INSTRUCTOR'S SOLUTIONS MANUAL

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Differential Equations

D1.1 Basic Ideas

D1.1.1 Second-order, because the highest-order derivative appearing in the equation is second order.

D1.1.2 Linear, because the unknown function and its derivatives appear only to the first power.

D1.1.3 The equation is second-order, so we expect two arbitrary constants in the general solution.

D1.1.4 We have $y(0) = C + 10 = 5$, so $C = -5$. The solution is $y(t) = -5e^{-3t} + 10$.

D1.1.5 Yes. Note that $y'''(t) = 0$ and $y'(t) = 2$.

D1.1.6 No. $y(0) = 6 \neq 3$.

D1.1.7 Yes, it is a solution. Note that $y'(t) = -5Ce^{-5t}$, so $y'(t) + 5y(t) = 0$.

D1.1.8 Yes, it is a solution. $y'(t) = -3Ct^{-4}$, so $ty'(t) + 3y(t) = -3Ct^{-3} + 3Ct^{-3} = 0$.

D1.1.9 Yes, it is a solution. $y'(t) = 4C_1 \cos 4t - 4C_2 \sin 4t$, so $y''(t) = -16C_1 \sin 4t - 16C_2 \cos 4t$, so $y''(t) + 16y(t) = 0$.

D1.1.10 Yes, it is a solution. $y'(x) = -C_1e^{-x} + C_2e^x$, so $y''(x) = C_1e^{-x} + C_2e^x$, so $y''(x) - y(x) = 0$.

D1.1.11 Yes, it is a solution. $y'(t) = 32e^{2t}$, so $y'(t) - 2y(t) = 32e^{2t} - (32e^{2t} - 20) = 20$. Also, $y(0) = 16 - 10 = 6$.

D1.1.12 Yes, it is a solution. $y'(t) = 48t^5$, so $ty'(t) - 6y(t) = 48t^6 - 48t^6 + 18 = 18$. Also, $y(1) = 8 - 3 = 5$.

D1.1.13 Yes, it is a solution. $y'(t) = 9 \sin 3t$, so $y''(t) = 27 \cos 3t$. Thus, $y''(t) + 9y(t) = 27 \cos 3t - 27 \cos 3t = 0$. Also, $y'(0) = 0$ and $y(0) = -3$.

D1.1.14 Yes, it is a solution. $y'(x) = \frac{1}{4}(2e^{2x} + 2e^{-2x})$ and $y''(x) = \frac{1}{4}(4e^{2x} - 4e^{-2x})$. So $y''(x) - 4y(x) = 0$. Also, $y(0) = 0$ and $y'(0) = 1$.

D1.1.15 $y(t) = \int (3 + e^{-2t})\, dt = 3t - \frac{1}{2}e^{-2t} + C$.

D1.1.16 $y(t) = \int (12t^5 - 20t^4 + 2 - 6t^{-2})\, dt = 2t^6 - 4t^5 + 2t + \frac{6}{t} + C$.

D1.1.17 $y(x) = \int (4 \tan 2x - 3 \cos x)\, dx = -2 \ln |\cos 2x| - 3 \sin x + C = 2 \ln |\sec 2x| - 3 \sin x + C$.

D1.1.18 $p(x) = \int (16x^{-9} - 5 + 14x^6)\, dx = -2x^{-8} - 5x + 2x^7 + C$.

D1.1.19 $y'(t) = \int (60t^4 - 4 + 12t^{-3})\, dt = 12t^5 - 4t - 6t^{-2} + C$. $y(t) = \int (12t^5 - 4t - 6t^{-2} + C)\, dt = 2t^6 - 2t^2 + 6t^{-1} + C_1t + C_2$.

D1.1.20 $y'(t) = \int (15e^{3t} + \sin 4t)\, dt = 5e^{3t} - \frac{1}{4} \cos 4t + C_1$. $y(t) = \int (5e^{3t} - \frac{1}{4} \cos 4t + C_1)\, dt = \frac{5}{3}e^{3t} - \frac{1}{16} \sin 4t + C_1t + C_2$.

