Instructor's Resource Guide
Explorations Solutions

Calculus

ELEVENTH EDITION

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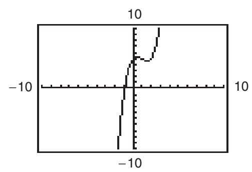
EXPLORATIONS

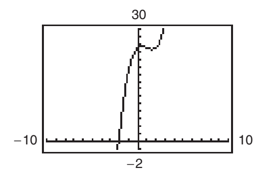
Chapter P, Section 1, page 3

a.



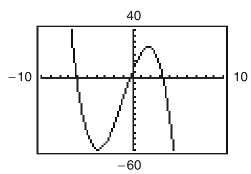
$y = x^3 - 3x^2 + 2x + 5$

b.



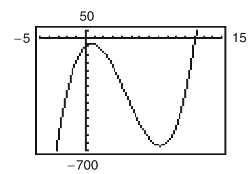
$y = x^3 - 3x^2 + 2x + 25$

c.



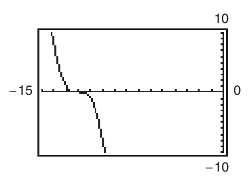
$y = -x^3 - 3x^2 + 20x + 5$

d.



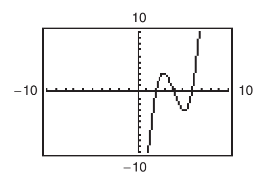
$y = 3x^3 - 40x^2 + 50x - 45$

e.



$y = -(x + 12)^3$

f.



$y = (x - 2)(x - 4)(x - 6)$

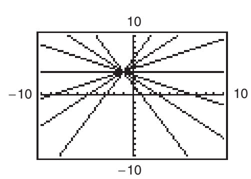
With an analytic approach, some other things to consider are:

(1) How many turns does the graph have?

(2) What are the x- and y-intercepts?

(3) Does the graph begin (and end) by going up or down?

Chapter P, Section 2, page 11



All seven lines have the point $(-1, 4)$ in common.
The coefficient of the quantity $(x + 1)$ is the slope.
An equation of the line passing through $(-1, 4)$
with a slope of m would be $y - 4 = m(x + 1)$.

Chapter 1, Section 1, page 49

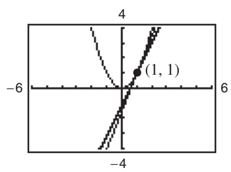
$P(1, 1)$ and $Q_1(1.5, f(1.5))$: $m = \dfrac{2.25 - 1}{1.5 - 1} = \dfrac{1.25}{0.5} = 2.5$

$P(1, 1)$ and $Q_2(1.1, f(1.1))$: $m = \dfrac{1.21 - 1}{1.1 - 1} = \dfrac{0.21}{0.1} = 2.1$

$P(1, 1)$ and $Q_3(1.01, f(1.01))$: $m = \dfrac{1.0201 - 1}{1.01 - 1} = \dfrac{0.0201}{0.01} = 2.01$

$P(1, 1)$ and $Q_4(1.001, f(1.001))$: $m = \dfrac{1.002001}{1.001 - 1} = \dfrac{0.002001}{0.001} = 2.001$

$P(1, 1)$ and $Q_5(1.0001, f(1.0001))$: $m = \dfrac{1.00020001}{1.0001 - 1} = \dfrac{0.00020001}{0.0001} = 2.0001$



$m = 2$

Chapter 1, Section 1, page 50

The width of each rectangle is $\dfrac{1}{5}$.

The area of the inscribed set of rectangles is:

$$\begin{aligned} A &= \frac{1}{5}\cdot f(0) + \frac{1}{5}\cdot f\left(\frac{1}{5}\right) + \frac{1}{5}\cdot f\left(\frac{2}{5}\right) + \frac{1}{5}\cdot f\left(\frac{3}{5}\right) + \frac{1}{5}\cdot f\left(\frac{4}{5}\right) \\ &= \frac{1}{5}\left[0 + \frac{1}{25} + \frac{4}{25} + \frac{9}{25} + \frac{16}{25}\right] \\ &= \frac{1}{5}\left[\frac{30}{25}\right] \\ &= \frac{6}{25} = 0.24. \end{aligned}$$

The area of the circumscribed set of rectangles is:

$$\begin{aligned} A &= \frac{1}{5}\cdot f\left(\frac{1}{5}\right) + \frac{1}{5}\cdot f\left(\frac{2}{5}\right) + \frac{1}{5}\cdot f\left(\frac{3}{5}\right) + \frac{1}{5}\cdot f\left(\frac{4}{5}\right) + \frac{1}{5}\cdot f(1) \\ &= \frac{1}{5}\left[\frac{1}{25} + \frac{4}{25} + \frac{9}{25} + \frac{16}{25} + 1\right] \\ &= \frac{1}{5}\left[\frac{55}{25}\right] \\ &= \frac{11}{25} = 0.44. \end{aligned}$$

The area of the region is approximately $\dfrac{1}{2}\left(\dfrac{6}{25} + \dfrac{11}{25}\right) = \dfrac{17}{50} = 0.34$.

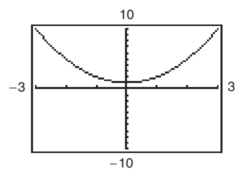
Chapter 1, Section 2, page 52

x	1.75	1.9	1.99	1.999	2	2.001	2.01	2.1	2.25
$f(x)$	0.75	0.9	0.99	0.999	1	1.001	1.01	1.1	1.25

The graph of $f(x) = \dfrac{x^2 - 3x + 2}{x - 2}$ agrees with the graph of $g(x) = x - 1$ at all points but one, where $x = 2$. If you trace along f getting closer and closer to $x = 2$, the value of y will get closer and closer to 1.

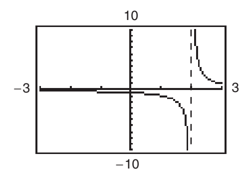
Chapter 1, Section 4, page 74

a.



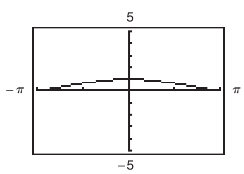
$y = x^2 + 1$ looks continuous on $(-3, 3)$.

b.



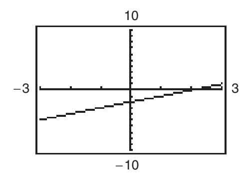
$y = \dfrac{1}{x - 2}$ does not look continuous on $(-3, 3)$.

c.



$y = \dfrac{\sin x}{x}$ looks continuous on $(-\pi, \pi)$.

d.

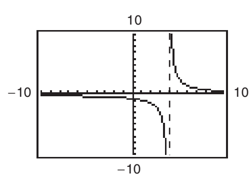


$y = \dfrac{x^2 - 4}{x + 2}$ looks continuous on $(-3, 3)$.

You cannot trust the results you obtain graphically. In these examples, only part **a.** is continuous. Part **b.** is discontinuous at $x = 2$; part **c.** is discontinuous at $x = 0$; part **d.** is discontinuous at $x = -2$.

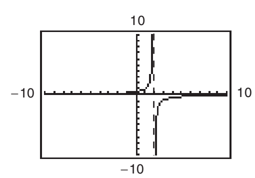
Chapter 1, Section 5, page 88

a.



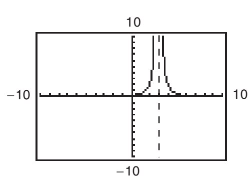
$c = 4;\ \lim\limits_{x \to 4^-} f(x) = -\infty;\ \lim\limits_{x \to 4^+} f(x) = +\infty$

b.



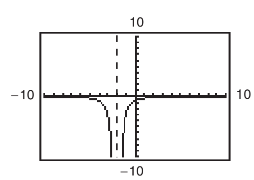
$c = 2;\ \lim\limits_{x \to 2^-} f(x) = +\infty;\ \lim\limits_{x \to 2^+} f(x) = -\infty$

c.



