2 LIMITS AND THE DERIVATIVE

EXERCISE 2-1

2. $x^2 - 64 = (x-8)(x+8)$

4. $x^2 + 5x - 36 = (x+9)(x-4)$

6. $x^3 + 15x^2 + 50x = x(x^2 + 15x + 50) = x(x+5)(x+10)$

8. $20x^2 + 11x - 3 = (4x+3)(5x-1)$

10. $f(0.5) = 2$

12. $f(2.25) = 2.25$

14. (A) $\lim_{x\to 1^-} f(x) = 2$ (B) $\lim_{x\to 1^+} f(x) = 2$ (C) $\lim_{x\to 1} f(x) = 2$ (D) $f(1) = 2$

16. (A) $\lim_{x\to 4^-} f(x) = 4$ (B) $\lim_{x\to 4^+} f(x) = 4$ (C) $\lim_{x\to 4} f(x) = 4$ (D) $f(4)$ does not exist

18. $g(2.1) = 1.9$

20. $g(2.5) = 1.5$

22. (A) $\lim_{x\to 2^-} g(x) = 2$ (B) $\lim_{x\to 2^+} g(x) = 2$ (C) $\lim_{x\to 2} g(x) = 2$ (D) $g(2) = 2$

24. (A) $\lim_{x\to 4^-} g(x) = 0$ (B) $\lim_{x\to 4^+} g(x) = 0$ (C) $\lim_{x\to 4} g(x) = 0$ (D) $g(4) = 0$

26. (A) $\lim_{x\to -2^+} f(x) = 3$ (B) $\lim_{x\to -2^-} f(x) = -3$

(C) Since $\lim_{x\to -2^+} f(x) \neq \lim_{x\to -2^-} f(x)$, $\lim_{x\to -2} f(x)$ does not exist.

(D) $f(-2) = -3$

28. (A) $\lim_{x\to 2^+} f(x) = -3$ (B) $\lim_{x\to 2^-} f(x) = 3$

(C) $\lim_{x\to 2} f(x)$ does not exist since $\lim_{x\to 2^+} f(x) \neq \lim_{x\to 2^-} f(x)$

(D) $f(2) = 3$

30. $3x \to -6$ as $x \to -2$; thus $\lim_{x\to -2} 3x = -6$

32. $x - 3 \to 5 - 3 = 2$ as $x \to 5$; thus $\lim_{x\to 5} (x-3) = 2$

34. $x(x+3) \to (-1)(-1+3) = -2$ as $x \to -1$; thus $\lim_{x\to -1} x(x+3) = -2$

36. $x - 2 \to 4 - 2 = 2$ as $x \to 4$; thus $\lim_{x\to 4} \frac{x-2}{x} = \frac{2}{4} = \frac{1}{2}$

38. $\sqrt{16-7x} \to \sqrt{16-7(0)} = \sqrt{16} = 4$ as $x \to 0$; thus $\lim_{x\to 0} \sqrt{16-7x} = 4$

40. $\lim_{x\to 1} 2g(x) = 2\lim_{x\to 1} g(x) = 2(4) = 8$

42. $\lim_{x\to 1} [g(x) - 3f(x)] = \lim_{x\to 1} g(x) - 3\lim_{x\to 1} f(x) = 4 - 3(-5) = 19$

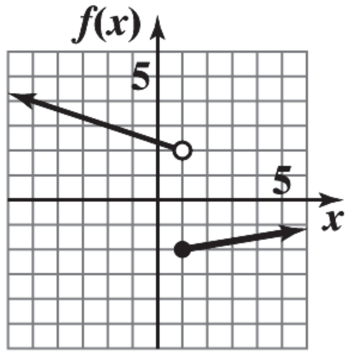
44. $\lim\limits_{x\to 1} \dfrac{3-f(x)}{1-4g(x)} = \dfrac{\lim\limits_{x\to 1}[3-f(x)]}{\lim\limits_{x\to 1}[1-4g(x)]} = \dfrac{3-\lim\limits_{x\to 1} f(x)}{1-4\lim\limits_{x\to 1} g(x)} = \dfrac{3-(-5)}{1-4(4)} = -\dfrac{8}{15}$

46. $\lim\limits_{x\to 1} \sqrt[3]{2x+2f(x)} = \sqrt[3]{\lim\limits_{x\to 1}[2x+2f(x)]}$

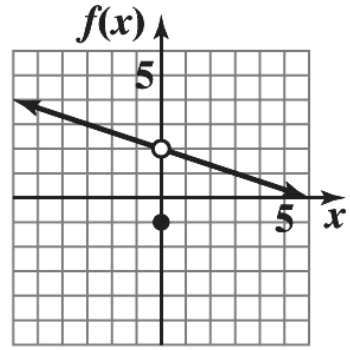
$= \sqrt[3]{2\lim\limits_{x\to 1} x + 2\lim\limits_{x\to 1} f(x)}$

$= \sqrt[3]{2-10} = -2$

48.



50.



52. $f(x) = \begin{cases} 2+x & \text{if } x \le 0 \\ 2-x & \text{if } x > 0 \end{cases}$

(A) $\lim\limits_{x\to 0^+} f(x) = \lim\limits_{x\to 0^+} (2-x) = 2$

(B) $\lim\limits_{x\to 0^-} f(x) = \lim\limits_{x\to 0^-} (2+x) = 2$

(C) $\lim\limits_{x\to 0} f(x) = 2$ since $\lim\limits_{x\to 0^+} f(x) = \lim\limits_{x\to 0^-} f(x) = 2$

(D) $f(0) = 2 + 0 = 2$

54. $f(x) = \begin{cases} x+3 & \text{if } x < -2 \\ \sqrt{x+2} & \text{if } x > -2 \end{cases}$

(A) $\lim\limits_{x\to -2^+} f(x) = \lim\limits_{x\to -2^+} \sqrt{x+2} = 0$

(B) $\lim\limits_{x\to -2^-} f(x) = \lim\limits_{x\to -2^-} (x+3) = 1$

(C) $\lim\limits_{x\to -2} f(x)$ does not exist since $\lim\limits_{x\to -2^+} f(x) \ne \lim\limits_{x\to -2^-} f(x)$

(D) $f(-2)$ does not exist; f is not defined at $x = -2$.

56. $f(x) = \begin{cases} \dfrac{x}{x+3} & \text{if } x < 0 \\ \dfrac{x}{x-3} & \text{if } x > 0 \end{cases}$

(A) $\lim\limits_{x\to -3} f(x) = \lim\limits_{x\to -3} \dfrac{x}{x+3}$ does not exist since $x = -3$ is a non-removable zero of the denominator.

(B) $\lim\limits_{x\to 0} f(x) = \lim\limits_{x\to 0^-} \dfrac{x}{x+3} = \lim\limits_{x\to 0^+} \dfrac{x}{x+3} = 0$

(C) $\lim\limits_{x\to 3} f(x)$ does not exist, since $\lim\limits_{x\to 3^+} f(x)$ does not exist.

58. $f(x) = \dfrac{x-3}{|x-3|} = \begin{cases} \dfrac{x-3}{-(x-3)} = -1 & \text{if} \quad x < 3 \\ \dfrac{x-3}{x-3} = 1 & \text{if} \quad x > 3 \end{cases}$

(Note: Observe that for $x < 3$, $|x-3| = 3 - x = -(x-3)$ and for $x > 3$, $|x-3| = x - 3$)

(A) $\lim_{x\to 3^+} f(x) = \lim_{x\to 3^+} 1 = 1$ (B) $\lim_{x\to 3^-} f(x) = \lim_{x\to 3^-} (-1) = -1$

(C) $\lim_{x\to 3} f(x)$ does not exist, since $\lim_{x\to 3^+} f(x) \neq \lim_{x\to 3^-} f(x)$

(D) $f(3)$ does not exist; f is not defined at $x = 3$.

60. $f(x) = \dfrac{x+3}{x^2+3x} = \dfrac{x+3}{x(x+3)}$

(A) $\lim_{x\to -3} \dfrac{x+3}{x(x+3)} = \lim_{x\to -3} \dfrac{1}{x} = -\dfrac{1}{3}$ (B) $\lim_{x\to 0} f(x) = \lim_{x\to 0} \dfrac{1}{x}$ does not exist. (C) $\lim_{x\to 3} \dfrac{1}{x} = \dfrac{1}{3}$

62. $f(x) = \dfrac{x^2+x-6}{x+3} = \dfrac{(x+3)(x-2)}{(x+3)}$

(A) $\lim_{x\to -3} f(x) = \lim_{x\to -3} \dfrac{(x+3)(x-2)}{(x+3)} = \lim_{x\to -3} (x-2) = -5$

(B) $\lim_{x\to 0} f(x) = \lim_{x\to 0} \dfrac{x^2+x-6}{x+3} = \dfrac{-6}{3} = -2$

(C) $\lim_{x\to 2} f(x) = \lim_{x\to 2} \dfrac{x^2+x-6}{x+3} = \dfrac{0}{5} = 0$

64. $f(x) = \dfrac{x^2-1}{(x+1)^2} = \dfrac{(x-1)(x+1)}{(x+1)^2}$

(A) $\lim_{x\to -1} f(x) = \lim_{x\to -1} \dfrac{(x-1)(x+1)}{(x+1)^2} = \lim_{x\to -1} \dfrac{x-1}{x+1}$ does not exist since $\lim_{x\to -1} (x-1) = -2$ but $\lim_{x\to -1} (x+1) = 0$.

(B) $\lim_{x\to 0} f(x) = \lim_{x\to 0} \dfrac{x^2-1}{(x+1)^2} = \dfrac{-1}{1} = -1$

(C) $\lim_{x\to 1} f(x) = \lim_{x\to 1} \dfrac{x^2-1}{(x+1)^2} = \dfrac{0}{4} = 0$

66. $f(x) = \dfrac{3x^2+2x-1}{x^2+3x+2} = \dfrac{(3x-1)(x+1)}{(x+2)(x+1)}$

(A) $\lim_{x\to -3} f(x) = \lim_{x\to -3} \dfrac{3x^2+2x-1}{x^2+3x+2} = \dfrac{20}{2} = 10$

(B) $\lim_{x\to -1} f(x) = \lim_{x\to -1} \dfrac{(3x-1)(x+1)}{(x+2)(x+1)} = \lim_{x\to -1} \dfrac{3x-1}{x+2} = \dfrac{-4}{1} = -4$

(C) $\lim_{x\to 2} f(x) = \lim_{x\to 2} \dfrac{3x^2+2x-1}{x^2+3x+2} = \dfrac{15}{12} = \dfrac{5}{4}$

68. True: $\lim\limits_{x\to 1}\dfrac{f(x)}{g(x)}=\dfrac{\lim\limits_{x\to 1}f(x)}{\lim\limits_{x\to 1}g(x)}=\dfrac{1}{1}=1$

70. Not always true. For example, the statement is false for $f(x)=\begin{cases}-1 & x\le 0\\ 1 & x>0\end{cases}$

72. Not always true. For example, the statement is false for $f(x)=\dfrac{1}{x}$.

74. $\lim\limits_{x\to -3}\dfrac{x-2}{x+3}$ does not have the form $\dfrac{0}{0}$; the limit does not exist since

$\lim\limits_{x\to -3^-}\dfrac{x-2}{x+3}=\infty,\ \lim\limits_{x\to -3^+}\dfrac{x-2}{x+3}=-\infty.$

76. $\lim\limits_{x\to 3}\dfrac{(x+1)(x-3)}{(x-3)(x-4)}$ has the form $\dfrac{0}{0}$; $\dfrac{(x+1)(x-3)}{(x-3)(x-4)}=\dfrac{x+1}{x-4}$ provided $x\ne 3$.

Therefore $\lim\limits_{x\to 3}\dfrac{(x+1)(x-3)}{(x-3)(x-4)}=\lim\limits_{x\to 3}\dfrac{x+1}{x-4}=-4.$

78. $\lim\limits_{x\to 5}\dfrac{x^2-7x+10}{x^2-4x-5}$ has the form $\dfrac{0}{0}$; $\dfrac{x^2-7x+10}{x^2-4x-5}=\dfrac{(x-5)(x+2)}{(x-5)(x+1)}=\dfrac{x-5}{x+1}$, provided $x\ne 5$.

Therefore $\lim\limits_{x\to 5}\dfrac{x^2-7x+10}{x^2-4x-5}=\lim\limits_{x\to 5}\dfrac{x-2}{x+1}=\dfrac{1}{2}.$

80. $\lim\limits_{x\to 2}\dfrac{x^2+2x+1}{x^2-2x+1}=\lim\limits_{x\to 2}\dfrac{(x+1)^2}{(x-1)^2}$ does not have the form $\dfrac{0}{0}$; $\lim\limits_{x\to 2}\dfrac{x^2+2x+1}{x^2-2x+1}=9.$

82. $f(x)=5x-1$

$$\lim_{h\to 0}\frac{f(2+h)-f(2)}{h}=\lim_{h\to 0}\frac{5(2+h)-1-(10-1)}{h}=\lim_{h\to 0}\frac{10+5h-1-9}{h}=\lim_{h\to 0}\frac{5h}{h}=\lim_{h\to 0}5=5$$

84. $f(x)=x^2-2$

$$\lim_{h\to 0}\frac{f(2+h)-f(2)}{h}=\lim_{h\to 0}\frac{(2+h)^2-2-(4-2)}{h}=\lim_{h\to 0}\frac{4+4h+h^2-2-2}{h}$$

$$=\lim_{h\to 0}\frac{4h+h^2}{h}=\lim_{h\to 0}(4+h)=4$$

86. $f(x)=-4x+13$

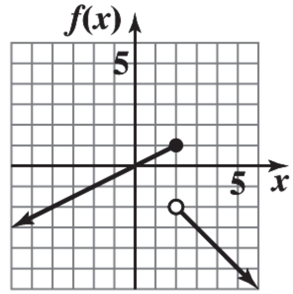
$$\lim_{h\to 0}\frac{f(2+h)-f(2)}{h}=\lim_{h\to 0}\frac{-4(2+h)+13-[-4(2)+13]}{h}=\lim_{h\to 0}\frac{-4h}{h}=-4$$

88. $f(x)=-3|x|$

$$\lim_{h\to 0}\frac{f(2+h)-f(2)}{h}=\lim_{h\to 0}\frac{-3|2+h|-[-3(2)]}{h}=\lim_{h\to 0}\frac{-3(2+h)+6}{h}=\lim_{h\to 0}\frac{-3h}{h}=-3$$

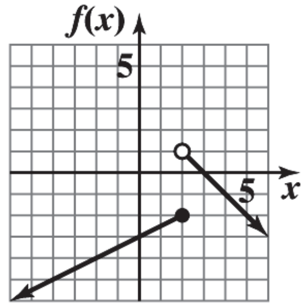
90. (A) $\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-} (0.5x) = 1$

$\lim_{x\to 2^+} f(x) = \lim_{x\to 2^+} (-x) = -2$



(B) $\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-} (-3 + 0.5x) = -2$

$\lim_{x\to 2^+} f(x) = \lim_{x\to 2^+} (3 - x) = 1$



(C) $\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-} (-3m + 0.5x) = -3m + 1$

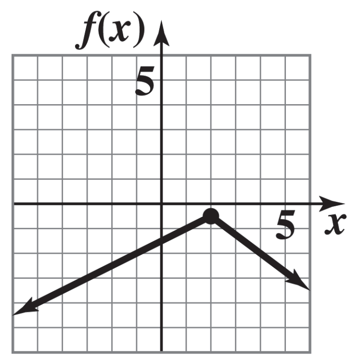
$\lim_{x\to 2^+} f(x) = \lim_{x\to 2^+} (3m - x) = 3m - 2$

$-3m + 1 = 3m - 2$

$6m = 3$

$m = \frac{1}{2} = 0.5$

$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^+} f(x) = -0.5$

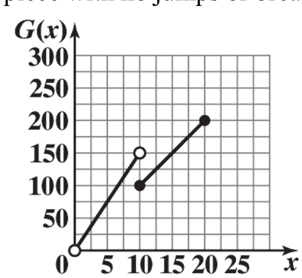


(D) The graph in (A) is broken when it jumps from (2, 1) down to (2, –2), the graph in (B) is also broken when it jumps from (2, –2) up to (2, 1), while the graph in (C) is one continuous piece with no jumps or breaks.

92. (A) For car sharing of not more than 10 hours, the charge per hour is *15x*. The charge per hour for sharing more than 10 hours is 10*x*. Thus,

$$G(x) = \begin{cases} 15x & \text{if } \quad 0 < x \le 10 \\ 10x & \text{if } \quad x > 10 \end{cases}$$

(B)



(C) As x approaches 10 from the left, $G(x)$ approaches 150, thus, the left limit of $G(x)$ at $x = 10$ exists, $\lim_{x\to 10^-} G(x) = 150$.

Similarly, $\lim_{x\to 10^+} G(x) = 100$. However, $\lim_{x\to 10} G(x)$ does not exist, since $\lim_{x\to 10^-} G(x) \neq \lim_{x\to 10^+} G(x)$.

94. For car sharing of more more than 10 hours per month, the charge for the service given in Problem 91 is *9x*-40 while the charge for in Problem 92 is 10*x*. It is clear that the latter is more expensive than the former.

96. (A) Let x be the volume of a purchase before the discount is applied. Then $P(x)$ is given by:

$$P(x) = \begin{cases} x & \text{if } 0 \le x < 300 \\ 300 + 0.97(x - 300) = 0.97x + 9 & \text{if } 300 \le x < 1,000 \\ 0.97(1,000) + 9 + 0.95(x - 1,000) = 0.95x + 29 & \text{if } 1,000 \le x < 3,000 \\ 0.95(3,000) + 29 + 0.93(x - 3,000) = 0.93x + 89 & \text{if } 3,000 \le x < 5,000 \\ 0.93(5,000) + 89 + 0.90(x - 5,000) = 0.90x + 239 & \text{if } x \ge 5,000 \end{cases}$$

(B) $\lim_{x \to 1,000^-} P(x) = 0.97(1,000) + 9 = 979$

$\lim_{x \to 1,000^+} P(x) = 0.95(1,000) + 29 = 979$

Thus, $\lim_{x \to 1,000} P(x) = 979$

$\lim_{x \to 3,000^-} P(x) = 0.95(3,000) + 29 = 2,879$

$\lim_{x \to 3,000^+} P(x) = 0.93(3,000) + 89 = 2,879$

Thus, $\lim_{x \to 3,000} P(x) = 2,879$

(C) For $0 \le x < 300$, they produce the same price. For $x \ge 300$, the one in Problem 95 produces a lower price.

98. From Problem 97, we have:

$$F(x) = \begin{cases} 20x & \text{if } 0 < x \le 4,000 \\ 80,000 & \text{if } x > 4,000 \end{cases}$$

Thus

$$A(x) = \frac{F(x)}{x} = \begin{cases} 20 & \text{if } 0 < x \le 4,000 \\ \dfrac{80,000}{x} & \text{if } x > 4,000 \end{cases}$$

$$\lim_{x \to 4,000^-} A(x) = \lim_{x \to 4,000^+} A(x) = 20 = \lim_{x \to 4,000} A(x)$$

$$\lim_{x \to 8,000^-} A(x) = \lim_{x \to 8,000^+} A(x) = \frac{80,000}{8,000} = 10 = \lim_{x \to 8,000} A(x)$$

EXERCISE 2-2

2. $x = 5$

4. $y = 1$

6. $y + 4 = -3(x - 8)$ (point-slope form); $3x + y = 20$

8. Slope: $m = \dfrac{30 - 20}{1 - (-1)} = 5$; $y - 20 = 5[x - (-1)]$ (point-slope form); $-5x + y = 25$

10. $\lim_{x \to -\infty} f(x) = \infty$

12. $\lim_{x \to -2^-} f(x) = \infty$

14. $\lim_{x \to 2^+} f(x) = \infty$

16. $\lim_{x \to 2} f(x)$ does not exist

18. $f(x) = \frac{x^2}{x+3}$

(A) $\lim_{x\to -3^-} \frac{x^2}{x+3} = -\infty$; as x approaches –3 from the left, the denominator is negatively approaching 0 and the numerator is positively approaching $(-3)^2 = 9$.

(B) $\lim_{x\to -3^+} \frac{x^2}{x+3} = \infty$; numerator approaches $(-3)^2 = 9$ and denominator is positively approaching 0.

(C) Since left and right limits at –3 are not equal, $\lim_{x\to -3} f(x)$ does not exist.

20. $f(x) = \frac{2x+2}{(x+2)^2}$

(A) $\lim_{x\to -2^-} \frac{2x+2}{(x+2)^2} = -\infty$; as x approaches –2 from the left, the denominator is positively approaching 0 and the numerator is negatively approaching $2(-2) + 2 = -2$.

(B) $\lim_{x\to -2^+} \frac{2x+2}{(x+2)^2} = -\infty$; as x approaches –2 from the right, the denominator is positively approaching 0 and the numerator is negatively approaching $2(-2) + 2 = -2$.

(C) Since $\lim_{x\to -2^-} f(x) = \lim_{x\to -2^+} f(x) = -\infty$, we can say that $\lim_{x\to -2} f(x) = -\infty$.

22. $f(x) = \frac{x^2+x+2}{x-1}$

(A) $\lim_{x\to 1^-} \frac{x^2+x+2}{x-1} = -\infty$; as x approaches 1, the numerator approaches 4 and the denominator negatively approaches 0.

(B) $\lim_{x\to 1^+} \frac{x^2+x+2}{x-1} = \infty$; in this case the denominator positively approaches 0.

(C) $\lim_{x\to 1} \frac{x^2+x+2}{x-1}$ does not exist.

