

# CHAPTER ONE

## Solutions for Section 1.1

### Exercises

1. Since  $t$  represents the number of years since 1970, we see that  $f(35)$  represents the population of the city in 2005. In 2005, the city's population was 12 million.
2. Since  $T = f(P)$ , we see that  $f(200)$  is the value of  $T$  when  $P = 200$ ; that is, the thickness of pelican eggs when the concentration of PCBs is 200 ppm.
3. If there are no workers, there is no productivity, so the graph goes through the origin. At first, as the number of workers increases, productivity also increases. As a result, the curve goes up initially. At a certain point the curve reaches its highest level, after which it goes downward; in other words, as the number of workers increases beyond that point, productivity decreases. This might, for example, be due either to the inefficiency inherent in large organizations or simply to workers getting in each other's way as too many are crammed on the same line. Many other reasons are possible.
4. The slope is  $(1 - 0)/(1 - 0) = 1$ . So the equation of the line is  $y = x$ .
5. The slope is  $(3 - 2)/(2 - 0) = 1/2$ . So the equation of the line is  $y = (1/2)x + 2$ .
6. Using the points  $(-2, 1)$  and  $(2, 3)$ , we have

$$\text{Slope} = \frac{3 - 1}{2 - (-2)} = \frac{2}{4} = \frac{1}{2}.$$

Now we know that  $y = (1/2)x + b$ . Using the point  $(-2, 1)$ , we have  $1 = -2/2 + b$ , which yields  $b = 2$ . Thus, the equation of the line is  $y = (1/2)x + 2$ .

7. Slope  $= \frac{6 - 0}{2 - (-1)} = 2$  so the equation is  $y - 6 = 2(x - 2)$  or  $y = 2x + 2$ .
8. Rewriting the equation as  $y = -\frac{5}{2}x + 4$  shows that the slope is  $-\frac{5}{2}$  and the vertical intercept is 4.
9. Rewriting the equation as

$$y = -\frac{12}{7}x + \frac{2}{7}$$

shows that the line has slope  $-12/7$  and vertical intercept  $2/7$ .

10. Rewriting the equation of the line as

$$\begin{aligned} -y &= \frac{-2}{4}x - 2 \\ y &= \frac{1}{2}x + 2, \end{aligned}$$

we see the line has slope  $1/2$  and vertical intercept 2.

11. Rewriting the equation of the line as

$$\begin{aligned} y &= \frac{12}{6}x - \frac{4}{6} \\ y &= 2x - \frac{2}{3}, \end{aligned}$$

we see that the line has slope 2 and vertical intercept  $-2/3$ .

12. (a) is (V), because slope is positive, vertical intercept is negative  
 (b) is (IV), because slope is negative, vertical intercept is positive  
 (c) is (I), because slope is 0, vertical intercept is positive  
 (d) is (VI), because slope and vertical intercept are both negative  
 (e) is (II), because slope and vertical intercept are both positive  
 (f) is (III), because slope is positive, vertical intercept is 0

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13. (a) is (V), because slope is negative, vertical intercept is 0  
 (b) is (VI), because slope and vertical intercept are both positive  
 (c) is (I), because slope is negative, vertical intercept is positive  
 (d) is (IV), because slope is positive, vertical intercept is negative  
 (e) is (III), because slope and vertical intercept are both negative  
 (f) is (II), because slope is positive, vertical intercept is 0

14. The intercepts appear to be (0, 3) and (7.5, 0), giving

$$\text{Slope} = \frac{-3}{7.5} = -\frac{6}{15} = -\frac{2}{5}.$$

The  $y$ -intercept is at (0, 3), so a possible equation for the line is

$$y = -\frac{2}{5}x + 3.$$

(Answers may vary.)

15.  $y - c = m(x - a)$

16. Given that the function is linear, choose any two points, for example (5.2, 27.8) and (5.3, 29.2). Then

$$\text{Slope} = \frac{29.2 - 27.8}{5.3 - 5.2} = \frac{1.4}{0.1} = 14.$$

Using the point-slope formula, with the point (5.2, 27.8), we get the equation

$$y - 27.8 = 14(x - 5.2)$$

which is equivalent to

$$y = 14x - 45.$$

17.  $y = 5x - 3$ . Since the slope of this line is 5, we want a line with slope  $-\frac{1}{5}$  passing through the point (2, 1). The equation is  $(y - 1) = -\frac{1}{5}(x - 2)$ , or  $y = -\frac{1}{5}x + \frac{7}{5}$ .

