INSTRUCTOR'S SOLUTIONS MANUAL

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CALCULUS
EARLY TRANSCENDENTALS
THIRD EDITION

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Chapter 1

Functions

1.1 Review of Functions

1.1.1 A function is a rule that assigns each to each value of the independent variable in the domain a unique value of the dependent variable in the range.

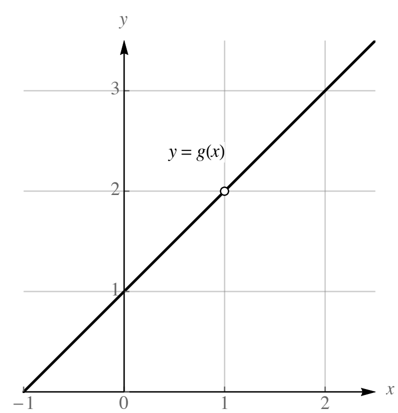
1.1.2 The independent variable belongs to the domain, while the dependent variable belongs to the range.

1.1.3 Graph A does not represent a function, while graph B does. Note that graph A fails the vertical line test, while graph B passes it.

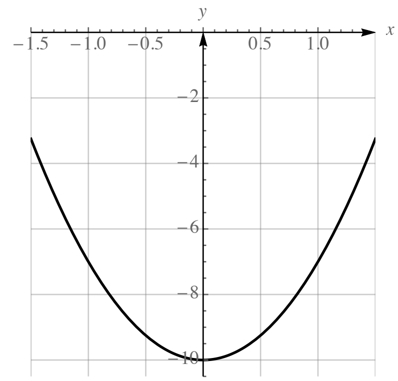
1.1.4 The domain of f is $[1, 4)$, while the range of f is $(1, 5]$. Note that the domain is the "shadow" of the graph on the x-axis, while the range is the "shadow" of the graph on the y-axis.

1.1.5 Item i. is true while item ii. isn't necessarily true. In the definition of function, item i. is stipulated. However, item ii. need not be true – for example, the function $f(x) = x^2$ has two different domain values associated with the one range value 4, because $f(2) = f(-2) = 4$.

1.1.6 $g(x) = \frac{x^2+1}{x-1} = \frac{(x+1)(x-1)}{x-1} = x + 1,\ x \neq 1$. The domain is $\{x : x \neq 1\}$ and the range is $\{x : x \neq 2\}$.



1.1.7 The domain of this function is the set of a real numbers. The range is $[-10, \infty)$.



1.1.8 The independent variable t is elapsed time and the dependent variable d is distance above the ground. The domain in context is $[0, 8]$

1.1.9 The independent variable h is the height of the water in the tank and the dependent variable V is the volume of water in the tank. The domain in context is $[0, 50]$

1.1.10 $f(2) = \dfrac{1}{2^3+1} = \dfrac{1}{9}$. $f(y^2) = \frac{1}{(y^2)^3+1} = \frac{1}{y^6+1}$.

1.1.11 $f(g(1/2)) = f(-2) = -3$; $g(f(4)) = g(9) = \frac{1}{8}$; $g(f(x)) = g(2x+1) = \dfrac{1}{(2x+1)-1} = \dfrac{1}{2x}$.

1.1.12 One possible answer is $g(x) = x^2 + 1$ and $f(x) = x^5$, because then $f(g(x)) = f(x^2+1) = (x^2+1)^5$. Another possible answer is $g(x) = x^2$ and $f(x) = (x+1)^5$, because then $f(g(x)) = f(x^2) = (x^2+1)^5$.

1.1.13 The domain of $f \circ g$ consists of all x in the domain of g such that $g(x)$ is in the domain of f.

1.1.14 $(f \circ g)(3) = f(g(3)) = f(25) = \sqrt{25} = 5$.
$(f \circ f)(64) = f(\sqrt{64}) = f(8) = \sqrt{8} = 2\sqrt{2}$.
$(g \circ f)(x) = g(f(x)) = g(\sqrt{x}) = x^{3/2} - 2$.
$(f \circ g)(x) = f(g(x)) = f(x^3 - 2) = \sqrt{x^3 - 2}$

1.1.15

a. $(f \circ g)(2) = f(g(2)) = f(2) = 4$.

b. $g(f(2)) = g(4) = 1$.

c. $f(g(4)) = f(1) = 3$.

d. $g(f(5)) = g(6) = 3$.

e. $f(f(8)) = f(8) = 8$.

f. $g(f(g(5))) = g(f(2)) = g(4) = 1$.

1.1.16

a. $h(g(0)) = h(0) = -1$.

b. $g(f(4)) = g(-1) = -1$.

c. $h(h(0)) = h(-1) = 0$.

d. $g(h(f(4))) = g(h(-1)) = g(0) = 0$.

e. $f(f(f(1))) = f(f(0)) = f(1) = 0$.

f. $h(h(h(0))) = h(h(-1)) = h(0) = -1$.

g. $f(h(g(2))) = f(h(3)) = f(0) = 1$.

h. $g(f(h(4))) = g(f(4)) = g(-1) = -1$.

i. $g(g(g(1))) = g(g(2)) = g(3) = 4$.

j. $f(f(h(3))) = f(f(0)) = f(1) = 0$.

1.1.17 $\dfrac{f(5) - f(0)}{5 - 0} = \dfrac{83 - 6}{5} = 15.4$; the radiosonde rises at an average rate of 15.4 ft/s during the first 5 seconds after it is released.

1.1.18 $f(0) = 0$. $f(34) = 127852.4 - 109731 = 18121.4$. $f(64) = 127852.4 - 75330.4 = 52522$.

$$\frac{f(64) - f(34)}{64 - 34} = \frac{52522 - 18121.4}{30} \approx 1146.69\,\text{ft/s}.$$

1.1.19 $f(-2) = f(2) = 2$; $g(-2) = -g(2) = -(-2) = 2$; $f(g(2)) = f(-2) = f(2) = 2$; $g(f(-2)) = g(f(2)) = g(2) = -2$.

1.1.20 The graph would be the result of leaving the portion of the graph in the first quadrant, and then also obtaining a portion in the third quadrant which would be the result of reflecting the portion in the first quadrant around the y-axis and then the x-axis.

1.1.21 Function A is symmetric about the y-axis, so is even. Function B is symmetric about the origin so is odd. Function C is symmetric about the y-axis, so is even.

1.1.22 Function A is symmetric about the y-axis, so is even. Function B is symmetric about the origin, so is odd. Function C is also symmetric about the origin, so is odd.

1.1.23 $f(x) = \dfrac{x^2 - 5x + 6}{x - 2} = \dfrac{(x-2)(x-3)}{x-2} = x - 3$, $x \neq 2$. The domain of f is $\{x : x \neq 2\}$. The range is $\{y : y \neq -1\}$.

1.1.24 $f(x) = \frac{x-2}{2-x} = \frac{x-2}{-(x-2)} = -1$, $x \neq 2$. The domain is $\{x : x \neq 2\}$. The range is $\{-1\}$.

1.1.25 The domain of the function is the set of numbers x which satisfy $7 - x^2 \geq 0$. This is the interval $[-\sqrt{7}, \sqrt{7}]$. Note that $f(\sqrt{7}) = 0$ and $f(0) = \sqrt{7}$. The range is $[0, \sqrt{7}]$.

1.1.26 The domain of the function is the set of numbers x which satisfy $25 - x^2 \geq 0$. This is the interval $[-5, 5]$. Note that $f(0) = -5$ and $f(5) = 0$. The range is $[-5, 0]$.

1.1.27 Because the cube root function is defined for all real numbrs, the domain is $\mathbb{R}$, the set of all real numbers.

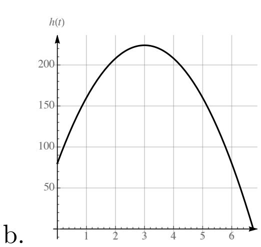
1.1.28 The domain consists of the set of numbers w for which $2 - w \geq 0$, so the interval $(-\infty, 2]$.

1.1.29 The domain consists of the set of numbers x for which $9 - x^2 \geq 0$, so the interval $[-3, 3]$.

1.1.30 Because $1 + t^2$ is never zero for any real numbered value of t, the domain of this function is $\mathbb{R}$, the set of all real numbers.

1.1.31

a. The formula for the height of the rocket is valid from $t = 0$ until the rocket hits the ground, which is the positive solution to $-16t^2 + 96t + 80 = 0$, which the quadratic formula reveals is $t = 3 + \sqrt{14}$. Thus, the domain is $[0, 3 + \sqrt{14}]$.

b. 

The maximum appears to occur at $t = 3$. The height at that time would be 224.

1.1.32

a. $d(0) = (10 - (2.2) \cdot 0)^2 = 100$.

b. The tank is first empty when $d(t) = 0$, which is when $10 - (2.2)t = 0$, or $t = 50/11$.

c. An appropriate domain would $[0, 50/11]$.

1.1.33 $g(1/z) = (1/z)^3 = \frac{1}{z^3}$

1.1.34 $F(y^4) = \frac{1}{y^4-3}$

1.1.35 $F(g(y)) = F(y^3) = \frac{1}{y^3-3}$

1.1.36 $f(g(w)) = f(w^3) = (w^3)^2 - 4 = w^6 - 4$

1.1.37 $g(f(u)) = g(u^2-4) = (u^2-4)^3$

1.1.38 $\dfrac{f(2+h)-f(2)}{h} = \dfrac{(2+h)^2-4-0}{h} = \dfrac{4+4h+h^2-4}{h} = \dfrac{4h+h^2}{h} = 4+h$

1.1.39 $F(F(x)) = F\left(\dfrac{1}{x-3}\right) = \dfrac{1}{\frac{1}{x-3}-3} = \dfrac{1}{\frac{1}{x-3}-\frac{3(x-3)}{x-3}} = \dfrac{1}{\frac{10-3x}{x-3}} = \dfrac{x-3}{10-3x}$

1.1.40 $g(F(f(x))) = g(F(x^2-4)) = g\left(\dfrac{1}{x^2-4-3}\right) = \left(\dfrac{1}{x^2-7}\right)^3$

1.1.41 $f(\sqrt{x+4}) = (\sqrt{x+4})^2 - 4 = x+4-4 = x$.

1.1.42 $F((3x+1)/x) = \dfrac{1}{\frac{3x+1}{x}-3} = \dfrac{1}{\frac{3x+1-3x}{x}} = \dfrac{x}{3x+1-3x} = x$.

1.1.43 $g(x) = x^3 - 5$ and $f(x) = x^{10}$.

1.1.44 $g(x) = x^6 + x^2 + 1$ and $f(x) = \frac{2}{x^2}$.

1.1.45 $g(x) = x^4 + 2$ and $f(x) = \sqrt{x}$.

1.1.46 $g(x) = x^3 - 1$ and $f(x) = \frac{1}{\sqrt{x}}$.

1.1.47 $(f \circ g)(x) = f(g(x)) = f(x^2-4) = |x^2-4|$. The domain of this function is the set of all real numbers.

1.1.48 $(g \circ f)(x) = g(f(x)) = g(|x|) = |x|^2 - 4 = x^2 - 4$. The domain of this function is the set of all real numbers.

1.1.49 $(f \circ G)(x) = f(G(x)) = f\left(\frac{1}{x-2}\right) = \left|\frac{1}{x-2}\right| = \frac{1}{|x-2|}$. The domain of this function is the set of all real numbers except for the number 2.

1.1.50 $(f \circ g \circ G)(x) = f(g(G(x))) = f\left(g\left(\frac{1}{x-2}\right)\right) = f\left(\left(\frac{1}{x-2}\right)^2 - 4\right) = \left|\left(\frac{1}{x-2}\right)^2 - 4\right|$. The domain of this function is the set of all real numbers except for the number 2.

1.1.51 $(G \circ g \circ f)(x) = G(g(f(x))) = G(g(|x|)) = G(x^2-4) = \frac{1}{x^2-4-2} = \frac{1}{x^2-6}$. The domain of this function is the set of all real numbers except for the numbers $\pm\sqrt{6}$.

1.1.52 $(g \circ F \circ F)(x) = g(F(F(x))) = g(F(\sqrt{x})) = g(\sqrt{\sqrt{x}}) = \sqrt{x} - 4$. The domain is $[0, \infty)$.

1.1.53 $(g \circ g)(x) = g(g(x)) = g(x^2-4) = (x^2-4)^2 - 4 = x^4 - 8x^2 + 16 - 4 = x^4 - 8x^2 + 12$. The domain is the set of all real numbers.

1.1.54 $(G \circ G)(x) = G(G(x)) = G(1/(x-2)) = \dfrac{1}{\frac{1}{x-2}-2} = \dfrac{1}{\frac{1-2(x-2)}{x-2}} = \dfrac{x-2}{1-2x+4} = \dfrac{x-2}{5-2x}$. Then $G \circ G$ is defined except where the denominator vanishes, so its domain is the set of all real numbers except for $x = \frac{5}{2}$.

1.1.55 Because $(x^2+3) - 3 = x^2$, we may choose $f(x) = x - 3$.

1.1.56 Because the reciprocal of x^2+3 is $\frac{1}{x^2+3}$, we may choose $f(x) = \frac{1}{x}$.

1.1.57 Because $(x^2+3)^2 = x^4 + 6x^2 + 9$, we may choose $f(x) = x^2$.

1.1.58 Because $(x^2+3)^2 = x^4+6x^2+9$, and the given expression is 11 more than this, we may choose $f(x) = x^2+11$.

1.1.59 Because $(x^2)^2+3 = x^4+3$, this expression results from squaring x^2 and adding 3 to it. Thus we may choose $f(x) = x^2$.

1.1.60 Because $x^{2/3}+3 = (\sqrt[3]{x})^2+3$, we may choose $f(x) = \sqrt[3]{x}$.

1.1.61

a. True. A real number z corresponds to the domain element $z/2+19$, because $f(z/2+19) = 2(z/2+19)-38 = z+38-38 = z$.

b. False. The definition of function does not require that each range element comes from a unique domain element, rather that each domain element is paired with a unique range element.

c. True. $f(1/x) = \frac{1}{1/x} = x$, and $\frac{1}{f(x)} = \frac{1}{1/x} = x$.

d. False. For example, suppose that f is the straight line through the origin with slope 1, so that $f(x) = x$. Then $f(f(x)) = f(x) = x$, while $(f(x))^2 = x^2$.

e. False. For example, let $f(x) = x+2$ and $g(x) = 2x-1$. Then $f(g(x)) = f(2x-1) = 2x-1+2 = 2x+1$, while $g(f(x)) = g(x+2) = 2(x+2)-1 = 2x+3$.

f. True. This is the definition of $f \circ g$.

g. True. If f is even, then $f(-z) = f(z)$ for all z, so this is true in particular for $z = ax$. So if $g(x) = cf(ax)$, then $g(-x) = cf(-ax) = cf(ax) = g(x)$, so g is even.

h. False. For example, $f(x) = x$ is an odd function, but $h(x) = x+1$ isn't, because $h(2) = 3$, while $h(-2) = -1$ which isn't $-h(2)$.

i. True. If $f(-x) = -f(x) = f(x)$, then in particular $-f(x) = f(x)$, so $0 = 2f(x)$, so $f(x) = 0$ for all x.

1.1.62 $\dfrac{f(x+h)-f(x)}{h} = \dfrac{10-10}{h} = \dfrac{0}{h} = 0.$

1.1.63 $\dfrac{f(x+h)-f(x)}{h} = \dfrac{3(x+h)-3x}{h} = \dfrac{3x+3h-3x}{h} = \dfrac{3h}{h} = 3.$

1.1.64 $\dfrac{f(x+h)-f(x)}{h} = \dfrac{4(x+h)-3-(4x-3)}{h} = \dfrac{4x+4h-3-4x+3}{h} = \dfrac{4h}{h} = 4.$

1.1.65 $\dfrac{f(x+h)-f(x)}{h} = \dfrac{(x+h)^2-x^2}{h} = \dfrac{(x^2+2hx+h^2)-x^2}{h} = \dfrac{h(2x+h)}{h} = 2x+h.$

1.1.66 $\dfrac{f(x+h)-f(x)}{h} = \dfrac{2(x+h)^2-3(x+h)+1-(2x^2-3x+1)}{h} =$
$\dfrac{2x^2+4xh+2h^2-3x-3h+1-2x^2+3x-1}{h} = \dfrac{4xh+2h^2-3h}{h} = \dfrac{h(4x+2h-3)}{h} = 4x+2h-3.$

1.1.67 $\dfrac{f(x+h)-f(x)}{h} = \dfrac{\frac{2}{x+h}-\frac{2}{x}}{h} = \dfrac{\frac{2x-2(x+h)}{x(x+h)}}{h} = \dfrac{2x-2x-2h}{hx(x+h)} = -\dfrac{2h}{hx(x+h)} = -\dfrac{2}{x(x+h)}.$

1.1.68 $\dfrac{f(x+h)-f(x)}{h} = \dfrac{\frac{x+h}{x+h+1}-\frac{x}{x+1}}{h} = \dfrac{\frac{(x+h)(x+1)-x(x+h+1)}{(x+1)(x+h+1)}}{h} = \dfrac{x^2+x+hx+h-x^2-xh-x}{h(x+1)(x+h+1)} =$
$\dfrac{h}{h(x+1)(x+h+1)} = \dfrac{1}{(x+1)(x+h+1)}$

1.1.69 $\frac{f(x)-f(a)}{x-a} = \frac{x^2+x-(a^2+a)}{x-a} = \frac{(x^2-a^2)+(x-a)}{x-a} = \frac{(x-a)(x+a)+(x-a)}{x-a} = \frac{(x-a)(x+a+1)}{x-a} = x+a+1.$

1.1.70

$$\frac{f(x)-f(a)}{x-a} = \frac{4-4x-x^2-(4-4a-a^2)}{x-a} = \frac{-4(x-a)-(x^2-a^2)}{x-a} = \frac{-4(x-a)-(x-a)(x+a)}{x-a}$$
$$= \frac{(x-a)(-4-(x+a))}{x-a} = -4-x-a.$$

1.1.71 $\frac{f(x)-f(a)}{x-a} = \frac{x^3-2x-(a^3-2a)}{x-a} = \frac{(x^3-a^3)-2(x-a)}{x-a} = \frac{(x-a)(x^2+ax+a^2)-2(x-a)}{x-a} = \frac{(x-a)(x^2+ax+a^2-2)}{x-a} = x^2+ax+a^2-2.$

1.1.72 $\frac{f(x)-f(a)}{x-a} = \frac{x^4-a^4}{x-a} = \frac{(x^2-a^2)(x^2+a^2)}{x-a} = \frac{(x-a)(x+a)(x^2+a^2)}{x-a} = (x+a)(x^2+a^2).$

1.1.73 $\frac{f(x)-f(a)}{x-a} = \frac{\frac{-4}{x^2}-\frac{-4}{a^2}}{x-a} = \frac{\frac{-4a^2+4x^2}{a^2x^2}}{x-a} = \frac{4(x^2-a^2)}{(x-a)a^2x^2} = \frac{4(x-a)(x+a)}{(x-a)a^2x^2} = \frac{4(x+a)}{a^2x^2}.$

1.1.74 $\frac{f(x)-f(a)}{x-a} = \frac{\frac{1}{x}-x^2-(\frac{1}{a}-a^2)}{x-a} = \frac{\frac{1}{x}-\frac{1}{a}}{x-a} - \frac{x^2-a^2}{x-a} = \frac{\frac{a-x}{ax}}{x-a} - \frac{(x-a)(x+a)}{x-a} = -\frac{1}{ax}-(x+a).$

1.1.75

a. The slope is $\frac{12227-10499}{3-1} = 864$ ft/h. The hiker's elevation increases at an average rate of 874 feet per hour.

b. The slope is $\frac{12144-12631}{5-4} = -487$ ft/h. The hiker's elevation decreases at an average rate of 487 feet per hour.

c. The hiker might have stopped to rest during this interval of time or the trail is level on this portion of the hike.

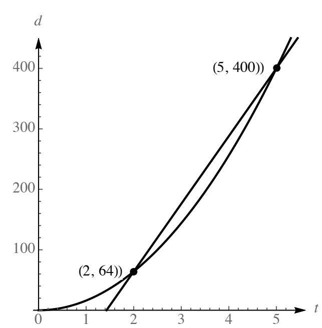
1.1.76

a. The slope is $\frac{11302-9954}{3-1} = 674$ ft/m. The elevation of the trail increases by an average of 674 feet per mile for $1 \le d \le 3$.

b. The slope is $\frac{12237-12357}{6-5} = -120$ ft/m. The elevation of the trail decreases by an average of 120 feet per mile for $5 \le d \le 6$.

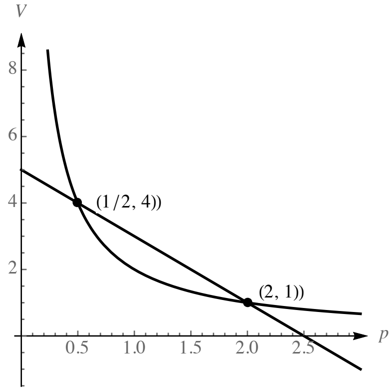
c. The elevation of the trail doesn't change much for $4.5 \le d \le 5$.

1.1.77

a. 

b. The slope of the secant line is given by $\frac{400-64}{5-2} = \frac{336}{3} = 112$ feet per second. The object falls at an average rate of 112 feet per second over the interval $2 \le t \le 5$.

1.1.78

a. 

b. The slope of the secant line is given by $\frac{4-1}{.5-2} = \frac{3}{-1.5} = -2$ cubic centimeters per atmosphere. The volume decreases by an average of 2 cubic centimeters per atmosphere over the interval $0.5 \le p \le 2$.

1.1.79 This function is symmetric about the y-axis, because $f(-x) = (-x)^4 + 5(-x)^2 - 12 = x^4 + 5x^2 - 12 = f(x)$.

1.1.80 This function is symmetric about the origin, because $f(-x) = 3(-x)^5 + 2(-x)^3 - (-x) = -3x^5 - 2x^3 + x = -(3x^5 + 2x^3 - x) = f(x)$.

1.1.81 This function has none of the indicated symmetries. For example, note that $f(-2) = -26$, while $f(2) = 22$, so f is not symmetric about either the origin or about the y-axis, and is not symmetric about the x-axis because it is a function.

1.1.82 This function is symmetric about the y-axis. Note that $f(-x) = 2|-x| = 2|x| = f(x)$.

1.1.83 This curve (which is not a function) is symmetric about the x-axis, the y-axis, and the origin. Note that replacing either x by $-x$ or y by $-y$ (or both) yields the same equation. This is due to the fact that $(-x)^{2/3} = ((-x)^2)^{1/3} = (x^2)^{1/3} = x^{2/3}$, and a similar fact holds for the term involving y.

1.1.84 This function is symmetric about the origin. Writing the function as $y = f(x) = x^{3/5}$, we see that $f(-x) = (-x)^{3/5} = -(x)^{3/5} = -f(x)$.

1.1.85 This function is symmetric about the origin. Note that $f(-x) = (-x)|(-x)| = -x|x| = -f(x)$.

1.1.86 This curve (which is not a function) is symmetric about the x-axis, the y-axis, and the origin. Note that replacing either x by $-x$ or y by $-y$ (or both) yields the same equation. This is due to the fact that $|-x| = |x|$ and $|-y| = |y|$.

1.1.87

a. $f(g(-2)) = f(-g(2)) = f(-2) = 4$

b. $g(f(-2)) = g(f(2)) = g(4) = 1$

c. $f(g(-4)) = f(-g(4)) = f(-1) = 3$

d. $g(f(5) - 8) = g(-2) = -g(2) = -2$

e. $g(g(-7)) = g(-g(7)) = g(-4) = -1$

f. $f(1 - f(8)) = f(-7) = 7$

1.1.88

a. $f(g(-1)) = f(-g(1)) = f(3) = 3$

b. $g(f(-4)) = g(f(4)) = g(-4) = -g(4) = 2$

c. $f(g(-3)) = f(-g(3)) = f(4) = -4$

d. $f(g(-2)) = f(-g(2)) = f(1) = 2$

e. $g(g(-1)) = g(-g(1)) = g(3) = -4$

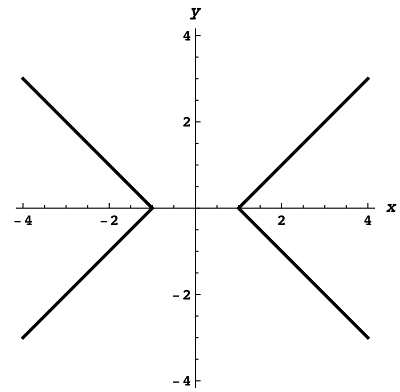
f. $f(g(0) - 1) = f(-1) = f(1) = 2$

g. $f(g(g(-2))) = f(g(-g(2))) = f(g(1)) = f(-3) = 3$

h. $g(f(f(-4))) = g(f(-4)) = g(-4) = 2$

i. $g(g(g(-1))) = g(g(-g(1))) = g(g(3)) = g(-4) = 2$

1.1.89 We will make heavy use of the fact that $|x|$ is x if $x > 0$, and is $-x$ if $x < 0$. In the first quadrant where x and y are both positive, this equation becomes $x - y = 1$ which is a straight line with slope 1 and y-intercept -1. In the second quadrant where x is negative and y is positive, this equation becomes $-x - y = 1$, which is a straight line with slope -1 and y-intercept -1. In the third quadrant where both x and y are negative, we obtain the equation $-x - (-y) = 1$, or $y = x + 1$, and in the fourth quadrant, we obtain $x + y = 1$. Graphing these lines and restricting them to the appropriate quadrants yields the following curve:



1.1.90 We have $y = 10 + \sqrt{-x^2 + 10x - 9}$, so by subtracting 10 from both sides and squaring we have $(y - 10)^2 = -x^2 + 10x - 9$, which can be written as

$$x^2 - 10x + (y - 10)^2 = -9.$$

To complete the square in x, we add 25 to both sides, yielding

$$x^2 - 10x + 25 + (y - 10)^2 = -9 + 25,$$

or

$$(x - 5)^2 + (y - 10)^2 = 16.$$

This is the equation of a circle of radius 4 centered at $(5, 10)$. Because $y \geq 10$, we see that the graph of f is the upper half of this circle. The domain of the function is $[1, 9]$ and the range is $[10, 14]$.