24. $f(x) = \frac{x^2+x-2}{(x+2)}$

$f(x) = \frac{(x-1)(x+2)}{(x+2)}$

(A) $\lim_{x\to -2^-} \frac{(x-1)(x+2)}{(x+2)} = \lim_{x\to -2^-} (x-1) = -3$

(B) $\lim_{x\to -2^+} \frac{(x-1)(x+2)}{(x+2)} = \lim_{x\to -2^+} (x-1) = -3$

(C) $\lim_{x\to -2} \frac{(x-1)(x+2)}{(x+2)} = \lim_{x\to -2} (x-1) = -3$ or we can say that left and right limits at $x = -2$ exist and are equal, therefore $\lim_{x\to -2} f(x)$ exists and is equal to the common value –3.

26. $p(x) = 10 - x^6 + 7x^3 = -x^6 + 7x^3 + 10$

(A) Leading term: $-x^6$ (B) $\lim_{x\to\infty} p(x) = \lim_{x\to\infty}(-x^6) = -\infty$ (C) $\lim_{x\to-\infty} p(x) = \lim_{x\to-\infty}(-x^6) = -\infty$

28. $p(x) = -x^5 + 2x^3 + 9x$

(A) Leading term: $-x^5$ (B) $\lim_{x\to\infty} p(x) = \lim_{x\to\infty}(-x^5) = -\infty$ (C) $\lim_{x\to-\infty} p(x) = \lim_{x\to-\infty}(-x^5) = \infty$

30. $p(x) = 5x + x^3 - 8x^2 = x^3 - 8x^2 + 5x$

(A) Leading term: x^3 (B) $\lim_{x\to\infty} p(x) = \lim_{x\to\infty}(x^3) = \infty$ (C) $\lim_{x\to-\infty} p(x) = \lim_{x\to-\infty}(x^3) = -\infty$

32. $p(x) = 1 + 4x^2 + 4x^4 = 4x^4 + 4x^2 + 1$

(A) Leading term: $4x^4$ (B) $\lim_{x\to\infty} p(x) = \lim_{x\to\infty}(4x^4) = \infty$ (C) $\lim_{x\to-\infty} p(x) = \lim_{x\to-\infty}(4x^4) = \infty$

34. (A) $f(5) = \frac{2-3(5)^3}{7+4(5)^3} = -\frac{373}{507} \approx -0.736$

(B) $f(10) = \frac{2-3(10)^3}{7+4(10)^3} = -\frac{2,998}{4,007} \approx -0.748$

(C) $\lim_{x\to\infty} \frac{2-3x^3}{7+4x^3} = \lim_{x\to\infty} \frac{-3x^3}{4x^3} = \lim_{x\to\infty} \frac{\frac{2}{x^3}-3}{\frac{7}{x^3}+4}$ (Divide numerator and denominator by x^3.)

$$= \frac{0-3}{0+4} = \frac{-3}{4}.$$

36. (A) $f(-8) = \frac{5(-8)+11}{7(-8)^3-2} = \frac{-29}{-3,586} = \frac{29}{3,586} \approx 0.008$

(B) $f(-16) = \frac{5(-16)+11}{7(-16)^3-2} = \frac{-69}{-28,674} = \frac{69}{28,674} \approx 0.002$

(C) $\lim_{x\to\infty} \frac{5x+11}{7x^3-2} = \lim_{x\to\infty} \frac{\frac{5}{x^2}+\frac{11}{x^3}}{7-\frac{2}{x^3}}$ (Divide numerator and denominator by x^3.)

$$= \frac{0+0}{7-0} = 0$$

38. (A) $f(-3) = \frac{4(-3)^7 - 8(-3)}{6(-3)^4 + 9(-3)^2} = -\frac{8,724}{567} \approx -15.386$

(B) $f(-6) = \frac{4(-6)^7 - 8(-6)}{6(-6)^4 + 9(-6)^2} = -\frac{1,119,696}{8,100} \approx -138.234$

(C) $\lim_{x\to-\infty} \frac{4x^7 - 8x}{6x^4 + 9x^2} = \lim_{x\to-\infty} \frac{4x^3 - \frac{8}{x^3}}{6 + \frac{9}{x^2}}$ (Divide numerator and denominator by x^4.)

As $x \to -\infty$, $4x^3 - \frac{8}{x^3} \to -\infty$ and $6 + \frac{9}{x^2} \to 6$. Therefore, $\lim_{x\to-\infty} \frac{4x^7 - 8x}{6x^4 + 9x^2} = -\infty$.

40. (A) $f(-50) = \frac{3 + (-50)}{5 + 4(-50)} = \frac{47}{195} \approx 0.241$

(B) $f(-100) = \frac{3 + (-100)}{5 + 4(-100)} = \frac{97}{395} \approx 0.246$

(C) $\lim_{x\to-\infty} \frac{3 + x}{5 + 4x} = \lim_{x\to\infty} \frac{\frac{3}{x} + 1}{\frac{5}{x} + 4}$ (Divide numerator and denominator by x.)

$= \frac{0 + 1}{0 + 4} = \frac{1}{4}$

42. $f(x) = \frac{2x}{x - 5}$; $\lim_{x\to5^-} f(x) = -\infty$, $\lim_{x\to5^+} f(x) = \infty$; $x = 5$ is a vertical asymptote.

44. $f(x) = \frac{x + 2}{x^2 + 3}$, the denominator has no zeros; no vertical asymptotes.

46. $f(x) = \frac{x - 5}{x^2 - 16} = \frac{x - 5}{(x + 4)(x - 4)}$; $\lim_{x\to-4^-} f(x) = -\infty$, $\lim_{x\to-4^+} f(x) = \infty$,

$\lim_{x\to4^-} f(x) = \infty$, $\lim_{x\to4^+} f(x) = -\infty$; $x = -4$ and $x = 4$ are vertical asymptotes.

48. $f(x) = \frac{x^2 - 1}{x^3 + 2x^2 + 3x} = \frac{(x + 1)(x - 1)}{x(x + 2)(x + 1)} = \frac{x - 1}{x(x + 2)}$, $x \neq -1$

$\lim_{x\to-2^-} f(x) = -\infty$, $\lim_{x\to-2^+} f(x) = \infty$, $\lim_{x\to-1} f(x) = 2$, $\lim_{x\to0^-} f(x) = \infty$, $\lim_{x\to0^+} f(x) = -\infty$;

$x = -2$ and $x = 0$ are vertical asymptotes.

50. $f(x) = \frac{x^2 + 2x - 15}{x^2 + 2x - 8} = \frac{(x + 5)(x - 3)}{(x + 4)(x - 2)}$;

$\lim_{x\to-4^-} f(x) = -\infty$, $\lim_{x\to-4^+} f(x) = \infty$, $\lim_{x\to2^-} f(x) = \infty$, $\lim_{x\to2^+} f(x) = -\infty$;

$x = -4$ and $x = 2$ are vertical asymptotes.

52. $f(x) = \frac{3x + 2}{x - 4}$

$\lim_{x\to\infty} f(x) = \lim_{x\to\infty} \frac{3x + 2}{x - 4} = \lim_{x\to\infty} \frac{3 + \frac{2}{x}}{1 - \frac{4}{x}} = \frac{3 + 0}{1 - 0} = 3$

So $y = 3$ is the horizontal asymptote.
Vertical asymptote: $x = 4$ (since $n(4) = 14$, $d(4) = 0$).

54. $f(x) = \dfrac{x^2-1}{x^2+2}$.

$$\lim_{x\to\infty} f(x) = \lim_{x\to\infty} \frac{x^2-1}{x^2+2} = \lim_{x\to\infty} \frac{1-\frac{1}{x^2}}{1+\frac{2}{x^2}}$$ (Dividing the numerator and denominator by x^2.)

$$= \frac{1-0}{1+0} = 1$$

So, the horizontal asymptote is: $y = 1$.
$d(x) = x^2 + 2 > 0$ so, there are no vertical asymptotes.

56. $f(x) = \dfrac{x}{x^2-4} = \dfrac{x}{(x-2)(x+2)}$

$$\lim_{x\to\infty} f(x) = \lim_{x\to\infty} \frac{x}{x^2-4} = \lim_{x\to\infty} \frac{\frac{1}{x}}{1-\frac{4}{x^2}} = \frac{0}{1-0} = 0,$$

so the horizontal asymptote is: $y = 0$.
Since $n(-2) = -2$, $n(2) = 2$, $d(-2) = d(2) = 0$, we have two vertical asymptotes: $x = -2$, $x = 2$.

58. $f(x) = \dfrac{x^2+9}{x}$

$$\lim_{x\to\infty} f(x) = \lim_{x\to\infty} \frac{x^2+9}{x} = \lim_{x\to\infty} \frac{1+\frac{9}{x^2}}{\frac{1}{x}} = \frac{1+0}{0} = \infty$$

So, there are no horizontal asymptotes. Since $n(0) = 9$, $d(0) = 0$,
$x = 0$ is the only vertical asymptote.

60. $f(x) = \dfrac{x+5}{x^2}$.

$$\lim_{x\to\infty} f(x) = \lim_{x\to\infty} \frac{x+5}{x^2} = \lim_{x\to\infty} \frac{\frac{1}{x}+\frac{5}{x^2}}{1} = \frac{0+0}{1} = 0,$$

so the horizontal asymptote is: $y = 0$.
Since $n(0) = 5$, $d(0) = 0$, $x = 0$ is the vertical asymptote.

62. $f(x) = \dfrac{2x^2+7x+12}{2x^2+5x-12}$.

$$\lim_{x\to\infty} f(x) = \lim_{x\to\infty} \frac{2x^2+7x+12}{2x^2+5x-12} = \lim_{x\to\infty} \frac{2x^2}{2x^2} = 1,$$

so, $y = 1$ is the horizontal asymptote.
Since $n(-4) = 16$, $n\left(\frac{3}{2}\right) = 27$, $d(-4) = d\left(\frac{3}{2}\right) = 0$, $x = -4$ and $x = \frac{3}{2}$ are the vertical asymptotes.

64. $f(x) = \dfrac{x^2 - x - 12}{2x^2 + 5x - 12}$

$\lim_{x\to\infty} f(x) = \lim_{x\to\infty} \dfrac{x^2 - x - 12}{2x^2 + 5x - 12} = \lim_{x\to\infty} \dfrac{x^2}{2x^2} = \dfrac{1}{2}$, so $y = \dfrac{1}{2}$ is the horizontal asymptote. Since $n(-4) = 8$, $n\left(\dfrac{3}{2}\right) = -11.25$, $d(-4) = d\left(\dfrac{3}{2}\right) = 0$, $x = -4$ and $x = \dfrac{3}{2}$ are the vertical asymptotes.

66. $f(x) = \dfrac{3 + 4x + x^2}{5 - x}$; $\lim_{x\to\infty} f(x) = \lim_{x\to\infty} \dfrac{3 + 4x + x^2}{5 - x} = \lim_{x\to\infty} \dfrac{x^2}{-x} = \lim_{x\to\infty} (-x) = -\infty$

68. $f(x) = \dfrac{4x + 1}{5x - 7}$; $\lim_{x\to\infty} f(x) = \lim_{x\to\infty} \dfrac{4x + 1}{5x - 7} = \lim_{x\to\infty} \dfrac{4x}{5x} = \dfrac{4}{5}$

70. $f(x) = \dfrac{2x + 3}{x^2 - 1}$; $\lim_{x\to-\infty} f(x) = \lim_{x\to-\infty} \dfrac{2x + 3}{x^2 - 1} = \lim_{x\to-\infty} \dfrac{2x}{x^2} = 0$

72. $f(x) = \dfrac{6 - x^4}{1 + 2x}$; $\lim_{x\to-\infty} f(x) = \lim_{x\to-\infty} \dfrac{6 - x^4}{1 + 2x} = \lim_{x\to-\infty} \dfrac{-x^4}{2x} = \infty$

74. False: $f(x) = \dfrac{1}{(x-2)(x+2)} = \dfrac{1}{x^2 - 4}$ has two vertical asymptotes.

76. True: Theorem 4 gives three possible cases, two of which give exactly one horizontal asymptote and one of which gives no horizontal asymptote.

78. False: $f(x) = \dfrac{x^2 + 2x}{x^2 + x + 2}$ crosses the horizontal asymptote $y = 1$ at $x = 2$.

80. $\lim_{x\to-\infty} (a_n x^n + a_{n-1} x^{n-1} + \ldots + a_0) = \infty$ if $a_n > 0$ and n an even positive integer, or $a_n < 0$ and n an odd positive integer.

$\lim_{x\to-\infty} (a_n x^n + a_{n-1} x^{n-1} + \ldots + a_0) = -\infty$ if $a_n > 0$ and n is an odd positive integer or $a_n < 0$ and n is an even positive integer.

82. (A) Since $C(x)$ is a linear function of x, it can be written in the form

$C(x) = mx + b$

Since the fixed costs are \$300, $b = 300$.

Also, $C(20) = 5100$, so

$5100 = m(20) + 300$

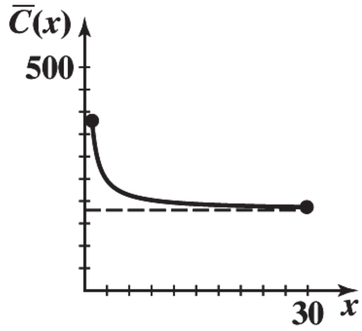
$20m = 4800$

$m = 240$

Therefore, $C(x) = 240x + 300$

(B) $\overline{C}(x) = \dfrac{C(x)}{x} = \dfrac{240x + 300}{x}$

(C)



(D) $\overline{C}(x) = \dfrac{240x + 300}{x}$

$= \dfrac{240 + \dfrac{300}{x}}{1}$

As x increases, the numerator tends to 240 and the denominator is 1. Therefore, $\overline{C}(x)$ tends to 240 or \$240 per board. Therefore, $\overline{C}(x)$ tends to \$240 per board.

$$\lim_{x\to\infty} \overline{C}_c(x) = \lim_{x\to\infty} \left(\frac{2{,}700}{x} + 1{,}332\right) = 0 + 1{,}332 = 1{,}332$$

84. $P(t) = \dfrac{99t^2}{t^2 + 50}$

(A) $P(5) = \dfrac{99(5)^2}{5^2 + 50} = \dfrac{2475}{75} = 33$ or 33%; $P(10) = \dfrac{99(10)^2}{10^2 + 50} = \dfrac{9900}{150} = 66$ or 66%

$P(20) = \dfrac{99(20)^2}{20^2 + 50} = \dfrac{39{,}600}{450} = 88$ or 88%

(B) $\lim\limits_{t\to\infty} P(t) = \lim\limits_{t\to\infty} \dfrac{99t^2}{t^2 + 50} = \lim\limits_{t\to\infty} \dfrac{99}{1 + \dfrac{50}{t^2}}$ (divide numerator and denominator by t^2)

$= \dfrac{99}{1+0} = 99,\ P(t) \to 99\%$

86. $C(t) = \dfrac{5t(t+50)}{t^3 + 100}$

$\lim\limits_{t\to\infty} C(t) = \lim\limits_{t\to\infty} \dfrac{5t^2 + 250t}{t^3 + 100}$ (Divide numerator and denominator by t^3.)

$= \lim\limits_{t\to\infty} \dfrac{\dfrac{5}{t} + \dfrac{250}{t^2}}{1 + \dfrac{100}{t^3}} = \dfrac{0+0}{1+0} = 0$

The long term drug concentration is 0 mg/ml.

88. $N(t) = \dfrac{100t}{t+9},\ t \ge 0$

(A) $N(6) = \dfrac{100(6)}{6+9} = \dfrac{600}{15} \approx 40$ components/day

(B) $70 = \dfrac{100t}{t+9}$ or

$70t + 630 = 100t$

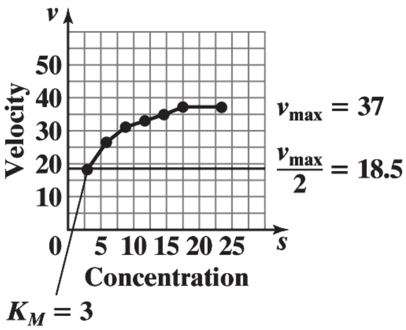
$30t = 630$

$t = \dfrac{630}{30} = 21$ days

(C) $\lim_{t\to\infty} N(t) = \lim_{t\to\infty} \frac{100t}{t+9} = \lim_{t\to\infty} \frac{100}{1+\frac{9}{t}} = \frac{100}{1+0} = 100$

The maximum number of components an employee can produce in consecutive days is 100 components.

90. (A) $v_{max} = 37, K_M = 3$



(B) $v(s) = \frac{37s}{3+s}$

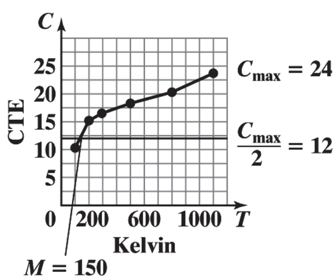
(C) For $s = 9$, $v = \frac{37(9)}{3+9} = 27.75$

For $v = 32$, $32 = \frac{37s}{3+s}$

or $96 + 32s = 37s$

and $s = \frac{96}{5} = 19.2$

92. (A) $C_{max} = 24, M = 150$



(B) $C(T) = \frac{24T}{150+T}$

(C) For $T = 600$,

$C = \frac{(24)(600)}{150+600} = 19.2$

For $C = 12$, $12 = \frac{24T}{150+T}$ or

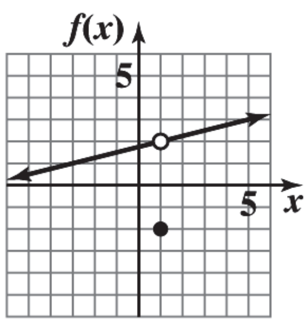
$1800 + 12T = 24T$,

$T = 150$.

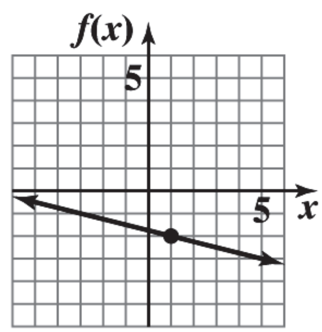
EXERCISE 2-3

2. $(-8,-4]$ **4.** $[0.1,0.3]$ **6.** $(-\infty,-4]\cup[4,\infty)$ **8.** $(-\infty,-6)\cup[9,\infty)$

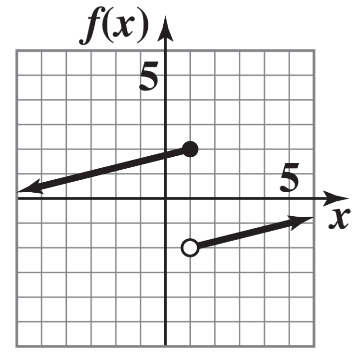
10. f is discontinuous at $x = 1$ since $\lim_{x\to1} f(x) \neq f(1)$



12. f is discontinuous at $x = 1$ since $\lim_{x\to1} f(x) = f(1)$



14. f is discontinuous at $x = 1$, since $\lim_{x \to 1} f(x)$ does not exist



16. $f(-2.1) = 1$

18. $f(-1.9) = 0.9$

20. (A) $\lim_{x \to 2^-} f(x) = 2$ (B) $\lim_{x \to 2^+} f(x) = 2$ (C) $\lim_{x \to 2} f(x) = 2$

$\left(\lim_{x \to 2^-} f(x) = \lim_{x \to 2^+} f(x) = 2 \right)$

(D) $f(2)$ does not exist; f is not defined at $x = 2$.(E) No, since f is not even defined at $x = 2$.

22. (A) $\lim_{x \to -1^-} f(x) = 0$

(B) $\lim_{x \to -1^+} f(x) = 0$

(C) $\lim_{x \to -1} f(x) = 0$ $\left(\lim_{x \to -1^-} f(x) = \lim_{x \to -1^+} f(x) = 0 \right)$

(D) $f(-1) = 0$

(E) Yes, since $\lim_{x \to -1} f(x) = f(0)$.

24. $g(-2.1) = 0.9$

26. $g(-1.9) = 2.95$

28. (A) $\lim_{x \to -2^-} g(x) = 1$

(B) $\lim_{x \to -2^+} g(x) = 3$

(C) $\lim_{x \to -2} g(x)$ does not exist, since $\lim_{x \to -2^-} g(x) \neq \lim_{x \to -2^+} g(x)$

(D) $g(-2)$ does not exist; g is not defined at $x = -2$. (E) No, since g is not even defined at $x = -2$.

30. (A) $\lim_{x \to 4^-} g(x) = 0$ (B) $\lim_{x \to 4^+} g(x) = 0$ (C) $\lim_{x \to 4} g(x) = 0$

$\left(\lim_{x \to 4^-} g(x) = \lim_{x \to 4^+} g(x) = 0 \right)$

(D) $g(4) = 0$. (E) Yes, since $\lim_{x \to 4} f(x) = f(4)$

32. $h(x) = 4 - 2x$ is a polynomial function. Therefore, f is continuous for all x [Theorem 1(C)].

34. $k(x) = \frac{2x}{x-4}$ is a rational function and the denominator $x - 4$ is 0 when $x = 4$. Thus, k is continuous for all x except $x = 4$ [Theorem 1(D)].

36. $n(x) = \frac{x-2}{(x-3)(x+1)}$ is a rational function and the denominator $(x - 3)(x + 1)$ is 0 when $x = 3$ or $x = -1$. Thus, n is continuous for all x except $x = 3, x = -1$ [Theorem 1(D)].

38. $G(x) = \frac{1-x^2}{x^2+1}$

$G(x)$ is a rational function and its denominator is never zero, hence by Theorem 1(D), $G(x)$ is continuous for all x.

40. $N(x) = \frac{x^2+4}{4-25x^2}$

$N(x)$ is a rational function and according to Theorem 1(D), $N(x)$ is continuous for all x except $x = \pm\frac{2}{5}$ which make the denominator 0.

42. $f(x) = \frac{2x+7}{5x-1}$; f is discontinuous at $x = \frac{1}{5}$; $f(x) = 0$ at $x = \frac{-7}{2}$. Partition numbers $\frac{1}{5}, \frac{-7}{2}$.

44. $f(x) = \frac{x^2+4}{x^2-9}$; f is discontinuous at $x = 3, -3$; $f(x) \neq 0$ for all x. Partition numbers $3, -3$.

