Chapter R Test

1. Use the compound interest formula with $i = 0.65$, $n = 1$, and $A = 798.75$

$$A = P\left(1+\frac{i}{n}\right)^{nt}$$

$$798.75 = P\left(1+\frac{0.065}{1}\right)^{1\cdot 1}$$

$$798.75 = P(1+0.065)^{1}$$

$$798.75 = P(1.065)$$

$$750 = P$$

Cecelia should deposit \$750 in order to have \$798.75 at the end of 1 year.

2. $f(x) = -x^2 + 5$

a) $f(-3) = -(-3)^2 + 5$
$= -9 + 5$
$= -4$

b) $f(x+h) = -(x+h)^2 + 5$
$= -(x^2 + 2xh + h^2) + 5$
$= -x^2 - 2xh - h^2 + 5$

3. Given the equation $y = \frac{4}{5}x - \frac{2}{3}$, it is already in the slope–intercept form, thus $m = \frac{4}{5}$ and the y–intercept is $\left(0, -\frac{2}{3}\right)$.

4. We are given the slope and a point, so use the point–slope form to determine the equation of the line:

$$y - y_1 = m(x - x_1)$$

$$y - 7 = \frac{1}{4}(x+3)$$

$$y - 7 = \frac{1}{4}x + \frac{3}{4}$$

$$y = \frac{1}{4}x + \frac{31}{4}$$

5. We are given two points, so first find the slope using the slope formula:

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-4-2}{3-(-9)} = \frac{-6}{12} = -\frac{1}{2}$$

6. The average rate of change will be the slope of the line. Use the points (0, 30) and (3, 9) and the slope formula:

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{9-30}{3-0} = \frac{-21}{3} = -7$$

Thus the average rate of change is –7 hundred dollars per year or –\$700/year.

7. The average rate of change will be the slope of the line. Use the points (0, 0) and (6, 3) and the slope formula:

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3-0}{6-0} = \frac{3}{6} = \frac{1}{2}$$

Thus the average rate of change is 0.5lbs/bag.

8. Using the direct proportion equation;

$$F = kW$$

$$120 = k(180)$$

$$\frac{120}{180} = k$$

$$\frac{2}{3} = k$$

Thus the equation will be $F = \frac{2}{3}W$.

9. **a)** The cost of producing the cards will be: $C(x) = 0.08x + 8000$.

b) The revenue from selling the cards will be $R(x) = 0.50x$.

c) The profit from selling x cards will be:

$$P(x) = R(x) - C(x)$$
$$= (0.50x) - (0.08x + 8000)$$
$$= 0.42x - 8000$$

d) At breakeven, the profit is 0, therefore

$$0 = 0.42x - 8000$$
$$8000 = 0.42x$$
$$19{,}047.62 = x$$

Thus, approximately 19,048 cards must be produced and sold to break even.

10. The equilibrium point is where supply and demand are the same.

$$(q-8)^2 = q^2 + q + 13$$

$$q^2 - 16q + 64 = q^2 + q + 13$$

$$51 = 17q$$

$$3 = q$$

Now substitute the value of q into either function to find the equilibrium price.

$$p = (3-8)^2$$
$$= (-5)^2$$
$$= 25$$

Thus the equilibrium point is (3, 25).

11. Yes, the graph is a function. Based on the vertical line test, any vertical line drawn will only cross the graph in exactly one point.

12. No, the graph is not a function. Based on the vertical line test, a vertical line drawn at $x = 0$ will cross the graph in more than one point.

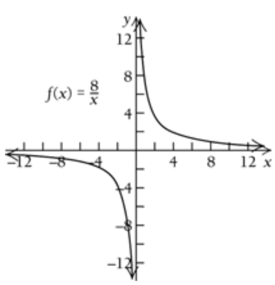
13. **a)** Based on the graph, the function value when $x = 1$ is –4.

b) The domain of the function is all real numbers or $(-\infty, \infty)$ in interval notation.

c) The function has a value of 4 when $x = -3$ and when $x = 3$.

d) The range of the function is all real numbers greater than or equal to –5 or $[-5, \infty)$ in interval notation.

14.



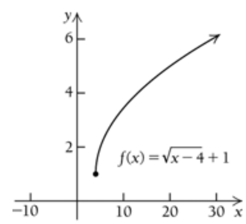
15. $\dfrac{1}{\sqrt{t}} = \dfrac{1}{t^{1/2}} = t^{-1/2}$

16. $t^{-3/5} = \dfrac{1}{t^{3/5}} = \dfrac{1}{\sqrt[5]{t^3}}$

17. Using the logarithm to exponential theorem:

$$\log_9 \frac{1}{3} = x$$
$$9^x = \frac{1}{3}$$
$$3^{2x} = 3^{-1}$$
$$2x = -1$$
$$x = -\frac{1}{2}$$

18.



19. The graph in interval notation is $[c, d)$.

20. The function is defined where the denominator is not equal to 0 and when the radical expression is greater than 0 or

$$3x + 6 \ge 0$$
$$3x \ge -6$$
$$x \ge -2$$

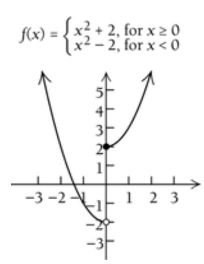
Therefore, the domain of the function, in interval notation is $(-2, \infty)$.

21. The function is not defined when the denominator is equal to 0 or

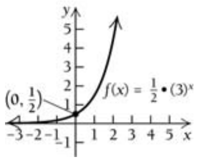
$$x^2 + 5x - 14 = 0$$
$$(x+7)(x-2) = 0$$
$$x = -7, 2$$

Therefore, the domain of the function, in interval notation is $(-\infty, -7) \cup (-7, 2) \cup (2, \infty)$.

22.



23.



The y –intercept, based on the graph, is $\left(0, \frac{1}{2}\right)$.

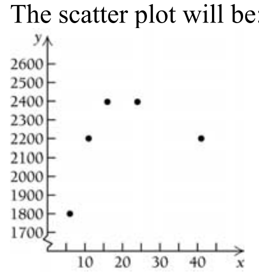
24. **a)** The annual percent increase is 0.041 or 4.1%.

b) $75,000 = 50,000(1.041)^t$

$$1.5 = 1.041^t$$
$$\ln 1.5 = t \ln 1.041$$
$$\frac{\ln 1.5}{\ln 1.041} = t$$
$$10.09 \approx t$$

It will take approximately 10.09 years for the investment to reach $75,000.

25. **a)** The scatter plot will be:



b) Yes, based on the scatter plot from part a.

c) Using the quadreg function on the calculator, the function that fits the data is $y = -1.94x^2 + 102.74x + 1253.49$.

d) $y = -1.94(30)^2 + 102.74(30) + 1253.49$

$$= -1.94(900) + 102.74(30) + 1253.49$$
$$= 2589.69$$

A 30-year-old physically active woman will need approximately 2589.69 calories.

e) Answer will vary.

26. Use the power rule and then simplify the result.

$$\left(64^{4/3}\right)^{-1/2} = 64^{-2/3} = \frac{1}{64^{2/3}} = \frac{1}{\sqrt[3]{64^2}} = \frac{1}{16}$$

27. The domain of the function is where

$$5 - 3x \geq 0$$
$$5 \geq 3x$$
$$\frac{5}{3} \geq x$$

The zero of the function is

$$0 = (5 - 3x)^{0.25} - 7$$
$$7 = (5 - 3x)^{0.25}$$
$$7^4 = 5 - 3x$$
$$2401 - 5 = -3x$$
$$-\frac{2396}{3} = x$$
$$-798\frac{2}{3} = x$$

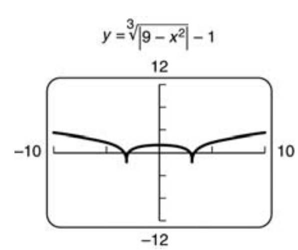
Therefore, the domain is $\left(-\infty, \frac{5}{3}\right]$ and the zero is $\left(-798\frac{2}{3}, 0\right)$.

28. Answers vary, but one possible solution is: $y = (x+3)(x-1)(x-4)$.

29. The average rate of change is similar to finding the slope. Therefore, we have a slope of $-\frac{3}{7}$ and points of (1, 9) and (5, y).

$$\frac{-3}{7} = \frac{y-9}{5-1}$$
$$7(y-9) = -3(4)$$
$$7y - 63 = -12$$
$$7y = 51$$
$$y = \frac{51}{7}$$

30.



Based on the graph above;

Zeros: $\pm\sqrt{8}, \pm\sqrt{10}$

Domain: $(-\infty, \infty)$

Range: $[-1, \infty)$

31. **a)** Using the data from exercise 5 and the QUADREG function on the calculator, the equation of the functions will be
$y = -1.51x^2 + 79.98x + 1436.93$.

b) $y = -1.51(30)^2 + 79.98(30) + 1436.93$
$= -1.51(900) + 79.98(30) + 1436.93$
$= 2477.33$
A 30–year old physically active woman will need approximately 2477.33 calories.

c) Answers will vary.

Chapter 1

Differentiation

Exercise Set 1.1

1. The limit of the sequence is 0.3 and using the notation will give the expression $x \to 0.3^{-}$.

2. The limit of the sequence is 1.7 and using the notation will give the expression $x \to 1.7^{+}$.

3. The limit of the sequence is -3 and using the notation will give the expression $x \to -3^{+}$.

4. The limit of the sequence is -4.9 and using the notation will give the expression $x \to -4.9^{+}$.

5. The limit of the sequence is $\frac{2}{3}$ and using the notation will give the expression $x \to \frac{2}{3}^{-}$.

6. The limit of the sequence is $\frac{4}{3}$ and using the notation will give the expression $x \to \frac{4}{3}^{-}$.

7. The limit of the sequence is 0.3 and using the notation will give the expression $x \to 0.3^{-}$.

8. The limit of the sequence is 1.12 and using the notation will give the expression $x \to 1.12^{-}$.

9. The limit of the sequence is 1 and using the notation will give the expression $x \to 1^{+}$.

10. The limit of the sequence is 0 and using the notation will give the expression $x \to 0^{-}$.

11. We solve the equation:

$$-3x = 6$$
$$x = -2$$

Therefore, As x approaches -2, the value of $-3x$ approaches 6.

12. As x approaches 7, the value of $x-2$ approaches 5.

13. The notation $\lim\limits_{x \to 2^{+}}$ is read "the limit, as x approaches 2 from the right."

14. The notation $\lim\limits_{x \to 3^{-}}$ is read "the limit, as x approaches 3 from the left".

15. The notation $\lim\limits_{x \to 5}$ is read "the limit, as x approaches 5".

16. The notation $\lim\limits_{x \to \frac{1}{2}}$ is read "the limit, as x approaches $\frac{1}{2}$ ".

17. The notation $\lim\limits_{x \to 4} f(x)$ is read "the limit, as x approaches 4, of $f(x)$."

18. The notation $\lim\limits_{x \to 1} g(x)$ is read "the limit, as x approaches 1, of $g(x)$."

19. The notation $\lim\limits_{x \to 5^{-}} F(x)$ is read "the limit, as x approaches 5 from the left, of $F(x)$."

20. The notation $\lim\limits_{x \to 4^{+}} G(x)$ is read "the limit, as x approaches 4 from the right, of $G(x)$."

21. a) As inputs x approach -1 from the left, outputs $f(x)$ approach -3. Thus the limit from the left is -3. That is, $\lim\limits_{x \to -1^{-}} f(x) = -3.$

b) As inputs x approach -1 from the right, outputs $f(x)$ approach -3. That is, $\lim\limits_{x \to -1^{+}} f(x) = -3.$

c) From parts (a) and (b) we find $\lim\limits_{x \to -1} f(x) = -3$.

22. a) As inputs x approach 3 from the left, outputs $f(x)$ approach 1. That is, $\lim_{x\to 3^-} f(x)=1$.

b) As inputs x approach 3 from the right, **o**utputs $f(x)$ approach 2. Thus the limit from the right is 2. That is, $\lim_{x\to 3^+} f(x)=2$.

c) From parts (a) and (b) we k now that $\lim_{x\to 3^-} f(x)=1$ and $\lim_{x\to 3^+} f(x)=2$. Since the limit from the left, 1, is not the same as the limit from the right, 2, $\lim_{x\to 3} f(x)$ *does not exist.*

23. a) As inputs x approach 4 from the right, outputs $g(x)$ approach -1. Thus the limit from the right is -1. That is, $\lim_{x\to 4^+} g(x)=-1$.

b) $\lim_{x\to 4^-} g(x)=-1$.

c) Since the limit from the left, -1, is the same as the limit from the right, -1, we have $\lim_{x\to 4} g(x)=-1$.

24. a) As inputs x approach -2 from the left, outputs $g(x)$ approach 4. Thus the limit from the left is 4. That is, $\lim_{x\to -2^-} g(x)=4$.

b) $\lim_{x\to -2^+} g(x)=2$.

c) $\lim_{x\to -2} g(x)$ *does not exist.*

25. As inputs x approach -3 from the left, outputs $F(x)$ approach 5. Thus the limit from the left is 5. That is, $\lim_{x\to -3^-} F(x)=5$.
As inputs x approach -3 from the right, outputs $F(x)$ approach 5. Thus the limit from the right is 5. That is, $\lim_{x\to -3^+} F(x)=5$.
Since the limit from the left, 5, is the same as the limit from the right, 5, we have $\lim_{x\to -3} F(x)=5$.

26. We have $\lim_{x\to 2^-} F(x)=4$ and $\lim_{x\to 2^+} F(x)=4$. Therefore, $\lim_{x\to 2} F(x)=4$.

27. As inputs x approach -2 from the left, outputs $F(x)$ approach 4. Thus the limit from the left is 4. That is, $\lim_{x\to -2^-} F(x)=4$.
As inputs x approach -2 from the right, outputs $F(x)$ approach 2. Thus the limit from the right is 2. That is, $\lim_{x\to -2^+} F(x)=2$.
Since the limit from the left, 4, is not the same as the limit from the right, 2, we have $\lim_{x\to -2} F(x)$ *does not exist.*

28. We have $\lim_{x\to -5^-} F(x)=0$ and $\lim_{x\to -5^+} F(x)=0$. Therefore, $\lim_{x\to -5} F(x)=0$.

29. As inputs x approach 4 from the left, outputs $F(x)$ approach 2. Thus the limit from the left is 2. That is, $\lim_{x\to 4^-} F(x)=2$.
As inputs x approach 4 from the right, outputs $F(x)$ approach 2. Thus the limit from the right is 2. That is, $\lim_{x\to 4^+} F(x)=2$.
Since the limit from the left, 2, is the same as the limit from the right, 2, we have $\lim_{x\to 4} F(x)=2$.

30. We have $\lim_{x\to 6^-} F(x)=0$ and $\lim_{x\to 6^+} F(x)=0$. Therefore, $\lim_{x\to 6} F(x)=0$.

31. As inputs x approach -2 from the right, outputs $F(x)$ approach 2. Thus the limit from the right is 2. That is, $\lim_{x\to -2^+} F(x)=2$.

32. As inputs x approach -2 from the left, outputs $F(x)$ approach 4. Thus the limit from the left is 4. That is, $\lim_{x\to -2^-} F(x)=4$.

33. As inputs x approach -2 from the left, outputs $G(x)$ approach 1. Thus the limit from the left is 1. That is, $\lim_{x\to -2^-} G(x)=1$.
As inputs x approach -2 from the right, outputs $G(x)$ approach 1. Thus the limit from the right is 1. That is, $\lim_{x\to -2^+} G(x)=1$.
Since the limit from the left, 1, is the same as the limit from the right, 1, we have $\lim_{x\to -2} G(x)=1$.

34. We have $\lim_{x\to 0^-} G(x)=3$ and $\lim_{x\to 0^+} G(x)=3$.
Therefore, $\lim_{x\to 0} G(x)=3$.

35. As inputs x approach 1 from the left, outputs $G(x)$ approach 4. Thus the limit from the left is 4. That is, $\lim_{x\to 1^-} G(x)=4$.

36. $\lim_{x\to 1^+} G(x)=-1$

37. $\lim_{x\to 3^-} G(x)=0$

38. As inputs x approach 1 from the left, outputs $G(x)$ approach 4. Thus the limit from the left is 4. That is, $\lim_{x\to 1^-} G(x)=4$.
As inputs x approach 1 from the right, outputs $G(x)$ approach -1. Thus the limit from the right is -1. That is, $\lim_{x\to 1^+} G(x)=-1$.
Since the limit from the left, 4, is not the same as the limit from the right, -1, we have $\lim_{x\to 1} G(x)$ *does not exist.*

39. As inputs x approach 3 from the right, outputs $G(x)$ approach 0. Thus the limit from the right is 0. That is, $\lim_{x\to 3^+} G(x)=0$.

40. We have $\lim_{x\to 3^-} G(x)=0$ and $\lim_{x\to 3^+} G(x)=0$.
Therefore, $\lim_{x\to 3} G(x)=0$.

41. As inputs x approach -3 from the left, outputs $H(x)$ approach 0. Thus the limit from the left is 0. That is, $\lim_{x\to -3^-} H(x)=0$.
As inputs x approach -3 from the right, outputs $H(x)$ approach 0. Thus the limit from the right is 0. That is, $\lim_{x\to -3^+} H(x)=0$.
Since the limit from the left, 0, is the same as the limit from the right, 0, we have $\lim_{x\to -3} H(x)=0$.

42. $\lim_{x\to -2^-} H(x)=1$.

43. As inputs x approach -2 from the right, outputs $H(x)$ approach 1. Thus the limit from the right is 1. That is, $\lim_{x\to -2^+} H(x)=1$.

44. We have $\lim_{x\to -2^-} H(x)=1$ and $\lim_{x\to -2^+} H(x)=1$.
Therefore, $\lim_{x\to -2} H(x)=1$.

45. As inputs x approach 1 from the left, outputs $H(x)$ approach 4. Thus the limit from the left is 4. That is, $\lim_{x\to 1^-} H(x)=4$.

46. $\lim_{x\to -1^+} H(x)=2$.

47. $\lim_{x\to 3^-} H(x)=1$.

48. As inputs x approach 1 from the left, outputs $H(x)$ approach 4. Thus the limit from the left is 4. That is, $\lim_{x\to 1^-} H(x)=4$.
The solution is continued on the next page.
As inputs x approach 1 from the right, outputs $H(x)$ approach 2. Thus the limit from the right is 2. That is, $\lim_{x\to 1^+} H(x)=2$.
Since the limit from the left, 4, is not the same as the limit from the right, 2, we have $\lim_{x\to 1} H(x)$ *does not exist.*

49. As inputs x approach 3 from the right, outputs $H(x)$ approach 1. Thus the limit from the right is 1. That is,
$\lim_{x\to 3^+} H(x) = 1$.

50. We have $\lim_{x\to 3^-} H(x) = 1$ and $\lim_{x\to 3^+} H(x) = 1$.
Therefore, $\lim_{x\to 3} H(x) = 1$.

51. We have $\lim_{x\to 2^-} f(x) = -1$ and $\lim_{x\to 2^+} f(x) = -1$.
Therefore, $\lim_{x\to 2} f(x) = -1$.

52. As inputs x approach -1 from the left, outputs $f(x)$ approach 1. Thus the limit from the left is 1. That is, $\lim_{x\to -1^-} f(x) = 1$.
As inputs x approach -1 from the right, outputs $f(x)$ approach 1. Thus the limit from the right is 1. That is, $\lim_{x\to -1^+} f(x) = 1$.
Since the limit from the left, 1, is the same as the limit from the right, 1, we have
$\lim_{x\to -1} f(x) = 1$.

53. We have $\lim_{x\to 0^-} f(x) = 2$ and $\lim_{x\to 0^+} f(x) = 2$.
Therefore, $\lim_{x\to 0} f(x) = 2$.

54. As inputs x approach -3 from the left, outputs $f(x)$ increase without bound. We say that the limit from the left is infinity. That is,
$\lim_{x\to -3^-} f(x) = \infty$.
As inputs x approach -3 from the right, outputs $f(x)$ decrease without bound. We say that limit from the right is negative infinity. That is,
$\lim_{x\to -3^+} f(x) = -\infty$.
Since the function values as $x \to 3$ from the left increase without bound, and the function values as $x \to 3$ from the right decrease without bound, the limit does not exist. We have,
$\lim_{x\to -3} f(x)$ *does not exist.*

55. We have $\lim_{x\to 1^-} f(x) = \infty$ and $\lim_{x\to 1^+} f(x) = -\infty$.
Therefore, $\lim_{x\to 1} f(x)$ *does not exist*.

56. As inputs x approach 3 from the left, outputs $f(x)$ approach 0. Thus the limit from the left is 0. That is,
$\lim_{x\to 3^-} f(x) = 0$.
As inputs x approach 3 from the right, outputs $f(x)$ approach 0.
Thus the limit from the right is 0. That is,
$\lim_{x\to 3^+} f(x) = 0$.
Since the limit from the left, 0, is the same as the limit from the right, 0, we have
$\lim_{x\to 3} f(x) = 0$.

57. We have $\lim_{x\to -2^-} f(x) = 0$ and $\lim_{x\to -2^+} f(x) = 0$.
Therefore, $\lim_{x\to -2} f(x) = 0$.

58. As inputs x approach -4 from the left, outputs $f(x)$ approach 3. Thus the limit from the left is 3. That is,
$\lim_{x\to -4^-} f(x) = 3$.
As inputs x approach -4 from the right, outputs $f(x)$ approach 3. Thus the limit from the right is 3. That is,
$\lim_{x\to -4^+} f(x) = 3$.
Since the limit from the left, 3, is the same as the limit from the right, 3, we have
$\lim_{x\to -4} f(x) = 3$.

59. As inputs x get more and more negative, output $f(x)$ get closer and closer to 2. $\lim_{x\to -\infty} f(x) = 2$.

60. As inputs x get larger and larger, outputs $f(x)$ get closer and closer to 1. We have
$\lim_{x\to \infty} f(x) = 1$.

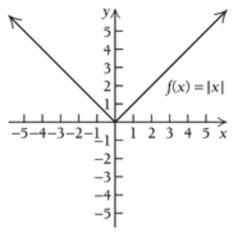
61. Defining $f(x)=|x|$ as a piecewise defined function we have:

$$f(x)=\begin{cases}-x, & x<0\\ x, & x\geq 0\end{cases}.$$

We graph the function by creating an input-output table.

x	-2	-1	0	1	2
$f(x)$	2	1	0	1	2

Next, we plot the points from the table and draw the graph.



Find $\lim_{x\to 0} f(x)$.

As inputs x approach 0 from the left, outputs $f(x)$ approach 0. We have, $\lim_{x\to 0^-} f(x)=0$.

As inputs x approach 0 from the right, outputs $f(x)$ approach 0. We have, $\lim_{x\to 0^+} f(x)=0$

Since the limit from the left is the same as the limit from the right, we have $\lim_{x\to 0} f(x)=0$.

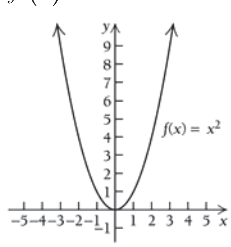
Find $\lim_{x\to -2} f(x)$.

As inputs x approach -2 from the left, outputs $f(x)$ approach 2. We have, $\lim_{x\to -2^-} f(x)=2$.

As inputs x approach -2 from the right, outputs $f(x)$ approach 2. We have, $\lim_{x\to -2^+} f(x)=2$

Since the limit from the left is the same as the limit from the right, we have $\lim_{x\to -2} f(x)=2$.

62. $f(x)=x^2$



Find $\lim_{x\to -1} f(x)$.

We have $\lim_{x\to -1^-} f(x)=1$ and $\lim_{x\to -1^+} f(x)=1$.

Therefore, $\lim_{x\to -1} f(x)=1$.

Find $\lim_{x\to 0} f(x)$.

We have $\lim_{x\to 0^-} f(x)=0$ and $\lim_{x\to 0^+} f(x)=0$.

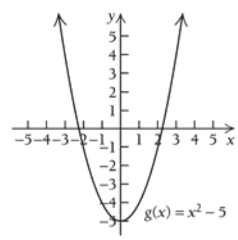
Therefore, $\lim_{x\to 0} f(x)=0$.

63. $g(x)=x^2-5$

We graph the function by creating an input-output table.

x	-2	-1	0	1	2
$g(x)$	-1	-4	-5	-4	-1

Next, we plot the points from the table and draw the graph.



Find $\lim_{x\to 0} g(x)$.

As inputs x approach 0 from the left, outputs $g(x)$ approach -5. We have, $\lim_{x\to 0^-} g(x)=-5$.

As inputs x approach 0 from the right, outputs $g(x)$ approach -5. We have, $\lim_{x\to 0^+} g(x)=-5$

Since the limit from the left is the same as the limit from the right, we have $\lim_{x\to 0} g(x)=-5$.

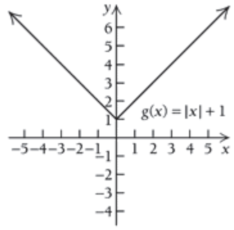
Find $\lim_{x\to -1} g(x)$.

As inputs x approach -1 from the left, outputs $g(x)$ approach -4. We have, $\lim_{x\to -1^-} g(x)=-4$.

As inputs x approach -1 from the right, outputs $g(x)$ approach -4. We have, $\lim_{x\to -1^+} g(x)=-4$

Since the limit from the left is the same as the limit from the right, we have $\lim_{x\to -1} g(x)=-4$.

64. $g(x)=|x|+1$



Find $\lim\limits_{x\to -3} g(x)$.

We have $\lim\limits_{x\to -3^-} g(x)=4$ and $\lim\limits_{x\to -3^+} g(x)=4$.

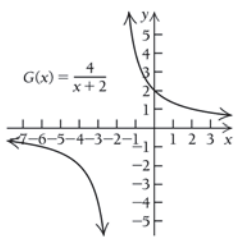
Therefore, $\lim\limits_{x\to -3} g(x)=4$.

Find $\lim\limits_{x\to 0} g(x)$.

We have $\lim\limits_{x\to 0^-} g(x)=1$ and $\lim\limits_{x\to 0^+} g(x)=1$.

Therefore, $\lim\limits_{x\to 0} g(x)=1$.

65. $G(x)=\dfrac{4}{x+2}$



Find $\lim\limits_{x\to -1} G(x)$.

We have $\lim\limits_{x\to -1^-} G(x)=4$ and $\lim\limits_{x\to -1^+} G(x)=4$.

Therefore, $\lim\limits_{x\to -1} G(x)=4$.

Find $\lim\limits_{x\to -2} G(x)$

We have $\lim\limits_{x\to -2^-} G(x)=-\infty$ and $\lim\limits_{x\to -2^+} G(x)=\infty$

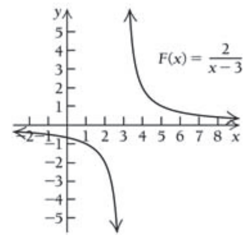
. Therefore, $\lim\limits_{x\to -2} G(x)$ *does not exist.*

66. $F(x)=\dfrac{2}{x-3}$

Since $x=3$ makes the denominator zero, we exclude the value 3 from the domain. Creating an input-output table we have

x	1	2	2.5	2.9	3.1	3.5	4	5
$F(x)$	−1	−2	−4	−20	20	4	2	1

Next we plot the points and draw the graph.



Find $\lim\limits_{x\to 3} F(x)$.

As inputs x approach 3 from the left, outputs $F(x)$ decrease without bound . We have,

$\lim\limits_{x\to 3^-} F(x)=-\infty$.

As inputs x approach 3 from the right, outputs $F(x)$ increase without bound. We have,

$\lim\limits_{x\to 3^+} F(x)=\infty$

Since the function values as $x\to 3$ from the left decrease without bound, and the function values as $x\to 3$ from the right increase without bound, the limit does not exist. We have, $\lim\limits_{x\to 3} F(x)$ *does not exist.*

Find $\lim\limits_{x\to 4} F(x)$.

As inputs x approach 4 from the left, outputs $F(x)$ approach 1. We have, $\lim\limits_{x\to 4^-} F(x)=2$.

As inputs x approach 4 from the right, outputs $F(x)$ approach 1. We have, $\lim\limits_{x\to 4^+} F(x)=2$

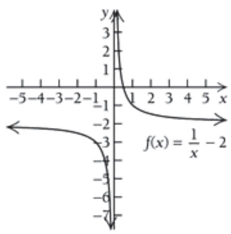
Since the limit from the left is the same as the limit from the right, we have $\lim\limits_{x\to 4} F(x)=2$.

67. $f(x)=\dfrac{1-2x}{x}$

Since $x=0$ makes the denominator zero, we exclude the value 0 from the domain. Creating an input-output table we have

x	-1	-0.5	-0.1	0.1	0.5	1
$f(x)$	-3	-4	-12	8	0	-1

Next we plot the points and draw the graph.



Find $\lim\limits_{x\to\infty} f(x)$.

As inputs x get larger and larger, outputs $f(x)$ get closer and closer to -2. We have $\lim\limits_{x\to\infty} f(x)=-2$.

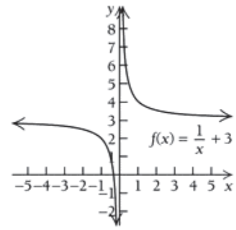
Find $\lim\limits_{x\to 0} f(x)$.

As inputs x approach 0 from the left, outputs $f(x)$ decrease without bound. We have, $\lim\limits_{x\to 0^-} f(x)=-\infty$.

As inputs x approach 0 from the right, outputs $f(x)$ increase without bound. We have, $\lim\limits_{x\to 0^+} f(x)=\infty$

Since the function values as $x\to 0$ from the left decrease without bound, and the function values as $x\to 0$ from the right increase without bound, the limit does not exist. We have, $\lim\limits_{x\to 0} f(x)$ *does not exist.*

68. $f(x)=\dfrac{1+3x}{x}$



Find $\lim\limits_{x\to\infty} f(x)$.

As inputs x get larger and larger, outputs $f(x)$ get closer and closer to 3. We have $\lim\limits_{x\to\infty} f(x)=3$.

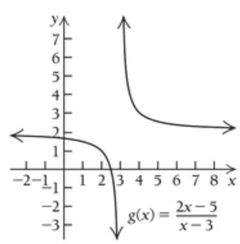
Find $\lim\limits_{x\to 0} f(x)$.

As inputs x approach 0 from the left, outputs $f(x)$ decrease without bound. We have, $\lim\limits_{x\to 0^-} f(x)=-\infty$.

As inputs x approach 0 from the right, outputs $f(x)$ increase without bound. We have, $\lim\limits_{x\to 0^+} f(x)=\infty$

Since the function values as $x\to 0$ from the left decrease without bound, and the function values as $x\to 0$ from the right increase without bound, the limit does not exist. We have, $\lim\limits_{x\to 0} f(x)$ *does not exist.*

69. $g(x)=\dfrac{2x-5}{x-3}$



Find $\lim\limits_{x\to\infty} g(x)$.

As inputs x get larger and larger, outputs $g(x)$ get closer and closer to 2. We have $\lim\limits_{x\to\infty} g(x)=2$.

Find $\lim_{x\to 3} g(x)$.

As inputs *x* approach 3 from the left, outputs $g(x)$ decrease without bound. We have,

$\lim_{x\to 3^-} g(x) = -\infty$.

As inputs *x* approach 3 from the right, outputs $g(x)$ increase without bound. We have,

$\lim_{x\to 3^+} g(x) = \infty$.

Since the function values as $x \to 3$ from the left decrease without bound, and the function values as $x \to 3$ from the right increase without bound, the limit does not exist. We have,

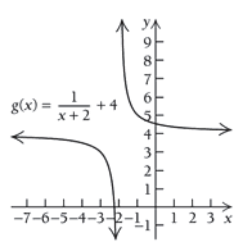
$\lim_{x\to 3} g(x)$ *does not exist.*

70. $g(x) = \dfrac{4x+9}{x+2}$

Since $x = -2$ makes the denominator zero, we exclude the value -2 from the domain. Creating an input-output table we have

x	-3	-2.5	-2.1	-1.9	-1.5	-1	0
$g(x)$	3	2	-6	14	6	5	$\frac{9}{2}$

Next we plot the points and draw the graph.



Find $\lim_{x\to\infty} g(x)$.

As inputs *x* get larger and larger, outputs $g(x)$ get closer and closer to 4. We have

$\lim_{x\to\infty} g(x) = 4$.

Find $\lim_{x\to -2} g(x)$.

As inputs *x* approach -2 from the left, outputs $g(x)$ decrease without bound. We have,

$\lim_{x\to -2^-} g(x) = -\infty$.

As inputs *x* approach -2 from the right, outputs $g(x)$ increase without bound. We have,

$\lim_{x\to -2^+} g(x) = \infty$.

Since the function values as $x \to 2$ from the left decrease without bound, and the function values as $x \to 2$ from the right increase without bound, the limit does not exist. We have,

$\lim_{x\to -2} g(x)$ *does not exist.*

71. $F(x) = \begin{cases} 2x+1, & \text{for } x < 1 \\ x, & \text{for } x \geq 1. \end{cases}$

We create an input-output table for each piece of the function.

For $x < 1$

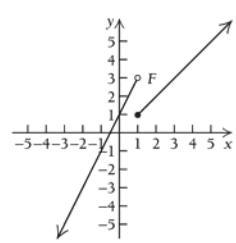
x	-1	0	0.9
$F(x)$	-1	1	2.8

We plot the points and draw the graph. Notice we draw an open circle at the point $(1,3)$ to indicate that the point is not part of the graph.

For $x \geq 1$

x	1	2	3
$F(x)$	1	2	3

We plot the points and draw the graph. Notice we draw a solid circle at the point $(1,1)$ to indicate that the point is part of the graph.



Find $\lim_{x\to 1^-} F(x)$.

As inputs *x* approach 1 from the left, outputs $F(x)$ approach 3. That is,

$\lim_{x\to 1^-} F(x) = 3$.

Find $\lim_{x\to 1^+} F(x)$.

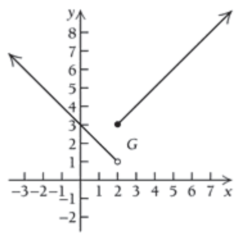
As inputs *x* approach 1 from the right, outputs $F(x)$ approach 1. That is,

$\lim_{x\to 1^+} F(x) = 1$.

Find $\lim_{x\to1} F(x)$

Since the limit from the left, 3, is not the same as the limit from the right, 1, we have $\lim_{x\to1} F(x)$ *does not exist*.

72. $G(x)=\begin{cases}-x+3, & \text{for } x<2\\ x+1, & \text{for } x\ge 2.\end{cases}$



We have $\lim_{x\to2^-} G(x)=1$ and $\lim_{x\to2^+} G(x)=3$.

Therefore, $\lim_{x\to2} G(x)$ *does not exist*.

73. $g(x)=\begin{cases}-x+4, & \text{for } x<3\\ x-3, & \text{for } x>3.\end{cases}$

We create an input-output table for each piece of the function.

For $x<3$

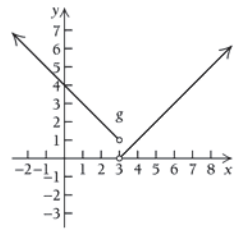
x	0	1	2	2.9
$g(x)$	4	3	2	1.1

We plot the points from the table draw the graph. Notice we draw an open circle at the point $(3,1)$ to indicate that the point is not part of the graph.

For $x>3$

x	3.1	4	5	6
$g(x)$	0.1	1	2	3

We plot the points and draw the graph. Notice we draw an open circle at the point $(3,0)$ to indicate that the point is not part of the graph.



Find $\lim_{x\to3^-} g(x)$.

As inputs x approach 3 from the left, outputs $g(x)$ approach 1. That is,

$\lim_{x\to3^-} g(x)=1$.

Find $\lim_{x\to3^+} g(x)$.

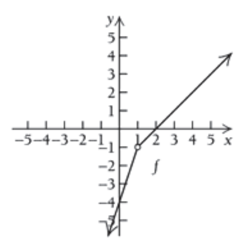
As inputs x approach 3 from the right, outputs $g(x)$ approach 0. That is,

$\lim_{x\to3^+} g(x)=0$.

Find $\lim_{x\to3} g(x)$

Since the limit from the left, 1, is not the same as the limit from the right, 0, we have $\lim_{x\to3} g(x)$ *does not exist*.

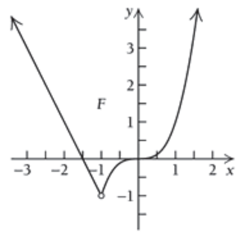
74. $f(x)=\begin{cases}3x-4, & \text{for } x<1\\ x-2, & \text{for } x>1.\end{cases}$



We have $\lim_{x\to1^-} f(x)=-1$ and $\lim_{x\to1^+} f(x)=-1$.

Therefore, $\lim_{x\to1} f(x)=-1$.

75. $F(x)=\begin{cases}-2x-3, & \text{for } x<-1\\ x^3, & \text{for } x>-1.\end{cases}$



We have $\lim_{x\to-1^-} F(x)=-1$ and $\lim_{x\to-1^+} F(x)=-1$.

Therefore, $\lim_{x\to-1} F(x)=-1$.

76. $G(x)=\begin{cases}x^2, & \text{for } x<-1\\ x+2, & \text{for } x>-1.\end{cases}$

We create an input-output table for each piece of the function.

For $x<-1$

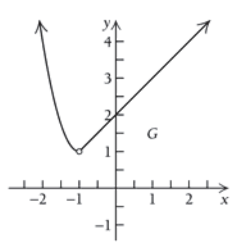
x	−3	−2	−1.1
$G(x)$	9	4	−1.21

We plot the points and draw the graph. Notice we draw an open circle at the point $(-1,1)$ to indicate that the point is not part of the graph.

For $x>-1$

x	−0.9	0	1
$G(x)$	1.1	2	3

We plot the points and draw the graph. Notice we draw an open circle at the point $(-1,1)$ to indicate that the point is not part of the graph.



Find $\lim_{x\to-1} G(x)$.

As inputs x approach -1 from the left, outputs $G(x)$ approach 1. We have,

$\lim_{x\to-1^-} G(x)=1$.

As inputs x approach -1 from the right, outputs $G(x)$ approach 1. We have,

$\lim_{x\to-1^+} G(x)=1$

Since the limit from the left is the same as the limit from the right, we have

$\lim_{x\to-1} G(x)=1$.

77. $H(x)=\begin{cases}x+1, & \text{for } x<0\\ 2, & \text{for } 0\le x<1\\ 3-x, & \text{for } x\ge 1.\end{cases}$

We create an input-output table for each piece of the function.

For $x<0$

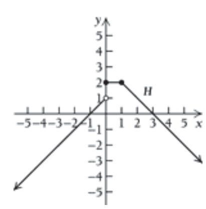
x	−1	−0.5	−0.1
$H(x)$	0	0.5	0.9

We plot the points and draw the graph. Notice we draw an open circle at the point $(0,1)$ to indicate that the point is not part of the graph. For $0\le x<1$, the function has value of 2. We draw a solid circle at the point $(0,2)$ to indicate the point is part of the graph and we draw an open circle at $(1,2)$ to indicate that the point is not part of the graph.

For $x\ge 1$

x	1	2	3
$H(x)$	2	1	−1

We plot the points and draw the graph. Notice we draw a solid circle at the point $(1,2)$ to indicate that the point is part of the graph.



Find $\lim_{x\to 0} H(x)$

As inputs x approach 0 from the left, outputs $H(x)$ approach 1. That is,

$\lim_{x\to 0^-} H(x)=1$.

As inputs x approach 0 from the right, outputs $H(x)$ approach 2. That is,

$\lim_{x\to 0^+} H(x)=2$.

Since the limit from the left, 1, is not the same as the limit from the right, 2, we have

$\lim_{x\to 0} H(x)$ *does not exist.*

Find $\lim_{x\to 1} H(x)$

As inputs x approach 1 from the left, outputs $H(x)$ approach 2. That is,

$\lim_{x\to 1^-} H(x)=2$.

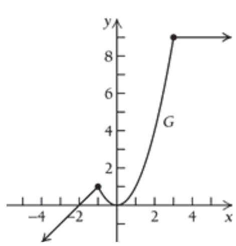
As inputs x approach 1 from the right, outputs $H(x)$ approach 2. That is,

$\lim_{x\to 1^+} H(x)=2$.

Since the limit from the left, 2, is the same as the limit from the right, 2, we have

$\lim_{x\to 0} H(x)=2$.

78. $G(x)=\begin{cases} 2+x, & \text{for } x\le -1 \\ x^2, & \text{for } -1<x<3 \\ 9, & \text{for } x\ge 3. \end{cases}$



We have $\lim_{x\to -1^-} G(x)=1$ and $\lim_{x\to -1^+} G(x)=1$.

Therefore, $\lim_{x\to -1} G(x)=1$.

We have $\lim_{x\to 3^-} G(x)=9$ and $\lim_{x\to 3^+} G(x)=9$.

Therefore, $\lim_{x\to 3} G(x)=9$.

79. $\lim_{x\to 0.25^-} C(x)=\$3.50$

$\lim_{x\to 0.25^+} C(x)=\$3.50$

$\lim_{x\to 0.25} C(x)=\$3.50$

80. $\lim_{x\to 0.2^-} C(x)=\$3.00$

$\lim_{x\to 0.2^+} C(x)=\$3.50$

$\lim_{x\to 0.2} C(x)$ *does not exist.*

81. $\lim_{x\to 0.6^-} C(x)=\$4.00$.

$\lim_{x\to 0.6^+} C(x)=\$4.50$.

$\lim_{x\to 0.6} C(x)$ *does not exist.*

82. $\lim_{x\to 1^-} p(x)=\$1.00$.

$\lim_{x\to 1^+} p(x)=\$1.21$.

$\lim_{x\to 1} p(x)$ *does not exist.*

83. $\lim_{x\to 2^-} p(x)=\$1.21$.

$\lim_{x\to 2^+} p(x)=\$1.42$.

$\lim_{x\to 2} p(x)$ *does not exist.*

84. $\lim_{x\to 2.6^-} p(x)=\$1.42$.

$\lim_{x\to 2.6^+} p(x)=\$1.42$.

$\lim_{x\to 2.6} p(x)=\$1.42$.

85. $\lim_{x\to 3^-} p(x)=\$1.42$.

$\lim_{x\to 3^+} p(x)=\$1.63$.

$\lim_{x\to 3} p(x)$ *does not exist.*

86. $\lim_{x\to 3.4^-} p(x)=\$1.63$.

$\lim_{x\to 3.4^+} p(x)=\$1.63$.

$\lim_{x\to 3.4} p(x)=\$1.63$.

87. $\lim_{x\to 9525^-} r(x) = 10\%.$

$\lim_{x\to 9525^+} r(x) = 12\%.$

$\lim_{x\to 9525} r(x)$ *does not exist.*

88. $\lim_{x\to 10{,}000^-} r(x) = 12\%.$

$\lim_{x\to 10{,}000^+} r(x) = 12\%.$

$\lim_{x\to 10{,}000} r(x) = 12\%.$

89. $\lim_{x\to 50{,}000^-} r(x) = 22\%.$

$\lim_{x\to 50{,}000^+} r(x) = 22\%.$

$\lim_{x\to 50{,}000} r(x) = 22\%.$

$\lim_{x\to 82{,}500^-} r(x) = 25\%.$

$\lim_{x\to 82{,}500^+} r(x) = 28\%.$

$\lim_{x\to 82{,}500} r(x)$ *does not exist.*

90. $\lim_{x\to 9{,}000^-} r(x) = 10\%.$

$\lim_{x\to 9{,}000^+} r(x) = 10\%.$

$\lim_{x\to 9{,}000} r(x) = 10\%.$

91. $\lim_{x\to 51{,}800^-} r(x) = 12\%.$

$\lim_{x\to 51{,}800^+} r(x) = 22\%.$

$\lim_{x\to 51{,}800} r(x)$ *does not exist.*

92. $\lim_{x\to 60{,}000^-} r(x) = 22\%.$

$\lim_{x\to 60{,}000^+} r(x) = 22\%.$

$\lim_{x\to 60{,}000} r(x) = 22\%.$

$\lim_{x\to 13{,}600^-} r(x) = 10\%.$

$\lim_{x\to 13{,}600^+} r(x) = 22\%.$

$\lim_{x\to 13{,}600} r(x)$ *does not exist.*

93. As inputs x approach 2 from the right, outputs $f(x)$ approach 4. We have, $\lim_{x\to 2^+} f(x) = 4$. In order for $\lim_{x\to 2} f(x)$ to exist we need $\lim_{x\to 2^-} f(x) = 4$. We will use the letter c for the unknown in the equation; therefore,

$\lim_{x\to 2^-} \frac{1}{2}(x) + c = 4$.

Substitute 2 in for x to get the equation:

$\frac{1}{2}(2) + c = 4$ and solving for c we get

$1 + c = 4$

$c = 3.$

Therefore, in order for the limit to exist as x approaches 2, the function must be:

$$f(x) = \begin{cases} \frac{1}{2}x + \underline{3} & \text{for } x < 2 \\ -x + 6 & \text{for } x > 2. \end{cases}$$

94. As inputs x approach 2 from the left, outputs $f(x)$ approach 0. We have, $\lim_{x\to 2^-} f(x) = 0$. In order for $\lim_{x\to 2} f(x)$ to exist we need $\lim_{x\to 2^+} f(x) = 0$. We will use the letter c for the unknown in the equation and this gives us

$\lim_{x\to 2^+} \frac{3}{2}(x) + c = 0$

Substitute 2 in for x to get the equation:

$\frac{3}{2}(2) + c = 0$ and solving for c we get

$3 + c = 0$

$c = -3.$

Therefore, in order for the limit to exist as x approaches 2, the function must be:

$$f(x) = \begin{cases} -\frac{1}{2}x + 1 & \text{for } x < 2 \\ \frac{3}{2}x + \underline{-3} & \text{for } x > 2. \end{cases}$$

95. As inputs x approach 2 from the left, outputs $f(x)$ approach -5. We have, $\lim_{x\to 2^-} f(x) = -5$. In order for $\lim_{x\to 2} f(x)$ to exist we need $\lim_{x\to 2^+} f(x) = -5$. We will use the letter c for the unknown in the equation and this gives us

$\lim_{x\to 2^+} \left(-x^2 + c\right) = -5$

Substitute 2 in for x to get the equation:

$-(2)^2 + c = -5$ and solving for c we get

$-(4) + c = -5$

$c = -1.$

Therefore, in order for the limit to exist as x approaches 2, the function must be:

$$f(x) = \begin{cases} x^2 - 9 & \text{for } x < 2 \\ -x^2 + \underline{-1} & \text{for } x > 2. \end{cases}$$

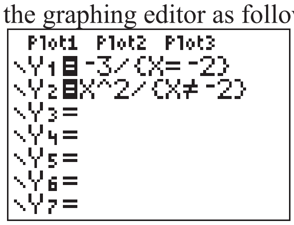
96. a) The limit of the sequence is 0.

b) The limit of the sequence is 2.

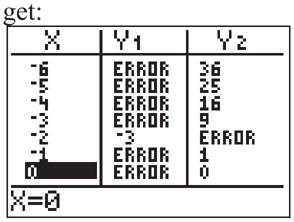
c) Answers will vary.

97. Graph $f(x) = \begin{cases} -3, & \text{for } x = -2 \\ x^2, & \text{for } x \neq -2. \end{cases}$

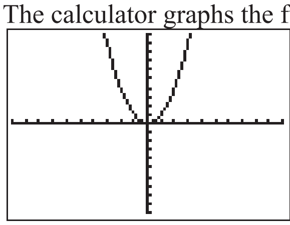
Using the calculator we enter the function into the graphing editor as follows:



Notice, when you select the table feature you get:



The calculator graphs the function:



Using the trace feature, we find the limits.

a) $\lim_{x \to -2^+} f(x) = 4$

b) $\lim_{x \to -2^-} f(x) = 4$

c) $\lim_{x \to -2} f(x) = 4$

d) $\lim_{x \to 2^+} f(x) = 4$

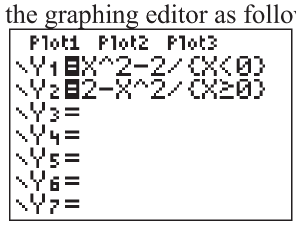
e) $\lim_{x \to 2^-} f(x) = 4$

f) $\lim_{x \to -2} f(x) = 4 \neq -3 = f(-2)$; no

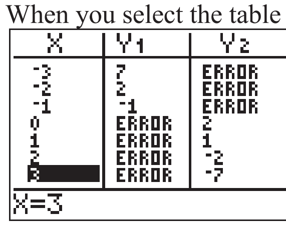
g) $\lim_{x \to 2} f(x) = 4 = f(2)$; yes

98. Graph $f(x) = \begin{cases} x^2 - 2, & \text{for } x < 0 \\ 2 - x^2, & \text{for } x \geq 0. \end{cases}$

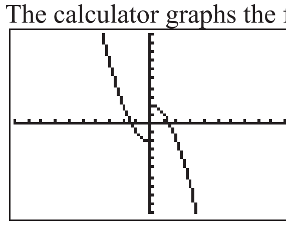
Using the calculator we enter the function into the graphing editor as follows:



When you select the table feature you get:



The calculator graphs the function



Using the trace feature, we find the limits.

Find $\lim_{x \to 0} f(x)$.

As inputs x approach 0 from the left, outputs $f(x)$ approach -2. We have,

$\lim_{x \to 0^-} f(x) = -2$.

As inputs x approach 0 from the right, outputs $f(x)$ approach 2. We have,

$\lim_{x \to 0^+} f(x) = 2$

Since the limit from the left, -2, is not the same as the limit from the right, 2, we have $\lim_{x \to 0} f(x)$ *does not exist.*

Find $\lim_{x \to -2} f(x)$.

As inputs x approach -2 from the left, outputs $f(x)$ approach 2. We have,

$\lim_{x \to -2^-} f(x) = 2$.

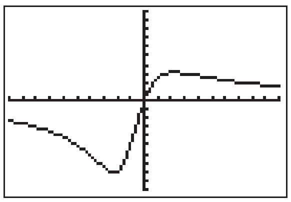
As inputs x approach -2 from the right, outputs $f(x)$ approach 2. We have,

$$\lim_{x\to -2^+} f(x) = 2$$

Since the limit from the left is the same as the limit from the right, we have

$$\lim_{x\to -2} f(x) = 2.$$

99. Graph $g(x) = \dfrac{20x^2}{x^3 + 2x^2 + 5x}$



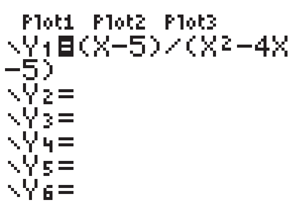
Using the trace feature on the graph, we have:

$$\lim_{x\to\infty} g(x) = 0$$

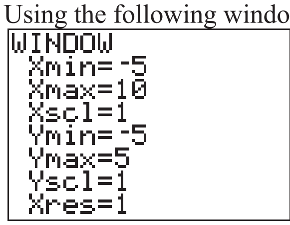
$$\lim_{x\to -\infty} g(x) = 0.$$

100. Graph $f(x) = \dfrac{x-5}{x^2 - 4x - 5}$

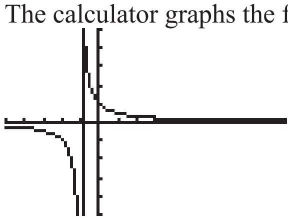
Using the calculator we enter the function into the graphing editor.



Using the following window:



The calculator graphs the function:



Using the trace feature on the calculator we find the limits.

Find $\lim_{x\to -1} f(x)$.

As inputs x approach -1 from the left, outputs $f(x)$ increase without bound. We have,

$$\lim_{x\to -1^-} f(x) = -\infty.$$

As inputs x approach -1 from the right, outputs $f(x)$ decrease without bound. We have,

$$\lim_{x\to -1^+} f(x) = \infty$$

Since the function values as $x \to -1$ from the left increase without bound, and the function values as $x \to -1$ from the right decrease without bound, the limit does not exist.

$\lim_{x\to -1} f(x)$ *does not exist.*

Find $\lim_{x\to 5} f(x)$.

As inputs x approach 5 from the left, outputs $f(x)$ decrease to the value $\frac{1}{6}$. We have,

$$\lim_{x\to 5^-} f(x) = \frac{1}{6}.$$

As inputs x approach 5 from the right, outputs $f(x)$ increase to the value $\frac{1}{6}$. We have,

$$\lim_{x\to 5^+} f(x) = \frac{1}{6}.$$

Since the function values as $x \to 5$ from the left and the function values as $x \to 5$ from the right are the same value, the limit is the value $\frac{1}{6}$. We have, $\lim_{x\to 5} f(x) = \frac{1}{6}$.

Exercise Set 1.2

1. By limit principle L2, the statement is true.

2. By limit principle L1, $\lim_{x\to 3} 7 = 7$
Therefore, the statement is a false.

3. By limit principle L2,
$\lim_{x\to 1}[g(x)]^2 = \left[\lim_{x\to 1} g(x)\right]^2 = [5]^2 = 25.$
Therefore, the statement is true.

4. By limit principle L6, the statement is true.

5. The statement is true.

6. By the definition of continuity, in order for f to be continuous at $x = 2$, $f(2)$ must exist.
Therefore, the statement is false.

7. This statement is false. If $\lim_{x\to 4} F(x)$ exists but is not equal to $F(4)$, then F is not continuous.

8. By the definition of continuity, the statement is true.

9. It follows from the Theorem on Limits of Rational Functions that we can find the limit by substitution:
$$\lim_{x\to 1}(3x+2) = 3(1)+2 = 5.$$

10. $\lim_{x\to 2}(4x-5) = 4(2)-5 = 3$.

11. It follows from the Theorem on Limits of Rational Functions that we can find the limit by substitution:
$$\lim_{x\to -1}(x^2-4) = (-1)^2-4 = 1-4 = -3.$$

12. $\lim_{x\to -2}(x^2+3) = (-2)^2+3 = 7$.

13. It follows from the Theorem on Limits of Rational Functions that we can find the limit by substitution:
$$\lim_{x\to 3}(x^2-4x+7) = (3)^2-4(3)+7 = 9-12+7 = 4.$$

14. $\lim_{x\to 5}(x^2-6x+9) = (5)^2-6(5)+9 = 4$.

15. It follows from the Theorem on Limits of Rational Functions that we can find the limit by substitution:
$$\lim_{x\to 2}(2x^4-3x^3+4x-1)$$
$$= 2(2)^4-3(2)^3+4(2)-1$$
$$= 2(16)-3(8)+8-1$$
$$= 32-24+8-1$$
$$= 15.$$

16. $\lim_{x\to -1}(3x^5+4x^4-3x+6)$
$$= 3(-1)^5+4(-1)^4-3(-1)+6 = 10.$$

17. It follows from the Theorem on Limits of Rational Functions that we can find the limit by substitution:
$$\lim_{x\to 3}\left(\frac{x^2-8}{x-2}\right) = \frac{(3)^2-8}{3-2} = \frac{9-8}{3-2} = 1.$$

18. $\lim_{x\to 3}\frac{x^2-25}{x^2-5} = \frac{(3)^2-25}{(3)^2-5} = \frac{9-25}{9-5} = \frac{-16}{4} = -4$.

19. $\lim_{x\to 5}\frac{x^2-25}{x-5}$
First we will simplify the function by factoring the numerator and canceling common factors.
$$\lim_{x\to 5}\frac{x^2-25}{x-5} = \lim_{x\to 5}\frac{(x-5)(x+5)}{x-5} = \lim_{x\to 5}(x+5) = 5+5 = 10.$$

20. We verify the expression yields an indeterminate form by substitution:

$$\lim_{x\to 3}\frac{x^2-9}{x-3}=\frac{(3)^2-9}{(3)-3}=\frac{0}{0}.$$

This is an indeterminate form.

In order to find the limit we will simplify the function by factoring the numerator and canceling common factors. Then we will apply the Theorem on Limits of Rational Functions to the simplified function.

$$\begin{aligned}\lim_{x\to 3}\frac{x^2-9}{x-3}&=\lim_{x\to 3}\frac{(x-3)(x+3)}{x-3}\\&=\lim_{x\to 3}(x+3) \quad \text{simplifying, assuming } x\neq -3\\&=3+3 \quad \text{substitution}\\&=6.\end{aligned}$$

21. We verify the expression yields an indeterminate form by substitution:

$$\lim_{x\to 1}\frac{x^2+5x-6}{x^2-1}=\frac{(1)^2+5(1)-6}{(1)^2-1}=\frac{0}{0}.$$

This is an indeterminate form. In order to find the limit we will simplify the function by factoring the numerator and denominator and canceling common factors. Then we will apply the Theorem on Limits of Rational Functions to the simplified function.

$$\begin{aligned}\lim_{x\to 1}\frac{x^2+5x-6}{x^2-1}&=\lim_{x\to 1}\frac{(x-1)(x+6)}{(x-1)(x+1)}\\&=\lim_{x\to 1}\frac{(x+6)}{(x+1)} \quad \text{simplifying, assuming } x\neq 1\\&=\frac{1+6}{1+1} \quad \text{substitution}\\&=\frac{7}{2}.\end{aligned}$$

22. $\displaystyle\lim_{x\to -2}\frac{x^2-2x-8}{x^2-4}$

First we will simplify the function by factoring the numerator and canceling common factors.

$$\begin{aligned}\lim_{x\to -2}\frac{x^2-2x-8}{x^2-4}&=\frac{(x-4)(x+2)}{(x-2)(x+2)}\\&=\lim_{x\to -2}\left(\frac{x-4}{x-2}\right)\\&=\frac{-2-4}{-2-2}\\&=\frac{-6}{-4}\\&=\frac{3}{2}.\end{aligned}$$

23. $\displaystyle\lim_{x\to -3}\frac{2x^2-x-21}{9-x^2}$

First we will simplify the function by factoring the numerator and denominator and canceling common factors.

$$\begin{aligned}\lim_{x\to -3}\frac{2x^2-x-21}{9-x^2}&=\lim_{x\to -3}\frac{(2x-7)(x+3)}{(3-x)(3+x)}\\&=\lim_{x\to -3}\frac{(2x-7)}{3-x}\\&=\frac{2(-3)-7}{3-(-3)}\\&=-\frac{13}{6}.\end{aligned}$$

24. We verify the expression yields an indeterminate form by substitution:

$$\lim_{x\to 2}\frac{3x^2+x-14}{x^2-4}=\frac{3(2)^2+(2)-14}{(2)^2-4}=\frac{0}{0}.$$

This is an indeterminate form. In order to find the limit we will simplify the function by factoring the numerator and denominator and canceling common factors. Then we will apply the Theorem on Limits of Rational Functions to the simplified function.

$$\lim_{x\to 2}\frac{3x^2+x-14}{x^2-4}=\lim_{x\to 2}\frac{(3x+7)(x-2)}{(x+2)(x-2)}$$
$$=\lim_{x\to 2}\frac{(3x+7)}{(x+2)} \quad \text{simplifying, assuming } x\neq 2$$
$$=\frac{3(2)+7}{2+2} \quad \text{substitution}$$
$$=\frac{13}{4}.$$

25. We verify the expression yields an indeterminate form by substitution:

$$\lim_{x\to 1}\frac{x^3-1}{x-1}=\frac{(1)^3-1}{(1)-1}$$
$$=\frac{0}{0}.$$

This is an indeterminate form. In order to find the limit we will simplify the function by factoring the numerator and canceling common factors. Then we will apply the Theorem on Limits of Rational Functions to the simplified function.

$$\lim_{x\to 1}\frac{x^3-1}{x-1}=\lim_{x\to 1}\frac{(x-1)(x^2+x+1)}{(x-1)}$$
$$=\lim_{x\to 1}(x^2+x+1) \quad \text{simplifying, assuming } x\neq 1$$
$$=(1)^2+(1)+1 \quad \text{substitution}$$
$$=3.$$

26. $\lim_{x\to 2}\frac{x^3-8}{2-x}$

First we will simplify the function by factoring the numerator and canceling common factors.

$$\lim_{x\to 2}\frac{x^3-8}{2-x}=\lim_{x\to 2}\frac{(x-2)(x^2+2x+4)}{-(x-2)}$$
$$=\lim_{x\to 2}\left[-(x^2+2x+4)\right]$$
$$=-\left((2)^2+2(2)+4\right)$$
$$=-12.$$

27. $\lim_{x\to 2}\frac{9-x}{\sqrt{x}-3}$

First we will simplify the function by factoring the numerator and canceling common factors.

$$\lim_{x\to 9}\frac{9-x}{\sqrt{x}-3}=\lim_{x\to 9}\frac{-(x-9)}{\sqrt{x}-3}$$
$$=\lim_{x\to 9}\left[\frac{-(\sqrt{x}-3)(\sqrt{x}+3)}{\sqrt{x}-3}\right]$$
$$=\lim_{x\to 9}\left[-(\sqrt{x}+3)\right]$$
$$=-(\sqrt{9}+3)$$
$$=-6.$$

28. We verify the expression yields an indeterminate form by substitution:

$$\lim_{x\to 25}\frac{\sqrt{x}-5}{x-25}=\frac{\sqrt{25}-5}{(25)-25}$$
$$=\frac{0}{0}.$$

This is an indeterminate form. In order to find the limit we will simplify the function by factoring the denominator and canceling common factors. Then we will apply the Theorem on Limits of Rational Functions to the simplified function.

$$\lim_{x\to 25}\frac{\sqrt{x}-5}{x-25}=\lim_{x\to 25}\frac{\sqrt{x}-5}{(\sqrt{x}-5)(\sqrt{x}+5)}$$
$$=\lim_{x\to 25}\frac{1}{(\sqrt{x}+5)} \quad \text{simplifying, assuming } x\neq 25$$
$$=\frac{1}{\sqrt{25}+5} \quad \text{substitution}$$
$$=\frac{1}{10}.$$

29. We verify the expression yields an indeterminate form by substitution:

$$\lim_{x\to -1}\frac{x^2+5x+4}{x^2+2x+1}=\frac{(-1)^2+5(-1)+4}{(-1)^2+2(-1)+1}$$
$$=\frac{0}{0}.$$

This is an indeterminate form. In order to find the limit we will simplify the function by factoring the numerator and denominator and canceling common factors. Then we will apply the Theorem on Limits of Rational Functions to the simplified function.

$$\lim_{x\to -1}\frac{x^2+5x+4}{x^2+2x+1}=\lim_{x\to -1}\frac{(x+1)(x+4)}{(x+1)(x+1)}$$

$$=\lim_{x\to -1}\left(\frac{x+4}{x+1}\right)\quad \text{simplifying, assuming } x\neq -1$$

$$=\frac{-1+4}{-1+1}\quad \text{substitution}$$

$$=\frac{3}{0}.$$

Substitution yields division by zero. Therefore,

$\lim_{x\to -1}\frac{x^2+5x+4}{x^2+2x+1}$ *does not exist.*

30. $\lim_{x\to 2}\frac{x^2+3x-10}{x^2-4x+4}$

First we will simplify the function by factoring the numerator and canceling common factors.

$$\lim_{x\to 2}\frac{x^2+3x-10}{x^2-4x+4}=\lim_{x\to 2}\frac{(x-2)(x+5)}{(x-2)(x-2)}$$

$$=\lim_{x\to 2}\left[\frac{x+5}{x-2}\right]$$

$$=\frac{7}{0}.$$

Therefore,

$\lim_{x\to 2}\frac{x^2+3x-10}{x^2-4x+4}$ *does not exist.*

31. $\lim_{x\to \infty}\left(\frac{5x-2}{4x+1}\right)$

First we will divide all terms by the highest power in the denominator, or x, and use the fact that $\lim_{x\to\infty}\left(\frac{1}{x}\right)=0$.

$$\lim_{x\to\infty}\frac{5x-2}{4x+1}=\lim_{x\to\infty}\frac{\frac{5x}{x}-\frac{2}{x}}{\frac{4x}{x}+\frac{1}{x}}$$

$$=\lim_{x\to\infty}\left[\frac{5-\frac{2}{x}}{4+\frac{1}{x}}\right]$$

$$=\frac{5}{4}.$$

Therefore,

$\lim_{x\to\infty}\left(\frac{5x-2}{4x+1}\right)=\frac{5}{4}$.