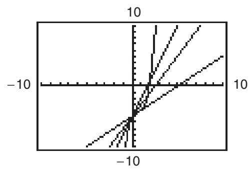
$c = 3;\ \lim\limits_{x \to 3^-} f(x) = +\infty;\ \lim\limits_{x \to 3^+} f(x) = +\infty$

d.



$c = -2;\ \lim\limits_{x \to 2^-} f(x) = -\infty;\ \lim\limits_{x \to -2^+} f(x) = -\infty$

Chapter 2, Section 1, page 100



The line $y = x - 5$ appears to be tangent to the graph of f at the point $(0, -5)$ because it seems to intersect the graph at only that point.

Chapter 2, Section 2, page 110

a. $f(x) = x^1$

$$\begin{aligned}f'(x) &= \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} \\ &= \lim_{\Delta x \to 0} \frac{x + \Delta x - x}{\Delta x} \\ &= \lim_{\Delta x \to 0} \frac{\cancel{\Delta x}}{\cancel{\Delta x}} \\ &= \lim_{\Delta x \to 0} 1 \\ &= 1\end{aligned}$$

b. $f(x) = x^2$

$$\begin{aligned}f'(x) &= \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} \\ &= \lim_{\Delta x \to 0} \frac{(x + \Delta x)^2 - x^2}{\Delta x} \\ &= \lim_{\Delta x \to 0} \frac{x^2 + 2x\Delta x + (\Delta x)^2 - x^2}{\Delta x} \\ &= \lim_{\Delta x \to 0} \frac{2x\Delta x + (\Delta x)^2}{\Delta x} \\ &= \lim_{\Delta x \to 0} \frac{\cancel{\Delta x}(2x + \Delta x)}{\cancel{\Delta x}} \\ &= \lim_{\Delta x \to 0} (2x + \Delta x) \\ &= 2x\end{aligned}$$

c. $f(x) = x^3$

$$\begin{aligned}f'(x) &= \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} \\ &= \lim_{\Delta x \to 0} \frac{(x + \Delta x)^3 - x^3}{\Delta x} \\ &= \lim_{\Delta x \to 0} \frac{x^3 + 3x^2\Delta x + 3x(\Delta x)^2 + (\Delta x)^3 - x^3}{\Delta x} \\ &= \lim_{\Delta x \to 0} \frac{3x^2\Delta x + 3x(\Delta x)^2 + (\Delta x)^3}{\Delta x} \\ &= \lim_{\Delta x \to 0} \frac{\cancel{\Delta x}\left[3x^2 + 3x\Delta x + (\Delta x)^2\right]}{\cancel{\Delta x}} \\ &= \lim_{\Delta x \to 0} \left[3x^2 + 3x\Delta x + (\Delta x)^2\right] \\ &= 3x^2\end{aligned}$$

d. $f(x) = x^4$

$$\begin{aligned}f'(x) &= \lim_{\Delta x\to 0}\frac{f(x+\Delta x)-f(x)}{\Delta x}\\ &= \lim_{\Delta x\to 0}\frac{(x+\Delta x)^4 - x^4}{\Delta x}\\ &= \lim_{\Delta x\to 0}\frac{x^4 + 4x^3\Delta x + 6x^2(\Delta x)^2 + 4x(\Delta x)^3 + (\Delta x)^4 - x^4}{\Delta x}\\ &= \lim_{\Delta x\to 0}\frac{4x^3\Delta x + 6x^2(\Delta x)^2 + 4x(\Delta x)^3 + (\Delta x)^4}{\Delta x}\\ &= \lim_{\Delta x\to 0}\frac{\cancel{\Delta x}\left[4x^3 + 6x^2\Delta x + 4x(\Delta x)^2 + (\Delta x)^3\right]}{\cancel{\Delta x}}\\ &= \lim_{\Delta x\to 0}\left[4x^3 + 6x^2\Delta x + 4x(\Delta x)^2 + (\Delta x)^3\right]\\ &= 4x^3\end{aligned}$$

e. $f(x) = x^{1/2} = \sqrt{x}$

$$\begin{aligned}f'(x) &= \lim_{\Delta x\to 0}\frac{f(x+\Delta x)-f(x)}{\Delta x}\\ &= \lim_{\Delta x\to 0}\frac{\sqrt{x+\Delta x}-\sqrt{x}}{\Delta x}\\ &= \lim_{\Delta x\to 0}\left(\frac{\sqrt{x+\Delta x}-\sqrt{x}}{\Delta x}\right)\left(\frac{\sqrt{x+\Delta x}+\sqrt{x}}{\sqrt{x+\Delta x}+\sqrt{x}}\right)\\ &= \lim_{\Delta x\to 0}\frac{(x+\Delta x)-x}{\Delta x\left(\sqrt{x+\Delta x}+\sqrt{x}\right)}\\ &= \lim_{\Delta x\to 0}\frac{\cancel{\Delta x}}{\cancel{\Delta x}\left(\sqrt{x+\Delta x}+\sqrt{x}\right)}\\ &= \lim_{\Delta x\to 0}\frac{1}{\sqrt{x+\Delta x}+\sqrt{x}}\\ &= \frac{1}{2\sqrt{x}}\\ &= \frac{1}{2}x^{-1/2}\end{aligned}$$

f. $f(x) = x^{-1} = \dfrac{1}{x}$

$$\begin{aligned}f'(x) &= \lim_{\Delta x\to 0}\frac{f(x+\Delta x)-f(x)}{\Delta x}\\ &= \lim_{\Delta x\to 0}\frac{\frac{1}{x+\Delta x}-\frac{1}{x}}{\Delta x}\\ &= \lim_{\Delta x\to 0}\frac{\frac{x-(x+\Delta x)}{x(x+\Delta x)}}{\Delta x}\\ &= \lim_{\Delta x\to 0}\frac{-\cancel{\Delta x}}{\cancel{\Delta x}(x)(x+\Delta x)}\\ &= \lim_{\Delta x\to 0}\frac{-1}{x(x+\Delta x)}\\ &= -\frac{1}{x^2}\\ &= -x^{-2}\end{aligned}$$

The exponent of f becomes the coefficient of f' and the power of x decreases by 1.

$(x^n)' = n(x^{n-1})$

Chapter 2, Section 4, page 134

a. Using the Quotient Rule: $y' = \dfrac{(3x+1)(0) - 2(3)}{(3x+1)^2} = -\dfrac{6}{(3x+1)^2}$

Using the Chain Rule:
$$\begin{aligned} y' &= \left[2(3x+1)^{-1}\right]' \\ &= 2(-1)(3x+1)^{-2}(3) \\ &= -\frac{6}{(3x+1)^2} \end{aligned}$$

b. Using algebra before differentiating: $y' = \left[x^3 + 6x^2 + 12x + 8\right]' = 3x^2 + 12x + 12$

Using the Chain Rule: $y' = 3(x+2)^2(1) = 3x^2 + 12x + 12$

c. Using a trigonometric identity and the Product Rule:

$$\begin{aligned} y &= \sin 2x = 2\sin x\cos x \\ y' &= 2\left[(\sin x)(-\sin x) + (\cos x)(\cos x)\right] \\ &= 2\left[\cos^2 x - \sin^2 x\right] \\ &= 2\cos 2x \end{aligned}$$

Using the Chain Rule:
$$\begin{aligned} y' &= (\cos 2x)(2) \\ &= 2\cos 2x \end{aligned}$$

In general, the Chain Rule is simpler.

Chapter 2, Section 6, page 152

$$\begin{aligned} V &= \frac{\pi}{3}r^2h \\ \frac{dV}{dt} &= \frac{\pi}{3}\left(r^2\frac{dh}{dt} + 2rh\frac{dr}{dt}\right) \\ &= \frac{\pi}{3}\left[(1\text{ ft})^2(-0.2\text{ ft/min}) + 2(1\text{ ft})(2\text{ ft})(-0.1\text{ ft/min})\right] \\ &= \frac{\pi}{3}\left(-0.2\text{ ft}^3/\text{min} - 0.4\text{ ft}^3/\text{min}\right) \\ &= \frac{\pi}{3}\left(-0.6\text{ ft}^3/\text{min}\right) \\ &= -\frac{\pi}{5}\text{ ft}^3/\text{min} \end{aligned}$$

Given:
$$\begin{aligned} \frac{dh}{dt} &= -0.2\text{ ft/min} \\ \frac{dr}{dt} &= -0.1\text{ ft/min} \\ r &= 1\text{ ft} \\ h &= 2\text{ ft} \end{aligned}$$

The rate of change in the volume does depend on the values of r and h because both variables are in the function $\dfrac{dV}{dt}$.