18. The line  $y + 4x = 7$  has slope  $-4$ . Therefore the parallel line has slope  $-4$  and equation  $y - 5 = -4(x - 1)$  or  $y = -4x + 9$ . The perpendicular line has slope  $\frac{-1}{(-4)} = \frac{1}{4}$  and equation  $y - 5 = \frac{1}{4}(x - 1)$  or  $y = 0.25x + 4.75$ .

19. The line parallel to  $y = mx + c$  also has slope  $m$ , so its equation is

$$y = m(x - a) + b.$$

The line perpendicular to  $y = mx + c$  has slope  $-1/m$ , so its equation will be

$$y = -\frac{1}{m}(x - a) + b.$$

20. Since the function goes from  $x = 0$  to  $x = 4$  and between  $y = 0$  and  $y = 2$ , the domain is  $0 \leq x \leq 4$  and the range is  $0 \leq y \leq 2$ .

21. Since  $x$  goes from 1 to 5 and  $y$  goes from 1 to 6, the domain is  $1 \leq x \leq 5$  and the range is  $1 \leq y \leq 6$ .

22. Since the function goes from  $x = -2$  to  $x = 2$  and from  $y = -2$  to  $y = 2$ , the domain is  $-2 \leq x \leq 2$  and the range is  $-2 \leq y \leq 2$ .

23. Since the function goes from  $x = 0$  to  $x = 5$  and between  $y = 0$  and  $y = 4$ , the domain is  $0 \leq x \leq 5$  and the range is  $0 \leq y \leq 4$ .

24. The domain is all numbers. The range is all numbers  $\geq 2$ , since  $x^2 \geq 0$  for all  $x$ .

25. The domain is all  $x$ -values, as the denominator is never zero. The range is  $0 < y \leq \frac{1}{2}$ .

26. The value of  $f(t)$  is real provided  $t^2 - 16 \geq 0$  or  $t^2 \geq 16$ . This occurs when either  $t \geq 4$ , or  $t \leq -4$ . Solving  $f(t) = 3$ , we have

$$\begin{aligned} \sqrt{t^2 - 16} &= 3 \\ t^2 - 16 &= 9 \\ t^2 &= 25 \end{aligned}$$

so

$$t = \pm 5.$$

27. We have  $V = kr^3$ . You may know that  $V = \frac{4}{3}\pi r^3$ .

28. If distance is  $d$ , then  $v = \frac{d}{t}$ .

29. For some constant  $k$ , we have  $S = kh^2$ .

30. We know that  $E$  is proportional to  $v^3$ , so  $E = kv^3$ , for some constant  $k$ .

31. We know that  $N$  is proportional to  $1/t^2$ , so

$$N = \frac{k}{t^2}, \quad \text{for some constant } k.$$

### Problems

32. The year 1983 was 25 years before 2008 so 1983 corresponds to  $t = 25$ . Thus, an expression that represents the statement is:

$$f(25) = 7.019$$

33. The year 2008 was 0 years before 2008 so 2008 corresponds to  $t = 0$ . Thus, an expression that represents the statement is:

$$f(0) \text{ meters.}$$

34. The year 1965 was  $2008 - 1865 = 143$  years before 2008 so 1965 corresponds to  $t = 143$ . Similarly, we see that the year 1911 corresponds to  $t = 97$ . Thus, an expression that represents the statement is:

$$f(143) = f(97)$$

35. Since  $t = 1$  means one year before 2008, then  $t = 1$  corresponds to the year 2007. Similarly,  $t = 0$  corresponds to the year 2008. Thus,  $f(1)$  and  $f(0)$  are the average annual sea level values, in meters, in 2007 and 2008, respectively. Because 1 millimeter is the same as 0.001 meters, an expression that represents the statement is:

$$f(0) = f(1) + 0.001.$$

Note that there are other possible equivalent expressions, such as:  $f(1) - f(0) = 0.001$ .

