Complete Solutions Manual to Accompany

Applied Calculus for the Managerial, Life and Social Sciences

TENTH EDITION

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PREFACE

This *Complete Solutions Manual* contains solutions to all of the exercises in my textbook *Applied Calculus for the Managerial, Life, and Social Sciences, Tenth Edition*. The corresponding *Student Solutions Manual* contains solutions to the odd-numbered exercises and the even-numbered exercises in the "Before Moving On" quizzes. It also offers problem-solving tips for many sections.

I would like to thank Tao Guo for checking the accuracy of the answers to the new exercises in this edition of the text and the *Complete Solutions Manual*, and Andy Bulman-Fleming for rendering the art and typesetting this manual. I also wish to thank my editor Rita Lombard of Cengage Learning for her help and support in bringing this supplement to market.

Please submit any errors in the solutions manual or suggestions for improvements to me in care of the publisher: Math Editorial, Cengage Learning, 20 Channel Center Street, Boston, MA, 02210.

Soo T. Tan

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1 PRELIMINARIES

1.1 Precalculus Review I

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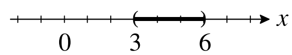
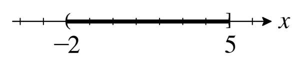
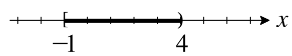
1. The interval $(3, 6)$ is shown on the number line below. Note that this is an open interval indicated by "(" and ")".



2. The interval $(-2, 5]$ is shown on the number line below.

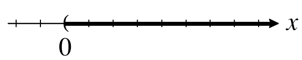


3. The interval $[-1, 4)$ is shown on the number line below. Note that this is a half-open interval indicated by "**[**'' (closed) and "**)**"(open).

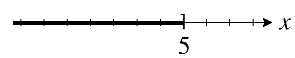


4. The closed interval $\left[-\frac{6}{5}, -\frac{1}{2}\right]$ is shown on the number line below.

5. The infinite interval $(0, \infty)$ is shown on the number line below.



6. The infinite interval $(-\infty, 5]$ is shown on the number line below.



7. $27^{2/3} = \left(3^3\right)^{2/3} = 3^2 = 9.$

8. $8^{-4/3} = \left(\dfrac{1}{8^{4/3}}\right) = \dfrac{1}{2^4} = \dfrac{1}{16}.$

9. $\left(\frac{1}{\sqrt{3}}\right)^0 = 1$. Recall that any number raised to the zeroth power is 1.

10. $\left(7^{1/2}\right)^4 = 7^{4/2} = 7^2 = 49.$

11. $\left[\left(\frac{1}{8}\right)^{1/3}\right]^{-2} = \left(\frac{1}{2}\right)^{-2} = \left(2^2\right) = 4.$

12. $\left[\left(-\frac{1}{3}\right)^2\right]^{-3} = \left(\frac{1}{9}\right)^{-3} = (9)^3 = 729.$

13. $\left(\dfrac{7^{-5} \cdot 7^2}{7^{-2}}\right)^{-1} = \left(7^{-5+2+2}\right)^{-1} = \left(7^{-1}\right)^{-1} = 7^1 = 7.$

14. $\left(\frac{9}{16}\right)^{-1/2} = \left(\frac{16}{9}\right)^{1/2} = \frac{4}{3}.$

15. $\left(125^{2/3}\right)^{-1/2} = 125^{(2/3)(-1/2)} = 125^{-1/3} = \dfrac{1}{125^{1/3}}$

$= \frac{1}{5}.$

16. $\sqrt[3]{2^6} = \left(2^6\right)^{1/3} = 2^{6(1/3)} = 2^2 = 4.$

17. $\dfrac{\sqrt{32}}{\sqrt{8}} = \sqrt{\dfrac{32}{8}} = \sqrt{4} = 2.$

18. $\sqrt[3]{\dfrac{-8}{27}} = \dfrac{\sqrt[3]{-8}}{\sqrt[3]{27}} = -\dfrac{2}{3}.$

19. $\dfrac{16^{5/8}16^{1/2}}{16^{7/8}} = 16^{(5/8)+(1/2)-(7/8)} = 16^{1/4} = 2.$

20. $\left(\dfrac{9^{-3}\cdot 9^5}{9^{-2}}\right)^{-1/2} = 9^{(-3+5+2)(-1/2)} = 9^{4(-1/2)} = \dfrac{1}{81}.$

21. $16^{1/4}\cdot 8^{-1/3} = 2\cdot\left(\frac{1}{8}\right)^{1/3} = 2\cdot\frac{1}{2} = 1.$

22. $\dfrac{6^{2.5}\cdot 6^{-1.9}}{6^{-1.4}} = 6^{2.5-1.9-(-1.4)} = 6^{2.5-1.9+1.4} = 6^2$

$= 36.$

23. True.

24. True. $3^2\times 2^2 = (3\times 2)^2 = 6^2 = 36.$

25. False. $x^3\times 2x^2 = 2x^{3+2} = 2x^5 \neq 2x^6.$

26. False. $3^3+3 = 27+3 = 30 \neq 3^4.$

27. False. $\dfrac{2^{4x}}{1^{3x}} = \dfrac{2^{4x}}{1} = 2^{4x}.$

28. True. $\left(2^2\times 3^2\right)^2 = (4\times 9)^2 = 36^2 = \left(6^2\right)^2 = 6^4.$

29. False. $\dfrac{1}{4^{-3}} = 4^3 = 64.$

30. True. $\dfrac{4^{3/2}}{2^4} = \dfrac{8}{16} = \dfrac{1}{2}.$

31. False. $\left(1.2^{1/2}\right)^{-1/2} = (1.2)^{-1/4} \neq 1.$

32. True.
$5^{2/3}\times 25^{2/3} = 5^{2/3}\left(5^2\right)^{2/3} = 5^{2/3}\times 5^{4/3} = 5^2 = 25.$

33. $(xy)^{-2} = \dfrac{1}{(xy)^2}.$

34. $3s^{1/3}\cdot s^{-7/3} = 3s^{(1/3)-(7/3)} = 3s^{-6/3} = 3s^{-2} = \dfrac{3}{s^2}.$

35. $\dfrac{x^{-1/3}}{x^{1/2}} = x^{(-1/3)-(1/2)} = x^{-5/6} = \dfrac{1}{x^{5/6}}.$

36. $\sqrt{x^{-1}}\cdot\sqrt{9x^{-3}} = x^{-1/2}\cdot 3x^{-3/2} = 3x^{(-1/2)+(-3/2)}$

$= 3x^{-2} = \dfrac{3}{x^2}.$

37. $12^0\,(s+t)^{-3} = 1\cdot\dfrac{1}{(s+t)^3} = \dfrac{1}{(s+t)^3}.$

38. $(x-y)\left(x^{-1}+y^{-1}\right) = (x-y)\left(\dfrac{1}{x}+\dfrac{1}{y}\right)$

$= (x-y)\left(\dfrac{y+x}{xy}\right) = \dfrac{(x-y)(x+y)}{xy} = \dfrac{x^2-y^2}{xy}.$

39. $\dfrac{x^{7/3}}{x^{-2}} = x^{(7/3)+2} = x^{(7/3)+(6/3)} = x^{13/3}.$

40. $\left(49x^{-2}\right)^{-1/2} = (49)^{-1/2}\,x^{(-2)(-1/2)} = \frac{1}{7}x.$

41. $\left(x^2y^{-3}\right)\left(x^{-5}y^3\right) = x^{2-5}y^{-3+3} = x^{-3}y^0 = x^{-3} = \dfrac{1}{x^3}.$

42. $\dfrac{5x^6y^3}{2x^2y^7} = \frac{5}{2}x^{6-2}y^{3-7} = \frac{5}{2}x^4y^{-4} = \dfrac{5x^4}{2y^4}.$

43. $\dfrac{x^{3/4}}{x^{-1/4}} = x^{(3/4)-(-1/4)} = x^{4/4} = x.$

44. $\left(\dfrac{x^3y^2}{z^2}\right)^2 = \dfrac{x^{3\cdot 2}y^{2\cdot 2}}{z^{2(2)}} = \dfrac{x^6y^4}{z^4}.$

45. $\left(\dfrac{x^3}{-27y^{-6}}\right)^{-2/3} = x^{3(-2/3)}\left(-\dfrac{1}{27}\right)^{-2/3}y^{6(-2/3)}$

$= x^{-2}\left(-\dfrac{1}{3}\right)^{-2}y^{-4} = \dfrac{9}{x^2y^4}.$

46. $\left(\dfrac{e^x}{e^{x-2}}\right)^{-1/2} = e^{[x-(x-2)](-1/2)} = e^{-1} = \dfrac{1}{e}.$

47. $\left(\dfrac{x^{-3}}{y^{-2}}\right)^2\left(\dfrac{y}{x}\right)^4 = \dfrac{x^{-3\cdot 2}y^4}{y^{-2\cdot 2}x^4} = \dfrac{y^{4+4}}{x^{4+6}} = \dfrac{y^8}{x^{10}}.$

48. $\dfrac{(r^n)^4}{r^{5-2n}} = r^{4n-(5-2n)} = r^{4n+2n-5} = r^{6n-5}.$

49. $\sqrt[3]{x^{-2}} \cdot \sqrt{4x^5} = x^{-2/3} \cdot 4^{1/2} \cdot x^{5/2} = x^{(-2/3)+(5/2)} \cdot 2 = 2x^{11/6}$.

50. $\sqrt{81x^6y^{-4}} = (81)^{1/2} \cdot x^{6/2} \cdot y^{-4/2} = \dfrac{9x^3}{y^2}$.

51. $-\sqrt[4]{16x^4y^8} = -\left(16^{1/4} \cdot x^{4/4} \cdot y^{8/4}\right) = -2xy^2$.

52. $\sqrt[3]{x^{3a+b}} = x^{(3a+b)(1/3)} = x^{a+(b/3)}$.

53. $\sqrt[6]{64x^8y^3} = 64^{1/6} \cdot x^{8/6}y^{3/6} = 2x^{4/3}y^{1/2}$.

54. $\sqrt[3]{27r^6} \cdot \sqrt{s^2t^4} = 27^{1/3}\left(r^6\right)^{1/3}\left(s^2\right)^{1/2}\left(t^4\right)^{1/2} = 3r^2st^2$.

55. $2^{3/2} = 2\left(2^{1/2}\right) \approx 2\,(1.414) = 2.828$.

56. $8^{1/2} = \left(2^3\right)^{1/2} = 2^{3/2} = 2\left(2^{1/2}\right) \approx 2.828$.

57. $9^{3/4} = \left(3^2\right)^{3/4} = 3^{6/4} = 3^{3/2} = 3 \cdot 3^{1/2} \approx 3\,(1.732) = 5.196$.

58. $6^{1/2} = (2 \cdot 3)^{1/2} = 2^{1/2} \cdot 3^{1/2} \approx (1.414)\,(1.732) \approx 2.449$.

59. $10^{3/2} = 10^{1/2} \cdot 10 \approx (3.162)\,(10) = 31.62$.

60. $1000^{3/2} = \left(10^3\right)^{3/2} = 10^{9/2} = 10^4 \times 10^{1/2} \approx (10000)\,(3.162) = 31{,}620$.

61. $10^{2.5} = 10^2 \cdot 10^{1/2} \approx 100\,(3.162) = 316.2$.

62. $(0.0001)^{-1/3} = \left(10^{-4}\right)^{-1/3} = 10^{4/3} = 10 \cdot 10^{1/3} \approx 10\,(2.154) = 21.54$.

63. $\dfrac{3}{2\sqrt{x}} \cdot \dfrac{\sqrt{x}}{\sqrt{x}} = \dfrac{3\sqrt{x}}{2x}$.

64. $\dfrac{3}{\sqrt{xy}} \cdot \dfrac{\sqrt{xy}}{\sqrt{xy}} = \dfrac{3\sqrt{xy}}{xy}$.

65. $\dfrac{2y}{\sqrt{3y}} \cdot \dfrac{\sqrt{3y}}{\sqrt{3y}} = \dfrac{2y\sqrt{3y}}{3y} = \dfrac{2\sqrt{3y}}{3}$.

66. $\dfrac{5x^2}{\sqrt{3x}} \cdot \dfrac{\sqrt{3x}}{\sqrt{3x}} = \dfrac{5x^2\sqrt{3x}}{3x} = \dfrac{5x\sqrt{3x}}{3}$.

67. $\dfrac{1}{\sqrt[3]{x}} \cdot \dfrac{\sqrt[3]{x^2}}{\sqrt[3]{x^2}} = \dfrac{\sqrt[3]{x^2}}{\sqrt[3]{x^3}} = \dfrac{\sqrt[3]{x^2}}{x}$.

68. $\sqrt{\dfrac{2x}{y}} = \dfrac{\sqrt{2x}}{\sqrt{y}} \cdot \dfrac{\sqrt{y}}{\sqrt{y}} = \dfrac{\sqrt{2xy}}{y}$.

69. $\dfrac{2\sqrt{x}}{3} \cdot \dfrac{\sqrt{x}}{\sqrt{x}} = \dfrac{2x}{3\sqrt{x}}$.

70. $\dfrac{\sqrt[3]{x}}{24} \cdot \dfrac{\sqrt[3]{x^2}}{\sqrt[3]{x^2}} = \dfrac{x}{24\sqrt[3]{x^2}}$.

71. $\sqrt{\dfrac{2y}{x}} = \dfrac{\sqrt{2y}}{\sqrt{x}} \cdot \dfrac{\sqrt{2y}}{\sqrt{2y}} = \dfrac{2y}{\sqrt{2xy}}$.

72. $\sqrt[3]{\dfrac{2x}{3y}} = \dfrac{\sqrt[3]{2x}}{\sqrt[3]{3y}} \cdot \dfrac{\sqrt[3]{(2x)^2}}{\sqrt[3]{(2x)^2}} = \dfrac{2x}{\sqrt[3]{12x^2y}}$.

73. $\dfrac{\sqrt[3]{x^2z}}{y} \cdot \dfrac{\sqrt[3]{xz^2}}{\sqrt[3]{xz^2}} = \dfrac{\sqrt[3]{x^3z^3}}{y\sqrt[3]{xz^2}} = \dfrac{xz}{y\sqrt[3]{xz^2}}$.

74. $\dfrac{\sqrt[3]{x^2y}}{2x} \cdot \dfrac{\sqrt[3]{xy^2}}{\sqrt[3]{xy^2}} = \dfrac{xy}{2x\sqrt[3]{xy^2}} = \dfrac{y}{2\sqrt[3]{xy^2}}$.

75. $\left(7x^2 - 2x + 5\right) + \left(2x^2 + 5x - 4\right) = 7x^2 - 2x + 5 + 2x^2 + 5x - 4 = 9x^2 + 3x + 1$.