Chapter 3, Section 1, page 166

a. $f(x) = x^2 - 4x + 5, [-1, 3]$

Minimum: $x = 1.9999994, y = 1$

Maximum: $x = -0.9999958, y = 9.999975$

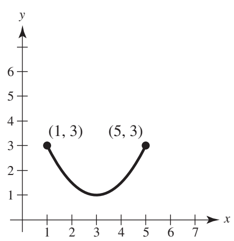
b. $f(x) = x^3 - 2x^2 - 3x - 2, [-1, 3]$

Minimum: $x = 1.8685171, y = -8.064605$

Maximum: $x = -0.5351846, y = -1.12058$

In each case, the x-values are approximate. For the graphing utility to come up with the exact answer, the approximation must be rational.

Chapter 3, Section 2, page 174

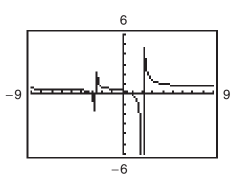


One possible solution is shown. There must be at least one point on the graph for which the derivative is zero. If the function is differentiable, then it is continuous.

Chapter 3, Section 4, page 194

The value of c has a bearing on the shape of the graph, but has no bearing on the value of the second derivative at given values of x. Graphically, this is true because changes in the value of c change the location of the extrema but do not affect the location of the point of inflection.

Chapter 3, Section 5, page 200



There are vertical asymptotes at $x = 2$ and $x = -\frac{8}{3}$.

There are no extrema.

There is a point of inflection at $(1, 0)$.

The graph is decreasing on the intervals $\left(-\infty, -\frac{8}{3}\right), \left(-\frac{8}{3}, 2\right)$, and $(2, \infty)$.

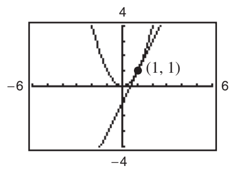
No one viewing window shows all features. For example, you need to zoom in to see the point of inflection. The horizontal asymptote is $y = \frac{2}{3}$. If

$x = 890, y = 0.6676664 < 0.667\overline{6}$ and, therefore, within 0.001 unit of its horizontal asymptote.

Chapter 3, Section 7, page 223

The minimum area occurs when $x = \dfrac{4}{\pi + 4} \approx 0.56$. Then, $r = \dfrac{2(1 - x)}{\pi} = \dfrac{2}{\pi + 4}$.

Chapter 3, Section 9, page 235



Answers will vary. *Sample answer*:

The TI-83 distinguishes between the two graphs after zooming in twice. As the x-values get closer to 1, the y-values do not change until the 5th decimal place.

Chapter 4, Section 1, page 248

a. $F(x) = x^2$ because $\frac{d}{dx}\left[x^2\right] = 2x.$

b. $F(x) = \frac{1}{2}x^2$ because $\frac{d}{dx}\left[\frac{1}{2}x^2\right] = x.$

c. $F(x) = \frac{1}{3}x^3$ because $\frac{d}{dx}\left[\frac{1}{3}x^3\right] = x^2.$

d. $F(x) = -\frac{1}{x}$ because $\frac{d}{dx}\left[-\frac{1}{x}\right] = \frac{1}{x^2}.$

e. $F(x) = -\frac{1}{2x^2}$ because $\frac{d}{dx}\left[-\frac{1}{2x^2}\right] = \frac{1}{x^3}.$

f. $F(x) = \sin x$ because $\frac{d}{dx}\left[\sin x\right] = \cos x.$

One way to find F is to guess, check, and revise.

Chapter 4, Section 2, page 264

n	10	100	1000
$s(n)$	$\frac{684}{300} = 2.28$	$\frac{78{,}804}{30{,}000} = 2.6268$	$\frac{7{,}988{,}004}{3{,}000{,}000} = 2.662668$
$S(n)$	$\frac{924}{300} = 3.08$	$\frac{81{,}204}{30{,}000} = 2.7068$	$\frac{8{,}012{,}004}{3{,}000{,}000} = 2.670668$

Because $s(n) \le \text{Area} \le S(n)$, Area $= \frac{8}{3}.$

Chapter 4, Section 3, page 272

One example to show that the converse of Theorem 4.4 is false is $f(x) = \begin{cases} 0 & \text{if } x \ne 3 \\ 7 & \text{if } x = 3 \end{cases}.$

Although f is not continuous at 3, f is integrable on the closed interval $[2, 4]$ and $\int_2^4 f(x)\,dx = 0.$

To see this, let $\varepsilon > 0$ and choose $\delta = \varepsilon/8$. Any Riemann sum $\sum_{i=1}^{n} f(c_i)\,\Delta x_i$ of f for a partition Δ of $[2, 4]$ can have at most one nonzero term and its value is between 0 and $7\|\Delta\|$.

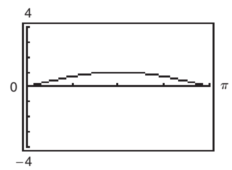
Hence, if $\|\Delta\| < \delta$ then $\left|0 - \sum_{i=1}^{n} f(c_i)\,\Delta x_i\right| = \sum_{i=1}^{n} f(c_i)\,\Delta x_i \le 7\|\Delta\| < 7 \cdot \frac{\varepsilon}{8} < \varepsilon.$

A second example is $g(x) = \begin{cases} 4 & \text{if } x \le 1 \\ 5 & \text{if } x > 1 \end{cases}$. As intuition suggests, $\int_0^2 g(x)\,dx = 9.$

If a function is differentiable on a closed interval $[a, b]$ as defined in Section 2.1, then the function is continuous on $[a, b]$. (The proof of Theorem 2.1 is easily extended to include the one-sided limits that occur at the endpoints a and b.) By Theorem 4.4, if a function is differentiable on $[a, b]$, then the function is integrable there. That is, differentiability on $[a, b]$ is stronger than continuity on $[a, b]$, which is stronger than integrability on $[a, b]$.

The example given here show that integrability is weaker than continuity on $[a, b]$. Examples in Section 2.1 show that a continuous function can have a sharp turn or a vertical tangent line and not be differentiable. Thus, continuity is weaker than differentiability.

Chapter 4, Section 4, page 287



The graph of F is the graph of the sine function. The antiderivative of $\cos t$ is $\sin t$ and when evaluated on the interval $[0, x]$ using the Fundamental Theorem of Calculus, the result is $\sin x$.

Chapter 4, Section 5, page 297

a.
$$f(u) = u^4$$
$$u = g(x) = x^2 + 1$$
$$g'(x) = 2x$$
$$\int 2x(x^2 + 1)^4\,dx = \frac{1}{5}(x^2 + 1)^5 + C$$

b.
$$f(u) = \sqrt{u}$$
$$u = g(x) = x^3 + 1$$
$$g'(x) = 3x^2$$
$$\int 3x^2\sqrt{x^3 + 1}\,dx = \frac{2}{3}(x^3 + 1)^{3/2} + C$$

c.
$$f(u) = u^1$$
$$u = g(x) = \tan x + 3$$
$$g'(x) = \sec^2 x$$
$$\int \sec^2 x(\tan x + 3)\,dx = \frac{1}{2}(\tan x + 3)^2 + C$$

d.
$$\int x(x^2 + 1)^4\,dx = \frac{1}{2}\int 2x(x^2 + 1)^4\,dx$$
$$= \frac{1}{2}\left[\frac{1}{5}(x^2 + 1)^5\right] + C$$
$$= \frac{1}{10}(x^2 + 1)^5 + C$$

e.
$$\int x^2\sqrt{x^3 + 1}\,dx = \frac{1}{3}\int 3x^2\sqrt{x^3 + 1}\,dx$$
$$= \frac{1}{3}\left[\frac{2}{3}(x^3 + 1)^{3/2}\right] + C$$
$$= \frac{2}{9}(x^3 + 1)^{3/2} + C$$