32. $\lim_{x\to \infty}\left(\frac{7x+5}{3x}\right)$

First we will divide all terms by the highest power in the denominator, or x, and use the fact that $\lim_{x\to\infty}\left(\frac{1}{x}\right)=0$.

$$\lim_{x\to\infty}\frac{7x+5}{3x}=\lim_{x\to\infty}\frac{\frac{7x}{x}+\frac{5}{x}}{\frac{3x}{x}}$$

$$=\lim_{x\to\infty}\left[\frac{7+\frac{5}{x}}{3}\right]$$

$$=\frac{7}{3}.$$

Therefore,

$\lim_{x\to\infty}\left(\frac{7x+5}{3x}\right)=\frac{7}{3}$.

33. $\lim_{x\to -\infty}\left(\frac{x+2}{6x-10}\right)$

First we will divide all terms by the highest power in the denominator, or x, and use the fact that $\lim_{x\to-\infty}\left(\frac{1}{x}\right)=0$.

$$\lim_{x\to-\infty}\frac{x+2}{6x-10}=\lim_{x\to-\infty}\frac{\frac{x}{x}+\frac{2}{x}}{\frac{6x}{x}-\frac{10}{x}}$$

$$=\lim_{x\to-\infty}\left[\frac{1+\frac{2}{x}}{6-\frac{10}{x}}\right]$$

$$=\frac{1}{6}.$$

Therefore,

$\lim_{x\to-\infty}\left(\frac{x+2}{6x-10}\right)=\frac{1}{6}$.

34. $\lim_{x\to -\infty}\left(\frac{12x}{4x-7}\right)$

First we will divide all terms by the highest power in the denominator, or x, and use the fact that $\lim_{x\to-\infty}\left(\frac{1}{x}\right)=0$.

$$\lim_{x\to-\infty}\frac{12x}{4x-7}=\lim_{x\to-\infty}\frac{\frac{12x}{x}}{\frac{4x}{x}-\frac{7}{x}}$$

$$=\lim_{x\to-\infty}\left[\frac{12}{4-\frac{7}{x}}\right]$$

$$=\frac{12}{4}=3.$$

Therefore,

$$\lim_{x\to-\infty}\left(\frac{12x}{4x-7}\right)=3.$$

35. $\lim_{x\to\infty}\left(\frac{x^2+2x+3}{3x^2+5}\right)$

First we will divide all terms by the highest power in the denominator, or x^2, and use the fact that $\lim_{x\to\infty}\left(\frac{1}{x^n}\right)=0$.

$$\lim_{x\to\infty}\frac{x^2+2x+3}{3x^2+5}=\lim_{x\to\infty}\frac{\frac{x^2}{x^2}+\frac{2x}{x^2}+\frac{3}{x^2}}{\frac{3x^2}{x^2}+\frac{5}{x^2}}$$

$$=\lim_{x\to\infty}\left[\frac{1+\frac{2}{x}+\frac{3}{x^2}}{3+\frac{5}{x^2}}\right]$$

$$=\frac{1+0+0}{3+0}=\frac{1}{3}.$$

Therefore,

$$\lim_{x\to\infty}\left(\frac{x^2+2x+3}{3x^2+5}\right)=\frac{1}{3}.$$

36. $\lim_{x\to\infty}\left(\frac{5x^2+5x+2}{10x^2+x+1}\right)$

First we will divide all terms by the highest power in the denominator, or x^2, and use the fact that $\lim_{x\to\infty}\left(\frac{1}{x^n}\right)=0$.

$$\lim_{x\to\infty}\frac{5x^2+5x+2}{10x^2+x+1}=\lim_{x\to\infty}\frac{\frac{5x^2}{x^2}+\frac{5x}{x^2}+\frac{2}{x^2}}{\frac{10x^2}{x^2}+\frac{x}{x^2}+\frac{1}{x^2}}$$

$$=\lim_{x\to\infty}\left[\frac{5+\frac{5}{x}+\frac{2}{x^2}}{10+\frac{1}{x}+\frac{1}{x^2}}\right]$$

$$=\frac{5+0+0}{10+0+0}=\frac{1}{2}.$$

Therefore,

$$\lim_{x\to\infty}\left(\frac{5x^2+5x+2}{10x^2+x+1}\right)=\frac{1}{2}.$$

37. $\lim_{x\to\infty}\left(\frac{8x+1}{x^2}\right)$

First we will divide all terms by the highest power in the denominator, or x^2, and use the fact that $\lim_{x\to\infty}\left(\frac{1}{x^n}\right)=0$.

$$\lim_{x\to\infty}\frac{8x+1}{x^2}=\lim_{x\to\infty}\frac{\frac{8x}{x^2}+\frac{1}{x^2}}{\frac{x^2}{x^2}}$$

$$=\lim_{x\to\infty}\left[\frac{\frac{8}{x}+\frac{1}{x^2}}{1}\right]$$

$$=\frac{0+0}{1}=0.$$

Therefore,

$$\lim_{x\to\infty}\left(\frac{8x+1}{x^2}\right)=0.$$

38. $\lim_{x\to\infty}\left(\frac{2x^2+6x-1}{x^3+x+7}\right)$

First we will divide all terms by the highest power in the denominator, or x^3, and use the fact that $\lim_{x\to\infty}\left(\frac{1}{x^n}\right)=0$.

$$\lim_{x\to\infty}\frac{2x^2+6x-1}{x^3+x+7}=\lim_{x\to\infty}\frac{\frac{2x^2}{x^3}+\frac{6x}{x^3}-\frac{1}{x^3}}{\frac{x^3}{x^3}+\frac{x}{x^3}+\frac{7}{x^3}}$$

$$=\lim_{x\to\infty}\left[\frac{\frac{2}{x}+\frac{6}{x^2}-\frac{1}{x^3}}{1+\frac{1}{x^2}+\frac{7}{x^3}}\right]$$

$$=\frac{0+0+0}{1+0+0}=0.$$

Therefore,

$$\lim_{x\to\infty}\left(\frac{2x^2+6x-1}{x^3+x+7}\right)=0.$$

39. $\lim_{x\to-\infty}\left(\frac{x^3+x^2+x+1}{x^2+x+1}\right)$

First we will divide all terms by the highest power in the denominator, or x^2, and use the fact that $\lim_{x\to\infty}\left(\frac{1}{x^n}\right)=0$.

$$\lim_{x\to-\infty}\frac{x^3+x^2+x+1}{x^2+x+1}=\lim_{x\to-\infty}\frac{\frac{x^3}{x^2}+\frac{x^2}{x^2}+\frac{x}{x^2}+\frac{1}{x^2}}{\frac{x^2}{x^2}+\frac{x}{x^2}+\frac{1}{x^2}}$$

$$=\lim_{x\to-\infty}\left[\frac{x+1+\frac{1}{x}+\frac{1}{x^2}}{1+\frac{1}{x}+\frac{1}{x^2}}\right]$$

$$=\frac{-\infty+1+0+0}{1+0+0}=-\infty.$$

Therefore,

$$\lim_{x\to-\infty}\left(\frac{x^3+x^2+x+1}{x^2+x+1}\right)=\text{DNE}.$$

40. $\lim_{x\to\infty}\left(\frac{x^5}{x^3+4x+10}\right)$

First we will divide all terms by the highest power in the denominator, or x^3, and use the fact that $\lim_{x\to\infty}\left(\frac{1}{x^n}\right)=0$.

$$\lim_{x\to-\infty}\frac{x^5}{x^3+4x+10}=\lim_{x\to\infty}\frac{\frac{x^5}{x^3}}{\frac{x^3}{x^3}+\frac{4x}{x^3}+\frac{10}{x^3}}$$

$$=\lim_{x\to\infty}\left[\frac{x^2}{1+\frac{4}{x^2}+\frac{10}{x^3}}\right]$$

$$=\frac{(\infty)^2}{1+0+0}=\infty.$$

Therefore,

$$\lim_{x\to\infty}\left(\frac{x^5}{x^3+4x+10}\right)=\text{DNE}.$$

41. $\lim_{x\to5}\sqrt{x^2-16}=\sqrt{\lim_{x\to5}\left(x^2-16\right)}$ By L2

$=\sqrt{\lim_{x\to5}x^2-\lim_{x\to5}16}$ By L3

$=\sqrt{\left(\lim_{x\to5}x\right)^2-16}$ By L2 and L1

$=\sqrt{(5)^2-16}$

$=\sqrt{25-16}$

$=\sqrt{9}$

$=3.$

42. $\lim_{x\to4}\sqrt{x^2-9}$

By limit principle L2,

$\lim_{x\to4}\sqrt{x^2-9}=\sqrt{\lim_{x\to4}\left(x^2-9\right)}$

$=\sqrt{\lim_{x\to4}x^2-\lim_{x\to4}9}$ By L3

$=\sqrt{\left(\lim_{x\to4}x\right)^2-9}$ By L2 and L1

$=\sqrt{(4)^2-9}$

$=\sqrt{16-9}$

$=\sqrt{7}.$

43. $\lim_{x\to2}\sqrt{x^2-9}$

By limit principle L2,

$\lim_{x\to2}\sqrt{x^2-9}=\sqrt{\lim_{x\to2}\left(x^2-9\right)}$

$=\sqrt{\lim_{x\to2}x^2-\lim_{x\to2}9}$ By L3

$=\sqrt{\left(\lim_{x\to2}x\right)^2-9}$ By L2 and L1

$=\sqrt{(2)^2-9}$

$=\sqrt{4-9}$

$=\sqrt{-5}.$

Therefore, $\lim_{x\to2}\sqrt{x^2-9}$ *does not exist.*

44. $\lim_{x\to3}\sqrt{x^2-16}=\sqrt{\lim_{x\to3}\left(x^2-16\right)}$ By L2

$=\sqrt{\lim_{x\to3}x^2-\lim_{x\to3}16}$ By L3

$=\sqrt{\left(\lim_{x\to3}x\right)^2-16}$ By L2 and L1

$=\sqrt{(3)^2-16}$

$=\sqrt{9-16}$

$=\sqrt{-7}.$

Therefore, $\lim_{x\to3}\sqrt{x^2-16}$ *does not exist.*

45. $\lim_{x\to -4^-}\sqrt{x^2-16}=\sqrt{\lim_{x\to -4^-}\left(x^2-16\right)}$ By L2

$=\sqrt{\lim_{x\to -4^-}x^2-\lim_{x\to -4^-}16}$ By L3

$=\sqrt{\left(\lim_{x\to -4^-}x\right)^2-16}$ By L2 and L1

$=\sqrt{(4)^2-16}$

$=\sqrt{16-16}$

$=\sqrt{0}$

$=0.$

46. $\lim_{x\to 3^+}\sqrt{x^2-9}$

By limit principle L2,

$\lim_{x\to 3^+}\sqrt{x^2-9}=\sqrt{\lim_{x\to 3^+}\left(x^2-9\right)}$

$=\sqrt{\lim_{x\to 3^+}x^2-\lim_{x\to 3^+}9}$ By L3

$=\sqrt{\left(\lim_{x\to 3^+}x\right)^2-9}$ By L2 and L1

$=\sqrt{(3)^2-9}$

$=\sqrt{9-9}$

$=\sqrt{0}$

$=0.$

47. The function is not continuous over the interval, because it is not continuous at $x=-2$.

48. The function is not continuous over the interval, because $f(x)$ is not continuous at $x=1$. As x approaches 1 from the left, $f(x)$ approaches 2. However, as x approaches 1 from the right $f(x)$ approaches -1. Therefore, $f(x)$ is not continuous at 1.

49. The function is continuous over the interval.

50. The function is not continuous over the interval, because $k(x)$ is not continuous at $x=-1$. The function is not defined at $x=-1$, in other words $k(-1)$ does not exist. Therefore, $k(x)$ is not continuous at -1.

51. The function is not continuous over the interval, because it is not continuous at $x=-2$. The limit does not exist as x approaches -2, furthermore, $t(-2)$ does not exist. Therefore the function is not continuous at $x=-2$.

52. a) As inputs x approach 1 from the right, outputs $g(x)$ approach -2. Thus, the limit from the right is -2. $\lim_{x\to 1^+}g(x)=-2$.

As inputs x approach 1 from the left, outputs $g(x)$ approach -2. Thus, the limit from the left is -2. $\lim_{x\to 1^-}g(x)=-2$.

Since the limit from the left, -2, is the same as the limit from the right, -2, we have. $\lim_{x\to 1}g(x)=-2$.

b) When the input is 1, the output $g(1)$ is -2. That is $g(1)=-2$.

c) The function $g(x)$ is continuous at $x=1$, because

1) $g(1)$ exists, $g(1)=-2$

2) $\lim_{x\to 1}g(x)$ exists, $\lim_{x\to 1}g(x)=-2$, and

3) $\lim_{x\to 1}g(x)=-2=g(1)$.

d) As inputs x approach -2 from the right, outputs $g(x)$ approach -3. Thus, the limit from the right is -3. $\lim_{x\to -2^+}g(x)=-3$.

As inputs x approach -2 from the left, outputs $g(x)$ approach 4. Thus, the limit from the left is 4. $\lim_{x\to -2^-}g(x)=4$.

Since the limit from the left, 4, is not the same as the limit from the right, -3, we say $\lim_{x\to -2}g(x)$ does not exist.

e) When the input is -2, the output $g(-2)$ is -3. That is $g(-2)=-3$.

f) Since the limit of $g(x)$ as x approaches -2 does not exist, the function is not continuous at $x=-2$.

53. a) $\lim_{x\to 1^+}f(x)=-1$ and $\lim_{x\to 1^-}f(x)=2$.

Therefore, $\lim_{x\to 1}f(x)$ *does not exist*

b) $f(1)=-1$

c) Since the limit of $f(x)$ as x approaches 1 does not exist, the function is not continuous at $x=1$.

d) $\lim_{x\to -2^+} f(x)=3$ and $\lim_{x\to -2^-} f(x)=3$

Therefore, $\lim_{x\to -2} f(x)=3$.

e) $f(-2)=3$

f) The function $f(x)$ is continuous at $x=-2$, because

1) $f(-2)$ exists, $f(-2)=3$,

2) $\lim_{x\to -2} f(x)$ exists, $\lim_{x\to -2} f(x)=3$, and

3) $\lim_{x\to -2} f(x)=3=f(-2)$.

54. a) As inputs x approach 1 from the right, outputs $h(x)$ approach 2. Thus, the limit from the right is 2. $\lim_{x\to 1^+} h(x)=2$.

As inputs x approach 1 from the left, outputs $h(x)$ approach 2. Thus, the limit from the left is 2. $\lim_{x\to 1^-} h(x)=2$.

Since the limit from the left, 2, is the same as the limit from the right, 2, we have. $\lim_{x\to 1} h(x)=2$.

b) When the input is 1, the output $h(1)$ is 2. That is $h(1)=2$.

c) The function $h(x)$ is continuous at $x=1$, because

1) $h(1)$ exists, $h(1)=2$

2) $\lim_{x\to 1} h(x)$ exists, $\lim_{x\to 1} h(x)=2$, and

3) $\lim_{x\to 1} h(x)=2=h(1)$.

d) As inputs x approach -2 from the right, outputs $h(x)$ approach 0. Thus, the limit from the right is 0. $\lim_{x\to -2^+} h(x)=0$.

As inputs x approach -2 from the left, outputs $h(x)$ approach 0. Thus, the limit from the left is 0. $\lim_{x\to -2^-} h(x)=0$.

Since the limit from the left, 0, is the same as the limit from the right, 0, we say $\lim_{x\to -2} h(x)=0$.

e) When the input is -2, the output $h(-2)$ is 0. That is $h(-2)=0$.

f) The function $h(x)$ is continuous at $x=-2$, because

1) $h(-2)$ exists, $h(-2)=0$

2) $\lim_{x\to -2} h(x)$ exists, $\lim_{x\to -2} h(x)=0$, and

3) $\lim_{x\to -2} h(x)=0=h(-2)$.

55. a) $\lim_{x\to -1^+} k(x)=2$ and $\lim_{x\to -1^-} k(x)=2$.

Therefore, $\lim_{x\to -1} k(x)=2$.

b) $k(-1)$ is not defined, or does not exist.

c) Since the value of $k(-1)$ does not exist, the function is not continuous at $x=-1$.

d) $\lim_{x\to 3^+} k(x)=-2$ and $\lim_{x\to 3^-} k(x)=-2$

Therefore, $\lim_{x\to 3} k(x)=-2$

e) $k(3)=-2$

f) The function $k(x)$ is continuous at $x=3$, because

1) $k(3)$ exists,

2) $\lim_{x\to 3} k(x)$ exists, and

3) $\lim_{x\to 3} k(x)=-2=k(3)$.

56. a) $\lim_{x\to 1^+} t(x)=0.25$ and $\lim_{x\to 1^-} t(x)=0.25$.

Therefore, $\lim_{x\to 1} t(x)=0.25$.

b) $t(1)=0.25$

c) The function $t(x)$ is continuous at $x=1$, because

1) $t(1)$ exists,

2) $\lim_{x\to 1} t(x)$ exists, and

3) $\lim_{x\to 1} t(x)=0.25=t(1)$.

d) $\lim_{x\to -2^+} t(x)=\infty$ and $\lim_{x\to -2^-} f(x)=\infty$

Therefore, $\lim_{x\to -2} f(x)=\infty$, which means that the limit does not exist because as x gets closer to -2 the function values increase without bound

e) $t(-2)$ is undefined or does not exist.

f) Since $t(-2)$ is undefined and the limit of $t(x)$ as x approaches -2 does not exist, the function is not continuous at $x=-2$.

57. a) As inputs x approach 3 from the right, outputs $G(x)$ approach 3. Thus, $\lim_{x\to 3^+} G(x) = 3$.

b) As inputs x approach 3 from the left, outputs $G(x)$ approach 1. Thus, $\lim_{x\to 3^-} G(x) = 1$.

c) Since the limit from the left, 1, is not the same as the limit from the right, 3, the limit does not exist. $\lim_{x\to 3} G(x)$ does not exist.

d) $G(3) = 1$

e) The function $G(x)$ is not continuous at $x = 3$ because the limit does not exist as x approaches 3.

f) The function $G(x)$ is continuous at $x = 0$, because
1) $G(0)$ exists,
2) $\lim_{x\to 0} G(x)$ exists, and
3) $\lim_{x\to 0} G(x) = G(0)$.

g) The function $G(x)$ is continuous at $x = 2.9$, because
1) $G(2.9)$ exists,
2) $\lim_{x\to 2.9} G(x)$ exists, and
3) $\lim_{x\to 2.9} G(x) = G(2.9)$.

58. a) $\lim_{x\to 2^+} C(x) = 1$

b) $\lim_{x\to 2^-} C(x) = -1$

c) $\lim_{x\to 2} C(x)$ does not exist.

d) $C(2) = 1$

e) The function $C(x)$ is not continuous at $x = 2$ because the limit does not exist as x approaches 2.

f) The function $C(x)$ is continuous at $x = 1.95$, because $\lim_{x\to 1.95} C(x) = -1 = C(1.95)$.

59. The function $g(x)$ is continuous at $x = 4$, because:
1) $g(4)$ exists, $g(4) = 4$
2) $\lim_{x\to 4} g(x)$ exists, $\lim_{x\to 4} g(x) = 4$, and
3) $\lim_{x\to 4} g(x) = 4 = g(4)$.

60. First we find the function value when $x = 5$.
$f(5) = 3(5) - 2 = 13$. Hence, $f(5)$ exists.
Next, we find the limit as x approaches 5. It follows from the Theorem on Limits of Rational Functions that we can find the limit by substitution: $\lim_{x\to 5} f(x) = 3(5) - 2 = 13$
Therefore, $\lim_{x\to 5} f(x) = 13 = f(5)$ and the function is continuous at $x = 5$.

61. The function $G(x) = \frac{1}{x}$ is not continuous at $x = 0$ because $G(0) = \frac{1}{0}$ is undefined.

62. The function $F(x) = \sqrt{x}$ is not continuous at $x = -1$ because $F(-1) = \sqrt{-1}$ is undefined on the real numbers.

63. The function $f(x)$ is continuous at $x = 4$, because:
1) $f(4)$ exists,
2) $\lim_{x\to 4} f(x)$ exists, and
3) $\lim_{x\to 4} f(x) = 3 = f(4)$.

64. First we find the function value when $x = 3$.
$g(3) = \frac{1}{3}(3) + 4 = 1 + 4 = 5$. Hence, $g(3)$ exists.
Next, we find the limit as x approaches 3. As the inputs x approach 3 from the right, the outputs $g(x)$ approach 5, that is,
$\lim_{x\to 3^-} g(x) = \frac{1}{3}(3) + 4 = 5$.
As the inputs x approach 3 from the left, the outputs $g(x)$ approach 5, that is,
$\lim_{x\to 3^+} g(x) = 2(3) - 1 = 5$.

Since the limit from the left, 5, is the same as the limit from the right, 5. The limit exists. We have:
$\lim_{x\to 3} g(x) = 5$.
Therefore, we have
$\lim_{x\to 3} g(x) = 5 = g(3)$.
Thus the function is continuous at $x = 3$.

65. The function is not continuous at $x = 3$ because the limit does not exist as x approaches 3. To verify this we take the limit as x approaches 3 from the left and the limit as x approaches 3 from the right.
As x approaches 3 from the left we have
$\lim_{x\to 3^-} F(x) = \frac{1}{3}(3)+4 = 5$.
As x approaches 3 from the right we have
$\lim_{x\to 3^+} F(x) = 2(3)-5 = 1$.

The solution is continued on the next page.
Since the limit from the left, 5, is not the same as the limit from the right, 1, the limit does not exist. $\lim_{x\to 3} F(x)$ does not exist.

66. The function is not continuous at $x = 4$ because the limit does not exist as x approaches 4.
$\lim_{x\to 4^-} G(x) = \frac{1}{2}(4)+1 = 3$
$\lim_{x\to 4^+} G(x) = -(4)+5 = 1$
$\lim_{x\to 4^-} G(x) \neq \lim_{x\to 4^+} G(x)$
Therefore,
$\lim_{x\to 4} G(x)$ does not exist.
Furthermore, the function is not defined at $x = 4$, so $G(4)$ does not exist.

67. The function is not continuous at $x = 4$ because the function is not defined at $x = 4$. Therefore, $g(4)$ does not exist.

68. First we find the function value when $x = 3$.
$f(3) = 2(3)-1 = 5$. Hence, $f(3)$ exists.
Next, we find the limit as x approaches 3. As the inputs x approach 3 from the left, the outputs $f(x)$ approach 5, that is,
$\lim_{x\to 3^-} f(x) = \frac{1}{3}(3)+4 = 5$.
As the inputs x approach 3 from the left, the outputs $f(x)$ approach 5, that is,
$\lim_{x\to 3^+} f(x) = 2(3)-1 = 5$.
Since the limit from the left, 5, is the same as the limit from the right, 5. The limit exists. We have:
$\lim_{x\to 3} f(x) = 5$.
Therefore, we have
$\lim_{x\to 3} f(x) = 5 = f(3)$.
Thus the function is continuous at $x = 3$.

69. The function is not continuous at $x = 2$. To verify this, we take the limit as x approaches 2. Using the Theorem on Limits of Rational Functions, we simplify the function near 2 by factoring the numerator and canceling common factors.

$$\begin{aligned}\lim_{x\to 2} G(x) &= \lim_{x\to 2}\frac{x^2-4}{x-2}\\ &= \lim_{x\to 2}\frac{(x-2)(x+2)}{x-2}\\ &= \lim_{x\to 2}(x+2)\\ &= 2+2 = 4\end{aligned}$$

Therefore,
$\lim_{x\to 2} G(x) = 4$.
However, when $x = 2$, the output $G(2)$ is defined to be 5. That is, $G(2) = 5$. Therefore, $\lim_{x\to 2} G(x) = 4 \neq 5 = G(2)$. Thus the function is not continuous at $x = 2$.

70. The function is not continuous at $x = 1$ because $\lim_{x\to 1} F(x) = 2 \neq 4 = F(1)$.

71. The function $G(x)$ is continuous at $x = 4$, because:
1) $G(4)$ exists,
2) $\lim_{x\to 4} G(x)$ exists, and
3) $\lim_{x\to 4} G(x) = 5 = G(4)$.

72. First we find the function value when $x = 5$.
$f(5) = (5)+1 = 6,\ f(5)$ exists.
Next we find the limit as x approaches 5. To find the limit as x approaches 5 from the left, we first simplify the rational function by factoring the numerator and canceling common factors.

$$\lim_{x\to 5^-} f(x) = \lim_{x\to 5^-} \frac{x^2-4x-5}{x-5}$$
$$= \lim_{x\to 5^-} \frac{(x-5)(x+1)}{x-5}$$
$$= \lim_{x\to 5^-} (x+1)$$
$$= 5+1$$
$$= 6$$

To find the limit as x approaches 5 from the right, we can use substitution.

$$\lim_{x\to 5^+} f(x) = \lim_{x\to 5^+} x+1$$
$$= 5+1$$
$$= 6$$

Therefore, the limit exists.

$\lim_{x\to 5} f(x) = 6$.

Thus we have,

$\lim_{x\to 5} f(x) = 6 = f(5)$.

Therefore, the function in continuous at $x = 5$.

73. The function is not continuous at $x = 5$ because $g(5)$ does not exist.

$$g(5) = \frac{1}{(5)^2 - 7(5) + 10}$$
$$= \frac{1}{25 - 35 + 10}$$
$$= \frac{1}{0}$$

74. The function $f(x)$ is continuous at $x = 3$, because:

1) $f(3)$ exists,

2) $\lim_{x\to 3} f(x)$ exists, and

3) $\lim_{x\to 3} f(x) = -1 = f(3)$.

75. The function is not continuous at $x = 2$, because $G(2)$ does not exist.

$$G(2) = \frac{1}{(2)^2 - 6(2) + 8}$$
$$= \frac{1}{0}$$

76. First we find the function value when $x = 4$.

$$F(4) = \frac{1}{(4)^2 - 7(4) + 10}$$
$$= \frac{1}{16 - 28 + 10}$$
$$= \frac{1}{-2}$$
$$= -\frac{1}{2}$$

Hence, $F(4)$ exists.

Next we find the limit. Applying the Theorem on Limits of Rational Functions we have:

$\lim_{x\to 4} F(x) = \frac{1}{(4)^2 - 7(4) + 10} = -\frac{1}{2}$.

We now have,

$\lim_{x\to 4} F(x) = \frac{1}{(4)^2 - 7(4) + 10} = -\frac{1}{2} = F(4)$.

Therefore, the function is continuous at $x = 4$.

77. Yes, the function is continuous over the interval $(-4, 4)$. Since the function is defined for every value in the interval, the Theorem on Limits of Rational Functions tells us $\lim_{x\to a} g(x) = g(a)$ for all values a in the interval. Thus $g(x)$ is continuous over the interval.

78. Yes, the function is continuous over the interval $(-5, 5)$. Since the function is defined for every value in the interval, the Theorem on Limits of Rational Functions tells us $\lim_{x\to a} F(x) = F(a)$ for all values a in the interval. Thus $F(x)$ is continuous over the interval.

79. Yes, because $\lim_{x\to a} g(x) = g(a)$ for all a such that $1 < a < \infty$, and $\lim_{x\to 1^+} g(x) = g(1)$.

80. Yes, because $\lim_{x\to a} h(x) = h(a)$ for all a such that $-3 < a < \infty$, and $\lim_{x\to -3^+} h(x) = h(-3)$.

81. Yes, because $\lim_{x\to a} F(x) = F(a)$ for all a such that $-5 < a < 5$, and $\lim_{x\to -5^+} F(x) = F(-5)$ and $\lim_{x\to 5^-} F(x) = F(5)$.

82. Yes, because $\lim_{x\to a} G(x) = G(a)$ for all a such that $-3 < a < 3$, and $\lim_{x\to -3^+} G(x) = G(-3)$ and $\lim_{x\to 3^-} G(x) = G(3)$.

83. $\lim_{x\to 20^-} p(x) = 1.50(20) = 30$

$\lim_{x\to 20^+} p(x) = 1.25(20) = 25$

$\lim_{x\to 20} p(x)$ does not exist.

84. $\lim_{x\to 100^-} p(x) = 0.08(100) = 8$

$\lim_{x\to 100^+} p(x) = 0.06(100) = 6$

$\lim_{x\to 100} p(x)$ does not exist.

85. $\lim_{x\to 60^-} T(t) = 2(60) = 120$

$\lim_{x\to 60^+} T(t) = 300 - 3(60) = 120$

$\lim_{x\to 60} T(t) = 120$

86. Yes, because the function is continuous over the domain $\left(-\infty, \frac{1}{2}\right) \cup \left(\frac{1}{2}, \infty\right)$.

87. Yes, because the function is continuous over the domain $(-\infty, 2) \cup (2, \infty)$.

88. Yes, because the function is continuous over the domain $(-\infty, \infty)$.

89. Yes, because the function is continuous over the domain $(-\infty, -1) \cup (1, \infty)$.

90. No, because the function is not continuous at the value $x = 0$.

91. No, because the function is not continuous at the value $x = 1$.

92. Yes, because the function is continuous over the domain $(-\infty, \infty)$.

93. Yes, because the function is continuous over the domain $(-\infty, \infty)$.

94. In order for the function to be continuous at $x = 20$ the limit as *x* approaches 20 from the left must equal the limit as *x* approaches 20 from the right. The limit from the left of the function is:
$\lim_{x\to 20^-} (1.5x) = 1.50(20) = 30$. Therefore, the limit of *p(x)* as *x* approaches 20 from the right must be equal to 30. We set the right hand limit equal to 30:
$\lim_{x\to 20^+} (1.25x + k) = 30$.
We allow *x* to approach 20. By Limit Property L3, we have:
$1.25(20) + k = 30$
Solving this equation for *k* yields:
$25 + k = 30$
$k = 5$.
Therefore, *k* must equal 5 in order for the price function to be continuous at $x = 20$.

95. $\lim_{t\to 100^-} = 0.08(100) = 8$. In order for $T(t)$ to be continuous, the limit from the right must equal 8. Therefore,
$\lim_{t\to 100^+} 0.06x + k = 8$
$0.06(100) + k = 8$ By Limit Property L3
$6 + k = 8$
$k = 2$
The constant *k* must equal 2 in order for the function to be continuous at $t = 100$.

96. a) As the inputs *x* approach 0 from the left, the outputs approach -1. We see this by looking at a table:

x	-0.1	-0.01	-0.001
$\frac{\lvert x\rvert}{x}$	-1	-1	-1

$\lim_{x\to 0^-} \frac{|x|}{x} = -1$.
As the inputs *x* approach 0 from the right, the outputs approach 1. We see this by looking at a table:

x	0.001	0.01	0.1
$\frac{\lvert x\rvert}{x}$	1	1	1

$\lim_{x\to 0^+} \frac{|x|}{x} = 1$.

Since the limit from the left, -1, is not the same as the limit from the right, 1, the limit does not exist. $\lim_{x\to 0}\frac{|x|}{x}$ does not exist.

b) The limit as x approaches -2 from the left is: $\lim_{x\to -2^-}\frac{x^3+8}{x^2-4}=-3$.
The limit as x approaches -2 from the right is: $\lim_{x\to -2^+}\frac{x^3+8}{x^2-4}=-3$.
Since the limit from the left is the same as the limit from the right, the limit exists and is: $\lim_{x\to -2}\frac{x^3+8}{x^2-4}=-3$.

Another approach would be to simplify the function, by factoring the numerator and denominator and canceling common factors.

$$\begin{aligned}\lim_{x\to -2}\frac{x^3+8}{x^2-4} &= \lim_{x\to -2}\frac{(x+2)(x^2-2x+4)}{(x+2)(x-2)}\\ &= \lim_{x\to -2}\frac{x^2-2x+4}{x-2} \quad \text{assuming } x\neq -2\\ &= \frac{(-2)^2-2(-2)+4}{(-2)-2}\\ &= \frac{12}{-4}\\ &= -3.\end{aligned}$$

97. For the function to be continuous, $y=-3$ when $x=0$ and $y=16$ when $x=4$. Using the first condition yields the equation:

$$\begin{aligned}a(0)+b&=-3\\ b&=-3\end{aligned}$$

Using the second condition and the value for b yields the equation:

$$\begin{aligned}a(4)-3&=16\\ 4a&=19\\ a&=\frac{19}{4}\end{aligned}$$

98. $c=e^2$

99. 0.5, or $\frac{1}{2}$

100. 6

101. 0.5, or $\frac{1}{2}$

102. -0.2887, or $-\frac{1}{2\sqrt{3}}$

103. 0.378, or $\frac{1}{\sqrt{7}}$

104. 0.75, or $\frac{3}{4}$

105. 0

106. 0.25, or $\frac{1}{4}$

Exercise Set 1.3

1. The average rate of change is $\frac{78-72}{5-3}=\frac{6}{2}=3$ or the temperature rose 3 degrees/hr.

2. The average rate of change is $\frac{6}{2}=3$ or Jennifer hiked 3 mi/hr.

3. The average rate of change is $\frac{28}{4}=7$ or Marcus delivered 7 packages/hr.

4. The average rate of change is $\frac{3000-2500}{2017-2012}=\frac{500}{5}=100$ or the population of Felton grew by 100 people/yr.

5. The average rate of change is $\frac{125}{5}=25$ or Tanya scored 25 points/game.

6. The average rate of change is $\frac{180-150}{17-13}=\frac{30}{4}=7.5$ or Chris grew 7.5 cm/yr.

7. The average rate of change is $\frac{20,000,000}{4}=5,000,000$ or Burnham industries had revenue of \$5,000,000/month.

8. The average rate of change is $\frac{33.75}{15}=2.25$ or Juan spent \$2.25/gallon on gasoline.

9. The average rate of change is $\frac{4-6}{6}=\frac{-2}{6}=-0.333$ or unemployment changed by -0.333 percentage points/month.

10. The average rate of change is $\frac{93}{30}=3.1$ or Shannon spent \$3.10/day on electricity for April.

11. We must first find the y values that correspond to the given x values, therefore:
$f(4)=3(4)+2=14$ and $f(6)=3(6)+2=20$
So the average rate of change will be
$\frac{20-14}{6-4}=\frac{6}{2}=3$

12. We must first find the y values that correspond to the given x values, therefore:
$g(8)=5(8)-4=36$ and $g(11)=5(11)-4=51$
So the average rate of change will be
$\frac{51-36}{11-8}=\frac{15}{3}=5$

13. We must first find the y values that correspond to the given x values, therefore:
$F(-1)=2(-1)^2=2$ and $F(2)=2(2)^2=8$
So the average rate of change will be
$\frac{8-2}{2-(-1)}=\frac{6}{3}=2$

14. We must first find the y values that correspond to the given x values, therefore:
$G(-2)=-3(-2)^2=-12$ and $G(0)=-3(0)^2=0$
So the average rate of change will be
$\frac{0-(-12)}{0-(-2)}=\frac{12}{2}=6$

15. We must first find the y values that correspond to the given x values, therefore:
$h(4)=\frac{1}{4}$ and $h(8)=\frac{1}{8}$
So the average rate of change will be
$\frac{\frac{1}{8}-\frac{1}{4}}{8-4}=\frac{-\frac{1}{8}}{4}=-\frac{1}{32}$

16. We must first find the y values that correspond to the given x values, therefore:
$k(3)=\frac{3}{3}=1$ and $k(5)=\frac{3}{5}$
So the average rate of change will be
$\frac{\frac{3}{5}-3}{5-1}=\frac{-\frac{12}{5}}{4}=-\frac{12}{20}=-\frac{3}{5}$

17. We must first find the y values that correspond to the given x values, therefore:

$f(0) = (0)^2 + 2(0) = 0$ and $f(6) = (6)^2 + 2(6) = 48$

So the average rate of change will be

$$\frac{48-0}{6-0} = \frac{48}{6} = 8$$

18. We must first find the y values that correspond to the given x values, therefore:

$g(-4) = -(-4)^2 + 4(-4) = -32$ and

$g(0) = -(0)^2 + 4(0) = 0$

So the average rate of change will be

$$\frac{0-(-32)}{0-(-4)} = \frac{32}{4} = 8$$

19. We must first find the y values that correspond to the given x values, therefore:

$r(3) = \frac{1}{2}(3)^2 + \frac{1}{4}(3) = \frac{21}{4}$ and $r(5) = \frac{1}{2}(5)^2 + \frac{1}{4}(5) = \frac{55}{4}$

So the average rate of change will be

$$\frac{\frac{55}{4} - \frac{21}{4}}{5-3} = \frac{\frac{34}{4}}{2} = \frac{34}{8} = 4.25$$

20. We must first find the y values that correspond to the given x values, therefore:

$s(4) = \frac{1}{10}(4)^2 + \frac{1}{2}(4) = \frac{18}{5}$ and

$s(7) = \frac{1}{10}(7)^2 + \frac{1}{2}(7) = \frac{84}{10}$

So the average rate of change will be

$$\frac{\frac{84}{10} - \frac{18}{5}}{7-4} = \frac{\frac{48}{10}}{3} = \frac{48}{30} = 1.6$$

21. a) $f(x) = 5x^2$

so,

$$\begin{aligned}\frac{f(x+h)-f(x)}{h} &= \frac{5(x+h)^2 - 5x^2}{h} \\ &= \frac{(5x^2 + 10xh + 5h^2) - 5x^2}{h} \\ &= \frac{10xh + 5h^2}{h} \\ &= \frac{h(10x+5h)}{h} \\ &= \frac{h}{h}\cdot(10x+5h) \\ &= 10x + 5h = 5(2x+h)\end{aligned}$$

b) The difference quotient column in the table can be completed using the simplified difference quotient.

$10x + 5h$ Simplified difference quotient

$= 10(5) + 5(2) = 60$

substitiuting 5 for x and 2 for h;

$= 10(5) + 5(1) = 55$

substitiuting 5 for x and 1 for h;

$= 10(5) + 5(0.1) = 50.5$

substitiuting 5 for x and 0.1 for h;

$= 10(5) + 5(0.01) = 50.05$

substitiuting 5 for x and 0.01 for h.

The completed table is:

x	h	$\frac{f(x+h)-f(x)}{h}$
5	2	60
5	1	55
5	0.1	50.5
5	0.01	50.05

22. a) $f(x) = 4x^2$

so,

$$\begin{aligned}f(x+h) &= 4(x+h)^2 \quad \text{substituing } x+h \text{ for } x \\ &= 4(x^2 + 2xh + h^2) \\ &= 4x^2 + 8xh + 4h^2\end{aligned}$$

Then

$$\frac{f(x+h)-f(x)}{h} \quad \text{Difference quotient}$$

$$= \frac{\left(4x^2+8xh+4h^2\right)-4x^2}{h} \quad \text{Substituting}$$

$$= \frac{8xh+4h^2}{h}$$

$$= \frac{h(8x+4h)}{h} \quad \text{Factoring the numerator}$$

$$= \frac{h}{h}\cdot(8x+4h) \quad \text{Removing a factor} = 1.$$

$$= 8x+4h, \quad \text{Simplified difference quotient}$$

or $4(2x+h)$

b) The difference quotient column in the table can be completed using the simplified difference quotient.

$8x+4h$ Simplified difference quotient

$= 8(5)+4(2) = 48$

substitiuting 5 for x and 2 for h;

$= 8(5)+4(1) = 44$

substitiuting 5 for x and 1 for h;

$= 8(5)+4(0.1) = 40.4$

substitiuting 5 for x and 0.1 for h;

$= 8(5)+4(0.01) = 40.04$

substitiuting 5 for x and 0.01 for h.

The completed table is:

x	h	$\frac{f(x+h)-f(x)}{h}$
5	2	48
5	1	44
5	0.1	40.4
5	0.01	40.04

23. a) $f(x) = -5x^2$

so,

$$\frac{f(x+h)-f(x)}{h} = \frac{-5(x+h)^2-\left(-5x^2\right)}{h}$$

$$= \frac{\left(-5x^2-10xh-5h^2\right)+5x^2}{h}$$

$$= \frac{-10xh-5h^2}{h}$$

$$= \frac{h(-10x-5h)}{h}$$

$$= \frac{h}{h}\cdot(10x+5h)$$

$$= -10x-5h = -5(2x+h)$$

b) The difference quotient column in the table can be completed using the simplified difference quotient.

$-10x-5h$ Simplified difference quotient

$= -10(5)-5(2) = -60$

substitiuting 5 for x and 2 for h;

$= -10(5)-5(1) = -55$

substitiuting 5 for x and 1 for h;

$= -10(5)-5(0.1) = -50.5$

substitiuting 5 for x and 0.1 for h;

$= -10(5)-5(0.01) = -50.05$

substitiuting 5 for x and 0.01 for h.

The completed table is:

x	h	$\frac{f(x+h)-f(x)}{h}$
5	2	–60
5	1	–55
5	0.1	–50.5
5	0.01	–50.05

24. a) $f(x) = -4x^2$

so,

$f(x+h) = -4(x+h)^2$ substituing $x + h$ for x

$= -4(x^2 + 2xh + h^2)$

$= -4x^2 - 8xh - 4h^2$

Then

$\frac{f(x+h) - f(x)}{h}$ Difference quotient

$= \frac{(-4x^2 - 8xh - 4h^2) - (-4x^2)}{h}$ Substituting

$= \frac{-8xh - 4h^2}{h}$

$= \frac{h(-8x - 4h)}{h}$ Factoring the numerator

$= \frac{h}{h} \cdot (-8x - 4h)$ Removing a factor = 1.

$= -8x - 4h,$ Simplified difference quotient

or $-4(2x + h)$

b) The difference quotient column in the table can be completed using the simplified difference quotient.

$-8x - 4h$ Simplified difference quotient

$= -8(5) - 4(2) = -48$

substitiuting 5 for x and 2 for h;

$= -8(5) - 4(1) = -44$

substitiuting 5 for x and 1 for h;

$= -8(5) - 4(0.1) = -40.4$

substitiuting 5 for x and 0.1 for h;

$= -8(5) - 4(0.01) = -40.04$

substitiuting 5 for x and 0.01 for h.

The completed table is:

x	h	$\frac{f(x+h) - f(x)}{h}$
5	2	−48
5	1	−44
5	0.1	−40.4
5	0.01	−40.04

25. a) $f(x) = x^2 - x$

Then

$\frac{f(x+h) - f(x)}{h}$ Difference quotient

$= \frac{((x+h)^2 - (x+h)) - (x^2 - x)}{h}$

$= \frac{(x^2 + 2xh + h^2 - x - h) - (x^2 - x)}{h}$

$= \frac{2xh + h^2 - h}{h}$

$= \frac{h(2x + h - 1)}{h}$

$= \frac{h}{h} \cdot (2x + h - 1)$

$= 2x + h - 1$ Simplified difference quotient

b) The difference quotient column in the table can be completed using the simplified difference quotient.

$2x + h - 1$ Simplified difference quotient

$= 2(5) + (2) - 1 = 11$

substitiuting 5 for x and 2 for h;

$= 2(5) + (1) - 1 = 10$

substitiuting 5 for x and 1 for h;

$= 2(5) + (0.1) - 1 = 9.1$

substitiuting 5 for x and 0.1 for h;

$= 2(5) + (0.01) - 1 = 9.01$

substitiuting 5 for x and 0.01 for h.

The completed table is:

x	h	$\frac{f(x+h) - f(x)}{h}$
5	2	11
5	1	10
5	0.1	9.1
5	0.01	9.01

26. a) $f(x)=x^2+x$

We substitute $x+h$ for x

$$f(x+h)=(x+h)^2+(x+h)$$
$$=(x^2+2xh+h^2)+x+h$$
$$=x^2+2xh+h^2+x+h$$

Then

$$\frac{f(x+h)-f(x)}{h} \quad \text{Difference quotient}$$
$$=\frac{(x^2+2xh+h^2+x+h)-(x^2+x)}{h}$$
$$=\frac{2xh+h^2+h}{h}$$
$$=\frac{h(2x+h+1)}{h} \quad \text{Factoring the numerator}$$
$$=\frac{h}{h}\cdot(2x+h+1) \quad \text{Removing a factor} = 1.$$
$$=2x+h+1 \quad \text{Simplified difference quotient}$$

b) The difference quotient column in the table can be completed using the simplified difference quotient.

$2x+h+1$ Simplified difference quotient

$=2(5)+(2)+1=13$

substitiuting 5 for x and 2 for h;

$=2(5)+(1)+1=12$

substitiuting 5 for x and 1 for h;

$=2(5)+(0.1)+1=11.1$

substitiuting 5 for x and 0.1 for h;

$=2(5)+(0.01)+1=11.01$

substitiuting 5 for x and 0.01 for h.

The completed table is:

x	h	$\frac{f(x+h)-f(x)}{h}$
5	2	13
5	1	12
5	0.1	11.1
5	0.01	11.01

27. a) $f(x)=\frac{9}{x}$

Then

$$\frac{f(x+h)-f(x)}{h} \quad \text{Difference quotient}$$
$$=\frac{\left(\frac{9}{x+h}\right)-\left(\frac{9}{x}\right)}{h}$$
$$=\frac{\left(\frac{9}{x+h}\cdot\frac{x}{x}\right)-\left(\frac{9}{x}\cdot\frac{(x+h)}{(x+h)}\right)}{h}$$
$$=\frac{\left(\frac{9x}{x(x+h)}\right)-\left(\frac{9(x+h)}{x(x+h)}\right)}{h}$$
$$=\frac{\frac{-9h}{x(x+h)}}{h}$$
$$=\frac{-9h}{x(x+h)}\cdot\frac{1}{h}$$
$$=\frac{h}{h}\cdot\left(\frac{-9}{x(x+h)}\right)$$
$$=\frac{-9}{x(x+h)} \quad \text{Simplified difference quotient}$$

b) The difference quotient column in the table can be completed using the simplified difference quotient.

$-\frac{9}{x(x+h)}$ Simplified difference quotient

$=-\frac{9}{5(5+2)}=-\frac{9}{35}$

substitiuting 5 for x and 2 for h;

$=-\frac{9}{5(5+1)}=-\frac{9}{30}=-\frac{3}{10}$

substitiuting 5 for x and 1 for h;

$=-\frac{9}{5(5+0.1)}=-\frac{9}{25.5}=-\frac{6}{17}$

substitiuting 5 for x and 0.1 for h;

$=-\frac{9}{5(5+0.01)}=-\frac{9}{25.05}=-\frac{60}{167}$

substitiuting 5 for x and 0.01 for h.

The completed table is:

x	h	$\frac{f(x+h)-f(x)}{h}$
5	2	$-\frac{9}{35}$
5	1	$-\frac{3}{10}$
5	0.1	$-\frac{6}{17}$
5	0.01	$-\frac{60}{167}$

28. a) $f(x)=\frac{2}{x}$

We substitute $x+h$ for x

$f(x+h)=\frac{2}{x+h}$

Then

$\frac{f(x+h)-f(x)}{h}$ Difference quotient

$=\frac{\left(\frac{2}{x+h}\right)-\left(\frac{2}{x}\right)}{h}$

$=\frac{\left(\frac{2}{x+h}\cdot\frac{x}{x}\right)-\left(\frac{2}{x}\cdot\frac{(x+h)}{(x+h)}\right)}{h}$ multiplying by 1

$=\frac{\left(\frac{2x}{x(x+h)}\right)-\left(\frac{2(x+h)}{x(x+h)}\right)}{h}$

$=\frac{\frac{-2h}{x(x+h)}}{h}$ adding fractions

$=\frac{\frac{-2h}{x(x+h)}}{\frac{h}{1}}$ $h=\frac{h}{1}$

$=\frac{-2h}{x(x+h)}\cdot\frac{1}{h}$ multiplying by the reciprocal

$=\frac{h}{h}\cdot\left(\frac{-2}{x(x+h)}\right)$ Removing a factor = 1.

$=\frac{-2}{x(x+h)}$ Simplified difference quotient

b) The difference quotient column in the table can be completed using the simplified difference quotient.

$-\frac{2}{x(x+h)}$ Simplified difference quotient

$=-\frac{2}{5(5+2)}=-\frac{2}{35}$

substitiuting 5 for x and 2 for h;

$=-\frac{2}{5(5+1)}=-\frac{2}{30}=-\frac{1}{15}$

substitiuting 5 for x and 1 for h;

$=-\frac{2}{5(5+0.1)}=-\frac{2}{25.5}=-\frac{4}{51}$

substitiuting 5 for x and 0.1 for h;

$=-\frac{2}{5(5+0.01)}=-\frac{2}{25.05}=-\frac{40}{501}$

substitiuting 5 for x and 0.01 for h.

The completed table is:

x	h	$\frac{f(x+h)-f(x)}{h}$
5	2	$-\frac{2}{35}$
5	1	$-\frac{1}{15}$
5	0.1	$-\frac{4}{51}$
5	0.01	$-\frac{40}{501}$

29. a) $f(x)=2x+3$

$\frac{f(x+h)-f(x)}{h}$ Difference quotient

$=\frac{(2(x+h)+3)-(2x+3)}{h}$

$=\frac{(2x+2h+3)-(2x+3)}{h}$

$=\frac{2h}{h}$

$=2$ Simplified difference quotient

b) The difference quotient is 2 for all values of x and h. Therefore, the completed table is:

x	h	$\frac{f(x+h)-f(x)}{h}$
5	2	2
5	1	2
5	0.1	2
5	0.01	2

30. a) $f(x)=-2x+5$

We substitute $x+h$ for x

$f(x+h)=-2(x+h)+5$

$-2x-2h+5$

Then

$\frac{f(x+h)-f(x)}{h}$ Difference quotient

$=\frac{(-2x-2h+5)-(-2x+5)}{h}$

$=\frac{-2h}{h}$

$=-2$ Simplified difference quotient

b) The difference quotient is -2 for all values of x and h. Therefore, the completed table is shown at the top of the next column.

x	h	$\frac{f(x+h)-f(x)}{h}$
5	2	−2
5	1	−2
5	0.1	−2
5	0.01	−2

31. a) $f(x)=12x^3$

Then

$\frac{f(x+h)-f(x)}{h}$ Difference quotient

$=\frac{\left(12(x+h)^3\right)-\left(12x^3\right)}{h}$

$=\frac{12\left(x^3+3x^2h+3xh^2+h^3\right)-\left(12x^3\right)}{h}$

$=\frac{36x^2h+36xh^2+12h^3}{h}$

$=\frac{h\left(36x^2+36xh+12h^2\right)}{h}$

$=\frac{h}{h}\cdot\left(36x^2+36xh+12h^2\right)$

$=36x^2+36xh+12h^2$

Simplified difference quotient

b) The difference quotient column in the table can be completed using the simplified difference quotient.

$36x^2+36xh+12h^2$

$=36(5)^2+36(5)(2)+12(2)^2=1308$

substitiuting 5 for x and 2 for h;

$=36(5)^2+36(5)(1)+12(1)^2=1092$

substitiuting 5 for x and 1 for h;

$=36(5)^2+36(5)(0.1)+12(0.1)^2=918.12$

substitiuting 5 for x and 0.1 for h;

$=36(5)^2+36(5)(0.01)+12(0.01)^2=901.8012$

substitiuting 5 for x and 0.01 for h.

The completed table is:

x	h	$\frac{f(x+h)-f(x)}{h}$
5	2	1308
5	1	1092
5	0.1	918.12
5	0.01	901.8012

32. a) $f(x)=1-x^3$

so,

$f(x+h)=1-(x+h)^3$ substituing $x+h$ for x

$=1-\left(x^3+3x^2h+3xh^2+h^3\right)$

$=1-x^3-3x^2h-3xh^2-h^3$

Then

$\dfrac{f(x+h)-f(x)}{h}$ Difference quotient

$=\dfrac{\left(1-x^3-3x^2h-3xh^2-h^3\right)-\left(1-x^3\right)}{h}$

$=\dfrac{-3x^2h-3xh^2-h^3}{h}$

$=\dfrac{h\left(-3x^2-3xh-h^2\right)}{h}$ Factoring the numerator

$=\dfrac{h}{h}\cdot\left(-3x^2-3xh-h^2\right)$ Removing a factor = 1.

$=-3x^2-3xh-h^2$

Simplified difference quotient

b) The difference quotient column in the table can be completed using the simplified difference quotient.

$-3x^2-3xh-h^2$

$=-3(5)^2-3(5)(2)-(2)^2=-109$

substitiuting 5 for x and 2 for h;

$=-3(5)^2-3(5)(1)-(1)^2=-91$

substitiuting 5 for x and 1 for h;

$=-3(5)^2-3(5)(0.1)-(0.1)^2=-76.51$

substitiuting 5 for x and 0.1 for h;

$=-3(5)^2-3(5)(0.01)-(0.01)^2=-75.1501$

substitiuting 5 for x and 0.01 for h.

The completed table is:

x	h	$\dfrac{f(x+h)-f(x)}{h}$
5	2	−109
5	1	−91
5	0.1	−76.51
5	0.01	−75.1501

33. a) $f(x)=x^2-4x$

Then

$\dfrac{f(x+h)-f(x)}{h}$ Difference quotient

$=\dfrac{\left((x+h)^2-4(x+h)\right)-\left(x^2-4x\right)}{h}$

$=\dfrac{\left(x^2+2xh+h^2-4x-4h\right)-\left(x^2-4x\right)}{h}$

$=\dfrac{2xh+h^2-4h}{h}$

$=\dfrac{h(2x+h-4)}{h}$

$=\dfrac{h}{h}\cdot(2x+h-4)$

$=2x+h-4$ Simplified difference quotient

b) The difference quotient column in the table can be completed using the simplified difference quotient.

$2x+h-4$ Simplified difference quotient

$=2(5)+(2)-4=8$

substitiuting 5 for x and 2 for h;

$=2(5)+(1)-4=7$

substitiuting 5 for x and 1 for h;

$=2(5)+(0.1)-4=6.1$

substitiuting 5 for x and 0.1 for h;

$=2(5)+(0.01)-4=6.01$

substitiuting 5 for x and 0.01 for h.

The completed table is:

x	h	$\dfrac{f(x+h)-f(x)}{h}$
5	2	8
5	1	7
5	0.1	6.1
5	0.01	6.01

34. a) $f(x)=x^2-3x$

We substitute $x+h$ for x

$f(x+h)=(x+h)^2-3(x+h)$

$=\left(x^2+2xh+h^2\right)-3x-3h$

$=x^2+2xh+h^2-3x-3h$

Then

$\dfrac{f(x+h)-f(x)}{h}$ Difference quotient

$$= \frac{\left(x^2 + 2xh + h^2 - 3x - 3h\right) - \left(x^2 - 3x\right)}{h}$$

$$= \frac{2xh + h^2 - 3h}{h}$$

$$= \frac{h(2x + h - 3)}{h} \quad \text{Factoring the numerator}$$

$$= \frac{h}{h} \cdot (2x + h - 3) \quad \text{Removing a factor} = 1.$$

$$= 2x + h - 3 \quad \text{Simplified difference quotient}$$

b) The difference quotient column in the table can be completed using the simplified difference quotient.

$2x + h - 3$ Simplified difference quotient

$= 2(5) + (2) - 3 = 9$

substitiuting 5 for x and 2 for h;

$= 2(5) + (1) - 3 = 8$

substitiuting 5 for x and 1 for h;

$= 2(5) + (0.1) - 3 = 7.1$

substitiuting 5 for x and 0.1 for h;

$= 2(5) + (0.01) - 3 = 7.01$

substitiuting 5 for x and 0.01 for h.

Using the values from the previous page, the completed table is:

x	h	$\frac{f(x+h) - f(x)}{h}$
5	2	9
5	1	8
5	0.1	7.1
5	0.01	7.01

35. a) $f(x) = x^2 - 3x + 5$

Then

$$\frac{f(x+h) - f(x)}{h} \quad \text{Difference quotient}$$

$$= \frac{\left((x+h)^2 - 3(x+h) + 5\right) - \left(x^2 - 3x + 5\right)}{h}$$

$$= \frac{\left(x^2 + 2xh + h^2 - 3x - 3h + 5\right) - \left(x^2 - 3x + 5\right)}{h}$$

$$= \frac{2xh + h^2 - 3h}{h}$$

$$= \frac{h(2x + h - 3)}{h}$$

$$= \frac{h}{h} \cdot (2x + h - 3)$$

$$= 2x + h - 3 \quad \text{Simplified difference quotient}$$

b) The difference quotient column in the table can be completed using the simplified difference quotient.

