

# STUDENT'S SOLUTIONS MANUAL

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AND

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# Differential Equations

## D1.1 Basic Ideas

**D1.1.1** Second-order, because the highest-order derivative appearing in the equation is second order.

**D1.1.3** The equation is second-order, so we expect two arbitrary constants in the general solution.

**D1.1.5** Yes. Note that  $y'''(t) = 0$  and  $y'(t) = 2$ .

**D1.1.7** Yes, it is a solution. Note that  $y'(t) = -5Ce^{-5t}$ , so  $y'(t) + 5y(t) = 0$ .

**D1.1.9** Yes, it is a solution.  $y'(t) = 4C_1 \cos 4t - 4C_2 \sin 4t$ , so  $y''(t) = -16C_1 \sin 4t - 16C_2 \cos 4t$ , so  $y''(t) + 16y(t) = 0$ .

**D1.1.11** Yes, it is a solution.  $y'(t) = 32e^{2t}$ , so  $y'(t) - 2y(t) = 32e^{2t} - (32e^{2t} - 20) = 20$ . Also,  $y(0) = 16 - 10 = 6$ .

**D1.1.13** Yes, it is a solution.  $y'(t) = 9 \sin 3t$ , so  $y''(t) = 27 \cos 3t$ . Thus,  $y''(t) + 9y(t) = 27 \cos 3t - 27 \cos 3t = 0$ . Also,  $y'(0) = 0$  and  $y(0) = -3$ .

**D1.1.15**  $y(t) = \int (3 + e^{-2t}) dt = 3t - \frac{1}{2}e^{-2t} + C$ .

**D1.1.17**  $y(x) = \int (4 \tan 2x - 3 \cos x) dx = -2 \ln |\cos 2x| - 3 \sin x + C = 2 \ln |\sec 2x| - 3 \sin x + C$ .

**D1.1.19**  $y'(t) = \int (60t^4 - 4 + 12t^{-3}) dt = 12t^5 - 4t - 6t^{-2} + C$ .  $y(t) = \int (12t^5 - 4t - 6t^{-2} + C) dt = 2t^6 - 2t^2 + 6t^{-1} + C_1t + C_2$ .

**D1.1.21**  $u'(x) = \int (55x^9 + 36x^7 - 21x^5 + 10x^{-3}) dx = 5.5x^{10} + \frac{9}{2}x^8 - \frac{7}{2}x^6 - 5x^{-2} + C_1$ .  
 $u(x) = \int (5.5x^{10} + \frac{9}{2}x^8 - \frac{7}{2}x^6 - 5x^{-2} + C) dx = \frac{1}{2}x^{11} + \frac{1}{2}x^9 - \frac{1}{2}x^7 + 5x^{-1} + C_1x + C_2$ .

**D1.1.23**  $y(t) = \int (1 + e^t) dt = t + e^t + C$ . Because  $y(0) = 4 = 1 + C$ , we have  $C = 3$ . Thus,  $y(t) = t + e^t + 3$ .

**D1.1.25**  $y(x) = \int (3x^2 - 3x^{-4}) dx = x^3 + x^{-3} + C$ . Because  $y(1) = 0 = 1 + 1 + C$ , we have  $C = -2$ . So  $y(x) = x^3 + x^{-3} - 2$ .

**D1.1.27**  $y'(t) = \int (12t - 20t^3) dt = 6t^2 - 5t^4 + C_1$ . Because  $y'(0) = 0 = 0 + C_1$ , we have  $C_1 = 0$ .  
 $y(t) = \int (6t^2 - 5t^4) dt = 2t^3 - t^5 + C_2$ . Because  $y(0) = 1 = 0 - 0 + C_2$ , we have  $C_2 = 1$ . Thus,  $y(t) = 2t^3 - t^5 + 1$ .

**D1.1.29**

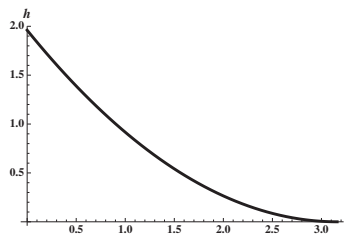
a.  $v(t) = -9.8t + 29.4$ .  $s(t) = -4.9t^2 + 29.4t + 30$ .

b. The object reaches its high point when  $-9.8t + 29.4 = 0$ , or  $t = \frac{29.4}{9.8} = 3$ . At that time its position is  $s(3) \approx 74.1$  meters.

