Chapter 4 Introduction to Probability Solutions

* 1. An *a priori* probability, because it is based on logical analysis rather than on observation or personal judgment.
  2. A subjective probability, because it is drawn on personal judgment.
  3. An empirical probability, as 0.40 represents the relative frequency of Republicans in the room, that is,



* 1. = 1.



The odds fortails are 1 to 1 (1:1).

* 1. = 9.



The odds assessed by John for receiving straight A’s are 9 to 1 (9:1).

* 1. =0.67.



The odds assessed by the reporter for a Republican coming out of the room are 0.67 to 1 (0.67:1).

* 1. (The events are equally likely)



* 1. First define



Then .











the events *A* and *B* are not mutually exclusive.



* 1. ;



; the events A and C are mutually exclusive.



* 1. ; the events



; the events *A* and *D* are mutually exclusive.



* 1. ;



; the events *B* and *C* are not mutually exclusive.



* 1. Events *A* and *B* are not exhaustive because you may not have get an offer from either firm.
  2. Events *A* and *B* are not mutually exclusive because you may get an offer from both firms.



* 1. Using the complement rule,



* 1. The events “overweight” and “obese” are not exhaustive. The US adult population also comprises individuals that are neither overweight nor obese. Furthermore,

.



* 1. According to the BMI classification, an individual is either “overweight” or “obese”. As a result, the two events are mutually exclusive.

1. 1. In 1971,



In 2006*,*



* 1. In 1971,



In 2006,



* 1. The data indicate that relative to 1971, younger workers are less attracted to municipal positions in 2006.

1. 1. .



* 1. = 0.115.



* 1. According to the data, Cape Cod appears to have a higher level of moderately severe to severe depression with 11.5% of its residents compared to 6.7% at the national level.
  2. An odd of 15:8 means that an individual who, prior to the final, bet $15 on Spain winning the World Cup would have won $8 in gains if Spain did win. Therefore, net gain for the $1000 bet is: $533.33.



If Spain had lost, your net loss would be your entire bet of $1,000.

* 1. .



|  |  |  |
| --- | --- | --- |
| Best Actress | Movie | Probability |
| Anne Hathaway | Rachel Getting Married |  |
| Angelina Jolie | Changeling | 0.048 |
| Melissa Leo | Frozen River | 0.029 |

|  |  |  |
| --- | --- | --- |
| Meryl Streep | Doubt | 0.231 |
| Kate Winslet | The Reader | 0.714 |

* 1. Kate Winslet is credited with a 0.714 probability of winning the Oscar. She did indeed win the Oscar, which is in accordance with the prediction.



* 1. =0.815



* 1. (Note that if *B* occurred then *A* cannot occur since the two are mutually exclusive.)



Therefore,



* 1. No, because



* 1. ≠ 0.



*A* and *B* are not mutually exclusive events.



;



Therefore,



* 1. *A* and *B* are not independent events.



* 1. ; *A* and *B* are not mutually exclusive events.



.











* 1. (*A*|)



* 1. (*A*)



* 1. ;



(*A*|); Events *A* and are not independent. Therefore, *A* and *B* are not independent either.



* 1. (|)



* 1. =0.86.







1. .



* 1. Events *A* and *B* are not mutually exclusive since



* 1. Events *A* and *B* are not independent since



1. Let event *R* be “Reduction in unemployment in the US” and event *E* be “Recession in Europe”.



* 1. *P*



* 1. P



1. Let event *A* be “Do homework regularly” and *B* be “Pass the course”.



* 1. *P*



* 1. *P*



* 1. No, because *P*



* 1. No, because P



Let *F* be “Foreign student” and *S* be “Smoke”.



From , we derive Therefore, 10% of the student body at the university is foreign.



1. Let event *A* be “Default on a seven-year AA bond” and *B* be “Default on a seven-year A bond”.



1. Let event *A* be “Experienced problems shopping online”, *B* be “abandoned the transaction or switched to a competitor’s website”, and *C* be “contacted customer-service representatives”

, ,







1. Let event *D* be “Dave shows up”, and event *M* be “Mike shows up”



1. Let event *S* be “Women face sexual harassment”, and event *T* be “Women use public transportation”



1. Let event *U* be “Unemployed”, and event *C* be “labor force has a college degree”



1. Let event *F* be “Foreclosed”, event *H* be “centered in Arizona, California, Florida, and Nevada". Therefore,



1. Let event *S* be “The patient had surgery”, and event *I* be “The patient experienced an improvement”.







* 1. =



1. 1. Joint probability table:

|  |  |  |  |
| --- | --- | --- | --- |
|  | *B* |  | Total |
| *A* | 0.26 | 0.34 | 0.60 |
|  | 0.14 | 0.26 | 0.40 |
| Total | 0.40 | 0.60 | 1 |



* 1. No, because .



