Complete Solutions Manual to Accompany

Calculus: An Applied Approach

TENTH EDITION

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Functions, Graphs, and Limits

Section 1.1 The Cartesian Plane and the Distance Formula

Skills Warm Up

1. $\sqrt{(3-6)^2+[1-(-5)]^2} = \sqrt{(-3)^2+6^2}$
$= \sqrt{9+36}$
$= \sqrt{45}$
$= 3\sqrt{5}$

2. $\sqrt{(-2-0)^2+[-7-(-3)]^2} = \sqrt{(-2)^2+(-4)^2}$
$= \sqrt{4+16}$
$= \sqrt{20}$
$= 2\sqrt{5}$

3. $\dfrac{5+(-4)}{2} = \dfrac{1}{2}$

4. $\dfrac{-3+(-1)}{2} = \dfrac{-4}{2} = -2$

5. $\sqrt{27}+\sqrt{12} = 3\sqrt{3}+2\sqrt{3} = 5\sqrt{3}$

6. $\sqrt{8}-\sqrt{18} = 2\sqrt{2}-3\sqrt{2} = -\sqrt{2}$

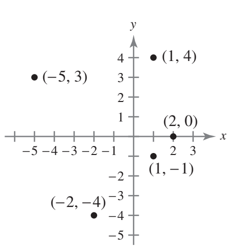
7. $\dfrac{x+(-5)}{2} = 7$
$x+(-5) = 14$
$x = 19$

8. $\dfrac{-7+y}{2} = -3$
$-7+y = -6$
$y = 1$

9. $\sqrt{(3-x)^2+(7-4)^2} = \sqrt{45}$
$\left(\sqrt{(3-x)^2+(7-4)^2}\right)^2 = \left(\sqrt{45}\right)^2$
$(3-x)^2+(7-4)^2 = 45$
$(3-x)^2+3^2 = 45$
$(3-x)^2+9 = 45$
$(3-x)^2 = 36$
$3-x = \pm 6$
$-x = -3 \pm 6$
$x = 3 \mp 6$
$x = -3, 9$

10. $\sqrt{(6-2)^2+(-2-y)^2} = \sqrt{52}$
$\left(\sqrt{(6-2)^2+(-2-y)^2}\right)^2 = \left(\sqrt{52}\right)^2$
$(6-2)^2+(-2-y)^2 = 52$
$4^2+(-2-y)^2 = 52$
$16+(-2-y)^2 = 52$
$(-2-y)^2 = 36$
$-2-y = \pm 6$
$-y = \pm 6 + 2$
$y = \mp 6 - 2$
$y = -8, 4$

1.



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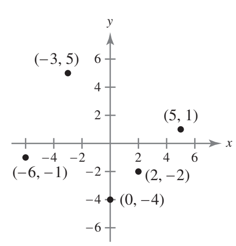
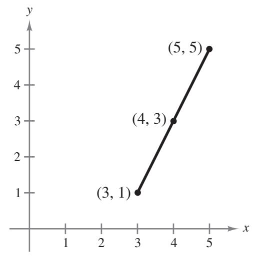


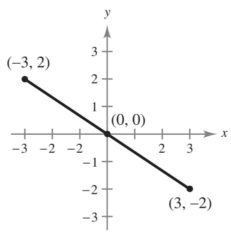
3. (a)



(b) $d = \sqrt{(5-3)^2 + (5-1)^2} = \sqrt{4+16} = 2\sqrt{5}$

(c) Midpoint $= \left(\dfrac{3+5}{2}, \dfrac{1+5}{2}\right) = (4, 3)$

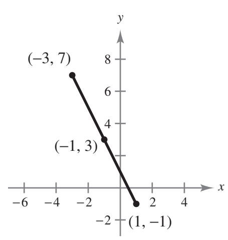
4. (a)



(b) $d = \sqrt{(-3-3)^2 + (2+2)^2} = \sqrt{36+16} = 2\sqrt{13}$

(c) Midpoint $= \left(\dfrac{-3+3}{2}, \dfrac{2+(-2)}{2}\right) = (0, 0)$

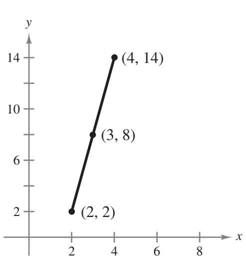
5. (a)



(b) $d = \sqrt{(-3-1)^2 + (7+1)^2} = \sqrt{16+64} = 4\sqrt{5}$

(c) Midpoint $= \left(\dfrac{-3+1}{2}, \dfrac{7-1}{2}\right) = (-1, 3)$

6. (a)



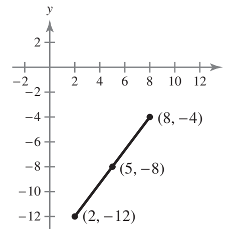
(b) $d = \sqrt{(4-2)^2 + (14-2)^2}$

$= \sqrt{4+144}$

$= 2\sqrt{37}$

(c) Midpoint $= \left(\dfrac{2+4}{2}, \dfrac{2+14}{2}\right) = (3, 8)$

7. (a)



(b) $d = \sqrt{(8-2)^2 + \left(-4-(-12)^2\right)}$

$= \sqrt{6^2 + 8^2}$

$= \sqrt{36+64}$

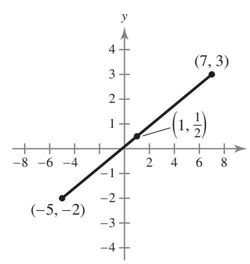
$= \sqrt{100} = 10$

(c) Midpoint $= \left(\dfrac{2+8}{2}, \dfrac{(-12)+(-4)}{2}\right)$

$= \left(\dfrac{10}{2}, \dfrac{-16}{2}\right)$

$= (5, -8)$

8. (a)



(b) $d = \sqrt{\left(7-(-5)\right)^2 + \left(3-(-2)^2\right)}$

$= \sqrt{12^2 + 5^2}$

$= \sqrt{144+25}$

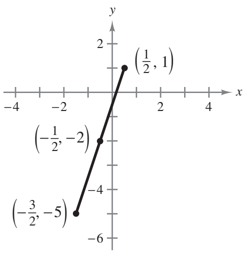
$= \sqrt{169} = 13$

(c) Midpoint $= \left(\dfrac{7+(-5)}{2}, \dfrac{3+(-2)}{2}\right)$

$= \left(\dfrac{2}{2}, \dfrac{1}{2}\right)$

$= \left(1, \dfrac{1}{2}\right)$

9. (a)



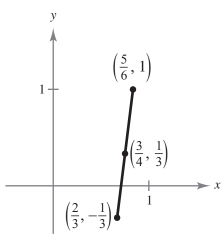
(b) $d = \sqrt{\left[(3/2) - (1/2)\right]^2 + (5 - 1)^2}$

$= \sqrt{4 + 36}$

$= 2\sqrt{10}$

(c) $\text{Midpoint} = \left(\dfrac{(1/2) + (-3/2)}{2}, \dfrac{1 + (-5)}{2}\right) = \left(-\dfrac{1}{2}, -2\right)$

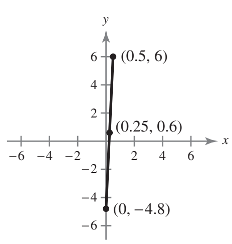
10. (a)



(b) $d = \sqrt{\left(\dfrac{5}{6} - \dfrac{2}{3}\right)^2 + \left(1 + \dfrac{1}{3}\right)^2} = \sqrt{\dfrac{1}{36} + \dfrac{16}{9}} = \dfrac{\sqrt{65}}{6}$

(c) $\text{Midpoint} = \left(\dfrac{(5/6) + (2/3)}{2}, \dfrac{1 - (1/3)}{2}\right) = \left(\dfrac{3}{4}, \dfrac{1}{3}\right)$

11. (a)



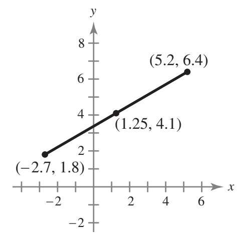
(b) $d = \sqrt{(0.5 - 0)^2 + (6 - (-4.8))^2}$

$= \sqrt{0.25 + 116.64}$

$= \sqrt{116.89}$

(c) $\text{Midpoint} = \left(\dfrac{0 + 0.5}{2}, \dfrac{-4.8 + 6}{2}\right) = (0.25, 0.6)$

12. (a)



(b) $d = \sqrt{(-2.7 - 5.2)^2 + (1.8 - 6.4)^2}$

$= \sqrt{62.41 + 21.16}$

$= \sqrt{83.57}$

(c) $\text{Midpoint} = \left(\dfrac{5.2 + (-2.7)}{2}, \dfrac{6.4 + 1.8}{2}\right)$

$= (1.25, 4.1)$

13. (a) $a = 4$

$b = 3$

$c = \sqrt{(4 - 0)^2 + (3 - 0)^2} = \sqrt{16 + 9} = 5$

(b) $a^2 + b^2 = 16 + 9 = 25 = c^2$

14. (a) $a = \sqrt{(13 - 1)^2 + (1 - 1)^2} = \sqrt{144 + 0} = 12$

$b = \sqrt{(13 - 13)^2 + (6 - 1)^2} = \sqrt{0 + 25} = 5$

$c = \sqrt{(13 - 1)^2 + (6 - 1)^2} = \sqrt{144 + 25} = 13$

(b) $a^2 + b^2 = 144 + 25 = 169 = c^2$

15. (a) $a = 10$

$b = 3$

$c = \sqrt{(7 + 3)^2 + (4 - 1)^2} = \sqrt{100 + 9} = \sqrt{109}$

(b) $a^2 + b^2 = 100 + 9 = 109 = c^2$

16. (a) $a = \sqrt{(6 - 2)^2 + (-2 + 2)^2} = \sqrt{16 + 0} = 4$

$b = \sqrt{(2 - 2)^2 + (5 + 2)^2} = \sqrt{0 + 49} = 7$

$c = \sqrt{(2 - 6)^2 + (5 + 2)^2} = \sqrt{16 + 49} = \sqrt{65}$

(b) $a^2 + b^2 = 16 + 49 = 65 = c^2$

17. $d_1 = \sqrt{(3 - 0)^2 + (7 - 1)^2}$

$= \sqrt{9 + 36}$

$= \sqrt{45}$

$= 3\sqrt{5}$

$d_2 = \sqrt{(4 - 0)^2 + (-1 - 1)^2}$

$= \sqrt{16 + 4}$

$= \sqrt{20}$

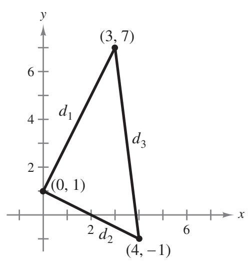
$= 2\sqrt{5}$

$d_3 = \sqrt{(3 - 4)^2 + [7 - (-1)]^2}$

$= \sqrt{1 + 64}$

$= \sqrt{65}$

Because $d_1^2 + d_2^2 = d_3^2$, the figure is a right triangle.



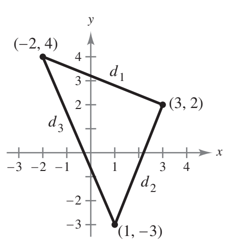
18. $a = \sqrt{(-2 - 3)^2 + (4 - 2)^2} = \sqrt{25 + 4} = \sqrt{29}$

$b = \sqrt{(3 - 1)^2 + (2 + 3)^2} = \sqrt{4 + 25} = \sqrt{29}$

$c = \sqrt{(-2 - 1)^2 + (4 + 3)^2} = \sqrt{9 + 49} = \sqrt{58}$

Because $a = b$ the figure is an isosceles triangle.

[**Note:** It is also a right triangle since $a^2 + b^2 = c^2$.]



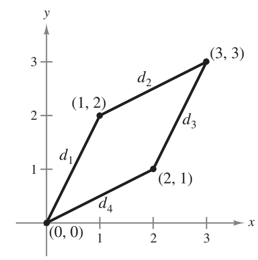
19. $d_1 = \sqrt{(1 - 0)^2 + (2 - 0)^2} = \sqrt{1 + 4} = \sqrt{5}$

$d_2 = \sqrt{(3 - 1)^2 + (3 - 2)^2} = \sqrt{4 + 1} = \sqrt{5}$

$d_3 = \sqrt{(2 - 3)^2 + (1 - 3)^2} = \sqrt{1 + 4} = \sqrt{5}$

$d_4 = \sqrt{(0 - 2)^2 + (0 - 1)^2} = \sqrt{4 + 1} = \sqrt{5}$

Because $d_1 = d_2 = d_3 = d_4$, the figure is a parallelogram.



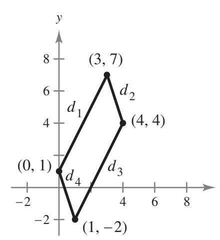
20. $a = \sqrt{(3 - 0)^2 + (7 - 1)^2} = \sqrt{9 + 36} = 3\sqrt{5}$

$b = \sqrt{(3 - 4)^2 + (7 - 4)^2} = \sqrt{1 + 9} = \sqrt{10}$

$c = \sqrt{(4 - 1)^2 + (4 + 2)^2} = \sqrt{9 + 36} = 3\sqrt{5}$

$d = \sqrt{(1 - 0)^2 + (-2 - 1)^2} = \sqrt{1 + 9} = \sqrt{10}$

Because $a = c$ and $b = d$, the figure is a parallelogram.



21. $d = \sqrt{(x - 1)^2 + (-4 - 0)^2} = 5$

$\sqrt{x^2 - 2x + 17} = 5$

$x^2 - 2x + 17 = 25$

$x^2 - 2x - 8 = 0$

$(x - 4)(x + 2) = 0$

$x = 4, -2$

22. $d = \sqrt{(x - 2)^2 + (2 + 1)^2} = 5$

$\sqrt{x^2 - 4x + 13} = 5$

$x^2 - 4x + 13 = 25$

$x^2 - 4x - 12 = 0$

$(x + 2)(x - 6) = 0$

$x = -2, 6$

23. $d = \sqrt{(-3-(-5))^2 + (y-0)^2} = 8$

$$\sqrt{4+y^2} = 8$$
$$4+y^2 = 64$$
$$y^2 = 60$$
$$y = \pm\sqrt{60}$$
$$y = \pm 2\sqrt{15}$$

24. $d = \sqrt{(4-4)^2 + (y-(-6))^2} = 8$

$$\sqrt{(y+6)^2} = 8$$
$$(y+6)^2 = 64$$
$$y+6 = \pm 8$$
$$y = -6 \pm 8$$
$$y = -14,\ 2$$

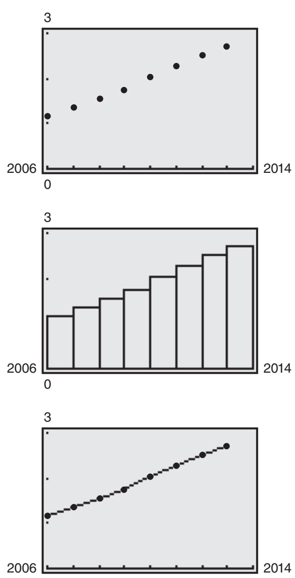
25. $d = \sqrt{(50-12)^2 + (42-18)^2}$

$$= \sqrt{38^2 + 24^2}$$
$$= \sqrt{2020}$$
$$= 2\sqrt{505} \approx 44.9 \text{ yd}$$

26. $d = \sqrt{(33-12)^2 + (37-18)^2}$

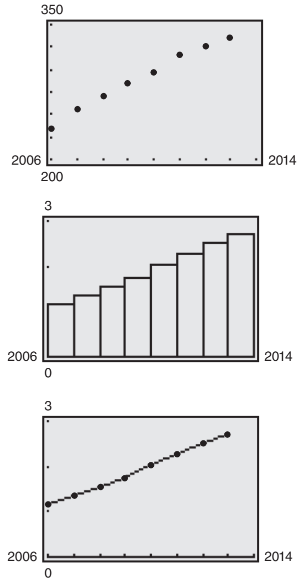
$$= \sqrt{21^2 + 19^2}$$
$$= \sqrt{441+361}$$
$$= \sqrt{802} \approx 28.3 \text{ yd}$$

27.



The numbers of individuals using the Internet increased each year from 2006 through 2013.

28.



The numbers of cellular telephone subscribers increased each year from 2006 through 2013.

29. (a) March 2013: 14,500

July 2013: 15,500

July 2014: 16,500

(b) December 2013: 16,600

January 2014: 15,750

Decrease: $|16{,}600 - 15{,}750| = 850$

Percent decrease: $\dfrac{850}{16{,}600} \approx 0.051 = 5.1\%$

30. (a) 2007: \$218,000

2009: \$172,000

2012: \$178,000

(b) 2011: \$168,000

2012: \$178,000

Increase: $178{,}000 - 168{,}000 = 10{,}000$

Percent increase: $\dfrac{10{,}000}{168{,}000} \approx 0.0595$

$$\approx 6.0\%$$

31. (a) Revenue $= \left(\dfrac{2011 + 2013}{2}, \dfrac{784.5 + 1266.7}{2}\right)$

$= (2012, 1025.6)$

Revenue estimate for 2012: \$1025.6 million

Profit $= \left(\dfrac{2011 + 2013}{2}, \dfrac{50.4 + 71.6}{2}\right)$

$= (2012, 61.0)$

Profit estimate for 2012: \$61.0 million

(b) Actual 2012 revenue: \$1040.5 million

Actual 2012 profit: \$57.3 million

(c) Yes, the revenue and profit increased in a linear pattern from 2011 to 2013.

(d) 2011 expense: $784.5 - 50.4 = \$734.1$ million

2012 expense: $1040.5 - 57.3 = \$983.2$ million

2013 expense: $1266.7 - 71.6 = \$1195.1$ million

(e) Answers will vary.

32. (a) Revenue $= \left(\dfrac{2011 + 2013}{2}, \dfrac{40.9 + 45.0}{2}\right)$

$= (2012, 42.95)$

Revenue estimate for 2012: \$42.95 billion

Profit $= \left(\dfrac{2011 + 2013}{2}, \dfrac{4.8 + 6.1}{2}\right)$

$= (2012, 5.45)$

Profit estimate for 2012: \$5.45 billion

(b) Actual 2012 revenue: \$42.3 billion

Actual 2012 profit: \$5.7 billion

(c) Yes, the revenue and profit increased in a linear pattern from 2011 to 2013.

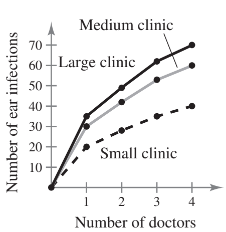
(d) 2011 expense: $40.9 - 4.8 = \$36.1$ billion

2012 expense: $42.3 - 5.7 = \$36.6$ billion

2013 expense: $45.0 - 6.1 = \$38.9$ billion

(e) Answers will vary.

33. (a)



(b) The larger the clinic, the more patients a doctor can treat.

34. (a) 500 pickups were sold in 2011.

(b) About 400 pickups were sold in 2013.

(c) The number of pickups sold each year is decreasing.

35. The vertex $(-3, -1)$ is translated to $(-6, -6)$.

The vertex $(0, 0)$ is translated to $(-3, -5)$.

The vertex $(-1, -2)$ is translated to $(-4, -7)$.

36. The vertex $(0, 2)$ is translated to $(2, 6)$.

The vertex $(1, 3)$ is translated to $(3, 7)$.

The vertex $(3, 1)$ is translated to $(5, 5)$.

The vertex $(2, 0)$ is translated to $(4, 4)$.

37. Midpoint $= \left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)$

The point one-fourth of the way between (x_1, y_1) and (x_2, y_2) is the midpoint of the line segment from (x_1, y_1) to $\left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)$, which is

$$\left(\frac{x_1 + \frac{x_1 + x_2}{2}}{2}, \frac{y_1 + \frac{y_1 + y_2}{2}}{2}\right) = \left(\frac{3x_1 + x_2}{4}, \frac{3y_1 + y_2}{4}\right).$$

The point three-fourths of the way between (x_1, y_1) and (x_2, y_2) is the midpoint of the line segment from $\left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)$ to (x_2, y_2), which is

$$\left(\frac{\frac{x_1 + x_2}{2} + x_2}{2}, \frac{\frac{y_1 + y_2}{2} + y_2}{2}\right) = \left(\frac{x_1 + 3x_2}{4}, \frac{y_1 + 3y_2}{4}\right).$$

Thus,

$$\left(\frac{3x_1 + x_2}{4}, \frac{3y_1 + y_2}{4}\right), \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right), \text{and}$$

$$\left(\frac{x_1 + 3x_2}{4}, \frac{y_1 + 3y_2}{4}\right)$$

are the three points that divide the line segment joining (x_1, y_1) and (x_2, y_2) into four equal parts.

38. (a) $\left(\frac{3(1)+4}{4}, \frac{3(-2)-1}{4}\right) = \left(\frac{7}{4}, -\frac{7}{4}\right)$

$\left(\frac{1+4}{2}, \frac{-2-1}{2}\right) = \left(\frac{5}{2}, -\frac{3}{2}\right)$

$\left(\frac{1+3(4)}{4}, \frac{-2+3(-1)}{4}\right) = \left(\frac{13}{4}, -\frac{5}{4}\right)$

(b) $\left(\frac{3(-2)+0}{4}, \frac{3(-3)+0}{4}\right) = \left(-\frac{3}{2}, -\frac{9}{4}\right)$

$\left(\frac{-2+0}{2}, \frac{-3+0}{2}\right) = \left(-1, -\frac{3}{2}\right)$

$\left(\frac{-2+3(0)}{4}, \frac{-3+3(0)}{4}\right) = \left(-\frac{1}{2}, -\frac{3}{4}\right)$

39. To show $\left(\frac{2x_1+x_2}{3}, \frac{2y_1+y_2}{3}\right)$ is a point of trisection of the line segment joining (x_1, y_1) and (x_2, y_2), we must show that $d_1 = \frac{1}{2}d_2$ and $d_1 + d_2 = d_3$.

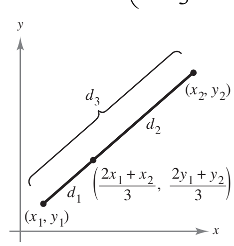
$$d_1 = \sqrt{\left(\frac{2x_1+x_2}{3} - x_1\right)^2 + \left(\frac{2y_1+y_2}{3} - y_1\right)^2}$$
$$= \sqrt{\left(\frac{x_2-x_1}{3}\right)^2 + \left(\frac{y_2-y_1}{3}\right)^2}$$
$$= \frac{1}{3}\sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$$

$$d_2 = \sqrt{\left(x_2 - \frac{2x_1+x_2}{3}\right)^2 + \left(y_2 - \frac{2y_1+y_2}{3}\right)^2}$$
$$= \sqrt{\left(\frac{2x_2-2x_1}{3}\right)^2 + \left(\frac{2y_2-2y_1}{3}\right)^2}$$
$$= \frac{2}{3}\sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$$

$$d_3 = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$$

Therefore, $d_1 = \frac{1}{2}d_2$ and $d_1 + d_2 = d_3$. The midpoint of the line segment joining $\left(\frac{2x_1+x_2}{3}, \frac{2y_1+y_2}{3}\right)$ and (x_2, y_2) is

$$\text{Midpoint} = \left(\frac{\frac{2x_1+x_2}{3}+x_2}{2}, \frac{\frac{2y_1+y_2}{3}+y_2}{2}\right)$$
$$= \left(\frac{x_1+2x_2}{3}, \frac{y_1+2y_2}{3}\right).$$



40. (a) $\left(\frac{2(1)+4}{3}, \frac{2(-2)+1}{3}\right) = (2, -1)$

$\left(\frac{1+2(4)}{3}, \frac{-2+2(1)}{3}\right) = (3, 0)$

(b) $\left(\frac{2(-2)+0}{3}, \frac{2(-3)+0}{3}\right) = \left(-\frac{4}{3}, -2\right)$

$\left(\frac{-2+2(0)}{3}, \frac{-3+2(0)}{3}\right) = \left(-\frac{2}{3}, -1\right)$

Section 1.2 Graphs of Equations

Skills Warm Up

1. $$\begin{aligned} 5y - 12 &= x \\ 5y &= x + 12 \\ y &= \frac{x + 12}{5} \end{aligned}$$

2. $$\begin{aligned} -y &= 15 - x \\ y &= x - 15 \end{aligned}$$

3. $$\begin{aligned} x^3y + 2y &= 1 \\ y(x^3 + 2) &= 1 \\ y &= \frac{1}{x^3 + 2} \end{aligned}$$

4. $$\begin{aligned} x^2 + x - y^2 - 6 &= 0 \\ -y^2 &= 6 - x^2 - x \\ y^2 &= x^2 + x - 6 \\ y &= \sqrt{x^2 + x - 6} \end{aligned}$$

5. $$\begin{aligned} (x - 2)^2 + (y + 1)^2 &= 9 \\ (y + 1)^2 &= 9 - (x - 2)^2 \\ y + 1 &= \sqrt{9 - (x - 2)^2} \\ y &= \left(\sqrt{9 - (x - 2)^2}\right) - 1 \\ &= \sqrt{9 - (x^2 - 4x + 4)} - 1 \\ &= \sqrt{5 + 4x - x^2} - 1 \end{aligned}$$

6. $$\begin{aligned} (x + 6)^2 + (y - 5)^2 &= 81 \\ (y - 5)^2 &= 81 - (x + 6)^2 \\ y - 5 &= \sqrt{81 - (x + 6)^2} \\ y &= 5 + \sqrt{81 - (x + 6)^2} \\ &= 5 + \sqrt{81 - (x^2 + 12x + 36)} \\ &= 5 + \sqrt{45 - 12x - x^2} \end{aligned}$$

7. $y = 5(-2) = -10$

8. $y = 3(3) - 4 = 5$

9. $$\begin{aligned} y &= 4(0.5)^2 - 7 \\ &= 4(0.25) - 7 \\ &= 1 - 7 \\ &= -6 \end{aligned}$$

10. $$\begin{aligned} y &= 9\left(\tfrac{1}{3}\right)^2 + 9\left(\tfrac{1}{3}\right) - 5 \\ &= 9\left(\tfrac{1}{9}\right) + 9\left(\tfrac{1}{3}\right) - 5 \\ &= 1 + 3 - 5 = -1 \end{aligned}$$

11. $x^2 - 3x + 2$
$(x - 1)(x - 2)$

12. $x^2 + 5x + 6$
$(x + 2)(x + 3)$

13. $y^2 - 3y + \frac{9}{4}$
$\left(y - \frac{3}{2}\right)^2$

14. $y^2 - 7y + \frac{49}{4}$
$\left(y - \frac{7}{2}\right)^2$

---

1. The graph of $y = x - 2$ is a straight line with y-intercept at $(0, -2)$. So, it matches (e).

2. The graph of $y = -\frac{1}{2}x + 2$ is a straight line with y-intercept at $(0, 2)$. So, it matches (b).

3. The graph of $y = x^2 + 2x$ is a parabola opening up with vertex at $(-1, -1)$. So, it matches (c).

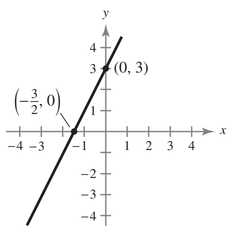
4. The graph of $y = \sqrt{9 - x^2}$ is a semicircle with intercepts $(0, 3)$, $(3, 0)$, and $(-3, 0)$. So, it matches (f).

5. The graph of $y = |x| - 2$ has a y-intercept at $(0, -2)$ and has x-intercepts at $(-2, 0)$ and $(2, 0)$.
So, it matches (a).

6. The graph of $y = x^3 - x$ has intercepts at $(0, 0)$, $(1, 0)$, and $(-1, 0)$. So, it matches (d).

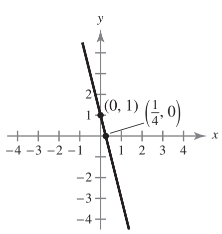
7. $y = 2x + 3$

x	-2	$-\frac{3}{2}$	-1	0	1	2
y	-1	0	1	3	5	7



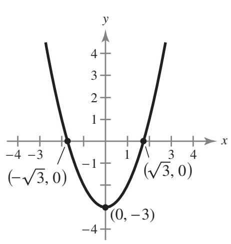
8. $y = 1 - 4x$

x	-1	0	$\frac{1}{4}$	1	2
y	5	1	0	-3	-7



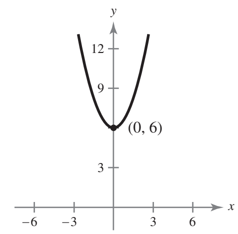
9. $y = x^2 - 3$

x	-2	-1	0	1	2	3
y	1	-2	-3	-2	1	6



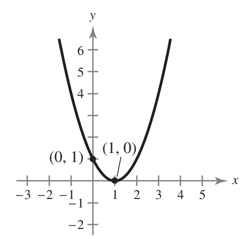
10. $y = x^2 + 6$

x	-2	-1	0	1	2
y	10	7	6	7	10



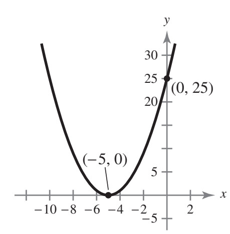
11. $y = (x - 1)^2$

x	-2	-1	0	1	2
y	9	4	1	0	1



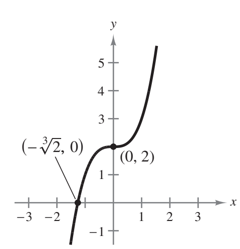
12. $y = (x + 5)^2$

x	-6	-5	-4	-3	-2
y	1	0	1	2	9



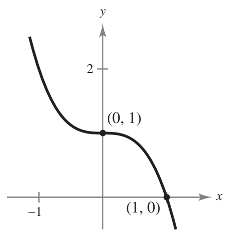
13. $y = x^3 + 2$

x	-2	-1	0	1	2
y	-6	1	2	3	10



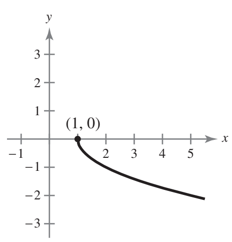
14. $y = 1 - x^3$

x	0	1	−1	2
y	1	0	2	−7



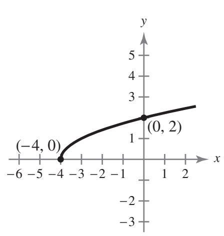
15. $y = -\sqrt{x - 1}$

x	1	2	3	4	5
y	0	−1	−1.41	−1.73	−2



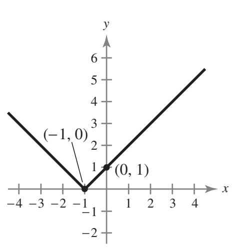
16. $y = \sqrt{x + 4}$

x	−4	−3	−2	−1	0
y	0	1	$\sqrt{2}$	$\sqrt{3}$	2



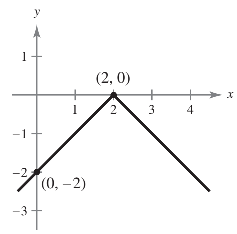
17. $y = |x + 1|$

x	−3	−2	−1	0	1
y	2	1	0	1	2



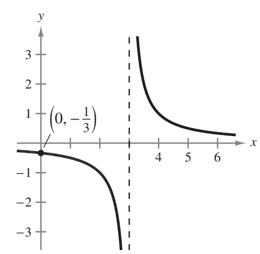
18. $y = -|x - 2|$

x	2	0	1	3	4
y	0	−2	−1	−1	−2



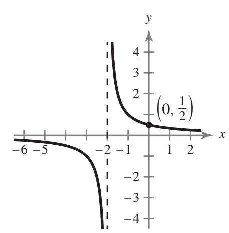
19. $y = \dfrac{1}{x - 3}$

x	−1	0	1	2	2.5	3.5	4	5	6
y	$-\frac{1}{4}$	$-\frac{1}{3}$	$-\frac{1}{2}$	−1	−2	2	1	$\frac{1}{2}$	$\frac{1}{3}$



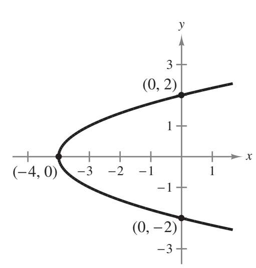
20. $y = \dfrac{1}{x + 2}$

x	−4	−3	−1	0	1	2
y	$-\frac{1}{2}$	−1	1	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$



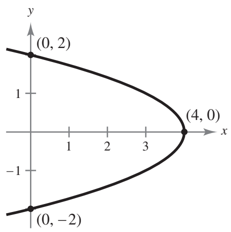
21. $x = y^2 - 4$

x	5	0	−3	−4
y	±3	±2	±1	0



22. $x = 4 - y^2$

x	0	3	4
y	± 2	± 1	0



23. Let $y = 0$. Then,

$2x - (0) - 3 = 0$

$x = \frac{3}{2}$.

Let $x = 0$. Then,

$2(0) - y - 3 = 0$

$y = -3$.

x-intercept: $\left(\frac{3}{2}, 0\right)$

y-intercept: $(0, -3)$

24. Let $y = 0$. Then,

$4x - 3(0) - 6 = 0$

$4x = 6$

$x = \frac{3}{2}$

Let $x = 0$. Then,

$4(0) - 3y - 6 = 0$

$-3y = 6$

$y = -2$

x-intercept: $\left(\frac{3}{2}, 0\right)$

y-intercept: $(0, -2)$

25. Let $y = 0$. Then,

$0 = x^2 + x - 2$

$0 = (x + 2)(x - 1)$

$x = -2, 1$.

Let $x = 0$. Then,

$y = (0)^2 + (0) - 2$

$y = -2$

x-intercepts: $(-2, 0), (1, 0)$

y-intercept: $(0, -2)$

26. Let $y = 0$. Then,

$0 = x^2 - 4x + 3$

$0 = (x - 3)(x - 1)$

$x = 1, 3$.

Let $x = 0$. Then,

$y = (0)^2 - 4(0) + 3$

$y = 3$.

x-intercepts: $(1, 0), (3, 0)$

y-intercept: $(0, 3)$

27. Let $y = 0$. Then,

$0 = x^3 + 7x^2$

$0 = x^2(x + 7)$

$x^2 = 0 \rightarrow x = 0$

$x + 7 = 0 \rightarrow x = -7$

Let $x = 0$. Then,

$y = (0)^3 + 7(0)^2$

$y = 0$

x-intercepts: $(0, 0), (-7, 0)$

y-intercept: $(0, 0)$

28. Let $y = 0$. Then,

$0 = x^3 - 9x^2$

$0 = x^2(x - 9)$

$x^2 = 0 \rightarrow x = 0$

$x - 9 = 0 \rightarrow x = 9$

Let $x = 0$. Then,

$y = (0)^3 - 9(0)^2$

$y = 0$

x-intercept: $(0, 0), (9, 0)$

y-intercept: $(0, 0)$

29. Let $y = 0$. Then,

$$0 = \frac{x^2 - 4}{x - 2}$$
$$0 = (x - 2)(x + 2)$$
$$x = \pm 2.$$

Let $x = 0$. Then,

$$y = \frac{(0)^2 - 4}{(0) - 2}$$
$$y = 2.$$

x-intercept: Because the equation is undefined when $x = 2$, the only x-intercept is $(-2, 0)$.

y-intercept: $(0, 2)$

30. Let $y = 0$. Then,

$$0 = \frac{x^2 + 3x}{2x}$$
$$0 = x(x + 3)$$
$$x = -3, 0.$$

Let $x = 0$. Then,

$$y = \frac{(0)^2 + 3(0)}{2(0)}$$
$$y = \text{undefined}.$$

x-intercept: Because the equation is undefined when $x = 0$, the only x-intercept is $(-3, 0)$.

y-intercept: Because the equation is undefined when $y = 0$, there is no y-intercept.

31. Let $y = 0$. Then,

$$x^2(0) - x^2 + 4(0) = 0$$
$$x^2 = 0$$
$$x = 0.$$

Let $x = 0$. Then,

$$(0)^2 y - (0)^2 + 4y = 0$$
$$y = 0.$$

x-intercept: $(0, 0)$

y-intercept: $(0, 0)$

32. Let $y = 0$. Then,

$$2x^2(0) + 8(0) - x^2 = 1$$
$$x^2 = -1$$
$$x = \pm\sqrt{-1}.$$

Let $x = 0$. Then,

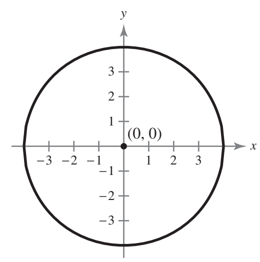
$$2(0)^2 y + 8y - (0)^2 = 1$$
$$y = \tfrac{1}{8}.$$

x-intercept: Because the equation has no real roots when $y = 0$, there is no x-intercept.

y-intercept: $\left(0, \tfrac{1}{8}\right)$

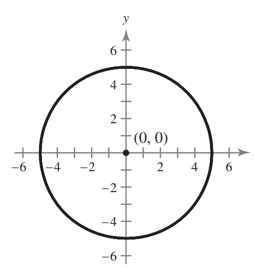
33. $(x - 0)^2 + (y - 0)^2 = 4^2$

$$x^2 + y^2 = 16$$



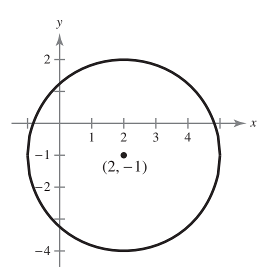
34. $(x - 0)^2 + (y - 0)^2 = 5^2$

$$x^2 + y^2 = 25$$



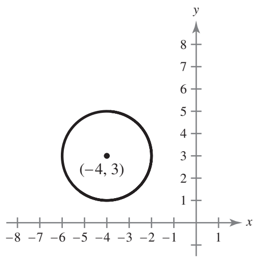
35. $(x - 2)^2 + (y - (-1))^2 = 3^2$

$$(x - 2)^2 + (y + 1)^2 = 9$$



36. $(x - (-4))^2 + (y - 3)^2 = 2^2$

$(x + 4)^2 + (y - 3)^2 = 4$



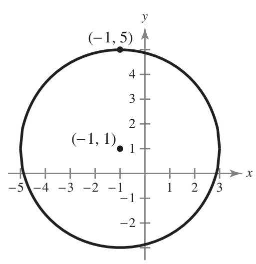
37. The radius is the distance between $(-1, 5)$ and $(-1, 1)$.

$$r = \sqrt{(-1 - (-1))^2 + (5 - 1)^2} = \sqrt{0^2 + 4^2} = \sqrt{16} = 4$$

Using the center $(-1, 1)$ and the radius $r = 4$:

$(x - (-1))^2 + (y - 1)^2 = 4^2$

$(x + 1)^2 + (y - 1)^2 = 16$



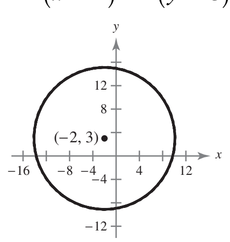
38. The radius is the distance between $(-2, 3)$ and $(5, -7)$.

$$r = \sqrt{(5 - (-2))^2 + (-7 - 3)^2} = \sqrt{7^2 + (-10)^2} = \sqrt{49 + 100} = \sqrt{149}$$

Using the center $(-2, 3)$ and the radius $r = \sqrt{149}$:

$(x - (-2))^2 + (y - 3)^2 = (\sqrt{149})^2$

$(x + 2)^2 + (y - 3)^2 = 149$



39. The diameter is the distance between $(-6, -8)$ and $(6, 8)$.

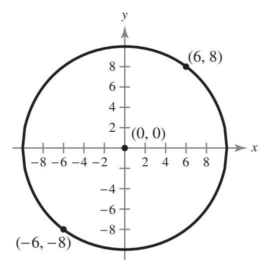
$$d = \sqrt{(6 - (-6))^2 + (8 - (-8))^2} = \sqrt{12^2 + 16^2} = \sqrt{144 + 256} = \sqrt{400} = 20$$

The radius is one-half the diameter: $r = \frac{20}{2} = 10$.

The center is the midpoint of the diameter: $\left(\frac{-6 + 6}{2}, \frac{-8 + 8}{2}\right) = (0, 0)$.

$(x - 0)^2 + (y - 0)^2 = 10^2$

$x^2 + y^2 = 100$



40. The diameter is the distance between $(0, -4)$ and $(6, 4)$.

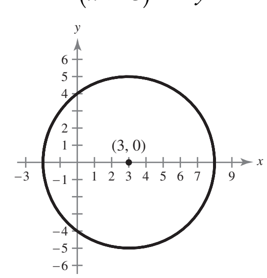
$$d = \sqrt{(6 - 0)^2 + (4 - (-4))^2} = \sqrt{6^2 + 8^2} = \sqrt{36 + 64} = \sqrt{100} = 10$$

The radius is one-half the diameter: $r = \frac{10}{2} = 5$.

The center is the midpoint of the diameter: $\left(\frac{0 + 6}{2}, \frac{4 + 4}{2}\right) = (3, 0)$

$(x - 3)^2 + (y - 0)^2 = 5^2$

$(x - 3)^2 + y^2 = 25$



41. Set the two equations equal to each other.

$-x + 2 = 2x - 1$

$-3x = -3$

$x = 1$

Substitute $x = 1$ into one of the equations.

$y = (-1) + 2 = 1$

The point of intersection is $(1, 1)$.

42. Set the two equations equal to each other.

$-x + 7 = \frac{3}{2}x - 8$

$-2x + 14 = 3x - 16$

$-5x = -30$

$x = 6$

Substitute $x = 6$ into one of the equations.

$y = (-6) + 7 = 1$

The point of intersection is $(6, 1)$.

43. Set the two equations equal to each other.

$-x^2 + 15 = 3x + 11$

$-x^2 - 3x + 4 = 0$

$x^2 + 3x - 4 = 0$

$(x + 4)(x - 1) = 0$

$x + 4 = 0 \qquad x - 1 = 0$

$x = -4 \qquad x = 1$

Substitute $x = -4$:

$y = -(-4)^2 + 15$

$y = -16 + 15$

$y = -1$

Substitute $x = 1$:

$y = -(1)^2 + 15$

$y = -1 + 15$

$y = 14$

The points of intersection are $(-4, -1)$ and $(1, 14)$.

44. Set the two equations equal to each other.

$x^2 - 5 = x + 1$

$x^2 - x - 6 = 0$

$(x - 3)(x + 2) = 0$

$x - 3 = 0 \qquad x + 2 = 0$

$x = 3 \qquad x = -2$

Substitute $x = 3$:

$y = (3)^2 - 5$

$y = 4$

Substitute $x = -2$:

$y = (-2)^2 - 5$

$y = -1$

The points of intersection are $(3, 4)$ and $(-2, -1)$.

45. Set the two equations equal to each other.

$x^3 = 2x$

$x^3 - 2x = 0$

$x(x^2 - 2) = 0$

$x = 0, \pm\sqrt{2}$

Substitute $x = 0$:

$y = 2(0)$

$y = 0$

Substitute $x = \sqrt{2}$:

$y = 2(\sqrt{2})$

$y = 2\sqrt{2}$

Substitute $x = -\sqrt{2}$:

$y = 2(-\sqrt{2})$

$y = -2\sqrt{2}$

The points of intersection are $(0, 0)$, $(-\sqrt{2}, -2\sqrt{2})$, and $(\sqrt{2}, 2\sqrt{2})$.

46. Set the two equations equal to each other.

$\sqrt{x} = x$

$x = x^2$

$0 = x(x - 1)$

$x = 0, 1$

Substitute $x = 0$:

$y = 0$

Substitute $x = 1$:

$y = 1$

The points of intersection are (0, 0) and (1, 1).

47. Set the two equations equal to each other.

$x^4 - 2x^2 + 1 = 1 - x^2$

$x^4 - x^2 = 0$

$x^2(x + 1)(x - 1) = 0$

$x = 0, \pm 1$

Substitute $x = 0$:

$y = 1 - (0)^2$

$y = 1$

Substitute $x = 1$:

$y = 1 - (1)^2$

$y = 0$

Substitute $x = -1$:

$y = 1 - (-1)^2$

$y = 1 - 1$

$y = 0$

The points of intersection are $(-1, 0)$, $(0, 1)$, and $(1, 0)$.

48. Set the two equations equal to each other.

$$x^3 - 2x^2 + x - 1 = -x^2 + 3x - 1$$
$$x^3 - x^2 - 2x = 0$$
$$x(x+1)(x-2) = 0$$
$$x = 0, -1, 2$$

Substitute $x = 0$:

$$y = -(0)^2 + 3(0) - 1$$
$$y = 0 + 0 - 1$$
$$y = -1$$

Substitute $x = -1$:

$$y = -(-1)^2 + 3(-1) - 1$$
$$y = -1 - 3 - 1$$
$$y = -5$$

Substitute $x = 2$:

$$y = -(2)^2 + 3(2) - 1$$
$$y = -4 + 6 - 1$$
$$y = 1$$

The points of intersection are $(0, -1)$, $(-1, -5)$, and $(2, 1)$.

49. To find the break-even point, set $R = C$.

$$1.55x = 0.85x + 35{,}000$$
$$0.7x = 35{,}000$$
$$x = \frac{35{,}000}{0.7} = 50{,}000 \text{ units}$$

50. To find the break-even point, set $R = C$.

$$35x = 6x + 500{,}000$$
$$29x = 500{,}000$$
$$x = \frac{500{,}000}{29} \approx 17{,}242 \text{ units}$$

51. To find the break-even point, set $R = C$.

$$9950x = 8650x + 250{,}000$$
$$1300x = 250{,}000$$
$$x = \frac{250{,}000}{1300} \approx 193 \text{ units}$$

52. To find the break-even point, set $R = C$.

$$4.9x = 2.5x + 10{,}000$$
$$2.4x = 10{,}000$$
$$x = \frac{10{,}000}{2.4} \approx 4167 \text{ units}$$

53. To find the break-even point, set $R = C$.

$$10x = 6x + 5000$$
$$4x = 5000$$
$$x = \frac{5000}{4} \approx 1250 \text{ units}$$

54. To find the break-even point, set $R = C$.

$$200x = 130x + 12{,}600$$
$$70x = 12{,}600$$
$$x = \frac{12{,}600}{70} \approx 180 \text{ units}$$

55. (a) $C = 11.5x + 21{,}000$

$R = 19.90x$

(b)
$$C = R$$
$$11.5x + 21{,}000 = 19.90x$$
$$21{,}000 = 8.4x$$
$$x = 2500 \text{ units}$$

(c)
$$P = R - C$$
$$1000 = 19.9x - (11.5x + 21{,}000)$$
$$22{,}000 = 8.4x$$
$$x \approx 2619 \text{ units}$$

So, 2619 units would yield a profit of \$1000.

56. (a) The cost C_g to drive x miles is the cost of the car itself plus the cost of gasoline per mile, which is the cost of gasoline per gallon divided by the number of gallons per mile.

$$C_g = 33{,}500 + \frac{2.759}{31}x$$

Similarly, the cost C_h to drive x miles is the cost of the car itself plus the cost of gasoline per mile.

$$C_h = 36{,}775 + \frac{2.759}{39}x$$

(b) To find the break-even point, set the cost equations equal to each other.

$$33{,}500 + \frac{2.759}{31}x = 36{,}775 + \frac{2.759x}{39}$$

Multiply both sides of the equation by $(31)(39)$.

$$40{,}501{,}000 + 107.601x = 44{,}460{,}975 + 85.529x$$
$$22.072x = 3{,}959{,}975$$
$$x = \frac{3{,}959{,}975}{22.072} \approx 179{,}412 \text{ mi}$$

57.
$$205 - 4x = 135 + 3x$$
$$70 = 7x$$
$$10 = x$$

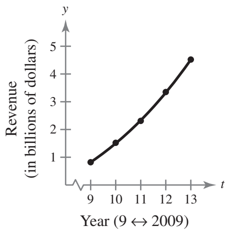
Equilibrium point $(x, p) = (10, 165)$

58.
$$190 - 15x = 75 + 8x$$
$$115 = 23x$$
$$x = 5$$

Equilibrium point $(x, p) = (5, 115)$

59. (a)

Year	2009	2010	2011	2012	2013
Revenue	0.82	1.52	2.31	3.35	4.52
Model	0.82	1.50	2.33	3.33	4.52



The model fits the data well.

(b) Let $t = 18$ (2018).

$$y = 0.00333(18)^3 - 0.0250(18)^2 + 0.252(18) - 1.85 \approx \$14.0 \text{ billion}$$

60. (a) If 10,000 units are sold, the company breaks even.

(b) If less than 10,000 units are sold, the company loses money.

(c) If more than 10,000 units are sold, the company makes a profit.

61. (a)

Year	2008	2009	2010	2011	2012	2016
Degrees	747	793	854	931	1024	1550

(b) Answers will vary.

(c) Let $t = 20$ (2020).

$$y = 7.79(20)^2 - 86.6(20) + 941$$
$$= 2325 \text{ degrees}$$

The prediction is valid because the number of associate's degrees should keep increasing over time.

62. (a) and (b)

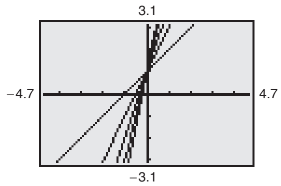
Year	2009	2010	2011	2012	2013
Transplants (model)	2213.79	2323.0	2336.79	2369.16	2534.11
Transplants (actual)	2211	2332	2322	2378	2531

(c) For 2019, let $t = 19$.

$$y = 19.000(19)^3 - 617.71(19)^2 + 6696.7(19) - 21{,}873 \approx 12{,}692$$

The prediction seems high. Answers will vary.

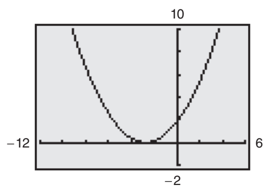
63.



The greater the value of c, the steeper the line.

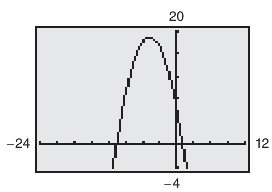
64. If C and R represent the cost and revenue for a business, the break-even point is that value of x for which $C = R$. For example, if $C = 100{,}000 + 10x$ and $R = 20x$, then the break-even point is $x = 10{,}000$ units.

65.



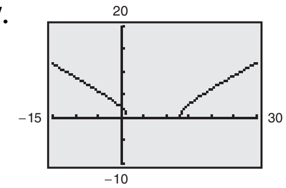
Intercepts: $(-2.75,\ 0), (0, 1.815)$

66.



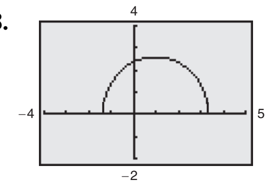
Intercepts: $(0,\ 6.25)$, $(1.0539,\ 0)$, $(-10.5896,\ 0)$

67.



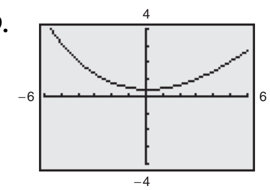
Intercepts: $(1.4780,\ 0)$, $(12.8553,\ 0)$, $(0,\ 2.3875)$

68.



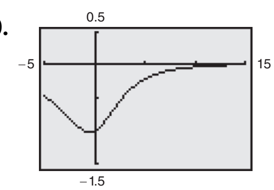
Intercepts: $(3.3256,\ 0)$, $(-1.3917,\ 0)$, $(0,\ 2.3664)$

69.



Intercept: $\left(0,\ \frac{5}{12}\right) \approx (0,\ 0.4167)$

70.



Intercepts: $(0,\ -1)$, $(13.25,\ 0)$

71. Answers will vary.

Section 1.3 Lines in the Plane and Slope

Skills Warm Up

1. $\dfrac{5 - (-2)}{-3 - 4} = \dfrac{7}{-7} = -1$

2. $\dfrac{-4 - (-10)}{7 - 5} = \dfrac{6}{2} = 3$

3. $-\dfrac{1}{m},\ m = -3$

$-\dfrac{1}{-3} = \dfrac{1}{3}$

4. $-\dfrac{1}{m},\ m = \dfrac{6}{7}$

$-\dfrac{1}{\frac{6}{7}} = -\dfrac{7}{6}$

5. $-4x + y = 7$

$y = 4x + 7$

6. $3x - y = 7$

$-y = 7 - 3x$

$y = 3x - 7$

Skills Warm Up —*continued*—

7. $y - 2 = 3(x - 4)$

$y = 3(x - 4) + 2$

$y = 3x - 12 + 2$

$y = 3x - 10$

8. $y - (-5) = -1[x - (-2)]$

$y + 5 = -x - 2$

$y = -x - 7$

9. $y - (-3) = \dfrac{4 - (-2)}{11 - 3}(x - 12)$

$y + 3 = \dfrac{6}{8}(x - 12)$

$y + 3 = \dfrac{3}{4}(x - 12)$

$y + 3 = \dfrac{3}{4}x - 9$

$y = \dfrac{3}{4}x - 12$

10. $y - 1 = \dfrac{-3 - 1}{-7 - (-1)}[x - (-1)]$

$y - 1 = \dfrac{-4}{-6}(x + 1)$

$y - 1 = \dfrac{2}{3}(x + 1)$

$y - 1 = \dfrac{2}{3}x + \dfrac{2}{3}$

$y = \dfrac{2}{3}x + \dfrac{5}{3}$

1. The slope is $m = 1$ because the line rises one unit vertically for each unit the line moves to the right.

2. The slope is 2 because the line rises two units vertically for each unit the line moves to the right.

3. The slope is $m = 0$ because the line is horizontal.

4. The slope is -1 because the line falls one unit vertically for each unit the line moves to the right.

5. $y = x + 7$

So, the slope is $m = 1$, and the y-intercept is $(0, 7)$.

6. $y = 4x + 3$

So, the slope is $m = 4$, and the y-intercept is $(0, 3)$.

7. $5x + y = 20$

$y = -5x + 20$

So, the slope is $m = -5$, and the y-intercept is $(0, 20)$.

8. $2x + y = 40$

$y = -2x + 40$

So, the slope is $m = -2$, and the y-intercept is $(0, 40)$.

9. $7x + 6y = 30$

$y = -\frac{7}{6}x + 5$

So, the slope is $m = -\frac{7}{6}$, and the y-intercept is $(0, 5)$.

10. $8x + 3y = 12$

$3y = -8x + 12$

$y = -\frac{8}{3}x + 4$

So, the slope is $m = -\frac{8}{3}$, and the y-intercept is $(0, 4)$.

11. $3x - y = 15$

$y = 3x - 15$

So, the slope is $m = 3$, and the y-intercept is $(0, -15)$.

12. $2x - 3y = 24$

$y = \frac{2}{3}x - 8$

So, the slope is $m = \frac{2}{3}$, and the y-intercept is $(0, -8)$.

13. $x = 4$

Because the line is vertical, the slope is undefined. There is no y-intercept.

14. $x + 5 = 0$

$x = -5$

Because the line is vertical, the slope is undefined. There is no y-intercept.

15. $y - 9 = 0$

$y = 9$

So, the slope is $m = 0$, and the y-intercept is $(0, 9)$.

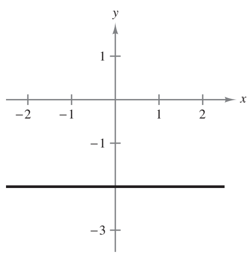
16. $y + 1 = 0$

$y = -1$

So, the slope is $m = 0$, and the y-intercept is $(0, -1)$.

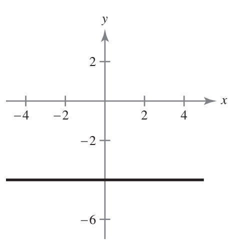
17. $y = -2$

x	-2	-1	0	1
y	-2	-2	-2	-2



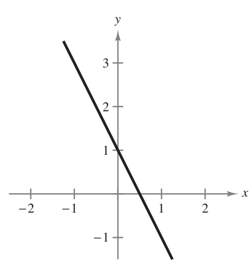
18. $y = -4$

x	-4	-2	0	2
y	-4	-4	-4	-4



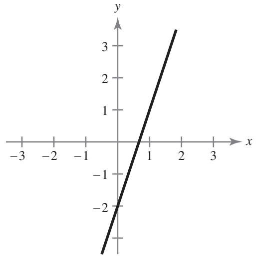
19. $y = -2x + 1$

x	-1	0	1	2
y	3	1	-1	-3



20. $y = 3x - 2$

x	-1	0	1	2
y	-5	-2	1	4

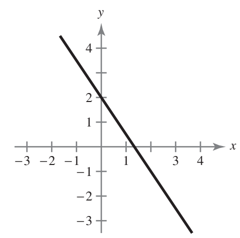


21. $3x + 2y = 4$

$2y = -3x + 4$

$y = -\frac{3}{2}x + 2$

x	-4	-2	0	2	4
y	8	5	2	-1	-4

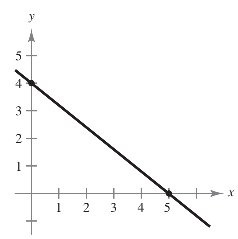


22. $4x + 5y = 20$

$5y = -4x + 20$

$y = -\frac{4}{5}x + 4$

x	0	2	4	5
y	4	$\frac{12}{5}$	$\frac{4}{5}$	0

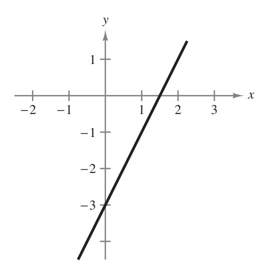


23. $2x - y - 3 = 0$

$-y = -2x + 3$

$y = 2x - 3$

x	-1	0	1	2
y	-5	-3	-1	1

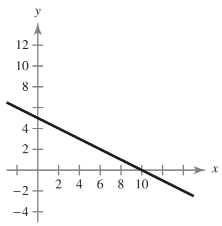


24. $x + 2y + 10 = 0$

$$2y = -x + 10$$

$$y = -\tfrac{1}{2}x + 5$$

x	-4	-2	0	2	4
y	7	6	5	4	3

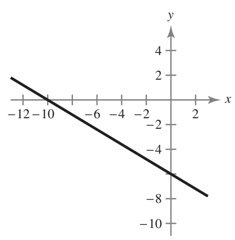


25. $3x + 5y + 30 = 0$

$$5y = -3x - 30$$

$$y = -\tfrac{3}{5}x - 6$$

x	-10	-5	0	5	10
y	0	-3	-6	-9	-12

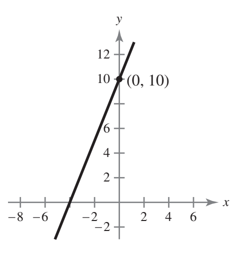


26. $-5x + 2y - 20 = 0$

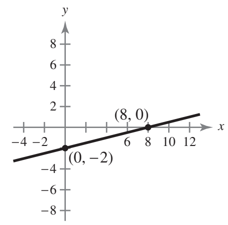
$$2y = 5x + 20$$

$$y = \tfrac{5}{2}x + 10$$

x	-4	-2	0	2
y	0	5	10	15

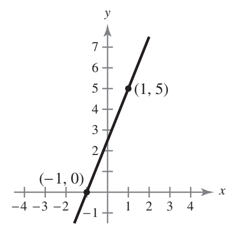


27.



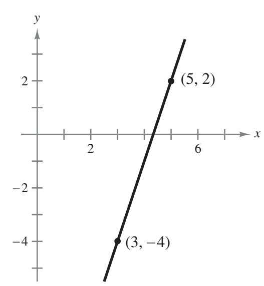
The slope is $m = \dfrac{0 - (-2)}{8 - 0} = \dfrac{2}{8} = \dfrac{1}{4}$.

28.



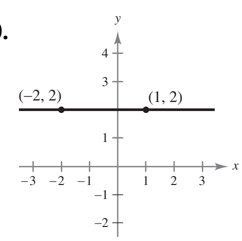
The slope is $m = \dfrac{5 - 0}{1 - (-1)} = \dfrac{5}{2}$.

29.



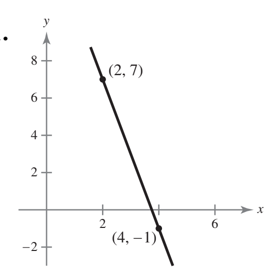
The slope is $m = \dfrac{2 - (-4)}{5 - 3} = 3$.

30.



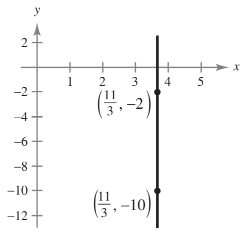
The slope is $m = \dfrac{2 - 2}{1 - (-2)} = 0$.

31.



The slope of $m = \dfrac{7 - (-1)}{2 - 4} = \dfrac{8}{-2} = -4$.

32.

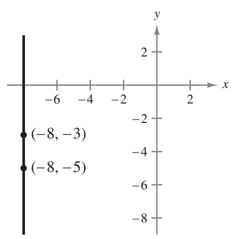


The slope is $m = \dfrac{-10 - (-2)}{\dfrac{11}{3} - \dfrac{11}{3}} = \dfrac{-8}{0}$,

which is undefined.

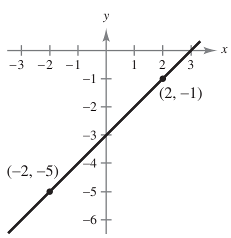
So, the line is vertical.

33.



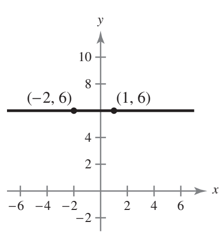
The slope is undefined because $m = \dfrac{-5 - (-3)}{-8 - (-8)}$ and division by zero is undefined. So, the line is vertical.

34.



The slope is $m = \dfrac{-1 - (-5)}{2 - (-2)} = \dfrac{4}{4} = 1.$

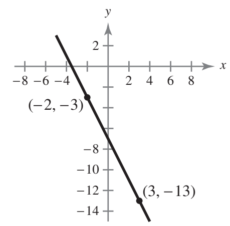
35.



The slope is $m = \dfrac{6 - 6}{1 - (-2)} = \dfrac{0}{3} = 0.$

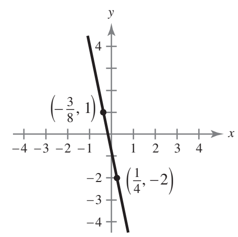
So, the line is horizontal.

36.



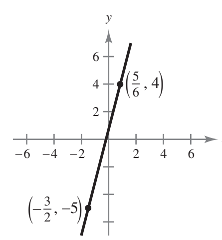
The slope is $m = \dfrac{-3 - (-13)}{-2 - 3} = \dfrac{10}{-5} = -2.$

37.



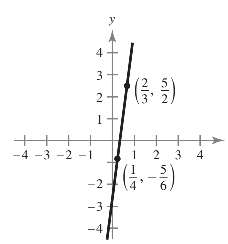
The slope is $m = \dfrac{1 - (-2)}{-\frac{3}{8} - \frac{1}{4}} = \dfrac{3}{-\frac{5}{8}} = -\dfrac{24}{5}.$

38.



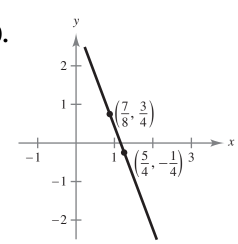
The slope is $m = \dfrac{4 + 5}{(5/6) + (3/2)} = \dfrac{27}{7}.$

39.



The slope is $m = \dfrac{\frac{5}{2} - \left(-\frac{5}{6}\right)}{\frac{2}{3} - \frac{1}{4}} = \dfrac{\frac{10}{3}}{\frac{5}{12}} = \dfrac{10}{3} \cdot \dfrac{12}{5} = 8.$

40.



The slope is $m = \dfrac{(-1/4) - (3/4)}{(5/4) - (7/8)} = -\dfrac{8}{3}.$

41. The equation of this horizontal line is $y = 1$. So, three additional points are $(0, 1)$, $(1, 1)$, and $(3, 1)$.

42. The equation of this horizontal line is $y = -3$. So, three additional points are $(0, -3)$, $(1, -3)$, and $(2, -3)$.

43. The equation of the line is

$$y - 7 = -3(x - 1)$$
$$y = -3x + 10.$$

So, three additional points are $(0, 10)$, $(2, 4)$, and $(3, 1)$.

44. The equation of this line is

$$y + 2 = 2(x - 7)$$
$$y = 2x - 16.$$

So, three additional points are $(0, -16)$, $(1, -14)$, and $(2, -12)$.

45. The equation of this line is

$$y + 4 = \frac{2}{3}(x - 6)$$
$$y = \frac{2}{3}x - 8.$$

So, three additional points are $(3, -6)$, $(9, -2)$, and $(12, 0)$.

46. The equation of this line is

$$y + 6 = -\tfrac{1}{2}(x + 1)$$
$$y = -\tfrac{1}{2}x - \tfrac{13}{2}.$$

So, three additional points are $(1, -7)$, $(3, -8)$, and $(5, -9)$.

