CHAPTER 2
Differentiation

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CHAPTER 2
Differentiation

Section 2.1 The Derivative and the Tangent Line Problem

1. The problem of finding the tangent line at a point P is essentially finding the slope of the tangent line at point P. To do so for a function f, if f is defined on an open interval containing c, and if the limit

$$\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} \frac{f(c + \Delta x) - f(c)}{\Delta x} = m$$

exists, then the line passing through the point $P(c, f(c))$ with slope m is the tangent line to the graph of f at the point P.

2. Some alternative notations for $f'(x)$ are

$\frac{dy}{dx}, y', \frac{d}{dx}[f(x)]$, and $D_x[y]$.

3. The limit used to define the slope of a tangent line is also used to define differentiation. The key is to rewrite the difference quotient so that Δx does not occur as a factor of the denominator.

4. If a function f is differentiable at a point $x = c$, then f is continuous at $x = c$. The converse is not true. That is, a function could be continuous at a point, but not differentiable there. For example, the function $y = |x|$ is continuous at $x = 0$, but is not differentiable there.

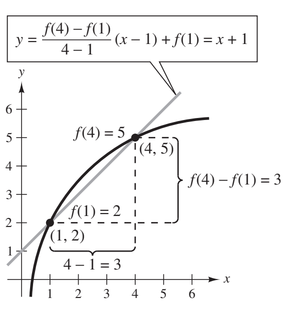
5. At (x_1, y_1), slope $= 0$.

At (x_2, y_2), slope $= \frac{5}{2}$.

6. At (x_1, y_1), slope $= \frac{2}{3}$.

At (x_2, y_2), slope $= -\frac{2}{5}$.

7. (a)–(c)



(d) $$y = \frac{f(4) - f(1)}{4 - 1}(x - 1) + f(1)$$
$$= \frac{3}{3}(x - 1) + 2$$
$$= 1(x - 1) + 2$$
$$= x + 1$$

8. (a) $$\frac{f(4) - f(1)}{4 - 1} = \frac{5 - 2}{3} = 1$$
$$\frac{f(4) - f(3)}{4 - 3} \approx \frac{5 - 4.75}{1} = 0.25$$
So, $\frac{f(4) - f(1)}{4 - 1} > \frac{f(4) - f(3)}{4 - 3}$.

(b) The slope of the tangent line at $(1, 2)$ equals $f'(1)$. This slope is steeper than the slope of the line through $(1, 2)$ and $(4, 5)$. So, $\frac{f(4) - f(1)}{4 - 1} < f'(1)$.

9. $f(x) = 3 - 5x$ is a line. Slope $= -5$

10. $g(x) = \frac{3}{2}x + 1$ is a line. Slope $= \frac{3}{2}$

11. Slope at $(2, 5)$
$$= \lim_{\Delta x \to 0} \frac{f(2 + \Delta x) - f(2)}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{2(2 + \Delta x)^2 - 3 - \left[2(2)^2 - 3\right]}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{2\left[4 + 4\Delta x + (\Delta x)^2\right] - 3 - (5)}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{8\Delta x + 2(\Delta x)^2}{\Delta x}$$
$$= \lim_{\Delta x \to 0} (8 + 2\Delta x) = 8$$

12. Slope at $(3, -4)$
$$= \lim_{\Delta x \to 0} \frac{f(3 + \Delta x) - f(3)}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{5 - (3 + \Delta x)^2 - (-4)}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{5 - 9 - 6(\Delta x) - (\Delta x)^2 + 4}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{-6(\Delta x) - (\Delta x)^2}{\Delta x}$$
$$= \lim_{\Delta x \to 0} (-6 - \Delta x) = -6$$

13. Slope at $(0, 0)$
$$= \lim_{\Delta t \to 0} \frac{f(0 + \Delta t) - f(0)}{\Delta t}$$
$$= \lim_{\Delta t \to 0} \frac{3(\Delta t) - (\Delta t)^2 - 0}{\Delta t}$$
$$= \lim_{\Delta t \to 0} (3 - \Delta t) = 3$$

14. Slope at $(1, 5)$ $= \lim_{\Delta t \to 0} \frac{h(1 + \Delta t) - h(1)}{\Delta t}$

$$= \lim_{\Delta t \to 0} \frac{(1 + \Delta t)^2 + 4(1 + \Delta t) - 5}{\Delta t}$$

$$= \lim_{\Delta t \to 0} \frac{1 + 2(\Delta t) + (\Delta t)^2 + 4 + 4(\Delta t) - 5}{\Delta t}$$

$$= \lim_{\Delta t \to 0} \frac{6(\Delta t) + (\Delta t)^2}{\Delta t}$$

$$= \lim_{\Delta t \to 0} (6 + \Delta t) = 6$$

15. $f(x) = 7$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{7 - 7}{\Delta x}$$

$$= \lim_{\Delta x \to 0} 0 = 0$$

16. $g(x) = -3$

$$g'(x) = \lim_{\Delta x \to 0} \frac{g(x + \Delta x) - g(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{-3 - (-3)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{0}{\Delta x} = 0$$

17. $f(x) = -5x$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{-5(x + \Delta x) - (-5x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{-5x - 5\Delta x + 5x}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{-5\Delta x}{\Delta x}$$

$$= \lim_{\Delta x \to 0} (-5) = -5$$

18. $f(x) = 7x - 3$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{7(x + \Delta x) - 3 - (7x - 3)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{7x + 7\Delta x - 3 - 7x + 3}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{7(\Delta x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} 7 = 7$$

19. $h(s) = 3 + \frac{2}{3}s$

$$h'(s) = \lim_{\Delta s \to 0} \frac{h(s + \Delta s) - h(s)}{\Delta s}$$

$$= \lim_{\Delta s \to 0} \frac{3 + \frac{2}{3}(s + \Delta s) - \left(3 + \frac{2}{3}s\right)}{\Delta s}$$

$$= \lim_{\Delta s \to 0} \frac{3 + \frac{2}{3}s + \frac{2}{3}\Delta s - 3 - \frac{2}{3}s}{\Delta s}$$

$$= \lim_{\Delta s \to 0} \frac{\frac{2}{3}\Delta s}{\Delta s} = \frac{2}{3}$$

20. $f(x) = 5 - \frac{2}{3}x$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{5 - \frac{2}{3}(x + \Delta x) - \left(5 - \frac{2}{3}x\right)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{5 - \frac{2}{3}x - \frac{2}{3}\Delta x - 5 + \frac{2}{3}x}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{-\frac{2}{3}(\Delta x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \left(-\frac{2}{3}\right) = -\frac{2}{3}$$

21. $f(x) = x^2 + x - 3$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{(x + \Delta x)^2 + (x + \Delta x) - 3 - (x^2 + x - 3)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{x^2 + 2x(\Delta x) + (\Delta x)^2 + x + \Delta x - 3 - x^2 - x + 3}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{2x(\Delta x) + (\Delta x)^2 + \Delta x}{\Delta x}$$

$$= \lim_{\Delta x \to 0} (2x + \Delta x + 1) = 2x + 1$$

22. $f(x) = x^2 - 5$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{(x + \Delta x)^2 - 5 - (x^2 - 5)}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{x^2 + 2x(\Delta x) + (\Delta x)^2 - 5 - x^2 + 5}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{2x(\Delta x) + (\Delta x)^2}{\Delta x}$$
$$= \lim_{\Delta x \to 0} (2x + \Delta x) = 2x$$

23. $f(x) = x^3 - 12x$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{\left[(x + \Delta x)^3 - 12(x + \Delta x)\right] - \left[x^3 - 12x\right]}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{x^3 + 3x^2\Delta x + 3x(\Delta x)^2 + (\Delta x)^3 - 12x - 12\,\Delta x - x^3 + 12x}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{3x^2\Delta x + 3x(\Delta x)^2 + (\Delta x)^3 - 12\,\Delta x}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \left(3x^2 + 3x\,\Delta x + (\Delta x)^2 - 12\right) = 3x^2 - 12$$

24. $g(t) = t^3 + 4t$

$$g'(t) = \lim_{\Delta t \to 0} \frac{g(t + \Delta t) - g(t)}{\Delta t}$$
$$= \lim_{\Delta t \to 0} \frac{\left[(t + \Delta t)^3 + 4(t + \Delta t)\right] - \left[t^3 + 4t\right]}{\Delta t}$$
$$= \lim_{\Delta t \to 0} \frac{t^3 + 3t^3\Delta t + 3t(\Delta t)^2 + (\Delta t)^3 + 4t + 4\Delta t - t^3 - 4t}{\Delta t}$$
$$= \lim_{\Delta t \to 0} \frac{3t^2\Delta t + 3t(\Delta t)^2 + (\Delta t)^3 + 4\Delta t}{\Delta t}$$
$$= \lim_{\Delta t \to 0} \left(3t^2 + 3t(\Delta t) + (\Delta t)^2 + 4\right)$$
$$= 3t^2 + 4$$

25. $f(x) = \dfrac{1}{x-1}$

$$\begin{aligned} f'(x) &= \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} \\ &= \lim_{\Delta x \to 0} \frac{\dfrac{1}{x+\Delta x-1} - \dfrac{1}{x-1}}{\Delta x} \\ &= \lim_{\Delta x \to 0} \frac{(x-1) - (x+\Delta x-1)}{\Delta x(x+\Delta x-1)(x-1)} \\ &= \lim_{\Delta x \to 0} \frac{-\Delta x}{\Delta x(x+\Delta x-1)(x-1)} \\ &= \lim_{\Delta x \to 0} \frac{-1}{(x+\Delta x-1)(x-1)} \\ &= -\frac{1}{(x-1)^2} \end{aligned}$$

26. $f(x) = \dfrac{1}{x^2}$

$$\begin{aligned} f'(x) &= \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} \\ &= \lim_{\Delta x \to 0} \frac{\dfrac{1}{(x+\Delta x)^2} - \dfrac{1}{x^2}}{\Delta x} \\ &= \lim_{\Delta x \to 0} \frac{x^2 - (x+\Delta x)^2}{\Delta x(x+\Delta x)^2 x^2} \\ &= \lim_{\Delta x \to 0} \frac{-2x\,\Delta x - (\Delta x)^2}{\Delta x(x+\Delta x)^2 x^2} \\ &= \lim_{\Delta x \to 0} \frac{-2x - \Delta x}{(x+\Delta x)^2 x^2} \\ &= \frac{-2x}{x^4} \\ &= -\frac{2}{x^3} \end{aligned}$$

27. $f(x) = \sqrt{x+4}$

$$\begin{aligned} f'(x) &= \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} \\ &= \lim_{\Delta x \to 0} \frac{\sqrt{x+\Delta x+4} - \sqrt{x+4}}{\Delta x} \cdot \left(\frac{\sqrt{x+\Delta x+4} + \sqrt{x+4}}{\sqrt{x+\Delta x+4} + \sqrt{x+4}}\right) \\ &= \lim_{\Delta x \to 0} \frac{(x+\Delta x+4) - (x+4)}{\Delta x\left[\sqrt{x+\Delta x+4} + \sqrt{x+4}\right]} \\ &= \lim_{\Delta x \to 0} \frac{1}{\sqrt{x+\Delta x+4} + \sqrt{x+4}} = \frac{1}{\sqrt{x+4} + \sqrt{x+4}} = \frac{1}{2\sqrt{x+4}} \end{aligned}$$

28. $h(s) = -2\sqrt{s}$

$$\begin{aligned} h'(s) &= \lim_{\Delta s \to 0} \frac{h(s+\Delta s) - h(s)}{\Delta s} \\ &= \lim_{\Delta s \to 0} \frac{-2\sqrt{s+\Delta s} - \left(-2\sqrt{s}\right)}{\Delta s} \\ &= \lim_{\Delta s \to 0} \frac{-2\left(\sqrt{s+\Delta s} - \sqrt{s}\right)}{\Delta s} \cdot \frac{\sqrt{s+\Delta s} + \sqrt{s}}{\sqrt{s+\Delta s} + \sqrt{s}} \\ &= \lim_{\Delta s \to 0} \frac{-2(s+\Delta s-s)}{\Delta s\left(\sqrt{s+\Delta s} + \sqrt{s}\right)} \\ &= \lim_{\Delta s \to 0} \frac{-2}{\sqrt{s+\Delta s} + \sqrt{s}} \\ &= -\frac{2}{2\sqrt{s}} = -\frac{1}{\sqrt{s}} \end{aligned}$$

29. (a) $f(x) = x^2 + 3$

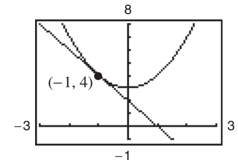
$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{\left[(x + \Delta x)^2 + 3\right] - (x^2 + 3)}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{x^2 + 2x\Delta x + (\Delta x)^2 + 3 - x^2 - 3}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{2x\Delta x + (\Delta x)^2}{\Delta x}$$
$$= \lim_{\Delta x \to 0} (2x + \Delta x) = 2x$$

At $(-1, 4)$, the slope of the tangent line is $m = 2(-1) = -2$.

The equation of the tangent line is

$$y - 4 = -2(x + 1)$$
$$y - 4 = -2x - 2$$
$$y = -2x + 2$$

(b)



(c) Graphing utility confirms $\frac{dy}{dx} = -2$ at $(-1, 4)$.

30. (a) $f(x) = x^2 + 2x - 1$

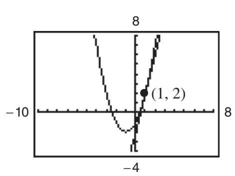
$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{\left[(x + \Delta x)^2 + 2(x + \Delta x) - 1\right] - \left[x^2 + 2x - 1\right]}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{\left[x^2 + 2x\Delta x + (\Delta x)^2 + 2x + 2\Delta x - 1\right] - \left[x^2 + 2x - 1\right]}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{2x\Delta x + (\Delta x)^2 + 2\Delta x}{\Delta x}$$
$$= \lim_{\Delta x \to 0} (2x + \Delta x + 2) = 2x + 2$$

At $(1, 2)$, the slope of the tangent line is $m = 2(1) + 2 = 4$.

The equation of the tangent line is

$$y - 2 = 4(x - 1)$$
$$y - 2 = 4x - 4$$
$$y = 4x - 2.$$

(b)



(c) Graphing utility confirms $\frac{dy}{dx} = 4$ at $(1, 2)$.

31. (a) $f(x) = x^3$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} = \lim_{\Delta x \to 0} \frac{(x + \Delta x)^3 - x^3}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{x^3 + 3x^2\Delta x + 3x(\Delta x)^2 + (\Delta x)^3}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{3x^2\Delta x + 3x(\Delta x)^2 + (\Delta x)^3}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \left(3x^2 + 3x\,\Delta x + (\Delta x)^2\right) = 3x^2$$

At $(2, 8)$, the slope of the tangent is $m = 3(2)^2 = 12$.

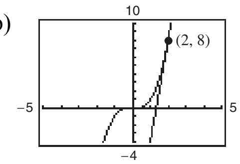
The equation of the tangent line is

$y - 8 = 12(x - 2)$

$y - 8 = 12x - 24$

$y = 12x - 16.$

(b)



(c) Graphing utility confirms $\frac{dy}{dx} = 12$ at $(2, 8)$.

32. (a) $f(x) = x^3 + 1$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\left[(x + \Delta x)^3 + 1\right] - (x^3 + 1)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{x^3 + 3x^2(\Delta x) + 3x(\Delta x)^2 + (\Delta x)^3 + 1 - x^3 - 1}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \left[3x^2 + 3x(\Delta x) + (\Delta x)^2\right] = 3x^2$$

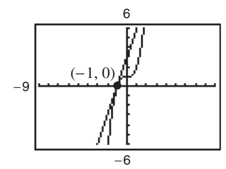
At $(-1, 0)$, the slope of the tangent line is $m = 3(-1)^2 = 3$.

The equation of the tangent line is

$y - 0 = 3(x + 1)$

$y = 3x + 3.$

(b)



(c) Graphing utility confirms $\frac{dy}{dx} = 3$ at $(-1, 0)$.

33. (a) $f(x) = \sqrt{x}$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\sqrt{x + \Delta x} - \sqrt{x}}{\Delta x} \cdot \frac{\sqrt{x + \Delta x} + \sqrt{x}}{\sqrt{x + \Delta x} + \sqrt{x}}$$

$$= \lim_{\Delta x \to 0} \frac{(x + \Delta x) - x}{\Delta x\left(\sqrt{x + \Delta x} + \sqrt{x}\right)}$$

$$= \lim_{\Delta x \to 0} \frac{1}{\sqrt{x + \Delta x} + \sqrt{x}} = \frac{1}{2\sqrt{x}}$$

At $(1, 1)$, the slope of the tangent line is $m = \frac{1}{2\sqrt{1}} = \frac{1}{2}$.

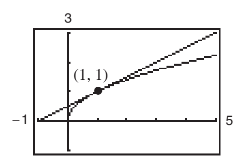
The equation of the tangent line is

$y - 1 = \frac{1}{2}(x - 1)$

$y - 1 = \frac{1}{2}x - \frac{1}{2}$

$y = \frac{1}{2}x + \frac{1}{2}.$

(b)



(c) Graphing utility confirms $\frac{dy}{dx} = \frac{1}{2}$ at $(1, 1)$.

34. (a) $f(x) = \sqrt{x-1}$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\sqrt{x + \Delta x - 1} - \sqrt{x-1}}{\Delta x} \cdot \left(\frac{\sqrt{x + \Delta x - 1} + \sqrt{x-1}}{\sqrt{x + \Delta x - 1} + \sqrt{x-1}}\right)$$

$$= \lim_{\Delta x \to 0} \frac{(x + \Delta x - 1) - (x - 1)}{\Delta x\left(\sqrt{x + \Delta x - 1} + \sqrt{x-1}\right)}$$

$$= \lim_{\Delta x \to 0} \frac{1}{\sqrt{x + \Delta x - 1} + \sqrt{x-1}} = \frac{1}{2\sqrt{x-1}}$$

At $(5, 2)$, the slope of the tangent line is $m = \dfrac{1}{2\sqrt{5-1}} = \dfrac{1}{4}.$

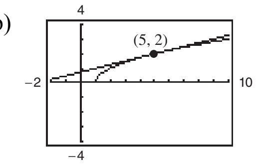
The equation of the tangent line is

$$y - 2 = \frac{1}{4}(x - 5)$$

$$y - 2 = \frac{1}{4}x - \frac{5}{4}$$

$$y = \frac{1}{4}x + \frac{3}{4}.$$

(b)



(c) Graphing utility confirms $\dfrac{dy}{dx} = \dfrac{1}{4}$ at $(5, 2)$.

35. (a) $f(x) = x + \dfrac{4}{x}$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{(x + \Delta x) + \dfrac{4}{x + \Delta x} - \left(x + \dfrac{4}{x}\right)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{x(x + \Delta x)(x + \Delta x) + 4x - x^2(x + \Delta x) - 4(x + \Delta x)}{x(\Delta x)(x + \Delta x)}$$

$$= \lim_{\Delta x \to 0} \frac{x^3 + 2x^2(\Delta x) + x(\Delta x)^2 - x^3 - x^2(\Delta x) - 4(\Delta x)}{x(\Delta x)(x + \Delta x)}$$

$$= \lim_{\Delta x \to 0} \frac{x^2(\Delta x) + x(\Delta x)^2 - 4(\Delta x)}{x(\Delta x)(x + \Delta x)}$$

$$= \lim_{\Delta x \to 0} \frac{x^2 + x(\Delta x) - 4}{x(x + \Delta x)}$$

$$= \frac{x^2 - 4}{x^2} = 1 - \frac{4}{x^2}$$

At $(-4, -5)$, the slope of the tangent line is $m = 1 - \dfrac{4}{(-4)^2} = \dfrac{3}{4}.$

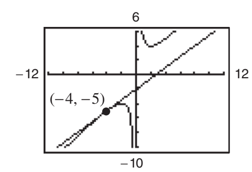
The equation of the tangent line is

$$y + 5 = \frac{3}{4}(x + 4)$$

$$y + 5 = \frac{3}{4}x + 3$$

$$y = \frac{3}{4}x - 2.$$

(b)



(c) Graphing utility confirms $\dfrac{dy}{dx} = \dfrac{3}{4}$ at $(-4, -5)$.

36. (a) $f(x) = x - \dfrac{1}{x}$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\left(x + \Delta x - \dfrac{1}{x + \Delta x}\right) - \left(x - \dfrac{1}{x}\right)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{(x + \Delta x)(x + \Delta x)x - x - x^2(x + \Delta x) + (x + \Delta x)}{\Delta x(x + \Delta x)x}$$

$$= \lim_{\Delta x \to 0} \frac{x^3 + 2x^2(\Delta x) + x(\Delta x)^2 - x - x^3 - x^2(\Delta x) + x + \Delta x}{\Delta x(x + \Delta x)x}$$

$$= \lim_{\Delta x \to 0} \frac{2x^2(\Delta x) + x(\Delta x)^2 - x^2(\Delta x) + \Delta x}{\Delta x(x + \Delta x)x}$$

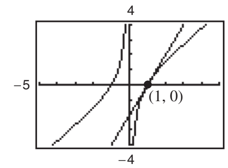
$$= \lim_{\Delta x \to 0} \frac{2x^2 + x(\Delta x) - x^2 + 1}{(x + \Delta x)x}$$

$$= \frac{x^2 + 1}{x^2} = 1 + \frac{1}{x^2}$$

At $(1, 0)$, the slope of the tangent line is $m = f'(1) = 2$. The equation of the tangent line is

$$y - 0 = 2(x - 1)$$
$$y = 2x - 2.$$

(b)



(c) Graphing utility confirms $\dfrac{dy}{dx} = 2$ at $(1, 0)$.

37. Using the limit definition of a derivative, $f'(x) = -\dfrac{1}{2}x$.

Because the slope of the given line is -1, you have

$$-\frac{1}{2}x = -1$$
$$x = 2.$$

At the point $(2, -1)$, the tangent line is parallel to $x + y = 0$. The equation of this line is

$$y - (-1) = -1(x - 2)$$
$$y = -x + 1.$$

38. Using the limit definition of derivative, $f'(x) = 4x$.

Because the slope of the given line is –4, you have

$$4x = -4$$
$$x = -1.$$

At the point $(-1, 2)$ the tangent line is parallel to $4x + y + 3 = 0$. The equation of this line is

$$y - 2 = -4(x + 1)$$
$$y = -4x - 2.$$

39. From Exercise 31 we know that $f'(x) = 3x^2$.

Because the slope of the given line is 3, you have

$$3x^2 = 3$$
$$x = \pm 1.$$

Therefore, at the points $(1, 1)$ and $(-1, -1)$ the tangent lines are parallel to $3x - y + 1 = 0$.

These lines have equations

$$y - 1 = 3(x - 1) \quad \text{and} \quad y + 1 = 3(x + 1)$$
$$y = 3x - 2 \qquad\qquad y = 3x + 2.$$

40. Using the limit definition of derivative, $f'(x) = 3x^2$.

Because the slope of the given line is 3, you have

$$3x^2 = 3$$
$$x^2 = 1 \Rightarrow x = \pm 1.$$

Therefore, at the points $(1, 3)$ and $(-1, 1)$ the tangent lines are parallel to $3x - y - 4 = 0$. These lines have equations

$$y - 3 = 3(x - 1) \quad \text{and} \quad y - 1 = 3(x + 1)$$
$$y = 3x \qquad\qquad y = 3x + 4.$$

41. Using the limit definition of derivative,

$$f'(x) = \frac{-1}{2x\sqrt{x}}.$$

Because the slope of the given line is $-\frac{1}{2}$, you have

$$-\frac{1}{2x\sqrt{x}} = -\frac{1}{2}$$
$$x = 1.$$

Therefore, at the point $(1, 1)$ the tangent line is parallel to $x + 2y - 6 = 0$. The equation of this line is

$$y - 1 = -\frac{1}{2}(x - 1)$$
$$y - 1 = -\frac{1}{2}x + \frac{1}{2}$$
$$y = -\frac{1}{2}x + \frac{3}{2}.$$

42. Using the limit definition of derivative,

$$f'(x) = \frac{-1}{2(x-1)^{3/2}}.$$

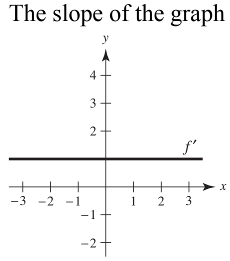
Because the slope of the given line is $-\frac{1}{2}$, you have

$$\frac{-1}{2(x-1)^{3/2}} = -\frac{1}{2}$$
$$1 = (x-1)^{3/2}$$
$$1 = x - 1 \Rightarrow x = 2.$$

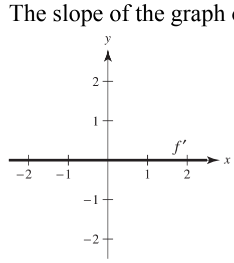
At the point $(2, 1)$, the tangent line is parallel to $x + 2y + 7 = 0$. The equation of the tangent line is

$$y - 1 = -\frac{1}{2}(x - 2)$$
$$y = -\frac{1}{2}x + 2.$$

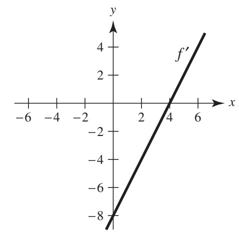
43. The slope of the graph of *f* is 1 for all *x*-values.



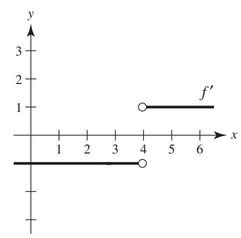
44. The slope of the graph of *f* is 0 for all *x*-values.



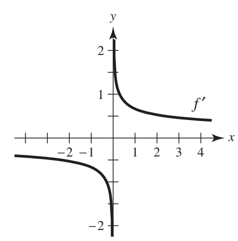
45. The slope of the graph of *f* is negative for $x < 4$, positive for $x > 4$, and 0 at $x = 4$.



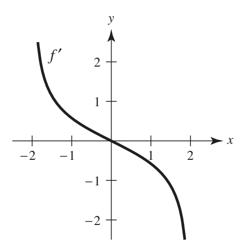
46. The slope of the graph of *f* is –1 for $x < 4$, 1 for $x > 4$, and undefined at $x = 4$.



47. The slope of the graph of *f* is negative for $x < 0$ and positive for $x > 0$. The slope is undefined at $x = 0$.

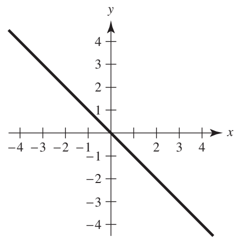


48. The slope is positive for $-2 < x < 0$ and negative for $0 < x < 2$. The slope is undefined at $x = \pm 2$, and 0 at $x = 0$.



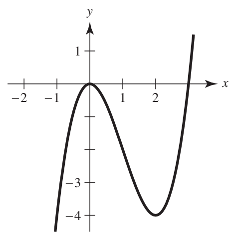
49. Answers will vary.

Sample answer: $y = -x$



The derivative of $y = -x$ is $y' = -1$. So, the derivative is always negative.

50. Answers will vary. *Sample answer*: $y = x^3 - 3x^2$



Note that $y' = 3x^2 - 6x = 3x(x - 2)$.

So, $y' = 0$ at $x = 0$ and $x = 2$.

51. No. For example, the domain of $f(x) = \sqrt{x}$ is $x \geq 0$, whereas the domain of $f'(x) = \dfrac{1}{2\sqrt{x}}$ is $x > 0$.

52. No. For example, $f(x) = x^3$ is symmetric with respect to the origin, but its derivative, $f'(x) = 3x^2$, is symmetric with respect to the y-axis.

53. $g(4) = 5$ because the tangent line passes through $(4, 5)$.

$$g'(4) = \frac{5 - 0}{4 - 7} = -\frac{5}{3}$$

54. $h(-1) = 4$ because the tangent line passes through $(-1, 4)$.

$$h'(-1) = \frac{6 - 4}{3 - (-1)} = \frac{2}{4} = \frac{1}{2}$$

55. $f(x) = 5 - 3x$ and $c = 1$

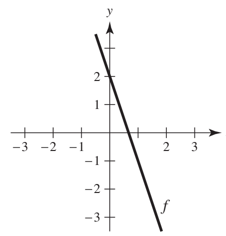
56. $f(x) = x^3$ and $c = -2$

57. $f(x) = -x^2$ and $c = 6$

58. $f(x) = 2\sqrt{x}$ and $c = 9$

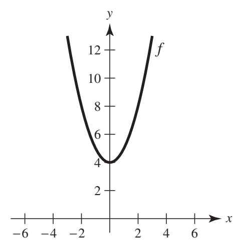
59. $f(0) = 2$ and $f'(x) = -3, -\infty < x < \infty$

$f(x) = -3x + 2$



60. $f(0) = 4, f'(0) = 0; f'(x) < 0$ for $x < 0, f'(x) > 0$ for $x > 0$

Answers will vary: *Sample answer:* $f(x) = x^2 + 4$

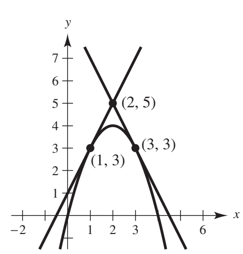


61. Let (x_0, y_0) be a point of tangency on the graph of f. By the limit definition for the derivative, $f'(x) = 4 - 2x$. The slope of the line through $(2, 5)$ and (x_0, y_0) equals the derivative of f at x_0:

$$\frac{5 - y_0}{2 - x_0} = 4 - 2x_0$$
$$5 - y_0 = (2 - x_0)(4 - 2x_0)$$
$$5 - (4x_0 - x_0^2) = 8 - 8x_0 + 2x_0^2$$
$$0 = x_0^2 - 4x_0 + 3$$
$$0 = (x_0 - 1)(x_0 - 3) \Rightarrow x_0 = 1, 3$$

Therefore, the points of tangency are $(1, 3)$ and $(3, 3)$, and the corresponding slopes are 2 and –2. The equations of the tangent lines are:

$$y - 5 = 2(x - 2) \qquad y - 5 = -2(x - 2)$$
$$y = 2x + 1 \qquad y = -2x + 9$$



62. Let (x_0, y_0) be a point of tangency on the graph of f.

By the limit definition for the derivative, $f'(x) = 2x$.

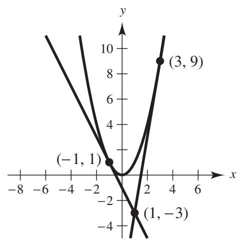
The slope of the line through $(1, -3)$ and (x_0, y_0) equals the derivative of f at x_0:

$$\frac{-3 - y_0}{1 - x_0} = 2x_0$$
$$-3 - y_0 = (1 - x_0)2x_0$$
$$-3 - x_0^2 = 2x_0 - 2x_0^2$$
$$x_0^2 - 2x_0 - 3 = 0$$
$$(x_0 - 3)(x_0 + 1) = 0 \Rightarrow x_0 = 3, -1$$

Therefore, the points of tangency are $(3, 9)$ and $(-1, 1)$, and the corresponding slopes are 6 and –2. The equations of the tangent lines are:

$y + 3 = 6(x - 1)$ $y + 3 = -2(x - 1)$

$y = 6x - 9$ $y = -2x - 1$



63. (a) $f(x) = x^2$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{(x + \Delta x)^2 - x^2}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{x^2 + 2x(\Delta x) + (\Delta x)^2 - x^2}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{\Delta x(2x + \Delta x)}{\Delta x}$$
$$= \lim_{\Delta x \to 0} (2x + \Delta x) = 2x$$

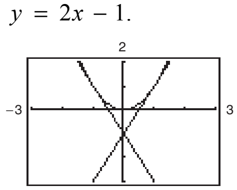
At $x = -1$, $f'(-1) = -2$ and the tangent line is

$y - 1 = -2(x + 1)$ or $y = -2x - 1$.

At $x = 0$, $f'(0) = 0$ and the tangent line is $y = 0$.

At $x = 1$, $f'(1) = 2$ and the tangent line is

$y = 2x - 1$.



For this function, the slopes of the tangent lines are always distinct for different values of x.

(b) $$g'(x) = \lim_{\Delta x \to 0} \frac{g(x + \Delta x) - g(x)}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{(x + \Delta x)^3 - x^3}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{x^3 + 3x^2(\Delta x) + 3x(\Delta x)^2 + (\Delta x)^3 - x^3}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{\Delta x\left(3x^2 + 3x(\Delta x) + (\Delta x)^2\right)}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \left(3x^2 + 3x(\Delta x) + (\Delta x)^2\right) = 3x^2$$

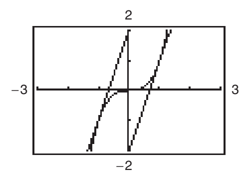
At $x = -1$, $g'(-1) = 3$ and the tangent line is

$y + 1 = 3(x + 1)$ or $y = 3x + 2$.

At $x = 0$, $g'(0) = 0$ and the tangent line is $y = 0$.

At $x = 1$, $g'(1) = 3$ and the tangent line is

$y - 1 = 3(x - 1)$ or $y = 3x - 2$.



For this function, the slopes of the tangent lines are sometimes the same.

64. (a) $g'(0) = -3$

(b) $g'(3) = 0$

(c) Because $g'(1) = -\frac{8}{3}$, g is decreasing (falling) at $x = 1$.

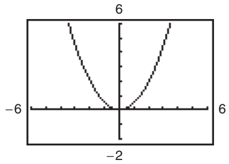
(d) Because $g'(-4) = \frac{7}{3}$, g is increasing (rising) at $x = -4$.

(e) Because $g'(4)$ and $g'(6)$ are both positive, $g(6)$ is greater than $g(4)$, and $g(6) - g(4) > 0$.

(f) No, it is not possible. All you can say is that g is decreasing (falling) at $x = 2$.

65. $f(x) = \frac{1}{2}x^2$

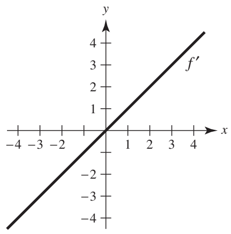
(a)



$f'(0) = 0, f'(1/2) = 1/2, f'(1) = 1, f'(2) = 2$

(b) By symmetry: $f'(-1/2) = -1/2, f'(-1) = -1, f'(-2) = -2$

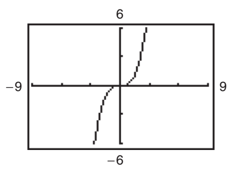
(c)



(d) $f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} = \lim_{\Delta x \to 0} \frac{\frac{1}{2}(x + \Delta x)^2 - \frac{1}{2}x^2}{\Delta x} = \lim_{\Delta x \to 0} \frac{\frac{1}{2}\left(x^2 + 2x(\Delta x) + (\Delta x)^2\right) - \frac{1}{2}x^2}{\Delta x} = \lim_{\Delta x \to 0}\left(x + \frac{\Delta x}{2}\right) = x$

66. $f(x) = \frac{1}{3}x^3$

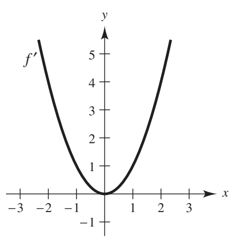
(a)



$f'(0) = 0, f'(1/2) = 1/4, f'(1) = 1, f'(2) = 4, f'(3) = 9$

(b) By symmetry: $f'(-1/2) = 1/4, f'(-1) = 1, f'(-2) = 4, f'(-3) = 9$

(c)



(d) $f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$

$= \lim_{\Delta x \to 0} \frac{\frac{1}{3}(x + \Delta x)^3 - \frac{1}{3}x^3}{\Delta x}$

$= \lim_{\Delta x \to 0} \frac{\frac{1}{3}\left(x^3 + 3x^2(\Delta x) + 3x(\Delta x)^2 + (\Delta x)^3\right) - \frac{1}{3}x^3}{\Delta x}$

$= \lim_{\Delta x \to 0}\left[x^2 + x(\Delta x) + \frac{1}{3}(\Delta x)^2\right] = x^2$

67. $f(2) = 2(4 - 2) = 4, f(2.1) = 2.1(4 - 2.1) = 3.99$

$$f'(2) \approx \frac{3.99 - 4}{2.1 - 2} = -0.1 \quad [\text{Exact: } f'(2) = 0]$$

68. $f(2) = \frac{1}{4}(2^3) = 2, f(2.1) = 2.31525$

$$f'(2) \approx \frac{2.31525 - 2}{2.1 - 2} = 3.1525 [\text{Exact: } f'(2) = 3]$$

69. $f(x) = x^3 + 2x^2 + 1, c = -2$

$$f'(-2) = \lim_{x \to -2} \frac{f(x) - f(-2)}{x + 2} = \lim_{x \to -2} \frac{(x^3 + 2x^2 + 1) - 1}{x + 2} = \lim_{x \to -2} \frac{x^2(x + 2)}{x + 2} = \lim_{x \to -2} x^2 = 4$$

70. $g(x) = x^2 - x, c = 1$

$$g'(1) = \lim_{x \to 1} \frac{g(x) - g(1)}{x - 1} = \lim_{x \to 1} \frac{x^2 - x - 0}{x - 1} = \lim_{x \to 1} \frac{x(x - 1)}{x - 1} = \lim_{x \to 1} x = 1$$

71. $g(x) = \sqrt{|x|}, c = 0$

$g'(0) = \lim_{x \to 0} \frac{g(x) - g(0)}{x - 0} = \lim_{x \to 0} \frac{\sqrt{|x|}}{x}$. Does not exist.

As $x \to 0^-, \frac{\sqrt{|x|}}{x} = \frac{-1}{\sqrt{|x|}} \to -\infty$.

As $x \to 0^+, \frac{\sqrt{|x|}}{x} = \frac{1}{\sqrt{x}} \to \infty$.

Therefore $g(x)$ is not differentiable at $x = 0$.

72. $f(x) = \frac{3}{x}, c = 4$

$$f'(4) = \lim_{x \to 4} \frac{f(x) - f(4)}{x - 4} = \lim_{x \to 4} \frac{\frac{3}{x} - \frac{3}{4}}{x - 4} = \lim_{x \to 4} \frac{12 - 3x}{4x(x - 4)} = \lim_{x \to 4} \frac{-3(x - 4)}{4x(x - 4)} = \lim_{x \to 4} -\frac{3}{4x} = -\frac{3}{16}$$

73. $f(x) = (x - 6)^{2/3}, c = 6$

$$f'(6) = \lim_{x \to 6} \frac{f(x) - f(6)}{x - 6} = \lim_{x \to 6} \frac{(x - 6)^{2/3} - 0}{x - 6} = \lim_{x \to 6} \frac{1}{(x - 6)^{1/3}}.$$

Does not exist.

Therefore $f(x)$ is not differentiable at $x = 6$.

74. $g(x) = (x + 3)^{1/3}, c = -3$

$$g'(-3) = \lim_{x \to -3} \frac{g(x) - g(-3)}{x - (-3)} = \lim_{x \to -3} \frac{(x + 3)^{1/3} - 0}{x + 3} = \lim_{x \to -3} \frac{1}{(x + 3)^{2/3}}.$$

Does not exist.

Therefore $g(x)$ is not differentiable at $x = -3$.

75. $h(x) = |x + 7|, c = -7$

$$h'(-7) = \lim_{x \to -7} \frac{h(x) - h(-7)}{x - (-7)} = \lim_{x \to -7} \frac{|x + 7| - 0}{x + 7} = \lim_{x \to -7} \frac{|x + 7|}{x + 7}.$$

Does not exist.

Therefore $h(x)$ is not differentiable at $x = -7$.

76. $f(x) = |x - 6|, c = 6$

$$f'(6) = \lim_{x \to 6} \frac{f(x) - f(6)}{x - 6} = \lim_{x \to 6} \frac{|x - 6| - 0}{x - 6} = \lim_{x \to 6} \frac{|x - 6|}{x - 6}.$$

Does not exist.

Therefore $f(x)$ is not differentiable at $x = 6$.

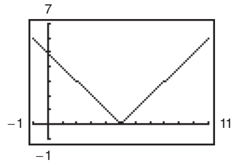
77. $f(x)$ is differentiable everywhere except at $x = -4$. (Sharp turn in the graph)

78. $f(x)$ is differentiable everywhere except at $x = \pm 2$. (Discontinuities)

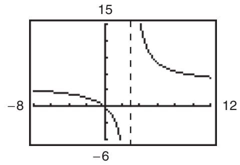
79. $f(x)$ is differentiable on the interval $(-1, \infty)$. (At $x = -1$ the tangent line is vertical.)

80. $f(x)$ is differentiable everywhere except at $x = 0$. (Discontinuity)

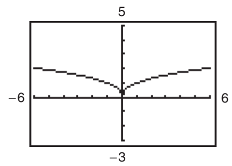
81. $f(x) = |x - 5|$ is differentiable everywhere except at $x = 5$. There is a sharp corner at $x = 5$.



82. $f(x) = \dfrac{4x}{x - 3}$ is differentiable everywhere except at $x = 3$. f is not defined at $x = 3$. (Vertical asymptote)

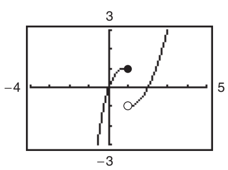


83. $f(x) = x^{2/5}$ is differentiable for all $x \neq 0$. There is a sharp corner at $x = 0$.



84. f is differentiable for all $x \neq 1$.

f is not continuous at $x = 1$.



85. $f(x) = |x - 1|$

The derivative from the left is

$$\lim_{x \to 1^-} \frac{f(x) - f(1)}{x - 1} = \lim_{x \to 1^-} \frac{|x - 1| - 0}{x - 1} = -1.$$

The derivative from the right is

$$\lim_{x \to 1^+} \frac{f(x) - f(1)}{x - 1} = \lim_{x \to 1^+} \frac{|x - 1| - 0}{x - 1} = 1.$$

The one-sided limits are not equal. Therefore, f is not differentiable at $x = 1$.