1.1.91 We have $y = 2 - \sqrt{-x^2 + 6x + 16}$, so by subtracting 2 from both sides and squaring we have $(y - 2)^2 = -x^2 + 6x + 16$, which can be written as

$$x^2 - 6x + (y - 2)^2 = 16.$$

To complete the square in x, we add 9 to both sides, yielding

$$x^2 - 6x + 9 + (y - 2)^2 = 16 + 9,$$

or

$$(x - 3)^2 + (y - 2)^2 = 25.$$

This is the equation of a circle of radius 5 centered at $(3, 2)$. Because $y \leq 2$, we see that the graph of f is the lower half of this circle. The domain of the function is $[-2, 8]$ and the range is $[-3, 2]$.

1.1.92

a. No. For example $f(x) = x^2 + 3$ is an even function, but $f(0)$ is not 0.

b. Yes. because $f(-x) = -f(x)$, and because $-0 = 0$, we must have $f(-0) = f(0) = -f(0)$, so $f(0) = -f(0)$, and the only number which is its own additive inverse is 0, so $f(0) = 0$.

1.1.93 Because the composition of f with itself has first degree, f has first degree as well, so let $f(x) = ax+b$. Then $(f \circ f)(x) = f(ax + b) = a(ax + b) + b = a^2x + (ab + b)$. Equating coefficients, we see that $a^2 = 9$ and $ab + b = -8$. If $a = 3$, we get that $b = -2$, while if $a = -3$ we have $b = 4$. So the two possible answers are $f(x) = 3x - 2$ and $f(x) = -3x + 4$.

1.1.94 Since the square of a linear function is a quadratic, we let $f(x) = ax+b$. Then $f(x)^2 = a^2x^2+2abx+b^2$. Equating coefficients yields that $a = \pm 3$ and $b = \pm 2$. However, a quick check shows that the middle term is correct only when one of these is positive and one is negative. So the two possible such functions f are $f(x) = 3x - 2$ and $f(x) = -3x + 2$.

1.1.95 Let $f(x) = ax^2 + bx + c$. Then $(f \circ f)(x) = f(ax^2 + bx + c) = a(ax^2 + bx + c)^2 + b(ax^2 + bx + c) + c$. Expanding this expression yields $a^3x^4 + 2a^2bx^3 + 2a^2cx^2 + ab^2x^2 + 2abcx + ac^2 + abx^2 + b^2x + bc + c$, which simplifies to $a^3x^4 + 2a^2bx^3 + (2a^2c + ab^2 + ab)x^2 + (2abc + b^2)x + (ac^2 + bc + c)$. Equating coefficients yields $a^3 = 1$, so $a = 1$. Then $2a^2b = 0$, so $b = 0$. It then follows that $c = -6$, so the original function was $f(x) = x^2 - 6$.

1.1.96 Because the square of a quadratic is a quartic, we let $f(x) = ax^2 + bx + c$. Then the square of f is $c^2 + 2bcx + b^2x^2 + 2acx^2 + 2abx^3 + a^2x^4$. By equating coefficients, we see that $a^2 = 1$ and so $a = \pm 1$. Because the coefficient on x^3 must be 0, we have that $b = 0$. And the constant term reveals that $c = \pm 6$. A quick check shows that the only possible solutions are thus $f(x) = x^2 - 6$ and $f(x) = -x^2 + 6$.

1.1.97 $\dfrac{f(x+h)-f(x)}{h} = \dfrac{\sqrt{x+h}-\sqrt{x}}{h} = \dfrac{\sqrt{x+h}-\sqrt{x}}{h} \cdot \dfrac{\sqrt{x+h}+\sqrt{x}}{\sqrt{x+h}+\sqrt{x}} = \dfrac{(x+h)-x}{h(\sqrt{x+h}+\sqrt{x})} = \dfrac{1}{\sqrt{x+h}+\sqrt{x}}$.

$$\frac{f(x)-f(a)}{x-a} = \frac{\sqrt{x}-\sqrt{a}}{x-a} = \frac{\sqrt{x}-\sqrt{a}}{x-a} \cdot \frac{\sqrt{x}+\sqrt{a}}{\sqrt{x}+\sqrt{a}} = \frac{x-a}{(x-a)(\sqrt{x}+\sqrt{a})} = \frac{1}{\sqrt{x}+\sqrt{a}}.$$

1.1.98 $\dfrac{f(x+h)-f(x)}{h} = \dfrac{\sqrt{1-2(x+h)}-\sqrt{1-2x}}{h} = \dfrac{\sqrt{1-2(x+h)}-\sqrt{1-2x}}{h} \cdot \dfrac{\sqrt{1-2(x+h)}+\sqrt{1-2x}}{\sqrt{1-2(x+h)}+\sqrt{1-2x}} = \dfrac{1-2(x+h)-(1-2x)}{h(\sqrt{1-2(x+h)}+\sqrt{1-2x})} = -\dfrac{2}{\sqrt{1-2(x+h)}+\sqrt{1-2x}}$.

$$\frac{f(x)-f(a)}{x-a} = \frac{\sqrt{1-2x}-\sqrt{1-2a}}{x-a} = \frac{\sqrt{1-2x}-\sqrt{1-2a}}{x-a} \cdot \frac{\sqrt{1-2x}+\sqrt{1-2a}}{\sqrt{1-2x}+\sqrt{1-2a}} = \frac{(1-2x)-(1-2a)}{(x-a)(\sqrt{1-2x}+\sqrt{1-2a})} = \frac{(-2)(x-a)}{(x-a)(\sqrt{1-2x}+\sqrt{1-2a})} = -\frac{2}{(\sqrt{1-2x}+\sqrt{1-2a})}.$$

1.1.99 $\dfrac{f(x+h)-f(x)}{h} = \dfrac{\frac{-3}{\sqrt{x+h}} - \frac{-3}{\sqrt{x}}}{h} = \dfrac{-3(\sqrt{x}-\sqrt{x+h})}{h\sqrt{x}\sqrt{x+h}} = \dfrac{-3(\sqrt{x}-\sqrt{x+h})}{h\sqrt{x}\sqrt{x+h}} \cdot \dfrac{\sqrt{x}+\sqrt{x+h}}{\sqrt{x}+\sqrt{x+h}} = \dfrac{-3(x-(x+h))}{h\sqrt{x}\sqrt{x+h}(\sqrt{x}+\sqrt{x+h})} = \dfrac{3}{\sqrt{x}\sqrt{x+h}(\sqrt{x}+\sqrt{x+h})}$.

$$\frac{f(x)-f(a)}{x-a} = \frac{\frac{-3}{\sqrt{x}} - \frac{-3}{\sqrt{a}}}{x-a} = \frac{-3\left(\frac{\sqrt{a}-\sqrt{x}}{\sqrt{a}\sqrt{x}}\right)}{x-a} = \frac{(-3)(\sqrt{a}-\sqrt{x})}{(x-a)\sqrt{a}\sqrt{x}} \cdot \frac{\sqrt{a}+\sqrt{x}}{\sqrt{a}+\sqrt{x}} = \frac{(3)(x-a)}{(x-a)(\sqrt{a}\sqrt{x})(\sqrt{a}+\sqrt{x})} = \frac{3}{\sqrt{ax}(\sqrt{a}+\sqrt{x})}.$$

1.1.100 $\dfrac{f(x+h)-f(x)}{h} = \dfrac{\sqrt{(x+h)^2+1}-\sqrt{x^2+1}}{h} = \dfrac{\sqrt{(x+h)^2+1}-\sqrt{x^2+1}}{h} \cdot \dfrac{\sqrt{(x+h)^2+1}+\sqrt{x^2+1}}{\sqrt{(x+h)^2+1}+\sqrt{x^2+1}} = \dfrac{(x+h)^2+1-(x^2+1)}{h(\sqrt{(x+h)^2+1}+\sqrt{x^2+1})} = \dfrac{x^2+2hx+h^2-x^2}{h(\sqrt{(x+h)^2+1}+\sqrt{x^2+1})} = \dfrac{2x+h}{\sqrt{(x+h)^2+1}+\sqrt{x^2+1}}$.

$$\frac{f(x)-f(a)}{x-a} = \frac{\sqrt{x^2+1}-\sqrt{a^2+1}}{x-a} = \frac{\sqrt{x^2+1}-\sqrt{a^2+1}}{x-a} \cdot \frac{\sqrt{x^2+1}+\sqrt{a^2+1}}{\sqrt{x^2+1}+\sqrt{a^2+1}} =$$
$$\frac{x^2+1-(a^2+1)}{(x-a)(\sqrt{x^2+1}+\sqrt{a^2+1})} = \frac{(x-a)(x+a)}{(x-a)(\sqrt{x^2+1}+\sqrt{a^2+1})} = \frac{x+a}{\sqrt{x^2+1}+\sqrt{a^2+1}}.$$

1.1.101 This would not necessarily have either kind of symmetry. For example, $f(x) = x^2$ is an even function and $g(x) = x^3$ is odd, but the sum of these two is neither even nor odd.

1.1.102 This would be an odd function, so it would be symmetric about the origin. Suppose f is even and g is odd. Then $(f \cdot g)(-x) = f(-x)g(-x) = f(x) \cdot (-g(x)) = -(f \cdot g)(x)$.

1.1.103 This would be an even function, so it would be symmetric about the y-axis. Suppose f is even and g is odd. Then $g(f(-x)) = g(f(x))$, because $f(-x) = f(x)$.

1.1.104 This would be an even function, so it would be symmetric about the y-axis. Suppose f is even and g is odd. Then $f(g(-x)) = f(-g(x)) = f(g(x))$.

1.2 Representing Functions

1.2.1 Functions can be defined and represented by a formula, through a graph, via a table, and by using words.

1.2.2 The domain of every polynomial is the set of all real numbers.

1.2.3 The slope of the line shown is $m = \frac{-3-(-1)}{3-0} = -2/3$. The y-intercept is $b = -1$. Thus the function is given by $f(x) = -\frac{2}{3}x - 1$.

1.2.4 Because it is to be parallel to a line with slope 2, it must also have slope 2. Using the point-slope form of the equation of the line, we have $y - 0 = 2(x - 5)$, or $y = 2x - 10$.

1.2.5 The domain of a rational function $\frac{p(x)}{q(x)}$ is the set of all real numbers for which $q(x) \neq 0$.

1.2.6 A piecewise linear function is one which is linear over intervals in the domain.

1.2.7 For $x < 0$, the graph is a line with slope 1 and y- intercept 3, while for $x > 0$, it is a line with slope $-1/2$ and y-intercept 3. Note that both of these lines contain the point $(0, 3)$. The function shown can thus be written

$$f(x) = \begin{cases} x+3 & \text{if } x < 0; \\ -\frac{1}{2}x + 3 & \text{if } x \geq 0. \end{cases}$$

1.2.8 The transformed graph would have equation $y = \sqrt{x-2} + 3$.

1.2.9 Compared to the graph of $f(x)$, the graph of $f(x + 2)$ will be shifted 2 units to the left.

1.2.10 Compared to the graph of $f(x)$, the graph of $-3f(x)$ will be scaled vertically by a factor of 3 and flipped about the x axis.

1.2.11 Compared to the graph of $f(x)$, the graph of $f(3x)$ will be compressed horizontally by a factor of $\frac{1}{3}$.

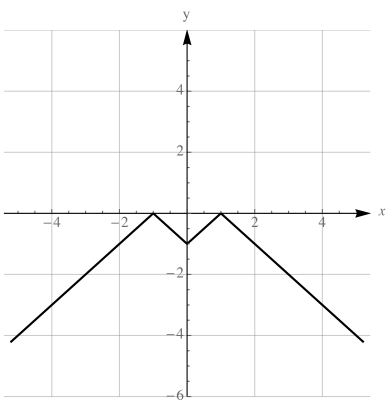
1.2.12 To produce the graph of $y = 4(x + 3)^2 + 6$ from the graph of x^2, one must

1. shift the graph horizontally by 3 units to left
2. scale the graph vertically by a factor of 4
3. shift the graph vertically up 6 units.

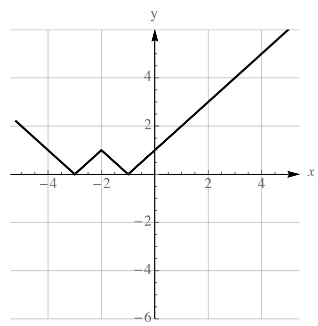
1.2.13 $f(x) = |x-2|+3$, because the graph of f is obtained from that of $|x|$ by shifting 2 units to the right and 3 units up.

$g(x) = -|x+2|-1$, because the graph of g is obtained from the graph of $|x|$ by shifting 2 units to the left, then reflecting about the x-axis, and then shifting 1 unit down.

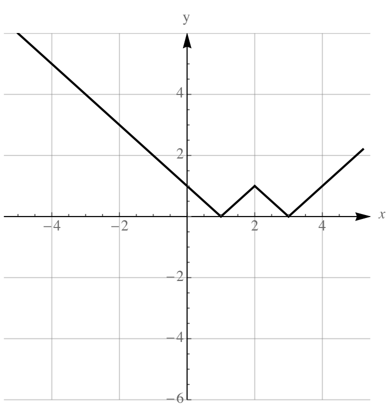
1.2.14



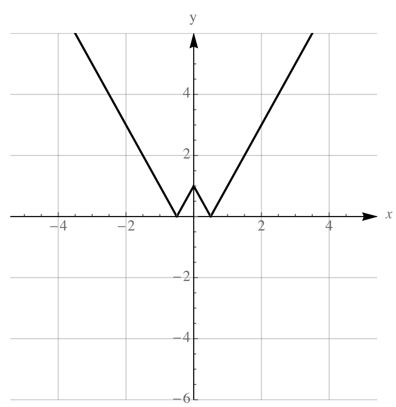
a.



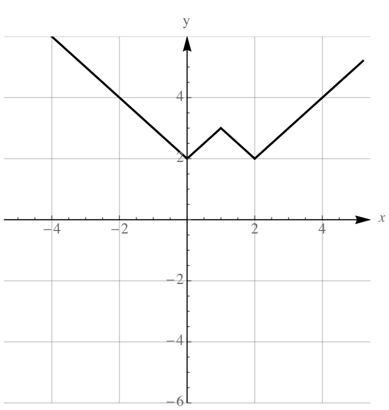
b.



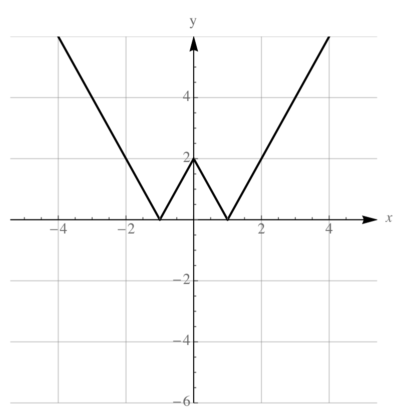
c.



d.



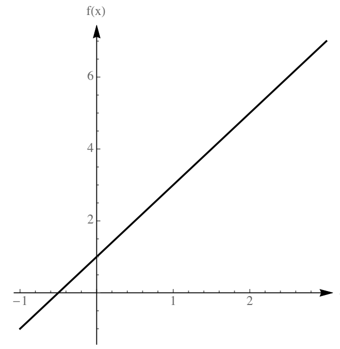
e.



f.

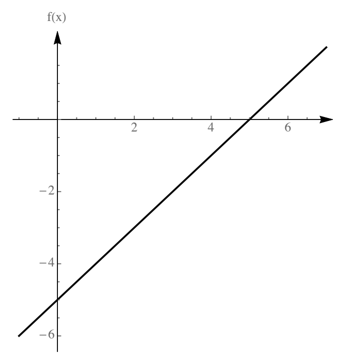
1.2.15

The slope is given by $\frac{5-3}{2-1} = 2$, so the equation of the line is $y - 3 = 2(x - 1)$, which can be written as $f(x) = 2x - 2 + 3$, or $f(x) = 2x + 1$.



1.2.16

The slope is given by $\frac{0-(-3)}{5-2} = 1$, so the equation of the line is $y - 0 = 1(x - 5)$, or $f(x) = x - 5$.



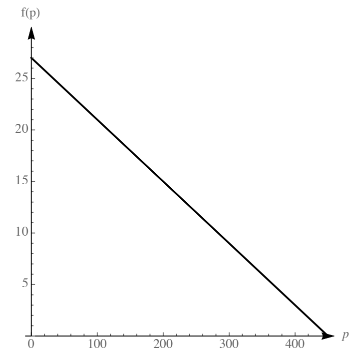
1.2.17 We are looking for the line with slope 3 that goes through the point $(3, 2)$. Using the point-slope form of the equation of a line, we have $y - 2 = 3(x - 3)$, which can be written as $y = 2 + 3x - 9$, or $y = 3x - 7$.

1.2.18 We are looking for the line with slope -4 which goes through the point $(-1, 4)$. Using the point-slope form of the equation of a line, we have $y - 4 = -4(x - (-1))$, which can be written as $y = 4 - 4x - 4$, or $y = -4x$.

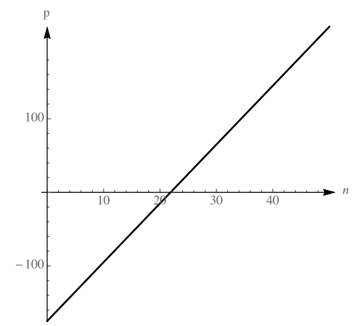
1.2.19 We have $571 = C_s(100)$, so $C_s = 5.71$. Therefore $N(150) = 5.71(150) = 856.5$ million.

1.2.20 We have $226 = C_s(100)$, so $C_s = 2.26$. Therefore $N(150) = 2.26(150) = 339$ million.

1.2.21 Using price as the independent variable p and the average number of units sold per day as the dependent variable d, we have the ordered pairs $(250, 12)$ and $(200, 15)$. The slope of the line determined by these points is $m = \frac{15-12}{200-250} = \frac{3}{-50}$. Thus the demand function has the form $d(p) = (-3/50)p + b$ for some constant b. Using the point $(200, 15)$, we find that $15 = (-3/50) \cdot 200 + b$, so $b = 27$. Thus the demand function is $d = (-3p/50) + 27$. While the domain of this linear function is the set of all real numbers, the formula is only likely to be valid for some subset of the interval $(0, 450)$, because outside of that interval either $p \leq 0$ or $d \leq 0$.



1.2.22 The profit is given by $p = f(n) = 8n - 175$. The break-even point is when $p = 0$, which occurs when $n = 175/8 = 21.875$, so they need to sell at least 22 tickets to not have a negative profit.

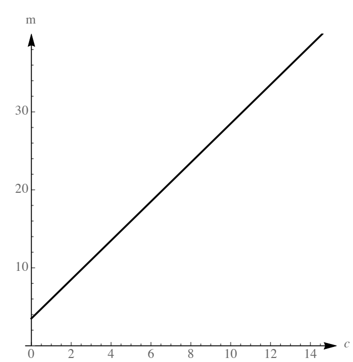


1.2.23

a. Using the points $(1986, 1875)$ and $(2000, 6471)$ we see that the slope is about 328.3. At $t = 0$, the value of p is 1875. Therefore a line which reasonably approximates the data is $p(t) = 328.3t + 1875$.

b. Using this line, we have that $p(9) = 4830$ breeding pairs.

1.2.24 The cost per mile is the slope of the desired line, and the intercept is the fixed cost of 3.5. Thus, the cost per mile is given by $c(m) = 2.5m + 3.5$. When $m = 9$, we have $c(9) = (2.5)(9) + 3.5 = 22.5 + 3.5 = 26$ dollars.



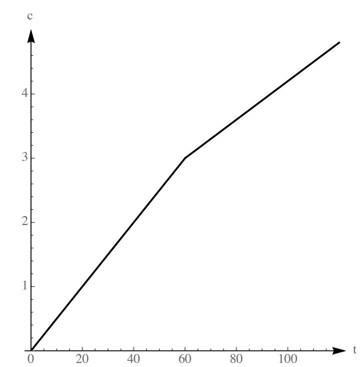
1.2.25 For $x \leq 3$, we have the constant function 3. For $x \geq 3$, we have a straight line with slope 2 that contains the point $(3, 3)$. So its equation is $y - 3 = 2(x - 3)$, or $y = 2x - 3$. So the function can be written as $f(x) = \begin{cases} 3 & \text{if } x \leq 3; \\ 2x - 3 & \text{if } x > 3 \end{cases}$

1.2.26 For $x < 3$ we have straight line with slope 1 and y-intercept 1, so the equation is $y = x+1$. For $x \geq 3$, we have a straight line with slope $-\frac{1}{3}$ which contains the point $(3, 2)$, so its equation is $y - 2 = -\frac{1}{3}(x - 3)$, or $y = -\frac{1}{3}x + 3$. Thus the function can be written as $f(x) = \begin{cases} x+1 & \text{if } x < 3; \\ -\frac{1}{3}x + 3 & \text{if } x \geq 3 \end{cases}$

1.2.27

The cost is given by

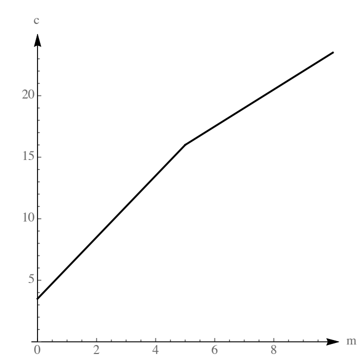
$$c(t) = \begin{cases} 0.05t & \text{for } 0 \leq t \leq 60 \\ 1.2 + 0.03t & \text{for } 60 < t \leq 120 \end{cases}.$$



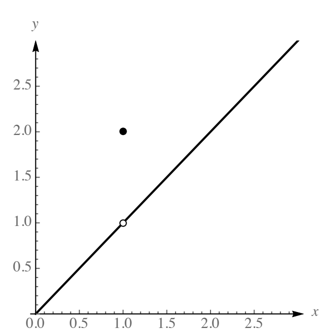
1.2.28

The cost is given by

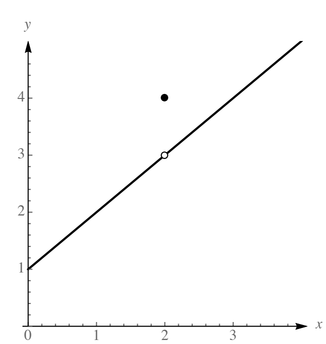
$$c(m) = \begin{cases} 3.5 + 2.5m & \text{for } 0 \leq m \leq 5 \\ 8.5 + 1.5m & \text{for } m > 5 \end{cases}.$$



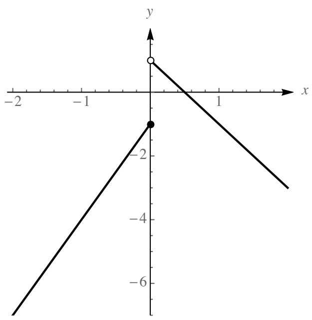
1.2.29



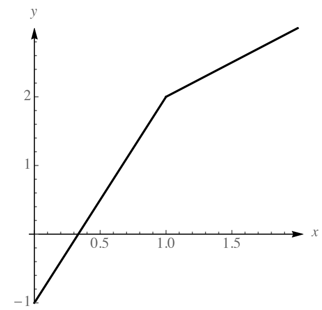
1.2.30



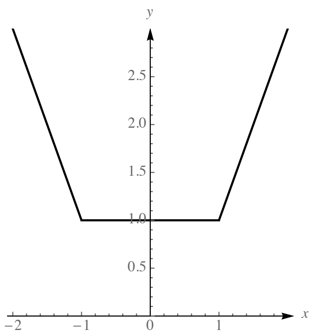
1.2.31



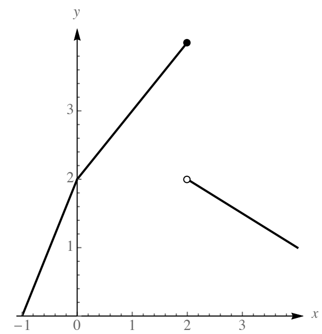
1.2.32



1.2.33

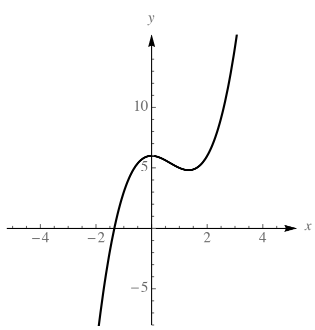


1.2.34



1.2.35

a.

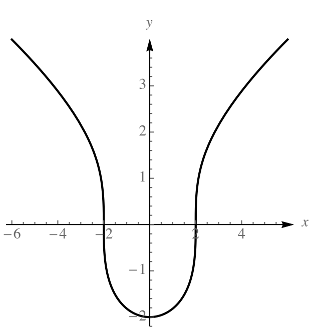


b. The function is a polynomial, so its domain is the set of all real numbers.

c. It has one peak near its y-intercept of $(0, 6)$ and one valley between $x = 1$ and $x = 2$. Its x-intercept is near $x = -4/3$.

1.2.36

a.

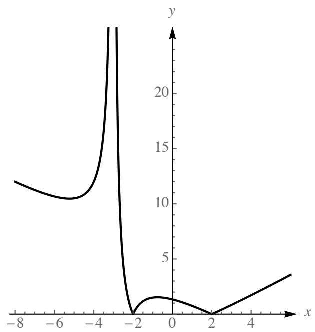


b. The function's domain is the set of all real numbers.

c. It has a valley at the y-intercept of $(0, -2)$, and is very steep at $x = -2$ and $x = 2$ which are the x-intercepts. It is symmetric about the y-axis.

1.2.37

a.

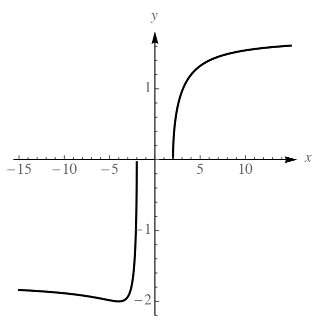


b. The domain of the function is the set of all real numbers except -3.

c. There is a valley near $x = -5.2$ and a peak near $x = -0.8$. The x-intercepts are at -2 and 2, where the curve does not appear to be smooth. There is a vertical asymptote at $x = -3$. The function is never below the x-axis. The y-intercept is $(0, 4/3)$.

