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Preface

The *Complete Solutions Manual* for *Calculus I with Precalculus: A One-Year Course*, Third Edition, is a supplement to the text by Ron Larson and Bruce H. Edwards. Solutions to every exercise in the text are given with all essential algebraic steps included.

CHAPTER P
Prerequisites

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Section P.1 Solving Equations

1. equation

2. $ax + b = 0$

3. extraneous

4. factoring; extracting square roots; completing the square; Quadratic Formula

5. $4(x + 1) = 4x + 4$ is an *identity* by the Distributive Property. It is true for all real values at x.

6. $-6(x - 3) + 5 = -2x + 10$ is *conditional*. There are real values of x for which the equation is not true.

7. $4(x + 1) - 2x = 4x + 4 - 2x = 2x + 4 = 2(x + 2)$

   This is an *identity* by simplification. It is true for all real values of x.

8. $x^2 + 2(3x - 2) = x^2 + 6x - 4$ is an *identity* by simplification. It is true for all real values of x.

9. $3 + \frac{1}{x+1} = \frac{4x}{x+1}$ is *conditional*. There are real values of x for which the equation is not true.

10. $\frac{5}{x} + \frac{3}{x} = 24$ is *conditional*. There are real values of x for which the equation is not true (for example, $x = 1$).

11. $$\begin{aligned} x + 11 &= 15 \\ x + 11 - 11 &= 15 - 11 \\ x &= 4 \end{aligned}$$

12. $$\begin{aligned} 7 - x &= 19 \\ 7 - x + x &= 19 + x \\ 7 &= 19 + x \\ 7 - 19 &= 19 + x - 19 \\ -12 &= x \end{aligned}$$

13. $$\begin{aligned} 7 - 2x &= 25 \\ 7 - 7 - 2x &= 25 - 7 \\ -2x &= 18 \\ \frac{-2x}{-2} &= \frac{18}{-2} \\ x &= -9 \end{aligned}$$

14. $$\begin{aligned} 7x + 2 &= 23 \\ 7x + 2 - 2 &= 23 - 2 \\ 7x &= 21 \\ \frac{7x}{7} &= \frac{21}{7} \\ x &= 3 \end{aligned}$$

15. $$\begin{aligned} 8x - 5 &= 3x + 20 \\ 8x - 3x - 5 &= 3x - 3x + 20 \\ 5x - 5 &= 20 \\ 5x - 5 + 5 &= 20 + 5 \\ 5x &= 25 \\ \frac{5x}{5} &= \frac{25}{5} \\ x &= 5 \end{aligned}$$

16. $$\begin{aligned} 7x + 3 &= 3x - 17 \\ 7x + 3 - 3 - 3x &= 3x - 17 - 3 - 3x \\ 4x &= -20 \\ x &= -5 \end{aligned}$$

17. $$\begin{aligned} 4y + 2 - 5y &= 7 - 6y \\ 2 - y &= 7 - 6y \\ 2 - y + 6y &= 7 - 6y + 6y \\ 2 + 5y &= 7 \\ 2 - 2 + 5y &= 7 - 2 \\ \frac{5y}{5} &= \frac{5}{5} \\ y &= 1 \end{aligned}$$

18. $$\begin{aligned} 3(x + 3) &= 5(1 - x) - 1 \\ 3x + 9 &= 5 - 5x - 1 \\ 3x + 9 &= 4 - 5x \\ 3x + 9 + 5x - 9 &= 4 - 5x + 5x - 9 \\ 8x &= -5 \\ x &= -\tfrac{5}{8} \end{aligned}$$

19. $$\begin{aligned} x - 3(2x + 3) &= 8 - 5x \\ x - 6x - 9 &= 8 - 5x \\ -5x - 9 &= 8 - 5x \\ -5x + 5x - 9 &= 8 - 5x + 5x \\ -9 &\neq 8 \end{aligned}$$

    No solution.

20. $9x - 10 = 5x + 2(2x - 5)$
$9x - 10 = 5x + 4x - 10$
$9x - 10 = 9x - 10$
The solution is the set of all real numbers.

21. $\frac{3x}{8} - \frac{4x}{3} = 4$
$24\left(\frac{3x}{8} - \frac{4x}{3}\right) = 24(4)$
$9x - 32x = 96$
$\frac{-23x}{-23} = \frac{96}{-23}$
$x = \frac{-96}{23}$

22. $\frac{x}{5} - \frac{x}{2} = 3 + \frac{3x}{10}$
$10\left(\frac{x}{5} - \frac{x}{2}\right) = 10\left(3 + \frac{3x}{10}\right)$
$2x - 5x = 30 + 3x$
$-6x = 30$
$x = -5$

23. $\frac{3}{2}(z + 5) - \frac{1}{4}(z + 24) = 0$
$4\left[\frac{3}{2}(z + 5) - \frac{1}{4}(z + 24)\right] = 4(0)$
$4\left(\frac{3}{2}\right)(z + 5) - 4\left(\frac{1}{4}\right)(z + 24) = 4(0)$
$6(z + 5) - (z + 24) = 0$
$6z + 30 - z - 24 = 0$
$5z = -6$
$z = -\frac{6}{5}$

24. $0.60x + 0.40(100 - x) = 50$
$0.60x + 40 - 0.40x = 50$
$0.20x = 10$
$x = 50$

25. $x + 8 = 2(x - 2) - x$
$x + 8 = 2x - 4 - x$
$x + 8 = x - 4$
$8 \neq -4$
Contradiction; no solution

26. $8(x + 2) - 3(2x + 1) = 2(x + 5)$
$8x + 16 - 6x - 3 = 2x + 10$
$2x + 13 = 2x + 10$
$13 = 10$
Contradiction; no solution

27. $\frac{100 - 4x}{3} = \frac{5x + 6}{4} + 6$
$12\left(\frac{100 - 4x}{3}\right) = 12\left(\frac{5x + 6}{4}\right) + 12(6)$
$4(100 - 4x) = 3(5x + 6) + 72$
$400 - 16x = 15x + 18 + 72$
$-31x = -310$
$x = 10$

28. $\frac{17 + y}{y} + \frac{32 + y}{y} = 100$
$(y)\frac{17 + y}{y} + (y)\frac{32 + y}{y} = 100(y)$
$17 + y + 32 + y = 100y$
$49 + 2y = 100y$
$49 = 98y$
$\frac{1}{2} = y$

29. $\frac{5x - 4}{5x + 4} = \frac{2}{3}$
$3(5x - 4) = 2(5x + 4)$
$15x - 12 = 10x + 8$
$5x = 20$
$x = 4$

30. $\frac{15}{x} - 4 = \frac{6}{x} + 3$
$\frac{15}{x} - \frac{6}{x} = 7$
$\frac{9}{x} = 7$
$9 = 7x$
$\frac{9}{7} = x$

31. $3 = 2 + \frac{2}{z + 2}$
$3(z + 2) = \left(2 + \frac{2}{z + 2}\right)(z + 2)$
$3z + 6 = 2z + 4 + 2$
$z = 0$

32. $\frac{1}{x} + \frac{2}{x - 5} = 0$ Multiply both sides by $x(x - 5)$.
$1(x - 5) + 2x = 0$
$3x - 5 = 0$
$3x = 5$
$x = \frac{5}{3}$

33. $\dfrac{x}{x+4} + \dfrac{4}{x+4} + 2 = 0$ Multiply both sides by $(x + 4)$.

$$x + 4 + 2(x + 4) = 0$$
$$x + 4 + 2x + 8 = 0$$
$$3x + 12 = 0$$
$$3x = -12$$
$$x = -4$$

A check reveals that $x = -4$ is an extraneous solution because it makes the denominator zero. There is no real solution.

34. $\dfrac{7}{2x+1} - \dfrac{8x}{2x-1} = -4$ Multiply both sides by $(2x + 1)(2x - 1)$.

$$7(2x - 1) - 8x(2x + 1) = -4(2x + 1)(2x - 1)$$
$$14x - 7 - 16x^2 - 8x = -16x^2 + 4$$
$$6x = 11$$
$$x = \frac{11}{6}$$

35. $\dfrac{3}{x^2 - 3x} + \dfrac{4}{x} = \dfrac{1}{x-3}$

$\dfrac{3}{x(x-3)} + \dfrac{4}{x} = \dfrac{1}{x-3}$ Multiply both sides by $x(x - 3)$.

$$3 + 4(x - 3) = x$$
$$3 + 4x - 12 = x$$
$$3x = 9$$
$$x = 3$$

A check reveals that $x = 3$ is an extraneous solution because it makes the denominator zero. There is no solution.

36. $\dfrac{6}{x} - \dfrac{2}{x+3} = \dfrac{3(x+5)}{x(x+3)}$ Multiply both sides by $x(x + 3)$.

$$6(x + 3) - 2x = 3(x + 5)$$
$$6x + 18 - 2x = 3x + 15$$
$$4x + 18 = 3x + 15$$
$$x = -3$$

A check reveals that $x = -3$ is an extraneous solution because it makes the denominator zero. There is no solution.

37. $(x + 2)^2 + 5 = (x + 3)^2$

$$x^2 + 4x + 4 + 5 = x^2 + 6x + 9$$
$$4x + 9 = 6x + 9$$
$$-2x = 0$$
$$x = 0$$

38. $(2x + 1)^2 = 4(x^2 + x + 1)$

$$4x^2 + 4x + 1 = 4x^2 + 4x + 4$$
$$1 = 4$$

This is a contradiction. So, the equation has no solution.

39. $2x^2 = 3 - 8x$

General form: $2x^2 + 8x - 3 = 0$

40. $13 - 3(x + 7)^2 = 0$

$$13 - 3(x^2 + 14x + 49) = 0$$
$$13 - 3x^2 - 42x - 147 = 0$$

General form:

$$-3x^2 - 42x - 134 = 0$$

41. $\frac{1}{5}(3x^2 - 10) = 18x$

$3x^2 - 10 = 90x$

General form: $3x^2 - 90x - 10 = 0$

42. $x(x + 2) = 5x^2 + 1$

$x^2 + 2x = 5x^2 + 1$

$-4x^2 + 2x - 1 = 0$

$(-1)(-4x^2 + 2x - 1) = -1(0)$

General form: $4x^2 - 2x + 1 = 0$

43. $6x^2 + 3x = 0$

$3x(2x + 1) = 0$

$3x = 0$ or $2x + 1 = 0$

$x = 0$ or $x = -\frac{1}{2}$

44. $9x^2 - 4 = 0$

$(3x + 2)(3x - 2) = 0$

$3x + 2 = 0$ or $3x - 2 = 0$

$x = -\frac{2}{3}$ $\quad x = \frac{2}{3}$

45. $x^2 - 2x - 8 = 0$

$(x - 4)(x + 2) = 0$

$x - 4 = 0$ or $x + 2 = 0$

$x = 4$ or $x = -2$

46. $x^2 - 10x + 9 = 0$

$(x - 9)(x - 1) = 0$

$x - 9 = 0 \Rightarrow x = 9$

$x - 1 = 0 \Rightarrow x = 1$

47. $x^2 - 12x + 35 = 0$

$(x - 7)(x - 5) = 0$

$x - 7 = 0$ or $x - 5 = 0$

$x = 7$ or $x = 5$

48. $4x^2 + 12x + 9 = 0$

$(2x + 3)(2x + 3) = 0$

$2x + 3 = 0 \Rightarrow x = -\frac{3}{2}$

49. $3 + 5x - 2x^2 = 0$

$(3 - x)(1 + 2x) = 0$

$3 - x = 0$ or $1 + 2x = 0$

$x = 3$ or $x = -\frac{1}{2}$

50. $2x^2 = 19x + 33$

$2x^2 - 19x - 33 = 0$

$(2x + 3)(x - 11) = 0$

$2x + 3 = 0 \Rightarrow x = -\frac{3}{2}$

$x - 11 = 0 \Rightarrow x = 11$

51. $x^2 + 4x = 12$

$x^2 + 4x - 12 = 0$

$(x + 6)(x - 2) = 0$

$x + 6 = 0$ or $x - 2 = 0$

$x = -6$ or $x = 2$

52. $\frac{1}{8}x^2 - x - 16 = 0$

$x^2 - 8x - 128 = 0$

$(x - 16)(x + 8) = 0$

$x - 16 = 0 \Rightarrow x = 16$

$x + 8 = 0 \Rightarrow x = -8$

53. $x^2 + 2ax + a^2 = 0$

$(x + a)^2 = 0$

$x + a = 0$

$x = -a$

54. $(x + a)^2 - b^2 = 0$

$[(x + a) + b][(x + a) - b] = 0$

$[x + (a + b)][x + (a - b)] = 0$

$x + (a + b) = 0 \Rightarrow x = -a - b$

$x + (a - b) = 0 \Rightarrow x = -a + b$

55. $x^2 = 49$

$x = \pm 7$

56. $x^2 = 32$

$x = \pm\sqrt{32} = \pm 4\sqrt{2}$

57. $3x^2 = 81$

$x^2 = 27$

$x = \pm 3\sqrt{3}$

58. $9x^2 = 36$

$x^2 = 4$

$x = \pm\sqrt{4} = \pm 2$

59. $(x-12)^2 = 16$

$x - 12 = \pm 4$

$x = 12 \pm 4$

$x = 16$ or $x = 8$

60. $(x+13)^2 = 25$

$x + 13 = \pm\sqrt{25}$

$x + 13 = \pm 5$

$x = -13 \pm 5 = -8, -18$

61. $(x+2)^2 = 14$

$x + 2 = \pm\sqrt{14}$

$x = -2 \pm \sqrt{14}$

62. $(x-5)^2 = 30$

$x - 5 = \pm\sqrt{30}$

$x = 5 \pm \sqrt{30}$

63. $(2x-1)^2 = 18$

$2x - 1 = \pm\sqrt{18}$

$2x = 1 \pm 3\sqrt{2}$

$x = \dfrac{1 \pm 3\sqrt{2}}{2}$

64. $(2x+3)^2 - 27 = 0$

$(2x+3)^2 = 27$

$2x + 3 = \pm\sqrt{27}$

$2x + 3 = \pm 3\sqrt{3}$

$2x = -3 \pm 3\sqrt{3}$

$x = \dfrac{-3 \pm 3\sqrt{3}}{2}$

65. $(x-7)^2 = (x+3)^2$

$x - 7 = \pm(x+3)$

$x - 7 = x + 3$ or $x - 7 = -x - 3$

$-7 \neq 3$ or $2x = 4$

$x = 2$

The only solution to the equation is $x = 2$.

66. $(x+5)^2 = (x+4)^2$

$x + 5 = \pm(x+4)$

$x + 5 = +(x+4)$ or $x + 5 = -(x+4)$

$5 \neq 4$ or $x + 5 = -x - 4$

$2x = -9$

$x = -\frac{9}{2}$

The only solution to the equation is $x = -\frac{9}{2}$.

67. $x^2 + 4x - 32 = 0$

$x^2 + 4x = 32$

$x^2 + 4x + 2^2 = 32 + 2^2$

$(x+2)^2 = 36$

$x + 2 = \pm 6$

$x = -2 \pm 6$

$x = 4$ or $x = -8$

68. $x^2 + 6x + 2 = 0$

$x^2 + 6x = -2$

$x^2 + 6x + 3^2 = -2 + 3^2$

$(x+3)^2 = 7$

$x + 3 = \pm\sqrt{7}$

$x = -3 \pm \sqrt{7}$

69. $x^2 + 12x + 25 = 0$

$x^2 + 12x = -25$

$x^2 + 12x + 6^2 = -25 + 6^2$

$(x+6)^2 = 11$

$x + 6 = \pm\sqrt{11}$

$x = -6 \pm \sqrt{11}$

70. $x^2 + 8x + 14 = 0$

$x^2 + 8x = -14$

$x^2 + 8x + 4^2 = -14 + 16$

$(x+4)^2 = 2$

$x + 4 = \pm\sqrt{2}$

$x = -4 \pm \sqrt{2}$

71.
$$8 + 4x - x^2 = 0$$
$$-x^2 + 4x + 8 = 0$$
$$x^2 - 4x - 8 = 0$$
$$x^2 - 4x = 8$$
$$x^2 - 4x + 2^2 = 8 + 2^2$$
$$(x - 2)^2 = 12$$
$$x - 2 = \pm\sqrt{12}$$
$$x = 2 \pm 2\sqrt{3}$$

72.
$$9x^2 - 12x = 14$$
$$x^2 - \tfrac{4}{3}x = \tfrac{14}{9}$$
$$x^2 - \tfrac{4}{3}x + \left(-\tfrac{2}{3}\right)^2 = \tfrac{14}{9} + \tfrac{4}{9}$$
$$\left(x - \tfrac{2}{3}\right)^2 = \tfrac{18}{9}$$
$$\left(x - \tfrac{2}{3}\right)^2 = 2$$
$$x - \tfrac{2}{3} = \pm\sqrt{2}$$
$$x = \tfrac{2}{3} \pm \sqrt{2}$$

73.
$$2x^2 + 5x - 8 = 0$$
$$2x^2 + 5x = 8$$
$$x^2 + \frac{5}{2}x = 4$$
$$x^2 + \frac{5}{2}x + \left(\frac{5}{4}\right)^2 = 4 + \left(\frac{5}{4}\right)^2$$
$$\left(x + \frac{5}{4}\right)^2 = \frac{89}{16}$$
$$x + \frac{5}{4} = \pm\frac{\sqrt{89}}{4}$$
$$x = -\frac{5}{4} \pm \frac{\sqrt{89}}{4}$$
$$x = \frac{-5 \pm \sqrt{89}}{4}$$

74.
$$4x^2 - 4x - 99 = 0$$
$$x^2 - x = \tfrac{99}{4}$$
$$x^2 - x + \left(-\tfrac{1}{2}\right)^2 = \tfrac{99}{4} + \tfrac{1}{4}$$
$$\left(x - \tfrac{1}{2}\right)^2 = \tfrac{100}{4}$$
$$\left(x - \tfrac{1}{2}\right)^2 = 25$$
$$x - \tfrac{1}{2} = \pm\sqrt{25}$$
$$x = \tfrac{1}{2} \pm 5 = \tfrac{11}{2}, -\tfrac{9}{2}$$

75.
$$5x^2 - 15x + 7 = 0$$
$$x^2 - 3x + \frac{7}{5} = 0$$
$$x^2 - 3x = \frac{-7}{5}$$
$$x^2 - 3x + \left(\frac{3}{2}\right)^2 = \frac{-7}{5} + \left(\frac{3}{2}\right)^2$$
$$\left(x - \frac{3}{2}\right)^2 = \frac{17}{20}$$
$$x - \frac{3}{2} = \pm\sqrt{\frac{17}{20}}$$
$$x - \frac{3}{2} = \frac{\pm\sqrt{17}}{2\sqrt{5}}$$
$$x = \frac{3}{2} \pm \frac{\sqrt{17}}{2\sqrt{5}}$$
$$x = \frac{3}{2} \pm \frac{\sqrt{85}}{10}$$
$$x = \frac{15 \pm \sqrt{85}}{10}$$

76.
$$3x^2 + 9x + 5 = 0$$
$$x^2 + 3x + \frac{5}{3} = 0$$
$$x^2 + 3x = -\frac{5}{3}$$
$$x^2 + 3x + \left(\frac{3}{2}\right)^2 = -\frac{5}{3} + \left(\frac{3}{2}\right)^2$$
$$\left(x + \frac{3}{2}\right)^2 = -\frac{5}{3} + \frac{9}{4}$$
$$\left(x + \frac{3}{2}\right)^2 = \frac{7}{12}$$
$$x + \frac{3}{2} = \pm\sqrt{\frac{7}{12}}$$
$$x + \frac{3}{2} = \pm\frac{\sqrt{7}}{2\sqrt{3}}$$
$$x = -\frac{3}{2} \pm \frac{\sqrt{7}}{2\sqrt{3}}$$
$$x = -\frac{3}{2} \pm \frac{\sqrt{21}}{6}$$
$$x = \frac{-9 \pm \sqrt{21}}{6}$$

77. $2x^2 + x - 1 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-1 \pm \sqrt{1^2 - 4(2)(-1)}}{2(2)} = \frac{-1 \pm 3}{4} = \frac{1}{2}, -1$$

78. $25x^2 - 20x + 3 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-20) \pm \sqrt{(-20)^2 - 4(25)(3)}}{2(25)} = \frac{20 \pm \sqrt{400 - 300}}{50} = \frac{20 \pm 10}{50} = \frac{3}{5}, \frac{1}{5}$$

79. $2 + 2x - x^2 = 0$

$$-x^2 + 2x + 2 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-2 \pm \sqrt{2^2 - 4(-1)(2)}}{2(-1)} = \frac{-2 \pm 2\sqrt{3}}{-2} = 1 \pm \sqrt{3}$$

80. $x^2 - 10x + 22 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-10) \pm \sqrt{(-10)^2 - 4(1)(22)}}{2(1)} = \frac{10 \pm \sqrt{100 - 88}}{2} = \frac{10 \pm 2\sqrt{3}}{2} = 5 \pm \sqrt{3}$$

81. $x^2 + 14x + 44 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-14 \pm \sqrt{14^2 - 4(1)(44)}}{2(1)} = \frac{-14 \pm 2\sqrt{5}}{2} = -7 \pm \sqrt{5}$$

82. $6x = 4 - x^2$

$$x^2 + 6x - 4 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-6 \pm \sqrt{6^2 - 4(1)(-4)}}{2(1)} = \frac{-6 \pm \sqrt{36 + 16}}{2} = \frac{-6 \pm 2\sqrt{13}}{2} = -3 \pm \sqrt{13}$$

83. $12x - 9x^2 = -3$

$$-9x^2 + 12x + 3 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-12 \pm \sqrt{12^2 - 4(-9)(3)}}{2(-9)} = \frac{-12 \pm 6\sqrt{7}}{-18} = \frac{2}{3} \pm \frac{\sqrt{7}}{3}$$

84. $4x^2 - 4x - 4 = 0$

$$x^2 - x - 1 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(-1)}}{2(1)} = \frac{1 \pm \sqrt{1 + 4}}{2} = \frac{1}{2} \pm \frac{\sqrt{5}}{2}$$

85. $9x^2 + 24x + 16 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-24 \pm \sqrt{24^2 - 4(9)(16)}}{2(9)} = \frac{-24 \pm 0}{18} = -\frac{4}{3}$$

86. $16x^2 - 40x + 5 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-(-40) \pm \sqrt{(-40)^2 - 4(16)(5)}}{2(16)}$$
$$= \frac{40 \pm \sqrt{1600 - 320}}{32}$$
$$= \frac{40 \pm 16\sqrt{5}}{32}$$
$$= \frac{5}{4} \pm \frac{\sqrt{5}}{2}$$

87. $28x - 49x^2 = 4$

$$-49x^2 + 28x - 4 = 0$$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-28 \pm \sqrt{28^2 - 4(-49)(-4)}}{2(-49)}$$
$$= \frac{-28 \pm 0}{-98} = \frac{2}{7}$$

88. $3x + x^2 - 1 = 0$

$$x^2 + 3x - 1 = 0$$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-3 \pm \sqrt{3^2 - 4(1)(-1)}}{2(1)}$$
$$= \frac{-3 \pm \sqrt{13}}{2} = -\frac{3}{2} \pm \frac{\sqrt{13}}{2}$$

89. $8t = 5 + 2t^2$

$$-2t^2 + 8t - 5 = 0$$
$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-8 \pm \sqrt{8^2 - 4(-2)(-5)}}{2(-2)} = \frac{-8 \pm 2\sqrt{6}}{-4} = 2 \pm \frac{\sqrt{6}}{2}$$

90. $25h^2 + 80h + 61 = 0$

$$h = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-80 \pm \sqrt{80^2 - 4(25)(61)}}{2(25)}$$
$$= \frac{-80 \pm \sqrt{6400 - 6100}}{50}$$
$$= -\frac{8}{5} \pm \frac{10\sqrt{3}}{50}$$
$$= -\frac{8}{5} \pm \frac{\sqrt{3}}{5}$$

91. $(y - 5)^2 = 2y$

$$y^2 - 12y + 25 = 0$$
$$y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-(-12) \pm \sqrt{(-12)^2 - 4(1)(25)}}{2(1)}$$
$$= \frac{12 \pm 2\sqrt{11}}{2} = 6 \pm \sqrt{11}$$

92. $\left(\frac{5}{7}x - 14\right)^2 = 8x$

$$\frac{25}{49}x^2 - 20x + 196 = 8x$$
$$\frac{25}{49}x^2 - 28x + 196 = 0$$
$$25x^2 - 1372x + 9604 = 0$$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-(-1372) \pm \sqrt{(-1372)^2 - 4(25)(9604)}}{2(25)}$$
$$= \frac{1372 \pm \sqrt{921{,}984}}{50}$$
$$= \frac{686 \pm 196\sqrt{6}}{25}$$

93. $0.1x^2 + 0.2x - 0.5 = 0$

$$x = -b \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{-0.2 \pm \sqrt{(0.2)^2 - 4(0.1)(-0.5)}}{2(0.1)}$$
$$x \approx 1.449, -3.449$$

94. $-0.005x^2 + 0.101x - 0.193 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-0.101 \pm \sqrt{(0.101)^2 - 4(-0.005)(-0.193)}}{2(-0.005)}$$
$$= \frac{-0.101 \pm \sqrt{0.006341}}{-0.01}$$
$$\approx 2.137, 18.063$$

95. $422x^2 - 506x - 347 = 0$

$$x = \frac{506 \pm \sqrt{(-506)^2 - 4(422)(-347)}}{2(422)}$$
$$x \approx 1.687, -0.488$$

96. $-3.22x^2 - 0.08x + 28.651 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-(-0.08) \pm \sqrt{(-0.08)^2 - 4(-3.22)(28.651)}}{2(-3.22)}$$
$$= \frac{0.08 \pm \sqrt{369.031}}{-6.44} \approx -2.995, 2.971$$

97. $x^2 - 2x - 1 = 0$ Complete the square.

$$x^2 - 2x = 1$$
$$x^2 - 2x + 1^2 = 1 + 1^2$$
$$(x - 1)^2 = 2$$
$$x - 1 = \pm\sqrt{2}$$
$$x = 1 \pm \sqrt{2}$$

98. $11x^2 + 33x = 0$ Factor.

$$11(x^2 + 3x) = 0$$
$$x(x + 3) = 0$$
$$x = 0 \quad \text{or} \quad x + 3 = 0$$
$$x = -3$$

99. $(x + 3)^2 = 81$ Extract square roots.

$$x + 3 = \pm 9$$
$$x + 3 = 9 \quad \text{or} \quad x + 3 = -9$$
$$x = 6 \quad \text{or} \quad x = -12$$

100. $x^2 - 14x + 49 = 0$ Extract square roots.

$$(x - 7)^2 = 0$$
$$x - 7 = 0$$
$$x = 7$$

101. $x^2 - x - \frac{11}{4} = 0$ Complete the square.

$$x^2 - x = \frac{11}{4}$$
$$x^2 - x + \left(\frac{1}{2}\right)^2 = \frac{11}{4} + \left(\frac{1}{2}\right)^2$$
$$\left(x - \frac{1}{2}\right)^2 = \frac{12}{4}$$
$$x - \frac{1}{2} = \pm\sqrt{\frac{12}{4}}$$
$$x = \frac{1}{2} \pm \sqrt{3}$$

102. $3x + 4 = 2x^2 - 7$ Quadratic Formula

$$0 = 2x^2 - 3x - 11$$
$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(2)(-11)}}{2(2)}$$
$$= \frac{3 \pm \sqrt{97}}{4}$$
$$= \frac{3}{4} \pm \frac{\sqrt{97}}{4}$$

103. $4x^2 + 2x + 4 = 2x + 8$ Factor.

$$4x^2 - 4 = 0$$
$$4(x^2 - 1) = 0$$
$$(x + 1)(x - 1) = 0$$
$$x + 1 = 0 \quad \text{or} \quad x - 1 = 0$$
$$x = -1 \qquad x = 1$$

104. $a^2x^2 - b^2 = 0$ Factor.

$$(ax + b)(ax - b) = 0$$
$$ax + b = 0 \Rightarrow x = -\frac{b}{a}$$
$$ax - b = 0 \Rightarrow x = \frac{b}{a}$$

105. $2x^4 - 50x^2 = 0$

$$2x^2(x^2 - 25) = 0$$
$$2x^2(x + 5)(x - 5) = 0$$
$$2x^2 = 0 \quad \text{or} \quad x + 5 = 0 \quad \text{or} \quad x - 5 = 0$$
$$x = 0 \quad \text{or} \quad x = -5 \quad \text{or} \quad x = 5$$

106.
$$20x^3 - 125x = 0$$
$$5x(4x^2 - 25) = 0$$
$$5x(2x + 5)(2x - 5) = 0$$
$$5x = 0 \Rightarrow x = 0$$
$$2x + 5 = 0 \Rightarrow x = -\tfrac{5}{2}$$
$$2x - 5 = 0 \Rightarrow x = \tfrac{5}{2}$$

107.
$$x^4 - 81 = 0$$
$$(x^2 + 9)(x + 3)(x - 3) = 0$$
$x^2 + 9 = 0 \Rightarrow$ No real solution
$$x + 3 = 0 \Rightarrow x = -3$$
$$x - 3 = 0 \Rightarrow x = 3$$

108.
$$x^6 - 64 = 0$$
$$(x^3 - 8)(x^3 + 8) = 0$$
$$(x - 2)(x^2 + 2x + 4)(x + 2)(x^2 - 2x + 4) = 0$$
$$x - 2 = 0 \Rightarrow x = 2$$
$x^2 + 2x + 4 = 0 \Rightarrow$ No real solution (by the Quadratic Formula)
$$x + 2 = 0 \Rightarrow x = -2$$
$x^2 - 2x + 4 = 0 \Rightarrow$ No real solution (by the Quadratic Formula)

109.
$$x^3 + 216 = 0$$
$$x^3 + 6^3 = 0$$
$$(x + 6)(x^2 - 6x + 36) = 0$$
$$x + 6 = 0 \Rightarrow x = -6$$
$x^2 - 6x + 36 = 0 \Rightarrow$ No real solution (by the Quadratic Formula)

110.
$$9x^4 - 24x^3 + 16x^2 = 0$$
$$x^2(9x^2 - 24x + 16) = 0$$
$$x^2(3x - 4)^2 = 0$$
$$x^2 = 0 \Rightarrow x = 0$$
$$3x - 4 = 0 \Rightarrow x = \tfrac{4}{3}$$

111.
$$x^3 - 3x^2 - x + 3 = 0$$
$$x^2(x - 3) - (x - 3) = 0$$
$$(x - 3)(x^2 - 1) = 0$$
$$(x - 3)(x + 1)(x - 1) = 0$$
$$x - 3 = 0 \Rightarrow x = 3$$
$$x + 1 = 0 \Rightarrow x = -1$$
$$x - 1 = 0 \Rightarrow x = 1$$

112.
$$x^3 + 2x^2 + 3x + 6 = 0$$
$$x^2(x + 2) + 3(x + 2) = 0$$
$$(x + 2)(x^2 + 3) = 0$$
$$x + 2 = 0 \Rightarrow x = -2$$
$x^2 + 3 = 0 \Rightarrow$ No real solution

113.
$$x^4 + x = x^3 + 1$$
$$x^4 - x^3 + x - 1 = 0$$
$$x^3(x - 1) + (x - 1) = 0$$
$$(x - 1)(x^3 + 1) = 0$$
$$(x - 1)(x + 1)(x^2 - x + 1) = 0$$
$$x - 1 = 0 \Rightarrow x = 1$$
$$x + 1 = 0 \Rightarrow x = -1$$
$x^2 - x + 1 = 0 \Rightarrow$ No real solution (by the Quadratic Formula)

114.
$$\begin{aligned} x^4 - 2x^3 &= 16 + 8x - 4x^3 \\ x^4 + 2x^3 - 8x - 16 &= 0 \\ x^3(x + 2) - 8(x + 2) &= 0 \\ (x^3 - 8)(x + 2) &= 0 \\ (x - 2)(x^2 + 2x + 4)(x + 2) &= 0 \\ x - 2 = 0 &\Rightarrow x = 2 \\ x^2 + 2x + 4 = 0 &\Rightarrow \text{No real solution (by the Quadratic Formula)} \\ x + 2 = 0 &\Rightarrow x = -2 \end{aligned}$$

115.
$$\begin{aligned} x^4 - 4x^2 + 3 &= 0 \\ (x^2 - 3)(x^2 - 1) &= 0 \\ (x + \sqrt{3})(x - \sqrt{3})(x + 1)(x - 1) &= 0 \\ x + \sqrt{3} = 0 &\Rightarrow x = -\sqrt{3} \\ x - \sqrt{3} = 0 &\Rightarrow x = \sqrt{3} \\ x + 1 = 0 &\Rightarrow x = -1 \\ x - 1 = 0 &\Rightarrow x = 1 \end{aligned}$$

116.
$$\begin{aligned} 36t^4 + 29t^2 - 7 &= 0 \\ (36t^2 - 7)(t^2 + 1) &= 0 \\ (6t + \sqrt{7})(6t - \sqrt{7})(t^2 + 1) &= 0 \\ 6t + \sqrt{7} = 0 &\Rightarrow t = -\frac{\sqrt{7}}{6} \\ 6t - \sqrt{7} = 0 &\Rightarrow t = \frac{\sqrt{7}}{6} \\ t^2 + 1 = 0 &\Rightarrow \text{No real solution} \end{aligned}$$

117.
$$\begin{aligned} x^6 + 7x^3 - 8 &= 0 \\ (x^3 + 8)(x^3 - 1) &= 0 \\ (x + 2)(x^2 - 2x + 4)(x - 1)(x^2 + x + 1) &= 0 \\ x + 2 = 0 &\Rightarrow x = -2 \\ x^2 - 2x + 4 = 0 &\Rightarrow \text{No real solution (by the Quadratic Formula)} \\ x - 1 = 0 &\Rightarrow x = 1 \\ x^2 + x + 1 = 0 &\Rightarrow \text{No real solution (by the Quadratic Formula)} \end{aligned}$$

118.
$$\begin{aligned} x^6 + 3x^3 + 2 &= 0 \\ (x^3 + 2)(x^3 + 1) &= 0 \\ (x + \sqrt[3]{2})\left[x^2 - \sqrt[3]{2}x + (\sqrt[3]{2})^2\right](x + 1)(x^2 - x + 1) &= 0 \\ x + \sqrt[3]{2} = 0 &\Rightarrow x = -\sqrt[3]{2} \\ x^2 - \sqrt[3]{2}x + (\sqrt[3]{2})^2 = 0 &\Rightarrow \text{No real solution (by the Quadratic Formula)} \\ x + 1 = 0 &\Rightarrow x = -1 \\ x^2 - x + 1 = 0 &\Rightarrow \text{No real solution (by the Quadratic Formula)} \end{aligned}$$

119.
$$\begin{aligned} \sqrt{2x} - 10 &= 0 \\ \sqrt{2x} &= 10 \\ 2x &= 100 \\ x &= 50 \end{aligned}$$

120.
$$\begin{aligned} 7\sqrt{x} - 6 &= 0 \\ 7\sqrt{x} &= 6 \\ 49x &= 36 \\ x &= \tfrac{36}{49} \end{aligned}$$

121.
$$\begin{aligned} \sqrt{x - 10} - 4 &= 0 \\ \sqrt{x - 10} &= 4 \\ x - 10 &= 16 \\ x &= 26 \end{aligned}$$

122.
$$\begin{aligned} \sqrt{5 - x} - 3 &= 0 \\ \sqrt{5 - x} &= 3 \\ 5 - x &= 9 \\ x &= -4 \end{aligned}$$

123. $\sqrt{2x+5}+3=0$

$\sqrt{2x+5}=-3$

$2x+5=9$

$2x=4$

$x=2$

No solution.

124. $\sqrt{3-2x}-2=0$

$\sqrt{3-2x}=2$

$3-2x=4$

$-2x=1$

$x=-\frac{1}{2}$

125. $\sqrt[3]{2x+1}+8=0$

$\sqrt[3]{2x+1}=-8$

$2x+1=-512$

$2x=-513$

$x=-\frac{513}{2}$

126. $\sqrt[3]{4x-3}+2=0$

$\sqrt[3]{4x-3}=-2$

$4x-3=-8$

$4x=-5$

$x=-\frac{5}{4}$

127. $\sqrt{5x-26}+4=x$

$\sqrt{5x-26}=x-4$

$5x-26=x^2-8x+16$

$x^2-13x+42=0$

$(x-6)(x-7)=0$

$x=6$ or $x=7$

128. $\sqrt{x+5}=\sqrt{2x-5}$

$x+5=2x-5$

$x=10$

129. $(x-6)^{3/2}=8$

$x-6=8^{2/3}$

$x-6=4$

$x=10$

130. $(x+3)^{3/2}=8$

$(x+3)^3=8^2$

$(x+3)^3=64$

$x+3=\sqrt[3]{64}$

$x=-3+4=1$

131. $(x+3)^{2/3}=5$

$x+3=5^{3/2}$

$x+3=\pm5\sqrt{5}$

$x=-3\pm5\sqrt{5}$

132. $(x^2-x-22)^{4/3}=16$

$x^2-x-22=16^{3/4}$

$x^2-x-22=8$ or $x^2-x-22=-8$

$x^2-x-30=0$ $\quad$ $x^2-x-14=0$

$(x-6)(x+5)=0$ $\quad$ $x=\frac{1\pm\sqrt{1^2-4(1)(-14)}}{2(1)}$

$x=6$ or $x=-5$ $\quad$ $x=\frac{1\pm\sqrt{57}}{2}$

133. $3x(x-1)^{1/2}+2(x-1)^{3/2}=0$

$(x-1)^{1/2}[3x+2(x-1)]=0$

$(x-1)^{1/2}(5x-2)=0$

$(x-1)^{1/2}=0 \Rightarrow x-1=0 \Rightarrow x=1$

$5x-2=0 \Rightarrow x=\frac{2}{5}$, extraneous

134. $4x^2(x-1)^{1/3}+6x(x-1)^{4/3}=0$

$2x\left[2x(x-1)^{1/3}+3(x-1)^{4/3}\right]=0$

$2x(x-1)^{1/3}[2x+3(x-1)]=0$

$2x(x-1)^{1/3}(5x-3)=0$

$2x=0 \Rightarrow x=0$

$x-1=0 \Rightarrow x=1$

$5x-3=0 \Rightarrow x=\frac{3}{5}$

135.
$$\begin{aligned} x &= \frac{3}{x} + \frac{1}{2} \\ (2x)(x) &= (2x)\left(\frac{3}{x}\right) + (2x)\left(\frac{1}{2}\right) \\ 2x^2 &= 6 + x \\ 2x^2 - x - 6 &= 0 \\ (2x + 3)(x - 2) &= 0 \\ 2x + 3 = 0 &\Rightarrow x = -\frac{3}{2} \\ x - 2 = 0 &\Rightarrow x = 2 \end{aligned}$$

136.
$$\begin{aligned} \frac{4}{x+1} - \frac{3}{x+2} &= 1 \\ 4(x + 2) - 3(x + 1) &= (x + 1)(x + 2) \\ 4x + 8 - 3x - 3 &= x^2 + 3x + 2 \\ x^2 + 2x - 3 &= 0 \\ (x - 1)(x + 3) &= 0 \\ x - 1 = 0 &\Rightarrow x = 1 \\ x + 3 = 0 &\Rightarrow x = -3 \end{aligned}$$

137.
$$\begin{aligned} \frac{20 - x}{x} &= x \\ 20 - x &= x^2 \\ 0 &= x^2 + x - 20 \\ 0 &= (x + 5)(x - 4) \\ x + 5 = 0 &\Rightarrow x = -5 \\ x - 4 = 0 &\Rightarrow x = 4 \end{aligned}$$

138.
$$\begin{aligned} 4x + 1 &= \frac{3}{x} \\ (x)4x + (x)1 &= (x)\frac{3}{x} \\ 4x^2 + x &= 3 \\ 4x^2 + x - 3 &= 0 \\ (4x - 3)(x + 1) &= 0 \\ 4x - 3 = 0 &\Rightarrow x = \frac{3}{4} \\ x + 1 = 0 &\Rightarrow x = -1 \end{aligned}$$

139.
$$\begin{aligned} |2x - 1| &= 5 \\ 2x - 1 = 5 &\Rightarrow x = 3 \\ -(2x - 1) = 5 &\Rightarrow x = -2 \end{aligned}$$

140.
$$\begin{aligned} |13x + 1| &= 12 \\ 13x + 1 = 12 &\Rightarrow x = \tfrac{11}{13} \\ -(13x + 1) &= 12 \\ -13x - 1 = 12 &\Rightarrow x = -1 \end{aligned}$$

141. $|x| = x^2 + x - 3$

First equation:
$$\begin{aligned} x &= x^2 + x - 3 \\ x^2 - 3 &= 0 \\ x &= \pm\sqrt{3} \end{aligned}$$

Second equation:
$$\begin{aligned} -x &= x^2 + x - 3 \\ x^2 + 2x - 3 &= 0 \\ (x - 1)(x + 3) &= 0 \\ x - 1 = 0 &\Rightarrow x = 1 \\ x + 3 = 0 &\Rightarrow x = -3 \end{aligned}$$

Only $x = \sqrt{3}$ and $x = -3$ are solutions to the original equation. $x = -\sqrt{3}$ and $x = 1$ are extraneous

142. $|x^2 + 6x| = 3x + 18$

First equation:
$$\begin{aligned} x^2 + 6x &= 3x + 18 \\ x^2 + 3x - 18 &= 0 \\ (x - 3)(x + 6) &= 0 \\ x - 3 = 0 &\Rightarrow x = 3 \\ x + 6 = 0 &\Rightarrow x = -6 \end{aligned}$$

Second equation:
$$\begin{aligned} -(x^2 + 6x) &= 3x + 18 \\ 0 &= x^2 + 9x + 18 \\ 0 &= (x + 3)(x + 6) \\ 0 = x + 3 &\Rightarrow x = -3 \\ 0 = x + 6 &\Rightarrow x = -6 \end{aligned}$$

The solutions to the original equation are $x = \pm 3$ and $x = -6$.

143. $|x + 1| = x^2 - 5$

First equation:

$$x + 1 = x^2 - 5$$
$$x^2 - x - 6 = 0$$
$$(x - 3)(x + 2) = 0$$
$$x - 3 = 0 \Rightarrow x = 3$$
$$x + 2 = 0 \Rightarrow x = -2$$

Second equation:

$$-(x + 1) = x^2 - 5$$
$$-x - 1 = x^2 - 5$$
$$x^2 + x - 4 = 0$$
$$x = \frac{-1 \pm \sqrt{17}}{2}$$

Only $x = 3$ and $x = \frac{-1 - \sqrt{17}}{2}$ are solutions to the original equation. $x = -2$ and $x = \frac{-1 + \sqrt{17}}{2}$ are extraneous.

144. $|x - 10| = x^2 - 10x$

First equation:

$$x - 10 = x^2 - 10x$$
$$0 = x^2 - 11x + 10$$
$$0 = (x - 1)(x - 10)$$
$$0 = x - 1 \Rightarrow x = 1$$
$$0 = x - 10 \Rightarrow x = 10$$

Second equation:

$$-(x - 10) = x^2 - 10x$$
$$0 = x^2 - 9x - 10$$
$$0 = (x - 10)(x + 1)$$
$$0 = x - 10 \Rightarrow x = 10$$
$$0 = x + 1 \Rightarrow x = -1$$

The solutions to the original equation are $x = 10$ and $x = -1$. $x = 1$ is extraneous.

145. The student should have subtracted $15x$ from both sides so that the equation is equal to zero. By factoring out an x, there are **two** solutions.

$x = 0$ or $x = 6$

146. The **zero-factor** property states that if the product of two factors is **zero,** then one (or both) of the factors must be zero. To solve $4x^2 + 4x = 15$, a student factors $4x$ from the left side of the equation and sets each factor equal to 15. The resulting **incorrect** solutions are $x = \frac{15}{4}$ and $x = 14$. To attempt to solve a quadratic equation by factoring, the equation should be in **general form** first. If it factors, then the zero-factor property can be employed to solve the equation.

$$4x^2 + 4x = 15$$
$$4x^2 + 4x - 15 = 0$$
$$(2x - 3)(2x + 5) = 0$$
$$2x - 3 = 0 \Rightarrow x = \tfrac{3}{2}$$
$$2x + 5 = 0 \Rightarrow x = -\tfrac{5}{2}$$

147. Equivalent equations are derived from the substitution principle and simplification techniques. They have the same solution(s).

$2x + 3 = 8$ and $2x = 5$ are equivalent equations.

148. Remove symbols of grouping, combine like terms, reduce fractions.

Add(or subtract) the same quantity to (from) both sides of the equation.

Multiply (or divide) both sides of the equation by the same nonzero quantity.

Interchange the two sides of the equation.

149. Female: $y = 0.432x - 10.44$

For $y = 16$: $16 = 0.432x - 10.44$

$$26.44 = 0.432x$$
$$\frac{26.44}{0.432} = x$$
$$x \approx 61.2 \text{ inches}$$

150. Male: $y = 0.449x - 12.15$

For $y = 19$: $19 = 0.449x - 12.15$

$$31.15 = 0.449x$$
$$69.4 \approx x$$

Yes, it is likely that both bones came from the same person because the estimated height of a male with a 19-inch thigh bone is 69.4 inches.

151. (a) $P = 200$ million when

$$200 = \frac{182.17 - 1.542t}{1 - 0.018t}$$
$$200(1 - 0.018t) = 182.17 - 1.542t$$
$$200 - 3.6t = 182.17 - 1.542t$$
$$2.058t = 17.83$$
$$t \approx 8.7 \text{ years}$$

So, the total voting-age population reached 200 million during 1998.

(b) $$241 = \frac{182.17 - 1.542t}{1 - 0.018t}$$
$$241(1 - 0.018t) = 182.17 - 1.542t$$
$$241 - 4.338t = 182.17 - 1.542t$$
$$2.796t = 58.83$$
$$t \approx 21 \text{ years}$$

The model predicts the total voting-age population will reach 241 million during 2011. This value is reasonable.

152. When $C = 2.5$:

$$2.5 = \sqrt{0.2x + 1}$$
$$6.25 = 0.2x + 1$$
$$5.25 = 0.2x$$
$$x = 26.25 = 26{,}250 \text{ passengers}$$

153. False—See Example 14 on page A58.

154. False. $|x| = 0$ has only one solution to check, 0.

155. –3 and 6

One possible equation is:

$$(x - (-3))(x - 6) = 0$$
$$(x + 3)(x - 6) = 0$$
$$x^2 - 3x - 18 = 0$$

Any non-zero multiple of this equation would also have these solutions.

156. $$(x - (-4))(x - (-11)) = 0$$
$$(x + 4)(x + 11) = 0$$
$$x^2 + 15x + 44 = 0$$

157. $1 + \sqrt{2}$ and $1 - \sqrt{2}$

One possible equation is:

$$\left[x - \left(1 + \sqrt{2}\right)\right]\left[x - \left(1 - \sqrt{2}\right)\right] = 0$$
$$\left[(x - 1) - \sqrt{2}\right]\left[(x - 1) + \sqrt{2}\right] = 0$$
$$(x - 1)^2 - \left(\sqrt{2}\right)^2 = 0$$
$$x^2 - 2x + 1 - 2 = 0$$
$$x^2 - 2x - 1 = 0$$

Any non-zero multiple of this equation would also have these solutions.

158. $x = -3 + \sqrt{5}, x = -3 - \sqrt{5},$ so:

$$\left(x - \left(-3 + \sqrt{5}\right)\right)\left(x - \left(-3 - \sqrt{5}\right)\right) = 0$$
$$\left(x + 3 - \sqrt{5}\right)\left(x + 3 + \sqrt{5}\right) = 0$$
$$x^2 + 6x + 4 = 0$$

159. $$9 + |9 - a| = b$$
$$|9 - a| = b - 9$$

$9 - a = b - 9$ OR $9 - a = -(b - 9)$

$-a = b - 18$ | $9 - a = -b + 9$

$a = 18 - b$ | $-a = -b$

$a = b$

So, $a = 18 - b$ or $a = b$. From the original equation you know that $b \geq 9$.

Some possibilities are: $b = 9, a = 9$

$b = 10, a = 8$ or $a = 10$

$b = 11, a = 7$ or $a = 11$

$b = 12, a = 6$ or $a = 12$

$b = 13, a = 5$ or $a = 13$

$b = 14, a = 4$ or $a = 14$

160. Isolate the absolute value by subtracting x from both sides of the equation. The expression inside the absolute value signs can be positive or negative, so two separate equations must be solved. Each solution must be checked because extraneous solutions may be included.

161. (a) $$ax^2 + bx = 0$$
$$x(ax + b) = 0$$
$$x = 0$$
$$ax + b = 0 \Rightarrow x = -\frac{b}{a}$$

(b) $$ax^2 - ax = 0$$
$$ax(x - 1) = 0$$
$$ax = 0 \Rightarrow x = 0$$
$$x - 1 = 0 \Rightarrow x = 1$$

162. Sample answer:

(a) An identity is true for all real numbers, whereas a conditional equation is true for just some (or none) of the real numbers in the domain.

(b) Sample answer: $|x - 4| = 0$

(c) The opposite of b plus or minus the square root of the quantity b^2 minus the product of 4, a, and c, all divided by the product of 2 and a.

(d) No. For instance, $x = 2$ is not equivalent to $x^2 = 4$ because $x^2 = 4$ has solutions $x = 2$ and $x = -2$.

Section P.2 Solving Inequalities

1. solution set

2. graph

3. negative

4. union

5. key; test intervals

6. zeros; undefined values

7. Interval: $[0, 9)$

(a) Inequality: $0 \le x \le 9$

(b) The interval is bounded.

8. Interval: $(-7, 4)$

(a) Inequality: $-7 \le x \le 4$

(b) The interval is bounded.

9. Interval: $[-1, 5]$

(a) Inequality: $-1 \le x \le 5$

(b) The interval is bounded.

10. Interval: $(2, 10]$

(a) Inequality: $2 < x \le 10$

(b) The interval is bounded.

11. Interval: $(11, \infty)$

(a) Inequality: $x > 11$

(b) The interval is unbounded.

12. Interval: $[-5, \infty)$

(a) Inequality: $-5 \le x < \infty$ or $x \ge -5$

(b) The interval is unbounded.

13. Interval: $(-\infty, -2)$

(a) Inequality: $x < -2$

(b) The interval is unbounded.

14. Interval: $(-\infty, 7]$

(a) Inequality: $-\infty < x \le 7$ or $x \le -7$

(b) The interval is unbounded.

15. $x < 3$

Matches (b).

16. $x \ge 5$

Matches (h).

17. $-3 < x \le 4$

Matches (e).

18. $0 \le x \le \frac{9}{2}$

Matches (d).

19. $|x| < 3 \Rightarrow -3 < x < 3$

Matches (f).

20. $|x| > 4 \Rightarrow x > 4$ or $x < -4$

Matches (a).

21. $-1 \le x \le \frac{5}{2}$

Matches (g).

22. $-1 < x < \frac{5}{2}$

Matches (c).

23. $5x - 12 > 0$

(a) $x = 3$

$5(3) - 12 \overset{?}{>} 0$

$3 > 0$

Yes, $x = 3$ *is* a solution.

(b) $x = -3$

$5(-3) - 12 \overset{?}{>} 0$

$-27 \not> 0$

No, $x = -3$ *is not* a solution.

(c) $x = \frac{5}{2}$

$5\left(\frac{5}{2}\right) - 12 \overset{?}{>} 0$

$\frac{1}{2} > 0$

Yes, $x = \frac{5}{2}$ *is* a solution.

(d) $x = \frac{3}{2}$

$5\left(\frac{3}{2}\right) - 12 \overset{?}{>} 0$

$-\frac{9}{2} \not> 0$

No, $x = \frac{3}{2}$ *is not* a solution.

24. $2x + 1 < -3$

(a) $x = 0$

$2(0) + 1 \overset{?}{<} -3$

$1 \not< -3$

No, $x = 0$ *is not* a solution.

(b) $x = -\frac{1}{4}$

$2\left(-\frac{1}{4}\right) + 1 \overset{?}{<} -3$

$\frac{1}{2} \not< -3$

No, $x = -\frac{1}{4}$ *is not* a solution.

(c) $x = -4$

$2(-4) + 1 \overset{?}{<} -3$

$-7 < -3$

Yes, $x = -4$ *is* a solution.

(d) $x = -\frac{3}{2}$

$2\left(-\frac{3}{2}\right) + 1 \overset{?}{<} -3$

$-2 \not< -3$

No, $x = -\frac{3}{2}$ *is not* a solution.

25. $0 < \dfrac{x - 2}{4} < 2$

(a) $x = 4$

$0 \overset{?}{<} \dfrac{4 - 2}{4} \overset{?}{<} 2$

$0 < \dfrac{1}{2} < 2$

Yes, $x = 4$ *is* a solution.

(b) $x = 10$

$0 \overset{?}{<} \dfrac{10 - 2}{4} \overset{?}{<} 2$

$0 < 2 \not< 2$

No, $x = 10$ *is not* a solution.

(c) $x = 0$

$0 \overset{?}{<} \dfrac{0 - 2}{4} \overset{?}{<} 2$

$0 \not< -\dfrac{1}{2} < 2$

No, $x = 0$ *is not* a solution.

(d) $x = \dfrac{7}{2}$

$0 \overset{?}{<} \dfrac{(7/2) - 2}{4} \overset{?}{<} 2$

$0 < \dfrac{3}{8} < 2$

Yes, $x = \dfrac{7}{2}$ *is* a solution.

26. $-5 < 2x - 1 \le 1$

(a) $x = -\frac{1}{2}$

$-5 \overset{?}{<} 2\left(-\frac{1}{2}\right) - 1 \overset{?}{\le} 1$

$-5 < -1 - 1 \overset{?}{\le} 1$

$-5 < -2 \le 1$

Yes, $x = -\frac{1}{2}$ *is* a solution.

(b) $x = -\frac{5}{2}$

$-5 \overset{?}{<} 2\left(-\frac{5}{2}\right) - 1 \overset{?}{\le} 1$

$-5 \overset{?}{<} -5 - 1 \overset{?}{\le} 1$

$-5 \not< -6 \not\le 1$

No, $x = -\frac{5}{2}$ *is not* a solution.

(c) $x = \frac{4}{3}$

$-5 \overset{?}{<} 2\left(\frac{4}{3}\right) - 1 \overset{?}{\le} 1$

$-5 \overset{?}{<} \frac{8}{3} - 1 \overset{?}{\le} 1$

$-5 < \frac{5}{3} \not< 1$

No, $x = \frac{4}{3}$ *is not* a solution.

(d) $x = 0$

$-5 \overset{?}{<} 2(0) - 1 \overset{?}{\le} 1$

$-5 \le -1 \le 1$

Yes, $x = 0$ *is* a solution.

27. $|x - 10| \ge 3$

(a) $x = 13$

$|13 - 10| \overset{?}{\ge} 3$

$3 \ge 3$

Yes, $x = 13$ *is* a solution.

(b) $x = -1$

$|-1 - 10| \overset{?}{\ge} 3$

$11 \ge 3$

Yes, $x = -1$ *is* a solution.

(c) $x = 14$

$|14 - 10| \overset{?}{\ge} 3$

$4 \ge 3$

Yes, $x = 14$ *is* a solution.

(d) $x = 9$

$|9 - 10| \overset{?}{\ge} 3$

$1 \not\ge 3$

No, $x = 9$ *is not* a solution.

28. $|2x - 3| < 15$

(a) $x = -6$

$|2(-6) - 3| \overset{?}{<} 15$

$15 \nless 15$

No, $x = -6$ *is not* a solution.

(b) $x = 0$

$|2(0) - 3| \overset{?}{<} 15$

$3 < 15$

Yes, $x = 0$ *is* a solution.

(c) $x = 12$

$|2(12) - 3| \overset{?}{<} 15$

$21 \nless 15$

No, $x = 12$ *is not* a solution.

(d) $x = 7$

$|2(7) - 3| \overset{?}{<} 15$

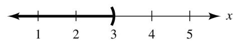
$11 < 15$

Yes, $x = 7$ *is* a solution.

29. $4x < 12$

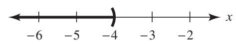
$\frac{1}{4}(4x) < \frac{1}{4}(12)$

$x < 3$



30. $10x < -40$

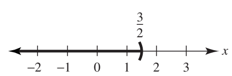
$x < -4$



31. $-2x > -3$

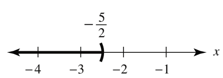
$-\frac{1}{2}(-2x) < \left(-\frac{1}{2}\right)(-3)$

$x < \frac{3}{2}$



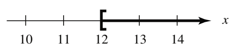
32. $-6x > 15$

$x < -\frac{15}{6}$ or $x < -\frac{5}{2}$



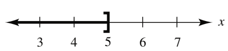
33. $x - 5 \geq 7$

$x \geq 12$



34. $x + 7 \leq 12$

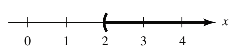
$x \leq 5$



35. $2x + 7 < 3 + 4x$

$-2x < -4$

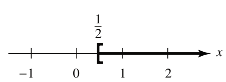
$x > 2$



36. $3x + 1 \geq 2 + x$

$2x \geq 1$

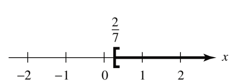
$x \geq \frac{1}{2}$



37. $2x - 1 \geq 1 - 5x$

$7x \geq 2$

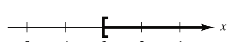
$x \geq \frac{2}{7}$



38. $6x - 4 \leq 2 + 8x$

$-2x \leq 6$

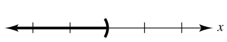
$x \geq -3$



39. $4 - 2x < 3(3 - x)$

$4 - 2x < 9 - 3x$

$x < 5$

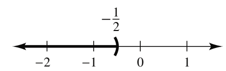


40. $4(x + 1) < 2x + 3$

$4x + 4 < 2x + 3$

$2x < -1$

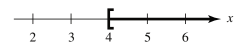
$x < -\frac{1}{2}$



41. $\frac{3}{4}x - 6 \leq x - 7$

$-\frac{1}{4}x \leq -1$

$x \geq 4$

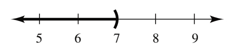


42. $3 + \frac{2}{7}x > x - 2$

$21 + 2x > 7x - 14$

$-5x > -35$

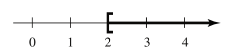
$x < 7$



43. $\frac{1}{2}(8x + 1) \geq 3x + \frac{5}{2}$

$4x + \frac{1}{2} \geq 3x + \frac{5}{2}$

$x \geq 2$

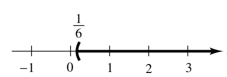


44. $9x - 1 < \frac{3}{4}(16x - 2)$

$36x - 4 < 48x - 6$

$-12x < -2$

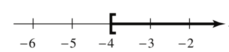
$x > \frac{1}{6}$



45. $3.6x + 11 \geq -3.4$

$3.6x \geq 14.4$

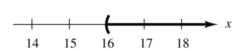
$x \geq -4$



46. $15.6 - 1.3x < -5.2$

$-1.3x < -20.8$

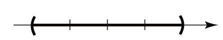
$x > 16$



47. $1 < 2x + 3 < 9$

$-2 < 2x < 6$

$-1 < x < 3$

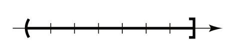


48. $-8 \leq -(3x + 5) < 13$

$-8 \leq -3x - 5 < 13$

$-3 \leq -3x < 18$

$-6 < x \leq 1$



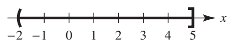
49. $-8 \le 1 - 3(x - 2) < 13$

$-8 \le 1 - 3x + 6 < 13$

$-8 \le 7 - 3x < 13$

$-15 \le -3x < 6$

$5 \ge x > -2$



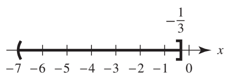
50. $0 \le 2 - 3(x + 1) < 20$

$0 \le 2 - 3x - 3 < 20$

$0 \le -1 - 3x < 20$

$1 \le -3x < 21$

$-\frac{1}{3} \ge x > -7$

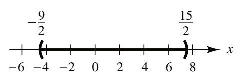


51. $-4 < \dfrac{2x - 3}{3} < 4$

$-12 < 2x - 3 < 12$

$-9 < 2x < 15$

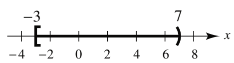
$-\dfrac{9}{2} < x < \dfrac{15}{2}$



52. $0 \le \dfrac{x + 3}{2} < 5$

$0 \le x + 3 < 10$

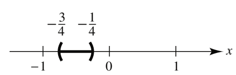
$-3 \le x < 7$



53. $\frac{3}{4} > x + 1 > \frac{1}{4}$

$-\frac{1}{4} > x > -\frac{3}{4}$

$-\frac{3}{4} < x < -\frac{1}{4}$

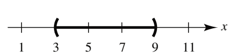


54. $-1 < 2 - \dfrac{x}{3} < 1$

$-3 < 6 - x < 3$

$-9 < -x < -3$

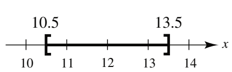
$3 < x < 9$



55. $3.2 \le 0.4x - 1 \le 4.4$

$4.2 \le 0.4x \le 5.4$

$10.5 \le x \le 13.5$



56. $4.5 > \dfrac{1.5x + 6}{2} > 10.5$

$9 > 1.5x + 6 > 21$

$3 > 1.5x > 15$

$2 > x > 10$

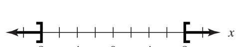
There is no solution.

57. $|x| < 5$

$-5 < x < 5$

58. $|x| \ge 8$

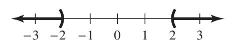
$x \ge 8$ or $x \le -8$



59. $\left|\dfrac{x}{2}\right| > 1$

$\dfrac{x}{2} < -1$ or $\dfrac{x}{2} > 1$

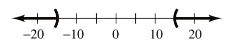
$x < -2 \qquad x > 2$



60. $\left|\dfrac{x}{5}\right| > 3$

$\dfrac{x}{5} < -3$ or $\dfrac{x}{5} > 3$

$x < -15 \qquad x > 15$



61. $|x - 5| < -1$

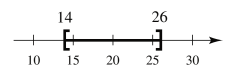
No solution. The absolute value of a number cannot be less than a negative number.

62. No solution. The absolute value of a number cannot be less than a negative number.

63. $|x - 20| \le 6$

$-6 \le x - 20 \le 6$

$14 \le x \le 26$



64. $|x - 8| \ge 0$

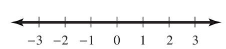
$x - 8 \ge 0$ or $-(x - 8) \ge 0$

$x \ge 8 \qquad -x + 8 \ge 0$

$-x \ge -8$

$x \le 8$

All real numbers x.

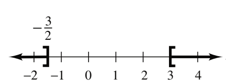


65. $|3 - 4x| \ge 9$

$3 - 4x \le -9$ or $3 - 4x \ge 9$

$-4x \le -12 \qquad -4x \ge 6$

$x \ge 3 \qquad x \le -\frac{3}{2}$



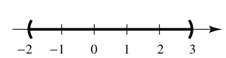
66. $|1 - 2x| < 5$

$-5 < 1 - 2x < 5$

$-6 < -2x < 4$

$3 > x > -2$

$-2 < x < 3$

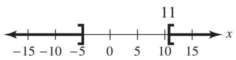


67. $\left|\dfrac{x-3}{2}\right| \geq 4$

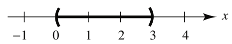
$\dfrac{x-3}{2} \leq -4$ or $\dfrac{x-3}{2} \geq 4$

$x - 3 \leq -8 \qquad x - 3 \geq 8$

$x \leq -5 \qquad x \geq 11$



68. $\left|1 - \dfrac{2x}{3}\right| < 1$



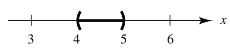
$-1 < 1 - \dfrac{2x}{3} < 1$

$-2 < -\dfrac{2x}{3} < 0$

$3 > x > 0$

$0 < x < 3$

69. $|9 - 2x| - 2 < -1$



$|9 - 2x| < 1$

$-1 < 9 - 2x < 1$

$-10 < -2x < -8$

$5 > x > 4$

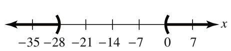
$4 < x < 5$

70. $|x + 14| + 3 > 17$

$|x + 14| > 14$

$x + 14 < -14$ or $x + 14 > 14$

$x < -28 \qquad x > 0$

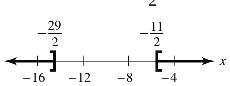


71. $2|x + 10| \geq 9$

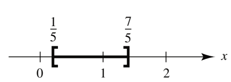
$|x + 10| \geq \frac{9}{2}$

$x + 10 \leq -\frac{9}{2}$ or $x + 10 \geq \frac{9}{2}$

$x \leq -\frac{29}{2} \qquad x \geq -\frac{11}{2}$



72. $3|4 - 5x| \leq 9$



$|4 - 5x| \leq 3$

$-3 \leq 4 - 5x \leq 3$

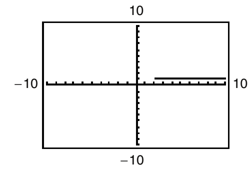
$-7 \leq -5x \leq -1$

$\frac{7}{5} \geq x \geq \frac{1}{5}$

$\frac{1}{5} \leq x \leq \frac{7}{5}$

73. $6x > 12$

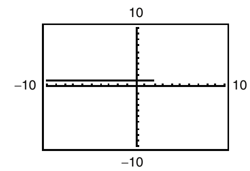
$x > 2$



74. $3x - 1 \leq 5$

$3x \leq 6$

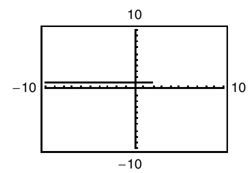
$x \leq 2$



75. $5 - 2x \geq 1$

$-2x \geq -4$

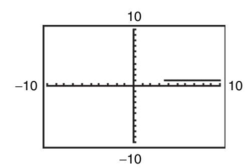
$x \leq 2$



76. $20 < 6x - 1$

$21 < 6x$

$\frac{7}{2} < x$

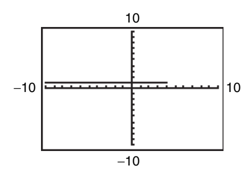


77. $4(x - 3) \leq 8 - x$

$4x - 12 \leq 8 - x$

$5x \leq 20$

$x \leq 4$

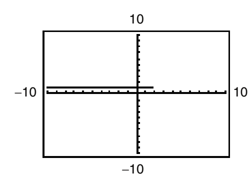


78. $3(x + 1) < x + 7$

$3x + 3 < x + 7$

$2x < 4$

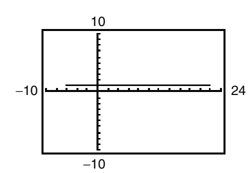
$x < 2$



79. $|x - 8| \leq 14$

$-14 \leq x - 8 \leq 14$

$-6 \leq x \leq 22$

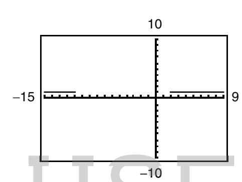


80. $|2x + 9| > 13$

$2x + 9 < -13$ or $2x + 9 > 13$

$2x < -22 \qquad 2x > 4$

$x < -11 \qquad x > 2$

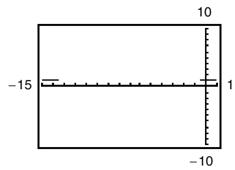


81. $2|x + 7| \geq 13$

$|x + 7| \geq \frac{13}{2}$

$x + 7 \leq -\frac{13}{2}$ or $x + 7 \geq \frac{13}{2}$

$x \leq -\frac{27}{2}$ $\quad x \geq -\frac{1}{2}$

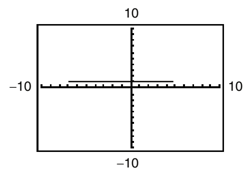


82. $\frac{1}{2}|x + 1| \leq 3$

$|x + 1| \leq 6$

$-6 \leq x + 1 \leq 6$

$-7 \leq x \leq 5$



83. $x - 5 \geq 0$

$x \geq 5$

$[5, \infty)$

84. $\sqrt{x - 10}$

$x - 10 \geq 0$

$x \geq 10$

$[10, \infty)$

85. $x + 3 \geq 0$

$x \geq -3$

$[-3, \infty)$

86. $\sqrt{3 - x}$

$3 - x \geq 0$

$3 \geq x$

$(-\infty, 3]$

87. $7 - 2x \geq 0$

$-2x \geq -7$

$x \leq \frac{7}{2}$

$\left(-\infty, \frac{7}{2}\right]$

88. $\sqrt[4]{6x + 15}$

$6x + 15 \geq 0$

$6x \geq -15$

$x \geq -\frac{5}{2}$

$\left[-\frac{5}{2}, \infty\right)$

89. $|x - 10| < 8$

All real numbers within 8 units of 10.

90. $|x - 8| > 4$

All real numbers more than 4 units from 8.

91. The midpoint of the interval $[-3, 3]$ is 0. The interval represents all real numbers x no more than 3 units from 0.

$|x - 0| \leq 3$

$|x| \leq 3$

92. The graph shows all real numbers more than 3 units from 0.

$|x - 0| > 3$

$|x| > 3$

93. The graph shows all real numbers at least 3 units from 7.

$|x - 7| \geq 3$

94. The graph shows all real numbers no more than 4 units from -1.

$|x + 1| \leq 4$

95. All real numbers within 10 units of 12.

$|x - 12| < 10$

96. All real numbers at least 5 units from 8.

$|x - 8| \geq 5$

97. All real numbers more than 4 units from -3.

$|x - (-3)| > 4$

$|x + 3| > 4$

98. All real numbers no more than 7 units from -6.

$|x + 6| \leq 7$

99. $x^2 - 3 < 0$

(a) $x = 3$

$(3)^2 - 3 \stackrel{?}{<} 0$

$6 \not< 0$

No, $x = 3$ *is not* a solution.

(b) $x = 0$

$(0)^2 - 3 \stackrel{?}{<} 0$

$-3 < 0$

Yes, $x = 0$ *is* a solution.

(c) $x = \frac{3}{2}$

$\left(\frac{3}{2}\right)^2 - 3 \stackrel{?}{<} 0$

$-\frac{3}{4} < 0$

Yes, $x = \frac{3}{2}$ *is* a solution.

(d) $x = -5$

$(-5)^2 - 3 \stackrel{?}{<} 0$

$22 \not< 0$

No, $x = -5$ *is not* a solution.

100. $x^2 - x - 12 \ge 0$

(a) $x = 5$

$(5)^2 - (5) - 12 \stackrel{?}{\ge} 0$

$8 \ge 0$

Yes, $x = 5$ *is* a solution.

(b) $x = 0$

$(0)^2 - 0 - 12 \stackrel{?}{\ge} 0$

$-12 \not\ge 0$

No, $x = 0$ *is not* a solution.

(c) $x = -4$

$(-4)^2 - (-4) - 12 \stackrel{?}{\ge} 0$

$16 + 4 - 12 \stackrel{?}{\ge} 0$

$8 \ge 0$

Yes, $x = -4$ *is* a solution.

(d) $x = -3$

$(-3)^2 - (-3) - 12 \stackrel{?}{\ge} 0$

$9 + 3 - 12 \stackrel{?}{\ge} 0$

$0 \ge 0$

Yes, $x = -3$ *is* a solution.

101. $\dfrac{x + 2}{x - 4} \ge 3$

(a) $x = 5$

$\dfrac{5 + 2}{5 - 4} \stackrel{?}{\ge} 3$

$7 \ge 3$

Yes, $x = 5$ *is* a solution.

(b) $x = 4$

$\dfrac{4 + 2}{4 - 4} \stackrel{?}{\ge} 3$

$\dfrac{6}{0}$ is undefined.

No, $x = 4$ *is not* a solution.

(c) $x = -\dfrac{9}{2}$

$\dfrac{-\frac{9}{2} + 2}{-\frac{9}{2} - 4} \stackrel{?}{\ge} 3$

$\dfrac{5}{17} \not\ge 3$

No, $x = -\dfrac{9}{2}$ *is not* a solution.

(d) $x = \dfrac{9}{2}$

$\dfrac{\frac{9}{2} + 2}{\frac{9}{2} - 4} \stackrel{?}{\ge} 3$

$13 \ge 3$

Yes, $x = \dfrac{9}{2}$ *is* a solution.

102. $\dfrac{3x^2}{x^2 + 4} < 1$

(a) $x = -2$

$\dfrac{3(-2)^2}{(-2)^2 + 4} \stackrel{?}{<} 1$

$\dfrac{12}{8} \not< 1$

No, $x = -2$ *is not* a solution.

(b) $x = -1$

$\dfrac{3(-1)^2}{(-1)^2 + 4} \stackrel{?}{<} 1$

$\dfrac{3}{5} < 1$

Yes, $x = -1$ *is* a solution.

(c) $x = 0$

$\dfrac{3(0)^2}{(0)^2 + 4} \stackrel{?}{<} 1$

$0 < 1$

Yes, $x = 0$ *is* a solution.

(d) $x = 3$

$\dfrac{3(3)^2}{(3)^2 + 4} \stackrel{?}{<} 1$

$\dfrac{27}{13} \not< 1$

No, $x = 3$ *is not* a solution.

103. $3x^2 - x - 2 = (3x + 2)(x - 1)$

$3x + 2 = 0 \Rightarrow x = -\frac{2}{3}$

$x - 1 = 0 \Rightarrow x = 1$

The key numbers are $-\frac{2}{3}$ and 1.

104. $9x^3 - 25x^2 = 0$

$x^2(9x - 25) = 0$

$x^2 = 0 \Rightarrow x = 0$

$9x - 25 = 0 \Rightarrow x = \frac{25}{9}$

The key numbers are 0 and $\frac{25}{9}$.

105. $\dfrac{1}{x-5}+1=\dfrac{1+1(x-5)}{x-5}$

$=\dfrac{x-4}{x-5}$

$x-4=0 \Rightarrow x=4$

$x-5=0 \Rightarrow x=5$

The key numbers are 4 and 5.

106. $\dfrac{x}{x+2}-\dfrac{2}{x-1}=\dfrac{x(x-1)-2(x+2)}{(x+2)(x-1)}$

$=\dfrac{x^2-x-2x-4}{(x+2)(x-1)}$

$=\dfrac{(x-4)(x+1)}{(x+2)(x-1)}$

$(x-4)(x+1)=0$

$x-4=0 \Rightarrow x=4$

$x+1=0 \Rightarrow x=-1$

$(x+2)(x-1)=0$

$x+2=0 \Rightarrow x=-2$

$x-1=0 \Rightarrow x=1$

The key numbers are $-2, -1, 1,$ and 4.

107. $x^2 < 9$

$x^2-9<0$

$(x+3)(x-3)<0$

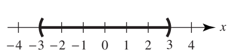
Key numbers: $x=\pm3$

Test intervals: $(-\infty,-3), (-3,3), (3,\infty)$

Test: Is $(x+3)(x-3)<0$?

Interval	x-Value	Value of x^2-9	Conclusion
$(-\infty,-3)$	-4	7	Positive
$(-3,3)$	0	-9	Negative
$(3,\infty)$	4	7	Positive

Solution set: $(-3,3)$



108. $x^2 \le 16$

$x^2-16 \le 0$

$(x+4)(x-4) \le 0$

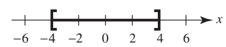
Key numbers: $x=\pm4$

Test intervals: $(-\infty,-4), (-4,4), (4,\infty)$

Test: Is $(x+4)(x-4) \le 0$?

Interval	x-Value	Value of x^2-16	Conclusion
$(-\infty,-4)$	-5	9	Positive
$(-4,4)$	0	-16	Negative
$(4,\infty)$	5	9	Positive

Solution set: $[-4,4]$



109. $(x+2)^2 \le 25$

$x^2+4x+4 \le 25$

$x^2+4x-21 \le 0$

$(x+7)(x-3) \le 0$

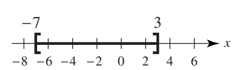
Key numbers: $x=-7, x=3$

Test intervals: $(-\infty,-7), (-7,3), (3,\infty)$

Test: Is $(x+7)(x-3) \le 0$?

Interval	x-Value	Value of $(x+7)(x-3)$	Conclusion
$(-\infty,-7)$	-8	$(-1)(-11)=11$	Positive
$(-7,3)$	0	$(7)(-3)=-21$	Negative
$(3,\infty)$	4	$(11)(1)=11$	Positive

Solution set: $[-7,3]$



110. $(x-3)^2 \ge 1$

$x^2-6x+8 \ge 0$

$(x-2)(x-4) \ge 0$

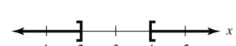
Key numbers: $x=2, x=4$

Test intervals: $(-\infty,2) \Rightarrow (x-2)(x-4)>0$

$(2,4) \Rightarrow (x-2)(x-4)<0$

$(4,\infty) \Rightarrow (x-2)(x-4)>0$

Solution set: $(-\infty,2] \cup [4,\infty)$



111. $x^2 + 4x + 4 \ge 9$

$x^2 + 4x - 5 \ge 0$

$(x + 5)(x - 1) \ge 0$

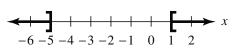
Key numbers: $x = -5, x = 1$

Test intervals: $(-\infty, -5), (-5, 1), (1, \infty)$

Test: Is $(x + 5)(x - 1) \ge 0$?

Interval	x-Value	Value of $(x + 5)(x - 1)$	Conclusion
$(-\infty, -5)$	-6	$(-1)(-7) = 7$	Positive
$(-5, 1)$	0	$(5)(-1) = -5$	Negative
$(1, \infty)$	2	$(7)(1) = 7$	Positive

Solution set: $(-\infty, -5] \cup [1, \infty)$



112. $x^2 - 6x + 9 < 16$

$x^2 - 6x - 7 < 0$

$(x + 1)(x - 7) < 0$

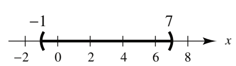
Key numbers: $x = -1, x = 7$

Test intervals: $(-\infty, -1) \Rightarrow (x + 1)(x - 7) > 0$

$(-1, 7) \Rightarrow (x + 1)(x - 7) < 0$

$(7, \infty) \Rightarrow (x + 1)(x - 7) > 0$

Solution set: $(-1, 7)$



113. $x^2 + x < 6$

$x^2 + x - 6 < 0$

$(x + 3)(x - 2) < 0$

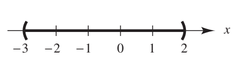
Key numbers: $x = -3, x = 2$

Test intervals: $(-\infty, -3), (-3, 2), (2, \infty)$

Test: Is $(x + 3)(x - 2) < 0$?

Interval	x-Value	Value of $(x + 3)(x - 2)$	Conclusion
$(-\infty, -3)$	-4	$(-1)(-6) = 6$	Positive
$(-3, 2)$	0	$(3)(-2) = -6$	Negative
$(2, \infty)$	3	$(6)(1) = 6$	Positive

Solution set: $(-3, 2)$



114. $x^2 + 2x > 3$

$x^2 + 2x - 3 > 0$

$(x + 3)(x - 1) > 0$

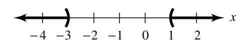
Key numbers: $x = -3, x = 1$

Test intervals: $(-\infty, -3) \Rightarrow (x + 3)(x - 1) > 0$

$(-3, 1) \Rightarrow (x + 3)(x - 1) < 0$

$(1, \infty) \Rightarrow (x + 3)(x - 1) > 0$

Solution set: $(-\infty, -3) \cup (1, \infty)$



115. $x^2 + 2x - 3 < 0$

$(x + 3)(x - 1) < 0$

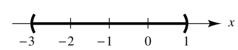
Key numbers: $x = -3, x = 1$

Test intervals: $(-\infty, -3), (-3, 1), (1, \infty)$

Test: Is $(x + 3)(x - 1) < 0$?

Interval	x-Value	Value of $(x + 3)(x - 1)$	Conclusion
$(-\infty, -3)$	-4	$(-1)(-5) = 5$	Positive
$(-3, 1)$	0	$(3)(-1) = -3$	Negative
$(1, \infty)$	2	$(5)(1) = 5$	Positive

Solution set: $(-3, 1)$



116. $x^2 > 2x + 8$

$x^2 - 2x - 8 > 0$

$(x - 4)(x + 2) > 0$

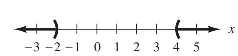
Key numbers: $x = -2, x = 4$

Test intervals: $(-\infty, -2), (-2, 4), (4, \infty)$

Test: Is $(x - 4)(x + 2) > 0$?

Interval	x-Value	Value of $(x - 4)(x + 2)$	Conclusion
$(-\infty, -2)$	-3	$(-7)(-1) = 7$	Positive
$(-2, 4)$	0	$(-4)(2) = -8$	Negative
$(4, \infty)$	5	$(1)(7) = 7$	Positive

Solution set: $(-\infty, -2) \cup (4, \infty)$



117. $3x^2 - 11x > 20$

$3x^2 - 11x - 20 > 0$

$(3x + 4)(x - 5) > 0$

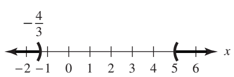
Key numbers: $x = 5, x = -\frac{4}{3}$

Test intervals: $\left(-\infty, -\frac{4}{5}\right), \left(-\frac{4}{3}, 5\right), (5, \infty)$

Test: Is $(3x + 4)(x - 5) > 0$?

Interval	x-Value	Value of $(3x + 4)(x - 5)$	Conclusion
$\left(-\infty, -\frac{4}{3}\right)$	–3	$(-5)(-8) = 40$	Positive
$\left(-\frac{4}{3}, 5\right)$	0	$(4)(-5) = -20$	Negative
$(5, \infty)$	6	$(22)(1) = 22$	Positive

Solution set: $\left(-\infty, -\frac{4}{3}\right) \cup (5, \infty)$



118. $-2x^2 + 6x + 15 \le 0$

$2x^2 - 6x - 15 \ge 0$

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(2)(-15)}}{2(2)}$$

$$= \frac{6 \pm \sqrt{156}}{4}$$

$$= \frac{6 \pm 2\sqrt{39}}{4}$$

$$= \frac{3}{2} \pm \frac{\sqrt{39}}{2}$$

Key numbers: $x = \frac{3}{2} - \frac{\sqrt{39}}{2}, x = \frac{3}{2} + \frac{\sqrt{39}}{2}$

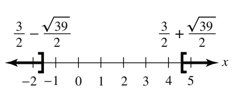
Test intervals:

$\left(-\infty, \frac{3}{2} - \frac{\sqrt{39}}{2}\right) \Rightarrow -2x^2 + 6x + 15 < 0$

$\left(\frac{3}{2} - \frac{\sqrt{39}}{2}, \frac{3}{2} + \frac{\sqrt{39}}{2}\right) \Rightarrow -2x^2 + 6x + 15 > 0$

$\left(\frac{3}{2} + \frac{\sqrt{39}}{2}, \infty\right) \Rightarrow -2x^2 + 6x + 15 < 0$

Solution set: $\left(-\infty, \frac{3}{2} - \frac{\sqrt{39}}{2}\right] \cup \left[\frac{3}{2} + \frac{\sqrt{39}}{2}, \infty\right)$



119. $x^2 - 3x - 18 > 0$

$(x + 3)(x - 6) > 0$

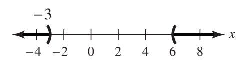
Key numbers: $x = -3, x = 6$

Test intervals: $(-\infty, -3), (-3, 6), (6, \infty)$

Test: Is $(x + 3)(x - 6) > 0$?

Interval	x-Value	Value of $(x + 3)(x - 6)$	Conclusion
$(-\infty, -3)$	–4	$(-1)(-10) = 10$	Positive
$(-3, 6)$	0	$(3)(-6) = -18$	Negative
$(6, \infty)$	7	$(10)(1) = 10$	Positive

Solution set: $(-\infty, -3) \cup (6, \infty)$



120. $x^3 + 2x^2 - 4x - 8 \le 0$

$x^2(x + 2) - 4(x + 2) \le 0$

$(x + 2)(x^2 - 4) \le 0$

$(x + 2)^2(x - 2) \le 0$

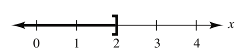
Key numbers: $x = -2, x = 2$

Test intervals: $(-\infty, -2) \Rightarrow x^3 + 2x^2 - 4x - 8 < 0$

$(-2, 2) \Rightarrow x^3 + 2x^2 - 4x - 8 < 0$

$(2, \infty) \Rightarrow x^3 + 2x^2 - 4x - 8 > 0$

Solution set: $(-\infty, 2]$



121. $x^3 - 3x^2 - x > -3$

$x^3 - 3x^2 - x + 3 > 0$

$x^2(x - 3) - (x - 3) > 0$

$(x - 3)(x^2 - 1) > 0$

$(x - 3)(x + 1)(x - 1) > 0$

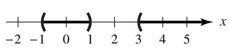
Key numbers: $x = -1, x = 1, x = 3$

Test intervals: $(-\infty, -1), (-1, 1), (1, 3), (3, \infty)$

Test: Is $(x - 3)(x + 1)(x - 1) > 0$?

Interval	x-Value	Value of $(x - 3)(x + 1)(x - 1)$	Conclusion
$(-\infty, -1)$	-2	$(-5)(-1)(-3) = -15$	Negative
$(-1, 1)$	0	$(-3)(1)(-1) = 3$	Positive
$(1, 3)$	2	$(-1)(3)(1) = -3$	Negative
$(3, \infty)$	4	$(1)(5)(3) = 15$	Positive

Solution set: $(-1, 1) \cup (3, \infty)$



122. $2x^3 + 13x^2 - 8x - 46 \ge 6$

$2x^3 + 13x^2 - 8x - 52 \ge 0$

$x^2(2x + 13) - 4(2x + 13) \ge 0$

$(2x + 13)(x^2 - 4) \ge 0$

$(2x + 13)(x + 2)(x - 2) \ge 0$

Key numbers: $x = -\frac{13}{2}, x = -2, x = 2$

Test intervals:

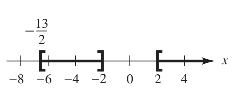
$\left(-\infty, -\frac{13}{2}\right) \Rightarrow 2x^3 + 13x^2 - 8x - 52 < 0$

$\left(-\frac{13}{2}, -2\right) \Rightarrow 2x^3 + 13x^2 - 8x - 52 > 0$

$(-2, 2) \Rightarrow 2x^3 + 13x^2 - 8x - 52 < 0$

$(2, \infty) \Rightarrow 2x^3 + 13x^2 - 8x - 52 > 0$

Solution set: $\left[-\frac{13}{2}, -2\right], [2, \infty)$



123. $4x^2 - 4x + 1 \le 0$

$(2x - 1)^2 \le 0$

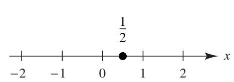
Key number: $x = \frac{1}{2}$

Test intervals: $\left(-\infty, \frac{1}{2}\right), \left(\frac{1}{2}, \infty\right)$

Test: Is $(2x - 1)^2 \le 0$?

Interval	x-Value	Value of $(2x - 1)^2$	Conclusion
$\left(-\infty, \frac{1}{2}\right)$	0	$(-1)^2 = 1$	Positive
$\left(\frac{1}{2}, \infty\right)$	1	$(1)^2 = 1$	Positive

Solution set: $x = \frac{1}{2}$

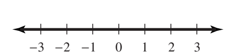


124. $x^2 + 3x + 8 > 0$

The key numbers are imaginary:

$-\frac{3}{2} \pm \frac{i\sqrt{23}}{2}$

So the set of real numbers is the solution set.



125. $4x^3 - 6x^2 < 0$

$2x^2(2x - 3) < 0$

Key numbers: $x = 0, x = \frac{3}{2}$

Test intervals: $(-\infty, 0) \Rightarrow 2x^2(2x - 3) < 0$

$\left(0, \frac{3}{2}\right) \Rightarrow 2 \Rightarrow 2x^2(2x - 3) < 0$

$\left(\frac{3}{2}, \infty\right) \Rightarrow 2x^2(2x - 3) > 0$

Solution set: $(-\infty, 0) \cup \left(0, \frac{3}{2}\right)$

126. $4x^3 - 12x^2 > 0$

$4x^2(x - 3) > 0$

Key numbers: $x = 0, x = 3$

Test intervals: $(-\infty, 0) \Rightarrow 4x^2(x - 3) < 0$

$(0, 3) \Rightarrow 4x^2(x - 3) < 0$

$(3, \infty) \Rightarrow 4x^2(x - 3) > 0$

Solution set: $(3, \infty)$

127. $x^3 - 4x \geq 0$

$x(x + 2)(x - 2) \geq 0$

Key numbers: $x = 0, x = \pm 2$

Test intervals: $(-\infty, -2) \Rightarrow x(x + 2)(x - 2) < 0$

$(-2, 0) \Rightarrow x(x + 2)(x - 2) > 0$

$(0, 2) \Rightarrow x(x + 2)(x - 2) < 0$

$(2, \infty) \Rightarrow x(x + 2)(x - 2) > 0$

Solution set: $[-2, 0] \cup [2, \infty)$

128. $2x^3 - x^4 \leq 0$

$x^3(2 - x) \leq 0$

Key numbers: $x = 0, x = 2$

Test intervals: $(-\infty, 0) \Rightarrow x^3(2 - x) < 0$

$(0, 2) \Rightarrow x^3(2 - x) > 0$

$(2, \infty) \Rightarrow x^3(2 - x) < 0$

Solution set: $(-\infty, 0] \cup [2, \infty)$

129. $(x - 1)^2(x + 2)^3 \geq 0$

Key numbers: $x = 1, x = -2$

Test intervals: $(-\infty, -2) \Rightarrow (x - 1)^2(x + 2)^3 < 0$

$(-2, 1) \Rightarrow (x - 1)^2(x + 2)^3 > 0$

$(1, \infty) \Rightarrow (x - 1)^2(x + 2)^3 > 0$

Solution set: $[-2, \infty)$

130. $x^4(x - 3) \leq 0$

Key numbers: $x = 0, x = 3$

Test intervals: $(-\infty, 0) \Rightarrow x^4(x - 3) < 0$

$(0, 3) \Rightarrow x^4(x - 3) < 0$

$(3, \infty) \Rightarrow x^4(x - 3) > 0$

Solution set: $(-\infty, 3]$

131. $\dfrac{4x - 1}{x} > 0$

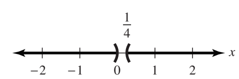
Key numbers: $x = 0, x = \dfrac{1}{4}$

Test intervals: $(-\infty, 0), \left(0, \frac{1}{4}\right), \left(\frac{1}{4}, \infty\right)$

Test: Is $\dfrac{4x - 1}{x} > 0$?

Interval	x-Value	Value of $\dfrac{4x - 1}{x}$	Conclusion
$(-\infty, 0)$	-1	$\dfrac{-5}{-1} = 5$	Positive
$\left(0, \dfrac{1}{4}\right)$	$\dfrac{1}{8}$	$\dfrac{-\frac{1}{2}}{\frac{1}{8}} = -4$	Negative
$\left(\dfrac{1}{4}, \infty\right)$	1	$\dfrac{3}{1} = 3$	Positive

Solution set: $(-\infty, 0) \cup \left(\dfrac{1}{4}, \infty\right)$



132. $\dfrac{x^2 - 1}{x} < 0$

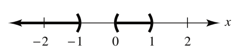
$\dfrac{(x-1)(x+1)}{x} < 0$

Key numbers: $x = -1, x = 0, x = 1$

Test intervals: $(-\infty, -1), (-1, 0), (0, 1), (1, \infty)$

Interval	x-Value	Value of $\dfrac{(x-1)(x+1)}{x}$	Conclusion
$(-\infty, -1)$	-2	$\dfrac{(-3)(-1)}{-2} = -\dfrac{3}{2}$	Negative
$(-1, 0)$	$-\dfrac{1}{2}$	$\dfrac{\left(-\frac{3}{2}\right)\left(\frac{1}{2}\right)}{-\frac{1}{2}} = \dfrac{3}{2}$	Positive
$(0, 1)$	$\dfrac{1}{2}$	$\dfrac{\left(-\frac{1}{2}\right)\left(\frac{3}{2}\right)}{\frac{1}{2}} = -\dfrac{3}{2}$	Negative
$(1, \infty)$	2	$\dfrac{(1)(3)}{2} = \dfrac{3}{2}$	Positive

Solution set: $(-\infty, -1) \cup (0, 1)$



133. $\dfrac{3x - 5}{x - 5} \geq 0$

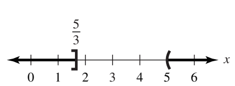
Key numbers: $x = \dfrac{5}{3}, x = 5$

Test intervals: $\left(-\infty, \dfrac{5}{3}\right), \left(\dfrac{5}{3}, 5\right), (5, \infty)$

Test: Is $\dfrac{3x - 5}{x - 5} \geq 0$?

Interval	x-Value	Value of $\dfrac{3x-5}{x-5}$	Conclusion
$\left(-\infty, \dfrac{5}{3}\right)$	0	$\dfrac{-5}{-5} = 1$	Positive
$\left(\dfrac{5}{3}, 5\right)$	2	$\dfrac{6-5}{2-5} = -\dfrac{1}{3}$	Negative
$(5, \infty)$	6	$\dfrac{18-5}{6-5} = 13$	Positive

Solution set: $\left(-\infty, \dfrac{5}{3}\right] \cup (5, \infty)$



134. $\dfrac{5 + 7x}{1 + 2x} \leq 4$

$\dfrac{5 + 7x - 4(1 + 2x)}{1 + 2x} \leq 0$

$\dfrac{1 - x}{1 + 2x} \leq 0$

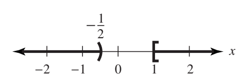
Key numbers: $x = -\dfrac{1}{2}, x = 1$

Test intervals: $\left(-\infty, -\dfrac{1}{2}\right), \left(-\dfrac{1}{2}, 1\right), (1, \infty)$

Test: Is $\dfrac{1 - x}{1 + 2x} \leq 0$?

Interval	x-Value	Value of $\dfrac{1-x}{1+2x}$	Conclusion
$\left(-\infty, -\dfrac{1}{2}\right)$	-1	$\dfrac{2}{-1} = -2$	Negative
$\left(-\dfrac{1}{2}, 1\right)$	0	$\dfrac{1}{1} = 1$	Positive
$(1, \infty)$	2	$\dfrac{-1}{5} = -\dfrac{1}{5}$	Negative

Solution set: $\left(-\infty, -\dfrac{1}{2}\right) \cup [1, \infty)$



135. $\dfrac{x + 6}{x + 1} - 2 < 0$

$\dfrac{x + 6 - 2(x + 1)}{x + 1} < 0$

$\dfrac{4 - x}{x + 1} < 0$

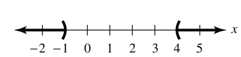
Key numbers: $x = -1, x = 4$

Test intervals: $(-\infty, -1) \Rightarrow \dfrac{4 - x}{x + 1} < 0$

$(-1, 4) \Rightarrow \dfrac{4 - x}{x + 1} > 0$

$(4, \infty) \Rightarrow \dfrac{4 - x}{x + 1} < 0$

Solution set: $(-\infty, -1) \cup (4, \infty)$



136. $\dfrac{x+12}{x+2} - 3 \ge 0$

$\dfrac{x + 12 - 3(x+2)}{x+2} \ge 0$

$\dfrac{6-2x}{x+2} \ge 0$

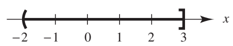
Key numbers: $x = -2,\ x = 3$

Test intervals: $(-\infty, -2) \Rightarrow \dfrac{6-2x}{x+2} < 0$

$(-2, 3) \Rightarrow \dfrac{6-2x}{x+2} > 0$

$(3, \infty) \Rightarrow \dfrac{6-2x}{x+2} < 0$

Solution interval: $(-2, 3]$



137. $\dfrac{2}{x+5} > \dfrac{1}{x-3}$

$\dfrac{2}{x+5} - \dfrac{1}{x-3} > 0$

$\dfrac{2(x-3) - 1(x+5)}{(x+5)(x-3)} > 0$

$\dfrac{x-11}{(x+5)(x-3)} > 0$

Key numbers: $x = -5,\ x = 3,\ x = 11$

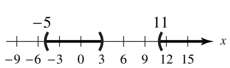
Test intervals: $(-\infty, -5) \Rightarrow \dfrac{x-11}{(x+5)(x-3)} < 0$

$(-5, 3) \Rightarrow \dfrac{x-11}{(x+5)(x-3)} > 0$

$(3, 11) \Rightarrow \dfrac{x-11}{(x+5)(x-3)} < 0$

$(11, \infty) \Rightarrow \dfrac{x-11}{(x+5)(x-3)} > 0$

Solution set: $(-5, 3) \cup (11, \infty)$



138. $\dfrac{5}{x-6} > \dfrac{3}{x+2}$

$\dfrac{5(x+2) - 3(x-6)}{(x-6)(x+2)} > 0$

$\dfrac{2x+28}{(x-6)(x+2)} > 0$

Key numbers: $x = -14,\ x = -2,\ x = 6$

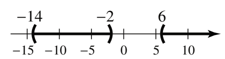
Test intervals: $(-\infty, -14) \Rightarrow \dfrac{2x+28}{(x-6)(x+2)} < 0$

$(-14, -2) \Rightarrow \dfrac{2x+28}{(x-6)(x+2)} > 0$

$(-2, 6) \Rightarrow \dfrac{2x+28}{(x-6)(x+2)} < 0$

$(6, \infty) \Rightarrow \dfrac{2x+28}{(x-6)(x+2)} > 0$

Solution intervals: $(-14, -2) \cup (6, \infty)$



139. $\dfrac{1}{x-3} \le \dfrac{9}{4x+3}$

$\dfrac{1}{x-3} - \dfrac{9}{4x+3} \le 0$

$\dfrac{4x + 3 - 9(x-3)}{(x-3)(4x+3)} \le 0$

$\dfrac{30-5x}{(x-3)(4x+3)} \le 0$

Key numbers: $x = 3,\ x = -\dfrac{3}{4},\ x = 6$

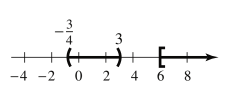
Test intervals: $\left(-\infty, -\dfrac{3}{4}\right) \Rightarrow \dfrac{30-5x}{(x-3)(4x+3)} > 0$

$\left(-\dfrac{3}{4}, 3\right) \Rightarrow \dfrac{30-5x}{(x-3)(4x+3)} < 0$

$(3, 6) \Rightarrow \dfrac{30-5x}{(x-3)(4x+3)} > 0$

$(6, \infty) \Rightarrow \dfrac{30-5x}{(x-3)(4x+3)} < 0$

Solution set: $\left(-\dfrac{3}{4}, 3\right) \cup [6, \infty)$



140. $$\frac{1}{x} \geq \frac{1}{x+3}$$

$$\frac{1(x+3) - 1(x)}{x(x+3)} \geq 0$$

$$\frac{3}{x(x+3)} \geq 0$$

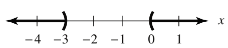
Key numbers: $x = -3, x = 0$

Test intervals: $(-\infty, -3) \Rightarrow \frac{3}{x(x+3)} > 0$

$(-3, 0) \Rightarrow \frac{3}{x(x+3)} < 0$

$(0, \infty) \Rightarrow \frac{3}{x(x+3)} > 0$

Solution intervals: $(-\infty, -3) \cup (0, \infty)$



141. $$\frac{x^2 + 2x}{x^2 - 9} \leq 0$$

$$\frac{x(x+2)}{(x+3)(x-3)} \leq 0$$

Key numbers: $x = 0, x = -2, x = \pm 3$

Test intervals: $(-\infty, -3) \Rightarrow \frac{x(x+2)}{(x+3)(x-3)} > 0$

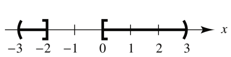
$(-3, -2) \Rightarrow \frac{x(x+2)}{(x+3)(x-3)} < 0$

$(-2, 0) \Rightarrow \frac{x(x+2)}{(x+3)(x-3)} > 0$

$(0, 3) \Rightarrow \frac{x(x+2)}{(x+3)(x-3)} < 0$

$(3, \infty) \Rightarrow \frac{x(x+2)}{(x+3)(x-3)} > 0$

Solution set: $(-3, -2] \cup [0, 3)$



142. $$\frac{x^2 + x - 6}{x} \geq 0$$

$$\frac{(x+3)(x-2)}{x} \geq 0$$

Key numbers: $x = -3, x = 0, x = 2$

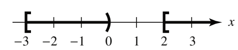
Test intervals: $(-\infty, -3) \Rightarrow \frac{(x+3)(x-2)}{x} < 0$

$(-3, 0) \Rightarrow \frac{(x+3)(x-2)}{x} > 0$

$(0, 2) \Rightarrow \frac{(x+3)(x-2)}{x} < 0$

$(2, \infty) \Rightarrow \frac{(x+3)(x-2)}{x} > 0$

Solution set: $[-3, 0) \cup [2, \infty)$



143. $$\frac{3}{x-1} + \frac{2x}{x+1} > -1$$

$$\frac{3(x+1) + 2x(x-1) + 1(x+1)(x-1)}{(x-1)(x+1)} > 0$$

$$\frac{3x^2 + x + 2}{(x-1)(x+1)} > 0$$

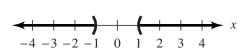
Key numbers: $x = -1, x = 1$

Test intervals: $(-\infty, -1) \Rightarrow \frac{3x^2 + x + 2}{(x-1)(x+1)} > 0$

$(-1, 1) \Rightarrow \frac{3x^2 + x + 2}{(x-1)(x+1)} < 0$

$(1, \infty) \Rightarrow \frac{3x^2 + x + 2}{(x-1)(x+1)} > 0$

Solution set: $(-\infty, -1) \cup (1, \infty)$



144. $$\frac{3x}{x-1} \le \frac{x}{x+4} + 3$$

$$\frac{3x(x+4) - x(x-1) - 3(x+4)(x-1)}{(x-1)(x+4)} \le 0$$

$$\frac{-x^2 + 4x + 12}{(x-1)(x+4)} \le 0$$

$$\frac{-(x-6)(x+2)}{(x-1)(x+4)} \le 0$$

Key numbers: $x = -4, x = -2, x = 1, x = 6$

Test: intervals $(-\infty, -4) \Rightarrow \frac{-(x-6)(x+2)}{(x-1)(x+4)} < 0$

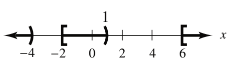
$(-4, -2) \Rightarrow \frac{-(x-6)(x+2)}{(x-1)(x+4)} > 0$

$(-2, 1) \Rightarrow \frac{-(x-6)(x+2)}{(x-1)(x+4)} < 0$

$(1, 6) \Rightarrow \frac{-(x-6)(x+2)}{(x-1)(x+4)} > 0$

$(6, \infty) \Rightarrow \frac{-(x-6)(x+2)}{(x-1)(x+4)} < 0$

Solution set: $(-\infty, -4) \cup [-2, 1) \cup [6, \infty)$



145. $4 - x^2 \ge 0$

$(2 + x)(2 - x) \ge 0$

Key numbers: $x = \pm 2$

Test intervals: $(-\infty, -2) \Rightarrow 4 - x^2 < 0$

$(-2, 2) \Rightarrow 4 - x^2 > 0$

$(2, \infty) \Rightarrow 4 - x^2 < 0$

Domain: $[-2, 2]$

146. $x^2 - 4 \ge 0$

$(x + 2)(x - 2) \ge 0$

Key numbers: $x = -2, x = 2$

Test intervals: $(-\infty, -2) \Rightarrow (x + 2)(x - 2) > 0$

$(-2, 2) \Rightarrow (x + 2)(x - 2) < 0$

$(2, \infty) \Rightarrow (x + 2)(x - 2) > 0$

Domain: $(-\infty, -2] \cup [2, \infty)$

147. $x^2 - 9x + 20 \ge 0$

$(x - 4)(x - 5) \ge 0$

Key numbers: $x = 4, x = 5$

Test intervals: $(-\infty, 4), (4, 5), (5, \infty)$

Interval	x-Value	Value of $(x-4)(x-5)$	Conclusion
$(-\infty, 4)$	0	$(-4)(-5) = 20$	Positive
$(4, 5)$	$\frac{9}{2}$	$\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right) = -\frac{1}{4}$	Negative
$(5, \infty)$	6	$(2)(1) = 2$	Positive

Domain: $(-\infty, 4] \cup [5, \infty)$

148. $81 - 4x^2 \ge 0$

$(9 - 2x)(9 + 2x) \ge 0$

Key numbers: $x = \pm\frac{9}{2}$

Test intervals: $\left(-\infty, -\frac{9}{2}\right), \left(-\frac{9}{2}, \frac{9}{2}\right), \left(\frac{9}{2}, \infty\right)$

Interval	x-Value	Value of $(9-2x)(9+2x)$	Conclusion
$\left(-\infty, -\frac{9}{2}\right)$	−5	$(19)(-1) = -19$	Negative
$\left(-\frac{9}{2}, \frac{9}{2}\right)$	0	$(9)(9) = 81$	Positive
$\left(\frac{9}{2}, \infty\right)$	5	$(-1)(19) = -19$	Negative

Domain: $\left[-\frac{9}{2}, \frac{9}{2}\right]$

149. $$\frac{x}{x^2 - 2x - 35} \ge 0$$

$$\frac{x}{(x+5)(x-7)} \ge 0$$

Key numbers: $x = 0, x = -5, x = 7$

Test intervals: $(-\infty, -5) \Rightarrow \frac{x}{(x+5)(x-7)} < 0$

$(-5, 0) \Rightarrow \frac{x}{(x+5)(x-7)} > 0$

$(0, 7) \Rightarrow \frac{x}{(x+5)(x-7)} < 0$

$(7, \infty) \Rightarrow \frac{x}{(x+5)(x-7)} > 0$

Domain: $(-5, 0] \cup (7, \infty)$

150. $\dfrac{x}{x^2 - 9} \geq 0$

$\dfrac{x}{(x+3)(x-3)} \geq 0$

Key numbers: $x = -3, x = 0, x = 3$

Test intervals: $(-\infty, -3) \Rightarrow \dfrac{x}{(x+3)(x-3)} < 0$

$(-3, 0) \Rightarrow \dfrac{x}{(x+3)(x-3)} > 0$

$(0, 3) \Rightarrow \dfrac{x}{(x+3)(x-3)} < 0$

$(3, \infty) \Rightarrow \dfrac{x}{(x+3)(x-3)} > 0$

Domain: $(-3, 0] \cup (3, \infty)$

151. $0.4x^2 + 5.26 < 10.2$

$0.4x^2 - 4.94 < 0$

$0.4(x^2 - 12.35) < 0$

Key numbers: $x \approx \pm 3.51$

Test intervals: $(-\infty, -3.51), (-3.51, 3.51), (3.51, \infty)$

Solution set: $(-3.51, 3.51)$

152. $-1.3x^2 + 3.78 > 2.12$

$-1.3x^2 + 1.66 > 0$

Key numbers: $x \approx \pm 1.13$

Test intervals: $(-\infty, -1.13), (-1.13, 1.13), (1.13, \infty)$

Solution set: $(-1.13, 1.13)$

153. $-0.5x^2 + 12.5x + 1.6 > 0$

Key numbers: $x \approx -0.13, x \approx 25.13$

Test intervals: $(-\infty, -0.13), (-0.13, 25.13), (25.13, \infty)$

Solution set: $(-0.13, 25.13)$

154. $1.2x^2 + 4.8x + 3.1 < 5.3$

$1.2x^2 + 4.8x - 2.2 < 0$

Key numbers: $x \approx -4.42, x \approx 0.42$

Test intervals: $(-\infty, -4.42), (-4.42, 0.42), (0.42, \infty)$

Solution set: $(-4.42, 0.42)$

155. $\dfrac{1}{2.3x - 5.2} > 3.4$

$\dfrac{1}{2.3x - 5.2} - 3.4 > 0$

$\dfrac{1 - 3.4(2.3x - 5.2)}{2.3x - 5.2} > 0$

$\dfrac{-7.82x + 18.68}{2.3x - 5.2} > 0$

Key numbers: $x \approx 2.39, x \approx 2.26$

Test intervals: $(-\infty, 2.26), (2.26, 2.39), (2.39, \infty)$

Solution set: $(2.26, 2.39)$

156. $\dfrac{2}{3.1x - 3.7} > 5.8$

$\dfrac{2 - 5.8(3.1x - 3.7)}{3.1x - 3.7} > 0$

$\dfrac{23.46 - 17.98x}{3.1x - 3.7} > 0$

Key numbers: $x \approx 1.19, x \approx 1.30$

Test intervals: $(-\infty, 1.19) \Rightarrow \dfrac{23.46 - 17.98x}{3.1x - 3.7} < 0$

$(1.19, 1.30) \Rightarrow \dfrac{23.46 - 17.98x}{3.1x - 3.7} > 0$

$(1.30, \infty) \Rightarrow \dfrac{23.46 - 17.98x}{3.1x - 3.7} < 0$

Solution set: $(1.19, 1.30)$

157. $|x - a| \geq 2$

$x - a \leq -2$ or $x - a \geq 2$

$x \leq a - 2$ or $x \geq a + 2$

Matches graph (b)

158. $|x - b| < 4$

$-4 < x - b < 4$

$b - 4 < x < b + 4$

Matches graph (b)

159. $|ax - b| \leq c \Rightarrow c$ must be greater than or equal to zero.

$-c \leq ax - b \leq c$

$b - c \leq ax \leq b + c$

Let $a = 1$, then $b - c = 0$ and $b + c = 10$.

This is true when $b = c = 5$.

One set of values is: $a = 1, b = 5, c = 5$.

(Note: This solution is not unique. The following are also solutions.

$a = 2, b = c = 10$

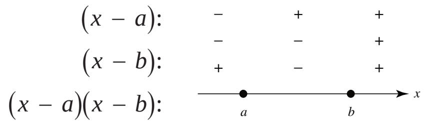
$a = 3, b = c = 15$.)

In general, $a = k, b = c = 5k, k \geq 0$ or

$a = k, b = 5k, c = -5k, k < 0$

160. $(x - a)(x - b)$

(a) The polynomial is zero when $x = a$ or $x = b$.

(b)


(c) A polynomial changes signs at its zeros.

161. $9.00 + 0.75x > 13.50$

$0.75x > 4.50$

$x > 6$

You must produce at least 6 units each hour in order to yield a greater hourly wage at the second job.

162. Let $x =$ gross sales per month

$1000 + 0.04x > 3000$

$0.04x > 2000$

$x > \$50,000$

You must earn at least \$50,000 each month in order to earn a greater monthly wage at the second job.

163. $1000(1 + r(2)) > 1062.50$

$1 + 2r > 1.0625$

$2r > 0.0625$

$r > 0.03125$

$r > 3.125\%$

164. $825 < 750(1 + r(2))$

$825 < 750(1 + 2r)$

$825 < 750 + 1500r$

$75 < 1500r$

$0.05 < r$

The rate must be more than 5%.

165. $E = 1.52t + 68.0$

(a) $70 \le 1.52t + 68.0 \le 80$

$2.0 \le 1.52t \le 12.0$

$1.32 \le t \le 7.89$

The annual egg production was between 70 and 80 billion eggs between 1991 and 1997.

(b) $1.52t + 68.0 > 100$

$1.52t > 32.0$

$t > 21.05$

The annual egg production will exceed 100 billion eggs sometime during 2011.

166. Let $x =$ number of dozens of doughnuts sold per day.

Revenue: $R = 4.50x$

Cost: $C = 2.75x + 220$

Profit: $P = R - C$

$= 4.50x - (2.75x + 220)$

$= 1.75x - 220$

$60 \le 1.75x - 220 \le 270$

$280 \le 1.75x \le 490$

$160 \le x \le 280$

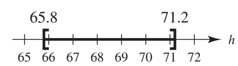
The daily sales vary between 160 and 280 dozen doughnuts per day.

167. $\left|\dfrac{h - 68.5}{2.7}\right| \le 1$

$-1 \le \dfrac{h - 68.5}{2.7} \le 1$

$-2.7 \le h - 68.5 \le 2.7$

65.8 inches $\le h \le$ 71.2 inches



168. $|h - 50| \le 30$

$-30 \le h - 50 \le 30$

$20 \le h \le 80$

The minimum relative humidity is 20 and the maximum is 80.

169. $2L + 2W = 100 \Rightarrow W = 50 - L$

$LW \ge 500$

$L(50 - L) \ge 500$

$-L^2 + 50L - 500 \ge 0$

By the Quadratic Formula you have:

Key numbers: $L = 25 \pm 5\sqrt{5}$

Test: Is $-L^2 + 50L - 500 \ge 0$?

Solution set: $25 - 5\sqrt{5} \le L \le 25 + 5\sqrt{5}$

13.8 meters $\le L \le$ 36.2 meters

170. $2L + 2W = 440 \Rightarrow W = 220 - L$

$LW \ge 8000$

$L(220 - L) \ge 8000$

$-L^2 + 220L - 8000 \ge 0$

By the Quadratic Formula we have:

Key numbers: $L = 110 \pm 10\sqrt{41}$

Test: Is $-L^2 + 220L - 8000 \ge 0$?

Solution set: $110 - 10\sqrt{41} \le L \le 110 + 10\sqrt{41}$

45.97 feet $\le L \le$ 174.03 feet

171. $1000(1 + r)^2 > 1100$

$(1 + r)^2 > 1.1$

$1 + 2r + r^2 - 1.1 > 0$

$r^2 + 2r - 0.1 > 0$

By the Quadratic Formula we have:

Critical Numbers: $r = -1 \pm \sqrt{1.1}$

Since r cannot be negative, $r = -1 \pm \sqrt{1.1} \approx 0.0488$

$= 4.88\%.$

Thus, $r > 4.88\%$.

172. $R = x(50 - 0.0002x)$ and $C = 12x + 150{,}000$

$P = R - C$

$= (50x - 0.0002x^2) - (12x + 150{,}000)$

$= -0.0002x^2 + 38x - 150{,}000$

$P \geq 1{,}650{,}000$

$-0.0002x^2 + 38x - 150{,}000 \geq 1{,}650{,}000$

$-0.0002x^2 + 38x - 1{,}800{,}000 \geq 0$

Key numbers: $x = 90{,}000$ and $x = 100{,}000$

Test intervals:
$(0, 90{,}000), (90{,}000, 100{,}000), (100{,}000, \infty)$

The solution set is $[90{,}000, 100{,}000]$ or $90{,}000 \leq x \leq 100{,}000$. The price per unit is

$p = \frac{R}{x} = 50 - 0.0002x.$

For $x = 90{,}000$, $p = \$32$. For $x = 100{,}000$, $p = \$30$. So, for $90{,}000 \leq x \leq 100{,}000$, $\$30 \leq p \leq \$32$.

173. $s = -16t^2 + v_0t + s_0 = -16t^2 + 160t$

(a) $-16t^2 + 160t = 0$

$-16t(t - 10) = 0$

$t = 0, t = 10$

It will be back on the ground in 10 seconds.

(b) $-16t^2 + 160t > 384$

$-16t^2 + 160t - 384 > 0$

$-16(t^2 - 10t + 24) > 0$

$t^2 - 10t + 24 < 0$

$(t - 4)(t - 6) < 0$

Key numbers: $t = 4, t = 6$

Test intervals: $(-\infty, 4), (4, 6), (6, \infty)$

Solution set: 4 seconds $< t <$ 6 seconds

174. $s = -16t^2 + v_0t + s_0 = -16t^2 + 128t$

(a) $-16t^2 + 128t = 0$

$-16t(t - 8) = 0$

$-16t = 0 \Rightarrow t = 0$

$t - 8 = 0 \Rightarrow t = 8$

It will be back on the ground in 8 seconds.

(b) $-16t^2 + 128t < 128$

$-16t^2 + 128t - 128 < 0$

$-16(t^2 - 8t + 8) < 0$

$t^2 - 8t + 8 > 0$

Key numbers: $t = 4 - 2\sqrt{2}, t = 4 + 2\sqrt{2}$

Test intervals:
$(-\infty, 4 - 2\sqrt{2}), (4 - 2\sqrt{2}, 4 + 2\sqrt{2}),$
$(4 + 2\sqrt{2}, \infty)$

Solution set: 0 seconds $\leq t < 4 - 2\sqrt{2}$ seconds and $4 + 2\sqrt{2}$ seconds $< t \leq 8$ seconds

175. $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{2}$

$2R_1 = 2R + RR_1$

$2R_1 = R(2 + R_1)$

$\frac{2R_1}{2 + R_1} = R$

Because $R \geq 1$,

$\frac{2R_1}{2 + R_1} \geq 1$

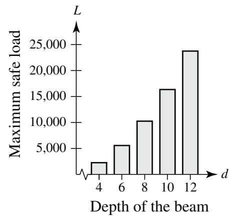
$\frac{2R_1}{2 + R_1} - 1 \geq 0$

$\frac{R_1 - 2}{2 + R_1} \geq 0.$

Because $R_1 > 0$, the only key number is $R_1 = 2$. The inequality is satisfied when $R_1 \geq 2$ ohms.

176. (a)

d	4	6	8	10	12
Load	2223.9	5593.9	10,312	16,378	23,792



(b)

$$2000 \le 168.5d^2 - 472.1$$
$$2472.1 \le 168.5d^2$$
$$14.67 \le d^2$$
$$3.83 \le d$$

The minimum depth is 3.83 inches.

177. False. If c is negative, then $ac \ge bc$.

178. False. If $-10 \le x \le 8$, then $10 \ge -x$ and $-x \ge -8$.

179. True

The y-values are greater than zero for all values of x.

180. When each side of an inequality is multiplied or divided by a negative number the direction of the inequality symbol must be reversed.