47. The equation of this vertical line is $x = -8$. So, three additional points are $(-8, 0)$, $(-8, 2)$, and $(-8, 3)$.

48. The equation of this vertical line is $x = -3$. So, three additional points are $(-3, 0)$, $(-3, 1)$, and $(-3, 2)$.

49. The slope of the line joining $(-2, 1)$ and $(-1, 0)$ is

$$\frac{1 - 0}{-2 - (-1)} = \frac{1}{-1} = -1.$$

The slope of the line joining $(-1, 0)$ and $(2, -2)$ is

$$\frac{0 - (-2)}{-1 - 2} = \frac{2}{-3} = -\frac{2}{3}.$$

Because the slopes are different, the points are not collinear.

50. The slope of the line joining $(-5, 11)$ and $(0, 4)$ is

$$\frac{11 - 4}{-5 - 0} = \frac{7}{-5} = -\frac{7}{5}.$$

The slope of the line joining $(0, 4)$ and $(7, -6)$ is

$$\frac{4 - (-6)}{0 - 7} = -\frac{10}{7}.$$

Because the slopes are different, the points are not collinear.

51. The slope of the line joining $(2, 7)$ and $(-2, -1)$ is

$$\frac{-1 - 7}{-2 - 2} = 2.$$

The slope of the line joining $(0, 3)$ and $(-2, -1)$ is

$$\frac{-1 - 3}{-2 - 0} = 2.$$

Because the slopes are equal and both lines pass through $(-2, -1)$, the three points are collinear.

52. The slope of the line joining $(4, 1)$ and $(-2, -2)$ is

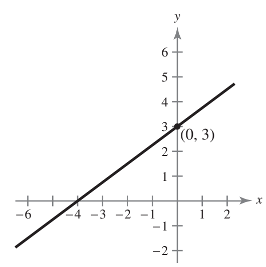
$$\frac{-2 - 1}{-2 - 4} = \frac{1}{2}.$$

The slope of the line joining $(8, 3)$ and $(-2, -2)$ is

$$\frac{-2 - 3}{-2 - 8} = \frac{1}{2}.$$

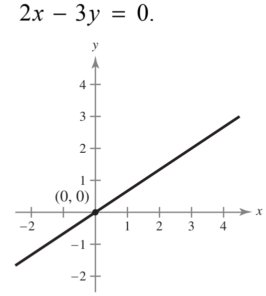
Because the slopes are equal and both lines pass through $(-2, -2)$, the three points are collinear.

53. Using the slope-intercept form, we have $y = \frac{3}{4}x + 3$.

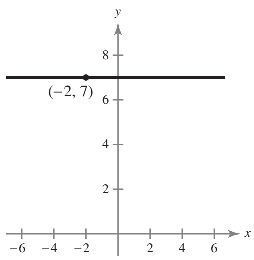


54. Using the slope-intercept form, we have

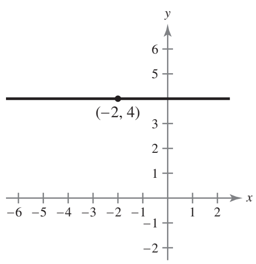
$$y = \tfrac{2}{3}x + 0$$
$$2x - 3y = 0.$$



55. Because the slope is 0, the line is horizontal and its equation is $y = 7$.

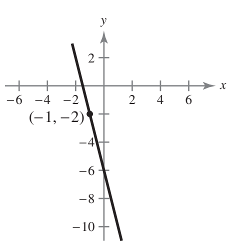


56. Because the slope is 0, the line is horizontal and its equation is $y = 4$.



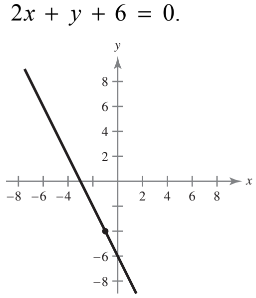
57. Using the point-slope form, you have

$$y - (-2) = -4(x - (-1))$$
$$y + 2 = -4(x + 1)$$
$$y + 2 = -4x - 4$$
$$y = -4x - 6$$
$$4x + y + 6 = 0.$$



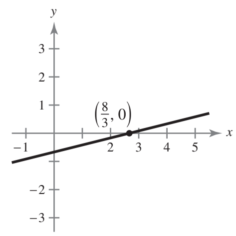
58. Using the point-slope form, you have

$$y + 4 = -2(x + 1)$$
$$y = -2x - 6$$
$$2x + y + 6 = 0.$$



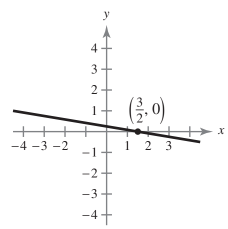
59. Using the point-slope form, you have

$$y - 0 = \tfrac{1}{4}\left(x - \tfrac{8}{3}\right)$$
$$y = \tfrac{1}{4}x - \tfrac{2}{3}$$
$$3x - 12y - 8 = 0.$$



60. Using the point-slope form, you have

$$y - 0 = -\tfrac{1}{6}\left(x - \tfrac{3}{2}\right)$$
$$y = -\tfrac{1}{6}x + \tfrac{1}{4}$$
$$2x + 12y - 3 = 0.$$

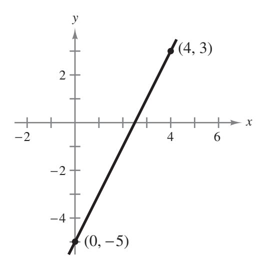


61. The slope of the line is

$$m = \frac{3 - (-5)}{4 - 0} = 2.$$

Using the point-slope form, you have

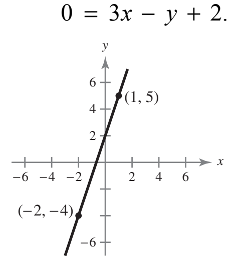
$$y + 5 = 2(x - 0)$$
$$y = 2x - 5$$
$$0 = 2x - y - 5.$$



62. The slope of the line is $m = \dfrac{5 - (-4)}{1 - (-2)} = \dfrac{9}{3} = 3.$

Using the point-slope form, you have

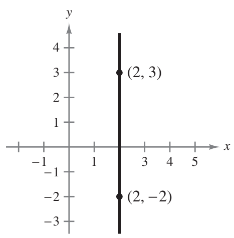
$$y - 5 = 3(x - 1)$$
$$y - 5 = 3x - 3$$
$$0 = 3x - y + 2.$$



63. The slope of the line is $m = \dfrac{-2 - 3}{2 - 2} = \text{undefined.}$

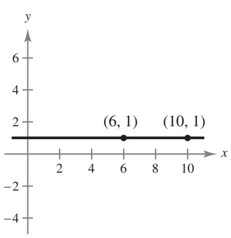
So, the line is vertical, and its equation is

$$x = 2$$
$$x - 2 = 0.$$



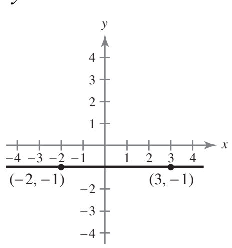
64. The slope of the line is $m = \dfrac{1 - 1}{10 - 6} = 0$. So, the line is horizontal, and its equation is

$$y = 1$$
$$y - 1 = 0.$$



65. The slope of the line is $m = \dfrac{-1 - (-1)}{-2 - 3} = 0$. So, the line is horizontal, and its equation is

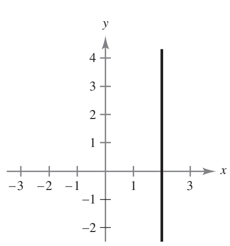
$$y = -1$$
$$y + 1 = 0.$$



66. The slope of the line is $m = \dfrac{-10 - 5}{2 - 2} = \text{undefined.}$

So, the line is vertical, and its equation is

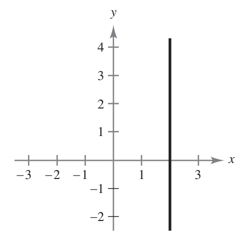
$$x = 2$$
$$x - 2 = 0.$$



67. The slope of the line is $m = \dfrac{8 - 4}{1/2 + 1/2} = 4.$

Using the point-slope form, you have

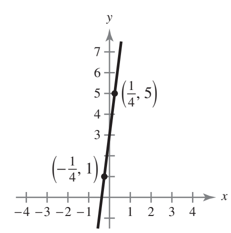
$$y - 8 = 4\left(x - \frac{1}{2}\right)$$
$$y = 4x + 6$$
$$0 = 4x - y + 6.$$



68. The slope is $m = \dfrac{5 - 1}{\frac{1}{4} - \left(-\frac{1}{4}\right)} = \dfrac{4}{\frac{1}{2}} = 8.$

Using the point-slope form, you have

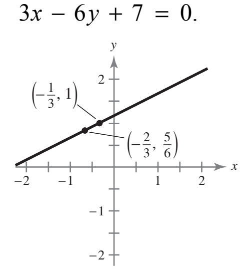
$$y - 1 = 8\left(x - \left(-\frac{1}{4}\right)\right)$$
$$y - 1 = 8x + 2$$
$$y = 8x + 3$$
$$8x - y + 3 = 0.$$



69. The slope of the line is $m = \dfrac{1 - 5/6}{(-1/3) + 2/3} = \dfrac{1}{2}.$

Using the point-slope form, you have

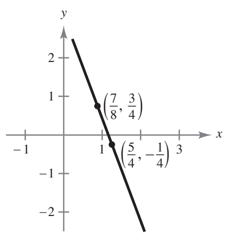
$$y - 1 = \frac{1}{2}\left(x + \frac{1}{3}\right)$$
$$y = \frac{1}{2}x + \frac{7}{6}$$
$$3x - 6y + 7 = 0.$$



70. The slope of the line is $m = \dfrac{(-1/4) - (3/4)}{(5/4) - (7/8)} = -\dfrac{8}{3}$.

Using the point-slope form, you have

$$y - \frac{3}{4} = -\frac{8}{3}\left(x - \frac{7}{8}\right)$$
$$y - \frac{3}{4} = -\frac{8}{3}x + \frac{7}{3}$$
$$12y - 9 = -32x + 28$$
$$32x + 12y - 37 = 0.$$



71. Because the line is vertical, it has an undefined slope, and its equation is

$$x = 3$$
$$x - 3 = 0.$$

72. Because the line is horizontal, it has a slope of $m = 0$, and its equation is

$$y = 0x + (-5)$$
$$y = -5.$$

73. Because the line is parallel to all horizontal lines, it has a slope of $m = 0$, and its equation is

$$y = -10.$$

74. Because the line is parallel to all vertical lines, it has an undefined slope, and its equation is

$$x = -5.$$

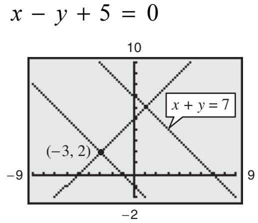
75. Given line: $y = -x + 7$, $m = -1$

(a) Parallel: $m_1 = -1$

$$y - 2 = -1(x + 3)$$
$$x + y + 1 = 0$$

(b) Perpendicular: $m_2 = 1$

$$y - 2 = 1(x + 3)$$
$$x - y + 5 = 0$$



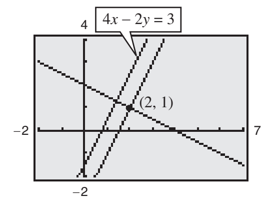
76. Given line: $y = 2x - \frac{3}{2}$, $m = 2$

(a) Parallel: $m_1 = 2$

$$y - 1 = 2(x - 2)$$
$$0 = 2x - y - 3$$

(b) Perpendicular: $m_2 = -\frac{1}{2}$

$$y - 1 = -\tfrac{1}{2}(x - 2)$$
$$2y - 2 = -x + 2$$
$$x + 2y - 4 = 0$$



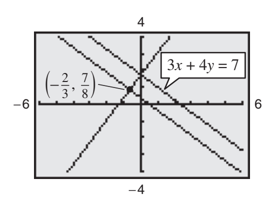
77. Given line: $y = -\frac{3}{4}x + \frac{7}{4}$, $m = -\frac{3}{4}$

(a) Parallel: $m_1 = -\frac{3}{4}$

$$y - \tfrac{7}{8} = -\tfrac{3}{4}\left(x + \tfrac{2}{3}\right) = -\tfrac{3}{4}x - \tfrac{1}{2}$$
$$8y - 7 = -6x - 4$$
$$6x + 8y - 3 = 0$$

(b) Perpendicular: $m_2 = \frac{4}{3}$

$$y - \tfrac{7}{8} = \tfrac{4}{3}\left(x + \tfrac{2}{3}\right) = \tfrac{4}{3}x + \tfrac{8}{9}$$
$$72y - 63 = 96x + 64$$
$$96x - 72y + 127 = 0$$



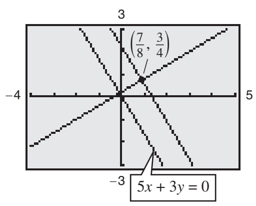
78. Given line: $y = -\frac{5}{3}x$, $m = -\frac{5}{3}$

(a) Parallel: $m_1 = -\frac{5}{3}$

$$y - \frac{3}{4} = -\frac{5}{3}\left(x - \frac{7}{8}\right)$$
$$y - \frac{3}{4} = -\frac{5}{3}x + \frac{35}{24}$$
$$24y - 18 = -40x + 35$$
$$40x + 24y - 53 = 0$$

(b) Perpendicular: $m_2 = \frac{3}{5}$

$$y - \frac{3}{4} = \frac{3}{5}\left(x - \frac{7}{8}\right)$$
$$y - \frac{3}{4} = \frac{3}{5}x - \frac{21}{40}$$
$$40y - 30 = 24x - 21$$
$$0 = 24x - 40y + 9$$



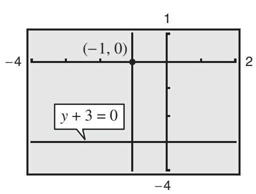
79. Given line: $y = -3$ is horizontal, $m = 0$

(a) Parallel: $m_1 = 0$

$$y - 0 = 0(x + 1)$$
$$y = 0$$

(b) Perpendicular: m_2 is undefined

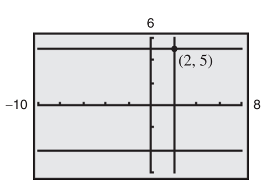
$$x = -1$$



80. Given line: $y + 4 = 0$ is horizontal, $m = 0$

(a) Parallel: $m_1 = 0$, $y - 5 = 0(x - 2)$, $y = 5$

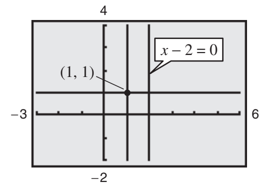
(b) Perpendicular: m_2 is undefined, $x = 2$



81. Given line: $x - 2 = 0$ is vertical, m is undefined

(a) Parallel: m_1 is undefined, $x = 1$

(b) Perpendicular: $m_2 = 0$, $y - 1 = 0(x - 1)$, $y = 1$

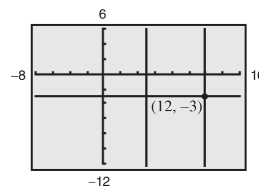


82. Given line: $x - 5 = 0$ is vertical, m is undefined.

(a) Parallel: m_1 is undefined, $x = 12$.

(b) Perpendicular:

$m_2 = 0$, $y + 3 = 0(x - 12)$, $y = -3$



83. (a) The average salary increased the most from 2008 to 2009 and increased the least from 2010 to 2011.

(b) 2008: $(8, 92{,}920)$ and 2013: $(13, 100{,}600)$

$$m = \frac{100{,}600 - 92{,}920}{13 - 8} = \frac{7680}{5} = \$1536/\text{yr}$$

(c) The average salary increased $1536 per year over the 5 years between 2008 and 2013.

84. (a) The revenue increased the greatest from 2010 to 2011 and increased the least from 2011 to 2012.

(b) 2009: $(9, 123.0)$ and 2013: $(13, 128.8)$

$$m = \frac{128.8 - 123.0}{13 - 9} = \frac{5.8}{4} = \$1.45 \text{ billion/yr}$$

(c) From 2009 to 2013, the revenue for AT&T increased $1.45 billion per year.

85. $\dfrac{6}{100} = \dfrac{x}{200}$

$12 = x$

Since the grade of the road is $\frac{6}{100}$, if you drive 200 feet, the vertical rise in the road will be 12 feet.

86. (a) $(0, 32), (100, 212)$

$$F - 32 = \frac{212 - 32}{100 - 0}(C - 0)$$
$$F = 1.8C + 32 = \frac{9}{5}C + 32$$

or

$$C = \frac{5}{9}(F - 32)$$

(b) Use $C = \frac{5}{9}(F - 32)$. If $F = 102.2°\text{F}$, then

$$C = \frac{5}{9}(102.2 - 32) = 39°\text{C}.$$

(c) Use $C = \frac{5}{9}(F - 32)$. If $F = 76°\text{F}$, then

$$C = \frac{5}{9}(76 - 32) = 24.4°\text{C}.$$

87. (a) 2009: $(9, 5655)$ and 2013: $(13, 5743)$

$$m = \frac{5743 - 5655}{13 - 9} = \frac{88}{4} = 22$$
$$y - y_1 = m(t - t_1)$$
$$y - 5655 = 22(t - 9)$$
$$y - 5655 = 22t - 198$$
$$y = 22t + 5457$$

The slope is 22.0 and indicates that the population increases 22 thousand per year from 2009 to 2013.

(b) Let $t = 11$.

$$y = 22(11) + 5457$$
$$y = 5699$$

The population was 5699 thousand or 5,699,000 in 2011.

(c) The actual population in 2011 was 5,709,000.

The model's estimate was very close to the actual population.

(d) The model could possibly be used to predict the population in 2018 if the population continues to grow at the same linear rate.

88. (a) 2008: $(8, 12{,}430)$ and 2013: $(13, 14{,}167)$

$$m = \frac{14{,}167 - 12{,}430}{13 - 8} = \frac{1737}{5} = 347.4$$
$$y - y_1 = m(t - t_1)$$
$$y - 12{,}430 = 347.4(t - 8)$$
$$y - 12{,}430 = 347.4t - 2779.2$$
$$y = 347.4t + 9650.8$$

The slope is 347.4 and indicates that the personal income increases \$347.3 billion per year from 2008 to 2013.

(b) Let $t = 11$.

$$y = 347.4(11) + 9650.8$$
$$y = 13{,}472.2$$

The personal income was \$13,472.2 billion in 2011.

Let $t = 14$.

$$y = 347.4(14) + 9650.8$$
$$y = 14{,}514.4$$

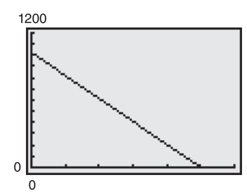
The personal income was \$14,514.4 billion in 2014.

(c) The actual personal income was \$13,202.0 billion in 2011 and \$14,728.6 billion in 2014.

The model's estimates were very close to the actual personal incomes in 2011 and 2014.

89. (a) The equipment depreciates $\frac{1025}{5} = \$205$ per year, so the value is $y = 1025 - 205t$, where $0 \le t \le 5$.

(b)



(c) When $t = 3$, the value is \$410.00.

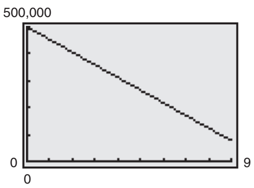
(d) The value is \$600 when $t = 2.07$ years.

90. (a) The slope is

$$\frac{77{,}000 - 500{,}000}{9} = \frac{-423{,}000}{9} = -47{,}000.$$

The equipment depreciates \$47,000 per year, so the value is $y = 500{,}000 - 47{,}000t$, where $0 \le t \le 9$.

(b)



(c) When $t = 5$, the value is

$$y = 500{,}000 - 47{,}000(5) = \$265{,}000.$$

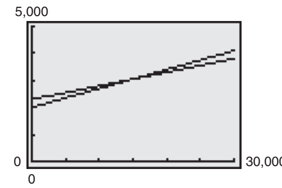
(d) The value is \$160,000 when

$$160{,}000 = 500{,}000 - 47{,}000t$$
$$47{,}000t = 340{,}000$$
$$t \approx 7.2 \text{ years.}$$

91. (a) Current wage: $W_c = 0.07s + 2000$

New offer wage: $W_N = 0.05s + 2300$

(b)



The lines intersect at $(15{,}000, 3050)$. If you sell \$15,000, then both jobs would yield wages of \$3050.

(c) No. Your current job would yield wages of \$3400 as compared to the new job, which would yield wages of \$3300 if your sales are \$20,000.

92. (a) Matches (ii); $y = -10x + 100$.

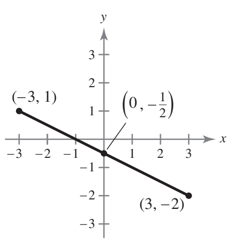
(b) Matches (iii); $y = 1.50x + 12.50$.

(c) Matches (i); $y = 0.51x + 30$.

(d) Matches (iv); $y = -100x + 600$.

Chapter 1 Quiz Yourself

1. (a)



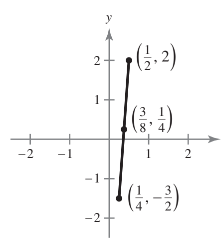
(b) $d = \sqrt{(-3 - 3)^2 + (1 - (-2))^2}$

$= \sqrt{36 + 9}$

$= 3\sqrt{5}$

(c) $\text{Midpoint} = \left(\frac{-3 + 3}{2}, \frac{1 - 2}{2}\right) = \left(0, -\frac{1}{2}\right)$

2. (a)



(b) $d = \sqrt{\left(\frac{1}{2} - \frac{1}{4}\right)^2 + \left(2 - \left(-\frac{3}{2}\right)\right)^2}$

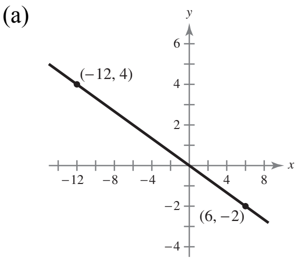
$= \sqrt{\frac{1}{16} + \frac{49}{4}}$

$= \sqrt{\frac{197}{16}}$

$= \frac{1}{4}\sqrt{197}$

(c) $\text{Midpoint} = \left(\frac{\frac{1}{2} + \frac{1}{4}}{2}, \frac{2 - \frac{3}{2}}{2}\right) = \left(\frac{3}{8}, \frac{1}{4}\right)$

3. (a)



(b) $d = \sqrt{(6-(-12))^2 + (-2-4)^2}$

$= \sqrt{18^2 + (-6)^2}$

$= \sqrt{324 + 36}$

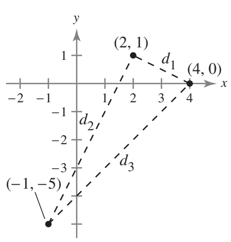
$= \sqrt{360}$

$= 6\sqrt{10}$

≈ 18.97

(c) Midpoint $= \left(\dfrac{-12+6}{2}, \dfrac{4+(-2)}{2}\right) = (-3, 1)$

4.



$a = \sqrt{(2-4)^2 + (1-0)^2} = \sqrt{5}$

$b = \sqrt{(2-(-1))^2 + (1-(-5))^2} = 3\sqrt{5}$

$c = \sqrt{(-1-4)^2 + (-5-0)^2} = 5\sqrt{2}$

$a^2 + b^2 = (\sqrt{5})^2 + (3\sqrt{5})^2 = (5\sqrt{2})^2 = c^2$

5. $(2011, 9810)$ and $(2013, 9992)$

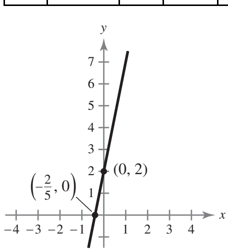
Midpoint $= \left(\dfrac{2011+2013}{2}, \dfrac{9810+9992}{2}\right)$

$= (2012, 9901)$

The population in 2012 was approximately 9901 thousand or 9,901,000.

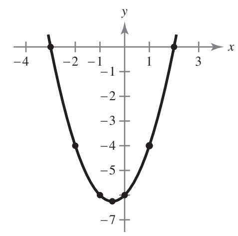
6. $y = 5x + 2$

x	$-\frac{2}{5}$	0	$\frac{1}{5}$	1
y	0	2	3	7



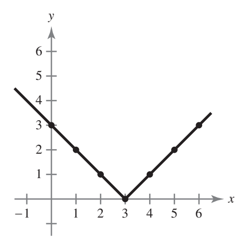
7. $y = x^2 + x - 6$

x	−3	−2	−1	−0.5	0	1	2
y	0	−4	−6	−6.25	−6	−4	0



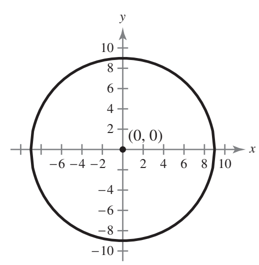
8. $y = |x - 3|$

x	0	1	2	3	4	5	6
y	3	2	1	0	1	2	3



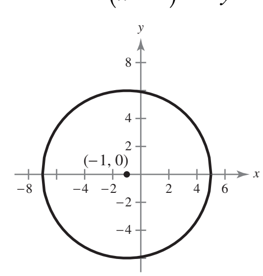
9. $(x-0)^2 + (y-0)^2 = 9^2$

$x^2 + y^2 = 81$



10. $(x-(-1))^2 + (y-0)^2 = 6^2$

$(x+1)^2 + y^2 = 36$



11. The radius is the distance between $(2, -2)$ and $(-1, 2)$.

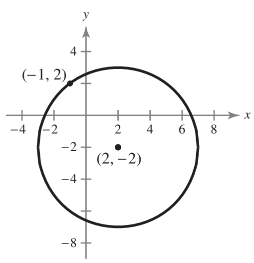
$$r = \sqrt{(-1-2)^2 + \left(2-(-2)^2\right)}$$

$$r = \sqrt{(-3)^2 + 4^2} = \sqrt{9+16} = \sqrt{25} = 5$$

Using the center $(2, -2)$ and radius $r = 5$:

$$(x-2)^2 + \left(y-(-2)\right)^2 = 5^2$$

$$(x-2)^2 + (y+2)^2 = 25$$



12. $C = 4.55x + 12{,}500$

$R = 7.19x$

$R = C$

$7.19x = 4.55x + 12{,}500$

$2.64x = 12{,}500$

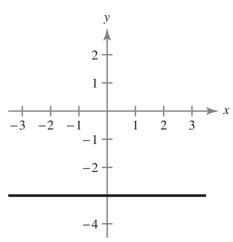
$x \approx 4734.8$

The company must sell 4735 units to break even.

13. $y = mx + b$

$y = 0x - 3$

$y = -3$

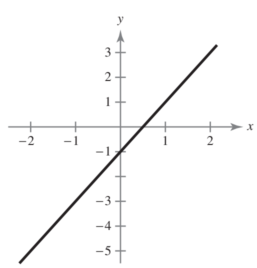


14. $y - y_1 = m(x - x_1)$

$y - 1 = 2(x - 1)$

$y - 1 = 2x - 2$

$y = 2x - 1$

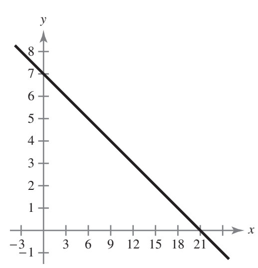


15. $y - y_1 = m(x - x_1)$

$y - 5 = -\frac{1}{3}(x - 6)$

$y - 5 = -\frac{1}{3}x + 2$

$y = -\frac{1}{3}x + 7$

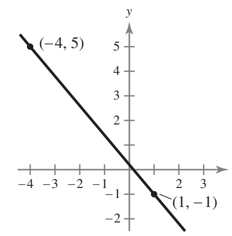


16. $(1, -1), (-4, 5)$

$$m = \frac{5+1}{-4-1} = -\frac{6}{5}$$

$$y + 1 = -\frac{6}{5}(x - 1)$$

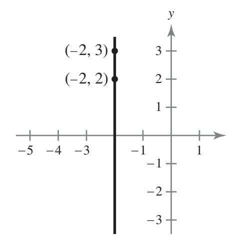
$$y = -\frac{6}{5}x + \frac{1}{5}$$



17. $(-2, 3), (-2, 2)$

$$m = \frac{2-3}{-2+2} = \text{undefined}$$

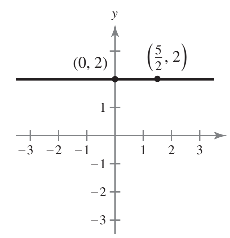
Because the slope is undefined, the line is vertical and its equation is $x = -2$.



18. $\left(\frac{5}{2}, 2\right), (0, 2)$

$$m = \frac{2-2}{0-\frac{5}{2}} = 0$$

Because the slope is 0, the line is horizontal and its equation is $y = 2$.



19. Given line: $y = -\frac{1}{4}x - \frac{1}{2}$, $m = -\frac{1}{4}$

(a) Parallel: $m_1 = -\frac{1}{4}$

$y + 5 = -\frac{1}{4}(x - 3)$

$y = -\frac{1}{4}x - \frac{17}{4}$

(b) Perpendicular: $m_2 = 4$

$y + 5 = 4(x - 3)$

$y = 4x - 17$

20. Let $t = 11$ correspond to 2011.

$(11, 1{,}330{,}000), (15, 1{,}800{,}000)$

$$m = \frac{1{,}800{,}000 - 1{,}330{,}000}{15 - 11} = \frac{470{,}000}{4} = 117{,}500$$

$y - 1{,}330{,}000 = 117{,}500(x - 11)$

$y - 1{,}330{,}000 = 117{,}500x - 1{,}292{,}500$

$y = 117{,}500x + 37{,}500$

For 2019, let $t = 19$.

$y = 117{,}500(19) + 37{,}500 = \$2{,}270{,}000$

For 2022, let $t = 22$.

$y = 117{,}500(22) + 37{,}500 = \$2{,}622{,}500$

21. The daily cost C equals the cost for lodging and meals plus the cost per mile driven, x.

$C = 218 + 0.56x$

22. (a) Let $t = 9$ correspond to 2013 and S equal salary.

2013: $(13, 35{,}700)$ and 2015: $(15, 39{,}100)$

$$m = \frac{39{,}100 - 35{,}700}{15 - 13} = \frac{3400}{2} = 1700$$

$$S - S_1 = m(t - t_1)$$

$$S - 35{,}700 = 1700(t - 13)$$

$$S - 35{,}700 = 1700t - 22{,}100$$

$$S = 1700t + 13{,}600$$

(b) For 2020, let $t = 20$.

$$S = 1700(20) + 13{,}600 = \$47{,}600$$

Section 1.4 Functions

Skills Warm Up

1. $5(-1)^2 - 6(-1) + 9 = 5(1) + 6 + 9 = 20$

2. $(-2)^3 + 4(-2)^2 - 12 = -8 + 4(4) - 12 = -8 + 16 - 12 = -4$

3. $(x - 2)^2 + 5x - 10 = x^2 - 4x + 4 + 5x - 10 = x^2 + x - 6$

4.
$$\begin{aligned}(3 - x) + (x + 3)^3 &= (3 - x) + (x + 3)(x^2 + 6x + 9)\\ &= (3 - x) + x^3 + 3x^2 + 6x^2 + 18x + 9x + 27\\ &= x^3 + 9x^2 + 26x + 30\end{aligned}$$

5. $\dfrac{1}{1 - (1 - x)} = \dfrac{1}{1 - 1 + x} = \dfrac{1}{x}$

6. $3 + \dfrac{2x - 7}{x} = \dfrac{3x}{x} + \dfrac{2x - 7}{x} = \dfrac{3x + 2x - 7}{x} = \dfrac{5x - 7}{x}$

7.
$$\begin{aligned}2x + y - 6 &= 11\\ y &= -2x + 17\end{aligned}$$

8.
$$\begin{aligned}5y - 6x^2 - 1 &= 0\\ 5y &= 6x^2 + 1\\ y &= \frac{6x^2 + 1}{5}\\ &= \frac{6}{5}x^2 + \frac{1}{5}\end{aligned}$$

9.
$$\begin{aligned}(y - 3)^2 &= 5 + (x + 1)^2\\ y - 3 &= \sqrt{5 + (x + 1)^2}\\ y - 3 &= \sqrt{5 + x^2 + 2x + 1}\\ y &= \sqrt{x^2 + 2x + 6} + 3\end{aligned}$$

10.
$$\begin{aligned}y^2 - 4x^2 &= 2\\ y^2 &= 2 + 4x^2\\ y &= \sqrt{2 + 4x^2}\end{aligned}$$

11.
$$\begin{aligned}x &= \frac{2y - 1}{4}\\ 4x &= 2y - 1\\ 4x + 1 &= 2y\\ \frac{4x + 1}{2} &= y\\ 2x + \frac{1}{2} &= y\end{aligned}$$

12.
$$\begin{aligned}x &= \sqrt[3]{2y - 1}\\ x^3 &= 2y - 1\\ -2y &= -x^3 - 1\\ y &= \tfrac{1}{2}x^3 + \tfrac{1}{2}\end{aligned}$$

1. $x^2 + y^2 = 16$

$$y^2 = 16 - x^2$$

$$y = \pm\sqrt{16 - x^2}$$

y is *not* a function of x since there are two values of y for some x.

2. $y = \pm\sqrt{4 - x}$

y is *not* a function of x since there are two values of y for some x.

3. $\frac{1}{2}x - 6y = -3$

$$y = \frac{1}{12}x + \frac{1}{2}$$

y *is* a function of x since there is only one value of y for each x.

4. $y = \dfrac{3x + 5}{2}$

y *is* a function of x since there is only one value of y for each x.

5. $y = 4 - x^2$

y *is* a function of x since there is only one value of y for each x.

6. $x^2 + y^2 + 2x = 0$

$$y^2 = -x^2 - 2x$$

$$y = \pm\sqrt{-x^2 - 2x}$$

y is *not* a function of x since there are two values of y for some x.

7. $y = |x + 2|$

y *is* a function of x since there is only one value of y for each x.

8. $x^2y^2 - 3x^2 + 4y^2 = 0$

$$x^2y^2 + 4y^2 = 3x^2$$

$$y^2(x^2 + 4) = 3x^2$$

$$y^2 = \frac{3x^2}{x^2 + 4}$$

$$y = \pm\sqrt{\frac{3x^2}{x^2 + 4}}$$

y is *not* a function of x since are two values of y for some x.

9. y is *not* a function of x.

10. y *is* a function of x.

11. y *is* a function of x.

12. y is *not* a function of x.

13. Domain: $(-\infty, \infty)$

Range: $(-\infty, \infty)$

14. Domain: $\left[\frac{3}{2}, \infty\right)$

Range: $[0, \infty)$

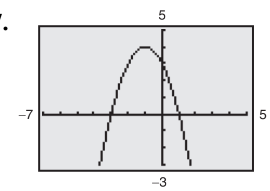
15. Domain: $[-2, 2]$

Range: $[0, 2]$

16. Domain: $(-\infty, \infty)$

Range: $[0, \infty)$

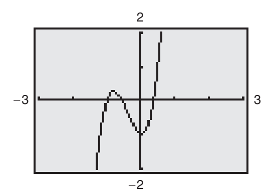
17.



Domain: $(-\infty, \infty)$

Range: $(-\infty, 4]$

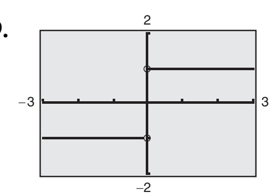
18.



Domain: $(-\infty, \infty)$

Range: $(-\infty, \infty)$

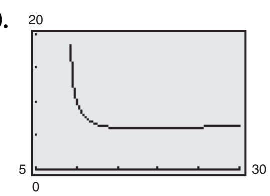
19.



Domain: $(-\infty, 0) \cup (0, \infty)$

Range: $\{-1, 1\}$

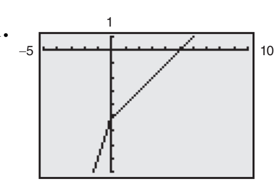
20.



Domain: $(9, \infty)$

Range: $[18, \infty)$

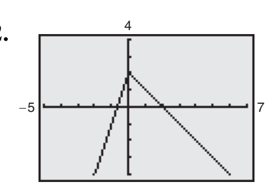
21.



Domain: $(-\infty, \infty)$

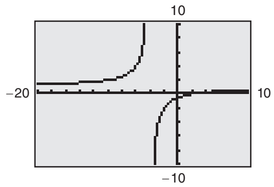
Range: $(-\infty, \infty)$

22.



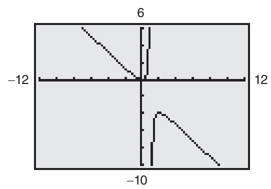
Domain: $(-\infty, \infty)$

Range: $(-\infty, 2]$

23. 

Domain: $(-\infty, -4) \cup (-4, \infty)$

Range: $(-\infty, 1) \cup (1, \infty)$

24. 

Domain: $(-\infty, 1) \cup (1, \infty)$

Range: $(-\infty, -4] \cup [0, \infty)$

25. $f(x) = 3x - 2$

(a) $f(0) = 3(0) - 2 = -2$

(b) $f(5) = 3(5) - 2 = 13$

(c) $f(x-1) = 3(x-1) - 2 = 3x - 3 - 2 = 3x - 5$

26. $f(x) = x^2 - 4x + 1$

(a) $f(-1) = (-1)^2 - 4(-1) + 1 = 6$

(b) $f\left(\frac{1}{2}\right) = \left(\frac{1}{2}\right)^2 - 4\left(\frac{1}{2}\right) + 1 = -\frac{3}{4}$

(c) $$\begin{aligned} f(c+2) &= (c+2)^2 - 4(c+2) + 1 \\ &= c^2 + 4c + 4 - 4c - 8 + 1 \\ &= c^2 - 3 \end{aligned}$$

27. $g(x) = \dfrac{1}{x}$

(a) $g\left(\dfrac{1}{5}\right) = \dfrac{1}{\frac{1}{5}} = 5$

(b) $g(-0.6) = \dfrac{1}{-0.6} = -\dfrac{5}{3}$

(c) $g(x+4) = \dfrac{1}{x+4}$

28. $f(x) = |x| + 4$

(a) $f(-3) = |-3| + 4 = 7$

(b) $f(0.8) = |0.8| + 4 = 4.8$

(c) $f(x+2) = |x+2| + 4$

29. $$\begin{aligned} &\frac{f(x+\Delta x) - f(x)}{\Delta x} \\ &= \frac{(x+\Delta x)^2 - 5(x+\Delta x) + 2 - (x^2 - 5x + 2)}{\Delta x} \\ &= \frac{[x^2 + 2x\Delta x + (\Delta x)^2 - 5x + 5\Delta x + 2] - [x^2 - 5x + 2]}{\Delta x} \\ &= \frac{2x\Delta x + (\Delta x)^2 + 5\Delta x}{\Delta x} \\ &= 2x + \Delta x + 5, \ \Delta x \neq 0 \end{aligned}$$

30. $$\begin{aligned} &\frac{h(x+\Delta x) - h(x)}{\Delta x} \\ &= \frac{(x+\Delta x)^2 + (x+\Delta x) + 3 - (x^2 + x + 3)}{\Delta x} \\ &= \frac{\left[x^2 + 2x\Delta x + (\Delta x)^2 + x + \Delta x + 3\right] - \left[x^2 + x + 3\right]}{\Delta x} \\ &= \frac{2x\Delta x + (\Delta x)^2 + \Delta x}{\Delta x} \\ &= \frac{\Delta x(2x + \Delta x + 1)}{\Delta x} \\ &= 2x + \Delta x + 1, \ \Delta x \neq 0 \end{aligned}$$

31. $$\begin{aligned} &\frac{g(4+\Delta x) - g(4)}{\Delta x} \\ &= \frac{\sqrt{4 + \Delta x + 1} - \sqrt{4+1}}{\Delta x} \\ &= \frac{\sqrt{\Delta x + 5} - \sqrt{5}}{\Delta x} \cdot \frac{\sqrt{\Delta x + 5} + \sqrt{5}}{\sqrt{\Delta x + 5} + \sqrt{5}} \\ &= \frac{(\Delta x + 5) - 5}{\Delta x\left[\sqrt{\Delta x + 5} + \sqrt{5}\right]} \\ &= \frac{\Delta x}{\Delta x\left[\sqrt{\Delta x + 5} + \sqrt{5}\right]} \\ &= \frac{1}{\sqrt{\Delta x + 5} + \sqrt{5}}, \ \Delta x \neq 0 \end{aligned}$$

32. $$\begin{aligned} \frac{f(x) - f(2)}{x-2} &= \frac{\frac{1}{\sqrt{x}} - \frac{1}{\sqrt{2}}}{x-2} \\ &= \frac{\frac{1}{\sqrt{x}} \cdot \frac{2\sqrt{x}}{2\sqrt{x}} - \frac{1}{\sqrt{2}} \cdot \frac{x\sqrt{2}}{x\sqrt{2}}}{x-2} \\ &= \frac{\frac{2\sqrt{x} - x\sqrt{2}}{2x}}{x-2} \\ &= \frac{2\sqrt{x} - x\sqrt{2}}{2x(x-2)} \end{aligned}$$

33. $$\frac{f(x+\Delta x)-f(x)}{\Delta x} = \frac{\dfrac{1}{x+\Delta x-2}-\dfrac{1}{x-2}}{\Delta x} = \frac{(x-2)-(x+\Delta x-2)}{(x+\Delta x-2)(x-2)\Delta x} = \frac{-1}{(x+\Delta x-2)(x-2)},\ \Delta x \neq 0$$

34. $$\frac{f(x+\Delta x)-f(x)}{\Delta x} = \frac{\dfrac{1}{x+\Delta x+4}-\dfrac{1}{x+4}}{\Delta x} = \frac{(x+4)-(x+\Delta x+4)}{\Delta x[x+\Delta x+4][x+4]} = \frac{-1}{(x+\Delta x+4)(x+4)},\ \Delta x \neq 0$$

35. (a) $f(x)+g(x) = (2x-5)+(4-3x) = -x-1$

(b) $f(x)-g(x) = (2x-5)-(4-3x) = 5x-9$

(c) $f(x)\cdot g(x) = (2x-5)(4-3x) = 8x-6x^2-20+15x = -6x^2+23x-20$

(d) $f(x)/g(x) = \dfrac{2x-5}{4-3x},\ x \neq \dfrac{3}{4}$

(e) $f(g(x)) = f(4-3x) = 2(4-3x)-5 = 8-6x-5 = -6x+3$

(f) $g(f(x)) = g(2x-5) = 4-3(2x-5) = 4-6x+15 = -6x+19$

36. (a) $f(x)+g(x) = x^2+5+\sqrt{1-x},\ x \leq 1$

(b) $f(x)-g(x) = x^2+5-\sqrt{1-x},\ x \leq 1$

(c) $f(x)\cdot g(x) = \left(\sqrt{1-x}\right)(x^2+5),\ x \leq 1$

(d) $f(x)/g(x) = \dfrac{x^2+5}{\sqrt{1-x}},\ x \leq 1$

(e) $f(g(x)) = 6-x,\ x \leq 1$

(f) $g(f(x))$ is undefined

37. (a) $f(x)+g(x) = x^2+1+x-1 = x^2+x$

(b) $f(x)-g(x) = x^2+1-x+1 = x^2-x+2$

(c) $f(x)\cdot g(x) = (x^2+1)(x-1) = x^3-x^2+x-1$

(d) $f(x)/g(x) = \dfrac{x^2+1}{x-1},\ x \neq 1$

(e) $f(g(x)) = (x-1)^2+1 = x^2-2x+2$

(f) $g(f(x)) = x^2+1-1 = x^2$

38. (a) $f(x)+g(x) = \dfrac{x^4+x^3+x}{x+1},\ x \neq -1$

(b) $f(x)-g(x) = \dfrac{x-x^3-x^4}{x+1},\ x \neq -1$

(c) $f(x)\cdot g(x) = \dfrac{x^4}{x+1},\ x \neq -1$

(d) $f(x)/g(x) = \dfrac{x}{x^4+x^3},\ x \neq 0, -1$

(e) $f(g(x)) = \dfrac{x^3}{x^3+1},\ x \neq -1$

(f) $g(f(x)) = \dfrac{x^3}{(x+1)^3},\ x \neq -1$

39. (a) $f(g(1)) = f(1^2-1) = f(0) = \sqrt{0} = 0$

(b) $g(f(1)) = g(\sqrt{1}) = g(1) = 1^2-1 = 0$

(c) $g\left(f\left(\frac{1}{2}\right)\right) = g\left(\sqrt{\frac{1}{2}}\right) = \left(\sqrt{\frac{1}{2}}\right)^2-1 = \frac{1}{2}-1 = -\frac{1}{2}$

(d) $f\left(g\left(-\sqrt{5}\right)\right) = f(4) = \sqrt{4} = 2$

(e) $f(g(x)) = f(x^2-1) = \sqrt{x^2-1}$

(f) $g(f(x)) = g(\sqrt{x}) = x-1,\ x \geq 0$

40. (a) $f(g(2)) = f(3) = \dfrac{1}{3}$

(b) $g(f(2)) = g\left(\dfrac{1}{2}\right) = -\dfrac{3}{4}$

(c) $f(g(-3)) = f(8) = \dfrac{1}{8}$

(d) $g\left(f\left(\dfrac{1}{\sqrt{2}}\right)\right) = g(\sqrt{2}) = 1$

(e) $f(g(x)) = \dfrac{1}{x^2-1},\ x \neq \pm 1$

(f) $g(f(x)) = \dfrac{1}{x^2}-1,\ x \neq 0$

41. $f(x) = 4x$

$f^{-1}(x) = \frac{1}{4}x$

$f(f^{-1}(x)) = 4\left(\frac{1}{4}x\right) = x$

$f^{-1}(f(x)) = \frac{1}{4}(4x) = x$

42. $f(x) = \frac{1}{3}x$

$f^{-1}(x) = 3x$

$f(f^{-1}(x)) = \frac{1}{3}(3x) = x$

$f^{-1}(f(x)) = 3\left(\frac{1}{3}x\right) = x$

43. $f(x) = x + 12$

$f^{-1}(x) = x - 12$

$f(f^{-1}(x)) = (x - 12) + 12 = x$

$f^{-1}(f(x)) = (x + 12) - 12 = x$

44. $f(x) = x - 3$

$f^{-1}(x) = x + 3$

$f(f^{-1}(x)) = (x + 3) - 3 = x$

$f^{-1}(f(x)) = (x - 3) + 3 = x$

45. $f(x) = 2x - 3 = y$

$2y - 3 = x$

$2y = x + 3$

$y = \frac{x + 3}{2}$

$f'(x) = \frac{1}{2}x + \frac{3}{2}$

46. $f(x) = 5 - \frac{3}{4}x = y$

$5 - \frac{3}{4}y = x$

$-\frac{3}{4}y = x - 5$

$-3y = 4x - 20$

$y = -\frac{3}{4}x + \frac{20}{3}$

$f'(x) = -\frac{4}{3}x + \frac{20}{3}$

47. $f(x) = \frac{3}{2}x + 1 = y$

$\frac{3}{2}y + 1 = x$

$3y + 2 = 2x$

$3y = 2x - 2$

$y = \frac{2}{3}x - \frac{2}{3}$

$f^{-1}(x) = \frac{2}{3}x - \frac{2}{3}$

48. $f(x) = -6x - 4 = y$

$-6y - 4 = x$

$-6y = x + 4$

$y = -\frac{1}{6}x - \frac{2}{3}$

$f^{-1}(x) = -\frac{1}{6}x - \frac{2}{3}$

49. $f(x) = x^5 = y$

$y^5 = x$

$y = \sqrt[5]{x}$

$f^{-1}(x) = \sqrt[5]{x}$

50. $f(x) = x^3 = y$

$y^3 = x$

$y = \sqrt[3]{x}$

$f^{-1}(x) = \sqrt[3]{x}$

51. $f(x) = \frac{1}{x} = y$

$\frac{1}{y} = x$

$y = \frac{1}{x}$

$f^{-1}(x) = \frac{1}{x}$

52. $f(x) = -\frac{2}{x} = y$

$-\frac{2}{y} = x$

$-\frac{1}{y} = \frac{1}{2}x$

$y = -\frac{2}{x}$

$f^{-1}(x) = -\frac{2}{x}$

53. $f(x) = \sqrt{9 - x^2} = y, 0 \le x \le 3$

$\sqrt{9 - y^2} = x$

$9 - y^2 = x^2$

$y^2 = 9 - x^2$

$y = \sqrt{9 - x^2}$

$f^{-1}(x) = \sqrt{9 - x^2},\ \ 0 \le x \le 3$

54. $f(x) = \sqrt{x^2 - 4} = y,\ x \ge 2$

$\sqrt{y^2 - 4} = x$

$y^2 = x^2 + 4$

$y = \sqrt{x^2 + 4},\ x \ge 0$

$f^{-1}(x) = \sqrt{x^2 + 4},\ x \ge 0$

55. $f(x) = x^{2/3} = y,\ x \geq 0$

$y^{2/3} = x$

$y = x^{3/2}$

$f^{-1}(x) = x^{3/2}$

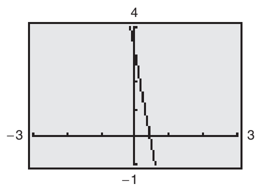
56. $f(x) = x^{3/5} = y$

$y^{3/5} = x$

$y = x^{5/3}$

$f^{-1}(x) = x^{5/3}$

57.



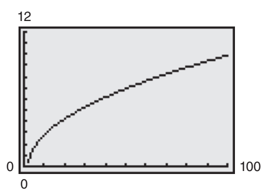
$f(x) = 3 - 7x$ is one-to-one.

$y = 3 - 7x$

$x = 3 - 7y$

$y = \dfrac{3 - x}{7}$

58.



$f(x) = \sqrt{x - 2}$ is one-to-one.

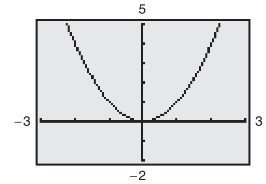
$y = \sqrt{x - 2}$

$x = \sqrt{y - 2}$

$x^2 = y - 2$

$y = x^2 + 2,\ x \geq 0$

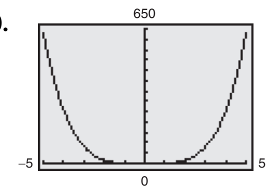
59.



$f(x) = x^2$

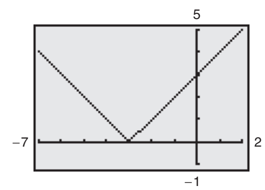
f is *not* one-to-one because $f(1) = 1 = f(-1)$.

60.



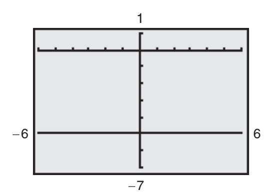
$f(x) = x^4$ is *not* one-to-one because $f(2) = 16 = f(-2)$.

61. $f(x) = |x + 3|$

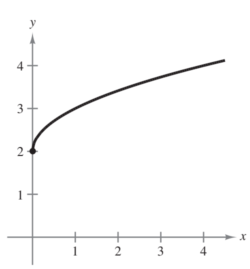


f is *not* one-to-one because $f(-5) = 2 = f(-1)$.

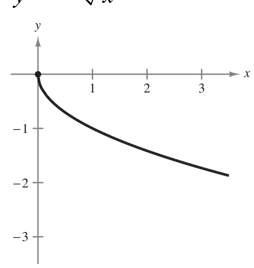
62. $f(x) = -5$ is *not* one-to-one because $f(1) = -5 = f(-1)$.



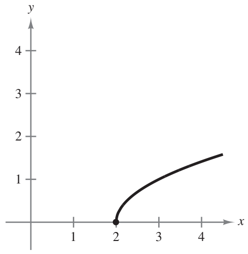
63. (a) $y = \sqrt{x} + 2$



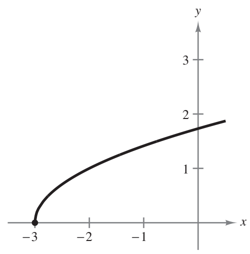
(b) $y = -\sqrt{x}$



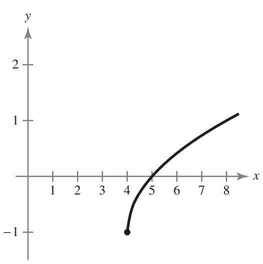
(c) $y = \sqrt{x - 2}$



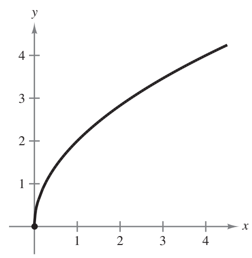
(d) $y = \sqrt{x + 3}$



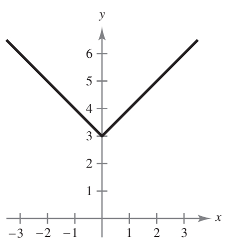
(e) $y = \sqrt{x - 4} - 1$



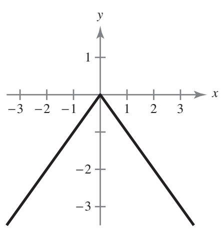
(f) $y = 2\sqrt{x}$



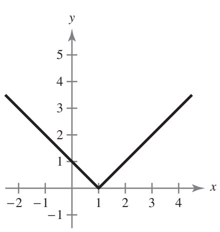
64. (a)



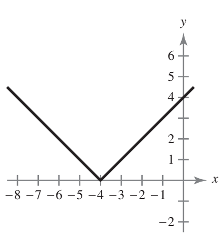
(b)



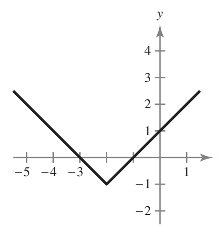
(c)



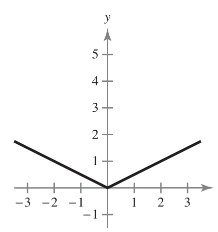
(d)



(e)



(f)



65. (a) Shifted three units to the left: $y = (x + 3)^2$

(b) Shifted six units to the left, three units downward, and reflected: $y = -(x + 6)^2 - 3$

66. (a) Stretched by a factor of $\frac{1}{8}$: $y = \frac{1}{8}x^3$

(b) Stretched by a factor of 2, and reflected: $y = -2x^3$

67. (a) 2004: \$190 billion
2006: \$225 billion
2013: \$270 billion

(b) 2004: $d(4) = 15.73(4) + 128.3 = \$191.22$ billion

2006: $d(6) = -0.620(6)^2 + 18.11(6) + 138.8 = \$225.14$ billion

2013: $d(13) = -0.620(13)^2 + 18.11(13) + 138.8 = \$269.45$ billion

The model fits the data well.

68. (a) T is a function of t because for any value of t (time of day), there is exactly one value of T (temperature in the house).

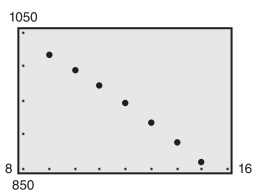
(b) $T(4) \approx 60°\text{F}$

$T(15) \approx 72°\text{F}$

(c) H is a horizontal shift of T, 1 hour to the right or 1 hour later.

(d) H is a vertical shift of T, 1 degree upward or 1 degree increase in setting.

69.
$$\begin{aligned} R_{\text{TOTAL}} &= R_1 + R_2 \\ &= 690 - 8t - 0.8t^2 + 458 + 0.78t \\ &= -0.8t^2 - 7.22t + 1148,\ t = 9, 10, \ldots, 15 \end{aligned}$$



70.
$$\begin{aligned} B(t) - D(t) &= 4.917t^3 - 124.71t^2 - 925.9t + 2308 - \left(-7.083t^3 + 222.64t^2 - 281.8t + 10{,}104\right) \\ &= 12.0t^3 - 347.35t^2 + 3207.7t - 7796 \end{aligned}$$

$B(t) - D(t)$ is the function that yields the increase and/or decrease in people living in the United States from 2008 to 2012.

71. (a) $C = 2.89x + 8000$

(b) $\overline{C} = \dfrac{C}{x} = \dfrac{2.89x + 8000}{x} = 2.89 + \dfrac{8000}{x}$

(c)
$$\begin{aligned} 2.89 + \frac{8000}{x} &< 6.89 \\ \frac{8000}{x} &< 4 \\ \frac{8000}{4} &< x \text{ because } x > 0. \\ 2000 &< x \end{aligned}$$

Must sell 2000 units before the average cost per unit falls below the selling price.

72. (a) $1 + 0.01x = \dfrac{14.75}{p}$

$$x = \frac{(14.75/p) - 1}{0.01} = \frac{14.75 - p}{0.01p} = \frac{100(14.75 - p)}{p} = \frac{1475}{p} - 100$$

(b) $x = \dfrac{100(14.75 - 10)}{10} = 47.5 \approx 48$ units

73. (a) $C(x) = 70x + 500$

$x(t) = 40t$

$C(x(t)) = 70(40t) + 500 = 2800t + 500$

C is the weekly cost per t hours of production.

(b) $C(x(4)) = 2800(4) + 500 = \$11{,}700$

(c) $C(x(t)) = 18{,}000$

$2800t + 500 = 18{,}000$

$2800t = 17{,}500$

$t = \dfrac{17{,}500}{2800} = 6.25$ hr

74. (a) Revenue $= R = rn = [15 - 0.05(n - 80)]n = 19n - 0.05n^2$

(b)

n	100	125	150	175	200	225	250
R	1400	1593.75	1725	1793.75	1800	1743.75	1625

(c) The revenue increases and then decreases as n gets larger, so it is not a good formula for the bus company to use.

75. (a) Cost $= C = 98{,}000 + 12.30x$

(b) Revenue $= R = 17.98x$

(c) Profit $= R - C = 17.98x - (12.30x + 98{,}000) = 5.68x - 98{,}000$

76. (a) If $0 \le x \le 100$, then $p = 90$. If $100 < x \le 1600$, then $p = 90 - 0.01(x - 100) = 91 - 0.01x$. If $x > 1600$, then $p = 75$. Thus,

$$p = \begin{cases} 90, & 0 \le x \le 100 \\ 91 - 0.01x, & 100 < x \le 1600. \\ 75, & x > 1600 \end{cases}$$

(b) $P = px - 60x$

$$P = \begin{cases} 90x - 60x, & 0 \le x \le 100 \\ (91 - 0.01x)x - 60x, & 100 < x \le 1600 \\ 75x - 60x, & x > 1600 \end{cases}$$

$$= \begin{cases} 30x, & 0 \le x \le 100 \\ 31x - 0.01x^2, & 100 < x \le 1600 \\ 15x, & x > 1600 \end{cases}$$

Answers for Exercises 77–84 are not unique. Sample answers are given.

77. $f(x) = (x - 1)^2,\ x \ge 1$

$y = (x - 1)^2$

$x = (y - 1)^2$

$\pm\sqrt{x} = y - 1$

$1 \pm \sqrt{x} = y$

$1 + \sqrt{x} = f^{-1}(x)$

Domain of f: $[1, \infty)$

Range of f: $[0, \infty)$

Domain of f^{-1}: $[0, \infty)$

Range of f^{-1}: $[1, \infty)$

78. $f(x) = (x + 2)^2,\ x \geq -2$

$$y = (x + 2)^2$$
$$x = (x + 2)^2$$
$$\pm\sqrt{x} = x + 2$$
$$-2 \pm \sqrt{x} = y$$
$$-2 + \sqrt{x} = f^{-1}(x)$$

Domain of f: $[-2, \infty)$

Range of f: $[0, \infty)$

Domain of f^{-1}: $[0, \infty)$

Range of f^{-1}: $[-2, \infty)$

79. $f(x) = |x + 4|,\ x \geq -4$

$$y = |x + 4|$$
$$x = |y + 4|$$
$$\pm x = y + 4$$
$$-4 \pm x = y$$
$$x - 4 = f^{-1}(x)$$

Domain of f: $[-4, \infty)$

Range of f: $[0, \infty)$

Domain of f^{-1}: $[0, \infty)$

Range of f^{-1}: $[-4, \infty)$

80. $f(x) = |x - 3|,\ x \geq 3$

$$y = |x - 3|$$
$$x = |y - 3|$$
$$\pm x = y - 3$$
$$3 \pm x = y$$
$$x + 3 = f^{-1}(x)$$

Domain of f: $[3, \infty)$

Range of f: $[0, \infty)$

Domain of f^{-1}: $[0, \infty)$

Range of f^{-1}: $[3, \infty)$

81. $f(x) = -2x^2 + 1,\ x \geq 0$

$$y = -2x^2 + 1$$
$$x = -2y^2 + 1$$
$$x - 1 = -2y^2$$
$$\frac{x - 1}{-2} = y^2$$
$$\frac{1 - x}{2} = y^2$$
$$\pm\sqrt{\frac{1 - x}{2}} = y$$
$$\sqrt{\frac{1 - x}{2}} = f^{-1}(x)$$
$$\frac{\sqrt{2(1 - x)}}{2} = f^{-1}(x)$$

Domain of f: $[0, \infty)$

Range of f: $(-\infty, 1]$

Domain of f^{-1}: $(-\infty, 1]$

Range of f^{-1}: $[0, \infty)$

82. $f(x) = \frac{1}{2}x^2 - 4,\ x \geq 0$

$$y = \tfrac{1}{2}x^2 - 4$$
$$x = \tfrac{1}{2}y^2 - 4$$
$$x + 4 = \tfrac{1}{2}y^2$$
$$2(x + 4) = y^2$$
$$\pm\sqrt{2(x + 4)} = y$$
$$\sqrt{2(x + 4)} = f^{-1}(x)$$

Domain of f: $[0, \infty)$

Range of f: $(-4, \infty]$

Domain of f^{-1}: $(-4, \infty]$

Range of f^{-1}: $[0, \infty)$

83.
$$\begin{aligned} f(x) &= |x+1| - 2,\ x \ge -1 \\ y &= |x+1| - 2 \\ x &= |y+1| - 2 \\ x+2 &= |y+1| \\ \pm(x+2) &= y+1 \\ -1 \pm (x+2) &= y \\ -1 + (x+2) &= f^{-1}(x) \\ x+1 &= f^{-1}(x) \end{aligned}$$

Domain of f: $[-1, \infty)$

Range of f: $[-2, \infty)$

Domain of f^{-1}: $[-2, \infty)$

Range of f^{-1}: $[-1, \infty)$

84.
$$\begin{aligned} f(x) &= -|x-2| + 3,\ x \ge 2 \\ y &= -|x-2| + 3 \\ x &= -|y-2| + 3 \\ x-3 &= -|y-2| \\ -(x-3) &= |y-2| \\ \mp(x-3) &= y-2 \\ 2 \mp (x-3) &= f^{-1}(x) \\ 5 - x &= f^{-1}(x) \end{aligned}$$

Domain of f: $[2, \infty)$

Range of f: $(-\infty, 3]$

Domain of f^{-1}: $(-\infty, 3]$

Range of f^{-1}: $[2, \infty)$

85. Answers will vary.

Section 1.5 Limits

Skills Warm Up

1. $\dfrac{2x^3 + x^2}{6x} = \dfrac{x^2(2x+1)}{6x} = \dfrac{x(2x+1)}{6} = \dfrac{1}{6}x(2x+1)$

2. $\dfrac{x^5 + 9x^4}{x^2} = \dfrac{x^4(x+9)}{x^2} = x^3(x+9)$

3. $\dfrac{x^2 - 3x - 28}{x-7} = \dfrac{(x-7)(x+4)}{x-7} = x+4$

4. $\dfrac{x^2 + 11x + 30}{x+5} = \dfrac{(x+6)(x+5)}{x+5} = x+6$

5. $f(x) = x^2 - 3x + 3$

(a) $f(-1) = (-1)^2 - 3(-1) + 3 = 1 + 3 + 3 = 7$

(b) $f(c) = c^2 - 3c + 3$

(c) $f(x+h) = (x+h)^2 - 3(x+h) + 3$
$= x^2 + 2xh + h^2 - 3x - 3h + 3$

6. $f(x) = \begin{cases} 2x - 2, & x < 1 \\ 3x + 1, & x \ge 1 \end{cases}$

(a) $f\left(-\frac{1}{2}\right) = 2\left(-\frac{1}{2}\right) - 2 = -1 - 2 = -3$

(b) $f(1) = 3(1) + 1 = 3 + 1 = 4$

(c) $f\left(t^2+1\right) = 3\left(t^2+1\right) + 1$
$= 3t^2 + 3 + 1$
$= 3t^2 + 4$

7. $f(x) = x^2 - 2x + 2$

$$\begin{aligned} &\frac{f(1+h) - f(1)}{h} \\ &= \frac{(1+h)^2 - 2(1+h) + 2 - \left(1^2 - 2(1) + 2\right)}{h} \\ &= \frac{1 + 2h + h^2 - 2 - 2h + 2 - 1 + 2 - 2}{h} \\ &= \frac{h^2}{h} \\ &= h \end{aligned}$$

8. $f(x) = 4x$

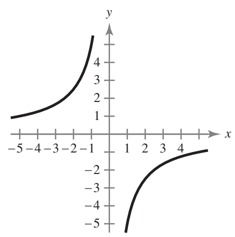
$$\begin{aligned} \frac{f(2+h) - f(2)}{h} &= \frac{4(2+h) - 4(2)}{h} \\ &= \frac{8 + 4h - 8}{h} \\ &= \frac{4h}{h} \\ &= 4 \end{aligned}$$

Skills Warm Up —*continued*—

9. $h(x) = -\dfrac{5}{x}$

Domain: $(-\infty, 0) \cup (0, \infty)$

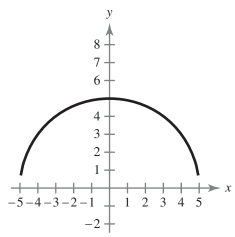
Range: $(-\infty, 0) \cup (0, \infty)$



10. $g(x) = \sqrt{25 - x^2}$

Domain: $[-5, 5]$

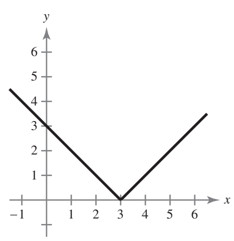
Range: $[0, 5]$



11. $f(x) = |x - 3|$

Domain: $(-\infty, \infty)$

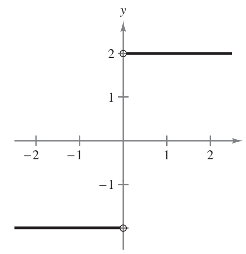
Range: $[0, \infty)$



12. $f(x) = \dfrac{2|x|}{x}$

Domain: $(-\infty, 0) \cup (0, \infty)$

Range: $y = -2,\ y = 2$



13.
$$9x^2 + 4y^2 = 49$$
$$4y^2 = 49 - 9x^2$$
$$y^2 = \frac{49 - 9x^2}{4}$$
$$y = \frac{\pm\sqrt{49 - 9x^2}}{2}$$

Not a function of x (fails the vertical line test).

