2 LIMITS AND THE DERIVATIVE

EXERCISE 2-1

2. $x^2 - 64 = (x-8)(x+8)$

4. $x^2 + 5x - 36 = (x+9)(x-4)$

6. $x^3 + 15x^2 + 50x = x(x^2 + 15x + 50) = x(x+5)(x+10)$

8. $20x^2 + 11x - 3 = (4x+3)(5x-1)$

10. $f(-1.5) = 2$

12. $f(1.25) = 1.75$

14. (A) $\lim_{x\to 1^-} f(x) = 2$ (B) $\lim_{x\to 1^+} f(x) = 2$ (C) $\lim_{x\to 1} f(x) = 2$ (D) $f(1) = 2$

16. (A) $\lim_{x\to 4^-} f(x) = 4$ (B) $\lim_{x\to 4^+} f(x) = 4$ (C) $\lim_{x\to 4} f(x) = 4$
(D) $f(4)$ does not exist (E) Yes, define $f(4) = 4$

18. $g(0.1) = 1$

20. $g(2.5) = 1.5$

22. (A) $\lim_{x\to 2^-} g(x) = 2$ (B) $\lim_{x\to 2^+} g(x) = 2$ (C) $\lim_{x\to 2} g(x) = 2$ (D) $g(2) = 2$

24. (A) $\lim_{x\to 4^-} g(x) = 0$ (B) $\lim_{x\to 4^+} g(x) = 0$ (C) $\lim_{x\to 4} g(x) = 0$ (D) $g(4) = 0$

26. (A) $\lim_{x\to -2^+} f(x) = 3$ (B) $\lim_{x\to -2^-} f(x) = -3$
(C) Since $\lim_{x\to -2^+} f(x) \neq \lim_{x\to -2^-} f(x)$, $\lim_{x\to -2} f(x)$ does not exist.
(D) $f(-2) = -3$
(E) No, the limit does not exist, so it cannot be equal to any possible value of $f(-2)$.

28. (A) $\lim_{x\to 2^+} f(x) = -3$ (B) $\lim_{x\to 2^-} f(x) = 3$
(C) $\lim_{x\to 2} f(x)$ does not exist since $\lim_{x\to 2^+} f(x) \neq \lim_{x\to 2^-} f(x)$
(D) $f(2) = 3$ (E) No, $\lim_{x\to 2} f(x)$ does not exist.

30. $3x \to -6$ as $x \to -2$; thus $\lim_{x\to -2} 3x = -6$

32. $x - 3 \to 5 - 3 = 2$ as $x \to 5$; thus $\lim_{x\to 5} (x - 3) = 2$

34. $x(x + 3) \to (-1)(-1 + 3) = -2$ as $x \to -1$; thus $\lim_{x\to -1} x(x + 3) = -2$

36. $x - 2 \to 4 - 2 = 2$ as $x \to 4$; thus $\lim_{x\to 4} \dfrac{x-2}{x} = \dfrac{2}{4} = \dfrac{1}{2}$

38. $\sqrt{16-7x} \to \sqrt{16-7(0)} = \sqrt{16} = 4$ as $x \to 0$; thus $\lim_{x\to 0} \sqrt{16-7x} = 4$

40. $\lim_{x\to 1} 2g(x) = 2\lim_{x\to 1} g(x) = 2(4) = 8$

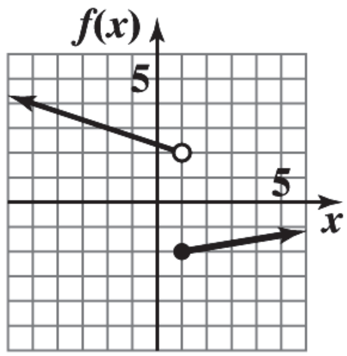
42. $\lim_{x\to1}[g(x) - 3f(x)] = \lim_{x\to1} g(x) - 3\lim_{x\to1} f(x) = 4 - 3(-5) = 19$

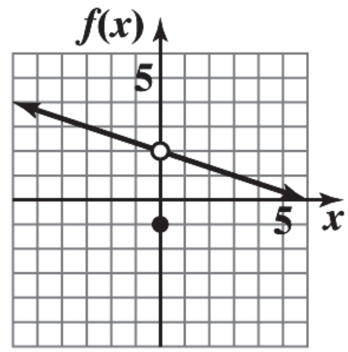
44. $\lim_{x\to1}\dfrac{3-f(x)}{1-4g(x)} = \dfrac{\lim_{x\to1}[3-f(x)]}{\lim_{x\to1}[1-4g(x)]} = \dfrac{3-\lim_{x\to1} f(x)}{1-4\lim_{x\to1} g(x)} = \dfrac{3-(-5)}{1-4(4)} = -\dfrac{8}{15}$

46. $\lim_{x\to1}\sqrt[3]{2x+2f(x)} = \sqrt[3]{\lim_{x\to1}[2x+2f(x)]}$

$= \sqrt[3]{2\lim_{x\to1} x + 2\lim_{x\to1} f(x)}$

$= \sqrt[3]{2-10} = -2$

48. 

50. 

52. $f(x) = \begin{cases} 2+x & \text{if } x \le 0 \\ 2-x & \text{if } x > 0 \end{cases}$

(A) $\lim_{x\to0^+} f(x) = \lim_{x\to0^+}(2-x) = 2$

(B) $\lim_{x\to0^-} f(x) = \lim_{x\to0^-}(2+x) = 2$

(C) $\lim_{x\to0} f(x) = 2$ since $\lim_{x\to0^+} f(x) = \lim_{x\to0^-} f(x) = 2$

(D) $f(0) = 2 + 0 = 2$

54. $f(x) = \begin{cases} x+3 & \text{if } x < -2 \\ \sqrt{x+2} & \text{if } x > -2 \end{cases}$

(A) $\lim_{x\to-2^+} f(x) = \lim_{x\to-2^+}\sqrt{x+2} = 0$

(B) $\lim_{x\to-2^-} f(x) = \lim_{x\to-2^-}(x+3) = 1$

(C) $\lim_{x\to-2} f(x)$ does not exist since $\lim_{x\to-2^+} f(x) \ne \lim_{x\to-2^-} f(x)$

(D) $f(-2)$ does not exist; f is not defined at $x = -2$.

56. $f(x) = \begin{cases} \dfrac{x}{x+3} & \text{if } x < 0 \\ \dfrac{x}{x-3} & \text{if } x > 0 \end{cases}$

(A) $\lim_{x\to-3} f(x) = \lim_{x\to-3}\dfrac{x}{x+3}$ does not exist since $x = -3$ is a non-removable zero of the denominator.

(B) $\lim_{x\to0} f(x) = \lim_{x\to0^-}\dfrac{x}{x+3} = \lim_{x\to0^+}\dfrac{x}{x+3} = 0$

(C) $\lim_{x\to3} f(x)$ does not exist, since $\lim_{x\to3^+} f(x)$ does not exist.

58. $f(x) = \dfrac{x-3}{|x-3|} = \begin{cases} \dfrac{x-3}{-(x-3)} = -1 & \text{if} \quad x < 3 \\ \dfrac{x-3}{x-3} = 1 & \text{if} \quad x > 3 \end{cases}$

(Note: Observe that for $x < 3$, $|x-3| = 3 - x = -(x-3)$ and for $x > 3$, $|x-3| = x-3$)

(A) $\lim\limits_{x\to 3^+} f(x) = \lim\limits_{x\to 3^+} 1 = 1$ (B) $\lim\limits_{x\to 3^-} f(x) = \lim\limits_{x\to 3^-} (-1) = -1$

(C) $\lim\limits_{x\to 3} f(x)$ does not exist, since $\lim\limits_{x\to 3^+} f(x) \neq \lim\limits_{x\to 3^-} f(x)$

(D) $f(3)$ does not exist; f is not defined at $x = 3$.

60. $f(x) = \dfrac{x+3}{x^2+3x} = \dfrac{x+3}{x(x+3)}$

(A) $\lim\limits_{x\to -3} \dfrac{x+3}{x(x+3)} = \lim\limits_{x\to -3} \dfrac{1}{x} = -\dfrac{1}{3}$ (B) $\lim\limits_{x\to 0} f(x) = \lim\limits_{x\to 0} \dfrac{1}{x}$ does not exist. (C) $\lim\limits_{x\to 3} \dfrac{1}{x} = \dfrac{1}{3}$

62. $f(x) = \dfrac{x^2+x-6}{x+3} = \dfrac{(x+3)(x-2)}{(x+3)}$

(A) $\lim\limits_{x\to -3} f(x) = \lim\limits_{x\to -3} \dfrac{(x+3)(x-2)}{(x+3)} = \lim\limits_{x\to -3} (x-2) = -5$

(B) $\lim\limits_{x\to 0} f(x) = \lim\limits_{x\to 0} \dfrac{x^2+x-6}{x+3} = \dfrac{-6}{3} = -2$

(C) $\lim\limits_{x\to 2} f(x) = \lim\limits_{x\to 2} \dfrac{x^2+x-6}{x+3} = \dfrac{0}{5} = 0$

64. $f(x) = \dfrac{x^2-1}{(x+1)^2} = \dfrac{(x-1)(x+1)}{(x+1)^2}$

(A) $\lim\limits_{x\to -1} f(x) = \lim\limits_{x\to -1} \dfrac{(x-1)(x+1)}{(x+1)^2} = \lim\limits_{x\to -1} \dfrac{x-1}{x+1}$ does not exist since $\lim\limits_{x\to -1} (x-1) = -2$ but $\lim\limits_{x\to -1} (x+1) = 0$.

(B) $\lim\limits_{x\to 0} f(x) = \lim\limits_{x\to 0} \dfrac{x^2-1}{(x+1)^2} = \dfrac{-1}{1} = -1$ (C) $\lim\limits_{x\to 1} f(x) = \lim\limits_{x\to 1} \dfrac{x^2-1}{(x+1)^2} = \dfrac{0}{4} = 0$

66. $f(x) = \dfrac{3x^2+2x-1}{x^2+3x+2} = \dfrac{(3x-1)(x+1)}{(x+2)(x+1)}$

(A) $\lim\limits_{x\to -3} f(x) = \lim\limits_{x\to -3} \dfrac{3x^2+2x-1}{x^2+3x+2} = \dfrac{20}{2} = 10$

(B) $\lim\limits_{x\to -1} f(x) = \lim\limits_{x\to -1} \dfrac{(3x-1)(x+1)}{(x+2)(x+1)} = \lim\limits_{x\to -1} \dfrac{3x-1}{x+2} = \dfrac{-4}{1} = -4$

(C) $\lim\limits_{x\to 2} f(x) = \lim\limits_{x\to 2} \dfrac{3x^2+2x-1}{x^2+3x+2} = \dfrac{15}{12} = \dfrac{5}{4}$

68. True: $\lim\limits_{x\to 1}\dfrac{f(x)}{g(x)}=\dfrac{\lim\limits_{x\to 1}f(x)}{\lim\limits_{x\to 1}g(x)}=\dfrac{1}{1}=1$

70. Not always true. For example, the statement is false for $f(x)=\dfrac{1}{x}$.

72. Not always true. For example, the statement is false for $f(x)=\begin{cases}-1 & x\le 0\\ 1 & x>0\end{cases}$

74. $\lim\limits_{x\to 2}\dfrac{x-5}{x+2}$ does not have the form $\dfrac{0}{0}$; $\lim\limits_{x\to 2}\dfrac{x-5}{x+2}=\dfrac{-3}{4}$.

76. $\lim\limits_{x\to 9}\dfrac{x^2-5x-36}{x-9}$ has the form $\dfrac{0}{0}$; $\dfrac{x^2-5x-36}{x-9}=\dfrac{(x-9)(x+4)}{x-9}=x+4$, provided $x\neq 9$.

Therefore $\lim\limits_{x\to 9}\dfrac{x^2-5x-36}{x-9}=\lim\limits_{x\to 9}(x+4)=13$.

78. $\lim\limits_{x\to 10}\dfrac{x^2-15x+50}{(x-10)^2}$ has the form $\dfrac{0}{0}$; $\dfrac{x^2-15x+50}{(x-10)^2}=\dfrac{(x-5)(x-10)}{(x-10)^2}=\dfrac{x-5}{x-10}$, provided $x\neq 10$.

Therefore $\lim\limits_{x\to 10}\dfrac{x^2-15x+50}{(x-10)^2}$ does not exist.

80. $\lim\limits_{x\to -3}\dfrac{x+3}{x-3}$ does not have the form $\dfrac{0}{0}$; $\lim\limits_{x\to -3}\dfrac{x+3}{x-3}=\dfrac{0}{-6}=0$.

82. $f(x) = 5x - 1$

$$\lim_{h\to 0}\frac{f(2+h)-f(2)}{h}=\lim_{h\to 0}\frac{5(2+h)-1-(10-1)}{h}=\lim_{h\to 0}\frac{10+5h-1-9}{h}=\lim_{h\to 0}\frac{5h}{h}=\lim_{h\to 0}5=5$$

84. $f(x) = x^2 - 2$

$$\lim_{h\to 0}\frac{f(2+h)-f(2)}{h}=\lim_{h\to 0}\frac{(2+h)^2-2-(4-2)}{h}=\lim_{h\to 0}\frac{4+4h+h^2-2-2}{h}$$

$$=\lim_{h\to 0}\frac{4h+h^2}{h}=\lim_{h\to 0}(4+h)=4$$

86. $f(x) = -4x + 13$

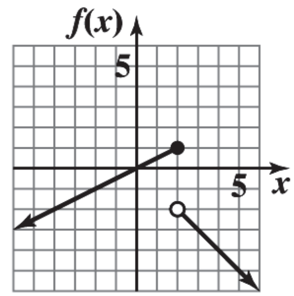
$$\lim_{h\to 0}\frac{f(2+h)-f(2)}{h}=\lim_{h\to 0}\frac{-4(2+h)+13-[-4(2)+13]}{h}=\lim_{h\to 0}\frac{-4h}{h}=-4$$

88. $f(x) = -3|x|$

$$\lim_{h\to 0}\frac{f(2+h)-f(2)}{h}=\lim_{h\to 0}\frac{-3|2+h|-[-3(2)]}{h}=\lim_{h\to 0}\frac{-3(2+h)+6}{h}=\lim_{h\to 0}\frac{-3h}{h}=-3$$

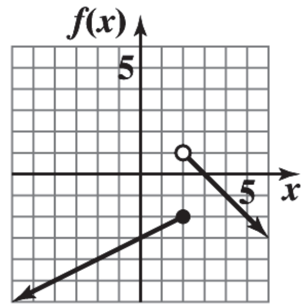
90. (A) $\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-} (0.5x) = 1$

$\lim_{x\to 2^+} f(x) = \lim_{x\to 2^+} (-x) = -2$



(B) $\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-} (-3 + 0.5x) = -2$

$\lim_{x\to 2^+} f(x) = \lim_{x\to 2^+} (3 - x) = 1$



(C) $\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-} (-3m + 0.5x) = -3m + 1$

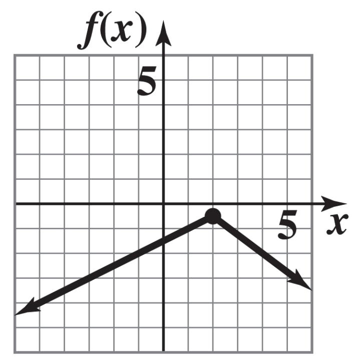
$\lim_{x\to 2^+} f(x) = \lim_{x\to 2^+} (3m - x) = 3m - 2$

$-3m + 1 = 3m - 2$

$6m = 3$

$m = \dfrac{1}{2} = 0.5$

$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^+} f(x) = -0.5$

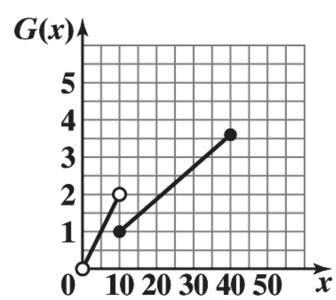


(D) The graph in (A) is broken when it jumps from (2, 1) down to (2, –2), the graph in (B) is also broken when it jumps from (2, –2) up to (2, 1), while the graph in (C) is one continuous piece with no jumps or breaks.

92. (A) If a state-to-state long distance call lasts x minutes, then for $0 < x < 10$, the charge will be $0.18x$ and for $x \geq 10$, the charge will be $0.09x$. Thus,

$$G(x) = \begin{cases} 0.18x & , \quad 0 < x < 10 \\ 0.09x & , \quad x \geq 10 \end{cases}$$

(B)



(C) As x approaches 10 from the left, $G(x)$ approaches 1.8, thus, the left limit of $G(x)$ at $x = 10$ exists, $\lim_{x\to 10^-} G(x) = 1.8$.