46. $f(x) = \frac{x^3+x}{x^2-x-42} = \frac{x(x^2+1)}{(x-7)(x+6)}$; f is discontinuous at $x = 7, -6$; $f(x) = 0$ at $x = 0$. Partition numbers $-6, 0, 7$.

48. $x^2 - 2x - 8 < 0$

Let $f(x) = x^2 - 2x - 8 = (x - 4)(x + 2)$.
Then f is continuous for all x and $f(-2) = f(4) = 0$.
Thus, $x = -2$ and $x = 4$ are partition numbers.

Test Numbers

x	$f(x)$
–3	$7(+)$
0	$-8(-)$
5	$7(+)$

Thus, $x^2 - 2x - 8 < 0$ for: $-2 < x < 4$ (inequality notation), $(-2, 4)$ (interval notation)

50. $x^2 + 7x > -10$ or $x^2 + 7x + 10 > 0$

Let $f(x) = x^2 + 7x + 10 = (x + 2)(x + 5)$.
Then f is continuous for all x and $f(-5) = f(-2) = 0$.

Thus, $x = -5$ and $x = -2$ are partition numbers.

Test Numbers

x	$f(x)$
-6	$4(+)$
-4	$-2(-)$
0	$10(+)$

Thus, $x^2 + 7x + 10 > 0$ for: $x < -5$ or $x > -2$ (inequality notation), $(-\infty, -5) \cup (-2, \infty)$ (interval notation)

52. $x^4 - 9x^2 > 0$
$x^4 - 9x^2 = x^2(x^2 - 9)$
Since $x^2 > 0$ for $x \neq 0$, then $x^4 - 9x^2 > 0$ if $x^2 - 9 > 0$ or $x^2 > 9$
or "$x < -3$ or $x > 3$" or $(-\infty, -3) \cup (3, \infty)$.

54. $\dfrac{x-4}{x^2+2x} < 0$

Let $f(x) = \dfrac{x-4}{x^2+2x} = \dfrac{x-4}{x(x+2)}$. Then f is discontinuous at $x = 0$ and $x = -2$ and $f(4) = 0$. Thus, $x = -2$, $x = 0$, and $x = 4$ are partition numbers.

Test Numbers

x	$f(x)$
-3	$-\frac{7}{3}(-)$
-1	$5(+)$
1	$-1(-)$
5	$\frac{1}{35}(+)$

Thus, $\dfrac{x-4}{x^2+2x} < 0$ for: $x < -2$ or $0 < x < 4$ (inequality notation), $(-\infty, -2) \cup (0, 4)$(interval notation)

56. (A) $g(x) > 0$ for $x < -4$ or $x > 4$; $(-\infty, -4) \cup (4, \infty)$.

(B) $g(x) < 0$ for $-4 < x < 1$ or $1 < x < 4$; $(-4, 1) \cup (1, 4)$.

58. $f(x) = x^4 - 4x^2 - 2x + 2$. Partition numbers: $x_1 \approx 0.5113$, $x_2 \approx 2.1209$

(A) $f(x) > 0$ on $(-\infty, 0.5113) \cup (2.1209, \infty)$

(B) $f(x) < 0$ on $(0.5113, 2.1209)$

60. $f(x) = \dfrac{x^3 - 5x + 1}{x^2 - 1}$. Partition numbers: $x_1 \approx -2.3301$, $x_2 \approx -1$, $x_3 \approx 0.2016$, $x_4 = 1$, $x_5 \approx 2.1284$

(A) $f(x) > 0$ on $(-2.3301, -1) \cup (0.2016, 1) \cup (2.1284, \infty)$.

(B) $f(x) < 0$ on $(-\infty, -2.3301) \cup (-1, 0.2016) \cup (1, 2.1284)$.

62. $\sqrt{7-x}$

Let $f(x) = 7 - x$. Then $\sqrt{7-x} = \sqrt[2]{f(x)}$ is continuous whenever $f(x)$ is continuous and nonnegative [Theorem 1(F)]. Since $f(x) = 7 - x$ is continuous for all x [Theorem 1(C)] and $f(x) \geq 0$ for $x \leq 7$, $\sqrt{7-x}$ is continuous on $(-\infty, 7]$.

64. $\sqrt[3]{x-8}$

Let $f(x) = x - 8$. Then $\sqrt[3]{x-8} = \sqrt[3]{f(x)}$ is continuous whenever $f(x)$ is continuous [Theorem 1(E)]. Since $f(x) = x - 8$ is continuous for all x [Theorem 1(C)], $\sqrt[3]{x-8}$ is continuous on $(-\infty, \infty)$.

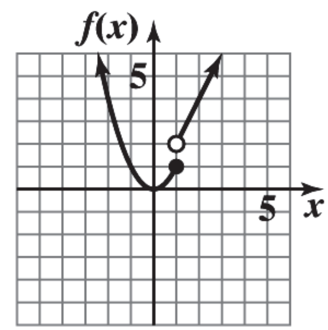
66. $\sqrt{4-x^2}$

Let $f(x) = 4 - x^2$. Then $\sqrt{4-x^2} = \sqrt[2]{f(x)}$ is continuous whenever $f(x)$ is continuous and nonnegative [Theorem 1(F)]. Since $f(x) = 4 - x^2$ is continuous for all x [Theorem 1(C)] and $f(x)$ is nonnegative on $[-2, 2]$, $\sqrt{4-x^2}$ is continuous on $[-2, 2]$.

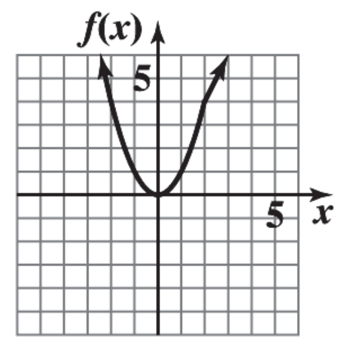
68. $\sqrt[3]{x^2+2}$

Let $f(x) = x^2 + 2$. Then $\sqrt[3]{x^2+2} = \sqrt[3]{f(x)}$ is continuous whenever $f(x)$ is continuous [Theorem 1(E)]. Since $f(x) = x^2 + 2$ is continuous for all x [Theorem 1(C)], $\sqrt[3]{x^2+2}$ is continuous on $(-\infty, \infty)$.

70. The graph of f is shown at the right. This function is discontinuous at $x = 1$.
[$\lim_{x\to 1^-} f(x) = 1$ and $\lim_{x\to 1^+} f(x) = 2$;
Thus, $\lim_{x\to 1} f(x)$ does not exist.]

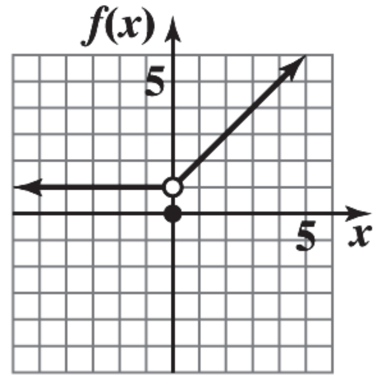


72. The graph of f is shown at the right.
This function is continuous for all x. [$\lim_{x\to 2} f(x) = f(2) = 4$]



74. The graph of f is shown at the right.

This function is discontinuous at $x = 0$, since $\lim_{x\to 0} f(x) = 1 \neq f(0) = 0$



76. (A) Since $\lim_{x\to 2^+} f(x) = f(2) = 2$, f is continuous from the right at $x = 2$.

(B) Since $\lim_{x\to 2^-} f(x) = 1 \neq f(2) = 2$, f is not continuous from the left at $x = 2$.

(C) f is continuous on the open interval (1, 2).

(D) f is *not* continuous on the closed interval [1, 2] since $\lim_{x\to 2^-} f(x) = 1 \neq f(2) = 2$, i.e., f is not continuous from the left at $x = 2$.

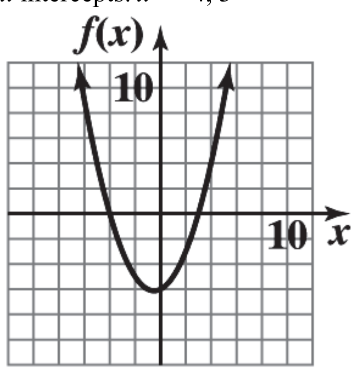
(E) f is continuous on the half-closed interval [1, 2).

78. True: If $r(x) = \dfrac{n(x)}{d(x)}$ is a rational function and $d(x)$ has degree n, then $r(x)$ has at most n points of discontinuity.

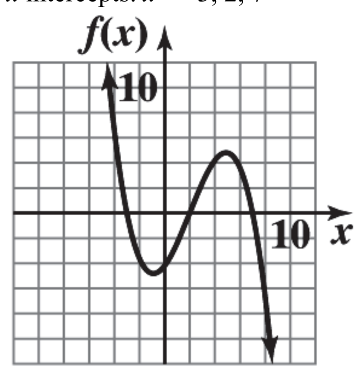
80. True: Continuous on (0, 2) means continuous at every real number x in (0, 2), including $x = 1$.

82. False. The greatest integer function has infinitely many points of discontinuity. See Prob. 75.

84. x intercepts: $x = -4, 3$



86. x intercepts: $x = -3, 2, 7$

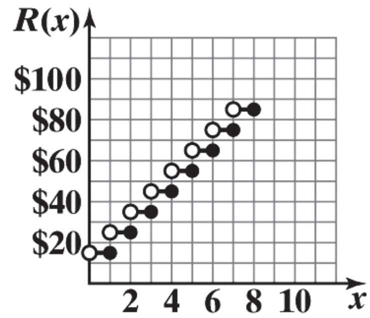


88. $f(x) = \dfrac{6}{x-4} \neq 0$ for all x. This does not contradict Theorem 2 because f is not continuous on (2, 7); f is discontinuous at $x = 4$.

90. (A)

$$\begin{cases} 15, & 0 \le x < 1 \\ 25, & 1 \le x < 2 \\ 35, & 2 \le x < 3 \\ 45, & 3 \le x < 4 \\ 55, & 4 \le x < 5 \\ 65, & 5 \le x < 6 \\ 75, & 6 \le x < 7 \\ 85, & 7 \le x < 8 \end{cases}$$

(B)



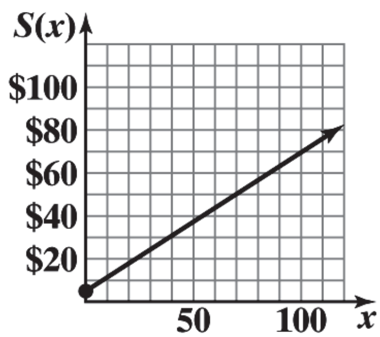
(C) $\lim_{x\to 3.5} R(x) = 45 = R(3.5)$; thus, $R(x)$ is continuous at $x = 3.5$.

$\lim_{x\to 4} R(x)$ does not exist; thus, $R(x)$ is not continuous at $x = 4$.

92. $S(x) = R(x)$.

94. (A) $S(x) = \begin{cases} 5 + 0.69x \text{ if } 0 \le x \le 5 \\ 5.2 + 0.65x \text{ if } 5 < x \le 50 \\ 6.2 + 0.63x \text{ if } 50 < x \end{cases}$

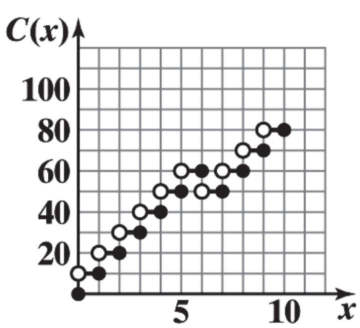
(B) The graph of S is:



(C) $\lim_{x\to 5} S(x) = 8.45 = S(5)$; thus, $S(x)$ is continuous at $x = 5$.

$\lim_{x\to 50} S(x) = 37.7 = S(50)$; thus, $S(x)$ is continuous at $x = 50$.

96. (A) The graph of $C(x)$ is:



(B) From the graph, $\lim_{x\to 4.5} C(x) = 50$ and $C(4.5) = 50$.

(C) From the graph, $\lim_{x\to 8} C(x)$ does not exist; $C(8) = 60$.

(D) Since $\lim_{x\to 4.5} C(x) = 50 = C(4.5)$, $C(x)$ is continuous at $x = 4.5$.
Since $\lim_{x\to 8} C(x)$ does not exist and $C(8) = 60$, $C(x)$ is not continuous at $x = 8$.

98. (A) From the graph, p is discontinuous at $t = t_2$, and $t = t_4$.

(B) $\lim_{t\to t_1} p(t) = 10$; $p(t_1) = 10$.

(C) $\lim_{t\to t_2} p(t) = 30$, $p(t_2) = 10$.

(D) $\lim_{t\to t_4} p(t)$ does not exist; $p(t_4) = 80$.

EXERCISE 2-4

2. Slope $m = \dfrac{8-11}{1-(-1)} = \dfrac{-3}{2},\ -1.5$

4. Slope $m = \dfrac{3-(-3)}{4-(-12)} = \dfrac{6}{16} = \dfrac{3}{8};\ 0.375$

6. $\dfrac{2}{\sqrt{5}} = \dfrac{1}{\sqrt{5}}\cdot\dfrac{\sqrt{5}}{\sqrt{5}} = \dfrac{2\sqrt{5}}{5}$

8. $\dfrac{1-\sqrt{2}}{5+\sqrt{2}} = \dfrac{1-\sqrt{2}}{5+\sqrt{2}}\cdot\dfrac{5-\sqrt{2}}{5-\sqrt{2}} = \dfrac{7-6\sqrt{2}}{23} = \dfrac{7}{23} - \dfrac{6}{23}\sqrt{2}$

10. (A) $\dfrac{f(-1)-f(-2)}{-1-(-2)} = \dfrac{4-1}{1} = 3$ is the slope of the secant line through $(-2, f(-2))$ and $(-1, f(-1))$.

(B) $\dfrac{f(-2+h)-f(-2)}{h} = \dfrac{5-(-2+h)^2-1}{h} = \dfrac{5-[4-4h+h^2]-1}{h}$

$= \dfrac{5-4+4h-h^2-1}{h} = \dfrac{4h-h^2}{h} = 4-h;$

slope of the secant line through $(-2, f(-2))$ and $(-2+h, f(-2+h))$

(C) $\lim_{h\to 0}\dfrac{f(-2+h)-f(-2)}{h} = \lim_{h\to 0}(4-h) = 4;$

slope of the tangent line at $(-2, f(-2))$

12. $f(x) = 3x^2$

(A) Slope of secant line through $(2, f(2))$ and $(5, f(5))$:

$\dfrac{f(5)-f(2)}{5-2} = \dfrac{3(5)^2-3(2)^2}{5-2} = \dfrac{75-12}{3} = \dfrac{63}{3} = 21$

(B) Slope of secant line through $(2, f(2))$ and $(2+h, f(2+h))$:

$\dfrac{3(2+h)^2-3(2)^2}{2+h-2} = \dfrac{3(4+4h+h^2)-12}{h} = \dfrac{12+12h+3h^2-12}{h} = \dfrac{12h+3h^2}{h} = 12+3h$

(C) Slope of the graph at $(2, f(2))$: $\lim_{h\to 0}\dfrac{f(2+h)-f(2)}{h} = \lim_{h\to 0}(12+3h) = 12.$

14. (A) Distance traveled for $0 \le t \le 4$: $352(1.5) = 528$; average velocity: $v = \dfrac{528}{4} = 132$ mph.

(B) $\dfrac{f(4)-f(0)}{4-0} = \dfrac{528}{4} = 132.$

(C) Slope at $x = 4$: $m = 150$. Equation of tangent line at $(4, f(4))$: $y - 528 = 150(x - 4)$ or $y = 150x - 72$.

16. $f(x) = \frac{1}{1+x^2}$; $f(2) = \frac{1}{5} = 0.2$. Equation of tangent line: $y - 0.2 = -0.16(x - 2)$ or $y = -0.16x + 3.4$.

18. $f(x) = x^4$; $f(-1) = 1$. Equation of tangent line: $y - 1 = -4(x + 1)$ or $y = -4x - 3$.

20. $f(x) = 9$

Step 1. Find $f(x + h)$.

$f(x + h) = 9$

Step 2. Find $f(x + h) - f(x)$.

$f(x + h) - f(x) = 9 - 9 = 0$

Step 3. Find $\frac{f(x+h)-f(x)}{h}$.

$$\frac{f(x+h)-f(x)}{h} = \frac{0}{h} = 0$$

Step 4. Find $f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} (0) = 0$

Thus, if $f(x) = 9$, then $f'(x) = 0$, $f'(1) = 0$, $f'(2) = 0$, $f'(3) = 0$.

22. $f(x) = 4 - 6x$

Step 1. $f(x + h) = 4 - 6(x + h) = 4 - 6x - 6h$

Step 2. $f(x + h) - f(x) = (4 - 6x - 6h) - (4 - 6x)$

$= 4 - 6x - 6h - 4 + 6x = -6h$

Step 3. $\frac{f(x+h)-f(x)}{h} = \frac{-6h}{h} = -6$

Step 4. $f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} (-6) = -6$

$f'(1) = -6$, $f'(2) = -6$, $f'(3) = -6$

24. $f(x) = 2x^2 + 8$

Step 1. $f(x + h) = 2(x + h)^2 + 8 = 2(x^2 + 2xh + h^2) + 8$

$= 2x^2 + 4xh + 2h^2 + 8$

Step 2. $f(x + h) - f(x) = (2x^2 + 4xh + 2h^2 + 8) - (2x^2 + 8)$

$= 2x^2 + 4xh + 2h^2 + 8 - 2x^2 - 8$

$= 4xh + 2h^2$

Step 3. $\frac{f(x+h)-f(x)}{h} = \frac{4xh+2h^2}{h} = 4x + 2h$

Step 4. $f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} (4x + 2h) = 4x$

$f'(1) = 4$, $f'(2) = 8$, $f'(3) = 12$

26. $f(x) = 3x^2 + 2x - 10$

Step 1. $f(x+h) = 3(x+h)^2 + 2(x+h) - 10 = 9x^2 + 6xh + 3h^2 + 2x + 2h - 10$

Step 2. $f(x+h) - f(x) = (3x^2 + 6xh + 3h^2 + 2x + 2h - 10) - (3x^2 + 2x - 10)$
$= 6xh + 3h^2 + 2h = h(6x + 3h + 2)$

Step 3. $\dfrac{f(x+h) - f(x)}{h} = \dfrac{h(6x+3h+2)}{h} = 6x + 3h + 2$

Step 4. $f'(x) = \lim_{h \to 0} \dfrac{f(x+h) - f(x)}{h} = \lim_{h \to 0} (6x + 3h + 2) = 6x + 2$

$f'(1) = 8, \quad f'(2) = 14, \quad f'(3) = 20$

28. $f(x) = x^2 - 4x + 7$

Step 1. $f(x+h) = (x+h)^2 - 4(x+h) + 7 = x^2 + 2xh + h^2 - 4x - 4h + 7$

Step 2. $f(x+h) - f(x) = x^2 + 2xh + h^2 - 4x - 4h + 7 - (x^2 - 4x + 7)$
$= 2xh + h^2 - 4h = h(2x + h - 4)$

Step 3. $\dfrac{f(x+h) - f(x)}{h} = \dfrac{h(2x+h-4)}{h} = 2x + h - 4$

Step 4. $f'(x) = \lim_{h \to 0} \dfrac{f(x+h) - f(x)}{h} = \lim_{h \to 0} (2x + h - 4) = 2x - 4$

$f'(1) = -2, \quad f'(2) = 0, \quad f'(3) = 2$

30. $f(x) = 6x^2 - 3x + 4$

Step 1. $f(x+h) = 6(x+h)^2 - 3(x+h) + 4 = 6x^2 + 12xh + 6h^2 - 3x - 3h + 4$

Step 2. $f(x+h) - f(x) = (6x^2 + 12xh + h^2 - 3x - 3h + 4) - (6x^2 - 3x + 4)$
$= 12xh + h^2 - 3h = h(12x + h - 3)$

Step 3. $\dfrac{f(x+h) - f(x)}{h} = \dfrac{h(12x+h-3)}{h} = 12x + h - 3$

Step 4. $f'(x) = \lim_{h \to 0} \dfrac{f(x+h) - f(x)}{h} = \lim_{h \to 0} (12x + h - 3) = 12x - 3$

$f'(1) = 9, \quad f'(2) = 21, \quad f'(3) = 33$

32. $f(x) = -x^2 + 3x + 2$

Step 1. $f(x+h) = -(x+h)^2 + 3(x+h) + 2 = -x^2 - 2xh - h^2 + 3x + 3h + 2$

Step 2. $f(x+h) - f(x) = (-x^2 - 2xh - h^2 + 3x + 3h + 2) - (-x^2 + 3x + 2)$
$= -2xh - h^2 + 3h = h(-2x - h + 3)$

Step 3. $\dfrac{f(x+h)-f(x)}{h} = \dfrac{h(-2x-h+3)}{h} = -2x-h+3$

Step 4. $f'(x) = \lim\limits_{h\to 0} \dfrac{f(x+h)-f(x)}{h} = = \lim\limits_{h\to 0}(-2x-h+3) = -2x+3$

$f'(1) = 1$, $f'(2) = -1$, $f'(3) = -3$

34. $f(x) = -2x^3 + 5$

Step 1. $f(x+h) = -2(x+h)^3 + 5 = -2(x^3 + 3x^2h + 3xh^2 + h^3) + 5$

$= -2x^3 - 6x^2h - 6xh^2 - 2h^3 + 5$

Step 2. $f(x+h) - f(x) = -2x^3 - 6x^2h - 6xh^2 - 2h^3 + 5 - (-2x^3 + 5)$

$= -6x^2h - 6xh^2 - 2h^3$

$= -2h(3x^2 + 3xh + h^2)$

Step 3. $\dfrac{f(x+h)-f(x)}{h} = \dfrac{-2h(3x^2+3xh+h^2)}{h} = -2(3x^2 + 3xh + h^2)$