86. $f(x) = \sqrt{1 - x^2}$

The derivative from the left does not exist because

$$\lim_{x \to 1^-} \frac{f(x) - f(1)}{x - 1} = \lim_{x \to 1^-} \frac{\sqrt{1 - x^2} - 0}{x - 1}$$

$$= \lim_{x \to 1^-} \frac{\sqrt{1 - x^2}}{x - 1} \cdot \frac{\sqrt{1 - x^2}}{\sqrt{1 - x^2}}$$

$$= \lim_{x \to 1^-} -\frac{1 + x}{\sqrt{1 - x^2}} = -\infty.$$

(Vertical tangent)

The limit from the right does not exist since f is undefined for $x > 1$. Therefore, f is not differentiable at $x = 1$.

87. $f(x) = \begin{cases} (x - 1)^3, & x \leq 1 \\ (x - 1)^2, & x > 1 \end{cases}$

The derivative from the left is

$$\lim_{x \to 1^-} \frac{f(x) - f(1)}{x - 1} = \lim_{x \to 1^-} \frac{(x - 1)^3 - 0}{x - 1}$$

$$= \lim_{x \to 1^-} (x - 1)^2 = 0.$$

The derivative from the right is

$$\lim_{x \to 1^+} \frac{f(x) - f(1)}{x - 1} = \lim_{x \to 1^+} \frac{(x - 1)^2 - 0}{x - 1}$$

$$= \lim_{x \to 1^+} (x - 1) = 0.$$

The one-sided limits are equal. Therefore, f is differentiable at $x = 1$. $(f'(1) = 0)$

88. $f(x) = (1 - x)^{2/3}$

The derivative from the left does not exist.

$$\lim_{x \to 1^-} \frac{f(x) - f(1)}{x - 1} = \lim_{x \to 1^-} \frac{(1 - x)^{2/3} - 0}{x - 1}$$

$$= \lim_{x \to 1^-} \frac{-1}{(1 - x)^{1/3}} = -\infty$$

Similarly, the derivative from the right does not exist because the limit is ∞.

Therefore, f is not differentiable at $x = 1$.

89. Note that f is continuous at $x = 2$.

$f(x) = \begin{cases} x^2 + 1, & x \leq 2 \\ 4x - 3, & x > 2 \end{cases}$

The derivative from the left is

$$\lim_{x \to 2^-} \frac{f(x) - f(2)}{x - 2} = \lim_{x \to 2^-} \frac{(x^2 + 1) - 5}{x - 2}$$

$$= \lim_{x \to 2^-} (x + 2) = 4.$$

The derivative from the right is

$$\lim_{x \to 2^+} \frac{f(x) - f(2)}{x - 2} = \lim_{x \to 2^+} \frac{(4x - 3) - 5}{x - 2} = \lim_{x \to 2^+} 4 = 4.$$

The one-sided limits are equal. Therefore, f is differentiable at $x = 2$. $(f'(2) = 4)$

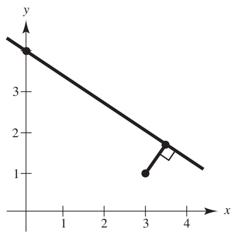
90. $f(x) = \begin{cases} \frac{1}{2}x + 2, & x < 2 \\ \sqrt{2x}, & x \geq 2 \end{cases}$

f is not differentiable at $x = 2$ because it is not continuous at $x = 2$.

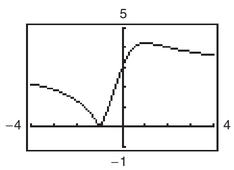
$$\lim_{x \to 2^-} f(x) = \frac{1}{2}(2) + 2 = 3$$

$$\lim_{x \to 2^+} f(x) = \sqrt{2(2)} = 2$$

91. (a) The distance from $(3, 1)$ to the line $mx - y + 4 = 0$ is $d = \dfrac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}} = \dfrac{|m(3) - 1(1) + 4|}{\sqrt{m^2 + 1}} = \dfrac{|3m + 3|}{\sqrt{m^2 + 1}}$.

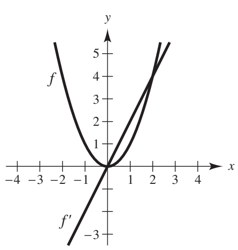


(b)

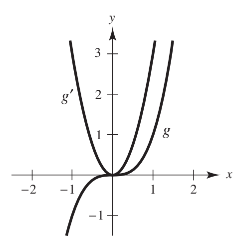


The function d is not differentiable at $m = -1$. This corresponds to the line $y = -x + 4$, which passes through the point $(3, 1)$.

92. (a) $f(x) = x^2$ and $f'(x) = 2x$



(b) $g(x) = x^3$ and $g'(x) = 3x^2$



(c) The derivative is a polynomial of degree 1 less than the original function. If $h(x) = x^n$, then $h'(x) = nx^{n-1}$.

(d) If $f(x) = x^4$, then

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{(x + \Delta x)^4 - x^4}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{x^4 + 4x^3(\Delta x) + 6x^2(\Delta x)^2 + 4x(\Delta x)^3 + (\Delta x)^4 - x^4}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\Delta x\left(4x^3 + 6x^2(\Delta x) + 4x(\Delta x)^2 + (\Delta x)^3\right)}{\Delta x} = \lim_{\Delta x \to 0}\left(4x^3 + 6x^2(\Delta x) + 4x(\Delta x)^2 + (\Delta x)^3\right) = 4x^3.$$

So, if $f(x) = x^4$, then $f'(x) = 4x^3$ which is consistent with the conjecture. However, this is not a proof because you must verify the conjecture for all integer values of n, $n \geq 2$.

93. False. The slope is $\displaystyle\lim_{\Delta x \to 0} \frac{f(2 + \Delta x) - f(2)}{\Delta x}$.

94. False. $y = |x - 2|$ is continuous at $x = 2$, but is not differentiable at $x = 2$. (Sharp turn in the graph)

95. False. If the derivative from the left of a point does not equal the derivative from the right of a point, then the derivative does not exist at that point. For example, if $f(x) = |x|$, then the derivative from the left at $x = 0$ is -1 and the derivative from the right at $x = 0$ is 1. At $x = 0$, the derivative does not exist.

96. True. See Theorem 2.1.

97. $f(x) = \begin{cases} x\sin(1/x), & x \neq 0 \\ 0, & x = 0 \end{cases}$

Using the Squeeze Theorem, you have $-|x| \leq x\sin(1/x) \leq |x|,\ x \neq 0$. So, $\lim_{x\to 0} x\sin(1/x) = 0 = f(0)$ and f is continuous at $x = 0$. Using the alternative form of the derivative, you have

$$\lim_{x\to 0}\frac{f(x) - f(0)}{x - 0} = \lim_{x\to 0}\frac{x\sin(1/x) - 0}{x - 0} = \lim_{x\to 0}\left(\sin\frac{1}{x}\right).$$

Because this limit does not exist ($\sin(1/x)$ oscillates between –1 and 1), the function is not differentiable at $x = 0$.

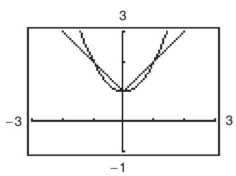
$g(x) = \begin{cases} x^2\sin(1/x), & x \neq 0 \\ 0, & x = 0 \end{cases}$

Using the Squeeze Theorem again, you have $-x^2 \leq x^2\sin(1/x) \leq x^2,\ x \neq 0$. So, $\lim_{x\to 0} x^2\sin(1/x) = 0 = g(0)$ and g is continuous at $x = 0$. Using the alternative form of the derivative again, you have

$$\lim_{x\to 0}\frac{g(x) - g(0)}{x - 0} = \lim_{x\to 0}\frac{x^2\sin(1/x) - 0}{x - 0} = \lim_{x\to 0} x\sin\frac{1}{x} = 0.$$

Therefore, g is differentiable at $x = 0$, $g'(0) = 0$.

98.



As you zoom in, the graph of $y_1 = x^2 + 1$ appears to be locally the graph of a horizontal line, whereas the graph of $y_2 = |x| + 1$ always has a sharp corner at $(0, 1)$. y_2 is not differentiable at $(0, 1)$.

Section 2.2 Basic Differentiation Rules and Rates of Change

1. The derivative of a constant function is 0.

$$\frac{d}{dx}[c] = 0$$

2. To find the derivative of $f(x) = cx^n$, multiply n by c, and reduce the power of x by 1.

$$f'(x) = ncx^{n-1}$$

3. The derivative of the sine function, $f(x) = \sin x$, is the cosine function, $f'(x) = \cos x$.

The derivative of the cosine function, $g(x) = \cos x$, is the negative of the sine function, $g'(x) = -\sin x$.

4. The average velocity of an object is the change in distance divided by the change in time. The velocity is the instantaneous change in velocity. It is the derivative of the position function.

5. (a) $y = x^{1/2}$

$y' = \frac{1}{2}x^{-1/2}$

$y'(1) = \frac{1}{2}$

(b) $y = x^3$

$y' = 3x^2$

$y'(1) = 3$

6. (a) $y = x^{-1/2}$

$y' = -\frac{1}{2}x^{-3/2}$

$y'(1) = -\frac{1}{2}$

(b) $y = x^{-1}$

$y' = -x^{-2}$

$y'(1) = -1$

7. $y = 12$
$y' = 0$

8. $f(x) = -9$
$f'(x) = 0$

9. $y = x^7$
$y' = 7x^6$

10. $y = x^{12}$
$y' = 12x^{11}$

11. $y = \frac{1}{x^5} = x^{-5}$
$y' = -5x^{-6} = -\frac{5}{x^6}$

12. $y = \frac{3}{x^7} = 3x^{-7}$
$y' = 3\left(-7x^{-8}\right) = -\frac{21}{x^8}$

13. $f(x) = \sqrt[9]{x} = x^{1/9}$
$f'(x) = \frac{1}{9}x^{-8/9} = \frac{1}{9x^{8/9}}$

14. $y = \sqrt[4]{x} = x^{1/4}$
$y' = \frac{1}{4}x^{-3/4} = \frac{1}{4x^{3/4}}$

15. $f(x) = x + 11$
$f'(x) = 1$

16. $g(x) = 6x + 3$
$g'(x) = 6$

17. $f(t) = -3t^2 + 2t - 6$
$f'(t) = -6t + 2$

18. $y = t^2 - 3t + 1$
$y' = 2t - 3$

19. $g(x) = x^2 + 4x^3$
$g'(x) = 2x + 12x^2$

20. $y = 4x - 3x^3$
$y' = 4 - 9x^2$

21. $s(t) = t^3 + 5t^2 - 3t + 8$
$s'(t) = 3t^2 + 10t - 3$

22. $y = 2x^3 + 6x^2 - 1$
$y' = 6x^2 + 12x$

23. $y = \frac{\pi}{2}\sin\theta$
$y' = \frac{\pi}{2}\cos\theta$

24. $g(t) = \pi\cos t$
$g'(t) = -\pi\sin t$

25. $y = x^2 - \frac{1}{2}\cos x$
$y' = 2x + \frac{1}{2}\sin x$

26. $y = 7x^4 + 2\sin x$
$y' = 7\left(4x^3\right) + 2\cos x = 28x^3 + 2\cos x$

	<u>Function</u>	<u>Rewrite</u>	<u>Differentiate</u>	<u>Simplify</u>
27.	$y = \frac{2}{7x^4}$	$y = \frac{2}{7}x^{-4}$	$y' = -\frac{8}{7}x^{-5}$	$y' = -\frac{8}{7x^5}$
28.	$y = \frac{8}{5x^{-5}}$	$y = \frac{8}{5}x^5$	$y' = \frac{8}{5}\left(5x^4\right)$	$y' = 8x^4$
29.	$y = \frac{6}{(5x)^3}$	$y = \frac{6}{125}x^{-3}$	$y' = -\frac{18}{125}x^{-4}$	$y' = -\frac{18}{125x^4}$
30.	$y = \frac{3}{(2x)^{-2}}$	$y = 12x^2$	$y' = 12(2x)$	$y' = 24x$

31. $f(x) = \dfrac{8}{x^2} = 8x^{-2}, (2, 2)$

$f'(x) = -16x^{-3} = -\dfrac{16}{x^3}$

$f'(2) = -2$

32. $f(t) = 2 - \dfrac{4}{t} = 2 - 4t^{-1}, (4, 1)$

$f'(t) = 4t^{-2} = \dfrac{4}{t^2}$

$f'(4) = \dfrac{1}{4}$

33. $f(x) = -\frac{1}{2} + \frac{7}{5}x^3, \left(0, -\frac{1}{2}\right)$

$f'(x) = \frac{21}{5}x^2$

$f'(0) = 0$

34. $y = 2x^4 - 3, (1, -1)$

$y' = 8x^3$

$y'(1) = 8$

35. $y = (4x + 1)^2, (0, 1)$

$= 16x^2 + 8x + 1$

$y' = 32x + 8$

$y'(0) = 32(0) + 8 = 8$

36. $f(x) = 2(x - 4)^2, (2, 8)$

$= 2x^2 - 16x + 32$

$f'(x) = 4x - 16$

$f'(2) = 8 - 16 = -8$

37. $f(\theta) = 4 \sin \theta - \theta, (0, 0)$

$f'(\theta) = 4 \cos \theta - 1$

$f'(0) = 4(1) - 1 = 3$

38. $g(t) = -2 \cos t + 5, (\pi, 7)$

$g'(t) = 2 \sin t$

$g'(\pi) = 0$

39. $f(x) = x^2 + 5 - 3x^{-2}$

$f'(x) = 2x + 6x^{-3} = 2x + \dfrac{6}{x^3}$

40. $f(x) = x^3 - 2x + 3x^{-3}$

$f'(x) = 3x^2 - 2 - 9x^{-4} = 3x^2 - 2 - \dfrac{9}{x^4}$

41. $g(t) = t^2 - \dfrac{4}{t^3} = t^2 - 4t^{-3}$

$g'(t) = 2t + 12t^{-4} = 2t + \dfrac{12}{t^4}$

42. $f(x) = 8x + \dfrac{3}{x^2} = 8x + 3x^{-2}$

$f'(x) = 8 - 6x^{-3} = 8 - \dfrac{6}{x^3}$

43. $f(x) = \dfrac{x^3 - 3x^2 + 4}{x^2} = x - 3 + 4x^{-2}$

$f'(x) = 1 - \dfrac{8}{x^3} = \dfrac{x^3 - 8}{x^3}$

44. $h(x) = \dfrac{4x^3 + 2x + 5}{x} = 4x^2 + 2 + 5x^{-1}$

$h'(x) = 8x - 5x^{-2} = 8x - \dfrac{5}{x^2}$

45. $g(t) = \dfrac{3t^2 + 4t - 8}{t^{3/2}} = 3t^{1/2} + 4t^{-1/2} - 8t^{-3/2}$

$g'(t) = \dfrac{3}{2}t^{-1/2} - 2t^{-3/2} + 12t^{-5/2}$

$= \dfrac{3t^2 - 4t + 24}{2t^{5/2}}$

46. $h(s) = \dfrac{s^5 + 2s + 6}{s^{1/3}} = s^{14/3} + 2s^{2/3} + 6s^{-1/3}$

$h'(s) = \dfrac{14}{3}s^{11/3} + \dfrac{4}{3}s^{-1/3} - 2s^{-4/3}$

$= \dfrac{14s^5 + 4s - 6}{3s^{4/3}}$

47. $y = x(x^2 + 1) = x^3 + x$

$y' = 3x^2 + 1$

48. $y = x^2(2x^2 - 3x) = 2x^4 - 3x^3$

$y' = 8x^3 - 9x^2 = x^2(8x - 9)$

49. $f(x) = \sqrt{x} - 6\sqrt[3]{x} = x^{1/2} - 6x^{1/3}$

$f'(x) = \dfrac{1}{2}x^{-1/2} - 2x^{-2/3} = \dfrac{1}{2\sqrt{x}} - \dfrac{2}{x^{2/3}}$

50. $f(t) = t^{2/3} - t^{1/3} + 4$

$f'(t) = \dfrac{2}{3}t^{-1/3} - \dfrac{1}{3}t^{-2/3} = \dfrac{2}{3t^{1/3}} - \dfrac{1}{3t^{2/3}}$

51. $f(x) = 6\sqrt{x} + 5 \cos x = 6x^{1/2} + 5 \cos x$

$f'(x) = 3x^{-1/2} - 5 \sin x = \dfrac{3}{\sqrt{x}} - 5 \sin x$

52. $f(x) = \dfrac{2}{\sqrt[3]{x}} + 3\cos x = 2x^{-1/3} + 3\cos x$

$f'(x) = -\dfrac{2}{3}x^{-4/3} - 3\sin x = -\dfrac{2}{3x^{4/3}} - 3\sin x$

53. $y = \dfrac{1}{(3x)^{-2}} - 5\cos x = (3x)^2 - 5\cos x = 9x^2 - 5\cos x$

$y' = 18x + 5\sin x$

54. $y = \dfrac{3}{(2x)^3} + 2\sin x = \dfrac{3}{8}x^{-3} + 2\sin x$

$y' = \dfrac{-9}{8}x^{-4} + 2\cos x$

$= -\dfrac{9}{8x^4} + 2\cos x$

55. (a) $f(x) = -2x^4 + 5x^2 - 3$

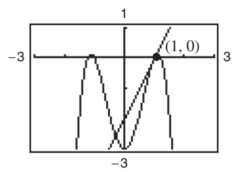
$f'(x) = -8x^3 + 10x$

At $(1, 0)$: $f'(1) = -8(1)^3 + 10(1) = 2$

Tangent line: $y - 0 = 2(x - 1)$

$y = 2x - 2$

(b) and (c)



56. (a) $y = x^3 - 3x$

$y' = 3x^2 - 3$

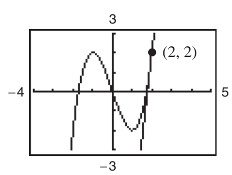
At $(2, 2)$: $y' = 3(2)^2 - 3 = 9$

Tangent line: $y - 2 = 9(x - 2)$

$y = 9x - 16$

$9x - y - 16 = 0$

(b) and (c)



57. (a) $f(x) = \dfrac{2}{\sqrt[4]{x^3}} = 2x^{-3/4}$

$f'(x) = -\dfrac{3}{2}x^{-7/4} = -\dfrac{3}{2x^{7/4}}$

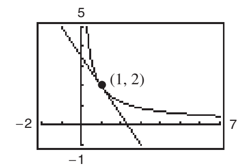
At $(1, 2)$: $f'(1) = -\dfrac{3}{2}$

Tangent line: $y - 2 = -\dfrac{3}{2}(x - 1)$

$y = -\dfrac{3}{2}x + \dfrac{7}{2}$

$3x + 2y - 7 = 0$

(b) and (c)



58. (a) $y = (x - 2)(x^2 + 3x) = x^3 + x^2 - 6x$

$y' = 3x^2 + 2x - 6$

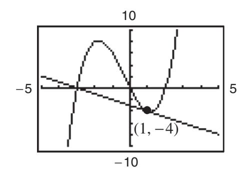
At $(1, -4)$: $y' = 3(1)^2 + 2(1) - 6 = -1$

Tangent line: $y - (-4) = -1(x - 1)$

$y = -x - 3$

$x + y + 3 = 0$

(b) and (c)



59. $y = x^4 - 2x^2 + 3$

$y' = 4x^3 - 4x$

$= 4x(x^2 - 1)$

$= 4x(x - 1)(x + 1)$

$y' = 0 \Rightarrow x = 0, \pm 1$

Horizontal tangents: $(0, 3)$, $(1, 2)$, $(-1, 2)$

60. $y = x^3 + x$

$y' = 3x^2 + 1 > 0$ for all x.

Therefore, there are no horizontal tangents.

61. $y = \dfrac{1}{x^2} = x^{-2}$

$y' = -2x^{-3} = -\dfrac{2}{x^3}$ cannot equal zero.

Therefore, there are no horizontal tangents.

62. $y = x^2 + 9$

$y' = 2x = 0 \Rightarrow x = 0$

At $x = 0, y = 1$.

Horizontal tangent: $(0, 9)$

63. $y = x + \sin x, 0 \le x < 2\pi$

$y' = 1 + \cos x = 0$

$\cos x = -1 \Rightarrow x = \pi$

At $x = \pi$: $y = \pi$

Horizontal tangent: (π, π)

64. $y = \sqrt{3}x + 2\cos x, 0 \le x < 2\pi$

$y' = \sqrt{3} - 2\sin x = 0$

$\sin x = \frac{\sqrt{3}}{2} \Rightarrow x = \frac{\pi}{3}$ or $\frac{2\pi}{3}$

At $x = \frac{\pi}{3}$: $y = \frac{\sqrt{3}\pi + 3}{3}$

At $x = \frac{2\pi}{3}$: $y = \frac{2\sqrt{3}\pi - 3}{3}$

Horizontal tangents: $\left(\frac{\pi}{3}, \frac{\sqrt{3}\pi + 3}{3}\right), \left(\frac{2\pi}{3}, \frac{2\sqrt{3}\pi - 3}{3}\right)$

65. $f(x) = k - x^2, y = -6x + 1$

$f'(x) = -2x$ and slope of tangent line is $m = -6$.

$$f'(x) = -6$$
$$-2x = -6$$
$$x = 3$$
$$y = -6(3) + 1 = -17$$
$$-17 = k - 3^2$$
$$8 = k$$

66. $f(x) = kx^2, y = -2x + 3$

$f'(x) = 2kx$ and slope of tangent line is $m = -2$.

$$f'(x) = -2$$
$$2kx = -2$$
$$x = -\frac{1}{k}$$
$$y = -2\left(-\frac{1}{k}\right) + 3 = \frac{2}{k} + 3$$
$$\frac{2}{k} + 3 = k\left(-\frac{1}{k}\right)^2$$
$$\frac{2}{k} + 3 = \frac{1}{k}$$
$$\frac{1}{k} = -3$$
$$k = -\frac{1}{3}$$

67. $f(x) = \frac{k}{x}, y = -\frac{3}{4}x + 3$

$f'(x) = -\frac{k}{x^2}$ and slope of tangent line is $m = -\frac{3}{4}$.

$$f'(x) = -\frac{3}{4}$$
$$-\frac{k}{x^2} = -\frac{3}{4}$$
$$x^2 = \frac{4k}{3}$$
$$x = \sqrt{\frac{4k}{3}}$$
$$y = -\frac{3}{4}x + 3 = -\frac{3}{4}\sqrt{\frac{4k}{3}} + 3$$
$$-\frac{3}{4}\sqrt{\frac{4k}{3}} + 3 = k\sqrt{\frac{3}{4k}}$$
$$k = 3$$

68. $f(x) = k\sqrt{x}$, $y = x + 4$

$f'(x) = \dfrac{k}{2\sqrt{x}}$ and slope of tangent line is $m = 1$.

$$f'(x) = 1$$
$$\frac{k}{2\sqrt{x}} = 1$$
$$\frac{k}{2} = \sqrt{x}$$
$$\frac{k^2}{4} = x$$
$$y = x + 4 = \frac{k^2}{4} + 4$$
$$\frac{k^2}{4} + 4 = k \cdot \frac{k}{2}$$
$$\frac{k^2}{4} - \frac{k^2}{2} = -4$$
$$-\frac{k^2}{4} = -4$$
$$k^2 = 16$$
$$k = 4$$

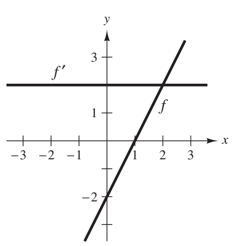
69. $g(x) = f(x) + 6 \Rightarrow g'(x) = f'(x)$

70. $g(x) = 2f(x) \Rightarrow g'(x) = 2f'(x)$

71. $g(x) = -5f(x) \Rightarrow g'(x) = -5f'(x)$

72. $g(x) = 3f(x) - 1 \Rightarrow g'(x) = 3f'(x)$

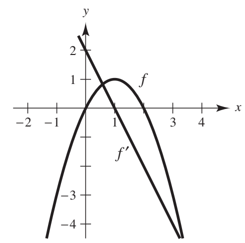
73.



If f is linear then its derivative is a constant function.

$$f(x) = ax + b$$
$$f'(x) = a$$

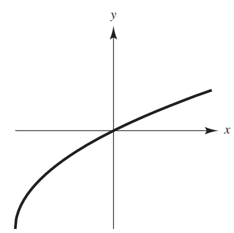
74.



If f is quadratic, then its derivative is a linear function.

$$f(x) = ax^2 + bx + c$$
$$f'(x) = 2ax + b$$

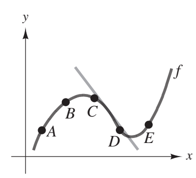
75. The graph of a function f such that $f' > 0$ for all x and the rate of change of the function is decreasing (i.e., as x increases, f' decreases) would, in general, look like the graph below.



76. (a) The slope appears to be steepest between A and B.

(b) The average rate of change between A and B is **greater** than the instantaneous rate of change at B.

(c)



77. Let (x_1, y_1) and (x_2, y_2) be the points of tangency on $y = x^2$ and $y = -x^2 + 6x - 5$, respectively.

The derivatives of these functions are:

$y' = 2x \Rightarrow m = 2x_1$ and $y' = -2x + 6 \Rightarrow m = -2x_2 + 6$

$m = 2x_1 = -2x_2 + 6$

$x_1 = -x_2 + 3$

Because $y_1 = x_1^2$ and $y_2 = -x_2^2 + 6x_2 - 5$:

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\left(-x_2^2 + 6x_2 - 5\right) - \left(x_1^2\right)}{x_2 - x_1} = -2x_2 + 6$$

$$\frac{\left(-x_2^2 + 6x_2 - 5\right) - \left(-x_2 + 3\right)^2}{x_2 - \left(-x_2 + 3\right)} = -2x_2 + 6$$

$$\left(-x_2^2 + 6x_2 - 5\right) - \left(x_2^2 - 6x_2 + 9\right) = \left(-2x_2 + 6\right)\left(2x_2 - 3\right)$$

$$-2x_2^2 + 12x_2 - 14 = -4x_2^2 + 18x_2 - 18$$

$$2x_2^2 - 6x_2 + 4 = 0$$

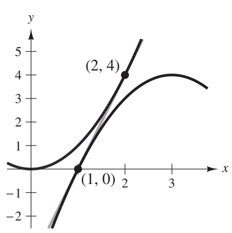
$$2\left(x_2 - 2\right)\left(x_2 - 1\right) = 0$$

$$x_2 = 1 \text{ or } 2$$

$x_2 = 1 \Rightarrow y_2 = 0, x_1 = 2$ and $y_1 = 4$

So, the tangent line through $(1, 0)$ and $(2, 4)$ is

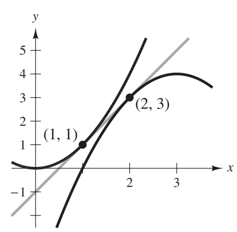
$$y - 0 = \left(\frac{4 - 0}{2 - 1}\right)(x - 1) \Rightarrow y = 4x - 4.$$



$x_2 = 2 \Rightarrow y_2 = 3, x_1 = 1$ and $y_1 = 1$

So, the tangent line through $(2, 3)$ and $(1, 1)$ is

$$y - 1 = \left(\frac{3 - 1}{2 - 1}\right)(x - 1) \Rightarrow y = 2x - 1.$$



78. m_1 is the slope of the line tangent to $y = x$. m_2 is the slope of the line tangent to $y = 1/x$. Because

$$y = x \Rightarrow y' = 1 \Rightarrow m_1 = 1 \text{ and } y = \frac{1}{x} \Rightarrow y' = -\frac{1}{x^2} \Rightarrow m_2 = -\frac{1}{x^2}.$$

The points of intersection of $y = x$ and $y = 1/x$ are

$$x = \frac{1}{x} \Rightarrow x^2 = 1 \Rightarrow x = \pm 1.$$

At $x = \pm 1$, $m_2 = -1$. Because $m_2 = -1/m_1$, these tangent lines are perpendicular at the points of intersection.

79. $f(x) = 3x + \sin x + 2$

$f'(x) = 3 + \cos x$

Because $|\cos x| \le 1$, $f'(x) \ne 0$ for all x and f does not have a horizontal tangent line.

80. $f(x) = x^5 + 3x^3 + 5x$

$f'(x) = 5x^4 + 9x^2 + 5$

Because $5x^4 + 9x^2 \ge 0$, $f'(x) \ge 5$. So, f does not have a tangent line with a slope of 3.

81. $f(x) = \sqrt{x}, (-4, 0)$

$$f'(x) = \frac{1}{2}x^{-1/2} = \frac{1}{2\sqrt{x}}$$

$$\frac{1}{2\sqrt{x}} = \frac{0 - y}{-4 - x}$$

$$4 + x = 2\sqrt{x}y$$

$$4 + x = 2\sqrt{x}\sqrt{x}$$

$$4 + x = 2x$$

$$x = 4, y = 2$$

The point $(4, 2)$ is on the graph of f.

Tangent line: $y - 2 = \frac{0 - 2}{-4 - 4}(x - 4)$

$$4y - 8 = x - 4$$

$$0 = x - 4y + 4$$

82. $f(x) = \frac{2}{x}, (5, 0)$

$$f'(x) = -\frac{2}{x^2}$$

$$-\frac{2}{x^2} = \frac{0 - y}{5 - x}$$

$$-10 + 2x = -x^2 y$$

$$-10 + 2x = -x^2\left(\frac{2}{x}\right)$$

$$-10 + 2x = -2x$$

$$4x = 10$$

$$x = \frac{5}{2}, y = \frac{4}{5}$$

The point $\left(\frac{5}{2}, \frac{4}{5}\right)$ is on the graph of f. The slope of the tangent line is $f'\left(\frac{5}{2}\right) = -\frac{8}{25}$.

Tangent line: $y - \frac{4}{5} = -\frac{8}{25}\left(x - \frac{5}{2}\right)$

$$25y - 20 = -8x + 20$$

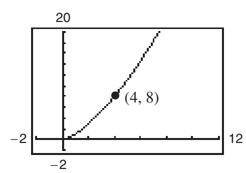
$$8x + 25y - 40 = 0$$

83. (a) One possible secant is between $(3.9, 7.7019)$ and $(4, 8)$:

$$y - 8 = \frac{8 - 7.7019}{4 - 3.9}(x - 4)$$

$$y - 8 = 2.981(x - 4)$$

$$y = S(x) = 2.981x - 3.924$$

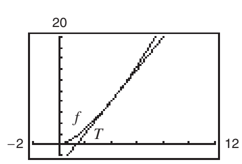


(b) $f'(x) = \frac{3}{2}x^{1/2} \Rightarrow f'(4) = \frac{3}{2}(2) = 3$

$$T(x) = 3(x - 4) + 8 = 3x - 4$$

The slope (and equation) of the secant line approaches that of the tangent line at $(4, 8)$ as you choose points closer and closer to $(4, 8)$.

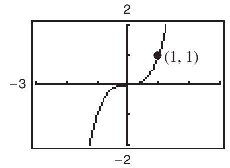
(c) As you move further away from $(4, 8)$, the accuracy of the approximation T gets worse.



(d)

Δx	−3	−2	−1	−0.5	−0.1	0	0.1	0.5	1	2	3
$f(4 + \Delta x)$	1	2.828	5.196	6.548	7.702	8	8.302	9.546	11.180	14.697	18.520
$T(4 + \Delta x)$	−1	2	5	6.5	7.7	8	8.3	9.5	11	14	17

84. (a) Nearby point: $(1.0073138, 1.0221024)$



Secant line: $y - 1 = \dfrac{1.0221024 - 1}{1.0073138 - 1}(x - 1)$

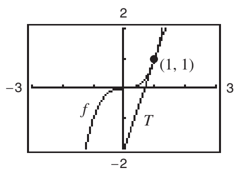
$y = 3.022(x - 1) + 1$

(Answers will vary.)

(b) $f'(x) = 3x^2$

$T(x) = 3(x - 1) + 1 = 3x - 2$

(c) The accuracy worsens as you move away from $(1, 1)$.



(d)

Δx	−3	−2	−1	−0.5	−0.1	0	0.1	0.5	1	2	3
$f(x)$	−8	−1	0	0.125	0.729	1	1.331	3.375	8	27	64
$T(x)$	−8	−5	−2	−0.5	0.7	1	1.3	2.5	4	7	10

The accuracy decreases more rapidly than in Exercise 85 because $y = x^3$ is less "linear" than $y = x^{3/2}$.

85. False. Let $f(x) = x$ and $g(x) = x + 1$. Then

$f'(x) = g'(x) = x$, but $f(x) \neq g(x)$.

86. True. If $y = x^{a+2} + bx$, then

$\dfrac{dy}{dx} = (a + 2)x^{(a+2)-1} + b = (a + 2)x^{a+1} + b.$

87. False. If $y = \pi^2$, then $dy/dx = 0$. (π^2 is a constant.)

88. True. If $f(x) = -g(x) + b$, then

$f'(x) = -g'(x) + 0 = -g'(x).$

89. False. If $f(x) = 0$, then $f'(x) = 0$ by the Constant Rule.

90. False. If $f(x) = \dfrac{1}{x_n} = x^{-n}$, then

$f'(x) = -nx^{-n-1} = \dfrac{-n}{x^{n+1}}.$

91. $f(t) = 3t + 5, \quad [1, 2]$

$f'(t) = 3$. So, $f'(1) = f'(2) = 3$.

Instantaneous rate of change is the constant 3.

Average rate of change:

$\dfrac{f(2) - f(1)}{2 - 1} = \dfrac{11 - 8}{1} = 3$

92. $f(t) = t^2 - 7, \quad [3, 3.1]$

$f'(t) = 2t$

Instantaneous rate of change:

At $(3, 2)$: $f'(3) = 6$

At $(3.1, 2.61)$: $f'(3.1) = 6.2$

Average rate of change:

$\dfrac{f(3.1) - f(3)}{3.1 - 3} = \dfrac{2.61 - 2}{0.1} = 6.1$

93. $f(x) = -\dfrac{1}{x}, \quad [1, 2]$

$f'(x) = \dfrac{1}{x^2}$

Instantaneous rate of change:

$(1, -1) \Rightarrow f'(1) = 1$

$\left(2, -\dfrac{1}{2}\right) \Rightarrow f'(2) = \dfrac{1}{4}$

Average rate of change:

$\dfrac{f(2) - f(1)}{2 - 1} = \dfrac{(-1/2) - (-1)}{2 - 1} = \dfrac{1}{2}$

94. $f(x) = \sin x, \quad \left[0, \frac{\pi}{6}\right]$

$f'(x) = \cos x$

Instantaneous rate of change:

$(0, 0) \Rightarrow f'(0) = 1$

$\left(\frac{\pi}{6}, \frac{1}{2}\right) \Rightarrow f'\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2} \approx 0.866$

Average rate of change:

$\frac{f(\pi/6) - f(0)}{(\pi/6) - 0} = \frac{(1/2) - 0}{(\pi/6) - 0} = \frac{3}{\pi} \approx 0.955$

95. (a) $s(t) = -4.9t^2 + 441$

$v(t) = -9.8t$

(b) $\frac{s(2) - s(1)}{2 - 1} = 421.4 - 436.1 = -14.7$ m/sec

(c) $v(t) = s'(t) = -9.8t$

When $t = 1$: $v(1) = -9.8$ m/sec

When $t = 2$: $v(2) = -19.6$ m/sec

(d) $-4.9t^2 + 441 = 0$

$t^2 = \frac{441}{4.9} \Rightarrow t = \sqrt{90} \approx 9.49$ sec

(e) $v(\sqrt{90}) = -9.8\sqrt{90}$

≈ -92.97 m/sec

96. (a) $s(t) = -4.9t^2 + v_0t + s_0 = -4.9t^2 + 214$

$s'(t) = v(t) = -9.8t$

(b) Average velocity $= \frac{s(5) - s(2)}{5 - 2}$

$= \frac{91.5 - 194.4}{3}$

$= -34.3$ m/sec

(c) $s'(2) = -9.8(2) = -19.6$ m/sec

$s'(5) = -9.8(5) = 49.0$ m/sec

(d) $s(t) = -4.9t^2 + 214 = 0$

$4.9t^2 = 214$

$t^2 = \frac{214}{4.9}$

$t \approx 6.61$ sec

(e) $v(6.61) = -9.8(6.61) \approx -64.8$ m/sec

97. $s(t) = -4.9t^2 + v_0t + s_0$

$= -4.9t^2 + 120t$

$v(t) = -9.8t + 120$

(a) $v(5) = -9.8(5) + 120 = 71$ m/sec

(b) $v(10) = -9.8(10) + 120 = 22$ m/sec

98 $s(t) = -4.9t^2 - 5t + 80$

$v(t) = -9.8t - 5$

(a) $v(2) = -24.6$ m/sec

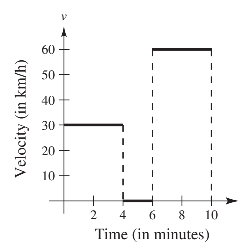
(b) $v(3) = -34.4$ m/sec

99. From $(0, 0)$ to $(4, 2)$, $s(t) = \frac{1}{2}t \Rightarrow v(t) = \frac{1}{2}$ km/min.

$v(t) = \frac{1}{2}(60) = 30$ km/h for $0 < t < 4$

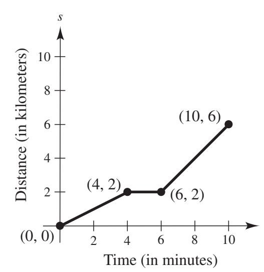
Similarly, $v(t) = 0$ for $4 < t < 6$. Finally, from $(6, 2)$ to $(10, 6)$,

$s(t) = t - 4 \Rightarrow v(t) = 1$ km/min. $= 60$ km/h.



(The velocity has been converted to kilometers per hour.)

100. This graph corresponds with Exercise 99.



101. $V = s^3, \frac{dV}{ds} = 3s^2$

When $s = 6$ cm, $\frac{dV}{ds} = 108$ cm^3 per cm change in s.

102. $A = s^2, \dfrac{dA}{ds} = 2s$

When $s = 6$ m, $\dfrac{dA}{ds} = 12$ m^2 per m change in s.

103. (a) Using a graphing utility,

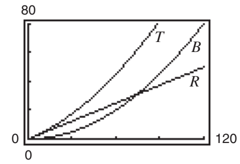
$R(v) = 0.417v - 0.02.$

(b) Using a graphing utility,

$B(v) = 0.0056v^2 + 0.001v + 0.04.$

(c) $T(v) = R(v) + B(v) = 0.0056v^2 + 0.418v + 0.02$

(d)



(e) $\dfrac{dT}{dv} = 0.0112v + 0.418$

For $v = 40, T'(40) \approx 0.866$

For $v = 80, T'(80) \approx 1.314$

For $v = 100, T'(100) \approx 1.538$

(f) For increasing speeds, the total stopping distance increases.

104. $C = (\text{liters of fuel used})(\text{cost per liter})$

$= \left(\dfrac{20{,}000}{x}\right)(1.21) = \dfrac{24{,}200}{x}$

$\dfrac{dC}{dx} = -\dfrac{24{,}200}{x^2}$

x	10	15	20	25	30	35	40
C	2420	1613.3	1210	968	806.7	691.4	605
dC/dx	–242	–107.6	–60.5	–38.7	–26.9	–19.8	–15.1

The driver who gets 25 kilometers per liter would benefit more. The rate of change at $x = 25$ is larger in absolute value than that at $x = 40$.

105. $s(t) = -\dfrac{1}{2}at^2 + c$ and $s'(t) = -at$

Average velocity: $\dfrac{s(t_0 + \Delta t) - s(t_0 - \Delta t)}{(t_0 + \Delta t) - (t_0 - \Delta t)} = \dfrac{\left[-(1/2)a(t_0 + \Delta t)^2 + c\right] - \left[-(1/2)a(t_0 - \Delta t)^2 + c)\right]}{2\Delta t}$

$$= \frac{-(1/2)a\left(t_0^2 + 2t_0\Delta t + (\Delta t)^2\right) + (1/2)a\left(t_0^2 - 2t_0\Delta t + (\Delta t)^2\right)}{2\,\Delta t}$$

$$= \frac{-2at_0\,\Delta t}{2\,\Delta t} = -at_0 = s'(t_0)$$ instantaneous velocity at $t = t_0$

106. $$C = \frac{1{,}008{,}000}{Q} + 6.3Q$$

$$\frac{dC}{dQ} = -\frac{1{,}008{,}000}{Q} + 6.3$$

$C(351) - C(350) \approx 5083.095 - 5085 \approx -\$1.91$

When $Q = 350, \dfrac{dC}{dQ} \approx -\$1.93.$

107. $y = ax^2 + bx + c$

Because the parabola passes through $(0, 1)$ and $(1, 0)$, you have:

$(0, 1)$: $1 = a(0)^2 + b(0) + c \Rightarrow c = 1$

$(1, 0)$: $0 = a(1)^2 + b(1) + 1 \Rightarrow b = -a - 1$

So, $y = ax^2 + (-a - 1)x + 1$. From the tangent line $y = x - 1$, you know that the derivative is 1 at the point $(1, 0)$.

$$y' = 2ax + (-a - 1)$$
$$1 = 2a(1) + (-a - 1)$$
$$1 = a - 1$$
$$a = 2$$
$$b = -a - 1 = -3$$

Therefore, $y = 2x^2 - 3x + 1$.

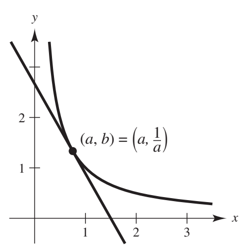
108. $y = \dfrac{1}{x}, x > 0$

$$y' = -\frac{1}{x^2}$$

At (a, b), the equation of the tangent line is $y - \dfrac{1}{a} = -\dfrac{1}{a^2}(x - a)$ or $y = -\dfrac{x}{a^2} + \dfrac{2}{a}$.

The x-intercept is $(2a, 0)$. The y-intercept is $\left(0, \dfrac{2}{a}\right)$.

The area of the triangle is $A = \dfrac{1}{2}bh = \dfrac{1}{2}(2a)\left(\dfrac{2}{a}\right) = 2$.