76. $\left(3x^2 + 5xy + 2y\right) + \left(4 - 3xy - 2x^2\right) = 3x^2 + 5xy + 2y + 4 - 3xy - 2x^2 = x^2 + 2xy + 2y + 4$.

77. $\left(5y^2 - 2y + 1\right) - \left(y^2 - 3y - 7\right) = 5y^2 - 2y + 1 - y^2 + 3y + 7 = 4y^2 + y + 8$.

78. $3\,(2a - b) - 4\,(b - 2a) = 6a - 3b - 4b + 8a = 14a - 7b = 7\,(2a - b)$.

79. $x-\{2x-[-x-(1-x)]\}=x-\{2x-[-x-1+x]\}=x-(2x+1)=x-2x-1=-x-1.$

80. $3x^2-\left\{x^2+1-x\left[x-(2x-1)\right]\right\}+2=3x^2-\left[x^2+1-x(x-2x+1)\right]+2$

$$=3x^2-\left[x^2+1-x(-x+1)\right]+2=3x^2-\left(x^2+1+x^2-x\right)+2$$

$$=3x^2-\left(2x^2-x+1\right)+2=x^2-1+x+2=x^2+x+1.$$

81. $\left(\frac{1}{3}-1+e\right)-\left(-\frac{1}{3}-1+e^{-1}\right)=\frac{1}{3}-1+e+\frac{1}{3}+1-\frac{1}{e}=\frac{2}{3}+e-\frac{1}{e}=\frac{3e^2+2e-3}{3e}.$

82. $-\frac{3}{4}y-\frac{1}{4}x+100+\frac{1}{2}x+\frac{1}{4}y-120=-\frac{3}{4}y+\frac{1}{4}y-\frac{1}{4}x+\frac{1}{2}x+100-120=-\frac{1}{2}y+\frac{1}{4}x-20.$

83. $3\sqrt{8}+8-2\sqrt{y}+\frac{1}{2}\sqrt{x}-\frac{3}{4}\sqrt{y}=3\sqrt{8}+8+\frac{1}{2}\sqrt{x}-\frac{11}{4}\sqrt{y}=6\sqrt{2}+8+\frac{1}{2}\sqrt{x}-\frac{11}{4}\sqrt{y}.$

84. $\frac{8}{9}x^2+\frac{2}{3}x+\frac{16}{3}x^2-\frac{16}{3}x-2x+2=\frac{8x^2+6x+48x^2-48x-18x+18}{9}=\frac{56x^2-60x+18}{9}$

$$=\frac{2}{9}\left(28x^2-30x+9\right).$$

85. $(x+8)(x-2)=x(x-2)+8(x-2)=x^2-2x+8x-16=x^2+6x-16.$

86. $(5x+2)(3x-4)=5x(3x-4)+2(3x-4)=15x^2-20x+6x-8=15x^2-14x-8.$

87. $(a+5)^2=(a+5)(a+5)=a(a+5)+5(a+5)=a^2+5a+5a+25=a^2+10a+25.$

88. $(3a-4b)^2=(3a-4b)(3a-4b)=3a(3a-4b)-4b(3a-4b)=9a^2-12ab-12ab+16b^2$

$$=9a^2-24ab+16b^2.$$

89. $(x+2y)^2=(x+2y)(x+2y)=x(x+2y)+2y(x+2y)=x^2+2xy+2yx+4y^2=x^2+4xy+4y^2.$

90. $(6-3x)^2=(6-3x)(6-3x)=6(6-3x)-3x(6-3x)=36-18x-18x+9x^2=36-36x+9x^2.$

91. $(2x+y)(2x-y)=2x(2x-y)+y(2x-y)=4x^2-2xy+2xy-y^2=4x^2-y^2.$

92. $(3x+2)(2-3x)=3x(2-3x)+2(2-3x)=6x-9x^2+4-6x=-9x^2+4.$

93. $\left(2x^2-1\right)\left(3x^2\right)+\left(x^2+3\right)(4x)=6x^4-3x^2+4x^3+12x=6x^4+4x^3-3x^2+12x=x\left(6x^3+4x^2-3x+12\right).$

94. $\left(x^2-1\right)(2x)-x^2(2x)=2x^3-2x-2x^3=-2x.$

95. $6x\left(\frac{1}{2}\right)\left(2x^2+3\right)^{-1/2}(4x)+6\left(2x^2+3\right)^{1/2}=3\left(2x^2+3\right)^{-1/2}\left[x(4x)+2\left(2x^2+3\right)\right]=\frac{6\left(4x^2+3\right)}{\left(2x^2+3\right)^{1/2}}.$

96. $\left(x^{1/2}+1\right)\left(\frac{1}{2}x^{-1/2}\right)-\left(x^{1/2}-1\right)\left(\frac{1}{2}x^{-1/2}\right)=\frac{1}{2}x^{-1/2}\left[\left(x^{1/2}+1\right)-\left(x^{1/2}-1\right)\right]=\frac{1}{2}x^{-1/2}(2)=\frac{1}{x^{1/2}}=\frac{1}{\sqrt{x}}.$

97. $100\left(-10te^{-0.1t}-100e^{-0.1t}\right)=-1000(10+t)e^{-0.1t}.$

98. $2\left(t+\sqrt{t}\right)^2-2t^2=2\left(t+\sqrt{t}\right)\left(t+\sqrt{t}\right)-2t^2=2\left(t^2+2t\sqrt{t}+t\right)-2t^2=2t^2+4t\sqrt{t}+2t-2t^2$

$$=4t\sqrt{t}+2t=2t\left(2\sqrt{t}+1\right).$$

99. $4x^5 - 12x^4 - 6x^3 = 2x^3\left(2x^2 - 6x - 3\right)$.

100. $4x^2y^2z - 2x^5y^2 + 6x^3y^2z^2 = 2x^2y^2\left(2z - x^3 + 3xz^2\right)$.

101. $7a^4 - 42a^2b^2 + 49a^3b = 7a^2\left(a^2 + 7ab - 6b^2\right)$.

102. $3x^{2/3} - 2x^{1/3} = x^{1/3}\left(3x^{1/3} - 2\right)$.

103. $e^{-x} - xe^{-x} = e^{-x}(1 - x)$.

104. $2ye^{xy^2} + 2xy^3e^{xy^2} = 2ye^{xy^2}\left(1 + xy^2\right)$.

105. $2x^{-5/2} - \frac{3}{2}x^{-3/2} = \frac{1}{2}x^{-5/2}(4 - 3x)$.

106. $\frac{1}{2}\left(\frac{2}{3}u^{3/2} - 2u^{1/2}\right) = \frac{1}{2} \cdot \frac{2}{3}u^{1/2}(u - 3) = \frac{1}{3}u^{1/2}(u - 3)$.

107. $6ac + 3bc - 4ad - 2bd = 3c(2a + b) - 2d(2a + b) = (2a + b)(3c - 2d)$.

108. $3x^3 - x^2 + 3x - 1 = x^2(3x - 1) + 1(3x - 1) = \left(x^2 + 1\right)(3x - 1)$.

109. $4a^2 - b^2 = (2a + b)(2a - b)$, a difference of two squares.

110. $12x^2 - 3y^2 = 3\left(4x^2 - y^2\right) = 3(2x + y)(2x - y)$.

111. $10 - 14x - 12x^2 = -2\left(6x^2 + 7x - 5\right) = -2(3x + 5)(2x - 1)$.

112. $x^2 - 2x - 15 = (x - 5)(x + 3)$.

113. $3x^2 - 6x - 24 = 3\left(x^2 - 2x - 8\right) = 3(x - 4)(x + 2)$.

114. $3x^2 - 4x - 4 = (3x + 2)(x - 2)$.

115. $12x^2 - 2x - 30 = 2\left(6x^2 - x - 15\right) = 2(3x - 5)(2x + 3)$.

116. $(x + y)^2 - 1 = (x + y - 1)(x + y + 1)$.

117. $9x^2 - 16y^2 = (3x)^2 - (4y)^2 = (3x - 4y)(3x + 4y)$.

118. $8a^2 - 2ab - 6b^2 = 2\left(4a^2 - ab - 3b^2\right) = 2(a - b)(4a + 3b)$.

119. $x^6 + 125 = \left(x^2\right)^3 + (5)^3 = \left(x^2 + 5\right)\left(x^4 - 5x^2 + 25\right)$.

120. $x^3 - 27 = x^3 - 3^3 = (x - 3)\left(x^2 + 3x + 9\right)$.

121. $\left(x^2 + y^2\right)x - xy(2y) = x^3 + xy^2 - 2xy^2 = x^3 - xy^2$.

122. $2kr(R - r) - kr^2 = 2kRr - 2kr^2 - kr^2 = 2kRr - 3kr^2 = kr(2R - 3r)$.

123. $$\begin{aligned}2(x-1)(2x+2)^3[4(x-1)+(2x+2)] &= 2(x-1)(2x+2)^3(4x-4+2x+2)\\ &= 2(x-1)(2x+2)^3(6x-2) = 4(x-1)(3x-1)(2x+2)^3\\ &= 32(x-1)(3x-1)(x+1)^3.\end{aligned}$$

124. $5x^2\left(3x^2+1\right)^4(6x)+\left(3x^2+1\right)^5(2x)=(2x)\left(3x^2+1\right)^4\left[15x^2+\left(3x^2+1\right)\right]=2x\left(3x^2+1\right)^4\left(18x^2+1\right)$.

125. $$\begin{aligned}4(x-1)^2(2x+2)^3(2)+(2x+2)^4(2)(x-1) &= 2(x-1)(2x+2)^3[4(x-1)+(2x+2)]\\ &= 2(x-1)(2x+2)^3(6x-2) = 4(x-1)(3x-1)(2x+2)^3\\ &= 32(x-1)(3x-1)(x+1)^3.\end{aligned}$$

126. $$\begin{aligned}\left(x^2+1\right)\left(4x^3-3x^2+2x\right)-\left(x^4-x^3+x^2\right)(2x) &= 4x^5-3x^4+2x^3+4x^3-3x^2+2x-2x^5+2x^4-2x^3\\ &= 2x^5-x^4+4x^3-3x^2+2x.\end{aligned}$$

127. $\left(x^2+2\right)^2\left[5\left(x^2+2\right)^2-3\right](2x)=\left(x^2+2\right)^2\left[5\left(x^4+4x^2+4\right)-3\right](2x)=(2x)\left(x^2+2\right)^2\left(5x^4+20x^2+17\right)$.

128. $$\begin{aligned}\left(x^2-4\right)\left(x^2+4\right)(2x+8)-\left(x^2+8x-4\right)\left(4x^3\right) &= \left(x^4-16\right)(2x+8)-4x^5-32x^4+16x^3\\ &= 2x^5+8x^4-32x-128-4x^5-32x^4+16x^3=-2x^5-24x^4+16x^3-32x-128\\ &= -2\left(x^5+12x^4-8x^3+16x+64\right).\end{aligned}$$

129. We factor the left-hand side of $x^2+x-12=0$ to obtain $(x+4)(x-3)=0$, so $x=-4$ or $x=3$. We conclude that the roots are $x=-4$ and $x=3$.

130. We factor the left-hand side of $3x^2-x-4=0$ to obtain $(3x-4)(x+1)=0$. Thus, $3x=4$ or $x=-1$, and we conclude that the roots are $x=\frac{4}{3}$ and $x=-1$.

131. $4t^2+2t-2=(2t-1)(2t+2)=0$. Thus, the roots are $t=\frac{1}{2}$ and $t=-1$.

132. $-6x^2+x+12=(3x+4)(-2x+3)=0$. Thus, $x=-\frac{4}{3}$ and $x=\frac{3}{2}$ are the roots of the equation.

133. $\frac{1}{4}x^2-x+1=\left(\frac{1}{2}x-1\right)\left(\frac{1}{2}x-1\right)=0$. Thus $\frac{1}{2}x=1$, and so $x=2$ is a double root of the equation.

134. $\frac{1}{2}a^2+a-12=a^2+2a-24=(a+6)(a-4)=0$. Thus, $a=-6$ and $a=4$ are the roots of the equation.

135. We use the quadratic formula to solve the equation $4x^2+5x-6=0$. In this case, $a=4$, $b=5$, and $c=-6$. Therefore, $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}=\dfrac{-5\pm\sqrt{5^2-4(4)(-6)}}{2(4)}=\dfrac{-5\pm\sqrt{121}}{8}=\dfrac{-5\pm11}{8}$. Thus, $x=-\frac{16}{8}=-2$ and $x=\frac{6}{8}=\frac{3}{4}$ are the roots of the equation.

136. We use the quadratic formula to solve the equation $3x^2-4x+1=0$. Here $a=3$, $b=-4$, and $c=1$, so $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}=\dfrac{-(-4)\pm\sqrt{(-4)^2-4(3)(1)}}{2(3)}=\dfrac{4\pm\sqrt{4}}{6}$. Thus, $x=\frac{6}{6}=1$ and $x=\frac{2}{6}=\frac{1}{3}$ are the roots of the equation.

137. We use the quadratic formula to solve the equation $8x^2 - 8x - 3 = 0$. Here $a = 8$, $b = -8$, and $c = -3$, so $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(8)(-3)}}{2(8)} = \frac{8 \pm \sqrt{160}}{16} = \frac{8 \pm 4\sqrt{10}}{16} = \frac{2 \pm \sqrt{10}}{4}$. Thus, $x = \frac{1}{2} + \frac{1}{4}\sqrt{10}$ and $x = \frac{1}{2} - \frac{1}{4}\sqrt{10}$ are the roots of the equation.

138. We use the quadratic formula to solve the equation $x^2 - 6x + 6 = 0$. Here $a = 1$, $b = -6$, and $c = 6$. Therefore, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(6)}}{2(1)} = \frac{6 \pm 2\sqrt{3}}{2} = 3 \pm \sqrt{3}$. Thus, the roots are $3 + \sqrt{3}$ and $3 - \sqrt{3}$.

139. We use the quadratic formula to solve $2x^2 + 4x - 3 = 0$. Here $a = 2$, $b = 4$, and $c = -3$, so $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-4 \pm \sqrt{4^2 - 4(2)(-3)}}{2(2)} = \frac{-4 \pm \sqrt{40}}{4} = \frac{-4 \pm 2\sqrt{10}}{4} = \frac{-2 \pm \sqrt{10}}{2}$. Thus, $x = -1 + \frac{1}{2}\sqrt{10}$ and $x = -1 - \frac{1}{2}\sqrt{10}$ are the roots of the equation.

140. We use the quadratic formula to solve the equation $2x^2 + 7x - 15 = 0$. Then $a = 2$, $b = 7$, and $c = -15$. Therefore, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-7 \pm \sqrt{7^2 - 4(2)(-15)}}{2(2)} = \frac{-7 \pm \sqrt{169}}{4} = \frac{-7 \pm 13}{4}$. We conclude that $x = \frac{3}{2}$ and $x = -5$ are the roots of the equation.