D1.1.21 $u'(x) = \int(55x^9 + 36x^7 - 21x^5 + 10x^{-3})\,dx = 5.5x^{10} + \frac{9}{2}x^8 - \frac{7}{2}x^6 - 5x^{-2} + C_1$.
$u(x) = \int(5.5x^{10} + \frac{9}{2}x^8 - \frac{7}{2}x^6 - 5x^{-2} + C)\,dx = \frac{1}{2}x^{11} + \frac{1}{2}x^9 - \frac{1}{2}x^7 + 5x^{-1} + C_1x + C_2$.

D1.1.22 $v'(x) = \int xe^x\,dx = xe^x - e^x + C_1$. $v(x) = \int(xe^x - e^x + C_1)\,dx = xe^x - e^x - e^x + C_1x + C_2 = xe^x - 2e^x + C_1x + C_2$.

D1.1.23 $y(t) = \int(1+e^t)\,dt = t + e^t + C$. Because $y(0) = 4 = 1 + C$, we have $C = 3$. Thus, $y(t) = t + e^t + 3$.

D1.1.24 $y(t) = \int(\sin t + \cos 2t)\,dt = -\cos t + \frac{1}{2}\sin 2t + C$. Because $y(0) = 4 = -1 + C$, we have $C = 5$. Thus, $y(t) = -\cos t + \frac{1}{2}\sin 2t + 5$.

D1.1.25 $y(x) = \int(3x^2 - 3x^{-4})\,dx = x^3 + x^{-3} + C$. Because $y(1) = 0 = 1 + 1 + C$, we have $C = -2$. So $y(x) = x^3 + x^{-3} - 2$.

D1.1.26 $y(x) = \int 4\sec^2 2x\,dx = 2\tan 2x + C$. Because $y(0) = 8 = 0 + C$, we have $C = 8$. Thus, $y(x) = 2\tan 2x + 8$.

D1.1.27 $y'(t) = \int(12t - 20t^3)\,dt = 6t^2 - 5t^4 + C_1$. Because $y'(0) = 0 = 0 + C_1$, we have $C_1 = 0$. $y(t) = \int(6t^2 - 5t^4)\,dt = 2t^3 - t^5 + C_2$. Because $y(0) = 1 = 0 - 0 + C_2$, we have $C_2 = 1$. Thus, $y(t) = 2t^3 - t^5 + 1$.

D1.1.28 $u'(x) = \int(4e^{2x} - 8e^{-2x})\,dx = 2e^{2x} + 4e^{-2x} + C_1$. Because $u'(0) = 3 = 2 + 4 + C_1$, we have $C_1 = -3$. $u(x) = \int(2e^{2x} + 4e^{-2x} - 3)\,dx = e^{2x} - 2e^{-2x} - 3x + C_2$. Because $u(0) = 1 = 1 - 2 - 0 + C_2$, we have $C_2 = 2$. Thus, $u(x) = e^{2x} - 2e^{-2x} - 3x + 2$.

D1.1.29

a. $v(t) = -9.8t + 29.4$. $s(t) = -4.9t^2 + 29.4t + 30$.

b. The object reaches its high point when $-9.8t + 29.4 = 0$, or $t = \frac{29.4}{9.8} = 3$. At that time its position is $s(3) \approx 74.1$ meters.

D1.1.30

a. $v(t) = -9.8t + 49$. $s(t) = -4.9t^2 + 49t + 60$.

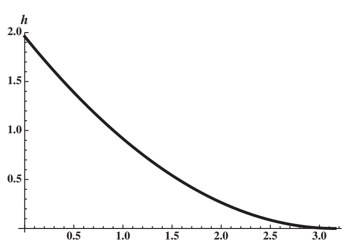
b. The object reaches its high point when $-9.8t + 49 = 0$, or $t = \frac{49}{9.8} = 5$. At that time its position is $s(5) = 182.5$ meters.

D1.1.31 We have $p(t) = (1500 - 20H)e^{.05t} + 20H$. The amount of resource is increasing when $1500 - 20H > 0$, which occurs for $H < 75$. The amount of resource is constant when $1500 - 20H = 0$, which occurs for $H = 75$. If $H = 100$, the resource is zero when $(1500 - 2000)e^{.05t} + 2000 = 0$, which occurs for $t = 20\ln 4 \approx 28$.