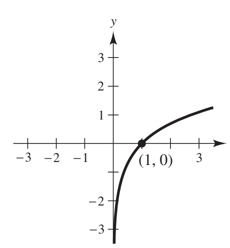
f.
$$\int 2\sec^2 x(\tan x + 3)\,dx = 2\int\left(\frac{1}{2}\right)(2\sec^2 x)(\tan x + 3)\,dx$$
$$= 2\left[\frac{1}{2}(\tan x + 3)^2\right] + C$$
$$= (\tan x + 3)^2 + C$$

Chapter 5, Section 1, page 314

Let $F(x) = \ln x = \int_1^x \frac{1}{t}\,dt$. Because $\frac{1}{t} > 0$ for $t > 0$, $\int_1^x \frac{1}{t}\,dt$ represents an area for $x > 1$ and, therefore, is positive. For $x < 1$, $\int_1^x \frac{1}{t}\,dt = -\int_x^1 \frac{1}{t}\,dt$ is negative.

Moreover, for $x = 1$, $F(1) = \ln(1) = \int_1^1 \frac{1}{t}\,dt = 0$. Therefore, $(1, 0)$ is an intercept.

By the Second Fundamental Theorem of Calculus, $F'(x) = (\ln x)' = \frac{1}{x}$, which is positive on the interval $(0, \infty)$. Also, $F''(x) = (\ln x)'' = -\frac{1}{x^2}$, which is negative everywhere. Therefore, the graph of $y = F(x) = \ln x$ is increasing and concave downward on the interval $(0, \infty)$. Combining all of these facts yields the graph at the right.

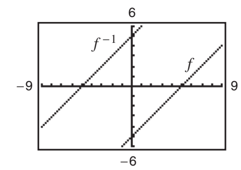


Chapter 5, Section 2, page 324

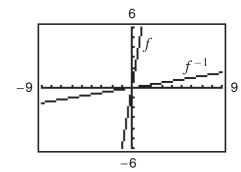
$y = \frac{1}{x^2 + 1}$, $y = \frac{x^2}{3x^2 + 5}$, and $y = \frac{x^3}{x^2 + 1}$ are examples of rational functions that cannot be integrated using the Log Rule. For each of these, if you let u equal the denominator, the numerator is not u'.

Chapter 5, Section 3, page 333

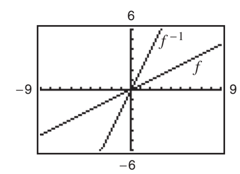
a. Add 5. $f^{-1}(x) = x + 5$



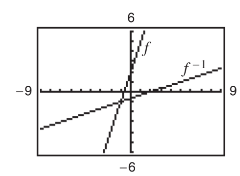
b. Divide by 6. $f^{-1}(x) = \dfrac{x}{6}$



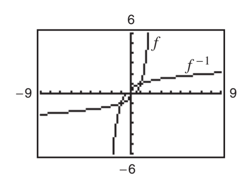
c. Multiply by 2. $f^{-1}(x) = 2x$



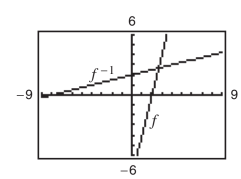
d. Subtract 2, then divide by 3. $f^{-1}(x) = \dfrac{x - 2}{3}$



e. Take the cube root. $f^{-1}(x) = \sqrt[3]{x}$



f. Divide by 4, then add 2. $f^{-1}(x) = \dfrac{x}{4} + 2$



The graphs of f and f^{-1} are symmetric with respect to the line $y = x$.

Chapter 5, Section 3, page 337

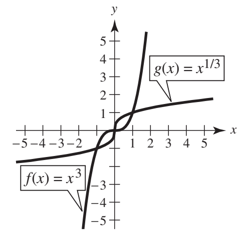
$f(x) = x^3$ $\quad$ $g(x) = x^{1/3}$

$f'(x) = 3x^2$ $\quad$ $g'(x) = \dfrac{1}{3x^{2/3}}$

$f'(1) = 3$ $\quad$ $g'(1) = \dfrac{1}{3}$

$f'(2) = 12$ $\quad$ $g'(8) = \dfrac{1}{12}$

$f'(3) = 27$ $\quad$ $g'(27) = \dfrac{1}{27}$



$$g'(x) = \frac{1}{f'(x)}$$

At $(0, 0)$, $f'(0) = 0$. Hence, $g'(0) = \dfrac{1}{f'(0)}$ is undefined.

Chapter 5, Section 6, page 364

a. $\displaystyle\lim_{x\to 0} \frac{2^{2x} - 1}{x} = 2 \ln 2 \approx 1.39$

b. $\displaystyle\lim_{x\to 0} \frac{3^{2x} - 1}{x} = 2 \ln 3 \approx 2.20$

c. $\displaystyle\lim_{x\to 0} \frac{4^{2x} - 1}{x} = 2 \ln 4 \approx 2.77$

d. $\displaystyle\lim_{x\to 0} \frac{5^{2x} - 1}{x} = 2 \ln 5 \approx 3.22$

The limit is of the form $2 \ln n$, where n is the base of the exponential term. An analytical approach gives you the exact limit, whereas a graphical or numerical approach gives you an approximation.

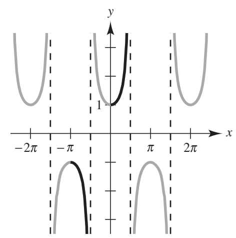
Chapter 5, Section 6, page 368

$$\lim_{x\to\infty} \frac{e^x}{x} = \lim_{x\to\infty} \frac{\frac{d}{dx}\left[e^x\right]}{\frac{d}{dx}[x]} = \lim_{x\to\infty} e^x = \infty$$

Chapter 5, Section 7, page 373

Many different domain restrictions yield an invertible function with the full range of the secant function. The textbook uses

$$\left[0, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \pi\right].$$

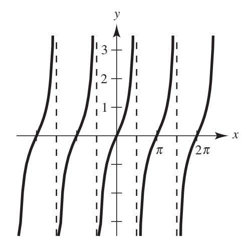


Another possibility is

$$\left[0, \frac{\pi}{2}\right) \cup \left[\pi, \frac{3\pi}{2}\right).$$

The graph at the right illustrates the choice

$$\left[-\pi, -\frac{\pi}{2}\right) \cup \left[0, \frac{\pi}{2}\right).$$



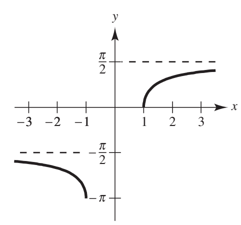
This choice makes sense for four reasons. First, why use big angles if small one will do? Second, in applications it is convenient to have the formula $\tan\theta = \sqrt{\sec^2\theta - 1}$, without any $(\pm)$ sign ambiguity. Comparing the graphs at the right shows how to restrict the domain of the secant function to values for which the tangent function is not negative. Third, it helps if inverse trigonometric functions evaluate to acute angles whenever possible, especially in working with right triangles. Thus,

$$\left(0, \frac{\pi}{2}\right)$$

should be part of the domain of the restricted secant function. Fourth, it should be easy to compute values of the inverse secant function using available technology.

Choosing this restricted domain for the secant function would mean that the inverse function satisfies

$$y = \operatorname{arcsec} x \text{ if } x = \sec y, -\pi \le y < -\frac{\pi}{2} \text{ or } 0 \le y < \frac{\pi}{2}.$$



A graph is shown at the right. One benefit is the derivative formula

$$\frac{d}{dx}\operatorname{arcsec} x = \frac{1}{x\sqrt{x^2 - 1}},$$

which is different from the one given in the textbook. Scientific and graphing calculators have a key for the inverse cosine function. Using the definition given in the textbook, it is easy to evaluate the inverse secant function on a calculator because $\operatorname{arcsec} x = \arccos 1/x$. By comparison, restricting the domain, as discussed above, means that

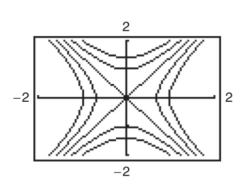
$$\operatorname{arcsec} x = \frac{x}{|x|}\arccos\frac{1}{x};$$

that is, after computing $\arccos 1/x$, adjust the sign to agree with x.

