x-1}$$
$$=\frac{1}{x}-\frac{1}{x-1}$$
$$=\frac{1}{x}\cdot\frac{x-1}{x-1}-\frac{1}{x-1}\cdot\frac{x}{x}$$
$$=\frac{x-1}{x(x-1)}-\frac{x}{x(x-1)}$$
$$=\frac{x-1-x}{x(x-1)}$$
$$=-\frac{1}{x(x-1)}$$

6. **(a)** If $f(x)=x^2-x+4$ then

$$f(x-2)=(x-2)^2-(x-2)+4$$
$$=x^2-5x+10.$$

(b) If $f(x)=\sqrt{x}+\dfrac{2}{x-1}$ then

$$f(x^2+1)=\sqrt{x^2+1}+\frac{2}{(x^2+1)-1}$$
$$=\sqrt{x^2+1}+\frac{2}{x^2}.$$

(c) If $f(x)=x^2$ then

$$f(x+1)-f(x)=(x+1)^2-x^2$$
$$=2x+1.$$

7. **(a)** One of many possible solutions is $g(u)=u^5$ and $h(x)=x^2+3x+4$.

Then,

$$g(h(x))=g(x^2+3x+4)$$
$$=(x^2+3x+4)^5$$
$$=f(x).$$

(b) One of many possible solutions is $g(u)=u^2+\dfrac{5}{2(u+1)^3}$ and $h(x) = 3x + 1$.

Then,

$$\begin{aligned} g(h(x)) &= g(3x+1) \\ &= (3x+1)^2 + \frac{5}{2((3x+1)+1)^3} \\ &= (3x+1)^2 + \frac{5}{2(3x+2)^3} \\ &= f(x). \end{aligned}$$

8. Answers may vary.

(a) Let $h(x) = x - 1$ and $g(u) = u^2 - 3u + 1.$

(b) Let $h(x) = x + 4$ and $g(u) = \dfrac{2u}{2u-11}.$

9. $f(x) = x^2 + 2x - 8 = (x+4)(x-2)$

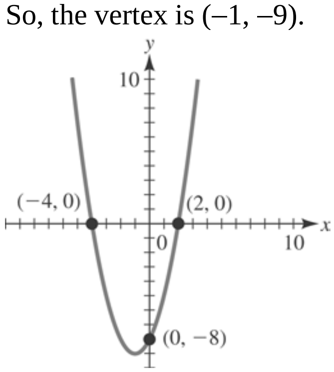
The intercepts of the function are (–4, 0), (2, 0) and (0, –8). Further, the vertex of the parabola is

$$x = -\frac{B}{2A}, \quad y = f\left(-\frac{B}{2A}\right)$$

$$x = -\frac{2}{2(1)} = -1$$

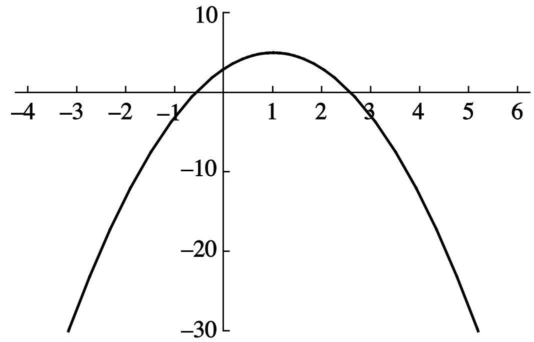
$$\begin{aligned} y = f(-1) &= (-1)^2 + 2(-1) - 8 \\ &= 1 - 2 - 8 \\ &= -9. \end{aligned}$$

So, the vertex is (–1, –9).

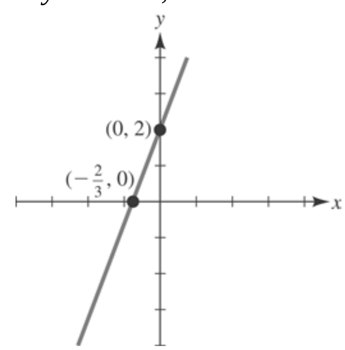


10. Some points on the graph of $y = 3 + 4x - 2x^2$ are shown below. Note that $y = -2x^2 + 4x + 3 = -2(x-1)^2 + 5$ is a parabola with vertex $(1, 5)$ and opening downward.

x	–3	–2	–1	0	1	2	3	4	5
y	–27	–13	–3	3	5	3	–3	–13	–27

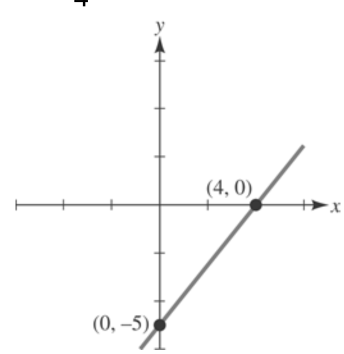


11. **(a)** If $y = 3x + 2$, $m = 3$ and $b = 2$.