141. The total revenue is given by $\left(0.2t^2 + 150t\right) + \left(0.5t^2 + 200t\right) = 0.7t^2 + 350t$ thousand dollars t months from now, where $0 \le t \le 12$.

142. In month t, the revenue of the second gas station will exceed that of the first gas station by $\left(0.5t^2 + 200t\right) - \left(0.2t^2 + 150t\right) = 0.3t^2 + 50t$ thousand dollars, where $0 < t \le 12$.

143. **a.** $f(30{,}000) = \left(5.6 \times 10^{11}\right)(30{,}000)^{-1.5} \approx 107{,}772$, or 107,772 families.

b. $f(60{,}000) = \left(5.6 \times 10^{11}\right)(60{,}000)^{-1.5} \approx 38{,}103$, or 38,103 families.

c. $f(150{,}000) = \left(5.6 \times 10^{11}\right)(150{,}000)^{-1.5} \approx 9639$, or 9639 families.

144. $-t^3 + 6t^2 + 15t = -t\left(t^2 - 6t - 15\right)$.

145. $8000x - 100x^2 = 100x(80 - x)$.

146. True. The two real roots are $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

147. True. If $b^2 - 4ac < 0$, then $\sqrt{b^2 - 4ac}$ is not a real number.

148. True, because $(a + b)(b - a) = b^2 - a^2$.

1.2 Precalculus Review II

Exercises page 23

1. $\dfrac{x^2+x-2}{x^2-4}=\dfrac{(x+2)(x-1)}{(x+2)(x-2)}=\dfrac{x-1}{x-2}.$

2. $\dfrac{2a^2-3ab-9b^2}{2ab^2+3b^3}=\dfrac{(2a+3b)(a-3b)}{b^2(2a+3b)}=\dfrac{a-3b}{b^2}.$

3. $\dfrac{12t^2+12t+3}{4t^2-1}=\dfrac{3\left(4t^2+4t+1\right)}{4t^2-1}=\dfrac{3(2t+1)(2t+1)}{(2t+1)(2t-1)}=\dfrac{3(2t+1)}{2t-1}.$

4. $\dfrac{x^3+2x^2-3x}{-2x^2-x+3}=\dfrac{x\left(x^2+2x-3\right)}{-\left(2x^2+x-3\right)}=\dfrac{x(x+3)(x-1)}{-(2x+3)(x-1)}=-\dfrac{x(x+3)}{2x+3}.$

5. $\dfrac{(4x-1)(3)-(3x+1)(4)}{(4x-1)^2}=\dfrac{12x-3-12x-4}{(4x-1)^2}=-\dfrac{7}{(4x-1)^2}.$

6. $\dfrac{\left(1+x^2\right)^2(2)-2x(2)\left(1+x^2\right)(2x)}{\left(1+x^2\right)^4}=\dfrac{\left(1+x^2\right)(2)\left(1+x^2-4x^2\right)}{\left(1+x^2\right)^4}=\dfrac{\left(1+x^2\right)(2)\left(-3x^2+1\right)}{\left(1+x^2\right)^4}=\dfrac{2\left(1-3x^2\right)}{\left(1+x^2\right)^3}.$

7. $\dfrac{2a^2-2b^2}{b-a}\cdot\dfrac{4a+4b}{a^2+2ab+b^2}=\dfrac{2(a+b)(a-b)4(a+b)}{-(a-b)(a+b)(a+b)}=-8.$

8. $\dfrac{x^2-6x+9}{x^2-x-6}\cdot\dfrac{3x+6}{2x^2-7x+3}=\dfrac{3(x-3)^2(x+2)}{(x-3)(x+2)(2x-1)(x-3)}=\dfrac{3}{2x-1}.$

9. $\dfrac{3x^2+2x-1}{2x+6}\div\dfrac{x^2-1}{x^2+2x-3}=\dfrac{(3x-1)(x+1)}{2(x+3)}\cdot\dfrac{(x+3)(x-1)}{(x+1)(x-1)}=\dfrac{3x-1}{2}.$

10. $\dfrac{3x^2-4xy-4y^2}{x^2y}\div\dfrac{(2y-x)^2}{x^3y}=\dfrac{(3x+2y)(x-2y)}{x^2y}\cdot\dfrac{x^3y}{(2y-x)(2y-x)}=\dfrac{x(3x+2y)}{x-2y}.$

11. $\dfrac{58}{3(3t+2)}+\dfrac{1}{3}=\dfrac{58+3t+2}{3(3t+2)}=\dfrac{3t+60}{3(3t+2)}=\dfrac{t+20}{3t+2}.$

12. $\dfrac{a+1}{3a}+\dfrac{b-2}{5b}=\dfrac{5b(a+1)+3a(b-2)}{15ab}=\dfrac{5ab+5b+3ab-6a}{15ab}=\dfrac{-6a+8ab+5b}{15ab}.$

13. $\dfrac{2x}{2x-1}-\dfrac{3x}{2x+5}=\dfrac{2x(2x+5)-3x(2x-1)}{(2x-1)(2x+5)}=\dfrac{4x^2+10x-6x^2+3x}{(2x-1)(2x+5)}=\dfrac{-2x^2+13x}{(2x-1)(2x+5)}$
$=-\dfrac{x(2x-13)}{(2x-1)(2x+5)}.$

14. $\dfrac{-xe^x}{x+1}+e^x=\dfrac{-xe^x+(x+1)e^x}{x+1}=\dfrac{-xe^x+xe^x+e^x}{x+1}=\dfrac{e^x}{x+1}.$

15. $\dfrac{4}{x^2-9}-\dfrac{5}{x^2-6x+9}=\dfrac{4}{(x+3)(x-3)}-\dfrac{5}{(x-3)^2}=\dfrac{4(x-3)-5(x+3)}{(x-3)^2(x+3)}=-\dfrac{x+27}{(x-3)^2(x+3)}.$

16. $\dfrac{x}{1-x}+\dfrac{2x+3}{x^2-1}=\dfrac{-x(x+1)+2x+3}{(x+1)(x-1)}=\dfrac{-x^2-x+2x+3}{x^2-1}=-\dfrac{x^2-x-3}{x^2-1}.$

17. $\dfrac{1+\frac{1}{x}}{1-\frac{1}{x}}=\dfrac{\frac{x+1}{x}}{\frac{x-1}{x}}=\dfrac{x+1}{x}\cdot\dfrac{x}{x-1}=\dfrac{x+1}{x-1}.$

18. $\dfrac{\frac{1}{x}+\frac{1}{y}}{1-\frac{1}{xy}}=\dfrac{\frac{x+y}{xy}}{\frac{xy-1}{xy}}=\dfrac{x+y}{xy}\cdot\dfrac{xy}{xy-1}=\dfrac{x+y}{xy-1}.$

19. $\dfrac{4x^2}{2\sqrt{2x^2+7}}+\sqrt{2x^2+7}=\dfrac{4x^2+2\sqrt{2x^2+7}\sqrt{2x^2+7}}{2\sqrt{2x^2+7}}=\dfrac{4x^2+4x^2+14}{2\sqrt{2x^2+7}}=\dfrac{4x^2+7}{\sqrt{2x^2+7}}.$

20. $6(2x+1)^2\sqrt{x^2+x}+\dfrac{(2x+1)^4}{2\sqrt{x^2+x}}=\dfrac{6(2x+1)^2\sqrt{x^2+x}\,(2)\sqrt{x^2+x}+(2x+1)^4}{2\sqrt{x^2+x}}$

$$=\frac{(2x+1)^2\left[12\left(x^2+x\right)+4x^2+4x+1\right]}{2\sqrt{x^2+x}}=\frac{(2x+1)^2\left(16x^2+16x+1\right)}{2\sqrt{x^2+x}}.$$

21. $5\left[\dfrac{\left(t^2+1\right)(1)-t(2t)}{\left(t^2+1\right)^2}\right]=\dfrac{5\left(t^2+1-2t^2\right)}{\left(t^2+1\right)^2}=\dfrac{5\left(1-t^2\right)}{\left(t^2+1\right)^2}=-\dfrac{5\left(t^2-1\right)}{\left(t^2+1\right)^2}.$

22. $\dfrac{2x(x+1)^{-1/2}-(x+1)^{1/2}}{x^2}=\dfrac{(x+1)^{-1/2}(2x-x-1)}{x^2}=\dfrac{(x+1)^{-1/2}(x-1)}{x^2}=\dfrac{x-1}{x^2\sqrt{x+1}}.$

23. $\dfrac{\left(x^2+1\right)^2(-2)+(2x)\,2\left(x^2+1\right)(2x)}{\left(x^2+1\right)^4}=\dfrac{2\left(x^2+1\right)\left[-\left(x^2+1\right)+4x^2\right]}{\left(x^2+1\right)^4}=\dfrac{2\left(3x^2-1\right)}{\left(x^2+1\right)^3}.$

24. $\dfrac{\left(x^2+1\right)^{1/2}-2x^2\left(x^2+1\right)^{-1/2}}{1-x^2}=\dfrac{\left(x^2+1\right)^{-1/2}\left(x^2+1-2x^2\right)}{1-x^2}=\dfrac{\left(x^2+1\right)^{-1/2}\left(-x^2+1\right)}{1-x^2}=\dfrac{1}{\sqrt{x^2+1}}.$

25. $3\left(\dfrac{2x+1}{3x+2}\right)^2\left[\dfrac{(3x+2)(2)-(2x+1)(3)}{(3x+2)^2}\right]=\dfrac{3(2x+1)^2(6x+4-6x-3)}{(3x+2)^4}=\dfrac{3(2x+1)^2}{(3x+2)^4}.$

26. $\dfrac{(2x+1)^{1/2}-(x+2)(2x+1)^{-1/2}}{2x+1}=\dfrac{(2x+1)^{-1/2}(2x+1-x-2)}{2x+1}=\dfrac{(2x+1)^{-1/2}(x-1)}{2x+1}=\dfrac{x-1}{(2x+1)^{3/2}}.$

27. $100\left[\dfrac{\left(t^2+20t+100\right)(2t+10)-\left(t^2+10t+100\right)(2t+20)}{\left(t^2+20t+100\right)^2}\right]$

$$=100\left[\frac{2t^3+40t^2+200t+10t^2+200t+1000-2t^3-20t^2-200t-20t^2-200t-2000}{\left(t^2+20t+100\right)^2}\right]$$

$$=\frac{100\left(10t^2-1000\right)}{\left(t^2+20t+100\right)^2}=\frac{1000(t-10)}{(t+10)^3}.$$

28. $$\frac{2(2x-3)^{1/3}-(x-1)(2x-3)^{-2/3}}{(2x-3)^{2/3}}=\frac{(2x-3)^{-2/3}[2(2x-3)-(x-1)]}{(2x-3)^{2/3}}=\frac{(2x-3)^{-2/3}(4x-6-x+1)}{(2x-3)^{2/3}}=\frac{3x-5}{(2x-3)^{4/3}}.$$

29. $\frac{1}{\sqrt{3}-1}\cdot\frac{\sqrt{3}+1}{\sqrt{3}+1}=\frac{\sqrt{3}+1}{3-1}=\frac{\sqrt{3}+1}{2}.$

30. $\frac{1}{\sqrt{x}+5}\cdot\frac{\sqrt{x}-5}{\sqrt{x}-5}=\frac{\sqrt{x}-5}{x-25}.$

31. $\frac{1}{\sqrt{x}-\sqrt{y}}\cdot\frac{\sqrt{x}+\sqrt{y}}{\sqrt{x}+\sqrt{y}}=\frac{\sqrt{x}+\sqrt{y}}{x-y}.$

32. $\frac{a}{1-\sqrt{a}}\cdot\frac{1+\sqrt{a}}{1+\sqrt{a}}=\frac{a\left(1+\sqrt{a}\right)}{1-a}.$

33. $\frac{\sqrt{a}+\sqrt{b}}{\sqrt{a}-\sqrt{b}}\cdot\frac{\sqrt{a}+\sqrt{b}}{\sqrt{a}+\sqrt{b}}=\frac{\left(\sqrt{a}+\sqrt{b}\right)^2}{a-b}.$

34. $\frac{2\sqrt{a}+\sqrt{b}}{2\sqrt{a}-\sqrt{b}}\cdot\frac{2\sqrt{a}+\sqrt{b}}{2\sqrt{a}+\sqrt{b}}=\frac{\left(2\sqrt{a}+\sqrt{b}\right)^2}{4a-b}.$

35. $\frac{\sqrt{x}}{3}\cdot\frac{\sqrt{x}}{\sqrt{x}}=\frac{x}{3\sqrt{x}}.$

36. $\frac{\sqrt[3]{y}}{x}\cdot\frac{\sqrt[3]{y^2}}{\sqrt[3]{y^2}}=\frac{y}{x\sqrt[3]{y^2}}.$

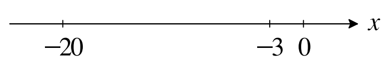
37. $\frac{1-\sqrt{3}}{3}\cdot\frac{1+\sqrt{3}}{1+\sqrt{3}}=\frac{1^2-\left(\sqrt{3}\right)^2}{3\left(1+\sqrt{3}\right)}=-\frac{2}{3\left(1+\sqrt{3}\right)}.$

38. $\frac{\sqrt{x}-1}{x}\cdot\frac{\sqrt{x}+1}{\sqrt{x}+1}=\frac{x-1}{x\left(\sqrt{x}+1\right)}.$

39. $\frac{1+\sqrt{x+2}}{\sqrt{x+2}}\cdot\frac{1-\sqrt{x+2}}{1-\sqrt{x+2}}=\frac{1-(x+2)}{\sqrt{x+2}\left(1-\sqrt{x+2}\right)}=-\frac{x+1}{\sqrt{x+2}\left(1-\sqrt{x+2}\right)}.$

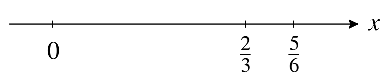
40. $\frac{\sqrt{x+3}-\sqrt{x}}{3}\cdot\frac{\sqrt{x+3}+\sqrt{x}}{\sqrt{x+3}+\sqrt{x}}=\frac{x+3-x}{3\left(\sqrt{x+3}+\sqrt{x}\right)}=\frac{1}{\sqrt{x+3}+\sqrt{x}}.$

41. The statement is false because -3 is greater than -20. See the number line below.



42. The statement is true because -5 is equal to -5.

43. The statement is false because $\frac{2}{3}=\frac{4}{6}$ is less than $\frac{5}{6}$.



44. The statement is false because $-\frac{5}{6}=-\frac{10}{12}$ is greater than $-\frac{11}{12}$.

45. We are given $2x+4<8$. Add -4 to each side of the inequality to obtain $2x<4$, then multiply each side of the inequality by $\frac{1}{2}$ to obtain $x<2$. We write this in interval notation as $(-\infty, 2)$.