Step 4. $f'(x) = \lim\limits_{h\to 0} \dfrac{f(x+h)-f(x)}{h} = \lim\limits_{h\to 0} \{-2(3x^2 + 3xh + h^2)\} = -6x^2$

$f'(1) = -6$, $f'(2) = -24$, $f'(3) = -54$

36. $f(x) = \dfrac{6}{x} - 2$

Step 1. $f(x+h) = \dfrac{6}{x+h} - 2$

Step 2. $f(x+h) - f(x) = \left(\dfrac{6}{x+h} - 2\right) - \left(\dfrac{6}{x} - 2\right)$

$= \dfrac{6}{x+h} - \dfrac{6}{x} = \dfrac{6x-6x-6h}{x(x+h)} = \dfrac{-6h}{x(x+h)}$

Step 3. $\dfrac{f(x+h)-f(x)}{h} = \dfrac{\frac{-6h}{x(x+h)}}{h} = -\dfrac{6}{x(x+h)}$

Step 4. $f'(x) = \lim\limits_{h\to 0} \dfrac{f(x+h)-f(x)}{h} = \lim\limits_{h\to 0} \dfrac{-6}{x(x+h)} = -\dfrac{6}{x^2}$

$f'(1) = -6$, $f'(2) = -\dfrac{6}{4} = -\dfrac{3}{2}$, $f'(3) = -\dfrac{6}{9} = -\dfrac{2}{3}$

38. $f(x) = 3 - 7\sqrt{x}$

Step 1. $f(x+h) = 3 - 7\sqrt{x+h}$

Step 2. $f(x+h) - f(x) = (3 - 7\sqrt{x+h}) - (3 - 7\sqrt{x}) = 7(\sqrt{x} - \sqrt{x+h})$

Step 3. $\dfrac{f(x+h)-f(x)}{h} = \dfrac{7(\sqrt{x}-\sqrt{x+h})}{h} = \dfrac{7(\sqrt{x}-\sqrt{x+h})}{h} \cdot \dfrac{(\sqrt{x}+\sqrt{x+h})}{(\sqrt{x}+\sqrt{x+h})}$

$= \dfrac{7(x-(x+h))}{h(\sqrt{x}+\sqrt{x+h})} = \dfrac{7(x-x-h)}{h(\sqrt{x}+\sqrt{x+h})}$

$= \dfrac{-7h}{h(\sqrt{x}+\sqrt{x+h})} = \dfrac{-7}{\sqrt{x}+\sqrt{x+h}}$

Step 4. $f'(x) = \lim_{h\to 0} \dfrac{f(x+h)-f(x)}{h} = \lim_{h\to 0}\left(\dfrac{-7}{\sqrt{x}+\sqrt{x+h}}\right) = \dfrac{-7}{2\sqrt{x}}$

$f'(1) = -\dfrac{7}{2}$, $f'(2) = -\dfrac{7}{2\sqrt{2}} = -\dfrac{7\sqrt{2}}{4}$, $f'(3) = -\dfrac{7}{2\sqrt{3}} = -\dfrac{7\sqrt{3}}{6}$

40. $f(x) = 16\sqrt{x+9}$

Step 1. $f(x+h) = 16\sqrt{x+h+9}$

Step 2. $f(x+h) - f(x) = 16\sqrt{x+h+9} - 16\sqrt{x+9}$
$= 16(\sqrt{x+h+9} - \sqrt{x+9})$

Step 3. $\dfrac{f(x+h)-f(x)}{h} = \dfrac{16(\sqrt{x+h+9}-\sqrt{x+9})}{h}$

$= \dfrac{16(\sqrt{x+h+9}-\sqrt{x+9})}{h} \cdot \dfrac{(\sqrt{x+h+9}+\sqrt{x+9})}{(\sqrt{x+h+9}+\sqrt{x+9})}$

$= \dfrac{16((x+h+9)-(x+9))}{h(\sqrt{x+h+9}+\sqrt{x+9})}$

$= \dfrac{16h}{h(\sqrt{x+h+9}+\sqrt{x+9})} = \dfrac{16}{\sqrt{x+h+9}+\sqrt{x+9}}$

Step 4. $f'(x) = \lim_{h\to 0} \dfrac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \dfrac{16}{\sqrt{x+h+9}+\sqrt{x+9}} = \dfrac{16}{2\sqrt{x+9}} = \dfrac{8}{\sqrt{x+9}}$

$f'(1) = \dfrac{8}{\sqrt{10}} = \dfrac{4\sqrt{10}}{5}$, $f'(2) = \dfrac{8}{\sqrt{11}} = \dfrac{8\sqrt{11}}{11}$, $f'(3) = \dfrac{8}{\sqrt{12}} = \dfrac{4\sqrt{3}}{3}$

42. $f(x) = \dfrac{1}{x+4}$.

Step 1. $f(x+h) = \dfrac{1}{x+4+h}$

Step 2. $f(x+h) - f(x) = \dfrac{1}{x+4+h} - \dfrac{1}{x+4} = \dfrac{x+4-(x+4+h)}{(x+4+h)(x+4)} = \dfrac{-h}{(x+4+h)(x+4)}$

Step3. $\dfrac{f(x+h)-f(x)}{h} = \dfrac{-h}{h(x+4+h)(x+4)} = \dfrac{-1}{(x+4+h)(x+4)}$

Step 4. $f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to 0}\frac{-1}{(x+4+h)(x+4)}=\frac{-1}{(x+4)^2}.$

$f'(1)=\frac{-1}{25},\quad f'(2)=\frac{-1}{36},\quad f'(3)=\frac{-1}{49}$

44. $f(x)=\frac{x}{x+2}$

Step1. $f(x+h)=\frac{x+h}{x+2+h}$

Step 2. $f(x+h)-f(x)=\frac{x+h}{x+2+h}-\frac{x}{x+2}=\frac{(x+h)(x+2)-x(x+2+h)}{(x+2+h)(x+2)}=\frac{2h}{(x+2+h)(x+2)}$

Step 3. $\frac{f(x+h)-f(x)}{h}=\frac{2h}{h(x+2+h)(x+2)}=\frac{2}{(x+2+h)(x+2)}$

Step 4. $f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to 0}\frac{2}{(x+2+h)(x+2)}=\frac{2}{(x+2)^2}.$

$f'(1)=\frac{2}{9},\quad f'(2)=\frac{2}{16}=\frac{1}{8},\quad f'(3)=\frac{2}{25}$

46. $y=f(x)=x^2+x$

(A) $f(2)=2^2+2=6, f(4)=4^2+4=20$

Slope of secant line: $\frac{f(4)-f(2)}{4-2}=\frac{20-6}{2}=\frac{14}{2}=7$

(B) $f(2)=6, f(2+h)=(2+h)^2+(2+h)=4+4h+h^2+2+h$
$=6+5h+h^2$

Slope of secant line: $\frac{f(2+h)-f(2)}{h}=\frac{6+5h+h^2-6}{h}$

$=\frac{5h+h^2}{h}=5+h$

(C) Slope of tangent line at $(2, f(2))$:

$\lim_{h\to 0}\frac{f(2+h)-f(2)}{h}=\lim_{h\to 0}(5+h)=5$

(D) Equation of tangent line at $(2, f(2))$:
$y-f(2)=f'(2)(x-2)$ or $y-6=5(x-2)$ and $y=5x-4$.

48. $f(x)=x^2+x$

(A) Average velocity: $\frac{f(4)-f(2)}{4-2}=\frac{(4)^2+4-((2)^2+2)}{2}=\frac{16+4-6}{2}=7$ meters per second

(B) Average velocity: $\frac{f(2+h)-f(2)}{h}=\frac{(2+h)^2+(2+h)-6}{h}=\frac{4+4h+h^2+2+h-6}{h}$

$=\frac{5h+h^2}{h}=5+h$ meters per second

(C) Instantaneous velocity: $\lim_{h\to 0}\frac{f(2+h)-f(2)}{h}=\lim_{h\to 0}(5+h)=5$ meters per second

50. $F'(x)$ does not exist at $x = b$.

52. $F'(x)$ does exist at $x = d$.

54. $F'(x)$ does not exist at $x = f$.

56. $F'(x)$ does not exist at $x = h$.

58. (A) To find f' use the two step process for the given function $f(x) = 4x - x^2 + 1$.

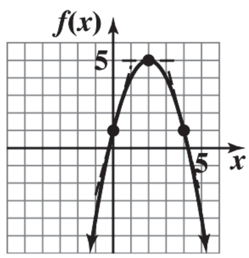
Step 1.

$$\begin{aligned}\frac{f(x+h)-f(x)}{h} &= \frac{[4(x+h)-(x+h)^2+1]-[4x-x^2+1]}{h}\\ &= \frac{(4x+4h-x^2-2xh-h^2+1)-(4x-x^2+1)}{h}\\ &= \frac{4h-2xh-h^2}{h} = 4-2x-h\end{aligned}$$

Step 2. $f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to 0}(4-2x-h) = 4-2x.$

(B) Slopes: at $x = 0$, $f'(0) = 4$; $x = 2$, $f'(2) = 0$; $x = 4$, $f'(4) = -4$

(C)



60. To find $v = f'(x)$, use the two-step process for the given distance function, $f(x) = 8x^2 - 4x$.

Step 1.

$$\begin{aligned}\frac{f(x+h)-f(x)}{h} &= \frac{8(x+h)^2-4(x+h)-(8x^2-4x)}{h}\\ &= \frac{8(x^2+2xh+h^2)-4x-4h-8x^2+4x}{h}\\ &= \frac{8x^2+16xh+8h^2-4x-4h-8x^2+4x}{h}\\ &= \frac{16xh-4h+8h^2}{h} = 16x-4+8h\end{aligned}$$

Step 2. $\lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to 0}(16x-4+8h) = 16x-4$

Thus, the velocity, $v = f'(x) = 16x - 4$

$f'(1) = 12$ feet per second, $f'(3) = 44$ feet per second, $f'(5) = 76$ feet per second

62. (A) The graphs of g and h are vertical translations of the graph of f. All Three functions should have the same derivatives; they differ from each other by a constant.

(B) $m(x) = -x^2 + c$

Step 1. $m(x + h) = -(x + h)^2 + c = -x^2 - 2xh - h^2 + c$

Step 2. $m(x + h) - m(x) = (-x^2 - 2xh - h^2 + c) - (-x^2 + c)$

$= -x^2 - 2xh - h^2 + c + x^2 - c = -2xh - h^2$

Step 3. $\dfrac{m(x+h)-m(x)}{h} = \dfrac{-2xh-h^2}{h} = -2x - h$

Step 4. $m'(x) = \lim\limits_{h\to 0} \dfrac{m(x+h)-m(x)}{h} = \lim\limits_{h\to 0} (-2x - h) = -2x$

64. True: $\lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h} = \lim\limits_{h\to 0}\dfrac{m(x+h)+b-(mx+b)}{h}$

$= \lim\limits_{h\to 0}\dfrac{mx+mh+b-mx-b}{h} = \lim\limits_{h\to 0}\dfrac{mh}{h} = \lim\limits_{h\to 0} m = m$

66. Let $c \in (a,b)$. We wish to show that $\lim\limits_{x\to c} f(x) = f(c)$. If we let $h = x - c$, then $x = h + c$, and this statement is equivalent to $\lim\limits_{h\to 0} f(c+h) = f(c)$, which is in turn equivalent to $\lim\limits_{h\to 0}\left(f(c+h) - f(c)\right) = 0$. Since $f'(x)$ exists at every point in the interval, we know that $f'(c)$ is defined and

$\lim\limits_{h\to 0}\dfrac{f(c+h)-f(c)}{h} = f'(c)$

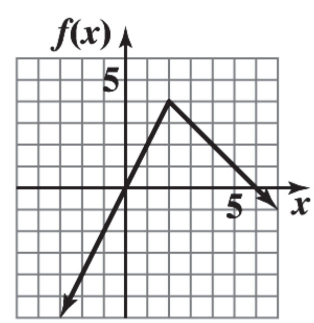
$\left(\lim\limits_{h\to 0} h\right)\left(\lim\limits_{h\to 0}\dfrac{f(c+h)-f(c)}{h}\right) = \left(\lim\limits_{h\to 0} h\right) f'(c)$

$\lim\limits_{h\to 0} h\left(\dfrac{f(c+h)-f(c)}{h}\right) = 0$

$\lim\limits_{h\to 0}\left(f(c+h) - f(c)\right) = 0$

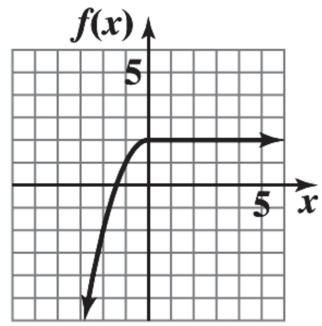
68. False. For example, $f(x) = |x|$ has a sharp corner at $x = 0$, but is continuous there.

70. The graph of $f(x) = \begin{cases} 2x & \text{if } x < 2 \\ 6 - x & \text{if } x \geq 2 \end{cases}$ is:



f is not differentiable at $x = 2$ because the graph of f has a sharp corner at this point.

72. $f(x) = \begin{cases} 2 - x^2 & \text{if } x \le 0 \\ 2 & \text{if } x > 0 \end{cases}$



It is clear that $f'(x) = \begin{cases} -2x & \text{if } x < 0 \\ 0 & \text{if } x > 2 \end{cases}$

Thus, the only question is $f'(0)$.
Since $\lim_{x\to 0^-} f'(x) = \lim_{x\to 0^-} (-2x) = 0$ and $\lim_{x\to 0^+} f'(x) = \lim_{x\to 0^+} (0) = 0$, f is differentiable at 0 as well;
f is differentiable for all real numbers.

74. $f(x) = 1 - |x|$

$$\lim_{h\to 0} \frac{f(0+h) - f(0)}{h} = \lim_{h\to 0} \frac{1 - |0+h| - (1 - |0|)}{h} = \lim_{h\to 0} -\frac{|h|}{h}$$

The limit does not exist. Thus, f is not differentiable at $x = 0$.

76. $f(x) = x^{2/3}$

$$\lim_{h\to 0} \frac{f(0+h) - f(0)}{h} = \lim_{h\to 0} \frac{(0+h)^{2/3} - 0^{2/3}}{h} = \lim_{h\to 0} \frac{h^{2/3}}{h} = \lim_{h\to 0} \frac{1}{h^{1/3}}$$

The limit does not exist. Thus, f is not differentiable at $x = 0$.

78. $f(x) = \sqrt{1 + x^2}$

$$\lim_{h\to 0} \frac{f(0+h) - f(0)}{h} = \lim_{h\to 0} \frac{\sqrt{1 + (0+h)^2} - \sqrt{1 + 0^2}}{h} = \lim_{h\to 0} \frac{\sqrt{1+h^2} - 1}{h}$$

$$= \lim_{h\to 0} \frac{\sqrt{1+h^2} - 1}{h} \cdot \frac{\sqrt{1+h^2} + 1}{\sqrt{1+h^2} + 1} = \lim_{h\to 0} \frac{1 + h^2 - 1}{h\left[\sqrt{1+h^2} + 1\right]} = \lim_{h\to 0} \frac{h}{\sqrt{1+h^2} + 1} = \frac{0}{2} = 0$$

f is differentiable at $x = 0$ and $f'(0) = 0$.

80. $y = 16x^2$

Now, if $y = 1{,}024$ ft, then

$$16x^2 = 1{,}024$$
$$x^2 = \frac{1{,}024}{16} = 64$$
$$x = 8 \text{ sec.}$$

$y' = 32x$ and at $x = 8$, $y' = 32(8) = 256$ ft/sec.

82. $P(x) = 45x - 0.025x^2 - 5{,}000,\ 0 \le x \le 2{,}400.$

(A) Average change $= \dfrac{P(850) - P(800)}{850 - 800}$

$$= \frac{[45(850) - 0.025(850)^2 - 5{,}000] - [45(800) - 0.025(800)^2 - 5{,}000]}{50}$$

$$= \frac{45(850) - 0.025(850)^2 - 45(800) + 0.025(800)^2}{50}$$

$$= \frac{54{,}250 - 54{,}062.5}{50} = \frac{187.5}{50} = \$3.75$$

(B) $P(x) = 45x - 0.025x^2 - 5{,}000$

Step 1. $P(x + h) = 45(x + h) - 0.025(x + h)^2 - 5{,}000$
$= 45x + 45h - 0.025x^2 - 0.05xh - 0.025h^2 - 5{,}000$

Step 2. $P(x + h) - P(x) = (45x + 45h - 0.025x^2 - 0.05xh - 0.025h^2 - 5{,}000) - (45x - 0.025x^2 - 5{,}000)$
$= 45h - 0.05xh - 0.025h^2$

Step 3. $\dfrac{P(x+h) - P(x)}{h} = \dfrac{45h - 0.05xh - 0.025h^2}{h} = 45 - 0.05x - 0.025h$

Step 4. $P'(x) = \lim\limits_{h\to 0} \dfrac{P(x+h) - P(x)}{h} = \lim\limits_{h\to 0} (45 - 0.05x - 0.025h) = 45 - 0.05x$

(C) $P(800) = 45(800) - 0.025(800)^2 - 5{,}000 = 15{,}000$
$P'(800) = 45 - 0.05(800) = 5;$
At a production level of 800 car seats, the profit is \$15,000 and is increasing at the rate of \$5 per seat.

84. $S(t) = \sqrt{t} + 8$

(A) Step 1. $S(t + h) = \sqrt{t+h} + 8$

Step 2. $S(t + h) - S(t) = \left(\sqrt{t+h} + 8\right) - \left(\sqrt{t} + 8\right) = \sqrt{t+h} - \sqrt{t}$

$$= \left(\sqrt{t+h} - \sqrt{t}\right) \cdot \frac{\sqrt{t+h} + \sqrt{t}}{\sqrt{t+h} + \sqrt{t}}$$

$$= \frac{(t+h) - t}{\sqrt{t+h} + \sqrt{t}} = \frac{h}{\sqrt{t+h} + \sqrt{t}}$$

Step 3. $\dfrac{S(t+h) - S(t)}{h} = \dfrac{\dfrac{h}{\sqrt{t+h} + \sqrt{t}}}{h} = \dfrac{1}{\sqrt{t+h} + \sqrt{t}}$

Step 4. $S'(t) = \lim\limits_{h\to 0} \dfrac{S(t+h) - S(t)}{h} = \lim\limits_{h\to 0} \dfrac{1}{\sqrt{t+h} + \sqrt{t}} = \dfrac{1}{2\sqrt{t}}$

(B) $S(9) = \sqrt{9} + 8 = 11$; $S'(9) = \dfrac{1}{2\sqrt{9}} = \dfrac{1}{6} \approx 0.167$

After 9 months, the total sales are \$11 million and are increasing at the rate of \$0.167 million = \$167,000 per month.

(C) The estimated total sales are \$11.167 million after 10 months and \$11.334 million after 11 months.

86. (A) $p(t) = 48t^2 - 37t + 1{,}698$

Step 1. $p(t+h) = 48(t+h)^2 - 37(t+h) + 1{,}698$
$= 48(t^2 + 2th + h^2) - 37t - 37h + 1{,}698$
$= 48t^2 + 96th + 48h^2 - 37t - 37h + 1{,}698$

Step 2. $p(t+h) - p(t) = 48t^2 + 96th + 48h^2 - 37t - 37h + 1{,}698 - (48t^2 - 37t + 1{,}698)$
$= 96th + 48h^2 - 37h$

Step 3. $\dfrac{p(t+h) - p(t)}{h} = \dfrac{96th + 48h^2 - 37h}{h} = 96t + 48h - 37$

Step 4. $p'(t) = \lim_{h\to 0} \dfrac{p(t+h) - p(t)}{h} = \lim_{h\to 0}(96t + 48h - 37) = 96t - 37$

(B) 2027 corresponds to $t = 17$. Thus
$p(17) = 48(17)^2 - 37(17) + 1{,}698 = 14{,}941$
$p'(17) = 96(17) - 37 = 1{,}595$
In 2027, 14,941 thousand tons of copper will be consumed and this quantity is increasing at the rate of 1,595 thousand tons/year.

88. (A) Quadratic regression model

```
QuadReg
 y=ax²+bx+c
 a=-1.763888889
 b=44.61071429
 c=1068.607143
```

$C(x) \approx -1.764x^2 + 44.611x + 1068.607, \quad C'(x) \approx -3.528x + 44.611.$

(B) $C(30) \approx -1.764(30)^2 + 44.611(30) + 1068.607 \approx 819.337$;
$C'(30) \approx -3.526(30) + 44.611 = -61.169$
In 2030, 819.3 billion kilowatts will be sold and the amount sold is decreasing at the rate of 61.2 billion kilowatts per year.

90. (A) $F(t) = 98 + \dfrac{4}{t+1}$

Step 1. $F(t+h) = 98 + \dfrac{4}{t+h+1}$

Step 2. $F(t+h) - F(t) = \left(98 + \dfrac{4}{t+h+1}\right) - \left(98 + \dfrac{4}{t+1}\right) = \dfrac{4}{t+h+1} - \dfrac{4}{t+1}$
$= 4\left[\dfrac{(t+1) - (t+h+1)}{(t+h+1)(t+1)}\right] = \dfrac{-4h}{(t+h+1)(t+1)}$

Step 3. $\dfrac{F(t+h)-F(t)}{h} = \dfrac{\dfrac{-4h}{(t+h+1)(t+1)}}{h} = \dfrac{-4}{(t+h+1)(t+1)}$

Step 4. $F'(t) = \lim_{h\to 0} \dfrac{F(t+h)-F(t)}{h} = \lim_{h\to 0} \dfrac{-4}{(t+h+1)(t+1)} = \dfrac{-4}{(t+1)^2}$

(B) $F(3) = 99$, $F'(3) = \dfrac{-4}{16} = \dfrac{-1}{4}$. The body temperature 3 hours after taking the medicine is 99° and is decreasing at the rate of 0.25° per hour.