14.
$$2x^2y + 8x = 7y$$
$$2x^2y - 7y = -8x$$
$$y(2x^2 - 7) = -8x$$
$$y = -\frac{8x}{2x^2 - 7}$$

Yes, y is a function of x.

1. (a) $\lim_{x\to 2} f(x) = 4$

(b) $\lim_{x\to -1} f(x) = 1$

2. (a) $\lim_{x\to 1} f(x) = -2$

(b) $\lim_{x\to 3} f(x) = 0$

3. (a) $\lim_{x\to 0} g(x) = 1$

(b) $\lim_{x\to -1} g(x) = 3$

4. (a) $\lim_{x\to -2} h(x) = -5$

(b) $\lim_{x\to 0} h(x) = -3$

5.

x	5.9	5.99	5.999	6	6.001	6.01	6.1
$f(x)$	2.96	2.996	2.9996	?	3.0004	3.004	3.04

$\lim_{x\to 6} \dfrac{2x + 3}{5} = 3$

6.

x	0.9	0.99	0.999	1	1.001	1.01	1.1
$f(x)$	−1.79	−1.9799	−1.997999	?	−2.001999	−2.0199	−2.19

$\lim_{x\to 1}(x^2 - 4x + 1) = -2$

7.

x	3.9	3.99	3.999	4	4.001	4.01	4.1
$f(x)$	0.3448	0.3344	0.3334	?	0.3332	0.3322	0.3226

$\lim_{x\to 4} \dfrac{x - 4}{x^2 - 5x + 4} = \dfrac{1}{3}$

8.

x	1.9	1.99	1.999	2	2.001	2.01	2.1
$f(x)$	0.2564	0.2506	0.2501	?	0.2499	0.2494	0.2439

$$\lim_{x\to 2} \frac{x-2}{x^2-4} = \frac{1}{4}$$

9.

x	–0.1	–0.01	–0.001	0	0.001	0.01	0.1
$f(x)$	0.1252	0.1250	0.1250	?	0.1250	0.1250	0.1248

$$\lim_{x\to 0} \frac{\sqrt{x+16}-4}{x} = 0.125 = \frac{1}{8}$$

10.

x	–0.1	–0.01	–0.001	0	0.001	0.01	0.1
$f(x)$	0.3581	0.3540	0.3536	?	0.3535	0.3531	0.3492

$$\lim_{x\to 0} \frac{\sqrt{x+2}-\sqrt{2}}{x} = \frac{1}{2\sqrt{2}} \approx 0.3536$$

11.

x	–4.1	–4.01	–4.001	–4	–3.999	–3.99	–3.9
$f(x)$	2.5	25	250	?	–250	–25	–2.5

The limit does not exist.

12.

x	–2.1	–2.01	–2.001	–2	–1.999	–1.99	–1.9
$f(x)$	–2.5	–25	–250	?	250	25	2.5

The limit does not exist.

13. $\lim_{x\to 3} 6 = 6$

14. $\lim_{x\to 5} 4 = 4$

15. $\lim_{x\to -2} x = -2$

16. $\lim_{x\to 10} x = 10$

17. $\lim_{x\to 7} x^2 = (7)^2 = 49$

18. $\lim_{x\to 3} x^3 = (3)^3 = 27$

19. $\lim_{x\to 36} \sqrt{x} = \sqrt{36} = 6$

20. $\lim_{x\to -1} \sqrt[3]{x} = \sqrt[3]{-1} = -1$

21. (a) $\lim_{x\to c}\left[f(x)+g(x)\right] = \lim_{x\to c} f(x) + \lim_{x\to c} g(x)$
$= 3 + 9$
$= 12$

(b) $\lim_{x\to c}\left[f(x)g(x)\right] = \left[\lim_{x\to c} f(x)\right]\left[\lim_{x\to c} g(x)\right]$
$= 3 \cdot 9$
$= 27$

(c) $\lim_{x\to c} \frac{f(x)}{g(x)} = \frac{\lim_{x\to c} f(x)}{\lim_{x\to c} g(x)} = \frac{3}{9} = \frac{1}{3}$

22. (a) $\lim_{x\to c}\left[f(x) + g(x)\right] = \lim_{x\to c} f(x) + \lim_{x\to c} g(x)$

$$= \frac{3}{2} + \frac{1}{2}$$

$$= 2$$

(b) $\lim_{x\to c}\left[f(x)\cdot g(x)\right] = \left[\lim_{x\to c} f(x)\right]\left[\lim_{x\to c} g(x)\right]$

$$= \left(\frac{3}{2}\right)\left(\frac{1}{2}\right)$$

$$= \frac{3}{4}$$

(c) $\lim_{x\to c}\frac{f(x)}{g(x)} = \frac{\lim_{x\to c} f(x)}{\lim_{x\to c} g(x)} = \frac{3/2}{1/2} = 3$

23. (a) $\lim_{x\to c}\sqrt{f(x)} = \sqrt{16} = 4$

(b) $\lim_{x\to c}\left[3f(x)\right] = 3(16) = 48$

(c) $\lim_{x\to c}\left[f(x)\right]^2 = 16^2 = 256$

24. (a) $\lim_{x\to c}\sqrt{f(x)} = \sqrt{9} = 3$

(b) $\lim_{x\to c}\left(3f(x)\right) = 3(9) = 27$

(c) $\lim_{x\to c}\left[f(x)\right]^2 = 9^2 = 81$

25. $\lim_{x\to -3}(2x + 5) = \lim_{x\to -3} 2x + \lim_{x\to -3} 5 = 2(-3) + 5 = -1$

26. $\lim_{x\to -4}(4x + 3) = \lim_{x\to -4} 4x + \lim_{x\to -4} 3 = 4(-4) + 3 = -13$

27. $\lim_{x\to 1}\left(1 - x^2\right) = \lim_{x\to 1} 1 - \lim_{x\to 1} x^2 = 1 - 1^2 = 0$

28. $\lim_{x\to 2}\left(-x^2 + x - 2\right) = -\lim_{x\to 2} x^2 + \lim_{x\to 2} x - \lim_{x\to 2} 2$

$$= -4 + 2 - 2 = -4$$

29. $\lim_{x\to 3}\sqrt{x + 6} = \sqrt{3 + 6} = 3$

30. $\lim_{x\to 5}\sqrt[3]{x - 5} = \sqrt[3]{5 - 5} = \sqrt[3]{0} = 0$

31. $\lim_{x\to -3}\frac{2}{x + 2} = \frac{2}{-3 + 2} = -2$

32. $\lim_{x\to -2}\frac{3x + 1}{2 - x} = \frac{3(-2) + 1}{2 - (-2)} = \frac{-5}{4} = -\frac{5}{4}$

33. $\lim_{x\to -2}\frac{x^2 - 1}{2x} = \frac{(-2)^2 - 1}{2(-2)} = \frac{3}{-4} = -\frac{3}{4}$

34. $\lim_{x\to -8}\frac{3x}{x + 2} = \frac{3(-8)}{(-8) + 2} = \frac{-24}{-6} = 4$

35. $\lim_{x\to 5}\frac{\sqrt{x + 11} + 6}{x} = \frac{\sqrt{5 + 11} + 6}{5}$

$$= \frac{\sqrt{16} + 6}{5}$$

$$= \frac{4 + 6}{5} = \frac{10}{5} = 2$$

36. $\lim_{x\to 12}\frac{\sqrt{x - 3} - 2}{x} = \frac{\sqrt{12 - 3} - 2}{12}$

$$= \frac{\sqrt{9} - 2}{12} = \frac{3 - 2}{12} = \frac{1}{12}$$

37. $\lim_{x\to -3}\frac{x^2 - 9}{x + 3} = \lim_{x\to -3}\frac{(x + 3)(x - 3)}{x + 3}$

$$= \lim_{x\to -3}(x - 3) = -6$$

38. $\lim_{x\to -1}\frac{2x^2 - x - 3}{x + 1} = \lim_{x\to -1}\frac{(x + 1)(2x - 3)}{x + 1}$

$$= \lim_{x\to -1}(2x - 3) = -5$$

39. $\lim_{x\to 2}\frac{x^2 + 3x - 10}{x^2 - 4} = \lim_{x\to 2}\frac{(x + 5)(x - 2)}{(x + 2)(x - 2)}$

$$= \lim_{x\to 2}\frac{x + 5}{x + 2} = \frac{2 + 5}{2 + 2} = \frac{7}{4}$$

40. $\lim_{t\to 1}\frac{t^2 + t - 2}{t^2 - 1} = \lim_{t\to 1}\frac{(t - 1)(t + 2)}{(t + 1)(t - 1)}$

$$= \lim_{t\to 1}\frac{t + 2}{t + 1} = \frac{3}{2}$$

41. $\lim_{x\to -2}\frac{x^3 + 8}{x + 2} = \lim_{x\to -2}\frac{(x + 2)\left(x^2 - 2x + 4\right)}{x + 2}$

$$= \lim_{x\to -2}\left(x^2 - 2x + 4\right) = 12$$

42. $\lim_{x\to -3}\frac{x^3 + 27}{x + 3} = \lim_{x\to -3}\frac{(x + 3)\left(x^2 - 3x + 9\right)}{x + 3}$

$$= \lim_{x\to -3}\left(x^2 - 3x + 9\right) = (-3)^2 - 3(-3) + 9 = 9$$

43. $\lim_{\Delta x \to 0} \frac{2(x + \Delta x) - 2x}{\Delta x} = \lim_{\Delta x \to 0} \frac{2x + 2\Delta x - 2x}{\Delta x}$

$= \lim_{\Delta x \to 0} 2 = 2$

44. $\lim_{\Delta x \to 0} \frac{4(x + \Delta x) - 5 - (4x - 5)}{\Delta x} = \lim_{\Delta x \to 0} \frac{4\Delta x}{\Delta x} = 4$

45. $\lim_{\Delta t \to 0} \frac{(t + \Delta t)^2 - 5(t + \Delta t) - (t^2 - 5t)}{\Delta t} = \lim_{\Delta t \to 0} \frac{t^2 + 2t(\Delta t) + (\Delta t)^2 - 5t - 5(\Delta t) - t^2 + 5t}{\Delta t}$

$= \lim_{\Delta t \to 0} \frac{2t(\Delta t) + (\Delta t)^2 - 5(\Delta t)}{\Delta t}$

$= \lim_{\Delta t \to 0} 2t + (\Delta t) - 5$

$= 2t - 5$

46. $\lim_{\Delta t \to 0} \frac{(t + \Delta t)^2 - 4(t + \Delta t) + 2 - (t^2 - 4t + 2)}{\Delta t} = \lim_{\Delta t \to 0} \frac{t^2 + 2t\Delta t + (\Delta t)^2 - 4t - 4\Delta t - t^2 + 4t}{\Delta t}$

$= \lim_{\Delta t \to 0} \frac{2t\Delta t + (\Delta t)^2 - 4\Delta t}{\Delta t}$

$= \lim_{\Delta t \to 0} (2t + \Delta t - 4)$

$= 2t - 4$

47. $\lim_{x \to 4} \frac{\sqrt{x + 5} - 3}{x - 4} = \lim_{x \to 4} \frac{\sqrt{x + 5} - 3}{x - 4} \cdot \frac{\sqrt{x + 5} + 3}{\sqrt{x + 5} + 3}$

$= \lim_{x \to 4} \frac{(x + 5) - 9}{(x - 4)\sqrt{x + 5} + 3}$

$= \lim_{x \to 4} \frac{x - 4}{(x - 4)(\sqrt{x + 5} + 3)}$

$= \lim_{x \to 4} \frac{1}{\sqrt{x + 5} + 3} = \frac{1}{6}$

48. $\lim_{x \to 3} \frac{\sqrt{x + 1} - 2}{x - 3} = \lim_{x \to 3} \frac{\sqrt{x + 1} - 2}{x - 3} \cdot \lim_{x \to 3} \frac{\sqrt{x + 1} + 2}{\sqrt{x + 1} + 2}$

$= \lim_{x \to 3} \frac{(x + 1) - 4}{(x - 3)(\sqrt{x + 1} + 2)}$

$= \lim_{x \to 3} \frac{x - 3}{(x - 3)(\sqrt{x + 1} + 2)}$

$= \lim_{x \to 3} \frac{1}{\sqrt{x + 1} + 2} = \frac{1}{4}$

49. $\lim_{x \to 0} x = \frac{\sqrt{x + 5} - \sqrt{5}}{x} = \lim_{x \to 0} \frac{\sqrt{x + 5} - \sqrt{5}}{x} \cdot \frac{\sqrt{x + 5} + \sqrt{5}}{\sqrt{x + 5} + \sqrt{5}}$

$= \lim_{x \to 0} \frac{(x + 5) - 5}{x(\sqrt{x + 5} + \sqrt{5})}$

$= \lim_{x \to 0} \frac{x}{x(\sqrt{x + 5} + \sqrt{5})}$

$= \lim_{x \to 0} \frac{1}{\sqrt{x + 5} + \sqrt{5}} = \frac{1}{2\sqrt{5}}$

50. $\lim_{x\to 0} x = \frac{\sqrt{x+2}-\sqrt{2}}{x} = \lim_{x\to 0} \frac{\sqrt{x+2}-\sqrt{2}}{x} \cdot \frac{\sqrt{x+2}+\sqrt{2}}{\sqrt{x+2}+\sqrt{2}}$

$= \lim_{x\to 0} \frac{(x+2)-2}{x\left(\sqrt{x+2}+\sqrt{2}\right)}$

$= \lim_{x\to 0} \frac{x}{x\left(\sqrt{x+2}+\sqrt{2}\right)}$

$= \lim_{x\to 0} \frac{1}{\sqrt{x+2}+\sqrt{2}} = \frac{1}{2\sqrt{2}}$

51. $\lim_{x\to 2^-} (4-x) = 2$

$\lim_{x\to 2^+} (4-x) = 2$

So, $\lim_{x\to 2} f(x) = 2$

52. $\lim_{x\to 1^-} \left(x^2+2\right) = 3$

$\lim_{x\to 1^+} \left(x^2+2\right) = 3$

So, $\lim_{x\to 1} f(x) = 3$

53. $\lim_{x\to 3^-} f(x) = \lim_{x\to 3^-} \left(\frac{1}{3}x - 5\right) = -4$

$\lim_{x\to 3^+} f(x) = \lim_{x\to 3^+} (-3x+7) = -2$

So, $\lim_{x\to 3} f(x)$ does not exist.

54. $\lim_{s\to 4^-} f(s) = \lim_{s\to 4^-} (3s-4) = 8$

$\lim_{s\to 4^+} f(s) = \lim_{s\to 4^+} \left(5 - \frac{1}{2}s\right) = 3$

So, $\lim_{s\to 4} f(s)$ does not exist.

55. $\lim_{x\to -4} \frac{2}{x+4} = \frac{2}{0}$

The limit does not exist.

56. $\lim_{x\to 5} \frac{4}{x-5} = \frac{4}{0}$

The limit does not exist.

57. $\lim_{x\to 2} \frac{x-2}{x^2-4x+4} = \lim_{x\to 2} \frac{x-2}{(x-2)(x-2)}$

$= \lim_{x\to 2} \frac{1}{x-2}$

The limit does not exist.

58. $\lim_{t\to -6} \frac{t+6}{t^2+12t+36} = \lim_{t\to -6} \frac{t+6}{(t+6)(t+6)}$

$= \lim_{t\to -6} \frac{1}{t+6} = \frac{1}{0}$

The limit does not exist.

59. (a) $\lim_{x\to 3^+} f(x) = 1$

(b) $\lim_{x\to 3^-} f(x) = 1$

(c) $\lim_{x\to 3} f(x) = 1$

60. (a) $\lim_{x\to -2^+} f(x) = -2$

(b) $\lim_{x\to -2^-} f(x) = -2$

(c) $\lim_{x\to -2} f(x) = -2$

61. (a) $\lim_{x\to 2^-} f(x) = -1$

(b) $\lim_{x\to 2^+} f(x) = -1$

(c) $\lim_{x\to 2} f(x) = -1$

62. (a) $\lim_{x\to 1^-} f(x) = 3$

(b) $\lim_{x\to 1^+} f(x) = 3$

(c) $\lim_{x\to 1} f(x) = 3$

63. (a) $\lim_{x\to 6^-} f(x) = -6$

(b) $\lim_{x\to 6^+} f(x) = 6$

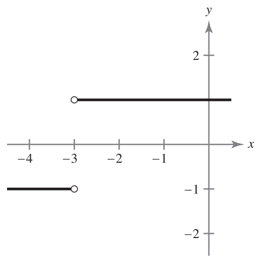
(c) $\lim_{x\to 6} f(x)$ does not exist.

64. (a) $\lim_{x \to -1^+} f(x) = 0$

(b) $\lim_{x \to -1^-} f(x) = 2$

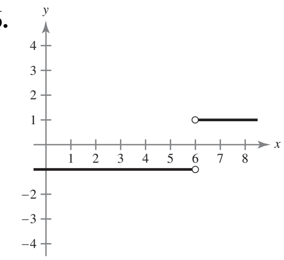
(c) $\lim_{x \to -1} f(x)$ does not exist.

65.



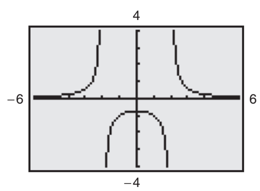
$\lim_{x \to -3^-} \frac{|x+3|}{x+3} = -1$ and $\lim_{x \to -3^+} \frac{|x+3|}{x+3} = 1$

66.



$\lim_{x \to 6^-} \frac{|x-6|}{x-6} = -1$ and $\lim_{x \to 6^+} \frac{|x-6|}{x-6} = 1$

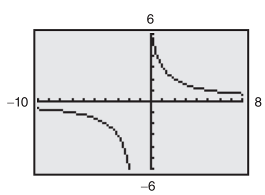
67.



x	3	2.5	2.1	2.01	2.001	2.0001	2
$f(x)$	0.6	1.33	7.32	74.81	749.81	7499.81	Undefined

The limit does not exist because *f* is unbounded as *x* approaches 2 from the right.

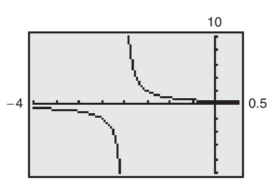
68.



x	−1	−0.999	−0.99	−0.9	−0.5	0	1
$f(x)$	Undefined	6000	600	60	12	6	3

Because $f(x) = \frac{6}{x+1}$ decreases without bound as *x* approaches -1 from the right, the limit does not exist.

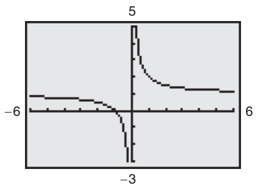
69.



x	−3	−2.5	−2.1	−2.01	−2.001	−2.0001	−2
$f(x)$	−1	−2	−10	−100	−1000	−10,000	undefined

Because $f(x) = \frac{1}{x+2}$ decreases without bound as *x* approaches -2 from the left, the limit does not exist.

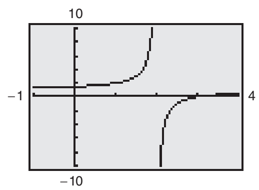
70.



x	−1	−0.5	−0.1	−0.01	−0.001	−0.0001	0
$f(x)$	0	−1	−9	−99	−999	−9999	undefined

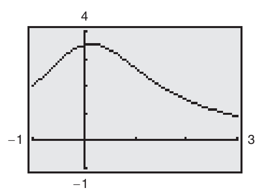
Because $f(x) = \dfrac{x+1}{x}$ decreases without bound as x approaches 0 from the left, the limit does not exist.

71.



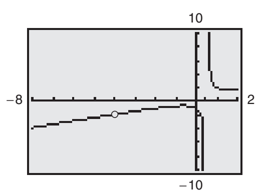
$\displaystyle\lim_{x\to 2} \frac{x^2 - 5x + 6}{x^2 - 4x + 4}$ does not exist.

72.



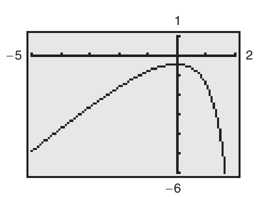
$$\lim_{x\to 1} \frac{x^2 + 6x - 7}{x^3 - x^2 + 2x - 2} \approx 2.667$$

73.



$$\lim_{x\to -4} \frac{x^3 + 4x^2 + x + 4}{2x^2 + 7x - 4} \approx -1.889$$

74.



$$\lim_{x\to -2} \frac{4x^3 + 7x^2 + x + 6}{3x^2 - x - 14} \approx -1.615$$

75. $C = \dfrac{25p}{100 - p}, \ 0 \le p < 100$

(a) $C(50) = \dfrac{25(50)}{100 - 50} = \$25$ thousand

(b) Find p for $C = 100$.

$$\begin{aligned} 100 &= \frac{25p}{100 - p} \\ 100(100 - p) &= 25p \\ 10{,}000 - 100p &= 25p \\ 10{,}000 &= 125p \\ 80 &= p, \text{ or } 80\% \end{aligned}$$

(c) $\displaystyle\lim_{p\to 100^-} = \lim_{p\to 100^-} \frac{25p}{100 - p} = \infty$

The cost function increases without bound as x approaches 100 from the left. Therefore, according to the model, it is not possible to remove 100% of the pollutants.

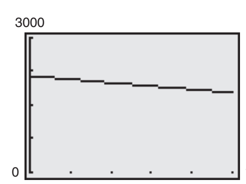
76. (a) $\displaystyle\lim_{x\to 50} C$ does not exist. The two one-sided limits are not equal.

$\displaystyle\lim_{x\to 50^-} C \approx \$7.50$ and $\displaystyle\lim_{x\to 50^+} C \approx \$5$

(b) $\displaystyle\lim_{x\to 150} C \approx \$10.50$

(c) It would be less expensive to make 201 copies, since $\displaystyle\lim_{x\to 200^-} C \approx \$14$ and $\displaystyle\lim_{x\to 200^+} C \approx \$10.$

77. (a)



(b) When $x = 0.25$: $A = 2685.06$

When $x = \frac{1}{365}$: $A = 2717.91$

(c) Using the *zoom* and *trace* features, $\displaystyle\lim_{x\to 0^+} A \approx \$2718.28$. Because x, the length of the compounding period, is approaching 0, this limit represents the balance with continuous compounding.

Section 1.6 Continuity

Skills Warm Up

1. $\dfrac{x^2 + 6x + 8}{x^2 - 6x - 16} = \dfrac{(x + 4)(x + 2)}{(x - 8)(x + 2)} = \dfrac{x + 4}{x - 8}$

2. $\dfrac{x^2 - 5x - 6}{x^2 - 9x + 18} = \dfrac{(x - 6)(x + 1)}{(x - 6)(x - 3)} = \dfrac{x + 1}{x - 3}$

3. $\dfrac{2x^2 - 2x - 12}{4x^2 - 24x + 36} = \dfrac{2(x^2 - x - 6)}{4(x^2 - 6x + 9)}$
$= \dfrac{2(x - 3)(x + 2)}{4(x - 3)(x - 3)}$
$= \dfrac{x + 2}{2(x - 3)}$

4. $\dfrac{x^3 - 16x}{x^3 + 2x^2 - 8x} = \dfrac{x(x^2 - 16)}{x(x^2 + 2x - 8)}$
$= \dfrac{x(x^2 - 16)}{x(x + 4)(x - 2)}$
$= \dfrac{x(x + 4)(x - 4)}{x(x + 4)(x - 2)}$
$= \dfrac{x - 4}{x - 2}$

5. $x^2 + 7x = 0$
$x(x + 7) = 0$
$x = 0$
$x + 7 = 0 \Rightarrow x = -7$

6. $x^2 + 4x - 5 = 0$
$(x + 5)(x - 1) = 0$
$x + 5 = 0 \Rightarrow x = -5$
$x - 1 = 0 \Rightarrow x = 1$

7. $3x^2 + 8x + 4 = 0$
$(3x + 2)(x + 2) = 0$
$3x + 2 = 0 \Rightarrow x = -\frac{2}{3}$
$x + 2 = 0 \Rightarrow x = -2$

8. $3x^3 - x^2 - 24x = 0$
$x(3x^2 - x - 24) = 0$
$x(3x + 8)(x - 3) = 0$
$x = 0$
$3x + 8 = 0 \Rightarrow x = -\frac{8}{3}$
$x - 3 = 0 \Rightarrow x = 3$

9. $\lim_{x\to 3} (2x^2 - 3x + 4) = 2(3^2) - 3(3) + 4$
$= 2(9) - 9 + 4$
$= 13$

10. $\lim_{x\to -2} \sqrt{x^2 - x + 3} = \sqrt{(-2)^3 - (-2) + 3}$
$= 3$

1. Continuous; The function is a polynomial.

2. Continuous; The function is a polynomial.

3. Not continuous; The rational function is not defined at $x = \pm 4$.

4. Not continuous; The rational function is not defined at $x = \pm 3$.

5. Continuous; The rational function's domain is the entire real line.

6. Continuous; The rational function's domain is the entire real line.

7. Not continuous; The rational function is not defined at $x = 3$ or $x = 5$.

8. Not continuous; The rational function is not defined at $x = 1$ or $x = 5$.

9. Not continuous; The rational function is not defined at $x = \pm 6$.

10. Not continuous; The rational function is not defined at $x = \pm 5$.

11. $f(x) = \dfrac{x^2 - 1}{x}$ is continuous on $(-\infty, 0)$ and $(0, \infty)$ because the domain of f consists of all real numbers except $x = 0$. There is a discontinuity at $x = 0$ because $f(0)$ is not defined and $\lim_{x\to 0} f(x)$ does not exist.

12. $f(x) = \frac{1}{x^2 - 4}$ is continuous on $(-\infty, -2), (-2, 2)$ and $(2, \infty)$ because the domain of f consists of all real numbers except $x = \pm 2$. There are discontinuities at $x = \pm 2$ because $f(2)$ and $f(-2)$ are not defined and $\lim_{x\to 2} f(x)$ and $\lim_{x\to -2} f(x)$ do not exist.

13. $f(x) = \frac{x^2 - 1}{x + 1}$ is continuous on $(-\infty, -1)$ and $(-1, \infty)$ because the domain of f consists of all real numbers except $x = -1$. There is a discontinuity at $x = -1$ because $f(-1)$ is not defined and $\lim_{x\to -1} f(x) \neq f(-1)$.

14. $f(x) = \frac{x^3 - 27}{x - 3}$ is continuous on $(-\infty, 3)$ and $(3, \infty)$ because the domain of f consists of all real numbers except $x = 3$. There is a discontinuity at $x = 3$ because $f(3)$ is not defined and $\lim_{x\to 3} f(x) \neq f(3)$.

15. $f(x) = x^2 - 9x + 14$ is continuous on $(-\infty, \infty)$ because the domain of f consists of all real numbers.

16. $f(x) = 3 - 2x - x^2$ is continuous on $(-\infty, \infty)$ because the domain of f consists of all real numbers.

17. $f(x) = \frac{x}{x^2 - 1} = \frac{x}{(x + 1)(x - 1)}$ is continuous on $(-\infty, -1), (-1, 1)$, and $(1, \infty)$ because the domain of f consists of all real numbers except $x = \pm 1$. There are discontinuities at $x = \pm 1$ because $f(1)$ and $f(-1)$ are not defined and $\lim_{x\to 1} f(x)$ and $\lim_{x\to -1} f(x)$ do not exist.

18. $f(x) = \frac{x - 3}{x^2 - 9}$ is continuous on $(-\infty, -3), (-3, 3)$, and $(3, \infty)$ because the domain of f consists of all real numbers except $x = \pm 3$. There are discontinuities at $x = \pm 3$ because $f(-3)$ and $f(3)$ are not defined, $\lim_{x\to -3}$ does not exist, and $\lim_{x\to 3} f(x) \neq f(3)$.

19. $f(x) = \frac{7x}{x^2 + 5}$ is continuous on $(-\infty, \infty)$ because the domain of f consists of all real numbers.

20. $f(x) = \frac{6}{x^2 + 3}$ is continuous on $(-\infty, \infty)$ because the domain of f consists of all real numbers.

21. $f(x) = \frac{x - 5}{x^2 - 9x + 20} = \frac{x - 5}{(x - 5)(x - 4)}$ is continuous on $(-\infty, 4), (4, 5)$, and $(5, \infty)$ because the domain of f consists of all real numbers except $x = 4$ and $x = 5$. There is a discontinuity at $x = 4$ and $x = 5$ because $f(4)$ and $f(5)$ are not defined and $\lim_{x\to 4} f(x)$ does not exist and $\lim_{x\to 5} f(x) \neq f(5)$.

22. $f(x) = \frac{x - 1}{x^2 + x - 2} = \frac{x - 1}{(x - 1)(x + 2)}$ is continuous on $(-\infty, -2), (-2, 1)$, and $(1, \infty)$ because the domain of f consists of all real number except $x = -2$ and $x = 1$. There is discontinuity at $x = -2$ and $x = 1$ because $f(-2)$ and $f(1)$ are not defined, $\lim_{x\to -2} f(x)$ does not exist, and $\lim_{x\to 1} f(x) \neq f(1)$.

23. $f(x) = \sqrt{4 - x}$ is continuous on $(-\infty, 4]$ because the domain of f consists of all real $x \leq 4$.

24. $f(x) = \sqrt{x - 1}$ is continuous on $[1, \infty)$ because the domain of f consists of all real $x \geq 1$.

25. $f(x) = \sqrt{x} + 2$ is continuous on $[0, \infty)$ because the domain of f consists of all real $x \geq 0$.

26. $f(x) = 3 - \sqrt{x}$ is continuous on $[0, \infty)$ because the domain of f consists of all real $x \geq 0$.

27. $f(x) = \begin{cases} -2x + 3, & -1 \leq x \leq 1 \\ x^2, & 1 < x \leq 3 \end{cases}$ is continuous on $[-1, 3]$.

28. $f(x) = \begin{cases} \frac{1}{2}x + 1, & -3 \leq x \leq 2 \\ 3 - x, & 2 < x \leq 4 \end{cases}$ is continuous on $[-3, 2), (2, 4]$. f is discontinuous at $x = 2$ because $\lim_{x\to 2} f(x)$ does not exist.

$\lim_{x\to 2^-} f(x) = 2$ and $\lim_{x\to 2^+} f(x) = 1$.

29. $f(x) = \begin{cases} 4 - 2x, & x \leq 2 \\ x^2 - 3, & x > 2 \end{cases}$

is continuous on $(-\infty, 2)$ and $(2, \infty)$. There is a discontinuity at $x = 2$ because $\lim_{x\to 2} f(x)$ does not exist.

30. $f(x) = \begin{cases} x^2 - 2, & x \le -1 \\ 3x + 2, & x > -1 \end{cases}$ is continuous on $(-\infty, \infty.)$

31. $f(x) = \dfrac{|x + 1|}{x + 1}$ is continuous on $(-\infty, -1)$ and $(-1, \infty)$ because the domain of f consists of all real numbers except $x = -1$. There is a discontinuity at $x = -1$ because $f(-1)$ is not defined, and $\lim_{x \to -1} f(x)$ does not exist.

32. $f(x) = \dfrac{|4 - x|}{4 - x}$ is continuous on $(-\infty, 4)$ and $(4, \infty)$ because the domain of f consists of all real numbers except $x = 4$. There is a discontinuity at $x = 4$ because $f(4)$ is not defined and $\lim_{x \to 4} f(x)$ does not exist.

33. $f(x) = x\sqrt{x + 3}$ is continuous on $[-3, \infty)$.

34. $f(x) = \dfrac{x + 1}{\sqrt{x}}$ is continuous on $(0, \infty)$.

35. $f(x) = [\![2x]\!] + 1$ is continuous on all intervals of the form $\left(\frac{1}{2}c, \frac{1}{2}c + \frac{1}{2}\right)$, where c is an integer. That is, f is continuous on $\ldots, \left(-\frac{1}{2}, 0\right), \left(0, \frac{1}{2}\right), \left(\frac{1}{2}, 1\right), \ldots$. f is not continuous at all points $\frac{1}{2}c$, where c is an integer. There are discontinuities at $x = \dfrac{c}{2}$, where c is an integer, because $\lim_{x \to c/2} f(x)$ does not exist.

36. $f(x) = \dfrac{[\![x]\!]}{2} + x$ is continuous on all intervals of the form $(c, c + 1)$ where c is an integer. There are discontinuities at all integer values c because $\lim_{x \to c} f(x)$ does not exist.

37. $f(x) = [\![x - 1]\!]$ is continuous on all intervals $(c, c + 1)$. There are discontinuities at $x = c$, where c is an integer, because $\lim_{x \to c} f(x)$ does not exist.

38. $f(x) = x - [\![x]\!]$ is continuous on all intervals $(c, c + 1)$. There are discontinuities at all integer values c because $\lim_{x \to c} f(x)$ does not exist.

39. $h(x) = f(g(x)) = f(x - 1) = \dfrac{1}{\sqrt{x - 1}}, \; x > 1$

h is continuous on its entire domain $(1, \infty)$.

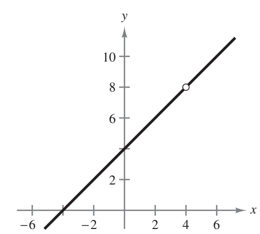
40. $h(x) = f(g(x)) = f(x^2 + 5)$

$$= \frac{1}{(x^2 + 5) - 1} = \frac{1}{x^2 + 4}$$

h is continuous on $(-\infty, \infty)$.

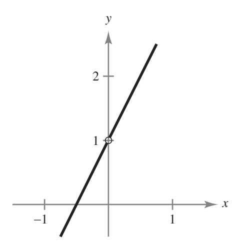
41. $f(x) = \dfrac{x^2 - 16}{x - 4} = \dfrac{(x + 4)(x - 4)}{x - 4} = x + 4, \; x \ne 4$

f has a removable discontinuity at $x = 4$;
Continuous on $(-\infty, 4)$ and $(4, \infty)$.



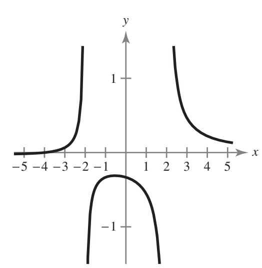
42. $f(x) = \dfrac{2x^2 + x}{x} = \dfrac{x(2x + 1)}{x}$

f has a removable discontinuity at $x = 0$;
Continuous on $(-\infty, 0)$ and $(0, \infty)$.



43. $f(x) = \dfrac{x + 4}{3x^2 - 12}$

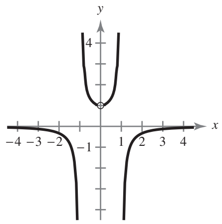
Continuous on $(-\infty, 2)$, $(-2, 2)$, and $(2, \infty)$.



44. $f(x) = \dfrac{-x}{x^3 - x} = \dfrac{-x}{x(x+1)(x-1)} = -\dfrac{1}{(x+1)(x-1)},\ x \neq 0$

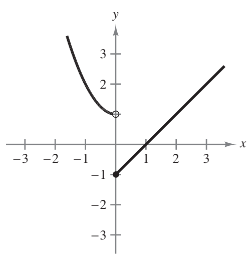
f has a removable discontinuity at $x = 0$; *f* has nonremovable discontinuities at $x = -1$ and $x = 1$;

Continuous on $(-\infty, -1)$, $(-1, 0)$, $(0, 1)$, and $(1, \infty)$.



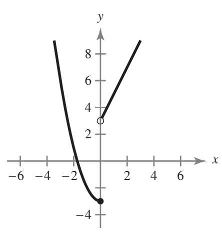
45. $f(x) = \begin{cases} x^2 + 1, & x < 0 \\ x - 1, & x \geq 0 \end{cases}$

f has a nonremovable discontinuity at $x = 0$;
Continuous on $(-\infty, 0)$ and $(0, \infty)$.



46. $f(x) = \begin{cases} x^2 - 3, & x \leq 0 \\ 2x + 3, & x > 0 \end{cases}$

f has a nonremovable discontinuity at $x = 0$; Continuous on $(-\infty, 0)$ and $(0, \infty)$.



47. Continuous on $[-1, 5]$ because $f(x) = x^2 - 4x - 5$ is a polynomial.

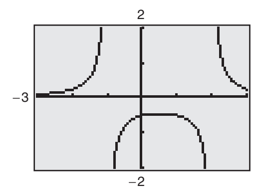
48. Continuous on $[-2, 2]$ because $f(x) = \dfrac{5}{x^2 + 1}$ is defined on the entire interval.

49. Continuous on $[1, 2)$ and $(2, 4]$ because $f(x) = \dfrac{1}{x - 2}$ has a nonremovable discontinuity at $x = 2$.

50. Continuous on $[0, 1)$, $(1, 3)$, and $(3, 4]$ because

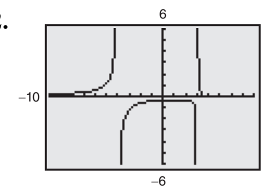
$f(x) = \dfrac{x - 1}{(x - 1)(x - 3)} = \dfrac{1}{x - 3},\ x \neq 1$, has a removable discontinuity at $x = 1$ and a nonremoveable discontinuity at $x = 3$.

51.



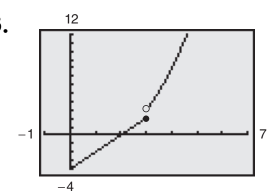
From the graph, you can see that $h(2)$ and $h(-1)$ are not defined, so *h* is not continuous at $x = 2$ and $x = -1$.

52.



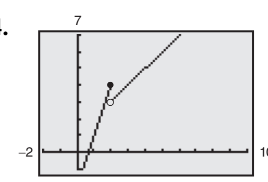
From the graph, you can see that $k(-3)$ is not defined, so *k* is not continuous at $x = -3$. [**Note:** There is a hole at $x = 4$.]

53.



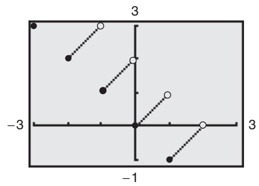
From the graph, you can see that $\lim_{x \to 3} f(x)$ does not exist, so *f* is not continuous at $x = 3$.

54.



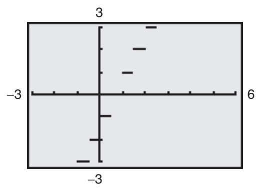
From the graph, you can see that $\lim_{x \to 2} f(x)$ does not exist, so *f* is not continuous at $x = 2$.

55.



From the graph, you can see that $\lim_{x\to c}(x - 2[\![x]\!])$, where c is an integer, does not exist. So f is not continuous at all integers c.

56.



From the graph, you can see that $\lim_{x\to c/2}[\![2x - 1]\!]$, where c is an integer, does not exist. So f is not continuous at all integers c.

57. $\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-} x^3 = 8$

$\lim_{x\to 2^+} f(x) = \lim_{x\to 2^+} ax^2 = 4a$

So, $8 = 4a$ and $a = 2$.

58. $\lim_{x\to -1^-} f(x) = 2$

$\lim_{x\to -1^+} f(x) = -a + b$

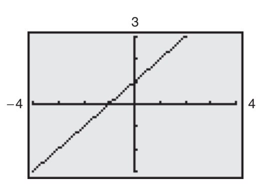
$\lim_{x\to 3^-} f(x) = 3a + b$

$\lim_{x\to 3^+} f(x) = -2$

So,

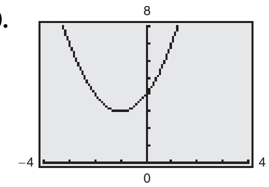
$$\begin{aligned} -a + b &= 2 \\ 3a + b &= -2 \\ \hline -4a \quad &= 4 \\ a &= -1 \\ b &= 1. \end{aligned}$$

59.



$f(x) = \dfrac{x^2 + x}{x} = \dfrac{x(x + 1)}{x}$ appears to be continuous on $[-4, 4]$. But it is not continuous at $x = 0$ (removable discontinuity). Examining a function analytically can reveal removable discontinuities that are difficult to find just from analyzing its graph.

60.



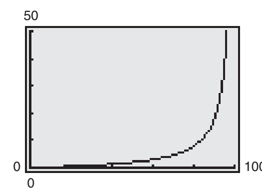
$f(x) = \dfrac{x^3 - 8}{x - 2} = \dfrac{(x - 2)(x^2 + 2x + 4)}{(x - 2)}$ appears to be continuous on $[-4, 4]$. But, it is not continuous at $x = 2$ (removable discontinuity). Examining a function analytically can reveal removable discontinuities that are difficult to find just from analyzing its graph.

61. (a) $[0, 100)$; Negative x-values and values greater than 100 do not make sense in this context. Also, $C(100)$ is undefined.

(b) C is continuous on its domain because all rational functions are continuous on their domains.

(c) For $x = 75$,

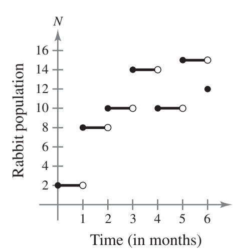
$$C = \frac{2(75)}{100 - 75} = \frac{150}{25} = 6 \text{ million dollars.}$$



62. (a) The graph of G is not continuous on day 8 and day 22.

(b) On these days, the person fills his or her gas tank.

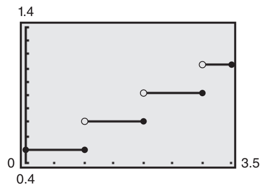
63.



There are nonremovable discontinuities at $t = 1$, 2, 3, 4, 5, and 6.

64. Yes, a linear model is a continuous function. No, actual revenue would probably not be continuous because revenue is usually recorded over larger units of time (hourly, daily, or monthly). In these cases, the revenue may jump between different units of time.

65. (a)

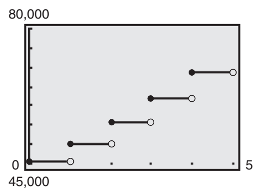


There are nonremovable discontinuities at $x = 1, 2,$ and 3. Explanations will vary.

(b) $C(2.5) = \$0.91$

A 2.5-ounce letter costs \$0.91.

66. (a)



Nonremovable discontinuities at $t = 1, 2, 3, 4, 5$

S is not continuous at $t = 1, 2, 3, 4,$ or 5.

(b) For $t = 5$, $S(5) = 45{,}300(1.11)^{[\![5]\!]} = 76{,}333.13$.

The salary during the fifth year is \$76,333.13.

67. (a) The graph has nonremovable discontinuities at $t = \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1, \frac{5}{4}, \ldots$.

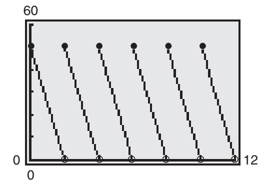
(b) Let $t = 2$.

$A = 7500(1.015)^{[\![4\cdot 2]\!]} \approx \$8448.69$

(c) Let $t = 7$.

$A = 7500(1.015)^{[\![4\cdot 7]\!]} \approx \$11{,}379.17$

68. (a)



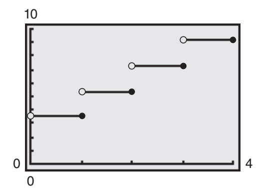
Nonremovable discontinuities at $t = 2, 4, 6, 8, \ldots$; N is not continuous at $t = 2, 4, 6, 8, \ldots$.

(b) For $t = 7$, $N = 25\left(2\left[\!\left[\dfrac{7+2}{2}\right]\!\right] - 7\right) = 25$. During the seventh month, there are 25 units in inventory.

(c) $N \to 0$ when $t \to 2^-, 4^-, 6^-, 8^-, \ldots$, so the inventory is replenished every two months.

69. (a) $C(x) = 3.50 - 1.90[\![1 - x]\!], x > 0$

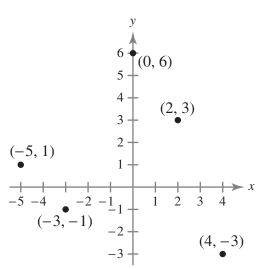
(b)



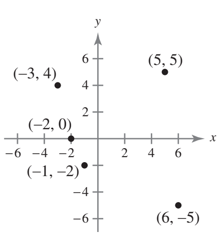
C is not continuous at all integers.

Review Exercises for Chapter 1

1.



2.



3. $\text{Distance} = \sqrt{(0-5)^2 + (0-2)^2} = \sqrt{25+4} = \sqrt{29}$

4. $\text{Distance} = \sqrt{(3-0)^2 + (4-2)^2} = \sqrt{9+4} = \sqrt{13}$

5. $\text{Distance} = \sqrt{[-1-(-4)]^2 + (3-6)^2} = \sqrt{9+9} = 3\sqrt{2}$

6. $\text{Distance} = \sqrt{[(-2)-(-8)]^2 + (7-5)^2} = \sqrt{36+4} = \sqrt{40} = 2\sqrt{10}$

7. $d = \sqrt{\left(\dfrac{3}{4} - \dfrac{1}{4}\right)^2 + \left(-6 - (-8)\right)^2} = \sqrt{\left(\dfrac{1}{2}\right)^2 + (2)^2} = \sqrt{\dfrac{1}{4} + 4} = \sqrt{\dfrac{17}{4}} = \dfrac{\sqrt{17}}{2}$

8. $d = \sqrt{\left(4 - (-0.6)\right)^2 + (-1.8 - 3)^2}$

$= \sqrt{(4.6)^2 + (-4.8)^2} = \sqrt{21.16 + 23.04} = \sqrt{44.2} \approx 6.65$

9. Midpoint $= \left(\dfrac{5 + 9}{2}, \dfrac{6 + 2}{2}\right) = (7, 4)$

10. Midpoint $= \left(\dfrac{-7 + 3}{2}, \dfrac{0 + 6}{2}\right) = (-2, 3)$

11. Midpoint $= \left(\dfrac{-10 - 6}{2}, \dfrac{4 + 8}{2}\right) = (-8, 6)$

12. Midpoint $= \left(\dfrac{7 - 3}{2}, \dfrac{-9 + 5}{2}\right) = (2, -2)$

13. Midpoint $= \left(\dfrac{-2 + 4.3}{2}, \dfrac{0.1 + (-3)}{2}\right)$

$= \left(\dfrac{2.3}{2}, \dfrac{-2.9}{5}\right)$

$= (1.15, -1.45)$

14. Midpoint $= \left(\dfrac{\frac{1}{2} + 1}{2}, \dfrac{\frac{5}{2} + \left(-\frac{3}{4}\right)}{2}\right)$

$= \left(\dfrac{\frac{3}{2}}{2}, \dfrac{\frac{7}{4}}{5}\right)$

$= \left(\dfrac{3}{4}, \dfrac{7}{20}\right)$

15. $P = R - C$. The tallest bars represent revenues. The middle bars represent costs. The bars on the left of each group represent profits because $P = R - C$.

16. 2009: $R \approx \$24.0$ billion
$C \approx \$16.0$ billion
$P \approx \$8.0$ billion

2010: $R \approx \$30.0$ billion
$C \approx \$22.0$ billion
$P \approx \$8.0$ billion

2011: $R \approx \$38.0$ billion
$C \approx \$30.0$ billion
$P \approx \$8.0$ billion

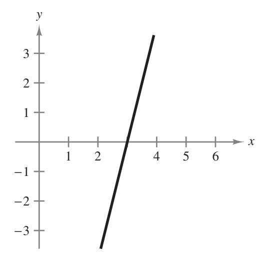
2012: $R \approx \$50.0$ billion
$C \approx \$40.0$ billion
$P \approx \$10.0$ billion

2013: $R \approx \$58.0$ billion
$C \approx \$48.0$ billion
$P \approx \$10.0$ billion

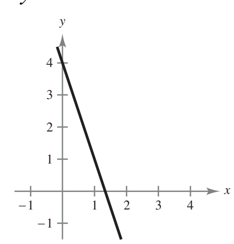
17. $(1, 3)$ translates to $(3, 6)$.
$(2, 4)$ translates to $(4, 7)$.
$(4, 1)$ translates to $(6, 4)$.

18. $(-2, 1)$ translates to $(-7, -1)$.
$(-1, 2)$ translates to $(-6, 0)$.
$(1, 0)$ translates to $(-4, -2)$.
$(0, -1)$ translates to $(-5, -3)$.

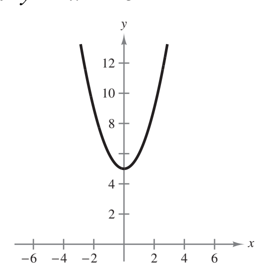
19. $y = 4x - 12$



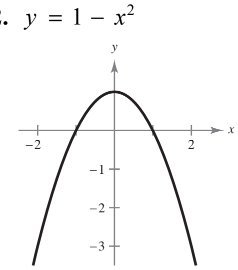
20. $y = 4 - 3x$



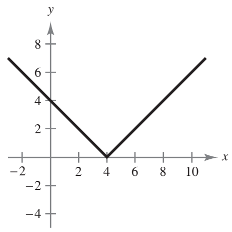
21. $y = x^2 + 5$



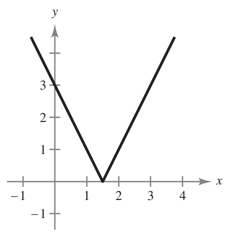
22. $y = 1 - x^2$



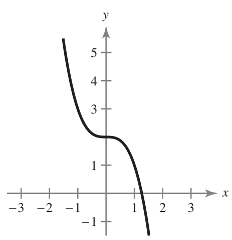
23. $y = |4 - x|$



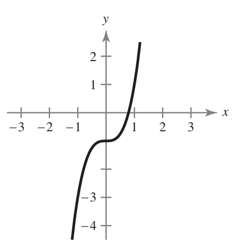
24. $y = |2x - 3|$



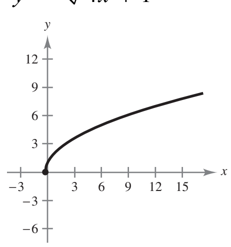
25. $y = 2 - x^3$



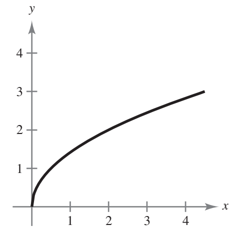
26. $y = 2x^3 - 1$



27. $y = \sqrt{4x + 1}$



28. $y = \sqrt{2x}$



29. Let $y = 0$. Then,

$$4x + 0 + 3 = 0$$
$$x = -\tfrac{3}{4}.$$

Let $x = 0$. Then,

$$4(0) + y + 3 = 0$$
$$y = -3.$$

x-intercept: $\left(-\tfrac{3}{4}, 0\right)$

y-intercept: $(0, -3)$

30. Let $y = 0$. Then,

$$3x - (0) + 6 = 0$$
$$3x = -6$$
$$x = -2.$$

Let $x = 0$. Then,

$$3(0) - y + 6 = 0$$
$$-y = -6$$
$$y = 6.$$

x-intercept: $(-2, 0)$

y-intercept: $(0, 6)$

31. Let $y = 0$. Then,

$$0 = x^2 + 2x - 8$$
$$0 = (x + 4)(x - 2)$$
$$x + 4 = 0 \quad \text{or} \quad x - 2 = 0$$
$$x = -4 \qquad\qquad x = 2.$$

Let $x = 0$. Then,

$$y = (0)^2 + 2(0) - 8$$
$$y = -8.$$

x-intercepts: $(-4, 0), (2, 0)$

y-intercept: $(0, -8)$

32. Let $y = 0$. Then,

$$0 = (x - 1)^3 + 2(x - 1)^2$$
$$0 = (x - 1)^2(x + 1)$$
$$x = \pm 1.$$

Let $x = 0$. Then,

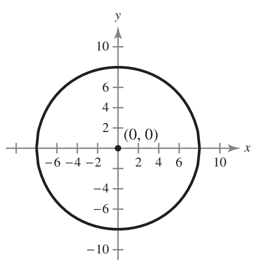
$$y = (0 - 1)^3 + 2(0 - 1)^2$$
$$y = 1.$$

x-intercepts: $(-1, 0)$, $(1, 0)$

y-intercept: $(0, 1)$

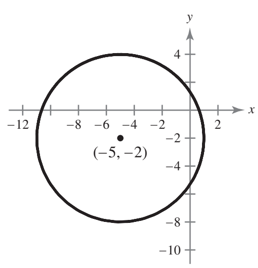
33. $(x - 0)^2 + (y - 0)^2 = 8^2$

$$x^2 + y^2 = 64$$



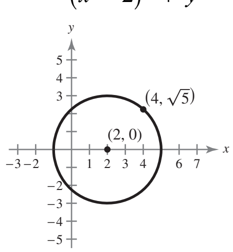
34. $(x - (-5))^2 + (y - (-2))^2 = 6^2$

$$(x + 5)^2 + (y + 2)^2 = 36$$



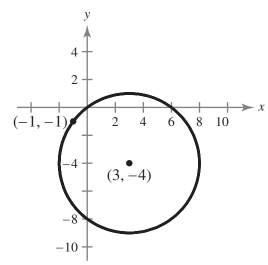
35. $(x - 2)^2 + (y - 0)^2 = r^2$

$$(x - 2)^2 + y^2 = r^2$$
$$(4 - 2)^2 + (\sqrt{5})^2 = r^2$$
$$4 + 5 = r^2$$
$$9 = r^2$$
$$(x - 2)^2 + y^2 = 9$$



36. $(x - 3)^2 + (y - (-4))^2 = r^2$

$$(x - 3)^2 + (y + 4)^2 = r^2$$
$$(-1 - 3)^2 + (-1 + 4)^2 = r^2$$
$$(-4)^2 + (3)^2 = r^2$$
$$16 + 9 = r^2$$
$$25 = r^2$$
$$(x - 3)^2 + (y + 4)^2 = 25$$



37. $y = 2x + 13$ and $y = -5x - 1$

Set the two equations equal to each other.

$$2x + 13 = -5x - 1$$
$$7x = -14$$
$$x = -2$$

Substitute $x = -2$ into one of the equations.

$$y = 2(-2) + 13 = 9$$

Point of intersection: $(-2, 9)$

38. $y = x^2 + 3$ and $y = 9 - x$

Set the two equations equal to each other.

$$x^2 + 3 = 9 - x$$
$$x^2 + x - 6 = 0$$
$$(x + 3)(x - 2) = 0$$
$$x + 3 = 0 \quad \text{or} \quad x - 2 = 0$$
$$x = -3 \qquad x = 2$$

Substitute $x = -3$ and $x = 2$ into one of the equations.

$$y = (-3)^2 + 3 = 12$$
$$y = (2)^2 + 3 = 7$$

Points of intersection: $(-3, 12)$ and $(2, 7)$

39. By equating the y-values for the two equations, you have

$$x^3 = x$$
$$x(x^2 - 1) = 0$$
$$x = -1, 0, 1.$$

The corresponding y-values are $y = -1$, $y = 0$, and $y = 1$, so the points of intersection are $(-1, -1)$, $(0, 0)$, and $(1, 1)$.

40. $y = x^3 + 4x^2 - 3$ and $y = -2x^2 + 27x - 3$

Set the two equations equal to each other.

$$x^3 + 4x^2 - 3 = -2x^2 + 27x - 3$$
$$x^3 + 6x^2 - 27x = 0$$
$$x(x^2 + 6x - 27) = 0$$
$$x(x + 9)(x - 3) = 0$$
$$x = 0$$
$$x + 9 = 0 \to x = -9$$
$$x - 3 = 0 \to x = 3$$

Substitute $x = 0$, $x = -9$, and $x = 3$ into one of the equations.

$$y = (0)^3 + 4(0)^2 - 3 = -3$$
$$y = (-9)^3 + 4(-9)^2 - 3 = -408$$
$$y = (3)^3 + 4(3)^2 - 3 = 60$$

Points of intersection: $(0, -3)$, $(-9, -408)$, and $(3, 60)$

41. (a) $C = 200 + 2x + 8x = 200 + 10x$

$R = 14x$

(b)
$$C = R$$
$$200 + 10x = 14x$$
$$200 = 4x$$
$$x = 50 \text{ shirts}$$

$(x, R) = (x, C) = (50, 700)$

(c) $P = R - C$

$P = 14x - (200 + 10x)$

$P = 4x - 200$

To find the number of shirts that yields a profit of \$600, set $P = 600$ and solve for x.

$$600 = 4x - 200$$
$$800 = 4x$$
$$200 = x$$

So, 200 shirts will yield a profit of \$600.

42. (a) $C = 6000 + 6.50x$

$R = 13.90x$

(b)
$$C = R$$
$$6000 + 6.5x = 13.9x$$
$$6000 = 7.4x$$
$$x \approx 810.81, \text{ or } 811 \text{ units}$$

(c) $P = R - C$

$P = 13.9x - (6000 + 6.5x)$

$P = 7.4x - 6000$

To find the number of units that yields a profit of \$1500, set $P = 1500$ and solve for x.

$$1500 = 7.4x - 6000$$
$$7500 = 7.4x$$
$$x \approx 1014 \text{ units}$$

So, about 1014 units will yield a profit of \$1500.

43. $p = 91.4 - 0.009x = 6.4 + 0.008x$

$$85 = 0.017x$$
$$x = 5000 \text{ units}$$

Equilibrium point $(x, p) = (5000, 46.40)$

44. (a)

Year	2008	2009	2010	2011	2012	2013
Wind (actual)	546	721	923	1168	1340	1595
Wind (model)	542.0	727.2	928.0	1140.7	1361.5	1586.6

The model fits the data well for the years 2008 through 2013.

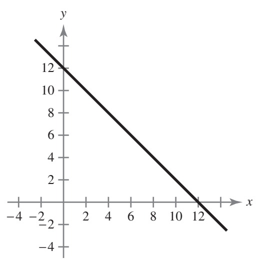
(b) Let $t = 19$.

$y \approx 2815.7$ trillion Btu

45. $y = -x + 12$

Slope: $m = -1$

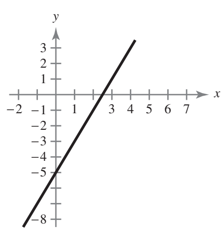
y-intercept: $(0, 12)$



46. $y = 2x - 5$

Slope: $m = 2$

y-intercept: $(0, -5)$

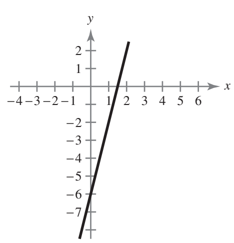


47. $4x - y = 6$

$y = 4x - 6$

Slope: $m = 4$

y-intercept: $(0, -6)$



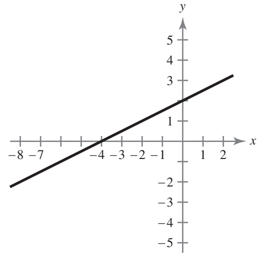
48. $2x - 4y = -8$

$-4y = -2x - 8$

$y = \frac{1}{2}x + 2$

Slope: $m = \frac{1}{2}$

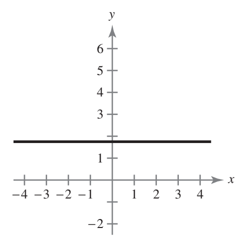
y-intercept: $(0, 2)$



49. $y = \frac{7}{4}$

Slope: $m = 0$ (horizontal line)

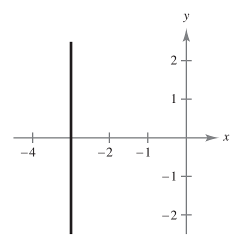
y-intercept: $\left(0, \frac{7}{4}\right)$



50. $x = -3$

Slope: undefined (vertical line)

No y-intercept



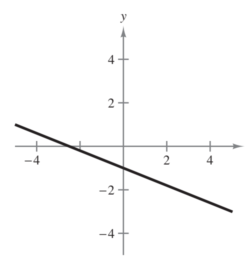
51. $-2x - 5y - 5 = 0$

$5y = -2x - 5$

$y = -\frac{2}{5}x - 1$

Slope: $m = -\frac{2}{5}$

y-intercept: $(0, -1)$



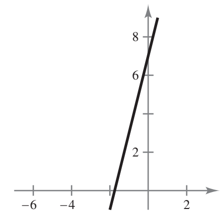
52. $3.2x - 0.8y + 5.6 = 0$

$8y = 32x + 56$

$y = 4x + 7$

Slope: $m = 4$

y-intercept: $(0, 7)$



53. Slope $= \dfrac{6-0}{7-0} = \dfrac{6}{7}$

54. Slope $= \dfrac{7-5}{-5-(-1)} = -\dfrac{1}{2}$

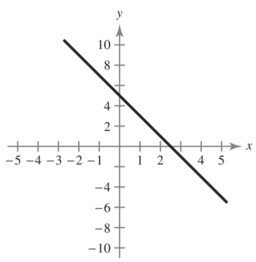
55. Slope $= \dfrac{29-29}{-3-5} = \dfrac{0}{-8} = 0$

56. Slope $= \dfrac{-3-(-3)}{-1-(-11)} = 0$ (horizontal line)

57. $y-(-1) = -2(x-3)$

$y+1 = -2x+6$

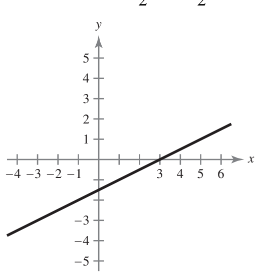
$y = -2x+5$



58. $y-(-3) = \frac{1}{2}(x-(-3))$

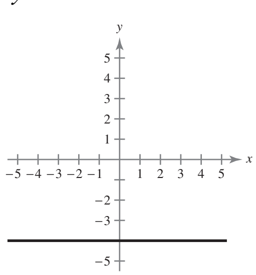
$y+3 = \frac{1}{2}x+\frac{3}{2}$

$y = \frac{1}{2}x-\frac{3}{2}$



59. $m = 0$: horizontal line through $(1.5, -4)$

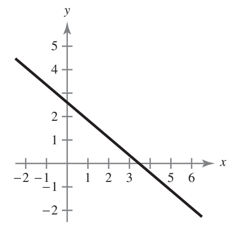
$y = -4$



60. $y-2 = -\frac{3}{4}\left(x-\frac{4}{5}\right)$

$y-2 = -\frac{3}{4}x+\frac{3}{5}$

$y = -\frac{3}{4}x+\frac{13}{5}$

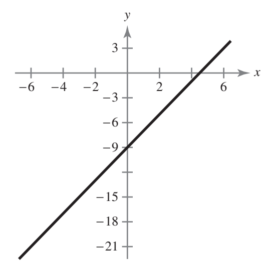


61. $m = \dfrac{5-(-7)}{7-1} = \dfrac{12}{6} = 2$

$y-(-7) = 2(x-1)$

$y+7 = 2x-2$

$y = 2x-9$



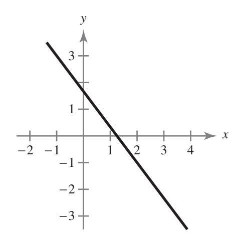
62. $m = \dfrac{-9-7}{8-(-4)} = \dfrac{-16}{12} = -\dfrac{4}{3}$

$y-7 = -\dfrac{4}{3}(x-(-4))$

$y-7 = -\dfrac{4}{3}(x+4)$

$y-7 = -\dfrac{4}{3}x-\dfrac{16}{3}$

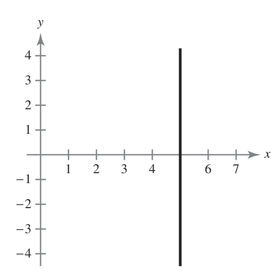
$y = -\dfrac{4}{3}x+\dfrac{5}{3}$



63. $m = \dfrac{14-7}{5-5} = \dfrac{7}{0} \Rightarrow m$ is undefined.