Similarly, $\lim_{x\to 10^+} G(x) = 0.90$. However, $\lim_{x\to 10} G(x)$ does not exist, since $\lim_{x\to 10^-} G(x) \neq \lim_{x\to 10^+} G(x)$.

94. For calls lasting more than 20 minutes, the charge for the service given in Problem 91 is $0.07x - 0.41$ whereas for that of Problem 92 is $0.09x$. It is clear that the latter is more expensive than the former.

96. (A) Let x be the volume of a purchase before the discount is applied. Then $P(x)$ is given by:

$$P(x) = \begin{cases} x & \text{if } 0 \le x < 300 \\ 300 + 0.97(x-300) = 0.97x + 9 & \text{if } 300 \le x < 1{,}000 \\ 0.97(1{,}000) + 9 + 0.95(x - 1{,}000) = 0.95x + 29 & \text{if } 1{,}000 \le x < 3{,}000 \\ 0.95(3{,}000) + 29 + 0.93(x - 3{,}000) = 0.93x + 89 & \text{if } 3{,}000 \le x < 5{,}000 \\ 0.93(5{,}000) + 89 + 0.90(x - 5{,}000) = 0.90x + 239 & \text{if } x \ge 5{,}000 \end{cases}$$

(B) $\lim_{x \to 1{,}000^-} P(x) = 0.97(1{,}000) + 9 = 979$

$\lim_{x \to 1{,}000^+} P(x) = 0.95(1{,}000) + 29 = 979$

Thus, $\lim_{x \to 1{,}000} P(x) = 979$

$\lim_{x \to 3{,}000^-} P(x) = 0.95(3{,}000) + 29 = 2{,}879$

$\lim_{x \to 3{,}000^+} P(x) = 0.93(3{,}000) + 89 = 2{,}879$

Thus, $\lim_{x \to 3{,}000} P(x) = 2{,}879$

(C) For $0 \le x < 300$, they produce the same price. For $x \ge 300$, the one in Problem 95 produces a lower price.

98. From Problem 97, we have:

$$F(x) = \begin{cases} 20x & \text{if } 0 < x \le 4{,}000 \\ 80{,}000 & \text{if } x > 4{,}000 \end{cases}$$

Thus

$$A(x) = \frac{F(x)}{x} = \begin{cases} 20 & \text{if } 0 < x \le 4{,}000 \\ \dfrac{80{,}000}{x} & \text{if } x > 4{,}000 \end{cases}$$

$$\lim_{x \to 4{,}000^-} A(x) = \lim_{x \to 4{,}000^+} A(x) = 20 = \lim_{x \to 4{,}000} A(x)$$

$$\lim_{x \to 8{,}000^-} A(x) = \lim_{x \to 8{,}000^+} A(x) = \frac{80{,}000}{8{,}000} = 10 = \lim_{x \to 8{,}000} A(x)$$

EXERCISE 2-2

2. $x = 5$

4. $y = 1$

6. $y + 4 = -3(x - 8)$ (point-slope form); $3x + y = 20$

8. Slope: $m = \dfrac{30 - 20}{1 - (-1)} = 5$; $y - 20 = 5[x - (-1)]$ (point-slope form); $-5x + y = 25$

10. $\lim_{x \to -\infty} f(x) = \infty$

12. $\lim_{x \to -2^-} f(x) = \infty$

14. $\lim_{x \to 2^+} f(x) = \infty$

16. $\lim_{x \to 2} f(x)$ does not exist

18. $f(x) = \dfrac{x^2}{x+3}$

(A) $\lim_{x\to -3^-} \dfrac{x^2}{x+3} = -\infty$; as x approaches –3 from the left, the

denominator is negatively approaching 0 and the numerator is positively approaching $(-3)^2 = 9$.

(B) $\lim_{x\to -3^+} \dfrac{x^2}{x+3} = \infty$; numerator approaches $(-3)^2 = 9$ and denominator

is positively approaching 0.

(C) Since left and right limits at –3 are not equal,
$\lim_{x\to -3} f(x)$ does not exist.

20. $f(x) = \dfrac{2x+2}{(x+2)^2}$

(A) $\lim_{x\to -2^-} \dfrac{2x+2}{(x+2)^2} = -\infty$; as x approaches –2 from the left, the denominator is positively approaching 0

and the numerator is negatively approaching $2(-2) + 2 = -2$.

(B) $\lim_{x\to -2^+} \dfrac{2x+2}{(x+2)^2} = -\infty$; as x approaches –2 from the right, the denominator is positively approaching 0

and the numerator is negatively approaching $2(-2) + 2 = -2$.

(C) Since $\lim_{x\to -2^-} f(x) = \lim_{x\to -2^+} f(x) = -\infty$, we can say that $\lim_{x\to -2} f(x) = -\infty$.

22. $f(x) = \dfrac{x^2+x+2}{x-1}$

(A) $\lim_{x\to 1^-} \dfrac{x^2+x+2}{x-1} = -\infty$; as x approaches 1, the numerator approaches 4 and the denominator negatively

approaches 0.

(B) $\lim_{x\to 1^+} \dfrac{x^2+x+2}{x-1} = \infty$; in this case the denominator positively approaches 0.

(C) $\lim_{x\to 1} \dfrac{x^2+x+2}{x-1}$ does not exist.

24. $f(x) = \dfrac{x^2+x-2}{(x+2)}$

$f(x) = \dfrac{(x-1)(x+2)}{(x+2)}$

(A) $\lim_{x\to -2^-} \dfrac{(x-1)(x+2)}{(x+2)} = \lim_{x\to -2^-} (x-1) = -3$

(B) $\lim_{x\to -2^+} \dfrac{(x-1)(x+2)}{(x+2)} = \lim_{x\to -2^+} (x-1) = -3$

(C) $\lim_{x\to -2} \dfrac{(x-1)(x+2)}{(x+2)} = \lim_{x\to -2} (x-1) = -3$ or we can say that left and right limits at $x = -2$ exist and are

equal, therefore
$\lim_{x\to -2} f(x)$ exists and is equal to the common value –3.

26. $p(x) = 10 - x^6 + 7x^3 = -x^6 + 7x^3 + 10$

(A) Leading term: $-x^6$ (B) $\lim_{x\to\infty} p(x) = \lim_{x\to\infty} (-x^6) = -\infty$ (C) $\lim_{x\to-\infty} p(x) = \lim_{x\to-\infty} (-x^6) = -\infty$

28. $p(x) = -x^5 + 2x^3 + 9x$

(A) Leading term: $-x^5$ (B) $\lim_{x\to\infty} p(x) = \lim_{x\to\infty} (-x^5) = -\infty$ (C) $\lim_{x\to-\infty} p(x) = \lim_{x\to-\infty} (-x^5) = \infty$

30. $p(x) = 5x + x^3 - 8x^2 = x^3 - 8x^2 + 5x$

(A) Leading term: x^3 (B) $\lim_{x\to\infty} p(x) = \lim_{x\to\infty} (x^3) = \infty$ (C) $\lim_{x\to-\infty} p(x) = \lim_{x\to-\infty} (x^3) = -\infty$

32. $p(x) = 1 + 4x^2 + 4x^4 = 4x^4 + 4x^2 + 1$

(A) Leading term: $4x^4$ (B) $\lim_{x\to\infty} p(x) = \lim_{x\to\infty} (4x^4) = \infty$ (C) $\lim_{x\to-\infty} p(x) = \lim_{x\to-\infty} (4x^4) = \infty$

34. $g(x) = \dfrac{x}{4-x}$.

Note that $n(4) = 4$, $d(4) = 0$, so $x = 4$ is a point of discontinuity (in fact it is the only point at which $g(x)$ is discontinuous.)

As x approaches 4 from the left, $g(x)$ approaches ∞ and as x approaches 4 from the right, $g(x)$ approaches $-\infty$.

The only vertical asymptote is $x = 4$.

36. $k(x) = \dfrac{x^2 - 9}{x^2 + 9}$.

$k(x)$ does not have any discontinuity point since $d(x) > 0$ for all x and hence no vertical asymptotes.

38. $G(x) = \dfrac{x^2 + 9}{9 - x^2}$

$G(x) = \dfrac{x^2 + 9}{(3-x)(3+x)}$

Since $n(-3) = (-3)^2 + 9 = 18$, $n(3) = (3)^2 + 9 = 18$;

$d(-3) = d(3) = 0$. $G(x)$ is discontinuous at $x = -3$ and at $x = 3$.

$\lim_{x\to-3^-} G(x) = -\infty$, $\lim_{x\to-3^+} G(x) = \infty$, $\lim_{x\to3^-} G(x) = \infty$, $\lim_{x\to3^+} G(x) = -\infty$

Vertical asymptotes: $x = -3$, $x = 3$.

40. $K(x) = \dfrac{x^2 + 2x - 3}{x^2 - 4x + 3} = \dfrac{(x+3)(x-1)}{(x-3)(x-1)}$. Discontinuous at $x = 1$ and $x = 3$.

$\lim_{x\to1} K(x) = \lim_{x\to1} \dfrac{x+3}{x-3} = -2$

$\lim_{x\to3^-} K(x) = -\infty$, $\lim_{x\to3^+} K(x) = \infty$

Vertical asymptote: $x = 3$

42. $S(x) = \dfrac{6x+9}{x^4+6x^3+9x^2}$.

$S(x) = \dfrac{3(2x+3)}{x^2(x^2+6x+9)} = \dfrac{3(2x+3)}{x^2(x+3)^2}$

$n(0) = 9$, $n(-3) = 3(-6+3) = -9$, $d(0) = 0$, $d(-3) = 0$, so $S(x)$ is discontinuous at $x = -3$ and at $x = 0$.

$\lim_{x\to 0^-} S(x) = \infty$, $\lim_{x\to 0^+} S(x) = \infty$, $\lim_{x\to 0} S(x) = \infty$

$\lim_{x\to -3^-} S(x) = -\infty$, $\lim_{x\to -3^+} S(x) = -\infty$, $\lim_{x\to -3} S(x) = -\infty$

Vertical asymptotes: $x = -3$, $x = 0$.

44. (A) $f(5) = \dfrac{2-3(5)^3}{7+4(5)^3} = -\dfrac{373}{507} \approx -0.736$

(B) $f(10) = \dfrac{2-3(10)^3}{7+4(10)^3} = -\dfrac{2{,}998}{4{,}007} \approx -0.748$

(C) $\lim_{x\to\infty} \dfrac{2-3x^3}{7+4x^3} = \lim_{x\to\infty} \dfrac{-3x^3}{4x^3} = \lim_{x\to\infty} \dfrac{\frac{2}{x^3}-3}{\frac{7}{x^3}+4}$ (Divide numerator and denominator by x^3.)

$= \dfrac{0-3}{0+4} = \dfrac{-3}{4}$.

46. (A) $f(-8) = \dfrac{5(-8)+11}{7(-8)^3-2} = \dfrac{-29}{-3{,}586} = \dfrac{29}{3{,}586} \approx 0.008$

(B) $f(-16) = \dfrac{5(-16)+11}{7(-16)^3-2} = \dfrac{-69}{-28{,}674} = \dfrac{69}{28{,}674} \approx 0.002$

(C) $\lim_{x\to\infty} \dfrac{5x+11}{7x^3-2} = \lim_{x\to\infty} \dfrac{\frac{5}{x^2}+\frac{11}{x^3}}{7-\frac{2}{x^3}}$ (Divide numerator and denominator by x^3.)

$= \dfrac{0+0}{7-0} = 0$

48. (A) $f(-3) = \dfrac{4(-3)^7-8(-3)}{6(-3)^4+9(-3)^2} = -\dfrac{8{,}724}{567} \approx -15.386$

(B) $f(-6) = \dfrac{4(-6)^7-8(-6)}{6(-6)^4+9(-6)^2} = -\dfrac{1{,}119{,}696}{8{,}100} \approx -138.234$

(C) $\lim_{x\to-\infty} \dfrac{4x^7-8x}{6x^4+9x^2} = \lim_{x\to-\infty} \dfrac{4x^3-\frac{8}{x^3}}{6+\frac{9}{x^2}}$ (Divide numerator and denominator by x^4.)

As $x \to -\infty$, $4x^3 - \dfrac{8}{x^3} \to -\infty$ and $6 + \dfrac{9}{x^2} \to 6$. Therefore, $\lim_{x\to-\infty} \dfrac{4x^7-8x}{6x^4+9x^2} = -\infty$.

50. (A) $f(-50) = \frac{3+(-50)}{5+4(-50)} = \frac{47}{195} \approx 0.241$

(B) $f(-100) = \frac{3+(-100)}{5+4(-100)} = \frac{97}{395} \approx 0.246$

(C) $\lim_{x\to-\infty} \frac{3+x}{5+4x} = \lim_{x\to\infty} \frac{\frac{3}{x}+1}{\frac{5}{x}+4}$ (Divide numerator and denominator by x.)

$= \frac{0+1}{0+4} = \frac{1}{4}$

52. $f(x) = \frac{3x+2}{x-4}$

$\lim_{x\to\infty} f(x) = \lim_{x\to\infty} \frac{3x+2}{x-4} = \lim_{x\to\infty} \frac{3+\frac{2}{x}}{1-\frac{4}{x}} = \frac{3+0}{1-0} = 3$

So $y = 3$ is the horizontal asymptote.
Vertical asymptote: $x = 4$ (since $n(4) = 14$, $d(4) = 0$).

54. $f(x) = \frac{x^2-1}{x^2+2}$.

$\lim_{x\to\infty} f(x) = \lim_{x\to\infty} \frac{x^2-1}{x^2+2} = \lim_{x\to\infty} \frac{1-\frac{1}{x^2}}{1+\frac{2}{x^2}}$ (Dividing the numerator and denominator by x^2.)

$= \frac{1-0}{1+0} = 1$

So, the horizontal asymptote is: $y = 1$.
$d(x) = x^2 + 2 > 0$ so, there are no vertical asymptotes.

56. $f(x) = \frac{x}{x^2-4} = \frac{x}{(x-2)(x+2)}$

$\lim_{x\to\infty} f(x) = \lim_{x\to\infty} \frac{x}{x^2-4} = \lim_{x\to\infty} \frac{\frac{1}{x}}{1-\frac{4}{x^2}} = \frac{0}{1-0} = 0,$

so the horizontal asymptote is: $y = 0$.
Since $n(-2) = -2$, $n(2) = 2$, $d(-2) = d(2) = 0$, we have two vertical asymptotes: $x = -2$, $x = 2$.

58. $f(x) = \frac{x^2+9}{x}$

$\lim_{x\to\infty} f(x) = \lim_{x\to\infty} \frac{x^2+9}{x} = \lim_{x\to\infty} \frac{1+\frac{9}{x^2}}{\frac{1}{x}} = \frac{1+0}{0} = \infty$

So, there are no horizontal asymptotes. Since $n(0) = 9$, $d(0) = 0$,
$x = 0$ is the only vertical asymptote.

60. $f(x) = \dfrac{x+5}{x^2}$.

$\lim\limits_{x\to\infty} f(x) = \lim\limits_{x\to\infty} \dfrac{x+5}{x^2} = \lim\limits_{x\to\infty} \dfrac{\frac{1}{x}+\frac{5}{x^2}}{1} = \dfrac{0+0}{1} = 0,$

so the horizontal asymptote is: $y = 0$.

Since $n(0) = 5$, $d(0) = 0$, $x = 0$ is the vertical asymptote.

62. $f(x) = \dfrac{2x^2+7x+12}{2x^2+5x-12}$.

$\lim\limits_{x\to\infty} f(x) = \lim\limits_{x\to\infty} \dfrac{2x^2+7x+12}{2x^2+5x-12} = \lim\limits_{x\to\infty} \dfrac{2x^2}{2x^2} = 1,$

so, $y = 1$ is the horizontal asymptote.

Since $n(-4) = 16$, $n\left(\dfrac{3}{2}\right) = 27$, $d(-4) = d\left(\dfrac{3}{2}\right) = 0$, $x = -4$ and $x = \dfrac{3}{2}$ are the vertical asymptotes.

64. $f(x) = \dfrac{x^2-x-12}{2x^2+5x-12}$

$\lim\limits_{x\to\infty} f(x) = \lim\limits_{x\to\infty} \dfrac{x^2-x-12}{2x^2+5x-12} = \lim\limits_{x\to\infty} \dfrac{x^2}{2x^2} = \dfrac{1}{2}$, so $y = \dfrac{1}{2}$ is the horizontal asymptote. Since $n(-4) = 8$, $n\left(\dfrac{3}{2}\right) = -11.25$, $d(-4) = d\left(\dfrac{3}{2}\right) = 0$, $x = -4$ and $x = \dfrac{3}{2}$ are the vertical asymptotes.