EXERCISE 2-5

2. $\sqrt[3]{x} = x^{1/3}$

4. $\dfrac{1}{x} = x^{-1}$

6. $\dfrac{1}{(x^5)^2} = \dfrac{1}{x^{10}} = x^{-10}$

8. $\dfrac{1}{\sqrt[5]{x}} = \dfrac{1}{x^{1/5}} = x^{-1/5}$

10. $\dfrac{d}{dx}(5) = 0$ (Derivative of a constant rule.)

12. $y = x^8$

$y' = 8x^{8-1} = 8x^7$ (Power rule)

14. $g(x) = x^9$

$g'(x) = 9x^{9-1} = 9x^8$ (Power rule)

16. $y = x^{-5}$

$\dfrac{dy}{dx} = -5x^{-5-1} = -5x^{-6}$ (Power rule)

18. $f(x) = x^{5/2}$

$f'(x) = \dfrac{5}{2}x^{5/2-1} = \dfrac{5}{2}x^{3/2}$ (Power rule)

20. $y = \dfrac{1}{x^7} = x^{-7}$

$y' = -7x^{-7-1} = -7x^{-8} = \dfrac{-7}{x^8}$ (Power rule)

22. $\dfrac{d}{dx}(-3x^2) = -3(2x) = -6x$ (constant times a function rule)

24. $f(x) = 0.7x^3$

$f'(x) = 0.7(3x^2) = 2.1x^2$

26. $y = \dfrac{x^3}{9}$

$y' = \dfrac{1}{9}(3x^2) = \dfrac{x^2}{3}$

28. $h(x) = 5g(x)$; $h'(2) = 5g'(2) = 5(-1) = -5$

30. $h(x) = g(x) - f(x)$; $h'(2) = g'(2) - f'(2) = -1 - 3 = -4$

32. $h(x) = -4f(x) + 5g(x) - 9$; $h'(2) = -4f'(2) + 5g'(2) = -4(3) + 5(-1) = -17$

34. $\dfrac{d}{dx}(-4x + 9) = \dfrac{d}{dx}(-4x) + \dfrac{d}{dx}(9) = -4 + 0 = -4$

36. $y = 2 + 5t - 8t^3$

$\frac{dy}{dt} = 0 + 5 - 24t^2 = 5 - 24t^2$

38. $g(x) = 5x^{-7} - 2x^{-4}$

$g'(x) = (5) \cdot (-7)x^{-8} - (2) \cdot (-4)x^{-5}$

$= -35x^{-8} + 8x^{-5}$

40. $\frac{d}{du}(2u^{4.5} - 3.1u + 13.2) = (2) \cdot (4.5)u^{3.5} - 3.1 + 0 = 9u^{3.5} - 3.1$

42. $F(t) = 0.2t^3 - 3.1t + 13.2$

$F'(t) = (0.2) \cdot (3)t^2 - 3.1 + 0 = 0.6t^2 - 3.1$

44. $w = \frac{7}{5u^2} = \frac{7}{5}u^{-2}$

$w' = \left(\frac{7}{5}\right) \cdot (-2)u^{-3} = -\frac{14}{5}u^{-3}$

46. $\frac{d}{dx}\left(\frac{5x^3}{4} - \frac{2}{5x^3}\right) = \frac{d}{dx}\left(\left(\frac{5}{4}\right)x^3 - \left(\frac{2}{5}\right)x^{-3}\right) = \left(\frac{5}{4}\right) \cdot (3)x^2 - \left(\frac{2}{5}\right) \cdot (-3)x^{-4} = \frac{15}{4}x^2 + \frac{6}{5}x^{-4}$

48. $H(w) = \frac{5}{w^6} - 2\sqrt{w} = 5w^{-6} - 2w^{1/2}$

$H'(w) = (5) \cdot (-6)w^{-7} - (2) \cdot \left(\frac{1}{2}\right)w^{-1/2} = -30w^{-7} - w^{-1/2}$

50. $\frac{d}{du}(8u^{3/4} + 4u^{-1/4}) = (8) \cdot \left(\frac{3}{4}\right)u^{-1/4} + (4) \cdot \left(-\frac{1}{4}\right)u^{-5/4} = 6u^{-1/4} - u^{-5/4}$

52. $F(t) = \frac{5}{t^{1/5}} - \frac{8}{t^{3/2}} = 5t^{-1/5} - 8t^{-3/2}$

$F'(t) = (5) \cdot \left(-\frac{1}{5}\right)t^{-6/5} - (8) \cdot \left(-\frac{3}{2}\right)t^{-5/2} = -t^{-6/5} + 12t^{-5/2}$

54. $w = \frac{10}{\sqrt[5]{u}} = 10u^{-1/5}$

$w' = (10) \cdot \left(-\frac{1}{5}\right)u^{-6/5} = -2u^{-6/5}$

56. $\frac{d}{dx}\left(2.8x^{-3} - \frac{0.6}{\sqrt[3]{x^2}} + 7\right) = \frac{d}{dx}(2.8x^{-3} - 0.6x^{-2/3} + 7) = (2.8) \cdot (-3)x^{-4} - (0.6) \cdot \left(-\frac{2}{3}\right)x^{-5/3} + 0$

$= -8.4x^{-4} + 0.4x^{-5/3}$

58. $f(x) = 2x^2 + 8x$

(A) $f'(x) = 4x + 8$

(B) Slope of the graph of f at $x = 2$: $f'(2) = 4(2) + 8 = 16$
Slope of the graph of f at $x = 4$: $f'(4) = 4(4) + 8 = 24$

(C) Tangent line at $x = 2$: $y - y_1 = m(x - x_1)$
$x_1 = 2$
$y_1 = f(2) = 2(2)^2 + 8(2) = 24$
$m = f'(2) = 16$
Thus, $y - 24 = 16(x - 2)$ or $y = 16x - 8$
Tangent line at $x = 4$: $y - y_1 = m(x - x_1)$
$x_1 = 4$
$y_1 = f(4) = 2(4)^2 + 8(4) = 64$
$m = f'(4) = 24$
Thus, $y - 64 = 24(x - 4)$ or $y = 24x - 32$

(D) The tangent line is horizontal at the values $x = c$ such that $f'(c) = 0$. Thus, we must solve the following:
$f'(x) = 4x + 8 = 0$
$4x = -8$
$x = -2$

60. $f(x) = x^4 - 32x^2 + 10$

(A) $f'(x) = 4x^3 - 64x$

(B) Slope of the graph of f at $x = 2$: $f'(2) = 4(2)^3 - 64(2) = -96$
Slope of the graph of f at $x = 4$: $f'(4) = 4(4)^3 - 64(4) = 0$

(C) Tangent line at $x = 2$: $y - y_1 = m(x - x_1)$, where
$x_1 = 2$, $y_1 = f(2) = (2)^4 - 32(2)^2 + 10 = -102$, $m = -96$
$y + 102 = -96(x - 2)$ or $y = -96x + 90$
Tangent line at $x = 4$ is a horizontal line since the slope $m = 0$. Therefore, the equation of the tangent line at $x = 4$ is: $y = f(4) = (4)^4 - 32(4)^2 + 10 = -246$

(D) Solve $f'(x) = 0$ for x:
$4x^3 - 64x = 0$
$4x(x^2 - 16) = 0$
$4x(x + 4)(x - 4) = 0$
$x = -4$, $x = 0$, $x = 4$

62. $f(x) = 80x - 10x^2$

(A) $v = f'(x) = 80 - 20x$

(B) $v\Big|_{x=0} = f'(0) = 80$ ft/sec.

$v\Big|_{x=3} = f'(3) = 80 - 20(3) = 20$ ft/sec.

(C) Solve $v = f'(x) = 0$ for x:

$80 - 20x = 0$

$20x = 80$

$x = 4$ seconds

64. $f(x) = x^3 - 9x^2 + 24x$

(A) $v = f'(x) = 3x^2 - 18x + 24$

(B) $v\Big|_{x=0} = f'(0) = 24$ ft/sec.

$v\Big|_{x=3} = f'(3) = 3(3)^2 - 18(3) + 24 = -3$ ft/sec.

(C) Solve $v = f'(x) = 0$ for x:

$3x^2 - 18x + 24 = 0$ or $x^2 - 6x + 8 = 0$

$(x-2)(x-4) = 0$

$x = 2,\ x = 4$ seconds

66. $f'(x) = 2x + 1 - \dfrac{5}{\sqrt{x}}$; $f'(x) = 0$ at $x \approx 1.5247$.

68. $f'(x) = 4x^{1/3} - 4x + 4$; $f'(x) = 0$ at $x \approx 2.3247$.

70. $f'(x) = 0.08x^3 - 0.18x^2 - 1.56x + 0.94$; $f'(x) = 0$ at $x \approx -3.7626,\ 0.5742,\ 5.4384$.

72. $f'(x) = x^3 - 7.8x^2 + 16.2x - 10$; $f'(x) = 0$ at $x \approx 1.2391,\ 1.6400,\ 4.9209$.

74. The tangent line to the graph of a parabola at the vertex is a horizontal line. Therefore, to find the x coordinate of the vertex,
we solve $f'(x) = 0$ for x.

76. No. The derivative is a quadratic function which can have at most two zeros.

78. $y = (2x-5)^2$; $y' = (2)(2x-5)(2) = 8x - 20$

80. $y = \dfrac{x^2+25}{x^2} = 1 + \dfrac{25}{x^2} = 1 + 25x^{-2}$; $\dfrac{dy}{dx} = 0 + (25)\cdot(-2)x^{-3} = -50x^{-3}$

82. $f(x) = \dfrac{2x^5 - 4x^3 + 2x}{x^3} = \dfrac{2x^5}{x^3} - \dfrac{4x^3}{x^3} + \dfrac{2x}{x^3} = 2x^2 - 4 + 2x^{-2}$; $f'(x) = 4x - 4x^{-3}$

84. False: The function $f(x) = \dfrac{1}{x}$ is a counter-example.

86. False: The function $f(x) = 2x$ is a counter-example.

88. $f(x) = u(x) - v(x)$

Step1. $f(x+h) = u(x+h) - v(x+h)$

Step 2. $f(x+h) - f(x) = u(x+h) - v(x+h) - [u(x) - v(x)] = u(x+h) - u(x) - [v(x+h) - v(x)]$

Step 3. $\frac{f(x+h) - f(x)}{h} = \frac{u(x+h) - u(x) - [v(x+h) - v(x)]}{h} = \frac{u(x+h) - u(x)}{h} - \frac{v(x+h) - v(x)}{h}$

Step 4. $\lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0}\left[\frac{u(x+h) - u(x)}{h} - \frac{v(x+h) - v(x)}{h}\right]$

$= \lim_{h\to 0}\frac{u(x+h) - u(x)}{h} - \lim_{h\to 0}\frac{v(x+h) - v(x)}{h} = u'(x) - v'(x)$

90. $S(t) = 0.015t^4 + 0.4t^3 + 3.4t^2 + 10t - 3$

(A) $S'(t) = (0.015)\cdot(4)t^3 + (0.4)\cdot(3)t^2 + (3.4)(2)t + 10 - 0 = 0.06t^3 + 1.2t^2 + 6.8t + 10$

(B) $S(4) = 0.015(4)^4 + 0.4(4)^3 + 3.4(4)^2 + 10(4) - 3 = 120.84$,
$S'(4) = 0.06(4)^3 + 1.2(4)^2 + 6.8(4) + 10 = 60.24$.

After 4 months, sales are \$120.84 million and are increasing at the rate of \$60.24 million per month.

(C) $S(8) = 0.015(8)^4 + 0.4(8)^3 + 3.4(8)^2 + 10(8) - 3 = 560.84$,
$S'(8) = 0.06(8)^3 + 1.2(8)^2 + 6.8(8) + 10 = 171.92$.

After 8 months, sales are \$560.84 million and are increasing at the rate of \$171.92 million per month.

92. $x = 10 + \frac{180}{p}$, $2 \le p \le 10$

For $p = 5$, $x = 10 + \frac{180}{5} = 10 + 36 = 46$

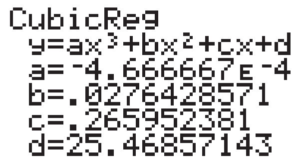
$x = 10 + \frac{180}{p} = 10 + 180p^{-1}$

$\frac{dx}{dp} = -180p^{-2} = -\frac{180}{p^2}$

For $p = 5$, $\left.\frac{dx}{dp}\right|_{p=5} = -\frac{180}{25} = -7.2$

At the \$5 price level, the demand is 46 pounds and is decreasing at the rate of 7.2 pounds per dollar increase in price.

94 . (A) Cubic Regression model



$F(x) \approx -0.000467x^3 + 0.027643x^2 + 0.265952x + 25.468751$

(B) $F'(x) \approx -0.001401x^2 + 0.055286x + 0.265952$

$F(55) \approx 46.1, \quad F'(55) \approx -0.9$

In 2025, 46.1% of female high-school graduates enroll in college and the percentage is decreasing at the rate of 0.9% per year.

96. $C(x) = \dfrac{0.1}{x^2} = 0.1x^{-2}$

$C'(x) = -0.2x^{-3} = -\dfrac{0.2}{x^3}$, the instantaneous rate of change of concentration at x miles.

(A) At $x = 1$, $C'(1) = -0.2$ parts per million per mile.

(B) At $x = 2$, $C'(2) = -\dfrac{0.2}{8} = -0.025$ parts per million per mile.

98. $y = 21\sqrt[3]{x^2}$, $0 \le x \le 8$.

First, find $y = 21\sqrt[3]{x^2} = 21x^{2/3}$.

Then $y' = 21\left(\dfrac{2}{3}x^{-1/3}\right) = 14x^{-1/3} = \dfrac{14}{x^{1/3}} = \dfrac{14}{\sqrt[3]{x}}$, is the rate of learning at the end of x hours.

(A) Rate of learning at the end of 1 hour:

$\dfrac{14}{\sqrt[3]{1}}$ = 14 items per hour.

(B) Rate of learning at the end of 8 hours:

$\dfrac{14}{\sqrt[3]{8}} = \dfrac{14}{2}$ = 7 items per hour.

EXERCISE 2-6

2. $f(x) = 0.1x + 3; \quad f(7) = 0.1(7) + 3 = 3.7, \quad f(7.1) = 0.1(7.1) + 3 = 3.71$

4. $f(x) = 0.1x + 3; \quad f(-10) = 0.1(-10) + 3 = 2, \quad f(-10.1) = 0.1(-10.1) + 3 = 1.99$

6. $g(x) = x^2; \quad g(1) = 1^2 = 1, \quad g(1.1) = (1.1)^2 = 1.21$

8. $g(x) = x^2; \quad g(5) = 5^2 = 25, \quad g(4.9) = (4.9)^2 = (5 - 0.1)^2 = 24.01$

10. $\Delta x = x_2 - x_1 = 5 - 2 = 3, \quad \Delta y = f(x_2) - f(x_1) = 5(5)^2 - 5(2)^2 = 125 - 20 = 105$

$$\frac{\Delta y}{\Delta x} = \frac{105}{3} = 35$$

12. $\dfrac{f(x_1+\Delta x) - f(x_1)}{\Delta x} = \dfrac{f(2+1) - f(2)}{1} = \dfrac{f(3) - f(2)}{1} = \dfrac{5(3)^2 - 5(2)^2}{1} = 45 - 20 = 25$

14. $\Delta y = f(x_2) - f(x_1) = f(3) - f(2) = 5(3)^2 - 5(2)^2 = 45 - 20 = 25$

$\Delta x = x_2 - x_1 = 3 - 2 = 1; \quad \dfrac{\Delta y}{\Delta x} = \dfrac{25}{1} = 25$

16. $y = 200x - \dfrac{x^2}{30}, \quad dy = \left(200x - \dfrac{x^2}{30}\right)' dx = \left(200 - \dfrac{x}{15}\right) dx$

18. $y = x^3(60 - x) = 60x^3 - x^4, \quad dy = (180x^2 - 4x^3)\,dx$

20. $y = 52\sqrt{x} = 52x^{1/2}, \quad dy = (52x^{1/2})'dx = (26x^{-1/2})dx$

22. (A) $\dfrac{f(3+\Delta x) - f(3)}{\Delta x} = \dfrac{3(3+\Delta x)^2 - 3(3)^2}{\Delta x} = \dfrac{3(9 + 6\Delta x + (\Delta x)^2) - 27}{\Delta x}$

$$= \frac{27 + 18\Delta x + 3(\Delta x)^2 - 27}{\Delta x} = \frac{18\Delta x + 3(\Delta x)^2}{\Delta x} = 18 + 3\Delta x$$

(B) As Δx tends to zero, then, clearly, $18 + 3\Delta x$ tends to 18.
Note the values in the following table:

Δx	$18 + 3\Delta x$
1	21
0.1	18.3
0.01	18.03
0.001	18.003

24. $y = (2x+3)^2 = 4x^2 + 12x + 9, \quad dy = (8x + 12)dx = 4(2x+3)dx$

26. $y = \dfrac{x^2 - 9}{x^2} = 1 - \dfrac{9}{x^2} = 1 - 9x^{-2}, \quad dy = 18x^{-3}\,dx = \dfrac{18}{x^3}\,dx.$

28. $y = f(x) = 30 + 12x^2 - x^3$

$\Delta y = f(2 + 0.1) - f(2) = f(2.1) - f(2) = [30 + 12(2.1)^2 - (2.1)^3] - [(30 + 12(2)^2 - 2^3]$

$= 30 + 52.92 - 9.261 - 30 - 48 + 8 = 3.66$

$dy = (30 + 12x^2 - x^3)'\big|_{x=2}\; dx = (24x - 3x^2)\big|_{x=2}\; (0.1) = (24(2) - 3(2)^2)(0.1) = (48 - 12)(0.1) = 3.6$

30. $y = f(x) = 100\left(x - \frac{4}{x^2}\right)$

$$\Delta y = f(2 - 0.1) - f(2) = f(1.9) - f(2) = 100\left(1.9 - \frac{4}{(1.9)^2}\right) - 100\left(2 - \frac{4}{2^2}\right) = 79.197 - 100 = -20.803$$

$$dy = \left(100\left(x - \frac{4}{x^2}\right)\right)'\Big|_{x=2} dx = 100\left(1 + \frac{8}{x^3}\right)\Big|_{x=2}(-0.1) = 100\left(1 + \frac{8}{2^3}\right)(-0.1) = -20$$

32. $V = \frac{4}{3}\pi r^3$, $r = 5$ cm, $dr = \Delta r = 0.1$ cm.

$$dV = \left(\frac{4}{3}\pi r^3\right)'\Big|_{r=5} dr = 4\pi r^2\Big|_{r=5}(0.1) = 31.4 \text{ cm}^3.$$

34. $f(x) = x^2 + 2x + 3$; $f'(x) = 2x + 2$; $x = -2$; $\Delta x = dx$

(A) $\Delta y = f(-2 + \Delta x) - f(-2)$

$= [(-2 + \Delta x)^2 + 2(-2 + \Delta x) + 3] - [(-2)^2 + 2(-2) + 3]$

$= 4 - 4\Delta x + (\Delta x)^2 - 4 + 2\Delta x + 3 - 4 + 4 - 3$

$= -2\Delta x + (\Delta x)^2$

$dy = f'(-2)dx = -2\, dx$

(B) $\Delta y(-0.1) = -2(-0.1) + (-0.1)^2 = 0.21$

$dy(-0.1) = -2(-0.1) = 0.2$

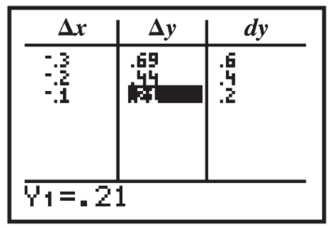
$\Delta y(-0.2) = -2(-0.2) + (-0.2)^2 = 0.44$

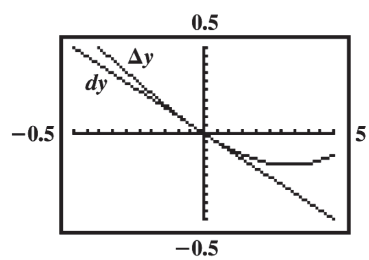
$dy(-0.2) = -2(-0.2) = 0.4$

$\Delta y(-0.3) = -2(-0.3) + (-0.3)^2 = 0.69$

$dy(-0.3) = -2(-0.3) = 0.6$

(C)





36. $f(x) = x^3 - 2x^2$; $f'(x) = 3x^2 - 4x$; $x = 2$, $\Delta x = dx$

(A) $\Delta y = f(2 + \Delta x) - f(2)$

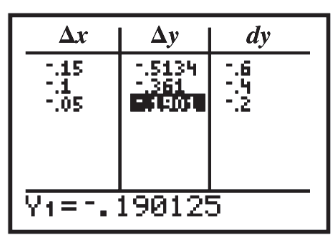
$= [(2 + \Delta x)^3 - 2(2 + \Delta x)^2] - [2^3 - 2(2)^2]$

$= 8 + 12\Delta x + 6(\Delta x)^2 + (\Delta x)^3 - 8 - 8\Delta x - 2(\Delta x)^2 - 8 + 8$

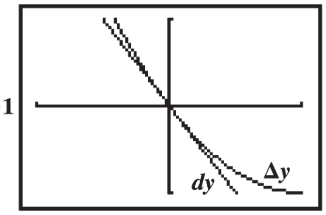
$= 4\Delta x + 4(\Delta x)^2 + (\Delta x)^3$

$dy = f'(2)dx = 4\, dx$

(C)



(B) $\Delta y(-0.05) = 4(-0.05) + 4(-0.05)^2 + (-0.05)^3$
$= -0.1901$
$dy(-0.05) = 4(-0.05) = -0.2$
$\Delta y(-0.10) = 4(-0.10) + 4(-0.10)^2 + (-0.10)^3$
$= -0.361$
$dy(-0.10) = 4(-0.10) = -0.4$
$\Delta y(-0.15) = 4(-0.15) + 4(-0.15)^2 + (-0.15)^3 = -0.5134$
$dy(-0.15) = 4(-0.15) = -0.6$



38. False.
Example. Let $y = f(x) = x^2 + 1$. Then
$\Delta y = f(0 + \Delta x) - f(0) = f(\Delta x) - f(0) = (\Delta x)^2 + 1 - 1 = (\Delta x)^2$
$dy = f'(0)dx = 0\, dx = 0.$

40. True.
$\Delta y = f(2 + \Delta x) - f(2) = 0$ implies that
$f(2 + \Delta x) = f(2)$
Since this is true for every increment and since the right-hand side of this equation is a constant, the function $f(x)$ must be a constant function.