1.2.38

a.

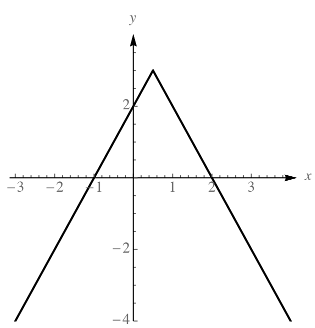


b. The domain of the function is $(-\infty, -2] \cup [2, \infty)$

c. x-intercepts are at -2 and 2. Because 0 isn't in the domain, there is no y-intercept. The function has a valley at $x = -4$.

1.2.39

a.

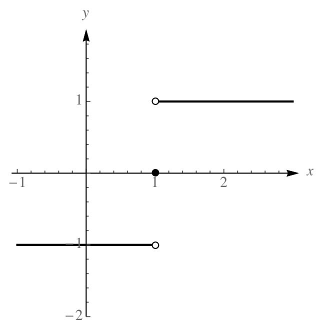


b. The domain of the function is $(-\infty, \infty)$

c. The function has a maximum of 3 at $x = 1/2$, and a y-intercept of 2.

1.2.40

a.



b. The domain of the function is $(-\infty, \infty)$

c. The function contains a jump at $x = 1$. The maximum value of the function is 1 and the minimum value is -1.

1.2.41

a. The zeros of f are the points where the graph crosses the x-axis, so these are points A, D, F, and I.

b. The only high point, or peak, of f occurs at point E, because it appears that the graph has larger and larger y values as x increases past point I and decreases past point A.

c. The only low points, or valleys, of f are at points B and H, again assuming that the graph of f continues its apparent behavior for larger values of x.

d. Past point H, the graph is rising, and is rising faster and faster as x increases. It is also rising between points B and E, but not as quickly as it is past point H. So the marked point at which it is rising most rapidly is I.

e. Before point B, the graph is falling, and falls more and more rapidly as x becomes more and more negative. It is also falling between points E and H, but not as rapidly as it is before point B. So the marked point at which it is falling most rapidly is A.

1.2.42

a. The zeros of g appear to be at $x = 0$, $x = 1$, $x = 1.6$, and $x \approx 3.15$.

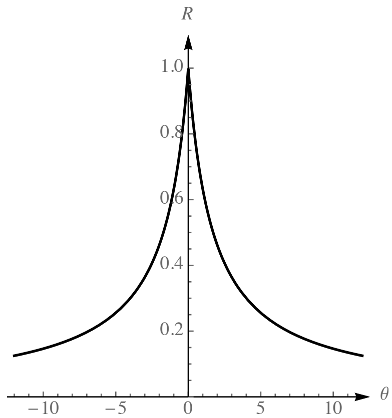
b. The two peaks of g appear to be at $x \approx 0.5$ and $x \approx 2.6$, with corresponding points $\approx (0.5, 0.4)$ and $\approx (2.6, 3.4)$.

c. The only valley of g is at $\approx (1.3, -0.2)$.

d. Moving right from $x \approx 1.3$, the graph is rising more and more rapidly until about $x = 2$, at which point it starts rising less rapidly (because, by $x \approx 2.6$, it is not rising at all). So the coordinates of the point at which it is rising most rapidly are approximately $(2.1, g(2)) \approx (2.1, 2)$. Note that while the curve is also rising between $x = 0$ and $x \approx 0.5$, it is not rising as rapidly as it is near $x = 2$.

e. To the right of $x \approx 2.6$, the curve is falling, and falling more and more rapidly as x increases. So the point at which it is falling most rapidly in the interval $[0, 3]$ is at $x = 3$, which has the approximate coordinates $(3, 1.4)$. Note that while the curve is also falling between $x \approx 0.5$ and $x \approx 1.3$, it is not falling as rapidly as it is near $x = 3$.

1.2.43

a. 

b. This appears to have a maximum when $\theta = 0$. Our vision is sharpest when we look straight ahead.

c. For $|\theta| \leq .19°$. We have an extremely narrow range where our eyesight is sharp.

1.2.44 Because the line is horizontal, the slope is constantly 0. So $S(x) = 0$.

1.2.45 The slope of this line is constantly 2, so the slope function is $S(x) = 2$.

1.2.46 The function can be written as $|x| = \begin{cases} -x & \text{if } x \leq 0 \\ x & \text{if } x > 0 \end{cases}$.

The slope function is $S(x) = \begin{cases} -1 & \text{if } x < 0 \\ 1 & \text{if } x > 0 \end{cases}$.

1.2.47 The slope function is given by $S(x) = \begin{cases} 1 & \text{if } x < 0; \\ -1/2 & \text{if } x > 0. \end{cases}$

1.2.48 The slope function is given by $s(x) = \begin{cases} 1 & \text{if } x < 3; \\ -1/3 & \text{if } x > 3. \end{cases}$

1.2.49

a. Because the area under consideration is that of a rectangle with base 2 and height 6, $A(2) = 12$.

b. Because the area under consideration is that of a rectangle with base 6 and height 6, $A(6) = 36$.

c. Because the area under consideration is that of a rectangle with base x and height 6, $A(x) = 6x$.

1.2.50

a. Because the area under consideration is that of a triangle with base 2 and height 1, $A(2) = 1$.

b. Because the area under consideration is that of a triangle with base 6 and height 3, the $A(6) = 9$.

c. Because $A(x)$ represents the area of a triangle with base x and height $(1/2)x$, the formula for $A(x)$ is $\frac{1}{2} \cdot x \cdot \frac{x}{2} = \frac{x^2}{4}$.

1.2.51

a. Because the area under consideration is that of a trapezoid with base 2 and heights 8 and 4, we have $A(2) = 2 \cdot \frac{8+4}{2} = 12$.

b. Note that $A(3)$ represents the area of a trapezoid with base 3 and heights 8 and 2, so $A(3) = 3 \cdot \frac{8+2}{2} = 15$. So $A(6) = 15 + (A(6) - A(3))$, and $A(6) - A(3)$ represents the area of a triangle with base 3 and height 2. Thus $A(6) = 15 + 6 = 21$.

c. For x between 0 and 3, $A(x)$ represents the area of a trapezoid with base x, and heights 8 and $8 - 2x$. Thus the area is $x \cdot \frac{8+8-2x}{2} = 8x - x^2$. For $x > 3$, $A(x) = A(3) + A(x) - A(3) = 15 + 2(x - 3) = 2x + 9$. Thus

$$A(x) = \begin{cases} 8x - x^2 & \text{if } 0 \le x \le 3; \\ 2x + 9 & \text{if } x > 3. \end{cases}$$

1.2.52

a. Because the area under consideration is that of trapezoid with base 2 and heights 3 and 1, we have $A(2) = 2 \cdot \frac{3+1}{2} = 4$.

b. Note that $A(6) = A(2) + (A(6) - A(2)$, and that $A(6) - A(2)$ represents a trapezoid with base $6 - 2 = 4$ and heights 1 and 5. The area is thus $4 + \left(4 \cdot \frac{1+5}{2}\right) = 4 + 12 = 16$.

c. For x between 0 and 2, $A(x)$ represents the area of a trapezoid with base x, and heights 3 and $3 - x$. Thus the area is $x \cdot \frac{3+3-x}{2} = 3x - \frac{x^2}{2}$. For $x > 2$, $A(x) = A(2) + A(x) - A(2) = 4 + (A(x) - A(2))$. Note that $A(x) - A(2)$ represents the area of a trapezoid with base $x - 2$ and heights 1 and $x - 1$. Thus $A(x) = 4 + (x - 2) \cdot \frac{1+x-1}{2} = 4 + (x - 2)\left(\frac{x}{2}\right) = \frac{x^2}{2} - x + 4$. Thus

$$A(x) = \begin{cases} 3x - \frac{x^2}{2} & \text{if } 0 \le x \le 2; \\ \frac{x^2}{2} - x + 4 & \text{if } x > 2. \end{cases}$$

1.2.53

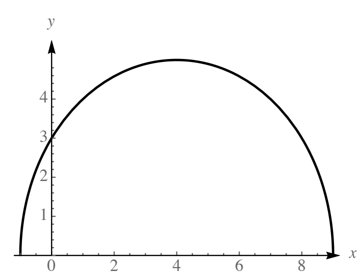
a. True. A polynomial $p(x)$ can be written as the ratio of polynomials $\frac{p(x)}{1}$, so it is a rational function. However, a rational function like $\frac{1}{x}$ is not a polynomial.

b. False. For example, if $f(x) = 2x$, then $(f \circ f)(x) = f(f(x)) = f(2x) = 4x$ is linear, not quadratic.

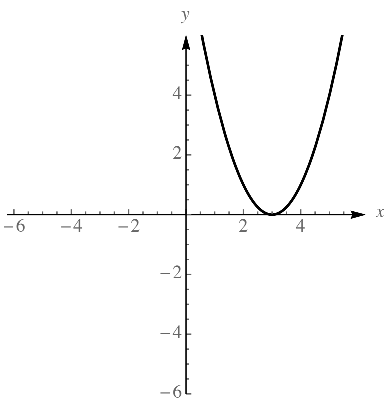
c. True. In fact, if f is degree m and g is degree n, then the degree of the composition of f and g is $m \cdot n$, regardless of the order they are composed.

d. False. The graph would be shifted two units to the left.

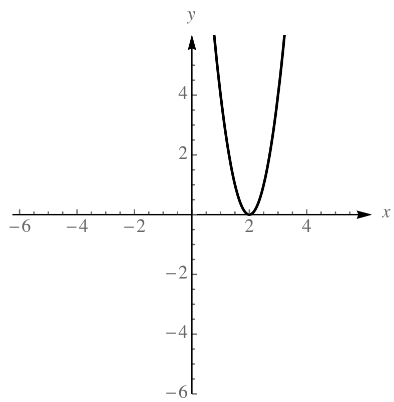
1.2.54 We complete the square for $-x^2 + 8x + 9$. Call this quantity z. Then $z = -(x^2 - 8x - 9)$, so $z = -(x^2 - 8x + 16 + (-16 - 9)) = -((x-4)^2 - 25 = 25 - (x-4)^2$. Thus $f(x)$ is obtained from the graph of $g(x) = \sqrt{25 - x^2}$ by shifting 4 units to the right. Thus the graph of f is the upper half of a circle of radius 5 centered at $(4, 0)$.



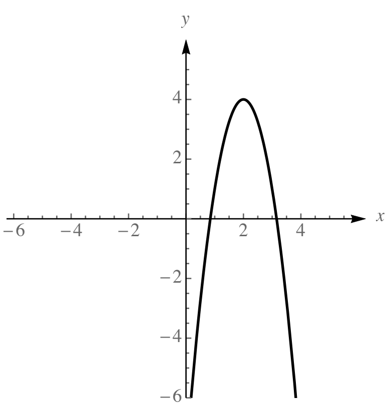
1.2.55



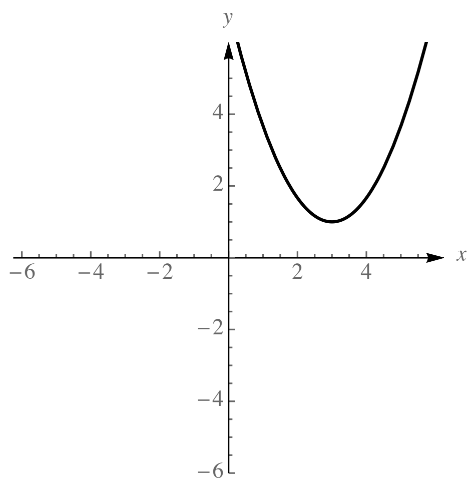
a.
Shift 3 units to the right.



b.
Horizontal compression by a factor of $\frac{1}{2}$, then shift 2 units to the right.

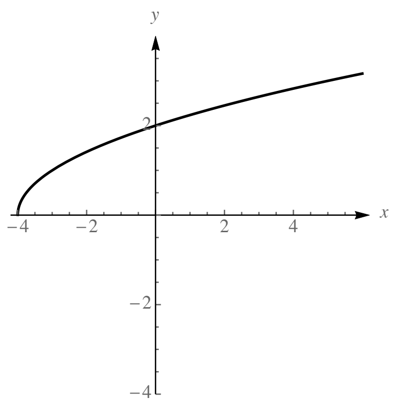


c.
Shift to the right 2 units, vertically stretch by a factor of 3, reflect across the x-axis, and shift up 4 units.

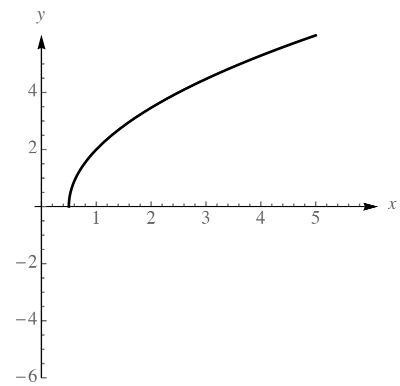


d.
Horizontal stretch by a factor of 3, horizontal shift right 2 units, vertical stretch by a factor of 6, and vertical shift up 1 unit.

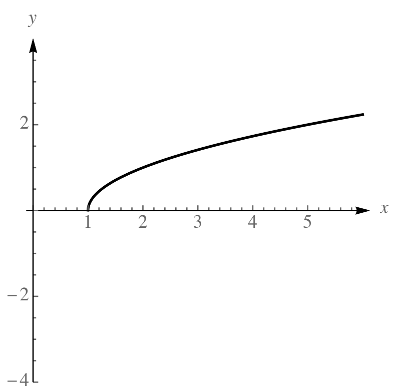
1.2.56



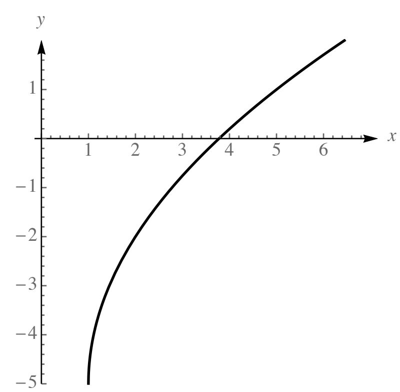
a. Shift 4 units to the left.



b. Horizontal compression by a factor of $\frac{1}{2}$, then shift $\frac{1}{2}$ units to the right. Then stretch vertically by a factor of 2.

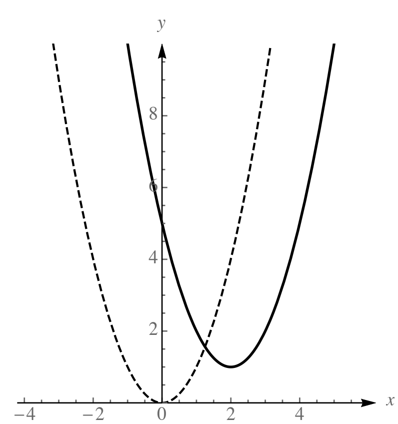


c. Shift 1 unit to the right.

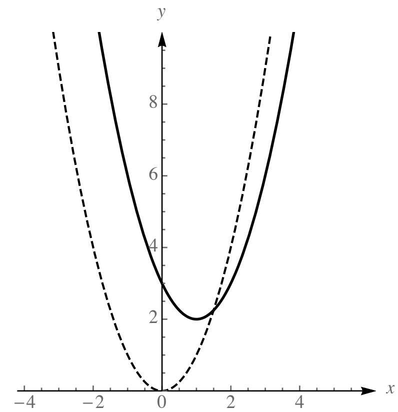


d. Shift 1 unit to the right, then stretch vertically by a factor of 3, then shift down 5 units.

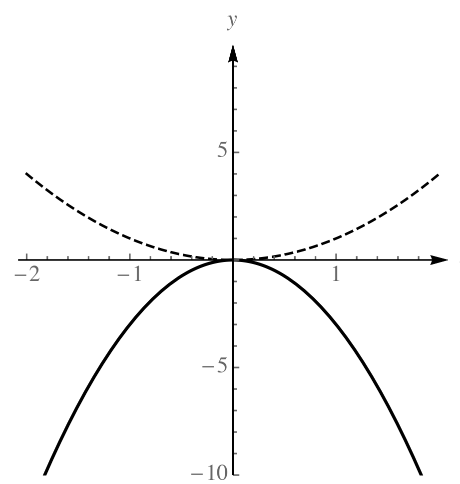
1.2.57 The graph is obtained by shifting the graph of x^2 two units to the right and one unit up.



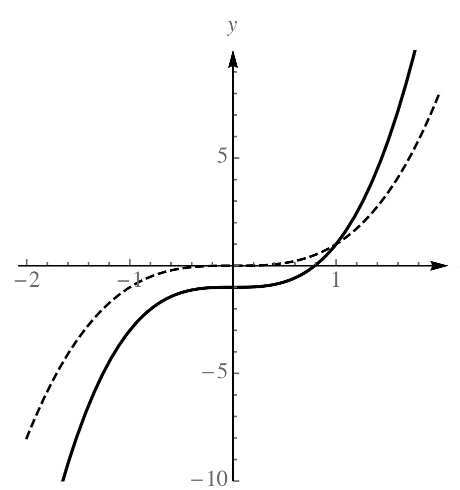
1.2.58 Write x^2-2x+3 as $(x^2-2x+1)+2=(x-1)^2+2$. The graph is obtained by shifting the graph of x^2 one unit to the right and two units up.

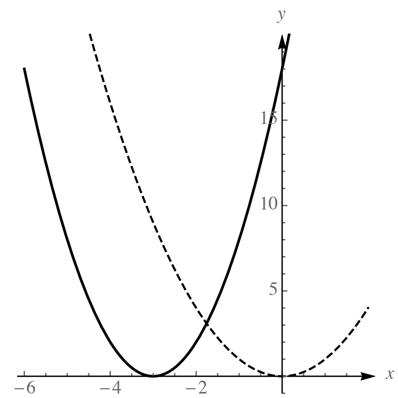


1.2.59 Stretch the graph of $y=x^2$ vertically by a factor of 3 and then reflect across the x-axis.

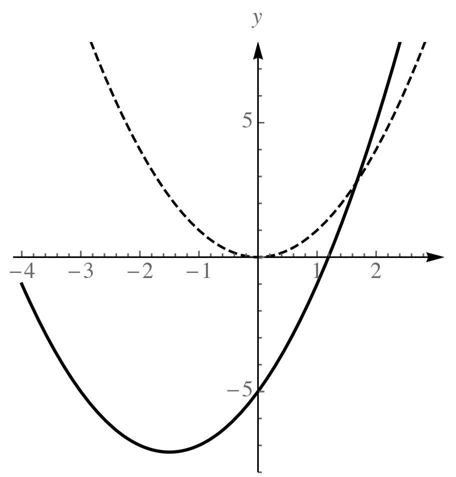


1.2.60 Scale the graph of $y=x^3$ vertically by a factor of 2, and then shift down 1 unit.

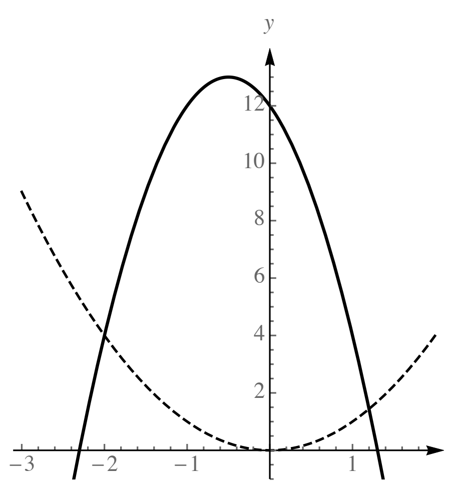




1.2.61 Shift the graph of $y = x^2$ left 3 units and stretch vertically by a factor of 2.

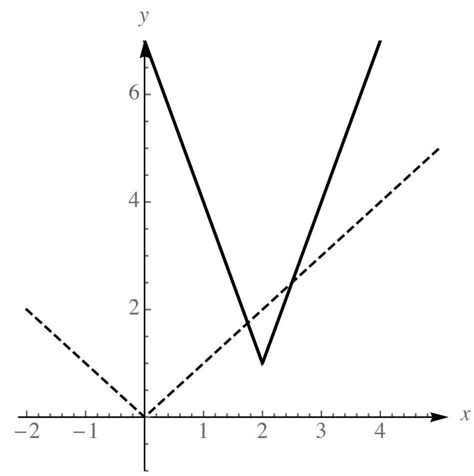


1.2.62 By completing the square, we have that $p(x) = x^2 + 3x - 5 = x^2 + 3x + \frac{9}{4} - 5 - \frac{9}{4} = (x + \frac{3}{2})^2 - \frac{29}{4}$. So it is $f(x + \frac{3}{2}) - (\frac{29}{4})$ where $f(x) = x^2$. The graph is shifted $\frac{3}{2}$ units to the left and then down $\frac{29}{4}$ units.



1.2.63 By completing the square, we have that $h(x) = -4(x^2 + x - 3) = -4\left(x^2 + x + \frac{1}{4} - \frac{1}{4} - 3\right) = -4(x + \frac{1}{2})^2 + 13$. So it is $-4f(x + (\frac{1}{2})) + 13$ where $f(x) = x^2$. The graph is shifted $\frac{1}{2}$ unit to the left, stretched vertically by a factor of 4, then reflected about the x-axis, then shifted up 13 units.

1.2.64 Because $|3x-6|+1 = 3|x-2|+1$, this is $3f(x-2)+1$ where $f(x) = |x|$. The graph is shifted 2 units to the right, then stretched vertically by a factor of 3, and then shifted up 1 unit.

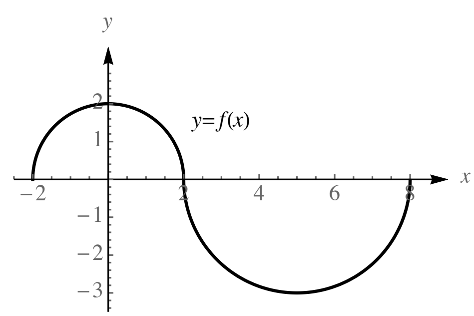


1.2.65 The curves intersect where $4\sqrt{2x} = 2x^2$. If we square both sides, we have $32x = 4x^4$, which can be written as $4x(8-x^3) = 0$, which has solutions at $x = 0$ and $x = 2$. So the points of intersection are $(0,0)$ and $(2,8)$.

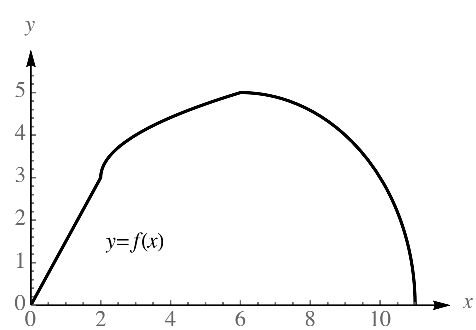
1.2.66 The points of intersection are found by solving $x^2 + 2 = x + 4$. This yields the quadratic equation $x^2 - x - 2 = 0$ or $(x-2)(x+1) = 0$. So the x-values of the points of intersection are 2 and -1. The actual points of intersection are $(2,6)$ and $(-1,3)$.

1.2.67 The points of intersection are found by solving $x^2 = -x^2 + 8x$. This yields the quadratic equation $2x^2 - 8x = 0$ or $(2x)(x-4) = 0$. So the x-values of the points of intersection are 0 and 4. The actual points of intersection are $(0,0)$ and $(4,16)$.

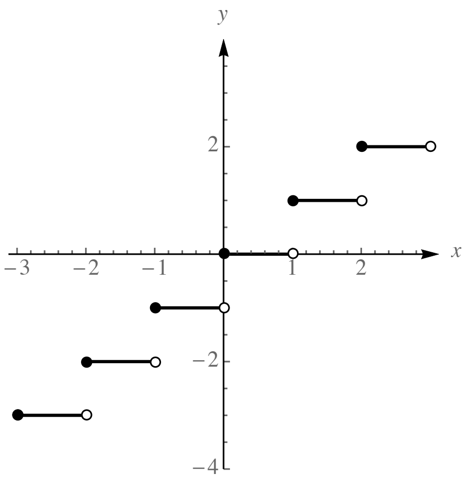
1.2.68 $f(x) = \begin{cases} \sqrt{4-x^2} & \text{if } -2 \le x \le 2 \\ -\sqrt{9-(x-5)^2} & \text{if } 2 < x \le 6. \end{cases}$.