$2x + h - 3$ Simplified difference quotient

$= 2(5) + (2) - 3 = 9$

substitiuting 5 for x and 2 for h;

$= 2(5) + (1) - 3 = 8$

substitiuting 5 for x and 1 for h;

$= 2(5) + (0.1) - 3 = 7.1$

substitiuting 5 for x and 0.1 for h;

$= 2(5) + (0.01) - 3 = 7.01$

substitiuting 5 for x and 0.01 for h.

The completed table is:

x	h	$\frac{f(x+h) - f(x)}{h}$
5	2	9
5	1	8
5	0.1	7.1
5	0.01	7.01

36. a) $f(x) = x^2 + 4x - 3$

We substitute $x + h$ for x

$$f(x+h) = (x+h)^2 + 4(x+h) - 3$$

$$= \left(x^2 + 2xh + h^2\right) + 4x + 4h - 3$$

$$= x^2 + 2xh + h^2 + 4x + 4h - 3$$

Then

$$\frac{f(x+h) - f(x)}{h} \quad \text{Difference quotient}$$

$$= \frac{\left(x^2 + 2xh + h^2 + 4x + 4h - 3\right) - \left(x^2 + 4x - 3\right)}{h}$$

$$= \frac{2xh + h^2 + 4h}{h}$$

$$= \frac{h(2x + h + 4)}{h}$$ Factoring the numerator

$$= \frac{h}{h} \cdot (2x + h + 4)$$ Removing a factor = 1

$$= 2x + h + 4$$ Simplified difference quotient

b) The difference quotient column in the table can be completed using the simplified difference quotient.

$2x + h + 4$ Simplified difference quotient

$= 2(5) + (2) + 4 = 16$

substitiuting 5 for x and 2 for h;

$= 2(5) + (1) + 4 = 15$

substitiuting 5 for x and 1 for h;

$= 2(5) + (0.1) + 4 = 14.1$

substitiuting 5 for x and 0.1 for h;

$= 2(5) + (0.01) + 4 = 14.01$

substitiuting 5 for x and 0.01 for h.

The completed table is:

x	h	$\frac{f(x+h) - f(x)}{h}$
5	2	16
5	1	15
5	0.1	14.1
5	0.01	14.01

(NOTE: 37 – 44 answers vary based on estimation of points used)

37. To find the average rate of change from 2008 to 2012, we locate the corresponding point $(2008, 0.05)$ and $(2012, 1.5)$. Using these two points we calculate the average rate of change

$$\frac{1.5 - 0.05}{2012 - 2008} = \frac{1.45}{4} = 0.36$$

The average rate of change of total employment from 2008 to 2012 increased approximately 0.36% per year.

To find the average rate of change from 2012 to 2017, we locate the corresponding points $(2012, 1.5)$ and $(2017, 1.0)$. Using these two points we calculate the average rate of change

$$\frac{1.0 - 1.5}{2017 - 2012} = \frac{-0.5}{5} = -0.1$$

The average rate of change of total employment from 2012 to 2017 increased approximately −0.1% per year.

To find the average rate of change from 2008 to 2017, we locate the corresponding points $(2008, 0.05)$ and $(2017, 1.0)$. Using these two points we calculate the average rate of change

$$\frac{1.0 - 0.05}{2017 - 2008} = \frac{0.95}{9} \approx 0.11$$

The average rate of change of total employment from 2000 to 2009 increased approximately −0.07% per year.

38. To find the average rate of change from 2008 to 2012, we locate the corresponding point $(2008, -4)$ and $(2012, 4)$. Using these two points we calculate the average rate of change

$$\frac{4 - (-4)}{2012 - 2008} = \frac{8}{4} = 2$$

The average rate of change of total employment from 2008 to 2012 increased approximately 2% per year.

To find the average rate of change from 2012 to 2017, we locate the corresponding points $(2012, 4)$ and $(2017, 2)$. Using these two points we calculate the average rate of change

$$\frac{2 - 4}{2017 - 2012} = \frac{-2}{5} = -0.4$$

The average rate of change of total employment from 2012 to 2017 increased approximately −0.4% per year.

To find the average rate of change from 2008 to 2017, we locate the corresponding points $(2008, -4)$ and $(2017, 2)$. Using these two points we calculate the average rate of change

$$\frac{2 - (-4)}{2017 - 2008} = \frac{6}{9} \approx 0.67$$

The average rate of change of total employment from 2008 to 2017 increased approximately 0.67% per year.

39. To find the average rate of change from 2008 to 2012, we locate the corresponding point $(2008,1)$ and $(2012,4)$. Using these two points we calculate the average rate of change

$$\frac{4-1}{2012-2008}=\frac{3}{4}=0.75$$

The average rate of change of total employment from 2008 to 2012 increased approximately 075% per year.

To find the average rate of change from 2012 to 2017, we locate the corresponding points $(2012,4)$ and $(2017,3)$. Using these two points we calculate the average rate of change

$$\frac{3-4}{2017-2012}=\frac{-1}{5}=-0.2$$

The average rate of change of total employment from 2012 to 2017 increased approximately -0.2% per year.

To find the average rate of change from 2008 to 2017, we locate the corresponding points $(2008,1)$ and $(2017,3)$. Using these two points we calculate the average rate of change

$$\frac{3-1}{2017-2008}=\frac{2}{9}\approx 0.22$$

The average rate of change of total employment from 2008 to 2017 increased approximately 0.22% per year.

40. To find the average rate of change from 2008 to 2012, we locate the corresponding point $(2008,3.1)$ and $(2012,1.9)$. Using these two points we calculate the average rate of change

$$\frac{1.9-3.1}{2012-2008}=\frac{-1.2}{4}=-0.3$$

The average rate of change of total employment from 2008 to 2012 increased approximately -0.3% per year.

To find the average rate of change from 2012 to 2017, we locate the corresponding points $(2012,1.9)$ and $(2017,2.4)$. Using these two points we calculate the average rate of change

$$\frac{2.4-1.9}{2017-2012}=\frac{0.5}{5}=0.1$$

The average rate of change of total employment from 2012 to 2017 increased approximately 0.1% per year.

To find the average rate of change from 2008 to 2017, we locate the corresponding points $(2008,3.1)$ and $(2017,2.4)$. Using these two points we calculate the average rate of change

$$\frac{2.4-3.1}{2017-2008}=\frac{-0.7}{9}\approx -0.078$$

The average rate of change of total employment from 2008 to 2017 increased approximately -0.078% per year.

41. To find the average rate of change from 2008 to 2012, we locate the corresponding point $(2008,2.5)$ and $(2012,2.9)$. Using these two points we calculate the average rate of change

$$\frac{2.9-2.5}{2012-2008}=\frac{0.4}{4}=0.1$$

The average rate of change of total employment from 2008 to 2012 increased approximately 0.1% per year.

To find the average rate of change from 2012 to 2017, we locate the corresponding points $(2012,2.9)$ and $(2017,2.6)$. Using these two points we calculate the average rate of change

$$\frac{2.6-2.9}{2017-2012}=\frac{-0.3}{5}=-0.6$$

The average rate of change of total employment from 2012 to 2017 increased approximately -0.6% per year.

To find the average rate of change from 2008 to 2017, we locate the corresponding points $(2008,2.5)$ and $(2017,2.6)$. Using these two points we calculate the average rate of change

$$\frac{2.6-2.5}{2017-2008}=\frac{0.1}{9}\approx 0.011$$

The average rate of change of total employment from 2008 to 2017 increased approximately 0.011% per year.

42. To find the average rate of change from 2008 to 2012, we locate the corresponding point $(2008,1.3)$ and $(2012,-1.4)$. Using these two points we calculate the average rate of change

$$\frac{-1.4-1.3}{2012-2008}=\frac{-2.7}{4}=-0.68$$

The average rate of change of total employment from 2008 to 2012 increased approximately -0.68% per year.

To find the average rate of change from 2012 to 2017, we locate the corresponding points $(2012,-1.4)$ and $(2017,0.8)$. Using these two points we calculate the average rate of change

$$\frac{0.8-(-1.4)}{2017-2012}=\frac{2.2}{5}=0.44$$

The average rate of change of total employment from 2012 to 2017 increased approximately 0.44% per year.

To find the average rate of change from 2008 to 2017, we locate the corresponding points $(2008,1.3)$ and $(2017,0.8)$. Using these two points we calculate the average rate of change

$$\frac{0.8-1.3}{2017-2008}=\frac{-0.5}{9}\approx -0.056$$

The average rate of change of total employment from 2008 to 2017 increased approximately -0.056% per year.

43. To find the average rate of change from 2008 to 2012, we locate the corresponding point $(2008,5)$ and $(2012,14)$. Using these two points we calculate the average rate of change

$$\frac{14-5}{2012-2008}=\frac{9}{4}=2.25$$

The average rate of change of total employment from 2008 to 2012 increased approximately 2.25% per year.

To find the average rate of change from 2012 to 2017, we locate the corresponding points $(2012,14)$ and $(2017,-8)$. Using these two points we calculate the average rate of change

$$\frac{-8-14}{2017-2012}=\frac{-22}{5}=-4.4$$

The average rate of change of total employment from 2012 to 2017 increased approximately -4.4% per year.

To find the average rate of change from 2008 to 2017, we locate the corresponding points $(2008,5)$ and $(2017,-8)$. Using these two points we calculate the average rate of change

$$\frac{-8-5}{2017-2008}=\frac{-13}{9}\approx -1.44$$

The average rate of change of total employment from 2008 to 2017 increased approximately -1.44% per year.

44. To find the average rate of change from 2008 to 2012, we locate the corresponding point $(2008,-2)$ and $(2012,2)$. Using these two points we calculate the average rate of change

$$\frac{2-(-2)}{2012-2008}=\frac{4}{4}=1$$

The average rate of change of total employment from 2008 to 2012 increased approximately 1% per year.

To find the average rate of change from 2012 to 2017, we locate the corresponding points $(2012,2)$ and $(2017,-0.5)$. Using these two points we calculate the average rate of change

$$\frac{-0.5-2}{2017-2012}=\frac{-2.5}{5}=-0.5$$

The average rate of change of total employment from 2012 to 2017 increased approximately -0.5% per year.

To find the average rate of change from 2008 to 2017, we locate the corresponding points $(2008,-2)$ and $(2017,-0.5)$. Using these two points we calculate the average rate of change

$$\frac{-0.5-(-2)}{2017-2008}=\frac{1.5}{9}\approx 0.17$$

The average rate of change of total employment from 2008 to 2017 increased approximately 0.17% per year.

45. In order to find the average rate of change from 2010 to 2012, we use the data points $(2010,53600)$ and $(2012,52700)$. The average rate of change is

$$\frac{52700-53600}{2012-2010}=\frac{-900}{2}=-450.$$

Between the years 2010 to 2012 the median household income in the U.S. decreased by about $450 per year.

In order to find the average rate of change from 2012 to 2015, we use the data points $(2012,52700)$ and $(2015,56500)$. The average rate of change is

$$\frac{56500-52700}{2015-2012}=\frac{3800}{3}=1266.67.$$

Between the years 2012 to 2015 the median household income in the U.S. increased by about $1,266.67 per year.

In order to find the average rate of change from 2010 to 2015, we use the data points $(2010, 53600)$ and $(2015, 56500)$. The average rate of change is

$$\frac{56500-53600}{2015-2010}=\frac{2900}{5}=580.$$

Between the years 2010 to 2015 the median household income in the U.S. increased by about \$580 per year.

46. Using the points $(2011, 63.2)$ and $(2014, 67.6)$ the average rate of change is

$$\frac{67.6-63.2}{2014-2011}=\frac{4.4}{3}\approx 1.467$$

Between 2011 and 2014, the average rate of change of the U.S. trade deficit with Japan was about \$1.467 billion per year.

Using the points $(2014, 67.6)$ and $(2017, 68.8)$ the average rate of change is

$$\frac{68.8-67.6}{2017-2014}=\frac{1.2}{3}=0.4$$

Between 2014 and 2017, the average rate of change of the U.S. trade deficit with Japan was about \$0.4 billion dollars per year.

Using the points $(2011, 63.2)$ and $(2017, 68.8)$ the average rate of change is

$$\frac{68.8-63.2}{2017-2011}=\frac{5.6}{6}\approx 0.93$$

Between 2001 and 2017, the average rate of change of the U.S. trade deficit with Japan was about \$0.93 billion dollars per year.

47. a) From 0 units to 1 unit the average rate of change is

$$\frac{70-0}{1-0}=70 \text{ pleasure units per unit.}$$

From 1 unit to 2 units the average rate of change is

$$\frac{109-70}{2-1}=\frac{39}{1}=39 \text{ pleasure units per unit.}$$

From 2 units to 3 units the average rate of change is

$$\frac{138-109}{3-2}=\frac{29}{1}=29 \text{ pleasure units per unit.}$$

From 3 units to 4 units the average rate of change is

$$\frac{161-138}{4-3}=\frac{23}{1}=23 \text{ pleasure units per unit.}$$

b) Answers will vary. As you consume more and more of a good, the additional utility, or amount of pleasure, associated with that good will start to fall. The additional satisfaction from the 1st to the 2nd slice of pizza is greater than the additional satisfaction from the 9th to the 10th slice of pizza.

48. a) $N(0)=0,\ N(1)=300$

$$\frac{300-0}{1-0}=300 \text{ units per thousand dollars.}$$

$N(1)=300,\ N(2)=480$

$$\frac{480-300}{2-1}=180 \text{ units per thousand dollars.}$$

$N(2)=480,\ N(3)=600$

$$\frac{600-480}{3-2}=120 \text{ units per thousand dollars.}$$

$N(3)=600,\ N(4)=700$

$$\frac{700-600}{4-3}=100 \text{ units per thousand dollars.}$$

b) As spending on advertising increases, there are few increases in sales from each additional amount of spending.

49. $p(x)=0.0278x^3-0.436x^2+2.524x+21.716$

a) $p(4)=0.0278(4)^3-0.436(4)^2+2.524(4)+21.716$
$\approx \$26.62$

b) $p(10)=0.0278(10)^3-0.436(10)^2+2.524(10)+21.716$
$\approx \$31.16$

c) $p(10)-p(4)=31.16-26.62=\$4.54$

d) $\dfrac{p(10)-p(4)}{10-4}=\dfrac{4.54}{6}=0.7567$

This result implies that the average price of a ticket between 2010 (x =4) and 2016 (x =10) grew at an average rate of \$0.76 per year.

50. $A(t)=2000(1.015)^{4t}$

a) $A(3)=2000(1.015)^{4(3)}\approx 2391.24$

b) $A(5)=2000(1.015)^{4(5)}\approx 2693.71$

c) $A(5)-A(3)=2693.710013-2391.236343$
$=302.47$

d) $\frac{A(5)-A(3)}{5-3}=\frac{302.4736702}{2}=151.236835$

The value of the account grew at a rate of \$151.24 per year between the 3rd and 5th year of the investment.

51. $C(x)=-0.05x^2+50x$

First substitute 301 for x.

$$C(305)=-0.05(305)^2+50(305)$$
$$=-4651.25+15,250$$
$$=10,598.75$$

The total cost of producing 305 units is \$10,598.75.

Next substitute 300 for x.

$$C(300)=-0.05(300)^2+50(300)$$
$$=-4500+15,000$$
$$=10,500.00$$

The total cost of producing 300 units is \$10,500.00.

Now we can substitute to find the average rate of change.

$$\frac{C(305)-C(300)}{305-300}=\frac{10,598.75-10,500}{305-300}$$
$$=\frac{98.75}{5}$$
$$=19.75$$

Total cost will increase \$19.75 if the company produces the 305th unit.

52. $R(x)=-0.001x^2+150x$

$$R(305)=-0.001(305)^2+150(305)$$
$$=45,656.975$$
$$R(300)=-0.001(300)^2+150(300)$$
$$=44,910.00$$

Therefore,

$$\frac{R(305)-R(300)}{305-300}=\frac{45,656.975-44,910}{5}$$
$$=149.40$$

The average increase in revenue from selling the 305th unit is \$149.40.

53. $R(x)=1.2x^2+11.62x+61.22$

$$R(4)=1.2(4)^2+11.62(4)+61.22$$
$$=126.9$$
$$R(1)=1.2(1)^2+11.62(1)+61.22$$
$$=74.04$$

Therefore,

$$\frac{R(4)-R(1)}{4-1}=\frac{126.9-74.04}{3}$$
$$=\frac{52.86}{3}$$
$$=17.62$$

The average rate of change in Amazon's revenue between 2014 and 2017 was \$17.62 billion per year.

54. $m(x)=7.34x^3-44.68x^2+67.85x+409.88$

$$m(5)=7.34(5)^3-44.68(5)^2$$
$$+67.85(5)+409.88$$
$$=549.63$$
$$m(2)=7.34(2)^3-44.68(2)^2$$
$$+67.85(2)+409.88$$
$$=425.58$$

Therefore,

$$\frac{m(5)-m(2)}{5-2}=\frac{549.63-425.58}{3}$$
$$=\frac{124.05}{3}$$
$$=41.35$$

The average rate of change in Panera Bread Co.'s income between 2014 and 2017 was \$41.35 million per year.

55. $H(w)=0.11w^{1.36}$

a) First we substitute 500 and 700 in for w to find the home range at the respective weights.

$$H(500)=0.11(500)^{1.36}$$
$$=0.11(4683.809314)$$
$$=515.2190246$$
$$\approx 515.22$$
$$H(700)=0.11(700)^{1.36}$$
$$=0.11(7401.731628)$$
$$=814.1904791$$
$$\approx 814.19$$

Next we use the function values to find the average rate at which the mammal's home range will increase

$$\frac{H(700)-H(500)}{700-500}=\frac{814.19-515.22}{700-500}$$
$$=\frac{298.97}{200}$$
$$\approx 1.49485$$

The average rate at which a carnivorous mammal's home range increases as the animal's weight grows from 500 g to 700 g is approximately 1.49 hectares per gram.

b) First we substitute 200 and 300 in for w to find the home range at the respective weights.

$$\begin{aligned}H(200) &= 0.11(200)^{1.36}\\ &= 0.11(1347.102971)\\ &= 148.1813269\\ &\approx 148.18\end{aligned}$$

$$\begin{aligned}H(300) &= 0.11(300)^{1.36}\\ &= 0.11(2338.217499)\\ &= 257.2039249\\ &\approx 257.20\end{aligned}$$

Next we use the function values to find the average rate at which the mammal's home range will increase

$$\begin{aligned}\frac{H(300)-H(200)}{300-200} &= \frac{257.20-148.18}{300-200}\\ &= \frac{109.02}{100}\\ &\approx 1.0902\end{aligned}$$

The average rate at which a carnivorous mammal's home range increases as the animal's weight grows from 200 g to 300 g is approximately 1.09 hectares per gram.

56. $P(t) = 2.8t^{1.87}$

a) First we substitute 10 and 17 in for t to find the condor population in the Grand Canyon in the respective years.

$$\begin{aligned}P(10) &= 2.8(10)^{1.87}\\ &= 2.8(74.13102413)\\ &= 207.5668676\\ &\approx 207.567\end{aligned}$$

$$\begin{aligned}P(17) &= 2.8(17)^{1.87}\\ &= 2.8(199.9583217)\\ &= 559.8833008\\ &\approx 559.883\end{aligned}$$

Next we use the function values to find the average rate at which the condor population will increase

$$\begin{aligned}\frac{P(17)-P(10)}{17-10} &= \frac{559.883-207.567}{17-10}\\ &= \frac{352.316}{7}\\ &\approx 50.53\end{aligned}$$

The average rate at which the condor population in the Grand Canyon was increasing between 2010 and 2017 was approximately 50 condors per year.

b) First we substitute 15 and 7 in for t to find the condor population in the Grand Canyon in the respective years.

$$\begin{aligned}P(15) &= 2.8(15)^{1.87}\\ &= 2.8(158.2306654)\\ &= 443.0458632\\ &\approx 443.046\end{aligned}$$

$$\begin{aligned}P(7) &= 2.8(7)^{1.87}\\ &= 2.8(38.04813158)\\ &= 106.5347684\\ &\approx 106.535\end{aligned}$$

Next we use the function values to find the average rate at which the condor population will increase

$$\begin{aligned}\frac{P(15)-P(7)}{15-7} &= \frac{443.046-106.535}{15-7}\\ &= \frac{336.511}{8}\\ &\approx 42.065\end{aligned}$$

The average rate at which the condor population in the Grand Canyon was increasing between 2007 and 2015 was approximately 42 condors per year.

57. a) We locate the points $(0,0)$ and $(8,10)$ on the graph and use them to calculate the average rate of change.

$$\frac{10-0}{8-0} = \frac{10}{8} = \frac{5}{4} = 1.25$$

The average rate of change is 1.25 words per minute.

We locate the points $(8,10)$ and $(16,20)$ on the graph and use them to calculate the average rate of change.

$$\frac{20-10}{16-8} = \frac{10}{8} = \frac{5}{4} = 1.25$$

The average rate of change is 1.25 words per minute.

We locate the points $(16,20)$ and $(24,25)$ on the graph and use them to calculate the average rate of change.
$\frac{25-20}{24-16}=\frac{5}{8}=0.625$
The average rate of change is 0.625 words per minute.
We locate the points $(24,25)$ and $(32,25)$ on the graph and use them to calculate the average rate of change.
$\frac{25-25}{32-24}=\frac{0}{8}=0$
The average rate of change is 0 words per minute.
We locate the points $(32,25)$ and $(36,25)$ on the graph and use them to calculate the average rate of change.
$\frac{25-25}{36-32}=\frac{0}{4}=0$
The average rate of change is 0 words per minute.

b) Answers will vary. The person has reached a saturation point after 24 minutes. They cannot memorize any more words.

58. a) We use the two points $(0,30680)$ and $(13.5,31077)$.
$\frac{31,077-30,680}{13.5-0}=\frac{397}{13.5}\approx 29.4$
The car averaged 29.4 miles per gallon on the trip.

b) To find the average rate of gas consumption in gallons per mile, we take the reciprocal of part (a).
$\frac{13.5-0}{31,077-30,680}=\frac{13.5}{397}\approx 0.034$
The car consumed approximately 0.034 gallons per mile on the trip.

59. $s(t)=16t^2$

a) First, we find the function values by substituting 3 and 5 in for t respectively.
$s(3)=16(3)^2=16(9)=144$
$s(5)=16(5)^2=16(25)=400$
Next we subtract the function values.
$s(5)-s(3)=400-144=256$
The object will fall 256 feet in the two second time period between $t=3$ and $t=5$.

b) The average rate of change is calculated as
$$\frac{s(5)-s(3)}{5-3}=\frac{400-144}{5-3}=\frac{256}{2}=128$$
The average velocity of the object during the two second time period from $t=3$ to $t=5$ is 128 feet per second.

60. $s(t)=40t^{1.2}$

a) First, we find the function values by substituting 2 and 5 in for t respectively.
$s(2)=40(2)^{1.2}\approx 91.896$
$s(5)=40(5)^{1.2}\approx 275.946$
Next we subtract the function values.
$s(5)-s(2)=275.946-91.896$
$=184.05$
The truck will travel 184.05 miles in the three hour time period between $t=2$ and $t=5$.

b) The average rate of change is calculated as
$$\frac{s(5)-s(2)}{5-2}=\frac{275.946-91.896}{5-2}=\frac{184.05}{3}=61.35$$
The average velocity of the truck during the Three hour time period from $t=2$ to $t=5$ is 61.35 miles per hour.

61. a) For each curve, as t changes from 0 to 4, $P(t)$ changes from 0 to 500. Thus, the average growth rate for each country is
$\frac{500-0}{4-0}=\frac{500}{4}=125$.
The average growth rate for each country is approximately 125 million people per year.

b) No; and average rate of change does not give an indication of the growth patterns of the two populations during the 4 years for which it was calculated.

c) For Country A:

As t changes from 0 to 1, $P(t)$ changes from 0 to 290. Thus the average growth rate is

$$\frac{290-0}{1-0} = 290 \text{ thousand people per year.}$$

As t changes from 1 to 2, $P(t)$ changes from 290 to 250. Thus the average growth rate is

$$\frac{250-290}{2-1} = -40 \text{ thousand people per year.}$$

As t changes from 2 to 3, $P(t)$ changes from 250 to 200. Thus the average growth rate is

$$\frac{200-250}{3-2} = -50 \text{ thousand people per year.}$$

As t changes from 3 to 4, $P(t)$ changes from 200 to 500. Thus the average growth rate is

$$\frac{500-200}{4-3} = 300 \text{ thousand people per year.}$$

For Country B:

As t changes from 0 to 1, $P(t)$ changes from 0 to 125. Thus the average growth rate is

$$\frac{125-0}{1-0} = 125 \text{ thousand people per year.}$$

As t changes from 1 to 2, $P(t)$ changes from 125 to 250. Thus the average growth rate is

$$\frac{250-125}{2-1} = 125 \text{ thousand people per year.}$$

As t changes from 2 to 3, $P(t)$ changes from 250 to 375. Thus the average growth rate is

$$\frac{375-250}{3-2} = 125 \text{ thousand people per year.}$$

As t changes from 3 to 4, $P(t)$ changes from 375 to 500. Thus the average growth rate is

$$\frac{500-375}{4-3} = 125 \text{ thousand people per year.}$$

d) The statement most accurately reflects the information found in Country B. Country B shows linear growth, which implies a constant rate of change.

62. $P(t) = 5400(0.975)^t$

$$P(8) = 5400(0.975)^8 = 4409.91974$$

$$P(5) = 5400(0.975)^5 = 4757.916744$$

$$\frac{P(8)-P(5)}{8-5} = \frac{-347.9970041}{3} \approx -116$$

Payton County lost an average of 116 people per year between the 5th and 8th years.

63. $P(t) = 17{,}000(1.042)^t$

$$P(6) = 17{,}000(1.042)^6 = 21{,}759.81683$$

$$P(2) = 17{,}000(1.042)^2 = 18{,}457.988$$

$$\frac{P(6)-P(2)}{6-2} = \frac{3301.82883}{4} \approx 825.46$$

Harbor University's undergraduate population was increasing at the rate of 825.46 students per year between the 2nd and 6th years.

64. $f(x) = mx + b$

$$\frac{f(x+h)-f(x)}{h} = \frac{(m(x+h)+b)-(mx+b)}{h}$$

$$= \frac{mx+mh+b-mx-b}{h}$$

$$= \frac{mh}{h}$$

$$= m$$

65. $f(x) = ax^2 + bx + c$

Substituting $x+h$ for x we have,

$$f(x+h) = a(x+h)^2 + b(x+h) + c$$

$$= a\left(x^2 + 2xh + h^2\right) + bx + bh + c$$

$$= ax^2 + 2axh + ah^2 + bx + bh + c$$

Thus,

$$\frac{f(x+h)-f(x)}{h} \quad \text{Difference quotient}$$

$$= \frac{\left(ax^2 + 2axh + ah^2 + bx + bh + c\right) - \left(ax^2 + bx + c\right)}{h}$$

$$= \frac{2axh + ah^2 + bh}{h}$$

$$= \frac{h(2ax + ah + b)}{h} \quad \text{Factoring the numerator}$$

$$= 2ax + ah + b \quad \text{Simplified difference quotient}$$

66. $f(x) = ax^3 + bx^2$

$$\frac{f(x+h) - f(x)}{h}$$
$$= \frac{a(x+h)^3 + b(x+h)^2 - (ax^3 + bx^2)}{h}$$
$$= \frac{a(x^3 + 3x^2h + 3xh^2 + h^3) + b(x^2 + 2hx + h^2) - ax^3 - bx^2}{h}$$
$$= \frac{ax^3 + 3ax^2h + 3axh^2 + ah^3 + bx^2 + 2bhx + bh^2 - ax^3 - bx^2}{h}$$
$$= \frac{3ax^2h + 3axh^2 + ah^3 + 2bhx + bh^2}{h}$$
$$= \frac{h(3ax^2 + 3axh + ah^2 + 2bx + bh)}{h}$$
$$= 3ax^2 + 3axh + ah^2 + 2bx + bh$$

67. $f(x) = x^4$

Substituting $x + h$ for x we have,

$$f(x+h) = (x+h)^4$$
$$= x^4 + 4x^3h + 6x^2h^2 + 4xh^3 + h^4$$

Thus,

$$\frac{f(x+h) - f(x)}{h} \quad \text{Difference quotient}$$
$$= \frac{(x^4 + 4x^3h + 6x^2h^2 + 4xh^3 + h^4) - (x^4)}{h}$$
$$= \frac{4x^3h + 6x^2h^2 + 4xh^3 + h^4}{h}$$
$$= \frac{h(4x^3 + 6x^2h + 4xh^2 + h^3)}{h}$$
$$= 4x^3 + 6x^2h + 4xh^2 + h^3$$

68. $f(x) = x^5$

$$\frac{f(x+h) - f(x)}{h}$$
$$= \frac{(x+h)^5 - x^5}{h}$$
$$= \frac{(x^5 + 5x^4h + 10x^3h^2 + 10x^2h^3 + 5xh^4 + h^5) - (x^4)}{h}$$
$$= \frac{5x^4h + 10x^3h^2 + 10x^2h^3 + 5xh^4 + h^5}{h}$$
$$= \frac{h(5x^4 + 10x^3h + 10x^2h^2 + 5xh^3 + h^4)}{h}$$
$$= 5x^4 + 10x^3h + 10x^2h^2 + 5xh^3 + h^4$$

69. $f(x)=ax^5+bx^4$

Substituting $x+h$ for x we have,

$f(x+h)=a(x+h)^5+b(x+h)^4$

$= ax^5+5ax^4h+10ax^3h^2+10ax^2h^3+5axh^4+ah^5+bx^4+4bx^3h+6bx^2h^2+4bxh^3+bh^4$

Thus,

$\dfrac{f(x+h)-f(x)}{h}$ Difference quotient

$=\dfrac{ax^5+5ax^4h+10ax^3h^2+10ax^2h^3+5axh^4+ah^5+bx^4+4bx^3h+6bx^2h^2+4bxh^3+bh^4-\left(ax^5+bx^4\right)}{h}$

$=\dfrac{5ax^4h+10ax^3h^2+10ax^2h^3+5axh^4+ah^5+4bx^3h+6bx^2h^2+4bxh^3+bh^4}{h}$

$=\dfrac{h\left(5ax^4+10ax^3h+10ax^2h^2+5axh^3+ah^4+4bx^3+6bx^2h^1+4bxh^2+bh^3\right)}{h}$ Factoring the numerator

$=5ax^4+10ax^3h+10ax^2h^2+5axh^3+ah^4+4bx^3+6bx^2h+4bxh^2+bh^3$ Simplified difference quotient

70. $f(x)=\dfrac{1}{x^2}$

Substituting $x+h$ for x we have,

$f(x+h)=\dfrac{1}{(x+h)^2}$

Thus,

$\dfrac{f(x+h)-f(x)}{h}$ Difference quotient

$=\dfrac{\left(\dfrac{1}{(x+h)^2}\right)-\left(\dfrac{1}{x^2}\right)}{h}$

Next, we find a common denominator in the numerator.

$=\dfrac{\left(\dfrac{1}{(x+h)^2}\cdot\dfrac{x^2}{x^2}\right)-\left(\dfrac{1}{x^2}\cdot\dfrac{(x+h)^2}{(x+h)^2}\right)}{h}$

$=\dfrac{\dfrac{x^2}{x^2(x+h)^2}-\dfrac{x^2+2xh+h^2}{x^2(x+h)^2}}{h}$

$=\dfrac{\dfrac{-2xh-h^2}{x^2(x+h)^2}}{h}$

$$= \frac{-2xh - h^2}{x^2(x+h)^2} \cdot \frac{1}{h}$$

$$= \frac{h(-2x-h)}{x^2(x+h)^2} \cdot \frac{1}{h}$$

$$= \frac{-2x-h}{x^2(x+h)^2}$$ Simplified difference quotient

71. $f(x) = \dfrac{1}{1-x}$

$$\frac{f(x+h) - f(x)}{h}$$

$$= \frac{\dfrac{1}{1-(x+h)} - \left(\dfrac{1}{1-x}\right)}{h}$$

$$= \frac{\dfrac{1}{1-x-h} \cdot \dfrac{1-x}{1-x} - \left(\dfrac{1}{1-x} \cdot \dfrac{1-x-h}{1-x-h}\right)}{h}$$

$$= \frac{\dfrac{1-x}{(1-x-h)(1-x)} - \dfrac{1-x-h}{(1-x-h)(1-x)}}{h}$$

$$= \frac{\dfrac{h}{(1-x-h)(1-x)}}{h}$$

$$= \frac{h}{(1-x-h)(1-x)} \cdot \frac{1}{h}$$

$$= \frac{1}{(1-x-h)(1-x)}$$

72. $f(x) = \sqrt{x}$

The difference quotient is:

$$\frac{f(x+h) - f(x)}{h} = \frac{\sqrt{x+h} - \sqrt{x}}{h}$$

a) $\dfrac{\sqrt{x+h} - \sqrt{x}}{h} \cdot \dfrac{\sqrt{x+h} + \sqrt{x}}{\sqrt{x+h} + \sqrt{x}}$ Reason: Multiplying by 1.

b) $\dfrac{x + h + \sqrt{x}\sqrt{x+h} - \sqrt{x}\sqrt{x+h} - x}{h\left(\sqrt{x+h} + \sqrt{x}\right)}$ Reason: Expanding the numerator.

c) $\dfrac{h}{h\left(\sqrt{x+h} + \sqrt{x}\right)}$ Reason: $x - x = 0$.

d) $\dfrac{1}{\sqrt{x+h} + \sqrt{x}}$ Reason: $\dfrac{h}{h} = 1$.

73. $f(x)=\sqrt{2x+1}$

$\dfrac{f(x+h)-f(x)}{h}$ Difference quotient

$=\dfrac{\sqrt{2(x+h)+1}-\sqrt{2x+1}}{h}$

$=\dfrac{\sqrt{2(x+h)+1}-\sqrt{2x+1}}{h}\cdot\dfrac{\sqrt{2(x+h)+1}+\sqrt{2x+1}}{\sqrt{2(x+h)+1}+\sqrt{2x+1}}$

$=\dfrac{2(x+h)+1-(2x+1)}{h\left(\sqrt{2(x+h)+1}+\sqrt{2x+1}\right)}$

$=\dfrac{2x-2h+1-2x-1}{h\left(\sqrt{2(x+h)+1}+\sqrt{2x+1}\right)}$

$=\dfrac{-2h}{h\left(\sqrt{2(x+h)+1}+\sqrt{2x+1}\right)}$

$=\dfrac{-2}{\sqrt{2(x+h)+1}+\sqrt{2x+1}}$

74. $f(x)=\dfrac{1}{\sqrt{x}}$

Substituting $x+h$ for x we have,

$f(x+h)=\dfrac{1}{\sqrt{x+h}}$

Thus,

$\dfrac{f(x+h)-f(x)}{h}$ Difference quotient

$=\dfrac{\left(\dfrac{1}{\sqrt{x+h}}\right)-\left(\dfrac{1}{\sqrt{x}}\right)}{h}$

Next, we find a common denominator in the numerator.

$=\dfrac{\left(\dfrac{1}{\sqrt{x+h}}\cdot\dfrac{\sqrt{x}}{\sqrt{x}}\right)-\left(\dfrac{1}{\sqrt{x}}\cdot\dfrac{\sqrt{x+h}}{\sqrt{x+h}}\right)}{h}$

$=\dfrac{\dfrac{\sqrt{x}}{\sqrt{x}\sqrt{x+h}}-\dfrac{\sqrt{x+h}}{\sqrt{x}\sqrt{x+h}}}{h}$

$=\dfrac{\dfrac{\sqrt{x}-\sqrt{x+h}}{\sqrt{x}\sqrt{x+h}}}{h}$

$=\dfrac{\sqrt{x}-\sqrt{x+h}}{\sqrt{x}\sqrt{x+h}}\cdot\dfrac{1}{h}$ Simplifying the complex fraction

$\dfrac{f(x+h)-f(x)}{h}=\dfrac{\sqrt{x}-\sqrt{x+h}}{h\sqrt{x}\sqrt{x+h}}$

Next, we rationalize the numerator.

$\dfrac{f(x+h)-f(x)}{h}=\dfrac{\sqrt{x}-\sqrt{x+h}}{h\sqrt{x}\sqrt{x+h}}\cdot\dfrac{\sqrt{x}+\sqrt{x+h}}{\sqrt{x}+\sqrt{x+h}}$

$=\dfrac{x-(x+h)}{h\sqrt{x}\sqrt{x+h}\left(\sqrt{x}+\sqrt{x+h}\right)}$

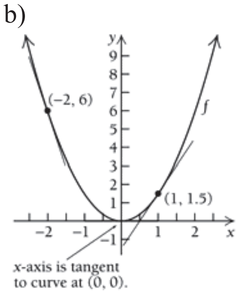
$=\dfrac{-h}{h\sqrt{x}\sqrt{x+h}\left(\sqrt{x}+\sqrt{x+h}\right)}$

$=\dfrac{-1}{\sqrt{x}\sqrt{x+h}\left(\sqrt{x}+\sqrt{x+h}\right)}$

Exercise Set 1.4

1. $f(x)=\frac{3}{2}x^2$

a), b)



c) Find the simplified difference quotient first.

$$\frac{f(x+h)-f(x)}{h}$$
$$=\frac{\frac{3}{2}(x+h)^2-\frac{3}{2}x^2}{h}$$
$$=\frac{\frac{3}{2}(x^2+2xh+h^2)-\frac{3}{2}x^2}{h}$$
$$=\frac{\frac{3}{2}x^2+3xh+\frac{3}{2}h^2-\frac{3}{2}x^2}{h}$$
$$=\frac{3xh+\frac{3}{2}h^2}{h}$$
$$=\frac{h\left(3x+\frac{3}{2}h\right)}{h}$$
$$=3x+\frac{3}{2}h \quad \text{Simplified difference quotient}$$

Now we will find the limit of the difference quotient as $h\to 0$ using the simplified difference quotient.

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to 0}\left(3x+\frac{3}{2}h\right)$$
$$=3x$$

Thus, $f'(x)=3x$.

d) Find the values of the derivative by making the appropriate substitutions.

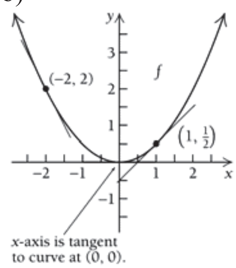
$f'(-2)=3(-2)=-6$ Substituting -2 for x

$f'(0)=3(0)=0$ Substituting 0 for x

$f'(1)=3(1)=3$ Substituting 1 for x

2. $f(x)=\frac{1}{2}x^2$

a), b)



c) Find the simplified difference quotient first.

$$\frac{f(x+h)-f(x)}{h}$$
$$=\frac{\frac{1}{2}(x+h)^2-\frac{1}{2}x^2}{h}$$
$$=\frac{\frac{1}{2}(x^2+2xh+h^2)-\frac{1}{2}x^2}{h}$$
$$=\frac{\frac{1}{2}x^2+xh+\frac{1}{2}h^2-\frac{1}{2}x^2}{h}$$
$$=\frac{xh+\frac{1}{2}h^2}{h}$$
$$=\frac{h\left(x+\frac{1}{2}h\right)}{h}$$
$$=x+\frac{1}{2}h \quad \text{Simplified difference quotient}$$

Now we will find the limit of the difference quotient as $h\to 0$ using the simplified difference quotient.

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to 0}\left(x+\frac{1}{2}h\right)$$
$$=x$$

Thus, $f'(x)=x$.

d) Find the values of the derivative by making the appropriate substitutions.

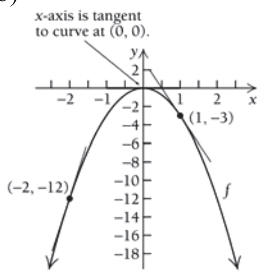
$f'(-2)=(-2)=-2$ Substituting -2 for x

$f'(0)=(0)=0$ Substituting 0 for x

$f'(1)=(1)=1$ Substituting 1 for x

3. $f(x) = -3x^2$

a), b)



c) Find the simplified difference quotient first.

$$\frac{f(x+h)-f(x)}{h}$$
$$= \frac{-3(x+h)^2 - (-3x^2)}{h}$$
$$= \frac{-3(x^2+2xh+h^2)-(-3x^2)}{h}$$
$$= \frac{-3x^2-6xh-3h^2+3x^2}{h}$$
$$= \frac{-6xh-3h^2}{h}$$
$$= \frac{h(-6x-3h)}{h}$$
$= -6x - 3h$ Simplified difference quotient

Now we will find the limit of the difference quotient as $h \to 0$ using the simplified difference quotient.

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to 0}(-6x-3h) = -6x$$

Thus, $f'(x) = -6x$.

d) Find the values of the derivative by making the appropriate substitutions.

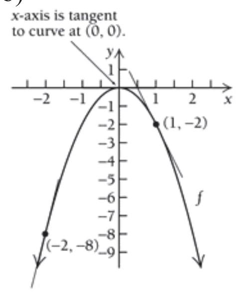
$f'(-2) = -6(-2) = 12$ Substituting −2 for x

$f'(0) = -6(0) = 0$ Substituting 0 for x

$f'(1) = -6(1) = -6$ Substituting 1 for x

4. $f(x) = -2x^2$

a), b)



c) Find the simplified difference quotient first.

$$\frac{f(x+h)-f(x)}{h}$$
$$= \frac{-2(x+h)^2 - (-2x^2)}{h}$$
$$= \frac{-2(x^2+2xh+h^2)-(-2x^2)}{h}$$
$$= \frac{-2x^2-4xh-2h^2+2x^2}{h}$$
$$= \frac{-4xh-2h^2}{h}$$
$$= \frac{h(-4x-2h)}{h}$$
$= -4x - 2h$ Simplified difference quotient

Now we will find the limit of the difference quotient as $h \to 0$ using the simplified difference quotient.

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to 0}(-4x-2h) = -4x$$

Thus, $f'(x) = -4x$.

d) Find the values of the derivative by making the appropriate substitutions.

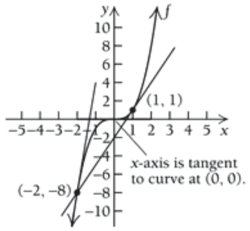
$f'(-2) = -4(-2) = 8$ Substituting −2 for x

$f'(0) = -4(0) = 0$ Substituting 0 for x

$f'(1) = -4(1) = -4$ Substituting 1 for x

5. $f(x) = x^3$

a), b)



c) Find the simplified difference quotient first.

$$\frac{f(x+h)-f(x)}{h}$$
$$=\frac{(x+h)^3-(x^3)}{h}$$
$$=\frac{(x^3+3x^2h+3xh^2+h^3)-(x^3)}{h}$$
$$=\frac{3x^2h+3xh^2+h^3}{h}$$
$$=\frac{h(3x^2+3xh+h^2)}{h}$$
$$=3x^2+3xh+h^2 \quad \text{Simplified difference quotient}$$

Now we will find the limit of the difference quotient as $h \to 0$ using the simplified difference quotient.

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to 0}(3x^2+3xh+h^2)$$
$$=3x^2$$

Thus, $f'(x) = 3x^2$.

d) Find the values of the derivative by making the appropriate substitutions.

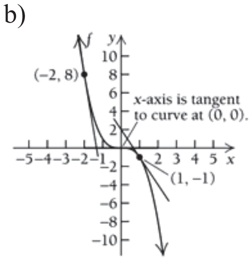
$f'(-2) = 3(-2)^2 = 12$ Substituting -2 for x

$f'(0) = 3(0)^2 = 0$ Substituting 0 for x

$f'(1) = 3(1)^2 = 3$ Substituting 1 for x

6. $f(x) = -x^3$

a), b)



c) Find the simplified difference quotient first.

$$\frac{f(x+h)-f(x)}{h}$$
$$=\frac{-(x+h)^3-(-x^3)}{h}$$
$$=\frac{-(x^3+3x^2h+3xh^2+h^3)-(-x^3)}{h}$$
$$=\frac{-x^3-3x^2h-3xh^2-h^3+x^3}{h}$$
$$=\frac{-3x^2h-3xh^2-h^3}{h}$$
$$=\frac{h(-3x^2-3xh-h^2)}{h}$$
$$=-3x^2-3xh-h^2 \quad \text{Simplified difference quotient}$$

Now we will find the limit of the difference quotient as $h \to 0$ using the simplified difference quotient.

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to 0}(-3x^2-3xh-h^2)$$
$$=-3x^2$$

Thus, $f'(x) = -3x^2$.

d) Find the values of the derivative by making the appropriate substitutions.

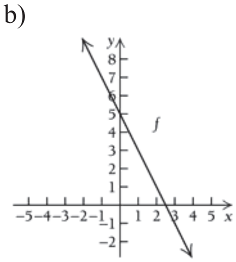
$f'(-2) = -3(-2)^2 = -12$ Substituting -2 for x

$f'(0) = -3(0)^2 = 0$ Substituting 0 for x

$f'(1) = -3(1)^2 = -3$ Substituting 1 for x

7. $f(x) = -2x + 5$

a), b)



c) Find the simplified difference quotient first.

$$\frac{f(x+h)-f(x)}{h}$$

$= \dfrac{-2(x+h)+5-(-2x+5)}{h}$

$= \dfrac{-2x-2h+5+2x-5}{h}$

$= \dfrac{-2h}{h}$

$= -2$ Simplified difference quotient

Now we will find the limit of the difference quotient as $h \to 0$ using the simplified difference quotient.

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to 0}(-2)$$
$$= -2$$

Thus, $f'(x) = -2$.

d) Since the derivative is a constant, the value of the derivative will be -2 regardless of the value of x.

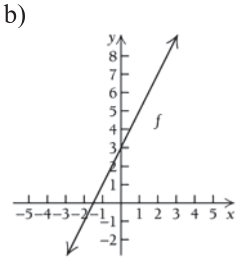
$f'(-2) = -2$ Substituting -2 for x

$f'(0) = -2$ Substituting 0 for x

$f'(1) = -2$ Substituting 1 for x

8. $f(x) = 2x+3$

a), b)



c) Find the simplified difference quotient first.

$\dfrac{f(x+h)-f(x)}{h}$

$= \dfrac{2(x+h)+3-(2x+3)}{h}$

$= \dfrac{2x+2h+3-2x-3}{h}$

$= \dfrac{2h}{h}$

$= 2$ Simplified difference quotient

Now we will find the limit of the difference quotient as $h \to 0$ using the simplified difference quotient.

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to 0}(2)$$
$$= 2$$

Thus, $f'(x) = 2$.

d) Since the derivative is a constant, the value of the derivative will be 2 regardless of the value of x.

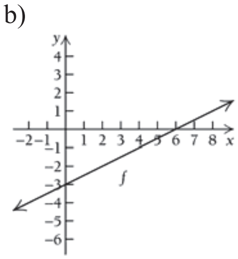
$f'(-2) = 2$ Substituting -2 for x

$f'(0) = 2$ Substituting 0 for x

$f'(1) = 2$ Substituting 1 for x

9. $f(x) = \dfrac{1}{2}x - 3$

a), b)



c) Find the simplified difference quotient first.

$\dfrac{f(x+h)-f(x)}{h}$

$= \dfrac{\frac{1}{2}(x+h)-3-\left(\frac{1}{2}x-3\right)}{h}$

$= \dfrac{\frac{1}{2}x+\frac{1}{2}h-3-\frac{1}{2}x+3}{h}$

$= \dfrac{\frac{1}{2}h}{h}$

$= \dfrac{1}{2}$ Simplified difference quotient

Now we will find the limit of the difference quotient as $h \to 0$ using the simplified difference quotient.

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to 0}\left(\frac{1}{2}\right)$$
$$= \frac{1}{2}$$

Thus, $f'(x) = \dfrac{1}{2}$.

d) Since the derivative is a constant, the value of the derivative will be $\frac{1}{2}$ regardless of the value of x.

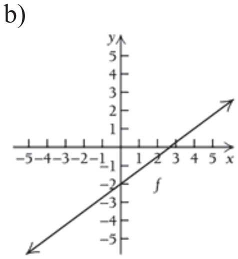
$f'(-2)=\frac{1}{2}$ Substituting -2 for x

$f'(0)=\frac{1}{2}$ Substituting 0 for x

$f'(1)=\frac{1}{2}$ Substituting 1 for x

10. $f(x)=\frac{3}{4}x-2$

a), b)



c) Find the simplified difference quotient first.

$$\frac{f(x+h)-f(x)}{h}$$
$$=\frac{\frac{3}{4}(x+h)-2-\left(\frac{3}{4}x-2\right)}{h}$$
$$=\frac{\frac{3}{4}x+\frac{3}{4}h-2-\frac{3}{4}x+2}{h}$$
$$=\frac{\frac{3}{4}h}{h}$$
$$=\frac{3}{4} \quad \text{Simplified difference quotient}$$

Now we will find the limit of the difference quotient as $h\to 0$ using the simplified difference quotient.

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to 0}\left(\frac{3}{4}\right)=\frac{3}{4}$$

Thus, $f'(x)=\frac{3}{4}$.

d) Since the derivative is a constant, the value of the derivative will be $\frac{3}{4}$ regardless of the value of x.

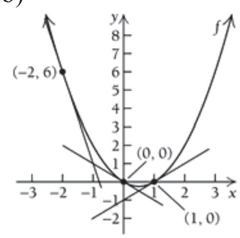
$f'(-2)=\frac{3}{4}$ Substituting -2 for x

$f'(0)=\frac{3}{4}$ Substituting 0 for x

$f'(1)=\frac{3}{4}$ Substituting 1 for x

11. $f(x)=x^2-x$

a), b)



c) Find the simplified difference quotient first. Referring to Exercise Set 1.3, Exercise 6 we know the simplified difference quotient is:

$$\frac{f(x+h)-f(x)}{h}=2x+h-1$$

Now we will find the limit of the difference quotient as $h\to 0$ using the simplified difference quotient.

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to 0}(2x+h-1)=2x-1$$

Thus, $f'(x)=2x-1$.

d) Find the values of the derivative by making the appropriate substitutions.

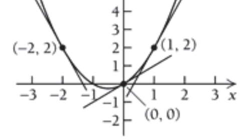
$f'(-2)=2(-2)-1=-5$

$f'(0)=2(0)-1=-1$

$f'(1)=2(1)-1=1$

12. $f(x)=x^2+x$

a), b)



c) Find the simplified difference quotient first.

$$\frac{f(x+h)-f(x)}{h}$$
$$=\frac{(x+h)^2+(x+h)-(x^2+x)}{h}$$
$$=\frac{x^2+2xh+h^2+x+h-x^2-x}{h}$$
$$=\frac{2xh+h^2+h}{h}$$
$$=\frac{h(2x+h+1)}{h}$$
$$=2x+h+1 \quad \text{Simplified difference quotient}$$

Now we will find the limit of the difference quotient as $h \to 0$ using the simplified difference quotient.

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to 0}(2x+h+1)$$
$$=2x+1$$

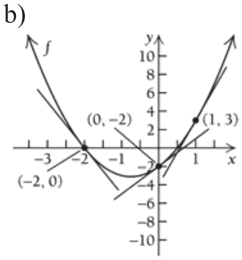
Thus, $f'(x)=2x+1$.

d) Find the values of the derivative by making the appropriate substitutions.

$$f'(-2)=2(-2)+1=-3$$
$$f'(0)=2(0)+1=1$$
$$f'(1)=2(1)+1=3$$

13. $f(x)=2x^2+3x-2$

a), b)



c) Find the simplified difference quotient first.

$$\frac{f(x+h)-f(x)}{h}$$
$$=\frac{\left(2(x+h)^2+3(x+h)-2\right)-\left(2x^2+3x-2\right)}{h}$$
$$=\frac{2\left(x^2+2xh+h^2\right)+3x+3h-2-2x^2-3x+2}{h}$$
$$=\frac{2x^2+4xh+2h^2+3x+3h-2-2x^2-3x+2}{h}$$
$$=\frac{4xh+2h^2+3h}{h}$$
$$=\frac{h(4x+2h+3)}{h}$$
$$=4x+2h+3 \quad \text{Simplified difference quotient}$$

Now we will find the limit of the difference quotient as $h \to 0$ using the simplified difference quotient.

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to 0}(4x+2h+3)$$
$$=4x+3$$

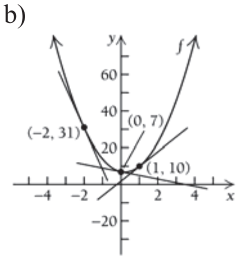
Thus, $f'(x)=4x+3$.

d) Find the values of the derivative by making the appropriate substitutions.

$$f'(-2)=4(-2)+3=-5$$
$$f'(0)=4(0)+3=3$$
$$f'(1)=4(1)+3=7$$

14. $f(x)=5x^2-2x+7$

a), b)



c) Find the simplified difference quotient first.

$$\frac{f(x+h)-f(x)}{h}$$

$$= \frac{\left(5(x+h)^2 - 2(x+h) + 7\right) - \left(5x^2 - 2x + 7\right)}{h}$$

$$= \frac{5\left(x^2 + 2xh + h^2\right) - 2x - 2h + 7 - 5x^2 + 2x - 7}{h}$$

$$= \frac{5x^2 + 10xh + 5h^2 - 2x - 2h + 7 - 5x^2 + 2x - 7}{h}$$

$$= \frac{10xh + 5h^2 - 2h}{h}$$

$$= \frac{h(10x + 5h - 2)}{h}$$

$= 10x + 5h - 2$ Simplified difference quotient

Now we will find the limit of the difference quotient as $h \to 0$ using the simplified difference quotient.

$$\lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0}(10x + 5h - 2)$$
$$= 10x - 2$$

Thus, $f'(x) = 10x - 2$.

d) Find the values of the derivative by making the appropriate substitutions.

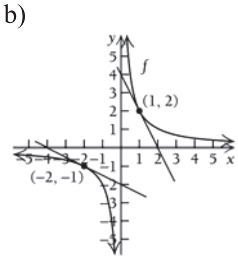
$f'(-2) = 10(-2) - 2 = -22$

$f'(0) = 10(0) - 2 = -2$

$f'(1) = 10(1) - 2 = 8$

15. $f(x) = \dfrac{2}{x}$

a), b)



There is no tangent line for $x = 0$

c) Find the simplified difference quotient first. Referring to Exercise Set 1.3, Exercise 7, we know that the simplified difference quotient is:

$$\frac{f(x+h) - f(x)}{h} = \frac{-2}{x(x+h)}$$

Now we will find the limit of the difference quotient as $h \to 0$ using the simplified difference quotient.

$$\lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0}\left(\frac{-2}{x(x+h)}\right)$$
$$= \frac{-2}{x^2}$$

Thus, $f'(x) = \dfrac{-2}{x^2}$.

d) Find the values of the derivative by making the appropriate substitutions.

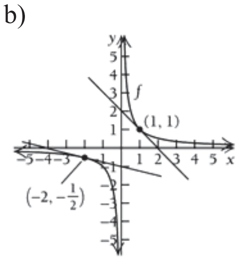
$f'(-2) = \dfrac{-2}{(-2)^2} = -\dfrac{1}{2}$

$f'(0) = \dfrac{-2}{(0)^2}$; Thus, $f'(0)$ does not exist.

$f'(1) = \dfrac{-2}{(1)^2} = -2$

16. $f(x) = \dfrac{1}{x}$

a), b)



There is no tangent line for $x = 0$

c) Find the simplified difference quotient first.

$$\frac{f(x+h) - f(x)}{h}$$

$$= \frac{\left(\frac{1}{x+h}\right) - \left(\frac{1}{x}\right)}{h}$$

$$= \frac{\left(\frac{1}{x+h}\cdot\frac{x}{x}\right) - \left(\frac{1}{x}\cdot\frac{(x+h)}{(x+h)}\right)}{h}$$

$$= \frac{\left(\frac{x}{x(x+h)}\right) - \left(\frac{(x+h)}{x(x+h)}\right)}{h}$$

$$= \frac{\frac{-h}{x(x+h)}}{h}$$

$$= \frac{-h}{x(x+h)} \cdot \frac{1}{h}$$

$= \dfrac{-1}{x(x+h)}$ Simplified difference quotient

Now we will find the limit of the difference quotient as $h \to 0$ using the simplified difference quotient.

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to 0}\left(\frac{-1}{x(x+h)}\right)$$
$$= \frac{-1}{x(x+0)}$$
$$= \frac{-1}{x^2}$$

Thus, $f'(x) = \frac{-1}{x^2}$.

d) Find the values of the derivative by making the appropriate substitutions.

$$f'(-2) = \frac{-1}{(-2)^2} = -\frac{1}{4}$$

$f'(0) = \frac{-1}{(0)^2}$; Thus, $f'(0)$ does not exist.

$$f'(1) = \frac{-1}{(1)^2} = -1$$

17. We know that $f'(x) = 3x^2$.

a) At $(-2,-8)$: $f'(-2) = 3(-2)^2 = 12$

$$y - y_1 = m(x - x_1)$$
$$y-(-8) = 12(x-(-2))$$
$$y+8 = 12x+24$$
$$y = 12x+16$$

b) At $(0,0)$: $f'(0) = 3(0)^2 = 0$

$$y - y_1 = m(x - x_1)$$
$$y-(0) = 0(x-(0))$$
$$y = 0$$

c) At $(4,64)$: $f'(4) = 3(4)^2 = 48$

$$y - y_1 = m(x - x_1)$$
$$y-64 = 48(x-4)$$
$$y-64 = 48x-192$$
$$y = 48x-128$$

18. a) We know that $f'(x) = 2x$.

