STUDENT'S SOLUTIONS MANUAL

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CALCULUS

AND

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EARLY TRANSCENDENTALS

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Contents

Differential Equations

D1.1 Basic Ideas

D1.1.1 Second-order, because the highest-order derivative appearing in the equation is second order.

D1.1.3 The equation is second-order, so we expect two arbitrary constants in the general solution.

D1.1.5 Yes. Note that $y'''(t) = 0$ and $y'(t) = 2$.

D1.1.7 Yes, it is a solution. Note that $y'(t) = -5Ce^{-5t}$, so $y'(t) + 5y(t) = 0$.

D1.1.9 Yes, it is a solution. $y'(t) = 4C_1 \cos 4t - 4C_2 \sin 4t$, so $y''(t) = -16C_1 \sin 4t - 16C_2 \cos 4t$, so $y''(t) + 16y(t) = 0$.

D1.1.11 Yes, it is a solution. $y'(t) = 32e^{2t}$, so $y'(t) - 2y(t) = 32e^{2t} - (32e^{2t} - 20) = 20$. Also, $y(0) = 16 - 10 = 6$.

D1.1.13 Yes, it is a solution. $y'(t) = 9 \sin 3t$, so $y''(t) = 27 \cos 3t$. Thus, $y''(t)+9y(t) = 27 \cos 3t-27 \cos 3t = 0$. Also, $y'(0) = 0$ and $y(0) = -3$.

D1.1.15 $y(t) = \int(3 + e^{-2t})\, dt = 3t - \frac{1}{2}e^{-2t} + C$.

D1.1.17 $y(x) = \int(4 \tan 2x - 3 \cos x)\, dx = -2 \ln |\cos 2x| - 3 \sin x + C = 2 \ln |\sec 2x| - 3 \sin x + C$.

D1.1.19 $y'(t) = \int(60t^4 - 4 + 12t^{-3})\, dt = 12t^5 - 4t - 6t^{-2} + C$. $y(t) = \int(12t^5 - 4t - 6t^{-2} + C)\, dt = 2t^6 - 2t^2 + 6t^{-1} + C_1t + C_2$.

D1.1.21 $u'(x) = \int(55x^9 + 36x^7 - 21x^5 + 10x^{-3})\, dx = 5.5x^{10} + \frac{9}{2}x^8 - \frac{7}{2}x^6 - 5x^{-2} + C_1$.
$u(x) = \int(5.5x^{10} + \frac{9}{2}x^8 - \frac{7}{2}x^6 - 5x^{-2} + C)\, dx = \frac{1}{2}x^{11} + \frac{1}{2}x^9 - \frac{1}{2}x^7 + 5x^{-1} + C_1x + C_2$.

D1.1.23 $y(t) = \int(1 + e^t)\, dt = t + e^t + C$. Because $y(0) = 4 = 1 + C$, we have $C = 3$. Thus, $y(t) = t + e^t + 3$.

D1.1.25 $y(x) = \int(3x^2 - 3x^{-4})\, dx = x^3 + x^{-3} + C$. Because $y(1) = 0 = 1 + 1 + C$, we have $C = -2$. So $y(x) = x^3 + x^{-3} - 2$.

D1.1.27 $y'(t) = \int(12t - 20t^3)\, dt = 6t^2 - 5t^4 + C_1$. Because $y'(0) = 0 = 0 + C_1$, we have $C_1 = 0$. $y(t) = \int(6t^2-5t^4)\, dt = 2t^3-t^5+C_2$. Because $y(0) = 1 = 0-0+C_2$, we have $C_2 = 1$. Thus, $y(t) = 2t^3-t^5+1$.

D1.1.29

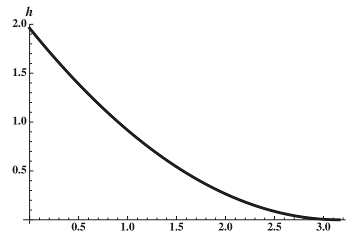
a. $v(t) = -9.8t + 29.4$. $s(t) = -4.9t^2 + 29.4t + 30$.

b. The object reaches its high point when $-9.8t + 29.4 = 0$, or $t = \frac{29.4}{9.8} = 3$. At that time its position is $s(3) \approx 74.1$ meters.