36. (a) Each date,  $t$ , has a unique daily snowfall,  $S$ , associated with it. So snowfall is a function of date.  
 (b) On December 12, the snowfall was approximately 5 inches.  
 (c) On December 11, the snowfall was above 10 inches.  
 (d) Looking at the graph we see that the largest increase in the snowfall was between December 10 to December 11.
37. (a) When the car is 5 years old, it is worth \$6000.  
 (b) Since the value of the car decreases as the car gets older, this is a decreasing function. A possible graph is in Figure 1.1:

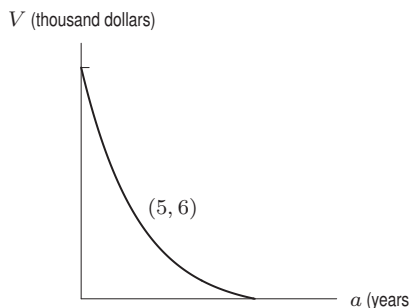


Figure 1.1

- (c) The vertical intercept is the value of  $V$  when  $a = 0$ , or the value of the car when it is new. The horizontal intercept is the value of  $a$  when  $V = 0$ , or the age of the car when it is worth nothing.

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38. (a) The story in (a) matches Graph (IV), in which the person forgot her books and had to return home.  
 (b) The story in (b) matches Graph (II), the flat tire story. Note the long period of time during which the distance from home did not change (the horizontal part).  
 (c) The story in (c) matches Graph (III), in which the person started calmly but sped up later.  
 The first graph (I) does not match any of the given stories. In this picture, the person keeps going away from home, but his speed decreases as time passes. So a story for this might be: *I started walking to school at a good pace, but since I stayed up all night studying calculus, I got more and more tired the farther I walked.*
39. (a)  $f(30) = 10$  means that the value of  $f$  at  $t = 30$  was 10. In other words, the temperature at time  $t = 30$  minutes was  $10^\circ\text{C}$ . So, 30 minutes after the object was placed outside, it had cooled to  $10^\circ\text{C}$ .  
 (b) The intercept  $a$  measures the value of  $f(t)$  when  $t = 0$ . In other words, when the object was initially put outside, it had a temperature of  $a^\circ\text{C}$ . The intercept  $b$  measures the value of  $t$  when  $f(t) = 0$ . In other words, at time  $b$  the object's temperature is  $0^\circ\text{C}$ .
40. (a) The height of the rock decreases as time passes, so the graph falls as you move from left to right. One possibility is shown in Figure 1.2.

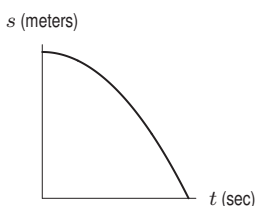


Figure 1.2

- (b) The statement  $f(7) = 12$  tells us that 7 seconds after the rock is dropped, it is 12 meters above the ground.  
 (c) The vertical intercept is the value of  $s$  when  $t = 0$ ; that is, the height from which the rock is dropped. The horizontal intercept is the value of  $t$  when  $s = 0$ ; that is, the time it takes for the rock to hit the ground.
41. (a) We find the slope  $m$  and intercept  $b$  in the linear equation  $C = b + mw$ . To find the slope  $m$ , we use

$$m = \frac{\Delta C}{\Delta w} = \frac{12.32 - 8}{68 - 32} = 0.12 \text{ dollars per gallon.}$$

We substitute to find  $b$ :

$$\begin{aligned} C &= b + mw \\ 8 &= b + (0.12)(32) \\ b &= 4.16 \text{ dollars.} \end{aligned}$$

The linear formula is  $C = 4.16 + 0.12w$ .

- (b) The slope is 0.12 dollars per gallon. Each additional gallon of waste collected costs 12 cents.  
 (c) The intercept is \$4.16. The flat monthly fee to subscribe to the waste collection service is \$4.16. This is the amount charged even if there is no waste.
42. We are looking for a linear function  $y = f(x)$  that, given a time  $x$  in years, gives a value  $y$  in dollars for the value of the refrigerator. We know that when  $x = 0$ , that is, when the refrigerator is new,  $y = 950$ , and when  $x = 7$ , the refrigerator is worthless, so  $y = 0$ . Thus  $(0, 950)$  and  $(7, 0)$  are on the line that we are looking for. The slope is then given by

$$m = \frac{950}{-7}$$

It is negative, indicating that the value decreases as time passes. Having found the slope, we can take the point  $(7, 0)$  and use the point-slope formula:

$$y - y_1 = m(x - x_1).$$

So,

$$\begin{aligned} y - 0 &= -\frac{950}{7}(x - 7) \\ y &= -\frac{950}{7}x + 950. \end{aligned}$$