109. $y = x^3 - 9x$

$$y' = 3x^2 - 9$$

Tangent lines through $(1, -9)$:

$$y + 9 = (3x^2 - 9)(x - 1)$$
$$(x^3 - 9x) + 9 = 3x^3 - 3x^2 - 9x + 9$$
$$0 = 2x^3 - 3x^2 = x^2(2x - 3)$$
$$x = 0 \text{ or } x = \tfrac{3}{2}$$

The points of tangency are $(0, 0)$ and $\left(\frac{3}{2}, -\frac{81}{8}\right)$. At $(0, 0)$, the slope is $y'(0) = -9$. At $\left(\frac{3}{2}, -\frac{81}{8}\right)$, the slope is $y'\left(\frac{3}{2}\right) = -\frac{9}{4}$.

Tangent Lines:

$y - 0 = -9(x - 0)$ and $\quad y + \frac{81}{8} = -\frac{9}{4}\left(x - \frac{3}{2}\right)$

$y = -9x \quad\quad y = -\frac{9}{4}x - \frac{27}{4}$

$9x + y = 0 \quad\quad 9x + 4y + 27 = 0$

110. $y = x^2$

$y' = 2x$

(a) Tangent lines through $(0, a)$:

$$\begin{aligned} y - a &= 2x(x - 0) \\ x^2 - a &= 2x^2 \\ -a &= x^2 \\ \pm\sqrt{-a} &= x \end{aligned}$$

The points of tangency are $\left(\pm\sqrt{-a}, -a\right)$. At $\left(\sqrt{-a}, -a\right)$, the slope is $y'\left(\sqrt{-a}\right) = 2\sqrt{-a}$.

At $\left(-\sqrt{-a}, -a\right)$, the slope is $y'\left(-\sqrt{-a}\right) = -2\sqrt{-a}$.

Tangent lines: $y + a = 2\sqrt{-a}\left(x - \sqrt{-a}\right)$ and $y + a = -2\sqrt{-a}\left(x + \sqrt{-a}\right)$

$y = 2\sqrt{-a}x + a \qquad y = -2\sqrt{-a}x + a$

Restriction: *a* must be negative.

(b) Tangent lines through $(a, 0)$:

$$\begin{aligned} y - 0 &= 2x(x - a) \\ x^2 &= 2x^2 - 2ax \\ 0 &= x^2 - 2ax = x(x - 2a) \end{aligned}$$

The points of tangency are $(0, 0)$ and $\left(2a, 4a^2\right)$. At $(0, 0)$, the slope is $y'(0) = 0$. At $\left(2a, 4a^2\right)$, the slope is $y'(2a) = 4a$.

Tangent lines: $y - 0 = 0(x - 0)$ and $y - 4a^2 = 4a(x - 2a)$

$y = 0 \qquad y = 4ax - 4a^2$

Restriction: None, *a* can be any real number.

111. $f(x) = \begin{cases} ax^3, & x \le 2 \\ x^2 + b, & x > 2 \end{cases}$

f must be continuous at $x = 2$ to be differentiable at $x = 2$.

$$\left.\begin{aligned} \lim_{x\to 2^-} f(x) &= \lim_{x\to 2^-} ax^3 = 8a \\ \lim_{x\to 2^+} f(x) &= \lim_{x\to 2^+} \left(x^2 + b\right) = 4 + b \end{aligned}\right\} \quad \begin{aligned} 8a &= 4 + b \\ 8a - 4 &= b \end{aligned}$$

$$f'(x) = \begin{cases} 3ax^2, & x < 2 \\ 2x, & x > 2 \end{cases}$$

For *f* to be differentiable at $x = 2$, the left derivative must equal the right derivative.

$$\begin{aligned} 3a(2)^2 &= 2(2) \\ 12a &= 4 \\ a &= \tfrac{1}{3} \\ b &= 8a - 4 = -\tfrac{4}{3} \end{aligned}$$

112. $f(x) = \begin{cases} \cos x, & x < 0 \\ ax + b, & x \ge 0 \end{cases}$

$f(0) = b = \cos(0) = 1 \Rightarrow b = 1$

$f'(x) = \begin{cases} -\sin x, & x < 0 \\ a, & x > 0 \end{cases}$

So, $a = 0$.

Answer: $a = 0, b = 1$

113. $f_1(x) = |\sin x|$ is differentiable for all $x \ne n\pi$, n an integer.

$f_2(x) = \sin|x|$ is differentiable for all $x \ne 0$.

You can verify this by graphing f_1 and f_2 and observing the locations of the sharp turns.

114. Let $f(x) = \cos x$.

$$\begin{aligned} f'(x) &= \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} \\ &= \lim_{\Delta x \to 0} \frac{\cos x \cos \Delta x - \sin x \sin \Delta x - \cos x}{\Delta x} \\ &= \lim_{\Delta x \to 0} \frac{\cos x(\cos \Delta x - 1)}{\Delta x} - \lim_{\Delta x \to 0} \sin x\left(\frac{\sin \Delta x}{\Delta x}\right) \\ &= 0 - \sin x(1) = -\sin x \end{aligned}$$

115. You are given $f : R \to R$ satisfying

$(*) f'(x) = \dfrac{f(x + n) - f(x)}{n}$ for all real numbers x and all positive integers n. You claim that $f(x) = mx + b$, $m, b \in R$.

For this case,

$$f'(x) = m = \frac{[m(x + n) + b] - [mx + b]}{n} = m.$$

Furthermore, these are the only solutions:

Note first that $f'(x + 1) = \dfrac{f(x + 2) - f(x + 1)}{1}$, and $f'(x) = f(x + 1) - f(x)$. From $(*)$ you have

$$\begin{aligned} 2f'(x) &= f(x + 2) - f(x) \\ &= [f(x + 2) - f(x + 1)] + [f(x + 1) - f(x)] \\ &= f'(x + 1) + f'(x). \end{aligned}$$

Thus, $f'(x) = f'(x + 1)$.

Let $g(x) = f(x + 1) - f(x)$.

Let $m = g(0) = f(1) - f(0)$.

Let $b = f(0)$. Then

$g'(x) = f'(x + 1) - f'(x) = 0$

$g(x) = \text{constant} = g(0) = m$

$f'(x) = f(x + 1) - f(x) = g(x) = m$

$\Rightarrow f(x) = mx + b.$

Section 2.3 Product and Quotient Rules and Higher-Order Derivatives

1. To find the derivative of the product of two differentiable functions f and g, multiply the first function f by the derivative of the second function g, and then add the second function g times the derivative of the first function f.

2. To find the derivative of the quotient of two differentiable functions f and g, where $g(x) \ne 0$, multiply the denominator by the derivative of the numerator minus the numerator times the derivative of the denominator, all of which is divided by the square of the denominator.

3. $\dfrac{d}{dx} \tan x = \sec^2 x$

$\dfrac{d}{dx} \cot x = -\csc^2 x$

$\dfrac{d}{dx} \sec x = \sec x \tan x$

$\dfrac{d}{dx} \csc x = -\csc x \cot x$

4. Higher-order derivatives are successive derivatives of a function.

5. $g(x) = (2x - 3)(1 - 5x)$

$$\begin{aligned} g'(x) &= (2x - 3)(-5) + (1 - 5x)(2) \\ &= -10x + 15 + 2 - 10x \\ &= -20x + 17 \end{aligned}$$

6. $y = (3x - 4)(x^3 + 5)$

$$\begin{aligned} y' &= (3x - 4)(3x^2) + (x^3 + 5)(3) \\ &= 9x^3 - 12x^2 + 3x^3 + 15 \\ &= 12x^3 - 12x^2 + 15 \end{aligned}$$

7. $h(t) = \sqrt{t}(1 - t^2) = t^{1/2}(1 - t^2)$

$$\begin{aligned} h'(t) &= t^{1/2}(-2t) + (1 - t^2)\frac{1}{2}t^{-1/2} \\ &= -2t^{3/2} + \frac{1}{2t^{1/2}} - \frac{1}{2}t^{3/2} \\ &= -\frac{5}{2}t^{3/2} + \frac{1}{2t^{1/2}} \\ &= \frac{1 - 5t^2}{2t^{1/2}} = \frac{1 - 5t^2}{2\sqrt{t}} \end{aligned}$$

8. $g(s) = \sqrt{s}\left(s^2 + 8\right) = s^{1/2}\left(s^2 + 8\right)$

$g'(s) = s^{1/2}(2s) + \left(s^2 + 8\right)\frac{1}{2}s^{-1/2}$

$= 2s^{3/2} + \frac{1}{2}s^{3/2} + 4s^{-1/2}$

$= \frac{5}{2}s^{3/2} + \frac{4}{s^{1/2}}$

$= \frac{5s^2 + 8}{2\sqrt{s}}$

9. $f(x) = x^3 \cos x$

$f'(x) = x^3(-\sin x) + \cos x\left(3x^2\right)$

$= 3x^2 \cos x - x^3 \sin x$

$= x^2(3 \cos x - x \sin x)$

10. $g(x) = \sqrt{x} \sin x$

$g'(x) = \sqrt{x} \cos x + \sin x\left(\frac{1}{2\sqrt{x}}\right)$

$= \sqrt{x} \cos x + \frac{1}{2\sqrt{x}} \sin x$

11. $f(x) = \frac{x}{x - 5}$

$f'(x) = \frac{(x - 5)(1) - x(1)}{(x - 5)^2} = \frac{x - 5 - x}{(x - 5)^2} = -\frac{5}{(x - 5)^2}$

12. $g(t) = \frac{3t^2 - 1}{2t + 5}$

$g'(t) = \frac{(2t + 5)(6t) - \left(3t^2 - 1\right)(2)}{(2t + 5)^2}$

$= \frac{12t^2 + 30t - 6t^2 + 2}{(2t + 5)^2}$

$= \frac{6t^2 + 30t + 2}{(2t + 5)^2}$

13. $h(x) = \frac{\sqrt{x}}{x^3 + 1} = \frac{x^{1/2}}{x^3 + 1}$

$h'(x) = \frac{\left(x^3 + 1\right)\frac{1}{2}x^{-1/2} - x^{1/2}\left(3x^2\right)}{\left(x^3 + 1\right)^2}$

$= \frac{x^3 + 1 - 6x^3}{2x^{1/2}\left(x^3 + 1\right)^2}$

$= \frac{1 - 5x^3}{2\sqrt{x}\left(x^3 + 1\right)^2}$

14. $f(x) = \frac{x^2}{2\sqrt{x} + 1}$

$f'(x) = \frac{\left(2\sqrt{x} + 1\right)(2x) - x^2\left(x^{-1/2}\right)}{\left(2\sqrt{x} + 1\right)^2}$

$= \frac{4x^{3/2} + 2x - x^{3/2}}{\left(2\sqrt{x} + 1\right)^2}$

$= \frac{3x^{3/2} + 2x}{\left(2\sqrt{x} + 1\right)^2}$

$= \frac{x\left(3\sqrt{x} + 2\right)}{\left(2\sqrt{x} + 1\right)^2}$

15. $g(x) = \frac{\sin x}{x^2}$

$g'(x) = \frac{x^2(\cos x) - \sin x(2x)}{\left(x^2\right)^2} = \frac{x \cos x - 2 \sin x}{x^3}$

16. $f(t) = \frac{\cos t}{t^3}$

$f'(t) = \frac{t^3(-\sin t) - \cos t\left(3t^2\right)}{\left(t^3\right)^2} = -\frac{t \sin t + 3 \cos t}{t^4}$

17. $f(x) = \left(x^3 + 4x\right)\left(3x^2 + 2x - 5\right)$

$f'(x) = \left(x^3 + 4x\right)(6x + 2) + \left(3x^2 + 2x - 5\right)\left(3x^2 + 4\right)$

$= 6x^4 + 24x^2 + 2x^3 + 8x + 9x^4 + 6x^3 - 15x^2 + 12x^2 + 8x - 20$

$= 15x^4 + 8x^3 + 21x^2 + 16x - 20$

$f'(0) = -20$

18. $f(x) = (2x^2 - 3x)(9x + 4)$

$= (2x^2 - 3x)(9) + (9x + 4)(4x - 3)$

$= 18x^2 - 27x + 36x^2 + 16x - 27x - 12$

$= 54x^2 - 38x - 12$

$f'(-1) = 54(-1)^2 - 38(-1) - 12 = 80$

19. $f(x) = \dfrac{x^2 - 4}{x - 3}$

$f'(x) = \dfrac{(x - 3)(2x) - (x^2 - 4)(1)}{(x - 3)^2}$

$= \dfrac{2x^2 - 6x - x^2 + 4}{(x - 3)^2}$

$= \dfrac{x^2 - 6x + 4}{(x - 3)^2}$

$f'(1) = \dfrac{1 - 6 + 4}{(1 - 3)^2} = -\dfrac{1}{4}$

20. $f(x) = \dfrac{x - 4}{x + 4}$

$f'(x) = \dfrac{(x + 4)(1) - (x - 4)(1)}{(x + 4)^2}$

$= \dfrac{x + 4 - x + 4}{(x + 4)^2}$

$= \dfrac{8}{(x + 4)^2}$

$f'(3) = \dfrac{8}{(3 + 4)^2} = \dfrac{8}{49}$

21. $f(x) = x \cos x$

$f'(x) = (x)(-\sin x) + (\cos x)(1) = \cos x - x \sin x$

$f'\left(\dfrac{\pi}{4}\right) = \dfrac{\sqrt{2}}{2} - \dfrac{\pi}{4}\left(\dfrac{\sqrt{2}}{2}\right) = \dfrac{\sqrt{2}}{8}(4 - \pi)$

22. $f(x) = \dfrac{\sin x}{x}$

$f'(x) = \dfrac{(x)(\cos x) - (\sin x)(1)}{x^2}$

$= \dfrac{x \cos x - \sin x}{x^2}$

$f'\left(\dfrac{\pi}{6}\right) = \dfrac{(\pi/6)(\sqrt{3}/2) - (1/2)}{\pi^2/36}$

$= \dfrac{3\sqrt{3}\pi - 18}{\pi^2}$

$= \dfrac{3(\sqrt{3}\pi - 6)}{\pi^2}$

	<u>*Function*</u>	<u>*Rewrite*</u>	<u>*Differentiate*</u>	<u>*Simplify*</u>
23.	$y = \dfrac{x^3 + 6x}{3}$	$y = \dfrac{1}{3}x^3 + 2x$	$y' = \dfrac{1}{3}(3x^2) + 2$	$y' = x^2 + 2$
24.	$y = \dfrac{5x^2 - 3}{4}$	$y = \dfrac{5}{4}x^2 - \dfrac{3}{4}$	$y' = \dfrac{10}{4}x$	$y' = \dfrac{5x}{2}$
25.	$y = \dfrac{6}{7x^2}$	$y = \dfrac{6}{7}x^{-2}$	$y' = -\dfrac{12}{7}x^{-3}$	$y' = -\dfrac{12}{7x^3}$
26.	$y = \dfrac{10}{3x^3}$	$y = \dfrac{10}{3}x^{-3}$	$y' = -\dfrac{30}{3}x^{-4}$	$y' = -\dfrac{10}{x^4}$
27.	$y = \dfrac{4x^{3/2}}{x}$	$y = 4x^{1/2}, x > 0$	$y' = 2x^{-1/2}$	$y' = \dfrac{2}{\sqrt{x}}, x > 0$
28.	$y = \dfrac{2x}{x^{1/3}}$	$y = 2x^{2/3}$	$y' = \dfrac{4}{3}x^{-1/3}$	$y' = \dfrac{4}{3x^{1/3}}$

29. $f(x) = \dfrac{4 - 3x - x^2}{x^2 - 1}$

$$f'(x) = \frac{(x^2 - 1)(-3 - 2x) - (4 - 3x - x^2)(2x)}{(x^2 - 1)^2}$$
$$= \frac{-3x^2 + 3 - 2x^3 + 2x - 8x + 6x^2 + 2x^3}{(x^2 - 1)^2}$$
$$= \frac{3x^2 - 6x + 3}{(x^2 - 1)^2}$$
$$= \frac{3(x^2 - 2x + 1)}{(x^2 - 1)^2}$$
$$= \frac{3(x - 1)^2}{(x - 1)^2(x + 1)^2} = \frac{3}{(x + 1)^2},\ x \neq 1$$

30. $f(x) = \dfrac{x^2 + 5x + 6}{x^2 - 4}$

$$f'(x) = \frac{(x^2 - 4)(2x + 5) - (x^2 + 5x + 6)(2x)}{(x^2 - 4)^2}$$
$$= \frac{2x^3 + 5x^2 - 8x - 20 - 2x^3 - 10x^2 - 12x}{(x^2 - 4)^2}$$
$$= \frac{-5x^2 - 20x - 20}{(x^2 - 4)^2}$$
$$= \frac{-5(x^2 + 4x + 4)}{(x - 2)^2(x + 2)^2}$$
$$= \frac{-5(x + 2)^2}{(x - 2)^2(x + 2)^2}$$
$$= -\frac{5}{(x - 2)^2},\ x \neq 2, -2$$

Alternate solution:

$$f(x) = \frac{x^2 + 5x + 6}{x^2 - 4}$$
$$= \frac{(x + 3)(x + 2)}{(x + 2)(x - 2)}$$
$$= \frac{x + 3}{x - 2},\ x \neq -2$$
$$f'(x) = \frac{(x - 2)(1) - (x + 3)(1)}{(x - 2)^2}$$
$$= -\frac{5}{(x - 2)^2}$$

31. $f(x) = x\left(1 - \dfrac{4}{x + 3}\right) = x - \dfrac{4x}{x + 3}$

$$f'(x) = 1 - \frac{(x + 3)4 - 4x(1)}{(x + 3)^2}$$
$$= \frac{(x^2 + 6x + 9) - 12}{(x + 3)^2}$$
$$= \frac{x^2 + 6x - 3}{(x + 3)^2}$$

32. $f(x) = x^4\left[1 - \dfrac{2}{x + 1}\right] = x^4\left[\dfrac{x - 1}{x + 1}\right]$

$$f'(x) = x^4\left[\frac{(x + 1) - (x - 1)}{(x + 1)^2}\right] + \left[\frac{x - 1}{x + 1}\right](4x^3)$$
$$= x^4\left[\frac{2}{(x + 1)^2}\right] + \left[\frac{x^2 - 1}{(x + 1)^2}\right](4x^3)$$
$$= 2x^3\left[\frac{2x^2 + x - 2}{(x + 1)^2}\right]$$

33. $f(x) = \dfrac{3x - 1}{\sqrt{x}} = 3x^{1/2} - x^{-1/2}$

$$f'(x) = \frac{3}{2}x^{-1/2} + \frac{1}{2}x^{-3/2} = \frac{3x + 1}{2x^{3/2}}$$

Alternate solution:

$$f(x) = \frac{3x - 1}{\sqrt{x}} = \frac{3x - 1}{x^{1/2}}$$
$$f'(x) = \frac{x^{1/2}(3) - (3x - 1)\left(\frac{1}{2}\right)(x^{-1/2})}{x}$$
$$= \frac{\frac{1}{2}x^{-1/2}(3x + 1)}{x}$$
$$= \frac{3x + 1}{2x^{3/2}}$$

34. $f(x) = \sqrt[3]{x}(\sqrt{x} + 3) = x^{1/3}(x^{1/2} + 3)$

$$f'(x) = x^{1/3}\left(\frac{1}{2}x^{-1/2}\right) + (x^{1/2} + 3)\left(\frac{1}{3}x^{-2/3}\right)$$
$$= \frac{5}{6}x^{-1/6} + x^{-2/3}$$
$$= \frac{5}{6x^{1/6}} + \frac{1}{x^{2/3}}$$

Alternate solution:

$$f(x) = \sqrt[3]{x}(\sqrt{x} + 3) = x^{5/6} + 3x^{1/3}$$
$$f'(x) = \frac{5}{6}x^{-1/6} + x^{-2/3} = \frac{5}{6x^{1/6}} + \frac{1}{x^{2/3}}$$

35. $f(x) = \dfrac{2 - (1/x)}{x - 3} = \dfrac{2x - 1}{x(x - 3)} = \dfrac{2x - 1}{x^2 - 3x}$

$$f'(x) = \frac{(x^2 - 3x)2 - (2x - 1)(2x - 3)}{(x^2 - 3x)^2}$$
$$= \frac{2x^2 - 6x - 4x^2 + 8x - 3}{(x^2 - 3x)^2}$$
$$= \frac{-2x^2 + 2x - 3}{(x^2 - 3x)^2} = \frac{2x^2 - 2x + 3}{x^2(x - 3)^2}$$

36. $h(x) = \dfrac{\frac{1}{x^2} + 5x}{x + 1} = \dfrac{1 + 5x^3}{x^3 + x^2}$

$$h'(x) = \frac{(x^3 + x^2)(15x^2) - (1 + 5x^3)(3x^2 + 2x)}{(x^3 + x^2)^2}$$
$$= \frac{15x^5 + 15x^4 - 3x^2 - 2x - 15x^5 - 10x^4}{x^4(x + 1)^2}$$
$$= \frac{5x^4 - 3x^2 - 2x}{x^4(x + 1)^2}$$
$$= \frac{5x^3 - 3x - 2}{x^3(x + 1)^2}$$

37. $g(s) = s^3\left(5 - \dfrac{s}{s + 2}\right) = 5s^3 - \dfrac{s^4}{s + 2}$

$$g'(s) = 15s^2 - \frac{(s + 2)4s^3 - s^4(1)}{(s + 2)^2}$$
$$= 15s^2 - \frac{3s^4 + 8s^3}{(s + 2)^2}$$
$$= \frac{15s^2(s + 2)^2 - (3s^4 + 8s^3)}{(s + 2)^2}$$
$$= \frac{15s^2(s^2 + 4s + 4) - 3s^4 - 8s^3}{(s + 2)^2}$$
$$= \frac{12s^4 + 52s^3 + 60s^2}{(s + 2)^2}$$
$$= \frac{4s^2(3s^2 + 13s + 15)}{(s + 2)^2}$$

38. $g(x) = x^2\left(\dfrac{2}{x} - \dfrac{1}{x + 1}\right) = 2x - \dfrac{x^2}{x + 1}$

$$g'(x) = 2 - \frac{(x + 1)2x - x^2(1)}{(x + 1)^2} = \frac{2(x^2 + 2x + 1) - x^2 - 2x}{(x + 1)^2} = \frac{x^2 + 2x + 2}{(x + 1)^2}$$

39. $f(x) = (2x^3 + 5x)(x - 3)(x + 2)$

$$f'(x) = (6x^2 + 5)(x - 3)(x + 2) + (2x^3 + 5x)(1)(x + 2) + (2x^3 + 5x)(x - 3)(1)$$
$$= (6x^2 + 5)(x^2 - x - 6) + (2x^3 + 5x)(x + 2) + (2x^3 + 5x)(x - 3)$$
$$= (6x^4 + 5x^2 - 6x^3 - 5x - 36x^2 - 30) + (2x^4 + 4x^3 + 5x^2 + 10x) + (2x^4 + 5x^2 - 6x^3 - 15x)$$
$$= 10x^4 - 8x^3 - 21x^2 - 10x - 30$$

Note: You could simplify first: $f(x) = (2x^3 + 5x)(x^2 - x - 6)$

40. $f(x) = (x^3 - x)(x^2 + 2)(x^2 + x - 1)$

$$f'(x) = (3x^2 - 1)(x^2 + 2)(x^2 + x - 1) + (x^3 - x)(2x)(x^2 + x - 1) + (x^3 - x)(x^2 + 2)(2x + 1)$$
$$= (3x^4 + 5x^2 - 2)(x^2 + x - 1) + (2x^4 - 2x^2)(x^2 + x - 1) + (x^5 + x^3 - 2x)(2x + 1)$$
$$= (3x^6 + 5x^4 - 2x^2 + 3x^5 + 5x^3 - 2x - 3x^4 - 5x^2 + 2)$$
$$+ (2x^6 - 2x^4 + 2x^5 - 2x^3 - 2x^4 + 2x^2)$$
$$+ (2x^6 + 2x^4 - 4x^2 + x^5 + x^3 - 2x)$$
$$= 7x^6 + 6x^5 + 4x^3 - 9x^2 - 4x + 2$$

41. $f(t) = t^2 \sin t$

$f'(t) = t^2 \cos t + 2t \sin t = t(t \cos t + 2 \sin t)$

42. $f(\theta) = (\theta + 1)\cos\theta$

$f'(\theta) = (\theta + 1)(-\sin\theta) + (\cos\theta)(1)$

$= \cos\theta - (\theta + 1)\sin\theta$

43. $f(t) = \dfrac{\cos t}{t}$

$f'(t) = \dfrac{-t \sin t - \cos t}{t^2} = -\dfrac{t \sin t + \cos t}{t^2}$

44. $f(x) = \dfrac{\sin x}{x^3}$

$f'(x) = \dfrac{x^3 \cos x - \sin x(3x^2)}{(x^3)^2} = \dfrac{x \cos x - 3 \sin x}{x^4}$

45. $f(x) = -x + \tan x$

$f'(x) = -1 + \sec^2 x = \tan^2 x$

46. $y = x + \cot x$

$y' = 1 - \csc^2 x = -\cot^2 x$

47. $g(t) = \sqrt[4]{t} + 6 \csc t = t^{1/4} + 6 \csc t$

$g'(t) = \dfrac{1}{4}t^{-3/4} - 6 \csc t \cot t$

$= \dfrac{1}{4t^{3/4}} - 6 \csc t \cot t$

48. $h(x) = \dfrac{1}{x} - 12 \sec x = x^{-1} - 12 \sec x$

$h'(x) = -x^{-2} - 12 \sec x \tan x$

$= \dfrac{-1}{x^2} - 12 \sec x \tan x$

49. $y = \dfrac{3(1 - \sin x)}{2 \cos x} = \dfrac{3 - 3 \sin x}{2 \cos x}$

$y' = \dfrac{(-3 \cos x)(2 \cos x) - (3 - 3 \sin x)(-2 \sin x)}{(2 \cos x)^2}$

$= \dfrac{-6 \cos^2 x + 6 \sin x - 6 \sin^2 x}{4 \cos^2 x}$

$= \dfrac{3}{2}(-1 + \tan x \sec x - \tan^2 x)$

$= \dfrac{3}{2} \sec x(\tan x - \sec x)$

50. $y = \dfrac{\sec x}{x}$

$y' = \dfrac{x \sec x \tan x - \sec x}{x^2}$

$= \dfrac{\sec x(x \tan x - 1)}{x^2}$

51. $y = -\csc x - \sin x$

$y' = \csc x \cot x - \cos x$

$= \dfrac{\cos x}{\sin^2 x} - \cos x$

$= \cos x(\csc^2 x - 1)$

$= \cos x \cot^2 x$

52. $y = x \sin x + \cos x$

$y' = x \cos x + \sin x - \sin x = x \cos x$

53. $f(x) = x^2 \tan x$

$f'(x) = x^2 \sec^2 x + 2x \tan x = x(x \sec^2 x + 2 \tan x)$

54. $f(x) = \sin x \cos x$

$f'(x) = \sin x(-\sin x) + \cos x(\cos x) = \cos 2x$

55. $y = 2x \sin x + x^2 \cos x$

$y' = 2x \cos x + 2 \sin x + x^2(-\sin x) + 2x \cos x$

$= 4x \cos x + (2 - x^2) \sin x$

56. $h(\theta) = 5\theta \sec\theta + \theta \tan\theta$

$h'(\theta) = 5\theta \sec\theta \tan\theta + 5 \sec\theta + \theta \sec^2\theta + \tan\theta$

57. $g(x) = \left(\dfrac{x + 1}{x + 2}\right)(2x - 5)$

$g'(x) = \left(\dfrac{x + 1}{x + 2}\right)(2) + (2x - 5)\left[\dfrac{(x + 2)(1) - (x + 1)(1)}{(x + 2)^2}\right]$

$= \dfrac{2x^2 + 8x - 1}{(x + 2)^2}$

(Form of answer may vary.)

58. $f(x) = \dfrac{\cos x}{1 - \sin x}$

$$f'(x) = \frac{(1 - \sin x)(-\sin x) - (\cos x)(-\cos x)}{(1 - \sin x)^2}$$
$$= \frac{-\sin x + \sin^2 x + \cos^2 x}{(1 - \sin x)^2}$$
$$= \frac{1 - \sin x}{(1 - \sin x)^2}$$
$$= \frac{1}{1 - \sin x}$$

(Form of answer may vary.)

59. $y = \dfrac{1 + \csc x}{1 - \csc x}$

$$y' = \frac{(1 - \csc x)(-\csc x \cot x) - (1 + \csc x)(\csc x \cot x)}{(1 - \csc x)^2} = \frac{-2 \csc x \cot x}{(1 - \csc x)^2}$$

$$y'\left(\frac{\pi}{6}\right) = \frac{-2(2)\left(\sqrt{3}\right)}{(1 - 2)^2} = -4\sqrt{3}$$

60. $f(x) = \tan x \cot x = 1$

$f'(x) = 0$

$f'(1) = 0$

61. $h(t) = \dfrac{\sec t}{t}$

$$h'(t) = \frac{t(\sec t \tan t) - (\sec t)(1)}{t^2} = \frac{\sec t(t \tan t - 1)}{t^2}$$

$$h'(\pi) = \frac{\sec \pi(\pi \tan \pi - 1)}{\pi^2} = \frac{1}{\pi^2}$$

62. $f(x) = \sin x(\sin x + \cos x)$

$$f'(x) = \sin x(\cos x - \sin x) + (\sin x + \cos x)\cos x$$
$$= \sin x \cos x - \sin^2 x + \sin x \cos x + \cos^2 x$$
$$= \sin 2x + \cos 2x$$

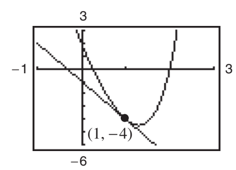
$$f'\left(\frac{\pi}{4}\right) = \sin \frac{\pi}{2} + \cos \frac{\pi}{2} = 1$$

63. (a) $f(x) = (x^3 + 4x - 1)(x - 2), \quad (1, -4)$

$$f'(x) = (x^3 + 4x - 1)(1) + (x - 2)(3x^2 + 4)$$
$$= x^3 + 4x - 1 + 3x^3 - 6x^2 + 4x - 8$$
$$= 4x^3 - 6x^2 + 8x - 9$$

$f'(1) = -3$; Slope at $(1, -4)$

Tangent line: $y + 4 = -3(x - 1) \Rightarrow y = -3x - 1$

(b) 

(c) Graphing utility confirms $\dfrac{dy}{dx} = -3$ at $(1, -4)$.

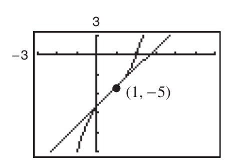
64. (a) $f(x) = (x - 2)(x^2 + 4), (1, -5)$

$$f'(x) = (x - 2)(2x) + (x^2 + 4)(1)$$
$$= 2x^2 - 4x + x^2 + 4$$
$$= 3x^2 - 4x + 4$$

$f'(1) = -3$; Slope at $(1, -5)$

Tangent line:

$y - (-5) = 3(x - 1) \Rightarrow y = 3x - 8$

(b) 

(c) Graphing utility confirms $\dfrac{dy}{dx} = 3$ at $(1, -5)$.

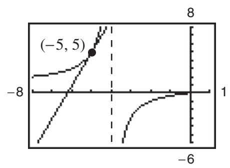
65. (a) $f(x) = \dfrac{x}{x+4}, \quad (-5, 5)$

$$f'(x) = \frac{(x+4)(1) - x(1)}{(x+4)^2} = \frac{4}{(x+4)^2}$$

$$f'(-5) = \frac{4}{(-5+4)^2} = 4; \quad \text{Slope at } (-5, 5)$$

Tangent line: $y - 5 = 4(x+5) \Rightarrow y = 4x + 25$

(b)



(c) Graphing utility confirms $\dfrac{dy}{dx} = 4$ at $(-5, 5)$.

66. (a) $f(x) = \dfrac{x+3}{x-3}, \quad (4, 7)$

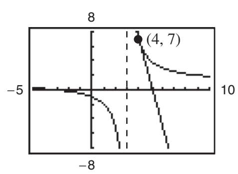
$$f'(x) = \frac{(x-3)(1) - (x+3)(1)}{(x-3)^2} = -\frac{6}{(x-3)^2}$$

$$f'(4) = \frac{-6}{1} = -6; \quad \text{Slope at } (4, 7)$$

Tangent line:

$y - 7 = -6(x-4) \Rightarrow y = -6x + 31$

(b)



(c) Graphing utility confirms $\dfrac{dy}{dx} = -6$ at $(4, 7)$.

67. (a) $f(x) = \tan x, \quad \left(\dfrac{\pi}{4}, 1\right)$

$$f'(x) = \sec^2 x$$

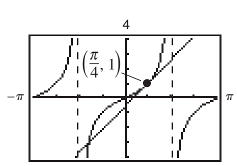
$$f'\left(\frac{\pi}{4}\right) = 2; \quad \text{Slope at } \left(\frac{\pi}{4}, 1\right)$$

Tangent line: $\quad y - 1 = 2\left(x - \dfrac{\pi}{4}\right)$

$$y - 1 = 2x - \frac{\pi}{2}$$

$$4x - 2y - \pi + 2 = 0$$

(b)



(c) Graphing utility confirms $\dfrac{dy}{dx} = 2$ at $\left(\dfrac{\pi}{4}, 1\right)$.

68. (a) $f(x) = \sec x, \quad \left(\dfrac{\pi}{3}, 2\right)$

$$f'(x) = \sec x \tan x$$

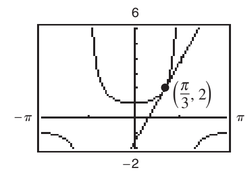
$$f'\left(\frac{\pi}{3}\right) = 2\sqrt{3}; \quad \text{Slope at } \left(\frac{\pi}{3}, 2\right)$$

Tangent line:

$$y - 2 = 2\sqrt{3}\left(x - \frac{\pi}{3}\right)$$

$$6\sqrt{3}x - 3y + 6 - 2\sqrt{3}\pi = 0$$

(b)



(c) Graphing utility confirms $\dfrac{dy}{dx} = 2\sqrt{3}$ at $\left(\dfrac{\pi}{3}, 2\right)$.

69. $f(x) = \dfrac{8}{x^2 + 4}; \quad (2, 1)$

$$f'(x) = \frac{(x^2+4)(0) - 8(2x)}{(x^2+4)^2} = \frac{-16x}{(x^2+4)^2}$$

$$f'(2) = \frac{-16(2)}{(4+4)^2} = -\frac{1}{2}$$

$$y - 1 = -\frac{1}{2}(x - 2)$$

$$y = -\frac{1}{2}x + 2$$

$$2y + x - 4 = 0$$

70. $f(x) = \dfrac{27}{x^2 + 9}; \quad \left(-3, \dfrac{3}{2}\right)$

$$f'(x) = \frac{(x^2+9)(0) - 27(2x)}{(x^2+9)^2} = \frac{-54x}{(x^2+9)^2}$$

$$f'(-3) = \frac{-54(-3)}{(9+9)^2} = \frac{1}{2}$$

$$y - \frac{3}{2} = \frac{1}{2}(x + 3)$$

$$y = \frac{1}{2}x + 3$$

$$2y - x - 6 = 0$$

71. $f(x) = \dfrac{16x}{x^2 + 16}; \quad \left(-2, -\dfrac{8}{5}\right)$

$$f'(x) = \frac{(x^2 + 16)(16) - 16x(2x)}{(x^2 + 16)^2} = \frac{256 - 16x^2}{(x^2 + 16)^2}$$

$$f'(-2) = \frac{256 - 16(4)}{20^2} = \frac{12}{25}$$

$$y + \frac{8}{5} = \frac{12}{25}(x + 2)$$

$$y = \frac{12}{25}x - \frac{16}{25}$$

$$25y - 12x + 16 = 0$$

72. $f(x) = \dfrac{4x}{x^2 + 6}; \left(2, \dfrac{4}{5}\right)$

$$f'(x) = \frac{(x^2 + 6)(4) - 4x(2x)}{(x^2 + 6)^2} = \frac{24 - 4x^2}{(x^2 + 6)^2}$$

$$f'(2) = \frac{24 - 16}{10^2} = \frac{2}{25}$$

$$y - \frac{4}{5} = \frac{2}{25}(x - 2)$$

$$y = \frac{2}{25}x + \frac{16}{25}$$

$$25y - 2x - 16 = 0$$

73. $f(x) = \dfrac{2x - 1}{x^2} = 2x^{-1} - x^{-2}$

$$f'(x) = -2x^{-2} + 2x^{-3} = \frac{2(-x + 1)}{x^3}$$

$f'(x) = 0$ when $x = 1$, and $f(1) = 1$.

Horizontal tangent at $(1, 1)$.

74. $f(x) = \dfrac{x^2}{x^2 + 1}$

$$f'(x) = \frac{(x^2 + 1)(2x) - (x^2)(2x)}{(x^2 + 1)^2}$$

$$= \frac{2x}{(x^2 + 1)^2}$$

$f'(x) = 0$ when $x = 0$.

Horizontal tangent is at $(0, 0)$.

75. $f(x) = \dfrac{x^2}{x - 1}$

$$f'(x) = \frac{(x - 1)(2x) - x^2(1)}{(x - 1)^2}$$

$$= \frac{x^2 - 2x}{(x - 1)^2} = \frac{x(x - 2)}{(x - 1)^2}$$

$f'(x) = 0$ when $x = 0$ or $x = 2$.

Horizontal tangents are at $(0, 0)$ and $(2, 4)$.

76. $f(x) = \dfrac{x - 4}{x^2 - 7}$

$$f'(x) = \frac{(x^2 - 7)(1) - (x - 4)(2x)}{(x^2 - 7)^2}$$

$$= \frac{x^2 - 7 - 2x^2 + 8x}{(x^2 - 7)^2}$$

$$= -\frac{x^2 - 8x + 7}{(x^2 - 7)^2} = -\frac{(x - 7)(x - 1)}{(x^2 - 7)^2}$$

$f'(x) = 0$ for $x = 1, 7$; $f(1) = \dfrac{1}{2}$, $f(7) = \dfrac{1}{14}$

f has horizontal tangents at $\left(1, \dfrac{1}{2}\right)$ and $\left(7, \dfrac{1}{14}\right)$.

77. $f(x) = \dfrac{x + 1}{x - 1}$

$$f'(x) = \frac{(x - 1) - (x + 1)}{(x - 1)^2} = \frac{-2}{(x - 1)^2}$$

$2y + x = 6 \Rightarrow y = -\dfrac{1}{2}x + 3$; Slope: $-\dfrac{1}{2}$

$$\frac{-2}{(x - 1)^2} = -\frac{1}{2}$$

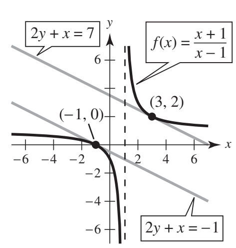
$$(x - 1)^2 = 4$$

$$x - 1 = \pm 2$$

$$x = -1, 3; \; f(-1) = 0, \; f(3) = 2$$

$$y - 0 = -\frac{1}{2}(x + 1) \Rightarrow y = -\frac{1}{2}x - \frac{1}{2}$$

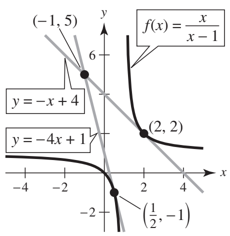
$$y - 2 = -\frac{1}{2}(x - 3) \Rightarrow y = -\frac{1}{2}x + \frac{7}{2}$$



78. $f(x) = \dfrac{x}{x-1}$

$$f'(x) = \frac{(x-1) - x}{(x-1)^2} = \frac{-1}{(x-1)^2}$$

Let $(x, y) = (x, x/(x-1))$ be a point of tangency on the graph of f.



$$\frac{5 - (x/(x-1))}{-1 - x} = \frac{-1}{(x-1)^2}$$

$$\frac{4x - 5}{(x-1)(x+1)} = \frac{1}{(x-1)^2}$$

$$(4x - 5)(x - 1) = x + 1$$

$$4x^2 - 10x + 4 = 0$$

$$(x - 2)(2x - 1) = 0 \Rightarrow x = \frac{1}{2}, 2$$

$$f\left(\frac{1}{2}\right) = -1, f(2) = 2; f'\left(\frac{1}{2}\right) = -4, f'(2) = -1$$

Two tangent lines:

$$y + 1 = -4\left(x - \frac{1}{2}\right) \Rightarrow y = -4x + 1$$

$$y - 2 = -1(x - 2) \Rightarrow y = -x + 4$$

79. $f'(x) = \dfrac{(x+2)3 - 3x(1)}{(x+2)^2} = \dfrac{6}{(x+2)^2}$

$$g'(x) = \frac{(x+2)5 - (5x+4)(1)}{(x+2)^2} = \frac{6}{(x+2)^2}$$

$$g(x) = \frac{5x+4}{(x+2)} = \frac{3x}{(x+2)} + \frac{2x+4}{(x+2)} = f(x) + 2$$

f and g differ by a constant.

80. $f'(x) = \dfrac{x(\cos x - 3) - (\sin x - 3x)(1)}{x^2} = \dfrac{x\cos x - \sin x}{x^2}$

$$g'(x) = \frac{x(\cos x + 2) - (\sin x + 2x)(1)}{x^2} = \frac{x\cos x - \sin x}{x^2}$$

$$g(x) = \frac{\sin x + 2x}{x} = \frac{\sin x - 3x + 5x}{x} = f(x) + 5$$

f and g differ by a constant.