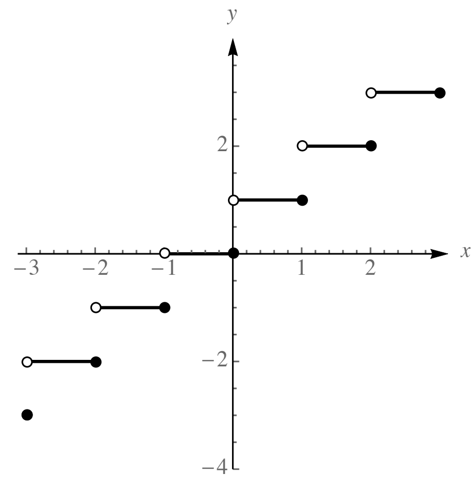
1.2.69



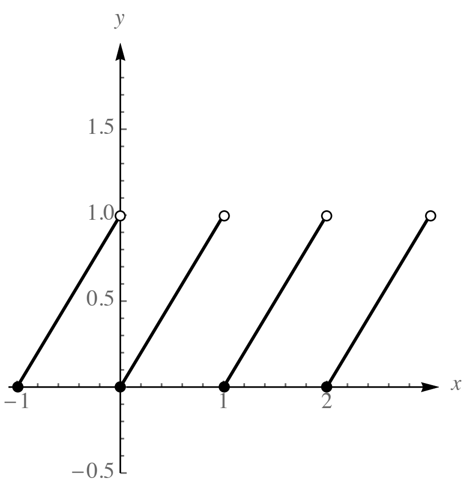
1.2.70



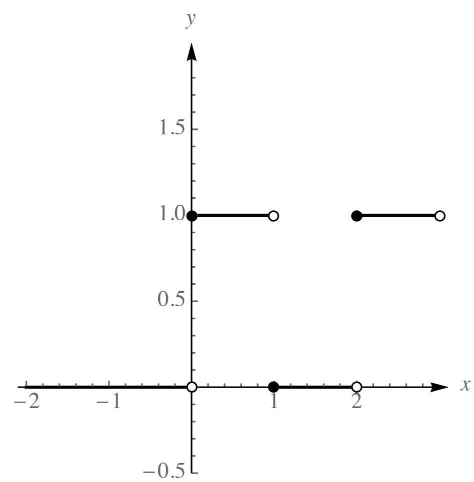
1.2.71



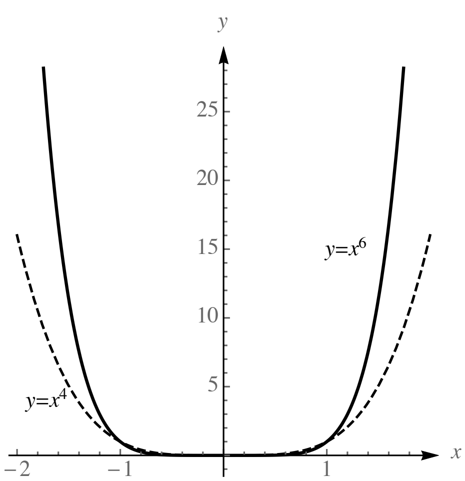
1.2.72



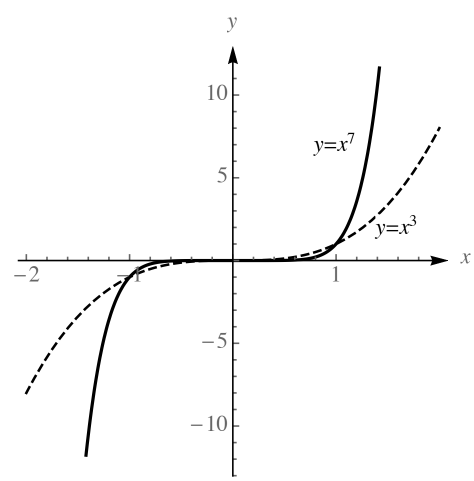
1.2.73



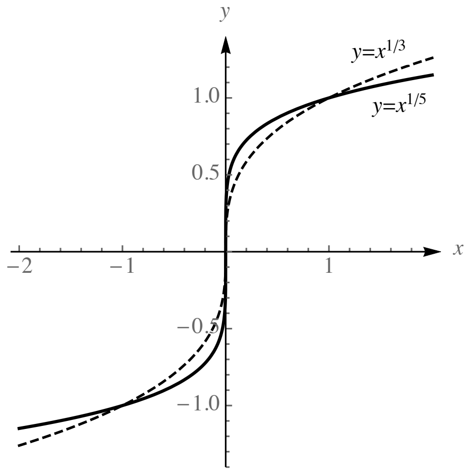
1.2.74



1.2.75



1.2.76



1.2.77

a. $f(0.75) = \frac{.75^2}{1-2(.75)(.25)} = .9$. There is a 90% chance that the server will win from deuce if they win 75% of their service points.

b. $f(0.25) = \frac{.25^2}{1-2(.25)(.75)} = .1$. There is a 10% chance that the server will win from deuce if they win 25% of their service points.

1.2.78

a. We know that the points $(32, 0)$ and $(212, 100)$ are on our line. The slope of our line is thus $\frac{100-0}{212-32} = \frac{100}{180} = \frac{5}{9}$. The function $f(F)$ thus has the form $C = (5/9)F + b$, and using the point $(32, 0)$ we see that $0 = (5/9)32 + b$, so $b = -(160/9)$. Thus $C = (5/9)F - (160/9)$

b. Solving the system of equations $C = (5/9)F - (160/9)$ and $C = F$, we have that $F = (5/9)F - (160/9)$, so $(4/9)F = -160/9$, so $F = -40$ when $C = -40$.

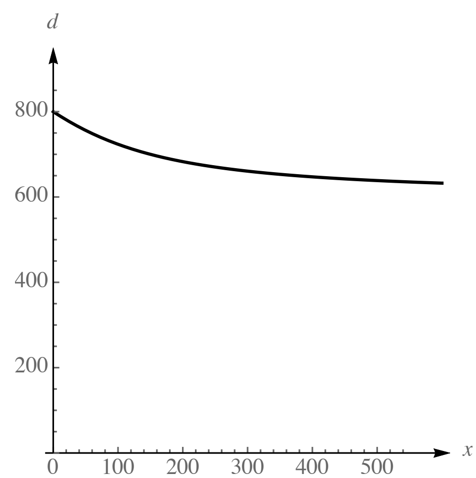
1.2.79

a. Because you are paying \$350 per month, the amount paid after m months is $y = 350m + 1200$.

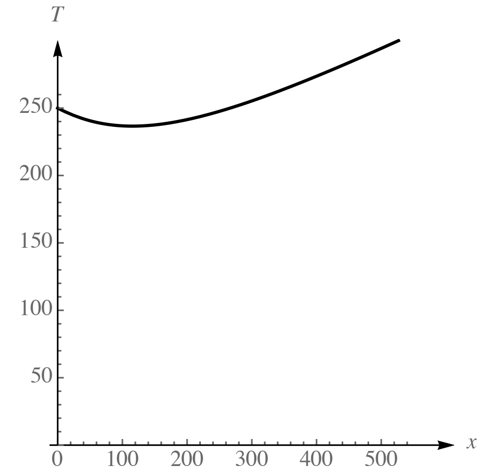
b. After 4 years (48 months) you have paid $350 \cdot 48 + 1200 = 18000$ dollars. If you then buy the car for \$10,000, you will have paid a total of \$28,000 for the car instead of \$25,000. So you should buy the car instead of leasing it.

1.2.80

a. Note that the island, the point P on shore, and the point down shore x units from P form a right triangle. By the Pythagorean theorem, the length of the hypotenuse is $\sqrt{40000 + x^2}$. So Kelly must row this distance and then jog $600-x$ meters to get home. So her total distance $d(x) = \sqrt{40000 + x^2} + (600 - x)$.



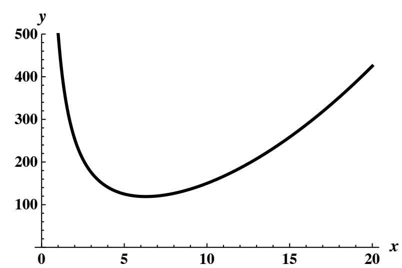
b. Because distance is rate times time, we have that time is distance divided by rate. Thus $T(x) = \frac{\sqrt{40000+x^2}}{2} + \frac{600-x}{4}$.



c. By inspection, it looks as though she should head to a point about 115 meters down shore from P. This would lead to a time of about 236.6 seconds.

1.2.81

a. The volume of the box is x^2h, but because the box has volume 125 cubic feet, we have that $x^2h = 125$, so $h = \frac{125}{x^2}$. The surface area of the box is given by x^2 (the area of the base) plus $4 \cdot hx$, because each side has area hx. Thus $S = x^2 + 4hx = x^2 + \frac{4 \cdot 125 \cdot x}{x^2} = x^2 + \frac{500}{x}$.



b. By inspection, it looks like the value of x which minimizes the surface area is about 6.3.

1.2.82 Let $f(x) = a_nx^n +$ smaller degree terms and let $g(x) = b_mx^m +$ some smaller degree terms.

a. The largest degree term in $f \cdot f$ is $a_nx^n \cdot a_nx^n = a_n^2x^{n+n}$, so the degree of this polynomial is $n+n = 2n$.

b. The largest degree term in $f \circ f$ is $a_n \cdot (a_nx^n)^n$, so the degree is n^2.

c. The largest degree term in $f \cdot g$ is $a_nb_mx^{m+n}$, so the degree of the product is $m + n$.

d. The largest degree term in $f \circ g$ is $a_n \cdot (b_mx^m)^n$, so the degree is mn.

1.2.83 Suppose that the parabola f crosses the x-axis at a and b, with $a < b$. Then a and b are roots of the polynomial, so $(x - a)$ and $(x - b)$ are factors. Thus the polynomial must be $f(x) = c(x - a)(x - b)$ for some non-zero real number c. So $f(x) = cx^2 - c(a + b)x + abc$. Because the vertex always occurs at the x value which is $\frac{-\text{coefficient on x}}{2 \cdot \text{coefficient on } x^2}$ we have that the vertex occurs at $\frac{c(a+b)}{2c} = \frac{a+b}{2}$, which is halfway between a and b.

1.2.84

a. We complete the square to rewrite the function f. Write $f(x) = ax^2 + bx + c$ as $f(x) = a(x^2 + \frac{b}{a}x + \frac{c}{a})$. Completing the square yields

$$a\left(\left(x^2 + \frac{b}{a}x + \frac{b^2}{4a}\right) + \left(\frac{c}{a} - \frac{b^2}{4a}\right)\right) = a\left(x + \frac{b}{2a}\right)^2 + \left(c - \frac{b^2}{4}\right).$$

Thus the graph of f is obtained from the graph of x^2 by shifting $\frac{b}{2a}$ units horizontally (and then doing some scaling and vertical shifting) – moving the vertex from 0 to $-\frac{b}{2a}$. The vertex is therefore $\left(\frac{-b}{2a}, c - \frac{b^2}{4}\right)$.

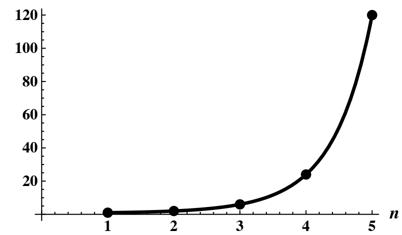
b. We know that the graph of f touches the x-axis twice if the equation $ax^2 + bx + c = 0$ has two real solutions. By the quadratic formula, we know that this occurs exactly when the discriminant $b^2 - 4ac$ is positive. So the condition we seek is for $b^2 - 4ac > 0$, or $b^2 > 4ac$.

1.2.85

a.

n	1	2	3	4	5
$n!$	1	2	6	24	120

b.



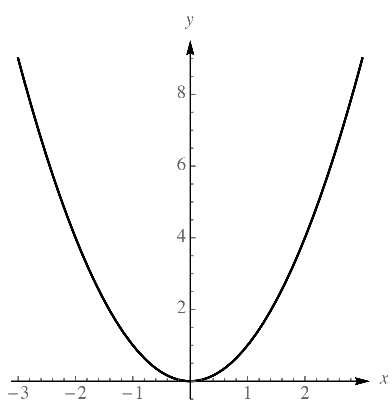
c. Using trial and error and a calculator yields that 10! is more than a million, but 9! isn't.

1.3 Inverse, Exponential and Logarithmic Functions

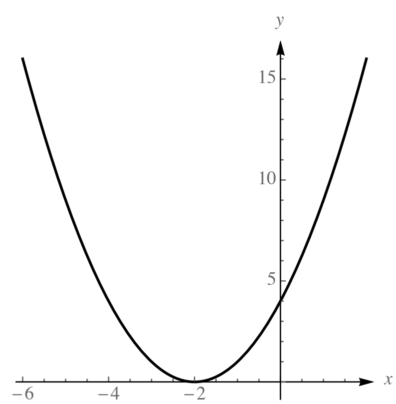
1.3.1 $D = \mathbb{R}, R = (0, \infty)$.

1.3.2 $f(x) = 2x + 1$ is one-to-one on all of $\mathbb{R}$. If $f(a) = f(b)$, then $2a + 1 = 2b + 1$, so it must follow that $a = b$.

1.3.3



1.3.4



1.3.5 f is one-to-one on $(-\infty, -1]$, on $[-1, 1]$, and on $[1, \infty)$.

1.3.6 f is one-to-one on $(-\infty, -2]$, on $[-2, 0]$, on $[0, 2]$, and on $[2, \infty)$.

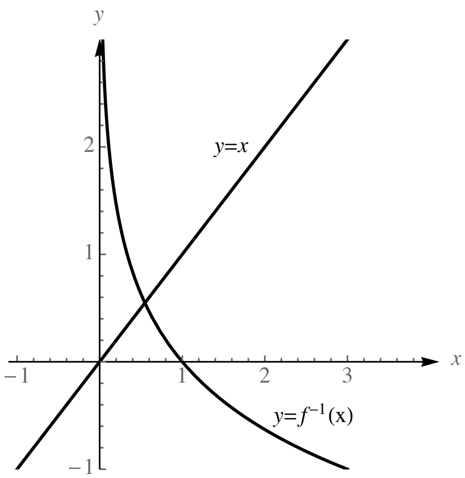
1.3.7 If a function f is not one-to-one, then there are domain values $x_1 \neq x_2$ with $f(x_1) = f(x_2)$. If f^{-1} were to exist, then $f^{-1}(f(x_1)) = f^{-1}(f(x_2))$ which would imply that $x_1 = x_2$, a contradiction.

1.3.8 Because $f(1) = 2$, $f^{-1}(2) = 1$. Because $f(5) = 9$, $f^{-1}(9) = 5$. Because $f(7) = 12$, $f^{-1}(12) = 7$.

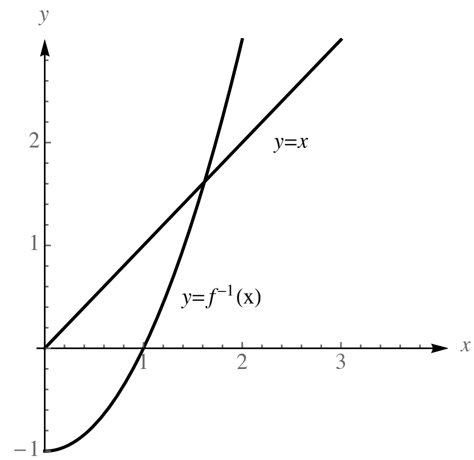
1.3.9 Suppose $x = 2y$, then $y = \frac{1}{2}x$, so the inverse of f is $f^{-1}(x) = \frac{1}{2}x$. Then $f(f^{-1}(x)) = f(\frac{1}{2}x) = 2 \cdot \frac{1}{2}x = x$. Also, $f^{-1}(f(x)) = f^{-1}(2x) = \frac{1}{2} \cdot 2x = x$.

1.3.10 Suppose $x = \sqrt{y}$. Then $y = x^2$. So the inverse of f is $f^{-1}(x) = x^2$, $x \geq 0$. Then $f(f^{-1}(x)) = f(x^2) = \sqrt{x^2} = |x| = x$ for $x \geq 0$. Also, $f^{-1}(f(x)) = f^{-1}(\sqrt{x}) = (\sqrt{x})^2 = x$.

1.3.11



1.3.12



1.3.13 $g_1(x)$ is the right side of the standard parabola shifted up one unit. So $g_1(x) = x^2 + 1$, $x \geq 0$. The domain for g_1 is $[0, \infty)$ and the range is $[1, \infty)$. The inverse of g_1 is therefore the square root function shifted one unit to the right. So $g_1^{-1}(x) = \sqrt{x-1}$, and its domain is $[1, \infty)$ and its range is $[0, \infty)$.

1.3.14 $g_2(x)$ is the left side of the standard parabola shifted up one unit. So $g_2(x) = x^2 + 1$, $x \leq 0$. The domain for g_1 is $(\infty, 0]$ and the range is $[1, \infty)$. The inverse of g_2 is therefore the square root function shifted one unit to the right and reflected across the x-axis. So $g_2^{-1}(x) = -\sqrt{x-1}$, and its domain is $[1, \infty)$ and its range is $(\infty, 0]$.

1.3.15 $\log_b x$ represents the power to which b must be raised in order to obtain x. So, $b^{\log_b x} = x$.

1.3.16 The properties are related in that each can be used to derive the other. Assume $b^{x+y} = b^x b^y$, for all real numbers x and y. Then applying this rule to the numbers $\log_b x$ and $\log_b y$ gives $b^{\log_b x + \log_b y} = b^{\log_b x} b^{\log_b y} = xy$. Taking logs of the leftmost and rightmost sides of this equation yields $\log_b x + \log_b y = \log_b(xy)$.

Now assume that $\log_b(xy) = \log_b x + \log_b y$ for all positive numbers x and y. Applying this rule to the product $b^x b^y$, we have $\log_b(b^x b^y) = \log_b b^x + \log_b b^y = x + y$. Now looking at the leftmost and rightmost sides of this equality and applying the definition of logarithm yields $b^{x+y} = b^x b^y$, as was desired.

1.3.17 Because the domain of b^x is $\mathbb{R}$ and the range of b^x is $(0, \infty)$, and because $\log_b x$ is the inverse of b^x, the domain of $\log_b x$ is $(0, \infty)$ and the range is $\mathbb{R}$.

1.3.18 Let $2^5 = z$. Then $\ln(2^5) = \ln(z)$, so $\ln(z) = 5\ln(2)$. Taking the exponential function of both sides gives $z = e^{5\ln(2)}$. Therefore, $2^5 = e^{5\ln(2)}$.

1.3.19

a. Because $10^3 = 1000$, $\log_{10} 1000 = 3$.

b. Because $2^4 = 16$, $\log_2 16 = 4$.

c. Because $10^{-2} = \frac{1}{100} = 0.01$, $\log_{10} 0.01 = -2$.

d. Because e^x and $\ln x$ are inverses, $\ln e^3 = 3$.

e. Because e^x and $\ln x$ are inverses, $\ln \sqrt{e} = \ln e^{1/2} = \frac{1}{2}$.

1.3.20 $\log_2 a = \frac{\ln a}{\ln 2} \approx 5.4923$, and $\log_a 2 = \frac{\ln 2}{\ln a} \approx 0.1821$.

1.3.21 f is one-to-one on $(-\infty, \infty)$, so it has an inverse on $(-\infty, \infty)$.

1.3.22 f is one-to-one on $[-1/2, \infty)$, so it has an inverse on that set. (Alternatively, it is one-to-one on the interval $(-\infty, -1/2]$, so that interval could be used as well.)

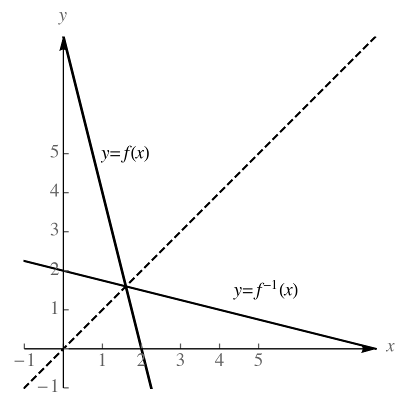
1.3.23 f is one-to-one on its domain, which is $(-\infty, 5) \cup (5, \infty)$, so it has an inverse on that set.

1.3.24 f is one-to-one on the set $(-\infty, 6]$, so it has an inverse on that set. (Alternatively, it is one-to-one on the interval $[6, \infty)$, so that interval could be used as well.)

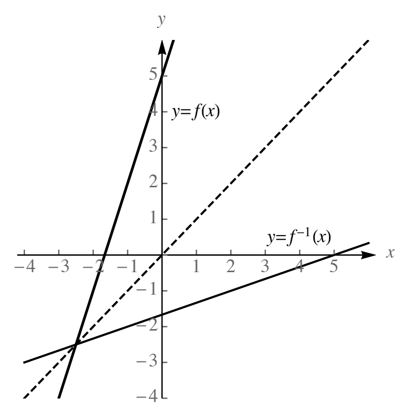
1.3.25 f is one-to-one on the interval $(0, \infty)$, so it has an inverse on that interval. (Alternatively, it is one-to-one on the interval $(-\infty, 0)$, so that interval could be used as well.)

1.3.26 Note that f can be written as $f(x) = x^2 - 2x + 8 = x^2 - 2x + 1 + 7 = (x-1)^2 + 7$. It is one-to-one on the interval $(1, \infty)$, so it has an inverse on that interval. (Alternatively, it is one-to-one on the interval $(-\infty, 1)$, so that interval could be used as well.)

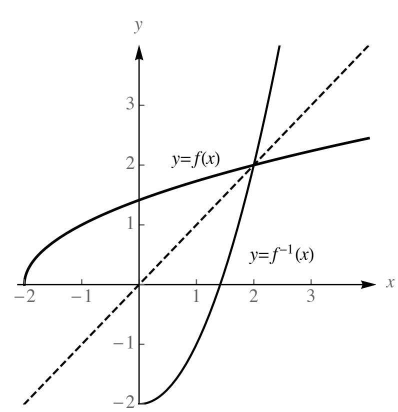
1.3.27 Switching x and y gives $x = 8 - 4y$. Solving this for y yields $y = f^{-1}(x) = \frac{8-x}{4}$.



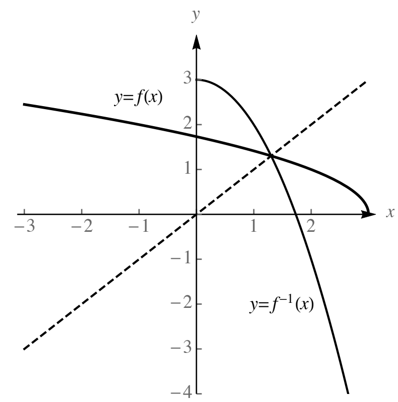
1.3.28 Switching x and y, we have $x = 3y+5$. Solving for y in terms of x we have $y = \frac{x-5}{3}$, so $y = f^{-1}(x) = \frac{x-5}{3}$.



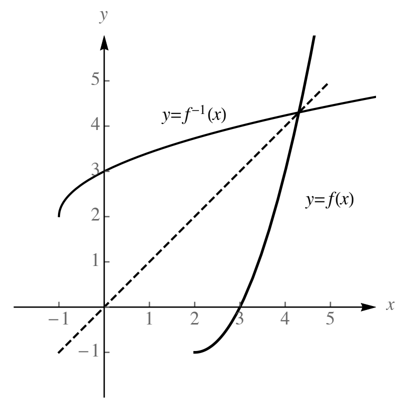
1.3.29 Switching x and y, we have $x = \sqrt{y+2}$. Solving for y in terms of x we have $y = f^{-1}(x) = x^2 - 2$. Note that because the range of f is $[0, \infty)$, that is also the domain of f^{-1}.



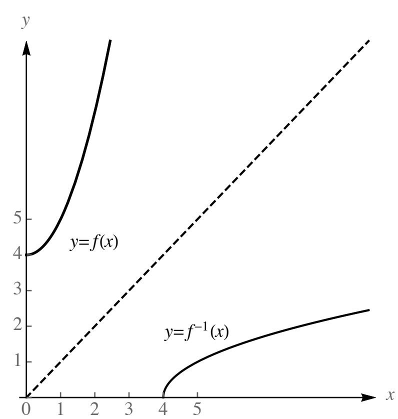
1.3.30 Switching x and y gives $x = \sqrt{3-y}$. Solving for y gives $3 - y = x^2$, or $y = 3 - x^2$. Since the domain of f is $(-\infty, 3]$ and the range of f is $[0, \infty)$, the domain of f^{-1} is $[0, \infty)$ and the range is $(-\infty, 3]$.



1.3.31 Switching x and y gives $x = (y-2)^2 - 1$. Then $x + 1 = (y-2)^2$, so $\sqrt{x+1} = |y-2|$, but because in the original function the variable is greater than or equal to 2, we choose the the positive portion of the graph of $|y-2|$. So we have $y = 2 + \sqrt{x+1}$.



1.3.32 Switching x and y, we have $x = y^2 + 4$. Solving for y in terms of x we have $y^2 = x - 4$, so $|y| = \sqrt{x-4}$. But because we are given that the domain of f is $\{x\colon x \geq 0\}$, we know that the range of f^{-1} is also non-negative. So $y = f^{-1}(x) = \sqrt{x-4}$.



1.3.33 Switching x and y, we have $x = \frac{2}{y^2+1}$. Then $y^2 + 1 = \frac{2}{x}$, so $y^2 = \frac{2}{x} - 1$. So $|y| = \sqrt{\frac{2}{x} - 1}$. We choose the positive portion, so that $y = \sqrt{\frac{2}{x} - 1}$. Note that the domain of f is $[0, \infty)$ while the range of f is $(0, 2]$. So the domain of f^{-1} is $(0, 2]$ and the range is $[0, \infty)$.

1.3.34 Switching x and y gives $x = \frac{6}{y^2-9}$. Solving yields $y^2 - 9 = \frac{6}{x}$, or $|y| = \sqrt{\frac{6}{x} + 9}$, but because the domain of f is positive, the range of f^{-1} must be positive as well, so we have $f^{-1}(x) = \sqrt{\frac{6}{x} + 9}$.

1.3.35 Switching x and y, we have $x = e^{2y+6}$. Then $\ln x = 2y + 6$, so $2y = \ln x - 6$, and $y = f^{-1}(x) = \frac{1}{2} \ln x - 3$.

1.3.36 Switching x and y, we have $x = 4e^{5y}$. Then $e^{5y} = \frac{x}{4}$, so $5y = \ln \frac{x}{4}$, and $y = f^{-1}(x) = .\frac{1}{5} \ln \frac{x}{4}$.

1.3.37 Switching x and y, we have $x = \ln(3y+1)$. Then $e^x = 3y+1$, so $3y = e^x - 1$ and $y = f^{-1}(x) = \frac{e^x-1}{3}$.

1.3.38 Switching x and y, we have $x = \log_{10} 4y$. Then $10^x = 4y$, so $y = f^{-1}(x) = \frac{10^x}{4}$.

1.3.39 Switching x and y, we have $x = 10^{-2y}$. Then $\log_{10} x = -2y$, so $y = f^{-1}(x) = -\frac{1}{2}\log_{10} x$.

1.3.40 Switching x and y, we have $x = \frac{1}{e^y+1}$. Then $e^y + 1 = \frac{1}{x}$. so $e^y = \frac{1}{x} - 1$, and $y = f^{-1}(x) = \ln\left(\frac{1}{x} - 1\right)$.

1.3.41 Switching x and y, we have $x = \frac{e^y}{e^y+2}$. Taking the reciprocal of both sides, we have $\frac{1}{x} = \frac{e^y+2}{e^y} = 1 + 2e^{-y}$. Then $2e^{-y} = \frac{1}{x} - 1$, and $e^{-y} = \frac{1}{2x} - \frac{1}{2}$. So $-y = \ln\left(\frac{1}{2x} - \frac{1}{2}\right)$, and $y = f^{-1}(x) = -\ln\left(\frac{1}{2x} - \frac{1}{2}\right) = -\ln\left(\frac{1-x}{2x}\right) = \ln\left(\frac{2x}{1-x}\right)$.