(b) If $5x - 4y = 20$ then $y = \dfrac{5}{4}x - 5,$

$m = \dfrac{5}{4},$ and $b = -5.$

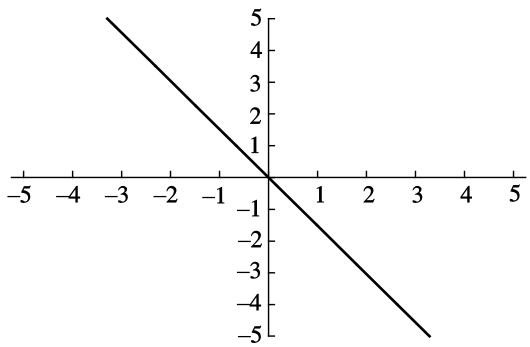


12. **(a)** $2y+3x=0$

$2y=-3x$

$y=-\frac{3}{2}x$

slope $m=-\frac{3}{2}$; y intercept (0, 0)

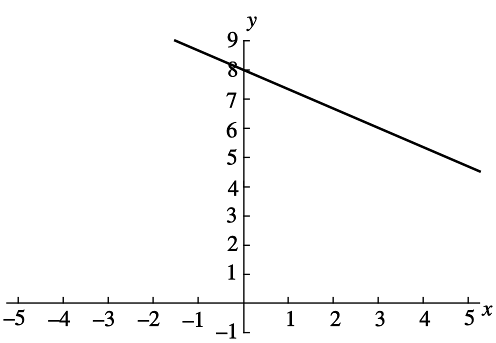


(b) $\frac{x}{3}+\frac{y}{2}=4$

$\frac{y}{2}=-\frac{x}{3}+4$

$y=-\frac{2}{3}x+8$

slope $m=-\frac{2}{3}$; y intercept (0, 8)



13. **(a)** $m = 5$ and y-intercept $b = -4$, so $y = 5x - 4$.

(b) $m = -2$ and $P(1, 3)$, so $y - 3 = -2(x - 1)$, or $y = -2x + 5$.

(c) $2x + y = 3 \rightarrow y = -2x + 3$, so $m = -2$ and $P(5, 4)$.
$y - 4 = -2(x - 5)$, or $y = -2x + 14$ $\rightarrow 2x + y = 14$.

14. **(a)** $m=\frac{1-3}{4-(-1)}=\frac{-2}{5}=-\frac{2}{5}$

$y-y_1=m(x-x_1)$

$y-3=-\frac{2}{5}[x-(-1)]$

$y=-\frac{2}{5}x+\frac{13}{5}$

(b) $m=\frac{-\frac{2}{3}-0}{0-3}=\frac{-\frac{2}{3}}{-3}=\frac{2}{9}$

$y=\frac{2}{9}x-\frac{2}{3}$

(c) $5x-3y=7$

$-3y=-5x+7$

$y=\frac{5}{3}x-\frac{7}{3}$

Slope of a perpendicular line is $-\frac{3}{5}$.

$y-y_1=m(x-x_1)$

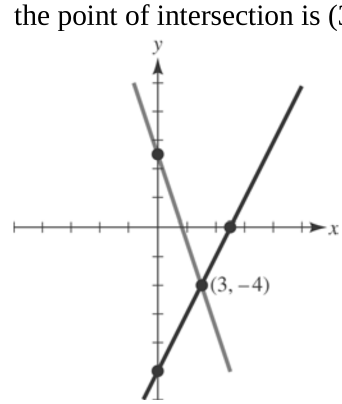
$y-3=-\frac{3}{5}[x-(-1)]$

$y=-\frac{3}{5}x+\frac{12}{5}$

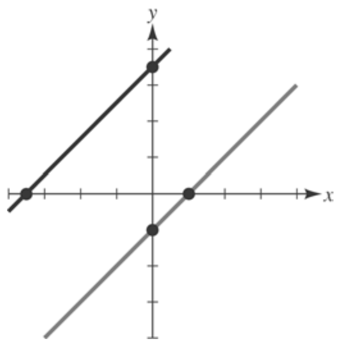
or

$3x+5y=12$

15. **(a)** The graphs of $y = -3x + 5$ and $y = 2x - 10$ intersect when $-3x + 5 = 2x - 10$, or $x = 3$. When $x = 3$, $y = -3(3) + 5 = -4$. So the point of intersection is (3, –4).

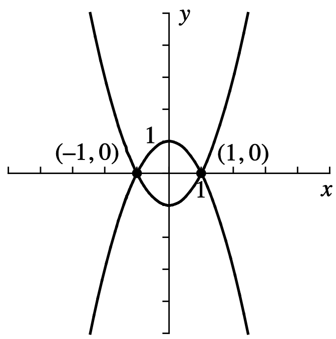


(b) The graphs of $y = x + 7$ and $y = -2 + x$ are lines having the same slope, so they are parallel lines and there are no points of intersection.



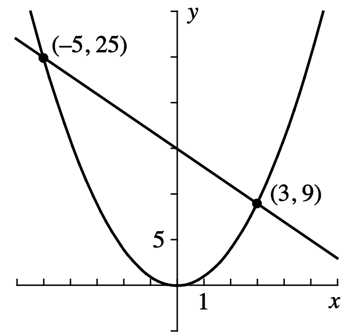
16. (a) The graphs of $y = x^2 - 1$ and $y = 1 - x^2$ intersect when $x^2 - 1 = 1 - x^2$, $2x^2 = 2$, $x^2 = 1$, or $x = \pm 1$.