Section P.3 Graphical Representation of Data

1. (a) **v** horizontal real number line

(b) **vi** vertical real number line

(c) **i** point of intersection of vertical axis and horizontal axis

(d) **iv** four regions of the coordinate plane

(e) **iii** directed distance from the y-axis

(f) **ii** directed distance from the x-axis

2. Cartesian

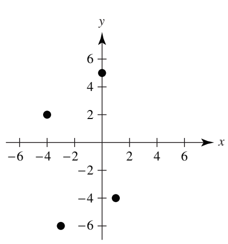
3. Distance Formula

4. Midpoint Formula

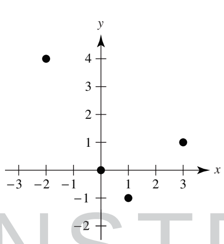
5. $A: (2, 6)$, $B: (-6, -2)$, $C: (4, -4)$, $D: (-3, 2)$

6. $A: \left(\frac{3}{2}, -4\right)$; $B: (0, -2)$; $C: \left(-3, \frac{5}{2}\right)$, $D: (-6, 0)$

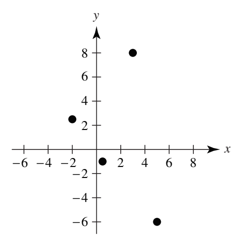
7.



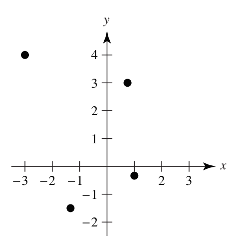
8.



9.



10.



11. $(-3, 4)$

12. $(4, -8)$

13. $(-5, -5)$

14. $(-12, 0)$

15. $x > 0$ and $y < 0$ in Quadrant IV.

16. $x < 0$ and $y < 0$ in Quadrant III.

17. $x = -4$ and $y > 0$ in Quadrant II.

18. $x > 2$ and $y = 3$ in Quadrant I.

19. $y < -5$ in Quadrant III or IV.

20. $x > 4$ in Quadrant I or IV.

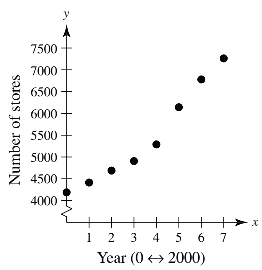
21. $(x, -y)$ is in the second Quadrant means that (x, y) is in Quadrant III.

22. If $(-x, y)$ is in Quadrant IV, then (x, y) must be in Quadrant III.

23. $(x, y), xy > 0$ means x and y have the same signs. This occurs in Quadrant I or III.

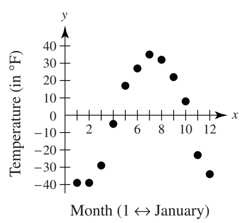
24. If $xy < 0$, then x and y have opposite signs. This happens in Quadrant II or IV.

25.



26.

Month, x	Temperature, y
1	−39
2	−39
3	−29
4	−5
5	17
6	27
7	35
8	32
9	22
10	8
11	−23
12	−34



27. $d = |5 - (-3)| = 8$

28. $d = |1 - 8| = |-7| = 7$

29. $d = |2 - (-3)| = 5$

30. $d = |-4 - 6| = |-10| = 10$

31. $$\begin{aligned} d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(3 - (-2))^2 + (-6 - 6)^2} \\ &= \sqrt{(5)^2 + (-12)^2} \\ &= \sqrt{25 + 144} \\ &= 13 \text{ units} \end{aligned}$$

32. $$\begin{aligned} d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(0 - 8)^2 + (20 - 5)^2} \\ &= \sqrt{(-8)^2 + (15)^2} \\ &= \sqrt{64 + 225} \\ &= \sqrt{289} \\ &= 17 \text{ units} \end{aligned}$$

33. $$\begin{aligned} d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(-5 - 1)^2 + (-1 - 4)^2} \\ &= \sqrt{(-6)^2 + (-5)^2} \\ &= \sqrt{36 + 25} \\ &= \sqrt{61} \text{ units} \end{aligned}$$

34. $$\begin{aligned} d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(3 - 1)^2 + (-2 - 3)^2} \\ &= \sqrt{(2)^2 + (-5)^2} \\ &= \sqrt{4 + 25} \\ &= \sqrt{29} \text{ units} \end{aligned}$$

35. $$\begin{aligned} d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{\left(2 - \frac{1}{2}\right)^2 + \left(-1 - \frac{4}{3}\right)^2} \\ &= \sqrt{\left(\frac{3}{2}\right)^2 + \left(-\frac{7}{3}\right)^2} \\ &= \sqrt{\frac{9}{4} + \frac{49}{9}} \\ &= \sqrt{\frac{277}{36}} \\ &= \frac{\sqrt{277}}{6} \text{ units} \end{aligned}$$

36. $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{\left(-1 - \left(-\frac{2}{3}\right)\right)^2 + \left(\frac{5}{4} - 3\right)^2}$

$= \sqrt{\left(-\frac{1}{3}\right)^2 + \left(-\frac{7}{4}\right)^2}$

$= \sqrt{\frac{1}{9} + \frac{49}{16}}$

$= \sqrt{\frac{457}{144}}$

$= \frac{\sqrt{457}}{12}$ units

37. $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(-12.5 - (-4.2))^2 + (4.8 - 3.1)^2}$

$= \sqrt{(-8.3)^2 + (1.7)^2}$

$= \sqrt{68.89 + 2.89}$

$= \sqrt{71.78}$

≈ 8.47 units

38. $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(-3.9 - 9.5)^2 + (8.2 - (-2.6))^2}$

$= \sqrt{(-13.4)^2 + (10.8)^2}$

$= \sqrt{179.56 + 116.64}$

$= \sqrt{296.2}$

≈ 17.21 units

39. (a) The distance between $(0, 2)$ and $(4, 2)$ is 4.

The distance between $(4, 2)$ and $(4, 5)$ is 3.

The distance between $(0, 2)$ and $(4, 5)$ is

$\sqrt{(4 - 0)^2 + (5 - 2)^2} = \sqrt{16 + 9} = \sqrt{25} = 5.$

(b) $4^2 + 3^2 = 16 + 9 = 25 = 5^2$

40. (a) $(1, 0), (13, 5)$

Distance $= \sqrt{(13 - 1)^2 + (5 - 0)^2}$

$= \sqrt{12^2 + 5^2} = \sqrt{169} = 13$

$(13, 5), (13, 0)$

Distance $= |5 - 0| = |5| = 5$

$(1, 0), (13, 0)$

Distance $= |1 - 13| = |-12| = 12$

(b) $5^2 + 12^2 = 25 + 144 = 169 = 13^2$

41. (a) The distance between $(-1, 1)$ and $(9, 1)$ is 10.

The distance between $(9, 1)$ and $(9, 4)$ is 3.

The distance between $(-1, 1)$ and $(9, 4)$ is

$\sqrt{(9 - (-1))^2 + (4 - 1)^2} = \sqrt{100 + 9} = \sqrt{109}.$

(b) $10^2 + 3^2 = 109 = \left(\sqrt{109}\right)^2$

42. (a) $(1, 5), (5, -2)$

Distance $= \sqrt{(1 - 5)^2 + (5 - (-2))^2}$

$= \sqrt{(-4)^2 + (7)^2} = \sqrt{16 + 49} = \sqrt{65}$

$(1, 5), (1, -2)$

Distance $= |5 - (-2)| = |5 + 2| = |7| = 7$

$(1, -2), (5, -2)$

Distance $= |1 - 5| = |-4| = 4$

(b) $4^2 + 7^2 = 16 + 49 = 65 = \left(\sqrt{65}\right)^2$

43. $d_1 = \sqrt{(4 - 2)^2 + (0 - 1)^2} = \sqrt{4 + 1} = \sqrt{5}$

$d_2 = \sqrt{(4 + 1)^2 + (0 + 5)^2} = \sqrt{25 + 25} = \sqrt{50}$

$d_3 = \sqrt{(2 + 1)^2 + (1 + 5)^2} = \sqrt{9 + 36} = \sqrt{45}$

$\left(\sqrt{5}\right)^2 + \left(\sqrt{45}\right)^2 = \left(\sqrt{50}\right)^2$

44. $d_1 = \sqrt{(3 - (-1))^2 + (5 - 3)^2} = \sqrt{16 + 4} = \sqrt{20}$

$d_2 = \sqrt{(5 - 3)^2 + (1 - 5)^2} = \sqrt{4 + 16} = \sqrt{20}$

$d_3 = \sqrt{(5 - (-1))^2 + (1 - 3)^2} = \sqrt{36 + 4} = \sqrt{40}$

$\left(\sqrt{20}\right)^2 + \left(\sqrt{20}\right)^2 = \left(\sqrt{40}\right)^2$

45. $d_1 = \sqrt{(1 - 3)^2 + (-3 - 2)^2} = \sqrt{4 + 25} = \sqrt{29}$

$d_2 = \sqrt{(3 + 2)^2 + (2 - 4)^2} = \sqrt{25 + 4} = \sqrt{29}$

$d_3 = \sqrt{(1 + 2)^2 + (-3 - 4)^2} = \sqrt{9 + 49} = \sqrt{58}$

$d_1 = d_2$

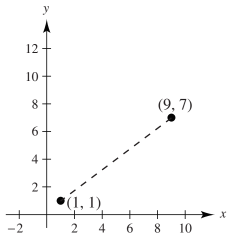
46. $d_1 = \sqrt{(4 - 2)^2 + (9 - 3)^2} = \sqrt{4 + 36} = \sqrt{40}$

$d_2 = \sqrt{(-2 - 4)^2 + (7 - 9)^2} = \sqrt{36 + 4} = \sqrt{40}$

$d_3 = \sqrt{(2 - (-2))^2 + (3 - 7)^2} = \sqrt{16 + 16} = \sqrt{32}$

$d_1 = d_2$

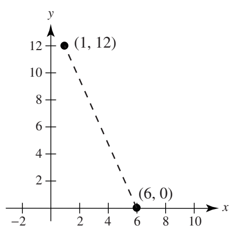
47. (a)



(b) $d = \sqrt{(9-1)^2 + (7-1)^2} = \sqrt{64+36} = 10$

(c) $\left(\dfrac{9+1}{2}, \dfrac{7+1}{2}\right) = (5, 4)$

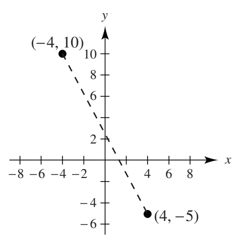
48. (a)



(b) $d = \sqrt{(1-6)^2 + (12-0)^2} = \sqrt{25+144} = 13$

(c) $\left(\dfrac{1+6}{2}, \dfrac{12+0}{2}\right) = \left(\dfrac{7}{2}, 6\right)$

49. (a)

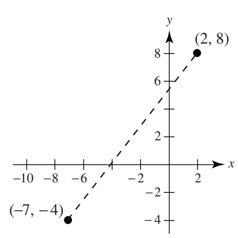


(b) $d = \sqrt{(4+4)^2 + (-5-10)^2}$

$= \sqrt{64+225} = 17$

(c) $\left(\dfrac{4-4}{2}, \dfrac{-5+10}{2}\right) = \left(0, \dfrac{5}{2}\right)$

50. (a)

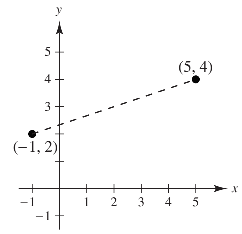


(b) $d = \sqrt{(-7-2)^2 + (-4-8)^2}$

$= \sqrt{81+144} = 15$

(c) $\left(\dfrac{-7+2}{2}, \dfrac{-4+8}{2}\right) = \left(-\dfrac{5}{2}, 2\right)$

51. (a)

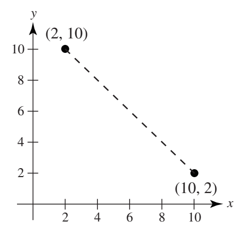


(b) $d = \sqrt{(5+1)^2 + (4-2)^2}$

$= \sqrt{36+4} = 2\sqrt{10}$

(c) $\left(\dfrac{-1+5}{2}, \dfrac{2+4}{2}\right) = (2, 3)$

52. (a)

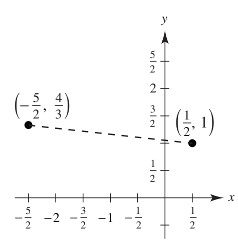


(b) $d = \sqrt{(2-10)^2 + (10-2)^2}$

$= \sqrt{64+64} = 8\sqrt{2}$

(c) $\left(\dfrac{2+10}{2}, \dfrac{10+2}{2}\right) = (6, 6)$

53. (a)

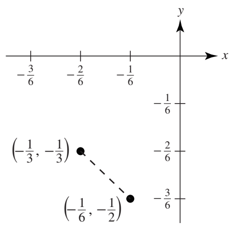


(b) $d = \sqrt{\left(\dfrac{1}{2} + \dfrac{5}{2}\right)^2 + \left(1 - \dfrac{4}{3}\right)^2}$

$= \sqrt{9 + \dfrac{1}{9}} = \dfrac{\sqrt{82}}{3}$

(c) $\left(\dfrac{-(5/2) + (1/2)}{2}, \dfrac{(4/3)+1}{2}\right) = \left(-1, \dfrac{7}{6}\right)$

54. (a)

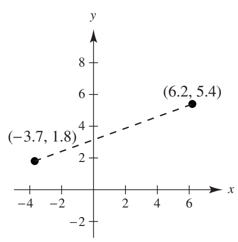


(b) $d = \sqrt{\left(-\frac{1}{3} + \frac{1}{6}\right)^2 + \left(-\frac{1}{3} + \frac{1}{2}\right)^2}$

$= \sqrt{\frac{1}{36} + \frac{1}{36}} = \frac{\sqrt{2}}{6}$

(c) $\left(\frac{-\frac{1}{3} + \left(-\frac{1}{6}\right)}{2}, \frac{-\frac{1}{3} + \left(-\frac{1}{2}\right)}{2}\right) = \left(-\frac{1}{4}, -\frac{5}{12}\right)$

55. (a)

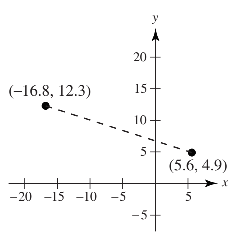


(b) $d = \sqrt{(6.2 + 3.7)^2 + (5.4 - 1.8)^2}$

$= \sqrt{98.01 + 12.96} = \sqrt{110.97}$

(c) $\left(\frac{6.2 - 3.7}{2}, \frac{5.4 + 1.8}{2}\right) = (1.25, 3.6)$

56. (a)



(b) $d = \sqrt{(-16.8 - 5.6)^2 + (12.3 - 4.9)^2}$

$= \sqrt{501.76 + 54.76} = \sqrt{556.52}$

(c) $\left(\frac{-16.8 + 5.6}{2}, \frac{12.3 + 4.9}{2}\right) = (-5.6, 8.6)$

57. $d = \sqrt{120^2 + 150^2}$

$= \sqrt{36{,}900}$

$= 30\sqrt{41}$

≈ 192.09

The plane flies about 192 kilometers.

58. $d = \sqrt{(42 - 18)^2 + (50 - 12)^2}$

$= \sqrt{24^2 + 38^2}$

$= \sqrt{2020}$

$= 2\sqrt{505}$

≈ 45

The pass is about 45 yards.

59. $\text{midpoint} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

$= \left(\frac{2003 + 2007}{2}, \frac{4174 + 4656}{2}\right)$

$= (2005, 4415)$

In 2005, the sales for Big Lots were about $4415 million.

60. $\text{midpoint} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

$= \left(\frac{2003 + 2007}{2}, \frac{2800 + 4243}{2}\right)$

$= (2005, 3521.50)$

In 2005, the sales for the Dollar Tree were about $3521.50 million.

61. $(-2 + 2, -4 + 5) = (0, 1)$

$(2 + 2, -3 + 5) = (4, 2)$

$(-1 + 2, -1 + 5) = (1, 4)$

62. $(-3 + 6, 6 - 3) = (3, 3)$

$(-5 + 6, 3 - 3) = (1, 0)$

$(-3 + 6, 0 - 3) = (3, -3)$

$(-1 + 6, 3 - 3) = (5, 0)$

63. $(-7 + 4, -2 + 8) = (-3, 6)$

$(-2 + 4, 2 + 8) = (2, 10)$

$(-2 + 4, -4 + 8) = (2, 4)$

$(-7 + 4, -4 + 8) = (-3, 4)$

64. $(5 - 10, 8 - 6) = (-5, 2)$

$(3 - 10, 6 - 6) = (-7, 0)$

$(7 - 10, 6 - 6) = (-3, 0)$

$(5 - 10, 2 - 6) = (-5, -4)$

65. To reflect the vertices in the *y*-axis, negate each *x*-coordinate.

Original Point	*Reflected Point*
$(1, 5)$	$(-1, 5)$
$(5, 4)$	$(-5, 4)$
$(2, 2)$	$(-2, 2)$

66. Negate each *x*-coordinate.

Original Point	*Reflected Point*
$(-4, 5)$	$(4, 5)$
$(-2, 3)$	$(2, 3)$
$(-5, 1)$	$(5, 1)$

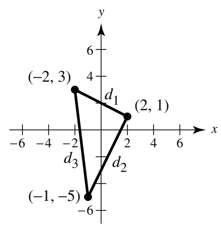
67. Negate each *x*-coordinate.

Original Point	*Reflected Point*
$(0, 3)$	$(0, 3)$
$(3, -2)$	$(-3, -2)$
$(6, 3)$	$(-6, 3)$
$(3, 8)$	$(-3, 8)$

68. Negate each *x*-coordinate.

Original Point	*Reflected Point*
$(-7, 1)$	$(7, 1)$
$(-5, 4)$	$(5, 4)$
$(-1, 4)$	$(1, 4)$
$(-3, 1)$	$(3, 1)$

69.



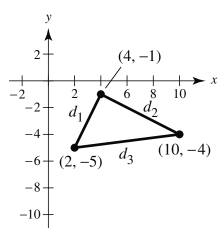
$$d_1 = \sqrt{\left(2 - (-2)\right)^2 + (1 - 3)^2} = \sqrt{(4)^2 + (-2)^2} = \sqrt{20} = 2\sqrt{5}$$

$$d_2 = \sqrt{(-1 - 2)^2 + (-5 - 1)^2} = \sqrt{(-3)^2 + (-6)^2} = \sqrt{45} = 3\sqrt{5}$$

$$d_3 = \sqrt{\left(-2 - (-1)\right)^2 + \left(3 - (-5)\right)^2} = \sqrt{(-1)^2 + (8)^2} = \sqrt{65}$$

Since $d_1^2 + d_2^2 = 20 + 45 = 65 = d_3^2$, the triangle is a right triangle.

70.



$$d_1 = \sqrt{(2 - 4)^2 + \left(-5 - (-1)\right)^2} = \sqrt{(-2)^2 + (-4)^2} = \sqrt{20} = 2\sqrt{5}$$

$$d_2 = \sqrt{(10 - 4)^2 + \left(-4 - (-1)\right)^2} = \sqrt{(6)^2 + (-3)^2} = \sqrt{45} = 3\sqrt{5}$$

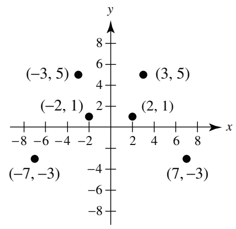
$$d_3 = \sqrt{(10 - 2)^2 + \left(-4 - (-5)\right)^2} = \sqrt{(8)^2 + (1)^2} = 65$$

Since $d_1^2 + d_2^2 = 20 + 45 = 65 = d_3^2$, the triangle is a right triangle.

71. On the *x*-axis, $y = 0$

On the *y*-axis, $x = 0$

72.



(a) The point is reflected through the *y*-axis.

(b) The point is reflected through the *x*-axis.

(c) The point is reflected through the origin.

73. The highest price of milk is approximately \$3.87. This occurred in 2007.

74. Price of milk in 1996 $\approx$ \$2.73

Highest price of milk = \$3.87 in 2007

Percent change $= \dfrac{3.87 - 2.73}{2.73} \approx 41.8\%$

75. (a) Cost during Super Bowl XXXVIII (2004) $\approx$ \$2,302,000

Cost during Super Bowl XXXIV (2000) $\approx$ \$2,100,000

Increase = \$2,302,000 − \$2,100,000 = \$202,000

Percent increase $= \dfrac{\$202{,}000}{\$2{,}100{,}000} \approx 0.096$ or 9.6%

(b) Cost during Super Bowl XLII (2008) = \$2,700,000

Cost during Super Bowl XXXIV (2000) = \$2,100,000

Increase = \$2,700,000 − \$2,100,000 = \$600,000

Percent increase $= \dfrac{\$600{,}000}{\$2{,}100{,}000} \approx 0.286$ or 28.6%

76. (a) Cost during 2002 awards = \$1,290,000

Cost during 1996 awards = \$795,000

Increase = \$1,290,000 − \$795,000 = \$495,000

Percent increase $= \dfrac{\$495{,}000}{\$795{,}000} \approx 0.623$ or 62.3%

(b) Cost during 2007 awards = \$1,700,000

Cost during 1996 awards = \$795,000

Increase = \$1,700,000 − \$795,000 = \$905,000

Percent increase $= \dfrac{\$905{,}000}{\$795{,}000} \approx 1.138$ or 113.8%

77. The number of performers elected each year seems to be nearly steady except for the middle years. Five performers will be elected in 2010.

78. (a) The minimum wage had the greatest increase in the 2000s.

(b) Minimum wage in 1990: \$3.80

Minimum wage in 1995: \$4.25

Percent increase: $\left(\dfrac{\$4.25 - \$3.80}{\$3.80}\right)(100) \approx 11.8\%$

Minimum wage in 1995: \$4.25

Minimum wage in 2009: \$7.25

Percent increase: $\left(\dfrac{\$7.25 - \$4.25}{\$4.25}\right)(100) \approx 70.6\%$

(c) $\$7.25 + 0.706(\$7.25) \approx \$12.37$

The minimum wage will be approximately \$12.37 in the year 2013.

(d) Answers will vary. *Sample answer:* No, the prediction is too high because it is likely that the percent increase over a 4-year period (2009–2013) will be less than the percent increase over a 14-year period (1995–2009).

79. midpoint $= \left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)$

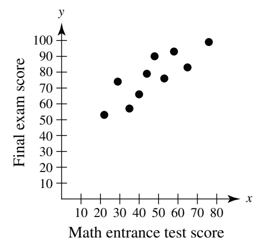
$$= \left(\frac{1999 + 2007}{2}, \frac{19{,}805 + 28{,}857}{2}\right)$$

$$= (2003, 24{,}331)$$

In 2003, the sales for the Coca-Cola Company were about $24,331 million.

80. (a)

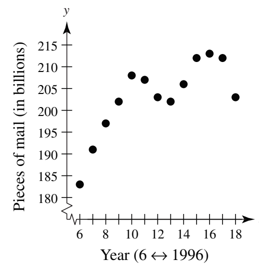
x	y
22	53
29	74
35	57
40	66
44	79
48	90
53	76
58	93
65	83
76	99



(b) The point $(65, 83)$ represents an entrance exam score of 65.

(c) No. There are many variables that will affect the final exam score.

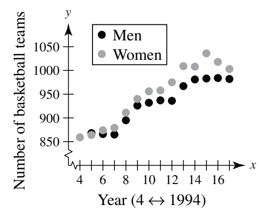
81. (a)



(b) The greatest decrease occurred in 2008.

(c) Answers will vary. *Sample answer:* Technology now enables us to transport information in ways other than by mail. The Internet is one example.

82. (a)



(b) In 1994, the number of men's and women's teams were nearly equal.

(c) In 2005, the difference between the number of men's and women's teams was the greatest: $1036 - 983 = 53$ teams.

83. Because $x_m = \dfrac{x_1 + x_2}{2}$ and $y_m = \dfrac{y_1 + y_2}{2}$ we have:

$$2x_m = x_1 + x_2 \qquad 2y_m = y_1 + y_2$$

$$2x_m - x_1 = x_2 \qquad 2y_m - y_1 = y_2$$

So, $(x_2, y_2) = (2x_m - x_1, 2y_m - y_1)$.

84. (a) $(x_2, y_2) = (2x_m - x_1, 2y_m - y_1) = (2 \cdot 4 - 1, 2(-1) - (-2)) = (7, 0)$

(b) $(x_2, y_2) = (2x_m - x_1, 2y_m - y_1) = (2 \cdot 2 - (-5), 2 \cdot 4 - 11) = (9, -3)$

85. The midpoint of the given line segment is $\left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)$.

The midpoint between (x_1, y_1) and $\left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)$ is $\left(\dfrac{x_1 + \frac{x_1 + x_2}{2}}{2}, \dfrac{y_1 + \frac{y_1 + y_2}{2}}{2}\right) = \left(\dfrac{3x_1 + x_2}{4}, \dfrac{3y_1 + y_2}{4}\right)$.

The midpoint between $\left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)$ and (x_2, y_2) is $\left(\dfrac{\frac{x_1 + x_2}{2} + x_2}{2}, \dfrac{\frac{y_1 + y_2}{2} + y_2}{2}\right) = \left(\dfrac{x_1 + 3x_2}{4}, \dfrac{y_1 + 3y_2}{4}\right)$.

So, the three points are

$$\left(\frac{3x_1 + x_2}{4}, \frac{3y_1 + y_2}{4}\right), \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right), \text{ and } \left(\frac{x_1 + 3x_2}{4}, \frac{y_1 + 3y_2}{4}\right).$$

86. (a) $\left(\frac{3x_1 + x_2}{4}, \frac{3y_1 + y_2}{4}\right) = \left(\frac{3 \cdot 1 + 4}{4}, \frac{3(-2) - 1}{4}\right) = \left(\frac{7}{4}, -\frac{7}{4}\right)$

$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{1 + 4}{2}, \frac{-2 - 1}{2}\right) = \left(\frac{5}{2}, -\frac{3}{2}\right)$

$\left(\frac{x_1 + 3x_2}{4}, \frac{y_1 + 3y_2}{4}\right) = \left(\frac{1 + 3 \cdot 4}{4}, \frac{-2 + 3(-1)}{4}\right) = \left(\frac{13}{4}, -\frac{5}{4}\right)$

(b) $\left(\frac{3x_1 + x_2}{4}, \frac{3y_1 + y_2}{4}\right) = \left(\frac{3(-2) + 0}{4}, \frac{3(-3) + 0}{4}\right) = \left(-\frac{3}{2}, -\frac{9}{4}\right)$

$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{-2 + 0}{2}, \frac{-3 + 0}{2}\right) = \left(-1, -\frac{3}{2}\right)$

$\left(\frac{x_1 + 3x_2}{4}, \frac{y_1 + 3y_2}{4}\right) = \left(\frac{-2 + 0}{4}, \frac{-3 + 0}{4}\right) = \left(-\frac{1}{2}, -\frac{3}{4}\right)$

87. No. It depends on the magnitude of the quantities measured.

88. (a) **First Set**

$d(A, B) = \sqrt{(2 - 2)^2 + (3 - 6)^2} = \sqrt{9} = 3$

$d(B, C) = \sqrt{(2 - 6)^2 + (6 - 3)^2} = \sqrt{16 + 9} = 5$

$d(A, C) = \sqrt{(2 - 6)^2 + (3 - 3)^2} = \sqrt{16} = 4$

Because $3^2 + 4^2 = 5^2$, A, B, and C are the vertices of a right triangle.

Second Set

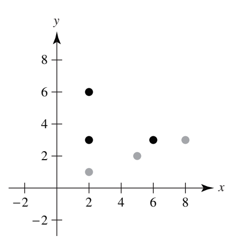
$d(A, B) = \sqrt{(8 - 5)^2 + (3 - 2)^2} = \sqrt{10}$

$d(B, C) = \sqrt{(5 - 2)^2 + (2 - 1)^2} = \sqrt{10}$

$d(A, C) = \sqrt{(8 - 2)^2 + (3 - 1)^2} = \sqrt{40}$

A, B, and C are the vertices of an isosceles triangle or are collinear: $\sqrt{10} + \sqrt{10} = 2\sqrt{10} = \sqrt{40}$.

(b)



First set: Not collinear
Second set: The points are collinear.

(c) If A, B, and C are collinear, then two of the distances will add up to the third distance.

89. False, you would have to use the Midpoint Formula 15 times.

90. True. Two sides of the triangle have lengths $\sqrt{149}$ and the third side has a length of $\sqrt{18}$.

91. Use the Midpoint Formula to prove the diagonals of the parallelogram bisect each other.

$\left(\frac{b + a}{2}, \frac{c + 0}{2}\right) = \left(\frac{a + b}{2}, \frac{c}{2}\right)$

$\left(\frac{a + b + 0}{2}, \frac{c + 0}{2}\right) = \left(\frac{a + b}{2}, \frac{c}{2}\right)$

92. (a) Because (x_0, y_0) lies in Quadrant II, $(x_0, -y_0)$ must lie in Quadrant III. Matches (ii).

(b) Because (x_0, y_0) lies in Quadrant II, $(-2x_0, y_0)$ must lie in Quadrant I. Matches (iii).

(c) Because (x_0, y_0) lies in Quadrant II, $\left(x_0, \frac{1}{2}y_0\right)$ must lie in Quadrant II. Matches (iv).

(d) Because (x_0, y_0) lies in Quadrant II, $(-x_0, -y_0)$ must lie in Quadrant IV. Matches (i).

Section P.4 Graphs of Equations

1. solution or solution point

2. graph

3. intercepts

4. y-axis

5. circle; (h, k); r

6. numerical

7. $y = \sqrt{x + 4}$

(a) $(0, 2)$: $2 \stackrel{?}{=} \sqrt{0 + 4}$

$2 = 2$

Yes, the point *is* on the graph.

(b) $(5, 3)$: $3 \stackrel{?}{=} \sqrt{5 + 4}$

$3 \stackrel{?}{=} \sqrt{9}$

$3 = 3$

Yes, the point *is* on the graph.

8. (a) $(1, 2)$: $2 \stackrel{?}{=} \sqrt{5 - 1}$

$2 \stackrel{?}{=} \sqrt{4}$

$2 = 2$

Yes, the point *is* on the graph.

(b) $(5, 0)$: $0 \stackrel{?}{=} \sqrt{5 - 5}$

$0 = 0$

Yes, the point *is* on the graph.

9. $y = x^2 - 3x + 2$

(a) $(2, 0)$: $(2)^2 - 3(2) + 2 \stackrel{?}{=} 0$

$4 - 6 + 2 \stackrel{?}{=} 0$

$0 = 0$

Yes, the point *is* on the graph.

(b) $(-2, 8)$: $(-2)^2 - 3(-2) + 2 \stackrel{?}{=} 8$

$4 + 6 + 2 \stackrel{?}{=} 8$

$12 \neq 8$

No, the point *is not* on the graph.

10. $y = 4 - |x - 2|$

(a) $(1, 5)$: $5 \stackrel{?}{=} 4 - |1 - 2|$

$5 \stackrel{?}{=} 4 - 1$

$5 \neq 3$

No, the point *is not* on the graph.

(b) $(6, 0)$: $0 \stackrel{?}{=} 4 - |6 - 2|$

$0 \stackrel{?}{=} 4 - 4$

$0 = 0$

Yes, the point *is* on the graph.

11. (a) $(2, 3)$: $3 \stackrel{?}{=} |2 - 1| + 2$

$3 \stackrel{?}{=} 1 + 2$

$3 = 3$

Yes, the point *is* on the graph.

(b) $(-1, 0)$: $0 \stackrel{?}{=} |-1 - 1| + 2$

$0 \stackrel{?}{=} 2 + 2$

$0 \neq 4$

No, the point *is not* on the graph.

12. (a) $(1, 2)$: $2(1) - 2 - 3 \stackrel{?}{=} 0$

$-3 \neq 0$

No, the point *is not* on the graph.

(b) $(1, -1)$: $2(1) - (-1) - 3 \stackrel{?}{=} 0$

$2 + 1 - 3 \stackrel{?}{=} 0$

$0 = 0$

Yes, the point *is* on the graph.

13. (a) $(3, -2)$: $(3)^2 + (-2)^2 \stackrel{?}{=} 20$

$9 + 4 \stackrel{?}{=} 20$

$13 \neq 20$

No, the point *is not* on the graph.

(b) $(-4, 2)$: $(-4)^2 + (2)^2 \stackrel{?}{=} 20$

$16 + 4 \stackrel{?}{=} 20$

$20 = 20$

Yes, the point *is* on the graph.

14. $y = \frac{1}{3}x^3 - 2x^2$

(a) $\left(2, -\frac{16}{3}\right)$: $\frac{1}{3}(2)^3 - 2(2)^2 \stackrel{?}{=} -\frac{16}{3}$

$\frac{1}{3} \cdot 8 - 2 \cdot 4 \stackrel{?}{=} -\frac{16}{3}$

$\frac{8}{3} - 8 \stackrel{?}{=} -\frac{16}{3}$

$\frac{8}{3} - \frac{24}{3} \stackrel{?}{=} -\frac{16}{3}$

$-\frac{16}{3} = -\frac{16}{3}$

Yes, the point *is* on the graph.

(b) $(-3, 9)$: $\frac{1}{3}(-3)^3 - 2(-3)^2 \stackrel{?}{=} 9$

$\frac{1}{3}(-27) - 2(9) \stackrel{?}{=} 9$

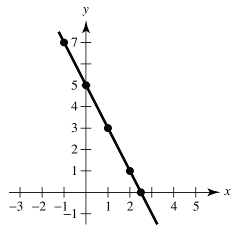
$-9 - 18 \stackrel{?}{=} 9$

$-27 \neq 9$

No, the point *is not* on the graph.

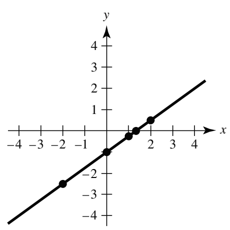
15. $y = -2x + 5$

x	-1	0	1	2	$\frac{5}{2}$
y	7	5	3	1	0
(x, y)	$(-1, 7)$	$(0, 5)$	$(1, 3)$	$(2, 1)$	$\left(\frac{5}{2}, 0\right)$



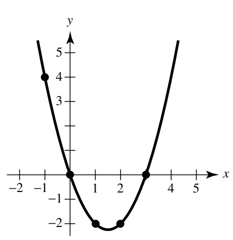
16. $y = \frac{3}{4}x - 1$

x	-2	0	1	$\frac{4}{3}$	2
y	$-\frac{5}{2}$	-1	$-\frac{1}{4}$	0	$\frac{1}{2}$
x, y	$\left(-2, -\frac{5}{2}\right)$	$(0, -1)$	$\left(1, -\frac{1}{4}\right)$	$\left(\frac{4}{3}, 0\right)$	$\left(2, \frac{1}{2}\right)$



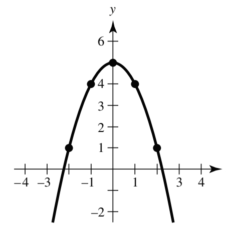
17. $y = x^2 - 3x$

x	-1	0	1	2	3
y	4	0	-2	-2	0
(x, y)	$(-1, 4)$	$(0, 0)$	$(1, -2)$	$(2, -2)$	$(3, 0)$



18. $y = 5 - x^2$

x	-2	-1	0	1	2
y	1	4	5	4	1
x, y	$(-2, 1)$	$(-1, 4)$	$(0, 5)$	$(1, 4)$	$(2, 1)$



19. $y = (x - 3)^2$

x-intercept: $0 = (x - 3)^2$

$0 = x - 3$

$3 = x$

$(3, 0)$

y-intercept: $y = (0 - 3)^2$

$y = (-3)^2$

$y = 9$

$(0, 9)$

20. $y = 16 - 4x^2$

x-intercepts: $0 = 16 - 4x^2$

$4x^2 = 16$

$x^2 = 4$

$x = \pm 2$

$(-2, 0), (2, 0)$

y-intercept: $y = 16 - 4(0)^2 = 16$

$(0, 16)$

21. $y = |x + 2|$

x-intercept: $0 = |x + 2|$

$0 = x + 2$

$x = -2$

$(-2, 0)$

y-intercept: $y = |0 + 2|$

$y = |2|$

$y = 2$

$(0, 2)$

22. $y^2 = 4 - x$

x-intercept: $0 = 4 - x$

$x = 4$

$(4, 0)$

y-intercepts: $y^2 = 4 - 0$

$y^2 = 4$

$y = \pm 2$

$(0, 2), (0, -2)$

23. $y = 5x - 6$

x-intercept: $0 = 5x - 6$

$6 = 5x$

$\frac{6}{5} = x$

$\left(\frac{6}{5}, 0\right)$

y-intercept: $y = 5(0) - 6 = -6$

$(0, -6)$

24. $y = 8 - 3x$

x-intercept: $0 = 8 - 3x$

$3x = 8$

$x = \frac{8}{3}$

$\left(\frac{8}{3}, 0\right)$

y-intercept: $y = 8 - 3(0) = 8$

$(0, 8)$

25. $y = \sqrt{x + 4}$

x-intercept: $0 = \sqrt{x + 4}$

$0 = x + 4$

$-4 = x$

$(-4, 0)$

y-intercept: $y = \sqrt{0 + 4} = 2$

$(0, 2)$

26. $y = \sqrt{2x - 1}$

x-intercept: $0 = \sqrt{2x - 1}$

$2x - 1 = 0$

$x = \frac{1}{2}$

$\left(\frac{1}{2}, 0\right)$

y-intercept: $y = \sqrt{2(0) - 1}$

$= \sqrt{-1}$ There is no real solution.

There is no y-intercept.

27. $y = |3x - 7|$

x-intercept: $0 = |3x - 7|$

$0 = 3x - 7$

$\frac{7}{3} = 0$

$\left(\frac{7}{3}, 0\right)$

y-intercept: $y = |3(0) - 7| = 7$

$(0, 7)$

28. $y = -|x + 10|$

x-intercept: $0 = -|x + 10|$

$x + 10 = 0$

$x = -10$

$(-10, 0)$

y-intercept: $y = -|0 + 10|$

$= -|10| = -10$

$(0, -10)$

29. $y = 2x^3 - 4x^2$

x-intercepts: $0 = 2x^3 - 4x^2$

$0 = 2x^2(x - 2)$

$x = 0$ or $x = 2$

$(0, 0), (2, 0)$

y-intercept: $y = 2(0)^3 - 4(0)^2$

$y = 0$

$(0, 0)$

30. $y = x^4 - 25$

x-intercept: $0 = x^4 - 25$

$x^4 = 25$

$x = \pm\sqrt[4]{5^2} = \pm\sqrt{5}$

$\left(\sqrt{5}, 0\right), \left(-\sqrt{5}, 0\right)$

y-intercept: $y = (0)^4 - 25 = -25$

$(0, -25)$

31. $y^2 = 6 - x$

x-intercept: $0 = 6 - x$

$x = 6$

$(6, 0)$

y-intercepts: $y^2 = 6 - 0$

$y = \pm\sqrt{6}$

$\left(0, \sqrt{6}\right), \left(0, -\sqrt{6}\right)$

32. $y^2 = x + 1$

x-intercept: $0 = x + 1$

$x = -1$

$(-1, 0)$

y-intercepts: $y^2 = 0 + 1$

$y = \pm 1$

$(0, 1), (0, -1)$

33. $x^2 - y = 0$

$(-x)^2 - y = 0 \Rightarrow x^2 - y = 0 \Rightarrow$ y-axis symmetry

$x^2 - (-y) = 0 \Rightarrow x^2 + y = 0 \Rightarrow$ No x-axis symmetry

$(-x)^2 - (-y) = 0 \Rightarrow x^2 + y = 0 \Rightarrow$ No origin symmetry

34. $x - y^2 = 0$

$x - (-y)^2 = 0$

$x - y^2 = 0$

x-axis symmetry

35. $y = x^3$

$y = (-x)^3 \Rightarrow y = -x^3 \Rightarrow$ No y-axis symmetry

$-y = x^3 \Rightarrow y = -x^3 \Rightarrow$ No x-axis symmetry

$-y = (-x)^3 \Rightarrow -y = -x^3 \Rightarrow y = x^3 \Rightarrow$ Origin symmetry

36. $y = x^4 - x^2 + 3$

$y = (-x)^4 - (-x)^2 + 3 \Rightarrow y = x^4 - x^2 + 3 \Rightarrow$ y-axis symmetry

$-y = x^4 - x^2 + 3 \Rightarrow y = -x^4 + x^2 - 3 \Rightarrow$ No x-axis symmetry

$-y = (-x)^4 - (-x)^2 + 3 \Rightarrow y = -x^4 + x^2 - 3 \Rightarrow$ No origin symmetry

37. $y = \dfrac{x}{x^2 + 1}$

$y = \dfrac{-x}{(-x)^2 + 1} \Rightarrow y = \dfrac{-x}{x^2 + 1} \Rightarrow$ No y-axis symmetry

$-y = \dfrac{x}{x^2 + 1} \Rightarrow y = \dfrac{-x}{x^2 + 1} \Rightarrow$ No x-axis symmetry

$-y = \dfrac{-x}{(-x)^2 + 1} \Rightarrow -y = \dfrac{-x}{x^2 + 1} \Rightarrow y = \dfrac{x}{x^2 + 1} \Rightarrow$ Origin symmetry

38. $y = \dfrac{1}{1 + x^2}$

$y = \dfrac{1}{1 + (-x)^2} \Rightarrow y = \dfrac{1}{1 + x^2} \Rightarrow$ y-axis symmetry

$-y = \dfrac{1}{1 + x^2} \Rightarrow y = \dfrac{-1}{1 + x^2} \Rightarrow$ No x-axis symmetry

$-y = \dfrac{1}{1 + (-x)^2} \Rightarrow y = \dfrac{-1}{1 + x^2} \Rightarrow$ No origin symmetry

39. $xy^2 + 10 = 0$

$(-x)y^2 + 10 = 0 \Rightarrow -xy^2 + 10 = 0 \Rightarrow$ No y-axis symmetry

$x(-y)^2 + 10 = 0 \Rightarrow xy^2 + 10 = 0 \Rightarrow$ x-axis symmetry

$(-x)(-y)^2 + 10 = 0 \Rightarrow -xy^2 + 10 = 0 \Rightarrow$ No origin symmetry

40. $xy = 4$

$(-x)y = 4 \Rightarrow xy = -4 \Rightarrow$ No y-axis symmetry

$x(-y) = 4 \Rightarrow xy = -4 \Rightarrow$ No x-axis symmetry

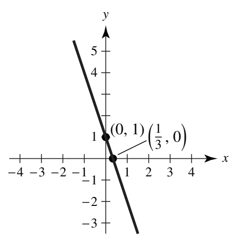
$(-x)(-y) = 4 \Rightarrow xy = 4 \Rightarrow$ Origin symmetry

41. $y = -3x + 1$

x-intercept: $\left(\frac{1}{3}, 0\right)$

y-intercept: $(0, 1)$

No symmetry

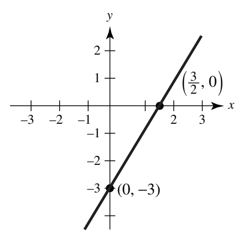


42. $y = 2x - 3$

x-intercept: $\left(\frac{3}{2}, 0\right)$

y-intercept: $(0, -3)$

No symmetry



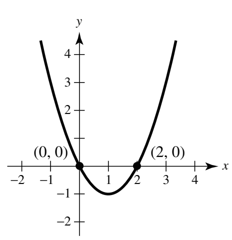
43. $y = x^2 - 2x$

x-intercepts: $(0, 0), (2, 0)$

y-intercept: $(0, 0)$

No symmetry

x	–1	0	1	2	3
y	3	0	–1	0	3

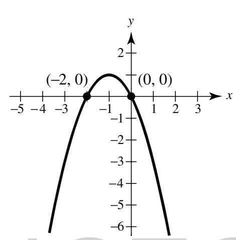


44. $y = -x^2 - 2x$

x-intercept: $(-2, 0), (0, 0)$

y-intercept: $(0, 0)$

No symmetry



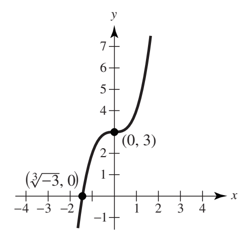
45. $y = x^3 + 3$

x-intercept: $\left(\sqrt[3]{-3}, 0\right)$

y-intercept: $(0, 3)$

No symmetry

x	–2	–1	0	1	2
y	–5	2	3	4	11

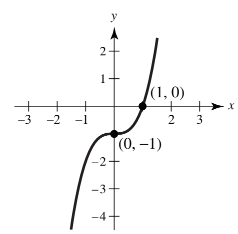


46. $y = x^3 - 1$

x-intercept: $(1, 0)$

y-intercept: $(0, -1)$

No symmetry



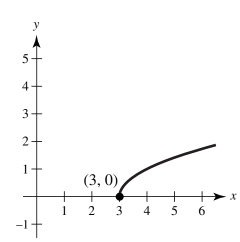
47. $y = \sqrt{x - 3}$

x-intercept: $(3, 0)$

y-intercept: none

No symmetry

x	3	4	7	12
y	0	1	2	3

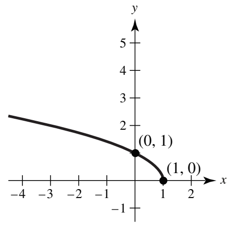


48. $y = \sqrt{1 - x}$

x-intercept: $(1, 0)$

y-intercept: $(0, 1)$

No symmetry



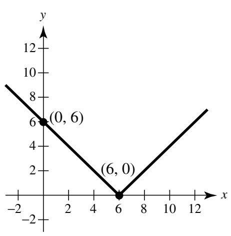
49. $y = |x - 6|$

x-intercept: $(6, 0)$

y-intercept: $(0, 6)$

No symmetry

x	–2	0	2	4	6	8	10
y	8	6	4	2	0	2	4

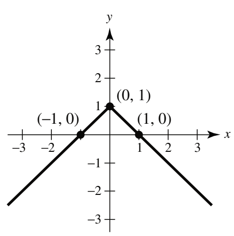


50. $y = 1 - |x|$

x-intercepts: $(1, 0), (-1, 0)$

y-intercept: $(0, 1)$

y-axis symmetry



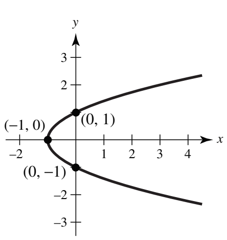
51. $x = y^2 - 1$

x-intercept: $(-1, 0)$

y-intercepts: $(0, -1), (0, 1)$

x-axis symmetry

x	–1	0	3
y	0	±1	±2

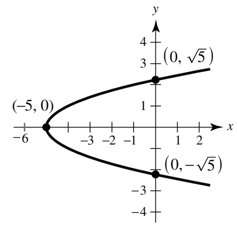


52. $x = y^2 - 5$

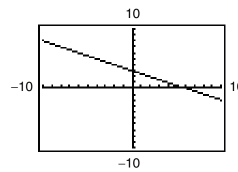
x-intercept: $(-5, 0)$

y-intercepts: $(0, \sqrt{5}), (0, -\sqrt{5})$

x-axis symmetry

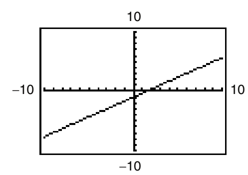


53. $y = 3 - \frac{1}{2}x$



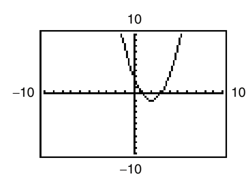
Intercepts: $(6, 0), (0, 3)$

54. $y = \frac{2}{3}x - 1$



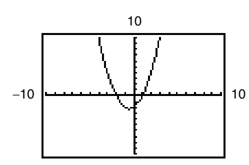
Intercepts: $(0, -1), \left(\frac{3}{2}, 0\right)$

55. $y = x^2 - 4x + 3$



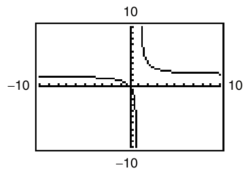
Intercepts: $(3, 0), (1, 0), (0, 3)$

56. $y = x^2 + x - 2$



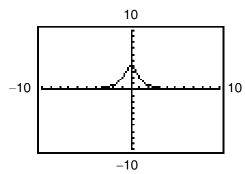
Intercepts: $(-2, 0), (1, 0), (0, -2)$

57. $y = \dfrac{2x}{x - 1}$



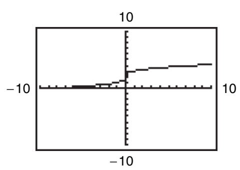
Intercept: $(0, 0)$

58. $y = \dfrac{4}{x^2 + 1}$



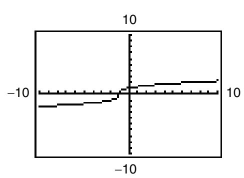
Intercept: $(0, 4)$

59. $y = \sqrt[3]{x} + 2$



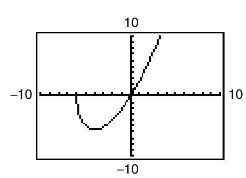
Intercepts: $(-8, 0), (0, 2)$

60. $y = \sqrt[3]{x + 1}$



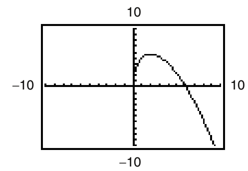
Intercepts: $(-1, 0), (0, 1)$

61. $y = x\sqrt{x + 6}$



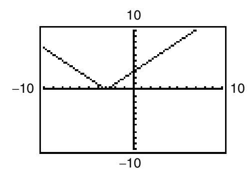
Intercepts: $(0, 0), (-6, 0)$

62. $y = (6 - x)\sqrt{x}$



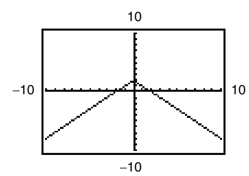
Intercepts: $(0, 0), (6, 0)$

63. $y = |x + 3|$



Intercepts: $(-3, 0), (0, 3)$

64. $y = 2 - |x|$



Intercepts: $(\pm 2, 0), (0, 2)$

65. Center: $(0, 0)$; Radius: 4

$$(x - 0)^2 + (y - 0)^2 = 4^2$$
$$x^2 + y^2 = 16$$

66. $(x - 0)^2 + (y - 0)^2 = 5^2$

$$x^2 + y^2 = 25$$

67. Center: $(2, -1)$; Radius: 4

$$(x - 2)^2 + (y - (-1))^2 = 4^2$$
$$(x - 2)^2 + (y + 1)^2 = 16$$

68. $(x - (-7))^2 + (y - (-4))^2 = 7^2$

$$(x + 7)^2 + (y + 4)^2 = 49$$

69. Center: $(-1, 2)$; Solution point: $(0, 0)$

$$(x - (-1))^2 + (y - 2)^2 = r^2$$
$$(0 + 1)^2 + (0 - 2)^2 = r^2 \Rightarrow 5 = r^2$$
$$(x + 1)^2 + (y - 2)^2 = 5$$

70. $r = \sqrt{(3 - (-1))^2 + (-2 - 1)^2}$

$= \sqrt{4^2 + (-3)^2} = \sqrt{25} = 5$

$(x - 3)^2 + (y - (-2))^2 = 5^2$

$(x - 3)^2 + (y + 2)^2 = 25$

71. Endpoints of a diameter: $(0, 0), (6, 8)$

Center: $\left(\dfrac{0 + 6}{2}, \dfrac{0 + 8}{2}\right) = (3, 4)$

$(x - 3)^2 + (y - 4)^2 = r^2$

$(0 - 3)^2 + (0 - 4)^2 = r^2 \Rightarrow 25 = r^2$

$(x - 3)^2 + (y - 4)^2 = 25$

72. $r = \dfrac{1}{2}\sqrt{(-4 - 4)^2 + (-1 - 1)^2}$

$= \dfrac{1}{2}\sqrt{(-8)^2 + (-2)^2}$

$= \dfrac{1}{2}\sqrt{64 + 4}$

$= \dfrac{1}{2}\sqrt{68} = \left(\dfrac{1}{2}\right)(2)\sqrt{17} = \sqrt{17}$

Midpoint of diameter (center of circle):

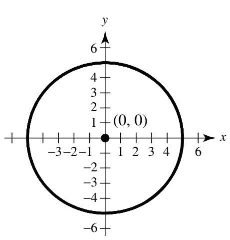
$\left(\dfrac{-4 + 4}{2}, \dfrac{-1 + 1}{2}\right) = (0, 0)$

$(x - 0)^2 + (y - 0)^2 = \left(\sqrt{17}\right)^2$

$x^2 + y^2 = 17$

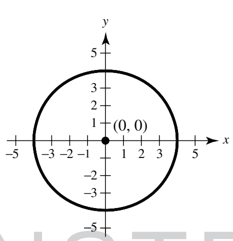
73. $x^2 + y^2 = 25$

Center: $(0, 0)$, Radius: 5



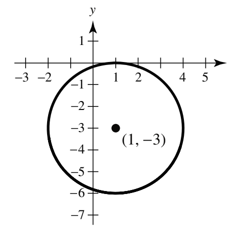
74. $x^2 + y^2 = 16$

Center: $(0, 0)$, Radius: 4



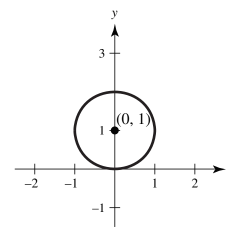
75. $(x - 1)^2 + (y + 3)^2 = 9$

Center: $(1, -3)$, Radius: 3



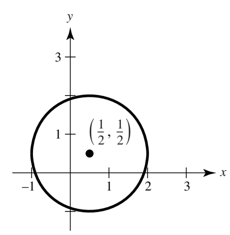
76. $x^2 + (y - 1)^2 = 1$

Center: $(0, 1)$, Radius: 1



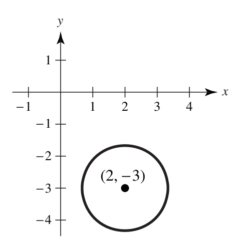
77. $\left(x - \frac{1}{2}\right)^2 + \left(y - \frac{1}{2}\right)^2 = \frac{9}{4}$

Center: $\left(\frac{1}{2}, \frac{1}{2}\right)$, Radius: $\frac{3}{2}$

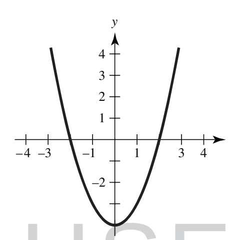


78. $(x - 2)^2 + (y + 3)^2 = \frac{16}{9}$

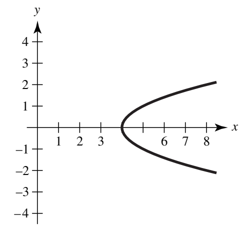
Center: $(2, -3)$, Radius: $\frac{4}{3}$



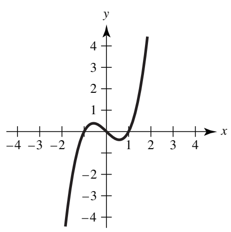
79. y-axis symmetry



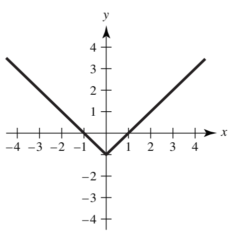
80.



81. Origin symmetry



82.



83. Answers will vary.

One possible equation with x-intercepts at $x = -2, x = 4$, and $x = 6$ is:

$$\begin{aligned} y &= (x + 2)(x - 4)(x - 6) \\ &= (x + 2)(x^2 - 10x + 24) \\ &= x^3 - 10x^2 + 24x + 2x^2 - 20x + 48 \\ &= x^3 - 8x^2 + 4x + 48 \end{aligned}$$

Any non-zero multiple of the right side of this equation, $y = k(x^3 - 8x^2 + 4x + 48)$, would also have these x-intercepts.

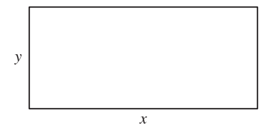
84. Answers will vary.

One possible equation with x-intercepts at $x = -\frac{5}{2}, x = 2$, and $x = \frac{3}{2}$ is:

$$\begin{aligned} y &= (2x + 5)(x - 2)(2x - 3) \\ &= (2x + 5)(2x^2 - 7x + 6) \\ &= 4x^3 - 14x^2 + 12x + 10x^2 - 35x + 30 \\ &= 4x^3 - 4x^2 - 23x + 30 \end{aligned}$$

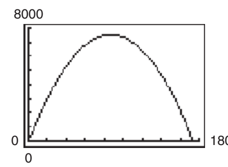
Any non-zero multiple of the right side of this equation, $y = k(4x^3 - 4x^2 - 23x + 30)$, would also have these x-intercepts.

85. (a)



(b)
$$\begin{aligned} 2x + 2y &= \frac{1040}{3} \\ 2y &= \frac{1040}{3} - 2x \\ y &= \frac{520}{3} - x \\ A = xy &= x\left(\frac{520}{3} - x\right) \end{aligned}$$

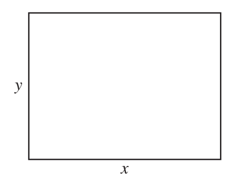
(c)



(d) When $x = y = 86\frac{2}{3}$ yards, the area is a maximum of $7511\frac{1}{9}$ square yards.

(e) A regulation NFL playing field is 120 yards long and $53\frac{1}{3}$ yards wide. The actual area is 6400 square yards.

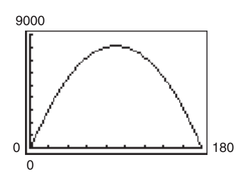
86. (a)



(b) $P = 360$ meters so:

$$\begin{aligned} 2x + 2y &= 360 \\ w = y &= 180 - x \\ A = lw &= x(180 - x) \end{aligned}$$

(c)

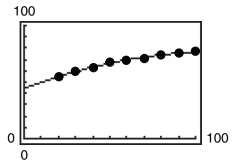


(d) $x = 90$ and $y = 90$

A square will give the maximum area of 8100 square meters.

(e) Answers will vary. *Sample answer:* The dimensions of a Major League Soccer field can vary between 110 and 120 yards in length and between 70 and 80 yards in width. A field of length 115 yards and width 75 yards would have an area of 8625 square yards.

87. (a)



The model fits the data very well.

Answers will vary.

(b) graphically: 75.4 years

algebraically:

$y = -0.0025(90)^2 + 0.574(90) + 44.25$

$y = 75.66$ years

(c) $76 = -0.0025t^2 + 0.574t + 44.25$

$0 = -0.0025t^2 + 0.574t - 31.75$

$$t = \frac{-0.574 \pm \sqrt{(0.574)^2 - 4(-0.0025)(-31.75)}}{2(-0.0025)}$$

$$t = \frac{-0.574 \pm \sqrt{0.011976}}{-0.005}$$

$t \approx 92.9$, or about 93 years

In the year $1900 + 93 = 1993$ the life expectancy was approximately 76 years.

(d) $\text{Year} = 2015 \rightarrow t = 115$

$y = -0.0025(115)^2 + 0.574(115) + 44.25$

$y \approx 77.2$

The model predicts the life expectancy to be 77.2 years in 2015.

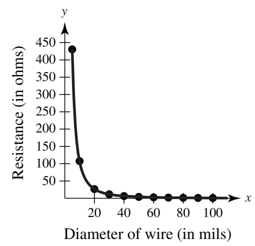
(e) Answers will vary. *Sample answer:* In 50 years the t-value will exceed 100 (since 1920), so the model will no longer provide accurate predictions.

88. (a)

x	5	10	20	30	40
y	430.43	107.33	26.56	11.60	6.36

x	50	60	70	80	90	100
y	3.94	2.62	1.83	1.31	0.96	0.71

(b)



When $x = 85.5$, the resistance is 1.1 ohms.

(c) When $x = 85.5$,

$$y = \frac{10{,}770}{85.5^2} - 0.37 = 1.10327.$$

(d) As the diameter of the copper wire increases, the resistance decreases.

89. $y = ax^2 + bx^3$

(a) $y = a(-x)^2 + b(-x)^3$

$= ax^2 - bx^3$

To be symmetric with respect to the y-axis; a can be any non-zero real number, b must be zero.

(b) $-y = a(-x)^2 + b(-x)^3$

$-y = ax^2 - bx^3$

$y = -ax^2 + bx^3$

To be symmetric with respect to the origin; a must be zero, b can be any non-zero real number.

90. (a) (v)

(b) (i)

(c) None

(d) (ii), (vi)

(e) (i), (iv)

(f) (i), (iv)

91. Assuming that the graph does not go beyond the vertical limits of the display, you will see the graph for the larger values of x.

Section P.5 Linear Equations in Two Variables

1. linear

2. slope

3. parallel

4. perpendicular

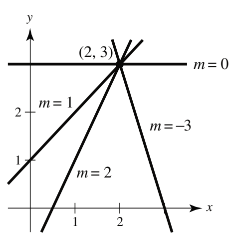
5. rate or rate of change

6. linear extrapolation

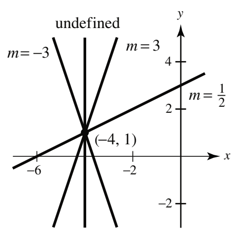
7. general

8. (a) $Ax + By + C = 0$ (iii) general form
(b) $x = a$ (i) vertical line
(c) $y = b$ (v) horizontal line
(d) $y = mx + b$ (ii) slope-intercept form
(e) $y - y_1 = m(x - x_1)$ (iv) point-slope form

9.



10.



11. Two points on the line: $(0, 0)$ and $(4, 6)$

$$\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{6}{4} = \frac{3}{2}$$

12. The line appears to go through $(1, 0)$ and $(3, 5)$.

$$\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5 - 0}{3 - 1} = \frac{5}{2}$$

13. Two points on the line: $(0, 8)$ and $(2, 0)$

$$\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-8}{2} = -4$$

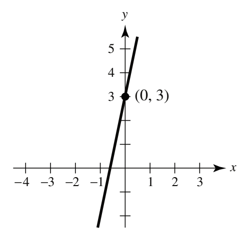
14. The line appears to go through $(0, 7)$ and $(7, 0)$.

$$\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0 - 7}{7 - 0} = -1$$

15. $y = 5x + 3$

Slope: $m = 5$

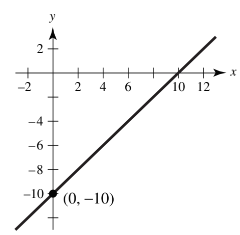
y-intercept: $(0, 3)$



16. $y = x - 10$

Slope: $m = 1$

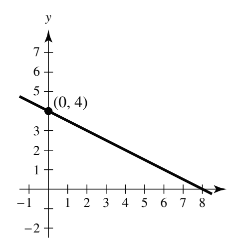
y-intercept: $(0, -10)$



17. $y = -\frac{1}{2}x + 4$

Slope: $m = -\frac{1}{2}$

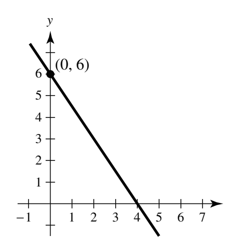
y-intercept: $(0, 4)$



18. $y = -\frac{3}{2}x + 6$

Slope: $m = -\frac{3}{2}$

y-intercept: $(0, 6)$

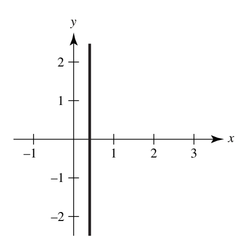


19. $5x - 2 = 0$

$x = \frac{2}{5}$, vertical line

Slope: undefined

No y-intercept



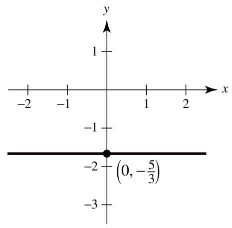
20. $3y + 5 = 0$

$3y = -5$

$y = -\frac{5}{3}$

Slope: $m = 0$

y-intercept: $\left(0, -\frac{5}{3}\right)$

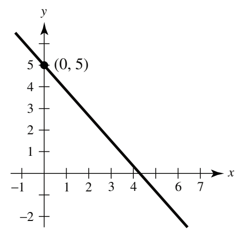


21. $7x + 6y = 30$

$y = -\frac{7}{6}x + 5$

Slope: $m = -\frac{7}{6}$

y-intercept: $(0, 5)$



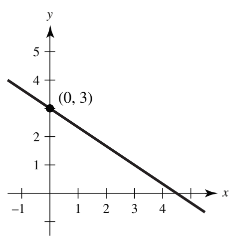
22. $2x + 3y = 9$

$3y = -2x + 9$

$y = -\frac{2}{3}x + 3$

Slope: $m = -\frac{2}{3}$

y-intercept: $(0, 3)$

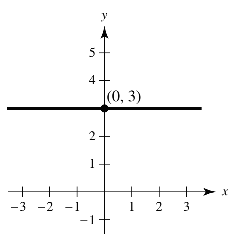


23. $y - 3 = 0$

$y = 3$, horizontal line

Slope: $m = 0$

y-intercept: $(0, 3)$

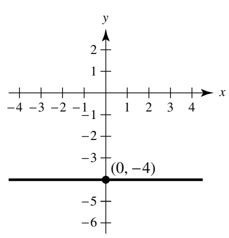


24. $y + 4 = 0$

$y = -4$

Slope: $m = 0$

y-intercept: $(0, -4)$

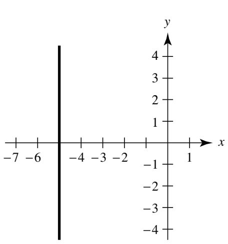


25. $x + 5 = 0$

$x = -5$

Slope: undefined (vertical line)

No y-intercept

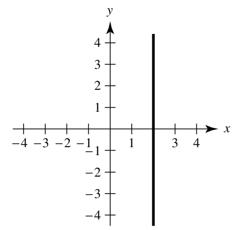


26. $x - 2 = 0$

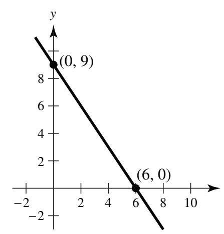
$x = 2$

Slope: undefined (vertical line)

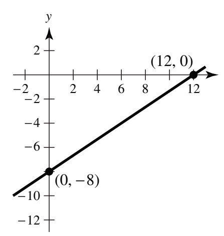
No y-intercept



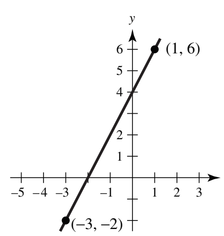
27. $m = \dfrac{0 - 9}{6 - 0} = \dfrac{-9}{6} = -\dfrac{3}{2}$



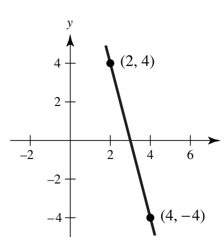
28. $m = \dfrac{-8 - 0}{0 - 12} = \dfrac{8}{12} = \dfrac{2}{3}$



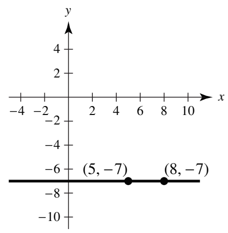
29. $m = \dfrac{6 - (-2)}{1 - (-3)} = \dfrac{8}{4} = 2$



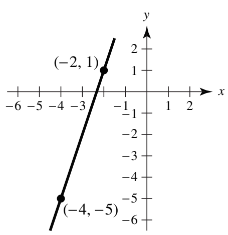
30. $m = \dfrac{-4 - 4}{4 - 2} = -4$



31. $m = \dfrac{-7 - (-7)}{8 - 5} = \dfrac{0}{3} = 0$

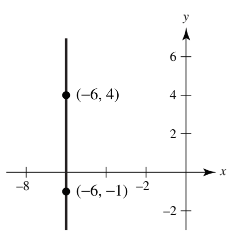


32. $m = \dfrac{-5 - 1}{-4 - (-2)} = 3$

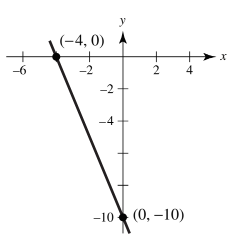


33. $m = \dfrac{4 - (-1)}{-6 - (-6)} = \dfrac{5}{0}$

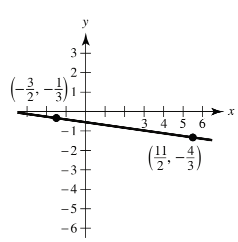
m is undefined.



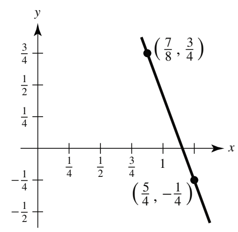
34. $m = \dfrac{0 - (-10)}{-4 - 0} = -\dfrac{5}{2}$



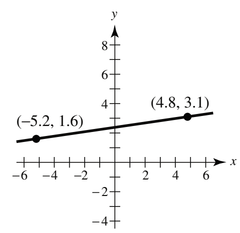
35. $m = \dfrac{-\frac{1}{3} - \left(-\frac{4}{3}\right)}{-\frac{3}{2} - \frac{11}{2}} = -\dfrac{1}{7}$



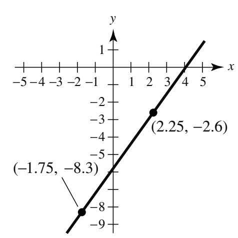
36. Slope $= \dfrac{-\frac{1}{4} - \frac{3}{4}}{\frac{5}{4} - \frac{7}{8}} = \dfrac{-1}{\frac{3}{8}} = -\dfrac{8}{3}$



37. $m = \dfrac{1.6 - 3.1}{-5.2 - 4.8} = \dfrac{-1.5}{-10} = 0.15$



38. Slope $= \dfrac{-2.6 - (-8.3)}{2.25 - (-1.75)} = 1.425$



39. (a) $m = \frac{2}{3}$. Since the slope is positive, the line rises. Matches L_2.

(b) m is undefined. The line is vertical. Matches L_3.

(c) $m = -2$. The line falls. Matches L_1.

40. (a) $m = 0$. The line is horizontal. Matches L_2.

(b) $m = -\frac{3}{4}$. Because the slope is negative, the line falls. Matches L_1.

(c) $m = 1$. Because the slope is positive, the line rises. Matches L_3.

41. Point: $(2, 1)$, Slope: $m = 0$

Because $m = 0$, y does not change. Three points are $(0, 1)$, $(3, 1)$, and $(-1, 1)$.

42. Point: $(3, -2)$, Slope: $m = 0$

Because $m = 0$, y does not change. Three other points are: $(-4, -2), (0, -2), (5, -2)$

43. Point: $(5, -6)$, Slope: $m = 1$

Because $m = 1$, y increases by 1 for every one unit increase in x. Three points are $(6, -5), (7, -4)$, and $(8, -3)$.

44. Point: $(10, -6)$, Slope: $m = -1$

Because $m = -1$, y decreases by 1 for every one unit increase in x. Three other points are: $(0, 4), (9, -5)$ $(11, -7)$.

45. Point: $(-8, 1)$, Slope is undefined.

Because m is undefined, x does not change. Three points are $(-8, 0), (-8, 2)$, and $(-8, 3)$.

46. Point: $(1, 5)$, Slope is undefined.

Because m is undefined, x does not change. Three other points are: $(1, -3), (1, 1), (1, 7)$

47. Point: $(-5, 4)$, Slope: $m = 2$

Because $m = 2 = \frac{2}{1}$, y increases by 2 for every one unit increase in x. Three additional points are $(-4, 6)$, $(-3, 8)$, and $(-2, 10)$.

48. Point: $(0, -9)$, Slope: $m = -2$

Because $m = -2$, y decreases by 2 for every one unit increase in x. Three other points are: $(-2, -5)$, $(1, -11), (3, -15)$.

49. Point: $(7, -2)$, Slope: $m = \frac{1}{2}$

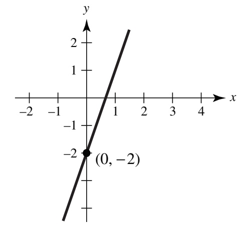
Because $m = \frac{1}{2}$, y increases by 1 unit for every two unit increase in x. Three additional points are $(9, -1)$, $(11, 0)$, and $(13, 1)$.

50. Point: $(-1, -6)$, Slope: $m = -\frac{1}{2}$

Because $m = -\frac{1}{2}$, y decreases by 1 for every 2 unit increase in x. Three other points are: $(-3, -5)$, $(1, -7), (5, -9)$.

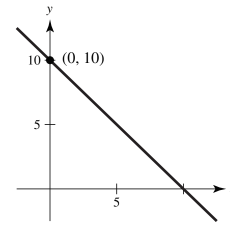
51. Point $(0, -2)$; $m = 3$

$$y + 2 = 3(x - 0)$$
$$y = 3x - 2$$



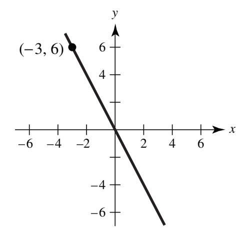
52. Point $(0, 10)$; $m = -1$

$$y - 10 = -1(x - 0)$$
$$y - 10 = -x$$
$$y = -x + 10$$



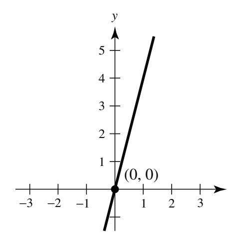
53. Point $(-3, 6)$; $m = -2$

$$y - 6 = -2(x + 3)$$
$$y = -2x$$



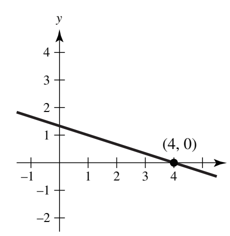
54. Point $(0, 0)$; $m = 4$

$$y - 0 = 4(x - 0)$$
$$y = 4x$$



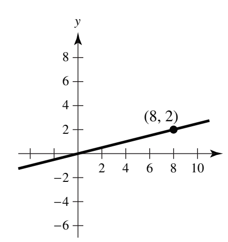
55. Point $(4, 0)$; $m = -\frac{1}{3}$

$$y - 0 = -\tfrac{1}{3}(x - 4)$$
$$y = -\tfrac{1}{3}x + \tfrac{4}{3}$$



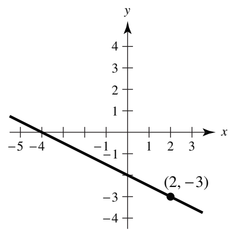
56. Point $(8, 2)$; $m = \frac{1}{4}$

$$y - 2 = \tfrac{1}{4}(x - 8)$$
$$y - 2 = \tfrac{1}{4}x - 2$$
$$y = \tfrac{1}{4}x$$



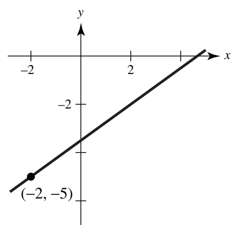
57. Point $(2, -3)$; $m = -\frac{1}{2}$

$$y - (-3) = -\frac{1}{2}(x - 2)$$
$$y + 3 = -\frac{1}{2}x + 1$$
$$y = -\frac{1}{2}x - 2$$



58. Point $(-2, -5)$; $m = \frac{3}{4}$

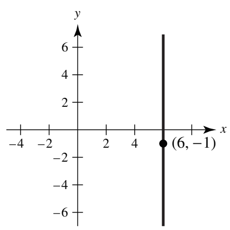
$$y + 5 = \frac{3}{4}(x + 2)$$
$$4y + 20 = 3x + 6$$
$$4y = 3x - 14$$
$$y = \frac{3}{4}x - \frac{7}{2}$$



59. Point $(6, -1)$; m is undefined.

Because the slope is undefined, the line is a vertical line.

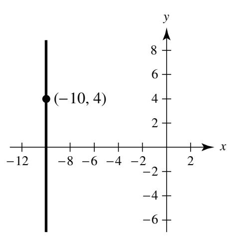
$x = 6$



60. Point $(-10, 4)$; m is undefined.

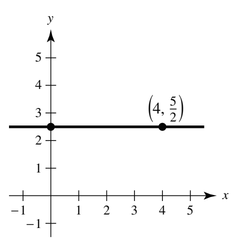
Because the slope is undefined, the line is a vertical line.

$x = -10$



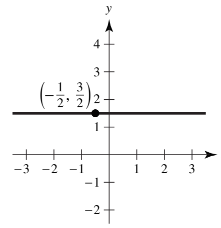
61. Point $\left(4, \frac{5}{2}\right)$; $m = 0$

$$y - \frac{5}{2} = 0(x - 4)$$
$$y - \frac{5}{2} = 0$$
$$y = \frac{5}{2}$$



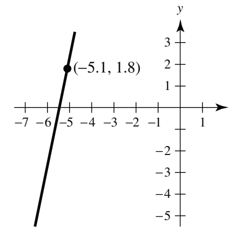
62. Point $\left(-\frac{1}{2}, \frac{3}{2}\right)$; $m = 0$

$$y - \frac{3}{2} = 0\left(x + \frac{1}{2}\right)$$
$$y - \frac{3}{2} = 0$$
$$y = \frac{3}{2}$$



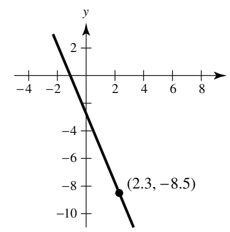
63. Point $(-5.1, 1.8)$; $m = 5$

$$y - 1.8 = 5(x - (-5.1))$$
$$y = 5x + 27.3$$



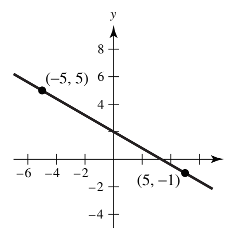
64. Point $(2.3, -8.5)$; $m = -2.5$

$$y - (-8.5) = -2.5(x - 2.3)$$
$$y + 8.5 = -2.5x + 5.75$$
$$y = -2.5x - 2.75$$



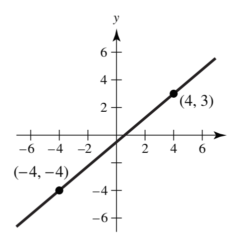
65. $(5, -1), (-5, 5)$

$$y + 1 = \frac{5 + 1}{-5 - 5}(x - 5)$$
$$y = -\frac{3}{5}(x - 5) - 1$$
$$y = -\frac{3}{5}x + 2$$



66. $(4, 3), (-4, -4)$

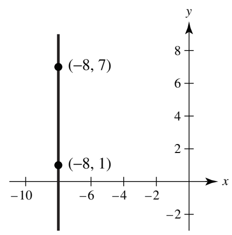
$$y - 3 = \frac{-4 - 3}{-4 - 4}(x - 4)$$
$$y - 3 = \frac{7}{8}(x - 4)$$
$$y - 3 = \frac{7}{8}x - \frac{7}{2}$$
$$y = \frac{7}{8}x - \frac{1}{2}$$



67. $(-8, 1), (-8, 7)$

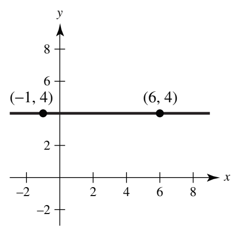
Because both points have $x = -8$, the slope is undefined, and the line is vertical.

$x = -8$



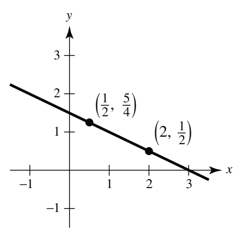
68. $(-1, 4), (6, 4)$

$$y - 4 = \frac{4 - 4}{6 - (-1)}(x + 1)$$
$$y - 4 = 0(x + 1)$$
$$y - 4 = 0$$
$$y = 4$$



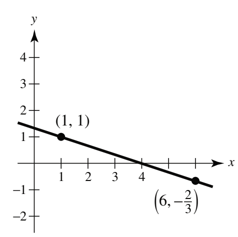
69. $\left(2, \frac{1}{2}\right), \left(\frac{1}{2}, \frac{5}{4}\right)$

$$y - \frac{1}{2} = \frac{\frac{5}{4} - \frac{1}{2}}{\frac{1}{2} - 2}(x - 2)$$
$$y = -\frac{1}{2}(x - 2) + \frac{1}{2}$$
$$y = -\frac{1}{2}x + \frac{3}{2}$$



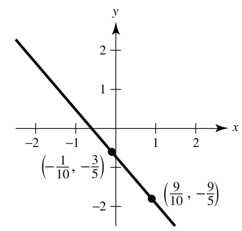
70. $(1, 1), \left(6, -\frac{2}{3}\right)$

$$y - 1 = \frac{-\frac{2}{3} - 1}{6 - 1}(x - 1)$$
$$y - 1 = -\frac{1}{3}(x - 1)$$
$$y - 1 = -\frac{1}{3}x + \frac{1}{3}$$
$$y = -\frac{1}{3}x + \frac{4}{3}$$



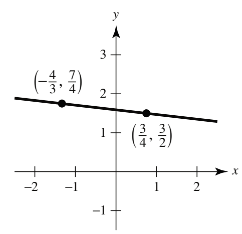
71. $\left(-\frac{1}{10}, -\frac{3}{5}\right), \left(\frac{9}{10}, -\frac{9}{5}\right)$

$$y - \left(-\frac{3}{5}\right) = \frac{-\frac{9}{5} - \left(-\frac{3}{5}\right)}{\frac{9}{10} - \left(-\frac{1}{10}\right)}\left(x - \left(-\frac{1}{10}\right)\right)$$
$$y = -\frac{6}{5}\left(x + \frac{1}{10}\right) - \frac{3}{5}$$
$$y = -\frac{6}{5}x - \frac{18}{28}$$



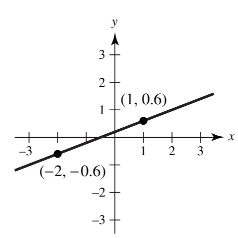
72. $\left(\frac{3}{4}, \frac{3}{2}\right), \left(-\frac{4}{3}, \frac{7}{4}\right)$

$$y - \frac{3}{2} = \frac{\frac{7}{4} - \frac{3}{2}}{-\frac{4}{3} - \frac{3}{4}}\left(x - \frac{3}{4}\right)$$
$$y - \frac{3}{2} = \frac{\frac{1}{4}}{-\frac{25}{12}}\left(x - \frac{3}{4}\right)$$
$$y - \frac{3}{2} = -\frac{3}{25}\left(x - \frac{3}{4}\right)$$
$$y - \frac{3}{2} = -\frac{3}{25}x + \frac{9}{100}$$
$$y = -\frac{3}{25}x + \frac{159}{100}$$



73. $(1, 0.6), (-2, -0.6)$

$$y - 0.6 = \frac{-0.6 - 0.6}{-2 - 1}(x - 1)$$
$$y = 0.4(x - 1) + 0.6$$
$$y = 0.4x + 0.2$$



74. $(-8, 0.6), (2, -2.4)$

$$y - 0.6 = \frac{-2.4 - 0.6}{2 - (-8)}(x + 8)$$

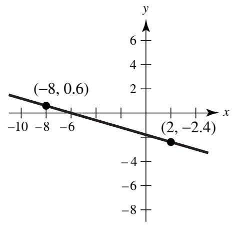
$$y - 0.6 = -\frac{3}{10}(x + 8)$$

$$10y - 6 = -3(x + 8)$$

$$10y - 6 = -3x - 24$$

$$10y = -3x - 18$$

$$y = -\frac{3}{10}x - \frac{9}{5} \quad \text{or} \quad y = -0.3x - 1.8$$



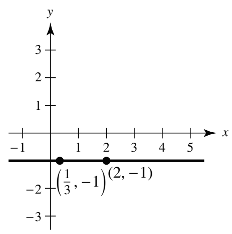
75. $(2, -1), \left(\frac{1}{3}, -1\right)$

$$y + 1 = \frac{-1 - (-1)}{\frac{1}{3} - 2}(x - 2)$$

$$y + 1 = 0$$

$$y = -1$$

The line is horizontal.



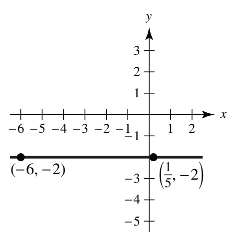
76. $\left(\frac{1}{5}, -2\right), (-6, -2)$

$$y + 2 = \frac{-2 - (-2)}{-6 - \frac{1}{5}}(x + 6)$$

$$y + 2 = \frac{0}{-6 - \frac{1}{5}}(x + 6)$$

$$y + 2 = 0$$

$$y = -2$$

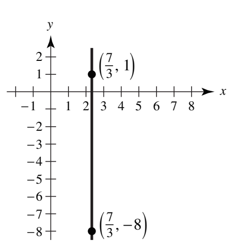


77. $\left(\frac{7}{3}, -8\right), \left(\frac{7}{3}, 1\right)$

$$m = \frac{1 - (-8)}{\frac{7}{3} - \frac{7}{3}} = \frac{9}{0} \text{ and is undefined.}$$

$$x = \frac{7}{3}$$

The line is vertical.



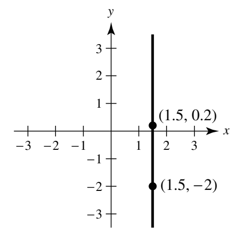
78. $(1.5, -2), (1.5, 0.2)$

$$y + 2 = \frac{-2 - 0.2}{1.5 - 1.5}(x - 1.5)$$

$$y + 2 = \frac{-2 - 0.2}{0}(x - 1.5)$$

The slope is undefined. The line is vertical.

$x = 1.5$



79. $L_1: y = \frac{1}{3}x - 2$

$m_1 = \frac{1}{3}$

$L_2: y = \frac{1}{3}x + 3$

$m_2 = \frac{1}{3}$

The lines are parallel.

80. $L_1: y = 4x - 1$

$m_1 = 4$

$L_2: y = 4x + 7$

$m_2 = 4$

The lines are parallel.

81. $L_1: y = \frac{1}{2}x - 3$

$m_1 = \frac{1}{2}$

$L_2: y = -\frac{1}{2}x + 1$

$m_2 = -\frac{1}{2}$

The lines are neither parallel nor perpendicular.

82. $L_1: y = -\frac{4}{5}x - 5$

$m_1 = -\frac{4}{5}$

$L_2: y = \frac{5}{4}x + 1$

$m_2 = \frac{5}{4}$

The lines are perpendicular.

83. $L_1: (0, -1), (5, 9)$

$$m_1 = \frac{9+1}{5-0} = 2$$

$L_2: (0, 3), (4, 1)$

$$m_2 = \frac{1-3}{4-0} = -\frac{1}{2}$$

The lines are perpendicular.

84. $L_1: (-2, -1), (1, 5)$

$$m_1 = \frac{5-(-1)}{1-(-2)} = \frac{6}{3} = 2$$

$L_2: (1, 3), (5, -5)$

$$m_2 = \frac{-5-3}{5-1} = \frac{-8}{4} = -2$$

The lines are neither parallel nor perpendicular.

85. $L_1: (3, 6), (-6, 0)$

$$m_1 = \frac{0-6}{-6-3} = \frac{2}{3}$$

$L_2: (0, -1), \left(5, \frac{7}{3}\right)$

$$m_2 = \frac{\frac{7}{3}+1}{5-0} = \frac{2}{3}$$

The lines are parallel.

86. $L_1: (4, 8), (-4, 2)$

$$m_1 = \frac{2-8}{-4-4} = \frac{-6}{-8} = \frac{3}{4}$$

$L_2: (3, -5), \left(-1, \frac{1}{3}\right)$

$$m_2 = \frac{\frac{1}{3}-(-5)}{-1-3} = \frac{\frac{16}{3}}{-4} = -\frac{4}{3}$$

The lines are perpendicular.

87. $4x - 2y = 3$

$$y = 2x - \tfrac{3}{2}$$

Slope: $m = 2$

(a) $(2, 1), m = 2$

$$y - 1 = 2(x - 2)$$
$$y = 2x - 3$$

(b) $(2, 1), m = -\frac{1}{2}$

$$y - 1 = -\tfrac{1}{2}(x - 2)$$
$$y = -\tfrac{1}{2}x + 2$$

88. $x + y = 7$

$$y = -x + 7$$

Slope: $m = -1$

(a) $m = -1, (-3, 2)$

$$y - 2 = -1(x + 3)$$
$$y - 2 = -x - 3$$
$$y = -x - 1$$

(b) $m = 1, (-3, 2)$

$$y - 2 = 1(x + 3)$$
$$y = x + 5$$

89. $3x + 4y = 7$

$$y = -\tfrac{3}{4}x + \tfrac{7}{4}$$

Slope: $m = -\frac{3}{4}$

(a) $\left(-\frac{2}{3}, \frac{7}{8}\right), m = -\frac{3}{4}$

$$y - \tfrac{7}{8} = -\tfrac{3}{4}\left(x - \left(-\tfrac{2}{3}\right)\right)$$
$$y = -\tfrac{3}{4}x + \tfrac{3}{8}$$

(b) $\left(-\frac{2}{3}, \frac{7}{8}\right), m = \frac{4}{3}$

$$y - \tfrac{7}{8} = \tfrac{4}{3}\left(x - \left(-\tfrac{2}{3}\right)\right)$$
$$y = \tfrac{4}{3}x + \tfrac{127}{72}$$

90. $5x + 3y = 0$

$$3y = -5x$$
$$y = -\tfrac{5}{3}x$$

Slope: $m = -\frac{5}{3}$

(a) $m = -\frac{5}{3}, \left(\frac{7}{8}, \frac{3}{4}\right)$

$$y - \tfrac{3}{4} = -\tfrac{5}{3}\left(x - \tfrac{7}{8}\right)$$
$$24y - 18 = -40\left(x - \tfrac{7}{8}\right)$$
$$24y - 18 = -40x + 35$$
$$24y = -40x + 53$$
$$y = -\tfrac{5}{3}x + \tfrac{53}{24}$$

(b) $m = \frac{3}{5}, \left(\frac{7}{8}, \frac{3}{4}\right)$

$$y - \tfrac{3}{4} = \tfrac{3}{5}\left(x - \tfrac{7}{8}\right)$$
$$40y - 30 = 24\left(x - \tfrac{7}{8}\right)$$
$$40y - 30 = 24x - 21$$
$$40y = 24x + 9$$
$$y = \tfrac{3}{5}x + \tfrac{9}{40}$$

91. $y + 3 = 0$

$y = -3$

Slope: $m = 0$

(a) $(-1, 0), m = 0$

$y = 0$

(b) $(-1, 0), m$ is undefined.

$x = -1$

92. $y - 2 = 0$

$y = 2$

Slope: $m = 0$

(a) $(-4, 1), m = 0$

$y - 1 = 0(x - (-4))$

$y - 1 = 0$

$y = 1$

(b) $(-4, 1), m$ is undefined.

$x = -4$

93. $x - 4 = 0$

$x = 4$

Slope: m is undefined.

(a) $(3, -2), m$ is undefined.

$x = 3$

(b) $(3, -2), m = 0$

$y = -2$

94. $x + 2 = 0$

$x = -2$

Slope: m is undefined.

(a) The original line is the vertical line through $x = -2$. The line parallel to this line containing $(-5, 1)$ is the vertical line $x = -5$.

(b) A perpendicular to a vertical line is a horizontal line, whose slope is 0. The horizontal line containing $(-5, 1)$ is the line $y = 1$.

95. $x - y = 4$

$y = x - 4$

Slope: $m = 1$

(a) $(2.5, 6.8), m = 1$

$y - 6.8 = 1(x - 2.5)$

$y = x + 4.3$

(b) $(2.5, 6.8), m = -1$

$y - 6.8 = (-1)(x - 2.5)$

$y = -x + 9.3$

96. $6x + 2y = 9$

$2y = -6x + 9$

$y = -3x + \frac{9}{2}$

Slope: $m = -3$

(a) $(-3.9, -1.4), m = -3$

$y - (-1.4) = -3(x - (-3.9))$

$y + 1.4 = -3x - 11.7$

$y = -3x - 13.1$

(b) $(-3.9, -1.4), m = \frac{1}{3}$

$y - (-1.4) = \frac{1}{3}(x - (-3.9))$

$y + 1.4 = \frac{1}{3}x + 1.3$

$y = \frac{1}{3}x - 0.1$

97. $\frac{x}{2} + \frac{y}{3} = 1$

$3x + 2y - 6 = 0$

98. $(-3, 0), (0, 4)$

$\frac{x}{-3} + \frac{y}{4} = 1$

$(-12)\frac{x}{-3} + (-12)\frac{y}{4} = (-12) \cdot 1$

$4x - 3y + 12 = 0$

99. $\frac{x}{-1/6} + \frac{y}{-2/3} = 1$

$6x + \frac{3}{2}y = -1$

$12x + 3y + 2 = 0$

100. $\left(\frac{2}{3}, 0\right), (0, -2)$

$\frac{x}{2/3} + \frac{y}{-2} = 1$

$\frac{3x}{2} - \frac{y}{2} = 1$

$3x - y - 2 = 0$

101. $\frac{x}{c} + \frac{y}{c} = 1, c \neq 0$

$x + y = c$

$1 + 2 = c$

$3 = c$

$x + y = 3$

$x + y - 3 = 0$

102. $(d, 0), (0, d), (-3, 4)$

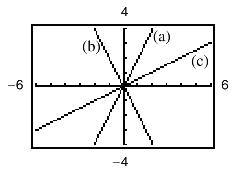
$$\frac{x}{d} + \frac{y}{d} = 1$$
$$x + y = d$$
$$-3 + 4 = d$$
$$1 = d$$
$$x + y = 1$$
$$x + y - 1 = 0$$

103. (a) $y = 2x$

(b) $y = -2x$

(c) $y = \frac{1}{2}x$

(b) and (c) are perpendicular.

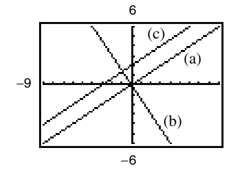


104. (a) $y = \frac{2}{3}x$

(b) $y = -\frac{3}{2}x$

(c) $y = \frac{2}{3}x + 2$

(a) is parallel to (c). (b) is perpendicular to (a) and (c).

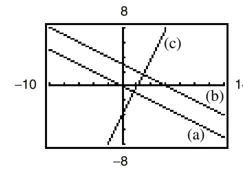


105. (a) $y = -\frac{1}{2}x$

(b) $y = -\frac{1}{2}x + 3$

(c) $y = 2x - 4$

(a) and (b) are parallel. (c) is perpendicular to (a) and (b).

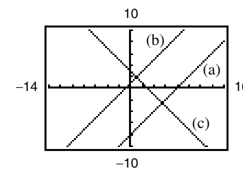


106. (a) $y = x - 8$

(b) $y = x + 1$

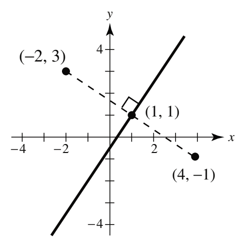
(c) $y = -x + 3$

(a) is parallel to (b). (c) is perpendicular to (a) and (b).



107. Set the distance between $(4, -1)$ and (x, y) equal to the distance between $(-2, 3)$ and (x, y).

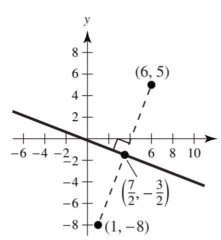
$$\sqrt{(x-4)^2 + [y-(-1)]^2} = \sqrt{[x-(-2)]^2 + (y-3)^2}$$
$$(x-4)^2 + (y+1)^2 = (x+2)^2 + (y-3)^2$$
$$x^2 - 8x + 16 + y^2 + 2y + 1 = x^2 + 4x + 4 + y^2 - 6y + 9$$
$$-8x + 2y + 17 = 4x - 6y + 13$$
$$0 = 12x - 8y - 4$$
$$0 = 4(3x - 2y - 1)$$
$$0 = 3x - 2y - 1$$



This line is the perpendicular bisector of the line segment connecting $(4, -1)$ and $(-2, 3)$.

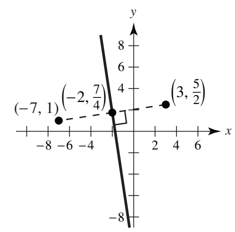
108. Set the distance between $(6, 5)$ and (x, y) equal to the distance between $(1, -8)$ and (x, y).

$$\sqrt{(x-6)^2 + (y-5)^2} = \sqrt{(x-1)^2 + (y-(-8))^2}$$
$$(x-6)^2 + (y-5)^2 = (x-1)^2 + (y+8)^2$$
$$x^2 - 12x + 36 + y^2 - 10y + 25 = x^2 - 2x + 1 + y^2 + 16y + 64$$
$$x^2 + y^2 - 12x - 10y + 61 = x^2 + y^2 - 2x + 16y + 65$$
$$-12x - 10y + 61 = -2x + 16y + 65$$
$$-10x - 26y - 4 = 0$$
$$-2(5x + 13y + 2) = 0$$
$$5x + 13y + 2 = 0$$



109. Set the distance between $\left(3, \frac{5}{2}\right)$ and (x, y) equal to the distance between $(-7, 1)$ and (x, y).

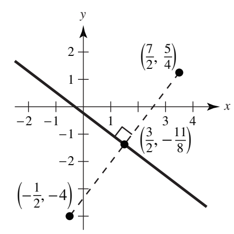
$$\sqrt{(x-3)^2 + \left(y - \frac{5}{2}\right)^2} = \sqrt{[x-(-7)]^2 + (y-1)^2}$$
$$(x-3)^2 + \left(y-\frac{5}{2}\right)^2 = (x+7)^2 + (y-1)^2$$
$$x^2 - 6x + 9 + y^2 - 5y + \frac{25}{4} = x^2 + 14x + 49 + y^2 - 2y + 1$$
$$-6x - 5y + \frac{61}{4} = 14x - 2y + 50$$
$$-24x - 20y + 61 = 56x - 8y + 200$$
$$80x + 12y + 139 = 0$$



This line is the perpendicular bisector of the line segment connecting $\left(3, \frac{5}{2}\right)$ and $(-7, 1)$.

110. Set the distance between $\left(-\frac{1}{2}, -4\right)$ and (x, y) equal to the distance between $\left(\frac{7}{2}, \frac{5}{4}\right)$ and (x, y).

$$\sqrt{\left(x - \left(-\frac{1}{2}\right)\right)^2 + (y-(-4))^2} = \sqrt{\left(x-\frac{7}{2}\right)^2 + \left(y-\frac{5}{4}\right)^2}$$
$$\left(x+\frac{1}{2}\right)^2 + (y+4)^2 = \left(x-\frac{7}{2}\right)^2 + \left(y-\frac{5}{4}\right)^2$$
$$x^2 + x + \frac{1}{4} + y^2 + 8y + 16 = x^2 - 7x + \frac{49}{4} + y^2 - \frac{5}{2}y + \frac{25}{16}$$
$$x^2 + y^2 + x + 8y + \frac{65}{4} = x^2 + y^2 - 7x - \frac{5}{2}y + \frac{221}{16}$$
$$x + 8y + \frac{65}{4} = -7x - \frac{5}{2}y + \frac{221}{16}$$
$$8x + \frac{21}{2}y + \frac{39}{16} = 0$$
$$128x + 168y + 39 = 0$$



111. (a) $m = 135$. The sales are increasing 135 units per year.

(b) $m = 0$. There is no change in sales during the year.

(c) $m = -40$. The sales are decreasing 40 units per year.

112. (a) $m = 400$. The revenues are increasing \$400 per day.

(b) $m = 100$. The revenues are increasing \$100 per day.

(c) $m = 0$. There is no change in revenue during the day. (Revenue remains constant.)

113. (a) greatest increase = largest slope

$(18, 97{,}486), (16, 90{,}260)$

$$m_1 = \frac{97{,}486 - 90{,}260}{18 - 16} = 3{,}613$$

The salary increased the greatest amount between 2006 and 2008.

least increase = smallest slope

$(14, 86{,}160), (12, 83{,}944)$

$$m_2 = \frac{86{,}160 - 83{,}944}{14 - 12} = 1108$$

The salary increased the least amount between 2002 and 2004.

(b) $m = \dfrac{97{,}486 - 69{,}277}{18 - 6} = \dfrac{9403}{12}$ or 2350.75

(c) The average salary of a senior high school principal increased \$2350.75 per year between 1996 and 2008.

114. (a) greatest increase = largest slope

$(5, 13.93), (4, 8.28)$

$$m = \frac{13.93 - 8.28}{5 - 4} = 5.65$$

Sales showed the greatest increase between 2004 and 2005.

least increase = smallest slope

$(2, 5.74), (1, 5.36)$

$$m = \frac{5.74 - 5.36}{2 - 1} = 0.38$$

Sales showed the least increase between 2001 and 2002.

(b) $m = \dfrac{24.01 - 5.36}{7 - 1} \approx 3.11$

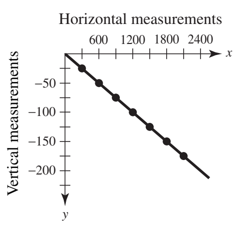
(c) The average sales increased about 3.11 billion dollars per year between 2001 and 2007.

115. $y = \frac{6}{100}x$

$y = \frac{6}{100}(200) = 12$ feet

116. (a) and (b)

x	300	600	900	1200	1500	1800	2100
y	–25	–50	–75	–100	–125	–150	–175



(c) $m = \dfrac{-50 - (-25)}{600 - 300} = \dfrac{-25}{300} = -\dfrac{1}{12}$

$$y - (-50) = -\frac{1}{12}(x - 600)$$

$$y + 50 = -\frac{1}{12}x + 50$$

$$y = -\frac{1}{12}x$$

(d) Because $m = -\dfrac{1}{12}$, for every change in the horizontal measurement of 12 feet, the vertical measurement decreases by 1 foot.

(e) $\dfrac{1}{12} \approx 0.083 = 8.3\%$ grade

117. $(10, 2540), m = -125$

$$V - 2540 = -125(t - 10)$$
$$V - 2540 = -125t + 1250$$
$$V = -125t + 3790, 5 \le t \le 10$$

118. $(10, 156), m = 4.50$

$$V - 156 = 4.50(t - 10)$$
$$V - 156 = 4.50t - 45$$
$$V = 4.5t + 111, 5 \le t \le 10$$

119. The V-intercept measures the value of the molding machine at the time of purchase (when $t = 0$).
The slope measures the amount the value of the machine depreciates per year.

120. The C-intercept measures the fixed costs of manufacturing when zero bags are produced.
The slope measures the cost to produce one laptop bag.

121. Using the points $(0, 875)$ and $(5, 0)$, where the first coordinate represents the year t and the second coordinate represents the value V, you have

$$m = \frac{0 - 875}{5 - 0} = -175$$
$$V = -175t + 875, 0 \le t \le 5.$$

122. $(0, 25{,}000)$ and $(10, 2000)$

$$m = \frac{2000 - 25000}{10 - 0} = -2300$$
$$V = -2300t + 25{,}000, \quad 0 \le t \le 10$$

123. Sales price $=$ List price $-$ 20% discount

$$S = L - 0.20L$$
$$S = 0.80L$$

124. $W = 0.75x + 12.25$

125. $W = 0.07S + 2500$

126. $C = 0.55x + 120$

127. $(17, 1.46), (9, 1.21)$

$$m = \frac{1.46 - 1.21}{17 - 9} = 0.03125$$
$$y - 1.46 = 0.03125(t - 17)$$
$$y = 0.03125t + 0.92875$$

For 2012, use $t = 22$

$$y = 0.03125(22) + 0.92875$$
$$y \approx \$1.62$$

For 2014, use $t = 24$

$$y = 0.03125(24) + 0.92875$$
$$y \approx \$1.68$$

128. $(3, 1078), (7, 1067)$

$$m = \frac{1067 - 1078}{7 - 3} = -\frac{11}{4}$$
$$y - 1078 = -\frac{11}{4}(t - 3)$$
$$y = -\frac{11}{4}t + \frac{4345}{4} \text{ or } y = -2.75t + 1086.25$$

For 2012, use $t = 12$

$$y = -2.75(12) + 1086.25$$
$$y = 1053.25$$
$$y \approx 1053 \text{ stores}$$

For 2014, use $t = 14$

$$y = -2.75(14) + 1086.25$$
$$y = 1047.75$$
$$y \approx 1047 \text{ stores}$$

Answers will vary.

129. (a) $(0, 40{,}571), (8, 44{,}112)$

$$m = \frac{44{,}112 - 40{,}571}{8 - 0} = 442.625$$
$$y - 40{,}571 = 442.625(t - 0)$$
$$y = 442.625t + 40{,}571$$

(b) For 2010, use $t = 10$

$$y = 442.625(10) + 40{,}571$$
$$y = 44997.25$$
$$y \approx 44{,}997 \text{ students}$$

For 2015, use $t = 15$

$$y = 442.625(15) + 40{,}571$$
$$y = 47{,}210.375$$
$$y \approx 47{,}210 \text{ students}$$

(c) $m = 442.625$; Each year, enrollment increases by about 443 students.

130. (a) $(2000, 46{,}107), (2008, 51{,}413)$

Average annual change:
$$\frac{51{,}413 - 46{,}107}{2008 - 2000} = \frac{5306}{8} = 663.25$$

So the average annual change in enrollment from 2000 to 2008 was 663.25.

(b) 2002: $46{,}107 + 2(663.25) \approx 47{,}434$ students

2004: $46{,}107 + 4(663.25) = 48{,}760$ students

2006: $46{,}107 + 6(663.25) \approx 50{,}087$ students

(c) $m = 663.25$, so $N(t) = 663.25 + 46{,}107$

Each year, enrollment increases by about 663 students.

(d) Answers will vary.

131. (a) Cost = cost of operation (per hr) + cost of operator (per hour) + cost of machine

$C = 6.50t + 11.50t + 42{,}000$

$C = 18t + 42{,}000$

(b) Revenue = charge per hour

$R = 30t$

(c) $P = R - C$

$P = 30t - (18t + 42{,}000)$

$P = 12t - 42{,}000$

(d) $0 = 12t - 42{,}000$

$\frac{42{,}000}{12} = \frac{12t}{12}$

$t = 3500.$

To break even the company must use the equipment for 3500 hours.

132. $(580, 50)$ and $(625, 47)$

(a) $m = \frac{47 - 50}{625 - 580} = \frac{-3}{45} = -\frac{1}{15}$

$x - 50 = -\frac{1}{15}(p - 580)$

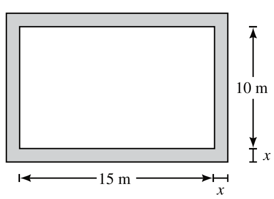
$x - 50 = -\frac{1}{15}p + \frac{116}{3}$

$x = -\frac{1}{15}p + \frac{266}{3}$

(b) $x = -\frac{1}{15}(655) + \frac{266}{3} = 45$ units

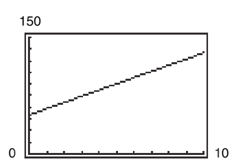
(c) $x = -\frac{1}{15}(595) + \frac{266}{3} = 49$ units

133. (a)



(b) $y = 2(15 + 2x) + 2(10 + 2x) = 8x + 50$

(c)



(d) Because $m = 8$, each 1-meter increase in x will increase y by 8 meters.

134. Answers will vary. *Sample answer:*

Choosing the points $(1, 2.1)$ and $(7, 2.8)$

$m = \frac{2.8 - 2.1}{7 - 1} \approx 0.1167.$

$y - y_1 = m(t - t_1)$

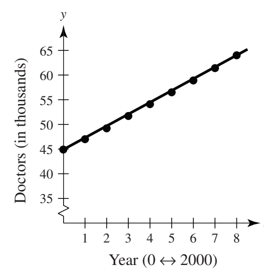
$y - 2.1 = 0.1167(t - 1)$

$y = 0.1167t + 1.9833$

Using a calculator the regression line is

$y = 0.1167t + 1.9667$

135. (a) and (b)



(c) Answers will vary.

Sample answer: Using technology:

$y = 2.3726x + 44.658$

Using 2 points to estimate:

$(3, 51.7), (4, 54.1)$

$m = \frac{54.1 - 51.7}{4 - 3} = 2.4$

$y - y_1 = m(x - x_1)$

$y - 51.7 = 2.4(x - 3)$

$y = 2.4x + 44.5$

(d) Answers will vary.

Sample answer: The slope describes the change (increase or decrease) in the number of osteopathic doctors per year. The y-intercept describes the number of doctors in the year 2000 (when $x = 0$).

(e) The model is a good fit to the data.

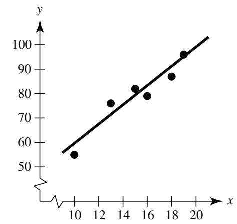
(f) Sample answer: For 2012, use $x = 12$

$y = 2.3726(12) + 44.658$

$y \approx 73.1$

In the year 2012, there will be approximately 73.1 thousand osteopathic doctors.

136. (a) and (b)



(c) Two approximate points on the line are $(10, 55)$ and $(19, 96)$.

$$m = \frac{96 - 55}{19 - 10} = \frac{41}{9}$$

$$y - 55 = \frac{41}{9}(x - 10)$$

$$y = \frac{41}{9}x + \frac{85}{9}$$

(d) $y = \frac{41}{9}(17) + \frac{85}{9} \approx 87$

(e) Each point will shift four units upward, so the best-fitting line will move four units upward. The slope remains the same, as the new line is parallel to the old, but the y-intercept becomes

$$\left(0, \frac{85}{9} + 4\right) = \left(0, \frac{121}{9}\right)$$

so the new equation is $y = \frac{41}{9}x + \frac{121}{9}$.

137. False. The slope with the greatest magnitude corresponds to the steepest line.

138. False, the lines are not parallel.

$(-8, 2)$ and $(-1, 4)$: $m_1 = \frac{4 - 2}{-1 - (-8)} = \frac{2}{7}$

$(0, -4)$ and $(-7, 7)$: $m_2 = \frac{7 - (-4)}{-7 - 0} = \frac{11}{-7}$

139. Using the Distance Formula, we have $AB = 6$, $BC = \sqrt{40}$, and $AC = 2$. Since $6^2 + 2^2 = \left(\sqrt{40}\right)^2$, the triangle is a right triangle.

140. On a vertical line, all the points have the same x-value, so when you evaluate $m = \frac{y_2 - y_1}{x_2 - x_1}$, you would have a zero in the denominator, and division by zero is undefined.

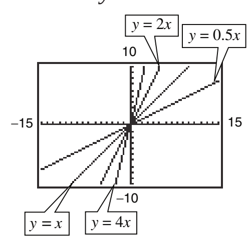
141. No. The slope cannot be determined without knowing the scale on the y-axis. The slopes will be the same if the scale on the y-axis of (a) is $2\frac{1}{2}$ and the scale on the y-axis of (b) is 1. Then the slope of both is $\frac{5}{4}$.

142. Because $|-4| > \left|\frac{5}{2}\right|$, the steeper line is the one with a slope of -4. The slope with the greatest magnitude corresponds to the steepest line.

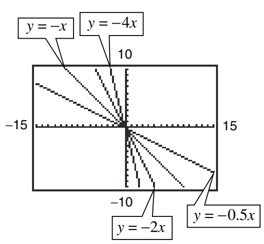
143. Both lines have positive slope, but their y-intercepts differ in sign. Matches (c).

144. The lines intersect in the first quadrant at a point (x, y) where $x < y$. Matches (a).

145. The line $y = 4x$ rises most quickly.



The line $y = -4x$ falls most quickly.



The greater the magnitude of the slope (the absolute value of the slope), the faster the line rises or falls.

146.
$$d_1 = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(1 - 0)^2 + (m_1 - 0)^2} = \sqrt{1 + (m_1)^2}$$
$$d_2 = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(1 - 0)^2 + (m_2 - 0)^2} = \sqrt{1 + (m_2)^2}$$

Using the Pythagorean Theorem:

$$(d_1)^2 + (d_2)^2 = \left(\text{distance between } (1, m_1), \text{ and } (1, m_2)\right)^2$$
$$\left(\sqrt{1 + (m_1)^2}\right)^2 + \left(\sqrt{1 + (m_2)^2}\right)^2 = \left(\sqrt{(1 - 1)^2 + (m_2 - m_1)^2}\right)^2$$
$$1 + (m_1)^2 + 1 + (m_2)^2 = (m_2 - m_1)^2$$
$$(m_1)^2 + (m_2)^2 + 2 = (m_2)^2 - 2m_1m_2 + (m_1)^2$$
$$2 = -2m_1m_2$$
$$-\frac{1}{m_2} = m_1$$

147. No, the slopes of two perpendicular lines have opposite signs. (Assume that neither line is vertical or horizontal.)

148. (a) Matches graph (ii).

The slope is –20, which represents the decrease in the amount of the loan each week. The y-intercept is $(0, 200)$, which represents the original amount of the loan.

(b) Matches graph (iii).

The slope is 2, which represents the increase in the hourly wage for each unit produced. The y-intercept is $(0, 8.5)$, which represents the hourly rate if the employee produces no units.

(c) Matches graph (i).

The slope is 0.32, which represents the increase in travel cost for each mile driven. The y-intercept is $(0, 30)$, which represents the fixed cost of $30 per day for meals. This amount does not depend on the number of miles driven.

(d) Matches graph (iv).

The slope is –100, which represents the amount by which the computer depreciates each year. The y-intercept is $(0, 750)$, which represents the original purchase price.

Review Exercises for Chapter P

1. $6 - (x-2)^2 = 6 - (x^2 - 4x + 4)$

$= 2 + 4x - x^2$

The equation is an identity.

2. $3(x-2) + 2x = 2(x+3)$

$3x - 6 + 2x = 2x + 6$

$3x = 12$

Conditional equation

3. $-x^3 + x(7 - x) + 3 = -x^3 - x^2 + 7x + 3$

$x(-x^2 - x) + 7(x+1) - 4 = -x^3 - x^2 + 7x + 3$

The equation is an identity.

4. $3(x^2 - 4x + 8) = -10(x+2) - 3x^2 + 6$

$3x^2 - 12x + 24 = -10x - 20 - 3x^2 + 6$

$6x^2 - 2x + 38 = 0$

Conditional equation

5. $3x - 2(x+5) = 10$

$3x - 2x - 10 = 10$

$x = 20$

6. $4x + 2(7 - x) = 5$

$4x + 14 - 2x = 5$

$2x = -9$

$x = -\frac{9}{2}$

7. $4(x+3) - 3 = 2(4 - 3x) - 4$

$4x + 12 - 3 = 8 - 6x - 4$

$4x + 9 = -6x + 4$

$10x = -5$

$x = -\frac{1}{2}$

8. $\frac{1}{2}(x+3) - 2(x+1) = 5$

$x - 3 - 4x - 4 = 10$

$-3x = 17$

$x = -\frac{17}{3}$

9. $2x^2 - x - 28 = 0$

$(2x + 7)(x - 4) = 0$

$2x + 7 = 0$ or $x - 4 = 0$

$x = -\frac{7}{2}$ or $x = 4$

10. $15 + x - 2x^2 = 0$

$2x^2 - x - 15 = 0$

$(2x + 5)(x - 3) = 0$

$x = -\frac{5}{2}, 3$

11. $16x^2 = 25$

$x^2 = \frac{25}{16}$

$x = \pm\sqrt{\frac{25}{16}} = \pm\frac{5}{4}$

12. $6 = 3x^2$

$x^2 = 2$

$x = \pm\sqrt{2}$

13. $(x - 8)^2 = 15$

$x - 8 = \pm\sqrt{15}$

$x = 8 \pm \sqrt{15}$

14. $(x + 4)^2 = 18$

$x + 4 = \pm 3\sqrt{2}$

$x = -4 \pm 3\sqrt{2}$

15. $x^2 + 6x - 3 = 0$

$a = 1, b = 6, c = -3$

$$x = \frac{-6 \pm \sqrt{6^2 - 4(1)(-3)}}{2(1)}$$

$$= \frac{-6 \pm \sqrt{48}}{2} = -3 \pm 2\sqrt{3}$$

16. $x^2 - 12x + 30 = 0$

$$x = \frac{-(-12) \pm \sqrt{(-12)^2 - 4(1)(30)}}{2(1)}$$

$$= 6 \pm \sqrt{6}$$

17. $-20 - 3x + 3x^2 = 0$

$3x^2 - 3x - 20 = 0$

$a = 3, b = -3, c = -20$

$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(3)(-20)}}{2(3)}$$

$$= \frac{3 \pm \sqrt{249}}{6} = \frac{1}{2} \pm \frac{\sqrt{249}}{6}$$

18. $-2x^2 - 5x + 27 = 0$

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(-2)(27)}}{2(-2)}$$

$$= \frac{-5 \pm \sqrt{241}}{4}$$

19. $4x^3 - 6x^2 = 0$

$x^2(4x - 6) = 0$

$x^2 = 0 \Rightarrow x = 0$

$4x - 6 = 0 \Rightarrow x = \frac{3}{2}$

20. $5x^4 - 12x^3 = 0$

$x^3(5x - 12) = 0$

$x = 0, \frac{12}{5}$

21. $9x^4 + 27x^3 - 4x^2 - 12x = 0$

$9x^3(x + 3) - 4x(x + 3) = 0$

$(9x^3 - 4x)(x + 3) = 0$

$x(9x^2 - 4)(x + 3) = 0$

$x(3x + 2)(3x - 2)(x + 3) = 0$

$x = 0$

$3x + 2 = 0 \Rightarrow x = -\frac{2}{3}$

$3x - 2 = 0 \Rightarrow x = \frac{2}{3}$

$x + 3 = 0 \Rightarrow x = -3$

22. $x^4 - 5x^2 + 6 = 0$

Let $u = x^2$.

$u^2 - 5u + 6 = 0$

$(u - 2)(u - 3) = 0$

$u = 2, 3$

$x^2 = 2, 3$

$x = \pm\sqrt{2}, \pm\sqrt{3}$

23. $\sqrt{x - 2} - 8 = 0$

$\sqrt{x - 2} = 8$

$x - 2 = 64$

$x = 66$

24. $\sqrt{x + 4} = 3$

$x + 4 = 9$

$x = 5$

25. $\sqrt{3x - 2} = 4 - x$

$3x - 2 = (4 - x)^2$

$3x - 2 = 16 - 8x + x^2$

$0 = 18 - 11x + x^2$

$0 = (x - 9)(x - 2)$

$0 = x - 9 \Rightarrow x = 9$, extraneous

$0 = x - 2 \Rightarrow x = 2$

26. $2\sqrt{x} - 5 = x$

$2\sqrt{x} = x + 5$

$4x = x^2 + 10x + 25$

$x^2 + 6x + 25 = 0$

$b^2 - 4ac = 6a^2 - 4(1)(25) = -64 < 0 \Rightarrow$ no real solutions

Original equation has no real solutions.