66. $f(x) = \dfrac{3+4x+x^2}{5-x}$; $\lim\limits_{x\to\infty} f(x) = \lim\limits_{x\to\infty} \dfrac{3+4x+x^2}{5-x} = \lim\limits_{x\to\infty} \dfrac{x^2}{-x} = \lim\limits_{x\to\infty} (-x) = -\infty$

68. $f(x) = \dfrac{4x+1}{5x-7}$; $\lim\limits_{x\to\infty} f(x) = \lim\limits_{x\to\infty} \dfrac{4x+1}{5x-7} = \lim\limits_{x\to\infty} \dfrac{4x}{5x} = \dfrac{4}{5}$

70. $f(x) = \dfrac{2x+3}{x^2-1}$; $\lim\limits_{x\to-\infty} f(x) = \lim\limits_{x\to-\infty} \dfrac{2x+3}{x^2-1} = \lim\limits_{x\to-\infty} \dfrac{2x}{x^2} = 0$

72. $f(x) = \dfrac{6-x^4}{1+2x}$; $\lim\limits_{x\to-\infty} f(x) = \lim\limits_{x\to-\infty} \dfrac{6-x^4}{1+2x} = \lim\limits_{x\to-\infty} \dfrac{-x^4}{2x} = \infty$

74. $f(x) = 4-5x-x^3$;

$\lim\limits_{x\to\infty} f(x) = \lim\limits_{x\to\infty} (4-5x-x^3) = \lim\limits_{x\to\infty} (-x^3) = -\infty$; $\lim\limits_{x\to-\infty} f(x) = \lim\limits_{x\to-\infty} (4-5x-x^3) = \lim\limits_{x\to-\infty} (-x^3) = \infty$

76. $f(x) = \dfrac{9x^2+6x+1}{4x^2+4x+1}$;

$\lim\limits_{x\to\infty} f(x) = \lim\limits_{x\to\infty} \dfrac{9x^2+6x+1}{4x^2+4x+1} = \lim\limits_{x\to\infty} \dfrac{9x^2}{4x^2} = \dfrac{9}{4}$; $\lim\limits_{x\to-\infty} f(x) = \lim\limits_{x\to-\infty} \dfrac{9x^2+6x+1}{4x^2+4x+1} = \lim\limits_{x\to-\infty} \dfrac{9x^2}{4x^2} = \dfrac{9}{4}$

78. False: $f(x) = \dfrac{1}{(x-2)(x+2)} = \dfrac{1}{x^2-4}$ has two vertical asymptotes.

80. True: Theorem 4 gives three possible cases, two of which give exactly one horizontal asymptote and one of which gives no horizontal asymptote.

82. False: $f(x)=\dfrac{x^2+2x}{x^2+x+2}$ crosses the horizontal asymptote $y=1$ at $x=2$.

84. $\lim\limits_{x\to-\infty}(a_nx^n+a_{n-1}x^{n-1}+\ldots+a_0)=\infty$ if $a_n>0$ and n an even positive integer, or $a_n<0$ and n an odd positive integer.

$\lim\limits_{x\to-\infty}(a_nx^n+a_{n-1}x^{n-1}+\ldots+a_0)=-\infty$ if $a_n>0$ and n is an odd positive integer or $a_n<0$ and n is an even positive integer.

86. (A) Since $C(x)$ is a linear function of x, it can be written in the form

$C(x)=mx+b$

Since the fixed costs are \$300, $b=300$.

Also, $C(20)=5100$, so

$5100=m(20)+300$

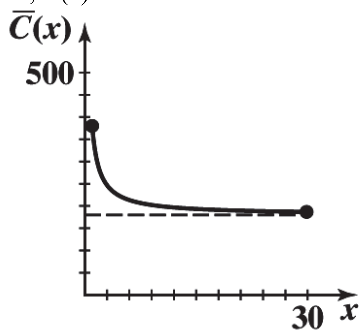
$20m=4800$

$m=240$

Therefore, $C(x)=240x+300$

(B) $\overline{C}(x)=\dfrac{C(x)}{x}=\dfrac{240x+300}{x}$

(C)



(D) $\overline{C}(x)=\dfrac{240x+300}{x}=\dfrac{240+\dfrac{300}{x}}{1}$

As x increases, the numerator tends to 240 and the denominator is 1. Therefore, $\overline{C}(x)$ tends to 240 or \$240 per board. Therefore, $\overline{C}(x)$ tends to \$240 per board.

88. (A) $C_e(x)=4{,}000+932x$; $\overline{C}_e(x)=\dfrac{4{,}000}{x}+932$

(B) $C_c(x)=2{,}700+1{,}332x$; $\overline{C}_c(x)=\dfrac{2{,}700}{x}+1{,}332$

(C) $C_e(x)=C_c(x)$ implies that $4{,}000+932x=2{,}700+1{,}332x$ or $400x=1300$ and

$x=\dfrac{1{,}300}{400}=3.25$ years

(D) $\overline{C}_e(x)=\overline{C}_c(x)$ implies that

$$\frac{4{,}000}{x}+932=\frac{2{,}700}{x}+1{,}332$$

Multiplying both sides by x results in the same equation as in (C) and hence the answer is $x=3.25$ years.

(E) $\lim\limits_{x\to\infty}\overline{C}_e(x)=\lim\limits_{x\to\infty}\left(\dfrac{4{,}000}{x}+932\right)=0+932=932$

$\lim\limits_{x\to\infty}\overline{C}_c(x)=\lim\limits_{x\to\infty}\left(\dfrac{2{,}700}{x}+1{,}332\right)=0+1{,}332=1{,}332$

90. $C(t) = \dfrac{5t(t+50)}{t^3+100}$

$\lim\limits_{t\to\infty} C(t) = \lim\limits_{t\to\infty} \dfrac{5t^2+250t}{t^3+100}$ (Divide numerator and denominator by t^3.)

$= \lim\limits_{t\to\infty} \dfrac{\frac{5}{t}+\frac{250}{t^2}}{1+\frac{100}{t^3}} = \dfrac{0+0}{1+0} = 0$

The long term drug concentration is 0 mg/ml.

92. $N(t) = \dfrac{100t}{t+9}, t \ge 0$

(A) $N(6) = \dfrac{100(6)}{6+9} = \dfrac{600}{15} \approx 40$ components/day

(B) $70 = \dfrac{100t}{t+9}$ or

$70t + 630 = 100t$

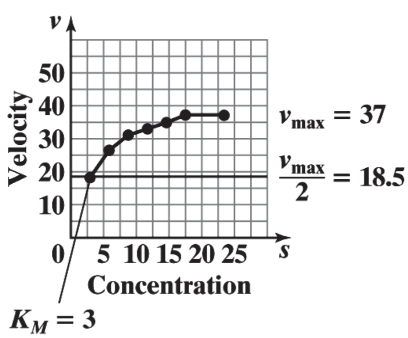
$30t = 630$

$t = \dfrac{630}{30} = 21$ days

(C) $\lim\limits_{t\to\infty} N(t) = \lim\limits_{t\to\infty} \dfrac{100t}{t+9} = \lim\limits_{t\to\infty} \dfrac{100}{1+\frac{9}{t}} = \dfrac{100}{1+0} = 100$

The maximum number of components an employee can produce in consecutive days is 100 components.

94. (A) $v_{max} = 37, K_M = 3$



(B) $v(s) = \dfrac{37s}{3+s}$

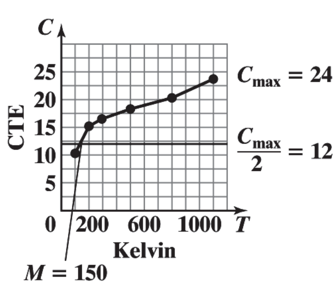
(C) For $s = 9$, $v = \dfrac{37(9)}{3+9} = 27.75$

For $v = 32$, $32 = \dfrac{37s}{3+s}$

or $96 + 32s = 37s$

and $s = \dfrac{96}{5} = 19.2$

96. (A) $C_{max} = 24, M = 150$



(B) $C(T) = \dfrac{24T}{150+T}$

(C) For $T = 600$,

$C = \dfrac{(24)(600)}{150+600} = 19.2$

For $C = 12$, $12 = \dfrac{24T}{150+T}$ or

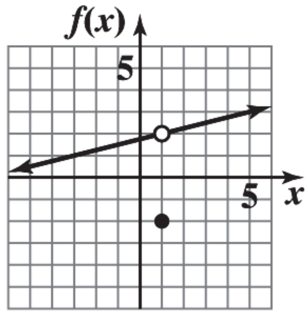
$1800 + 12T = 24T$,

$T = 150$.

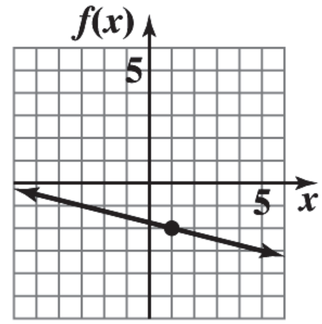
EXERCISE 2-3

2. $(-8,-4]$ **4.** $[0.1, 0.3]$ **6.** $(-\infty,-4]\cup[4,\infty)$ **8.** $(-\infty,-6)\cup[9,\infty)$

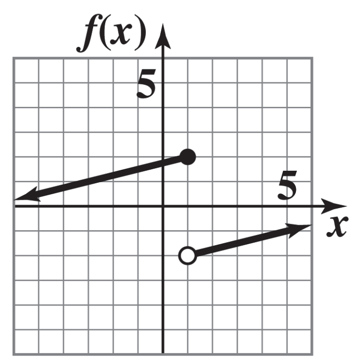
10. f is discontinuous at $x = 1$ since $\lim_{x\to1} f(x) \neq f(1)$



12. f is discontinuous at $x = 1$ since $\lim_{x\to1} f(x) = f(1)$



14. f is discontinuous at $x = 1$, since $\lim_{x\to1} f(x)$ does not exist



16. $f(0.1) = 1.1$

18. $f(-0.9) = 0.1$

20. (A) $\lim_{x\to2^-} f(x) = 2$ (B) $\lim_{x\to2^+} f(x) = 2$ (C) $\lim_{x\to2} f(x) = 2$

$\left(\lim_{x\to2^-} f(x) = \lim_{x\to2^+} f(x) = 2\right)$

(D) $f(2)$ does not exist; f is not defined at $x = 2$. (E) No, since f is not even defined at $x = 2$.

22. (A) $\lim_{x\to-1^-} f(x) = 0$ (B) $\lim_{x\to-1^+} f(x) = 0$

(C) $\lim_{x\to-1} f(x) = 0$ $\left(\lim_{x\to-1^-} f(x) = \lim_{x\to-1^+} f(x) = 0\right)$

(D) $f(-1) = 0$ (E) Yes, since $\lim_{x\to-1} f(x) = f(0)$.

24. $g(-2.1) = 1.9$

26. $g(-1.9) = 3.95$

28. (A) $\lim_{x \to -2^-} g(x) = 2$

(B) $\lim_{x \to -2^+} g(x) = 4$

(C) $\lim_{x \to -2} g(x)$ does not exist, since $\lim_{x \to -2^-} g(x) \neq \lim_{x \to -2^+} g(x)$

(D) $g(-2)$ does not exist; g is not defined at $x = -2$.

(E) No, since g is not even defined at $x = -2$.

30. (A) $\lim_{x \to 4^-} g(x) = 1$ (B) $\lim_{x \to 4^+} g(x) = 1$ (C) $\lim_{x \to 4} g(x) = 1$

$\left(\lim_{x \to 4^-} g(x) = \lim_{x \to 4^+} g(x) = 1\right)$

(D) $g(4) = 1$.

(E) Yes, since $\lim_{x \to 4} f(x) = f(4)$

32. $h(x) = 4 - 2x$ is a polynomial function. Therefore, f is continuous for all x [Theorem 1(C)].

34. $k(x) = \dfrac{2x}{x-4}$ is a rational function and the denominator $x - 4$ is 0 when $x = 4$. Thus, k is continuous for all x except $x = 4$ [Theorem 1(D)].

36. $n(x) = \dfrac{x-2}{(x-3)(x+1)}$ is a rational function and the denominator $(x-3)(x+1)$ is 0 when $x = 3$ or $x = -1$. Thus, n is continuous for all x except $x = 3$, $x = -1$ [Theorem 1(D)].

38. $G(x) = \dfrac{1-x^2}{x^2+1}$

$G(x)$ is a rational function and its denominator is never zero, hence by Theorem 1(D), $G(x)$ is continuous for all x.

40. $N(x) = \dfrac{x^2+4}{4-25x^2}$

$N(x)$ is a rational function and according to Theorem 1(D), $N(x)$ is continuous for all x except $x = \pm\dfrac{2}{5}$ which make the denominator 0.

42. $f(x) = \dfrac{2x+7}{5x-1}$; f is discontinuous at $x = \dfrac{1}{5}$; $f(x) = 0$ at $x = \dfrac{-7}{2}$. Partition numbers $\dfrac{1}{5}, \dfrac{-7}{2}$.

44. $f(x) = \dfrac{x^2+4}{x^2-9}$; f is discontinuous at $x = 3, -3$; $f(x) \neq 0$ for all x. Partition numbers $3, -3$.

46. $f(x) = \dfrac{x^3+x}{x^2-x-42} = \dfrac{x(x^2+1)}{(x-7)(x+6)}$; f is discontinuous at $x = 7, -6$; $f(x) = 0$ at $x = 0$. Partition numbers $-6, 0, 7$.

48. $x^2 - 2x - 8 < 0$
Let $f(x) = x^2 - 2x - 8 = (x - 4)(x + 2)$.
Then f is continuous for all x and $f(-2) = f(4) = 0$.
Thus, $x = -2$ and $x = 4$ are partition numbers.

Test Numbers

x	$f(x)$
-3	$7(+)$
0	$-8(-)$
5	$7(+)$

Thus, $x^2 - 2x - 8 < 0$ for: $-2 < x < 4$ (inequality notation), $(-2, 4)$ (interval notation)

50. $x^2 + 7x > -10$ or $x^2 + 7x + 10 > 0$
Let $f(x) = x^2 + 7x + 10 = (x + 2)(x + 5)$.
Then f is continuous for all x and $f(-5) = f(-2) = 0$.

Thus, $x = -5$ and $x = -2$ are partition numbers.

Test Numbers

x	$f(x)$
-6	$4(+)$
-4	$-2(-)$
0	$10(+)$

Thus, $x^2 + 7x + 10 > 0$ for: $x < -5$ or $x > -2$ (inequality notation), $(-\infty, -5) \cup (-2, \infty)$ (interval notation)

52. $x^4 - 9x^2 > 0$
$x^4 - 9x^2 = x^2(x^2 - 9)$
Since $x^2 > 0$ for $x \neq 0$, then $x^4 - 9x^2 > 0$ if $x^2 - 9 > 0$ or $x^2 > 9$
or "$x < -3$ or $x > 3$" or $(-\infty, -3) \cup (3, \infty)$.

54. $\dfrac{x-4}{x^2+2x} < 0$

Let $f(x) = \dfrac{x-4}{x^2+2x} = \dfrac{x-4}{x(x+2)}$. Then f is discontinuous at $x = 0$ and $x = -2$ and $f(4) = 0$. Thus, $x = -2$, $x = 0$, and $x = 4$ are partition numbers.

Test Numbers

x	$f(x)$
-3	$-\frac{7}{3}(-)$
-1	$5(+)$
1	$-1(-)$
5	$\frac{1}{35}(+)$

Thus, $\dfrac{x-4}{x^2+2x} < 0$ for: $x < -2$ or $0 < x < 4$ (inequality notation), $(-\infty, -2) \cup (0, 4)$ (interval notation)

56. (A) $g(x) > 0$ for $x < -4$ or $x > 4$; $(-\infty, -4) \cup (4, \infty)$.

(B) $g(x) < 0$ for $-4 < x < 1$ or $1 < x < 4$; $(-4, 1) \cup (1, 4)$.

58. $f(x) = x^4 - 4x^2 - 2x + 2$. Partition numbers: $x_1 \approx 0.5113$, $x_2 \approx 2.1209$

(A) $f(x) > 0$ on $(-\infty, 0.5113) \cup (2.1209, \infty)$

(B) $f(x) < 0$ on $(0.5113, 2.1209)$

60. $f(x) = \dfrac{x^3 - 5x + 1}{x^2 - 1}$. Partition numbers: $x_1 \approx -2.3301$, $x_2 \approx -1$, $x_3 \approx 0.2016$, $x_4 = 1$, $x_5 \approx 2.1284$

(A) $f(x) > 0$ on $(-2.3301, -1) \cup (0.2016, 1) \cup (2.1284, \infty)$.