46. We are given $-6>4+5x$. Add -4 to each side of the inequality to obtain $-6-4>5x$, so $-10>5x$. Dividing by 2, we obtain $-2>x$, so $x<-2$. We write this in interval notation as $(-\infty,-2)$.

47. We are given the inequality $-4x\geq 20$. Multiply both sides of the inequality by $-\frac{1}{4}$ and reverse the sign of the inequality to obtain $x\leq -5$. We write this in interval notation as $(-\infty,-5]$.

48. $-12 \le -3x \Rightarrow 4 \ge x$, or $x \le 4$. We write this in interval notation as $(-\infty, 4]$.

49. We are given the inequality $-6 < x - 2 < 4$. First add 2 to each member of the inequality to obtain $-6 + 2 < x < 4 + 2$ and $-4 < x < 6$, so the solution set is the open interval $(-4, 6)$.

50. We add -1 to each member of the given double inequality $0 \le x + 1 \le 4$ to obtain $-1 \le x \le 3$, and the solution set is $[-1, 3]$.

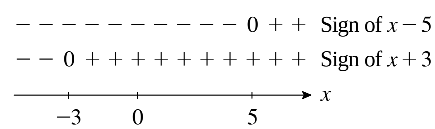
51. We want to find the values of x that satisfy at least one of the inequalities $x + 1 > 4$ and $x + 2 < -1$. Adding -1 to both sides of the first inequality, we obtain $x + 1 - 1 > 4 - 1$, so $x > 3$. Similarly, adding -2 to both sides of the second inequality, we obtain $x + 2 - 2 < -1 - 2$, so $x < -3$. Therefore, the solution set is $(-\infty, -3) \cup (3, \infty)$.

52. We want to find the values of x that satisfy at least one of the inequalities $x + 1 > 2$ and $x - 1 < -2$. Solving these inequalities, we find that $x > 1$ or $x < -1$, and the solution set is $(-\infty, -1) \cup (1, \infty)$.

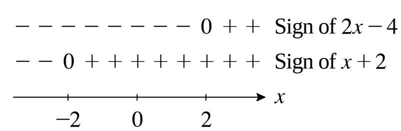
53. We want to find the values of x that satisfy the inequalities $x + 3 > 1$ and $x - 2 < 1$. Adding -3 to both sides of the first inequality, we obtain $x + 3 - 3 > 1 - 3$, or $x > -2$. Similarly, adding 2 to each side of the second inequality, we obtain $x - 2 + 2 < 1 + 2$, so $x < 3$. Because both inequalities must be satisfied, the solution set is $(-2, 3)$.

54. We want to find the values of x that satisfy the inequalities $x - 4 \le 1$ and $x + 3 > 2$. Solving these inequalities, we find that $x \le 5$ and $x > -1$, and the solution set is $(-1, 5]$.

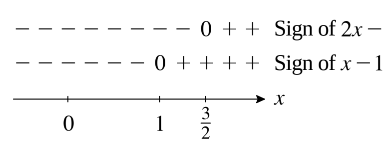
55. We want to find the values of x that satisfy the inequality $(x + 3)(x - 5) \le 0$. From the sign diagram, we see that the given inequality is satisfied when $-3 \le x \le 5$, that is, when the signs of the two factors are different or when one of the factors is equal to zero.



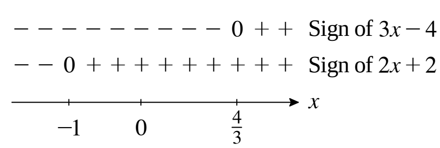
56. We want to find the values of x that satisfy the inequality $(2x - 4)(x + 2) \ge 0$. From the sign diagram, we see that the given inequality is satisfied when $x \le -2$ or $x \ge 2$; that is, when the signs of both factors are the same or one of the factors is equal to zero.



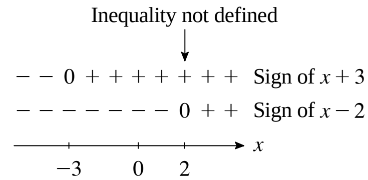
57. We want to find the values of x that satisfy the inequality $(2x - 3)(x - 1) \ge 0$. From the sign diagram, we see that the given inequality is satisfied when $x \le 1$ or $x \ge \frac{3}{2}$; that is,when the signs of both factors are the same, or one of the factors is equal to zero.



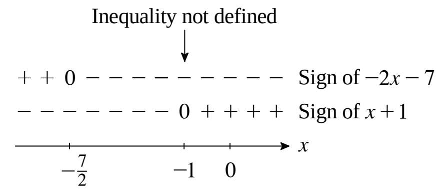
58. We want to find the values of x that satisfy the inequalities $(3x - 4)(2x + 2) \le 0$.
From the sign diagram, we see that the given inequality is satisfied when $-1 \le x \le \frac{4}{3}$, that is, when the signs of the two factors are different or when one of the factors is equal to zero.



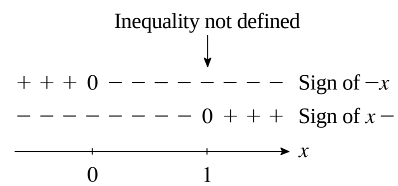
59. We want to find the values of x that satisfy the inequalities $\frac{x+3}{x-2} \geq 0$. From the sign diagram, we see that the given inequality is satisfied when $x \leq -3$ or $x > 2$, that is, when the signs of the two factors are the same. Notice that $x = 2$ is not included because the inequality is not defined at that value of x.



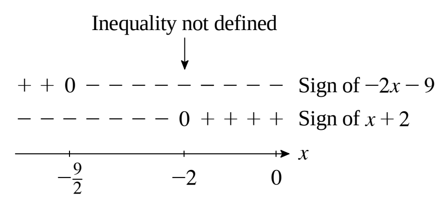
60. We want to find the values of x that satisfy the inequality $\frac{2x-3}{x+1} \geq 4$. We rewrite the inequality as $\frac{2x-3}{x+1} - 4 \geq 0$, $\frac{2x-3-4x-4}{x+1} \geq 0$, and $\frac{-2x-7}{x+1} \geq 0$. From the sign diagram, we see that the given inequality is satisfied when $-\frac{7}{2} \leq x < -1$; that is, when the signs of the two factors are the same. The solution set is $\left[-\frac{7}{2}, -1\right)$. Notice that $x = -1$ is not included because the inequality is not defined at that value of x.



61. We want to find the values of x that satisfy the inequality $\frac{x-2}{x-1} \leq 2$. Subtracting 2 from each side of the given inequality and simplifying gives $\frac{x-2}{x-1} - 2 \leq 0$, $\frac{x-2-2(x-1)}{x-1} \leq 0$, and $-\frac{x}{x-1} \leq 0$. From the sign diagram, we see that the given inequality is satisfied when $x \leq 0$ or $x > 1$; that is, when the signs of the two factors differ. The solution set is $(-\infty, 0] \cup (1, \infty)$. Notice that $x = 1$ is not included because the inequality is undefined at that value of x.



62. We want to find the values of x that satisfy the inequality $\frac{2x-1}{x+2} \leq 4$. Subtracting 4 from each side of the given inequality and simplifying gives $\frac{2x-1}{x+2} - 4 \leq 0$, $\frac{2x-1-4(x+2)}{x+2} \leq 0$, $\frac{2x-1-4x-8}{x+2} \leq 0$, and finally $\frac{-2x-9}{x+2} \leq 0$. From the sign diagram, we see that the given inequality is satisfied when $x \leq -\frac{9}{2}$ or $x > -2$. The solution set is $\left(-\infty, -\frac{9}{2}\right] \cup (-2, \infty)$.



63. $|-6+2| = 4$.

64. $4 + |-4| = 4 + 4 = 8$.

65. $\frac{|-12+4|}{|16-12|} = \frac{|-8|}{|4|} = 2$.

66. $\left|\frac{0.2-1.4}{1.6-2.4}\right| = \left|\frac{-1.2}{-0.8}\right| = 1.5$.

67. $\sqrt{3}\,|-2| + 3\left|-\sqrt{3}\right| = \sqrt{3}\,(2) + 3\sqrt{3} = 5\sqrt{3}$.

68. $|-1| + \sqrt{2}\,|-2| = 1 + 2\sqrt{2}$.

69. $|\pi - 1| + 2 = \pi - 1 + 2 = \pi + 1$.

70. $|\pi - 6| - 3 = 6 - \pi - 3 = 3 - \pi$.

71. $\left|\sqrt{2}-1\right|+\left|3-\sqrt{2}\right|=\sqrt{2}-1+3-\sqrt{2}=2.$

72. $\left|2\sqrt{3}-3\right|-\left|\sqrt{3}-4\right|=2\sqrt{3}-3-\left(4-\sqrt{3}\right)=3\sqrt{3}-7.$

73. False. If $a>b$, then $-a<-b$, $-a+b<-b+b$, and $b-a<0$.

74. False. Let $a=-2$ and $b=-3$. Then $a/b=\frac{-2}{-3}=\frac{2}{3}<1$.

75. False. Let $a=-2$ and $b=-3$. Then $a^2=4$ and $b^2=9$, and $4<9$. Note that we need only to provide a counterexample to show that the statement is not always true.

76. False. Let $a=-2$ and $b=-3$. Then $\dfrac{1}{a}=-\dfrac{1}{2}$ and $\dfrac{1}{b}=-\dfrac{1}{3}$, and $-\dfrac{1}{2}<-\dfrac{1}{3}$.

77. True. There are three possible cases.
Case 1: If $a>0$ and $b>0$, then $a^3>b^3$, since $a^3-b^3=(a-b)\left(a^2+ab+b^2\right)>0$.
Case 2: If $a>0$ and $b<0$, then $a^3>0$ and $b^3<0$, and it follows that $a^3>b^3$.
Case 3: If $a<0$ and $b<0$, then $a^3-b^3=(a-b)\left(a^2+ab+b^2\right)>0$, and we see that $a^3>b^3$. (Note that $a-b>0$ and $ab>0$.)

78. True. If $a>b$, then it follows that $-a<-b$ because an inequality symbol is reversed when both sides of the inequality are multiplied by a negative number.

79. False. If we take $a=-2$, then $|-a|=|-(-2)|=|2|=2\neq a$.

80. True. If $b<0$, then $b^2>0$, and $\left|b^2\right|=b^2$.

81. True. If $a-4<0$, then $|a-4|=4-a=|4-a|$. If $a-4>0$, then $|4-a|=a-4=|a-4|$.

82. False. If we let $a=-2$, then $|a+1|=|-2+1|=|-1|=1\neq|-2|+1=3$.

83. False. If we take $a=3$ and $b=-1$, then $|a+b|=|3-1|=2\neq|a|+|b|=3+1=4$.

84. False. If we take $a=3$ and $b=-1$, then $|a-b|=4\neq|a|-|b|=3-1=2$.

85. If the car is driven in the city, then it can be expected to cover $(18.1)(20)=362\ \frac{\text{miles}}{\text{gal}}\cdot\text{gal}$, or 362 miles, on a full tank. If the car is driven on the highway, then it can be expected to cover $(18.1)(27)=488.7\ \frac{\text{miles}}{\text{gal}}\cdot\text{gal}$, or 488.7 miles, on a full tank. Thus, the driving range of the car may be described by the interval [362, 488.7].

86. Simplifying $5(C-25)\geq 1.75+2.5C$, we obtain $5C-125\geq 1.75+2.5C$, $5C-2.5C\geq 1.75+125$, $2.5C\geq 126.75$, and finally $C\geq 50.7$. Therefore, the minimum cost is \$50.70.

87. $6(P-2500)\leq 4(P+2400)$ can be rewritten as $6P-15{,}000\leq 4P+9600$, $2P\leq 24{,}600$, or $P\leq 12{,}300$. Therefore, the maximum profit is \$12,300.

88. a. We want to find a formula for converting Centigrade temperatures to Fahrenheit temperatures. Thus, $C=\frac{5}{9}(F-32)=\frac{5}{9}F-\frac{160}{9}$. Therefore, $\frac{5}{9}F=C+\frac{160}{9}$, $5F=9C+160$, and $F=\frac{9}{5}C+32$. Calculating the lower temperature range, we have $F=\frac{9}{5}(-15)+32=5$, or 5 degrees. Calculating the upper temperature range, $F=\frac{9}{5}(-5)+32=23$, or 23 degrees. Therefore, the temperature range is $5^\circ<F<23^\circ$.

b. For the lower temperature range, $C = \frac{5}{9}(63 - 32) = \frac{155}{9} \approx 17.2$, or 17.2 degrees. For the upper temperature range, $C = \frac{5}{9}(80 - 32) = \frac{5}{9}(48) \approx 26.7$, or 26.7 degrees. Therefore, the temperature range is $17.2^\circ < C < 26.7^\circ$.

89. Let x represent the salesman's monthly sales in dollars. Then $0.15(x - 12{,}000) \geq 6000$, $15(x - 12{,}000) \geq 600{,}000$, $15x - 180{,}000 \geq 600{,}000$, $15x \geq 780{,}000$, and $x \geq 52{,}000$. We conclude that the salesman must have sales of at least \$52,000 to reach his goal.

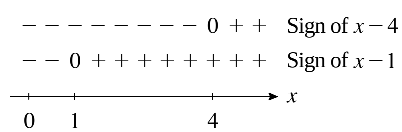
90. Let x represent the wholesale price of the car. Then $\frac{\text{Selling price}}{\text{Wholesale price}} - 1 \geq \text{Markup}$; that is, $\frac{11{,}200}{x} - 1 \geq 0.30$, whence $\frac{11{,}200}{x} \geq 1.30$, $1.3x \leq 11{,}200$, and $x \leq 8615.38$. We conclude that the maximum wholesale price is \$8615.38.

91. The rod is acceptable if $0.49 \leq x \leq 0.51$ or $-0.01 \leq x - 0.5 \leq 0.01$. This gives the required inequality, $|x - 0.5| \leq 0.01$.

92. $|x - 0.1| \leq 0.01$ is equivalent to $-0.01 \leq x - 0.1 \leq 0.01$ or $0.09 \leq x \leq 0.11$. Therefore, the smallest diameter a ball bearing in the batch can have is 0.09 inch, and the largest diameter is 0.11 inch.

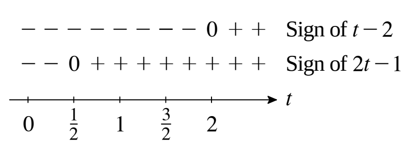
93. We want to solve the inequality $-6x^2 + 30x - 10 \geq 14$. (Remember that x is expressed in thousands.) Adding -14 to both sides of this inequality, we have $-6x^2 + 30x - 10 - 14 \geq 14 - 14$, or $-6x^2 + 30x - 24 \geq 0$. Dividing both sides of the inequality by -6 (which reverses the sign of the inequality), we have $x^2 - 5x + 4 \leq 0$. Factoring this last expression, we have $(x - 4)(x - 1) \leq 0$.