27. $(x+2)^{3/4} = 27$

$x + 2 = 27^{4/3}$

$x + 2 = 81$

$x = 79$

28. $(x-1)^{2/3} - 25 = 0$

$x - 1 = 25^{3/2}$

$x - 1 = \pm 125$

$x = 126, -124$

29. $8x^2(x^2-4)^{1/3} + (x^2-4)^{4/3} = 0$

$(x^2-4)^{1/3}[8x^2 + x^2 - 4] = 0$

$(x^2-4)^{1/3}(9x^2-4) = 0$

$(x-2)^{1/3}(x+2)^{1/3}(3x-2)(3x+2) = 0$

$x - 2 = 0 \Rightarrow x = 2$

$x + 2 = 0 \Rightarrow x = -2$

$3x - 2 = 0 \Rightarrow x = \frac{2}{3}$

$3x + 2 = 0 \Rightarrow x = -\frac{2}{3}$

30. $(x+4)^{1/2} + 5x(x+4)^{3/2} = 0$

$(x+4)^{1/2}[1 + 5x(x+4)] = 0$

$(x+4)^{1/2}[5x^2 + 20x + 1] = 0$

$(x+4)^{1/2} = 0$

$x = -4$

$5x^2 + 20x + 1 = 0$

$$x = \frac{-20 \pm \sqrt{20^2 - 4(5)(1)}}{2(5)} = \frac{-10 \pm \sqrt{95}}{5}$$

$$x = -4, \frac{-10 \pm \sqrt{95}}{5}$$

31. $|2x + 3| = 7$

$2x + 3 = 7$ or $2x + 3 = -7$

$2x = 4$ or $2x = -10$

$x = 2$ $\quad x = -5$

32. $|x - 5| = 10$

First Equation	*Second Equation*
$x - 5 = 10$	$x - 5 = -10$
$x = 15$	$x = -5$

33. $|x^2 - 6| = x$

$x^2 - 6 = x$

$x^2 - x - 6 = 0$

$(x-3)(x+2) = 0$

$x - 3 = 0 \Rightarrow x = 3$

$x + 2 = 0 \Rightarrow x = -2$, extraneous

or

$-(x^2 - 6) = x$

$x^2 + x - 6 = 0$

$(x+3)(x-2) = 0$

$x - 2 = 0 \Rightarrow x = 2$

$x + 3 = 0 \Rightarrow x = -3$, extraneous

34. $|x^2 - 3| = 2x$

First Equation

$x^2 - 3 = 2x$

$x^2 - 2x - 3 = 0$

$(x-3)(x+1) = 0$

$x - 3 = 0 \Rightarrow x = 3$

$x + 1 = 0 \Rightarrow x = -1$, not a solution

Second Equation

$(-x^2 - 3) = 2x$

$x^2 + 2x - 3 = 0$

$(x+3)(x-1) = 0$

$x + 3 = 0 \Rightarrow x = -3$, not a solution

$x - 1 = 0 \Rightarrow x = 1$

35. Let x = the number of liters of pure antifreeze.

30% of $(10 - x)$ + 100% of x = 50% of 10

$0.30(10 - x) + 1.00x = 0.50(10)$

$3 - 0.30x + 1.00x = 5$

$0.70x = 2$

$$x = \frac{2}{0.70} = \frac{20}{7} = 2\frac{6}{7} \text{ liters}$$

36. $42 - \sqrt{0.001x + 2} = 29.95$

$42 - 29.95 = \sqrt{0.001x + 2}$

$(12.05)^2 = 0.001x + 2$

$x = 143{,}202.5$

Number of units demanded per day: 143,203

37. $6x - 17 > 0$

(a) $x = 3$

$6(3) - 17 \overset{?}{>} 0$

$1 > 0$

Yes, $x = 3$ is a solution.

(b) $x = -4$

$6(-4) - 17 \overset{?}{>} 0$

$-41 \not> 0$

No, $x = -4$ is not a solution.

38. $-3 \le \dfrac{x - 3}{5} < 2$

(a) $x = 3$

$-3 \overset{?}{\le} \dfrac{3 - 3}{5} \overset{?}{<} 2$

$-3 \le 0 < 2$

Yes, $x = 3$ is a solution.

(b) $x = -12$

$-3 \overset{?}{\le} \dfrac{-12 - 3}{5} \overset{?}{<} 2$

$-3 \le -3 < 2$

Yes, $x = -12$ is a solution.

39. $9x - 8 \le 7x + 16$

$2x \le 24$

$x \le 12$

$(-\infty, 12]$

40. $4(5 - 2x) \le \frac{1}{2}(8 - x)$

$20 - 8x \le 4 - \frac{1}{2}x$

$-\frac{15}{2}x \le -16$

$x \ge \frac{32}{15}$

$\left[\frac{32}{15}, \infty\right)$

41. $\frac{15}{2}x + 4 > 3x - 5$

$15x + 8 > 6x - 10$

$9x > -18$

$x > -2$

$(-2, \infty]$

42. $\frac{1}{2}(3 - x) > \frac{1}{3}(2 - 3x)$

$3(3 - x) > 2(2 - 3x)$

$9 - 3x > 4 - 6x$

$3x > -5$

$x > -\frac{5}{3}$

$\left(-\frac{5}{3}, \infty\right)$

43. $-19 < \dfrac{3x - 17}{2} \le 34$

$-38 < 3x - 17 \le 68$

$-21 < 3x \le 85$

$-7 < x \le \dfrac{85}{3}$

$-7 < x \le 28\dfrac{1}{3}$ or $\left(-7, 28\dfrac{1}{3}\right]$

44. $-3 \le \dfrac{2x - 5}{3} < 5$

$-9 \le 2x - 5 < 15$

$-4 \le 2x < 20$

$-2 \le x < 10$

$[-2, 10)$

45. $|x + 1| \le 5$

$-5 \le x + 1 \le 5$

$-6 \le x \le 4$ or $[-6, 4)$

46. $|x - 2| < 1$

$-1 < x - 2 < 1$

$1 < x < 3$

$(1, 3)$

47. $|x - 3| > 4$

$x - 3 > 4$ or $x - 3 < -4$

$x > 7$ $\quad$ $x < -1$

$(-\infty, -1) \cup (7, \infty)$

48. $\left|x - \frac{3}{2}\right| \ge \frac{3}{2}$

$x - \frac{3}{2} \le -\frac{3}{2}$ $\quad$ $x - \frac{3}{2} \ge \frac{3}{2}$

$x \le 0$ $\quad$ $x \ge 3$

$(-\infty, 0], [3, \infty)$

49. $R > C$

$125.33x > 92x + 1200$

$33.33x > 1200$

$x > \dfrac{1200}{33.33} \approx 36$ units

50. $A = s^2$

$s = 19.3 \pm 0.5$ cm

$A = (19.3)^2 \approx 372\text{ cm}^2$

Smallest area: $A = (18.8)^2 \approx 353\text{ cm}^2$

Largest area: $A = (19.8)^2 \approx 392\text{ cm}^2$

Interval containing area of square:
$353\text{ cm}^2 < A < 392\text{ cm}^2$

51. $x^2 - 6x - 27 < 0$

$(x + 3)(x - 9) < 0$

Critical numbers: $x = -3, x = 9$

Test intervals: $(-\infty, -3), (-3, 9), (9, \infty)$

Test: Is $(x + 3)(x - 9) < 0$?

Solution set: $(-3, 9)$

52. $x^2 - 2x \geq 3$

$x^2 - 2x - 3 \geq 0$

$(x - 3)(x + 1) \geq 0$

Critical numbers: $-1, 3$

Test intervals: $(-\infty, -1) \Rightarrow (x - 3)(x + 1) > 0$

$(-1, 3) \Rightarrow (x - 3)(x + 1) < 0$

$(3, \infty) \Rightarrow (x - 3)(x + 1) > 0$

Solution intervals: $(-\infty, -1] \cup [3, \infty)$

53. $6x^2 + 5 < 4$

$6x^2 + 5x - 4 < 0$

$(3x + 4)(2x - 1) < 0$

Critical numbers: $x = -\frac{4}{3}, x = \frac{1}{2}$

Test intervals: $\left(-\infty, -\frac{4}{3}\right), \left(-\frac{4}{3}, \frac{1}{2}\right), \left(\frac{1}{2}, \infty\right)$

Test: Is $(3x + 4)(2x - 1) < 0$?

Solution set: $\left(-\frac{4}{3}, \frac{1}{2}\right)$

54. $2x^2 + x \geq 15$

$2x^2 + x - 15 \geq 0$

$(2x - 5)(x + 3) \geq 0$

Critical numbers: $-3, \frac{5}{2}$

Test intervals: $(-\infty, -3) \Rightarrow (2x - 5)(x + 3) > 0$

$\left(-3, \frac{5}{2}\right) \Rightarrow (2x - 5)(x + 3) < 0$

$\left(\frac{5}{2}, \infty\right) \Rightarrow (2x - 5)(x + 3) > 0$

Solution intervals: $(-\infty, -3] \cup \left[\frac{5}{2}, \infty\right)$

55. $\dfrac{2}{x + 1} \leq \dfrac{3}{x - 1}$

$\dfrac{2(x - 1) - 3(x + 1)}{(x + 1)(x - 1)} \leq 0$

$\dfrac{2x - 2 - 3x - 3}{(x + 1)(x - 1)} \leq 0$

$\dfrac{-(x + 5)}{(x + 1)(x - 1)} \leq 0$

Critical numbers: $x = -5, x = \pm 1$

Test intervals: $(-\infty, -5), (-5, -1), (-1, 1), (1, \infty)$

Test: Is $\dfrac{-(x + 5)}{(x + 1)(x - 1)} \leq 0$?

Solution set: $[-5, -1) \cup (1, \infty)$

56. $\dfrac{x - 5}{3 - x} < 0$

Critical numbers: $3, 5$

Test intervals: $(-\infty, 3) \Rightarrow \dfrac{x - 5}{3 - x} < 0$

$(3, 5) \Rightarrow \dfrac{x - 5}{3 - x} > 0$

$(5, \infty) \Rightarrow \dfrac{x - 5}{3 - x} < 0$

Solution intervals: $(-\infty, 3) \cup (5, \infty)$

57. $\dfrac{x^2 + 7x + 12}{x} \geq 0$

$\dfrac{(x + 4)(x + 3)}{x} \geq 0$

Critical numbers: $x = -4, x = -3, x = 0$

Test intervals: $(-\infty, -4), (-4, -3), (-3, 0), (0, \infty)$

Test: Is $\dfrac{(x + 4)(x + 3)}{x} \geq 0$?

Solution set: $[-4, -3] \cup (0, \infty)$

58. $$\frac{1}{x-2} > \frac{1}{x}$$
$$\frac{1}{x-2} - \frac{1}{x} > 0$$
$$\frac{2}{x(x-2)} > 0$$

Critical numbers: 0, 2

Test intervals: $(-\infty, 0) \Rightarrow \frac{2}{x(x-2)} > 0$

$(0, 2) \Rightarrow \frac{2}{x(x-2)} < 0$

$(2, \infty) \Rightarrow \frac{2}{x(x-2)} > 0$

Solution intervals: $(-\infty, 0) \cup (2, \infty)$

59. $$5000(1+r)^2 > 5500$$
$$(1+r)^2 > 1.1$$
$$1 + r > 1.0488$$
$$r > 0.0488$$
$$r > 4.9\%$$

60. $$P = \frac{1000(1+3t)}{5+t} \geq 2000$$
$$\frac{1000(1+3t)}{5+t} - 2000 \geq 0$$
$$\frac{1000t - 9000}{t+5} \geq 0$$

t must be greater than zero, so the critical value to check is $t = 9$.

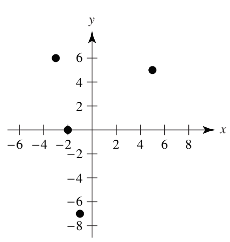
$(0, 9) \Rightarrow \frac{1000t - 9000}{t+5} < 0$

$(9, \infty) \Rightarrow \frac{1000t - 9000}{t+5} > 0$

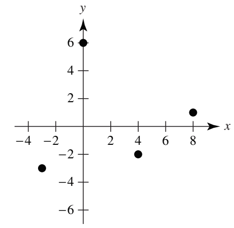
$t \geq 9$

So, the required time is at least 9 days.

61.



62.



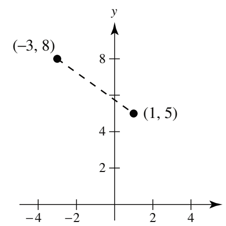
63. $x > 0$ and $y = -2$ in Quadrant IV.

64. Because the products positive, x and y must have the same sign.

$x > 0$ and $y > 0$ in Quadrant I.

$x < 0$ and $y < 0$ in Quadrant III.

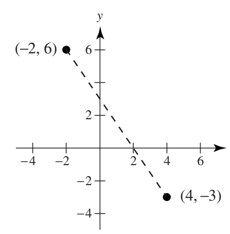
65. (a)



(b) $d = \sqrt{(-3-1)^2 + (8-5)^2} = \sqrt{16+9} = 5$

(c) Midpoint: $\left(\frac{-3+1}{2}, \frac{8+5}{2}\right) = \left(-1, \frac{13}{2}\right)$

66. (a)

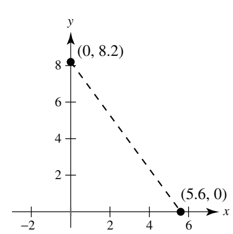


(b) $d = \sqrt{(-2-4)^2 + (6+3)^2}$

$= \sqrt{36+81} = \sqrt{117} = 3\sqrt{13}$

(c) Midpoint: $\left(\frac{-2+4}{2}, \frac{6-3}{2}\right) = \left(1, \frac{3}{2}\right)$

67. (a)

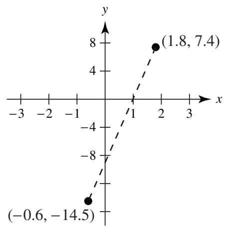


(b) $d = \sqrt{(5.6-0)^2 + (0-8.2)^2}$

$= \sqrt{31.36 + 67.24} = \sqrt{98.6} \approx 9.9$

(c) Midpoint: $\left(\frac{0+5.6}{2}, \frac{8.2+0}{2}\right) = (2.8, 4.1)$

68. (a)



(b) $d = \sqrt{\left(1.8 - (-0.6)\right)^2 + \left(7.4 - (-14.5)\right)^2}$

$d = \sqrt{(2.4)^2 + (21.9)^2}$

$d = \sqrt{485.37}$

$d \approx 22.03$ units

(c) Midpoint:

$\left(\dfrac{1.8 - 0.6}{2}, \dfrac{7.4 - 14.5}{2}\right)$

$(0.6, -3.55)$

69. New vertices:

$(4 - 4, 8 - 8) = (0, 0)$

$(6 - 4, 8 - 8) = (2, 0)$

$(4 - 4, 3 - 8) = (0, -5)$

$(6 - 4, 3 - 8) = (2, -5)$

70. New vertices:

$(0 - 2, 1 + 3) = (-2, 4)$

$(3 - 2, 3 + 3) = (1, 6)$

$(0 - 2, 5 + 3) = (-2, 8)$

$(-3 - 2, 3 + 3) = (-5, 6)$

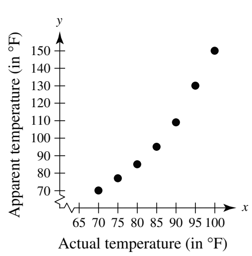
71. Midpoint:

$\left(\dfrac{2000 + 2008}{2}, \dfrac{2.17 + 10.38}{2}\right)$

$(2004, 6.275)$

In 2004, the sales were $6.275 billion.

72. (a)

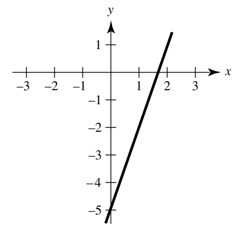


(b) Change in apparent temperature $= 150°\text{F} - 70°\text{F}$

$= 80°\text{F}$

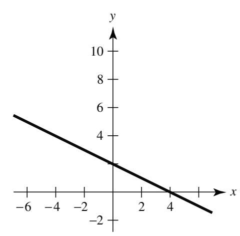
73. $y = 3x - 5$

x	−2	−1	0	1	2
y	−11	−8	−5	−2	1



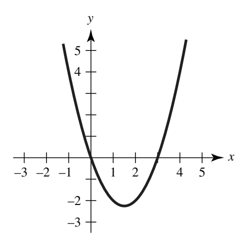
74. $y = -\frac{1}{2}x + 2$

x	−4	−2	0	2	4
y	4	3	2	1	0



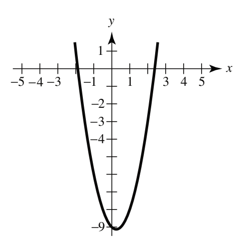
75. $y = x^2 - 3x$

x	−1	0	1	2	3	4
y	4	0	−2	−2	0	4



76. $y = 2x^2 - x - 9$

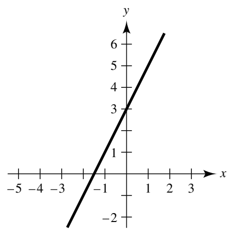
x	−2	−1	0	1	2	3
y	1	−6	−9	−8	−3	6



77. $y - 2x - 3 = 0$

$y = 2x + 3$

Line with x-intercept $\left(-\frac{3}{2}, 0\right)$ and y-intercept $(0, 3)$

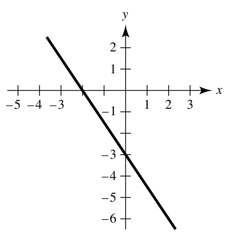


78. $3x + 2y + 6 = 0$

$2y = -3x - 6$

$y = -\frac{3}{2}x - 3$

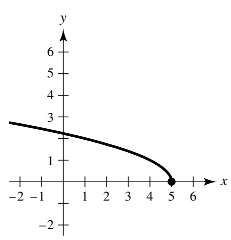
Line with x-intercept $(-2, 0)$ and y-intercept $(0, -3)$



79. $y = \sqrt{5 - x}$

Domain: $(-\infty, 5]$

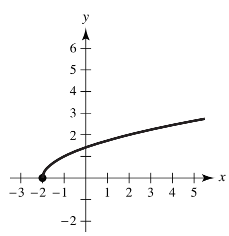
x	5	4	1	–4
y	0	1	2	3



80. $y = \sqrt{x + 2}$

Domain: $[-2, \infty)$

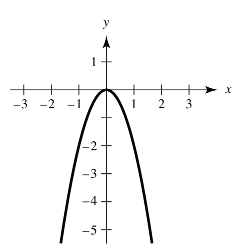
x	–2	0	2	7
y	0	$\sqrt{2}$	2	3



81. $y + 2x^2 = 0$

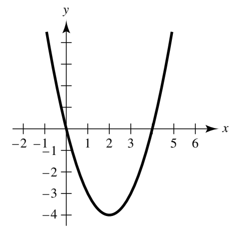
$y = -2x^2$ is a parabola.

x	0	±1	±2
y	0	–2	–8



82. $y = x^2 - 4x$ is a parabola.

x	–1	0	1	2	3	4
y	5	0	–3	–4	–3	0



83. $y = 2x + 7$

x-intercept: Let $y = 0$.

$0 = 2x + 7$

$x = -\frac{7}{2}$

$\left(-\frac{7}{2}, 0\right)$

y-intercept: Let $x = 0$.

$y = 2(0) + 7$

$y = 7$

$(0, 7)$

84. x-intercept: Let $y = 0$.

$y = |x + 1| - 3$

$0 = |x + 1| - 3$

For $x + 1 > 0, 0 = x + 1 - 3$, or $2 = x$.

For $x + 1 < 0, 0 = -(x + 1) - 3$, or $-4 = x$.

$(2, 0), (4, 0)$

y-intercept: Let $x = 0$.

$y = |x + 1| - 3$

$y = |0 + 1| - 3$ or $y = -2$

$(0, -2)$

85. $y = (x - 3)^2 - 4$

x-intercepts: $0 = (x - 3)^2 - 4 \Rightarrow (x - 3)^2 = 4$

$\Rightarrow x - 3 = \pm 2$

$\Rightarrow x = 3 \pm 2$

$\Rightarrow x = 5$ or $x = 1$

$(5, 0), (1, 0)$

y-intercept: $y = (0 - 3)^2 - 4$

$y = 9 - 4$

$y = 5$

$(0, 5)$

86. $y = x\sqrt{4 - x^2}$

x-intercepts: $0 = x\sqrt{4 - x^2}$

$$x = 0 \quad \sqrt{4 - x^2} = 0$$
$$4 - x^2 = 0$$
$$x = \pm 2$$
$$(0, 0), (-2, 0), (2, 0)$$

y-intercept: $y = 0 \cdot \sqrt{4 - 0} = 0$

$(0, 0)$

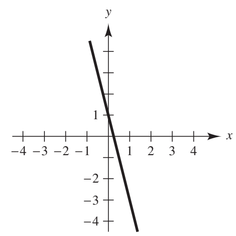
87. $y = -4x + 1$

Intercepts: $\left(\frac{1}{4}, 0\right), (0, 1)$

$y = -4(-x) + 1 \Rightarrow y = 4x + 1 \Rightarrow$ No y-axis symmetry

$-y = -4x + 1 \Rightarrow y = 4x - 1 \Rightarrow$ No x-axis symmetry

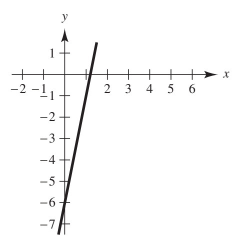
$-y = -4(-x) + 1 \Rightarrow y = -4x - 1 \Rightarrow$ No origin symmetry



88. $y = 5x - 6$

Intercepts: $\left(\frac{6}{5}, 0\right), (0, -6)$

No symmetry



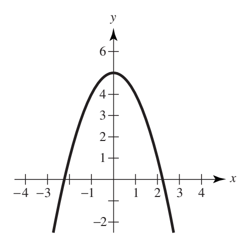
89. $y = 5 - x^2$

Intercepts: $\left(\pm\sqrt{5}, 0\right), (0, 5)$

$y = 5 - (-x)^2 \Rightarrow y = 5 - x^2 \Rightarrow$ y-axis symmetry

$-y = 5 - x^2 \Rightarrow y = -5 + x^2 \Rightarrow$ No x-axis symmetry

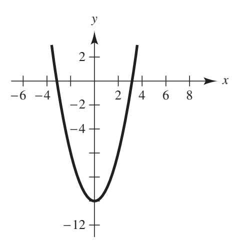
$-y = 5 - (-x)^2 \Rightarrow y = -5 + x^2 \Rightarrow$ No origin symmetry



90. $y = x^2 - 10$

Intercepts: $\left(\pm\sqrt{10}, 0\right), (0, -10)$

y-axis symmetry



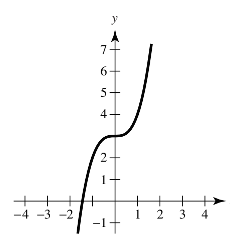
91. $y = x^3 + 3$

Intercepts: $\left(\sqrt[3]{-3}, 0\right), (0, 3)$

$y = (-x)^3 + 3 \Rightarrow y = -x^3 + 3 \Rightarrow$ No y-axis symmetry

$-y = x^3 + 3 \Rightarrow y = -x^3 - 3 \Rightarrow$ No x-axis symmetry

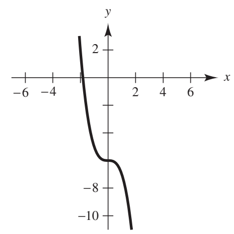
$-y = (-x)^3 + 3 \Rightarrow y = x^3 - 3 \Rightarrow$ No origin symmetry



92. $y = -6 - x^3$

Intercepts: $\left(\sqrt[3]{-6}, 0\right), (0, -6)$

No symmetry



93. $y = \sqrt{x + 5}$

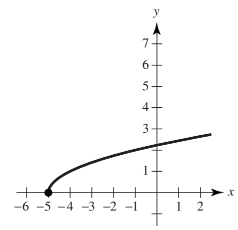
Domain: $[-5, \infty)$

Intercepts: $(-5, 0), \left(0, \sqrt{5}\right)$

$y = \sqrt{-x + 5} \Rightarrow$ No y-axis symmetry

$-y = \sqrt{x + 5} \Rightarrow y = -\sqrt{x + 5} \Rightarrow$ No x-axis symmetry

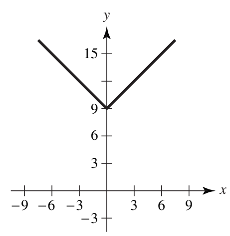
$-y = \sqrt{-x + 5} \Rightarrow y = -\sqrt{-x + 5} \Rightarrow$ No origin symmetry



94. $y = |x| + 9$

Intercepts: $(0, 9)$

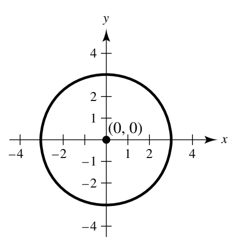
y-axis symmetry



95. $x^2 + y^2 = 9$

Center: $(0, 0)$

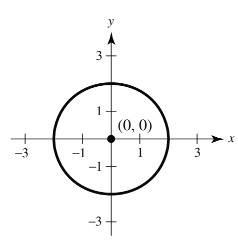
Radius: 3



96. $x^2 + y^2 = 4$

Center: $(0, 0)$

Radius: 2

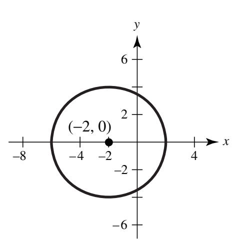


97. $(x + 2)^2 + y^2 = 16$

$(x - (-2))^2 + (y - 0)^2 = 4^2$

Center: $(-2, 0)$

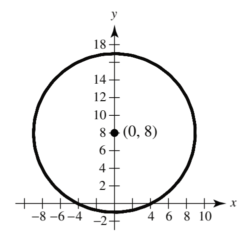
Radius: 4



98. $x^2 + (y - 8)^2 = 81$

Center: $(0, 8)$

Radius: 9

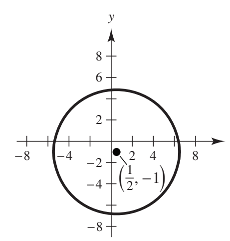


99. $\left(x - \frac{1}{2}\right)^2 + (y + 1)^2 = 36$

$\left(x - \frac{1}{2}\right)^2 + (y - (-1))^2 = 6^2$

Center: $\left(\frac{1}{2}, -1\right)$

Radius: 6

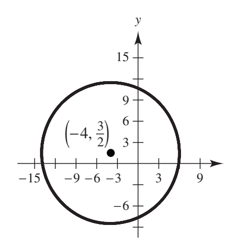


100. $(x + 4)^2 + \left(y - \frac{3}{2}\right)^2 = 100$

$(x - (-4))^2 + \left(y - \frac{3}{2}\right)^2 = 100$

Center: $\left(-4, \frac{3}{2}\right)$

Radius: 10



101. Endpoints of a diameter: $(0, 0)$ and $(4, -6)$

Center: $\left(\dfrac{0 + 4}{2}, \dfrac{0 + (-6)}{2}\right) = (2, -3)$

Radius:

$r = \sqrt{(2 - 0)^2 + (-3 - 0)^2} = \sqrt{4 + 9} = \sqrt{13}$

Standard form: $(x - 2)^2 + (y - (-3))^2 = \left(\sqrt{13}\right)^2$

$(x - 2)^2 + (y + 3)^2 = 13$

102. Endpoints of a diameter: $(-2, -3)$ and $(4, -10)$

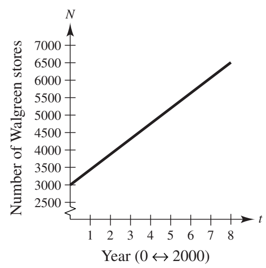
Center: $\left(\frac{-2+4}{2}, \frac{-3+(-10)}{2}\right) = \left(1, -\frac{13}{2}\right)$

Radius: $r = \sqrt{(1-(-2))^2 + \left(-\frac{13}{2} - (-3)\right)^2} = \sqrt{9 + \frac{49}{4}} = \sqrt{\frac{85}{4}}$

Standard form: $(x-1)^2 + \left(y - \left(-\frac{13}{2}\right)\right)^2 = \left(\sqrt{\frac{85}{4}}\right)^2$

$$(x-1)^2 + \left(y + \frac{13}{2}\right)^2 = \frac{85}{4}$$

103. (a)



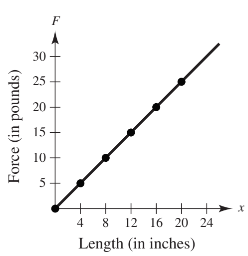
(b) 2008

104. $F = \frac{5}{4}x, 0 \le x \le 20$

(a)

x	0	4	8	12	16	20
F	0	5	10	15	20	25

(b)

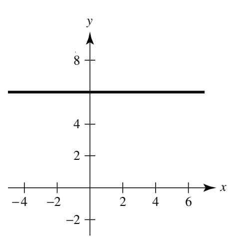


(c) When $x = 10$, $F = \frac{50}{4} = 12.5$ pounds.

105. $y = 6$

Slope: $m = 0$

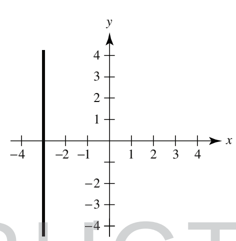
y-intercept: $(0, 6)$



106. $x = -3$

Slope: m is undefined.

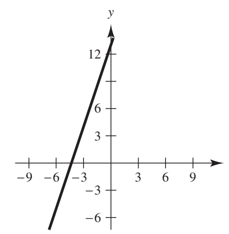
y-intercept: none



107. $y = 3x + 13$

Slope: $m = 3$

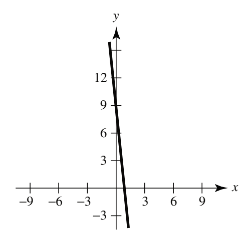
y-intercept: $(0, 13)$



108. $y = -10x + 9$

Slope: $m = -10$

y-intercept: $(0, 9)$



109. (a) $m = \frac{3}{2} > 0 \Rightarrow$ The line rises.

Matches L_2.

(b) $m = 0 \Rightarrow$ The line is horizontal.

Matches L_3.

(c) $m = -3 < 0 \Rightarrow$ The line falls.

Matches L_1.

(d) $m = -\frac{1}{5} < 0 \Rightarrow$ The line gradually falls.

Matches L_4.

110. (a) $m = -\frac{5}{2} \Rightarrow$ The line falls. Matches L_3.

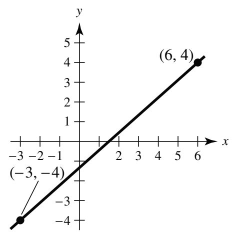
(b) m is undefined. $\Rightarrow$ The line is vertical. Matches L_1.

(c) $m = 0 \Rightarrow$ The line is horizontal. Matches L_4.

(d) $m = \frac{1}{2} \Rightarrow$ The line rises. Matches L_2.

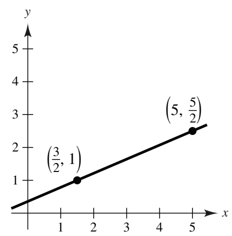
111. $(6, 4), (-3, -4)$

$$m = \frac{4 - (-4)}{6 - (-3)} = \frac{4 + 4}{6 + 3} = \frac{8}{9}$$



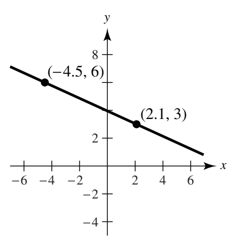
112. $\left(\frac{3}{2}, 1\right), \left(5, \frac{5}{2}\right)$

$$m = \frac{\frac{5}{2} - 1}{5 - \frac{3}{2}} = \frac{\frac{5}{2} - \frac{2}{2}}{\frac{10}{2} - \frac{3}{2}} = \frac{\frac{3}{2}}{\frac{7}{2}} = \frac{6}{14} \text{ or } \frac{3}{7}$$



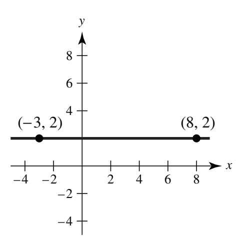
113. $(-4.5, 6), (2.1, 3)$

$$m = \frac{3 - 6}{2.1 - (-4.5)} = \frac{-3}{6.6} = -\frac{30}{66} = -\frac{5}{11}$$



114. $(-3, 2), (8, 2)$

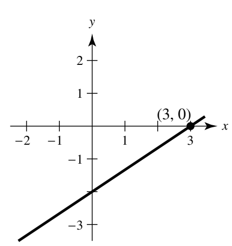
$$m = \frac{2 - 2}{-3 - 8} = \frac{0}{-11} = 0$$



115. $(3, 0), \quad m = \frac{2}{3}$

$$y - 0 = \frac{2}{3}(x - 3)$$

$$y = \frac{2}{3}x - 2$$

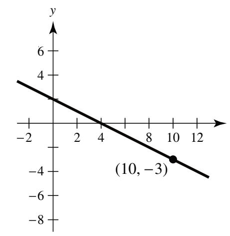


116. $(10, -3), m = -\frac{1}{2}$

$$y - (-3) = -\frac{1}{2}(x - 10)$$

$$y + 3 = -\frac{1}{2}x + 5$$

$$y = -\frac{1}{2}x + 2$$

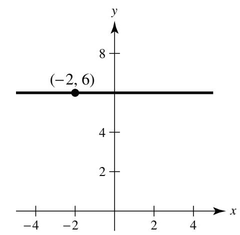


117. Point: $(-2, 6)$

Slope: $m = 0$

$$y - 6 = 0(x - (-2))$$

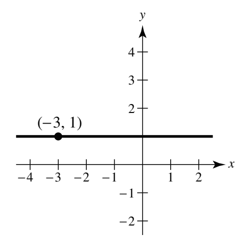
$$y - 6 = 0$$



118. $(-3, 1), m = 0$

$$y - 1 = 0(x + 3)$$

$$y - 1 = 0 \text{ or } y = 1$$

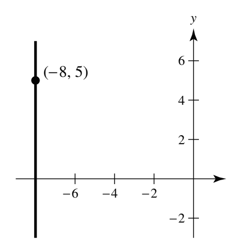


119. Point: $(-8, 5)$

Slope: Undefined

$$x = -8$$

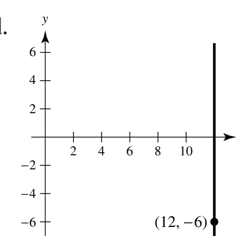
$$x + 8 = 0$$



120. $(12, -6)$, m is undefined.

The line is vertical.

$$x = 12$$



121. $(0, 0), (0, 10)$

$$m = \frac{10 - 0}{0 - 0} = \frac{10}{0}, \text{ undefined}$$

The line is vertical.

$$x = 0$$

122. $(2, -1), (4, -1)$

$$m = \frac{-1 - (-1)}{4 - 2} = \frac{0}{2} = 0$$

$$y - (-1) = 0(x - 2)$$

$$y + 1 = 0$$

$$y = -1$$

123. $(-1, 0), (6, 2)$

$$m = \frac{2 - (0)}{6 - (-1)} = \frac{2}{7}$$

$$y - 0 = \frac{2}{7}(x - (-1))$$

$$y = \frac{2}{7}(x + 1)$$

$$y = \frac{2}{7}x + \frac{2}{7}$$

124. $(11, -2), (6, -1)$

$$m = \frac{-1 - (-2)}{6 - 11} = -\frac{1}{5}$$

$$y - (-2) = -\frac{1}{5}(x - 11)$$

$$5y + 10 = -x + 11$$

$$5y = -x + 1$$

$$y = -\frac{1}{5}x + \frac{1}{5}$$

125. $x - 5 = 0 \Rightarrow x = 5$ and m is undefined.

(a) $(2, -1)$, m is undefined.

$x = 2$ or $x - 2 = 0$

(b) $(2, -1)$, $m = 0$

$y = -1$ or $y + 1 = 0$

126. $x + 4 = 0 \Rightarrow x = -4$ and m is undefined.

(a) $(3, 2)$, m is undefined.

$x = 3$ or $x - 3 = 0$

(b) $(3, 2)$, $m = 0$

$y = 2$ or $y - 2 = 0$

127. $y + 6 = 0 \Rightarrow y = -6$ and $m = 0$

(a) $(-2, 1)$, $m = 0$

$y = 1$ or $y - 1 = 0$

(b) $(-2, 1)$, m is undefined.

$x = -2$ or $x + 2 = 0$

128. $y - 1 = 0 \Rightarrow y = 1$ and $m = 0$

(a) $(3, 4)$, $m = 0$

$y = 4$ or $y - 4 = 0$

(b) $(3, 4)$, m is undefined.

$x = 3$ or $x - 3 = 0$

129. Point: $(3, -2)$

$$5x - 4y = 8$$

$$y = \tfrac{5}{4}x - 2$$

(a) Parallel slope: $m = \frac{5}{4}$

$$y - (-2) = \tfrac{5}{4}(x - 3)$$

$$y + 2 = \tfrac{5}{4}x - \tfrac{15}{4}$$

$$y = \tfrac{5}{4}x - \tfrac{23}{4}$$

(b) Perpendicular slope: $m = -\frac{4}{5}$

$$y - (-2) = -\tfrac{4}{5}(x - 3)$$

$$y + 2 = -\tfrac{4}{5}x + \tfrac{12}{5}$$

$$y = -\tfrac{4}{5}x + \tfrac{2}{5}$$

130. Point: $(-8, 3)$, $2x + 3y = 5$

$$3y = 5 - 2x$$

$$y = \tfrac{5}{3} - \tfrac{2}{3}x$$

(a) Parallel slope: $m = -\frac{2}{3}$

$$y - 3 = -\tfrac{2}{3}(x + 8)$$

$$3y - 9 = -2x - 16$$

$$3y = -2x - 7$$

$$y = -\tfrac{2}{3}x - \tfrac{7}{3}$$

(b) Perpendicular slope: $m = \frac{3}{2}$

$$y - 3 = \tfrac{3}{2}(x + 8)$$

$$2y - 6 = 3x + 24$$

$$2y = 3x + 30$$

$$y = \tfrac{3}{2}x + 15$$

131. $(10, 12{,}500)$, $m = -850$

$$V - 12{,}500 = -850(t - 10)$$

$$V - 12{,}500 = -850t + 8500$$

$$V = -850t + 21{,}000$$

132. $(10, 72.95)$, $m = 5.15$

$$V - 72.95 = 5.15(t - 10)$$

$$V - 72.95 = 5.15t - 51.5$$

$$V = 5.15t + 21.45, 10 \le t \le 15$$

133. Sample answer:

When $x = 20$

$$x + \sqrt{x - a} = b$$
$$20 + \sqrt{20 - a} = b$$

If $a = 20$,

$$20 + \sqrt{0} = b$$
$$b = 20$$

So, $a = 20, b = 20$

134. Isolate the radical by subtracting x from both sides of the equation. Then square both sides to eliminate the radical, and rewrite the equation in standard form. The solutions can now be found by using the Quadratic Formula. Each solution must be checked because extraneous solutions may be included.

Chapter Test for Chapter P

1.
$$\frac{2}{3}(x - 1) + \frac{1}{4}x = 10$$
$$12\left[\frac{2}{3}(x - 1) + \frac{1}{4}x\right] = 12(10)$$
$$8(x - 1) + 3x = 120$$
$$8x - 8 + 3x = 120$$
$$11x = 128$$
$$x = \frac{128}{11}$$

2.
$$\frac{x - 2}{x + 2} + \frac{4}{x + 2} + 4 = 0$$
$$\frac{x + 2}{x + 2} = -4$$
$1 \neq -4 \Rightarrow$ No solution

3.
$$(x - 3)(x + 2) = 14$$
$$x^2 - x - 6 = 14$$
$$x^2 - x - 20 = 0$$
$$(x + 4)(x - 5) = 0$$
$$x = -4 \quad \text{or} \quad x = 5$$

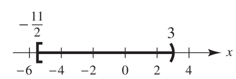
4.
$$x^4 + x^2 - 6 = 0$$
$$(x^2 - 2)(x^2 + 3) = 0$$
$$x^2 = 2 \Rightarrow x = \pm\sqrt{2}$$
$$x^2 = -3 \Rightarrow x = \pm\sqrt{3}i$$

5.
$$x - \sqrt{2x + 1} = 1$$
$$x - 1 = \sqrt{2x + 1}$$
$$x^2 - 2x + 1 = 2x + 1$$
$$x^2 - 4x = 0$$
$$x(x - 4) = 0$$
$$x = 0$$
$$x - 4 = 0 \Rightarrow x = 4$$

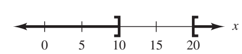
Only $x = 4$ is a solution to the original equation. $x = 0$ is extraneous.

6.
$$|3x - 1| = 7$$
$$3x - 1 = 7 \quad \text{or} \quad 3x - 1 = -7$$
$$3x = 8 \qquad 3x = -6$$
$$x = \frac{8}{3} \qquad x = -2$$

7.
$$-3 \le 2(x + 4) < 14$$
$$-3 \le 2x + 8 < 14$$
$$-11 \le 2x < 6$$
$$-\frac{11}{2} \le x < 3$$



8.
$$|x - 15| \ge 5$$
$$x - 15 \le -5 \quad \text{or} \quad x - 15 \ge 5$$
$$x \le 10 \qquad x \ge 20$$



9.
$$2x^2 + 5x > 12$$
$$2x^2 + 5x - 12 > 0$$
$$(2x - 3)(x + 4) > 0$$

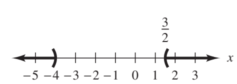
Critical numbers: $x = \frac{3}{2}, x = -4$

Test intervals: $(-\infty, -4), \left(-4, \frac{3}{2}\right), \left(\frac{3}{2}, \infty\right)$

Test: Is $(2x - 3)(x + 4) > 0$?

Solution set: $(-\infty, -4) \cup \left(\frac{3}{2}, \infty\right)$

In inequality notation: $x < -4$ or $x > \frac{3}{2}$



10.
$$\frac{2}{x} > \frac{5}{x+6}$$
$$\frac{2}{x} - \frac{5}{x+6} > 0$$
$$\frac{2(x+6) - 5x}{x(x+6)} > 0$$
$$\frac{-3x + 12}{x(x+6)} > 0$$
$$\frac{-3(x-4)}{x(x+6)} > 0$$

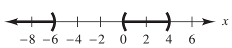
Critical numbers: $x = 4,\ x = 0,\ x = -6$

Test intervals: $(-\infty, -6), (-6, 0), (0, 4), (4, \infty)$

Test: Is $\frac{-3(x-4)}{x(x+6)} > 0$?

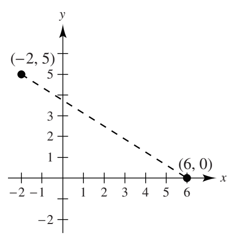
Solution set: $(-\infty, -6) \cup (0, 4)$

In inequality notation: $x < -6$ or $0 < x < 4$



11. Midpoint: $\left(\frac{-2+6}{2}, \frac{5+0}{2}\right) = \left(2, \frac{5}{2}\right)$

Distance: $d = \sqrt{(-2-6)^2 + (5-0)^2}$
$= \sqrt{64 + 25}$
$= \sqrt{89}$



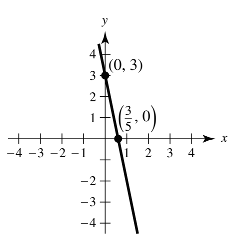
12. $(-2 - 2, 1 - 3) = (-4, -2)$
$(4 - 2, -1 - 3) = (2, -4)$
$(5 - 2, 2 - 3) = (3, -1)$

13. $y = 3 - 5x$

x-intercept: $\left(\frac{3}{5}, 0\right)$

y-intercept: $(0, 3)$

No axis or origin symmetry

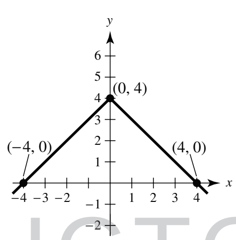


14. $y = 4 - |x|$

x-intercepts: $(\pm 4, 0)$

y-intercept: $(0, 4)$

y-axis symmetry

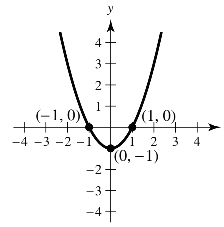


15. $y = x^2 - 1$

x-intercepts: $(\pm 1, 0)$

y-intercept: $(0, -1)$

y-axis symmetry

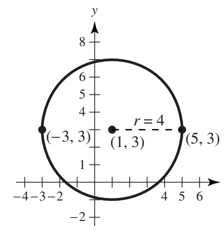


16. Center: $(1, 3)$

Radius: 4

Standard form:

$(x - 1)^2 + (y - 3)^2 = 16$



17. $(2, -3)$ and $(-4, 9)$

$$m = \frac{9 - (-3)}{-4 - 2} = -2$$
$$y - (-3) = -2(x - 2)$$
$$y + 3 = -2x + 4$$
$$y = -2x + 1$$

18. $(3, 0.8)$ and $(7, -6)$

$$m = \frac{-6 - 0.8}{7 - 3} = -1.7$$
$$y - (-6) = -1.7(x - 7)$$
$$y + 6 = -1.7x + 11.9$$
$$y = -1.7x + 5.9$$

19. $5x + 2y = 3$
$$2y = -5x + 3$$
$$y = -\frac{5}{2}x + \frac{3}{2}$$

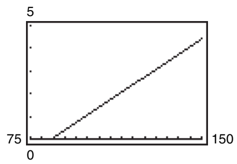
(a) $m = -\frac{5}{2}, (0, 4)$

$$y - 4 = -\frac{5}{2}(x - 0)$$
$$y - 4 = -\frac{5}{2}x$$
$$y = -\frac{5}{2}x + 4 \quad \text{or} \quad 5x + 2y - 8 = 0$$

(b) $m = \frac{2}{5}, (0, 4)$

$$y - 4 = \frac{2}{5}(x - 0)$$
$$y - 4 = \frac{2}{5}x$$
$$y = \frac{2}{5}x + 4 \quad \text{or} \quad -2x + 5y - 20 = 0$$

20. (a) $y = 0.067x - 5.638$



(b) From the graph you see that $y \geq 3$ when $x \geq 129$.

Algebraically:

$$3 \leq 0.067x - 5.638$$
$$8.638 \leq 0.067x$$
$$x \geq 129$$

IQ scores are not a good predictor of GPAs. Other factors include study habits, class attendance, and attitude.

21. $r = 220 - A = 220 - 20 = 200$ beats per minute

$$0.50(200) \leq r \leq 0.85(200)$$
$$100 \leq r \leq 170$$

The target heartrate is at least 100 beats per minute and at most 170 beats per minute.

Problem Solving for Chapter P

1. (a) $3(x + 4)^2 + (x + 4) - 2 = 0$

Let $u = x + 4$.

$$3u^2 + u - 2 = 0$$
$$(3u - 2)(u + 1) = 0$$
$$u = \tfrac{2}{3}, -1$$
$$x = u - 4 = -\tfrac{10}{3}, -5$$

(b) $3(x^2 + 8x + 16) + x + 4 - 2 = 0$

$$3x^2 + 24x + 48 + x + 4 - 2 = 0$$
$$3x^2 + 25x + 50 = 0$$
$$(3x + 10)(x + 5) = 0$$
$$x = -\tfrac{10}{3}, -5$$

2. (a) $ax^2 + bx = 0, a \neq 0, b \neq 0$

$$x(ax + b) = 0$$
$$x = 0 \quad \text{or} \quad ax + b = 0$$
$$ax = -b$$
$$x = -\frac{b}{a}$$

(b) $ax^2 - (a - b)x - b = 0, a \neq 0, b \neq 0$

$$ax^2 - ax + bx - b = 0$$
$$ax(x - 1) + b(x - 1) = 0$$
$$(ax + b)(x - 1) = 0$$
$$ax + b = 0 \quad \text{or} \quad x - 1 = 0$$
$$ax = -b \qquad x = 1$$
$$x = -\frac{b}{a}$$

3. (a) $x^2 + bx + 4 = 0$

To have at least one real solution,

$$b^2 - 4(1)(4) \geq 0$$
$$b^2 - 16 \geq 0.$$

Critical numbers: $b = \pm 4$

Test intervals: $(-\infty, -4), (-4, 4), (4, \infty)$

Test: Is $b^2 - 16 \geq 0$?

By testing values in each test interval, we see that $b^2 - 16$ is greater than or equal to zero on the intervals $(-\infty, -4] \cup [4, \infty)$.

(b) $x^2 + bx - 4 = 0$

To have at least one real solution,

$$b^2 - 4(1)(-4) \geq 0$$
$$b^2 + 16 \geq 0.$$

This is true for all values of b:

$$-\infty < b < \infty$$

(c) $3x^2 + bx + 10 = 0$

To have at least one real solution,

$$b^2 - 4(3)(10) \geq 0$$
$$b^2 - 120 \geq 0.$$

Critical numbers: $b = \pm\sqrt{120} = \pm 2\sqrt{30}$

Test intervals: $\left(-\infty, -2\sqrt{30}\right), \left(-2\sqrt{30}, 2\sqrt{30}\right), \left(2\sqrt{30}, \infty\right)$

Test: Is $b^2 - 120 \geq 0$?

By testing values in each test interval, we see that $b^2 - 120$ is greater than or equal to zero on the intervals $\left(-\infty, -2\sqrt{30}\right) \cup \left[2\sqrt{30}, \infty\right)$.

(d) $2x^2 + bx + 5 = 0$

To have at least one real solution,

$$b^2 - 4(2)(5) \geq 0$$
$$b^2 - 40 \geq 0.$$

This is true for $b \leq -2\sqrt{10}$ or $b \geq 2\sqrt{10}$, $\left(-\infty, -2\sqrt{10}\right] \cup \left[2\sqrt{10}, \infty\right)$.

(e) If $a > 0$ and $c \leq 0$, then b can be any real number since $b^2 - 4ac$ would always be positive.

If $a > 0$ and $c > 0$, then $b \leq -2\sqrt{ac}$ or $b \geq 2\sqrt{ac}$, as in parts (a), (e), and (d).

(f) Since the intervals for b are symmetric about $b = 0$, the center of the interval is $b = 0$.

4. (a) Estimate from the graph: when the plate thickness is 2 millimeters, the frequency is approximately 330 vibrations per second.

(b) Estimate from the graph: when the frequency is 600, the plate thickness is approximately 3.6 millimeters.

(c) Estimate from the graph: when the frequency is between 200 and 400 vibrations per second, the plate thickness is between 1.2 and 2.4 millimeter.

(d) Estimate from the graph: when the plate thickness is less than 3 millimeters, the frequency is less than 500 vibrations per second.

5. (a) Since there are three solutions, the equation is not linear nor is it quadratic. Neither

(b) Since there is only one solution, the equation could have been either linear or quadratic. Both

(c) Since there are two solutions, the equation is not linear but could be quadratic. Quadratic

(d) Since there are four solutions, the equation is not linear nor is it quadratic. Neither

6. (a)
$$x^2 - 6x + y^2 - 8y = 0$$
$$\left(x^2 - 6x + 9\right) + \left(y^2 - 8y + 16\right) = 9 + 16$$
$$(x - 3)^2 + (y - 4)^2 = 25$$

Center: $(3, 4)$ Radius: 5

(b) Slope of line from $(0, 0)$ to $(3, 4)$ is $\frac{4}{3}$. Slope of tangent line is $-\frac{3}{4}$. Hence,

$y - 0 = -\frac{3}{4}(x - 0) \Rightarrow y = -\frac{3}{4}x$. Tangent line

(c) Slope of line from $(6, 0)$ to $(3, 4)$ is $\frac{4 - 0}{3 - 6} = -\frac{4}{3}$.

Slope of tangent line is $\frac{3}{4}$. Hence,

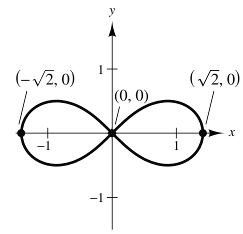
$y - 0 = \frac{3}{4}(x - 6) \Rightarrow y = \frac{3}{4}x - \frac{9}{2}$. Tangent line

(d)
$$-\frac{3}{4}x = \frac{3}{4}x - \frac{9}{2}$$
$$\frac{3}{2}x = \frac{9}{2}$$
$$x = 3$$

Intersection: $\left(3, -\frac{9}{4}\right)$

7.

$$d_1 d_2 = 1$$
$$\left[(x+1)^2 + y^2\right]\left[(x-1)^2 + y^2\right] = 1$$
$$(x+1)^2(x-1)^2 + y^2\left[(x+1)^2 + (x-1)^2\right] + y^4 = 1$$
$$\left(x^2 - 1\right)^2 + y^2\left[2x^2 + 2\right] + y^4 = 1$$
$$x^4 - 2x^2 + 1 + 2x^2y^2 + 2y^2 + y^4 = 1$$
$$\left(x^4 + 2x^2y^2 + y^4\right) - 2x^2 + 2y^2 = 0$$
$$\left(x^2 + y^2\right)^2 = 2\left(x^2 - y^2\right)$$



Let $y = 0$. Then $x^4 = 2x^2 \Rightarrow x = 0$ or $x^2 = 2$. Thus, $(0, 0)$, $(\sqrt{2}, 0)$ and $(-\sqrt{2}, 0)$ are on the curve.

8. The original point is (x, y).

(a) The transformed point $(-x, y)$ is a reflection through the y-axis.

(b) The transformed point $(x, -y)$ is a reflection through the x-axis.

(c) The transformed point $(-x, -y)$ is a reflection through the origin.

9. (a) $\dfrac{7000 - 5500}{10} = 150$ students per year

(b) 2003: $5500 + 3(150) = 5950$ students

2007: $5500 + 7(150) = 6550$ students

2009: $5500 + 9(150) = 6850$ students

(c) Equation: $y = 150x + 5500$ where $x = 0$ represents 2000.

Slope: $m = 150$

This means that enrollments increase by approximately 150 students per year.

10. Milk: $M = -0.23t + 22.3$

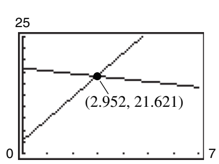
Bottled Water: $B = 1.87t + 16.1$

(a) $B = M$

$$1.87t + 16.1 = -0.23t + 22.3$$
$$2.1t = 6.2$$
$$t \approx 2.952$$

Point of intersection: $(2.952, 21.621)$

(b) and (c)

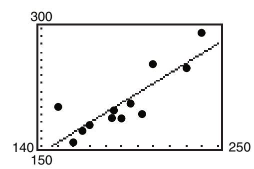


(d) Per capita consumption of milk was equal to per capita consumption of bottled water in 2002.

11. (a) and (b)

x	165	184	150	210	196	240
y	170	185	200	255	205	295
$1.3x - 36$	179	203	159	237	219	276

x	202	170	185	190	230	160
y	190	175	195	185	250	155
$1.3x - 36$	227	185	205	211	263	172



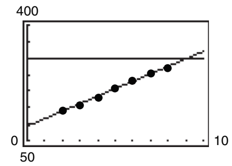
(c) One estimate is $x \geq 182$ pounds.

(d) $1.3x - 36 \geq 200$

$$1.3x \geq 236$$
$$x \geq 181.5385 \approx 181.54 \text{ pounds}$$

(e) An athlete's weight is not a particularly good indicator of the athlete's maximum bench-press weight. Other factors, such as muscle tone and exercise habits, influence maximum bench press weight.

12. $S \approx 22.6t + 94$



t	s
2	140.8
3	158.7
4	182.1
5	207.9
6	233.0
7	255.4
8	270.3

$s = 300$ when $t \approx 9$, which corresponds to 2009.

13. (a) Choice 1: $W = 3000 + 0.07s$

Choice 2: $W = 3400 + 0.05s$

(b) $$\begin{aligned} 3000 + 0.07s &= 3400 + 0.05s \\ 0.02s &= 400 \\ s &= \$20{,}000 \end{aligned}$$

The salaries are the same ($4400 per month) when sales equal $20,000.

(c) An ambitious salesperson who call sell more than $20,000 per month would be wise to select choice 1. A more conservative choice for a salesperson who is unsure of the market for this product would be choice 2.

CHAPTER 1
Functions and Their Graphs

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Functions and Their Graphs

Section 1.1 Functions

1. domain; range; function

2. verbally; numerically; graphically; algebraically

3. independent; dependent

4. piecewise-defined

5. implied domain

6. difference quotient

7. Yes, the relationship is a function. Each domain value is matched with exactly one range value.

8. No, the relationship is not a function. The domain value of –1 is matched with two output values.

9. No, the relationship is not a function. The domain values are each matched with two range values.

10. Yes, it is a function. Each domain value is matched with only one range value.

11. No, the relationship is not a function. The domain values are each matched with three range values.

12. Yes, the relationship is a function. Each domain value is matched with exactly one range value.

13. Yes, it does represent a function. Each input value is matched with exactly one output value.

14. No, the table does not represent a function. The input values of 0 and 1 are each matched with two different output values.

15. No, it does not represent a function. The input values of 10 and 7 are each matched with two output values.

16. Yes, the table does represent a function. Each input value is matched with exactly one output value.

17. (a) Each element of A is matched with exactly one element of B, so it does represent a function.
    (b) The element 1 in A is matched with two elements, –2 and 1 of B, so it does not represent a function.
    (c) Each element of A is matched with exactly one element of B, so it does represent a function.
    (d) The element 2 in A is not matched with an element of B, so the relation does not represent a function.

18. (a) The element c in A is matched with two elements, 2 and 3 of B, so it is not a function.
    (b) Each element of A is matched with exactly one element of B, so it does represent a function.
    (c) This is not a function from A to B (it represents a function from B to A instead).
    (d) Each element of A is matched with exactly one element of B, so it does represent a function.

19. Each is a function. For each year there corresponds one and only one circulation.

20. Reading from the graph, $f(2002)$ is approximately 9 million.

21. $x^2 + y^2 = 4 \Rightarrow y = \pm\sqrt{4 - x^2}$

    No, y *is not* a function of x.

22. $x^2 - y^2 = 16$

    $y = \pm\sqrt{16 - x^2}$

    No, y *is not* a function of x.

23. $x^2 + y = 4 \Rightarrow y = 4 - x^2$

    Yes, y *is* a function of x.

24. $y - 4x^2 = 36$

    $y = 4x^2 + 36$

    Yes, y *is* a function of x.

25. $2x + 3y = 4 \Rightarrow y = \frac{1}{3}(4 - 2x)$

    Yes, y *is* a function of x.

26. $2x + 5y = 10$

    $y = -\frac{2}{5}x + 2$

    Yes, y *is* a function of x.

27. $(x + 2)^2 + (y - 1)^2 = 25$

    $y = \pm\sqrt{25 - (x + 2)^2} + 1$

    No, y *is not* a function of x.

28. $(x-2)^2+y^2=4$

$y=\pm\sqrt{4-(x-2)^2}$

No, y *is not* a function of x.

29. $y^2=x^2-1 \Rightarrow y=\pm\sqrt{x^2-1}$

No, y *is not* a function of x.

30. $x+y^2=4$

$y=\pm\sqrt{4-x}$

No, y *is not* a function of x.

31. $y=\sqrt{16-x^2}$

Yes, y *is* a function of x.

32. $y=\sqrt{x+5}$

Yes, y *is* a function of x.

33. $y=|4-x|$

Yes, y *is* a function of x.

34. $|y|=4-x \Rightarrow y=4-x$ or $y=-(4-x)$

No, y *is not* a function of x.

35. $x=14$

No, this *is not* a function of x.

36. $y=-75$ or $y=-75+0x$

Yes, y *is* a function of x.

37. $y+5=0$

$y=-5$ or $y=0x-5$

Yes, y *is* a function of x.

38. $x-1=0$

$x=1$

No, this *is not* a function of x.

39. $f(x)=2x-3$

(a) $f(1)=2(1)-3=-1$

(b) $f(-3)=2(-3)-3=-9$

(c) $f(x-1)=2(x-1)-3=2x-5$

40. $g(y)=7-3y$

(a) $g(0)=7-3(0)=7$

(b) $g\left(\frac{7}{3}\right)=7-3\left(\frac{7}{3}\right)=0$

(c) $g(s+2)=7-3(s+2)$

$=7-3s-6=1-3s$

41. $V(r)=\frac{4}{3}\pi r^3$

(a) $V(3)=\frac{4}{3}\pi(3)^3=\frac{4}{3}\pi(27)=36\pi$

(b) $V\left(\frac{3}{2}\right)=\frac{4}{3}\pi\left(\frac{3}{2}\right)^3=\frac{4}{3}\pi\left(\frac{27}{8}\right)=\frac{9}{2}\pi$

(c) $V(2r)=\frac{4}{3}\pi(2r)^3=\frac{4}{3}\pi(8r^3)=\frac{32}{3}\pi r^3$

42. $S(r)=4\pi r^2$

(a) $S(2)=4\pi(2)^2$

$=16\pi$

(b) $S\left(\frac{1}{2}\right)=4\pi\left(\frac{1}{2}\right)^2$

$=\pi$

(c) $S(3r)=4\pi(3r)^2$

$=36\pi r^2$

43. $g(t)=4t^2-3t+5$

(a) $g(2)=4(2)^2-3(2)+5$

$=15$

(b) $g(t-2)=4(t-2)^2-3(t-2)+5$

$=4t^2-19t+27$

(c) $g(t)-g(2)=4t^2-3t+5-15$

$=4t^2-3t-10$

44. $h(t)=t^2-2t$

(a) $h(2)=2^2-2(2)=0$

(b) $h(1.5)=(1.5)^2-2(1.5)=-0.75$

(c) $h(x+2)=(x+2)^2-2(x+2)=x^2+2x$

45. $f(y)=3-\sqrt{y}$

(a) $f(4)=3-\sqrt{4}=1$

(b) $f(0.25)=3-\sqrt{0.25}=2.5$

(c) $f(4x^2)=3-\sqrt{4x^2}=3-2|x|$

46. $f(x)=\sqrt{x+8}+2$

(a) $f(-8)=\sqrt{(-8)+8}+2=2$

(b) $f(1)=\sqrt{(1)+8}+2=5$

(c) $f(x-8)=\sqrt{(x-8)+8}+2=\sqrt{x}+2$

47. $q(x) = \dfrac{1}{x^2 - 9}$

(a) $q(0) = \dfrac{1}{0^2 - 9} = -\dfrac{1}{9}$

(b) $q(3) = \dfrac{1}{3^2 - 9}$ is undefined.

(c) $q(y + 3) = \dfrac{1}{(y + 3)^2 - 9} = \dfrac{1}{y^2 + 6y}$

48. $q(t) = \dfrac{2t^2 + 3}{t^2}$

(a) $q(2) = \dfrac{2(2)^2 + 3}{(2)^2} = \dfrac{8 + 3}{4} = \dfrac{11}{4}$

(b) $q(0) = \dfrac{2(0)^2 + 3}{(0)^2}$

Division by zero is undefined.

(c) $q(-x) = \dfrac{2(-x)^2 + 3}{(-x)^2} = \dfrac{2x^2 + 3}{x^2}$

49. $f(x) = \dfrac{|x|}{x}$

(a) $f(2) = \dfrac{|2|}{2} = 1$

(b) $f(-2) = \dfrac{|-2|}{-2} = -1$

(c) $f(x - 1) = \dfrac{|x - 1|}{x - 1} = \begin{cases} -1, & \text{if } x < 1 \\ 1, & \text{if } x > 1 \end{cases}$

50. $f(x) = |x| + 4$

(a) $f(2) = |2| + 4 = 6$

(b) $f(-2) = |-2| + 4 = 6$

(c) $f(x^2) = |x^2| + 4 = x^2 + 4$

51. $f(x) = \begin{cases} 2x + 1, & x < 0 \\ 2x + 2, & x \ge 0 \end{cases}$

(a) $f(-1) = 2(-1) + 1 = -1$

(b) $f(0) = 2(0) + 2 = 2$

(c) $f(2) = 2(2) + 2 = 6$

52. $f(x) = \begin{cases} x^2 + 2, & x \le 1 \\ 2x^2 + 2, & x > 1 \end{cases}$

(a) $f(-2) = (-2)^2 + 2 = 6$

(b) $f(1) = (1)^2 + 2 = 3$

(c) $f(2) = 2(2)^2 + 2 = 10$

53. $f(x) = \begin{cases} 3x - 1, & x < -1 \\ 4, & -1 \le x \le 1 \\ x^2, & x > 1 \end{cases}$

(a) $f(-2) = 3(-2) - 1 = -7$

(b) $f\left(-\frac{1}{2}\right) = 4$

(c) $f(3) = 3^2 = 9$

54. $f(x) = \begin{cases} 4 - 5x, & x \le -2 \\ 0, & -2 < x \le 2 \\ x^2 + 1, & x > 2 \end{cases}$

(a) $f(-3) = 4 - 5(-3) = 19$

(b) $f(4) = (4)^2 + 1 = 17$

(c) $f(-1) = 0$

55. $f(x) = x^2 - 3$

$f(-2) = (-2)^2 - 3 = 1$

$f(-1) = (-1)^2 - 3 = -2$

$f(0) = (0)^2 - 3 = -3$

$f(1) = (1)^2 - 3 = -2$

$f(2) = (2)^2 - 3 = 1$

x	-2	-1	0	1	2
$f(x)$	1	-2	-3	-2	1

56. $g(x) = \sqrt{x - 3}$

$g(3) = \sqrt{3 - 3} = 0$

$g(4) = \sqrt{4 - 3} = 1$

$g(5) = \sqrt{5 - 3} = \sqrt{2}$

$g(6) = \sqrt{6 - 3} = \sqrt{3}$

$g(7) = \sqrt{7 - 3} = 2$

x	3	4	5	6	7
$g(x)$	0	1	$\sqrt{2}$	$\sqrt{3}$	2

57. $h(t) = \frac{1}{2}|t + 3|$

$h(-5) = \frac{1}{2}|-5 + 3| = 1$

$h(-4) = \frac{1}{2}|-4 + 3| = \frac{1}{2}$

$h(-3) = \frac{1}{2}|-3 + 3| = 0$

$h(-2) = \frac{1}{2}|-2 + 3| = \frac{1}{2}$

$h(-1) = \frac{1}{2}|-1 + 3| = 1$

t	–5	–4	–3	–2	–1
$h(t)$	1	$\frac{1}{2}$	0	$\frac{1}{2}$	1

58. $f(s) = \dfrac{|s - 2|}{s - 2}$

$f(0) = \dfrac{|0 - 2|}{0 - 2} = \dfrac{2}{-2} = -1$

$f(1) = \dfrac{|1 - 2|}{1 - 2} = \dfrac{1}{-1} = -1$

$f\left(\dfrac{3}{2}\right) = \dfrac{|(3/2) - 2|}{(3/2) - 2} = \dfrac{1/2}{-1/2} = -1$

$f\left(\dfrac{5}{2}\right) = \dfrac{|(5/2) - 2|}{(5/2) - 2} = \dfrac{1/2}{1/2} = 1$

$f(4) = \dfrac{|4 - 2|}{4 - 2} = \dfrac{2}{2} = 1$

s	0	1	$\frac{3}{2}$	$\frac{5}{2}$	4
$f(s)$	–1	–1	–1	1	1

59. $f(x) = \begin{cases} -\frac{1}{2}x + 4, & x \le 0 \\ (x - 2)^2, & x > 0 \end{cases}$

$f(-2) = -\frac{1}{2}(-2) + 4 = 5$

$f(-1) = -\frac{1}{2}(-1) + 4 = 4\frac{1}{2} = \frac{9}{2}$

$f(0) = -\frac{1}{2}(0) + 4 = 4$

$f(1) = (1 - 2)^2 = 1$

$f(2) = (2 - 2)^2 = 0$

x	–2	–1	0	1	2
$f(x)$	5	$\frac{9}{2}$	4	1	0

60. $f(x) = \begin{cases} 9 - x^2, & x < 3 \\ x - 3, & x \ge 3 \end{cases}$

$f(1) = 9 - (1)^2 = 8$

$f(2) = 9 - (2)^2 = 5$

$f(3) = (3) - 3 = 0$

$f(4) = (4) - 3 = 1$

$f(5) = (5) - 3 = 2$

x	1	2	3	4	5
$f(x)$	8	5	0	1	2

61. $15 - 3x = 0$

$3x = 15$

$x = 5$

62. $f(x) = 5x + 1$

$5x + 1 = 0$

$x = -\frac{1}{5}$

63. $\dfrac{3x - 4}{5} = 0$

$3x - 4 = 0$

$x = \dfrac{4}{3}$

64. $f(x) = \dfrac{12 - x^2}{5}$

$\dfrac{12 - x^2}{5} = 0$

$x^2 = 12$

$x = \pm\sqrt{12} = \pm 2\sqrt{3}$

65. $x^2 - 9 = 0$

$x^2 = 9$

$x = \pm 3$

66. $f(x) = x^2 - 8x + 15$

$x^2 - 8x + 15 = 0$

$(x - 5)(x - 3) = 0$

$x - 5 = 0 \Rightarrow x = 5$

$x - 3 = 0 \Rightarrow x = 3$

67. $x^3 - x = 0$

$x(x^2 - 1) = 0$

$x(x + 1)(x - 1) = 0$

$x = 0, x = -1,$ or $x = 1$

68.
$$f(x) = x^3 - x^2 - 4x + 4$$
$$x^3 - x^2 - 4x + 4 = 0$$
$$x^2(x - 1) - 4(x - 1) = 0$$
$$(x - 1)(x^2 - 4) = 0$$
$$x - 1 = 0 \Rightarrow x = 1$$
$$x^2 - 4 = 0 \Rightarrow x = \pm 2$$

69.
$$f(x) = g(x)$$
$$x^2 = x + 2$$
$$x^2 - x - 2 = 0$$
$$(x - 2)(x + 1) = 0$$
$$x - 2 = 0 \quad x + 1 = 0$$
$$x = 2 \quad x = -1$$

70.
$$f(x) = g(x)$$
$$x^2 + 2x + 1 = 7x - 5$$
$$x^2 - 5x + 6 = 0$$
$$(x - 3)(x - 2) = 0$$
$$x - 3 = 0 \quad x - 2 = 0$$
$$x = 3 \quad x = 2$$

71.
$$f(x) = g(x)$$
$$x^4 - 2x^2 = 2x^2$$
$$x^4 - 4x^2 = 0$$
$$x^2(x^2 - 4) = 0$$
$$x^2(x + 2)(x - 2) = 0$$
$$x^2 = 0 \Rightarrow x = 0$$
$$x + 2 = 0 \Rightarrow x = -2$$
$$x - 2 = 0 \Rightarrow x = 2$$

72.
$$f(x) = g(x)$$
$$\sqrt{x} - 4 = 2 - x$$
$$x + \sqrt{x} - 6 = 0$$
$$(\sqrt{x} + 3)(\sqrt{x} - 2) = 0$$

$\sqrt{x} + 3 = 0 \Rightarrow \sqrt{x} = -3$, which is a contradiction, since $\sqrt{x}$ represents the principal square root.

$\sqrt{x} - 2 = 0 \Rightarrow \sqrt{x} = 2 \Rightarrow x = 4$

73. $f(x) = 5x^2 + 2x - 1$

Because $f(x)$ is a polynomial, the domain is all real numbers x.

74. $f(x) = 1 - 2x^2$

Because $f(x)$ is a polynomial, the domain is all real numbers x.

75. $h(t) = \dfrac{4}{t}$

The domain is all real numbers t except $t = 0$.

76. $s(y) = \dfrac{3y}{y + 5}$

$$y + 5 \neq 0$$
$$y \neq -5$$

The domain is all real numbers y except $y = -5$.

77. $g(y) = \sqrt{y - 10}$

Domain: $y - 10 \geq 0$

$y \geq 10$

The domain is all real numbers y such that $y \geq 10$.

78. $f(t) = \sqrt[3]{t + 4}$

Because $f(t)$ is a cube root, the domain is all real numbers t.

79. $g(x) = \dfrac{1}{x} - \dfrac{3}{x + 2}$

The domain is all real numbers x except $x = 0, x = -2$.

80. $h(x) = \dfrac{10}{x^2 - 2x}$

$$x^2 - 2x \neq 0$$
$$x(x - 2) \neq 0$$

The domain is all real numbers x except $x = 0, x = 2$.

81. $f(s) = \dfrac{\sqrt{s - 1}}{s - 4}$

Domain: $s - 1 \geq 0 \Rightarrow s \geq 1$ and $s \neq 4$

The domain consists of all real numbers s, such that $s \geq 1$ and $s \neq 4$.

82. $f(x) = \dfrac{\sqrt{x+6}}{6+x}$

Domain: $x + 6 \geq 0 \Rightarrow x \geq -6$ and $x \neq -6$

The domain is all real numbers x such that $x > -6$ or $(-6, \infty)$.

83. $f(x) = \dfrac{x-4}{\sqrt{x}}$

The domain is all real numbers such that $x > 0$ or $(0, \infty)$.

84. $f(x) = \dfrac{x+2}{\sqrt{x-10}}$

$x - 10 > 0$

$x > 10$

The domain is all real numbers x such that $x > 10$.

85. $f(x) = x^2$

$f(-2) = (-2)^2 = 4$

$f(-1) = (-1)^2 = 1$

$f(0) = 0^2 = 0$

$f(1) = 1^2 = 1$

$f(2) = 2^2 = 4$

$\{(-2, 4), (-1, 1), (0, 0), (1, 1), (2, 4)\}$

86. $f(x) = (x-3)^2$

$f(-2) = (-2-3)^2 = 25$

$f(-1) = (-1-3)^2 = 16$

$f(0) = (0-3)^2 = 9$

$f(1) = (1-3)^2 = 4$

$f(2) = (2-3)^2 = 1$

$\{(-2, 25), (-1, 16), (0, 9), (1, 4), (2, 1)\}$

87. $f(x) = |x| + 2$

$f(-2) = |-2| + 2 = 4$

$f(-1) = |-1| + 2 = 3$

$f(0) = |0| + 2 = 2$

$f(1) = |1| + 2 = 3$

$f(2) = |2| + 2 = 4$

$\{(-2, 4), (-1, 3), (0, 2), (1, 3), (2, 4)\}$

88. $f(x) = |x+1|$

$f(-2) = |-2+1| = 1$

$f(-1) = |-1+1| = 0$

$f(0) = |0+1| = 1$

$f(1) = |1+1| = 2$

$f(2) = |2+1| = 3$

$\{(-2, 1), (-1, 0), (0, 1), (1, 2), (2, 3)\}$

89. No. The element 3 in the domain corresponds to two elements in the range.

90. An advantage to function notation is that it gives a name to the relationship so it can be easily referenced. When evaluating a function, you see both the input and output values.

91. $A = s^2$ and $P = 4s \Rightarrow \dfrac{P}{4} = s$

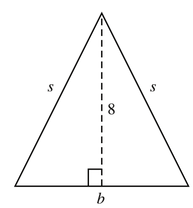
$A = \left(\dfrac{P}{4}\right)^2 = \dfrac{P^2}{16}$

92. $A = \pi r^2, C = 2\pi r$

$r = \dfrac{C}{2\pi}$

$A = \pi\left(\dfrac{C}{2\pi}\right)^2 = \dfrac{C^2}{4\pi}$

93. $8^2 + \left(\dfrac{b}{2}\right)^2 = s^2$



$\dfrac{b^2}{4} = s^2 - 64$

$b^2 = 4(s^2 - 64)$

$b = 2\sqrt{s^2 - 64}$

Thus, $A = \dfrac{1}{2}bh$

$= \dfrac{1}{2}\left(2\sqrt{s^2 - 64}\right)(8)$

$= 8\sqrt{s^2 - 64}$ square inches.

94. $A = \dfrac{1}{2}bh$, and in an equilateral triangle $b = s$

$s^2 = h^2 + \left(\dfrac{s}{2}\right)^2$

$h = \sqrt{s^2 - \left(\dfrac{s}{2}\right)^2}$

$h = \sqrt{\dfrac{4s^2}{4} - \dfrac{s^2}{4}} = \dfrac{\sqrt{3}s}{2}$

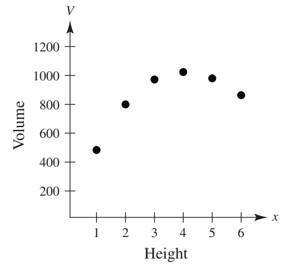
$A = \dfrac{1}{2}s \cdot \dfrac{\sqrt{3}s}{2} = \dfrac{\sqrt{3}s^2}{4}$

95. (a)

Height, x	Volume, V
1	484
2	800
3	972
4	1024
5	980
6	864

The volume is maximum when $x = 4$ and $V = 1024$ cubic centimeters.

(b)



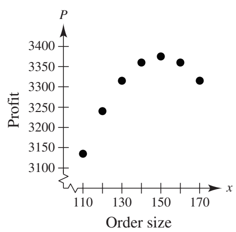
V is a function of x.

(c) $V = x(24 - 2x)^2$

Domain: $0 < x < 12$

96. (a) The maximum profit is $3375.

(b)



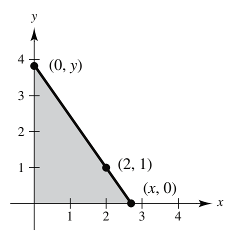
Yes, P is a function of x.

(c)
$$\begin{aligned}\text{Profit} &= \text{Revenue} - \text{Cost}\\ &= (\text{price per unit})(\text{number of units}) - (\text{cost})(\text{number of units})\\ &= [90 - (x - 100)(0.15)]x - 60x,\ x > 100\\ &= (90 - 0.15x + 15)x - 60x\\ &= (105 - 0.15x)x - 60x\\ &= 105x - 0.15x^2 - 60x\\ &= 45x - 0.15x^2,\ x > 100\end{aligned}$$

97. $A = \frac{1}{2}bh = \frac{1}{2}xy$

Because $(0, y)$, $(2, 1)$, and $(x, 0)$ all lie on the same line, the slopes between any pair are equal.

$$\frac{1 - y}{2 - 0} = \frac{0 - 1}{x - 2}$$
$$\frac{1 - y}{2} = \frac{-1}{x - 2}$$
$$y = \frac{2}{x - 2} + 1$$
$$y = \frac{x}{x - 2}$$



So, $A = \frac{1}{2}x\left(\frac{x}{x - 2}\right) = \frac{x^2}{2(x - 2)}$.

The domain of A includes x-values such that $x^2/[2(x - 2)] > 0$. By solving this inequality, the domain is $x > 2$.

98. $A = \ell \cdot w = (2x)y = 2xy$

But $y = \sqrt{36 - x^2}$, so $A = 2x\sqrt{36 - x^2}$. The domain is $0 < x < 6$.

99. $y = -\frac{1}{10}x^2 + 3x + 6$

$y(30) = -\frac{1}{10}(30)^2 + 3(30) + 6 = 6$ feet

If the child holds a glove at a height of 5 feet, then the ball *will* be over the child's head because it will be at a height of 6 feet

100. $d(t) = \begin{cases} 10.6t + 699, & 0 \le t \le 4 \\ 15.5t + 637, & 5 \le t \le 7 \end{cases}$

2000: Use $t = 0$ and find $d(0)$. $d(0) = 10.6(0) + 699 = 699.0$ million

2001: Use $t = 1$ and find $d(1)$. $d(1) = 10.6(1) + 699 = 709.6$ million

2002: Use $t = 2$ and find $d(2)$. $d(2) = 10.6(2) + 699 = 720.2$ million

2003: Use $t = 3$ and find $d(3)$. $d(3) = 10.6(3) + 699 = 730.8$ million

2004: Use $t = 4$ and find $d(4)$. $d(4) = 10.6(4) + 699 = 741.4$ million

2005: Use $t = 5$ and find $d(5)$. $d(5) = 15.5(5) + 637 = 714.5$ million

2006: Use $t = 6$ and find $d(6)$. $d(6) = 15.5(6) + 637 = 730.0$ million

2007: Use $t = 7$ and find $d(7)$. $d(7) = 15.5(7) + 637 = 745.5$ million

101. $p(t) = \begin{cases} 1.011t^2 - 12.38t + 170.5, & 8 \le t \le 13 \\ -6.950t^2 + 222.55t - 1557.6, & 14 \le t \le 17 \end{cases}$

1998: Use $t = 8$ and find $p(8)$. $p(8) = 1.011(8)^2 - 12.38(t) + 170.5 = 136.164$ thousand $= \$136,164$

1999: Use $t = 9$ and find $p(9)$. $p(9) = 1.011(9)^2 - 12.38(9) + 170.5 = 140.971$ thousand $= \$140,971$

2000: Use $t = 10$ and find $p(10)$. $p(10) = 1.011(10)^2 - 12.38(10) + 170.5 = 147.800$ thousand $= \$147,800$

2001: Use $t = 11$ and find $p(11)$. $p(11) = 1.011(11)^2 - 12.38(11) + 170.5 = 156.651$ thousand $= \$156,651$

2002: Use $t = 12$ and find $p(12)$. $p(12) = 1.011(12)^2 - 12.38(12) + 170.5 = 167.524$ thousand $= \$167,524$

2003: Use $t = 13$ and find $p(13)$. $p(13) = 1.011(13)^2 - 12.38(13) + 170.5 = 180.419$ thousand $= \$180,419$

2004: Use $t = 14$ and find $p(14)$. $p(14) = -6.950(14)^2 + 222.55(14) - 1557.6 = 195.900$ thousand $= \$195,900$

2005: Use $t = 15$ and find $p(15)$. $p(15) = -6.950(15)^2 + 222.55(15) - 1557.6 = 216.900$ thousand $= \$216,900$

2006: Use $t = 16$ and find $p(16)$. $p(16) = -6.950(16)^2 + 222.55(16) - 1557.6 = 224.000$ thousand $= \$224,000$

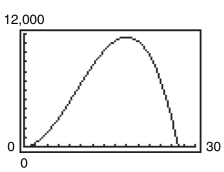
2007: Use $t = 17$ and find $p(17)$. $p(17) = -6.950(17)^2 + 222.55(17) - 1557.6 = 217.200$ thousand $= \$217,200$

102. (a) $V = \ell \cdot w \cdot h = x \cdot y \cdot x = x^2 y$ where $4x + y = 108$. So, $y = 108 - 4x$ and

$V = x^2(108 - 4x) = 108x^2 - 4x^3.$

Domain: $0 < x < 27$

(b)



(c) The dimensions that will maximize the volume of the package are $18 \times 18 \times 36$. From the graph, the maximum volume occurs when $x = 18$. To find the dimension for y, use the equation $y = 108 - 4x$.

$y = 108 - 4x = 108 - 4(18) = 108 - 72 = 36$

103. (a) Cost = variable costs + fixed costs

$C = 12.30x + 98,000$

(b) Revenue = price per unit × number of units

$R = 17.98x$

(c) Profit = Revenue − Cost

$P = 17.98x - (12.30x + 98,000)$

$P = 5.68x - 98,000$

104. (a) *Model:*

(Total cost) = (Fixed costs) + (Variable costs)

Labels: Total cost $= C$

Fixed cost $= 6000$

Variable costs $= 0.95x$

Equation: $C = 6000 + 0.95x$

(b) $\overline{C} = \dfrac{C}{x} = \dfrac{6000 + 0.95x}{x} = \dfrac{6000}{x} + 0.95$

105. (a) $R = n(\text{rate}) = n[8.00 - 0.05(n - 80)],\ n \geq 80$

$R = 12.00n - 0.05n^2 = 12n - \dfrac{n^2}{20} = \dfrac{240n - n^2}{20},\ n \geq 80$

(b)

n	90	100	110	120	130	140	150
$R(n)$	\$675	\$700	\$715	\$720	\$715	\$700	\$675

The revenue is maximum when 120 people take the trip.

106. $F(y) = 149.76\sqrt{10}y^{5/2}$

(a)

y	5	10	20	30	40
$F(y)$	26,474.08	149,760.00	847,170.49	2,334,527.36	4,792,320

The force, in tons, of the water against the dam increases with the depth of the water.

(b) It appears that approximately 21 feet of water would produce 1,000,000 tons of force.

(c) $1{,}000{,}000 = 149.76\sqrt{10}y^{5/2}$

$\dfrac{1{,}000{,}000}{149.76\sqrt{10}} = y^{5/2}$

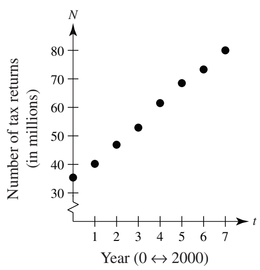
$2111.56 \approx y^{5/2}$

$21.37 \text{ feet} \approx y$

107. (a) $\dfrac{f(2007) - f(2000)}{2007 - 2000} = \dfrac{80.0 - 35.4}{2007 - 2000} \approx 6.37$

Approximately 6.37 million more tax returns were made through e-file each year from 2000 to 2007.

(b)



(c) Use the points $(0, 35.4)$ and $(7, 80.0)$.

$m = \dfrac{80.0 - 35.4}{7 - 0} = 6.37$

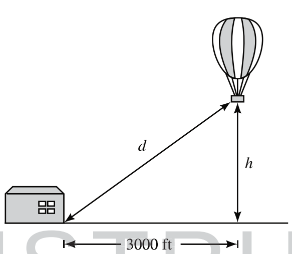
$N = 6.37t + 35.4$

(d)

t	0	1	2	3	4	5	6	7
N	35.4	41.8	48.1	54.5	60.9	67.3	73.6	80.0

(e) Using a graphing utility yields the model $N = 6.56t + 34.4$. Compared to the model in part (c), the model generated by the graphing utility produces values that reflect the data more accurately.

108. (a)



(b) $(3000)^2 + h^2 = d^2$

$h = \sqrt{d^2 - (3000)^2}$

Domain: $d \geq 3000$ (because both $d \geq 0$ and $d^2 - (3000)^2 \geq 0$)

109.
$$\begin{aligned} f(x) &= x^2 - x + 1 \\ f(2+h) &= (2+h)^2 - (2+h) + 1 \\ &= 4 + 4h + h^2 - 2 - h + 1 \\ &= h^2 + 3h + 3 \\ f(2) &= (2)^2 - 2 + 1 = 3 \\ f(2+h) - f(2) &= h^2 + 3h \\ \frac{f(2+h) - f(2)}{h} &= \frac{h^2 + 3h}{h} = h + 3, h \neq 0 \end{aligned}$$

110.
$$\begin{aligned} f(x) &= 5x - x^2 \\ f(5+h) &= 5(5+h) - (5+h)^2 \\ &= 25 + 5h - (25 + 10h + h^2) \\ &= 25 + 5h - 25 - 10h - h^2 \\ &= -h^2 - 5h \\ f(5) &= 5(5) - (5)^2 \\ &= 25 - 25 = 0 \\ \frac{f(5+h) - f(5)}{h} &= \frac{-h^2 - 5h}{h} \\ &= \frac{-h(h+5)}{h} = -(h+5), h \neq 0 \end{aligned}$$

111.
$$\begin{aligned} f(x) &= x^3 + 2x - 1 \\ \frac{f(x+c) - f(x)}{c} &= \frac{\left[(x+c)^3 + 2(x+c) - 1\right] - (x^3 + 2x - 1)}{c} \\ &= \frac{x^3 + 3x^2c + 3xc^2 + c^3 + 2x + 2c - 1 - x^3 - 2x + 1}{c} \\ &= \frac{3x^2c + 3xc^2 + c^3 + 2c}{c} = \frac{c(3x^2 + 3xc + c^2 + 2)}{c} \\ &= 3x^2 + 3xc + c^2 + 2, \quad c \neq 0 \end{aligned}$$

112.
$$\begin{aligned} f(x) &= x^3 - x + 1 \\ \frac{f(x+c) - f(x)}{c} &= \frac{\left[(x+c)^3 - (x+c) + 1\right] - (x^3 - x + 1)}{c} \\ &= \frac{x^3 + 3x^2c + 3xc^2 + c^3 - x - c + 1 - x^3 + x - 1}{c} \\ &= \frac{3x^2c + 3xc^2 + c^3 - c}{c} = \frac{c(3x^2 + 3xc + c^2 - 1)}{c} \\ &= 3x^2 + 3xc + c^2 - 1, \quad c \neq 0 \end{aligned}$$

113. $g(x) = 3x - 1$

$$\frac{g(x) - g(3)}{x - 3} = \frac{(3x - 1) - 8}{x - 3} = \frac{3x - 9}{x - 3} = \frac{3(x - 3)}{x - 3} = 3, x \neq 3$$

114.
$$\begin{aligned} f(t) &= \frac{1}{t} \\ f(1) &= \frac{1}{1} = 1 \end{aligned}$$

$$\frac{f(t) - f(1)}{t - 1} = \frac{\frac{1}{t} - 1}{t - 1} = \frac{\frac{1}{t} - \frac{t}{t}}{t - 1} = \frac{\frac{1 - t}{t}}{t - 1} = \frac{\left(-\frac{1}{t}\right)(t - 1)}{t - 1} = -\frac{1}{t}, \qquad t \neq 1$$

115. $f(x) = \sqrt{5x}$

$$\frac{f(x) - f(5)}{x - 5} = \frac{\sqrt{5x} - 5}{x - 5}$$

116.
$$\begin{aligned} f(x) &= x^{2/3} + 1 \\ f(8) &= 8^{2/3} + 1 = 5 \\ \frac{f(x) - f(8)}{x - 8} &= \frac{x^{2/3} + 1 - 5}{x - 8} = \frac{x^{2/3} - 4}{x - 8} \end{aligned}$$

117. By plotting the points, we have a parabola, so $g(x) = cx^2$. Because $(-4, -32)$ is on the graph, you have $-32 = c(-4)^2 \Rightarrow c = -2$. So, $g(x) = -2x^2$.

118. By plotting the data, you can see that they represent a line, or $f(x) = cx$. Because $(0, 0)$ and $\left(1, \frac{1}{4}\right)$ are on the line, the slope is $\frac{1}{4}$. So, $f(x) = \frac{1}{4}x$.

119. Because the function is undefined at 0, we have $r(x) = c/x$. Because $(-4, -8)$ is on the graph, you have $-8 = c/-4 \Rightarrow c = 32$. So, $r(x) = 32/x$.

120. By plotting the data, you can see that they represent $h(x) = c\sqrt{|x|}$. Because $\sqrt{|-4|} = 2$ and $\sqrt{|-1|} = 1$, and the corresponding y-values are 6 and 3, $c = 3$ and $h(x) = 3\sqrt{|x|}$.

121. False. The equation $y^2 = x^2 + 4$ is a relation between x and y. However, $y = \pm\sqrt{x^2 + 4}$ does not represent a function.

122. True. A function is a relation by definition.

123. True.

As long as **all** elements in the domain are matched with elements in the range, even if it is the same element, then the relation is a function.

124. False.

Each element in the domain is matched with exactly **one** element in the range in a function.

125. False. The range is $[-1, \infty)$.

126. True. The set represents a function. Each x-value is mapped to exactly one y-value.

127. $f(x) = \sqrt{x - 1}$ Domain: $x \geq 1$

$g(x) = \dfrac{1}{\sqrt{x - 1}}$ Domain: $x > 1$

The value 1 may be included in the domain of $f(x)$ as it is possible to find the square root of 0. However, 1 cannot be included in the domain of $g(x)$ as it causes a zero to occur in the denominator which results in the function being undefined.

128. Because $f(x)$ is a function of an even root, the radicand cannot be negative. $g(x)$ is an odd root, therefore the radicand can be any real number. So, the domain of g is all real numbers x and the domain of f is all real numbers x such that $x \geq 2$.

129. No; x is the independent variable, f is the name of the function.

130. (a) Answers will vary.
Sample answer: A relation is a rule of correspondence between two variables. A function is a particular relation that assigns exactly one output (y) to each input (x).

(b) The domain of a function is the set of all inputs of the independent variable for which the function is defined. The range of a function is the set of all possible outputs.

131. (a) Yes. The amount that you pay in sales tax will increase as the price of the item purchased increases.

(b) No. The length of time that you study the night before an exam does not necessarily determine your score on the exam.

132. (a) No. During the course of a year, for example, your salary may remain constant while your savings account balance may vary. That is, there may be two or more outputs (savings account balances) for one input (salary).

(b) Yes. The greater the height from which the ball is dropped, the greater the speed with which the ball will strike the ground.

Section 1.2 Analyzing Graphs of Functions

1. ordered pairs

2. Vertical Line Test

3. zeros

4. decreasing

5. maximum

6. step; greatest integer

7. odd

8. even

9. Domain: $(-\infty, -1] \cup [1, \infty)$

Range: $[0, \infty)$

10. Domain: $(-\infty, \infty)$

Range: $[0, \infty)$

11. Domain: $[-4, 4]$

Range: $[0, 4]$

12. Domain: $(-\infty, 1) \cup (1, \infty)$

Range: $-1, 1$

13. Domain: $(-\infty, \infty)$; Range: $[-4, \infty)$

(a) $f(-2) = 0$

(b) $f(-1) = -1$

(c) $f\left(\frac{1}{2}\right) = 0$

(d) $f(1) = -2$

14. Domain: $(-\infty, \infty)$; Range: $(-\infty, \infty)$

(a) $f(-1) = 4$

(b) $f(2) = 4$

(c) $f(0) = 2$

(d) $f(1) = 0$

15. Domain: $(-\infty, \infty)$; Range: $(-2, \infty)$

(a) $f(2) = 0$

(b) $f(1) = 1$

(c) $f(3) = 2$

(d) $f(-1) = 3$

16. Domain: $(-\infty, \infty)$; Range: $(-\infty, 1]$

(a) $f(-2) = -3$

(b) $f(1) = 0$

(c) $f(0) = 1$

(d) $f(2) = -3$

17. $y = \frac{1}{2}x^2$

A vertical line intersects the graph at most once, so *y is* a function of *x*.

18. $y = \frac{1}{4}x^3$

A vertical line intersects the graph at most once, so *y is* a function of *x*.

19. $x - y^2 = 1 \Rightarrow y = \pm\sqrt{x - 1}$

y is not a function of *x*. Some vertical lines intersect the graph twice.

20. $x^2 + y^2 = 25$

A vertical line intersects the graph more than once, so *y is not* a function of *x*.

21. $x^2 = 2xy - 1$

A vertical line intersects the graph at most once, so *y is* a function of *x*.

22. $x = |y + 2|$

A vertical line intersects the graph more than once, so *y is not* a function of *x*.

23. $f(x) = 2x^2 - 7x - 30$

$$2x^2 - 7x - 30 = 0$$
$$(2x + 5)(x - 6) = 0$$
$$2x + 5 = 0 \quad \text{or} \quad x - 6 = 0$$
$$x = -\tfrac{5}{2} \quad \text{or} \quad x = 6$$

24. $f(x) = 3x^2 + 22x - 16$

$$3x^2 + 22x - 16 = 0$$
$$(3x - 2)(x + 8) = 0$$
$$3x - 2 = 0 \Rightarrow x = \tfrac{2}{3}$$
$$x + 8 = 0 \Rightarrow x = -8$$

25. $f(x) = \dfrac{x}{9x^2 - 4}$

$$\frac{x}{9x^2 - 4} = 0$$
$$x = 0$$

26. $f(x) = \dfrac{x^2 - 9x + 14}{4x}$

$$\frac{x^2 - 9x + 14}{4x} = 0$$
$$(x - 7)(x - 2) = 0$$
$$x - 7 = 0 \Rightarrow x = 7$$
$$x - 2 = 0 \Rightarrow x = 2$$

27. $f(x) = \frac{1}{2}x^3 - x$

$$\tfrac{1}{2}x^3 - x = 0$$
$$x^3 - 2x = 2(0)$$
$$x(x^2 - 2) = 0$$
$$x = 0 \quad \text{or} \quad x^2 - 2 = 0$$
$$x^2 = 2$$
$$x = \pm\sqrt{2}$$

28. $f(x) = x^3 - 4x^2 - 9x + 36$

$$x^3 - 4x^2 - 9x + 36 = 0$$
$$x^2(x - 4) - 9(x - 4) = 0$$
$$(x - 4)(x^2 - 9) = 0$$
$$x - 4 = 0 \Rightarrow x = 4$$
$$x^2 - 9 = 0 \Rightarrow x = \pm 3$$

29. $f(x) = 4x^3 - 24x^2 - x + 6$

$$4x^3 - 24x^2 - x + 6 = 0$$
$$4x^2(x - 6) - 1(x - 6) = 0$$
$$(x - 6)(4x^2 - 1) = 0$$
$$(x - 6)(2x + 1)(2x - 1) = 0$$
$$x - 6 = 0, \quad 2x + 1 = 0, \quad 2x - 1 = 0$$
$$x = 6, \quad x = -\tfrac{1}{2}, \quad x = \tfrac{1}{2}$$

30. $f(x) = 9x^4 - 25x^2$

$$9x^4 - 25x^2 = 0$$
$$x^2(9x^2 - 25) = 0$$
$$x^2 = 0 \Rightarrow x = 0$$
$$9x^2 - 25 = 0 \Rightarrow x = \pm\tfrac{5}{3}$$

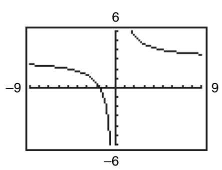
31. $f(x) = \sqrt{2x} - 1$

$$\sqrt{2x} - 1 = 0$$
$$\sqrt{2x} = 1$$
$$2x = 1$$
$$x = \tfrac{1}{2}$$

32. $f(x) = \sqrt{3x + 2}$

$$\sqrt{3x + 2} = 0$$
$$3x + 2 = 0$$
$$-\tfrac{2}{3} = x$$

33. (a)

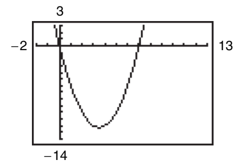


Zero: $x = -\dfrac{5}{3}$

(b) $f(x) = 3 + \dfrac{5}{x}$

$$3 + \frac{5}{x} = 0$$
$$3x + 5 = 0$$
$$x = -\frac{5}{3}$$

34. (a)

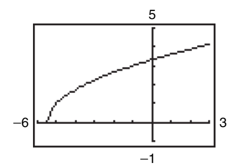


Zeros: $x = 0, x = 7$

(b) $f(x) = x(x - 7)$

$$x(x - 7) = 0$$
$$x = 0$$
$$x - 7 = 0 \Rightarrow x = 7$$

35. (a)

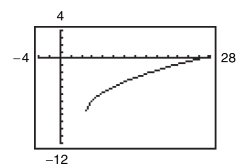


Zero: $x = -\tfrac{11}{2}$

(b) $f(x) = \sqrt{2x + 11}$

$$\sqrt{2x + 11} = 0$$
$$2x + 11 = 0$$
$$x = -\tfrac{11}{2}$$

36. (a)

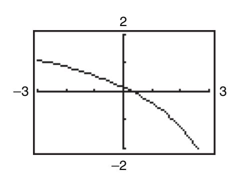


Zero: $x = 26$

(b) $f(x) = \sqrt{3x - 14} - 8$

$$\sqrt{3x - 14} - 8 = 0$$
$$\sqrt{3x - 14} = 8$$
$$3x - 14 = 64$$
$$x = 26$$

37. (a)

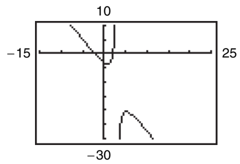


Zero: $x = \dfrac{1}{3}$

(b) $f(x) = \dfrac{3x - 1}{x - 6}$

$$\frac{3x - 1}{x - 6} = 0$$
$$3x - 1 = 0$$
$$x = \frac{1}{3}$$

38. (a)



Zeros: $x = \pm 2.1213$

(b) $f(x) = \dfrac{2x^2 - 9}{3 - x}$

$\dfrac{2x^2 - 9}{3 - x} = 0$

$2x^2 - 9 = 0 \Rightarrow x = \pm\dfrac{3\sqrt{2}}{2} = \pm 2.1213$

39. $f(x) = \frac{3}{2}x$

The function is increasing on $(-\infty, \infty)$.

40. $f(x) = x^2 - 4x$

The function is decreasing on $(-\infty, 2)$ and increasing on $(2, \infty)$.

41. $f(x) = x^3 - 3x^2 + 2$

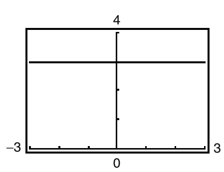
The function is increasing on $(-\infty, 0)$ and $(2, \infty)$ and decreasing on $(0, 2)$.

42. $f(x) = \sqrt{x^2 - 1}$

The function is decreasing on $(-\infty, -1)$ and increasing on $(1, \infty)$.

43. $f(x) = 3$

(a)



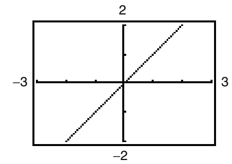
Constant on $(-\infty, \infty)$

(b)

x	−2	−1	0	1	2
$f(x)$	3	3	3	3	3

44. $g(x) = x$

(a)



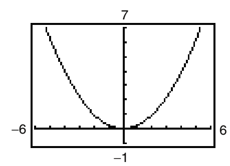
Increasing on $(-\infty, \infty)$

(b)

x	−2	−1	0	1	2
$g(x)$	−2	−1	0	1	2

45. $g(s) = \dfrac{s^2}{4}$

(a)



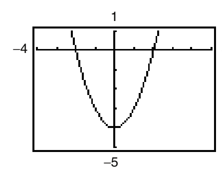
Decreasing on $(-\infty, 0)$; Increasing on $(0, \infty)$

(b)

s	−4	−2	0	2	4
$g(s)$	4	1	0	1	4

46. $h(x) = x^2 - 4$

(a)



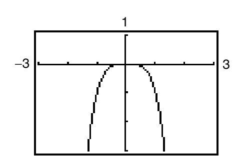
Decreasing on $(-\infty, 0)$; Increasing on $(0, \infty)$

(b)

x	−2	−1	0	1	2
$h(x)$	0	−3	−4	−3	0

47. $f(t) = -t^4$

(a)



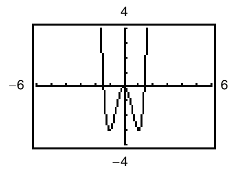
Increasing on $(-\infty, 0)$; Decreasing on $(0, \infty)$

(b)

t	−2	−1	0	1	2
$f(t)$	−16	−1	0	−1	−16

48. $f(x) = 3x^4 - 6x^2$

(a)



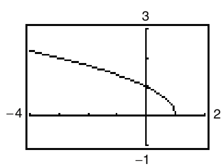
Increasing on $(-1, 0), (1, \infty)$; Decreasing on $(-\infty, -1), (0, 1)$

(b)

x	−2	−1	0	1	2
$f(x)$	24	−3	0	−3	24

49. $f(x) = \sqrt{1 - x}$

(a)



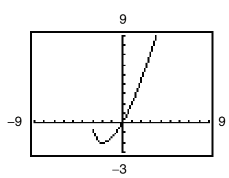
Decreasing on $(-\infty, 1)$

(b)

x	−3	−2	−1	0	1
$f(x)$	2	$\sqrt{3}$	$\sqrt{2}$	1	0

50. $f(x) = x\sqrt{x + 3}$

(a)



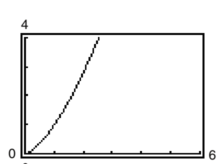
Increasing on $(-2, \infty)$; Decreasing on $(-3, -2)$

(b)

x	−3	−2	−1	0	1
$f(x)$	0	−2	−1.414	0	2

51. $f(x) = x^{3/2}$

(a)



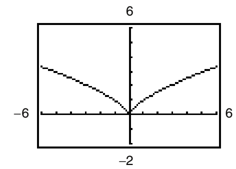
Increasing on $(0, \infty)$

(b)

x	0	1	2	3	4
$f(x)$	0	1	2.8	5.2	8

52. $f(x) = x^{2/3}$

(a)



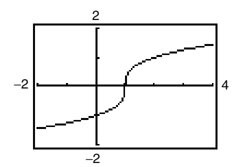
Decreasing on $(-\infty, 0)$; Increasing on $(0, \infty)$

(b)

x	−2	−1	0	1	2
$f(x)$	1.59	1	0	1	1.59

53. $g(t) = \sqrt[3]{t - 1}$

(a)



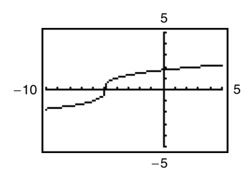
Increasing on $(-\infty, \infty)$

(b)

t	−2	−1	0	1	2
$g(t)$	−1.44	−1.26	−1	0	1

54. $f(x) = \sqrt[3]{x + 5}$

(a)



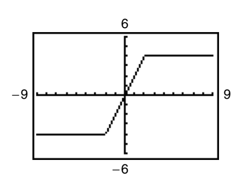
Increasing on $(-\infty, \infty)$

(b)

x	−13	−6	−5	−4	3
$f(x)$	−2	−1	0	1	2

55. $f(x) = |x + 2| - |x - 2|$

(a)



Increasing on $(-2, 2)$

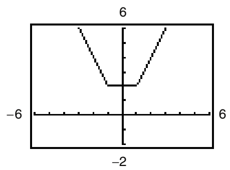
Constant on $(-\infty, -2)$ and $(2, \infty)$

(b)

x	−2	−1	0	1	4
$f(x)$	−4	−2	0	2	4

56. $f(x) = |x + 1| + |x - 1|$

(a)



Decreasing on $(-\infty, -1)$

Constant on $(-1, 1)$

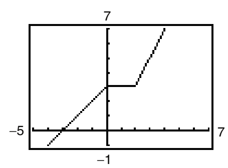
Increasing on $(1, \infty)$

(b)

x	−3	−2	−1	0	1	2	3
$f(x)$	6	4	2	2	2	4	6

57. $f(x) = \begin{cases} x + 3, & x \le 0 \\ 3, & 0 < x \le 2 \\ 2x - 1, & x > 2 \end{cases}$

(a)



Increasing on $(-\infty, 0)$ and $(2, \infty)$

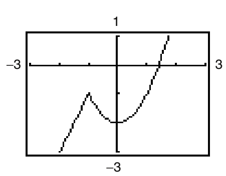
Constant on $(0, 2)$

(b)

x	−2	−1	0	1	2	3	4
$f(x)$	1	2	3	3	3	5	7

58. $f(x) = \begin{cases} 2x + 1, & x \le -1 \\ x^2 - 2, & x > -1 \end{cases}$

(a)



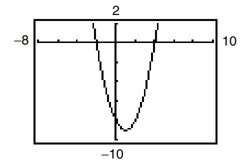
Increasing on $(-\infty, -1)$ and $(0, \infty)$

Decreasing on $(-1, 0)$

(b)

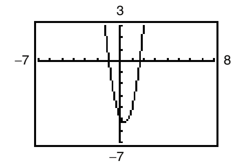
x	−2	−1	$-\frac{1}{2}$	0	1	2
$f(x)$	−3	−1	−1.75	−2	−1	2

59. $f(x) = (x - 4)(x + 2)$



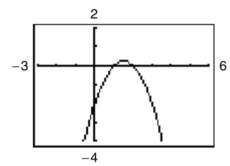
Relative minimum: $(1, -9)$

60. $f(x) = 3x^2 - 2x - 5$



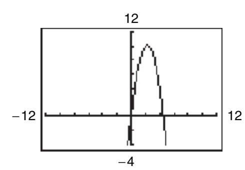
Relative minimum: $\left(\frac{1}{3}, -\frac{16}{3}\right)$ or $(0.33, -5.33)$

61. $f(x) = -x^2 + 3x - 2$



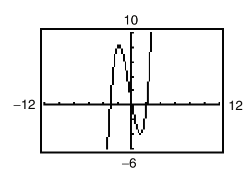
Relative maximum: $(1.5, 0.25)$

62. $f(x) = -2x^2 + 9x$



Relative maximum: $(2.25, 10.125)$

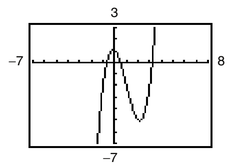
63. $f(x) = x(x - 2)(x + 3)$



Relative minimum: $(1.12, -4.06)$

Relative maximum: $(-1.79, 8.21)$

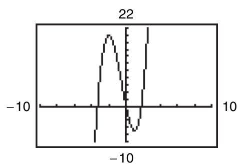
64. $f(x) = x^3 - 3x^2 - x + 1$



Relative maximum: $(-0.15, 1.08)$

Relative minimum: $(2.15, -5.08)$

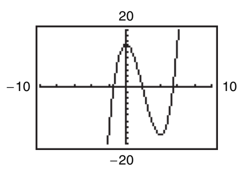
65.



Relative minimum: $(1, -7)$

Relative maximum: $(-2, 20)$

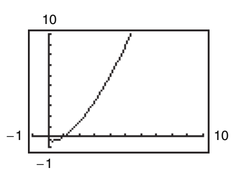
66.



Relative minimum: $(4, -17)$

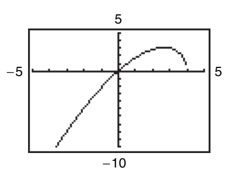
Relative maximum: $(0, 15)$

67.



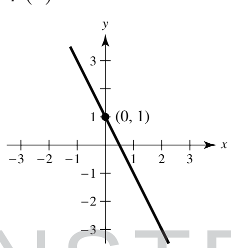
Relative minimum: $(0.33, -0.38)$

68.

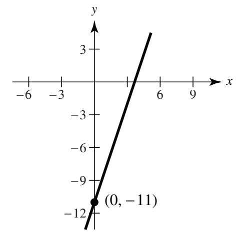


Relative maximum: $(2.67, 3.08)$

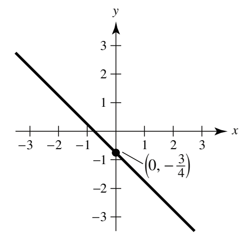
69. $f(x) = 1 - 2x$



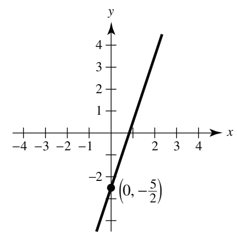
70. $f(x) = 3x - 11$



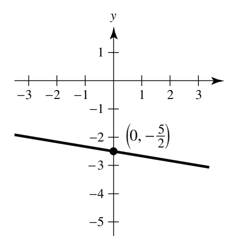
71. $f(x) = -x - \frac{3}{4}$



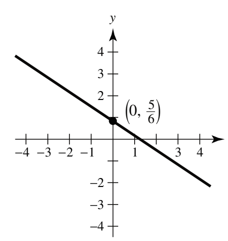
72. $f(x) = 3x - \frac{5}{2}$



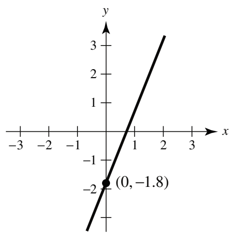
73. $f(x) = -\frac{1}{6}x - \frac{5}{2}$



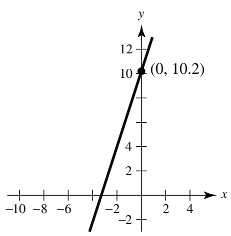
74. $f(x) = \frac{5}{6} - \frac{2}{3}x$



75. $f(x) = -1.8 + 2.5x$



76. $f(x) = 10.2 + 3.1x$



77. (a) $f(1) = 4,\ f(0) = 6$

$(1, 4), (0, 6)$

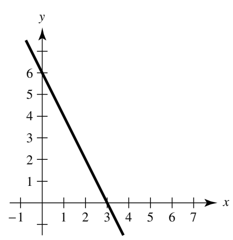
$$m = \frac{6 - 4}{0 - 1} = -2$$

$$y - 6 = -2(x - 0)$$

$$y = -2x + 6$$

$$f(x) = -2x + 6$$

(b)



78. (a) $f(-3) = -8,\ f(1) = 2$

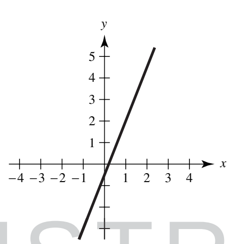
$(-3, -8), (1, 2)$

$$m = \frac{2 - (-8)}{1 - (-3)} = \frac{10}{4} = \frac{5}{2}$$

$$f(x) - 2 = \frac{5}{2}(x - 1)$$

$$f(x) = \frac{5}{2}x - \frac{1}{2}$$

(b)



79. (a) $f(5) = -4,\ f(-2) = 17$

$(5, -4), (-2, 17)$

$$m = \frac{17 - (-4)}{-2 - 5} = \frac{21}{-7} = -3$$

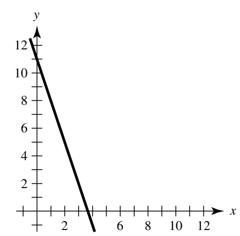
$$y - (-4) = -3(x - 5)$$

$$y + 4 = -3x + 15$$

$$y = -3x + 11$$

$$f(x) = -3x + 11$$

(b)



80. (a) $f(3) = 9,\ f(-1) = -11$

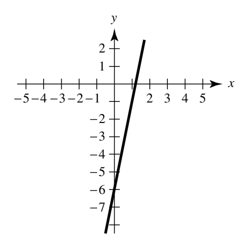
$(3, 9), (-1, -11)$

$$m = \frac{-11 - 9}{-1 - 3} = \frac{-20}{-4} = 5$$

$$f(x) - 9 = 5(x - 3)$$

$$f(x) = 5x - 6$$

(b)



81. (a) $f(-5) = -1,\ f(5) = -1$

$(-5, -1), (5, -1)$

$$m = \frac{-1 - (-1)}{5 - (-5)} = \frac{0}{10} = 0$$

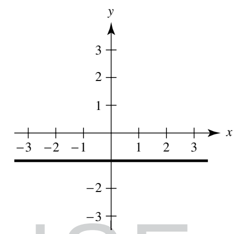
$$y - (-1) = 0(x - (-5))$$

$$y + 1 = 0$$

$$y = -1$$

$$f(x) = -1$$

(b)



82. (a) $f\left(\frac{2}{3}\right) = -\frac{15}{2},\ f(-4) = -11$

$\left(\frac{2}{3}, -\frac{15}{2}\right), (-4, -11)$

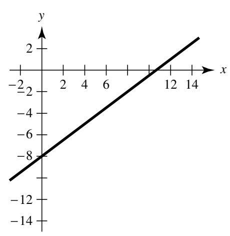
$$m = \frac{-11 - (-15/2)}{-4 - (2/3)}$$

$$= \frac{-7/2}{-14/3} = \left(-\frac{7}{2}\right)\cdot\left(-\frac{3}{14}\right) = \frac{3}{4}$$

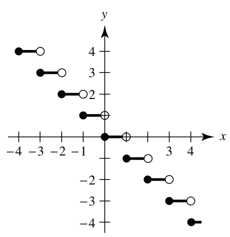
$$f(x) - (-11) = \frac{3}{4}(x - (-4))$$

$$f(x) = \frac{3}{4}x - 8$$

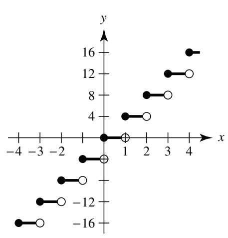
(b)



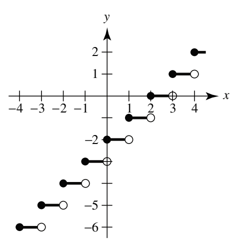
83. $g(x) = -[\![x]\!]$



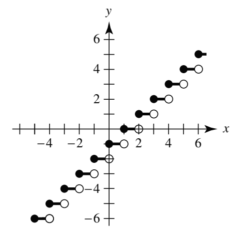
84. $g(x) = 4[\![x]\!]$



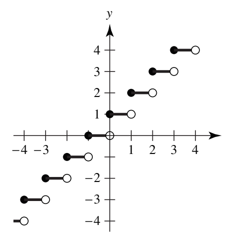
85. $g(x) = [\![x]\!] - 2$



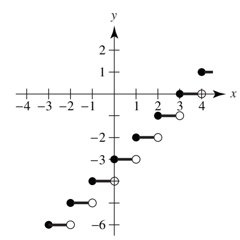
86. $g(x) = [\![x]\!] - 1$



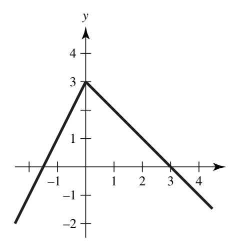
87. $g(x) = [\![x + 1]\!]$



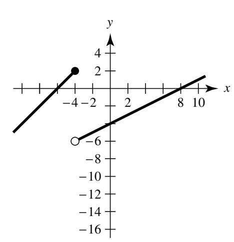
88. $g(x) = [\![x - 3]\!]$



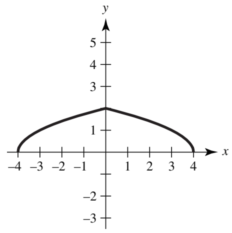
89. $f(x) = \begin{cases} 2x + 3, & x < 0 \\ 3 - x, & x \ge 0 \end{cases}$



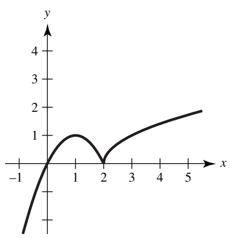
90. $g(x) = \begin{cases} x + 6, & x \le -4 \\ \frac{1}{2}x - 4, & x > -4 \end{cases}$



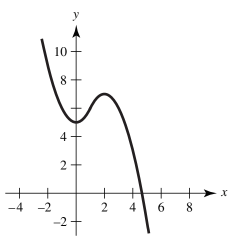
91. $f(x) = \begin{cases} \sqrt{4 + x}, & x < 0 \\ \sqrt{4 - x}, & x \geq 0 \end{cases}$



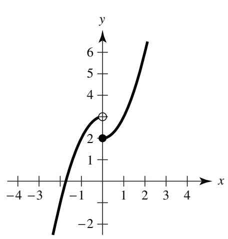
92. $f(x) = \begin{cases} 1 - (x - 1)^2, & x \leq 2 \\ \sqrt{x - 2}, & x > 2 \end{cases}$



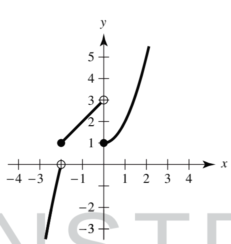
93. $f(x) = \begin{cases} x^2 + 5, & x \leq 1 \\ -x^2 + 4x + 3, & x > 1 \end{cases}$



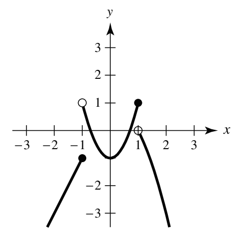
94. $h(x) = \begin{cases} 3 - x^2, & x < 0 \\ x^2 + 2, & x \geq 0 \end{cases}$



95. $h(x) = \begin{cases} 4 - x^2, & x < -2 \\ 3 + x, & -2 \leq x < 0 \\ x^2 + 1, & x \geq 0 \end{cases}$

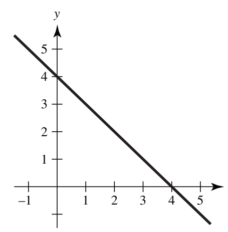


96. $k(x) = \begin{cases} 2x + 1, & x \leq -1 \\ 2x^2 - 1, & -1 < x \leq 1 \\ 1 - x^2, & x > 1 \end{cases}$



97. $f(x) = 4 - x$

$f(x) \geq 0$ on $(-\infty, 4]$.