81. (a) $p'(x) = f'(x)g(x) + f(x)g'(x)$

$$p'(1) = f'(1)g(1) + f(1)g'(1) = 1(4) + 6\left(-\frac{1}{2}\right) = 1$$

(b) $q'(x) = \dfrac{g(x)f'(x) - f(x)g'(x)}{g(x)^2}$

$$q'(4) = \frac{3(-1) - 7(0)}{3^2} = -\frac{1}{3}$$

82. (a) $p'(x) = f'(x)g(x) + f(x)g'(x)$

$$p'(4) = \frac{1}{2}(8) + 1(0) = 4$$

(b) $q'(x) = \dfrac{g(x)f'(x) - f(x)g'(x)}{g(x)^2}$

$$q'(7) = \frac{4(2) - 4(-1)}{4^2} = \frac{12}{16} = \frac{3}{4}$$

83. Area $= A(t) = (6t + 5)\sqrt{t} = 6t^{3/2} + 5t^{1/2}$

$$A'(t) = 9t^{1/2} + \frac{5}{2}t^{-1/2} = \frac{18t + 5}{2\sqrt{t}} \text{ cm}^2/\text{sec}$$

84. $V = \pi r^2 h = \pi(t + 2)\left(\frac{1}{2}\sqrt{t}\right) = \frac{1}{2}\left(t^{3/2} + 2t^{1/2}\right)\pi$

$$V'(t) = \frac{1}{2}\left(\frac{3}{2}t^{1/2} + t^{-1/2}\right)\pi = \frac{3t + 2}{4t^{1/2}}\pi \text{ cm}^3/\text{sec}$$

85. $C = 100\left(\dfrac{200}{x^2} + \dfrac{x}{x + 30}\right), \; 1 \le x$

$$\frac{dC}{dx} = 100\left(-\frac{400}{x^3} + \frac{30}{(x+30)^2}\right)$$

(a) When $x = 10$: $\dfrac{dC}{dx} = -\$38.13$ thousand/100 components

(b) When $x = 15$: $\dfrac{dC}{dx} = -\$10.37$ thousand/100 components

(c) When $x = 20$: $\dfrac{dC}{dx} = -\$3.80$ thousand/100 components

As the order size increases, the cost per item decreases.

86. $P(t) = 500\left[1 + \dfrac{4t}{50 + t^2}\right]$

$$P'(t) = 500\left[\frac{(50 + t^2)(4) - (4t)(2t)}{(50 + t^2)^2}\right] = 500\left[\frac{200 - 4t^2}{(50 + t^2)^2}\right] = 2000\left[\frac{50 - t^2}{(50 + t^2)^2}\right]$$

$P'(2) \approx 31.55$ bacteria/h

87. (a) $\sec x = \dfrac{1}{\cos x}$

$$\frac{d}{dx}[\sec x] = \frac{d}{dx}\left[\frac{1}{\cos x}\right] = \frac{(\cos x)(0) - (1)(-\sin x)}{(\cos x)^2} = \frac{\sin x}{\cos x \cos x} = \frac{1}{\cos x} \cdot \frac{\sin x}{\cos x} = \sec x \tan x$$

(b) $\csc x = \dfrac{1}{\sin x}$

$$\frac{d}{dx}[\csc x] = \frac{d}{dx}\left[\frac{1}{\sin x}\right] = \frac{(\sin x)(0) - (1)(\cos x)}{(\sin x)^2} = -\frac{\cos x}{\sin x \sin x} = -\frac{1}{\sin x} \cdot \frac{\cos x}{\sin x} = -\csc x \cot x$$

(c) $\cot x = \dfrac{\cos x}{\sin x}$

$$\frac{d}{dx}[\cot x] = \frac{d}{dx}\left[\frac{\cos x}{\sin x}\right] = \frac{\sin x(-\sin x) - (\cos x)(\cos x)}{(\sin x)^2} = -\frac{\sin^2 x + \cos^2 x}{\sin^2 x} = -\frac{1}{\sin^2 x} = -\csc^2 x$$

88. $f(x) = \sec x$

$g(x) = \csc x,\ [0, 2\pi)$

$f'(x) = g'(x)$

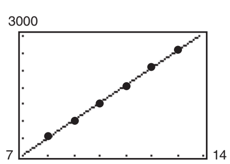
$$\sec x \tan x = -\csc x \cot x \Rightarrow \frac{\sec x \tan x}{\csc x \cot x} = -1 \Rightarrow \frac{\frac{1}{\cos x} \cdot \frac{\sin x}{\cos x}}{\frac{1}{\sin x} \cdot \frac{\cos x}{\sin x}} = -1 \Rightarrow \frac{\sin^3 x}{\cos^3 x} = -1 \Rightarrow \tan^3 x = -1 \Rightarrow \tan x = -1$$

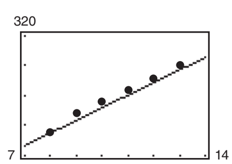
$x = \dfrac{3\pi}{4}, \dfrac{7\pi}{4}$

89. (a) $h(t) = 101.7t + 1593$

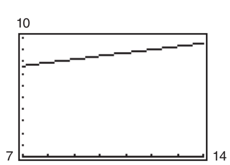
$p(t) = 2.1t + 287$

(b)





(c) $A = \dfrac{101.7t + 1593}{2.1t + 287}$



A represents the average health care expenditures per person (in thousands of dollars).

(d) $A'(t) \approx \dfrac{25{,}842.6}{4.41t^2 + 1205.4t + 82{,}369}$

$A'(t)$ represents the rate of change of the average health care expenditures per person for the given year t.

90. (a) $\sin\theta = \dfrac{r}{r + h}$

$r + h = r\csc\theta$

$h = r\csc\theta - r = r(\csc\theta - 1)$

(b) $h'(\theta) = r(-\csc\theta \cdot \cot\theta)$

$h'(30^\circ) = h'\left(\dfrac{\pi}{6}\right)$

$= -6371\left(2 \cdot \sqrt{3}\right) = -12{,}742\sqrt{3}$ km/rad

91. $f(x) = x^2 + 7x - 4$

$f'(x) = 2x + 7$

$f''(x) = 2$

92. $f(x) = 4x^5 - 2x^3 + 5x^2$

$f'(x) = 20x^4 - 6x^2 + 10x$

$f''(x) = 80x^3 - 12x + 10$

93. $f(x) = 4x^{3/2}$

$f'(x) = 6x^{1/2}$

$f''(x) = 3x^{-1/2} = \dfrac{3}{\sqrt{x}}$

94. $f(x) = x^2 + 3x^{-3}$

$f'(x) = 2x - 9x^{-4}$

$f''(x) = 2 + 36x^{-5} = 2 + \dfrac{36}{x^5}$

95. $f(x) = \dfrac{x}{x - 1}$

$f'(x) = \dfrac{(x - 1)(1) - x(1)}{(x - 1)^2} = \dfrac{-1}{(x - 1)^2}$

$f''(x) = \dfrac{2}{(x - 1)^3}$

96. $f(x) = \dfrac{x^2 + 3x}{x - 4}$

$$f'(x) = \frac{(x - 4)(2x + 3) - (x^2 + 3x)(1)}{(x - 4)^2}$$

$$= \frac{2x^2 - 5x - 12 - x^2 - 3x}{(x - 4)^2} = \frac{x^2 - 8x - 12}{x^2 - 8x + 16}$$

$$f''(x) = \frac{(x - 4)^2(2x - 8) - (x^2 - 8x - 12)(2x - 8)}{(x - 4)^4}$$

$$= \frac{(x - 4)\left[(x - 4)(2x - 8) - 2(x^2 - 8x - 12)\right]}{(x - 4)^4}$$

$$= \frac{(x - 4)(2x - 8) - 2(x^2 - 8x - 12)}{(x - 4)^3}$$

$$= \frac{2x^2 - 16x + 32 - 2x^2 + 16x + 24}{(x - 4)^3}$$

$$= \frac{56}{(x - 4)^3}$$

97. $f(x) = x \sin x$

$f'(x) = x \cos x + \sin x$

$f''(x) = x(-\sin x) + \cos x + \cos x$

$= -x \sin x + 2 \cos x$

98. $f(x) = x \cos x$

$f'(x) = \cos x - x \sin x$

$f''(x) = -\sin x - (\sin x + x \cos x) = -x \cos x - 2 \sin x$

99. $f(x) = \csc x$

$f'(x) = -\csc x + \cot x$

$f''(x) = -\csc x(-\csc^2 x) - \cot x(-\csc x \cot x)$

$= \csc^3 x + \cot^2 x \csc x$

100. $f(x) = \sec x$

$f'(x) = \sec x \tan x$

$f''(x) = \sec x(\sec^2 x) + \tan x(\sec x \tan x)$

$= \sec x(\sec^2 x + \tan^2 x)$

101. $f'(x) = x^3 - x^{2/5}$

$f''(x) = 3x^2 - \dfrac{2}{5}x^{-3/5}$

$f'''(x) = 6x + \dfrac{6}{25}x^{-8/5} = 6x + \dfrac{6}{25x^{8/5}}$

102. $f^{(3)}(x) = \sqrt[5]{x^4} = x^{4/5}$

$f^{(4)}(x) = \dfrac{4}{5}x^{-1/5} = \dfrac{4}{5x^{1/5}}$

103. $f''(x) = -\sin x$

$f^{(3)}(x) = -\cos x$

$f^{(4)}(x) = \sin x$

$f^{(5)}(x) = \cos x$

$f^{(6)}(x) = -\sin x$

$f^{(7)}(x) = -\cos x$

$f^{(8)}(x) = \sin x$

104. $f^{(4)}(t) = t \cos t$

$f^{(5)}(t) = \cos t - t \sin t$

105. $f(x) = 2g(x) + h(x)$

$f'(x) = 2g'(x) + h'(x)$

$f'(2) = 2g'(2) + h'(2)$

$= 2(-2) + 4$

$= 0$

106. $f(x) = 4 - h(x)$

$f'(x) = -h'(x)$

$f'(2) = -h'(2) = -4$

107. $f(x) = \dfrac{g(x)}{h(x)}$

$f'(x) = \dfrac{h(x)g'(x) - g(x)h'(x)}{[h(x)]^2}$

$f'(2) = \dfrac{h(2)g'(2) - g(2)h'(2)}{[h(2)]^2}$

$= \dfrac{(-1)(-2) - (3)(4)}{(-1)^2}$

$= -10$

108. $f(x) = g(x)h(x)$

$f'(x) = g(x)h'(x) + h(x)g'(x)$

$f'(2) = g(2)h'(2) + h(2)g'(2)$

$= (3)(4) + (-1)(-2)$

$= 14$

109. Polynomials of degree $n - 1$ (or lower) satisfy $f^{(n)}(x) = 0$. The derivative of a polynomial of the 0th degree (a constant) is 0.

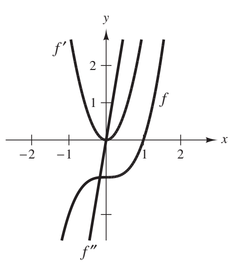
110. To differentiate a piecewise function, separate the function into its pieces, and differentiate each piece.

If $f(x) = x|x|$, then on $(-\infty, 0)$ you have $f(x) = -x^2$, $f'(x) = -2x$, and $f''(x) = -2$.

On $(0, \infty)$ you have $f(x) = x^2$, $f'(x) = 2x$, and $f''(x) = 2$.

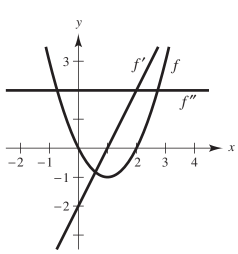
Notice that $f(0) = 0$, $f'(0) = 0$, but $f''(0)$ does not exist (the left-hand limit is -2, whereas the right-hand limit is 2).

111.



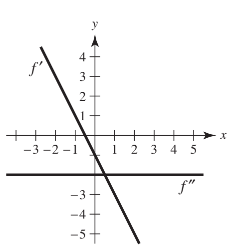
It appears that f is cubic, so f' would be quadratic and f'' would be linear.

112.

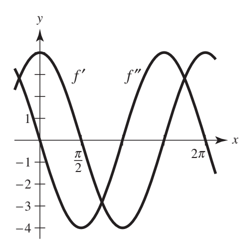


It appears that f is quadratic so f' would be linear and f'' would be constant.

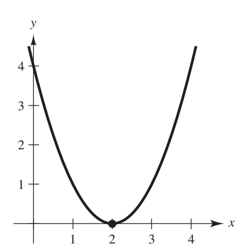
113.



114.

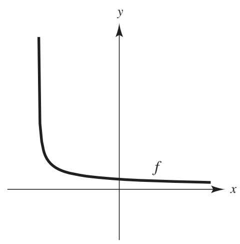


115. The graph of a differentiable function f such that $f(2) = 0$, $f' < 0$ for $-\infty < x < 2$, and $f' > 0$ for $2 < x < \infty$ would, in general, look like the graph below.



One such function is $f(x) = (x - 2)^2$.

116. The graph of a differentiable function f such that $f > 0$ and $f' < 0$ for all real numbers x would, in general, look like the graph below.



117. $v(t) = 36 - t^2, 0 \le t \le 6$

$a(t) = v'(t) = -2t$

$v(3) = 27$ m/sec

$a(3) = -6$ m/sec^2

The speed of the object is decreasing.

118. $v(t) = \dfrac{60t}{2t + 15}$

$$a(t) = v'(t) = \frac{(2t + 15)(60) - (60t)(2)}{(2t + 15)^2} = \frac{900}{(2t + 15)^2}$$

(a) $a(5) = \dfrac{900}{[2(5) + 15]^2} = 1.44$ m/sec^2

(b) $a(10) = \dfrac{900}{[2(10) + 15]^2} \approx 0.73$ m/sec^2

(c) $a(20) = \dfrac{900}{[2(20) + 15]^2} \approx 0.30$ m/sec^2

119. $s(t) = -2.5t^2 + 20t$

$v(t) = s'(t) = -5t + 20$

$a(t) = v'(t) = -5$

t (sec)	0	1	2	3	4
$s(t)$ (m)	0	17.5	30	37.5	40
$v(t) = s'(t)$ (m/sec)	20	15	10	5	0
$a(t) = v'(t)$ (m/sec^2)	–5	–5	–5	–5	–5

Average velocity on:

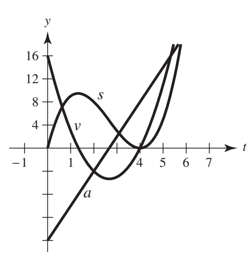
$[0, 1]$ is $\dfrac{17.5 - 0}{1 - 0} = 17.5$

$[1, 2]$ is $\dfrac{30 - 17.5}{2 - 1} = 12.5$

$[2, 3]$ is $\dfrac{37.5 - 30}{3 - 2} = 7.5$

$[3, 4]$ is $\dfrac{40 - 37.5}{4 - 3} = 2.5$

120. (a)

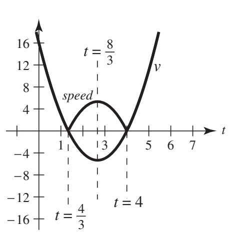


s position function

v velocity function

a acceleration function

(b) The speed of the particle is the absolute value of its velocity. So, the particle's speed is slowing down on the intervals $(0, 4/3)$ and $(8/3, 4)$ and it speeds up on the intervals $(4/3, 8/3)$ and $(4, 6)$.



121. $f(x) = x^n$

$f^{(n)}(x) = n(n-1)(n-2)\cdots(2)(1) = n!$

Note: $n! = n(n-1)\cdots 3\cdot 2\cdot 1$ (read "n factorial")

122. $f(x) = \dfrac{1}{x}$

$$f^{(n)}(x) = \frac{(-1)^n(n)(n-1)(n-2)\cdots(2)(1)}{x^{n+1}} = \frac{(-1)^n n!}{x^{n+1}}$$

123. $f(x) = g(x)h(x)$

(a)
$$\begin{aligned}
f'(x) &= g(x)h'(x) + h(x)g'(x)\\
f''(x) &= g(x)h''(x) + g'(x)h'(x) + h(x)g''(x) + h'(x)g'(x)\\
&= g(x)h''(x) + 2g'(x)h'(x) + h(x)g''(x)\\
f'''(x) &= g(x)h'''(x) + g'(x)h''(x) + 2g'(x)h''(x) + 2g''(x)h'(x) + h(x)g'''(x) + h'(x)g''(x)\\
&= g(x)h'''(x) + 3g'(x)h''(x) + 3g''(x)h'(x) + g'''(x)h(x)\\
f^{(4)}(x) &= g(x)h^{(4)}(x) + g'(x)h'''(x) + 3g'(x)h'''(x) + 3g''(x)h''(x) + 3g''(x)h''(x) + 3g'''(x)h'(x)\\
&\quad + g'''(x)h'(x) + g^{(4)}(x)h(x)\\
&= g(x)h^{(4)}(x) + 4g'(x)h'''(x) + 6g''(x)h''(x) + 4g'''(x)h'(x) + g^{(4)}(x)h(x)
\end{aligned}$$

(b)
$$\begin{aligned}
f^{(n)}(x) &= g(x)h^{(n)}(x) + \frac{n(n-1)(n-2)\cdots(2)(1)}{1[(n-1)(n-2)\cdots(2)(1)]}g'(x)h^{(n-1)}(x) + \frac{n(n-1)(n-2)\cdots(2)(1)}{(2)(1)[(n-2)(n-3)\cdots(2)(1)]}g''(x)h^{(n-2)}(x)\\
&\quad + \frac{n(n-1)(n-2)\cdots(2)(1)}{(3)(2)(1)[(n-3)(n-4)\cdots(2)(1)]}g'''(x)h^{(n-3)}(x) + \cdots\\
&\quad + \frac{n(n-1)(n-2)\cdots(2)(1)}{[(n-1)(n-2)\cdots(2)(1)](1)}g^{(n-1)}(x)h'(x) + g^{(n)}(x)h(x)\\
&= g(x)h^{(n)}(x) + \frac{n!}{1!(n-1)!}g'(x)h^{(n-1)}(x) + \frac{n!}{2!(n-2)!}g''(x)h^{(n-2)}(x) + \cdots\\
&\quad + \frac{n!}{(n-1)!1!}g^{(n-1)}(x)h'(x) + g^{(n)}(x)h(x)
\end{aligned}$$

Note: $n! = n(n-1)\cdots 3\cdot 2\cdot 1$ (read "n factorial")

124. $[xf(x)]' = xf'(x) + f(x)$

$[xf(x)]'' = xf''(x) + f'(x) + f'(x) = xf''(x) + 2f'(x)$

$[xf(x)]''' = xf'''(x) + f''(x) + 2f''(x) = xf'''(x) + 3f''(x)$

In general, $[xf(x)]^{(n)} = xf^{(n)}(x) + nf^{(n-1)}(x)$.

125. $f(x) = x^n \sin x$

$f'(x) = x^n \cos x + nx^{n-1}\sin x$

When $n = 1$: $f'(x) = x\cos x + \sin x$

When $n = 2$: $f'(x) = x^2\cos x + 2\sin x$

When $n = 3$: $f'(x) = x^3\cos x + 3x^2\sin x$

When $n = 4$: $f'(x) = x^4\cos x + 4x^3\sin x$

For general n, $f'(x) = x^n\cos x + nx^{n-1}\sin x$.

126. $f(x) = \dfrac{\cos x}{x^n} = x^{-n}\cos x$

$$\begin{aligned} f'(x) &= -x^{-n}\sin x - nx^{-n-1}\cos x \\ &= -x^{-n-1}(x\sin x + n\cos x) \\ &= -\frac{x\sin x + n\cos x}{x^{n+1}} \end{aligned}$$

When $n = 1$: $f'(x) = -\dfrac{x\sin x + \cos x}{x^2}$

When $n = 2$: $f'(x) = -\dfrac{x\sin x + 2\cos x}{x^3}$

When $n = 3$: $f'(x) = -\dfrac{x\sin x + 3\cos x}{x^4}$

When $n = 4$: $f'(x) = -\dfrac{x\sin x + 4\cos x}{x^5}$

For general n, $f'(x) = -\dfrac{x\sin x + n\cos x}{x^{n+1}}$.

127. $y = \dfrac{1}{x},\ y' = -\dfrac{1}{x^2},\ y'' = \dfrac{2}{x^3}$

$$x^3y'' + 2x^2y' = x^3\left[\frac{2}{x^3}\right] + 2x^2\left[-\frac{1}{x^2}\right] = 2 - 2 = 0$$

128.

$$\begin{aligned} y &= 2x^3 - 6x + 10 \\ y' &= 6x^2 - 6 \\ y'' &= 12x \\ y''' &= 12 \end{aligned}$$

$$-y''' - xy'' - 2y' = -12 - x(12x) - 2(6x^2 - 6) = -24x^2$$

129.

$$\begin{aligned} y &= 2\sin x + 3 \\ y' &= 2\cos x \\ y'' &= -2\sin x \\ y'' + y &= -2\sin x + (2\sin x + 3) = 3 \end{aligned}$$

130.

$$\begin{aligned} y &= 3\cos x + \sin x \\ y' &= -3\sin x + \cos x \\ y'' &= -3\cos x - \sin x \\ y'' + y &= (-3\cos x - \sin x) + (3\cos x + \sin x) = 0 \end{aligned}$$

131. False. If $y = f(x)g(x)$, then

$$\frac{dy}{dx} = f(x)g'(x) + g(x)f'(x).$$

132. True. y is a fourth-degree polynomial.

$$\frac{d^ny}{dx^n} = 0 \text{ when } n > 4.$$

133. True

$$\begin{aligned} h'(c) &= f(c)g'(c) + g(c)f'(c) \\ &= f(c)(0) + g(c)(0) \\ &= 0 \end{aligned}$$

134. True

135. True

136. True

137.

$$\begin{aligned} \frac{d}{dx}\left[f(x)g(x)h(x)\right] &= \frac{d}{dx}\left[\left(f(x)g(x)\right)h(x)\right] \\ &= \frac{d}{dx}\left[f(x)g(x)\right]h(x) + f(x)g(x)h'(x) \\ &= \left[f(x)g'(x) + g(x)f'(x)\right]h(x) + f(x)g(x)h'(x) \\ &= f'(x)g(x)h(x) + f(x)g'(x)h(x) + f(x)g(x)h'(x) \end{aligned}$$

138. (a)

$$\begin{aligned} (fg' - f'g)' &= fg'' + f'g' - f'g' - f''g \\ &= fg'' - f''g \qquad \text{True} \end{aligned}$$

(b)

$$\begin{aligned} (fg)'' &= (fg' + f'g)' \\ &= fg'' + f'g' + f'g' + f''g \\ &= fg'' + 2f'g' + f''g \\ &\neq fg'' + f''g \qquad \text{False} \end{aligned}$$

Section 2.4 The Chain Rule

1. To find the derivative of the composition of two differentiable functions, take the derivative of the outer function and keep the inner function the same. Then multiply this by the derivative of the inner function.

$$\left[f(g(x))\right]' = f'(g(x))\,g'(x)$$

2. The (Simple) Power Rule is $\dfrac{d}{dx}(x^n) = nx^{n-1}$. The General Power Rule uses the Chain Rule:

$$\frac{d}{dx}(u^n) = nu^{n-1}\frac{du}{dx}.$$

$y = f(g(x))$	$u = g(x)$	$y = f(u)$
3. $y = (6x - 5)^4$	$u = 6x - 5$	$y = u^4$
4. $y = \sqrt[3]{4x + 3}$	$u = 4x + 3$	$y = u^{1/3}$
5. $y = \dfrac{1}{3x + 5}$	$u = 3x + 5$	$y = \dfrac{1}{u}$
6. $y = \dfrac{2}{\sqrt{x^2 + 10}}$	$u = x^2 + 10$	$y = \dfrac{2}{\sqrt{u}}$
7. $y = \csc^3 x$	$u = \csc x$	$y = u^3$
8. $y = \sin \dfrac{5x}{2}$	$u = \dfrac{5x}{2}$	$y = \sin u$

9. $y = (2x - 7)^3$

$$y' = 3(2x - 7)^2(2) = 6(2x - 7)^2$$

10. $y = 5(2 - x^3)^4$

$$y' = 5(4)(2 - x^3)^3(-3x^2) = -60x^2(2 - x^3)^3 = 60x^2(x^3 - 2)^3$$

11. $g(x) = 3(4 - 9x)^{5/6}$

$$g'(x) = 3\left(\frac{5}{6}\right)(4 - 9x)^{-1/6}(-9) = \frac{-45}{2}(4 - 9x)^{-1/6} = -\frac{45}{2(4 - 9x)^{1/6}}$$

12. $f(t) = (9t + 2)^{2/3}$

$$f'(t) = \frac{2}{3}(9t + 2)^{-1/3}(9) = \frac{6}{\sqrt[3]{9t + 2}}$$

13. $h(s) = -2\sqrt{5s^2 + 3} = -2(5s^2 + 3)^{1/2}$

$$h'(s) = -2\left(\frac{1}{2}\right)(5s^2 + 3)^{-1/2}(10s) = \frac{-10s}{(5s^2 + 3)^{1/2}} = -\frac{10s}{\sqrt{5s^2 + 3}}$$

14. $g(x) = \sqrt{4 - 3x^2} = (4 - 3x^2)^{1/2}$

$$g'(x) = \frac{1}{2}(4 - 3x^2)^{-1/2}(-6x) = -\frac{3x}{\sqrt{4 - 3x^2}}$$

15. $y = \sqrt[3]{6x^2 + 1} = (6x^2 + 1)^{1/3}$

$$y' = \frac{1}{3}(6x^2 + 1)^{-2/3}(12x) = \frac{4x}{(6x^2 + 1)^{2/3}} = \frac{4x}{\sqrt[3]{(6x^2 + 1)^2}}$$

16. $y = 2\sqrt[4]{9 - x^2} = 2(9 - x^2)^{1/4}$

$$y' = 2\left(\frac{1}{4}\right)(9 - x^2)^{-3/4}(-2x) = \frac{-x}{(9 - x^2)^{3/4}} = \frac{-x}{\sqrt[4]{(9 - x^2)^3}}$$

17. $y = (x - 2)^{-1}$

$$y' = -1(x - 2)^{-2}(1) = \frac{-1}{(x - 2)^2}$$

18. $s(t) = \dfrac{1}{4 - 5t - t^2} = (4 - 5t - t^2)^{-1}$

$$s'(t) = -(4 - 5t - t^2)^{-2}(-5 - 2t) = \frac{5 + 2t}{(4 - 5t - t^2)^2} = \frac{2t + 5}{(t^2 + 5t - 4)^2}$$

19. $g(s) = \dfrac{6}{(s^3 - 2)^3} = 6(s^3 - 2)^{-3}$

$$g'(s) = 6(-3)(s^3 - 2)^{-4}(3s^2) = -\frac{54s^2}{(s^3 - 2)^4}$$

20. $y = -\dfrac{3}{(t-2)^4} = -3(t-2)^{-4}$

$y' = 12(t-2)^{-5} = \dfrac{12}{(t-2)^5}$

21. $y = \dfrac{1}{\sqrt{3x+5}} = (3x+5)^{-1/2}$

$y' = -\dfrac{1}{2}(3x+5)^{-3/2}(3)$

$= \dfrac{-3}{2(3x+5)^{3/2}}$

$= -\dfrac{3}{2\sqrt{(3x+5)^3}}$

22. $g(t) = \dfrac{1}{\sqrt{t^2-2}} = (t^2-2)^{-1/2}$

$g'(t) = -\dfrac{1}{2}(t^2-2)^{-3/2}(2t)$

$= \dfrac{-t}{(t^2-2)^{3/2}}$

$= -\dfrac{t}{\sqrt{(t^2-2)^3}}$

23. $f(x) = x^2(x-2)^7$

$f'(x) = 2x(x-2)^7 + 7(x-2)^6x^2$

$= x(x-2)^6[2(x-2)+7x]$

$= x(x-2)^6(9x-4)$

24. $f(x) = x(2x-5)^3$

$f'(x) = x(3)(2x-5)^2(2) + (2x-5)^3(1)$

$= (2x-5)^2[6x + (2x-5)]$

$= (2x-5)^2(8x-5)$

25. $y = x\sqrt{1-x^2} = x(1-x^2)^{1/2}$

$y' = x\left[\dfrac{1}{2}(1-x^2)^{-1/2}(-2x)\right] + (1-x^2)^{1/2}(1)$

$= -x^2(1-x^2)^{-1/2} + (1-x^2)^{1/2}$

$= (1-x^2)^{-1/2}\left[-x^2 + (1-x^2)\right]$

$= \dfrac{1-2x^2}{\sqrt{1-x^2}}$

26. $y = x^2\sqrt{16-x^2} = x^2(16-x^2)^{1/2}$

$y' = 2x(16-x^2)^{1/2} + \dfrac{1}{2}(16-x^2)^{-1/2}(-2x)x^2$

$= \dfrac{x}{(16-x^2)^{1/2}}\left[2(16-x^2) - x^2\right]$

$= \dfrac{x(32-3x^2)}{\sqrt{16-x^2}}$

27. $y = \dfrac{x}{\sqrt{x^2+1}} = \dfrac{x}{(x^2+1)^{1/2}}$

$y' = \dfrac{(x^2+1)^{1/2}(1) - x\left(\frac{1}{2}\right)(x^2+1)^{-1/2}(2x)}{\left[(x^2+1)^{1/2}\right]^2}$

$= \dfrac{(x^2+1)^{1/2} - x^2(x^2+1)^{-1/2}}{x^2+1}$

$= \dfrac{(x^2+1)^{-1/2}\left[x^2+1-x^2\right]}{x^2+1}$

$= \dfrac{1}{(x^2+1)^{3/2}} = \dfrac{1}{\sqrt{(x^2+1)^3}}$

28. $y = \dfrac{x}{\sqrt{x^4+4}}$

$y' = \dfrac{(x^4+4)^{1/2}(1) - x\frac{1}{2}(x^4+4)^{-1/2}(4x^3)}{x^4+4}$

$= \dfrac{x^4+4-2x^4}{(x^4+4)^{3/2}} = \dfrac{4-x^4}{(x^4+4)^{3/2}} = \dfrac{4-x^4}{\sqrt{(x^4+4)^3}}$

29. $g(x) = \left(\dfrac{x+5}{x^2+2}\right)^2$

$g'(x) = 2\left(\dfrac{x+5}{x^2+2}\right)\left(\dfrac{(x^2+2)-(x+5)(2x)}{(x^2+2)^2}\right)$

$= \dfrac{2(x+5)(2-10x-x^2)}{(x^2+2)^3}$

$= \dfrac{-2(x+5)(x^2+10x-2)}{(x^2+2)^3}$

30. $h(t) = \left(\dfrac{t^2}{t^3 + 2}\right)^2$

$$h'(t) = 2\left(\frac{t^2}{t^3 + 2}\right)\left(\frac{(t^3 + 2)(2t) - t^2(3t^2)}{(t^3 + 2)^2}\right)$$

$$= \frac{2t^2(4t - t^4)}{(t^3 + 2)^3} = \frac{2t^3(4 - t^3)}{(t^3 + 2)^3}$$

31. $s(t) = \left(\dfrac{1 + t}{t + 3}\right)^4$

$$s'(t) = 4\left(\frac{1 + t}{t + 3}\right)^3\left[\frac{(t + 3)(1) - (1 + t)(1)}{(t + 3)^2}\right]$$

$$= \frac{4(1 + t)^3(2)}{(t + 3)^5}$$

$$= \frac{8(t + 1)^3}{(t + 3)^5}$$

32. $g(x) = \left(\dfrac{3x^2 - 2}{2x + 3}\right)^{-2} = \left(\dfrac{2x + 3}{3x^2 - 2}\right)^2$

$$g'(x) = 2\left(\frac{2x + 3}{3x^2 - 2}\right)\left[\frac{(3x^2 - 2)(2) - (2x + 3)(6x)}{(3x^2 - 2)^2}\right]$$

$$= \frac{2(2x + 3)(-6x^2 - 18x - 4)}{(3x^2 - 2)^3}$$

$$= -\frac{4(2x + 3)(3x^2 + 9x + 2)}{(3x^2 - 2)^3}$$

33. $f(x) = \left((x^2 + 3)^5 + x\right)^2$

$$f'(x) = 2\left((x^2 + 3)^5 + x\right)\left(5(x^2 + 3)^4(2x) + 1\right)$$

$$= 2\left[10x(x^2 + 3)^9 + (x^2 + 3)^5 + 10x^2(x^2 + 3)^4 + x\right] = 20x(x^2 + 3)^9 + 2(x^2 + 3)^5 + 20x^2(x^2 + 3)^4 + 2x$$

34. $g(x) = \left(2 + (x^2 + 1)^4\right)^3$

$$g'(x) = 3\left(2 + (x^2 + 1)^4\right)^2\left(4(x^2 + 1)^3(2x)\right) = 24x(x^2 + 1)^3\left(2 + (x^2 + 1)^4\right)^2$$

35. $y = \cos 4x$

$$\frac{dy}{dx} = -4\sin 4x$$

36. $y = \sin \pi x$

$$\frac{dy}{dx} = \pi \cos \pi x$$

37. $g(x) = 5\tan 3x$

$$g'(x) = 15\sec^2 3x$$

38. $h(x) = \sec 6x$

$$h'(x) = \sec 6x \tan 6x\,(6)$$

$$= 6\sec 6x \tan 6x$$

39. $y = \sin(\pi x)^2 = \sin(\pi^2 x^2)$

$$y' = \cos(\pi^2 x^2)\left[2\pi^2 x\right] = 2\pi^2 x \cos(\pi^2 x^2)$$

$$= 2\pi^2 x \cos(\pi x)^2$$

40. $y = \csc(1 - 2x)^2$

$$y' = -\csc(1 - 2x)^2 \cot(1 - 2x)^2 \left[2(1 - 2x)(-2)\right]$$

$$= 4(1 - 2x)\csc(1 - 2x)^2 \cot(1 - 2x)^2$$

41. $h(x) = \sin 2x \cos 2x$

$$h'(x) = \sin 2x(-2\sin 2x) + \cos 2x(2\cos 2x)$$

$$= 2\cos^2 2x - 2\sin^2 2x$$

$$= 2\cos 4x$$

Alternate solution: $h(x) = \frac{1}{2}\sin 4x$

$$h'(x) = \tfrac{1}{2}\cos 4x(4) = 2\cos 4x$$

42. $g(\theta) = \sec \frac{1}{2}\theta \tan \frac{1}{2}\theta$

$$g'(\theta) = \sec\left(\tfrac{1}{2}\theta\right)\sec^2\left(\tfrac{1}{2}\theta\right)\tfrac{1}{2} + \tan\left(\tfrac{1}{2}\theta\right)\sec\left(\tfrac{1}{2}\theta\right)\tan\left(\tfrac{1}{2}\theta\right)\tfrac{1}{2}$$

$$= \tfrac{1}{2}\sec\left(\tfrac{1}{2}\theta\right)\left[\sec^2\left(\tfrac{1}{2}\theta\right) + \tan^2\left(\tfrac{1}{2}\theta\right)\right]$$

43. $f(x) = \dfrac{\cot x}{\sin x} = \dfrac{\cos x}{\sin^2 x}$

$f'(x) = \dfrac{\sin^2 x(-\sin x) - \cos x(2\sin x\cos x)}{\sin^4 x}$

$= \dfrac{-\sin^2 x - 2\cos^2 x}{\sin^3 x} = \dfrac{-1 - \cos^2 x}{\sin^3 x}$

44. $g(v) = \dfrac{\cos v}{\csc v} = \cos v \cdot \sin v$

$g'(v) = \cos v(\cos v) + \sin v(-\sin v)$

$= \cos^2 v - \sin^2 v = \cos 2v$

45. $y = 4\sec^2 x$

$y' = 8\sec x \cdot \sec x\tan x = 8\sec^2 x\tan x$

46. $g(t) = 5\cos^2 \pi t = 5(\cos \pi t)^2$

$g'(t) = 10\cos \pi t(-\sin \pi t)(\pi)$

$= -10\pi(\sin \pi t)(\cos \pi t)$

$= -5\pi\sin 2\pi t$

47. $f(\theta) = \frac{1}{4}\sin^2 2\theta = \frac{1}{4}(\sin 2\theta)^2$

$f'(\theta) = 2\left(\frac{1}{4}\right)(\sin 2\theta)(\cos 2\theta)(2)$

$= \sin 2\theta\cos 2\theta = \frac{1}{2}\sin 4\theta$

48. $h(t) = 2\cot^2(\pi t + 2)$

$h'(t) = 4\cot(\pi t + 2)\left[-\csc^2(\pi t + 2)(\pi)\right]$

$= -4\pi\cot(\pi t + 2)\csc^2(\pi t + 2)$

49. $f(t) = 3\sec(\pi t - 1)^2$

$f'(t) = 3\sec(\pi t - 1)^2\tan(\pi t - 1)^2(2)(\pi t - 1)(\pi)$

$= 6\pi(\pi t - 1)\sec(\pi t - 1)^2\tan(\pi t - 1)^2$

50. $y = 5\cos(\pi x)^2$

$y' = -5\sin(\pi x)^2(2)(\pi x)(\pi)$

$= -10\pi^2 x\sin(\pi x)^2$

51. $y = \sin(3x^2 + \cos x)$

$y' = \cos(3x^2 + \cos x)(6x - \sin x)$

52. $y = \cos(5x + \csc x)$

$y' = -\sin(5x + \csc x)(5 - \csc x\cot x)$

53. $y = \sin\sqrt{\cot 3\pi x} = \sin(\cot 3\pi x)^{1/2}$

$y' = \cos(\cot 3\pi x)^{1/2}\left[\dfrac{1}{2}(\cot 3\pi x)^{-1/2}(-\csc^2 3\pi x)(3\pi)\right]$

$= -\dfrac{3\pi\cos\left(\sqrt{\cot 3\pi x}\right)\csc^2(3\pi x)}{2\sqrt{\cot 3\pi x}}$

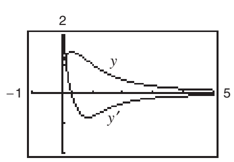
54. $y = \cos\sqrt{\sin(\tan \pi x)}$

$y' = -\sin\sqrt{\sin(\tan \pi x)} \cdot \dfrac{1}{2}(\sin(\tan \pi x))^{-1/2}\cos(\tan \pi x)\sec^2 \pi x(\pi) = \dfrac{-\pi\sin\sqrt{\sin(\tan \pi x)}\cos(\tan \pi x)\sec^2 \pi x}{2\sqrt{\sin(\tan \pi x)}}$

55. $y = \dfrac{\sqrt{x} + 1}{x^2 + 1}$

$y' = \dfrac{1 - 3x^2 - 4x^{3/2}}{2\sqrt{x}(x^2 + 1)^2}$

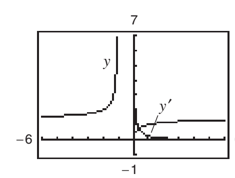
The zero of y' corresponds to the point on the graph of y where the tangent line is horizontal.



56. $y = \sqrt{\dfrac{2x}{x + 1}}$

$y' = \dfrac{1}{\sqrt{2x}(x + 1)^{3/2}}$

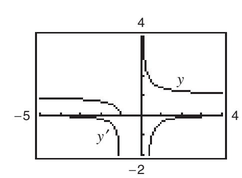
y' has no zeros.



57. $y = \sqrt{\dfrac{x + 1}{x}}$

$y' = -\dfrac{\sqrt{(x + 1)/x}}{2x(x + 1)}$

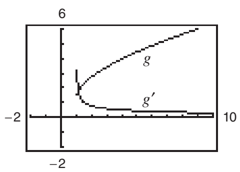
y' has no zeros.



58. $g(x) = \sqrt{x-1} + \sqrt{x+1}$

$g'(x) = \dfrac{1}{2\sqrt{x-1}} + \dfrac{1}{2\sqrt{x+1}}$

g' has no zeros.

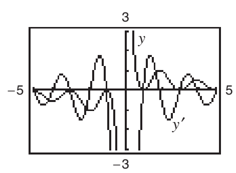


59. $y = \dfrac{\cos \pi x + 1}{x}$

$\dfrac{dy}{dx} = \dfrac{-\pi x \sin \pi x - \cos \pi x - 1}{x^2}$

$= -\dfrac{\pi x \sin \pi x + \cos \pi x + 1}{x^2}$

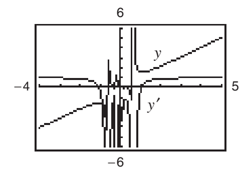
The zeros of y' correspond to the points on the graph of y where the tangent lines are horizontal.



60. $y = x^2 \tan \dfrac{1}{x}$

$\dfrac{dy}{dx} = 2x \tan \dfrac{1}{x} - \sec^2 \dfrac{1}{x}$

The zeros of y' correspond to the points on the graph of y where the tangent lines are horizontal.



61. $y = \sin 3x$

$y' = 3 \cos 3x$

$y'(0) = 3$

3 cycles in $[0, 2\pi]$

62. $y = \sin \dfrac{x}{2}$

$y' = \dfrac{1}{2} \cos \dfrac{x}{2}$

$y'(0) = \dfrac{1}{2}$

$\dfrac{1}{2}$ cycle in $[0, 2\pi]$

63. $y = \sqrt{x^2 + 8x} = \left(x^2 + 8x\right)^{1/2}, (1, 3)$

$y' = \dfrac{1}{2}\left(x^2 + 8x\right)^{-1/2}(2x + 8) = \dfrac{2(x+4)}{2\left(x^2 + 8x\right)^{1/2}} = \dfrac{x+4}{\sqrt{x^2 + 8x}}$

$y'(1) = \dfrac{1+4}{\sqrt{1^2 + 8(1)}} = \dfrac{5}{\sqrt{9}} = \dfrac{5}{3}$

64. $y = \left(3x^3 + 4x\right)^{1/5}, \quad (2, 2)$

$y' = \dfrac{1}{5}\left(3x^3 + 4x\right)^{-4/5}\left(9x^2 + 4\right)$

$= \dfrac{9x^2 + 4}{5\left(3x^3 + 4x\right)^{4/5}}$

$y'(2) = \dfrac{1}{2}$

65. $f(x) = 5\left(x^3 - 2\right)^{-1}, \quad \left(-2, -\dfrac{1}{2}\right)$

$f'(x) = -5\left(x^3 - 2\right)^{-2}\left(3x^2\right) = \dfrac{-15x^2}{\left(x^3 - 2\right)^2}$

$f'(-2) = -\dfrac{60}{100} = -\dfrac{3}{5}$

66. $f(x) = \dfrac{1}{\left(x^2 - 3x\right)^2} = \left(x^2 - 3x\right)^{-2}, \quad \left(4, \dfrac{1}{16}\right)$

$f'(x) = -2\left(x^2 - 3x\right)^{-3}(2x - 3) = \dfrac{-2(2x - 3)}{\left(x^2 - 3x\right)^3}$

$f'(4) = -\dfrac{5}{32}$

67. $y = \dfrac{4}{(x+2)^2} = 4(x+2)^{-2}, (0, 1)$

$y' = -8(x+2)^{-3} = \dfrac{-8}{(x+2)^3}$

$y'(0) = \dfrac{-8}{8} = -1$

68. $y = \dfrac{4}{\left(x^2 - 2x\right)^3} = 4\left(x^2 - 2x\right)^{-3}, (1, -4)$

$y' = -12\left(x^2 - 2x\right)^{-4}(2x - 2)$

$y'(1) = -12(-1)^{-4}(0) = 0$

69. $y = 26 - \sec^3 4x, \quad (0, 25)$

$y' = -3\sec^2 4x \sec 4x \tan 4x\, 4$

$= -12\sec^3 4x \tan 4x$

$y'(0) = 0$

70. $y = \dfrac{1}{x} + \sqrt{\cos x} = x^{-1} + (\cos x)^{1/2}, \quad \left(\dfrac{\pi}{2}, \dfrac{2}{\pi}\right)$

$y' = -x^{-2} + \dfrac{1}{2}(\cos x)^{-1/2}(-\sin x) = -\dfrac{1}{x^2} - \dfrac{\sin x}{2\sqrt{\cos x}}$

$y'(\pi/2)$ is undefined.

