Chapter 6 Continuous Probability Distributions

Solutions

1. Note that for a continuous distribution, .



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* 1. Since 4 is the upper bound,



1. Note that for a continuous distribution, .



* 1. ; The probability that a continuous random variable assumes a particular value is 0.























1. Let *X* equal the price of electricity in New England.
   1. .



1. Let *X* equal the arrival time of an elevator.



* 1. (Note that



1. Let *X* equal the height of a tulip.



1. Let *X* equal the arrival time for a daily flight from Boston to New York.
   1. We first convert the time data to a minute scale. So, the interval from 9:15 am to 9:55am translates to an interval from 0 minutes to 40 minutes.

Therefore 11.55 minutes.



* 1. In order to find the probability that a flight arrives is late (later than 9:25am), we specify the problem in minutes as We first find the probability density function: .























1. Let *X* represent the return on the portfolio.
   1. A return of 20% is one standard deviation above the mean (20 = 8 + 12). Since about 68% of the observations fall within one standard deviation of the mean, half of 32% (100 ‒ 68) = 16% of the returns were greater than 20%.
   2. A return of ‒16% is two standard deviations below the mean (‒16 = 8 ‒2×12). Since about 95% of the observations fall within two standard deviations, half of 5% (100 ‒ 95) = 2.5% of the returns were below ‒16%.
2. Let *X* represent the IQ score.
   1. Since 84 and 116 represent plus or minus one standard deviation from the mean, about 68% of people scored between 84 and 116.
   2. An IQ score of 68 is two standard deviations below the mean (68 = 100 ‒ 32). Since about 95% of the observations fall within two standard deviations of the mean, half of 5% (100 ‒ 95) = 2.5% of people scored less than 68.
3. Let *X* represent rent in a city.
   1. Since $1250 and $1750 represent plus or minus one standard deviation from the mean, about 68% of rents are between $1,250 and $1,750.
   2. A rent of $1250 is one standard deviation below the mean (1250 = 1500 ‒ 250). Since about 68% of the observations fall within one standard deviation of the mean, half of 32% (100 ‒ 68) = 16% of rents are below $1250.
   3. A rent of $2000 is two standard deviations above the mean (2000 = 1500 + 2×250). Since 95% of the observations fall within two standard deviations of the mean, half of 5% (100 ‒ 95) = 2.5% of rents are greater than $2000.
4. Let *X* equal points scored in a game.
   1. Since 60 and 100 represent plus or minus two standard deviations from the mean, about 95% of scores are between 60 and 100 points.
   2. A score of 100 is two standard deviations above the mean (100 = 80 + 2×10). Since about 95% of the observations fall within two standard deviations of the mean., half of 5% (100 ‒ 95) = 2.5% of scores are more than 100 points. If there are 82 games in the regular season, we expect the team to score more than 100 points in approximately 2 games (82×0.025 = 2.05).



















* 1. Given , we find



* 1. Given Therefore,











* 1. Given T



* 1. using



1. 1. Given , we find



* 1. Given we find



* 1. ; using



* 1. Given , we find







1. Let *X* represent high-school teacher salary.







1. Let *X* representsleep time on weekdays.



1. Let *X* represent the weight of a turkey.



1. For both distributions let *X* represent the number of weeks to find a job.



1. Let *X* represent the rate of seriously delinquent loans.







1. Let *X* represent the time required to assemble an electronic component.







1. Let *X* represent the number of calls made per day.







1. We convert into = , to find We then use the inverse transformation to solve for as . Therefore,



1. Let *X* represent the cumulative debt of a recent college graduate.

= 0.1423. Therefore about 1,800,000 × 0.1423 = 256,140 students have accumulated a student loan of more than $30,000.



1. Let *X* represent the score on a marketing exam.



* 1. 0



1. Let *X* equal the talk time between charges of a cell phone.



1. We first use this cumulative probability to find the corresponding We then use the inverse transformation to solve for as . Therefore,



1. Let *X* represent the price of a condominium.



* 1. Here X represents the artist's condo.



1. Let *X* represent the return on a mutual fund.



You should pick the less risky fund because it gives you a lower likelihood of earning a negative return (21.29% < 28.43%).

* 1. For the riskier fund:



For the less risky fund:



You should pick the riskier fund because it gives you a higher likelihood of earning a return above 8% (50% > 21.19%).




















1. We f







1. Let *X* represent the time between customer purchases.



* 1. No, Jack is wrong in his belief since a noted feature of the exponential distribution is that it is memoryless. In this example, the probability that a new customer arrives in the next five minutes is independent of whether or not a customer has just been serviced.







1. Let *X* represent the time drivers wait in line to pay the toll.



1. 1.  5















1. Let *X* represent delivery time.



* 1. . 25% of deliveries are made after 4 PM.



* 1. . 37.5% of deliveries are made prior to 2.30 PM.



1. Let *X* equal the weight of a bag.



1. Let *X* equal diastolic (part a) and systolic readings (part b).



1. Let *X* represent the amount spent annually on a debit card.
   1. . The proportion of consumers who spend over $8,000 is only about 0.34, which is not the majority.



1. Let *X* represent the amount spent on St. Patrick’s Day.



* 1. Women are slightly more likely to spend over $50, with a 3.14% likelihood as opposed to 2.07% likelihood for men.

1. Let *X* represent the number of customers signed up over a month.



* 1. At first glance, the results seem somewhat surprising. Brad, on average, signs in more customers than Lisa (56 compared to 48), which would make one think that he had a better chance of earning the bonus. However, Lisa has a higher probability of earning the bonus because of a higher standard deviation (22 compared to 17).

1. Let *X* represent the time that elapses between successive speeders.



1. Let *X* represent the time between violent crimes.



* 1. The probability will be the same because the exponential distribution has a “memoryless” property which means that the probability is independent of whether or not a violent crime occurred in the previous minute.

1. Let *X* represent the relief time.



* 1. , and  



* 1. There is a slightly higher likelihood that the relief will last less than 4 hours with the normal distribution as compared to the lognormal distribution.



**Case Study 6.1**



Healthy Weight:



Overweight:



Obese:



1. The concern of the health officials is justified. As the following table shows, 34.30% of their 10-year boys are overweight and 12.51% are obese.

|  |  |
| --- | --- |
| Weight Status Category | Proportion of Boys in town |
| Underweight | 0.0274 |
| Healthy Weight | 0.5045 |
| Overweight | 0.3430 |
| Obese | 0.1251 |

**Case Study 6.2**



The Income fund is better for minimizing the probability of earning a negative return.



The Income fund is better for maximizing the probability of earning a return between 0 to 10%.



The Metals fund is better for maximizing the probability of earning a return greater than 10%.

**Case Study 6.3**

1. With the Normal Distribution:



1. With the Lognormal Distribution:



, and 







1. Compared to the normal distribution, with the lognormal distribution, there is a slightly lower probability of the temperature going below 20C but a higher probability of the temperature going above 80C.