D1.1.31 We have $p(t) = (1500-20H)e^{.05t}+20H$. The amount of resource is increasing when $1500-20H > 0$, which occurs for $H < 75$. The amount of resource is constant when $1500-20H = 0$, which occurs for $H = 75$. If $H = 100$, the resource is zero when $(1500 - 2000)e^{.05t} + 2000 = 0$, which occurs for $t = 20 \ln 4 \approx 28$.

D1.1.33

The height function is given by $h(t) = \left(\sqrt{1.96} - \frac{.3\sqrt{2\cdot 9.8}}{1.5}\cdot\frac{t}{2}\right)^2 \approx (1.4-0.44t)^2$. The tank is empty when $h(t)=0$, which occurs after about 3.16 seconds.



D1.1.35

a. False. That is a specific solution. The general solution is $t+C$.

b. False. It is second order, but is not linear.

c. True. First find the general solution, and then find the specific solution which satisfies the initial condition.

D1.1.37 $u(x) = \int \frac{2x}{x^2+4}\,dx - \int \frac{2}{x^2+4}\,dx = \ln(x^2+4) - \tan^{-1}(x/2) + C$.

D1.1.39 $y'(x) = \int \frac{x}{(1-x^2)^{3/2}}\,dx$. Let $u = 1-x^2$, so that $du = -2x\,dx$. Substituting gives $y'(x) = \frac{-1}{2}\int u^{-3/2}\,du = u^{-1/2} + C_1 = \frac{1}{\sqrt{1-x^2}} + C_1\,dx$. $y(x) = \int\left(\frac{1}{\sqrt{1-x^2}} + C_1\right)dx = \sin^{-1}(x) + C_1x + C_2$.

D1.1.41 $u(x) = \int\left(\frac{1}{x^2+4^2} - 4\right)dx = \frac{1}{4}\tan^{-1}(x/4) - 4x + C$. Because $u(0) = 2 = 0-0+C$, we have $C=2$. Thus, $u(x) = \frac{1}{4}\tan^{-1}(x/4) - 4x + 2$.

D1.1.43 Using the result of number 40 above, we have $y'(t) = te^t - e^t + C_1$, and because $y'(0) = 1 = 0-1+C_1$, we have $C_1 = 2$. Thus $y'(t) = te^t - e^t + 2$. $y(t) = \int y'(t)\,dt = \int(te^t - e^t + 2)\,dt = te^t - e^t - e^t + 2t + C_2 = te^t - 2e^t + 2t + C_2$. Because $y(0) = 0 = 0 - 2 + 0 + C_2$, we have $C_2 = 2$. Thus, $y(t) = te^t - 2e^t + 2t + 2$.

D1.1.45 $u'(t) = C_1e^t + C_2e^t + C_2te^t$, and $u''(t) = C_1e^t + C_2e^t + C_2e^t + C_2te^t = C_1e^t + 2C_2e^t + C_2te^t$. Thus, $u''(t) - 2u'(t) + u(t) = (C_1e^t + 2C_2e^t + C_2te^t) - 2(C_1e^t + C_2e^t + C_2te^t) + C_1e^t + C_2te^t = 0$.

D1.1.47 $u'(t) = 2C_1t + 3C_2t^2$, so $u''(t) = 2C_1 + 6C_2t$. Thus,

$$t^2u''(t) - 4tu'(t) + 6u(t) = 2C_1t^2 + 6C_2t^3 - 4(2C_1t^2 + 3C_2t^3) + 6C_1t^2 + 6C_2t^3 = 0.$$