71. (a) $f(x) = \left(2x^2 - 7\right)^{1/2}, \quad (4, 5)$

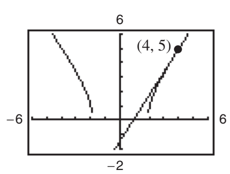
$f'(x) = \dfrac{1}{2}\left(2x^2 - 7\right)^{-1/2}(4x) = \dfrac{2x}{\sqrt{2x^2 - 7}}$

$f'(4) = \dfrac{8}{5}$

Tangent line:

$y - 5 = \dfrac{8}{5}(x - 4) \Rightarrow 8x - 5y - 7 = 0$

(b)



72. (a) $f(x) = \dfrac{1}{3}x\sqrt{x^2 + 5} = \dfrac{1}{3}x\left(x^2 + 5\right)^{1/2}, (2, 2)$

$f'(x) = \dfrac{1}{3}x\left[\dfrac{1}{2}\left(x^2 + 5\right)^{-1/2}(2x)\right] + \dfrac{1}{3}\left(x^2 + 5\right)^{1/2}$

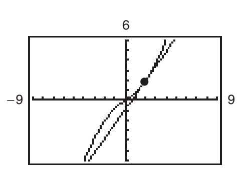
$= \dfrac{x^2}{2\sqrt{x^2 + 5}} + \dfrac{1}{3}\sqrt{x^2 + 5}$

$f'(2) = \dfrac{4}{3(3)} + \dfrac{1}{3}(3) = \dfrac{13}{9}$

Tangent line:

$y - 2 = \dfrac{13}{9}(x - 2) \Rightarrow 13x - 9y - 8 = 0$

(b)



73. (a) $y = \left(4x^3 + 3\right)^2, \quad (-1, 1)$

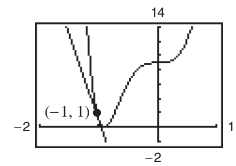
$y' = 2\left(4x^3 + 3\right)\left(12x^2\right) = 24x^2\left(4x^3 + 3\right)$

$y'(-1) = -24$

Tangent line:

$y - 1 = -24(x + 1) \Rightarrow 24x + y + 23 = 0$

(b)



74. (a) $f(x) = \left(9 - x^2\right)^{2/3}, \quad (1, 4)$

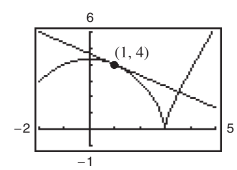
$f'(x) = \dfrac{2}{3}\left(9 - x^2\right)^{-1/3}(-2x) = \dfrac{-4x}{3\left(9 - x^2\right)^{1/3}}$

$f'(1) = \dfrac{-4}{3(8)^{1/3}} = -\dfrac{2}{3}$

Tangent line:

$y - 4 = -\dfrac{2}{3}(x - 1) \Rightarrow 2x + 3y - 14 = 0$

(b)



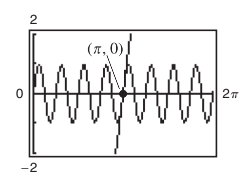
75. (a) $f(x) = \sin 8x, \quad (\pi, 0)$

$f'(x) = 8\cos 8x$

$f'(\pi) = 8$

Tangent line: $y = 8(x - \pi) = 8x - 8\pi$

(b)



76. (a) $y = \cos 3x, \quad \left(\frac{\pi}{4}, -\frac{\sqrt{2}}{2}\right)$

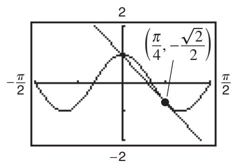
$y' = -3 \sin 3x$

$y'\left(\frac{\pi}{4}\right) = -3 \sin\left(\frac{3\pi}{4}\right) = \frac{-3\sqrt{2}}{2}$

Tangent line: $y + \frac{\sqrt{2}}{2} = \frac{-3\sqrt{2}}{2}\left(x - \frac{\pi}{4}\right)$

$y = \frac{-3\sqrt{2}}{2}x + \frac{3\sqrt{2}\pi}{8} - \frac{\sqrt{2}}{2}$

(b)



77. (a) $f(x) = \tan^2 x, \quad \left(\frac{\pi}{4}, 1\right)$

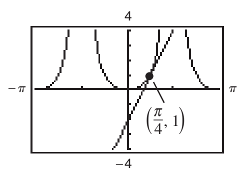
$f'(x) = 2 \tan x \sec^2 x$

$f'\left(\frac{\pi}{4}\right) = 2(1)(2) = 4$

Tangent line:

$y - 1 = 4\left(x - \frac{\pi}{4}\right) \Rightarrow 4x - y + (1 - \pi) = 0$

(b)



78. (a) $y = 2 \tan^3 x, \quad \left(\frac{\pi}{4}, 2\right)$

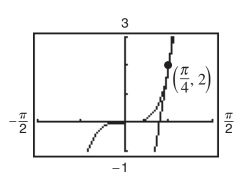
$y' = 6 \tan^2 x \cdot \sec^2 x$

$y'\left(\frac{\pi}{4}\right) = 6(1)(2) = 12$

Tangent line:

$y - 2 = 12\left(x - \frac{\pi}{4}\right) \Rightarrow 12x - y + (2 - 3\pi) = 0$

(b)



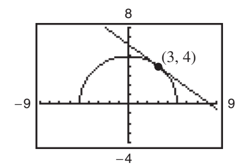
79. $f(x) = \sqrt{25 - x^2} = \left(25 - x^2\right)^{1/2}, \quad (3, 4)$

$f'(x) = \frac{1}{2}\left(25 - x^2\right)(-2x) = \frac{-x}{\sqrt{25 - x^2}}$

$f'(3) = -\frac{3}{4}$

Tangent line:

$y - 4 = -\frac{3}{4}(x - 3) \Rightarrow 3x + 4y - 25 = 0$

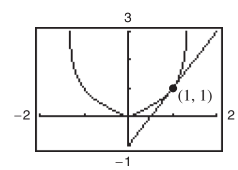


80. $f(x) = \frac{|x|}{\sqrt{2 - x^2}} = |x|\left(2 - x^2\right)^{-1/2}, \quad (1, 1)$

$f'(x) = \frac{2}{\left(2 - x^2\right)^{3/2}}$ for $x > 0$

$f'(1) = 2$

Tangent line: $y - 1 = 2(x - 1) \Rightarrow 2x - y - 1 = 0$



81.

$$f(x) = 2\cos x + \sin 2x, \quad 0 < x < 2\pi$$
$$f'(x) = -2\sin x + 2\cos 2x$$
$$= -2\sin x + 2 - 4\sin^2 x = 0$$
$$2\sin^2 x + \sin x - 1 = 0$$
$$(\sin x + 1)(2\sin x - 1) = 0$$
$$\sin x = -1 \Rightarrow x = \frac{3\pi}{2}$$
$$\sin x = \frac{1}{2} \Rightarrow x = \frac{\pi}{6}, \frac{5\pi}{6}$$

Horizontal tangents at $x = \frac{\pi}{6}, \frac{3\pi}{2}, \frac{5\pi}{6}$

Horizontal tangent at the points $\left(\frac{\pi}{6}, \frac{3\sqrt{3}}{2}\right), \left(\frac{3\pi}{2}, 0\right)$, and $\left(\frac{5\pi}{6}, -\frac{3\sqrt{3}}{2}\right)$

82. $f(x) = \dfrac{-4x}{\sqrt{2x-1}}$

$$f'(x) = \frac{(2x-1)^{1/2}(-4) + (4x)\left(\frac{1}{2}\right)(2x-1)^{-1/2}(2)}{(2x-1)}$$
$$= \frac{(2x-1)(-4) + 4x}{(2x-1)^{3/2}}$$
$$= \frac{4-4x}{(2x-1)^{3/2}}$$
$$f'(x) = 0 = \frac{4-4x}{(2x-1)^{2/2}} \Rightarrow x = 1$$

Horizontal tangent at $(1, -4)$

83. $f(x) = 5(2-7x)^4$

$$f'(x) = 20(2-7x)^3(-7) = -140(2-7x)^3$$
$$f''(x) = -420(2-7x)^2(-7) = 2940(2-7x)^2$$

84. $f(x) = 6(x^3+4)^3$

$$f'(x) = 18(x^3+4)^2(3x^2) = 54x^2(x^3+4)^2$$
$$f''(x) = 54x^2(2)(x^3+4)(3x^2) + 108x(x^3+4)^2$$
$$= 108x(x^3+4)\left[3x^3 + x^3 + 4\right]$$
$$= 432x(x^3+4)(x^3+1)$$

85. $f(x) = \dfrac{1}{11x-6} = (11x-6)^{-1}$

$$f'(x) = -(11x-6)^{-2}(11)$$
$$f''(x) = -22(11x-6)^{-3}(11)$$
$$= 242(11x-6)^{-3}$$
$$= \frac{242}{(11x-6)^3}$$

86. $f(x) = \dfrac{8}{(x-2)^2} = 8(x-2)^{-2}$

$$f'(x) = -16(x-2)^{-3}$$
$$f''(x) = 48(x-2)^{-4} = \frac{48}{(x-2)^4}$$

87. $f(x) = \sin x^2$

$$f'(x) = 2x\cos x^2$$
$$f''(x) = 2x\left[2x(-\sin x^2)\right] + 2\cos x^2$$
$$= 2(\cos x^2 - 2x^2\sin x^2)$$

88. $f(x) = \sec^2 \pi x$

$$f'(x) = 2\sec \pi x(\pi \sec \pi x \tan \pi x)$$
$$= 2\pi \sec^2 \pi x \tan \pi x$$
$$f''(x) = 2\pi \sec^2 \pi x(\sec^2 \pi x)(\pi) + 2\pi \tan \pi x(2\pi \sec^2 \pi x \tan \pi x)$$
$$= 2\pi^2 \sec^4 \pi x + 4\pi^2 \sec^2 \pi x \tan^2 \pi x$$
$$= 2\pi^2 \sec^2 \pi x(\sec^2 \pi x + 2\tan^2 \pi x)$$
$$= 2\pi^2 \sec^2 \pi x(3\sec^2 \pi x - 2)$$

89. $h(x) = \frac{1}{9}(3x + 1)^3, \quad \left(1, \frac{64}{9}\right)$

$h'(x) = \frac{1}{9}3(3x + 1)^2(3) = (3x + 1)^2$

$h''(x) = 2(3x + 1)(3) = 18x + 6$

$h''(1) = 24$

90. $f(x) = \dfrac{1}{\sqrt{x + 4}} = (x + 4)^{-1/2}, \quad \left(0, \dfrac{1}{2}\right)$

$f'(x) = -\dfrac{1}{2}(x + 4)^{-3/2}$

$f''(x) = \dfrac{3}{4}(x + 4)^{-5/2} = \dfrac{3}{4(x + 4)^{5/2}}$

$f''(0) = \dfrac{3}{128}$

91. $f(x) = \cos x^2, \quad (0, 1)$

$f'(x) = -\sin(x^2)(2x) = -2x \sin(x^2)$

$f''(x) = -2x \cos(x^2)(2x) - 2 \sin(x^2)$

$= -4x^2 \cos(x^2) - 2 \sin(x^2)$

$f''(0) = 0$

92. $g(t) = \tan 2t, \quad \left(\dfrac{\pi}{6}, \sqrt{3}\right)$

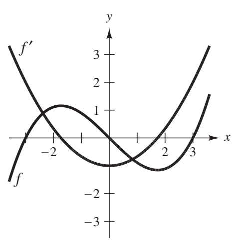
$g'(t) = 2 \sec^2(2t)$

$g''(t) = 4 \sec(2t) \cdot \sec(2t) \tan(2t)2$

$= 8 \sec^2(2t) \tan(2t)$

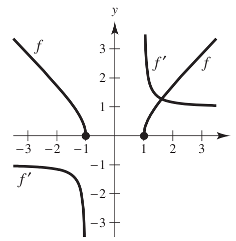
$g''\left(\dfrac{\pi}{6}\right) = 32\sqrt{3}$

93.



The zeros of f' correspond to the points where the graph of f has horizontal tangents.

94.



f is decreasing on $(-\infty, -1)$ so f' must be negative there.

f is increasing on $(1, \infty)$ so f' must be positive there.

95. (a) $g(x) = f(3x)$

$g'(x) = f'(3x)(3) \Rightarrow g'(x) = 3f'(3x)$

The rate of change of g is three times as fast as the rate of change of f.

(b) $g(x) = f(x^2)$

$g'(x) = f'(x^2)(2x) \Rightarrow g'(x) = 2x\, f'(x^2)$

The rate of change of g is $2x$ times as fast as the rate of change of f.

96. $r(x) = \dfrac{2x - 5}{(3x + 1)^2}$

(a) If $h(x) = \dfrac{f(x)}{g(x)}$, then write $h(x) = f(x)\,(g(x))^{-1}$ and use the Product Rule.

(b) $r(x) = (2x - 5)(3x + 1)^{-2}$

$r'(x) = (2x - 5)(-2)(3x + 1)^{-3}(3) + (3x + 1)^{-2}(2)$

$= \dfrac{-6(2x - 5) + 2(3x + 1)}{(3x + 1)^3}$

$= \dfrac{-6x + 32}{(3x + 1)^3}$

(c) $r'(x) = \dfrac{(3x + 1)^2(2) - (2x - 5)(2)(3x + 1)(3)}{(3x + 1)^4}$

$= \dfrac{(3x + 1)(2) - 6(2x - 5)}{(3x + 1)^3}$

$= \dfrac{-6x + 32}{(3x + 1)^3}$

(d) Answers will vary.

97. (a) $g(x) = f(x) - 2 \Rightarrow g'(x) = f'(x)$

(b) $h(x) = 2f(x) \Rightarrow h'(x) = 2f'(x)$

(c) $r(x) = f(-3x) \Rightarrow r'(x) = f'(-3x)(-3) = -3f'(-3x)$

So, you need to know $f'(-3x)$.

$r'(0) = -3f'(0) = (-3)\left(-\frac{1}{3}\right) = 1$

$r'(-1) = -3f'(3) = (-3)(-4) = 12$

(d) $s(x) = f(x + 2) \Rightarrow s'(x) = f'(x + 2)$

So, you need to know $f'(x + 2)$.

$s'(-2) = f'(0) = -\frac{1}{3}$, etc.

x	-2	-1	0	1	2	3
$f'(x)$	4	$\frac{2}{3}$	$-\frac{1}{3}$	-1	-2	-4
$g'(x)$	4	$\frac{2}{3}$	$-\frac{1}{3}$	-1	-2	-4
$h'(x)$	8	$\frac{4}{3}$	$-\frac{2}{3}$	-2	-4	-8
$r'(x)$		12	1			
$s'(x)$	$-\frac{1}{3}$	-1	-2	-4		

98. (a) $f(x) = g(x)h(x)$

$f'(x) = g(x)h'(x) + g'(x)h(x)$

$f'(5) = (-3)(-2) + (6)(3) = 24$

(b) $f(x) = g(h(x))$

$f'(x) = g'(h(x))h'(x)$

$f'(5) = g'(3)(-2) = -2g'(3)$

Not possible, you need $g'(3)$ to find $f'(5)$.

(c) $f(x) = \dfrac{g(x)}{h(x)}$

$$f'(x) = \frac{h(x)g'(x) - g(x)h'(x)}{\left[h(x)\right]^2}$$

$$f'(5) = \frac{(3)(6) - (-3)(-2)}{(3)^2} = \frac{12}{9} = \frac{4}{3}$$

(d) $f(x) = \left[g(x)\right]^3$

$f'(x) = 3\left[g(x)\right]^2 g'(x)$

$f'(5) = 3(-3)^2(6) = 162$

99. (a) $h(x) = f(g(x)),\ g(1) = 4,\ g'(1) = -\frac{1}{2},\ f'(4) = -1$

$h'(x) = f'(g(x))g'(x)$

$h'(1) = f'(g(1))g'(1) = f'(4)g'(1) = (-1)\left(-\frac{1}{2}\right) = \frac{1}{2}$

(b) $s(x) = g(f(x)),\ f(5) = 6,\ f'(5) = -1,\ g'(6)$ does not exist.

$s'(x) = g'(f(x))\,f'(x)$

$s'(5) = g'(f(5))f'(5) = g'(6)(-1)$

$s'(5)$ does not exist because g is not differentiable at 6.

100. (a) $h(x) = f(g(x))$

$h'(x) = f'(g(x))g'(x)$

$h'(3) = f'(g(3))g'(3) = f'(5)(1) = \frac{1}{2}$

(b) $s(x) = g(f(x))$

$s'(x) = g'(f(x))f'(x)$

$s'(9) = g'(f(9))f'(9) = g'(8)(2) = (-1)(2) = -2$

101. (a) $F = 132{,}400(331 - v)^{-1}$

$F' = (-1)(132{,}400)(331 - v)^{-2}(-1) = \dfrac{132{,}400}{(331 - v)^2}$

When $v = 30$, $F' \approx 1.461$.

(b) $F = 132{,}400(331 + v)^{-1}$

$F' = (-1)(132{,}400)(331 + v)^{-2}(-1) = \dfrac{-132{,}400}{(331 + v)^2}$

When $v = 30$, $F' \approx -1.016$.

102. $y = 4\cos 12t - 3\sin 12t$

$v = y' = 4[-12\sin 12t] - 3[12\cos 12t]$

$= -48\sin 12t - 36\cos 12t$

When $t = \pi/8$, $y = 3$ cm and $v = 48$ cm/sec.

103. $\theta = 0.2\cos 8t$

The maximum angular displacement is $\theta = 0.2$ (because $-1 \le \cos 8t \le 1$).

$\dfrac{d\theta}{dt} = 0.2[-8\sin 8t] = -1.6\sin 8t$

When $t = 3$, $d\theta/dt = -1.6\sin 24 \approx 1.4489$ rad/sec.

104. $y = A\cos\omega t$

(a) Amplitude: $A = \dfrac{2.2}{2} = 1.1$

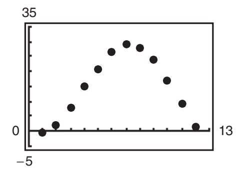
$y = 1.1\cos\omega t$

Period: $10 \Rightarrow \omega = \dfrac{2\pi}{10} = \dfrac{\pi}{5}$

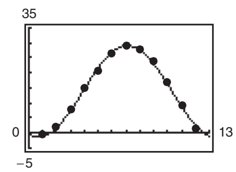
$y = 1.1\cos\dfrac{\pi t}{5}$

(b) $v = y' = 1.1\left[-\dfrac{\pi}{5}\sin\dfrac{\pi t}{5}\right] = -0.22\pi\sin\dfrac{\pi t}{5}$

105. (a) $T(t) = 15.16\sin(0.49t - 1.90) + 13.9$

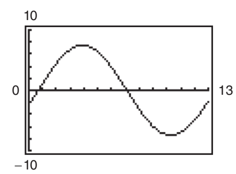


(b)



The model is a good fit.

(c) $T'(t) = 7.4284\cos(0.49t - 1.90)$



(d) The temperature changes most rapidly around spring (March–May) and fall (Oct.–Nov.). The temperature changes most slowly around winter (Dec.–Feb.) and summer (Jun.–Aug.). Yes. Explanations will vary.

106. (a) According to the graph $C'(4) > C'(1)$.

(b) Answers will vary.

107. $N = 400\left[1 - \dfrac{3}{(t^2 + 2)^2}\right] = 400 - 1200(t^2 + 2)^{-2}$

$N'(t) = 2400(t^2 + 2)^{-3}(2t) = \dfrac{4800t}{(t^2 + 2)^3}$

(a) $N'(0) = 0$ bacteria/day

(b) $N'(1) = \dfrac{4800(1)}{(1 + 2)^3} = \dfrac{4800}{27} \approx 177.8$ bacteria/day

(c) $N'(2) = \dfrac{4800(2)}{(4 + 2)^3} = \dfrac{9600}{216} \approx 44.4$ bacteria/day

(d) $N'(3) = \dfrac{4800(3)}{(9 + 2)^3} = \dfrac{14{,}400}{1331} \approx 10.8$ bacteria/day

(e) $N'(4) = \dfrac{4800(4)}{(16 + 2)^3} = \dfrac{19{,}200}{5832} \approx 3.3$ bacteria/day

(f) The rate of change of the population is decreasing as $t \to \infty$.

108. (a) $V = \dfrac{k}{\sqrt{t+1}}$

$V(0) = 10{,}000 = \dfrac{k}{\sqrt{0+1}} = k$

$V = \dfrac{10{,}000}{\sqrt{t+1}} = 10{,}000(t+1)^{-1/2}$

(b) $\dfrac{dV}{dt} = 10{,}000\left(-\dfrac{1}{2}\right)(t+1)^{-3/2} = \dfrac{-5000}{(t+1)^{3/2}}$

$V'(1) = \dfrac{-5000}{2^{3/2}} \approx -1767.77$ dollars/year

(c) $V'(3) = \dfrac{-5000}{4^{3/2}} = \dfrac{-5000}{8} = -625$ dollars/year

109. $f(x) = \sin \beta x$

(a) $f'(x) = \beta \cos \beta x$

$f''(x) = -\beta^2 \sin \beta x$

$f'''(x) = -\beta^3 \cos \beta x$

$f^{(4)} = \beta^4 \sin \beta x$

(b) $f''(x) + \beta^2 f(x) = -\beta^2 \sin \beta x + \beta^2(\sin \beta x) = 0$

(c) $f^{(2k)}(x) = (-1)^k \beta^{2k} \sin \beta x$

$f^{(2k-1)}(x) = (-1)^{k+1}\beta^{2k-1} \cos \beta x$

110. (a) Yes, if $f(x+p) = f(x)$ for all x, then $f'(x+p) = f'(x)$, which shows that f is periodic as well.

(b) Yes, if $g(x) = f(2x)$, then $g'(x) = 2f'(2x)$. Because f' is periodic, so is g'.

111. (a) $r'(x) = f'(g(x))g'(x)$

$r'(1) = f'(g(1))g'(1)$

Note that $g(1) = 4$ and $f'(4) = \dfrac{5-0}{6-2} = \dfrac{5}{4}$.

Also, $g'(1) = 0$. So, $r'(1) = 0$.

(b) $s'(x) = g'(f(x))f'(x)$

$s'(4) = g'(f(4))f'(4)$

Note that $f(4) = \dfrac{5}{2}, g'\left(\dfrac{5}{2}\right) = \dfrac{6-4}{6-2} = \dfrac{1}{2}$ and

$f'(4) = \dfrac{5}{4}$. So, $s'(4) = \dfrac{1}{2}\left(\dfrac{5}{4}\right) = \dfrac{5}{8}$.

112. (a) $g(x) = \sin^2 x + \cos^2 x = 1 \Rightarrow g'(x) = 0$

$g'(x) = 2 \sin x \cos x + 2 \cos x(-\sin x) = 0$

(b) $\tan^2 x + 1 = \sec^2 x$

$g(x) + 1 = f(x)$

Taking derivatives of both sides, $g'(x) = f'(x)$. Equivalently,

$f'(x) = 2 \sec x \cdot \sec x \tan x = 2\sec^2 x \tan x$ and

$g'(x) = 2 \tan x \cdot \sec^2 x = 2 \sec^2 x \tan x$, which are the same.

113. (a) If $f(-x) = -f(x)$, then

$\dfrac{d}{dx}[f(-x)] = \dfrac{d}{dx}[-f(x)]$

$f'(-x)(-1) = -f'(x)$

$f'(-x) = f'(x)$.

So, $f'(x)$ is even.

(b) If $f(-x) = f(x)$, then

$\dfrac{d}{dx}[f(-x)] = \dfrac{d}{dx}[f(x)]$

$f'(-x)(-1) = f'(x)$

$f'(-x) = -f'(x)$.

So, f' is odd.

114. $|u| = \sqrt{u^2}$

$\dfrac{d}{dx}[|u|] = \dfrac{d}{dx}\left[\sqrt{u^2}\right] = \dfrac{1}{2}(u^2)^{-1/2}(2uu')$

$= \dfrac{uu'}{\sqrt{u^2}} = u'\dfrac{u}{|u|}, \quad u \neq 0$

115. $g(x) = |3x - 5|$

$g'(x) = 3\left(\dfrac{3x-5}{|3x-5|}\right), \quad x \neq \dfrac{5}{3}$

116. $f(x) = |x^2 - 9|$

$f'(x) = 2x\left(\dfrac{x^2-9}{|x^2-9|}\right), \quad x \neq \pm 3$

117. $h(x) = |x| \cos x$

$h'(x) = -|x| \sin x + \dfrac{x}{|x|}\cos x, \quad x \neq 0$

118. $f(x) = |\sin x|$

$f'(x) = \cos x\left(\dfrac{\sin x}{|\sin x|}\right), x \neq k\pi$

119. (a) $f(x) = \tan x \qquad f(\pi/4) = 1$

$f'(x) = \sec^2 x \qquad f'(\pi/4) = 2$

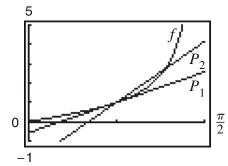
$f''(x) = 2\sec^2 x \tan x \qquad f''(\pi/4) = 4$

$P_1(x) = 2(x - \pi/4) + 1$

$P_2(x) = \frac{1}{2}(4)(x - \pi/4)^2 + 2(x - \pi/4) + 1$

$= 2(x - \pi/4)^2 + 2(x - \pi/4) + 1$

(b)



(c) P_2 is a better approximation than P_1.

(d) The accuracy worsens as you move away from $x = \pi/4$.

120. (a) $f(x) = \sec x \qquad f(\pi/6) = \frac{2}{\sqrt{3}}$

$f'(x) = \sec x \tan x \qquad f'(\pi/6) = \frac{2}{3}$

$f''(x) = \sec x(\sec^2 x) + \tan x(\sec x \tan x) \qquad f''(\pi/6) = \frac{10\sqrt{3}}{9}$

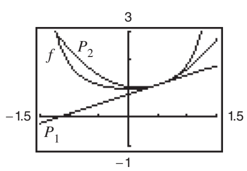
$= \sec^3 x + \sec x \tan^2 x$

$P_1(x) = \frac{2}{3}(x - \pi/6) + \frac{2}{\sqrt{3}}$

$P_2(x) = \frac{1}{2} \cdot \left(\frac{10}{3\sqrt{3}}\right)\left(x - \frac{\pi}{6}\right)^2 + \frac{2}{3}\left(x - \frac{\pi}{6}\right) + \frac{2}{\sqrt{3}}$

$= \left(\frac{5}{3\sqrt{3}}\right)\left(x - \frac{\pi}{6}\right)^2 + \frac{2}{3}\left(x - \frac{\pi}{6}\right) + \frac{2}{\sqrt{3}}$

(b)



(c) P_2 is a better approximation than P_1.

(d) The accuracy worsens as you move away from $x = \pi/6$.

121. True

122. False. $f'(x) = -b \sin x$ and $f'(0) = 0$

123. True

124. True

125.

$$f(x) = a_1 \sin x + a_2 \sin 2x + \cdots + a_n \sin nx$$

$$f'(x) = a_1 \cos x + 2a_2 \cos 2x + \cdots + na_n \cos nx$$

$$f'(0) = a_1 + 2a_2 + \cdots + na_n$$

$$\left|a_1 + 2a_2 + \cdots + na_n\right| = \left|f'(0)\right| = \lim_{x\to 0}\left|\frac{f(x) - f(0)}{x - 0}\right| = \lim_{x\to 0}\left|\frac{f(x)}{\sin x}\right| \cdot \left|\frac{\sin x}{x}\right| = \lim_{x\to 0}\left|\frac{f(x)}{\sin x}\right| \le 1$$

126. $\frac{d}{dx}\left[\frac{P_n(x)}{(x^k - 1)^{n+1}}\right] = \frac{(x^k - 1)^{n+1}P_n'(x) - P_n(x)(n+1)(x^k - 1)^n kx^{k-1}}{(x^k - 1)^{2n+2}} = \frac{(x^k - 1)P_n'(x) - (n+1)kx^{k-1}P_n(x)}{(x^k - 1)^{n+2}}$

$P_n(x) = (x^k - 1)^{n+1}\frac{d^n}{dx^n}\left[\frac{1}{x^k - 1}\right] \Rightarrow$

$P_{n+1}(x) = (x^k - 1)^{n+2}\frac{d}{dx}\left[\frac{d^n}{dx^n}\left[\frac{1}{x^k - 1}\right]\right] = (x^k - 1)P_n'(x) - (n+1)kx^{k-1}P_n(x)$

$P_{n+1}(1) = -(n+1)kP_n(1)$

For $n = 1$, $\frac{d}{dx}\left[\frac{1}{x^k - 1}\right] = \frac{-kx^{k-1}}{(x^k - 1)^2} = \frac{P_1(x)}{(x^k - 1)^2} \Rightarrow P_1(1) = -k$. Also, $P_0(1) = 1$.

You now use mathematical induction to verify that $P_n(1) = (-k)^n n!$ for $n \geq 0$. Assume true for n. Then

$P_{n+1}(1) = -(n+1)k\,P_n(1) = -(n+1)k(-k)^n n! = (-k)^{n+1}(n+1)!$.

Section 2.5 Implicit Differentiation

1. Answers will vary. *Sample answer:* In the explicit form of a function, the variable is explicitly written as a function of x. In an implicit equation, the function is only implied by an equation. An example of an implicit function is $x^2 + xy = 5$. In explicit form it would be $y = (5 - x^2)/x$.

2. Answers will vary. *Sample answer:* Given an implicit equation, first differentiate both sides with respect to x. Collect all terms involving y' on the left, and all other terms to the right. Factor out y' on the left side. Finally, divide both sides by the left-hand factor that does not contain y'.

3. You use implicit differentiation to find the derivative y' when it is difficult to express y explicitly as a function of x.

4. If y is an implicit function of x, then to compute y', you differentiate the equation with respect to x. For example, if $xy^2 = 1$, then $y^2 + 2xyy' = 0$. Here, the derivative of y^2 is $2yy'$.

5.
$$x^2 + y^2 = 9$$
$$2x + 2yy' = 0$$
$$y' = -\frac{x}{y}$$

6.
$$x^2 - y^2 = 25$$
$$2x - 2yy' = 0$$
$$y' = \frac{x}{y}$$

7.
$$x^5 + y^5 = 16$$
$$5x^4 + 5y^4y' = 0$$
$$5y^4y' = -5x^4$$
$$y' = -\frac{x^4}{y^4}$$

8.
$$2x^3 + 3y^3 = 64$$
$$6x^2 + 9y^2y' = 0$$
$$9y^2y' = -6x^2$$
$$y' = \frac{-6x^2}{9y^2} = -\frac{2x^2}{3y^2}$$

9.
$$x^3 - xy + y^2 = 7$$
$$3x^2 - xy' - y + 2yy' = 0$$
$$(2y - x)y' = y - 3x^2$$
$$y' = \frac{y - 3x^2}{2y - x}$$

10.
$$x^2y + y^2x = -2$$
$$x^2y' + 2xy + y^2 + 2yxy' = 0$$
$$(x^2 + 2xy)y' = -(y^2 + 2xy)$$
$$y' = \frac{-y(y + 2x)}{x(x + 2y)}$$

11.
$$x^3y^3 - y - x = 0$$
$$3x^3y^2y' + 3x^2y^3 - y' - 1 = 0$$
$$(3x^3y^2 - 1)y' = 1 - 3x^2y^3$$
$$y' = \frac{1 - 3x^2y^3}{3x^3y^2 - 1}$$

12.
$$\sqrt{xy} = x^2y + 1$$
$$\frac{1}{2}(xy)^{-1/2}(xy' + y) = 2xy + x^2y'$$
$$\frac{x}{2\sqrt{xy}}y' + \frac{y}{2\sqrt{xy}} = 2xy + x^2y'$$
$$\left(\frac{x}{2\sqrt{xy}} - x^2\right)y' = 2xy - \frac{y}{2\sqrt{xy}}$$
$$y' = \frac{2xy - \dfrac{y}{2\sqrt{xy}}}{\dfrac{x}{2\sqrt{xy}} - x^2}$$
$$y' = \frac{4xy\sqrt{xy} - y}{x - 2x^2\sqrt{xy}}$$

13.
$$x^3 - 3x^2y + 2xy^2 = 12$$
$$3x^2 - 3x^2y' - 6xy + 4xyy' + 2y^2 = 0$$
$$(4xy - 3x^2)y' = 6xy - 3x^2 - 2y^2$$
$$y' = \frac{6xy - 3x^2 - 2y^2}{4xy - 3x^2}$$

14.
$$x^4y - 8xy + 3xy^2 = 9$$
$$x^4y' + 4x^3y - 8xy' - 8y + 6xyy' + 3y^2 = 0$$
$$(x^4 - 8x + 6xy)y' = 8y - 4x^3y - 3y^2$$
$$y' = \frac{8y - 4x^3y - 3y^2}{x^4 - 8x + 6xy}$$

15.
$$\sin x + 2\cos 2y = 1$$
$$\cos x - 4(\sin 2y)y' = 0$$
$$y' = \frac{\cos x}{4\sin 2y}$$

16.
$$(\sin \pi x + \cos \pi y)^2 = 2$$
$$2(\sin \pi x + \cos \pi y)[\pi \cos \pi x - \pi(\sin \pi y)y'] = 0$$
$$\pi \cos \pi x - \pi(\sin \pi y)y' = 0$$
$$y' = \frac{\cos \pi x}{\sin \pi y}$$

17.
$$\csc x = x(1 + \tan y)$$
$$-\csc x \cot x = (1 + \tan y) + x(\sec^2 y)y'$$
$$y' = -\frac{\csc x \cot x + 1 + \tan y}{x \sec^2 y}$$

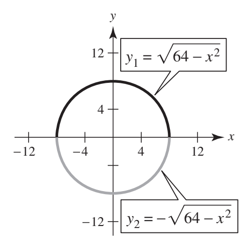
18.
$$\cot y = x - y$$
$$(-\csc^2 y)y' = 1 - y'$$
$$y' = \frac{1}{1 - \csc^2 y} = \frac{1}{-\cot^2 y} = -\tan^2 y$$

19.
$$y = \sin xy$$
$$y' = [xy' + y]\cos(xy)$$
$$y' - x\cos(xy)y' = y\cos(xy)$$
$$y' = \frac{y\cos(xy)}{1 - x\cos(xy)}$$

20.
$$x = \sec\frac{1}{y}$$
$$1 = -\frac{y'}{y^2}\sec\frac{1}{y}\tan\frac{1}{y}$$
$$y' = \frac{-y^2}{\sec(1/y)\tan(1/y)} = -y^2\cos\left(\frac{1}{y}\right)\cot\left(\frac{1}{y}\right)$$

21. (a)
$$x^2 + y^2 = 64$$
$$y^2 = 64 - x^2$$
$$y = \pm\sqrt{64 - x^2}$$

(b)



(c) Explicitly:
$$\frac{dy}{dx} = \pm\frac{1}{2}(64 - x^2)^{-1/2}(-2x) = \frac{\mp x}{\sqrt{64 - x^2}}$$
$$= \frac{-x}{\pm\sqrt{64 - x^2}} = -\frac{x}{y}$$

(d) Implicitly: $2x + 2yy' = 0$
$$y' = -\frac{x}{y}$$

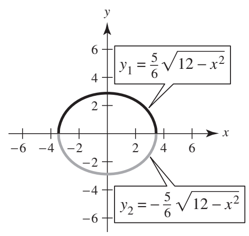
22. (a) $25x^2 + 36y^2 = 300$

$$36y^2 = 300 - 25x^2 = 25(12 - x^2)$$

$$y^2 = \frac{25}{36}(12 - x^2)$$

$$y = \pm\frac{5}{6}\sqrt{12 - x^2}$$

(b)



(c) Explicitly:

$$\frac{dy}{dx} = \pm\frac{5}{6}\left(\frac{1}{2}\right)(12 - x^2)^{-1/2}(-2x)$$

$$= \mp\frac{5x}{6\sqrt{12 - x^2}}$$

$$= -\frac{25x}{36y}$$

(d) Implicitly: $50x + 72y \cdot y' = 0$

$$y' = \frac{-50x}{72y} = -\frac{25x}{36y}$$

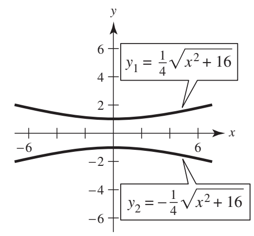
23. (a) $16y^2 - x^2 = 16$

$$16y^2 = x^2 + 16$$

$$y^2 = \frac{x^2}{16} + 1 = \frac{x^2 + 16}{16}$$

$$y = \frac{\pm\sqrt{x^2 + 16}}{4}$$

(b)



(c) Explicitly:

$$\frac{dy}{dx} = \frac{\pm\frac{1}{2}(x^2 + 16)^{-1/2}(-2x)}{4}$$

$$= \frac{\pm x}{4\sqrt{x^2 + 16}} = \frac{\pm x}{4(\pm 4y)} = \frac{x}{16y}$$

(d) Implicitly: $16y^2 - x^2 = 16$

$$32yy' - 2x = 0$$

$$32yy' = 2x$$

$$y' = \frac{2x}{32y} = \frac{x}{16y}$$

24. (a) $x^2 + y^2 - 4x + 6y + 9 = 0$

$$(x^2 - 4x + 4) + (y^2 + 6y + 9) = -9 + 4 + 9$$

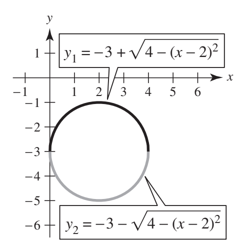
$$(x - 2)^2 + (y + 3)^2 = 4$$

$$(y + 3)^2 = 4 - (x - 2)^2$$

$$y + 3 = \pm\sqrt{4 - (x - 2)^2}$$

$$y = -3 \pm\sqrt{4 - (x - 2)^2}$$

(b)



(c) Explicitly:

$$\frac{dy}{dx} = \pm\frac{1}{2}\left[4 - (x - 2)^2\right]^{-1/2}\left[-2(x - 2)\right]$$

$$= \mp\frac{x - 2}{\sqrt{4 - (x - 2)^2}}$$

$$= -\frac{x - 2}{y + 3}$$

(d) Implicitly:

$$2x + 2yy' - 4 + 6y' = 0$$

$$2yy' + 6y' = -2x + 4$$

$$y'(2y + 6) = -2(x - 2)$$

$$y' = \frac{-2(x - 2)}{2(y + 3)} = -\frac{x - 2}{y + 3}$$

25.
$$xy = 6$$
$$xy' + y(1) = 0$$
$$xy' = -y$$
$$y' = -\frac{y}{x}$$

At $(-6, -1)$: $y' = -\frac{1}{6}$

26.
$$3x^3y = 6$$
$$x^3y = 2$$
$$3x^2y + x^3y' = 0$$
$$x^3y' = -3x^2y$$
$$y' = \frac{-3x^2y}{x^3} = -\frac{3y}{x}$$

At $(1, 2)$: $y' = \frac{-3(2)}{1} = -6$

27.
$$y^2 = \frac{x^2 - 49}{x^2 + 49}$$
$$2yy' = \frac{(x^2 + 49)(2x) - (x^2 - 49)(2x)}{(x^2 + 49)^2}$$
$$2yy' = \frac{196x}{(x^2 + 49)^2}$$
$$y' = \frac{98x}{y(x^2 + 49)^2}$$

At $(7, 0)$: y' is undefined.

28.
$$4y^3 = \frac{x^2 - 36}{x^3 + 36}$$
$$12y^2y' = \frac{(x^3 + 36)(2x) - (x^2 - 36)(3x^2)}{(x^3 + 36)^2}$$
$$y' = \frac{72x + 108x^2 - x^4}{12y^2(x^3 + 36)^2}$$

At $(6, 0)$: y' is undefined (division by 0).