From the sign diagram, we see that x must lie between 1 and 4. (The inequality is satisfied only when the two factors have opposite signs.) Because x is expressed in thousands of units, we see that the manufacturer must produce between 1000 and 4000 units of the commodity.



94. We solve the inequality $\frac{0.2t}{t^2 + 1} \geq 0.08$, obtaining $0.08t^2 + 0.08 \leq 0.2t$, $0.08t^2 - 0.2t + 0.08 \leq 0$, $2t^2 - 5t + 2 \leq 0$, and $(2t - 1)(t - 2) \leq 0$.

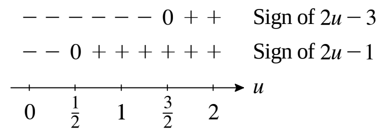
From the sign diagram, we see that the required solution is $\left[\frac{1}{2}, 2\right]$, so the concentration of the drug is greater than or equal to 0.08 mg/cc between $\frac{1}{2}$ hr and 2 hr after injection.



95. We solve the inequalities $25 \leq \frac{0.5x}{100 - x} \leq 30$, obtaining $2500 - 25x \leq 0.5x \leq 3000 - 30x$, which is equivalent to $2500 - 25x \leq 0.5x$ and $0.5x \leq 3000 - 30x$. Simplifying further, $25.5x \geq 2500$ and $30.5x \leq 3000$, so $x \geq \frac{2500}{25.5} \approx 98.04$ and $x \leq \frac{3000}{30.5} \approx 98.36$. Thus, the city could expect to remove between 98.04% and 98.36% of the toxic pollutant.

96. We simplify the inequality $20t - 40\sqrt{t} + 50 \le 35$ to $20t - 40\sqrt{t} + 15 \le 0$ (1). Let $u = \sqrt{t}$. Then $u^2 = t$, so we have $20u^2 - 40u + 15 \le 0$, $4u^2 - 8u + 3 \le 0$, and $(2u-3)(2u-1) \le 0$.

From the sign diagram, we see that we must have u in $\left[\frac{1}{2}, \frac{3}{2}\right]$.



Because $t = u^2$, we see that the solution to Equation (1) is $\left[\frac{1}{4}, \frac{9}{4}\right]$.

Thus, the average speed of a vehicle is less than or equal to 35 miles per hour between 6:15 a.m. and 8:15 a.m.

97. We solve the inequality $\dfrac{136}{1 + 0.25(t-4.5)^2} + 28 \ge 128$ or $\dfrac{136}{1 + 0.25(t-4.5)^2} \ge 100$. Next, $136 \ge 100\left[1 + 0.25(t-4.5)^2\right]$, so $136 \ge 100 + 25(t-4.5)^2$, $36 \ge 25(t-4.5)^2$, $(t-4.5)^2 \le \frac{36}{25}$, and $t - 4.5 \le \pm\frac{6}{5}$. Solving this last inequality, we have $t \le 5.7$ and $t \ge 3.3$. Thus, the amount of nitrogen dioxide is greater than or equal to 128 PSI between 10:18 a.m. and 12:42 p.m.

98. False. Take $a = 2$, $b = 3$, and $c = 4$. Then $\dfrac{a}{b+c} = \dfrac{2}{3+4} = \dfrac{2}{7}$, but $\dfrac{a}{b} + \dfrac{a}{c} = \dfrac{2}{3} + \dfrac{2}{4} = \dfrac{8+6}{12} = \dfrac{14}{12} = \dfrac{7}{6}$.

99. False. Take $a = 1$, $b = 2$, and $c = 3$. Then $a < b$, but $a - c = 1 - 3 = -2 \not> 2 - 3 = -1 = b - c$.

100. True. $|b-a| = |(-1)(a-b)| = |-1|\,|a-b| = |a-b|$.

101. True. $|a-b| = |a+(-b)| \le |a| + |-b| = |a| + |b|$.

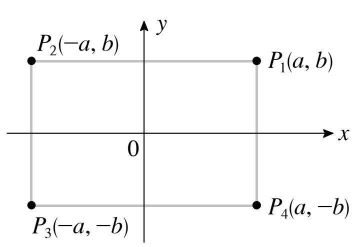
102. False. Take $a = 3$ and $b = 1$. Then $\sqrt{a^2 - b^2} = \sqrt{9-1} = \sqrt{8} = 2\sqrt{2}$, but $|a| - |b| = 3 - 1 = 2$.

1.3 The Cartesian Coordinate System

Concept Questions page 29

1. a. $a < 0$ and $b > 0$ **b.** $a < 0$ and $b < 0$ **c.** $a > 0$ and $b < 0$

2. a.



b. $d(P_1(a,b), (0,0)) = \sqrt{(0-a)^2 + (0-b)^2} = \sqrt{a^2+b^2}$,
$d(P_2(-a,b), (0,0)) = \sqrt{[0-(-a)]^2 + (0-b)^2} = \sqrt{a^2+b^2}$,
$d(P_3(-a,-b), (0,0)) = \sqrt{[0-(-a)]^2 + [0-(-b)]^2} = \sqrt{a^2+b^2}$,
and $d(P_4(a,-b), (0,0)) = \sqrt{(0-a)^2 + [0-(-b)]^2} = \sqrt{a^2+b^2}$,
so the points $P_1(a,b)$, $P_2(-a,b)$, $P_3(-a,-b)$, and $P_4(a,-b)$ are all the same distance from the origin.

Exercises page 30

1. The coordinates of A are $(3, 3)$ and it is located in Quadrant I.

2. The coordinates of B are $(-5, 2)$ and it is located in Quadrant II.

3. The coordinates of C are $(2, -2)$ and it is located in Quadrant IV.

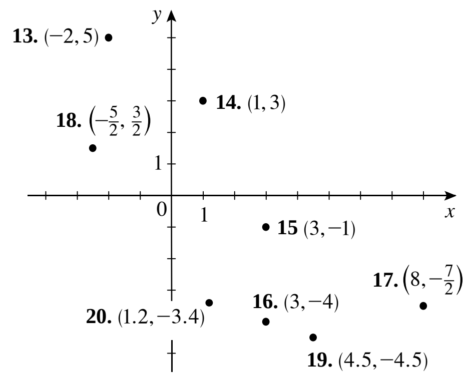
4. The coordinates of D are $(-2, 5)$ and it is located in Quadrant II.

5. The coordinates of E are $(-4, -6)$ and it is located in Quadrant III.

6. The coordinates of F are $(8, -2)$ and it is located in Quadrant IV.

7. A **8.** $(-5, 4)$ **9.** E, F, and G **10.** E **11.** F **12.** D

For Exercises 13—20, refer to the following figure.



21. Using the distance formula, we find that $\sqrt{(4-1)^2+(7-3)^2} = \sqrt{3^2+4^2} = \sqrt{25} = 5$.

22. Using the distance formula, we find that $\sqrt{(4-1)^2+(4-0)^2} = \sqrt{3^2+4^2} = \sqrt{25} = 5$.

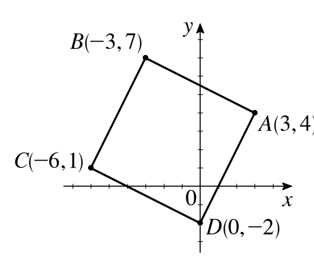
23. Using the distance formula, we find that $\sqrt{[4-(-1)]^2+(9-3)^2} = \sqrt{5^2+6^2} = \sqrt{25+36} = \sqrt{61}$.

24. Using the distance formula, we find that $\sqrt{[10-(-2)]^2+(6-1)^2} = \sqrt{12^2+5^2} = \sqrt{144+25} = \sqrt{169} = 13$.

25. The coordinates of the points have the form $(x, -6)$. Because the points are 10 units away from the origin, we have $(x-0)^2+(-6-0)^2 = 10^2$, $x^2 = 64$, or $x = \pm 8$. Therefore, the required points are $(-8, -6)$ and $(8, -6)$.

26. The coordinates of the points have the form $(3, y)$. Because the points are 5 units away from the origin, we have $(3-0)^2+(y-0)^2 = 5^2$, $y^2 = 16$, or $y = \pm 4$. Therefore, the required points are $(3, 4)$ and $(3, -4)$.

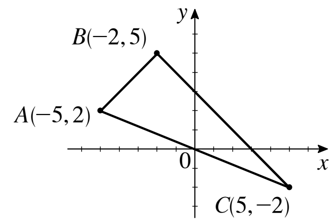
27. The points are shown in the diagram. To show that the four sides are equal, we compute



$d(A, B) = \sqrt{(-3-3)^2+(7-4)^2} = \sqrt{(-6)^2+3^2} = \sqrt{45}$,
$d(B, C) = \sqrt{[-6-(-3)]^2+(1-7)^2} = \sqrt{(-3)^2+(-6)^2} = \sqrt{45}$,
$d(C, D) = \sqrt{[0-(-6)]^2+[(-2)-1]^2} = \sqrt{(6)^2+(-3)^2} = \sqrt{45}$,
and $d(A, D) = \sqrt{(0-3)^2+(-2-4)^2} = \sqrt{(3)^2+(-6)^2} = \sqrt{45}$.

Next, to show that ΔABC is a right triangle, we show that it satisfies the Pythagorean Theorem. Thus, $d(A, C) = \sqrt{(-6-3)^2+(1-4)^2} = \sqrt{(-9)^2+(-3)^2} = \sqrt{90} = 3\sqrt{10}$ and $[d(A, B)]^2 + [d(B, C)]^2 = 90 = [d(A, C)]^2$. Similarly, $d(B, D) = \sqrt{90} = 3\sqrt{10}$, so ΔBAD is a right triangle as well. It follows that $\angle B$ and $\angle D$ are right angles, and we conclude that $ADCB$ is a square.

28. The triangle is shown in the figure. To prove that ΔABC is a right triangle, we show that $[d(A,C)]^2 = [d(A,B)]^2 + [d(B,C)]^2$ and the result will then follow from the Pythagorean Theorem. Now $[d(A,C)]^2 = (-5-5)^2 + [2-(-2)]^2 = 100 + 16 = 116$. Next, we find



$[d(A,B)]^2 + [d(B,C)]^2 = [-2-(-5)]^2 + (5-2)^2 + [5-(-2)]^2 + (-2-5)^2 = 9+9+49+49 = 116$, and the result follows.

29. The equation of the circle with radius 5 and center $(2,-3)$ is given by $(x-2)^2 + [y-(-3)]^2 = 5^2$, or $(x-2)^2 + (y+3)^2 = 25$.

30. The equation of the circle with radius 3 and center $(-2,-4)$ is given by $[x-(-2)]^2 + [y-(-4)]^2 = 9$, or $(x+2)^2 + (y+4)^2 = 9$.

31. The equation of the circle with radius 5 and center $(0,0)$ is given by $(x-0)^2 + (y-0)^2 = 5^2$, or $x^2 + y^2 = 25$.

32. The distance between the center of the circle and the point $(2,3)$ on the circumference of the circle is given by $d = \sqrt{(3-0)^2 + (2-0)^2} = \sqrt{13}$. Therefore $r = \sqrt{13}$ and the equation of the circle centered at the origin that passes through $(2,3)$ is $x^2 + y^2 = 13$.

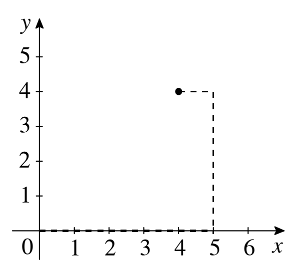
33. The distance between the points $(5,2)$ and $(2,-3)$ is given by $d = \sqrt{(5-2)^2 + [2-(-3)]^2} = \sqrt{3^2+5^2} = \sqrt{34}$. Therefore $r = \sqrt{34}$ and the equation of the circle passing through $(5,2)$ and $(2,-3)$ is $(x-2)^2 + [y-(-3)]^2 = 34$, or $(x-2)^2 + (y+3)^2 = 34$.

34. The equation of the circle with center $(-a,a)$ and radius $2a$ is given by $[x-(-a)]^2 + (y-a)^2 = (2a)^2$, or $(x+a)^2 + (y-a)^2 = 4a^2$.

35. **a.** The coordinates of the suspect's car at its final destination are $x = 4$ and $y = 4$.

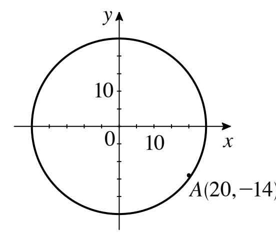
b. The distance traveled by the suspect was $5+4+1$, or 10 miles.

c. The distance between the original and final positions of the suspect's car was $d = \sqrt{(4-0)^2 + (4-0)^2} = \sqrt{32} = 4\sqrt{2}$, or approximately 5.66 miles.

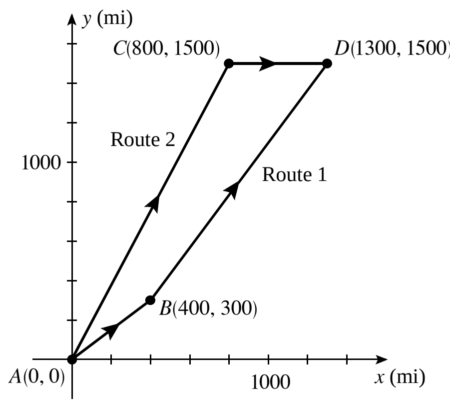


36. Referring to the diagram on page 31 of the text, we see that the distance from A to B is given by $d(A,B) = \sqrt{400^2 + 300^2} = \sqrt{250{,}000} = 500$. The distance from B to C is given by $d(B,C) = \sqrt{(-800-400)^2 + (800-300)^2} = \sqrt{(-1200)^2 + (500)^2} = \sqrt{1{,}690{,}000} = 1300$. The distance from C to D is given by $d(C,D) = \sqrt{[-800-(-800)]^2 + (800-0)^2} = \sqrt{0+800^2} = 800$. The distance from D to A is given by $d(D,A) = \sqrt{[(-800)-0]^2 + (0-0)} = \sqrt{640{,}000} = 800$. Therefore, the total distance covered on the tour is $d(A,B) + d(B,C) + d(C,D) + d(D,A) = 500 + 1300 + 800 + 800 = 3400$, or 3400 miles.

37. Suppose that the furniture store is located at the origin O so that your house is located at $A(20, -14)$. Because $d(O, A) = \sqrt{20^2 + (-14)^2} = \sqrt{596} \approx 24.4$, your house is located within a 25-mile radius of the store and you will not incur a delivery charge.



38.