1.3.42 Switching x and y, we have $x = \frac{y}{y-2}$. Cross multiplying yields $xy - 2x = y$, so $xy - y = 2x$, and $y(x-1) = 2x$. Then $y = \frac{2x}{x-1}$. Note that the domain of f is given to be $(2, \infty)$, and the range is $(1, \infty)$. So the domain of f^{-1} must be restricted to be $(1, \infty)$.

1.3.43 First note that because the expression is symmetric, switching x and y doesn't change the expression. Solving for y gives $|y| = \sqrt{1-x^2}$. To get the four one-to-one functions, we restrict the domain and choose either the upper part or lower part of the circle as follows:

a. $f_1(x) = \sqrt{1-x^2},\ 0 \le x \le 1$
$f_2(x) = \sqrt{1-x^2},\ -1 \le x \le 0$
$f_3(x) = -\sqrt{1-x^2},\ -1 \le x \le 0$
$f_4(x) = -\sqrt{1-x^2},\ 0 \le x \le 1$

b. Reflecting these functions across the line $y = x$ yields the following:
$f_1^{-1}(x) = \sqrt{1-x^2},\ 0 \le x \le 1$
$f_2^{-1}(x) = -\sqrt{1-x^2},\ 0 \le x \le 1$
$f_3^{-1}(x) = -\sqrt{1-x^2},\ -1 \le x \le 0$
$f_4^{-1}(x) = \sqrt{1-x^2},\ -1 \le x \le 0$

1.3.44 First note that because the expression is symmetric, switching x and y doesn't change the expression. Solving for y gives $|y| = \sqrt{2|x|}$. To get the four one-to-one functions, we restrict the domain and choose either the upper part or lower part of the parabola as follows:

a. $f_1(x) = \sqrt{2x},\ x \ge 0$
$f_2(x) = \sqrt{-2x},\ x \le 0$
$f_3(x) = -\sqrt{-2x},\ x \le 0$
$f_4(x) = -\sqrt{2x},\ x \ge 0$

b. Reflecting these functions across the line $y = x$ yields the following:
$f_1^{-1}(x) = x^2/2,\ x \ge 0$
$f_2^{-1}(x) = -x^2/2,\ x \ge 0$
$f_3^{-1}(x) = -x^2/2,\ x \le 0$
$f_4^{-1}(x) = x^2/2,\ x \le 0$

1.3.45 $\log_b\left(\frac{x}{y}\right) = \log_b x - \log_b y = 0.36 - .056 = -0.2$.

1.3.46 $\log_b x^2 = 2\log_b x = 2(0.36) = 0.72$.

1.3.47 $\log_b xz = \log_b x + \log_b z = 0.36 + 0.83 = 1.19$.

1.3.48 $\log_b \frac{\sqrt{xy}}{z} = \log_b(xy)^{1/2} - \log_b z = \frac{1}{2}(\log_b x + \log_b y) - \log_b z = (0.36)/2 + (0.56)/2 - 0.83 = -0.37.$

1.3.49 $\log_b \frac{\sqrt{x}}{\sqrt[3]{z}} = \log_b x^{1/2} - \log_b z^{1/3} = (1/2)\log_b x - (1/3)\log_b z = (0.36)/2 - (0.83)/3 = -.09\overline{6}.$

1.3.50 $\log_b \frac{b^2 x^{5/2}}{\sqrt{y}} = \log_b b^2 x^{5/2} - \log_b y^{1/2} = \log_b b^2 + (5/2)\log_b x - (1/2)\log_b y = 2 + (5/2)(0.36) - (1/2)(0.56) = 2.62.$

1.3.51 If $\log_{10} x = 3$, then $10^3 = x$, so $x = 1000$.

1.3.52 If $\log_5 x = -1$, then $5^{-1} = x$, so $x = 1/5$.

1.3.53 If $\log_8 x = 1/3$, then $x = 8^{1/3} = 2$.

1.3.54 If $\log_b 125 = 3$, then $b^3 = 125$, so $b = 5$ because $5^3 = 125$.

1.3.55 $\ln x = -1$, then $e^{-1} = x$, so $x = \frac{1}{e}$.

1.3.56 If $\ln y = 3$, then $y = e^3$.

1.3.57 Since $7^x = 21$, we have that $\ln 7^x = \ln 21$, so $x \ln 7 = \ln 21$, and $x = \frac{\ln 21}{\ln 7}$.

1.3.58 Since $2^x = 55$, we have that $\ln 2^x = \ln 55$, so $x \ln 2 = \ln 55$, and $x = \frac{\ln 55}{\ln 2}$.

1.3.59 Since $3^{3x-4} = 15$, we have that $\ln 3^{3x-4} = \ln 15$, so $(3x-4)\ln 3 = \ln 15$. Thus, $3x - 4 = \frac{\ln 15}{\ln 3}$, so $x = \frac{(\ln 15)/(\ln 3)+4}{3} = \frac{\ln 15 + 4\ln 3}{3\ln 3} = \frac{\ln 5 + \ln 3 + 4\ln 3}{3 \ln 3} = \frac{\ln 5}{3\ln 3} + \frac{5}{3}$.

1.3.60 Since $5^{3x} = 29$, we have that $\ln 5^{3x} = \ln 29$, so $(3x)\ln 5 = \ln 29$. Solving for x gives $x = \frac{\ln 29}{3 \ln 5}$.

1.3.61 We are seeking t so that $50 = 100e^{-t/650}$. This occurs when $e^{-t/650} = \frac{1}{2}$, which is when $-\frac{t}{650} = \ln(1/2)$, so $t = 650 \ln 2 \approx 451$ years.

1.3.62 We need to solve $150 = 64e^{0.004t}$ for t. We have $\frac{150}{64} = e^{0.004t}$. So $0.004t = \ln \frac{150}{64}$, so $t = \frac{\ln \frac{150}{64}}{0.004} \approx 212.938$, so about 213 days.

1.3.63 We need to solve $1100 = 1000\left(1 + \frac{0.01}{12}\right)^{12t}$ for t. We have $1.1 = \left(1 + \frac{0.01}{12}\right)^{12t}$, so $\ln 1.1 = 12t \ln\left(1 + \frac{0.01}{12}\right)$. Then $t = \frac{\ln 1.1}{12 \ln\left(1 + \frac{0.01}{12}\right)} \approx 9.53$ years.

1.3.64 We need to solve $22000 = 20000\left(1 + \frac{0.025}{12}\right)^{12t}$ for t. We have $1.1 = \left(1 + \frac{0.025}{12}\right)^{12t}$, so $\ln 1.1 = 12t \ln\left(1 + \frac{0.025}{12}\right)$. Then $t = \frac{\ln 1.1}{12 \ln\left(1 + \frac{0.025}{12}\right)} \approx 3.82$ years.

1.3.65

a. No. The function takes on the values from 0 to 64 as t varies from 0 to 2, and then takes on the values from 64 to 0 as t varies from 2 to 4, so h is not one-to-one.

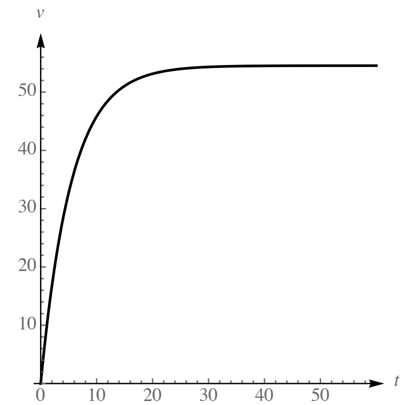
b. Solving for h in terms of t we have $h = 64t - 16t^2$, so (completing the square) we have $h - 64 = -16(t^2 - 4t + 4)$. Thus, $h - 64 = -16(t-2)^2$, and $(t-2)^2 = \frac{64-h}{16}$. Therefore $|t - 2| = \frac{\sqrt{64-h}}{4}$. When the ball is on the way up we know that $t < 2$, so the inverse of f is $f^{-1}(h) = 2 - \frac{\sqrt{64-h}}{4}$.

c. Using the work from the previous part of this problem, we have that when the ball is on the way down (when $t > 2$) we have that the inverse of f is $f^{-1}(h) = 2 + \frac{\sqrt{64-h}}{4}$.

d. On the way up, the ball is at a height of 30 ft at $2 - \frac{\sqrt{64-30}}{4} \approx 0.542$ seconds.

e. On the way down, the ball is at a height of 10 ft at $2 + \frac{\sqrt{64-10}}{4} \approx 3.837$ seconds.

1.3.66 The terminal velocity for $k = 11$ is $\frac{600}{11}$.



1.3.67 $\log_2 15 = \frac{\ln 15}{\ln 2} \approx 3.9069$.

1.3.68 $\log_3 30 = \frac{\ln 30}{\ln 3} \approx 3.0959$.

1.3.69 $\log_4 40 = \frac{\ln 40}{\ln 4} \approx 2.6610$.

1.3.70 $\log_6 60 = \frac{\ln 60}{\ln 6} \approx 2.2851$.

1.3.71 Let $2^x = z$. Then $\ln 2^x = \ln z$, so $x \ln 2 = \ln z$. Taking the exponential function of both sides gives $z = e^{x \ln 2}$.

1.3.72 Let $3^{\sin x} = z$. Then $\ln 3^{\sin x} = \ln z$, so $(\sin x) \ln 3 = \ln z$. Taking the exponential function of both sides gives $z = e^{(\sin x) \ln 3}$.

1.3.73 Let $z = \ln |x|$. Then $e^z = |x|$. Taking logarithms with base 5 of both sides gives $\log_5 e^z = \log_5 |x|$, so $z \cdot \log_5 e = \log_5 |x|$, and thus $z = \frac{\log_5 |x|}{\log_5 e}$.

1.3.74 Using the change of base formula, $\log_2(x^2 + 1) = \frac{\ln(x^2+1)}{\ln 2}$.

1.3.75 Let $z = a^{1/\ln a}$. Then $\ln z = \ln \left(a^{1/\ln a}\right) = \frac{1}{\ln a} \cdot \ln a = 1$. Thus $z = e$.

1.3.76 Let $z = a^{1/\log a}$. Then $\log z = \log \left(a^{1/\log a}\right) = \frac{1}{\log a} \cdot \log a = 1$. Thus $z = 10$.

1.3.77

a. False. For example, $3 = 3^1$, but $1 \neq \sqrt[3]{3}$.

b. False. For example, suppose $x = y = b = 2$. Then the left-hand side of the equation is equal to 1, but the right-hand side is 0.

c. False. $\log_5 4^6 = 6 \log_5 4 > 4 \log_5 6$.

d. True. This follows because 10^x and $\log_{10}$ are inverses of each other.

e. False. $\ln 2^e = e \ln 2 < 2$.

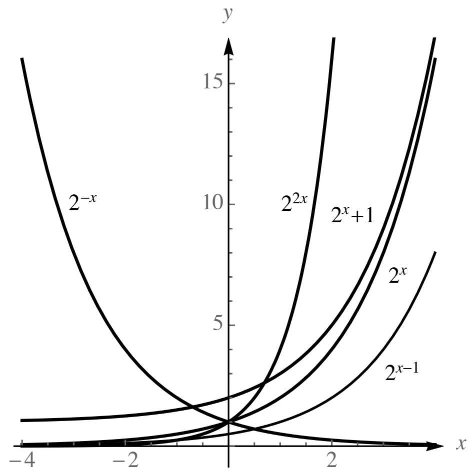
f. False. For example $f(0) = 1$, but the alleged inverse function evaluated at 1 is not 0 (rather, it has value 1/2.)

g. True. f is its own inverse because $f(f(x)) = f(1/x) = \frac{1}{1/x} = x$.

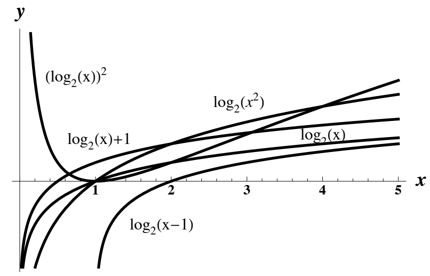
1.3.78 A is 2^{-x}, B is 3^{-x}, C is 3^x and D is 2^x.

1.3.79 A is $\log_2 x$, B is $\log_4 x$, C is $\log_{10} x$.

1.3.80



1.3.81



Note: need better pic from back of book

1.3.82

a. $p(0) = 150(2^{0/12}) = 150$.

b. At a given time t, let the population be $z = 150(2^{t/12})$. Then 12 hours later, the time is $12 + t$, and the population is $150(2^{(t+12)/12}) = 150(2^{(t/12)+1}) = 150(2^{t/12} \cdot 2) = 2z$.

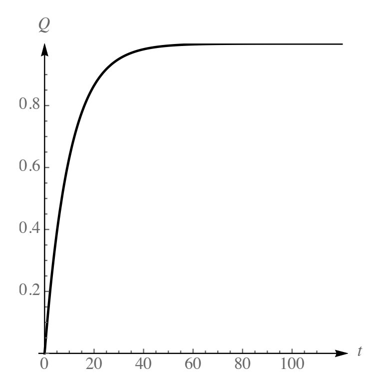
c. Since 4 days is 96 hours, we have $p(96) = 150(2^{96/12}) = 150(2^8) = 38{,}400$.

d. We can find the time to triple by solving $450 = 150(2^{t/12})$, which is equivalent to $3 = 2^{t/12}$. By taking logs of both sides we have $\ln 3 = \frac{t}{12} \cdot \ln 2$, so $t = \frac{12 \ln 3}{\ln 2} \approx 19.0$ hours.

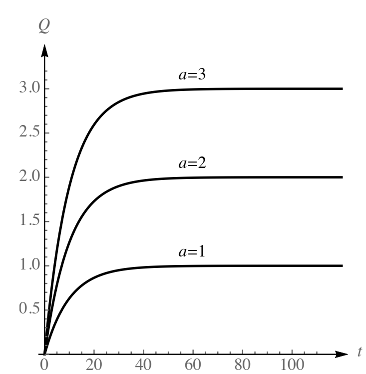
e. The population will reach 10,000 when $10,000 = 150(2^{12/t})$, which is equivalent to $\frac{200}{3} = 2^{t/12}$. By taking logs of both sides we have $\ln(200/3) = \frac{t}{12} \ln 2$, so $= \frac{12 \cdot \ln(200/3)}{\ln 2} \approx 72.7$ hours.

1.3.83

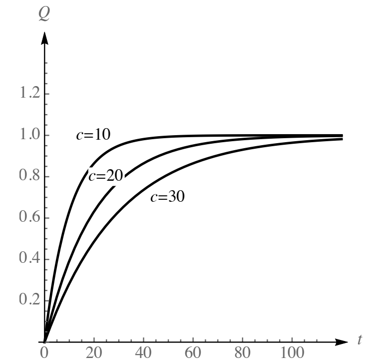
a. The relevant graph is:



b. Varying a while holding c constant scales the curve vertically. It appears that the steady-state charge is equal to a.



c. Varying c while holding a constant scales the curve horizontally. It appears that the steady-state charge does not vary with c.



d. As t grows large, the term $ae^{-t/c}$ approaches zero for any fixed c and a. So the steady-state charge for $a - ae^{-t/c}$ is a.

1.3.84 Since $e^x = x^{123}$, we have $x = \ln(x^{123})$, so $x = 123 \ln x$. Consider the function $f(x) = x - 123 \ln x$. Plotting this function using a computer or calculator reveals a graph which crosses the x axis twice, near $x = 1$ and near $x = 826$. (Try graphing it using the domain $(0, 900)$). Using a calculator and some trial and error reveals that the roots of f are approximately 1.0082 and 826.1659.

1.3.85 Begin by completing the square: $f(x) = x^2 - 2x + 6 = (x^2 - 2x + 1) + 5 = (x - 1)^2 + 5$. Switching x and y yields $x = (y - 1)^2 + 5$. Solving for y gives $|y - 1| = \sqrt{x - 5}$. Choosing the principal square root (because the original given interval has x positive) gives $y = f^{-1}(x) = \sqrt{x - 5} + 1$, $x \geq 5$.

1.3.86 Begin by completing the square: $f(x) = -x^2 - 4x - 3 = -(x^2 + 4x + 3) = -(x^2 + 4x + 4 - 1) = -((x + 2)^2 - 1) = 1 - (x + 2)^2$. Switching x and y yields $x = 1 - (y + 2)^2$, and solving for y gives $|y + 2| = \sqrt{1 - x}$. Since the given domain of f was negative, the range of f^{-1} must be negative, so we must have $y + 2 = -\sqrt{1 - x}$, so the inverse function is $f^{-1}(x) = -\sqrt{1 - x} - 2$.

1.3.87 Note that f is one-to-one, so there is only one inverse. Switching x and y gives $x = (y + 1)^3$. Then $\sqrt[3]{x} = y + 1$, so $y = f^{-1}(x) = \sqrt[3]{x} - 1$. The domain of f^{-1} is $\mathbb{R}$.

1.3.88 Note that to get a one-to-one function, we should restrict the domain to either $[4, \infty)$ or $(-\infty, 4]$. Switching x and y yields $x = (y - 4)^2$, so $\sqrt{x} = |y - 4|$. So $y = 4 \pm \sqrt{x}$. So the inverse of f when the domain of f is restricted to $[4, \infty)$ is $f^{-1}(x) = 4 + \sqrt{x}$, while if the domain of f is restricted to $(-\infty, 4]$ the inverse is $f^{-1}(x) = 4 - \sqrt{x}$. In either case, the domain of f^{-1} is $[0, \infty)$.

1.3.89 Note that to get a one-to-one function, we should restrict the domain to either $[0, \infty)$ or $(-\infty, 0]$. Switching x and y yields $x = \frac{2}{y^2+2}$, so $y^2 + 2 = (2/x)$. So $y = \pm\sqrt{(2/x) - 2}$. So the inverse of f when the domain of f is restricted to $[0, \infty)$ is $f^{-1}(x) = \sqrt{(2/x) - 2}$, while if the domain of f is restricted to $(-\infty, 0]$ the inverse is $f^{-1}(x) = -\sqrt{(2/x) - 2}$. In either case, the domain of f^{-1} is $(0, 1]$.

1.3.90 Note that f is one-to-one. Switching x and y yields $x = \frac{2y}{y+2}$, so $x(y + 2) = 2y$. Thus $xy + 2x = 2y$, so $2x = 2y - xy = y(2 - x)$. Thus, $y = \frac{2x}{2-x}$. The domain of $f^{-1}(x) = \frac{2x}{2-x}$ is $(-\infty, 2) \cup (2, \infty)$.

1.3.91 Using the change of base formula, we have $\log_{1/b} x = \frac{\ln x}{\ln 1/b} = \frac{\ln x}{\ln 1 - \ln b} = \frac{\ln x}{-\ln b} = -\frac{\ln x}{\ln b} = -\log_b x$.

1.3.92

a. Given $x = b^p$, we have $p = \log_b x$, and given $y = b^q$, we have $q = \log_b y$.

b. $xy = b^p b^q = b^{p+q}$.

c. $\log_b xy = \log_b b^{p+q} = p + q = \log_b x + \log_b y$.

1.3.93 Let $x = b^p$ and $y = b^q$. Then
$\frac{x}{y} = \frac{b^p}{b^q} = b^{p-q}$. Thus $\log_b \frac{x}{y} = \log_b b^{p-q} = p - q = \log_b x - \log_b y$.

1.3.94

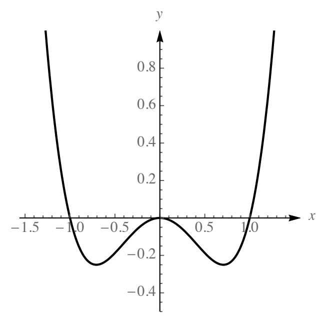
a. Given $x = b^p$, we have $p = \log_b x$.

b. $x^y = (b^p)^y = b^{yp}$.

c. $\log_b x^y = \log_b b^{yp} = yp = y \log_b x$.

1.3.95

a. f is one-to-one on $(-\infty, -\sqrt{2}/2]$, on $[-\sqrt{2}/2, 0]$, on $[0, \sqrt{2}/2]$, and on $[\sqrt{2}/2, \infty)$.



b. If $u = x^2$, then our function becomes $y = u^2 - u$. Completing the square gives $y + (1/4) = u^2 - u + (1/4) = (u - (1/2))^2$. Thus $|u - (1/2)| = \sqrt{y + (1/4)}$, so $u = (1/2) \pm \sqrt{y + (1/4)}$, with the "+" applying for $u = x^2 > (1/2)$ and the "−" applying when $u = x^2 < (1/2)$. Now letting $u = x^2$, we have $x^2 = (1/2) \pm \sqrt{y + (1/4)}$, so $x = \pm\sqrt{(1/2) \pm \sqrt{y + (1/4)}}$. Now switching the x and y gives the following inverses:

Domain of f	$(-\infty, -\sqrt{2}/2]$	$[-\sqrt{2}/2, 0]$	$[0, \sqrt{2}/2]$	$[\sqrt{2}/2, \infty)$
Range of f	$[-1/4, \infty)$	$[-1/4, 0]$	$[-1/4, 0]$	$[-1/4, \infty)$
Inverse of f	$-\sqrt{(1/2) + \sqrt{x + (1/4)}}$	$-\sqrt{(1/2) - \sqrt{x + (1/4)}}$	$\sqrt{(1/2) - \sqrt{x + (1/4)}}$	$\sqrt{(1/2) + \sqrt{x + (1/4)}}$

1.3.96

a. $f(x) = g(h(x)) = g(x^3) = 2x^3 + 3$. To find the inverse of f, we switch x and y to obtain $x = 2y^3 + 3$, so that $y^3 = \frac{x-3}{2}$, so $f^{-1}(x) = \sqrt[3]{\frac{x-3}{2}}$. Note that $g^{-1}(x) = \frac{x-3}{2}$, and $h^{-1}(x) = \sqrt[3]{x}$, and so $f^{-1}(x) = h^{-1}(g^{-1}(x))$.

b. $f(x) = g(h(x)) = g(\sqrt{x}) = (\sqrt{x})^2 + 1 = x + 1$. so the inverse of f is $f^{-1}(x) = x - 1$. Note that $g^{-1}(x) = \sqrt{x-1}$, and $h^{-1}(x) = x^2$, and so $f^{-1}(x) = h^{-1}(g^{-1}(x))$.

c. If h and g are one-to-one, then their inverses exist, and $f^{-1}(x) = h^{-1}(g^{-1}(x))$, because $f(f^{-1}(x)) = g(h(h^{-1}(g^{-1}(x))) = g(g^{-1}(x)) = x$ and likewise, $f^{-1}(f(x)) = h^{-1}(g^{-1}(g(h(x))) = h^{-1}(h(x)) = x$.

1.3.97 Using the change of base formulas $\log_b c = \frac{\ln c}{\ln b}$ and $\log_c b = \frac{\ln b}{\ln c}$ we have

$$(\log_b c) \cdot (\log_c b) = \frac{\ln c}{\ln b} \cdot \frac{\ln b}{\ln c} = 1.$$

1.4 Trigonometric Functions and Their Inverses

1.4.1 Let O be the length of the side opposite the angle x, let A be length of the side adjacent to the angle x, and let H be the length of the hypotenuse. Then $\sin x = \dfrac{O}{H}$, $\cos x = \dfrac{A}{H}$, $\tan x = \dfrac{O}{A}$, $\csc x = \dfrac{H}{O}$, $\sec x = \dfrac{H}{A}$, and $\cot x = \dfrac{A}{O}$.

1.4.2 Note that the distance from the origin to the point $(-4, -3)$ is $\sqrt{(-4)^2 + (-3)^2} = \sqrt{16+9} = \sqrt{25} = 5$. Then we have $\sin\theta = -\dfrac{3}{5}$, $\cos\theta = -\dfrac{4}{5}$, $\tan\theta = \dfrac{3}{4}$, $\cot\theta = \dfrac{4}{3}$, $\sec\theta = -\dfrac{5}{4}$, $\csc\theta = -\dfrac{5}{3}$.

1.4.3 We have $t = \dfrac{v\sin\theta}{16} = \dfrac{96\sin\dfrac{\pi}{6}}{16} = \dfrac{96/2}{16} = \dfrac{48}{16} = 3$ seconds.

1.4.4

a. Because $\tan\theta = \dfrac{50}{d}$, we have $d = \frac{50}{\tan\theta}$.

b. Because $\sin\theta = \dfrac{50}{L}$, we have $L = \frac{50}{\sin\theta}$.

1.4.5 The radian measure of an angle θ is the length of the arc s on the unit circle associated with θ.

1.4.6 The period of a function is the smallest positive real number k so that $f(x+k) = f(x)$ for all x in the domain of the function. The sine, cosine, secant, and cosecant function all have period 2π. The tangent and cotangent functions have period π.

1.4.7 $\sin^2 x + \cos^2 x = 1$, $1 + \cot^2 x = \csc^2 x$, and $\tan^2 x + 1 = \sec^2 x$.

1.4.8
$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{1/\sqrt{5}}{-2/\sqrt{5}} = -\dfrac{1}{2}$.
$\cot\theta = \frac{1}{\tan\theta} = -2$.
$\sec\theta = \frac{1}{\cos\theta} = \frac{1}{-2/\sqrt{5}} = -\frac{\sqrt{5}}{2}$.
$\csc\theta = \frac{1}{\sin\theta} = \frac{1}{1/\sqrt{5}} = \sqrt{5}$.

1.4.9 The only point on the unit circle whose second coordinate is -1 is the point $(0,-1)$, which is the point associated with $\theta = \frac{3\pi}{2}$. So that is the only solution for $0 \le \theta < 2\pi$.

1.4.10 Note that if $0 \le \theta < 2\pi$, then $0 \le 2\theta < 4\pi$. So we must consider "two trips" around the unit circle. The second coordinate on the unit circle is 1 at the point $(0,1)$, which is associated with $\frac{\pi}{2}$ and $\frac{5\pi}{2}$. When $2\theta = \frac{\pi}{2}$ we have $\theta = \frac{\pi}{4}$, and when $2\theta = \frac{5\pi}{2}$ we have $\theta = \frac{5\pi}{4}$.

1.4.11 The tangent function is undefined where $\cos x = 0$, which is at all real numbers of the form $\dfrac{\pi}{2} + k\pi$, k an integer.