(B) $f(x) < 0$ on $(-\infty, -2.3301) \cup (-1, 0.2016) \cup (1, 2.1284)$.

62. $\sqrt{7-x}$

Let $f(x) = 7 - x$. Then $\sqrt{7-x} = \sqrt[2]{f(x)}$ is continuous whenever $f(x)$ is continuous and nonnegative [Theorem 1(F)]. Since $f(x) = 7 - x$ is continuous for all x [Theorem 1(C)] and $f(x) \geq 0$ for $x \leq 7$, $\sqrt{7-x}$ is continuous on $(-\infty, 7]$.

64. $\sqrt[3]{x-8}$

Let $f(x) = x - 8$. Then $\sqrt[3]{x-8} = \sqrt[3]{f(x)}$ is continuous whenever $f(x)$ is continuous [Theorem 1(E)]. Since $f(x) = x - 8$ is continuous for all x [Theorem 1(C)], $\sqrt[3]{x-8}$ is continuous on $(-\infty, \infty)$.

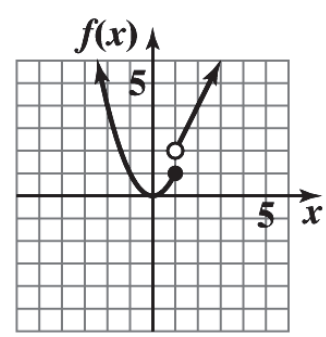
66. $\sqrt{4-x^2}$

Let $f(x) = 4 - x^2$. Then $\sqrt{4-x^2} = \sqrt[2]{f(x)}$ is continuous whenever $f(x)$ is continuous and nonnegative [Theorem 1(F)]. Since $f(x) = 4 - x^2$ is continuous for all x [Theorem 1(C)] and $f(x)$ is nonnegative on $[-2, 2]$, $\sqrt{4-x^2}$ is continuous on $[-2, 2]$.

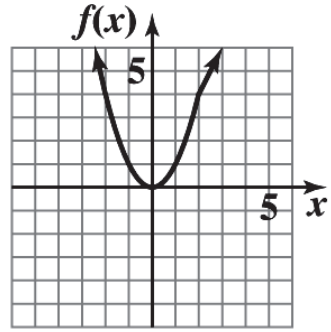
68. $\sqrt[3]{x^2+2}$

Let $f(x) = x^2 + 2$. Then $\sqrt[3]{x^2+2} = \sqrt[3]{f(x)}$ is continuous whenever $f(x)$ is continuous [Theorem 1(E)]. Since $f(x) = x^2 + 2$ is continuous for all x [Theorem 1(C)], $\sqrt[3]{x^2+2}$ is continuous on $(-\infty, \infty)$.

70. The graph of f is shown at the right. This function is discontinuous at $x = 1$.
[$\lim_{x\to 1^-} f(x) = 1$ and $\lim_{x\to 1^+} f(x) = 2$;
Thus, $\lim_{x\to 1} f(x)$ does not exist.]

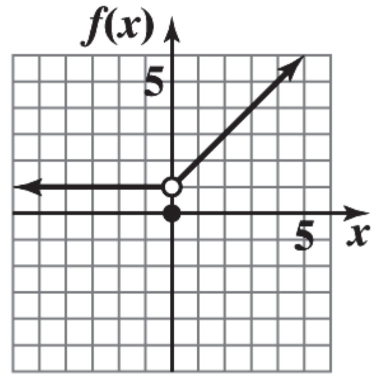


72. The graph of f is shown at the right.
This function is continuous for all x. [$\lim_{x\to 2} f(x) = f(2) = 4$]



74. The graph of f is shown at the right.

This function is discontinuous at $x = 0$,
since $\lim_{x\to 0} f(x) = 1 \neq f(0) = 0$



76. (A) Since $\lim_{x\to 2^+} f(x) = f(2) = 2$, f is continuous from the right at $x = 2$.

(B) Since $\lim_{x\to 2^-} f(x) = 1 \neq f(2) = 2$, f is not continuous from the left at $x = 2$.

(C) f is continuous on the open interval (1, 2).

(D) f is *not* continuous on the closed interval [1, 2] since $\lim_{x\to 2^-} f(x) = 1 \neq f(2) = 2$, i.e., f is not continuous from the left at $x = 2$.

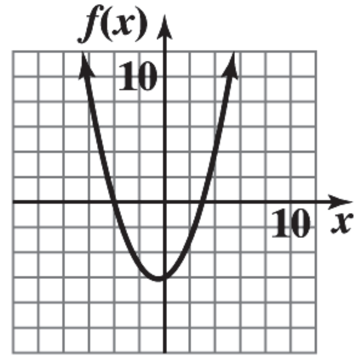
(E) f is continuous on the half-closed interval [1, 2).

78. True: If $r(x) = \frac{n(x)}{d(x)}$ is a rational function and $d(x)$ has degree n, then $r(x)$ has at most n points of discontinuity.

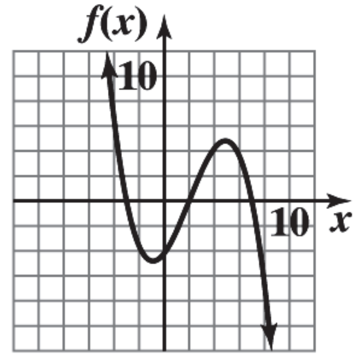
80. True: Continuous on (0, 2) means continuous at every real number x in (0, 2), including $x = 1$.

82. False. The greatest integer function has infinitely many points of discontinuity. See Prob. 75.

84. x intercepts: $x = -4, 3$



86. x intercepts: $x = -3, 2, 7$

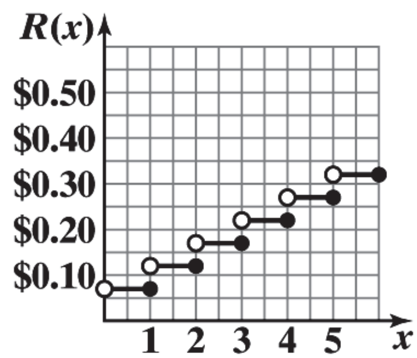


88. $f(x) = \dfrac{6}{x-4} \neq 0$ for all x. This does not contradict Theorem 2 because f is not continuous on (2, 7); f is discontinuous at $x = 4$.

90. (A)

$$R(x) = \begin{cases} 0.07 & \text{if} \quad 0 < x \le 1 \\ 0.12 & \text{if} \quad 1 < x \le 2 \\ 0.17 & \text{if} \quad 2 < x \le 3 \\ 0.22 & \text{if} \quad 3 < x \le 4 \\ 0.27 & \text{if} \quad 4 < x \le 5 \\ 0.32 & \text{if} \quad 5 < x \le 6 \\ \vdots & \quad\quad \vdots \end{cases}$$

(B)



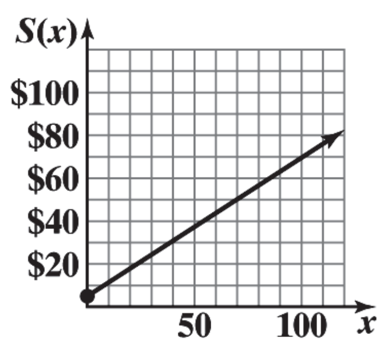
(C) $\lim_{x \to 3.5} R(x) = 0.22 = R(3.5)$; Thus, $R(x)$ is continuous at $x = 3.5$.

$\lim_{x \to 3} R(x)$ does not exist; $R(3) = 0.17$; Thus, $R(x)$ is not continuous at $x = 3$.

92. If x is a positive integer, then $S(x) = R(x) + 0.05$.
$S(x) = R(x)$ for all other values of x in the domain of R.

94. (A) $$S(x) = \begin{cases} 5 + 0.69x \text{ if } & 0 \le x \le 5 \\ 5.2 + 0.65x \text{ if } & 5 < x \le 50 \\ 6.2 + 0.63x \text{ if } & 50 < x \end{cases}$$

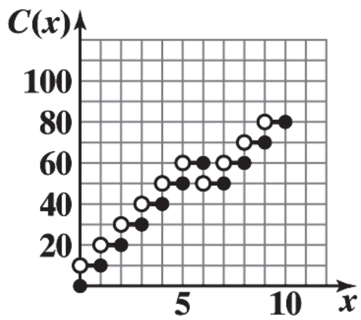
(B) The graph of S is:



(C) $\lim_{x\to 5} S(x) = 8.45 = S(5)$; thus, $S(x)$ is continuous at $x = 5$.

$\lim_{x\to 50} S(x) = 37.7 = S(50)$; thus, $S(x)$ is continuous at $x = 50$.

96. (A) The graph of $C(x)$ is:



(B) From the graph, $\lim_{x\to 4.5} C(x) = 50$ and $C(4.5) = 50$.

(C) From the graph, $\lim_{x\to 8} C(x)$ does not exist; $C(8) = 60$.

(D) Since $\lim_{x\to 4.5} C(x) = 50 = C(4.5)$, $C(x)$ is continuous at $x = 4.5$. Since $\lim_{x\to 8} C(x)$ does not exist and $C(8) = 60$, $C(x)$ is not continuous at $x = 8$.

98. (A) From the graph, p is discontinuous at $t = t_2$, and $t = t_4$.

(B) $\lim_{t\to t_1} p(t) = 10$; $p(t_1) = 10$.

(C) $\lim_{t\to t_2} p(t) = 30$, $p(t_2) = 10$.

(D) $\lim_{t\to t_4} p(t)$ does not exist; $p(t_4) = 80$.

EXERCISE 2-4

2. Slope $m = \frac{8-11}{1-(-1)} = \frac{-3}{2}, \ -1.5$

4. Slope $m = \frac{3-(-3)}{4-(-12)} = \frac{6}{16} = \frac{3}{8}; \ 0.375$

6. $\frac{2}{\sqrt{5}} = \frac{1}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$

8. $\frac{1-\sqrt{2}}{5+\sqrt{2}} = \frac{1-\sqrt{2}}{5+\sqrt{2}} \cdot \frac{5-\sqrt{2}}{5-\sqrt{2}} = \frac{7-6\sqrt{2}}{23} = \frac{7}{23} - \frac{6}{23}\sqrt{2}$

10. (A) $\frac{f(-1)-f(-2)}{-1-(-2)} = \frac{4-1}{1} = 3$ is the slope of the secant line through $(-2, f(-2))$ and $(-1, f(-1))$.

(B) $$\frac{f(-2+h)-f(-2)}{h} = \frac{5-(-2+h)^2-1}{h} = \frac{5-[4-4h+h^2]-1}{h}$$
$$= \frac{5-4+4h-h^2-1}{h} = \frac{4h-h^2}{h} = 4-h;$$

slope of the secant line through $(-2, f(-2))$ and $(-2+h, f(-2+h))$

(C) $\lim\limits_{h \to 0} \dfrac{f(-2+h) - f(-2)}{h} = \lim\limits_{h \to 0} (4 - h) = 4;$

slope of the tangent line at $(-2, f(-2))$

12. $f(x) = 3x^2$

(A) Slope of secant line through $(2, f(2))$ and $(5, f(5))$:

$$\frac{f(5)-f(2)}{5-2} = \frac{3(5)^2-3(2)^2}{5-2} = \frac{75-12}{3} = \frac{63}{3} = 21$$

(B) Slope of secant line through $(2, f(2))$ and $(2+h, f(2+h))$:

$$\frac{3(2+h)^2-3(2)^2}{2+h-2} = \frac{3(4+4h+h^2)-12}{h} = \frac{12+12h+3h^2-12}{h} = \frac{12h+3h^2}{h} = 12 + 3h$$

(C) Slope of the graph at $(2, f(2))$: $\lim\limits_{h\to 0}\frac{f(2+h)-f(2)}{h} = \lim\limits_{h\to 0}(12+3h) = 12.$

14. (A) Distance traveled for $0 \le t \le 4$: $352(1.5) = 528$; average velocity: $v = \frac{528}{4} = 132$ mph .

(B) $\frac{f(4)-f(0)}{4-0} = \frac{528}{4} = 132.$

(C) Slope at $x = 4$: $m = 150$. Equation of tangent line at $(4, f(4))$: $y - 528 = 150(x-4)$ or

$y = 150x - 72.$

16. $f(x) = \frac{1}{1+x^2}$; $f(2) = \frac{1}{5} = 0.2.$ Equation of tangent line: $y - 0.2 = -0.16(x-2)$ or

$y = -0.16x + 3.4.$

18. $f(x) = x^4$; $f(-1) = 1.$ Equation of tangent line: $y - 1 = -4(x+1)$ or $y = -4x - 3.$

20. $f(x) = 9$

Step 1. Find $f(x + h)$.

$f(x + h) = 9$

Step 2. Find $f(x + h) - f(x)$.

$f(x + h) - f(x) = 9 - 9 = 0$

Step 3. Find $\frac{f(x+h)-f(x)}{h}$.

$$\frac{f(x+h)-f(x)}{h} = \frac{0}{h} = 0$$

Step 4. Find $f'(x) = \lim\limits_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim\limits_{h\to 0}(0) = 0$

Thus, if $f(x) = 9$, then $f'(x) = 0$, $f'(1) = 0$, $f'(2) = 0$, $f'(3) = 0$.

22. $f(x) = 4 - 6x$

Step 1. $f(x + h) = 4 - 6(x + h) = 4 - 6x - 6h$

Step 2. $f(x + h) - f(x) = (4 - 6x - 6h) - (4 - 6x)$

$= 4 - 6x - 6h - 4 + 6x = -6h$

Step 3. $\frac{f(x+h)-f(x)}{h} = \frac{-6h}{h} = -6$

Step 4. $f'(x) = \lim\limits_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim\limits_{h\to 0}(-6) = -6$

$f'(1) = -6$, $f'(2) = -6$, $f'(3) = -6$

24. $f(x) = 2x^2 + 8$

Step 1. $f(x + h) = 2(x + h)^2 + 8 = 2(x^2 + 2xh + h^2) + 8$
$= 2x^2 + 4xh + 2h^2 + 8$

Step 2. $f(x + h) - f(x) = (2x^2 + 4xh + 2h^2 + 8) - (2x^2 + 8)$
$= 2x^2 + 4xh + 2h^2 + 8 - 2x^2 - 8$
$= 4xh + 2h^2$

Step 3. $\dfrac{f(x+h)-f(x)}{h} = \dfrac{4xh+2h^2}{h} = 4x + 2h$

Step 4. $f'(x) = \lim_{h\to 0} \dfrac{f(x+h)-f(x)}{h} = \lim_{h\to 0} (4x + 2h) = 4x$
$f'(1) = 4, \quad f'(2) = 8, \quad f'(3) = 12$

26. $f(x) = x^2 + 4x + 7$

Step 1. $f(x + h) = (x + h)^2 + 4(x + h) + 7 = x^2 + 2xh + h^2 + 4x + 4h + 7$

Step 2. $f(x + h) - f(x) = (x^2 + 2xh + h^2 + 4x + 4h + 7) - (x^2 + 4x + 7)$
$= 2xh + 4h + h^2 = h(2x + 4 + h)$

Step 3. $\dfrac{f(x+h)-f(x)}{h} = \dfrac{h(2x+4+h)}{h} = 2x + 4 + h$

Step 4. $f'(x) = \lim_{h\to 0} \dfrac{f(x+h)-f(x)}{h} = \lim_{h\to 0} (2x + 4 + h) = 2x + 4$
$f'(1) = 6, \quad f'(2) = 8, \quad f'(3) = 10$

28. $f(x) = 2x^2 + 5x + 1$

Step 1. $f(x + h) = 2(x + h)^2 + 5(x + h) + 1$
$= 2(x^2 + 2xh + h^2) + 5x + 5h + 1$
$= 2x^2 + 4xh + 2h^2 + 5x + 5h + 1$

Step 2. $f(x + h) - f(x) = (2x^2 + 4xh + 2h^2 + 5x + 5h + 1) - (2x^2 + 5x + 1)$
$= h(4x + 5 + 2h)$

Step 3. $\dfrac{f(x+h)-f(x)}{h} = \dfrac{h(4x+5+2h)}{h} = 4x + 5 + 2h$

Step 4. $f'(x) = \lim_{h\to 0} \dfrac{f(x+h)-f(x)}{h} = \lim_{h\to 0} (4x + 5 + 2h) = 4x + 5$
$f'(1) = 9, \quad f'(2) = 13, \quad f'(3) = 17$

30. $f(x) = -x^2 + 9x - 2$

Step 1. $f(x+h) = -(x+h)^2 + 9(x+h) - 2$
$= -(x^2 + 2xh + h^2) + 9x + 9h - 2$
$= -x^2 - 2xh - h^2 + 9x + 9h - 2$