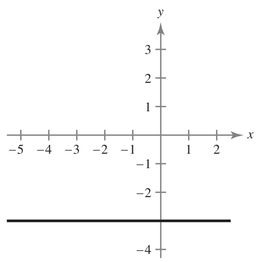
Vertical line through $(5, 7)$

$x = 5$



64. $m = \dfrac{-3 - (-3)}{-2 - 4} = \dfrac{0}{-6} = 0 \Rightarrow$ horizontal line through $(4, -3)$

$y = -3$



65. (a)
$$y - 6 = \tfrac{5}{8}[x - (-3)]$$
$$y - 6 = \tfrac{5}{8}(x + 3)$$
$$y - 6 = \tfrac{5}{8}x + \tfrac{15}{8}$$
$$y = \tfrac{5}{8}x + \tfrac{63}{8}$$
$$5x - 8y + 63 = 0$$

(b) $y = -5x - 3$

The given line's slope is $m = -5$, so all the lines perpendicular to the given line have slope $m = \frac{1}{5}$.

$$y - 6 = \tfrac{1}{5}[x - (-3)]$$
$$y - 6 = \tfrac{1}{5}(x + 3)$$
$$y - 6 = \tfrac{1}{5}x + \tfrac{3}{5}$$
$$y = \tfrac{1}{5}x + \tfrac{33}{5}$$
$$x - 5y + 33 = 0$$

(c) $4x + 2y = 7 \Rightarrow y = -2x + \frac{7}{2}$

The given line's slope is $m = -2$, so all the lines parallel to the given line have slope $m = -2$.

$$y - 6 = -2[x - (-3)]$$
$$y = -2x$$
$$2x + y = 0$$

(d) $3x - 2y = 2 \Rightarrow y = \frac{3}{2}x - 1$

The given line's slope is $m = \frac{3}{2}$, so all the lines perpendicular to the given line have slope $m = -\frac{2}{3}$.

$$y - 6 = -\tfrac{2}{3}[x - (-3)]$$
$$y = -\tfrac{2}{3}x + 4$$
$$2x + 3y - 12 = 0$$

66. (a) Slope $= 0$
$$y = -3$$
$$y + 3 = 0$$

(b) Slope undefined
$$x = 1$$
$$x - 1 = 0$$

(c) $-4x + 5y = -3$
$$y = \tfrac{4}{5}x - \tfrac{3}{5}$$
$$y - (-3) = \tfrac{4}{5}(x - 1)$$
$$y = \tfrac{4}{5}x - \tfrac{19}{5}$$
$$-\tfrac{4}{5}x + y + \tfrac{19}{5} = 0$$

(d) $5x - 2y = 3$
$$y = \tfrac{5}{2}x - \tfrac{3}{2}$$

Slope of perpendicular is $-\frac{2}{5}$.

$$y - (-3) = -\tfrac{2}{5}(x - 1)$$
$$y = -\tfrac{2}{5}x - \tfrac{13}{5}$$
$$\tfrac{2}{5}x + y + \tfrac{13}{5} = 0$$

67. $(32, 750), (37, 700)$

$$m = \frac{750 - 700}{32 - 37} = \frac{50}{-5} = -10$$

(a) $x - 750 = -10(p - 32)$
$$x = -10p + 1070$$

(b) If $p = 34.50$, $x = -10(34.50) + 1070 = 725$ units

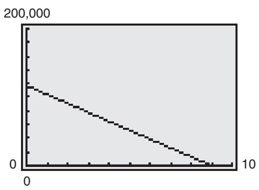
(c) If $p = 42.00$, $x = -10(42.00) + 1070 = 650$ units

68. $(0,\ 117{,}000),\ (9,\ 0)$

$m = \dfrac{117{,}000}{-9} = -13{,}000$

(a) $v = -13{,}000(t - 9) = -13{,}000t + 117{,}000$

(b) Graphing utility



(c) $v(4) = \$65{,}000$

(d) $v = 84{,}000$ when $t \approx 2.54$ years

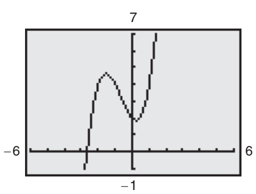
69. Yes, $y = -x^2 + 2$ is a function of x.

70. No, $x = y^2 - 2$ is not a function of x.

71. No, $y^2 - \frac{1}{4}x^2 = 4$ is not a function of x.

72. Yes, $y = |x + 4|$ is a function of x.

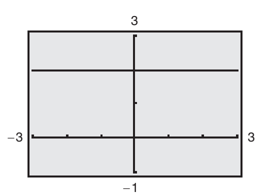
73.



Domain: $(-\infty,\ \infty)$

Range: $(-\infty,\ \infty)$

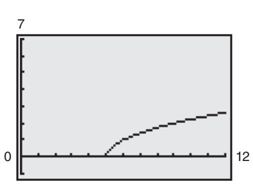
74.



Domain: $(-\infty,\ \infty)$

Range: $\{2\}$

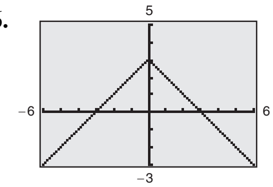
75.



Domain: $[-5,\ \infty)$

Range: $[0,\ \infty)$

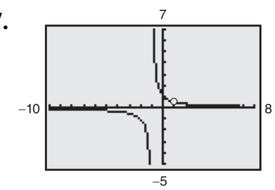
76.



Domain: $(-\infty,\ \infty)$

Range: $(-\infty,\ 3]$

77.

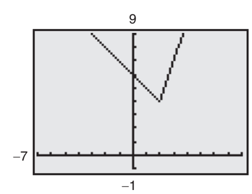


$$\begin{aligned} f(x) &= \frac{x - 1}{x^2 - 1} \\ &= \frac{x - 1}{(x - 1)(x + 1)} \\ &= \frac{1}{x + 1},\ x \neq 1 \end{aligned}$$

Domain: $(-\infty,\ -1) \cup (-1,\ 1) \cup (1,\ \infty)$

Range: $(-\infty,\ 0) \cup \left(0,\ \dfrac{1}{2}\right) \cup \left(\dfrac{1}{2},\ \infty\right)$

78. $f(x) = \begin{cases} 6 - x, & x < 2 \\ 3x - 2, & x \geq 2 \end{cases}$



Domain: $(-\infty,\ \infty)$

Range: $(-\infty,\ \infty)$

79. $f(x) = 3x + 4$

(a) $f(1) = 3(1) + 4 = 7$

(b) $f(-5) = 3(-5) + 4 = -11$

(c) $f(x + 1) = 3(x + 1) + 4 = 3x + 7$

80. $f(x) = x^2 + 4x + 3$

(a) $f(0) = (0)^2 + 4(0) + 3 = 3$

(b) $f(3) = (3)^2 + 4(3) + 3 = 24$

(c) $$\begin{aligned} f(x - 1) &= (x - 1)^2 + 4(x - 1) + 3 \\ &= x^2 - 2x + 1 + 4x - 4 + 3 \\ &= x^2 + 2x \end{aligned}$$

81. (a) $f(x) + g(x) = (6 + x^2) + (3x - 5)$
$= x^2 + 3x + 1$

(b) $f(x) - g(x) = (6 + x^2) - (3x - 5)$
$= x^2 - 3x + 11$

(c) $f(x)g(x) = (6 + x^2)(3x - 5)$
$= 3x^3 - 5x^2 + 18x - 30$

(d) $\dfrac{f(x)}{g(x)} = \dfrac{6 + x^2}{3x - 5}$

(e) $f(g(x)) = f(3x - 5)$
$= 6 + (3x - 5)^2$
$= 9x^2 - 30x + 31$

(f) $g(f(x)) = g(6 + x^2)$
$= 3(6 + x^2) - 5$
$= 3x^2 + 13$

82. (a) $f(x) + g(x) = 2x - 3 + \sqrt{x + 1}$

(b) $f(x) - g(x) = 2x - 3 - \sqrt{x + 1}$

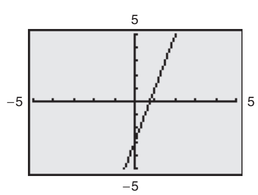
(c) $f(x)g(x) = (2x - 3)\sqrt{x + 1}$

(d) $\dfrac{f(x)}{g(x)} = \dfrac{2x - 3}{\sqrt{x + 1}}$

(e) $f(g(x)) = f(\sqrt{x + 1})$
$= 2\sqrt{x + 1} - 3$

(f) $g(f(x)) = g(2x - 3)$
$= \sqrt{(2x - 3) + 1}$
$= \sqrt{2x - 2}$

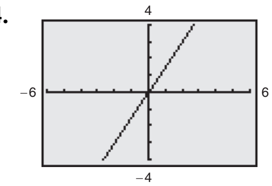
83.



$f(x)$ is one-to-one.

$f(x) = 4x - 3 = y$
$4y - 3 = x$
$4y = x + 3$
$y = \frac{1}{4}x + \frac{3}{4}$
$f^{-1}(x) = \frac{1}{4}x + \frac{3}{4}$

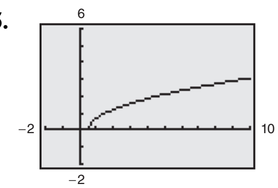
84.



$f(x)$ is one-to-one.

$f(x) = \frac{3}{2}x = y$
$\frac{3}{2}y = x$
$3y = 2x$
$y = \frac{2}{3}x$
$f^{-1}(x) = \frac{2}{3}x$

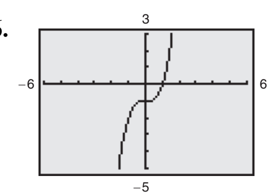
85.



$f(x)$ is one-to-one.

$f(x) = \sqrt{x - \frac{1}{2}}$
$y = \sqrt{x - \frac{1}{2}}$
$x = \sqrt{y - \frac{1}{2}}$
$x^2 = y - \frac{1}{2}$
$y = x^2 + \frac{1}{2}$
$f^{-1}(x) = x^2 + \frac{1}{2}, x \ge 0$

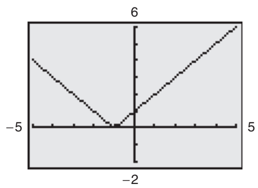
86.



$f(x)$ is one-to-one.

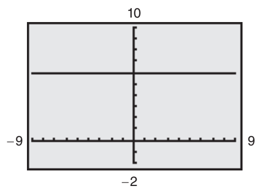
$f(x) = x^3 - 1 = y$
$y^3 - 1 = x$
$y^3 = x + 1$
$y = \sqrt[3]{x + 1}$
$f^{-1}(x) = \sqrt[3]{x + 1}$

87.



$f(x)$ does not have an inverse function.

88.



$f(x)$ does not have an inverse function.

89.

x	0.9	0.99	0.999	1	1.001	1.01	1.1
$f(x)$	0.6	0.96	0.996	?	1.004	1.04	1.4

$\lim_{x\to 1}(4x-3) = 1$

90.

x	2.9	2.99	2.999	3
$f(x)$	0.2564	0.2506	0.2501	?

x	3.001	3.01	3.1
$f(x)$	0.2499	0.2494	0.2439

$\lim_{x\to 3}\dfrac{x-3}{x^2-2x-3} = 0.25$

91.

x	−0.1	−0.01	−0.001	0
$f(x)$	0.2050	0.2042	0.2041	?

x	0.001	0.01	0.1
$f(x)$	0.2041	0.2040	0.2033

$\lim_{x\to 0}\dfrac{\sqrt{x+6}-\sqrt{6}}{x} \approx 0.204$

Note: $\lim_{x\to 0}\dfrac{\sqrt{x+6}-\sqrt{6}}{x} = \dfrac{1}{2\sqrt{6}}$

92.

x	6.9	6.99	6.999	7
$f(x)$	−1.47	−14.33	−142.90	?

x	7.001	7.01	7.1
$f(x)$	142.82	14.24	1.39

$\lim_{x\to 7}\dfrac{\dfrac{1}{x-7}-\dfrac{1}{7}}{x}$ does not exist.

93. $\lim_{x\to 3} 8 = 8$

94. $\lim_{x\to 6} x^4 = (6)^4 = 1296$

95. $\lim_{x\to 2}(5x-3) = 5(2) - 3 = 7$

96. $\lim_{x\to 5}(3x^2+4) = 3(5)^2 + 4 = 79$

97. $\lim_{x\to -1}\dfrac{x+3}{6x+1} = \dfrac{-1+3}{6(-1)+1} = -\dfrac{2}{5}$

98. $\lim_{t\to -3}\dfrac{6t+5}{t+8} = \dfrac{6(-3)+5}{(-3)+8} = \dfrac{-13}{5} = -\dfrac{13}{5}$

99. $\lim_{t\to 0^-}\dfrac{t^2+1}{t} = -\infty$

$\lim_{t\to 0^+}\dfrac{t^2+1}{t} = \infty$

$\lim_{t\to 0}\dfrac{t^2+1}{t}$ does not exist.

100. $\lim_{t\to 2^-}\dfrac{t+1}{t-2} = -\infty$

$\lim_{t\to 2^+}\dfrac{t+1}{t-2} = \infty$

$\lim_{t\to 2}\dfrac{t+1}{t-2}$ does not exist.

101. $\lim_{x\to -2}\dfrac{x+2}{x^2-4} = \lim_{x\to -2}\dfrac{x+2}{(x+2)(x-2)}$

$= \lim_{x\to -2}\dfrac{1}{x-2}$

$= -\dfrac{1}{4}$

102. $\lim_{x\to 4}\dfrac{x^2-6x-8}{x-4} = \lim_{x\to 4}\dfrac{(x-4)(x+2)}{x-4}$

$= \lim_{x\to 4}(x+2)$

$= 6$

103. $\lim_{x\to 0^+}\left(x - \frac{1}{x}\right) = \lim_{x\to 0^+}\frac{x^2 - 1}{x} = -\infty$

104. $\lim_{x\to 1/2^-}\frac{-x}{6x - 3} = \frac{\frac{1}{2}}{6\left(\frac{1}{2}\right) - 3} = \frac{\frac{1}{2}}{3 - 3} = \frac{\frac{1}{2}}{0} = \infty$

105.
$$\begin{aligned}\lim_{x\to 0}\frac{\sqrt{3} - \sqrt{x + 3}}{x} &= \lim_{x\to 0}\frac{\sqrt{3} - \sqrt{x + 3}}{x}\cdot\frac{\sqrt{3} + \sqrt{x + 3}}{\sqrt{3} + \sqrt{x + 3}}\\ &= \lim_{x\to 0}\frac{3 - (x + 3)}{x\left(\sqrt{3} + \sqrt{x + 3}\right)}\\ &= \lim_{x\to 0}\frac{-x}{x\left(\sqrt{3} + \sqrt{x + 3}\right)}\\ &= \lim_{x\to 0}\frac{-1}{\sqrt{3} + \sqrt{x + 3}}\\ &= -\frac{1}{2\sqrt{3}} = -\frac{\sqrt{3}}{6}\end{aligned}$$

106.
$$\begin{aligned}\lim_{s\to 0}\frac{\left(1/\sqrt{1 + s}\right) - 1}{s} &= \lim_{s\to 0}\frac{1 - \sqrt{1 + s}}{s\sqrt{1 + s}}\cdot\frac{1 + \sqrt{1 + s}}{1 + \sqrt{1 + s}}\\ &= \lim_{s\to 0}\frac{1 - (1 + s)}{s\sqrt{1 + s}\left(1 + \sqrt{1 + s}\right)}\\ &= \lim_{s\to 0}\frac{-1}{\sqrt{1 + s}\left(1 + \sqrt{1 + s}\right)}\\ &= -\frac{1}{2}\end{aligned}$$

107. $\lim_{x\to 0} f(x)$, where $f(x) = \begin{cases} x + 5, & x \neq 0\\ 3, & x = 0\end{cases}$

$\lim_{x\to 0} f(x) = \lim_{x\to 0}(x + 5) = 0 + 5 = 5$

108. $\lim_{x\to -2} f(x)$, where $f(x) = \begin{cases}\frac{1}{2}x + 5, & x < -2\\ -x + 2, & x \geq -2\end{cases}$

$\lim_{x\to -2} f(x) = -x + 2 = -(-2) + 4 = 4$

109.
$$\begin{aligned}\lim_{\Delta x\to 0}\frac{(x + \Delta x)^3 - (x + \Delta x) - (x^3 - x)}{\Delta x} &= \lim_{\Delta x\to 0}\frac{x^3 + 3x^2\Delta x + 3x(\Delta x)^2 + (\Delta x)^3 - x - \Delta x - x^3 + x}{\Delta x}\\ &= \lim_{\Delta x\to 0}\frac{3x^2\Delta x + 3x(\Delta x)^2 + (\Delta x)^3 - \Delta x}{\Delta x}\\ &= \lim_{\Delta x\to 0}\left[3x^2 + 3x\Delta x + (\Delta x)^2 - 1\right]\\ &= 3x^2 - 1\end{aligned}$$

110.
$$\begin{aligned}\lim_{\Delta x\to 0}\frac{\left[1 - (x + \Delta x)^2\right] - (1 - x^2)}{\Delta x} &= \lim_{\Delta x\to 0}\frac{1 - x^2 - 2x\Delta x - (\Delta x)^2 - 1 + x^2}{\Delta x}\\ &= \lim_{\Delta x\to 0}\frac{\Delta x(-2x - \Delta x)}{\Delta x}\\ &= \lim_{\Delta x\to 0}(-2x - \Delta x) = -2x\end{aligned}$$

111. $f(x) = x + b$ is continuous on $(-\infty, \infty)$ because the domain of f consists of all real x.

112. $f(x) = x^2 + 3x + 2$ is continuous on $(-\infty, \infty)$ because the domain of f consists of all real x.

113. $f(x) = \frac{1}{(x + 4)^2}$ is continuous on the intervals $(-\infty, -4)$ and $(-4, \infty)$ because the domain of f consists of all real numbers except $x = -4$. There is a discontinuity at $x = -4$ because $f(4)$ is not defined.

114. $f(x) = \dfrac{x+2}{x}$ is continuous on the intervals $(-\infty, 0)$ and $(0, \infty)$ because the domain of f consists of all real numbers except $x = 0$. There is a discontinuity at $x = 0$ because $f(0)$ is not defined.

115. $f(x) = \dfrac{3}{x+1}$ is continuous on the intervals $(-\infty, -1)$ and $(-1, \infty)$ because the domain of f consists of all real numbers except $x = -1$. There is a discontinuity at $x = -1$ because $f(-1)$ is not defined.

116. $f(x) = \dfrac{x+1}{2x+2}$ is continuous on the intervals $(-\infty, -1)$ and $(-1, \infty)$ because the domain of f consists of all real number except $x = -1$. There is a discontinuity at $x = -1$ because $f(-1)$ is not defined.

117. $f(x) = \sqrt{x-8}$ is continuous on the interval $[8, \infty)$ because the domain of f consists of all real numbers $x \ge 8$. For all values of $c > 8$, $F(c)$ is defined, the limit exists as $x \to c$, and $f(c) = \lim_{x\to c} f(x)$.

$\left[\text{Note: } f \text{ is continuous at } x = 8 \text{ since } f(8) = \lim_{x\to 8^+} f(x).\right]$

118. $f(x) = \sqrt{5-x}$ is continuous on the interval $(-\infty, 5]$ because the domain of f consists of all real numbers $x \le 5$. For all values of $c < 5$, $F(c)$ is defined, the limit exists as $x \to c$, and $f(c) = \lim_{x\to c} f(x)$.

$\left[\left[\text{Note: } f \text{ is continuous at } x = 5 \text{ since } \lim_{x\to 5^-} f(x).\right]\right.$

119. $f(x) = [\![x+3]\!]$ is continuous on all intervals of the form $(c, c+1)$, where c is an integer. There are discontinuities at all integer values c because $\lim_{x\to c} f(x)$ does not exist.

120. $f(x) = [\![x]\!] - 2$ is continuous on all intervals of the form $(c, c+1,)$ where c is an integer. There are discontinuities at all integer values c because $\lim_{x\to c} f(x)$ does not exist.

121. $f(x) = \begin{cases} x, & x \le 0 \\ x+1, & x > 0 \end{cases}$ is continuous on the intervals $(-\infty, 0)$ and $(0, \infty)$. There is a discontinuity at $x = 0$ because $\lim_{x\to 0} f(x)$ does not exist.

122. $f(x) = \begin{cases} x, & x \le 0 \\ x^2, & x > 0 \end{cases}$ is continuous on $(-\infty, \infty)$ because $f(0)$ is defined, $\lim_{x\to 0} f(x)$ exists, and $\lim_{x\to 0} f(x) = f(0)$.

123. $\lim_{x\to 3^-} f(x) = \lim_{x\to 3^-} (-x+1) = -2$

$\lim_{x\to 3^+} f(x) = \lim_{x\to 3^+} (ax-8) = 3a - 8$

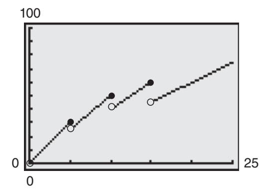
So, $-2 = 3a - 8$ and $a = 2$.

124. $\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} (x+1) = 2$

$\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} (2x+a) = 2 + a$

So, $2 = 2 + a$ and $a = 0$.

125. (a)



Explanations will vary. The function is defined for all values of x greater than zero. The function is not continuous at $x = 5$, $x = 10$, and $x = 15$.

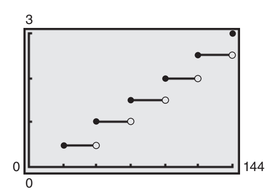
(b) $C(10) = 4.99(10) = \$49.90$

126. $\lim_{t\to 2^-} S(t) = 41{,}400$

$\lim_{t\to 2^+} S(t) = 42{,}849$

The limit of S as t approaches 2 does not exist.

127. (a)

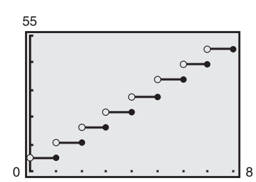


The function is not continuous at $x = 24n$, where n is a positive integer.

(b) When $x = 1500$, $A = \$31$.

128. (a) $C = 32.3 - 2.9[\![1-x]\!]$

(b)



Chapter 1 Test Yourself

1. (a) $d = \sqrt{(-4-1)^2 + (4-(-1))^2}$

$= \sqrt{(-5)^2 + (5)^2} = \sqrt{50} = 5\sqrt{2}$

(b) $\text{Midpoint} = \left(\frac{1+(-4)}{2}, \frac{-1+4}{2}\right) = \left(-\frac{3}{2}, \frac{3}{2}\right)$

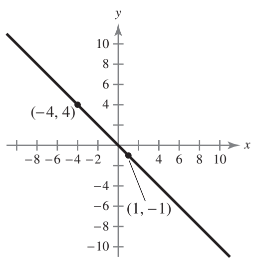
(c) $m = \frac{4-(-1)}{-4-1} = \frac{5}{-5} = -1$

(d) $y - (-1) = -1(x-1)$

$y + 1 = -x + 1$

$y = -x$

(e)



2. (a) $d = \sqrt{\left(0 - \frac{5}{2}\right)^2 + (2-2)^2}$

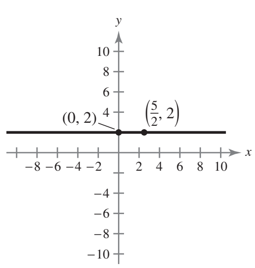
$= \sqrt{\left(-\frac{5}{2}\right)^2 + 0^2} = \frac{5}{2}$

(b) $\text{Midpoint} = \left(\frac{\frac{5}{2}+0}{2}, \frac{2+2}{2}\right) = \left(\frac{5}{4}, 2\right)$

(c) $m = \frac{2-2}{0-\frac{5}{2}} = 0$

(d) Horizontal line: $y = 2$

(e)



3. (a) $d = \sqrt{(-4-2)^2 + (1-3)^2}$

$= \sqrt{(-6)^2 + (-2)^2} = \sqrt{40}$

$= 2\sqrt{10}$

(b) $\text{Midpoint} = \left(\frac{2+(-4)}{2}, \frac{3+1}{2}\right)$

$= \left(-\frac{2}{2}, \frac{4}{2}\right)$

$= (-1, 2)$

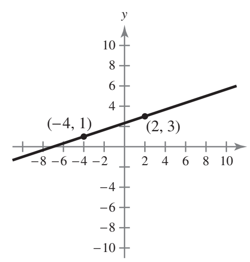
(c) $m = \frac{1-3}{-4-2} = \frac{-2}{-6} = \frac{1}{3}$

(d) $y - 3 = \frac{1}{3}(x-2)$

$y - 3 = \frac{1}{3}x - \frac{2}{3}$

$y = \frac{1}{3}x + \frac{7}{3}$

(e)



4. $65 - 2.1x = 43 + 1.9x$

$-4x = -22$

$x = 5.5$

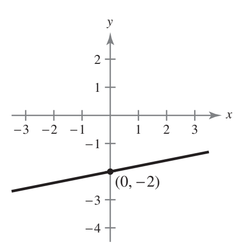
Equilibrium point $(x, p) = (5.5, 5500)$

The equilibrium point occurs when the demand and supply are each 5500 units.

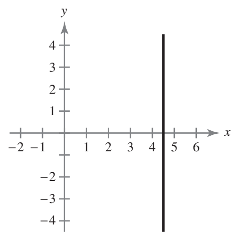
5. $m = \frac{1}{5}$

When $x = 0$: $y = \frac{1}{5}(0) - 2 = -2$

y-intercept: $(0, -2)$



6. The line $x - \frac{9}{2} = 0 \Rightarrow x = \frac{9}{2}$ is vertical, so its slope is undefined, and it has no y-intercept.

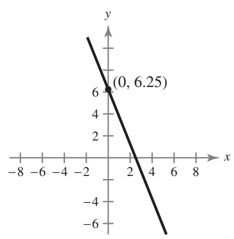


7. $y = -2.5x + 6.25$

$m = -2.5$

When $x = 0$: $y = -2.5(0) + 6.25 = 6.25$

y-intercept: $(0, 6.25)$



8. The slope of the given line $-6x + y = 3 \Rightarrow y = 6x + 3$ is $m = 4$. The slope of the perpendicular line is $m = -\frac{1}{6}$.

Using the point $(-3, -1)$ and $m = -\frac{1}{4}$, the equation is:

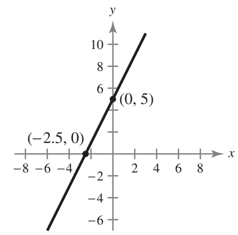
$$y - (-1) = -\tfrac{1}{6}(x - (-3))$$
$$y + 1 = -\tfrac{1}{6}(x + 3)$$
$$y + 1 = -\tfrac{1}{6}x - \tfrac{1}{2}$$
$$y = -\tfrac{1}{6}x - \tfrac{3}{2}.$$

9. The slope of the given line $5x - 2y = 8 \Rightarrow -2y = -5x + 8 \Rightarrow y = \frac{5}{2}x - 4$ is $m = \frac{5}{2}$. The slope of the line parallel is $m = \frac{5}{2}$.

Using the point $(2, 1)$ and $m = \frac{5}{2}$, the equation is:

$$y - 1 = \tfrac{5}{2}(x - 2)$$
$$y - 1 = \tfrac{5}{2}x - 5$$
$$y = \tfrac{5}{2}x - 4$$

10. (a)



(b) Domain: $(-\infty, \infty)$

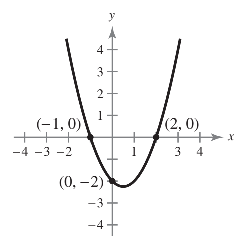
Range: $(-\infty, \infty)$

(c)

x	-3	-2	3
$f(x)$	-1	1	11

(d) The function is one-to-one.

11. (a)



(b) Domain: $(-\infty, \infty)$

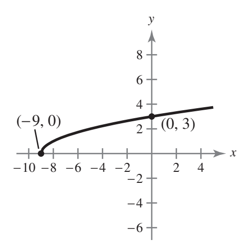
Range: $\left[-\frac{9}{4}, \infty\right)$

(c)

x	-3	-2	3
$f(x)$	10	4	4

(d) The function is not one-to-one.

12. (a)



(b) Domain: $[-9, \infty)$

Range: $[0, \infty)$

(c)

x	-3	-2	3
$f(x)$	$\sqrt{6}$	$\sqrt{7}$	$\sqrt{12} = 2\sqrt{3}$

(d) The function is one-to-one.

13. $f(x) = 4x + 6 = y$

$$4y + 6 = x$$
$$4y = x - 6$$
$$y = \tfrac{1}{4}x - \tfrac{3}{2}$$
$$f^{-1}(x) = \tfrac{1}{4}x - \tfrac{3}{2}$$

14. $f(x) = \sqrt[3]{8 - 3x} = y$

$$\sqrt[3]{8 - 3y} = x$$
$$8 - 3y = x^3$$
$$-3y = x^3 - 8$$
$$y = -\tfrac{1}{3}x^3 + \tfrac{8}{3}$$
$$f^{-1}(x) = -\tfrac{1}{3}x^3 + \tfrac{8}{3}$$

15. $\lim_{x \to 0} \dfrac{x - 2}{x + 2} = \dfrac{0 - 2}{0 + 2} = -1$

16.

x	4.9	4.99	4.999	5.001	5.01	5.1
$f(x)$	–99	–999	–9999	10,001	1001	101

$$\lim_{x \to 5^-} \frac{x + 5}{x - 5} = -\infty$$
$$\lim_{x \to 5^+} \frac{x + 5}{x - 5} = \infty$$

$\lim_{x \to 5} f(x)$ does not exist.

17. $\lim_{x \to -3} \dfrac{x^2 + 2x - 3}{x^2 + 4x + 3} = \lim_{x \to -3} \dfrac{(x - 1)(x + 3)}{(x + 1)(x + 3)}$

$$= \lim_{x \to -3} \frac{x - 1}{x + 1}$$
$$= 2$$

18.

x	–0.01	–0.001	–0.0001
$f(x)$	0.16671	0.16667	0.16667

x	0.0001	0.001	0.01
$f(x)$	0.16667	0.16666	0.16662

$$\lim_{x \to 0} \frac{\sqrt{x + 9} - 3}{x} \approx 0.16667$$

19. $f(x) = \dfrac{x^2 - 36}{x - 6}$ is continuous on the intervals $(-\infty, 6)$ and $(6, \infty)$ because the domain of *f* consists of all real numbers except $x = 6$. There is a discontinuity at $x = 6$ because $f(6)$ is not defined.

20. $f(x) = \sqrt{5 - x}$ is continuous on the interval $(5, \infty)$ because the domain of *f* consists of all $x > 5$.

21. $\lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} (1 - x) = 0$

$\lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} (x - x^2) = 0$

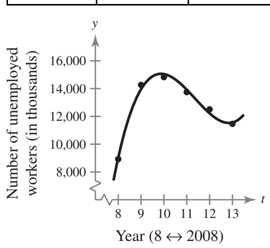
So, $\lim_{x \to 1} f(x) = 0$.

Because $f(1)$ is defined, $\lim_{x \to 1} f(x)$ exists, and $\lim_{x \to 1} f(x) = f(1)$, the function is continuous on the interval $(-\infty, \infty)$.

22. (a)

Year	2008	2009	2010
Actual	8924	14,265	14,825
Model	9026	13,956	15,046

Year	2011	2012	2013
Actual	13,747	12,506	11,460
Model	13,924	12,219	11,558



The model fits the data well.

(b) Let $t = 18$.

$$y = 271.343(18)^3 - 9246.20(18)^2 + 103{,}234.1(18) - 364{,}018$$
$$\approx 80{,}899.376 \text{ thousand}$$

In 2018, the number of unemployed workers will by 80,899,376. This prediction is invalid because this would represent an increase of over 600% in a five-year period, which is unreasonable.

C H A P T E R 2
Differentiation

CHAPTER 2
Differentiation

Section 2.1 The Derivative and the Slope of a Graph

Skills Warm Up

1. $P(3, 1), Q(3, 6)$

$m = \dfrac{6 - 1}{3 - 3}$; m is undefined.

$x = 3$

2. $P(2, 2), Q = (-5, 2)$

$m = \dfrac{2 - 2}{-5 - 2} = 0$

$y - 2 = 0(x - 2)$

$y = 2$

3. $P(1, 5), Q(4, -1)$

$m = \dfrac{-1 - 5}{4 - 1} = \dfrac{-6}{3} = -2$

$y - 5 = -2(x - 1)$

$y - 5 = -2x + 2$

$y = -2x + 7$

4. $P(3, 5), Q(-1, -7)$

$m = \dfrac{-7 - 5}{-1 - 3} = \dfrac{-12}{-4} = 3$

$y - 5 = 3(x - 3)$

$y = 3x - 4$

5. $\displaystyle\lim_{\Delta x \to 0} \frac{2x\Delta x + (\Delta x)^2}{\Delta x} = \lim_{\Delta x \to 0} \frac{\Delta x(2x + \Delta x)}{\Delta x}$

$= \displaystyle\lim_{\Delta x \to 0} 2x + \Delta x$

$= 2x$

6. $\displaystyle\lim_{\Delta x \to 0} \frac{3x^2\Delta x + 3x(\Delta x)^2 + (\Delta x)^3}{\Delta x}$

$= \displaystyle\lim_{\Delta x \to 0} \frac{\Delta x\left[3x^2 + 3x\Delta x + (\Delta x)^2\right]}{\Delta x}$

$= \displaystyle\lim_{\Delta x \to 0} 3x^2 + 3x\Delta x + (\Delta x)^2$

$= 3x^2$

7. $\displaystyle\lim_{\Delta x \to 0} \frac{1}{x(x + \Delta x)} = \frac{1}{x^2}$

8. $\displaystyle\lim_{\Delta x \to 0} \frac{(x + \Delta x)^2 - x^2}{\Delta x}$

$= \displaystyle\lim_{\Delta x \to 0} \frac{x^2 + 2x\Delta x + (\Delta x)^2 - x^2}{\Delta x}$

$= \displaystyle\lim_{\Delta x \to 0} \frac{2x\Delta x + (\Delta x)^2}{\Delta x}$

$= \displaystyle\lim_{\Delta x \to 0} \frac{\Delta x(2x + \Delta x)}{\Delta x}$

$= 2x$

9. $f(x) = 3x$

Domain: $(-\infty, \infty)$

10. $f(x) = \dfrac{1}{x - 1}$

Domain: $(-\infty, 1) \cup (1, \infty)$

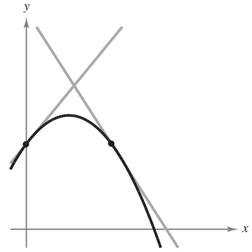
11. $f(x) = \frac{1}{5}x^3 - 2x^2 + \frac{1}{3}x - 1$

Domain: $(-\infty, \infty)$

12. $f(x) = \dfrac{6x}{x^3 + x}$

Domain: $(-\infty, 0) \cup (0, \infty)$

1.



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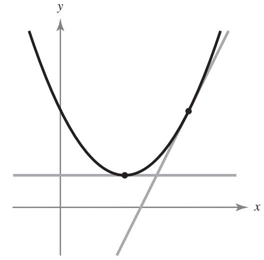

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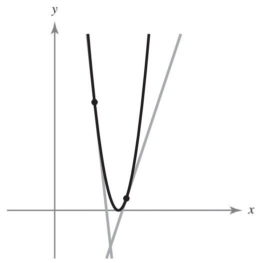

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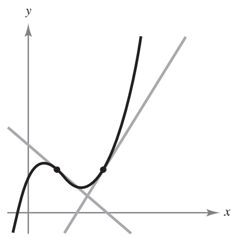

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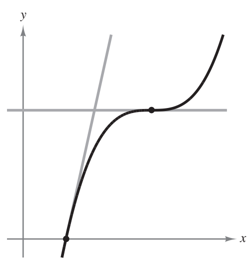
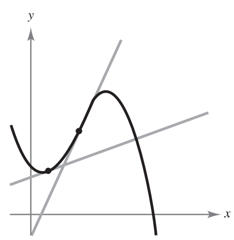


6.



7. The slope is $m = 1$.

8. The slope is $m = \frac{4}{3}$.

9. The slope is $m = 0$.

10. The slope is $m = \frac{1}{4}$.

11. The slope is $m = -\frac{1}{3}$.

12. The slope is $m = -3$.

13. 2009: $m \approx 118$

2011: $m \approx 375$

The slope is the rate of change in millions of dollars per year of revenue for the years 2009 and 2011 for Under Armour.

14. 2010: $m \approx 500$

2012: $m \approx 500$

The slope is the rate of change in millions of dollars per year of sales for the years 2010 and 2012 for Fossil.

15. $t = 3: m \approx 8$

$t = 7: m \approx 1$

$t = 10: m \approx -10$

The slope is the rate of change of the average temperature in degrees Fahrenheit per month in Bland, Virginia, for March, July, and October.

16. (a) At t_1, $f'(t_1) > g'(t_1)$, so the runner given by f is running faster.

(b) At t_2, $g'(t_2) > f'(t_2)$, so the runner given by g is running faster. The runner given by f has traveled farther.

(c) At t_3, the runners are at the same location, but the runner given by g is running faster.

(d) The runner given by g will finish first because that runner finishes the distance at a lesser value of t.

17. $f(x) = -1$ at $(0, -1)$

$$m_{sec} = \frac{f(0 + \Delta x) - f(0)}{\Delta x} = \frac{-1 - (-1)}{\Delta x} = \frac{0}{\Delta x} = 0$$

$$m = \lim_{\Delta x \to 0} m_{sec} = \lim_{\Delta x \to 0} 0 = 0$$

18. $f(x) = 6$ at $(-2, 6)$

$$m_{sec} = \frac{f(-2 + \Delta x) - f(-2)}{\Delta x} = \frac{6 - 6}{\Delta x} = \frac{0}{\Delta x} = 0$$

$$m = \lim_{\Delta x \to 0} m_{sec} = \lim_{\Delta x \to 0} 0 = 0$$

19. $f(x) = 13 - 4x$ at $(3, 1)$

$$\begin{aligned} m_{\text{sec}} &= \frac{f(3 + \Delta x) - f(3)}{\Delta x} \\ &= \frac{\left[13 - 4(3 + \Delta x)\right] - 1}{\Delta x} \\ &= \frac{13 - 12 - 4\Delta x - 1}{\Delta x} \\ &= \frac{-4\Delta x}{\Delta x} \\ &= -4 \end{aligned}$$

$$m = \lim_{\Delta x \to 0} m_{\text{sec}} = \lim_{\Delta x \to 0} (-4) = -4$$

20. $f(x) = 6x + 3$ at $(1, 9)$

$$\begin{aligned} m_{\text{sec}} &= \frac{f(1 + \Delta x) - f(1)}{\Delta x} \\ &= \frac{\left[6(1 + \Delta x) + 3\right] - 9}{\Delta x} \\ &= \frac{6 + 6\Delta x + 3 - 9}{\Delta x} \\ &= \frac{6\Delta x}{\Delta x} \\ &= 6 \end{aligned}$$

$$m = \lim_{\Delta x \to 0} m_{\text{sec}} = \lim_{\Delta x \to 0} 6 = 6$$

21. $f(x) = 2x^2 - 3$ at $(2, 5)$

$$\begin{aligned} m_{\text{sec}} &= \frac{f(2 + \Delta x) - f(2)}{\Delta x} \\ &= \frac{\left[2(2 + \Delta x)^2 - 3\right] - 5}{\Delta x} \\ &= \frac{\left[2\left(4 + 4\Delta x + (\Delta x)^2\right) - 3\right] - 5}{\Delta x} \\ &= \frac{\left[8 + 8\Delta x + 2(\Delta x)^2 - 3\right] - 5}{\Delta x} \\ &= \frac{2\Delta x(4 + \Delta x)}{\Delta x} \\ &= 2(4 + \Delta x) \end{aligned}$$

$$m = \lim_{\Delta x \to 0} m_{\text{sec}} = \lim_{\Delta x \to 0} \left(2(4 + \Delta x)\right) = 8$$

22. $f(x) = 11 - x^2$ at $(3, 2)$

$$\begin{aligned} m_{\text{sec}} &= \frac{f(3 + \Delta x) - f(3)}{\Delta x} \\ &= \frac{11 - (3 + \Delta x)^2 - \left[11 - (3)^2\right]}{\Delta x} \\ &= \frac{11 - \left(9 + 6\Delta x + \Delta x^2\right) - 2}{\Delta x} \\ &= \frac{11 - 9 - 6\Delta x + (\Delta x)^2 - 2}{\Delta x} \\ &= \frac{-6\Delta x - (\Delta x)^2}{\Delta x} \\ &= \frac{\Delta x(-6 - \Delta x)}{\Delta x} \\ &= -6 - \Delta x \end{aligned}$$

$$m = \lim_{\Delta x \to 0} m_{\text{sec}} = \lim_{\Delta x \to 0} (-6 - \Delta x) = -6$$

23. $f(x) = x^3 - 4x$ at $(-1, 3)$

$$\begin{aligned} m_{\text{sec}} &= \frac{f(-1 + \Delta x) - f(-1)}{\Delta x} \\ &= \frac{(-1 + \Delta x)^3 - 4(-1 + \Delta x) - \left[(-1)^3 - 4(-1)\right]}{\Delta x} \\ &= \frac{-1 + 3\Delta x - 3(\Delta x)^2 + (\Delta x)^3 + 4 - 4\Delta x - 3}{\Delta x} \\ &= \frac{-\Delta x - 3(\Delta x)^2 + (\Delta x)^3}{\Delta x} \\ &= \frac{\Delta x\left(-1 - 3\Delta x + (\Delta x)^2\right)}{\Delta x} \\ &= -1 - 3\Delta x + (\Delta x)^2 \end{aligned}$$

$$m = \lim_{\Delta x \to 0} m_{\text{sec}} = \lim_{\Delta x \to 0} \left(-1 - 3\Delta x + (\Delta x)^2\right) = -1$$

24. $f(x) = 7x - x^3$ at $(-3, 6)$

$$\begin{aligned} m_{\text{sec}} &= \frac{f(-3 + \Delta x) - f(-3)}{\Delta x} \\ &= \frac{7(-3 + \Delta x) - (-3 + \Delta x)^3 - \left[7(-3) - (-3)^3\right]}{\Delta x} \\ &= \frac{-21 + 7\Delta x - \left(-27 + 27\Delta x - 9(\Delta x)^2 + (\Delta x)^3\right) - 6}{\Delta x} \\ &= \frac{-20\Delta x + 9(\Delta x)^2 - (\Delta x)^3}{\Delta x} \\ &= \frac{\Delta x\left(-20 + 9\Delta x - (\Delta x)^2\right)}{\Delta x} \\ &= -20 + 9\Delta x - (\Delta x)^2 \end{aligned}$$

$$m = \lim_{\Delta x \to 0} m_{\text{sec}} = \lim_{\Delta x \to 0} \left(-20 + 9\Delta x - (\Delta x)^2\right) = -20$$

25. $f(x) = 2\sqrt{x}$ at $(4, 4)$

$$\begin{aligned} m_{\text{sec}} &= \frac{f(4 + \Delta x) - f(4)}{\Delta x} \\ &= \frac{2\sqrt{4 + \Delta x} - 2\sqrt{4}}{\Delta x} \\ &= \frac{2\sqrt{4 + \Delta x} - 4}{\Delta x} \cdot \frac{2\sqrt{4 + \Delta x} + 4}{2\sqrt{4 + \Delta x} + 4} \\ &= \frac{4(4 + \Delta x) - 16}{\Delta x\left(2\sqrt{4 + \Delta x} + 4\right)} \\ &= \frac{16 + 4\Delta x - 16}{\Delta x\left(2\sqrt{4 + \Delta x} + 4\right)} \\ &= \frac{4\Delta x}{\Delta x\left(2\sqrt{4 + \Delta x} + 4\right)} \\ &= \frac{4}{2\sqrt{4 + \Delta x} + 4} \end{aligned}$$

$$\begin{aligned} m = \lim_{\Delta x \to 0} m_{\text{sec}} &= \lim_{\Delta x \to 0} \frac{4}{2\sqrt{4 + \Delta x + 4}} \\ &= \frac{4}{2\sqrt{4} + 4} = \frac{1}{2} \end{aligned}$$

26. $f(x) = \sqrt{x + 1}$ at $(8, 3)$

$$\begin{aligned} m_{\text{sec}} &= \frac{f(8 + \Delta x) - f(8)}{\Delta x} \\ &= \frac{\sqrt{8 + \Delta x + 1} - \sqrt{8 + 1}}{\Delta x} \\ &= \frac{\sqrt{9 + \Delta x} - 3}{\Delta x} \\ &= \frac{\sqrt{9 + \Delta x} - 3}{\Delta x} \cdot \frac{\sqrt{9 + \Delta x} + 3}{\sqrt{9 + \Delta x} + 3} \\ &= \frac{9 + \Delta x - 9}{\Delta x\left(\sqrt{9 + \Delta x} + 3\right)} \\ &= \frac{\Delta x}{\Delta x\left(\sqrt{9 + \Delta x} + 3\right)} \\ &= \frac{1}{\sqrt{9 + \Delta x} + 3} \end{aligned}$$

$$\begin{aligned} m &= \lim_{\Delta x \to 0} m_{\text{sec}} \\ &= \lim_{\Delta x \to 0} \frac{1}{\sqrt{9 + \Delta x} + 3} \\ &= \frac{1}{\sqrt{9} + 3} \\ &= \frac{1}{6} \end{aligned}$$

27. $f(x) = 3$

$$\begin{aligned} f'(x) &= \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} \\ &= \lim_{\Delta x \to 0} \frac{3 - 3}{\Delta x} \\ &= \lim_{\Delta x \to 0} \frac{0}{\Delta x} \\ &= \lim_{\Delta x \to 0} 0 \\ &= 0 \end{aligned}$$

28. $f(x) = -2$

$$\begin{aligned} f'(x) &= \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} \\ &= \lim_{\Delta x \to 0} \frac{-2 - (-2)}{\Delta x} \\ &= \lim_{\Delta x \to 0} \frac{0}{\Delta x} \\ &= \lim_{\Delta x \to 0} 0 \\ &= 0 \end{aligned}$$

29. $f(x) = -5x$

$$\begin{aligned} f'(x) &= \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} \\ &= \lim_{\Delta x \to 0} \frac{-5(x + \Delta x) - (-5x)}{\Delta x} \\ &= \lim_{\Delta x \to 0} \frac{-5x - 5\Delta x + 5x}{\Delta x} \\ &= \lim_{\Delta x \to 0} \frac{-5\Delta x}{\Delta x} \\ &= \lim_{\Delta x \to 0} -5 \\ &= -5 \end{aligned}$$

30. $f(x) = 4x + 1$

$$\begin{aligned} f'(x) &= \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} \\ &= \lim_{\Delta x \to 0} \frac{4(x + \Delta x) + 1 - (4x + 1)}{\Delta x} \\ &= \lim_{\Delta x \to 0} \frac{4x + 4\Delta x + 1 - 4x - 1}{\Delta x} \\ &= \lim_{\Delta x \to 0} \frac{4\Delta x}{\Delta x} \\ &= \lim_{\Delta x \to 0} 4 \\ &= 4 \end{aligned}$$

31. $g(s) = \frac{1}{3}s + 2$

$$\begin{aligned} g'(s) &= \lim_{\Delta s \to 0} \frac{g(s + \Delta s) - g(s)}{\Delta s} \\ &= \lim_{\Delta s \to 0} \frac{\frac{1}{3}(s + \Delta s) + 2 - \left(\frac{1}{3}s + 2\right)}{\Delta s} \\ &= \lim_{\Delta s \to 0} \frac{\frac{1}{3}s + \frac{1}{3}\Delta s + 2 - \frac{1}{3}s - 2}{\Delta s} \\ &= \lim_{\Delta s \to 0} \frac{\frac{1}{3}\Delta s}{\Delta s} \\ &= \lim_{\Delta s \to 0} \frac{1}{3} \\ &= \frac{1}{3} \end{aligned}$$

32. $h(t) = 6 - \frac{1}{2}t$

$$\begin{aligned} h'(t) &= \lim_{\Delta t \to 0} \frac{h(t + \Delta t) - h(t)}{\Delta t} \\ &= \lim_{\Delta t \to 0} \frac{6 - \frac{1}{2}(t + \Delta t) - \left(6 - \frac{1}{2}t\right)}{\Delta t} \\ &= \lim_{\Delta t \to 0} \frac{6 - \frac{1}{2}t - \frac{1}{2}\Delta t - 6 + \frac{1}{2}t}{\Delta t} \\ &= \lim_{\Delta t \to 0} \frac{-\frac{1}{2}\Delta t}{\Delta t} \\ &= \lim_{\Delta t \to 0} -\frac{1}{2} \\ &= -\frac{1}{2} \end{aligned}$$

33. $f(x) = 4x^2 - 5x$

$$\begin{aligned} f'(x) &= \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} \\ &= \lim_{\Delta x \to 0} \frac{\left[4(x + \Delta x)^2 - 5(x + \Delta x)\right] - \left(4x^2 - 5x\right)}{\Delta x} \\ &= \lim_{\Delta x \to 0} \frac{\left[4\left(x^2 + 2x\Delta x + (\Delta x)^2\right) - 5x - 5\Delta x\right] - 4x^2 + 5x}{\Delta x} \\ &= \lim_{\Delta x \to 0} \frac{4x^2 + 8x\Delta x + 4(\Delta x)^2 - 5x - 5\Delta x - 4x^2 + 5x}{\Delta x} \\ &= \lim_{\Delta x \to 0} \frac{4\Delta x\left(2x + \Delta x - \frac{5}{4}\right)}{\Delta x} \\ &= \lim_{\Delta x \to 0} 4\left(2x + \Delta x - \frac{5}{4}\right) = 8x - 5 \end{aligned}$$

34. $f(x) = 2x^2 + 7x$

$$\begin{aligned} f'(x) &= \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} \\ &= \lim_{\Delta x \to 0} \frac{\left[2(x + \Delta x)^2 + 7(x + \Delta x)\right] - \left(2x^2 + 7x\right)}{\Delta x} \\ &= \lim_{\Delta x \to 0} \frac{\left[2\left(x^2 + 2x\Delta x + (\Delta x)^2\right) + 7x + 7\Delta x\right] - 2x^2 - 7x}{\Delta x} \\ &= \lim_{\Delta x \to 0} \frac{2x^2 + 4x\Delta x + 2(\Delta x)^2 + 7x + 7\Delta x - 2x^2 - 7x}{\Delta x} \\ &= \lim_{\Delta x \to 0} \frac{2\Delta x\left(2x + \Delta x + \frac{7}{2}\right)}{\Delta x} \\ &= \lim_{\Delta x \to 0} 2\left(2x + \Delta x + \frac{7}{2}\right) = 4x + 7 \end{aligned}$$

35. $h(t) = \sqrt{t - 3}$

$$h'(t) = \lim_{\Delta t \to 0} \frac{h(t + \Delta t) - h(t)}{t}$$
$$= \lim_{\Delta t \to 0} \frac{\sqrt{t + \Delta t - 3} - \sqrt{t - 3}}{\Delta t}$$
$$= \lim_{\Delta t \to 0} \frac{\sqrt{t + \Delta t - 3} - \sqrt{t - 3}}{\Delta t} \cdot \frac{\sqrt{t + \Delta t - 3} + \sqrt{t - 3}}{\sqrt{t + \Delta t - 3} + \sqrt{t - 3}}$$
$$= \lim_{\Delta t \to 0} \frac{t + \Delta t - 3 - (t - 3)}{\Delta t\left(\sqrt{t + \Delta t - 3} + \sqrt{t - 3}\right)}$$
$$= \lim_{\Delta t \to 0} \frac{\Delta t}{\Delta t\left(\sqrt{t + \Delta t - 3} + \sqrt{t - 3}\right)}$$
$$= \lim_{\Delta t \to 0} \frac{1}{\sqrt{t + \Delta t - 3} + \sqrt{t - 3}}$$
$$= \frac{1}{2\sqrt{t - 1}3}$$
$$= \frac{\sqrt{t - 3}}{2(t - 3)}$$

36. $f(x) = \sqrt{x + 2}$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{\sqrt{x + \Delta x + 2} - \sqrt{x + 2}}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{\sqrt{x + \Delta x + 2} - \sqrt{x + 2}}{\Delta x} \cdot \frac{\sqrt{x + \Delta x + 2} + \sqrt{x + 2}}{\sqrt{x + \Delta x + 2} + \sqrt{x + 2}}$$
$$= \lim_{\Delta x \to 0} \frac{x + \Delta x + 2 - (x + 2)}{\Delta x\left(\sqrt{x + \Delta x + 2} + \sqrt{x + 2}\right)}$$
$$= \lim_{\Delta x \to 0} \frac{\Delta x}{\Delta x\left(\sqrt{x + \Delta x + 2} + \sqrt{x + 2}\right)}$$
$$= \lim_{\Delta x \to 0} \frac{1}{\sqrt{x + \Delta x + 2} + \sqrt{x + 2}}$$
$$= \frac{1}{2\sqrt{x + 2}}$$

37. $f(t) = t^3 - 12t$

$$f'(t) = \lim_{\Delta t \to 0} \frac{f(t + \Delta t) - f(t)}{\Delta t}$$
$$= \lim_{\Delta t \to 0} \frac{(t + \Delta t)^3 - 12(t + \Delta t) - (t^3 - 12t)}{\Delta t}$$
$$= \lim_{\Delta t \to 0} \frac{t^3 + 3t^2\Delta t + 3t(\Delta t)^2 + (\Delta t)^3 - 12t - 12\Delta t - t^3 + 12t}{\Delta t}$$
$$= \lim_{\Delta t \to 0} \frac{3t^2\Delta t + 3t(\Delta t)^2 + (\Delta t)^3 - 12\Delta t}{\Delta t}$$
$$= \lim_{\Delta t \to 0} \frac{\Delta t\left(3t^2 + 3t\Delta t + (\Delta t)^2 - 12\right)}{\Delta t}$$
$$= \lim_{\Delta t \to 0} \left(3t^2 + 3t\Delta t + (\Delta t)^2 - 12\right)$$
$$= 3t^2 - 12$$

38. $f(t) = t^3 + t^2$

$$f'(t) = \lim_{\Delta t \to 0} \frac{f(t + \Delta t) - f(t)}{\Delta t}$$
$$= \lim_{\Delta t \to 0} \frac{(t + \Delta t)^3 + (t + \Delta t)^2 - (t^3 + t^2)}{\Delta t}$$
$$= \lim_{\Delta t \to 0} \frac{t^3 + 3t^2\Delta t + 3t(\Delta t)^2 + (\Delta t)^3 + t^2 + 2t\Delta t + (\Delta t)^2 - t^3 - t^2}{\Delta t}$$
$$= \lim_{\Delta t \to 0} \frac{3t^2\Delta t + 3t(\Delta t)^2 + (\Delta t)^3 + 2t\Delta t + (\Delta t)^2}{\Delta t}$$
$$= \lim_{\Delta t \to 0} \frac{\Delta t\left(3t^2 + 3t\Delta t + (\Delta t)^2 + 2t + \Delta t\right)}{\Delta t}$$
$$= \lim_{\Delta t \to 0} \left(3t^2 + 3t\Delta t + (\Delta t)^2 + 2t + \Delta t\right)$$
$$= 3t^2 + 2t$$

39. $f(x) = \dfrac{1}{x + 2}$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{\dfrac{1}{x + \Delta x + 2} - \dfrac{1}{x + 2}}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{\dfrac{1}{x + \Delta x + 2} \cdot \dfrac{x + 2}{x + 2} - \dfrac{1}{x + 2} \cdot \dfrac{x + \Delta x + 2}{x + \Delta x + 2}}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{\dfrac{x + 2 - (x + \Delta x + 2)}{(x + \Delta x + 2)(x + 2)}}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{-\Delta x}{\Delta x(x + \Delta x + 2)(x + 2)}$$
$$= \lim_{\Delta x \to 0} \frac{-1}{(x + \Delta x + 2)(x + 2)}$$
$$= -\frac{1}{(x + 2)^2}$$

40. $g(s) = \dfrac{1}{s - 4}$

$$g'(s) = \lim_{\Delta s \to 0} \frac{g(s + \Delta s) - g(s)}{\Delta s}$$

$$= \lim_{\Delta s \to 0} \frac{\dfrac{1}{s + \Delta s - 4} - \dfrac{1}{s - 4}}{\Delta s}$$

$$= \lim_{\Delta s \to 0} \frac{\dfrac{s - 4 - (s + \Delta s - 4)}{(s + \Delta s - 4)(s - 4)}}{\Delta s}$$

$$= \lim_{\Delta s \to 0} \frac{\dfrac{s - 4 - (s + \Delta s - 4)}{(s + \Delta s - 4)(s - 4)}}{(s + \Delta s - 4)(s - 4)} \cdot \frac{1}{\Delta s}$$

$$= \lim_{\Delta s \to 0} \frac{-\Delta s}{\Delta s(s + \Delta s - 4)(s - 4)}$$

$$= \lim_{\Delta s \to 0} \frac{-1}{(s + \Delta s - 4)(s - 4)}$$

$$= -\frac{1}{(s - 4)^2}$$

41. $f(x) = \frac{1}{2}x^2$ at $(2, 2)$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\frac{1}{2}(x + \Delta x)^2 - \frac{1}{2}x^2}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\frac{1}{2}\left(x^2 + 2x\Delta x + (\Delta x)^2\right) - \frac{1}{2}x^2}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{x\Delta x + (\Delta x)^2}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\Delta x(x + \Delta x)}{\Delta x}$$

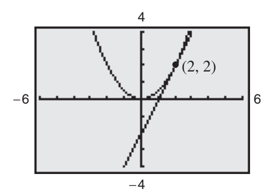
$$= \lim_{\Delta x \to 0} (x + \Delta x)$$

$$= x$$

$m = f'(2) = 2$

$y - 2 = 2(x - 2)$

$y = 2x - 2$



42. $f(x) = -\frac{1}{8}x^2$ at $(-4, -2)$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{-\frac{1}{8}(x + \Delta x)^2 - \left(-\frac{1}{8}x^2\right)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{-\frac{1}{8}\left(x^2 + 2x\Delta x + (\Delta x)^2\right) + \frac{1}{8}x^2}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{-\frac{1}{8}x^2 - \frac{1}{4}x\Delta x - \frac{1}{8}(\Delta x)^2 + \frac{1}{8}x^2}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{-\frac{1}{4}x\Delta x - \frac{1}{8}(\Delta x)^2}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\Delta x\left(-\frac{1}{4}x - \frac{1}{8}\Delta x\right)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \left(-\frac{1}{4}x - \frac{1}{8}\Delta x\right)$$

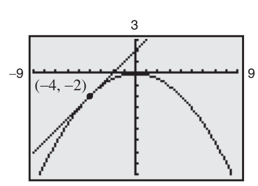
$$= -\frac{1}{4}x$$

$m = f'(-4) = -\frac{1}{4}(-4) = 1$

$y - (-2) = 1[x - (-4)]$

$y + 2 = x + 4$

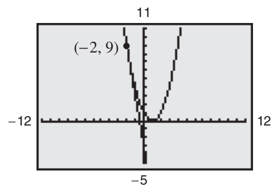
$y = x + 2$



43. $f(x) = (x - 1)^2$ at $(-2, 9)$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{(x + \Delta x - 1)^2 - (x - 1)^2}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{x^2 + 2x\Delta x - 2x + (\Delta x)^2 - 2\Delta x + 1 - x^2 + 2x - 1}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{2x\Delta x + (\Delta x)^2 - 2\Delta x}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{\Delta x(2x + \Delta x - 2)}{\Delta x}$$
$$= \lim_{\Delta x \to 0} (2x + \Delta x - 2)$$
$$= 2x - 2$$

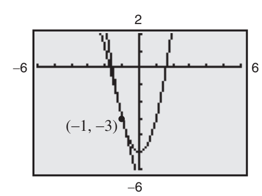
$$m = f'(-2) = 2(-2) - 2 = -6$$
$$y - 9 = -6[x - (-2)]$$
$$y = -6x - 3$$



44. $f(x) = 2x^2 - 5$ at $(-1, -3)$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{2(x + \Delta x)^2 - 5 - (2x^2 - 5)}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{2x^2 + 4x\Delta x + 2(\Delta x)^2 - 5 - 2x^2 + 5}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{4x\Delta x + 2(\Delta x)^2}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{\Delta x(4x + 2\Delta x)}{\Delta x}$$
$$= \lim_{\Delta x \to 0} (4x + 2\Delta x)$$
$$= 4x$$

$$m = f'(-1) = 4(-1) = -4$$
$$y - (-3) = -4(x - (-1))$$
$$y + 3 = -4x - 4$$
$$y = -4x - 7$$



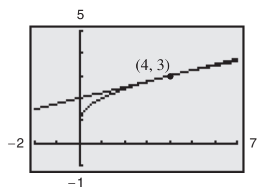
45. $f(x) = \sqrt{x} + 1$ at $(4, 3)$

$$\begin{aligned}
f'(x) &= \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} \\
&= \lim_{\Delta x \to 0} \frac{\sqrt{x + \Delta x} + 1 - \left(\sqrt{x} + 1\right)}{\Delta x} \\
&= \lim_{\Delta x \to 0} \frac{\sqrt{x + \Delta x} - \sqrt{x}}{\Delta x} \cdot \frac{\sqrt{x + \Delta x} + \sqrt{x}}{\sqrt{x + \Delta x} + \sqrt{x}} \\
&= \lim_{\Delta x \to 0} \frac{x + \Delta x - x}{\Delta x\left(\sqrt{x + \Delta x} + \sqrt{x}\right)} \\
&= \lim_{\Delta x \to 0} \frac{\Delta x}{\Delta x\left(\sqrt{x + \Delta x} + \sqrt{x}\right)} \\
&= \lim_{\Delta x \to 0} \frac{1}{\sqrt{x + \Delta x} + \sqrt{x}} \\
&= \frac{1}{2\sqrt{x}}
\end{aligned}$$

$$m = f'(4) = \frac{1}{2\sqrt{4}} = \frac{1}{4}$$

$$y - 3 = \frac{1}{4}(x - 4)$$

$$y = \frac{1}{4}x + 2$$



46. $f(x) = \sqrt{x + 3}$ at $(6, 3)$

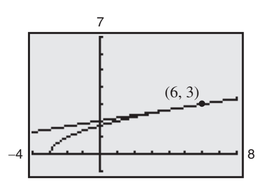
$$\begin{aligned}
f'(x) &= \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} \\
&= \lim_{\Delta x \to 0} \frac{\sqrt{x + \Delta x + 3} - \sqrt{x + 3}}{\Delta x} \\
&= \lim_{\Delta x \to 0} \frac{\sqrt{x + \Delta x + 3} - \sqrt{x + 3}}{\Delta x} \cdot \frac{\sqrt{x + \Delta x + 3} + \sqrt{x + 3}}{\sqrt{x + \Delta x + 3} + \sqrt{x + 3}} \\
&= \lim_{\Delta x \to 0} \frac{x + \Delta x + 3 - (x + 3)}{\Delta x\left(\sqrt{x + \Delta x + 3} + \sqrt{x + 3}\right)} \\
&= \lim_{\Delta x \to 0} \frac{\Delta x}{\Delta x\left(\sqrt{x + \Delta x + 3} + \sqrt{x + 3}\right)} \\
&= \lim_{\Delta x \to 0} \frac{1}{\sqrt{x + \Delta x + 3} + \sqrt{x + 3}} \\
&= \frac{1}{2\sqrt{x + 3}} \\
&= \frac{\sqrt{x + 3}}{2(x + 3)}
\end{aligned}$$

$$m = f'(6) = \frac{1}{2\sqrt{6 + 3}} = \frac{1}{6}$$

$$y - 3 = \frac{1}{6}(x - 6)$$

$$y - 3 = \frac{1}{6}x - 1$$

$$y = \frac{1}{6}x + 2$$



47. $f(x) = \frac{1}{5x}$ at $\left(-\frac{1}{5}, -1\right)$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\frac{1}{5(x + \Delta x)} - \frac{1}{5x}}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\frac{x - (x + \Delta x)}{5x(x + \Delta x)}}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{x - (x + \Delta x)}{5x(x + \Delta x)} \cdot \frac{1}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{-\Delta x}{5x \cdot \Delta x \cdot (x + \Delta x)}$$

$$= \lim_{\Delta x \to 0} \frac{-1}{5x(x + \Delta x)}$$

$$= -\frac{1}{5x^2}$$

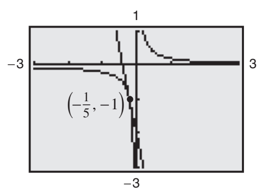
$$m = f'\left(-\frac{1}{5}\right) = -\frac{1}{5\left(-\frac{1}{5}\right)^2} = -\frac{1}{5\left(\frac{1}{25}\right)} = -5$$

$$y - (-1) = -5\left(x - \left(-\frac{1}{5}\right)\right)$$

$$y + 1 = -5\left(x + \frac{1}{5}\right)$$

$$y + 1 = -5x - 1$$

$$y = -5x - 2$$



48. $f(x) = \frac{1}{x - 3}$ at $(2, -1)$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\frac{1}{x + \Delta x - 3} - \frac{1}{x - 3}}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\frac{1}{x + \Delta x - 3} \cdot \frac{x - 3}{x - 3} - \frac{1}{x - 3} \cdot \frac{x + \Delta x - 3}{x + \Delta x - 3}}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\frac{x - 3 - (x + \Delta x - 3)}{(x + \Delta x - 3)(x - 3)}}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{-\Delta x}{(x + \Delta x - 3)(x - 3)\Delta x}$$

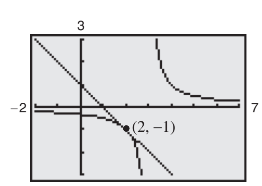
$$= \lim_{\Delta x \to 0} \frac{-1}{(x + \Delta x - 3)(x - 3)} = -\frac{1}{(x - 3)^2}$$

$$m = f'(2) = -\frac{1}{(2 - 3)^2} = -1$$

$$y - (-1) = -1(x - 2)$$

$$y + 1 = -x + 2$$

$$y = -x + 1$$



49. $f(x) = -\frac{1}{4}x^2$

$$\begin{aligned} f'(x) &= \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} \\ &= \lim_{\Delta x \to 0} \frac{-\frac{1}{4}(x + \Delta x)^2 - \left(-\frac{1}{4}x^2\right)}{\Delta x} \\ &= \lim_{\Delta x \to 0} \frac{-\frac{1}{4}x^2 - \frac{1}{2}x\Delta x - \frac{1}{4}(\Delta x)^2 + \frac{1}{4}x^2}{\Delta x} \\ &= \lim_{\Delta x \to 0} \frac{-\frac{1}{2}x\Delta x - \frac{1}{4}(\Delta x)^2}{\Delta x} \\ &= \lim_{\Delta x \to 0} \frac{\Delta x\left(-\frac{1}{2}x - \frac{1}{4}\Delta x\right)}{\Delta x} \\ &= \lim_{\Delta x \to 0}\left(-\frac{1}{2}x - \frac{1}{4}\Delta x\right) \\ &= -\frac{1}{2}x \end{aligned}$$

Since the slope of the given line is -1,

$$\begin{aligned} -\tfrac{1}{2}x &= -1 \\ x &= 2 \text{ and } f(2) = -1. \end{aligned}$$

At the point $(2, -1)$, the tangent line parallel to $x + y = 0$ is $y - (-1) = -1(x - 2)$

$$y = -x + 1.$$

50. $f(x) = x^2 - 7$

$$\begin{aligned} f'(x) &= \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} \\ &= \lim_{\Delta x \to 0} \frac{\left[(x + \Delta x)^2 - 7\right] - (x^2 - 7)}{\Delta x} \\ &= \lim_{\Delta x \to 0} \frac{x^2 + 2x\Delta x + (\Delta x)^2 - 7 - x^2 + 7}{\Delta x} \\ &= \lim_{\Delta x \to 0} \frac{\Delta x(2x + \Delta x)}{\Delta x} \\ &= \lim_{\Delta x \to 0} (2x + \Delta x) = 2x \end{aligned}$$

Since the slope of the given line is -2,

$$\begin{aligned} 2x &= -2 \\ x &= -1 \text{ and } f(-1) = -6. \end{aligned}$$

At the point $(-1, -6)$, the tangent line parallel to $2x + y = 0$ is

$$\begin{aligned} y - (-6) &= -2(x - (-1)) \\ y &= -2x - 8. \end{aligned}$$

51. $f(x) = -\frac{1}{3}x^3$

$$\begin{aligned} f'(x) &= \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} \\ &= \lim_{\Delta x \to 0} \frac{-\frac{1}{3}(x + \Delta x)^3 - \left(-\frac{1}{3}x^3\right)}{\Delta x} \\ &= \lim_{\Delta x \to 0} \frac{-\frac{1}{3}\left(x^3 + 3x^2\Delta x + 3x(\Delta x)^2 + (\Delta x)^3\right) + \frac{1}{3}x^3}{\Delta x} \\ &= \lim_{\Delta x \to 0} \frac{-\frac{1}{3}x^3 - x^2\Delta x - x(\Delta x)^2 - \frac{1}{3}(\Delta x)^3 + \frac{1}{3}x^3}{\Delta x} \\ &= \lim_{\Delta x \to 0} \frac{\Delta x\left(-x^2 - x\Delta x - \frac{1}{3}(\Delta x)^2\right)}{\Delta x} \\ &= \lim_{\Delta x \to 0} \left(-x^2 - x\Delta x - \frac{1}{3}(\Delta x)^2\right) = -x^2 \end{aligned}$$

Since the slope of the given line is -9,

$$\begin{aligned} -x^2 &= -9 \\ x^2 &= 9 \\ x &= \pm 3 \text{ and } f(3) = -9 \text{ and } f(-3) = 9. \end{aligned}$$

At the point $(3, -9)$, the tangent line parallel to $9x + y - 6 = 0$ is

$$\begin{aligned} y - (-9) &= -9(x - 3) \\ y &= -9x + 18. \end{aligned}$$

At the point $(-3, 9)$, the tangent line parallel to $9x + y - 6 = 0$ is

$$\begin{aligned} y - 9 &= -9(x - (-3)) \\ y &= -9x - 18. \end{aligned}$$

52. $f(x) = x^3 + 2$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{(x + \Delta x)^3 + 2 - (x^3 + 2)}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{x^3 + 3x^2\Delta x + 3x(\Delta x)^2 + (\Delta x)^3 + 2 - x^3 - 2}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{3x^2\Delta x + 3(\Delta x)^2 + (\Delta x)^3}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{\Delta x\left(3x^2 + 3x\Delta x + (\Delta x)^2\right)}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \left(3x^2 + 3x\Delta x + (\Delta x)^2\right)$$
$$= 3x^2$$

The slope of the given line is

$$3x - y - 4 = 0$$
$$y = 3x - 4$$
$$m = 3.$$

$$3x^2 = 3$$
$$x^2 = 1$$
$$x = \pm 1$$
$$x = 1 \text{ and } f(1) = 3$$
$$x = -1 \text{ and } f(-1) = 1$$

At the point $(1, 3)$, the tangent line parallel to $3x - y - 4 = 0$ is

$$y - 3 = 3(x - 1)$$
$$y - 3 = 3x - 3$$
$$y = 3x.$$

At the point $(-1, 1)$, the tangent line parallel to $3x - y - 4 = 0$ is

$$y - 1 = 3(x - (-1))$$
$$y - 1 = 3(x + 1)$$
$$y - 1 = 3x + 3$$
$$y = 3x + 4.$$

53. y is differentiable for all $x \neq -3$.