When $x = \pm 1$, $y = (\pm 1)^2 - 1 = 0$. So the points of intersection are (–1, 0) and (1, 0).



(b) The graphs of $y = x^2$ and $y = 15 - 2x$ intersect when $x^2 = 15 - 2x$.

$(x + 5)(x - 3) = 0$, or $x = -5$ and $x = 3$.
When $x = -5$, $y = 25$, and when $x = 3$, $y = 9$. So, the points of intersection are (–5, 25) and (3, 9).



17. If the graph of $y = 3x^2 - 2x + c$ passes through the point (2, 4),
$4 = 3(2)^2 - 2(2) + c$ or $c = -4$.

18. If $y = 4 - x - cx^2$ passes through (–2, 1), then:

$$1 = 4 - (-2) - c(-2)^2$$
$$1 = 4 + 2 - 4c$$
$$-5 = -4c$$
$$\frac{5}{4} = c$$

19.
$$\lim_{x\to 1} \frac{x^2 + x - 2}{x^2 - 1} = \lim_{x\to 1} \frac{(x+2)(x-1)}{(x+1)(x-1)} = \lim_{x\to 1} \frac{x+2}{x+1} = \frac{\lim_{x\to 1}(x+2)}{\lim_{x\to 1}(x+1)} = \frac{1+2}{1+1} = \frac{3}{2}$$

20.
$$\lim_{x\to 2} \frac{x^2 - 3x}{x+1} = \frac{\lim_{x\to 2}\left(x^2 - 3x\right)}{\lim_{x\to 2}\left(x+1\right)} = -\frac{2}{3}$$

21. $\lim_{x\to 2}\frac{x^3-8}{2-x}=\lim_{x\to 2}\frac{(x-2)(x^2+2x+4)}{-(x-2)}$
$=\lim_{x\to 2}-(x^2+2x+4)$
$=-(2^2+2(2)+4)$
$=-12$

22. $\lim_{x\to 1}\left(\frac{1}{x^2}-\frac{1}{x}\right)=1-1=0$

23. $\lim_{x\to 0}\left(2-\frac{1}{x^3}\right)=\lim_{x\to 0}2-\lim_{x\to 0}\frac{1}{x^3}$

Now, $\lim_{x\to 0}2=2;$ but $\lim_{x\to 0^+}\frac{1}{x^3}=+\infty$ and $\lim_{x\to 0^-}\frac{1}{x^3}=-\infty.$ Since $\lim_{x\to 0}\frac{1}{x^3}$ does not exist, $\lim_{x\to 0}\left(x^3-\frac{1}{x^3}\right)$ does not exist.

24. $\lim_{x\to -\infty}\left(2+\frac{1}{x^2}\right)=2+0=2$

25. $\lim_{x\to -\infty}\frac{x}{x^2+5}=\lim_{x\to -\infty}\frac{\frac{1}{x}}{1+\frac{5}{x^2}}=0$

26. $\lim_{x\to 0^+}\left(x^3-\frac{1}{x^2}\right)=0-\infty=-\infty$

27. $\lim_{x\to -\infty}\frac{x^4+3x^2-2x+7}{x^3+x+1}$
$=\lim_{x\to -\infty}\frac{x+\frac{3}{x}-\frac{2}{x^2}+\frac{7}{x^3}}{1+\frac{1}{x^2}+\frac{1}{x^3}}$

Since $\lim_{x\to -\infty}\left(x+\frac{3}{x}-\frac{2}{x^2}+\frac{7}{x^3}\right)=-\infty$ and $\lim_{x\to -\infty}\left(1+\frac{1}{x^2}+\frac{1}{x^3}\right)=1,$ then $\lim_{x\to -\infty}\frac{x^4+3x^2-2x+7}{x^3+x+1}=-\infty.$

28. $\lim_{x\to -\infty}\frac{x^3-3x+5}{2x+3}=\lim_{x\to -\infty}\frac{1-\frac{3}{x^2}+\frac{5}{x^3}}{\frac{2}{x^2}-\frac{3}{x^3}}$
$=+\infty$

29. Since $\lim_{x\to -\infty}\left(1+\frac{1}{x}+\frac{1}{x^2}\right)=1$ and $\lim_{x\to -\infty}(x^3+x+1)=-\infty,$ $\lim_{x\to -\infty}\frac{1+\frac{1}{x}+\frac{1}{x^2}}{x^3+x+1}=0.$

30. $\lim_{x\to -\infty}\frac{x(x-3)}{7-x^2}=\lim_{x\to -\infty}\frac{1-\frac{3}{x}}{\frac{7}{x^2}-1}=-1$

31. $\lim_{x\to 0^-}x\sqrt{1-\frac{1}{x}}=\left(\lim_{x\to 0^-}x\right)\left(\lim_{x\to 0^-}\sqrt{1-\frac{1}{x}}\right)$

Since $\lim_{x\to 0^-}x=0,$ and $\lim_{x\to 0^-}\left(1-\frac{1}{x}\right)=\infty$ implies $\lim_{x\to 0^-}\sqrt{1-\frac{1}{x}}=\infty,$ then $\lim_{x\to 0^-}x\sqrt{1-\frac{1}{x}}=0.$

32. $\lim_{x\to 0^+}\sqrt{x\left(1+\frac{1}{x^2}\right)}=\lim_{x\to 0^+}\sqrt{x+\frac{1}{x}}=+\infty$

33. $f(x)=\frac{x^2-1}{x+3}$ is not continuous at $x=-3$ since $f(-3)=\frac{10}{0}$ and division by 0 is undefined.