Step 2. $f(x+h) - f(x) = (-x^2 - 2xh - h^2 + 9x + 9h - 2) - (-x^2 + 9x - 2)$
$= -2xh + 9h - h^2 = h(-2x + 9 - h)$

Step 3. $\dfrac{f(x+h)-f(x)}{h} = \dfrac{h(-2x+9-h)}{h} = -2x + 9 - h$

Step 4. $f'(x) = \lim\limits_{h\to 0} \dfrac{f(x+h)-f(x)}{h} = \lim\limits_{h\to 0} (-2x + 9 - h) = -2x + 9$ f'

$f'(1) = 7, \quad f'(2) = 5, \quad f'(3) = 3$

32. $f(x) = -2x^3 + 5$

Step 1. $f(x+h) = -2(x+h)^3 + 5 = -2(x^3 + 3x^2h + 3xh^2 + h^3) + 5$
$= -2x^3 - 6x^2h - 6xh^2 - 2h^3 + 5$

Step 2. $f(x+h) - f(x) = -2x^3 - 6x^2h - 6xh^2 - 2h^3 + 5 - (-2x^3 + 5)$
$= -6x^2h - 6xh^2 - 2h^3$
$= -2h(3x^2 + 3xh + h^2)$

Step 3. $\dfrac{f(x+h)-f(x)}{h} = \dfrac{-2h(3x^2+3xh+h^2)}{h} = -2(3x^2 + 3xh + h^2)$

Step 4. $f'(x) = \lim\limits_{h\to 0} \dfrac{f(x+h)-f(x)}{h} = \lim\limits_{h\to 0} \{-2(3x^2 + 3xh + h^2)\} = -6x^2$
$f'(1) = -6, \quad f'(2) = -24, \quad f'(3) = -54$

34. $f(x) = \dfrac{6}{x} - 2$

Step 1. $f(x+h) = \dfrac{6}{x+h} - 2$

Step 2. $f(x+h) - f(x) = \left(\dfrac{6}{x+h} - 2\right) - \left(\dfrac{6}{x} - 2\right)$
$= \dfrac{6}{x+h} - \dfrac{6}{x} = \dfrac{6x - 6x - 6h}{x(x+h)} = \dfrac{-6h}{x(x+h)}$

Step 3. $\dfrac{f(x+h)-f(x)}{h} = \dfrac{\frac{-6h}{x(x+h)}}{h} = -\dfrac{6}{x(x+h)}$

Step 4. $f'(x) = \lim\limits_{h\to 0} \dfrac{f(x+h)-f(x)}{h} = \lim\limits_{h\to 0} \dfrac{-6}{x(x+h)} = -\dfrac{6}{x^2}$

$f'(1) = -6, \quad f'(2) = -\dfrac{6}{4} = -\dfrac{3}{2}, \quad f'(3) = -\dfrac{6}{9} = -\dfrac{2}{3}$

36. $f(x) = 3 - 7\sqrt{x}$

Step 1. $f(x + h) = 3 - 7\sqrt{x+h}$

Step 2. $f(x + h) - f(x) = (3 - 7\sqrt{x+h}\,) - (3 - 7\sqrt{x}\,) = 7(\sqrt{x} - \sqrt{x+h}\,)$

Step 3. $\dfrac{f(x+h)-f(x)}{h} = \dfrac{7(\sqrt{x}-\sqrt{x+h})}{h} = \dfrac{7(\sqrt{x}-\sqrt{x+h})}{h} \cdot \dfrac{(\sqrt{x}+\sqrt{x+h})}{(\sqrt{x}+\sqrt{x+h})}$

$= \dfrac{7(x-(x+h))}{h(\sqrt{x}+\sqrt{x+h})} = \dfrac{7(x-x-h)}{h(\sqrt{x}+\sqrt{x+h})}$

$= \dfrac{-7h}{h(\sqrt{x}+\sqrt{x+h})} = \dfrac{-7}{\sqrt{x}+\sqrt{x+h}}$

Step 4. $f'(x) = \lim\limits_{h\to 0} \dfrac{f(x+h)-f(x)}{h} = \lim\limits_{h\to 0}\left(\dfrac{-7}{\sqrt{x}+\sqrt{x+h}}\right) = \dfrac{-7}{2\sqrt{x}}$

$f'(1) = -\dfrac{7}{2}$, $f'(2) = -\dfrac{7}{2\sqrt{2}} = -\dfrac{7\sqrt{2}}{4}$, $f'(3) = -\dfrac{7}{2\sqrt{3}} = -\dfrac{7\sqrt{3}}{6}$

38. $f(x) = 16\sqrt{x+9}$

Step 1. $f(x + h) = 16\sqrt{x+h+9}$

Step 2. $f(x + h) - f(x) = 16\sqrt{x+h+9} - 16\sqrt{x+9}$
$= 16(\sqrt{x+h+9} - \sqrt{x+9}\,)$

Step 3. $\dfrac{f(x+h)-f(x)}{h} = \dfrac{16(\sqrt{x+h+9}-\sqrt{x+9})}{h}$

$= \dfrac{16(\sqrt{x+h+9}-\sqrt{x+9})}{h} \cdot \dfrac{(\sqrt{x+h+9}+\sqrt{x+9})}{(\sqrt{x+h+9}+\sqrt{x+9})}$

$= \dfrac{16((x+h+9)-(x+9))}{h(\sqrt{x+h+9}+\sqrt{x+9})}$

$= \dfrac{16h}{h(\sqrt{x+h+9}+\sqrt{x+9})} = \dfrac{16}{\sqrt{x+h+9}+\sqrt{x+9}}$

Step 4. $f'(x) = \lim\limits_{h\to 0} \dfrac{f(x+h)-f(x)}{h} = \lim\limits_{h\to 0} \dfrac{16}{\sqrt{x+h+9}+\sqrt{x+9}} = \dfrac{16}{2\sqrt{x+9}} = \dfrac{8}{\sqrt{x+9}}$

$f'(1) = \dfrac{8}{\sqrt{10}} = \dfrac{4\sqrt{10}}{5}$, $f'(2) = \dfrac{8}{\sqrt{11}} = \dfrac{8\sqrt{11}}{11}$, $f'(3) = \dfrac{8}{\sqrt{12}} = \dfrac{4\sqrt{3}}{3}$

40. $f(x) = \dfrac{1}{x+4}$.

Step 1. $f(x+h) = \dfrac{1}{x+4+h}$

Step 2. $f(x+h) - f(x) = \dfrac{1}{x+4+h} - \dfrac{1}{x+4} = \dfrac{x+4-(x+4+h)}{(x+4+h)(x+4)} = \dfrac{-h}{(x+4+h)(x+4)}$

Step3. $\dfrac{f(x+h)-f(x)}{h} = \dfrac{-h}{h(x+4+h)(x+4)} = \dfrac{-1}{(x+4+h)(x+4)}$

Step 4. $f'(x) = \lim\limits_{h\to 0} \dfrac{f(x+h)-f(x)}{h} = \lim\limits_{h\to 0} \dfrac{-1}{(x+4+h)(x+4)} = \dfrac{-1}{(x+4)^2}$.

$f'(1) = \dfrac{-1}{25}, \quad f'(2) = \dfrac{-1}{36}, \quad f'(3) = \dfrac{-1}{49}$

42. $f(x) = \dfrac{x}{x+2}$

Step1. $f(x+h) = \dfrac{x+h}{x+2+h}$

Step 2. $f(x+h) - f(x) = \dfrac{x+h}{x+2+h} - \dfrac{x}{x+2} = \dfrac{(x+h)(x+2) - x(x+2+h)}{(x+2+h)(x+2)} = \dfrac{2h}{(x+2+h)(x+2)}$

Step 3. $\dfrac{f(x+h)-f(x)}{h} = \dfrac{2h}{h(x+2+h)(x+2)} = \dfrac{2}{(x+2+h)(x+2)}$

Step 4. $f'(x) = \lim\limits_{h\to 0} \dfrac{f(x+h)-f(x)}{h} = \lim\limits_{h\to 0} \dfrac{2}{(x+2+h)(x+2)} = \dfrac{2}{(x+2)^2}$.

$f'(1) = \dfrac{2}{9}, \quad f'(2) = \dfrac{2}{16} = \dfrac{1}{8}, \quad f'(3) = \dfrac{2}{25}$

44. $y = f(x) = x^2 + x$

(A) $f(2) = 2^2 + 2 = 6, f(4) = 4^2 + 4 = 20$

Slope of secant line: $\dfrac{f(4)-f(2)}{4-2} = \dfrac{20-6}{2} = \dfrac{14}{2} = 7$

(B) $f(2) = 6, f(2+h) = (2+h)^2 + (2+h) = 4 + 4h + h^2 + 2 + h$

$= 6 + 5h + h^2$

Slope of secant line: $\dfrac{f(2+h)-f(2)}{h} = \dfrac{6+5h+h^2-6}{h}$

$= \dfrac{5h+h^2}{h} = 5 + h$

(C) Slope of tangent line at $(2, f(2))$:

$\lim\limits_{h\to 0} \dfrac{f(2+h)-f(2)}{h} = \lim\limits_{h\to 0} (5+h) = 5$

(D) Equation of tangent line at $(2, f(2))$:

$y - f(2) = f'(2)(x-2)$ or $y - 6 = 5(x-2)$ and $y = 5x - 4$.

46. $f(x) = x^2 + x$

(A) Average velocity: $\dfrac{f(4)-f(2)}{4-2} = \dfrac{(4)^2+4-((2)^2+2)}{2} = \dfrac{16+4-6}{2} = 7$ meters per second

(B) Average velocity: $\dfrac{f(2+h)-f(2)}{h} = \dfrac{(2+h)^2+(2+h)-6}{h} = \dfrac{4+4h+h^2+2+h-6}{h}$

$$= \frac{5h+h^2}{h} = 5 + h \text{ meters per second}$$

(C) Instantaneous velocity: $\lim\limits_{h\to 0} \dfrac{f(2+h)-f(2)}{h} = \lim\limits_{h\to 0}(5+h) = 5$ meters per second

48. $F'(x)$ does not exist at $x = b$.

50. $F'(x)$ does exist at $x = d$.

52. $F'(x)$ does not exist at $x = f$.

54. $F'(x)$ does not exist at $x = h$.

56. $f(x) = x^2 + 2x$

(A) Step 1. Simplify $\dfrac{f(x+h)-f(x)}{h}$.

$$\frac{f(x+h)-f(x)}{h} = \frac{(x+h)^2+2(x+h)-(x^2+2x)}{h}$$

$$= \frac{x^2+2xh+h^2+2x+2h-x^2-2x}{h}$$

$$= \frac{2xh+h^2+2h}{h} = 2x + 2 + h$$

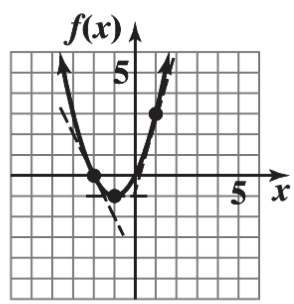
Step 2. Evaluate $\lim\limits_{h\to 0} \dfrac{f(x+h)-f(x)}{h}$.

$$\lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0}(2x+2+h) = 2x+2$$

Therefore, $f'(x) = 2x + 2$.

(B) $f'(-2) = -2$, $f'(-1) = 0$,

(C) $f'(1) = 4$



58. To find $v = f'(x)$, use the two-step process for the given distance function, $f(x) = 8x^2 - 4x$.

Step 1. $\dfrac{f(x+h)-f(x)}{h} = \dfrac{8(x+h)^2-4(x+h)-(8x^2-4x)}{h}$

$$= \frac{8(x^2+2xh+h^2)-4x-4h-8x^2+4x}{h}$$

$$= \frac{8x^2 + 16xh + 8h^2 - 4x - 4h - 8x^2 + 4x}{h}$$

$$= \frac{16xh - 4h + 8h^2}{h} = 16x - 4 + 8h$$

Step 2. $\lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} (16x - 4 + 8h) = 16x - 4$

Thus, the velocity, $v = f'(x) = 16x - 4$

$f'(1) = 12$ feet per second, $f'(3) = 44$ feet per second, $f'(5) = 76$ feet per second

60. (A) The graphs of g and h are vertical translations of the graph of f. All Three functions should have the same derivatives; they differ from each other by a constant.

(B) $m(x) = -x^2 + c$

Step 1. $m(x + h) = -(x + h)^2 + c = -x^2 - 2xh - h^2 + c$

Step 2. $m(x + h) - m(x) = (-x^2 - 2xh - h^2 + c) - (-x^2 + c)$
$= -x^2 - 2xh - h^2 + c + x^2 - c = -2xh - h^2$

Step 3. $\frac{m(x+h) - m(x)}{h} = \frac{-2xh - h^2}{h} = -2x - h$

Step 4. $m'(x) = \lim_{h\to 0} \frac{m(x+h) - m(x)}{h} = \lim_{h\to 0} (-2x - h) = -2x$

62. True: $\lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{m(x+h) + b - (mx + b)}{h}$

$$= \lim_{h\to 0} \frac{mx + mh + b - mx - b}{h} = \lim_{h\to 0} \frac{mh}{h} = \lim_{h\to 0} m = m$$

64. Let $c \in (a,b)$**.** We wish to show that $\lim_{x\to c} f(x) = f(c)$. If we let $h = x - c$, then $x = h + c$, and this statement is equivalent to $\lim_{h\to 0} f(c+h) = f(c)$, which is in turn equivalent to $\lim_{h\to 0} (f(c+h) - f(c)) = 0$.

Since $f'(x)$ exists at every point in the interval, we know that $f'(c)$ is defined and

$$\lim_{h\to 0} \frac{f(c+h) - f(c)}{h} = f'(c)$$

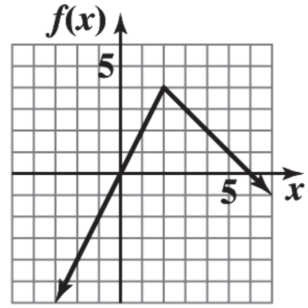
$$\left(\lim_{h\to 0} h\right)\left(\lim_{h\to 0} \frac{f(c+h) - f(c)}{h}\right) = \left(\lim_{h\to 0} h\right) f'(c)$$

$$\lim_{h\to 0} h\left(\frac{f(c+h) - f(c)}{h}\right) = 0$$

$$\lim_{h\to 0} (f(c+h) - f(c)) = 0$$

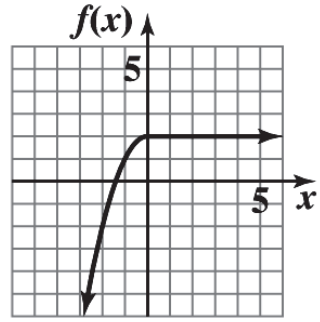
66. False. For example, $f(x) = |x|$ has a sharp corner at $x = 0$, but is continuous there.

68. The graph of $f(x) = \begin{cases} 2x & \text{if } x < 2 \\ 6 - x & \text{if } x \geq 2 \end{cases}$ is:



f is not differentiable at $x = 2$ because the graph of f has a sharp corner at this point.

70. $f(x) = \begin{cases} 2 - x^2 & \text{if } x \leq 0 \\ 2 & \text{if } x > 0 \end{cases}$



It is clear that $f\,'(x) = \begin{cases} -2x & \text{if } x < 0 \\ 0 & \text{if } x > 2 \end{cases}$

Thus, the only question is $f\,'(0)$.
Since $\lim_{x \to 0^-} f\,'(x) = \lim_{x \to 0^-} (-2x) = 0$ and

$\lim_{x \to 0^+} f\,'(x) = \lim_{x \to 0^+} (0) = 0$, f is differentiable at 0 as well;
f is differentiable for all real numbers.

72. $f(x) = 1 - |x|$

$$\lim_{h \to 0} \frac{f(0+h) - f(0)}{h} = \lim_{h \to 0} \frac{1 - |0+h| - (1 - |0|)}{h} = \lim_{h \to 0} -\frac{|h|}{h}$$

The limit does not exist. Thus, f is not differentiable at $x = 0$.

74. $f(x) = x^{2/3}$

$$\lim_{h \to 0} \frac{f(0+h) - f(0)}{h} = \lim_{h \to 0} \frac{(0+h)^{2/3} - 0^{2/3}}{h} = \lim_{h \to 0} \frac{h^{2/3}}{h} = \lim_{h \to 0} \frac{1}{h^{1/3}}$$

The limit does not exist. Thus, f is not differentiable at $x = 0$.