Chapter 6, Section 2, page 415

The solutions are hyperbolas.

Yes, $y' = 2\left(\frac{x}{y}\right)$.



To see this, use implicit differentiation.

$$\begin{aligned} y^2 - 2x^2 &= C \\ 2yy' - 4x &= 0 \\ 2yy' &= 4x \\ y' &= \frac{2x}{y} \end{aligned}$$

Yes, all curves for which this statement is true are represented by the general solution.

Chapter 6, Section 3, page 427

The graph of $y = \dfrac{L}{1 + be^{-kt}}$ has horizontal asymptotes at $y = 0$ and $y = L$ and its range is $(0, L)$. So, a change in L changes the horizontal asymptote $y = L$ and the range accordingly. If $L < 0$, then the graph is reflected about the x-axis.

The value of b affects the y-intercept, which is the point $\left(0, \dfrac{L}{1 + b}\right)$. As positive values of b increase, the intercept approaches $(0, 0)$. Whereas, as b decreases, the intercept approaches $(0, L)$. If $b < 0$, the graph has a vertical asymptote.

The value of k affects the steepness of the graph. As k increases, the graph increases more quickly. As k decreases, the graph increases more slowly. If the sign of k changes, then the graph is reflected about the y-axis.

Chapter 7, Section 3, page 467

$$y = \begin{cases} 1, & 0 \le x \le 1/e \\ \sqrt{-\ln x}, & 1/e < x \le 1 \end{cases}$$

$$\begin{aligned} V_1 &= \pi \int_0^{1/e} dx \\ &= \pi[x]_0^{1/e} \\ &= \frac{\pi}{e} \end{aligned}$$

$$\begin{aligned} V_2 &= \pi \int_{1/e}^{1} -\ln x \, dx \\ &= -\pi \int_{1/e}^{1} \ln x \, dx \\ &= -\pi\left[-x + x \ln x\right]_{1/e}^{1} \\ &= -\pi\left(-1 + \frac{1}{e} + \frac{1}{e}\right) \\ &= \pi - \frac{2\pi}{e} \end{aligned}$$

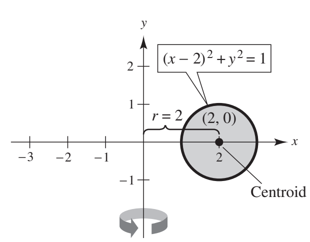
$$V = V_1 + V_2 = \frac{\pi}{e} + \pi - \frac{2\pi}{e} \approx 1.986$$

Chapter 7, Section 6, page 501

For the shell method $V = 2\pi \int_a^b ph \, dx$,

where

$$\begin{aligned} a &= 1 \\ b &= 3 \\ p &= x \\ h &= \sqrt{1 - (x - 2)^2} - \left[-\sqrt{1 - (x - 2)^2}\right] \\ &= 2\sqrt{1 - (x - 2)^2}. \end{aligned}$$



$$\begin{aligned} \text{Hence, } V &= 2\pi \int_1^3 x\left[2\sqrt{1 - (x - 2)^2}\right] dx \\ &= 4\pi \int_1^3 x\sqrt{1 - (x - 2)^2} \, dx \end{aligned}$$

A graphing utility gives 39.47842289 as a value for this integral, which agrees with the one in Example 7.

Chapter 8, Section 1, page 517

Parts **a** and **b** can be evaluated using the basic integration rules.

a. Use rule 18.

$$\int \frac{3}{\sqrt{1-x^2}}\,dx = 3\int \frac{1}{\sqrt{1-x^2}}\,dx$$
$$= 3 \arcsin x + C$$

b. Use rule 4.

$$\int \frac{3x}{\sqrt{1-x^2}}\,dx = 3\int x\left(1-x^2\right)^{-1/2}\,dx$$
$$= 3\left(-\frac{1}{2}\right)\int (-2x)\left(1-x^2\right)^{-1/2}\,dx$$
$$= \frac{-\frac{3}{2}\left(1-x^2\right)^{1/2}}{\frac{1}{2}} + C$$
$$= -3\sqrt{1-x^2} + C$$

Part **c** cannot be evaluated using the basic integration rules because it does not fit any of the forms listed.

Chapter 8, Section 2, page 523

In the special case when f and g are both increasing functions, this graph gives the definite integral formulation of integration by parts.

$$[uv]_{(p,\,r)}^{(q,\,s)}$$

is unfamiliar notation. (p, r) and (q, s) are ordered pairs of numbers where the first components are values of u and the second components are values of v.

Chapter 8, Section 4, page 541

The exact value of the integral $\int_{-1}^{1} \sqrt{1 - x^2}\, dx$ is $\frac{1}{2}\pi$. From geometry, you know that $y = \sqrt{1 - x^2}$ is a semicircle with radius 1. Therefore, $\text{Area} = \frac{1}{2}\pi r^2 = \frac{1}{2}\pi$.

Let $x = \sin\theta$

$dx = \cos\theta\, d\theta$

At $x = 1, 1 = \sin\theta \Rightarrow \theta = \frac{\pi}{2}$.

At $x = -1, -1 = \sin\theta \Rightarrow \theta = \frac{\pi}{2}$.

$$\begin{aligned}
\int_{-1}^{1} \sqrt{1 - x^2}\, dx &= \int_{-\pi/2}^{\pi/2} \sqrt{1 - \sin^2\theta} \cos\theta\, d\theta \\
&= \int_{-\pi/2}^{\pi/2} \cos^2\theta\, d\theta \\
&= \frac{1}{2}\int_{-\pi/2}^{\pi/2} [1 + \cos 2\theta]\, d\theta \\
&= \frac{1}{2}\left[\theta + \frac{1}{2}\sin 2\theta\right]_{-\pi/2}^{\pi/2} \\
&= \frac{1}{2}\left[\frac{\pi}{2} + \frac{1}{2}\sin\pi\right] - \frac{1}{2}\left[-\frac{\pi}{2} + \frac{1}{2}\sin(-\pi)\right] \\
&= \frac{\pi}{4} + \frac{\pi}{4} \\
&= \frac{\pi}{2}
\end{aligned}$$

Chapter 8, Section 7, page 567

Yes, because
$$\begin{aligned}
\int \frac{2\, du}{u^2 + 1} &= 2 \arctan u + C \\
&= 2 \arctan \sqrt{x - 1} + C.
\end{aligned}$$

Chapter 9, Section 1, page 588

a. The denominators are powers of 2.

$$a_n = \frac{1}{2^{n-1}}, n \geq 1$$

As $n \to \infty, \frac{1}{2^{n-1}} \to 0$ because the numerator stays constant as the denominator increases.

b. The denominators are factorials.

$$a_n = \frac{1}{n!}, n \geq 1$$

As $n \to \infty, \frac{1}{n!} \to 0$ because the numerator stays constant as the denominator increases.

c. The denominator is the sum $1 + 2 + 3 + \cdots + n$.

$$a_n = \frac{10}{\frac{1}{2}(n)(n+1)}, n \geq 1$$

As $n \to \infty, \frac{10}{\frac{1}{2}(n)(n+1)} \to 0$ because the numerator stays constant as the denominator increases.

d. The numerator is an integer squared and the corresponding denominator is that integer plus one squared.

$$a_n = \frac{n^2}{(n+1)^2}, n \geq 1$$

As $n \to \infty, \frac{n^2}{(n+1)^2} \to 1$ because the numerator and the denominator are equal degrees.

e. The numerator is an odd integer starting with 3 and each denominator is three more than the previous one.

$$a_n = \frac{2n+1}{4+3n}, n \geq 1$$

As $n \to \infty, \frac{2n+1}{4+3n} \to \frac{2}{3}$ because the numerator and the denominator are equal degrees.