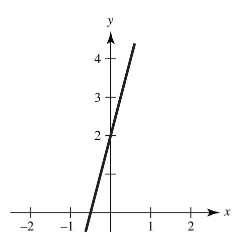


98. $f(x) = 4x + 2$

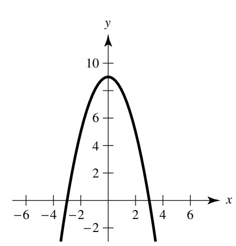
$f(x) \geq 0$ on $\left[-\frac{1}{2}, \infty\right)$.

$$4x + 2 \geq 0$$
$$4x \geq -2$$
$$x \geq -\frac{1}{2}$$

$\left[-\frac{1}{2}, \infty\right)$



99. $f(x) = 9 - x^2$

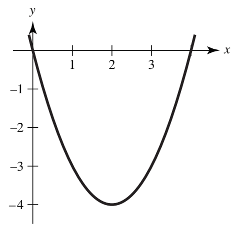


$f(x) \geq 0$ on $[-3, 3]$

100. $f(x) = x^2 - 4x$

$f(x) \geq 0$ on $(-\infty, 0]$ and $[4, \infty)$.

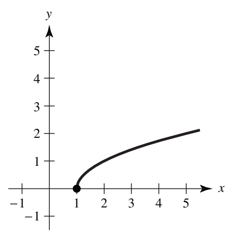
$$x^2 - 4x \geq 0$$
$$x(x - 4) \geq 0$$
$$(-\infty, 0], [4, \infty)$$



101. $f(x) = \sqrt{x - 1}$

$f(x) \geq 0$ on $[1, \infty)$.

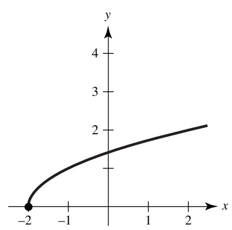
$$\sqrt{x - 1} \geq 0$$
$$x - 1 \geq 0$$
$$x \geq 1$$
$$[1, \infty)$$



102. $f(x) = \sqrt{x + 2}$

$f(x) \geq 0$ on $[-2, \infty)$.

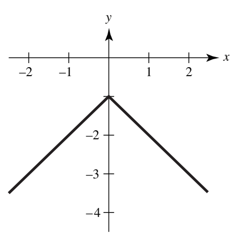
$$\sqrt{x + 2} \geq 0$$
$$x + 2 \geq 0$$
$$x \geq -2$$
$$[-2, \infty)$$



103. $f(x) = -(1 + |x|)$

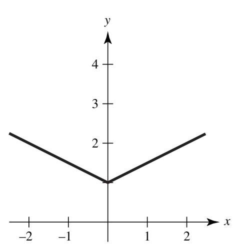
$f(x)$ is never greater than 0.

$(f(x) < 0$ for all $x.)$



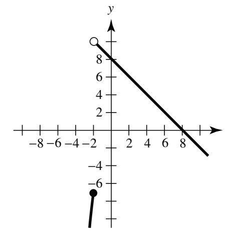
104. $f(x) = \frac{1}{2}(2 + |x|)$

$f(x)$ is always greater than 0. $(-\infty, \infty)$

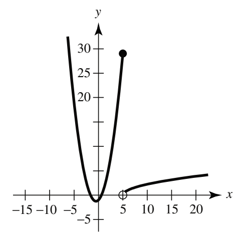


105. $f(x) = \begin{cases} 1 - 2x^2, & x \leq -2 \\ -x + 8, & x > -2 \end{cases}$

$f(x) \geq 0$ on $(-2, 8]$



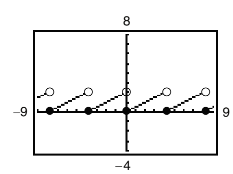
106. $f(x) = \begin{cases} \sqrt{x - 5}, & x > 5 \\ x^2 + x - 1, & x \leq 5 \end{cases}$



$$f(x) \geq 0$$
$$\sqrt{x - 5} \geq 0$$
$$x - 5 \geq 0$$
$$x \geq 5$$
$$x^2 + x - 1 \geq 0$$
$$x = \frac{-1 \pm \sqrt{1^2 - 4(1)(-1)}}{2(1)}$$
$$= \frac{-1 \pm \sqrt{5}}{2}$$
$$\left(-\infty, \frac{-1 - \sqrt{5}}{2}\right], \left[\frac{-1 + \sqrt{5}}{2}, \infty\right)$$

107. $s(x) = 2\left(\frac{1}{4}x - [\![\frac{1}{4}x]\!]\right)$

(a)



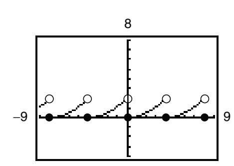
(b) Domain: $(-\infty, \infty)$

Range: $[0, 2)$

(c) Sawtooth pattern

108. $g(x) = 2\left(\frac{1}{4}x - [\![\frac{1}{4}x]\!]\right)^2$

(a)



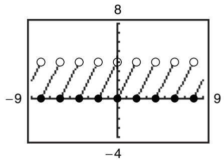
(b) Domain: $(-\infty, \infty)$

Range: $[0, 2)$

(c) Sawtooth pattern

109. $h(x) = 4\left(\frac{1}{2}x - \left[\!\left[\frac{1}{2}x\right]\!\right]\right)$

(a)



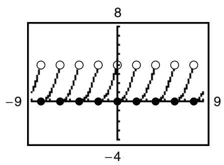
(b) Domain: $(-\infty, \infty)$

Range: $[0, 4)$

(c) Sawtooth pattern

110. $k(x) = 4\left(\frac{1}{2}x - \left[\!\left[\frac{1}{2}x\right]\!\right]\right)^2$

(a)

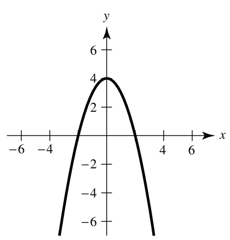


(b) Domain: $(-\infty, \infty)$

Range: $[0, 4)$

(c) Sawtooth pattern

111.



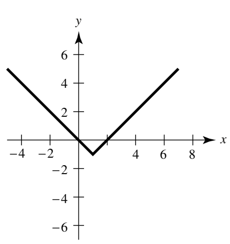
(a) Domain: All real numbers or $(-\infty, \infty)$

(b) Range: $(-\infty, 4]$

(c) Increasing on $(-\infty, 0)$

Decreasing on $(0, \infty)$

112.



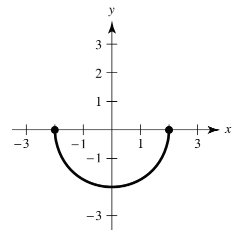
(a) Domain: All real numbers or $(-\infty, \infty)$

(b) Range: $[-1, \infty)$

(c) Increasing on $(1, \infty)$

Decreasing on $(-\infty, 1)$

113.



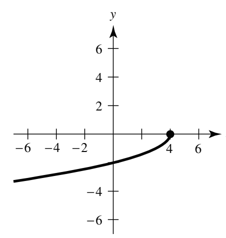
(a) Domain: $[-2, 2]$

(b) Range: $[-2, 0]$

(c) Increasing on $(0, 2)$

Decreasing on $(-2, 0)$

114.

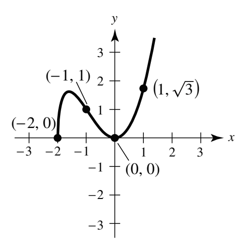


(a) Domain: $(-\infty, 4]$

(b) Range: $(-\infty, 0]$

(c) Increasing on $(-\infty, 4)$

115.



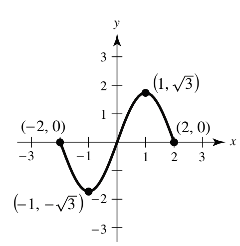
(a) $f(-1) = 1$

(b) $f(1) = \sqrt{3}$

(c) f is increasing on $(-2, -1.6)$ and $(0, \infty)$

f is decreasing on $(-1.6, 0)$.

116.



(a) $g(-1) = -\sqrt{3}$

(b) $g(1) = \sqrt{3}$

(c) g is increasing on $(-1, 1)$.

g is decreasing on $(-2, -1)$ and $(1, 2)$.

117. $f(x) = x^6 - 2x^2 + 3$

$f(-x) = (-x)^6 - 2(-x)^2 + 3$

$= x^6 - 2x^2 + 3$

$= f(x)$

The function is even. y-axis symmetry.

118. $h(x) = x^3 - 5$

$h(-x) = (-x)^3 - 5$

$= -x^3 - 5$

$\neq h(x)$

$\neq -h(x)$

The function is neither odd nor even. No symmetry.

119. $g(x) = x^3 - 5x$

$g(-x) = (-x)^3 - 5(-x)$

$= -x^3 + 5x$

$= -g(x)$

The function is odd. Origin symmetry.

120. $f(t) = t^2 + 2t - 3$

$f(-t) = (-t)^2 + 2(-t) - 3$

$= t^2 - 2t - 3$

$\neq f(t), \neq -f(t)$

The function is neither even nor odd. No symmetry.

121. $h(x) = x\sqrt{x + 5}$

$h(-x) = (-x)\sqrt{-x + 5}$

$= -x\sqrt{5 - x}$

$\neq h(x)$

$\neq -h(x)$

The function is neither odd nor even. No symmetry.

122. $f(x) = x\sqrt{1 - x^2}$

$f(-x) = -x\sqrt{1 - (-x)^2}$

$= -x\sqrt{1 - x^2}$

$= -f(x)$

The function is odd. Origin symmetry.

123. $f(s) = 4s^{3/2}$

$= 4(-s)^{3/2}$

$\neq f(s)$

$\neq -f(s)$

The function is neither odd nor even. No symmetry.

124. $g(s) = 4s^{2/3}$

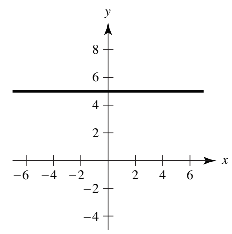
$g(-s) = 4(-s)^{2/3}$

$= 4s^{2/3}$

$= g(s)$

The function is even. y-axis symmetry.

125.



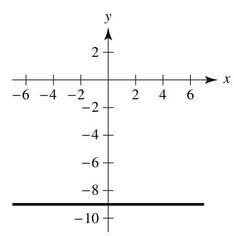
The graph of $f(x) = 5$ is symmetric to the y-axis, which implies $f(x)$ is even.

$f(-x) = 5$

$= f(x)$

The function is even.

126.



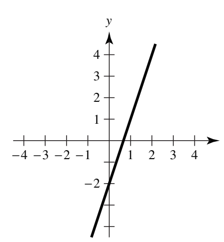
The graph of $f(x) = -9$ is symmetric to the y-axis, which implies $f(x)$ is even.

$f(-x) = -9$

$= f(x)$

The function is even.

127. $f(x) = 3x - 2$



The graph displays no symmetry, which implies $f(x)$ is neither odd nor even.

$f(-x) = 3(-x) - 2$

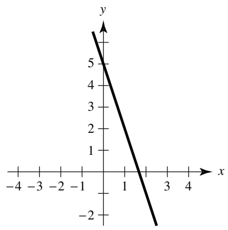
$= -3x - 2$

$\neq f(x)$

$\neq -f(x)$

The function is neither even nor odd.

128. $f(x) = 5 - 3x$

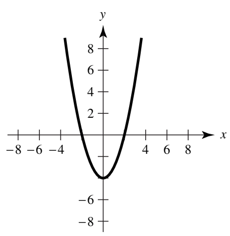


The graph displays no symmetry, which implies $f(x)$ is neither odd nor even.

$$\begin{aligned} f(-x) &= 5 - 3(-x) \\ &= 5 + 3x \\ &\neq f(x) \\ &\neq -f(x) \end{aligned}$$

The function is neither even nor odd.

129. $h(x) = x^2 - 4$

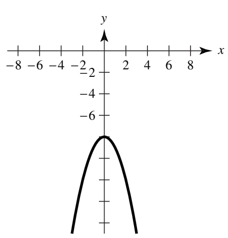


The graph displays y-axis symmetry, which implies $h(x)$ is even.

$$h(-x) = (-x)^2 - 4 = x^2 - 4 = h(x)$$

The function is even.

130. $f(x) = -x^2 - 8$

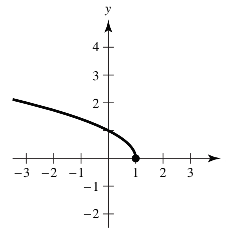


The graph displays y-axis symmetry, which implies $f(x)$ is even.

$$f(-x) = -(-x)^2 - 8 = -x^2 - 8 = f(x)$$

The function is even.

131. $f(x) = \sqrt{1 - x}$

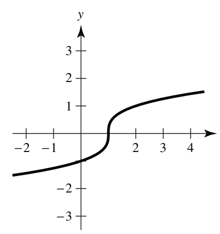


The graph displays no symmetry, which implies $f(x)$ is neither odd nor even.

$$\begin{aligned} f(-x) &= \sqrt{1 - (-x)} \\ &= \sqrt{1 + x} \\ &\neq f(x) \\ &\neq -f(x) \end{aligned}$$

The function is neither even nor odd.

132. $g(t) = \sqrt[3]{t - 1}$

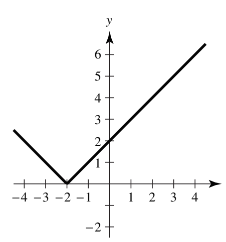


The graph displays no symmetry, which implies $g(t)$ is neither odd nor even.

$$\begin{aligned} g(-t) &= \sqrt[3]{(-t) - 1} \\ &= \sqrt[3]{-t - 1} \\ &\neq g(t) \\ &\neq -g(t) \end{aligned}$$

The function is neither even nor odd.

133. $f(x) = |x + 2|$

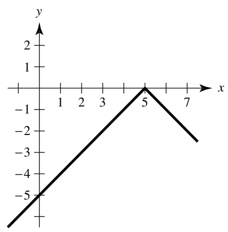


The graph displays no symmetry, which implies $f(x)$ is neither odd nor even.

$$f(-x) = |-x + 2| \neq f(x) \neq -f(x)$$

The function is neither even nor odd.

134. $f(x) = -|x - 5|$



The graph displays no symmetry, which implies $f(x)$ is neither odd nor even.

$$f(x) = -|(-x) - 5|$$
$$= -|-x - 5|$$
$$\neq f(x)$$
$$\neq -f(x)$$

The function is neither even nor odd.

135. $h = \text{top} - \text{bottom}$
$$= (-x^2 + 4x - 1) - 2$$
$$= -x^2 + 4x - 3$$

136. $h = \text{top} - \text{bottom}$
$$= 3 - (4x - x^2)$$
$$= 3 - 4x + x^2$$

137. $h = \text{top} - \text{bottom}$
$$= (4x - x^2) - 2x$$
$$= 2x - x^2$$

138. $h = \text{top} - \text{bottom}$
$$= 2 - \sqrt[3]{x}$$

139. $L = \text{right} - \text{left}$
$$= \tfrac{1}{2}y^2 - 0 = \tfrac{1}{2}y^2$$

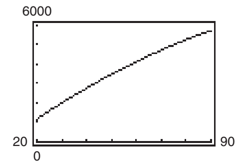
140. $L = \text{right} - \text{left}$
$$= 2 - \sqrt[3]{2y}$$

141. $L = \text{right} - \text{left}$
$$= 4 - y^2$$

142. $L = \text{right} - \text{left}$
$$= \frac{2}{y} - 0$$
$$= \frac{2}{y}$$

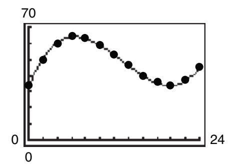
143. $L = -0.294x^2 + 97.744x - 664.875,\ 20 \le x \le 90$

(a)



(b) $L = 2000$ when $x \approx 29.9645 \approx 30$ watts.

144. (a)



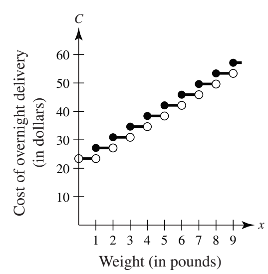
(b) The model is an excellent fit.

(c) The temperature is increasing from 6 A.M. until noon $(x = 0 \text{ to } x = 6)$. Then it decreases until 2 A.M. $(x = 6 \text{ to } x = 20)$. Then the temperature increases until 6 A.M. $(x = 20 \text{ to } x = 24)$.

(d) The maximum temperature according to the model is about 63.93°F. According to the data, it is 64°F. The minimum temperature according to the model is about 33.98°F. According to the data, it is 34°F.

(e) Answers may vary. Temperatures will depend upon the weather patterns, which usually change from day to day.

145. (a)



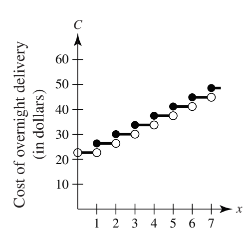
(b)
$$C(9.25) = 23.40 + 3.75[\![9.25]\!]$$
$$= 23.40 + 3.75(9)$$
$$= 57.15$$

It costs \$57.15 to mail a 9.25 pound package.

146. (a) Cost = Flat fee + fee per pound

$$C(x) = 22.65 + 3.70[\![x]\!]$$

(b)

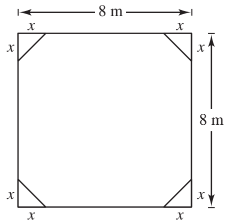


147. (a) For the average salaries of college professors, a scale of \$10,000 would be appropriate.

(b) For the population of the United States, use a scale of 10,000,000.

(c) For the percent of the civilian workforce that is unemployed, use a scale of 1%.

148.

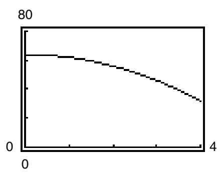


(a) $A = (8)(8) - 4\left(\frac{1}{2}\right)(x)(x)$

$= 64 - 2x^2$

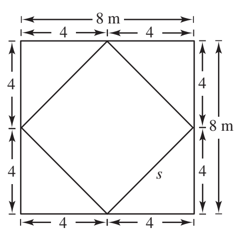
Domain: $0 \le x \le 4$

(b)



Range: $32 \le A \le 64$

(c) When $x = 4$, the resulting figure is a square.



By the Pythagorean Theorem,
$4^2 + 4^2 = s^2 \Rightarrow s = \sqrt{32} = 4\sqrt{2}$ meters.

149. False. The function $f(x) = \sqrt{x^2 + 1}$ has a domain of all real numbers.

150. False. A piecewise-defined function is a function that is defined by two or more equations over a specified domain. That domain may or may not include *x*- and *y*-intercepts.

151. (a) Even. The graph is a reflection in the *x*-axis.

(b) Even. The graph is a reflection in the *y*-axis.

(c) Even. The graph is a vertical translation of *f*.

(d) Neither. The graph is a horizontal translation of *f*.

152. Yes, the graph of $x = y^2 + 1$ in Exercise 19 does represent *x* as a function of *y*. Each *y*-value corresponds to only one *x*-value.

153. $\left(-\frac{3}{2}, 4\right)$

(a) If *f* is even, another point is $\left(\frac{3}{2}, 4\right)$.

(b) If *f* is odd, another point is $\left(\frac{3}{2}, -4\right)$.

154. $\left(-\frac{5}{3}, -7\right)$

(a) If *f* is even, another point is $\left(\frac{5}{3}, -7\right)$.

(b) If *f* is odd, another point is $\left(\frac{5}{3}, 7\right)$.

155. $(4, 9)$

(a) If *f* is even, another point is $(-4, 9)$.

(b) If *f* is odd, another point is $(-4, -9)$.

156. $(5, -1)$

(a) If *f* is even, another point is $(-5, -1)$.

(b) If *f* is odd, another point is $(-5, 1)$.

157. (a) $(-x, -y)$ (b) $(-x, y)$

158. (a) $(-2a, 2c)$ (b) $(-2a, -2c)$

159. $f(x) = \begin{cases} x + 2, & x < -2 \\ 0, & -2 \le x \le 2 \\ ax + b, & x > 2 \end{cases}$

(a) If $f(x)$ is odd, then $f(-x) = -f(x)$

$f(-3) = -f(3) \Rightarrow -1 = -(3a + b)$

$f(-4) = -f(4) \Rightarrow -2 = -(4a + b)$

Solving the system: $4a + b = 2$

$3a + b = 1$

yields $a = 1$ and $b = -2$

(b) If $f(x)$ is even, then $f(-x) = f(x)$

$f(-3) = f(3) \Rightarrow -1 = 3a + b$

$f(-4) = f(4) \Rightarrow -2 = 4a + b$

Solving this system yields $a = -1$ and $b = 2$.

160. (a) Domain: $[-4, 5)$; Range: $[0, 9]$

(b) $(3, 0)$

(c) Increasing $(-4, 0) \cup (3, 5)$

Decreasing $(0, 3)$

(d) Relative Minimum $(3, 0)$

Relative Maximum $(0, 9)$

(e) Neither

Section 1.3 Transformations of Functions

1. rigid

2. $-f(x)$; $f(-x)$

3. nonrigid

4. horizontal shrink; horizontal stretch

5. vertical stretch; vertical shrink

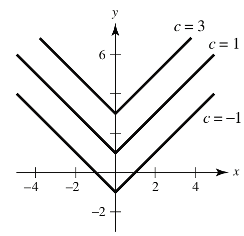
6. (a) iv
(b) ii
(c) iii
(d) i

7. (a) $f(x) = |x| + c$ Vertical shifts

$c = -1: f(x) = |x| - 1$ 1 unit down

$c = 1: f(x) = |x| + 1$ 1 unit up

$c = 3: f(x) = |x| + 3$ 3 units up

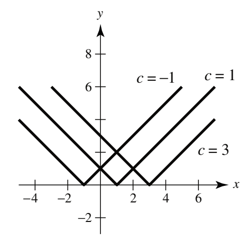


(b) $f(x) = |x - c|$ Horizontal shifts

$c = -1: f(x) = |x + 1|$ 1 unit left

$c = 1: f(x) = |x - 1|$ 1 unit right

$c = 3: f(x) = |x - 3|$ 3 units right

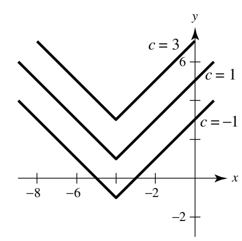


(c) $f(x) = |x + 4| + c$ Horizontal shift four units left and a vertical shift

$c = -1: f(x) = |x + 4| - 1$ 1 unit down

$c = 1: f(x) = |x + 4| + 1$ 1 unit up

$c = 3: f(x) = |x + 4| + 3$ 3 units up



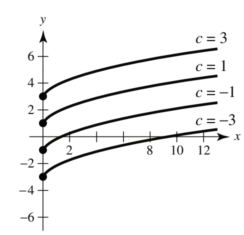
8. (a) $f(x) = \sqrt{x} + c$ Vertical shifts

$c = -3: f(x) = \sqrt{x} - 3$ 3 units down

$c = -1: f(x) = \sqrt{x} - 1$ 1 unit down

$c = 1: f(x) = \sqrt{x} + 1$ 1 unit up

$c = 3: f(x) = \sqrt{x} + 3$ 3 units up



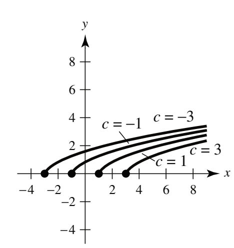
(b) $f(x) = \sqrt{x - c}$ Horizontal shifts

$c = -3: f(x) = \sqrt{x + 3}$ 3 units left

$c = -1: f(x) = \sqrt{x + 1}$ 1 unit left

$c = 1: f(x) = \sqrt{x - 1}$ 1 unit right

$c = 3: f(x) = \sqrt{x - 3}$ 3 units right



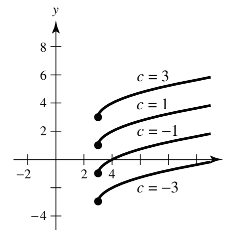
(c) $f(x) = \sqrt{x-3} + c$ — Horizontal shift 3 units right and a vertical shift

$c = -3: f(x) = \sqrt{x-3} - 3$ — 3 units down

$c = -1: f(x) = \sqrt{x-3} - 1$ — 1 unit down

$c = 1: f(x) = \sqrt{x-3} + 1$ — 1 unit up

$c = 3: f(x) = \sqrt{x-3} + 3$ — 3 units up

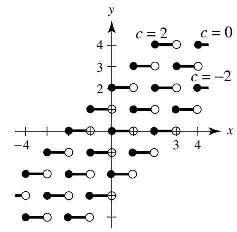


9. (a) $f(x) = [\![x]\!] + c$ — Vertical shifts

$c = -2: f(x) = [\![x]\!] - 2$ — 2 units down

$c = 0: f(x) = [\![x]\!]$ — Parent function

$c = 2: f(x) = [\![x]\!] + 2$ — 2 units up

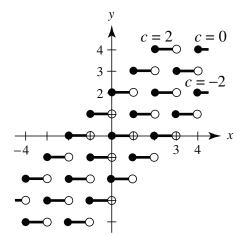


(b) $f(x) = [\![x + c]\!]$ — Horizontal shifts

$c = -2: f(x) = [\![x - 2]\!]$ — 2 units right

$c = 0: f(x) = [\![x]\!]$ — Parent function

$c = 2: f(x) = [\![x + 2]\!]$ — 2 units left

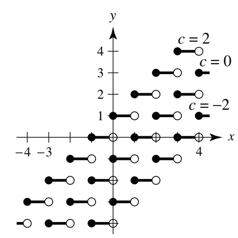


(c) $f(x) = [\![x - 1]\!] + c$ — Horizontal shift 1 unit right and a vertical shift

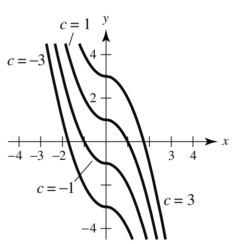
$c = -2: f(x) = [\![x - 1]\!] - 2$ — 2 units down

$c = 0: f(x) = [\![x - 1]\!]$

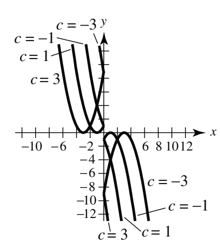
$c = 2: f(x) = [\![x - 1]\!] + 2$ — 2 units up



10. (a) $f(x) = \begin{cases} x^2 + c, & x < 0 \\ -x^2 + c, & x \ge 0 \end{cases}$



(b) $f(x) = \begin{cases} (x + c)^2, & x < 0 \\ -(x + c)^2, & x \ge 0 \end{cases}$

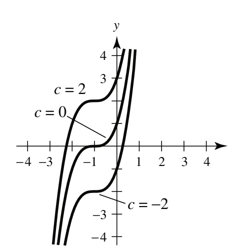


(c) $f(x) = (x + 1)^3 + c$ — Horizontal shift 1 unit to the left and a vertical shift

$c = -2: f(x) = (x + 1)^3 - 2$ — 2 units down

$c = 0: f(x) = (x + 1)^3$

$c = 2: f(x) = (x + 1)^3 + 2$ — 2 units up



11. Parent function: $f(x) = x^2$

(a) Vertical shift 1 unit downward

$g(x) = x^2 - 1$

(b) Reflection in the x-axis, horizontal shift 1 unit to the left, and a vertical shift 1 unit upward

$g(x) = -(x + 1)^2 + 1$

(c) Reflection in the x-axis, horizontal shift 2 units to the right, and a vertical shift 6 units upward

$g(x) = -(x - 2)^2 + 6$

(d) Horizontal shift 5 units to the right and a vertical shift 3 units downward

$g(x) = (x - 5)^2 - 3$

12. Parent function: $f(x) = x^3$

(a) Reflected in the x-axis and shifted upward 1 unit

$g(x) = -x^3 + 1 = 1 - x^3$

(b) Shifted to the right 1 unit and upward 1 unit

$g(x) = (x - 1)^3 + 1$

(c) Reflected in the x-axis and shifted to the left 3 units and downward 1 unit

$g(x) = -(x + 3)^3 - 1$

(d) Shifted to the right 10 units and downward 4 units

$g(x) = (x - 10)^3 - 4$

13. Parent function: $f(x) = |x|$

(a) Vertical shift 5 units upward

$g(x) = |x| + 5$

(b) Reflection in the x-axis and a horizontal shift 3 units to the left

$g(x) = -|x + 3|$

(c) Horizontal shift 2 units to the right and a vertical shift 4 units downward

$g(x) = |x - 2| - 4$

(d) Reflection in the x-axis, horizontal shift 6 units to the right, and a vertical shift 1 unit downward

$g(x) = -|x - 6| - 1$

14. Parent function: $f(x) = \sqrt{x}$

(a) Shifted down 3 units

$g(x) = \sqrt{x} - 3$

(b) Shifted downward 7 units and to the left 1 unit

$g(x) = \sqrt{x + 1} - 7$

(c) Reflected in the x-axis and shifted to the right 5 units and upward 5 units

$g(x) = -\sqrt{x - 5} + 5$

(d) Reflected about the x- and y-axis and shifted to the right 3 units and downward 4 units

$g(x) = -\sqrt{-x + 3} - 4$

15. Parent function: $f(x) = x^3$

Horizontal shift 2 units to the right

$y = (x - 2)^3$

16. Parent function: $y = x$

Vertical shrink

$y = \frac{1}{2}x$

17. Parent function: $f(x) = x^2$

Reflection in the x-axis

$y = -x^2$

18. Parent function: $y = [\![x]\!]$

Vertical shift

$y = [\![x]\!] + 4$

19. Parent function: $f(x) = \sqrt{x}$

Reflection in the x-axis and a vertical shift 1 unit upward

$y = -\sqrt{x} + 1$

20. Parent function: $y = |x|$

Horizontal shift

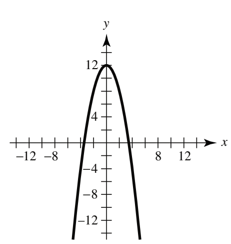
$y = |x + 2|$

21. $g(x) = 12 - x^2$

(a) Parent function: $f(x) = x^2$

(b) Reflection in the x-axis and a vertical shift 12 units upward

(c)



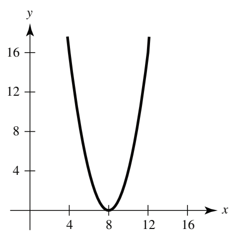
(d) $g(x) = 12 - f(x)$

22. $g(x) = (x - 8)^2$

(a) Parent function: $f(x) = y = x^2$

(b) Horizontal shift of 8 units to the right

(c)



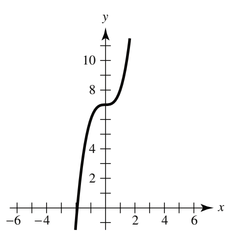
(d) $g(x) = f(x - 8)$

23. $g(x) = x^3 + 7$

(a) Parent function: $f(x) = x^3$

(b) Vertical shift 7 units upward

(c)



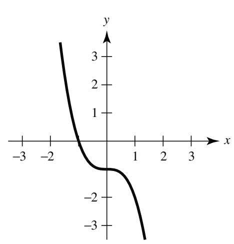
(d) $g(x) = f(x) + 7$

24. $g(x) = -x^3 - 1$

(a) Parent function: $f(x) = x^3$

(b) Reflection in the x-axis; vertical shift of 1 unit downward

(c)



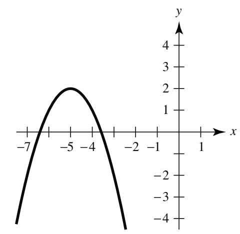
(d) $g(x) = -f(x) - 1$

25. $g(x) = 2 - (x + 5)^2$

(a) Parent function: $f(x) = x^2$

(b) Reflection in the x-axis, horizontal shift 5 units to the left, and a vertical shift 2 units upward

(c)



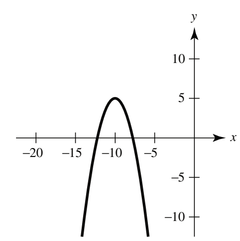
(d) $g(x) = 2 - f(x + 5)$

26. $g(x) = -(x + 10)^2 + 5$

(a) Parent function: $f(x) = x^2$

(b) Reflection in the x-axis, horizontal shift 10 units to the left; vertical shift 5 units upward

(c)



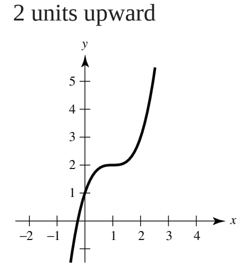
(d) $g(x) = -f(x + 10) + 5$

27. $g(x) = (x - 1)^3 + 2$

(a) Parent function: $f(x) = x^3$

(b) Horizontal shift 1 unit to the right and a vertical shift 2 units upward

(c)



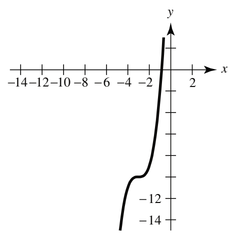
(d) $g(x) = f(x - 1) + 2$

28. $g(x) = (x + 3)^3 - 10$

(a) Parent function: $f(x) = x^3$

(b) Horizontal shift of 3 units to the left; vertical shift of 10 units downward

(c)



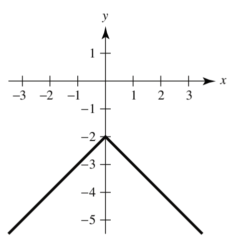
(d) $g(x) = f(x + 3) - 10$

29. $g(x) = -|x| - 2$

(a) Parent function: $f(x) = |x|$

(b) Reflection in the x-axis; vertical shift 2 units downward

(c)



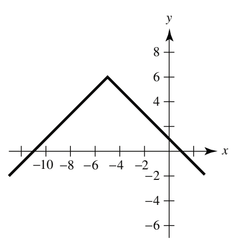
(d) $g(x) = -f(x) - 2$

30. $g(x) = 6 - |x + 5|$

(a) Parent function: $f(x) = |x|$

(b) Reflection in the x-axis; horizontal shift of 5 units to the left; vertical shift of 6 units upward

(c)



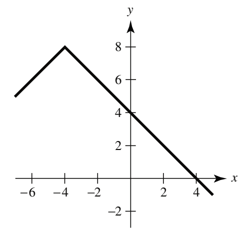
(d) $g(x) = 6 - f(x + 5)$

31. $g(x) = -|x + 4| + 8$

(a) Parent function: $f(x) = |x|$

(b) Reflection in the x-axis; horizontal shift 4 units to the left; and a vertical shift 8 units upward

(c)



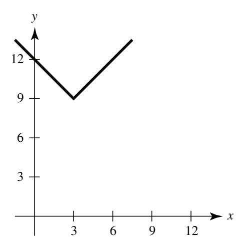
(d) $g(x) = -f(x + 4) + 8$

32. $g(x) = |-x + 3| + 9$

(a) Parent function: $f(x) = |x|$

(b) Reflection in the y-axis; horizontal shift of 3 units to the right; vertical shift of 9 units upward

(c)



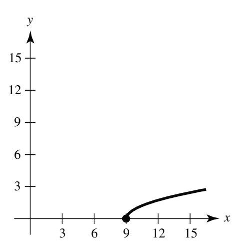
(d) $g(x) = f(-(x - 3)) + 9$

33. $g(x) = \sqrt{x - 9}$

(a) Parent function: $f(x) = \sqrt{x}$

(b) Horizontal shift 9 units to the right

(c)



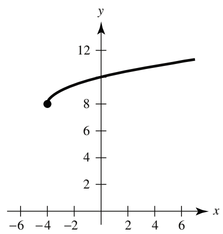
(d) $g(x) = f(x - 9)$

34. $g(x) = \sqrt{x + 4} + 8$

(a) Parent function: $f(x) = \sqrt{x}$

(b) Horizontal shift of 4 units to the left; vertical shift of 8 units upward

(c)



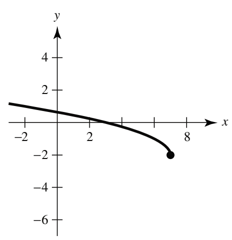
(d) $g(x) = f(x + 4) + 8$

35. $g(x) = \sqrt{7 - x} - 2$ or $g(x) = \sqrt{-(x - 7)} - 2$

(a) Parent function: $f(x) = \sqrt{x}$

(b) Reflection in the y-axis, horizontal shift 7 units to the right, and a vertical shift 2 units downward

(c)



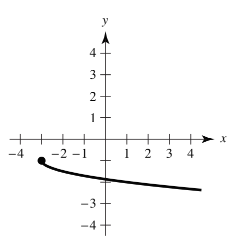
(d) $g(x) = f(7 - x) - 2$

36. $g(x) = -\frac{1}{2}\sqrt{x + 3} - 1$

(a) Parent function: $f(x) = \sqrt{x}$

(b) Horizontal shift 3 units to the left, vertical shrink, reflection in the x-axis, vertical shift one unit down.

(c)



(d) $g(x) = -\frac{1}{2}f(x + 3) - 1$

37. $f(x) = (x - 3)^2 - 7$

38. $f(x) = -(x + 2)^2 + 9$

39. $f(x) = x^3$ moved 13 units to the right.

$g(x) = (x - 13)^3$

40. $f(x) = x^3$ moved 6 units to the left, 6 units downward, and reflected in the y-axis (in that order)

$g(x) = (-x + 6)^3 - 6$

41. $f(x) = -|x| + 12$

42. $f(x) = |x + 4| - 8$

43. $f(x) = \sqrt{x}$ moved 6 units to the left and reflected in both the x- and y-axes.

$g(x) = -\sqrt{-x + 6}$

44. $f(x) = \sqrt{x}$ moved 9 units downward and reflected in both the x-axis and the y-axis

$g(x) = -\left(\sqrt{-x} - 9\right)$

45. $f(x) = x^2$

(a) Reflection in the x-axis and a vertical stretch (each y-value is multiplied by 3)

$g(x) = -3x^2$

(b) Vertical shift 3 units upward and a vertical stretch (each y-value is multiplied by 4)

$g(x) = 4x^2 + 3$

46. $f(x) = x^3$

(a) Vertical shrink (each y-value is multiplied by $\frac{1}{4}$)

$g(x) = \frac{1}{4}x^3$

(b) Reflection in the x-axis and a vertical stretch (each y-value is multiplied by 2)

$g(x) = -2x^3$

47. $f(x) = |x|$

(a) Reflection in the x-axis and a vertical shrink (each y-value is multiplied by $\frac{1}{2}$)

$g(x) = -\frac{1}{2}|x|$

(b) Vertical stretch (each y-value is multiplied by 3) and a vertical shift 3 units downward

$g(x) = 3|x| - 3$

48. $f(x) = \sqrt{x}$

(a) Vertical stretch (each y-value is multiplied by 8)

$g(x) = 8\sqrt{x}$

(b) Reflection in the x-axis and a vertical shrink (each y-value is multiplied by $\frac{1}{4}$)

$g(x) = -\frac{1}{4}\sqrt{x}$

49. Parent function: $f(x) = x^3$

Vertical stretch (each y-value is multiplied by 2)

$g(x) = 2x^3$

50. Parent function: $f(x) = |x|$

Vertical stretch (each y-value is multiplied by 6)

$g(x) = 6|x|$

51. Parent function: $f(x) = x^2$

Reflection in the x-axis; vertical shrink $\left(\text{each } y\text{-value is multiplied by } \frac{1}{2}\right)$

$g(x) = -\frac{1}{2}x^2$

52. Parent function: $y = [\![x]\!]$

Horizontal stretch (each x-value is multiplied by 2)

$g(x) = [\![\frac{1}{2}x]\!]$

53. Parent function: $f(x) = x^3$

Reflection in the x-axis, horizontal shift 2 units to the right and a vertical shift 2 units upward

$g(x) = -(x - 2)^3 + 2$

54. Parent function: $f(x) = |x|$

Horizontal shift of 4 units to the left and a vertical shift of 2 units downward

$g(x) = |x + 4| - 2$

55. Parent function: $f(x) = \sqrt{x}$

Reflection in the x-axis and a vertical shift 3 units downward

$g(x) = -\sqrt{x} - 3$

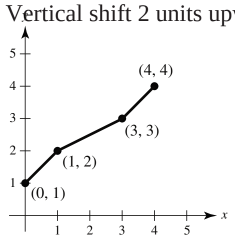
56. Parent function: $f(x) = x^2$

Horizontal shift of 2 units to the right and a vertical shift of 4 units upward.

$g(x) = (x - 2)^2 + 4$

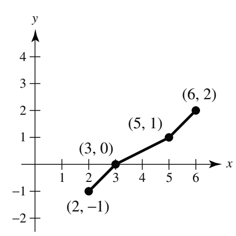
57. (a) $y = f(x) + 2$

Vertical shift 2 units upward



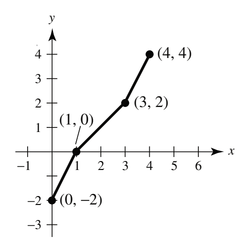
(b) $y = f(x - 2)$

Horizontal shift 2 units to the right



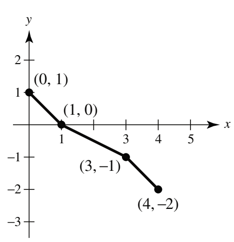
(c) $y = 2f(x)$

Vertical stretch by a factor of 2.



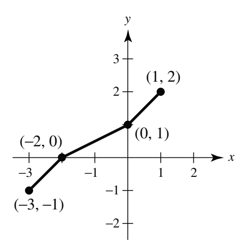
(d) $y = -f(x)$

Reflection in the x-axis



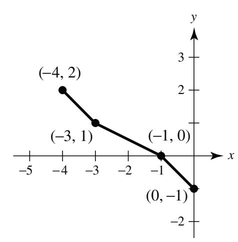
(e) $y = f(x + 3)$

Horizontal shift 3 units to the left



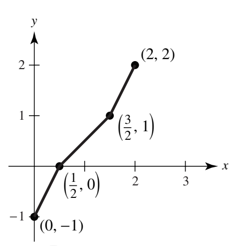
(f) $y = f(-x)$

Reflection in the y-axis

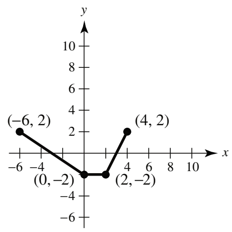


(g) $y = f(2x)$

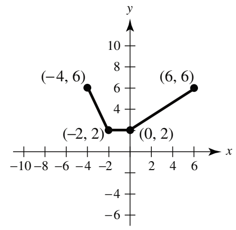
Horizontal shink
Each x-value is divided by 2.



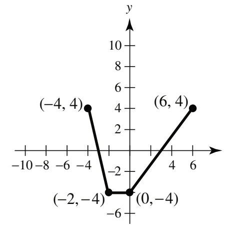
58. (a) $y = f(-x)$



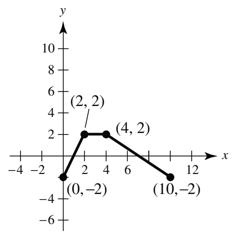
(b) $y = f(x) + 4$



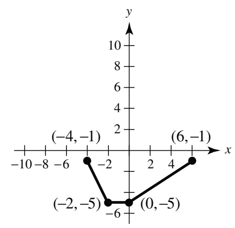
(c) $y = 2f(x)$



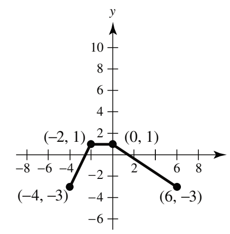
(d) $y = -f(x - 4)$



(e) $y = f(x) - 3$

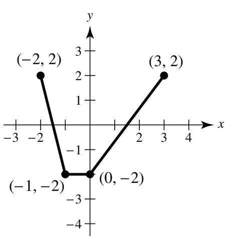


(f) $y = -f(x) - 1$



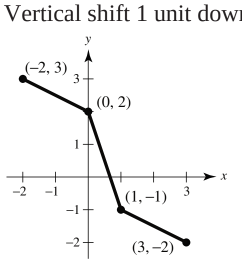
(g) $y = f(2x)$

Horizontal shrink. Each x-value is divided by 2.



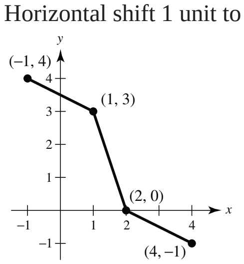
59. (a) $y = f(x) - 1$

Vertical shift 1 unit downward



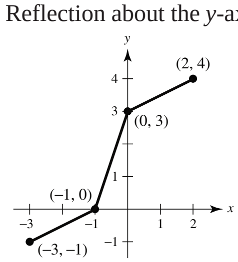
(b) $y = f(x - 1)$

Horizontal shift 1 unit to the right



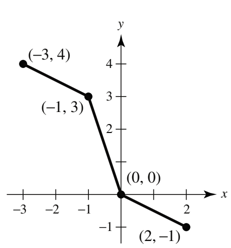
(c) $y = f(-x)$

Reflection about the y-axis



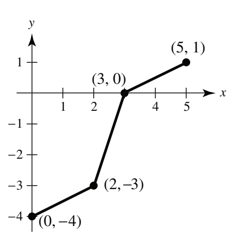
(d) $y = f(x + 1)$

Horizontal shift 1 unit to the left



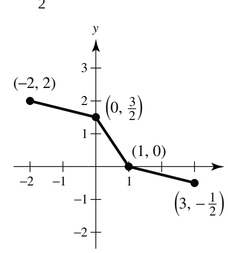
(e) $y = -f(x - 2)$

Reflection about the x-axis and a horizontal shift 2 units to the right



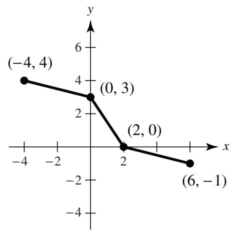
(f) $y = f(-x)$

Vertical shrink by a factor of $\frac{1}{2}$

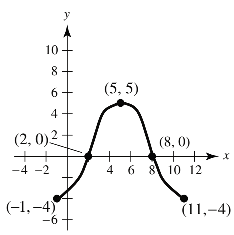


(g) $y = f\left(\frac{1}{2}x\right)$

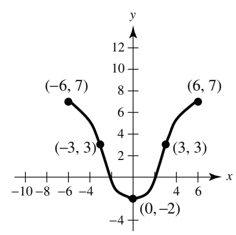
Horizontal stretch Each x-value is multiplied by 2.



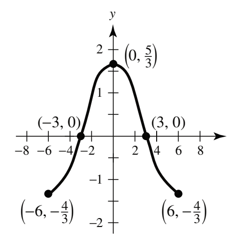
60. (a) $y = f(x - 5)$



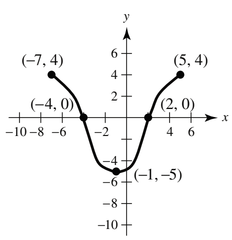
(b) $y = -f(x) + 3$



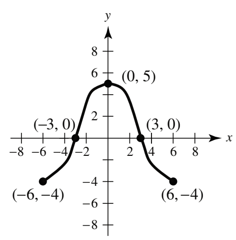
(c) $y = \frac{1}{3}f(x)$



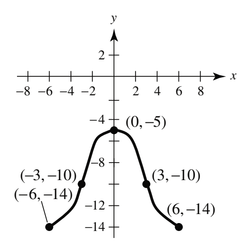
(d) $y = -f(x + 1)$



(e) $y = f(-x)$

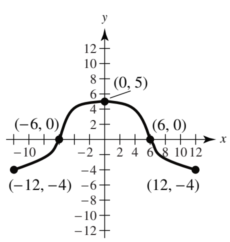


(f) $y = f(x) - 10$



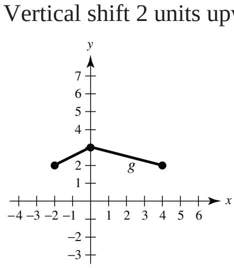
(g) $y = f\left(\frac{1}{2}x\right)$

Horizontal stretch. Each x-value is multiplied by 2.



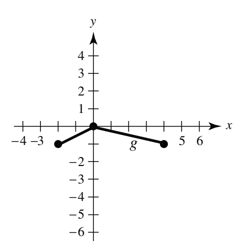
61. (a) $g(x) = f(x) + 2$

Vertical shift 2 units upward



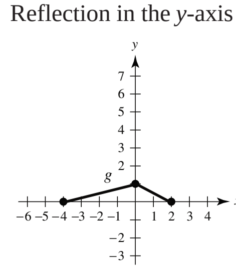
(b) $g(x) = f(x) - 1$

Vertical shift 1 unit downward



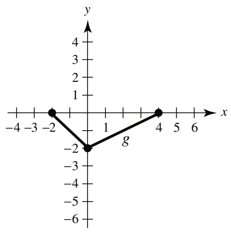
(c) $g(x) = f(-x)$

Reflection in the y-axis



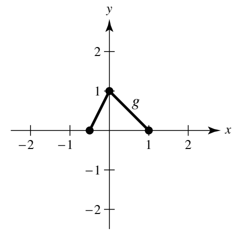
(d) $g(x) = -2f(x)$

Reflection in the x-axis and a vertical stretch by a factor of 2



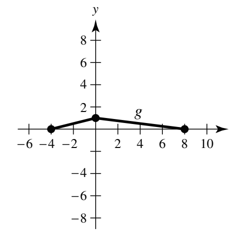
(e) $g(x) = f(2x)$

Horizontal shrink
Each x-value is divided by 2.

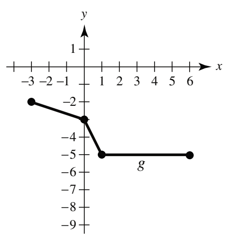


(f) $g(x) = f\left(\frac{1}{2}x\right)$

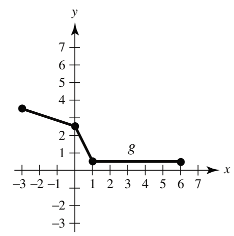
Horizontal stretch
Each x-value is multiplied by 2.



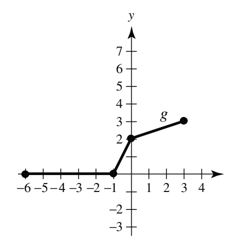
62. (a) $g(x) = f(x) - 5$



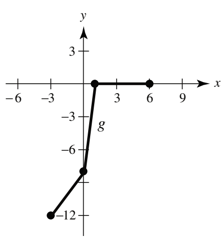
(b) $g(x) = f(x) + \frac{1}{2}$



(c) $g(x) = f(-x)$

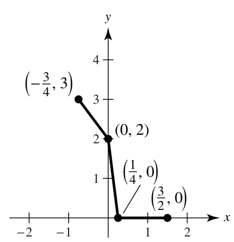


(d) $g(x) = -4f(x)$



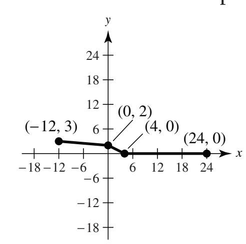
(e) $g(x) = f(4x)$

Horizontal shrink
Each x-value is divided by 4.

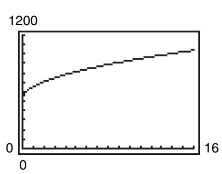


(f) $g(x) = f\left(\frac{1}{4}x\right)$

Horizontal stretch
Each x-value is multiplied by 4.

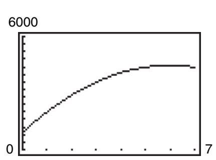


63. (a) Vertical stretch of 128.0 and vertical shift 527 units up



(b) $M = 527 + 128.0\sqrt{t + 10}$; The graph is shifted 10 units to the left.

64. (a) Horizontal shift 5.99 units to the right, vertical stretch, vertical shift 5617 units up, reflection in the x-axis.



(b) 2015: Use $t = 15$ $\quad N(15) = -24.70(15 - 5.99)^2 + 5617 \approx 3611.85$

In the year 2015, there will be about 3,612,000 couples with stay-at-home mothers.

Answers will vary. *Sample answer:* No, because the number of stay-at-home mothers has been increasing on average.

65. True, because $|x| = |-x|$, the graphs of $f(x) = |x| + 6$ and $f(x) = |-x| + 6$ are identical.

66. False. The point $(-2, -61)$ lies on the transformation.

67. True.

$$\begin{aligned} \text{Let} \quad g(x) &= f(x) + c \\ \text{Then } g(-x) &= f(-x) + c \\ &= f(x) + c \quad \text{Since } f \text{ is even} \\ &= g(x) \end{aligned}$$

Thus, $g(x) = f(x) + c$ is also even.

68. (a) Increasing: $(-2, 1)$

Decreasing: $(-\infty, -2)$ and $(1, \infty)$

(b) Increasing: $(-1, 2)$

Decreasing: $(-\infty, -1)$ and $(2, \infty)$

(c) Increasing: $(-\infty, -1)$ and $(2, \infty)$

Decreasing: $(-1, 2)$

(d) Increasing: $(0, 3)$

Decreasing: $(-\infty, 0)$ and $(3, \infty)$

(e) Increasing: $(-\infty, 1)$ and $(4, \infty)$

Decreasing: $(1, 4)$

Section 1.4 Combinations of Functions

1. addition; subtraction; multiplication; division

2. composition

3. $g(x)$

4. inner; outer

5. $f(x) = x + 2, g(x) = x - 2$

(a) $$\begin{aligned}(f + g)(x) &= f(x) + g(x) \\ &= (x + 2) + (x - 2) \\ &= 2x\end{aligned}$$

(b) $$\begin{aligned}(f - g)(x) &= f(x) - g(x) \\ &= (x + 2) - (x - 2) \\ &= 4\end{aligned}$$

(c) $$\begin{aligned}(fg)(x) &= f(x) \cdot g(x) \\ &= (x + 2)(x - 2) \\ &= x^2 - 4\end{aligned}$$

(d) $$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{x + 2}{x - 2}$$

Domain: all real numbers x except $x = 2$

6. $f(x) = 2x - 5, g(x) = 2 - x$

(a) $(f + g)(x) = 2x - 5 + 2 - x = x - 3$

(b) $$\begin{aligned}(f - g)(x) &= 2x - 5 - (2 - x) \\ &= 2x - 5 - 2 + x \\ &= 3x - 7\end{aligned}$$

(c) $$\begin{aligned}(fg)(x) &= (2x - 5)(2 - x) \\ &= 4x - 2x^2 - 10 + 5x \\ &= -2x^2 + 9x - 10\end{aligned}$$

(d) $$\left(\frac{f}{g}\right)(x) = \frac{2x - 5}{2 - x}$$

Domain: all real numbers x except $x = 2$

7. $f(x) = x^2, g(x) = 4x - 5$

(a) $$\begin{aligned}(f + g)(x) &= f(x) + g(x) \\ &= x^2 + (4x - 5) \\ &= x^2 + 4x - 5\end{aligned}$$

(b) $$\begin{aligned}(f - g)(x) &= f(x) - g(x) \\ &= x^2 - (4x - 5) \\ &= x^2 - 4x + 5\end{aligned}$$

(c) $$\begin{aligned}(fg)(x) &= f(x) \cdot g(x) \\ &= x^2(4x - 5) \\ &= 4x^3 - 5x^2\end{aligned}$$

(d) $$\begin{aligned}\left(\frac{f}{g}\right)(x) &= \frac{f(x)}{g(x)} \\ &= \frac{x^2}{4x - 5}\end{aligned}$$

Domain: all real numbers x except $x = \frac{5}{4}$

8. $f(x) = 3x + 1, g(x) = 5x - 4$

(a) $(f + g)(x) = f(x) + g(x)$
$= 3x + 1 + 5x - 4$
$= 8x - 3$

(b) $(f - g)(x) = f(x) - g(x)$
$= 3x + 1 - (5x - 4)$
$= -2x + 5$

(c) $(fg)(x) = f(x) \cdot g(x)$
$= (3x + 1)(5x - 4)$
$= 15x^2 - 7x - 4$

(d) $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{3x + 1}{5x - 4}$

Domain: all real numbers x except $x = \frac{4}{5}$.

9. $f(x) = x^2 + 6, g(x) = \sqrt{1 - x}$

(a) $(f + g)(x) = f(x) + g(x) = x^2 + 6 + \sqrt{1 - x}$

(b) $(f - g)(x) = f(x) - g(x) = x^2 + 6 - \sqrt{1 - x}$

(c) $(fg)(x) = f(x) \cdot g(x) = (x^2 + 6)\sqrt{1 - x}$

(d) $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{x^2 + 6}{\sqrt{1 - x}} = \frac{(x^2 + 6)\sqrt{1 - x}}{1 - x},$

Domain: $x < 1$

10. $f(x) = \sqrt{x^2 - 4}, g(x) = \frac{x^2}{x^2 + 1}$

(a) $(f + g)(x) = \sqrt{x^2 - 4} + \frac{x^2}{x^2 + 1}$

(b) $(f - g)(x) = \sqrt{x^2 - 4} - \frac{x^2}{x^2 + 1}$

(c) $(fg)(x) = \sqrt{x^2 - 4}\left(\frac{x^2}{x^2 + 1}\right) = \frac{x^2\sqrt{x^2 - 4}}{x^2 + 1}$

(d) $\left(\frac{f}{g}\right)(x) = \sqrt{x^2 - 4} \div \frac{x^2}{x^2 + 1}$
$= \frac{(x^2 + 1)\sqrt{x^2 - 4}}{x^2}$

Domain: $x^2 - 4 \ge 0$
$x^2 \ge 4 \Rightarrow x \ge 2$ or $x \le -2$

Domain: $|x| \ge 2$

11. $f(x) = \frac{1}{x}, g(x) = \frac{1}{x^2}$

(a) $(f + g)(x) = f(x) + g(x) = \frac{1}{x} + \frac{1}{x^2} = \frac{x + 1}{x^2}$

(b) $(f - g)(x) = f(x) - g(x) = \frac{1}{x} - \frac{1}{x^2} = \frac{x - 1}{x^2}$

(c) $(fg)(x) = f(x) \cdot g(x) = \frac{1}{x}\left(\frac{1}{x^2}\right) = \frac{1}{x^3}$

(d) $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{1/x}{1/x^2} = \frac{x^2}{x} = x$

Domain: all real numbers x except $x = 0$

12. $f(x) = \frac{x}{x + 1}, g(x) = x^3$

(a) $(f + g)(x) = \frac{x}{x + 1} + x^3 = \frac{x + x^4 + x^3}{x + 1}$

(b) $(f - g)(x) = \frac{x}{x + 1} - x^3 = \frac{x - x^4 - x^3}{x + 1}$

(c) $(fg)(x) = \frac{x}{x + 1} \cdot x^3 = \frac{x^4}{x + 1}$

(d) $\left(\frac{f}{g}\right)(x) = \frac{x}{x + 1} \div x^3 = \frac{x}{x + 1} \cdot \frac{1}{x^3} = \frac{1}{x^2(x + 1)}$

Domain: all real numbers x except $x = 0$ and $x = -1$

For Exercises 13–23, $f(x) = x^2 + 1$ and $g(x) = x - 4$.

13. $(f + g)(2) = f(2) + g(2) = (2^2 + 1) + (2 - 4) = 3$

14. $(f - g)(-1) = f(-1) - g(-1)$
$= (-1)^2 + 1 - (-1 - 4)$
$= 1 + 1 - (-5)$
$= 7$

15. $(f - g)(0) = f(0) - g(0)$
$= (0^2 + 1) - (0 - 4)$
$= 5$

16. $(f + g)(1) = f(1) + g(1)$
$= (1)^2 + 1 + (1) - 4$
$= -1$

17. $(f - g)(3t) = f(3t) - g(3t)$
$= \left[(3t)^2 + 1\right] - (3t - 4)$
$= 9t^2 - 3t + 5$

18. $(f+g)(t-2) = f(t-2) + g(t-2)$
$= (t-2)^2 + 1 + (t-2) - 4$
$= t^2 - 4t + 4 + 1 + t - 2 - 4$
$= t^2 - 3t - 1$

19. $(fg)(6) = f(6)g(6)$
$= (6^2 + 1)(6 - 4)$
$= 74$

20. $(fg)(-6) = f(-6) \cdot g(-6)$
$= \left[(-6)^2 + 1\right]\left[(-6) - 4\right]$
$= (37)(-10)$
$= -370$

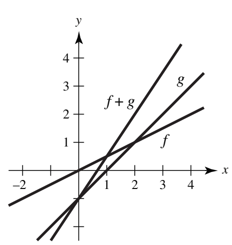
21. $\left(\dfrac{f}{g}\right)(5) = \dfrac{f(5)}{g(5)} = \dfrac{5^2 + 1}{5 - 4} = 26$

22. $\left(\dfrac{f}{g}\right)(0) = \dfrac{f(0)}{g(0)} = \dfrac{0^2 + 1}{0 - 4} = -\dfrac{1}{4}$

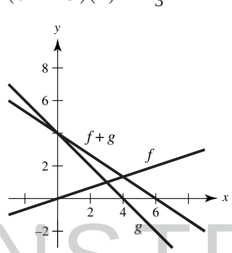
23. $\left(\dfrac{f}{g}\right)(-1) - g(3) = \dfrac{f(-1)}{g(-1)} - g(3)$
$= \dfrac{(-1)^2 + 1}{-1 - 4} - (3 - 4)$
$= -\dfrac{2}{5} + 1 = \dfrac{3}{5}$

24. $(fg)(5) + f(4) = f(5)g(5) + f(4)$
$= (5^2 + 1)(5 - 4) + (4^2 + 1)$
$= 26 \cdot 1 + 17$
$= 43$

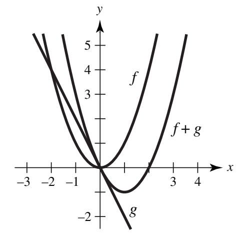
25. $f(x) = \frac{1}{2}x,\ g(x) = x - 1$
$(f + g)(x) = \frac{3}{2}x - 1$



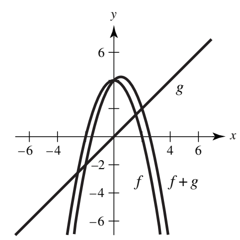
26. $f(x) = \frac{1}{3}x,\ g(x) = -x + 4$
$(f + g)(x) = \frac{1}{3}x - x + 4 = -\frac{2}{3}x + 4$



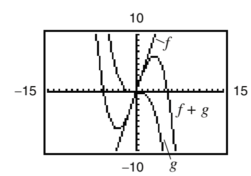
27. $f(x) = x^2,\ g(x) = -2x$
$(f + g)(x) = x^2 - 2x$



28. $f(x) = 4 - x^2,\ g(x) = x$
$(f + g)(x) = 4 - x^2 + x = 4 + x - x^2$

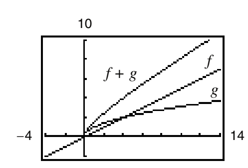


29. $f(x) = 3x,\ g(x) = -\dfrac{x^3}{10}$
$(f + g)(x) = 3x - \dfrac{x^3}{10}$



For $0 \le x \le 2$, $f(x)$ contributes most to the magnitude.
For $x > 6$, $g(x)$ contributes most to the magnitude.

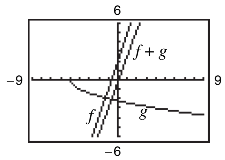
30. $f(x) = \dfrac{x}{2},\ g(x) = \sqrt{x}$
$(f + g)(x) = \dfrac{x}{2} + \sqrt{x}$



$g(x)$ contributes most to the magnitude of the sum for $0 \le x \le 2$. $f(x)$ contributes most to the magnitude of the sum for $x > 6$.

31. $f(x) = 3x + 2,\ g(x) = -\sqrt{x+5}$

$(f + g)x = 3x - \sqrt{x+5} + 2$

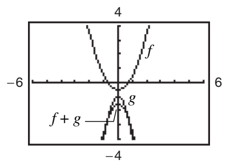


For $0 \le x \le 2$, $f(x)$ contributes most to the magnitude.

For $x > 6$, $f(x)$ contributes most to the magnitude.

32. $f(x) = x^2 - \frac{1}{2},\ g(x) = -3x^2 - 1$

$(f + g)(x) = -2x^2 - \frac{3}{2}$



For $0 \le x \le 2$, $g(x)$ contributes most to the magnitude.

For $x > 6$, $g(x)$ contributes most to the magnitude.

33. $f(x) = x^2,\ g(x) = x - 1$

(a) $(f \circ g)(x) = f(g(x)) = f(x-1) = (x-1)^2$

(b) $(g \circ f)(x) = g(f(x)) = g(x^2) = x^2 - 1$

(c) $(g \circ g)(x) = g(g(x)) = g(x-1) = x - 2$

34. $f(x) = 3x + 5,\ g(x) = 5 - x$

(a) $(f \circ g)(x) = f(g(x))$
$= f(5-x) = 3(5-x) + 5$
$= 20 - 3x$

(b) $(g \circ f)(x) = g(f(x))$
$= g(3x+5) = 5 - (3x+5)$
$= -3x$

(c) $(g \circ g)(x) = g(g(x)) = g(5-x) = x$

35. $f(x) = \sqrt[3]{x-1},\ g(x) = x^3 + 1$

(a) $(f \circ g)(x) = f(g(x))$
$= f(x^3+1)$
$= \sqrt[3]{(x^3+1) - 1}$
$= \sqrt[3]{x^3} = x$

(b) $(g \circ f)(x) = g(f(x))$
$= g(\sqrt[3]{x-1})$
$= (\sqrt[3]{x-1})^3 + 1$
$= (x-1) + 1 = x$

(c) $(g \circ g)(x) = g(g(x))$
$= g(x^3+1)$
$= (x^3+1)^3 + 1$
$= x^9 + 3x^6 + 3x^3 + 2$

36. $f(x) = x^3,\ g(x) = \dfrac{1}{x}$

(a) $(f \circ g)(x) = f(g(x)) = f\left(\dfrac{1}{x}\right) = \left(\dfrac{1}{x}\right)^3 = \dfrac{1}{x^3}$

(b) $(g \circ f)(x) = g(f(x)) = g(x^3) = \dfrac{1}{x^3}$

(c) $(g \circ g)(x) = g(g(x)) = g\left(\dfrac{1}{x}\right) = x$

37. $f(x) = \sqrt{x+4}$ Domain: $x \ge -4$

$g(x) = x^2$ Domain: all real numbers x

(a) $(f \circ g)(x) = f(g(x)) = f(x^2) = \sqrt{x^2+4}$

Domain: all real numbers x

(b) $(g \circ f)(x) = g(f(x))$
$= g(\sqrt{x+4}) = (\sqrt{x+4})^2 = x + 4$

Domain: $x \ge -4$

38. $f(x) = \sqrt[3]{x-5}$ Domain: all real numbers x

$g(x) = x^3 + 1$ all real numbers x

(a) $(f \circ g)(x) = f(g(x))$
$= f(x^3+1)$
$= \sqrt[3]{x^3 + 1 - 5}$
$= \sqrt[3]{x^3 - 4}$

Domain: all real numbers x

(b) $(g \circ f)(x) = g(f(x))$
$= g(\sqrt[3]{x-5})$
$= (\sqrt[3]{x-5})^3 + 1$
$= x - 5 + 1 = x - 4$

Domain: all real numbers x

39. $f(x) = x^2 + 1$ Domain: all real numbers x

$g(x) = \sqrt{x}$ Domain: $x \geq 0$

(a) $(f \circ g)(x) = f(g(x))$

$= f(\sqrt{x})$

$= (\sqrt{x})^2 + 1$

$= x + 1$

Domain: $x \geq 0$

(b) $(g \circ f)(x) = g(f(x)) = g(x^2 + 1) = \sqrt{x^2 + 1}$

Domain: all real numbers x

40. $f(x) = x^{2/3}$ Domain: all real numbers x

$g(x) = x^6$ Domain: all real numbers x

(a) $(f \circ g)(x) = f(g(x)) = f(x^6) = (x^6)^{2/3} = x^4$

Domain: all real numbers x

(b) $(g \circ f)(x) = g(f(x)) = g(x^{2/3}) = (x^{2/3})^6 = x^4$

Domain: all real numbers x

41. $f(x) = |x|$ Domain: all real numbers x

$g(x) = x + 6$ Domain: all real numbers x

(a) $(f \circ g)(x) = f(g(x)) = f(x + 6) = |x + 6|$

Domain: all real numbers x

(b) $(g \circ f)(x) = g(f(x)) = g(|x|) = |x| + 6$

Domain: all real numbers x

42. $f(x) = |x - 4|$ Domain: all real numbers x

$g(x) = 3 - x$ Domain: all real numbers x

(a) $(f \circ g)(x) = f(g(x)) = f(3 - x) = |(3 - x) - 4| = |-x - 1|$

Domain: all real numbers x

(b) $(g \circ f)(x) = g(f(x)) = g(|x - 4|) = 3 - (|x - 4|) = 3 - |x - 4|$

Domain: all real numbers x

43. $f(x) = \dfrac{1}{x}$ Domain: all real numbers x except $x = 0$

$g(x) = x + 3$ Domain: all real numbers x

(a) $(f \circ g)(x) = f(g(x)) = f(x + 3) = \dfrac{1}{x + 3}$

Domain: all real numbers x except $x = -3$

(b) $(g \circ f)(x) = g(f(x)) = g\left(\dfrac{1}{x}\right) = \dfrac{1}{x} + 3$

Domain: all real numbers x except $x = 0$

44. $f(x) = \dfrac{3}{x^2 - 1}$ Domain: all real numbers x except $x = \pm 1$

$g(x) = x + 1$ Domain: all real numbers x

(a) $(f \circ g)(x) = f(g(x)) = f(x + 1) = \dfrac{3}{(x + 1)^2 - 1} = \dfrac{3}{x^2 + 2x + 1 - 1} = \dfrac{3}{x^2 + 2x}$

Domain: all real numbers x except $x = 0$ and $x = -2$

(b) $(g \circ f)(x) = g(f(x)) = g\left(\dfrac{3}{x^2 - 1}\right) = \dfrac{3}{x^2 - 1} + 1 = \dfrac{3 + x^2 - 1}{x^2 - 1} = \dfrac{x^2 + 2}{x^2 - 1}$

Domain: all real numbers x except $x = \pm 1$

45. $h(x) = (2x + 1)^2$

One possibility: Let $f(x) = x^2$ and $g(x) = 2x + 1$, then $(f \circ g)(x) = h(x)$.

46. $h(x) = (1 - x)^3$

One possibility: Let $g(x) = 1 - x$ and $f(x) = x^3$, then $(f \circ g)(x) = h(x)$.

47. $h(x) = \sqrt[3]{x^2 - 4}$

One possibility: Let $f(x) = \sqrt[3]{x}$ and $g(x) = x^2 - 4$, then $(f \circ g)(x) = h(x)$.

48. $h(x) = \sqrt{9 - x}$

One possibility: Let $g(x) = 9 - x$ and $f(x) = \sqrt{x}$, then $(f \circ g)(x) = h(x)$.

49. $h(x) = \dfrac{1}{x + 2}$

One possibility: Let $f(x) = 1/x$ and $g(x) = x + 2$, then $(f \circ g)(x) = h(x)$.

50. $h(x) = \dfrac{4}{(5x + 2)^2}$

One possibility: Let $g(x) = 5x + 2$ and $f(x) = \dfrac{4}{x^2}$, then $(f \circ g)(x) = h(x)$.

51. $h(x) = \dfrac{-x^2 + 3}{4 - x^2}$

One possibility: Let $f(x) = \dfrac{x + 3}{4 + x}$ and $g(x) = -x^2$, then $(f \circ g)(x) = h(x)$.

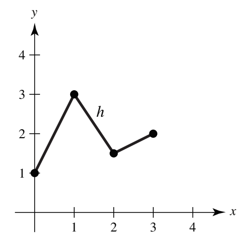
52. $h(x) = \dfrac{27x^3 + 6x}{10 - 27x^3}$

One possibility: Let $g(x) = x^3$ and

$f(x) = \dfrac{27x + 6\sqrt[3]{x}}{10 - 27x}$, then $(f \circ g)(x) = h(x)$.

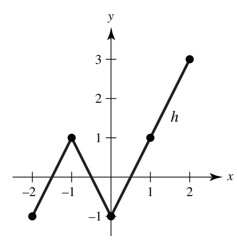
53.

x	0	1	2	3
f	2	3	1	2
g	–1	0	$\frac{1}{2}$	0
$f + g$	1	3	$\frac{3}{2}$	2



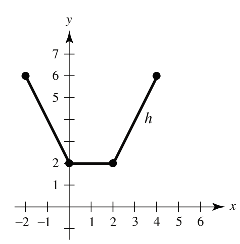
54.

x	–2	–1	0	1	2
$f(x)$	–2	0	–1	–1	1
$g(x)$	1	1	0	2	2
$h(x) = f(x) + g(x)$	–1	1	–1	1	3



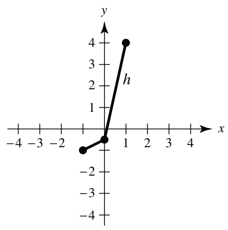
55.

x	–2	0	1	2	4
f	2	0	1	2	4
g	4	2	1	0	2
$f + g$	6	2	2	2	6



56. The domain common to both functions is $[-1, 1]$, which is the domain of the sum.

x	-1	0	1
$f(x)$	0	1.5	3
$g(x)$	-1	-2	1
$h(x) = f(x) + g(x)$	-1	-0.5	4



57. (a) $(f + g)(3) = f(3) + g(3) = 2 + 1 = 3$

(b) $\left(\frac{f}{g}\right)(2) = \frac{f(2)}{g(2)} = \frac{0}{2} = 0$

58. (a) $(f - g)(1) = f(1) - g(1) = 2 - 3 = -1$

(b) $(fg)(4) = f(4) \cdot g(4) = 4 \cdot 0 = 0$

59. (a) $(f \circ g)(2) = f(g(2)) = f(2) = 0$

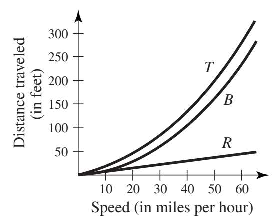
(b) $(g \circ f)(2) = g(f(2)) = g(0) = 4$

60. (a) $(f \circ g)(1) = f(g(1)) = f(3) = 2$

(b) $(g \circ f)(3) = g(f(3)) = g(2) = 2$

61. (a) $T(x) = R(x) + B(x) = \frac{3}{4}x + \frac{1}{15}x^2$

(b)



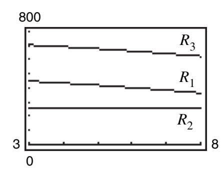
(c) $B(x)$; As x increases, $B(x)$ increases at a faster rate.

62. (a) $R_3 = R_1 + R_2$

$= 480 - 8t - 0.8t^2 + 254 + 0.78t$

$= 734 - 7.22t - 0.8t^2$

(b)



63. $(B - D)(t) = B(t) - D(t) = -0.197t^3 + 10.17t^2 - 128t + 2043$

This represents the number of births (in millions) more than the number of deaths in the United States from 1990 to 2006, where $t = 0$ corresponds to 1990.

64. 2010: Use $t = 20$

$B(20) = 4388$ million

This represents the total number of births in the United States in 2010.

$D(20) = 2413$ million

This represents the total number of deaths in the United States in 2010.

$(B - D)(20) = 1975$ million

This represents the number of births more than the number of deaths in the United States in 2010.

2012: Use $t = 22$

$B(22) = 4438.98$ million

This represents the total number of births in the United States in 2012.

$D(22) = 2387.36$ million

This represents the total number of deaths in the United States in 2012.

$(B - D)(22) = 2051.62$ million

This represents the number of births more than deaths in the United States in 2012.

65. (a) $h(t) = \dfrac{T(t)}{P(t)} = \dfrac{0.0233t^4 - 0.3408t^3 + 1.556t^2 - 1.86t + 22.8}{2.78t + 282.5}$

$h(t)$ represents the number of people (in millions) playing tennis in the United States compared to the number of people (in millions) in the United States, or the percent of the population that plays tennis expressed as a decimal.

(b) $h(0) = 0.0807$ million

$h(3) = 0.0822$ million

$h(6) = 0.0810$ million

66. (a) T is a function of t since for each time t there corresponds one and only one temperature T.

(b) $T(4) \approx 60°$; $T(15) \approx 72°$

(c) $H(t) = T(t - 1)$; All the temperature changes would be one hour later.

(d) $H(t) = T(t) - 1$; The temperature would be decreased by one degree.

(e) The points at the endpoints of the individual functions that form each "piece" appear to be $(0, 60), (6, 60), (7, 72), (20, 72), (21, 60),$ and $(24, 60)$. Note that the value $t = 24$ is chosen for the last ordered pair because that is when the day ends and the cycle starts over.

From $t = 0$ to $t = 6$: This is the constant function $T(t) = 60$.

From $t = 6$ to $t = 7$: Use the points $(6, 60)$ and $(7, 72)$.

$$m = \frac{72 - 60}{7 - 6} = 12$$

$y - 60 = 12(x - 6) \Rightarrow y = 12x - 12,$ or $T(t) = 12t - 12$

From $t = 7$ to $t = 20$: This is the constant function $T(t) = 72$.

From $t = 20$ to $t = 21$: Use the points $(20, 72)$ and $(21, 60)$.

$$m = \frac{72 - 60}{20 - 21} = -12$$

$y - 60 = -12(x - 21) \Rightarrow y = -12x + 312,$ or $T(t) = -12t + 312$

From $t = 21$ to $t = 24$: This is the constant function $T(t) = 60$.

A piecewise-defined function is $T(t) = \begin{cases} 60, & 0 \le t \le 6 \\ 12t - 12, & 6 < t < 7 \\ 72, & 7 \le t \le 20 \\ -12t + 312, & 20 < t < 21 \\ 60, & 21 \le t \le 24 \end{cases}$.

67. (a) $r(x) = \dfrac{x}{2}$

(b) $A(r) = \pi r^2$

(c) $(A \circ r)(x) = A(r(x)) = A\left(\dfrac{x}{2}\right) = \pi\left(\dfrac{x}{2}\right)^2$

$(A \circ r)(x)$ represents the area of the circular base of the tank on the square foundation with side length x.

68. $(A \circ r)(t) = A(r(t)) = A(0.6t) = \pi(0.6t)^2 = 0.36\pi t^2$

$A \circ r$ represents the area of the circle at the time t.

69. $C(x) = 60x + 750, x(t) = 50t$

(a) $(C \circ x)(t) = C(x(t))$
$= C(50t)$
$= 60(50t) + 750$
$= 3000t + 750$

$(C \circ x)(t)$ represents the cost of production as a function of time.

(b) Use $t = 4$

$x(t) = 50(4) = 200$

In 4 hours, 200 units are produced.

(c) Find t when $(C \circ x)(t) = 15{,}000$.

$$15{,}000 = 3000t + 750$$
$$t = 4.75 \text{ hours}$$

The cost of production for 4 hours 45 minutes is $15,000.

70. (a) $f(g(x)) = f(0.03x) = 0.03x - 500{,}000$

(b) $g(f(x)) = g(x - 500{,}000) = 0.03(x - 500{,}000)$

$g(f(x))$ represents your bonus of 3% of an amount over $500,000.

71. False. $(f \circ g)(x) = 6x + 1$ and $(g \circ f)(x) = 6x + 6$.

72. True. The range of g must be a subset of the domain of f for $(f \circ g)(x)$ to be defined.

73. (a) $g(x) = \frac{1}{2}\left[f(x) + f(-x)\right]$

To determine if $g(x)$ is even, show $g(-x) = g(x)$.

$$\begin{aligned} g(-x) &= \frac{1}{2}\left[f(-x) + f(-(-x))\right] \\ &= \frac{1}{2}\left[f(-x) + f(x)\right] \\ &= \frac{1}{2}\left[f(x) + f(-x)\right] \\ &= g(x) \checkmark \end{aligned}$$

$$h(x) = \frac{1}{2}\left[f(x) - f(-x)\right]$$

To determine if $h(x)$ is odd show $h(-x) = -h(x)$

$$\begin{aligned} h(-x) &= \frac{1}{2}\left[f(-x) - f(-(-x))\right] \\ &= \frac{1}{2}\left[f(-x) - f(x)\right] \\ &= -\frac{1}{2}\left[f(x) - f(-x)\right] \\ &= -h(x) \checkmark \end{aligned}$$

(b) Let $f(x) = a$ function

$f(x) =$ even function + odd function

Using the result from part (a) $g(x)$ is an even function and $h(x)$ is an odd function.

$$\begin{aligned} f(x) &= g(x) + h(x) \\ &= \frac{1}{2}\left[f(x) + f(-x)\right] + \frac{1}{2}\left[f(x) - f(-x)\right] \\ &= \frac{1}{2}f(x) + \frac{1}{2}f(-x) + \frac{1}{2}f(x) - \frac{1}{2}f(-x) \\ &= f(x) \checkmark \end{aligned}$$

(c) $f(x) = x^2 - 2x + 1$

$$f(x) = g(x) + h(x)$$

$$\begin{aligned} g(x) &= \frac{1}{2}\left[f(x) + f(-x)\right] \\ &= \frac{1}{2}\left[x^2 - 2x + 1 + (-x)^2 - 2(-x) + 1\right] \\ &= \frac{1}{2}\left[x^2 - 2x + 1 + x^2 + 2x + 1\right] \\ &= \frac{1}{2}\left[2x^2 + 2\right] = x^2 + 1 \end{aligned}$$

$$\begin{aligned} h(x) &= \frac{1}{2}\left[f(x) - f(-x)\right] \\ &= \frac{1}{2}\left[x^2 - 2x + 1 - \left((-x)^2 - 2(-x) + 1\right)\right] \\ &= \frac{1}{2}\left[x^2 - 2x + 1 - x^2 - 2x - 1\right] \\ &= \frac{1}{2}\left[-4x\right] = -2x \end{aligned}$$

$$f(x) = \left(x^2 + 1\right) + (-2x)$$

$$k(x) = \frac{1}{x + 1}$$

$$k(x) = g(x) + h(x)$$

$$\begin{aligned} g(x) &= \frac{1}{2}\left[k(x) + k(-x)\right] \\ &= \frac{1}{2}\left[\frac{1}{x + 1} + \frac{1}{-x + 1}\right] \\ &= \frac{1}{2}\left[\frac{1 - x + x + 1}{(x + 1)(1 - x)}\right] \\ &= \frac{1}{2}\left[\frac{2}{(x + 1)(1 - x)}\right] \\ &= \frac{1}{(x + 1)(1 - x)} \\ &= \frac{-1}{(x + 1)(x - 1)} \end{aligned}$$

$$\begin{aligned} h(x) &= \frac{1}{2}\left[k(x) - k(-x)\right] \\ &= \frac{1}{2}\left[\frac{1}{x + 1} - \frac{1}{1 - x}\right] \\ &= \frac{1}{2}\left[\frac{1 - x - (x + 1)}{(x + 1)(1 - x)}\right] \\ &= \frac{1}{2}\left[\frac{-2x}{(x + 1)(1 - x)}\right] \\ &= \frac{-x}{(x + 1)(1 - x)} \\ &= \frac{x}{(x + 1)(x - 1)} \end{aligned}$$

$$k(x) = \left(\frac{-1}{(x + 1)(x - 1)}\right) + \left(\frac{x}{(x + 1)(x - 1)}\right)$$

74. (a) $\left(\dfrac{f}{g}\right)(x) = x^{3/2}$; Domain: $x > 0$

(b) $(f \circ g)(x) = f(g(x)) = x$; Domain: $x \geq 0$

$(g \circ f)(x) = g(f(x)) = x$; Domain: all real numbers x.

No, they are not the same because negative numbers are in the domain of $g \circ f$ but not in the domain of $f \circ g$.

Section 1.5 Inverse Functions

1. inverse

2. f^{-1}

3. range; domain

4. $y = x$

5. one-to-one

6. Horizontal

7. $f(x) = 6x$

$f^{-1}(x) = \dfrac{x}{6} = \dfrac{1}{6}x$

$f(f^{-1}(x)) = f\left(\dfrac{x}{6}\right) = 6\left(\dfrac{x}{6}\right) = x$

$f^{-1}(f(x)) = f^{-1}(6x) = \dfrac{6x}{6} = x$

8. $f(x) = \frac{1}{3}x$

$f^{-1}(x) = 3x$

$f(f^{-1}(x)) = f(3x) = \frac{1}{3}(3x) = x$

$f^{-1}(f(x)) = f^{-1}\left(\frac{1}{3}x\right) = 3\left(\frac{1}{3}x\right) = x$

9. $f(x) = x + 9$

$f^{-1}(x) = x - 9$

$f(f^{-1}(x)) = f(x - 9) = (x - 9) + 9 = x$

$f^{-1}(f(x)) = f^{-1}(x + 9) = (x + 9) - 9 = x$

10. $f(x) = x - 4$

$f^{-1}(x) = x + 4$

$f(f^{-1}(x)) = f(x + 4) = x + 4 - 4 = x$

$f^{-1}(f(x)) = f^{-1}(x - 4) = x - 4 + 4 = x$

11. $f(x) = 3x + 1$

$f^{-1}(x) = \dfrac{x - 1}{3}$

$f(f^{-1}(x)) = f\left(\dfrac{x - 1}{3}\right) = 3\left(\dfrac{x - 1}{3}\right) + 1 = x$

$f^{-1}(f(x)) = f^{-1}(3x + 1) = \dfrac{(3x + 1) - 1}{3} = x$

12. $f(x) = -2x - 9$

$y = -2x - 9$

$x = -2y - 9$

$-\dfrac{1}{2}(x + 9) = y$

$f^{-1}(x) = -\dfrac{1}{2}(x + 9)$

$f(f^{-1}(x)) = f\left[-\dfrac{1}{2}(x + 9)\right] = -2\left[-\dfrac{1}{2}(x + 9)\right] - 9$

$= (x + 9) - 9 = x$

$f^{-1}(f(x)) = f^{-1}(-2x - 9) = -\dfrac{1}{2}\left[(-2x - 9) + 9\right]$

$= -\dfrac{1}{2}(-2x) = x$

13. $f(x) = \dfrac{x - 1}{5}$

$f^{-1}(x) = 5x + 1$

$f(f^{-1}(x)) = f(5x + 1) = \dfrac{5x + 1 - 1}{5} = \dfrac{5x}{5} = x$

$f^{-1}(f(x)) = f^{-1}\left(\dfrac{x - 1}{5}\right) = 5\left(\dfrac{x - 1}{5}\right) + 1$

$= x - 1 + 1 = x$

14. $f(x) = \dfrac{4x + 7}{2}$

$y = \dfrac{4x + 7}{2}$

$x = \dfrac{4y + 7}{2}$

$2x = 4y + 7$

$\dfrac{1}{4}(2x - 7) = y$

$f^{-1}(x) = \dfrac{1}{4}(2x - 7)$

$f\left(f^{-1}(x)\right) = f\left[\dfrac{1}{4}(2x - 7)\right] = \dfrac{4\left[1/4(2x - 7)\right] + 7}{2} = \dfrac{(2x - 7) + 7}{2} = x$

$f^{-1}\left(f(x)\right) = f^{-1}\left(\dfrac{4x + 7}{2}\right) = \dfrac{1}{4}\left[2\left(\dfrac{4x + 7}{2}\right) - 7\right] = \dfrac{1}{4}\left[(4x + 7) - 7\right] = x$

15. $f(x) = \sqrt[3]{x}$

$f^{-1}(x) = x^3$

$f\left(f^{-1}(x)\right) = f\left(x^3\right) = \sqrt[3]{x^3} = x$

$f^{-1}\left(f(x)\right) = f^{-1}\left(\sqrt[3]{x}\right) = \left(\sqrt[3]{x}\right)^3 = x$

16. $f(x) = x^5$

$f^{-1}(x) = \sqrt[5]{x}$

$f\left(f^{-1}(x)\right) = f\left(\sqrt[5]{x}\right) = \left(\sqrt[5]{x}\right)^5 = x$

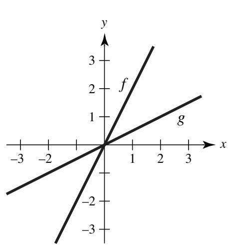
$f^{-1}\left(f(x)\right) = f^{-1}\left(x^5\right) = \sqrt[5]{x^5} = x$

17. $f(x) = 2x, g(x) = \dfrac{x}{2}$

(a) $f\left(g(x)\right) = f\left(\dfrac{x}{2}\right) = 2\left(\dfrac{x}{2}\right) = x$

$g\left(f(x)\right) = g(2x) = \dfrac{2x}{2} = x$

(b)

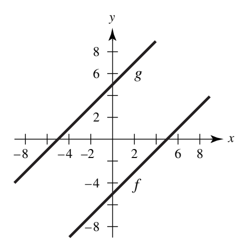


18. $f(x) = x - 5, g(x) = x + 5$

(a) $f\left(g(x)\right) = f(x + 5) = (x + 5) - 5 = x$

$g\left(f(x)\right) = g(x - 5) = (x - 5) + 5 = x$

(b)

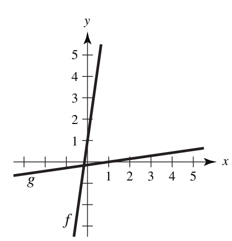


19. $f(x) = 7x + 1, g(x) = \dfrac{x - 1}{7}$

(a) $f\left(g(x)\right) = f\left(\dfrac{x - 1}{7}\right) = 7\left(\dfrac{x - 1}{7}\right) + 1 = x$

$g\left(f(x)\right) = g(7x + 1) = \dfrac{(7x + 1) - 1}{7} = x$

(b)



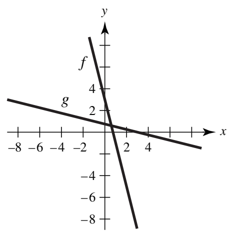
20. $f(x) = 3 - 4x,\ g(x) = \dfrac{3 - x}{4}$

(a) $f(g(x)) = f\left(\dfrac{3 - x}{4}\right) = 3 - 4\left(\dfrac{3 - x}{4}\right)$

$= 3 - (3 - x) = x$

$g(f(x)) = g(3 - 4x) = \dfrac{3 - (3 - 4x)}{4} = \dfrac{4x}{4} = x$

(b)

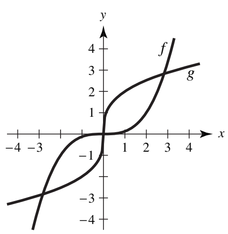


21. $f(x) = \dfrac{x^3}{8},\ g(x) = \sqrt[3]{8x}$

(a) $f(g(x)) = f\left(\sqrt[3]{8x}\right) = \dfrac{\left(\sqrt[3]{8x}\right)^3}{8} = \dfrac{8x}{8} = x$

$g(f(x)) = g\left(\dfrac{x^3}{8}\right) = \sqrt[3]{8\left(\dfrac{x^3}{8}\right)} = \sqrt[3]{x^3} = x$

(b)

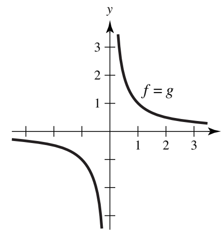


22. $f(x) = \dfrac{1}{x},\ g(x) = \dfrac{1}{x}$

(a) $f(g(x)) = f\left(\dfrac{1}{x}\right) = \dfrac{1}{1/x} = 1 \div \dfrac{1}{x} = 1 \cdot \dfrac{x}{1} = x$

$g(f(x)) = g\left(\dfrac{1}{x}\right) = \dfrac{1}{1/x} = 1 \div \dfrac{1}{x} = 1 \cdot \dfrac{x}{1} = x$

(b)



23. $f(x) = \sqrt{x - 4},\ g(x) = x^2 + 4,\ x \geq 0$

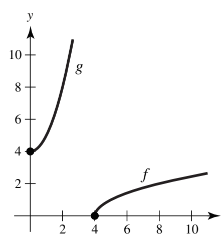
(a) $f(g(x)) = f(x^2 + 4),\ x \geq 0$

$= \sqrt{(x^2 + 4) - 4} = x$

$g(f(x)) = g\left(\sqrt{x - 4}\right)$

$= \left(\sqrt{x - 4}\right)^2 + 4 = x$

(b)



24. $f(x) = 1 - x^3,\ g(x) = \sqrt[3]{1 - x}$

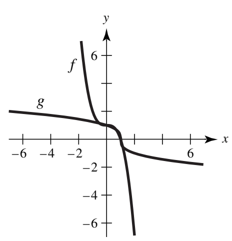
(a) $f(g(x)) = f\left(\sqrt[3]{1 - x}\right) = 1 - \left(\sqrt[3]{1 - x}\right)^3$

$= 1 - (1 - x) = x$

$g(f(x)) = g(1 - x^3) = \sqrt[3]{1 - (1 - x^3)}$

$= \sqrt[3]{x^3} = x$

(b)

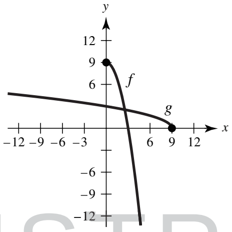


25. $f(x) = 9 - x^2,\ x \geq 0;\ g(x) = \sqrt{9 - x},\ x \leq 9$

(a) $f(g(x)) = f\left(\sqrt{9 - x}\right),\ x \leq 9 = 9 - \left(\sqrt{9 - x}\right)^2 = x$

$g(f(x)) = g(9 - x^2),\ x \geq 0 = \sqrt{9 - (9 - x^2)} = x$

(b)

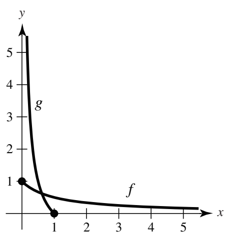


26. $f(x) = \dfrac{1}{1+x}, x \ge 0; g(x) = \dfrac{1-x}{x}, 0 < x \le 1$

(a) $f(g(x)) = f\left(\dfrac{1-x}{x}\right) = \dfrac{1}{1+\left(\dfrac{1-x}{x}\right)} = \dfrac{1}{\dfrac{x}{x}+\dfrac{1-x}{x}} = \dfrac{1}{\dfrac{1}{x}} = x$

$g(f(x)) = g\left(\dfrac{1}{1+x}\right) = \dfrac{1-\left(\dfrac{1}{1+x}\right)}{\left(\dfrac{1}{1+x}\right)} = \dfrac{\dfrac{1+x}{1+x}-\dfrac{1}{1+x}}{\dfrac{1}{1+x}} = \dfrac{\dfrac{x}{1+x}}{\dfrac{1}{1+x}} = \dfrac{x}{1+x}\cdot\dfrac{x+1}{1} = x$

(b)

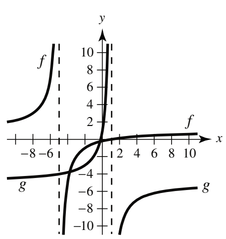


27. $f(x) = \dfrac{x-1}{x+5}, g(x) = -\dfrac{5x+1}{x-1}$

(a) $f(g(x)) = f\left(-\dfrac{5x+1}{x-1}\right) = \dfrac{\left(-\dfrac{5x+1}{x-1}-1\right)}{\left(-\dfrac{5x+1}{x-1}+5\right)}\cdot\dfrac{x-1}{x-1} = \dfrac{-(5x+1)-(x-1)}{-(5x+1)+5(x-1)} = \dfrac{-6x}{-6} = x$

$g(f(x)) = g\left(\dfrac{x-1}{x+5}\right) = -\dfrac{\left[5\left(\dfrac{x-1}{x+5}\right)+1\right]}{\left[\dfrac{x-1}{x+5}-1\right]}\cdot\dfrac{x+5}{x+5} = -\dfrac{5(x-1)+(x+5)}{(x-1)-(x+5)} = -\dfrac{6x}{-6} = x$

(b)

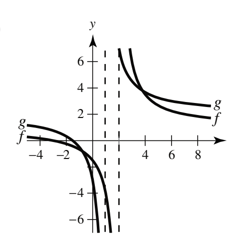


28. $f(x) = \dfrac{x+3}{x-2}, g(x) = \dfrac{2x+3}{x-1}$

(a) $f(g(x)) = f\left(\dfrac{2x+3}{x-1}\right) = \dfrac{\dfrac{2x+3}{x-1}+3}{\dfrac{2x+3}{x-1}-2} = \dfrac{\dfrac{2x+3+3x-3}{x-1}}{\dfrac{2x+3-2x+2}{x-1}} = \dfrac{5x}{5} = x$

$g(f(x)) = g\left(\dfrac{x+3}{x-2}\right) = \dfrac{2\left(\dfrac{x+3}{x-2}\right)+3}{\dfrac{x+3}{x-2}-1} = \dfrac{\dfrac{2x+6+3x-6}{x-2}}{\dfrac{x+3-x+2}{x-2}} = \dfrac{5x}{5} = x$

(b)



29. No, $\{(-2,-1), (1,0), (2,1), (1,2), (-2,3), (-6,4)\}$ does not represent a function. -2 and 1 are paired with two different values.

30. Yes, $\{(10,-3), (6,-2), (4,-1), (1,0), (-3,2), (10,2)\}$ does represent a function.

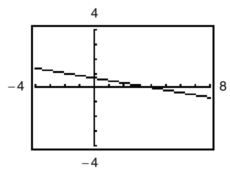
31. Yes, because no horizontal line crosses the graph of *f* at more than one point, *f has* an inverse.

32. No, because some horizontal lines intersect the graph of *f* twice, *f does not* have an inverse.

33. No, because some horizontal lines cross the graph of *f* twice, *f does not* have an inverse.

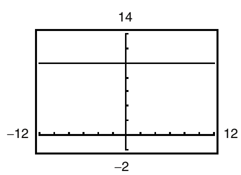
34. Yes, because no horizontal lines intersect the graph, of *f* at more than one point, *f has* an inverse.

35. $g(x) = \dfrac{4 - x}{6}$



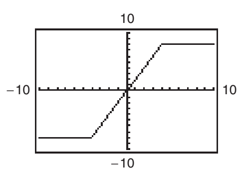
g passes the horizontal line test, so *g has* an inverse.

36. $f(x) = 10$



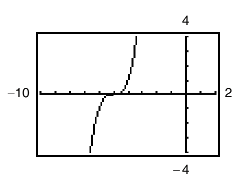
f does not pass the horizontal line test, so *f does not* have an inverse.

37. $h(x) = |x + 4| - |x - 4|$



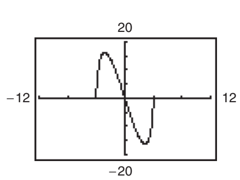
h does not pass the horizontal line test, so *h does not* have an inverse.

38. $g(x) = (x + 5)^3$



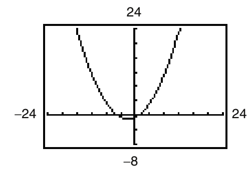
g passes the horizontal line test, so *g has* an inverse.

39. $f(x) = -2x\sqrt{16 - x^2}$



f does not pass the horizontal line test, so *f does not* have an inverse.

40. $f(x) = \frac{1}{8}(x + 2)^2 - 1$

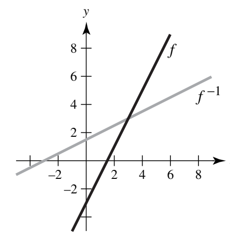


f does not pass the horizontal line test, so *f does not* have an inverse.

41. (a)

$$\begin{aligned} f(x) &= 2x - 3 \\ y &= 2x - 3 \\ x &= 2y - 3 \\ y &= \frac{x + 3}{2} \\ f^{-1}(x) &= \frac{x + 3}{2} \end{aligned}$$

(b)



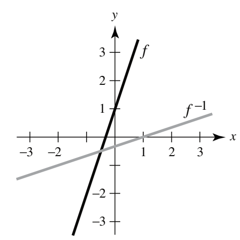
(c) The graph of f^{-1} is the reflection of the graph of *f* in the line $y = x$.

(d) The domains and ranges of *f* and f^{-1} are all real numbers.

42. (a)

$$\begin{aligned} f(x) &= 3x + 1 \\ y &= 3x + 1 \\ x &= 3y + 1 \\ \frac{x - 1}{3} &= y \\ f^{-1}(x) &= \frac{x - 1}{3} \end{aligned}$$

(b)



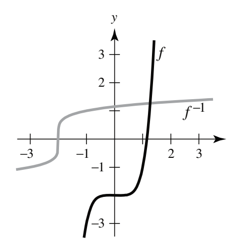
(c) The graph of f^{-1} is the reflection of *f* in the line $y = x$.

(d) The domains and ranges of *f* and f^{-1} are all real numbers.

43. (a)

$$\begin{aligned} f(x) &= x^5 - 2 \\ y &= x^5 - 2 \\ x &= y^5 - 2 \\ y &= \sqrt[5]{x + 2} \\ f^{-1}(x) &= \sqrt[5]{x + 2} \end{aligned}$$

(b)

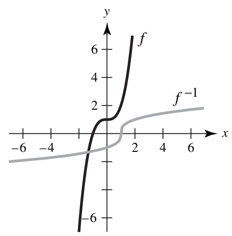


(c) The graph of f^{-1} is the reflection of the graph of *f* in the line $y = x$.

(d) The domains and ranges of *f* and f^{-1} are all real numbers.

44. (a)
$$f(x) = x^3 + 1$$
$$y = x^3 + 1$$
$$x = y^3 + 1$$
$$x - 1 = y^3$$
$$\sqrt[3]{x-1} = y$$
$$f^{-1}(x) = \sqrt[3]{x-1}$$

(b)

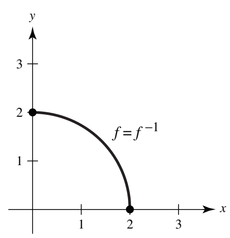


(c) The graph of f^{-1} is the reflection of f in the line $y = x$.

(d) The domains and ranges of f and f^{-1} are all real numbers.

45. (a)
$$f(x) = \sqrt{4 - x^2},\ 0 \le x \le 2$$
$$y = \sqrt{4 - x^2}$$
$$x = \sqrt{4 - y^2}$$
$$x^2 = 4 - y^2$$
$$y^2 = 4 - x^2$$
$$y = \sqrt{4 - x^2}$$
$$f^{-1}(x) = \sqrt{4 - x^2},\ 0 \le x \le 2$$

(b)

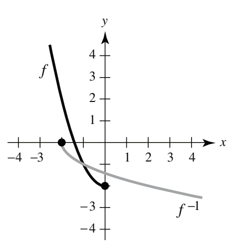


(c) The graph of f^{-1} is the same as the graph of f.

(d) The domains and ranges of f and f^{-1} are all real numbers x such that $0 \le x \le 2$.

46. (a)
$$f(x) = x^2 - 2,\ x \le 0$$
$$y = x^2 - 2$$
$$x = y^2 - 2$$
$$\pm\sqrt{x+2} = y$$
$$f^{-1}(x) = -\sqrt{x+2}$$

(b)



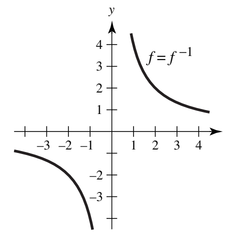
(c) The graph of f^{-1} is the reflection of f in the line $y = x$.

(d) $[-2, \infty)$ is the range of f and domain of f^{-1}.

$(-\infty, 0]$ is the domain of f and the range of f^{-1}.

47. (a)
$$f(x) = \frac{4}{x}$$
$$y = \frac{4}{x}$$
$$x = \frac{4}{y}$$
$$xy = 4$$
$$y = \frac{4}{x}$$
$$f^{-1}(x) = \frac{4}{x}$$

(b)

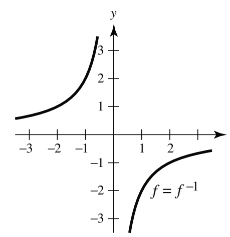


(c) The graph of f^{-1} is the same as the graph of f.

(d) The domains and ranges of f and f^{-1} are all real numbers except for 0.

48. (a)
$$f(x) = -\frac{2}{x}$$
$$y = -\frac{2}{x}$$
$$x = -\frac{2}{y}$$
$$y = -\frac{2}{x}$$
$$f^{-1}(x) = -\frac{2}{x}$$

(b)

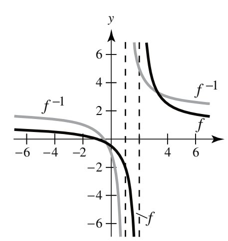


(c) The graphs are the same.

(d) The domains and ranges of f and f^{-1} are all real numbers except for 0.

49. (a)
$$f(x) = \frac{x+1}{x-2}$$
$$y = \frac{x+1}{x-2}$$
$$x = \frac{y+1}{y-2}$$
$$x(y-2) = y + 1$$
$$xy - 2x = y + 1$$
$$xy - y = 2x + 1$$
$$y(x-1) = 2x + 1$$
$$y = \frac{2x+1}{x-1}$$
$$f^{-1}(x) = \frac{2x+1}{x-1}$$

(b)



(c) The graph of f^{-1} is the reflection of graph of f in the line $y = x$.

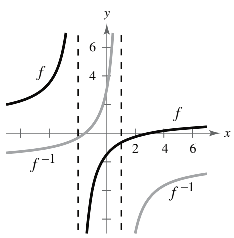
(d) The domain of f and the range of f^{-1} is all real numbers except 2.

The range of f and the domain of f^{-1} is all real numbers except 1.

50. (a)

$$\begin{aligned} f(x) &= \frac{x-3}{x+2} \\ y &= \frac{x-3}{x+2} \\ x &= \frac{y-3}{y+2} \\ xy + 2x - y + 3 &= 0 \\ y(x-1) &= -2x-3 \\ y &= \frac{-2x-3}{x-1} \\ f^{-1}(x) &= \frac{-2x-3}{x-1} \end{aligned}$$

(b)



(c) The graph of f^{-1} is the reflection of the graph of f in the line $y = x$.

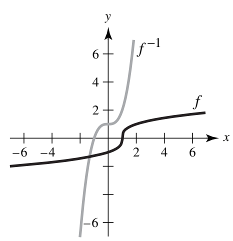
(d) The domain of f and the range of f^{-1} is all real numbers except $x = -2$.

The range of f and the domain of f^{-1} is all real numbers x except $x = 1$.

51. (a)

$$\begin{aligned} f(x) &= \sqrt[3]{x-1} \\ y &= \sqrt[3]{x-1} \\ x &= \sqrt[3]{y-1} \\ x^3 &= y-1 \\ y &= x^3+1 \\ f^{-1}(x) &= x^3+1 \end{aligned}$$

(b)



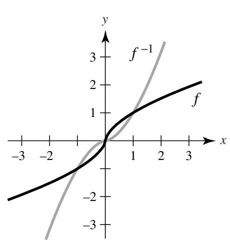
(c) The graph of f^{-1} is the reflection of the graph of f in the line $y = x$.

(d) The domains and ranges of f and f^{-1} are all real numbers.

52. (a)

$$\begin{aligned} f(x) &= x^{3/5} \\ y &= x^{3/5} \\ x &= y^{3/5} \\ x^{5/3} &= \left(y^{3/5}\right)^{5/3} \\ x^{5/3} &= y \\ f^{-1}(x) &= x^{5/3} \end{aligned}$$

(b)



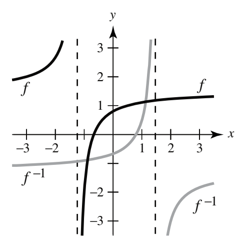
(c) The graph of f^{-1} is the reflection of the graph of f in the line $y = x$.

(d) The domains and ranges of f and f^{-1} are all real numbers.

53. (a)

$$\begin{aligned} f(x) &= \frac{6x+4}{4x+5} \\ y &= \frac{6x+4}{4x+5} \\ x &= \frac{6y+4}{4y+5} \\ x(4y+5) &= 6y+4 \\ 4xy+5x &= 6y+4 \\ 4xy-6y &= -5x+4 \\ y(4x-6) &= -5x+4 \\ y &= \frac{-5x+4}{4x-6} \\ f^{-1}(x) &= \frac{-5x+4}{4x-6} = \frac{5x-4}{6-4x} \end{aligned}$$

(b)



(c) The graph of f^{-1} is the graph of f reflected in the line $y = x$.

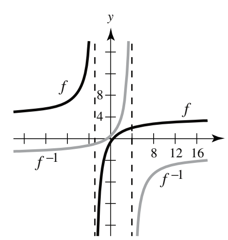
(d) The domain of f and the range of f^{-1} is all real numbers except $-\frac{5}{4}$.

The range of f and the domain of f^{-1} is all real numbers except $\frac{3}{2}$.

54. (a)

$$\begin{aligned} f(x) &= \frac{8x-4}{2x+6} \\ y &= \frac{8x-4}{2x+6} \\ x &= \frac{8y-4}{2y+6} \\ 2xy+6x &= 8y-4 \\ y(2x-8) &= -6x-4 \\ y &= \frac{-6x-4}{2x-8} \\ &= \frac{-3x-2}{x-4} \end{aligned}$$

(b)



(c) The graph of f^{-1} is the graph of f reflected in the line $y = x$.

(d) The domain of f and the range of f^{-1} is the set of all real numbers x except $x = -3$.

The domain of f^{-1} and the range of f is the set of all real numbers x except $x = 4$.

55. $f(x) = x^4$

$y = x^4$

$x = y^4$

$y = \pm\sqrt[4]{x}$

This does not represent *y* as a function of *x*. *f* does not have an inverse.

56. $f(x) = \dfrac{1}{x^2}$

$y = \dfrac{1}{x^2}$

$x = \dfrac{1}{y^2}$

$y^2 = \dfrac{1}{x}$

$y = \pm\sqrt{\dfrac{1}{x}}$

This does not represent *y* as a function of *x*. *f* does not have an inverse.

57. $g(x) = \dfrac{x}{8}$

$y = \dfrac{x}{8}$

$x = \dfrac{y}{8}$

$y = 8x$

This is a function of *x*, so *g* has an inverse.

$g^{-1}(x) = 8x$

58. $f(x) = 3x + 5$

$y = 3x + 5$

$x = 3y + 5$

$x - 5 = 3y$

$\dfrac{x - 5}{3} = y$

This is a function of *x*, so *f* has an inverse.

$f^{-1}(x) = \dfrac{x - 5}{3}$

59. $p(x) = -4$

$y = -4$

Because $y = -4$ for all *x*, the graph is a horizontal line and fails the Horizontal Line Test. *p* does not have an inverse.

60. $f(x) = \dfrac{3x + 4}{5}$

$y = \dfrac{3x + 4}{5}$

$x = \dfrac{3y + 4}{5}$

$5x = 3y + 4$

$5x - 4 = 3y$

$\dfrac{5x - 4}{3} = y$

This is a function of *x*, so *f* has an inverse.

$f^{-1}(x) = \dfrac{5x - 4}{3}$

61. $f(x) = (x + 3)^2,\ x \ge -3 \Rightarrow y \ge 0$

$y = (x + 3)^2,\ x \ge -3,\ y \ge 0$

$x = (y + 3)^2,\ y \ge -3,\ x \ge 0$

$\sqrt{x} = y + 3,\ y \ge -3,\ x \ge 0$

$y = \sqrt{x} - 3,\ x \ge 0,\ y \ge -3$

This is a function of *x*, so *f* has an inverse.

$f^{-1}(x) = \sqrt{x} - 3,\ x \ge 0$

62. $q(x) = (x - 5)^2$

$y = (x - 5)^2$

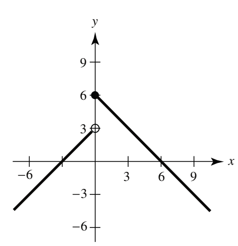
$x = (y - 5)^2$

$\pm\sqrt{x} = y - 5$

$5 \pm \sqrt{x} = y$

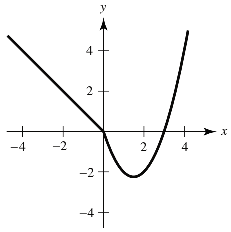
This does not represent *y* as a function of *x*, so *q* does not have an inverse.

63. $f(x) = \begin{cases} x + 3, & x < 0 \\ 6 - x, & x \ge 0 \end{cases}$



This graph fails the Horizontal Line Test, so *f* does not have an inverse.

64. $f(x) = \begin{cases} -x, & x \le 0 \\ x^2 - 3x, & x > 0 \end{cases}$



The graph fails the Horizontal Line Test, so f does not have an inverse.

65. $h(x) = -\dfrac{4}{x^2}$

The graph fails the Horizontal Line Test so h does not have an inverse.

66. $f(x) = |x - 2|, x \le 2 \Rightarrow y \ge 0$

$$y = |x - 2|, x \le 2, y \ge 0$$
$$x = |y - 2|, y \le 2, x \ge 0$$
$$x = y - 2 \quad \text{or} \quad -x = y - 2$$
$$2 + x = y \quad \text{or} \quad 2 - x = y$$

The portion that satisfies the conditions $y \le 2$ and $x \ge 0$ is $2 - x = y$. This is a function of x, so f has an inverse.

$f^{-1}(x) = 2 - x, x \ge 0$

67. $f(x) = \sqrt{2x + 3} \Rightarrow x \ge -\dfrac{3}{2}, y \ge 0$

$$y = \sqrt{2x + 3}, x \ge -\frac{3}{2}, y \ge 0$$
$$x = \sqrt{2y + 3}, y \ge -\frac{3}{2}, x \ge 0$$
$$x^2 = 2y + 3, x \ge 0, y \ge -\frac{3}{2}$$
$$y = \frac{x^2 - 3}{2}, x \ge 0, y \ge -\frac{3}{2}$$

This is a function of x, so f has an inverse.

$f^{-1}(x) = \dfrac{x^2 - 3}{2}, x \ge 0$

68. $f(x) = \sqrt{x - 2} \Rightarrow x \ge 2, y \ge 0$

$$y = \sqrt{x - 2}, x \ge 2, y \ge 0$$
$$x = \sqrt{y - 2}, y \ge 2, x \ge 0$$
$$x^2 = y - 2, x \ge 0, y \ge 2$$
$$x^2 + 2 = y, x \ge 0, y \ge 2$$

This is a function of x, so f has an inverse.

$f^{-1}(x) = x^2 + 2, x \ge 0$

In Exercises 69–74, $f(x) = \frac{1}{8}x - 3$, $f^{-1}(x) = 8(x + 3)$, $g(x) = x^3$, $g^{-1}(x) = \sqrt[3]{x}$.

69. $(f^{-1} \circ g^{-1})(1) = f^{-1}(g^{-1}(1))$
$$= f^{-1}(\sqrt[3]{1})$$
$$= 8(\sqrt[3]{1} + 3) = 32$$

70. $(g^{-1} \circ f^{-1})(-3) = g^{-1}(f^{-1}(-3))$
$$= g^{-1}(8(-3 + 3))$$
$$= g^{-1}(0) = \sqrt[3]{0} = 0$$

71. $(f^{-1} \circ f^{-1})(6) = f^{-1}(f^{-1}(6))$
$$= f^{-1}(8[6 + 3])$$
$$= 8[8(6 + 3) + 3] = 600$$

72. $(g^{-1} \circ g^{-1})(-4) = g^{-1}(g^{-1}(-4))$
$$= g^{-1}(\sqrt[3]{-4})$$
$$= \sqrt[3]{\sqrt[3]{-4}} = \sqrt[9]{-4}$$

73. $(f \circ g)(x) = f(g(x)) = f(x^3) = \frac{1}{8}x^3 - 3$
$$y = \frac{1}{8}x^3 - 3$$
$$x = \frac{1}{8}y^3 - 3$$
$$x + 3 = \frac{1}{8}y^3$$
$$8(x + 3) = y^3$$
$$\sqrt[3]{8(x + 3)} = y$$
$$(f \circ g)^{-1}(x) = 2\sqrt[3]{x + 3}$$

74. $g^{-1} \circ f^{-1} = g^{-1}(f^{-1}(x))$
$$= g^{-1}(8(x + 3))$$
$$= \sqrt[3]{8(x + 3)}$$
$$= 2\sqrt[3]{x + 3}$$

In Exercises 75–78, $f(x) = x + 4$, $f^{-1}(x) = x - 4$, $g(x) = 2x - 5$, $g^{-1}(x) = \dfrac{x + 5}{2}$.

75. $(g^{-1} \circ f^{-1})(x) = g^{-1}(f^{-1}(x))$
$$= g^{-1}(x - 4)$$
$$= \frac{(x - 4) + 5}{2}$$
$$= \frac{x + 1}{2}$$

76. $(f^{-1} \circ g^{-1})(x) = f^{-1}(g^{-1}(x))$

$$= f^{-1}\left(\frac{x+5}{2}\right)$$

$$= \frac{x+5}{2} - 4$$

$$= \frac{x+5-8}{2}$$

$$= \frac{x-3}{2}$$

77. $(f \circ g)(x) = f(g(x))$

$$= f(2x-5)$$

$$= (2x-5)+4$$

$$= 2x-1$$

$$(f \circ g)^{-1}(x) = \frac{x+1}{2}$$

Note: Comparing Exercises 75 and 77,
$(f \circ g)^{-1}(x) = (g^{-1} \circ f^{-1})(x)$.