76. $f(x) = \sqrt{1 + x^2}$

$$\lim_{h \to 0} \frac{f(0+h) - f(0)}{h} = \lim_{h \to 0} \frac{\sqrt{1 + (0+h)^2} - \sqrt{1 + 0^2}}{h} = \lim_{h \to 0} \frac{\sqrt{1+h^2} - 1}{h}$$

$$= \lim_{h \to 0} \frac{\sqrt{1+h^2} - 1}{h} \cdot \frac{\sqrt{1+h^2} + 1}{\sqrt{1+h^2} + 1} = \lim_{h \to 0} \frac{1 + h^2 - 1}{h\left[\sqrt{1+h^2} + 1\right]} = \lim_{h \to 0} \frac{h}{\sqrt{1+h^2} + 1} = \frac{0}{2} = 0$$

f is differentiable at $x = 0$ and $f\,'(0) = 0$.

78. $y = 16x^2$

Now, if $y = 1{,}024$ ft, then

$$16x^2 = 1{,}024$$
$$x^2 = \frac{1{,}024}{16} = 64$$
$$x = 8 \text{ sec.}$$

$y' = 32x$ and at $x = 8$, $y' = 32(8) = 256$ ft/sec.

80. $P(x) = 45x - 0.025x^2 - 5{,}000$, $0 \le x \le 2{,}400$.

(A) Average change $= \dfrac{P(850) - P(800)}{850 - 800}$

$$= \frac{[45(850) - 0.025(850)^2 - 5{,}000] - [45(800) - 0.025(800)^2 - 5{,}000]}{50}$$

$$= \frac{45(850) - 0.025(850)^2 - 45(800) + 0.025(800)^2}{50}$$

$$= \frac{54{,}250 - 54{,}062.5}{50} = \frac{187.5}{50} = \$3.75$$

(B) $P(x) = 45x - 0.025x^2 - 5{,}000$

Step 1. $P(x + h) = 45(x + h) - 0.025(x + h)^2 - 5{,}000$
$= 45x + 45h - 0.025x^2 - 0.05xh - 0.025h^2 - 5{,}000$

Step 2. $P(x + h) - P(x) = (45x + 45h - 0.025x^2 - 0.05xh - 0.025h^2 - 5{,}000) - (45x - 0.025x^2 - 5{,}000)$
$= 45h - 0.05xh - 0.025h^2$

Step 3. $\dfrac{P(x+h) - P(x)}{h} = \dfrac{45h - 0.05xh - 0.025h^2}{h} = 45 - 0.05x - 0.025h$

Step 4. $P'(x) = \lim\limits_{h\to 0} \dfrac{P(x+h) - P(x)}{h} = \lim\limits_{h\to 0} (45 - 0.05x - 0.025h) = 45 - 0.05x$

(C) $P(800) = 45(800) - 0.025(800)^2 - 5{,}000 = 15{,}000$
$P'(800) = 45 - 0.05(800) = 5$;

At a production level of 800 car seats, the profit is \$15,000 and is increasing at the rate of \$5 per seat.

82. $S(t) = 2\sqrt{t+6}$

(A) Step 1. $S(t + h) = 2\sqrt{t+h+6}$

Step 2. $S(t + h) - S(t) = 2[\sqrt{t+h+6} - \sqrt{t+6}\,]$

$$= 2[\sqrt{t+h+6} - \sqrt{t+6}\,] \cdot \frac{\sqrt{t+h+6} + \sqrt{t+6}}{\sqrt{t+h+6} + \sqrt{t+6}}$$

$$= \frac{2[(t+h+6) - (t+6)]}{\sqrt{t+h+6} + \sqrt{t+6}} = \frac{2h}{\sqrt{t+h+6} + \sqrt{t+6}}$$

Step 3. $\dfrac{S(t+h) - S(t)}{h} = \dfrac{\dfrac{2h}{\sqrt{t+h+6} + \sqrt{t+6}}}{h} = \dfrac{2}{\sqrt{t+h+6} + \sqrt{t+6}}$

Step 4. $S'(t) = \lim\limits_{h\to 0} \dfrac{S(t+h) - S(t)}{h} = \lim\limits_{h\to 0} \dfrac{2}{\sqrt{t+h+6} + \sqrt{t+6}} = \dfrac{2}{2\sqrt{t+6}} = \dfrac{1}{\sqrt{t+6}}$

(B) $S(10) = 2\sqrt{10+6} = 2\sqrt{16} = 2(4) = 8$;

$S'(10) = \dfrac{1}{\sqrt{10+6}} = \dfrac{1}{\sqrt{16}} = \dfrac{1}{4} = 0.25$

After 10 months, the total sales are \$8 million and are increasing at the rate of \$0.25 million = \$250,000 per month.

(C) The estimated total sales are \$8.25 million after 11 months and \$8.5 million after 12 months.

84. (A) $p(t) = 48t^2 - 37t + 1{,}698$

Step 1. $p(t+h) = 48(t+h)^2 - 37\left(t+h\right) + 1{,}698$

$$= 48\left(t^2 + 2th + h^2\right) - 37t - 37h + 1{,}698$$

$$= 48t^2 + 96th + 48h^2 - 37t - 37h + 1{,}698$$

Step 2. $p(t+h) - p(t) = 48t^2 + 96th + 48h^2 - 37t - 37h + 1{,}698 - (48t^2 - 37t + 1{,}698)$

$$= 96th + 48h^2 - 37h$$

Step 3. $\dfrac{p(t+h) - p(t)}{h} = \dfrac{96th + 48h^2 - 37h}{h} = 96t + 48h - 37$

Step 4. $p'(t) = \lim\limits_{h\to 0} \dfrac{p(t+h) - p(t)}{h} = \lim\limits_{h\to 0}\left(96t + 48h - 37\right) = 96t - 37$

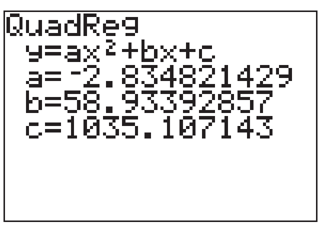
(B) 2022 corresponds to $t = 12$. Thus

$p(12) = 48\left(12\right)^2 - 37(12) + 1{,}698 = 8{,}166$

$p'(12) = 96(12) - 37 = 1{,}115$

In 2022, 8,166 thousand tons of copper will be consumed and this quantity is increasing at the rate of 1,115 thousand tons/year.

86. (A) Quadratic regression model



$C(x) \approx -2.835x^2 + 58.934x + 1035.107, \quad C'(x) \approx -5.670x + 58.934.$

(B) $C(20) = -2.835(20)^2 + 58.934(20) + 1035.107 \approx 1{,}079.9$;

$C'(20) = -5.670(20) + 58.934 = -54.5$

In 2020, 1,079.9 billion kilowatts will be sold and the amount sold is decreasing at the rate of 54.5 billion kilowatts per year.

88. (A) $F(t) = 98 + \dfrac{4}{t+1}$

Step 1. $F(t+h) = 98 + \dfrac{4}{t+h+1}$

Step 2. $F(t+h) - F(t) = \left(98 + \dfrac{4}{t+h+1}\right) - \left(98 + \dfrac{4}{t+1}\right) = \dfrac{4}{t+h+1} - \dfrac{4}{t+1}$

$$= 4\left[\frac{(t+1) - (t+h+1)}{(t+h+1)(t+1)}\right] = \frac{-4h}{(t+h+1)(t+1)}$$

Step 3. $\dfrac{F(t+h)-\mathrm{F}(t)}{h}=\dfrac{\dfrac{-4h}{(t+h+1)(t+1)}}{h}=\dfrac{-4}{(t+h+1)(t+1)}$

Step 4. $F'(t)=\lim\limits_{h\to 0}\dfrac{F(t+h)-\mathrm{F}(t)}{h}=\lim\limits_{h\to 0}\dfrac{-4}{(t+h+1)(t+1)}=\dfrac{-4}{(t+1)^2}$

(B) $F(3)=99$, $F'(3)=\dfrac{-4}{16}=\dfrac{-1}{4}$. The body temperature 3 hours after taking the medicine is 99° and is decreasing at the rate of 0.25° per hour.

EXERCISE 2-5

2. $\sqrt[3]{x}=x^{1/3}$

4. $\dfrac{1}{x}=x^{-1}$

6. $\dfrac{1}{(x^5)^2}=\dfrac{1}{x^{10}}=x^{-10}$

8. $\dfrac{1}{\sqrt[5]{x}}=\dfrac{1}{x^{1/5}}=x^{-1/5}$

10. $\dfrac{d}{dx}3=0$ (Derivative of a constant rule.)

12. $y=x^6$
$y'=6x^{6-1}=6x^5$ (Power rule)

14. $g(x)=x^5$
$g'(x)=5x^{5-1}=5x^4$ (Power rule)

16. $y=x^{-8}$
$\dfrac{dy}{dx}=-8x^{-8-1}=-8x^{-9}$ (Power rule)

18. $f(x)=x^{9/2}$
$f'(x)=\dfrac{9}{2}x^{9/2-1}=\dfrac{9}{2}x^{7/2}$ (Power rule)

20. $y=\dfrac{1}{x^{12}}=x^{-12}$

$y'=-12x^{-12-1}=-12x^{-13}$ (Power rule)

22. $\dfrac{d}{dx}(-2x^3)=-2(3x^2)$ (constant times a function rule)

$=-6x^2$

24. $f(x)=.8x^4$
$f'(x)=(.8)(4x^3)=3.2x^3$

26. $y=\dfrac{x^5}{25}$
$y'=\dfrac{1}{25}(5x^4)=\dfrac{x^4}{5}$

28. $h(x)=5g(x)$; $h'(2)=5g'(2)=5(-1)=-5$

30. $h(x)=g(x)-f(x)$; $h'(2)=g'(2)-f'(2)=-1-3=-4$

32. $h(x)=-4f(x)+5g(x)-9$; $h'(2)=-4f'(2)+5g'(2)=-4(3)+5(-1)=-17$

34. $\frac{d}{dx}(-4x+9) = \frac{d}{dx}(-4x) + \frac{d}{dx}(9) = -4 + 0 = -4$

36. $y = 2 + 5t - 8t^3$

$\frac{dy}{dt} = 0 + 5 - 24t^2 = 5 - 24t^2$

38. $g(x) = 5x^{-7} - 2x^{-4}$

$g'(x) = (5) \cdot (-7)x^{-8} - (2) \cdot (-4)x^{-5}$

$= -35x^{-8} + 8x^{-5}$

40. $\frac{d}{du}(2u^{4.5} - 3.1u + 13.2) = (2) \cdot (4.5)u^{3.5} - 3.1 + 0 = 9u^{3.5} - 3.1$

42. $F(t) = 0.2t^3 - 3.1t + 13.2$

$F'(t) = (0.2) \cdot (3)t^2 - 3.1 + 0 = 0.6t^2 - 3.1$

44. $w = \frac{7}{5u^2} = \frac{7}{5}u^{-2}$

$w' = \left(\frac{7}{5}\right) \cdot (-2)u^{-3} = -\frac{14}{5}u^{-3}$

46. $\frac{d}{dx}\left(\frac{5x^3}{4} - \frac{2}{5x^3}\right) = \frac{d}{dx}\left(\left(\frac{5}{4}\right)x^3 - \left(\frac{2}{5}\right)x^{-3}\right) = \left(\frac{5}{4}\right) \cdot (3)x^2 - \left(\frac{2}{5}\right) \cdot (-3)x^{-4} = \frac{15}{4}x^2 + \frac{6}{5}x^{-4}$

48. $H(w) = \frac{5}{w^6} - 2\sqrt{w} = 5w^{-6} - 2w^{1/2}$

$H'(w) = (5) \cdot (-6)w^{-7} - (2) \cdot \left(\frac{1}{2}\right)w^{-1/2} = -30w^{-7} - w^{-1/2}$

50. $\frac{d}{du}(8u^{3/4} + 4u^{-1/4}) = (8) \cdot \left(\frac{3}{4}\right)u^{-1/4} + (4) \cdot \left(-\frac{1}{4}\right)u^{-5/4} = 6u^{-1/4} - u^{-5/4}$

52. $F(t) = \frac{5}{t^{1/5}} - \frac{8}{t^{3/2}} = 5t^{-1/5} - 8t^{-3/2}$

$F'(t) = (5) \cdot \left(-\frac{1}{5}\right)t^{-6/5} - (8) \cdot \left(-\frac{3}{2}\right)t^{-5/2} = -t^{-6/5} + 12t^{-5/2}$

54. $w = \frac{10}{\sqrt[5]{u}} = 10u^{-1/5}$

$w' = (10) \cdot \left(-\frac{1}{5}\right)u^{-6/5} = -2u^{-6/5}$

56. $\frac{d}{dx}\left(2.8x^{-3} - \frac{0.6}{\sqrt[3]{x^2}} + 7\right) = \frac{d}{dx}(2.8x^{-3} - 0.6x^{-2/3} + 7) = (2.8) \cdot (-3)x^{-4} - (0.6) \cdot \left(-\frac{2}{3}\right)x^{-5/3} + 0$

$= -8.4x^{-4} + 0.4x^{-5/3}$

58. $f(x) = 2x^2 + 8x$

(A) $f'(x) = 4x + 8$

(B) Slope of the graph of f at $x = 2$: $f'(2) = 4(2) + 8 = 16$
Slope of the graph of f at $x = 4$: $f'(4) = 4(4) + 8 = 24$

(C) Tangent line at $x = 2$: $y - y_1 = m(x - x_1)$
$x_1 = 2$
$y_1 = f(2) = 2(2)^2 + 8(2) = 24$
$m = f'(2) = 16$
Thus, $y - 24 = 16(x - 2)$ or $y = 16x - 8$
Tangent line at $x = 4$: $y - y_1 = m(x - x_1)$
$x_1 = 4$
$y_1 = f(4) = 2(4)^2 + 8(4) = 64$
$m = f'(4) = 24$
Thus, $y - 64 = 24(x - 4)$ or $y = 24x - 32$

(D) The tangent line is horizontal at the values $x = c$ such that $f'(c) = 0$. Thus, we must solve the following:
$f'(x) = 4x + 8 = 0$
$4x = -8$
$x = -2$

60. $f(x) = x^4 - 32x^2 + 10$

(A) $f'(x) = 4x^3 - 64x$

(B) Slope of the graph of f at $x = 2$: $f'(2) = 4(2)^3 - 64(2) = -96$
Slope of the graph of f at $x = 4$: $f'(4) = 4(4)^3 - 64(4) = 0$

(C) Tangent line at $x = 2$: $y - y_1 = m(x - x_1)$, where
$x_1 = 2$, $y_1 = f(2) = (2)^4 - 32(2)^2 + 10 = -102$, $m = -96$
$y + 102 = -96(x - 2)$ or $y = -96x + 90$
Tangent line at $x = 4$ is a horizontal line since the slope $m = 0$. Therefore, the equation of the tangent
line at $x = 4$ is: $y = f(4) = (4)^4 - 32(4)^2 + 10 = -246$

(D) Solve $f'(x) = 0$ for x:
$4x^3 - 64x = 0$
$4x(x^2 - 16) = 0$
$4x(x + 4)(x - 4) = 0$
$x = -4$, $x = 0$, $x = 4$

62. $f(x) = 80x - 10x^2$

(A) $v = f'(x) = 80 - 20x$

(B) $v\big|_{x=0} = f'(0) = 80$ ft/sec.

$v\big|_{x=3} = f'(3) = 80 - 20(3) = 20$ ft/sec.

(C) Solve $v = f'(x) = 0$ for x:

$80 - 20x = 0$

$20x = 80$

$x = 4$ seconds

64. $f(x) = x^3 - 9x^2 + 24x$

(A) $v = f'(x) = 3x^2 - 18x + 24$

(B) $v\big|_{x=0} = f'(0) = 24$ ft/sec.

$v\big|_{x=3} = f'(3) = 3(3)^2 - 18(3) + 24 = -3$ ft/sec.

(C) Solve $v = f'(x) = 0$ for x:

$3x^2 - 18x + 24 = 0$ or $x^2 - 6x + 8 = 0$

$(x - 2)(x - 4) = 0$

$x = 2,\ x = 4$ seconds

66. $f'(x) = 2x + 1 - \frac{5}{\sqrt{x}}$; $f'(x) = 0$ at $x \approx 1.5247$.

68. $f'(x) = 4x^{1/3} - 4x + 4$; $f'(x) = 0$ at $x \approx 2.3247$.

70. $f'(x) = 0.08x^3 - 0.18x^2 - 1.56x + 0.94$; $f'(x) = 0$ at $x \approx -3.7626,\ 0.5742,\ 5.4384$.