At $(-3, 0)$, the graph has a node.

54. y is differentiable for all $x \neq \pm 3$.

At $(\pm 3, 0)$, the graph has a cusp.

55. y is differentiable for all $x \neq -\frac{1}{2}$.

At $\left(-\frac{1}{2}, 0\right)$, the graph has a vertical tangent line.

56. y is differentiable for all $x > 1$.

The derivative does not exist at endpoints.

57. y is differentiable for all $x \neq \pm 2$.

The function is not defined at $x = \pm 2$.

58. y is differentiable for all $x \neq 0$.

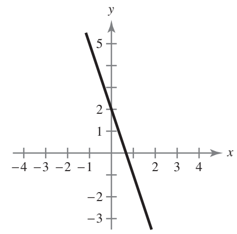
The function is discontinuous at $x = 0$.

59. Since $f'(x) = -3$ for all x, f is a line of the form

$f(x) = -3x + b.$

$f(0) = 2$, so $2 = (-3)(0) + b$, or $b = 2$.

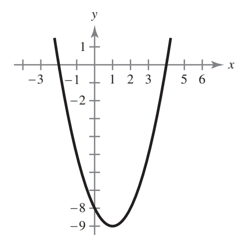
Thus, $f(x) = -3x + 2$.



60. Sample answer: Since $f(-2) = f(4) = 0$, $(x + 2)(x - 4) = 0$.

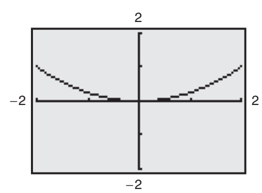
A function with these zeros is $f(x) = x^2 - 2x - 8$.

$$\begin{aligned}
\text{Then } f'(x) &= \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} \\
&= \lim_{\Delta x \to 0} \frac{\left[(x + \Delta x)^2 - 2(x + \Delta x) - 8\right] - \left(x^2 - 2x - 8\right)}{\Delta x} \\
&= \lim_{\Delta x \to 0} \frac{x^2 + 2x\Delta x + (\Delta x)^2 - 2x - 2\Delta x - 8 - x^2 + 2x + 8}{\Delta x} \\
&= \lim_{\Delta x \to 0} \frac{2x\Delta x + (\Delta x)^2 - 2\Delta x}{\Delta x} \\
&= \lim_{\Delta x \to 0} 2x + \Delta x - 2 \\
&= 2x - 2.
\end{aligned}$$



So $f'(1) = 2(1) - 2 = 0$. Sketching $f(x)$ shows that $f'(x) < 0$ for $x < 1$ and $f'(0) > 0$ for $x > 1$.

61.

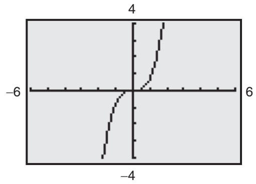


x	-2	$-\frac{3}{2}$	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	1	$\frac{3}{2}$	2
$f(x)$	1	0.5625	0.25	0.0625	0	0.0625	0.25	0.5625	1
$f'(x)$	-1	-0.75	-0.5	-0.25	0	0.25	0.5	0.75	1

Analytically, the slope of $f(x) = \frac{1}{4}x^2$ is

$$\begin{aligned}
m &= \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} \\
&= \lim_{\Delta x \to 0} \frac{\frac{1}{4}(x + \Delta x)^2 - \frac{1}{4}x^2}{\Delta x} \\
&= \lim_{\Delta x \to 0} \frac{\frac{1}{4}\left[x^2 + 2x(\Delta x)^2 + (\Delta x)^2\right] - \frac{1}{4}x^2}{\Delta x} \\
&= \lim_{\Delta x \to 0} \frac{\frac{1}{4}x^2 + \frac{1}{2}x\Delta x + \frac{1}{4}(\Delta x)^2 - \frac{1}{4}x^2}{\Delta x} \\
&= \lim_{\Delta x \to 0} \frac{\frac{1}{2}x\Delta x + \frac{1}{4}(\Delta x)^2}{\Delta x} \\
&= \lim_{\Delta x \to 0} \frac{\Delta x\left(\frac{1}{2}x\Delta x + \frac{1}{4}\Delta x\right)}{\Delta x} \\
&= \lim_{\Delta x \to 0} \left(\frac{1}{2}x + \frac{1}{4}\Delta x\right) \\
&= \frac{1}{2}x.
\end{aligned}$$

62.

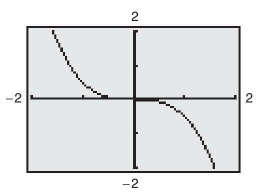


x	-2	$-\frac{3}{2}$	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	1	$\frac{3}{2}$	2
$f(x)$	-6	-2.53	-0.75	-0.1	0	0.1	0.75	2.53	6
$f'(x)$	9	5.0625	2.25	0.5625	0	0.5625	2.25	5.0625	9

Analytically, the slope of $f(x) = \frac{3}{4}x^3$ is

$$\begin{aligned}
m &= \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} \\
&= \lim_{\Delta x \to 0} \frac{\frac{3}{4}(x + \Delta x)^3 - \frac{3}{4}x^3}{\Delta x} \\
&= \lim_{\Delta x \to 0} \frac{\frac{3}{4}\left(x^3 + 3x^2\Delta x + 3x(\Delta x)^2 + (\Delta x)^3\right) - \frac{3}{4}x^3}{\Delta x} \\
&= \lim_{\Delta x \to 0} \frac{\frac{3}{4}x^3 + \frac{9}{4}x^2\Delta x + \frac{9}{4}x(\Delta x)^2 + \frac{3}{4}(\Delta x)^3 - \frac{3}{4}x^3}{\Delta x} \\
&= \lim_{\Delta x \to 0} \frac{\frac{9}{4}x^2\Delta x + \frac{9}{4}x(\Delta x)^2 + \frac{3}{4}x(\Delta x)^3}{\Delta x} \\
&= \lim_{\Delta x \to 0} \frac{\Delta x\left(\frac{9}{4}x^2 + \frac{9}{4}x\Delta x + \frac{3}{4}(\Delta x)^2\right)}{\Delta x} \\
&= \lim_{\Delta x \to 0} \left(\frac{9}{4}x^2 + \frac{9}{4}x\Delta x + \frac{3}{4}(\Delta x)^2\right) \\
&= \frac{9}{4}x^2.
\end{aligned}$$

63.

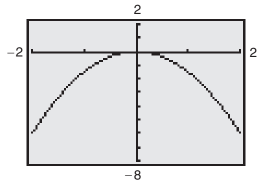


x	-2	$-\frac{3}{2}$	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	1	$\frac{3}{2}$	2
$f(x)$	4	1.6875	0.5	0.0625	0	-0.0625	-0.5	-1.6875	-4
$f'(x)$	-6	-3.375	-1.5	-0.375	0	-0.375	-1.5	-3.375	-6

Analytically, the slope of $f(x) = -\frac{1}{2}x^3$ is

$$\begin{aligned}
m &= \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} \\
&= \lim_{\Delta x \to 0} \frac{-\frac{1}{2}(x + \Delta x)^3 + \frac{1}{2}x^3}{\Delta x} \\
&= \lim_{\Delta x \to 0} \frac{-\frac{1}{2}\left[x^3 + 3x^2\Delta x + 3x(\Delta x)^2 + (\Delta x)^3\right] + \frac{1}{2}x^3}{\Delta x} \\
&= \lim_{\Delta x \to 0} \frac{-\frac{1}{2}\left[3x^2\Delta x + 3x(\Delta x)^2 + (\Delta x)^3\right]}{\Delta x} \\
&= \lim_{\Delta x \to 0} -\frac{1}{2}\left[3x^2 + 3x(\Delta x) + (\Delta x)^2\right] \\
&= -\frac{3}{2}x^2.
\end{aligned}$$

64.

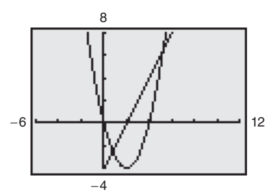


x	-2	$-\frac{3}{2}$	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	1	$\frac{3}{2}$	2
$f(x)$	-6	-3.375	-1.5	-0.375	0	-0.375	-1.5	-3.375	-6
$f'(x)$	6	4.5	3	1.5	0	-1.5	-3	-4.5	-6

Analytically, the slope of $f(x) = -\frac{3}{2}x^2$ is

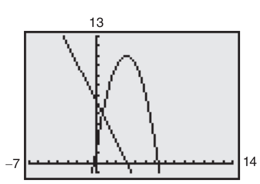
$$\begin{aligned} m &= \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} \\ &= \lim_{\Delta x \to 0} \frac{-\frac{3}{2}(x + \Delta x)^2 - \left(-\frac{3}{2}x^2\right)}{\Delta x} \\ &= \lim_{\Delta x \to 0} \frac{-\frac{3}{2}\left[x^2 + 2x\Delta x + (\Delta x)^2\right] + \frac{3}{2}x^2}{\Delta x} \\ &= \lim_{\Delta x \to 0} \frac{-\frac{3}{2}\left[2x\Delta x + (\Delta x)^2\right]}{\Delta x} \\ &= \lim_{\Delta x \to 0} -\frac{3}{2}(2x + \Delta x) \\ &= -3x. \end{aligned}$$

65. $$\begin{aligned} f'(x) &= \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} \\ &= \lim_{\Delta x \to 0} \frac{(x + \Delta x)^2 - 4(x + \Delta x) - (x^2 - 4x)}{\Delta x} \\ &= \lim_{\Delta x \to 0} \frac{2x\Delta x + (\Delta x)^2 - 4\Delta x}{\Delta x} \\ &= \lim_{\Delta x \to 0} (2x + \Delta x - 4) \\ &= 2x - 4 \end{aligned}$$



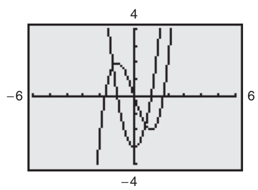
The x-intercept of the derivative indicates a point of horizontal tangency for f.

66. $$\begin{aligned} f'(x) &= \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} = \lim_{\Delta x \to 0} \frac{2 + 6(x + \Delta x) - (x + \Delta x)^2 - (2 + 6x - x^2)}{\Delta x} \\ &= \lim_{\Delta x \to 0} \frac{6\Delta x - 2x\Delta x - (\Delta x)^2}{\Delta x} = \lim_{\Delta x \to 0} (6 - 2x - \Delta x) = 6 - 2x \end{aligned}$$



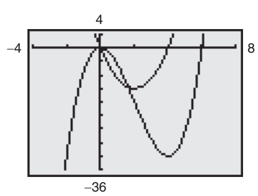
The x-intercept of the derivative indicates a point of horizontal tangency for f.

67. $$\begin{aligned} f'(x) &= \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} \\ &= \lim_{\Delta x \to 0} \frac{(x + \Delta x)^3 - 3(x + \Delta x) - (x^3 - 3x)}{\Delta x} \\ &= \lim_{\Delta x \to 0} \frac{x^3 + 3x^2\Delta x + 3x(\Delta x)^2 + (\Delta x)^3 - 3x - 3\Delta x - x^3 + 3x}{\Delta x} \\ &= \lim_{\Delta x \to 0} \frac{3x^2\Delta x + 3x(\Delta x)^2 + (\Delta x)^3 - 3\Delta x}{\Delta x} \\ &= \lim_{\Delta x \to 0} \left(3x^2 + 3x\Delta x + (\Delta x)^2 - 3\right) \\ &= 3x^2 - 3 \end{aligned}$$



The x-intercepts of the derivative indicate points of horizontal tangency for f.

68. $$\begin{aligned} f'(x) &= \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} \\ &= \lim_{\Delta x \to 0} \frac{(x + \Delta x)^3 - 6(x + \Delta x)^2 - (x^3 - 6x^2)}{\Delta x} \\ &= \lim_{\Delta x \to 0} \frac{x^3 - 3x^2\Delta x + 3x(\Delta x)^2 + (\Delta x)^3 - 6\left(x^2 + 2x\Delta x + (\Delta x)^2\right) - x^3 + 6x^2}{\Delta x} \\ &= \lim_{\Delta x \to 0} \left(3x^2 + 3x\Delta x + (\Delta x)^2 - 12x - 6\Delta x\right) \\ &= 3x^2 - 12x \end{aligned}$$



The x-intercepts of the derivative indicate points of horizontal tangency for f.

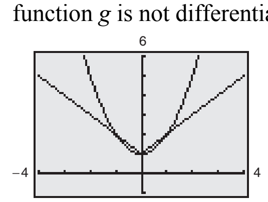
69. True. The slope of the graph is given by $f'(x) = 2x$, which is different for each different x value.

70. False. $f(x) = |x|$ is continuous, but not differentiable at $x = 0$.

71. True. See page 122.

72. True. See page 115.

73. The graph of $f(x) = x^2 + 1$ is smooth at (0, 1), but the graph of $g(x) = |x| + 1$ has a node at (0, 1). The function g is not differentiable at (0, 1).



Section 2.2 Some Rules for Differentiation

Skills Warm Up

1. (a) $2x^2,\ x = 2$

$2(2^2) = 2(4) = 8$

(b) $(5x)^2,\ x = 2$

$[5(2)]^2 = 10^2 = 100$

(c) $6x^{-2},\ x = 2$

$6(2)^{-2} = 6\left(\frac{1}{4}\right) = \frac{3}{2}$

2. (a) $\dfrac{1}{(3x)^2},\ x = 2$

$\dfrac{1}{[3(2)]^2} = \dfrac{1}{6^2} = \dfrac{1}{36}$

(b) $\dfrac{1}{4x^3},\ x = 2$

$\dfrac{1}{4(2^3)} = \dfrac{1}{4(8)} = \dfrac{1}{32}$

(c) $\dfrac{(2x)^{-3}}{4x^{-2}},\ x = 2$

$\dfrac{[2(2)]^{-3}}{4(2)^{-2}} = \dfrac{4^{-3}}{4(2)^{-2}} = \dfrac{2^2}{4(4^3)} = \dfrac{1}{64}$

3. $4(3)x^3 + 2(2)x = 12x^3 + 4x = 4x(3x^2 + 1)$

4. $\frac{1}{2}(3)x^2 - \frac{3}{2}x^{1/2} = \frac{3}{2}x^2 - \frac{3}{2}\sqrt{x} = \frac{3}{2}x^{1/2}(x^{3/2} - 1)$

5. $\left(\dfrac{1}{4}\right)x^{-3/4} = \dfrac{1}{4x^{3/4}}$

6. $\dfrac{1}{3}(3)x^2 - 2\left(\dfrac{1}{2}\right)x^{-1/2} + \dfrac{1}{3}x^{-2/3} = x^2 - x^{-1/2} + \dfrac{1}{3}x^{-2/3}$

$= x^2 - \dfrac{1}{\sqrt{x}} + \dfrac{1}{3\sqrt[3]{x^2}}$

7. $3x^2 + 2x = 0$

$x(3x + 2) = 0$

$x = 0$

$3x + 2 = 0 \rightarrow x = -\frac{2}{3}$

8. $x^3 - x = 0$

$x(x^2 - 1) = 0$

$x(x + 1)(x - 1) = 0$

$x = 0$

$x + 1 = 0 \rightarrow x = -1$

$x - 1 = 0 \rightarrow x = 1$

9. $x^2 + 8x - 20 = 0$

$(x + 10)(x - 2) = 0$

$x + 10 = 0 \rightarrow x = -10$

$x - 2 = 0 \rightarrow x = 2$

10. $3x^2 - 10x + 8 = 0$

$(3x - 4)(x - 2) = 0$

$3x - 4 = 0 \rightarrow x = \frac{4}{3}$

$x - 2 = 0 \rightarrow x = 2$

1. $y = 3$

$y' = 0$

2. $f(x) = -8$

$f'(x) = 0$

3. $y = x^5$

$y' = 5x^4$

4. $f(x) = \dfrac{1}{x^6} = x^{-6}$

$f'(x) = -6x^{-7} = -\dfrac{6}{x^7}$

5. $h(x) = 3x^3$

$h'(x) = 9x^2$

6. $h(x) = 6x^5$

$h'(x) = 30x^4$

7. $y = \dfrac{5x^4}{3}$

$y' = \dfrac{20}{6}x^3 = \dfrac{10}{3}x^3$

8. $g(t) = \dfrac{3t^2}{4}$

$g'(t) = \dfrac{3}{2}t$

9. $f(x) = 4x$

$f'(x) = 4$

10. $g(x) = \frac{x}{3} = \frac{1}{3}x$

$g'(x) = \frac{1}{3}$

11. $y = 8 - x^3$

$y = -3x^2$

12. $y = t^2 - 6$

$y' = 2t$

13. $f(x) = 4x^2 - 3x$

$f'(x) = 8x - 3$

14. $g(x) = 3x^2 + 5x^3$

$g'(x) = 6x + 15x^2 = 15x^2 + 6x$

15. $f(t) = -3t^2 + 2t - 4$

$f'(t) = -6t + 2$

16. $y = 7x^3 - 9x^2 + 8$

$y' = 21x^2 - 18x$

17. $s(t) = 4t^4 - 2t + t + 3$

$s'(t) = 16t^2 - 4t + 1$

18. $y = 2x^3 - x^2 + 3x - 1$

$y' = 6x^2 - 2x + 3$

19. $g(x) = x^{2/3}$

$g'(x) = \frac{2}{3}x^{-1/3} = \frac{2}{3x^{1/3}}$

20. $h(x) = x^{5/2}$

$h'(x) = \frac{5}{2}x^{3/2}$

21. $y = 4t^{4/3}$

$y' = 4\left(\frac{4}{3}\right)t^{1/3} = \frac{16}{3}t^{1/3}$

22. $f(x) = 10x^{1/6}$

$f'(x) = \frac{5}{3}x^{-5/6} = \frac{5}{3x^{5/6}} = \frac{5}{3\sqrt[6]{x^5}}$

23. $y = 4x^{-2} + 2x^2$

$y' = -8x^{-3} + 4x^1 = -\frac{8}{x^3} + 4x$

24. $s(t) = 8t^{-4} + t$

$s'(t) = 8\left(-4t^{-6}\right) + 1 = -\frac{32}{t^6} + 1$

	Function	*Rewrite*	*Differentiate*	*Simplify*
25.	$y = \frac{2}{7x^4}$	$y = \frac{2}{7}x^{-4}$	$y' = \frac{-8}{7}x^{-5}$	$y' = -\frac{8}{7x^5}$
26.	$y = \frac{2}{3x^2}$	$y = \frac{2}{3}x^{-2}$	$y' = -\frac{4}{3}x^{-3}$	$y' = -\frac{4}{3x^3}$
27.	$y = \frac{1}{(4x)^3}$	$y = \frac{1}{64}x^{-3}$	$y' = -\frac{3}{64}x^{-4}$	$y' = -\frac{3}{64x^4}$
28.	$y = \frac{\pi}{(2x)^6}$	$y = \frac{\pi}{64}x^{-6}$	$y' = -\frac{6\pi}{64}x^{-7}$	$y' = -\frac{3\pi}{32x^7}$
29.	$y = \frac{4}{(2x)^{-5}}$	$y = 128x^5$	$y' = 128(5)x^4$	$y' = 640x^4$
30.	$y = \frac{4x}{x^{-3}}$	$y = 4x^4$	$y' = 4(4)x^3$	$y' = 16x^3$
31.	$y = 6\sqrt{x}$	$y = 6x^{1/2}$	$y' = 6\left(\frac{1}{2}\right)x^{-1/2}$	$y' = \frac{3}{\sqrt{x}}$
32.	$y = \frac{3\sqrt{x}}{4}$	$y = \frac{3}{4}x^{1/2}$	$y' = \frac{3}{4}\left(\frac{1}{2}\right)x^{-1/2}$	$y' = \frac{3}{8\sqrt{x}}$

	Function	*Rewrite*	*Differentiate*	*Simplify*
33.	$y = \dfrac{1}{7\sqrt[6]{x}}$	$y = \dfrac{1}{7}x^{-1/6}$	$y' = \dfrac{1}{7}\left(-\dfrac{1}{6}\right)x^{-7/6}$	$y' = -\dfrac{1}{42\sqrt[6]{x^7}}$
34.	$y = \dfrac{3}{2\sqrt[4]{x^3}}$	$y = \dfrac{3}{2}x^{-3/4}$	$y' = \dfrac{3}{2}\left(-\dfrac{3}{4}\right)x^{-7/4}$	$y' = -\dfrac{9}{8\sqrt[4]{x^7}}$
35.	$y = \sqrt[5]{8x}$	$y = (8x)^{1/5} = 8^{1/5}x^{1/5}$	$y' = 8^{1/5}\left(\dfrac{1}{5}x^{-4/5}\right)$	$y' = 8^{1/5}\dfrac{1}{5x^{4/5}} = \dfrac{\sqrt[5]{8}}{5\sqrt[5]{x^4}} = \dfrac{\sqrt[5]{8}}{5x}$
36.	$y = \sqrt[3]{6x^2}$	$y = \sqrt[3]{6}(x)^{2/3}$	$y' = \sqrt[3]{6}\left(\dfrac{2}{3}\right)x^{-1/3}$	$y' = \dfrac{2\sqrt[3]{6}}{3\sqrt[3]{x}}$

37. $y = x^{3/2}$

$y' = \frac{3}{2}x^{1/2}$

At the point $(1, 1)$, $y' = \frac{3}{2}(1)^{1/2} = \frac{3}{2} = m.$

38. $y = x^{-1}$

$y' = x^{-2} = -\dfrac{1}{x^2}$

At the point $\left(\dfrac{3}{4}, \dfrac{4}{3}\right)$, $y' = -\dfrac{1}{\left(\frac{3}{4}\right)^2} = -\dfrac{16}{9} = m.$

39. $f(t) = t^{-4}$

$f'(t) = -4t^{-5} = -\dfrac{4}{t^5}$

At the point

$\left(\dfrac{1}{2}, 16\right)$, $f'\left(\dfrac{1}{2}\right) = -\dfrac{4}{\left(\dfrac{1}{2}\right)^5} = -\dfrac{4}{\dfrac{1}{32}} = -128 = m.$

40. $f(x) = x^{-1/3}$

$f'(x) = -\dfrac{1}{3}x^{-4/3} = -\dfrac{1}{3x^{4/3}}$

At the point $\left(8, \dfrac{1}{2}\right)$, $f'(8) = -\dfrac{1}{3(8)^{4/3}} = -\dfrac{1}{48} = m.$

41. $f(x) = 2x^3 + 8x^2 - x - 4$

$f'(x) = 6x^2 + 16x - 1$

At the point

$(-1, 3)$, $f'(-1) = 6(-1)^2 + 16(-1) - 1 = -11 = m.$

42. $f(x) = x^4 - 2x^3 + 5x^2 - 7x$

$f'(x) = 4x^3 - 6x^2 + 10x - 7$

At the point

$(-1, 15)$, $f'(-1) = 4(-1)^3 - 6(-1)^2 + 10(-1) - 7$

$= -4 - 6 - 10 - 7 = -27 = m.$

43. $f(x) = -\frac{1}{2}x(1 + x^2) = -\frac{1}{2}x - \frac{1}{2}x^3$

$f'(x) = -\frac{1}{2} - \frac{3}{2}x^2$

$f'(1) = -\frac{1}{2} - \frac{3}{2} = -2$

44. $f(x) = 3(5 - x)^2 = 75 - 30x + 3x^2$

$f'(x) = -30 + 6x$

$f'(5) = -30 + (6)(5) = 0$

45. (a) $y = -2x^4 + 5x^2 - 3$

$y' = -8x^3 + 10x$

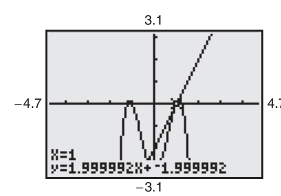
$m = y'(1) = -8 + 10 = 2$

The equation of the tangent line is

$y - 0 = 2(x - 1)$

$y = 2x - 2.$

(b) and (c)



46. (a) $y = x^3 + x + 4$

$y' = 3x^2 + 1$

$m = y'(-2) = 3(-2)^2 + 1 = 13$

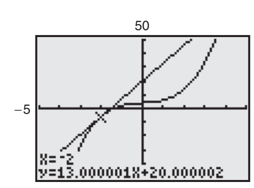
The equation of the tangent line is

$y - (-6) = 13[x - (-2)]$

$y + 6 = 13x + 26$

$y = 13x + 20.$

(b) and (c)



47. (a) $f(x) = \sqrt[3]{x} + \sqrt[5]{x} = x^{1/3} + x^{1/5}$

$$f'(x) = \frac{1}{3}x^{-2/3} + \frac{1}{5}x^{-4/5} = \frac{1}{3x^{2/3}} + \frac{1}{5x^{4/5}}$$

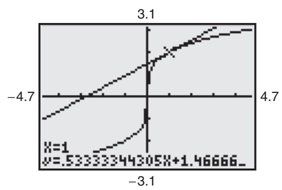
$$m = f'(1) = \frac{1}{3} + \frac{1}{5} = \frac{8}{15}$$

The equation of the tangent line is

$$y - 2 = \frac{8}{15}(x - 1)$$

$$y = \frac{8}{15}x + \frac{22}{15}.$$

(b) and (c)



48. (a) $f(x) = \dfrac{1}{\sqrt[3]{x^2}} - x = x^{-2/3} - x$

$$f'(x) = -\frac{2}{3}x^{-5/3} - 1$$

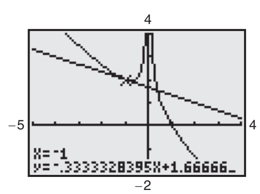
$$m = f'(-1) = \frac{2}{3} - 1 = -\frac{1}{3}$$

The equation of the tangent line is

$$y - 2 = -\tfrac{1}{3}(x + 1)$$

$$y = -\tfrac{1}{3}x + \tfrac{5}{3}.$$

(b) and (c)



49. (a) $y = 3x\left(x^2 - \dfrac{2}{x}\right)$

$$y = 3x^3 - 6$$

$$y' = 9x^2$$

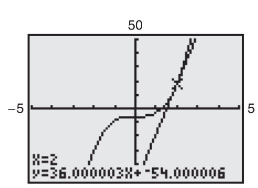
$$m = y' = 9(2)^2 = 36$$

The equation of the tangent line is

$$y - 18 = 36(x - 2)$$

$$y = 36x - 54.$$

(b) and (c)



50. (a) $y = (2x + 1)^2$

$$y = 4x^2 + 4x + 1$$

$$y' = 8x + 4$$

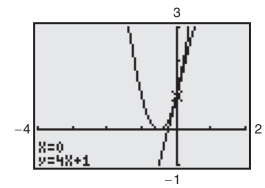
$$m = y' = 8(0) + 4 = 4$$

The equation of the tangent line is

$$y - 1 = 4(x - 0)$$

$$y = 4x + 1.$$

(b) and (c)



51. $f(x) = x^2 - 4x^{-1} - 3x^{-2}$

$$f'(x) = 2x + 4x^{-2} + 6x^{-3} = 2x + \frac{4}{x^2} + \frac{6}{x^3}$$

52. $f(x) = 6x^2 - 5x^{-2} + 7x^{-3}$

$$f'(x) = 12x + 10x^{-3} - 21x^{-4} = 12x + \frac{10}{x^3} - \frac{21}{x^4}$$

53. $f(x) = x^2 - 2x - \dfrac{2}{x^4} = x^2 - 2x - 2x^{-4}$

$$f'(x) = 2x - 2 + 8x^{-5} = 2x - 2 + \frac{8}{x^5}$$

54. $f(x) = x^2 + 4x + \dfrac{1}{x} = x^2 + 4x + x^{-1}$

$$f'(x) = 2x + 4 - x^{-2} = 2x + 4 - \frac{1}{x^2}$$

55. $f(x) = x^{4/5} + x$

$$f'(x) = \frac{4}{5}x^{-1/5} + 1 = \frac{4}{5x^{1/5}} + 1$$

56. $f(x) = x^{1/3} - 1$

$$f'(x) = \frac{1}{3}x^{-2/3} = \frac{1}{3x^{2/3}}$$

57. $f(x) = x(x^2 + 1) = x^3 + x$

$$f'(x) = 3x^2 + 1$$

58. $f(x) = (x^2 + 2x)(x + 1) = x^3 + 3x^2 + 2x$

$$f'(x) = 3x^2 + 6x + 2$$

59. $f(x) = \dfrac{2x^3 - 4x^2 + 3}{x^2} = 2x - 4 + 3x^{-2}$

$f'(x) = 2 - 6x^{-3} = 2 - \dfrac{6}{x^3} = \dfrac{2x^3 - 6}{x^3} = \dfrac{2(x^3 - 3)}{x^3}$

60. $f(x) = \dfrac{2x^2 - 3x + 1}{x} = 2x - 3 + x^{-1}$

$f'(x) = 2 - x^{-2} = 2 - \dfrac{1}{x^2} = \dfrac{2x^2 - 1}{x^2}$

61. $f(x) = \dfrac{4x^3 - 3x^2 + 2x + 5}{x^2} = 4x - 3 + 2x^{-1} + 5x^{-2}$

$f'(x) = 4 - 2x^{-2} - 10x^{-3} = 4 - \dfrac{2}{x^2} - \dfrac{10}{x^3} = \dfrac{4x^3 - 2x - 10}{x^3}$

62. $f(x) = \dfrac{-6x^3 + 3x^2 - 2x + 1}{x} = -6x^2 + 3x - 2 + x^{-1}$

$f'(x) = -12x + 3 - x^{-2} = -12x + 3 - \dfrac{1}{x^2}$

63. $y = x^4 - 2x + 3$

$y' = 4x^3 - 4x = 4x(x^2 - 1) = 0$ when $x = 0, \pm 1$

If $x = \pm 1$, then $y = (\pm 1)^4 - 2(\pm 1)^2 + 3 = 2$.

The function has horizontal tangent lines at the points $(0, 3)$, $(1, 2)$, and $(-1, 2)$.

64. $y = x^3 + 3x^2$

$y' = 3x^2 + 6x = 3x(x + 2) = 0$ when $x = 0, -2$.

The function has horizontal tangent lines at the points (0, 0) and (–2, 4).

65. $y = \frac{1}{2}x^2 + 5x$

$y' = x + 5 = 0$ when $x = -5$.

The function has a horizontal tangent line at the point $\left(-5, -\frac{25}{2}\right)$.

66. $y = x^2 + 2x$

$y' = 2x + 2 = 0$ when $x = -1$.

The function has a horizontal tangent line at the point $(-1, -1)$.

67. $y = x^2 + 3$

$y' = 2x$

Set $y' = 4$.

$2x = 4$

$x = 2$

If $x = 2$, $y = (2)^2 + 3 = 7 \rightarrow (2, 7)$.

The graph of $y = x^2 + 3$ has a tangent line with slope $m = 4$ at the point $(2, 7)$.

68. $y = x^2 + 2x$

$y' = 2x + 2$

Set $y' = 10$.

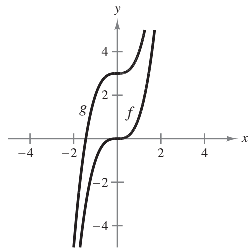
$2x + 2 = 10$

$x = 4$

If $x = 4$, $y = (4)^2 + 2(4) = 24 \rightarrow (4, 24)$.

The graph of $y = x^2 + 2x$ has a tangent line with slope $m = 10$ at the point $(4, 24)$.

69. (a)



(b) $f'(x) = g'(x) = 3x^2$

$f'(1) = g'(1) = 3$

(c) Tangent line to f at $x = 1$:

$f(1) = 1$

$y - 1 = 3(x - 1)$

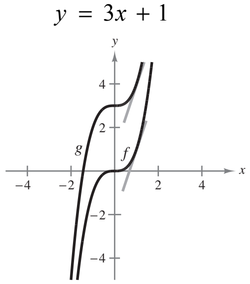
$y = 3x - 2$

Tangent line to g at $x = 1$:

$g(1) = 4$

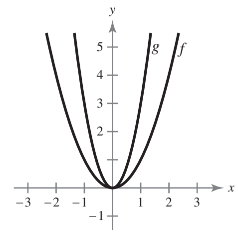
$y - 4 = 3(x - 1)$

$y = 3x + 1$



(d) f' and g' are the same.

70. (a)



(b) $f'(x) = 2x$

$f'(1) = 2$

$g'(x) = 6x$

$g'(1) = 6$

(c) Tangent line to f at $x = 1$:

$f(1) = 1$

$y - 1 = 2(x - 1)$

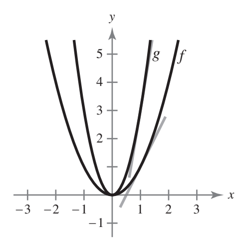
$y = 2x - 1$

Tangent line to g at $x = 1$:

$g(1) = 3$

$y - 3 = 6(x - 1)$

$y = 6x - 3$



(d) g' is 3 times f'.

71. If $g(x) = f(x) + 6$, then $g'(x) = f'(x)$ because the derivative of a constant is $0, g'(x) = f'(x)$.

72. If $g(x) = 2f(x)$, then $g'(x) = 2f'(x)$ because of the Constant Multiple Rule.

73. If $g(x) = -5f(x)$, then $g'(x) = -5f'(x)$ because of the Constant Multiple Rule.

74. If $g(x) = 3f(x) - 1$, then $g'(x) = 3f'(x)$ because of the Constant Multiple Rule and the derivative of a constant is 0.

75. (a) $R = -4.1685t^3 + 175.037^2 - 1950.88t + 7265.3$

$R' = -12.5055t^2 + 350.074t - 1950.88$

2009: $R'(9) = -12.5055(9)^2 + 350.074(9) - 1950.88 \approx \$186.8$ million per year

2011: $R'(11) = -12.5055(11)^2 + 350.074(1) - 1950.88 \approx \$386.8$ million per year

(b) These results are close to the estimates in Exercise 13 in Section 2.1.

(c) The slope of the graph at time t is the rate at which sales are increasing in millions of dollars per year.

76. (a) $R = -2.67538t^4 + 94.0568t^3 - 1155.203t^2 + 6002.42t - 9794.2$

$R' = -10.70152t^3 + 282.1704t^2 - 2310.406t + 6002.42$

2010: $R'(10) = -10.70152(10)^3 + 282.1704(10)^2 - 2310.406(10) + 6002 \approx \$413.88$ million per year

2012: $R'(12) = -10.70152(12)^3 + 282.1704(12)^2 - 2310.406(12) + 6002 \approx \$417.86$ million per year

(b) These results are close to the estimates in Exercise 14 in Section 2.1.

(c) The slope of the graph at time t is the rate at which sales are increasing in millions of dollars per year.

77. (a) More men and women seem to suffer from migraines between 30 and 40 years old. More females than males suffer from migraines. Fewer people whose income is greater than or equal to \$30,000 suffer from migraines than people whose income is less than \$10,000.

(b) The derivatives are positive up to approximately 37 years old and negative after about 37 years of age. The percent of adults suffering from migraines increases up to about 37 years old, then decreases. The units of the derivative are percent of adults suffering from migraines per year.

78. (a) The attendance rate for football games, $g'(t)$, is greater at game 1.

(b) The attendance rate for basketball games, $f'(t)$, is greater than the rate for football games, $g'(t)$, at game 3.

(c) The attendance rate for basketball games, $f'(t)$, is greater than the rate for football games, $g'(t)$, at game 4. In addition, the attendance rate for football games is decreasing at game 4.

(d) At game 5, the attendance rate for football continues to increase, while the attendance rate for basketball continues to decrease.

79. $C = 7.75x + 500$

$C' = 7.75$, which equals the variable cost.

80. $C = 150x + 7000$

$P = R - C$

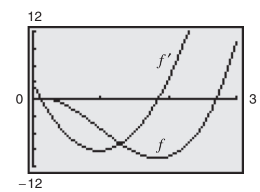
$P = 500x - (150x + 7000)$

$P = 350x - 7000$

$P' = 350$, which equals the profit on each dinner sold.

81. $f(x) = 4.1x^3 - 12x^2 + 2.5x$

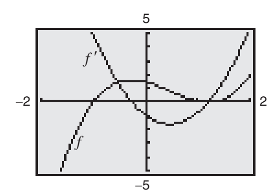
$f'(x) = 12.3x^2 - 24x + 2.5$



f has horizontal tangents at $(0.110, 0.135)$ and $(1.841, -10.486)$.

82. $f(x) = x^3 - 1.4x^2 - 0.96x + 1.44$

$f'(x) = 3x^2 - 2.8x - 0.96$



f has horizontal tangents at $(1.2, 0)$ and $(-0.267, 1.577)$.

83. False. Let $f(x) = x$ and $g(x) = x + 1$.

Then $f'(x) = g'(x) = 1$, but $f(x) \neq g(x)$.

84. True. c is a constant.

Section 2.3 Rates of Change: Velocity and Marginals

Skills Warm Up

1. $\dfrac{-63-(-105)}{21-7} = \dfrac{42}{14} = 3$

2. $\dfrac{-43-35}{6-(-7)} = \dfrac{-78}{13} = -6$

3. $\dfrac{24-33}{9-6} = \dfrac{-9}{3} = -3$

4. $\dfrac{40-16}{18-8} = \dfrac{24}{10} = \dfrac{12}{5}$

5. $y = 4x^2 - 2x + 7$
   $y' = 8x - 2$

6. $s = -2t^3 + 8t^2 - 7t$
   $s' = -6t^2 + 16t - 7$

7. $s = -16t^2 + 24t + 30$
   $s' = -32t + 24$

8. $y = -16x^2 + 54x + 70$
   $y' = -32x + 54$

9. $A = \frac{1}{10}\left(-2r^3 + 3r^2 + 5r\right)$
   $A' = \frac{1}{10}\left(-6r^2 + 6r + 5\right)$
   $A' = -\frac{3}{5}r^2 + \frac{3}{5}r + \frac{1}{2}$

10. $y = \frac{1}{9}\left(6x^3 - 18x^2 + 63x - 15\right)$
    $y' = \frac{1}{9}\left(18x^2 - 36x + 63\right)$
    $y' = 2x^2 - 4x + 7$

11. $y = 12x - \dfrac{x^2}{5000}$
    $y' = 12 - \dfrac{2x}{5000}$
    $y' = 12 - \dfrac{x}{2500}$

12. $y = 138 + 74x - \dfrac{x^3}{10{,}000}$
    $y' = 74 - \dfrac{3x^2}{10{,}000}$

1. (a) 1980–1986: $\dfrac{120-63}{6-0} = \$9.5$ billion/yr

(b) 1986–1992: $\dfrac{165-120}{12-6} = \$7.5$ billion/yr

(c) 1992–1998: $\dfrac{226-165}{18-12} \approx \$10.2$ billion/yr

(d) 1998–2004: $\dfrac{305-226}{24-18} \approx \$13.2$ billion/yr

(e) 2004–2010: $\dfrac{408-305}{30-24} \approx \$17.2$ billion/yr

(f) 1980–2012: $\dfrac{453-63}{32-0} \approx \$12.2$ billion/yr

(g) 1990–2012: $\dfrac{453-152}{32-10} \approx \$13.7$ billion/yr

(h) 2000–2012: $\dfrac{453-269}{32-20} \approx \$15.3$ billion/yr

2. (a) Imports:

1980–1990: $\dfrac{495 - 245}{10 - 0} = \$25$ billion/yr

(b) Exports:

1980–1990: $\dfrac{394 - 226}{10 - 0} = \$16.8$ billion/yr

(c) Imports:

1990–2000: $\dfrac{1218 - 495}{20 - 10} \approx \$72.3$ billion/yr

(d) Exports:

1990–2000: $\dfrac{782 - 394}{20 - 10} = \$38.8$ billion/yr

(e) Imports:

2000–2010: $\dfrac{1560 - 1218}{29 - 20} = \$38.0$ billion/yr

(f) Exports:

2000–2010: $\dfrac{1056 - 782}{29 - 20} = \$30.4$ billion/yr

(g) Imports:

1980–2013: $\dfrac{2268 - 245}{33 - 0} \approx \$61.3$ billion/yr

(h) Exports:

1980–2013: $\dfrac{1580 - 226}{33 - 0} \approx \$41.0$ billion/yr

3. $f(t) = 3t + 5; [1, 2]$

Average rate of change:

$$\frac{\Delta y}{\Delta t} = \frac{f(2) - f(1)}{2 - 1} = \frac{11 - 8}{1} = 3$$

$f'(t) = 3$

Instantaneous rates of change: $f'(1) = 3,\ f'(2) = 3$

4. $h(x) = 7 - 2x; [1, 3]$

Average rate of change:

$$\frac{\Delta h}{\Delta t} = \frac{h(3) - h(1)}{3 - 1} = \frac{1 - 5}{2} = -2$$

$h'(t) = -2$

Instantaneous rates of change: $h(1) = -2,\ h(3) = -2$

5. $h(x) = x^2 - 4x + 2; [-2, 2]$

Average rate of change:

$$\frac{\Delta h}{\Delta x} = \frac{h(2) - h(-2)}{2 - (-2)} = \frac{-2 - 14}{4} = -4$$

$h'(x) = 2x - 4$

Instantaneous rates of change: $h'(-2) = -8,\ h'(2) = 0$

6. $f(x) = -x^2 - 6x - 5; [-3, 1]$

Average rate of change:

$$\frac{\Delta f}{\Delta x} = \frac{f(1) - f(-3)}{1 - (-3)} = \frac{-12 - 4}{4} = -4$$

$f'(x) = -2x - 6$

Instantaneous rates of change: $f'(-3) = 0,\ f'(1) = -8$

7. $f(x) = 3x^{4/3}; [1, 8]$

Average rate of change:

$$\frac{\Delta y}{\Delta x} = \frac{f(8) - f(1)}{8 - 1} = \frac{48 - 3}{7} = \frac{45}{7}$$

$f'(x) = 4x^{1/3}$

Instantaneous rates of change: $f'(1) = 4,\ f'(8) = 8$

8. $f(x) = x^{3/2}; [1, 4]$

Average rate of change:

$$\frac{\Delta y}{\Delta x} = \frac{f(4) - f(1)}{4 - 1} = \frac{8 - 1}{3} = \frac{7}{3}$$

$f'(x) = \frac{3}{2}x^{1/2}$

Instantaneous rates of change: $f'(1) = \frac{3}{2},\ f'(4) = 3$

9. $f(x) = \frac{1}{x}; [1, 5]$

Average rate of change:

$$\frac{\Delta y}{\Delta x} = \frac{f(5) - f(1)}{5 - 1} = \frac{\frac{1}{5} - 1}{3} = \frac{-\frac{4}{5}}{4} = -\frac{1}{5}$$

$f'(x) = -\frac{1}{x^2}$

Instantaneous rates of change:

$f'(1) = -1,\ f'(5) = -\frac{1}{25}$

10. $f(x) = \frac{1}{\sqrt{x}}; [1, 9]$

Average rate of change:

$$\frac{\Delta y}{\Delta x} = \frac{f(9) - f(1)}{9 - 1} = \frac{\frac{1}{3} - 1}{8} = \frac{-\frac{2}{3}}{4} = -\frac{1}{6}$$

$f'(x) = \frac{1}{2x^{3/2}}$

Instantaneous rates of change:

$f'(1) = -\frac{1}{2},\ f'(9) = \frac{1}{54}$

11. $f(t) = t^4 - 2t^2; [-2, -1]$

Average rate of change:

$$\frac{\Delta y}{\Delta x} = \frac{f(-1) - f(-2)}{-1 - (-2)} = \frac{-1 - 8}{1} = -9$$

$$f'(t) = 4t^3 - 4t$$

Instantaneous rates of change:

$$f'(-2) = -24, f'(-1) = 0$$

12. $g(x) = x^3 - 1; [-1, 1]$

Average rate of change:

$$\frac{\Delta y}{\Delta x} = \frac{g(1) - g(-1)}{1 - (-1)} = \frac{0 - (-2)}{2} = 1$$

$$g'(x) = 3x^2$$

Instantaneous rates of change:

$$g'(-1) = 3, g'(1) = 3$$

13. (a) $\approx \frac{0 - 1400}{3} \approx -467$

The number of visitors to the park is decreasing at an average rate of 467 people per month from September to December.

(b) Answers will vary. Sample answer: $[4, 11]$

Both the instantaneous rate of change at $t = 8$ and the average rate of change on $[4, 11]$ are about zero.

14. (a) $\frac{\Delta M}{\Delta t} = \frac{800 - 200}{3 - 1} = \frac{600}{2} = 300 \text{ mg/hr}$

(b) Answers will vary. Sample answer: $[2, 5]$

Both the instantaneous rate of change at $t = 4$ and the average rate of change on $[2, 5]$ is about zero.

15. $s = -16t^2 + 30t + 250$

Instantaneous: $v(t) = s'(t) = -32t + 30$

(a) Average: $[0, 1]$:

$$\frac{s(1) - s(0)}{1 - 0} = \frac{264 - 250}{1} = 14 \text{ ft/sec}$$

$$v(0) = s'(0) = 30 \text{ ft/sec}$$

$$v(1) = s'(1) = -2 \text{ ft/sec}$$

(b) Average: $[1, 2]$:

$$\frac{s(2) - s(1)}{2 - 1} = \frac{246 - 264}{1} = -18 \text{ ft/sec}$$

$$v(1) = s'(1) = -2 \text{ ft/sec}$$

$$v(2) = s'(2) = -34 \text{ ft/sec}$$

(c) Average: $[2, 3]$:

$$\frac{s(3) - s(2)}{3 - 2} = \frac{196 - 246}{1} = -50 \text{ ft/sec}$$

$$v(2) = s'(2) = -34 \text{ ft/sec}$$

$$v(3) = s'(3) = -66 \text{ ft/sec}$$

(d) Average: $[3, 4]$:

$$\frac{s(4) - s(3)}{4 - 3} = \frac{114 - 196}{1} = -82 \text{ ft/sec}$$

$$v(3) = s'(3) = -66 \text{ ft/sec}$$

$$v(4) = s'(4) = -98 \text{ ft/sec}$$

16. (a) $H'(v) = 33\left[10\left(\frac{1}{2}v^{-1/2}\right) - 1\right] = 33\left[\frac{5}{\sqrt{v}} - 1\right]$

Rate of change of heat loss with respect to wind speed.

(b)
$$\begin{aligned}
H'(2) &= 33\left[\frac{5}{\sqrt{2}} - 1\right] \\
&\approx 83.673 \frac{\text{kcal/m}^2\text{/hr}}{\text{m/sec}} \\
&= 83.673 \frac{\text{kcal}}{\text{m}^3} \cdot \frac{\text{sec}}{\text{hr}} \\
&= 83.673 \frac{\text{kcal}}{\text{m}^3} \cdot \frac{1}{3600} \\
&= 0.023 \text{ kcal/m}^3
\end{aligned}$$

$$\begin{aligned}
H'(5) &= 33\left[\frac{5}{\sqrt{5}} - 1\right] \\
&\approx 40.790 \frac{\text{kcal/m}^2\text{/hr}}{\text{m/sec}} \\
&= 40.790 \frac{\text{kcal}}{\text{m}^3} \cdot \frac{\text{sec}}{\text{hr}} \\
&= 40.790 \frac{\text{kcal}}{\text{m}^3} \cdot \frac{1}{3600} \\
&= 0.11 \text{ kcal/m}^3
\end{aligned}$$

17. $s = -16t^2 + 555$

(a) Average velocity $= \frac{s(3) - s(2)}{3 - 2}$

$$= \frac{411 - 491}{1}$$

$$= -80 \text{ ft/sec}$$

(b) $v = s'(t) = -32t,\ v(2) = -64 \text{ ft/sec},$

$v(3) = -96 \text{ ft/sec}$

(c)
$$\begin{aligned}
s &= -16t^2 + 555 = 0 \\
16t^2 &= 555 \\
t^2 &= \frac{555}{16} \\
t &\approx 5.89 \text{ seconds}
\end{aligned}$$

(d) $v(5.89) \approx -188.5 \text{ ft/sec}$

18. (a) $s(t) = -16t^2 - 18t + 210$

$v(t) = s'(t) = -32t - 18$

(b) $[1, 2]: \dfrac{s(2) - s(1)}{2 - 1} = \dfrac{110 - 176}{1} = -66$ ft/sec

(c) $v(1) = -50$ ft/sec

$v(2) = -82$ ft/sec

(d) Set $s(t) = 0$.

$-16t^2 - 18t + 210 = 0$

$$t = -\frac{(-18) \pm \sqrt{(-18)^2 - 4(-16)(210)}}{2(-16)} = \frac{18 \pm \sqrt{13{,}764}}{-32} \approx 3.10 \text{ sec}$$

(e) $v(3.10) = -117.2$ ft/sec

19. $C = 205{,}000 + 9800x$

$\dfrac{dC}{dx} = 9800$

20. $C = 150{,}000 + 7x^3$

$\dfrac{dC}{dx} = 21x^2$

21. $C = 55{,}000 + 470x - 0.25x^2, \ 0 \le x \le 940$

$\dfrac{dC}{dx} = 470 - 0.5x$

22. $C = 100\left(9 + 3\sqrt{x}\right)$

$\dfrac{dC}{dx} = 100\left[0 + 3\left(\dfrac{1}{2}x^{-1/2}\right)\right] = \dfrac{150}{\sqrt{x}}$

23. $R = 50x - 0.5x^2$

$\dfrac{dR}{dx} = 50 - x$

24. $R = 30x - x^2$

$\dfrac{dR}{dx} = 30 - 2x$

25. $R = -6x^3 + 8x^2 + 200x$

$\dfrac{dR}{dx} = -18x^2 + 16x + 200$

26. $R = 50\left(20x - x^{3/2}\right)$

$\dfrac{dR}{dx} = 50\left[20 - \dfrac{3}{2}x^{1/2}\right] = 1000 - 75\sqrt{x}$

27. $P = -2x^2 + 72x - 145$

$\dfrac{dP}{dx} = -4x + 72$

28. $P = -0.25x^2 + 2000x - 1{,}250{,}000$

$\dfrac{dP}{dx} = -0.5x + 2000$

29. $P = 0.0013x^3 + 12x$

$\dfrac{dP}{dx} = 0.0039x^2 + 12$

30. $P = -0.5x^3 + 30x^2 - 164.25x - 1000$

$\dfrac{dP}{dx} = -1.5x^2 + 60x - 164.25$

31. $C = 3.6\sqrt{x} + 500$

(a) $C'(x) = 1.8/\sqrt{x}$

$C'(9) = \$0.60$ per unit.

(b) $C(10) - C(9) \approx \$0.584$

(c) The answers are close.

32. $R = 2x\left(900 + 32x - x^2\right)$

(a) $R = 1800x + 64x^2 - 2x^3$

$R'(x) = 1800 + 128x - 6x^2$

$R'(14) = \$2416$

(b) $R(15) - R(14) = 2(15)\left[900 + 32(15) - 15^2\right] - 2(14)\left[900 + 32(14) - 14^2\right]$

$= 34{,}650 - 32{,}256 = \$2394$

(c) The answers are close.

33. $P = -0.04x^2 + 25x - 1500$

(a) $\frac{dP}{dx} = -0.08x + 25 = P'(x)$

$P'(150) = \$13$

(b) $\frac{\Delta P}{\Delta x} = \frac{P(151) - P(150)}{151 - 150} = \frac{1362.96 - 1350}{1} = \$12.96$

(c) The results are close.

34. $P = 36{,}000 + 2048\sqrt{x} - \frac{1}{8x^2}, 150 \le x \le 275$

$\frac{dP}{dx} = 2048\left(\frac{1}{2}x^{-1/2}\right) - \frac{1}{8}\left(-2x^{-3}\right)$

$= \frac{1024}{\sqrt{x}} + \frac{1}{4x^3}$

(a) When $x = 150$, $\frac{dP}{dx} \approx \$83.61$.

(b) When $x = 175$, $\frac{dP}{dx} \approx \$77.41$.

(c) When $x = 200$, $\frac{dP}{dx} \approx \$72.41$.

(d) When $x = 225$, $\frac{dP}{dx} \approx \$68.27$.

(e) When $x = 250$, $\frac{dP}{dx} \approx \$64.76$.

(f) When $x = 275$, $\frac{dP}{dx} \approx \$61.75$.

35. $P = 1.73t^2 + 190.6t + 16{,}994$

(a) $P(0) = 16{,}994$ thousand people

$P(3) = 17{,}581.37$ thousand people

$P(6) = 18{,}199.88$ thousand people

$P(9) = 18{,}849.53$ thousand people

$P(12) = 19{,}530.32$ thousand people

$P(15) = 20{,}242.25$ thousand people

$P(18) = 20{,}985.32$ thousand people

$P(21) = 21{,}759.53$ thousand people

The population is increasing from 1990 to 2011.

(b) $\frac{dP}{dt} = P'(t) = 3.46t + 190.6$

$\frac{dP}{dt}$ represents the population growth rate.

(c) $P'(0) = 190.6$ thousand people per year

$P'(3) = 200.98$ thousand people per year

$P'(6) = 211.36$ thousand people per year

$P'(9) = 221.74$ thousand people per year

$P'(12) = 232.12$ thousand people per year

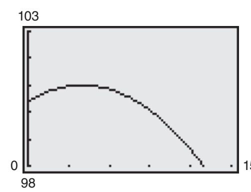
$P'(15) = 242.5$ thousand people per year

$P'(18) = 252.88$ thousand people per year

$P'(21) = 263.26$ thousand people per year

The rate of growth is increasing.

36. (a)



(b) For $t < 4$, the slopes are positive, and the fever is increasing. For $t > 4$, the slopes are negative, and the fever is decreasing.

(c) $T(0) = 100.4°\text{F}$

$T'(4) = 101°\text{F}$

$T(8) = 100.4°\text{F}$

$T(12) = 98.6°\text{F}$

(d) $\frac{dT}{dt} = -0.075t + 0.3$; the rate of change of temperature with respect to time

(e) $T'(0) = 0.3°\text{F}$ per hour

$T'(4) = 0°\text{F}$ per hour

$T'(8) = -0.3°\text{F}$ per hour

$T(12) = -0.6°\text{F}$ per hour

For $0 \le t < 4$, the rate of change of the temperature is positive; therefore, the temperature is increasing. For $4 < t \le 12$, the rate of change of the temperature is decreasing; therefore, the temperature is decreasing back to a normal temperature of $98.6°\text{F}$.

37. (a) $TR = -10Q^2 + 160Q$

(b) $(TR)' = MR = -20Q + 160$

(c)

Q	0	2	4	6	8	10
Model	160	120	80	40	0	–40
Table	–	130	90	50	10	–30

38. (a) $R = xp = x(5 - 0.001x) = 5x - 0.001x^2$

(b) $P = R - C = (5x - 0.001x^2) - (35 + 1.5x)$

$= -0.001x^2 + 3.5x - 35$

(c) $\dfrac{dR}{dx} = 5 - 0.002x$

$\dfrac{dP}{dx} = 3.5 - 0.002x$

x	600	1200	1800	2400	3000
dR/dx	3.8	2.6	1.4	0.2	–1.0
dP/dx	2.3	1.1	–0.1	–1.3	–2.5
P	1705	2725	3025	2605	1465

39. (a) $(400, 1.75), (500, 1.50)$

$$\text{Slope} = \frac{1.50 - 1.75}{500 - 400} = -0.0025$$

$$p - 1.75 = -0.0025(x - 400)$$

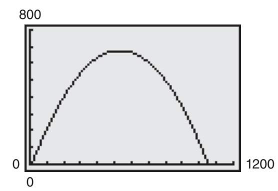
$$p = -0.0025x + 2.75$$

$$P = R - C = xp - c$$

$$= x(-0.0025x + 2.75) - (0.1x + 25)$$

$$= -0.0025x^2 + 2.65x - 25$$

(b)



At $x = 300$, P has a positive slope.

At $x = 530$, P has a 0 slope.

At $x = 700$, P has a negative slope.

(c) $P'(x) = -0.005x + 2.65$

$P'(300) = \$1.15$ per unit

$P'(530) = \$0$ per unit

$P'(700) = -\$.85$ per unit

40. $(36{,}000, 30), (32{,}000, 35)$

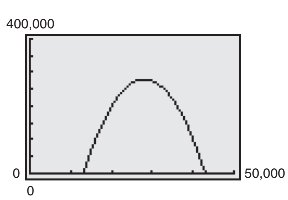
$$\text{Slope} = \frac{35 - 30}{32{,}000 - 36{,}000} = -\frac{5}{4000} = \frac{1}{800}$$

$$p - 30 = -\frac{1}{800}(x - 36{,}000)$$

$$p = -\frac{1}{800}x + 75 \text{ (demand function)}$$

(a) $P = R - C = xp - C = x\left(-\dfrac{1}{800}x + 75\right) - (5x + 700{,}000) = -\dfrac{1}{800}x^2 + 70x - 700{,}000$

(b)



At $x = 18{,}000$, P has a positive slope.

At $x = 28{,}000$, P has a 0 slope.

At $x = 36{,}000$, P has a negative slope.

(c) $P'(x) = -\dfrac{1}{400}x + 70$

$P'(18{,}000) = \$25$ per ticket

$P'(28{,}000) = \$0$ per ticket

$P'(36{,}000) = -\$20$ per ticket

41. (a) $C(x) = \left(\dfrac{15{,}000 \text{ mi}}{\text{yr}}\right)\left(\dfrac{1 \text{ gal}}{x \text{ mi}}\right)\left(\dfrac{2.60 \text{ dollars}}{1 \text{ gal}}\right)$

$C(x) = \dfrac{39{,}000}{x} \dfrac{\text{dollars}}{\text{yr}}$

(b) $\dfrac{dC}{dx} = -\dfrac{39{,}000}{x^2} \dfrac{\frac{\text{dollars}}{\text{yr}}}{\text{mpg}}$

The marginal cost is the change of savings for a 1-mile per gallon increase in fuel efficiency.