$f'(3) = 2(3) = 6$, so the slope of the line tangent to the curve at $(3,9)$ is 6. We substitute the point and the slope into the point-slope equation to find the equation of the tangent line.

$$y - y_1 = m(x - x_1)$$
$$y-9 = 6(x-3)$$
$$y-9 = 6x-18$$
$$y = 6x-9$$

b) $f'(-1) = 2(-1) = -2$, so the slope of the line tangent to the curve at $(-1,1)$ is -2. We substitute the point and the slope into the point-slope equation to find the equation of the tangent line.

$$y - y_1 = m(x - x_1)$$
$$y-1 = -2(x-(-1))$$
$$y-1 = -2x-2$$
$$y = -2x-1$$

c) $f'(10) = 2(10) = 20$, so the slope of the line tangent to the curve at $(10,100)$ is 20. We substitute the point and the slope into the point-slope equation to find the equation of the tangent line.

$$y - y_1 = m(x - x_1)$$
$$y-100 = 20(x-10)$$
$$y-100 = 20x-200$$
$$y = 20x-100$$

19. From Exercise 15 we know that $f'(x) = \frac{-2}{x^2}$.

a) $f'(1) = \frac{-2}{(1)^2} = -2$, so the slope of the line tangent to the curve at $(1,2)$ is -2. We substitute the point and the slope into the point-slope equation to find the equation of the tangent line.

$$y - y_1 = m(x - x_1)$$
$$y-2 = -2(x-1)$$
$$y-2 = -2x+2$$
$$y = -2x+4$$

b) $f'(-1)=\frac{-2}{(-1)^2}=-2$, so the slope of the line tangent to the curve at $(-1,2)$ is -2. We substitute the point and the slope into the point-slope equation to find the equation of the tangent line.

$$\begin{aligned} y-y_1&=m(x-x_1)\\ y-(-2)&=-2(x-(-1))\\ y+2&=-2x-2\\ y&=-2x-4 \end{aligned}$$

c) $f'(100)=\frac{-2}{(100)^2}=-0.0002$, so the slope of the line tangent to the curve at $(100,0.02)$ is -0.002. We substitute the point and the slope into the point-slope equation to find the equation of the tangent line.

$$\begin{aligned} y-y_1&=m(x-x_1)\\ y-0.02&=-0.0002(x-100)\\ y-0.02&=-0.0002x+0.02\\ y&=-0.0002x+0.04 \end{aligned}$$

20. First, we find $f'(x)$.

$$\begin{aligned} \frac{f(x+h)-f(x)}{h}&=\frac{\left(\frac{-1}{x+h}\right)-\left(\frac{-1}{x}\right)}{h}\\ &=\frac{\left(\frac{-1}{x+h}\cdot\frac{x}{x}\right)-\left(\frac{-1}{x}\cdot\frac{(x+h)}{(x+h)}\right)}{h}\\ &=\frac{\left(\frac{-x}{x(x+h)}\right)-\left(\frac{-x-h}{x(x+h)}\right)}{h}\\ &=\frac{\frac{h}{x(x+h)}}{h}\\ &=\frac{h}{x(x+h)}\cdot\frac{1}{h}\\ &=\frac{1}{x(x+h)} \end{aligned}$$

Taking the limit as $h\to 0$ we have:

$$\begin{aligned} \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}&=\lim_{h\to 0}\frac{1}{x(x+h)}\\ &=\frac{1}{x^2} \end{aligned}$$

Thus, . $f'(x)=\frac{1}{x^2}$ . .

a) At $(-1,1)$: $f'(-1)=\frac{1}{(-1)^2}=1$

$$\begin{aligned} y-y_1&=m(x-x_1)\\ y-1&=1(x-(-1))\\ y-1&=x+1\\ y&=x+2 \end{aligned}$$

b) At $\left(2,-\frac{1}{2}\right)$: $f'(2)=\frac{1}{(2)^2}=\frac{1}{4}$

$$\begin{aligned} y-y_1&=m(x-x_1)\\ y-\left(-\frac{1}{2}\right)&=\frac{1}{4}(x-2)\\ y+\frac{1}{2}&=\frac{1}{4}x-\frac{1}{2}\\ y&=\frac{1}{4}x-1 \end{aligned}$$

c) At $\left(5,-\frac{1}{5}\right)$: $f'(5)=\frac{1}{(5)^2}=\frac{1}{25}$

$$\begin{aligned} y-y_1&=m(x-x_1)\\ y-\left(-\frac{1}{5}\right)&=\frac{1}{25}(x-5)\\ y+\frac{1}{5}&=\frac{1}{25}x-\frac{1}{5}\\ y&=\frac{1}{25}x-\frac{2}{5} \end{aligned}$$

21. First, we will find $f'(x)$

$$\begin{aligned} &\frac{f(x+h)-f(x)}{h}\\ &=\frac{\left((x+h)^2-2(x+h)\right)-\left(x^2-2x\right)}{h}\\ &=\frac{x^2+2xh+h^2-2x-2h-x^2+2x}{h}\\ &=\frac{2xh+h^2-2h}{h}\\ &=\frac{h(2x+h-2)}{h}\\ &=2x+h-2 \quad \text{Simplified difference quotient} \end{aligned}$$

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$
$$=\lim_{h\to 0}(2x+h-2)$$
$$=2x-2$$

Thus, $f'(x)=2x-2$.

a) At $(-2,8)$: $f'(-2)=2(-2)-2=-6$

$$y-y_1=m(x-x_1)$$
$$y-8=-6(x-(-2))$$
$$y-8=-6x-12$$
$$y=-6x-4$$

b) At $(1,-1)$: $f'(1)=2(1)-2=0$

$$y-y_1=m(x-x_1)$$
$$y-(-1)=0(x-1)$$
$$y+1=0$$
$$y=-1$$

c) At $(4,8)$: $f'(4)=2(4)-2=6$

$$y-y_1=m(x-x_1)$$
$$y-8=6(x-4)$$
$$y-8=6x-24$$
$$y=6x-16$$

22. First, we find $f'(x)$:

$$\frac{f(x+h)-f(x)}{h}$$
$$=\frac{\left(4-(x+h)^2\right)-\left(4-x^2\right)}{h}$$
$$=\frac{4-\left(x^2+2xh+h^2\right)-4+x^2}{h}$$
$$=\frac{4-x^2-2xh-h^2-4+x^2}{h}$$
$$=\frac{-2xh-h^2}{h}$$
$$=\frac{h(-2x-h)}{h}$$
$$=-2x-h \quad \text{Simplified difference quotient}$$

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$
$$=\lim_{h\to 0}(-2x-h)$$
$$=-2x$$

a) $f'(-1)=-2(-1)=2$, so the slope of the line tangent to the curve at $(-1,3)$ is 2. We substitute the point and the slope into the point-slope equation to find the equation of the tangent line.

$$y-y_1=m(x-x_1)$$
$$y-3=2(x-(-1))$$
$$y-3=2x+2$$
$$y=2x+5$$

b) $f'(0)=-2(0)=0$, so the slope of the line tangent to the curve at $(0,4)$ is 0. We substitute the point and the slope into the point-slope equation to find the equation of the tangent line at the top of the next column.

Using the point-slope equation we have:

$$y-y_1=m(x-x_1)$$
$$y-4=0(x-0)$$
$$y-4=0$$
$$y=4$$

c) $f'(5)=-2(5)=-10$, so the slope of the line tangent to the curve at $(5,-21)$ is -10. We substitute the point and the slope into the point-slope equation to find the equation of the tangent line.

$$y-y_1=m(x-x_1)$$
$$y-(-21)=-10(x-5)$$
$$y+21=-10x+50$$
$$y=-10x+29$$

23. Find the simplified difference quotient for $f(x)=mx+b$ first.

$$\frac{f(x+h)-f(x)}{h}$$
$$=\frac{m(x+h)+b-(mx+b)}{h}$$
$$=\frac{mx+mh+b-mx-b}{h}$$
$$=\frac{mh}{h}$$
$$=m \quad \text{Simplified difference quotient}$$

Now we will find the limit of the difference quotient as $h \to 0$ using the simplified difference quotient.

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to 0}(m) = m$$

Thus, $f'(x) = m$.

24. $f(x) = ax^2 + bx$

$$\frac{f(x+h)-f(x)}{h}$$
$$= \frac{\left(a(x+h)^2 + b(x+h)\right) - \left(ax^2 + bx\right)}{h}$$
$$= \frac{a\left(x^2 + 2hx + h^2\right) + bx + bh - ax^2 - bx}{h}$$
$$= \frac{ax^2 + 2ahx + ah^2 + bx + bh - ax^2 - bx}{h}$$
$$= \frac{2ahx + ah^2 + bh}{h}$$
$$= \frac{h(2ax + ah + b)}{h}$$
$$= 2ax + ah + b$$

Next we take the limit as $h \to 0$

$$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to 0}(2ax + ah + b) = 2ax + b$$

25. The function is not differentiable at x_1, x_3, x_5 because there is a "corner" at each of those points. The function is not differentiable at x_4, x_7 because the function is discontinuous at those points.

26. The function is not differentiable at x_2, x_5 because there is a "corner" at each of those points. The function is not differentiable at x_0, x_4, x_6, x_7 because the function is discontinuous at those points.

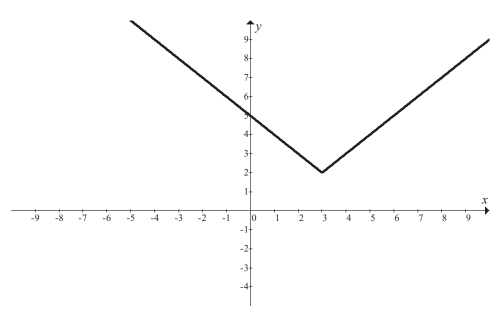
27. If a function has a "corner," it will not be differentiable at that point. Thus, the function is not differentiable at x_3, x_4, x_6. The function has a vertical tangent at x_{12}. Vertical lines have undefined slope, hence the function is not differentiable at x_{12}. Also, if a function is discontinuous at some point *a*, then it is not differentiable at *a*. The function is discontinuous at the point x_0, thus it is not differentiable at x_0.
Therefore, the graph is not differentiable at the points x_0, x_3, x_4, x_6, x_{12}.

28. The function is not differentiable at x_2, x_4, x_5 because there is a "corner" at each of those points. The function is not differentiable at x_7, x_8 because the function is discontinuous at those points.

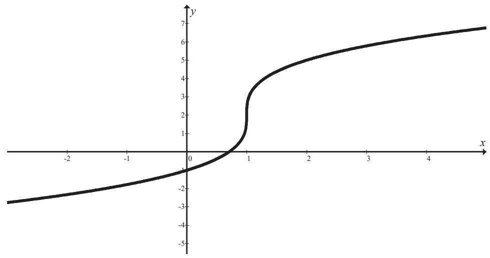
29. If a function has a "corner," it will not be differentiable at that point. Thus, the function is not differentiable at x_3. The function has a vertical tangent at x_1. Vertical lines have undefined slope, hence the function is not differentiable at x_1. Also, if a function is discontinuous at some point *a*, then it is not differentiable at *a*. The function is discontinuous at the point x_1, x_2,x_4, thus it is not differentiable at x_1, x_2, x_4.
Therefore, the graph is not differentiable at the points x_1, x_2, x_3, x_4.

30. The function is not differentiable at $x_1, x_2, x_3, x_4, x_6, x_9, x_{10}$ because there is a "corner" at each of those points. The function is not differentiable at x_5, x_7 because the function is discontinuous at those points.

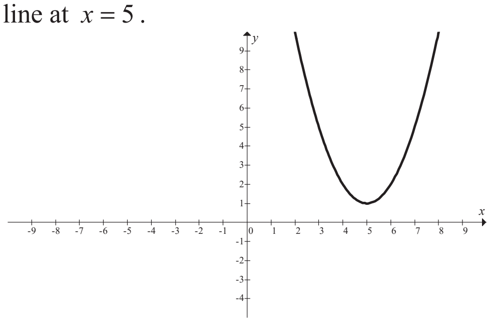
31. The following graph is continuous but not differentiable, at $x = 3$.



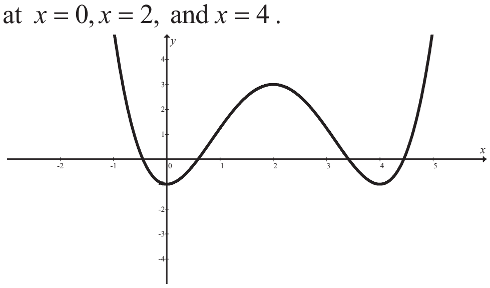
32. The following graph is smooth for all x but is not differentiable at $x = 1$.



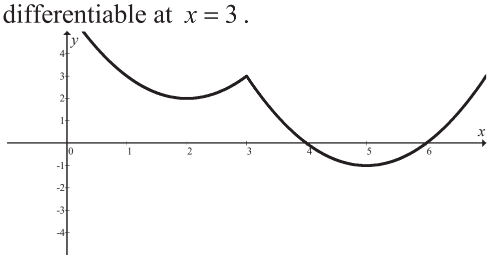
33. The following graph has a horizontal tangent line at $x = 5$.



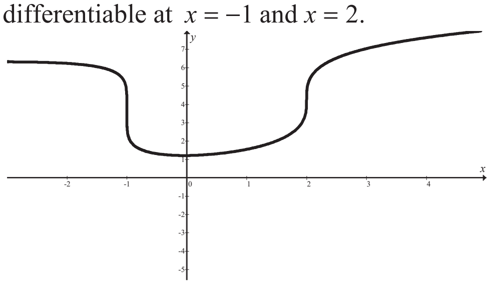
34. The following graph has horizontal tangent line at $x = 0, x = 2,$ and $x = 4$.



35. The following graph has horizontal tangent lines at $x = 2$ and $x = 5$ and is continuous but not differentiable at $x = 3$.



36. The following graph is smooth for all x but not differentiable at $x = -1$ and $x = 2$.



37. Answer will vary

38. Answers will vary

39. False

40. True

41. False

42. False

43. The lines L_2, L_3, L_4, L_6 appear to be tangent lines. The slopes appear to be the same as the instantaneous rate of change of the function at the indicated points.

44. The graph is left to the student. As the points Q approach P, the slopes of the secant lines approach the slope of the tangent line at P.

45. $f(x) = \dfrac{1}{1-x}$

At the top of the next page, we find the limit of the difference quotient as $h \to 0$.

Taking the limit, we have:

$$\lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{1}{(1-x)(1-x-h)}$$
$$= \frac{1}{(1-x)(1-x-0)}$$
$$= \frac{1}{(1-x)^2}$$

Thus, $f'(x) = \dfrac{1}{(1-x)^2}$.

46. $f(x) = x^5$

We now find the limit of the difference quotient as $h \to 0$.

Thus,

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$
$$= \lim_{h\to 0} 5x^4 + 10x^3h + 10x^2h^2 + 5xh^3 + h^4$$
$$= 5x^4$$

47. $f(x) = \dfrac{1}{x^2}$

Find the difference quotient first.

$$\frac{f(x+h) - f(x)}{h}$$

$$\frac{f(x+h)-f(x)}{h}=\frac{\frac{1}{(x+h)^2}-\frac{1}{x^2}}{h}$$

$$=\frac{\frac{1}{(x+h)^2}\cdot\frac{x^2}{x^2}-\frac{1}{x^2}\cdot\frac{(x+h)^2}{(x+h)^2}}{h}$$

$$=\frac{\frac{x^2-(x+h)^2}{x^2(x+h)^2}}{h}$$

$$=\frac{x^2-x^2-2xh-h^2}{x^2(x+h)^2}\cdot\frac{1}{h}$$

$$=\frac{-2xh-h^2}{x^2(x+h)^2}\cdot\frac{1}{h}$$

$$=\frac{-2x-h}{x^2(x+h)^2}$$

Next, we will find the limit of the difference quotient as $h\to 0$.

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$=\lim_{h\to 0}\frac{-2x-h}{x^2(x+h)^2}$$

$$=\frac{-2x}{x^2(x+0)^2}$$

$$=\frac{-2x}{x^4}$$

$$=\frac{-2}{x^3}$$

Thus, $f'(x)=\frac{-2}{x^3}$.

48. $f(x)=\sqrt{x}$

We now find the limit of the difference quotient as $h\to 0$.

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to 0}\frac{1}{\sqrt{x+h}+\sqrt{x}}$$

$$=\frac{1}{\sqrt{x+0}+\sqrt{x}}$$

$$=\frac{1}{2\sqrt{x}}$$

Thus, $f'(x)=\frac{1}{2\sqrt{x}}$

49. $f(x)=\sqrt{2x+1}$

We found the simplified difference quotient in Exercise 73 of Exercise Set 1.3. We now find the limit of the difference quotient as $h\to 0$.

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$=\lim_{h\to 0}\frac{2}{\sqrt{2(x+h)+1}+\sqrt{2x+1}}$$

$$=\frac{2}{\sqrt{2(x+0)+1}+\sqrt{2x+1}}$$

$$=\frac{2}{\sqrt{2x+1}+\sqrt{2x+1}}$$

$$=\frac{2}{2\sqrt{2x+1}}$$

$$=\frac{1}{\sqrt{2x+1}}$$

Thus, $f'(x)=\frac{1}{\sqrt{2x+1}}$.

50. $f(x)=\frac{1}{\sqrt{x}}$

We found the simplified difference quotient in Exercise 74 of Exercise Set 1.3. We now find the limit of the difference quotient as $h\to 0$.

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$=\lim_{h\to 0}\frac{-1}{\sqrt{x}\sqrt{x+h}\left(\sqrt{x}+\sqrt{x+h}\right)}$$

$$=\frac{-1}{\sqrt{x}\sqrt{x+0}\left(\sqrt{x}+\sqrt{x+0}\right)}$$

$$=\frac{-1}{x\left(2\sqrt{x}\right)}$$

$$=\frac{-1}{2x\sqrt{x}}$$

Thus, $f'(x)=\frac{-1}{2x\sqrt{x}}$.

51. $f(x)=ax^2+bx+c$

Find the difference quotient first.

$$\frac{f(x+h)-f(x)}{h}$$

$$\frac{f(x+h)-f(x)}{h} = \frac{a(x+h)^2 + b(x+h) + c - \left(ax^2 + bx + c\right)}{h}$$
$$= \frac{ax^2 + 2axh + ah^2 + bx + bh + c - ax^2 - bx - c}{h}$$
$$= \frac{2axh + ah^2 + bh}{h}$$
$$= 2ax + ah + b$$

Next, we will find the limit of the difference quotient as $h \to 0$.

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$
$$= \lim_{h\to 0} (2ax + ah + b)$$
$$= 2ax + a(0) + b$$
$$= 2ax + b$$

Thus, $f'(x) = 2ax + b$.

52. The error was made when the student did not determine the implied domain of the function.
$$f(x) = \frac{x^2 + 4x + 3}{x+1}$$
is undefined at $x = -1$. Therefore, $f(x)$ is not differentiable at $x = -1$. Once the domain is properly defined, the student can find the derivative of the function.

53. a) The domain of the rational function is restricted to those input values that do not result in division by 0. The domain for $f(x) = \frac{x^2 - 9}{x+3}$ consists of all real numbers except -3. Since $f(-3)$ does not exist, the function is not continuous at -3. Thus, the function is not differentiable at $x = -3$.

b) No, the function is differentiable for all values in the domain.

c) ✎ Answers will vary. The simplest way of finding $f'(4)$ is to use the nDeriv function on your calculator. However, without using advanced technology the easiest way is to approximate the difference quotient using a very small value of h, and allowing the calculator to perform the basic computations. We illustrate using $h = 0.0001$. The difference quotient will be
$$\frac{f(x+h)-f(x)}{h} = \frac{f(4+0.0001)-f(4)}{0.0001}$$
Using the calculator to evaluate the function we have:
$$f(4.0001) = 1.0001$$
$$f(4) = 1$$
Plugging in these values we have:
$$\frac{f(4+0.0001)-f(4)}{0.0001} = \frac{1.0001-1}{0.0001} = 1$$
Therefore, $f'(4) = 1$.

54. a) The domain of the rational function is restricted to those input values that do not result in division by 0. The domain for $g(x) = \frac{x^2 + x}{2x}$ consists of all real numbers except 0. Since $f(0)$ does not exist, the function is not continuous at 0. Thus, the function is not differentiable at $x = 0$.

b) No, the function is differentiable for all values in the domain.

c) ✎ Answers will vary. The simplest way of finding $g'(3)$ is to use the nDeriv function on your calculator. However, without using advanced technology the easiest way is to approximate the difference quotient using a very small value of h, and allowing the calculator to perform the basic computations. We illustrate using $h = 0.0001$. The difference quotient will be
$$\frac{g(x+h)-g(x)}{h} = \frac{g(3+0.0001)-g(3)}{0.0001}$$
Using the calculator to evaluate the function we have:
$$g(3.0001) = 2.00005$$
$$g(3) = 2$$

Plugging in these values we have:

$$\frac{g(3+0.0001)-g(3)}{0.0001} \approx \frac{2.00005-2}{0.0001} \approx 0.5$$

Therefore, $g'(3) = 0.5$.

55. a) Looking at the graph of the function, we see there is a "corner" when $x = 3$. Therefore, $h(x) = |x-3| + 2$ is not differentiable at $x = 3$.

b) No, the function is differentiable for all values in the domain

c) Using the piecewise definition of

$$h(x) = |x-3| + 2 = \begin{cases} -(x-3)+2, & \text{for } x < 3 \\ (x-3)+2, & \text{for } x \geq 3 \end{cases}$$

We notice that:

$$h'(x) = \begin{cases} -1, & \text{for } x < 3 \\ 1, & \text{for } x > 3 \end{cases}$$

Therefore,

$h'(0) = -1;\ h'(1) = -1;$

$h'(4) = 1;\quad h'(10) = 1$

The "shortcut" is noticing that this function is a linear function with slope $m = -1$ for $x < 3$ and slope $m = 1$ for $x > 3$.

56. a) Looking at the graph of the function, we see there is a "corner" when $x = -5$. Therefore, $k(x) = 2|x+5|$ is not differentiable at $x = -5$.

b) No, the function is differentiable for all values in the domain

c) Using the piecewise definition of

$$k(x) = 2|x+5| = \begin{cases} -2(x+5), & \text{for } x < -5 \\ 2(x+5), & \text{for } x \geq -5 \end{cases}$$

We notice that:

$$h'(x) = \begin{cases} -2, & \text{for } x < -5 \\ 2, & \text{for } x > -5 \end{cases}$$

Therefore,

$h'(-10) = -2;\quad h'(-7) = -2$

$h'(-2) = 2;\quad h'(0) = 2$

The "shortcut" is noticing that this function is a linear function with slope $m = -2$ for $x < -5$ and slope $m = 2$ for $x > 5$.

57. The function $g(x) = \sqrt[3]{x}$ has a vertical slope at $x = 0$. Therefore, $g'(x)$ is not defined at $x = 0$. The correct conclusion is that $g(x)$ is differentiable for all real numbers x except $x = 0$.

58. a) The function $F(x)$ is continuous at $x = 2$, because

1) $F(2)$ exists, $F(2) = 5$

2) $\lim_{x \to 2^-} F(x) = 5$ and $\lim_{x \to 2^+} F(x) = 5$,

Therefore,

$\lim_{x \to 2} F(x) = 5$

3) $\lim_{x \to 2} F(x) = 5 = F(2)$.

b) The function $F(x)$ is not differentiable at $x = 2$ because there is a "corner" at $x = 2$.

59. a) The function $G(x)$ is continuous at $x = 1$, because

1) $G(1)$ exists, $G(1) = 1$

2) $\lim_{x \to 1} G(x) = 1$

3) $\lim_{x \to 1} G(x) = 1 = G(1)$.

b) The function $G(x)$ is differentiable at $x = 1$. $G'(1) = 3$.

60. In order for $H(x)$ to be differentiable at $x = 3$. $H(x)$ must be continuous at $x = 3$. It must also be "smooth" at $x = 3$ which means the slope as x approaches 3 from the left must equal the slope as x approaches 3 from the right.

For $x \leq 3$

$H'(3) = 4(3) - 1 = 11$.

Using this information, we know

$m = 11$.

We also know:

$\lim_{x \to 3^+} H(x) = 15$ in order for $H(x)$ to be continuous.

Using the above information and substituting $m = 11$ we have:

$$\begin{aligned} \lim_{x \to 3^+} 11(x) + b &= 15 \\ 11(3) + b &= 15 \\ 33 + b &= 15 \\ b &= -18 \end{aligned}$$

Therefore, the values $m = 11$ and $b = -18$ will make $H(x)$ differentiable at $x = 3$.

61. We must first find the values for the function at the given domain values:

$$f(4) = (4)^2 = 16 \quad \text{and} \quad f(-1) = (-1)^2 = 1$$

Substitute these values into the equation yields:

$$f'(c) = \frac{16-1}{4-(-1)} = \frac{15}{5} = 3.$$

To find the value of c, substitute into the derivative of the function:

$$3 = 2c$$

$$\frac{3}{2} = c$$

62. We must first find the values for the function at the given domain values:

$$f(5) = -(5)^3 = -125 \quad \text{and} \quad f(0) = -(0)^3 = 0$$

Substitute these values into the equation yields:

$$f'(c) = \frac{-125-0}{5-0} = \frac{-125}{5} = -25.$$

To find the value of c, substitute into the derivative of the function:

$$-25 = -3c^2$$

$$\frac{25}{3} = c^2$$

$$\frac{5}{\sqrt{3}} = c$$

63. The trucker's average speed was $\frac{290}{4} = 72.5$, and his distance function is differentiable for the entire 4–hour drive, so he must have driven 72.5 miles per hour at least once in that 4–hour period.

64. The Man Value Theorem does not apply because g is not differentiable everywhere in the interval.

Exercise Set 1.5

1. The four (4) ways to represent the derivative are:
a) The derivative of u with respect to v is $f'(v)$
b) The derivative of u with respect to v is u'
c) The derivative of u with respect to v is $\frac{du}{dv}$
d) The derivative of u with respect to v is
$\frac{d}{dv}f(v)$

2. The four (4) ways to represent the derivative are:
a) The derivative of s with respect to t is $g'(t)$
b) The derivative of s with respect to t is s'
c) The derivative of s with respect to t is $\frac{ds}{dt}$
d) The derivative of s with respect to t is
$\frac{d}{dt}g(t)$

3. The four (4) ways to represent the derivative are:
a) The derivative of p with respect to q is $R'(q)$
b) The derivative of p with respect to q is p'
c) The derivative of p with respect to q is $\frac{dp}{dq}$
d) The derivative of p with respect to q is
$\frac{d}{dq}R(q)$

4. The four (4) ways to represent the derivative are:
a) The derivative of m with respect to n is $G'(n)$
b) The derivative of m with respect to n is m'
c) The derivative of m with respect to n is $\frac{dm}{dn}$
d) The derivative of m with respect to n is
$\frac{d}{dn}G(n)$

5. The four (4) ways to represent the derivative are:
a) The derivative of h with respect to k is $m'(k)$
b) The derivative of h with respect to k is h'
c) The derivative of h with respect to k is $\frac{dh}{dk}$
d) The derivative of h with respect to k is
$\frac{d}{dk}m(k)$

6. The four (4) ways to represent the derivative are:
a) The derivative of C with respect to z is $T'(z)$
b) The derivative of C with respect to z is C'
c) The derivative of C with respect to z is $\frac{dC}{dz}$
d) The derivative of C with respect to z is
$\frac{d}{dz}T(z)$

7. $y = x^7$

$$\frac{dy}{dx} = \frac{d}{dx}x^7$$
$$= 7x^{7-1} \qquad \text{Theorem 1}$$
$$= 7x^6$$

8. $y = x^8$

$$\frac{dy}{dx} = 8x^{8-1} = 8x^7$$

9. $y = -3x$

$$\frac{dy}{dx} = \frac{d}{dx}(-3x)$$
$$= -3\frac{d}{dx}x \qquad \text{Theorem 3}$$
$$= -3\left(1x^{1-1}\right) \qquad \text{Theorem 1}$$
$$= -3\left(x^0\right)$$
$$= -3 \qquad \left[a^0 = 1\right]$$

10. $y = -0.5x$

$$\frac{dy}{dx} = -0.5$$

11. $y = 12$ Constant function

$\frac{dy}{dx} = \frac{d}{dx}12$

$= 0$ Theorem 2

12. $y = 7$

$\frac{dy}{dx} = 0$

13. $y = 2x^{15}$

$\frac{dy}{dx} = \frac{d}{dx}\left(2x^{15}\right)$

$= 2\frac{d}{dx}\left(x^{15}\right)$ Theorem 3

$= 2\left(15x^{15-1}\right)$ Theorem 1

$= 30x^{14}$

14. $y = 3x^{10}$

$\frac{dy}{dx} = 3\left(10x^{10-1}\right) = 30x^9$

15. $y = x^{-6}$

$\frac{dy}{dx} = \frac{d}{dx}x^{-6}$

$= -6x^{-6-1}$ Theorem 1

$= -6x^{-7}$

16. $y = x^{-8}$

$\frac{dy}{dx} = -8x^{-8-1} = -8x^{-9}$

17. $y = 4x^{-2}$

$\frac{dy}{dx} = \frac{d}{dx}\left(4x^{-2}\right)$

$= 4\frac{d}{dx}\left(x^{-2}\right)$ Theorem 3

$= 4\left(-2x^{-2-1}\right)$ Theorem 1

$= -8x^{-3}$

18. $y = 3x^{-5}$

$\frac{dy}{dx} = 3\left(-5x^{-5-1}\right) = -15x^{-6}$

19. $y = x^3 + 3x^2$

$\frac{dy}{dx} = \frac{d}{dx}\left(x^3 + 3x^2\right)$

$= \frac{d}{dx}x^3 + \frac{d}{dx}3x^2$ Theorem 4

$= \frac{d}{dx}x^3 + 3\frac{d}{dx}x^2$ Theorem 3

$= 3x^{3-1} + 3\left(2x^{2-1}\right)$ Theorem 1

$= 3x^2 + 6x$

20. $y = x^4 - 7x$

$\frac{dy}{dx} = 4x^{4-1} - 7\cdot 1x^{1-1} = 4x^3 - 7$

21. $y = 8\sqrt{x} = 8x^{1/2}$

$\frac{dy}{dx} = \frac{d}{dx}8x^{1/2}$

$= 8\frac{d}{dx}x^{1/2}$ Theorem 3

$\frac{dy}{dx} = 8\left(\frac{1}{2}x^{1/2-1}\right)$ Theorem 1

$= 4x^{-1/2}$

$= \frac{4}{x^{1/2}} = \frac{4}{\sqrt{x}}$ Properties of exponents

22. $y = 4\sqrt{x} = 4x^{1/2}$

$\frac{dy}{dx} = 4\left(\frac{1}{2}x^{1/2-1}\right) = 2x^{-1/2} = \frac{2}{x^{1/2}} = \frac{2}{\sqrt{x}}$

23. $y = x^{0.9}$

$\frac{dy}{dx} = \frac{d}{dx}x^{0.9}$

$= 0.9x^{0.9-1}$ Theorem 1

$= 0.9x^{-0.1}$

24. $y = x^{1.7}$

$\frac{dy}{dx} = 1.7x^{1.7-1} = 1.7x^{0.7}$

25. $y=\frac{1}{2}x^{4/5}$

$\frac{dy}{dx}=\frac{d}{dx}\left(\frac{1}{2}x^{4/5}\right)$

$=\frac{1}{2}\cdot\frac{d}{dx}\left(x^{4/5}\right)$ Theorem 3

$=\frac{1}{2}\left(\frac{4}{5}x^{4/5-1}\right)$ Theorem 1

$=\frac{2}{5}x^{-1/5}$

26. $y=-4.8x^{1/3}$

$\frac{dy}{dx}=-4.8\left(\frac{1}{3}x^{1/3-1}\right)=-1.6x^{-2/3}$

27. $y=\frac{7}{x^3}=7x^{-3}$

$\frac{dy}{dx}=\frac{d}{dx}\left(7x^{-3}\right)$

$=7\frac{d}{dx}\left(x^{-3}\right)$ Theorem 3

$=7\left(-3x^{-3-1}\right)$ Theorem 1

$=-21x^{-4}$

$=-\frac{21}{x^4}$ Properties of exponents

28. $y=\frac{6}{x^4}=6x^{-4}$

$\frac{dy}{dx}=6\left(-4x^{-4-1}\right)=-24x^{-5}=-\frac{24}{x^5}$

29. $\frac{d}{dx}\left(\sqrt[4]{x}-\frac{3}{x}\right)$

$=\frac{d}{dx}\sqrt[4]{x}-\frac{d}{dx}\frac{3}{x}$ Theorem 4

$=\frac{d}{dx}x^{1/4}-\frac{d}{dx}3x^{-1}$ Properties of exponents

$=\frac{d}{dx}x^{1/4}-3\frac{d}{dx}x^{-1}$ Theorem 3

$=\frac{1}{4}x^{1/4-1}-3\left(-1x^{-1-1}\right)$ Theorem 1

$=\frac{1}{4}x^{-3/4}+3x^{-2}$

$=\frac{1}{4x^{3/4}}+\frac{3}{x^2}$

$=\frac{1}{4\sqrt[4]{x^3}}+\frac{3}{x^2}$

30. $\frac{d}{dx}\left(\sqrt[3]{x}+\frac{4}{\sqrt{x}}\right)$

$=\frac{d}{dx}\sqrt[3]{x}+\frac{d}{dx}\frac{4}{\sqrt{x}}$

$=\frac{d}{dx}x^{1/3}+\frac{d}{dx}4x^{-1/2}$

$=\frac{1}{3}x^{-2/3}-2x^{-3/2}$

$=\frac{1}{3x^{2/3}}-\frac{2}{x^{3/2}}$

$=\frac{1}{3\sqrt[3]{x^2}}-\frac{2}{\sqrt{x^3}}=\frac{1}{3\sqrt[3]{x^2}}-\frac{2}{x\sqrt{x}}$

31. $\frac{d}{dx}\left(-2\sqrt[3]{x^5}\right)$

$=-2\frac{d}{dx}\left(\sqrt[3]{x^5}\right)$ Theorem 3

$=-2\frac{d}{dx}\left(x^{5/3}\right)$

$=-2\left(\frac{5}{3}x^{5/3-1}\right)$ Theorem 1

$=-\frac{10}{3}x^{2/3}=-\frac{10\sqrt[3]{x^2}}{3}$

32. $\frac{d}{dx}\left(-\sqrt[4]{x^3}\right)$

$=-\frac{d}{dx}\left(x^{3/4}\right)=-\frac{3}{4}x^{-1/4}=-\frac{3}{4x^{1/4}}=-\frac{3}{4\sqrt[4]{x}}$

33. $f(x)=\frac{5x}{11}=\frac{5}{11}x$

$f'(x)=\frac{d}{dx}\left(\frac{5}{11}x\right)$

$=\frac{5}{11}\frac{d}{dx}(x)$

$=\frac{5}{11}\left(1x^{1-1}\right)$

$=\frac{5}{11}$

34. $f(x)=\frac{2x}{3}=\frac{2}{3}x$

$f'(x)=\frac{d}{dx}\left(\frac{2}{3}x\right)$

$=\frac{2}{3}\frac{d}{dx}(x)$

$=\frac{2}{3}\left(1x^{1-1}\right)$

$=\frac{2}{3}$

35. $f(x)=\frac{2}{5x^6}=\frac{2}{5}x^{-6}$

$f'(x)=\frac{2}{5}\left(-6x^{-6-1}\right)=\frac{-12}{5}x^{-7}=-\frac{12}{5x^7}$

36. $f(x)=\frac{4}{7x^3}=\frac{4x^{-3}}{7}=\frac{4}{7}x^{-3}$

$f'(x)=\frac{d}{dx}\left(\frac{4}{7}x^{-3}\right)$

$=\frac{4}{7}\frac{d}{dx}\left(x^{-3}\right)$

$=\frac{4}{7}\left(-3x^{-3-1}\right)$

$=\frac{-12}{7}x^{-4}$

$=-\frac{12}{7x^4}$

37. $f(x)=\frac{4}{x}-x^{3/5}=4x^{-1}-x^{3/5}$

$f'(x)=4\left(-1x^{-1-1}\right)-\left(\frac{3}{5}x^{3/5-1}\right)$

$=-4x^{-2}-\frac{3}{5}x^{-2/5}$

$=-\frac{4}{x^2}-\frac{3}{5}x^{-2/5}$

38. $f(x)=\frac{5}{x}-x^{2/3}=5x^{-1}-x^{2/3}$

$f'(x)=\frac{d}{dx}\left(5x^{-1}-x^{2/3}\right)$

$=\frac{d}{dx}\left(5x^{-1}\right)-\frac{d}{dx}\left(x^{2/3}\right)$

$=5\frac{d}{dx}\left(x^{-1}\right)-\frac{d}{dx}\left(x^{2/3}\right)$

$=5\left(-1x^{-1-1}\right)-\frac{2}{3}x^{2/3-1}$

$=-5x^{-2}-\frac{2}{3}x^{-1/3}$

$=-\frac{5}{x^2}-\frac{2}{3}x^{-1/3}$

39. $f(x)=7x-14$

$f'(x)=7\left(1x^{1-1}\right)-0=7$

40. $f(x)=4x-7$

$f'(x)=\frac{d}{dx}(4x-7)$

$=\frac{d}{dx}(4x)-\frac{d}{dx}(7)$

$f'(x)=4\frac{d}{dx}(x)-\frac{d}{dx}(7)$

$=4\left(1x^{1-1}\right)-0$

$=4$

41. $f(x)=\frac{x^{3/2}}{3}=\frac{1}{3}x^{3/2}$

$f'(x)=\frac{1}{3}\left(\frac{3}{2}x^{3/2-1}\right)=\frac{1}{2}x^{1/2}$, or $\frac{1}{2}\sqrt{x}$

42. $f(x) = \frac{x^{4/3}}{4} = \frac{1}{4}x^{4/3}$

$$f'(x) = \frac{d}{dx}\left(\frac{1}{4}x^{4/3}\right) = \frac{1}{4}\frac{d}{dx}\left(x^{4/3}\right) = \frac{1}{4}\left(\frac{4}{3}x^{4/3-1}\right) = \frac{1}{3}x^{1/3}, \text{ or } \frac{1}{3}\sqrt[3]{x}$$

43. $f(x) = -0.01x^2 + 0.4x + 50$

$$f'(x) = -0.01\left(2x^{2-1}\right) + 0.4\left(1x^{1-1}\right) + 0 = -0.02x + 0.4$$

44. $f(x) = -0.01x^2 - 0.5x + 70$

$$f'(x) = \frac{d}{dx}\left(-0.01x^2 - 0.5x + 70\right)$$
$$= \frac{d}{dx}\left(-0.01x^2\right) - \frac{d}{dx}(0.5x) + \frac{d}{dx}(70)$$
$$= -0.01\frac{d}{dx}\left(x^2\right) - 0.5\frac{d}{dx}(x) + \frac{d}{dx}(70)$$
$$= -0.01\left(2x^{2-1}\right) - 0.5\left(1x^{1-1}\right) + 0$$
$$= -0.02x - 0.5$$

45. $y = x^{-3/4} - 3x^{2/3} + x^{5/4} + \frac{2}{x^4}$

$$y = x^{-3/4} - 3x^{2/3} + x^{5/4} + 2x^{-4}$$
$$y' = \left(\frac{-3}{4}x^{-3/4-1}\right) - 3\left(\frac{2}{3}x^{2/3-1}\right) + \left(\frac{5}{4}x^{5/4-1}\right) + 2\left(-4x^{-4-1}\right)$$
$$= \frac{-3}{4}x^{-7/4} - 2x^{-1/3} + \frac{5}{4}x^{1/4} - 8x^{-5}$$
$$= \frac{-3}{4}x^{-7/4} - 2x^{-1/3} + \frac{5}{4}x^{1/4} - \frac{8}{x^5}$$

46. $y = 3x^{-2/3} + x^{3/4} + x^{6/5} + \frac{8}{x^3}$

$$y = 3x^{-2/3} + x^{3/4} + x^{6/5} + 8x^{-3}$$
$$y' = \frac{d}{dx}\left(3x^{-2/3} + x^{3/4} + x^{6/5} + 8x^{-3}\right)$$
$$= \frac{d}{dx}\left(3x^{-2/3}\right) + \frac{d}{dx}\left(x^{3/4}\right) + \frac{d}{dx}\left(x^{6/5}\right) + \frac{d}{dx}\left(8x^{-3}\right)$$
$$= 3\left(\frac{-2}{3}x^{-2/3-1}\right) + \left(\frac{3}{4}x^{3/4-1}\right) + \left(\frac{6}{5}x^{6/5-1}\right) + 8\left(-3x^{-3-1}\right)$$
$$= -2x^{-5/3} + \frac{3}{4}x^{-1/4} + \frac{6}{5}x^{1/5} - 24x^{-4}$$
$$= -2x^{-5/3} + \frac{3}{4}x^{-1/4} + \frac{6}{5}x^{1/5} - \frac{24}{x^4}$$

47. $y = \frac{x}{7} + \frac{7}{x} = \frac{1}{7}x + 7x^{-1}$

$$y' = \frac{1}{7}\left(1x^{1-1}\right) + 7\left(-1x^{-1-1}\right) = \frac{1}{7} - 7x^{-2} = \frac{1}{7} - \frac{7}{x^2}$$

48. $y = \frac{2}{x} - \frac{x}{2} = 2x^{-1} - \frac{1}{2}x$

$$y' = \frac{d}{dx}\left(2x^{-1} - \frac{1}{2}x\right)$$
$$= \frac{d}{dx}\left(2x^{-1}\right) - \frac{d}{dx}\left(\frac{1}{2}x\right)$$
$$= 2\frac{d}{dx}\left(x^{-1}\right) - \frac{1}{2}\frac{d}{dx}(x)$$
$$= 2\left(-1x^{-1-1}\right) - \frac{1}{2}\left(1x^{1-1}\right)$$
$$= -2x^{-2} - \frac{1}{2}$$
$$= -\frac{2}{x^2} - \frac{1}{2}$$

49. $f(x) = \sqrt{x} = x^{1/2}$

$$f'(x) = \frac{1}{2}x^{1/2-1} = \frac{1}{2}x^{-1/2} = \frac{1}{2x^{1/2}} = \frac{1}{2\sqrt{x}}$$

Therefore,

$$f'(4) = \frac{1}{2\sqrt{4}} = \frac{1}{2\cdot 2} = \frac{1}{4}$$

50. $f(x) = x^2 + 4x - 5$

First, we find $f'(x)$

$$f'(x) = \frac{d}{dx}\left(x^2 + 4x - 5\right)$$
$$= \frac{d}{dx}\left(x^2\right) + 4\frac{d}{dx}(x) - \frac{d}{dx}5$$
$$= \left(2x^{2-1}\right) + 4\left(1x^{1-1}\right) - 0$$
$$= 2x + 4$$

Therefore,

$$f'(10) = 2(10) + 4$$
$$= 24$$

51. $y = x + \dfrac{2}{x^3} = x + 2x^{-3}$

$$\frac{dy}{dx} = 1x^{1-1} + 2\left(-3x^{-3-1}\right) = 1 - 6x^{-4} = 1 - \frac{6}{x^4}$$

Therefore,

$$\frac{dy}{dx} = 1 - \frac{6}{(1)^4}$$
$$= 1 - 6 = -5$$

52. $y = \dfrac{4}{x^2} = 4x^{-2}$

Find $\dfrac{dy}{dx}$ first.

$$\frac{dy}{dx} = \frac{d}{dx}\left(4x^{-2}\right)$$
$$= 4\frac{d}{dx}\left(x^{-2}\right)$$
$$= 4\left(-2x^{-2-1}\right)$$
$$= -8x^{-3}$$
$$= -\frac{8}{x^3}$$

Therefore,

$$\frac{dy}{dx} = -\frac{8}{(-2)^3}$$
$$= -\frac{8}{(-8)}$$
$$= 1$$

53. $y = \sqrt[3]{x} + \sqrt{x} = x^{\frac{1}{3}} + x^{\frac{1}{2}}$

$$\frac{dy}{dx} = \frac{1}{3}x^{\frac{1}{3}-1} + \frac{1}{2}x^{\frac{1}{2}-1}$$
$$= \frac{1}{3\sqrt[3]{x^2}} + \frac{1}{2\sqrt{x}}$$

Therefore,

$$\frac{dy}{dx} = \frac{1}{3\sqrt[3]{(64)^2}} + \frac{1}{2\sqrt{64}}$$
$$= \frac{1}{48} + \frac{1}{16}$$
$$= \frac{1}{12}$$

54. $y = x^3 + 2x - 5$

Find $\dfrac{dy}{dx}$ first.

$$\frac{dy}{dx} = \frac{d}{dx}\left(x^3 + 2x - 5\right)$$
$$= \frac{d}{dx}\left(x^3\right) + 2\frac{d}{dx}(x) - 5$$
$$= 3x^{3-1} + 2\left(x^{1-1}\right) - 0$$
$$= 3x^2 + 2$$

Therefore,

$$\frac{dy}{dx} = 3(-2)^2 + 2$$
$$= 3(4) + 2$$
$$= 14$$

55. $y = \dfrac{2}{5x^3} = \dfrac{2}{5}x^{-3}$

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{2}{5}x^{-3}\right)$$
$$= \frac{2}{5}\left(-3x^{-3-1}\right)$$
$$= -\frac{6}{5x^4}$$

Therefore,

$$\frac{dy}{dx} = -\frac{6}{5(4)^4}$$
$$= -\frac{3}{640}$$

56. $y=\frac{1}{3x^4}=\frac{1}{3}x^{-4}$

Find $\frac{dy}{dx}$ first.

$$\frac{dy}{dx}=\frac{d}{dx}\left(\frac{1}{3}x^{-4}\right)$$
$$=\frac{1}{3}\frac{d}{dx}\left(x^{-4}\right)$$
$$=\frac{1}{3}\left(-4x^{-4-1}\right)$$
$$=-\frac{4}{3}x^{-5}$$
$$=-\frac{4}{3x^5}$$

Therefore,

$$\frac{dy}{dx}=-\frac{4}{3(-1)^5}$$
$$=-\frac{4}{(-3)}$$
$$=\frac{4}{3}$$

57. $f(x)=x^2-\sqrt{x}=x^2-x^{1/2}$

$$f'(x)=2x^{2-1}-\frac{1}{2}x^{1/2-1}$$
$$=2x-\frac{1}{2x^{1/2}}$$
$$=2x-\frac{1}{2\sqrt{x}}$$

a) At $(1,0)$: $f'(1)=2(1)-\frac{1}{2\sqrt{1}}=2-\frac{1}{2}=\frac{3}{2}$

The equation of the tangent line is:

$$y-y_1=m(x-x_1)$$
$$y-0=\frac{3}{2}(x-1)$$
$$y=\frac{3}{2}x-\frac{3}{2}$$

b) At $(4,14)$:

$$f'(1)=2(4)-\frac{1}{2\sqrt{4}}=8-\frac{1}{4}=\frac{31}{4}$$

The equation of the tangent line is:

$$y-y_1=m(x-x_1)$$
$$y-14=\frac{31}{4}(x-4)$$
$$y-14=\frac{31}{4}x-31$$
$$y=\frac{31}{4}x-17$$

c) At $(9,78)$:

$$f'(9)=2(9)-\frac{1}{2\sqrt{9}}=18-\frac{1}{6}=\frac{107}{6}$$

The equation of the tangent line is:

$$y-y_1=m(x-x_1)$$
$$y-78=\frac{107}{6}(x-9)$$
$$y-78=\frac{107}{6}x-\frac{321}{2}$$
$$y=\frac{107}{6}x-\frac{165}{2}$$

58. We will need the derivative to find the slope of the tangent line at each of the indicated points. We find the derivative first.

$$f(x)=x^3-2x+1$$
$$f'(x)=\frac{d}{dx}\left(x^3-2x+1\right)$$
$$=\frac{d}{dx}\left(x^3\right)-\frac{d}{dx}(2x)+\frac{d}{dx}(1)$$
$$=\left(3x^{3-1}\right)-2\left(1x^{1-1}\right)+0$$
$$=3x^2-2$$

a) Using the derivative, we find the slope of the line tangent to the curve at point $(2,5)$ by evaluating the derivative at $x=2$. $f'(2)=3(2)^2-2=10$. Therefore the slope of the tangent line is 10. We use the point-slope equation to find the equation of the tangent line.

$$y-y_1=m(x-x_1)$$
$$y-5=10(x-2)$$
$$y-5=10x-20$$
$$y=10x-15$$

b) Using the derivative, we find the slope of the line tangent to the curve at point $(-1,2)$ by evaluating the derivative at $x=-1$.
$f'(-1)=3(-1)^2-2=1$. Therefore the slope of the tangent line is 1. We use the point-slope equation to find the equation of the tangent line.

$$y-y_1=m(x-x_1)$$
$$y-2=1(x-(-1))$$
$$y-2=x+1$$
$$y=x+3$$

c) Using the derivative, we find the slope of the line tangent to the curve at point $(0,1)$ by evaluating the derivative at $x=0$.
$f'(0)=3(0)^2-2=-2$. Therefore the slope of the tangent line is -2. We use the point-slope equation to find the equation of the tangent line.

$$y-y_1=m(x-x_1)$$
$$y-1=-2(x-0)$$
$$y-1=-2x$$
$$y=-2x+1$$

59. $f(x)=\sqrt[3]{x^2}=x^{\frac{2}{3}}$

$$f'(x)=\frac{2}{3}x^{\frac{2}{3}-1}$$
$$=\frac{2}{3\sqrt[3]{x}}$$

a) At $(-1,1)$: $f'(-1)=\frac{2}{3\sqrt[3]{-1}}=-\frac{2}{3}$

The equation of the tangent line is:

$$y-y_1=m(x-x_1)$$
$$y-1=-\frac{2}{3}(x+1)$$
$$y=-\frac{2}{3}x+\frac{1}{3}$$

b) At $(1,1)$:

$$f'(1)=\frac{2}{3\sqrt[3]{1}}=\frac{2}{3}$$

The equation of the tangent line is:

$$y-y_1=m(x-x_1)$$
$$y-1=\frac{2}{3}(x-1)$$
$$y=\frac{2}{3}x+\frac{1}{3}$$

c) At $(8,4)$:

$$f'(8)=\frac{2}{3\sqrt[3]{8}}=\frac{1}{3}$$

The equation of the tangent line is:

$$y-y_1=m(x-x_1)$$
$$y-4=\frac{1}{3}(x-8)$$
$$y=\frac{1}{3}x+\frac{4}{3}$$

60. We will need the derivative to find the slope of the tangent line at each of the indicated points. We find the derivative first.

$$f(x)=\frac{1}{x^2}=x^{-2}$$
$$f'(x)=\frac{d}{dx}\left(x^{-2}\right)$$
$$=-2x^{-2-1}$$
$$=-2x^{-3}$$
$$=-\frac{2}{x^3}$$

a) Using the derivative, we find the slope of the line tangent to the curve at point $(1,1)$ by evaluating the derivative at $x=1$.
$f'(1)=-\frac{2}{(1)^3}=-2$. Therefore the slope of the tangent line is -2. We use the point-slope equation to find the equation of the tangent line.

$$y-y_1=m(x-x_1)$$
$$y-1=-2(x-1)$$
$$y-1=-2x+2$$
$$y=-2x+3$$

b) Using the derivative, we find the slope of the line tangent to the curve at point $\left(3,\frac{1}{9}\right)$ by evaluating the derivative at $x = 3$.

$f'(3) = -\frac{2}{(3)^3} = -\frac{2}{27}$. Therefore the slope of the tangent line is $-\frac{2}{27}$. We use the point-slope equation to find the equation of the tangent line.

$$y - y_1 = m(x - x_1)$$
$$y - \frac{1}{9} = -\frac{2}{27}(x - 3)$$
$$y - \frac{1}{9} = -\frac{2}{27}x + \frac{2}{9}$$
$$y = -\frac{2}{27}x + \frac{1}{3}$$

c) Using the derivative, we find the slope of the line tangent to the curve at point $\left(-2,\frac{1}{4}\right)$ by evaluating the derivative at $x = -2$.

$f'(-2) = -\frac{2}{(-2)^3} = \frac{1}{4}$. Therefore the slope of the tangent line is $\frac{1}{4}$.

We use the point-slope equation to find the equation of the tangent line.

$$y - y_1 = m(x - x_1)$$
$$y - \frac{1}{4} = \frac{1}{4}(x - (-2))$$
$$y - \frac{1}{4} = \frac{1}{4}x + \frac{1}{2}$$
$$y = \frac{1}{4}x + \frac{3}{4}$$

61. $y = -x^2 + 4$

$$\frac{dy}{dx} = -2x$$

Solve:

$$\frac{dy}{dx} = 0$$
$$-2x = 0$$
$$x = 0$$

For $x = 0,\ y = -(0)^2 + 4 = 4$.

Therefore, there is a horizontal tangent at the point $(0, 4)$.

62. $y = x^2 - 3$

A horizontal tangent line has slope equal to 0, so we first find the values of x that make $\frac{dy}{dx} = 0$.

First, we find the derivative.

$$\frac{dy}{dx} = \frac{d}{dx}\left(x^2 - 3\right)$$
$$\frac{dy}{dx} = \frac{d}{dx}x^2 - \frac{d}{dx}3$$
$$= 2x - 0$$
$$= 2x$$

Next, we set the derivative equal to zero and solve for x.

$$\frac{dy}{dx} = 0$$
$$2x = 0$$
$$x = \frac{0}{2} = 0$$

So the horizontal tangent will occur when $x = 0$

Next we find the point on the graph. For, so there is a horizontal tangent at the point $x = 0,\ y = (0)^2 - 3 = -3\ (0, -3)$.

63. $y = x^3 - 2$

$$\frac{dy}{dx} = 3x^2$$

Solve:

$$\frac{dy}{dx} = 0$$
$$3x^2 = 0$$
$$x = 0$$

For $x = 0,\ y = (0)^3 - 2 = -2$.

Therefore, there is a horizontal tangent at the point $(0, -2)$.

64. $y = -x^3 + 1$

A horizontal tangent line has slope equal to 0, so we first find the values of x that make $\frac{dy}{dx} = 0$.

First, we find the derivative.

$$\frac{dy}{dx}=\frac{d}{dx}\left(-x^3+1\right)$$
$$=-\frac{d}{dx}x^3+\frac{d}{dx}1$$
$$=-3x^2-0$$
$$=-3x^2$$

Next, we set the derivative equal to zero and solve for x.

$$\frac{dy}{dx}=0$$
$$-3x^2=0$$
$$x^2=0$$
$$x=0$$

So the horizontal tangent will occur when $x=0$. Next we find the point on the graph. For $x=0,\ y=-(0)^3+1=1$, so there is a horizontal tangent at the point $(0,1)$.

65. $y=5x^2-3x+8$

$$\frac{dy}{dx}=10x-3$$

Solve:

$$\frac{dy}{dx}=0$$
$$10x-3=0$$
$$10x=3$$
$$x=\frac{3}{10}$$

For $x=\frac{3}{10}$,

$$y=5\left(\frac{3}{10}\right)^2-3\left(\frac{3}{10}\right)+8$$
$$=5\left(\frac{9}{100}\right)-\frac{9}{10}+8$$
$$=\frac{45}{100}-\frac{90}{100}+\frac{800}{100}$$
$$=\frac{755}{100}$$

Therefore, there is a horizontal tangent at the point $\left(\frac{3}{10},\frac{755}{100}\right)$, or $(0.3,7.55)$.

66. $y=3x^2-5x+4$

A horizontal tangent line has slope equal to 0, so we need to find the values of x that make $\frac{dy}{dx}=0$.

First, we find the derivative.

$$\frac{dy}{dx}=\frac{d}{dx}\left(3x^2-5x+4\right)$$
$$=\frac{d}{dx}3x^2-\frac{d}{dx}5x+\frac{d}{dx}4$$
$$=3\left(2x^{2-1}\right)-5\left(1x^{1-1}\right)+0$$
$$=6x-5$$

Next, we set the derivative equal to zero and solve for x.

$$\frac{dy}{dx}=0$$
$$6x-5=0$$
$$6x=5$$
$$x=\frac{5}{6}$$

The solution is continued on the next page.

So the horizontal tangent will occur when $x=\frac{5}{6}$. Next we find the point on the graph.

For $x=\frac{5}{6}$,

$$y=3\left(\frac{5}{6}\right)^2-5\left(\frac{5}{6}\right)+4$$
$$=3\left(\frac{25}{36}\right)-\frac{25}{6}+4$$
$$=\frac{25}{12}-\frac{25}{6}+4$$
$$=\frac{25}{12}-\frac{25}{6}\cdot\frac{2}{2}+\frac{4}{1}\cdot\frac{12}{12}$$
$$=\frac{25}{12}-\frac{50}{12}+\frac{48}{12}$$
$$=\frac{25-50+48}{12}$$
$$=\frac{23}{12}$$

Therefore, there is a horizontal tangent at the point $\left(\frac{5}{6},\frac{23}{12}\right)$.

67. $y = -0.01x^2 + 0.4x + 50$

$\frac{dy}{dx} = -0.02x + 0.4$ See Exercise 46

$$\frac{dy}{dx} = 0$$
$$-0.02x + 0.4 = 0$$
$$-0.02x = -0.4$$
$$x = \frac{-0.4}{-0.02}$$
$$x = 20$$

For $x = 20$,

$$y = -0.01(20)^2 + 0.4(20) + 50$$
$$= -0.01(400) + 8 + 50$$
$$= -4 + 8 + 50$$
$$= 54$$

Therefore, there is a horizontal tangent at the point $(20, 54)$.

68. $y = -0.01x^2 - 0.5x + 70$

A horizontal tangent line has slope equal to 0, so we need to find the values of x that make $\frac{dy}{dx} = 0$.

First, we find the derivative.

$$\frac{dy}{dx} = \frac{d}{dx}\left(-0.01x^2 - 0.5x + 70\right)$$
$$= -0.02x - 0.5$$ See Exercise 45.

Next, we set the derivative equal to zero and solve for x.

$$\frac{dy}{dx} = 0$$
$$-0.02x - 0.5 = 0$$
$$-0.02x = 0.5$$
$$x = \frac{0.5}{-0.02}$$
$$x = -25$$

So the horizontal tangent will occur when $x = -25$. Next we find the point on the graph.

For $x = -25$,

$$y = -0.01(-25)^2 - 0.5(-25) + 70$$
$$= -0.01(625) + 12.5 + 70$$
$$= -6.25 + 12.5 + 70$$
$$= 76.25$$

Therefore, there is a horizontal tangent at the point $(-25, 76.25)$.

69. $y = -2x + 5$

$\frac{dy}{dx} = -2$

There are no values of x for which $\frac{dy}{dx} = 0$, so there are no points on the graph at which there is a horizontal tangent.

70. $y = 2x + 4$ Linear function

$\frac{dy}{dx} = 2$ Slope is 2

There are no values of x for which $\frac{dy}{dx} = 0$, so there are no points on the graph at which there is a horizontal tangent.

71. $y = -3$

$\frac{dy}{dx} = 0$ Theorem 2

$\frac{dy}{dx} = 0$ for all values of x, so the tangent line is horizontal for all points on the graph.

72. $y = 4$ Constant Function

$\frac{dy}{dx} = 0$ Theorem 2

$\frac{dy}{dx} = 0$ for all values of x, so the tangent line is horizontal for all points on the graph.

73. $y = -\frac{1}{3}x^3 + 6x^2 - 11x - 50$

$\frac{dy}{dx} = -x^2 + 12x - 11$

Solve:

$$\frac{dy}{dx} = 0$$
$$-x^2 + 12x - 11 = 0$$
$$x^2 - 12x + 11 = 0$$
$$(x-1)(x-11) = 0$$
$$x - 1 = 0 \quad \text{or} \quad x - 11 = 0$$
$$x = 1 \quad \text{or} \quad x = 11$$

The solution is continued on the next page.

For $x=1$,

$$y=-\frac{1}{3}(1)^3+6(1)^2-11(1)-50$$
$$=-\frac{1}{3}+6-11-50$$
$$=-\frac{166}{3}=-55\tfrac{1}{3}$$

For $x=11$,

$$y=-\frac{1}{3}(11)^3+6(11)^2-11(11)-50$$
$$=-\frac{1331}{3}+726-121-50$$
$$=\frac{334}{3}=111\tfrac{1}{3}$$

Therefore, there are horizontal tangents at the points $\left(1,-55\tfrac{1}{3}\right)$ and $\left(11,111\tfrac{1}{3}\right)$.

74. $y=-x^3+x^2+5x-1$

A horizontal tangent line has slope equal to 0, so we need to find the values of *x* that make $\frac{dy}{dx}=0$.

First, we find the derivative.

$$\frac{dy}{dx}=\frac{d}{dx}\left(-x^3+x^2+5x-1\right)$$
$$=-\frac{d}{dx}\left(x^3\right)+\frac{d}{dx}\left(x^2\right)+\frac{d}{dx}(5x)-\frac{d}{dx}(1)$$
$$=-3x^2+2x+5$$

Next, we set the derivative equal to zero and solve for *x*.

$$\frac{dy}{dx}=0$$
$$-3x^2+2x+5=0$$
$$3x^2-2x-5=0 \quad \text{Multiply both sides by -1.}$$
$$(3x-5)(x+1)=0 \quad \text{Factor the left hand side.}$$
$$3x-5=0 \quad \text{or} \quad x+1=0$$
$$3x=5 \quad \text{or} \quad x=-1$$
$$x=\frac{5}{3} \quad \text{or} \quad x=-1$$

There are two horizontal tangents. One at $x=\frac{5}{3}$ and one at $x=-1$.

Next we find the points on the graph where the horizontal tangents occur.