43. (a) The first company's price for a day's rental with  $m$  miles on it is  $C_1(m) = 40 + 0.15m$ . Its competitor's price for a day's rental with  $m$  miles on it is  $C_2(m) = 50 + 0.10m$ .  
 (b) See Figure 1.3.

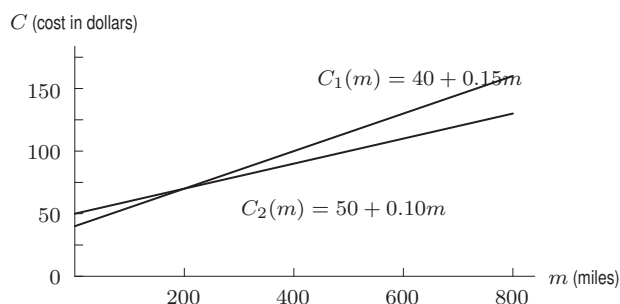


Figure 1.3

- (c) To find which company is cheaper, we need to determine where the two lines intersect. We let  $C_1 = C_2$ , and thus

$$\begin{aligned} 40 + 0.15m &= 50 + 0.10m \\ 0.05m &= 10 \\ m &= 200. \end{aligned}$$

If you are going more than 200 miles a day, the competitor is cheaper. If you are going less than 200 miles a day, the first company is cheaper.

44. (a) Charge per cubic foot =  $\frac{\Delta\$}{\Delta \text{ cu. ft.}} = \frac{55 - 40}{1600 - 1000} = \$0.025/\text{cu. ft.}$   
 Alternatively, if we let  $c = \text{cost}$ ,  $w = \text{cubic feet of water}$ ,  $b = \text{fixed charge}$ , and  $m = \text{cost/cubic feet}$ , we obtain  $c = b + mw$ . Substituting the information given in the problem, we have

$$\begin{aligned} 40 &= b + 1000m \\ 55 &= b + 1600m. \end{aligned}$$

Subtracting the first equation from the second yields  $15 = 600m$ , so  $m = 0.025$ .

- (b) The equation is  $c = b + 0.025w$ , so  $40 = b + 0.025(1000)$ , which yields  $b = 15$ . Thus the equation is  $c = 15 + 0.025w$ .  
 (c) We need to solve the equation  $100 = 15 + 0.025w$ , which yields  $w = 3400$ . It costs \$100 to use 3400 cubic feet of water.
45. See Figure 1.4.

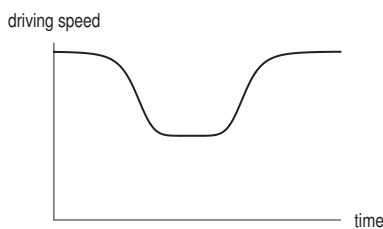


Figure 1.4

46. See Figure 1.5.

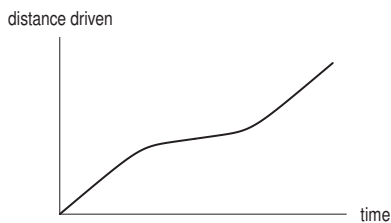


Figure 1.5

47. See Figure 1.6.

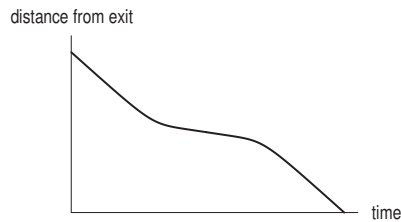


Figure 1.6

48. See Figure 1.7.

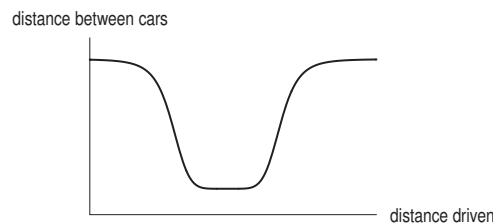


Figure 1.7

49. (a) (i)  $f(1985) = 13$   
(ii)  $f(1990) = 99$   
(b) The average yearly increase is the rate of change.