42. $y = (2x^2 - 4)\sqrt{x} = (2x^2 - 4)(x)^{1/2} = 2x^{5/2} - 4x^{1/2}, \quad dy = (5x^{3/2} - 2x^{-1/2})\,dx.$

44. $y = f(x) = \dfrac{590}{\sqrt{x}} = 590x^{-1/2}; \; x = 64, \; \Delta x = dx = 1.$

$\Delta y = f(x + \Delta x) - f(x) = f(64 + 1) - f(64) = f(65) - f(64) = \dfrac{590}{\sqrt{65}} - \dfrac{590}{\sqrt{64}} = -0.57$

$y = f(x) = \dfrac{590}{\sqrt{x}} = 590x^{-1/2}, \qquad f'(x) = -295x^{-3/2}$

$dy = f'(64)dx = f'(64)(1) = -295(64)^{-3/2} = -\dfrac{295}{512} = -0.576$

46. Given $D(x) = 1{,}000 - 40x^2$, $1 \le x \le 5$. Then, $D'(x) = -80x$.
The approximate change in demand dD corresponding to a change $\Delta x = dx$ in the price x is:
$dD = D'(x)dx$
Thus, letting $x = 3$ and $dx = 0.20$, we get
$dD = D'(3)(0.20) = -80(3)(0.20) = -48.$
There will be a 48-pound decrease in demand (approximately) when the price is increased from \$3.00 to \$3.20.

48. $R(x) = 200x - \dfrac{x^2}{30}; \quad R'(x) = 200 - \dfrac{x}{15}$

Profit $P(x) = R(x) - C(x) = 200x - \dfrac{x^2}{30} - 72{,}000 - 60x = 140x - \dfrac{x^2}{30} - 72{,}000$

$P'(x) = 140 - \dfrac{x}{15}$

Now, for $x = 1{,}500$, $\Delta x = dx = 10$, we get

$$dR = R'(1{,}500)(10) = \left(200 - \frac{1{,}500}{15}\right)(10) = 1{,}000$$

$$dP = P'(1{,}500)(10) = \left(140 - \frac{1{,}500}{15}\right)(10) = 400$$

Thus, the approximate change in revenue is \$1,000 and the approximate change in profit is \$400 if the production is increased from 1,500 to 1,510 televisions.

For $x = 4{,}500$, $\Delta x = dx = 10$, we have:

$$dR = R'(4{,}500)(10) = \left(200 - \frac{4{,}500}{15}\right)(10) = -1{,}000$$

$$dP = P'(4{,}500)(10) = \left(140 - \frac{4{,}500}{15}\right)(10) = -1{,}600.$$

Thus, the approximate change in revenue is –\$1,000 and and the approximate change in profit is –\$1,600 if the production is increased from 4,500 to 4,510 televisions.

50. $V = \frac{4}{3}\pi r^3$; $V' = 4\pi r^2$.

The approximate volume of the shell for a radius change from 5 mm to 5.3 mm is given by:

$$dV = 4\pi r^2\Big|_{r=5}\ dx = 4\pi(5)^2(0.3) \quad \text{(Note: } \Delta x = dx = 0.3 \text{ mm)}$$

$$= 94.2 \text{ cubic millimeters}$$

52. $T = x^2\left(1 - \frac{x}{9}\right) = x^2 - \frac{x^3}{9}$, $0 \le x \le 6$; $T' = 2x - \frac{x^2}{3}$.

(A) For $x = 2$, $\Delta x = dx = 0.1$,

$$dT = \left(2x - \frac{x^2}{3}\right)\Big|_{x=2}\ dx = \left(2(2) - \frac{2^2}{3}\right)(0.1) = 0.27 \text{ degrees}$$

(B) For $x = 3$, $\Delta x = dx = 0.1$

$$dT = \left(2x - \frac{x^2}{3}\right)\Big|_{x=3}\ dx = \left(2(3) - \frac{3^2}{3}\right)(0.1) = 0.3 \text{ degrees}$$

(C) For $x = 4$, $\Delta x = dx = 0.1$

$$dT = \left(2x - \frac{x^2}{3}\right)\Big|_{x=4}\ dx = \left(2(4) - \frac{4^2}{3}\right)(0.1) = 0.27 \text{ degrees}$$

54. $y = 52\sqrt{x}$, $0 \le x \le 9$; $y = 52x^{1/2}$ and hence $y' = \frac{52}{2}x^{-1/2} = 26x^{-1/2}$.

For $x = 1$ and $\Delta x = dx = 0.1$ the approximate increase in the number of items learned is given by

$$dy = y'\Big|_{x=1}\ dx = 26(1)^{-1/2}(0.1) = 2.6 \text{ items.}$$

Similarly, for $x = 4$, $\Delta x = dx = 0.1$, we have

$$dy = y'\Big|_{x=4}\ dx = 26(4)^{-1/2}(0.1) = 1.3 \text{ items.}$$

EXERCISE 2-7

In Problems 2 – 8, $C(x) = 10{,}000 + 150x - 0.2x^2$.

2. $C(100) = 10{,}000 + 150(100) - 0.2(100)^2 = 25{,}000 - 2{,}000 = 23{,}000$, \$23,000

4. $C(199) = 10{,}000 + 150(199) - 0.2(199)^2 = 39{,}850 - 7{,}920.20 = 31{,}929.80$, \$31,929.80

6. Using the results in Problems 4 and 5, $C(200) - C(199) = 32{,}000 - 31{,}929.80 = 70.20$, \$70.20

8. Average cost of producing 200 bicycles: $\frac{C(200)}{200} = \frac{32{,}000}{200} = 160$, \$160

10. $C'(x) = 6$

12. $C'(x) = 12 - 0.2x$

14. $R'(x) = 36 - 0.06x$

16. $R'(x) = 25 - 0.10x$

18. $P'(x) = (36 - 0.06x) - 6 = 30 - 0.06x$

20. $P'(x) = (25 - 0.1x) - (12 - 0.2x) = 13 + 0.1x$

22. $\overline{R}(x) = \frac{5x - 0.02x^2}{x} = 5 - 0.02x$

24. $\overline{R}'(x) = -0.02$

26. $P'(x) = 3.9 - 0.04x$

28. $\overline{P}'(x) = -0.02 + \frac{145}{x^2}$

30. True: If $p = b - mx$ then $R(x) = xp = bx - mx^2$, and $R'(x) = b - 2mx$.

32. False: If $C(x) = 5x + 10$, then the marginal cost is $C'(x) = 5$. In this case, the average marginal cost over any interval is 5. However, the average cost is $\overline{C}(x) = 5 + \frac{10}{x}$ so the marginal average cost is $\overline{C}'(x) = -\frac{10}{x^2}$, which is not equal to 5 over the interval [1,2], for example.

34. $C(x) = 1{,}000 + 100x - 0.25x^2$

(A) The exact cost of producing the 51st guitar is:

$C(51) - C(50)$

$= 1{,}000 + 100(51) - 0.25(51)^2 - [1{,}000 + 100(50) - 0.25(50)^2]$

$= 100 - 0.25(51)^2 + 0.25(50)^2 = 74.75$ or \$74.75

(B) $C'(x) = 100 - 0.5x$

$C'(50) = 100 - 0.5(50) = 75$ or \$75.

36. $C(x) = 10{,}000 + 20x$

(A) $\overline{C}(x) = \frac{10{,}000 + 20x}{x} = \frac{10{,}000}{x} + 20$

$\overline{C}(1{,}000) = \frac{10{,}000}{1000} + 20 = 30 = 30$ or \$30

(B) $\overline{C}'(x) = -10{,}000x^{-2} = \dfrac{-10{,}000}{x^2}$

$\overline{C}'(1{,}000) = \dfrac{-10{,}000}{(1{,}000)^2} = -0.01$ or $-1¢$

At a production level of 1,000 dictionaries, average cost is decreasing at the rate of 1¢ per game.

(C) The average cost per game if 1,001 are produced is approximately \$30.00 – \$0.01 = \$29.99.

38. $P(x) = 22x - 0.2x^2 - 400,\ 0 \le x \le 100$

(A) The exact profit from the sale of the 41st calendar is

$P(41) - P(40) = 22(41) - 0.2(41)^2 - 400 - [22(40) - 0.2(40)^2 - 400]$

$= 22 - 0.2(41)^2 + 0.2(40)^2 = 5.80$ or \$5.80

(B) $P'(x) = 22 - 0.4x$

$P'(40) = 22 - 0.4(40) = 22 - 16 = 6$ or \$6

40. $P(x) = 12x - 0.02x^2 - 1{,}000,\ 0 \le x \le 600;\quad P'(x) = 12 - 0.04x$

(A) $P'(200) = 12 - 0.04(200) = 12 - 8 = 4$ or \$4;

at a production level of 200 cameras, profit is increasing at the rate of \$4 per camera.

(B) $P'(350) = 12 - 0.04(350) = 12 - 14 = -2$ or –\$2;

at a production level of 350 cameras, profit is decreasing at the rate of \$2 per camera.

42. $P(x) = 20x - 0.02x^2 - 320,\ 0 \le x \le 1{,}000$

Average profit: $\overline{P}(x) = \dfrac{P(x)}{x} = 20 - 0.02x - \dfrac{320}{x} = 20 - 0.02x - 320x^{-1}$

(A) At $x = 40$, $\overline{P}(40) = 20 - 0.02(40) - \dfrac{320}{40} = 11.20$ or \$11.20.

(B) $\overline{P}'(x) = -0.02 + 320x^{-2} = -0.02 + \dfrac{320}{x^2}$

$\overline{P}'(40) = -0.02 + \dfrac{320}{(40)^2} = 0.18$ or \$0.18;

at a production level of 40 grills, the average profit per grill is increasing at the rate of \$0.18 per grill.

(C) The average profit per grill if 41 grills are produced is approximately \$11.20 + \$0.18 = \$11.38.

44. $x = 1{,}000 - 20p$

(A) $20p = 1{,}000 - x$, $p = 50 - 0.05x$, $0 \le x \le 1{,}000$

(B) $R(x) = x(50 - 0.05x) = 50x - 0.05x^2,\ 0 \le x \le 1{,}000$

(C) $R'(x) = 50 - 0.10x$

$R'(400) = 50 - 0.10(400) = 50 - 40 = 10$;

at a production level of 400 steam irons, revenue is increasing at the rate of \$10 per steam iron.

(D) $R'(650) = 50 - 0.10(650) = 50 - 65 = -15$;

at a production level of 650 steam irons, revenue is decreasing at the rate of \$15 per steam iron.

46. $x = 9{,}000 - 30p$ and $C(x) = 150{,}000 + 30x$

(A) $30p = 9{,}000 - x,\quad p = 300 - \frac{1}{30}x,\ 0 \le x \le 9{,}000$

(B) $C'(x) = 30$

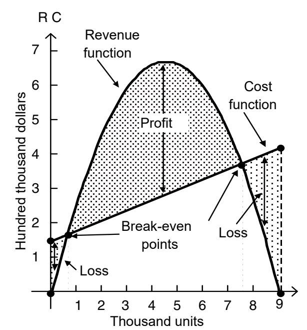
(C) $R(x) = x\left(300 - \frac{1}{30}x\right) = 300x - \frac{1}{30}x^2,\ 0 \le x \le 9{,}000$

(D) $R'(x) = 300 - \frac{1}{15}x$

(E) $R'(3{,}000) = 300 - \frac{1}{15}(3{,}000) = 100$; at a production level of 3,000 sets, revenue is increasing at the rate of \$100 per set.

$R'(6000) = 300 - \frac{1}{15}(6{,}000) = 300 - 400 = -100$; at a production level of 6,000 sets, revenue is decreasing at the rate of \$100 per set.

(F) The graphs of $C(x)$ and $R(x)$ are shown at the right.



To find the break-even points, set $C(x) = R(x)$:

$$150{,}000 + 30x = 300x - \frac{1}{30}x^2$$
$$x^2 - 8{,}100x + 4{,}500{,}000 = 0$$
$$(x - 600)(x - 7{,}500) = 0$$
$$x = 600 \quad \text{or} \quad x = 7{,}500$$

Now, $C(600) = 150{,}000 + 30(600) = 168{,}000$;

$C(7{,}500) = 150{,}000 + 30(7{,}500) = 375{,}000$

Thus, the break-even points are:
(600, 168,000) and (7,500, 375,000).

(G) $P(x) = R(x) - C(x) = 300x - \frac{1}{30}x^2 - (150{,}000 + 30x)$

$= -\frac{1}{30}x^2 + 270x - 150{,}000$

(H) $P'(x) = -\frac{1}{15}x + 270$

(I) $P'(1{,}500) = -\frac{1}{15}(1{,}500) + 270 = 170$; at a production level of 1,500 sets, profit is increasing at the rate of \$170 per set.

$P'(4{,}500) = -\frac{1}{15}(4{,}500) + 270 = -30$; at a production level of 4,500 sets, profit is decreasing at the rate of \$30 per set.

48. (A) We are given $p = 25$ when $x = 300$ and $p = 20$ when $x = 400$. Thus, we have the pair of equations:

$25 = 300m + b$

$20 = 400\,m + b$

Subtracting the second equation from the first, we get $-100m = 5$. Thus, $m = -\frac{1}{20}$.

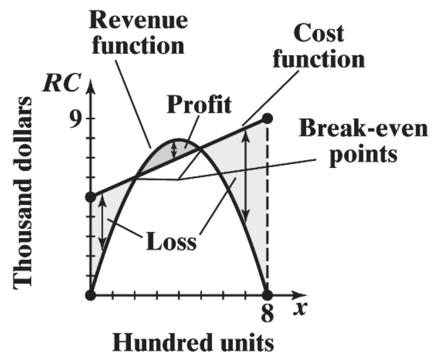
Substituting this into either equation yields $b = 40$. Therefore,

$p = -\frac{1}{20}x + 40 = 40 - \frac{x}{20}$, $0 \le x \le 800$

(B) $R(x) = x\left(40 - \frac{x}{20}\right) = 40x - \frac{x^2}{20}$, $0 \le x \le 800$

(C) From the financial department's estimates, $m = 5$ and $b = 5{,}000$. Thus, $C(x) = 5x + 5{,}000$.

(D) The graphs of $R(x)$ and $C(x)$ are shown at the right.



To find the break-even points, set $C(x) = R(x)$:

$$5x + 5{,}000 = 40x - \frac{x^2}{20}$$

$$x^2 - 700x + 100{,}000 = 0$$

$$(x - 200)(x - 500) = 0$$

$$x = 200 \quad \text{or} \quad x = 500$$

Now, $C(200) = 5(200) + 5{,}000 = 6{,}000$ and

$C(500) = 5(500) + 5{,}000 = 7{,}500$

Thus, the break-even points are: (200, 6,000) and (500, 7,500).

(E) $P(x) = R(x) - C(x) = 40x - \frac{x^2}{20} - (5x + 5{,}000)$

$= 35x - \frac{x^2}{20} - 5{,}000$

(F) $P'(x) = 35 - \frac{x}{10}$

$P'(325) = 35 - \frac{325}{10} = 2.5$; at a production level of 325 toasters, profit is increasing at the rate of $2.50 per toaster.

$P'(425) = 35 - \frac{425}{10} = -7.5$; at a production level of 425 toasters, profit is decreasing at the rate of $7.50 per toaster.

50. Total cost: $C(x) = 5x + 2{,}340$

Total revenue: $R(x) = 40x - 0.1x^2$, $0 \le x \le 400$

(A) $R'(x) = 40 - 0.2x$

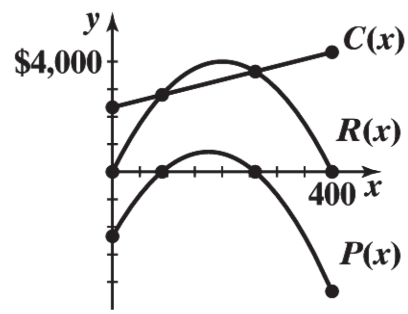
The graph of R has a horizontal tangent line at the value(s) of x where $R'(x) = 0$, i.e.

$40 - 0.2x = 0$
or $x = 200$

(B) $P(x) = R(x) - C(x) = 40x - 0.1x^2 - (5x + 2{,}340)$
$= 35x - 0.1x^2 - 2{,}340$

(C) $P'(x) = 35 - 0.2x$. Setting $P'(x) = 0$, we have
$35 - 0.2x = 0$
or $x = 175$

(D) The graphs of $C(x)$, $R(x)$ and $P(x)$ are shown at the right.



Break-even points: $R(x) = C(x)$

$$\begin{aligned} 40x - 0.1x^2 &= 5x + 2{,}340 \\ x^2 - 350x + 23{,}400 &= 0 \\ (x - 90)(x - 260) &= 0 \\ x = 90 &\quad \text{or } x = 260 \end{aligned}$$

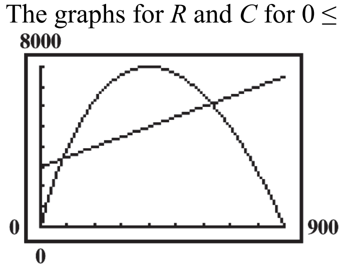
Thus, the break-even points are:
(90, 2,790) and (260, 3,640).

x intercepts for P: $-0.1x^2 + 35x - 2{,}340 = 0$ or
$x^2 - 350x + 23{,}400 = 0$
which is the same as the above equation. Thus, $x = 90$ and $x = 260$ are x intercepts of P.

52. Demand equation: $p = 60 - 2\sqrt{x} = 60 - 2x^{1/2}$
Cost equation: $C(x) = 3{,}000 + 5x$

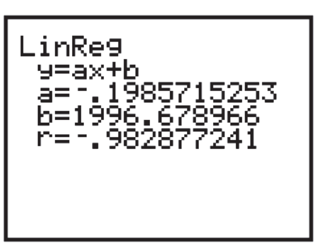
(A) Revenue $R(x) = xp = x(60 - 2x^{1/2})$
$= 60x - 2x^{3/2}$

(B) The graphs for R and C for $0 \le x \le 900$ are shown below:

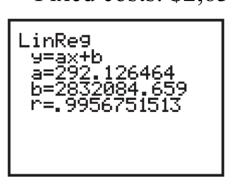


Break-even points: (81, 3,405), (631, 6,155)

54. (A)



(B) Fixed costs: \$2,832,085; variable cost: \$292



(C) Let $y = p(x)$ be the linear regression equation found in part (A) and let $y = C(x)$ be the linear regression equation found in part (B). Then revenue $R(x) = xp(x)$, and the break-even points are

$R(x) = C(x)$.

Break-even points: (2,253, 3,490,130), (6,331, 4,681,675).

(D) The company will make a profit when $2,253 \le x \le 6,331$. From part A), $p(2,253) = 740$ and $p(6,331) = 1,549$. Thus, the company will make a profit for the price range $\$740 \le p \le \$1,549$.

CHAPTER 2 REVIEW

1. $f(x) = 2x^2 + 5$

(A) $f(3) - f(1) = 2(3)^2 + 5 - [2(1)^2 + 5] = 16$

(B) Average rate of change: $\frac{f(3)-f(1)}{3-1} = \frac{16}{2} = 8$

(C) Slope of secant line: $\frac{f(3)-f(1)}{3-1} = \frac{16}{2} = 8$

(D) Instantaneous rate of change at $x = 1$:

Step 1. $\frac{f(1+h)-f(1)}{h} = \frac{2(1+h)^2+5-[2(1)^2+5]}{h} = \frac{2(1+2h+h^2)+5-7}{h} = \frac{4h+2h^2}{h} = 4 + 2h$

Step 2. $\lim_{h\to 0}\frac{f(1+h)-f(1)}{h} = \lim_{h\to 0}(4+2h) = 4$

(E) Slope of the tangent line at $x = 1$: 4

(F) $f'(1) = 4$ (2-2)

2. $f(x) = -3x + 2$

Step 1. Find $f(x+h)$

$f(x+h) = -3(x+h)+2 = -3x-3h+2$

Step 2. Find $f(x+h)-f(x)$

$f(x+h)-f(x) = -3x-3h+2-(-3x+2) = -3x-3h+2+3x-2 = -3h$

Step 3. Find $\frac{f(x+h)-f(x)}{h}$

$\frac{f(x+h)-f(x)}{h} = \frac{-3h}{h} = -3$

Step 4. Find $\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$.