For $x=-1$

$$y=-(-1)^3+(-1)^2+5(-1)-1$$
$$y=-(-1)+(1)-5-1$$
$$y=-4$$

For $x=\frac{5}{3}$

$$y=-\left(\frac{5}{3}\right)^3+\left(\frac{5}{3}\right)^2+5\left(\frac{5}{3}\right)-1$$
$$y=-\left(\frac{125}{27}\right)+\left(\frac{25}{9}\right)+\frac{25}{3}-1$$
$$y=-\frac{125}{27}+\frac{75}{27}+\frac{225}{27}-\frac{27}{27}$$
$$y=\frac{148}{27}=5\tfrac{13}{27}$$

Therefore, there are horizontal tangents at the points $\left(\frac{5}{3},5\tfrac{13}{27}\right)$ and $(-1,-4)$.

75. $y=x^3-6x+1$

$$\frac{dy}{dx}=3x^2-6$$

Solve:

$$\frac{dy}{dx}=0$$
$$3x^2-6=0$$
$$x^2=2$$
$$x=\pm\sqrt{2}$$

For $x=-\sqrt{2}$,

$$y=\left(-\sqrt{2}\right)^3-6\left(-\sqrt{2}\right)+1$$
$$=-2\sqrt{2}+6\sqrt{2}+1$$
$$=1+4\sqrt{2}$$

For $x=\sqrt{2}$,

$$y=\left(\sqrt{2}\right)^3-6\left(\sqrt{2}\right)+1$$
$$=2\sqrt{2}-6\sqrt{2}+1$$
$$=1-4\sqrt{2}$$

Therefore, there are horizontal tangents at the points $\left(-\sqrt{2},1+4\sqrt{2}\right)$ and $\left(\sqrt{2},1-4\sqrt{2}\right)$.

76. $y=\frac{1}{3}x^3-3x+2$

A horizontal tangent line has slope equal to 0, so we need to find the values of x that make $\frac{dy}{dx}=0$.

First, we find the derivative.

$$\frac{dy}{dx}=\frac{d}{dx}\left(\frac{1}{3}x^3-3x+2\right)$$
$$=\frac{d}{dx}\left(\frac{1}{3}x^3\right)-\frac{d}{dx}(3x)+\frac{d}{dx}(2)$$
$$=x^2-3$$

Next, we set the derivative equal to zero and solve for x.

$$\frac{dy}{dx}=0$$
$$x^2-3=0$$
$$x^2=3$$
$$x=\pm\sqrt{3}$$

There are two horizontal tangents. One at $x=-\sqrt{3}$ and one at $x=\sqrt{3}$. Next we find the points on the graph where the horizontal tangents occur.

For $x=-\sqrt{3}$

$$y=\frac{1}{3}\left(-\sqrt{3}\right)^3-3\left(-\sqrt{3}\right)+2$$
$$=\frac{1}{3}\left(-3\sqrt{3}\right)-3\left(-\sqrt{3}\right)+2$$
$$=-\sqrt{3}+3\sqrt{3}+2$$
$$=2+2\sqrt{3}$$

For $x=\sqrt{3}$

$$y=\frac{1}{3}\left(\sqrt{3}\right)^3-3\left(\sqrt{3}\right)+2$$
$$=\frac{1}{3}\left(3\sqrt{3}\right)-3\left(\sqrt{3}\right)+2$$
$$=+\sqrt{3}-3\sqrt{3}+2$$
$$=2-2\sqrt{3}$$

Therefore, there are horizontal tangents at the points $\left(-\sqrt{3},2+2\sqrt{3}\right)$ and $\left(\sqrt{3},2-2\sqrt{3}\right)$.

77. $y=\frac{1}{3}x^3-3x^2+9x-9$

$$\frac{dy}{dx}=x^2-6x+9$$

Solve:

$$\frac{dy}{dx}=0$$
$$x^2-6x+9=0$$
$$(x-3)^2=0$$
$$x-3=0$$
$$x=3$$

For $x=3$,

$$y=\frac{1}{3}(3)^3-3(3)^2+9(3)-9$$
$$=9-27+27-9=0$$

Therefore, there is horizontal tangent at the point $(3,0)$.

78. $y=\frac{1}{3}x^3+\frac{1}{2}x^2-2$

A horizontal tangent line has slope equal to 0, so we need to find the values of x that make $\frac{dy}{dx}=0$.

First, we find the derivative.

$$\frac{dy}{dx}=\frac{d}{dx}\left(\frac{1}{3}x^3+\frac{1}{2}x^2-2\right)$$
$$=\frac{d}{dx}\left(\frac{1}{3}x^3\right)+\frac{d}{dx}\left(\frac{1}{2}x^2\right)-\frac{d}{dx}(2)$$
$$=x^2+x$$

The solution is continued on the next page.

Next, we set the derivative equal to zero and solve for x.

$$\frac{dy}{dx}=0$$
$$x^2+x=0$$
$$x(x+1)=0$$
$$x=0 \quad\text{or}\quad x+1=0$$
$$x=0 \quad\text{or}\quad x=-1$$

There are two horizontal tangents. One at $x=0$ and one at $x=-1$.

Next we find the points on the graph where the horizontal tangents occur.

For $x=0$

$$y=\frac{1}{3}(0)^3+\frac{1}{2}(0)^2-2$$
$$y=-2$$

For $x=-1$

$$y=\frac{1}{3}(-1)^3+\frac{1}{2}(-1)^2-2$$
$$=-\frac{1}{3}+\frac{1}{2}-2$$
$$=-\frac{2}{6}+\frac{3}{6}-\frac{12}{6}$$
$$=-\frac{11}{6}$$

Therefore, there are horizontal tangents at the points $(0,-2)$ and $\left(-1,-\frac{11}{6}\right)$.

79. $y=6x-x^2$; $\frac{dy}{dx}=6-2x$

Solve:

$$\frac{dy}{dx}=1$$
$$6-2x=1$$
$$-2x=-5$$
$$x=\frac{5}{2}$$

For $x=\frac{5}{2}$,

$$y=6\left(\frac{5}{2}\right)-\left(\frac{5}{2}\right)^2$$
$$=15-\frac{25}{4}=\frac{35}{4}$$

The tangent line has slope 1 at $(2.5, 8.75)$.

80. $y=20x-x^2$

To find the tangent line that has slope equal to 1, so we need to find the values of x that make $\frac{dy}{dx}=1$.

First, we find the derivative.

$$\frac{dy}{dx}=\frac{d}{dx}\left(20x-x^2\right)$$
$$=\frac{d}{dx}20x-\frac{d}{dx}x^2$$
$$=20-2x$$

Next, we set the derivative equal to 1 and solve for x.

$$\frac{dy}{dx}=1$$
$$20-2x=1$$
$$-2x=1-20$$
$$-2x=-19$$
$$x=\frac{19}{2}$$

So the tangent will occur when $x=\frac{19}{2}$.

Next we find the point on the graph.

For $x=\frac{19}{2}$,

$$y=20\left(\frac{19}{2}\right)-\left(\frac{19}{2}\right)^2$$
$$y=190-\left(\frac{361}{4}\right)$$
$$=\frac{760}{4}-\frac{361}{4}$$
$$=\frac{399}{4}$$

The tangent line has slope 1 at the point $(9.5, 99.75)$.

81. $y=-0.01x^2+2x$; $\frac{dy}{dx}=-0.02x+2$

Solve:

$$\frac{dy}{dx}=1$$
$$-0.02x+2=1$$
$$-0.02x=-1$$
$$x=50$$

For $x=50$,

$$y=-0.01(50)^2+2(50)=-25+100=75$$

The tangent line has slope 1 at $(50, 75)$.

82. $y=-0.025x^2+4x$

To find the tangent line that has slope equal to 1, we need to find the values of x that make $\frac{dy}{dx}=1$.

First, we find the derivative.

$$\frac{dy}{dx} = \frac{d}{dx}\left(-0.025x^2 + 4x\right)$$
$$= \frac{d}{dx} - 0.025x^2 + \frac{d}{dx}4x$$
$$= -0.025(2x) + 4$$
$$= -0.05x + 4$$

Next, we set the derivative equal to 1 and solve for *x*.

$$\frac{dy}{dx} = 1$$
$$-0.05x + 4 = 1$$
$$-0.05x = -3$$
$$x = \frac{-3}{-0.05}$$
$$x = 60$$

So the tangent will occur when $x = 60$. Next we find the point on the graph.

For $x = 60$,

$$y = -0.025(60)^2 + 4(60)$$
$$= -0.025(3600) + 240$$
$$= -90 + 240$$
$$= 150$$

The tangent line has slope 1 at the point $(60, 150)$.

83. $y = \frac{1}{3}x^3 - x^2 - 4x + 1$

$$\frac{dy}{dx} = x^2 - 2x - 4$$

Solve:

$$\frac{dy}{dx} = 1$$
$$x^2 - 2x - 4 = 1$$
$$x^2 - 2x - 5 = 0$$

Using the quadratic formula,

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-5)}}{2(1)}$$
$$= \frac{2 \pm \sqrt{4 + 20}}{2}$$
$$= \frac{2 \pm \sqrt{24}}{2}$$
$$= \frac{2 \pm 2\sqrt{6}}{2}$$
$$= 1 \pm \sqrt{6}$$

For $x = 1 + \sqrt{6}$,

$$y = \frac{1}{3}\left(1+\sqrt{6}\right)^3 - \left(1+\sqrt{6}\right)^2 - 4\left(1+\sqrt{6}\right) + 1$$
$$= \frac{1}{3}\left(19 + 9\sqrt{6}\right) - \left(7 + 2\sqrt{6}\right) - 4 - 4\sqrt{6} + 1$$
$$= \frac{19}{3} + 3\sqrt{6} - 7 - 2\sqrt{6} - 4 - 4\sqrt{6} + 1$$
$$= -\frac{11}{3} - 3\sqrt{6}$$

For $x = 1 - \sqrt{6}$,

$$y = \frac{1}{3}\left(1-\sqrt{6}\right)^3 - \left(1-\sqrt{6}\right)^2 - 4\left(1-\sqrt{6}\right) + 1$$
$$= \frac{1}{3}\left(19 - 9\sqrt{6}\right) - \left(7 - 2\sqrt{6}\right) - 4 + 4\sqrt{6} + 1$$
$$= \frac{19}{3} - 3\sqrt{6} - 7 + 2\sqrt{6} - 4 + 4\sqrt{6} + 1$$
$$= -\frac{11}{3} + 3\sqrt{6}$$

Therefore, the tangent line has slope 1 at the points

$\left(1+\sqrt{6}, -\frac{11}{3} - 3\sqrt{6}\right)$ and $\left(1-\sqrt{6}, -\frac{11}{3} + 3\sqrt{6}\right)$.

84. $y = \frac{1}{3}x^3 + 2x^2 + 2x$

To find the tangent line that has slope equal to 1, we need to find the values of *x* that make $\frac{dy}{dx} = 1$.

First, we find the derivative.

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{1}{3}x^3 + 2x^2 + 2x\right)$$
$$= \frac{d}{dx}\left(\frac{1}{3}x^3\right) + \frac{d}{dx}2x^2 + \frac{d}{dx}2x$$
$$= x^2 + 4x + 2$$

Next, we set the derivative equal to 1 and solve for *x*.

$$\frac{dy}{dx} = 1$$
$$x^2 + 4x + 2 = 1$$
$$x^2 + 4x + 1 = 0$$

This is a quadratic equation, not readily factorable, so we use the quadratic formula where $a = 1, b = 4,$ and, $c = 1$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(4) \pm \sqrt{(4)^2 - 4(1)(1)}}{2(1)} \quad \text{Substituting}$$

$$= \frac{-4 \pm \sqrt{12}}{2}$$

$$= \frac{-4 \pm 2\sqrt{3}}{2} \qquad \left[\sqrt{12} = \sqrt{4 \cdot 3} = 2\sqrt{3}\right]$$

$$= \frac{2\left(-2 \pm \sqrt{3}\right)}{2}$$

$$= -2 \pm \sqrt{3}$$

The solution is continued on the next page.
From the previous page, we know there are two tangent lines that have slope equal to 1. The first one occurs at $x = -2 + \sqrt{3}$ and the second one occurs at $x = -2 - \sqrt{3}$. We use the original equation to find the point on the graph.

For $x = -2 + \sqrt{3}$,

$$y = \frac{1}{3}\left(-2+\sqrt{3}\right)^3 + 2\left(-2+\sqrt{3}\right)^2 + 2\left(-2+\sqrt{3}\right)$$

$$= \frac{1}{3}\left(-26+15\sqrt{3}\right) + 2\left(7-4\sqrt{3}\right) - 4 + 2\sqrt{3}$$

$$= -\frac{26}{3} + 5\sqrt{3} + 14 - 8\sqrt{3} - 4 + 2\sqrt{3}$$

$$= \frac{4}{3} - \sqrt{3}$$

For $x = -2 - \sqrt{3}$,

$$y = \frac{1}{3}\left(-2-\sqrt{3}\right)^3 + 2\left(-2-\sqrt{3}\right)^2 + 2\left(-2-\sqrt{3}\right)$$

$$= \frac{1}{3}\left(-26-15\sqrt{3}\right) + 2\left(7+4\sqrt{3}\right) - 4 - 2\sqrt{3}$$

$$= -\frac{26}{3} - 5\sqrt{3} + 14 + 8\sqrt{3} - 4 - 2\sqrt{3}$$

$$= \frac{4}{3} + \sqrt{3}$$

The tangent lines have slope 1 at the points $\left(-2+\sqrt{3}, \frac{4}{3} - \sqrt{3}\right)$ and $\left(-2-\sqrt{3}, \frac{4}{3} + \sqrt{3}\right)$.

85. $y = \sqrt[3]{x} - \frac{1}{3}x = x^{\frac{1}{3}} - \frac{1}{3}x$;

$$\frac{dy}{dx} = \frac{1}{3}x^{-\frac{2}{3}} - \frac{1}{3} = \frac{1}{3\sqrt[3]{x^2}} - \frac{1}{3}$$

Solve:

$$\frac{dy}{dx} = 1$$

$$\frac{1}{3\sqrt[3]{x^2}} - \frac{1}{3} = 1$$

$$\frac{1}{3\sqrt[3]{x^2}} = \frac{4}{3}$$

$$4 = x^{\frac{2}{3}}$$

$$8 = x$$

For $x = 8$,

$$y = \sqrt[3]{x} - \frac{1}{3}x = \sqrt[3]{8} - \frac{1}{3}(8) = -\frac{2}{3}$$

The tangent line has slope 1 at $\left(8, -\frac{2}{3}\right)$.

86. $y = \sqrt{x} + \frac{1}{2}x = x^{\frac{1}{2}} + \frac{1}{2}x$;

$$\frac{dy}{dx} = \frac{1}{2}x^{-\frac{1}{2}} + \frac{1}{2} = \frac{1}{2\sqrt{x}} + \frac{1}{2}$$

Solve:

$$\frac{dy}{dx} = 1$$

$$\frac{1}{2\sqrt{x}} + \frac{1}{2} = 1$$

$$\frac{1}{2\sqrt{x}} = \frac{1}{2}$$

$$1 = x^{\frac{1}{2}}$$

$$1 = x$$

For $x = 1$,

$$y = \sqrt{x} + \frac{1}{2}x = \sqrt{1} + \frac{1}{2}(1) = \frac{3}{2}$$

The tangent line has slope 1 at $\left(1, \frac{3}{2}\right)$.

87. $w(t) = 8.15 + 1.82t - 0.0596t^2 + 0.000758t^3$

a) In order to find the rate of change of weight with respect to time, we take the derivative of the function with respect to *t*.

$w'(t)$

$= \frac{d}{dt}\left(8.15 + 1.82t - 0.0596t^2 + 0.000758t^3\right)$

$= 0 + 1.82 - 0.0596(2t) + 0.000758(3t^2)$

$= 1.82 - 0.1192t + 0.002274t^2$

Therefore, the rate of change of weight with respect to time is given by:

$w'(t) = 1.82 - 0.1192t + 0.002274t^2$

b) The weight of the baby at age 10 months can be found by evaluating the function when $t = 10$.

$w(10) = 8.15 + 1.82(10) - 0.0596(10)^2 + 0.000758(10)^3$

≈ 21.148 Using a calculator

Therefore, a 10 month old boy weighs approximately 21.148 pounds.

c) The rate of change of the baby's weight with respect to time at age of 10 months can be found by evaluating the derivative when $t = 10$.

$w'(10)$

$= 1.82 - 0.1192(10) + 0.002274(10)^2$

≈ 0.8554

A 10 month old boys weight will be increasing at a rate of 0.86 pounds per month.

88. $T(t) = -0.1t^2 + 1.2t + 98.6$

a) $T'(t) = -0.2t + 1.2$

b) Evaluate *T* when $t = 1.5$

$T(1.5) = -0.1(1.5)^2 + 1.2(1.5) + 98.6$

$= 100.175$

The temperature of the ill person after 1.5 days is 100.2 degrees Fahrenheit.

c) Evaluate $T'(t)$ when $t = 1.5$.

$T'(1.5) = -0.2(1.5) + 1.2$

$= 0.9$

The ill person's temperature is increasing 0.9 degrees Fahrenheit per day after 1.5 days.

89. $R(v) = \frac{6000}{v} = 6000v^{-1}$

a) Using the power rule, we take the derivative of *R* with respect to *v*.

$R'(v) = 6000\left(-1v^{-1-1}\right)$

$= -6000v^{-2}$

$= -\frac{6000}{v^2}$

The rate of change of heart rate with respect to the output per beat is

$R'(v) = -\frac{6000}{v^2}$.

b) To find the heart rate at $v = 80$ ml per beat, we evaluate the function $R(v)$ when $v = 80$.

$R(80) = \frac{6000}{80} = 75$.

The heart rate is 75 beats per minute when the output per beat is 80 ml per beat.

c) To find the rate of change of the heart beat at $v = 80$ ml per beat, we evaluate the derivative $R'(v)$ at $v = 80$.

$R'(80) = -\frac{6000}{80^2}$

$R'(80) = -\frac{15}{16}$

$= -0.9375$

The heart rate is decreasing at a rate of 0.94 beats per minute when the output per beat is 80 mL per beat.

90. $S(r) = \frac{1}{r^4} = r^{-4}$

a) $S'(r) = \frac{d}{dr}\left(r^{-4}\right) = -4r^{-5} = -\frac{4}{r^5}$

b) $S(1.2) = \frac{1}{(1.2)^4} \approx 0.48225309$

The resistance when $r = 1.2$ mm is 0.48225309.

c) $S'(r) = -\frac{4}{(0.8)^5} \approx -12.20703125$

The resistance, *S*, is changing with respect to *r* at an approximate rate of -12.2 per mm when $r = 0.8$ mm

91. a) Using the power rule, we find the growth rate $\frac{dP}{dt}$.

$$\frac{dP}{dt} = \frac{d}{dt}\left(100,000 + 2000t^2\right)$$
$$= 0 + 2000(2t)$$
$$= 4000t$$

b) Evaluate the function P when $t = 10$.

$$P(10) = 100,000 + 2000(10)^2$$
$$= 100,000 + 2000(100)$$
$$= 300,000$$

The population of the city will be 300,000 people after 10 years.

c) Evaluate the derivative $P'(t)$ when $t = 10$.

$$\left.\frac{dP}{dt}\right|_{t=10} = P'(10) = 4000(10) = 40,000$$

The population's growth rate after 10 years is 40,000 people per year.

d) Answers will vary. $P'(10) = 40,000$ means that after 10 years, the city's population is growing at a rate of 40,000 people per year.

92. $A(t) = 0.08t + 19.7$

a) $A'(t) = 0.08$

b) Answers will vary. The median age, *A,* of women marrying for the first time has been increasing at a rate 0.08 year per year since 1950.

93. $V = 1.22\sqrt{h} = 1.22h^{1/2}$

a) Using the power rule,

$$\frac{dV}{dh} = \frac{d}{dh}\left(1.22h^{1/2}\right)$$
$$= 1.22\left(\frac{1}{2}h^{1/2 - 1}\right)$$
$$= 0.61h^{-1/2}$$
$$= \frac{0.61}{h^{1/2}} = \frac{0.61}{\sqrt{h}}$$

b) Evaluate the function V when $h = 32,000$.

$$V = 1.22\sqrt{32,000}$$
$$\approx 218.24$$

A person would be able to see approximately 218.24 miles to the horizon from a height of 32,000 feet.

c) Evaluate the derivative $\frac{dV}{dh}$ when $h = 32,000$.

$$\frac{dV}{dh} = \frac{0.61}{\sqrt{32,000}}$$
$$= \frac{0.61}{178.8854}$$
$$\approx 0.0034$$

The rate of change at $h = 32,000$ is approximately 0.0034 miles per foot.

d) Answers will vary. From part (a), we find that the distance that one can see to the horizon from height h increases $\frac{0.61}{\sqrt{h}}$ miles for every one foot increase in height. From part (c) we find that, at a height of 32,000 feet, the distance that a person can see to the horizon increases at a rate of 0.0034 miles per foot.

94. $s = 3.1\sqrt{d} = 3.1d^{1/2}$

a) Using the power rule,

$$\frac{ds}{dd} = \frac{d}{dd}\left(3.1d^{1/2}\right)$$
$$= 3.1\left(\frac{1}{2}d^{1/2 - 1}\right)$$
$$= 1.55d^{-1/2}$$
$$= \frac{1.55}{d^{1/2}} = \frac{1.55}{\sqrt{d}}$$

b) Evaluate the function s when $d = 10$.

$$s = 3.1\sqrt{10}$$
$$\approx 9.8$$

The speed of the wave would be approximately 9.8 meters per second when the depth of the water is 10 feet.

c) Evaluate the derivative $\frac{ds}{dd}$ when $d = 10$.

$$\frac{dV}{dh} = \frac{1.55}{\sqrt{10}}$$
$$= \frac{1.55}{3.162278}$$
$$\approx 0.49$$

The rate of change at $d = 10$ is approximately 0.49meters per second.

95. $p(t) = 0.858t^2 - 18.864t + 78.354$

a) In 2020, $t = 2020 - 1967 = 53$. Evaluating the function at $t = 53$, we have

$$p(53) = 0.858(53)^2 - 18.864(53) + 78.354$$
$$= 1488.684$$

The average ticket price in 2020 will be approximately \$1490.

b) $\frac{dp}{dt} = p'(t) = 1.716t - 18.864$

c) Evaluate the derivative at $t = 53$.

$$p'(53) = 1.716(53) - 18.864 = 72.084$$

In 2020, the average ticket price is increasing at a rate of about \$72.08 per year.

96. $f(x) = x^2 - 4x + 1$

The derivative is positive when $f'(x) > 0$.

Find $f'(x)$.

$$f'(x) = \frac{d}{dx}\left(x^2 - 4x + 1\right)$$
$$= 2x - 4$$

Next, we solve the inequality

$$f'(x) > 0$$
$$2x - 4 > 0$$
$$2x > 4$$
$$x > 2$$

Therefore, the interval for which $f'(x)$ is positive is $(2, \infty)$.

97. $f(x) = \frac{1}{3}x^3 - x^2 - 3x + 5$

The derivative is positive when $f'(x) > 0$.

Find $f'(x)$.

$$f'(x) = \frac{d}{dx}\left(\frac{1}{3}x^3 - x^2 - 3x + 5\right)$$
$$= x^2 - 2x - 3$$

Next, we solve the inequality

$$f'(x) > 0$$
$$x^2 - 2x - 3 > 0$$

First we find where the quadratic is equal to zero, in order to determine the intervals that we will need to test.

$$x^2 - 2x - 3 = 0$$
$$(x-3)(x+1) = 0$$
$$x + 1 = 0 \quad \text{or} \quad x - 3 = 0$$
$$x = -1 \quad \text{or} \quad x = 3$$

Now we will test a value to the left of -1, between -1 and 3 and to the right of 3 to determine where the quadratic is positive or negative. We choose the values $x = -2,\ x = 0,$ and, $x = 4$ to test.

When $x = -2$, the derivative $f'(x)$ is

$$f'(-2) = (-2)^2 - 2(-2) - 3 = 5.$$

When $x = 0$, the derivative $f'(x)$ is

$$f'(0) = (0)^2 - 2(0) - 3 = -3.$$

When $x = 4$, the derivative $f'(x)$ is

$$f'(4) = (4)^2 - 2(4) - 3 = 5.$$

We organize the results in the table below.

Test point x	Test $x = -2$	-1	Test $x = 0$	3	Test $x = 4$
$f'(x)$	$f'(-2) = 5$	0	$f'(0) = -3$	0	$f'(4) = 5$

From the table, we can see that $f'(x)$ is positive on the interval $(-\infty, -1)$ and the interval $(3, \infty)$.

98. $y = x^4 - \frac{4}{3}x^2 - 4$

$$\frac{dy}{dx} = 4x^3 - \frac{8}{3}x$$

Solve $\frac{dy}{dx} = 0$

$$4x^3 - \frac{8}{3}x = 0$$
$$4x\left(x^2 - \frac{2}{3}\right) = 0$$
$$4x = 0 \quad \text{or} \quad x^2 - \frac{2}{3} = 0$$
$$x = 0 \quad \text{or} \quad x^2 = \frac{2}{3}$$
$$x = 0 \quad \text{or} \quad x = \pm\sqrt{\frac{2}{3}}$$

For $x = 0$,

$$y = (0)^4 - \frac{4}{3}(0)^2 - 4 = -4.$$

For $x=\sqrt{\frac{2}{3}}$

$$y=\left(\sqrt{\frac{2}{3}}\right)^4-\frac{4}{3}\left(\sqrt{\frac{2}{3}}\right)^2-4$$
$$=-\frac{40}{9}$$

For $x=-\sqrt{\frac{2}{3}}$

$$y=\left(-\sqrt{\frac{2}{3}}\right)^4-\frac{4}{3}\left(-\sqrt{\frac{2}{3}}\right)^2-4$$
$$=-\frac{40}{9}$$

There are three points on the graph for which the tangent line is horizontal.

$(0,-4)$, $\left(\sqrt{\frac{2}{3}},-\frac{40}{9}\right)$, and, $\left(-\sqrt{\frac{2}{3}},-\frac{40}{9}\right)$.

99. $y=2x^6-x^4-2$

A horizontal tangent line has slope equal to 0, so we need to find the values of x that make $\frac{dy}{dx}=0$.

First, we find the derivative.

$$\frac{dy}{dx}=\frac{d}{dx}\left(2x^6-x^4-2\right)$$
$$=\frac{d}{dx}2x^6-\frac{d}{dx}x^4-\frac{d}{dx}2$$
$$=2\left(6x^{6-1}\right)-\left(4x^{4-1}\right)+0$$
$$=12x^5-4x^3$$

Next, we set the derivative equal to zero and solve for x.

$$\frac{dy}{dx}=0$$
$$12x^5-4x^3=0$$
$$4x^3\left(3x^2-1\right)=0$$
$$4x^3=0 \quad \text{or} \quad 3x^2-1=0$$
$$x=0 \quad \text{or} \quad 3x^2=1$$
$$x=0 \quad \text{or} \quad x^2=\frac{1}{3}$$
$$x=0 \quad \text{or} \quad x=\pm\sqrt{\frac{1}{3}}=\pm\frac{1}{\sqrt{3}}$$

So the horizontal tangent will occur when $x=0$, $x=\frac{1}{\sqrt{3}}$, and $x=-\frac{1}{\sqrt{3}}$. Next we find the points on the graph.

For $x=0$,

$$y=2(0)^6-(0)-2$$
$$=-2$$

For $x=\frac{1}{\sqrt{3}}$,

$$y=2\left(\frac{1}{\sqrt{3}}\right)^6-\left(\frac{1}{\sqrt{3}}\right)^4-2$$
$$=2\left(\frac{1}{27}\right)-\frac{1}{9}-2$$
$$=\frac{2}{27}-\frac{3}{27}-\frac{54}{27}$$
$$=-\frac{55}{27}$$

For $x=-\frac{1}{\sqrt{3}}$,

$$y=2\left(-\frac{1}{\sqrt{3}}\right)^6-\left(-\frac{1}{\sqrt{3}}\right)^4-2$$
$$=2\left(\frac{1}{27}\right)-\frac{1}{9}-2$$
$$=\frac{2}{27}-\frac{3}{27}-\frac{54}{27}$$
$$=-\frac{55}{27}$$

Therefore, there are horizontal tangents at the points $(0,-2)$, $\left(\frac{1}{\sqrt{3}},-\frac{55}{27}\right)$, and $\left(-\frac{1}{\sqrt{3}},-\frac{55}{27}\right)$.

100. $f(x)=x^5+x^3$

Taking the derivative we have:

$$f'(x)=\frac{d}{dx}\left(x^5+x^3\right)$$
$$=\frac{d}{dx}x^5+\frac{d}{dx}x^3$$
$$=5x^4+3x^2$$

Notice that $f'(x)\geq 0$ for all values of x.

Therefore, $f(x)$ is always increasing.

101. $f(x)=-2x-x^3$

Taking the derivative we have:

$$f'(x)=\frac{d}{dx}\left(-2x-x^3\right)$$
$$=-2\frac{d}{dx}x-\frac{d}{dx}x^3$$
$$=-2x^{1-1}-3x^{3-1}$$
$$=-2-3x^2$$

Notice that $f'(x)\le 0$ for all values of x.

Therefore, $f(x)$ is always decreasing.

102. $f(x)=\frac{1}{x^2},\ \ x\neq 0$

Taking the derivative we have:

$$f'(x)=\frac{d}{dx}\left(\frac{1}{x^2}\right)$$
$$=-\frac{2}{x^3}$$

Notice that $f'(x)<0$ for all values of x over the interval $(-\infty,0)\cup(0,\infty)$ Therefore, $f(x)$ is always decreasing over the interval $(-\infty,0)\cup(0,\infty)$.

103. $f(x)=x^3+ax$

Finding the derivative, we have:

$f'(x)=3x^2+a$.

When a is positive, $f'(x)>0$ for all values of x and will be always increasing. However, if a is negative, then $f'(x)<0$ for some values of x. (most noticeably $x=0$) and therefore will not be always increasing.

104. $y=(x+3)(x-2)=x^2+x-6$

$$\frac{dy}{dx}=2x+1$$

105. $y=(x-1)(x+1)$

First, we multiply the two binomials.

$y=(x-1)(x+1)=x^2-1$

Therefore,

$$\frac{dy}{dx}=\frac{d}{dx}\left(x^2-1\right)$$
$$=\frac{d}{dx}\left(x^2\right)-\frac{d}{dx}(1)$$
$$=2x$$

106. $y=\frac{x^5-x^3}{x^2}$

First, we separate the fraction.

$$y=\frac{x^5}{x^2}-\frac{x^3}{x^2}$$
$$=x^{5-2}-x^{3-2} \qquad \left[\frac{a^m}{a^n}=a^{m-n}\right]$$
$$=x^3-x^1$$

Therefore,

$$\frac{dy}{dx}=\frac{d}{dx}\left(x^3-x\right)$$
$$=\frac{d}{dx}x^3-\frac{d}{dx}x$$
$$=3x^2-1$$

107. $y=\frac{x^5+x}{x^2}$

First, we separate the fraction.

$$y=\frac{x^5}{x^2}+\frac{x}{x^2}$$
$$=x^{5-2}+x^{1-2}$$
$$=x^3+x^{-1}$$

Therefore,

$$\frac{dy}{dx}=\frac{d}{dx}\left(x^3+x^{-1}\right)$$
$$=3x^2+\left(-x^{-1-1}\right)$$
$$=3x^2-x^{-2}$$
$$=3x^2-\frac{1}{x^2}$$

108. $y=\sqrt{7x}$

First, we simplify the radical.

$$y=\sqrt{7\cdot x}$$
$$=\sqrt{7}\sqrt{x} \qquad \left[\sqrt{m\cdot n}=\sqrt{m}\sqrt{n}\right]$$
$$=\sqrt{7}\left(x^{1/2}\right) \qquad \left[\sqrt[m]{a}=a^{1/m};m=2\right]$$

Therefore,

$$\frac{dy}{dx} = \frac{d}{dx}\left(\sqrt{7}\,(x)^{1/2}\right)$$
$$= \sqrt{7}\,\frac{d}{dx}\left(x^{1/2}\right)$$
$$= \sqrt{7}\left(\frac{1}{2}x^{1/2-1}\right)$$
$$= \frac{\sqrt{7}}{2}x^{-1/2}$$
$$= \frac{\sqrt{7}}{2x^{1/2}}$$
$$= \frac{\sqrt{7}}{2\sqrt{x}}$$

109. $y = \sqrt[3]{8x} = (8x)^{1/3} = 8^{1/3} \cdot x^{1/3} = 2x^{1/3}$

$$\frac{dy}{dx} = \frac{2}{3}x^{-\frac{2}{3}} = \frac{2}{3x^{2/3}} = \frac{2}{3\sqrt[3]{x^2}}$$

110. $y = \left(\sqrt{x} - \frac{1}{\sqrt{x}}\right)^2$

$$y = \left(x^{1/2} - x^{-1/2}\right)^2$$
$$= \left(x^{1/2} - x^{-1/2}\right)\left(x^{1/2} - x^{-1/2}\right)$$
$$= x - 2x^0 + x^{-1}$$
$$= x + x^{-1} - 2$$
$$\frac{dy}{dx} = 1 - x^{-2}$$
$$= 1 - \frac{1}{x^2}$$

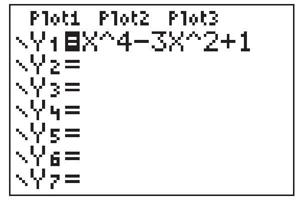
111. $y = (x+1)^3 = x^3 + 3x^2 + 3x + 1$

$$\frac{dy}{dx} = 3x^2 + 6x + 3$$

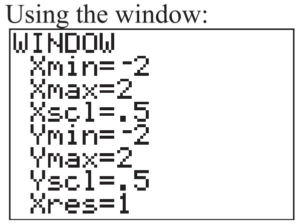
112. Answers will vary

113. $f(x) = x^4 - 3x^2 + 1$

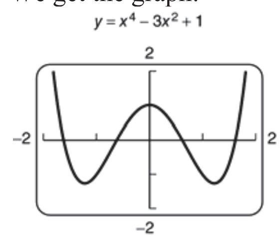
First we enter the equation into the graphing editor on the calculator.



Using the window:



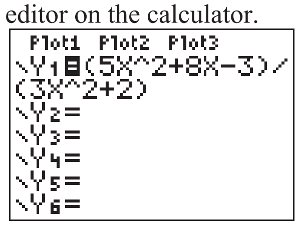
We get the graph:



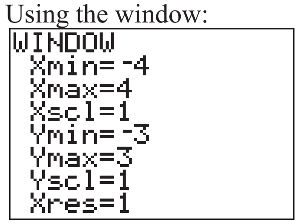
We estimate the x-values at which the tangent lines are horizontal are $x = -1.225,\ x = 0,$ and, $x = 1.225$.

114. $f(x) = \frac{5x^2 + 8x - 3}{3x^2 + 2}$

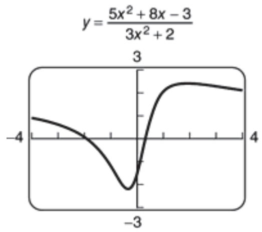
First we enter the equation into the graphing editor on the calculator.



Using the window:



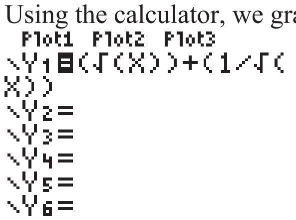
We get the graph:



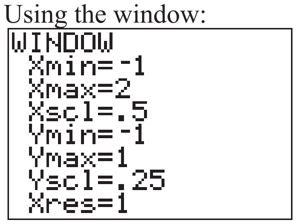
The horizontal tangents occur at the turning points of this function. Using the trace feature, or the minimum/maximum feature on the calculator, we find the turning points.
We estimate the x-values at which the tangent lines are horizontal are
$x = -0.346$ and $x = 1.929$.

115. $f(x) = \sqrt{x} + \frac{1}{\sqrt{x}}$

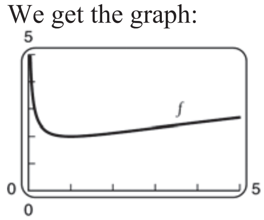
Using the calculator, we graph the function.



Using the window:

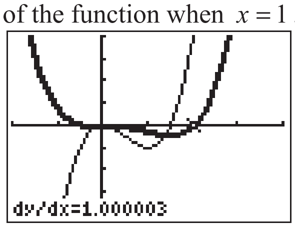


We get the graph:



Note, the function $f(x)$ is the thicker graph.

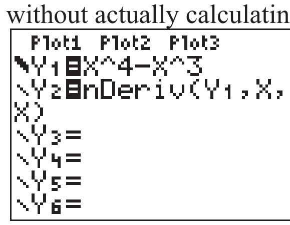
Using the calculator, we can find the derivative of the function when $x = 1$.



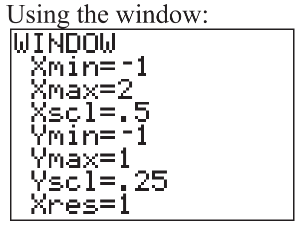
We have $f'(1) = 45$.

116. $f(x) = x^4 - x^3$

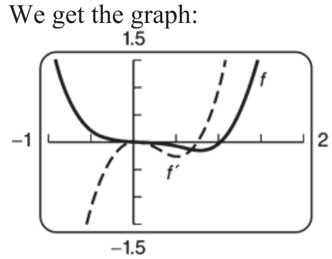
Using the calculator, we graph the function and the derivative in the same window. We can use the nDeriv feature to graph the derivative without actually calculating the derivative.



Using the window:

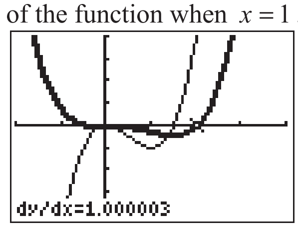


We get the graph:



Note, the function $f(x)$ is the thicker graph.

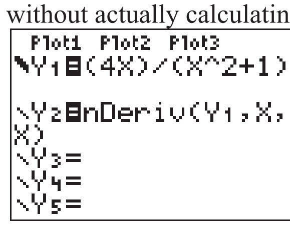
Using the calculator, we can find the derivative of the function when $x = 1$.



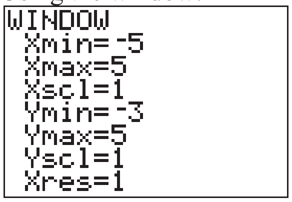
We have $f'(1) = 1$.

117. $f(x) = \frac{4x}{x^2 + 1}$

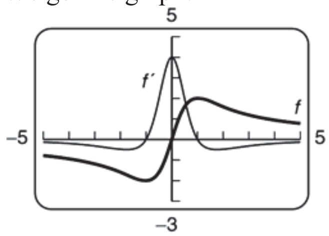
Using the calculator, we graph the function and the derivative in the same window. We can use the nDeriv feature to graph the derivative without actually calculating the derivative.



Using the window:

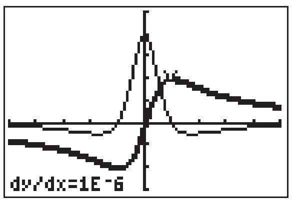


We get the graph:



Note, the function $f(x)$ is the thicker graph.

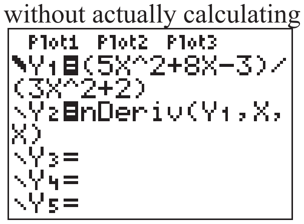
Using the calculator, we can find the derivative of the function when $x = 1$.



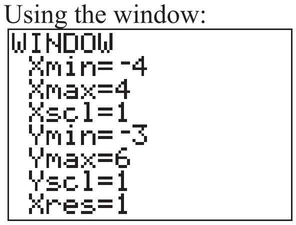
We have $f'(1) = 0$.

118. $f(x) = \dfrac{5x^2 + 8x - 3}{3x^2 + 2}$

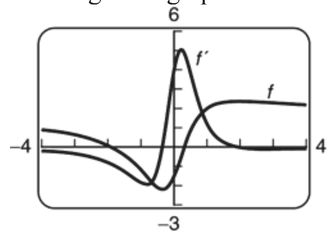
Using the calculator, we graph the function and the derivative in the same window. We can use the nDeriv feature to graph the derivative without actually calculating the derivative.



Using the window:

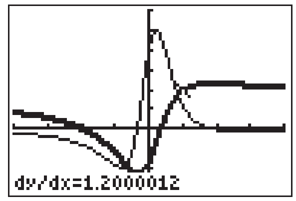


We get the graph:



Note, the function $f(x)$ is the thicker graph.

Using the calculator, we can find the derivative of the function when $x = 1$.



We have $f'(1) = 1.2$.

Exercise Set 1.6

1. Differentiate $y = x^5 \cdot x^6$ using the Product Rule (Theorem 5) .

$$\frac{dy}{dx} = \frac{d}{dx}\left(x^5 \cdot x^6\right)$$
$$= x^5 \cdot \frac{d}{dx}\left(x^6\right) + x^6 \frac{d}{dx}\left(x^5\right)$$
$$= x^5 \cdot 6x^5 + x^6 \cdot 5x^4$$
$$= 6x^{10} + 5x^{10}$$
$$= 11x^{10}$$

Differentiate $y = x^5 \cdot x^6 = x^{11}$ using the Power Rule (Theorem 1).

$$\frac{dy}{dx} = \frac{d}{dx} x^{11}$$
$$= 11x^{11-1}$$
$$= 11x^{10}$$

The two results are equivalent.

2. Using the Product Rule:

$$\frac{dy}{dx} = \frac{d}{dx}\left(x^9 \cdot x^4\right)$$
$$= x^9 \cdot 4x^3 + x^4 \cdot 9x^8$$
$$= 4x^{12} + 9x^{12}$$
$$= 13x^{12}$$

Using the Power Rule:

$$\frac{dy}{dx} = \frac{d}{dx}\left(x^4 \cdot x^9\right)$$
$$= \frac{d}{dx}\left(x^{13}\right)$$
$$= 13x^{12}$$

The two results are equivalent.

3. Differentiate $f(x) = (2x+5)(3x-4)$ using the Product Rule (Theorem 5).

$$f'(x) = \frac{d}{dx}\left[(2x+5)(3x-4)\right]$$
$$= (2x+5)\cdot\frac{d}{dx}(3x-4) + (3x-4)\cdot\frac{d}{dx}(2x+5)$$
$$= (2x+5)\cdot 3 + (3x-4)\cdot 2$$
$$= 6x + 15 + 6x - 8$$
$$= 12x + 7$$

Differentiate $f(x) = (2x+5)(3x-4)$ using the Power Rule (Theorem 1). First, we multiply the binomial terms in the function.

$$f(x) = (2x+5)(3x-4)$$
$$= 6x^2 + 7x - 20$$

Therefore, by Theorem 1 and Theorem 4 we have:

$$f'(x) = \frac{d}{dx}\left(6x^2 + 7x - 20\right)$$
$$= \frac{d}{dx}\left(6x^2\right) + \frac{d}{dx}(7x) - \frac{d}{dx}(20) \quad \text{Theorem 4}$$
$$= 12x + 7 \quad \text{Theorem 1}$$

The two results are equivalent.

4. Using the Product Rule:

$$g'(x) = \frac{d}{dx}\left[(3x-2)(4x+1)\right]$$
$$= (3x-2)\cdot 4 + (4x+1)\cdot 3$$
$$= 12x - 8 + 12x + 3$$
$$= 24x - 5$$

Using the Power Rule:

$$g(x) = (3x-2)(4x+1) = 12x^2 - 5x - 2$$
$$g'(x) = \frac{d}{dx}\left(12x^2 - 5x - 2\right)$$
$$= 24x - 5$$

The two results are equivalent.

5. Differentiate $G(x) = 4x^2\left(x^3 + 5x\right)$ using the Product Rule.

$$G'(x) = \frac{d}{dx}\left[4x^2\left(x^3 + 5x\right)\right]$$
$$= 4x^2 \cdot \frac{d}{dx}\left(x^3 + 5x\right) + \left(x^3 + 5x\right)\cdot\frac{d}{dx}\left(4x^2\right)$$
$$= 4x^2 \cdot \left(3x^2 + 5\right) + \left(x^3 + 5x\right)\cdot(8x)$$
$$= 12x^4 + 20x^2 + 8x^4 + 40x^2$$
$$= 20x^4 + 60x^2$$

The solution is continued on the next page.

Differentiate $G(x)=4x^2(x^3+5x)$ using the Power Rule. First, we multiply the function.

$$G(x)=4x^2(x^3+5x)$$
$$=4x^5+20x^3$$

Therefore, we have:

$$G'(x)=\frac{d}{dx}(4x^5+20x^3)$$
$$=\frac{d}{dx}(4x^5)+\frac{d}{dx}(20x^3) \quad \text{Theorem 4}$$
$$=4(5x^4)+20(3x^2) \quad \text{Theorem 1, Theorem 3}$$
$$=20x^4+60x^2$$

The two results are equivalent.

6. Using the Product Rule:

$$F'(x)=\frac{d}{dx}\left[3x^4(x^2-4x)\right]$$
$$=3x^4\cdot(2x-4)+(x^2-4x)\cdot 12x^3$$
$$=6x^5-12x^4+12x^5-48x^4$$
$$=18x^5-60x^4$$

Using the Power Rule:

$$F(x)=3x^4(x^2-4x)=3x^6-12x^5$$
$$F'(x)=\frac{d}{dx}(3x^6-12x^5)$$
$$=18x^5-60x^4$$

The two results are equivalent.

7. Differentiate $y=(3\sqrt{x}+2)x^2$ using the Product Rule.

$$\frac{dy}{dx}=\frac{d}{dx}\left[(3\sqrt{x}+2)x^2\right]$$
$$=(3\sqrt{x}+2)\cdot\frac{d}{dx}(x^2)+x^2\cdot\frac{d}{dx}(3\sqrt{x}+2)$$
$$=(3x^{1/2}+2)\cdot\frac{d}{dx}(x^2)+x^2\cdot\frac{d}{dx}(3x^{1/2}+2)$$
$$=(3x^{1/2}+2)\cdot 2x+x^2\cdot\frac{3}{2}x^{-1/2} \quad \text{Theorem 1}$$
$$=6x^{3/2}+4x+\frac{3}{2}x^{3/2}$$
$$=\frac{15}{2}x^{3/2}+4x$$

Differentiate $y=(3\sqrt{x}+2)x^2$ using the Power Rule. First, we multiply the function.

$$y=(3\sqrt{x}+2)x^2$$
$$=3x^{1/2+2}+2x^2$$
$$=3x^{5/2}+2x^2$$

Therefore, we have:

$$\frac{dy}{dx}=\frac{d}{dx}\left(3x^{5/2}+2x^2\right)$$
$$=\frac{d}{dx}\left(3x^{5/2}\right)+\frac{d}{dx}(2x^2) \quad \text{Theorem 4}$$
$$=3\left(\frac{5}{2}x^{5/2-1}\right)+2(2x^1) \quad \text{Theorem 1, Theorem 3}$$
$$=\frac{15}{2}x^{3/2}+4x$$

The two results are equivalent.

8. Using the Product Rule:

$$\frac{dy}{dx}=\frac{d}{dx}\left[(4\sqrt{x}+3)x^3\right]$$
$$=(4x^{1/2}+3)\cdot\frac{d}{dx}(x^3)+x^3\cdot\frac{d}{dx}(4x^{1/2}+3)$$
$$=(4x^{1/2}+3)\cdot 3x^2+x^2\cdot\frac{4}{2}x^{-1/2}$$
$$=14x^{5/2}+9x^2$$

Using the Power Rule.

$$y=(4\sqrt{x}+3)x^3=4x^{7/2}+3x^3$$

Therefore, we have:

$$\frac{dy}{dx}=\frac{d}{dx}\left(4x^{7/2}+3x^3\right)$$
$$=4\left(\frac{7}{2}x^{7/2-1}\right)+3(3x^2)$$
$$=14x^{5/2}+9x^2$$

The two results are equivalent.

9. Differentiate $g(x)=(4x-3)(2x^2+3x+5)$ using the Product Rule.

$$g'(x)=\frac{d}{dx}\left[(4x-3)(2x^2+3x+5)\right]$$
$$=(4x-3)\cdot\frac{d}{dx}(2x^2+3x+5)+(2x^2+3x+5)\cdot\frac{d}{dx}(4x-3)$$
$$=(4x-3)\cdot(4x+3)+(2x^2+3x+5)\cdot 4$$
$$=16x^2-9+8x^2+12x+20$$
$$=24x^2+12x+11$$

Differentiate $g(x)=(4x-3)(2x^2+3x+5)$ using the Power Rule. First, we multiply the terms in the function.

$$g(x)=(4x-3)(2x^2+3x+5)$$
$$=8x^3+6x^2+11x-15$$

Therefore, we have:

$$g'(x)=\frac{d}{dx}(8x^3+6x^2+11x-15)$$
$$=\frac{d}{dx}(8x^3)+\frac{d}{dx}(6x^2)+\frac{d}{dx}(11x)-\frac{d}{dx}(15)$$
$$=24x^2+12x+11$$

The two results are equivalent.

10. Using the Product Rule:

$$f'(x)=\frac{d}{dx}\left[(2x+5)(3x^2-4x+1)\right]$$
$$f'(x)=(2x+5)\cdot(6x-4)+(3x^2-4x+1)\cdot 2$$
$$=12x^2+22x-20+6x^2-8x+2$$
$$=18x^2+14x-18$$

Using the Power Rule:

$$f(x)=\left[(2x+5)(3x^2-4x+1)\right]$$
$$=6x^3+7x^2-18x+5$$
$$f'(x)=\frac{d}{dx}(6x^3+7x^2-18x+5)$$
$$=18x^2+14x-18$$

The two results are equivalent.

11. Differentiate $F(t)=(\sqrt{t}+2)(3t-4\sqrt{t}+7)$ using the Product Rule.

$$F'(t)=\frac{d}{dt}\left[(\sqrt{t}+2)(3t-4\sqrt{t}+7)\right]$$
$$=(t^{1/2}+2)\cdot\frac{d}{dt}(3t-4t^{1/2}+7)+(3t-4t^{1/2}+7)\cdot\frac{d}{dt}(t^{1/2}+2) \qquad \left[\sqrt{t}=t^{1/2}\right]$$
$$=(t^{1/2}+2)\cdot\left(3-4\left(\frac{1}{2}t^{-1/2}\right)\right)+(3t-4t^{1/2}+7)\cdot\left(\frac{1}{2}t^{-1/2}\right)$$

The solution is continued at the top of the next column.

$$F'(t)=(t^{1/2}+2)\cdot(3-2t^{-1/2})+(3t-4t^{1/2}+7)\cdot\left(\frac{1}{2}t^{-1/2}\right)$$
$$=3t^{1/2}-2+6-4t^{-1/2}+\frac{3}{2}t^{1/2}-2+\frac{7}{2}t^{-1/2}$$
$$=\frac{9}{2}t^{1/2}-\frac{1}{2}t^{-1/2}+2$$
$$=\frac{9\sqrt{t}}{2}-\frac{1}{2\sqrt{t}}+2$$

Differentiate $F(t)=(\sqrt{t}+2)(3t-4\sqrt{t}+7)$ using the Power Rule

$$F(t)=(\sqrt{t}+2)(3t-4\sqrt{t}+7)$$
$$=3t^{3/2}-4t+7t^{1/2}+6t-8t^{1/2}+14$$
$$=3t^{3/2}-t^{1/2}+2t+14$$

Therefore, we have:

$$F'(t)=\frac{d}{dt}(3t^{3/2}-t^{1/2}+2t+14)$$
$$=\frac{d}{dt}(3t^{3/2})-\frac{d}{dt}(t^{1/2})+\frac{d}{dt}(2t)+\frac{d}{dt}(14)$$
$$=\frac{9t^{1/2}}{2}-\frac{1}{2}t^{-1/2}+2$$
$$=\frac{9\sqrt{t}}{2}-\frac{1}{2\sqrt{t}}+2$$

The two results are equivalent.

12. Using the Product Rule:

$$G'(t)=\frac{d}{dt}\left[\left(2t+3\sqrt{t}+5\right)\left(\sqrt{t}+4\right)\right]$$
$$=\left(2t+3t^{1/2}+5\right)\cdot\left(\frac{1}{2}t^{-1/2}\right)+$$
$$\left(t^{1/2}+4\right)\cdot\left(2+\frac{3}{2}t^{-1/2}\right)$$
$$=t^{1/2}+\frac{3}{2}+\frac{5}{2}t^{-1/2}+2t^{1/2}+\frac{3}{2}+8+6t^{-1/2}$$
$$=3t^{1/2}+\frac{17}{2}t^{-1/2}+11$$
$$=3\sqrt{t}+\frac{17}{2\sqrt{t}}+11$$

Using the Power Rule:

$$G(x)=\left[\left(2t+3\sqrt{t}+5\right)\left(\sqrt{t}+4\right)\right]$$
$$=2t^{3/2}+17t^{1/2}+11t+20$$
$$G'(t)=\frac{d}{dt}\left(2t^{3/2}+17t^{1/2}+11t+20\right)$$
$$=3t^{1/2}+\frac{17}{2}t^{-1/2}+11$$
$$=3\sqrt{t}+\frac{17}{2\sqrt{t}}+11$$

The two results are equivalent.

13. Differentiate $y=\frac{x^7}{x^3}$ using the Quotient Rule (Theorem 6).

$$\frac{dy}{dx}=\frac{d}{dx}\left(\frac{x^7}{x^3}\right)$$
$$=\frac{x^3\frac{d}{dx}\left(x^7\right)-x^7\frac{d}{dx}\left(x^3\right)}{\left(x^3\right)^2}$$
$$=\frac{x^3\left(7x^6\right)-x^7\left(3x^2\right)}{x^6}$$
$$\frac{dy}{dx}=\frac{7x^9-3x^9}{x^6}$$
$$=\frac{4x^9}{x^6}$$
$$=4x^3,\quad \text{for } x\neq 0$$

Differentiate $y=\frac{x^7}{x^3}=x^4$ using the Power Rule.

$$\frac{dy}{dx}=\frac{d}{dx}x^4$$
$$=4x^{4-1}$$
$$=4x^3,\quad \text{for } x\neq 0$$

The two results are equivalent.

14. Using the Quotient Rule.

$$\frac{dy}{dx}=\frac{d}{dx}\left(\frac{x^6}{x^4}\right)$$
$$=\frac{x^4\frac{d}{dx}\left(x^6\right)-x^6\frac{d}{dx}\left(x^4\right)}{\left(x^4\right)^2}$$
$$=\frac{x^4\left(6x^5\right)-x^6\left(4x^3\right)}{x^8}$$
$$\frac{dy}{dx}=\frac{6x^9-4x^9}{x^8}$$
$$=\frac{2x^9}{x^8}$$
$$=2x,\quad \text{for } x\neq 0$$

Using the Power Rule.

$$y=\frac{x^6}{x^4}=x^2$$
$$\frac{dy}{dx}=\frac{d}{dx}x^2$$
$$=2x,\quad \text{for } x\neq 0$$

The two results are equivalent.

15. Using the Quotient Rule.

$$g'(x)=\frac{d}{dx}\left(\frac{3x^7-x^3}{x}\right)$$
$$=\frac{x\left(21x^6-3x^2\right)-\left(3x^7-x^3\right)(1)}{x^2}$$
$$=\frac{18x^7-2x^3}{x^2}$$
$$=\frac{x^2\left(18x^5-2x\right)}{x^2}$$
$$=18x^5-2x,\quad \text{for } x\neq 0$$

Using the Power Rule.

$$g(x)=\frac{3x^7-x^3}{x}=\frac{x\left(3x^6-x^2\right)}{x}=3x^6-x^2$$
$$g'(x)=\frac{d}{dx}\left(3x^6-x^2\right)$$
$$=18x^5-2x,\quad \text{for } x\neq 0$$

The two results are equivalent.

16. Differentiate $f(x)=\dfrac{2x^5+x^2}{x}$ using the Quotient Rule.

$$f'(x)=\frac{d}{dx}\left(\frac{2x^5+x^2}{x}\right)$$
$$=\frac{x\frac{d}{dx}\left(2x^5+x^2\right)-\left(2x^5+x^2\right)\frac{d}{dx}(x)}{(x)^2}$$
$$f'(x)=\frac{x\left(10x^4+2x\right)-\left(2x^5+x^2\right)(1)}{x^2}$$
$$=\frac{10x^5+2x^2-2x^5-x^2}{x^2}$$
$$=\frac{8x^5+x^2}{x^2}$$
$$=\frac{x^2\left(8x^3+1\right)}{x^2}$$
$$=8x^3+1, \quad \text{for } x\neq 0$$

Differentiate $f(x)=\dfrac{2x^5+x^2}{x}$ using the Power Rule.

First, factor the numerator and divide the common factors.

$$f(x)=\frac{2x^5+x^2}{x}$$
$$=\frac{x\left(2x^4+x\right)}{x}$$
$$=2x^4+x$$
$$f'(x)=\frac{d}{dx}\left(2x^4+x\right)$$
$$=8x^3+1, \quad \text{for } x\neq 0$$

The two results are equivalent.

17. Using the Quotient Rule.

$$F'(x)=\frac{d}{dx}\left(\frac{x^3+27}{x+3}\right)$$
$$=\frac{(x+3)\left(3x^2\right)-\left(x^3+27\right)(1)}{(x+3)^2}$$
$$=\frac{(x+3)\left(3x^2\right)-(x+3)\left(x^2-3x+9\right)}{(x+3)^2}$$
$$=\frac{(x+3)\left(3x^2-\left(x^2-3x+9\right)\right)}{(x+3)^2}$$
$$=\frac{\left(2x^2+3x-9\right)}{(x+3)}$$
$$=\frac{(x+3)(2x-3)}{(x+3)}$$
$$=2x-3, \quad \text{for } x\neq -3$$

Using the Power Rule.

$$F(x)=\frac{x^3+27}{x+3}$$
$$=\frac{(x+3)\left(x^2-3x+9\right)}{x+3}$$
$$=x^2-3x+9$$
$$F'(x)=\frac{d}{dx}\left(x^2-3x+9\right)$$
$$=2x-3, \quad \text{for } x\neq -3$$

The two results are equivalent.

18. Differentiate $G(x)=\dfrac{8x^3-1}{2x-1}$ using the Quotient Rule.

$$G'(x)=\frac{d}{dx}\left(\frac{8x^3-1}{2x-1}\right)$$
$$=\frac{(2x-1)\frac{d}{dx}(8x^3-1)-(8x^3-1)\frac{d}{dx}(2x-1)}{(2x-1)^2}$$
$$=\frac{(2x-1)(24x^2)-(8x^3-1)(2)}{(2x-1)^2}$$
$$=\frac{(2x-1)(24x^2)-(2x-1)(4x^2+2x+1)(2)}{(2x-1)^2}$$
$$=\frac{(2x-1)\left[(24x^2)-(4x^2+2x+1)(2)\right]}{(2x-1)^2}$$
$$=\frac{(2x-1)\left[(24x^2)-(8x^2+4x+2)\right]}{(2x-1)^2}$$
$$=\frac{\left[16x^2-4x-2\right]}{(2x-1)}$$
$$=\frac{(2x-1)(8x+2)}{(2x-1)}$$
$$=8x+2; \qquad x\neq\tfrac{1}{2}$$

Differentiate $G(x)=\dfrac{8x^3-1}{2x-1}$ using the Power Rule.

First, factor the numerator and divide the common factors.

$$G(x)=\frac{8x^3-1}{2x-1}$$
$$=\frac{(2x-1)(4x^2+2x+1)}{2x-1} \quad \text{Difference of cubes}$$
$$=4x^2+2x+1$$
$$G'(x)=\frac{d}{dx}(4x^2+2x+1)$$
$$=8x+2; \qquad x\neq\tfrac{1}{2}$$

The two results are equivalent.

19. Differentiate $y=\dfrac{t^2-16}{t+4}$ using the Quotient Rule.

$$\frac{dy}{dt}=\frac{d}{dt}\left(\frac{t^2-16}{t+4}\right)$$
$$=\frac{(t+4)\frac{d}{dt}(t^2-16)-(t^2-16)\frac{d}{dt}(t+4)}{(t+4)^2}$$
$$=\frac{(t+4)(2t)-(t^2-16)(1)}{(t+4)^2}$$
$$=\frac{2t^2+8t-t^2+16}{(t+4)^2}$$
$$=\frac{t^2+8t+16}{(t+4)^2}$$
$$\frac{dy}{dt}=\frac{(t+4)^2}{(t+4)^2}$$
$$=1; \qquad t\neq-4$$

Differentiate $y=\dfrac{t^2-16}{t+4}$ using the Power Rule.