29.
$$(x + y)^3 = x^3 + y^3$$
$$x^3 + 3x^2y + 3xy^2 + y^3 = x^3 + y^3$$
$$3x^2y + 3xy^2 = 0$$
$$x^2y + xy^2 = 0$$
$$x^2y' + 2xy + 2xyy' + y^2 = 0$$
$$(x^2 + 2xy)y' = -(y^2 + 2xy)$$
$$y' = -\frac{y(y + 2x)}{x(x + 2y)}$$

At $(-1, 1)$: $y' = -1$

30.
$$x^3 + y^3 = 6xy - 1$$
$$3x^2 + 3y^2y' = 6xy' + 6y$$
$$(3y^2 - 6x)y' = 6y - 3x^2$$
$$y' = \frac{6y - 3x^2}{3y^2 - 6x}$$

At $(2, 3)$: $y' = \frac{18 - 12}{27 - 12} = \frac{6}{15} = \frac{2}{5}$

31.
$$\tan(x + y) = x$$
$$(1 + y')\sec^2(x + y) = 1$$
$$y' = \frac{1 - \sec^2(x + y)}{\sec^2(x + y)}$$
$$= \frac{-\tan^2(x + y)}{\tan^2(x + y) + 1}$$
$$= -\sin^2(x + y)$$
$$= -\frac{x^2}{x^2 + 1}$$

At $(0, 0)$: $y' = 0$

32.
$$x\cos y = 1$$
$$x[-y'\sin y] + \cos y = 0$$
$$y' = \frac{\cos y}{x\sin y}$$
$$= \frac{1}{x}\cot y$$
$$= \frac{\cot y}{x}$$

At $\left(2, \frac{\pi}{3}\right)$: $y' = \frac{1}{2\sqrt{3}}$

33.
$$(x^2 + 4)y = 8$$
$$(x^2 + 4)y' + y(2x) = 0$$
$$y' = \frac{-2xy}{x^2 + 4}$$
$$= \frac{-2x\left[8/(x^2 + 4)\right]}{x^2 + 4}$$
$$= \frac{-16x}{(x^2 + 4)^2}$$

At $(2, 1)$: $y' = \frac{-32}{64} = -\frac{1}{2}$

$\left(\text{Or, you could just solve for } y\text{: } y = \frac{8}{x^2 + 4}\right)$

34. $(4 - x)y^2 = x^3$

$(4 - x)(2yy') + y^2(-1) = 3x^2$

$y' = \dfrac{3x^2 + y^2}{2y(4 - x)}$

At $(2, 2)$: $y' = 2$

35. $(x^2 + y^2)^2 = 4x^2y$

$2(x^2 + y^2)(2x + 2yy') = 4x^2y' + y(8x)$

$4x^3 + 4x^2yy' + 4xy^2 + 4y^3y' = 4x^2y' + 8xy$

$4x^2yy' + 4y^3y' - 4x^2y' = 8xy - 4x^3 - 4xy^2$

$4y'(x^2y + y^3 - x^2) = 4(2xy - x^3 - xy^2)$

$y' = \dfrac{2xy - x^3 - xy^2}{x^2y + y^3 - x^2}$

At $(1, 1)$: $y' = 0$

36. $x^3 + y^3 - 6xy = 0$

$3x^2 + 3y^2y' - 6xy' - 6y = 0$

$y'(3y^2 - 6x) = 6y - 3x^2$

$y' = \dfrac{6y - 3x^2}{3y^2 - 6x} = \dfrac{2y - x^2}{y^2 - 2x}$

At $\left(\dfrac{4}{3}, \dfrac{8}{3}\right)$: $y' = \dfrac{(16/3) - (16/9)}{(64/9) - (8/3)} = \dfrac{32}{40} = \dfrac{4}{5}$

37. $(y - 3)^2 = 4(x - 5)$, $(6, 1)$

$2(y - 3)y' = 4$

$y' = \dfrac{2}{y - 3}$

At $(6, 1)$: $y' = \dfrac{2}{1 - 3} = -1$

Tangent line: $y - 1 = -1(x - 6)$

$y = -x + 7$

38. $(x + 2)^2 + (y - 3)^2 = 37$, $(4, 4)$

$2(x + 2) + 2(y - 3)y' = 0$

$(y - 3)y' = -(x + 2)$

$y' = -\dfrac{(x + 2)}{y - 3}$

At $(4, 4)$: $y' = -\dfrac{6}{1} = -6$

Tangent line: $y - 4 = -6(x - 4)$

$y = -6x + 28$

39. $x^2y^2 - 9x^2 - 4y^2 = 0$, $(-4, 2\sqrt{3})$

$x^2 2yy' + 2xy^2 - 18x - 8yy' = 0$

$y' = \dfrac{18x - 2xy^2}{2x^2y - 8y}$

At $(-4, 2\sqrt{3})$: $y' = \dfrac{18(-4) - 2(-4)(12)}{2(16)(2\sqrt{3}) - 16\sqrt{3}}$

$= \dfrac{24}{48\sqrt{3}} = \dfrac{1}{2\sqrt{3}} = \dfrac{\sqrt{3}}{6}$

Tangent line: $y - 2\sqrt{3} = \dfrac{\sqrt{3}}{6}(x + 4)$

$y = \dfrac{\sqrt{3}}{6}x + \dfrac{8}{3}\sqrt{3}$

40. $x^{2/3} + y^{2/3} = 5$, $(8, 1)$

$\dfrac{2}{3}x^{-1/3} + \dfrac{2}{3}y^{-1/3}y' = 0$

$y' = \dfrac{-x^{-1/3}}{y^{-1/3}} = -\left(\dfrac{y}{x}\right)^{1/3}$

At $(8, 1)$: $y' = -\dfrac{1}{2}$

Tangent line: $y - 1 = -\dfrac{1}{2}(x - 8)$

$y = -\dfrac{1}{2}x + 5$

41. $3(x^2 + y^2)^2 = 100(x^2 - y^2)$, $(4, 2)$

$6(x^2 + y^2)(2x + 2yy') = 100(2x - 2yy')$

At $(4, 2)$:

$6(16 + 4)(8 + 4y') = 100(8 - 4y')$

$960 + 480y' = 800 - 400y'$

$880y' = -160$

$y' = -\frac{2}{11}$

Tangent line: $y - 2 = -\frac{2}{11}(x - 4)$

$11y + 2x - 30 = 0$

$y = -\frac{2}{11}x + \frac{30}{11}$

42. $y^2(x^2 + y^2) = 2x^2, \quad (1, 1)$

$y^2x^2 + y^4 = 2x^2$

$2yy'x^2 + 2xy^2 + 4y^3y' = 4x$

At $(1, 1)$:

$2y' + 2 + 4y' = 4$

$6y' = 2$

$y' = \frac{1}{3}$

Tangent line: $y - 1 = \frac{1}{3}(x - 1)$

$y = \frac{1}{3}x + \frac{2}{3}$

43. Answers will vary. *Sample answers*:

$xy = 2 \Rightarrow y = \dfrac{2}{x}$

$yx^2 + x = 2 \Rightarrow y = \dfrac{2 - x}{x^2}$

$x^2 + y^2 = 4$

$xy + y^2 = 2$

44. The equation $x^2 + y^2 + 2 = 1$ implies $x^2 + y^2 = -1$, which has no real solutions.

45. (a) $\dfrac{x^2}{2} + \dfrac{y^2}{8} = 1, \quad (1, 2)$

$x + \dfrac{yy'}{4} = 0$

$y' = -\dfrac{4x}{y}$

At $(1, 2)$: $y' = -2$

Tangent line: $y - 2 = -2(x - 1)$

$y = -2x + 4$

(b) $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1 \Rightarrow \dfrac{2x}{a^2} + \dfrac{2yy'}{b^2} = 0 \Rightarrow y' = \dfrac{-b^2x}{a^2y}$

$y - y_0 = \dfrac{-b^2x_0}{a^2y_0}(x - x_0)$, Tangent line at (x_0, y_0)

$\dfrac{y_0y}{b^2} - \dfrac{y_0^2}{b^2} = \dfrac{-x_0x}{a^2} + \dfrac{x_0^2}{a^2}$

Because $\dfrac{x_0^2}{a^2} + \dfrac{y_0^2}{b^2} = 1$, you have $\dfrac{y_0y}{b^2} + \dfrac{x_0x}{a^2} = 1$.

Note: From part (a),

$\dfrac{1(x)}{2} + \dfrac{2(y)}{8} = 1 \Rightarrow \dfrac{1}{4}y = -\dfrac{1}{2}x + 1 \Rightarrow y = -2x + 4$,

Tangent line.

46. (a) $\dfrac{x^2}{6} - \dfrac{y^2}{8} = 1, \quad (3, -2)$

$\dfrac{x}{3} - \dfrac{y}{4}y' = 0$

$\dfrac{y}{4}y' = \dfrac{x}{3}$

$y' = \dfrac{4x}{3y}$

At $(3, -2)$: $y' = \dfrac{4(3)}{3(-2)} = -2$

Tangent line: $y + 2 = -2(x - 3)$

$y = -2x + 4$

(b) $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1 \Rightarrow \dfrac{2x}{a^2} - \dfrac{2yy'}{b^2} = 0 \Rightarrow y' = \dfrac{xb^2}{ya^2}$

$y - y_0 = \dfrac{x_0b^2}{y_0a^2}(x - x_0)$, Tangent line at (x_0, y_0)

$\dfrac{yy_0}{b^2} - \dfrac{y_0^2}{b^2} = \dfrac{x_0x}{a^2} - \dfrac{x_0^2}{a^2}$

Because $\dfrac{x_0^2}{a^2} - \dfrac{y_0^2}{b^2} = 1$, you have $\dfrac{x_0x}{a^2} - \dfrac{yy_0}{b^2} = 1$.

Note: From part (a),

$\dfrac{3x}{6} - \dfrac{(-2)y}{8} = 1 \Rightarrow \dfrac{1}{2}x + \dfrac{y}{4} = 1 \Rightarrow y = -2x + 4$,

Tangent line.

47. $\tan y = x$

$y' \sec^2 y = 1$

$y' = \dfrac{1}{\sec^2 y} = \cos^2 y, -\dfrac{\pi}{2} < y < \dfrac{\pi}{2}$

$\sec^2 y = 1 + \tan^2 y = 1 + x^2$

$y' = \dfrac{1}{1 + x^2}$

48. $\cos y = x$

$-\sin y \cdot y' = 1$

$y' = \dfrac{-1}{\sin y}, \quad 0 < y < \pi$

$\sin^2 y + \cos^2 y = 1$

$\sin^2 y = 1 - \cos^2 y$

$\sin y = \sqrt{1 - \cos^2 y} = \sqrt{1 - x^2}$

$y' = \dfrac{-1}{\sqrt{1 - x^2}}, \quad -1 < x < 1$

49.
$$\begin{aligned} x^2 + y^2 &= 4 \\ 2x + 2yy' &= 0 \\ y' &= \frac{-x}{y} \\ y'' &= \frac{y(-1) + xy'}{y^2} \\ &= \frac{-y + x(-x/y)}{y^2} \\ &= \frac{-y^2 - x^2}{y^3} \\ &= -\frac{4}{y^3} \end{aligned}$$

50.
$$\begin{aligned} x^2y - 4x &= 5 \\ x^2y' + 2xy - 4 &= 0 \\ y' &= \frac{4 - 2xy}{x^2} \\ x^2y'' + 2xy' + 2xy' + 2y &= 0 \\ x^2y'' + 4x\left[\frac{4 - 2xy}{x^2}\right] + 2y &= 0 \\ x^4y'' + 4x(4 - 2xy) + 2x^2y &= 0 \\ x^4y'' + 16x - 8x^2y + 2x^2y &= 0 \\ x^4y'' &= 6x^2y - 16x \\ y'' &= \frac{6xy - 16}{x^3} \end{aligned}$$

51.
$$\begin{aligned} x^2y - 2 &= 5x + y \\ 2xy + x^2y' &= 5 + y' \\ \left(x^2 - 1\right)y' &= 5 - 2xy \\ y' &= \frac{5 - 2xy}{x^2 - 1} \\ 2xy' + \left(x^2 - 1\right)y'' &= -2y - 2xy' \\ \left(x^2 - 1\right)y'' &= -2y - 4xy' = -2y - 4x\left(\frac{5 - 2xy}{x^2 - 1}\right) \\ y'' &= \frac{-2y}{x^2 - 1} - \frac{4x(5 - 2xy)}{\left(x^2 - 1\right)^2} \\ &= \frac{-2y\left(x^2 - 1\right) - 20x + 8x^2y}{\left(x^2 - 1\right)^2} = \frac{6x^2y - 20x + 2y}{\left(x^2 - 1\right)^2} \end{aligned}$$

52.
$$\begin{aligned} xy - 1 &= 2x + y^2 \\ xy' + y &= 2 + 2yy' \\ xy' - 2yy' &= 2 - y \\ (x - 2y)y' &= 2 - y \\ y' &= \frac{2 - y}{x - 2y} \\ xy'' + y' + y' &= 2yy'' + 2(y')^2 \\ xy'' - 2yy'' &= 2(y')^2 - 2y' \\ (x - 2y)y'' &= 2(y')^2 - 2y' = 2\left(\frac{2 - y}{x - 2y}\right)^2 - 2\left(\frac{2 - y}{x - 2y}\right) \\ y'' &= \frac{2(2 - y)\left[(2 - y) - (x - 2y)\right]}{(x - 2y)^3} = \frac{2(2 - y)(2 - x + y)}{(x - 2y)^3} \\ &= \frac{2\left(4 - 2x + 2y - 2y + xy - y^2\right)}{(x - 2y)^3} = \frac{2\left(y^2 - xy + 2x - 4\right)}{(2y - x)^3} \\ &= \frac{2(-5)}{(2y - x)^3} = \frac{10}{(x - 2y)^3} \end{aligned}$$

53.

$$7xy + \sin x = 2$$

$$7xy' + 7y + \cos x = 0$$

$$y' = \frac{-7y - \cos x}{7x}$$

$$7xy'' + 7y' + 7y' - \sin x = 0$$

$$7xy'' = \sin x - 14y' = \sin x - 14\left(\frac{-7y - \cos x}{7x}\right)$$

$$7xy'' = \sin x + \frac{14y + 2\cos x}{x}$$

$$y'' = \frac{\sin x}{7x} + \frac{14y + 2\cos x}{7x^2}$$

$$y'' = \frac{x\sin x + 14y + 2\cos x}{7x^2}$$

54.

$$3xy - 4\cos x = -6$$

$$3xy' + 3y + 4\sin x = 0$$

$$y' = \frac{-4\sin x - 3y}{3x}$$

$$3xy'' + 3y' + 3y' + 4\cos x = 0$$

$$3xy'' = -6y' - 4\cos x = -6\left(\frac{-4\sin x - 3y}{3x}\right) - 4\cos x$$

$$= \frac{8\sin x + 6y - 4x\cos x}{x}$$

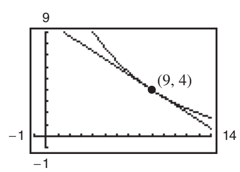
$$y'' = \frac{8\sin x + 6y - 4x\cos x}{3x^2}$$

55.

$$\sqrt{x} + \sqrt{y} = 5$$

$$\frac{1}{2}x^{-1/2} + \frac{1}{2}y^{-1/2}y' = 0$$

$$y' = \frac{-\sqrt{y}}{\sqrt{x}}$$



At $(9, 4)$: $y' = -\frac{2}{3}$

Tangent line: $y - 4 = -\frac{2}{3}(x - 9)$

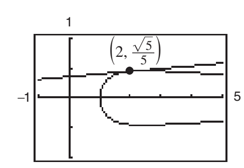
$$2x + 3y - 30 = 0$$

56.

$$y^2 = \frac{x-1}{x^2+1}$$

$$2yy' = \frac{(x^2+1)(1) - (x-1)(2x)}{(x^2+1)^2} = \frac{x^2 + 1 - 2x^2 + 2x}{(x^2+1)^2}$$

$$y' = \frac{1 + 2x - x^2}{2y(x^2+1)^2}$$



At $\left(2, \frac{\sqrt{5}}{5}\right)$: $y' = \dfrac{1 + 4 - 4}{\left[\frac{(2\sqrt{5})}{5}\right](4+1)^2} = \dfrac{1}{10\sqrt{5}}$

Tangent line: $y - \frac{\sqrt{5}}{5} = \frac{1}{10\sqrt{5}}(x - 2)$

$$10\sqrt{5}y - 10 = x - 2$$

$$x - 10\sqrt{5}y + 8 = 0$$

57. $x^2 + y^2 = 25$

$2x + 2yy' = 0$

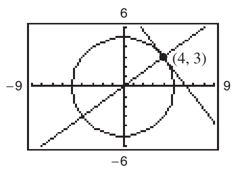
$y' = \frac{-x}{y}$

At $(4, 3)$:

Tangent line:

$y - 3 = \frac{-4}{3}(x - 4) \Rightarrow 4x + 3y - 25 = 0$

Normal line: $y - 3 = \frac{3}{4}(x - 4) \Rightarrow 3x - 4y = 0$

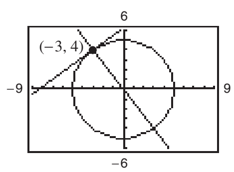


At $(-3, 4)$:

Tangent line:

$y - 4 = \frac{3}{4}(x + 3) \Rightarrow 3x - 4y + 25 = 0$

Normal line: $y - 4 = \frac{-4}{3}(x + 3) \Rightarrow 4x + 3y = 0$



58. $x^2 + y^2 = 36$

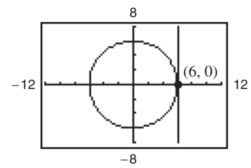
$2x + 2yy' = 0$

$y' = -\frac{x}{y}$

At $(6, 0)$; slope is undefined.

Tangent line: $x = 6$

Normal line: $y = 0$



At $(5, \sqrt{11})$, slope is $\frac{-5}{\sqrt{11}}$.

Tangent line: $y - \sqrt{11} = \frac{-5}{\sqrt{11}}(x - 5)$

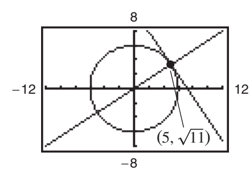
$\sqrt{11}y - 11 = -5x + 25$

$5x + \sqrt{11}y - 36 = 0$

Normal line: $y - \sqrt{11} = \frac{\sqrt{11}}{5}(x - 5)$

$5y - 5\sqrt{11} = \sqrt{11}x - 5\sqrt{11}$

$5y - \sqrt{11}x = 0$



59. $x^2 + y^2 = r^2$

$2x + 2yy' = 0$

$y' = \frac{-x}{y}$ = slope of tangent line

$\frac{y}{x}$ = slope of normal line

Let (x_0, y_0) be a point on the circle. If $x_0 = 0$, then the tangent line is horizontal, the normal line is vertical and, hence, passes through the origin. If $x_0 \neq 0$, then the equation of the normal line is

$y - y_0 = \frac{y_0}{x_0}(x - x_0)$

$y = \frac{y_0}{x_0}x$

which passes through the origin.

60. $y^2 = 4x$

$2yy' = 4$

$y' = \frac{2}{y} = 1$ at $(1, 2)$

Equation of normal line at $(1, 2)$ is $y - 2 = -1(x - 1)$, $y = 3 - x$. The centers of the circles must be on the normal line and at a distance of 4 units from $(1, 2)$. Therefore,

$$(x - 1)^2 + [(3 - x) - 2]^2 = 16$$
$$2(x - 1)^2 = 16$$
$$x = 1 \pm 2\sqrt{2}.$$

Centers of the circles: $(1 + 2\sqrt{2}, 2 - 2\sqrt{2})$ and $(1 - 2\sqrt{2}, 2 + 2\sqrt{2})$

Equations: $(x - 1 - 2\sqrt{2})^2 + (y - 2 + 2\sqrt{2})^2 = 16$

$(x - 1 + 2\sqrt{2})^2 + (y - 2 - 2\sqrt{2})^2 = 16$

61. $25x^2 + 16y^2 + 200x - 160y + 400 = 0$

$50x + 32yy' + 200 - 160y' = 0$

$y' = \frac{200 + 50x}{160 - 32y}$

Horizontal tangents occur when $x = -4$:

$25(16) + 16y^2 + 200(-4) - 160y + 400 = 0$

$y(y - 10) = 0 \Rightarrow y = 0, 10$

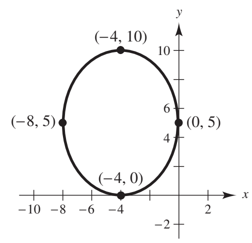
Horizontal tangents: $(-4, 0), (-4, 10)$

Vertical tangents occur when $y = 5$:

$25x^2 + 400 + 200x - 800 + 400 = 0$

$25x(x + 8) = 0 \Rightarrow x = 0, -8$

Vertical tangents: $(0, 5), (-8, 5)$



62. $4x^2 + y^2 - 8x + 4y + 4 = 0$

$8x + 2yy' - 8 + 4y' = 0$

$y' = \frac{8 - 8x}{2y + 4} = \frac{4 - 4x}{y + 2}$

Horizontal tangents occur when $x = 1$:

$4(1)^2 + y^2 - 8(1) + 4y + 4 = 0$

$y^2 + 4y = y(y + 4) = 0 \Rightarrow y = 0, -4$

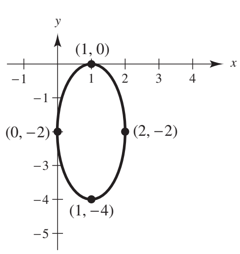
Horizontal tangents: $(1, 0), (1, -4)$

Vertical tangents occur when $y = -2$:

$4x^2 + (-2)^2 - 8x + 4(-2) + 4 = 0$

$4x^2 - 8x = 4x(x - 2) = 0 \Rightarrow x = 0, 2$

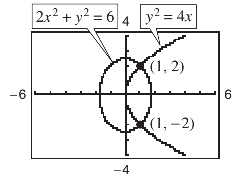
Vertical tangents: $(0, -2), (2, -2)$



63. Find the points of intersection by letting $y^2 = 4x$ in the equation $2x^2 + y^2 = 6$.

$2x^2 + 4x = 6$ and $(x + 3)(x - 1) = 0$

The curves intersect at $(1, \pm 2)$.



Ellipse:	*Parabola*:
$4x + 2yy' = 0$	$2yy' = 4$
$y' = -\dfrac{2x}{y}$	$y' = \dfrac{2}{y}$

At $(1, 2)$, the slopes are:

$y' = -1 \qquad y' = 1$

At $(1, -2)$, the slopes are:

$y' = 1 \qquad y' = -1$

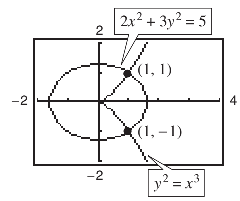
Tangents are perpendicular.

64. Find the points of intersection by letting $y^2 = x^3$ in the equation $2x^2 + 3y^2 = 5$.

$2x^2 + 3x^3 = 5$ and $3x^3 + 2x^2 - 5 = 0$

Intersect when $x = 1$.

Points of intersection: $(1, \pm 1)$



$y^2 = x^3$:	$2x^2 + 3y^2 = 5$:
$2yy' = 3x^2$	$4x + 6yy' = 0$
$y' = \dfrac{3x^2}{2y}$	$y' = -\dfrac{2x}{3y}$

At $(1, 1)$, the slopes are:

$y' = \dfrac{3}{2} \qquad y' = -\dfrac{2}{3}$

At $(1, -1)$, the slopes are:

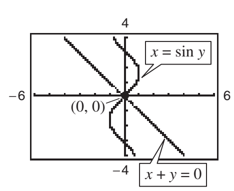
$y' = -\dfrac{3}{2} \qquad y' = \dfrac{2}{3}$

Tangents are perpendicular.

65. $y = -x$ and $x = \sin y$

Point of intersection: $(0, 0)$

$y = -x$:	$x = \sin y$:
$y' = -1$	$1 = y'\cos y$
	$y' = \sec y$



At $(0, 0)$, the slopes are:

$y' = -1 \qquad y' = 1$

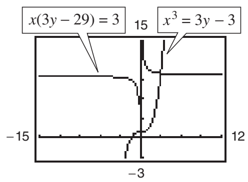
Tangents are perpendicular.

66. Rewriting each equation and differentiating:

$$x^3 = 3(y - 1) \qquad x(3y - 29) = 3$$

$$y = \frac{x^3}{3} + 1 \qquad y = \frac{1}{3}\left(\frac{3}{x} + 29\right)$$

$$y' = x^2 \qquad y' = -\frac{1}{x^2}$$



For each value of x, the derivatives are negative reciprocals of each other.
So, the tangent lines are orthogonal at both points of intersection.

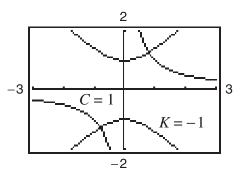
67.

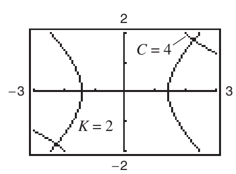
$$xy = C \qquad x^2 - y^2 = K$$

$$xy' + y = 0 \qquad 2x - 2yy' = 0$$

$$y' = -\frac{y}{x} \qquad y' = \frac{x}{y}$$

At any point of intersection (x, y) the product of the slopes is $(-y/x)(x/y) = -1$. The curves are orthogonal.





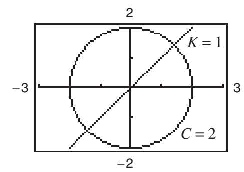
68.

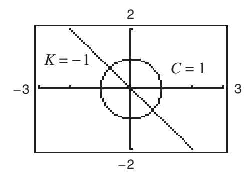
$$x^2 + y^2 = C^2 \qquad y = Kx$$

$$2x + 2yy' = 0 \qquad y' = K$$

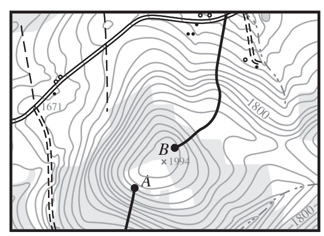
$$y' = -\frac{x}{y}$$

At the point of intersection (x, y), the product of the slopes is $(-x/y)(K) = (-x/Kx)(K) = -1$. The curves are orthogonal.





69.



Use starting point B.

70. (a) The slope is greater at $x = -3$.

(b) The graph has vertical tangent lines at about $(-2, 3)$ and $(2, 3)$.

(c) The graph has a horizontal tangent line at about $(0, 6)$.

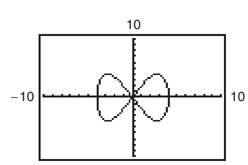
71. (a)

$$x^4 = 4(4x^2 - y^2)$$

$$4y^2 = 16x^2 - x^4$$

$$y^2 = 4x^2 - \frac{1}{4}x^4$$

$$y = \pm\sqrt{4x^2 - \frac{1}{4}x^4}$$

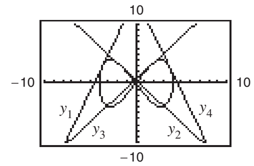


(b) $y = 3 \Rightarrow 9 = 4x^2 - \frac{1}{4}x^4$

$$36 = 16x^2 - x^4$$

$$x^4 - 16x^2 + 36 = 0$$

$$x^2 = \frac{16 \pm \sqrt{256 - 144}}{2} = 8 \pm \sqrt{28}$$



Note that $x^2 = 8 \pm \sqrt{28} = 8 \pm 2\sqrt{7} = \left(1 \pm \sqrt{7}\right)^2$. So, there are four values of x:

$-1 - \sqrt{7}, 1 - \sqrt{7}, -1 + \sqrt{7}, 1 + \sqrt{7}$

To find the slope, $2yy' = 8x - x^3 \Rightarrow y' = \frac{x\left(8 - x^2\right)}{2(3)}$.

For $x = -1 - \sqrt{7}$, $y' = \frac{1}{3}\left(\sqrt{7} + 7\right)$, and the line is

$$y_1 = \frac{1}{3}\left(\sqrt{7} + 7\right)\left(x + 1 + \sqrt{7}\right) + 3 = \frac{1}{3}\left[\left(\sqrt{7} + 7\right)x + 8\sqrt{7} + 23\right].$$

For $x = 1 - \sqrt{7}$, $y' = \frac{1}{3}\left(\sqrt{7} - 7\right)$, and the line is

$$y_2 = \frac{1}{3}\left(\sqrt{7} - 7\right)\left(x - 1 + \sqrt{7}\right) + 3 = \frac{1}{3}\left[\left(\sqrt{7} - 7\right)x + 23 - 8\sqrt{7}\right].$$

For $x = -1 + \sqrt{7}$, $y' = -\frac{1}{3}\left(\sqrt{7} - 7\right)$, and the line is

$$y_3 = -\frac{1}{3}\left(\sqrt{7} - 7\right)\left(x + 1 - \sqrt{7}\right) + 3 = -\frac{1}{3}\left[\left(\sqrt{7} - 7\right)x - \left(23 - 8\sqrt{7}\right)\right].$$

For $x = 1 + \sqrt{7}$, $y' = -\frac{1}{3}\left(\sqrt{7} + 7\right)$, and the line is

$$y_4 = -\frac{1}{3}\left(\sqrt{7} + 7\right)\left(x - 1 - \sqrt{7}\right) + 3 = -\frac{1}{3}\left[\left(\sqrt{7} + 7\right)x - \left(8\sqrt{7} + 23\right)\right].$$

(c) Equating y_3 and y_4:

$$-\frac{1}{3}\left(\sqrt{7} - 7\right)\left(x + 1 - \sqrt{7}\right) + 3 = -\frac{1}{3}\left(\sqrt{7} + 7\right)\left(x - 1 - \sqrt{7}\right) + 3$$

$$\left(\sqrt{7} - 7\right)\left(x + 1 - \sqrt{7}\right) = \left(\sqrt{7} + 7\right)\left(x - 1 - \sqrt{7}\right)$$

$$\sqrt{7}x + \sqrt{7} - 7 - 7x - 7 + 7\sqrt{7} = \sqrt{7}x - \sqrt{7} - 7 + 7x - 7 - 7\sqrt{7}$$

$$16\sqrt{7} = 14x$$

$$x = \frac{8\sqrt{7}}{7}$$

If $x = \frac{8\sqrt{7}}{7}$, then $y = 5$ and the lines intersect at $\left(\frac{8\sqrt{7}}{7}, 5\right)$.

72. $\sqrt{x} + \sqrt{y} = \sqrt{c}$

$$\frac{1}{2\sqrt{x}} + \frac{1}{2\sqrt{y}}\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = -\frac{\sqrt{y}}{\sqrt{x}}$$

Tangent line at (x_0, y_0): $y - y_0 = -\dfrac{\sqrt{y_0}}{\sqrt{x_0}}(x - x_0)$

x-intercept: $\left(x_0 + \sqrt{x_0}\sqrt{y_0}, 0\right)$

y-intercept: $\left(0, y_0 + \sqrt{x_0}\sqrt{y_0}\right)$

Sum of intercepts:

$$\left(x_0 + \sqrt{x_0}\sqrt{y_0}\right) + \left(y_0 + \sqrt{x_0}\sqrt{y_0}\right) = x_0 + 2\sqrt{x_0}\sqrt{y_0} + y_0 = \left(\sqrt{x_0} + \sqrt{y_0}\right)^2 = \left(\sqrt{c}\right)^2 = c$$

73. $y = x^{p/q}$; p, q integers and $q > 0$

$$y^q = x^p$$

$$qy^{q-1}y' = px^{p-1}$$

$$y' = \frac{p}{q} \cdot \frac{x^{p-1}}{y^{q-1}} = \frac{p}{q} \cdot \frac{x^{p-1}y}{y^q}$$

$$= \frac{p}{q} \cdot \frac{x^{p-1}}{x^p}x^{p/q} = \frac{p}{q}x^{p/q-1}$$

So, if $y = x^n$, $n = p/q$, then $y' = nx^{n-1}$.

74. $x^2 + y^2 = 100$, slope $= \dfrac{3}{4}$

$$2x + 2yy' = 0$$

$$y' = -\frac{x}{y} = \frac{3}{4} \Rightarrow y = -\frac{4}{3}x$$

$$x^2 + \left(\frac{16}{9}x^2\right) = 100$$

$$\frac{25}{9}x^2 = 100$$

$$x = \pm 6$$

Points: $(6, -8)$ and $(-6, 8)$

75. $\dfrac{x^2}{4} + \dfrac{y^2}{9} = 1, \quad (4, 0)$

$$\frac{2x}{4} + \frac{2yy'}{9} = 0$$

$$y' = \frac{-9x}{4y}$$

$$\frac{-9x}{4y} = \frac{y - 0}{x - 4}$$

$$-9x(x - 4) = 4y^2$$

But, $9x^2 + 4y^2 = 36 \Rightarrow 4y^2 = 36 - 9x^2$. So, $-9x^2 + 36x = 4y^2 = 36 - 9x^2 \Rightarrow x = 1$.

Points on ellipse: $\left(1, \pm\dfrac{3}{2}\sqrt{3}\right)$

At $\left(1, \dfrac{3}{2}\sqrt{3}\right)$: $y' = \dfrac{-9x}{4y} = \dfrac{-9}{4\left[(3/2)\sqrt{3}\right]} = -\dfrac{\sqrt{3}}{2}$

At $\left(1, -\dfrac{3}{2}\sqrt{3}\right)$: $y' = \dfrac{\sqrt{3}}{2}$

Tangent lines: $y = -\dfrac{\sqrt{3}}{2}(x - 4) = -\dfrac{\sqrt{3}}{2}x + 2\sqrt{3}$

$$y = \frac{\sqrt{3}}{2}(x - 4) = \frac{\sqrt{3}}{2}x - 2\sqrt{3}$$

76. $x = y^2$

$1 = 2yy'$

$y' = \dfrac{1}{2y}$, slope of tangent line

Consider the slope of the normal line joining $(x_0, 0)$ and $(x, y) = (y^2, y)$ on the parabola.

$$-2y = \frac{y - 0}{y^2 - x_0}$$

$$y^2 - x_0 = -\frac{1}{2}$$

$$y^2 = x_0 - \frac{1}{2}$$

(a) If $x_0 = \frac{1}{4}$, then $y^2 = \frac{1}{4} - \frac{1}{2} = -\frac{1}{4}$, which is impossible. So, the only normal line is the x-axis $(y = 0)$.

(b) If $x_0 = \frac{1}{2}$, then $y^2 = 0 \Rightarrow y = 0$. Same as part (a).

(c) If $x_0 = 1$, then $y^2 = \frac{1}{2} = x$ and there are three normal lines.

The x-axis, the line joining $(x_0, 0)$ and $\left(\frac{1}{2}, \frac{1}{\sqrt{2}}\right)$, and the line joining $(x_0, 0)$ and $\left(\frac{1}{2}, -\frac{1}{\sqrt{2}}\right)$

(d) If two normals are perpendicular, then their slopes are –1 and 1. So,

$$-2y = -1 = \frac{y - 0}{y^2 - x_0} \Rightarrow y = \frac{1}{2}$$

and

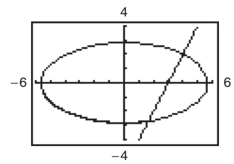
$$\frac{1/2}{(1/4) - x_0} = -1 \Rightarrow \frac{1}{4} - x_0 = -\frac{1}{2} \Rightarrow x_0 = \frac{3}{4}.$$

The perpendicular normal lines are $y = -x + \dfrac{3}{4}$ and $y = x - \dfrac{3}{4}$.

77. (a) $\dfrac{x^2}{32} + \dfrac{y^2}{8} = 1$

$$\frac{2x}{32} + \frac{2yy'}{8} = 0 \Rightarrow y' = \frac{-x}{4y}$$

(b)



At $(4, 2)$: $y' = \dfrac{-4}{4(2)} = -\dfrac{1}{2}$

Slope of normal line is 2.

$$y - 2 = 2(x - 4)$$

$$y = 2x - 6$$

(c) $$\frac{x^2}{32} + \frac{(2x - 6)^2}{8} = 1$$

$$x^2 + 4(4x^2 - 24x + 36) = 32$$

$$17x^2 - 96x + 112 = 0$$

$$(17x - 28)(x - 4) = 0 \Rightarrow x = 4, \frac{28}{17}$$

Second point: $\left(\dfrac{28}{17}, -\dfrac{46}{17}\right)$

Section 2.6 Related Rates

1. A related-rate equation is an equation that relates the rates of change of various quantities.

2. Answers will vary. See page 153.

3. $y = \sqrt{x}$

$$\frac{dy}{dt} = \left(\frac{1}{2\sqrt{x}}\right)\frac{dx}{dt}$$

$$\frac{dx}{dt} = 2\sqrt{x}\,\frac{dy}{dt}$$

(a) When $x = 4$ and $dx/dt = 3$:

$$\frac{dy}{dt} = \frac{1}{2\sqrt{4}}(3) = \frac{3}{4}$$

(b) When $x = 25$ and $dy/dt = 2$:

$$\frac{dx}{dt} = 2\sqrt{25}(2) = 20$$

4. $y = 3x^2 - 5x$

$$\frac{dy}{dt} = (6x - 5)\frac{dx}{dt}$$

$$\frac{dx}{dt} = \frac{1}{6x - 5}\frac{dy}{dt}$$

(a) When $x = 3$ and $\frac{dx}{dt} = 2$:

$$\frac{dy}{dt} = [6(3) - 5]2 = 26$$

(b) When $x = 2$ and $\frac{dy}{dt} = 4$:

$$\frac{dx}{dt} = \frac{1}{6(2) - 5}(4) = \frac{4}{7}$$

5. $xy = 4$

$$x\frac{dy}{dt} + y\frac{dx}{dt} = 0$$

$$\frac{dy}{dt} = \left(-\frac{y}{x}\right)\frac{dx}{dt}$$

$$\frac{dx}{dt} = \left(-\frac{x}{y}\right)\frac{dy}{dt}$$

(a) When $x = 8$, $y = 1/2$, and $dx/dt = 10$:

$$\frac{dy}{dt} = -\frac{1/2}{8}(10) = -\frac{5}{8}$$

(b) When $x = 1$, $y = 4$, and $dy/dt = -6$:

$$\frac{dx}{dt} = -\frac{1}{4}(-6) = \frac{3}{2}$$

6. $x^2 + y^2 = 25$

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$

$$\frac{dy}{dt} = \left(-\frac{x}{y}\right)\frac{dx}{dt}$$

$$\frac{dx}{dt} = \left(-\frac{y}{x}\right)\frac{dy}{dt}$$

(a) When $x = 3$, $y = 4$, and $dx/dt = 8$:

$$\frac{dy}{dt} = -\frac{3}{4}(8) = -6$$

(b) When $x = 4$, $y = 3$, and $dy/dt = -2$:

$$\frac{dx}{dt} = -\frac{3}{4}(-2) = \frac{3}{2}$$

7. $y = 2x^2 + 1$

$$\frac{dx}{dt} = 2$$

$$\frac{dy}{dt} = 4x\frac{dx}{dt}$$

(a) When $x = -1$:

$$\frac{dy}{dt} = 4(-1)(2) = -8 \text{ cm/sec}$$

(b) When $x = 0$:

$$\frac{dy}{dt} = 4(0)(2) = 0 \text{ cm/sec}$$

(c) When $x = 1$:

$$\frac{dy}{dt} = 4(1)(2) = 8 \text{ cm/sec}$$

8. $y = \frac{1}{1 + x^2}, \frac{dx}{dt} = 6$

$$\frac{dy}{dt} = \frac{-2x}{(1 + x^2)^2} \cdot \frac{dx}{dt}$$

$$= \frac{-2x}{(1 + x^2)^2}(6) = \frac{-12x}{(1 + x^2)^2}$$

(a) When $x = -2$:

$$\frac{dy}{dt} = \frac{(-12)(-2)}{\left[1 + (-2)^2\right]^2} = \frac{24}{25} \text{ mm/sec}$$

(b) When $x = 0$:

$$\frac{dy}{dt} = \frac{-12(0)}{(1 + 0)^2} = 0 \text{ mm/sec}$$

(c) When $x = 2$:

$$\frac{dy}{dt} = \frac{(-12)(2)}{(1 + 2^2)^2} = -\frac{24}{25} \text{ mm/sec}$$

9. $y = \tan x, \dfrac{dx}{dt} = 3$

$$\frac{dy}{dt} = \sec^2 x \cdot \frac{dx}{dt} = \sec^2 x(3) = 3\sec^2 x$$

(a) When $x = -\dfrac{\pi}{3}$:

$$\frac{dy}{dt} = 3\sec^2\left(-\frac{\pi}{3}\right) = 3(2)^2 = 12 \text{ mm/sec}$$

(b) When $x = -\dfrac{\pi}{4}$:

$$\frac{dy}{dt} = 3\sec^2\left(-\frac{\pi}{4}\right) = 3\left(\sqrt{2}\right)^2 = 6 \text{ mm/sec}$$

(c) When $x = 0$:

$$\frac{dy}{dt} = 3\sec^2(0) = 3 \text{ mm/sec}$$

10. $y = \cos x, \dfrac{dx}{dt} = 4$

$$\frac{dy}{dt} = -\sin x \cdot \frac{dx}{dt} = -\sin x(4) = -4\sin x$$

(a) When $x = \dfrac{\pi}{6}$:

$$\frac{dy}{dt} = -4\sin\left(\frac{\pi}{6}\right) = -4\left(\frac{1}{2}\right) = -2 \text{ cm/sec}$$

(b) When $x = \dfrac{\pi}{4}$:

$$\frac{dy}{dt} = -4\sin\left(\frac{\pi}{4}\right) = -4\left(\frac{\sqrt{2}}{2}\right) = -2\sqrt{2} \text{ cm/sec}$$

(c) When $x = \dfrac{\pi}{3}$:

$$\frac{dy}{dt} = -4\sin\left(\frac{\pi}{3}\right) = -4\left(\frac{\sqrt{3}}{2}\right) = -2\sqrt{3} \text{ cm/sec}$$

11. $A = \pi r^2$

$$\frac{dr}{dt} = 4$$

$$\frac{dA}{dt} = 2\pi r\frac{dr}{dt}$$

When $r = 37, \dfrac{dA}{dt} = 2\pi(37)(4) = 296\,\pi \text{ cm}^2/\text{min}.$

12. $A = \dfrac{s^2\sqrt{3}}{4}$

$$\frac{ds}{dt} = 13$$

$$\frac{dA}{dt} = \frac{\sqrt{3}}{4}(2s)\frac{ds}{dt} = \frac{\sqrt{3}s}{2}\frac{ds}{dt}$$

When $s = 41, \dfrac{dA}{dt} = \dfrac{\sqrt{3}}{2}(41)(13) = \dfrac{533\sqrt{3}}{2} \text{ m}^2/\text{min}.$

13. $V = \dfrac{4}{3}\pi r^3$

$$\frac{dr}{dt} = 3$$

$$\frac{dV}{dt} = 4\pi r^2\frac{dr}{dt}$$

(a) When $r = 9$,

$$\frac{dV}{dt} = 4\pi(9)^2(3) = 972\pi \text{ cm}^3/\text{min}.$$

When $r = 36$,

$$\frac{dV}{dt} = 4\pi(36)^2(3) = 15{,}552\pi \text{ cm}^3/\text{min}.$$

(b) If dr/dt is constant, dV/dt is proportional to r^2.