**D1.1.31** We have  $p(t) = (1500 - 20H)e^{0.05t} + 20H$ . The amount of resource is increasing when  $1500 - 20H > 0$ , which occurs for  $H < 75$ . The amount of resource is constant when  $1500 - 20H = 0$ , which occurs for  $H = 75$ . If  $H = 100$ , the resource is zero when  $(1500 - 2000)e^{0.05t} + 2000 = 0$ , which occurs for  $t = 20 \ln 4 \approx 28$ .

**D1.1.33**

The height function is given by  $h(t) = \left(\sqrt{1.96} - \frac{.3\sqrt{2 \cdot 9.8}}{1.5} \cdot \frac{t}{2}\right)^2 \approx (1.4 - 0.44t)^2$ . The tank is empty when  $h(t) = 0$ , which occurs after about 3.16 seconds.

**D1.1.35**

- False. That is a specific solution. The general solution is  $t + C$ .
- False. It is second order, but is not linear.
- True. First find the general solution, and then find the specific solution which satisfies the initial condition.

**D1.1.37**  $u(x) = \int \frac{2x}{x^2+4} dx - \int \frac{2}{x^2+4} dx = \ln(x^2+4) - \tan^{-1}(x/2) + C$ .

**D1.1.39**  $y'(x) = \int \frac{x}{(1-x^2)^{3/2}} dx$ . Let  $u = 1 - x^2$ , so that  $du = -2x dx$ . Substituting gives  $y'(x) = \frac{-1}{2} \int u^{-3/2} du = u^{-1/2} + C_1 = \frac{1}{\sqrt{1-x^2}} + C_1 dx$ .  $y(x) = \int \left(\frac{1}{\sqrt{1-x^2}} + C_1\right) dx = \sin^{-1}(x) + C_1x + C_2$ .

**D1.1.41**  $u(x) = \int \left(\frac{1}{x^2+4^2} - 4\right) dx = \frac{1}{4} \tan^{-1}(x/4) - 4x + C$ . Because  $u(0) = 2 = 0 - 0 + C$ , we have  $C = 2$ . Thus,  $u(x) = \frac{1}{4} \tan^{-1}(x/4) - 4x + 2$ .

**D1.1.43** Using the result of number 40 above, we have  $y'(t) = te^t - e^t + C_1$ , and because  $y'(0) = 1 = 0 - 1 + C_1$ , we have  $C_1 = 2$ . Thus  $y'(t) = te^t - e^t + 2$ .  $y(t) = \int y'(t) dt = \int (te^t - e^t + 2) dt = te^t - e^t - e^t + 2t + C_2 = te^t - 2e^t + 2t + C_2$ . Because  $y(0) = 0 = 0 - 2 + 0 + C_2$ , we have  $C_2 = 2$ . Thus,  $y(t) = te^t - 2e^t + 2t + 2$ .

**D1.1.45**  $u'(t) = C_1e^t + C_2e^t + C_2te^t$ , and  $u''(t) = C_1e^t + C_2e^t + C_2e^t + C_2te^t = C_1e^t + 2C_2e^t + C_2te^t$ . Thus,  $u''(t) - 2u'(t) + u(t) = (C_1e^t + 2C_2e^t + C_2te^t) - 2(C_1e^t + C_2e^t + C_2te^t) + C_1e^t + C_2te^t = 0$ .

**D1.1.47**  $u'(t) = 2C_1t + 3C_2t^2$ , so  $u''(t) = 2C_1 + 6C_2t$ . Thus,

$$t^2u''(t) - 4tu'(t) + 6u(t) = 2C_1t^2 + 6C_2t^3 - 4(2C_1t^2 + 3C_2t^3) + 6C_1t^2 + 6C_2t^3 = 0.$$