34. $f(x)=5x^3-3x+\sqrt{x}$ is not continuous for $x<0$ since square roots of negative numbers do not exist in the real numbers.

35. $h(x)=\begin{cases} x^3+2x-33 & \text{if } x\le 3 \\ \dfrac{x^2-6x+9}{x-3} & \text{if } x>3 \end{cases}$

The denominator in $\dfrac{x^2-6x+9}{x-3}$ will never be zero, since $x = 3$ is not included in its domain. However, in checking the break point (the only point in question),

$h(3)=(3)^3+2(3)-33=0.$

Further,

$\lim\limits_{x\to 3^-} h(x)=\lim\limits_{x\to 3^-}(x^3+2x-33)=0$ and

$$\begin{aligned}\lim_{x\to 3^+} h(x) &= \lim_{x\to 3^+}\frac{x^2-6x+9}{x-3} \\ &= \lim_{x\to 3^+}\frac{(x-3)(x-3)}{x-3} \\ &= \lim_{x\to 3^+}(x-3) \\ &= 3-3 \\ &= 0.\end{aligned}$$

Since $h(3)=\lim\limits_{x\to 3} h(x)$, h is continuous for all x.

36. $g(x)=\dfrac{x^3+5x}{(x-2)(2x+3)}$ is not continuous at $x=2$ and $x=-\dfrac{3}{2}$ since the denominator in the definition of $g(x)$ is 0.

37. $P(x)=40+\dfrac{30}{x+1}$

(a) $P(5)=40+\dfrac{30}{5+1}=40+5=\$45$

(b) Need $P(5) - P(4)$

$P(4)=40+\dfrac{30}{4+1}=40+6=\$46$

$P(5) - P(4) = 45 - 46 = -1$

Price drops one dollar during the 5th month.

(c) Find x so that $P(x) = 43$

$$\begin{aligned}40+\frac{30}{x+1}&=43\\ \frac{30}{x+1}&=\frac{3}{1}\\ 3(x+1)&=30\\ 3x+3&=30\\ 3x&=27\\ x&=9\end{aligned}$$

The price will be \$43 nine months from now.

(d)
$$\begin{aligned}\lim_{x\to+\infty} P(x) &= \lim_{x\to+\infty}\left(40+\frac{30}{x+1}\right)\\ &= \lim_{x\to+\infty}40+\lim_{x\to+\infty}\frac{30}{x+1}\\ &= 40+0\\ &= 40\end{aligned}$$

In the long run, the price will approach \$40.

38. **(a)** Since the smog level Q is related to p by the equation $Q(p)=\sqrt{0.5p+19.4}$ and p is related to t by $p(t)=8+0.2t^2$ it follows that the composite function

$$\begin{aligned}Q[p(t)]&=\sqrt{0.5\left(8+0.2t^2\right)+19.4}\\ &=\sqrt{23.4+0.1t^2}\end{aligned}$$

expresses the smog level as a function of the variable t.

(b) The smog level 3 years from now will be

$$\begin{aligned}Q[p(3)]&=\sqrt{23.4+0.1(3)^2}\\ &=\sqrt{24.3}\approx 4.93 \text{ units.}\end{aligned}$$

(c) Set $Q[p(t)]$ equal to 5 and solve for t to get

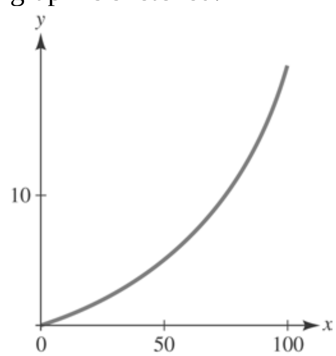
$5=\sqrt{23.4+0.1t^2}$

$1.6=0.1t^2$

$t^2=16$ or $t=4$ years from now.

39. The number of weeks needed to reach x percent of the fundraising goal is given by $f(x)=\frac{10x}{150-x}$.

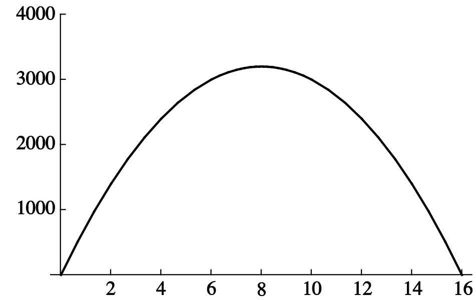
(a) Since x denotes a percentage, the function has a practical interpretation for $0 \le x \le 100$.
The corresponding portion of the graph is sketched.