78. $(g \circ f)(x) = g(f(x))$

$$= g(x+4)$$

$$= 2(x+4)-5$$

$$= 2x+8-5$$

$$= 2x+3$$

$$y = 2x+3$$

$$x = 2y+3$$

$$x-3 = 2y$$

$$\frac{x-3}{2} = y$$

$$(g \circ f)^{-1}(x) = \frac{x-3}{2}$$

79. The inverse is a line through $(-1, 0)$. Matches graph (c).

80. The inverse is a line through $(0, 6)$ and $(6, 0)$. Matches graph (b).

81. The inverse is half a parabola starting at $(1, 0)$. Matches graph (a).

82. The inverse is a third-degree equation through $(0, 0)$. Matches graph (d)

83.

x	–2	0	2	4	6	8
$f^{-1}(x)$	–2	–1	0	1	2	3

84.

x	–10	–7	–4	–1	2	5
$f^{-1}(x)$	–3	–2	–1	0	1	2

85. (a) $y = 10 + 0.75x$

$$x = 10 + 0.75y$$

$$x - 10 = 0.75y$$

$$\frac{x-10}{0.75} = y$$

So, $f^{-1}(x) = \dfrac{x-10}{0.75}$

x = hourly wage, y = number of units produced

(b) $y = \dfrac{24.25-10}{0.75} = 19$

So, 19 units are produced.

86. (a) (Total cost) = (Cost of first commodity) + (Cost of second commodity)

Labels: Total cost $= y$

Amount of first commodity $= x$

Amount of second commodity $= 50 - x$

Cost of first commodity $= 1.25x$

Cost of second commodity $= 1.60(50 - x)$

Equation: $y = 1.25x + 1.60(50 - x)$

(b) $x = 1.25y + 1.60(50 - y)$

$$x = 1.25y + 80 - 1.60y$$

$$x - 80 = -0.35y$$

$$\frac{x-80}{-0.35} = y$$

$$y = \frac{80-x}{0.35}$$

x = total cost

y = number of pounds of less expensive commodity

(c) $0 \le y \le 50$

$$0 \le \frac{80-x}{0.35} \le 50$$

$$0 \le 80 - x \le 17.5$$

$$-80 \le -x \le -62.5$$

$$62.5 \le x \le 80$$

(d) $\dfrac{80-73}{0.35} = y = 20$ pounds

87. (a) $y = 0.03x^2 + 245.50,\ 0 < x < 100$

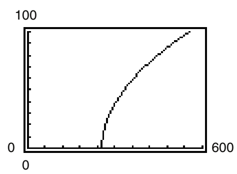
$\Rightarrow 245.50 < y < 545.50$

$$x = 0.03y^2 + 245.50$$
$$x - 245.50 = 0.03y^2$$
$$\frac{x - 245.50}{0.03} = y^2$$
$$\sqrt{\frac{x - 245.50}{0.03}} = y,\ 245.50 < x < 545.50$$
$$f^{-1}(x) = \sqrt{\frac{x - 245.50}{0.03}}$$

x = temperature in degrees Fahrenheit

y = percent load for a diesel engine

(b)



(c) $0.03x^2 + 245.50 \le 500$

$$0.03x^2 \le 254.50$$
$$x^2 \le 8483.33$$
$$x \le 92.10$$

Thus, $0 < x \le 92.10$.

88. (a) Yes.

(b) Given the population (in millions of people) you can determine the year.

(c) $P^{-1}(357.5) = 25$ because $P(25) = 357.5$

So, 357.5 million people are projected to be living in the United States in 2025.

(d) No. The function would no longer be one-to-one because the projected population would be 357.5 million people in 2025 and 2050.

89. (a) $f^{-1}(113.5) = 5$

(b) f^{-1} yields the year for a given amount spent on wireless communications services in the United States.

(c) $f(t) = 11.4t + 55.4$

(d) $f^{-1}(t) = \dfrac{t - 55.4}{11.4}$

(e) $f^{-1}(170.938) = 15$ which represents 2015.

90. (a) $f^{-1}(116{,}011) = 7$ which represents 2007.

(b) f^{-1} yields the year for a given number of households.

(c) $f(t) \approx 1140.32t + 107{,}512.86$

(d) $f^{-1}(t) = \dfrac{t - 107{,}512.86}{1140.32}$

(e) $f^{-1}(123{,}477) \approx 14$ which represents 2014.

91. False. $f(x) = x^2$ is even and does not have an inverse.

92. True. If $f(x)$ has an inverse and it has a y-intercept at $(0, b)$, then the point $(b, 0)$, must be a point on the graph of $f^{-1}(x)$.

93. Let $(f \circ g)(x) = y$. Then $x = (f \circ g)^{-1}(y)$. Also,

$$(f \circ g)(x) = y \Rightarrow f(g(x)) = y$$
$$g(x) = f^{-1}(y)$$
$$x = g^{-1}(f^{-1}(y))$$
$$x = (g^{-1} \circ f^{-1})(y).$$

Because f and g are both one-to-one functions, $(f \circ g)^{-1} = g^{-1} \circ f^{-1}$.

94. The reciprocal, not the inverse, of $f(x)$ was found.

Given $f(x) = \sqrt{x - 6}$, then

$f^{-1}(x) = x^2 + 6,\ x \ge 0$.

95. This situation could be represented by a one-to-one function if the runner does not stop to rest. The inverse function would represent the time in hours for a given number of miles completed.

96. This situation could be represented by a one-to-one function if the population continues to increase. The inverse function would represent the year for a given population.

97. No. The function oscillates.

98. No. After a certain age, height remains constant

Section 1.6 Mathematical Modeling and Variation

1. variation; regression
2. sum of square differences
3. least squares regression
4. correlation coefficient
5. directly proportional
6. constant of variation

7. directly proportional

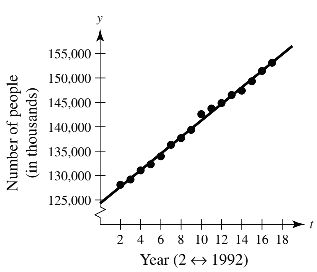
8. inverse

9. combined

10. jointly proportional

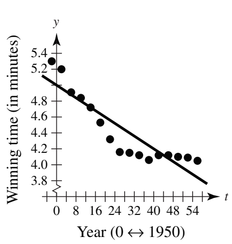
11.

Year	Actual Number (in thousands)	Model (in thousands)
1992	128,105	127,712
1993	129,200	129,408
1994	131,056	131,104
1995	132,304	132,800
1996	133,943	134,495
1997	136,297	136,191
1998	137,673	137,887
1999	139,368	139,583
2000	142,583	141,279
2001	143,734	142,975
2002	144,863	144,671
2003	146,510	146,367
2004	147,401	148,063
2005	149,320	149,759
2006	151,428	151,454
2007	153,124	153,150

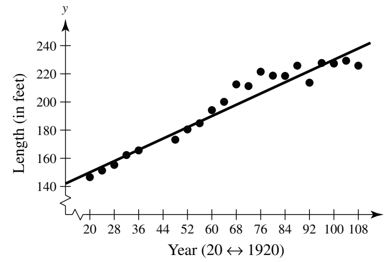


The model is a good fit for the actual data.

12. The model is not a good fit for the actual data.



13. (a)



(b) Using the points $(32, 162.3)$ and $(96, 227.7)$:

$$m = \frac{227.7 - 162.3}{96 - 32}$$
$$\approx 1.02$$
$$y - 162.3 = 1.02(t - 32)$$
$$y = 1.02t + 129.66$$

(c) $y \approx 1.01t + 130.82$

(d) The models are similar

(e) $2012 \rightarrow$ use $t = 112$

Model from part (b):

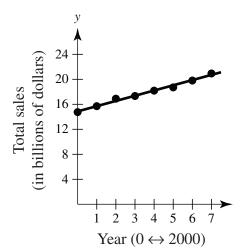
$y = 1.02(112) + 129.66 = 243.9$ feet

Model from part (c)

$y = 1.01(112) + 130.82 = 243.94$ feet

(f) Answers will vary.

14. (a)



(b) Using the points $(1, 15.700)$ and $(7, 20.936)$:

$$m = \frac{20.936 - 15.700}{7 - 1}$$
$$\approx 0.873$$
$$y - 15.700 = 0.873(x - 1)$$
$$y \approx 0.873x + 14.827$$

(c) $y = 0.835x + 14.868$

(d) The models are similar.

(e) $2008 \rightarrow$ use $t = 8$

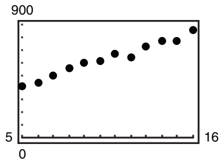
Model from part (b):

$y = 0.873(8) + 14.827 \approx \$21.811$ billion

Model from part (c):

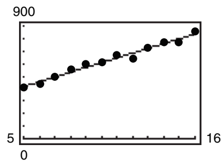
$y = 0.835(8) + 14.868 \approx \$21.548$ billion

15. (a)



(b) $S \approx 38.3t + 224$

(c)



The model is a good fit to the actual data.

(d) 2007 $\rightarrow$ use $t = 12$

$S(12) = 38.3(17) + 224 \approx \$875.1$ million

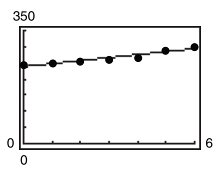
2009 $\rightarrow$ use $t = 14$

$S(14) = 38.3(19) + 224 \approx \$951.7$ million

(e) Each year the annual gross ticket sales for Broadway shows in New York City increase by \$38.3 million.

16. (a) $N \approx 9.29t + 238.29$

(b)



(c) 2008 $\rightarrow t = 8$

$N(8) = 9.29(8) + 238.29 \approx 312.6$ million

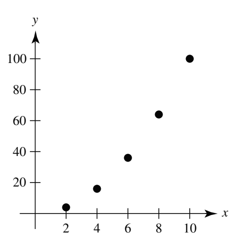
(d) Answers will vary.

17. The graph appears to represent $y = 4/x$, so y varies inversely as x.

18. The graph appears to represent $y = \frac{3}{2}x$, so y varies directly with x.

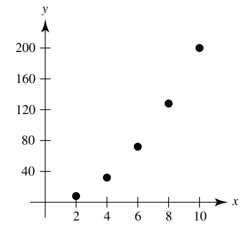
19. $k = 1$

x	2	4	6	8	10
$y = kx^2$	4	16	36	64	100



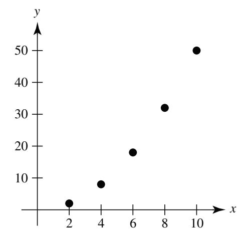
20. $k = 2$

x	2	4	6	8	10
$y = kx^2$	8	32	72	128	200



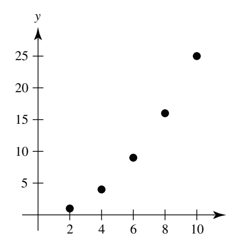
21. $k = \frac{1}{2}$

x	2	4	6	8	10
$y = kx^2$	2	8	18	32	50



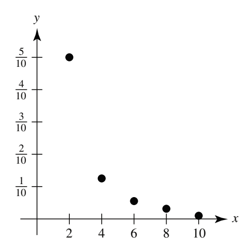
22. $k = \frac{1}{4}$

x	2	4	6	8	10
$y = kx^2$	1	4	9	16	25



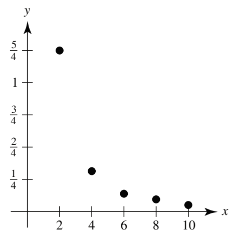
23. $k = 2$

x	2	4	6	8	10
$y = \dfrac{k}{x^2}$	$\frac{1}{2}$	$\frac{1}{8}$	$\frac{1}{18}$	$\frac{1}{32}$	$\frac{1}{50}$



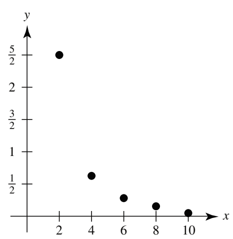
24. $k = 5$

x	2	4	6	8	10
$y = \dfrac{k}{x^2}$	$\dfrac{5}{4}$	$\dfrac{5}{16}$	$\dfrac{5}{36}$	$\dfrac{5}{64}$	$\dfrac{1}{20}$



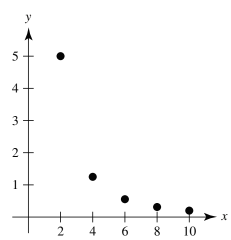
25. $k = 10$

x	2	4	6	8	10
$y = \dfrac{k}{x^2}$	$\dfrac{5}{2}$	$\dfrac{5}{8}$	$\dfrac{5}{18}$	$\dfrac{5}{32}$	$\dfrac{1}{10}$



26. $k = 20$

x	2	4	6	8	10
$y = \dfrac{k}{x^2}$	5	$\dfrac{5}{4}$	$\dfrac{5}{9}$	$\dfrac{5}{16}$	$\dfrac{1}{5}$



27. $y = \dfrac{k}{x}$

$1 = \dfrac{k}{5}$

$5 = k$

$y = \dfrac{5}{x}$

This equation checks with the other points given in the table.

28. $y = kx$

$2 = k5$

$\frac{2}{5} = k$

$y = \frac{2}{5}x$

This equation checks with the other points given in the table.

29. $y = kx$

$-7 = k(10)$

$-\frac{7}{10} = k$

$y = -\frac{7}{10}x$

This equation checks with the other points given in the table.

30. $y = \dfrac{k}{x}$

$24 = \dfrac{k}{5}$

$120 = k$

$y = 120/x$

This equation checks with the other points given in the table.

31. $y = kx$

$12 = k(5)$

$\frac{12}{5} = k$

$y = \frac{12}{5}x$

32. $y = kx$

$14 = k(2)$

$7 = k$

$y = 7x$

33. $y = kx$

$2050 = k(10)$

$205 = k$

$y = 205x$

34. $y = kx$

$580 = k(6)$

$\frac{290}{3} = k$

$y = \frac{290}{3}x$

35. $A = kr^2$

36. $V = ke^3$

37. $y = \dfrac{k}{x^2}$

38. $h = \dfrac{k}{\sqrt{s}}$

39. $F = \dfrac{kg}{r^2}$

40. $z = kx^2y^3$

41. $A = \frac{1}{2}bh$

The area of a triangle is jointly proportional to its base and height.

42. $S = 4\pi r^2$

The surface area of a sphere varies directly as the square of the radius r.

43. $V = \frac{4}{3}\pi r^3$

The volume of a sphere varies directly as the cube of its radius.

44. $V = \pi r^2 h$

The volume of a right circular cylinder is jointly proportional to the height and the square of the radius.

45. $r = \dfrac{d}{t}$

Average speed is directly proportional to the distance and inversely proportional to the time.

46. $\omega = \sqrt{\dfrac{kg}{W}}$

ω varies directly as the square root of g and inversely as the square root of W.

(**Note:** The constant of proportionality is $\sqrt{k}$.)

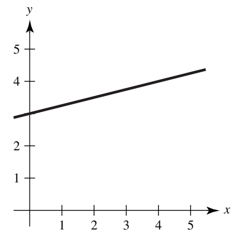
47. The data shown could be represented by a linear model which would be a good approximation.

48. The points do not follow a linear pattern. A linear model would be a poor approximation. A quadratic model would be better.

49. The points do not follow a linear pattern. A linear model would not be a good approximation.

50. The data shown could be represented by a linear model which would be a good approximation.

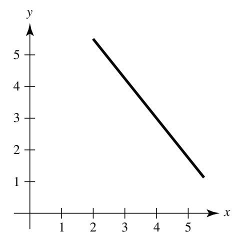
51.



Using the points $(0, 3)$ and $(4, 4)$, we have

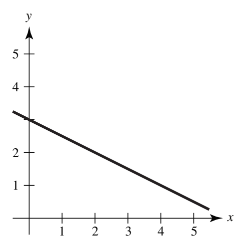
$y = \frac{1}{4}x + 3.$

52.



The line appears to pass through (2, 5.5) and (6, 0.5), so its equation is $y = -\frac{5}{4}x + 8.$

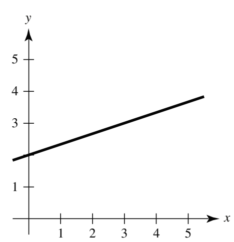
53.



Using the points $(2, 2)$ and $(4, 1)$, we have

$y = -\frac{1}{2}x + 3.$

54.



The line appears to pass through (0, 2) and (3, 3), so its equation is $y = \frac{1}{3}x + 2.$

55.

$$\begin{aligned} I &= kP \\ 113.75 &= k(3250) \\ 0.035 &= k \\ I &= 0.035P \end{aligned}$$

56. $I = kP$

$211.25 = k(6500)$

$0.0325 = k$

$I = 0.0325P$

57. $y = kx$

$33 = k(13)$

$\frac{33}{13} = k$

$y = \frac{33}{13}x$

When $x = 10$ inches, $y \approx 25.4$ centimeters.

When $x = 20$ inches, $y \approx 50.8$ centimeters.

58. $y = kx$

$53 = k(14)$

$\frac{53}{14} = k$

$y = \frac{53}{14}x$

5 gallons: $y = \frac{53}{14}(5) \approx 18.9$ liters

25 gallons: $y = \frac{53}{14}(5) \approx 94.6$ liters

59. $y = kx$

$5520 = k(150{,}000)$

$0.0368 = k$

$y = 0.0368x$

$y = 0.0368(225{,}000)$

$= \$8280$

The property tax is \$8280.

60. $y = kx$

$11.40 = k(189.99)$

$0.06 \approx k$

$y = 0.06x$

$y = 0.06(639.99)$

$\approx \$38.40$

The sales tax is \$38.40.

61. $P = \dfrac{k}{V}$

62. $R = k(T - T_e)$

63. $F = \dfrac{km_1m_2}{r^2}$

64. $R = kS(S - L)$

65. $d = kF$

$0.15 = k(265)$

$\frac{3}{5300} = k$

$d = \frac{3}{5300}F$

(a) $d = \frac{3}{5300}(90) \approx 0.05$ meter

(b) $0.1 = \frac{3}{5300}F$

$\frac{530}{3} = F$

$F = 176\frac{2}{3}$ newtons

66. $d = kF$

$0.12 = k(220)$

$\frac{3}{5500} = k$

$d = \frac{3}{5500}F$

$0.16 = \frac{3}{5500}F$

$\frac{880}{3} = F$

The required force is $293\frac{1}{3}$ newtons.

67. $d = kF$

$1.9 = k(25) \Rightarrow k = 0.076$

$d = 0.076F$

When the distance compressed is 3 inches, we have

$3 = 0.076F$

$F \approx 39.47.$

No child over 39.47 pounds should use the toy.

68. $d = kF$

$1 = k(15)$

$k = \frac{1}{15}$

$d = \frac{1}{15}F$

$\frac{8}{2} = \frac{1}{15}F$

$F = 60$ lb per spring

Combined lifting force $= 2F = 120$ lb

69. $A = kr^2$

$9\pi = k(3)^2$

$\pi = k$

$A = \pi r^2$

70. $y = \dfrac{k}{x}$

$3 = \dfrac{k}{25}$

$75 = k$

$y = \dfrac{75}{x}$

71. $y = \frac{k}{x}$

$7 = \frac{k}{4}$

$28 = k$

$y = \frac{28}{x}$

72. $z = kxy$

$64 = k(4)(8)$

$2 = k$

$z = 2xy$

73. $F = krs^3$

$4158 = k(11)(3)^3$

$k = 14$

$F = 14rs^3$

74. $P = \frac{kx}{y^2}$

$\frac{28}{3} = \frac{k(42)}{9^2}$

$\frac{28}{3} \cdot \frac{81}{42} = k$

$\frac{2 \cdot 27}{3} = k$

$18 = k$

$P = \frac{18x}{y^2}$

75. $z = \frac{kx^2}{y}$

$6 = \frac{k(6)^2}{4}$

$\frac{24}{36} = k$

$\frac{2}{3} = k$

$z = \frac{2/3x^2}{y} = \frac{2x^2}{3y}$

76. $v = \frac{kpq}{s^2}$

$1.5 = \frac{k(4.1)(6.3)}{(1.2)^2}$

$\frac{(1.5)(1.44)}{(4.1)(6.3)} = k$

$\frac{2.16}{25.83} = k$

$k = \frac{24}{287}$

$v = \frac{24pq}{287s^2}$

77. $d = kv^2$

$0.02 = k\left(\frac{1}{4}\right)^2$

$k = 0.32$

$d = 0.32v^2$

$0.12 = 0.32v^2$

$v^2 = \frac{0.12}{0.32} = \frac{3}{8}$

$v = \frac{\sqrt{3}}{2\sqrt{2}} = \frac{\sqrt{6}}{4} \approx 0.61$ mi/hr

78. $d = kv^2$

If the velocity is doubled:

$d = k(2v)^2$

$d = k \cdot 4v^2$

$\frac{4kv^2}{kv^2} = 4$

d increases by a factor of 4 when velocity is doubled.

79. $r = \frac{kl}{A}, A = \pi r^2 = \frac{\pi d^2}{4}$

$r = \frac{4kl}{\pi d^2}$

$66.17 = \frac{4(1000)k}{\pi\left(\frac{0.0126}{12}\right)^2}$

$k \approx 5.73 \times 10^{-8}$

$r = \frac{4(5.73 \times 10^{-8})l}{\pi\left(\frac{0.0126}{12}\right)^2}$

$33.5 = \frac{4(5.73 \times 10^{-8})l}{\pi\left(\frac{0.0126}{12}\right)^2}$

$\frac{33.5\pi\left(\frac{0.0126}{12}\right)^2}{4(5.73 \times 10^{-8})} = l$

$l \approx 506$ feet

80. From Exercise 79.

$$k \approx 5.73 \times 10^{-8}.$$

$$r = \frac{4(5.73 \times 10^{-8})l}{\pi d^2}$$

$$d = \sqrt{\frac{4(5.73 \times 10^{-8})l}{\pi r}}$$

$$d = \sqrt{\frac{4(5.73 \times 10^{-8})(14)}{\pi(0.05)}}$$

$$d \approx 0.0045 \text{ feet} = 0.054 \text{ inch}$$

81.

$$W = kmh$$

$$2116.8 = k(120)(1.8)$$

$$k = \frac{2116.8}{(120)(1.8)} = 9.8$$

$$W = 9.8mh$$

When $m = 100$ kilograms and $h = 1.5$ meters, we have $W = 9.8(100)(1.5) = 1470$ joules.

82. $f = k\dfrac{\sqrt{T}}{l}$ where f = frequency, T = tension, and l = length of string

$$440 = k\frac{\sqrt{T}}{l}$$

$$f = k\frac{\sqrt{1.25T}}{1.2l}$$

$$\frac{440l}{\sqrt{T}} = k \quad \text{and} \quad \frac{1.2\ fl}{\sqrt{1.25T}} = k$$

$$\frac{440l}{\sqrt{T}} = \frac{1.2\,fl}{\sqrt{1.25T}}$$

$$440l\sqrt{1.25T} = 1.2\,fl\sqrt{T} \qquad (l > 0)$$

$$440\sqrt{1.25T} = 1.2\,f\sqrt{T}$$

$$242{,}000T = 1.44\,f^2T \qquad (T > 0)$$

$$242{,}000 = 1.44\,f^2$$

$$168{,}055.56 = f^2$$

$$f \approx 409.95 \text{ vibrations per second}$$

83. (a) $v = \dfrac{k}{A}$

$$v = \frac{k}{0.75A} = \frac{4}{3}\left(\frac{k}{A}\right)$$

The velocity is increased by one-third.

(b) $v = \dfrac{k}{A}$

$$v = \frac{k}{\frac{4}{3}A} = \frac{3}{4}\left(\frac{k}{A}\right)$$

The velocity is decreased by one-fourth.

84. $\text{Load} = \dfrac{kwd^2}{l}$

(a) $\text{load} = \dfrac{k(2w)d^2}{2l} = \dfrac{kwd^2}{l}$

The safe load is unchanged.

(b) $\text{load} = \dfrac{k(2w)(2d)^2}{l} = \dfrac{8kwd^2}{l}$

The safe load is eight times as great.

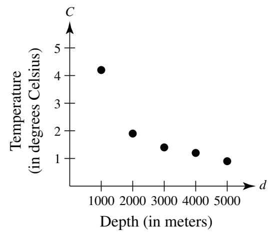
(c) $\text{load} = \dfrac{k(2w)(2d)^2}{2l} = \dfrac{4kwd^2}{l}$

The safe load is four times as great.

(d) $\text{load} = \dfrac{kw(d/2)^2}{l} = \dfrac{(1/4)kwd^2}{l}$

The safe load is one-fourth as great.

85. (a)



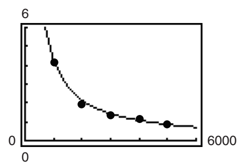
(b) Yes, the data appears to be modeled (approximately) by the inverse proportion model.

$4.2 = \frac{k_1}{1000}$ $\quad 1.9 = \frac{k_2}{2000}$ $\quad 1.4 = \frac{k_3}{3000}$ $\quad 1.2 = \frac{k_4}{4000}$ $\quad 0.9 = \frac{k_5}{5000}$

$4200 = k_1$ $\quad 3800 = k_2$ $\quad 4200 = k_3$ $\quad 4800 = k_4$ $\quad 4500 = k_5$

(c) Mean: $k = \dfrac{4200 + 3800 + 4200 + 4800 + 4500}{5} = 4300$, Model: $C = \dfrac{4300}{d}$

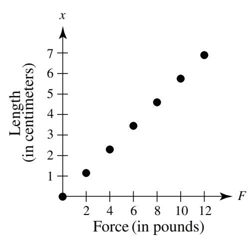
(d)



(e) $3 = \dfrac{4300}{d}$

$d = \dfrac{4300}{3} = 1433\frac{1}{3}$ meters

86. (a)



(b) It appears to fit Hooke's Law.

$k \approx \dfrac{6.9}{12} = 0.575$

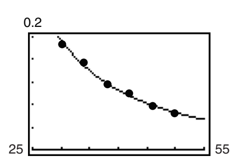
(c) $y = kF$

$9 = 0.575F$

$F \approx 15.7$ pounds

87. $y = \dfrac{262.76}{x^{2.12}}$

(a)



(b) $y = \dfrac{262.76}{(25)^{2.12}}$

≈ 0.2857 microwatts per sq. cm.

88. $I = \dfrac{k}{d^2}$

When the distance is doubled:

$I = \dfrac{k}{(2d)^2} = \dfrac{k}{4d^2}$.

The illumination is one-fourth as great. The model given in Exercise 85 is very close to $I = k/d^2$.

The difference is probably due to measurement error.

89. False.

"*y* varies directly as *x*" means $y = kx$ for some nonzero constant *k*.

"*y* is inversely proportional to *x*" means $y = \dfrac{k}{x}$ for some nonzero constant *k*.

90. False.

"*a* is jointly proportional to *y* and *z* with the constant of proportionality *k*" means $a = kyz$

91. False. *E* is jointly proportional (not "directly proportional") to the mass of an object and the square of its velocity.

92. False. The closer the value of $|r|$ is to 1, the better the fit.

93. The accuracy of the model in predicting prize winnings is questionable because the model is based on limited data.

94. As one variable increases, the other variable will also increase. Answers will vary.

95. (a) y will change by a factor of one-fourth.

(b) y will change by a factor of four.

96.
$$P = kA = k(\pi r^2) = k\pi\left(\frac{d}{2}\right)^2$$
$$8.78 = k\pi\left(\frac{9}{2}\right)^2$$
$$\frac{4(8.78)}{81\pi} = k$$
$$k \approx 0.138$$

However, we do not obtain $11.78 when $d = 12$ inches.

$$P = 0.138\pi\left(\frac{12}{2}\right)^2 \approx \$15.61$$

Instead, $k = \dfrac{11.78}{36\pi} \approx 0.104.$

For the 15-inch pizza, $k = \dfrac{4(14.18)}{225\pi} \approx 0.080.$

The price is not directly proportional to the surface area. The best buy is the 15-inch pizza.

Review Exercises for Chapter 1

1.
$$16x - y^4 = 0$$
$$y^4 = 16x$$
$$y = \pm 2\sqrt[4]{x}$$

No, y is not a function of x. Some x-values correspond to two y-values.

2.
$$2x - y - 3 = 0$$
$$2x - 3 = y$$

Yes, the equation represents y as a function of x.

3. $y = \sqrt{1 - x}$

Yes, the equation represents y as a function of x. Each x-value, $x \le 1$, corresponds to only one y-value.

4. $|y| = x + 2$ corresponds to $y = x + 2$ or $-y = x + 2.$

No, y is not a function of x. Some x-value correspond to two y-values.

5. $f(x) = x^2 + 1$

(a) $f(2) = (2)^2 + 1 = 5$

(b) $f(-4) = (-4)^2 + 1 = 17$

(c) $f(t^2) = (t^2)^2 + 1 = t^4 + 1$

(d) $f(t + 1) = (t + 1)^2 + 1$
$= t^2 + 2t + 2$

6. $h(x) = \begin{cases} 2x + 1, & x \le -1 \\ x^2 + 2, & x > -1 \end{cases}$

(a) $h(-2) = 2(-2) + 1 = -3$

(b) $h(-1) = 2(-1) + 1 = -1$

(c) $h(0) = 0^2 + 2 = 2$

(d) $h(2) = 2^2 + 2 = 6$

7. $f(x) = \sqrt{25 - x^2}$

Domain: $25 - x^2 \ge 0$

$(5 + x)(5 - x) \ge 0$

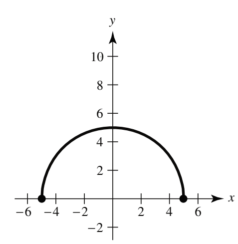
Critical numbers: $x = \pm 5$

Test intervals: $(-\infty, -5), (-5, 5), (5, \infty)$

Test: Is $25 - x^2 \ge 0$?

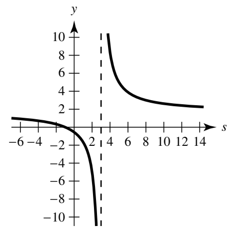
Solution set: $-5 \le x \le 5$

Domain: all real numbers x such that $-5 \le x \le 5$, or $[-5, 5]$.



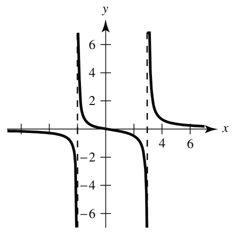
8. $g(s) = \dfrac{5s + 5}{3s - 9} = \dfrac{5s + 5}{3(s - 3)}$

Domain: All real numbers s except $s = 3$.



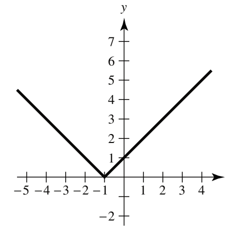
9. $h(x) = \dfrac{x}{x^2 - x - 6} = \dfrac{x}{(x + 2)(x - 3)}$

Domain: All real numbers x except $x = -2, 3$



10. $h(t) = |t + 1|$

Domain: all real numbers



11. $v(t) = -32t + 48$

(a) $v(1) = 16$ feet per second

(b) $0 = -32t + 48$

$t = \frac{48}{32} = 1.5$ seconds

(c) $v(2) = -16$ feet per second

12. (a) Model: (40% of $(50 - x)$) + (100% of x) = (amount of acid in final mixture)

Amount of acid in final mixture $= f(x)$

$f(x) = 0.4(50 - x) + 1.0x = 20 + 0.6x$

(b) Domain: $0 \le x \le 50$

Range: $20 \le y \le 50$

(c) $20 + 0.6x = 50\%(50)$

$20 + 0.6x = 25$

$0.6x = 5$

$x = 8\frac{1}{3}$ liters

13. $f(x) = 2x^2 + 3x - 1$

$$\frac{f(x+h) - f(x)}{h} = \frac{\left[2(x+h)^2 + 3(x+h) - 1\right] - \left(2x^2 + 3x - 1\right)}{h}$$
$$= \frac{2x^2 + 4xh + 2h^2 + 3x + 3h - 1 - 2x^2 - 3x + 1}{h}$$
$$= \frac{h(4x + 2h + 3)}{h}$$
$$= 4x + 2h + 3, \quad h \ne 0$$

14.

$$f(x) = x^3 - 5x^2 + x$$
$$f(x+h) = (x+h)^3 - 5(x+h)^2 + (x+h)$$
$$= x^3 + 3x^2h + 3xh^2 + h^3 - 5x^2 - 10xh - 5h^2 + x + h$$
$$\frac{f(x+h) - f(x)}{h} = \frac{x^3 + 3x^2h + 3xh^2 + h^3 - 5x^2 - 10xh - 5h^2 + x + h - x^3 + 5x^2 - x}{h}$$
$$= \frac{3x^2h + 3xh^2 + h^3 - 10xh - 5h^2 + h}{h}$$
$$= \frac{h(3x^2 + 3xh + h^2 - 10x - 5h + 1)}{h}$$
$$= 3x^2 + 3xh + h^2 - 10x - 5h + 1, \quad h \ne 0$$

15. $y = (x - 3)^2$

A vertical line intersects the graph no more than once, so *y is* a function of *x*.

16. $x = -|4 - y|$

A vertical line intersects the graph more than once, so *y is not* a function of *x*.

17. $f(x) = 3x^2 - 16x + 21$

$$3x^2 - 16x + 21 = 0$$
$$(3x - 7)(x - 3) = 0$$
$$3x - 7 = 0 \quad \text{or} \quad x - 3 = 0$$
$$x = \tfrac{7}{3} \quad \text{or} \quad x = 3$$

18. $f(x) = 5x^2 + 4x - 1$

$$5x^2 + 4x - 1 = 0$$
$$(5x - 1)(x + 1) = 0$$
$$5x - 1 = 0 \Rightarrow x = \tfrac{1}{5}$$
$$x + 1 = 0 \Rightarrow x = -1$$

19. $f(x) = \dfrac{8x + 3}{11 - x}$

$$\frac{8x + 3}{11 - x} = 0$$
$$8x + 3 = 0$$
$$x = -\tfrac{3}{8}$$

20. $f(x) = x^3 - x^2 - 25x + 25$

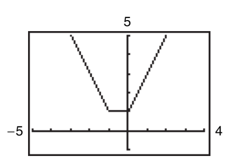
$$x^3 - x^2 - 25x + 25 = 0$$
$$x^2(x - 1) - 25(x - 1) = 0$$
$$(x - 1)(x^2 - 25) = 0$$
$$x - 1 = 0 \Rightarrow x = 1$$
$$x^2 - 25 = 0 \Rightarrow x = \pm 5$$

21. $f(x) = |x| + |x + 1|$

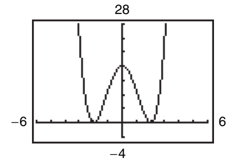
f is increasing on $(0, \infty)$.

f is decreasing on $(-\infty, -1)$.

f is constant on $(-1, 0)$.



22. $f(x) = (x^2 - 4)^2$

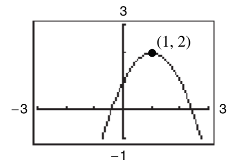


f is increasing on $(-2, 0)$ and $(2, \infty)$.

f is decreasing on $(-\infty, -2)$ and $(0, 2)$.

23. $f(x) = -x^2 + 2x + 1$

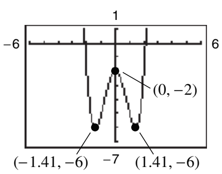
Relative maximum: $(1, 2)$



24. $f(x) = x^4 - 4x^2 - 2$

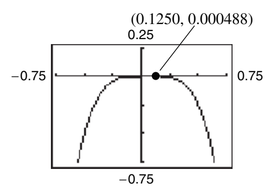
Relative minimum: $(-1.41, -6), (1.41, -6)$

Relative maximum: $(0, -2)$



25. $f(x) = x^3 - 6x^4$

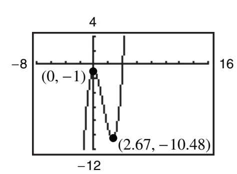
Relative maximum: $(0.125, 0.000488) \approx (0.13, 0.00)$



26. $f(x) = x^3 - 4x^2 - 1$

Relative minimum: $(2.67, -10.48)$.

Relative maximum: $(0, -1)$

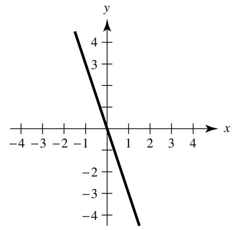


27. $f(2) = -6,\ f(-1) = 3$

Points: $(2, -6), (-1, 3)$

$$m = \frac{3 - (-6)}{-1 - 2} = \frac{9}{-3} = -3$$

$$y - (-6) = -3(x - 2)$$
$$y + 6 = -3x + 6$$
$$y = -3x$$
$$f(x) = -3x$$

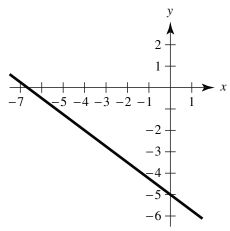


28. $f(0) = -5,\ f(4) = -8$

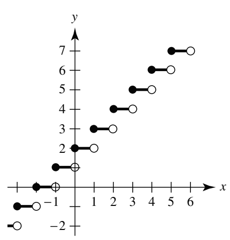
$(0, -5), (4, -8)$

$$m = \frac{-8 - (-5)}{4 - 0} = -\frac{3}{4}$$

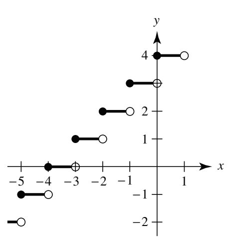
$$y - (-5) = -\frac{3}{4}(x - 0)$$
$$y = -\frac{3}{4}x - 5$$
$$f(x) = -\frac{3}{4}x - 5$$



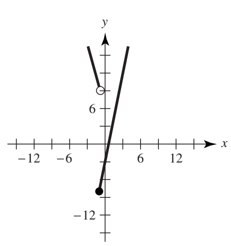
29. $f(x) = [\![x]\!] + 2$



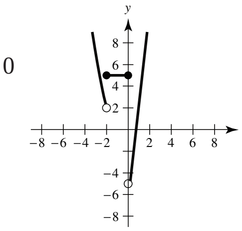
30. $g(x) = [\![x + 4]\!]$



31. $f(x) = \begin{cases} 5x - 3, & x \ge -1 \\ -4x + 5, & x < -1 \end{cases}$



32. $f(x) = \begin{cases} x^2 - 2, & x < -2 \\ 5, & -2 \le x \le 0 \\ 8x - 5, & x > 0 \end{cases}$



33. $f(x) = x^5 + 4x - 7$

$$f(-x) = (-x)^5 + 4(-x) - 7$$
$$= -x^5 - 4x - 7$$
$$\ne f(x)$$
$$\ne -f(x)$$

Neither even nor odd

34. $f(x) = x^4 - 20x^2$

$$f(-x) = (-x)^4 - 20(-x)^2 = x^4 - 20x^2 = f(x)$$

The function is even.

35. $f(x) = 2x\sqrt{x^2 + 3}$

$$f(-x) = 2(-x)\sqrt{(-x)^2 + 3}$$
$$= -2x\sqrt{x^2 + 3}$$
$$= -f(x)$$

The function is odd.

36. $f(x) = \sqrt[5]{6x^2}$

$$f(-x) = \sqrt[5]{6(-x)^2} = \sqrt[5]{6x^2} = f(x)$$

The function is even.

37. Parent function: $f(x) = x^3$

Horizontal shift 4 units to the left and a vertical shift 4 units upward

38. Parent function: $f(x) = \sqrt{x}$

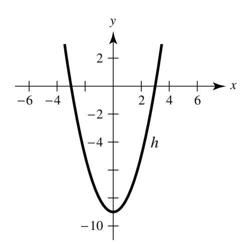
Vertical shift 4 units upward.

39. (a) $f(x) = x^2$

(b) $h(x) = x^2 - 9$

Vertical shift 9 units downward

(c)



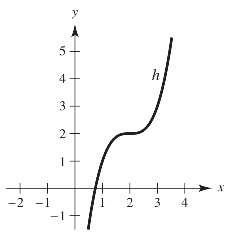
(d) $h(x) = f(x) - 9$

40. (a) $f(x) = x^3$

(b) $h(x) = (x - 2)^3 + 2$

Horizontal shift 2 units to the right; vertical shift 2 units upward

(c)



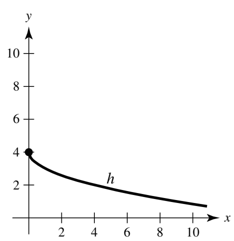
(d) $h(x) = f(x - 2) + 2$

41. $h(x) = -\sqrt{x} + 4$

(a) $f(x) = \sqrt{x}$

(b) Vertical shift 4 units upward, reflection in the x-axis

(c)



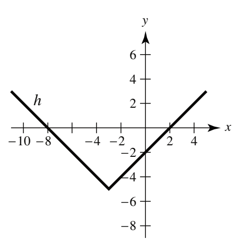
(d) $h(x) = -f(x) + 4$

42. (a) $f(x) = |x|$

(b) $h(x) = |x + 3| - 5$

Horizontal shift 3 units to the left; vertical shift 5 units downward

(c)



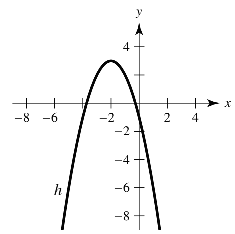
(d) $h(x) = f(x + 3) - 5$

43. $h(x) = -(x + 2)^2 + 3$

(a) $f(x) = x^2$

(b) Horizontal shift two units to the left, vertical shift 3 units upward, reflection in the x-axis.

(c)



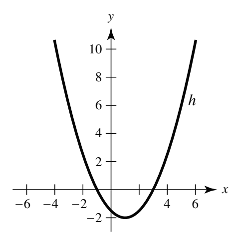
(d) $h(x) = -f(x + 2) + 3$

44. $h(x) = \frac{1}{2}(x - 1)^2 - 2$

(a) $f(x) = x^2$

(b) Horizontal shift one unit to the right, vertical shrink, vertical shift 2 units downward.

(c)



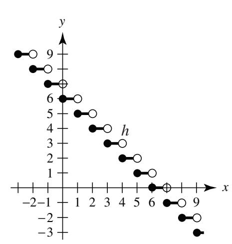
(d) $h(x) = \frac{1}{2}f(x - 1) - 2$

45. (a) $f(x) = [\![x]\!]$

(b) $h(x) = -[\![x]\!] + 6$

Reflection in the x-axis and a vertical shift 6 units upward

(c)



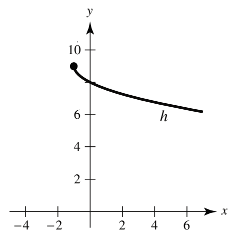
(d) $h(x) = -f(x) + 6$

46. (a) $f(x) = \sqrt{x}$

(b) $h(x) = -\sqrt{x + 1} + 9$

Reflection in the x-axis, a horizontal shift 1 unit to the left, and a vertical shift 9 units upward

(c)



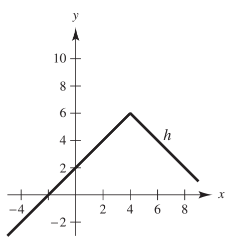
(d) $h(x) = -f(x + 1) + 9$

47. (a) $f(x) = |x|$

(b) $h(x) = -|-x + 4| + 6$

Reflection in both the x- and y-axes; horizontal shift of 4 unit to the right; vertical shift of 6 units upward

(c)



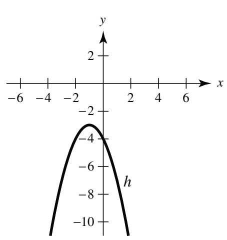
(d) $h(x) = -f(-(x - 4)) + 6 = -f(-x + 4) + 6$

48. (a) $f(x) = x^2$

(b) $h(x) = -(x + 1)^2 - 3$

Reflection in the x-axis; horizontal shift 1 unit to the left; vertical shift 3 units downward

(c)



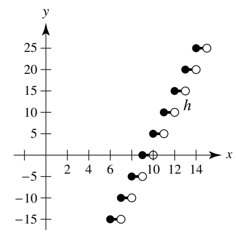
(d) $h(x) = -f(x + 1) - 3$

49. (a) $f(x) = [\![x]\!]$

(b) $h(x) = 5[\![x - 9]\!]$

Horizontal shift 9 units to the right and a vertical stretch (each y-value is multiplied by 5)

(c)



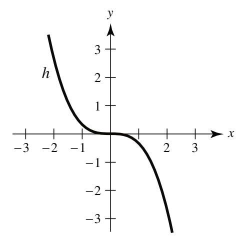
(d) $h(x) = 5f(x - 9)$

50. (a) $f(x) = x^3$

(b) $h(x) = -\frac{1}{3}x^3$

Reflection in the x-axis; vertical shrink $\left(\text{each } y\text{-value is multiplied by } \frac{1}{3}\right)$

(c)



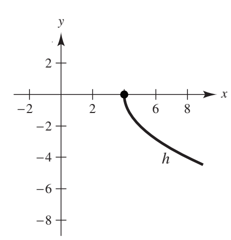
(d) $h(x) = -\frac{1}{3}f(x)$

51. (a) $f(x) = \sqrt{x}$

(b) $h(x) = -2\sqrt{x - 4}$

Reflection in the x-axis, a vertical stretch (each y-value is multiplied by 2), and a horizontal shift 4 units to the right

(c)



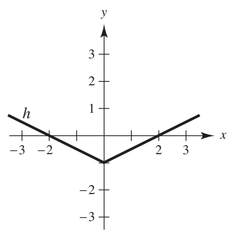
(d) $h(x) = -2f(x - 4)$

52. (a) $f(x) = |x|$

(b) $h(x) = \frac{1}{2}|x| - 1$

Vertical shrink (each y-value is multiplied by $\frac{1}{2}$); vertical shift 1 unit downward

(c)



(d) $h(x) = \frac{1}{2}f(x) - 1$

53. $f(x) = x^2 + 3$, $g(x) = 2x - 1$

(a) $(f + g)(x) = (x^2 + 3) + (2x - 1) = x^2 + 2x + 2$

(b) $(f - g)(x) = (x^2 + 3) - (2x - 1) = x^2 - 2x + 4$

(c) $(fg)(x) = (x^2 + 3)(2x - 1) = 2x^3 - x^2 + 6x - 3$

(d) $\left(\frac{f}{g}\right)(x) = \frac{x^2 + 3}{2x - 1}$, Domain: $x \neq \frac{1}{2}$

54. $f(x) = x^2 - 4$, $g(x) = \sqrt{3 - x}$

(a) $(f + g)(x) = f(x) + g(x) = x^2 - 4 + \sqrt{3 - x}$

(b) $(f - g)(x) = f(x) - g(x) = x^2 - 4 - \sqrt{3 - x}$

(c) $(fg)(x) = f(x)g(x) = (x^2 - 4)(\sqrt{3 - x})$

(d) $(f/g)(x) = \frac{f(x)}{g(x)} = \frac{x^2 - 4}{\sqrt{3 - x}}$, $x < 3$

55. $f(x) = \frac{1}{3}x - 3$, $g(x) = 3x + 1$

The domains of $f(x)$ and $g(x)$ are all real numbers.

(a)
$$\begin{aligned}(f \circ g)(x) &= f(g(x))\\ &= f(3x + 1)\\ &= \tfrac{1}{3}(3x + 1) - 3\\ &= x + \tfrac{1}{3} - 3\\ &= x - \tfrac{8}{3}\end{aligned}$$

Domain: all real numbers

(b)
$$\begin{aligned}(g \circ f)(x) &= g(f(x))\\ &= g\left(\tfrac{1}{3}x - 3\right)\\ &= 3\left(\tfrac{1}{3}x - 3\right) + 1\\ &= x - 9 + 1\\ &= x - 8\end{aligned}$$

Domain: all real numbers

56. $f(x) = x^3 - 4$, $g(x) = \sqrt[3]{x + 7}$

The domains of $f(x)$ and $g(x)$ are all real numbers.

(a)
$$\begin{aligned}(f \circ g)(x) &= f(g(x))\\ &= \left(\sqrt[3]{x + 7}\right)^3 - 4\\ &= x + 7 - 4\\ &= x + 3\end{aligned}$$

Domain: all real numbers

(b)
$$\begin{aligned}(g \circ f)(x) &= g(f(x))\\ &= \sqrt[3]{(x^3 - 4) + 7}\\ &= \sqrt[3]{x^3 + 3}\end{aligned}$$

Domain: all real numbers

57. $h(x) = (1 - 2x)^3$

Answer is not unique.

One possibility: Let $f(x) = x^3$ and $g(x) = 1 - 2x$.

$f(g(x)) = f(1 - 2x) = (1 - 2x)^3 = h(x)$.

58. $h(x) = \sqrt[3]{x + 2}$

Answer is not unique.

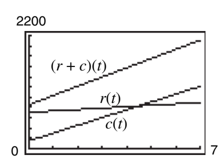
One possibility: Let $g(x) = x + 2$ and $f(x) = \sqrt[3]{x}$.

$f(g(x)) = f(x + 2) = \sqrt[3]{x + 2} = h(x)$

59. (a) $(r + c)(t) = r(t) + c(t) = 178.8t + 856$

This represents the average annual expenditures for both residential and cellular phone services from 2001 to 2006.

(b)



(c) $(r + c)(13) = 178.8(13) + 856 = \$3180.40$

60. (a) $N(T(t)) = 25(2t+1)^2 - 50(2t+1) + 300,\ 2 \le t \le 20$

$$= 25(4t^2 + 4t + 1) - 100t - 50 + 300$$
$$= 100t^2 + 100t + 25 - 100t + 250$$
$$= 100t^2 + 275$$

The composition $N(T(t))$ represents the number of bacteria in the food as a function of time.

(b) When $N = 750$,

$$750 = 100t^2 + 275$$
$$100t^2 = 475$$
$$t^2 = 4.75$$
$$t = 2.18 \text{ hours.}$$

After about 2.18 hours, the bacterial count will reach 750.

61. $f(x) = 3x + 8$

$$y = 3x + 8$$
$$x = 3y + 8$$
$$x - 8 = 3y$$
$$y = \frac{x-8}{3}$$
$$y = \frac{1}{3}(x-8)$$

So $f^{-1}(x) = \frac{1}{3}(x-8)$

$$f(f^{-1}(x)) = f\left(\frac{1}{3}(x-8)\right) = 3\left(\frac{1}{3}(x-8)\right) + 8 = x - 8 + 8 = x$$
$$f^{-1}(f(x)) = f^{-1}(3x+8) = \frac{1}{3}(3x + 8 - 8) = \frac{1}{3}(3x) = x$$

62. $f(x) = \frac{x-4}{5}$

$$y = \frac{x-4}{5}$$
$$x = \frac{y-4}{5}$$
$$5x = y - 4$$
$$y = 5x + 4$$

So, $f^{-1}(x) = 5x + 4$

$$f(f^{-1}(x)) = f(5x+4) = \frac{5x + 4 - 4}{5} = \frac{5x}{5} = x$$
$$f^{-1}(f(x)) = f^{-1}\left(\frac{x-4}{5}\right) = 5\left(\frac{x-4}{5}\right) + 4 = x - 4 + 4 = x$$

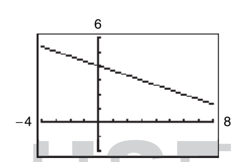
63. Yes, the function has an inverse because no horizontal lines intersect the graph at more than one point.

The function has an inverse.

64. No, the function does not have an inverse because some horizontal lines intersect the graph twice.

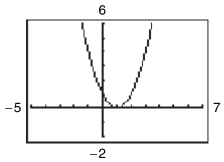
65. $f(x) = 4 - \frac{1}{3}x$

Yes, the function has an inverse because no horizontal lines intersect the graph at more than one point. The function has an inverse.



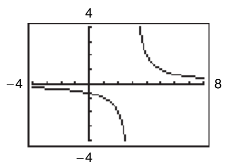
66. $f(x) = (x - 1)^2$

No, the function does not have an inverse because some horizontal lines intersect the graph twice.



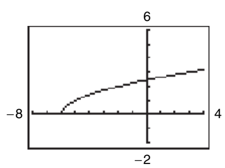
67. $h(t) = \dfrac{2}{t - 3}$

Yes, the function has an inverse because no horizontal lines intersect the graph at more than one point. The function has an inverse.



68. $g(x) = \sqrt{x + 6}$

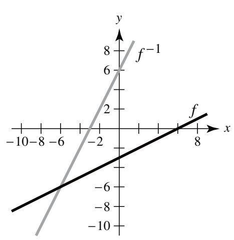
Yes, the function has an inverse because no horizontal lines intersect the graph at more than one point.



69. (a)

$$\begin{aligned} f(x) &= \tfrac{1}{2}x - 3 \\ y &= \tfrac{1}{2}x - 3 \\ x &= \tfrac{1}{2}y - 3 \\ x + 3 &= \tfrac{1}{2}y \\ 2(x + 3) &= y \\ f^{-1}(x) &= 2x + 6 \end{aligned}$$

(b)



(c) The graph of f^{-1} is the reflection of the graph of f in the line $y = x$.

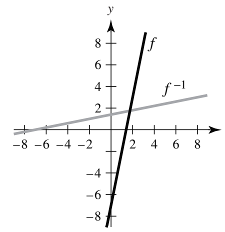
(d) The domains and ranges of f and f^{-1} are the set of all real numbers.

70. $f(x) = 5x - 7$

(a)

$$\begin{aligned} y &= 5x - 7 \\ x &= 5y - 7 \\ x + 7 &= 5y \\ \frac{x + 7}{5} &= y \\ f^{-1}(x) &= \frac{x + 7}{5} \end{aligned}$$

(b)



(c) The graph of f^{-1} is the reflection of the graph of f in the line $y = x$.

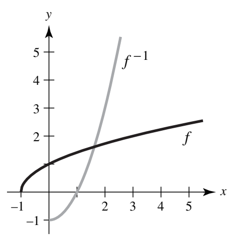
(d) The domains and ranges of f and f^{-1} are the set of all real numbers.

71. (a)

$$\begin{aligned} f(x) &= \sqrt{x + 1} \\ y &= \sqrt{x + 1} \\ x &= \sqrt{y + 1} \\ x^2 &= y + 1 \\ x^2 - 1 &= y \\ f^{-1}(x) &= x^2 - 1, \quad x \ge 0 \end{aligned}$$

Note: The inverse must have a restricted domain.

(b)



(c) The graph of f^{-1} is the reflection of the graph of f in the line $y = x$.

(d) The domain of f and the range of f^{-1} is $[-1, \infty)$.

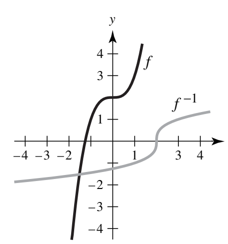
The range of f and the domain of f^{-1} is $[0, \infty)$.

72. $f(x) = x^3 + 2$

(a)

$$\begin{aligned} y &= x^3 + 2 \\ x &= y^3 + 2 \\ x - 2 &= y^3 \\ \sqrt[3]{x - 2} &= y \\ f^{-1}(x) &= \sqrt[3]{x - 2} \end{aligned}$$

(b)



(c) The graph of f^{-1} is the reflection of the graph of f in the line $y = x$.

(d) The domains and ranges of f and f^{-1} are the set of all real numbers.

73. $f(x) = 2(x - 4)^2$ is increasing on $(4, \infty)$.

Let $f(x) = 2(x - 4)^2$, $x > 4$ and $y > 0$.

$$y = 2(x - 4)^2$$
$$x = 2(y - 4)^2, x > 0, y > 4$$
$$\frac{x}{2} = (y - 4)^2$$
$$\sqrt{\frac{x}{2}} = y - 4$$
$$\sqrt{\frac{x}{2}} + 4 = y$$
$$f^{-1}(x) = \sqrt{\frac{x}{2}} + 4, x > 0$$

74. $f(x) = |x - 2|$

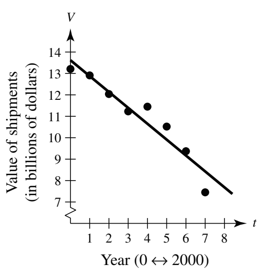
Increasing on $(2, \infty)$

Let $f(x) = x - 2, x > 2, y > 0$.

$$y = x - 2$$
$$x = y - 2, x > 0, y > 2$$
$$x + 2 = y, x > 0, y > 2$$
$$f^{-1}(x) = x + 2, x > 0$$

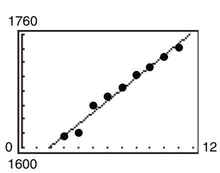
75. $V = -0.742t + 13.62$

(a)



(b) The model is a good fit to the actual data.

76. (a)



(b) $H = 16.22t + 1568.8$

The model is a good fit to the actual data.

(c) $2020 \rightarrow$ use $t = 20$

$H(20) = 16.22(20) + 1568.8 = 1893.2$ hours

(d) The projected number of hours of television usage in the United States increases by about 16.22 hours per year.

77.
$$D = km$$
$$4 = 2.5k$$
$$1.6 = k$$
$$D = \tfrac{8}{5}m \text{ or } D = 1.6m$$

In 2 miles:

$D = 1.6(2) = 3.2$ kilometers

In 10 miles:

$D = 1.6(10) = 16$ kilometers

78.
$$P = kS^3$$
$$750 = k(27)^3$$
$$k = 0.03810395$$
$$P = 0.03810395(40)^3$$
$$= 2438.7 \text{ kilowatts}$$

79. $F = ks^2$

If speed is doubled,

$$F = k(2s)^2$$
$$F = 4ks^2.$$

So, the force will be changed by a factor of 4.

80.
$$x = \frac{k}{p}$$
$$800 = \frac{k}{5}$$
$$k = 4000$$
$$x = \frac{4000}{6} \approx 667 \text{ boxes}$$

81.
$$T = \frac{k}{r}$$
$$3 = \frac{k}{65}$$
$$k = 3(65) = 195$$
$$T = \frac{195}{r}$$

When $r = 80$ mph,

$$T = \frac{195}{80} = 2.4375 \text{ hours}$$
$$\approx 2 \text{ hours, } 26 \text{ minutes.}$$

82.
$$C = khw^2$$
$$28.80 = k(16)(6)^2$$
$$k = 0.05$$
$$C = (0.05)(14)(8)^2$$
$$= \$44.80$$

83. y is inversely proportional to x: $y = \dfrac{k}{x}$

$y = 9$ when $x = 5.5$

$9 = \dfrac{k}{5.5} \Rightarrow k = (9)(5.5) = 49.5$

So, $y = \dfrac{49.5}{x}$

84. $F = cx\sqrt{y}$

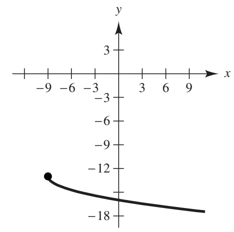
$6 = c(9)\sqrt{4}$

$6 = 18c$

$\frac{1}{3} = c$

$F = \frac{1}{3}x\sqrt{y}$

85. False. The graph is reflected in the x-axis, shifted 9 units to the left, then shifted 13 units down.



86. True. If $f(x) = x^3$ and $g(x) = \sqrt[3]{x}$, then the domain of g is all real numbers, which is equal to the range of f and vice versa.

87. The Vertical Line Test is used to determine if a graph of y is a function of x. The Horizontal Line Test is used to determine if a function has an inverse function.

88. A function from a set A to a set B is a relation that assigns to each element x in the set A exactly one element y in the set B.

89. If $y = kx$, then the y-intercept is (0, 0).

Chapter Test for Chapter 1

1. $f(x) = \dfrac{\sqrt{x+9}}{x^2 - 81}$

(a) $f(7) = \dfrac{4}{-32} = -\dfrac{1}{8}$

(b) $f(-5) = \dfrac{2}{-56} = -\dfrac{1}{28}$

(c) $f(x-9) = \dfrac{\sqrt{x}}{(x-9)^2 - 81} = \dfrac{\sqrt{x}}{x^2 - 18x}$

2. $f(x) = 10 - \sqrt{3 - x}$

Domain: $3 - x \ge 0$

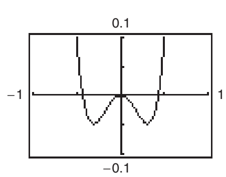
$-x \ge -3$

$x \le 3$

3. $f(x) = 2x^6 + 5x^4 - x^2$

(a) $0, \pm 0.4314$

(b)



(c) Increasing on $(-0.31, 0), (0.31, \infty)$

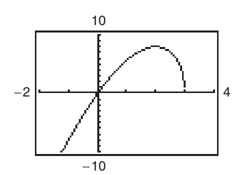
Decreasing on $(-\infty, -0.31), (0, 0.31)$

(d) y-axis symmetry $\Rightarrow$ The function is even.

4. $f(x) = 4x\sqrt{3 - x}$

(a) 0, 3

(b)



(c) Increasing on $(-\infty, 2)$

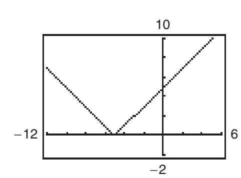
Decreasing on $(2, 3)$

(d) The function is neither odd nor even.

5. $f(x) = |x + 5|$

(a) -5

(b)

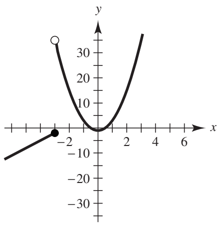


(c) Increasing on $(-5, \infty)$

Decreasing on $(-\infty, -5)$

(d) The function is neither odd nor even.

6. $f(x) = \begin{cases} 3x + 7, & x \le -3 \\ 4x^2 - 1, & x > -3 \end{cases}$



7. (a) $f(-10) = 12,\ f(16) = -1$

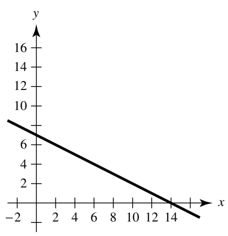
$(-10, 12), (16, -1)$

$$m = \frac{-1 - 12}{16 - (-10)} = \frac{-13}{26} = -\frac{1}{2}$$

$$f(x) - (-1) = -\frac{1}{2}(x - 16)$$

$$f(x) = -\frac{1}{2}x + 7$$

(b)



8. (a) $f\left(\frac{1}{2}\right) = -6,\ f(4) = -3$

$\left(\frac{1}{2}, -6\right), (4, -3)$

$$m = \frac{-3 - (-6)}{4 - (1/2)} = \frac{3}{7/2} = \frac{6}{7}$$

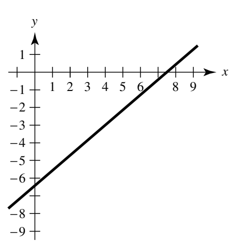
$$y - (-3) = \frac{6}{7}(x - 4)$$

$$y + 3 = \frac{6}{7}x - \frac{24}{7}$$

$$y = \frac{6}{7}x - \frac{45}{7}$$

$$f(x) = \frac{6}{7}x - \frac{45}{7}$$

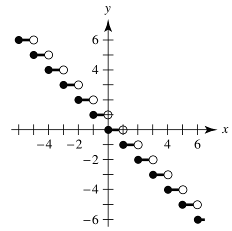
(b)



9. $h(x) = -[\![x]\!]$

Parent function: $f(x) = [\![x]\!]$

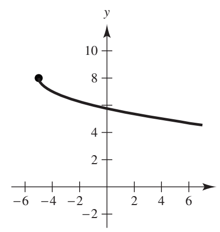
Transformation: Reflection in the *x*-axis



10. $h(x) = -\sqrt{x + 5} + 8$

Parent function: $f(x) = \sqrt{x}$

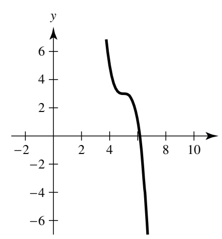
Transformation: Reflection in the *x*-axis, a horizontal shift 5 units to the left, and a vertical shift 8 units upward



11. $h(x) = -2(x - 5)^3 + 3$

Parent function: $f(x) = x^3$

Transformation: Reflection in the *x*-axis, vertical stretch, a horizontal shift 5 units to the right, and a vertical shift 3 units upward



12. $f(x) = 3x^2 - 7,\ g(x) = -x^2 - 4x + 5$

(a) $(f + g)(x) = (3x^2 - 7) + (-x^2 - 4x + 5) = 2x^2 - 4x - 2$

(b) $(f - g)(x) = (3x^2 - 7) - (-x^2 - 4x + 5) = 4x^2 + 4x - 12$

(c) $(fg)(x) = (3x^2 - 7)(-x^2 - 4x + 5) = -3x^4 - 12x^3 + 22x^2 + 28x - 35$

(d) $\left(\dfrac{f}{g}\right)(x) = \dfrac{3x^2 - 7}{-x^2 - 4x + 5},\ x \neq -5, 1$

(e) $(f \circ g)(x) = f(g(x)) = f(-x^2 - 4x + 5) = 3(-x^2 - 4x + 5)^2 - 7 = 3x^4 + 24x^3 + 18x^2 - 120x + 68$

(f) $(g \circ f)(x) = g(f(x)) = g(3x^2 - 7) = -(3x^2 - 7)^2 - 4(3x^2 - 7) + 5 = -9x^4 + 30x^2 - 16$

13. $f(x) = \dfrac{1}{x},\ g(x) = 2\sqrt{x}$

(a) $(f + g)(x) = \dfrac{1}{x} + 2\sqrt{x} = \dfrac{1 + 2x^{3/2}}{x},\ x > 0$

(b) $(f - g)(x) = \dfrac{1}{x} - 2\sqrt{x} = \dfrac{1 - 2x^{3/2}}{x},\ x > 0$

(c) $(fg)(x) = \left(\dfrac{1}{x}\right)(2\sqrt{x}) = \dfrac{2\sqrt{x}}{x},\ x > 0$

(d) $\left(\dfrac{f}{g}\right)(x) = \dfrac{\frac{1}{x}}{2\sqrt{x}} = \dfrac{1}{2x\sqrt{x}} = \dfrac{1}{2x^{3/2}},\ x > 0$

(e) $(f \circ g)(x) = f(g(x)) = f(2\sqrt{x}) = \dfrac{1}{2\sqrt{x}} = \dfrac{\sqrt{x}}{2x},\ x > 0$

(f) $(g \circ f)(x) = g(f(x)) = g\left(\dfrac{1}{x}\right) = 2\sqrt{\dfrac{1}{x}} = \dfrac{2}{\sqrt{x}} = \dfrac{2\sqrt{x}}{x},\ x > 0$

14. $f(x) = x^3 + 8$

Since f is one-to-one, f has an inverse.

$$\begin{aligned} y &= x^3 + 8 \\ x &= y^3 + 8 \\ x - 8 &= y^3 \\ \sqrt[3]{x - 8} &= y \\ f^{-1}(x) &= \sqrt[3]{x - 8} \end{aligned}$$

15. $f(x) = |x^2 - 3| + 6$

Since f is not one-to-one, f does not have an inverse.

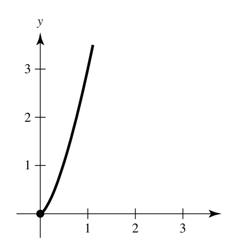
16. $f(x) = 3x\sqrt{x} = 3x^{3/2}$

Domain: $[0, \infty)$

Range: $[0, \infty)$

The graph of $f(x)$ passes the Horizontal Line Test, so $f(x)$ is one-to-one and has an inverse.

$$\begin{aligned} f(x) &= 3x^{3/2} \\ y &= 3x^{3/2} \\ x &= 3y^{3/2} \\ \frac{x}{3} &= y^{3/2} \\ \left(\frac{x}{3}\right)^{2/3} &= y \\ f^{-1}(x) &= \left(\frac{x}{3}\right)^{2/3},\ x \geq 0 \end{aligned}$$

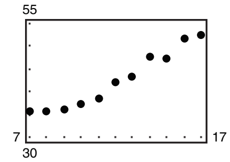


17.
$$v = k\sqrt{s}$$
$$24 = k\sqrt{16}$$
$$6 = k$$
$$v = 6\sqrt{s}$$

18
$$A = kxy$$
$$500 = k(15)(8)$$
$$500 = k(120)$$
$$\frac{25}{6} = k$$
$$A = \frac{25}{6}xy$$

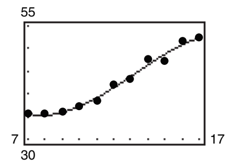
19.
$$b = \frac{k}{a}$$
$$32 = \frac{k}{1.5}$$
$$48 = k$$
$$b = \frac{48}{a}$$

20. (a)



(b) $S = -0.0297t^3 + 1.175t^2 - 12.96t + 79.0$

(c)



The model is a good fit for the data.

(d) For 2015, use $t = 25$: $S(25) \approx \$25.3$ billion

No, this does not seem reasonable. The model decreases sharply after 2009.

Problem Solving for Chapter 1

1. Mapping numbers onto letters is *not* a function. Each number between 2 and 9 is mapped to more than one letter.

$\{(2, A), (2, B), (2, C), (3, D), (3, E), (3, F), (4, G), (4, H), (4, I), (5, J), (5, K), (5, L),$
$(6, M), (6, N), (6, O), (7, P), (7, Q), (7, R), (7, S), (8, T), (8, U), (8, V), (9, W), (9, X), (9, Y), (9, Z)\}$

Mapping letters onto numbers *is* a function. Each letter is only mapped to one number.

$\{(A, 2), (B, 2), (C, 2), (D, 3), (E, 3), (F, 3), (G, 4), (H, 4), (I, 4), (J, 5), (K, 5), (L, 5),$
$(M, 6), (N, 6), (O, 6), (P, 7), (Q, 7), (R, 7), (S, 7), (T, 8), (U, 8), (V, 8), (W, 9), (X, 9), (Y, 9), (Z, 9)\}$

2. (a) Let $f(x)$ and $g(x)$ be two even functions.

Then define $h(x) = f(x) \pm g(x)$.

$$h(-x) = f(-x) \pm g(-x)$$
$$= f(x) \pm g(x) \text{ because } f \text{ and } g \text{ are even}$$
$$= h(x)$$

So, $h(x)$ is also even.

(b) Let $f(x)$ and $g(x)$ be two odd functions.

Then define $h(x) = f(x) \pm g(x)$.

$$h(-x) = f(-x) \pm g(-x)$$
$$= -f(x) \pm g(x) \text{ because } f \text{ and } g \text{ are odd}$$
$$= -h(x)$$

So, $h(x)$ is also odd. $(\text{If } f(x) \neq g(x))$

(c) Let $f(x)$ be odd and $g(x)$ be even. Then define $h(x) = f(x) \pm g(x)$.

$$h(-x) = f(-x) \pm g(-x)$$
$$= -f(x) \pm g(x) \text{ because } f \text{ is odd and } g \text{ is even}$$
$$\neq h(x)$$
$$\neq -h(x)$$

So, $h(x)$ is neither odd nor even.

3. $f(x) = a_{2n+1}x^{2n+1} + a_{2n-1}x^{2n-1} + \cdots + a_3x^3 + a_1x$

$$f(-x) = a_{2n+1}(-x)^{2n+1} + a_{2n-1}(-x)^{2n-1} + \cdots + a_3(-x)^3 + a_1(-x)$$
$$= -a_{2n+1}x^{2n+1} - a_{2n-1}x^{2n-1} - \cdots - a_3x^3 - a_1x = -f(x)$$

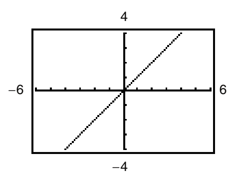
Therefore, $f(x)$ is odd.

4. $f(x) = a_{2n}x^{2n} + a_{2n-2}x^{2n-2} + \cdots + a_2x^2 + a_0$

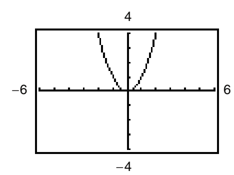
$$f(-x) = a_{2n}(-x)^{2n} + a_{2n-2}(-x)^{2n-2} + \cdots + a_2(-x)^2 + a_0$$
$$= a_{2n}x^{2n} + a_{2n-2}x^{2n-2} + \cdots + a_2x^2 + a_0$$
$$= f(x)$$

Therefore, $f(x)$ is even.

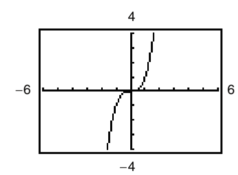
5. (a) $y = x$



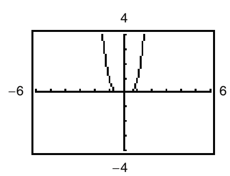
(b) $y = x^2$



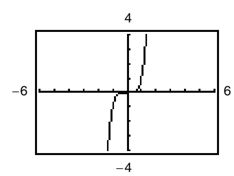
(c) $y = x^3$



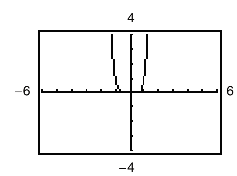
(d) $y = x^4$



(e) $y = x^5$



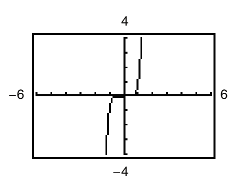
(f) $y = x^6$

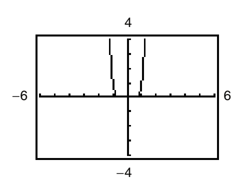


All the graphs pass through the origin. The graphs of the odd powers of x are symmetric with respect to the origin and the graphs of the even powers are symmetric with respect to the y-axis. As the powers increase, the graphs become flatter in the interval $-1 < x < 1$.

(g) The graph of $y = x^7$ will pass through the origin and will be symmetric with the origin.

The graph of $y = x^8$ will pass through the origin and will be symmetric with respect to the y-axis.





6. If you consider the x-axis to be a mirror, the graph of $y = -f(x)$ is the mirror image of the graph of $y = f(x)$.

7. $y = f(x + 2) - 1$

Horizontal shift 2 units to the left and a vertical shift 1 unit downward.

$$(0, 1) \to (0 - 2, 1 - 1) = (-2, 0)$$
$$(1, 2) \to (1 - 2, 2 - 1) = (-1, 1)$$
$$(2, 3) \to (2 - 2, 3 - 1) = (0, 2)$$

8. Let $f(x)$ and $g(x)$ be two odd functions and define $h(x) = f(x)g(x)$. Then

$$\begin{aligned} h(-x) &= f(-x)g(-x) \\ &= [-f(x)][-g(x)] \quad \text{Since } f \text{ and } g \text{ are odd} \\ &= f(x)g(x) \\ &= h(x) \end{aligned}$$

Thus, $h(x)$ is even.

Let $f(x)$ and $g(x)$ be two even functions and define $h(x) = f(x)g(x)$. Then

$$\begin{aligned} h(-x) &= f(-x)g(-x) \\ &= f(x)g(x) \quad \text{Since } f \text{ and } g \text{ are even} \\ &= h(x) \end{aligned}$$

Thus, $h(x)$ is even.

9. Let $f(x)$ be an odd function, $g(x)$ be an even function and define $h(x) = f(x)g(x)$. Then

$$\begin{aligned} h(-x) &= f(-x)g(-x) \\ &= [-f(x)]g(x) \quad \text{Since } f \text{ is odd and } g \text{ is even.} \\ &= -f(x)g(x) \\ &= -h(x) \end{aligned}$$

Thus, h is odd.

10. (a) The profits were only $\frac{3}{4}$ as large as expected:
$g(t) = \frac{3}{4}f(t)$

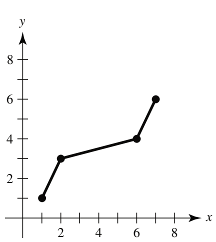
(b) The profits were \$10,000 greater than predicted:
$g(t) = f(t) + 10{,}000$

(c) There was a 2-year delay: $g(t) = f(t - 2)$

11.

x	1	3	4	6
f	1	2	6	7

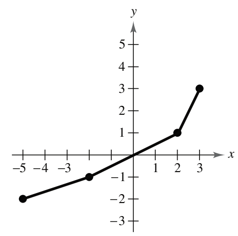
x	1	2	6	7
$f^{-1}(x)$	1	3	4	6



12.

x	$f(x)$
–2	–5
–1	–2
1	2
3	3

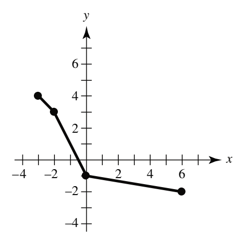
x	$f^{-1}(x)$
–5	–2
–2	–1
2	1
3	3



13.

x	–2	–1	3	4
f	6	0	–2	–3

x	–3	–2	0	6
$f^{-1}(x)$	4	3	–1	–2



14.

x	$f(x)$
–4	3
–2	4
0	0
3	–1

The graph does not pass the Horizontal Line Test, so $f^{-1}(x)$ does not exist.

15. If $f(x) = k(2 - x - x^3)$ has an inverse and $f^{-1}(3) = -2$, then $f(-2) = 3$. So,

$$f(-2) = k(2 - (-2) - (-2)^3) = 3$$

$$\begin{aligned} k(2 + 2 + 8) &= 3 \\ 12k &= 3 \\ k &= \frac{3}{12} = \frac{1}{4} \end{aligned}$$

So, $k = \frac{1}{4}$.

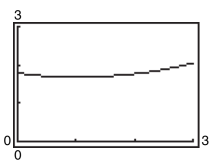
16. (a) The length of the trip in the water is $\sqrt{2^2 + x^2}$, and the length of the trip over land is $\sqrt{1 + (3 - x)^2}$.

The total time is

$$T(x) = \frac{\sqrt{4 + x^2}}{2} + \frac{\sqrt{1 + (3 - x)^2}}{4}$$
$$= \frac{1}{2}\sqrt{4 + x^2} + \frac{1}{4}\sqrt{x^2 - 6x + 10}$$

(b) Domain of $T(x)$: $0 \le x \le 3$

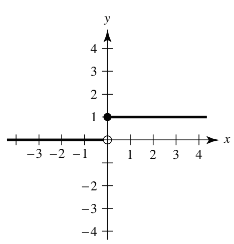
(c)



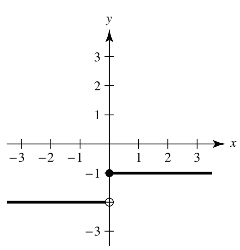
(d) $T(x)$ is a minimum when $x = 1$.

(e) Answers will vary. Sample answer: To reach point Q in the shortest amount of time, you should row to a point one mile down the coast, and then walk the rest of the way.

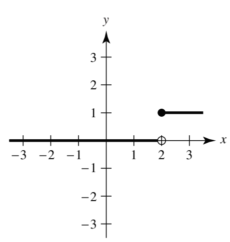
17. $H(x) = \begin{cases} 1, & x \ge 0 \\ 0, & x < 0 \end{cases}$



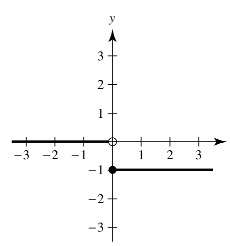
(a) $H(x) - 2$



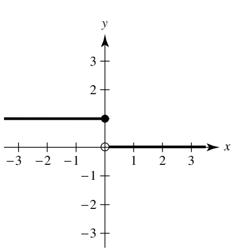
(b) $H(x - 2)$



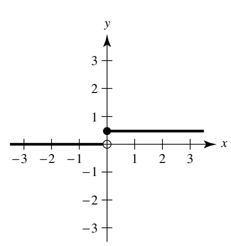
(c) $-H(x)$



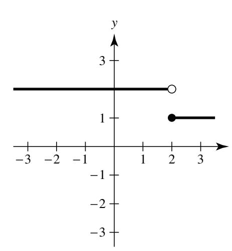
(d) $H(-x)$



(e) $\frac{1}{2}H(x)$



(f) $-H(x - 2) + 2$



18. $f(x) = y = \dfrac{1}{1-x}$

(a) Domain: all real numbers x except $x = 1$

Range: all real numbers y except $y = 0$

(b) $f(f(x)) = f\left(\dfrac{1}{1-x}\right)$

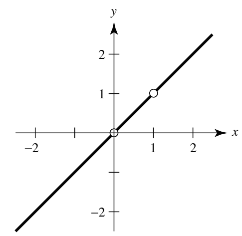
$$= \frac{1}{1-\left(\dfrac{1}{1-x}\right)} = \frac{1}{\dfrac{1-x-1}{1-x}}$$

$$= \frac{1-x}{-x} = \frac{x-1}{x}$$

Domain: all real numbers x except $x = 0$ and $x = 1$

(c) $f(f(f(x))) = f\left(\dfrac{x-1}{x}\right) = \dfrac{1}{1-\left(\dfrac{x-1}{x}\right)} = \dfrac{1}{\dfrac{1}{x}} = x$

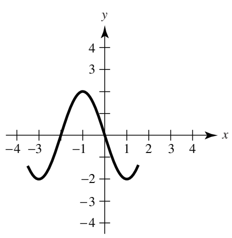
The graph is not a line. It has holes at $(0, 0)$ and $(1, 1)$.



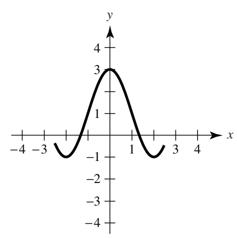
19. $(f \circ (g \circ h))(x) = f((g \circ h)(x)) = f(g(h(x))) = (f \circ g \circ h)(x)$

$((f \circ g) \circ h)(x) = (f \circ g)(h(x)) = f(g(h(x))) = (f \circ g \circ h)(x)$

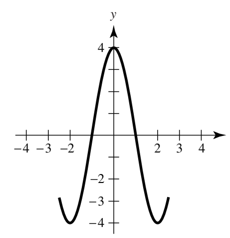
20. (a) $f(x+1)$



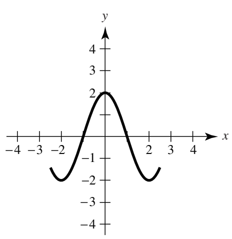
(b) $f(x) + 1$



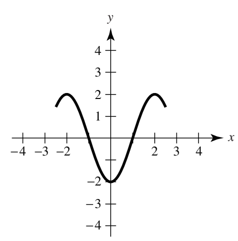
(c) $2f(x)$



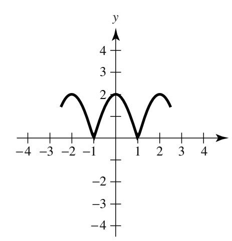
(d) $f(-x)$



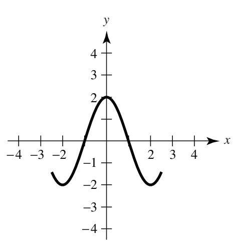
(e) $-f(x)$



(f) $|f(x)|$



(g) $f(|x|)$



21.

x	$f(x)$	$f^{-1}(x)$
−4	—	2
−3	4	1
−2	1	0
−1	0	—
0	−2	−1
1	−3	−2
2	−4	—
3	—	—
4	—	−3

(a)

x	$f(f^{-1}(x))$
−4	$f(f^{-1}(-4)) = f(2) = -4$
−2	$f(f^{-1}(-2)) = f(0) = -2$
0	$f(f^{-1}(0)) = f(-1) = 0$
4	$f(f^{-1}(4)) = f(-3) = 4$

(b)

x	$(f + f^{-1})(x)$
−3	$f(-3) + f^{-1}(-3) = 4 + 1 = 5$
−2	$f(-2) + f^{-1}(-2) = 1 + 0 = 1$
0	$f(0) + f^{-1}(0) = -2 + (-1) = -3$
1	$f(1) + f^{-1}(1) = -3 + (-2) = -5$

(c)

x	$(f \cdot f^{-1})(x)$
−3	$f(-3)f^{-1}(-3) = (4)(1) = 4$
−2	$f(-2)f^{-1}(-2) = (1)(0) = 0$
0	$f(0)f^{-1}(0) = (-2)(-1) = 2$
1	$f(1)f^{-1}(1) = (-3)(-2) = 6$

(d)

x	$\lvert f^{-1}(x) \rvert$
−4	$\lvert f^{-1}(-4) \rvert = \lvert 2 \rvert = 2$
−3	$\lvert f^{-1}(-3) \rvert = \lvert 1 \rvert = 1$
0	$\lvert f^{-1}(0) \rvert = \lvert -1 \rvert = 1$
4	$\lvert f^{-1}(4) \rvert = \lvert -3 \rvert = 3$

CHAPTER 2
Polynomial and Rational Functions

CHAPTER 2
Polynomial and Rational Functions

Section 2.1 Quadratic Functions and Models

1. polynomial

2. nonnegative integer; real

3. quadratic; parabola

4. axis

5. positive; minimum

6. negative; maximum

7. $f(x) = (x - 2)^2$ opens upward and has vertex $(2, 0)$. Matches graph (e).

8. $f(x) = (x + 4)^2$ opens upward and has vertex $(-4, 0)$. Matches graph (c).

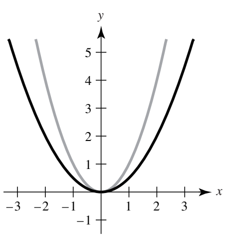
9. $f(x) = x^2 - 2$ opens upward and has vertex $(0, -2)$. Matches graph (b).

10. $f(x) = (x + 1)^2 - 2$ opens upward and has vertex $(-1, -2)$. Matches graph (a).

11. $f(x) = 4 - (x - 2)^2 = -(x - 2)^2 + 4$ opens downward and has vertex $(2, 4)$. Matches graph (f).

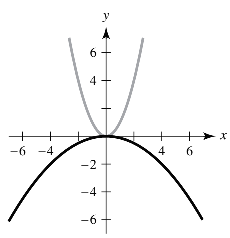
12. $f(x) = -(x - 4)^2$ opens downward and has vertex $(4, 0)$. Matches graph (d)

13. (a) $y = \frac{1}{2}x^2$



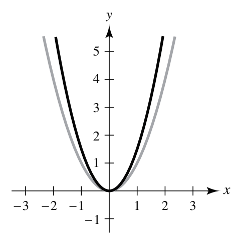
Vertical shrink

(b) $y = -\frac{1}{8}x^2$



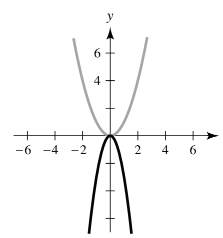
Vertical shrink and reflection in the x-axis

(c) $y = \frac{3}{2}x^2$



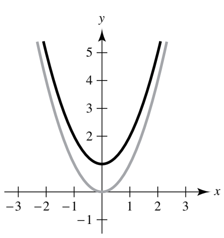
Vertical stretch

(d) $y = -3x^2$



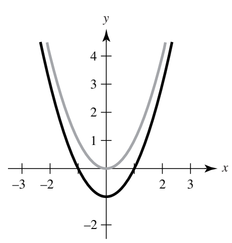
Vertical stretch and reflection in the x-axis

14. (a) $y = x^2 + 1$



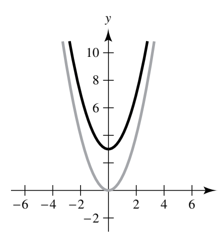
Vertical shift one unit upward

(b) $y = x^2 - 1$



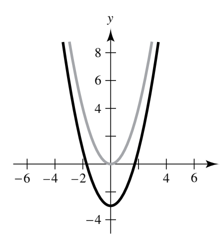
Vertical shift one unit downward

(c) $y = x^2 + 3$



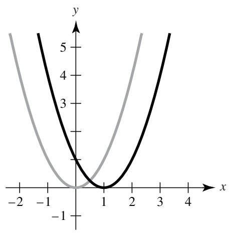
Vertical shift three units upward

(d) $y = x^2 - 3$



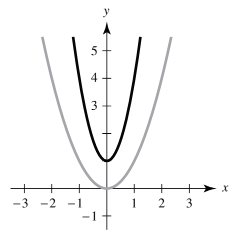
Vertical shift three units downward

15. (a) $y = (x - 1)^2$



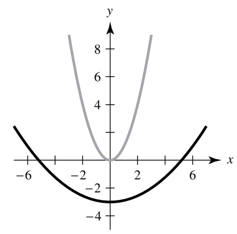
Horizontal shift one unit to the right

(b) $y = (3x)^2 + 1$



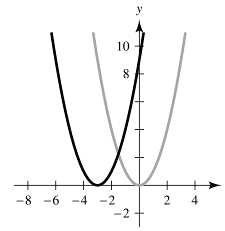
Horizontal shrink and a vertical shift one unit upward

(c) $y = \left(\frac{1}{3}x\right)^2 - 3$



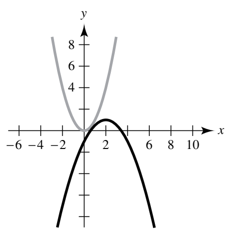
Horizontal stretch and a vertical shift three units downward

(d) $y = (x + 3)^2$



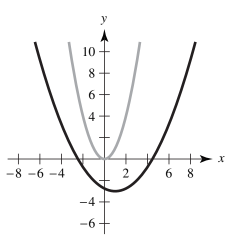
Horizontal shift three units to the left

16. (a) $y = -\frac{1}{2}(x - 2)^2 + 1$



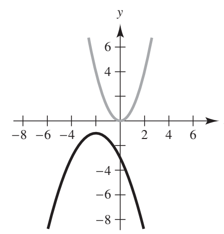
Horizontal shift two units to the right, vertical shrink (each y-value is multiplied by $\frac{1}{2}$), reflection in the x-axis, and vertical shift one unit upward

(b) $y = \left[\frac{1}{2}(x - 1)\right]^2 - 3$



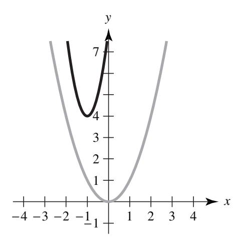
Horizontal shift one unit to the right, horizontal stretch (each x-value is multiplied by 2), and vertical shift three units downward

(c) $y = -\frac{1}{2}(x + 2)^2 - 1$



Horizontal shift two units to the left, vertical shrink (each y-value is multiplied by $\frac{1}{2}$), reflection in x-axis, and vertical shift one unit downward

(d) $y = \left[2(x + 1)\right]^2 + 4$



Horizontal shift one unit to the left, horizontal shrink (each x-value is multiplied by $\frac{1}{2}$), and vertical shift four units upward

17. $f(x) = 1 - x^2$

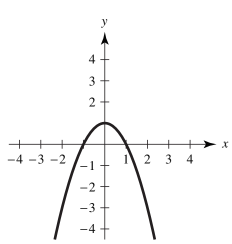
Vertex: $(0, 1)$

Axis of symmetry: $x = 0$ or the y-axis

Find x-intercepts:

$$1 - x^2 = 0$$
$$1 = x^2$$
$$\pm 1 = x$$

x-intercepts: $(1, 0), (-1, 0)$



18. $f(x) = x^2 - 8$

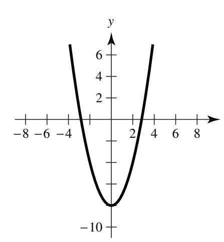
Vertex: $(0, -8)$

Axis of symmetry: $x = 0$ or the y-axis

Find x-intercepts:

$$x^2 - 8 = 0$$
$$x^2 = 8$$
$$x = \pm 2\sqrt{2}$$

x-intercepts: $(2\sqrt{2}, 0), (-2\sqrt{2}, 0)$



19. $f(x) = x^2 + 7$

Vertex: $(0, 7)$

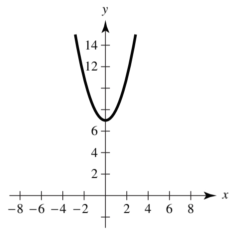
Axis of symmetry: $x = 0$ or the y-axis

Find x-intercepts:

$x^2 + 7 = 0$

$x^2 = -7$

x-intercepts: none



20. $f(x) = 12 - x^2$

Vertex: $(0, 12)$

Axis of symmetry: $x = 0$ or the y-axis

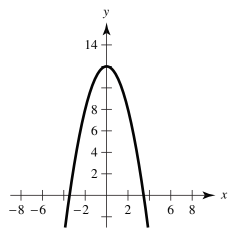
Find x-intercepts:

$12 - x^2 = 0$

$12 = x^2$

$x = \pm 2\sqrt{3}$

x-intercepts: $\left(2\sqrt{3}, 0\right), \left(-2\sqrt{3}, 0\right)$



21. $f(x) = \frac{1}{2}x^2 - 4 = \frac{1}{2}(x - 0)^2 - 4$

Vertex: $(0, -4)$

Axis of symmetry: $x = 0$ or the y-axis

Find x-intercepts:

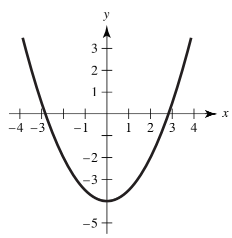
$\frac{1}{2}x^2 - 4 = 0$

$x^2 = 8$

$x = \pm\sqrt{8} = \pm 2\sqrt{2}$

x-intercepts:

$\left(-2\sqrt{2}, 0\right), \left(2\sqrt{2}, 0\right)$



22. $f(x) = 16 - \frac{1}{4}x^2 = -\frac{1}{4}x^2 + 16$

Vertex: $(0, 16)$

Axis of symmetry: $x = 0$ or the y-axis

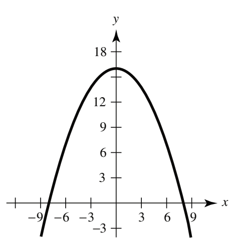
Find x-intercepts:

$16 - \frac{1}{4}x^2 = 0$

$x^2 = 64$

$x = \pm 8$

x-intercepts: $(8, 0), (-8, 0)$



23. $f(x) = (x + 4)^2 - 3$

Vertex: $(-4, -3)$

Axis of symmetry: $x = -4$

Find x-intercepts:

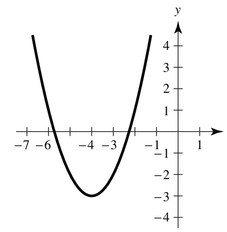
$0 = (x + 4)^2 - 3$

$3 = (x + 4)^2$

$\pm\sqrt{3} = x + 4$

$-4 \pm \sqrt{3} = x$

x-intercepts: $\left(-4 + \sqrt{3}, 0\right), \left(-4 - \sqrt{3}, 0\right)$



24. $f(x) = (x - 6)^2 + 8$

Vertex: $(6, 8)$

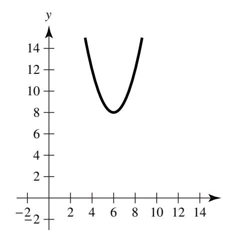
Axis of symmetry: $x = 6$

Find x-intercepts:

$0 = (x - 6)^2 + 8$

$-8 = (x - 6)^2$

x-intercepts: none



25. $h(x) = x^2 - 8x + 16 = (x - 4)^2$

Vertex: $(4, 0)$

Axis of symmetry: $x = 4$

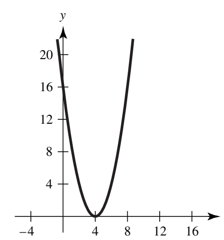
Find x-intercepts:

$(x - 4)^2 = 0$

$x - 4 = 0$

$x = 4$

x-intercept: $(4, 0)$



26. $g(x) = x^2 + 2x + 1 = (x + 1)^2$

Vertex: $(-1, 0)$

Axis of symmetry: $x = -1$

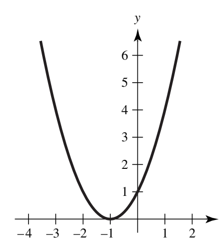
Find x-intercepts:

$(x + 1)^2 = 0$

$x + 1 = 0$

$x = -1$

x-intercept: $(-1, 0)$



27. $f(x) = x^2 - x + \frac{5}{4}$

$= \left(x^2 - x + \frac{1}{4}\right) - \frac{1}{4} + \frac{5}{4}$

$= \left(x - \frac{1}{2}\right)^2 + 1$

Vertex: $\left(\frac{1}{2}, 1\right)$

Axis of symmetry: $x = \frac{1}{2}$

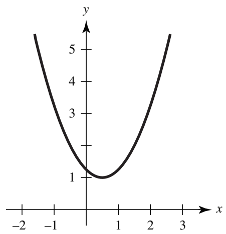
Find x-intercepts:

$x^2 - x + \frac{5}{4} = 0$

$x = \frac{1 \pm \sqrt{1 - 5}}{2}$

Not a real number

No x-intercepts



28. $f(x) = x^2 + 3x + \frac{1}{4}$

$= \left(x^2 + 3x + \frac{9}{4}\right) - \frac{9}{4} + \frac{1}{4}$

$= \left(x + \frac{3}{2}\right)^2 - 2$

Vertex: $\left(-\frac{3}{2}, -2\right)$

Axis of symmetry: $x = -\frac{3}{2}$

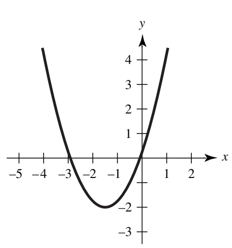
Find x-intercepts:

$x^2 + 3x + \frac{1}{4} = 0$

$x = \frac{-3 \pm \sqrt{9 - 1}}{2}$

$= -\frac{3}{2} \pm \sqrt{2}$

x-intercepts: $\left(-\frac{3}{2} - \sqrt{2}, 0\right), \left(-\frac{3}{2} + \sqrt{2}, 0\right)$



29. $f(x) = -x^2 + 2x + 5$

$= -\left(x^2 - 2x + 1\right) - (-1) + 5$

$= -(x - 1)^2 + 6$

Vertex: $(1, 6)$

Axis of symmetry: $x = 1$

Find x-intercepts:

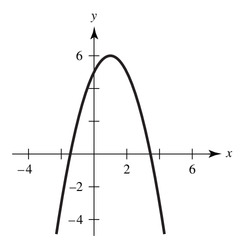
$-x^2 + 2x + 5 = 0$

$x^2 - 2x - 5 = 0$

$x = \frac{2 \pm \sqrt{4 + 20}}{2}$

$= 1 \pm \sqrt{6}$

x-intercepts: $\left(1 - \sqrt{6}, 0\right), \left(1 + \sqrt{6}, 0\right)$



30. $f(x) = -x^2 - 4x + 1 = -\left(x^2 + 4x\right) + 1$

$= -\left(x^2 + 4x + 4\right) - (-4) + 1$

$= -(x + 2)^2 + 5$

Vertex: $(-2, 5)$

Axis of symmetry: $x = -2$

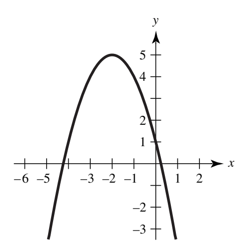
Find x-intercepts: $-x^2 - 4x + 1 = 0$

$x^2 + 4x - 1 = 0$

$x = \frac{-4 \pm \sqrt{16 + 4}}{2}$

$= -2 \pm \sqrt{5}$

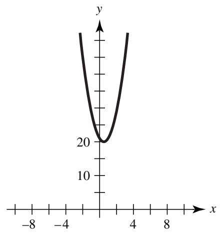
x-intercepts: $\left(-2 - \sqrt{5}, 0\right), \left(-2 + \sqrt{5}, 0\right)$



31. $h(x) = 4x^2 - 4x + 21$

$= 4\left(x^2 - x + \dfrac{1}{4}\right) - 4\left(\dfrac{1}{4}\right) + 21$

$= 4\left(x - \dfrac{1}{2}\right)^2 + 20$



Vertex: $\left(\dfrac{1}{2}, 20\right)$

Axis of symmetry: $x = \dfrac{1}{2}$

Find x-intercepts:

$4x^2 - 4x + 21 = 0$

$x = \dfrac{4 \pm \sqrt{16 - 336}}{2(4)}$

Not a real number

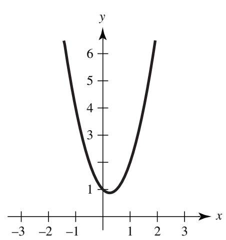
No x-intercepts

32. $f(x) = 2x^2 - x + 1$

$= 2\left(x^2 - \dfrac{1}{2}x\right) + 1$

$= 2\left(x - \dfrac{1}{4}\right)^2 - 2\left(\dfrac{1}{16}\right) + 1$

$= 2\left(x - \dfrac{1}{4}\right)^2 + \dfrac{7}{8}$



Vertex: $\left(\dfrac{1}{4}, \dfrac{7}{8}\right)$

Axis of symmetry: $x = \dfrac{1}{4}$

Find x-intercepts:

$2x^2 - x + 1 = 0$

$x = \dfrac{1 \pm \sqrt{1 - 8}}{2(2)}$

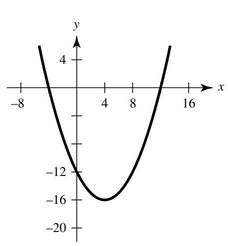
Not a real number

No x-intercepts

33. $f(x) = \frac{1}{4}x^2 - 2x - 12$

$= \frac{1}{4}(x^2 - 8x + 16) - \frac{1}{4}(16) - 12$

$= \frac{1}{4}(x - 4)^2 - 16$



Vertex: $(4, -16)$

Axis of symmetry: $x = 4$

Find x-intercepts:

$\frac{1}{4}x^2 - 2x - 12 = 0$

$x^2 - 8x - 48 = 0$

$(x + 4)(x - 12) = 0$

$x = -4$ or $x = 12$

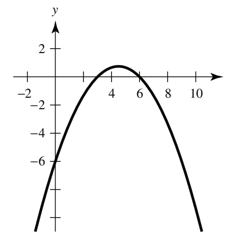
x-intercepts: $(-4, 0), (12, 0)$

34. $f(x) = -\frac{1}{3}x^2 + 3x - 6$

$= -\frac{1}{3}(x^2 - 9x) - 6$

$= -\frac{1}{3}\left(x^2 - 9x + \frac{81}{4}\right) + \frac{1}{3}\left(\frac{81}{4}\right) - 6$

$= -\frac{1}{3}\left(x - \frac{9}{2}\right)^2 + \frac{3}{4}$



Vertex: $\left(\frac{9}{2}, \frac{3}{4}\right)$

Axis of symmetry: $x = \frac{9}{2}$

Find x-intercepts:

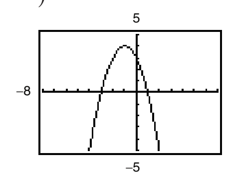
$-\frac{1}{3}x^2 + 3x - 6 = 0$

$x^2 - 9x + 18 = 0$

$(x - 3)(x - 6) = 0$

x-intercepts: $(3, 0), (6, 0)$

35. $f(x) = -(x^2 + 2x - 3) = -(x + 1)^2 + 4$



Vertex: $(-1, 4)$

Axis of symmetry: $x = -1$

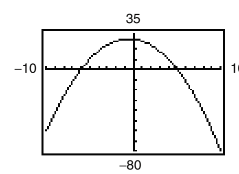
x-intercepts: $(-3, 0), (1, 0)$

36. $f(x) = -(x^2 + x - 30)$

$= -(x^2 + x) + 30$

$= -\left(x^2 + x + \frac{1}{4}\right) + \frac{1}{4} + 30$

$= -\left(x + \frac{1}{2}\right)^2 + \frac{121}{4}$

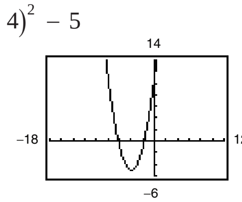


Vertex: $\left(-\frac{1}{2}, \frac{121}{4}\right)$

Axis of symmetry: $x = -\frac{1}{2}$

x-intercepts: $(-6, 0), (5, 0)$

37. $g(x) = x^2 + 8x + 11 = (x + 4)^2 - 5$



Vertex: $(-4, -5)$

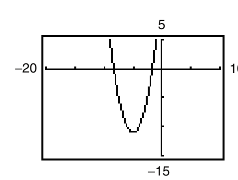
Axis of symmetry: $x = -4$

x-intercepts: $\left(-4 \pm \sqrt{5}, 0\right)$

38. $f(x) = x^2 + 10x + 14$

$= (x^2 + 10x + 25) - 25 + 14$

$= (x + 5)^2 - 11$



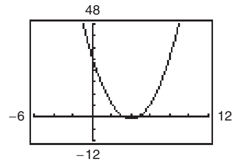
Vertex: $(-5, -11)$

Axis of symmetry: $x = -5$

x-intercepts: $\left(-5 \pm \sqrt{11}, 0\right)$

39. $f(x) = 2x^2 - 16x + 31$

$= 2(x - 4)^2 - 1$



Vertex: $(4, -1)$

Axis of symmetry: $x = 4$

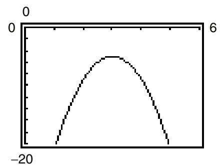
x-intercepts: $\left(4 \pm \frac{1}{2}\sqrt{2}, 0\right)$

40. $f(x) = -4x^2 + 24x - 41$

$= -4(x^2 - 6x) - 41$

$= -4(x^2 - 6x + 9) + 36 - 41$

$= -4(x - 3)^2 - 5$

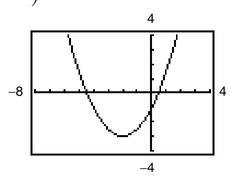


Vertex: $(3, -5)$

Axis of symmetry: $x = 3$

No x-intercepts

41. $g(x) = \frac{1}{2}(x^2 + 4x - 2) = \frac{1}{2}(x + 2)^2 - 3$



Vertex: $(-2, -3)$

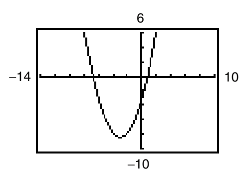
Axis of symmetry: $x = -2$

x-intercepts: $\left(-2 \pm \sqrt{6}, 0\right)$

42. $f(x) = \frac{3}{5}(x^2 + 6x - 5)$

$= \frac{3}{5}(x^2 + 6x + 9) - \frac{27}{5} - 3$

$= \frac{3}{5}(x + 3)^2 - \frac{42}{5}$



Vertex: $\left(-3, -\frac{42}{5}\right)$

Axis of symmetry: $x = -3$

x-intercepts: $\left(-3 \pm \sqrt{14}, 0\right)$

43. $(-1, 4)$ is the vertex.

$y = a(x + 1)^2 + 4$

Because the graph passes through $(1, 0)$,

$0 = a(1 + 1)^2 + 4$

$-4 = 4a$

$-1 = a.$

So, $y = -1(x + 1)^2 + 4 = -(x + 1)^2 + 4.$

44. $(-2, -1)$ is the vertex.

$f(x) = a(x + 2)^2 - 1$

Because the graph passes through $(0, 3)$,

$3 = a(0 + 2)^2 - 1$

$3 = 4a - 1$

$4 = 4a$

$1 = a.$

So, $y = (x + 2)^2 - 1.$

45. $(-2, 2)$ is the vertex.

$y = a(x + 2)^2 + 2$

Because the graph passes through $(-1, 0)$,

$0 = a(-1 + 2)^2 + 2$

$-2 = a.$

So, $y = -2(x + 2)^2 + 2.$

46. $(2, 0)$ is the vertex.

$f(x) = a(x - 2)^2 + 0 = a(x - 2)^2$

Because the graph passes through $(3, 2)$,

$2 = a(3 - 2)^2$

$2 = a.$

So, $y = 2(x - 2)^2.$

47. $(-2, 5)$ is the vertex.

$f(x) = a(x + 2)^2 + 5$

Because the graph passes through $(0, 9)$,

$9 = a(0 + 2)^2 + 5$

$4 = 4a$

$1 = a.$

So, $f(x) = 1(x + 2)^2 + 5 = (x + 2)^2 + 5.$

48. $(4, -1)$ is the vertex.

$f(x) = a(x - 4)^2 - 1$

Because the graph passes through $(2, 3)$,

$3 = a(2 - 4)^2 - 1$

$3 = 4a - 1$

$4 = 4a$

$1 = a.$

So, $f(x) = (x - 4)^2 - 1.$

49. $(1, -2)$ is the vertex.

$$f(x) = a(x - 1)^2 - 2$$

Because the graph passes through $(-1, 14)$,

$$14 = a(-1 - 1)^2 - 2$$
$$14 = 4a - 2$$
$$16 = 4a$$
$$4 = a.$$

So, $f(x) = 4(x - 1)^2 - 2.$

50. $(2, 3)$ is the vertex.

$$f(x) = a(x - 2)^2 + 3$$

Because the graph passes through $(0, 2)$,

$$2 = a(0 - 2)^2 + 3$$
$$2 = 4a + 3$$
$$-1 = 4a$$
$$-\tfrac{1}{4} = a.$$

So, $f(x) = -\frac{1}{4}(x - 2)^2 + 3.$

51. $(5, 12)$ is the vertex.

$$f(x) = a(x - 5)^2 + 12$$

Because the graph passes through $(7, 15)$,

$$15 = a(7 - 5)^2 + 12$$
$$3 = 4a \Rightarrow a = \tfrac{3}{4}.$$

So, $f(x) = \frac{3}{4}(x - 5)^2 + 12.$

52. $(-2, -2)$ is the vertex.

$$f(x) = a(x + 2)^2 - 2$$

Because the graph passes through $(-1, 0)$,

$$0 = a(-1 + 2)^2 - 2$$
$$0 = a - 2$$
$$2 = a.$$

So, $f(x) = 2(x + 2)^2 - 2.$

53. $\left(-\frac{1}{4}, \frac{3}{2}\right)$ is the vertex.

$$f(x) = a\left(x + \tfrac{1}{4}\right)^2 + \tfrac{3}{2}$$

Because the graph passes through $(-2, 0)$,

$$0 = a\left(-2 + \tfrac{1}{4}\right)^2 + \tfrac{3}{2}$$
$$-\tfrac{3}{2} = \tfrac{49}{16}a \Rightarrow a = -\tfrac{24}{49}.$$

So, $f(x) = -\frac{24}{49}\left(x + \frac{1}{4}\right)^2 + \frac{3}{2}.$

54. $\left(\frac{5}{2}, -\frac{3}{4}\right)$ is the vertex.

$$f(x) = a\left(x - \tfrac{5}{2}\right)^2 - \tfrac{3}{4}$$

Because the graph passes through $(-2, 4)$,

$$4 = a\left(-2 - \tfrac{5}{2}\right)^2 - \tfrac{3}{4}$$
$$4 = \tfrac{81}{4}a - \tfrac{3}{4}$$
$$\tfrac{19}{4} = \tfrac{81}{4}a$$
$$\tfrac{19}{81} = a.$$

So, $f(x) = \frac{19}{81}\left(x - \frac{5}{2}\right)^2 - \frac{3}{4}.$

55. $\left(-\frac{5}{2}, 0\right)$ is the vertex.

$$f(x) = a\left(x + \tfrac{5}{2}\right)^2$$

Because the graph passes through $\left(-\frac{7}{2}, -\frac{16}{3}\right)$,

$$-\tfrac{16}{3} = a\left(-\tfrac{7}{2} + \tfrac{5}{2}\right)^2$$
$$-\tfrac{16}{3} = a.$$

So, $f(x) = -\frac{16}{3}\left(x + \frac{5}{2}\right)^2.$

56. $(6, 6)$ is the vertex.

$$f(x) = a(x - 6)^2 + 6$$

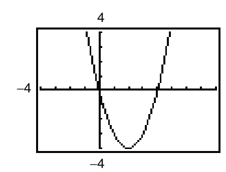
Because the graph passes through $\left(\frac{61}{10}, \frac{3}{2}\right)$,

$$\tfrac{3}{2} = a\left(\tfrac{61}{10} - 6\right)^2 + 6$$
$$\tfrac{3}{2} = \tfrac{1}{100}a + 6$$
$$-\tfrac{9}{2} = \tfrac{1}{100}a$$
$$-450 = a.$$

So, $f(x) = -450(x - 6)^2 + 6.$

57. $f(x) = x^2 - 4x$

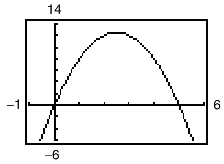
x-intercepts: $(0, 0)$, $(4, 0)$



$$0 = x^2 - 4x$$
$$0 = x(x - 4)$$
$$x = 0 \quad \text{or} \quad x = 4$$

The x-intercepts and the solutions of $f(x) = 0$ are the same.

58. $f(x) = -2x^2 + 10x$



x-intercepts: $(0, 0), (5, 0)$

$0 = -2x^2 + 10x$

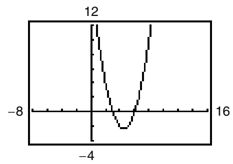
$0 = -2x(x - 5)$

$-2x = 0 \Rightarrow x = 0$

$x - 5 = 0 \Rightarrow x = 5$

The x-intercepts and the solutions of $f(x) = 0$ are the same.

59. $f(x) = x^2 - 9x + 18$



x-intercepts: $(3, 0), (6, 0)$

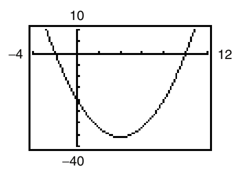
$0 = x^2 - 9x + 18$

$0 = (x - 3)(x - 6)$

$x = 3$ or $x = 6$

The x-intercepts and the solutions of $f(x) = 0$ are the same.

60. $f(x) = x^2 - 8x - 20$



x-intercepts: $(-2, 0), (10, 0)$

$0 = x^2 - 8x - 20$

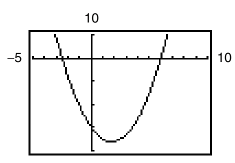
$0 = (x + 2)(x - 10)$

$x + 2 = 0 \Rightarrow x = -2$

$x - 10 = 0 \Rightarrow x = 10$

The x-intercepts and the solutions of $f(x) = 0$ are the same.

61. $f(x) = 2x^2 - 7x - 30$



x-intercepts: $\left(-\frac{5}{2}, 0\right), (6, 0)$

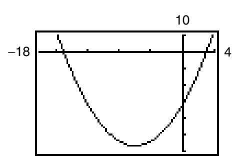
$0 = 2x^2 - 7x - 30$

$0 = (2x + 5)(x - 6)$

$x = -\frac{5}{2}$ or $x = 6$

The x-intercepts and the solutions of $f(x) = 0$ are the same.

62. $f(x) = \frac{7}{10}\left(x^2 + 12x - 45\right)$



x-intercepts: $(-15, 0), (3, 0)$

$0 = \frac{7}{10}\left(x^2 + 12x - 45\right)$

$0 = (x + 15)(x - 3)$

$x + 15 = 0 \Rightarrow x = -15$

$x - 3 = 0 \Rightarrow x = 3$

The x-intercepts and the solutions of $f(x) = 0$ are the same.

63. $f(x) = [x - (-1)](x - 3)$ opens upward

$= (x + 1)(x - 3)$

$= x^2 - 2x - 3$

$g(x) = -[x - (-1)](x - 3)$ opens downward

$= -(x + 1)(x - 3)$

$= -\left(x^2 - 2x - 3\right)$

$= -x^2 + 2x + 3$

Note: $f(x) = a(x + 1)(x - 3)$ has x-intercepts $(-1, 0)$ and $(3, 0)$ for all real numbers $a \neq 0$.

64. $f(x) = [x - (-5)](x - 5)$

$= (x + 5)(x - 5)$

$= x^2 - 25$, opens upward

$g(x) = -f(x)$, opens downward

$g(x) = -x^2 + 25$

Note: $f(x) = a\left(x^2 - 25\right)$ has x-intercepts $(-5, 0)$ and $(5, 0)$ for all real numbers $a \neq 0$.

65. $f(x) = (x - 0)(x - 10)$ opens upward

$= x^2 - 10x$

$g(x) = -(x - 0)(x - 10)$ opens downward

$= -x^2 + 10x$

Note: $f(x) = a(x - 0)(x - 10) = ax(x - 10)$ has x-intercepts $(0, 0)$ and $(10, 0)$ for all real numbers $a \neq 0$.

66. $f(x) = (x - 4)(x - 8)$

$= x^2 - 12x + 32$, opens upward

$g(x) = -f(x)$, opens downward

$g(x) = -x^2 + 12x - 32$

Note: $f(x) = a(x - 4)(x - 8)$ has x-intercepts $(4, 0)$ and $(8,0)$ for all real numbers $a \neq 0$.

67. $f(x) = [x - (-3)]\left[x - \left(-\frac{1}{2}\right)\right](2)$ opens upward

$= (x + 3)\left(x + \frac{1}{2}\right)(2)$

$= (x + 3)(2x + 1)$

$= 2x^2 + 7x + 3$

$g(x) = -\left(2x^2 + 7x + 3\right)$ opens downward

$= -2x^2 - 7x - 3$

Note: $f(x) = a(x + 3)(2x + 1)$ has x-intercepts $(-3, 0)$ and $\left(-\frac{1}{2}, 0\right)$ for all real numbers $a \neq 0$.

68. $f(x) = 2\left[x - \left(-\frac{5}{2}\right)\right](x - 2)$

$= 2\left(x + \frac{5}{2}\right)(x - 2)$

$= 2\left(x^2 + \frac{1}{2}x - 5\right)$

$= 2x^2 + x - 10$, opens upward

$g(x) = -f(x)$, opens downward

$g(x) = -2x^2 - x + 10$

Note: $f(x) = a\left(x + \frac{5}{2}\right)(x - 2)$ has x-intercepts $\left(-\frac{5}{2}, 0\right)$ and $(2, 0)$ for all real numbers $a \neq 0$.

69. $y = x^2 - 4x - 5$

(a) x-intercepts: $(5, 0), (-1, 0)$

(b) The x-intercepts and the solutions of the equation are the same.

(c) $0 = x^2 - 4x - 5$

$0 = (x - 5)(x + 1)$

$x = 5$ or $x = -1$

The x-intercepts are $(5, 0)$ and $(-1, 0)$.

70. $y = 2x^2 + 5x - 3$

(a) From the graph it appears that the x-intercepts are $\left(\frac{1}{2}, 0\right)$ and $(-3, 0)$.

(b) The x-intercepts and solutions of $2x^2 + 5x - 3 = 0$ are the same.

(c) $2x^2 + 5x - 3 = 0$

$(2x - 1)(x + 3) = 0$

$x = \frac{1}{2}$ or $x = -3 \Rightarrow$ The x-intercepts are $\left(\frac{1}{2}, 0\right)$ and $(-3, 0)$.