14. $V = \dfrac{4}{3}\pi r^3, \dfrac{dV}{dt} = 4\pi r^2\dfrac{dr}{dt}$

$$\frac{dV}{dt} = 800$$

(a) $\dfrac{dr}{dt} = \dfrac{dV/dt}{4\pi r^2}$

At $r = 30, \dfrac{dr}{dt} = \dfrac{800}{4\pi(30)^2} = \dfrac{2}{9\pi}$ cm/min.

At $r = 85, \dfrac{dr}{dt} = \dfrac{800}{4\pi(85)^2} = \dfrac{8}{289\pi}$ cm/min.

(b) $\dfrac{dr}{dt}$ depends on r^2, not r.

15. $V = x^3$

$$\frac{dx}{dt} = 6$$

$$\frac{dV}{dt} = 3x^2\frac{dx}{dt}$$

(a) When $x = 2$,

$$\frac{dV}{dt} = 3(2)^2(6) = 72 \text{ cm}^3/\text{sec}.$$

(b) When $x = 10$,

$$\frac{dV}{dt} = 3(10)^2(6) = 1800 \text{ cm}^3/\text{sec}.$$

16. $s = 6x^2$

$$\frac{dx}{dt} = 6$$

$$\frac{ds}{dt} = 12x\frac{dx}{dt}$$

(a) When $x = 2$,

$$\frac{ds}{dt} = 12(2)(6) = 144 \text{ cm}^2/\text{sec}.$$

(b) When $x = 10$,

$$\frac{ds}{dt} = 12(10)(6) = 720 \text{ cm}^2/\text{sec}.$$

17. $V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi\left(\frac{9}{4}h^2\right)h$ [because $2r = 3h$]

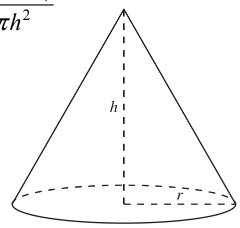
$= \frac{3\pi}{4}h^3$

$\frac{dV}{dt} = 10$

$\frac{dV}{dt} = \frac{9\pi}{4}h^2 \frac{dh}{dt} \Rightarrow \frac{dh}{dt} = \frac{4(dV/dt)}{9\pi h^2}$

When $h = 4$,

$\frac{dh}{dt} = \frac{4(10)}{9\pi(4)^2} = \frac{5}{18\pi}$ m/min.



18. $V = \pi r^2 h$

$\frac{dV}{dt} = 150$

$h = 10r \Rightarrow r = \frac{h}{10}$

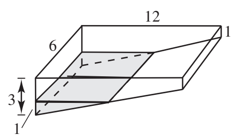
$V = \pi\left(\frac{h}{10}\right)^2 h = \frac{\pi}{100}h^3$

$\frac{dV}{dt} = \frac{3\pi}{100}h^2 \frac{dh}{dt}$

$\frac{dh}{dt} = \frac{100}{3\pi h^2}\frac{dV}{dt}$

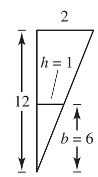
When $h = 35$, $\frac{dh}{dt} = \frac{100}{3\pi(35)^2}(150) = \frac{200}{49\pi}$ cm/sec.

19.



(a) Total volume of pool $= \frac{1}{2}(2)(12)(6) + (1)(6)(12) = 144$ m^3

Volume of 1 m of water $= \frac{1}{2}(1)(6)(6) = 18$ m^3 (see similar triangle diagram)



% pool filled $= \frac{18}{144}(100\%) = 12.5\%$

(b) Because for $0 \le h \le 2$, $b = 6h$, you have

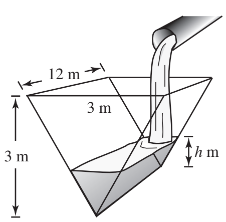
$V = \frac{1}{2}bh(6) = 3bh = 3(6h)h = 18h^2$

$\frac{dV}{dt} = 36h\frac{dh}{dt} = \frac{1}{4} \Rightarrow \frac{dh}{dt} = \frac{1}{144h} = \frac{1}{144(1)} = \frac{1}{144}$ m/min.

20. $V = \frac{1}{2}bh(12) = 6bh = 6h^2$ (since $b = h$)

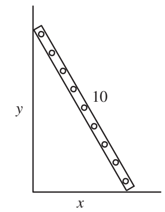
(a) $\frac{dV}{dt} = 12h\frac{dh}{dt} \Rightarrow \frac{dh}{dt} = \frac{1}{12h}\frac{dV}{dt}$

When $h = 1$ and $\frac{dV}{dt} = 2$, $\frac{dh}{dt} = \frac{1}{12(1)}(2) = \frac{1}{6}$ m/min.



(b) If $\frac{dh}{dt} = 2.5$ cm/min $= 0.025$ m/min and $h = 2$ m, then $\frac{dV}{dt} = (12)(2)(0.025) = 0.6$ m^3/min.

21. $x^2 + y^2 = 10^2$



$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$

$$\frac{dy}{dt} = \frac{-x}{y}\cdot\frac{dx}{dt} = \frac{-2x}{3y} \quad \text{because } \frac{dx}{dt} = \frac{2}{3}.$$

(a) When $x = 2, y = \sqrt{96} = 4\sqrt{6}, \dfrac{dy}{dt} = \dfrac{-2(2)}{3(4\sqrt{6})} = \dfrac{-\sqrt{6}}{18} \approx -0.136$ m/sec.

When $x = 6, y = \sqrt{64} = 8, \dfrac{dy}{dt} = \dfrac{-2(6)}{3(8)} = -\dfrac{1}{2}$ m/sec.

When $x = 8, y = 6, \dfrac{dy}{dt} = \dfrac{-2(8)}{3(6)} = -\dfrac{8}{9}$ m/sec.

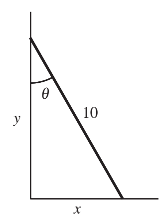
(b) $A = \dfrac{1}{2}xy$

$$\frac{dA}{dt} = \frac{1}{2}\left(x\frac{dy}{dt} + y\frac{dx}{dt}\right)$$

From part (a) you have $x = 6, y = 8, \dfrac{dx}{dt} = \dfrac{2}{3}$, and $\dfrac{dy}{dt} = -\dfrac{1}{2}$. So,

$$\frac{dA}{dt} = \frac{1}{2}\left[6\left(-\frac{1}{2}\right) + 8\left(\frac{2}{3}\right)\right] = \frac{7}{6} \text{ m}^2/\text{sec}.$$

(c) $\tan\theta = \dfrac{x}{y}$



$$\sec^2\theta\frac{d\theta}{dt} = \frac{1}{y}\cdot\frac{dx}{dt} - \frac{x}{y^2}\cdot\frac{dy}{dt}$$

$$\frac{d\theta}{dt} = \cos^2\theta\left[\frac{1}{y}\cdot\frac{dx}{dt} - \frac{x}{y^2}\cdot\frac{dy}{dt}\right]$$

Using $x = 6, y = 8, \dfrac{dx}{dt} = \dfrac{2}{3}, \dfrac{dy}{dt} = -\dfrac{1}{2}$ and $\cos\theta = \dfrac{8}{10}$, you have

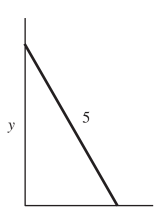
$$\frac{d\theta}{dt} = \left(\frac{8}{10}\right)^2\left[\frac{1}{8}\left(\frac{2}{3}\right) - \frac{6}{(8)^2}\left(-\frac{1}{2}\right)\right] = \frac{1}{12} \text{ rad/sec.}$$

22. $x^2 + y^2 = 25$

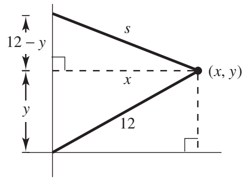
$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$

$$\frac{dx}{dt} = -\frac{y}{x}\cdot\frac{dy}{dt} = -\frac{0.15y}{x} \quad \left(\text{because } \frac{dy}{dt} = 0.15\right)$$

When $x = 2.5, y = \sqrt{18.75}, \dfrac{dx}{dt} = -\dfrac{\sqrt{18.75}}{2.5}0.15 \approx -0.26$ m/sec.



23. When $y = 6, x = \sqrt{12^2 - 6^2} = 6\sqrt{3}$, and $s = \sqrt{x^2 + (12 - y)^2} = \sqrt{108 + 36} = 12$.



$$x^2 + (12 - y)^2 = s^2$$

$$2x\frac{dx}{dt} + 2(12 - y)(-1)\frac{dy}{dt} = 2s\frac{ds}{dt}$$

$$x\frac{dx}{dt} + (y - 12)\frac{dy}{dt} = s\frac{ds}{dt}$$

Also, $x^2 + y^2 = 12^2$.

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0 \Rightarrow \frac{dy}{dt} = \frac{-x}{y}\frac{dx}{dt}$$

So, $x\frac{dx}{dt} + (y - 12)\left(\frac{-x}{y}\frac{dx}{dt}\right) = s\frac{ds}{dt}$.

$$\frac{dx}{dt}\left[x - x + \frac{12x}{y}\right] = s\frac{ds}{dt} \Rightarrow \frac{dx}{dt} = \frac{sy}{12x}\cdot\frac{ds}{dt} = \frac{(12)(6)}{(12)(6\sqrt{3})}(-0.2) = \frac{-1}{5\sqrt{3}} = \frac{-\sqrt{3}}{15} \text{ m/sec (horizontal)}$$

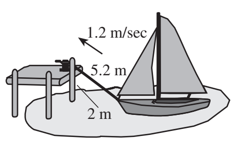
$$\frac{dy}{dt} = \frac{-x}{y}\frac{dx}{dt} = \frac{-6\sqrt{3}}{6}\cdot\frac{(-\sqrt{3})}{15} = \frac{1}{5} \text{ m/sec (vertical)}$$

24. Let L be the length of the rope.

(a) $$L^2 = 4 + x^2$$

$$2L\frac{dL}{dt} = 2x\frac{dx}{dt}$$

$$\frac{dx}{dt} = \frac{L}{x}\cdot\frac{dL}{dt} = -\frac{1.2L}{x} \quad \left(\text{since } \frac{dL}{dt} = -1.2 \text{ m/sec}\right)$$



When $L = 5.2$:

$$x = \sqrt{L^2 - 4} = \sqrt{27.04 - 4} = 4.8$$

$$\frac{dx}{dt} = -\frac{1.2(5.2)}{4.8} = -\frac{6.24}{4.8} = -1.3 \text{ m/sec}$$

Speed of the boat increases as it approaches the dock.

(b) If $\frac{dx}{dt} = -1.2$, and $L = 5.2$:

$$\frac{dL}{dt} = \frac{x}{L}\frac{dx}{dt} = \frac{4.8}{5.2}(-1.2) \approx 1.11 \text{ m/sec}$$

$$\frac{dL}{dt} = \frac{x}{L}\frac{dx}{dt} = \frac{\sqrt{L^2 - 4}}{L}(-1.2)$$

$$\lim_{L\to 2^+}\frac{dL}{dt} = \lim_{L\to 2^+}\frac{-1.2}{L}\sqrt{L^2 - 4} = 0$$

Speed of the winch decreases as the boat gets closer to the dock.

25. (a)

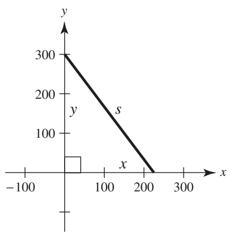
$$s^2 = x^2 + y^2$$

$$\frac{dx}{dt} = -450$$

$$\frac{dy}{dt} = -600$$

$$2s\frac{ds}{dt} = 2x\frac{dx}{dt} + 2y\frac{dy}{dt}$$

$$\frac{ds}{dt} = \frac{x(dx/dt) + y(dy/dt)}{s}$$



When $x = 225$ and $y = 300$, $s = 375$ and

$$\frac{ds}{dt} = \frac{225(-450) + 300(-600)}{375} = -750 \text{ km/h}.$$

(b) $t = \dfrac{375}{750} = \dfrac{1}{2}\text{ h} = 30 \text{ min}$

26.

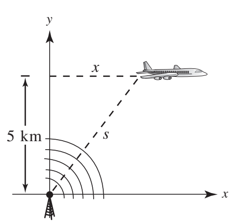
$$x^2 + y^2 = s^2$$

$$2x\frac{dx}{dt} + 0 = 2s\frac{ds}{dt} \qquad \left(\text{because } \frac{dy}{dt} = 0\right)$$

$$\frac{dx}{dt} = \frac{s}{x}\frac{ds}{dt}$$

When $s = 10$, $x = \sqrt{100 - 25} = \sqrt{75} = 5\sqrt{3}$,

$$\frac{dx}{dt} = \frac{10}{5\sqrt{3}}(480) = \frac{960}{\sqrt{3}} = 320\sqrt{3} \approx 554.26 \text{ km/h}.$$



27.

$$s^2 = 90^2 + x^2$$

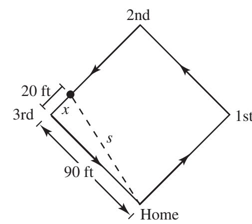
$$x = 20$$

$$\frac{dx}{dt} = -25$$

$$2s\frac{ds}{dt} = 2x\frac{dx}{dt} \Rightarrow \frac{ds}{dt} = \frac{x}{s}\cdot\frac{dx}{dt}$$

When $x = 20$, $s = \sqrt{90^2 + 20^2} = 10\sqrt{85}$,

$$\frac{ds}{dt} = \frac{20}{10\sqrt{85}}(-25) = \frac{-50}{\sqrt{85}} \approx -5.42 \text{ ft/sec}.$$



28.

$$s^2 = 90^2 + x^2$$

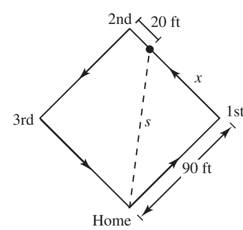
$$x = 90 - 20 = 70$$

$$\frac{dx}{dt} = 25$$

$$\frac{ds}{dt} = \frac{x}{s}\cdot\frac{dx}{dt}$$

When $x = 70$, $s = \sqrt{90^2 + 70^2} = 10\sqrt{130}$,

$$\frac{ds}{dt} = \frac{70}{10\sqrt{130}}(25) = \frac{175}{\sqrt{130}} \approx 15.35 \text{ ft/sec}.$$



29. (a)

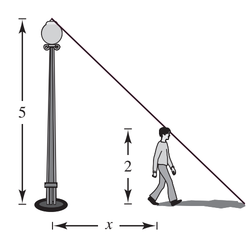
$$\frac{5}{2} = \frac{y}{y - x} \Rightarrow 5y - 5x = 2y$$

$$y = \frac{5}{3}x$$

$$\frac{dx}{dt} = 1.5$$

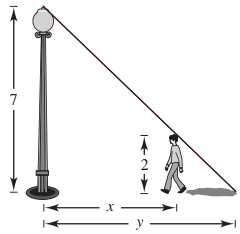
$$\frac{dy}{dt} = \frac{5}{3}\cdot\frac{dx}{dt} = \frac{5}{3}(1.5) = 2.5 \text{ m/sec}$$

(b) $\dfrac{d(y - x)}{dt} = \dfrac{dy}{dt} - \dfrac{dx}{dt} = 2.5 - 1.5 = 1 \text{ m/sec}$



30. (a)

$$\frac{7}{2} = \frac{y}{y-x}$$
$$7y - 7x = 2y$$
$$5y = 7x$$
$$y = \frac{7}{5}x$$
$$\frac{dx}{dt} = -1.5$$
$$\frac{dy}{dt} = \frac{7}{5}\frac{dx}{dt} = \frac{7}{5}(-1.5) = -2.1 \text{ m/sec}$$



(b)
$$\frac{d(y-x)}{dt} = \frac{dy}{dt} - \frac{dx}{dt} = -2.1 - (-1.5) = -0.6 \text{ m/sec}$$

31. $x(t) = \frac{1}{2}\sin\frac{\pi t}{6},\ x^2 + y^2 = 1$

(a) Period: $\frac{2\pi}{\pi/6} = 12$ seconds

(b) When $x = \frac{1}{2}$, $y = \sqrt{1^2 - \left(\frac{1}{2}\right)^2} = \frac{\sqrt{3}}{2}$ m.

Lowest point: $\left(0, \frac{\sqrt{3}}{2}\right)$

(c) When $x = \frac{1}{4}$,

$$y = \sqrt{1 - \left(\frac{1}{4}\right)^2} = \frac{\sqrt{15}}{4} \text{ and } t = 1:$$
$$\frac{dx}{dt} = \frac{1}{2}\left(\frac{\pi}{6}\right)\cos\frac{\pi t}{6} = \frac{\pi}{12}\cos\frac{\pi t}{6}$$
$$x^2 + y^2 = 1$$
$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0 \Rightarrow \frac{dy}{dt} = \frac{-x}{y}\frac{dx}{dt}$$

So, $\frac{dy}{dt} = -\frac{1/4}{\sqrt{15}/4} \cdot \frac{\pi}{12}\cos\left(\frac{\pi}{6}\right)$

$$= \frac{-\pi}{\sqrt{15}}\left(\frac{1}{12}\right)\frac{\sqrt{3}}{2} = \frac{-\pi}{24}\frac{1}{\sqrt{5}} = \frac{-\sqrt{5}\pi}{120}.$$

Speed $= \left|\frac{-\sqrt{5}\pi}{120}\right| = \frac{\sqrt{5}\pi}{120}$ m/sec

32. $x(t) = \frac{3}{5}\sin \pi t,\ x^2 + y^2 = 1$

(a) Period: $\frac{2\pi}{\pi} = 2$ seconds

(c) When $x = \frac{3}{10}$, $y = \sqrt{1 - \left(\frac{1}{4}\right)^2} = \frac{\sqrt{15}}{4}$ and

$$\frac{3}{10} = \frac{3}{5}\sin \pi t \Rightarrow \sin \pi t = \frac{1}{2} \Rightarrow t = \frac{1}{6}$$
$$\frac{dx}{dt} = \frac{3}{5}\pi\cos \pi t$$
$$x^2 + y^2 = 1$$
$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0 \Rightarrow \frac{dy}{dt} = \frac{-x}{y}\frac{dx}{dt}$$

So, $\frac{dy}{dt} = \frac{-3/10}{\sqrt{15}/4} \cdot \frac{3}{5}\pi\cos\left(\frac{\pi}{6}\right)$

$$= \frac{-9\pi}{25\sqrt{5}} = \frac{-9\sqrt{5}\pi}{125}.$$

Speed $= \left|\frac{-9\sqrt{5}\pi}{125}\right| \approx 0.5058$ m/sec

33. Because the evaporation rate is proportional to the surface area, $dV/dt = k(4\pi r^2)$. However, because $V = (4/3)\pi r^3$, you have $\frac{dV}{dt} = 4\pi r^2\frac{dr}{dt}$. Therefore, $k(4\pi r^2) = 4\pi r^2\frac{dr}{dt} \Rightarrow k = \frac{dr}{dt}$.

34. **(i)** (a) $\frac{dx}{dt}$ negative $\Rightarrow \frac{dy}{dt}$ positive

(b) $\frac{dy}{dt}$ positive $\Rightarrow \frac{dx}{dt}$ negative

(ii) (a) $\frac{dx}{dt}$ negative $\Rightarrow \frac{dy}{dt}$ negative

(b) $\frac{dy}{dt}$ positive $\Rightarrow \frac{dx}{dt}$ positive

35. (a) $dy/dt = 3(dx/dt)$ means that y changes three times as fast as x changes.

(b) y changes slowly when $x \approx 0$ or $x \approx L$. y changes more rapidly when x is near the middle of the interval.

36. No. $V = s^3$, $\dfrac{dV}{dt} = 3s^2\dfrac{ds}{dt}$

If $\dfrac{ds}{dt}$ is constant, then $\dfrac{dV}{dt}$ is $3s^2$ times that constant.

37.
$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$
$$\frac{dR_1}{dt} = 1$$
$$\frac{dR_2}{dt} = 1.5$$
$$\frac{1}{R^2}\cdot\frac{dR}{dt} = \frac{1}{R_1^2}\cdot\frac{dR_1}{dt} + \frac{1}{R_2^2}\cdot\frac{dR_2}{dt}$$

When $R_1 = 50$ and $R_2 = 75$:

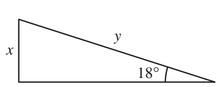
$R = 30$
$$\frac{dR}{dt} = (30)^2\left[\frac{1}{(50)^2}(1) + \frac{1}{(75)^2}(1.5)\right] = 0.6 \text{ ohm/sec}$$

38. $V = IR$
$$\frac{dV}{dt} = I\frac{dR}{dt} + R\frac{dI}{dt}$$
$$\frac{dI}{dt} = \frac{1}{R}\frac{dV}{dt} - \frac{I}{R}\frac{dR}{dt}$$

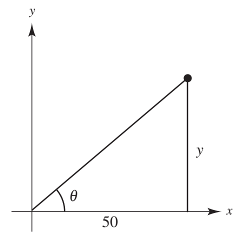
When $V = 12$, $R = 4$, $\dfrac{dV}{dt} = 3$, and

$\dfrac{dR}{dt} = 2$, $I = \dfrac{V}{R} = \dfrac{12}{4} = 3$ and
$$\frac{dI}{dt} = \frac{1}{4}(3) - \frac{3}{4}(2) = -\frac{3}{4} \text{ amperes/sec.}$$

39. $\sin 18^\circ = \dfrac{x}{y}$
$$0 = -\frac{x}{y^2}\cdot\frac{dy}{dt} + \frac{1}{y}\cdot\frac{dx}{dt}$$
$$\frac{dx}{dt} = \frac{x}{y}\cdot\frac{dy}{dt} = (\sin 18^\circ)(275) \approx 84.9797 \text{ km/h}$$

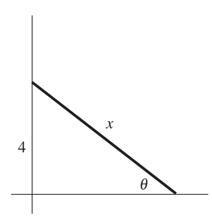


40.
$$\tan\theta = \frac{y}{50}$$
$$\frac{dy}{dt} = 4 \text{ m/sec}$$
$$\sec^2\theta\cdot\frac{d\theta}{dt} = \frac{1}{50}\frac{dy}{dt}$$
$$\frac{d\theta}{dt} = \frac{1}{50}\cos^2\theta\cdot\frac{dy}{dt}$$



When $y = 50$, $\theta = \dfrac{\pi}{4}$, and $\cos\theta = \dfrac{\sqrt{2}}{2}$. So,
$$\frac{d\theta}{dt} = \frac{1}{50}\left(\frac{\sqrt{2}}{2}\right)^2(4) = \frac{1}{25} \text{ rad/sec.}$$

41.
$$\sin\theta = \frac{4}{x}$$
$$\frac{dx}{dt} = (-0.4) \text{ m/sec}$$
$$\cos\theta\left(\frac{d\theta}{dt}\right) = \frac{-4}{x^2}\cdot\frac{dx}{dt}$$
$$\frac{d\theta}{dt} = \frac{-4}{x^2}\frac{dx}{dt}(\sec\theta)$$
$$= \frac{-4}{10^2}(-0.4)\frac{10}{\sqrt{10^2 - 4^2}}$$
$$= \frac{16}{100\sqrt{84}} = \frac{2}{25\sqrt{21}}$$
$$= \frac{2\sqrt{21}}{525} \approx 0.017 \text{ rad/sec}$$



42. $\tan\theta = \dfrac{y}{x},\ y = 5$

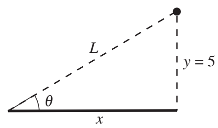
$$\frac{dx}{dt} = -600 \text{ km/h}$$

$$\left(\sec^2\theta\right)\frac{d\theta}{dt} = -\frac{5}{x^2}\cdot\frac{dx}{dt}$$

$$\frac{d\theta}{dt} = \cos^2\theta\left(-\frac{5}{x^2}\right)\frac{dx}{dt} = \frac{x^2}{L^2}\left(-\frac{5}{x^2}\right)\frac{dx}{dt}$$

$$= \left(-\frac{5^2}{L^2}\right)\left(\frac{1}{5}\right)\frac{dx}{dt}$$

$$= \left(-\sin^2\theta\right)\left(\frac{1}{5}\right)(-600) = 120\sin^2\theta$$



(a) When $\theta = 30°$,

$$\frac{d\theta}{dt} = \frac{120}{4} = 30 \text{ rad/h} = \frac{1}{2} \text{ rad/min.}$$

(b) When $\theta = 60°$,

$$\frac{d\theta}{dt} = 120\left(\frac{3}{4}\right) = 90 \text{ rad/h} = \frac{3}{2} \text{ rad/min.}$$

(c) When $\theta = 75°$,

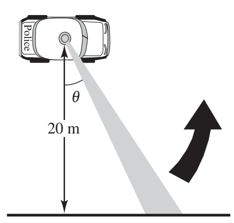
$$\frac{d\theta}{dt} = 120\sin^2 75° \approx 111.96 \text{ rad/h} \approx 1.87 \text{ rad/min.}$$

43. $\tan\theta = \dfrac{x}{20}$

$$\frac{d\theta}{dt} = 30(2\pi) = 60\pi \text{ rad/min} = \pi \text{ rad/sec}$$

$$\sec^2\theta\left(\frac{d\theta}{dt}\right) = \frac{1}{20}\left(\frac{dx}{dt}\right)$$

$$\frac{dx}{dt} = 20\sec^2\theta\left(\frac{d\theta}{dt}\right)$$



(a) When $\theta = 30°$, $\dfrac{dx}{dt} = \dfrac{80\pi}{3}$ m/sec.

(b) When $\theta = 60°$, $\dfrac{dx}{dt} = 80\pi$ m/sec.

(c) When $\theta = 70°$, $\dfrac{dx}{dt} \approx 170.97\pi$ m/sec.

44. $\dfrac{d\theta}{dt} = (10 \text{ rev/sec})(2\pi \text{ rad/rev}) = 20\pi \text{ rad/sec}$

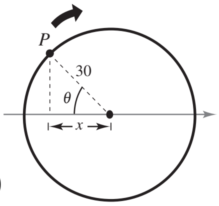
(a) $\cos\theta = \dfrac{x}{30}$

$$-\sin\theta\frac{d\theta}{dt} = \frac{1}{30}\frac{dx}{dt}$$

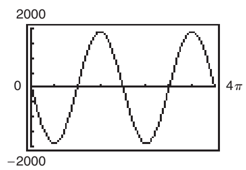
$$\frac{dx}{dt} = -30\sin\theta\frac{d\theta}{dt}$$

$$= -30\sin\theta(20\pi)$$

$$= -600\pi\sin\theta$$



(b)



(c) $|dx/dt| = |-600\pi\sin\theta|$ is greatest when

$|\sin\theta| = 1 \Rightarrow \theta = \dfrac{\pi}{2} + n\pi$ (or $90° + n\cdot 180°$).

$|dx/dt|$ is least when $\theta = n\pi$ (or $n\cdot 180°$).

(d) For $\theta = 30°$,

$$\frac{dx}{dt} = -600\pi\sin(30°) = -600\pi\frac{1}{2} = -300\pi \text{ cm/sec.}$$

For $\theta = 60°$,

$$\frac{dx}{dt} = -600\pi\sin(60°)$$

$$= -600\pi\frac{\sqrt{3}}{2} = -300\sqrt{3}\pi \text{ cm/sec.}$$

45. (a) $\sin\dfrac{\theta}{2} = \dfrac{(1/2)b}{s} \Rightarrow b = 2s\sin\dfrac{\theta}{2}$

$$\cos\frac{\theta}{2} = \frac{h}{s} \Rightarrow h = s\cos\frac{\theta}{2}$$

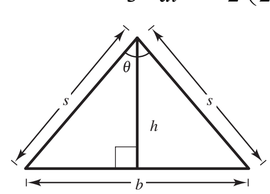
$$A = \frac{1}{2}bh = \frac{1}{2}\left(2s\sin\frac{\theta}{2}\right)\left(s\cos\frac{\theta}{2}\right)$$

$$= \frac{s^2}{2}\left(2\sin\frac{\theta}{2}\cos\frac{\theta}{2}\right) = \frac{s^2}{2}\sin\theta$$

(b) $\dfrac{dA}{dt} = \dfrac{s^2}{2}\cos\theta\dfrac{d\theta}{dt}$ where $\dfrac{d\theta}{dt} = \dfrac{1}{2}$ rad/min.

When $\theta = \dfrac{\pi}{6}$, $\dfrac{dA}{dt} = \dfrac{s^2}{2}\left(\dfrac{\sqrt{3}}{2}\right)\left(\dfrac{1}{2}\right) = \dfrac{\sqrt{3}s^2}{8}$.

When $\theta = \dfrac{\pi}{3}$, $\dfrac{dA}{dt} = \dfrac{s^2}{2}\left(\dfrac{1}{2}\right)\left(\dfrac{1}{2}\right) = \dfrac{s^2}{8}$.



(c) If s and $\dfrac{d\theta}{dt}$ is constant, $\dfrac{dA}{dt}$ is proportional to $\cos\theta$.

46. $\tan\theta = \dfrac{x}{10} \Rightarrow x = 10\tan\theta$

$$\frac{dx}{dt} = 10\sec^2\theta\frac{d\theta}{dt}$$

$$2 = 10\sec^2\theta\frac{d\theta}{dt}$$

$$\frac{d\theta}{dt} = \frac{1}{5}\cos^2\theta, \quad -\frac{\pi}{4} \le \theta \le \frac{\pi}{4}$$

47. (a) Using a graphing utility,

$r(f) = 0.0096f^3 - 0.559f^2 + 10.54f - 61.5.$

(b) $\dfrac{dr}{dt} = \dfrac{dr}{df}\dfrac{df}{dt} = \left(0.0288f^2 - 1.118f + 10.54\right)\dfrac{df}{dt}$

For $t = 9$, $f = 16.3$ from the table under the year 2009.

Also, $\dfrac{df}{dt} = 1.25$, so you have

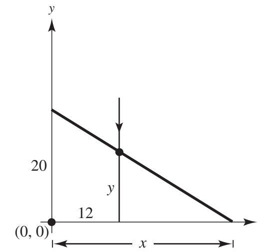
$$\frac{dr}{dt} = \left(0.0288(16.3)^2 - 1.118(16.3) + 10.54\right)(1.25)$$
$$= -0.03941 \text{ million participants per year.}$$

48. $y(t) = -4.9t^2 + 20$

$$\frac{dy}{dt} = -9.8t$$

$$y(1) = -4.9 + 20 = 15.1$$

$$y'(1) = -9.8$$



By similar triangles: $\dfrac{20}{x} = \dfrac{y}{x - 12}$

$$20x - 240 = xy$$

When $y = 15.1$: $20x - 240 = x(15.1)$

$$(20 - 15.1)x = 240$$

$$x = \frac{240}{4.9}$$

$$20x - 240 = xy$$

$$20\frac{dx}{dt} = x\frac{dy}{dt} + y\frac{dx}{dt}$$

$$\frac{dx}{dt} = \frac{x}{20 - y}\frac{dy}{dt}$$

At $t = 1$, $\dfrac{dx}{dt} = \dfrac{240/4.9}{20 - 15.1}(-9.8) \approx -97.96$ m/sec.

49. $x^2 + y^2 = 10^2$; acceleration of the top of the ladder $= \dfrac{d^2y}{dt^2}$

First derivative: $2x\dfrac{dx}{dt} + 2y\dfrac{dy}{dt} = 0$

$$x\frac{dx}{dt} + y\frac{dy}{dt} = 0$$

Second derivative: $x\dfrac{d^2x}{dt^2} + \dfrac{dx}{dt}\cdot\dfrac{dx}{dt} + y\dfrac{d^2y}{dt^2} + \dfrac{dy}{dt}\cdot\dfrac{dy}{dt} = 0$

$$\frac{d^2y}{dt^2} = \left(\frac{1}{y}\right)\left[-x\frac{d^2x}{dt^2} - \left(\frac{dx}{dt}\right)^2 - \left(\frac{dy}{dt}\right)^2\right]$$

When $x = 6$, $y = 8$, $\dfrac{dy}{dt} = -\dfrac{1}{2}$, and $\dfrac{dx}{dt} = \dfrac{2}{3}$ (see Exercise 21). Because $\dfrac{dx}{dt}$ is constant, $\dfrac{d^2x}{dt^2} = 0.$

$$\frac{d^2y}{dt^2} = \frac{1}{8}\left[-6(0) - \left(\frac{2}{3}\right)^2 - \left(-\frac{1}{2}\right)^2\right] = \frac{1}{8}\left[-\frac{4}{9} - \frac{1}{4}\right] = \frac{1}{8}\left[-\frac{25}{36}\right] \approx -0.0868 \text{ m/sec}^2$$

50. $L^2 = 4 + x^2$; acceleration of the boat $= \dfrac{d^2x}{dt^2}$

First derivative: $2L\dfrac{dL}{dt} = 2x\dfrac{dx}{dt}$

$$L\frac{dL}{dt} = x\frac{dx}{dt}$$

Second derivative: $L\dfrac{d^2L}{dt^2} + \dfrac{dL}{dt}\cdot\dfrac{dL}{dt} = x\dfrac{d^2x}{dt^2} + \dfrac{dx}{dt}\cdot\dfrac{dx}{dt}$

$$\frac{d^2x}{dt^2} = \left(\frac{1}{x}\right)\left[L\frac{d^2L}{dt^2} + \left(\frac{dL}{dt}\right)^2 - \left(\frac{dx}{dt}\right)^2\right]$$

When $L = 5.2$, $x = 4.8$, $\dfrac{dx}{dt} = -1.3$, and $\dfrac{dL}{dt} = -1.2$ (see Exercise 24). Because $\dfrac{dL}{dt}$ is constant, $\dfrac{d^2L}{dt^2} = 0$.

$$\frac{d^2x}{dt^2} = \frac{1}{4.8}\left[5.2(0) + (-1.2)^2 - (-1.3)^2\right]$$

$$= \frac{1}{4.8}[1.44 - 1.69] = \frac{1}{4.8}[-0.25] \approx -0.052 \text{ m/sec}^2$$

Review Exercises for Chapter 2

1. $f(x) = 12$

$$f'(x) = \lim_{\Delta x\to 0}\frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$= \lim_{\Delta x\to 0}\frac{12 - 12}{\Delta x}$$

$$= \lim_{\Delta x\to 0}\frac{0}{\Delta x} = 0$$

2. $f(x) = 5x - 4$

$$f'(x) = \lim_{\Delta x\to 0}\frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$= \lim_{\Delta x\to 0}\frac{\left[5(x+\Delta x) - 4\right] - (5x - 4)}{\Delta x}$$

$$= \lim_{\Delta x\to 0}\frac{5x + 5\Delta x - 4 - 5x + 4}{\Delta x}$$

$$= \lim_{\Delta x\to 0}\frac{5\Delta x}{\Delta x} = 5$$

3. $f(x) = x^3 - 2x + 1$

$$f'(x) = \lim_{\Delta x\to 0}\frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$= \lim_{\Delta x\to 0}\frac{\left[(x+\Delta x)^3 - 2(x+\Delta x) + 1\right] - \left[x^3 - 2x + 1\right]}{\Delta x}$$

$$= \lim_{\Delta x\to 0}\frac{x^3 + 3x^2(\Delta x) + 3x(\Delta x)^2 + (\Delta x)^3 - 2x - 2(\Delta x) + 1 - x^3 + 2x - 1}{\Delta x}$$

$$= \lim_{\Delta x\to 0}\frac{3x^2(\Delta x) + 3x(\Delta x)^2 + (\Delta x)^3 - 2(\Delta x)}{\Delta x}$$

$$= \lim_{\Delta x\to 0}\left[\frac{3x^2 + 3x(\Delta x) + (\Delta x)^2 - 2}{1}\right]$$

$$= 3x^2 - 2$$

4. $f(x) = \dfrac{6}{x}$

$f'(x) = \lim_{\Delta x \to 0} \dfrac{f(x+\Delta x) - f(x)}{\Delta x}$

$= \lim_{\Delta x \to 0} \dfrac{\dfrac{6}{x+\Delta x} - \dfrac{6}{x}}{\Delta x} = \lim_{\Delta x \to 0} \dfrac{6x - (6x + 6\,\Delta x)}{\Delta x(x+\Delta x)x} = \lim_{\Delta x \to 0} \dfrac{-6\,\Delta x}{\Delta x(x+\Delta x)x} = \lim_{\Delta x \to 0} \dfrac{-6}{(x+\Delta x)x} = \dfrac{-6}{x^2}$

5. $g(x) = 2x^2 - 3x,\ c = 2$

$g'(2) = \lim_{x \to 2} \dfrac{g(x) - g(2)}{x-2}$

$= \lim_{x \to 2} \dfrac{(2x^2 - 3x) - 2}{x-2}$

$= \lim_{x \to 2} \dfrac{(x-2)(2x+1)}{x-2}$

$= \lim_{x \to 2} (2x+1) = 2(2) + 1 = 5$

6. $f(x) = \dfrac{1}{x+4},\ c = 3$

$f'(3) = \lim_{x \to 3} \dfrac{f(x) - f(3)}{x-3}$

$= \lim_{x \to 3} \dfrac{\dfrac{1}{x+4} - \dfrac{1}{7}}{x-3}$

$= \lim_{x \to 3} \dfrac{7 - x - 4}{(x-3)(x+4)7}$

$= \lim_{x \to 3} \dfrac{-1}{(x+4)7} = -\dfrac{1}{49}$

7. f is differentiable for all $x \neq 3$.

8. f is differentiable for all $x \neq -1$.

9. $y = 25$

$y' = 0$

10. $f(t) = \dfrac{\pi}{6}$

$f'(t) = 0$

11. $f(x) = x^3 - 11x^2$

$f'(x) = 3x^2 - 22x$

12. $g(s) = 3s^5 - 2s^4$

$g'(s) = 15s^4 - 8s^3$

13. $h(x) = 6\sqrt{x} + 3\sqrt[3]{x} = 6x^{1/2} + 3x^{1/3}$

$h'(x) = 3x^{-1/2} + x^{-2/3} = \dfrac{3}{\sqrt{x}} + \dfrac{1}{\sqrt[3]{x^2}}$

14. $f(x) = x^{1/2} - x^{-5/6}$

$f'(x) = \dfrac{1}{2}x^{-1/2} + \dfrac{5}{6}x^{-11/6}$

15. $g(t) = \dfrac{2}{3}t^{-2}$

$g'(t) = \dfrac{-4}{3}t^{-3} = -\dfrac{4}{3t^3}$

16. $h(x) = \dfrac{8}{5x^4} = \dfrac{8}{5}x^{-4}$

$h'(x) = -\dfrac{32}{5}x^{-5} = -\dfrac{32}{5x^5}$

17. $f(\theta) = 4\theta - 5\sin\theta$

$f'(\theta) = 4 - 5\cos\theta$

18. $g(\alpha) = 4\cos\alpha + 6$

$g'(\alpha) = -4\sin\alpha$

19. $f(\theta) = 3\cos\theta - \dfrac{\sin\theta}{4}$

$f'(\theta) = -3\sin\theta - \dfrac{\cos\theta}{4}$

20. $g(\alpha) = \dfrac{5\sin\alpha}{3} - 2\alpha$

$g'(\alpha) = \dfrac{5\cos\alpha}{3} - 2$

21. $f(x) = \dfrac{27}{x^3} = 27x^{-3},\ (3, 1)$

$f'(x) = 27(-3)x^{-4} = -\dfrac{81}{x^4}$

$f'(3) = -\dfrac{81}{3^4} = -1$

22. $f(x) = 3x^2 - 4x,\ (1, -1)$

$f'(x) = 6x - 4$

$f'(1) = 6 - 4 = 2$

23. $f(x) = 4x^5 + 3x - \sin x, (0, 0)$

$f'(x) = 20x^4 + 3 - \cos x$

$f'(0) = 3 - 1 = 2$

24. $f(x) = 5\cos x - 9x, (0, 5)$

$f'(x) = -5\sin x - 9$

$f'(0) = -5\sin 0 - 9 = -9$

25. $F = 200\sqrt{T}$

$F'(t) = \dfrac{100}{\sqrt{T}}$

(a) When $T = 4$, $F'(4) = 50$ vibrations/sec/lb.

(b) When $T = 9$, $F'(9) = 33\frac{1}{3}$ vibrations/sec/lb.

26. $S = 6x^2$

$\dfrac{dS}{dx} = 12x$

When $x = 4$, $\dfrac{dS}{dx} = 12(4) = 48\ \text{cm}^2/\text{cm}$

27. $s(t) = -4.9t^2 + v_0 t + s_0;\ s_0 = 200, v_0 = -10$

(a) $s(t) = -4.9t^2 - 10t + 200$

$s'(t) = v(t) = -9.8t - 10$

(b) Average velocity $= \dfrac{s(3) - s(1)}{3 - 1}$

$= \dfrac{125.9 - 185.1}{2}$

$= -29.6$ m/sec

(c) $v(1) = -9.8(1) - 10 = -19.8$ m/sec

$v(3) = -9.8(3) - 10 = -39.4$ m/sec

(d) $s(t) = 0 = -4.9t^2 - 10t + 200$

Using a graphing utility or the Quadratic Formula, $t \approx 5.449$ seconds.

(e) When $t \approx 5.449$,

$v(t) \approx -9.8(5.449) - 10 \approx -63.40$ m/sec.