$$\text{Yearly increase} = \frac{f(1990) - f(1985)}{1990 - 1985} = \frac{99 - 13}{5} = 17.2 \text{ billionaires per year.}$$

- (c) Since we assume the rate of increase remains constant, we use a linear function with slope 17.2 billionaires per year. The equation is

$$f(t) = b + 17.2t$$

where  $f(1985) = 13$ , so

$$13 = b + 17.2(1985)$$

$$b = -34,129.$$

Thus,  $f(t) = 17.2t - 34,129$ .

50. (a) The largest time interval was 2008–2009 since the percentage growth rate increased from  $-11.7$  to  $7.3$  from 2008 to 2009. This means the US consumption of biofuels grew relatively more from 2008 to 2009 than from 2007 to 2008. (Note that the percentage growth rate was a decreasing function of time over 2005–2007.)  
(b) The largest time interval was 2005–2007 since the percentage growth rates were positive for each of these three consecutive years. This means that the amount of biofuels consumed in the US steadily increased during the three year span from 2005 to 2007, then decreased in 2008.
51. (a) The largest time interval was 2005–2007 since the percentage growth rate decreased from  $-1.9$  in 2005 to  $-45.4$  in 2007. This means that from 2005 to 2007 the US consumption of hydroelectric power shrunk relatively more with each successive year.  
(b) The largest time interval was 2004–2007 since the percentage growth rates were negative for each of these four consecutive years. This means that the amount of hydroelectric power consumed by the US industrial sector steadily decreased during the four year span from 2004 to 2007, then increased in 2008.
52. (a) The largest time interval was 2004–2006 since the percentage growth rate increased from  $-5.7$  in 2004 to  $9.7$  in 2006. This means that from 2004 to 2006 the US price per watt of a solar panel grew relatively more with each successive year.  
(b) The largest time interval was 2005–2006 since the percentage growth rates were positive for each of these two consecutive years. This means that the US price per watt of a solar panel steadily increased during the two year span from 2005 to 2006, then decreased in 2007.

53. (a) Since 2008 corresponds to  $t = 0$ , the average annual sea level in Aberdeen in 2008 was 7.094 meters.  
 (b) Looking at the table, we see that the average annual sea level was 7.019 fifty years before 2008, or in the year 1958. Similar reasoning shows that the average sea level was 6.957 meters 125 years before 2008, or in 1883.  
 (c) Because 125 years before 2008 the year was 1883, we see that the sea level value corresponding to the year 1883 is 6.957 (this is the sea level value corresponding to  $t = 125$ ). Similar reasoning yields the table:

Year	1883	1908	1933	1958	1983	2008
$S$	6.957	6.938	6.965	6.992	7.019	7.094

54. (a) We find the slope  $m$  and intercept  $b$  in the linear equation  $S = b + mt$ . To find the slope  $m$ , we use

$$m = \frac{\Delta S}{\Delta t} = \frac{66 - 113}{50 - 0} = -0.94.$$

When  $t = 0$ , we have  $S = 113$ , so the intercept  $b$  is 113. The linear formula is

$$S = 113 - 0.94t.$$

- (b) We use the formula  $S = 113 - 0.94t$ . When  $S = 20$ , we have  $20 = 113 - 0.94t$  and so  $t = 98.9$ . If this linear model were correct, the average male sperm count would drop below the fertility level during the year 2038.
55. (a) This could be a linear function because  $w$  increases by 5 as  $h$  increases by 1.  
 (b) We find the slope  $m$  and the intercept  $b$  in the linear equation  $w = b + mh$ . We first find the slope  $m$  using the first two points in the table. Since we want  $w$  to be a function of  $h$ , we take

$$m = \frac{\Delta w}{\Delta h} = \frac{171 - 166}{69 - 68} = 5.$$

Substituting the first point and the slope  $m = 5$  into the linear equation  $w = b + mh$ , we have  $166 = b + (5)(68)$ , so  $b = -174$ . The linear function is

$$w = 5h - 174.$$

The slope,  $m = 5$ , is in units of pounds per inch.