(b) The number of weeks needed to reach 50% of the goal is
$$f(50)=\frac{10(50)}{150-50}=5 \text{ weeks.}$$

(c) The number of weeks needed to reach 100% of the goal is
$$f(100)=\frac{10(100)}{150-100}=20 \text{ weeks.}$$

40. (a) The function
$D(p)=-50p+800$
is linear with slope –50 and y intercept 800. It represents demand for $0 \le p \le 16$.
The total monthly expenditure is
$$E(p)=(\text{price per unit})(\text{demand})$$
$$=p(-50p+800)$$
$$=-50p(p-16)$$
Since the expenditure is assumed to be non-negative, the relevant interval is $0 \le p \le 16$.



(b) The graph suggests that the expenditure will be greatest if $p=8$.

41. $S=4\pi r^2$, or $r=\sqrt{\frac{S}{4\pi}}=\left(\frac{S}{4\pi}\right)^{1/2}$

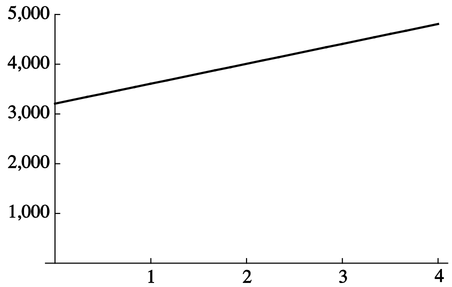
$$V(S)=\frac{4}{3}\pi\left(\left(\frac{S}{4\pi}\right)^{1/2}\right)^3=\frac{4}{3}\pi\left(\frac{S}{4\pi}\right)^{3/2}$$

$$V(S)=\frac{4}{3}\pi\frac{S^{3/2}}{4^{3/2}\pi^{3/2}}$$
$$=\frac{4}{3}\pi\frac{S^{3/2}}{8\pi^{3/2}}$$
$$=\frac{S^{3/2}}{6\pi^{1/2}}$$
$$=\frac{S^{3/2}}{6\sqrt{\pi}}$$

$V(2S)=\frac{(2S)^{3/2}}{6\sqrt{\pi}}=\frac{2^{3/2}S^{3/2}}{6\sqrt{\pi}}$, so volume increased by a factor of $2^{3/2}$, or $2\sqrt{2}$, when S is doubled.

42. (a) Suppose C is the circulation of the newspaper and t is time measured in months. Further suppose $t=0$ represents three months ago. Then C is a linear function of t passing through the points (0, 3200) and (3, 4400). The slope is
$\frac{4400-3200}{3}=400$ and the
C intercept is 3200, hence

$C = 400t + 3200$.



(b) Two months from now is represented by $t = 5$ and the circulation at that time will be
$C = 400(5) + 3200 = 5200$.

43. Let x denote the number of machines used and $C(x)$ the corresponding cost function. Then,

$$C(x) = (\text{set up cost}) + (\text{operating cost})$$
$$= 80(\text{number of machines})$$
$$+ 5.76(\text{number of hours}).$$

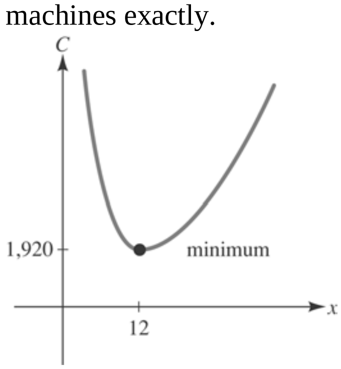
Since 400,000 medals are to be produced and each of the x machines can produce 200 medals per hour,

$$\text{number of hours} = \frac{400{,}000}{200x} = \frac{2{,}000}{x}.$$

So, $C(x) = 80x + 5.76\left(\frac{2{,}000}{x}\right)$

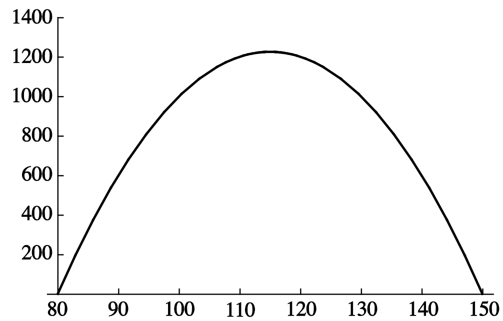
$$= 80x + \frac{11{,}520}{x}.$$

The graph suggests that the cost will be smallest when x is approximately 12.
Note: In chapter 3 you will learn how to use calculus to find the optimal number of machines exactly.



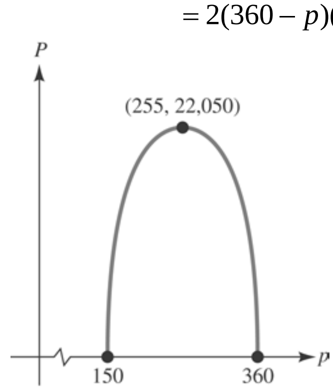
44. Let x denote the selling price of a bookcase (in dollars). The manufacturer's cost and revenue functions are
$C(x) = 80(150 - x)$ and $R(x) = x(150 - x)$
respectively. The profit function is then

$$P(x) = R(x) - C(x)$$
$$= x(150 - x) - 80(150 - x)$$
$$= -x^2 + 230x - 12{,}000.$$



From the graph, or by using the formula for the vertex of a parabola
$\frac{-B}{2A} = \frac{-230}{-2} = 115$, the maximum profit occurs at a price of $x = 115$ dollars.