1.4.12 $\sec x$ is defined wherever $\cos x \ne 0$, which is $\{x\colon x \ne \dfrac{\pi}{2} + k\pi, k \text{ an integer}\}$.

1.4.13 The sine function is not one-to-one over its whole domain, so in order to define an inverse, it must be restricted to an interval on which it is one-to-one.

1.4.14 In order to define an inverse for the cosine function, we restricted the domain to $[0,\pi]$ in order to get a one-to-one function. Because the range of the inverse of a function is the domain of the function, we have that the values of $\cos^{-1} x$ lie in the interval $[0,\pi]$.

1.4.15 $\cos\dfrac{5\pi}{4} = -\dfrac{\sqrt{2}}{2}$, so $\cos^{-1}\left(\cos\dfrac{5\pi}{4}\right) = \cos^{-1}\left(-\dfrac{\sqrt{2}}{2}\right) = \frac{3\pi}{4}$.

1.4.16 $\sin\dfrac{11\pi}{6} = -\dfrac{1}{2}$, so $\sin^{-1}\left(\sin\dfrac{11\pi}{6}\right) = \sin^{-1}\left(-\dfrac{1}{2}\right) = -\dfrac{\pi}{6}$.

1.4.17 The numbers $\pm\pi/2$ are not in the range of $\tan^{-1} x$. The range is $(-\pi/2, \pi/2)$. However, it is true that as x increases without bound, the values of $\tan^{-1} x$ get close to $\pi/2$, and as x decreases without bound, the values of $\tan^{-1} x$ get close to $-\pi/2$.

1.4.18 The domain of $\sec^{-1} x$ is $\{x\colon |x| \ge 1\}$. The range is $[0,\pi/2) \cup (\pi/2,\pi]$.

1.4.19 The point on the unit circle associated with $2\pi/3$ is $(-1/2, \sqrt{3}/2)$, so $\cos(2\pi/3) = -1/2$.

1.4.20 The point on the unit circle associated with $2\pi/3$ is $(-1/2, \sqrt{3}/2)$, so $\sin(2\pi/3) = \sqrt{3}/2$.

1.4.21 The point on the unit circle associated with $-3\pi/4$ is $(-\sqrt{2}/2, -\sqrt{2}/2)$, so $\tan(-3\pi/4) = 1$.

1.4.22 The point on the unit circle associated with $15\pi/4$ is $(\sqrt{2}/2, -\sqrt{2}/2)$, so $\tan(15\pi/4) = -1$.

1.4.23 The point on the unit circle associated with $-13\pi/3$ is $(1/2, -\sqrt{3}/2)$, so $\cot(-13\pi/3) = -1/\sqrt{3} = -\sqrt{3}/3$.

1.4.24 The point on the unit circle associated with $7\pi/6$ is $(-\sqrt{3}/2, -1/2)$, so $\sec(7\pi/6) = -2/\sqrt{3} = -2\sqrt{3}/3$.

1.4.25 The point on the unit circle associated with $-17\pi/3$ is $(1/2, \sqrt{3}/2)$, so $\cot(-17\pi/3) = 1/\sqrt{3} = \sqrt{3}/3$.

1.4.26 The point on the unit circle associated with $16\pi/3$ is $(-1/2, -\sqrt{3}/2)$, so $\sin(16\pi/3) = -\sqrt{3}/2$.

1.4.27 Because the point on the unit circle associated with $\theta = 0$ is the point $(1, 0)$, we have $\cos 0 = 1$.

1.4.28 Because $-\pi/2$ corresponds to a quarter circle clockwise revolution, the point on the unit circle associated with $-\pi/2$ is the point $(0, -1)$. Thus $\sin(-\pi/2) = -1$.

1.4.29 Because $-\pi$ corresponds to a half circle clockwise revolution, the point on the unit circle associated with $-\pi$ is the point $(-1, 0)$. Thus $\cos(-\pi) = -1$.

1.4.30 Because 3π corresponds to one and a half counterclockwise revolutions, the point on the unit circle associated with 3π is $(-1, 0)$, so $\tan 3\pi = \dfrac{0}{-1} = 0$.

1.4.31 Because $5\pi/2$ corresponds to one and a quarter counterclockwise revolutions, the point on the unit circle associated with $5\pi/2$ is the same as the point associated with $\pi/2$, which is $(0, 1)$. Thus $\sec 5\pi/2$ is undefined.

1.4.32 Because π corresponds to one half circle counterclockwise revolution, the point on the unit circle associated with π is $(-1, 0)$. Thus $\cot \pi$ is undefined.

1.4.33 Using the fact that $\dfrac{\pi}{12} = \dfrac{\pi/6}{2}$ and the half-angle identity for cosine:

$$\cos^2(\pi/12) = \frac{1 + \cos(\pi/6)}{2} = \frac{1 + \sqrt{3}/2}{2} = \frac{2 + \sqrt{3}}{4}.$$

Thus, $\cos(\pi/12) = \sqrt{\dfrac{2 + \sqrt{3}}{4}}$.

1.4.34 Using the fact that $\dfrac{3\pi}{8} = \dfrac{3\pi/4}{2}$ and the half-angle identities for sine, we have:

$$\sin^2\left(\frac{3\pi}{8}\right) = \frac{1 - \cos(3\pi/4)}{2} = \frac{1 - (-\sqrt{2}/2)}{2} = \frac{2 + \sqrt{2}}{4},$$

and using the fact that $3\pi/8$ is in the first quadrant (and thus has positive value for sine) we deduce that $\sin\left(\dfrac{3\pi}{8}\right) = \dfrac{\sqrt{2 + \sqrt{2}}}{2}$.

1.4.35 First note that $\tan x = 1$ when $\sin x = \cos x$. Using our knowledge of the values of the standard angles between 0 and 2π, we recognize that the sine function and the cosine function are equal at $\pi/4$. Then, because we recall that the period of the tangent function is π, we know that $\tan(\pi/4 + k\pi) = \tan(\pi/4) = 1$ for every integer value of k. Thus the solution set is $\{\pi/4 + k\pi, \text{where } k \text{ is an integer}\}$.

1.4.36 Given that $2\theta\cos(\theta)+\theta=0$, we have $\theta(2\cos(\theta)+1)=0$. Which means that either $\theta=0$, or $2\cos(\theta)+1=0$. The latter leads to the equation $\cos\theta=-1/2$, which occurs at $\theta=2\pi/3$ and $\theta=4\pi/3$. Using the fact that the cosine function has period 2π the entire solution set is thus

$$\{0\}\cup\{2\pi/3+2k\pi, \text{where } k \text{ is an integer}\}\cup\{4\pi/3+2l\pi, \text{where } l \text{ is an integer}\}.$$

1.4.37 Given that $\sin^2\theta=\dfrac{1}{4}$, we have $|\sin\theta|=\dfrac{1}{2}$, so $\sin\theta=\dfrac{1}{2}$ or $\sin\theta=-\dfrac{1}{2}$. It follows that $\theta=\pi/6, 5\pi/6, 7\pi/6, 11\pi/6$.

1.4.38 Given that $\cos^2\theta=\dfrac{1}{2}$, we have $|\cos\theta|=\dfrac{1}{\sqrt{2}}=\dfrac{\sqrt{2}}{2}$. Thus $\cos\theta=\dfrac{\sqrt{2}}{2}$ or $\cos\theta=-\dfrac{\sqrt{2}}{2}$. We have $\theta=\pi/4, 3\pi/4, 5\pi/4, 7\pi/4$.

1.4.39 The equation $\sqrt{2}\sin(x)-1=0$ can be written as $\sin x=\dfrac{1}{\sqrt{2}}=\dfrac{\sqrt{2}}{2}$. Standard solutions to this equation occur at $x=\pi/4$ and $x=3\pi/4$. Because the sine function has period 2π the set of all solutions can be written as:

$$\{\pi/4+2k\pi, \text{where } k \text{ is an integer}\}\cup\{3\pi/4+2l\pi, \text{where } l \text{ is an integer}\}.$$

1.4.40 $\sin^2(\theta)-1=0$ wherever $\sin^2(\theta)=1$, which is wherever $\sin(\theta)=\pm1$. This occurs for $\theta=\pi/2+k\pi$, where k is an integer.

1.4.41 If $\sin\theta\cos\theta=0$, then either $\sin\theta=0$ or $\cos\theta=0$. This occurs for $\theta=0,\pi/2,\pi,3\pi/2$.

1.4.42 Let $u=3x$. Note that because $0\le x<2\pi$, we have $0\le u<6\pi$. Because $\sin u=\sqrt{2}/2$ for $u=\pi/4$, $3\pi/4$, $9\pi/4$, $11\pi/4$, $17\pi/4$, and $19\pi/4$, we must have that $\sin 3x=\sqrt{2}/2$ for $3x=\pi/4$, $3\pi/4$, $9\pi/4$, $11\pi/4$, $17\pi/4$, and $19\pi/4$, which translates into

$$x=\pi/12, \pi/4, 3\pi/4, 11\pi/12, 17\pi/12, \text{and } 19\pi/12.$$

1.4.43 Let $u=3x$. Then we are interested in the solutions to $\cos u=\sin u$, for $0\le u<6\pi$. This would occur for $u=3x=\pi/4, 5\pi/4, 9\pi/4, 13\pi/4, 17\pi/4, \text{and } 21\pi/4$. Thus there are solutions for the original equation at

$$x=\pi/12, 5\pi/12, 3\pi/4, 13\pi/12, 17\pi/12, \text{and } 7\pi/4.$$

1.4.44 If $\tan^2 2\theta=1$, then $\sin^2 2\theta=\cos^2 2\theta$, so we have either $\sin 2\theta=\cos 2\theta$ or $\sin 2\theta=-\cos 2\theta$. This occurs for $2\theta=\pi/4, 3\pi/4, 5\pi/4, 7\pi/4$ for $0\le 2\theta\le 2\pi$, so the corresponding values for θ are $\pi/8$, $3\pi/8$, $5\pi/8$, $7\pi/8$, $0\le\theta\le\pi$.

1.4.45 Using a computer algebra system or graphing calculator, we find that the roots are approximately 0.1007 and 1.4701.

1.4.46 Using a computer algebra system or graphing calculator, we find that the roots are approximately 0.375962, 1.71843, and 2.47036.

1.4.47 We are seeking solutions to the equation $400=\frac{150^2}{32}\sin 2\theta$, or $\sin 2\theta=0.56\overline{8}$. Using a computer algebra system or graphing calculator, we find that the solutions are about 0.30257 radians or about 17.3 degrees, and about 1.2682 radians which is about 72.7 degrees.

1.4.48 We are seeking solutions to the equation $350=\frac{160^2}{32}\sin 2\theta$, or $\sin 2\theta=0.4375$. Using a computer algebra system or graphing calculator, we find that the solutions are about 0.2264 radians which is about 13 degrees, and about 1.3444 radians which is about 77 degrees.

1.4.49 Let $z=\sin^{-1}1$. Then $\sin z=1$, and because $\sin\pi/2=1$, and $\pi/2$ is in the desired interval, $z=\pi/2$.

1.4.50 Let $z=\cos^{-1}(-1)$. Then $\cos z=-1$, and because $\cos\pi=-1$ and π is in the desired interval, $z=\pi$.

1.4.51 Let $z = \sin^{-1}(-1/2)$. Then $\sin z = 1/2$, and because $\sin(-\pi/6) = -1/2$, and $-\pi/6$ is in the desired interval, $z = -\pi/6$.

1.4.52 Let $z = \cos^{-1}(-\sqrt{2}/2)$. Then $\cos z = -\sqrt{2}/2$. Because $\cos 3\pi/4 = -\sqrt{2}/2$ and $3\pi/4$ is in the desired interval, we have $z = 3\pi/4$. (Note that $\cos(-\pi/4)$ is also equal to $-\sqrt{2}/2$, but $-\pi/4$ isn't in the desired interval $[0, \pi]$.)

1.4.53 $\sin^{-1}(\sqrt{3}/2) = \pi/3$, because $\sin(\pi/3) = \sqrt{3}/2$.

1.4.54 $\cos^{-1} 2$ does not exist, because 2 is not in the domain of the inverse cosine function (because 2 is not in the range of the cosine function.)

1.4.55 $\cos^{-1}(-1/2) = 2\pi/3$, because $\cos(2\pi/3) = -1/2$.

1.4.56 $\sin^{-1}(-1) = -\pi/2$, because $\sin(-\pi/2) = -1$.

1.4.57 $\cos(\cos^{-1}(-1)) = \cos(\pi) = -1$.

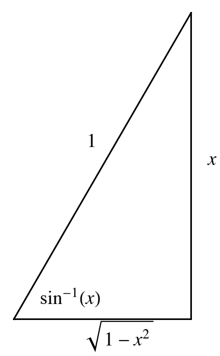
1.4.58 $\cos^{-1}(\cos(7\pi/6)) = \cos^{-1}(-\sqrt{3}/2) = 5\pi/6$. Note that the range of the inverse cosine function is $[0, \pi]$.

1.4.59 Because $\theta = \cos^{-1}(5/13)$, we know that $\cos\theta = 5/13$. The triangle in question has a leg of length 5 and a hypotenuse of length 13, so we can deduce using the Pythagorean theorem that the other leg as length 12. So $\sin\theta = 12/13$. Then $\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{12/13}{5/13} = \frac{12}{5}$.

1.4.60 Because $\theta = \tan^{-1}(4/3)$, we know that $\tan\theta = 4/3$. The triangle in question has legs of length 3 and 4, so the hypotenuse has length 5 by the Pythagorean theorem. Then $\sec\theta = \frac{1}{\cos\theta} = \frac{1}{3/5} = \dfrac{5}{3}$. And $\csc\theta = \frac{1}{\sin\theta} = \frac{1}{4/5} = \frac{5}{4}$.

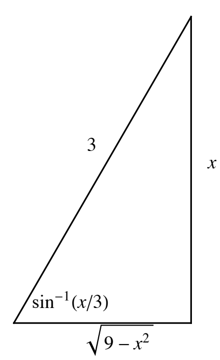
1.4.61

$$\cos(\sin^{-1}(x)) = \frac{\text{side adjacent to } \sin^{-1}(x)}{\text{hypotenuse}} = \frac{\sqrt{1-x^2}}{1} = \sqrt{1-x^2}.$$



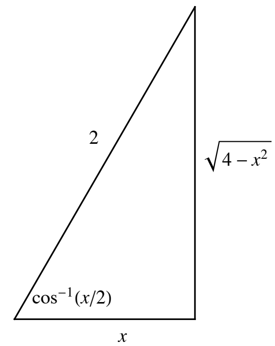
1.4.62

$$\cos(\sin^{-1}(x/3)) = \frac{\text{side adjacent to } \sin^{-1}(x/3)}{\text{hypotenuse}} = \frac{\sqrt{9-x^2}}{3}.$$



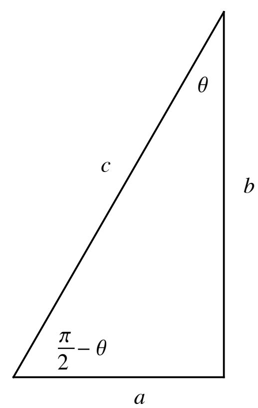
1.4.63

$$\sin(\cos^{-1}(x/2)) = \frac{\text{side opposite of } \cos^{-1}(x/2)}{\text{hypotenuse}} = \frac{\sqrt{4-x^2}}{2}.$$



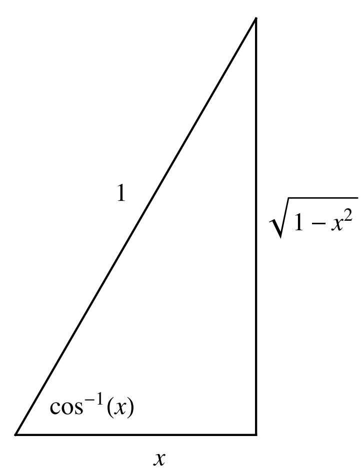
1.4.64

Note (from the triangle pictured) that $\cos\theta = \frac{b}{c} = \sin(\frac{\pi}{2} - \theta)$. Thus $\sin^{-1}(\cos\theta) = \sin^{-1}(\sin(\frac{\pi}{2} - \theta)) = \frac{\pi}{2} - \theta$.



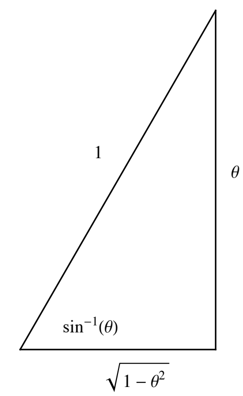
1.4.65

Using the identity given, we have $\sin(2\cos^{-1}(x)) = 2\sin(\cos^{-1}(x))\cos(\cos^{-1}(x)) = 2x\sin(\cos^{-1}(x)) = 2x\sqrt{1-x^2}$.



1.4.66

First note that $\cos(\sin^{-1}(\theta)) = \sqrt{1-\theta^2}$, as indicated in the triangle shown.
Using the identity given, we have $\cos(2\sin^{-1}(x)) = \cos^2((\sin^{-1}(x)) - \sin^2(\sin^{-1}(x)) = (\sqrt{1-x^2})^2 - x^2 = 1 - 2x^2$.



1.4.67 From our definitions of the trigonometric functions via a point $P(x,y)$ on a circle of radius $r = \sqrt{x^2+y^2}$, we have $\sec\theta = \dfrac{r}{x} = \dfrac{1}{x/r} = \dfrac{1}{\cos\theta}$.

1.4.68 From our definitions of the trigonometric functions via a point $P(x,y)$ on a circle of radius $r = \sqrt{x^2+y^2}$, we have $\tan\theta = \dfrac{y}{x} = \dfrac{y/r}{x/r} = \dfrac{\sin\theta}{\cos\theta}$.

1.4.69 We have already established that $\sin^2\theta + \cos^2\theta = 1$. Dividing both sides by $\cos^2\theta$ gives $\tan^2\theta + 1 = \sec^2\theta$.

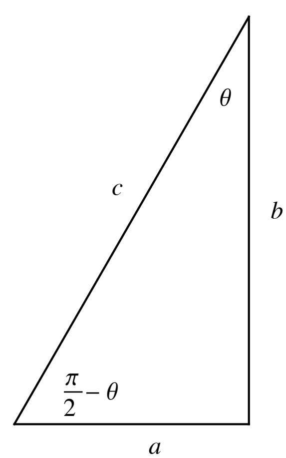
1.4.70 We have already established that $\sin^2\theta + \cos^2\theta = 1$. We can write this as $\frac{\sin\theta}{(1/\sin\theta)} + \frac{\cos\theta}{(1/\cos\theta)} = 1$, or $\frac{\sin\theta}{\csc\theta} + \frac{\cos\theta}{\sec\theta} = 1$.

1.4.71

Using the triangle pictured, we see that

$$\sec(\pi/2 - \theta) = \frac{c}{a} = \csc\theta.$$

This also follows from the sum identity $\cos(a+b) = \cos a\cos b - \sin a \sin b$ as follows: $\sec(\pi/2 - \theta) = \dfrac{1}{\cos(\pi/2 + (-\theta))} = \dfrac{1}{\cos(\pi/2)\cos(-\theta) - \sin(\pi/2)\sin(-\theta)} = \dfrac{1}{0 - (-\sin(\theta))} = \csc(\theta)$.

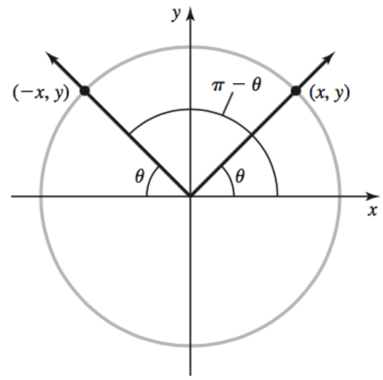


1.4.72 Using the trig identity for the cosine of a sum (mentioned in the previous solution) we have:

$$\sec(x+\pi) = \frac{1}{\cos(x+\pi)} = \frac{1}{\cos(x)\cos(\pi) - \sin(x)\sin(\pi)} = \frac{1}{\cos(x)\cdot(-1) - \sin(x)\cdot 0} = \frac{1}{-\cos(x)} = -\sec x.$$

1.4.73

Let $\theta = \cos^{-1}(x)$, and note from the diagram that it then follows that $\cos^{-1}(-x) = \pi - \theta$. So $\cos^{-1}(x) + \cos^{-1}(-x) = \theta + \pi - \theta = \pi$.



1.4.74 Let $\theta = \sin^{-1}(y)$. Then $\sin\theta = y$, and $\sin(-\theta) = -\sin(\theta) = -y$ (because the sine function is an odd function) and it then follows that $-\theta = \sin^{-1}(-y)$. Therefore, $\sin^{-1}(y) + \sin^{-1}(-y) = \theta + -\theta = 0$. It would be instructive for the reader to draw his or her own diagram like that in the previous solution.

1.4.75 $\tan^{-1}(\sqrt{3}) = \tan^{-1}\left(\dfrac{\sqrt{3}/2}{1/2}\right) = \pi/3$, because $\sin(\pi/3) = \sqrt{3}/2$ and $\cos(\pi/3) = 1/2$.

1.4.76 $\cot^{-1}(-1/\sqrt{3}) = \cot^{-1}\left(-\dfrac{1/2}{\sqrt{3}/2}\right) = 2\pi/3$, because $\sin(2\pi/3) = \sqrt{3}/2$ and $\cos(2\pi/3) = -1/2$.

1.4.77 $\sec^{-1}(2) = \sec^{-1}\left(\dfrac{1}{1/2}\right) = \pi/3$, because $\sec(\pi/3) = \dfrac{1}{\cos(\pi/3)} = \dfrac{1}{1/2} = 2$.

1.4.78 $\csc^{-1}(-1) = \sin^{-1}(-1) = -\pi/2$.

1.4.79 $\tan^{-1}(\tan(\pi/4)) = \tan^{-1}(1) = \pi/4$.

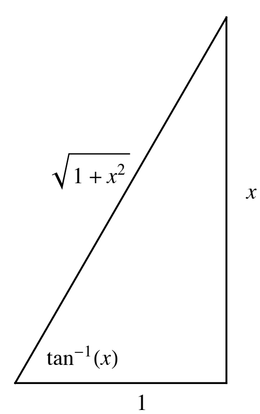
1.4.80 $\tan^{-1}(\tan(3\pi/4)) = \tan^{-1}(-1) = -\pi/4$.

1.4.81 Let $\csc^{-1}(\sec 2) = z$. Then $\csc z = \sec 2$, so $\sin z = \cos 2$. Now by applying the result of problem 64, we see that $z = \sin^{-1}(\cos 2) = \pi/2 - 2 = \dfrac{\pi - 4}{2}$.

1.4.82 $\tan(\tan^{-1}(1)) = \tan(\pi/4) = 1$.

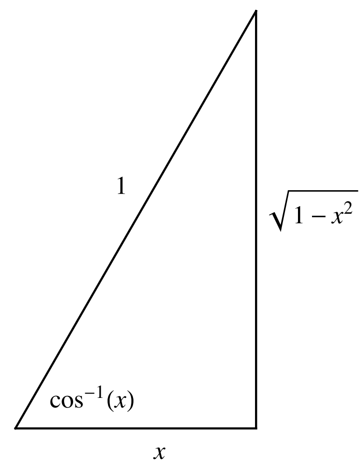
1.4.83

$$\cos(\tan^{-1}(x)) = \frac{\text{side adjacent to } \tan^{-1}(x)}{\text{hypotenuse}} = \frac{1}{\sqrt{1+x^2}}.$$



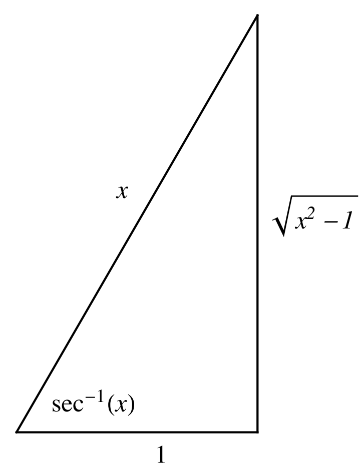
1.4.84

$$\tan(\cos^{-1}(x)) = \frac{\text{side opposite of } \cos^{-1}(x)}{\text{side adjacent to } \cos^{-1}(x)} = \frac{\sqrt{1-x^2}}{x}.$$



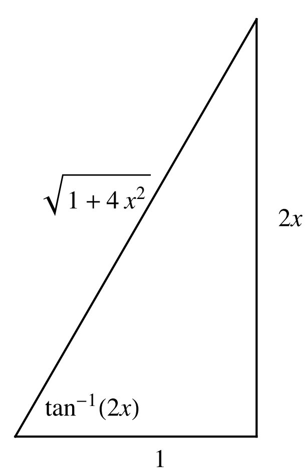
1.4.85

$$\cos(\sec^{-1}(x)) = \frac{\text{side adjacent to } \sec^{-1} x}{\text{hypotenuse}} = \frac{1}{x}.$$



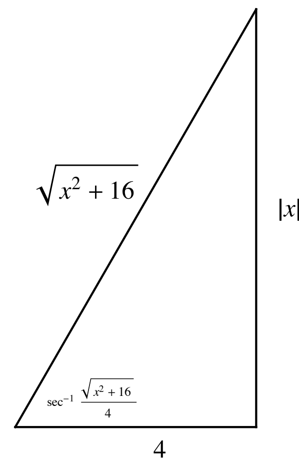
1.4.86

$$\cot(\tan^{-1} 2x) = \frac{\text{side adjacent to } \tan^{-1} 2x}{\text{side opposite of } \tan^{-1} 2x} = \frac{1}{2x}.$$



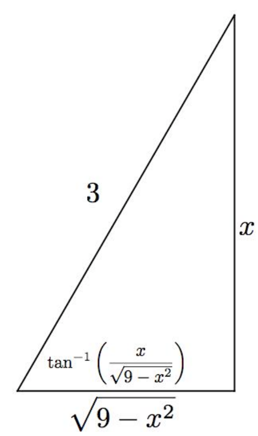
1.4.87

Assume $x > 0$. Then $\sin\left(\sec^{-1}\left(\frac{\sqrt{x^2+16}}{4}\right)\right) = \frac{\text{side opposite of } \sec^{-1}\left(\frac{\sqrt{x^2+16}}{4}\right)}{\text{hypotenuse}} = \frac{x}{\sqrt{x^2+16}}$.