71. $y = -x^2 - 2x - 1$

(a) From the graph it appears that the x-intercept is $(-1, 0)$.

(b) The x-intercept and the solution to $-x^2 - 2x - 1 = 0$ are the same.

(c) $-x^2 - 2x - 1 = 0$

$x^2 + 2x + 1 = 0$

$(x + 1)^2 = 0$

$x + 1 = 0$

$x = -1 \Rightarrow$ The x-intercept is at $(-1, 0)$.

72. $y = -x^2 - 3x - 3$

(a) From the graph it appears that there are no x-intercepts.

(b) There are no x-intercepts and there are no real solutions to the equation $-x^2 - 3x - 3 = 0$.

(c) $-x^2 - 3x - 3 = 0$

$x^2 + 3x + 3 = 0$

$$x = \frac{-3 \pm \sqrt{3^2 - 4(1)(3)}}{2} = \frac{-3 \pm \sqrt{-3}}{2}$$

Not a real number $\Rightarrow$ No x-intercepts

73. $f(x) = ax^2 + bx + c$

$$= a\left(x^2 + \frac{b}{a}x\right) + c$$

$$= a\left(x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} - \frac{b^2}{4a^2}\right) + c$$

$$= a\left(x^2 + \frac{b}{a}x + \frac{b^2}{4a^2}\right) - a\left(\frac{b^2}{4a^2}\right) + c$$

$$= a\left(x + \frac{b}{2a}\right)^2 - \frac{b^2}{4a} + \frac{4ac}{4a}$$

$$= a\left(x + \frac{b}{2a}\right)^2 + \frac{4ac - b^2}{4a}$$

The vertex is $\left(-\frac{b}{2a}, \frac{4ac - b^2}{4a}\right)$.

$$f\left(-\frac{b}{2a}\right) = a\left(-\frac{b}{2a}\right)^2 + b\left(-\frac{b}{2a}\right) + c$$

$$= a\left(\frac{b^2}{4a^2}\right) - \frac{b^2}{2a} + c$$

$$= \frac{b^2}{4a} - \frac{2b^2}{4a} + \frac{4ac}{4a}$$

$$= \frac{-b^2 + 4ac}{4a}$$

$$= \frac{4ac - b^2}{4a}$$

Thus, the vertex occurs at $\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$.

74. (a) Yes, it is possible for a quadratic equation to have only one x-intercept. That happens when the vertex is the x-intercept.

(b) Yes. If the vertex is above the x-axis and the parabola opens upward, or if the vertex is below the x-axis and the parabola opens downward, then the graph of the quadratic equation will have no x-intercepts.

Examples: $f(x) = x^2 + 4$; $g(x) = -x^2 - 1$

75. Let $x =$ the first number and $y =$ the second number.

Then the sum is

$x + y = 110 \Rightarrow y = 110 - x.$

The product is $P(x) = xy = x(110 - x) = 110x - x^2.$

$$\begin{aligned} P(x) &= -x^2 + 110x \\ &= -(x^2 - 110x + 3025 - 3025) \\ &= -\left[(x - 55)^2 - 3025\right] \\ &= -(x - 55)^2 + 3025 \end{aligned}$$

The maximum value of the product occurs at the vertex of $P(x)$ and is 3025. This happens when $x = y = 55.$

76. Let $x =$ the first number and $y =$ the second number. Then the sum is

$x + y = S \Rightarrow y = S - x.$

The product is $P(x) = xy = x(S - x) = Sx - x^2.$

$$\begin{aligned} P(x) &= Sx - x^2 \\ &= -x^2 + Sx \\ &= -\left(x^2 - Sx + \frac{S^2}{4} - \frac{S^2}{4}\right) \\ &= -\left(x - \frac{S}{2}\right)^2 + \frac{S^2}{4} \end{aligned}$$

The maximum value of the product occurs at the vertex of $P(x)$ and is $S^2/4.$ This happens when $x = y = S/2.$

77. Let $x =$ the first number and $y =$ the second number.

Then the sum is

$x + 2y = 24 \Rightarrow y = \dfrac{24 - x}{2}.$

The product is $P(x) = xy = x\left(\dfrac{24 - x}{2}\right).$

$$\begin{aligned} P(x) &= \frac{1}{2}(-x^2 + 24x) \\ &= -\frac{1}{2}(x^2 - 24x + 144 - 144) \\ &= -\frac{1}{2}\left[(x - 12)^2 - 144\right] = -\frac{1}{2}(x - 12)^2 + 72 \end{aligned}$$

The maximum value of the product occurs at the vertex of $P(x)$ and is 72. This happens when $x = 12$ and $y = (24 - 12)/2 = 6.$ So, the numbers are 12 and 6.

78. Let $x =$ the first number and $y =$ the second number.

Then the sum is $x + 3y = 42 \Rightarrow y = \dfrac{42 - x}{3}.$

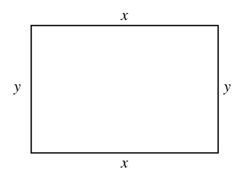
The product is $P(x) = xy = x\left(\dfrac{42 - x}{3}\right).$

$$\begin{aligned} P(x) &= \frac{1}{3}(-x^2 + 42x) \\ &= -\frac{1}{3}(x^2 - 42x + 441 - 441) \\ &= -\frac{1}{3}\left[(x - 21)^2 - 441\right] = -\frac{1}{3}(x - 21)^2 + 147 \end{aligned}$$

The maximum value of the product occurs at the vertex of $P(x)$ and is 147. This happens when $x = 21$

and $y = \dfrac{42 - 21}{3} = 7.$ So, the numbers are 21 and 7.

79.

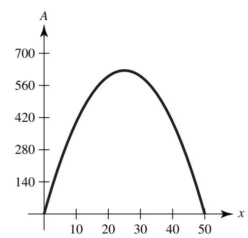


$$\begin{aligned} 2x + 2y &= 100 \\ y &= 50 - x \end{aligned}$$

(a) $A(x) = xy = x(50 - x)$

Domain: $0 < x < 50$

(b)



(c) The area is maximum (625 square feet) when $x = y = 25.$ The rectangle has dimensions 25 ft × 25 ft.

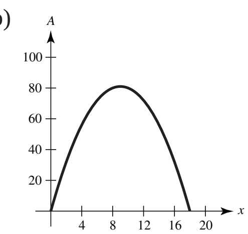
80. Let $x =$ length of rectangle and $y =$ width of rectangle.

$$\begin{aligned} 2x + 2y &= 36 \\ y &= 18 - x \end{aligned}$$

(a) $A(x) = xy = x(18 - x)$

Domain: $0 < x < 18$

(b)



(c) The area is maximum (81 square meters) when $x = y = 9$ meters. The rectangle has dimensions 9 meters × 9 meters.

81. $y = -\frac{4}{9}x^2 + \frac{24}{9}x + 12$

The vertex occurs at $-\frac{b}{2a} = \frac{-24/9}{2(-4/9)} = 3.$ The maximum height is $y(3) = -\frac{4}{9}(3)^2 + \frac{24}{9}(3) + 12 = 16$ feet.

82. $y = -\frac{16}{2025}x^2 + \frac{9}{5}x + 1.5$

(a) The ball height when it is punted is the y-intercept.

$y = -\frac{16}{2025}(0)^2 + \frac{9}{5}(0) + 1.5 = 1.5$ feet

(b) The vertex occurs at $x = -\frac{b}{2a} = -\frac{9/5}{2(-16/2025)} = \frac{3645}{32}.$

The maximum height is $f\left(\frac{3645}{32}\right) = -\frac{16}{2025}\left(\frac{3645}{32}\right)^2 + \frac{9}{5}\left(\frac{3645}{32}\right) + 1.5$

$= -\frac{6561}{64} + \frac{6561}{32} + 1.5 = -\frac{6561}{64} + \frac{13{,}122}{64} + \frac{96}{64} = \frac{6657}{64}$ feet ≈ 104.02 feet.

(c) The length of the punt is the positive x-intercept.

$0 = -\frac{16}{2025}x^2 + \frac{9}{5}x + 1.5$

$x = \frac{-(9/5) \pm \sqrt{(9/5)^2 - (4)(1.5)(-16/2025)}}{-32/2025} \approx \frac{1.8 \pm 1.81312}{-0.01580247}$

$x \approx -0.83031$ or $x \approx 228.64$

The punt is about 228.64 feet.

83. $C = 800 - 10x + 0.25x^2 = 0.25x^2 - 10x + 800$

The vertex occurs at $x = -\frac{b}{2a} = -\frac{-10}{2(0.25)} = 20.$

The cost is minimum when $x = 20$ fixtures.

84. $P = 230 + 20x - 0.5x^2$

The vertex occurs at $x = -\frac{b}{2a} = -\frac{20}{2(-0.5)} = 20.$

Because x is in hundreds of dollars, $20 \times 100 = 2000$ dollars is the amount spent on advertising that gives maximum profit.

85. $R(p) = -25p^2 + 1200p$

(a) $R(20) = \$14{,}000$ thousand $= \$14{,}000{,}000$

$R(25) = \$14{,}375$ thousand $= \$14{,}375{,}000$

$R(30) = \$13{,}500$ thousand $= \$13{,}500{,}000$

(b) The revenue is a maximum at the vertex.

$-\frac{b}{2a} = \frac{-1200}{2(-25)} = 24$

$R(24) = 14{,}400$

The unit price that will yield a maximum revenue of \$14,400 thousand is \$24.

86. $R(p) = -12p^2 + 150p$

(a) $R(\$4) = -12(\$4)^2 + 150(\$4) = \$408$

$R(\$6) = -12(\$6)^2 + 150(\$6) = \$468$

$R(\$8) = -12(\$8)^2 + 150(\$8) = \$432$

(b) The vertex occurs at

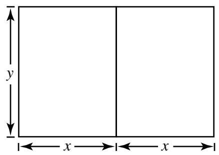
$p = -\frac{b}{2a} = -\frac{150}{2(-12)} = \$6.25.$

Revenue is maximum when price $= \$6.25$ per pet.

The maximum revenue is

$R(\$6.25) = -12(\$6.25)^2 + 150(\$6.25) = \$468.75.$

87. (a)



$4x + 3y = 200 \Rightarrow y = \frac{1}{3}(200 - 4x) = \frac{4}{3}(50 - x)$

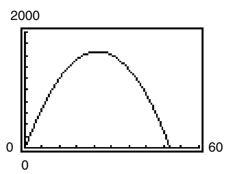
$A = 2xy = 2x\left[\frac{4}{3}(50 - x)\right] = \frac{8}{3}x(50 - x) = \frac{8x(50 - x)}{3}$

(b)

x	A
5	600
10	$1066\frac{2}{3}$
15	1400
20	1600
25	$1666\frac{2}{3}$
30	1600

This area is maximum when $x = 25$ feet and $y = \frac{100}{3} = 33\frac{1}{3}$ feet.

(c)



This area is maximum when $x = 25$ feet and $y = \frac{100}{3} = 33\frac{1}{3}$ feet.

(d) $A = \frac{8}{3}x(50 - x)$

$= -\frac{8}{3}(x^2 - 50x)$

$= -\frac{8}{3}(x^2 - 50x + 625 - 625)$

$= -\frac{8}{3}\left[(x - 25)^2 - 625\right]$

$= -\frac{8}{3}(x - 25)^2 + \frac{5000}{3}$

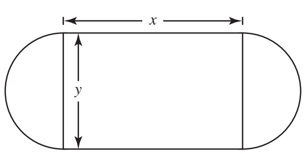
The maximum area occurs at the vertex and is $5000/3$ square feet. This happens when $x = 25$ feet and $y = (200 - 4(25))/3 = 100/3$ feet.

The dimensions are $2x = 50$ feet by $33\frac{1}{3}$ feet.

(e) They are all identical.

$x = 25$ feet and $y = 33\frac{1}{3}$ feet

88. (a)



(b) Radius of semicircular ends of track: $r = \frac{1}{2}y$

Distance around two semicircular parts of track:

$d = 2\pi r = 2\pi\left(\frac{1}{2}y\right) = \pi y$

(c) Distance traveled around track in one lap:

$d = \pi y + 2x = 200$

$\pi y = 200 - 2x$

$y = \frac{200 - 2x}{\pi}$

(d) Area of rectangular region:

$A = xy = x\left(\frac{200 - 2x}{\pi}\right)$

$= \frac{1}{\pi}(200x - 2x^2)$

$= -\frac{2}{\pi}(x^2 - 100x)$

$= -\frac{2}{\pi}(x^2 - 100x + 2500 - 2500)$

$= -\frac{2}{\pi}(x - 50)^2 + \frac{5000}{\pi}$

The area is maximum when $x = 50$ and $y = \frac{200 - 2(50)}{\pi} = \frac{100}{\pi}$.

89. (a) Revenue = (number of tickets sold)(price per ticket)

Let $y =$ attendance, or the number of tickets sold.

$m = -100, (20, 1500)$

$y - 1500 = -100(x - 20)$

$y - 1500 = -100x + 2000$

$y = -100x + 3500$

$R(x) = (y)(x)$

$R(x) = (-100x + 3500)(x)$

$R(x) = -100x^2 + 3500x$

(b) The revenue is at a maximum at the vertex.

$$-\frac{b}{2a} = \frac{-3500}{2(-100)} = 17.5$$

$$R(17.5) = -100(17.5)^2 + 3500(17.5) = \$30,625$$

A ticket price of \$17.50 will yield a maximum revenue of \$30,625.

90. (a) Area of window = Area of rectangle + Area of semicircle

$$= xy + \frac{1}{2}\pi(\text{radius})^2$$

$$= xy + \frac{1}{2}\pi\left(\frac{x}{2}\right)^2$$

$$= xy + \frac{\pi x^2}{8}$$

To eliminate the y in the equation for area, introduce a secondary equation.

Perimeter = perimeter of rectangle + perimeter of semicircle

$$16 = 2y + x + \frac{1}{2}(\text{circumference})$$

$$16 = 2y + x + \frac{1}{2}(2\pi \cdot \text{radius})$$

$$16 = 2y + x + \pi\left(\frac{x}{2}\right)$$

$$y = 8 - \frac{1}{2}x - \frac{\pi x}{4}$$

Substitute the secondary equation into the area equation.

$$\text{Area} = xy + \frac{\pi x^2}{8}$$

$$= x\left(8 - \frac{1}{2}x - \frac{\pi x}{4}\right) + \frac{\pi x^2}{8}$$

$$= 8x - \frac{1}{2}x^2 - \frac{\pi x^2}{4} + \frac{\pi x^2}{8}$$

$$= 8x - \frac{1}{2}x^2 - \frac{\pi x^2}{8}$$

$$= \frac{1}{8}\left(64x - 4x^2 - \pi x^2\right)$$

(b) The area is maximum at the vertex.

$$\text{Area} = 8x - \frac{1}{2}x^2 - \frac{\pi x^2}{8}$$

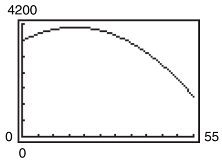
$$= \left(-\frac{1}{2} - \frac{\pi}{8}\right)x^2 + 8x$$

$$x = -\frac{b}{2a} = \frac{-8}{2\left(-\frac{1}{2} - \frac{\pi}{8}\right)} \approx 4.48$$

$$y = 8 - \frac{1}{2}(4.48) - \frac{\pi(4.48)}{4} \approx 2.24$$

The area will be at a maximum when the width is about 4.48 feet and the length is about 2.24 feet.

91. (a)



(b) The maximum annual consumption occurs at the point $(16.9, 4074.813)$.

4075 cigarettes

$1966 \rightarrow t = 16$

The maximum consumption occurred in 1966. After that year, the consumption decreases. It is likely that the warning was responsible for the decrease in consumption.

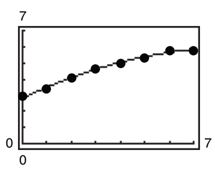
(c) $$\text{Annual Consumption per smoker} = \frac{\text{Annual consumption in 2005} \cdot \text{total population}}{\text{total number of smokers in 2005}} = \frac{1487.9(296{,}329{,}000)}{59{,}858{,}458} = 7365.8$$

About 7366 cigarettes per smoker annually

$$\text{Daily Consumption per smoker} = \frac{\text{Number of cigarettes per year}}{\text{Number of days per year}} = \frac{7366}{365} \approx 20.2$$

About 20 cigarettes per day

92. (a), (b)



$y = -0.0408x^2 + 0.715x + 2.82$

(c) The model is a good fit to the actual data.

(d) The greatest sales occurred in the year 2007.

(e) Sales will be at a maximum at the vertex.

$$x = -\frac{b}{2a} = \frac{-0.715}{2(-0.0408)} \approx 8.76$$

Sometime during 2008.

(f) $2011 \rightarrow$ Use $x = 11$.

$$y(11) = -0.0408(11)^2 + 0.715(11) + 2.82 \approx 5.75$$

Sales in the year 2011 will be about \$5.75 billion.

93. True. The equation $-12x^2 - 1 = 0$ has no real solution, so the graph has no x-intercepts.

94. True. The vertex of $f(x)$ is $\left(-\frac{5}{4}, \frac{53}{4}\right)$ and the vertex of $g(x)$ is $\left(-\frac{5}{4}, -\frac{71}{4}\right)$.

95. True. The negative leading coefficient causes the parabola to open downward, making the vertex the maximum point on the graph.

96. True. The positive leading coefficient causes the parabola to open upward, making the vertex the minimum point on the graph.

97. $f(x) = -x^2 + bx - 75$, maximum value: 25

The maximum value, 25, is the y-coordinate of the vertex.

Find the x-coordinate of the vertex:

$$x = -\frac{b}{2a} = -\frac{b}{2(-1)} = \frac{b}{2}$$

$$f(x) = -x^2 + bx - 75$$

$$f\left(\frac{b}{2}\right) = -\left(\frac{b}{2}\right)^2 + b\left(\frac{b}{2}\right) - 75$$

$$25 = -\frac{b^2}{4} + \frac{b^2}{2} - 75$$

$$100 = \frac{b^2}{4}$$

$$400 = b^2$$

$$\pm 20 = b$$

98. $f(x) = -x^2 + bx - 16$, maximum value: 48

The maximum value, 48, is the y-coordinate of the vertex.

Find the x-coordinate of the vertex:

$$x = -\frac{b}{2a} = -\frac{b}{2(-1)} = \frac{b}{2}$$

$$f(x) = -x^2 + bx - 16$$
$$f\left(\frac{b}{2}\right) = -\left(\frac{b}{2}\right)^2 + b\left(\frac{b}{2}\right) - 16$$
$$48 = -\frac{b^2}{4} + \frac{b^2}{2} - 16$$
$$64 = \frac{b^2}{4}$$
$$256 = b^2$$
$$\pm 16 = b$$

99. $f(x) = x^2 + bx + 26$, minimum value: 10

The minimum value, 10, is the y-coordinate of the vertex.

Find the x-coordinate of the vertex:

$$x = -\frac{b}{2a} = -\frac{b}{2(1)} = -\frac{b}{2}$$

$$f(x) = x^2 + bx + 26$$
$$f\left(-\frac{b}{2}\right) = \left(-\frac{b}{2}\right)^2 + b\left(-\frac{b}{2}\right) + 26$$
$$10 = \frac{b^2}{4} - \frac{b^2}{2} + 26$$
$$-16 = -\frac{b^2}{4}$$
$$64 = b^2$$
$$\pm 8 = b$$

100. $f(x) = x^2 + bx - 25$, minimum value: –50

The minimum value, –50, is the y-coordinate of the vertex.

Find the x-coordinate:

$$x = -\frac{b}{2a} = -\frac{b}{2(1)} = -\frac{b}{2}$$

$$f(x) = x^2 + bx - 25$$
$$f\left(-\frac{b}{2}\right) = \left(-\frac{b}{2}\right)^2 + b\left(-\frac{b}{2}\right) - 25$$
$$-50 = \frac{b^2}{4} - \frac{b^2}{2} - 25$$
$$-25 = \frac{-b^2}{4}$$
$$100 = b^2$$
$$\pm 10 = b$$

101. The graph of $f(x)$ is moved h units to the right. Every y value is adjusted by a factor of a, and the parabola becomes narrower or wider. Every point on the parabola is shifted up k units.

102. Conditions (a) and (d) are preferable because profits would be increasing.

103. If $f(x) = ax^2 + bx + c$ has two real zeros, then by the Quadratic Formula they are

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

The average of the zeros of f is

$$\frac{\dfrac{-b - \sqrt{b^2 - 4ac}}{2a} + \dfrac{-b + \sqrt{b^2 - 4ac}}{2a}}{2} = \frac{\dfrac{-2b}{2a}}{2} = -\frac{b}{2a}.$$

This is the x-coordinate of the vertex of the graph.

Section 2.2 Polynomial Functions of Higher Degree

1. continuous

2. Leading Coefficient Test

3. x^n

4. n; $n - 1$

5. (a) solution; (b) $(x - a)$; (c) x-intercept

6. repeated; multiplicity

7. touches; crosses

8. standard

9. $f(x) = -2x + 3$ is a line with y-intercept $(0, 3)$. Matches graph (c).

10. $f(x) = x^2 - 4x$ is a parabola with intercepts $(0, 0)$ and $(4, 0)$ and opens upward. Matches graph (g).

11. $f(x) = -2x^2 - 5x$ is a parabola with x-intercepts $(0, 0)$ and $\left(-\frac{5}{2}, 0\right)$ and opens downward. Matches graph (h).

12. $f(x) = 2x^3 - 3x + 1$ has intercepts $(0, 1)$, $(1, 0)$, $\left(-\frac{1}{2} - \frac{1}{2}\sqrt{3}, 0\right)$ and $\left(-\frac{1}{2} + \frac{1}{2}\sqrt{3}, 0\right)$. Matches graph (f).

13. $f(x) = -\frac{1}{4}x^4 + 3x^2$ has intercepts $(0, 0)$ and $(\pm 2\sqrt{3}, 0)$. Matches graph (a).

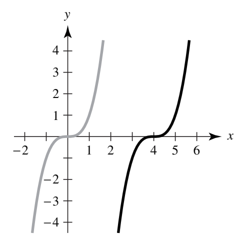
14. $f(x) = -\frac{1}{3}x^3 + x^2 - \frac{4}{3}$ has y-intercept $\left(0, -\frac{4}{3}\right)$. Matches graph (e).

15. $f(x) = x^4 + 2x^3$ has intercepts $(0, 0)$ and $(-2, 0)$. Matches graph (d).

16. $f(x) = \frac{1}{5}x^5 - 2x^3 + \frac{9}{5}x$ has intercepts $(0, 0), (1, 0), (-1, 0), (3, 0), (-3, 0)$. Matches graph (b).

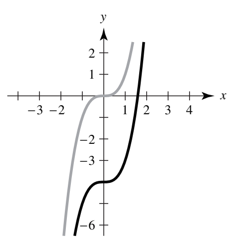
17. $y = x^3$

(a) $f(x) = (x - 4)^3$



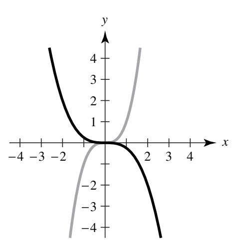
Horizontal shift four units to the right

(b) $f(x) = x^3 - 4$



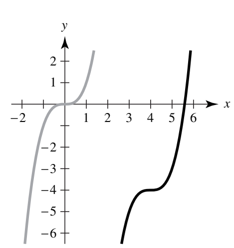
Vertical shift four units downward

(c) $f(x) = -\dfrac{1}{4}x^3$



Reflection in the x-axis and a vertical shrink (each y-value is multiplied by $\frac{1}{4}$)

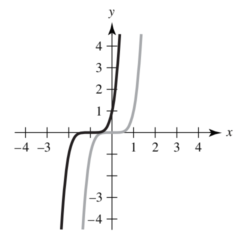
(d) $f(x) = (x - 4)^3 - 4$



Horizontal shift four units to the right and vertical shift four units downward

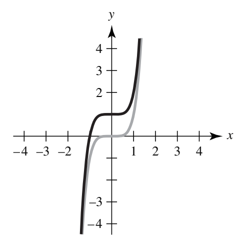
18. $y = x^5$

(a) $f(x) = (x + 1)^5$



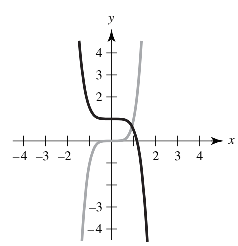
Horizontal shift one unit to the left

(b) $f(x) = x^5 + 1$



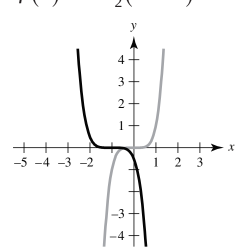
Vertical shift one unit upward

(c) $f(x) = 1 - \frac{1}{2}x^5$



Reflection in the x-axis, vertical shrink (each y-value is multiplied by $\frac{1}{2}$), and vertical shift one unit upward

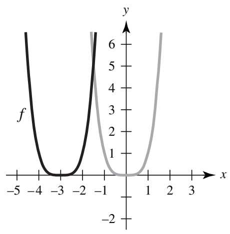
(d) $f(x) = -\frac{1}{2}(x + 1)^5$



Refection in the x-axis, vertical shrink (each y-value is multiplied by $\frac{1}{2}$), and horizontal shift one unit to the left

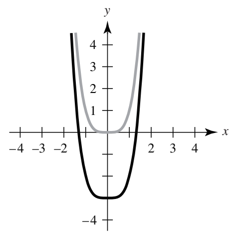
19. $y = x^4$

(a) $f(x) = (x + 3)^4$



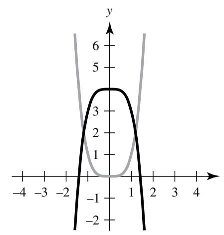
Horizontal shift three units to the left

(b) $f(x) = x^4 - 3$



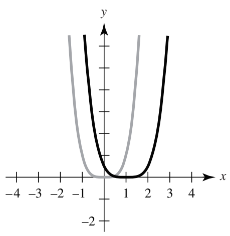
Vertical shift three units downward

(c) $f(x) = 4 - x^4$



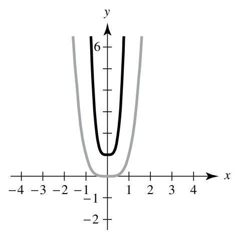
Reflection in the x-axis and then a vertical shift four units upward

(d) $f(x) = \frac{1}{2}(x - 1)^4$



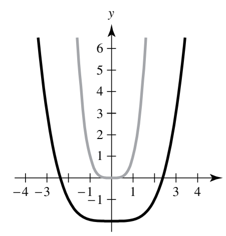
Horizontal shift one unit to the right and a vertical shrink $\left(\text{each } y\text{-value is multiplied by } \frac{1}{2}\right)$

(e) $f(x) = (2x)^4 + 1$



Vertical shift one unit upward and a horizontal shrink (each y-value is multiplied by 16)

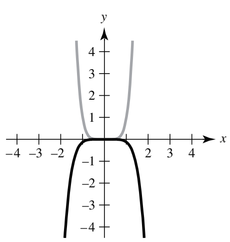
(f) $f(x) = \left(\frac{1}{2}x\right)^4 - 2$



Vertical shift two units downward and a horizontal stretch $\left(\text{each } y\text{-value is multipied by } \frac{1}{16}\right)$

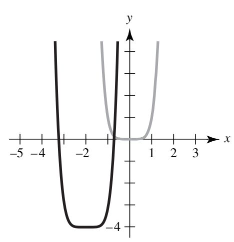
20. $y = x^6$

(a) $f(x) = -\frac{1}{8}x^6$



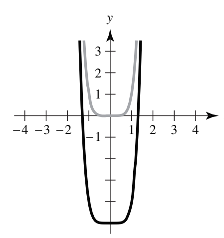
Vertical shrink $\left(\text{each } y\text{-value is multiplied by } \frac{1}{8}\right)$ and reflection in the x-axis

(b) $f(x) = (x + 2)^6 - 4$



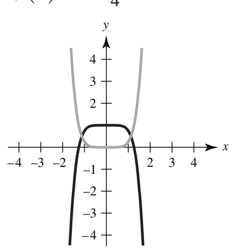
Horizontal shift two units to the left and a vertical shift four units downward

(c) $f(x) = x^6 - 5$



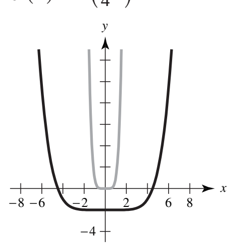
Vertical shift five units downward

(d) $f(x) = -\frac{1}{4}x^6 + 1$



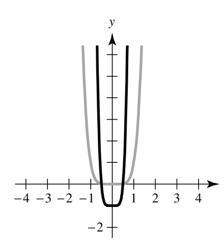
Reflection in the x-axis, vertical shrink $\left(\text{each } y\text{-value is multiplied by } \frac{1}{4}\right)$, and vertical shift one unit upward

(e) $f(x) = \left(\frac{1}{4}x\right)^6 - 2$



Horizontal stretch (each x-value is multiplied by 4), and vertical shift two units downward

(f) $f(x) = (2x)^6 - 1$



Horizontal shrink $\left(\text{each } x\text{-value is multiplied by } \frac{1}{2}\right)$, and vertical shift one unit downward

21. $f(x) = \frac{1}{5}x^3 + 4x$

Degree: 3

Leading coefficient: $\frac{1}{5}$

The degree is odd and the leading coefficient is positive. The graph falls to the left and rises to the right.

22. $f(x) = 2x^2 - 3x + 1$

Degree: 2

Leading coefficient: 2

The degree is even and the leading coefficient is positive.

The graph rises to the left and rises to the right.

23. $g(x) = 5 - \frac{7}{2}x - 3x^2$

Degree: 2

Leading coefficient: -3

The degree is even and the leading coefficient is negative. The graph falls to the left and falls to the right.

24. $h(x) = 1 - x^6$

Degree: 6

Leading coefficient: -1

The degree is even and the leading coefficient is negative. The graph falls to the left and falls to the right.

25. $f(x) = -2.1x^5 + 4x^3 - 2$

Degree: 5

Leading coefficient: -2.1

The degree is odd and the leading coefficient is negative.

The graph rises to the left and falls to the right.

26. $f(x) = 4x^5 - 7x + 6.5$

Degree: 5

Leading coefficient: 4

The degree is odd and the leading coefficient is positive. The graph falls to the left and rises to the right.

27. $f(x) = 6 - 2x + 4x^2 - 5x^3$

Degree: 3

Leading coefficient: -5

The degree is odd and the leading coefficient is negative.

The graph rises to the left and falls to the right.

28. $f(x) = \dfrac{3x^4 - 2x + 5}{4}$

Degree: 4

Leading coefficient: $\dfrac{3}{4}$

The degree is even and the leading coefficient is positive.

The graph rises to the left and rises to the right.

29. $f(x) = -\dfrac{3}{4}(t^2 - 3t + 6)$

Degree: 2

Leading coefficient: $-\dfrac{3}{4}$

The degree is even and the leading coefficient is negative. The graph falls to the left and falls to the right.

30. $f(s) = -\frac{7}{8}(s^3 + 5s^2 - 7s + 1)$

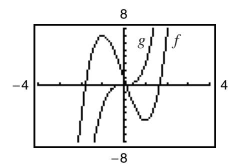
Degree: 3

Leading coefficient: $-\frac{7}{8}$

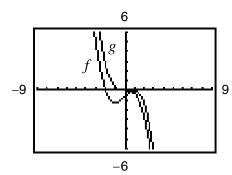
The degree is odd and the leading coefficient is negative.

The graph rises to the left and falls to the right.

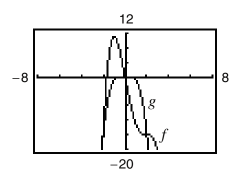
31. $f(x) = 3x^3 - 9x + 1$; $g(x) = 3x^3$



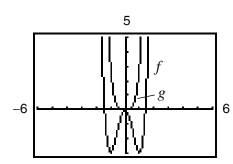
32. $f(x) = -\frac{1}{3}(x^3 - 3x + 2)$, $g(x) = -\frac{1}{3}x^3$



33. $f(x) = -(x^4 - 4x^3 + 16x)$; $g(x) = -x^4$



34. $f(x) = 3x^4 - 6x^2$, $g(x) = 3x^4$



35. $f(x) = x^2 - 36$

(a) $0 = x^2 - 36$

$0 = (x + 6)(x - 6)$

$x + 6 = 0 \qquad x - 6 = 0$

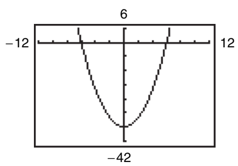
$x = -6 \qquad x = 6$

Zeros: ± 6

(b) Each zero has a multiplicity of one (odd multiplicity).

Turning points: 1 (the vertex of the parabola)

(c)



36. $f(x) = 81 - x^2$

(a) $0 = 81 - x^2$

$0 = (9 - x)(9 + x)$

$9 - x = 0 \qquad 9 + x = 0$

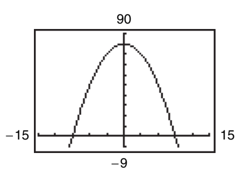
$9 = x \qquad x = -9$

Zeros: ± 9

(b) Each zero has a multiplicity of one (odd multiplicity).

Turning points: 1 (the vertex of the parabola)

(c)



37. $h(t) = t^2 - 6t + 9$

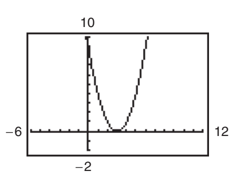
(a) $0 = t^2 - 6t + 9 = (t - 3)^2$

Zero: $t = 3$

(b) $t = 3$ has a multiplicity of 2 (even multiplicity).

Turning points: 1 (the vertex of the parabola)

(c)



38. $f(x) = x^2 + 10x + 25$

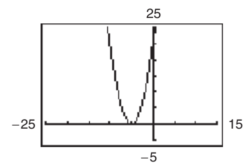
(a) $0 = x^2 + 10x + 25 = (x + 5)^2$

Zero: $x = -5$

(b) $x = -5$ has a multiplicity of 2 (even multiplicity).

Turning points: 1 (the vertex of the parabola)

(c)



39. $f(x) = \frac{1}{3}x^2 + \frac{1}{3}x - \frac{2}{3}$

(a) $0 = \frac{1}{3}x^2 + \frac{1}{3}x - \frac{2}{3}$

$= \frac{1}{3}(x^2 + x - 2)$

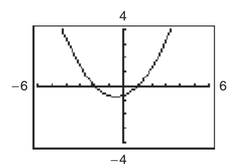
$= \frac{1}{3}(x + 2)(x - 1)$

Zeros: $x = -2, x = 1$

(b) Each zero has a multiplicity of 1 (odd multiplicity).

Turning points: 1 (the vertex of the parabola)

(c)



40. $f(x) = \dfrac{1}{2}x^2 + \dfrac{5}{2}x - \dfrac{3}{2}$

(a) For $\dfrac{1}{2}x^2 + \dfrac{5}{2}x - \dfrac{3}{2} = 0, a = \dfrac{1}{2}, b = \dfrac{5}{2}, c = -\dfrac{3}{2}$.

$$x = \frac{-\frac{5}{2} \pm \sqrt{\left(\frac{5}{2}\right)^2 - 4\left(\frac{1}{2}\right)\left(-\frac{3}{2}\right)}}{1}$$

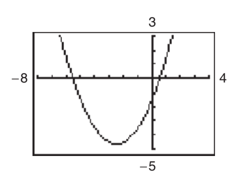
$$= -\frac{5}{2} \pm \sqrt{\frac{37}{4}}$$

Zeros: $x = \dfrac{-5 \pm \sqrt{37}}{2}$

(b) Each zero has a multiplicity of 1 (odd multiplicity).

Turning points: 1 (the vertex of the parabola)

(c)



41. $f(x) = 3x^3 - 12x^2 + 3x$

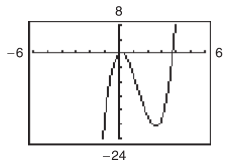
(a) $0 = 3x^3 - 12x^2 + 3x = 3x(x^2 - 4x + 1)$

Zeros: $x = 0, x = 2 \pm \sqrt{3}$ (by the Quadratic Formula)

(b) Each zero has a multiplicity of 1 (odd multiplicity).

Turning points: 2

(c)



42. $g(x) = 5x(x^2 - 2x - 1)$

(a) $0 = 5x(x^2 - 2x - 1)$

$0 = x(x^2 - 2x - 1)$

For $x^2 - 2x - 1 = 0, a = 1, b = -2, c = -1.$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-1)}}{2(1)}$$

$$= \frac{2 \pm \sqrt{8}}{2}$$

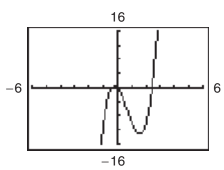
$$= 1 \pm \sqrt{2}$$

Zeros: $x = 0, x = 1 \pm \sqrt{2}$

(b) Each zero has a multiplicity of 1 (odd multiplicity).

Turning points: 2

(c)



43. $f(t) = t^3 - 8t^2 + 16t$

(a) $0 = t^3 - 8t^2 + 16t$

$0 = t(t^2 - 8t + 16)$

$0 = t(t - 4)(t - 4)$

$t = 0 \quad t - 4 = 0 \quad t - 4 = 0$

$t = 0 \quad t = 4 \quad t = 4$

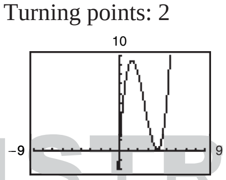
Zeros: $t = 0, t = 4$

(b) The multiplicity of $t = 0$ is 1 (odd multiplicity).

The multiplicity of $t = 4$ is 2 (even multiplicity).

Turning points: 2

(c)



44. $f(x) = x^4 - x^3 - 30x^2$

(a) $0 = x^4 - x^3 - 30x^2$

$0 = x^2(x^2 - x - 30)$

$0 = x^2(x - 6)(x + 5)$

$x^2 = 0 \quad x - 6 = 0 \quad x + 5 = 0$

$x = 0 \quad x = 6 \quad x = -5$

Zeros: $x = 0, x = 6, x = -5$

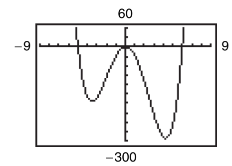
(b) The multiplicity of $x = 0$ is 2 (even multiplicity).

The multiplicity of $x = 6$ is 1 (odd multiplicity).

The multiplicity of $x = -5$ is 1 (odd multiplicity).

Turning points: 3

(c)



45. $g(t) = t^5 - 6t^3 + 9t$

(a) $0 = t^5 - 6t^3 + 9t = t(t^4 - 6t^2 + 9) = t(t^2 - 3)^2$

$= t(t + \sqrt{3})^2(t - \sqrt{3})^2$

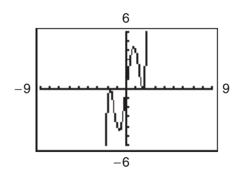
Zeros: $t = 0, t = \pm\sqrt{3}$

(b) $t = 0$ has a multiplicity of 1 (odd multiplicity).

$t = \pm\sqrt{3}$ each have a multiplicity of 2 (even multiplicity).

Turning points: 4

(c)



46. (a) $f(x) = x^5 + x^3 - 6x$

$0 = x(x^4 + x^2 - 6)$

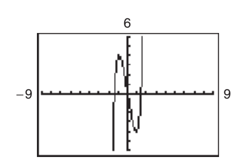
$0 = x(x^2 + 3)(x^2 - 2)$

Zeros: $x = 0, \pm\sqrt{2}$

(b) Each zero has a multiplicity of 1 (odd multiplicity).

Turning points: 2

(c)



47. $f(x) = 3x^4 + 9x^2 + 6$

(a) $0 = 3x^4 + 9x^2 + 6$

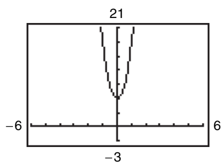
$0 = 3(x^4 + 3x^2 + 2)$

$0 = 3(x^2 + 1)(x^2 + 2)$

No real zeros

(b) Turning points: 1

(c)



48. $f(x) = 2x^4 - 2x^2 - 40$

(a) $0 = 2x^4 - 2x^2 - 40$

$0 = 2(x^4 - x^2 - 20)$

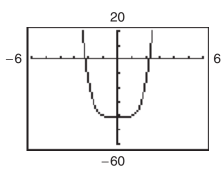
$0 = 2(x^2 + 4)(x^2 - 5)$

Zeros: $x = \pm\sqrt{5}$

(b) Each zero has a multiplicity of 1 (odd multiplicity).

Turning points: 3

(c)



49. $g(x) = x^3 + 3x^2 - 4x - 12$

(a) $0 = x^3 + 3x^2 - 4x - 12 = x^2(x + 3) - 4(x + 3)$

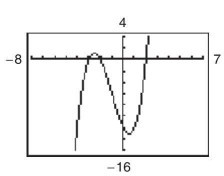
$= (x^2 - 4)(x + 3) = (x - 2)(x + 2)(x + 3)$

Zeros: $x = \pm 2, x = -3$

(b) Each zero has a multiplicity of 1 (odd multiplicity).

Turning points: 2

(c)



50. $f(x) = x^3 - 4x^2 - 25x + 100$

(a) $0 = x^2(x - 4) - 25(x - 4)$

$0 = (x^2 - 25)(x - 4)$

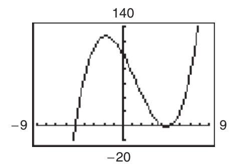
$0 = (x + 5)(x - 5)(x - 4)$

Zeros: $x = \pm 5, 4$

(b) Each zero has a multiplicity of 1 (odd multiplicity).

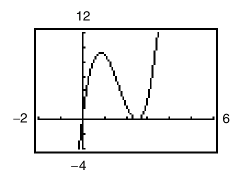
Turning points: 2

(c)



51. $y = 4x^3 - 20x^2 + 25x$

(a)



(b) x-intercepts: $(0, 0), \left(\frac{5}{2}, 0\right)$

(c) $0 = 4x^3 - 20x^2 + 25x$

$0 = x(4x^2 - 20x + 25)$

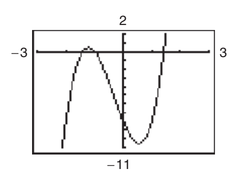
$0 = x(2x - 5)^2$

$x = 0, \frac{5}{2}$

(d) The solutions are the same as the x-coordinates of the x-intercepts.

52. $y = 4x^3 + 4x^2 - 8x - 8$

(a)



(b) $(-1, 0), (-\sqrt{2}, 0), (\sqrt{2}, 0)$

(c) $0 = 4x^3 + 4x^2 - 8x - 8$

$0 = 4x^2(x + 1) - 8(x + 1)$

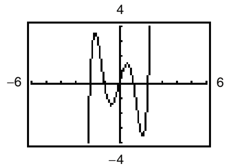
$0 = (4x^2 - 8)(x + 1)$

$0 = 4(x^2 - 2)(x + 1)$

$x = \pm\sqrt{2}, -1$

(d) The solutions are the same as the x-coordinates of the x-intercepts.

53. $y = x^5 - 5x^3 + 4x$

(a)


(b) x-intercepts: $(0, 0), (\pm 1, 0), (\pm 2, 0)$

(c) $0 = x^5 - 5x^3 + 4x$

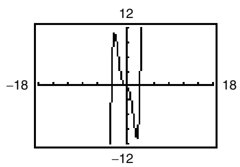
$0 = x(x^2 - 1)(x^2 - 4)$

$0 = x(x + 1)(x - 1)(x + 2)(x - 2)$

$x = 0, \pm 1, \pm 2$

(d) The solutions are the same as the x-coordinates of the x-intercepts.

54. $y = \frac{1}{4}x^3(x^2 - 9)$

(a)


(b) x-intercepts: $(0, 0), (3, 0), (-3, 0)$

(c) $0 = \frac{1}{4}x^3(x^2 - 9)$

$x = 0, \pm 3$

(d) The solutions are the same as the x-coordinates of the x-intercepts.

55. $f(x) = (x - 0)(x - 8)$

$= x^2 - 8x$

Note: $f(x) = ax(x - 8)$ has zeros 0 and 8 for all real numbers $a \neq 0$.

56. $f(x) = (x - 0)(x + 7)$

$= x^2 + 7x$

Note: $f(x) = ax(x + 7)$ has zeros 0 and -7 for all real numbers $a \neq 0$.

57. $f(x) = (x - 2)(x + 6)$

$= x^2 + 4x - 12$

Note: $f(x) = a(x - 2)(x + 6)$ has zeros 2 and -6 for all real numbers $a \neq 0$.

58. $f(x) = (x + 4)(x - 5)$

$= x^2 - x - 20$

Note: $f(x) = a(x + 4)(x - 5)$ has zeros -4 and 5 for all real numbers $a \neq 0$.

59. $f(x) = (x - 0)(x + 4)(x + 5)$

$= x(x^2 + 9x + 20)$

$= x^3 + 9x^2 + 20x$

Note: $f(x) = ax(x + 4)(x + 5)$ has zeros $0, -4,$ and -5 for all real numbers $a \neq 0$.

60. $f(x) = (x - 0)(x - 1)(x - 10)$

$= x(x^2 - 11x + 10)$

$= x^3 - 11x^2 + 10x$

Note: $f(x) = ax(x - 1)(x - 10)$ has zeros $0, 1,$ and 10 for all real numbers $a \neq 0$.

61. $f(x) = (x - 4)(x + 3)(x - 3)(x - 0)$

$= (x - 4)(x^2 - 9)x$

$= x^4 - 4x^3 - 9x^2 + 36x$

Note: $f(x) = a(x^4 - 4x^3 - 9x^2 + 36x)$ has zeros $4, -3, 3,$ and 0 for all real numbers $a \neq 0$.

62. $f(x) = (x - (-2))(x - (-1))(x - 0)(x - 1)(x - 2)$

$= x(x + 2)(x + 1)(x - 1)(x - 2)$

$= x(x^2 - 4)(x^2 - 1)$

$= x(x^4 - 5x^2 + 4)$

$= x^5 - 5x^3 + 4x$

Note: $f(x) = ax(x + 2)(x + 1)(x - 1)(x - 2)$ has zeros $-2, -1, 0, 1,$ and 2 for all real numbers $a \neq 0$.

63. $f(x) = \left[x - (1 + \sqrt{3})\right]\left[x - (1 - \sqrt{3})\right]$

$= \left[(x - 1) - \sqrt{3}\right]\left[(x - 1) + \sqrt{3}\right]$

$= (x - 1)^2 - (\sqrt{3})^2$

$= x^2 - 2x + 1 - 3$

$= x^2 - 2x - 2$

Note: $f(x) = a(x^2 - 2x - 2)$ has zeros $1 + \sqrt{3}$ and $1 - \sqrt{3}$ for all real numbers $a \neq 0$.

64. $f(x) = (x - 2)\left[x - (4 + \sqrt{5})\right]\left[x - (4 - \sqrt{5})\right]$

$= (x - 2)\left[(x - 4) - \sqrt{5}\right]\left[(x - 4) + \sqrt{5}\right]$

$= (x - 2)\left[(x - 4)^2 - 5\right]$

$= x(x - 4)^2 - 5x - 2(x - 4)^2 + 10$

$= x^3 - 8x^2 + 16x - 5x - 2x^2 + 16x - 32 + 10$

$= x^3 - 10x^2 + 27x - 22$

Note: $f(x) = a(x^3 - 10x^2 + 27x - 22)$ has zeros $2, 4 + \sqrt{5},$ and $4 - \sqrt{5}$ for all real numbers $a \neq 0$.

65. $f(x) = (x + 3)(x + 3)$
$= x^2 + 6x + 9$

Note: $f(x) = a(x^2 + 6x + 9), a \neq 0,$ has degree 2 and zero $x = -3$.

66. $f(x) = (x + 12)(x + 6)$
$= x^2 + 18x + 72$

Note: $f(x) = a(x^2 + 18x + 72), a \neq 0,$ has degree 2 and zeros $x = -12$ and -6.

67. $f(x) = (x - 0)(x + 5)(x - 1)$
$= x(x^2 + 4x - 5)$
$= x^3 + 4x^2 - 5x$

Note: $f(x) = ax(x^2 + 4x - 5), a \neq 0,$ has degree 3 and zeros $x = 0, -5,$ and 1.

68. $f(x) = (x + 2)(x - 4)(x - 7)$
$= (x + 2)(x^2 - 11x + 28) = x^3 - 9x^2 + 6x + 56$

Note: $f(x) = a(x^3 - 9x^2 + 6x + 56), a \neq 0,$ has degree 3 and zeros $x = -2, 4,$ and 7.

69. $f(x) = (x - 0)(x - \sqrt{3})(x - (-\sqrt{3}))$
$= x(x - \sqrt{3})(x + \sqrt{3}) = x^3 - 3x$

Note: $f(x) = a(x^3 - 3x), a \neq 0,$ has degree 3 and zeros $x = 0, \sqrt{3},$ and $-\sqrt{3}$.

70. $f(x) = (x - 0)(x - 2\sqrt{2})\left[x - (-2\sqrt{2})\right]$
$= x(x - 2\sqrt{2})(x + 2\sqrt{2})$
$= x(x^2 - 8)$
$= x^3 - 8x$

Note: $f(x) = a(x^3 - 8x)$ has these zeros for all real numbers $a \neq 0$.

71. $f(x) = (x - 1)\left[x - (-2)\right]\left[x - (1 + \sqrt{3})\right]\left[x - (1 - \sqrt{3})\right]$
$= (x - 1)(x + 2)\left[(x - 1) - \sqrt{3}\right]\left[(x - 1) + \sqrt{3}\right]$
$= (x^2 + x - 2)\left[(x - 1)^2 - 3\right]$
$= (x^2 + x - 2)(x^2 - 2x - 2)$
$= x^4 - x^3 - 6x^2 + 2x + 4$

Note: $f(x) = a(x^4 - x^3 - 6x^2 + 2x + 4)$ has these zeros for all real numbers $a \neq 0$.

72. $f(x) = (x - 3)\left[x - (-2)\right]\left[x - (2 + \sqrt{5})\right]\left[x - (2 - \sqrt{5})\right]$
$= (x - 3)(x + 2)\left[(x - 2) - \sqrt{5}\right]\left[(x - 2) + \sqrt{5}\right]$
$= (x^2 - x - 6)\left[(x - 2)^2 - 5\right]$
$= (x^2 - x - 6)(x^2 - 4x - 1)$
$= x^4 - 5x^3 - 3x^2 + 25x + 6$

Note: $f(x) = a(x^4 - 5x^3 - 3x^2 + 25x + 6)$ has these zeros for all real numbers $a \neq 0$.

73. $f(x) = x^4(x + 4) = x^5 + 4x^4$
or $f(x) = x^3(x + 4)^2 = x^5 + 8x^4 + 16x^3$
or $f(x) = x^2(x + 4)^3 = x^5 + 12x^4 + 48x^3 + 64x^2$
or $f(x) = x(x + 4)^4 = x^5 + 16x^4 + 96x^3 + 256x^2 + 256x$

Note: Any nonzero scalar multiple of these functions would also have degree 5 and zeros $x = 0$ and -4.

74. $f(x) = (x + 1)^2(x - 4)(x - 7)(x - 8) = x^5 - 17x^4 + 79x^3 - 11x^2 - 332x - 224$

or $f(x) = (x + 1)(x - 4)^2(x - 7)(x - 8) = x^5 - 22x^4 + 169x^3 - 496x^2 + 208x + 896$

or $f(x) = (x + 1)(x - 4)(x - 7)^2(x - 8) = x^5 - 25x^4 + 223x^3 - 787x^2 + 532x + 1568$

or $f(x) = (x + 1)(x - 4)(x - 7)(x - 8)^2 = x^5 - 26x^4 + 241x^3 - 884x^2 + 640x + 1792$

Note: Any nonzero scalar multiple of these functions would also have degree 5 and zeros $x = -1, 4, 7,$ and 8.

75. $f(x) = x^3 - 25x = x(x + 5)(x - 5)$

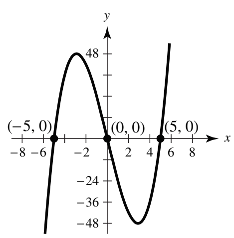
(a) Falls to the left; rises to the right

(b) Zeros: 0, −5, 5

(c)

x	−2	−1	0	1	2
$f(x)$	42	24	0	−24	−42

(d)



76. $f(x) = x^4 - 9x^2 = x^2(x + 3)(x - 3)$

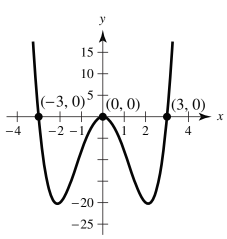
(a) Rises to the left; rises to the right

(b) Zeros: −3, 0, 3

(c)

x	−2	−1	0	1	2
$f(x)$	−24	−8	0	−8	−24

(d)



77. $f(t) = \frac{1}{4}(t^2 - 2t + 15) = \frac{1}{4}(t - 1)^2 + \frac{7}{2}$

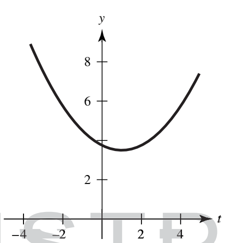
(a) Rises to the left; rises to the right

(b) No real zeros (no x-intercepts)

(c)

t	−1	0	1	2	3
$f(t)$	4.5	3.75	3.5	3.75	4.5

(d) The graph is a parabola with vertex $\left(1, \frac{7}{2}\right)$.



78. $g(x) = -x^2 + 10x - 16 = -(x - 2)(x - 8)$

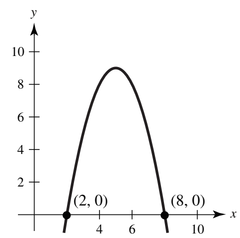
(a) Falls to the left; falls to the right

(b) Zeros: 2, 8

(c)

x	1	3	5	7	9
$g(x)$	−7	5	9	5	−7

(d)



79. $f(x) = x^3 - 2x^2 = x^2(x - 2)$

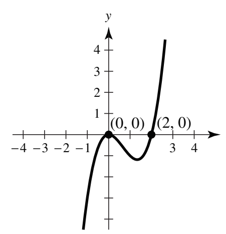
(a) Falls to the left; rises to the right

(b) Zeros: 0, 2

(c)

x	−1	0	$\frac{1}{2}$	1	2	3
$f(x)$	−3	0	$-\frac{3}{8}$	−1	0	9

(d)



80. $f(x) = 8 - x^3 = (2 - x)(4 + 2x + x^2)$

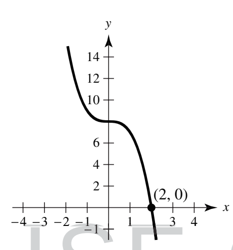
(a) Rises to the left; falls to the right

(b) Zero: 2

(c)

x	−2	−1	0	1	2
$f(x)$	16	9	8	7	0

(d)



81. $f(x) = 3x^3 - 15x^2 + 18x = 3x(x - 2)(x - 3)$

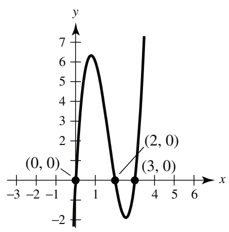
(a) Falls to the left; rises to the right

(b) Zeros: 0, 2, 3

(c)

x	0	1	2	2.5	3	3.5
$f(x)$	0	6	0	–1.875	0	7.875

(d)



82. $f(x) = -4x^3 + 4x^2 + 15x$

$= -x(4x^2 - 4x - 15)$

$= -x(2x - 5)(2x + 3)$

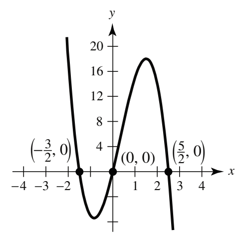
(a) Rises to the left; falls to the right

(b) Zeros: $-\frac{3}{2}, 0, \frac{5}{2}$

(c)

x	–3	–2	–1	0	1	2	3
$f(x)$	99	18	–7	0	15	14	–27

(d)



83. $f(x) = -5x^2 - x^3 = -x^2(5 + x)$

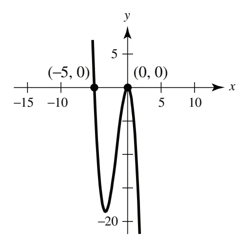
(a) Rises to the left; falls to the right

(b) Zeros: 0, –5

(c)

x	–5	–4	–3	–2	–1	0	1
$f(x)$	0	–16	–18	–12	–4	0	–6

(d)



84. $f(x) = -48x^2 + 3x^4 = 3x^2(x^2 - 16)$

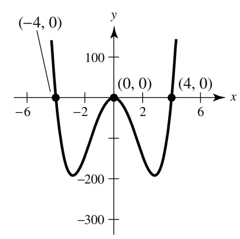
(a) Rises to the left; rises to the right

(b) Zeros; 0, ±4

(c)

x	–5	–4	–3	–2	–1	0	1	2	3	4	5
$f(x)$	675	0	–189	–144	–45	0	–45	–144	–189	0	675

(d)



85. $f(x) = x^2(x - 4)$

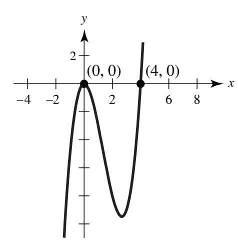
(a) Falls to the left; rises to the right

(b) Zeros: 0, 4

(c)

x	–1	0	1	2	3	4	5
$f(x)$	–5	0	–3	–8	–9	0	25

(d)



86. $h(x) = \frac{1}{3}x^3(x - 4)^2$

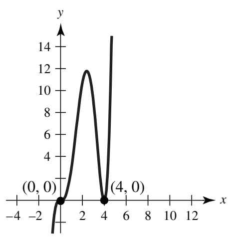
(a) Falls to the left; rises to the right

(b) Zeros: 0, 4

(c)

x	–1	0	1	2	3	4	5
$h(x)$	$-\frac{25}{3}$	0	3	$\frac{32}{3}$	9	0	$\frac{125}{3}$

(d)



87. $g(t) = -\frac{1}{4}(t - 2)^2(t + 2)^2$

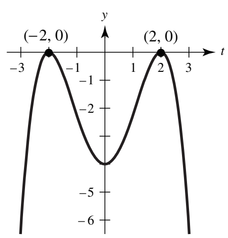
(a) Falls to the left; falls to the right

(b) Zeros: 2, – 2

(c)

t	–3	–2	–1	0	1	2	3
$g(t)$	$-\frac{25}{4}$	0	$-\frac{9}{4}$	–4	$-\frac{9}{4}$	0	$-\frac{25}{4}$

(d)



88. $g(x) = \frac{1}{10}(x + 1)^2(x - 3)^3$

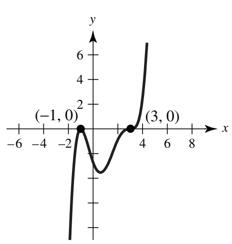
(a) Falls to the left; rises to the right

(b) Zeros: –1, 3

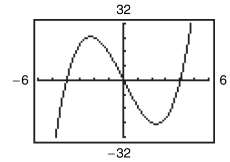
(c)

x	–2	–1	0	1	2	4
$g(x)$	–12.5	0	–2.7	–3.2	–0.9	2.5

(d)

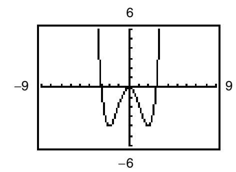


89. $f(x) = x^3 - 16x = x(x - 4)(x + 4)$



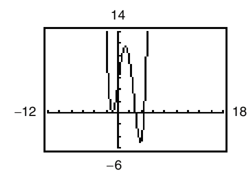
Zeros: 0 of multiplicity 1; 4 of multiplicity 1; and –4 of multiplicity 1.

90. $f(x) = \frac{1}{4}x^4 - 2x^2$



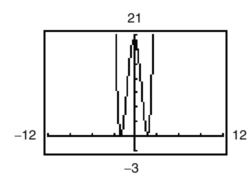
Zeros: –2.828 and 2.828 of multiplicity 1; 0 of multiplicity 2

91. $g(x) = \frac{1}{5}(x + 1)^2(x - 3)(2x - 9)$



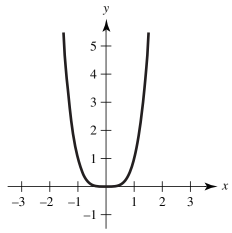
Zeros: –1 of multiplicity 2; 3 of multiplicity 1; $\frac{9}{2}$ of multiplicity 1

92. $h(x) = \frac{1}{5}(x + 2)^2(3x - 5)^2$



Zeros: $-2, \frac{5}{3}$, both with multiplicity 2

93. $f(x) = x^4$; $f(x)$ is even.



(a) $g(x) = f(x) + 2$

Vertical shift two units upward

$g(-x) = f(-x) + 2 = f(x) + 2 = g(x)$

Even

(b) $g(x) = f(x + 2)$

Horizontal shift two units to the left

Neither odd nor even

(c) $g(x) = f(-x) = (-x)^4 = x^4$

Reflection in the y-axis. The graph looks the same.

Even

(d) $g(x) = -f(x) = -x^4$

Reflection in the x-axis

Even

(e) $g(x) = f\left(\frac{1}{2}x\right) = \frac{1}{16}x^4$

Horizontal stretch

Even

(f) $g(x) = \frac{1}{2}f(x) = \frac{1}{2}x^4$

Vertical shrink

Even

(g) $g(x) = f\left(x^{3/4}\right) = \left(x^{3/4}\right)^4 = \left(\sqrt[4]{x^3}\right)^4 = x^3, x \geq 0$

Neither

(h)
$$\begin{aligned} g(x) &= (f \circ f)(x) \\ &= f(f(x)) \\ &= f(x^4) \\ &= (x^4)^4 \\ &= x^{16} \end{aligned}$$

Even

94. $R = \dfrac{1}{100{,}000}\left(-x^3 + 600x^2\right)$

The point of diminishing returns (where the graph changes from curving upward to curving downward) occurs when $x = 200$. The point is $(200, 160)$ which corresponds to spending $\$2{,}000{,}000$ on advertising to obtain a revenue of $\$160$ million.

95. (a) Volume $= l \cdot w \cdot h$

height $= x$

length $=$ width $= 36 - 2x$

So, $V(x) = (36 - 2x)(36 - 2x)(x) = x(36 - 2x)^2$.

(b) Domain: $0 < x < 18$

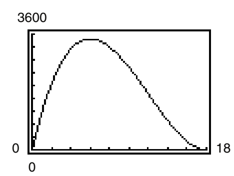
The length and width must be positive.

(c)

Box Height	Box Width	Box Volume, V
1	$36 - 2(1)$	$1[36 - 2(1)]^2 = 1156$
2	$36 - 2(2)$	$2[36 - 2(2)]^2 = 2048$
3	$36 - 2(3)$	$3[36 - 2(3)]^2 = 2700$
4	$36 - 2(4)$	$4[36 - 2(4)]^2 = 3136$
5	$36 - 2(5)$	$5[36 - 2(5)]^2 = 3380$
6	$36 - 2(6)$	$6[36 - 2(6)]^2 = 3456$
7	$36 - 2(7)$	$7[36 - 2(7)]^2 = 3388$

The volume is a maximum of 3456 cubic inches when the height is 6 inches and the length and width are each 24 inches. So the dimensions are $6 \times 24 \times 24$ inches.

(d)



The maximum point on the graph occurs at $x = 6$. This agrees with the maximum found in part (c).

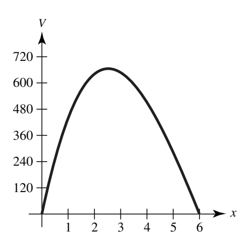
96. (a)
$$\begin{aligned} \text{Volume} &= l \cdot w \cdot h = (24 - 2x)(24 - 4x)x \\ &= 2(12 - x) \cdot 4(6 - x)x \\ &= 8x(12 - x)(6 - x) \end{aligned}$$

(b) $x > 0$, $\quad 12 - x > 0$, $\quad 6 - x > 0$

$x < 12 \qquad x < 6$

Domain: $0 < x < 6$

(c)



$x \approx 2.5$ corresponds to a maximum of 665 cubic inches.

97. False. A fifth-degree polynomial can have at most four turning points.

98. True. $f(x) = (x - 1)^6$ has one repeated solution.

99. True. A polynomial of degree 7 with a negative leading coefficient rises to the left and falls to the right.

100. (a) Degree: 3
Leading coefficient: Positive
(b) Degree: 2
Leading coefficient: Positive
(c) Degree: 4
Leading coefficient: Positive
(d) Degree: 5
Leading coefficient: Positive

Section 2.3 Polynomial and Synthetic Division

1. $f(x)$ is the dividend; $d(x)$ is the divisor: $q(x)$ is the quotient: $r(x)$ is the remainder

2. improper; proper

3. improper

4. synthetic division

5. Factor

6. Remainder

7. $y_1 = \dfrac{x^2}{x + 2}$ and $y_2 = x - 2 + \dfrac{4}{x + 2}$

$$\begin{array}{r} x - 2 \\ x + 2\overline{\big)x^2 + 0x + 0} \\ \underline{x^2 + 2x} \\ -2x + 0 \\ \underline{-2x - 4} \\ 4 \end{array}$$

So, $\dfrac{x^2}{x + 2} = x - 2 + \dfrac{4}{x + 2}$ and $y_1 = y_2$.

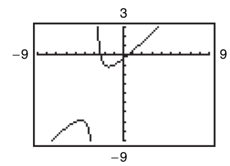
8. $y_1 = \dfrac{x^4 - 3x^2 - 1}{x^2 + 5}$ and $y_2 = x^2 - 8 + \dfrac{39}{x^2 + 5}$

$$\begin{array}{r} x^2 - 8 \\ x^2 + 5\overline{\big)x^4 - 3x^2 - 1} \\ \underline{x^4 + 5x^2} \\ -8x^2 - 1 \\ \underline{-8x^2 - 40} \\ 39 \end{array}$$

So, $\dfrac{x^4 - 3x^2 - 1}{x^2 + 5} = x^2 - 8 + \dfrac{39}{x^2 + 5}$ and $y_1 = y_2$.

9. $y_1 = \dfrac{x^2 + 2x - 1}{x + 3}$, $y_2 = x - 1 + \dfrac{2}{x + 3}$

(a) and (b)

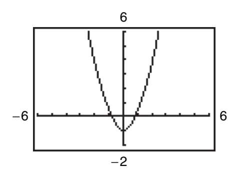


(c) $$\begin{array}{r} x - 1 \\ x + 3\overline{\big)x^2 + 2x - 1} \\ \underline{x^2 + 3x} \\ -x - 1 \\ \underline{-x - 3} \\ 2 \end{array}$$

So, $\dfrac{x^2 + 2x - 1}{x + 3} = x - 1 + \dfrac{2}{x + 3}$ and $y_1 = y_2$.

10. $y_1 = \dfrac{x^4 + x^2 - 1}{x^2 + 1}$, $y_2 = x^2 - \dfrac{1}{x^2 + 1}$

(a) and (b)



(c) $$\begin{array}{r} x^2 \\ x^2 + 0x + 1\overline{\big)x^4 + 0x^3 + x^2 + 0x - 1} \\ \underline{x^4 + 0x^3 + x^2 } \\ -1 \end{array}$$

So, $\dfrac{x^4 + x^2 - 1}{x^2 + 1} = x^2 - \dfrac{1}{x^2 + 1}$ and $y_1 = y_2$.

11. $$\begin{array}{r} 2x + 4 \\ x + 3\overline{\big)2x^2 + 10x + 12} \\ \underline{2x^2 + 6x } \\ 4x + 12 \\ \underline{4x + 12} \\ 0 \end{array}$$

$\dfrac{2x^2 + 10x + 12}{x + 3} = 2x + 4, x \neq 3$

12.
$$\begin{array}{r} 5x + 3 \\ x - 4\overline{)5x^2 - 17x - 12} \\ \underline{5x^2 - 20x} \\ 3x - 12 \\ \underline{3x - 12} \\ 0 \end{array}$$

$$\frac{5x^2 - 17x - 12}{x - 4} = 5x + 3,\ x \neq 4$$

13.
$$\begin{array}{r} x^2 - 3x + 1 \\ 4x + 5\overline{)4x^3 - 7x^2 - 11x + 5} \\ \underline{4x^3 + 5x^2} \\ -12x^2 - 11x \\ \underline{-12x^2 - 15x} \\ 4x + 5 \\ \underline{4x + 5} \\ 0 \end{array}$$

$$\frac{4x^3 - 7x^2 - 11x + 5}{4x + 5} = x^2 - 3x + 1,\ x \neq -\frac{5}{4}$$

14.
$$\begin{array}{r} 2x^2 - 4x + 3 \\ 3x - 2\overline{)6x^3 - 16x^2 + 17x - 6} \\ \underline{6x^3 - 4x^2} \\ -12x^2 + 17x \\ \underline{-12x^2 + 8x} \\ 9x - 6 \\ \underline{9x - 6} \\ 0 \end{array}$$

$$\frac{6x^3 - 16x^2 + 17x - 6}{3x - 2} = 2x^2 - 4x + 3,\ x \neq \frac{2}{3}$$

15.
$$\begin{array}{r} x^3 + 3x^2 \qquad -1 \\ x + 2\overline{)x^4 + 5x^3 + 6x^2 - x - 2} \\ \underline{x^4 + 2x^3} \\ 3x^3 + 6x^2 \\ \underline{3x^3 + 6x^2} \\ -x - 2 \\ \underline{-x - 2} \\ 0 \end{array}$$

$$\frac{x^4 + 5x^3 + 6x^2 - x - 2}{x + 2} = x^3 + 3x^2 - 1,\ x \neq -2$$

16.
$$\begin{array}{r} x^2 + 7x + 18 \\ x - 3\overline{)x^3 + 4x^2 - 3x - 12} \\ \underline{x^3 - 3x^2} \\ 7x^2 - 3x \\ \underline{7x^2 - 21x} \\ 18x - 12 \\ \underline{18x - 54} \\ 42 \end{array}$$

$$\frac{x^3 + 4x^2 - 3x - 12}{x - 3} = x^2 + 7x + 18 + \frac{42}{x - 3}$$

17.
$$\begin{array}{r} x^2 + 3x + 9 \\ x - 3\overline{)x^3 + 0x^2 + 0x - 27} \\ \underline{x^3 - 3x^2} \\ 3x^2 + 0x \\ \underline{3x^2 - 9x} \\ 9x - 27 \\ \underline{9x - 27} \\ 0 \end{array}$$

$$\frac{x^3 - 27}{x - 3} = x^2 + 3x + 9,\ x \neq 3$$

18.
$$\begin{array}{r} x^2 - 5x + 25 \\ x + 5\overline{)x^3 + 0x^2 + 0x + 125} \\ \underline{x^3 + 5x^2} \\ -5x^2 + 0x \\ \underline{-5x^2 - 25x} \\ 25x + 125 \\ \underline{25x + 125} \\ 0 \end{array}$$

$$\frac{x^3 + 125}{x + 5} = x^2 - 5x + 25,\ x \neq -5$$

19.
$$\begin{array}{r} 7 \\ x + 2\overline{)7x + 3} \\ \underline{7x + 14} \\ -11 \end{array}$$

$$\frac{7x + 3}{x + 2} = 7 - \frac{11}{x + 2}$$

20.
$$\begin{array}{r} 4 \\ 2x + 1\overline{)8x - 5} \\ \underline{8x + 4} \\ -9 \end{array}$$

$$\frac{8x - 5}{2x + 1} = 4 - \frac{9}{2x + 1}$$

21.
$$\begin{array}{r} x \\ x^2 + 0x + 1\overline{)x^3 + 0x^2 + 0x - 9} \\ \underline{x^3 + 0x^2 + x} \\ -x - 9 \end{array}$$

$$\frac{x^3 - 9}{x^2 + 1} = x - \frac{x + 9}{x^2 + 1}$$

22.
$$\begin{array}{r} x^2 \\ x^3 + 0x^2 + 0x - 1\overline{)x^5 + 0x^4 + 0x^3 + 0x^2 + 0x + 7} \\ \underline{x^5 + 0x^4 + 0x^3 - x^2 \qquad\quad} \\ x^2 \qquad + 7 \end{array}$$

$$\frac{x^5 + 7}{x^3 - 1} = x^2 + \frac{x^2 + 7}{x^3 - 1}$$

23.
$$\begin{array}{r} 2x - 8 \\ x^2 + 0x + 1\overline{)2x^3 - 8x^2 + 3x - 9} \\ \underline{2x^3 + 0x^2 + 2x} \\ -8x^2 + x - 9 \\ \underline{-8x^2 - 0x - 8} \\ x - 1 \end{array}$$

$$\frac{2x^3 - 8x^2 + 3x - 9}{x^2 + 1} = 2x - 8 + \frac{x - 1}{x^2 + 1}$$

24.
$$\begin{array}{r} x^2 + 6x + 9 \\ x^2 - x - 3\overline{)x^4 + 5x^3 + 0x^2 - 20x - 16} \\ \underline{x^4 - x^3 - 3x^2} \\ 6x^3 + 3x^2 - 20x \\ \underline{6x^3 - 6x^2 - 18x} \\ 9x^2 - 2x - 16 \\ \underline{9x^2 - 9x - 27} \\ 7x + 11 \end{array}$$

$$\frac{x^4 + 5x^3 - 20x - 16}{x^2 - x - 3} = x^2 + 6x + 9 + \frac{7x + 11}{x^2 - x - 3}$$

25.
$$\begin{array}{r} x + 3 \\ x^3 - 3x^2 + 3x - 1\overline{)x^4 + 0x^3 + 0x^2 + 0x + 0} \\ \underline{x^4 - 3x^3 + 3x^2 - x} \\ 3x^3 - 3x^2 + x + 0 \\ \underline{3x^3 - 9x^2 + 9x - 3} \\ 6x^2 - 8x + 3 \end{array}$$

$$\frac{x^4}{(x - 1)^3} = x + 3 + \frac{6x^2 - 8x + 3}{(x - 1)^3}$$

26.
$$\begin{array}{r} 2x \\ x^2 - 2x + 1\overline{)2x^3 - 4x^2 - 15x + 5} \\ \underline{2x^3 - 4x^2 + 2x} \\ -17x + 5 \end{array}$$

$$\frac{2x^3 - 4x^2 - 15x + 5}{(x - 1)^2} = 2x - \frac{17x - 5}{x^2 - 2x + 1}$$

27.
$$\begin{array}{r|rrrr} 5 & 3 & -17 & 15 & -25 \\ & & 15 & -10 & 25 \\ \hline & 3 & -2 & 5 & 0 \end{array}$$

$$\frac{3x^3 - 17x^2 + 15x - 25}{x - 5} = 3x^2 - 2x + 5, \; x \neq 5$$

28.
$$\begin{array}{r|rrrr} -3 & 5 & 18 & 7 & -6 \\ & & -15 & -9 & 6 \\ \hline & 5 & 3 & -2 & 0 \end{array}$$

$$\frac{5x^3 + 18x^2 + 7x - 6}{x + 3} = 5x^2 + 3x - 2, \; x \neq -3$$

29.
$$\begin{array}{r|rrrr} 3 & 6 & 7 & -1 & 26 \\ & & 18 & 75 & 222 \\ \hline & 6 & 25 & 74 & 248 \end{array}$$

$$\frac{6x^3 + 7x^2 - x + 26}{x - 3} = 6x^2 + 25x + 74 + \frac{248}{x - 3}$$

30.
$$\begin{array}{r|rrrr} -6 & 2 & 14 & -20 & 7 \\ & & -12 & -12 & 192 \\ \hline & 2 & 2 & -32 & 199 \end{array}$$

$$\frac{2x^3 + 14x^2 - 20x + 7}{x + 6} = 2x^2 + 2x - 32 + \frac{199}{x + 6}$$

31.
$$\begin{array}{r|rrrr} -2 & 4 & 8 & -9 & -18 \\ & & -8 & 0 & 18 \\ \hline & 4 & 0 & -9 & 0 \end{array}$$

$$\frac{4x^3 + 8x^2 - 9x - 18}{x + 2} = 4x^2 - 9, \; x \neq -2$$

32.
$$\begin{array}{r|rrrr} 2 & 9 & -18 & -16 & 32 \\ & & 18 & 0 & -32 \\ \hline & 9 & 0 & -16 & 0 \end{array}$$

$$\frac{9x^3 - 18x^2 - 16x + 32}{x - 2} = 9x^2 - 16, \; x \neq 2$$

33.
$$\begin{array}{r|rrrr} -10 & -1 & 0 & 75 & -250 \\ & & 10 & -100 & 250 \\ \hline & -1 & 10 & -25 & 0 \end{array}$$

$$\frac{-x^3 + 75x - 250}{x + 10} = -x^2 + 10x - 25, \; x \neq -10$$

34.
$$\begin{array}{r|rrrr} 6 & 3 & -16 & 0 & -72 \\ & & 18 & 12 & 72 \\ \hline & 3 & 2 & 12 & 0 \end{array}$$

$$\frac{3x^3 - 16x^2 - 72}{x - 6} = 3x^2 + 2x + 12, \; x \neq 6$$

35.
$$\begin{array}{r|rrrr} 4 & 5 & -6 & 0 & 8 \\ & & 20 & 56 & 224 \\ \hline & 5 & 14 & 56 & 232 \end{array}$$

$$\frac{5x^3 - 6x^2 + 8}{x - 4} = 5x^2 + 14x + 56 + \frac{232}{x - 4}$$

36.
$$\begin{array}{r|rrrr} -2 & 5 & 0 & 6 & 8 \\ & & -10 & 20 & -52 \\ \hline & 5 & -10 & 26 & -44 \end{array}$$

$$\frac{5x^3 + 6x + 8}{x + 2} = 5x^2 - 10x + 26 - \frac{44}{x + 2}$$

37.
$$\begin{array}{r|rrrrr} 6 & 10 & -50 & 0 & 0 & -800 \\ & & 60 & 60 & 360 & 2160 \\ \hline & 10 & 10 & 60 & 360 & 1360 \end{array}$$

$$\frac{10x^4 - 50x^3 - 800}{x - 6} = 10x^3 + 10x^2 + 60x + 360 + \frac{1360}{x - 6}$$

38.
$$\begin{array}{r|rrrrrr} -3 & 1 & -13 & 0 & 0 & -120 & 80 \\ & & -3 & 48 & -144 & 432 & -936 \\ \hline & 1 & -16 & 48 & -144 & 312 & -856 \end{array}$$

$$\frac{x^5 - 13x^4 - 120x + 80}{x + 3} = x^4 - 16x^3 + 48x^2 - 144x + 312 - \frac{856}{x + 3}$$

39.
$$\begin{array}{r|rrrr} -8 & 1 & 0 & 0 & 512 \\ & & -8 & 64 & -512 \\ \hline & 1 & -8 & 64 & 0 \end{array}$$

$$\frac{x^3 + 512}{x + 8} = x^2 - 8x + 64,\ x \neq -8$$

40.
$$\begin{array}{r|rrrr} 9 & 1 & 0 & 0 & -729 \\ & & 9 & 81 & 729 \\ \hline & 1 & 9 & 81 & 0 \end{array}$$

$$\frac{x^3 - 729}{x - 9} = x^2 + 9x + 81,\ x \neq 9$$

41.
$$\begin{array}{r|rrrrr} 2 & -3 & 0 & 0 & 0 & 0 \\ & & -6 & -12 & -24 & -48 \\ \hline & -3 & -6 & -12 & -24 & -48 \end{array}$$

$$\frac{-3x^4}{x - 2} = -3x^3 - 6x^2 - 12x - 24 - \frac{48}{x - 2}$$

42.
$$\begin{array}{r|rrrrr} -2 & -3 & 0 & 0 & 0 & 0 \\ & & 6 & -12 & 24 & -48 \\ \hline & -3 & 6 & -12 & 24 & -48 \end{array}$$

$$\frac{-3x^4}{x + 2} = -3x^3 + 6x^2 - 12x + 24 - \frac{48}{x + 2}$$

43.
$$\begin{array}{r|rrrrr} 6 & -1 & 0 & 0 & 180 & 0 \\ & & -6 & -36 & -216 & -216 \\ \hline & -1 & -6 & -36 & -36 & -216 \end{array}$$

$$\frac{180x - x^4}{x - 6} = -x^3 - 6x^2 - 36x - 36 - \frac{216}{x - 6}$$

44.
$$\begin{array}{r|rrrr} -1 & -1 & 2 & -3 & 5 \\ & & 1 & -3 & 6 \\ \hline & -1 & 3 & -6 & 11 \end{array}$$

$$\frac{5 - 3x + 2x^2 - x^3}{x + 1} = -x^2 + 3x - 6 + \frac{11}{x + 1}$$

45.
$$\begin{array}{r|rrrr} -\frac{1}{2} & 4 & 16 & -23 & -15 \\ & & -2 & -7 & 15 \\ \hline & 4 & 14 & -30 & 0 \end{array}$$

$$\frac{4x^3 + 16x^2 - 23x - 15}{x + \frac{1}{2}} = 4x^2 + 14x - 30,\ x \neq -\frac{1}{2}$$

46.
$$\begin{array}{r|rrrr} \frac{3}{2} & 3 & -4 & 0 & 5 \\ & & \frac{9}{2} & \frac{3}{4} & \frac{9}{8} \\ \hline & 3 & \frac{1}{2} & \frac{3}{4} & \frac{49}{8} \end{array}$$

$$\frac{3x^3 - 4x^2 + 5}{x - \frac{3}{2}} = 3x^2 + \frac{1}{2}x + \frac{3}{4} + \frac{49}{8x - 12}$$

47. $f(x) = x^3 - x^2 - 14x + 11,\ k = 4$

$$\begin{array}{r|rrrr} 4 & 1 & -1 & -14 & 11 \\ & & 4 & 12 & -8 \\ \hline & 1 & 3 & -2 & 3 \end{array}$$

$$f(x) = (x - 4)(x^2 + 3x - 2) + 3$$
$$f(4) = 4^3 - 4^2 - 14(4) + 11 = 3$$

48. $f(x) = x^3 - 5x^2 - 11x + 8,\ k = -2$

$$\begin{array}{r|rrrr} -2 & 1 & -5 & -11 & 8 \\ & & -2 & 14 & -6 \\ \hline & 1 & -7 & 3 & 2 \end{array}$$

$$f(x) = (x + 2)(x^2 - 7x + 3) + 2$$
$$f(-2) = (-2)^3 - 5(-2)^2 - 11(-2) + 8 = 2$$

49. $f(x) = 15x^4 + 10x^3 - 6x^2 + 14, k = -\frac{2}{3}$

$$\begin{array}{r|rrrrr} -\frac{2}{3} & 15 & 10 & -6 & 0 & 14 \\ & & -10 & 0 & 4 & -\frac{8}{3} \\ \hline & 15 & 0 & -6 & 4 & \frac{34}{3} \end{array}$$

$$f(x) = \left(x + \frac{2}{3}\right)\left(15x^3 - 6x + 4\right) + \frac{34}{3}$$

$$f\left(-\frac{2}{3}\right) = 15\left(-\frac{2}{3}\right)^4 + 10\left(-\frac{2}{3}\right)^3 - 6\left(-\frac{2}{3}\right)^2 + 14 = \frac{34}{3}$$

50. $f(x) = 10x^3 - 22x^2 - 3x + 4, k = \frac{1}{5}$

$$\begin{array}{r|rrrr} \frac{1}{5} & 10 & -22 & -3 & 4 \\ & & 2 & -4 & -\frac{7}{5} \\ \hline & 10 & -20 & -7 & \frac{13}{5} \end{array}$$

$$f(x) = \left(x - \frac{1}{5}\right)\left(10x^2 - 20x - 7\right) + \frac{13}{5}$$

$$f\left(\frac{1}{5}\right) = 10\left(\frac{1}{5}\right)^3 - 22\left(\frac{1}{5}\right)^2 - 3\left(\frac{1}{5}\right) + 4 = \frac{13}{5}$$

51. $f(x) = x^3 + 3x^2 - 2x - 14, k = \sqrt{2}$

$$\begin{array}{r|rrrr} \sqrt{2} & 1 & 3 & -2 & -14 \\ & & \sqrt{2} & 2 + 3\sqrt{2} & 6 \\ \hline & 1 & 3 + \sqrt{2} & 3\sqrt{2} & -8 \end{array}$$

$$f(x) = \left(x - \sqrt{2}\right)\left[x^2 + \left(3 + \sqrt{2}\right)x + 3\sqrt{2}\right] - 8$$

$$f\left(\sqrt{2}\right) = \left(\sqrt{2}\right)^3 + 3\left(\sqrt{2}\right)^2 - 2\sqrt{2} - 14 = -8$$

52. $f(x) = x^3 + 2x^2 - 5x - 4, k = -\sqrt{5}$

$$\begin{array}{r|rrrr} -\sqrt{5} & 1 & 2 & -5 & -4 \\ & & -\sqrt{5} & -2\sqrt{5} + 5 & 10 \\ \hline & 1 & 2 - \sqrt{5} & -2\sqrt{5} & 6 \end{array}$$

$$f(x) = \left(x + \sqrt{5}\right)\left[x^2 + \left(2 - \sqrt{5}\right)x - 2\sqrt{5}\right] + 6$$

$$f\left(-\sqrt{5}\right) = \left(-\sqrt{5}\right)^3 + 2\left(-\sqrt{5}\right)^2 - 5\left(-\sqrt{5}\right) - 4 = 6$$

53. $f(x) = -4x^3 + 6x^2 + 12x + 4, k = 1 - \sqrt{3}$

$$\begin{array}{r|rrrr} 1 - \sqrt{3} & -4 & 6 & 12 & 4 \\ & & -4 + 4\sqrt{3} & -10 + 2\sqrt{3} & -4 \\ \hline & -4 & 2 + 4\sqrt{3} & 2 + 2\sqrt{3} & 0 \end{array}$$

$$f(x) = \left(x - 1 + \sqrt{3}\right)\left[-4x^2 + \left(2 + 4\sqrt{3}\right)x + \left(2 + 2\sqrt{3}\right)\right]$$

$$f\left(1 - \sqrt{3}\right) = -4\left(1 - \sqrt{3}\right)^3 + 6\left(1 - \sqrt{3}\right)^2 + 12\left(1 - \sqrt{3}\right) + 4 = 0$$

54. $f(x) = -3x^3 + 8x^2 + 10x - 8, k = 2 + \sqrt{2}$

$$\begin{array}{r|rrrr} 2 + \sqrt{2} & -3 & 8 & 10 & -8 \\ & & -6 - 3\sqrt{2} & -2 - 4\sqrt{2} & 8 \\ \hline & -3 & 2 - 3\sqrt{2} & 8 - 4\sqrt{2} & 0 \end{array}$$

$$f(x) = \left(x - 2 - \sqrt{2}\right)\left[-3x^2 + \left(2 - 3\sqrt{2}\right)x + 8 - 4\sqrt{2}\right]$$

$$f\left(2 + \sqrt{2}\right) = -3\left(2 + \sqrt{2}\right)^3 + 8\left(2 + \sqrt{2}\right)^2 + 10\left(2 + \sqrt{2}\right) - 8 = 0$$

55. $f(x) = 2x^3 - 7x + 3$

(a) Using the Remainder Theorem:

$$f(1) = 2(1)^3 - 7(1) + 3 = -2$$

Using synthetic division:

$$\begin{array}{r|rrrr} 1 & 2 & 0 & -7 & 3 \\ & & 2 & 2 & -5 \\ \hline & 2 & 2 & -5 & -2 \end{array}$$

Verify using long division:

$$\begin{array}{r} 2x^2 + 2x - 5 \\ x - 1 \overline{\smash{)}2x^3 + 0x^2 - 7x + 3} \\ \underline{2x^3 - 2x^2} \qquad\qquad \\ 2x^2 - 7x \qquad \\ \underline{2x^2 - 2x} \qquad \\ -5x + 3 \\ \underline{-5x + 5} \\ -2 \end{array}$$

(b) Using the Remainder Theorem:

$$f(-2) = 2(-2)^3 - 7(-2) + 3 = 1$$

Using synthetic division:

$$\begin{array}{r|rrrr} -2 & 2 & 0 & -7 & 3 \\ & & -4 & 8 & -2 \\ \hline & 2 & -4 & 1 & 1 \end{array}$$

Verify using long division:

$$\begin{array}{r} 2x^2 - 4x + 1 \\ x + 2 \overline{\smash{)}2x^3 + 0x^2 - 7x + 3} \\ \underline{2x^3 + 4x^2} \qquad\qquad \\ -4x^2 - 7x \qquad \\ \underline{-4x^2 - 8x} \qquad \\ x + 3 \\ \underline{x + 2} \\ 1 \end{array}$$

(c) Using the Remainder Theorem:

$$f\left(\frac{1}{2}\right) = 2\left(\frac{1}{2}\right)^3 - 7\left(\frac{1}{2}\right) + 3 = -\frac{1}{4}$$

Using synthetic division:

$$\begin{array}{r|rrrr} \frac{1}{2} & 2 & 0 & -7 & 3 \\ & & 1 & \frac{1}{2} & -\frac{13}{4} \\ \hline & 2 & 1 & -\frac{13}{2} & -\frac{1}{4} \end{array}$$

Verify using long division:

$$\begin{array}{r} 2x^2 + x - \frac{13}{2} \\ x - \frac{1}{2} \overline{\smash{)}2x^3 + 0x^2 - 7x + 3} \\ \underline{2x^3 - x^2} \qquad\qquad \\ x^2 - 7x \qquad \\ \underline{x^2 - \frac{1}{2}x} \qquad \\ -\frac{13}{2}x + 3 \\ \underline{-\frac{13}{2}x + \frac{13}{4}} \\ -\frac{1}{4} \end{array}$$

(d) Using the Remainder Theorem:

$$f(2) = 2(2)^3 - 7(2) + 3 = 5$$

Using synthetic division:

$$\begin{array}{r|rrrr} 2 & 2 & 0 & -7 & 3 \\ & & 4 & 8 & 2 \\ \hline & 2 & 4 & 1 & 5 \end{array}$$

Verify using long division:

$$\begin{array}{r} 2x^2 + 4x + 1 \\ x - 2 \overline{\smash{)}2x^3 + 0x^2 - 7x + 3} \\ \underline{2x^3 - 4x^2} \qquad\qquad \\ 4x^2 - 7x \qquad \\ \underline{4x^2 - 8x} \qquad \\ x + 3 \\ \underline{x - 2} \\ 5 \end{array}$$

56. $g(x) = 2x^6 + 3x^4 - x^2 + 3$

(a) Using the Remainder Theorem:

$$g(2) = 2(2)^6 + 3(2)^4 - (2)^2 + 3 = 175$$

Using synthetic division:

$$\begin{array}{r|rrrrrrr} 2 & 2 & 0 & 3 & 0 & -1 & 0 & 3 \\ & & 4 & 8 & 22 & 44 & 86 & 172 \\ \hline & 2 & 4 & 11 & 22 & 43 & 86 & 175 \end{array}$$

Verify using long division:

$$\begin{array}{l} \quad\quad\quad\;\; 2x^5 + 4x^4 + 11x^3 + 22x^2 + 43x + 86 \\ x - 2\overline{)2x^6 + 0x^5 + 3x^4 + 0x^3 - x^2 + 0x + 3} \\ \quad\;\; \underline{2x^6 - 4x^5} \\ \quad\quad\quad 4x^5 + 3x^4 \\ \quad\quad\quad \underline{4x^5 - 8x^4} \\ \quad\quad\quad\quad 11x^4 + 0x^3 \\ \quad\quad\quad\quad \underline{11x^4 - 22x^3} \\ \quad\quad\quad\quad\quad 22x^3 - x^2 \\ \quad\quad\quad\quad\quad \underline{22x^3 - 44x^2} \\ \quad\quad\quad\quad\quad\quad 43x^2 + 0x \\ \quad\quad\quad\quad\quad\quad \underline{43x^2 - 86x} \\ \quad\quad\quad\quad\quad\quad\quad 86x + 3 \\ \quad\quad\quad\quad\quad\quad\quad \underline{86x - 172} \\ \quad\quad\quad\quad\quad\quad\quad\quad\;\; 175 \end{array}$$

(b) Using the Remainder Theorem:

$$g(1) = 2(1)^6 + 3(1)^4 - (1)^2 + 3 = 7$$

Using synthetic division:

$$\begin{array}{r|rrrrrrr} 1 & 2 & 0 & 3 & 0 & -1 & 0 & 3 \\ & & 2 & 2 & 5 & 5 & 4 & 4 \\ \hline & 2 & 2 & 5 & 5 & 4 & 4 & 7 \end{array}$$

Verify using long division:

$$\begin{array}{l} \quad\quad\quad\;\; 2x^5 + 2x^4 + 5x^3 + 5x^2 + 4x + 4 \\ x - 1\overline{)2x^6 + 0x^5 + 3x^4 + 0x^3 - x^2 + 0x + 3} \\ \quad\;\; \underline{2x^6 - 2x^5} \\ \quad\quad\quad 2x^5 + 3x^4 \\ \quad\quad\quad \underline{2x^5 - 2x^4} \\ \quad\quad\quad\quad 5x^4 + 0x^3 \\ \quad\quad\quad\quad \underline{5x^4 - 5x^3} \\ \quad\quad\quad\quad\quad 5x^3 - x^2 \\ \quad\quad\quad\quad\quad \underline{5x^3 - 5x^2} \\ \quad\quad\quad\quad\quad\quad 4x^2 + 0x \\ \quad\quad\quad\quad\quad\quad \underline{4x^2 - 4x} \\ \quad\quad\quad\quad\quad\quad\quad 4x + 3 \\ \quad\quad\quad\quad\quad\quad\quad \underline{4x - 4} \\ \quad\quad\quad\quad\quad\quad\quad\quad\;\; 7 \end{array}$$

(c) Using the Remainder Theorem:

$$g(3) = 2(3)^6 + 3(3)^4 - (3)^2 + 3 = 1695$$

Using synthetic division:

$$\begin{array}{r|rrrrrrr} 3 & 2 & 0 & 3 & 0 & -1 & 0 & 3 \\ & & 6 & 18 & 63 & 189 & 564 & 1692 \\ \hline & 2 & 6 & 21 & 63 & 188 & 564 & 1695 \end{array}$$

Verify using long division:

$$\begin{array}{l} \quad\quad\quad\;\; 2x^5 + 6x^4 + 21x^3 + 63x^2 + 188x + 564 \\ x - 3\overline{)2x^6 + 0x^5 + 3x^4 + 0x^3 - x^2 + 0x + 3} \\ \quad\;\; \underline{2x^6 - 6x^5} \\ \quad\quad\quad 6x^5 + 3x^4 \\ \quad\quad\quad \underline{6x^4 - 18x^4} \\ \quad\quad\quad\quad 21x^4 + 0x^3 \\ \quad\quad\quad\quad \underline{21x^4 - 63x^3} \\ \quad\quad\quad\quad\quad 63x^3 - x^2 \\ \quad\quad\quad\quad\quad \underline{63x^3 - 189x^2} \\ \quad\quad\quad\quad\quad\quad 188x^2 + 0x \\ \quad\quad\quad\quad\quad\quad \underline{188x^2 - 564x} \\ \quad\quad\quad\quad\quad\quad\quad 564x + 3 \\ \quad\quad\quad\quad\quad\quad\quad \underline{564x - 1692} \\ \quad\quad\quad\quad\quad\quad\quad\quad\;\; 1695 \end{array}$$

(d) Using the Remainder Theorem:

$$g(-1) = 2(-1)^6 + 3(-1)^4 - (-1)^2 + 3 = 7$$

Using synthetic division:

$$\begin{array}{r|rrrrrrr} -1 & 2 & 0 & 3 & 0 & -1 & 0 & 3 \\ & & -2 & 2 & -5 & 5 & -4 & 4 \\ \hline & 2 & -2 & 5 & -5 & 4 & -4 & 7 \end{array}$$

Verify using long division:

$$\begin{array}{l} \quad\quad\quad\;\; 2x^5 - 2x^4 + 5x^3 - 5x^2 + 4x - 4 \\ x + 1\overline{)2x^6 + 0x^5 + 3x^4 + 0x^3 - x^2 + 0x + 3} \\ \quad\;\; \underline{2x^6 + 2x^5} \\ \quad\quad\quad -2x^5 + 3x^4 \\ \quad\quad\quad \underline{-2x^5 - 2x^4} \\ \quad\quad\quad\quad 5x^4 + 0x^3 \\ \quad\quad\quad\quad \underline{5x^4 + 5x^3} \\ \quad\quad\quad\quad\quad -5x^3 - x^2 \\ \quad\quad\quad\quad\quad \underline{-5x^3 - 5x^2} \\ \quad\quad\quad\quad\quad\quad 4x^2 + 0x \\ \quad\quad\quad\quad\quad\quad \underline{4x^2 + 4x} \\ \quad\quad\quad\quad\quad\quad\quad -4x + 3 \\ \quad\quad\quad\quad\quad\quad\quad \underline{-4x - 4} \\ \quad\quad\quad\quad\quad\quad\quad\quad\;\; 7 \end{array}$$

57. $h(x) = x^3 - 5x^2 - 7x + 4$

(a) Using the Remainder Theorem:

$h(3) = (3)^3 - 5(3)^2 - 7(3) + 4 = -35$

Using synthetic division:

$$\begin{array}{r|rrrr} 3 & 1 & -5 & -7 & 4 \\ & & 3 & -6 & -39 \\ \hline & 1 & -2 & -13 & -35 \end{array}$$

Verify using long division:

$$\begin{array}{r} x^2 - 2x - 13 \\ x - 3\overline{\smash{)}x^3 - 5x^2 - 7x + 4} \\ \underline{x^3 - 3x^2} \qquad\qquad \\ -2x^2 - 7x \qquad \\ \underline{-2x^2 + 6x} \qquad \\ -13x + 4 \\ \underline{-13x + 39} \\ -35 \end{array}$$

(b) Using the Remainder Theorem:

$h(2) = (2)^3 - 5(2)^2 - 7(2) + 4 = -22$

Using synthetic division:

$$\begin{array}{r|rrrr} 2 & 1 & -5 & -7 & 4 \\ & & 2 & -6 & -26 \\ \hline & 1 & -3 & -13 & -22 \end{array}$$

Verify using long division:

$$\begin{array}{r} x^2 - 3x - 13 \\ x - 2\overline{\smash{)}x^3 - 5x^2 - 7x + 4} \\ \underline{x^3 - 2x^2} \qquad\qquad \\ -3x^2 - 7x \qquad \\ \underline{-3x^2 + 6x} \qquad \\ -13x + 4 \\ \underline{-13x + 26} \\ -22 \end{array}$$

(c) Using the Remainder Theorem:

$h(-2) = (-2)^3 - 5(-2)^2 - 7(-2) + 4 = -10$

Using synthetic division:

$$\begin{array}{r|rrrr} -2 & 1 & -5 & -7 & 4 \\ & & -2 & 14 & -14 \\ \hline & 1 & -7 & 7 & -10 \end{array}$$

Verify using long division:

$$\begin{array}{r} x^2 - 7x + 7 \\ x + 2\overline{\smash{)}x^3 - 5x^2 - 7x + 4} \\ \underline{x^3 + 2x^2} \qquad\qquad \\ -7x^2 - 7x \qquad \\ \underline{-7x^2 - 14x} \qquad \\ 7x + 4 \\ \underline{7x + 14} \\ -10 \end{array}$$

(d) Using the Remainder Theorem:

$h(-5) = (-5)^3 - 5(-5)^2 - 7(-5) + 4 = -211$

Using synthetic division:

$$\begin{array}{r|rrrr} -5 & 1 & -5 & -7 & 4 \\ & & -5 & 50 & -215 \\ \hline & 1 & -10 & 43 & -211 \end{array}$$

Verify using long division:

$$\begin{array}{r} x^2 - 10x + 43 \\ x + 5\overline{\smash{)}x^3 - 5x^2 - 7x + 4} \\ \underline{x^3 + 5x^2} \qquad\qquad \\ -10x^2 - 7x \qquad \\ \underline{-10x^2 - 50x} \qquad \\ 43x + 4 \\ \underline{43x + 215} \\ -211 \end{array}$$

58. $f(x) = 4x^4 - 16x^3 + 7x^2 + 20$

(a) Using the Remainder Theorem:

$f(1) = 4(1)^4 - 16(1)^3 + 7(1) + 20 = 15$

Using synthetic division:

$$\begin{array}{r|rrrrr} 1 & 4 & -16 & 7 & 0 & 20 \\ & & 4 & -12 & -5 & -5 \\ \hline & 4 & -12 & -5 & -5 & 15 \end{array}$$

Verify using long division:

$$\begin{array}{r} 4x^3 - 12x^2 - 5x - 5 \\ x - 1\overline{\smash{)}4x^4 - 16x^3 + 7x^2 + 0x + 20} \\ \underline{4x^4 - 4x^3} \qquad\qquad\qquad\qquad \\ -12x^3 + 7x^2 \qquad\qquad \\ \underline{-12x^3 + 12x^2} \qquad\qquad \\ -5x^2 + 0x \qquad \\ \underline{-5x^2 + 5x} \qquad \\ -5x + 20 \\ \underline{-5x + 5} \\ 15 \end{array}$$

(b) Using the Remainder Theorem:

$f(-2) = 4(-2)^4 - 16(-2)^3 + 7(-2)^2 + 20 = 240$

Using synthetic division:

$$\begin{array}{r|rrrrr} -2 & 4 & -16 & 7 & 0 & 20 \\ & & -8 & 48 & -110 & 220 \\ \hline & 4 & -24 & 55 & -110 & 240 \end{array}$$

Verify using long division:

$$\begin{array}{r} 4x^3 - 24x^2 + 55x - 110 \\ x + 2\overline{\smash{)}4x^4 - 16x^3 + 7x^2 + 0x + 20} \\ \underline{4x^4 + 8x^3} \qquad\qquad\qquad\qquad \\ -24x^3 + 7x^2 \qquad\qquad \\ \underline{-24x^3 - 48x^2} \qquad\qquad \\ 55x^2 + 0x \qquad \\ \underline{55x^2 + 110x} \qquad \\ -110x + 20 \\ \underline{-110x - 220} \\ 240 \end{array}$$

(c) Using the Remainder Theorem:

$$f(5) = 4(5)^4 - 16(5)^3 + 7(5)^2 + 20 = 695$$

Using synthetic division:

$$\begin{array}{r|rrrrr} 5 & 4 & -16 & 7 & 0 & 20 \\ & & 20 & 20 & 135 & 675 \\ \hline & 4 & 4 & 27 & 135 & 695 \end{array}$$

Verify using long division:

$$\begin{array}{r}
4x^3 + 4x^2 + 27x + 135 \\
x - 5 \overline{\smash{)}\, 4x^4 - 16x^3 + 7x^2 + 0x + 20} \\
\underline{4x^4 - 20x^3} \qquad\qquad\qquad\quad \\
4x^3 + 7x^2 \qquad\qquad\quad \\
\underline{4x^3 - 20x^2} \qquad\qquad\quad \\
27x^2 + 0x \qquad\quad \\
\underline{27x^2 - 135x} \qquad\quad \\
135x + 20 \\
\underline{135x - 675} \\
695
\end{array}$$

(d) Using the Remainder Theorem:

$$f(-10) = 4(-10)^4 - 16(-10)^3 + 7(-10)^2 + 20 = 56{,}720$$

Using synthetic division:

$$\begin{array}{r|rrrrr} -10 & 4 & -16 & 7 & 0 & 20 \\ & & -40 & 560 & -5670 & 56{,}700 \\ \hline & 4 & -56 & 567 & -5670 & 56{,}720 \end{array}$$

Verify using long division:

$$\begin{array}{r}
4x^3 - 56x^2 + 567x - 5670 \\
x + 10 \overline{\smash{)}\, 4x^4 - 16x^3 + 7x^2 + 0x + 20} \\
\underline{4x^4 + 40x^3} \qquad\qquad\qquad\quad \\
-56x^3 + 7x^2 \qquad\qquad\quad \\
\underline{-56x^3 - 560x^2} \qquad\qquad\quad \\
567x^2 + 0x \qquad\quad \\
\underline{567x^2 + 5670x} \qquad\quad \\
-5670x + 20 \\
\underline{-5670x - 56{,}700} \\
56{,}720
\end{array}$$

59.

$$\begin{array}{r|rrrr} 2 & 1 & 0 & -7 & 6 \\ & & 2 & 4 & -6 \\ \hline & 1 & 2 & -3 & 0 \end{array}$$

$$\begin{aligned} x^3 - 7x + 6 &= (x - 2)(x^2 + 2x - 3) \\ &= (x - 2)(x + 3)(x - 1) \end{aligned}$$

Zeros: $2, -3, 1$

60.

$$\begin{array}{r|rrrr} -4 & 1 & 0 & -28 & -48 \\ & & -4 & 16 & 48 \\ \hline & 1 & -4 & -12 & 0 \end{array}$$

$$\begin{aligned} x^3 - 28x - 48 &= (x + 4)(x^2 - 4x - 12) \\ &= (x + 4)(x - 6)(x + 2) \end{aligned}$$

Zeros: $-4, -2, 6$

61.

$$\begin{array}{r|rrrr} \frac{1}{2} & 2 & -15 & 27 & -10 \\ & & 1 & -7 & 10 \\ \hline & 2 & -14 & 20 & 0 \end{array}$$

$$\begin{aligned} 2x^3 - 15x^2 + 27x - 10 &= \left(x - \tfrac{1}{2}\right)(2x^2 - 14x + 20) \\ &= (2x - 1)(x - 2)(x - 5) \end{aligned}$$

Zeros: $\frac{1}{2}, 2, 5$

62.

$$\begin{array}{r|rrrr} \frac{2}{3} & 48 & -80 & 41 & -6 \\ & & 32 & -32 & 6 \\ \hline & 48 & -48 & 9 & 0 \end{array}$$

$$\begin{aligned} 48x^3 - 80x^2 + 41x - 6 &= \left(x - \tfrac{2}{3}\right)(48x^2 - 48x + 9) \\ &= \left(x - \tfrac{2}{3}\right)(4x - 3)(12x - 3) \\ &= (3x - 2)(4x - 3)(4x - 1) \end{aligned}$$

Zeros: $\frac{2}{3}, \frac{3}{4}, \frac{1}{4}$

63.

$$\begin{array}{r|rrrr} \sqrt{3} & 1 & 2 & -3 & -6 \\ & & \sqrt{3} & 3 + 2\sqrt{3} & 6 \\ \hline & 1 & 2 + \sqrt{3} & 2\sqrt{3} & 0 \end{array}$$

$$\begin{array}{r|rrr} -\sqrt{3} & 1 & 2 + \sqrt{3} & 2\sqrt{3} \\ & & -\sqrt{3} & -2\sqrt{3} \\ \hline & 1 & 2 & 0 \end{array}$$

$$x^3 + 2x^2 - 3x - 6 = \left(x - \sqrt{3}\right)\left(x + \sqrt{3}\right)(x + 2)$$

Zeros: $-\sqrt{3}, \sqrt{3}, -2$

64.

$$\begin{array}{r|rrrr} \sqrt{2} & 1 & 2 & -2 & -4 \\ & & \sqrt{2} & 2\sqrt{2} + 2 & 4 \\ \hline & 1 & 2 + \sqrt{2} & 2\sqrt{2} & 0 \end{array}$$

$$\begin{array}{r|rrr} -\sqrt{2} & 1 & 2 + \sqrt{2} & 2\sqrt{2} \\ & & -\sqrt{2} & -2\sqrt{2} \\ \hline & 1 & 2 & 0 \end{array}$$

$$x^3 + 2x^2 - 2x - 4 = \left(x - \sqrt{2}\right)(x + 2)\left(x + \sqrt{2}\right)$$

Zeros: $-2, -\sqrt{2}, \sqrt{2}$

65.
$$\begin{array}{r|cccc} 1+\sqrt{3} & 1 & -3 & 0 & 2 \\ & & 1+\sqrt{3} & 1-\sqrt{3} & -2 \\ \hline & 1 & -2+\sqrt{3} & 1-\sqrt{3} & 0 \end{array}$$

$$\begin{array}{r|ccc} 1-\sqrt{3} & 1 & -2+\sqrt{3} & 1-\sqrt{3} \\ & & 1-\sqrt{3} & -1+\sqrt{3} \\ \hline & 1 & -1 & 0 \end{array}$$

$$\begin{aligned} x^3 - 3x^2 + 2 &= \left[x - \left(1+\sqrt{3}\right)\right]\left[x - \left(1-\sqrt{3}\right)\right](x-1) \\ &= (x-1)\left(x-1-\sqrt{3}\right)\left(x-1+\sqrt{3}\right) \end{aligned}$$

Zeros: $1, 1-\sqrt{3}, 1+\sqrt{3}$

66.
$$\begin{array}{r|cccc} 2-\sqrt{5} & 1 & -1 & -13 & -3 \\ & & 2-\sqrt{5} & 7-3\sqrt{5} & 3 \\ \hline & 1 & 1-\sqrt{5} & -6-3\sqrt{5} & 0 \end{array}$$

$$\begin{array}{r|ccc} 2+\sqrt{5} & 1 & 1-\sqrt{5} & -6-3\sqrt{5} \\ & & 2+\sqrt{5} & 6+3\sqrt{5} \\ \hline & 1 & 3 & 0 \end{array}$$

$$x^3 - x^2 - 13x - 3 = \left(x-2+\sqrt{5}\right)\left(x-2-\sqrt{5}\right)(x+3)$$

Zeros: $2-\sqrt{5}, 2+\sqrt{5}, -3$

67. $f(x) = 2x^3 + x^2 - 5x + 2$; Factors: $(x+2), (x-1)$

(a)
$$\begin{array}{r|cccc} -2 & 2 & 1 & -5 & 2 \\ & & -4 & 6 & -2 \\ \hline & 2 & -3 & 1 & 0 \end{array}$$

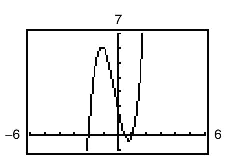
$$\begin{array}{r|ccc} 1 & 2 & -3 & 1 \\ & & 2 & -1 \\ \hline & 2 & -1 & 0 \end{array}$$

Both are factors of $f(x)$ because the remainders are zero.

(b) The remaining factor is $(2x-1)$.

(c) $f(x) = (2x-1)(x+2)(x-1)$

(d) Zeros: $\frac{1}{2}, -2, 1$

(e)


68. $f(x) = 3x^3 + 2x^2 - 19x + 6$;
Factors: $(x+3), (x-2)$

(a)
$$\begin{array}{r|cccc} -3 & 3 & 2 & -19 & 6 \\ & & -9 & 21 & -6 \\ \hline & 3 & -7 & 2 & 0 \end{array}$$

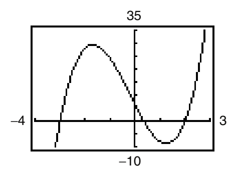
$$\begin{array}{r|ccc} 2 & 3 & -7 & 2 \\ & & 6 & -2 \\ \hline & 3 & -1 & 0 \end{array}$$

Both are factors of $f(x)$ because the remainders are zero.

(b) The remaining factor is $(3x-1)$.

(c)
$$\begin{aligned} f(x) &= 3x^3 + 2x^2 - 19x + 6 \\ &= (3x-1)(x+3)(x-2) \end{aligned}$$

(d) Zeros: $\frac{1}{3}, -3, 2$

(e)


69. $f(x) = x^4 - 4x^3 - 15x^2 + 58x - 40$;
Factors: $(x-5), (x+4)$

(a)
$$\begin{array}{r|ccccc} 5 & 1 & -4 & -15 & 58 & -40 \\ & & 5 & 5 & -50 & 40 \\ \hline & 1 & 1 & -10 & 8 & 0 \end{array}$$

$$\begin{array}{r|cccc} -4 & 1 & 1 & -10 & 8 \\ & & -4 & 12 & -8 \\ \hline & 1 & -3 & 2 & 0 \end{array}$$

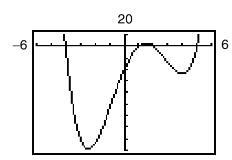
Both are factors of $f(x)$ because the remainders are zero.

(b) $x^2 - 3x + 2 = (x-1)(x-2)$

The remaining factors are $(x-1)$ and $(x-2)$

(c) $f(x) = (x-1)(x-2)(x-5)(x+4)$

(d) Zeros: $1, 2, 5, -4$

(e)


70. $f(x) = 8x^4 - 14x^3 - 71x^2 - 10x + 24;$

Factors: $(x + 2), (x - 4)$

(a)
$$\begin{array}{r|rrrrr} -2 & 8 & -14 & -71 & -10 & 24 \\ & & -16 & 60 & 22 & -24 \\ \hline & 8 & -30 & -11 & 12 & 0 \end{array}$$

$$\begin{array}{r|rrrr} 4 & 8 & -30 & -11 & 12 \\ & & 32 & 8 & -12 \\ \hline & 8 & 2 & -3 & 0 \end{array}$$

Both are factors of $f(x)$ because the remainders are zero.

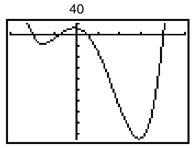
(b) $8x^2 + 2x - 3 = (4x + 3)(2x - 1)$

The remaining factors are $(4x + 3)$ and $(2x - 1)$.

(c) $f(x) = (4x + 3)(2x - 1)(x + 2)(x - 4)$

(d) Zeros: $-\frac{3}{4}, \frac{1}{2}, -2, 4$

(e)



71. $f(x) = 6x^3 + 41x^2 - 9x - 14;$

Factors: $(2x + 1), (3x - 2)$

(a)
$$\begin{array}{r|rrrr} -\frac{1}{2} & 6 & 41 & -9 & -14 \\ & & -3 & -19 & 14 \\ \hline & 6 & 38 & -28 & 0 \end{array}$$

$$\begin{array}{r|rrr} \frac{2}{3} & 6 & 38 & -28 \\ & & 4 & 28 \\ \hline & 6 & 42 & 0 \end{array}$$

Both are factors of $f(x)$ because the remainders are zero.

(b) $6x + 42 = 6(x + 7)$

This shows that $\dfrac{f(x)}{\left(x + \frac{1}{2}\right)\left(x - \frac{2}{3}\right)} = 6(x + 7),$

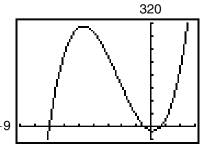
so $\dfrac{f(x)}{(2x + 1)(3x - 2)} = x + 7.$

The remaining factor is $(x + 7)$.

(c) $f(x) = (x + 7)(2x + 1)(3x - 2)$

(d) Zeros: $-7, -\dfrac{1}{2}, \dfrac{2}{3}$

(e)



72. $f(x) = 10x^3 - 11x^2 - 72x + 45;$

Factors: $(2x + 5), (5x - 3)$

(a)
$$\begin{array}{r|rrrr} -\frac{5}{2} & 10 & -11 & -72 & 45 \\ & & -25 & 90 & -45 \\ \hline & 10 & -36 & 18 & 0 \end{array}$$

$$\begin{array}{r|rrr} \frac{3}{5} & 10 & -36 & 18 \\ & & 6 & -18 \\ \hline & 10 & -30 & 0 \end{array}$$

Both are factors of $f(x)$ because the remainders are zero.

(b) $10x - 30 = 10(x - 3)$

This shows that $\dfrac{f(x)}{\left(x + \frac{5}{2}\right)\left(x - \frac{3}{5}\right)} = 10(x - 3),$

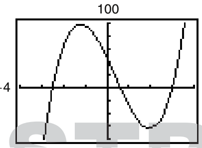
so $\dfrac{f(x)}{(2x + 5)(5x - 3)} = x - 3.$

The remaining factor is $(x - 3)$.

(c) $f(x) = (x - 3)(2x + 5)(5x - 3)$

(d) Zeros: $3, -\dfrac{5}{2}, \dfrac{3}{5}$

(e)



73. $f(x) = 2x^3 - x^2 - 10x + 5;$

Factors: $(2x - 1), \left(x + \sqrt{5}\right)$

(a)
$$\begin{array}{r|rrrr} \frac{1}{2} & 2 & -1 & -10 & 5 \\ & & 1 & 0 & -5 \\ \hline & 2 & 0 & -10 & 0 \end{array}$$

$$\begin{array}{r|rrr} -\sqrt{5} & 2 & 0 & -10 \\ & & -2\sqrt{5} & 10 \\ \hline & 2 & -2\sqrt{5} & 0 \end{array}$$

Both are factors of $f(x)$ because the remainders are zero.

(b) $2x - 2\sqrt{5} = 2\left(x - \sqrt{5}\right)$

This shows that $\dfrac{f(x)}{\left(x - \frac{1}{2}\right)\left(x + \sqrt{5}\right)} = 2\left(x - \sqrt{5}\right),$

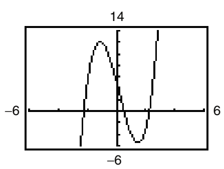
so $\dfrac{f(x)}{(2x - 1)\left(x + \sqrt{5}\right)} = x - \sqrt{5}.$

The remaining factor is $\left(x - \sqrt{5}\right)$.

(c) $f(x) = \left(x + \sqrt{5}\right)\left(x - \sqrt{5}\right)(2x - 1)$

(d) Zeros: $-\sqrt{5}, \sqrt{5}, \dfrac{1}{2}$

(e)



74. $f(x) = x^3 + 3x^2 - 48x - 144;$

Factors: $\left(x + 4\sqrt{3}\right), (x + 3)$

(a)
$$\begin{array}{r|rrrr} -3 & 1 & 3 & -48 & -144 \\ & & -3 & 0 & 144 \\ \hline & 1 & 0 & -48 & 0 \end{array}$$

$$\begin{array}{r|rrr} -4\sqrt{3} & 1 & 0 & -48 \\ & & -4\sqrt{3} & 48 \\ \hline & 1 & -4\sqrt{3} & 0 \end{array}$$

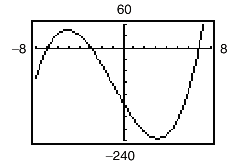
Both are factors of $f(x)$ because the remainders are zero.