- (c) We find the slope and intercept in the linear function  $h = b + mw$  using  $m = \Delta h / \Delta w$  to obtain the linear function

$$h = 0.2w + 34.8.$$

Alternatively, we could solve the linear equation found in part (b) for  $h$ . The slope,  $m = 0.2$ , has units inches per pound.

56. We will let

$T$  = amount of fuel for take-off,

$L$  = amount of fuel for landing,

$P$  = amount of fuel per mile in the air,

$m$  = the length of the trip in miles.

Then  $Q$ , the total amount of fuel needed, is given by

$$Q(m) = T + L + Pm.$$

57. (a) The variable costs for  $x$  acres are  $\$200x$ , or  $0.2x$  thousand dollars. The total cost,  $C$  (again in thousands of dollars), of planting  $x$  acres is:

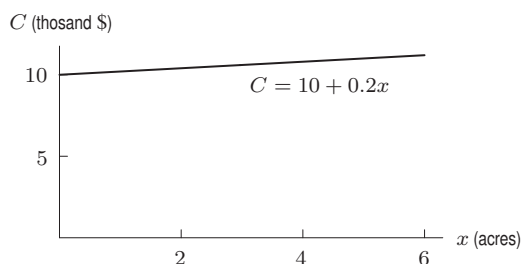
$$C = f(x) = 10 + 0.2x.$$

This is a linear function. See Figure 1.8. Since  $C = f(x)$  increases with  $x$ ,  $f$  is an increasing function of  $x$ . Look at the values of  $C$  shown in the table; you will see that each time  $x$  increases by 1,  $C$  increases by 0.2. Because  $C$  increases at a constant rate as  $x$  increases, the graph of  $C$  against  $x$  is a line.

(b) See Figure 1.8 and Table 1.1.

**Table 1.1**  
Cost of  
planting  
seed

$x$	$C$
0	10
2	10.4
3	10.6
4	10.8
5	11
6	11.2

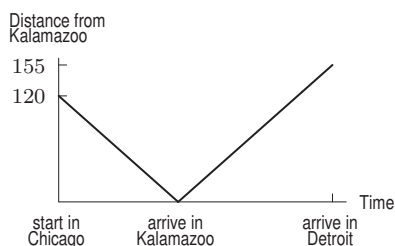


**Figure 1.8**

(c) The vertical intercept of 10 corresponds to the fixed costs. For  $C = f(x) = 10 + 0.2x$ , the intercept on the vertical axis is 10 because  $C = f(0) = 10 + 0.2(0) = 10$ . Since 10 is the value of  $C$  when  $x = 0$ , we recognize it as the initial outlay for equipment, or the fixed cost.

The slope 0.2 corresponds to the variable costs. The slope is telling us that for every additional acre planted, the costs go up by 0.2 thousand dollars. The rate at which the cost is increasing is 0.2 thousand dollars per acre. Thus the variable costs are represented by the slope of the line  $f(x) = 10 + 0.2x$ .

58. See Figure 1.9.



**Figure 1.9**

59. (a) The line given by  $(0, 2)$  and  $(1, 1)$  has slope  $m = \frac{2-1}{-1} = -1$  and  $y$ -intercept 2, so its equation is

$$y = -x + 2.$$

The points of intersection of this line with the parabola  $y = x^2$  are given by

$$x^2 = -x + 2$$

$$x^2 + x - 2 = 0$$

$$(x + 2)(x - 1) = 0.$$

The solution  $x = 1$  corresponds to the point we are already given, so the other solution,  $x = -2$ , gives the  $x$ -coordinate of  $C$ . When we substitute back into either equation to get  $y$ , we get the coordinates for  $C$ ,  $(-2, 4)$ .

(b) The line given by  $(0, b)$  and  $(1, 1)$  has slope  $m = \frac{b-1}{-1} = 1 - b$ , and  $y$ -intercept at  $(0, b)$ , so we can write the equation for the line as we did in part (a):

$$y = (1 - b)x + b.$$

We then solve for the points of intersection with  $y = x^2$  the same way:

$$x^2 = (1 - b)x + b$$

$$x^2 - (1 - b)x - b = 0$$

$$x^2 + (b - 1)x - b = 0$$

$$(x + b)(x - 1) = 0$$

Again, we have the solution at the given point  $(1, 1)$ , and a new solution at  $x = -b$ , corresponding to the other point of intersection  $C$ . Substituting back into either equation, we can find the  $y$ -coordinate for  $C$  is  $b^2$ , and thus  $C$  is given by  $(-b, b^2)$ . This result agrees with the particular case of part (a) where  $b = 2$ .