1.4.88

$\cos\left(\tan^{-1}\left(\frac{x}{\sqrt{9-x^2}}\right)\right) = \frac{\text{side adjacent to } \tan^{-1}\left(\frac{x}{\sqrt{9-x^2}}\right)}{\text{hypotenuse}} = \frac{\sqrt{9-x^2}}{3}$.

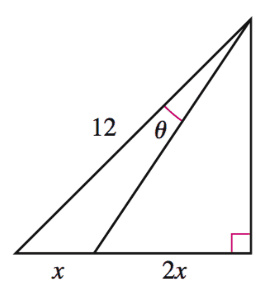


1.4.89 Because $\sin\theta = \frac{x}{6}$, $\theta = \sin^{-1}(x/6)$. Also, $\theta = \tan^{-1}\left(\frac{x}{\sqrt{36-x^2}}\right) = \sec^{-1}\left(\frac{6}{\sqrt{36-x^2}}\right)$.

1.4.90

Note that the vertical side in the adjacent diagram has length $\sqrt{144-9x^2}$ (by the Pythagorean theorem.) Let ψ be the angle in the adjacent diagram which is opposite the side labeled $2x$. First note that $\tan(\psi) = \frac{2x}{\sqrt{144-9x^2}}$, so $\psi = \tan^{-1}\left(\frac{2x}{\sqrt{144-9x^2}}\right)$. Also, $\sin(\theta+\psi) = \frac{3x}{12} = \frac{x}{4}$, so $\theta+\psi = \sin^{-1}(x/4)$. Therefore,

$$\theta = \sin^{-1}(x/4) - \psi = \sin^{-1}(x/4) - \tan^{-1}(2x/\sqrt{144-9x^2}).$$



1.4.91

a. False. For example, $\sin(\pi/2+\pi/2) = \sin(\pi) = 0 \neq \sin(\pi/2)+\sin(\pi/2) = 1+1 = 2$.

b. False. That equation has zero solutions, because the range of the cosine function is $[-1, 1]$.

c. False. It has infinitely many solutions of the form $\pi/6 + 2k\pi$, where k is an integer (among others.)

d. False. It has period $\dfrac{2\pi}{\pi/12} = 24$.

e. True. The others have a range of either $[-1, 1]$ or $(-\infty, -1] \cup [1, \infty)$.

f. False. For example, suppose $x = .5$. Then $\sin^{-1}(x) = \pi/6$ and $\cos^{-1}(x) = \pi/3$, so that $\dfrac{\sin^{-1}(x)}{\cos^{-1}(x)} = \dfrac{\pi/6}{\pi/3} = .5$. However, note that $\tan^{-1}(.5) \neq .5$,

g. True. Note that the range of the inverse cosine function is $[0, \pi]$.

h. False. For example, if $x = .5$, we would have $\sin^{-1}(.5) = \pi/6 \neq 1/\sin(.5)$.

1.4.92 If $\sin\theta = -4/5$, then the Pythagorean identity gives $|\cos\theta| = 3/5$. But if $\pi < \theta < 3\pi/2$, then the cosine of θ is negative, so $\cos\theta = -3/5$. Thus $\tan\theta = 4/3$, $\cot\theta = 3/4$, $\sec\theta = -5/3$, and $\csc\theta = -5/4$.

1.4.93 If $\cos\theta = 5/13$, then the Pythagorean identity gives $|\sin\theta| = 12/13$. But if $0 < \theta < \pi/2$, then the sine of θ is positive, so $\sin\theta = 12/13$. Thus $\tan\theta = 12/5$, $\cot\theta = 5/12$, $\sec\theta = 13/5$, and $\csc\theta = 13/12$.

1.4.94 If $\sec\theta = 5/3$, then $\cos\theta = 3/5$, and the Pythagorean identity gives $|\sin\theta| = 4/5$. But if $3\pi/2 < \theta < 2\pi$, then the sine of θ is negative, so $\sin\theta = -4/5$. Thus $\tan\theta = -4/3$, $\cot\theta = -3/4$, and $\csc\theta = -5/4$.

1.4.95 If $\csc\theta = 13/12$, then $\sin\theta = 12/13$, and the Pythagorean identity gives $|\cos\theta| = 5/13$. But if $0 < \theta < \pi/2$, then the cosine of θ is positive, so $\cos\theta = 5/13$. Thus $\tan\theta = 12/5$, $\cot\theta = 5/12$, and $\sec\theta = 13/5$.

1.4.96 The amplitude is 2, and the period is $\dfrac{2\pi}{2} = \pi$.

1.4.97 The amplitude is 3, and the period is $\dfrac{2\pi}{1/3} = 6\pi$.

1.4.98 The amplitude is 2.5, and the period is $\dfrac{2\pi}{1/2} = 4\pi$.

1.4.99 The amplitude is 3.6, and the period is $\dfrac{2\pi}{\pi/24} = 48$.

1.4.100 Using the given diagram, drop a perpendicular from the point $(b\cos\theta, b\sin\theta)$ to the x axis, and consider the right triangle thus formed whose hypotenuse has length c. By the Pythagorean theorem, $(b\sin\theta)^2 + (a - b\cos\theta)^2 = c^2$. Expanding the binomial gives $b^2\sin^2\theta + a^2 - 2ab\cos\theta + b^2\cos^2\theta = c^2$. Now because $b^2\sin^2\theta + b^2\cos^2\theta = b^2$, this reduces to $a^2 + b^2 - 2ab\cos\theta = c^2$.

1.4.101 Note that $\sin A = \dfrac{h}{c}$ and $\sin C = \dfrac{h}{a}$, so $h = c\sin A = a\sin C$. Thus

$$\frac{\sin A}{a} = \frac{\sin C}{c}.$$

Now drop a perpendicular from the vertex A to the line determined by $\overline{BC}$, and let h_2 be the length of this perpendicular. Then $\sin C = \dfrac{h_2}{b}$ and $\sin B = \dfrac{h_2}{C}$, so $h_2 = b\sin C = c\sin B$. Thus

$$\frac{\sin C}{c} = \frac{\sin B}{b}.$$

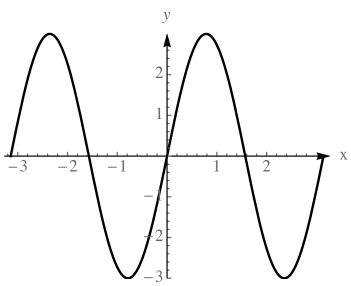
Putting the two displayed equations together gives

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}.$$

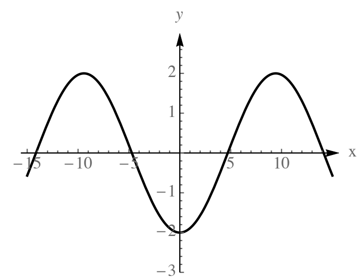
1.4.102 Consider the $\theta = \angle DOA$ where $D(300, 200)$ is the point where the Ditl is anchored, $O(0, 0)$ is the point where the observer is, and $A(300, 0)$ is the point on the x-axis closest to D. Then $\theta = \tan^{-1}(2/3)$. Then consider the angle $\phi = \angle WOB$ where $W(-100, 250)$ is the location of the Windborne and $B(-100, 0)$ is the point on the x-axis closest to W. Then $\phi = \tan^{-1}(250/100) = \tan^{-1}(5/2)$. The angle we are looking for has measure $\pi - \theta - \phi = \pi - \tan^{-1}(5/2) - \tan^{-1}(2/3) \approx 1.3633$ radians.

1.4.103 The area of the entire circle is πr^2. The ratio $\dfrac{\theta}{2\pi}$ represents the proportion of the area swept out by a central angle θ. Thus the area of a sector of a circle is this same proportion of the entire area, so it is $\dfrac{\theta}{2\pi} \cdot \pi r^2 = \dfrac{r^2\theta}{2}$.

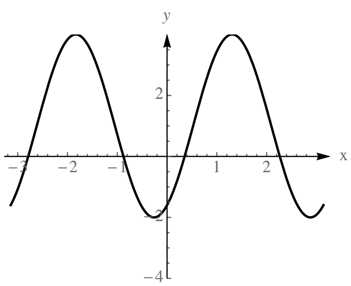
1.4.104



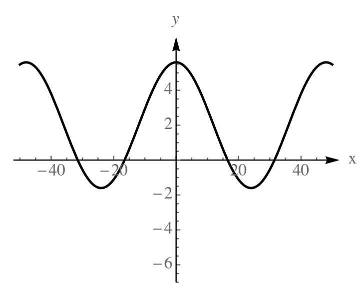
1.4.105



1.4.106



1.4.107



1.4.108 It is helpful to imagine first shifting the function horizontally so that the x intercept is where it should be, then stretching the function horizontally to obtain the correct period, and then stretching the function vertically to obtain the correct amplitude. Because the old x-intercept is at $x = 0$ and the new one should be at $x = 3$ (halfway between where the maximum and the minimum occur), we need to shift the function 3 units to the right. Then to get the right period, we need to multiply (before applying the sine function) by $\pi/6$ so that the new period is $\dfrac{2\pi}{\pi/6} = 12$. Finally, to get the right amplitude and to get the max and min at the right spots, we need to multiply on the outside by 4. Thus, the desired function is:

$$f(x) = 4\sin((\pi/6)(x - 3)) = 4\sin((\pi/6)x - \pi/2).$$

1.4.109 It is helpful to imagine first shifting the function horizontally so that the x intercept is where it should be, then stretching the function horizontally to obtain the correct period, and then stretching the function vertically to obtain the correct amplitude, and then shifting the whole graph up. Because the old x-intercept is at $x = 0$ and the new one should be at $x = 9$ (halfway between where the maximum and the minimum occur), we need to shift the function 9 units to the right. Then to get the right period, we need to multiply (before applying the sine function) by $\pi/12$ so that the new period is $\dfrac{2\pi}{\pi/12} = 24$. Finally, to get the right amplitude and to get the max and min at the right spots, we need to multiply on the outside by 3, and then shift the whole thing up 13 units. Thus, the desired function is:

$$f(x) = 3\sin((\pi/12)(x - 9)) + 13 = 3\sin((\pi/12)x - 3\pi/4) + 13.$$

1.4.110 Let C be the point on the end line so that segment $\overline{AC}$ is perpendicular to the endline. Then the distance $G_1C = 38.\overline{3}$, $G_2C = 15$, and $AC = 69$ and $BC = 84$, where all lengths are in feet. Thus

$$\mathrm{m}(\angle G_1AG_2) = \mathrm{m}(\angle G_1AC) - \mathrm{m}(\angle G_2AC) = \tan^{-1}\left(\frac{38.\overline{3}}{69}\right) - \tan^{-1}\left(\frac{15}{69}\right) \approx 16.79°,$$

while

$$\mathrm{m}(\angle G_1BG_2) = \mathrm{m}(\angle G_1BC) - \mathrm{m}(\angle G_2BC) = \tan^{-1}\left(\frac{38.\overline{3}}{84}\right) - \tan^{-1}\left(\frac{15}{84}\right) \approx 14.4°.$$

The kicking angle was not improved by the penalty.

1.4.111 Let C be the circumference of the earth. Then the first rope has radius $r_1 = \frac{C}{2\pi}$. The circle generated by the longer rope has circumference $C + 38$, so its radius is $r_2 = \frac{C+38}{2\pi} = \frac{C}{2\pi} + \frac{38}{2\pi} \approx r_1 + 6$, so the radius of the bigger circle is about 6 feet more than the smaller circle.

1.4.112

a. The period of this function is $\frac{2\pi}{2\pi/365} = 365$.

b. Because the maximum for the regular sine function is 1, and this function is scaled vertically by a factor of 2.8 and shifted 12 units up, the maximum for this function is $(2.8)(1) + 12 = 14.8$. Similarly, the minimum is $(2.8)(-1) + 12 = 9.2$. Because of the horizontal shift, the point at $t = 81$ is the midpoint between where the max and min occur. Thus the max occurs at $81 + (365/4) \approx 172$ and the min occurs approximately $(365/2)$ days later at about $t = 355$.

c. The solstices occur halfway between these points, at 81 and $81 + (365/2) \approx 264$.

1.4.113 We are seeking a function with amplitude 10 and period 1.5, and value 10 at time 0, so it should have the form $10\cos(kt)$, where $\frac{2\pi}{k} = 1.5$. Solving for k yields $k = \frac{4\pi}{3}$, so the desired function is $d(t) = 10\cos(4\pi t/3)$.

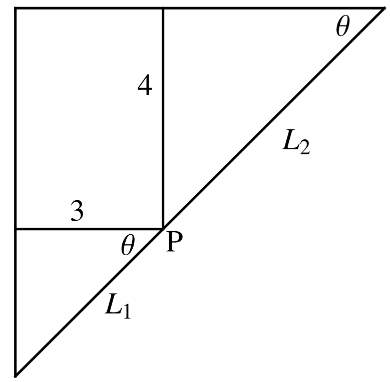
1.4.114 Let θ_1 be the viewing angle to the bottom of the television. Then $\theta_1 = \tan^{-1}\left(\frac{3}{x}\right)$. Now $\tan(\theta + \theta_1) = \frac{10}{x}$, so $\theta + \theta_1 = \tan^{-1}\left(\frac{10}{x}\right)$, so $\theta = \tan^{-1}\left(\frac{10}{x}\right) - \theta_1 = \tan^{-1}\left(\frac{10}{x}\right) - \tan^{-1}\left(\frac{3}{x}\right)$.

1.4.107 The area of the entire circle is πr^2. The ratio $\frac{\theta}{2\pi}$ represents the proportion of the area swept out by a central angle θ. Thus the area of a sector of a circle is this same proportion of the entire area, so it is $\frac{\theta}{2\pi} \cdot \pi r^2 = \frac{r^2\theta}{2}$.

1.4.115 Let L be the line segment connecting the tops of the ladders and let M be the horizontal line segment between the walls h feet above the ground. Now note that the triangle formed by the ladders and L is equilateral, because the angle between the ladders is 60 degrees, and the other two angles must be equal and add to 120, so they are 60 degrees as well. Now we can see that the triangle formed by L, M and the right wall is similar to the triangle formed by the left ladder, the left wall, and the ground, because they are both right triangles with one angle of 75 degrees and one of 15 degrees. Thus $M = h$ is the distance between the walls.

1.4.116

Let the corner point P divide the pole into two pieces, L_1 (which spans the 3-ft hallway) and L_2 (which spans the 4-ft hallway.) Then $L = L_1 + L_2$. Now $L_2 = \frac{4}{\sin\theta}$, and $\frac{3}{L_1} = \cos\theta$ (see diagram.) Thus $L = L_1 + L_2 = \frac{3}{\cos\theta} + \frac{4}{\sin\theta}$. When $L = 10$, $\theta \approx .9273$.



Chapter One Review

1

a. True. For example, $f(x) = x^2$ is such a function.

b. False. For example, $\cos(\pi/2 + \pi/2) = \cos(\pi) = -1 \neq \cos(\pi/2) + \cos(\pi/2) = 0 + 0 = 0$.

c. False. Consider $f(1+1) = f(2) = 2m + b \neq f(1) + f(1) = (m+b) + (m+b) = 2m + 2b$. (At least these aren't equal when $b \neq 0$.)

d. True. $f(f(x)) = f(1-x) = 1 - (1-x) = x$.

e. False. This set is the union of the disjoint intervals $(-\infty, -7)$ and $(1, \infty)$.

f. False. For example, if $x = y = 10$, then $\log_{10} xy = \log_{10} 100 = 2$, but $\log_{10} 10 \cdot \log_{10} 10 = 1 \cdot 1 = 1$.

g. True. $\sin^{-1}(\sin(2\pi)) = \sin^{-1}(0) = 0$.

2 B represents a function but A doesn't. A does not pass the vertical line test.

3 f is a one-to-one function, but g isn't (it fails the horizontal line test.)

4 Because the quantity under the radical must be non-zero, the domain of f is $[0, \infty)$. The range is also $[0, \infty)$.

5 The denominator must not be zero, so we must have $w \neq 2$. The domain is $\{w : w \neq 2\}$. Note that when $w \neq 2$, the function becomes $\frac{(w-2)(2w+1)}{w-2} = 2w + 1$. So the graph of f is a line of slope 2 with the point $(2, 5)$ missing, so the range is $\{y : y \neq 5\}$.

6 It is necessary that $x + 6 > 0$, so $x > -6$. The domain is $(-6, \infty)$ and the range is $(-\infty, \infty)$.

7 Because h can be written $h(z) = \sqrt{(z-3)(z+1)}$, we see that the domain is $(-\infty, -1] \cup [3, \infty)$. The range is $[0, \infty)$. (Note that as z gets large, $h(z)$ gets large as well.)

8 $f(g(2)) = f(-2) = f(2) = 2$, and $g(f(-2) = g(f(2)) = g(2) = -2$.

9 Yes, $\tan(\tan^{-1} x) = x$ because the range of the inverse tangent function is a subset of the domain of the tangent function, and the functions are inverses. However, $\tan^{-1}(\tan x)$ does not always equal x, for example: $\tan^{-1}(\tan \pi) = \tan^{-1} 0 = 0$.

10 $f(g(4)) = f(9) = 11$.

11 $g(f(4)) = g(5) = 8$.

12 $f^{-1}(10) = 8$ (Because $f(8) = 10$.)

13 $g^{-1}(5) = 7$ (Because $g(7) = 5$.)

14 $f^{-1}(g^{-1}(4) = f^{-1}(8) = 6$.

15 $g^{-1}(f(3)) = g^{-1}(4) = 8$.

16 Note that $f(6) = 8$ and recall that f is an odd function. Then $f(-6) = -f(6) = -8$, so $f^{-1}(-8) = -6$.

17 $f^{-1}(1 + f(-3)) = f^{-1}(1 - f(3)) = f^{-1}(1 - 4) = f^{-1}(-3) = -f(3) = -2$.

18 $g(1 - f(f^{-1}(-7))) = g(1 - (-7)) = g(8) = 4$.

19

a. $h(g(\pi/2)) = h(1) = 1$

b. $h(f(x)) = h(x^3) = x^{3/2}$.

c. $f(g(h(x))) = f(g(\sqrt{x})) = f(\sin(\sqrt{x})) = (\sin(\sqrt{x}))^3$.

d The domain of $g(f(x))$ is $\mathbb{R}$, because the domain of both functions is the set of all real numbers.

e. The range of $f(g(x))$ is $[-1, 1]$. This is because the range of g is $[-1, 1]$, and on the restricted domain $[-1, 1]$, the range of f is also $[-1, 1]$.

20

a. If $g(x) = x^2 + 1$ and $f(x) = \sin x$, then $f(g(x)) = f(x^2 + 1) = \sin(x^2 + 1)$.

b. If $g(x) = x^2 - 4$ and $f(x) = x^{-3}$ then $f(g(x)) = f(x^2 - 4) = (x^2 - 4)^{-3}$.

c. If $g(x) = \cos 2x$ and $f(x) = e^x$, then $f(g(x)) = f(\cos 2x) = e^{\cos 2x}$.

21

$$\frac{f(x+h) - f(x)}{h} = \frac{(x+h)^2 - 2(x+h) - (x^2 - 2x)}{h} = \frac{x^2 + 2hx + h^2 - 2x - 2h - x^2 + 2x}{h}$$
$$= \frac{2hx + h^2 - 2h}{h} = 2x + h - 2.$$

$$\frac{f(x) - f(a)}{x - a} = \frac{x^2 - 2x - (a^2 - 2a)}{x - a} = \frac{(x^2 - a^2) - 2(x - a)}{x - a} = \frac{(x - a)(x + a) - 2(x - a)}{x - a} = x + a - 2.$$

22 $\dfrac{f(x+h) - f(x)}{h} = \dfrac{4 - 5(x+h) - (4 - 5x)}{h} = \dfrac{4 - 5x - 5h - 4 + 5x}{h} = -\dfrac{5h}{h} = -5.$

$\dfrac{f(x) - f(a)}{x - a} = \dfrac{4 - 5x - (4 - 5a)}{x - a} = -\dfrac{5(x - a)}{x - a} = -5.$

23

$$\frac{f(x+h) - f(x)}{h} = \frac{(x+h)^2 + 2 - (x^3 + 2)}{h} = \frac{x^2 + 3x^2h + 3xh^2 + h^3 + 2 - x^3 - 2}{h}$$
$$= \frac{h(3x^2 + 3xh + h^2)}{h} = 3x^2 + 3xh + h^2.$$

$$\frac{f(x) - f(a)}{x - a} = \frac{x^3 + 2 - (a^3 + 2)}{x - a} = \frac{x^3 - a^3}{x - a} = \frac{(x - a)(x^2 + ax + a^2)}{x - a} = x^2 + ax + a^2.$$

24

$$\frac{f(x+h)-f(x)}{h} = \frac{\frac{7}{x+h+3}-\frac{7}{x+3}}{h} = \frac{\frac{7x+21-(7x+7h+21)}{(x+3)(x+h+3)}}{h}$$
$$= -\frac{7h}{(h)(x+3)(x+h+3)} = -\frac{7}{(x+3)(x+h+3)}.$$

$$\frac{f(x)-f(a)}{x-a} = \frac{\frac{7}{x+3}-\frac{7}{a+3}}{x-a} = \frac{\frac{7a+21-(7x+21)}{(x+3)(a+3)}}{x-a} = -\frac{7(x-a)}{(x-a)(x+3)(a+3)} = -\frac{7}{(x+3)(a+3)}.$$

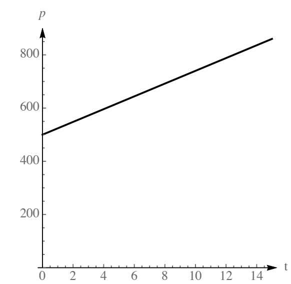
25

a. This line has slope $\frac{2-(-3)}{4-2} = \frac{5}{2}$. Therefore the equation of the line is $y-2 = \frac{5}{2}(x-4)$, so $y = \frac{5}{2}x - 8$.

b. This line has the form $y = \frac{3}{4}x + b$, and because $(-4, 0)$ is on the line, $0 = (3/4)(-4) + b$, so $b = 3$. Thus the equation of the line is given by $y = \frac{3}{4}x + 3$.

c. This line has slope $\frac{0-(-2)}{4-0} = \frac{1}{2}$, and the y-intercept is given to be -2, so the equation of this line is $y = \frac{1}{2}x - 2$.

26 If t is the number of years **after** 2018, then $p(t) = 24t + 500$. Because the year 2033 is 15 years after 2018, the population is predicted by $p(15) = 24 \cdot 15 + 500 = 360 + 500 = 860$.



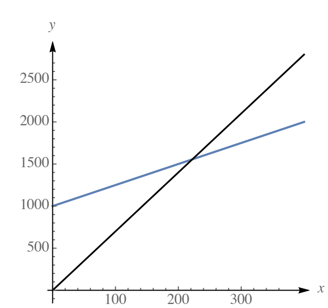
27 We are looking for the line between the points $(0, 212)$ and $(6000, 200)$. The slope is $\frac{212-200}{0-6000} = -\frac{12}{6000} = -\frac{1}{500}$. Because the intercept is given, we deduce that the line is $B = f(a) = -\frac{1}{500}a + 212$.

28

a. The cost of producing x books is $C(x) = 1000 + 2.5x$.

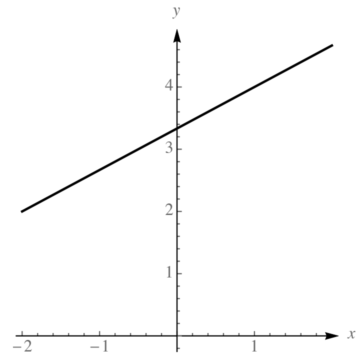
b. The revenue generated by selling x books is $R(x) = 7x$.

c. The break-even point is where $R(x) = C(x)$. This is where $7x = 1000 + 2.5x$, or $4.5x = 1000$. So $x = \frac{1000}{4.5} \approx 222$.

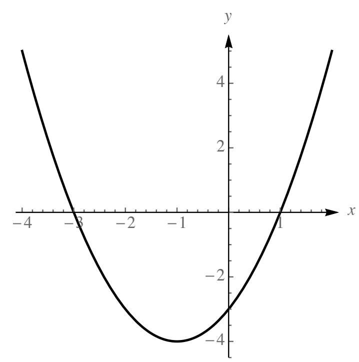


29

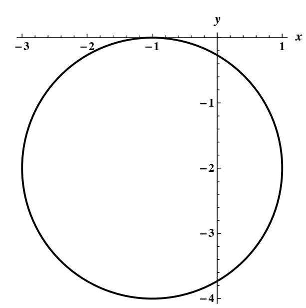
a. This is a straight line with slope 2/3 and y-intercept 10/3.



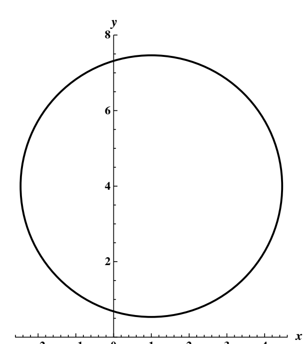
b. Completing the square gives $y = (x^2+2x+1)-4$, or $y = (x+1)^2-4$, so this is the standard parabola shifted one unit to the left and down 4 units.



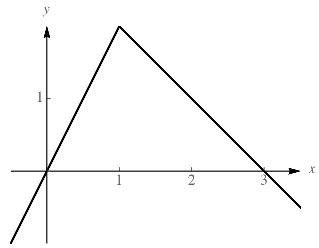
c. Completing the square, we have $x^2+2x+1+y^2+4y+4 = -1+1+4$, so we have $(x+1)^2+(y+2)^2 = 4$, a circle of radius 2 centered at $(-1,-2)$.



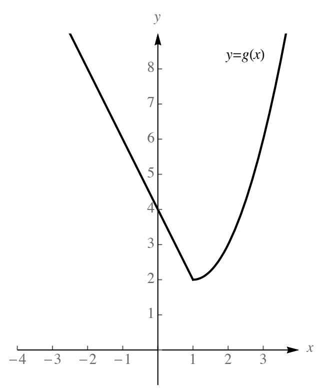
d. Completing the square, we have $x^2-2x+1+y^2-8y+16 = -5+1+16$, or $(x-1)^2+(y-4)^2 = 12$, which is a circle of radius $\sqrt{12}$ centered at $(1,4)$.