D1.1.49 $z'(t) = -C_1e^{-t} + 2C_2e^{2t} - 3C_3e^{-3t} - e^t$. So $z''(t) = C_1e^{-t} + 4C_2e^{2t} + 9C_3e^{-3t} - e^t$, and $z'''(t) = -C_1e^{-t} + 8C_2e^{2t} - 27C_3e^{-3t} - e^t$. Thus

$$\begin{aligned} z'''(t) + 2z''(t) - 5z'(t) - 6z(t) &= -C_1e^{-t} + 8C_2e^{2t} - 27C_3e^{-3t} - e^t \\ &\quad + 2C_1e^{-t} + 8C_2e^{2t} + 18C_3e^{-3t} - 2e^t \\ &\quad + 5C_1e^{-t} - 10C_2e^{2t} + 15C_3e^{-3t} + 5e^t \\ &\quad - 6C_1e^{-t} - 6C_2e^{2t} - 6C_3e^{-3t} + 6e^t \\ &= 8e^t \end{aligned}$$

D1.1.51

a. $y'(t) = C_1\cos t - C_2\sin t$, so $y''(t) = -C_1\sin t - C_2\cos t$. Thus, $y''(t) + y(t) = 0$.

b. $y'(t) = 2C_2\cos 2t - 2C_2\sin 2t$, so $y''(t) = -4C_2\sin 2t - 4C_2\cos 2t$. Thus, $y''(t) + 4y(t) = 0$.

c. A general solution appears to be $y(t) = C_1\sin kt + C_2\cos kt$. Then $y'(t) = kC_1\cos kt - kC_2\sin kt$, so $y''(t) = -k^2C_1\sin kt - k^2C_2\cos kt$. And then $y''(t) + k^2y(t) = 0$.

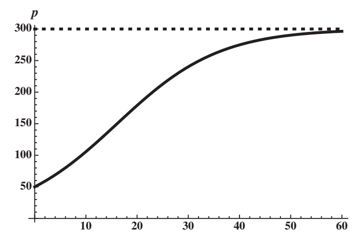
D1.1.53

a. Let $p(t) = \frac{K}{1+Ce^{-rt}}$. Note that $1 - \frac{P}{K} = 1 - \frac{1}{1+Ce^{-rt}} = \frac{Ce^{-rt}}{1+Ce^{-rt}}$. We have

$$p'(t) = \frac{KCre^{-rt}}{(1+Ce^{-rt})^2} = r \cdot \frac{K}{1+Ce^{-rt}} \cdot \frac{Ce^{-rt}}{1+Ce^{-rt}} = rp\left(1 - \frac{p}{K}\right).$$

b. If $p(0) = 50 = \frac{K}{1+C}$, then $50 + 50C = K$, so $C = \frac{K-50}{50}$.

c. We have $p(t) = \frac{300}{1+5e^{-.1t}}$.



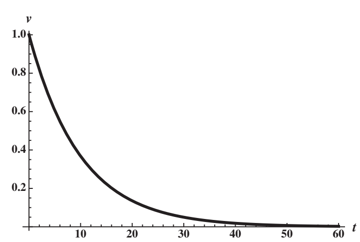
d. $\lim_{t\to\infty} \frac{300}{1+5e^{-.1t}} = \frac{300}{1+0} = 300$, which is consistent with the graph from part c.

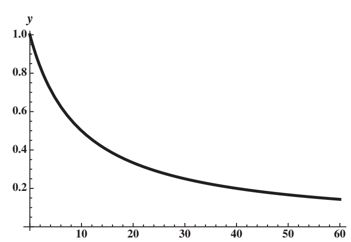
D1.1.55

a. If $y(t) = y_0 e^{-kt}$, then $y(0) = y_0$, and $y'(t) = -ky_0e^{-kt}$, so $y'(t) = -ky(t)$.

b. Let $y(t) = \frac{y_0}{y_0kt+1}$. Then $y(0) = y_0$, and $y'(t) = \frac{-y_0^2k}{(y_0kt+1)^2} = -k(y(t))^2$.

c. The first order reaction decays more quickly.





D1.2 Direction Fields and Euler's Method

D1.2.1 Choose a regular grid of points in the ty-plane, and for each point P, make a small line segment with slope $f(t, y)$.

D1.2.3 $u_0 = y(3) = 1$. $u_1 = u_0 + f(3, 1)(.1) = 1 + .6 = 1.6$.