45. If p represents the selling price, the monthly profit is
$P(p)$
= (number of cameras sold)(price – cost)
Since $\frac{340 - p}{5}$ represents the number of \$5 decreases, $40 + 10\left(\frac{340 - p}{5}\right) =$
represents the number of cameras that will sell. So, $P(p) = (720 - 2p)(p - 150)$
$= 2(360 - p)(p - 150)$



The graph suggests a maximum profit

when $p = 255$, that is when the selling price is \$255.

46. Let r denote the radius, h the height, and V the volume of the can. Then

$V = \pi r^2 h$

To write h in terms of r, use the fact that the cost of constructing the can is 80 cents. That is,

$80 = \text{cost of bottom} + \text{cost of side}$ where

$$\text{cost of bottom} = (\text{cost per in}^2)(\text{area}) = 3\pi r^2$$

and

$$\text{cost of side} = (\text{cost per in}^2)(\text{area}) = 2(2\pi rh) = 4\pi rh.$$

Hence $80 = 3\pi r^2 + 4\pi rh$, or $h = \dfrac{20}{\pi r} - \dfrac{3r}{4}$.

47. Taxes under Proposition A are $100 + .08a$, where a is the assessed value of the home. Taxes under Proposition B are $1,900 + .02a$. Taxes are the same when

$$100 + .08a = 1,900 + .02a$$
$$.06a = 1,800$$
$$a = 30,000$$

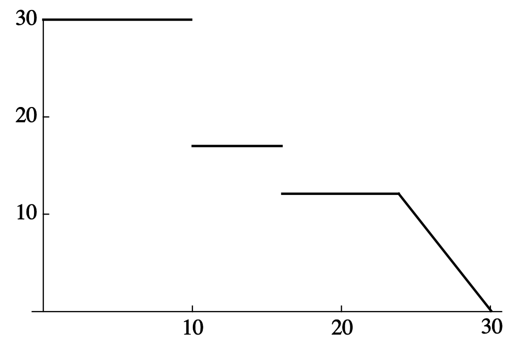
or for an assessed value of \$30,000. Since both tax functions are linear, it is only necessary to test one additional assessed value to determine which proposition is best for all assessed values. For $a = 20,000$,

$100 + .08(20,000) = \$1,700$

$1,900 + .02(20,000) = \$2,300$

So, for $0 < a < 30,000$, Proposition A is preferable while for $a > 30,000$, Proposition B is preferable.

48. Assume the inventory to be maintained at the same level, continuously, over a 24-hour period. A discontinuity occurs when the inventory drops, say, at midnight the appropriate days.



The graph is discontinuous at $t = 10$, $t = 16$ and $t = 24$.

49. **(a)** Let x denote the number of units manufactured and sold. $C(x)$ and $R(x)$ are the corresponding cost and revenue functions, respectively.

$C(x) = 4,500 + 50x$

$R(x) = 80x$

For the manufacturer to break even, since profit = revenue – cost, 0 = revenue – cost, or revenue = cost. That is, $4,500 + 50x = 80x$ or $x = 150$ units.

(b) Let $P(x)$ denote the profit from the manufacture and sale of x units. Then,

$$P(x) = R(x) - C(x) = 80x - (4,500 + 50x) = 30x - 4,500.$$

When 200 units are sold, the profit is $P(200) = 30(200) - 4,500 = \$1,500$.

(c) The profit will be \$900 when $900 = 30x - 4,500$ or $x = 180$, that is, when 180 units are manufactured and sold.

50. **(a)** The revenue is $R(x) = 275x$ and the cost is $C(x) = 125x + 1,500$ where x is the number of kayaks sold.

For the break even point, $275x = 125x + 1,500$ or $x = 10$.

(b) The profit is

$$P(x) = R(x) - C(x)$$
$$= 150x - 1{,}500$$

For $P(x) = 1{,}000$

$$15x = 250$$
$$x = 17$$

51. Let x denote the number of relevant facts recalled, n the total number of relevant facts in the person's memory, and $R(x)$ the rate of recall. Then $n - x$ is the number of relevant facts not recalled.
So, $R(x) = k(n - x)$, where k is a constant of proportionality.

52. Let the power plant be at E, the opposite point at O, the point at which the cable reaches the opposite bank at P, and the factory at F.

$$\overline{OP} = x$$
$$\overline{PF} = 3{,}000 - x$$
$$\overline{EP} = \sqrt{900^2 + x^2}$$

The cost of the cable in the river is
$C_r = 5\sqrt{900^2 + x^2}$.
and the cost of the cable on land is
$C_l = 4(3{,}000 - x)$.
Thus the total cost is
$C = 4(3{,}000 - x) + 5\sqrt{900^2 + x^2}$.

53. The cost for the clear glass is (area)(cost per sq ft) = $(2xy)(3)$, and similarly, the cost for the stained glass is $\left(\frac{1}{2}\pi x^2\right)(10)$.