D1.1.32 We have $p(t) = (p_0 - 10000)e^{.05t} + 10000$. The amount of resource is decreasing when $p_0 - 10000 < 0$, or $p_0 < 10,000$. The amount of resource is constant when $p_0 = 10,000$. If $p_0 = 9000$, the resource vanishes when $-1000e^{.05t} = -10000$, or $t = 20\ln 10 \approx 46$.

D1.1.33

The height function is given by $h(t) = \left(\sqrt{1.96} - \frac{.3\sqrt{2\cdot 9.8}}{1.5}\cdot\frac{t}{2}\right)^2 \approx (1.4 - 0.44t)^2$. The tank is empty when $h(t) = 0$, which occurs after about 3.16 seconds.



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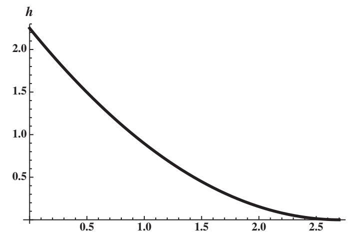
D1.1.34

The height function is given by $h(t) = \left(\sqrt{2.25} - \frac{.5\sqrt{2\cdot 9.8}}{2}\cdot\frac{t}{2}\right)^2 \approx (1.5 - .553t)^2$. The tank is empty when $h(t) = 0$, which occurs after about 2.71 seconds.



D1.1.35

a. False. That is a specific solution. The general solution is $t + C$.

b. False. It is second order, but is not linear.

c. True. First find the general solution, and then find the specific solution which satisfies the initial condition.

D1.1.36 $y(t) = \int (t\ln t + 1)\,dt = t + \int t\ln t\,dt$. Let $u = \ln t$ and $dv = t$, so that $du = \frac{dt}{t}$ and $v = t^2/2$. Then $y(t) = t + (t^2\ln t)/2 - \int t/2\,dt = t + (t^2\ln t)/2 - t^2/4 + C$.

D1.1.37 $u(x) = \int \frac{2x}{x^2+4}\,dx - \int \frac{2}{x^2+4}\,dx = \ln(x^2+4) - \tan^{-1}(x/2) + C$.

D1.1.38 Note that $\frac{4}{t^2-4} = \frac{1}{t-2} - \frac{1}{t+2}$. Thus, $v(t) = \int \frac{4}{t^2-4}\,dt = \int\left(\frac{1}{t-2} - \frac{1}{t+2}\right)\,dt = \ln\left|\frac{t-2}{t+2}\right| + C$.

D1.1.39 $y'(x) = \int \frac{x}{(1-x^2)^{3/2}}\,dx$. Let $u = 1 - x^2$, so that $du = -2x\,dx$. Substituting gives
$y'(x) = \frac{-1}{2}\int u^{-3/2}\,du = u^{-1/2} + C_1 = \frac{1}{\sqrt{1-x^2}} + C_1\,dx$. $y(x) = \int\left(\frac{1}{\sqrt{1-x^2}} + C_1\right)\,dx = \sin^{-1}(x) + C_1x + C_2$.

D1.1.40 Let $u = t$ and $dv = e^t\,dt$. Then $du = dt$ and $v = e^t$. Thus, $y(t) = \int te^t\,dt = te^t - \int e^t\,dt = te^t - e^t + C$. Because $y(0) = -1 = 0 - 1 + C$, we have $C = 0$. Thus $y(t) = te^t - e^t$.

D1.1.41 $u(x) = \int\left(\frac{1}{x^2+4^2} - 4\right)\,dx = \frac{1}{4}\tan^{-1}(x/4) - 4x + C$. Because $u(0) = 2 = 0 - 0 + C$, we have $C = 2$. Thus, $u(x) = \frac{1}{4}\tan^{-1}(x/4) - 4x + 2$.

D1.1.42 $p(x) = \int \frac{2}{x(x+1)}\,dx = \int\left(\frac{2}{x} - \frac{2}{x+1}\right)\,dx = 2\ln\left|\frac{x}{x+1}\right| + C$. Because $p(1) = 0 = 2\ln(1/2) + C$, we have $C = -2\ln(1/2) = 2\ln 2$. Thus, $p(x) = 2\ln\left|\frac{x}{x+1}\right| + 2\ln 2$.