28. $s(t) = -4.9t^2 + 150$

$v(t) = s'(t) = -9.8t$

$v(2) = -9.8(2) = -19.6$ m/sec

$v(5) = -9.8(5) = -49$ m/sec

29. $f(x) = (5x^2 + 8)(x^2 - 4x - 6)$

$f'(x) = (5x^2 + 8)(2x - 4) + (x^2 - 4x - 6)(10x)$

$= 10x^3 + 16x - 20x^2 - 32 + 10x^3 - 40x^2 - 60x$

$= 20x^3 - 60x^2 - 44x - 32$

$= 4(5x^3 - 15x^2 - 11x - 8)$

30. $g(x) = (2x^3 + 5x)(3x - 4)$

$g'(x) = (2x^3 + 5x)(3) + (3x - 4)(6x^2 + 5)$

$= 6x^3 + 15x + 18x^3 - 24x^2 + 15x - 20$

$= 24x^3 - 24x^2 + 30x - 20$

31. $f(x) = (9x - 1)\sin x$

$f'(x) = (9x - 1)\cos x + 9\sin x$

$= 9x\cos x - \cos x + 9\sin x$

32. $f(t) = 2t^5\cos t$

$f'(t) = 2t^5(-\sin t) + \cos t(10t^4)$

$= -2t^5\sin t + 10t^4\cos t$

33. $f(x) = \dfrac{x^2 + x - 1}{x^2 - 1}$

$f'(x) = \dfrac{(x^2 - 1)(2x + 1) - (x^2 + x - 1)(2x)}{(x^2 - 1)^2}$

$= \dfrac{-(x^2 + 1)}{(x^2 - 1)^2}$

34. $f(x) = \dfrac{2x + 7}{x^2 + 4}$

$f'(x) = \dfrac{(x^2 + 4)(2) - (2x + 7)(2x)}{(x^2 + 4)^2}$

$= \dfrac{2x^2 + 8 - 4x^2 - 14x}{(x^2 + 4)^2}$

$= \dfrac{-2x^2 - 14x + 8}{(x^2 + 4)^2} = \dfrac{-2(x^2 + 7x - 4)}{(x^2 + 4)^2}$

35. $y = \dfrac{x^4}{\cos x}$

$y' = \dfrac{(\cos x)\,4x^3 - x^4(-\sin x)}{\cos^2 x}$

$= \dfrac{4x^3\cos x + x^4\sin x}{\cos^2 x}$

36. $y = \dfrac{\sin x}{x^4}$

$y' = \dfrac{(x^4)\cos x - (\sin x)(4x^3)}{(x^4)^2} = \dfrac{x\cos x - 4\sin x}{x^5}$

37. $y = 3x^2 \sec x$

$y' = 3x^2 \sec x \tan x + 6x \sec x$

38. $y = -x^2 \tan x$

$y' = -x^2 \sec^2 x - 2x \tan x$

39. $y = x\cos x - \sin x$

$y' = -x\sin x + \cos x - \cos x = -x \sin x$

40. $g(x) = x^4 \cot x + 3x \cos x$

$g'(x) = 4x^3 \cot x + x^4(-\csc^2 x) + 3\cos x - 3x \sin x$

$= 4x^3 \cot x - x^4 \csc^2 x + 3\cos x - 3x\sin x$

41. $f(x) = (x+2)(x^2+5), (-1, 6)$

$f'(x) = (x+2)(2x) + (x^2+5)(1)$

$= 2x^2 + 4x + x^2 + 5 = 3x^2 + 4x + 5$

$f'(-1) = 3 - 4 + 5 = 4$

Tangent line: $y - 6 = 4(x+1)$

$y = 4x + 10$

42. $f(x) = (x-4)(x^2+6x-1), (0, 4)$

$f'(x) = (x-4)(2x+6) + (x^2+6x-1)(1)$

$= 2x^2 - 2x - 24 + x^2 + 6x - 1$

$= 3x^2 + 4x - 25$

$f'(0) = 0 + 0 - 25 = -25$

Tangent line: $y - 4 = -25(x - 0)$

$y = -25x + 4$

43. $f(x) = \dfrac{x+1}{x-1}, \left(\dfrac{1}{2}, -3\right)$

$f'(x) = \dfrac{(x-1)-(x+1)}{(x-1)^2} = \dfrac{-2}{(x-1)^2}$

$f'\left(\dfrac{1}{2}\right) = \dfrac{-2}{(1/4)} = -8$

Tangent line: $y + 3 = -8\left(x - \dfrac{1}{2}\right)$

$y = -8x + 1$

44. $f(x) = \dfrac{1+\cos x}{1-\cos x}, \left(\dfrac{\pi}{2}, 1\right)$

$f'(x) = \dfrac{(1-\cos x)(-\sin x) - (1+\cos x)(\sin x)}{(1-\cos x)^2}$

$= \dfrac{-2\sin x}{(1-\cos x)^2}$

$f'\left(\dfrac{\pi}{2}\right) = \dfrac{-2}{1} = -2$

Tangent line: $y - 1 = -2\left(x - \dfrac{\pi}{2}\right)$

$y = -2x + 1 + \pi$

45. $g(t) = -8t^3 - 5t + 12$

$g'(t) = -24t^2 - 5$

$g''(t) = -48t$

46. $h(x) = 6x^{-2} + 7x^2$

$h'(x) = -12x^{-3} + 14x$

$h''(x) = 36x^{-4} + 14 = \dfrac{36}{x^4} + 14$

47. $f(x) = 15x^{5/2}$

$f'(x) = \frac{75}{2}x^{3/2}$

$f''(x) = \frac{225}{4}x^{1/2} = \frac{225}{4}\sqrt{x}$

48. $f(x) = 20\sqrt[5]{x} = 20x^{1/5}$

$f'(x) = 4x^{-4/5}$

$f''(x) = \dfrac{-16}{5}x^{-9/5} = -\dfrac{16}{5x^{9/5}}$

49. $f(\theta) = 3\tan\theta$

$f'(\theta) = 3\sec^2\theta$

$f''(\theta) = 6\sec\theta(\sec\theta\tan\theta)$

$= 6\sec^2\theta\tan\theta$

50. $h(t) = 10\cos t - 15\sin t$

$h'(t) = -10\sin t - 15\cos t$

$h''(t) = -10\cos t + 15\sin t$

51. $g(x) = 4\cot x$

$g'(x) = -4\csc^2 x$

$g''(x) = -8\csc x(-\csc x\cot x)$

$= 8\csc^2 x\cot x$

52. $h(t) = -12 \csc t$

$h'(t) = -12(-\csc t \cot t) = 12 \csc t \cot t$

$h''(t) = 12 \csc t(-\csc^2 t) + 12 \cot t(-\csc t \cot t)$

$= -12(\csc^3 t + \csc t \cot^2 t)$

53. $v(t) = 20 - t^2, 0 \le t \le 6$

$a(t) = v'(t) = -2t$

$v(3) = 20 - 3^2 = 11$ m/sec

$a(3) = -2(3) = -6$ m/sec^2

54. $v(t) = \dfrac{300t}{4t + 80}$

$a(t) = \dfrac{(4t + 80)300 - 300t(4)}{(4t + 80)^2}$

$= \dfrac{24{,}000}{(4t + 80)^2} = \dfrac{1500}{(t + 20)^2}$

(a) $a(1) = \dfrac{1500}{21^2} \approx 3.40$ m/sec^2

(b) $a(5) = \dfrac{1500}{25^2} = 2.4$ m/sec^2

(c) $a(10) = \dfrac{1500}{900} \approx 1.67$ m/sec^2

55. $y = (7x + 3)^4$

$y' = 4(7x + 3)^3(7) = 28(7x + 3)^3$

56. $y = (x^2 - 6)^3$

$y' = 3(x^2 - 6)^2(2x) = 6x(x^2 - 6)^2$

57. $y = \dfrac{1}{(x^2 + 5)^3} = (x^2 + 5)^{-3}$

$y' = -3(x^2 + 5)^{-4}(2x)$

$= -\dfrac{6x}{(x^2 + 5)^4}$

58. $f(x) = \dfrac{1}{(5x + 1)^2} = (5x + 1)^{-2}$

$f'(x) = -2(5x + 1)^{-3}(5) = -\dfrac{10}{(5x + 1)^3}$

59. $y = 5 \cos(9x + 1)$

$y' = -5 \sin(9x + 1)(9) = -45 \sin(9x + 1)$

60. $y = -6 \sin 3x^4$

$y' = -6 \cos(3x^4)(12x^3) = -72x^3 \cos 3x^4$

61. $y = \dfrac{x}{2} - \dfrac{\sin 2x}{4}$

$y' = \dfrac{1}{2} - \dfrac{1}{4}\cos 2x(2) = \dfrac{1}{2}(1 - \cos 2x) = \sin^2 x$

62. $y = \dfrac{\sec^7 x}{7} - \dfrac{\sec^5 x}{5}$

$y' = \sec^6 x(\sec x \tan x) - \sec^4 x(\sec x \tan x)$

$= \sec^5 x \tan x(\sec^2 x - 1)$

$= \sec^5 x \tan^3 x$

63. $y = x(6x + 1)^5$

$y' = x\,5(6x + 1)^4(6) + (6x + 1)^5(1)$

$= 30x(6x + 1)^4 + (6x + 1)^5$

$= (6x + 1)^4(30x + 6x + 1)$

$= (6x + 1)^4(36x + 1)$

64. $f(s) = (s^2 - 1)^{5/2}(s^3 + 5)$

$f'(s) = (s^2 - 1)^{5/2}(3s^2) + (s^3 + 5)\left(\frac{5}{2}\right)(s^2 - 1)^{3/2}(2s)$

$= s(s^2 - 1)^{3/2}\left[3s(s^2 - 1) + 5(s^3 + 5)\right]$

$= s(s^2 - 1)^{3/2}(8s^3 - 3s + 25)$

65. $f(x) = \left(\dfrac{x}{\sqrt{x + 5}}\right)^3$

$f'(x) = 3\left(\dfrac{x}{\sqrt{x + 5}}\right)^2 \left[\dfrac{(x + 5)^{1/2}(1) - x\left(\frac{1}{2}\right)(x + 5)^{-1/2}}{x + 5}\right]$

$= \dfrac{3x^2}{x + 5}\left[\dfrac{2(x + 5) - x}{2(x + 5)^{3/2}}\right]$

$= \dfrac{3x^2(x + 10)}{2(x + 5)^{5/2}}$

66. $h(x) = \left(\dfrac{x+5}{x^2+3}\right)^2$

$$h'(x) = 2\left(\frac{x+5}{x^2+3}\right)\left(\frac{(x^2+3)(1)-(x+5)(2x)}{(x^2+3)^2}\right)$$
$$= \frac{2(x+5)(-x^2-10x+3)}{(x^2+3)^3}$$

67. $f(x) = \sqrt{1-x^3}, (-2, 3)$

$$f'(x) = \frac{1}{2}(1-x^3)^{-1/2}(-3x^2) = \frac{-3x^2}{2\sqrt{1-x^3}}$$
$$f'(-2) = \frac{-12}{2(3)} = -2$$

68. $f(x) = \sqrt[3]{x^2-1}, (3, 2)$

$$f'(x) = \frac{1}{3}(x^2-1)^{-2/3}(2x) = \frac{2x}{3(x^2-1)^{2/3}}$$
$$f'(3) = \frac{2(3)}{3(4)} = \frac{1}{2}$$

69. $f(x) = \dfrac{x+8}{(3x+1)^{1/2}}, \quad (0, 8)$

$$f'(x) = \frac{(3x+1)^{1/2}(1)-(x+8)\left(\frac{1}{2}\right)(3x+1)^{-1/2}(3)}{3x+1}$$
$$f'(0) = \frac{1-4(3)}{1} = -11$$

70. $f(x) = \dfrac{3x+1}{(4x-3)^3}, (1, 4)$

$$f'(x) = \frac{(4x-3)^3(3)-(3x+1)(3)(4x-3)^2(4)}{(4x-3)^6}$$
$$f'(1) = \frac{3-4(3)(4)}{1} = -45$$

71. $y = \dfrac{1}{2}\csc 2x, \left(\dfrac{\pi}{4}, \dfrac{1}{2}\right)$

$$y' = -\csc 2x \cot 2x$$
$$y'\left(\frac{\pi}{4}\right) = 0$$

72. $y = \csc 3x + \cot 3x, \left(\dfrac{\pi}{6}, 1\right)$

$$y' = -3\csc 3x \cot 3x - 3\csc^2 3x$$
$$y'\left(\frac{\pi}{6}\right) = 0 - 3 = -3$$

73. $y = (8x+5)^3$

$$y' = 3(8x+5)^2(8) = 24(8x+5)^2$$
$$y'' = 24(2)(8x+5)(8) = 384(8x+5)$$

74. $y = \dfrac{1}{5x+1} > (5x+1)^{-1}$

$$y' = (-1)(5x+1)^{-2}(5) = -5(5x+1)^{-2}$$
$$y'' = (-5)(-2)(5x+1)^{-3}(5) = \frac{50}{(5x+1)^3}$$

75. $f(x) = \cot x$

$$f'(x) = -\csc^2 x$$
$$f''(x) = -2\csc x(-\csc x \cdot \cot x)$$
$$= 2\csc^2 x \cot x$$

76. $y = x\sin^2 x$

$$y' = \sin^2 x + 2x\sin x\cos x$$
$$y'' = 2\sin x\cos x + 2\sin x\cos x + 2x\cos^2 x - 2x\sin^2 x$$
$$= 4\sin x\cos x + 2x(\cos^2 x - \sin^2 x)$$

77. $T = \dfrac{700}{t^2 + 4t + 10}$

$T = 700(t^2 + 4t + 10)^{-1}$

$T' = \dfrac{-1400(t + 2)}{(t^2 + 4t + 10)^2}$

(a) When $t = 1$,

$T' = \dfrac{-1400(1 + 2)}{(1 + 4 + 10)^2} \approx -18.667$ deg/h.

(b) When $t = 3$,

$T' = \dfrac{-1400(3 + 2)}{(9 + 12 + 10)^2} \approx -7.284$ deg/h.

(c) When $t = 5$,

$T' = \dfrac{-1400(5 + 2)}{(25 + 20 + 10)^2} \approx -3.240$ deg/h.

(d) When $t = 10$,

$T' = \dfrac{-1400(10 + 2)}{(100 + 40 + 10)^2} \approx -0.747$ deg/h.

78.
$$y = \frac{1}{4}\cos 8t - \frac{1}{4}\sin 8t$$
$$y' = \frac{1}{4}(-\sin 8t)8 - \frac{1}{4}(\cos 8t)8$$
$$= -2\sin 8t - 2\cos 8t$$

At time $t = \dfrac{\pi}{4}$,

$$y\left(\frac{\pi}{4}\right) = \frac{1}{4}\cos\left[8\left(\frac{\pi}{4}\right)\right] - \frac{1}{4}\sin\left[8\left(\frac{\pi}{4}\right)\right]$$
$$= \frac{1}{4}(1) = \frac{1}{4} \text{ meter.}$$
$$v(t) = y'\left(\frac{\pi}{4}\right) = -2\sin\left[8\left(\frac{\pi}{4}\right)\right] - 2\cos\left[8\left(\frac{\pi}{4}\right)\right]$$
$$= -2(0) - 2(1) = -2 \text{ m/sec}$$

79.
$$x^2 + y^2 = 64$$
$$2x + 2yy' = 0$$
$$2yy' = -2x$$
$$y' = -\frac{x}{y}$$

80.
$$x^2 + 4xy - y^3 = 6$$
$$2x + 4xy' + 4y - 3y^2y' = 0$$
$$(4x - 3y^2)y' = -2x - 4y$$
$$y' = \frac{2x + 4y}{3y^2 - 4x}$$

81.
$$x^3y - xy^3 = 4$$
$$x^3y' + 3x^2y - x3y^2y' - y^3 = 0$$
$$x^3y' - 3xy^2y' = y^3 - 3x^2y$$
$$y'(x^3 - 3xy^2) = y^3 - 3x^2y$$
$$y' = \frac{y^3 - 3x^2y}{x^3 - 3xy^2}$$
$$y' = \frac{y(y^2 - 3x^2)}{x(x^2 - 3y^2)}$$

82.
$$\sqrt{xy} = x - 4y$$
$$\frac{\sqrt{x}}{2\sqrt{y}}y' + \frac{\sqrt{y}}{2\sqrt{x}} = 1 - 4y'$$
$$xy' + y = 2\sqrt{xy} - 8\sqrt{xy}y'$$
$$x + 8\sqrt{xy}y' = 2\sqrt{xy} - y$$
$$y' = \frac{2\sqrt{xy} - y}{x + 8\sqrt{xy}}$$
$$= \frac{2(x - 4y) - y}{x + 8(x - 4y)}$$
$$= \frac{2x - 9y}{9x - 32y}$$

83.
$$x\sin y = y\cos x$$
$$(x\cos y)y' + \sin y = -y\sin x + y'\cos x$$
$$y'(x\cos y - \cos x) = -y\sin x - \sin y$$
$$y' = \frac{y\sin x + \sin y}{\cos x - x\cos y}$$

84.
$$\cos(x + y) = x$$
$$-(1 + y')\sin(x + y) = 1$$
$$-y'\sin(x + y) = 1 + \sin(x + y)$$
$$y' = -\frac{1 + \sin(x + y)}{\sin(x + y)} = -\csc(x + 1) - 1$$

85. $x^2 + y^2 = 10$

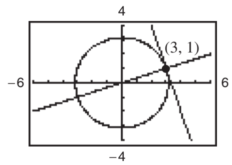
$2x + 2yy' = 0$

$y' = \frac{-x}{y}$

At $(3, 1)$, $y' = -3$

Tangent line: $y - 1 = -3(x - 3) \Rightarrow 3x + y - 10 = 0$

Normal line: $y - 1 = \frac{1}{3}(x - 3) \Rightarrow x - 3y = 0$



86. $x^2 - y^2 = 20$

$2x - 2yy' = 0$

$y' = \frac{x}{y}$

At $(6, 4)$, $y' = \frac{3}{2}$

Tangent line: $y - 4 = \frac{3}{2}(x - 6)$

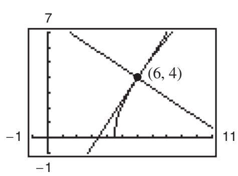
$y = \frac{3}{2}x - 5$

$2y - 3x + 10 = 0$

Normal line: $y - 4 = -\frac{2}{3}(x - 6)$

$y = -\frac{2}{3}x + 8$

$3y + 2x - 24 = 0$



87. $y = \sqrt{x}$

$\frac{dy}{dt} = 2$ units/sec

$\frac{dy}{dt} = \frac{1}{2\sqrt{x}}\frac{dx}{dt} \Rightarrow \frac{dx}{dt} = 2\sqrt{x}\frac{dy}{dt} = 4\sqrt{x}$

(a) When $x = \frac{1}{2}$, $\frac{dx}{dt} = 2\sqrt{2}$ units/sec.

(b) When $x = 1$, $\frac{dx}{dt} = 4$ units/sec.

(c) When $x = 4$, $\frac{dx}{dt} = 8$ units/sec.

88. Surface area $= A = 6x^2$, $x =$ length of edge

$\frac{dx}{dt} = 8$

$\frac{dA}{dt} = 12x\frac{dx}{dt} = 12(6.5)(8) = 624 \text{ cm}^2/\text{sec}$

89. $\tan\theta = x$

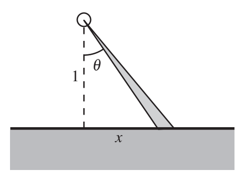
$\frac{d\theta}{dt} = 3(2\pi)$ rad/min

$\sec^2\theta\left(\frac{d\theta}{dt}\right) = \frac{dx}{dt}$

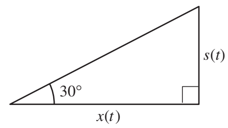
$\frac{dx}{dt} = (\tan^2\theta + 1)(6\pi) = 6\pi(x^2 + 1)$

When $x = \frac{1}{2}$,

$\frac{dx}{dt} = 6\pi\left(\frac{1}{4} + 1\right) = \frac{15\pi}{2}$ km/min $= 450\pi$ km/h.



90.
$$s(t) = 60 - 4.9t^2$$
$$s'(t) = -9.8t$$
$$s = 35 = 60 - 4.9t^2$$
$$4.9t^2 = 25$$
$$t = \frac{5}{\sqrt{4.9}}$$
$$\tan 30 = \frac{1}{\sqrt{3}} = \frac{s(t)}{x(t)}$$
$$x(t) = \sqrt{3}s(t)$$
$$\frac{dx}{dt} = \sqrt{3}\frac{ds}{dt} = \sqrt{3}(-9.8)\frac{5}{\sqrt{4.9}} \approx -38.34 \text{ m/sec}$$

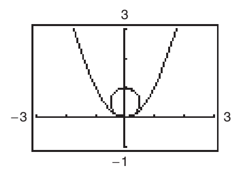


Problem Solving for Chapter 2

1. (a) $x^2 + (y - r)^2 = r^2$, Circle

$x^2 = y$, Parabola

Substituting:
$$(y - r)^2 = r^2 - y$$
$$y^2 - 2ry + r^2 = r^2 - y$$
$$y^2 - 2ry + y = 0$$
$$y(y - 2r + 1) = 0$$



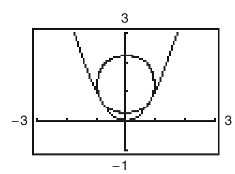
Because you want only one solution, let $1 - 2r = 0 \Rightarrow r = \frac{1}{2}$. Graph $y = x^2$ and $x^2 + \left(y - \frac{1}{2}\right)^2 = \frac{1}{4}$.

(b) Let (x, y) be a point of tangency:

$x^2 + (y - b)^2 = 1 \Rightarrow 2x + 2(y - b)y' = 0 \Rightarrow y' = \frac{x}{b - y}$, Circle

$y = x^2 \Rightarrow y' = 2x$, Parabola

Equating:
$$2x = \frac{x}{b - y}$$
$$2(b - y) = 1$$
$$b - y = \frac{1}{2} \Rightarrow b = y + \frac{1}{2}$$



Also, $x^2 + (y - b)^2 = 1$ and $y = x^2$ imply:

$$y + (y - b)^2 = 1 \Rightarrow y + \left[y - \left(y + \frac{1}{2}\right)\right]^2 = 1 \Rightarrow y + \frac{1}{4} = 1 \Rightarrow y = \frac{3}{4} \text{ and } b = \frac{5}{4}$$

Center: $\left(0, \frac{5}{4}\right)$

Graph $y = x^2$ and $x^2 + \left(y - \frac{5}{4}\right)^2 = 1$.

2. Let (a, a^2) and $(b, -b^2 + 2b - 5)$ be the points of tangency. For $y = x^2$, $y' = 2x$ and for $y = -x^2 + 2x - 5$, $y' = -2x + 2$. So, $2a = -2b + 2 \Rightarrow a + b = 1$, or $a = 1 - b$. Furthermore, the slope of the common tangent line is

$$\frac{a^2 - (-b^2 + 2b - 5)}{a - b} = \frac{(1 - b)^2 + b^2 - 2b + 5}{(1 - b) - b} = -2b + 2$$

$$\Rightarrow \frac{1 - 2b + b^2 + b^2 - 2b + 5}{1 - 2b} = -2b + 2$$

$$\Rightarrow 2b^2 - 4b + 6 = 4b^2 - 6b + 2$$

$$\Rightarrow 2b^2 - 2b - 4 = 0$$

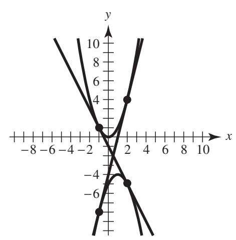
$$\Rightarrow b^2 - b - 2 = 0$$

$$\Rightarrow (b - 2)(b + 1) = 0$$

$$b = 2, -1$$

For $b = 2$, $a = 1 - b = -1$ and the points of tangency are $(-1, 1)$ and $(2, -5)$. The tangent line has slope -2: $y - 1 = -2(x = 1) \Rightarrow y = -2x - 1$

For $b = -1$, $a = 1 - b = 2$ and the points of tangency are $(2, 4)$ and $(-1, -8)$. The tangent line has slope 4: $y - 4 = 4(x - 2) \Rightarrow y = 4x - 4$



3. Let $p(x) = Ax^3 + Bx^2 + Cx + D$

$p'(x) = 3Ax^2 + 2Bx + C$.

At $(1, 1)$:

$A + B + C + D = 1$ Equation 1

$3A + 2B + C = 14$ Equation 2

At $(-1, -3)$:

$A + B - C + D = -3$ Equation 3

$3A + 2B + C = -2$ Equation 4

Adding Equations 1 and 3: $2B + 2D = -2$

Subtracting Equations 1 and 3: $2A + 2C = 4$ $D = \frac{1}{2}(-2 - 2B) = -5$.

Adding Equations 2 and 4: $6A + 2C = 12$

Subtracting Equations 2 and 4: $4B = 16$

So, $B = 4$ and $D = \frac{1}{2}(-2 - 2B) = -5$. Subtracting $2A + 2C = 4$ and $6A + 2C = 12$, you obtain $4A = 8 \Rightarrow A = 2$. Finally, $C = \frac{1}{2}(4 - 2A) = 0$. So, $p(x) = 2x^3 + 4x^2 - 5$.

4. $f(x) = a + b\cos cx$

$f'(x) = -bc\sin cx$

At $(0, 1)$: $a + b = 1$ Equation 1

At $\left(\frac{\pi}{4}, \frac{3}{2}\right)$: $a + b\cos\left(\frac{c\pi}{4}\right) = \frac{3}{2}$ Equation 2

$-bc\sin\left(\frac{c\pi}{4}\right) = 1$ Equation 3

From Equation 1, $a = 1 - b$. Equation 2 becomes

$$(1 - b) + b\cos\left(\frac{c\pi}{4}\right) = \frac{3}{2} \Rightarrow -b + b\cos\frac{c\pi}{4} = \frac{1}{2}.$$

From Equation 3, $b = \dfrac{-1}{c\sin(c\pi/4)}$. So:

$$\frac{1}{c\sin(c\pi/4)} + \frac{-1}{c\sin(c\pi/4)}\cos\left(\frac{c\pi}{4}\right) = \frac{1}{2}$$

$$1 - \cos\left(\frac{c\pi}{4}\right) = \frac{1}{2}c\sin\left(\frac{c\pi}{4}\right)$$

Graphing the equation

$$g(c) = \frac{1}{2}c\sin\left(\frac{c\pi}{4}\right) + \cos\left(\frac{c\pi}{4}\right) - 1,$$

you see that many values of c will work. One answer:

$$c = 2, b = -\frac{1}{2}, a = \frac{3}{2} \Rightarrow f(x) = \frac{3}{2} - \frac{1}{2}\cos 2x$$

5. (a) $y = x^2$, $y' = 2x$, Slope $= 4$ at $(2, 4)$

Tangent line: $y - 4 = 4(x - 2)$

$y = 4x - 4$

(b) Slope of normal line: $-\frac{1}{4}$

Normal line: $y - 4 = -\frac{1}{4}(x - 2)$

$$y = -\frac{1}{4}x + \frac{9}{2}$$

$$y = -\frac{1}{4}x + \frac{9}{2} = x^2$$

$$\Rightarrow 4x^2 + x - 18 = 0$$

$$\Rightarrow (4x + 9)(x - 2) = 0$$

$x = 2, -\frac{9}{4}$

Second intersection point: $\left(-\frac{9}{4}, \frac{81}{16}\right)$

(c) Tangent line: $y = 0$

Normal line: $x = 0$

(d) Let (a, a^2), $a \neq 0$, be a point on the parabola $y = x^2$.

Tangent line at (a, a^2) is $y = 2a(x - a) + a^2$.

Normal line at (a, a^2) is $y = -(1/2a)(x - a) + a^2$.

To find points of intersection, solve:

$$x^2 = -\frac{1}{2a}(x - a) + a^2$$

$$x^2 + \frac{1}{2a}x = a^2 + \frac{1}{2}$$

$$x^2 + \frac{1}{2a}x + \frac{1}{16a^2} = a^2 + \frac{1}{2} + \frac{1}{16a^2}$$

$$\left(x + \frac{1}{4a}\right)^2 = \left(a + \frac{1}{4a}\right)^2$$

$$x + \frac{1}{4a} = \pm\left(a + \frac{1}{4a}\right)$$

$$x + \frac{1}{4a} = a + \frac{1}{4a} \Rightarrow x = a \quad \text{(Point of tangency)}$$

$$x + \frac{1}{4a} = -\left(a + \frac{1}{4a}\right) \Rightarrow x = -a - \frac{1}{2a} = -\frac{2a^2 + 1}{2a}$$

The normal line intersects a second time at $x = -\dfrac{2a^2 + 1}{2a}$.

6. (a) $f(x) = \cos x$

$f(0) = 1$

$f'(0) = 0$

$P_1(x) = 1$

$P_1(x) = a_0 + a_1x$

$P_1(0) = a_0 \Rightarrow a_0 = 1$

$P_1'(0) = a_1 \Rightarrow a_1 = 0$

(b) $f(x) = \cos x$

$f(0) = 1$

$f'(0) = 0$

$f''(0) = -1$

$P_2(x) = 1 - \frac{1}{2}x^2$

$P_2(x) = a_0 + a_1x + a_2x^2$

$P_2(0) = a_0 \Rightarrow a_0 = 1$

$P_2'(0) = a_1 \Rightarrow a_1 = 0$

$P_2''(0) = 2a_2 \Rightarrow a_2 = -\frac{1}{2}$

(c)

x	-1.0	-0.1	-0.001	0	0.001	0.1	1.0
$\cos x$	0.5403	0.9950	≈ 1	1	≈ 1	0.9950	0.5403
$P_2(x)$	0.5	0.9950	≈ 1	1	≈ 1	0.9950	0.5

$P_2(x)$ is a good approximation of $f(x) = \cos x$ when x is near 0.

(d) $f(x) = \sin x$ $\quad$ $P_3(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3$

$f(0) = 0$ $\quad$ $P_3(0) = a_0 \Rightarrow a_0 = 0$

$f'(0) = 1$ $\quad$ $P_3'(0) = a_1 \Rightarrow a_1 = 1$

$f''(0) = 0$ $\quad$ $P_3''(0) = 2a_2 \Rightarrow a_2 = 0$

$f'''(0) = -1$ $\quad$ $P_3'''(0) = 6a_3 \Rightarrow a_3 = -\frac{1}{6}$

$P_3(x) = x - \frac{1}{6}x^3$

7. (a) $x^4 = a^2x^2 - a^2y^2$

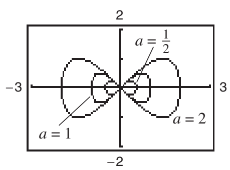
$a^2y^2 = a^2x^2 - x^4$

$$y = \frac{\pm\sqrt{a^2x^2 - x^4}}{a}$$

Graph: $y_1 = \dfrac{\sqrt{a^2x^2 - x^4}}{a}$

and $y_2 = -\dfrac{\sqrt{a^2x^2 - x^4}}{a}$.

(b)



$(\pm a, 0)$ are the x-intercepts, along with $(0, 0)$.

(c) Differentiating implicitly:

$4x^3 = 2a^2x - 2a^2yy'$

$$y' = \frac{2a^2x - 4x^3}{2a^2y} = \frac{x(a^2 - 2x^2)}{a^2y} = 0 \Rightarrow 2x^2 = a^2 \Rightarrow x = \frac{\pm a}{\sqrt{2}}$$

$$\left(\frac{a^2}{2}\right)^2 = a^2\left(\frac{a^2}{2}\right) - a^2y^2$$

$$\frac{a^4}{4} = \frac{a^4}{2} - a^2y^2$$

$$a^2y^2 = \frac{a^4}{4}$$

$$y^2 = \frac{a^2}{4}$$

$$y = \pm\frac{a}{2}$$

Four points: $\left(\frac{a}{\sqrt{2}}, \frac{a}{2}\right), \left(\frac{a}{\sqrt{2}}, -\frac{a}{2}\right), \left(-\frac{a}{\sqrt{2}}, \frac{a}{2}\right), \left(-\frac{a}{\sqrt{2}}, -\frac{a}{2}\right)$

8. (a) $b^2y^2 = x^3(a - x); \quad a, b > 0$

$$y^2 = \frac{x^3(a - x)}{b^2}$$

Graph $y_1 = \dfrac{\sqrt{x^3(a - x)}}{b}$ and $y_2 = -\dfrac{\sqrt{x^3(a - x)}}{b}$.

(b) a determines the x-intercept on the right: $(a, 0)$. b affects the height.

(c) Differentiating implicitly:

$$2b^2yy' = 3x^2(a - x) - x^3 = 3ax^2 - 4x^3$$

$$y' = \frac{(3ax^2 - 4x^3)}{2b^2y} = 0$$

$$\Rightarrow 3ax^2 = 4x^3$$

$$3a = 4x$$

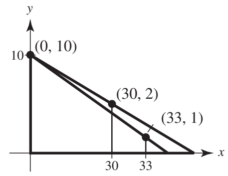
$$x = \frac{3a}{4}$$

$$b^2y^2 = \left(\frac{3a}{4}\right)^3\left(a - \frac{3a}{4}\right) = \frac{27a^3}{64}\left(\frac{1}{4}a\right)$$

$$y^2 = \frac{27a^4}{256b^2} \Rightarrow y = \pm\frac{3\sqrt{3}a^2}{16b}$$

Two points: $\left(\frac{3a}{4}, \frac{3\sqrt{3}a^2}{16b}\right), \left(\frac{3a}{4}, -\frac{3\sqrt{3}a^2}{16b}\right)$

9. (a)



Not drawn to scale

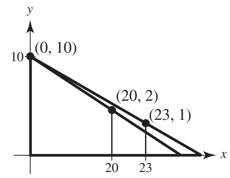
Line determined by $(0, 10)$ and $(30, 2)$:

$$y - 10 = \frac{10 - 2}{0 - 30}(x - 0) = -\frac{8}{30}x = -\frac{4}{15}x \Rightarrow y = -\frac{4}{15}x + 10$$

When $x = 33$: $y = -\frac{4}{15}(33) + 10 = \frac{6}{5} > 1$

As you can see from the figure, the shadow determined by the man extends beyond the shadow determined by the child.

(b)



Not drawn to scale

Line determined by $(0, 10)$ and $(20, 2)$:

$$y - 10 = \frac{10 - 2}{0 - 20}(x - 0) = -\frac{2}{5}x \Rightarrow y = -\frac{2}{5}x + 10$$

When $x = 23$: $y = -\frac{2}{5}(23) + 10 = \frac{4}{5} < 1$

As you can see from the figure, the shadow determined by the child extends beyond the shadow determined by the man.

(c) Need $(0, 10), (d, 2), (d + 3, 1)$ collinear.

$$\frac{10 - 2}{0 - d} = \frac{2 - 1}{d - (d + 3)} \Rightarrow \frac{8}{d} = \frac{1}{3} \Rightarrow d = 24 \text{ meters}$$

(d) Let y be the distance from the base of the street light to the tip of the man's shadow. You know that $dx/dt = \frac{-5}{6}$.

$$\frac{y}{10} = \frac{y - x}{2} \Rightarrow y = \frac{5}{4}x \text{ and } \frac{dy}{dt} = \frac{5}{4}\frac{dx}{dt} = \frac{-25}{24} \text{ meters per second.}$$

ALTERNATE SOLUTION for parts (a) and (b):

(a) As before, the line determined by the man's shadow is

$$y_m = -\frac{4}{15}x + 10$$

The line determined by the child's shadow is obtained by finding the line through $(0, 10)$ and $(33, 1)$:

$$y - 10 = \frac{10 - 1}{0 - 33}(x - 0) \Rightarrow y_c = -\frac{9}{33}x + 10$$

By setting $y_m = y_c = 0$, you can determine how far the shadows extend:

Man: $y_m = 0 \Rightarrow \frac{4}{15}x = 10 \Rightarrow x = 37\frac{1}{2}$

Child: $y_c = 0 \Rightarrow \frac{9}{33}x = 10 \Rightarrow x = 36\frac{2}{3}$

The man's shadow is $37\frac{1}{2} - 36\frac{2}{3} \approx \frac{5}{6}$ meter beyond the child's shadow.

(b) As before, the line determined by the man's shadow is

$$y_m = -\frac{2}{5}x + 10$$

For the child's shadow,

$$y - 10 = \frac{10 - 1}{0 - 23}(x - 0) \Rightarrow y_c = -\frac{9}{23}x + 10$$

Man: $y_m = 0 \Rightarrow \frac{2}{5}x = 10 \Rightarrow x = 25$

Child: $y_c = 0 \Rightarrow \frac{9}{23}x = 10 \Rightarrow x = \frac{230}{9} = 25\frac{5}{9}$

So the child's shadow is $25\frac{5}{9} - 25 = \frac{5}{9}$ meter beyond the man's shadow.

10. (a) $y = x^{1/3} \Rightarrow \frac{dy}{dt} = \frac{1}{3}x^{-2/3}\frac{dx}{dt}$

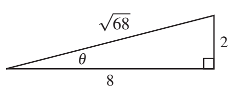
$$1 = \frac{1}{3}(8)^{-2/3}\frac{dx}{dt}$$

$$\frac{dx}{dt} = 12 \text{ cm/sec}$$

(b) $D = \sqrt{x^2 + y^2} \Rightarrow \frac{dD}{dt} = \frac{1}{2}(x^2 + y^2)\left(2x\frac{dx}{dt} + 2y\frac{dy}{dt}\right) = \frac{x(dx\ /\ dt) + y(dy\ /\ dt)}{\sqrt{x^2 + y^2}}$

$$= \frac{8(12) + 2(1)}{\sqrt{64 + 4}} = \frac{98}{\sqrt{68}} = \frac{49}{\sqrt{17}} \text{ cm/sec}$$

(c) $\tan\theta = \frac{y}{x} \Rightarrow \sec^2\theta \cdot \frac{d\theta}{dt} = \frac{x(dy/dt) - y(dx/dt)}{x^2}$



From the triangle, $\sec\theta = \sqrt{68}/8$. So $\frac{d\theta}{dt} = \frac{8(1) - 2(12)}{64(68/64)} = \frac{-16}{68} = -\frac{4}{17}$ rad/sec.

11. (a) $v(t) = -\frac{8}{5}t + 8$ m/sec

$a(t) = -\frac{8}{5}$ m/sec^2

(b) $v(t) = -\frac{8}{5}t + 8 = 0 \Rightarrow \frac{8}{5}t = 8 \Rightarrow t = 5$ seconds

$s(5) = -\frac{4}{5}(5)^2 + 8(5) + 2 = 22$ meters

(c) The acceleration due to gravity on Earth is greater in magnitude than that on the moon.

12. $E'(x) = \lim_{\Delta x \to 0}\frac{E(x + \Delta x) - E(x)}{\Delta x} = \lim_{\Delta x \to 0}\frac{E(x)E(\Delta x) - E(x)}{\Delta x} = \lim_{\Delta x \to 0}E(x)\left(\frac{E(\Delta x) - 1}{\Delta x}\right) = E(x)\lim_{\Delta x \to 0}\frac{E(\Delta x) - 1}{\Delta x}$

But, $E'(0) = \lim_{\Delta x \to 0}\frac{E(\Delta x) - E(0)}{\Delta x} = \lim_{\Delta x \to 0}\frac{E(\Delta x) - 1}{\Delta x} = 1$. So, $E'(x) = E(x)E'(0) = E(x)$ exists for all x.

For example: $E(x) = e^x$.

13. $L'(x) = \lim_{\Delta x \to 0} \frac{L(x + \Delta x) - L(x)}{\Delta x} = \lim_{\Delta x \to 0} \frac{L(x) + L(\Delta x) - L(x)}{\Delta x} = \lim_{\Delta x \to 0} \frac{L(\Delta x)}{\Delta x}$

Also, $L'(0) = \lim_{\Delta x \to 0} \frac{L(\Delta x) - L(0)}{\Delta x}$. But, $L(0) = 0$ because $L(0) = L(0 + 0) = L(0) + L(0) \Rightarrow L(0) = 0$.

So, $L'(x) = L'(0)$ for all x. The graph of L is a line through the origin of slope $L'(0)$.

14. (a)

z (degrees)	0.1	0.01	0.0001
$\frac{\sin z}{z}$	0.0174524	0.0174533	0.0174533

(b) $\lim_{z \to 0} \frac{\sin z}{z} \approx 0.0174533$

In fact, $\lim_{z \to 0} \frac{\sin z}{z} = \frac{\pi}{180}$.

(c)
$$\begin{aligned}\frac{d}{dz}(\sin z) &= \lim_{\Delta z \to 0} \frac{\sin(z + \Delta z) - \sin z}{\Delta z}\\ &= \lim_{\Delta z \to 0} \frac{\sin z \cdot \cos \Delta z + \sin \Delta z \cdot \cos z - \sin z}{\Delta z}\\ &= \lim_{\Delta z \to 0} \left[\sin z\left(\frac{\cos \Delta z - 1}{\Delta z}\right)\right] + \lim_{\Delta z \to 0} \left[\cos z\left(\frac{\sin \Delta z}{\Delta z}\right)\right]\\ &= (\sin z)(0) + (\cos z)\left(\frac{\pi}{180}\right) = \frac{\pi}{180} \cos z\end{aligned}$$

(d) $S(90) = \sin\left(\frac{\pi}{180} 90\right) = \sin \frac{\pi}{2} = 1$

$C(180) = \cos\left(\frac{\pi}{180} 180\right) = -1$

$\frac{d}{dz} S(z) = \frac{d}{dz} \sin(cz) = c \cdot \cos(cz) = \frac{\pi}{180} C(z)$

(e) The formulas for the derivatives are more complicated in degrees.

15. $j(t) = a'(t)$

(a) $j(t)$ is the rate of change of acceleration.

(b) $s(t) = -2.5t^2 + 20t$

$v(t) = -5t + 20$

$a(t) = -5$

$a'(t) = j(t) = 0$

The acceleration is constant, so $j(t) = 0$.

(c) a is position.
b is acceleration.
c is jerk.
d is velocity.