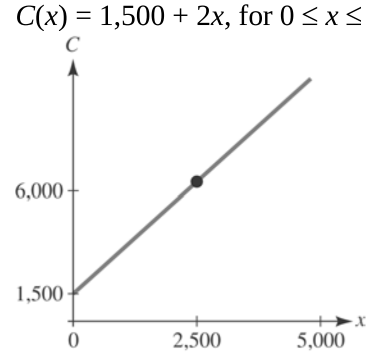
So, $C = 6xy + 5\pi x^2$. Now, the perimeter is $\frac{1}{2}(2\pi x) = 2x + 2y = 20$ so,
$\pi x + 2x + 2y = 20$, or $y = \frac{20 - \pi x - 2x}{2}$.

Cost as a function of x is

$$C(x) = 6x\left(\frac{20 - \pi x - 2x}{2}\right) + 5\pi x^2$$
$$= 3x(20 - \pi x - 2x) + 5\pi x^2$$
$$= 60x - 3\pi x^2 - 6x^2 + 5\pi x^2$$
$$= 60x - 6x^2 + 2\pi x^2$$
$$= 60x + (2\pi - 6)x$$

54. When 200 tables are sold the revenue is 200(\$125) = \$25,000 while the cost is 200(\$85) + A = \$17,000 + A where A is the overhead. Since revenue is equal to cost when 200 tables are sold, \$25,000 = \$17,000 + A, and so the overhead, A, is \$8,000.

55. (a) The fixed cost is \$1,500 and the cost per unit is \$2, so the cost is
$C(x) = 1{,}500 + 2x$, for $0 \le x \le 5{,}000$.



(b) The average cost is
$AC(x) = \frac{C(x)}{x} = \frac{1{,}500}{x} + 2$, so
$AC(3{,}000) = \frac{1{,}500}{3{,}000} + 2 = 2.5$. The average cost of producing 3,000 units per day is \$2.50 per unit.

(c) As to the question of continuity, the answer is both yes and no. Yes, if (as we normally do) x is any real number. No, if x is discrete (x = 0, 1, 2, ..., 5,000). The discontinuities of $C(x)$ occur wherever x is discontinuous.

56. Let x be the time in minutes for the hour hand to move from position 3 to the position at which the hands coincide.

The hour hand moves $\frac{1}{12}$ of a tick (distance between minute tick marks on the circumference) per minute while the minute hand moves one whole tick per minute. The minute has to cover 15 ticks before reaching position 3. Thus

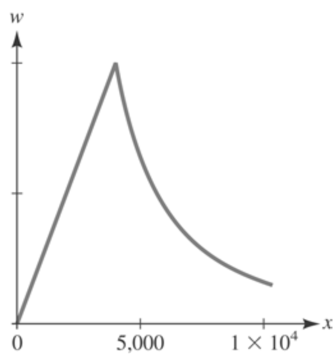
$15+\frac{x}{12}=x$, x = 16.36, or

x = 16 minutes and 22 seconds.

57. $w(x)=\begin{cases} Ax & \text{if } x \le 4{,}000 \\ \frac{B}{x^2} & \text{if } x > 4{,}000 \end{cases}$

For continuity $4{,}000A=\frac{B}{(4{,}000)^2}$ or

$B=A(4{,}000)^3$.



58. **(a)** $f(x)=\begin{cases} 2x+3 & \text{if } x<1 \\ Ax-1 & \text{if } x \ge 1 \end{cases}$

Then $f(x)$ is continuous everywhere except possibly at $x=1$ since $2x+3$ and $Ax-1$ are polynomials.
Since $f(1)=A-1$, in order for $f(x)$ to be continuous at $x=1$, A must be chosen so that
$\lim_{x\to 1} f(x)=A-1$
As x approaches 1 from the right,
$\lim_{x\to 1^+} f(x)=\lim_{x\to 1^+}(Ax-1)=A-1$
and as x approaches 1 from the left,
$\lim_{x\to 1^-} f(x)=\lim_{x\to 1^-}(2x+3)=5.$
$\lim_{x\to 1} f(x)$ exists whenever
$A-1=5$ or $A=6$. Furthermore, for
$A=6$, $\lim_{x\to 1} f(x)=5$, $f(1)=6-1=5.$
Thus, $f(x)$ is continuous at $x=1$ only when $A=6$.

(b) $f(x)=\begin{cases} \frac{x^2-1}{x+1} & \text{if } x<11 \\ Ax^2+x-3 & \text{if } x \ge -1 \end{cases}$

Then $f(x)$ is continuous everywhere except possibly at $x=-1$ since
$\frac{x^2-1}{x+1}$
is a rational function and Ax^2+x-3 is a polynomial.
Since $f(-1)=A-4$, in order for $f(x)$ to be continuous at $x=-1$, A must be chosen so that
$\lim_{x\to -1} f(x)=A-4$
As x approaches –1 from the right,
$\lim_{x\to 1^+} f(x)=\lim_{x\to -1^+}(Ax^2+x-3)=A-4$
and as x approaches –1 from the left,

$$\lim_{x\to 1^-} f(x)=\lim_{x\to 1^-}\left(\frac{x^2-1}{x+1}\right)$$
$$=\lim_{x\to 1^-}\frac{(x+1)(x-1)}{x+1}$$
$$=-2$$

$\lim_{x\to -1} f(x)$ exists whenever
$A-4=-2$ or $A=2$. Furthermore, for $A=2$,
$\lim_{x\to -1} f(x)=-2$, $f(-1)=2-4=-2.$
Thus, $f(x)$ is continuous at $x=-1$ only when $A=2$.