D1.1.43 Using the result of number 40 above, we have $y'(t) = te^t - e^t + C_1$, and because $y'(0) = 1 = 0 - 1 + C_1$, we have $C_1 = 2$. Thus $y'(t) = te^t - e^t + 2$. $y(t) = \int y'(t)\,dt = \int(te^t - e^t + 2)\,dt = te^t - e^t - e^t + 2t + C_2 = te^t - 2e^t + 2t + C_2$. Because $y(0) = 0 = 0 - 2 + 0 + C_2$, we have $C_2 = 2$. Thus, $y(t) = te^t - 2e^t + 2t + 2$.

D1.1.44 $u'(t) = Ce^{1/(4t^4)}\frac{-4}{4}t^{-5} = \frac{-u(t)}{t^5}$. Thus $u'(t) + \frac{u(t)}{t^5} = \frac{-u(t)}{t^5} + \frac{u(t)}{t^5} = 0$.

D1.1.45 $u'(t) = C_1e^t + C_2e^t + C_2te^t$, and $u''(t) = C_1e^t + C_2e^t + C_2e^t + C_2te^t = C_1e^t + 2C_2e^t + C_2te^t$. Thus, $u''(t) - 2u'(t) + u(t) = (C_1e^t + 2C_2e^t + C_2te^t) - 2(C_1e^t + C_2e^t + C_2te^t) + C_1e^t + C_2te^t = 0$.

D1.1.46 $g'(x) = -2C_1e^{-2x} + C_2e^{-2x} + -2C_2xe^{-2x}$, so $g''(x) = 4C_1e^{-2x} - 2C_2e^{-2x} + -2C_2e^{-2x} + 4C_2xe^{-2x} = 4C_1e^{-2x} - 4C_2e^{-2x} + 4C_2xe^{-2x}$. Thus, $g''(x) + 4g'(x) + 4g(x) = 4C_1e^{-2x} - 4C_2e^{-2x} + 4C_2xe^{-2x} + 4(-2C_1e^{-2x} + C_2e^{-2x} + -2C_2xe^{-2x}) + 4(C_1e^{-2x} + C_2xe^{-2x} + 2) = 8$.

D1.1.47 $u'(t) = 2C_1t + 3C_2t^2$, so $u''(t) = 2C_1 + 6C_2t$. Thus,

$$t^2u''(t) - 4tu'(t) + 6u(t) = 2C_1t^2 + 6C_2t^3 - 4(2C_1t^2 + 3C_2t^3) + 6C_1t^2 + 6C_2t^3 = 0.$$

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D1.1.48 $u'(t) = 5C_1t^4 - 4C_2t^{-5} - 3t^2$, so $u''(t) = 20C_1t^3 + 20C_2t^{-6} - 6t$. Thus,

$$t^2u''(t) - 20u(t) = 20C_1t^5 + 20C_2t^{-4} - 6t^3 - 20\left(C_1t^5 + C_2t^{-4} - t^3\right) = 14t^3.$$

D1.1.49 $z'(t) = -C_1e^{-t} + 2C_2e^{2t} - 3C_3e^{-3t} - e^t$. So $z''(t) = C_1e^{-t} + 4C_2e^{2t} + 9C_3e^{-3t} - e^t$, and $z'''(t) = -C_1e^{-t} + 8C_2e^{2t} - 27C_3e^{-3t} - e^t$. Thus

$$\begin{aligned} z'''(t) + 2z''(t) - 5z'(t) - 6z(t) &= -C_1e^{-t} + 8C_2e^{2t} - 27C_3e^{-3t} - e^t \\ &\quad + 2C_1e^{-t} + 8C_2e^{2t} + 18C_3e^{-3t} - 2e^t \\ &\quad + 5C_1e^{-t} - 10C_2e^{2t} + 15C_3e^{-3t} + 5e^t \\ &\quad - 6C_1e^{-t} - 6C_2e^{2t} - 6C_3e^{-3t} + 6e^t \\ &= 8e^t \end{aligned}$$

D1.1.50

a. $y'(t) = C_1e^t - C_2e^{-t}$, so $y''(t) = C_1e^t + C_2e^{-t}$. Thus, $y''(t) - y(t) = 0$.

b. $y'(t) = 2C_1e^{2t} - 2C_2e^{-2t}$, so $y''(t) = 4C_2e^{2t} + 4C_2e^{-2t}$. Thus, $y''(t) - 4y(t) = 0$.