Referring to the diagram, we see that the distance the salesman would cover if he took Route 1 is given by

$$d(A, B) + d(B, D) = \sqrt{400^2 + 300^2} + \sqrt{(1300 - 400)^2 + (1500 - 300)^2}$$
$$= \sqrt{250{,}000} + \sqrt{2{,}250{,}000} = 500 + 1500 = 2000$$

or 2000 miles. On the other hand, the distance he would cover if he took Route 2 is given by

$$d(A, C) + d(C, D) = \sqrt{800^2 + 1500^2} + \sqrt{(1300 - 800)^2} = \sqrt{2{,}890{,}000} + \sqrt{250{,}000}$$
$$= 1700 + 500 = 2200$$

or 2200 miles. Comparing these results, we see that he should take Route 1.

39. The cost of shipping by freight train is $(0.66)(2000)(100) = 132{,}000$, or \$132,000.
The cost of shipping by truck is $(0.62)(2200)(100) = 136{,}400$, or \$136,400.
Comparing these results, we see that the automobiles should be shipped by freight train. The net savings are $136{,}400 - 132{,}000 = 4400$, or \$4400.

40. The length of cable required on land is $d(S, Q) = 10{,}000 - x$ and the length of cable required under water is $d(Q, M) = \sqrt{(x^2 - 0) + (0 - 3000)^2} = \sqrt{x^2 + 3000^2}$. The cost of laying cable is thus $3(10{,}000 - x) + 5\sqrt{x^2 + 3000^2}$.
If $x = 2500$, then the total cost is given by $3(10{,}000 - 2500) + 5\sqrt{2500^2 + 3000^2} \approx 42{,}025.62$, or \$42,025.62.
If $x = 3000$, then the total cost is given by $3(10{,}000 - 3000) + 5\sqrt{3000^2 + 3000^2} \approx 42{,}213.20$, or \$42,213.20.

41. To determine the VHF requirements, we calculate $d = \sqrt{25^2 + 35^2} = \sqrt{625 + 1225} = \sqrt{1850} \approx 43.01$.
Models B, C, and D satisfy this requirement.
To determine the UHF requirements, we calculate $d = \sqrt{20^2 + 32^2} = \sqrt{400 + 1024} = \sqrt{1424} \approx 37.74$. Models C and D satisfy this requirement.
Therefore, Model C allows him to receive both channels at the least cost.

42. a. Let the positions of ships A and B after t hours be $A(0, y)$ and $B(x, 0)$, respectively. Then $x = 30t$ and $y = 20t$. Therefore, the distance in miles between the two ships is $D = \sqrt{(30t)^2 + (20t)^2} = \sqrt{900t^2 + 400t^2} = 10\sqrt{13}t$.

b. The required distance is obtained by letting $t = 2$, giving $D = 10\sqrt{13}(2)$, or approximately 72.11 miles.

43. a. Let the positions of ships A and B be $(0, y)$ and $(x, 0)$, respectively. Then $y = 25\left(t + \frac{1}{2}\right)$ and $x = 20t$. The distance D in miles between the two ships is

$$D = \sqrt{(x-0)^2 + (0-y)^2} = \sqrt{x^2 + y^2} = \sqrt{400t^2 + 625\left(t + \tfrac{1}{2}\right)^2} \quad (1).$$

b. The distance between the ships 2 hours after ship A has left port is obtained by letting $t = \frac{3}{2}$ in Equation (1), yielding $D = \sqrt{400\left(\frac{3}{2}\right)^2 + 625\left(\frac{3}{2} + \frac{1}{2}\right)^2} = \sqrt{3400}$, or approximately 58.31 miles.

44. a. The distance in feet is given by $\sqrt{(4000)^2 + x^2} = \sqrt{16{,}000{,}000 + x^2}$.

b. Substituting the value $x = 20{,}000$ into the above expression gives $\sqrt{16{,}000{,}000 + (20{,}000)^2} \approx 20{,}396$, or 20,396 ft.

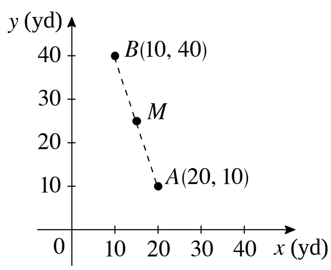
45. a. Suppose that $P = (x_1, y_1)$ and $Q = (x_2, y_2)$ are endpoints of the line segment and that the point $M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$ is the midpoint of the line segment PQ. The distance between P and Q is $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$. The distance between P and M is

$$\sqrt{\left(\frac{x_1 + x_2}{2} - x_1\right)^2 + \left(\frac{y_1 + y_2}{2} - y_1\right)^2} = \sqrt{\left(\frac{x_2 - x_1}{2}\right)^2 + \left(\frac{y_2 - y_1}{2}\right)^2} = \tfrac{1}{2}\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2},$$

which is one-half the distance from P to Q. Similarly, we obtain the same expression for the distance from M to P.

b. The midpoint is given by $\left(\frac{4-3}{2}, \frac{-5+2}{2}\right)$, or $\left(\frac{1}{2}, -\frac{3}{2}\right)$.

46. a.



b. The coordinates of the position of the prize are $x = \dfrac{20 + 10}{2}$ and $y = \dfrac{10 + 40}{2}$, or $x = 15$ yards and $y = 25$ yards.

c. The distance from the prize to the house is

$$d(M(15, 25), (0, 0)) = \sqrt{(15 - 0)^2 + (25 - 0)^2} = \sqrt{850} \approx 29.15 \text{ (yards)}.$$

47. True. Plot the points.

48. True. Plot the points.

49. False. The distance between $P_1(a, b)$ and $P_3(kc, kd)$ is

$$d = \sqrt{(kc - a)^2 + (kd - b)^2}$$
$$\neq |k|\, D = |k|\sqrt{(c - a)^2 + (d - b)^2} = \sqrt{k^2(c - a)^2 + k^2(d - b)^2} = \sqrt{[k(c - a)]^2 + [k(d - b)]^2}.$$

50. True. $kx^2 + ky^2 = a^2$ gives $x^2 + y^2 = \dfrac{a^2}{k} < a^2$ if $k > 1$. So the radius of the circle with equation $kx^2 + ky^2 = a^2$ is a circle of radius smaller than a centered at the origin if $k > 1$. Therefore, it lies inside the circle of radius a with equation $x^2 + y^2 = a^2$.

51. Referring to the figure in the text, we see that the distance between the two points is given by the length of the hypotenuse of the right triangle. That is, $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

52. $(x - h)^2 + (y - k)^2 = r^2$; $x^2 - 2xh + h^2 + y^2 - 2ky + k^2 = r^2$. This has the form $x^2 + y^2 + Cx + Dy + E = 0$, where $C = -2h$, $D = -2k$, and $E = h^2 + k^2 - r^2$.

1.4 Straight Lines

Concept Questions page 42

1. The slope is $m = \dfrac{y_2 - y_1}{x_2 - x_1}$, where $P(x_1, y_1)$ and $P(x_2, y_2)$ are any two distinct points on the nonvertical line. The slope of a vertical line is undefined.

2. a. $y - y_1 = m(x - x_1)$ **b.** $y = mx + b$ **c.** $ax + by + c = 0$, where a and b are not both zero.

3. a. $m_1 = m_2$ **b.** $m_2 = -\dfrac{1}{m_1}$

4. a. Solving the equation for y gives $By = -Ax - C$, so $y = -\dfrac{A}{B}x - \dfrac{C}{B}$. The slope of L is the coefficient of x, $-\dfrac{A}{B}$.

b. If $B = 0$, then the equation reduces to $Ax + C = 0$. Solving this equation for x, we obtain $x = -\dfrac{C}{A}$. This is an equation of a vertical line, and we conclude that the slope of L is undefined.

Exercises page 42

1. (e) **2.** (c) **3.** (a) **4.** (d) **5.** (f) **6.** (b)

7. Referring to the figure shown in the text, we see that $m = \dfrac{2 - 0}{0 - (-4)} = \dfrac{1}{2}$.

8. Referring to the figure shown in the text, we see that $m = \dfrac{4 - 0}{0 - 2} = -2$.

9. This is a vertical line, and hence its slope is undefined.

10. This is a horizontal line, and hence its slope is 0.

11. $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{8 - 3}{5 - 4} = 5$.

12. $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{8 - 5}{3 - 4} = \dfrac{3}{-1} = -3$.

13. $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{8 - 3}{4 - (-2)} = \dfrac{5}{6}$.

14. $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{-4 - (-2)}{4 - (-2)} = \dfrac{-2}{6} = -\dfrac{1}{3}$.

15. $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{d - b}{c - a}$, provided $a \neq c$.

16. $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{-b - (b-1)}{a + 1 - (-a+1)} = -\dfrac{-b - b + 1}{a + 1 + a - 1} = \dfrac{1 - 2b}{2a}$.

17. Because the equation is already in slope-intercept form, we read off the slope $m = 4$.

a. If x increases by 1 unit, then y increases by 4 units.

b. If x decreases by 2 units, then y decreases by $4(-2) = -8$ units.

18. Rewrite the given equation in slope-intercept form: $2x + 3y = 4$, $3y = 4 - 2x$, and so $y = \frac{4}{3} - \frac{2}{3}x$.

a. Because $m = -\frac{2}{3}$, we conclude that the slope is negative.

b. Because the slope is negative, y decreases as x increases.

c. If x decreases by 2 units, then y increases by $\left(-\frac{2}{3}\right)(-2) = \frac{4}{3}$ units.

19. The slope of the line through A and B is $\dfrac{-10 - (-2)}{-3 - 1} = \dfrac{-8}{-4} = 2$. The slope of the line through C and D is $\dfrac{1 - 5}{-1 - 1} = \dfrac{-4}{-2} = 2$. Because the slopes of these two lines are equal, the lines are parallel.

20. The slope of the line through A and B is $\dfrac{-2 - 3}{2 - 2}$. Because this slope is undefined, we see that the line is vertical. The slope of the line through C and D is $\dfrac{5 - 4}{-2 - (-2)}$. Because this slope is undefined, we see that this line is also vertical. Therefore, the lines are parallel.

21. The slope of the line through A and B is $\dfrac{2 - 5}{4 - (-2)} = -\dfrac{3}{6} = -\dfrac{1}{2}$. The slope of the line through C and D is $\dfrac{6 - (-2)}{3 - (-1)} = \dfrac{8}{4} = 2$. Because the slopes of these two lines are the negative reciprocals of each other, the lines are perpendicular.

22. The slope of the line through A and B is $\dfrac{-2 - 0}{1 - 2} = \dfrac{-2}{-1} = 2$. The slope of the line through C and D is $\dfrac{4 - 2}{-8 - 4} = \dfrac{2}{-12} = -\dfrac{1}{6}$. Because the slopes of these two lines are not the negative reciprocals of each other, the lines are not perpendicular.

23. The slope of the line through the point $(1, a)$ and $(4, -2)$ is $m_1 = \dfrac{-2 - a}{4 - 1}$ and the slope of the line through $(2, 8)$ and $(-7, a + 4)$ is $m_2 = \dfrac{a + 4 - 8}{-7 - 2}$. Because these two lines are parallel, m_1 is equal to m_2. Therefore, $\dfrac{-2 - a}{3} = \dfrac{a - 4}{-9}$, $-9(-2 - a) = 3(a - 4)$, $18 + 9a = 3a - 12$, and $6a = -30$, so $a = -5$.

24. The slope of the line through the point $(a, 1)$ and $(5, 8)$ is $m_1 = \dfrac{8 - 1}{5 - a}$ and the slope of the line through $(4, 9)$ and $(a + 2, 1)$ is $m_2 = \dfrac{1 - 9}{a + 2 - 4}$. Because these two lines are parallel, m_1 is equal to m_2. Therefore, $\dfrac{7}{5 - a} = \dfrac{-8}{a - 2}$, $7(a - 2) = -8(5 - a)$, $7a - 14 = -40 + 8a$, and $a = 26$.

25. An equation of a horizontal line is of the form $y = b$. In this case $b = -3$, so $y = -3$ is an equation of the line.

26. An equation of a vertical line is of the form $x = a$. In this case $a = 0$, so $x = 0$ is an equation of the line.

27. We use the point-slope form of an equation of a line with the point $(3, -4)$ and slope $m = 2$. Thus $y - y_1 = m(x - x_1)$ becomes $y - (-4) = 2(x - 3)$. Simplifying, we have $y + 4 = 2x - 6$, or $y = 2x - 10$.

28. We use the point-slope form of an equation of a line with the point $(2, 4)$ and slope $m = -1$. Thus $y - y_1 = m(x - x_1)$, giving $y - 4 = -1(x - 2)$, $y - 4 = -x + 2$, and finally $y = -x + 6$.

29. Because the slope $m = 0$, we know that the line is a horizontal line of the form $y = b$. Because the line passes through $(-3, 2)$, we see that $b = 2$, and an equation of the line is $y = 2$.

30. We use the point-slope form of an equation of a line with the point $(1, 2)$ and slope $m = -\frac{1}{2}$. Thus $y - y_1 = m(x - x_1)$ gives $y - 2 = -\frac{1}{2}(x - 1)$, $2y - 4 = -x + 1$, $2y = -x + 5$, and $y = -\frac{1}{2}x + \frac{5}{2}$.

31. We first compute the slope of the line joining the points $(2, 4)$ and $(3, 7)$ and find that $m = \dfrac{7 - 4}{3 - 2} = 3$. Using the point-slope form of an equation of a line with the point $(2, 4)$ and slope $m = 3$, we find $y - 4 = 3(x - 2)$, or $y = 3x - 2$.

32. We first compute the slope of the line joining the points $(2, 1)$ and $(2, 5)$ and find that $m = \dfrac{5 - 1}{2 - 2}$. Because this slope is undefined, we see that the line must be a vertical line of the form $x = a$. Because it passes through $(2, 5)$, we see that $x = 2$ is the equation of the line.

33. We first compute the slope of the line joining the points $(1, 2)$ and $(-3, -2)$ and find that $m = \dfrac{-2 - 2}{-3 - 1} = \dfrac{-4}{-4} = 1$. Using the point-slope form of an equation of a line with the point $(1, 2)$ and slope $m = 1$, we find $y - 2 = x - 1$, or $y = x + 1$.

34. We first compute the slope of the line joining the points $(-1, -2)$ and $(3, -4)$ and find that $m = \dfrac{-4 - (-2)}{3 - (-1)} = \dfrac{-2}{4} = -\dfrac{1}{2}$. Using the point-slope form of an equation of a line with the point $(-1, -2)$ and slope $m = -\frac{1}{2}$, we find $y - (-2) = -\frac{1}{2}[x - (-1)]$, $y + 2 = -\frac{1}{2}(x + 1)$, and finally $y = -\frac{1}{2}x - \frac{5}{2}$.