60. Looking at the given data, it seems that Galileo's hypothesis was incorrect. The first table suggests that velocity is not a linear function of distance, since the increases in velocity for each foot of distance are themselves getting smaller. Moreover, the second table suggests that velocity is instead proportional to *time*, since for each second of time, the velocity increases by 32 ft/sec.

### Strengthen Your Understanding

61. The line  $y = 0.5 - 3x$  has a negative slope and is therefore a decreasing function.
62. If  $y$  is directly proportional to  $x$  we have  $y = kx$ . Adding the constant 1 to give  $y = 2x + 1$  means that  $y$  is not proportional to  $x$ .
63. One possible answer is  $f(x) = 2x + 3$ .
64. One possible answer is  $q = \frac{8}{p^{1/3}}$ .
65. False. A line can be put through any two points in the plane. However, if the line is vertical, it is not the graph of a function.
66. True. Suppose we start at  $x = x_1$  and increase  $x$  by 1 unit to  $x_1 + 1$ . If  $y = b + mx$ , the corresponding values of  $y$  are  $b + mx_1$  and  $b + m(x_1 + 1)$ . Thus  $y$  increases by

$$\Delta y = b + m(x_1 + 1) - (b + mx_1) = m.$$

67. False. For example, let  $y = x + 1$ . Then the points  $(1, 2)$  and  $(2, 3)$  are on the line. However the ratios

$$\frac{2}{1} = 2 \quad \text{and} \quad \frac{3}{2} = 1.5$$

are different. The ratio  $y/x$  is constant for linear functions of the form  $y = mx$ , but not in general. (Other examples are possible.)

68. False. For example, if  $y = 4x + 1$  (so  $m = 4$ ) and  $x = 1$ , then  $y = 5$ . Increasing  $x$  by 2 units gives 3, so  $y = 4(3) + 1 = 13$ . Thus,  $y$  has increased by 8 units, not  $4 + 2 = 6$ . (Other examples are possible.)
69. (b) and (c). For  $g(x) = \sqrt{x}$ , the domain and range are all nonnegative numbers, and for  $h(x) = x^3$ , the domain and range are all real numbers.

## Solutions for Section 1.2

### Exercises

- The graph shows a concave up function.
- The graph shows a concave down function.
- This graph is neither concave up or down.
- The graph is concave up.
- Initial quantity = 5; growth rate =  $0.07 = 7\%$ .
- Initial quantity = 7.7; growth rate =  $-0.08 = -8\%$  (decay).
- Initial quantity = 3.2; growth rate =  $0.03 = 3\%$  (continuous).
- Initial quantity = 15; growth rate =  $-0.06 = -6\%$  (continuous decay).
- Since  $e^{0.25t} = (e^{0.25})^t \approx (1.2840)^t$ , we have  $P = 15(1.2840)^t$ . This is exponential growth since 0.25 is positive. We can also see that this is growth because  $1.2840 > 1$ .
- Since  $e^{-0.5t} = (e^{-0.5})^t \approx (0.6065)^t$ , we have  $P = 2(0.6065)^t$ . This is exponential decay since  $-0.5$  is negative. We can also see that this is decay because  $0.6065 < 1$ .
- $P = P_0(e^{0.2})^t = P_0(1.2214)^t$ . Exponential growth because  $0.2 > 0$  or  $1.2214 > 1$ .
- $P = 7(e^{-\pi})^t = 7(0.0432)^t$ . Exponential decay because  $-\pi < 0$  or  $0.0432 < 1$ .

13. (a) Let  $Q = Q_0a^t$ . Then  $Q_0a^5 = 75.94$  and  $Q_0a^7 = 170.86$ . So

$$\frac{Q_0a^7}{Q_0a^5} = \frac{170.86}{75.94} = 2.25 = a^2.$$

So  $a = 1.5$ .

- (b) Since  $a = 1.5$ , the growth rate is  $r = 0.5 = 50\%$ .