Chapter 9, Section 2, page 601

Because ΔPQR and ΔTSP are similar triangles, the ratios of corresponding sides are equal.

In ΔTSP, the ratio of the vertical side to the horizontal side is $\frac{1 + r + r^2 + r^3 + \cdots}{1}$.

In ΔPQR, the ratio of the vertical side to the horizontal side is $\frac{1}{1-r}$.

Therefore, $1 + r + r^2 + r^3 + \cdots = \frac{1}{1-r}$.

This result is a special case of Theorem 9.6, where $r > 0$ and $a = 1$.

Chapter 9, Section 5, page 628

$$S_4 = 1 - \frac{1}{2} + \frac{1}{3} + \frac{1}{5} \approx 1.033$$

$$S_7 = 1 - \frac{1}{2} + \frac{1}{3} + \frac{1}{5} - \frac{1}{4} + \frac{1}{7} + \frac{1}{9} \approx 1.037$$

$$S_{10} = 1 - \frac{1}{2} + \frac{1}{3} + \frac{1}{5} - \frac{1}{4} + \frac{1}{7} + \frac{1}{9} - \frac{1}{6} + \frac{1}{11} + \frac{1}{13} \approx 1.039$$

$$S_{13} = 1 - \frac{1}{2} + \frac{1}{3} + \frac{1}{5} - \frac{1}{4} + \frac{1}{7} + \frac{1}{9} - \frac{1}{6} + \frac{1}{11} + \frac{1}{13} - \frac{1}{8} + \frac{1}{15} + \frac{1}{17} \approx 1.039$$

$$S_{16} = 1 - \frac{1}{2} + \frac{1}{3} + \frac{1}{5} - \frac{1}{4} + \frac{1}{7} + \frac{1}{9} - \frac{1}{6} + \frac{1}{11} + \frac{1}{13} - \frac{1}{8} + \frac{1}{15} + \frac{1}{17} - \frac{1}{10} + \frac{1}{19} + \frac{1}{21} \approx 1.039$$

$$S_{19} = 1 - \frac{1}{2} + \frac{1}{3} + \frac{1}{5} - \frac{1}{4} + \frac{1}{7} + \frac{1}{9} - \frac{1}{6} + \frac{1}{11} + \frac{1}{13} - \frac{1}{8} + \frac{1}{15} + \frac{1}{17} - \frac{1}{10} + \frac{1}{19} + \frac{1}{21} - \frac{1}{12} + \frac{1}{23} + \frac{1}{25} \approx 1.039$$

The sum is approximately $\frac{3}{2}\ln 2 \approx 1.040$.

Chapter 9, Section 7, page 645

$$P_4(1.1) = (1.1 - 1) - \frac{1}{2}(1.1 - 1)^2 + \frac{1}{3}(1.1 - 1)^3 - \frac{1}{4}(1.1 - 1)^4$$
$$\approx 0.0953083$$

This is the same result as in Example 7.

Chapter 9, Section 8, page 651

a. e^{-x}

b. $\cos x$

c. $\sin x$

d. $\arctan x$

e. e^{2x}

Chapter 10, Section 3, page 711

$$\frac{dy}{dx} = -\tan t$$

At $(1, 0)$, $t = \frac{\pi}{2}$. So, the slope at $(1, 0)$ is undefined.

At $(0, 1)$, $t = 0$. So, the slope at $(0, 1)$ is 0.

Chapter 10, Section 6, page 738

If $a \neq 0$ and $b = 1$, the graph is a parabola.

If $a \neq 0$ and $0 < b < 1$, the graph is an ellipse.

If $a \neq 0$ and $b > 1$, the graph is a hyperbola.

Chapter 11, Section 3, page 770

At $30°$, the vector is

$$\frac{\sqrt{3}}{2}\mathbf{i} + \frac{1}{2}\mathbf{j}.$$

At $330°$, the vector is

$$\frac{\sqrt{3}}{2}\mathbf{i} + \frac{1}{2}\mathbf{j}.$$

The angle between these two vectors is $60°$.

$$\left(\frac{\sqrt{3}}{2}\mathbf{i} + \frac{1}{2}\mathbf{j}\right) \cdot \left(\frac{\sqrt{3}}{2}\mathbf{i} - \frac{1}{2}\mathbf{j}\right) = \frac{3}{4} - \frac{1}{4} = \frac{1}{2} \text{ and } \cos 60° = \frac{1}{2}$$

At $60°$, the vector is

$$\frac{1}{2}\mathbf{i} + \frac{\sqrt{3}}{2}\mathbf{j}.$$

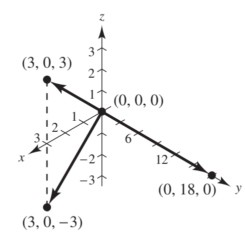
At $180°$, the vector is $-\mathbf{i}$. The angle between these two vectors is $120°$.

$$\left(\frac{1}{2}\mathbf{i} + \frac{\sqrt{3}}{2}\mathbf{j}\right) \cdot (-\mathbf{i}) = -\frac{1}{2} \text{ and } \cos 120° = -\frac{1}{2}$$

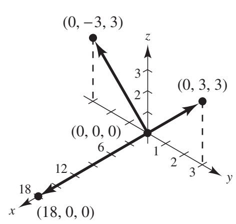
Conjecture: The dot product of two unit vectors is equal to the cosine of the angle between the vectors.

Chapter 11, Section 4, page 779

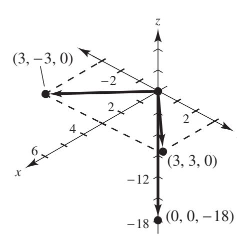
a. $$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & 0 & 3 \\ 3 & 0 & -3 \end{vmatrix} = -\mathbf{j}(-9 - 9) = 18\mathbf{j}$$



b. $$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 3 & 3 \\ 0 & -3 & 3 \end{vmatrix} = \mathbf{i}(9 + 9) = 18\mathbf{i}$$



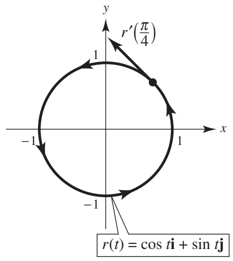
c. $$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & 3 & 0 \\ 3 & -3 & 0 \end{vmatrix} = \mathbf{k}(-9 - 9) = -18\mathbf{k}$$



In each of the cases, the vectors **u** and **v** lie on a plane parallel to one of the three coordinate planes and $\mathbf{u} \times \mathbf{v}$ is parallel to the axis perpendicular to the plane.

Conjecture: $\mathbf{u} \times \mathbf{v}$ is perpendicular to both **u** and **v**.

Chapter 12, Section 2, page 831



Let $x = \cos t$ and $y = \sin t$. Because $\cos^2 t + \sin^2 t = 1$ you obtain $x^2 + y^2 = 1$, the unit circle centered at the origin.

$$\mathbf{r}(t) = \cos t\,\mathbf{i} + \sin t\,\mathbf{j} \Rightarrow \mathbf{r}\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}\mathbf{i} + \frac{1}{\sqrt{2}}\mathbf{j}$$

$$\mathbf{r}'(t) = -\sin t\,\mathbf{i} + \cos t\,\mathbf{j} \Rightarrow \mathbf{r}'\left(\frac{\pi}{4}\right) = -\frac{1}{\sqrt{2}}\mathbf{i} + \frac{1}{\sqrt{2}}\mathbf{j}$$

$\mathbf{r}\left(\frac{\pi}{4}\right)$ and $\mathbf{r}'\left(\frac{\pi}{4}\right)$ are perpendicular.

$$\mathbf{r}(t) = \cos t\,\mathbf{i} + \sin t\,\mathbf{j}$$

$$\begin{aligned}\mathbf{r}(t) \cdot \mathbf{r}(t) &= (\cos t\,\mathbf{i} + \sin t\,\mathbf{j}) \cdot (\cos t\,\mathbf{i} + \sin t\,\mathbf{j})\\ &= \cos^2 t + \sin^2 t\\ &= 1, \text{ a constant}\end{aligned}$$

$$\begin{aligned}\mathbf{r}(t) \cdot \mathbf{r}'(t) &= (\cos t\,\mathbf{i} + \sin t\,\mathbf{j}) \cdot (-\sin t\,\mathbf{i} + \cos t\,\mathbf{j})\\ &= -\sin t \cos t + \sin t \cos t\\ &= 0\end{aligned}$$

This example is a direct application of Property 7 of Theorem 12.2 because $\mathbf{r}(t) \cdot \mathbf{r}(t) = 1$, a constant, and $\mathbf{r}(t) \cdot \mathbf{r}'(t) = 0$.