35. We use the slope-intercept form of an equation of a line: $y = mx + b$. Because $m = 3$ and $b = 4$, the equation is $y = 3x + 4$.

36. We use the slope-intercept form of an equation of a line: $y = mx + b$. Because $m = -2$ and $b = -1$, the equation is $y = -2x - 1$.

37. We use the slope-intercept form of an equation of a line: $y = mx + b$. Because $m = 0$ and $b = 5$, the equation is $y = 5$.

38. We use the slope-intercept form of an equation of a line:$y = mx + b$. Because $m = -\frac{1}{2}$, and $b = \frac{3}{4}$, the equation is $y = -\frac{1}{2}x + \frac{3}{4}$.

39. We first write the given equation in the slope-intercept form: $x - 2y = 0$, so $-2y = -x$, or $y = \frac{1}{2}x$. From this equation, we see that $m = \frac{1}{2}$ and $b = 0$.

40. We write the equation in slope-intercept form: $y - 2 = 0$, so $y = 2$. From this equation, we see that $m = 0$ and $b = 2$.

41. We write the equation in slope-intercept form: $2x - 3y - 9 = 0$, $-3y = -2x + 9$, and $y = \frac{2}{3}x - 3$. From this equation, we see that $m = \frac{2}{3}$ and $b = -3$.

42. We write the equation in slope-intercept form: $3x - 4y + 8 = 0$, $-4y = -3x - 8$, and $y = \frac{3}{4}x + 2$. From this equation, we see that $m = \frac{3}{4}$ and $b = 2$.

43. We write the equation in slope-intercept form: $2x + 4y = 14$, $4y = -2x + 14$, and $y = -\frac{2}{4}x + \frac{14}{4} = -\frac{1}{2}x + \frac{7}{2}$. From this equation, we see that $m = -\frac{1}{2}$ and $b = \frac{7}{2}$.

44. We write the equation in the slope-intercept form: $5x + 8y - 24 = 0$, $8y = -5x + 24$, and $y = -\frac{5}{8}x + 3$. From this equation, we conclude that $m = -\frac{5}{8}$ and $b = 3$.

45. We first write the equation $2x - 4y - 8 = 0$ in slope-intercept form: $2x - 4y - 8 = 0$, $4y = 2x - 8$, $y = \frac{1}{2}x - 2$. Now the required line is parallel to this line, and hence has the same slope. Using the point-slope form of an equation of a line with $m = \frac{1}{2}$ and the point $(-2, 2)$, we have $y - 2 = \frac{1}{2}[x - (-2)]$ or $y = \frac{1}{2}x + 3$.

46. We first write the equation $3x + 4y - 22 = 0$ in slope-intercept form: $3x + 4y - 22 = 0$, so $4y = -3x + 22$ and $y = -\frac{3}{4}x + \frac{11}{2}$ Now the required line is perpendicular to this line, and hence has slope $\frac{4}{3}$ (the negative reciprocal of $-\frac{3}{4}$). Using the point-slope form of an equation of a line with $m = \frac{4}{3}$ and the point $(2, 4)$, we have $y - 4 = \frac{4}{3}(x - 2)$, or $y = \frac{4}{3}x + \frac{4}{3}$.

47. The midpoint of the line segment joining $P_1(-2, -4)$ and $P_2(3, 6)$ is $M\left(\dfrac{-2+3}{2}, \dfrac{-4+6}{2}\right)$ or $M\left(\frac{1}{2}, 1\right)$.

Using the point-slope form of the equation of a line with $m = -2$, we have $y - 1 = -2\left(x - \frac{1}{2}\right)$ or $y = -2x + 2$.

48. The midpoint of the line segment joining $P_1(-1, -3)$ and $P_2(3, 3)$ is $M_1\left(\dfrac{-1+3}{2}, \dfrac{-3+3}{2}\right)$ or $M_1(1, 0)$.

The midpoint of the line segment joining $P_3(-2, 3)$ and $P_4(2, -3)$ is $M_2\left(\dfrac{-2+2}{2}, \dfrac{3-3}{2}\right)$ or $M_2(0, 0)$.

The slope of the required line is $m = \dfrac{0-0}{1-0} = 0$, so an equation of the line is $y - 0 = 0(x - 0)$ or $y = 0$.

49. A line parallel to the x-axis has slope 0 and is of the form $y = b$. Because the line is 6 units below the axis, it passes through $(0, -6)$ and its equation is $y = -6$.

50. Because the required line is parallel to the line joining $(2, 4)$ and $(4, 7)$, it has slope $m = \dfrac{7-4}{4-2} = \dfrac{3}{2}$. We also know that the required line passes through the origin $(0, 0)$. Using the point-slope form of an equation of a line, we find $y - 0 = \frac{3}{2}(x - 0)$, or $y = \frac{3}{2}x$.

51. We use the point-slope form of an equation of a line to obtain $y - b = 0(x - a)$, or $y = b$.

52. Because the line is parallel to the x-axis, its slope is 0 and its equation has the form $y = b$. We know that the line passes through $(-3, 4)$, so the required equation is $y = 4$.

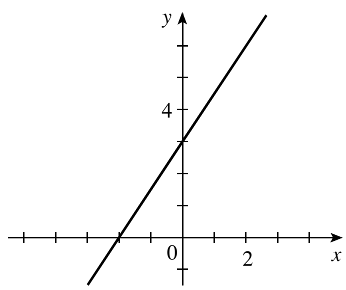
53. Because the required line is parallel to the line joining $(-3, 2)$ and $(6, 8)$, it has slope $m = \dfrac{8-2}{6-(-3)} = \dfrac{6}{9} = \dfrac{2}{3}$. We also know that the required line passes through $(-5, -4)$. Using the point-slope form of an equation of a line, we find $y - (-4) = \frac{2}{3}[x - (-5)]$, $y = \frac{2}{3}x + \frac{10}{3} - 4$, and finally $y = \frac{2}{3}x - \frac{2}{3}$.

54. Because the slope of the line is undefined, it has the form $x = a$. Furthermore, since the line passes through (a, b), the required equation is $x = a$.

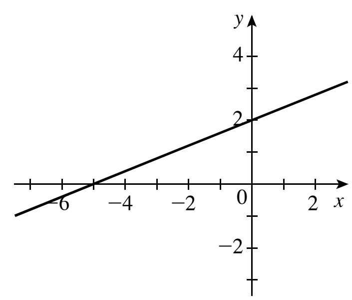
55. Because the point $(-3, 5)$ lies on the line $kx + 3y + 9 = 0$, it satisfies the equation. Substituting $x = -3$ and $y = 5$ into the equation gives $-3k + 15 + 9 = 0$, or $k = 8$.

56. Because the point $(2, -3)$ lies on the line $-2x + ky + 10 = 0$, it satisfies the equation. Substituting $x = 2$ and $y = -3$ into the equation gives $-2(2) + (-3)k + 10 = 0$, $-4 - 3k + 10 = 0$, $-3k = -6$, and finally $k = 2$.

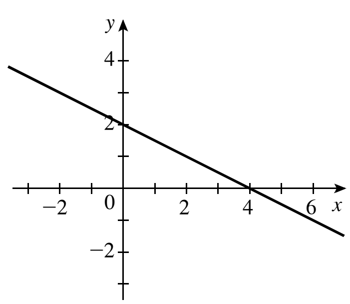
57. $3x - 2y + 6 = 0$. Setting $y = 0$, we have $3x + 6 = 0$ or $x = -2$, so the x-intercept is -2. Setting $x = 0$, we have $-2y + 6 = 0$ or $y = 3$, so the y-intercept is 3.



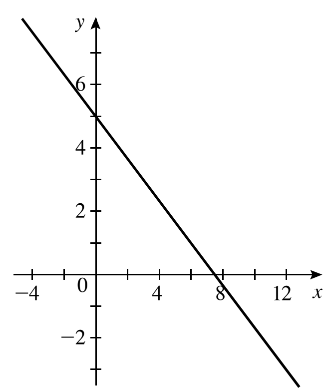
58. $2x - 5y + 10 = 0$. Setting $y = 0$, we have $2x + 10 = 0$ or $x = -5$, so the x-intercept is -5. Setting $x = 0$, we have $-5y + 10 = 0$ or $y = 2$, so the y-intercept is 2.



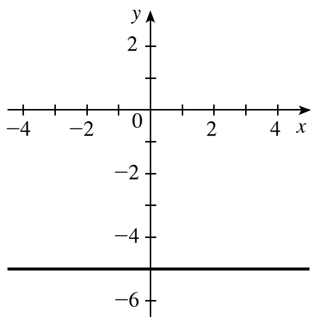
59. $x + 2y - 4 = 0$. Setting $y = 0$, we have $x - 4 = 0$ or $x = 4$, so the x-intercept is 4. Setting $x = 0$, we have $2y - 4 = 0$ or $y = 2$, so the y-intercept is 2.



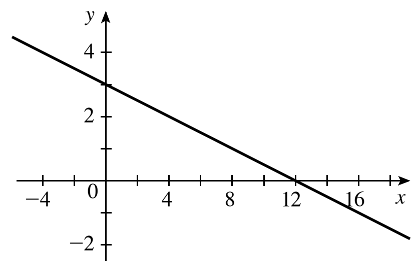
60. $2x + 3y - 15 = 0$. Setting $y = 0$, we have $2x - 15 = 0$, so the x-intercept is $\frac{15}{2}$. Setting $x = 0$, we have $3y - 15 = 0$, so the y-intercept is 5.



61. $y + 5 = 0$. Setting $y = 0$, we have $0 + 5 = 0$, which has no solution, so there is no x-intercept. Setting $x = 0$, we have $y + 5 = 0$ or $y = -5$, so the y-intercept is -5.



62. $-2x - 8y + 24 = 0$. Setting $y = 0$, we have $-2x + 24 = 0$ or $x = 12$, so the x-intercept is 12. Setting $x = 0$, we have $-8y + 24 = 0$ or $y = 3$, so the y-intercept is 3.



63. Because the line passes through the points $(a, 0)$ and $(0, b)$, its slope is $m = \dfrac{b-0}{0-a} = -\dfrac{b}{a}$. Then, using the point-slope form of an equation of a line with the point $(a, 0)$, we have $y - 0 = -\dfrac{b}{a}(x - a)$ or $y = -\dfrac{b}{a}x + b$, which may be written in the form $\dfrac{b}{a}x + y = b$. Multiplying this last equation by $\dfrac{1}{b}$, we have $\dfrac{x}{a} + \dfrac{y}{b} = 1$.

64. Using the equation $\dfrac{x}{a} + \dfrac{y}{b} = 1$ with $a = 3$ and $b = 4$, we have $\dfrac{x}{3} + \dfrac{y}{4} = 1$. Then $4x + 3y = 12$, so $3y = 12 - 4x$ and thus $y = -\frac{4}{3}x + 4$.

65. Using the equation $\dfrac{x}{a} + \dfrac{y}{b} = 1$ with $a = -2$ and $b = -4$, we have $-\dfrac{x}{2} - \dfrac{y}{4} = 1$. Then $-4x - 2y = 8$, $2y = -8 - 4x$, and finally $y = -2x - 4$.

66. Using the equation $\dfrac{x}{a} + \dfrac{y}{b} = 1$ with $a = -\frac{1}{2}$ and $b = \frac{3}{4}$, we have $\dfrac{x}{-1/2} + \dfrac{y}{3/4} = 1$, $\frac{3}{4}x - \frac{1}{2}y = \left(-\frac{1}{2}\right)\left(\frac{3}{4}\right)$, $\frac{1}{2}y = -\frac{3}{4}x - \frac{3}{8}$, and finally $y = 2\left(\frac{3}{4}x + \frac{3}{8}\right) = \frac{3}{2}x + \frac{3}{4}$.

67. Using the equation $\dfrac{x}{a} + \dfrac{y}{b} = 1$ with $a = 4$ and $b = -\frac{1}{2}$, we have $\dfrac{x}{4} + \dfrac{y}{-1/2} = 1$, $-\frac{1}{4}x + 2y = -1$, $2y = \frac{1}{4}x - 1$, and so $y = \frac{1}{8}x - \frac{1}{2}$.

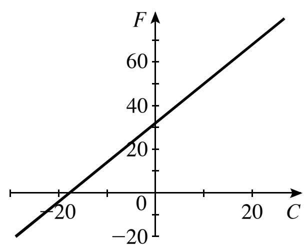
68. The slope of the line passing through A and B is $m = \dfrac{-2-7}{2-(-1)} = -\dfrac{9}{3} = -3$, and the slope of the line passing through B and C is $m = \dfrac{-9-(-2)}{5-2} = -\dfrac{7}{3}$. Because the slopes are not equal, the points do not lie on the same line.

69. The slope of the line passing through A and B is $m = \dfrac{7-1}{1-(-2)} = \dfrac{6}{3} = 2$, and the slope of the line passing through B and C is $m = \dfrac{13-7}{4-1} = \dfrac{6}{3} = 2$. Because the slopes are equal, the points lie on the same line.

70. The slope of the line L passing through $P_1\,(1.2, -9.04)$ and $P_2\,(2.3, -5.96)$ is $m = \dfrac{-5.96 - (-9.04)}{2.3 - 1.2} = 2.8$, so an equation of L is $y - (-9.04) = 2.8\,(x - 1.2)$ or $y = 2.8x - 12.4$.
Substituting $x = 4.8$ into this equation gives $y = 2.8\,(4.8) - 12.4 = 1.04$. This shows that the point $P_3\,(4.8, 1.04)$ lies on L. Next, substituting $x = 7.2$ into the equation gives $y = 2.8\,(7.2) - 12.4 = 7.76$, which shows that the point $P_4\,(7.2, 7.76)$ also lies on L. We conclude that John's claim is valid.

71. The slope of the line L passing through $P_1\,(1.8, -6.44)$ and $P_2\,(2.4, -5.72)$ is $m = \dfrac{-5.72 - (-6.44)}{2.4 - 1.8} = 1.2$, so an equation of L is $y - (-6.44) = 1.2\,(x - 1.8)$ or $y = 1.2x - 8.6$.
Substituting $x = 5.0$ into this equation gives $y = 1.2\,(5) - 8.6 = -2.6$. This shows that the point $P_3\,(5.0, -2.72)$ does not lie on L, and we conclude that Alison's claim is not valid.

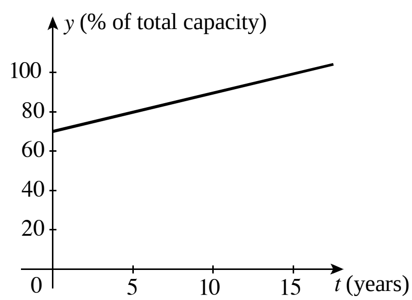
72. a.



b. The slope is $\frac{9}{5}$. It represents the change in °F per unit change in °C.

c. The F-intercept of the line is 32. It corresponds to 0°, so it is the freezing point in °F.