* 1. No, because



1. Joint probability table:

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  |  |  |  |  | Total |
| *A* | 0.09 | 0.22 | 0.15 | 0.20 | 0.66 |

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | 0.03 | 0.10 | 0.09 | 0.12 | 0.34 |
| Total | 0.12 | 0.32 | 0.24 | 0.32 | 1 |



* 1. Joint probability table (A slight inconsistency in the totals is due to rounding):

|  |  |  |  |
| --- | --- | --- | --- |
|  | IT Professional (*IT*) | Government Professional (*G*) | Total |
| Yes | 0.2214 | 0.3657 | 0.5871 |
| No | 0.2071 | 0.2057 | 0.4129 |
| Total | 0.4286 | 0.5714 | 1.00 |







Therefore, job category is not independent of whether or not a worker sleeps on the job.

1. Joint probability table:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | Hispanic (*H*) | Black (*B*) | White (*W*) | Total |
| Male (*M*) | 0.335 | 0.205 | 0.165 | 0.705 |
| Female (*F*) | 0.145 | 0.105 | 0.045 | 0.295 |

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Total | 0.480 | 0.310 | 0.210 | 1.000 |



Gender is not independent of race.

* 1. Joint probability table:

|  |  |  |  |
| --- | --- | --- | --- |
|  | Vaccinated (*V*) | Dummy Shot (*D*) | Total |
| Infected (*I*) | 0.016 | 0.014 | 0.030 |
| Not Infected (*N* ) | 0.477 | 0.493 | 0.970 |
| Total | 0.493 | 0.507 | 1.000 |



* 1. (up to 2 decimal places)



Being infected with HIV appears to be independent to getting vaccinated. The vaccine does not appear to be effective in preventing HIV infection. Given this finding, it is not a surprise that Merck & Co. ended the program.

1. Joint probability table:

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | Natural Gas (*A*) | Electricity (*B*) | Heating Oil (*C*) | Propane (*D*) | Total |

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Yes | 0.10 | 0.04 | 0.03 | 0.02 | 0.19 |
| No | 0.48 | 0.26 | 0.04 | 0.03 | 0.81 |
| Total | 0.58 | 0.30 | 0.07 | 0.05 | 1 |



* 1. ; delinquency in paying utility bill on time is not independent of the type of heating.



1. We cannot fit the given information in one contingency table. The totals included in the following table aid in the calculations.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Preference | Gender | America(*A*) | Europe (*B*) | Asia( *C*) | Total |
| Like it (*L*) | Men (*M*) | 210 | 150 | 120 | 480 |
| Women(*W*) | 370 | 310 | 180 | 860 |
| Don't like it () | Men (*M*) | 290 | 150 | 80 | 520 |
| Women(*W*) | 330 | 190 | 120 | 640 |

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | Total | 1200 | 800 | 500 | 2,500 |



* 1. Let the event *BW* represent a European woman.



* 1. Let the event *AM* represent an American man.



* 1. For all countries, we will determine if



1. America: ; not independent.



1. Europe: ; not independent.



1. Asia: ; independent.



* 1. We need to determine if



not independent. At the international level, the probability that a woman customer will like the perfume is greater.



* 1. 1



|  |  |  |  |
| --- | --- | --- | --- |
| Prior Probability | Conditional  Probability | Joint Probability | Posterior Probability |
| *P*(*B*)=0.85 | *P*(*A*|*B*)=0.05 | *P*(  = 0.0425 |  |
| *P*()=0.15 | *P*(*A*|)=0.80 | = 0.12 |  |

|  |  |  |  |
| --- | --- | --- | --- |
| Total=1.00 |  | *P*(*A*) = 0.0425 + 0.12  = 0.1625 | Total = 1.00 |

|  |  |  |  |
| --- | --- | --- | --- |
| Prior Probabilities | Conditional Probabilities | Joint Probabilities | Posterior Probabilities |

|  |  |  |  |
| --- | --- | --- | --- |
| *P*()=0.10 | *P*( |)=0.40 |  |  |
| *P*()=0.60 | *P*( |)=0.60 |  |  |
| *P*()=0.30 | *P*( |)=0.80 |  |  |
| Total = 1.00 |  | *P*(*A*)=0.04+0.36+0.24 =0.64 | Total = 1.00 |

1. Let event *F* be “Fail the course”, and event *T* be “Find a tutor”



1. Let event *D* be “Experience a decline”, and event *N* be “Ratio is negative”



In order to find we first compute the probability that the ratio will be negative:



Therefore, .