72. $f'(x) = x^3 - 7.8x^2 + 16.2x - 10$; $f'(x) = 0$ at $x \approx 1.2391,\ 1.6400,\ 4.9209$.

74. The tangent line to the graph of a parabola at the vertex is a horizontal line. Therefore, to find the x coordinate of the vertex,
we solve $f'(x) = 0$ for x.

76. No. The derivative is a quadratic function which can have at most two zeros.

78. $y = (2x - 5)^2$; $y' = (2)(2x - 5)(2) = 8x - 20$

80. $y = \frac{x^2 + 25}{x^2} = 1 + \frac{25}{x^2} = 1 + 25x^{-2}$; $\frac{dy}{dx} = 0 + (25)\cdot(-2)x^{-3} = -50x^{-3}$

82. $f(x) = \frac{2x^5 - 4x^3 + 2x}{x^3} = \frac{2x^5}{x^3} - \frac{4x^3}{x^3} + \frac{2x}{x^3} = 2x^2 - 4 + 2x^{-2}$; $f'(x) = 4x - 4x^{-3}$

84. False: The function $f(x) = \frac{1}{x}$ is a counter-example.

86. False: The function $f(x) = 2x$ is a counter-example.

88. $f(x) = u(x) - v(x)$

Step1. $f(x+h) = u(x+h) - v(x+h)$

Step 2. $f(x+h) - f(x) = u(x+h) - v(x+h) - [u(x) - v(x)] = u(x+h) - u(x) - [v(x+h) - v(x)]$

Step 3. $\dfrac{f(x+h) - f(x)}{h} = \dfrac{u(x+h) - u(x) - [v(x+h) - v(x)]}{h} = \dfrac{u(x+h) - u(x)}{h} - \dfrac{v(x+h) - v(x)}{h}$

Step 4. $\lim_{h\to 0} \dfrac{f(x+h) - f(x)}{h} = \lim_{h\to 0}\left[\dfrac{u(x+h) - u(x)}{h} - \dfrac{v(x+h) - v(x)}{h}\right]$

$$= \lim_{h\to 0} \frac{u(x+h) - u(x)}{h} - \lim_{h\to 0} \frac{v(x+h) - v(x)}{h} = u'(x) - v'(x)$$

90. $S(t) = 0.015t^4 + 0.4t^3 + 3.4t^2 + 10t - 3$

(A) $S'(t) = (0.015)\cdot(4)t^3 + (0.4)\cdot(3)t^2 + (3.4)(2)t + 10 - 0 = 0.06t^3 + 1.2t^2 + 6.8t + 10$

(B) $S(4) = 0.015(4)^4 + 0.4(4)^3 + 3.4(4)^2 + 10(4) - 3 = 120.84$,
$S'(4) = 0.06(4)^3 + 1.2(4)^2 + 6.8(4) + 10 = 60.24$.

After 4 months, sales are \$120.84 million and are increasing at the rate of \$60.24 million per month.

(C) $S(8) = 0.015(8)^4 + 0.4(8)^3 + 3.4(8)^2 + 10(8) - 3 = 560.84$,
$S'(8) = 0.06(8)^3 + 1.2(8)^2 + 6.8(8) + 10 = 171.92$.

After 8 months, sales are \$560.84 million and are increasing at the rate of \$171.92 million per month.

92. $x = 10 + \dfrac{180}{p}$, $2 \le p \le 10$

For $p = 5$, $x = 10 + \dfrac{180}{5} = 10 + 36 = 46$

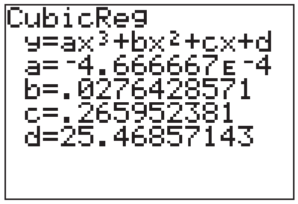
$x = 10 + \dfrac{180}{p} = 10 + 180p^{-1}$

$\dfrac{dx}{dp} = -180p^{-2} = -\dfrac{180}{p^2}$

For $p = 5$, $\left.\dfrac{dx}{dp}\right|_{p=5} = -\dfrac{180}{25} = -7.2$

At the \$5 price level, the demand is 46 pounds and is decreasing at the rate of 7.2 pounds per dollar increase in price.

94 . (A) Cubic Regression model



(B) $F(x) \approx -0.000467x^3 + 0.027643x^2 + 0.265952x + 25.468751$
$F(50) \approx 49.5, \quad F'(50) \approx -0.5$

In 2020, 49.5% of female high-school graduates enroll in college and the percentage is decreasing at the rate of 0.5% per year.

96. $C(x) = \dfrac{0.1}{x^2} = 0.1x^{-2}$

$C'(x) = -0.2x^{-3} = -\dfrac{0.2}{x^3}$, the instantaneous rate of change of concentration at x miles.

(A) At $x = 1$, $C'(1) = -0.2$ parts per million per mile.

(B) At $x = 2$, $C'(2) = -\dfrac{0.2}{8} = -0.025$ parts per million per mile.

98. $y = 21\sqrt[3]{x^2}$, $0 \le x \le 8$.

First, find $y = 21\sqrt[3]{x^2} = 21x^{2/3}$.

Then $y' = 21\left(\dfrac{2}{3}x^{-1/3}\right) = 14x^{-1/3} = \dfrac{14}{x^{1/3}} = \dfrac{14}{\sqrt[3]{x}}$, is the rate of learning at the end of x hours.

(A) Rate of learning at the end of 1 hour:
$\dfrac{14}{\sqrt[3]{1}} = 14$ items per hour.

(B) Rate of learning at the end of 8 hours:
$\dfrac{14}{\sqrt[3]{8}} = \dfrac{14}{2} = 7$ items per hour.

EXERCISE 2-6

2. $f(x) = 0.1x + 3; \quad f(7) = 0.1(7) + 3 = 3.7, \quad f(7.1) = 0.1(7.1) + 3 = 3.71$

4. $f(x) = 0.1x + 3; \quad f(-10) = 0.1(-10) + 3 = 2, \quad f(-10.1) = 0.1(-10.1) + 3 = 1.99$

6. $g(x) = x^2; \quad g(1) = 1^2 = 1, \quad g(1.1) = (1.1)^2 = 1.21$

8. $g(x) = x^2; \quad g(5) = 5^2 = 25, \quad g(4.9) = (4.9)^2 = (5-0.1)^2 = 24.01$

10. $\Delta x = x_2 - x_1 = 5 - 2 = 3, \quad \Delta y = f(x_2) - f(x_1) = 3(5)^2 - 3(2)^2 = 75 - 12 = 63$

$$\frac{\Delta y}{\Delta x} = \frac{63}{3} = 21$$

12. $\dfrac{f(x_1+\Delta x) - f(x_1)}{\Delta x} = \dfrac{f(2+1) - f(2)}{1} = \dfrac{f(3) - f(2)}{1} = \dfrac{3(3)^2 - 3(2)^2}{1} = 27 - 12 = 15$

14. $\Delta y = f(x_2) - f(x_1) = f(3) - f(2) = 3(3)^2 - 3(2)^2 = 27 - 12 = 15$

$\Delta x = x_2 - x_1 = 3 - 2 = 1$

$$\frac{\Delta y}{\Delta x} = \frac{15}{1} = 15$$

16. $y = 200x - \dfrac{x^2}{30}, \quad dy = \left(200x - \dfrac{x^2}{30}\right)' dx = \left(200 - \dfrac{x}{15}\right) dx$

18. $y = x^3(60 - x) = 60x^3 - x^4, \quad dy = (180x^2 - 4x^3)\,dx$

20. $y = 52\sqrt{x} = 52x^{1/2}, \quad dy = (52x^{1/2})'dx = (26x^{-1/2})dx$

22. (A) $\dfrac{f(3+\Delta x) - f(3)}{\Delta x} = \dfrac{3(3+\Delta x)^2 - 3(3)^2}{\Delta x} = \dfrac{3(9 + 6\Delta x + (\Delta x)^2) - 27}{\Delta x}$

$$= \frac{27 + 18\Delta x + 3(\Delta x)^2 - 27}{\Delta x} = \frac{18\Delta x + 3(\Delta x)^2}{\Delta x} = 18 + 3\Delta x$$

(B) As Δx tends to zero, then, clearly, $18 + 3\Delta x$ tends to 18.
Note the values in the following table:

Δx	$18 + 3\Delta x$
1	21
0.1	18.3
0.01	18.03
0.001	18.003

24. $y = (3x + 5)^2 = 9x^2 + 30x + 25, \quad dy = (18x + 30)\,dx = 6(3x+5)\,dx.$

26. $y = \dfrac{(x-1)^2}{x^2} = \dfrac{x^2 - 2x + 1}{x^2} = 1 - 2x^{-1} + x^{-2}, \quad dy = (2x^{-2} - 2x^{-3})\,dx.$

28. $y = f(x) = 30 + 12x^2 - x^3$

$\Delta y = f(2 + 0.1) - f(2) = f(2.1) - f(2) = [30 + 12(2.1)^2 - (2.1)^3] - [(30 + 12(2)^2 - 2^3]$

$= 30 + 52.92 - 9.261 - 30 - 48 + 8 = 3.66$

$dy = (30 + 12x^2 - x^3)'\big|_{x=2}\ dx = (24x - 3x^2)\big|_{x=2}\ (0.1) = (24(2) - 3(2)^2)(0.1) = (48 - 12)(0.1) = 3.6$

30. $y = f(x) = 100\left(x - \dfrac{4}{x^2}\right)$

$\Delta y = f(2 - 0.1) - f(2) = f(1.9) - f(2) = 100\left(1.9 - \dfrac{4}{(1.9)^2}\right) - 100\left(2 - \dfrac{4}{2^2}\right) = 79.197 - 100 = -20.803$

$dy = \left(100\left(x - \dfrac{4}{x^2}\right)\right)'\Big|_{x=2}\ dx = 100\left(1 + \dfrac{8}{x^3}\right)\Big|_{x=2}\ (-0.1) = 100\left(1 + \dfrac{8}{2^3}\right)(-0.1) = -20$

32. $V = \dfrac{4}{3}\pi r^3$, $r = 5$ cm, $dr = \Delta r = 0.1$ cm.

$dV = \left(\dfrac{4}{3}\pi r^3\right)'\Big|_{r=5}\ dr = 4\pi r^2\Big|_{r=5}\ (0.1) = 31.4\ \text{cm}^3.$

34. $f(x) = x^2 + 2x + 3$; $f'(x) = 2x + 2$; $x = -2$; $\Delta x = dx$

(A) $\Delta y = f(-2 + \Delta x) - f(-2)$

$= [(-2 + \Delta x)^2 + 2(-2 + \Delta x) + 3]$

$\quad - [(-2)^2 + 2(-2) + 3]$

$= 4 - 4\Delta x + (\Delta x)^2 - 4 + 2\Delta x + 3 - 4 + 4 - 3$

$= -2\Delta x + (\Delta x)^2$

$dy = f'(-2)dx = -2\ dx$

(B) $\Delta y(-0.1) = -2(-0.1) + (-0.1)^2 = 0.21$

$dy(-0.1) = -2(-0.1) = 0.2$

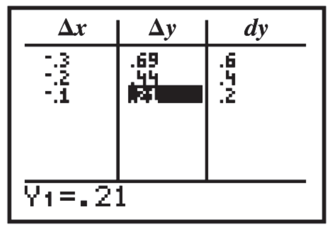
$\Delta y(-0.2) = -2(-0.2) + (-0.2)^2 = 0.44$

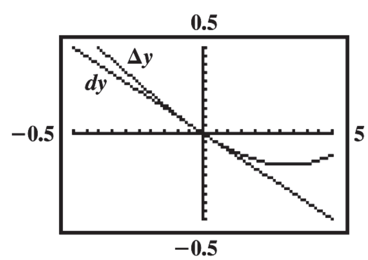
$dy(-0.2) = -2(-0.2) = 0.4$

$\Delta y(-0.3) = -2(-0.3) + (-0.3)^2 = 0.69$

$dy(-0.3) = -2(-0.3) = 0.6$

(C)





36. $f(x) = x^3 - 2x^2$; $f'(x) = 3x^2 - 4x$; $x = 2$, $\Delta x = dx$

(A) $\Delta y = f(2 + \Delta x) - f(2)$

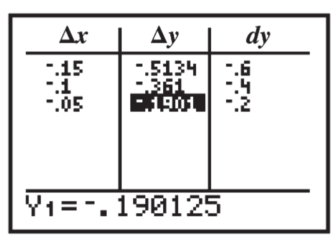
$= [(2 + \Delta x)^3 - 2(2 + \Delta x)^2] - [2^3 - 2(2)^2]$

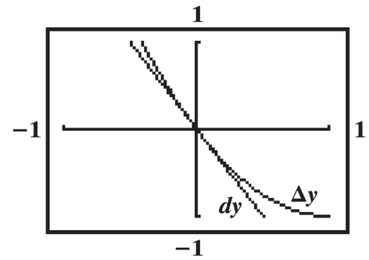
$= 8 + 12\Delta x + 6(\Delta x)^2 + (\Delta x)^3 - 8 - 8\Delta x - 2(\Delta x)^2 - 8 + 8$

$= 4\Delta x + 4(\Delta x)^2 + (\Delta x)^3$

$dy = f'(2)dx = 4\ dx$

(C)





(B) $\Delta y(-0.05) = 4(-0.05) + 4(-0.05)^2 + (-0.05)^3$
$= -0.1901$
$dy(-0.05) = 4(-0.05) = -0.2$
$\Delta y(-0.10) = 4(-0.10) + 4(-0.10)^2 + (-0.10)^3$
$= -0.361$
$dy(-0.10) = 4(-0.10) = -0.4$

$\Delta y(-0.15) = 4(-0.15) + 4(-0.15)^2 + (-0.15)^3 = -0.5134$
$dy(-0.15) = 4(-0.15) = -0.6$

38. False.
Example. Let $y = f(x) = x^2 + 1$. Then
$\Delta y = f(0 + \Delta x) - f(0) = f(\Delta x) - f(0) = (\Delta x)^2 + 1 - 1 = (\Delta x)^2$
$dy = f'(0)dx = 0\ dx = 0.$

40. True.
$\Delta y = f(2 + \Delta x) - f(2) = 0$ implies that
$f(2 + \Delta x) = f(2)$
Since this is true for every increment and since the right-hand side of this equation is a constant, the function $f(x)$ must be a constant function.

42. $y = (2x^2 - 4)\sqrt{x} = (2x^2 - 4)(x)^{1/2} = 2x^{5/2} - 4x^{1/2}$, $dy = (5x^{3/2} - 2x^{-1/2})dx.$

44. $y = f(x) = \dfrac{590}{\sqrt{x}} = 590x^{-1/2}$; $x = 64$, $\Delta x = dx = 1$.

$\Delta y = f(x + \Delta x) - f(x) = f(64 + 1) - f(64) = f(65) - f(64) = \dfrac{590}{\sqrt{65}} - \dfrac{590}{\sqrt{64}} = -0.57$

$y = f(x) = \dfrac{590}{\sqrt{x}} = 590x^{-1/2}$, $\quad f'(x) = -295x^{-3/2}$

$dy = f'(64)dx = f'(64)(1) = -295(64)^{-3/2} = -\dfrac{295}{512} = -0.576$

46. Given $D(x) = 1{,}000 - 40x^2$, $1 \le x \le 5$. Then, $D'(x) = -80x$.
The approximate change in demand dD corresponding to a change $\Delta x = dx$ in the price x is:
$dD = D'(x)dx$
Thus, letting $x = 3$ and $dx = 0.20$, we get
$dD = D'(3)(0.20) = -80(3)(0.20) = -48.$
There will be a 48-pound decrease in demand (approximately) when the price is increased from \$3.00 to \$3.20.

48. $R(x) = 200x - \dfrac{x^2}{30}$; $\quad R'(x) = 200 - \dfrac{x}{15}$

Profit $P(x) = R(x) - C(x) = 200x - \dfrac{x^2}{30} - 72{,}000 - 60x = 140x - \dfrac{x^2}{30} - 72{,}000$

$P'(x) = 140 - \dfrac{x}{15}$

Now, for $x = 1{,}500$, $\Delta x = dx = 10$, we get

$$dR = R'(1{,}500)(10) = \left(200 - \frac{1{,}500}{15}\right)(10) = 1{,}000$$

$$dP = P'(1{,}500)(10) = \left(140 - \frac{1{,}500}{15}\right)(10) = 400$$

Thus, the approximate change in revenue is \$1,000 and the approximate change in profit is \$400 if the production is increased from 1,500 to 1,510 televisions.