**D1.1.49**  $z'(t) = -C_1e^{-t} + 2C_2e^{2t} - 3C_3e^{-3t} - e^t$ . So  $z''(t) = C_1e^{-t} + 4C_2e^{2t} + 9C_3e^{-3t} - e^t$ , and  $z'''(t) = -C_1e^{-t} + 8C_2e^{2t} - 27C_3e^{-3t} - e^t$ . Thus

$$\begin{aligned} z'''(t) + 2z''(t) - 5z'(t) - 6z(t) &= -C_1e^{-t} + 8C_2e^{2t} - 27C_3e^{-3t} - e^t \\ &\quad + 2C_1e^{-t} + 8C_2e^{2t} + 18C_3e^{-3t} - 2e^t \\ &\quad + 5C_1e^{-t} - 10C_2e^{2t} + 15C_3e^{-3t} + 5e^t \\ &\quad - 6C_1e^{-t} - 6C_2e^{2t} - 6C_3e^{-3t} + 6e^t \\ &= 8e^t \end{aligned}$$

**D1.1.51**

- $y'(t) = C_1 \cos t - C_2 \sin t$ , so  $y''(t) = -C_1 \sin t - C_2 \cos t$ . Thus,  $y''(t) + y(t) = 0$ .
- $y'(t) = 2C_2 \cos 2t - 2C_2 \sin 2t$ , so  $y''(t) = -4C_2 \sin 2t - 4C_2 \cos 2t$ . Thus,  $y''(t) + 4y(t) = 0$ .
- A general solution appears to be  $y(t) = C_1 \sin kt + C_2 \cos kt$ . Then  $y'(t) = kC_1 \cos kt - kC_2 \sin kt$ , so  $y''(t) = -k^2C_1 \sin kt - k^2C_2 \cos kt$ . And then  $y''(t) + k^2y(t) = 0$ .

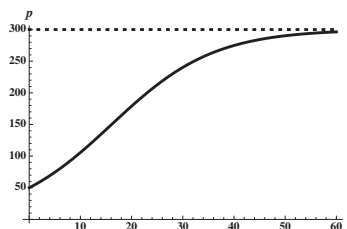
**D1.1.53**

a. Let  $p(t) = \frac{K}{1+Ce^{-rt}}$ . Note that  $1 - \frac{P}{K} = 1 - \frac{1}{1+Ce^{-rt}} = \frac{Ce^{-rt}}{1+Ce^{-rt}}$ . We have

$$p'(t) = \frac{KCre^{-rt}}{(1+Ce^{-rt})^2} = r \cdot \frac{K}{1+Ce^{-rt}} \cdot \frac{Ce^{-rt}}{1+Ce^{-rt}} = rp \left(1 - \frac{p}{K}\right).$$

b. If  $p(0) = 50 = \frac{K}{1+C}$ , then  $50 + 50C = K$ , so  $C = \frac{K-50}{50}$ .

c. We have  $p(t) = \frac{300}{1+5e^{-.1t}}$ .



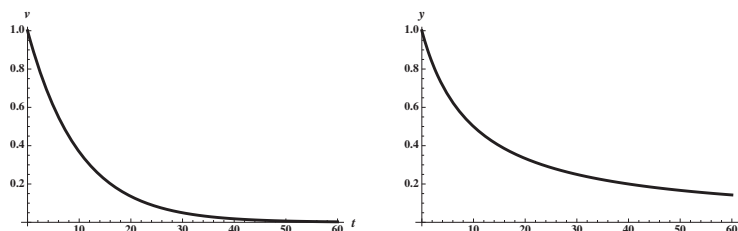
d.  $\lim_{t \rightarrow \infty} \frac{300}{1+5e^{-.1t}} = \frac{300}{1+0} = 300$ , which is consistent with the graph from part c.

**D1.1.55**

a. If  $y(t) = y_0e^{-kt}$ , then  $y(0) = y_0$ , and  $y'(t) = -ky_0e^{-kt}$ , so  $y'(t) = -ky(t)$ .

b. Let  $y(t) = \frac{y_0}{y_0kt+1}$ . Then  $y(0) = y_0$ , and  $y'(t) = \frac{-y_0^2k}{(y_0kt+1)^2} = -k(y(t))^2$ .

c. The first order reaction decays more quickly.

**D1.2 Direction Fields and Euler's Method**

**D1.2.1** Choose a regular grid of points in the  $ty$ -plane, and for each point  $P$ , make a small line segment with slope  $f(t, y)$ .

**D1.2.3**  $u_0 = y(3) = 1$ .  $u_1 = u_0 + f(3, 1)(.1) = 1 + .6 = 1.6$ .