1. Let event *D* be “Detect a speeder”, and event *S* be “Driver is speeding”



* 1. 0.1624



Let *F* = “Player is fully fit to play”, *S* = “Player is somewhat fit to play”, *N* = “Player is not able to play”, and *W* = “The Lakers win the game”

* 1. Consider the following probability table:

|  |  |  |
| --- | --- | --- |
| = 0.40 | =0.80 |  |
| = 0.30 | =0.60 |  |

|  |  |  |
| --- | --- | --- |
| = 0.30 | =0.40 |  |
| 1.00 |  | = 0.32+0.18+0.12=0.62 |

The Lakers have a 62% chance of winning the game.



* 1. P(“World Economy is neutral” *and* “Creative ideas is poor”)



* 1. Let event *A* be “World Economy is good”, *B* be “World Economy is neutral”, *C* be “World Economy is poor”, and *D* be “Creative Ideas are poor”



















1. We use the combination formula since the order does not matter;



a. We use the combination formula since the order does not matter;



b. Here we use the permutation formula since the order does matter;



a. ; so the probability that David and Valerie get picked is 1/28 = 0.0357.



b. ; so the probability that Valerie gets picked before David is 1/56 = 0.0179.



1. Let *U*: “US equity”, *F*: “Foreign equity”

, and



1. Let *X*: “Age”



1. Let *F*: “Fight during the game”



The odds for a fight occurring during the game are: 5.25 or 5.25 to 1.



* 1. (independence)



1. Let *U*: “Unemployment”, *F*: “Female”, and *M*: “Male”











1. Let *S*: “Biggest smilers”, “Biggest frowners”, *D*: “Divorced”



* 1. using



0.1818 or 18.18%.



* 1. or 7.75%.



|  |  |  |  |
| --- | --- | --- | --- |
|  | Survived for | Did not Survive |  |
|  | Discharge () | for Discharge () | Total |
| Day or Evening Shift () | 0.1338 | 0.5417 | 0.6755 |
| Graveyard Shift () | 0.0477 | 0.2768 | 0.3245 |
| Total | 0.1815 | 0.8185 | 1.00 |



* 1. 5



Whether or not a patient survives is not independent of the timing of a heart attack. These results suggest that hospitals should have an equally adequate qualified professionals and resources available to patients at all times.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Response by Women | "20 to 35" | "35 to 50" | "Over 50" | Total Women |
| "Yes" | 0.243 | 0.12 | 0.107 | 0.47 |
| "No" | 0.223 | 0.18 | 0.127 | 0.53 |
| Total Women | 0.466 | 0.30 | 0.234 | 1.00 |
|  |  |  |  |  |
| Response by Men | "20 to 35" | "35 to 50" | "Over 50" | Total Men |
| "Yes" | 0.193 | 0.113 | 0.127 | 0.433 |
| "No" | 0.307 | 0.153 | 0.107 | 0.567 |
| Total Men | 0.500 | 0.266 | 0.234 | 1.00 |

* 1. For women:



* 1. For men:



* 1. For Women: therefore satisfaction is not independent of age.



* 1. For Men: therefore satisfaction is not independent of age.



1. Let A: “US economy performs well” and B: “Asian countries will perform well”



1. Let A: “Depression”, and B: “Dementia”

, and



**Case Study 4.1**

|  |  |  |  |
| --- | --- | --- | --- |
| Return in Near Future? | First-time customer (*F*) | Established customer (*E*) | Total |
| Yes | 0.35 | 0.10 | 0.45 |
| No | 0.05 | 0.50 | 0.55 |
| Total | 0.40 | 0.60 | 1.00 |

1. ; ;



1. Also,



The type of customer is dependent on whether a customer decides to return or not.

However, compared to established customers, first-time customers are much more likely to return. At the same time, Starbuck has interest in not losing its older clientele which account for 60%. The decision to reintroduce the older brew probably aims at achieving that goal.

**Case Study 4.2**

Let *H*: “Women suffered hypothyroid during pregnancy”, *I*: “Child with impaired intelligence (IQ is 85 or lower)”



1. Number of children with IQ of 85 or lower if the thyroid gland is ignored: 19,000 children.



1. Number of children with IQ of 85 or lower if the thyroid gland is treated or not an issue: 5,000 children. In other words, 14,000 children could avoid potential learning impairments if thyroid gland is properly taken care of during pregnancy.



**Case Study 4.3**

1. Let *O*: “Find oil”, *S*: “New technology signals oil”

=0.05, = 0.80, and = 0.01. Note that = 1 ‒ 0.80 = 0.20 and = 1 ‒ 0.01 = 0.99. We have to determine the probability, . We complete the following probability table.



|  |  |  |  |
| --- | --- | --- | --- |
| Prior Probability | Conditional Probabilities | Joint Probabilities | Posterior Probabilities |
| 0.05 | =0.20 |  |  |
| 0.95 | =0.99 |  |  |
| Total = 1.00 |  |  | Total = 1.00 |

1. = 0.0105. In other words there is only a 1.05% chance of finding oil in the area where the new technology has detected no oil.