c. It appears that a general solution should be $C_1e^{kt} + C_2e^{-kt}$. Then $y'(t) = kC_1e^{kt} - kC_2e^{-kt}$, and $y''(t) = k^2C_1e^{kt} + k^2C_2e^{-kt}$. Thus, $y''(t) - k^2y(t) = 0$.

d. If $y(t) = C_1\cosh kt + C_2\sinh kt$, then $y'(t) = kC_1\sinh kt + kC_2\cosh kt$ and $y''(t) = k^2C_1\cosh kt + k^2C_2\sinh kt$. Thus $y''(t) - k^2y(t) = 0$.

D1.1.51

a. $y'(t) = C_1\cos t - C_2\sin t$, so $y''(t) = -C_1\sin t - C_2\cos t$. Thus, $y''(t) + y(t) = 0$.

b. $y'(t) = 2C_2\cos 2t - 2C_2\sin 2t$, so $y''(t) = -4C_2\sin 2t - 4C_2\cos 2t$. Thus, $y''(t) + 4y(t) = 0$.

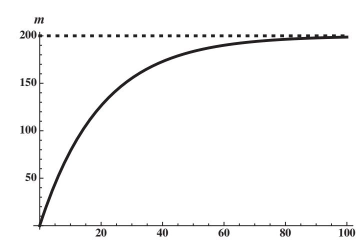
c. A general solution appears to be $y(t) = C_1\sin kt + C_2\cos kt$. Then $y'(t) = kC_1\cos kt - kC_2\sin kt$, so $y''(t) = -k^2C_1\sin kt - k^2C_2\cos kt$. And then $y''(t) + k^2y(t) = 0$.

D1.1.52

a. Let $m(t) = \frac{I}{k}(1 - e^{-kt})$. Note that $m(0) = 0$. Then $m'(t) = \frac{I}{k}(ke^{-kt})$. Therefore,

$$m'(t) + km(t) = \frac{I}{k}(ke^{-kt}) + \frac{kI}{k}(1 - e^{-kt}) = Ie^{-kt} + I - Ie^{-kt} = I.$$

b. We have $m(t) = 200(1 - e^{-.05t})$.



c. It appears that $\lim_{t\to\infty} m(t) = 200$.

D1.1.53

a. Let $p(t) = \frac{K}{1+Ce^{-rt}}$. Note that $1 - \frac{P}{K} = 1 - \frac{1}{1+Ce^{-rt}} = \frac{Ce^{-rt}}{1+Ce^{-rt}}$. We have

$$p'(t) = \frac{KCre^{-rt}}{(1+Ce^{-rt})^2} = r \cdot \frac{K}{1+Ce^{-rt}} \cdot \frac{Ce^{-rt}}{1+Ce^{-rt}} = rp\left(1 - \frac{p}{K}\right).$$

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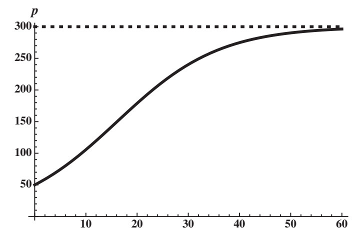
b. If $p(0) = 50 = \frac{K}{1+C}$, then $50 + 50C = K$, so $C = \frac{K-50}{50}$.

c. We have $p(t) = \frac{300}{1+5e^{-.1t}}$.



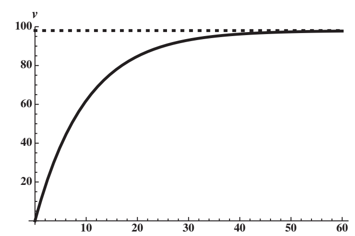
d. $\lim_{t\to\infty} \frac{300}{1+5e^{-.1t}} = \frac{300}{1+0} = 300$, which is consistent with the graph from part c.

D1.1.54

a. Let $v(t) = \frac{g}{b}(1 - e^{-bt})$. Then $v(0) = 0$, and

$$v'(t) = \frac{g}{b} \cdot be^{-bt} = ge^{-bt} = g - b \cdot \frac{g}{b}(1 - e^{-bt}) = g - bv.$$

b. With $b = 0.1$, we have $v(t) = 98(1 - e^{-.1t})$.