First, factor the numerator and divide the common factors.

$$y=\frac{t^2-16}{t+4}$$
$$=\frac{(t+4)(t-4)}{t+4} \quad \text{Difference of squares}$$
$$=t-4$$
$$\frac{dy}{dt}=\frac{d}{dt}(x-4)$$
$$=1, \quad \text{for } t\neq-4$$

The two results are equivalent.

20. Using the Quotient Rule.

$$\frac{dy}{dt}=\frac{d}{dt}\left(\frac{t^2-25}{t-5}\right)$$
$$=\frac{(t-5)(2t)-(t^2-25)(1)}{(t-5)^2}$$
$$=\frac{2t^2-10t-(t^2-25)}{(t-5)^2}$$

The solution is continued at the top of the next column.

$$\frac{dy}{dt}=\frac{t^2-10t+25}{(t-5)^2}$$
$$=\frac{(t-5)^2}{(t-5)^2}$$
$$=1, \qquad \text{for } t\neq 5$$

Using the Power Rule.

$$y=\frac{t^2-25}{t-5}$$
$$=\frac{(t-5)(t+5)}{t-5}$$
$$=t+5$$
$$\frac{dy}{dt}=\frac{d}{dt}(t+5)$$
$$=1, \qquad \text{for } t\neq 5$$

The two results are equivalent.

21. $f(x)=(3x^2-2x+5)(4x^2+3x-1)$

Using the Product Rule, we have:

$$f'(x)=\frac{d}{dx}\left[(3x^2-2x+5)(4x^2+3x-1)\right]$$
$$=(3x^2-2x+5)\cdot\frac{d}{dx}(4x^2+3x-1)+$$
$$(4x^2+3x-1)\cdot\frac{d}{dx}(3x^2-2x+5)$$
$$=(3x^2-2x+5)\cdot(8x+3)+$$
$$(4x^2+3x-1)\cdot(6x-2)$$

Simplifying, we get

$$=(24x^3-7x^2+34x+15)+$$
$$(24x^3+10x^2-12x+2)$$
$$=48x^3+3x^2+22x+17$$

22. $g(x)=(5x^2+4x-3)(2x^2-3x+1)$

Using the Product Rule, we have:

$$g'(x)=\frac{d}{dx}\left[(5x^2+4x-3)(2x^2-3x+1)\right]$$
$$=(5x^2+4x-3)\cdot(4x-3)+$$
$$(2x^2-3x+1)\cdot(10x+4)$$
$$=(20x^3+x^2-24x+9)+$$
$$(20x^3-22x^2-2x+4)$$
$$=40x^3-21x^2-26x+13$$

23. $y=\dfrac{5x^2-1}{2x^3+3}$

Using the Quotient Rule.

$$\frac{dy}{dx}=\frac{d}{dx}\left(\frac{5x^2-1}{2x^3+3}\right)$$
$$=\frac{(2x^3+3)\frac{d}{dx}(5x^2-1)-(5x^2-1)\frac{d}{dx}(2x^3+3)}{(2x^3+3)^2}$$
$$=\frac{(2x^3+3)(10x)-(5x^2-1)(6x^2)}{(2x^3+3)^2}$$
$$=\frac{20x^4+30x-30x^4+6x^2}{(2x^3+3)^2}$$
$$=\frac{-10x^4+6x^2+30x}{(2x^3+3)^2}$$
$$=\frac{-2x(5x^3-3x-15)}{(2x^3+3)^2}$$

24. $y=\dfrac{3x^4+2x}{x^3-1}$

Using the Quotient Rule.

$$\frac{dy}{dx}=\frac{d}{dx}\left(\frac{3x^4+2x}{x^3-1}\right)$$
$$=\frac{(x^3-1)\frac{d}{dx}(3x^4+2x)-(3x^4+2x)\frac{d}{dx}(x^3-1)}{(x^3-1)^2}$$
$$=\frac{(x^3-1)(12x^3+2)-(3x^4+2x)(3x^2)}{(x^3-1)^2}$$
$$=\frac{12x^6-10x^3-2-(9x^6+6x^3)}{(x^3-1)^2}$$
$$=\frac{3x^6-16x^3-2}{(x^3-1)^2}$$

25. $G(x)=(8x+\sqrt{x})(5x^2+3)$

$G(x)=(8x+x^{1/2})(5x^2+3) \qquad \left[\sqrt{x}=x^{1/2}\right]$

Using the Product Rule, we calculate the derivate at the top of the next column.

$$G'(x) = \frac{d}{dx}\left[\left(8x + x^{1/2}\right)\left(5x^2 + 3\right)\right]$$
$$= \left(8x + x^{1/2}\right) \cdot \frac{d}{dx}\left(5x^2 + 3\right) +$$
$$\left(5x^2 + 3\right) \cdot \frac{d}{dx}\left(8x + x^{1/2}\right)$$
$$= \left(8x + x^{1/2}\right) \cdot (10x) +$$
$$\left(5x^2 + 3\right) \cdot \left(8 + \frac{1}{2}x^{-1/2}\right)$$

Simplifying, we get

$$= \left(80x^2 + 10x^{3/2}\right) +$$
$$\left(40x^2 + \frac{5}{2}x^{3/2} + \frac{3}{2}x^{-1/2} + 24\right)$$
$$= 120x^2 + \frac{25}{2}x^{3/2} + \frac{3}{2}x^{-1/2} + 24$$

26. $F(x) = \left(-3x^2 + 4x\right)\left(7\sqrt{x} + 1\right)$

Using the Product Rule, we have:

$$F(x) = \frac{d}{dx}\left[\left(-3x^2 + 4x\right)\left(7\sqrt{x} + 1\right)\right]$$
$$= \left(-3x^2 + 4x\right) \cdot \frac{d}{dx}\left(7\sqrt{x} + 1\right) +$$
$$\left(7\sqrt{x} + 1\right) \cdot \frac{d}{dx}\left(-3x^2 + 4x\right)$$
$$= \left(-3x^2 + 4x\right) \cdot \left(\frac{7}{2}x^{-1/2}\right) +$$
$$\left(7x^{1/2} + 1\right) \cdot (-6x + 4)$$
$$= \left(-\frac{21}{2}x^{3/2} + 14x^{1/2}\right) +$$
$$\left(-42x^{3/2} - 6x + 28x^{1/2} + 4\right)$$
$$= -\frac{105}{2}x^{3/2} - 6x + 42x^{1/2} + 4$$

27. $g(t) = \frac{t}{3 - t} + 5t^3$

Differentiating we have:

$$g'(t) = \frac{d}{dt}\left(\frac{t}{3 - t} + 5t^3\right)$$
$$= \frac{d}{dt}\left(\frac{t}{3 - t}\right) + \frac{d}{dt}\left(5t^3\right)$$

Using the derivative from the previous page, we will apply the Quotient Rule to the first term, and the Power Rule to the second term.

$$g'(t) = \underbrace{\frac{(3 - t) \cdot \frac{d}{dt}(t) - t \cdot \frac{d}{dt}(3 - t)}{(3 - t)^2}}_{\text{Quotient Rule}} + 15t^2$$
$$= \frac{(3 - t)(1) - t(-1)}{(3 - t)^2} + 15t^2$$
$$= \frac{3}{(3 - t)^2} + 15t^2$$

28. $f(t) = \frac{t}{5 + 2t} - 2t^4$

$$f'(t) = \frac{d}{dt}\left(\frac{t}{5 + 2t} - 2t^4\right)$$
$$= \underbrace{\frac{(5 + 2t)(1) - t(2)}{(5 + 2t)^2}}_{\text{Quotient Rule}} - 8t^3$$
$$f'(t) = \frac{5}{(5 + 2t)^2} - 8t^3$$

29. $F(x) = (x + 3)^2 = (x + 3)(x + 3)$

Using the Product Rule, we have

$$F'(x) = \frac{d}{dx}\left[(x + 3)(x + 3)\right]$$
$$= (x + 3) \cdot \frac{d}{dx}(x + 3) + (x + 3) \cdot \frac{d}{dx}(x + 3)$$
$$= (x + 3) \cdot (1) + (x + 3) \cdot (1)$$
$$= 2x + 6$$
$$= 2(x + 3)$$

30. $G(x) = (5x - 4)^2 = (5x - 4)(5x - 4)$

Using the Product Rule, we have

$$G'(x) = \frac{d}{dx}\left[(5x - 4)(5x - 4)\right]$$
$$= (5x - 4) \cdot \frac{d}{dx}(5x - 4) +$$
$$(5x - 4) \cdot \frac{d}{dx}(5x - 4)$$
$$= (5x - 4) \cdot (5) + (5x - 4) \cdot (5)$$
$$= 50x - 40$$

31. $y=\left(x^3-4x\right)^2=\left(x^3-4x\right)\left(x^3-4x\right)$

Using the Product Rule, we have

$$\frac{dy}{dx}=\frac{d}{dx}\left[\left(x^3-4x\right)\left(x^3-4x\right)\right]$$
$$=\left(x^3-4x\right)\cdot\frac{d}{dx}\left(x^3-4x\right)+\left(x^3-4x\right)\cdot\frac{d}{dx}\left(x^3-4x\right)$$
$$=\left(x^3-4x\right)\cdot\left(3x^2-4\right)+\left(x^3-4x\right)\cdot\left(3x^2-4\right)$$
$$=2\left(x^3-4x\right)\left(3x^2-4\right)$$
$$=2x\left(x^2-4\right)\left(3x^2-4\right)$$

32. $y=\left(3x^2-4x+5\right)^2$

$$=\left(3x^2-4x+5\right)\left(3x^2-4x+5\right)$$

Using the Product Rule, we have

$$\frac{dy}{dx}=\frac{d}{dx}\left[\left(3x^2-4x+5\right)\left(3x^2-4x+5\right)\right]$$
$$=\left(3x^2-4x+5\right)\cdot\frac{d}{dx}\left(3x^2-4x+5\right)+\left(3x^2-4x+5\right)\cdot\frac{d}{dx}\left(3x^2-4x+5\right)$$
$$=\left(3x^2-4x+5\right)\cdot(6x-4)+\left(3x^2-4x+5\right)\cdot(6x-4)$$
$$=2\left(3x^2-4x+5\right)(6x-4)$$
$$=4(3x-2)\left(3x^2-4x+5\right)$$

33. $g(x)=5x^{-3}\left(x^4-5x^3+10x-2\right)$

Using the Product Rule:

$$g'(x)=5x^{-3}\frac{d}{dx}\left(x^4-5x^3+10x-2\right)+\left(x^4-5x^3+10x-2\right)\frac{d}{dx}\left(5x^{-3}\right)$$
$$=\left(5x^{-3}\right)\left(4x^3-15x^2+10\right)+\left(x^4-5x^3+10x-2\right)\left(-15x^{-4}\right)$$

Simplifying, we get

$$=20-75x^{-1}+50x^{-3}-15+75x^{-1}-150x^{-3}+30x^{-4}$$
$$=5-100x^{-3}+30x^{-4}$$

34. $f(x)=6x^{-4}\left(6x^3+10x^2-8x+3\right)$

Using the Product Rule:

$$f'(x)=\left(6x^{-4}\right)\left(18x^2+20x-8\right)+\left(6x^3+10x^2-8x+3\right)\left(-24x^{-5}\right)$$
$$=108x^{-2}+120x^{-3}-48x^{-4}-144x^{-2}-240x^{-3}+192x^{-4}-72x^{-5}$$
$$=-36x^{-2}-120x^{-3}+144x^{-4}-72x^{-5}$$

35. $F(t)=\left(t+\frac{2}{t}\right)\left(t^2-3\right)=\left(t+2t^{-1}\right)\left(t^2-3\right)$

Using the Product Rule, we have:

$$f'(t)=\frac{d}{dt}\left[\left(t+2t^{-1}\right)\left(t^2-3\right)\right]$$
$$=\left(t+2t^{-1}\right)\cdot\frac{d}{dt}\left(t^2-3\right)+\left(t^2-3\right)\cdot\frac{d}{dt}\left(t+2t^{-1}\right)$$
$$f'(t)=\left(t+2t^{-1}\right)\cdot(2t)+\left(t^2-3\right)\cdot\left(1-2t^{-2}\right)$$

Simplifying, we get

$$=2t^2+4+\left(t^2-2-3+6t^{-2}\right)$$
$$=3t^2-1+6t^{-2}$$
$$=3t^2-1+\frac{6}{t^2}$$

36. $G(t)=\left(3t^5-t^2\right)\left(t-\frac{5}{t}\right)=\left(3t^5-t^2\right)\left(t-5t^{-1}\right)$

Using the Product Rule, we have:

$$G'(t)=\frac{d}{dt}\left[\left(3t^5-t^2\right)\left(t-5t^{-1}\right)\right]$$
$$=\left(3t^5-t^2\right)\left(1+5t^{-2}\right)+\left(t-5t^{-1}\right)\left(15t^4-2t\right)$$

Simplifying, we get

$$=\left(3t^5+15t^3-t^2-5\right)+\left(15t^5-75t^3-2t^2+10\right)$$
$$=18t^5-60t^3-3t^2+5,\quad \text{for } t\neq 0$$

37. $y = \dfrac{x^2+1}{x^3-1} - 5x^2$

Differentiating we have:

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{x^2+1}{x^3-1} - 5x^2\right)$$

$$= \frac{d}{dx}\left(\frac{x^2+1}{x^3-1}\right) - \frac{d}{dx}\left(5x^2\right)$$

We will apply the Quotient Rule to the first term, and the Power Rule to the second term.

$$\frac{dy}{dx} = \underbrace{\frac{\left(x^3-1\right)\cdot(2x) - \left(x^2+1\right)\cdot\left(3x^2\right)}{\left(x^3-1\right)^2}}_{\text{Quotient Rule}} - 10x$$

Simplifying, we get

$$= \frac{2x^4 - 2x - \left(3x^4+3x^2\right)}{\left(x^3-1\right)^2} - 10x$$

$$= \frac{-x^4 - 3x^2 - 2x}{\left(x^3-1\right)^2} - 10x$$

38. $y = \dfrac{x^3-1}{x^2+1} + 4x^3$

Differentiating we have:

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{x^3-1}{x^2+1} + 4x^3\right)$$

$$\frac{dy}{dx} = \frac{\left(x^2+1\right)\cdot\left(3x^2\right) - \left(x^3-1\right)\cdot(2x)}{\left(x^2+1\right)^2} + 12x^2$$

$$= \frac{\left(3x^4+3x^2\right) - \left(2x^4-2x\right)}{\left(x^2+1\right)^2} + 12x^2$$

$$= \frac{x^4+3x^2+2x}{\left(x^2+1\right)^2} + 12x^2$$

39. $y = \dfrac{\sqrt[3]{x}-7}{\sqrt{x}+3} = \dfrac{x^{1/3}-7}{x^{1/2}+3}$

Using the Quotient Rule.

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{x^{1/3}-7}{x^{1/2}+3}\right)$$

$$= \frac{\left(x^{1/2}+3\right)\frac{d}{dx}\left(x^{1/3}-7\right) - \left(x^{1/3}-7\right)\frac{d}{dx}\left(x^{1/2}+3\right)}{\left(x^{1/2}+3\right)^2}$$

$$= \frac{\left(x^{1/2}+3\right)\left(\frac{1}{3}x^{-2/3}\right) - \left(x^{1/3}-7\right)\left(\frac{1}{2}x^{-1/2}\right)}{\left(x^{1/2}+3\right)^2}$$

Note, the previous derivative can be simplified as follows

$$\frac{dy}{dx} = \frac{\left(x^{1/2}+3\right)\left(\frac{1}{3}x^{-2/3}\right) - \left(x^{1/3}-7\right)\left(\frac{1}{2}x^{-1/2}\right)}{\left(x^{1/2}+3\right)^2}$$

$$= \frac{\frac{1}{3}x^{-1/6} + x^{-2/3} - \frac{1}{2}x^{-1/6} + \frac{7}{2}x^{-1/2}}{\left(x^{1/2}+3\right)^2}$$

$$= \frac{x^{-2/3} - \frac{1}{6}x^{-1/6} + \frac{7}{2}x^{-1/2}}{\left(x^{1/2}+3\right)^2} \cdot \frac{6x^{2/3}}{6x^{2/3}}$$

$$= \frac{6 - \sqrt{x} + 21x^{1/6}}{6x^{2/3}\left(\sqrt{x}+3\right)^2}$$

40. $y = \dfrac{\sqrt{x}+4}{\sqrt[3]{x}-5} = \dfrac{x^{1/2}+4}{x^{1/3}-5}$

Using the Quotient Rule.

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{x^{1/2}+4}{x^{1/3}-5}\right)$$

$$= \frac{\left(x^{1/3}-5\right)\left(\frac{1}{2}x^{-1/2}\right) - \left(x^{1/2}+4\right)\left(\frac{1}{3}x^{-2/3}\right)}{\left(x^{1/3}-5\right)^2}$$

Simplifying the derivative, we get

$$\frac{dy}{dx} == \frac{-8 + \sqrt{x} - 15x^{1/6}}{6x^{2/3}\left(x^{1/3}-5\right)^2}$$

41. $f(x)=\frac{x}{x^{-1}+1}$

Using the Quotient Rule, we have

$$f'(x)=\frac{d}{dx}\left(\frac{x}{x^{-1}+1}\right)$$
$$=\frac{\left(x^{-1}+1\right)\frac{d}{dx}(x)-(x)\frac{d}{dx}\left(x^{-1}+1\right)}{\left(x^{-1}+1\right)^2}$$
$$=\frac{\left(x^{-1}+1\right)(1)-(x)\left(-1x^{-2}\right)}{\left(x^{-1}+1\right)^2}$$
$$=\frac{x^{-1}+1+x^{-1}}{\left(x^{-1}+1\right)^2}$$
$$=\frac{2x^{-1}+1}{\left(x^{-1}+1\right)^2},\quad \text{for } x\neq 0 \text{ and } x\neq -1$$

Note, the previous derivative could be simplified as follows:

$$f'(x)=\frac{2x^{-1}+1}{\left(x^{-1}+1\right)^2}$$
$$=\frac{\frac{2}{x}+1}{\left(\frac{1}{x}+1\right)^2}$$
$$=\frac{\frac{2+x}{x}}{\left(\frac{1+x}{x}\right)^2}$$
$$f'(x)=\frac{2+x}{x}\cdot\frac{x^2}{(1+x)^2}$$
$$=\frac{x(x+2)}{(1+x)^2},\quad \text{for } x\neq 0 \text{ and } x\neq -1$$

42. $f(x)=\frac{x^{-1}}{x+x^{-1}}$

Using the Quotient Rule, we have

$$f'(x)=\frac{d}{dx}\left(\frac{x^{-1}}{x+x^{-1}}\right)$$
$$=\frac{\left(x+x^{-1}\right)\left(-1x^{-2}\right)-\left(x^{-1}\right)\left(1-x^{-2}\right)}{\left(x+x^{-1}\right)^2}$$
$$=\frac{-x^{-1}-x^{-3}-x^{-1}+x^{-3}}{\left(x^{-1}+1\right)^2}$$
$$=\frac{-2x^{-1}}{\left(x^{-1}+1\right)^2},\quad \text{for } x\neq 0$$

The previous derivative can be simplified as follows:

$$f'(x)=\frac{-2x}{\left(x^2+1\right)^2},\quad \text{for } x\neq 0$$

43. $F(t)=\frac{1}{t-4}$

Using the Quotient Rule, we have

$$F'(t)=\frac{d}{dt}\left(\frac{1}{t-4}\right)$$
$$=\frac{(t-4)\frac{d}{dt}(1)-(1)\frac{d}{dt}(t-4)}{(t-4)^2}$$
$$=\frac{(t-4)(0)-(1)(1)}{(t-4)^2}$$
$$=\frac{-1}{(t-4)^2}$$

44. $G(t)=\frac{1}{t+2}$

Using the Quotient Rule, we have

$$G'(t)=\frac{d}{dt}\left(\frac{1}{t+2}\right)$$
$$=\frac{(t+2)(0)-(1)(1)}{(t+2)^2}$$
$$=-\frac{1}{(t+2)^2}$$

45. $f(x)=\frac{3x^2+2x}{x^2+1}$

Using the Quotient Rule, we have

$$f'(x)=\frac{d}{dx}\left(\frac{3x^2+2x}{x^2+1}\right)$$
$$=\frac{(x^2+1)\frac{d}{dx}(3x^2+2x)-(3x^2+2x)\frac{d}{dx}(x^2+1)}{(x^2+1)^2}$$
$$=\frac{(x^2+1)(6x+2)-(3x^2+2x)(2x)}{(x^2+1)^2}$$
$$=\frac{6x^3+2x^2+6x+2-(6x^3+4x^2)}{(x^2+1)^2}$$
$$=\frac{-2x^2+6x+2}{(x^2+1)^2}$$
$$=\frac{-2(x^2-3x-1)}{(x^2+1)^2}$$

46. $f(x)=\frac{3x^2-5x}{x^2-1}$

Using the Quotient Rule, we have

$$f'(x)=\frac{d}{dx}\left(\frac{3x^2-5x}{x^2-1}\right)$$
$$=\frac{(x^2-1)(6x-5)-(3x^2-5x)(2x)}{(x^2-1)^2}$$
$$=\frac{6x^3-5x^2-6x+5-(6x^3-10x^2)}{(x^2-1)^2}$$
$$=\frac{5x^2-6x+5}{(x^2-1)^2}$$

47. $g(t)=\frac{-t^2+3t+5}{t^2-2t+4}$

Using the Quotient Rule, we have

$$g'(t)=\frac{d}{dt}\left(\frac{-t^2+3t+5}{t^2-2t+4}\right)$$
$$=\frac{(t^2-2t+4)\frac{d}{dt}(-t^2+3t+5)}{(t^2-2t+4)^2}-$$
$$\frac{(-t^2+3t+5)\frac{d}{dt}(t^2-2t+4)}{(t^2-2t+4)^2}$$
$$=\frac{(t^2-2t+4)(-2t+3)-(-t^2+3t+5)(2t-2)}{(t^2-2t+4)^2}$$

Note, the previous derivative could be simplified as follows:

$$g'(t)=\frac{-2t^3+7t^2-14t+12-(-2t^3+8t^2+4t-10)}{(t^2-2t+4)^2}$$
$$=-\frac{t^2+18t-22}{(t^2-2t+4)^2}$$

48. $f(t)=\frac{3t^2+2t-1}{-t^2+4t+1}$

Using the Quotient Rule, we have

$$f'(t)=\frac{d}{dt}\left(\frac{3t^2+2t-1}{-t^2+4t+1}\right)$$
$$=\frac{(-t^2+4t+1)(6t+2)-(3t^2+2t-1)(-2t+4)}{(-t^2+4t+1)^2}$$

The previous derivative can be simplified as follows:

$$f'(t)=\frac{14t^2+4t+6}{(-t^2+4t+1)^2}.$$

49. $y=\frac{\sqrt{x}}{x+1}=\frac{x^{1/2}}{x+1}$

$$\frac{dy}{dx}=\frac{(x+1)\cdot\left(\frac{1}{2}x^{-1/2}\right)-x^{1/2}\cdot(1)}{(x+1)^2}$$

$$=\frac{\frac{1}{2}x^{1/2}+\frac{1}{2}x^{-1/2}-x^{1/2}}{(x+1)^2}$$

$$=\frac{-x^{1/2}+x^{-1/2}}{2(x+1)^2}$$

a) When $x=1$, $y=\frac{\sqrt{1}}{(1+1)}=\frac{1}{2}$.

$$\frac{dy}{dx}=\frac{-(1)^{1/2}+(1)^{-1/2}}{2(1+1)^2}=0$$

Therefore, the slope of the tangent line at $\left(1,\frac{1}{2}\right)$ is 0. The equation of the horizontal line passing through $\left(1,\frac{1}{2}\right)$ is $y=\frac{1}{2}$.

b) When $x=\frac{1}{4}$,

$$y=\frac{\sqrt{\frac{1}{4}}}{\left(\frac{1}{4}\right)+1}=\frac{\frac{1}{2}}{\frac{5}{4}}=\frac{1}{2}\cdot\frac{4}{5}=\frac{2}{5}$$

$$\frac{dy}{dx}=\frac{-\left(\frac{1}{4}\right)^{1/2}+\left(\frac{1}{4}\right)^{-1/2}}{2\left(\frac{1}{4}+1\right)^2}=\frac{-\frac{1}{2}+2}{\frac{25}{8}}=\frac{3}{2}\cdot\frac{8}{25}=\frac{12}{25}$$

Therefore, the slope of the tangent line at $\left(\frac{1}{4},\frac{2}{5}\right)$ is $\frac{12}{25}$. Using the point-slope equation, we have:

$$y-y_1=m(x-x_1)$$
$$y-\frac{2}{5}=\frac{12}{25}\left(x-\frac{1}{4}\right)$$
$$y-\frac{2}{5}=\frac{12}{25}x-\frac{3}{25}$$
$$y=\frac{12}{25}x+\frac{7}{25}$$

50. $y=\frac{8}{x^2+4}$

$$\frac{dy}{dx}=\frac{(x^2+4)(0)-8(2x)}{(x^2+4)^2}$$

$$\frac{dy}{dx}=\frac{-16x}{(x^2+4)^2}$$

a) When $x=0$, $\frac{dy}{dx}=\frac{-16(0)}{(0^2+4)^2}=0$, so the slope of the tangent line at $(0,2)$ is 0. The equation of the horizontal line passing through $(0,2)$ is $y=2$.

b) When $x=-2$,
$\frac{dy}{dx}=\frac{-16(-2)}{((-2)^2+4)^2}=\frac{32}{64}=\frac{1}{2}$, so the slope of the tangent line at $(-2,1)$ is $\frac{1}{2}$. Using the point-slope equation, we have:

$$y-y_1=m(x-x_1)$$
$$y-1=\frac{1}{2}(x-(-2))$$
$$y-1=\frac{1}{2}x+1$$
$$y=\frac{1}{2}x+2$$

51. $y=\frac{4x}{1+x^2}$

$$\frac{dy}{dx}=\frac{(1+x^2)(4)-4x(2x)}{(1+x^2)^2}$$

$$=\frac{4+4x^2-8x^2}{(1+x^2)^2}$$

$$=\frac{4-4x^2}{(1+x^2)^2}$$

a) When $x=0$, $\frac{dy}{dx}=\frac{4-4(0)^2}{\left(1+(0)^2\right)^2}=4$.

Therefore, the slope of the tangent line at $(0,0)$ is 4.

Using the point-slope equation, we have:

$$y-y_1=m(x-x_1)$$
$$y-0=4(x-0)$$
$$y=4x$$

b) When $x=-1$, $\frac{dy}{dx}=\frac{4-4(-1)^2}{\left(1+(-1)^2\right)^2}=\frac{0}{4}=0$.

Therefore, the slope of the tangent line at $(-1,-2)$ is 0. The equation of the horizontal line passing through $(-1,-2)$ is $y=-2$.

52. $y=x^2+\frac{3}{x-1}$

$$\frac{dy}{dx}=2x+\frac{(x-1)(0)-3(1)}{(x-1)^2}$$
$$=2x-\frac{3}{(x-1)^2}$$

a) When $x=2$, $y=(2)^2+\frac{3}{2-1}=4+3=7$,

and $\frac{dy}{dx}=2(2)-\frac{3}{(2-1)^2}=4-3=1$.

Therefore, the slope of the tangent line at $(2,7)$ is 1.

Using the point-slope equation, we have:

$$y-y_1=m(x-x_1)$$
$$y-7=1(x-2)$$
$$y-7=x-2$$
$$y=x+5$$

b) When $x=3$, $y=(3)^2+\frac{3}{3-1}=9+\frac{3}{2}=\frac{21}{2}$,

and $\frac{dy}{dx}=2(3)-\frac{3}{(3-1)^2}=6-\frac{3}{4}=\frac{21}{4}$.

Therefore, the slope of the tangent line at $\left(3,\frac{21}{2}\right)$ is $\frac{21}{4}$.

Using the point-slope equation, we have:

$$y-y_1=m(x-x_1)$$
$$y-\frac{21}{2}=\frac{21}{4}(x-3)$$
$$y-\frac{21}{2}=\frac{21}{4}x-\frac{63}{4}$$
$$y=\frac{21}{4}x-\frac{21}{4}$$

53. The average cost of producing x items is $A_C(x)=\frac{C(x)}{x}$. Therefore,

$$A_C(x)=\frac{7550+34x-0.068x^2}{x}.$$

Next, we take the derivative using the Quotient Rule to find the rate at which average cost is changing.

$$A_C'(x)=\frac{d}{dx}\left(\frac{750+34x-0.068x^2}{x}\right)$$

$$A_C'(x)=\frac{x(34-0.136x)-\left(750+34x-0.068x^2\right)(1)}{(x)^2}$$
$$=\frac{34x-0.136x^2-750-34x+0.068x^2}{x^2}$$
$$=\frac{-0.068x^2-750}{x^2}$$

Substituting 175 for x, we have

$$A_C'(400)=\frac{-0.068(175)^2-750}{(175)^2}$$
$$=\frac{-2082.5-750}{30,625}$$
$$\approx -0.0925$$

Therefore, when 175 belts have been produced, average cost is changing at a rate of -0.0925 dollars per belt.

54. $A_C(x) = \dfrac{C(x)}{x}$

$A_C(x) = \dfrac{375 + 0.75x^{3/4}}{x}$

The derivative is calculated at the top of the next column.

$$A_C'(x) = \frac{x\left(0.75\left(\frac{3}{4}x^{-1/4}\right)\right) - \left(375 + 0.75x^{3/4}\right)(1)}{x^2}$$

$$= \frac{\frac{9}{16}x^{3/4} - 375 - \frac{3}{4}x^{3/4}}{x^2}$$

$$= \frac{-\frac{3}{16}x^{3/4} - 375}{x^2}$$

Substituting 81 for x, we have:

$$A_C'(81) = \frac{\frac{-3}{16}(81)^{3/4} - 375}{(81)^2}$$

$$= -0.05792753$$

$$\approx -0.0579$$

Therefore, when 81 bottles of barbecue sauce have been produced, the average cost is changing at a rate of -0.0579 dollars per bottle.

55. The average revenue of producing x items is $A_R(x) = \dfrac{R(x)}{x}$. Therefore,

$A_R(x) = \dfrac{45x^{9/10}}{x} = \dfrac{45}{x^{1/10}}$.

Next, we take the derivative using the Quotient Rule to find the rate at which average revenue is changing.

$$A_R'(x) = \frac{d}{dx}\left(\frac{45}{x^{1/10}}\right)$$

$$= \frac{x^{1/10}(0) - (45)\left(\frac{1}{10}x^{-9/10}\right)}{\left(x^{1/10}\right)^2}$$

$$= \frac{-\frac{45}{10}x^{-9/10}}{x^{1/5}}$$

$$= -\frac{9}{2x^{11/10}}$$

Substituting 175 for x, we have

$$A_R'(175) = -\frac{9}{2(175)^{11/10}}$$

$$= -\frac{9}{586.64023}$$

$$\approx -0.0153$$

Therefore, when 175 belts have been produced, average revenue is changing at a rate of -0.0153 dollars per belt.

56. $A_R(x) = \dfrac{R(x)}{x}$

$A_R(x) = \dfrac{7.5x^{0.7}}{x} = \dfrac{7.5}{x^{0.3}}$

$$A_R'(x) = \frac{x^{0.3}(0) - (7.5)\left(0.3x^{-0.7}\right)}{\left(x^{0.3}\right)^2}$$

$$= \frac{-2.25x^{-0.7}}{x^{0.6}}$$

$$= -\frac{2.25}{x^{1.3}}$$

Substituting 81 for x, we have:

$$A_R'(81) = -\frac{2.25}{(81)^{1.3}}$$

$$= -0.007432792$$

$$\approx -0.0074$$

Therefore, when 81 bottles of barbecue sauce have been produce, the average revenue is changing at rate of -0.0074 dollars per bottle.

57. $A_P(x) = \dfrac{P(x)}{x} = \dfrac{R(x) - C(x)}{x}$

From Exercises 53 and 55, we know that

$$A_P(x) = \frac{45x^{9/10} - \left(750 + 34x - 0.068x^2\right)}{x}$$

$$= \frac{45x^{9/10} - 750 - 34x + 0.068x^2}{x}$$

Using the Quotient Rule to take the derivative, we have:

$$A_P'(x) = \frac{-4.5x^{9/10} + 0.068x^2 + 750}{x^2}$$

Substituting 175 for x, we have:

$$A_P'(175) = \frac{-4.5(175)^{9/10} + 0.068(175)^2 + 750}{(175)^2}$$
$$= \frac{-469.837 + 2082.5 + 750}{30,625}$$
$$= \frac{2362.6635}{30,625}$$
$$\approx 0.0772$$

When 175 belts have been produced and sold, the average profit is changing at a rate of 0.0772 dollars per belt.

Alternatively, we could have used the information in Exercises 53 and 55 to find the rate of change of average profit when 175 jackets are produced and sold. Notice that

$$A_P'(x) = A_R'(x) - A_C'(x)$$
$$= -0.0153 - (-0.0925)$$
$$= 0.0772$$

58. $A_P'(x) = A_R'(x) - A_C'(x)$

$$= -0.0074 - (-0.0579)$$
$$= 0.0505$$

Therefore, when 81 bottles where produced and sold, average profit is changing at rate of 0.050 dollars per bottle.

59. The average profit of producing x items is $A_P(x) = \frac{R(x) - C(x)}{x}$. Therefore,

$$A_P(x) = \frac{65x^{0.9} - \left(4300 + 2.1x^{0.6}\right)}{x}$$
$$= \frac{65x^{0.9} - 2.1x^{0.6} - 4300}{x}.$$

Using the Quotient Rule to take the derivative, we have

$$A_P'(x)$$
$$= \frac{x\left(65\left(0.9x^{-0.1}\right) - 2.1\left(0.6x^{-0.4}\right)\right) - \left(65x^{0.9} - 2.1x^{0.6} - 4300\right)(1)}{(x)^2}$$
$$= \frac{58.5x^{0.9} - 1.26x^{0.6} - \left(65x^{0.9} - 2.1x^{0.6} - 4300\right)}{x^2}$$
$$= \frac{-6.5x^{0.9} + 0.84x^{0.6} + 4300}{x^2}$$

Substituting 50 for x, we have

$$A_P'(50) = \frac{-6.5(50)^{0.9} + 0.84(50)^{0.6} + 4300}{(50)^2}$$
$$= \frac{4089.00428745}{2500}$$
$$= 1.63560171$$
$$\approx 1.64$$

Therefore, when 50 vases have been produced and sold, the average profit is changing at rate of 1.64 dollars per vase.

60. $A_P(x) = \frac{R(x) - C(x)}{x}$.

Therefore,

$$A_P(x) = \frac{75x^{0.8} - \left(900 + 18x^{0.7}\right)}{x}$$
$$= \frac{75x^{0.8} - 18x^{0.7} - 900}{x}.$$

$$A_P'(x)$$
$$= \frac{x\left(60x^{-0.2} - 12.6x^{-0.3}\right) - \left(75x^{0.8} - 18x^{0.7} - 900\right)(1)}{(x)^2}$$
$$= \frac{-15x^{0.8} + 5.4x^{0.7} + 900}{x^2}$$

Substituting 20 for x, we have

$$A_P'(20) = \frac{-15(20)^{0.8} + 5.4(20)^{0.7} + 900}{(20)^2}$$
$$= \frac{779.18169591}{400}$$
$$= 1.94795424$$
$$\approx 1.95$$

Therefore, when 20 skateboards have been produced and sold, the average profit is changing at rate of 1.95 dollars per skateboard.

61. $P(t) = 75 + \frac{500t}{2t^2+9}$

a) $P'(t) = \frac{(2t^2+9)(500)-(500t)(4t)}{(2t^2+9)^2}$

$= \frac{1000t^2+4500-2000t^2}{(2t^2+9)^2}$

$= \frac{-1000t^2+4500}{(2t^2+9)^2}$

$= \frac{-500(2t^2-9)}{(2t^2+9)^2}$

b) $P(2) = 75 + \frac{500(2)}{2(2)^2+9}$

$= 75 + \frac{1000}{17}$

$= 133.824$

After 2 months, the city's population is approximately 133.824 thousand people or 133,824 people.

c) $P'(2) = \frac{-500(2(2)^2-9)}{(2(2)^2+9)^2}$

$P'(2) = \frac{500}{289}$

$= 1.730$

The city population is growing at rate of 1.730 thousand people per year. In other words, the city population is increasing at a rate of 1,730 people per year.

d) $P(12) = 75 + \frac{500(12)}{2(12)^2+9}$

$= 75 + \frac{6000}{297}$

$= 95.2020$

After 12 months, the city's population is approximately 95.2020 thousand people or 95,202 people.

e) $P'(12) = \frac{-500(2(12)^2-9)}{(2(12)^2+9)^2}$

$P'(12) = \frac{-139,500}{88209}$

$= -1.58147128$

The city population is growing at rate of -1.581 thousand people per year. In other words, the city population is declining at a rate of 1,581 people per year.

62. $T(t) = \frac{4t}{t^2+1} + 98.6$

a) $T'(t) = \frac{(t^2+1)(4)-(4t)(2t)}{(t^2+1)^2} + 0$

$= \frac{4t^2+4-8t^2}{(t^2+1)^2}$

$= \frac{-4t^2+4}{(t^2+1)^2}$

$= -\frac{4t^2-4}{(t^2+1)^2}$

b) $T(2) = \frac{4(2)}{(2)^2+1} + 98.6 = \frac{8}{5} + 98.6 = 100.2$

After 2 hours, the temperature of the ill person is approximately 100.2 degrees Fahrenheit.

c) $T'(2) = \frac{-4((2)^2-1)}{((2)^2+1)^2} = \frac{-12}{25} = -0.48$

After 2 hours, the person's temperature is changing at rate of -0.48 degrees per hour.

d) $T(24) = \frac{4(24)}{(24)^2+1} + 98.6$

$= \frac{96}{577} + 98.6 = 98.77$

After 1 day or 24 hours, the temperature of the ill person is approximately 98.77 degrees Fahrenheit.

e) $T'(24) = \frac{-4((24)^2-1)}{((24)^2+1)^2}$

$= \frac{-2300}{332,929} = -0.0069$

After 1 day (24 hours), the person's temperature is changing at rate of -0.0069 degrees per hour

63. a) $(f\cdot g)'(1)=0$

b) $\left(\frac{f}{g}\right)'(2)=\frac{3}{4}$

c) $(g\cdot f)'(3)=-2$

d) $\left(\frac{g}{f}\right)'(1)=-\frac{4}{3}$

64. a) $(f\cdot g)'(3)=0$

b) $\left(\frac{f}{g}\right)'(2)=\text{DNE}$

c) $(f\cdot g)'(1)=-4$

d) $\left(\frac{g}{f}\right)'(1)=\frac{3}{2}$

65. $f(x)=\dfrac{7-\frac{3}{2x}}{\frac{4}{x^2}+5}$

Simplifying the function we have:

$$f(x)=\frac{7-\frac{3}{2x}}{\frac{4}{x^2}+5}\cdot\frac{2x^2}{2x^2}$$
$$=\frac{7(2x^2)-3(x)}{4(2)+5(2x^2)}$$
$$=\frac{14x^2-3x}{8+10x^2}$$

We apply the quotient rule to take the derivative.

$$f'(x)=\frac{(8+10x^2)(28x-3)-(14x^2-3x)(20x)}{(8+10x^2)^2}$$
$$=\frac{280x^3-30x^2+224x-24-280x^3+60x^2}{(8+10x^2)^2}$$
$$=\frac{30x^2+224x-24}{(8+10x^2)^2}$$

The previous derivative can be simplified as follows:

$$f'(x)=\frac{15x^2+112x-12}{2(5x^2+4)^2};\quad x\neq 0.$$

66. $y(t)=5t(t-1)(2t+3)$

First, group the factors of $y(t)$ in order to apply the product rule.

$y(t)=[5t(t-1)]\cdot(2t+3)$

Now we calculate the derivative.

Notice that when we take the derivative of the first term, $[5t(t-1)]$ we will have to apply the Product Rule again.

$$y'(t)=[5t(t-1)](2)+(2t+3)\left[\underbrace{(5t)(1)+(t-1)(5)}_{\text{Product Rule for }[5t(t-1)]}\right]$$
$$=10t(t-1)+(2t+3)[5t+5t-5]$$
$$=10t(t-1)+(2t+3)(10t-5)$$

The previous derivative can be simplified as follows:

$$y'(t)=10t^2-10t+20t^2+30t-10t-15$$
$$=30t^2+10t-15$$

67. $f(x)=\left[x(3x^3+6x-2)\right](3x^4+7)$

$$f'(x)=\left[x(3x^3+6x-2)\right](12x^3)+$$
$$(3x^4+7)\left[x(9x^2+6)+(3x^3+6x-2)(1)\right]$$
$$=36x^7+72x^5-24x^4+$$
$$(3x^4+7)\left[9x^3+6x+3x^3+6x-2\right]$$
$$=36x^7+72x^5-24x^4+$$
$$(3x^4+7)\left[12x^3+12x-2\right]$$
$$=36x^7+72x^5-24x^4+36x^7+$$
$$36x^5-6x^4+84x^3+84x-14$$
$$=72x^7+108x^5-30x^4+84x^3+84x-14$$

68. $g(x)=(x^3-8)\cdot\dfrac{x^2+1}{x^2-1}$

We will begin by applying the Product Rule.

$$g'(x)=(x^3-8)\frac{d}{dx}\left(\frac{x^2+1}{x^2-1}\right)+\frac{x^2+1}{x^2-1}\cdot\frac{d}{dx}(x^3-8)$$

Notice, that we will have to apply the Quotient Rule to take the derivative of $\dfrac{x^2+1}{x^2-1}$.

$$g'(x)=(x^3-8)\frac{(x^2-1)(2x)-(x^2+1)(2x)}{(x^2-1)^2}+\left(\frac{x^2+1}{x^2-1}\right)\cdot(3x^2)$$

$$=(x^3-8)\frac{-4x}{(x^2-1)^2}+\left(\frac{x^2+1}{x^2-1}\right)\cdot(3x^2)$$

The derivative on the previous page can be simplified as follows.

$$g'(x)=\frac{-4x(x^3-8)}{(x^2-1)^2}+\frac{3x^2(x^2+1)}{x^2-1}$$

$$=\frac{-4x^4+32x}{(x^2-1)^2}+\frac{3x^4+3x^2}{x^2-1}\cdot\frac{x^2-1}{x^2-1}$$

$$=\frac{-4x^4+32x+3x^6-3x^2}{(x^2-1)^2}$$

$$=\frac{3x^6-4x^4-3x^2+32x}{(x^2-1)^2}$$

69. $f(x)=\frac{x}{x+1}$ and $g(x)=\frac{-1}{x+1}$

a) $f'(x)=\frac{(x+1)(1)-x(1)}{(x+1)^2}=\frac{1}{(x+1)^2}$

b) $g'(x)=\frac{(x+1)(0)-(-1)(1)}{(x+1)^2}=\frac{1}{(x+1)^2}$

c) ✎ The two functions have the same rate of change.

70. $f(x)=\frac{x^2}{x^2-1}$ and $g(x)=\frac{1}{x^2-1}$

a) $f'(x)=\frac{(x^2-1)(2x)-x^2(2x)}{(x^2-1)^2}=\frac{-2x}{(x^2-1)^2}$

b) $g'(x)=\frac{(x^2-1)(0)-1(2x)}{(x^2-1)^2}=\frac{-2x}{(x^2-1)^2}$

c) ✎ The two functions have the same rate of change.

71. ✎ Suppose $F(x)=f(x)\cdot g(x)\cdot h(x)$, where $f(x)$ is the "first" function, $g(x)$ is the "second" function and $h(x)$ is the third function. Using the associative law of multiplication, we can write $F(x)=[f(x)\cdot g(x)]\cdot h(x)$. Then using the Product Rule we have:

$$F'(x)=\frac{d}{dx}([f(x)\cdot g(x)]\cdot h(x))$$

$$=[f(x)g(x)]h'(x)+h(x)[f(x)g(x)]'$$

$$=[f(x)g(x)]h'(x)+h(x)[f(x)g'(x)+g(x)f'(x)]$$

$$=f(x)g(x)h'(x)+f(x)g'(x)h(x)+f'(x)g(x)h(x)$$

The derivative of the product of three functions is the first function times the second function times the derivative of the third functions plus the first function times the derivative of the second function times the third function plus the derivative of the first function times the second function times the third function.

72. ✎ In general the derivative of the reciprocal of a function is not the reciprocal of the derivative. Let $f'(x)$ be the derivative of $f(x)$. Therefore, the reciprocal of the derivative is $\frac{1}{f'(x)}$.

The reciprocal of the function is $\frac{1}{f(x)}$, using the Quotient Rule, we find the derivative of the reciprocal.

$$\frac{d}{dx}\left(\frac{1}{f(x)}\right)=\frac{f(x)(0)-(1)f'(x)}{(f(x))^2}=-\frac{f'(x)}{(f(x))^2}$$

Clearly, the derivative of the reciprocal is not equal to the reciprocal of the derivative.

73. $R(Q)=Q^2\left(\frac{k}{2}-\frac{Q}{3}\right)$

a) $\frac{dR}{dQ}=Q^2\left(-\frac{1}{3}\right)+\left(\frac{k}{2}-\frac{Q}{3}\right)(2Q)$

$$=-\frac{1}{3}Q^2+\frac{k}{2}2Q-\frac{2Q^2}{3}$$

$$=-Q^2+kQ$$

b) The derivative found in part (a) tells us the rate of change of the reaction with respect to a small change in the quantity of the dose. If the reaction is measured in blood pressure, then the derivative tells us at a dosage Q, how much change in millimeters of mercury is seen due to a small change in the dosage. If the reaction is measured in temperature, the derivative tells us at a dosage Q, how much change in degrees Fahrenheit is seen due to a small change in the dosage.

74. a) Definition of the derivative.
b) Adding and subtracting the same quantity is the same as adding 0.
c) The limit of a sum is the sum of the limits.
d) Factoring common factors.
e) The limit of a product is the product of the limits and $\lim_{h\to 0} f(x+h) = f(x)$.
f) Definition of the derivative.
g) Using Leibniz's notation.

75. The break-even point occurs when $P(x)=0$.

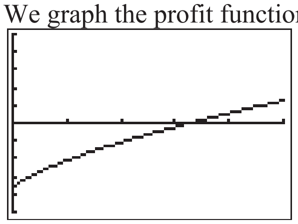
$P(x) = R(x) - C(x)$

$= 7.5x^{0.7} - 375 - 0.75x^{3/4}$

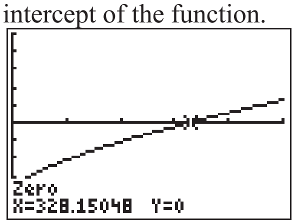
Using the window:

```
WINDOW
 Xmin=0
 Xmax=500
 Xscl=100
 Ymin=-500
 Ymax=500
 Yscl=100
 Xres=1
```

We graph the profit function on the calculator.



The break-even point will be the zero or x-intercept of the function.



We see that the break-even point occurs at $x = 328$ bottles.

The profit is changing at rate of

$P'(x) = 5.25x^{-0.3} - 0.5625x^{-1/4}$

Substituting 328 for x we have:

$P'(328) = 5.25(328)^{-0.3} - 0.5625(328)^{-1/4}$

$= 0.791239$

≈ 0.79

Therefore, at the break-even point, profit is increasing at a rate of 0.79 dollars per bottle, or 79 cents per bottle.

From Exercises 54, 56 and 58 we know that:

$A_P'(x) = A_R'(x) - A_C'(x)$

$$= -\frac{2.25}{x^{1.3}} - \frac{-\frac{3}{16}x^{3/4} - 375}{x^2}$$

$$= \frac{3x^{3/4} - 36x^{0.7} + 6000}{16x^2}$$

Substituting 328 for x we get:

$$A_P'(328) = \frac{3(328)^{3/4} - 36(328)^{0.7} + 6000}{16(328)^2}$$

≈ 0.00241342

≈ 0.0024

At the break-even point, average profit is changing at a rate of 0.0024 dollars per bottle.

76. The break-even point occurs when $P(x)=0$.

$P(x) = R(x) - C(x)$

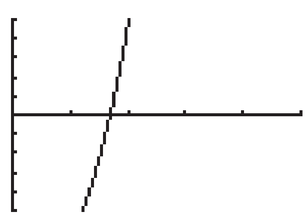
$= 45x^{9/10} - (750 + 34x - 0.068x^2)$

$= 45x^{9/10} - 750 - 34x + 0.068x^2$

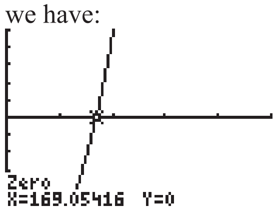
Using the window:

```
WINDOW
 Xmin=0
 Xmax=500
 Xscl=100
 Ymin=-500
 Ymax=500
 Yscl=100
 Xres=1
```

We graph the profit function on the calculator.



The break-even point will be the zero of the function. Using the zero finder on the calculator we have:



We see that the break-even point occurs at $x = 169$ jackets.

The profit is changing at rate of

$$P'(x) = 40.5x^{-1/10} - 34 + 0.136x$$

Substituting 169 for x we have:

$$P'(169) = 40.5(169)^{-1/10} - 34 + 0.136(169)$$
$$= 13.23146565$$
$$\approx 13.23$$

Therefore, at the break-even point, profit is increasing at a rate of 13.23 dollars per belt.

From Exercise 57 we know that:

$$A_P'(x) = \frac{-4.5x^{9/10} + 0.068x^2 + 750}{x^2}$$

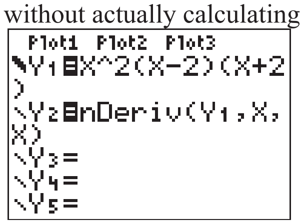
Substituting 184 for x we get:

$$A_P'(169) = \frac{-4.5(169)^{9/10} + 0.068(169)^2 + 750}{(169)^2}$$
$$= 0.0783177927$$
$$\approx 0.078$$

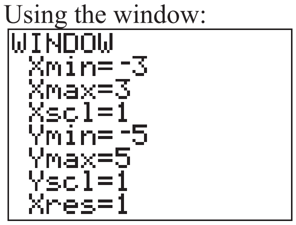
At the break-even point, average profit is changing at a rate of 0.078 dollars per belt.

77 $f(x) = x^2(x-2)(x+2)$

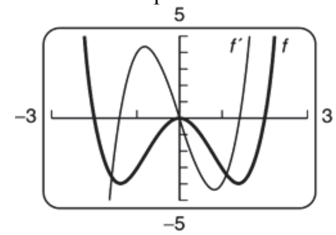
Using the calculator, we graph the function and the derivative in the same window. We can use the nDeriv feature to graph the derivative without actually calculating the derivative.



Using the window:



From the previous column we have:

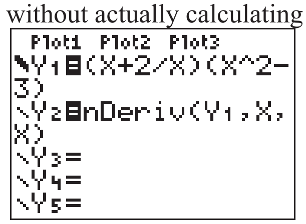


Note, the function $f(x)$ is the thicker graph.

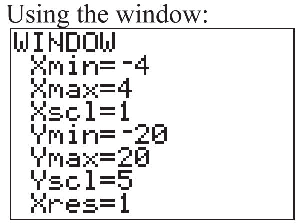
The horizontal tangents occur at the turning points of this function, or at the x-intercepts of the derivative. Using the trace feature, the minimum/maximum feature on the function, or the zero feature on the derivative on the calculator, we find the points of horizontal tangency. We estimate the points at which the tangent lines are horizontal are $(0,0), (-1.414,-4)$, and $(1.414,-4)$.

78. $f(x) = \left(x + \frac{2}{x}\right)(x^2 - 3)$

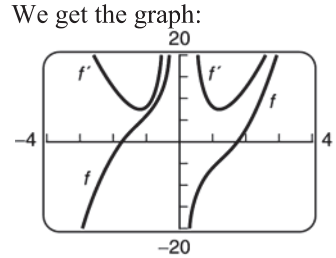
Using the calculator, we graph the function and the derivative in the same window. We can use the nDeriv feature to graph the derivative without actually calculating the derivative.



Using the window:



We get the graph:

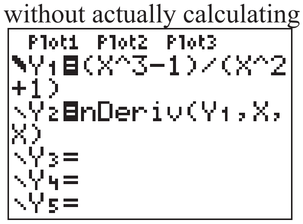


Note, the function $f(x)$ is the thicker graph.

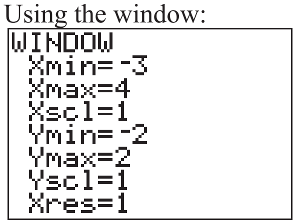
The horizontal tangents occur at the turning points of this function, or at the x-intercepts of the derivative. We can see that the derivative never intersects the x-axis, therefore, there are no points at which the tangent line is horizontal.

79. $f(x)=\dfrac{x^3-1}{x^2+1}$

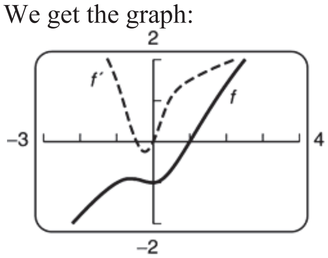
Using the calculator, we graph the function and the derivative in the same window. We can use the nDeriv feature to graph the derivative without actually calculating the derivative.



Using the window:



We get the graph:



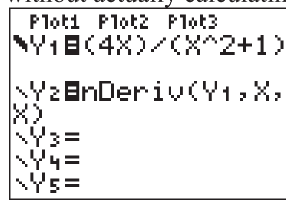
Note, the function $f(x)$ is the thicker graph.

The horizontal tangents occur at the turning points of this function, or at the *x*-intercepts of the derivative.

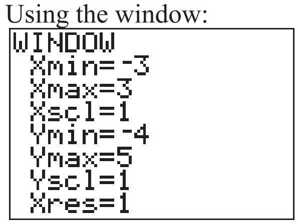
Using the trace feature, the minimum/maximum feature on the function, or the zero feature on the derivative on the calculator, we find the points of horizontal tangency. We estimate the points at which the tangent lines are horizontal are $(-0.596,-0.894)$ and $(0,-1)$.

80. $f(x)=\dfrac{4x}{x^2+1}$

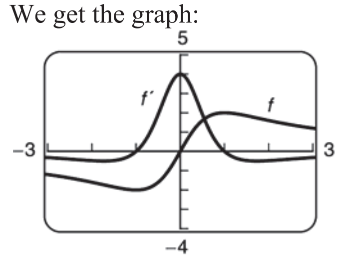
Using the calculator, we graph the function and the derivative in the same window. We can use the nDeriv feature to graph the derivative without actually calculating the derivative.



Using the window:



We get the graph:

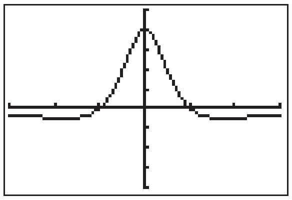


Note, the function $f(x)$ is the thicker graph.

The horizontal tangents occur at the turning points of this function, or at the *x*-intercepts of the derivative. Using the trace feature, the minimum/maximum feature on the function, or the zero feature on the derivative on the calculator, we find the points of horizontal tangency.

We estimate the points at which the tangent lines are horizontal are $(-1,-2)$ and $(1,2)$.

81. The graph of $y_3 = \dfrac{4-4x^2}{\left(x^2+1\right)^2}$ is:



This graph appears to be the correct derivative of the function in Exercise 131. Using the Quotient Rule we can verify this result.

$$f(x) = \frac{4x}{x^2+1}$$

$$f'(x) = \frac{4-4x^2}{\left(x^2+1\right)^2}$$

Exercise Set 1.7

1. $y = (3-2x)^2$

Using the Extended Power Rule:

$$\frac{dy}{dx} = \frac{d}{dx}\left[(3-2x)^2\right]$$
$$= 2(3-2x)^{2-1}(-2)$$
$$= 8x-12$$

Simplifying the function first, we have:

$$y = (3-2x)^2 = 4x^2 - 12x + 9$$

We take the derivative using the Power Rule.

$$\frac{dy}{dx} = \frac{d}{dx}\left(4x^2 - 12x + 9\right)$$
$$= 8x-12$$

The results are the same.

2. $y = (2x+1)^2$

Using the Extended Power Rule:

$$\frac{dy}{dx} = \frac{d}{dx}\left[(2x+1)^2\right]$$
$$= 2(2x+1)^{2-1}\cdot\frac{d}{dx}(2x+1)$$
$$= 2(2x+1)(2)$$
$$= 8x+4$$

Simplifying the function first, we have:

$$y = (2x+1)^2$$
$$= (2x+1)(2x+1)$$
$$= 4x^2 + 4x + 1$$

Now we take the derivative using the Power Rule.

$$\frac{dy}{dx} = \frac{d}{dx}\left(4x^2 + 4x + 1\right)$$
$$= \frac{d}{dx}\left(4x^2\right) + \frac{d}{dx}(4x) + \frac{d}{dx}(1)$$
$$= 8x+4$$

The results are the same.