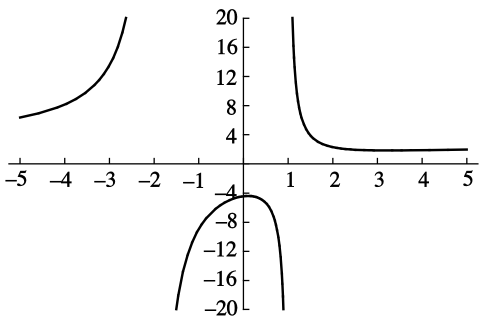
59. This limit does exist. The curve is bounded by the lines $y = mx$ and $y = -mx$. Since $-m|x| \le g(x) \le m|x|$, as x approaches

0, the bounding values on the right and the left of the inequality also approach 0. The function in the middle $g(x)$, is squeezed or sandwiched between 0 and 0. Its limit has to be 0.
Note:

$$\lim_{x\to 0^-} |x|\sin\left(\frac{1}{x}\right) = \lim_{x\to 0^+} |x|\sin\left(\frac{1}{x}\right) = 0$$

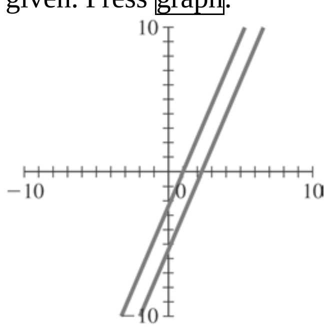
since $-1 \le \sin x \le 1$.

60.



The function is undefined at $x = -2$ and $x = 1$ which are the values of *x* satisfying $x^2 + x - 2 = 0$.

61. To graph $y = \frac{21}{9}x - \frac{84}{35}$ and $y = \frac{654}{279}x - \frac{54}{10}$, press y =.
Input (21*x*)/9 – 84/35 for $y_1 =$ and press enter.
Input (654*x*)/279 – 54/10 for $y_2 =$.
Use the z-standard function under the zoom menu to use the window dimensions given. Press graph.



It appears from the graph that the two lines are parallel. However, the difference in the slopes is $\frac{21}{9} - \frac{654}{279} = -.01$ which shows that, in fact, the lines are not parallel since they have different slopes.

62. $f(x) = \sqrt{x+3}, \quad g(x) = 5x^2 + 4$

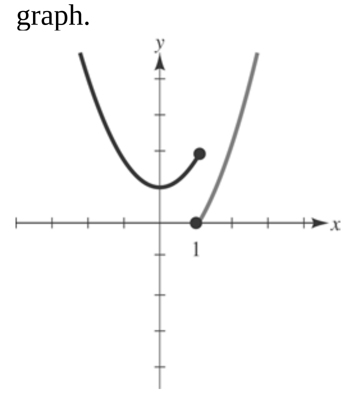
(a) $g(-1.28) = 5(-1.28)^2 + 4 = 12.192$

$$\begin{aligned} f(g(-1.28)) &= f(12.192) \\ &= \sqrt{15.192} \\ &= 3.898 \end{aligned}$$

(b) $f(\sqrt{2}) = \sqrt{\sqrt{2} + 3} \approx 2.101$

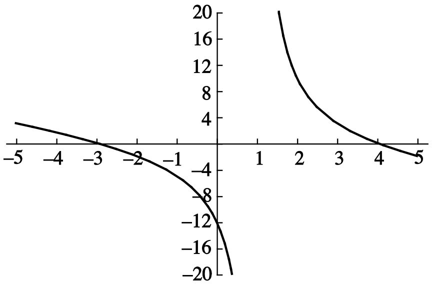
$$\begin{aligned} g(f(\sqrt{2})) &= g(2.101) \\ &= 5(2.101)^2 + 4 \\ &\approx 26.071 \end{aligned}$$

63. Press y =.
Input (*x* ^ 2 + 1)/(*x* ≤ 1) for $y_1 =$ and press enter. (You can obtain the ≤ from 2ND test and enter 6: ≤).
Input (*x* ^ 2 – 1)/(*x* > 1) for $y_2 =$ and press enter. (You can obtain the > from 2ND test and enter 3: >). Press graph.



The graph of *y* is discontinuous at *x* = 1.

64. $f(x) = \dfrac{x^2 - 3x - 10}{1 - x} - 2$



$f(x) = \dfrac{x^2 - 3x - 10}{1 - x} - 2 = 0$

when $x^2 - 3x - 10 = 2(1 - x)$ or $x^2 - x - 12 = (x - 4)(x + 3) = 0$. The x intercepts are then $(4, 0)$ and $(-3, 0)$. Since $f(0) = -12$, the y intercept is $(0, -12)$. The function is defined for all $x \neq 1$.