Chapter 12, Section 3, page 836

$$\mathbf{r}(t) = (\cos \omega t)\mathbf{i} + (\sin \omega t)\mathbf{j}$$

$$\mathbf{v}(t) = -(\omega \sin \omega t)\mathbf{i} + (\omega \cos \omega t)\mathbf{j}$$

$$\begin{aligned}\text{speed} = |\mathbf{v}(t)| &= \sqrt{\omega^2 \sin^2 \omega t + \omega^2 \cos^2 \omega t}\\ &= \sqrt{\omega^2(\sin^2 \omega t + \cos^2 \omega t)}\\ &= \sqrt{\omega^2(1)}\\ &= \omega, \text{ a constant}\end{aligned}$$

Acceleration $= \mathbf{v}'(t) = \mathbf{r}''(t) = -\omega^2 \cos \omega t\,\mathbf{i} - \omega^2 \sin \omega t\,\mathbf{j}$ is not a constant.

Chapter 12, Section 5, page 855

No, the arc length of a curve does not depend on the parameter being used. If it did, the arc length would not be the same, no matter which parameter was used.

With $\mathbf{r}(t) = t^2\mathbf{i} + \frac{4}{3}t^3\mathbf{j} + \frac{1}{2}t^4\mathbf{k}$, the arc length is

$$\begin{aligned} s &= \int_0^{\sqrt{2}} \sqrt{(2t)^2 + (4t^2)^2 + (2t^3)^2}\, dt \\ &= \int_0^{\sqrt{2}} \sqrt{4t^2 + 16t^4 + 4t^6}\, dt \\ &= \int_0^{\sqrt{2}} \sqrt{4t^2(1 + 4t^2 + t^4)}\, dt \\ &= \int_0^{\sqrt{2}} 2t\sqrt{(t^2 + 2) - 3}\, dt \\ &= \left[\frac{t^2 + 2}{2}\sqrt{(t^2 + 2)^2 - 3} - \frac{3}{2}\ln\left|(t^2 + 2) + \sqrt{(t^2 + 2)^2 - 3}\right|\right]_0^{\sqrt{2}} \\ &\approx 4.816 \end{aligned}$$

The result is the same as in Example 1.

Chapter 13, Section 1, page 872

a. Paraboloid

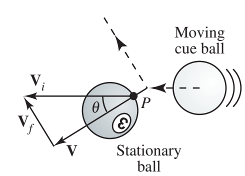
b. Plane

c. Cylinder

d. Cone (the half above the *xy*-plane)

e. Hyperboloid of One Sheet (the half above the *xy*-plane)

Chapter 13, Section 7, page 931



The diagram at the right depicts what happens if both balls have the same mass and the collision is elastic (absorbs no energy). The angle θ between the line of impact (the normal line to the stationary ball at the point P of collision) and the line of motion of the cue ball before impact can vary from 0 to $\pi/2$. The pre-impact velocity $\mathbf{v}_i$ of the cue ball is resolved into orthogonal components. The collision imparts a speed of $\|\mathbf{v}\| = \|\mathbf{v}_i\| \cos\theta$ to the stationary ball and its direction is away from P along the line of impact.

The post-impact speed of the cue ball is $\|\mathbf{v}_f\| = \|\mathbf{v}_i\| \sin\theta$, directed along a line that is perpendicular to the line of impact.

The top figure in the Exploration illustrates the case $\theta = 0$, where the stationary ball acquires the greatest possible speed—the same speed that the cue ball has before impact. As θ increases, the speed acquired decreases.

The stationary ball in the bottom figure would acquire the least speed.

Two physical principles are needed to justify this explanation: conservation of linear momentum and conservation of kinetic energy.

Chapter 13, Section 8, page 941

There appears to be an absolute minimum on the surface $z = f(x, y) = x^3 - 3xy + y^3$ at $f(1, 1) = -1$. However, this point is not an absolute minimum for the surface because there are points on the surface where $z < -1$. For example, $f(-1, -1) -5$.

Chapter 14, Section 2, page 980

Using the depth at the center of each square as the height, the volume of each section can be approximated by multiplying the length (10) by the width (10) by the height. So, the total volume is

$$\begin{aligned} V &\approx (10)(10)(100) + (10)(10)(90) + (10)(10)(70) + (10)(10)(70) + (10)(10)(70) + \\ &\quad (10)(10)(40) + (10)(10)(50) + (10)(10)(50) + (10)(10)(40) + (10)(10)(40) + \\ &\quad (10)(10)(50) + (10)(10)(30) \\ &= 7000 \text{ cubic yards.} \end{aligned}$$

Chapter 14, Section 2, page 983

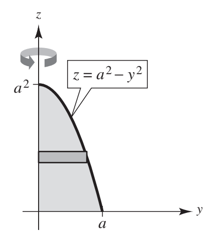
The volume of the circular paraboloid can be found in three ways.

(1) Using a double integral:

$$V = \int_{-a}^{a} \int_{-\sqrt{a^2-x^2}}^{\sqrt{a^2-x^2}} \left(a - x^2 - y^2\right) dy\, dx = \frac{\pi a^4}{2}$$

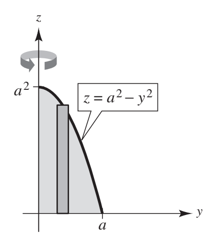
(2) Using the disk method for the volume of a solid of revolution:

$$\begin{aligned} V &= \pi \int_0^{a^2} R^2 - r^2\, dz & \text{where } R &= \sqrt{a^2 - z} \\ &= \pi \int_0^{a^2} \left(a^2 - z\right) dz & r &= 0 \\ &= \pi \left[a^2 z - \frac{z^2}{2}\right]_0^{a^2} \\ &= \frac{\pi a^4}{2} \end{aligned}$$



(3) Using the shell method for the volume of a solid of revolution:

$$\begin{aligned} V &= 2\pi \int_0^{a} ph\, dy & \text{where } p &= y \\ &= 2\pi \int_0^{a} y\left(a^2 - y^2\right) dy & h &= a^2 - y^2 \\ &= 2\pi \left[\frac{a^2 y^2}{2} - \frac{y^4}{4}\right]_0^{a} \\ &= \frac{\pi a^4}{2} \end{aligned}$$

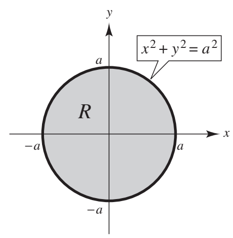


Chapter 14, Section 3, page 992

The volume of the circular paraboloid can be found using polar coordinates as follows.

$$\begin{aligned}
V &= \int_R\int z\,dA \\
&= \int_0^{2\pi}\int_0^a \left(a^2 - r^2\right) r\,dr\,d\theta \\
&= \int_0^{2\pi}\int_0^a \left(a^2 r - r^3\right) dr\,d\theta \\
&= \int_0^{2\pi}\left[\frac{a^2r^2}{2} - \frac{r^4}{4}\right]_0^a d\theta \\
&= \int_0^{2\pi}\left[\frac{a^4}{2} - \frac{a^4}{4}\right] d\theta \\
&= \int_0^{2\pi}\frac{a^4}{4}\,d\theta \\
&= \frac{a^4}{4}\left[\theta\right]_0^{2\pi} \\
&= \frac{a^4}{4}(2\pi) \\
&= \frac{\pi a^4}{2}
\end{aligned}$$

where $z = a^2 - \left(x^2 + y^2\right)$
$= a^2 - r^2$
and R is as shown



Chapter 14, Section 6, page 1014

The volume of the circular paraboloid can be found using a triple integral.