14. (a) Let  $Q = Q_0a^t$ . Then  $Q_0a^{0.02} = 25.02$  and  $Q_0a^{0.05} = 25.06$ . So

$$\frac{Q_0a^{0.05}}{Q_0a^{0.02}} = \frac{25.06}{25.02} = 1.001 = a^{0.03}.$$

So

$$a = (1.001)^{\frac{100}{3}} = 1.05.$$

- (b) Since  $a = 1.05$ , the growth rate is  $r = 0.05 = 5\%$ .

15. (a) The function is linear with initial population of 1000 and slope of 50, so  $P = 1000 + 50t$ .

- (b) This function is exponential with initial population of 1000 and growth rate of 5%, so  $P = 1000(1.05)^t$ .

16. (a) This is a linear function with slope  $-2$  grams per day and intercept 30 grams. The function is  $Q = 30 - 2t$ , and the graph is shown in Figure 1.10.

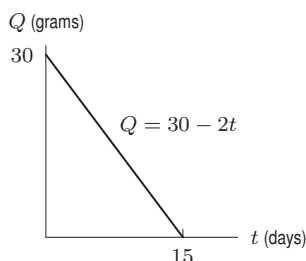


Figure 1.10

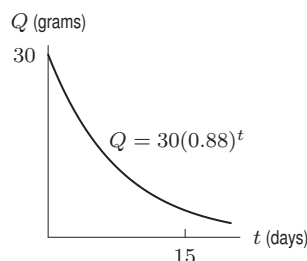


Figure 1.11

- (b) Since the quantity is decreasing by a constant percent change, this is an exponential function with base  $1 - 0.12 = 0.88$ . The function is  $Q = 30(0.88)^t$ , and the graph is shown in Figure 1.11.

17. The function is increasing and concave up between  $D$  and  $E$ , and between  $H$  and  $I$ . It is increasing and concave down between  $A$  and  $B$ , and between  $E$  and  $F$ . It is decreasing and concave up between  $C$  and  $D$ , and between  $G$  and  $H$ . Finally, it is decreasing and concave down between  $B$  and  $C$ , and between  $F$  and  $G$ .

18. (a) It was decreasing from March 2 to March 5 and increasing from March 5 to March 9.

- (b) From March 5 to 8, the average temperature increased, but the rate of increase went down, from  $12^\circ$  between March 5 and 6 to  $4^\circ$  between March 6 and 7 to  $2^\circ$  between March 7 and 8.

From March 7 to 9, the average temperature increased, and the rate of increase went up, from  $2^\circ$  between March 7 and 8 to  $9^\circ$  between March 8 and 9.

## Problems

19. (a) A linear function must change by exactly the same amount whenever  $x$  changes by some fixed quantity. While  $h(x)$  decreases by 3 whenever  $x$  increases by 1,  $f(x)$  and  $g(x)$  fail this test, since both change by different amounts between  $x = -2$  and  $x = -1$  and between  $x = -1$  and  $x = 0$ . So the only possible linear function is  $h(x)$ , so it will be given by a formula of the type:  $h(x) = mx + b$ . As noted,  $m = -3$ . Since the  $y$ -intercept of  $h$  is 31, the formula for  $h(x)$  is  $h(x) = 31 - 3x$ .
- (b) An exponential function must grow by exactly the same factor whenever  $x$  changes by some fixed quantity. Here,  $g(x)$  increases by a factor of 1.5 whenever  $x$  increases by 1. Since the  $y$ -intercept of  $g(x)$  is 36,  $g(x)$  has the formula  $g(x) = 36(1.5)^x$ . The other two functions are not exponential;  $h(x)$  is not because it is a linear function, and  $f(x)$  is not because it both increases and decreases.
20. Table A and Table B could represent linear functions of  $x$ . Table A could represent the constant linear function  $y = 2.2$  because all  $y$  values are the same. Table B could represent a linear function of  $x$  with slope equal to  $11/4$ . This is because  $x$  values that differ by 4 have corresponding  $y$  values that differ by 11, and  $x$  values that differ by 8 have corresponding  $y$  values that differ by 22. In Table C,  $y$  decreases and then increases as  $x$  increases, so the table cannot represent a linear function. Table D does not show a constant rate of change, so it cannot represent a linear function.