(b) The remaining factor is $\left(x - 4\sqrt{3}\right)$.

(c) $f(x) = \left(x - 4\sqrt{3}\right)\left(x + 4\sqrt{3}\right)(x + 3)$

(d) Zeros: $\pm 4\sqrt{3}, -3$

(e)



75. $f(x) = x^3 - 2x^2 - 5x + 10$

(a) The zeros of f are $x = 2$ and $x \approx \pm 2.236$.

(b) An exact zero is $x = 2$.

(c)
$$\begin{array}{r|rrrr} 2 & 1 & -2 & -5 & 10 \\ & & 2 & 0 & -10 \\ \hline & 1 & 0 & -5 & 0 \end{array}$$

$$\begin{aligned} f(x) &= (x - 2)\left(x^2 - 5\right) \\ &= (x - 2)\left(x - \sqrt{5}\right)\left(x + \sqrt{5}\right) \end{aligned}$$

76. $g(x) = x^3 - 4x^2 - 2x + 8$

(a) The zeros of g are $x = 4, x \approx -1.414, x \approx 1.414$.

(b) $x = 4$ is an exact zero.

(c)
$$\begin{array}{r|rrrr} 4 & 1 & -4 & -2 & 8 \\ & & 4 & 0 & -8 \\ \hline & 1 & 0 & -2 & 0 \end{array}$$

$$\begin{aligned} f(x) &= (x - 4)\left(x^2 - 2\right) \\ &= (x - 4)\left(x - \sqrt{2}\right)\left(x + \sqrt{2}\right) \end{aligned}$$

77. $h(t) = t^3 - 2t^2 - 7t + 2$

(a) The zeros of h are $t = -2, t \approx 3.732, t \approx 0.268$.

(b) An exact zero is $t = -2$.

(c)
$$\begin{array}{r|rrrr} -2 & 1 & -2 & -7 & 2 \\ & & -2 & 8 & -2 \\ \hline & 1 & -4 & 1 & 0 \end{array}$$

$h(t) = (t + 2)\left(t^2 - 4t + 1\right)$

By the Quadratic Formula, the zeros of $t^2 - 4t + 1$ are $2 \pm \sqrt{3}$. Thus,

$h(t) = (t + 2)\left[t - \left(2 + \sqrt{3}\right)\right]\left[t - \left(2 - \sqrt{3}\right)\right].$

78. $f(s) = s^3 - 12s^2 + 40s - 24$

(a) The zeros of f are $s = 6, s \approx 0.764, s \approx 5.236$

(b) $s = 6$ is an exact zero.

(c)

6	1	−12	40	−24
		6	−36	24
	1	−6	4	0

$f(s) = (s - 6)(s^2 - 6s + 4)$

By the Quadratic Formula, the zeros of $s^2 - 6s + 4$ are $3 \pm \sqrt{5}$. Thus,

$f(s) = (s - 6)\left[s - \left(3 + \sqrt{5}\right)\right]\left[s - \left(3 - \sqrt{5}\right)\right].$

79. $h(x) = x^5 - 7x^4 + 10x^3 + 14x^2 - 24x$

(a) The zeros of h are $x = 0, x = 3, x = 4,$ $x \approx 1.414, x \approx -1.414.$

(b) An exact zero is $x = 4.$

(c)

4	1	−7	10	14	−24
		4	−12	−8	24
	1	−3	−2	6	0

$h(x) = (x - 4)(x^4 - 3x^3 - 2x^2 + 6x)$

$= x(x - 4)(x - 3)\left(x + \sqrt{2}\right)\left(x - \sqrt{2}\right)$

80. $g(x) = 6x^4 - 11x^3 - 51x^2 + 99x - 27$

(a) The zeros of a are $x = 3, x = -3, x = 1.5,$ $x \approx 0.333.$

(b) An exact zero is $x = -3.$

(c)

−3	6	−11	−51	99	−27
		−18	87	−108	27
	6	−29	36	−9	0

$a(x) = (x + 3)(6x^3 - 29x^2 + 36x - 9)$

$= (x + 3)(x - 3)(2x - 3)(3x - 1)$

84. $\dfrac{x^4 + 9x^3 - 5x^2 - 36x + 4}{x^2 - 4} = \dfrac{x^4 + 9x^3 - 5x^2 - 36x + 4}{(x + 2)(x - 2)}$

2	1	9	−5	−36	4
		2	22	34	−4
	1	11	17	−2	0

−2	1	11	17	−2
		−2	−18	2
	1	9	−1	0

$\dfrac{x^4 + 9x^3 - 5x^2 - 36x + 4}{x^2 - 4} = x^2 + 9x - 1, x \neq \pm 2$

81. $\dfrac{4x^3 - 8x^2 + x + 3}{2x - 3}$

$\frac{3}{2}$	4	−8	1	3
		6	−3	−3
	4	−2	−2	0

$\dfrac{4x^3 - 8x^2 + x + 3}{x - \frac{3}{2}} = 4x^2 - 2x - 2 = 2(2x^2 - x - 1)$

So, $\dfrac{4x^3 - 8x^2 + x + 3}{2x - 3} = 2x^2 - x - 1, x \neq \dfrac{3}{2}.$

82. $\dfrac{x^3 + x^2 - 64x - 64}{x + 8}$

−8	1	1	−64	−64
		−8	56	64
	1	−7	−8	0

$\dfrac{x^3 + x^2 - 64x - 64}{x + 8} = x^2 - 7x - 8, x \neq -8$

83. $\dfrac{x^4 + 6x^3 + 11x^2 + 6x}{x^2 + 3x + 2} = \dfrac{x^4 + 6x^3 + 11x^2 + 6x}{(x + 1)(x + 2)}$

−1	1	6	11	6	0
		−1	−5	−6	0
	1	5	6	0	0

−2	1	5	6	0
		−2	−6	0
	1	3	0	0

$\dfrac{x^4 + 6x^3 + 11x^2 + 6x}{(x + 1)(x + 2)} = x^2 + 3x, x \neq -2, -1$

85.

$$\begin{array}{r} x^{2n} + 6x^n + 9 \\ x^n + 3 \overline{\smash{)} x^{3n} + 9x^{2n} + 27x^n + 27} \\ \underline{x^{3n} + 3x^{2n}} \qquad\qquad\quad \\ 6x^{2n} + 27x^n \qquad \\ \underline{6x^{2n} + 18x^n} \qquad \\ 9x^n + 27 \\ \underline{9x^n + 27} \\ 0 \end{array}$$

$$\frac{x^{3n} + 9x^{2n} + 27x^n + 27}{x^n + 3} = x^{2n} + 6x^n + 9,\ x^n \neq -3$$

86.

$$\begin{array}{r} x^{2n} - x^n + 3 \\ x^n - 2 \overline{\smash{)} x^{3n} - 3x^{2n} + 5x^n - 6} \\ \underline{x^{3n} - 2x^{2n}} \qquad\qquad \\ -x^{2n} + 5x^n \qquad \\ \underline{-x^{2n} + 2x^n} \qquad \\ 3x^n - 6 \\ \underline{3x^n - 6} \\ 0 \end{array}$$

$$\frac{x^{3n} - 3x^{2n} + 5x^n - 6}{x^n - 2} = x^{2n} - x^n + 3,\ x^n \neq 2$$

87. A divisor divides evenly into a dividend if the remainder is zero.

88. You can check polynomial division by multiplying the quotient by the divisor. This should yield the original dividend if the multiplication was performed correctly.

89.

$$\begin{array}{r|rrrr} 5 & 1 & 4 & -3 & c \\ & & 5 & 45 & 210 \\ \hline & 1 & 9 & 42 & c + 210 \end{array}$$

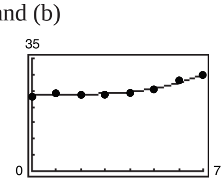
To divide evenly, $c + 210$ must equal zero. So, c must equal -210.

90.

$$\begin{array}{r|rrrrrr} -2 & 1 & 0 & 0 & -2 & 1 & c \\ & & -2 & 4 & -8 & 20 & -42 \\ \hline & 1 & -2 & 4 & -10 & 21 & c - 42 \end{array}$$

To divide evenly, $c - 42$ must equal zero. So, c must equal 42.

91. (a) and (b)



$$A \approx 0.0349t^3 - 0.168t^2 + 0.42t + 23.4$$

(c)

Year	Actual Value	Estimated Value
0	23.2	23.4
1	24.2	23.7
2	23.9	23.8
3	23.9	24.1
4	24.4	24.6
5	25.6	25.7
6	28.0	27.4
7	29.8	30.1

(d) $2010 \to t = 10$

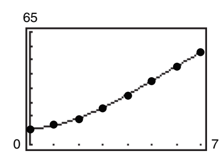
$$\begin{array}{r|rrrr} 10 & 0.0349 & -0.168 & 0.42 & 23.4 \\ & & 0.349 & 1.81 & 22.3 \\ \hline & 0.0349 & 0.181 & 2.23 & 45.7 \end{array}$$

$A(10) \approx \$45.7$

In 2010, the amount of money supporting higher education is about \$45.7 billion.

No, because the model will approach infinity quickly.

92. (a) and (b)



$$A \approx -0.0576t^3 + 0.913t^2 + 0.28t + 30.7$$

(c)

Year	Actual Value	Estimated Value
0	30.5	30.7
1	32.2	31.8
2	34.2	34.5
3	38.0	38.2
4	42.7	42.7
5	47.9	47.7
6	52.7	52.8
7	57.6	57.6

(d) $2010 \to t = 10$

$$\begin{array}{r|rrrr} 10 & -0.0576 & 0.913 & 0.28 & 30.7 \\ & & -0.576 & 3.37 & 36.5 \\ \hline & -0.0576 & 0.337 & 3.65 & 67.2 \end{array}$$

In 2010, the amount of money spent on health care is about \$67.2 billion.

93. False. If $(7x + 4)$ is a factor of f, then $-\frac{4}{7}$ is a zero of f.

94. True.

$\frac{1}{2}$	6	1	–92	45	184	4	–48
		3	2	–45	0	92	48
	6	4	–90	0	184	96	0

$f(x) = (2x - 1)(x + 1)(x - 2)(x - 3)(3x + 2)(x + 4)$

95. True. The degree of the numerator is greater than the degree of the denominator.

96. True. If $x = k$ is a zero of $f(x)$, then $(x - k)$ is a factor of $f(x)$, and $f(k) = 0$.

97. False.

To divide $x^4 - 3x^2 + 4x - 1$ by $x + 2$ using synthetic division, the set up would be:

–2	1	0	–3	4	–1

A zero must be included for the missing x^3 term.

98. $f(x) = (x - k)q(x) + r$

(a) $k = 2, r = 5, q(x) =$ any quadratic $ax^2 + bx + c$ where $a > 0$. One example:

$f(x) = (x - 2)x^2 + 5 = x^3 - 2x^2 + 5$

(b) $k = -3, r = 1, q(x) =$ any quadratic $ax^2 + bx + c$ where $a < 0$. One example:

$f(x) = (x + 3)(-x^2) + 1 = -x^3 - 3x^2 + 1$

99. If $x - 4$ is a factor of $f(x) = x^3 - kx^2 + 2kx - 8$, then $f(4) = 0$.

$$\begin{aligned} f(4) &= (4)^3 - k(4)^2 + 2k(4) - 8 \\ 0 &= 64 - 16k + 8k - 8 \\ -56 &= -8k \\ 7 &= k \end{aligned}$$

100. If $x - 3$ is a factor of $f(x) = x^3 - kx^2 + 2kx - 12$ then $f(3) = 0$.

$$\begin{aligned} f(3) &= (3)^3 - k(3)^2 + 2k(3) - 12 \\ 0 &= 27 - 9k + 6k - 12 \\ -15 &= -3k \\ 5 &= k \end{aligned}$$

101. (a)
$$\begin{array}{r} x + 1 \\ x - 1 \overline{)\, x^2 + 0x - 1} \\ \underline{x^2 - x \quad} \\ x - 1 \\ \underline{x - 1} \\ 0 \end{array}$$

$\dfrac{x^2 - 1}{x - 1} = x + 1, x \neq 1$

(b)
$$\begin{array}{r} x^2 + x + 1 \\ x - 1 \overline{)\, x^3 + 0x^2 + 0x - 1} \\ \underline{x^3 - x^2 \qquad\qquad} \\ x^2 + 0x \quad \\ \underline{x^2 - x \quad} \\ x - 1 \\ \underline{x - 1} \\ 0 \end{array}$$

$\dfrac{x^3 - 1}{x - 1} = x^2 + x + 1, x \neq 1$

(c)
$$\begin{array}{r} x^3 + x^2 + x + 1 \\ x - 1 \overline{)\, x^4 + 0x^3 + 0x^2 + 0x - 1} \\ \underline{x^4 - x^3 \qquad\qquad\qquad} \\ x^3 + 0x^2 \qquad\quad \\ \underline{x^3 - x^2 \qquad\quad} \\ x^2 + 0x \quad \\ \underline{x^2 - x \quad} \\ x - 1 \\ \underline{x - 1} \\ 0 \end{array}$$

$\dfrac{x^4 - 1}{x - 1} = x^3 + x^2 + x + 1, x \neq 1$

$\dfrac{x^n - 1}{x - 1} = x^{n-1} + x^{n-2} + \cdots + x + 1, x \neq 1$

102. (a) $f(-3) = 0$ because $x + 3$ is a factor of f.

(b) Because $f(x)$ is in factored form, it is easier to evaluate directly.

Section 2.4 Complex Numbers

1. (a) iii
(b) i
(c) ii

2. $\sqrt{-1}$; – 1

3. principal square

4. complex conjugates

5. $a + bi = -12 + 7i$
$a = -12$
$b = 7$

6. $a + bi = 13 + 4i$
$a = 13$
$b = 4$

7. $(a - 1) + (b + 3)i = 5 + 8i$
$a - 1 = 5 \Rightarrow a = 6$
$b + 3 = 8 \Rightarrow b = 5$

8. $(a + 6) + 2bi = 6 - 5i$
$a + 6 = 6 \Rightarrow a = 0$
$2b = -5 \Rightarrow b = -\frac{5}{2}$

9. $8 + \sqrt{-25} = 8 + 5i$

10. $2 - \sqrt{-27} = 2 - \sqrt{27}i$
$= 2 - 3\sqrt{3}i$

11. $\sqrt{-80} = 4\sqrt{5}i$

12. $\sqrt{-4} = 2i$

13. $\sqrt{-0.09} = \sqrt{0.09}i$
$= 0.3i$

14. $14 = 14 + 0i = 14$

15. $-10i + i^2 = -10i - 1 = -1 - 10i$

16. $-4i^2 + 2i = -4(-1) + 2i$
$= 4 + 2i$

17. $(7 + i) + (3 - 4i) = 10 - 3i$

18. $(13 - 2i) + (-5 + 6i) = 8 + 4i$

19. $(9 - i) - (8 - i) = 1$

20. $(3 + 2i) - (6 + 13i) = 3 + 2i - 6 - 13i$
$= -3 - 11i$

21. $(-2 + \sqrt{-8}) + (5 - \sqrt{-50}) = -2 + 2\sqrt{2}i + 5 - 5\sqrt{2}i$
$= 3 - 3\sqrt{2}i$

22. $(8 + \sqrt{-18}) - (4 + 3\sqrt{2}i) = 8 + 3\sqrt{2}i - 4 - 3\sqrt{2}i$
$= 4$

23. $13i - (14 - 7i) = 13i - 14 + 7i$
$= -14 + 20i$

24. $-\left(\frac{3}{2} + \frac{5}{2}i\right) + \left(\frac{5}{3} + \frac{11}{3}i\right) = -\frac{3}{2} - \frac{5}{2}i + \frac{5}{3} + \frac{11}{3}i$
$= -\frac{9}{6} - \frac{15}{6}i + \frac{10}{6} + \frac{22}{6}i$
$= \frac{1}{6} + \frac{7}{6}i$

25. $\sqrt{-5} \cdot \sqrt{-10} = (\sqrt{5}i)(\sqrt{10}i)$
$= \sqrt{50}i^2 = 5\sqrt{2}(-1) = -5\sqrt{2}$

26. $(\sqrt{-75})^2 = (\sqrt{75}i)^2 = 75i^2 = -75$

27. $(1 + i)(3 - 2i) = 3 - 2i + 3i - 2i^2$
$= 3 + i + 2 = 5 + i$

28. $(7 - 2i)(3 - 5i) = 21 - 35i - 6i + 10i^2$
$= 21 - 41i - 10$
$= 11 - 41i$

29. $12i(1 - 9i) = 12i - 108i^2$
$= 12i + 108$
$= 108 + 12i$

30. $-8i(9 + 4i) = -72i - 32i^2$
$= 32 - 72i$

31. $(\sqrt{14} + \sqrt{10}i)(\sqrt{14} - \sqrt{10}i) = 14 - 10i^2$
$= 14 + 10 = 24$

32. $(3 + \sqrt{-5})(7 - \sqrt{-10}) = (3 + \sqrt{5}i)(7 - \sqrt{10}i)$
$= 21 - 3\sqrt{10}i + 7\sqrt{5}i - \sqrt{50}i^2$
$= (21 + \sqrt{50}) + (7\sqrt{5} - 3\sqrt{10})i$
$= (21 + 5\sqrt{2}) + (7\sqrt{5} - 3\sqrt{10})i$

33. $(6 + 7i)^2 = 36 + 84i + 49i^2$
$= 36 + 84i - 49$
$= -13 + 84i$

34. $(5 - 4i)^2 = 25 - 40i + 16i^2$
$= 25 - 40i - 16$
$= 9 - 40i$

35. The complex conjugate of $9 + 2i$ is $9 - 2i$.

$$(9 + 2i)(9 - 2i) = 81 - 4i^2 = 81 + 4 = 85$$

36. The complex conjugate of $-1 - \sqrt{5}i$ is $-1 + \sqrt{5}i$.

$$\left(-1 - \sqrt{5}i\right)\left(-1 + \sqrt{5}i\right) = 1 - 5i^2 = 1 + 5 = 6$$

37. The complex conjugate of $\sqrt{-20} = 2\sqrt{5}i$ is $-2\sqrt{5}i$.

$$\left(2\sqrt{5}i\right)\left(-2\sqrt{5}i\right) = -20i^2 = 20$$

38. The complex conjugate of $\sqrt{6}$ is $\sqrt{6}$.

$$\left(\sqrt{6}\right)\left(\sqrt{6}\right) = 6$$

39. $\frac{3}{i} \cdot \frac{-i}{-i} = \frac{-3i}{-i^2} = -3i$

40. $-\frac{14}{2i} \cdot \frac{-2i}{-2i} = \frac{28i}{-4i^2} = \frac{28i}{4} = 7i$

41. $\frac{13}{1 - i} \cdot \frac{(1 + i)}{(1 + i)} = \frac{13 + 13i}{1 - i^2} = \frac{13 + 13i}{2} = \frac{13}{2} + \frac{13}{2}i$

42. $\frac{6 - 7i}{1 - 2i} \cdot \frac{1 + 2i}{1 + 2i} = \frac{6 + 12i - 7i - 14i^2}{1 - 4i^2} = \frac{20 + 5i}{5} = 4 + i$

43. $\frac{8 + 16i}{2i} \cdot \frac{-2i}{-2i} = \frac{-16i - 32i^2}{-4i^2} = 8 - 4i$

44. $$\frac{3i}{(4 - 5i)^2} = \frac{3i}{16 - 40i + 25i^2} = \frac{3i}{-9 - 40i} \cdot \frac{-9 + 40i}{-9 + 40i} = \frac{-27i + 120i^2}{81 + 1600} = \frac{-120 - 27i}{1681} = -\frac{120}{1681} - \frac{27}{1681}i$$

45. $$\frac{2}{1 + i} - \frac{3}{1 - i} = \frac{2(1 - i) - 3(1 + i)}{(1 + i)(1 - i)} = \frac{2 - 2i - 3 - 3i}{1 + 1} = \frac{-1 - 5i}{2} = -\frac{1}{2} - \frac{5}{2}i$$

46. $$\frac{2i}{2 + i} + \frac{5}{2 - i} = \frac{2i(2 - i) + 5(2 + i)}{(2 + i)(2 - i)} = \frac{4i - 2i^2 + 10 + 5i}{4 - i^2} = \frac{12 + 9i}{5} = \frac{12}{5} + \frac{9}{5}i$$

47. $$\frac{i}{3 - 2i} + \frac{2i}{3 + 8i} = \frac{i(3 + 8i) + 2i(3 - 2i)}{(3 - 2i)(3 + 8i)} = \frac{3i + 8i^2 + 6i - 4i^2}{9 + 24i - 6i - 16i^2} = \frac{4i^2 + 9i}{9 + 18i + 16} = \frac{-4 + 9i}{25 + 18i} \cdot \frac{25 - 18i}{25 - 18i} = \frac{-100 + 72i + 225i - 162i^2}{625 + 324} = \frac{62 + 297i}{949} = \frac{62}{949} + \frac{297}{949}i$$

48. $$\frac{1 + i}{i} - \frac{3}{4 - i} = \frac{(1 + i)(4 - i) - 3i}{i(4 - i)} = \frac{4 - i + 4i - i^2 - 3i}{4i - i^2} = \frac{5}{1 + 4i} \cdot \frac{1 - 4i}{1 - 4i} = \frac{5 - 20i}{1 - 16i^2} = \frac{5}{17} - \frac{20}{17}i$$

49. $x^2 - 2x + 2 = 0;\ a = 1, b = -2, c = 2$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(2)}}{2(1)} = \frac{2 \pm \sqrt{-4}}{2} = \frac{2 \pm 2i}{2} = 1 \pm i$$

50. $4x^2 + 16x + 17 = 0;\ a = 4, b = 16, c = 17$

$$x = \frac{-16 \pm \sqrt{(16)^2 - 4(4)(17)}}{2(4)} = \frac{-16 \pm \sqrt{-16}}{8} = \frac{-16 \pm 4i}{8} = -2 \pm \frac{1}{2}i$$

51. $9x^2 - 6x + 37 = 0;\ a = 9, b = -6, c = 37$

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(9)(37)}}{2(9)} = \frac{6 \pm \sqrt{-1296}}{18} = \frac{6 \pm 36i}{18} = \frac{1}{3} \pm 2i$$

52. $16t^2 - 4t + 3 = 0;\ a = 16, b = -4, c = 3$

$$t = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(16)(3)}}{2(16)} = \frac{4 \pm \sqrt{-176}}{32} = \frac{4 \pm 4\sqrt{11}i}{32} = \frac{1}{8} \pm \frac{\sqrt{11}}{8}i$$

53. $1.4x^2 - 2x - 10 = 0$ Multiply both sides by 5.

$7x^2 - 10x - 50 = 0$

$$x = \frac{-(-10) \pm \sqrt{(-10)^2 - 4(7)(-50)}}{2(7)} = \frac{10 \pm \sqrt{1500}}{14} = \frac{10 \pm 10\sqrt{15}}{14} = \frac{5}{7} \pm \frac{5\sqrt{15}}{7}$$

54. $\frac{3}{2}x^2 - 6x + 9 = 0$ Multiply both sides by 2.

$3x^2 - 12x + 18 = 0$

$$x = \frac{-(-12) \pm \sqrt{(-12)^2 - 4(3)(18)}}{2(3)} = \frac{12 \pm \sqrt{-72}}{6} = \frac{12 \pm 6\sqrt{2}i}{6} = 2 \pm \sqrt{2}i$$

55. $-6i^3 + i^2 = -6i^2i + i^2 = -6(-1)i + (-1) = 6i - 1 = -1 + 6i$

56. $4i^2 - 2i^3 = 4i^2 - 2i^2i = 4(-1) - 2(-1)i = -4 + 2i$

57. $(-i)^3 = (-1)(i^3) = (-1)i^2i = (-1)(-1)i = i$

58. $\left(\sqrt{-2}\right)^6 = \left(\sqrt{2}i\right)^6 = 8i^6 = 8i^2i^2i^2 = 8(-1)(-1)(-1) = -8$

59. $\frac{1}{i^3} = \frac{1}{i^2i} = \frac{1}{-i} = \frac{1}{-i} \cdot \frac{i}{i} = \frac{i}{-i^2} = i$

60. $\frac{1}{(2i)^3} = \frac{1}{8i^3} = \frac{1}{8i^2i} = \frac{1}{-8i} = \frac{1}{-8i} \cdot \frac{8i}{8i} = \frac{8i}{-64i^2} = \frac{1}{8}i$

61. $(a + bi)(a - bi) = a^2 + abi - abi - b^2i^2 = a^2 - b^2(-1) = a^2 + b^2$

which is a real number since a and b are real numbers. Thus, the product of a complex number and its conjugate is a real number.

62. $\sqrt{-6}\sqrt{-6} = \sqrt{6}i\sqrt{6}i = 6i^2 = -6$

63. $(a_1 + b_1i) + (a_2 + b_2i) = (a_1 + a_2) + (b_1 + b_2)i$

The complex conjugate of this sum is $(a_1 + a_2) - (b_1 + b_2)i$.

The sum of the complex conjugates is $(a_1 - b_1i) + (a_2 - b_2i) = (a_1 + a_2) - (b_1 + b_2)i$.

So, the complex conjugate of the sum of two complex numbers is the sum of their complex conjugates.

64. (a) $2^4 = 16$

(b) $(-2)^4 = 16$

(c) $(2i)^4 = 2^4 i^4 = 16i^2 i^2 = 16(-1)(-1) = 16$

(d) $(-2i)^4 = (-2)^4 i^4 = 16i^2 i^2 = 16(-1)(-1) = 16$

65. (a) $i^{40} = (i^2)^{20} = (-1)^{20} = 1$

(b) $i^{25} = (i^2)^{12} \cdot i = (-1)^{12} i = i$

(c) $i^{50} = (i^2)^{25} = (-1)^{25} = -1$

(d) $i^{67} = (i^2)^{33} i = (-1)^{33} i = -i$

66. $f(x) = 2(x-3)^2 - 4,\ g(x) = -2(x-3)^2 - 4$

(a) The graph of f is a parabola with vertex at the point $(3, -4)$.
The a value is positive, so the graph opens upward.
The graph of g is also a parabola with vertex at the point $(3, -4)$.
The a value is negative, so the graph opens downward.
f has an x-intercept and g does not because when $g(x) = 0$, x is a complex number.

(b)
$$\begin{aligned} f(x) &= 2(x-3)^2 - 4 \\ 0 &= 2(x-3)^2 - 4 \\ 4 &= 2(x-3)^2 \\ 2 &= (x-3)^2 \\ \pm\sqrt{2} &= x - 3 \\ 3 \pm \sqrt{2} &= x \end{aligned}$$

$$\begin{aligned} g(x) &= -2(x-3)^2 - 4 \\ 0 &= -2(x-3)^2 - 4 \\ 4 &= -2(x-3)^2 \\ -2 &= (x-3)^2 \\ \pm\sqrt{-2} &= x - 3 \\ 3 \pm \sqrt{2}i &= x \end{aligned}$$

(c) If all the zeros contain i, then the graph has no x-intercepts.

(d) If a and k have the same sign (both positive or both negative), then the graph of f has no x-intercepts and the zeros are complex. Otherwise, the graph of f has x-intercepts and the zeros are real.

67. False, if $b = 0$ then $a + bi = a - bi = a$.

That is, if the complex number is real, the number equals its conjugate.

68. True.

$$\begin{aligned} x^4 - x^2 + 14 &= 56 \\ \left(-i\sqrt{6}\right)^4 - \left(-i\sqrt{6}\right)^2 + 14 &\stackrel{?}{=} 56 \\ 36 + 6 + 14 &\stackrel{?}{=} 56 \\ 56 &= 56 \end{aligned}$$

69. False.

$$\begin{aligned} i^{44} + i^{150} - i^{74} - i^{109} + i^{61} &= (i^2)^{22} + (i^2)^{75} - (i^2)^{37} - (i^2)^{54} i + (i^2)^{30} i \\ &= (-1)^{22} + (-1)^{75} - (-1)^{37} - (-1)^{54} i + (-1)^{30} i \\ &= 1 - 1 + 1 - i + i = 1 \end{aligned}$$

70. (a) $z_1 = 9 + 16i,\ z_2 = 20 - 10i$

(b) $\dfrac{1}{z} = \dfrac{1}{z_1} + \dfrac{1}{z_2} = \dfrac{1}{9 + 16i} + \dfrac{1}{20 - 10i} = \dfrac{20 - 10i + 9 + 16i}{(9 + 16i)(20 - 10i)} = \dfrac{29 + 6i}{340 + 230i}$

$z = \left(\dfrac{340 + 230i}{29 + 6i}\right)\left(\dfrac{29 - 6i}{29 - 6i}\right) = \dfrac{11{,}240 + 4630i}{877} = \dfrac{11{,}240}{877} + \dfrac{4630}{877}i$

Section 2.5 The Fundamental Theorem of Algebra

1. Fundamental Theorem of Algebra

2. Linear Factorization Theorem

3. Rational Zero

4. conjugate

5. linear; quadratic; quadratic

6. irreducible; reals

7. $f(x) = x(x - 6)^2$

The zeros are: $x = 0, x = 6$

8. $f(x) = x^2(x + 3)(x^2 - 1) = x^2(x + 3)(x + 1)(x - 1)$

The zeros are: $x = 0, x = -3, x = 1, x = -1$

9. $g(x) = (x - 2)(x + 4)^3$

The zeros are: $x = 2, x = -4$

10. $f(x) = (x + 5)(x - 8)^2$

The zeros are: $x = -5, x = 8$

11. $f(x) = (x + 6)(x + i)(x - i)$

The zeros are: $x = -6, x = -i, x = i$

12. $h(t) = (t - 3)(t - 2)(t - 3i)(t + 3i)$

The zeros are: $x = 3, x = 2, x = 3i, x = -3i$

13. $f(x) = x^3 + 2x^2 - x - 2$

Possible rational zeros: $\pm1, \pm2$

Zeros shown on graph: $-2, -1, 1$

14. $f(x) = x^3 - 4x^2 - 4x + 16$

Possible rational zeros: $\pm1, \pm2, \pm4, \pm8, \pm16$

Zeros shown on graph: $-2, 2, 4$

15. $f(x) = 2x^4 - 17x^3 + 35x^2 + 9x - 45$

Possible rational zeros: $\pm1, \pm3, \pm5, \pm9, \pm15, \pm45, \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{5}{2}, \pm\frac{9}{2}, \pm\frac{15}{2}, \pm\frac{45}{2}$

Zeros shown on graph: $-1, \frac{3}{2}, 3, 5$

16. $f(x) = 4x^5 - 8x^4 - 5x^3 + 10x^2 + x - 2$

Possible rational zeros: $\pm1, \pm2, \pm\frac{1}{2}, \pm\frac{1}{4}$

Zeros shown on graph: $-1, -\frac{1}{2}, \frac{1}{2}, 1, 2$

17. $f(x) = x^3 - 6x^2 + 11x - 6$

Possible rational zeros: $\pm1, \pm2, \pm3, \pm6$

$$\begin{array}{r|rrrr} 1 & 1 & -6 & 11 & -6 \\ & & 1 & -5 & 6 \\ \hline & 1 & -5 & 6 & 0 \end{array}$$

$$\begin{aligned} x^3 - 6x^2 + 11x - 6 &= (x - 1)(x^2 - 5x + 6) \\ &= (x - 1)(x - 2)(x - 3) \end{aligned}$$

So, the rational zeros are 1, 2, and 3.

18. $f(x) = x^3 - 7x - 6$

Possible rational zeros: $\pm1, \pm2, \pm3, \pm6$

$$\begin{array}{r|rrrr} 3 & 1 & 0 & -7 & -6 \\ & & 3 & 9 & 6 \\ \hline & 1 & 3 & 2 & 0 \end{array}$$

$$\begin{aligned} f(x) &= (x - 3)(x^2 + 3x + 2) \\ &= (x - 3)(x + 2)(x + 1) \end{aligned}$$

So, the rational zeros are $-2, -1$, and 3.

19. $$\begin{aligned} g(x) &= x^3 - 4x^2 - x + 4 \\ &= x^2(x - 4) - 1(x - 4) \\ &= (x - 4)(x^2 - 1) \\ &= (x - 4)(x - 1)(x + 1) \end{aligned}$$

So, the rational zeros are 4, 1, and -1.

20. $h(x) = x^3 - 9x^2 + 20x - 12$

Possible rational zeros: $\pm1, \pm2, \pm3, \pm4, \pm6, \pm12$

$$\begin{array}{r|rrrr} 1 & 1 & -9 & 20 & -12 \\ & & 1 & -8 & 12 \\ \hline & 1 & -8 & 12 & 0 \end{array}$$

$$\begin{aligned} h(x) &= (x - 1)(x^2 - 8x + 12) \\ &= (x - 1)(x - 2)(x - 6) \end{aligned}$$

So, the rational zeros are 1, 2, and 6.

21. $h(t) = t^3 + 8t^2 + 13t + 6$

Possible rational zeros: $\pm1, \pm2, \pm3, \pm6$

$$\begin{array}{r|rrrr} -6 & 1 & 8 & 13 & 6 \\ & & -6 & -12 & -6 \\ \hline & 1 & 2 & 1 & 0 \end{array}$$

$$\begin{aligned} t^3 + 8t^2 + 13t + 6 &= (t + 6)(t^2 + 2t + 1) \\ &= (t + 6)(t + 1)(t + 1) \end{aligned}$$

So, the rational zeros are -1 and -6.

22. $p(x) = x^3 - 9x^2 + 27x - 27$

Possible rational zeros: $\pm1, \pm3, \pm9, \pm27$

$$\begin{array}{r|rrrr} 3 & 1 & -9 & 27 & -27 \\ & & 3 & -18 & 27 \\ \hline & 1 & -6 & 9 & 0 \end{array}$$

$$\begin{aligned} f(x) &= (x - 3)(x^2 - 6x + 9) \\ &= (x - 3)(x - 3)(x - 3) \end{aligned}$$

So, the rational zero is 3.

23. $C(x) = 2x^3 + 3x^2 - 1$

Possible rational zeros: $\pm1, \pm\frac{1}{2}$

$$\begin{array}{r|rrrr} -1 & 2 & 3 & 0 & -1 \\ & & -2 & -1 & 1 \\ \hline & 2 & 1 & -1 & 0 \end{array}$$

$$\begin{aligned} 2x^3 + 3x^2 - 1 &= (x + 1)(2x^2 + x - 1) \\ &= (x + 1)(x + 1)(2x - 1) \\ &= (x + 1)^2(2x - 1) \end{aligned}$$

So, the rational zeros are -1 and $\frac{1}{2}$.

24. $f(x) = 3x^3 - 19x^2 + 33x - 9$

Possible rational zeros: $\pm1, \pm3, \pm9, \pm\frac{1}{3}$

$$\begin{array}{r|rrrr} 3 & 3 & -19 & 33 & -9 \\ & & 9 & -30 & 9 \\ \hline & 3 & -10 & 3 & 0 \end{array}$$

$$\begin{aligned} f(x) &= (x - 3)(3x^2 - 10x + 3) \\ &= (x - 3)(3x - 1)(x - 3) \end{aligned}$$

So, the rational zeros are 3 and $\frac{1}{3}$.

25. $f(x) = 9x^4 - 9x^3 - 58x^2 + 4x + 24$

Possible rational zeros:
$\pm1, \pm2, \pm3, \pm4, \pm6, \pm8, \pm12, \pm24,$
$\pm\frac{1}{3}, \pm\frac{2}{3}, \pm\frac{4}{3}, \pm\frac{8}{3}, \pm\frac{1}{9}, \pm\frac{2}{9}, \pm\frac{4}{9}, \pm\frac{8}{9}$

$$\begin{array}{r|rrrrr} -2 & 9 & -9 & -58 & 4 & 24 \\ & & -18 & 54 & 8 & -24 \\ \hline & 9 & -27 & -4 & 12 & 0 \end{array}$$

$$\begin{array}{r|rrrr} 3 & 9 & -27 & -4 & 12 \\ & & 27 & 0 & -12 \\ \hline & 9 & 0 & -4 & 0 \end{array}$$

$$\begin{aligned} f(x) &= (x + 2)(x - 3)(9x^2 - 4) \\ &= (x + 2)(x - 3)(3x - 2)(3x + 2) \end{aligned}$$

So, the rational zeros are $-2, 3, \frac{2}{3},$ and $-\frac{2}{3}$.

26. $f(x) = 2x^4 - 15x^3 + 23x^2 + 15x - 25$

Possible rational zeros: $\pm1, \pm5, \pm25, \pm\frac{1}{2}, \pm\frac{5}{2}, \pm\frac{25}{2}$

$$\begin{array}{r|rrrrr} 5 & 2 & -15 & 23 & 15 & -25 \\ & & 10 & -25 & -10 & 25 \\ \hline & 2 & -5 & -2 & 5 & 0 \end{array}$$

$$\begin{array}{r|rrrr} 1 & 2 & -5 & -2 & 5 \\ & & 2 & -3 & -5 \\ \hline & 2 & -3 & -5 & 0 \end{array}$$

$$\begin{array}{r|rrr} -1 & 2 & -3 & -5 \\ & & -2 & 5 \\ \hline & 2 & -5 & 0 \end{array}$$

$$f(x) = (x - 5)(x - 1)(x + 1)(2x - 5)$$

So, the rational zeros are $5, 1, -1$ and $\frac{5}{2}$.

27. $z^4 + z^3 + z^2 + 3z - 6 = 0$

Possible rational zeros: $\pm1, \pm2, \pm3, \pm6$

$$\begin{array}{r|rrrrr} 1 & 1 & 1 & 1 & 3 & -6 \\ & & 1 & 2 & 3 & 6 \\ \hline & 1 & 2 & 3 & 6 & 0 \end{array}$$

$$\begin{aligned} (z - 1)(z^3 + 2z^2 + 3z + 6) &= 0 \\ (z - 1)(z^2 + 3)(z + 2) &= 0 \end{aligned}$$

So, the real zeros are -2 and 1.

28. $x^4 - 13x^2 - 12x = 0$

$x\left(x^3 - 13x - 12\right) = 0$

Possible rational zeros of $x^3 - 13x - 12$:

$\pm1, \pm2, \pm3, \pm4, \pm6, \pm12$

$$\begin{array}{r|rrrr} -1 & 1 & 0 & -13 & -12 \\ & & -1 & 1 & 12 \\ \hline & 1 & -1 & -12 & 0 \end{array}$$

$x(x + 1)\left(x^2 - x - 12\right) = 0$

$x(x + 1)(x - 4)(x + 3) = 0$

The real zeros are $0, -1, 4,$ and -3.

29. $2y^4 + 3y^3 - 16y^2 + 15y - 4 = 0$

Possible rational zeros: $\pm\frac{1}{2}, \pm1, \pm2, \pm4$

$$\begin{array}{r|rrrrr} 1 & 2 & 3 & -16 & 15 & -4 \\ & & 2 & 5 & -11 & 4 \\ \hline & 2 & 5 & -11 & 4 & 0 \end{array}$$

$$\begin{array}{r|rrrr} 1 & 2 & 5 & -11 & 4 \\ & & 2 & 7 & -4 \\ \hline & 2 & 7 & -4 & 0 \end{array}$$

$(y - 1)(y - 1)\left(2y^2 + 7y - 4\right) = 0$

$(y - 1)(y - 1)(2y - 1)(y + 4) = 0$

So, the real zeros are $-4, \frac{1}{2}$ and 1.

30. $x^5 - x^4 - 3x^3 + 5x^2 - 2x = 0$

$x\left(x^4 - x^3 - 3x^2 + 5x - 2\right) = 0$

Possible rational zeros of $x^4 - x^3 - 3x^2 + 5x - 2$:

$\pm1, \pm2$

$$\begin{array}{r|rrrrr} 1 & 1 & -1 & -3 & 5 & -2 \\ & & 1 & 0 & -3 & 2 \\ \hline & 1 & 0 & -3 & 2 & 0 \end{array}$$

$$\begin{array}{r|rrrr} -2 & 1 & 0 & -3 & 2 \\ & & -2 & 4 & -2 \\ \hline & 1 & -2 & 1 & 0 \end{array}$$

$x(x - 1)(x + 2)\left(x^2 - 2x + 1\right) = 0$

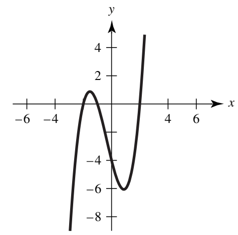
$x(x - 1)(x + 2)(x - 1)(x - 1) = 0$

The real zeros are $-2, 0,$ and 1.

31. $f(x) = x^3 + x^2 - 4x - 4$

(a) Possible rational zeros: $\pm1, \pm2, \pm4$

(b)

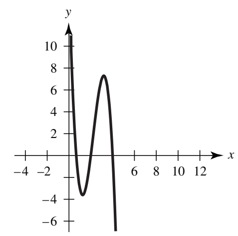


(c) Real zeros: $-2, -1, 2$

32. $f(x) = -3x^3 + 20x^2 - 36x + 16$

(a) Possible rational zeros: $\pm1, \pm2, \pm4, \pm8, \pm16, \pm\frac{1}{3},$
$\pm\frac{2}{3}, \pm\frac{4}{3}, \pm\frac{8}{3}, \pm\frac{16}{3}$

(b)

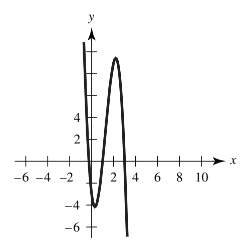


(c) Real zeros: $\frac{2}{3}, 2, 4$

33. $f(x) = -4x^3 + 15x^2 - 8x - 3$

(a) Possible rational zeros: $\pm1, \pm3, \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{1}{4}, \pm\frac{3}{4}$

(b)

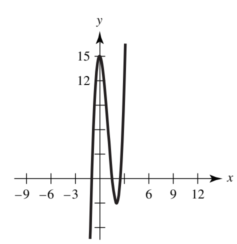


(c) Real zeros: $-\frac{1}{4}, 1, 3$

34. $f(x) = 4x^3 - 12x^2 - x + 15$

(a) Possible rational zeros: $\pm1, \pm3, \pm5, \pm15, \pm\frac{1}{2}, \pm\frac{3}{2},$
$\pm\frac{5}{2}, \pm\frac{15}{2}, \pm\frac{1}{4}, \pm\frac{3}{4}, \pm\frac{5}{4}, \pm\frac{15}{4}$

(b)

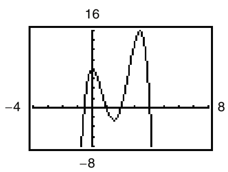


(c) Real zeros: $-1, \frac{3}{2}, \frac{5}{2}$

35. $f(x) = -2x^4 + 13x^3 - 21x^2 + 2x + 8$

(a) Possible rational zeros: $\pm1, \pm2, \pm4, \pm8, \pm\frac{1}{2}$

(b)

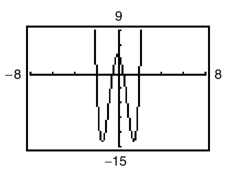


(c) Real zeros: $-\frac{1}{2}, 1, 2, 4$

36. $f(x) = 4x^4 - 17x^2 + 4$

(a) Possible rational zeros: $\pm1, \pm2, \pm4, \pm\frac{1}{2}, \pm\frac{1}{4}$

(b)



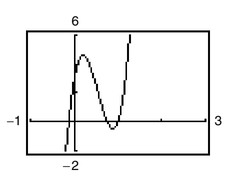
(c) Real zeros: $-2, -\frac{1}{2}, \frac{1}{2}, 2$

37. $f(x) = 32x^3 - 52x^2 + 17x + 3$

(a) Possible rational zeros: $\pm1, \pm3, \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{1}{4}, \pm\frac{3}{4},$

$\pm\frac{1}{8}, \pm\frac{3}{8}, \pm\frac{1}{16}, \pm\frac{3}{16}, \pm\frac{1}{32}, \pm\frac{3}{32}$

(b)



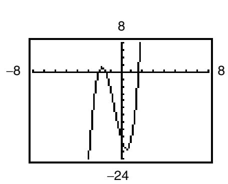
(c) Real zeros: $-\frac{1}{8}, \frac{3}{4}, 1$

38. $f(x) = 4x^3 + 7x^2 - 11x - 18$

(a) Possible rational zeros: $\pm1, \pm2, \pm3, \pm6, \pm9, \pm18,$

$\pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{9}{2}, \pm\frac{1}{4}, \pm\frac{3}{4}, \pm\frac{9}{4}$

(b)



(c) Real zeros: $-2, \frac{1}{8} \pm \frac{\sqrt{145}}{8}$

39. $f(x) = x^4 - 3x^2 + 2$

(a) $x = \pm1$, about ±1.414

(b) An exact zero is $x = 1$.

$$\begin{array}{r|rrrrr} 1 & 1 & 0 & -3 & 0 & 2 \\ & & 1 & 1 & -2 & -2 \\ \hline & 1 & 1 & -2 & -2 & 0 \end{array}$$

(c)
$$\begin{array}{r|rrrr} -1 & 1 & 1 & -2 & -2 \\ & & -1 & 0 & 2 \\ \hline & 1 & 0 & -2 & 0 \end{array}$$

$f(x) = (x - 1)(x + 1)(x^2 - 2)$

$= (x - 1)(x + 1)(x - \sqrt{2})(x + \sqrt{2})$

40. $P(t) = t^4 - 7t^2 + 12$

(a) $t = \pm2$, about ±1.732

(b) An exact zero is $t = 2$.

$$\begin{array}{r|rrrrr} 2 & 1 & 0 & -7 & 0 & 12 \\ & & 2 & 4 & -6 & -12 \\ \hline & 1 & 2 & -3 & -6 & 0 \end{array}$$

An exact zero is $t = -2$.

$$\begin{array}{r|rrrr} -2 & 1 & 2 & -3 & -6 \\ & & -2 & 0 & 6 \\ \hline & 1 & 0 & -3 & 0 \end{array}$$

(c) $P(t) = (t - 2)(t + 2)(t^2 - 3)$

$= (t - 2)(t + 2)(t - \sqrt{3})(t + \sqrt{3})$

41. $h(x) = x^5 - 7x^4 + 10x^3 + 14x^2 - 24x$

(a) $h(x) = x(x^4 - 7x^3 + 10x^2 + 14x - 24)$

$x = 0, 3, 4$, about ±1.414

(b) An exact zero is $x = 3$.

$$\begin{array}{r|rrrrr} 3 & 1 & -7 & 10 & 14 & -24 \\ & & 3 & -12 & -6 & 24 \\ \hline & 1 & -4 & -2 & 8 & 0 \end{array}$$

(c)
$$\begin{array}{r|rrrr} 4 & 1 & -4 & -2 & 8 \\ & & 4 & 0 & -8 \\ \hline & 1 & 0 & -2 & 0 \end{array}$$

$h(x) = x(x - 3)(x - 4)(x^2 - 2)$

$= x(x - 3)(x - 4)(x - \sqrt{2})(x + \sqrt{2})$

42. $g(x) = 6x^4 - 11x^3 - 51x^2 + 99x - 27$

(a) $x = \pm 3, 1.5$, about 0.333

(b) An exact zero is $x = 3$.

$$\begin{array}{r|rrrrr} 3 & 6 & -11 & -51 & 99 & -27 \\ & & 18 & 21 & -90 & 27 \\ \hline & 6 & 7 & -30 & 9 & 0 \end{array}$$

An exact zero is $x = -3$.

$$\begin{array}{r|rrrr} -3 & 6 & 7 & -30 & 9 \\ & & -18 & 33 & -9 \\ \hline & 6 & -11 & 3 & 0 \end{array}$$

(c) $g(x) = (x - 3)(x + 3)(6x^2 - 11x + 3)$
$= (x - 3)(x + 3)(3x - 1)(2x - 3)$

43. If $5i$ is a zero, so is its conjugate, $-5i$.

$$\begin{aligned} f(x) &= (x - 1)(x - 5i)(x + 5i) \\ &= (x - 1)(x^2 + 25) \\ &= x^3 - x^2 + 25x - 25 \end{aligned}$$

Note: $f(x) = a(x^3 - x^2 + 25x - 25)$, where a is any nonzero real number, has the zeros 1 and $\pm 5i$.

44. If $-3i$ is a zero, so is its conjugate, $3i$.

$$\begin{aligned} f(x) &= (x - 4)(x - 3i)(x + 3i) \\ &= (x - 4)(x^2 + 9) \\ &= x^3 - 4x^2 + 9x - 36 \end{aligned}$$

Note: $f(x) = a(x^3 - 4x^2 + 9x - 36)$, where a is any real number, has the zeros $4, 3i$, and $-3i$.

45. If $5 + i$ is a zero, so is its conjugate, $5 - i$.

$$\begin{aligned} f(x) &= (x - 2)(x - (5 + i))(x - (5 - i)) \\ &= (x - 2)(x^2 - 10x + 26) \\ &= x^3 - 12x^2 + 46x - 52 \end{aligned}$$

Note: $f(x) = a(x^3 - 12x^2 + 46x - 52)$, where a is any nonzero real number, has the zeros 2 and $5 \pm i$.

46. If $3 - 2i$ is a zero, so is its conjugate, $3 + 2i$.

$$\begin{aligned} f(x) &= (x - 5)(x - (3 - 2i))(x - (3 + 2i)) \\ &= (x - 5)(x^2 - 6x + 13) \\ &= x^3 - 11x^2 + 43x - 65 \end{aligned}$$

Note: $f(x) = a(x^3 - 11x^2 + 43x - 65)$, where a is any nonzero real number, has the zeros 5 and $3 \pm 2i$.

47. If $3 + \sqrt{2}i$ is a zero, so is its conjugate, $3 - \sqrt{2}i$.

$$\begin{aligned} f(x) &= (3x - 2)(x + 1)\left[x - (3 + \sqrt{2}i)\right]\left[x - (3 - \sqrt{2}i)\right] \\ &= (3x - 2)(x + 1)\left[(x - 3) - \sqrt{2}i\right]\left[(x - 3) + \sqrt{2}i\right] \\ &= (3x^2 + x - 2)\left[(x - 3)^2 - (\sqrt{2}i)^2\right] \\ &= (3x^2 + x - 2)(x^2 - 6x + 9 + 2) \\ &= (3x^2 + x - 2)(x^2 - 6x + 11) \\ &= 3x^4 - 17x^3 + 25x^2 + 23x - 22 \end{aligned}$$

Note: $f(x) = a(3x^4 - 17x^3 + 25x^2 + 23x - 22)$, where a is any nonzero real number, has the zeros $\frac{2}{3}, -1$, and $3 \pm \sqrt{2}i$.

48. If $1 + \sqrt{3}i$ is a zero, so is its conjugate, $1 - \sqrt{3}i$.

$$\begin{aligned} f(x) &= (x + 5)^2(x - 1 - \sqrt{3}i)(x - 1 + \sqrt{3}i) \\ &= (x^2 + 10x + 25)(x^2 - 2x + 4) \\ &= x^4 + 8x^3 + 9x^2 - 10x + 100 \end{aligned}$$

Note: $f(x) = a(x^4 + 8x^3 + 9x^2 - 10x + 100)$, where a is any real number, has the zeros $-5, -5$, and $1 \pm \sqrt{3}i$.

49. $f(x) = x^4 + 6x^2 - 27$

(a) $f(x) = (x^2 + 9)(x^2 - 3)$

(b) $f(x) = (x^2 + 9)(x + \sqrt{3})(x - \sqrt{3})$

(c) $f(x) = (x + 3i)(x - 3i)(x + \sqrt{3})(x - \sqrt{3})$

50. $f(x) = x^4 - 2x^3 - 3x^2 + 12x - 18$

$$\begin{array}{r} x^2 - 2x + 3 \\ x^2 - 6 \overline{) x^4 - 2x^3 - 3x^2 + 12x - 18} \\ \underline{x^4 \qquad\quad - 6x^2} \qquad\qquad \\ -2x^3 + 3x^2 + 12x \qquad \\ \underline{-2x^3 \qquad\quad + 12x} \qquad \\ 3x^2 \qquad\quad -18 \\ \underline{3x^2 \qquad\quad -18} \\ 0 \end{array}$$

(a) $f(x) = (x^2 - 6)(x^2 - 2x + 3)$

(b) $f(x) = (x + \sqrt{6})(x - \sqrt{6})(x^2 - 2x + 3)$

(c) $f(x) = (x + \sqrt{6})(x - \sqrt{6})(x - 1 - \sqrt{2}i)(x - 1 + \sqrt{2}i)$

Note: Use the Quadratic Formula for (c).

51. $f(x) = x^4 - 4x^3 + 5x^2 - 2x - 6$

$$\begin{array}{r} x^2 - 2x + 3 \\ x^2 - 2x - 2 \overline{) x^4 - 4x^3 + 5x^2 - 2x - 6} \\ \underline{x^4 - 2x^3 - 2x^2} \qquad\qquad \\ -2x^3 + 7x^2 - 2x \qquad \\ \underline{-2x^3 + 4x^2 + 4x} \qquad \\ 3x^2 - 6x - 6 \\ \underline{3x^2 - 6x - 6} \\ 0 \end{array}$$

(a) $f(x) = (x^2 - 2x - 2)(x^2 - 2x + 3)$

(b) $f(x) = (x - 1 + \sqrt{3})(x - 1 - \sqrt{3})(x^2 - 2x + 3)$

(c) $f(x) = (x - 1 + \sqrt{3})(x - 1 - \sqrt{3})(x - 1 + \sqrt{2}i)(x - 1 - \sqrt{2}i)$

Note: Use the Quadratic Formula for (b) and (c).

52. $f(x) = x^4 - 3x^3 - x^2 - 12x - 20$

$$\begin{array}{r} x^2 - 3x - 5 \\ x^2 + 4 \overline{) x^4 - 3x^3 - x^2 - 12x - 20} \\ \underline{x^4 \qquad\quad + 4x^2} \qquad\qquad \\ -3x^3 - 5x^2 - 12x \qquad \\ \underline{-3x^3 \qquad\quad - 12x} \qquad \\ -5x^2 \qquad\quad - 20 \\ \underline{-5x^2 \qquad\quad - 20} \\ 0 \end{array}$$

(a) $f(x) = (x^2 + 4)(x^2 - 3x - 5)$

(b) $f(x) = (x^2 + 4)\left(x - \dfrac{3 + \sqrt{29}}{2}\right)\left(x - \dfrac{3 - \sqrt{29}}{2}\right)$

(c) $f(x) = (x + 2i)(x - 2i)\left(x - \dfrac{3 + \sqrt{29}}{2}\right)\left(x - \dfrac{3 - \sqrt{29}}{2}\right)$

Note: Use the Quadratic Formula for (b).

53. $f(x) = x^3 - x^2 + 4x - 4$

Because $2i$ is a zero, so is $-2i$.

$$\begin{array}{r|rrrr} 2i & 1 & -1 & 4 & -4 \\ & & 2i & -4-2i & 4 \\ \hline & 1 & 2i-1 & -2i & 0 \end{array}$$

$$\begin{array}{r|rrr} -2i & 1 & 2i-1 & -2i \\ & & -2i & 2i \\ \hline & 1 & -1 & 0 \end{array}$$

$f(x) = (x - 2i)(x + 2i)(x - 1)$

The zeros of $f(x)$ are $x = 1, \pm 2i$.

Alternate Solution:

Because $x = \pm 2i$ are zeros of $f(x)$,

$(x + 2i)(x - 2i) = x^2 + 4$ is a factor of $f(x)$.

By long division, you have:

$$\begin{array}{r} x - 1 \\ x^2 + 0x + 4 \overline{\big)\, x^3 - x^2 + 4x - 4} \\ \underline{x^3 + 0x^2 + 4x} \quad\;\; \\ -x^2 + 0x - 4 \\ \underline{-x^2 + 0x - 4} \\ 0 \end{array}$$

$f(x) = (x^2 + 4)(x - 1)$

The zeros of $f(x)$ are $x = 1, \pm 2i$.

54. $f(x) = 2x^3 + 3x^2 + 18x + 27$

Because $3i$ is a zero, so is $-3i$.

$$\begin{array}{r|rrrr} 3i & 2 & 3 & 18 & 27 \\ & & 6i & 9i-18 & -27 \\ \hline & 2 & 3+6i & 9i & 0 \end{array}$$

$$\begin{array}{r|rrr} -3i & 2 & 3+6i & 9i \\ & & -6i & -9i \\ \hline & 2 & 3 & 0 \end{array}$$

$f(x) = (x - 3i)(x + 3i)(2x + 3)$

The zeros of $f(x)$ are $x = \pm 3i, -\frac{3}{2}$.

Alternate Solution:

Because $x = \pm 3i$ are zeros of $f(x)$,

$(x - 3i)(x + 3i) = x^2 + 9$ is a factor of $f(x)$.

By long division, you have:

$$\begin{array}{r} 2x + 3 \\ x^2 + 0x + 9 \overline{\big)\, 2x^3 + 3x^2 + 18x + 27} \\ \underline{2x^3 + 0x^2 + 18x} \quad\;\;\; \\ 3x^2 + 0x + 27 \\ \underline{3x^2 + 0x + 27} \\ 0 \end{array}$$

$f(x) = (x^2 + 9)(2x + 3)$

The zeros of $f(x)$ are $x = \pm 3i, -\frac{3}{2}$.

55. $f(x) = 2x^4 - x^3 + 49x^2 - 25x - 25$

Because $5i$ is a zero, so is $-5i$.

$$\begin{array}{r|rrrrr} 5i & 2 & -1 & 49 & -25 & -25 \\ & & 10i & -5i-50 & -5i+25 & 25 \\ \hline & 2 & -1+10i & -1-5i & -5i & 0 \end{array}$$

$$\begin{array}{r|rrrr} -5i & 2 & -1+10i & -1-5i & -5i \\ & & -10i & 5i & 5i \\ \hline & 2 & -1 & -1 & 0 \end{array}$$

$$\begin{aligned} f(x) &= (x - 5i)(x + 5i)(2x^2 - x - 1) \\ &= (x - 5i)(x + 5i)(2x + 1)(x - 1) \end{aligned}$$

The zeros of $f(x)$ are $x = \pm 5i, -\frac{1}{2}, 1$.

Alternate Solution:

Because $x = \pm 5i$ are zeros of $f(x)$,

$(x - 5i)(x + 5i) = x^2 + 25$ is a factor of $f(x)$.

By long division, you have:

$$\begin{array}{r} 2x^2 - x - 1 \\ x^2 + 0x + 25 \overline{\big)\, 2x^4 - x^3 + 49x^2 - 25x - 25} \\ \underline{2x^4 + 0x^3 + 50x^2} \qquad\qquad\quad \\ -x^3 - x^2 - 25x \qquad \\ \underline{-x^3 + 0x^2 - 25x} \qquad \\ -x^2 + 0x - 25 \\ \underline{-x^2 + 0x - 25} \\ 0 \end{array}$$

$f(x) = (x^2 + 25)(2x^2 - x - 1)$

The zeros of $f(x)$ are $x = \pm 5i, -\frac{1}{2}, 1$.

56. $g(x) = x^3 - 7x^2 - x + 87$

Because $5 + 2i$ is a zero, so is $5 - 2i$.

$$\begin{array}{r|rrrr} 5+2i & 1 & -7 & -1 & 87 \\ & & 5+2i & -14+6i & -87 \\ \hline & 1 & -2+2i & -15+6i & 0 \end{array}$$

$$\begin{array}{r|rrr} 5-2i & 1 & -2+2i & -15+6i \\ & & 5-2i & 15-6i \\ \hline & 1 & 3 & 0 \end{array}$$

The zero of $x + 3$ is $x = -3$. The zeros of $f(x)$ are $x = -3, 5 \pm 2i$.

57. $g(x) = 4x^3 + 23x^2 + 34x - 10$

Because $-3 + i$ is a zero, so is $-3 - i$.

$$\begin{array}{r|rrrr} -3+i & 4 & 23 & 34 & -10 \\ & & -12+4i & -37-i & 10 \\ \hline & 4 & 11+4i & -3-i & 0 \end{array}$$

$$\begin{array}{r|rrr} -3-i & 4 & 11+4i & -3-i \\ & & -12-4i & 3+i \\ \hline & 4 & -1 & 0 \end{array}$$

The zero of $4x - 1$ is $x = \frac{1}{4}$. The zeros of $g(x)$ are $x = -3 \pm i, \frac{1}{4}$.

Alternate Solution

Because $-3 \pm i$ are zeros of $g(x)$,

$$\begin{aligned} [x - (-3 + i)][x - (-3 - i)] &= [(x + 3) - i][(x + 3) + i] \\ &= (x + 3)^3 - i^2 \\ &= x^2 + 6x + 10 \end{aligned}$$

is a factor of $g(x)$. By long division, you have:

$$\begin{array}{r} 4x - 1 \\ x^2 + 6x + 10 \overline{\big) 4x^3 + 23x^2 + 34x - 10} \\ \underline{4x^3 + 24x^2 + 40x} \\ -x^2 - 6x - 10 \\ \underline{-x^2 - 6x - 10} \\ 0 \end{array}$$

$$g(x) = (x^2 + 6x + 10)(4x - 1)$$

The zeros of $g(x)$ are $x = -3 \pm i, \frac{1}{4}$.

58. $h(x) = 3x^3 - 4x^2 + 8x + 8$

Because $1 - \sqrt{3}i$ is a zero, so is $1 + \sqrt{3}i$.

$$\begin{array}{r|rrrr} 1-\sqrt{3}i & 3 & -4 & 8 & 8 \\ & & 3-3\sqrt{3}i & -10-2\sqrt{3}i & -8 \\ \hline & 3 & -1-3\sqrt{3}i & -2-2\sqrt{3}i & 0 \end{array}$$

$$\begin{array}{r|rrr} 1+\sqrt{3}i & 3 & -1-3\sqrt{3}i & -2-2\sqrt{3}i \\ & & 3+3\sqrt{3}i & 2+2\sqrt{3}i \\ \hline & 3 & 2 & 0 \end{array}$$

The zero of $3x + 2$ is $x = -\frac{2}{3}$. The zeros of $f(x)$ are $x = -\frac{2}{3}, 1 \pm \sqrt{3}i$.

59. $f(x) = x^4 + 3x^3 - 5x^2 - 21x + 22$

Because $-3 + \sqrt{2}i$ is a zero, so is $-3 - \sqrt{2}i$, and

$$\begin{aligned}\left[x - \left(-3 + \sqrt{2}i\right)\right]\left[x - \left(-3 - \sqrt{2}i\right)\right] &= \left[(x + 3) - \sqrt{2}i\right]\left[(x + 3) + \sqrt{2}i\right] \\ &= (x + 3)^2 - \left(\sqrt{2}i\right)^2 \\ &= x^2 + 6x + 11\end{aligned}$$

is a factor of $f(x)$. By long division, you have:

$$\begin{array}{r} x^2 - 3x + 2 \\ x^2 + 6x + 11 \overline{\smash{\big)} x^4 + 3x^3 - 5x^2 - 21x + 22} \\ \underline{x^4 + 6x^3 + 11x^2} \\ -3x^3 - 16x^2 - 21x \\ \underline{-3x^3 - 18x^2 - 33x} \\ 2x^2 + 12x + 22 \\ \underline{2x^2 + 12x + 22} \\ 0 \end{array}$$

$$\begin{aligned}f(x) &= \left(x^2 + 6x + 11\right)\left(x^2 - 3x + 2\right) \\ &= \left(x^2 + 6x + 11\right)(x - 1)(x - 2)\end{aligned}$$

The zeros of $f(x)$ are $x = -3 \pm \sqrt{2}i, 1, 2$.

60. $f(x) = x^3 + 4x^2 + 14x + 20$

Because $-1 - 3i$ is zero, so is $-1 + 3i$.

$$\begin{array}{r|rrrr} -1 - 3i & 1 & 4 & 14 & 20 \\ & & -1 - 3i & -12 - 6i & -20 \\ \hline & 1 & 3 - 3i & 2 - 6i & 0 \end{array}$$

$$\begin{array}{r|rrr} -1 + 3i & 1 & 3 - 3i & 2 - 6i \\ & & -1 + 3i & -2 + 6i \\ \hline & 1 & 2 & 0 \end{array}$$

The zero of $x + 2$ is $x = -2$.

The zeros of $f(x)$ are $x = -2, -1 \pm 3i$.

61. $f(x) = x^2 + 36$

$= (x + 6i)(x - 6i)$

The zeros of $f(x)$ are $x = \pm 6i$.

62. $f(x) = x^2 - x + 56$

By the Quadratic Formula, the zeros of $f(x)$ are

$$x = \frac{1 \pm \sqrt{1 - 224}}{2} = \frac{1 \pm \sqrt{223}i}{2}.$$

$$f(x) = \left(x - \frac{1 - \sqrt{223}i}{2}\right)\left(x - \frac{1 + \sqrt{223}i}{2}\right)$$

63. $h(x) = x^2 - 2x + 17$

By the Quadratic Formula, the zeros of $f(x)$ are

$$x = \frac{2 \pm \sqrt{4 - 68}}{2} = \frac{2 \pm \sqrt{-64}}{2} = 1 \pm 4i.$$

$$\begin{aligned}f(x) &= \left(x - (1 + 4i)\right)\left(x - (1 - 4i)\right) \\ &= (x - 1 - 4i)(x - 1 + 4i)\end{aligned}$$

64. $g(x) = x^2 + 10x + 17$

By the Quadratic Formula, the zeros of $f(x)$ are

$$x = \frac{-10 \pm \sqrt{100 - 68}}{2} = \frac{-10 \pm \sqrt{32}}{2} = -5 \pm 2\sqrt{2}.$$

$$\begin{aligned}f(x) &= \left(x - \left(-5 + 2\sqrt{2}\right)\right)\left(x - \left(-5 - 2\sqrt{2}\right)\right) \\ &= \left(x + 5 - 2\sqrt{2}\right)\left(x + 5 + 2\sqrt{2}\right)\end{aligned}$$

65. $f(x) = x^4 - 16$

$$\begin{aligned}&= \left(x^2 - 4\right)\left(x^2 + 4\right) \\ &= (x - 2)(x + 2)(x - 2i)(x + 2i)\end{aligned}$$

Zeros: $\pm 2, \pm 2i$

66. $f(y) = y^4 - 256$

$= (y^2 - 16)(y^2 + 16)$

$= (y - 4)(y + 4)(y - 4i)(y + 4i)$

Zeros: $\pm 4, \pm 4i$

67. $f(z) = z^2 - 2z + 2$

By the Quadratic Formula, the zeros of $f(z)$ are

$$z = \frac{2 \pm \sqrt{4 - 8}}{2} = 1 \pm i.$$

$f(z) = [z - (1 + i)][z - (1 - i)]$

$= (z - 1 - i)(z - 1 + i)$

68. $h(x) = x^3 - 3x^2 + 4x - 2$

Possible rational zeros: $\pm 1, \pm 2$

1	1	−3	4	−2
		1	−2	2
	1	−2	2	0

By the Quadratic Formula, the zeros of $x^2 - 2x + 2$ are $x = \dfrac{2 \pm \sqrt{4 - 8}}{2} = 1 \pm i.$

Zeros: $1, 1 \pm i$

$h(x) = (x - 1)(x - 1 - i)(x - 1 + i)$

69. $g(x) = x^3 - 3x^2 + x + 5$

Possible rational zeros: $\pm 1, \pm 5$

−1	1	−3	1	5
		−1	4	−5
	1	−4	5	0

By the Quadratic Formula, the zeros of $x^2 - 4x + 5$ are: $x = \dfrac{4 \pm \sqrt{16 - 20}}{2} = 2 \pm i$

Zeros: $-1, 2 \pm i$

$g(x) = (x + 1)(x - 2 - i)(x - 2 + i)$

70. $f(x) = x^3 - x^2 + x + 39$

Possible rational zeros: $\pm 1, \pm 3, \pm 13, \pm 39$

−3	1	−1	1	39
		−3	12	−39
	1	−4	13	0

By the Quadratic Formula, the zeros of $x^2 - 4x + 13$ are: $x = \dfrac{4 \pm \sqrt{16 - 52}}{2} = 2 \pm 3i$

Zeros: $-3, 2 \pm 3i$

$f(x) = (x + 3)(x - 2 - 3i)(x - 2 + 3i)$

71. $h(x) = x^3 - x + 6$

Possible rational zeros: $\pm 1, \pm 2, \pm 3, \pm 6$

−2	1	0	−1	6
		−2	4	−6
	1	−2	3	0

By the Quadratic Formula, the zeros of $x^2 - 2x + 3$ are

$$x = \frac{2 \pm \sqrt{4 - 12}}{2} = 1 \pm \sqrt{2}i.$$

Zeros: $-2, 1 \pm \sqrt{2}i.$

$h(x) = (x + 2)\left[x - (1 + \sqrt{2}i)\right]\left[x - (1 - \sqrt{2}i)\right]$

$= (x + 2)(x - 1 - \sqrt{2}i)(x - 1 + \sqrt{2}i)$

72. $h(x) = x^3 + 9x^2 + 27x + 35$

Possible rational zeros: $\pm 1, \pm 5, \pm 7, \pm 35$

−5	1	9	27	35
		−5	−20	−35
	1	4	7	0

By the Quadratic Formula, the zeros of $x^2 + 4x + 7$ are $x = \dfrac{-4 \pm \sqrt{16 - 28}}{2} = -2 \pm \sqrt{3}i.$

Zeros: $-5, -2 \pm \sqrt{3}i$

$h(x) = (x + 5)(x + 2 + \sqrt{3}i)(x + 2 - \sqrt{3}i)$

73. $f(x) = 5x^3 - 9x^2 + 28x + 6$

Possible rational zeros:

$\pm1, \pm2, \pm3, \pm6, \pm\frac{1}{5}, \pm\frac{2}{5}, \pm\frac{3}{5}, \pm\frac{6}{5}$

$$\begin{array}{r|rrrr} -\frac{1}{5} & 5 & -9 & 28 & 6 \\ & & -1 & 2 & -6 \\ \hline & 5 & -10 & 30 & 0 \end{array}$$

By the Quadratic Formula, the zeros of $5x^2 - 10x + 30 = 5(x^2 - 2x + 6)$ are

$$x = \frac{2 \pm \sqrt{4 - 24}}{2} = 1 \pm \sqrt{5}i.$$

Zeros: $-\frac{1}{5}, 1 \pm \sqrt{5}i.$

$$f(x) = \left[x - \left(-\frac{1}{5}\right)\right](5)\left[x - \left(1 + \sqrt{5}i\right)\right]\left[x - \left(1 - \sqrt{5}i\right)\right]$$
$$= (5x + 1)\left(x - 1 - \sqrt{5}i\right)\left(x - 1 + \sqrt{5}i\right)$$

74. $g(x) = 2x^3 - x^2 + 8x + 21$

Possible rational roots:

$\pm\frac{1}{2}, \pm1, \pm\frac{3}{2}, \pm3, \pm\frac{7}{2}, \pm7, \pm\frac{21}{2}, \pm21$

$$\begin{array}{r|rrrr} -\frac{3}{2} & 2 & -1 & 8 & 21 \\ & & -3 & 6 & -21 \\ \hline & 2 & -4 & 14 & 0 \end{array}$$

By the Quadratic Formula, the zeros of $2x^2 - 4x + 14$ are $x = \dfrac{4 \pm \sqrt{16 - 112}}{4} = \dfrac{4 \pm \sqrt{-96}}{4} = 1 \pm \sqrt{6}i.$

Zeros: $-\frac{3}{2}, 1 \pm \sqrt{6}i$

$$f(x) = \left(x + \frac{3}{2}\right)\left(x - 1 - \sqrt{6}i\right)\left(x - 1 + \sqrt{6}i\right)$$

75. $g(x) = x^4 - 4x^3 + 8x^2 - 16x + 16$

Possible rational zeros: $\pm1, \pm2, \pm4, \pm8, \pm16$

$$\begin{array}{r|rrrrr} 2 & 1 & -4 & 8 & -16 & 16 \\ & & 2 & -4 & 8 & -16 \\ \hline & 1 & -2 & 4 & -8 & 0 \end{array}$$

$$\begin{array}{r|rrrr} 2 & 1 & -2 & 4 & -8 \\ & & 2 & 0 & 8 \\ \hline & 1 & 0 & 4 & 0 \end{array}$$

$$g(x) = (x - 2)(x - 2)(x^2 + 4)$$
$$= (x - 2)^2(x + 2i)(x - 2i)$$

Zeros: $2, \pm2i$

76. $h(x) = x^4 + 6x^3 + 10x^2 + 6x + 9$

Possible rational zeros: $\pm1, \pm3, \pm9$

$$\begin{array}{r|rrrrr} -3 & 1 & 6 & 10 & 6 & 9 \\ & & -3 & -9 & -3 & -9 \\ \hline & 1 & 3 & 1 & 3 & 0 \end{array}$$

$$\begin{array}{r|rrrr} -3 & 1 & 3 & 1 & 3 \\ & & -3 & 0 & -3 \\ \hline & 1 & 0 & 1 & 0 \end{array}$$

The zeros of $x^2 + 1$ are $x = \pm i.$

Zeros: $-3, \pm i$

$$h(x) = (x + 3)^2(x + i)(x - i)$$

77. $f(x) = x^4 + 10x^2 + 9$
$$= (x^2 + 1)(x^2 + 9)$$
$$= (x + i)(x - i)(x + 3i)(x - 3i)$$

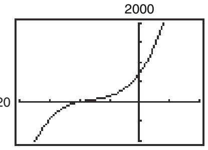
Zeros: $\pm i, \pm3i$

78. $f(x) = x^4 + 29x^2 + 100$
$$= (x^2 + 25)(x^2 + 4)$$
$$= (x + 2i)(x - 2i)(x + 5i)(x - 5i)$$

Zeros: $\pm2i, \pm5i$

79. $f(x) = x^3 + 24x^2 + 214x + 740$

Possible rational zeros: $\pm1, \pm2, \pm4, \pm5, \pm10, \pm20, \pm37, \pm74, \pm148, \pm185, \pm370, \pm740$



Based on the graph, try $x = -10.$

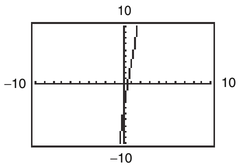
$$\begin{array}{r|rrrr} -10 & 1 & 24 & 214 & 740 \\ & & -10 & -140 & -740 \\ \hline & 1 & 14 & 74 & 0 \end{array}$$

By the Quadratic Formula, the zeros of $x^2 + 14x + 74$ are $x = \dfrac{-14 \pm \sqrt{196 - 296}}{2} = -7 \pm 5i.$

The zeros of $f(x)$ are $x = -10$ and $x = -7 \pm 5i.$

80. $f(s) = 2s^3 - 5s^2 + 12s - 5$

Possible rational zeros: $\pm 1, \pm 5, \pm\frac{1}{2}, \pm\frac{5}{2}$



Based on the graph, try $s = \frac{1}{2}$.

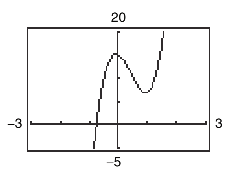
$\frac{1}{2}$	2	−5	12	−5
		1	−2	5
	2	−4	10	0

By the Quadratic Formula, the zeros of $2(s^2 - 2s + 5)$ are $s = \dfrac{2 \pm \sqrt{4 - 20}}{2} = 1 \pm 2i$.

The zeros of $f(s)$ are $s = \frac{1}{2}$ and $s = 1 \pm 2i$.

81. $f(x) = 16x^3 - 20x^2 - 4x + 15$

Possible rational zeros:
$\pm 1, \pm 3, \pm 5, \pm 15, \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{5}{2}, \pm\frac{15}{2}, \pm\frac{1}{4}, \pm\frac{3}{4},$
$\pm\frac{5}{4}, \pm\frac{15}{4}, \pm\frac{1}{8}, \pm\frac{3}{8}, \pm\frac{5}{8}, \pm\frac{15}{8}, \pm\frac{1}{16}, \pm\frac{3}{16}, \pm\frac{5}{16}, \pm\frac{15}{16}$



Based on the graph, try $x = -\frac{3}{4}$.

$-\frac{3}{4}$	16	−20	−4	15
		−12	24	−15
	16	−32	20	0

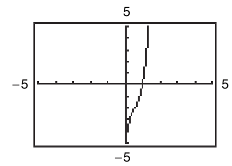
By the Quadratic Formula, the zeros of $16x^2 - 32x + 20 = 4(4x^2 - 8x + 5)$ are

$x = \dfrac{8 \pm \sqrt{64 - 80}}{8} = 1 \pm \frac{1}{2}i$.

The zeros of $f(x)$ are $x = -\frac{3}{4}$ and $x = 1 \pm \frac{1}{2}i$.

82. $f(x) = 9x^3 - 15x^2 + 11x - 5$

Possible rational zeros: $\pm 1, \pm 5, \pm\frac{1}{3}, \pm\frac{5}{3}, \pm\frac{1}{9}, \pm\frac{5}{9}$



Based on the graph, try $x = 1$.

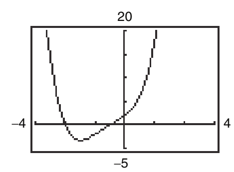
1	9	−15	11	−5
		9	−6	5
	9	−6	5	0

By the Quadratic Formula, the zeros of $9x^2 - 6x + 5$ are $x = \dfrac{6 \pm \sqrt{36 - 180}}{18} = \frac{1}{3} \pm \frac{2}{3}i$.

The zeros of $f(x)$ are $x = 1$ and $x = \frac{1}{3} \pm \frac{2}{3}i$.

83. $f(x) = 2x^4 + 5x^3 + 4x^2 + 5x + 2$

Possible rational zeros: $\pm 1, \pm 2, \pm\frac{1}{2}$



Based on the graph, try $x = -2$ and $x = -\frac{1}{2}$.

−2	2	5	4	5	2
		−4	−2	−4	−2
	2	1	2	1	0

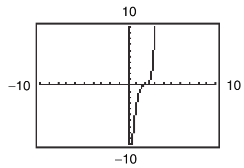
$-\frac{1}{2}$	2	1	2	1
		−1	0	−1
	2	0	2	0

The zeros of $2x^2 + 2 = 2(x^2 + 1)$ are $x = \pm i$.

The zeros of $f(x)$ are $x = -2, x = -\frac{1}{2},$ and $x = \pm i$.

84. $g(x) = x^5 - 8x^4 + 28x^3 - 56x^2 + 64x - 32$

Possible rational zeros: $\pm1, \pm2, \pm4, \pm8, \pm16, \pm32$



Based on the graph, try $x = 2$.

$$\begin{array}{r|rrrrrr} 2 & 1 & -8 & 28 & -56 & 64 & -32 \\ & & 2 & -12 & 32 & -48 & 32 \\ \hline & 1 & -6 & 16 & -24 & 16 & 0 \end{array}$$

$$\begin{array}{r|rrrrr} 2 & 1 & -6 & 16 & -24 & 16 \\ & & 2 & -8 & 16 & -16 \\ \hline & 1 & -4 & 8 & -8 & 0 \end{array}$$

$$\begin{array}{r|rrrr} 2 & 1 & -4 & 8 & -8 \\ & & 2 & -4 & 8 \\ \hline & 1 & -2 & 4 & 0 \end{array}$$

By the Quadratic Formula, the zeros of $x^2 - 2x + 4$ are $x = \dfrac{2 \pm \sqrt{4 - 16}}{2} = 1 \pm \sqrt{3}i.$

The zeros of $g(x)$ are $x = 2$ and $x = 1 \pm \sqrt{3}i.$

85. $f(x) = 4x^3 - 3x - 1$

Possible rational zeros: $\pm1, \pm\frac{1}{2}, \pm\frac{1}{4}$

$$\begin{array}{r|rrrr} 1 & 4 & 0 & -3 & -1 \\ & & 4 & 4 & 1 \\ \hline & 4 & 4 & 1 & 0 \end{array}$$

$$\begin{aligned} 4x^3 - 3x - 1 &= (x - 1)(4x^2 + 4x + 1) \\ &= (x - 1)(2x + 1)^2 \end{aligned}$$

So, the real zeros are 1 and $-\dfrac{1}{2}$.

86. $f(z) = 12z^3 - 4z^2 - 27z + 9$

Possible rational zeros: $\pm1, \pm3, \pm9, \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{9}{2}, \pm\frac{1}{3}, \pm\frac{1}{4}, \pm\frac{3}{4}, \pm\frac{9}{4}, \pm\frac{1}{6}, \pm\frac{1}{12}$

$$\begin{array}{r|rrrr} \frac{3}{2} & 12 & -4 & -27 & 9 \\ & & 18 & 21 & -9 \\ \hline & 12 & 14 & -6 & 0 \end{array}$$

$$\begin{aligned} f(z) &= 2\left(z - \tfrac{3}{2}\right)(6z^2 + 7z - 3) \\ &= (2z - 3)(3z - 1)(2z + 3) \end{aligned}$$

So, the real zeros are $-\frac{3}{2}, \frac{1}{3},$ and $\frac{3}{2}$.

87. $f(y) = 4y^3 + 3y^2 + 8y + 6$

Possible rational zeros: $\pm1, \pm2, \pm3, \pm6, \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{1}{4}, \pm\frac{3}{4}$

$$\begin{array}{r|rrrr} -\frac{3}{4} & 4 & 3 & 8 & 6 \\ & & -3 & 0 & -6 \\ \hline & 4 & 0 & 8 & 0 \end{array}$$

$$\begin{aligned} 4y^3 + 3y^2 + 8y + 6 &= \left(y + \tfrac{3}{4}\right)(4y^2 + 8) \\ &= \left(y + \tfrac{3}{4}\right)4(y^2 + 2) \\ &= (4y + 3)(y^2 + 2) \end{aligned}$$

So, the only real zero is $-\frac{3}{4}$.

88. $g(x) = 3x^3 - 2x^2 + 15x - 10$

Possible rational zeros:
$\pm1, \pm2, \pm5, \pm10, \pm\frac{1}{3}, \pm\frac{2}{3}, \pm\frac{5}{3}, \pm\frac{10}{3}$

$$\begin{array}{r|rrrr} \frac{2}{3} & 3 & -2 & 15 & -10 \\ & & 2 & 0 & 10 \\ \hline & 3 & 0 & 15 & 0 \end{array}$$

$g(x) = \left(x - \frac{2}{3}\right)(3x^2 + 15) = (3x - 2)(x^2 + 5)$

So, the only real zero is $\frac{2}{3}$.

89.
$$\begin{aligned} P(x) &= x^4 - \tfrac{25}{4}x^2 + 9 \\ &= \tfrac{1}{4}(4x^4 - 25x^2 + 36) \\ &= \tfrac{1}{4}(4x^2 - 9)(x^2 - 4) \\ &= \tfrac{1}{4}(2x + 3)(2x - 3)(x + 2)(x - 2) \end{aligned}$$

The rational zeros are $\pm\frac{3}{2}$ and ±2.

90. $f(x) = \frac{1}{2}(2x^3 - 3x^2 - 23x + 12)$

Possible rational zeros: $\pm1, \pm2, \pm3, \pm4, \pm6, \pm12, \pm\frac{1}{2}, \frac{3}{2}$

$$\begin{array}{r|rrrr} 4 & 2 & -3 & -23 & 12 \\ & & 8 & 20 & -12 \\ \hline & 2 & 5 & -3 & 0 \end{array}$$

$$\begin{aligned} f(x) &= \tfrac{1}{2}(x - 4)(2x^2 + 5x - 3) \\ &= \tfrac{1}{2}(x - 4)(2x - 1)(x + 3) \end{aligned}$$

The rational zeros are $-3, \frac{1}{2},$ and 4.

91. $f(x) = x^3 - \frac{1}{4}x^2 - x + \frac{1}{4}$

$= \frac{1}{4}(4x^3 - x^2 - 4x + 1)$

$= \frac{1}{4}[x^2(4x - 1) - 1(4x - 1)]$

$= \frac{1}{4}(4x - 1)(x^2 - 1)$

$= \frac{1}{4}(4x - 1)(x + 1)(x - 1)$

The rational zeros are $\frac{1}{4}$ and ± 1.

92. $f(z) = \frac{1}{6}(6z^3 + 11z^2 - 3z - 2)$

Possible rational zeros: $\pm 1, \pm 2, \pm\frac{1}{2}, \pm\frac{1}{3}, \pm\frac{2}{3}, \pm\frac{1}{6}$

–2	6	11	–3	–2
		–12	2	2
	6	–1	–1	0

$f(x) = \frac{1}{6}(x + 2)(6x^2 - x - 1)$

$= \frac{1}{6}(x + 2)(3x + 1)(2x - 1)$

The rational zeros are $-2, -\frac{1}{3}$, and $\frac{1}{2}$.

93. $f(x) = x^3 - 1 = (x - 1)(x^2 + x + 1)$

Rational zeros: $1\ (x = 1)$

Irrational zeros: 0

Matches (d).

94. $f(x) = x^3 - 2$

$= (x - \sqrt[3]{2})(x^2 + \sqrt[3]{2}x + \sqrt[3]{4})$

Rational zeros: 0

Irrational zeros: $1\ (x = \sqrt[3]{2})$

Matches (a).

95. $f(x) = x^3 - x = x(x + 1)(x - 1)$

Rational zeros: $3\ (x = 0, \pm 1)$

Irrational zeros: 0

Matches (b).

96. $f(x) = x^3 - 2x$

$= x(x^2 - 2)$

$= x(x + \sqrt{2})(x - \sqrt{2})$

Rational zeros: $1\ (x = 0)$

Irrational zeros: $2\ (x = \pm\sqrt{2})$

Matches (c).

97. Zeros: $-2, \frac{1}{2}, 3$

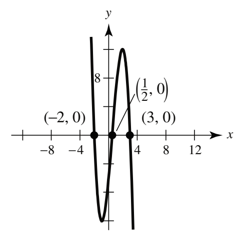
$f(x) = -(x + 2)(2x - 1)(x - 3)$

$= -2x^3 + 3x^2 + 11x - 6$

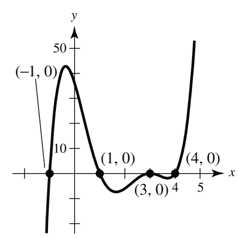
Any nonzero scalar multiple of f would have the same three zeros.

Let $g(x) = af(x), a > 0$.

There are infinitely many possible functions for f.



98.



99. Interval: $(-\infty, -2), (-2, 1), (1, 4), (4, \infty)$

Value of *f(x)*: Positive Negative Negative Positive

(a) Zeros of $f(x)$: $x = -2, x = 1, x = 4$.

(b) The graph touches the x-axis at $x = 1$.

(c) The least possible degree of the function is 4 because there are at least four real zeros (1 is repeated) and a function can have at most the number of real zeros equal to the degree of the function. The degree cannot be odd by the behavior at $\pm\infty$.

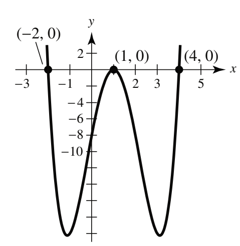
(d) The leading coefficient of f is positive. From the information in the table, you can conclude that the graph will eventually rise to the left and to the right.

(e) $f(x) = (x + 2)(x - 1)^2(x - 4)$

$= x^4 - 4x^3 - 3x^2 + 14x - 8$

(Any nonzero multiple of *f*(*x*) is also a solution.)

(f)



100. (a) $-2, 0, 2$

(b) The graph touches the x-axis at $x = 0$ and at $x = 2$.

(c) The least possible degree of f is 5 because there are at least 5 real zeros (0 and 2 are repeated) and a function can have at most the number of real zeros equal to the degree of the function. The degree cannot be even by the definition of multiplicity.

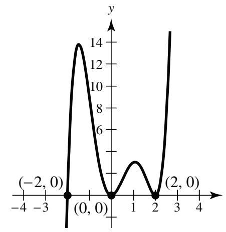
(d) The leading coefficient of f is positive. From the information in the table, you can conclude that the graph will eventually fall to the left and rise to the right.

(e) $f(x) = (x - 0)^2[x - (-2)](x - 2)^2$

$= x^2(x + 2)(x - 2)^2$

$= x^5 - 2x^4 - 4x^3 + 8x^2$

(f)



101. (a) Combined length and width:

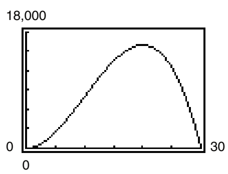
$4x + y = 120 \Rightarrow y = 120 - 4x$

$\text{Volume} = l \cdot w \cdot h = x^2y$

$= x^2(120 - 4x)$

$= 4x^2(30 - x)$

(b)



Dimensions with maximum volume: 20 in. × 20 in. × 40 in.

(c) $13{,}500 = 4x^2(30 - x)$

$4x^3 - 120x^2 + 13{,}500 = 0$

$x^3 - 30x^2 + 3375 = 0$

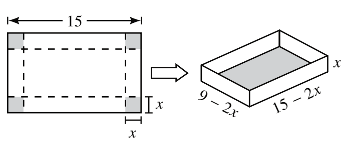
15	1	–30	0	3375
		15	–225	–3375
	1	–15	–225	0

$(x - 15)(x^2 - 15x - 225) = 0$

Using the Quadratic Formula, $x = 15, \dfrac{15 \pm 15\sqrt{5}}{2}$.

The value of $\dfrac{15 - 15\sqrt{5}}{2}$ is not possible because it is negative.

102. (a)

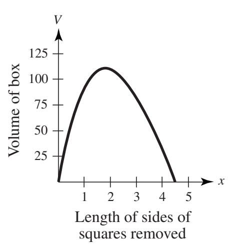


(b) $V = l \cdot w \cdot h = (15 - 2x)(9 - 2x)x$

$= x(9 - 2x)(15 - 2x)$

Because length, width, and height must be positive, you have $0 < x < \frac{9}{2}$ for the domain.

(c)



The volume is maximum when $x \approx 1.82$.

The dimensions are: length $\approx 15 - 2(1.82) = 11.36$

width $\approx 9 - 2(1.82) = 5.36$

height $= x \approx 1.82$

1.82 cm × 5.36 cm × 11.36 cm

(d) $56 = x(9 - 2x)(15 - 2x)$

$56 = 135x - 48x^2 + 4x^3$

$0 = 4x^3 - 48x^2 + 135x - 56$

The zeros of this polynomial are $\frac{1}{2}, \frac{7}{2}$, and 8.

x cannot equal 8 because it is not in the domain of V. [The length cannot equal -1 and the width cannot equal -7. The product of $(8)(-1)(-7) = 56$ so it showed up as an extraneous solution.]

So, the volume is 56 cubic centimeters when $x = \frac{1}{2}$ centimeter or $x = \frac{7}{2}$ centimeters.

103. $P = -76x^3 + 4830x^2 - 320{,}000,\ 0 \le x \le 60$

$$2{,}500{,}000 = -76x^3 + 4830x^2 - 320{,}000$$

$$76x^3 - 4830x^2 + 2{,}820{,}000 = 0$$

The zeros of this equation are $x \approx 46.1$, $x \approx 38.4$, and $x \approx -21.0$. Because $0 \le x \le 60$, we disregard $x \approx -21.0$. The smaller remaining solution is $x \approx 38.4$. The advertising expense is $384,000.

104. $P = -45x^3 + 2500x^2 - 275{,}000$

$$800{,}000 = -45x^3 + 2500x^2 - 275{,}000$$
$$0 = 45x^3 - 2500x^2 + 1{,}075{,}000$$
$$0 = 9x^3 - 500x^2 + 215{,}000$$

The zeros of this equation are $x \approx -18.0$, $x \approx 31.5$, and $x \approx 42.0$. Because $0 \le x \le 50$, disregard $x \approx -18.02$. The smaller remaining solution is $x \approx 31.5$, or an advertising expense of $315,000.

105. (a) Current bin: $V = 2 \times 3 \times 4 = 24$ cubic feet

New bin: $V = 5(24) = 120$ cubic feet

$$V(x) = (2 + x)(3 + x)(4 + x) = 120$$

(b) $x^3 + 9x^2 + 26x + 24 = 120$

$x^3 + 9x^2 + 26x - 96 = 0$

The only real zero of this polynomial is $x = 2$. All the dimensions should be increased by 2 feet, so the new bin will have dimensions of 4 feet by 5 feet by 6 feet.

106. (a) $A(x) = (250 + x)(160 + x) = (1.5)(160)(250) = 60{,}000$

(b) $60{,}000 = x^2 + 410x + 40{,}000$

$0 = x^2 + 410x - 20{,}000$

$$x = \frac{-410 \pm \sqrt{410^2 - (4)(1)(-20{,}000)}}{2(1)} = \frac{-410 \pm \sqrt{248{,}100}}{2}$$

x must be positive, so $x = \dfrac{-410 + \sqrt{248{,}100}}{2} \approx 44.05.$

The new length is $250 + 44.05 = 294.05$ ft and the new width is $160 + 44.05 = 204.05$ ft, so the new dimensions are $294.05\text{ ft} \times 204.05\text{ ft}$.

(c) $A(x) = (250 + 2x)(160 + x) = 60{,}000$

$2x^2 + 570x - 20{,}000 = 0$

$$x = \frac{-570 \pm \sqrt{570^2 - (4)(2)(-20{,}000)}}{2(2)} = \frac{-570 \pm \sqrt{484{,}900}}{4}$$

x must be positive, so $x = \dfrac{-570 + \sqrt{484{,}900}}{4} \approx 31.6.$

The new length is $250 + 2(31.6) = 313.2$ ft and the new width is $160 + (31.6) = 191.6$ ft, so the new dimensions are $313.2\text{ ft} \times 191.6$ ft.

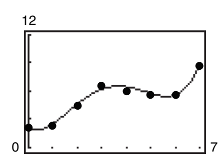
107. $C = 100\left(\dfrac{200}{x^2} + \dfrac{x}{x + 30}\right),\ x \ge 1$

C is minimum when

$3x^3 - 40x^2 - 2400x - 36000 = 0.$

The only real zero is $x \approx 40$ or 4000 units.

108. (a)



(b) $A \approx 0.01676t^4 - 0.2152t^3 + 0.794t^2 - 0.44t + 8.7$

(c) The model is a good fit to the actual data.

(d) $A \ge 10$ when $t = 3, 4,$ and 7, which corresponds to the years 2003, 2004, and 2007.

(e) Yes. The degree of A is even and the leading coefficient is positive, so as t increases, A will increase. This implies that attendance will continue to grow.

109. $h(t) = -16t^2 + 48t + 6$

Let $h = 64$ and solve for t.

$$64 = -16t^2 + 48t + 6$$

$$16t^2 - 48t + 58 = 0$$

By the Quadratic Formula, $t = \dfrac{48 \pm i\sqrt{1408}}{32}$.

Because the equation yields only imaginary zeros, it is *not* possible for the ball to have reached a height of 64 feet.

110.
$$P = R - C = xp - C$$
$$= x(140 - 0.0001x) - (80x + 150,000)$$
$$= -0.0001x^2 + 60x - 150,000$$
$$9,000,000 = -0.0001x^2 + 60x - 150,000$$

Thus, $0 = 0.0001x^2 - 60x + 9,150,000$.

$$x = \frac{60 \pm \sqrt{-60}}{0.0002} = 300,000 \pm 10,000\sqrt{15}i$$

Because the solutions are both complex, it is not possible to determine a price p that would yield a profit of 9 million dollars.

111. False. The most complex zeros it can have is two, and the Linear Factorization Theorem guarantees that there are three linear factors, so one zero must be real.

112. False. f does not have real coefficients.

113. $g(x) = -f(x)$. This function would have the same zeros as $f(x)$, so r_1, r_2, and r_3 are also zeros of $g(x)$.

114. $g(x) = 3f(x)$. This function has the same zeros as f because it is a vertical stretch of f. The zeros of g are r_1, r_2, and r_3.

115. $g(x) = f(x - 5)$. The graph of $g(x)$ is a horizontal shift of the graph of $f(x)$ five units of the right, so the zeros of $g(x)$ are $5 + r_1, 5 + r_2$, and $5 + r_3$.

116. $g(x) = f(2x)$. Note that x is a zero of g if and only if $2x$ is a zero of f. The zeros of g are $\dfrac{r_1}{2}, \dfrac{r_2}{2}$, and $\dfrac{r_3}{2}$.

117. $g(x) = 3 + f(x)$. Because $g(x)$ is a vertical shift of the graph of $f(x)$, the zeros of $g(x)$ cannot be determined.

118. $g(x) = f(-x)$. Note that x is a zero of g if and only if $-x$ is a zero of f. The zeros of g are $-r_1, -r_2$, and $-r_3$.

119. Because $1 + i$ is a zero of f, so is $1 - i$. From the graph, 1 is also a zero.

$$f(x) = (x - (1 + i))(x - (1 - i))(x - 1)$$
$$= (x^2 - 2x + 2)(x - 1)$$
$$= x^3 - 3x^2 + 4x - 2$$

120. Because $1 + i$ is a zero of f, so is $1 - i$. From the graph, -1 is also a zero.

$$f(x) = (x - (1 + i))(x - (1 - i))(x + 1)$$
$$= (x^2 - 2x + 2)(x + 1)$$
$$= x^3 - x^2 + 2$$

Because the graph rises to the left and falls to the right, $a = -1$, and $f(x) = -x^3 + x^2 - 2$.

121. Because $f(i) = f(2i) = 0$, then i and $2i$ are zeros of f. Because i and $2i$ are zeros of f, so are $-i$ and $-2i$.

$$f(x) = (x - i)(x + i)(x - 2i)(x + 2i)$$
$$= (x^2 + 1)(x^2 + 4)$$
$$= x^4 + 5x^2 + 4$$

122. Because $f(2) = 0$, 2 is a zero of f. Because $f(i) = 0$, i is a zero of f. Because i is a zero of f, so is $-i$.

$$f(x) = -1(x - 2)(x + i)(x - i)$$
$$= -1(x - 2)(x^2 + 1)$$
$$= -x^3 + 2x^2 - x + 2$$

123. Answers will vary. Some of the factoring techniques are:

1. Factor out the greatest common factor.
2 Use special product formulas.

$$a^2 - b^2 = (a + b)(a - b)$$
$$a^2 + 2ab + b^2 = (a + b)^2$$
$$a^2 - 2ab + b^2 = (a - b)^2$$
$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$
$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

3. Factor by grouping, if possible.
4. Factor general trinomials with binomial factors by "guess-and-test" or by the grouping method.
5. Use the Rational Zero Test together with synthetic division to factor a polynomial.
6. Use Descartes's Rule of Signs to determine the number of real zeros. Then find any zeros and use them to factor the polynomial.
7. Find any upper and lower bounds for the real zeros to eliminate some of the possible rational zeros. Then test the remaining candidates by synthetic division and use any zeros to factor the polynomial.

124. $f(x) = x^4 - 4x^2 + k$

$$x^2 = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(k)}}{2(1)} = \frac{4 \pm 2\sqrt{4-k}}{2} = 2 \pm \sqrt{4-k}$$

$$x = \pm\sqrt{2 \pm \sqrt{4-k}}$$

(a) For there to be four distinct real roots, both $4 - k$ and $2 \pm \sqrt{4-k}$ must be positive. This occurs when $0 < k < 4$.
So, some possible k-values are
$k = 1, k = 2, k = 3, k = \frac{1}{2}, k = \sqrt{2}$, etc.

(b) For there to be two real roots, each of multiplicity 2, $4 - k$ must equal zero. So, $k = 4$.

(c) For there to be two real zeros and two complex zeros, $2 + \sqrt{4-k}$ must be positive and $2 - \sqrt{4-k}$ must be negative. This occurs when $k < 0$. So, some possible k-values are
$k = -1, k = -2, k = -\frac{1}{2}$, etc.

(d) For there to be four complex zeros, $2 \pm \sqrt{4-k}$ must be nonreal. This occurs when $k > 4$. Some possible k-values are $k = 5, k = 6, k = 7.4$, etc.

(e) $g(x) = f(x - 2)$

No. This function is a horizontal shift of $f(x)$. Note that x is a zero of g if and only if $x - 2$ is a zero of f; the number of real and complex zeros is not affected by a horizontal shift.

(f) $g(x) = f(2x)$

No. Because x is a zero of g if and only if $2x$ is a zero of f, the number of real and complex zeros of g is the same as the number of real and complex zeros of f.

125. (a) $f(x) = \left(x - \sqrt{b}i\right)\left(x + \sqrt{b}i\right) = x^2 + b$

(b)
$$\begin{aligned} f(x) &= \left[x - (a + bi)\right]\left[x - (a - bi)\right] \\ &= \left[(x - a) - bi\right]\left[(x - a) + bi\right] \\ &= (x - a)^2 - (bi)^2 \\ &= x^2 - 2ax + a^2 + b^2 \end{aligned}$$

126. (a) $f(x)$ cannot have this graph because it also has a zero at $x = 0$.

(b) $g(x)$ cannot have this graph because it is a quadratic function. Its graph is a parabola.

(c) $h(x)$ is the correct function. It has two real zeros, $x = 2$ and $x = 3.5$, and it has a degree of four, needed to yield three turning points.

(d) $k(x)$ cannot have this graph because it also has a zero at $x = -1$. In addition, because it is only of degree three, it would have at most two turning points.

Section 2.6 Rational Functions

1. rational functions

2. vertical asymptote

3. horizontal asymptote

4. slant asymptote

5. $f(x) = \frac{1}{x-1}$

(a)

x	$f(x)$
0.5	–2
0.9	–10
0.99	–100
0.999	–1000

x	$f(x)$
1.5	2
1.1	10
1.01	100
1.001	1000

x	$f(x)$
5	0.25
10	$0.\overline{1}$
100	$0.\overline{01}$
1000	$0.\overline{001}$

(b) The zero of the denominator is $x = 1$, so $x = 1$ is a vertical asymptote. The degree of the numerator is less than the degree of the denominator, so the x-axis, or $y = 0$, is a horizontal asymptote.

(c) The domain is all real numbers x except $x = 1$.

6. $f(x) = \dfrac{5x}{x - 1}$

(a)

x	$f(x)$
0.5	–5
0.9	–45
0.99	–495
0.999	–4995

x	$f(x)$
1.5	15
1.1	55
1.01	505
1.001	5005

x	$f(x)$
5	6.25
10	$5.\overline{5}$
100	$5.\overline{05}$
1000	$5.\overline{005}$

(b) The zero of the denominator is $x = 1$, so $x = 1$ is a vertical asymptote. The degree of the numerator is equal to the degree of the denominator, so the line $y = \frac{5}{1} = 5$ is a horizontal asymptote.

(c) The domain is all real numbers x except $x = 1$.

7. $f(x) = \dfrac{3x^2}{x^2 - 1}$

(a)

x	$f(x)$
0.5	–1
0.9	–12.79
0.99	–147.8
0.999	–1498

x	$f(x)$
1.5	5.4
1.1	17.29
1.01	152.3
1.001	1502

x	$f(x)$
5	3.125
10	$3.\overline{03}$
100	$3.\overline{0003}$
1000	3

(b) The zeros of the denominator are $x = \pm 1$, so both $x = 1$ and $x = -1$ are vertical asymptotes. The degree of the numerator equals the degree of the denominator, so $y = \frac{3}{1} = 3$ is a horizontal asymptote.

(c) The domain is all real numbers x except $x = \pm 1$.

8. $f(x) = \dfrac{4x}{x^2 - 1}$

(a)

x	$f(x)$
0.5	$-2.\overline{6}$
0.9	–18.95
0.99	–199
0.999	–1999

x	$f(x)$
1.5	4.8
1.1	20.95
1.01	201
1.001	2001

x	$f(x)$
5	$0.8\overline{3}$
10	$0.\overline{40}$
100	0.04
1000	0.004

(b) The zeros of the denominator are $x = \pm 1$, so both $x = 1$ and $x = -1$ are vertical asymptotes. The degree of the numerator is less than the degree of the denominator, so the x-axis, or $y = 0$, is a horizontal asymptote.

(c) The domain is all real numbers x except $x = \pm 1$.

9. $f(x) = \dfrac{4}{x^2}$

Domain: all real numbers x except $x = 0$

Vertical asymptote: $x = 0$

Horizontal asymptote: $y = 0$

$\left[\text{Degree of } N(x) < \text{degree of } D(x)\right]$

10. $f(x) = \dfrac{4}{(x - 2)^3}$

Domain: all real numbers x except $x = 2$

Vertical asymptote: $x = 2$

Horizontal asymptote: $y = 0$

$\left[\text{Degree of } N(x) < \text{degree of } D(x)\right]$

11. $f(x) = \dfrac{5 + x}{5 - x} = \dfrac{x + 5}{-x + 5}$

Domain: all real numbers x except $x = 5$

Vertical asymptote: $x = 5$

Horizontal asymptote: $y = -1$

$\left[\text{Degree of } N(x) = \text{degree of } D(x)\right]$

12. $f(x) = \dfrac{3 - 7x}{3 + 2x} = \dfrac{-7x + 3}{2x + 3}$

Domain: all real numbers x except $x = -\dfrac{3}{2}$

Vertical asymptote: $x = -\dfrac{3}{2}$

Horizontal asymptote: $y = -\dfrac{7}{2}$

$\left[\text{Degree of } N(x) = \text{degree of } D(x)\right]$

13. $f(x) = \dfrac{x^3}{x^2 - 1}$

Domain: all real numbers x except $x = \pm 1$

Vertical asymptotes: $x = \pm 1$

Horizontal asymptote: None

$\left[\text{Degree of } N(x) > \text{ degree of } D(x)\right]$

14. $f(x) = \dfrac{4x^2}{x + 2}$

Domain: all real numbers x except $x = -2$

Vertical asymptote: $x = -2$

Horizontal asymptote: None

$\left[\text{Degree of } N(x) = \text{ degree of } D(x)\right]$

15. $f(x) = \dfrac{3x^2 + 1}{x^2 + x + 9}$

Domain: All real numbers x. The denominator has no real zeros. [Try the Quadratic Formula on the denominator.]

Vertical asymptote: None

Horizontal asymptote: $y = 3$

$\left[\text{Degree of } N(x) = \text{ degree of } D(x)\right]$

16. $f(x) = \dfrac{3x^2 + x - 5}{x^2 + 1}$

Domain: All real numbers x. The denominator has no real zeros. [Try the Quadratic Formula on the denominator.]

Vertical asymptote: None

Horizontal asymptote: $y = 3$

$\left[\text{Degree of } N(x) = \text{ degree of } D(x)\right]$

17. $f(x) = \dfrac{4}{x + 5}$

Vertical asymptote: $x = -5$

Horizontal asymptote: $y = 0$

Matches graph (d).

18. $f(x) = \dfrac{5}{x - 2}$

Vertical asymptote: $x = 2$

Horizontal asymptote: $y = 0$

Matches graph (a).

19. $f(x) = \dfrac{x - 1}{x - 4}$

Vertical asymptote: $x = 4$

Horizontal asymptote: $y = 1$

Matches graph (c).

20. $f(x) = -\dfrac{x + 2}{x + 4}$

Vertical asymptote: $x = -4$

Horizontal asymptote: $y = -1$

Matches graph (b).

21. $g(x) = \dfrac{x^2 - 9}{x + 3} = \dfrac{(x + 3)(x - 3)}{x + 3}$

The only zero of $g(x)$ is $x = 3$. $x = -3$ makes $g(x)$ undefined.

22.

$$\begin{aligned} h(x) &= 4 + \frac{10}{x^2 + 5} \\ 0 &= 4 + \frac{10}{x^2 + 5} \\ -4 &= \frac{10}{x^2 + 5} \\ -4(x^2 + 5) &= 10 \\ -4x^2 &= 30 \\ x^2 &= -\frac{15}{2} \end{aligned}$$

No real solution, $h(x)$ has no real zeros.

23.

$$\begin{aligned} f(x) &= 1 - \frac{2}{x - 7} \\ 0 &= 1 - \frac{2}{x - 7} \\ \frac{2}{x - 7} &= 1 \\ x - 7 &= 2 \\ x &= 9 \end{aligned}$$

$x = 9$ is a zero of $f(x)$.

24.

$$\begin{aligned} g(x) &= \frac{x^3 - 8}{x^2 + 1} \\ \frac{x^3 - 8}{x^2 + 1} &= 0 \\ x^3 - 8 &= 0 \\ x^3 &= 8 \\ x &= 2 \end{aligned}$$

$x = 2$ is a real zero of $g(x)$.

25. $f(x) = \dfrac{x - 4}{x^2 - 16} = \dfrac{x - 4}{(x - 4)(x + 4)} = \dfrac{1}{x + 4}, x \neq 4$

Domain: all real numbers x except $x = \pm 4$

Vertical asymptote: $x = -4$ (Because $x - 4$ is a common factor of $N(x)$ and $D(x)$, $x = 4$ is not a vertical asymptote of $f(x)$.)

Horizontal asymptote: $y = 0$

[Degree of $N(x) <$ degree of $D(x)$]

26. $f(x) = \dfrac{x + 1}{x^2 - 1} = \dfrac{x + 1}{(x + 1)(x - 1)} = \dfrac{1}{x - 1}, x \neq -1$

Domain: all real numbers x except $x = \pm 1$

Vertical asymptote: $x = 1$ (Because $x + 1$ is a common factor of $N(x)$ and $D(x)$, $x = -1$ is not a vertical asymptote of $f(x)$.)

Horizontal asymptote: $y = 0$

[Degree of $N(x) <$ degree of $D(x)$]

27. $f(x) = \dfrac{x^2 - 25}{x^2 - 4x - 5} = \dfrac{(x + 5)(x - 5)}{(x - 5)(x + 1)} = \dfrac{x + 5}{x + 1}, x \neq 5$

Domain: all real numbers x except $x = 5$ and $x = -1$

Vertical asymptote: $x = -1$ (Because $x - 5$ is a common factor of $N(x)$ and $D(x)$, $x = 5$ is not a vertical asymptote of $f(x)$.)

Horizontal asymptote: $y = 1$

[Degree of $N(x) =$ degree of $D(x)$]

28. $f(x) = \dfrac{x^2 - 4}{x^2 - 3x + 2}$

$= \dfrac{(x - 2)(x + 2)}{(x - 2)(x - 1)} = \dfrac{x + 2}{x - 1}, x \neq 2$

Domain: all real numbers x except $x = 1$ and $x = 2$

Vertical asymptote: $x = 1$ (Because $x - 2$ is a common factor of $N(x)$ and $D(x)$, $x = 2$ is not a vertical asymptote of $f(x)$.)

Horizontal asymptote: $y = 1$

[Degree of $N(x) =$ degree of $D(x)$]

29. $f(x) = \dfrac{x^2 - 3x - 4}{2x^2 + x - 1}$

$= \dfrac{(x + 1)(x - 4)}{(2x - 1)(x + 1)} = \dfrac{x - 4}{2x - 1}, x \neq -1$

Domain: all real numbers x except $x = \dfrac{1}{2}$ and $x = -1$

Vertical asymptote: $x = \dfrac{1}{2}$ (Because $x + 1$ is a common factor of $N(x)$ and $D(x)$, $x = -1$ is not a vertical asymptote of $f(x)$.)

Horizontal asymptote: $y = \dfrac{1}{2}$

[Degree of $N(x) =$ degree of $D(x)$]

30. $f(x) = \dfrac{6x^2 - 11x + 3}{6x^2 - 7x - 3}$

$= \dfrac{(2x - 3)(3x - 1)}{(2x - 3)(3x + 1)} = \dfrac{3x - 1}{3x + 1}, x \neq \dfrac{3}{2}$

Domain: all real numbers x except

$x = \dfrac{3}{2}$ or $x = -\dfrac{1}{3}$

Vertical asymptote: $x = -\dfrac{1}{3}$ (Because $2x - 3$ is a common factor of $N(x)$ and $D(x)$, $x = \dfrac{3}{2}$ is not a vertical asymptote of $f(x)$.)