$$\begin{aligned}
V &= \int_{-a}^{a}\int_{-\sqrt{x^2-x^2}}^{\sqrt{a^2-x^2}}\int_0^{a^2-x^2-y^2} dz\,dy\,dx \\
&= \int_{-a}^{a}\int_{-\sqrt{a^2-x^2}}^{\sqrt{a^2-x^2}} \left(a^2 - x^2 - y^2\right) dy\,dx \\
&= \int_{-a}^{a}\left[\left(a^2 - x^2\right)y - \frac{y^3}{3}\right]_{-\sqrt{a^2-x^2}}^{\sqrt{a^2-x^2}} dy\,dx \\
&= \int_{-a}^{a}\frac{4}{3}\left(a^2 - x^2\right)^{3/2} dx && \text{let } x = a\sin\theta \\
&= \frac{4}{3}\int_{-\pi/2}^{\pi/2}\left(a^2\cos^2\theta\right)^{3/2}\cdot a\cos\theta\,d\theta && dx = a\cos\theta\,d\theta \\
&= \frac{8}{3}a^4\int_0^{\pi/2}\cos^4\theta\,d\theta \\
&= \frac{8}{3}a^4\left(\frac{1}{2}\right)\left(\frac{3}{4}\right)\left(\frac{\pi}{2}\right) && \text{Wallis's Formula} \\
&= \frac{\pi a^4}{2}
\end{aligned}$$

Chapter 14, Section 7, page 1025

The volume of the circular paraboloid can be found using cylindrical coordinates.

Because $z = a^2 - x^2 - y^2 = a^2 - r^2$, the bounds on z are $0 \le z \le a^2 - r^2$.

$$\begin{aligned} V &= \int_0^{2\pi} \int_0^a \int_0^{a^2 - r^2} r \, dz \, dr \, d\theta \\ &= \int_0^{2\pi} \int_0^a \left[rz\right]_0^{a^2 - r^2} dr \, d\theta \\ &= \int_0^{2\pi} \int_0^a r\left(a^2 - r^2\right) dr \, d\theta \\ &= \int_0^{2\pi} \left[\frac{a^2}{2} r^2 - \frac{1}{4} r^4\right]_0^a d\theta \\ &= \int_0^{2\pi} \frac{a^4}{4} \, d\theta \\ &= \left[\frac{a^4}{4}\theta\right]_0^{2\pi} \\ &= \frac{\pi a^4}{2} \end{aligned}$$

(For details, refer to the Exploration for Chapter 14, Section 3, page 992.)

Chapter 15, Section 2, page 1064

You can use a line integral to find the surface area of the piece of tin by integrating the height function over the path of the base of the cylinder. Begin by writing a parametric form for the circular base, $x^2 + y^2 = 9$.

$$\begin{aligned} x &= 3 \cos t \text{ and } y = 3 \sin t \\ \mathbf{r}(t) &= 3 \cos t \, \mathbf{i} + 3 \sin t \, \mathbf{j} \\ \mathbf{r}'(t) &= -3 \sin t \, \mathbf{i} + 3 \cos t \, \mathbf{j} \\ \left\|\mathbf{r}'(t)\right\| &= 3 \end{aligned}$$

With this parametrization,

$$ds = \left\|\mathbf{r}'(t)\right\| dt = 3 \, dt \text{ and } f(x, y) = 1 + \cos \frac{\pi x}{4} = 1 + \cos\left(\frac{3\pi \cos t}{4}\right)$$

Therefore,

$$\begin{aligned} \text{lateral surface area} &= \int_c f(x, y) \, ds \\ &= \int_0^{2\pi} \left[1 + \cos \frac{3\pi \cos t}{4}\right](3 \, dt). \end{aligned}$$

Using the integration capabilities of a graphing utility gives a value of approximately 19.330.

Chapter 15, Section 5, page 1094

If u is fixed, then $\mathbf{r}(u, v) = C_1 \cos v\mathbf{i} + C_2 \sin v\mathbf{j} + C_2\mathbf{k}$, where $C_1 = 2 + \cos u$ and $C_2 = \sin u$. Then $z = C_2$ and, because $x = C_1 \cos v$ and $y - C_1 \sin v$, $x^2 + y^2 = C_1^2$. This is a circle with center at $(0, 0, C_2)$ on a plane parallel to the xy-plane and C_2 units from it.

If v is fixed, then $\mathbf{r}(u, v) = C_1(2 + \cos u)\mathbf{i} + C_2(2 + \cos u)\mathbf{j} + \sin u\mathbf{k}$, where

$$\frac{x}{C_1} = 2 + \cos u = \frac{y}{C_2} \text{ and } z = \sin u.$$

Hence,

$$\cos u = \frac{x}{C_1} - 2 = \frac{y}{C_2} - 2$$

and you obtain the equations

$$\left(\frac{x}{C_1} - 2\right)^2 + z^2 = 1 \text{ and } \left(\frac{y}{C_2} - 2\right)^2 + z^2 = 1.$$

The first equation is the generating curve of an elliptic cylinder with rulings parallel to the y-axis. The second equation is also the generating curve of an elliptic cylinder, but with rulings parallel to the x-axis. Thus, the surface is two elliptic cylinders that intersect each other at right angles.

Chapter 16, Section 1, page 1133

$$\frac{dy}{dx} + P(x)y = Q(x)$$

$$dy + P(x)y\,dx = Q(x)\,dx$$

$$\left[P(x)y - Q(x)\right]dx + dy = 0$$

$$M(x, y) = P(x)y - Q(x) \text{ and } N(x, y) = 1$$

$$\frac{M_y(x, y) - N_x(x, y)}{N(x, y)} = \frac{\frac{d}{dy}\left[P(x)y - Q(x)\right] - \frac{d}{dx}[1]}{1}$$

$$= P(x)$$

By Theorem 16.2, $u(x) = e^{\int P(x)\,dx}$ is an integrating factor.

Chapter 16, Section 2, page 1138

(a) $y'' - 9y = 0$: $m^2 - 9 = 0$

$$m = \pm 3$$

General solution: $y = C_1e^{3x} + C_2e^{-3x}$

(b) $y'' - 6y' + 8y = 0$: $m^2 - 6m + 8 = 0$

$$(m - 4)(m - 2) = 0$$

$$m = 2, 4$$

General solution: $y = C_1e^{2x} + C_2e^{4x}$

The general solution of a second-order linear differential equation $y'' + ay' + by = 0$ having distinct real roots m_1 and m_2 is $y = C_1e^{m_1x} + C_2e^{m_2x}$.

Chapter 16, Section 3, page 1150

Assume that $u_1 = -\frac{x}{2} + a_1$ and $u_2 = \ln\sqrt{x} + a_2$.

Then $y_p = u_1y_1 + u_2y_2$

$$= \left(-\frac{x}{2} + a_1\right)e^x + \left(\ln\sqrt{x} + a_2\right)xe^x.$$

and $y = y_h + y_p$

$$= \left(C_1e^x + C_2xe^x\right) + \left(-\frac{x}{2} + a_1\right)e^x + \left(\ln\sqrt{x} + a_2\right)xe^x$$

$$= k_1e^x + k_2xe^x - \frac{x}{2}e^x + xe^x\ln\sqrt{x}.$$

Here, $k_1 = C_1 + a_1$ and $k_2 = C_2 + a_2$. This solution is the same as that obtained in Example 5.

Chapter 16, Section 4, page 1153

The differential equation $y' - 2y = 0$ is both linear and separable. Using separation, you have

$$\frac{dy}{dx} = 2y$$

$$\frac{dy}{y} = 2\,dx$$

$$\ln|y| = 2x + C_1$$

$$y = e^{2x+C_1} = Ce^{2x}.$$

This is the same solution as obtained in Example 1.