For $x = 4{,}500$, $\Delta x = dx = 10$, we have:

$$dR = R'(4{,}500)(10) = \left(200 - \frac{4{,}500}{15}\right)(10) = -1{,}000$$

$$dP = P'(4{,}500)(10) = \left(140 - \frac{4{,}500}{15}\right)(10) = -1{,}600.$$

Thus, the approximate change in revenue is –\$1,000 and and the approximate change in profit is –\$1,600 if the production is increased from 4,500 to 4,510 televisions.

50. $V = \frac{4}{3}\pi r^3$; $V' = 4\pi r^2$.

The approximate volume of the shell for a radius change from 5 mm to 5.3 mm is given by:

$$dV = 4\pi r^2\Big|_{r=5}\ dx = 4\pi(5)^2(0.3) \quad \text{(Note: } \Delta x = dx = 0.3 \text{ mm)}$$

$$= 94.2 \text{ cubic millimeters}$$

52. $T = x^2\left(1 - \frac{x}{9}\right) = x^2 - \frac{x^3}{9}, \quad 0 \le x \le 6; \quad T' = 2x - \frac{x^2}{3}.$

(A) For $x = 2$, $\Delta x = dx = 0.1$,

$$dT = \left(2x - \frac{x^2}{3}\right)\Big|_{x=2}\ dx = \left(2(2) - \frac{2^2}{3}\right)(0.1) = 0.27 \text{ degrees}$$

(B) For $x = 3$, $\Delta x = dx = 0.1$

$$dT = \left(2x - \frac{x^2}{3}\right)\Big|_{x=3}\ dx = \left(2(3) - \frac{3^2}{3}\right)(0.1) = 0.3 \text{ degrees}$$

(C) For $x = 4$, $\Delta x = dx = 0.1$

$$dT = \left(2x - \frac{x^2}{3}\right)\Big|_{x=4}\ dx = \left(2(4) - \frac{4^2}{3}\right)(0.1) = 0.27 \text{ degrees}$$

54. $y = 52\sqrt{x}$, $0 \le x \le 9$; $y = 52x^{1/2}$ and hence $y' = \frac{52}{2}x^{-1/2} = 26x^{-1/2}$.

For $x = 1$ and $\Delta x = dx = 0.1$ the approximate increase in the number of items learned is given by

$$dy = y'\Big|_{x=1}\ dx = 26(1)^{-1/2}(0.1) = 2.6 \text{ items.}$$

Similarly, for $x = 4$, $\Delta x = dx = 0.1$, we have

$$dy = y'\Big|_{x=4}\ dx = 26(4)^{-1/2}(0.1) = 1.3 \text{ items.}$$

EXERCISE 2-7

In Problems 2 – 8, $C(x) = 10{,}000 + 150x - 0.2x^2$.

2. $C(100) = 10{,}000 + 150(100) - 0.2(100)^2 = 25{,}000 - 2{,}000 = 23{,}000$, \$23,000

4. $C(199) = 10{,}000 + 150(199) - 0.2(199)^2 = 39{,}850 - 7{,}920.20 = 31{,}929.80$, \$31,929.80

6. Using the results in Problems 4 and 5, $C(200) - C(199) = 32{,}000 - 31{,}929.80 = 70.20$, \$70.20

8. Average cost of producing 200 bicycles: $\dfrac{C(200)}{200} = \dfrac{32{,}000}{200} = 160$, \$160

10. $C'(x) = 9.5$

12. $C'(x) = 13 - 0.4x$

14. $R'(x) = 36 - 0.06x$

16. $R'(x) = 25 - 0.10x$

18. $P'(x) = (36 - 0.06x) - 9.5 = 26.5 - 0.06x$

20. $P'(x) = (25 - 0.10x) - (13 - 0.4x) = 12 + 0.3x$

22. $\overline{R}(x) = \dfrac{5x - 0.02x^2}{x} = 5 - 0.02x$

24. $\overline{R}'(x) = -0.02$

26. $P'(x) = 3.9 - 0.04x$

28. $\overline{P}'(x) = -0.02 + \dfrac{145}{x^2}$

30. True: If $p = b - mx$ then $R(x) = xp = bx - mx^2$, and $R'(x) = b - 2mx$.

32. False: If $C(x) = 5x + 10$, then the marginal cost is $C'(x) = 5$. In this case, the average marginal cost over any interval is 5. However, the average cost is $\overline{C}(x) = 5 + \dfrac{10}{x}$ so the marginal average cost is $\overline{C}'(x) = -\dfrac{10}{x^2}$, which is not equal to 5 over the interval [1,2], for example.

34. $C(x) = 1{,}000 + 100x - 0.25x^2$

(A) The exact cost of producing the 51st guitar is:
$C(51) - C(50)$
$= 1{,}000 + 100(51) - 0.25(51)^2 - [1{,}000 + 100(50) - 0.25(50)^2]$
$= 100 - 0.25(51)^2 + 0.25(50)^2 = 74.75$ or \$74.75

(B) $C'(x) = 100 - 0.5x$
$C'(50) = 100 - 0.5(50) = 75$ or \$75.

36. $C(x) = 20{,}000 + 10x$

(A) $\overline{C}(x) = \dfrac{20{,}000 + 10x}{x} = \dfrac{20{,}000}{x} + 10 = 20{,}000x^{-1} + 10$

$\overline{C}(1{,}000) = \dfrac{20{,}000 + 10(1{,}000)}{1{,}000} = \dfrac{30{,}000}{1{,}000} = 30$ or \$30

(B) $\overline{C}'(x) = -20{,}000x^{-2} = \dfrac{-20{,}000}{x^2}$

$\overline{C}'(1{,}000) = \dfrac{-20{,}000}{(1{,}000)^2} = -0.02$ or $-2¢$

At a production level of 1,000 dictionaries, average cost is decreasing at the rate of 2¢ per dictionary.

(C) The average cost per dictionary if 1,001 are produced is approximately \$30.00 – \$0.02 = \$29.98.

38. $P(x) = 22x - 0.2x^2 - 400,\ 0 \le x \le 100$

(A) The exact profit from the sale of the 41st calendar is

$P(41) - P(40) = 22(41) - 0.2(41)^2 - 400 - [22(40) - 0.2(40)^2 - 400]$

$= 22 - 0.2(41)^2 + 0.2(40)^2 = 5.80$ or \$5.80

(B) $P'(x) = 22 - 0.4x$

$P'(40) = 22 - 0.4(40) = 22 - 16 = 6$ or \$6

40. $P(x) = 12x - 0.02x^2 - 1{,}000,\ 0 \le x \le 600;\quad P'(x) = 12 - 0.04x$

(A) $P'(200) = 12 - 0.04(200) = 12 - 8 = 4$ or \$4;

at a production level of 200 cameras, profit is increasing at the rate of \$4 per camera.

(B) $P'(350) = 12 - 0.04(350) = 12 - 14 = -2$ or –\$2;

at a production level of 350 cameras, profit is decreasing at the rate of \$2 per camera.

42. $P(x) = 20x - 0.02x^2 - 320,\ 0 \le x \le 1{,}000$

Average profit: $\overline{P}(x) = \dfrac{P(x)}{x} = 20 - 0.02x - \dfrac{320}{x} = 20 - 0.02x - 320x^{-1}$

(A) At $x = 40$, $\overline{P}(40) = 20 - 0.02(40) - \dfrac{320}{40} = 11.20$ or \$11.20.

(B) $\overline{P}'(x) = -0.02 + 320x^{-2} = -0.02 + \dfrac{320}{x^2}$

$\overline{P}'(40) = -0.02 + \dfrac{320}{(40)^2} = 0.18$ or \$0.18;

at a production level of 40 grills, the average profit per grill is increasing at the rate of \$0.18 per grill.

(C) The average profit per grill if 41 grills are produced is approximately \$11.20 + \$0.18 = \$11.38.

44. $x = 1{,}000 - 20p$

(A) $20p = 1{,}000 - x,\ p = 50 - 0.05x,\ 0 \le x \le 1{,}000$

(B) $R(x) = x(50 - 0.05x) = 50x - 0.05x^2,\ 0 \le x \le 1{,}000$

(C) $R'(x) = 50 - 0.10x$

$R'(400) = 50 - 0.10(400) = 50 - 40 = 10$;

at a production level of 400 steam irons, revenue is increasing at the rate of \$10 per steam iron.

(D) $R'(650) = 50 - 0.10(650) = 50 - 65 = -15$;
at a production level of 650 steam irons, revenue is decreasing at the rate of \$15 per steam iron.

46. $x = 9{,}000 - 30p$ and $C(x) = 150{,}000 + 30x$

(A) $30p = 9{,}000 - x, \quad p = 300 - \frac{1}{30}x, 0 \le x \le 9{,}000$

(B) $C'(x) = 30$

(C) $R(x) = x\left(300 - \frac{1}{30}x\right) = 300x - \frac{1}{30}x^2, 0 \le x \le 9{,}000$

(D) $R'(x) = 300 - \frac{1}{15}x$

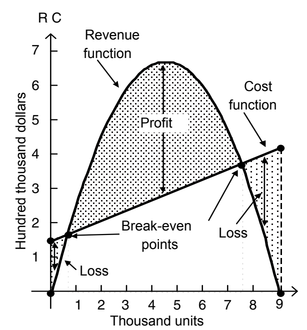
(E) $R'(3{,}000) = 300 - \frac{1}{15}(3{,}000) = 100$; at a production level of

3,000 sets, revenue is increasing at the rate of \$100 per set.

$R'(6000) = 300 - \frac{1}{15}(6{,}000) = 300 - 400 = -100$; at a production level of 6,000 sets, revenue is

decreasing at the rate of \$100 per set.

(F) The graphs of $C(x)$ and $R(x)$ are shown at the right.



To find the break-even points, set $C(x) = R(x)$:

$$150{,}000 + 30x = 300x - \frac{1}{30}x^2$$
$$x^2 - 8{,}100x + 4{,}500{,}000 = 0$$
$$(x - 600)(x - 7{,}500) = 0$$
$$x = 600 \quad \text{or} \quad x = 7{,}500$$

Now, $C(600) = 150{,}000 + 30(600) = 168{,}000$;

$C(7{,}500) = 150{,}000 + 30(7{,}500) = 375{,}000$

Thus, the break-even points are:
(600, 168,000) and (7,500, 375,000).

(G) $P(x) = R(x) - C(x) = 300x - \frac{1}{30}x^2 - (150{,}000 + 30x)$

$= -\frac{1}{30}x^2 + 270x - 150{,}000$

(H) $P'(x) = -\frac{1}{15}x + 270$

(I) $P'(1{,}500) = -\frac{1}{15}(1{,}500) + 270 = 170$; at a production level of 1,500 sets, profit is increasing at the

rate of \$170 per set.

$P'(4{,}500) = -\frac{1}{15}(4{,}500) + 270 = -30$; at a production level of 4,500 sets, profit is decreasing at the rate of \$30 per set.

48. (A) We are given $p = 25$ when $x = 300$ and $p = 20$ when $x = 400$. Thus, we have the pair of equations:
$25 = 300m + b$
$20 = 400\,m + b$

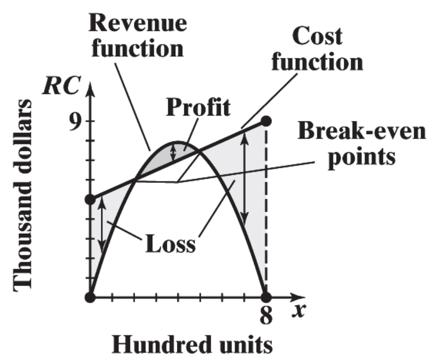
Subtracting the second equation from the first, we get $-100m = 5$. Thus, $m = -\frac{1}{20}$.

Substituting this into either equation yields $b = 40$. Therefore,
$p = -\frac{1}{20}x + 40 = 40 - \frac{x}{20},\ 0 \le x \le 800$

(B) $R(x) = x\left(40 - \frac{x}{20}\right) = 40x - \frac{x^2}{20},\ 0 \le x \le 800$

(C) From the financial department's estimates, $m = 5$ and $b = 5{,}000$. Thus, $C(x) = 5x + 5{,}000$.

(D) The graphs of $R(x)$ and $C(x)$ are shown at the right.



To find the break-even points, set $C(x) = R(x)$:

$$\begin{aligned} 5x + 5{,}000 &= 40x - \frac{x^2}{20} \\ x^2 - 700x + 100{,}000 &= 0 \\ (x - 200)(x - 500) &= 0 \\ x = 200 \quad &\text{or} \quad x = 500 \end{aligned}$$

Now, $C(200) = 5(200) + 5{,}000 = 6{,}000$ and
$C(500) = 5(500) + 5{,}000 = 7{,}500$

Thus, the break-even points are: (200, 6,000) and (500, 7,500).

(E) $P(x) = R(x) - C(x) = 40x - \frac{x^2}{20} - (5x + 5{,}000)$

$= 35x - \frac{x^2}{20} - 5{,}000$

(F) $P'(x) = 35 - \frac{x}{10}$

$P'(325) = 35 - \frac{325}{10} = 2.5$; at a production level of 325 toasters, profit is increasing at the rate of \$2.50 per toaster.

$P'(425) = 35 - \frac{425}{10} = -7.5$; at a production level of 425 toasters, profit is decreasing at the rate of \$7.50 per toaster.

50. Total cost: $C(x) = 5x + 2{,}340$
Total revenue: $R(x) = 40x - 0.1x^2,\ 0 \le x \le 400$

(A) $R'(x) = 40 - 0.2x$

The graph of R has a horizontal tangent line at the value(s) of x where $R'(x) = 0$, i.e.

$40 - 0.2x = 0$

or $x = 200$

(B) $P(x) = R(x) - C(x) = 40x - 0.1x^2 - (5x + 2{,}340)$

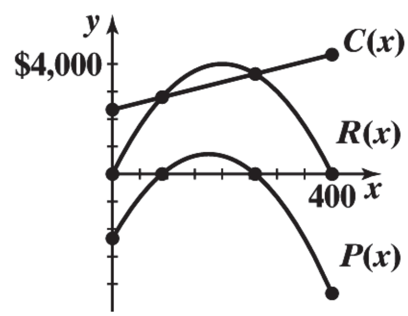
$= 35x - 0.1x^2 - 2{,}340$

(C) $P'(x) = 35 - 0.2x$. Setting $P'(x) = 0$, we have

$35 - 0.2x = 0$

or $x = 175$

(D) The graphs of $C(x)$, $R(x)$ and $P(x)$ are shown at the right.



Break-even points: $R(x) = C(x)$

$$\begin{aligned} 40x - 0.1x^2 &= 5x + 2{,}340 \\ x^2 - 350x + 23{,}400 &= 0 \\ (x - 90)(x - 260) &= 0 \\ x = 90 &\quad \text{or } x = 260 \end{aligned}$$

Thus, the break-even points are:
(90, 2,790) and (260, 3,640).

x intercepts for P: $-0.1x^2 + 35x - 2{,}340 = 0$ or

$x^2 - 350x + 23{,}400 = 0$

which is the same as the above equation. Thus, $x = 90$ and $x = 260$ are x intercepts of P.

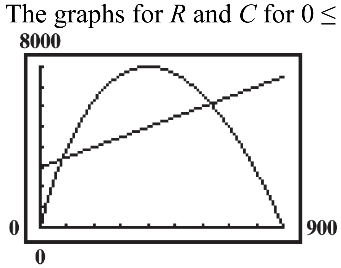
52. Demand equation: $p = 60 - 2\sqrt{x} = 60 - 2x^{1/2}$

Cost equation: $C(x) = 3{,}000 + 5x$

(A) Revenue $R(x) = xp = x(60 - 2x^{1/2})$

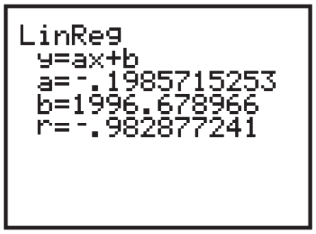
$= 60x - 2x^{3/2}$

(B) The graphs for R and C for $0 \le x \le 900$ are shown below:

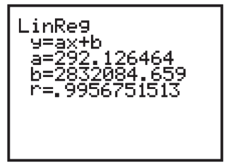


Break-even points: (81, 3,405), (631, 6,155)

54. (A)



(B) Fixed costs: \$2,832,085; variable cost: \$292



(C) Let $y = p(x)$ be the linear regression equation found in part (A) and let $y = C(x)$ be the linear regression equation found in part (B). Then revenue $R(x) = xp(x)$, and the break-even points are

$R(x) = C(x)$.

Break-even points: (2,253, 3,490,130), (6,331, 4,681,675).

(D) The company will make a profit when $2{,}253 \le x \le 6{,}331$. From part A), $p(2{,}253) = 740$ and $p(6{,}331) = 1{,}549$. Thus, the company will make a profit for the price range $\$740 \le p \le \$1{,}549$.