3. $y = (7-x)^{55}$

Using the Extended Power Rule:

$$\frac{dy}{dx} = \frac{d}{dx}\left[(7-x)^{55}\right]$$
$$= 55(7-x)^{55-1}\cdot\frac{d}{dx}(7-x)$$
$$\frac{dy}{dx} = 55(7-x)^{54}(-1)$$
$$= -55(7-x)^{54}$$

4. $y = (8-x)^{100}$

$$\frac{dy}{dx} = 100(8-x)^{99}(-1)$$
$$= -100(8-x)^{99}$$

5. $y = \sqrt{3x^2-4} = \left(3x^2-4\right)^{1/2}$

Using the Extended Power Rule

$$\frac{dy}{dx} = \frac{d}{dx}\left[\left(3x^2-4\right)^{1/2}\right]$$
$$= \frac{1}{2}\left(3x^2-4\right)^{-1/2}\frac{d}{dx}\left(3x^2-4\right)$$
$$= \frac{1}{2\left(3x^2-4\right)^{1/2}}\cdot(6x)$$
$$= \frac{3x}{\sqrt{3x^2-4}}$$

6. $y = \sqrt{4x^2+1} = \left(4x^2+1\right)^{1/2}$

$$\frac{dy}{dx} = \frac{d}{dx}\left[\left(4x^2+1\right)^{1/2}\right]$$
$$\frac{dy}{dx} = \frac{1}{2}\left(4x^2+1\right)^{-1/2}(8x)$$
$$= \frac{8x}{2\left(4x^2+1\right)^{1/2}}$$
$$= \frac{4x}{\sqrt{4x^2+1}}$$

7. $y = \sqrt{1-x} = (1-x)^{1/2}$

$$\frac{dy}{dx} = \frac{1}{2}(1-x)^{-1/2}(-1)$$
$$= \frac{-1}{2(1-x)^{1/2}}$$
$$= \frac{-1}{2\sqrt{1-x}}$$

8. $y=\sqrt{1+8x}=(1+8x)^{1/2}$

Using the Extended Power Rule

$$\frac{dy}{dx}=\frac{d}{dx}\left[(1+8x)^{1/2}\right]$$
$$=\frac{1}{2}(1+8x)^{-1/2}\frac{d}{dx}(1+8x)$$
$$=\frac{1}{2(1+8x)^{1/2}}\cdot(8)$$
$$=\frac{4}{\sqrt{1+8x}}$$

9. $y=\left(4x^2+1\right)^{-50}$

$$\frac{dy}{dx}=\frac{d}{dx}\left[\left(4x^2+1\right)^{-50}\right]$$
$$=-50\left(4x^2+1\right)^{-51}\cdot(8x)$$
$$=-400x\left(4x^2+1\right)^{-51}$$
$$=\frac{-400x}{\left(4x^2+1\right)^{51}}$$

10. $y=\left(8x^2-6\right)^{-40}$

Using the Extended Power Rule

$$\frac{dy}{dx}=\frac{d}{dx}\left[\left(8x^2-6\right)^{-40}\right]$$
$$=-40\left(8x^2-6\right)^{-40-1}\frac{d}{dx}\left(8x^2-6\right)$$
$$=-40\left(8x^2-6\right)^{-41}\cdot(16x)$$
$$=-640x\left(8x^2-6\right)^{-41}$$
$$=\frac{-640x}{\left(8x^2-6\right)^{41}}$$

11. $y=(x-4)^8(2x+3)^6$

Using the Product Rule, we have

$$\frac{dy}{dx}=\frac{d}{dx}\left[(x-4)^8(2x+3)^6\right]$$
$$=(x-4)^8\frac{d}{dx}(2x+3)^6+(2x+3)^6\frac{d}{dx}(x-4)^8$$

Next, we will apply the Extended Power Rule.

$$\frac{dy}{dx}=(x-4)^8\left[6(2x+3)^{6-1}\frac{d}{dx}(2x+3)\right]+$$
$$(2x+3)^6\left[8(x-4)^{8-1}\frac{d}{dx}(x-4)\right]$$
$$=(x-4)^8\left[6(2x+3)^5(2)\right]+$$
$$(2x+3)^6\left[8(x-4)^7(1)\right]$$
$$=12(x-4)^8(2x+3)^5+8(2x+3)^6(x-4)^7$$

Factoring out common factors, we have:

$$\frac{dy}{dx}=4(x-4)^7(2x+3)^5\left[3(x-4)+2(2x+3)\right]$$
$$=4(x-4)^7(2x+3)^5\left[3x-12+4x+6\right]$$
$$=4(x-4)^7(2x+3)^5(7x-6)$$

12. $y=(x+5)^7(4x-1)^{10}$

$$\frac{dy}{dx}=(x+5)^7\left[10(4x-1)^9(4)\right]+$$
$$(4x-1)^{10}\left[7(x+5)^6(1)\right]$$
$$=40(x+5)^7(4x-1)^9+7(4x-1)^{10}(x+5)^6$$
$$=(x+5)^6(4x-1)^9\left[40(x+5)+7(4x-1)\right]$$
$$=(x+5)^6(4x-1)^9\left[40x+200+28x-7\right]$$
$$=(x+5)^6(4x-1)^9(68x+193)$$

13. $y=\dfrac{1}{(4x+5)^2}=(4x+5)^{-2}$

$$\frac{dy}{dx}=-2(4x+5)^{-3}(4)$$
$$\frac{dy}{dx}=-8(4x+5)^{-3}$$
$$=\frac{-8}{(4x+5)^3}$$

14. $y=\frac{1}{(3x+8)^2}=(3x+8)^{-2}$

Using the Extended Power Rule

$$\frac{dy}{dx}=\frac{d}{dx}\left[(3x+8)^{-2}\right]$$
$$=-2(3x+8)^{-2-1}\frac{d}{dx}(3x+8)$$
$$=-2(3x+8)^{-3}\cdot(3)$$
$$=-6(3x+8)^{-3}$$
$$=\frac{-6}{(3x+8)^3}$$

15. $y=\frac{4x^2}{(7-5x)^3}$

First, we use the Quotient Rule.

$$\frac{dy}{dx}=\frac{d}{dx}\left[\frac{4x^2}{(7-5x)^3}\right]$$
$$=\frac{(7-5x)^3\frac{d}{dx}(4x^2)-4x^2\frac{d}{dx}(7-5x)^3}{\left((7-5x)^3\right)^2}$$

Next, using the Extended Power Rule, we have:

$$\frac{dy}{dx}=\frac{(7-5x)^3(8x)-4x^2\left[3(7-5x)^2(-5)\right]}{(7-5x)^6}$$
$$=\frac{8x(7-5x)^3+60x^2(7-5x)^2}{\left((7-5x)^3\right)^2}$$
$$=\frac{(7-5x)^2\left[8x(7-5x)+60x^2\right]}{(7-5x)^6}\quad\text{Factoring}$$
$$=\frac{56x-40x^2+60x^2}{(7-5x)^4}\quad\text{Dividing common factors}$$
$$=\frac{20x^2+56x}{(7-5x)^4}$$
$$=\frac{4x(5x+14)}{(7-5x)^4}$$

16. $y=\frac{7x^3}{(4-9x)^5}$

$$\frac{dy}{dx}=\frac{(4-9x)^5(21x^2)-7x^3\left[5(4-9x)^4(-9)\right]}{\left((4-9x)^5\right)^2}$$
$$=\frac{21x^2(4-9x)^5+315x^3(4-9x)^4}{(4-9x)^{10}}$$
$$=\frac{21x^2(4-9x)^4\left[(4-9x)+15x\right]}{(4-9x)^{10}}$$
$$\frac{dy}{dx}=\frac{21x^2(4+6x)}{(4-9x)^6}$$
$$=\frac{42x^2(3x+2)}{(4-9x)^6}$$

17. $f(x)=-5x(2x-3)^4$

Using the Product Rule, we have

$$f'(x)=\frac{d}{dx}\left[-5x(2x-3)^4\right]$$
$$=-5x\frac{d}{dx}\left[(2x-3)^4\right]+(2x-3)^4\frac{d}{dx}(-5x)$$

Using the Extended Power Rule, we have

$$f'(x)=-5x\left[4(2x-3)^3\left(\frac{d}{dx}(2x-3)\right)\right]+$$
$$(2x-3)^4(-5)$$
$$=-5x\left[4(2x-3)^3(2)\right]+(2x-3)^4(-5)$$
$$=-40x(2x-3)^3-5(2x-3)^4$$
$$=-5(2x-3)^3\left[8x+(2x-3)\right]\quad\text{Factoring}$$
$$=-5(2x-3)^3(10x-3)$$

18. $f(x)=-3x(5x+4)^6$

$$f'(x)=-3x\left[6(5x+4)^{6-1}(5)\right]+(5x+4)^6(-3)$$
$$=-90x(5x+4)^5-3(5x+4)^6$$
$$=-3(5x+4)^5\left[30x+5x+4\right]$$
$$=-3(5x+4)^5(35x+4)$$

19. $F(x) = (5x+2)^4(2x-3)^8$

$$F'(x) = (5x+2)^4\left[8(2x-3)^7(2)\right] + (2x-3)^8\left[4(5x+2)^3(5)\right]$$
$$= 16(5x+2)^4(2x-3)^7 + 20(2x-3)^8(5x+2)^3$$
$$= 4(5x+2)^3(2x-3)^7\left[4(5x+2)+5(2x-3)\right]$$
$$= 4(5x+2)^3(2x-3)^7(30x-7)$$

20. $g(x) = (3x-1)^7(2x+1)^5$

Using the Product Rule and the Extended Power Rule, we have

$$g'(x) = \frac{d}{dx}\left[(3x-1)^7(2x+1)^5\right]$$
$$= (3x-1)^7\frac{d}{dx}(2x+1)^5 + (2x+1)^5\frac{d}{dx}(3x-1)^7$$
$$= (3x-1)^7\left[5(2x+1)^4(2)\right] + (2x+1)^5\left[7(3x-1)^6(3)\right]$$
$$= 10(3x-1)^7(2x+1)^4 + 21(2x+1)^5(3x-1)^6$$
$$= (3x-1)^6(2x+1)^4\left[10(3x-1)+21(2x+1)\right]$$
$$= (3x-1)^6(2x+1)^4\left[30x-10+42x+21\right]$$
$$= (3x-1)^6(2x+1)^4(72x+11)$$

21. $f(x) = x^2\sqrt{4x-1} = x^2(4x-1)^{1/2}$

Using the Product Rule and the Extended Power Rule, we have

$$f'(x) = \frac{d}{dx}\left[x^2(4x-1)^{1/2}\right]$$
$$= x^2\left[\frac{1}{2}(4x-1)^{-1/2}(4)\right] + (4x-1)^{1/2}(2x)$$
$$f'(x) = \frac{2x^2}{(4x-1)^{1/2}} + 2x(4x-1)^{1/2}$$
$$= \frac{2x^2}{\sqrt{(4x-1)}} + 2x\sqrt{(4x-1)}$$

The derivative on the previous page can be simplified as follows:

$$f'(x) = \frac{2x^2}{\sqrt{4x-1}} + \frac{2x\sqrt{4x-1}}{1}\cdot\frac{\sqrt{4x-1}}{\sqrt{4x-1}}$$
$$= \frac{2x^2}{\sqrt{4x-1}} + \frac{2x(4x-1)}{\sqrt{4x-1}}$$
$$= \frac{2x^2}{\sqrt{4x-1}} + \frac{8x^2-2x}{\sqrt{4x-1}}$$
$$= \frac{10x^2-2x}{\sqrt{4x-1}}$$
$$= \frac{2x(5x-1)}{\sqrt{4x-1}}$$

22. $f(x) = x^3\sqrt{5x+2} = x^3(5x+2)^{1/2}$

$$f'(x) = x^3\left[\frac{1}{2}(5x+2)^{-1/2}(5)\right] + (5x+2)^{1/2}(3x^2)$$
$$= \frac{5}{2}x^3(5x+2)^{-1/2} + 3x^2(5x+2)^{1/2}$$
$$= \frac{5x^3}{2\sqrt{5x+2}} + 3x^2\sqrt{5x+2}$$

The previous derivative can be simplified as follows:

$$f'(x) = \frac{x^2(35x+12)}{2\sqrt{5x+2}}$$

23. $F(x) = \sqrt[4]{x^2-5x+2} = \left(x^2-5x+2\right)^{1/4}$

$$F'(x) = \frac{1}{4}\left(x^2-5x+2\right)^{1/4-1}(2x-5)$$

Continued at the top of the next column.

$$F'(x) = \frac{1}{4}\left(x^2-5x+2\right)^{-3/4}(2x-5)$$
$$= \frac{2x-5}{4\left(x^2-5x+2\right)^{3/4}}$$

24. $G(x)=\sqrt[3]{x^5+6x}=\left(x^5+6x\right)^{1/3}$

Using the Extended Power Rule, we have

$$G'(x)=\frac{d}{dx}\left[\left(x^5+6x\right)^{1/3}\right]$$
$$=\frac{1}{3}\left(x^5+6x\right)^{1/3-1}\frac{d}{dx}\left(x^5+6x\right)$$
$$=\frac{1}{3}\left(x^5+6x\right)^{-2/3}\left(5x^4+6\right)$$
$$=\frac{5x^4+6}{3\left(x^5+6x\right)^{2/3}}$$
$$=\frac{5x^4+6}{3\cdot\sqrt[3]{\left(x^5+6x\right)^2}}$$

25. $f(x)=\left(\frac{3x-1}{5x+2}\right)^4$

Using the Extended Power Rule, we have

$$f'(x)=4\left(\frac{3x-1}{5x+2}\right)^3\frac{d}{dx}\left[\frac{3x-1}{5x+2}\right]$$

Using the Quotient Rule, we have

$$f'(x)=4\left(\frac{3x-1}{5x+2}\right)^3\left[\frac{(5x+2)(3)-(3x-1)(5)}{(5x+2)^2}\right]$$
$$=4\left(\frac{3x-1}{5x+2}\right)^3\left[\frac{15x+6-15x+5}{(5x+2)^2}\right]$$
$$=4\left(\frac{3x-1}{5x+2}\right)^3\left[\frac{11}{(5x+2)^2}\right]$$
$$=\frac{44(3x-1)^3}{(5x+2)^5}$$

26. $f(x)=\left(\frac{2x}{x^2+1}\right)^3$

$$f'(x)=3\left(\frac{2x}{x^2+1}\right)^2\left[\frac{\left(x^2+1\right)(2)-2x(2x)}{\left(x^2+1\right)^2}\right]$$
$$=3\left(\frac{2x}{x^2+1}\right)^2\left[\frac{2x^2+2-4x^2}{\left(x^2+1\right)^2}\right]$$
$$=3\left(\frac{2x}{x^2+1}\right)^2\left[\frac{2-2x^2}{\left(x^2+1\right)^2}\right]$$
$$=\frac{3(2x)^2\left(2-2x^2\right)}{\left(x^2+1\right)^4}$$
$$=\frac{24x^2\left(1-x^2\right)}{\left(x^2+1\right)^4}$$
$$=\frac{-24x^2\left(x^2-1\right)}{\left(x^2+1\right)^4}$$

27. $h(x)=\left(\frac{1-3x}{2-7x}\right)^{-5}=\left(\frac{2-7x}{1-3x}\right)^5$

$h'(x)$

$$=5\left(\frac{2-7x}{1-3x}\right)^4\left(\frac{(1-3x)(-7)-(2-7x)(-3)}{(1-3x)^2}\right)$$
$$=5\left(\frac{2-7x}{1-3x}\right)^4\left(\frac{-7+21x+6-21x}{(1-3x)^2}\right)$$
$$=5\left(\frac{2-7x}{1-3x}\right)^4\left(\frac{-1}{(1-3x)^2}\right)$$
$$=\frac{-5(2-7x)^4}{(1-3x)^6}=\frac{-5(7x-2)^4}{(3x-1)^6}$$

28. $g(x)=\left(\frac{2x+3}{5x-1}\right)^{-4}=\left(\frac{5x-1}{2x+3}\right)^4$

Using the Extended Power Rule, we have

$$g'(x)=\frac{d}{dx}\left[\left(\frac{5x-1}{2x+3}\right)^4\right]$$
$$=4\left(\frac{5x-1}{2x+3}\right)^{4-1}\left[\frac{d}{dx}\left(\frac{5x-1}{2x+3}\right)\right]$$

Next, using the Quotient Rule, we have

$$g'(x)=4\left(\frac{5x-1}{2x+3}\right)^3\left[\frac{(2x+3)(5)-(5x-1)(2)}{(2x+3)^2}\right]$$
$$=4\left(\frac{5x-1}{2x+3}\right)^3\left[\frac{10x+15-10x+2}{(2x+3)^2}\right]$$
$$=4\left(\frac{5x-1}{2x+3}\right)^3\left[\frac{17}{(2x+3)^2}\right]$$
$$=\frac{68(5x-1)^3}{(2x+3)^5}$$

29. $f(x)=\frac{(5x-4)^7}{(6x+1)^3}$

$f'(x)$
$$=\frac{(6x+1)^3\left[7(5x-4)^6(5)\right]-(5x-4)^7\left[3(6x+1)^2(6)\right]}{\left((6x+1)^3\right)^2}$$

$$f'(x)=\frac{35(5x-4)^6(6x+1)^3-18(5x-4)^7(6x+1)^2}{(6x+1)^6}$$
$$=\frac{(5x-4)^6(6x+1)^2\left[35(6x+1)-18(5x-4)\right]}{(6x+1)^6}$$
$$=\frac{(5x-4)^6[210x+35-90x+72]}{(6x+1)^4}$$
$$=\frac{(5x-4)^6[120x+107]}{(6x+1)^4}$$
$$f'(x)=\frac{(5x-4)^6(120x+107)}{(6x+1)^4}$$

30. $f(x)=\frac{(2x+3)^4}{(3x-2)^5}$

Using the Quotient Rule and the Extended Power Rule, we have:

$$f'(x)=\frac{d}{dx}\left[\frac{(2x+3)^4}{(3x-2)^5}\right]$$
$$=\frac{(3x-2)^5\left[4(2x+3)^3(2)\right]-(2x+3)^4\left[5(3x-2)^4(3)\right]}{\left((3x-2)^5\right)^2}$$
$$=\frac{8(2x+3)^3(3x-2)^5-15(2x+3)^4(3x-2)^4}{(3x-2)^{10}}$$
$$=\frac{(2x+3)^3(3x-2)^4\left[8(3x-2)-15(2x+3)\right]}{(3x-2)^{10}}$$
$$=\frac{(2x+3)^3[24x-16-30x-45]}{(3x-2)^6}$$
$$=\frac{(2x+3)^3[-6x-61]}{(3x-2)^6}$$
$$=\frac{-(2x+3)^3[6x+61]}{(3x-2)^6}$$
$$f'(x)=\frac{-(2x+3)^3[6x+61]}{(3x-2)^6}$$

31. $g(x)=\sqrt{\frac{3+2x}{5-x}}=\left(\frac{3+2x}{5-x}\right)^{1/2}$

$g'(x)$
$$=\frac{1}{2}\left(\frac{3+2x}{5-x}\right)^{-1/2}\left[\frac{(5-x)(2)-(3+2x)(-1)}{(5-x)^2}\right]$$
$$=\frac{1}{2}\left(\frac{5-x}{3+2x}\right)^{1/2}\left[\frac{10-2x+3+2x}{(5-x)^2}\right]$$
$$=\frac{1}{2}\left(\frac{5-x}{3+2x}\right)^{1/2}\left[\frac{13}{(5-x)^2}\right]$$
$$=\frac{13}{2(5-x)^{3/2}(3+2x)^{1/2}}$$
$$=\frac{13}{2\sqrt{(5-x)^3}\cdot\sqrt{(3+2x)}}$$

32. $g(x)=\sqrt{\dfrac{4-x}{3+x}}=\left(\dfrac{4-x}{3+x}\right)^{1/2}$

Using the Extended Power Rule, we have

$$g'(x)=\frac{1}{2}\left(\frac{4-x}{3+x}\right)^{1/2-1}\frac{d}{dx}\left[\frac{4-x}{3+x}\right]$$

Using the Quotient Rule, we have

$$g'(x)=\frac{1}{2}\left(\frac{4-x}{3+x}\right)^{-1/2}\left[\frac{(3+x)(-1)-(4-x)(1)}{(3+x)^2}\right]$$

$$=\frac{1}{2}\left(\frac{4-x}{3+x}\right)^{-1/2}\left[\frac{-3-x-4+x}{(3+x)^2}\right]$$

$$=\frac{1}{2}\left(\frac{3+x}{4-x}\right)^{1/2}\left[\frac{-7}{(3+x)^2}\right]$$

$$=\frac{-7}{2(3+x)^{3/2}(4-x)^{1/2}}$$

$$=\frac{-7}{2\sqrt{(3+x)^3}\cdot\sqrt{4-x}}$$

33. $y=\dfrac{15}{u^3}=15u^{-3}$ and $u=2x+1$

$$\frac{dy}{du}=15\left(-3u^{-3-1}\right)=-45u^{-4}=\frac{-45}{u^4}$$

$$\frac{du}{dx}=2$$

$$\frac{dy}{dx}=\frac{dy}{du}\cdot\frac{du}{dx}$$

$$=\frac{-45}{u^4}\cdot 2$$

$$=\frac{-90}{(2x+1)^4}$$

34. $y=\sqrt{u}=u^{1/2}$ and $u=x^2-1$

$$\frac{dy}{du}=\frac{1}{2}u^{1/2-1}=\frac{1}{2}u^{-1/2}=\frac{1}{2\sqrt{u}}$$

$$\frac{du}{dx}=2x^{2-1}=2x$$

Applying the Chain Rule, we have:

$$\frac{dy}{dx}=\frac{dy}{du}\cdot\frac{du}{dx}$$

$$=\frac{1}{2\sqrt{u}}\cdot 2x$$

$$=\frac{2x}{2\sqrt{x^2-1}}\quad\text{Subtituting } x^2-1 \text{ for } u.$$

$$=\frac{x}{\sqrt{x^2-1}}\quad\text{Simplifying}$$

35. $y=u^{50}$ and $u=4x^3-2x^2$

$$\frac{dy}{du}=50u^{50-1}=50u^{49}$$

$$\frac{du}{dx}=4\left(3x^{3-1}\right)-2\left(2x^{2-1}\right)=12x^2-4x$$

$$\frac{dy}{dx}=\frac{dy}{du}\cdot\frac{du}{dx}$$

$$=50u^{49}\cdot\left(12x^2-4x\right)$$

Subtituting $4x^3-2x^2$ for u.

$$=50\left(4x^3-2x^2\right)^{49}\cdot\left(12x^2-4x\right)$$

$$=200x(3x-1)\left(4x^3-2x^2\right)^{49}\quad\text{Simplifying}$$

36. $y=\dfrac{u+1}{u-1}$ and $u=1+\sqrt{x}=1+x^{1/2}$

$$\frac{dy}{du}=\frac{(u-1)(1)-(u+1)(1)}{(u-1)^2}=\frac{-2}{(u-1)^2}$$

$$\frac{du}{dx}=\frac{1}{2}x^{-1/2}=\frac{1}{2\sqrt{x}}$$

$$\frac{dy}{dx}=\frac{dy}{du}\cdot\frac{du}{dx}$$

$$=\frac{-2}{(u-1)^2}\cdot\frac{1}{2\sqrt{x}}$$

$$=\frac{-1}{\sqrt{x}\left(\left(1+\sqrt{x}\right)-1\right)^2}\quad\text{Subtituting } 1+\sqrt{x} \text{ for } u.$$

$$=\frac{-1}{\sqrt{x}\left(\sqrt{x}\right)^2}\quad\text{Simplifying}$$

$$=\frac{-1}{x^{3/2}}$$

37. $y = 5u^2 + 3u$ and $u = x^3 + 1$

$$\frac{dy}{du} = 10u + 3$$

$$\frac{du}{dx} = 3x^2$$

Applying the Chain Rule, we have:

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

$$= (10u + 3) \cdot (3x^2)$$

$$= (10(x^3 + 1) + 3) \cdot (3x^2) \quad \text{Substituting for } u.$$

$$= 3x^2((10x^3 + 10) + 3)$$

$$= 3x^2(10x^3 + 13)$$

38. $y = u^3 - 7u^2$ and $u = x^2 + 3$

$$\frac{dy}{du} = 3u^2 - 14u$$

$$\frac{du}{dx} = 2x$$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

$$= (3u^2 - 14u) \cdot (2x)$$

$$= (3(x^2 + 3)^2 - 14(x^2 + 3)) \cdot (2x)$$

$$\frac{dy}{dx} = (3x^4 + 18x^2 + 27 - 14x^2 - 42)(2x)$$

$$= (3x^4 + 4x^2 - 15)(2x)$$

$$= 2x(x^2 + 3)(3x^2 - 5)$$

39. $y = \frac{1}{u^2 + u}$ and $u = 5 + 3t$

$$\frac{dy}{du} = \frac{(u^2 + u)(0) - (1)(2u + 1)}{(u^2 + u)^2} \quad \text{Quotient Rule}$$

$$= \frac{-(2u + 1)}{(u^2 + u)^2}$$

$$\frac{du}{dt} = 3$$

We apply the Chain Rule in the next column.

$$\frac{dy}{dt} = \frac{dy}{du} \cdot \frac{du}{dt}$$

$$= \left(\frac{-(2u + 1)}{(u^2 + u)^2}\right) \cdot (3)$$

$$= \left(\frac{-(2(5 + 3t) + 1)}{((5 + 3t)^2 + (5 + 3t))^2}\right) \cdot (3) \quad \text{Substituting}$$

$$= \frac{-3(10 + 6t + 1)}{((5 + 3t)^2 + (5 + 3t))^2}$$

$$= \frac{-3(6t + 11)}{(5 + 3t)^2((5 + 3t) + 1)^2} \quad \text{Factoring}$$

$$= \frac{-3(6t + 11)}{(5 + 3t)^2(6 + 3t)^2}$$

$$= \frac{-6t - 11}{3(3t + 5)^2(t + 2)^2}$$

40. $y = \frac{1}{3u^5 - 7}$ and $u = 7t^2 + 1$

$$\frac{dy}{du} = \frac{(3u^5 - 7)(0) - (1)(15u^4)}{(3u^5 - 7)^2} \quad \text{Quotient Rule}$$

$$= \frac{-15u^4}{(3u^5 - 7)^2}$$

$$\frac{du}{dt} = 14t$$

$$\frac{dy}{dt} = \frac{dy}{du} \cdot \frac{du}{dt}$$

$$= \left(\frac{-15u^4}{(3u^5 - 7)^2}\right) \cdot (14t)$$

$$= \left(\frac{-15(7t^2 + 1)^4}{(3(7t^2 + 1)^5 - 7)^2}\right) \cdot (14t)$$

$$= \frac{-210t(7t^2 + 1)^4}{(3(7t^2 + 1)^5 - 7)^2}$$

41. $y = \left(x^3 - 4x\right)^{10}$

$\frac{dy}{dx} = 10\left(x^3 - 4x\right)^9\left(3x^2 - 4\right)$

When $x = 2$,

$\frac{dy}{dx} = 10\left((2)^3 - 4(2)\right)^9\left(3(2)^2 - 4\right)$

$= 10(0)^9(8)$

$= 0$

Thus, the slope of the tangent line at the point, $(2,0)$ is 0. The equation of the horizontal line passing through the point $(2,0)$ is $y = 0$.

42. $y = \sqrt{x^2 + 3x} = \left(x^2 + 3x\right)^{1/2}$

First, we find the derivative using the Extended Power Rule.

$\frac{dy}{dx} = \frac{1}{2}\left(x^2 + 3x\right)^{1/2 - 1}(2x + 3)$

$= \frac{2x + 3}{2\sqrt{x^2 + 3x}}$

When $x = 1$,

$\frac{dy}{dx} = \frac{2(1) + 3}{2\sqrt{(1)^2 + 3(1)}} = \frac{5}{2\sqrt{4}} = \frac{5}{2 \cdot 2} = \frac{5}{4}$

Thus, the slope of the tangent line at $(1,2)$ is $\frac{5}{4}$.

Using the point-slope equation, we find the equation of the tangent line.

$y - y_1 = m(x - x_1)$

$y - 2 = \frac{5}{4}(x - 1)$

$y - 2 = \frac{5}{4}x - \frac{5}{4}$

$y = \frac{5}{4}x + \frac{3}{4}$

43. $y = x\sqrt{2x + 3} = x(2x + 3)^{1/2}$

First, we find the derivative using the Product Rule and the Extended Power Rule.

$\frac{dy}{dx} = x\left(\frac{1}{2}(2x + 3)^{1/2 - 1}(2)\right) + (2x + 3)^{1/2}(1)$

$= \frac{x}{\sqrt{2x + 3}} + \sqrt{2x + 3}$

When $x = 3$,

$\frac{dy}{dx} = \frac{(3)}{\sqrt{2(3) + 3}} + \sqrt{2(3) + 3}$

$= \frac{3}{\sqrt{9}} + \sqrt{9}$

$= \frac{3}{3} + 3$

$= 1 + 3$

$= 4$

Thus, the slope of the tangent line at $(3,9)$ is 4.

Using the point-slope equation, we find the equation of the tangent line.

$y - y_1 = m(x - x_1)$

$y - 9 = 4(x - 3)$

$y - 9 = 4x - 12$

$y = 4x - 3$

44. $y = \left(\frac{2x + 3}{x - 1}\right)^3$

$\frac{dy}{dx} = 3\left(\frac{2x + 3}{x - 1}\right)^2\left[\frac{(x - 1)(2) - (2x + 3)(1)}{(x - 1)^2}\right]$

$= 3\left(\frac{2x + 3}{x - 1}\right)^2\left[\frac{2x - 2 - 2x - 3}{(x - 1)^2}\right]$

$= 3\left(\frac{2x + 3}{x - 1}\right)^2\left[\frac{-5}{(x - 1)^2}\right]$

$= \frac{-15(2x + 3)^2}{(x - 1)^4}$

When $x = 2$,

$\frac{dy}{dx} = \frac{-15(2(2) + 3)^2}{((2) - 1)^4}$

$= \frac{-15(7)^2}{(1)^4}$

$= -15 \cdot 49$

$= -735$

Thus, the slope of the tangent line at $(2,343)$ is -735.

Using the point-slope equation, we find the equation of the tangent line.

$$y - y_1 = m(x - x_1)$$
$$y - 343 = -735(x - 2)$$
$$y - 343 = -735x + 1470$$
$$y = -735x + 1813$$

45. $g(x) = \left(\dfrac{6x+1}{2x-5}\right)^2$

a) Using Extended Power Rule, we have

$$g'(x) = 2\left(\frac{6x+1}{2x-5}\right)\left[\frac{(2x-5)(6)-(6x+1)(2)}{(2x-5)^2}\right]$$
$$= \frac{2(6x+1)}{2x-5}\left[\frac{12x-30-12x-2}{(2x-5)^2}\right]$$
$$= \frac{2(6x+1)}{2x-5}\left[\frac{-32}{(2x-5)^2}\right]$$
$$= \frac{-64(6x+1)}{(2x-5)^3}$$

b) Using the Quotient Rule on

$g(x) = \dfrac{36x^2+12x+1}{4x^2-20x+25}$, we have

$$g'(x) = \frac{(4x^2-20x+25)(72x+12)}{(4x^2-20x+25)^2} - \frac{(36x^2+12x+1)(8x-20)}{(4x^2-20x+25)^2}$$
$$= \frac{288x^3-1392x^2+1560x+300}{(4x^2-20x+25)^2} - \frac{288x^3-624x^2-232x-20}{(4x^2-20x+25)^2}$$

$$g'(x) = \frac{-768x^2+1792x+320}{(4x^2-20x+25)^2}$$
$$= \frac{-64(2x-5)(6x+1)}{(2x-5)^4}$$
$$= \frac{-64(6x+1)}{(2x-5)^3}$$

c) The results are the same. Which method is easier depends on the student. We believe that the Extended Power rule offers us a more efficient approach. It takes too much time to expand the function, and then factor back in the binomials.

46. $f(x) = \dfrac{x^2}{(1+x)^5}$

a) Using the Quotient Rule and the Extended Power Rule, we have:

$$f'(x) = \frac{(1+x)^5(2x) - x^2\left[5(1+x)^4(1)\right]}{\left((1+x)^5\right)^2}$$
$$= \frac{2x(1+x)^5 - 5x^2(1+x)^4}{(1+x)^{10}}$$
$$= \frac{(1+x)^4\left(2x(1+x)-5x^2\right)}{(1+x)^{10}} \quad \text{Factoring}$$
$$= \frac{2x+2x^2-5x^2}{(1+x)^6}$$
$$= \frac{2x-3x^2}{(1+x)^6}$$
$$= \frac{x(2-3x)}{(1+x)^6}$$

b) Using the Product Rule and the Extended Power Rule on $f(x) = x^2(1+x)^{-5}$, we have

$$f'(x) = x^2\left(-5(1+x)^{-6}(1)\right) + (1+x)^{-5}(2x)$$
$$= \frac{-5x^2}{(1+x)^6} + \frac{2x}{(1+x)^5}$$
$$= \frac{-5x^2}{(1+x)^6} + \frac{2x}{(1+x)^5}\cdot\frac{(1+x)}{(1+x)}$$
$$= \frac{-5x^2+2x+2x^2}{(1+x)^6}$$
$$= \frac{2x-3x^2}{(1+x)^6}$$
$$= \frac{x(2-3x)}{(1+x)^6}$$

c) The results are the same.

47. Using the Chain Rule:

$f(u)=u^3$, $g(x)=u=2x^4+1$

First find $f'(u)$ and $g'(x)$.

$f'(u)=3u^2$

$f'(g(x))=3(2x^4+1)^2$ Substituting $g(x)$ for u.

$g'(x)=8x^3$

The Chain Rule states

$(f\circ g)'(x)=f'(g(x))\cdot g'(x)$

Substituting, we have:

$$(f\circ g)'(x)=3(2x^4+1)^2\cdot(8x^3)$$
$$=24x^3(2x^4+1)^2$$

Therefore,

$$(f\circ g)'(-1)=24(-1)^3(2(-1)^4+1)^2$$
$$=-24(2+1)^2$$
$$=-24\cdot 9$$
$$=-216$$

Finding $f(g(x))$ first, we have:

$f(g(x))=f(2x^4+1)=(2x^4+1)^3$

By the Extended Power Rule:

$$f'(g(x))=3(2x^4+1)^2(8x^3)$$
$$=24x^3(2x^4+1)^2$$

Therefore, $f'(g(-1))=-216$ as above.

48. Using the Chain Rule:

$f(u)=\dfrac{u+1}{u-1}$, $g(x)=u=\sqrt{x}=x^{1/2}$

$$f'(u)=\frac{(u-1)(1)-(u+1)(1)}{(u-1)^2}=\frac{-2}{(u-1)^2}$$

$$f'(g(x))=\frac{-2}{(\sqrt{x}-1)^2}$$

$$g'(x)=\frac{1}{2\sqrt{x}}$$

$$(f\circ g)'(x)=f'(g(x))\cdot g'(x)$$
$$=\frac{-2}{(\sqrt{x}-1)^2}\cdot\frac{1}{2\sqrt{x}}$$
$$=\frac{-1}{\sqrt{x}(\sqrt{x}-1)^2}$$

Therefore,

$$(f\circ g)'(4)=\frac{-1}{\sqrt{4}(\sqrt{4}-1)^2}=\frac{-1}{2(1)^2}=-\frac{1}{2}.$$

Finding $f(g(x))$ first, we have:

$$f(g(x))=f(\sqrt{x})=\frac{\sqrt{x}+1}{\sqrt{x}-1}$$

Using the Quotient Rule:

$$f'(g(x))=\frac{(\sqrt{x}-1)\left(\frac{1}{2}x^{-1/2}\right)-(\sqrt{x}+1)\frac{1}{2}x^{-1/2}}{(\sqrt{x}-1)^2}$$

$$f'(g(x))=\frac{\frac{1}{2}-\frac{1}{2}x^{-1/2}-\frac{1}{2}-\frac{1}{2}x^{-1/2}}{(\sqrt{x}-1)^2}$$
$$=\frac{-1}{\sqrt{x}(\sqrt{x}-1)^2}$$

Therefore, $(f\circ g)'(4)=-\dfrac{1}{2}$ as above.

49. Using the Chain Rule:

$f(u)=\sqrt[3]{u}=u^{1/3}$, $g(x)=u=1+3x^2$

First find $f'(u)$ and $g'(x)$.

$$f'(u)=\frac{1}{3}u^{-2/3}=\frac{1}{3\cdot\sqrt[3]{u^2}}$$

$$f'(g(x))=\frac{1}{3\cdot\sqrt[3]{(1+3x^2)^2}}$$ Substituting $g(x)$ for u.

$g'(x)=6x$

The Chain Rule states

$(f\circ g)'(x)=f'(g(x))\cdot g'(x)$

Substituting, we have:

$$(f\circ g)'(x)=\frac{1}{3\cdot\sqrt[3]{(1+3x^2)^2}}\cdot(6x)$$
$$=\frac{2x}{\sqrt[3]{(1+3x^2)^2}}$$

Therefore,

$$(f \circ g)'(2) = \frac{2(2)}{\sqrt[3]{\left(1+3(2)^2\right)^2}}$$
$$= \frac{4}{\sqrt[3]{(13)^2}}$$
$$\approx 0.72348760$$

Finding $f(g(x))$ first, we have:

$$f(g(x)) = f\left(1+3x^2\right) = \left(1+3x^2\right)^{1/3}$$

By the Extended Power Rule:

$$f'(g(x)) = \frac{1}{3}\left(1+3x^2\right)^{-2/3}(6x)$$
$$= \frac{2x}{\left(1+3x^2\right)^{2/3}}$$

Therefore $(f \circ g)'(2) = \dfrac{4}{\sqrt[3]{(13)^2}} \approx 0.72$.

50. Using the Chain Rule:

$$f(u) = 2u^5, g(x) = u = \frac{3-x}{4+x}$$
$$f'(u) = 10u^4$$
$$f'(g(x)) = 10\left(\frac{3-x}{4+x}\right)^4$$
$$g'(x) = \frac{(4+x)(-1)-(3-x)(1)}{(4+x)^2}$$
$$= \frac{-7}{(4+x)^2}$$
$$(f \circ g)'(x) = f'(g(x)) \cdot g'(x)$$
$$= 10\left(\frac{3-x}{4+x}\right)^4 \cdot \left(\frac{-7}{(4+x)^2}\right)$$
$$= \frac{-70(3-x)^4}{(4+x)^6}$$

Therefore,

$$(f \circ g)'(-10) = \frac{-70\left(3-(-10)\right)^4}{\left(4+(-10)\right)^6}$$
$$= \frac{-70(13)^4}{(-6)^6} .$$
$$= -42.85129458$$

Finding $f(g(x))$ first, we have:

$$f(g(x)) = f\left(\frac{3-x}{4+x}\right) = 2\left(\frac{3-x}{4+x}\right)^5$$

Using the Extended Power Rule:

$$f'(g(x)) = 2 \cdot 5\left(\frac{3-x}{4+x}\right)^4\left[\frac{(4+x)(-1)-(3-x)(1)}{(4+x)^2}\right]$$
$$= 10\left(\frac{3-x}{4+x}\right)^4\left[\frac{-7}{(4+x)^2}\right]$$
$$f'(g(x)) = \frac{-70(3-x)^4}{(4+x)^6}$$

Therefore, $(f \circ g)'(-10) = -42.85$ as above.

51. a) $h(x) = \left(3x^2+2x\right)^5$

Therefore, $f(x) = x^5$ and $g(x) = 3x^2+2x$.

b) Using the Chain Rule:

$$f(u) = x^5, g(x) = u = 3x^2+2x$$
$$f'(u) = 5u^4$$
$$f'(g(x)) = 5\left(3x^2+2x\right)^4$$
$$g'(x) = 6x+2$$
$$(f \circ g)'(x) = f'(g(x)) \cdot g'(x)$$
$$= 5\left(3x^2+2x\right)^4 \cdot (6x+2)$$

Therefore,

$$(f \circ g)'(2) = 5\left(3(2)^2+2(2)\right)^4 \cdot (6(2)+2) .$$
$$= 5(16)^4(14)$$
$$= 4,587,520$$

52. a) $h(x) = \sqrt{1+5x^2}$

Therefore, $f(x) = \sqrt{x}$ and $g(x) = 1+5x^2$.

b) Using the Chain Rule:

$$f(u) = x^{1/2}, g(x) = u = 1+5x^2$$
$$f'(u) = \frac{1}{2}u^{-1/2}$$
$$f'(g(x)) = \frac{1}{2}\left(1+5x^2\right)^{-1/2}$$
$$g'(x) = 10x$$
$$(f \circ g)'(x) = f'(g(x)) \cdot g'(x)$$
$$= \frac{1}{2}\left(1+5x^2\right)^{-1/2} \cdot (10x)$$

Therefore,

$$(f\circ g)'(4)=\frac{5(4)}{\sqrt{1+5(4)^2}}.$$
$$=\frac{20}{\sqrt{81}}=\frac{20}{9}$$

53. a) $h(x)=\dfrac{x^3+1}{x^3+4}$

Therefore, $f(x)=\dfrac{x+1}{x+4}$ and $g(x)=x^3$.

b) Using the Chain Rule:

$$f(u)=\frac{x+1}{x+4},\ g(x)=u=x^3$$
$$f'(u)=\frac{(x+4)(1)-(x+1)(1)}{(x+4)^2}$$
$$f'(g(x))=\frac{3}{\left(x^3+4\right)^2}$$
$$g'(x)=3x^2$$
$$(f\circ g)'(x)=f'(g(x))\cdot g'(x)$$
$$=\frac{3}{\left(x^3+4\right)^2}\cdot\left(3x^2\right)$$

Therefore,

$$(f\circ g)'(1)=\frac{9(1)^2}{\left((1)^3+4\right)^2}.$$
$$=\frac{9}{(5)^2}=\frac{9}{25}$$

54. a) $h(x)=\dfrac{1}{x^2+2x}$

Therefore, $f(x)=\dfrac{1}{x}$ and $g(x)=x^2+2x$.

b) Using the Chain Rule:

$$f(u)=\frac{1}{x},\ g(x)=u=x^2+2x$$
$$f'(u)=-x^{-2}$$
$$f'(g(x))=-\frac{1}{\left(x^2+2x\right)^2}$$
$$g'(x)=2x+2$$
$$(f\circ g)'(x)=f'(g(x))\cdot g'(x)$$
$$=-\frac{1}{\left(x^2+2x\right)^2}\cdot(2x+2)$$

Therefore,

$$(f\circ g)'(-3)=-\frac{2(-3)+2}{\left((-3)^2+2(-3)\right)^2}.$$
$$=-\frac{-4}{(9-6)^2}=\frac{4}{9}$$

55. $f(x)=\left(2x^3+(4x-5)^2\right)^6$

Letting $u=2x^3+(4x-5)^2$ and applying the Chain Rule, we have:

$$f'(x)$$
$$=6\left(2x^3+(4x-5)^2\right)^{6-1}\bullet\frac{d}{dx}\left(2x^3+(4x-5)^2\right)$$

We will have to apply the chain rule again to find $\dfrac{d}{dx}\left(2x^3+(4x-5)^2\right)$.

Applying the Chain Rule again, we have:

$$\frac{d}{dx}\left(2x^3+(4x-5)^2\right)$$
$$=2\frac{d}{dx}x^3+\frac{d}{dx}(4x-5)^2$$
$$=2\left(3x^2\right)+2(4x-5)^{2-1}\bullet\frac{d}{dx}(4x-5)$$
$$=6x^2+2(4x-5)\bullet4$$
$$=6x^2+32x-40.$$

Therefore, the derivative is:

$$f'(x)=6\left(2x^3+(4x-5)^2\right)^5\left(6x^2+8(4x-5)\right).$$

56. $f(x)=\left(-x^5+4x+\sqrt{2x+1}\right)^3$

$$f'(x)$$
$$=3\left(-x^5+4x+\sqrt{2x+1}\right)^2\bullet$$
$$\frac{d}{dx}\left(-x^5+4x+\sqrt{2x+1}\right)$$

Applying the Chain Rule again, we have:

$$\frac{d}{dx}\left(-x^5+4x+\sqrt{2x+1}\right)$$
$$=-5x^4+4+\frac{1}{2\sqrt{2x+1}}\frac{d}{dx}(2x+1)$$
$$=-5x^4+4+\frac{1}{\sqrt{2x+1}}.$$

Therefore, the derivative is:

$$f'(x)=3\left(-x^5+4x+\sqrt{2x+1}\right)^2\bullet\left(-5x^4+4+\frac{1}{\sqrt{2x+1}}\right)$$

57. $f(x)=\sqrt{x^2+\sqrt{1-3x}}=\left(x^2+(1-3x)^{\frac{1}{2}}\right)^{\frac{1}{2}}$

Applying the Chain Rule, we have

$$\begin{aligned}f'(x)&=\frac{1}{2}\left(x^2+(1-3x)^{\frac{1}{2}}\right)^{\frac{1}{2}-1}\bullet\frac{d}{dx}\left(x^2+(1-3x)^{\frac{1}{2}}\right)\\&=\frac{1}{2}\left(x^2+(1-3x)^{\frac{1}{2}}\right)^{-\frac{1}{2}}\bullet\frac{d}{dx}\left(x^2+(1-3x)^{\frac{1}{2}}\right)\end{aligned}$$

Applying the Chain Rule again, we have

$$\begin{aligned}\frac{d}{dx}\left(x^2+(1-3x)^{\frac{1}{2}}\right)&=\frac{d}{dx}x^2+\frac{d}{dx}(1-3x)^{\frac{1}{2}}\\&=2x^{2-1}+\frac{1}{2}(1-3x)^{\frac{1}{2}-1}\bullet\frac{d}{dx}(1-3x)\\&=2x+\frac{1}{2\sqrt{1-3x}}\bullet(-3)\\&=2x-\frac{3}{2\sqrt{1-3x}}.\end{aligned}$$

Therefore, the derivative is:

$$\begin{aligned}f'(x)&=\frac{1}{2}\left(x^2+(1-3x)^{\frac{1}{2}}\right)^{-\frac{1}{2}}\left(2x-\frac{3}{2\sqrt{1-3x}}\right)\\&=\frac{1}{2\sqrt{x^2+\sqrt{1-3x}}}\left(2x-\frac{3}{2\sqrt{1-3x}}\right).\end{aligned}$$

58. $f(x)=\sqrt[3]{2x+\left(x^2+x\right)^4}$

Applying the Chain Rule, we have

$$f'(x)=\frac{1}{3}\left(2x+\left(x^2+x\right)^4\right)^{-\frac{2}{3}}\bullet\frac{d}{dx}\left(2x+\left(x^2+x\right)^4\right)$$

Applying the Chain Rule again, we have

$$\begin{aligned}\frac{d}{dx}\left(2x+\left(x^2+x\right)^4\right)&=2+4\left(x^2+x\right)^3\bullet\frac{d}{dx}\left(x^2+x\right)\\&=2+4\left(x^2+x\right)^3(2x+1)\end{aligned}$$

Therefore, the derivative is:

$$f'(x)=\frac{1}{3}\left(2x+\left(x^2+x\right)^4\right)^{-\frac{2}{3}}\bullet\left(2+4\left(x^2+x\right)^3(2x+1)\right)$$

59. $R(x)=1000\sqrt{x^2-0.1x}=1000\left(x^2-0.1x\right)^{1/2}$

Using the Extended Power Rule, we have

$$\begin{aligned}R'(x)&=1000\left[\frac{1}{2}\left(x^2-0.1x\right)^{-1/2}(2x-0.1)\right]\\&=500\left[\frac{2x-0.1}{\left(x^2-0.1x\right)^{1/2}}\right]\\&=\frac{500(2x-0.1)}{\sqrt{x^2-0.1x}}\end{aligned}$$

Substituting 20 for x, we have

$$\begin{aligned}R'(20)&=\frac{500(2(20)-0.1)}{\sqrt{(20)^2-0.1(20)}}\\&=1000.00314070\\&\approx 1000\end{aligned}$$

When 20 airplanes have been sold, revenue is changing at a rate of 1,000 thousand of dollars per airplane, or 1,000,000 dollars per airplane.

60. $C(x)=2000\left(x^2+2\right)^{1/3}+700$

$$\begin{aligned}C'(x)&=2000\left[\frac{1}{3}\left(x^2+2\right)^{-2/3}(2x)\right]\\&=\frac{4000x}{3}\left(x^2+2\right)^{-2/3}\\&=\frac{4000x}{3\left(x^2+2\right)^{2/3}}\end{aligned}$$

$$C'(20)=\frac{4000(20)}{3\left((20)^2+2\right)^{2/3}}\approx 489.57364458$$

When 20 airplanes are produced, the total cost is changing at a rate of 489.574 thousand of dollars per airplane, or 489,574 dollars per airplane.

61. $P(x)=R(x)-C(x)$ and $P'(x)=R'(x)-C'(x)$

Since we are trying to find the rate at which total profit is changing as a function of x, we can use the derivatives found in Exercise 59 and 60 to find the derivative of the profit function.

There is no need to find the Profit function first and then take the derivative.

$P'(x) = R'(x) - C'(x)$

$= \dfrac{500(2x-0.1)}{\sqrt{x^2-0.1x}} - \dfrac{4000x}{3(x^2+2)^{2/3}}$

Therefore,

$P'(20) = 1000 - 489.574$

$= 510.429$

62. $C(x) = \sqrt{5x^2+60}$ and $x(t) = 20t + 40$. To find the rate of change of the cost function with respect to the number of months, we must apply the Chain Rule.

$C'(x) = \dfrac{1}{2}(5x^2+60)^{-1/2}(10x) = 5x(5x^2+60)^{-1/2}$

$C'(x(t)) = 5(20t+40)\left(5(20t+40)^2+60\right)^{-1/2}$

$x'(t) = 20$

Using the Chain Rule, we have:

$\dfrac{dC}{dt} = C'(x(t)) \cdot x'(t)$

$= 5(20t+40)\left(5(20t+40)^2+60\right)^{-1/2} \cdot (20)$

$= 100(20t+40)\left(5(20t+40)^2+60\right)^{-1/2}$

$= \dfrac{100(20t+40)}{\left(5(20t+40)^2+60\right)^{1/2}}$

Substituting 4 in for t, we have:

$\dfrac{dC}{dt} = \dfrac{100(20(4)+40)}{\left(5(20(4)+40)^2+60\right)^{1/2}}$

≈ 44.70273729

After 4 months, the total cost is changing at a rate of 44.7 thousand of dollars per month, or 44,700 dollars per month.

63. $A = 1000\left(1+\dfrac{r}{4}\right)^{20}$

a) Using the Extended Power Rule, we have

$\dfrac{dA}{di} = 1000(20)\left(1+\dfrac{r}{4}\right)^{19}\left(\dfrac{1}{4}\right)$;

$= 5000\left(1+\dfrac{r}{4}\right)^{19}$

the units are dollars per interest rate.

b) $\dfrac{dA}{di}$ is the rate of change in the amount as the interest rate r changes.

64. $A = 1000\left(1+\dfrac{r}{12}\right)^{36}$

a) Using the Extended Power Rule, we have

$\dfrac{dA}{di} = 1000(36)\left(1+\dfrac{r}{12}\right)^{35}\left(\dfrac{1}{12}\right)$;

$= 3000\left(1+\dfrac{r}{12}\right)^{35}$

the units are dollars per interest rate.

b) $\dfrac{dA}{di}$ is the rate of change in the amount as the interest rate r changes.

65. $P(x) = 0.08x^2 + 80x$ and $x = 5t + 1$

a) Substituting $5t+1$ for x, we have

$P(t) = 0.08(5t+1)^2 + 80(5t+1)$

$= 0.08(25t^2+10t+1) + 400t + 80$

$= 2t^2 + 400.8t + 80.08$

b) $P'(t) = 4t + 400.8$

Therefore,

$P'(48) = 4(48) + 400.8 = 592.80$

After 48 months, profit is increasing at rate of 592.80 dollars per month.

66. $D(p) = \dfrac{80,000}{p}$ and $p = 1.6t + 9$

a) Substitute $1.6t+9$ in for p in the demand function.

$D(t) = \dfrac{80,000}{1.6t+9}$

b) Using the Quotient Rule, we have

$D'(t) = \dfrac{(1.6t+9)(0) - (80,000)(1.6)}{(1.6t+9)^2}$

$= \dfrac{-128,000}{(1.6t+9)^2}$

Substituting 100 for t into the derivative from the previous page, we have:

$$D'(100) = \frac{-128,000}{\left(1.6(100)+9\right)^2} = \frac{-128,000}{28,561} \approx -4.48163580$$

After 100 days, quantity demanded is changing -4.482 units per day.

67. $D = 0.85A(c+25)$ and $c = (140-y)\frac{w}{72x}$

a) Substituting 5 for A we have:

$$D(c) = 0.85(5)(c+25) = 4.25(c+25) = 4.25c + 106.25$$

Substituting 0.6 for x, 45 for y, we have:

$$c(w) = (140-45)\frac{w}{72(0.6)} = 95\frac{w}{43.2} = \frac{95w}{43.2} \approx 2.199w$$

b) $\frac{dD}{dc} = \frac{d}{dc}(4.25c + 106.25) = 4.25$

The dosage changes at a rate of 4.25 mg per unit of creatine clearance.

c) $\frac{dc}{dw} = \frac{d}{dw}\left(\frac{95w}{43.2}\right) = \frac{95}{43.2} \approx 2.199$

The creatine clearance changes at a rate of 2.199 unit of creatine clearance per kilogram.

d) By the Chain Rule:

$$\frac{dD}{dw} = \frac{dD}{dc}\cdot\frac{dc}{dw} = (4.25)\left(\frac{95}{43.2}\right) \approx 9.346$$

The dosage changes at a rate of 9.35 milligrams per kilogram.

e) $\frac{dD}{dw}$ represents the rate of change in dosage as the patient's weight, in kilograms, varies.

68. $s(p) = \frac{1}{14}p$ and $p(x) = 2.20462x$

a) Substituting 100 for x we have:

$$p(100) = 2.20462(100) = 220.462$$

Substituting 220.462 for p, we have:

$$s(220.462) = \frac{1}{14}(220.462); \approx 15.75$$

which means that an object with a weight of 100 kilograms at sea level has an equivalent weight of 15.75 stone.

b) $\frac{d}{dx}(s \circ p)(x) = \frac{d}{dx}\left(\frac{2.20462}{14}x\right) = \frac{2.20462}{14}$

Therefore;

$\frac{d}{dx}(s \circ p)(100) = \frac{2.20462}{14} \approx 0.157$ which means that an objects weight increases by approximately 0.157 stone per kilogram.

69. $C(m) = -\frac{1}{225}m + 100$ and $F(C) = \frac{9}{5}c + 32$

a) Substituting 3000 for m we have:

$$C(3000) = -\frac{1}{225}(3000) + 100 = 86.67$$

Substituting 86.67 for c, we have:

$$s(86.67) = \frac{9}{5}(86.67) + 32; \approx 188$$

which means that water at 3000 m boils at 188 degrees Fahrenheit.

b)

$$\frac{d}{dx}(F \circ C)(m) = \frac{d}{dx}\left(\frac{9}{5}\left(-\frac{1}{225}m + 100\right) + 32\right) = -\frac{9}{1125}$$

Therefore;

$\frac{d}{dx}(F \circ C)(3000) = -\frac{9}{1125} \approx 0.0098$

which means that at 3000 m in altitude, the boiling point of water drops by 0.008 degrees Fahrenheit per meter as altitude increases.

70. $U(x) = 3x - 21$ and $E = 1.35U + 28.25$

a) Substituting 9 for x we have:

$$U(9) = 3(9) - 21 = 6$$

Substituting 6 for U, we have:

$$E = 1.35(6) + 28.25; = 36.35$$

which means that a woman with a 9 inch foot wears a size 36.35 shoe in Europe.

b) $\frac{d}{dx}(E \circ U)(x) = \frac{d}{dx}(1.35(3x-21)+28.25)$
$= 4.05$

Therefore; $\frac{d}{dx}(E \circ U)(9) = 4.05$ which means that for every inch a woman's foot grows, her European shoe size increases by 4.05 units per inch.

71. $f(x) = x^2 + 1$

Note that $f'(x) = 2x$ and

$f'(f(x)) = 2(x^2+1)$.

Applying the Chain Rule to the iterated function, we have

$$\begin{aligned}\frac{d}{dx}[(f \circ f)(x)] &= \frac{d}{dx}[f(f(x))] \\ &= f'(f(x)) \cdot f'(x) \\ &= 2(x^2+1) \cdot 2x \\ &= 4x^3 + 4x. \\ &= 4x(x^2+1)\end{aligned}$$

72. $f(x) = x + \sqrt{x}$

Note that $f'(x) = 1 + \frac{1}{2\sqrt{x}}$ and

$f'(f(x)) = 1 + \frac{1}{2\sqrt{x+\sqrt{x}}}$.

Applying the Chain Rule to the iterated function, we have

$$\begin{aligned}&\frac{d}{dx}[(f \circ f)(x)] \\ &= \frac{d}{dx}[f(f(x))] \\ &= f'(f(x)) \cdot f'(x) \\ &= \left[1 + \frac{1}{2\sqrt{x+\sqrt{x}}}\right] \cdot \left[1 + \frac{1}{2\sqrt{x}}\right] \\ &= 1 + \frac{1}{2\sqrt{x}} + \left(\frac{1}{2\sqrt{x+\sqrt{x}}}\right)\left(1 + \frac{1}{2\sqrt{x}}\right)\end{aligned}$$

73. $f(x) = x^2 + 1$

Note that $f'(x) = 2x$,

$f'(f(x)) = 2(x^2+1)$, and

$f'(f(f(x))) = 2\left((x^2+1)^2+1\right)$

Applying the Chain Rule to the iterated function, we have

$$\begin{aligned}&\frac{d}{dx}[(f \circ f \circ f)(x)] \\ &= \frac{d}{dx}[f(f(f(x)))] \\ &= f'(f(f(x))) \cdot f'(f(x)) \cdot f'(x) \\ &= 2\left((x^2+1)^2+1\right) \cdot 2(x^2+1) \cdot 2x \\ &= 8x\left((x^2+1)^2+1\right)(x^2+1).\end{aligned}$$

74. $f(x) = \sqrt[3]{x} = x^{\frac{1}{3}}$

Note that $f'(x) = \frac{1}{3}x^{-\frac{2}{3}}$,

$f'(f(x)) = \frac{1}{3}x^{-\frac{2}{9}}$, and

$f'(f(f(x))) = \frac{1}{3}x^{-\frac{2}{27}}$

Applying the Chain Rule to the iterated function, we have

$$\begin{aligned}&\frac{d}{dx}[(f \circ f \circ f)(x)] \\ &= \frac{d}{dx}[f(f(f(x)))] \\ &= f'(f(f(x))) \cdot f'(f(x)) \cdot f'(x) \\ &= \frac{1}{3}x^{-\frac{2}{27}} \cdot \frac{1}{3}\left(x^{-\frac{2}{9}}\right) \cdot \frac{1}{3}\left(x^{-\frac{2}{3}}\right) \\ &= \frac{1}{27}x^{-\frac{2}{27}-\frac{2}{9}-\frac{2}{3}} \\ &= \frac{1}{27}x^{-\frac{26}{27}}.\end{aligned}$$

75. $y = \sqrt[3]{x^3+6x+1}\cdot x^5 = \left(x^3+6x+1\right)^{1/3}\cdot x^5$

Using the Product Rule and the Extended Power Rule, we have

$$\frac{dy}{dx} = \left(x^3+6x+1\right)^{1/3}\cdot\left(5x^4\right) + x^5\left[\frac{1}{3}\left(x^3+6x+1\right)^{-2/3}\left(3x^2+6\right)\right]$$

$$= 5x^4\left(x^3+6x+1\right)^{1/3} + \frac{3x^5\left(x^2+2\right)}{3\left(x^3+6x+1\right)^{2/3}}$$

The derivative can be further simplified by finding a common denominator and combining the fractions.

$$\frac{dy}{dx} = \frac{\left(x^3+6x+1\right)^{2/3}}{\left(x^3+6x+1\right)^{2/3}}\cdot\frac{5x^4\left(x^3+6x+1\right)^{1/3}}{1} + \frac{x^5\left(x^2+2\right)}{\left(x^3+6x+1\right)^{2/3}}$$

$$= \frac{5x^4\left(x^3+6x+1\right)}{\left(x^3+6x+1\right)^{2/3}} + \frac{x^5\left(x^2+2\right)}{\left(x^3+6x+1\right)^{2/3}}$$

$$= \frac{5x^7+30x^5+5x^4+x^7+2x^5}{\left(x^3+6x+1\right)^{2/3}}$$

$$= \frac{6x^7+32x^5+5x^4}{\left(x^3+6x+1\right)^{2/3}}$$

76. $y = \left(x\sqrt{1+x^2}\right)^3 = \left(x\left(1+x^2\right)^{1/2}\right)^3$

$$\frac{dy}{dx} = 3\left(x\left(1+x^2\right)^{1/2}\right)^2\cdot\left(x\left(\frac{1}{2}\left(1+x^2\right)^{-1/2}\cdot 2x\right)+1\left(1+x^2\right)^{1/2}\right)$$

$$= 3x^2\left(1+x^2\right)\cdot\left(\frac{x^2}{\left(1+x^2\right)^{1/2}}+\left(1+x^2\right)^{1/2}\right)$$

$$= 3x^2\left(1+x^2\right)\cdot\left(\frac{x^2}{\left(1+x^2\right)^{1/2}}+\frac{1+x^2}{\left(1+x^2\right)^{1/2}}\right)$$

$$= 3x^2\left(1+x^2\right)\cdot\left(\frac{1+2x^2}{\left(1+x^2\right)^{1/2}}\right)$$

$$= \left(3x^2+6x^4\right)\left(1+x^2\right)^{1/2}$$

$$= \left(3x^2+6x^4\right)\sqrt{1+x^2}$$

77. $y = \frac{\sqrt{1-x^2}}{1-x} = \frac{\left(1-x^2\right)^{1/2}}{1-x}$

Using the Quotient Rule and the Extended Power Rule, we have:

$$\frac{dy}{dx} = \frac{(1-x)\left(\frac{1}{2}\left(1-x^2\right)^{-1/2}(-2x)\right)-\left(1-x^2\right)^{1/2}(-1)}{(1-x)^2}$$

$$\frac{dy}{dx} = \frac{\frac{x(x-1)}{\left(1-x^2\right)^{1/2}}+\left(1-x^2\right)^{1/2}}{(1-x)^2}$$

$$= \frac{\frac{x^2-x}{\left(1-x^2\right)^{1/2}}+\frac{\left(1-x^2\right)^{1/2}}{1}\cdot\frac{\left(1-x^2\right)^{1/2}}{\left(1-x^2\right)^{1/2}}}{(1-x)^2}$$

$$= \frac{\frac{x^2-x}{\left(1-x^2\right)^{1/2}}+\frac{\left(1-x^2\right)}{\left(1-x^2\right)^{1/2}}}{(1-x)^2}$$

$$= \frac{1-x}{\left(1-x^2\right)^{1/2}(1-x)^2}$$

$$= \frac{1}{(1-x)\sqrt{1-x^2}}$$

78. $y=\left(\dfrac{x^2-x-1}{x^2+1}\right)^3$

Using the Extended Power Rule and the Quotient Rule, we have

$$\frac{dy}{dx}$$

$$=3\left(\frac{x^2-x-1}{x^2+1}\right)^2\left[\frac{(x^2+1)(2x-1)-(x^2-x-1)(2x)}{(x^2+1)^2}\right]$$

$$=3\left(\frac{x^2-x-1}{x^2+1}\right)^2\left[\frac{x^2+4x-1}{(x^2+1)^2}\right]$$

$$=\frac{3(x^2-x-1)^2(x^2+4x-1)}{(x^2+1)^4}$$

79. $g(x)=\sqrt{\dfrac{x^2-4x}{2x+1}}=\left(\dfrac{x^2-4x}{2x+1}\right)^{1/2}$

$$g'(x)$$

$$=\frac{1}{2}\left(\frac{x^2-4x}{2x+1}\right)^{-1/2}\left[\frac{(2x+1)(2x-4)-(x^2-4x)(2)}{(2x+1)^2}\right]$$

$$=\frac{1}{2}\left(\frac{2x+1}{x^2-4x}\right)^{1/2}\left[\frac{4x^2-6x-4-2x^2+8x}{(2x+1)^2}\right]$$

$$=\frac{1}{2}\left(\frac{2x+1}{x^2-4x}\right)^{1/2}\left[\frac{2x^2+2x-4}{(2x+1)^2}\right]$$

$$=\frac{1}{2}\left(\frac{2x+1}{x^2-4x}\right)^{1/2}\left[\frac{2(x^2+x-2)}{(2x+1)^2}\right]$$

$$=\frac{x^2+x-2}{(2x+1)^{3/2}(x^2-4x)^{1/2}}$$

$$=\frac{x^2+x-2}{\sqrt{(2x+1)^3(x^2-4x)}}$$

80. $f(t)=\sqrt{3t+\sqrt{t}}=\left(3t+t^{1/2}\right)^{1/2}$

Using the Extended Power Rule, we have

$$f'(t)=\frac{1}{2}\left(3t+t^{1/2}\right)^{-1/2}\left(3+\frac{1}{2}t^{-1/2}\right)$$

$$=\frac{3+\dfrac{1}{2\sqrt{t}}}{2\sqrt{3t+\sqrt{t}}}$$

$$=\frac{\dfrac{3}{1}\cdot\dfrac{2\sqrt{t}}{2\sqrt{t}}+\dfrac{1}{2\sqrt{t}}}{2\sqrt{3t+\sqrt{t}}}$$

Simplifying, we have:

$$f'(t)=\frac{\dfrac{6\sqrt{t}+1}{2\sqrt{t}}}{2\sqrt{3t+\sqrt{t}}}$$

$$=\frac{6\sqrt{t}+1}{4\sqrt{t}\sqrt{3t+\sqrt{t}}}$$

81. $F(x)=\left(6x(3-x)^5+2\right)^4$

$$F'(x)$$

$$=4\left(6x(3-x)^5+2\right)^3\left[6x\left(5(3-x)^4(-1)\right)+(3-x)^5(6)\right]$$

$$=4\left(6x(3-x)^5+2\right)^3\left[-30x(3-x)^4+6(3-x)^5\right]$$

$$=4\left(6x(3-x)^5+2\right)^3\left[6(3-x)^4\left(-5x+(3-x)\right)\right]$$

$$=24(3-x)^4(3-6x)\left(6x(3-x)^5+2\right)^3$$

$$=-72(3-x)^4(2x-1)\left(6x(3-x)^5+2\right)^3$$

82. **a)** Rewriting $\dfrac{1}{g(x)}=[g(x)]^{-1}$

b) Using Leibniz notation, differentiating both sides.

c) Using the Product Rule for differentiation.

d) Using the Extended Power Rule.

e) Simplifying by moving negative powers to denominator.

f) Multiplying the second fraction by $\dfrac{g(x)}{g(x)}=1.$

g) Writing over a common denominator and simplifying.