Horizontal asymptote: $y = 1$

[Degree of $N(x) =$ degree of $D(x)$]

31. $f(x) = \dfrac{1}{x + 2}$

(a) Domain: all real numbers x except $x = -2$

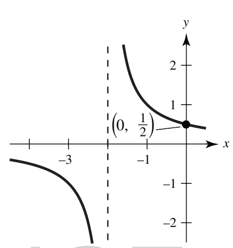
(b) y-intercept: $\left(0, \dfrac{1}{2}\right)$

(c) Vertical asymptote: $x = -2$

Horizontal asymptote: $y = 0$

(d)

x	-4	-3	-1	0	1
y	$-\frac{1}{2}$	-1	1	$\frac{1}{2}$	$\frac{1}{3}$



32. $f(x) = \dfrac{1}{x - 3}$

(a) Domain: all real numbers x except $x = 3$

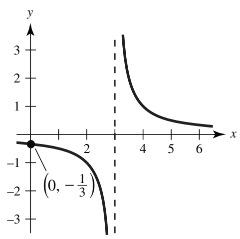
(b) y-intercept: $\left(0, -\dfrac{1}{3}\right)$

(c) Vertical asymptote: $x = 3$

Horizontal asymptote: $y = 0$

(d)

x	0	1	2	4	5	6
y	$-\frac{1}{3}$	$-\frac{1}{2}$	–1	1	$\frac{1}{2}$	$\frac{1}{3}$



33. $h(x) = \dfrac{-1}{x + 4}$

(a) Domain: all real numbers x except $x = -4$

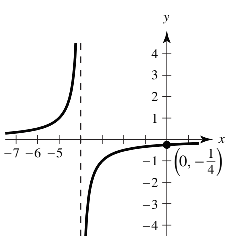
(b) y-intercept: $\left(0, -\dfrac{1}{4}\right)$

(c) Vertical asymptote: $x = -4$

Horizontal asymptote: $y = 0$

(d)

x	–5	–3	–1	1
y	1	–1	$-\frac{1}{3}$	$-\frac{1}{5}$



34. $g(x) = \dfrac{1}{6 - x} = -\dfrac{1}{x - 6}$

(a) Domain: all real numbers x except $x = 6$

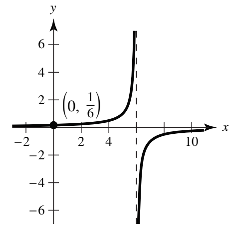
(b) y-intercept: $\left(0, \dfrac{1}{6}\right)$

(c) Vertical asymptote: $x = 6$

Horizontal asymptote: $y = 0$

(d)

x	4	5	7	8
y	$\frac{1}{2}$	1	–1	$-\frac{1}{2}$



35. $C(x) = \dfrac{7 + 2x}{2 + x} = \dfrac{2x + 7}{x + 2}$

(a) Domain: all real numbers x except $x = -2$

(b) x-intercept: $\left(-\dfrac{7}{2}, 0\right)$

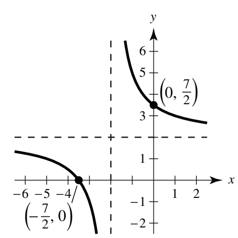
y-intercept: $\left(0, \dfrac{7}{2}\right)$

(c) Vertical asymptote: $x = -2$

Horizontal asymptote: $y = 2$

(d)

x	–3	–1	1	3
y	–1	5	3	$\frac{13}{5}$



36. $P(x) = \dfrac{1 - 3x}{1 - x} = \dfrac{3x - 1}{x - 1}$

(a) Domain: all real numbers x except $x = 1$

(b) x-intercept: $\left(\dfrac{1}{3}, 0\right)$

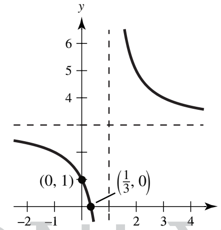
y-intercept: $(0, 1)$

(c) Vertical asymptote: $x = 1$

Horizontal asymptote: $y = 3$

(d)

x	–1	0	2	3
y	2	1	5	4



37. $f(x) = \dfrac{x^2}{x^2 + 9}$

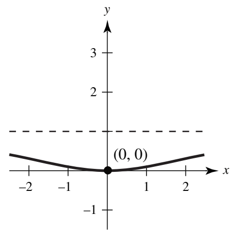
(a) Domain: all real numbers x

(b) Intercept: $(0, 0)$

(c) Horizontal asymptote: $y = 1$

(d)

x	± 1	± 2	± 3
y	$\frac{1}{10}$	$\frac{4}{13}$	$\frac{1}{2}$



38. $f(t) = \dfrac{1 - 2t}{t} = -\dfrac{2t - 1}{t}$

(a) Domain: all real numbers t except $t = 0$

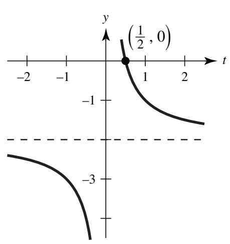
(b) t-intercept: $\left(\frac{1}{2}, 0\right)$

(c) Vertical asymptote: $t = 0$

Horizontal asymptote: $y = -2$

(d)

t	-2	-1	$\frac{1}{2}$	1	2
y	$-\frac{5}{2}$	-3	0	-1	$-\frac{3}{2}$



39. $g(s) = \dfrac{4s}{s^2 + 4}$

(a) Domain: all real numbers s

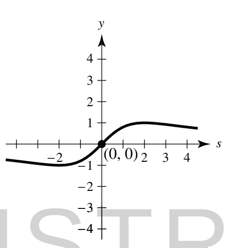
(b) Intercept: $(0, 0)$

(c) Vertical asymptote: none

Horizontal asymptote: $y = 0$

(d)

s	-2	-1	0	1	2
y	-1	$-\frac{4}{5}$	0	$\frac{4}{5}$	1



40. $f(x) = -\dfrac{1}{(x - 2)^2}$

(a) Domain: all real numbers x except $x = 2$

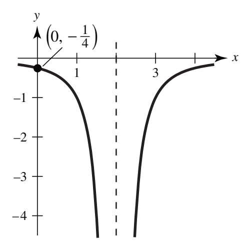
(b) y-intercept: $\left(0, -\frac{1}{4}\right)$

(c) Vertical asymptote: $x = 2$

Horizontal asymptote: $y = 0$

(d)

x	0	$\frac{1}{2}$	1	$\frac{3}{2}$	$\frac{5}{2}$	3	$\frac{7}{2}$	4
y	$-\frac{1}{4}$	$-\frac{4}{9}$	-1	-4	-4	-1	$-\frac{4}{9}$	$-\frac{1}{4}$



41. $h(x) = \dfrac{x^2 - 5x + 4}{x^2 - 4} = \dfrac{(x - 1)(x - 4)}{(x + 2)(x - 2)}$

(a) Domain: all real numbers x except $x = \pm 2$

(b) x-intercepts: $(1, 0), (4, 0)$

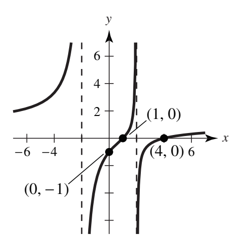
y-intercept: $(0, -1)$

(c) Vertical asymptotes: $x = -2, x = 2$

Horizontal asymptote: $y = 1$

(d)

x	-4	-3	-1	0	1	3	4
y	$\frac{10}{3}$	$\frac{28}{5}$	$-\frac{10}{3}$	-1	0	$-\frac{2}{5}$	0



42. $g(x) = \dfrac{x^2 - 2x - 8}{x^2 - 9} = \dfrac{(x-4)(x+2)}{(x-3)(x+3)}$

(a) Domain: all real numbers x except $x = \pm 3$

(b) x-intercepts: $(4, 0), (-2, 0)$

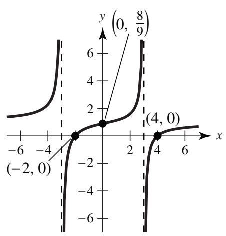
y-intercept: $\left(0, \dfrac{8}{9}\right)$

(c) Vertical asymptotes: $x = 3, x = -3$

Horizontal asymptote: $y = 1$

(d)

x	–5	–4	–2	0	2	4	5
y	$\frac{27}{16}$	$\frac{16}{7}$	0	$\frac{8}{9}$	$\frac{8}{5}$	0	$\frac{7}{16}$



43. $f(x) = \dfrac{2x^2 - 5x - 3}{x^2 - 2x^2 - x + 2} = \dfrac{(2x+1)(x-3)}{(x-2)(x+1)(x-1)}$

(a) Domain: all real numbers x except $x = 2, x = 1$, and $x = -1$

(b) x-intercept: $\left(-\dfrac{1}{2}, 0\right), (3, 0)$

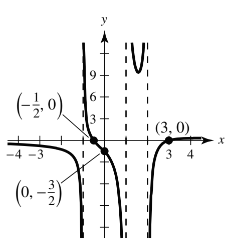
y-intercept: $\left(0, -\dfrac{3}{2}\right)$

(c) Vertical asymptotes: $x = 2, x = -1, x = 1$

Horizontal asymptote: $y = 0$

(d)

x	–3	–2	0	1.5	3	4
y	$-\frac{3}{4}$	$-\frac{5}{4}$	$-\frac{3}{2}$	$\frac{48}{5}$	0	$\frac{3}{10}$



44. $f(x) = \dfrac{x^2 - x - 2}{x^3 - 2x^2 - 5x + 6} = \dfrac{(x+1)(x-2)}{(x-1)(x+2)(x-3)}$

(a) Domain: all real numbers x except $x = 1, x = -2$, and $x = 3$

(b) x-intercepts: $(-1, 0), (2, 0)$

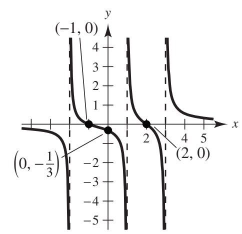
y-intercept: $\left(0, -\dfrac{1}{3}\right)$

(c) Vertical asymptotes: $x = -2, x = 1, x = 3$

Horizontal asymptote: $y = 0$

(d)

x	–4	–3	–1	0	2	4
y	$-\frac{9}{35}$	$-\frac{5}{12}$	0	$-\frac{1}{3}$	0	$\frac{5}{9}$



45. $f(x) = \dfrac{x^2 + 3x}{x^2 + x - 6} = \dfrac{x(x+3)}{(x+3)(x-2)} = \dfrac{x}{x-2}, x \neq -3$

(a) Domain: all real numbers x except $x = -3$ and $x = 2$

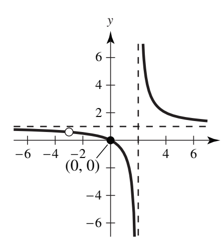
(b) Intercept: $(0, 0)$

(c) Vertical asymptote: $x = 2$

Horizontal asymptote: $y = 1$

(d)

x	–1	0	1	3	4
y	$\frac{1}{3}$	0	–1	3	2



46. $f(x) = \dfrac{5(x+4)}{x^2+x-12} = \dfrac{5(x+4)}{(x+4)(x-3)} = \dfrac{5}{x-3}, x \neq -4$

(a) Domain: all real numbers x except $x = -4$ and $x = 3$

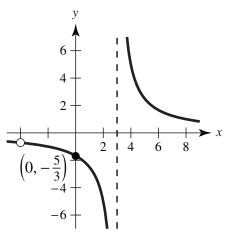
(b) y-intercept: $\left(0, -\dfrac{5}{3}\right)$

(c) Vertical asymptote: $x = 3$

Horizontal asymptote: $y = 0$

(d)

x	-2	0	2	5	7
y	-1	$-\frac{5}{3}$	-5	$\frac{5}{2}$	$\frac{5}{4}$



47. $f(x) = \dfrac{2x^2-5x+2}{2x^2-x-6}$

$= \dfrac{(2x-1)(x-2)}{(2x+3)(x-2)} = \dfrac{2x-1}{2x+3}, x \neq 2$

(a) Domain: all real numbers x except $x = 2$ and $x = -\dfrac{3}{2}$

(b) x-intercept: $\left(\dfrac{1}{2}, 0\right)$

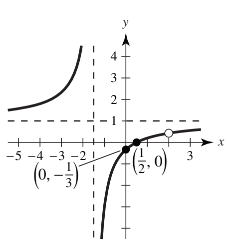
y-intercept: $\left(0, -\dfrac{1}{3}\right)$

(c) Vertical asymptote: $x = -\dfrac{3}{2}$

Horizontal asymptote: $y = 1$

(d)

x	-3	-2	-1	0	1
y	$\frac{7}{3}$	5	-3	$-\frac{1}{3}$	$\frac{1}{5}$



48. $f(x) = \dfrac{3x^2-8x+4}{2x^2-3x-2}$

$= \dfrac{(x-2)(3x-2)}{(x-2)(2x+1)} = \dfrac{3x-2}{2x+1}, x \neq 2$

(a) Domain: all real numbers x except $x = 2$ and $x = -\dfrac{1}{2}$

(b) x-intercept: $\left(\dfrac{2}{3}, 0\right)$

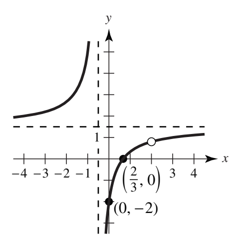
y-intercept: $(0, -2)$

(c) Vertical asymptote: $x = -\dfrac{1}{2}$

Horizontal asymptote: $y = \dfrac{3}{2}$

(d)

x	-3	-1	0	$\frac{2}{3}$	3
y	$\frac{11}{5}$	5	-2	0	1



49. $f(t) = \dfrac{t^2-1}{t-1} = \dfrac{(t-1)(t+1)}{t-1} = t+1; t \neq 1$

(a) Domain: all real numbers t except $t = 1$

(b) t-intercept: $(-1, 0)$

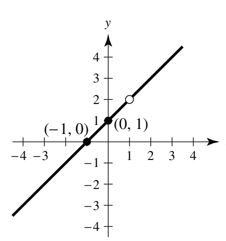
y-intercept: $(0, 1)$

(c) Vertical asymptote: none

Horizontal asymptote: none

(d)

t	-1	0	2	3
y	0	1	3	4



50. $f(x) = \dfrac{x^2 - 36}{x + 6} = \dfrac{(x + 6)(x - 6)}{x + 6} = x - 6;\ x \neq -6$

(a) Domain: all real numbers x except $x = -6$

(b) x-intercept: $(6, 0)$

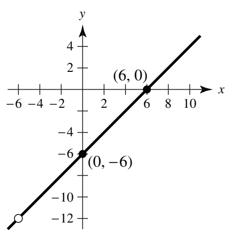
y-intercept: $(0, -6)$

(c) Vertical asymptote: none

Horizontal asymptote: none

(d)

x	1	2	3	4
y	–5	–4	–3	–2



51. $f(x) = \dfrac{x^2 - 1}{x + 1},\ g(x) = x - 1$

(a) Domain of f: all real numbers x except $x = -1$

Domain of g: all real numbers x

(b) $f(x) = \dfrac{x^2 - 1}{x + 1}$

$= \dfrac{(x + 1)(x - 1)}{x + 1}$

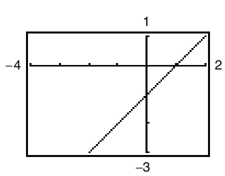
$= x - 1$

Because $x + 1$ is a factor of both the numerator and the denominator of f, $x = -1$ is not a vertical asymptote. So, f has no vertical asymptotes.

(c)

x	–3	–2	–1.5	–1	–0.5	0	1
$f(x)$	–4	–3	–2.5	Undef.	–1.5	–1	0
$g(x)$	–4	–3	–2.5	–2	–1.5	–1	0

(d)



(e) Because there are only a finite number of pixels, the utility may not attempt to evaluate the function where it does not exist.

52. $f(x) = \dfrac{x^2(x - 2)}{x^2 - 2x},\ g(x) = x$

(a) Domain of f: All real numbers x except $x = 0$ and $x = 2$

Domain of g: All real numbers x

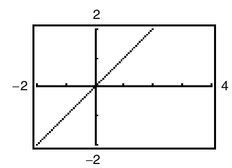
(b) $f(x) = \dfrac{x^2(x - 2)}{x^2 - 2x} = \dfrac{x^2(x - 2)}{x(x - 2)} = x$

Because $x(x - 2)$ is a factor of both the numerator and the denominator of f, neither $x = 0$ nor $x = 2$ is a vertial asymptote of f. So, f has no vertical asymptotes.

(c)

x	–1	0	1	1.5	2	2.5	3
$f(x)$	–1	Undef.	1	1.5	Undef.	2.5	3
$g(x)$	–1	0	1	1.5	2	2.5	3

(d)



(e) Because there are only a finite number of pixels, the utility may not attempt to evaluate the function where it does not exist.

53. $f(x) = \dfrac{x - 2}{x^2 - 2x},\ g(x) = \dfrac{1}{x}$

(a) Domain of f: All real numbers x except $x = 0$ and $x = 2$

Domain of g: All real numbers x except $x = 0$

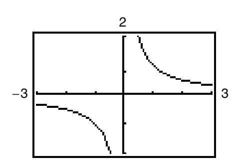
(b) $f(x) = \dfrac{x - 2}{x^2 - 2x} = \dfrac{x - 2}{x(x - 2)} = \dfrac{1}{x}$

Because $x - 2$ is a factor of both the numerator and the denominator of f, $x = 2$ is not a vertical asymptote. The only vertical asymptote of f is $x = 0$.

(c)

x	–0.5	0	0.5	1	1.5	2	3
$f(x)$	–2	Undef.	2	1	$\frac{2}{3}$	Undef.	$\frac{1}{3}$
$g(x)$	–2	Undef.	2	1	$\frac{2}{3}$	$\frac{1}{2}$	$\frac{1}{3}$

(d)



(e) Because there are only a finite number of pixels, the utility may not attempt to evaluate the function where it does not exist.

54. $f(x) = \dfrac{2x - 6}{x^2 - 7x + 12}, g(x) = \dfrac{2}{x - 4}$

(a) Domain of *f*: All real numbers *x* except $x = 3$ and $x = 4$

Domain of *g*: All real numbers *x* except $x = 4$

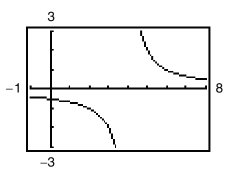
(b) $$f(x) = \frac{2x - 6}{x^2 - 7x + 12} = \frac{2(x - 3)}{(x - 3)(x - 4)} = \frac{2}{x - 4}$$

Because $x - 3$ is a factor of both the numerator and the denominator of f, $x = 3$ is not a vertical asymptote of *f*. So, *f* has $x = 4$ as its only vertical asymptote.

(c)

x	0	1	2	3	4	5	6
$f(x)$	$-\frac{1}{2}$	$-\frac{2}{3}$	–1	Undef.	Undef.	2	1
$g(x)$	$-\frac{1}{2}$	$-\frac{2}{3}$	–1	–2	Undef.	2	1

(d)



(e) Because there are only a finite number of pixels, the utility may not attempt to evaluate the function where it does not exist.

55. $h(x) = \dfrac{x^2 - 9}{x} = x - \dfrac{9}{x}$

(a) Domain: all real numbers *x* except $x = 0$

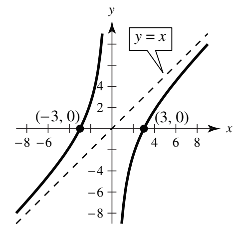
(b) *x*-intercepts: $(3, 0), (-3, 0)$

(c) Vertical asymptote: $x = 0$

Slant asymptote: $y = x$

(d)

x	–2	–1	1	2
y	$\frac{5}{2}$	8	–8	$-\frac{5}{2}$



56. $g(x) = \dfrac{x^2 + 5}{x} = x + \dfrac{5}{x}$

(a) Domain: all real numbers *x* except $x = 0$

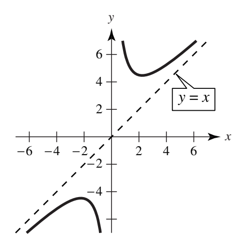
(b) No intercepts

(c) Vertical asymptote: $x = 0$

Slant asymptote: $y = x$

(d)

x	–3	–2	–1	1	2	3
y	$-\frac{14}{3}$	$-\frac{9}{2}$	–6	6	$\frac{9}{2}$	$\frac{14}{3}$



57. $f(x) = \dfrac{2x^2 + 1}{x} = 2x + \dfrac{1}{x}$

(a) Domain: all real numbers *x* except $x = 0$

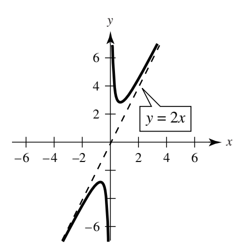
(b) No intercepts

(c) Vertical asymptote: $x = 0$

Slant asymptote: $y = 2x$

(d)

x	–4	–2	2	4	6
y	$-\frac{33}{4}$	$-\frac{9}{2}$	$\frac{9}{2}$	$\frac{33}{4}$	$\frac{73}{6}$



58. $f(x) = \frac{1 - x^2}{x} = -x + \frac{1}{x}$

(a) Domain: all real numbers x except $x = 0$

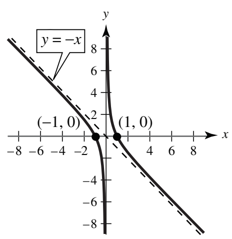
(b) x-intercepts: $(-1, 0), (1, 0)$

(c) Vertical asymptote: $x = 0$

Slant asymptote: $y = -x$

(d)

x	-6	-4	-2	2	4	6
y	$\frac{35}{6}$	$\frac{15}{4}$	$\frac{3}{2}$	$-\frac{3}{2}$	$-\frac{15}{4}$	$-\frac{35}{6}$



59. $g(x) = \frac{x^2 + 1}{x} = x + \frac{1}{x}$

(a) Domain: all real numbers x except $x = 0$

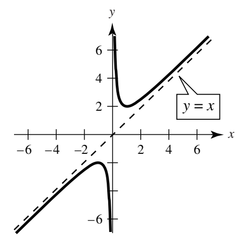
(b) No intercepts

(c) Vertical asymptote: $x = 0$

Slant asymptote: $y = x$

(d)

x	-4	-2	2	4	6
y	$-\frac{17}{4}$	$-\frac{5}{2}$	$\frac{5}{2}$	$\frac{17}{4}$	$\frac{37}{6}$



60. $h(x) = \frac{x^2}{x - 1} = x + 1 + \frac{1}{x - 1}$

(a) Domain: all real numbers x except $x = 1$

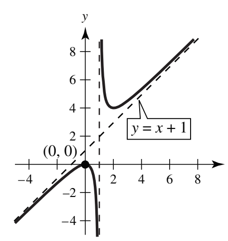
(b) Intercept: $(0, 0)$

(c) Vertical asymptote: $x = 1$

Slant asymptote: $y = x + 1$

(d)

x	-4	-2	2	4	6
y	$-\frac{16}{5}$	$-\frac{4}{3}$	4	$\frac{16}{3}$	$\frac{36}{5}$



61. $f(t) = -\frac{t^2 + 1}{t + 5} = -t + 5 - \frac{26}{t + 5}$

(a) Domain: all real numbers t except $t = -5$

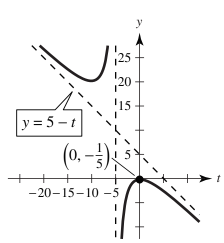
(b) y-intercept: $\left(0, -\frac{1}{5}\right)$

(c) Vertical asymptote: $t = -5$

Slant asymptote: $y = -t + 5$

(d)

t	-7	-6	-4	-3	0
y	25	37	-17	-5	$-\frac{1}{5}$



62. $f(x) = \frac{x^2}{3x + 1} = \frac{1}{3}x - \frac{1}{9} + \frac{1}{9(3x + 1)}$

(a) Domain: all real numbers x except $x = -\frac{1}{3}$

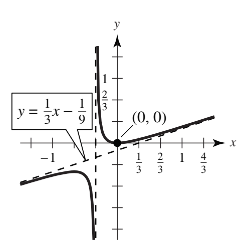
(b) Intercept: $(0, 0)$

(c) Vertical asymptote: $x = -\frac{1}{3}$

Slant asymptote: $y = \frac{1}{3}x - \frac{1}{9}$

(d)

x	-3	-2	-1	$-\frac{1}{2}$	0	2
y	$-\frac{9}{8}$	$-\frac{4}{5}$	$-\frac{1}{2}$	$-\frac{1}{2}$	0	$\frac{4}{7}$



63. $f(x) = \dfrac{x^3}{x^2 - 4} = x + \dfrac{4x}{x^2 - 4}$

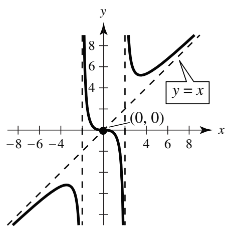
(a) Domain: all real numbers x except $x = \pm 2$

(b) Intercept: $(0, 0)$

(c) Vertical asymptotes: $x = 2$ and $x = -2$
Slant asymptote: $y = x$

(d)

x	-3	-1	1	3
y	$-\frac{27}{5}$	$\frac{1}{3}$	$-\frac{1}{3}$	$\frac{27}{5}$



64. $g(x) = \dfrac{x^3}{2x^2 - 8} = \dfrac{1}{2}x + \dfrac{4x}{2x^2 - 8}$

(a) Domain: all real numbers x except $x = \pm 2$

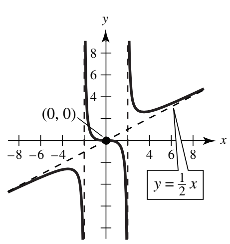
(b) Intercept: $(0, 0)$

(c) Vertical asymptote: $x = \pm 2$

Slant asymptote: $y = \dfrac{1}{2}x$

(d)

x	-6	-4	-1	1	4	6
y	$-\frac{27}{8}$	$-\frac{8}{3}$	$\frac{1}{6}$	$-\frac{1}{6}$	$\frac{8}{3}$	$\frac{27}{8}$



65. $f(x) = \dfrac{x^2 - x + 1}{x - 1} = x + \dfrac{1}{x - 1}$

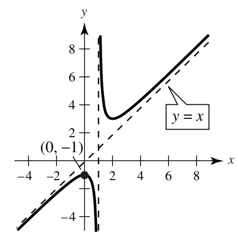
(a) Domain: all real numbers x except $x = 1$

(b) y-intercept: $(0, -1)$

(c) Vertical asymptote: $x = 1$
Slant asymptote: $y = x$

(d)

x	-4	-2	0	2	4
y	$-\frac{21}{5}$	$-\frac{7}{3}$	-1	3	$\frac{13}{3}$



66. $f(x) = \dfrac{2x^2 - 5x + 5}{x - 2} = 2x - 1 + \dfrac{3}{x - 2}$

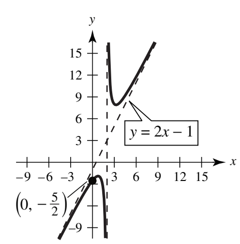
(a) Domain: all real numbers x except $x = 2$

(b) y-intercept: $\left(0, -\dfrac{5}{2}\right)$

(c) Vertical asymptote: $x = 2$
Slant asymptote: $y = 2x - 1$

(d)

x	-6	-3	1	3	6	7
y	$-\frac{107}{8}$	$-\frac{38}{5}$	-2	8	$\frac{47}{4}$	$\frac{68}{5}$



67. $f(x) = \dfrac{2x^3 - x^2 - 2x + 1}{x^2 + 3x + 2}$

$= \dfrac{(2x - 1)(x + 1)(x - 1)}{(x + 1)(x + 2)}$

$= \dfrac{(2x - 1)(x - 1)}{x + 2}, \quad x \neq -1$

$= \dfrac{2x^2 - 3x + 1}{x + 2}$

$= 2x - 7 + \dfrac{15}{x + 2}, \quad x \neq -1$

(a) Domain: all real numbers x except $x = -1$ and $x = -2$

(b) y-intercept: $\left(0, \frac{1}{2}\right)$

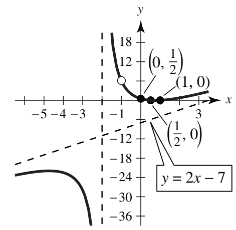
x-intercepts: $\left(\frac{1}{2}, 0\right), (1, 0)$

(c) Vertical asymptote: $x = -2$

Slant asymptote: $y = 2x - 7$

(d)

x	-4	-3	$-\frac{3}{2}$	0	1
y	$-\frac{45}{2}$	-28	20	$\frac{1}{2}$	0



68. $f(x) = \dfrac{2x^3 + x^2 - 8x - 4}{x^2 - 3x + 2}$

$= \dfrac{(x - 2)(x + 2)(2x + 1)}{(x - 2)(x - 1)}$

$= \dfrac{(x + 2)(2x + 1)}{x - 1}, x \neq 2$

$= \dfrac{2x^2 + 5x + 2}{x - 1}$

$= 2x + 7 + \dfrac{9}{x - 1}, x \neq 2$

(a) Domain: all real numbers x except $x = 1$ and $x = 2$

(b) y-intercept: $(0, -2)$

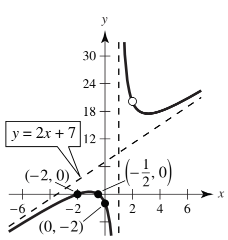
x-intercepts: $(-2, 0), \left(-\frac{1}{2}, 0\right)$

(c) Vertical asymptote: $x = 1$

Slant asymptote: $y = 2x + 7$

(d)

x	-3	-2	-1	0	$\frac{1}{2}$	$\frac{3}{2}$	3	4
y	$-\frac{5}{4}$	0	$\frac{1}{2}$	-2	-10	28	$\frac{35}{2}$	18



69. Domain: All real numbers

One possibility: $f(x) = \dfrac{1}{x^2 + 2}$

Domain: All real numbers except $x = 2$.

One possibility: $f(x) = \dfrac{1}{x - 2}$

(Answers are not unique).

70. An asymptote is a line to which a graph gets arbitrarily close to, but does not reach, as $|x|$ or $|y|$ increases without bound.

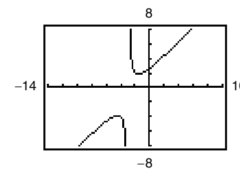
71. $f(x) = \dfrac{x^2 + 5x + 8}{x + 3} = x + 2 + \dfrac{2}{x + 3}$

Domain: all real numbers x except $x = -3$

Vertical asymptote: $x = -3$

Slant asymptote: $y = x + 2$

Line: $y = x + 2$



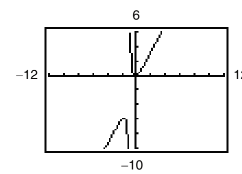
72. $f(x) = \dfrac{2x^2 + x}{x + 1} = 2x - 1 + \dfrac{1}{x + 1}$

Domain: all real numbers x except $x = -1$

Vertical asymptote: $x = -1$

Slant asymptote: $y = 2x - 1$

Line: $y = 2x - 1$



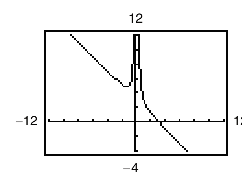
73. $g(x) = \dfrac{1 + 3x^2 - x^3}{x^2} = \dfrac{1}{x^2} + 3 - x = -x + 3 + \dfrac{1}{x^2}$

Domain: all real numbers x except $x = 0$

Vertical asymptote: $x = 0$

Slant asymptote: $y = -x + 3$

Line: $y = -x + 3$



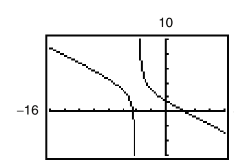
74. $h(x) = \dfrac{12 - 2x - x^2}{2(4 + x)} = -\dfrac{1}{2}x + 1 + \dfrac{2}{4 + x}$

Domain: all real numbers x except $x = -4$

Vertical asymptote: $x = -4$

Slant asymptote: $y = -\dfrac{1}{2}x + 1$

Line: $y = -\dfrac{1}{2}x + 1$



75. $y = \dfrac{x+1}{x-3}$

(a) x-intercept: $(-1, 0)$

(b) $0 = \dfrac{x+1}{x-3}$

$0 = x + 1$

$-1 = x$

76. $y = \dfrac{2x}{x-3}$

(a) x-intercept: $(0, 0)$

(b) $0 = \dfrac{2x}{x-3}$

$0 = 2x$

$0 = x$

77. $y = \dfrac{1}{x} - x$

(a) x-intercepts: $(1, 0), (-1, 0)$

(b) $0 = \dfrac{1}{x} - x$

$x = \dfrac{1}{x}$

$x^2 = 1$

$x = \pm 1$

78. $y = x - 3 + \dfrac{2}{x}$

(a) x-intercepts: $(1, 0), (2, 0)$

(b) $0 = x - 3 + \dfrac{2}{x}$

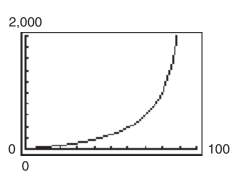
$0 = x^2 - 3x + 2$

$0 = (x-1)(x-2)$

$x = 1, x = 2$

79. $C = \dfrac{255p}{100 - p}, 0 \le p < 100$

(a)



(b) $C(10) = \dfrac{255(10)}{100 - 10} \approx 28.33$ million dollars

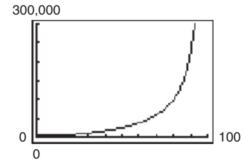
$C(40) = \dfrac{255(40)}{100 - 40} = 170$ million dollars

$C(75) = \dfrac{255(75)}{100 - 75} = 765$ million dollars

(c) $C \to \infty$ as $x \to 100$. No. The function is undefined at $p = 100$.

80. $C = \dfrac{25{,}000p}{100 - p}, 0 \le p < 100$

(a)



(b) $C(15) = \dfrac{25{,}000(15)}{100 - 15} \approx \$4411.76$

$C(50) = \dfrac{25{,}000(50)}{100 - 50} = \$25{,}000$

$C(90) = \dfrac{25{,}000(90)}{100 - 90} = \$225{,}000$

(c) $C \to \infty$ as $x \to 100$. No. The function is undefined for $p = 100$.

81. $N = \dfrac{20(5 + 3t)}{1 + 0.04t}, t \ge 0$

(a) $N(5) \approx 333$ deer

$N(10) = 500$ deer

$N(25) = 800$ deer

(b) The herd is limited by the horizontal asymptote:

$N = \dfrac{60}{0.04} = 1500$ deer

82. (a) $0.25(50) + 0.75(x) = C(50 + x)$

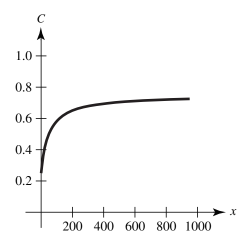
$C = \dfrac{12.50 + 0.75x}{50 + x} \cdot \dfrac{4}{4}$

$C = \dfrac{50 + 3x}{4(50 + x)} = \dfrac{3x + 50}{4(x + 50)}$

(b) Domain: $x \ge 0$ and $x \le 1000 - 50$

So, $0 \le x \le 950$. Using interval notation, the domain is $[0, 950]$.

(c)



(d) As the tank is filled, the concentration increases more slowly. It approaches the horizontal asymptote of $C = \dfrac{3}{4} = 0.75 = 75\%$.

83. (a) Let t_1 = time from Akron to Columbus and t_2 = time from Columbus back to Akron.

$$xt_1 = 100 \Rightarrow t_1 = \frac{100}{x}$$

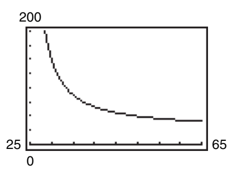
$$yt_2 = 100 \Rightarrow t_2 = \frac{100}{y}$$

$$50(t_1 + t_2) = 200$$
$$t_1 + t_2 = 4$$
$$\frac{100}{x} + \frac{100}{y} = 4$$
$$100y + 100x = 4xy$$
$$25y + 25x = xy$$
$$25x = xy - 25y$$
$$25x = y(x - 25)$$

So, $y = \dfrac{25x}{x - 25}$.

(b) Vertical asymptote: $x = 25$

Horizontal asymptote: $y = 25$

(c) 

(d)

x	30	35	40	45	50	55	60
y	150	87.5	66.7	56.3	50	45.8	42.9

(e) Sample answer: No. You might expect the average speed for the round trip to be the average of the average speeds for the two parts of the trip.

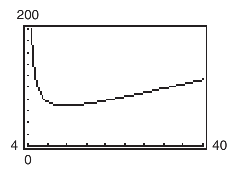
(f) No. At 20 miles per hour you would use more time in one direction than is required for the round trip at an average speed of 50 miles per hour.

84. (a) $A = xy$ and

$$(x - 4)(y - 2) = 30$$
$$y - 2 = \frac{30}{x - 4}$$
$$y = 2 + \frac{30}{x - 4} = \frac{2x + 22}{x - 4}$$

So, $A = xy = x\left(\dfrac{2x + 22}{x - 4}\right) = \dfrac{2x(x + 11)}{x - 4}$.

(b) Domain: Because the margins on the left and right are each 2 inches, $x > 4$. In interval notation, the domain is $(4, \infty)$.

(c) 

x	5	6	7	8	9	10
y_1(Area)	160	102	84	76	72	70

x	11	12	13	14	15
y_1(Area)	69.143	69	69.333	70	70.999

The area is minimum when $x \approx 11.75$ inches and $y \approx 5.87$ inches.

85. False. Polynomial functions do not have vertical asymptotes.

86. False. The graph of $f(x) = \dfrac{x}{x^2 + 1}$ crosses $y = 0$, which is a horizontal asymptote.

87. False. A graph can have a vertical asymptote and a horizontal asymptote or a vertical asymptote and a slant asymptote, but a graph cannot have both a horizontal asymptote and a slant asymptote.

A horizontal asymptote occurs when the degree of $N(x)$ is equal to the degree of $D(x)$ or when the degree of $N(x)$ is less than the degree of $D(x)$. A slant asymptote occurs when the degree of $N(x)$ is greater than the degree of $D(x)$ by one. Because the degree of a polynomial is constant, it is impossible to have both relationships at the same time.

88. (a) $f(x) = \dfrac{x - 1}{x^3 - 8}$

(b) $f(x) = \dfrac{x - 2}{x^3 + 1}$

(c) $f(x) = \dfrac{2(x^2 - 9)}{(x + 2)(x - 1)} = \dfrac{2x^2 - 18}{x^2 + x - 2}$

(d) $f(x) = \dfrac{-2(x + 2)(x - 3)}{(x + 1)(x - 2)} = \dfrac{-2x^2 + 2x + 12}{x^2 - x - 2}$

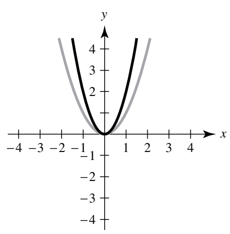
89. No; Yes;

Not every rational function is a polynomial because $g(x) = \dfrac{1}{x}$ and $h(x) = \dfrac{3}{x + 2}$ are rational functions, but they are not polynomials. Every polynomial $f(x)$ is a rational function because it can be written as $\dfrac{f(x)}{1}$.

Review Exercises for Chapter 2

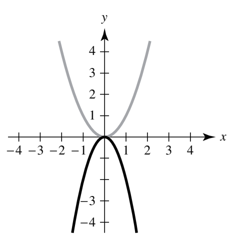
1. (a) $y = 2x^2$

Vertical stretch



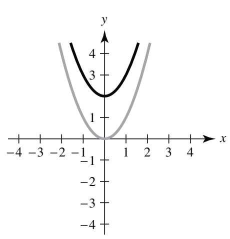
(b) $y = -2x^2$

Vertical stretch and a reflection in the x-axis



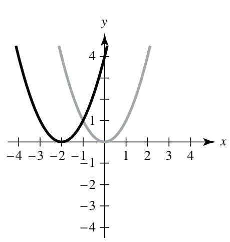
(c) $y = x^2 + 2$

Vertical shift two units upward



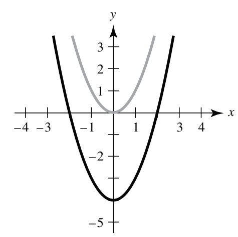
(d) $y = (x + 2)^2$

Horizontal shift two units to the left



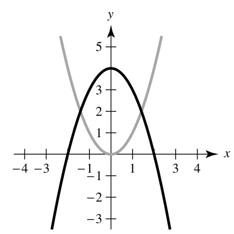
2. (a) $y = x^2 - 4$

Vertical shift four units downward



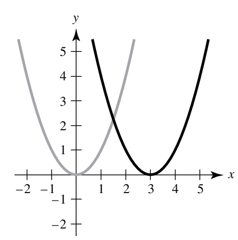
(b) $y = 4 - x^2$

Reflection in the x-axis and a vertical shift four units upward



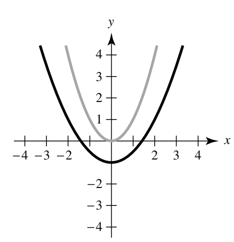
(c) $y = (x - 3)^2$

Horizontal shift three units to the right



(d) $y = \frac{1}{2}x^2 - 1$

Vertical shrink $\left(\text{each } y\text{-value is multiplied by } \frac{1}{2}\right)$, and a vertical shift one unit downward



3. $g(x) = x^2 - 2x$

$= x^2 - 2x + 1 - 1$

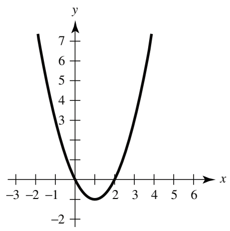
$= (x - 1)^2 - 1$

Vertex: $(1, -1)$

Axis of symmetry: $x = 1$

$0 = x^2 - 2x = x(x - 2)$

x-intercepts: $(0, 0), (2, 0)$



4. $f(x) = 6x - x^2$

$= -(x^2 - 6x + 9 - 9)$

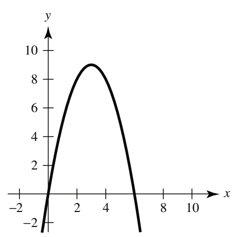
$= -(x - 3)^2 + 9$

Vertex: $(3, 9)$

Axis of symmetry: $x = 3$

$0 = 6x - x^2 = x(6 - x)$

x-intercepts: $(0, 0), (6, 0)$



5. $f(x) = x^2 + 8x + 10$

$= x^2 + 8x + 16 - 16 + 10$

$= (x + 4)^2 - 6$

Vertex: $(-4, -6)$

Axis of symmetry: $x = -4$

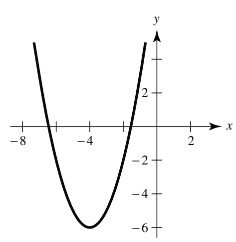
$0 = (x + 4)^2 - 6$

$(x + 4)^2 = 6$

$x + 4 = \pm\sqrt{6}$

$x = -4 \pm \sqrt{6}$

x-intercepts: $\left(-4 \pm \sqrt{6}, 0\right)$



6. $h(x) = 3 + 4x - x^2$

$= -(x^2 - 4x - 3)$

$= -(x^2 - 4x + 4 - 4 - 3)$

$= -\left[(x - 2)^2 - 7\right]$

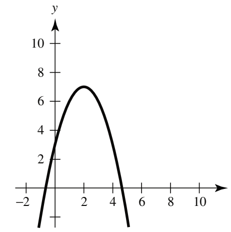
$= -(x - 2)^2 + 7$

Vertex: $(2, 7)$

Axis of symmetry: $x = 2$

$0 = 3 + 4x - x^2$

$0 = x^2 - 4x - 3$



$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(-3)}}{2(1)}$$

$$= \frac{4 \pm \sqrt{28}}{2} = 2 \pm \sqrt{7}$$

x-intercepts: $\left(2 \pm \sqrt{7}, 0\right)$

7. $f(t) = -2t^2 + 4t + 1$

$= -2(t^2 - 2t + 1 - 1) + 1$

$= -2\left[(t - 1)^2 - 1\right] + 1$

$= -2(t - 1)^2 + 3$

Vertex: $(1, 3)$

Axis of symmetry: $t = 1$

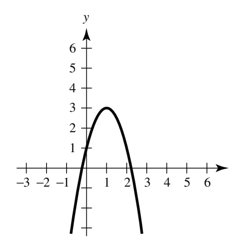
$0 = -2(t - 1)^2 + 3$

$2(t - 1)^2 = 3$

$$t - 1 = \pm\sqrt{\frac{3}{2}}$$

$$t = 1 \pm \frac{\sqrt{6}}{2}$$

t-intercepts: $\left(1 \pm \dfrac{\sqrt{6}}{2}, 0\right)$



8. $f(x) = x^2 - 8x + 12$

$= x^2 - 8x + 16 - 16 + 12$

$= (x - 4)^2 - 4$

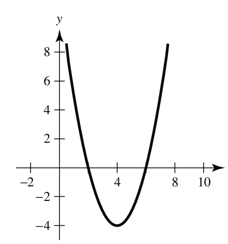
Vertex: $(4, -4)$

Axis of symmetry: $x = 4$

$0 = x^2 - 8x + 12$

$0 = (x - 2)(x - 6)$

x-intercepts: $(2, 0), (6, 0)$



9. $h(x) = 4x^2 + 4x + 13$

$= 4(x^2 + x) + 13$

$= 4\left(x^2 + x + \frac{1}{4} - \frac{1}{4}\right) + 13$

$= 4\left(x^2 + x + \frac{1}{4}\right) - 1 + 13$

$= 4\left(x + \frac{1}{2}\right)^2 + 12$

Vertex: $\left(-\frac{1}{2}, 12\right)$

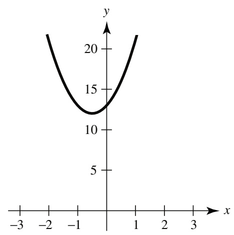
Axis of symmetry: $x = -\frac{1}{2}$

$0 = 4\left(x + \frac{1}{2}\right)^2 + 12$

$\left(x + \frac{1}{2}\right)^2 = -3$

No real zeros

x-intercepts: none



10. $f(x) = x^2 - 6x + 1$

$= x^2 - 6x + 9 - 9 + 1$

$= (x - 3)^2 - 8$

Vertex: $(3, -8)$

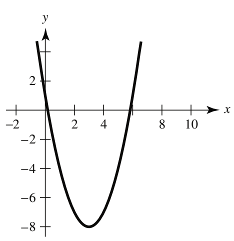
Axis of symmetry: $x = 3$

$0 = x^2 - 6x + 1$

$x = \dfrac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(1)}}{2(1)}$

$= \dfrac{6 \pm \sqrt{32}}{2} = 3 \pm 2\sqrt{2}$

x-intercepts: $\left(3 \pm 2\sqrt{2}, 0\right)$



11. $h(x) = x^2 + 5x - 4$

$= x^2 + 5x + \dfrac{25}{4} - \dfrac{25}{4} - 4$

$= \left(x + \dfrac{5}{2}\right)^2 - \dfrac{25}{4} - \dfrac{16}{4}$

$= \left(x + \dfrac{5}{2}\right)^2 - \dfrac{41}{4}$

Vertex: $\left(-\dfrac{5}{2}, -\dfrac{41}{4}\right)$

Axis of symmetry:

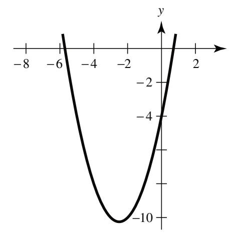
$x = -\dfrac{5}{2}$

$0 = x^2 + 5x - 4$

$x = \dfrac{-5 \pm \sqrt{5^2 - 4(1)(-4)}}{2(1)}$

$= \dfrac{-5 \pm \sqrt{41}}{2}$

x-intercepts: $\left(\dfrac{-5 \pm \sqrt{41}}{2}, 0\right)$



12. $f(x) = 4x^2 + 4x + 5$

$= 4\left(x^2 + x + \dfrac{1}{4} - \dfrac{1}{4} + \dfrac{5}{4}\right)$

$= 4\left[\left(x + \dfrac{1}{2}\right)^2 + 1\right]$

$= 4\left(x + \dfrac{1}{2}\right)^2 + 4$

Vertex: $\left(-\dfrac{1}{2}, 4\right)$

Axis of symmetry: $x = -\dfrac{1}{2}$

$0 = 4x^2 + 4x + 5$

$x = \dfrac{-4 \pm \sqrt{4^2 - 4(4)(5)}}{2(4)}$

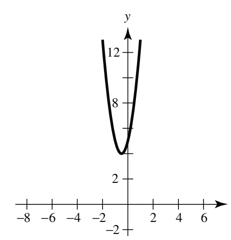
$= \dfrac{-4 \pm \sqrt{-64}}{8}$

$= \dfrac{-4 \pm 8i}{8}$

$= -\dfrac{1}{2} \pm i$

The equation has no real zeros.

x-intercepts: none



13. $f(x) = \frac{1}{3}(x^2 + 5x - 4)$

$$= \frac{1}{3}\left(x^2 + 5x + \frac{25}{4} - \frac{25}{4} - 4\right)$$

$$= \frac{1}{3}\left[\left(x + \frac{5}{2}\right)^2 - \frac{41}{4}\right]$$

$$= \frac{1}{3}\left(x + \frac{5}{2}\right)^2 - \frac{41}{12}$$

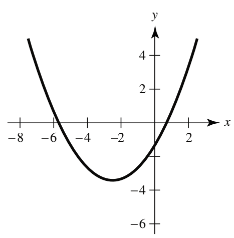
Vertex: $\left(-\frac{5}{2}, -\frac{41}{12}\right)$

Axis of symmetry: $x = -\frac{5}{2}$

$0 = x^2 + 5x - 4$

$$x = \frac{-5 \pm \sqrt{5^2 - 4(1)(-4)}}{2(1)} = \frac{-5 \pm \sqrt{41}}{2}$$

x-intercepts: $\left(\frac{-5 \pm \sqrt{41}}{2}, 0\right)$



14. $f(x) = \frac{1}{2}(6x^2 - 24x + 22)$

$= 3x^2 - 12x + 11$

$= 3(x^2 - 4x + 4 - 4) + 11$

$= 3(x - 2)^2 + 3(-4) + 11$

$= 3(x - 2)^2 - 1$

Vertex: $(2, -1)$

Axis of symmetry: $x = 2$

$0 = 3x^2 - 12x + 11$

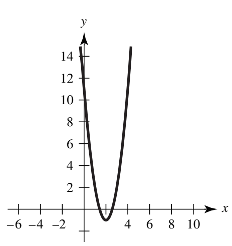
$$x = \frac{-(-12) \pm \sqrt{(-12)^2 - 4(3)(11)}}{2(3)}$$

$$= \frac{12 \pm \sqrt{12}}{6}$$

$$= 2 \pm \frac{\sqrt{3}}{3}$$

x-intercepts:

$\left(2 \pm \frac{\sqrt{3}}{3}, 0\right)$



15. Vertex: $(4, 1) \Rightarrow f(x) = a(x - 4)^2 + 1$

Point: $(2, -1) \Rightarrow -1 = a(2 - 4)^2 + 1$

$-2 = 4a$

$-\frac{1}{2} = a$

$f(x) = -\frac{1}{2}(x - 4)^2 + 1$

16. Vertex: $(2, 2) \Rightarrow f(x) = a(x - 2)^2 + 2$

Point: $(0, 3) \Rightarrow 3 = a(0 - 2)^2 + 2$

$3 = 4a + 2$

$1 = 4a$

$\frac{1}{4} = a$

$f(x) = \frac{1}{4}(x - 2)^2 + 2$

17. Vertex: $(1, -4) \Rightarrow f(x) = a(x - 1)^2 - 4$

Point: $(2, -3) \Rightarrow -3 = a(2 - 1)^2 - 4$

$1 = a$

$f(x) = (x - 1)^2 - 4$

18. Vertex: $(2, 3) \Rightarrow f(x) = a(x - 2)^2 + 3$

Point: $(-1, 6) \Rightarrow 6 = a(-1 - 2)^2 + 3$

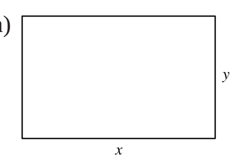
$6 = 9a + 3$

$3 = 9a$

$\frac{1}{3} = a$

$f(x) = \frac{1}{3}(x - 2)^2 + 3$

19. (a)



(b) $x + x + y + y = P$

$2x + 2y = 1000$

$y = 500 - x$

$A = xy$

$= x(500 - x)$

$= 500x - x^2$

(c) $A = 500x - x^2$

$= -(x^2 - 500x + 62{,}500) + 62{,}500$

$= -(x - 250)^2 + 62{,}500$

The maximum area occurs at the vertex when $x = 250$ and $y = 500 - 250 = 250$.

The dimensions with the maximum area are $x = 250$ meters and $y = 250$ meters.

20. $R = -10p^2 + 800p$

(a) $R(20) = \$12{,}000$

$R(25) = \$13{,}750$

$R(30) = \$15{,}000$

(b) The maximum revenue occurs at the vertex of the parabola.

$$-\frac{b}{2a} = \frac{-800}{2(-10)} = \$40$$

$R(40) = \$16{,}000$

The revenue is maximum when the price is \$40 per unit.

The maximum revenue is \$16,000.

Any price greater or less than \$40 per unit will not yield as much revenue.

21. $C = 70{,}000 - 120x + 0.055x^2$

The minimum cost occurs at the vertex of the parabola.

Vertex: $-\frac{b}{2a} = -\frac{-120}{2(0.055)} \approx 1091$ units

About 1091 units should be produced each day to yield a minimum cost.

22. $26 = -0.107x^2 + 5.68x - 48.5$

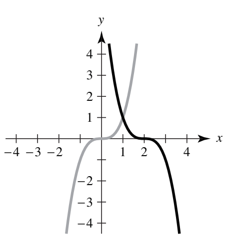
$0 = -0.107x^2 + 5.68x - 74.5$

$$x = \frac{-5.68 \pm \sqrt{(5.68)^2 - 4(-0.107)(-74.5)}}{2(-0.107)}$$

$x \approx 23.7, 29.4$

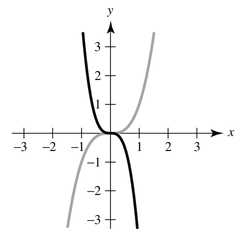
The age of the bride is about 24 years when the age of the groom is 26 years.

23. $y = x^3,\ f(x) = -(x-2)^3$



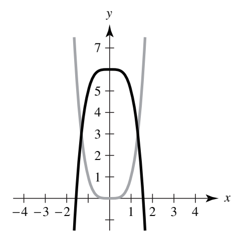
Transformation: Reflection in the x-axis and a horizontal shift two units to the right

24. $y = x^3,\ f(x) = -4x^3$



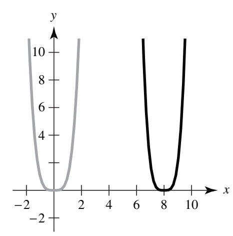
Transformation: Reflection in the x-axis and a vertical stretch

25. $y = x^4,\ f(x) = 6 - x^4$



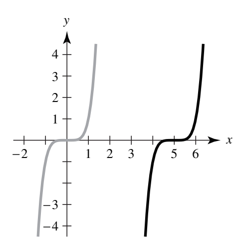
Transformation: Reflection in the x-axis and a vertical shift six units upward

26. $y = x^4,\ f(x) = 2(x-8)^4$



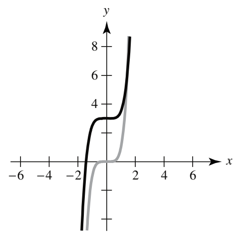
Transformation: Horizontal shift eight units to the right and a vertical stretch

27. $y = x^5,\ f(x) = (x-5)^5$



Transformation: Horizontal shift five units to the right

28. $y = x^5,\ f(x) = \frac{1}{2}x^5 + 3$



Transformation: Vertical shrink and a vertical shift three units upward

29. $f(x) = -2x^2 - 5x + 12$

The degree is even and the leading coefficient is negative. The graph falls to the left and falls to the right.

30. $f(x) = \frac{1}{2}x^3 + 2x$

The degree is odd and the leading coefficient is positive. The graph falls to the left and rises to the right.

31. $g(x) = \frac{3}{4}(x^4 + 3x^2 + 2)$

The degree is even and the leading coefficient is positive. The graph rises to the left and rises to the right.

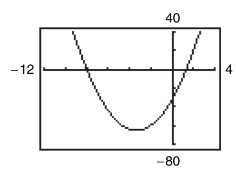
32. $f(x) = -x^7 + 8x^2 - 8x$

The degree is odd and the leading coefficient is negative. The graph rises to the left and falls to the right.

33. $f(x) = 3x^2 + 20x - 32$

$0 = 3x^2 + 20x - 32$

$0 = (3x - 4)(x + 8)$



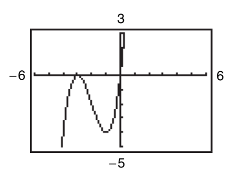
Zeros: $x = \frac{4}{3}$ and $x = -8$,

both of multiplicity 1 (odd multiplicity)

Turning points: 1

34. $f(x) = x(x + 3)^2$

$0 = x(x + 3)^2$



Zeros: $x = 0$ of multiplicity 1 (odd multiplicity)

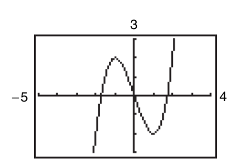
$x = -3$ of multiplicity 2 (even multiplicity)

Turning points: 2

35. $f(t) = t^3 - 3t$

$0 = t^3 - 3t$

$0 = t(t^2 - 3)$



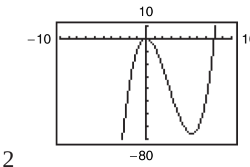
Zeros: $t = 0, \pm\sqrt{3}$, all of multiplicity 1 (odd multiplicity)

Turning points: 2

36. $f(x) = x^3 - 8x^2$

$0 = x^3 - 8x^2$

$0 = x^2(x - 8)$



Zeros: $x = 0$ of multiplicity 2 (even multiplicity)

$x = 8$ of multiplicity 1 (odd multiplicity)

Turning points: 2

37. $f(x) = -18x^3 + 12x^2$

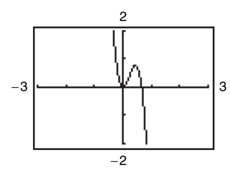
$0 = -18x^3 + 12x^2$

$0 = -6x^2(3x - 2)$

Zeros: $x = \frac{2}{3}$ of multiplicity 1 (odd multiplicity)

$x = 0$ of multiplicity 2 (even multiplicity)

Turning points: 2



38. $g(x) = x^4 + x^3 - 12x^2$

$0 = x^4 + x^3 - 12x^2$

$0 = x^2(x^2 + x - 12)$

$0 = x^2(x + 4)(x - 3)$

Zeros: $x = 0$ of multiplicity 2 (even multiplicity)

$x = -4$ of multiplicity 1 (odd multiplicity)

$x = 3$ of multiplicity 1 (odd multiplicity)

Turning points: 3

39. $f(x) = -x^3 + x^2 - 2$

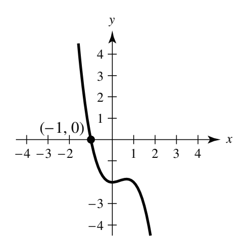
(a) The degree is odd and the leading coefficient is negative. The graph rises to the left and falls to the right.

(b) Zero: $x = -1$

(c)

x	−3	−2	−1	0	1	2
$f(x)$	34	10	0	−2	−2	−6

(d)



40. $g(x) = 2x^3 + 4x^2$

(a) The degree is odd and the leading coefficient, 2, is positive. The graph falls to the left and rises to the right.

(b) $g(x) = 2x^3 + 4x^2$

$0 = 2x^3 + 4x^2$

$0 = 2x^2(x + 2)$

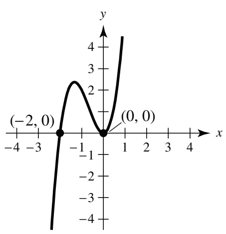
$0 = x^2(x + 2)$

Zeros: $x = -2, 0$

(c)

x	-3	-2	-1	0	1
$g(x)$	-18	0	2	0	6

(d)



41. $f(x) = x(x^3 + x^2 - 5x + 3)$

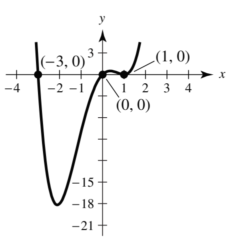
(a) The degree is even and the leading coefficient is positive. The graph rises to the left and rises to the right.

(b) Zeros: $x = 0, 1, -3$

(c)

x	-4	-3	-2	-1	0	1	2	3
$f(x)$	100	0	-18	-8	0	0	10	72

(d)



42. $h(x) = 3x^2 - x^4$

(a) The degree is even and the leading coefficient, -1, is negative. The graph falls to the left and falls to the right.

(b) $g(x) = 3x^2 - x^4$

$0 = 3x^2 - x^4$

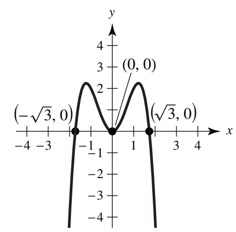
$0 = x^2(3 - x^2)$

Zeros: $x = 0, \pm\sqrt{3}$

(c)

x	-2	-1	0	1	2
$h(x)$	-4	2	0	2	-4

(d)



43.

$$\begin{array}{r}
6x + 3 \\
5x - 3\overline{)30x^2 - 3x + 8} \\
\underline{30x^2 - 18x} \\
15x + 8 \\
\underline{15x - 9} \\
17
\end{array}$$

$$\frac{30x^2 - 3x + 8}{5x - 3} = 6x + 3 + \frac{17}{5x - 3}$$

44.

$$\begin{array}{r}
\frac{4}{3} \\
3x - 2\overline{)4x + 7} \\
\underline{4x - \frac{8}{3}} \\
\frac{29}{3}
\end{array}$$

$$\frac{4x + 7}{3x - 2} = \frac{4}{3} + \frac{29}{3(3x - 2)}$$

45.

$$\begin{array}{r}
5x + 4 \\
x^2 - 5x - 1\overline{)5x^3 - 21x^2 - 25x - 4} \\
\underline{5x^3 - 25x^2 - 5x} \\
4x^2 - 20x - 4 \\
\underline{4x^2 - 20x - 4} \\
0
\end{array}$$

$$\frac{5x^3 - 21x^2 - 25x - 4}{x^2 - 5x - 1} = 5x + 4,\ x \neq \frac{5}{2} \pm \frac{\sqrt{29}}{2}$$

46.

$$\begin{array}{r}
3x^2 + 3 \\
x^2 - 1\overline{)3x^4 + 0x^3 + 0x^2 + 0x + 0} \\
\underline{3x^4 - 3x^2} \\
3x^2 + 0 \\
\underline{3x^2 - 3} \\
3
\end{array}$$

$$\frac{3x^4}{x^2 - 1} = 3x^2 + 3 + \frac{3}{x^2 - 1}$$

47.
$$\begin{array}{r} x^2 - 3x + 2 \\ x^2 + 0x + 2 \overline{\smash{)} x^4 - 3x^3 + 4x^2 - 6x + 3} \\ \underline{x^4 + 0x^3 + 2x^2} \qquad\qquad \\ -3x^3 + 2x^2 - 6x \quad \\ \underline{-3x^3 + 0x^2 - 6x} \quad \\ 2x^2 + 0x + 3 \\ \underline{2x^2 + 0x + 4} \\ -1 \end{array}$$

$$\frac{x^4 - 3x^3 + 4x^2 - 6x + 3}{x^2 + 2} = x^2 - 3x + 2 - \frac{1}{x^2 + 2}$$

48.
$$\begin{array}{r} 3x^2 + 5x + 8 \\ 2x^2 + 0x - 1 \overline{\smash{)} 6x^4 + 10x^3 + 13x^2 - 5x + 2} \\ \underline{6x^4 + 0x^3 - 3x^2} \qquad\qquad \\ 10x^3 + 16x^2 - 5x \quad \\ \underline{10x^3 + 0x^2 - 5x} \quad \\ 16x^2 + 0x + 2 \\ \underline{16x^2 + 0x - 8} \\ 10 \end{array}$$

$$\frac{6x^4 + 10x^3 + 13x^2 - 5x + 2}{2x^2 - 1} = 3x^2 + 5x + 8 + \frac{10}{2x^2 - 1}$$

49.
$$\begin{array}{r|rrrrr} 2 & 6 & -4 & -27 & 18 & 0 \\ & & 12 & 16 & -22 & -8 \\ \hline & 6 & 8 & -11 & -4 & -8 \end{array}$$

$$\frac{6x^4 - 4x^3 - 27x^2 + 18x}{x - 2} = 6x^3 + 8x^2 - 11x - 4 - \frac{8}{x - 2}$$

50.
$$\begin{array}{r|rrrr} 5 & 0.1 & 0.3 & 0 & -0.5 \\ & & 0.5 & 4 & 20 \\ \hline & 0.1 & 0.8 & 4 & 19.5 \end{array}$$

$$\frac{0.1x^3 + 0.3x^2 - 0.5}{x - 5} = 0.1x^2 + 0.8x + 4 + \frac{19.5}{x - 5}$$

51.
$$\begin{array}{r|rrrr} 8 & 2 & -25 & 66 & 48 \\ & & 16 & -72 & -48 \\ \hline & 2 & -9 & -6 & 0 \end{array}$$

$$\frac{2x^3 - 25x^2 + 66x + 48}{x - 8} = 2x^2 - 9x - 6, \quad x \neq 8$$

52.
$$\begin{array}{r|rrrr} -4 & 5 & 33 & 50 & -8 \\ & & -20 & -52 & 8 \\ \hline & 5 & 13 & -2 & 0 \end{array}$$

$$\frac{5x^3 + 33x^2 + 50x - 8}{x + 4} = 5x^2 + 13x - 2, x \neq -4$$

53. $f(x) = 20x^4 + 9x^3 - 14x^2 - 3x$

(a)
$$\begin{array}{r|rrrrr} -1 & 20 & 9 & -14 & -3 & 0 \\ & & -20 & 11 & 3 & 0 \\ \hline & 20 & -11 & -3 & 0 & 0 \end{array}$$

Yes, $x = -1$ is a zero of *f*.

(b)
$$\begin{array}{r|rrrrr} \frac{3}{4} & 20 & 9 & -14 & -3 & 0 \\ & & 15 & 18 & 3 & 0 \\ \hline & 20 & 24 & 4 & 0 & 0 \end{array}$$

Yes, $x = \frac{3}{4}$ is a zero of *f*.

(c)
$$\begin{array}{r|rrrrr} 0 & 20 & 9 & -14 & -3 & 0 \\ & & 0 & 0 & 0 & 0 \\ \hline & 20 & 9 & -14 & -3 & 0 \end{array}$$

Yes, $x = 0$ is a zero of *f*.

(d)
$$\begin{array}{r|rrrrr} 1 & 20 & 9 & -14 & -3 & 0 \\ & & 20 & 29 & 15 & 12 \\ \hline & 20 & 29 & 15 & 12 & 12 \end{array}$$

No, $x = 1$ is not a zero of *f*.

54. $f(x) = 3x^3 - 8x^2 - 20x + 16$

(a)

4	3	–8	–20	16
		12	16	–16
	3	4	–4	0

Yes, $x = 4$ is a zero of *f*.

(b)

–4	3	–8	–20	16
		–12	80	–240
	3	–20	60	–224

No, $x = -4$ is not a zero of *f*.

(c)

$\frac{2}{3}$	3	–8	–20	16
		2	–4	–16
	3	–6	–24	0

Yes, $x = \frac{2}{3}$ is a zero of *f*.

(d)

–1	3	–8	–20	16
		–3	11	9
	3	–11	–9	25

No, $x = -1$ is not a zero of *f*.

55. $f(x) = x^4 + 10x^3 - 24x^2 + 20x + 44$

(a) Remainder Theorem:

$$f(-3) = (-3)^4 + 10(-3)^3 - 24(-3)^2 + 20(-3) + 44$$
$$= -421$$

Synthetic Division:

–3	1	10	–24	20	44
		–3	–21	135	–465
	1	7	–45	155	–421

So, $f(-3) = -421$.

(b) Remainder Theorem:

$$f(-1) = (-1)^4 + 10(-1)^3 - 24(-1)^2 + 20(-1) + 44$$
$$= -9$$

Synthetic Division:

–1	1	10	–24	20	44
		–1	–9	33	–53
	1	9	–33	53	–9

So, $f(-1) = -9$.

56. $g(t) = 2t^5 - 5t^4 - 8t + 20$

(a) Remainder Theorem:

$$g(-4) = 2(-4)^5 - 5(-4)^4 - 8(-4) + 20 = -3276$$

Synthetic Division:

–4	2	–5	0	0	–8	20
		–8	52	–208	832	–3296
	2	–13	52	–208	824	–3276

So, $g(-4) = -3276$.

(b) Remainder Theorem:

$$g(\sqrt{2}) = 2(\sqrt{2})^5 - 5(\sqrt{2})^4 - 8(\sqrt{2}) + 20 = 0$$

Synthetic Division:

$\sqrt{2}$	2	–5	0	0	–8	20
		$2\sqrt{2}$	$-5\sqrt{2} + 4$	$-10 + 4\sqrt{2}$	$-10\sqrt{2} + 8$	–20
	2	$-5 + 2\sqrt{2}$	$-5\sqrt{2} + 4$	$-10 + 4\sqrt{2}$	$-10\sqrt{2}$	0

So, $g(\sqrt{2}) = 0$.

57. $f(x) = x^3 + 4x^2 - 25x - 28$; Factor: $(x - 4)$

(a)

4	1	4	–25	–28
		4	32	28
	1	8	7	0

Yes, $(x - 4)$ is a factor of $f(x)$.

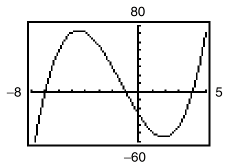
(b) $x^2 + 8x + 7 = (x + 7)(x + 1)$

The remaining factors are $(x + 7)$ and $(x + 1)$.

(c) $f(x) = x^3 + 4x^2 - 25x - 28$
$= (x + 7)(x + 1)(x - 4)$

(d) Zeros: $-7, -1, 4$

(e)



58. $f(x) = 2x^3 + 11x^2 - 21x - 90$; Factor: $(x + 6)$

(a)

–6	2	11	–21	–90
		–12	6	90
	2	–1	–15	0

Yes, $(x + 6)$ is a factor of $f(x)$.

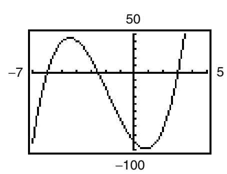
(b) $2x^2 - x - 15 = (2x + 5)(x - 3)$

The remaining factors are $(2x + 5)$ and $(x - 3)$.

(c) $f(x) = (2x + 5)(x - 3)(x + 6)$

(d) Zeros: $x = -\frac{5}{2}, 3, -6$

(e)



59. $f(x) = x^4 - 4x^3 - 7x^2 + 22x + 24$

Factors: $(x + 2), (x - 3)$

(a)

–2	1	–4	–7	22	24
		–2	12	–10	–24
	1	–6	5	12	0

3	1	–6	5	12
		3	–9	–12
	1	–3	–4	0

Yes, $(x + 2)$ and $(x - 3)$ are both factors of $f(x)$.

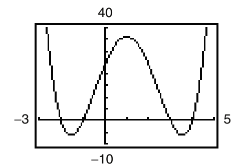
(b) $x^2 - 3x - 4 = (x + 1)(x - 4)$

The remaining factors are $(x + 1)$ and $(x - 4)$.

(c) $f(x) = (x + 1)(x - 4)(x + 2)(x - 3)$

(d) Zeros: $-2, -1, 3, 4$

(e)



60. $f(x) = x^4 - 11x^3 + 41x^2 - 61x + 30$

(a)

2	1	–11	41	–61	30
		2	–18	46	–30
	1	–9	23	–15	0

5	1	–9	23	–15
		5	–20	15
	1	–4	3	0

Yes, $(x - 2)$ and $(x - 5)$ are both factors of $f(x)$.

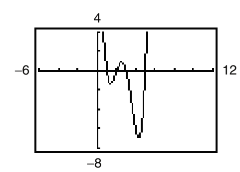
(b) $x^2 - 4x + 3 = (x - 1)(x - 3)$

The remaining factors are $(x - 1)$ and $(x - 3)$.

(c) $f(x) = (x - 1)(x - 3)(x - 2)(x - 5)$

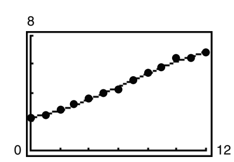
(d) Zeros: $x = 1, 2, 3, 5$

(e)



61. $A \approx -0.0022t^3 + 0.044t^2 + 0.17t + 2.3$

62.



The model is a good fit to the actual data.

63.

t	A, actual	A, cubic model
0	2.3	2.3
1	2.4	2.5
2	2.9	2.8
3	3.2	3.1
4	3.6	3.5
5	4.0	4.0
6	4.2	4.4
7	4.9	4.9
8	5.4	5.3
9	5.8	5.8
10	6.4	6.2
11	6.5	6.6
12	6.9	6.9

64.

18	–0.0022	0.044	0.17	2.3
		–0.0396	0.0792	4.4856
	–0.0022	0.0044	0.2492	6.7856

$A(18) \approx 6.8$ million

No, the model falls to the right as t increases since the degree is odd and the leading coefficient is negative.

65. $8 + \sqrt{-100} = 8 + 10i$

66. $5 - \sqrt{-49} = 5 - 7i$

67. $i^2 + 3i = -1 + 3i$

68. $-5i + i^2 = -1 - 5i$

69. $(7 + 5i) + (-4 + 2i) = (7 - 4) + (5i + 2i) = 3 + 7i$

70. $\left(\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i\right) - \left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i\right) = \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i - \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i = -2\left(\frac{\sqrt{2}}{2}i\right) = -\sqrt{2}i$

71. $7i(11 - 9i) = 77i - 63i^2 = 63 + 77i$

72.
$$\begin{aligned}(1 + 6i)(5 - 2i) &= 5 - 2i + 30i - 12i^2 \\ &= 5 + 28i + 12 \\ &= 17 + 28i\end{aligned}$$

73.
$$\begin{aligned}(10 - 8i)(2 - 3i) &= 20 - 30i - 16i + 24i^2 \\ &= 20 - 46i - 24 \\ &= -4 - 46i\end{aligned}$$

74.
$$\begin{aligned}i(6 + i)(3 - 2i) &= i(18 - 12i + 3i - 2i^2) \\ &= i(18 - 9i + 2) \\ &= i(20 - 9i) \\ &= 20i - 9i^2 \\ &= 9 + 20i\end{aligned}$$

75.
$$\begin{aligned}(8 - 5i)^2 &= 64 - 80i + 25i^2 \\ &= 64 - 80i - 25 \\ &= 39 - 80i\end{aligned}$$

76.
$$\begin{aligned}(4 + 7i)^2 + (4 - 7i)^2 &= 16 + 56i + 49i^2 + 16 - 56i + 49i^2 \\ &= 32 + 98i^2 \\ &= -66\end{aligned}$$

77.
$$\begin{aligned}\frac{6 + i}{4 - i} &= \frac{6 + i}{4 - i} \cdot \frac{4 + i}{4 + i} \\ &= \frac{24 + 10i + i^2}{16 + 1} \\ &= \frac{23 + 10i}{17} \\ &= \frac{23}{17} + \frac{10}{17}i\end{aligned}$$

78. $\frac{8 - 5i}{i} \cdot \frac{-i}{-i} = \frac{-8i + 5i^2}{-i^2} = \frac{-5 - 8i}{1} = -5 - 8i$

79.
$$\begin{aligned}\frac{4}{2 - 3i} + \frac{2}{1 + i} &= \frac{4}{2 - 3i} \cdot \frac{2 + 3i}{2 + 3i} + \frac{2}{1 + i} \cdot \frac{1 - i}{1 - i} \\ &= \frac{8 + 12i}{4 + 9} + \frac{2 - 2i}{1 + 1} \\ &= \frac{8}{13} + \frac{12}{13}i + 1 - i \\ &= \left(\frac{8}{13} + 1\right) + \left(\frac{12}{13}i - i\right) \\ &= \frac{21}{13} - \frac{1}{13}i\end{aligned}$$

80. $\frac{1}{2+i} - \frac{5}{1+4i} = \frac{(1+4i) - 5(2+i)}{(2+i)(1+4i)}$

$= \frac{1 + 4i - 10 - 5i}{2 + 8i + i + 4i^2}$

$= \frac{-9 - i}{-2 + 9i} \cdot \frac{(-2 - 9i)}{(-2 - 9i)}$

$= \frac{18 + 81i + 2i + 9i^2}{4 - 81i^2}$

$= \frac{9 + 83i}{85} = \frac{9}{85} + \frac{83}{85}i$

81. $5x^2 + 2 = 0$

$5x^2 = -2$

$x^2 = -\frac{2}{5}$

$x = \pm\sqrt{\frac{2}{5}}\, i$

$x = \pm\frac{\sqrt{10}}{5}\, i$

82. $2 + 8x^2 = 0$

$8x^2 = -2$

$x^2 = -\frac{1}{4}$

$x = \pm\frac{1}{2}i$

83. $x^2 - 2x + 10 = 0$

$x^2 - 2x + 1 = -10 + 1$

$(x - 1)^2 = -9$

$x - 1 = \pm\sqrt{-9}$

$x = 1 \pm 3i$

84. $6x^2 + 3x + 27 = 0$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-3 \pm \sqrt{3^2 - 4(6)(27)}}{2(6)}$

$= \frac{-3 \pm \sqrt{-639}}{12}$

$= \frac{-3 \pm 3i\sqrt{71}}{12} = -\frac{1}{4} \pm \frac{\sqrt{71}}{4}i$

85. $f(x) = 4x(x - 3)^2$

Zeros: $x = 0, 3$

86. $f(x) = (x - 4)(x + 9)^2$

Zeros: $x = -9, 4$

87. $f(x) = x^2 - 11x + 18$

$= (x - 2)(x - 9)$

Zeros: $x = 2, 9$

88. $f(x) = x^3 + 10x$

$= x(x^2 + 10)$

$= x(x + \sqrt{10}i)(x - \sqrt{10}i)$

Zeros: $x = 0, \pm\sqrt{10}i$

89. $f(x) = (x + 4)(x - 6)(x - 2i)(x + 2i)$

Zeros: $x = -4, 6, 2i, -2i$

90. $f(x) = (x - 8)(x - 5)^2(x - 3 + i)(x - 3 - i)$

Zeros: $x = 5, 8, 3 \pm i$

91. $f(x) = -4x^3 + 8x^2 - 3x + 15$

Possible rational zeros:
$\pm 1, \pm 3, \pm 5, \pm 15, \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{5}{2}, \pm\frac{15}{2}, \pm\frac{1}{4}, \pm\frac{3}{4}, \pm\frac{5}{4}, \pm\frac{15}{4}$

92. $f(x) = 3x^4 + 4x^3 - 5x^2 - 8$

Possible rational zeros: $\pm 1, \pm 2, \pm 4, \pm 8, \pm\frac{1}{3}, \pm\frac{2}{3}, \pm\frac{4}{3}, \pm\frac{8}{3}$

93. $f(x) = x^3 + 3x^2 - 28x - 60$

Possible rational zeros:
$\pm 1, \pm 2, \pm 3, \pm 4, \pm 5, \pm 6, \pm 10, \pm 12, \pm 15, \pm 20, \pm 30, \pm 60$

−2	1	3	−28	−60
		−2	−2	60
	1	1	−30	0

$x^3 + 3x^2 - 28x - 60 = (x + 2)(x^2 + x - 30)$

$= (x + 2)(x + 6)(x - 5)$

The zeros of $f(x)$ are $x = -2$, $x = -6$, and $x = 5$.

94. $f(x) = 4x^3 - 27x^2 + 11x + 42$

Possible rational zeros: $\pm\frac{1}{4}, \pm\frac{1}{2}, \pm\frac{3}{4}, \pm 1, \pm\frac{3}{2}, \pm\frac{7}{4},$

$\pm 2, \pm 3, \pm\frac{7}{2}, \pm\frac{21}{4}, \pm 6, \pm 7, \pm\frac{21}{2}, \pm 14, \pm 21, \pm 42$

−1	4	−27	11	42
		−4	31	−42
	4	−31	42	0

$4x^3 - 27x^2 + 11x + 42 = (x + 1)(4x^2 - 31x + 42)$

$= (x + 1)(x - 6)(4x - 7)$

The zeros of $f(x)$ are $x = -1$, $x = \frac{7}{4}$, and $x = 6$.

95. $f(x) = x^3 - 10x^2 + 17x - 8$

Possible rational zeros: $\pm 1, \pm 2, \pm 4, \pm 8$

$$\begin{array}{r|rrrr} 1 & 1 & -10 & 17 & -8 \\ & & 1 & -9 & 8 \\ \hline & 1 & -9 & 8 & 0 \end{array}$$

$$x^3 - 10x^2 + 17x - 8 = (x-1)(x^2 - 9x + 8) = (x-1)(x-1)(x-8) = (x-1)^2(x-8)$$

The zeros of $f(x)$ are $x = 1$ and $x = 8$.

96. $f(x) = x^3 + 9x^2 + 24x + 20$

Possible rational zeros: $\pm 1, \pm 2, \pm 4, \pm 5, \pm 10, \pm 20$

$$\begin{array}{r|rrrr} -5 & 1 & 9 & 24 & 20 \\ & & -5 & -20 & -20 \\ \hline & 1 & 4 & 4 & 0 \end{array}$$

$$\begin{aligned} x^3 + 9x^2 + 24x + 20 &= (x+5)(x^2 + 4x + 4) \\ &= (x+5)(x+2)^2. \end{aligned}$$

The zeros of $f(x)$ are $x = -5$ and $x = -2$.

97. $f(x) = x^4 + x^3 - 11x^2 + x - 12$

Possible rational zeros: $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12$

$$\begin{array}{r|rrrrr} 3 & 1 & 1 & -11 & 1 & -12 \\ & & 3 & 12 & 3 & 12 \\ \hline & 1 & 4 & 1 & 4 & 0 \end{array}$$

$$\begin{array}{r|rrrr} -4 & 1 & 4 & 1 & 4 \\ & & -4 & 0 & -4 \\ \hline & 1 & 0 & 1 & 0 \end{array}$$

$$x^4 + x^3 - 11x^2 + x - 12 = (x-3)(x+4)(x^2+1)$$

The zeros of $f(x)$ are $x = 3$ and $x = -4$.

98. $f(x) = 25x^4 + 25x^3 - 154x^2 - 4x + 24$

Possible rational zeros: $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 8, \pm 12, \pm 24, \pm\frac{1}{5}, \pm\frac{2}{5}, \pm\frac{3}{5}, \pm\frac{4}{5}, \pm\frac{6}{5}, \pm\frac{8}{5}, \pm\frac{12}{5},$
$\pm\frac{24}{5}, \pm\frac{1}{25}, \pm\frac{2}{25}, \pm\frac{3}{25}, \pm\frac{4}{25}, \pm\frac{6}{25}, \pm\frac{8}{25}, \pm\frac{12}{25}, \pm\frac{24}{25}$

$$\begin{array}{r|rrrrr} -3 & 25 & 25 & -154 & -4 & 24 \\ & & -75 & 150 & 12 & -24 \\ \hline & 25 & -50 & -4 & 8 & 0 \end{array}$$

$$\begin{array}{r|rrrr} 2 & 25 & -50 & -4 & 8 \\ & & 50 & 0 & -8 \\ \hline & 25 & 0 & -4 & 0 \end{array}$$

$$25x^4 + 25x^3 - 154x^2 - 4x + 24 = (x+3)(x-2)(25x^2 - 4) = (x+3)(x-2)(5x+2)(5x-2).$$

The zeros of $f(x)$ are $x = -3$, $x = 2$, and $x = \pm\dfrac{2}{5}$.

99. $f(x) = x^3 - 4x^2 + x - 4$, Zero: i

Because i is a zero, so is $-i$.

$$\begin{array}{r|rrrr} i & 1 & -4 & 1 & -4 \\ & & i & -1-4i & 4 \\ \hline & 1 & -4+i & -4i & 0 \end{array}$$

$$\begin{array}{r|rrr} -i & 1 & -4+i & -4i \\ & & -i & 4i \\ \hline & 1 & -4 & 0 \end{array}$$

$$f(x) = (x-i)(x+i)(x-4)$$

Zeros: $x = \pm i, 4$

100. $h(x) = -x^3 + 2x^2 - 16x + 32$

Because $-4i$ is a zero, so is $4i$.

$$\begin{array}{r|rrrr} -4i & -1 & 2 & -16 & 32 \\ & & 4i & 16-8i & -32 \\ \hline & -1 & 2+4i & -8i & 0 \end{array}$$

$$\begin{array}{r|rrr} 4i & -1 & 2+4i & -8i \\ & & -4i & 8i \\ \hline & -1 & 2 & 0 \end{array}$$

$$h(x) = (x+4i)(x-4i)(-x+2)$$

Zeros: $x = \pm 4i, 2$

101. $g(x) = 2x^4 - 3x^3 - 13x^2 + 37x - 15$, Zero: $2 + i$

Because $2 + i$ is a zero, so is $2 - i$.

$$\begin{array}{r|rrrrr} 2+i & 2 & -3 & -13 & 37 & -15 \\ & & 4+2i & 5i & -31-3i & 15 \\ \hline & 2 & 1+2i & -13+5i & 6-3i & 0 \end{array}$$

$$\begin{array}{r|rrrr} 2-i & 2 & 1+2i & -13+5i & 6-3i \\ & & 4-2i & 10-5i & -6+3i \\ \hline & 2 & 5 & -3 & 0 \end{array}$$

$$\begin{aligned} g(x) &= [x - (2 + i)][x - (2 - i)](2x^2 + 5x - 3) \\ &= (x - 2 - i)(x - 2 + i)(2x - 1)(x + 3) \end{aligned}$$

Zeros: $x = 2 \pm i, \frac{1}{2}, -3$

102. $$\begin{aligned} f(x) &= 4x^4 - 11x^3 + 14x^2 - 6x \\ &= x(4x^3 - 11x^2 + 14x - 6) \end{aligned}$$

One zero is $x = 0$. Because $1 - i$ is a zero, so is $1 + i$.

$$\begin{array}{r|rrrr} 1-i & 4 & -11 & 14 & -6 \\ & & 4-4i & -11+3i & 6 \\ \hline & 4 & -7-4i & 3+3i & 0 \end{array}$$

$$\begin{array}{r|rrr} 1+i & 4 & -7-4i & 3+3i \\ & & 4+4i & -3-3i \\ \hline & 4 & -3 & 0 \end{array}$$

$$\begin{aligned} f(x) &= x[x - (1 - i)][x - (1 + i)](4x - 3) \\ &= x(x - 1 + i)(x - 1 - i)(4x - 3) \end{aligned}$$

Zeros: $0, \frac{3}{4}, 1 + i, 1 - i$

103. $$\begin{aligned} f(x) &= x^3 + 4x^2 - 5x \\ &= x(x^2 + 4x - 5) \\ &= x(x + 5)(x - 1) \end{aligned}$$

Zeros: $x = 0, -5, 1$

104. $g(x) = x^3 - 7x^2 + 36$

$$\begin{array}{r|rrrr} -2 & 1 & -7 & 0 & 36 \\ & & -2 & 18 & -36 \\ \hline & 1 & -9 & 18 & 0 \end{array}$$

The zeros of $x^2 - 9x + 18 = (x - 3)(x - 6)$ are $x = 3, 6$. The zeros of $g(x)$ are $x = -2, 3, 6$.

$g(x) = (x + 2)(x - 3)(x - 6)$

105. $g(x) = x^4 + 4x^3 - 3x^2 + 40x + 208$, Zero: $x = -4$

$$\begin{array}{r|rrrrr} -4 & 1 & 4 & -3 & 40 & 208 \\ & & -4 & 0 & 12 & -208 \\ \hline & 1 & 0 & -3 & 52 & 0 \end{array}$$

$$\begin{array}{r|rrrr} -4 & 1 & 0 & -3 & 52 \\ & & -4 & 16 & -52 \\ \hline & 1 & -4 & 13 & 0 \end{array}$$

$g(x) = (x + 4)^2(x^2 - 4x + 13)$

By the Quadratic Formula the zeros of $x^2 - 4x + 13$ are $x = 2 \pm 3i$. The zeros of $g(x)$ are $x = -4$ and $x = 2 \pm 3i$.

$$\begin{aligned} g(x) &= (x + 4)^2[x - (2 + 3i)][x - (2 - 3i)] \\ &= (x + 4)^2(x - 2 - 3i)(x - 2 + 3i) \end{aligned}$$

106. $f(x) = x^4 + 8x^3 + 8x^2 - 72x - 153$

$$\begin{array}{r|rrrrr} 3 & 1 & 8 & 8 & -72 & -153 \\ & & 3 & 33 & 123 & 153 \\ \hline & 1 & 11 & 41 & 51 & 0 \end{array}$$

$$\begin{array}{r|rrrr} -3 & 1 & 11 & 41 & 51 \\ & & -3 & -24 & -51 \\ \hline & 1 & 8 & 17 & \end{array}$$

By the Quadratic Formula, the zeros of $x^2 + 8x + 17$ are

$$x = \frac{-8 \pm \sqrt{(8)^2 - 4(1)(17)}}{2(1)} = \frac{-8 \pm \sqrt{-4}}{2} = -4 \pm i.$$

The zeros of $f(x)$ are $-3, 3, -4 - i, -4 + i$.

$f(x) = (x + 3)(x - 3)(x + 4 - i)(x + 4 + i)$

107. Because $\sqrt{3}i$ is a zero, so is $-\sqrt{3}i$.

Multiply by 3 to clear the fraction.

$$\begin{aligned} f(x) &= 3\left(x - \tfrac{2}{3}\right)(x - 4)\left(x - \sqrt{3}i\right)\left(x + \sqrt{3}i\right) \\ &= (3x - 2)(x - 4)(x^2 + 3) \\ &= (3x^2 - 14x + 8)(x^2 + 3) \\ &= 3x^4 - 14x^3 + 17x^2 - 42x + 24 \end{aligned}$$

Note: $f(x) = a(3x^4 - 14x^3 + 17x^2 - 42x + 24)$, where a is any real nonzero number, has zeros $\frac{2}{3}$, 4, and $\pm\sqrt{3}i$.

108. Because $1 - 2i$ is a zero, so is $1 + 2i$.

$$\begin{aligned} f(x) &= (x - 2)(x + 3)(x - 1 + 2i)(x - 1 - 2i) \\ &= (x^2 + x - 6)\left[(x - 1)^2 + 4\right] \\ &= (x^2 + x - 6)(x^2 - 2x + 5) \\ &= x^4 - x^3 - 3x^2 + 17x - 30 \end{aligned}$$

109. $f(x) = \dfrac{3x}{x + 10}$

Domain: all real numbers x except $x = -10$

110. $f(x) = \dfrac{4x^3}{2 + 5x} = \dfrac{4x^3}{5x + 2}$

Domain: all real numbers x except $x = -\dfrac{2}{5}$

111. $f(x) = \dfrac{8}{x^2 - 10x + 24} = \dfrac{8}{(x - 4)(x - 6)}$

Domain: all real numbers x except $x = 4$ and $x = 6$

112. $f(x) = \dfrac{x^2 - x - 2}{x^2 + 4}$

Domain: all real numbers x

113. $f(x) = \dfrac{4}{x + 3}$

Vertical asymptote: $x = -3$

Horizontal asymptote: $y = 0$

114. $f(x) = \dfrac{2x^2 + 5x - 3}{x^2 + 2}$

Horizontal asymptote: $y = 2$

115. $f(x) = \dfrac{5x + 20}{x^2 - 2x - 24} = \dfrac{5(x + 4)}{(x - 6)(x + 4)} = \dfrac{5}{x - 6};\ x \neq -4$

Vertical asymptote: $x = 6$

Horizontal asymptote: $y = 0$

116. $h(x) = \dfrac{x^3 - 4x^2}{x^2 + 3x + 2} = \dfrac{x^2(x - 4)}{(x + 2)(x + 1)}$

Vertical asymptotes: $x = -2, x = -1$

117. $f(x) = \dfrac{-3}{2x^2}$

(a) Domain: all real numbers x except $x = 0$

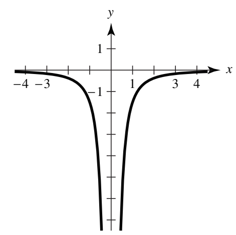
(b) No intercepts

(c) Vertical asymptote: $x = 0$

Horizontal asymptote: $y = 0$

(d)

x	-1	$-\frac{1}{2}$	$\frac{1}{2}$	1
y	$-\frac{3}{2}$	-6	6	$\frac{3}{2}$



118. $f(x) = \dfrac{4}{x}$

(a) Domain: all real numbers x except $x = 0$

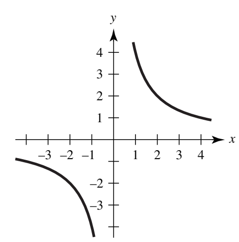
(b) No intercepts

(c) Vertical asymptote: $x = 0$

Horizontal asymptote: $y = 0$

(d)

x	-3	-2	-1	1	2	3
y	$-\frac{4}{3}$	-2	-4	4	2	$\frac{4}{3}$



119. $g(x) = \dfrac{2 + x}{1 - x} = -\dfrac{x + 2}{x - 1}$

(a) Domain: all real numbers x except $x = 1$

(b) x-intercept: $(-2, 0)$

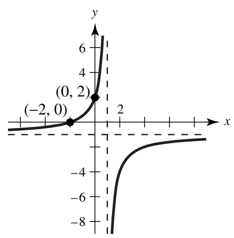
y-intercept: $(0, 2)$

(c) Vertical asymptote: $x = 1$

Horizontal asymptote: $y = -1$

(d)

x	-1	0	2	3
y	$\frac{1}{2}$	2	-4	$-\frac{5}{2}$



120. $f(x) = \dfrac{x - 4}{x - 7}$

(a) Domain: all real numbers x except $x = 7$

(b) x-intercept: $(4, 0)$

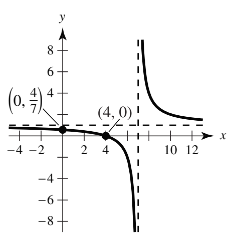
y-intercept: $\left(0, \dfrac{4}{7}\right)$

(c) Vertical asymptote: $x = 7$

Horizontal asymptote: $y = 1$

(d)

x	-2	-1	0	1	2
y	$\frac{2}{3}$	$\frac{5}{8}$	$\frac{4}{7}$	$\frac{1}{2}$	$\frac{2}{5}$



121. $f(x) = \dfrac{5x^2}{4x^2 + 1}$

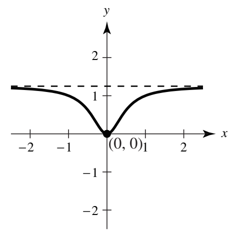
(a) Domain: all real numbers x

(b) Intercept: $(0, 0)$

(c) Horizontal asymptote: $y = \dfrac{5}{4}$

(d)

x	-2	-1	0	1	2
y	$\frac{20}{17}$	1	0	1	$\frac{20}{17}$



122. $f(x) = \dfrac{2x}{x^2 + 4}$

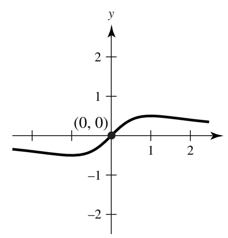
(a) Domain: all real numbers x

(b) Intercept: $(0, 0)$

(c) Horizontal asymptote: $y = 0$

(d)

x	-2	-1	0	1	2
y	$-\frac{1}{2}$	$-\frac{2}{5}$	0	$\frac{2}{5}$	$\frac{1}{2}$



123. $f(x) = \dfrac{x}{x^2 + 1}$

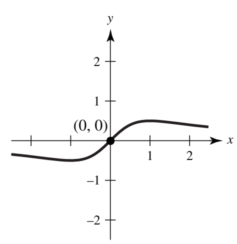
(a) Domain: all real numbers x

(b) Intercept: $(0, 0)$

(c) Horizontal asymptote: $y = 0$

(d)

x	-2	-1	0	1	2
y	$-\frac{2}{5}$	$-\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{2}{5}$



124. $h(x) = \dfrac{9}{(x-3)^2}$

(a) Domain: all real numbers x except $x = 3$

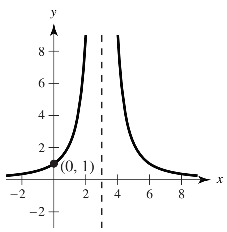
(b) y-intercept: $(0, 1)$

(c) Vertical asymptote: $x = 3$

Horizontal asymptote: $y = 0$

(d)

x	-3	-2	-1	0	1	2
y	$\frac{1}{4}$	$\frac{9}{25}$	$\frac{9}{16}$	1	$\frac{9}{4}$	9



125. $f(x) = \dfrac{-6x^2}{x^2+1}$

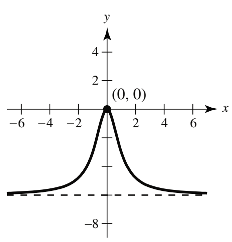
(a) Domain: all real numbers x

(b) Intercept: $(0, 0)$

(c) Horizontal asymptote: $y = -6$

(d)

x	± 3	± 2	± 1	0
y	$-\frac{27}{5}$	$-\frac{24}{5}$	-3	0



126. $y = \dfrac{2x^2}{x^2-4}$

(a) Domain: all real numbers x except $x = \pm 2$

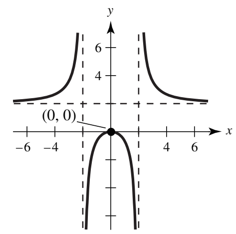
(b) Intercept: $(0, 0)$

(c) Vertical asymptotes: $x = 2, x = -2$

Horizontal asymptote: $y = 2$

(d)

x	± 5	± 4	± 3	± 1	0
y	$\frac{50}{21}$	$\frac{8}{3}$	$\frac{18}{5}$	$-\frac{2}{3}$	0



127. $f(x) = \dfrac{6x^2 - 11x + 3}{3x^2 - x}$

$= \dfrac{(3x-1)(2x-3)}{x(3x-1)} = \dfrac{2x-3}{x}, x \neq \dfrac{1}{3}$

(a) Domain: all real numbers x except $x = 0$ and $x = \dfrac{1}{3}$

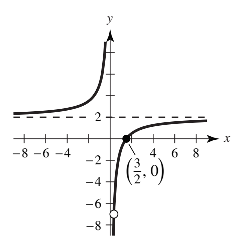
(b) x-intercept: $\left(\dfrac{3}{2}, 0\right)$

(c) Vertical asymptote: $x = 0$

Horizontal asymptote: $y = 2$

(d)

x	-2	-1	1	2	3	4
y	$\frac{7}{2}$	5	-1	$\frac{1}{2}$	1	$\frac{5}{4}$



128. $f(x) = \dfrac{6x^2 - 7x + 2}{4x^2 - 1}$

$= \dfrac{(2x - 1)(3x - 2)}{(2x - 1)(2x + 1)} = \dfrac{3x - 2}{2x + 1}, x \neq \dfrac{1}{2}$

(a) Domain: all real numbers x except $x = \pm\dfrac{1}{2}$

(b) x-intercept: $\left(\dfrac{2}{3}, 0\right)$

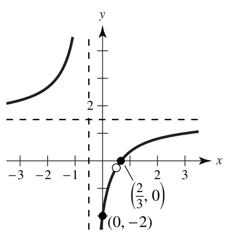
y-intercept: $(0, -2)$

(c) Vertical asymptote: $x = -\dfrac{1}{2}$

Horizontal asymptote: $y = \dfrac{3}{2}$

(d)

x	-3	-2	-1	0	$\frac{2}{3}$	1	2
y	$\frac{11}{5}$	$\frac{8}{3}$	5	-2	0	$\frac{1}{3}$	$\frac{4}{5}$



129. $f(x) = \dfrac{2x^3}{x^2 + 1} = 2x - \dfrac{2x}{x^2 + 1}$

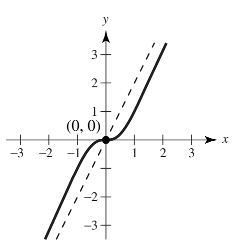
(a) Domain: all real numbers x

(b) Intercept: $(0, 0)$

(c) Slant asymptote: $y = 2x$

(d)

x	-2	-1	0	1	2
y	$-\frac{16}{5}$	-1	0	1	$\frac{16}{5}$



130. $f(x) = \dfrac{x^2 + 1}{x + 1} = x - 1 + \dfrac{2}{x + 1}$

(a) Domain: all real numbers x except $x = -1$

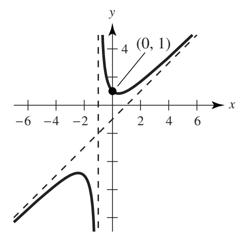
(b) y-intercept: $(0, 1)$

(c) Vertical asymptote: $x = -1$

Slant asymptote: $y = x - 1$

(d)

x	-6	-2	$-\frac{3}{2}$	$-\frac{1}{2}$	0	4
y	$-\frac{37}{5}$	-5	$-\frac{13}{2}$	$\frac{5}{2}$	1	$\frac{17}{5}$



131. $f(x) = \dfrac{3x^3 - 2x^2 - 3x + 2}{3x^2 - x - 4}$

$= \dfrac{(3x - 2)(x + 1)(x - 1)}{(3x - 4)(x + 1)}$

$= \dfrac{(3x - 2)(x - 1)}{3x - 4}$

$= x - \dfrac{1}{3} + \dfrac{2/3}{3x - 4}, \quad x \neq -1$

(a) Domain: all real numbers x except $x = -1$ and $x = \dfrac{4}{3}$

(b) x-intercepts: $(1, 0), \left(\dfrac{2}{3}, 0\right)$

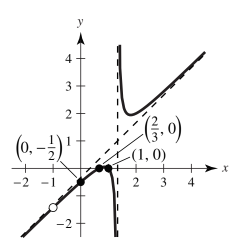
y-intercept: $\left(0, -\dfrac{1}{2}\right)$

(c) Vertical asymptote: $x = \dfrac{4}{3}$

Slant asymptote: $y = x - \dfrac{1}{3}$

(d)

x	-3	-2	0	1	2	3
y	$-\frac{44}{13}$	$-\frac{12}{5}$	$-\frac{1}{2}$	0	2	$\frac{14}{5}$



132. $f(x) = \dfrac{3x^3 - 4x^2 - 12x + 16}{3x^2 + 5x - 2}$

$= \dfrac{(x-2)(x+2)(3x-4)}{(x+2)(3x-1)}$

$= \dfrac{(x-2)(3x-4)}{3x-1}$

$= x - 3 + \dfrac{5}{3x-1},\ x \neq -2$

(a) Domain: all real numbers x except $x = -2$ and $x = \dfrac{1}{3}$

(b) x-intercepts: $\left(\dfrac{4}{3}, 0\right), (2, 0)$

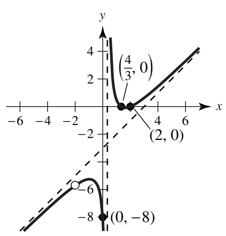
y-intercept: $(0, -8)$

(c) Vertical asymptote: $x = \dfrac{1}{3}$

Slant asymptote: $y = x - 3$

(d)

x	-4	-1	0	1	2	4
y	$-\frac{96}{13}$	$-\frac{21}{4}$	-8	$\frac{1}{2}$	0	$\frac{16}{11}$



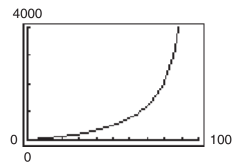
133. $\overline{C} = \dfrac{C}{x} = \dfrac{0.5x + 500}{x},\ 0 < x$

Horizontal asymptote: $\overline{C} = \dfrac{0.5}{1} = 0.5$

As x increases, the average cost per unit approaches the horizontal asymptote, $\overline{C} = 0.5 = \$0.50$.

134. $C = \dfrac{528p}{100 - p},\ 0 \le p < 100$

(a)



(b) When $p = 25$, $C = \dfrac{528(25)}{100 - 25} = \$176$ million.

When $p = 50$, $C = \dfrac{528(50)}{100 - 50} = \$528$ million.

When $p = 75$, $C = \dfrac{528(75)}{100 - 75} = \$1584$ million.

(c) As $p \to 100$, $C \to \infty$. No, it is not possible.

Chapter Test for Chapter 2

1. $f(x) = x^2$

(a) $g(x) = 2 - x^2$

Reflection in the x-axis followed by a vertical shift two units upward

(b) $g(x) = \left(x - \frac{3}{2}\right)^2$

Horizontal shift $\frac{3}{2}$ units to the right

2. Vertex: $(3, -6)$

$y = a(x - 3)^2 - 6$

Point on the graph: $(0, 3)$

$3 = a(0 - 3)^2 - 6$

$9 = 9a \Rightarrow a = 1$

So, $y = (x - 3)^2 - 6$.

3. (a) $y = -\frac{1}{20}x^2 + 3x + 5$

$= -\frac{1}{20}\left(x^2 - 60x + 900 - 900\right) + 5$

$= -\frac{1}{20}\left[(x - 30)^2 - 900\right] + 5$

$= -\frac{1}{20}(x - 30)^2 + 50$

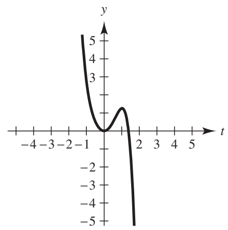
Vertex: $(30, 50)$

The maximum height is 50 feet.

(b) The constant term, $c = 5$, determines the height at which the ball was thrown. Changing this constant results in a vertical translation of the graph, and, therefore, changes the maximum height.

4. $h(t) = -\frac{3}{4}t^5 + 2t^2$

The degree is odd and the leading coefficient is negative. The graph rises to the left and falls to the right.



5. $$\begin{array}{r} 3x + \dfrac{x-1}{x^2+1} \\ x^2 + 0x + 1 \overline{)3x^3 + 0x^2 + 4x - 1} \\ \underline{3x^3 + 0x^2 + 3x} \\ x - 1 \end{array}$$

So, $\dfrac{3x^3 + 4x - 1}{x^2 + 1} = 3x + \dfrac{x - 1}{x^2 + 1}$.

6.

2	2	0	–5	0	–3
		4	8	6	12
	2	4	3	6	9

So, $\dfrac{2x^4 - 5x^2 - 3}{x - 2} = 2x^3 + 4x^2 + 3x + 6 + \dfrac{9}{x - 2}$.

7. $f(x) = 2x^3 - 5x^2 - 6x + 15$

$\frac{5}{2}$	2	–5	–6	15
		5	0	–15
	2	0	–6	0

$$\begin{aligned} 2x^3 - 5x^2 - 6x + 15 &= \left(x - \tfrac{5}{2}\right)\left(2x^2 - 6\right) \\ &= 2\left(x - \tfrac{5}{2}\right)\left(x^2 - 3\right) \\ &= 2\left(x - \tfrac{5}{2}\right)\left(x + \sqrt{3}\right)\left(x - \sqrt{3}\right) \end{aligned}$$

Zeros: $x = \frac{5}{2}, x = \pm\sqrt{3}$

8. (a) $$\begin{aligned} 10i - \left(3 + \sqrt{-25}\right) &= 10i - 3 - 5i \\ &= -3 + 5i \end{aligned}$$

(b) $$\begin{aligned} \left(2 + \sqrt{3}i\right)\left(2 - \sqrt{3}i\right) &= 4 - 3i^2 \\ &= 4 + 3 \\ &= 7 \end{aligned}$$

9. $$\begin{aligned} \frac{5}{2 + i} &= \frac{5}{2 + i} \cdot \frac{2 - i}{2 - i} \\ &= \frac{5(2 - i)}{4 + 1} \\ &= 2 - i \end{aligned}$$

10. Because $2 + i$ is a zero, so is $2 - i$.

$$\begin{aligned} f(x) &= (x - 0)(x - 3)\left(x - (2 + i)\right)\left(x - (2 - i)\right) \\ &= x(x - 3)\left((x - 2) - i\right)\left((x - 2) + i\right) \\ &= x(x - 3)\left(x^2 - 4x + 5\right) \\ &= x\left(x^3 - 7x^2 + 17x - 15\right) \\ &= x^4 - 7x^3 + 17x^2 - 15x \end{aligned}$$

Note: $f(x) = a\left(x^4 - 7x^3 + 17x^2 - 15x\right)$, where a is any non-zero real number, has the zeros 0, 3, and $2 + i$.

11. Because $1 - \sqrt{3}i$ is a zero, so is $1 + \sqrt{3}i$.

$$\begin{aligned} f(x) &= \left(x - \left(1 - \sqrt{3}i\right)\right)\left(x - \left(1 + \sqrt{3}i\right)\right)(x - 2)(x - 2) \\ &= \left((x - 1) + \sqrt{3}i\right)\left((x - 1) - \sqrt{3}i\right)(x - 2)(x - 2) \\ &= \left(x^2 - 2x + 4\right)\left(x^2 - 4x + 4\right) \\ &= x^4 - 6x^3 + 16x^2 - 24x + 16 \end{aligned}$$

Note: $f(x) = a\left(x^4 - 6x^3 + 16x^2 - 24x + 16\right)$, where a is any non-zero real number, has the zeros $1 - \sqrt{3}i$, 2, and 2.

12. $f(x) = 3x^3 + 14x^2 - 7x - 10$

Possible rational zeros:
$\pm\frac{1}{3}, \pm\frac{2}{3}, \pm1, \pm\frac{5}{3}, \pm2, \pm\frac{10}{3}, \pm5, \pm10$

1	3	14	–7	–10
		3	17	10
	3	17	10	0

$$\begin{aligned} 3x^3 + 14x^2 - 7x - 10 &= (x - 1)\left(3x^2 + 17x + 10\right) \\ &= (x - 1)(3x + 2)(x + 5) \end{aligned}$$

Zeros: $x = 1, -\frac{2}{3}, -5$

13. $f(x) = x^4 - 9x^2 - 22x - 24$

Possible rational zeros:
$\pm1, \pm2, \pm3, \pm4, \pm6, \pm8, \pm12, \pm24$

–2	1	0	–9	–22	–24
		–2	4	10	24
	1	–2	–5	–12	0

4	1	–2	–5	–12
		4	8	12
	1	2	3	0

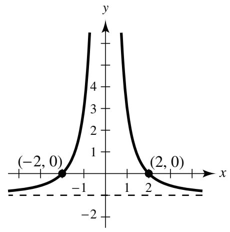
$f(x) = (x + 2)(x - 4)\left(x^2 + 2x + 3\right)$

By the Quadratic Formula, the zeros of $x^2 + 2x + 3$ are $x = -1 \pm \sqrt{2}i$. The zeros of f are: $x = -2, 4, -1 \pm \sqrt{2}i$.

14. $h(x) = \dfrac{4}{x^2} - 1$

$= \dfrac{4 - x^2}{x^2}$

$= \dfrac{(2 - x)(2 + x)}{x^2}$



x-intercepts: $(\pm 2, 0)$

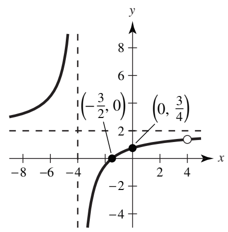
Vertical asymptote: $x = 0$

Horizontal asymptote: $y = -1$

15. $f(x) = \dfrac{2x^2 - 5x - 12}{x^2 - 16}$

$= \dfrac{(2x + 3)(x - 4)}{(x + 4)(x - 4)}$

$= \dfrac{2x + 3}{x + 4}, x \neq 4$



x-intercept: $\left(-\dfrac{3}{2}, 0\right)$

y-intercept: $\left(0, \dfrac{3}{4}\right)$

Vertical asymptote: $x = -4$

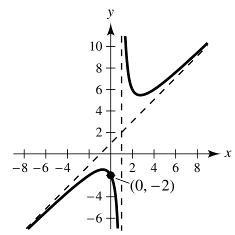
Horizontal asymptote: $y = 2$

16. $g(x) = \dfrac{x^2 + 2}{x - 1} = x + 1 + \dfrac{3}{x - 1}$

y-intercept: $(0, -2)$

Vertical asymptote: $x = 1$

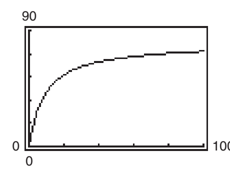
Slant asymptote: $y = x + 1$



17. $y = \dfrac{18.47x - 2.96}{0.23x + 1}, 0 < x$

The limiting amount of CO_2 uptake is determined by the horizontal asymptote.

$y = \dfrac{18.47}{0.23} \approx 80.3 \text{ mg/dm}^2\text{/hr.}$



Problem Solving for Chapter 2

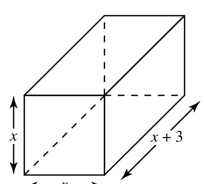
1. $V = l \cdot w \cdot h = x^2(x + 3)$

$x^2(x + 3) = 20$

$x^3 + 3x^2 - 20 = 0$

Possible rational zeros: $\pm 1, \pm 2, \pm 4, \pm 5, \pm 10, \pm 20$

2	1	3	0	−20
		2	10	20
	1	5	10	0



$(x - 2)(x^2 + 5x + 10) = 0$

$x = 2$ or $x = \dfrac{-5 \pm \sqrt{15}i}{2}$

Choosing the real positive value for x we have:
$x = 2$ and $x + 3 = 5$.

The dimensions of the mold are
2 inches × 2 inches × 5 inches.

2. False. Because $f(x) = d(x)q(x) + r(x)$, you have

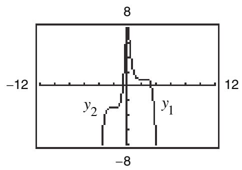
$\dfrac{f(x)}{d(x)} = q(x) + \dfrac{r(x)}{d(x)}.$

The statement should be corrected to read $f(-1) = 2$ because $\dfrac{f(x)}{x + 1} = q(x) + \dfrac{f(-1)}{x + 1}.$

3. If $h = 0$ and $k = 0$, then $a < -1$ produces a stretch that is reflected in the x-axis, and $-1 < a < 0$ produces a shrink that is reflected in the x-axis.

4. (a) $y_1 = -\frac{1}{3}(x - 2)^5 + 1$ is decreasing.

$y_2 = \frac{3}{5}(x + 2)^5 - 3$ is increasing.



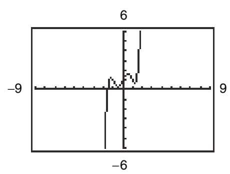
(b) The graph is either always increasing or always decreasing.
The behavior is determined by a.

If $a > 0$, $g(x)$ will always be increasing.

If $a < 0$, $g(x)$ will always be decreasing.

(c) $H(x) = x^5 - 3x^3 + 2x + 1$

Since $H(x)$ is not always increasing or always decreasing, $H(x) \neq a(x - h)^5 + k$.



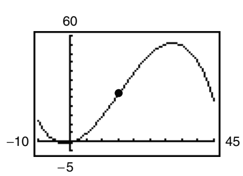
5. $f(x) = \dfrac{ax}{(x - b)^2}$

(a) $b \neq 0 \Rightarrow x = b$ is a vertical asymptote. a causes a vertical stretch if $|a| > 1$ and a vertical shrink if $0 < |a| < 1$. For $|a| > 1$, the graph becomes wider as $|a|$ increases. When a is negative, the graph is reflected about the x-axis.

(b) $a \neq 0$. Varying the value of b varies the vertical asymptote of the graph of f. For $b > 0$, the graph is translated to the right. For $b < 0$, the graph is reflected in the x-axis and is translated to the left

6. $G = -0.003t^3 + 0.137t^2 + 0.458t - 0.839,\ 2 \leq t \leq 34$

(a)



(b) The tree is growing most rapidly when it is approximately 15.2 years old.

(c) $y = -0.009t^2 + 0.274t + 0.458$

$-\dfrac{b}{2a} = \dfrac{-0.274}{2(-0.009)} \approx 15.2222$

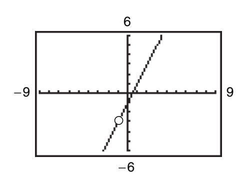
$y(15.2222) \approx 2.5434$

Vertex: $(15.2222, 2.5434)$

(d) In both (b) and (c) the point of diminishing returns occured when $t \approx 15.2$.

7. $f(x) = \dfrac{2x^2 + x - 1}{x + 1}$

(a)



The graph has a "hole" when $x = -1$. There are no vertical asymptotes.

(b) $\dfrac{2x^2 + x - 1}{x + 1} = \dfrac{(2x - 1)(x + 1)}{x + 1} = 2x - 1,\ x \neq -1$

(c) As $x \to -1$, $\dfrac{2x^2 + x - 1}{x + 1} \to -3$

8. Let $x =$ length of the wire used to form the square.

Then $100 - x =$ length of wire used to form the circle.

(a) let $s =$ the side of the square. Then $4s = x \Rightarrow s = \dfrac{x}{4}$ and the area of the square is $s^2 = \left(\dfrac{x}{4}\right)^2$.

Let $r =$ the radius of the circle. Then $2\pi r = 100 - x \Rightarrow r = \dfrac{100 - x}{2\pi}$ and the area of the circle

is $\pi r^2 = \pi\left(\dfrac{100 - x}{2\pi}\right)^2$.

The combined area is:

$$\begin{aligned}
A(x) &= \left(\frac{x}{4}\right)^2 + \pi\left(\frac{100 - x}{2\pi}\right)^2 \\
&= \frac{x^2}{16} + \pi\left(\frac{10{,}000 - 200x + x^2}{4\pi^2}\right) \\
&= \frac{x^2}{16} + \frac{2500}{\pi} - \frac{50x}{\pi} + \frac{x^2}{4\pi} \\
&= \left(\frac{1}{16} + \frac{1}{4\pi}\right)x^2 - \frac{50x}{\pi} + \frac{2500}{\pi} \\
&= \left(\frac{\pi + 4}{16\pi}\right)x^2 - \frac{50}{\pi}x + \frac{2500}{\pi}
\end{aligned}$$

(b) Domain: Since the wire is 100 cm, $0 \le x \le 100$.

(c)
$$\begin{aligned}
A(x) &= \left(\frac{\pi + 4}{16\pi}\right)x^2 - \frac{50}{\pi}x + \frac{2500}{\pi} \\
&= \left(\frac{\pi + 4}{16\pi}\right)\left(x^2 - \frac{800}{\pi + 4}x\right) + \frac{2500}{\pi} \\
&= \left(\frac{\pi + 4}{16\pi}\right)\left[x^2 - \frac{800}{\pi + 4}x + \left(\frac{400}{\pi + 4}\right)^2 - \left(\frac{400}{\pi + 4}\right)^2\right] + \frac{2500}{\pi} \\
&= \left(\frac{\pi + 4}{16\pi}\right)\left[x - \left(\frac{400}{\pi + 4}\right)\right]^2 - \left(\frac{\pi + 4}{16\pi}\right)\left(\frac{400}{\pi + 4}\right)^2 + \frac{2500}{\pi} \\
&= \left(\frac{\pi + 4}{16\pi}\right)\left[x - \left(\frac{400}{\pi + 4}\right)\right]^2 - \frac{10{,}000}{\pi(\pi + 4)} + \frac{2500}{\pi} \\
&= \left(\frac{\pi + 4}{16\pi}\right)\left[x - \left(\frac{400}{\pi + 4}\right)\right]^2 + \frac{2500}{\pi + 4}
\end{aligned}$$

The minimum occurs at the vertex when $x = \dfrac{400}{\pi + 4} \approx 56$ cm and $A(x) \approx 350$ cm^2.

The maximum occurs at one of the endpoints of the domain.

When $x = 0$, $A(x) \approx 796$ cm^2.

When $x = 100$, $A(x) = 625$ cm^2.

Thus, the area is maximum when $x = 0$ cm.

(d) Answers will vary. Graph $A(x)$ to see where the minimum and maximum values occur.

9. (a) $z_m = \frac{1}{z}$

$= \frac{1}{1+i} = \frac{1}{1+i} \cdot \frac{1-i}{1-i}$

$= \frac{1-i}{2} = \frac{1}{2} - \frac{1}{2}i$

(b) $z_m = \frac{1}{z}$

$= \frac{1}{3-i} = \frac{1}{3-i} \cdot \frac{3+i}{3+i}$

$= \frac{3+i}{10} = \frac{3}{10} + \frac{1}{10}i$

(c) $z_m = \frac{1}{z} = \frac{1}{-2+8i}$

$= \frac{1}{-2+8i} \cdot \frac{-2-8i}{-2-8i}$

$= \frac{-2-8i}{68} = -\frac{1}{34} - \frac{2}{17}i$

10. $y = ax^2 + bx + c$

(a) $(0, -4)$: $-4 = c$

$(1, 0)$: $0 = a + b + c \Rightarrow a + b = 4$

$(2, 2)$: $2 = 4a + 2b + c \Rightarrow 4a + 2b = 6$

Solve the system of equations:

$$\begin{aligned} 4a + 2b = 6 &\Rightarrow 2a + b = 3 \\ a + b = 4 &\Rightarrow -a - b = -4 \\ & \quad a = -1 \\ & \quad -1 + b = 4 \\ & \quad b = 5 \end{aligned}$$

Thus, $y = -x^2 + 5x - 4$.

Check: $(4, 0)$: $0 = -(4)^2 + 5(4) - 4$ ✓

$(6, -10)$: $-10 = -(6)^2 + 5(6) - 4$ ✓

(b)

L_1	L_2
0	–4
1	0
2	2
4	0
6	– 10

Use the "Quad Reg" feature of your graphing utility to obtain $y = -x^2 + 5x - 4$

11. (a) Slope $= \frac{9-4}{3-2} = 5$. Slope of tangent line is less than 5.

(b) Slope $= \frac{4-1}{2-1} = 3$. Slope of tangent line is greater than 3.

(c) Slope $= \frac{4.41-4}{2.1-2} = 4.1$. Slope of tangent line is less than 4.1.

(d) Slope $= \frac{f(2+h) - f(2)}{(2+h) - 2}$

$= \frac{(2+h)^2 - 4}{h}$

$= \frac{4h + h^2}{h}$

$= 4 + h, h \neq 0$

(e) Slope $= 4 + h, h \neq 0$

$4 + (-1) = 3$

$4 + 1 = 5$

$4 + 0.1 = 4.1$

The results are the same as in (a)–(c).

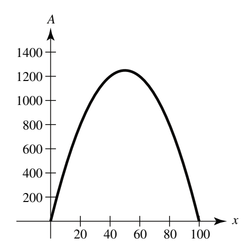
(f) Letting h get closer and closer to 0, the slope approaches 4. Hence, the slope at $(2, 4)$ is 4.

12. (a) $x + 2y = 100 \Rightarrow y = \frac{100 - x}{2}$

$A(x) = xy = x\left(\frac{100 - x}{2}\right) = -\frac{x^2}{2} + 50x$

Domain: $0 < x < 100$

(b)



Maximum of 1250 m^2 at $x = 50$ m, $y = 25$ m

(c) $A(x) = -\frac{1}{2}(x^2 - 100x)$

$= -\frac{1}{2}(x^2 - 100x + 2500) + 1250$

$= -\frac{1}{2}(x - 50)^2 + 1250$

$A(50) = 1250 \text{ m}^2$ is the maximum.

$x = 50$ m, $y = 25$ m

13. $f(x) = \dfrac{ax + b}{cx + d}$

$f(x)$ has a vertical asymptote at $x = -\dfrac{d}{c}$ and a horizontal asymptote at $y = \dfrac{a}{c}$.

(i) $a > 0$
$b < 0$
$c > 0$
$d < 0$

$x = -\dfrac{d}{c}$ is positive.

$y = \dfrac{a}{c}$ is positive.

Both asymptotes are positive on graph (d).

(ii) $a > 0$
$b > 0$
$c < 0$
$d < 0$

$x = -\dfrac{d}{c}$ is negative.

$y = \dfrac{a}{c}$ is negative.

Both asymptotes are negative on graph (b).

(iii) $a < 0$
$b > 0$
$c > 0$
$d < 0$

$x = -\dfrac{d}{c}$ is positive.

$y = \dfrac{a}{c}$ is negative.

The vertical asymptote is positive and the horizontal asymptote is negative on graph (a).

(iv) $a > 0$
$b < 0$
$c > 0$
$d > 0$

$x = -\dfrac{d}{c}$ is negative.

$y = \dfrac{a}{c}$ is positive.

The vertical asymptote is negative and the horizontal asymptote is positive on graph (c).