(c)

x	10	15	20	25	30	35	40
C	3900	2600	1950	1560	1300	1114.29	975
dC/dx	−390	−173.33	−97.5	−62.4	−43.33	−31.84	−24.38

(d) The driver who gets 15 miles per gallon would benefit more than the driver who gets 35 miles per gallon. The value of dC/dx is a greater savings for $x = 15$ than for $x = 35$.

42. (a) $f'(2.959)$ is the rate of change of the number of gallons of gasoline sold when the price is $\$2.959/\text{gallon}$.

(b) In general, it should be negative. Demand tends to decrease as price increases. Answers will vary.

43. (a) Average rate of change from 2000 to 2013: $\dfrac{\Delta p}{\Delta t} = \dfrac{16{,}576.66 - 10{,}786.85}{13 - 0} \approx \$445.37/\text{yr}$

(b) Average rate of change from 2003 to 2007: $\dfrac{\Delta p}{\Delta t} = \dfrac{13{,}264.82 - 10{,}453.92}{7 - 3} \approx \$702.73/\text{yr}$

So, the instantaneous rate of change for 2005 is $p'(5) \approx \$702.73/\text{yr}$.

(c) Average rate of change from 2004 to 2006: $\dfrac{\Delta p}{\Delta t} = \dfrac{12{,}463.15 - 10{,}783.01}{6 - 4} \approx \$840.07/\text{yr}$

So, the instantaneous rate of change for 2005 is $p'(5) \approx \$840.07/\text{yr}$.

(d) The average rate of change from 2004 to 2006 is a better estimate because the data is closer to the years in question.

44. Answers will vary. *Sample answer:*
The rate of growth in the lag phase is relatively slow when compared with the rapid growth in the acceleration phase. The population grows slower in the deceleration phase, and there is no growth at equilibrium. These changes could be explained by food supply or seasonal growth.

Section 2.4 The Product and Quotient Rules

Skills Warm Up

1. $$\begin{aligned}(x^2 + 1)(2) + (2x + 7)(2x) &= 2x^2 + 2 + 4x^2 + 14x \\ &= 6x^2 + 14x + 2 \\ &= 2(3x^2 + 7x + 1)\end{aligned}$$

2. $$\begin{aligned}(2x - x^3)(8x) + (4x^2)(2 - 3x^2) &= 16x^2 - 8x^4 + 8x^2 - 12x^4 \\ &= 24x^2 - 20x^4 \\ &= 4x^2(6 - 5x^2)\end{aligned}$$

3. $x(4)(x^2 + 2)^3(2x) + (x^2 + 4)(1) = 8x^2(x^2 + 2)^3(x^2 + 4)$

Skills Warm Up —*continued*—

4. $x^2(2)(2x+1)(2) + (2x+1)^4(2x) = 4x^2(2x+1) + 2x(2x+1)^4$

$= 2x(2x+1)\left[2x + (2x+1)^3\right]$

5. $\dfrac{(2x+7)(5) - (5x+6)(2)}{(2x+7)^2} = \dfrac{10x + 35 - 10x - 12}{(2x+7)^2}$

$= \dfrac{23}{(2x+7)^2}$

6. $\dfrac{(x^2-4)(2x+1) - (x^2+x)(2x)}{(x^2-4)^2} = \dfrac{2x^3 + x^2 - 8x - 4 - 2x^3 - 2x^2}{(x^2-4)^2}$

$= \dfrac{-x^2 - 8x - 4}{(x^2-4)^2}$

7. $\dfrac{(x^2+1)(2) - (2x+1)(2x)}{(x^2+1)^2} = \dfrac{2x^2 + 2 - 4x^2 - 2x}{(x^2+1)^2}$

$= \dfrac{-2x^2 - 2x + 2}{(x^2+1)^2}$

$= \dfrac{-2(x^2 + x - 1)}{(x^2+1)^2}$

8. $\dfrac{(1-x^4)(4) - (4x-1)(-4x^3)}{(1-x^4)^2} = \dfrac{4 - 4x^4 + 16x^4 - 4x^3}{(1-x^4)^2}$

$= \dfrac{12x^4 - 4x^3 + 4}{(1-x^4)^2}$

$= \dfrac{4(3x^4 - x^3 + 1)}{(1-x^4)^2}$

9. $(x^{-1}+x)(2) + (2x-3)(-x^{-2}+1) = 2x^{-1} + 2x + (-2x^{-1} + 2x + 3x^{-2} - 3)$

$= 4x + 3x^{-2} - 3$

$= 4x + \dfrac{3}{x^2} - 3$

$= \dfrac{4x^3 - 3x^2 + 3}{x^2}$

10. $\dfrac{(1-x^{-1})(1) - (x-4)(x^{-2})}{(1-x^{-1})^2} = \left(\dfrac{1 - x^{-1} - x^{-1} + 4x^{-2}}{1 - 2x^{-1} + x^{-2}}\right)\left(\dfrac{x^2}{x^2}\right)$

$= \dfrac{x^2 - 2x + 4}{x^2 - 2x + 1}$

$= \dfrac{x^2 - 2x + 4}{(x-1)^2}$

Skills Warm Up —*continued*—

11. $f(x) = 3x^2 - x + 4$

$f'(x) = 6x - 1$

$f'(2) = 6(2) - 1$

$= 12 - 1$

$= 11$

12. $f(x) = -x^3 + x^2 + 8x$

$f'(x) = -3x^2 + 2x + 8$

$f'(2) = -3(2^2) + 2(2) + 8$

$= -3(4) + 4 + 8$

$= 0$

13. $f(x) = \dfrac{2}{7x} = \dfrac{2}{7}x^{-1}$

$f'(x) = -\dfrac{2}{7}x^{-2} = -\dfrac{2}{7x^2}$

$f'(2) = -\dfrac{2}{7(2)^2}$

$= -\dfrac{1}{14}$

14. $f(x) = x^2 - \dfrac{1}{x^2}$

$f'(x) = 2x + \dfrac{2}{x^3}$

$f'(2) = 2(2) + \dfrac{2}{2^3}$

$= 4 + \dfrac{2}{8}$

$= 4 + \dfrac{1}{4}$

$= \dfrac{17}{4}$

1. $f(x) = (2x - 3)(1 - 5x)$

$f'(x) = (2x - 3)(-5) + (1 - 5x)(2)$

$= -10x + 15 + 2 - 10x$

$= -20x + 17$

2. $g(x) = (4x - 7)(3x + 1)$

$g'(x) = (4x - 7)(3) + (3x + 1)(4)$

$= 12x - 21 + 12x + 4$

$= 24x - 17$

3. $f(x) = (6x - x^2)(4 + 3x)$

$f'(x) = (6x - x^2)(3) + (4 + 3x)(6 - 2x)$

$= 18x - 3x^2 + 24 - 8x + 18x - 6x^2$

$= -9x^2 + 28x + 24$

4. $f(x) = (5x - x^3)(2x + 9)$

$f'(x) = (5x - x^3)(2) + (2x + 9)(5 - 3x^2)$

$= 10x - 2x^3 + 10 - 6x^3 + 45 - 27x^2$

$= -8x^3 - 27x^2 + 20x + 45$

5. $f(x) = x(x^2 + 3)$

$f'(x) = x(2x) + (x^2 + 3)(1)$

$= 2x^2 + x^2 + 3$

$= 3x^2 + 3$

6. $f(x) = x^2(3x^3 - 1)$

$f'(x) = x^2(9x^2) + (3x^3 - 1)(2x)$

$= 9x^4 + 6x^4 - 2x$

$= 15x^4 - 2x$

7. $h(x) = \left(\dfrac{2}{x} - 3\right)(x^2 + 7) = (2x^{-1} - 3)(x^2 + 7)$

$h'(x) = (2x^{-1} - 3)(2x) + (x^2 + 7)(-2x^{-2})$

$= 4 - 6x - 2 - 14x^{-2}$

$= -6x + 2 - \dfrac{14}{x^2}$

8. $f(x) = (3 - x)\left(\dfrac{4}{x^2} - 5\right) = (3 - x)(4x^{-2} - 5)$

$f'(x) = (3 - x)(-8x^{-3}) + (4x^{-2} - 5)(-1)$

$= -24x^{-3} + 8x^{-2} - 4x^{-2} + 5$

$= -\dfrac{24}{x^3} + \dfrac{4}{x^2} + 5$

9. $g(x) = (x^2 - 4x + 3)(x - 2)$

$$g'(x) = (x^2 - 4x + 3)(1) + (x - 2)(2x - 4)$$
$$= x^2 - 4x + 3 + 2x^2 - 4x - 4x + 8$$
$$= 3x^2 - 12x + 11$$

10. $g(x) = (x^2 - 2x + 1)(x^3 - 1)$

$$g'(x) = (x^2 - 2x + 1)(3x^2) + (x^3 - 1)(2x - 2)$$
$$= 3x^4 - 6x^3 + 3x^2 + 2x^4 - 2x^3 - 2x + 2$$
$$= 5x^4 - 8x^3 + 3x^2 - 2x + 2$$

11. $h(x) = \dfrac{x}{x - 5}$

$$h'(x) = \frac{(x - 5)(1) - x(1)}{(x - 5)^2} = \frac{x - 5 - x}{(x - 5)^2} = -\frac{5}{(x - 5)^2}$$

12. $h(x) = \dfrac{x^2}{x + 3}$

$$h'(x) = \frac{(x + 3)(2x) - x^2(1)}{(x + 3)^2}$$
$$= \frac{2x^2 + 6x - x^2}{(x + 3)^2}$$
$$= \frac{x^2 + 6x}{(x + 3)^2}$$

13. $f(t) = \dfrac{2t^2 - 3}{3t + 1}$

$$f'(t) = \frac{(3t + 1)(4t) - (2t^2 - 3)(3)}{(3t + 1)^2}$$
$$= \frac{12t^2 + 4t - 6t^2 + 9}{(3t + 1)^2}$$
$$= \frac{6t^2 + 4t + 9}{(3t + 1)^2}$$

14. $f(x) = \dfrac{7x + 3}{4x - 9}$

$$f'(x) = \frac{(4x - 9)(7) - (7x + 3)(4)}{(4x - 9)^2}$$
$$= \frac{28x - 63 - 28x - 12}{(x - 1)^2}$$
$$= -\frac{75}{(4x - 9)^2}$$

15. $f(t) = \dfrac{t + 6}{t^2 - 8}$

$$f'(t) = \frac{(t^2 - 8)(1) - (t + 6)(2t)}{(t^2 - 8)^2}$$
$$= \frac{t^2 - 8t - 2t^2 - 12t}{(t^2 - 8)^2}$$
$$= \frac{-t^2 - 12t - 8}{(t^2 - 8)^2}$$

16. $g(x) = \dfrac{4x - 5}{x^2 - 1}$

$$g'(x) = \frac{(x^2 - 1)(4) - (4x - 5)(2x)}{(x^2 - 1)^2}$$
$$= \frac{4x^2 - 4 - 8x^2 + 10x}{(x^2 - 1)^2}$$
$$= \frac{-4x^2 + 10x - 4}{(x^2 - 1)^2}$$

17. $f(x) = \dfrac{x^2 + 6x + 5}{2x - 1}$

$$f'(x) = \frac{(2x - 1)(2x + 6) - (x^2 + 6x + 5)(2)}{(2x - 1)^2}$$
$$= \frac{4x^2 + 12x - 2x - 6 - 2x^2 - 12x - 10}{(2x - 1)^2}$$
$$= \frac{2x^2 - 2x - 16}{(2x - 1)^2}$$

18. $f(x) = \dfrac{4x^2 - x + 2}{3 - 4x}$

$$f'(x) = \frac{(3 - 4x)(8x - 1) - (4x^2 - x + 2)(-4)}{(3 - 4x)^2}$$
$$= \frac{24x - 3 - 32x^2 + 4x + 16x^2 - 4x + 8}{(3 - 4x)^2}$$
$$= \frac{-16x^2 + 24x + 5}{(3 - 4x)^2}$$

19. $f(x) = \dfrac{6 + 2x^{-1}}{3x - 1}$

$$f'(x) = \frac{(3x - 1)(-2x^{-2}) - (6 + 2x^{-1})(3)}{(3x - 1)^2}$$
$$= \frac{-6x^{-1} + 2x^{-2} - 18 - 6x^{-1}}{(3x - 1)^2}$$
$$= \frac{2x^{-2} - 12x^{-1} - 18}{(3x - 1)^2}$$
$$= \frac{\frac{2}{x^2} - \frac{12}{x} - 18}{(3x - 1)^2} = \frac{2 - 12x - 18x^2}{x^2(3x - 1)^2}$$

20. $f(x) = \dfrac{5 - x^{-2}}{x + 2}$

$$f'(x) = \frac{(x + 2)(2x^{-3}) - (5 - x^{-2})(1)}{x + 2}$$
$$= \frac{2x^{-2} + 4x^{-3} - 5 + x^{-2}}{(x + 2)^2}$$
$$= \frac{4x^{-3} + 3x^{-2} - 5}{(x + 2)^2}$$
$$= \frac{\frac{4}{x^3} + \frac{3}{x^2} - 5}{(x + 2)^2}$$
$$= \frac{4 + 2x - 5x^3}{x^3(x + 2)^2}$$

	Function	*Rewrite*	*Differentiate*	*Simplify*
21.	$f(x) = \dfrac{x^3 + 6x}{3}$	$f(x) = \dfrac{1}{3}x^3 + 2x$	$f'(x) = x^2 + 2$	$f'(x) = x^2 + 2$
22.	$f(x) = \dfrac{x^3 + 2x^2}{10}$	$f(x) = \dfrac{1}{10}x^3 + \dfrac{1}{5}x^2$	$f'(x) = \dfrac{3}{10}x^2 + \dfrac{2}{5}x$	$f'(x) = \dfrac{3}{10}x^2 + \dfrac{2}{5}x$
23.	$y = \dfrac{7x^2}{5}$	$y = \dfrac{7}{5}x^2$	$y' = \dfrac{7}{5} \cdot 2x$	$y' = \dfrac{14}{5}x$
24.	$y = \dfrac{2x^4}{9}$	$y = \dfrac{2}{9}x^4$	$y' = \dfrac{2}{9} \cdot 4x^3$	$y' = \dfrac{8}{9}x^3$
25.	$y = \dfrac{7}{3x^3}$	$y = \dfrac{7}{3}x^{-3}$	$y' = -7x^{-4}$	$y' = -\dfrac{7}{x^4}$
26.	$y = \dfrac{4}{5x^2}$	$y = \dfrac{4}{5}x^{-2}$	$y' = -\dfrac{8}{5}x^{-3}$	$y' = -\dfrac{8}{5x^3}$
27.	$y = \dfrac{4x^2 - 3x}{8\sqrt{x}}$	$y = \dfrac{1}{2}x^{3/2} - \dfrac{3}{8}x^{1/2},\ x \neq 0$	$y' = \dfrac{3}{4}x^{1/2} - \dfrac{3}{16}x^{-1/2}$	$y' = \dfrac{3}{4}\sqrt{x} - \dfrac{3}{16\sqrt{x}}$
28.	$y = \dfrac{5(3x^2 + 2x)}{6\sqrt[3]{x}}$	$y = \dfrac{5}{2}x^{5/3} + \dfrac{5}{3}x^{5/3},\ x \neq 0$	$y' = \dfrac{25}{6}x^{2/3} + \dfrac{10}{9}x^{-1/3},\ x \neq 0$	$y' = \dfrac{25}{6}\sqrt[3]{x^2} + \dfrac{10}{9\sqrt[3]{x}}$
29.	$y = \dfrac{x^2 - 4x + 3}{2(x - 1)}$	$y = \dfrac{1}{2}(x - 3),\ x \neq 1$	$y' = \dfrac{1}{2}(1),\ x \neq 1$	$y' = \dfrac{1}{2},\ x \neq 1$
30.	$y = \dfrac{x^2 - 4}{4(x + 2)}$	$y = \dfrac{1}{4}(x - 2),\ x \neq -2$	$y' = \dfrac{1}{4}(1),\ x \neq -2$	$y' = \dfrac{1}{4},\ x \neq -2$

31. $f'(x) = (x^3 - 3x)(4x + 3) + (3x^2 - 3)(2x^2 + 3x + 5)$

$$= 4x^4 + 3x^3 - 12x^2 - 9x + 6x^4 + 9x^3 + 9x^2 - 9x - 15$$
$$= 10x^4 + 12x^3 - 3x^2 - 18x - 15$$

Product Rule and Simple Power Rule

32. $h'(t) = (t^5 - 1)(8t - 7) + (5t^4)(4t^2 - 7t - 3)$

$= 8t^6 - 7t^5 - 8t + 7 + 20t^6 - 35t^5 - 15t^4$

$= 28t^6 - 42t^5 - 15t^4 - 8t + 7$

Product Rule and Simple Power Rule

33. $h(t) = \frac{1}{3}(6t - 4)$

$h'(t) = \frac{1}{3}(6) = 2$

Constant Multiple and Simple Power Rules

34. $f(x) = \frac{1}{2}(3x - 8)$

$f'(x) = \frac{1}{2}(3) = \frac{3}{2}$

Constant Multiple and Simple Power Rules

35. $f'(x) = \dfrac{(x^2 - 1)(3x^2 + 3) - (x^3 + 3x + 2)(2x)}{(x^2 - 1)^2}$

$= \dfrac{3x^4 - 3 - 2x^4 - 6x^2 - 4x}{(x^2 - 1)^2}$

$= \dfrac{x^4 - 6x^2 - 4x - 3}{(x^2 - 1)^2}$

Quotient Rule and Simple Power Rule

36. $f(x) = \dfrac{2x^3 - 4x^2 - 9}{x^3 - 5}$

$f'(x) = \dfrac{(x^3 - 5)(3x^2 - 8x) - (2x^3 - 4x^2 - 9)(3x^2)}{(x^3 - 5)^2}$

$= \dfrac{3x^5 - 8x^4 - 15x^2 + 40x - 6x^5 + 12x^4 - 27x^2}{(x^3 - 5)^2}$

$= \dfrac{-3x^5 + 4x^4 - 42x^2 + 40x}{(x^3 - 5)^2}$

Quotient Rule and Simple Power Rule

37. $f(x) = \dfrac{x^2 - x - 20}{x + 4} = \dfrac{(x - 5)(x + 4)}{(x + 4)} = x - 5, x \neq -4$

$f'(x) = 1$

Simple Power Rule

38. $h(t) = \dfrac{3t^2 + 22t + 7}{t + 7} = \dfrac{(3t + 1)(1 + 7)}{t + 7} = 3t + 1, t \neq -7$

$h'(t) = 3, t \neq -7$

Simple Power Rule

39. $g(t) = (2t^3 - 1)^2 = (2t^3 - 1)(2t^3 - 1)$

$g'(t) = (2t^3 - 1)(6t^2) + (2t^3 - 1)(6t^2)$

$= 12t^2(2t^3 - 1)$

Product Rule and Simple Power Rule

40. $f(x) = (4x^3 - 2x - 3)^2 = (4x^3 - 2x - 3)(4x^3 - 2x - 3)$

$f'(x) = (4x^3 - 2x - 3)(12x^2 - 2) + (4x^3 - 2x - 3)(12x^2 - 2)$

$= 48x^5 - 24x^3 - 36x^2 - 8x^3 + 4x + 6 + 48x^5 - 24x^3 - 36x^2 - 8x^3 + 4x + 6$

$= 96x^5 - 48x^3 - 72x^2 - 16x^3 + 8x + 12$

Product Rule and Simple Power Rule

41. $g(s) = \dfrac{s^2 - 2s + 5}{\sqrt{s}} = \dfrac{s^2 - 2s + 5}{s^{1/2}}$

$g'(s) = \dfrac{s^{1/2}(2s - 2) - (s^2 - 2s + 5)\left(\frac{1}{2}s^{-1/2}\right)}{s}$

$= \dfrac{2s^{3/2} - 2s^{1/2} - \frac{1}{2}s^{3/2} + s^{1/2} - \frac{5}{2}s^{-1/2}}{s}$

$= \dfrac{3}{2}s^{1/2} - s^{-1/2} - \dfrac{5}{2}s^{-3/2} = \dfrac{3s^2 - 2s - 5}{2s^{3/2}}$

Quotient Rule and Simple Power Rule

42. $f(x) = \dfrac{x^3 - 5x^2 - 6x}{\sqrt{x}} = \dfrac{x^3 - 5x^2 - 6x}{x^{1/2}}$

$= x^{5/2} - 5x^{3/2} - 6x^{1/2}$

$f'(x) = \dfrac{5}{2}x^{3/2} - \dfrac{15}{2}x^{1/2} - 3x^{-1/2}$

$= \dfrac{5}{2}x^{3/2} - \dfrac{15}{2}x^{1/2} - \dfrac{3}{x^{1/2}}$

$= \dfrac{5x^2 - 15x - 6}{2x^{1/2}}$

Constant Multiple and Simple Power Rules

43. $f(x) = \dfrac{(x - 2)(3x + 1)}{4x + 2} = \dfrac{3x^2 - 5x - 2}{4x + 2}$

$f'(x) = \dfrac{(4x + 2)(6x - 5) - (3x^2 - 5x - 2)(4)}{(4x + 2)^2}$

$= \dfrac{24x^2 - 8x - 10 - 12x^2 + 20x + 8}{(4x + 2)^2}$

$= \dfrac{12x^2 + 12x - 2}{4(2x + 1)^2}$

$= \dfrac{2(6x^2 + 6x - 1)}{2(2x + 1)^2}$

$= \dfrac{6x^2 + 6x - 1}{2(2x + 1)^2}$

Quotient Rule and Simple Power Rule

44. $f(x) = \dfrac{(x + 1)(2x - 7)}{2x + 1} = \dfrac{2x^2 - 5x - 7}{2x + 1}$

$f'(x) = \dfrac{(2x + 1)(4x - 5) - (2x^2 - 5x - 7)(2)}{(2x + 1)^2}$

$= \dfrac{8x^2 - 6x - 5 - 4x^2 + 10x + 14}{(2x + 1)^2}$

$= \dfrac{4x^2 + 4x + 9}{(2x + 1)^2}$

Quotient Rule and Simple Power Rule

45. $f(x) = (x + 4)(2x + 9)(x - 3)$

$= (2x^2 + 17x + 36)(x - 3)$

$f'(x) = (2x^2 + 17x + 36)(1) + (x - 3)(4x + 17)$

$= (2x^2 + 17x + 36) + (4x^2 + 5x - 51)$

$= 6x^2 + 22x - 15$

Product Rule and Simple Power Rule

46. $f(x) = (3x^3 + 4x)(x - 5)(x + 1)$

$= (3x^3 + 4x)(x^2 - 4x - 5)$

$f'(x) = (3x^3 + 4x)(2x - 4) + (x^2 - 4x - 5)(9x^2 + 4)$

$= (6x^4 - 12x^3 + 8x^2 - 16x)$

$+ (9x^4 - 36x^3 - 41x^2 - 16x - 20)$

$= 15x^4 - 48x^3 - 33x^2 - 32x - 20$

Product Rule and Simple Power Rule

47. $f(x) = (5x + 2)(x^2 + x)$

$f'(x) = (5x + 2)(2x + 1) + (x^2 + x)(5)$

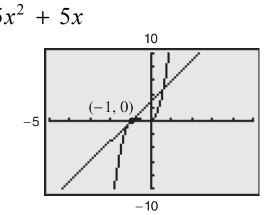
$= 10x^2 + 9x + 2 + 5x^2 + 5x$

$= 15x^2 + 14x + 2$

$m = f'(-1) = 3$

$y - 0 = 3(x - (-1))$

$y = 3x + 3$



48. $f(x) = (x^2 - 1)(x^3 - 3x)$

$f'(x) = (x^2 - 1)(3x^2 - 3) + (x^2 - 3x)(2x)$

$= 3x^4 - 6x^2 + 3 + 2x^4 - 6x^2$

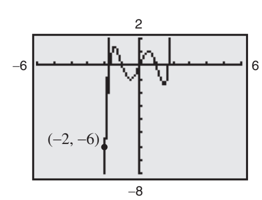
$= 5x^4 - 12x^2 + 3$

$m = f'(-2) = 5(-2)^4 - 12(-2)^2 + 3 = 35$

$y - (-6) = 35(x - (-2))$

$y + 6 = 35x + 70$

$y = 35x + 64$



49. $f(x) = x^3(x^2 - 4)$

$f'(x) = x^2(2x) + (x^2 - 4)(3x^2)$

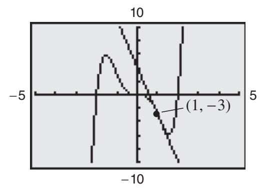
$= 2x^4 + 3x^4 - 12x^2$

$= 5x^4 - 12x^2$

$m = f'(1) = -7$

$y - (-3) = -7(x - 1)$

$y = -7x + 4$



50. $f(x) = \sqrt{x}(x - 3) = x^{1/2}(x - 3)$

$f'(x) = x^{1/2}(1) + (x - 3)\left(\frac{1}{2}x^{-1/2}\right)$

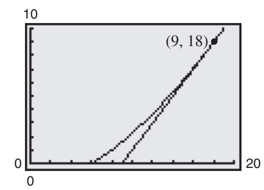
$= x^{1/2} + \frac{1}{2}x^{1/2} - \frac{3}{2}x^{-1/2}$

$= \frac{3}{2}x^{1/2} - \frac{3}{2}x^{-1/2}$

$m = f'(9) = \frac{3}{2}(9)^{1/2} - \frac{3}{2}(9)^{-1/2} = \frac{9}{2} - \frac{1}{2} = 4$

$y - 18 = 4(x - 9)$

$y = 4x - 18$



51. $f(x) = \dfrac{3x - 2}{x + 1}$

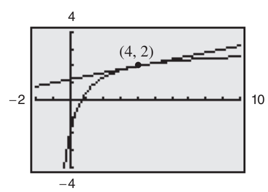
$f'(x) = \dfrac{(x + 1)(3) - (3x - 2)(1)}{(x + 1)^2} = \dfrac{3x + 3 - 3x + 2}{(x + 1)^2} = \dfrac{5}{(x + 1)^2}$

$f'(4) = \dfrac{1}{5}$

$y - 2 = \dfrac{1}{5}(x - 4)$

$y - 2 = \dfrac{1}{5}x - \dfrac{4}{5}$

$y = \dfrac{1}{5}x + \dfrac{6}{5}$

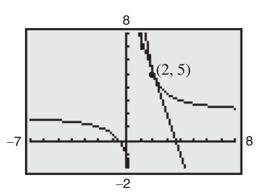


52. $f'(x) = \dfrac{(x - 1)2 - (2x + 1)}{(x - 1)^2} = \dfrac{-3}{(x - 1)^2}$

$f'(2) = -3$

$y - 5 = -3(x - 2)$

$y = -3x + 11$



53. $f(x) = \dfrac{(3x - 2)(6x + 5)}{2x - 3} = \dfrac{18x^2 + 3x - 10}{2x - 3}$

$f'(x) = \dfrac{(2x - 3)(36x + 3) - (18x^2 + 3x - 10)(2)}{(2x - 3)^2}$

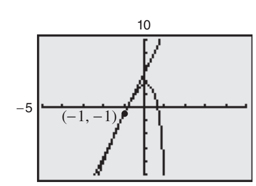
$= \dfrac{72x^2 - 102x - 9 - 36x^2 - 6x - 20}{(2x - 3)^2}$

$= \dfrac{36x^2 - 108x + 11}{(2x - 3)^2}$

$m = f'(-1) = \dfrac{36(-1)^2 - 108(-1) + 11}{(2(-1) - 3)^2} = \dfrac{31}{5}$

$y - (-1) = \dfrac{31}{5}(x - (-1))$

$y = \dfrac{31}{5}x + \dfrac{26}{5}$



54. $f(x) = \dfrac{(x+2)(x^2+x)}{x-4} = \dfrac{x^3+3x^2+2x}{x-4}$

$$f'(x) = \frac{(x-4)(3x^2+6x+2)-(x^3+3x^2+2x)(1)}{(x-4)^2}$$

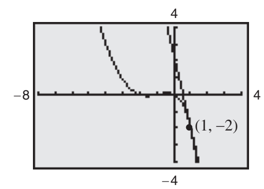
$$= \frac{3x^3-12x^2+6x^2-24x+2x-8-x^3-3x^2-2x}{(x-4)^2}$$

$$= \frac{2x^3-9x^2-24x-8}{(x-4)^2}$$

$$m = f'(1) = \frac{2(1)^3-9(1)^2-24(1)-8}{(1-4)^2} = -\frac{13}{3}$$

$$y-(-2) = -\frac{13}{3}(x-1)$$

$$y = \frac{13}{3}x + \frac{7}{3}$$



55. $f'(x) = \dfrac{(x-1)(2x)-x^2(1)}{(x-1)^2} = \dfrac{x^2-2x}{(x-1)^2}$

$f'(x) = 0$ when $x^2-2x = x(x-2) = 0$, which implies that $x = 0$ or $x = 2$. Thus, the horizontal tangent lines occur at $(0, 0)$ and $(2, 4)$.

56. $f'(x) = \dfrac{(x^2+1)(2x)-(x^2)(2x)}{(x^2+1)^2} = \dfrac{2x}{(x^2+1)^2}$

$f'(x) = 0$ when $2x = 0$, which implies that $x = 0$. Thus, the horizontal tangent line occurs at $(0, 0)$.

57. $f'(x) = \dfrac{(x^3+1)(4x^3)-x^4(3x^2)}{(x^3+1)^2} = \dfrac{x^6+4x^3}{(x^3+1)^2}$

$f'(x) = 0$ when $x^6+4x^3 = x^3(x^3+4) = 0$, which implies that $x = 0$ or $x = \sqrt[3]{-4}$. Thus, the horizontal tangent lines occur at $(0, 0)$ and $\left(\sqrt[3]{-4}, -2.117\right)$.

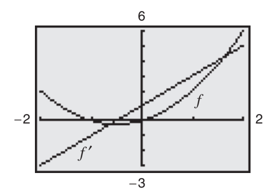
58. $f'(x) = \dfrac{(x^2+1)(4x^3)-(x^4+3)(2x)}{(x^2+1)^2}$

$$= \frac{2x(x^2+3)(x^2-1)}{(x^2+1)^2}$$

$f'(x) = 0$ when $2x(x^2+3)(x^2-1) = 0$, which implies that $x = 0$ or $x = \pm1$. Thus, the horizontal tangent lines occur at (0, 3), (1, 2), and (−1, 2).

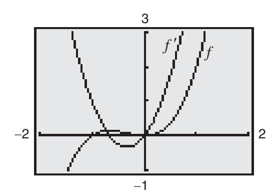
59. $f(x) = x(x+1) = x^2+x$

$f'(x) = 2x+1$



60. $f(x) = x^2(x+1) = x^3+x^2$

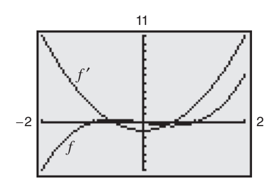
$f'(x) = 3x^2+2x = x(3x+2)$



61. $f(x) = x(x+1)(x-1)$

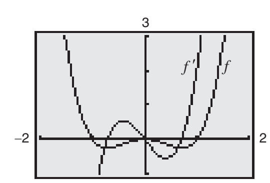
$= x^3 - x$

$f'(x) = 3x^2 - 1$



62. $f(x) = x^2(x+1)(x-1) = x^4 - x^2$

$f'(x) = 4x^3 - 2x = 2x(2x^2-1)$



63. $x = 275\left(1 - \frac{3p}{5p + 1}\right)$

$$\frac{dx}{dp} = -275\left[\frac{(5p + 1)3 - (3p)(5)}{(5p + 1)^2}\right] = -275\left[\frac{3}{(5p + 1)^2}\right]$$

When $p = 4$, $\frac{dx}{dp} = -275\left[\frac{3}{(21)^2}\right] \approx -1.87$ units per dollar.

64. $\frac{dx}{dp} = 0 - 1 - \frac{(p + 1)(2) - (2p)(1)}{(p + 1)^2}$

$$= -1 - \frac{2}{(p + 1)^2}$$

$$= \frac{-(p + 1)^2 - 2}{(p + 1)^2}$$

$$= \frac{-p^2 - 2p - 3}{(p + 1)^2}$$

When $p = 3$, $\frac{dx}{dp} = \frac{-9 - 6 - 3}{16} \approx -1.13$ units per dollar.

65. $P' = 500\left[\frac{(50 + t^2)(4) - (4t)(2t)}{(50 + t^2)^2}\right] = 500\left[\frac{200 - 4t^2}{(50 + t^2)^2}\right]$

When $t = 2$, $P' = 500\left[\frac{184}{(54)^2}\right] \approx 31.55$ bacteria/hour.

66. $\frac{dP}{dt} = \frac{50(t + 2)(1) - (t + 1750)(50)}{\left[50(t + 2)\right]^2}$

$$= \frac{50\left[(t + 2) - (t + 1750)\right]}{2500(t + 2)^2}$$

$$= \frac{-1748}{50(t + 2)^2}$$

$$= \frac{-874}{25(t + 2)^2}$$

(a) When $t = 1$, $\frac{dP}{dt} = \frac{-874}{225} \approx -3.88$ percent/day.

(b) When $t = 10$, $\frac{dP}{dt} = \frac{-874}{3600}$

$$= -\frac{437}{1800}$$

$$\approx -0.24 \text{ percent/day.}$$

67. $P = \frac{t^2 - t + 1}{t^2 + 1}$

$$P' = \frac{(t^2 + 1)(2t - 1) - (t^2 - t + 1)(2t)}{(t^2 + 1)^2} = \frac{t^2 - 1}{(t^2 + 1)^2}$$

(a) $P'(0.5) = -0.480$/week

(b) $P'(2) = 0.120$/week

(c) $P'(8) = 0.015$/week

Each rate in parts (a), (b), and (c) is the rate at which the level of oxygen in the pond is changing at that particular time.

68. $T = 10\left(\frac{4t^2 + 16t + 75}{t^2 + 4t + 10}\right)$

Initial temperature: $T(0) = 75°\text{F}$

$$T'(t) = 10\left(\frac{(t^2 + 4t + 10)(8t + 16) - (4t^2 + 16t + 75)(2t + 4)}{(t^2 + 4t + 10)^2}\right) = \frac{-700(t + 2)}{(t^2 + 4t + 10)^2}$$

(a) $T'(1) \approx -9.33°\text{F/hr}$

(b) $T'(3) \approx -3.64°\text{F/hr}$

(c) $T'(5) \approx -1.62°\text{F/hr}$

(d) $T'(10) \approx -0.37°\text{F/hr}$

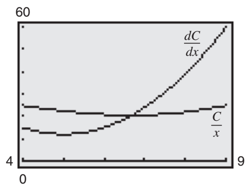
Each rate in parts (a), (b), (c), and (d) is the rate at which the temperature of the food in the refrigerator is changing at that particular time.

69. $C = x^3 - 15x^2 + 87x - 73,\ 4 \le x \le 9$

Marginal cost: $\dfrac{dC}{dx} = 3x^2 - 30x + 87$

Average cost: $\dfrac{C}{x} = x^2 - 15x + 87 - \dfrac{73}{x}$

(a)



(b) Point of intersection:

$$3x^2 - 30x + 87 = x^2 - 15x + 87 - \frac{73}{x}$$

$$2x^2 - 15x + \frac{73}{x} = 0$$

$$2x^3 - 15x^2 + 73 = 0$$

$$x \approx 6.683$$

When $x = 6.683$, $\dfrac{C}{x} = \dfrac{dC}{dx} \approx 20.50$.

Thus, the point of intersection is (6.683, 20.50). At this point average cost is at a minimum.

70. (a) As time passes, the percent of people aware of the product approaches approximately 95%.

(b) As time passes, the rate of change of the percent of people aware of the product approaches zero.

71. $C = 100\left(\dfrac{200}{x^2} + \dfrac{x}{x+30}\right),\ x \ge 1$

$$C' = 100\left[-2\left(200x^{-3}\right) + \frac{(x+30) - x}{(x+30)^2}\right]$$

$$= 100\left[-\frac{400}{x^3} + \frac{30}{(x+30)^2}\right]$$

(a) $C'(10) = 100\left(-\dfrac{400}{10^3} + \dfrac{30}{40^2}\right) = -38.125$

(b) $C'(15) \approx -10.37$

(c) $C'(20) \approx -3.8$

Increasing the order size reduces the cost per item. An order size of 2000 should be chosen since the cost per item is the smallest at $x = 20$.

72. (a) $P = ax^2 + bx + c$

When $x = 10$, $P = 50$: $50 = 100a + 10b + c$.

When $x = 12$, $P = 60$: $60 = 144a + 12b + c$.

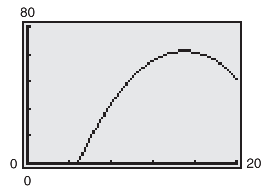
When $x = 14$, $P = 65$: $65 = 196a + 14b + c$.

Solving this system, we have

$a = -\frac{5}{8}$, $b = \frac{75}{4}$, and $c = -75$.

Thus, $P = -\frac{5}{8}x^2 + \frac{75}{4}x - 75$.

(b)



(c) Marginal profit: $P' = -\frac{5}{4}x + \frac{75}{4} = 0 \Rightarrow x = 15$

This is the maximum point on the graph of P, so selling 15 units will maximize the profit.

73. $f(x) = 2g(x) + h(x)$

$f'(x) = 2g'(x) + h'(x)$

$f'(2) = 2(-2) + 4 = 0$

74. $f(x) = 3 - g(x)$

$f'(x) = -g'(x)$

$f'(2) = -(-2) = 2$

75. $f(x) = g(x)h(x)$

$f'(x) = g(x)h'(x) + h(x)g'(x)$

$f'(2) = g(2)h'(2) + h(2)g'(2)$

$= (3)(4) + (-1)(-2)$

$= 14$

76. $f(x) = \dfrac{g(x)}{h(x)}$

$$f'(x) = \frac{h(x)g'(x) - g(x)h'(x)}{\left[h(x)\right]^2}$$

$$f'(2) = \frac{(-1)(-2) - (3)(4)}{(-1)^2} = -10$$

77. Answers will vary.

Chapter 2 Quiz Yourself

1. $f(x) = 5x + 3$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{[5(x + \Delta x) + 3] - (5x + 3)}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{5x + 5\Delta x + 3 - 5x - 3}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{5\Delta x}{\Delta x}$$
$$= \lim_{\Delta x \to 0} 5 = 5$$

At $(-2, -7)$: $m = 5$

2. $f(x) = \sqrt{x + 3}$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{\sqrt{x + \Delta x + 3} - \sqrt{x + 3}}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{x + \Delta x + 3 - (x + 3)}{\Delta x\left(\sqrt{x + \Delta x + 3} + \sqrt{x + 3}\right)}$$
$$= \lim_{\Delta x \to 0} \frac{1}{\sqrt{x + \Delta x + 3} + \sqrt{x + 3}}$$
$$= \frac{1}{2\sqrt{x + 3}}$$

At $(1, 2)$: $m = \dfrac{1}{2\sqrt{1 + 3}} = \dfrac{1}{4}$

3. $f(x) = 3x - x^2$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{\left[3(x + \Delta x) - (x + \Delta x)^2\right] - \left(3x - x^2\right)}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{3x + 3\Delta x - \left(x^2 + 2x(\Delta x) + (\Delta x)^2\right) - \left(3x - x^2\right)}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{3x + 3\Delta x - x^2 - 2x(\Delta x) - (\Delta x)^2 - 3x + x^2}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{3x - 2x(\Delta x) - (\Delta x)^2}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{\Delta x(3 - 2x - \Delta x)}{\Delta x}$$
$$= \lim_{\Delta x \to 0} (3 - 2x - \Delta) = 3 - 2x$$

At $(4, -4)$: $m = 3 - 2(4) = 3 - 8 = -5$

4. $f(x) = 12$

$f'(x) = 0$

5. $f(x) = 19x + 9$

$f'(x) = 19$

6. $f(x) = x^4 - 3x^3 - 5x^2 + 8$

$f'(x) = 4x^3 - 9x^2 - 10x$

7. $f(x) = 12x^{1/4}$

$f'(x) = 3x^{-3/4} = \dfrac{3}{x^{3/4}}$

8. $f(x) = 4x^{-2}$

$f'(x) = -8x^{-3} = -\dfrac{8}{x^3}$

9. $f(x) = 10x^{-1/5} + x^{-3}$

$f'(x) = -2x^{-6/5} - 3x^{-4} = -\dfrac{2}{x^{-6/5}} - \dfrac{3}{x^4}$

10. $f(x) = \dfrac{2x + 3}{3x + 2}$

$f'(x) = \dfrac{(3x + 2)(2) - (2x + 3)(3)}{(3x + 2)^2}$

$= \dfrac{6x + 4 - 6x - 9}{(3x + 2)^2}$

$= -\dfrac{5}{(3x + 2)^2}$

11. $f(x) = (x^2 + 1)(-2x + 4)$

$f'(x) = (x^2 + 1)(-2) + (-2x + 4)(2x)$

$= -6x^2 + 8x - 2$

12. $f(x) = (x^2 + 3x + 4)(5x - 2)$

$f'(x) = (x^2 + 3x + 4)(5) + (5x - 2)(2x + 3)$

$= 5x^2 + 15x + 20 + 10x^2 + 11x - 6$

$= 15x^2 + 26x + 14$

13. $f(x) = \dfrac{4x}{x^2 + 3}$

$f'(x) = \dfrac{(x^2 + 3)(4) - 4x(2x)}{(x^2 + 3)^2}$

$= \dfrac{4x^2 + 12 - 8x^2}{(x^2 + 3)^2}$

$= \dfrac{-4x^2 + 12}{(x^2 + 3)^2}$

$= \dfrac{-4(x^2 - 3)}{(x^2 + 3)^2}$

14. $f(x) = x^2 - 3x + 1; [0, 3]$

Average rate of change:

$\dfrac{\Delta y}{\Delta x} = \dfrac{f(3) - f(0)}{3 - 0} = \dfrac{1 - 1}{3} = 0$

$f'(x) = 2x - 3$

Instantaneous rates of change: $f'(0) = -3,\ f'(3) = 3$

15. $f(x) = 2x^3 + x^3 - x + 4; [-1, 1]$

Average rate of change:

$\dfrac{\Delta y}{\Delta x} = \dfrac{f(1) - f(-1)}{1 - (-1)} = \dfrac{6 - 4}{2} = 1$

$f'(x) = 6x^2 + 2x - 1$

Instantaneous rates of change: $f'(-1) = 3,\ f'(1) = 7$

16. $f(x) = \dfrac{1}{3x}; [-5, -2]$

Average rate of change:

$\dfrac{\Delta y}{\Delta x} = \dfrac{f(-2) - f(5)}{-2 - (-5)} = \dfrac{-\frac{1}{6} - \left(-\frac{1}{15}\right)}{3} = \dfrac{-\frac{3}{30}}{3} = -\dfrac{1}{30}$

$f'(x) = -\dfrac{1}{3x^2}$

Instantaneous rates of change:

$f'(-2) = -\dfrac{1}{12},\ f'(-5) = -\dfrac{1}{75}$

17. $f(x) = \sqrt[3]{x}; [8, 27]$

Average rate of change:

$\dfrac{\Delta y}{\Delta x} = \dfrac{f(27) - f(8)}{27 - 8} = \dfrac{3 - 2}{19} = \dfrac{1}{19}$

$f(x) = \sqrt[3]{x} = x^{1/3}$

$f'(x) = \dfrac{1}{3}x^{-2/3} = \dfrac{1}{3x^{2/3}}$

Instantaneous rates of change: $f'(8) = \dfrac{1}{12}$,

$f'(27) = \dfrac{1}{27}$

18. $P = -0.0125x^2 + 16x - 600$

(a) $\dfrac{dP}{dx} = -0.025x + 16$

When $x = 175,\ \dfrac{dP}{dx} = \$11.625$.

(b) $P(176) - P(175) = 1828.8 - 1817.1875$

$= \$11.6125$

(c) The results are approximately equal.

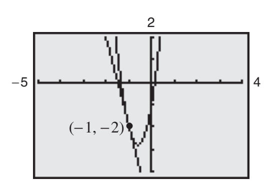
19. $f(x) = 5x^2 + 6x - 1$

$f'(x) = 10x + 6$

At $(-1, -2)$, $m = -4$.

$y + 2 = -4(x + 1)$

$y = -4x - 6$



20. $f(x) = \dfrac{8}{\sqrt{x^3}} = 8x^{-3/2}$

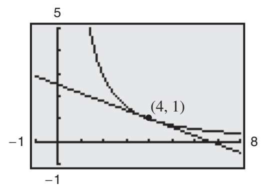
$f'(x) = -12x^{-5/2} = -\dfrac{12}{x^{5/2}} = -\dfrac{12}{x^2\sqrt{x}}$

$m = f'(4) = -\dfrac{12}{(4)^2\sqrt{4}} = -\dfrac{3}{8}$

$y - 1 = -\dfrac{3}{8}(x - 4)$

$y - 1 = -\dfrac{3}{8}x + \dfrac{3}{2}$

$y = -\dfrac{3}{8}x + \dfrac{5}{2}$



21. $f(x) = (x^2 + 1)(4x - 3)$

$f'(x) = (x^2 + 1)(4) + (4x - 3)(2x)$

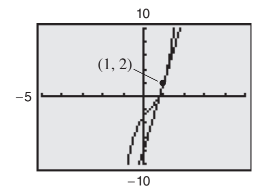
$= 4x^2 + 4 + 8x^2 - 6x$

$= 12x^2 - 6x + 4$

$m = f'(1) = 12(1)^2 - 6(1) + 4 = 10$

$y - 2 = 10(x - 1)$

$y = 10x - 8$



22. $f(x) = \dfrac{5x + 4}{2 - 3x}$

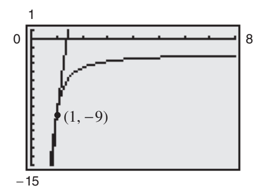
$f'(x) = \dfrac{(2 - 3x)(5) - (5x + 4)(-3)}{(2 - 3x)^2} = \dfrac{10 - 15x + 15x + 12}{(2 - 3x)^2} = \dfrac{22}{(2 - 3x)^2}$

$m = f'(1) = \dfrac{22}{(2 - 3(1))^2} = 22$

$y - (-9) = 22(x - 1)$

$y + 9 = 22x - 22$

$y = 22x - 31$



23. $S = -0.01722t^3 + 0.7333t^2 - 7.657t + 45.47,\ 7 \le t \le 13$

(a) $\dfrac{dS}{dt} = S'(t) = -0.051666t^2 + 1.4666t - 7.657$

(b) 2008: $S'(8) \approx \$0.77/\text{yr}$

2011: $S'(11) \approx \$2.22/\text{yr}$

2012: $S'(12) \approx \$2.50/\text{yr}$

Section 2.5 The Chain Rule

Skills Warm Up

1. $\sqrt[5]{(1 - 5x)^2} = (1 - 5x)^{2/5}$

2. $\sqrt[4]{(2x - 1)^3} = (2x - 1)^{3/4}$

3. $\dfrac{1}{\sqrt{4x^2 + 1}} = (4x^2 + 1)^{-1/2}$

4. $\dfrac{1}{\sqrt[6]{2x^3 + 9}} = (2x^3 + 9)^{-1/6}$

5. $\dfrac{\sqrt{x}}{\sqrt[3]{1 - 2x}} = x^{1/2}(1 - 2x)^{-1/3}$

6. $\dfrac{\sqrt{(3 - 7x)^3}}{2x} = \dfrac{(3 - 7x)^{3/2}}{2x} = (2x)^{-1}(3 - 7x)^{3/2}$

7. $3x^3 - 6x^2 + 5x - 10 = 3x^2(x - 2) + 5(x - 2)$

$= (3x^2 + 5)(x - 2)$

Skills Warm Up —*continued*—

8. $5x\sqrt{x} - x - 5\sqrt{x} + 1 = x(5\sqrt{x} - 1) - 1(5\sqrt{x} - 1)$
$= (x - 1)(5\sqrt{x} - 1)$

9. $4(x^2 + 1)^2 - x(x^2 + 1)^3 = (x^2 + 1)^2[4 - x(x^2 + 1)]$
$= (x^2 + 1)^2(4 - x^3 - x)$

10. $-x^5 + 6x^3 + 7x^2 - 42 = -x^3(x^2 - 6) + 7(x^2 - 6)$
$= (-x^3 + 7)(x^2 - 6)$
$= -(x^3 - 7)(x^2 - 6)$

	$y = f(g(x))$	$u = g(x)$	$y = f(u)$
1.	$y = (6x - 5)^4$	$u = 6x - 5$	$y = u^4$
2.	$y = (x^2 - 2x + 3)^3$	$u = x^2 - 2x + 3$	$y = u^3$
3.	$y = \sqrt{5x - 2}$	$u = 5x - 2$	$y = \sqrt{u}$
4.	$y = \sqrt[3]{9 - x^2}$	$u = 9 - x^2$	$y = \sqrt[3]{u}$
5.	$y = (3x + 1)^{-1}$	$u = 3x + 1$	$y = u^{-1}$
6.	$y = (x^2 - 3)^{-1/2}$	$u = x^2 - 3$	$y = u^{-1/2}$

7. $y = (4x + 7)^2$
$y' = 2(4x + 7)^1(4)$
$y' = 8(4x + 7)$
$= 32x + 56$

8. $y = (3x^2 - 2)^3$
$y' = 3(3x^2 - 2)^2(6x)$
$y' = 18x(3x^2 - 2)^2$

9. $y = \sqrt{3 - x^2} = (3 - x^2)^{1/2}$
$y' = \frac{1}{2}(3 - x^2)^{-1/2}(-2x)$
$y' = -x(3 - x^2)^{-1/2} = -\frac{x}{(3 - x^2)^{1/2}}$
$y' = -\frac{x}{\sqrt{3 - x^2}}$

10. $y = 4\sqrt[4]{6x + 5} = 4(6x + 5)^{1/4}$
$y' = 4\left(\frac{1}{4}\right)(6x + 5)^{-5/4}(6)$
$y' = 6(6x + 5)^{-5/4} = \frac{6}{(6x + 5)^{5/4}}$

11. $y = (5x^4 - 2x)^{2/3}$
$y' = \left(\frac{2}{3}\right)(5x^4 - 2x)^{-1/3}(20x^3 - 2)$
$y' = \left(\frac{2}{3}\right)(5x^4 - 2x)^{-1/3}(2)(10x^3 - 1)$
$y' = \left(\frac{4}{3}\right)(5x^4 - 2x)^{-1/3}(10x^3 - 1)$
$y' = \frac{4(10x^3 - 1)}{3(5x^4 - 2x)^{1/3}} = \frac{40x^3 - 4}{3\sqrt[3]{5x^4 - 2x}}$

12. $y = (x^3 + 2x^2)^{-1}$
$y' = (-1)(x^3 + 2x^2)^{-2}(3x^2 + 4x)$
$y' = -\frac{3x^2 + 4x}{(x^3 + 2x^2)^2}$

13. $f(x) = \frac{2}{1 - x^3} = 2(1 - x^3)^{-1}$; (c) General Power Rule

14. $f(x) = \frac{7}{(1 - x)^3} = 7(1 - x)^{-3}$; (c) General Power Rule

15. $f(x) = \sqrt[3]{8^2}$; (b) Constant Rule

16. $f(x) = \sqrt[3]{x^2} = x^{2/3}$; (a) Simple Power Rule

17. $f(x) = \frac{x^2 + 9}{x^3 + 4x^2 - 6}$; (d) Quotient Rule

18. $f(x) = \frac{x^{1/2}}{x^3 + 2x - 5}$; (d) Quotient Rule

19. $y' = 3(2x - 7)^2(2) = 6(2x - 7)^2$

20. $y = (3 - 5x)^4$
$y' = 4(3 - 5x)^3(-5) = -20(3 - 5x)^3$

21. $h'(x) = 2(6x - x^3)(6 - 3x^2) = 6x(6 - x^2)(2 - x^2)$

22. $f(x) = (2x^3 - 6x)^{4/3}$

$$f'(x) = \left(\frac{4}{3}\right)(2x^3 - 6x)^{1/3}(6x^2 - 6)$$

$$f'(x) = \left(\frac{4}{3}\right)(2x^3 - 6x)^{1/3}(6)(x^2 - 1)$$

$$f'(x) = 8(2x^3 - 6x)^{1/3}(x^2 - 1)$$

23. $f(t) = \sqrt{t + 1} = (t + 1)^{1/2}$

$$f'(t) = \frac{1}{2}(t + 1)^{-1/2}(1) = \frac{1}{2\sqrt{t + 1}}$$

24. $g(x) = \sqrt{5 - 3x} = (5 - 3x)^{1/2}$

$$g'(x) = \frac{1}{2}(5 - 3x)^{-1/2}(-3) = -\frac{3}{2\sqrt{5 - 3x}}$$

25. $s(t) = \sqrt{2t^2 + 5t + 2} = (2t^2 + 5t + 2)^{1/2}$

$$s'(t) = \frac{1}{2}(2t^2 + 5t + 2)^{-1/2}(4t + 5) = \frac{4t + 5}{2\sqrt{2t^2 + 5t + 2}}$$

26. $y = 9\sqrt[3]{4x^2 + 3} = 9(4x^2 + 3)^{1/3}$

$$y' = 9\left(\frac{1}{3}\right)(4x^2 + 3)^{-2/3}(8x)$$

$$y' = 24x(4x^2 + 3)^{-2/3}$$

$$y' = \frac{24x}{(4x^2 + 3)^{2/3}}$$

27. $f(x) = 2(2 - 9x)^{-3}$

$$f'(x) = 2(-3)(2 - 9x)^{-4}(-9) = \frac{54}{(2 - 9x)^4}$$

28. $g(x) = \dfrac{3}{(7x^2 + 6x)^5} = 3(7x^2 + 6x)^{-5}$

$$g'(x) = 3(-5)(7x^2 + 6x)^{-6}(14x + 6)$$

$$g'(x) = -15(7x^2 + 6x)^{-6}(14x + 6)$$

$$g'(x) = -\frac{15(14x + 6)}{(7x^2 + 6x)^6}$$

29. $f(x) = \dfrac{1}{\sqrt{(x^2 + 11)^7}} = (x^2 + 11)^{-7/2}$

$$f'(x) = \left(-\frac{7}{2}\right)(x^2 + 11)^{-9/2}(2x)$$

$$f'(x) = -7x(x^2 + 11)^{-9/2}$$

$$f'(x) = -\frac{7x}{(x^2 + 11)^{9/2}} = -\frac{7x}{\sqrt{(x^2 + 11)^9}}$$

30. $y = (4 - x^3)^{-4/3}$

$$y' = \left(-\frac{4}{3}\right)(4 - x^3)^{-7/3}(-3x^2) = \frac{4x^2}{3(4 - x^2)^{7/3}}$$

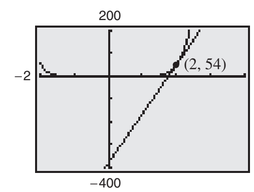
31. $f'(x) = 2(3)(x^2 - 1)^2(2x) = 12x(x^2 - 1)^2$

$f'(2) = 24(3^2) = 216$

$f(2) = 54$

$y - 54 = 216(x - 2)$

$y = 216x - 378$



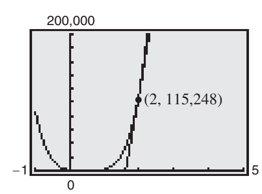
32. $f'(x) = 12(9x - 4)^3(9) = 108(9x - 4)^3$

$f'(2) = 12(14)^3(9) = 296{,}352$

$f(2) = 3(14)^4 = 115{,}248$

$y - 115{,}248 = 296{,}352(x - 2)$

$y = 296{,}352x - 477{,}456$



33. $f(x) = \sqrt{4x^2 - 7} = (4x^2 - 7)^{1/2}$

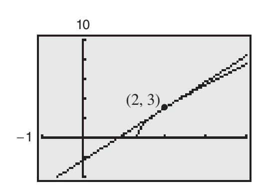
$$f'(x) = \frac{1}{2}(4x^2 - 7)^{-1/2}(8x) = \frac{4x}{\sqrt{4x^2 - 7}}$$

$f'(2) = \dfrac{8}{3}$

$f(2) = 3$

$y - 3 = \dfrac{8}{3}(x - 2)$

$y = \dfrac{8}{3}x - \dfrac{7}{3}$



34. $f(x) = x\sqrt{x^2 + 5} = x(x^2 + 5)^{1/2}$

$$f'(x) = x\left[\frac{1}{2}(x^2 + 5)^{-1/2}(2x)\right] + (x^2 + 5)^{1/2}(1)$$

$$= x^2(x^2 + 5)^{-1/2} + (x^2 + 5)^{1/2}$$

$$= (x^2 + 5)^{-1/2}\left[x^2 + (x^2 + 5)\right]$$

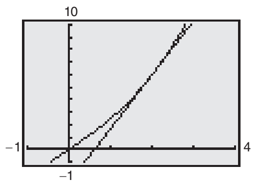
$$= \frac{2x^2 + 5}{\sqrt{x^2 + 5}}$$

$f'(2) = \frac{13}{3}$

$f(2) = 6$

$y - 6 = \frac{13}{3}(x - 2)$

$y = \frac{13}{3}x - \frac{8}{3}$



35. $f(x) = \sqrt{x^2 - 2x + 1} = (x^2 - 2x + 1)^{1/2}$

$$f'(x) = \frac{1}{2}(x^2 - 2x + 1)^{-1/2}(2x - 2)$$

$$= \frac{x - 1}{\sqrt{x^2 - 2x + 1}}$$

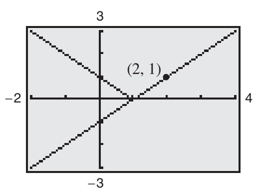
$$= \frac{x - 1}{|x - 1|}$$

$f'(2) = 1$

$f(2) = 1$

$y - 1 = 1(x - 2)$

$y = x - 1$



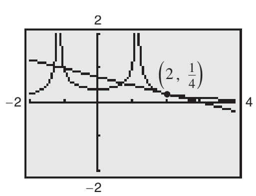
36. $f'(x) = -\frac{2}{3}(4 - 3x^2)^{-5/3}(-6x) = \frac{4x}{(4 - 3x^2)^{5/3}}$

$f'(2) = \frac{4(2)}{(-8)^{5/3}} = \frac{8}{-32} = -\frac{1}{4}$

$f(2) = (-8)^{-2/3} = \frac{1}{4}$

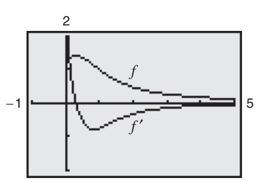
$y - \frac{1}{4} = -\frac{1}{4}(x - 2)$

$y = -\frac{1}{4}x + \frac{3}{4}$



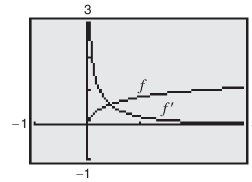
37. $f'(x) = \frac{1 - 3x^2 - 4x^{3/2}}{2\sqrt{x}(x^2 + 1)^2}$

f has a horizontal tangent when $f' = 0$.



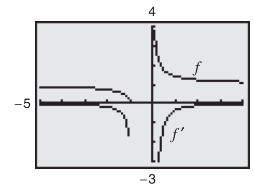
38. $f'(x) = \frac{\sqrt{2}}{2\sqrt{x}(x + 1)^{3/2}}$

f' is never 0.



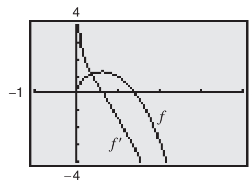
39. $f'(x) = -\frac{\sqrt{(x + 1)/x}}{2x(x + 1)}$

f' is never 0.



40. $f'(x) = \frac{2 - 5x^2}{2\sqrt{x}}$

f has a horizontal tangent when $f' = 0$.