$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to 0}(-3) = -3$ (2-2)

3. (A) $\lim_{x\to 1}(5f(x)+3g(x)) = 5\lim_{x\to 1}f(x) + 3\lim_{x\to 1}g(x) = 5\cdot 2 + 3\cdot 4 = 22$

(B) $\lim_{x\to 1}[f(x)g(x)] = [\lim_{x\to 1}f(x)][\lim_{x\to 1}g(x)] = 2\cdot 4 = 8$

(C) $\lim_{x\to 1}\frac{g(x)}{f(x)} = \frac{\lim_{x\to 1} g(x)}{\lim_{x\to 1} f(x)} = \frac{4}{2} = 2$

(D) $\lim_{x\to 1}[5 + 2x - 3g(x)] = \lim_{x\to 1} 5 + \lim_{x\to 1} 2x - 3\lim_{x\to 1} g(x) = 5 + 2 - 3(4) = -5$ (2-1)

4. $f(1.5) \approx 1.5$ (2-1)

5. $f(2.5) \approx 3.5$ (2-1)

6. $f(2.75) \approx 3.75$ (2-1)

7. $f(3.25) \approx 3.75$ (2-1)

8. (A) $\lim_{x\to 1^-} f(x) = 1$ (B) $\lim_{x\to 1^+} f(x) = 1$ (C) $\lim_{x\to 1} f(x) = 1$ (D) $f(1) = 1$ (2-1)

9. (A) $\lim_{x\to 2^-} f(x) = 2$ (B) $\lim_{x\to 2^+} f(x) = 3$ (C) $\lim_{x\to 2} f(x)$ does not exist (D) $f(2) = 3$ (2-1)

10. (A) $\lim_{x\to 3^-} f(x) = 4$ (B) $\lim_{x\to 3^+} f(x) = 4$ (C) $\lim_{x\to 3} f(x) = 4$ (D) $f(3)$ does not exist (2-1)

11. (A) From the graph, $\lim_{x\to 1} f(x)$ does not exist since
$\lim_{x\to 1^-} f(x) = 2 \neq \lim_{x\to 1^+} f(x) = 3$.

(B) $f(1) = 3$

(C) f is NOT continuous at $x = 1$, since $\lim_{x\to 1} f(x)$ does not exist. (2-3)

12. (A) $\lim_{x\to 2} f(x) = 2$ (B) $f(2)$ is not defined

(C) f is NOT continuous at $x = 2$ since $f(2)$ is not defined. (2-3)

13. (A) $\lim_{x\to 3} f(x) = 1$ (B) $f(3) = 1$

(C) f is continuous at $x = 3$ since $\lim_{x\to 3} f(x) = f(3)$. (2-3)

14. $\lim_{x\to\infty} f(x) = 10$ (2-2)

15. $\lim_{x\to -\infty} f(x) = 5$ (2-2)

16. $\lim_{x\to 2^+} f(x) = \infty$ (2-2)

17. $\lim_{x\to 2^-} f(x) = -\infty$ (2-2)

18. $\lim_{x\to 6^-} f(x) = \infty$ (2-2)

19. $\lim_{x\to 6^+} f(x) = \infty$ (2-2)

20. $\lim_{x\to 6} f(x) = \infty$ (2-2)

21. $x = 2$ and $x = 6$ (2-2)

22. $y = 5$ and $y = 10$ (2-2)

23. $x = 2$ and $x = 6$ (2-3)

24. $f(x) = 3x^2 - 5$

Step 1. Find $f(x+h)$:
$f(x+h) = 3(x+h)^2 - 5 = 3x^2 + 6xh + 3h^2 - 5$

Step 2. Find $f(x+h) - f(x)$:
$f(x+h) - f(x) = 3x^2 + 6xh + 3h^2 - 5 - (3x^2 - 5) = 6xh + 3h^2$

Step 3. Find $\frac{f(x+h)-f(x)}{h}$:
$\frac{f(x+h)-f(x)}{h} = \frac{6xh + 3h^2}{h} = \frac{h(6x+3h)}{h} = 6x + 3h,\ h \neq 0$

Step 4. Find $\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$
$\lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0}(6x + 3h) = 6x$
Thus, $f'(x) = 6x$. (2-4)

25. (A) $h'(x) = (3f(x))' = 3f'(x)$; $h'(5) = 3f'(5) = 3(-1) = -3$

(B) $h'(x) = (-2g(x))' = -2g'(x)$; $h'(5) = -2g'(5) = -2(-3) = 6$

(C) $h'(x) = 2f'(x)$; $h'(5) = 2(-1) = -2$

(D) $h'(x) = -g'(x)$; $h'(5) = -(-3) = 3$

(E) $h'(x) = 2f'(x) + 3g'(x)$; $h'(5) = 2(-1) + 3(-3) = -11$ (2-5)

26. $f(x) = \frac{1}{3}x^3 - 5x^2 + 1$; $f'(x) = x^2 - 10$ (2-5)

27. $f(x) = 2x^{1/2} - 3x$; $f'(x) = 2\cdot\frac{1}{2}x^{-1/2} - 3 = \frac{1}{x^{1/2}} - 3$ (2-5)

28. $f(x) = 5$
$f'(x) = 0$ (2-5)

29. $f(x) = \frac{3}{2x} + \frac{5x^3}{4} = \frac{3}{2}x^{-1} + \frac{5}{4}x^3$;

$f'(x) = -\frac{3}{2}x^{-2} + \frac{15}{4}x^2 = -\frac{3}{2x^2} + \frac{15}{4}x^2$ (2-5)

30. $f(x) = \frac{0.5}{x^4} + 0.25x^4 = 0.5x^{-4} + 0.25x^4$

$f'(x) = 0.5(-4)x^{-5} + 0.25(4x^3) = -2x^{-5} + x^3 = -\frac{2}{x^5} + x^3$ (2-5)

31. $f(x) = (3x^3 - 2)(x+1) = 3x^4 + 3x^3 - 2x - 2$
$f'(x) = 12x^3 + 9x^2 - 2$ (2-5)

For Problems 32 – 35, $f(x) = x^2 + x$.

32. $\Delta x = x_2 - x_1 = 3 - 1 = 2$, $\Delta y = f(x_2) - f(x_1) = 12 - 2 = 10$,

$\dfrac{\Delta y}{\Delta x} = \dfrac{10}{2} = 5.$ (2-6)

33. $\dfrac{f(x_1 + \Delta x) - f(x_1)}{\Delta x} = \dfrac{f(1+2) - f(1)}{2} = \dfrac{f(3) - f(1)}{2} = \dfrac{12 - 2}{2} = 5$ (2-6)

34. $dy = f'(x)dx = (2x + 1)dx$. For $x_1 = 1$, $x_2 = 3$,

$dx = \Delta x = 3 - 1 = 2$, $dy = (2\cdot 1 + 1)\cdot 2 = 3\cdot 2 = 6$ (2-6)

35. $\Delta y = f(x + \Delta x) - f(x)$; at $x = 1$, $\Delta x = 0.2$,

$\Delta y = f(1.2) - f(1) = 0.64$

$dy = f'(x)dx$ where $f'(x) = 2x + 1$; at $x = 1$

$dy = 3(0.2) = 0.6$ (2-6)

36. From the graph:

(A) $\lim_{x\to 2^-} f(x) = 4$ (B) $\lim_{x\to 2^+} f(x) = 6$

(C) $\lim_{x\to 2} f(x)$ does not exist since $\lim_{x\to 2^-} f(x) \neq \lim_{x\to 2^+} f(x)$

(D) $f(2) = 6$ (E) No, since $\lim_{x\to 2} f(x)$ does not exist. (2-3)

37. From the graph:

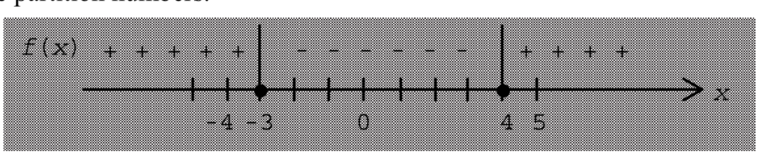
(A) $\lim_{x\to 5^-} f(x) = 3$ (B) $\lim_{x\to 5^+} f(x) = 3$ (C) $\lim_{x\to 5} f(x) = 3$ (D) $f(5) = 3$

(E) Yes, since $\lim_{x\to 5} f(x) = f(5) = 3$. (2-3)

38. (A) $f(x) < 0$ on $(8, \infty)$ (B) $f(x) \geq 0$ on $[0, 8]$ (2-3)

39. $x^2 - x < 12$ or $x^2 - x - 12 < 0$

Let $f(x) = x^2 - x - 12 = (x + 3)(x - 4)$. Then f is continuous for all x and $f(-3) = f(4) = 0$. Thus, $x = -3$ and $x = 4$ are partition numbers.



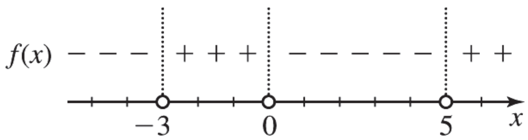
Test Numbers

x	$f(x)$
–4	8 (+)
0	–12 (–)
5	8 (+)

Thus, $x^2 - x < 12$ for: $-3 < x < 4$ or $(-3, 4)$. (2-3)

40. $\frac{x-5}{x^2+3x} > 0$ or $\frac{x-5}{x(x+3)} > 0$

Let $f(x) = \frac{x-5}{x(x+3)}$. Then f is discontinuous at $x = 0$ and $x = -3$, and $f(5) = 0$. Thus, $x = -3$, $x = 0$, and $x = 5$ are partition numbers.



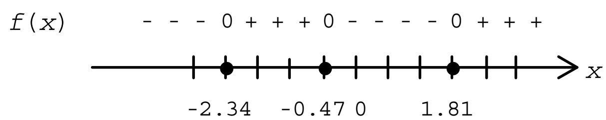
Test Numbers

x	$f(x)$
−4	$-\frac{9}{4}(-)$
−1	$3(+)$
1	$-1(-)$
6	$\frac{1}{54}(+)$

Thus, $\frac{x-5}{x^2+3x} > 0$ for $-3 < x < 0$ or $x > 5$, or $(-3, 0) \cup (5, \infty)$. (2-3)

41. $x^3 + x^2 - 4x - 2 > 0$

Let $f(x) = x^3 + x^2 - 4x - 2$. Then f is continuous for all x and $f(x) = 0$ at $x = -2.3429$, -0.4707 and 1.8136.



Thus, $x^3 + x^2 - 4x - 2 > 0$ for $-2.3429 < x < -0.4707$ or $1.8136 < x < \infty$, or $(-2.3429, -0.4707) \cup (1.8136, \infty)$. (2-3)

42. $f(x) = 0.5x^2 - 5$

(A) $\frac{f(4)-f(2)}{4-2} = \frac{0.5(4)^2 - 5 - [0.5(2)^2 - 5]}{2} = \frac{8-2}{2} = 3$

(B) $\frac{f(2+h)-f(2)}{h} = \frac{0.5(2+h)^2 - 5 - [0.5(2)^2 - 5]}{h} = \frac{0.5(4+4h+h^2) - 5 + 3}{h}$

$= \frac{2h + 0.5h^2}{h} = \frac{h(2+0.5h)}{h} = 2 + 0.5h$

(C) $\lim_{h\to 0} \frac{f(2+h)-f(2)}{h} = \lim_{h\to 0} (2 + 0.5h) = 2$ (2-4)

43. $y = \frac{1}{3}x^{-3} - 5x^{-2} + 1;$

$\frac{dy}{dx} = \frac{1}{3}(-3)x^{-4} - 5(-2)x^{-3} = -x^{-4} + 10x^{-3}$ (2-5)

44. $y = \frac{3\sqrt{x}}{2} + \frac{5}{3\sqrt{x}} = \frac{3}{2}x^{1/2} + \frac{5}{3}x^{-1/2};$

$y' = \frac{3}{2}\left(\frac{1}{2}x^{-1/2}\right) + \frac{5}{3}\left(-\frac{1}{2}x^{-3/2}\right) = \frac{3}{4x^{1/2}} - \frac{5}{6x^{3/2}} = \frac{3}{4\sqrt{x}} - \frac{5}{6\sqrt{x^3}}$ (2-5)

45. $g(x) = 1.8\sqrt[3]{x} + \dfrac{0.9}{\sqrt[3]{x}} = 1.8x^{1/3} + 0.9x^{-1/3}$

$g'(x) = 1.8\left(\dfrac{1}{3}x^{-2/3}\right) + 0.9\left(-\dfrac{1}{3}x^{-4/3}\right) = 0.6x^{-2/3} - 0.3x^{-4/3} = \dfrac{0.6}{x^{2/3}} - \dfrac{0.3}{x^{4/3}}$ (2-5)

46. $y = \dfrac{2x^3 - 3}{5x^3} = \dfrac{2}{5} - \dfrac{3}{5}x^{-3}$; $y' = -\dfrac{3}{5}(-3x^{-4}) = \dfrac{9}{5x^4}$ (2-5)

47. $f(x) = x^2 + 4$
$f'(x) = 2x$

(A) The slope of the graph at $x = 1$ is $m = f'(1) = 2$.

(B) $f(1) = 1^2 + 4 = 5$
The tangent line at (1, 5), where the slope $m = 2$, is:
$(y - 5) = 2(x - 1)$ [Note: $(y - y_1) = m(x - x_1)$.]

$y = 5 + 2x - 2$
$y = 2x + 3$ (2-4, 2-5)

48. $f(x) = 10x - x^2$
$f'(x) = 10 - 2x$
The tangent line is horizontal at the values of x such that
$f'(x) = 0$:
$10 - 2x = 0$
$x = 5$ (2-4)

49. $f(x) = x^3 + 3x^2 - 45x - 135$
$f'(x) = 3x^2 + 6x - 45$
Set $f'(x) = 0$:
$3x^2 + 6x - 45 = 0$
$x^2 + 2x - 15 = 0$
$(x - 3)(x + 5) = 0$
$x = 3, \quad x = -5$ (2-5)

50. $f(x) = x^4 - 2x^3 - 5x^2 + 7x$
$f'(x) = 4x^3 - 6x^2 - 10x + 7$
Set $f'(x) = 4x^3 - 6x^2 - 10x + 7 = 0$ and solve for x using a root-approximation routine on a graphing utility:
$f'(x) = 0$ at $x = -1.34, \quad x = 0.58, \quad x = 2.26$ (2-5)

51. $f(x) = x^5 - 10x^3 - 5x + 10$
$f'(x) = 5x^4 - 30x^2 - 5 = 5(x^4 - 6x^2 - 1)$
Let $f'(x) = 5(x^4 - 6x^2 - 1) = 0$ and solve for x using a root-approximation routine on a graphing utility; that is, solve $x^4 - 6x^2 - 1 = 0$ for x.
$f'(x) = 0$ at $x = \pm 2.4824$ (2-5)

52. $y = f(x) = 8x^2 - 4x + 1$

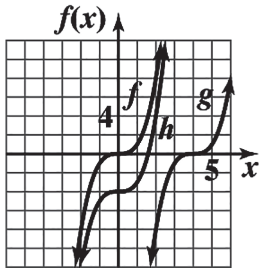
(A) Instantaneous velocity function; $v(x) = f'(x) = 16x - 4$.
(B) $v(3) = 16(3) - 4 = 44$ ft/sec. (2-5)

53. $y = f(x) = -5x^2 + 16x + 3$

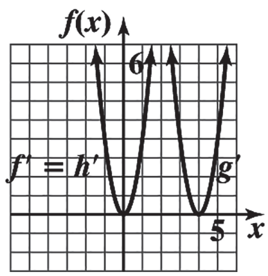
(A) Instantaneous velocity function: $v(x) = f'(x) = -10x + 16$.
(B) $v(x) = 0$ when $-10x + 16 = 0$

$$10x = 16$$
$$x = 1.6 \text{ sec}$$
(2-5)

54. (A) The graph of g is the graph of f shifted 4 units to the right, and the graph of h is the graph of *f* shifted 4 units down.



(B) The graph of g' is the graph of f' shifted 4 units to the right, and the graph of h' is the graph of f'.



(2-4)

55. $f(x) = x^2 - 4$ is a polynomial function; *f* is continuous on $(-\infty, \infty)$. (2-3)

56. $f(x) = \dfrac{x+1}{x-2}$ is a rational function and the denominator $x - 2$ is 0 at $x = 2$. Thus *f* is continuous for all *x* such that $x \neq 2$, i.e., on $(-\infty, 2) \cup (2, \infty)$. (2-3)

57. $f(x) = \dfrac{x+4}{x^2+3x-4}$ is a rational function and the denominator $x^2 + 3x - 4 = (x + 4)(x - 1)$ is 0 at $x = -4$ and $x = 1$. Thus, *f* is continuous for all *x* except $x = -4$ and $x = 1$, i.e., on $(-\infty, -4) \cup (-4, 1) \cup (1, \infty)$. (2-2)

58. $f(x) = \sqrt[3]{4-x^2}$; $g(x) = 4 - x^2$ is continuous for all *x* since it is a polynomial function. Therefore, $f(x) = \sqrt[3]{g(x)}$ is continuous for all *x*, i.e., on $(-\infty, \infty)$. (2-3)

59. $f(x) = \sqrt{4-x^2}$; $g(x) = 4 - x^2$ is continuous for all x and $g(x)$ is nonnegative for $-2 \le x \le 2$. Therefore, $f(x) = \sqrt{g(x)}$ is continuous for $-2 \le x \le 2$, i.e., on $[-2, 2]$. (2-3)

60. $f(x) = \dfrac{2x}{x^2 - 3x} = \dfrac{2x}{x(x-3)} = \dfrac{2}{x-3}, x \ne 0$

(A) $\lim\limits_{x\to 1} f(x) = \lim\limits_{x\to 1} \dfrac{2}{x-3} = \dfrac{\lim\limits_{x\to 1} 2}{\lim\limits_{x\to 1}(x-3)} = \dfrac{2}{-2} = -1$

(B) $\lim\limits_{x\to 3} f(x) = \lim\limits_{x\to 3} \dfrac{2}{x-3}$ does not exist since $\lim\limits_{x\to 3} 2 = 2$ and $\lim\limits_{x\to 3} (x-3) = 0$

(C) $\lim\limits_{x\to 0} f(x) = \lim\limits_{x\to 0} \dfrac{2}{x-3} = -\dfrac{2}{3}$ (2-1)

61. $f(x) = \dfrac{x+1}{(3-x)^2}$

(A) $\lim\limits_{x\to 1} \dfrac{x+1}{(3-x)^2} = \dfrac{\lim\limits_{x\to 1}(x+1)}{\lim\limits_{x\to 1}(3-x)^2} = \dfrac{2}{2^2} = \dfrac{1}{2}$

(B) $\lim\limits_{x\to -1} \dfrac{x+1}{(3-x)^2} = \dfrac{\lim\limits_{x\to -1}(x+1)}{\lim\limits_{x\to -1}(3-x)^2} = \dfrac{0}{4^2} = 0$

(C) $\lim\limits_{x\to 3} \dfrac{x+1}{(3-x)^2}$ does not exist since $\lim\limits_{x\to 3} (x+1) = 4$ and $\lim\limits_{x\to 3} (3-x)^2 = 0$ (2-1)

62. $f(x) = \dfrac{|x-4|}{x-4} = \begin{cases} -1 & \text{if } x < 4 \\ 1 & \text{if } x > 4 \end{cases}$

(A) $\lim\limits_{x\to 4^-} f(x) = -1$ (B) $\lim\limits_{x\to 4^+} f(x) = 1$ (C) $\lim\limits_{x\to 4} f(x)$ does not exist. (2-1)

63. $f(x) = \dfrac{x-3}{9-x^2} = \dfrac{x-3}{(3+x)(3-x)} = \dfrac{-(3-x)}{(3+x)(3-x)} = \dfrac{-1}{3+x}, x \ne 3$

(A) $\lim\limits_{x\to 3} f(x) = \lim\limits_{x\to 3} \dfrac{-1}{3+x} = -\dfrac{1}{6}$

(B) $\lim\limits_{x\to -3} f(x) = \lim\limits_{x\to -3} \dfrac{-1}{3+x}$ does not exist

(C) $\lim\limits_{x\to 0} f(x) = \lim\limits_{x\to 0} \dfrac{-1}{3+x} = -\dfrac{1}{3}$ (2-1)

64. $f(x) = \dfrac{x^2 - x - 2}{x^2 - 7x + 10} = \dfrac{(x-2)(x+1)}{(x-2)(x-5)} = \dfrac{x+1}{x-5}, x \neq 2$

(A) $\lim_{x\to -1} f(x) = \lim_{x\to -1} \dfrac{x+1}{x-5} = 0$

(B) $\lim_{x\to 2} f(x) = \lim_{x\to 2} \dfrac{x+1}{x-5} = \dfrac{3}{-3} = -1$

(C) $\lim_{x\to 5} f(x) = \lim_{x\to 5} \dfrac{x+1}{x-5}$ does not exist (2-1)

65. $f(x) = \dfrac{2x}{3x-6} = \dfrac{2x}{3(x-2)}$

(A) $\lim_{x\to\infty} \dfrac{2x}{3x-6} = \lim_{x\to\infty} \dfrac{2x}{3x} = \dfrac{2}{3}$

(B) $\lim_{x\to -\infty} \dfrac{2x}{3x-6} = \lim_{x\to -\infty} \dfrac{2x}{3x} = \dfrac{2}{3}$

(C) $\lim_{x\to 2^-} \dfrac{2x}{3x-6} = \lim_{x\to 2^-} \dfrac{2x}{3(x-2)} = -\infty$

$\lim_{x\to 2^+} \dfrac{2x}{3(x-2)} = \infty$; $\lim_{x\to 2} \dfrac{2x}{3x-6}$ does not exist. (2-2)

66. $f(x) = \dfrac{2x^3}{3(x-2)^2} = \dfrac{2x^3}{3x^2 - 12x + 12}$

(A) $\lim_{x\to\infty} \dfrac{2x^3}{3x^2 - 12x + 12} = \lim_{x\to\infty} \dfrac{2x^3}{3x^2} = \lim_{x\to\infty} \dfrac{2x}{3} = \infty$

(B) $\lim_{x\to -\infty} \dfrac{2x^3}{3x^2 - 12x + 12} = \lim_{x\to -\infty} \dfrac{2x^3}{3x^2} = \lim_{x\to -\infty} \dfrac{2x}{3} = -\infty$

(C) $\lim_{x\to 2^-} \dfrac{2x^3}{3(x-2)^2} = \lim_{x\to 2^+} \dfrac{2x^3}{3(x-2)^2} = \infty$; $\lim_{x\to 2} \dfrac{2x^3}{3(x-2)^2} = \infty$ (2-2)

67. $f(x) = \dfrac{2x}{3(x-2)^3}$

(A) $\lim_{x\to\infty} \dfrac{2x}{3(x-2)^3} = \lim_{x\to\infty} \dfrac{2x}{3x^3} = \lim_{x\to\infty} \dfrac{2}{3x^2} = 0$

(B) $\lim_{x\to -\infty} \dfrac{2x}{3(x-2)^3} = \lim_{x\to -\infty} \dfrac{2x}{3x^3} = \lim_{x\to -\infty} \dfrac{2}{3x^2} = 0$

(C) $\lim_{x\to 2^-} \dfrac{2x}{3(x-2)^3} = -\infty$, $\lim_{x\to 2^+} \dfrac{2x}{3(x-2)^3} = \infty$; $\lim_{x\to 2} \dfrac{2x}{3(x-2)^3}$ does not exist. (2-2)

68. $f(x) = x^2 + 4$

$$\lim_{h\to 0}\frac{f(2+h)-f(2)}{h} = \lim_{h\to 0}\frac{[(2+h)^2+4]-[2^2+4]}{h} = \lim_{h\to 0}\frac{4+4h+h^2+4-8}{h} = \lim_{h\to 0}\frac{4h+h^2}{h}$$

$$= \lim_{h\to 0}(4+h) = 4 \qquad \text{(2-1)}$$

69. Let $f(x) = \dfrac{1}{x+2}$

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to 0}\frac{\frac{1}{(x+h)+2}-\frac{1}{x+2}}{h} = \lim_{h\to 0}\frac{x+2-(x+h+2)}{h(x+h+2)(x+2)} = \lim_{h\to 0}\frac{-h}{h(x+h+2)(x+2)}$$

$$= \lim_{h\to 0}\frac{-1}{(x+h+2)(x+2)} = \frac{-1}{(x+2)^2} \qquad \text{(2-1)}$$

70. $f(x) = x^2 - x$

Step 1. Find $f(x+h)$.