83. a) Applying the product rule, we have:

$$\frac{d}{dx}\left[f(x)\right]^3$$
$$=\frac{d}{dx}\left[\left[f(x)\right]^2\cdot\left[f(x)\right]\right]$$
$$=\left[f(x)\right]^2\frac{d}{dx}\left(f(x)\right)+\frac{d}{dx}\left[f(x)\right]^2 f(x)$$
$$=\left[f(x)\right]^2 f'(x)+\frac{d}{dx}\left[f(x)\right]^2 f(x)$$

We apply the Product Rule to $\frac{d}{dx}\left[f(x)\right]^2$.

$$\frac{d}{dx}\left[f(x)\right]^2=f(x)f'(x)+f'(x)f(x)$$
$$=2f(x)f'(x).$$

Therefore, we now have:

$$\frac{d}{dx}\left[f(x)\right]^3=\left[f(x)\right]^2 f'(x)+2\left[f(x)\right]^2 f'(x)$$
$$=3\left[f(x)\right]^2 f'(x)$$

b) Applying the Product Rule we have:

$$\frac{d}{dx}\left[f(x)\right]^4$$
$$=\frac{d}{dx}\left[\left[f(x)\right]^3\cdot\left[f(x)\right]\right]$$
$$=\left[f(x)\right]^3\frac{d}{dx}\left(f(x)\right)+\frac{d}{dx}\left[f(x)\right]^3 f(x)$$
$$=\left[f(x)\right]^2 f'(x)+\frac{d}{dx}\left[f(x)\right]^3 f(x).$$

From part (a) we know that:

$$\frac{d}{dx}\left[f(x)\right]^3=3\left[f(x)\right]^2 f'(x)$$

Therefore, substituting into the derivative we have:

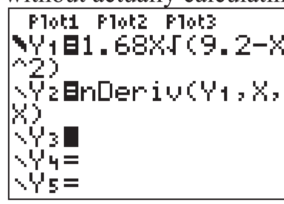
$$\frac{d}{dx}\left[f(x)\right]^4$$
$$=\left[f(x)\right]^3 f'(x)+3\left[f(x)\right]^2 f'(x)f(x)$$
$$=4\left[f(x)\right]^3 f'(x)$$

c) The generalized pattern that is established will be:

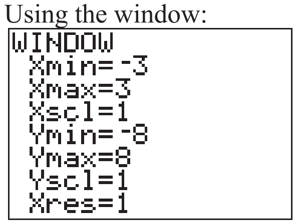
$$\frac{d}{dx}\left[f(x)\right]^n$$
$$=n\left[f(x)\right]^{n-1} f'(x)$$

84. $f(x)=1.68x\sqrt{9.2-x^2}$; $[-3,3]$

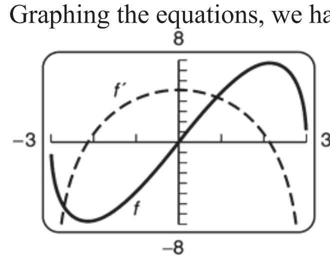
Using the calculator, we graph the function and the derivative in the same window. We can use the nDeriv feature to graph the derivative without actually calculating the derivative.



Using the window:

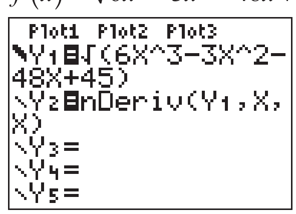


Graphing the equations, we have:

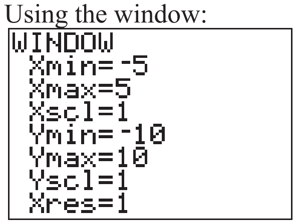


Note, the function $f(x)$ is the solid graph. The horizontal tangents occur at the turning points of this function, or at the x-intercepts of the derivative. Using the trace feature, the minimum/maximum feature on the function, or the zero feature on the derivative on the calculator, we find the points of horizontal tangency. We estimate the points at which the tangent lines are horizontal are $(-2.14476,-7.728)$ and $(2.14476,7.728)$.

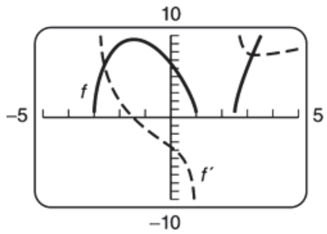
85. $f(x)=\sqrt{6x^3-3x^2-48x+45}$, $[-5,5]$



Using the window:



We get the graph:



Note, the function $f(x)$ is the solid graph.

The horizontal tangents occur at the turning points of this function, or at the x-intercepts of the derivative. Using the trace feature, the minimum/maximum feature on the function, or the zero feature on the derivative on the calculator, we find the points of horizontal tangency. We estimate the point at which the tangent line is horizontal to be $(-1.47481, 9.4878)$.

Exercise Set 1.8

1. $y = x^5 + 9$

$\frac{dy}{dx} = 5x^{5-1} = 5x^4$ First Derivative

$\frac{d^2y}{dx^2} = 5\left(4x^{4-1}\right) = 20x^3$ Second Derivative

2. $y = x^4 - 7$

$\frac{dy}{dx} = 4x^3$

$\frac{d^2y}{dx^2} = 12x^2$

3. $y = 5x^3 + 4x$

$\frac{dy}{dx} = 15x^2 + 4$

$\frac{d^2y}{dx^2} = 30x$

4. $y = 2x^4 - 5x$

$\frac{dy}{dx} = 2\left(4x^3\right) - 5 =$

$= 8x^3 - 5$ First Derivative

$\frac{d^2y}{dx^2} = 8\left(3x^2\right)$

$= 24x^2$ Second Derivative

5. $y = 4x^2 + 3x - 1$

$\frac{dy}{dx} = 4\left(2x^{2-1}\right) + 3 - 0$

$= 8x + 3$ First Derivative

$\frac{d^2y}{dx^2} = 8$ Second Derivative

6. $y = 4x^2 - 5x + 7$

$\frac{dy}{dx} = 8x - 5$

$\frac{d^2y}{dx^2} = 8$

7. $y = 6x - 3$

$\frac{dy}{dx} = 6$

$\frac{d^2y}{dx^2} = 0$

8. $y = 7x + 2$

$\frac{dy}{dx} = 7$ First Derivative

$\frac{d^2y}{dx^2} = 0$ Second Derivative

9. $y = \frac{1}{x^2} = x^{-2}$

$\frac{dy}{dx} = -2x^{-2-1}$

$= -2x^{-3}$

$= \frac{-2}{x^3}$ First Derivative

$\frac{d^2y}{dx^2} = -2\left(-3x^{-3-1}\right)$

$= 6x^{-4}$

$= \frac{6}{x^4}$ Second Derivative

10. $y = \frac{1}{x^3} = x^{-3}$

$\frac{dy}{dx} = -3x^{-4} = \frac{-3}{x^4}$

$\frac{d^2y}{dx^2} = 12x^{-5} = \frac{12}{x^5}$

11. $y = \sqrt[4]{x} = x^{1/4}$

$\frac{dy}{dx} = \frac{1}{4}x^{1/4-1} = \frac{1}{4}x^{-3/4} = \frac{1}{4x^{3/4}} = \frac{1}{4\cdot\sqrt[4]{x^3}}$

$\frac{d^2y}{dx^2} = \frac{1}{4}\left(-\frac{3}{4}\right)x^{-3/4-1}$

$= -\frac{3}{16}x^{-7/4}$

$= -\frac{3}{16x^{7/4}} = -\frac{3}{16\sqrt[4]{x^7}}$

12. $y = \sqrt{x} = x^{1/2}$

$\dfrac{dy}{dx} = \dfrac{1}{2}x^{1/2-1}$

$= \dfrac{1}{2}x^{-1/2}$

$= \dfrac{1}{2x^{1/2}} = \dfrac{1}{2\sqrt{x}}$ First Derivative

$\dfrac{d^2y}{dx^2} = \dfrac{1}{2}\cdot\left(-\dfrac{1}{2}x^{-1/2-1}\right)$

$= -\dfrac{1}{4}x^{-3/2}$

$= -\dfrac{1}{4x^{3/2}} = -\dfrac{1}{4\sqrt{x^3}}$ Second Derivative

13. $f(x) = x^4 + \dfrac{3}{x}$

$f'(x) = 4x^{4-1} + 3\left(-1x^{-1-1}\right)$

$= 4x^3 - 3x^{-2}$

$= 4x^3 - \dfrac{3}{x^2}$ First Derivative

$f''(x) = 4\left(3x^{3-1}\right) - 3\left(-2x^{-2-1}\right)$

$= 12x^2 + 6x^{-3}$

$= 12x^2 + \dfrac{6}{x^3}$ Second Derivative

14. $f(x) = x^3 - \dfrac{5}{x} = x^3 - 5x^{-1}$

$f'(x) = 3x^2 + 5x^{-2}$

$= 3x^2 + \dfrac{5}{x^2}$

$f''(x) = 6x - 10x^{-3}$

$= 6x - \dfrac{10}{x^3}$

15. $f(x) = x^{1/3}$

$f'(x) = \dfrac{1}{3}x^{-2/3} = \dfrac{1}{3x^{2/3}}$

$f''(x) = \dfrac{1}{3}\left(-\dfrac{2}{3}\right)x^{-5/3} = -\dfrac{2}{9x^{5/3}}$

16. $f(x) = x^{1/5}$

$f'(x) = \dfrac{1}{5}x^{1/5-1}$

$= \dfrac{1}{5}x^{-4/5}$

$= \dfrac{1}{5x^{4/5}}$ First Derivative

$f''(x) = \dfrac{1}{5}\left(-\dfrac{4}{5}\right)x^{-4/5-1}$

$= -\dfrac{4}{25}x^{-9/5}$

$= -\dfrac{4}{25x^{9/5}}$ Second Derivative

17. $f(x) = 4x^{-3}$

$f'(x) = 4\left(-3x^{-3-1}\right)$

$= -12x^{-4}$

$= -\dfrac{12}{x^4}$ First Derivative

$f''(x) = -12\left(-4x^{-4-1}\right)$

$= 48x^{-5}$

$= \dfrac{48}{x^5}$ Second Derivative

18. $f(x) = 2x^{-2}$

$f'(x) = -4x^{-3} = -\dfrac{4}{x^3}$

$f''(x) = 12x^{-4} = \dfrac{12}{x^4}$

19. $f(x)=(x^3+2x)^6$

$$f'(x)=6(x^3+2x)^5(3x^2+2)$$

$$f''(x)=6\left[(x^3+2x)^5(6x)\right]+$$

$$6\left[(3x^2+2)\cdot 5(x^3+2x)^4(3x^2+2)\right]$$

$$=36x(x^3+2x)^5+30(3x^2+2)^2(x^3+2x)^4$$

$$=6(x^3+2x)^4\left[6x(x^3+2x)+5(3x^2+2)^2\right]$$

$$=6(x^3+2x)^4\left[6x^4+12x^2+45x^4+60x^2+20\right]$$

$$=6(x^3+2x)^4(51x^4+72x^2+20)$$

20. $f(x)=(x^2+3x)^7$

$$f'(x)=7(x^2+3x)^{7-1}(2x+3) \quad \text{Theorem 7}$$

$$=7(2x+3)(x^2+3x)^6 \quad \text{First Derivative}$$

$$f''(x)=7(2x+3)\left(6(x^2+3x)^{6-1}(2x+3)\right)+$$

$$7(x^2+6x)^6(2) \quad \text{Theorem 5}$$

$$=42(2x+3)^2(x^2+3x)^5+14(x^2+3x)^6$$

We can simplify the second derivative by factoring out common factors.

$f''(x)$

$$=14(x^2+3x)^5\left[3(2x+3)^2+(x^2+3x)\right]$$

$$=14(x^2+3x)^5\left[3(4x^2+12x+9)+(x^2+3x)\right]$$

$$=14(x^2+3x)^5\left[12x^2+36x+27+x^2+3x\right]$$

$$=14(x^2+3x)^5(13x^2+39x+27) \quad \text{Second Derivative}$$

21. $f(x)=\sqrt[4]{(x^2+1)^3}=(x^2+1)^{3/4}$

$$f'(x)=\frac{3}{4}(x^2+1)^{-1/4}(2x) \quad \text{Theorem 7}$$

$$=\frac{3}{2}x(x^2+1)^{-1/4} \quad \text{First Derivative}$$

$$f''(x)=\frac{3}{2}x\cdot\frac{-1}{4}(x^2+1)^{-5/4}(2x)+$$

$$\frac{3}{2}(x^2+1)^{-1/4}(1) \quad \text{Theorem 5}$$

$$=-\frac{3}{4}x^2(x^2+1)^{-5/4}+\frac{3}{2}(x^2+1)^{-1/4}$$

$$=\frac{-3x^2}{4(x^2+1)^{5/4}}+\frac{3}{2(x^2+1)^{1/4}}$$

We can simplify the second derivative by finding a common denominator and combining the fractions.

$$f''(x)=\frac{-3x^2}{4(x^2+1)^{5/4}}+\frac{3}{2(x^2+1)^{1/4}}\cdot\frac{2(x^2+1)}{2(x^2+1)}$$

$$=\frac{-3x^2}{4(x^2+1)^{5/4}}+\frac{6(x^2+1)}{4(x^2+1)^{5/4}}$$

$$=\frac{3x^2+6}{4(x^2+1)^{5/4}}$$

$$=\frac{3(x^2+2)}{4(x^2+1)^{5/4}}$$

22. $f(x)=\sqrt[3]{(x^2-1)^2}=(x^2-1)^{2/3}$

$$f'(x)=\frac{2}{3}(x^2-1)^{-1/3}(2x)$$

$$=\frac{4}{3}x(x^2-1)^{-1/3}$$

$$f''(x)=\frac{4}{3}x\cdot\frac{-1}{3}\left(x^2-1\right)^{-4/3}(2x)$$
$$+\frac{4}{3}\left(x^2-1\right)^{-1/3}(1)$$
$$=\frac{-8}{9}x^2\left(x^2-1\right)^{-4/3}+\frac{4}{3}\left(x^2-1\right)^{-1/3}$$
$$=\frac{-8x^2}{9\left(x^2-1\right)^{4/3}}+\frac{4}{3\left(x^2-1\right)^{1/3}}$$
$$=\frac{-8x^2}{9\left(x^2-1\right)^{4/3}}+\frac{12\left(x^2-1\right)}{9\left(x^2-1\right)^{4/3}}$$
$$=\frac{4x^2-12}{9\left(x^2-1\right)^{4/3}}$$
$$=\frac{4\left(x^2-3\right)}{9\left(x^2-1\right)^{4/3}}$$

23. $y=x^{3/2}-5x$

$$y'=\frac{3}{2}x^{1/2}-5$$
$$y''=\frac{3}{4}x^{-1/2}=\frac{3}{4\sqrt{x}}$$

24. $y=x^{2/3}+4x$

$$y'=\frac{2}{3}x^{2/3-1}+4$$
$$=\frac{2}{3}x^{-1/3}+4 \qquad \text{First Derivative}$$
$$y''=\frac{2}{3}\cdot\frac{-1}{3}x^{-1/3-1}$$
$$=-\frac{2}{9}x^{-4/3}$$
$$=-\frac{2}{9x^{4/3}} \qquad \text{Second Derivative}$$

25. $y=\left(x^3-x\right)^{3/4}$

$$y'=\frac{3}{4}\left(x^3-x\right)^{3/4-1}\left(3x^2-1\right) \qquad \text{Theorem 7}$$
$$=\frac{3}{4}\left(x^3-x\right)^{-1/4}\left(3x^2-1\right) \qquad \text{First Derivative}$$
$$y''=\frac{3}{4}\left(x^3-x\right)^{-1/4}(6x)+ \qquad \text{Theorem 5}$$
$$\frac{3}{4}\left(3x^2-1\right)\cdot\frac{-1}{4}\left(x^3-x\right)^{-1/4-1}\left(3x^2-1\right)$$
$$=\frac{9}{2}x\left(x^3-x\right)^{-1/4}+$$
$$\frac{-3}{16}\left(3x^2-1\right)^2\left(x^3-x\right)^{-5/4}$$
$$=\frac{9x}{2\left(x^3-x\right)^{1/4}}-\frac{3\left(3x^2-1\right)^2}{16\left(x^3-x\right)^{5/4}}$$

The second derivative can be simplified by finding a common denominator and combining the fractions.

$$y''=\frac{9x}{2\left(x^3-x\right)^{1/4}}\cdot\frac{8\left(x^3-x\right)}{8\left(x^3-x\right)}-\frac{3\left(3x^2-1\right)^2}{16\left(x^3-x\right)^{5/4}}$$
$$=\frac{72x^4-72x^2}{16\left(x^3-x\right)^{5/4}}-\frac{3\left(9x^4-6x^2+1\right)}{16\left(x^3-x\right)^{5/4}}$$
$$=\frac{72x^4-72x^2-27x^4+18x^2-3}{16\left(x^3-x\right)^{5/4}}$$
$$=\frac{45x^4-54x^2-3}{16\left(x^3-x\right)^{5/4}} \qquad \text{Second Derivative}$$

26. $y=\left(x^4+x\right)^{2/3}$

$$y' = \frac{2}{3}\left(x^4 + x\right)^{-1/3}\left(4x^3 + 1\right)$$

Differentiating the derivative, we find the second derivative.

$$y'' = \frac{2}{3}\left[\left(x^4 + x\right)^{-1/3}\left(12x^2\right) + \left(4x^3+1\right)\left(-\frac{1}{3}\left(x^4+x\right)^{-4/3}\left(4x^3+1\right)\right)\right]$$

$$= \frac{2}{3}\left[\frac{12x^2}{\left(x^4+x\right)^{1/3}} - \frac{\left(4x^3+1\right)^2}{3\left(x^4+x\right)^{4/3}}\right]$$

$$= \frac{2}{3}\left[\frac{36x^2\left(x^4+x\right)}{3\left(x^4+x\right)^{4/3}} - \frac{16x^6+8x^3+1}{3\left(x^4+x\right)^{4/3}}\right]$$

$$= \frac{2}{3}\left[\frac{20x^6+28x^3-1}{3\left(x^4+x\right)^{4/3}}\right]$$

$$= \frac{40x^6+56x^3-2}{9\left(x^4+x\right)^{4/3}}$$

27. $y = \frac{3x+1}{2x-3}$

$y' = \frac{(2x-3)(3)-(3x+1)(2)}{(2x-3)^2}$ Theorem 6

$= \frac{6x-9-6x-2}{(2x-3)^2}$

$= \frac{-11}{(2x-3)^2}$ First Derivative

Differentiating the derivative, we find the second derivative.

$$y'' = \frac{(2x-3)^2(0)-(-11)\left(2(2x-3)^{2-1}(2)\right)}{\left((2x-3)^2\right)^2}$$

Theorem 6 and Theorem 7

$= \frac{44(2x-3)}{(2x-3)^4}$

$= \frac{44}{(2x-3)^3}$ Second Derivative

28. $y = \frac{2x+3}{5x-1}$

$$y' = \frac{(5x-1)(2)-(2x+3)(5)}{(5x-1)^2}$$

$$= \frac{-17}{(5x-1)^2} = -17(5x-1)^{-2}$$

$$y'' = -17\left(-2(5x-1)^{-3}(5)\right)$$

$$= 170(5x-1)^{-3}$$

$$= \frac{170}{(5x-1)^3}$$

29. $y = x^5$

$$\frac{dy}{dx} = 5x^4$$

$$\frac{d^2y}{dx^2} = 20x^3$$

$$\frac{d^3y}{dx^3} = 60x^2$$

$$\frac{d^4y}{dx^4} = 120x$$

30. $y = x^4$

$\frac{dy}{dx} = 4x^{4-1} = 4x^3$ First Derivative

$\frac{d^2y}{dx^2} = 4\left(3x^{3-1}\right) = 12x^2$ Second Derivative

$\frac{d^3y}{dx^3} = 12\left(2x^{2-1}\right) = 24x$ Third Derivative

$\frac{d^4y}{dx^4} = 24$ Fourth Derivative

31. $y = x^6 - x^3 + 2x$

$$\frac{dy}{dx} = 6x^{6-1} - 3x^{3-1} + 2$$
$$= 6x^5 - 3x^2 + 2 \quad \text{First Derivative}$$
$$\frac{d^2y}{dx^2} = 6\left(5x^{5-1}\right) - 3\left(2x^{2-1}\right)$$
$$= 30x^4 - 6x \quad \text{Second Derivative}$$
$$\frac{d^3y}{dx^3} = 30\left(4x^{4-1}\right) - 6$$
$$= 120x^3 - 6 \quad \text{Third Derivative}$$
$$\frac{d^4y}{dx^4} = 120\left(3x^{3-1}\right)$$
$$= 360x^2 \quad \text{Fourth Derivative}$$
$$\frac{d^5y}{dx^5} = 360\left(2x^{2-1}\right)$$
$$= 720x \quad \text{Fifth Derivative}$$

32. $y = x^7 - 8x^2 + 2$

$$\frac{dy}{dx} = 7x^6 - 16x$$
$$\frac{d^2y}{dx^2} = 42x^5 - 16$$
$$\frac{d^3y}{dx^3} = 210x^4$$
$$\frac{d^4y}{dx^4} = 840x^3$$
$$\frac{d^5y}{dx^5} = 2520x^2$$
$$\frac{d^6y}{dx^6} = 5040x$$

33. $f(x) = x^{-3} + 2x^{1/3}$

$$f'(x) = -3x^{-4} + \frac{2}{3}x^{-2/3}$$
$$f''(x) = 12x^{-5} - \frac{4}{9}x^{-5/3}$$
$$f'''(x) = -60x^{-6} + \frac{20}{27}x^{-8/3}$$
$$f^{(4)}(x) = 360x^{-7} - \frac{160}{81}x^{-11/3}$$
$$f^{(5)}(x) = -2520x^{-8} + \frac{1760}{243}x^{-14/3}$$

34. $f(x) = x^{-2} - x^{1/2}$

$$f'(x) = -2x^{-2-1} - \frac{1}{2}x^{1/2-1}$$
$$= -2x^{-3} - \frac{1}{2}x^{-1/2} \quad \text{First Derivative}$$
$$f''(x) = -2\left(-3x^{-3-1}\right) - \frac{1}{2}\cdot\frac{-1}{2}x^{-1/2-1}$$
$$= 6x^{-4} + \frac{1}{4}x^{-3/2} \quad \text{Second Derivative}$$
$$f'''(x) = 6\left(-4x^{-4-1}\right) + \frac{1}{4}\cdot\frac{-3}{2}x^{-3/2-1}$$
$$= -24x^{-5} - \frac{3}{8}x^{-5/2} \quad \text{Third Derivative}$$
$$f^{(4)}(x) = -24\left(-5x^{-5-1}\right) - \frac{3}{8}\cdot\frac{-5}{2}x^{-5/2-1}$$
$$= 120x^{-6} + \frac{15}{16}x^{-7/2} \quad \text{Fourth Derivative}$$

35. $s(t) = t^3 + t$

a) $v(t) = s'(t) = 3t^2 + 1$

b) $a(t) = v'(t) = s''(t) = 6t$

c) When $t = 4$

$v(4) = 3(4)^2 + 1 = 49$

$a(4) = 6(4) = 24$

After 4 seconds, the velocity is 49 feet per second, and the acceleration is 24 feet per second squared.

36. $s(t) = -10t^2 + 2t + 5$

a) $v(t) = s'(t) = -20t + 2$

b) $a(t) = v'(t) = s''(t) = -20$

c) When $t = 1$,

$v(1) = -20(1) + 2 = -18\,\frac{\text{m}}{\text{sec}}$

$a(1) = -20\,\frac{\text{m}}{\text{sec}^2}$

37. $s(t) = t^2 - \frac{1}{2}t + 3$

a) $v(t) = s'(t) = 2t - \frac{1}{2}$

b) $a(t) = v'(t) = s''(t) = 2$

c) When $t = 1$,

$v(1) = 2(1) - \frac{1}{2} = \frac{3}{2} = 1.5\,\frac{\text{m}}{\text{sec}}$

$a(1) = 2\,\frac{\text{m}}{\text{sec}^2}$

38. $s(t) = 3t + 10$

a) $v(t)=s'(t)=3$

b) $a(t)=v'(t)=s''(t)=0$

c) When $t=2$

$v(2)=3$

$a(2)=0$

After 2 hours, the velocity is 3 miles per hour, and the acceleration is 0 miles per hour squared.

d) ✎ Answers will vary. Uniform motion means that the object is moving with a constant velocity. Since the objects velocity is not changing, the objects acceleration will be zero.

39. $s(t)=16t^2$

a) When $t=3$, $s(3)=16(3)^2=144$.

The hammer falls 144 feet in 3 seconds.

b) $v(t)=s'(t)=32t$

When $t=3$, $v(3)=32(3)=96$

the hammer is falling at 96 feet per second after 3 seconds.

c) $a(t)=v'(t)=s''(t)=32$

When $t=3$, $a(3)=32$

the hammer is accelerating at 32 feet per second squared after 3 seconds.

40. $s(t)=16t^2$

a) When $t=2$, $s(2)=16(2)^2=64$.

The bolt falls 64 feet in 2 seconds.

b) $v(t)=s'(t)=32t$

When $t=2$, $v(2)=32(2)=64$

the bolt is falling at 64 feet per second after 2 seconds.

c) $a(t)=v'(t)=s''(t)=32$

When $t=2$, $a(3)=32$

the bolt is accelerating at 32 feet per second squared after 2 seconds.

41. $s(t)=4.905t^2$

The velocity and acceleration are given by:

$v(t)=s'(t)=9.81t$

$a(t)=v'(t)=s''(t)=9.81$

After 2 seconds, we have

$v(2)=9.81(2)=19.62$

The stone is falling at 19.62 meters per second.

$a(2)=9.81$

The stone is accelerating at 9.81 meters per second squared.

42. $s(t)=4.905t^2$

$v(t)=s'(t)=9.81t$

$a(t)=v'(t)=s''(t)=9.81$

After 3 seconds, we have

$v(3)=9.81(3)=29.43$

The stone is falling at 29.43 meters per second.

$a(3)=9.81$

The stone is accelerating at 9.81 meters per second squared.

43. a) The bicyclist's velocity is the greatest at time $t=0$. The tangent line at $t=0$ has the greatest slope.

b) The bicyclist's acceleration is negative, since the slopes of the tangent line are decreasing with time.

44. a) The plane's velocity is greater at $t=20$ seconds. We know this because the slope of the tangent line is greater at $t=20$ then it is at $t=6$.

b) The plane's acceleration is positive, since the velocity (slope of the tangent lines) is increasing over time.

45. **a)** The open interval where the first derivative equals 0 is (7,11).

b) The open intervals where the second derivative equals 0 are (2,4), (7,11), and (13,15).

c) The open intervals where the second derivative is greater than 0 are (0,2) and (11,13).

d) The open interval where the second derivative is less than 0 is (4,7).

e) Answer will vary.

46. **a)** The open intervals where Jesse's car is accelerating are (0,2) and (8,9).

b) The open intervals where Jesse's car is decelerating are (4,5) and (11,13).

c) The open intervals where Jesse's car is maintaining constant velocity are (2,4), (5,8), (9,11), and (13,18).

47. $S(t)=2t^3-40t^2+220t+160$

a) $S'(t) = 6t^2 - 80t + 220$

When $t = 1$,

$S'(1) = 6(1)^2 - 80(1) + 220 = 146$

After 1 month, sales are increasing at 146 thousand (146,000) dollars per month.

When $t = 2$,

$S'(2) = 6(2)^2 - 80(2) + 220 = 84$

After 2 month, sales are increasing at 84 thousand (84,000) dollars per month.

When $t = 4$,

$S'(4) = 6(4)^2 - 80(4) + 220 = -4$

After 4 months, sales are changing at a rate of -4 thousand (-4000) dollars per month.

b) $S''(t) = 12t - 80$

When $t = 1$, $S''(1) = 12(1) - 80 = -68$

After 1 month, the rate of change of sales are changing at a rate of -68 thousand $(-68,000)$ dollars per month squared.

When $t = 2$,

$S''(2) = 12(2) - 80 = -56$

After 2 months, the rate of change of sales are changing at a rate of -56 thousand $(-56,000)$ dollars per month squared.

When $t = 4$, $S''(4) = 12(4) - 80 = -32$

After 4 months, the rate of change of sales are changing at a rate of -32 thousand $(-32,000)$ dollars per month squared.

c) Answers will vary. The first derivative found in part (a) determined the rate at which sales were changing *t* months after the product was marketed. We saw that for the first 2 months, sales were increasing. However, in the 4th month, sales had started to decrease. The second derivative found in part (b) determined how fast the rate of change was changing. We saw that the rate at which sales were changing was negative. Which means that in the first two months when sales were increasing, they were doing so at a decreasing rate.

48. $N(t) = 2t^3 - 3t^2 + 2t$

a) $N'(t) = 6t^2 - 6t + 2$

When $t = 1$, $N'(1) = 6(1)^2 - 6(1) + 2 = 2$

After 1 day, the number of items sold was increasing by 2 items per day.

When $t = 2$, $N'(2) = 6(2)^2 - 6(2) + 2 = 14$

After 2 days, the number of items sold was increasing by 14 items per day.

When $t = 4$, $N'(4) = 6(4)^2 - 6(4) + 2 = 74$

After 4 days, the number of items sold was increasing by 74 items per day.

b) $N''(t) = 12t - 6$

When $t = 1$, $N''(1) = 12(1) - 6 = 6$

After 1 day, the rate of change of the number of items sold was increasing by 6 items per day squared.

When $t = 2$, $N''(2) = 12(2) - 6 = 18$

After 2 days, the rate of change of the number of items sold was increasing by 18 items per day squared.

When $t = 4$, $N''(4) = 12(4) - 6 = 42$

After 4 days, the rate of change of the number of items sold was increasing by 42 items per day squared.

c) Answers will vary. The first derivative found in part (a) determined the rate at which items were sold t days after the new sales promotion was launched. The second derivative found in part (b) determined the rate at which the rates in part (a) where changing. The information tells us that after the new sales promotion was launched, the number of items sold were increasing at a increasing rate with respect to time.

49. a) $p(t) = \dfrac{2000t}{4t + 75}$

First find the derivative of the population function. Using the quotient rule, we have:

$$p'(t)$$
$$= \frac{d}{dt}\left(\frac{2000t}{4t+75}\right)$$
$$= \frac{(4t+75)\frac{d}{dt}(2000t)-(2000t)\frac{d}{dt}(4t+75)}{(4t+75)^2}$$
$$= \frac{(4t+75)(2000)-(2000t)(4)}{(4t+75)^2}$$
$$= \frac{8000t+150{,}000-8000t}{(4t+75)^2}$$
$$= \frac{150{,}000}{(4t+75)^2}.$$

Next we substitute the appropriate values in for t.

$$p'(10) = \frac{150{,}000}{(4(10)+75)^2} = \frac{150{,}000}{13{,}225} \approx 11.34 .$$

$$p'(50) = \frac{150{,}000}{(4(50)+75)^2} = \frac{150{,}000}{75{,}625} \approx 1.98 .$$

$$p'(100) = \frac{150{,}000}{(4(100)+75)^2} = \frac{150{,}000}{225{,}625} \approx 0.665 .$$

b) First find the second derivative.

$$p''(t) = \frac{d}{dt}\left(\frac{150{,}000}{(4t+75)^2}\right)$$
$$= \frac{d}{dt}\left(150{,}000(4t+75)^{-2}\right)$$
$$= -300{,}000(4t+75)^{-3}\cdot 4$$
$$= -\frac{1{,}200{,}000}{(4t+75)^{-3}}$$

Next we substitute the appropriate values in for t on the next page.

From the previous page, we have:

$$p''(10) = -\frac{1{,}200{,}000}{(4(10)+75)^3} = -\frac{1{,}200{,}000}{1{,}520{,}875} \approx -0.789 .$$

$$p''(50) = -\frac{1{,}200{,}000}{(4(50)+75)^3} = -\frac{1{,}200{,}000}{20{,}796{,}875} \approx -0.0577 .$$

$$p''(100) = -\frac{1{,}200{,}000}{(4(100)+75)^3} = -\frac{1{,}200{,}000}{107{,}171{,}875} \approx -0.0112 .$$

c) The population of deer is increasing but at a decreasing rate. The growth of the deer population will eventually stagnate and find a stable population around 500 deer.

50. a) $p(t) = \frac{2.5t}{t^2+1}$

$$p'(t)$$
$$= \frac{d}{dt}\left(\frac{2.5t}{t^2+1}\right)$$
$$= \frac{(t^2+1)\frac{d}{dt}(2.5t)-(2.5t)\frac{d}{dt}(t^2+1)}{(t^2+1)^2}$$
$$= \frac{(2.5t^2+2.5)-(5t^2)}{(t^2+1)^2}$$
$$= \frac{-2.5t^2+2.5}{(t^2+1)^2}.$$

Next we substitute the appropriate values in for t.

$$p'(0.5) = \frac{-2.5(0.5)^2+2.5}{\left((0.5)^2+1\right)^2} = 1.2 .$$

$$p'(1) = \frac{-2.5(1)^2+2.5}{\left((1)^2+1\right)^2} = 0 .$$

$$p'(5) = \frac{-2.5(5)^2 + 2.5}{\left((5)^2+1\right)^2}$$
$$\approx -0.888 .$$
$$p'(30) = \frac{-2.5(30)^2 + 2.5}{\left((30)^2+1\right)^2}$$
$$\approx -0.0028 .$$

b) First find the second derivative.

$$p''(t) = \frac{d}{dt}\left(\frac{-2.5t^2+2.5}{\left(t^2+1\right)^2}\right)$$
$$= \frac{5t^3 - 15t}{\left(t^2+1\right)^3}$$

Substitute the appropriate values in for *t*.

$$p''(0.5) = \frac{5(0.5)^3 - 15(0.5)}{\left((0.5)^2+1\right)^3} = -3.52 .$$

$$p''(1) = \frac{5(1)^3 - 15(1)}{\left((1)^2+1\right)^3} = -1.25 .$$

$$p''(5) = \frac{5(5)^3 - 15(5)}{\left((5)^2+1\right)^3} \approx 0.031 .$$

$$p''(30) = \frac{5(30)^3 - 15(30)}{\left((30)^2+1\right)^3} \approx 0.00018 .$$

c) ✎The medication initially increases, however it quickly begins to decrease at an increasing rate. The amount of medicine in the bloodstream will eventually be negligible.

51. **a)** To find the height, substitute the value 2 in for *t* in the height function, *y*.

$$y(2) = -4.9(2)^2 + 22.15(2) + 50$$
$$= 74.7$$

To find the horizontal distance, substitute the value 2 in for *t* in the horizontal distance function, *x*.

$$x(2) = 27.25(2) = 54.5$$

Therefore the height will be 74.7 meters and the horizontal distance will be 54.5 meters.

b) The find the velocities, we must first find the derivatives of the given functions:

$$y'(t) = -9.8t + 22.15$$
$$x'(t) = 27.25$$

Now substitute the value 2 into both derivatives:

$$y'(2) = -9.8(2) + 22.15 = 2.55$$
$$x'(2) = 27.25$$

Therefore, the vertical velocity is 2.55 meters per second and the horizontal velocity is 27.25 meters per second.

c) The find the accelerations, we must first find the second derivatives of the given functions:

$$y''(t) = -9.8$$
$$x''(t) = 0$$

Now substitute the value 2 into both derivatives:

$$y''(2) = -9.8$$
$$x''(2) = 0$$

Therefore, the vertical acceleration is –9.8 meters per second squared and the horizontal acceleration is 0 meters per second squared.

d) The rock is moving upward since $v(2) > 0$.

e) The vertical acceleration is the same throughout the duration of the rocks flight because the only force acting on it is gravity.

f) There is no acceleration in the horizontal direction because there is no force affecting the horizontal distance.

52. **a)** To find the height, substitute the value 1 in for *t* in the height function, *y*.

$$y(1) = -4.9(1)^2 + 10$$
$$= 5.1$$

To find the horizontal distance, substitute the value 1 in for *t* in the horizontal distance function, *x*.

$$x(1) = 2(1) = 2$$

Therefore the height will be 5.1 meters and the horizontal distance will be 2 meters.

b) The find the velocities, we must first find the derivatives of the given functions:

$$y'(t) = -9.8t$$
$$x'(t) = 2$$

Now substitute the value 1 into both derivatives:

$$y'(1) = -9.8(1) = -9.8$$
$$x'(1) = 2$$

Therefore, the vertical velocity is –9.8 meters per second and the horizontal velocity is 2 meters per second.

c) The find the accelerations, we must first find the second derivatives of the given functions:
$y''(t) = -9.8$
$x''(t) = 0$
Now substitute the value 1 into both derivatives:
$y''(1) = -9.8$
$x''(1) = 0$
Therefore, the vertical acceleration is –9.8 meters per second squared and the horizontal acceleration is 0 meters per second squared.

d) The initial velocity was zero because the ball was at rest before it started to roll.

53. Graph I

54. Graph IV

55. Graph II

56. Graph III

57. False

58. True

59. False

60. $f(x) = \dfrac{1}{x} = x^{-1}$

$f'(x) = -x^{-2} = -\dfrac{1}{x^2}$

$f''(x) = 2x^{-3} = \dfrac{2}{x^3}$

$f'''(x) = -6x^{-4} = -\dfrac{6}{x^4}$

$f^{(4)}(x) = 24x^{-5} = \dfrac{24}{x^5}$

Signs alternate, and odd-order derivatives are negative. The denominator's power is always one higher than the order of the derivative. The numerator is –1 for the first derivative, $(-1)(-2) = 2$ for the second derivative, $(-1)(-2)(-3) = -6$ for the third derivative, and so on. Thus, the 12th derivative will have $(-1)(-2)\cdots(-12) = 479,001,600$ as the numerator. $f^{(12)}(x) = \dfrac{479,001,600}{x^{13}}$

61. $f(x) = \dfrac{x-1}{x+2}$

$f'(x) = \dfrac{(x+2)(1)-(x-1)(1)}{(x+2)^2}$

$= \dfrac{3}{(x+2)^2}$

Notice, $f'(x) = \dfrac{3}{(x+2)^2} = 3(x+2)^{-2}$

$f''(x) = 3(-2)(x+2)^{-2-1}$

$= -6(x+2)^{-3}$

$= -\dfrac{6}{(x+2)^3}$

Notice, $f''(x) = -\dfrac{6}{(x+2)^3} = -6(x+2)^{-3}$

$f'''(x) = -6(-3)(x+2)^{-3-1}(1)$

$= 18(x+2)^{-4}$

$= \dfrac{18}{(x+2)^4}$

Notice, $f'''(x) = \dfrac{18}{(x+2)^4} = 18(x+2)^{-4}$

$f^{(4)}(x) = 18(-4)(x+2)^{-4-1}(1)$

$= -72(x+2)^{-5}$

$= -\dfrac{72}{(x+2)^5}$

62. $f(x) = \dfrac{x+3}{x-2}$

$f'(x) = \dfrac{(x-2)(1)-(x+3)(1)}{(x-2)^2}$

$= \dfrac{-5}{(x-2)^2} = -5(x-2)^{-2}$

$f''(x) = -5(-2)(x-2)^{-3}(1)$

$= 10(x-2)^{-3}$

$= \dfrac{10}{(x-2)^3}$

$$f'''(x) = 10(-3)(x-2)^{-3-1}(1)$$
$$= -30(x-2)^{-4}$$
$$= -\frac{30}{(x-2)^4}$$
$$f^{(4)}(x) = -30(-4)(x-2)^{-4-1}(1)$$
$$= 120(x-2)^{-5}$$
$$= \frac{120}{(x-2)^5}$$

63. a) $s(2) = 0.81(2)^2 \approx 3.24$

The object has fallen 3.24 meters after 2 seconds.

b) To determine the speed, we find the first derivative of the position function, which is

$s'(t) = 1.62t$.

Therefore,

$s'(2) = 1.62(2) = 3.24$.

The object is traveling at 3.24 meters per second after 2 seconds.

c) To determine the acceleration, we find the second derivative of the position function, which is:

$s''(t) = 1.62$.

The object is accelerating at 1.62 meters per second squared.

d) The second derivative represents the acceleration due to gravity on the moon. It is a constant $1.62\frac{m}{\sec^2}$.

64. Answers will vary. 2 seconds is impossible, the top high jumpers clear about 2 meters in height. So it is possible for a human to have a "hang time" of 1 second, maybe even 1.5 seconds.

65. First we must find out how long it took to fall 28ft. Solving the equation

$$s(t) = 28$$
$$16t^2 = 28$$
$$t^2 = 1.75$$
$$t = \pm 1.3229$$

We find out it took about 1.3229 seconds to fall to the ramp.

Next, we find the velocity function which is the first derivative of the position function. The first derivative is

$s'(t) = 32t$.

Substituting 1.3229 in for t, we have

$s'(1.32) = 32(1.3229) = 42.33$

Danny Way was traveling around 42.33 feet per second when he touched down on the ramp.

66. Answers will vary.

67. **a)** The vacuum cleaner is stationary over the interval $6 < x < 8$.

b) The vacuum cleaner has positive velocity and negative acceleration over the interval $3 < x < 6$.

c) The vacuum cleaner has negative velocity and positive acceleration over the interval $10 < x < 12$.

d) The vacuum cleaner has negative velocity and negative acceleration over the interval $8 < x < 10$.

e) The vacuum cleaner is returning to it's base over the interval $8 < x < 12$.

f) Answers will vary.

68. **a)** The drone is moving upward over the interval $0 < x < 9$.

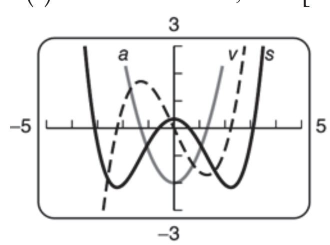
b) The drone's acceleration is positive over the intervals $0 < x < 4$ or $10.5 < x < 12$.

c) The drone's acceleration is zero over the interval $4 < x < 6$.

d) The drone's velocity is negative over the interval $9 < x < 12$.

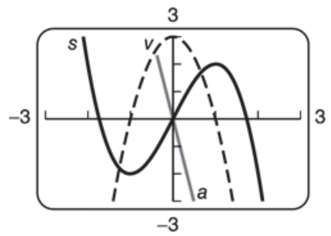
e) Answers will vary.

69. $s(t) = 0.1t^4 - t^2 + 0.4$; $[-5, 5]$



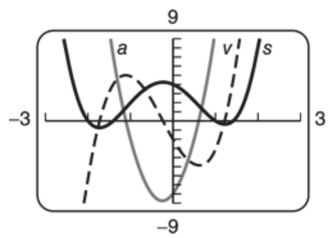
From the graph we see that $v(t)$ switches at $t = -1.29$ and $t = 1.29$.

70. $s(t) = -t^3 + 3t$; $[-3, 3]$



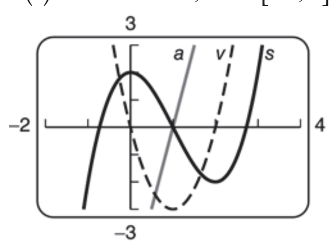
From the graph we see that $v(t)$ switches at $t = 0$.

71. $s(t) = t^4 + t^3 - 4t^2 - 2t + 4;\qquad [-3, 3]$



From the graph we see that $v(t)$ switches at $t = 0.604$ and $t = -1.104$.

72. $s(t) = t^3 - 3t^2 + 2;\qquad [-2, 4]$



From the graph we see that $v(t)$ switches at $t = 1$.

Chapter 1 Test

1. a) Substitute the given values into the function:

$x \to 6^-$	$f(x) = \frac{x^2 - 36}{x - 6}$
5.9	11.9
5.99	11.99
5.999	11.999

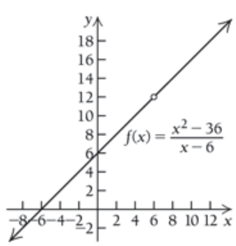
$x \to 6^+$	$f(x) = \frac{x^2 - 36}{x - 6}$
6.1	12.1
6.01	12.01
6.001	12.001

b) $\lim_{x \to 6^-} f(x) = 12$

$\lim_{x \to 6^+} f(x) = 12$

$\lim_{x \to 6} f(x) = 12$

2.



3.

$$\begin{aligned}\lim_{x \to 6} f(x) &= \lim_{x \to 6}\left(\frac{x^2 - 36}{x - 6}\right)\\ &= \lim_{x \to 6}\left(\frac{(x-6)(x+6)}{x - 6}\right)\\ &= \lim_{x \to 6}(x+6)\\ &= 6 + 6\\ &= 12\end{aligned}$$

4. $\lim_{x \to -5^-} f(x) = \infty$

$\lim_{x \to -5^+} f(x) = -\infty$

$\lim_{x \to -5} f(x) =$ does not exist because the left and right limits are not the same.

5. $\lim_{x \to -4^-} f(x) = 0$

$\lim_{x \to -4^+} f(x) = 0$

$\lim_{x \to -4} f(x) = 0$

6. $\lim_{x \to -3^-} f(x) = 1$

$\lim_{x \to -3^+} f(x) = 2$

$\lim_{x \to -3} f(x) =$ does not exist because the left and right limits are not the same.

7. $\lim_{x \to -2^-} f(x) = 2$

$\lim_{x \to -2^+} f(x) = 2$

$\lim_{x \to -2} f(x) = 2$

8. $\lim_{x \to -1^-} f(x) = 4$

$\lim_{x \to -1^+} f(x) = 4$

$\lim_{x \to -1} f(x) = 4$

9. $\lim_{x \to 1^-} f(x) = 1$

$\lim_{x \to 1^+} f(x) = 1$

$\lim_{x \to 1} f(x) = 1$

10. $\lim_{x \to 2^-} f(x) = 1$

$\lim_{x \to 2^+} f(x) = 1$

$\lim_{x \to 2} f(x) = 1$

11. $\lim_{x \to 4^-} f(x) = 2$

$\lim_{x \to 4^+} f(x) = 2$

$\lim_{x \to 4} f(x) = 2$

12. $f'(2) = 0$

13. $f'(-6) = 0$

14. The values for which the function is not continuous are $-5, -3, -2, 1, 4$.

15. The values for which $f'(x)$ is not defined are $-5, -3, -2, -1, 1, 3, 4$.

16. The function is continuous.

17. The function is not continuous because $\lim_{x\to 3} f(x)$ does not exist.

18. **a)** $\lim_{x\to 3^-} f(x) = -4$

$\lim_{x\to 3^+} f(x) = 1$

$\lim_{x\to 3} f(x) =$ does not exist

b) $f(3) = 1$

c) No, because the limit does not exist.

19. $\lim_{x\to 4^-} f(x) = 3$

$\lim_{x\to 4^+} f(x) = 3$

$\lim_{x\to 4} f(x) = 3$

20.
$$\begin{aligned}\lim_{x\to 1}\left(3x^4 - 2x^2 + 5\right) &= 3(1)^4 - 2(1)^2 + 5\\ &= 3 - 2 + 5\\ &= 6\end{aligned}$$

21.
$$\begin{aligned}\lim_{x\to 2^+}\left(\frac{x-2}{x(x^2-4)}\right) &= \lim_{x\to 2^+}\left(\frac{x-2}{x(x-2)(x+2)}\right)\\ &= \lim_{x\to 2^+}\left(\frac{1}{x(x+2)}\right)\\ &= \frac{1}{2(2+2)}\\ &= \frac{1}{8}\end{aligned}$$

22. $\lim_{x\to 0^-}\left(\frac{7}{x}\right) = -\infty$

$\lim_{x\to 0^+}\left(\frac{7}{x}\right) = \infty$

$\lim_{x\to 0}\left(\frac{7}{x}\right) =$ does not exist because the left and right limits are not the same.

23.
$$\begin{aligned}&\frac{f(x+h) - f(x)}{h}\\ &= \frac{2x^2 + 4xh + 2h^2 + 3x + 3h - 9 - 2x^2 - 3x + 9}{h}\\ &= \frac{4xh + 2h^2 + 3h}{h}\\ &= 4x + 2h + 3\end{aligned}$$

24. Find the first derivative of the function to determine the slope of the line:

$$\begin{aligned}y &= x + 4x^{-1}\\ y' &= 1 - 4x^{-2}\\ y' &= 1 - \frac{4}{x^2}\\ y' &= 1 - \frac{4}{16}\\ y' &= \frac{3}{4}\end{aligned}$$

Use the slope found and the given point:

$$\begin{aligned}y - 5 &= \frac{3}{4}(x - 4)\\ y - 5 &= \frac{3}{4}x - 3\\ y &= \frac{3}{4}x + 2\end{aligned}$$

25. The point(s) where the tangent line(s) are horizontal occur when the first derivative has a value of 0.

$$\begin{aligned}y &= x^3 - 3x^2\\ y' &= 3x^2 - 6x\\ 0 &= 3x(x-2)\\ x &= 0, 2\end{aligned}$$

Substitute these domain values to find the corresponding range values.

$y = (0)^3 - 3(0)^2 = 0$

$y = (2)^3 - 3(2)^2 = -4$

Thus, the points are (0, 0) and (2, –4).

26.
$$\begin{aligned}y &= x^{23}\\ \frac{dy}{dx} &= 23x^{22}\end{aligned}$$

27.
$$\begin{aligned}y &= 4\sqrt[3]{x} + 5\sqrt{x}\\ y &= 4x^{1/3} + 5x^{1/2}\\ \frac{dy}{dx} &= \frac{4}{3}x^{-2/3} + \frac{5}{2}x^{-1/2}\end{aligned}$$

28.
$$\begin{aligned}y &= \frac{-10}{x}\\ y &= -10x^{-1}\\ \frac{dy}{dx} &= 10x^{-2} = \frac{10}{x^2}\end{aligned}$$

29. $y = x^{5/4}$

$\frac{dy}{dx} = \frac{5}{4}x^{1/4}$

30. $y = -0.5x^2 + 0.61x + 90$

$\frac{dy}{dx} = -x + 0.61$

31. $y = \frac{1}{3}x^3 - x^2 + 2x + 4$

$\frac{dy}{dx} = x^2 - 2x + 2$

32. $y = \left(3\sqrt{x} + 1\right)\left(x^2 - x\right)$

$y = \left(3x^{1/2} + 1\right)\left(x^2 - x\right)$

$\frac{dy}{dx} = \left(3x^{1/2} + 1\right)(2x - 1) + \left(x^2 - x\right)\left(\frac{3}{2}x^{-1/2}\right)$

$\frac{dy}{dx} = \left(3\sqrt{x} + 1\right)(2x - 1) + \left(x^2 - x\right)\left(\frac{3}{2\sqrt{x}}\right)$

33. $y = \frac{x}{5 - x}$

$\frac{dy}{dx} = \frac{(5 - x)(1) - (x)(-1)}{(5 - x)^2}$

$\frac{dy}{dx} = \frac{5}{(5 - x)^2}$

34. $y = (x + 3)^4 (7 - x)^5$

$\frac{dy}{dx} = (x + 3)^4 5(7 - x)^4 (-1)$

$\quad + (7 - x)^5 4(x + 3)^3 (1)$

$\frac{dy}{dx} = (x + 3)^3 (7 - x)^4 (13 - 9x)$

35. $y = \left(x^5 - 4x^3 + x\right)^{-5}$

$\frac{dy}{dx} = -5\left(x^5 - 4x^3 + x\right)^{-6}\left(5x^4 - 12x^2 + 1\right)$

36. $y = x\sqrt{x^2 + 5}$

$y = x\left(x^2 + 5\right)^{1/2}$

$\frac{dy}{dx} = x\frac{1}{2}\left(x^2 + 5\right)^{-1/2}(2x) + \left(x^2 + 5\right)^{1/2}(1)$

$\frac{dy}{dx} = \left(x^2 + 5\right)^{-1/2}\left(x^2 + x^2 + 5\right)$

$\frac{dy}{dx} = \frac{2x^2 + 5}{\sqrt{x^2 + 5}}$

37. $y = x^4 - 3x^2$

$\frac{dy}{dx} = 4x^3 - 6x$

$\frac{d^2y}{dx^2} = 12x^2 - 6$

$\frac{d^3y}{dx^3} = 24x$

38. **a)** $M(t) = -0.001t^3 + 0.1t^2$

$M'(t) = -0.003t^2 + 0.2t$

b) $M(10) = -0.001(10)^3 + 0.1(10)^2$

$= -0.001(1000) + 0.1(100)$

$= -1 + 10$

$= 9$

c) $M'(10) = -0.003(10)^2 + 0.2(10)$

$= -0.003(100) + 0.2(10)$

$= -0.3 + 2$

$= 1.7$ words/minute

39. **a)** $\bar{C}(x) = \frac{x^{2/3} + 750}{x}$

$\bar{R}(x) = \frac{50x}{x} = 50$

$\bar{P}(x) = 50 - \frac{x^{2/3} + 750}{x}$

b) $\bar{C}(x) = x^{-1/3} + 750x^{-1}$

$\bar{C}'(x) = -\frac{1}{3}x^{-2/3} - 750x^{-2}$

$\bar{C}'(8) = -\frac{1}{3}(8)^{-2/3} - 750(8)^{-2}$

$= -\frac{1}{3 \cdot 16} - \frac{750}{64}$

≈ -11.74 / speaker

40. Using the Chain Rule:

$$f(u)=u^2-u,\ g(x)=u=2x^3$$
$$f'(u)=2u-1$$
$$f'(g(x))=2(2x^3)-1$$
$$g'(x)=6x^2$$
$$(f\circ g)'(x)=f'(g(x))\cdot g'(x)$$
$$=(2(2x^3)-1)\cdot(6x^2)$$
$$=24x^5-6x^2$$

41. Using the Chain Rule:

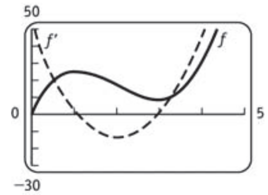
$$g(u)=2u^3,\ f(x)=u=x^2-x$$
$$g'(u)=6u^2$$
$$g'(f(x))=6(x^2-x)^2$$
$$f'(x)=2x-1$$
$$(g\circ f)'(x)=g'(f(x))\cdot f'(x)$$
$$=6(x^2-x)^2\cdot(2x-1)$$

42. Graph A

43.
$$y=\left((1-3x)^{2/3}(1+3x)^{1/3}\right)^{1/2}$$
$$y'=\frac{1}{2}\left((1-3x)^{2/3}(1+3x)^{1/3}\right)^{-1/2}$$
$$\cdot\left(\begin{array}{l}(1-3x)^{2/3}\,\tfrac{1}{3}(1+3x)^{-2/3}(3)\\ \quad+(1+3x)^{1/3}\,\tfrac{2}{3}(1-3x)^{-1/3}(-3)\end{array}\right)$$
$$y'=-\frac{1+9x}{2(1-3x)^{2/3}(1+3x)^{5/6}}$$

44.
$$\lim_{x\to3}\left(\frac{x^3-27}{x-3}\right)=\lim_{x\to3}\left(\frac{(x-3)(x^2+3x+9)}{x-3}\right)$$
$$=\lim_{x\to3}(x^2+3x+9)$$
$$=(3)^2+3(3)+9$$
$$=9+9+9$$
$$=27$$

45.



Using the trace function, the points at which the tangent line to f is horizontal are (1.0835, 25.1029) and (2.9502, 8.6247).

46. By the use of technology, use the table feature to determine $\lim_{x\to0}\left(\frac{\sqrt{5x+25}-5}{x}\right)=0.5$.