41. $y = (4 - x^2)^{-1}$

$$y' = (-1)(4 - x^2)^{-2}(-2x)$$

$$= \frac{2x}{(4 - x^2)^2}$$

General Power Rule

42. $s(t) = \frac{1}{t^2 + 3t - 1} = (t^2 + 3t - 1)^{-1}$

$$s'(t) = -1(t^2 + 3t - 1)^{-2}(2t + 3)$$

$$= -\frac{2t + 3}{(t^2 + 3t - 1)^2}$$

General Power Rule

43. $y = -\frac{5t}{(t + 8)^2}$

$$y' = \frac{(t + 8)^2(5) - (5t)(2)(t + 8)(1)}{\left((t + 8)^2\right)^2}$$

$$y' = \frac{5(t + 8)\left[(t + 8) - 2t\right]}{(t^2 + 8)^4}$$

$$y' = \frac{5(t + 8)(-t + 8)}{(t + 8)^4} = \frac{5(t - 8)}{(t + 8)^3}$$

Quotient Rule and Chain Rule

44. $f(x) = 3x(x^3 - 4)^{-2}$

$$f'(x) = 3x\left[(-2)(x^3 - 4)^{-3}(3x^2)\right] + (x^3 - 4)^{-2}(3)$$
$$= -18x^3(x^3 - 4)^{-3} + 3(x^3 - 4)^{-2}$$
$$= -3(x^3 - 4)^{-3}\left[6x^3 - (x^3 - 4)\right]$$
$$= \frac{-3(5x^3 + 4)}{(x^3 - 4)^3}$$

Product Rule and Chain Rule

45. $f(x) = (2x - 1)(9 - 3x^2)$

$$f'(x) = (2x - 1)(-6x) + (9 - 3x^2)(2)$$
$$= -12x^2 + 6x + 18 - 6x^2$$
$$= 18 + 6x - 18x^2$$
$$= -6(3x^2 - 2x - 3)$$

Product Rule and Simple Power Rule

46. $y = (7x + 4)(x^3 - 2x^2)$

$$y' = (7x + 4)(3x^2 - 4x) + (x^3 - 2x^2)(7)$$
$$= 21x^3 - 16x^2 - 16x + 7x^3 - 14x^2$$
$$= 28x^3 - 30x^2 - 16x$$

Product Rule and Simple Power Rule

47. $y = \dfrac{1}{\sqrt{x + 2}} = (x + 2)^{-1/2}$

$$y' = -\frac{1}{2}(x + 2)^{-3/2} = -\frac{1}{2(x + 2)^{3/2}}$$

General Power Rule

48. $g(x) = \dfrac{3}{\sqrt[3]{x^3 - 1}} = 3(x^3 - 1)^{-1/3}$

$$g'(x) = 3\left(-\frac{1}{3}\right)(x^3 - 1)^{-4/3}(3x^2) = -\frac{3x^2}{(x^3 - 1)^{4/3}}$$

General Power Rule

49. $f(x) = x(3x - 9)^3$

$$f'(x) = x(3)(3x - 9)^2(3) + (3x - 9)^3(1)$$
$$= (3x - 9)^2\left[9x + (3x - 9)\right]$$
$$= 9(x - 3)^2(12x - 9)$$
$$= 27(x - 3)^2(4x - 3)$$

Product and General Power Rule

50. $f(x) = x^3(x - 4)^2$

$$= x^3(x^2 - 8x + 16)$$
$$= x^5 - 8x^4 + 16x^3$$
$$f'(x) = 5x^4 - 32x^3 + 48x^2$$
$$= x^2(5x^2 - 32x + 48)$$
$$= x^2(5x - 12)(x - 4)$$

Simple Power Rule

51. $y = x\sqrt{2x + 3} = x(2x + 3)^{1/2}$

$$y' = x\left[\frac{1}{2}(2x + 3)^{-1/2}(2)\right] + (2x + 3)^{1/2}$$
$$= (2x + 3)^{-1/2}\left[x + (2x + 3)\right]$$
$$= \frac{3(x + 1)}{\sqrt{2x + 3}}$$

Product and General Power Rule

52. $y = 2t\sqrt{t + 6} = 2t(t + 6)^{1/2}$

$$y' = 2t\left[\frac{1}{2}(t + 6)^{-1/2}(1)\right] + (t + 6)^{1/2}(2)$$
$$= t(t + 6)^{-1/2} + 2(t + 6)^{1/2}$$
$$= (t + 6)^{-1/2}\left[t + 2(t + 6)\right]$$
$$= (t + 6)^{-1/2}(3t + 12)$$
$$= \frac{3t + 12}{\sqrt{t + 6}} = \frac{3(t + 4)}{\sqrt{t + 6}}$$

Product and General Power Rule

53. $y = t^2\sqrt{t - 2} = t^2(t - 2)^{1/2}$

$$y' = t^2\left[\frac{1}{2}(t - 2)^{-1/2}(1)\right] + 2t(t - 2)^{1/2}$$
$$= \frac{1}{2}(t - 2)^{-1/2}\left[t^2 + 4t(t - 2)\right]$$
$$= \frac{t^2 + 4t(t - 2)}{2\sqrt{t - 2}}$$
$$= \frac{t(5t - 8)}{2\sqrt{t - 2}}$$

Product and General Power Rule

54. $y = \sqrt{x}(x-2)^2 = x^{1/2}(x-2)^2$

$$y' = x^{1/2}\left[2(x-2)^1(1)\right] + (x-2)^2\left(\frac{1}{2}x^{-1/2}\right)$$

$$= 2\sqrt{x}(x-2) + \frac{(x-2)^2}{2\sqrt{x}}$$

$$= \frac{4x(x-2) + (x-2)^2}{2\sqrt{x}}$$

$$= \frac{(x-2)\left[4x + (x-2)\right]}{2\sqrt{x}}$$

$$= \frac{(x-2)(5x-2)}{2\sqrt{x}}$$

Product and General Power Rule

55. $y = \left(\dfrac{6-5x}{x^2-1}\right)^2$

$$y' = 2\left(\frac{6-5x}{x^2-1}\right)\left[\frac{(x^2-1)(-5) - (6-5x)(2x)}{(x^2-1)^2}\right]$$

$$= \frac{2(6-5x)(5x^2-12x+5)}{(x^2-1)^3}$$

Quotient and General Power Rule

56. $y = \left(\dfrac{4x^2-5}{2-x}\right)^3$

$$y' = 3\left(\frac{4x^2-5}{2-x}\right)^2\left[\frac{(2-x)(8x) - (4x^2-5)(-1)}{(2-x)^2}\right]$$

$$= 3\left(\frac{4x^2-5}{2-x}\right)^2\left[\frac{16x - 8x^2 + 4x^2 - 5}{(2-x)^2}\right]$$

$$= 3\left(\frac{4x^2-5}{2-x}\right)^2\left[\frac{-4x^2+16x-5}{(2-x)^2}\right]$$

$$= \frac{3(4x^2-5)^2(-4x^2+16x-5)}{(2-x)^3}$$

Quotient and General Power Rule

57. $y = (x^3 - 2x^2 - x + 1)^2$

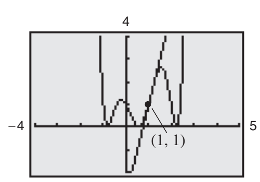
$$y' = 2(x^3 - 2x^2 - x + 1)(3x^2 - 4x - 1)$$

$$m = y'(1) = 2\left((1)^3 - 2(1)^2 - (1) + 1\right)\left(3(1)^2 - 4(1) - 1\right)$$

$$= 2(-1)(-2) = 4$$

$$y - 1 = 4(x-1)$$

$$y = 4x - 3$$



58. $f(x) = (3x^3 + 4x)^{1/5}$

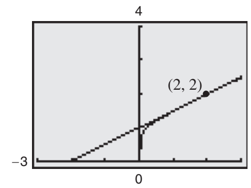
$$f'(x) = \frac{1}{5}(3x^3+4x)^{-4/5}(9x^2+4)$$

$$= \frac{9x^2+4}{5(3x^3+4x)^{4/5}}$$

$$m = f'(2) = \frac{1}{2}$$

$$y - 2 = \frac{1}{2}(x-2)$$

$$y = \frac{1}{2}x + 1$$



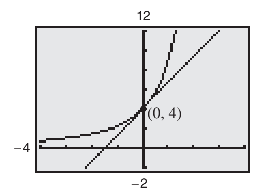
59. $f(t) = \dfrac{36}{(3-t)^2} = 36(3-t)^{-2}$

$$f'(t) = -72(3-t)^{-3}(-1) = \frac{72}{(3-t)^3}$$

$$f'(0) = \frac{72}{27} = \frac{8}{3}$$

$$y - 4 = \frac{8}{3}(t-0)$$

$$y = \frac{8}{3}t + 4$$



60. $s(x) = \dfrac{1}{\sqrt{x^2-3x+4}} = (x^2-3x+4)^{-1/2}$

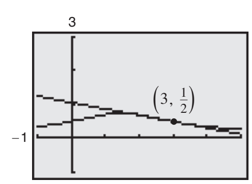
$$s'(x) = -\frac{1}{2}(x^2-3x+4)^{-3/2}(2x-3)$$

$$= \frac{3-2x}{2(x^2-3x+4)^{3/2}}$$

$$s'(3) = \frac{3-6}{2(4)^{3/2}} = -\frac{3}{16}$$

$$y - \frac{1}{2} = -\frac{3}{16}(x-3)$$

$$y = -\frac{3}{16}x + \frac{17}{16}$$



61. $f(t) = (t^2 - 9)\sqrt{t + 2} = (t^2 - 9)(t + 2)^{1/2}$

$$f'(t) = (t^2 - 9)\left[\frac{1}{2}(t + 2)^{-1/2}\right] + (t + 2)^{1/2}(2t)$$

$$= \frac{1}{2}(t^2 - 9)(t + 2)^{-1/2} + 2t(t + 2)^{1/2}$$

$$= (t + 2)^{-1/2}\left[\frac{1}{2}(t^2 - 9) + 2t(t + 2)\right]$$

$$= (t + 2)^{-1/2}\left[\frac{1}{2}t^2 - \frac{9}{2} + 2t^2 + 4t\right]$$

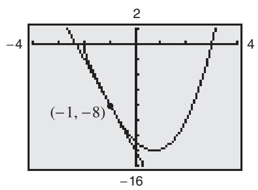
$$= (t + 2)^{-1/2}\left(\frac{5}{2}t^2 + 4t - \frac{9}{2}\right)$$

$$= \frac{\frac{5}{2}t^2 + 4t - \frac{9}{2}}{\sqrt{t + 2}}$$

$f'(-1) = -6$

$y - (-8) = -6[t - (-1)]$

$y = -6t - 14$



62. $y = -\dfrac{2x}{\sqrt{1 - x}} = -\dfrac{2x}{(1 - x)^{1/2}}$

$$y' = -\left[\frac{(1 - x)^{1/2}(2) - \left(\frac{1}{2}\right)(1 - x)^{-1/2}(-1)(2x)}{\left((1 - x)^{1/2}\right)^2}\right]$$

$$= -\left[\frac{2(1 - x)^{1/2} + x(1 - x)^{-1/2}}{1 - x}\right]$$

$$= -\left[\frac{(1 - x)^{-1/2}(2(1 - x) + x)}{1 - x}\right]$$

$$= -\left[\frac{(1 - x)^{-1/2}(2 - 2x + x)}{1 - x}\right]$$

$$= -\left[\frac{(1 - x)^{-1/2}(2 - x)}{1 - x}\right]$$

$$= -\left[\frac{(2 - x)}{(1 - x)^{3/2}}\right]$$

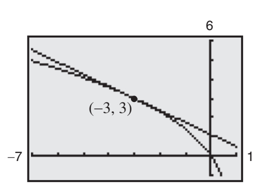
$$= \frac{x - 2}{(1 - x)^{3/2}}$$

$$y'(-3) = \frac{(-3) - 2}{(1 - (-3))^{3/2}} = \frac{-5}{4^{3/2}} = -\frac{5}{8}$$

$y - 3 = -\frac{5}{8}(x - (-3))$

$y - 3 = -\frac{5}{8}x - \frac{15}{8}$

$y = -\frac{5}{8}x + \frac{9}{8}$



63. $f(x) = \dfrac{x + 1}{\sqrt{2x - 3}} = \dfrac{x + 1}{(2x - 3)^{1/2}}$

$$f'(x) = \frac{(2x - 3)^{1/2}(1) - (x + 1)\left(\frac{1}{2}\right)(2x - 3)^{-1/2}(2)}{(2x - 3)}$$

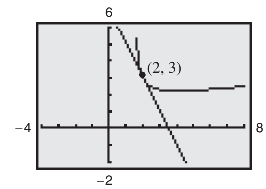
$$= \frac{(2x - 3) - (x + 1)}{(2x - 3)^{3/2}}$$

$$= \frac{x - 4}{(2x - 3)^{3/2}}$$

$f'(2) = \dfrac{1 - 3}{1} = -2$

$y - 3 = -2(x - 2)$

$y = -2x + 7$



64. $y = \dfrac{x}{\sqrt{25 + x^2}} = x(25 + x^2)^{-1/2}$

$$y' = x\left[-\tfrac{1}{2}(25 + x^2)^{-3/2}(2x)\right] + (25 + x^2)^{-1/2}(1)$$

$$= -x^2(25 + x^2)^{-3/2} + (25 + x^2)^{-1/2}$$

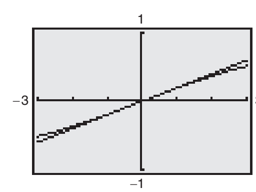
$$= (25 + x^2)^{-3/2}\left[-x^2 + (25 + x^2)\right]$$

$$= \frac{25}{(25 + x^2)^{3/2}}$$

$y'(0) = \dfrac{1}{5}$

$y - 0 = \frac{1}{5}(x - 0)$

$y = \frac{1}{5}x$



65. $f(x) = \sqrt[3]{x^2 + 4} = (x^2 + 4)^{1/3}$

$$f'(x) = \frac{1}{3}(x^2 + 4)^{-2/3}(2x)$$

$$f'(x) = \frac{2x}{3(x^2 + 4)^{2/3}}$$

Set $f'(x) = \dfrac{2x}{3(x^2 + 4)^{2/3}} = 0.$

$2x = 0$

$x = 0 \rightarrow y = f(0) = \sqrt[3]{4}$

Horizontal tangent at: $\left(0, \sqrt[3]{4}\right)$

66. $f(x) = \sqrt{5x^2 + x - 3} = (5x^2 + x - 3)^{1/2}$

$f'(x) = \frac{1}{2}(5x^2 + x - 3)^{-1/2}(10x + 1)$

$f'(x) = \frac{10x + 1}{2(5x^2 + x - 3)^{1/2}}$

Set $f'(x) = \frac{10x + 1}{2(5x^2 + x - 3)^{1/2}} = 0.$

$10x + 1 = 0$

$x = -\frac{1}{10} \to y = f\left(-\frac{1}{10}\right) = \sqrt{-\frac{61}{20}}$

Because $\sqrt{-\frac{61}{20}}$ is not a real number, there is no point of horizontal tangency.

67. $f(x) = \frac{x}{\sqrt{2x - 1}} = \frac{x}{(2x - 1)^{1/2}}$

$f'(x) = \frac{(2x - 1)^{1/2}(1) - x\left(\frac{1}{2}(2x - 1)^{-1/2}(2)\right)}{\left[(2x + 1)^{1/2}\right]^2}$

$f'(x) = \frac{(2x - 1)^{1/2} - x(2x - 1)^{-1/2}}{(2x + 1)}$

$f'(x) = \frac{(2x - 1)^{-1/2}\left[(2x - 1) - x\right]}{(2x - 1)}$

$f'(x) = \frac{x - 1}{(2x - 1)^{3/2}}$

Set $f'(x) = \frac{x - 1}{(2x - 1)^{3/2}} = 0.$

$x - 1 = 0$

$x = 1 \to y = f(1) = \frac{1}{\sqrt{1}} = 1$

Horizontal tangent at: $(1, 1)$

68. $f(x) = \frac{5x}{\sqrt{3x - 2}} = \frac{5x}{(3x - 2)^{1/2}}$

$f'(x) = \frac{(3x - 2)^{1/2}(5) - 5x\left(\frac{1}{2}(3x - 2)^{-1/2}(3)\right)}{\left[(3x - 2)^{1/2}\right]^2}$

$f'(x) = \frac{5(3x - 2)^{1/2} - \frac{15}{2}x(3x - 2)^{-1/2}}{(2x + 1)}$

$f'(x) = \frac{\frac{5}{2}(3x - 2)^{-1/2}\left[2(3 - x2) - 3x\right]}{(2x - 1)}$

$f'(x) = \frac{5(3x - 4)}{2(3x - 2)^{3/2}}$

Set $f'(x) = \frac{5(3x - 4)}{2(3x - 2)^{3/2}} = 0.$

$5(3x - 4) = 0$

$3x - 4 = 0$

$x = \frac{4}{3} \to y = f\left(\frac{4}{3}\right) = \frac{20}{3\sqrt{2}}$

Horizontal tangent at: $\left(\frac{4}{3}, \frac{10}{3\sqrt{2}}\right)$

69. $A' = 1000(60)\left(1 + \frac{r}{12}\right)^{59}\left(\frac{1}{12}\right) = 5000\left(1 + \frac{r}{12}\right)^{59}$

(a) $A'(0.08) = 50\left(1 + \frac{0.08}{12}\right)^{59}$

$\approx$ \$74.00 per percentage point

(b) $A'(0.10) = 50\left(1 + \frac{0.10}{12}\right)^{59}$

$\approx$ \$81.59 per percentage point

(c) $A'(0.12) = 50\left(1 + \frac{0.12}{12}\right)^{59}$

$\approx$ \$89.94 per percentage point

70. $N = 400\left[1 - 3(t^2 + 2)^{-2}\right]$

$$\frac{dN}{dt} = N'(t) = 400\left[(-3)(-2)(t^2 + 2)^{-3}(2t)\right]$$
$$= \frac{4800t}{(t^2 + 2)^3}$$

(a) $N'(0) = 0$ bacteria/day

(b) $N'(1) \approx 177.8$ bacteria/day

(c) $N'(2) \approx 44.4$ bacteria/day

(d) $N'(3) \approx 10.8$ bacteria/day

(e) $N'(4) \approx 3.3$ bacteria/day

(f) The rate of change of the population is decreasing as time passes.

71. $V = \dfrac{k}{\sqrt{t + 1}}$

When $t = 0$, $V = 10{,}000$.

$$10{,}000 = \frac{k}{\sqrt{0 + 1}} \Rightarrow k = 10{,}000$$
$$V = \frac{10{,}000}{\sqrt{t + 1}}$$
$$V = 10{,}000(t + 1)^{-1/2}$$
$$\frac{dV}{dt} = -5000(t + 1)^{-3/2}(1) = -\frac{5000}{(t + 1)^{3/2}}$$

When $t = 1$,

$$\frac{dV}{dt} = -\frac{5000}{(2)^{3/2}} = -\frac{2500}{\sqrt{2}} \approx -\$1767.77 \text{ per year.}$$

When $t = 3$, $\dfrac{dV}{dt} = -\dfrac{5000}{(4)^{3/2}} = -\$625.00$ per year.

72. (a) From the graph, the tangent line at $t = 4$ is steeper than the tangent line at $t = 1$. So, the rate of change after 4 hours is greater.

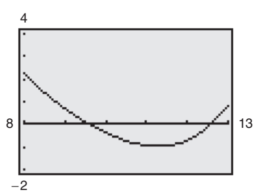
(b) The cost function is a composite function of x units, which is a function of the number of hours, which is not a linear function.

73. (a) $r = \left(0.3017t^4 - 9.657t^3 + 97.35t^2 - 266.8t - 242\right)^{1/2}$

$$\frac{dr}{dt} = r'(t) = \frac{1}{2}\left(0.3017t^4 - 9.657t^3 + 97.35t^2 - 266.8t - 242\right)^{-1/2} \cdot \left(1.2068t^3 - 28.971t^2 + 194.7t - 266.8\right)$$
$$= \frac{1.2068t^3 - 28.971t^2 + 194.7t - 266.8}{2\sqrt{0.3017t^4 - 9.657t^3 + 97.35t^2 - 266.8t - 242}}$$

Chain Rule

(b)



(c) The rate of change appears to be the greatest when $t = 8$ or 2008.

The rate of change appears to be the least when $t \approx 9.60$, or 2009, and when $t \approx 12.57$, or 2012.

Section 2.6 Higher-Order Derivatives

Skills Warm Up

1.
$$-16t^2 + 292 = 0$$
$$-16t^2 = -292$$
$$t^2 = \frac{73}{4}$$
$$t = \pm\frac{\sqrt{73}}{2}$$

2.
$$-16t^2 + 88t = 0$$
$$-8t(2t - 11) = 0$$
$$-8t = 0 \rightarrow t = 0$$
$$2t - 11 = 0 \rightarrow t = \tfrac{11}{2}$$

Skills Warm Up —*continued*—

3. $-16t^2 + 128t + 320 = 0$

$-16(t^2 - 8t - 20) = 0$

$-16(t - 10)(t + 2) = 0$

$t - 10 = 0 \rightarrow t = 10$

$t + 2 = 0 \rightarrow t = -2$

4. $-16t^2 + 9t + 1440 = 0$

$$t = \frac{-9 \pm \sqrt{9^2 - 4(-16)(1440)}}{2(-16)}$$

$$= \frac{-9 \pm \sqrt{92241}}{-32}$$

$$= \frac{9 \pm 3\sqrt{10249}}{32}$$

$t \approx -9.21$ and $t \approx 9.77$

5. $y = x^2(2x + 7)$

$$\frac{dy}{dx} = x^2(2) + 2x(2x + 7)$$

$$= 2x^2 + 4x^2 + 14x$$

$$= 6x^2 + 14x$$

6. $y = (x^2 + 3x)(2x^2 - 5)$

$$\frac{dy}{dx} = (x^2 + 3x)(4x) + (2x + 3)(2x^2 - 5)$$

$$= 4x^3 + 12x^2 + 4x^3 - 10x + 6x^2 - 15$$

$$= 8x^3 + 18x^2 - 10x - 15$$

7. $y = \dfrac{x^2}{2x + 7}$

$$\frac{dy}{dx} = \frac{(2x + 7)(2x) - (x^2)(2)}{(2x + 7)^2}$$

$$= \frac{4x^2 + 14x - 2x^2}{(2x + 7)^2}$$

$$= \frac{2x^2 + 14x}{(2x + 7)^2}$$

$$= \frac{2x(x + 7)}{(2x + 7)^2}$$

8. $y = \dfrac{x^2 + 3x}{2x^2 - 5}$

$$\frac{dy}{dx} = \frac{(2x^2 - 5)(2x + 3) - (x^2 + 3x)(4x)}{(2x^2 - 5)^2}$$

$$= \frac{4x^3 + 6x^2 - 10x - 15 - 4x^3 - 12x^2}{(2x^2 - 5)^2}$$

$$= \frac{-6x^2 - 10x - 15}{(2x^2 - 5)^2}$$

9. $f(x) = x^2 - 4$

Domain: $(-\infty, \infty)$

Range: $[-4, \infty)$

10. $f(x) = \sqrt{x - 7}$

Domain: $[7, \infty)$

Range: $[0, \infty)$

1. $f(x) = 9 - 2x$

$f'(x) = -2$

$f''(x) = 0$

2. $f(x) = 4x + 15$

$f'(x) = 4$

$f''(x) = 0$

3. $f(x) = x^2 + 7x - 4$

$f'(x) = 2x + 7$

$f''(x) = 2$

4. $f(x) = 3x^2 + 4x$

$f'(x) = 6x + 4$

$f''(x) = 6$

5. $g(t) = \frac{1}{3}t^3 - 4t^2 + 2t$

$g'(t) = t^2 - 8t + 2$

$g''(t) = 2t - 8$

6. $f(x) = -\frac{5}{4}x^4 + 3x^2 - 6x$

$f'(x) = -5x^3 + 6x - 6$

$f''(x) = -15x^2 + 6$

7. $f(t) = \frac{2}{t^3} = 2t^{-3}$

$f'(t) = -6t^{-4}$

$f''(t) = 24t^{-5} = \frac{24}{t^5}$

8. $g(t) = \frac{5}{6t^4} = \frac{5}{6}t^{-4}$

$g'(t) = -\frac{10}{3}t^{-5}$

$g''(t) = \frac{50}{3}t^{-6} = \frac{50}{3t^6}$

9. $f(x) = 3(2 - x^2)^3$

$f'(x) = 9(2 - x^2)^2(-2x) = -18x(2 - x^2)^2$

$f''(x) = (-18x)2(2 - x^2)(-2x) + (2 - x^2)^2(-18)$

$= 18(2 - x^2)\left[4x^2 - (2 - x^2)\right]$

$= 18(2 - x^2)(5x^2 - 2)$

10. $y = 4(x^2 + 5x)^3$

$y' = 4(3)(x^2 + 5x)^2(2x + 5)$

$= (24x + 60)(x^4 + 10x^3 + 25x^2)$

$= 24x^5 + 300x^4 + 1200x^3 + 1500x^2$

$y'' = 120x^4 + 1200x^3 + 3600x^2 + 3000x$

11. $f(x) = \frac{x + 1}{x - 1}$

$f'(x) = \frac{(x - 1)(1) - (x + 1)(1)}{(x - 1)^2}$

$= -\frac{2}{(x - 1)^2} = -2(x - 1)^{-2}$

$f''(x) = 4(x - 1)^{-3}(1) = \frac{4}{(x - 1)^3}$

12. $g(x) = \frac{1 - 4x}{x - 3}$

$g'(x) = \frac{(x - 3)(-4) - (1 - 4x)(1)}{(x - 3)^2}$

$= \frac{-4x + 12 - 1 + 4x}{(x - 3)^2}$

$= \frac{11}{(x - 3)^2} = 11(x - 3)^{-2}$

$g''(x) = -22(x - 3)^{-3}(1) = -\frac{22}{(x - 3)^3}$

13. $f(x) = x^5 - 3x^4$

$f'(x) = 5x^4 - 12x^3$

$f''(x) = 20x^3 - 36x^2$

$f'''(x) = 60x^2 - 72x$

14. $f(x) = x^4 - 2x^3$

$f'(x) = 4x^3 - 6x^2$

$f''(x) = 12x^2 - 12x$

$f'''(x) = 24x - 12 = 12(2x - 1)$

15. $f(x) = 5x(x + 4)^3$

$= 5x(x^3 + 12x^2 + 48x + 64)$

$= 5x^4 + 60x^3 + 240x^2 + 320x$

$f'(x) = 20x^3 + 180x^2 + 480x + 320$

$f''(x) = 60x^2 + 360x + 480$

$f'''(x) = 120x + 360$

16. $f(x) = (x^3 - 6)^4$

$f'(x) = 4(x^3 - 6)^3(3x^2)$

$= 12x^{11} - 216x^8 + 1296x^5 - 2592x^2$

$f''(x) = 132x^{10} - 1728x^7 + 6480x^4 - 5184x$

$f'''(x) = 1320x^9 - 12{,}096x^6 + 25{,}920x^3 - 5184$

17. $f(x) = \frac{3}{8x^4} = \frac{3}{8}x^{-4}$

$f'(x) = -\frac{3}{2}x^{-5}$

$f''(x) = \frac{15}{2}x^{-6}$

$f'''(x) = -45x^{-7} = -\frac{45}{x^7}$

18. $f(x) = -\frac{2}{25x^5}$

$f(x) = -\frac{2}{25}x^{-5}$

$f'(x) = \frac{2}{5}x^{-6}$

$f''(x) = -\frac{12}{5}x^{-7}$

$f'''(x) = \frac{84}{5}x^{-8} = \frac{84}{5x^8}$

19. $g(t) = 5t^4 + 10t^2 + 3$

$g'(t) = 20t^3 + 20t$

$g''(t) = 60t^2 + 20$

$g''(2) = 60(4) + 20 = 260$

20. $f(x) = 9 - x^2$

$f'(x) = -2x$

$f''(x) = -2$

$f''(-\sqrt{5}) = -2$

21. $f(x) = \sqrt{4 - x} = (4 - x)^{1/2}$

$f'(x) = -\frac{1}{2}(4 - x)^{-1/2}$

$f''(x) = -\frac{1}{4}(4 - x)^{-3/2}$

$f'''(x) = -\frac{3}{8}(4 - x)^{-5/2} = \frac{-3}{8(4 - x)^{5/2}}$

$f'''(-5) = \frac{-3}{8(9)^{5/2}} = -\frac{1}{648}$

22. $f(t) = \sqrt{2t + 3} = (2t + 3)^{1/2}$

$f'(t) = \frac{1}{2}(2t + 3)^{-1/2}(2) = (2t + 3)^{-1/2}$

$f''(t) = -\frac{1}{2}(2t + 3)^{-3/2}(2) = -(2t + 3)^{-3/2}$

$f'''(t) = \frac{3}{2}(2t + 3)^{-5/2}(2) = \frac{3}{(2t + 3)^{5/2}}$

$f'''\left(\frac{1}{2}\right) = \frac{3}{32}$

23. $f(x) = (x^3 - 2x)^3 = x^9 - 6x^7 + 12x^5 - 8x^3$

$f'(x) = 9x^8 - 42x^6 + 60x^4 - 24x^2$

$f''(x) = 72x^7 - 252x^5 + 240x^3 - 48x$

$f''(1) = 12$

24. $g(x) = (x^2 + 3x)^4 = x^8 + 12x^7 + 54x^6 + 108x^5 + 81x^4$

$g'(x) = 8x^7 + 84x^6 + 324x^5 + 540x^4 + 324x^3$

$g''(x) = 56x^6 + 504x^5 + 1620x^4 + 2160x^3 + 972x^2$

$g''(-1) = -16$

25. $f'(x) = 2x^2$

$f''(x) = 4x$

26. $f''(x) = 20x^3 - 36x^2$

$f'''(x) = 60x^2 - 72x = 12x(5x - 6)$

27. $f'''(x) = 4x^{-4}$

$f^{(4)}(x) = -16x^{-5}$

$f^{(5)}(x) = 80x^{-6} = \frac{80}{x^6}$

28. $f''(x) = 4\sqrt{x - 2} = 4(x - 2)^{1/2}$

$f'''(x) = 4\left(\frac{1}{2}\right)(x - 2)^{-1/2}(1) = 2(x - 2)^{-1/2}$

$f^{(4)}(x) = 2\left(-\frac{1}{2}\right)(x - 2)^{-3/2}(1) = -(x - 2)^{-3/2}$

$f^{(5)}(x) = \frac{3}{2}(x - 2)^{-5/2}(1) = \frac{3}{2(x - 2)^{5/2}}$

29. $f^{(5)}(x) = 2(x^2 + 1)(2x)$

$= 4x^3 + 4x$

$f^{(6)}(x) = 12x^2 + 4$

30. $f'''(x) = 4x + 7$

$f^{(4)}(x) = 4$

$f^{(5)}(x) = 0$

31. $f'(x) = 3x^2 - 18x + 27$

$f''(x) = 6x - 18$

$f''(x) = 0 \Rightarrow 6x = 18$

$x = 3$

32. $f(x) = (x + 2)(x - 2)(x + 3)(x - 3)$

$= (x^2 - 4)(x^2 - 9)$

$= x^4 - 13x^2 + 36$

$f'(x) = 4x^3 - 26x$

$f''(x) = 12x^2 - 26$

$f''(x) = 0 \Rightarrow 12x^2 = 26$

$x = \pm\sqrt{\frac{13}{6}} = \pm\frac{\sqrt{78}}{6}$

33. $f(x) = x\sqrt{x^2 - 1} = x(x^2 - 1)^{1/2}$

$$f'(x) = x\frac{1}{2}(x^2 - 1)^{-1/2}(2x) + (x^2 - 1)^{1/2} = \frac{x^2}{(x^2 - 1)^{1/2}} + (x^2 - 1)^{1/2}$$

$$f''(x) = \frac{(x^2 - 1)^{1/2}(2x) - x^2\left(\frac{1}{2}\right)(x^2 - 1)^{-1/2}(2x)}{x^2 - 1} + \frac{1}{2}(x^2 - 1)^{-1/2}(2x)$$

$$= \frac{(x^2 - 1)(2x) - x^3}{(x^2 - 1)^{3/2}} + \frac{x}{(x^2 - 1)^{1/2}} \cdot \frac{x^2 - 1}{x^2 - 1}$$

$$= \frac{2x^3 - 3x}{(x^2 - 1)^{3/2}}$$

$$f''(x) = 0 \Rightarrow 2x^3 - 3x = x(2x^2 - 3) = 0$$

$$x = \pm\sqrt{\frac{3}{2}} = \pm\frac{\sqrt{6}}{2}$$

$x = 0$ is not in the domain of f.

34. $$f'(x) = \frac{(x^2 + 3)(1) - (x)(2x)}{(x^2 + 3)^2} = \frac{3 - x^2}{(x^2 + 3)^2} = (3 - x^2)(x^2 + 3)^{-2}$$

$$f''(x) = (3 - x^2)\left[-2(x^2 + 3)^{-3}(2x)\right] + (x^2 + 3)^{-2}(-2x)$$

$$= -2x(x^2 + 3)^{-3}\left[2(3 - x^2) + (x^2 + 3)\right]$$

$$= \frac{-2x(9 - x^2)}{(x^2 + 3)^3}$$

$$= \frac{2x(x^2 - 9)}{(x^2 + 3)^3}$$

$$f''(x) = 0 \Rightarrow 2x(x^2 - 9) = 0$$

$$x = 0, \pm 3$$

35. (a) $s(t) = -16t^2 + 144t$

$v(t) = s'(t) = -32t + 144$

$a(t) = v'(t) = s''(t) = -32$

(b) $s(3) = 288$ ft

$v(3) = 48$ ft/sec

$a(3) = -32$ ft/sec^2

(c) $v(t) = 0$

$-32t + 144 = 0$

$-32t = -144$

$t = 4.5$ sec

$s(4.5) = 324$ ft

(d) $s(t) = 0$

$-16t^2 + 144t = 0$

$-16t(t - 9) = 0$

$t = 0$ sec $\quad t = 9$ sec

$v(9) = -32(9) + 144 = -144$ ft/sec

This is the same speed as the initial velocity.

36. (a) $s(t) = -16t^2 + 1250$

$v(t) = s'(t) = -32t$

$a(t) = v'(t) = -32$

(b) $s(t) = 0$ when $16t^2 = 1250$, or

$t = \sqrt{78.125} \approx 8.8$ sec.

(c) $v(8.8) \approx -282.8$ ft/sec

37. $\dfrac{d^2s}{dt^2} = \dfrac{(t + 10)(90) - (90t)(1)}{(t + 10)^2} = \dfrac{900}{(t + 10)^2}$

t	0	10	20	30	40	50	60
$\dfrac{ds}{dt}$	0	45	60	67.5	72	75	77.14
$\dfrac{d^2s}{dt^2}$	9	2.25	1	0.56	0.36	0.25	0.18

As time increases, the acceleration decreases. After 1 minute, the automobile is traveling at about 77.14 feet per second.

38. $s(t) = -8.25t^2 + 66t$

$v(t) = s'(t) = -16.50t + 66$

$a(t) = s''(t) = -16.50$

t	0	1	2	3	4	5
$s(t)$	0	57.75	99	123.75	132	123.75
$v(t)$	66	49.50	33	16.50	0	–16.50
$a(t)$	–16.50	–16.50	–16.50	–16.50	–16.50	–16.50

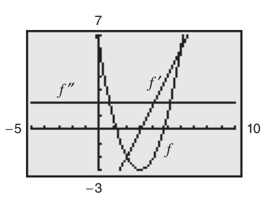
It takes 4 seconds for the car to stop, at which time it has traveled 132 feet.

39. $f(x) = x^2 - 6x + 6$

$f'(x) = 2x - 6$

$f''(x) = 2$

(a)

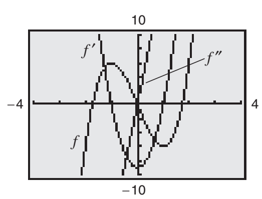


(b) The degree decreased by 1 for each successive derivative.

(c) $f(x) = 3x^2 - 9x$

$f'(x) = 6x - 9$

$f''(x) = 6$



(d) The degree decreases by 1 for each successive derivative.

40. Graph A is the position function. Graph B is the velocity function. Graph C is the acceleration function. Explanations will vary. Sample explanation:
The position function appears to be a third-degree function, while the velocity is a second-degree function, and the acceleration is a linear function.

41. (a) $y(t) = -21.944t^3 + 701.75t^2 - 6969.4t + 27{,}164$

(b) $y'(t) = -65.832t^2 + 1403.5t - 6969.4$

$y''(t) = -131.664t + 1403.5$

(c) Over the interval $8 \le t \le 13$, $y'(t) > 0$; therefore, y is increasing over $8 \le t \le 13$, or from 2008 to 20013.

(d)
$$\begin{aligned} y''(t) &= 0 \\ -131.664t + 1403.5 &= 0 \\ -131.664t &= -1403.5 \\ t &\approx 10.66 \text{ or } 2010 \end{aligned}$$

42. Let $y = xf(x)$.

Then,
$$\begin{aligned} y' &= xf'(x) + f(x) \\ y'' &= xf''(x) + f'(x) + f'(x) \\ &= xf''(x) + 2f'(x) \\ y''' &= xf'''(x) + f''(x) + 2f''(x) \\ &= xf'''(x) + 3f''(x). \end{aligned}$$

In general $y^{(n)} = \left[xf(x)\right]^{(n)} = xf^{(n)}(x) + nf^{(n-1)}(x)$.

43. True. If $y = (x + 1)(x + 2)(x + 3)(x + 4)$, then y is a fourth-degree polynomial function and its fifth derivative $\dfrac{d^5y}{dx^5}$ equals o.

44. True. The second derivative represents the rate of change of the first derivative, the same way that the first derivative represents the rate of change of the function.

45. Answers will vary.

Section 2.7 Implicit Differentiation

Skills Warm Up

1.
$$\begin{aligned} x - \frac{y}{x} &= 2 \\ x^2 - y &= 2x \\ -y &= 2x - x^2 \\ y &= x^2 - 2x \end{aligned}$$

2.
$$\begin{aligned} \frac{4}{x-3} &= \frac{1}{y} \\ 4y &= x - 3 \\ y &= \frac{x-3}{4} \end{aligned}$$

3.
$$\begin{aligned} xy - x + 6y &= 6 \\ xy + 6y &= 6 + x \\ y(x+6) &= 6 + x \\ y &= \frac{6+x}{x+6} \\ y &= 1,\ x \neq -6 \end{aligned}$$

4.
$$\begin{aligned} 7 + 4y &= 3x^2 + x^2y \\ 4y - x^2y &= 3x^2 - 7 \\ y(4 - x^2) &= 3x^2 - 7 \\ y &= \frac{3x^2 - 7}{4 - x^2},\ x \neq \pm 2 \end{aligned}$$

5.
$$\begin{aligned} x^2 + y^2 &= 5 \\ y^2 &= 5 - x^2 \\ y &= \pm\sqrt{5 - x^2} \end{aligned}$$

6.
$$\begin{aligned} x &= \pm\sqrt{6 - y^2} \\ x^2 &= 6 - y^2 \\ x^2 - 6 &= -y^2 \\ 6 - x^2 &= y^2 \\ \pm\sqrt{6 - x^2} &= y \end{aligned}$$

7. $\dfrac{3x^2 - 4}{3y^2}$, $(2, 1)$

$$\frac{3(2^2) - 4}{3(1^2)} = \frac{3(4) - 4}{3} = \frac{8}{3}$$

8. $\dfrac{x^2 - 2}{1 - y}$, $(0, -3)$

$$\frac{0^2 - 2}{1 - (-3)} = \frac{-2}{4} = -\frac{1}{2}$$

9. $\dfrac{7x}{4y^2 + 13y + 3}$, $\left(-\dfrac{1}{7}, -2\right)$

$$\frac{7\left(-\frac{1}{7}\right)}{4(-2)^2 + 13(-2) + 3} = \frac{-1}{16 - 26 + 3} = \frac{-1}{-7} = \frac{1}{7}$$

1.
$$\begin{aligned} x^3y &= 6 \\ x^3\frac{dy}{dx} + 3x^2y &= 0 \\ x^3\frac{dy}{dx} &= -3x^2y \\ \frac{dy}{dx} &= -\frac{3x^2}{x^3}y = -\frac{3}{x}y \end{aligned}$$

2.
$$\begin{aligned} 3x^2 - y &= 8x \\ 6x - \frac{dy}{dx} &= 8 \\ -\frac{dy}{dx} &= 8 - 6x \\ \frac{dy}{dx} &= 6x - 8 \end{aligned}$$

3.
$$\begin{aligned} y^2 &= 1 - x^2 \\ 2y\frac{dy}{dx} &= -2x \\ \frac{dy}{dx} &= -\frac{x}{y} \end{aligned}$$

4.
$$y^3 = 5x^3 + 8x$$
$$3y^2\frac{dy}{dx} = 15x^2 + 8$$
$$\frac{dy}{dx} = \frac{15x^2 + 8}{3y^2}$$

5.
$$y^4 - y^2 + 7y - 6x = 9$$
$$4y^3\frac{dy}{dx} - 2y\frac{dy}{dx} + 7\frac{dy}{dx} - 6 = 0$$
$$\left(4y^3 - 2y + 7\right)\frac{dy}{dx} = 6$$
$$\frac{dy}{dx} = \frac{6}{4y^3 - 2y + 7}$$

6.
$$4y^3 + 5y^2 - y - 3x^3 = 8x$$
$$12y^2\frac{dy}{dx} + 10y\frac{dy}{dx} - \frac{dy}{dx} - 9x^2 = 8$$
$$\left(12y^2 + 10y - 1\right)\frac{dy}{dx} = 8 + 9x^2$$
$$\frac{dy}{dx} = \frac{8 + 9x^2}{12y^2 + 10y - 1}$$

7.
$$xy^2 + 4xy = 10$$
$$y^2 + 2xy\frac{dy}{dx} + 4y + 4x\frac{dy}{dx} = 0$$
$$(2xy + 4x)\frac{dy}{dx} = -y^2 - 4y$$
$$\frac{dy}{dx} = -\frac{y^2 + 4y}{2xy + 4x}$$

8.
$$2xy^3 - x^2y = 2$$
$$2y^3 + 6xy^2\frac{dy}{dx} - 2xy - x^2\frac{dy}{dx} = 0$$
$$\left(6xy^2 - x^2\right)\frac{dy}{dx} = 2xy - 2y^3$$
$$\frac{dy}{dx} = \frac{2xy - 2y^3}{6xy^2 - x^2}$$

9.
$$\frac{2x + y}{x - 5y} = 1$$
$$2x + y = x - 5y$$
$$6y = -x$$
$$y = -\frac{1}{6}x$$
$$\frac{dy}{dx} = -\frac{1}{6}$$

10.
$$\frac{xy - y^2}{y - x} = 1$$
$$xy - y^2 = y - x$$
$$y(x - y) = -(x - y)$$
$$y = -1$$
$$\frac{dy}{dx} = 0$$

11.
$$\frac{2y}{y^2 + 3} = 4x$$
$$2y = 4x\left(y^2 + 3\right)$$
$$2y = 4xy^2 + 12x$$
$$2\frac{dy}{dx} = 8xy\frac{dy}{dx} + 4y^2 + 12$$
$$2\frac{dy}{dx} - 8xy\frac{dy}{dx} = 4y^2 + 12$$
$$\frac{dy}{dx} = \frac{4y^2 + 12}{2 - 8xy}$$

12.
$$\frac{4y^2}{y^2 - 9} = x^2$$
$$\frac{\left(y^2 - 9\right)\left(8y\frac{dy}{dx}\right) - 4y^2\left(2y\frac{dy}{dx}\right)}{\left(y^2 - 9\right)^2} = 2x$$
$$\frac{8y\frac{dy}{dx}\left(y^2 - 9 - y^2\right)}{\left(y^2 - 9\right)^2} = 2x$$
$$\frac{-72y\frac{dy}{dx}}{\left(y^2 - 9\right)^2} = 2x$$
$$\frac{dy}{dx} = \frac{2x\left(y^2 - 9\right)^2}{-72y}$$
$$\frac{dy}{dx} = -\frac{x\left(y^2 - 9\right)^2}{36y}$$

13.
$$x^2 + y^2 = 16$$
$$2x + 2y\frac{dy}{dx} = 0$$
$$2y\frac{dy}{dx} = -2x$$
$$\frac{dy}{dx} = -\frac{x}{y}$$

At (0, 4), $\frac{dy}{dx} = -\frac{0}{4} = 0.$

14.
$$x^2 - y^2 = 25$$
$$2x - 2y\frac{dy}{dx} = 0$$
$$-2y\frac{dy}{dx} = -2x$$
$$\frac{dy}{dx} = \frac{x}{y}$$

At (5, 0), $\frac{dy}{dx}$ is undefined.

15.
$$y + xy = 4$$
$$\frac{dy}{dx} + x\frac{dy}{dx} + y = 0$$
$$\frac{dy}{dx}(1 + x) = -y$$
$$\frac{dy}{dx} = -\frac{y}{x + 1}$$

At $(-5, -1)$, $\frac{dy}{dx} = -\frac{1}{4}$.

16.
$$xy - 3y^2 = 2$$
$$x\frac{dy}{dx} + y - 6y\frac{dy}{dx} = 0$$
$$\frac{dy}{dx}(x - 6y) = -y$$
$$\frac{dy}{dx} = -\frac{y}{x - 6y}$$

At $(7, 2)$, $\frac{dy}{dx} = -\frac{2}{7 - 6(2)} = \frac{2}{5}$.

17.
$$x^2 - xy + y^2 = 4$$
$$2x - \left(x\frac{dy}{dx} + y\right) + 2y\frac{dy}{dx} = 0$$
$$2x - x\frac{dy}{dx} - y + 2y\frac{dy}{dx} = 0$$
$$\frac{dy}{dx}(2y - x) = y - 2x$$
$$\frac{dy}{dx} = \frac{y - 3x^2}{2y - x}$$

At $(-2, -1)$, $\frac{dy}{dx} = \frac{(-1) - 3(-2)^2}{2(-1) - (-2)} = \frac{-7}{0}$; $\frac{dy}{dx}$ is undefined.

18.
$$x^2y + y^3x = -6$$
$$x^2\frac{dy}{dx} + 2xy + y^3 + 3y^2\frac{dy}{dx}x = 0$$
$$\frac{dy}{dx}\left(x^2 + 2xy\right) = -2xy - y^2$$
$$\frac{dy}{dx} = \frac{-\left(2xy + y^3\right)}{x^2 + 3xy^2}$$
$$\frac{dy}{dx} = \frac{y\left(2x + y^2\right)}{x\left(x + 3y^2\right)}$$

At $(2, -1)$, $\frac{dy}{dx} = -\frac{(-1)\left(2(2) + (-1)^2\right)}{(2)\left((2) + 3(-1)^2\right)} = -\frac{-5}{10} = \frac{1}{2}$.

19.
$$xy - x = y$$
$$x\frac{dy}{dx} + y - 1 = \frac{dy}{dx}$$
$$x\frac{dy}{dx} - \frac{dy}{dx} = 1 - y$$
$$\frac{dy}{dx}(x - 1) = 1 - y$$
$$\frac{dy}{dx} = \frac{1 - y}{x - 1} = -\frac{y - 1}{x - 1}$$

At $\left(\frac{3}{2}, 3\right)$, $\frac{dy}{dx} = -\frac{3 - 1}{\frac{3}{2} - 1} = -\frac{2}{\frac{1}{2}} = -4$.

20.
$$x^3 + y^3 = 6xy$$
$$3x^2 + 3y^2\frac{dy}{dx} = 6\left(x\frac{dy}{dx} + y\right)$$
$$3x^2 + 3y^2\frac{dy}{dx} = 6x\frac{dy}{dx} + 6y$$
$$\left(3y^2 - 6x\right)\frac{dy}{dx} = 6y - 3x^2$$
$$\frac{dy}{dx} = \frac{6y - 3x^2}{3y^2 - 6x}$$
$$\frac{dy}{dx} = \frac{3\left(2y - x^2\right)}{3\left(y^2 - 2x\right)}$$
$$\frac{dy}{dx} = \frac{2y - x^2}{y^2 - 2x}$$

At $\left(\frac{4}{3}, \frac{8}{3}\right)$, $\frac{dy}{dx} = \frac{4}{5}$.

21.
$$x^{1/2} + y^{1/2} = 9$$
$$\frac{1}{2}x^{-1/2} + \frac{1}{2}y^{-1/2}\frac{dy}{dx} = 0$$
$$x^{-1/2} + y^{-1/2}\frac{dy}{dx} = 0$$
$$\frac{dy}{dx} = \frac{-x^{-1/2}}{y^{-1/2}} = -\sqrt{\frac{y}{x}}$$

At (16, 25), $\frac{dy}{dx} = -\frac{5}{4}$.

22.
$$x^{2/3} + y^{2/3} = 5$$
$$\frac{2}{3}x^{-1/3} + \frac{2}{3}y^{-1/3}\frac{dy}{dx} = 0$$
$$\frac{dy}{dx} = \frac{-x^{-1/3}}{y^{-1/3}} = -\frac{y^{1/3}}{x^{1/3}} = -\sqrt[3]{\frac{y}{x}}$$

At (8, 1), $\frac{dy}{dx} = -\frac{1}{2}$.

23.
$$\sqrt{xy} = x - 2y$$
$$\sqrt{x}\sqrt{y} = x - 2y$$
$$\sqrt{x}\left(\frac{1}{2}y^{-1/2}\frac{dy}{dx}\right) + \sqrt{y}\left(\frac{1}{2}x^{-1/2}\right) = 1 - 2\frac{dy}{dx}$$
$$\frac{\sqrt{x}}{2\sqrt{y}}\frac{dy}{dx} + 2\frac{dy}{dx} = 1 - \frac{\sqrt{y}}{2\sqrt{x}}$$
$$\frac{dy}{dx} = \frac{1 - \frac{\sqrt{y}}{2\sqrt{x}}}{\frac{\sqrt{x}}{2\sqrt{y}} + 2} \cdot \frac{2\sqrt{x}\sqrt{y}}{2\sqrt{x}\sqrt{y}}$$
$$= \frac{2\sqrt{xy} - y}{x + 4\sqrt{xy}}$$
$$= \frac{2(x - 2y) - y}{x + 4(x - 2y)}$$
$$= \frac{2x - 5y}{5x - 8y}$$

At $(4, 1)$, $\frac{dy}{dx} = \frac{1}{4}$.

24.
$$(x + y)^3 = x^3 + y^3$$
$$3(x + y)^2\left(1 + \frac{dy}{dx}\right) = 3x^2 + 3y^2\frac{dy}{dx}$$
$$3(x + y)^2 + 3(x + y)^2\frac{dy}{dx} = 3x^2 + 3y^2\frac{dy}{dx}$$
$$(x + y)^2\frac{dy}{dx} - y^2\frac{dy}{dx} = x^2 - (x + y)^2$$
$$\frac{dy}{dx}\left[(x + y)^2 - y^2\right] = x^2 - \left(x^2 + 2xy + y^2\right)$$
$$\frac{dy}{dx} = \frac{-\left(2xy + y^2\right)}{x^2 + 2xy} = -\frac{y(2x + y)}{x(x + 2y)}$$

At $(-1, 1)$, $\frac{dy}{dx} = -1$.

25.
$$y^2\left(x^2 + y^2\right) = 2x^2$$
$$y^2\left(2x + 2y\frac{dy}{dx}\right) + \left(x^2 + y^2\right)\left(2y\frac{dy}{dx}\right) = 4x$$
$$2xy^2 + 2y^3\frac{dy}{dx} + 2x^2y\frac{dy}{dx} + 2y^3\frac{dy}{dx} = 4x$$
$$\frac{dy}{dx}\left(4y^3 + 2x^2y\right) = 4x - 2xy^2$$
$$\frac{dy}{dx} = \frac{2x\left(2 - y^2\right)}{2y\left(2y^2 + x^2\right)}$$
$$\frac{dy}{dx} = \frac{x\left(2 - y^2\right)}{y\left(2y^2 + x^2\right)}$$

At $(1, 1)$, $\frac{dy}{dx} = \frac{1}{3}$.

26.
$$\left(x^2 + y^2\right)^2 = 8x^2y$$
$$2\left(x^2 + y^2\right)\left(2x + 2y\frac{dy}{dx}\right) = 8x^2\frac{dy}{dx} + y(16x)$$
$$4x^3 + 4x^2y\frac{dy}{dx} + 4xy^2 + 4y^3\frac{dy}{dx} = 8x^2\frac{dy}{dx} + 16xy$$
$$\frac{dy}{dx}\left(4x^2y + 4y^3 - 8x^2\right) = 16xy - 4x^3 - 4xy^2$$
$$\frac{dy}{dx} = \frac{4\left(4xy - x^3 - xy^2\right)}{4\left(x^2y + y^3 - 2x^2\right)}$$
$$\frac{dy}{dx} = \frac{x\left(4y - x^2 - y^2\right)}{x^2y + y^3 - 2x^2}$$

At $(2, 2)$, $\frac{dy}{dx} = 0$.

27. $3x^2 - 2y + 5 = 0$
$$6x - 2\frac{dy}{dx} = 0$$
$$\frac{dy}{dx} = 3x$$

At (1, 4), $\frac{dy}{dx} = 3$.

28. $4x^2 + 2y - 1 = 0$
$$8x + 2\frac{dy}{dx} = 0$$
$$\frac{dy}{dx} = -\frac{8x}{2} = -4x$$
$$\frac{dy}{dx}(-1) = -4(-1) = 4$$

29. $x^2 + y^2 = 4$

$$2x + 2y\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = -\frac{x}{y}$$

At $\left(\sqrt{3}, 1\right)$, $\dfrac{dy}{dx} = -\dfrac{\sqrt{3}}{1} = -\sqrt{3}$.

30. $4x^2 + 9y^2 = 36$

$$8x + 18\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = -\frac{4x}{9y}$$

At $\left(\sqrt{5}, \dfrac{4}{3}\right)$, $\dfrac{dy}{dx} = -\dfrac{4\sqrt{5}}{9(4/3)} = -\dfrac{\sqrt{5}}{3}$.

31. $x^2 - y^3 = 0$

$$2x - 3y^2\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{2x}{3y^2}$$

At $(-1, 1)$, $\dfrac{dy}{dx} = -\dfrac{2}{3}$.

32. $(4 - x)y^2 = x^3$

$$y^2 = \frac{x^3}{4 - x}$$

$$2y\frac{dy}{dx} = \frac{(4 - x)(3x^2) - x^3(-1)}{(4 - x)^2}$$

$$2y\frac{dy}{dx} = \frac{12x^2 - 3x^3 + x^3}{(4 - x)^2}$$

$$2y\frac{dy}{dx} = \frac{12x^2 - 2x^3}{(4 - x)^2}$$

$$\frac{dy}{dx} = -\frac{2x^2(x - 6)}{2y(4 - x)^2}$$

$$\frac{dy}{dx} = -\frac{x^2(x - 6)}{y(4 - x)^2}$$

At $(2, 7)$, $\dfrac{dy}{dx} = 2$.

33. Implicitly: $1 - 2y\dfrac{dy}{dx} = 0$

$$\frac{dy}{dx} = \frac{1}{2y}$$

Explicitly: $y = \pm\sqrt{x - 1}$

$$= \pm(x - 1)^{1/2}$$

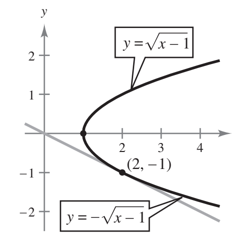
$$\frac{dy}{dx} = \pm\frac{1}{2}(x - 1)^{-1/2}(1)$$

$$= \pm\frac{1}{2\sqrt{x - 1}}$$

$$= \frac{1}{2\left(\pm\sqrt{x - 1}\right)}$$

$$= \frac{1}{2y}$$

At $(2, -1)$, $\dfrac{dy}{dx} = -\dfrac{1}{2}$.



34. Implicitly: $8y\dfrac{dy}{dx} - 2x = 0$

$$\frac{dy}{dx} = \frac{x}{4y}$$

Explicitly: $y = \pm\dfrac{1}{2}\sqrt{x^2 + 7}$

$$= \pm\frac{1}{2}(x^2 + 7)^{1/2}$$

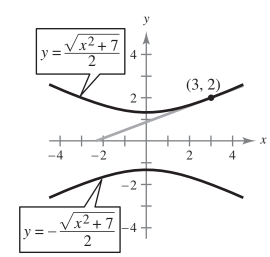
$$\frac{dy}{dx} = \pm\frac{1}{4}(x^2 + 7)^{-1/2}(2x)$$

$$= \pm\frac{x}{2\sqrt{x^2 + 7}}$$

$$= \frac{x}{4\left(\pm\frac{1}{2}\sqrt{x^2 + 7}\right)}$$

$$= \frac{x}{4y}$$

At (3, 2), $\dfrac{dy}{dx} = \dfrac{3}{8}$.



35. $x^2 + y^2 = 100$

$$2x + 2y\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = -\frac{x}{y}$$

At (8, 6):

$$m = -\frac{4}{3}$$

$$y - 6 = -\frac{4}{3}(x - 8)$$

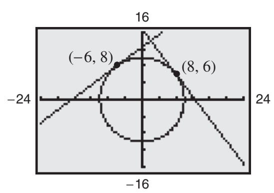
$$y = -\frac{4}{3}x + \frac{50}{3}$$

At $(-6, 8)$:

$$m = \frac{3}{4}$$

$$y - 8 = \frac{3}{4}(x + 6)$$

$$y = \frac{3}{4}x + \frac{25}{2}$$



36. $x^2 + y^2 = 9$

$$2x + 2y\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = -\frac{x}{y}$$

At (0, 3):

$$m = 0$$

$$y - 3 = 0(x - 0)$$

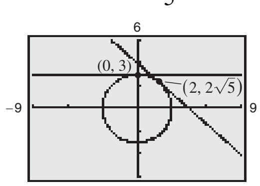
$$y = 3$$

At $\left(2, \sqrt{5}\right)$:

$$m = -\frac{2}{\sqrt{5}} = -\frac{2\sqrt{5}}{5}$$

$$y - \sqrt{5} = -\frac{2\sqrt{5}}{5}(x - 2)$$

$$y = -\frac{2\sqrt{5}}{5}x + \frac{9\sqrt{5}}{5}$$



37. $y^2 = 5x^3$

$$2y\frac{dy}{dx} = 15x^2$$

$$\frac{dy}{dx} = \frac{15x^2}{2y}$$

At $\left(1, \sqrt{5}\right)$:

$$m = \frac{15}{2\sqrt{5}} = \frac{3\sqrt{5}}{2}$$

$$y - \sqrt{5} = \frac{3\sqrt{5}}{2}(x - 1)$$

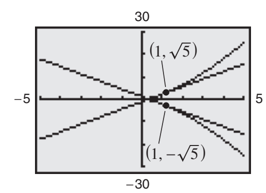
$$y = \frac{3\sqrt{5}}{2}x - \frac{\sqrt{5}}{2}$$

At $\left(1, -\sqrt{5}\right)$:

$$m = \frac{-15}{2\sqrt{5}} = -\frac{3\sqrt{5}}{2}$$

$$y + \sqrt{5} = -\frac{3\sqrt{5}}{2}(x - 1)$$

$$y = -\frac{3\sqrt{5}}{2}x + \frac{\sqrt{5}}{2}$$



38. $4xy + x^2 = 5$

$$4x\frac{dy}{dx} + 4y + 2x = 0$$

$$\frac{dy}{dx} = -\frac{4y + 2x}{4x} = -\frac{2y + x}{2x}$$

At (1, 1):

$$m = -\frac{3}{2}$$

$$y - 1 = -\frac{3}{2}(x - 1)$$

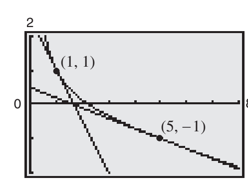
$$y = -\frac{3}{2}x + \frac{5}{2}$$

At $(5, -1)$:

$$m = -\frac{3}{10}$$

$$y + 1 = -\frac{3}{10}(x - 5)$$

$$y = -\frac{3}{10}x + \frac{1}{2}$$



39.

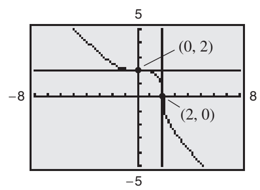
$$x^3 + y^3 = 8$$
$$3x^2 + 3y^2\frac{dy}{dx} = 0$$
$$3y^2\frac{dy}{dx} = -3x^2$$
$$\frac{dy}{dx} = -\frac{x^2}{y^2}$$

At $(0, 2)$:

$$m = \frac{dy}{dx} = 0$$
$$y - 2 = 0(x - 0)$$
$$y = 2$$

At $(2, 0)$:

$m = \frac{dy}{dx}$ is undefined.



The tangent line is $x = 2$.

40.

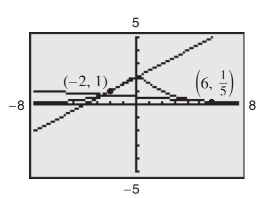
$$x^2y - 8 = -4y$$
$$x^2y + 4y = 8$$
$$y(x^2 + 4) = 8$$
$$y = \frac{8}{x^2 + 4} = 8(x^2 + 4)^{-1}$$
$$\frac{dy}{dx} = 8(-1)(x^2 + 4)^{-2}(2x)$$
$$\frac{dy}{dx} = -\frac{16x}{(x^2 + 4)^2}$$

At $(-2, 1)$:

$$m = \frac{dy}{dx} = -\frac{16(-2)}{((-2)^2 + 4)^2} = \frac{32}{64} = \frac{1}{2}$$
$$y - 1 = \frac{1}{2}(x - (-2))$$
$$y = \frac{1}{2}x + 2$$

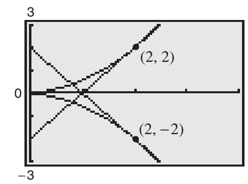
At $\left(6, \frac{1}{5}\right)$:

$$m = \frac{dy}{dx} = -\frac{16(6)}{\left[(6)^2 + 4\right]^2} = -\frac{96}{1600} = -\frac{3}{50}$$
$$y - \frac{1}{5} = -\frac{3}{50}(x - 6)$$
$$y - \frac{1}{5} = -\frac{3}{50}x + \frac{9}{25}$$
$$y = -\frac{3}{50}x + \frac{14}{25}$$



41.

$$y^2 = \frac{x^3}{4 - x}$$
$$2y\frac{dy}{dx} = \frac{(4 - x)(3x^2) - (x^3)(-1)}{(4 - x)^2}$$
$$2y\frac{dy}{dx} = \frac{2x^2(6 - x)}{(4 - x)^2}$$
$$\frac{dy}{dx} = \frac{x^2(6 - x)}{y(4 - x)^2}$$



At $(2, 2)$:

$$m = 2$$
$$y - 2 = 2(x - 2)$$
$$y = 2x - 2$$

At $(2, -2)$:

$$m = -2$$
$$y + 2 = -2(x - 2)$$
$$y = -2x + 2$$

42.

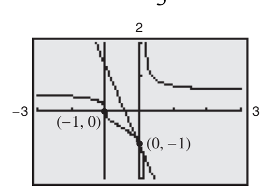
$$x + y^3 = 6xy^3 - 1$$
$$y^3 - 6xy^3 = -1 - x$$
$$y^3(1 - 6x) = -(1 + x)$$
$$y^3 = \frac{x + 1}{6x - 1}$$
$$3y^2\frac{dy}{dx} = \frac{(6x - 1)(1) - (x + 1)(6)}{(6x - 1)^2}$$
$$3y^2\frac{dy}{dx} = \frac{6x - 1 - 6x - 6}{(6x - 1)^2}$$
$$\frac{dy}{dx} = -\frac{7}{3y^2(6x - 1)^2}$$

At $(-1, 0)$:

$m = \frac{dy}{dx}$ is undefined. The tangent line is $x = -1$.

At $(0, -1)$:

$$m = \frac{dy}{dx} = -\frac{7}{3}$$
$$y - (-1) = -\frac{7}{3}(x - 0)$$
$$y = -\frac{7}{3}x - 1$$



43. $p = \dfrac{2}{0.00001x^3 + 0.1x}, x \geq 0$

$$0.00001x^3 + 0.1x = \frac{2}{p}$$
$$0.00003x^2\frac{dx}{dp} + 0.1\frac{dx}{dp} = -\frac{2}{p^2}$$
$$\left(0.00003x^2 + 0.1\right)\frac{dx}{dp} = -\frac{2}{p^2}$$
$$\frac{dx}{dp} = -\frac{2}{p^2\left(0.00003x^2 + 0.1\right)}$$

44. $p = \dfrac{4}{0.000001x^2 + 0.05x + 1}, x \geq 0$

$$0.000001x^2 + 0.05x + 1 = \frac{4}{p}$$
$$0.000002x\frac{dx}{dp} + 0.05\frac{dx}{dp} = -\frac{4}{p^2}$$
$$(0.000002x + 0.05)\frac{dx}{dp} = -\frac{4}{p^2}$$
$$\frac{dx}{dp} = -\frac{4}{p^2(0.000002x + 0.05)}$$

45. $p = \sqrt{\dfrac{200 - x}{2x}}, 0 < x \leq 200$

$$2xp^2 = 200 - x$$
$$2x(2p) + p^2\left(2\frac{dx}{dp}\right) = -\frac{dx}{dp}$$
$$\left(2p^2 + 1\right)\frac{dx}{dp} = -4xp$$
$$\frac{dx}{dp} = -\frac{4xp}{2p^2 + 1}$$

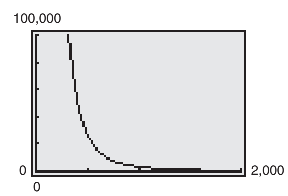
46. $p = \sqrt{\dfrac{500 - x}{2x}}, 0 < x \leq 500$

$$2xp^2 = 500 - x$$
$$2x(2p) + p^2\left(2\frac{dx}{dp}\right) = -\frac{dx}{dp}$$
$$\frac{dx}{dp}\left(2p^2 + 1\right) = -4xp$$
$$\frac{dx}{dp} = -\frac{4xp}{2p^2 + 1}$$

47. (a)
$$100x^{0.75}y^{0.25} = 135{,}540$$
$$100x^{0.75}\left(0.25y^{-0.75}\frac{dy}{dx}\right) + y^{0.25}\left(75x^{-0.25}\right) = 0$$
$$\frac{25x^{0.75}}{y^{0.75}} \cdot \frac{dy}{dx} = -\frac{75y^{0.25}}{x^{0.25}}$$
$$\frac{dy}{dx} = -\frac{3y}{x}$$

When $x = 1500$ and $y = 1000$, $\dfrac{dy}{dx} = -2$.

(b)

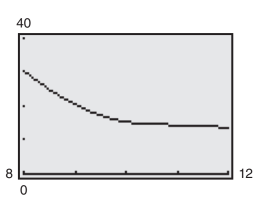


If more labor is used, then less capital is available.
If more capital is used, then less labor is available.

48. (a) As price increases, the demand decreases.

(b) For $x > 0$, the rate of change of demand, x, with respect to the price, p, is always decreasing; that is, for $x > 0$, $\dfrac{dx}{dp}$ is never increasing.

49. (a)
$$y^2 - 35{,}892.5 = -27.0021t^3 + 888.789t^2 - 9753.25t$$
$$y^2 = -27.0021t^3 + 888.789t^2 - 9753.25t + 35{,}892.5$$
$$y = \pm\sqrt{-27.0021t^3 + 888.789t^2 - 9753.25t + 35{,}892.5}$$



The numbers of cases of Chickenpox decreases from 2008 to 2012.

(b) It appears that the number of reported cases was decreasing at the greatest rate during 2008, $t = 8$.

(c) $$y^2 - 35{,}892.5 = -27.0021t^3 + 888.789t^2 - 9753.25t$$
$$2y\frac{dy}{dt} = -81.0063t^2 + 1777.578t - 9753.25t$$
$$y' = \frac{dy}{dt} = \frac{-81.0063t^2 + 1777.578t - 9753.25}{2y}$$

t	8	9	10	11	12
y	30.40	20.51	15.39	14.51	13.40
y'	−11.79	−7.71	−2.54	−0.06	−3.26

The table of values for y' agrees with the answer in part (b) when the greatest value of y' is −11.79 thousand cases per year.