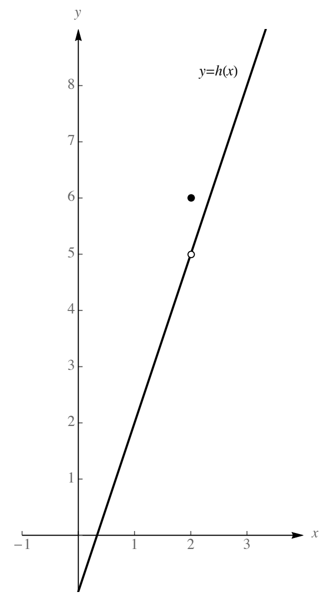
30



31

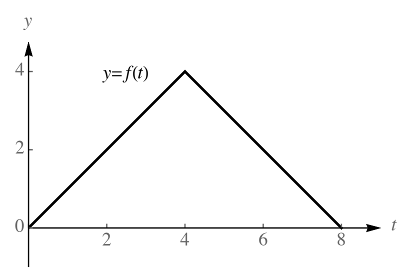


32



33

a.



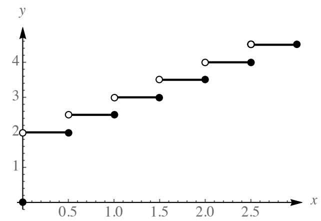
b. $A(2) = \frac{1}{2} \cdot 2 \cdot 2 = 2$ and $A(6) = 16 - \frac{1}{2} \cdot 2 \cdot 2 = 14$.

c. Note that for $0 \le x \le 4$, the area is that of a triangle with base x and height x. For $4 \le x \le 8$, the area is given by the difference of 16 (the total area under the curve from 0 to 8) minus the area of a triangle with base $8 - x$ and height $8 - x$. So the area for x in that range is $16 - \frac{(8-x)^2}{2} = 16 - 32 + 8x - \frac{x^2}{2}$. Therefore,

$$A(x) = \begin{cases} \frac{x^2}{2} & \text{if } 0 \le x \le 4 \\ -\frac{x^2}{2} + 8x - 16 & \text{if } 4 \le x \le 8. \end{cases}$$

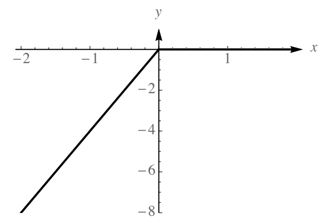
34

The function is a piecewise step function which jumps up by one every half-hour step.



35

Because $|x| = \begin{cases} -x & \text{if } x < 0; \\ x & \text{if } x \geq 0, \end{cases}$
we have
$2(x - |x|) = \begin{cases} 2(x - (-x)) = 4x & \text{if } x < 0; \\ 2(x - x) = 0 & \text{if } x \geq 0. \end{cases}$



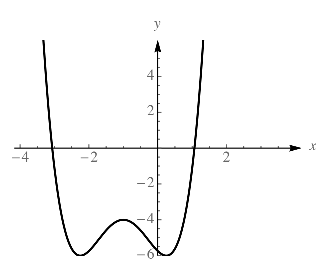
36 To solve $x^{1/3} = x^{1/4}$ we raise each side to the 12th power, yielding $x^4 = x^3$. This gives $x^4 - x^3 = 0$, or $x^3(x-1) = 0$, so the only solutions are $x = 0$ and $x = 1$ (which can be easily verified as solutions.) Between 0 and 1, $x^{1/4} > x^{1/3}$, but for $x > 1$, $x^{1/3} > x^{1/4}$.

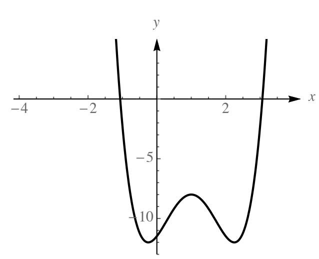
37 The domain of $x^{1/7}$ is the set of all real numbers, as is its range. The domain of $x^{1/4}$ is the set of non-negative real numbers, as is its range.

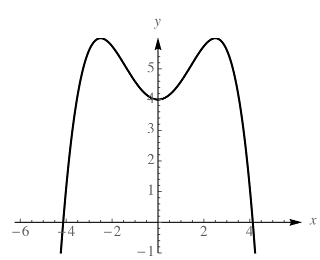
38 Completing the square in the second equation, we have $x^2 + y^2 - 7y + \frac{49}{4} = -8 + \frac{49}{4}$, which can be written as $x^2 + (y - (7/2))^2 = \frac{17}{4}$. Thus we have a circle of radius $\sqrt{17}/2$ centered at $(0, 7/2)$, along with the standard parabola. These intersect when $y = 7y - y^2 - 8$, which occurs for $y^2 - 6y + 8 = 0$, so for $y = 2$ and $y = 4$, with corresponding x values of ± 2 and $\pm\sqrt{2}$.

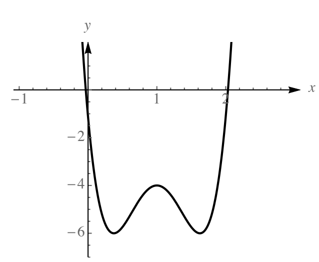
39 Completing the square, we can write $x^2 + 6x - 3 = x^2 + 6x + 9 - 3 - 9 = (x+3)^2 - 12$, so the graph is obtained by shifting $y = x^2$ 3 units right and 12 units down.

40

a. 

b. 

c. 

d. 

41

a. Because $f(-x) = \cos -3x = \cos 3x = f(x)$, this is an even function, and is symmetric about the y-axis.

b. Because $f(-x) = 3(-x)^4 - 3(-x)^2 + 1 = 3x^4 - 3x^2 + 1 = f(x)$, this is an even function, and is symmetric about the y-axis.

c. Because replacing x by $-x$ and/or replacing y by $-y$ gives the same equation, this represents a curve which is symmetric about the y-axis and about the origin and about the x-axis.

42 We have $8 = e^{4k}$, and so $\ln 8 = 4k$, so $k = \frac{\ln 8}{4}$.

43 If $\log x^2 + 3\log x = \log 32$, then $\log(x^2 \cdot x^3) = \log(32)$, so $x^5 = 32$ and $x = 2$. The answer does not depend on the base of the log.

44 $\ln 3x + \ln(x+2) = \ln(3x(x+2))$. This is zero when $3x(x+2) = 1$, or $3x^2 + 6x - 1 = 0$. By the quadratic formula, we have $\frac{-6\pm\sqrt{36-4\cdot 3\cdot(-1)}}{6} = \frac{-6\pm\sqrt{48}}{6} = -1 \pm \frac{2\sqrt{3}}{3}$. The original equation isn't defined for $-1 - \frac{2\sqrt{3}}{3}$ so the only solution is $-1 + \frac{2\sqrt{3}}{3}$.

45 If $3\ln(5t+4) = 12$, then $\ln(5t+4) = 4$, and $e^4 = 5t+4$. It then follows that $5t = e^4 - 4$ and so $t = \frac{e^4-4}{5}$.

46 If $7^{y-3} = 50$, then $\ln(7^{y-3}) = \ln 50$, so $(y-3)\ln 7 = \ln 50$ and $y - 3 = \frac{\ln 50}{\ln 7}$. Then $y = 3 + \frac{\ln 50}{\ln 7}$.

47 If $1 - 2\sin^2\theta = 0$, then $\sin^2\theta = \frac{1}{2}$, so $|\sin\theta| = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$. So $\theta = \frac{\pi}{4}, \frac{3\pi}{4} \frac{5\pi}{4} \frac{7\pi}{4}$.

48 First note that if θ is between $-\pi/2$ and $\pi/2$, that 2θ is then between $-\pi$ and π. If $\sin^2 2\theta = \frac{1}{2}$, then $|\sin 2\theta| = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$. So $2\theta = -\frac{3\pi}{4}, -\frac{\pi}{4}, \frac{\pi}{4}, \frac{3\pi}{4}$. Thus $\theta = -\frac{3\pi}{8}, -\frac{\pi}{8}, \frac{\pi}{8}, \frac{3\pi}{8}$

49 First note that if θ is between $-\pi/2$ and $\pi/2$, that 2θ is then between $-\pi$ and π. If $4\cos^2 2\theta = 3$, then $\cos^2 2\theta = \frac{3}{4}$, and $|\cos 2\theta| = \frac{\sqrt{3}}{2}$. Thus $2\theta = \pm\frac{\pi}{6}, \pm\frac{5\pi}{6}$, and $\theta = \pm\frac{\pi}{12}, \pm\frac{5\pi}{12}$.

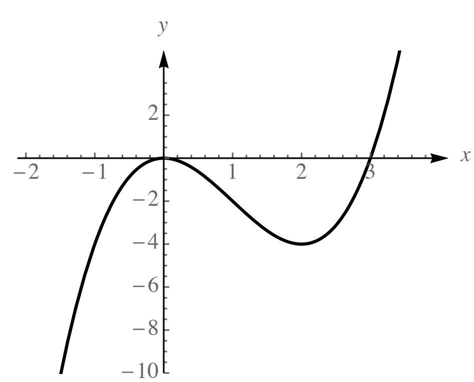
50 First note that if θ is between 0 and π that 3θ is then between 0 and 3π. If $\sqrt{2}\sin 3\theta + 1 = 2$, then $\sin 3\theta = \frac{1}{\sqrt{2}}$. Then $3\theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{9\pi}{4}, \frac{11\pi}{4}$, so $\theta = \frac{\pi}{12}, \frac{\pi}{4}, \frac{3\pi}{4}, \frac{11\pi}{12}$.

51 In 2010 (when $t = 0$), the population is $P(0) = 100$. So we are seeking t so that $200 = 100e^{t/50}$, or $e^{t/50} = 2$. Taking the natural logarithm of both sides yields $\frac{t}{50} = \ln 2$, or $t = 50\ln 2 \approx 35$ years.

52 Curve A is $y = 3^{-x}$, curve B is $y = 2^x$, and curve C is $y = -\ln x$.

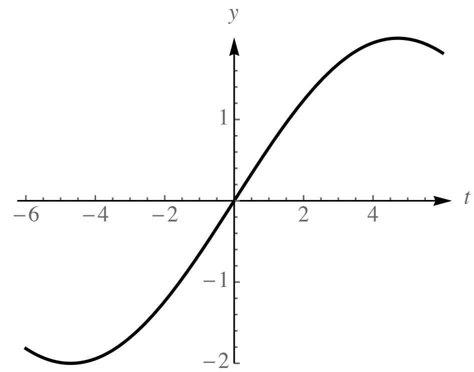
53

By graphing, it is clear that this function is not one-to-one on its whole domain, but it is one-to-one on the interval $(-\infty, 0]$, on the interval $[0, 2]$, and on the interval $[2, \infty)$, so it would have an inverse if we restricted it to any of these particular intervals.



54

This function is a stretched version of the sine function, it is one-to-one on the interval $[-3\pi/2, 3\pi/2]$ (and on other intervals as well ...)



55 Switching x and y gives $x = 6 - 4y$. Then $x - 6 = -4y$, so

$$y = \frac{x-6}{-4} = \frac{6-x}{4} = -\frac{1}{4}x + \frac{3}{2}.$$

56 Switching x and y gives $x = 3y - 4$, so $x + 4 = 3y$ and $y = \frac{x+4}{3}$.

57 Completing the square gives $f(x) = x^2 - 4x + 4 + 1 = (x-2)^2 + 1$. Switching the x and y and solving for y yields $(y-2)^2 = x - 1$, so $|y - 2| = \sqrt{x-1}$, and thus $y = f^{-1}(x) = 2 + \sqrt{x-1}$ (we choose the "+" rather than the "−" because the domain of f is $x \geq 2$, so the range of f^{-1} must also consist of numbers greater than or equal to 2).

58 Switching x and y gives $x = \frac{4y^2}{y^2+10}$. Then $x(y^2+10) = 4y^2$, so $xy^2 - 4y^2 = -10x$. Then $y^2(x-4) = -10x$, so $y^2 = \frac{10x}{4-x}$, and $|y| = \sqrt{\frac{10x}{4-x}}$, so $y = \sqrt{\frac{10x}{4-x}}$ (we choose the "+" rather than the "−" because the domain of f is $x \geq 0$, so the range of f^{-1} must also consist of numbers greater than or equal to 0).

59 Switching x and y gives $x = 3y^2 + 1$, so $3y^2 = x - 1$, and $y^2 = \frac{x-1}{3}$. Then $|y| = \sqrt{\frac{x-1}{3}}$, and $y = -\sqrt{\frac{x-1}{3}}$ (we choose the "−" rather than the "+" because the domain of f is $x \leq 0$, so the range of f^{-1} must also consist of numbers less than or equal to 0).

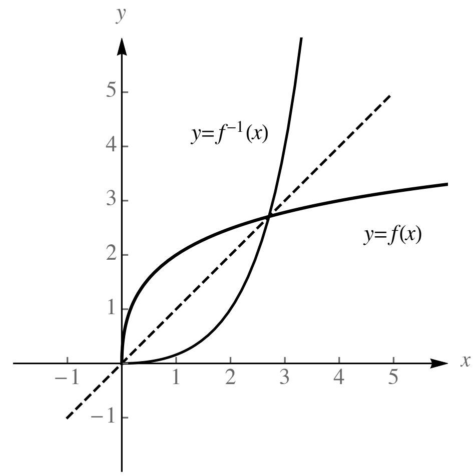
60 If $y = 1/x^2$, then switching x and y gives $x = 1/y^2$, so $y = f^{-1}(x) = 1/\sqrt{x}$.

61 Switching x and y gives $x = e^{y^2+1}$. Then $\ln x = y^2 + 1$, so $y^2 = \ln x - 1$, and $|y| = \sqrt{\ln x - 1}$, so $y = \sqrt{\ln x - 1}$ (we choose the "+" rather than the "−" because the domain of f is $x \geq 0$, so the range of f^{-1} must also consist of numbers greater than or equal to 0).

62 Switching x and y gives $x = \ln(y^2+1)$, so $e^x = y^2 + 1$, and $y^2 = e^x - 1$, so $|y| = \sqrt{e^x - 1}$, so $y = \sqrt{e^x - 1}$ (we choose the "+" rather than the "−" because the domain of f is $x \geq 0$, so the range of f^{-1} must also consist of numbers greater than or equal to 0.)

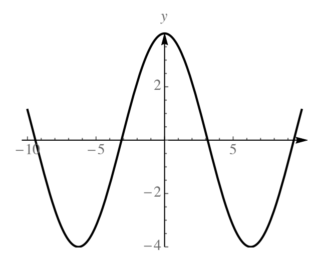
63

Switching x and y gives $x = \frac{6\sqrt{y}}{\sqrt{y}+2}$. Then $x\sqrt{y} + 2x = 6\sqrt{y}$, and so $x\sqrt{y} - 6\sqrt{y} = -2x$. Then $\sqrt{y}(x-6) = -2x$, so $\sqrt{y} = \frac{2x}{6-x}$, and $y = \left(\frac{2x}{6-x}\right)^2 = \frac{4x^2}{(6-x)^2}$. Because the range of f must match the domain of f^{-1}, we must restrict our inverse function to $[0, 6]$.

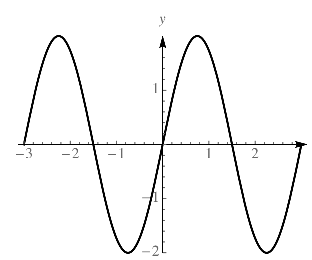


64

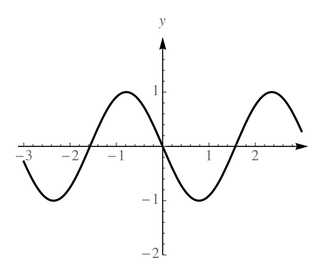
a. This function has period $\frac{2\pi}{1/2} = 4\pi$ and amplitude 4.



b. This function has period $\frac{2\pi}{2\pi/3} = 3$ and amplitude 2.



c. This function has period $\frac{2\pi}{2} = \pi$ and amplitude 1. Compared to the ordinary cosine function it is compressed horizontally, flipped about the x-axis, and shifted $\pi/4$ units to the right.



65

a. We need to scale the ordinary cosine function so that its period is 6, and then shift it 3 units to the right, and multiply it by 2. So the function we seek is $y = 2\cos((\pi/3)(t-3)) = -2\cos(\pi t/3)$.

b. We need to scale the ordinary cosine function so that its period is 24, and then shift it to the right 6 units. We then need to change the amplitude to be half the difference between the maximum and minimum, which would be 5. Then finally we need to shift the whole thing up by 15 units. The function we seek is thus $y = 15 + 5\cos((\pi/12)(t-6)) = 15 + 5\sin(\pi t/12)$.

66 The pictured function has a period of π, an amplitude of 2, and a maximum of 3 and a minimum of -1. It can be described by $y = 1 + 2\cos(2(x - \pi/2))$.

67

a. $-\sin x$ is pictured in F.

b. $\cos 2x$ is pictured in E.

c. $\tan(x/2)$ is pictured in D.

d. $-\sec x$ is pictured in B.

e. $\cot 2x$ is pictured in C.

f. $\sin^2 x$ is pictured in A.

68 If $\sec x = 2$, then $\cos x = \frac{1}{2}$. This occurs for $x = -\pi/3$ and $x = \pi/3$, so the intersection points are $(-\pi/3, 2)$ and $(\pi/3, 2)$.

69 $\sin x = -\frac{1}{2}$ for $x = 7\pi/6$ and for $x = 11\pi/6$, so the intersection points are $(7\pi/6, -1/2)$ and $(11\pi/6, -1/2)$.

70 Note that $\frac{5\pi}{8} = \frac{5\pi/4}{2}$. Using a half-angle identity,

$$\sin\left(\frac{5\pi/4}{2}\right) = \sqrt{\frac{1-\cos 5\pi/4}{2}} = \sqrt{\frac{1+\sqrt{2}/2}{2}} = \sqrt{\frac{2+\sqrt{2}}{4}} = \frac{\sqrt{2+\sqrt{2}}}{2}.$$

71 Note that $\frac{7\pi}{8} = \frac{7\pi/4}{2}$. Using the half-angle identity,

$$\cos\left(\frac{7\pi/4}{2}\right) = -\sqrt{\frac{1+\cos 7\pi/4}{2}} = -\sqrt{\frac{1+\sqrt{2}/2}{2}} = -\sqrt{\frac{2+\sqrt{2}}{4}} = -\frac{\sqrt{2+\sqrt{2}}}{2}.$$

72 Because $\sin(\pi/3) = \sqrt{3}/2$, $\sin^{-1}(\sqrt{3}/2) = \pi/3$.

73 Because $\cos(\pi/6) = \sqrt{3}/2$, $\cos^{-1}(\sqrt{3}/2) = \pi/6$.

74 Because $\cos(2\pi/3) = -1/2$, $\cos^{-1}(-1/2) = 2\pi/3$.

75 Because $\sin(-\pi/2) = -1$, $\sin^{-1}(-1) = -\pi/2$.

76 $\cos(\cos^{-1}(-1)) = \cos(\pi) = -1$.

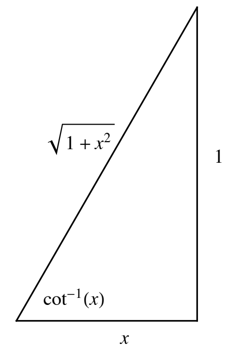
77 $\sin(\sin^{-1}(x)) = x$, for all x in the domain of the inverse sine function.

78 $\cos^{-1}(\sin 3\pi) = \cos^{-1}(0) = \pi/2$.

79 If $\theta = \sin^{-1}(12/13)$, then $0 < \theta < \pi/2$, and $\sin\theta = 12/13$. Then (using the Pythagorean identity) we can deduce that $\cos\theta = 5/13$. It must follow that $\tan\theta = 12/5$, $\cot\theta = 5/12$, $\sec\theta = 13/5$, and $\csc\theta = 13/12$.

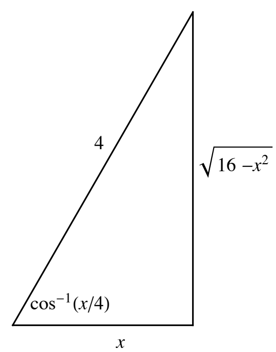
80

$$\csc(\cot^{-1} x) = \frac{\text{hypotenuse}}{\text{side opposite of } \cot^{-1} x} = \sqrt{1+x^2}$$



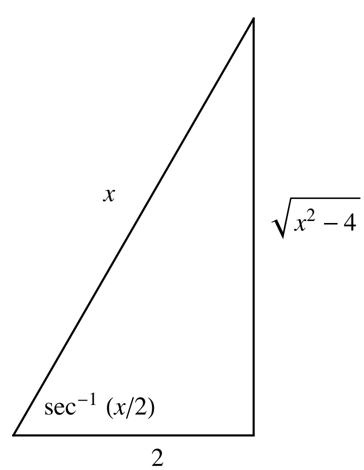
81

$$\sin(\cos^{-1}(x/4)) = \frac{\text{side opposite of } \cos^{-1}(x/4)}{\text{hypotenuse}} = \frac{\sqrt{16-x^2}}{4}.$$



82

$$\tan(\sec^{-1}(x/2)) = \frac{\text{side opposite of } \sec^{-1}(x/2)}{\text{side adjacent to } \sec^{-1}(x/2)} = \frac{\sqrt{x^2-4}}{2}.$$



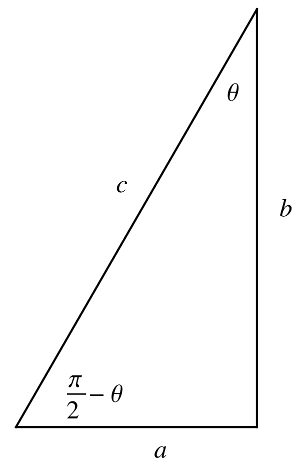
83

Note that

$$\tan\theta = \frac{a}{b} = \cot(\pi/2 - \theta).$$

Thus,

$$\cot^{-1}(\tan\theta) = \cot^{-1}(\cot(\pi/2 - \theta)) = \pi/2 - \theta.$$



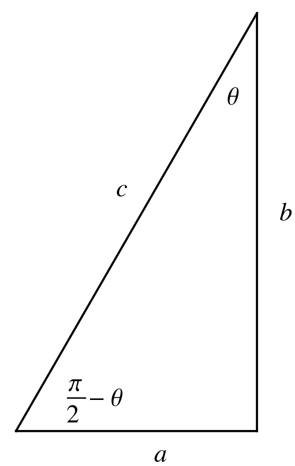
84

Note that

$$\sec\theta = \frac{c}{b} = \csc(\pi/2 - \theta).$$

Thus,

$$\csc^{-1}(\sec\theta) = \csc^{-1}(\csc(\pi/2 - \theta)) = \pi/2 - \theta.$$



85 Let $\theta = \sin^{-1}(x)$. Then $\sin\theta = x$ and note that then $\sin(-\theta) = -\sin\theta = -x$, so $-\theta = \sin^{-1}(-x)$. Then $\sin^{-1}(x) + \sin^{-1}(-x) = \theta + -\theta = 0$.

86 We multiply the quantity $\frac{\sin\theta}{1+\cos\theta}$ by the conjugate of the denominator over itself:

$$\frac{\sin\theta}{1+\cos\theta} = \frac{\sin\theta}{1+\cos\theta} \cdot \frac{1-\cos\theta}{1-\cos\theta} = \frac{\sin\theta(1-\cos\theta)}{1-\cos^2\theta} = \frac{\sin\theta(1-\cos\theta)}{\sin^2\theta} = \frac{1-\cos\theta}{\sin\theta}.$$

87 Using the definition of the tangent function in terms of sine and cosine, we have:

$$\tan 2\theta = \frac{\sin 2\theta}{\cos 2\theta} = \frac{2\sin\theta\cos\theta}{\cos^2\theta - \sin^2\theta}.$$

If we divide both the numerator and denominator of this last expression by $\cos^2\theta$, we obtain

$$\frac{2\tan\theta}{1-\tan^2\theta}.$$

88 Let N be the north pole, and C the center of the given circle, and consider the angle CNP. This angle measures $\frac{\pi-\varphi}{2}$. (Note that the triangle CNP is isosceles.) Now consider the triangle NOX where O is the origin and X is the point $(x, 0)$. Using triangle NOX, we have $\tan\left(\frac{\pi-\varphi}{2}\right) = \frac{x}{2R}$, so $x = 2R\tan\left(\frac{\pi-\varphi}{2}\right)$.

89

a.

n	1	2	3	4	5	6	7	8	9	10
$T(n)$	1	5	14	30	55	91	140	204	285	385

b. The domain of this function consists of the positive integers.

c. Using trial and error and a calculator yields that $T(n) > 1000$ for the first time for $n = 14$.

90

a.

n	1	2	3	4	5	6	7	8	9	10
$S(n)$	1	3	6	10	15	21	28	36	45	55

b. The domain of this function consists of the positive integers. The range is a subset of the set of positive integers.

c. Using trial and error and a calculator yields that $S(n) > 1000$ for the first time for $n = 45$.

91 To find $s(t)$ note that we are seeking a periodic function with period 365, and with amplitude 87.5 (which is half of the number of minutes between 7:25 and 4:30). We need to shift the function 4 days plus one fourth of 365, which is about 95 days so that the max and min occur at $t = 4$ days and at half a year later. Also, to get the right value for the maximum and minimum, we need to multiply by negative one and add 117.5 (which represents 30 minutes plus half the amplitude, because $s = 0$ corresponds to 4:00 AM.) Thus we have

$$s(t) = 117.5 - 87.5 \sin\left(\frac{\pi}{182.5}(t - 95)\right).$$

A similar analysis leads to the formula

$$S(t) = 844.5 + 87.5 \sin\left(\frac{\pi}{182.5}(t - 67)\right).$$

The graph pictured shows $D(t) = S(t) - s(t)$, the length of day function, which has its max at the summer solstice which is about the 172nd day of the year, and its min at the winter solstice.