$f(x+h) = (x+h)^2 - (x+h) = x^2 + 2xh + h^2 - x - h$

Step 2. Find $f(x+h) - f(x)$

$f(x+h) - f(x) = x^2 + 2xh + h^2 - x - h - (x^2 - x) = x^2 + 2xh + h^2 - x - h - (x^2 - x)$

$= x^2 + 2xh + h^2 - x - h - x^2 + x = 2xh + h^2 - h$

Step 3. Find $\dfrac{f(x+h)-f(x)}{h}$.

$\dfrac{f(x+h)-f(x)}{h} = \dfrac{2xh+h^2-h}{h} = 2x + h - 1$

Step 4. Find $\lim_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$.

$\lim_{h\to 0}\dfrac{f(x+h)-f(x)}{h} = \lim_{h\to 0}(2x + h - 1) = 2x - 1$

Thus, $f'(x) = 2x - 1$. (2-4)

71. $f(x) = \sqrt{x} - 3$

Step 1. Find $f(x+h)$.

$f(x+h) = \sqrt{x+h} - 3$

Step 2. Find $f(x+h) - f(x)$

$f(x+h) - f(x) = \sqrt{x+h} - 3 - (\sqrt{x} - 3) = \sqrt{x+h} - 3 - \sqrt{x} + 3 = \sqrt{x+h} - \sqrt{x}$

Step 3. Find $\dfrac{f(x+h)-f(x)}{h}$.

$$\frac{f(x+h)-f(x)}{h} = \frac{\sqrt{x+h}-\sqrt{x}}{h} = \frac{\sqrt{x+h}-\sqrt{x}}{h}\cdot\frac{\sqrt{x+h}+\sqrt{x}}{\sqrt{x+h}+\sqrt{x}} = \frac{1}{\sqrt{x+h}+\sqrt{x}}$$

Step 4. Find $\lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$.

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to 0}\frac{1}{\sqrt{x+h}+\sqrt{x}} = \frac{1}{2\sqrt{x}}$$ (2-4)

72. Yes, $f'(-1)=0$. (2-4)

73. No. f is not differentiable at $x=0$ since it is not continuous at $x=0$. (2-4)

74. No. f has a vertical tangent at $x=1$. (2-4)

75. No. f is not differentiable at $x=2$; the curve has a "corner" at this point. (2-4)

76. Yes. f is differentiable at $x=3$. In fact, $f'(3)=0$. (2-4)

77. Yes. f is differentiable at $x=4$. (2-4)

78. $f(x)=\dfrac{5x}{x-7}$; f is discontinuous at $x=7$

$\lim\limits_{x\to 7^-}\dfrac{5x}{x-7}=-\infty$, $\lim\limits_{x\to 7^+}\dfrac{5x}{x-7}=\infty$; $x=7$ is a vertical asymptote

$\lim\limits_{x\to\infty}f(x)=\lim\limits_{x\to\infty}\dfrac{5x}{x-7}=\lim\limits_{x\to\infty}\dfrac{5x}{x}=5$; $y=5$ is a horizontal asymptote. (2-2, 2-3)

79. $f(x)=\dfrac{-2x+5}{(x-4)^2}$; f is discontinuous at $x=4$.

$\lim\limits_{x\to 4^-}\dfrac{-2x+5}{(x-4)^2}=-\infty$, $\lim\limits_{x\to 4^+}\dfrac{-2x+5}{(x-4)^2}=-\infty$; $x=4$ is a vertical asymptote.

$\lim\limits_{x\to\infty}\dfrac{-2x+5}{(x-4)^2}=\lim\limits_{x\to\infty}\dfrac{-2x}{x^2}=\lim\limits_{x\to\infty}\dfrac{-2}{x}=0$; $y=0$ is a horizontal asymptote. (2-2)

80. $f(x)=\dfrac{x^2+9}{x-3}$; f is discontinuous at $x=3$.

$\lim\limits_{x\to 3^-}\dfrac{x^2+9}{x-3}=-\infty$, $\lim\limits_{x\to 3^+}\dfrac{x^2+9}{x-3}=\infty$; $x=3$ is a vertical asymptote.

$\lim\limits_{x\to\infty}\dfrac{x^2+9}{x-3}=\lim\limits_{x\to\infty}\dfrac{x^2}{x}=\lim\limits_{x\to\infty}x=\infty$; no horizontal asymptotes. (2-2)

81. $f(x)=\dfrac{x^2-9}{x^2+x-2}=\dfrac{x^2-9}{(x+2)(x-1)}$; f is discontinuous at $x=-2$, $x=1$.

At $x=-2$:

$\lim\limits_{x\to -2^-}\dfrac{x^2-9}{(x+2)(x-1)}=-\infty$, $\lim\limits_{x\to -2^+}\dfrac{x^2-9}{(x+2)(x-1)}=\infty$; $x=-2$ is a vertical asymptote.

At $x = 1$

$$\lim_{x\to 1^-} \frac{x^2-9}{(x+2)(x-1)} = \infty, \quad \lim_{x\to 1^+} \frac{x^2-9}{(x+2)(x-1)} = -\infty;$$ $x = 1$ is a vertical asymptote.

$$\lim_{x\to\infty} \frac{x^2-9}{x^2+x-2} = \lim_{x\to\infty} \frac{x^2}{x^2} = \lim_{x\to\infty} 1 = 1;$$ $y = 1$ is a horizontal asymptote. (2-2)

82. $f(x) = \dfrac{x^3-1}{x^3-x^2-x+1} = \dfrac{(x-1)(x^2+x+1)}{(x-1)(x^2-1)} = \dfrac{(x-1)(x^2+x+1)}{(x-1)^2(x+1)} = \dfrac{x^2+x+1}{(x-1)(x+1)}, x \neq 1.$

f is discontinuous at $x = 1$, $x = -1$.

At $x = 1$:

$$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} \frac{x^2+x+1}{(x-1)(x+1)} = -\infty, \quad \lim_{x\to 1^+} f(x) = \infty;$$ $x = 1$ is a vertical asymptote.

At $x = -1$:

$$\lim_{x\to -1^-} \frac{x^2+x+1}{(x-1)(x+1)} = \infty, \quad \lim_{x\to -1^+} \frac{x^2+x+1}{(x-1)(x+1)} = -\infty;$$ $x = -1$ is a vertical asymptote.

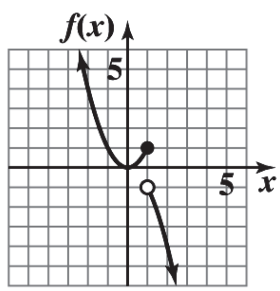
$$\lim_{x\to\infty} \frac{x^3-1}{x^3-x^2-x+1} = \lim_{x\to\infty} \frac{x^3}{x^3} = \lim_{x\to\infty} 1 = 1;$$ $y = 1$ is a horizontal asymptote. (2-2)

83. $f(x) = x^{1/3};\ f'(x) = \dfrac{1}{3}x^{-2/3} = \dfrac{1}{3x^{2/3}}$

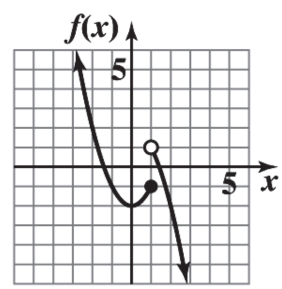
The domain of $f'(x)$ is all real numbers except $x = 0$. At $x = 0$, the graph of $f(x)$ is smooth, but it has a vertical tangent. (2-4)

84. $f(x) = \begin{cases} x^2 - m & \text{if } x \le 1 \\ -x^2 + m & \text{if } x > 1 \end{cases}$

(A)



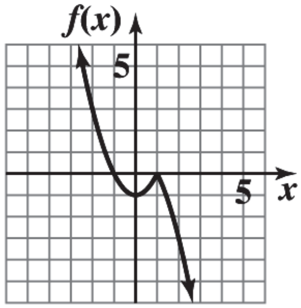
$\lim_{x\to 1^-} f(x) = 1, \quad \lim_{x\to 1^+} f(x) = -1$



$\lim_{x\to 1^-} f(x) = -1, \quad \lim_{x\to 1^+} f(x) = 1$

(C) $\lim_{x\to 1^-} f(x) = 1 - m, \quad \lim_{x\to 1^+} f(x) = -1 + m$

We want $1 - m = -1 + m$ which implies $m = 1$.



(D) The graphs in (A) and (B) have jumps at $x = 1$; the graph in (C) does not. (2-2)

85. $f(x) = 1 - |x - 1|, 0 \le x \le 2$

(A) $\lim_{h\to 0^-} \frac{f(1+h)-f(1)}{h} = \lim_{h\to 0^-} \frac{1-|1+h-1|-1}{h} = \lim_{h\to 0^-} \frac{-|h|}{h} = \lim_{h\to 0^-} \frac{h}{h} = 1$ ($|h| = -h$ if $h < 0$)

(B) $\lim_{h\to 0^+} \frac{f(1+h)-f(1)}{h} = \lim_{h\to 0^+} \frac{1-|1+h-1|-1}{h} = \lim_{h\to 0^+} \frac{-|h|}{h} = \lim_{h\to 0^+} \frac{-h}{h} = -1$ ($|h| = h$ if $h > 0$)

(C) $\lim_{h\to 0} \frac{f(1+h)-f(1)}{h}$ does not exist, since the left limit and the right limit are not equal.

(D) $f'(1)$ does not exist. (2-4)

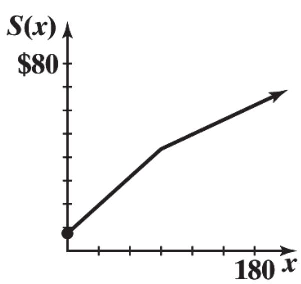
86. (A) $S(x) = 7.47 + 0.4000x$ for $0 \le x \le 90$; $S(90) = 43.47$;

$S(x) = 43.47 + 0.2076(x - 90) = 24.786 + 0.2076x, x > 90$

Therefore,

$$S(x) = \begin{cases} 7.47 + 0.4000x & \text{if } 0 \le x \le 90 \\ 24.786 + 0.2076x & \text{if } x > 90 \end{cases}$$

(B)



(C) $\lim_{x\to 90^-} S(x) = \lim_{x\to 90^+} S(x) = 43.47 = S(90)$;

$S(x)$ is continuous at $x = 90$.

(2-2)

87. $C(x) = 10{,}000 + 200x - 0.1x^2$

(A) $C(101) - C(100) = 10{,}000 + 200(101) - 0.1(101)^2 - [10{,}000 + 200(100) - 0.1(100)^2]$

$= 29{,}179.90 - 29{,}000 = \$179.90$

(B) $C'(x) = 200 - 0.2x$

$C'(100) = 200 - 0.2(100) = 200 - 20 = \$180$ (2-7)

88. $C(x) = 5{,}000 + 40x + 0.05x^2$

(A) Cost of producing 100 bicycles:

$C(100) = 5{,}000 + 40(100) + 0.05(100)^2 = 9{,}000 + 500 = 9{,}500$

Marginal cost:

$C'(x) = 40 + 0.1x$

$C'(100) = 40 + 0.1(100) = 40 + 10 = 50$

Interpretation: At a production level of 100 bicycles, the total cost is \$9,500 and is increasing at the rate of \$50 per additional bicycle.

(B) Average cost: $\overline{C}(x) = \frac{C(x)}{x} = \frac{5{,}000}{x} + 40 + 0.05x$

$$\overline{C}(100) = \frac{5{,}000}{100} + 40 + 0.05(100) = 50 + 40 + 5 = 95$$

Marginal average cost: $\overline{C}'(x) = -\frac{5{,}000}{x^2} + 0.05$ and

$$\overline{C}'(100) = -\frac{5{,}000}{(100)^2} + 0.05 = -0.5 + 0.05 = -0.45$$

Interpretation: At a production level of 100 bicycles, the average cost is \$95 and the average cost is decreasing at a rate of \$0.45 per additional bicycle. (2-7)

89. The approximate cost of producing the 201st printer is greater than that of producing the 601st printer (the slope of the tangent line at $x = 200$ is greater than the slope of the tangent line at $x = 600$). Since the marginal costs are decreasing, the manufacturing process is becoming more efficient. (2-7)

90. $p = 25 - 0.01x$, $C(x) = 2x + 9{,}000$

(A) Marginal cost: $C'(x) = 2$

Average cost: $\overline{C}(x) = \frac{C(x)}{x} = 2 + \frac{9{,}000}{x}$

Marginal average cost: $\overline{C}' = -\frac{9{,}000}{x^2}$

(B) Revenue: $R(x) = xp = 25x - 0.01x^2$

Marginal revenue: $R'(x) = 25 - 0.02x$

Average revenue: $\overline{R}(x) = \frac{R(x)}{x} = 25 - 0.01x$

Marginal average revenue: $\overline{R}'(x) = -0.01$

(C) Profit: $P(x) = R(x) - C(x) = 25x - 0.01x^2 - (2x + 9{,}000) = 23x - 0.01x^2 - 9{,}000$

Marginal profit: $P'(x) = 23 - 0.02x$

Average profit: $\overline{P}(x) = \frac{P(x)}{x} = 23 - 0.01x - \frac{9{,}000}{x}$

Marginal average profit: $\overline{P}'(x) = -0.01 + \frac{9{,}000}{x^2}$

(D) Break-even points: $R(x) = C(x)$

$$25x - 0.01x^2 = 2x + 9{,}000$$
$$0.01x^2 - 23x + 9{,}000 = 0$$
$$x^2 - 2{,}300x + 900{,}000 = 0$$
$$(x - 500)(x - 1{,}800) = 0$$

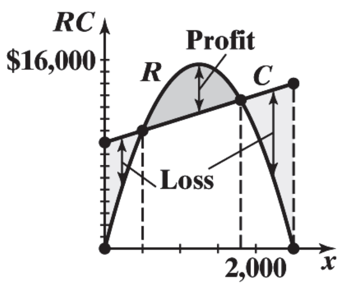
Thus, the break-even points are at $x = 500$, $x = 1{,}800$;
break-even points: (500, 10,000), (1,800, 12,600).

(E) $P'(1{,}000) = 23 - 0.02(1000) = 3$; profit is increasing at the rate of \$3 per umbrella.

$P'(1{,}150) = 23 - 0.02(1{,}150) = 0$; profit is flat.

$P'(1{,}400) = 23 - 0.02(1{,}400) = -5$; profit is decreasing at the rate of \$5 per umbrella.

(F)



(2-7)

91. $N(t) = \dfrac{40t - 80}{t} = 40 - \dfrac{80}{t}$, $t \geq 2$

(A) Average rate of change from $t = 2$ to $t = 5$:

$$\frac{N(5) - N(2)}{5 - 2} = \frac{\frac{40(5) - 80}{5} - \frac{40(2) - 80}{2}}{3} = \frac{120}{15} = 8 \text{ components per day.}$$

(B) $N(t) = 40 - \dfrac{80}{t} = 40 - 80t^{-1}$; $N'(t) = 80t^{-2} = \dfrac{80}{t^2}$.

$N'(2) = \dfrac{80}{4} = 20$ components per day. (2-5)

(C) $\displaystyle\lim_{t \to \infty} \frac{40t - 80}{t} = \lim_{t \to \infty}\left(\frac{40t}{t} - \frac{80}{t}\right) = \lim_{t \to \infty}\left(40 - \frac{80}{t}\right) = 40$

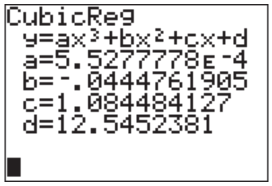
Long-term employees should near 40 components per day. (2-2)

92. $N(t) = 2t + \dfrac{1}{3}t^{3/2}$, $N'(t) = 2 + \dfrac{1}{2}t^{1/2} = \dfrac{4 + \sqrt{t}}{2}$

$N(9) = 18 + \dfrac{1}{3}(9)^{3/2} = 27$, $N'(9) = \dfrac{4 + \sqrt{9}}{2} = \dfrac{7}{2} = 3.5$

After 9 months, 27,000 pools have been sold and the total sales are increasing at the rate of 3,500 pools per month. (2-5)

93. (A)

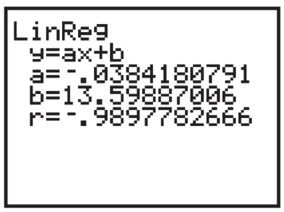


(B) $N(x) \approx 0.0005528x^3 - 0.044x^2 + 1.084x + 12.545$

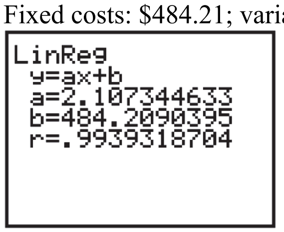
$N'(x) \approx 0.0016584x^2 - 0.088x + 1.084$

$N(60) \approx 36.9$, $N'(60) \approx 1.7$. In 2020, natural gas consumption will be 36.9 trillion cubic feet and will be INCREASING at the rate of 1.7 trillion cubic feet per year. (2-4)

94. (A)

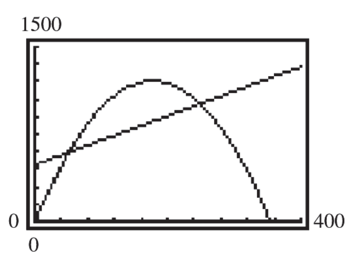


(B) Fixed costs: $484.21; variable cost per kringle: $2.11.



(C) Let $p(x)$ be the linear regression equation found in part (A) and let $C(x)$ be the linear regression equation found in part (B). Then revenue $R(x) = xp(x)$ and the break-even points are the points where $R(x) = C(x)$.

Using an intersection routine on a graphing utility, the break-even points are: (51, 591.15) and (248, 1,007.62).



(D) The bakery will make a profit when $51 < x < 248$. From the regression equation in part (A), $p(51) = 11.64$ and $p(248) = 4.07$. Thus, the bakery will make a profit for the price range $\$4.07 < p < \$11.64$. (2-7)

95. $C(x) = \dfrac{500}{x^2} = 500x^{-2}, x \ge 1.$

The instantaneous rate of change of concentration at x meters is:

$$C'(x) = 500(-2)x^{-3} = \frac{-1,000}{x^3}$$

The rate of change of concentration at 10 meters is:

$$C'(10) = \frac{-1,000}{10^3} = -1 \text{ parts per million per meter}$$

The rate of change of concentration at 100 meters is:

$$C'(100) = \frac{-1,000}{(100)^3} = \frac{-1,000}{100,000,000} = -\frac{1}{1,000} = -0.001 \text{ part per million per meter.}$$ (2-5)

96. $F(t) = 0.16t^2 - 1.6t + 102, F'(t) = 0.32t - 1.6$

$F(4) = 98.16, F'(4) = -0.32.$

After 4 hours the patient's temperature is 98.16˚F and is decreasing at the rate of 0.32˚F per hour. (2-5)

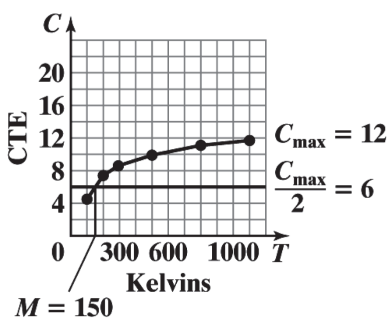
97. $N(t) = 20\sqrt{t} = 20t^{1/2}$

The rate of learning is $N'(t) = 20\left(\dfrac{1}{2}\right)t^{-1/2} = 10t^{-1/2} = \dfrac{10}{\sqrt{t}}$.

(A) The rate of learning after one hour is $N'(1) = \dfrac{10}{\sqrt{1}} = 10$ items per hour.

(B) The rate of learning after four hours is $N'(4) = \dfrac{10}{\sqrt{4}} = \dfrac{10}{2} = 5$ items per hour. (2-5)

98. (A)



(B) $C(T) = \dfrac{12T}{150 + T}$

(C) $C(600) = \dfrac{12(600)}{150 + 600} = 9.6$

To find T when $C = 10$, solve $\dfrac{12T}{150 + T} = 10$ for T.

$$\frac{12T}{150+T} = 10$$
$$12T = 1500 + 10T$$
$$2T = 1500$$
$$T = 750$$

$T = 750$ when $C = 10$. (2-3)