Section 2.8 Related Rates

Skills Warm Up

1. $A = \pi r^2$

2. $V = \frac{4}{3}\pi r^3$

3. $SA = 6s^2$

4. $V = s^3$

5. $V = \frac{1}{3}\pi r^2 h$

6. $A = \frac{1}{2}bh$

7. $$\begin{aligned} x^2 + y^2 &= 9 \\ \frac{d}{dx}\left[x^2 + y^2\right] &= \frac{d}{dx}[9] \\ 2x + 2y\frac{dy}{dx} &= 0 \\ 2y\frac{dy}{dx} &= -2x \\ \frac{dy}{dx} &= \frac{-2x}{2y} \\ &= \frac{-x}{y} \end{aligned}$$

8. $$\begin{aligned} 3xy - x^2 &= 6 \\ \frac{d}{dx}\left[3xy - x^2\right] &= \frac{d}{dx}[6] \\ 3y + 3x\frac{dy}{dx} - 2x &= 0 \\ 3x\frac{dy}{dx} &= 2x - 3y \\ \frac{dy}{dx} &= \frac{2x - 3y}{3x} \end{aligned}$$

9. $$\begin{aligned} x^2 + 2y + xy &= 12 \\ \frac{d}{dx}\left[x^2 + 2y + xy\right] &= \frac{d}{dx}(12) \\ 2x + 2\frac{dy}{dx} + y + x\frac{dy}{dx} &= 0 \\ 2\frac{dy}{dx} + x\frac{dy}{dx} &= -y - 2x \\ \frac{dy}{dx}(2 + x) &= -y - 2x \\ \frac{dy}{dx} &= \frac{-y - 2x}{2 + x} \end{aligned}$$

10. $$\begin{aligned} x + xy^2 - y^2 &= xy \\ \frac{d}{dx}\left[x + xy^2 - y^2\right] &= \frac{d}{dx}[xy] \\ 1 + y^2 + 2xy\frac{dy}{dx} - 2y\frac{dy}{dx} &= y + x\frac{dy}{dx} \\ 2xy\frac{dy}{dx} - 2y\frac{dy}{dx} - x\frac{dy}{dx} &= y - y^2 - 1 \\ \frac{dy}{dx}(2xy - 2y - x) &= y - y^2 - 1 \\ \frac{dy}{dx} &= \frac{y - y^2 - 1}{2xy - 2y - x} \end{aligned}$$

1. $y = \sqrt{x}$, $\dfrac{dy}{dt} = \dfrac{1}{2}x^{-1/2}\dfrac{dx}{dt} = \dfrac{1}{2\sqrt{x}}\dfrac{dx}{dt}$, $\dfrac{dx}{dt} = 2\sqrt{x}\dfrac{dy}{dt}$

(a) When $x = 4$ and $\dfrac{dx}{dt} = 3$, $\dfrac{dy}{dt} = \left(\dfrac{1}{2\sqrt{4}}\right)(3) = \dfrac{3}{4}$.

(b) When $x = 25$ and $\dfrac{dy}{dt} = 2$, $\dfrac{dx}{dt} = 2\sqrt{25}(2) = 20$.

2. $y = 3x^2 - 5x$, $\dfrac{dy}{dt} = 6x\dfrac{dx}{dt} - 5\dfrac{dx}{dt}$, $\dfrac{dy}{dt} = (6x - 5)\dfrac{dx}{dt}$, $\dfrac{\frac{dy}{dt}}{6x - 5} = \dfrac{dx}{dt}$

(a) When $x = 3$ and $\dfrac{dx}{dt} = 2$, $\dfrac{dy}{dt} = \big(6(3) - 5(2)\big) = 26$.

(b) When $x = 2$ and $\dfrac{dy}{dt} = 4$, $\dfrac{4}{6(2) - 5} = \dfrac{4}{7} = \dfrac{dx}{dt}$.

3. $xy = 4$, $x\dfrac{dy}{dt} + y\dfrac{dx}{dt} = 0$, $\dfrac{dy}{dt} = \left(-\dfrac{y}{x}\right)\dfrac{dx}{dt}$, $\dfrac{dx}{dt} = \left(-\dfrac{x}{y}\right)\dfrac{dy}{dt}$

(a) When $x = 8$, $y = \dfrac{1}{2}$, and $\dfrac{dx}{dt} = 10$, $\dfrac{dy}{dt} = -\dfrac{1/2}{8}(10) = -\dfrac{5}{8}$.

(b) When $x = 1$, $y = 4$, and $\dfrac{dy}{dt} = -6$, $\dfrac{dx}{dt} = -\dfrac{1}{4}(-6) = \dfrac{3}{2}$.

4. $x^2 + y^2 = 25$, $2x\dfrac{dx}{dt} + 2y\dfrac{dy}{dt} = 0$, $\dfrac{dy}{dt} = -\dfrac{x}{y}\dfrac{dx}{dt}$, $\dfrac{dx}{dt} = -\dfrac{y}{x}\dfrac{dy}{dt}$

(a) When $x = 3$, $y = 4$, and $\dfrac{dx}{dt} = 8$, $\dfrac{dy}{dt} = -\dfrac{3}{4}(8) = -6$.

(b) When $x = 4$, $y = 3$, and $\dfrac{dy}{dt} = -2$, $\dfrac{dx}{dt} = -\dfrac{3}{4}(-2) = \dfrac{3}{2}$.

5. $A = \pi r^2$, $\dfrac{dr}{dt} = 3$, $\dfrac{dA}{dt} = 2\pi r\dfrac{dr}{dt} = 6\pi r$

(a) When $r = 6$, $\dfrac{dA}{dt} = 2\pi(6)(3) = 36\pi$ in.^{2}/min.

(b) When $r = 24$, $\dfrac{dA}{dt} = 2\pi(24)(3) = 144\pi$ in.^{2}/min.

6. $V = \dfrac{4}{3}\pi r^3$, $\dfrac{dr}{dt} = 3$, $\dfrac{dV}{dt} = 4\pi r^2\dfrac{dr}{dt} = 12\pi r^2$

(a) When $r = 9$, $\dfrac{dV}{dt} = 12\pi(9)^2 = 972\pi$ in.^{3}/min.

(b) When $r = 16$, $\dfrac{dV}{dt} = 12\pi(16)^2 = 3072\pi$ in.^{3}/min.

7. $A = \pi r^2$, $\dfrac{dA}{dt} = 2\pi r\dfrac{dr}{dt}$

If $\dfrac{dr}{dt}$ is constant, then $\dfrac{dA}{dt}$ is not constant; $\dfrac{dA}{dt}$ is proportional to r.

8. $V = \dfrac{4}{3}\pi r^3$, $\dfrac{dV}{dt} = 4\pi r^2\dfrac{dr}{dt}$

If $\dfrac{dr}{dt}$ is constant, $\dfrac{dV}{dt}$ is *not* constant since it is proportional to the square of r.

9. $V = \dfrac{4}{3}\pi r^3$, $\dfrac{dV}{dt} = 10$, $\dfrac{dV}{dt} = 4\pi r^2\dfrac{dr}{dt}$,

$\dfrac{dr}{dt} = \left(\dfrac{1}{4\pi r^2}\right)\dfrac{dV}{dt}$

(a) When $r = 1$, $\dfrac{dr}{dt} = \dfrac{1}{4\pi(1)^2}(10) = \dfrac{5}{2\pi}$ ft/min.

(b) When $r = 2$, $\dfrac{dr}{dt} = \dfrac{1}{4\pi(2)^2}(10) = \dfrac{5}{8\pi}$ ft/min.

10. $V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi r^2(3r) = \pi r^3$

$\frac{dV}{dt} = 3\pi r^2 \frac{dr}{dt} = 6\pi r^2$

(a) When $r = 6$, $\frac{dV}{dt} = 6\pi(6)^2 = 216\pi$ in.^{3}/min.

(b) When $r = 24$, $\frac{dV}{dt} = 6\pi(24)^2 = 3456\pi$ in.^{3}/min.

11. (a) $\frac{dC}{dt} = 0.75\frac{dx}{dt} = 0.75(150)$
$= 112.5$ dollars per week

(b) $\frac{dR}{dt} = 250\frac{dx}{dt} - \frac{1}{5}x\frac{dx}{dt}$
$= 250(150) - \frac{1}{5}(1000)(150)$
$= 7500$ dollars per week

(c) $P = R - C$
$\frac{dP}{dt} = \frac{dR}{dt} - \frac{dC}{dt} = 7500 - 112.5$
$= 7387.5$ dollars per week

12. (a) $\frac{dC}{dt} = 1.05\frac{dx}{dt} = 1.05(250) = 262.5$ dollars/week

(b) $\frac{dR}{dt} = \left(500 - \frac{2x}{25}\right)\frac{dx}{dt} = \left(500 - \frac{2(5000)}{25}\right)(250)$
$= 25{,}000$ dollars/week

(c) $P = R - C$
$\frac{dP}{dt} = \frac{dR}{dt} - \frac{dC}{dt} = 25{,}000 - 262.5$
$= 24{,}737.5$ dollars/week

13. $R = 1200x - x^2$, $\frac{dR}{dt} = 1200\frac{dx}{dt} - 2x\frac{dx}{dt}$,

$\frac{dR}{dt} = (1200 - 2x)\frac{dx}{dt}$

(a) When $\frac{dx}{dt} = 23$ units/day and $x = 300$ units,

$\frac{dR}{dt} = [1200 - 2(300)](23) = \$13{,}800$ per day.

(b) When $\frac{dx}{dt} = 23$ units/day and $x = 450$ units,

$\frac{dR}{dt} = [1200 - 2(450)](23) = \$6900$ per day.

14. $R = 510x - 0.3x^2$, $\frac{dR}{dt} = 510\frac{dx}{dt} - 0.6x\frac{dx}{dt}$,

$\frac{dR}{dt} = (510 - 0.6x)\frac{dx}{dt}$

(a) When $\frac{dx}{dt} = 9$ units/day and $x = 400$ units,

$\frac{dR}{dt} = [510 - 0.6(400)](9) = \$2430$ per day.

(b) When $\frac{dx}{dt} = 9$ units/day and $x = 600$ units,

$\frac{dR}{dt} = [510 - 0.6(600)](9) = \$1350$ per day.

15. $V = x^3$, $\frac{dx}{dt} = 6$, $\frac{dV}{dt} = 3x^2\frac{dx}{dt}$

(a) When $x = 2$, $\frac{dV}{dt} = 3(2)^2(6) = 72$ cm^3/sec.

(b) When $x = 10$, $\frac{dV}{dt} = 3(10)^2(6) = 1800$ cm^3/sec.

16. $A = 6x^2$, $\frac{dx}{dt} = 6$, $\frac{dA}{dt} = 12x\frac{dx}{dt}$

(a) When $x = 2$, $\frac{dA}{dt} = 12(2)(6) = 144$ cm^2/sec.

(b) When $x = 10$, $\frac{dA}{dt} = 12(10)(6) = 720$ cm^2/sec.

17. Let x be the distance from the boat to the dock and y be the length of the rope.

$12^2 + x^2 = y^2$

$\frac{dy}{dt} = -4$

$2x\frac{dx}{dt} = 2y\frac{dy}{dt}$

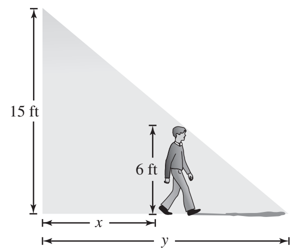
$\frac{dx}{dt} = \frac{y}{x}\frac{dy}{dt}$

When $y = 13$, $x = 5$ and

$\frac{dx}{dt} = \frac{13}{5}(-4) = -10.4$ ft/sec.

As $x \to 0$, $\frac{dx}{dt}$ increases.

18.



(a) $\dfrac{15}{6} = \dfrac{y}{y - x} \Rightarrow 15y - 15x = 6y$

$9y = 15x$

$y = \dfrac{5}{3}x$

Find $\dfrac{dy}{dt}$ if $\dfrac{dx}{dt} = 5$ ft/sec when $x = 10$ ft.

$\dfrac{dy}{dt} = \dfrac{5}{3}\dfrac{dx}{dt}$

$\dfrac{dy}{dt} = \dfrac{5}{3}(5) = \dfrac{25}{3}$ ft/sec

(b) Find $\dfrac{d}{dt}(y - x)$ if $\dfrac{dx}{dt} = 5$ ft/sec and

$\dfrac{dy}{dt} = \dfrac{25}{3}$ ft/sec when $x = 10$ ft.

$\dfrac{d}{dt}(y - x) = \dfrac{dy}{dt} - \dfrac{dx}{dt}$

$= \dfrac{25}{3} - 5 = \dfrac{10}{3}$ ft/sec

19. $x^2 + 6^2 = s^2$

$2x\dfrac{dx}{dt} = 2s\dfrac{ds}{dt}$

$\dfrac{dx}{dt} = \dfrac{s}{x}\dfrac{ds}{dt}$

When $s = 10$, $x = 8$ and $\dfrac{ds}{dt} = 240$:

$\dfrac{dx}{dt} = \dfrac{10}{8}(-240) = 300$ mi/hr.

20. $s^2 = 90^2 + x^2$, $x = 26$, $\dfrac{dx}{dt} = -30$

$2s\dfrac{ds}{dt} = 2x\dfrac{dx}{dt}$

$\dfrac{ds}{dt} = \dfrac{x}{s}\dfrac{dx}{dt}$

When $x = 26$,

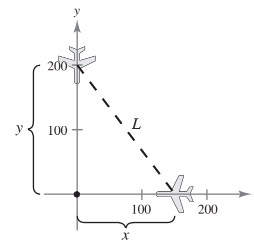
$\dfrac{ds}{dt} = \dfrac{26}{\sqrt{90^2 + 26^2}}(-30) \approx -8.33$ ft/sec.

21. (a) $L^2 = x^2 + y^2$, $\dfrac{dx}{dt} = -450$, $\dfrac{dy}{dt} = -600$, and

$\dfrac{dL}{dt} = \dfrac{x(dx/dt) + y(dy/dt)}{L}$

When $x = 150$ and $y = 200$, $L = 250$ and

$\dfrac{dL}{dt} = \dfrac{150(-450) + 200(-600)}{250} = -750$ mph.



(b) $t = \dfrac{250}{750} = \dfrac{1}{3}$ hr $= 20$ min

22. $S = 2250 + 50x + 0.35x^2$

$\dfrac{dS}{dt} = 50\dfrac{dx}{dt} + 0.70x\dfrac{dx}{dt}$

$\dfrac{dS}{dt} = 50(125) + 0.70(1500)(125)$

$= \$137{,}500$ per week

23. $V = \pi r^2 h$, $h = 0.08$, $V = 0.08\pi r^2$, $\dfrac{dV}{dt} = 0.16\pi r\dfrac{dr}{dt}$

When $r = 150$ and $\dfrac{dr}{dt} = \dfrac{1}{2}$,

$\dfrac{dV}{dt} = 0.16\pi(150)\left(\dfrac{1}{2}\right) = 12\pi = 37.70$ ft^3/min.

24. $P = R - C$

$= xp - C$

$= x(50 - 0.01x) - (4000 + 40x - 0.02x^2)$

$= 50x - 0.01x^2 - 4000 - 40x + 0.02x^2$

$= 0.01x^2 + 10x - 4000$

$\dfrac{dP}{dt} = 0.02x\dfrac{dx}{dt} + 10\dfrac{dx}{dt}$

When $x = 800$ and $\dfrac{dx}{dt} = 25$,

$\dfrac{dP}{dt} = 0.02(800)(25) + (10)(25) = \$650$/week.

25. $P = R - C = xp - C = x(6000 - 25x) - (2400x + 5200)$

$= -25x^2 + 3600x - 5200$

$$\frac{dP}{dt} = -50x\frac{dx}{dt} + 3600\frac{dx}{dt}$$

$$\frac{dx}{dt} = \frac{1}{3600 - 50x}\frac{dP}{dt}$$

When $x = 44$ and $\frac{dP}{dt} = 5600$, $\frac{dx}{dt} = \frac{1}{3600 - 50(44)}(5600) = 4$ units per week.

26. (a) For supply, if $\frac{dx}{dt}$ is negative, then $\frac{dp}{dt}$ is negative. For demand, if $\frac{dx}{dt}$ is negative, then $\frac{dp}{dt}$ is positive.

(b) For supply, if $\frac{dp}{dt}$ is positive, then $\frac{dx}{dt}$ is positive. For demand, if $\frac{dp}{dt}$ is positive, then $\frac{dx}{dt}$ is negative.

Review Exercises for Chapter 2

1. Slope $\approx \frac{-4}{2} = -2$

2. Slope $\approx \frac{4}{2} = 2$

3. Slope ≈ 0

4. Slope $\approx \frac{-2}{4} = -\frac{1}{2}$

5. Answers will vary. Sample answer:

$t = 8$; slope $\approx$ \$225 million/yr; Revenue was increasing by about \$225 million per year in 2008.

$t = 10$; slope $\approx$ \$350 million/yr; Revenue was increasing by about \$350 million per year in 2010.

6. Answers will vary. Sample answer:

$t = 10$; slope ≈ -20 thousand/year; The number of farms was decreasing by about 20 thousand per year in 2010.

$t = 12$; slope ≈ -10 thousand/year; The number of farms was decreasing by about 10 thousand per year in 2012.

7. Answers will vary. Sample answer:

$t = 1$: $m \approx 65$ hundred thousand visitors/month; The number of visitors to the national park is increasing at about 65,000,000/per month in January.

$t = 8$: $m \approx 0$ visitors/month; The number of visitors to the national park is neither increasing nor decreasing in August.

$t = 12$: $m \approx -1000$ hundred thousand/month; The number of visitors to the national park is decreasing at about 1,000,000,000 visitors per month in December.

8. (a) At t_1, the slope of $g(t)$ is greater than the slope of $f(t)$, so the rafter whose progress is given by $g(t)$ is traveling faster.

(b) At t_2, the slope of $f(t)$ is greater than the slope of $g(t)$, so the rafter whose progress is given by $f(t)$ is traveling faster.

(c) At t_3, the slope of $f(t)$ is greater than the slope of $g(t)$, so the rafter whose progress is given by $f(t)$ is traveling faster.

(d) The rafter whose progress is given by $f(t)$ finishes first. The value of t where $f(t) = 9$ is smaller than the value of t where $g(t) = 9$.

9. $f(x) = -3x - 5; (-2, 1)$

$$\begin{aligned} f'(x) &= \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} \\ &= \lim_{\Delta x \to 0} \frac{-3(x + \Delta x) - 5 - (-3x - 5)}{\Delta x} \\ &= \lim_{\Delta x \to 0} \frac{-3\Delta x}{\Delta x} = -3 \end{aligned}$$

$f'(-2) = -3$

10. $f(x) = 7x + 3; (-1, -4)$

$$\begin{aligned} f'(x) &= \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} \\ &= \lim_{\Delta x \to 0} \frac{7(x + \Delta x) + 3 - (7x + 3)}{\Delta x} \\ &= \lim_{\Delta x \to 0} \frac{7\Delta x}{\Delta x} = 7 \end{aligned}$$

$f'(-1) = 7$

11. $f(x) = x^2 + 9;\ (3, 18)$

$$\begin{aligned}
f'(x) &= \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} \\
&= \lim_{\Delta x \to 0} \frac{(x + \Delta x)^2 + 9 - (x^2 + 9)}{\Delta x} \\
&= \lim_{\Delta x \to 0} \frac{x^2 + 2x\Delta x + (\Delta x)^2 + 9 - x^2 - 9}{\Delta x} \\
&= \lim_{\Delta x \to 0} \frac{2x\Delta x + (\Delta x)^2}{\Delta x} \\
&= \lim_{\Delta x \to 0} \frac{\Delta x(2x + \Delta x)}{\Delta x} \\
&= \lim_{\Delta x \to 0} (2x + \Delta x) = 2x
\end{aligned}$$

$f'(3) = 2(3) = 6$

12. $f(x) = x^2 - 7x;\ (1, -6)$

$$\begin{aligned}
f'(x) &= \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} \\
&= \lim_{\Delta x \to 0} \frac{(x + \Delta x)^2 - 7(x + \Delta x) - (x^2 - 7x)}{\Delta x} \\
&= \lim_{\Delta x \to 0} \frac{x^2 + 2x\Delta x + (\Delta x)^2 - 7x - 7\Delta x - x^2 + 7x}{\Delta x} \\
&= \lim_{\Delta x \to 0} \frac{2x\Delta x + (\Delta x)^2 - 7\Delta x}{\Delta x} \\
&= \lim_{\Delta x \to 0} \frac{\Delta x(2x + \Delta x - 7)}{\Delta x} \\
&= \lim_{\Delta x \to 0} (2x + \Delta x - 7) = 2x - 7
\end{aligned}$$

$f'(1) = 2(1) - 7 = -5$

13. $f(x) = \sqrt{x + 9};\ (-5, 2)$

$$\begin{aligned}
f'(x) &= \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} \\
&= \lim_{\Delta x \to 0} \frac{\sqrt{x + \Delta x + 9} - \sqrt{x + 9}}{\Delta x} \cdot \frac{\sqrt{x + \Delta x + 9} + \sqrt{x + 9}}{\sqrt{x + \Delta x + 9} + \sqrt{x + 9}} \\
&= \lim_{\Delta x \to 0} \frac{(x + \Delta x + 9) - (x + 9)}{\Delta x\left[\sqrt{x + \Delta x + 9} + \sqrt{x + 9}\right]} \\
&= \lim_{\Delta x \to 0} \frac{1}{\sqrt{x + \Delta x + 9} + \sqrt{x + 9}} = \frac{1}{2\sqrt{x + 9}}
\end{aligned}$$

$f'(-5) = \dfrac{1}{4}$

14. $f(x) = \sqrt{x - 1};\ (10, 3)$

$$\begin{aligned}
f'(x) &= \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} \\
&= \lim_{\Delta x \to 0} \frac{\sqrt{x + \Delta x - 1} - \sqrt{x - 1}}{\Delta x} \cdot \frac{\sqrt{x + \Delta x - 1} + \sqrt{x - 1}}{\sqrt{x + \Delta x - 1} + \sqrt{x - 1}} \\
&= \lim_{\Delta x \to 0} \frac{(x + \Delta x - 1) - (x - 1)}{\Delta x\left[\sqrt{x + \Delta x - 1} + \sqrt{x - 1}\right]} \\
&= \lim_{\Delta x \to 0} \frac{1}{\sqrt{x + \Delta x - 1} + \sqrt{x - 1}} = \frac{1}{2\sqrt{x - 1}}
\end{aligned}$$

$f'(10) = \dfrac{1}{6}$

15. $f(x) = \dfrac{1}{x-5}$; $(6, 1)$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\dfrac{1}{x+\Delta x-5} - \dfrac{1}{x-5}}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{(x-5) - (x+\Delta x-5)}{\Delta x(x+\Delta x-5)(x-5)}$$

$$= \lim_{\Delta x \to 0} \frac{-1}{(x+\Delta x-5)(x-5)} = -\frac{1}{(x-5)^2}$$

$$f'(6) = -1$$

16. $f(x) = \dfrac{1}{x+6}$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\dfrac{1}{x+\Delta x+6} - \dfrac{1}{x+6}}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{(x+6) - (x+\Delta x+6)}{\Delta x\left[(x+\Delta x+6)(x+6)\right]}$$

$$= \lim_{\Delta x \to 0} \frac{-1}{(x+\Delta x+6)(x+6)}$$

$$= -\frac{1}{(x+6)^2}$$

$$f'(-3) = -\frac{1}{(-2+6)^2} = -\frac{1}{16}$$

17. $f(x) = 9x + 1$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\left[9(x+\Delta x)+1\right] - (9x+1)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{9x + 9\Delta x + 1 - 9x - 1}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{9\Delta x}{\Delta x}$$

$$= \lim_{\Delta x \to 0} 9 = 9$$

18. $f(x) = 1 - 4x$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\left[1 - 4(x+\Delta x)\right] - (1-4x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{1 - 4x - 4\Delta x - 1 + 4x}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{-4\Delta x}{\Delta x}$$

$$= \lim_{\Delta x \to 0} -4 = -4$$

19. $f(x) = -\dfrac{1}{2}x^2 + 2x$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\left[-\frac{1}{2}(x+\Delta x)^2 + 2(x+\Delta x)\right] - \left(-\frac{1}{2}x^2 + 2x\right)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{-\frac{1}{2}x^2 - x(\Delta x) - \frac{1}{2}(\Delta x)^2 + 2x + 2\Delta x + \frac{1}{2}x^2 - 2x}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\Delta x\left(-x - \frac{1}{2}\Delta x + 2\right)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \left(-x - \tfrac{1}{2}\Delta x + 2\right) = -x + 2$$

20. $f(x) = 3x^2 - \frac{1}{4}x$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\left[3(x + \Delta x)^2 - \frac{1}{4}(x + \Delta x)\right] - \left(3x^2 - \frac{1}{4}x\right)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{3x^2 + 6x(\Delta x) + 3(\Delta x)^2 - \frac{1}{4}x - \frac{1}{4}\Delta x - 3x^2 + \frac{1}{4}x}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{6x(\Delta x) + 3(\Delta x)^2 - \frac{1}{4}\Delta x}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\Delta x\left(6x + 3(\Delta x) - \frac{1}{4}\right)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \left(6x + 3(\Delta x) - \frac{1}{4}\right) = 6x - \frac{1}{4}$$

21. $f(x) = \sqrt{x - 5}$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\sqrt{x + \Delta x - 5} - \sqrt{x - 5}}{\Delta x} \cdot \frac{\sqrt{x + \Delta x - 5} + \sqrt{x - 5}}{\sqrt{x + \Delta x - 5} + \sqrt{x - 5}}$$

$$= \lim_{\Delta x \to 0} \frac{(x + \Delta x - 5) - (x - 5)}{\Delta x\left(\sqrt{x + \Delta x - 5} + \sqrt{x - 5}\right)}$$

$$= \lim_{\Delta x \to 0} \frac{1}{\sqrt{x + \Delta x - 5} + \sqrt{x - 5}} = \frac{1}{2\sqrt{x - 5}}$$

22. $f(x) = \sqrt{x} + 3$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\left[\sqrt{x + \Delta x} + 3\right] - \left(\sqrt{x} + 3\right)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\sqrt{x + \Delta x} - \sqrt{x}}{\Delta x} \cdot \frac{\sqrt{x + \Delta x} + \sqrt{x}}{\sqrt{x + \Delta x} + \sqrt{x}}$$

$$= \lim_{\Delta x \to 0} \frac{(x + \Delta x) - x}{\Delta x\left(\sqrt{x + \Delta x} + \sqrt{x}\right)}$$

$$= \lim_{\Delta x \to 0} \frac{1}{\sqrt{x + \Delta x} + \sqrt{x}} = \frac{1}{2\sqrt{x}}$$

23. $f(x) = \frac{5}{x}$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\frac{5}{x + \Delta x} - \frac{5}{x}}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\frac{5x - 5(x + \Delta x)}{x(x + \Delta x)}}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{-5\Delta x}{\Delta x\left[x(x + \Delta x)\right]}$$

$$= \lim_{\Delta x \to 0} -\frac{5}{x(x + \Delta x)} = -\frac{5}{x^2}$$

24. $f(x) = \dfrac{1}{x+4}$

$$f'(x) = \lim_{\Delta x\to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$= \lim_{\Delta x\to 0} \frac{\dfrac{1}{x+\Delta x+4} - \dfrac{1}{x+4}}{\Delta x}$$

$$= \lim_{\Delta x\to 0} \frac{\dfrac{(x+4)-(x+\Delta x+4)}{(x+4)(x+\Delta x+4)}}{\Delta x}$$

$$= \lim_{\Delta x\to 0} \frac{-\Delta x}{\Delta x\left[(x+4)(x+\Delta x+4)\right]}$$

$$= \lim_{\Delta x\to 0} -\frac{1}{(x+4)(x+\Delta x+4)} = -\frac{1}{(x+4)^2}$$

25. y is not differentiable at $x = -1$. At $(-1, 0)$, the graph has a vertical tangent line.

26. y is not differentiable at $x = 0$. At $(0, 3)$, the graph has a node.

27. y is not differentiable at $x = 0$. The function is discontinuous at $x = 0$.

28. y is not differentiable at $x = -1$. At $(-1, 0)$, the graph has a cusp.

29. $y = -6$
$y' = 0$

30. $f(x) = 5$
$f'(x) = 0$

31. $f(x) = x^7$
$f'(x) = 7x^6$

32. $h(x) = \dfrac{1}{x^{-4}}$
$h(x) = x^{-4}$
$h'(x) = -4x^{-5}$
$h'(x) = \dfrac{-4}{x^5}$

33. $f(x) = 4x^2$
$f'(x) = 8x$

34. $g(t) = 8t^6$
$g'(t) = 48t^5$

35. $f(x) = \dfrac{5x^3}{4}$
$f'(x) = \dfrac{15x^2}{4}$

36. $y = 3x^{2/3}$
$y' = 2x^{-1/3}$
$y' = \dfrac{2}{x^{1/3}}$

37. $g(x) = 2x^4 + 3x^2$
$g'(x) = 8x^3 + 6x$

38. $f(x) = 6x^2 - 4x$
$f'(x) = 12x - 4$

39. $y = x^2 + 6x - 7$
$y' = 2x + 6$

40. $y = 2x^4 - 3x^3 + x$
$y' = 8x^3 - 9x^2 + 1$

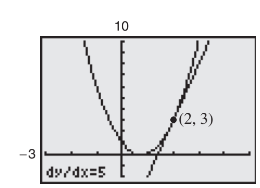
41. $f(x) = 2x^{-1/2}$; $(4, 1)$
$f'(x) = -x^{-3/2}$
$f'(4) = -(4)^{-3/2} = -0.125$

42. $y = \dfrac{3}{2x} + 3$; $\left(\dfrac{1}{2}, 6\right)$
$y = \dfrac{3}{2}x^{-1} + 3$
$y' = -\dfrac{3}{2}x^{-2} = -\dfrac{3}{2x^2}$
$y'\left(\dfrac{1}{2}\right) = \dfrac{3}{2\left(\frac{1}{2}\right)^2} = 6$

43. $g(x) = x^3 - 4x^2 - 6x + 8$; $(-1, 9)$
$g'(x) = 3x^2 - 8x - 6$
$g'(-1) = 3(-1)^2 - 8(-1) - 6 = 5$

44. $y = 2x^4 - 5x^3 + 6x^2 - x$; $(1, 2)$
$y' = 8x^3 - 15x^2 + 12x - 1$
$y'(1) = 8(1)^3 - 15(1)^2 + 12(1) - 1 = 4$

45. $f'(x) = 4x - 3$
$f'(2) = 5$
$y - 3 = 5(x - 2)$
$y = 5x - 7$

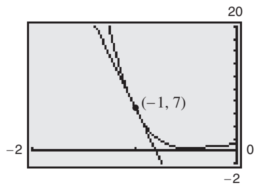


46. $y' = 44x^3 - 10x$

$y'(-1) = -34$

$y - 7 = -34(x + 1)$

$y = -34x - 27$



47. $f(x) = \sqrt{x} - \dfrac{1}{\sqrt{x}} = x^{1/2} - x^{-1/2}$

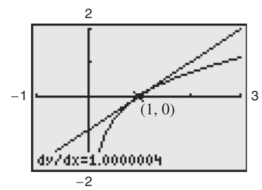
$f'(x) = \dfrac{1}{2}x^{-1/2} + \dfrac{1}{2}x^{-3/2}$

$= \dfrac{1}{2\sqrt{x}} + \dfrac{1}{2x^{3/2}}$

$f'(1) = 1$

$y - 0 = 1(x - 1)$

$y = x - 1$



48. $f(x) = \sqrt[3]{x} - x = x^{1/3} - x$

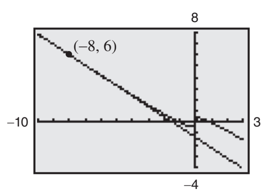
$f'(x) = \dfrac{1}{3}x^{-2/3} - 1 = \dfrac{1}{3\sqrt[3]{x^2}} - 1$

$f'(-8) = \dfrac{1}{3\sqrt[3]{(-8)^2}} - 1 = \dfrac{1}{12} - 1 = -\dfrac{11}{12}$

$y - 6 = -\dfrac{11}{12}(x + 8)$

$y - 6 = -\dfrac{11}{12}x - \dfrac{22}{3}$

$y = -\dfrac{11}{12}x - \dfrac{4}{3}$



49. $R = -0.5972t^3 + 51.187t^2 - 485.54t + 2199.0$

$\dfrac{dR}{dt} = R'(t) = -1.7916t^2 + 102.374t - 485.54$

(a) 2008: $R'(8) = m \approx 218.8$

2010: $R'(10) = m \approx 359.0$

(b) Results should be similar.

(c) The slope shows the rate at which sales were increasing or decreasing in that particular year, or value of t.

In 2008, the revenue was increasing about \$218.8 million per year, and in 2010, revenue was increasing about \$359.0 million per year.

50. $N = 0.2083t^4 - 7.954t^3 + 11.96t^2 - 706.5t + 3891$

$\dfrac{dN}{dt} = N'(t) = 0.8332t^3 - 23.862t^2 + 23.92t - 706.5$

(a) 2010: $N'(10) = m \approx -2020.33$

2012: $N'(12) = m \approx -2415.85$

(b) Results should be similar.

(c) The slope shows the rate at which the number of farms was increasing or decreasing in that particular year, or value of t.

In 2010, the number of farms was decreasing about 2020.33 thousand per year, and in 2012, the number of farms was decreasing about 2415.85 thousand per year.

51. $f(t) = 4t + 3; [-3, 1]$

Average rate of change:
$\dfrac{f(1) - f(-3)}{1 - (-3)} = \dfrac{7 - (-9)}{4} = 4$

Instantaneous rate of change:

$f'(t) = 4$

$f'(1) = 4$

$f'(-3) = 4$

52. $f(x) = x^2 + 3x - 4; [0, 1]$

Average rate of change: $\dfrac{f(1) - f(0)}{1 - 0} = \dfrac{0 - (-4)}{1} = 4$

Instantaneous rate of change:

$f'(x) = 2x + 3$

$f'(1) = 5$

$f'(0) = 3$

53. $f(x) = x^{2/3}; [1, 8]$

Average rate of change: $\dfrac{f(8) - f(1)}{8 - 1} = \dfrac{4 - 1}{7} = \dfrac{3}{7}$

Instantaneous rate of change:

$f'(x) = \dfrac{2}{3x^{1/3}}$

$f'(8) = \dfrac{1}{3}$

$f'(1) = \dfrac{2}{3}$

54. $f(x) = x^3 - x^2 + 3; [-2, 2]$

Average rate of change: $\dfrac{f(2) - f(-2)}{2 - (-2)} = \dfrac{7 - (-9)}{4} = 4$

Instantaneous rate of change: $f'(x) = 3x^2 - 2x$

$f'(2) = 8$

$f'(-2) = 16$

55. $s(t) = -16t^2 - 30t + 600$

(a) Average velocity $= \dfrac{s(3) - s(1)}{3 - 1} = \dfrac{366 - 554}{2} = -94$ ft/sec

(b) $v(t) = s'(t) = -32t - 30$

$v(1) = -62$ ft/sec

$v(3) = -126$ ft/sec

(c) $s(t) = 0$

$-16t^2 - 30 + 600 = 0$

$16t^2 + 30t - 600 = 0$

$$t = \frac{-(30) \pm \sqrt{(30)^2 - 4(16)(-600)}}{2(16)} = \frac{-30 \pm \sqrt{39{,}300}}{32}$$

$t \approx 5.26$ sec

(d) $v(t) = s'(5.26) = -32(5.26) - 30 = -198.32$ ft/sec

56. (a) $s(t) = -16t^2 + 276$

$v(t) = s'(t) = -32t$

(b) Average velocity $= \dfrac{s(2) - 2(0)}{2 - 0}$

$= \dfrac{212 - 276}{2}$

$= -32$ ft/sec

(c) $v(t) = -32t$

$v(2) = -64$ ft /sec

$v(3) = -96$ ft /sec

(d) $s(t) = 0$

$-16t^2 + 276 = 0$

$16t^2 = 276$

$t^2 = \dfrac{276}{16}$

$t \approx 4.15$ sec

(e) $v(4.15) = -132.8$ ft/sec

57. $C = 2500 + 320x$

$\dfrac{dC}{dx} = 320$

58. $C = 24{,}000 + 450x - x^2, 0 \le x \le 225$

$\dfrac{dC}{dx} = 450 - 2x$

59. $C = 370 + 2.55\sqrt{x} = 370 + 2.25x^{1/2}$

$\dfrac{dC}{dx} = \dfrac{1}{2}(2.55)\left(x^{-1/2}\right) = \dfrac{1.275}{\sqrt{x}}$

60. $C = 475 + 5.25x^{2/3}$

$\dfrac{dC}{dx} = 5.25\left(\dfrac{2}{3}x^{-1/3}\right) = \dfrac{3.5}{\sqrt[3]{x}}$

61. $R = 150x - 0.6x^2$

$\dfrac{dR}{dx} = 150 - 1.2x$

62. $R = 150x - \dfrac{3}{4}x^2$

$\dfrac{dR}{dx} = 150 - \dfrac{3}{2}x$

63. $R = -4x^3 + 2x^2 + 100x$

$\dfrac{dR}{dx} = -12x^2 + 4x + 100$

64. $R = 4x + 10x^{1/2}$

$\frac{dR}{dx} = 4 + \frac{5}{x^{1/2}}$

65. $P = -0.0002x^3 + 6x^2 - x - 2000$

$\frac{dP}{dx} = -0.0006x^2 + 12x - 1$

66. $P = -\frac{1}{15}x^3 + 4000x^2 - 120x - 144{,}000$

$\frac{dP}{dx} = -\frac{1}{5}x^2 + 8000x - 120$

67. $P = -0.05x^2 + 20x - 1000$

(a) Find $\frac{dP}{dx}$ when $x = 100$.

$\frac{dP}{dx} = -0.1x + 20 = P'(x)$

When $x = 100$, $\frac{dP}{dx} = P'(100) = \$10$.

(b) Find $\frac{\Delta P}{\Delta x}$ for $100 \le x \le 101$.

$\frac{P(101) - P(100)}{101 - 100} = 509.95 - 500 = \$9.95$

(c) Parts (a) and (b) differ by only \$0.05.

68. $P = -0.021t^2 + 2.77t + 148.9$

(a) $P(0) = 148.9$

$P(4) = 159.644$

$P(8) = 169.716$

$P(12) = 179.116$

$P(16) = 187.844$

$P(20) = 195.9$

$P(23) = 201.501$

These values are the populations in millions for Brazil from 1990 to 2013.

(b) $\frac{dP}{dt} = -0.042t + 2.77 = P'(t)$

(c) $P'(0) = 2.77$

$P'(4) = 2.602$

$P'(8) = 2.434$

$P'(12) = 2.266$

$P'(16) = 2.098$

$P'(20) = 1.93$

$P'(23) = 1.804$

These are the rates at which the population of Brazil is changing in millions per year from 1990 to 2013.

69. $f(x) = x^3(5 - 3x^2) = 5x^3 - 3x^5$

$f'(x) = 15x^2 - 15x^4 = 15x^2(1 - x^2)$

Simple Power Rule

70. $f(x) = 4x^2(2x^2 - 5) = 8x^4 - 20x^2$

$f'(x) = 32x^3 - 40x = 8x(4x^2 - 5)$

Simple Power Rule

71. $y = (4x - 3)(x^3 - 2x^2)$

$y' = (4x - 3)(3x^2 - 4x) + 4(x^3 - 2x^2)$

$= 12x^3 - 25x^2 + 12x + 4x^3 - 8x^2$

$= 16x^3 - 33x^2 + 12x$

Product Rule and Simple Power Rule

72. $s = \left(4 - \frac{1}{t^2}\right)(t^2 - 3t) = (4 - t^{-2})(t^2 - 3t)$

$s' = (4 - t^{-2})(2t - 3) + (t^2 - 3t)(2t^{-3})$

$= 8t - 12 - 2t^{-1} + 3t^{-2} + 2t^{-1} - 6t^{-2}$

$= 8t - 12 - 3t^{-2}$

Product Rule and Simple Power Rule

73. $g(x) = \frac{x}{x + 3}$

$g'(x) = \frac{(x + 3)(1) - x(1)}{(x + 3)^2}$

$g'(x) = \frac{3}{(x + 3)^2}$

Quotient Rule and Simple Power Rule

74. $f(x) = \frac{2 - 5x}{3x + 1}$

$f'(x) = \frac{(3x + 1)(-5) - (2 - 5x)(3)}{(3x + 1)^2}$

$f'(x) = \frac{-15x - 5 - 6 + 15x}{(3x + 1)^2}$

$f'(x) = -\frac{11}{(3x + 1)^2}$

Quotient Rule and Simple Power Rule

75. $f(x) = \dfrac{6x - 5}{x^2 + 1}$`

$$f'(x) = \frac{(x^2 + 1)(6) - (6x - 5)(2x)}{(x^2 + 1)^2}$$

$$= \frac{6 + 10x - 6x^2}{(x^2 + 1)^2}$$

$$= \frac{2(3 + 5x - 3x^2)}{(x^2 + 1)^2}$$

Quotient Rule and Simple Power Rule

76. $f(x) = \dfrac{x^2 + x - 1}{x^2 - 1}$

$$f'(x) = \frac{(x^2 - 1)(2x + 1) - (x^2 + x - 1)(2x)}{(x^2 - 1)^2}$$

$$= \frac{2x^3 + x^2 - 2x - 1 - 2x^3 - 2x^2 + 2x}{(x^2 - 1)^2}$$

$$= \frac{-x^2 - 1}{(x^2 - 1)^2} = -\frac{x^2 + 1}{(x^2 - 1)^2}$$

Quotient Rule and Simple Power Rule

77. $f(x) = (5x^2 + 2)^3$

$$f'(x) = 3(5x^2 + 2)^2(10x)$$

$$= 30x(5x^2 + 2)^2$$

General Power Rule

78. $f(x) = \sqrt[3]{x^2 - 1} = (x^2 - 1)^{1/3}$

$$f'(x) = \frac{1}{3}(x^2 - 1)^{-2/3}(2x) = \frac{2x}{3(x^2 - 1)^{2/3}}$$

General Power Rule

79. $h(x) = \dfrac{2}{\sqrt{x + 1}} = 2(x + 1)^{-1/2}$

$$h'(x) = 2\left(-\frac{1}{2}\right)(x + 1)^{-3/2}$$

$$= -\frac{1}{(x + 1)^{3/2}}$$

General Power Rule

80. $g(x) = \dfrac{6}{(3x^2 - 5x)^4} = 6(3x^2 - 5x)^{-4}$

$$g'(x) = 6(-4)(3x^2 - 5x)^{-5}(6x - 5)$$

$$= -\frac{24(6x - 5)}{(3x^2 - 5x)^5}$$

General Power Rule

81. $g(x) = x\sqrt{x^2 + 1} = x(x^2 + 1)^{1/2}$

$$g'(x) = x\left[\frac{1}{2}(x^2 + 1)^{-1/2}(2x)\right] + (1)(x^2 + 1)^{1/2}$$

$$= (x^2 + 1)^{-1/2}\left[x^2 + (x^2 + 1)\right]$$

$$= \frac{2x^2 + 1}{\sqrt{x^2 + 1}}$$

Product and General Power Rule

82. $g(t) = \dfrac{t}{(1 - t)^3}$

$$g'(t) = \frac{(1 - t)^3(1) - t(3)(1 - t)^2(-1)}{(1 - t)^6}$$

$$= \frac{(1 - t)^3 + 3t(1 - t)^2}{(1 - t)^6}$$

$$= \frac{(1 - t) + 3t}{(1 - t)^4} = \frac{2t + 1}{(1 - t)^4}$$

Quotient Rule and General Power Rule

83. $f(x) = x(1 - 4x^2)^2$

$$f'(x) = x(2)(1 - 4x^2)(-8x) + (1 - 4x^2)^2$$

$$= -16x^2(1 - 4x^2) + (1 - 4x^2)^2$$

$$= (1 - 4x^2)\left[-16x^2 + (1 - 4x^2)\right]$$

$$= (1 - 4x^2)(1 - 20x^2)$$

Product and General Power Rule

84. $f(x) = \left(x^2 + \dfrac{1}{x}\right)^5 = (x^2 + x^{-1})^5$

$$f'(x) = 5(x^2 + x^{-1})^4(2x - x^{-2})$$

$$= 5\left(x^2 + \frac{1}{x}\right)^4\left(2x - \frac{1}{x^2}\right)$$

General Power Rule

85. $h(x) = \left[x^2(2x+3)\right]^3 = x^6(2x+3)^3$

$$h'(x) = x^6\left[3(2x+3)^2(2)\right] + 6x^5(2x+3)^3$$
$$= 6x^5(2x+3)^2\left[x + (2x+3)\right]$$
$$= 18x^5(2x+3)^2(x+1)$$

Product and General Power Rule

86. $f(x) = \left[(x-2)(x+4)\right]^2$

$$f'(x) = 2\left[(x-2)(x+4)\right]\left[(x-2)+(x+4)\right]$$
$$= 2(x-2)(x+4)(2x+2)$$
$$= 4(x-2)(x+4)(x+1)$$

Product and General Power Rule

87. $f(x) = x^2(x-1)^5$

$$f'(x) = 5x^2(x-1)^4 + 2x(x-1)^5$$
$$= x(x-1)^4\left[5x + 2(x-1)\right]$$
$$= x(x-1)^4(7x-2)$$

Product and General Power Rule

88. $f(s) = s^3\left(s^2-1\right)^{5/2}$

$$f'(s) = s^3\left(\tfrac{5}{2}\right)\left(s^2-1\right)^{3/2}(2s) + 3s^2\left(s^2-1\right)^{5/2}$$
$$= s^2\left(s^2-1\right)^{3/2}\left[5s^2 + 3\left(s^2-1\right)\right]$$
$$= s^2\left(s^2-1\right)^{3/2}\left(8s^2-3\right)$$

Product and General Power Rule

89. $h(t) = \dfrac{\sqrt{3t+1}}{(1-3t)^2} = \dfrac{(3t+1)^{1/2}}{(1-3t)^2}$

$$h'(t) = \frac{(1-3t)^2(1/2)(3t+1)^{-1/2}(3) - (3t+1)^{1/2}(2)(1-3t)(-3)}{(1-3t)^4}$$
$$= \frac{(3t+1)^{-1/2}\left[(1-3t)(3/2) + (3t+1)6\right]}{(1-3t)^3}$$
$$= \frac{3(9t+5)}{2\sqrt{3t+1}(1-3t)^3}$$

Quotient and General Power Rule

90. $g(x) = \left(\dfrac{3x+1}{x^2+1}\right)^2 = \dfrac{(3x+1)^2}{\left(x^2+1\right)^2}$

$$g'(x) = \frac{\left(x^2+1\right)^2(2)(3x+1)(3) - (3x+1)^2 2\left(x^2+1\right)2x}{\left(x^2+1\right)^4}$$
$$= \frac{6\left(x^2+1\right)^2(3x+1) - 4x(3x+1)^2\left(x^2+1\right)}{\left(x^2+1\right)^4}$$
$$= \frac{2(3x+1)\left(x^2+1\right)\left[3\left(x^2+1\right) - 2x(3x+1)\right]}{\left(x^2+1\right)^4}$$
$$= \frac{2(3x+1)\left(x^2+1\right)\left(-3x^2-2x+3\right)}{\left(x^2+1\right)^4}$$
$$= \frac{2(3x+1)\left(-3x^2-2x+3\right)}{\left(x^2+1\right)^3}$$

Quotient and General Power Rule.

91. $T = \dfrac{1300}{t^2 + 2t + 25} = 1300\left(t^2 + 2t + 25\right)^{-1}$

$T'(t) = -1300\left(t^2 + 2t + 25\right)^{-2}(2t + 2)$

$= -\dfrac{2600(t + 1)}{\left(t^2 + 2t + 25\right)^2}$

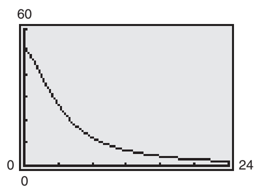
(a) $T'(1) = -\dfrac{325}{49} \approx -6.63°\text{F/hr}$

$T'(3) = -\dfrac{13}{2} \approx -6.5°\text{F/hr}$

$T'(5) = -\dfrac{13}{3} \approx -4.33°\text{F/hr}$

$T'(10) = -\dfrac{1144}{841} \approx -1.36°\text{F/hr}$

(b)



The rate of decrease is approaching zero.

92. When $L = 12$,

$V = \dfrac{L}{16}(D - 4)^2 = \dfrac{12}{16}(D - 4)^2 = \dfrac{3}{4}(D - 4)^2$

$\dfrac{dV}{dD} = \dfrac{3}{2}(D - 4).$

(a) When $D = 8$, $\dfrac{dV}{dD} = \left(\dfrac{3}{2}\right)(8 - 4) = 6$ board ft/in.

(b) When $D = 16$,

$\dfrac{dV}{dD} = \left(\dfrac{3}{2}\right)(16 - 4) = 18$ board ft/in.

(c) When $D = 24$,

$\dfrac{dV}{dD} = \left(\dfrac{3}{2}\right)(24 - 4) = 30$ board ft/in.

(d) When $D = 36$,

$\dfrac{dV}{dD} = \left(\dfrac{3}{2}\right)(36 - 4) = 48$ board ft/in.

93. $f(x) = 3x^2 + 7x + 1$

$f'(x) = 6x + 7$

$f''(x) = 6$

94. $f'(x) = 5x^4 - 6x^2 + 2x$

$f''(x) = 20x^3 - 12x + 2$

$f'''(x) = 60x^2 - 12 = 12\left(5x^2 - 1\right)$

95. $f'''(x) = -\dfrac{3}{x^4} = -3x^{-4}$

$f^{(4)}(x) = 12x^{-5}$

$f^{(5)}(x) = -60x^{-6}$

$f^{(6)}(x) = 360x^{-7} = \dfrac{360}{x^7}$

96. $f(x) = \sqrt{x} = x^{1/2}$

$f'(x) = \dfrac{1}{2}x^{-1/2}$

$f''(x) = -\dfrac{1}{4}x^{-3/2}$

$f'''(x) = \dfrac{3}{8}x^{-5/2}$

$f^{(4)}(x) = -\dfrac{15}{16}x^{-7/2} = -\dfrac{15}{16x^{7/2}}$

97. $f'(x) = 8x^{5/2}$

$f''(x) = 20x^{3/2}$

$f'''(x) = 30x^{1/2}$

$f^{(4)}(x) = 15x^{-1/2} = \dfrac{15}{x^{1/2}}$

98. $f''(x) = 9\sqrt[3]{x} = 9x^{1/3}$

$f'''(x) = 3x^{-2/3}$

$f^{(4)}(x) = -2x^{-5/3}$

$f^{(5)}(x) = \dfrac{10}{3}x^{-8/3} = \dfrac{10}{3x^{8/3}}$

99. $f(x) = x^2 + \dfrac{3}{x} = x^2 + 3x^{-1}$

$f'(x) = 2x - 3x^{-2}$

$f''(x) = 2 + 6x^{-3} = 2 + \dfrac{6}{x^3}$

100. $f'''(x) = 20x^4 - \dfrac{2}{x^3} = 20x^4 - 2x^{-3}$

$f^{(4)}(x) = 80x^3 + 6x^{-4}$

$f^{(5)}(x) = 240x^2 - 24x^{-5} = 240x^2 - \dfrac{24}{x^5}$

101. (a) $s(t) = -16t^2 + 5t + 30$

$v(t) = s'(t) = -32t + 5$

$a(t) = v'(t) = s''(t) = -32$

(b) $s(t) = 0 = -16t^2 + 5t + 30$

Using the Quadratic Formula, $t \approx 1.534$ seconds.

(c) $v(t) = s'(t) = -32t + 5$

$v(1.534) \approx -44.09$ ft/sec

(d) $a(t) = v'(t) = -32$ ft/sec^2

102. $s(t) = \dfrac{1}{t^2 + 2t + 1} = (t+1)^{-2}$

$v(t) = s'(t) = -2(t+1)^{-3} = -\dfrac{2}{(t+1)^3}$

$a(t) = v'(t) = 6(t+1)^{-4} = \dfrac{6}{(t+1)^4}$

103.
$$x^2 + 3xy + y^3 = 10$$
$$2x + 3x\frac{dy}{dx} + 3y + 3y^2\frac{dy}{dx} = 0$$
$$\frac{dy}{dx}(3x + 3y^2) = -2x - 3y$$
$$\frac{dy}{dx} = \frac{-2x - 3y}{3x + 3y^2} = -\frac{2x + 3y}{3(x + y^2)}$$

104.
$$x^2 + 9xy + y^2 = 0$$
$$2x + 9y + 9x\frac{dy}{dx} + 2y\frac{dy}{dx} = 0$$
$$(9x + 2y)\frac{dy}{dx} = -2x - 9y$$
$$\frac{dy}{dx} = \frac{-2x - 9y}{9x + 2y} = -\frac{2x + 9y}{9x + 2y}$$

105. $y^2 - x^2 + 8x - 9y - 1 = 0$
$$2y\frac{dy}{dx} - 2x + 8 - 9\frac{dy}{dx} = 0$$
$$(2y - 9)\frac{dy}{dx} = 2x - 8$$
$$\frac{dy}{dx} = \frac{2x - 8}{2y - 9}$$

106. $y^2 + x^2 - 6y - 2x - 5 = 0$
$$2y\frac{dy}{dx} + 2x - 6\frac{dy}{dx} - 2 = 0$$
$$\frac{dy}{dx}(2y - 6) = 2 - 2x$$
$$\frac{dy}{dx} = \frac{2 - 2x}{2y - 6} = \frac{1 - x}{y - 3}$$

107.
$$y^2 = x - y$$
$$2y\frac{dy}{dx} = 1 - \frac{dy}{dx}$$
$$2y\frac{dy}{dx} + \frac{dy}{dx} = 1$$
$$(2y + 1)\frac{dy}{dx} = 1$$
$$\frac{dy}{dx} = \frac{1}{2y + 1}$$

At $(2, 1)$, $\dfrac{dy}{dx} = \dfrac{1}{3}$.
$$y - 1 = \frac{1}{3}(x - 2)$$
$$y = \frac{1}{3}x + \frac{1}{3}$$

108.
$$2x^{1/3} + 3y^{1/2} = 10$$
$$\frac{2}{3}x^{-2/3} + \frac{3}{2}y^{-1/2}\frac{dy}{dx} = 0$$
$$\frac{dy}{dx} = -\frac{4y^{1/2}}{9x^{2/3}}$$

At $(8, 4)$, $\dfrac{dy}{dx} = -\dfrac{2}{9}$.
$$y - 4 = -\frac{2}{9}(x - 8)$$
$$y = -\frac{2}{9}x + \frac{52}{9}$$

109.
$$y^2 - 2x = xy$$
$$2y\frac{dy}{dx} - 2 = x\frac{dy}{dx} + y$$
$$(2y - x)\frac{dy}{dx} = y + 2$$
$$\frac{dy}{dx} = \frac{y + 2}{2y - x}$$

At $(1, 2)$, $\dfrac{dy}{dx} = \dfrac{4}{3}$.
$$y - 2 = \frac{4}{3}(x - 1)$$
$$y = \frac{4}{3}x + \frac{2}{3}$$

110.

$$y^3 - 2x^2y + 3xy^2 = -1$$

$$3y^2\frac{dy}{dx} - 2x^2\frac{dy}{dx} - 4xy + 6xy\frac{dy}{dx} + 3y^2 = 0$$

$$\frac{dy}{dx}\left(3y^2 - 2x^2 + 6xy\right) = 4xy - 3y^2$$

$$\frac{dy}{dx} = \frac{4xy - 3y^2}{3y^2 - 2x^2 + 6xy}$$

At $(0, -1)$, $\frac{dy}{dx} = -1$.

$$y + 1 = -1(x - 0)$$

$$y = -x - 1$$

111. $A = \pi r^2$

$$\frac{dA}{dt} = 2\pi r\frac{dr}{dt}$$

(a) Find $\frac{dA}{dt}$ when $r = 3$ in. and $\frac{dr}{dt} = 2$ in./min.

$$\frac{dA}{dt} = 2\pi(3)(2) = 12\pi \text{ in.}^2/\text{min} \approx 37.7 \text{ in.}^2/\text{min}$$

(b) Find $\frac{dA}{dt}$ when $r = 10$ in. and $\frac{dr}{dt} = 2$ in./min.

$$\frac{dA}{dt} = 2\pi(10)(2) = 40\pi \text{ in.}^2/\text{min} \approx 125.7 \text{ in.}^2/\text{min}$$

112. $P = 375x - 1.5x^2$

$$\frac{dP}{dt} = 375\frac{dx}{dt} - 3.0x\frac{dx}{dt}$$

$$\frac{dP}{dt} = (375 - 3.0x)\frac{dx}{dt}$$

(a) $\frac{dP}{dt} = \left[375 - 3.0(50)\right](2) = \$450/\text{day}$

(b) $\frac{dP}{dt} = \left[375 - 3.0(100)\right](2) = \$150/\text{day}$

113. Let b be the horizontal distance of the water and h be the depth of the water at the deep end.

Then $b = 8h$ for $0 \le h \le 5$.

$$V = \frac{1}{2}bh(20) = 10bh = 10(8h)h = 80h^2$$

$$\frac{dV}{dt} = 160h\frac{dh}{dt}$$

$$\frac{dh}{dt} = \frac{1}{160h}\frac{dV}{dt} = \frac{1}{160h}(10) = \frac{1}{16h}$$

When $h = 4$, $\frac{dh}{dt} = \frac{1}{16(4)} = \frac{1}{64}$ ft/min.

114.

$$\begin{aligned} P &= R - C \\ &= xp - C \\ &= x(211 - 0.002x) - (30x + 1{,}500{,}000) \\ &= 181x - 0.002x^2 - 1{,}500{,}000 \end{aligned}$$

$$\frac{dP}{dt} = 181\frac{dx}{dt} - 0.004x\frac{dx}{dt}$$

$$\frac{dP}{dt} = (181 - 0.004x)\frac{dx}{dt}$$

$$\frac{dP}{dt} = \left[181 - 0.004(1600)\right](15) = \$2619/\text{week}$$

Chapter 2 Test Yourself

1. $f(x) = x^2 + 3;\ (3, 12)$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{(x + \Delta x)^2 + 3 - (x^2 + 3)}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{x^2 + 2x\Delta x + (\Delta x)^2 + 3 - (x^2 + 3)}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{2x\Delta x + (\Delta x)^2}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{\Delta x[2x + (\Delta x)]}{\Delta x}$$
$$= \lim_{\Delta x \to 0} 2x + \Delta x$$
$$= 2x$$

At $(3, 12)$: $m = 2(3) = 6$

2. $f(x) = \sqrt{x} - 2;\ (4, 0)$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{\sqrt{x + \Delta x} - 2 - (\sqrt{x} - 2)}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{\sqrt{x + \Delta x} - \sqrt{x}}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{1}{\sqrt{x + \Delta x} + \sqrt{x}}$$
$$= \frac{1}{2\sqrt{x}}$$

At $(4, 0)$: $m = \dfrac{1}{2\sqrt{4}} = \dfrac{1}{4}$

3. $f(t) = t^3 + 2t$

$f'(t) = 3t^2 + 2$

4. $f(x) = 4x^2 - 8x + 1$

$f'(x) = 8x - 8$

5. $f(x) = x^{3/2} + 6x^{1/2}$

$f'(x) = \dfrac{3}{2}x^{1/2} + 3x^{-1/2} = \dfrac{3\sqrt{x}}{2} + \dfrac{3}{\sqrt{x}}$

6. $f(x) = 5x^2 - \dfrac{3}{x^3} = 5x^2 - 3x^{-3}$

$f'(x) = 10 + 9x^{-4} = 10x + \dfrac{9}{x^4}$

7. $f(x) = (x + 3)(x^2 + 2x)$

$f(x) = x^3 + 5x^2 + 6x$

$f'(x) = 3x^2 + 10x + 6$

(Or use the Product Rule.)

8. $f(x) = \sqrt{x}(5 + x) = 5x^{1/2} + x^{3/2}$

$f'(x) = \dfrac{5}{2}x^{-1/2} + \dfrac{3}{2}x^{1/2} = \dfrac{5}{2\sqrt{x}} + \dfrac{3\sqrt{x}}{2}$

9. $f(x) = (3x^2 + 4)^2$

$f'(x) = 2(3x^2 + 4)(6x)$

$= 36x^3 + 48x$

10. $f(x) = \sqrt{1 - 2x} = (1 - 2x)^{1/2}$

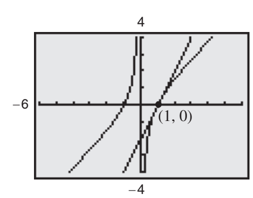
$f'(x) = \dfrac{1}{2}(1 - 2x)^{-1/2}(-2)$

$= -\dfrac{1}{\sqrt{1 - 2x}}$

11. $f(x) = \dfrac{(5x - 1)^3}{x}$

$$f'(x) = \frac{x(3)(5x - 1)^2(5) - (5x - 1)^3}{x^2}$$
$$= \frac{(5x - 1)^2[15x - (5x - 1)]}{x^2}$$
$$= \frac{(5x - 1)^2(10x + 1)}{x^2}$$

12. $f(x) = x - \dfrac{1}{x}$



$f'(x) = 1 + \dfrac{1}{x^2}$

$f'(1) = 1 + \dfrac{1}{1^2} = 2$

$y - 0 = 2(x - 1)$

$y = 2x - 2$

13. $S = -2.1083t^3 + 70.811t^2 - 777.05t + 2893.6$

(a) $\dfrac{\Delta S}{\Delta t}$ for $10 \le t \le 12$

$$\frac{S(12) - S(10)}{12 - 10} = \frac{122.6416 - 95.9}{2} = \$13.3708 \text{ billion/yr}$$

(b) $S'(t) = -6.3249t^2 + 141.622t - 777.05$

2010: $S'(10) = \$6.68$ billion/yr

2012: $S'(12) = \$11.6284$ billion/yr

(c) The annual sales of CVS Caremark from 2010 to 2012 increased by an average of about \$13.37 billion per year, and the instantaneous rates of change for 2010 and 2012 are \$6.68 billion per year and \$11.63 billion per year, respectively.

14. $P = 1700 - 0.016x$, $C = 715{,}000 + 240x$

Profit = Revenue − Cost

(a) Revenue: $R = xp$

$$R = x(1700 - 0.016x)$$
$$R = 1700x - 0.016x^2$$
$$P = R - C$$
$$P = (1700x - 0.016x^2) - (715{,}000 + 240x)$$
$$P = -0.016x^2 + 1460x - 715{,}000$$

(b) $\dfrac{dP}{dx} = -0.032x + 1460 = P'(x)$

$P'(700) = \$1437.60$

15. $f(x) = 2x^2 + 3x + 1$

$f'(x) = 4x + 3$

$f''(x) = 4$

$f'''(x) = 0$

16. $f(x) = \sqrt{3 - x} = (3 - x)^{1/2}$

$$f'(x) = \frac{1}{2}(3 - x)^{-1/2}(-1) = -\frac{1}{2}(3 - x)^{-1/2}$$
$$f''(x) = -\frac{1}{2}\left(-\frac{1}{2}\right)(3 - x)^{-3/2}(-1) = -\frac{1}{4}(3 - x)^{-3/2}$$
$$f'''(x) = -\frac{1}{4}\left(-\frac{3}{2}\right)(3 - x)^{-5/2}(-1)$$
$$= -\frac{3}{8}(3 - x)^{-5/2}$$
$$= -\frac{3}{8(3 - x)^{5/2}}$$

17. $f(x) = \dfrac{2x + 1}{2x - 1}$

$$f'(x) = \frac{(2x - 1)(2) - (2x + 1)(2)}{(2x - 1)^2}$$
$$= \frac{4}{(2x - 1)^2}$$
$$= -4(2x - 1)^{-2}$$
$$f''(x) = 8(2x - 1)^{-3}(2) = 16(2x - 1)^{-3}$$
$$f'''(x) = -48(2x - 1)^{-4}(2) = -\frac{96}{(2x - 1)^4}$$

18. $s(t) = -16t^2 + 30t + 75$

$v(t) = s'(t) = -32t + 30$

$a(t) = v'(t) = s''(t) = -32$

At $t = 2$: $s(2) = 71$ ft

$v(2) = -34$ ft/sec

$a(2) = -32$ ft/sec^2

19. $x + xy = 6$

$$1 + x\frac{dy}{dx} + y = 0$$
$$x\frac{dy}{dx} = -y - 1$$
$$\frac{dy}{dx} = -\frac{y + 1}{x}$$

20. $y^2 + 2x - 2y + 1 = 0$

$$2y\frac{dy}{dx} + 2 - 2\frac{dy}{dx} = 0$$
$$\frac{dy}{dx}(2y - 2) = -2$$
$$\frac{dy}{dx} = -\frac{1}{y - 1}$$

21. $4x^2 - 3y^2 + x^3y = 5$

$$8x - 6y\frac{dy}{dx} + x^3\frac{dy}{dx}3x^2y = 0$$
$$-6y\frac{dy}{dx} + x^3\frac{dy}{dx} = -8x - 3x^2y$$
$$(x^3 - 6y)\frac{dy}{dx} = -(8x + 3x^2y)$$
$$\frac{dy}{dx} = -\frac{8x + 3x^2y}{x^3 - 6y}$$
$$\frac{dy}{dx} = \frac{8x + 3x^2y}{6y - x^3} = \frac{x(8 + 3xy)}{6y - x^3}$$

22. $V = \pi r^2 h = 20\pi r^3$

$$\frac{dV}{dt} = 60\pi r^2 \frac{dr}{dt}$$

(a) $\frac{dV}{dt} = 60\pi(0.5)^2(0.25) = 3.75\pi \text{ cm}^3/\text{min}$

(b) $\frac{dV}{dt} = 60\pi(1)^2(0.25) = 15\pi \text{ cm}^3/\text{min}$

CHAPTER 3
Applications of the Derivative