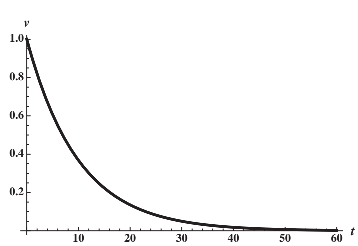
c. $\lim_{t\to\infty} v(t) = 98$.

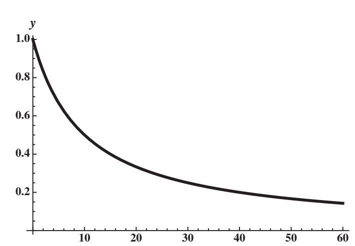
D1.1.55

a. If $y(t) = y_0e^{-kt}$, then $y(0) = y_0$, and $y'(t) = -ky_0e^{-kt}$, so $y'(t) = -ky(t)$.

b. Let $y(t) = \frac{y_0}{y_0kt+1}$. Then $y(0) = y_0$, and $y'(t) = \frac{-y_0^2k}{(y_0kt+1)^2} = -k(y(t))^2$.

c. The first order reaction decays more quickly.





D1.1.56

a. Let $M(t) = K\left(\frac{M_0}{K}\right)^{e^{-rt}}$. Note that $\ln(M(t)/K) = e^{-rt}\ln(M_0/K)$.

$M'(t) = K\left(\frac{M_0}{K}\right)^{e^{-rt}} \ln(M_0/K)\,(-re^{-rt}) = -rM(t)\ln(M(t)/K)$. Also, $M(0) = K(M_0/K) = M_0$.

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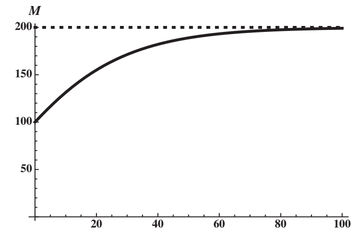
b. Using $K = 200$, $M_0 = 100$, and $r = .05$, we have $M(t) = K\left(\frac{M_0}{K}\right)^{e^{-rt}} = 200(1/2)^{e^{-.05t}}$.



c. $\lim_{t\to\infty} M(t) = 200 = K$.

D1.2 Direction Fields and Euler's Method

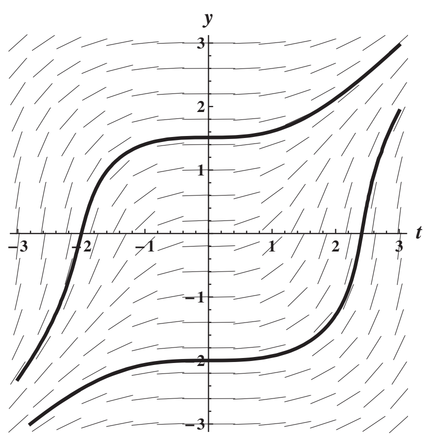
D1.2.1 Choose a regular grid of points in the ty-plane, and for each point P, make a small line segment with slope $f(t, y)$.

D1.2.2 It will have slope $3^2 - 3(1)^2 = 6$.

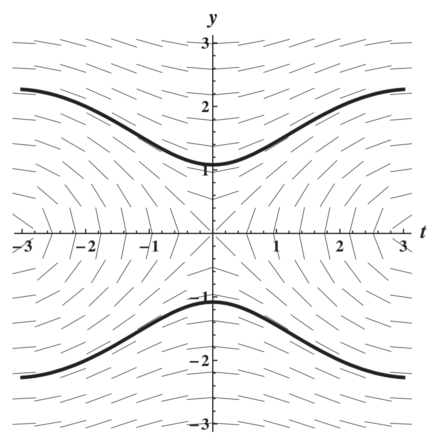
D1.2.3 $u_0 = y(3) = 1$. $u_1 = u_0 + f(3, 1)(.1) = 1 + .6 = 1.6$.

D1.2.4 Because the differential equation is giving the slope at a given point, we can approximate the solution to the differential equation by starting at the initial point, and using the slope to guide where the next iteration should be. In essence, we are numerically "following the direction field" to estimate the solution to the differential equation.

D1.2.5



D1.2.6



D1.2.7

a. This matches with D.

b. This matches with B.

c. This matches with A.

d. This matches with C.

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