73. a.



b. The slope is 1.9467 and the y-intercept is 70.082.

c. The output is increasing at the rate of 1.9467% per year. The output at the beginning of 1990 was 70.082%.

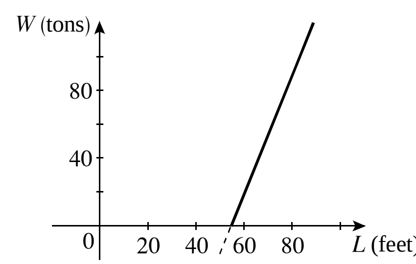
d. We solve the equation $1.9467t + 70.082 = 100$, obtaining $t \approx 15.37$. We conclude that the plants were generating at maximum capacity during April 2005.

74. a. $y = 0.0765x$ **b.** \$0.0765 **c.** $0.0765\,(65{,}000) = 4972.50$, or \$4972.50.

75. a. $y = 0.55x$ **b.** Solving the equation $1100 = 0.55x$ for x, we have $x = \dfrac{1100}{0.55} = 2000$.

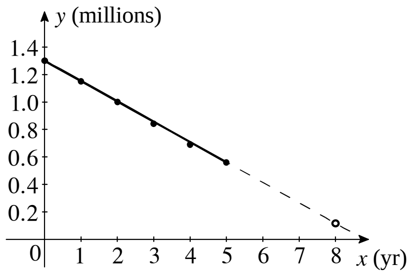
76. a. Substituting $L = 80$ into the given equation, we have $W = 3.51\,(80) - 192 = 280.8 - 192 = 88.8$, or 88.8 British tons.

b.



77. Using the points $(0, 0.68)$ and $(10, 0.80)$, we see that the slope of the required line is $m = \dfrac{0.80 - 0.68}{10 - 0} = \dfrac{0.12}{10} = 0.012$. Next, using the point-slope form of the equation of a line, we have $y - 0.68 = 0.012\,(t - 0)$ or $y = 0.012t + 0.68$. Therefore, when $t = 18$, we have $y = 0.012\,(18) + 0.68 = 0.896$, or 89.6%. That is, in 2008 women's wages were expected to be 89.6% of men's wages.

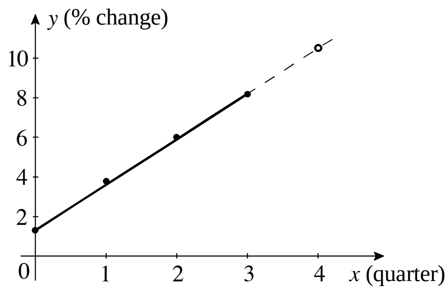
78. a, b.



c. The slope of L is $m = \dfrac{0.56 - 1.30}{5 - 0} = -0.148$, so an equation of L is $y - 1.3 = -0.148\,(x - 0)$ or $y = -0.148x + 1.3$.

d. The number of pay phones in 2012 is estimated to be $-0.148\,(8) + 1.3$, or approximately 116,000.

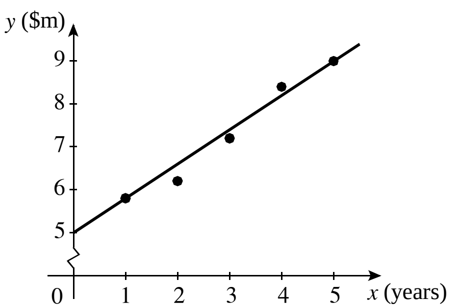
79. a, b.



c. The slope of L is $m = \dfrac{8.2 - 1.3}{3 - 0} = 2.3$, so an equation of L is $y - 1.3 = 2.3\,(x - 0)$ or $y = 2.3x + 1.3$.

d. The change in spending in the first quarter of 2014 is estimated to be $2.3\,(4) + 1.3$, or 10.5%.

80. a, b.

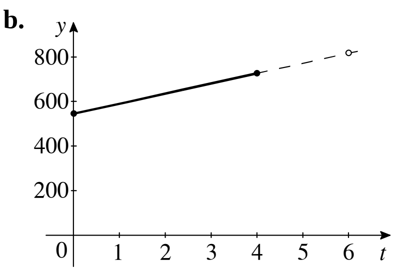


c. The slope of L is $m = \dfrac{9.0 - 5.8}{5 - 1} = \dfrac{3.2}{4} = 0.8$. Using the point-slope form of an equation of a line, we have $y - 5.8 = 0.8\,(x - 1) = 0.8x - 0.8$, or $y = 0.8x + 5$.

d. Using the equation from part (c) with $x = 9$, we have $y = 0.8\,(9) + 5 = 12.2$, or \$12.2 million.

81. a. The slope of the line L passing through $A\,(0, 545)$ and $B\,(4, 726)$ is $m = \dfrac{726 - 545}{4 - 0} = \dfrac{181}{4}$, so an equation of L is $y - 545 = \frac{181}{4}\,(t - 0)$ or $y = \frac{181}{4}t + 545$.

b.



c. The number of corporate fraud cases pending at the beginning of 2016 is estimated to be $\frac{181}{4}\,(8) + 545$, or approximately 907.

82. a. The slope of the line through $P_1\,(0, 27)$ and $P_2\,(1, 29)$ is $m_1 = \dfrac{29 - 27}{1 - 0} = 2$, and it is equal to the slope of the line through $P_2\,(1, 29)$ and $P_3\,(2, 31)$, which is $m_2 = \dfrac{31 - 29}{1 - 0} = 2$. Thus, the three points lie on the line L.

b. The percentage is of moviegoers who use social media to chat about movies in 2014 is estimated to be $31 + 2\,(2)$, or 35%.

c. $y - 27 = 2\,(x - 0)$, so $y = 2x + 27$. The estimate for 2014 ($t = 4$) is $2\,(4) + 27 = 35$, as found in part (b).

83. True. The slope of the line is given by $-\frac{2}{4} = -\frac{1}{2}$.

84. True. The slope of the line $Ax + By + C = 0$ is $-\frac{A}{B}$. (Write it in slope-intercept form.) Similarly, the slope of the line $ax + by + c = 0$ is $-\frac{a}{b}$. They are parallel if and only if $-\frac{A}{B} = -\frac{a}{b}$, that is, if $Ab = aB$, or $Ab - aB = 0$.

85. False. Let the slope of L_1 be $m_1 > 0$. Then the slope of L_2 is $m_2 = -\frac{1}{m_1} < 0$.

86. True. The slope of the line $ax + by + c_1 = 0$ is $m_1 = -\frac{a}{b}$. The slope of the line $bx - ay + c_2 = 0$ is $m_2 = \frac{b}{a}$. Because $m_1 m_2 = -1$, the straight lines are indeed perpendicular.

87. True. Set $y = 0$ and we have $Ax + C = 0$ or $x = -C/A$, and this is where the line intersects the x-axis.

88. Yes. A straight line with slope zero ($m = 0$) is a horizontal line, whereas a straight line whose slope does not exist is a vertical line (m cannot be computed).

89. Writing each equation in the slope-intercept form, we have $y = -\frac{a_1}{b_1}x - \frac{c_1}{b_1}$ $(b_1 \neq 0)$ and $y = -\frac{a_2}{b_2}x - \frac{c_2}{b_2}$ $(b_2 \neq 0)$. Because two lines are parallel if and only if their slopes are equal, we see that the lines are parallel if and only if $-\frac{a_1}{b_1} = -\frac{a_2}{b_2}$, or $a_1 b_2 - b_1 a_2 = 0$.

90. The slope of L_1 is $m_1 = \frac{b - 0}{1 - 0} = b$. The slope of L_2 is $m_2 = \frac{c - 0}{1 - 0} = c$. Applying the Pythagorean theorem to ΔOAC and ΔOCB gives $(OA)^2 = 1^2 + b^2$ and $(OB)^2 = 1^2 + c^2$. Adding these equations and applying the Pythagorean theorem to ΔOBA gives $(AB)^2 = (OA)^2 + (OB)^2 = 1^2 + b^2 + 1^2 + c^2 = 2 + b^2 + c^2$. Also, $(AB)^2 = (b - c)^2$, so $(b - c)^2 = 2 + b^2 + c^2$, $b^2 - 2bc + c^2 = 2 + b^2 + c^2$, and $-2bc = 2$, $1 = -bc$. Finally, $m_1 m_2 = b \cdot c = bc = -1$, as was to be shown.

CHAPTER 1 Concept Review Questions page 48

1. ordered, abscissa or x-coordinate, ordinate or y-coordinate

2. a. x, y **b.** third

3. $\sqrt{(c - a)^2 + (d - b)^2}$

4. $(x - a)^2 + (y - b)^2 = r^2$

5. a. $\frac{y_2 - y_1}{x_2 - x_1}$ **b.** undefined **c.** 0 **d.** positive

6. $m_1 = m_2$, $m_1 = -\frac{1}{m_2}$

7. a. $y - y_1 = m(x - x_1)$, point-slope form **b.** $y = mx + b$, slope-intercept

8. a. $Ax + By + C = 0$, where A and B are not both zero **b.** $-a/b$

CHAPTER 1 Review Exercises page 48

1. Adding x to both sides yields $3 \le 3x + 9$, $3x \ge -6$, or $x \ge -2$. We conclude that the solution set is $[-2, \infty)$.

2. $-2 \le 3x + 1 \le 7$ implies $-3 \le 3x \le 6$, or $-1 \le x \le 2$, and so the solution set is $[-1, 2]$.

3. The inequalities imply $x > 5$ or $x < -4$, so the solution set is $(-\infty, -4) \cup (5, \infty)$.

4. $2x^2 > 50$ is equivalent to $x^2 > 25$, so either $x > 5$ or $x < -5$ and the solution set is $(-\infty, -5) \cup (5, \infty)$.

5. $|-5 + 7| + |-2| = |2| + |-2| = 2 + 2 = 4$.

6. $\left|\dfrac{5 - 12}{-4 - 3}\right| = \dfrac{|5 - 12|}{|-7|} = \dfrac{|-7|}{7} = \dfrac{7}{7} = 1$.

7. $|2\pi - 6| - \pi = 2\pi - 6 - \pi = \pi - 6$.

8. $\left|\sqrt{3} - 4\right| + \left|4 - 2\sqrt{3}\right| = \left(4 - \sqrt{3}\right) + \left(4 - 2\sqrt{3}\right) = 8 - 3\sqrt{3}$.

9. $\left(\dfrac{9}{4}\right)^{3/2} = \dfrac{9^{3/2}}{4^{3/2}} = \dfrac{27}{8}$.

10. $\dfrac{5^6}{5^4} = 5^{6-4} = 5^2 = 25$.

11. $(3 \cdot 4)^{-2} = 12^{-2} = \dfrac{1}{12^2} = \dfrac{1}{144}$.

12. $(-8)^{5/3} = \left(-8^{1/3}\right)^5 = (-2)^5 = -32$.

13. $\dfrac{\left(3 \cdot 2^{-3}\right)\left(4 \cdot 3^5\right)}{2 \cdot 9^3} = \dfrac{3 \cdot 2^{-3} \cdot 2^2 \cdot 3^5}{2 \cdot \left(3^2\right)^3} = \dfrac{2^{-1} \cdot 3^6}{2 \cdot 3^6} = \dfrac{1}{4}$.

14. $\dfrac{3\sqrt[3]{54}}{\sqrt[3]{18}} = \dfrac{3 \cdot \left(2 \cdot 3^3\right)^{1/3}}{\left(2 \cdot 3^2\right)^{1/3}} = \dfrac{3^2 \cdot 2^{1/3}}{2^{1/3} \cdot 3^{2/3}} = 3^{4/3} = 3\sqrt[3]{3}$.

15. $\dfrac{4\left(x^2 + y\right)^3}{x^2 + y} = 4\left(x^2 + y\right)^2$.

16. $\dfrac{a^6 b^{-5}}{\left(a^3 b^{-2}\right)^{-3}} = \dfrac{a^6 b^{-5}}{a^{-9} b^6} = \dfrac{a^{15}}{b^{11}}$.

17. $\dfrac{\sqrt[4]{16x^5yz}}{\sqrt[4]{81xyz^5}} = \dfrac{\left(2^4x^5yz\right)^{1/4}}{\left(3^4xyz^5\right)^{1/4}} = \dfrac{2x^{5/4}y^{1/4}z^{1/4}}{3x^{1/4}y^{1/4}z^{5/4}} = \dfrac{2x}{3z}$.

18. $\left(2x^3\right)\left(-3x^{-2}\right)\left(\frac{1}{6}x^{-1/2}\right) = -x^{1/2}$.

19. $\left(\dfrac{3xy^2}{4x^3y}\right)^{-2}\left(\dfrac{3xy^3}{2x^2}\right)^3 = \left(\dfrac{3y}{4x^2}\right)^{-2}\left(\dfrac{3y^3}{2x}\right)^3 = \left(\dfrac{4x^2}{3y}\right)^2\left(\dfrac{3y^3}{2x}\right)^3 = \dfrac{\left(16x^4\right)\left(27y^9\right)}{\left(9y^2\right)\left(8x^3\right)} = 6xy^7$.

20. $\sqrt[3]{81x^5y^{10}}\sqrt[3]{9xy^2} = \sqrt[3]{\left(3^4x^5y^{10}\right)\left(3^2xy^2\right)} = \left(3^6x^6y^{12}\right)^{1/3} = 3^2x^2y^4 = 9x^2y^4$.

21. $-2\pi^2r^3 + 100\pi r^2 = -2\pi r^2(\pi r - 50)$.

22. $2v^3w + 2vw^3 + 2u^2vw = 2vw\left(v^2 + w^2 + u^2\right)$.

23. $16 - x^2 = 4^2 - x^2 = (4 - x)(4 + x)$.

24. $12t^3 - 6t^2 - 18t = 6t\left(2t^2 - t - 3\right) = 6t(2t - 3)(t + 1)$.

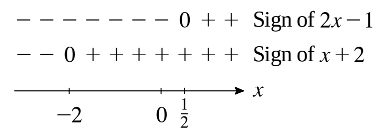
25. $8x^2 + 2x - 3 = (4x + 3)(2x - 1) = 0$, so $x = -\frac{3}{4}$ and $x = \frac{1}{2}$ are the roots of the equation.

26. $-6x^2 - 10x + 4 = 0$, $3x^2 + 5x - 2 = (3x - 1)(x + 2) = 0$, and so $x = -2$ or $x = \frac{1}{3}$.

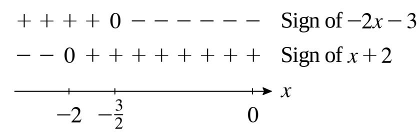
27. $-x^3 - 2x^2 + 3x = -x\left(x^2 + 2x - 3\right) = -x\,(x+3)\,(x-1) = 0$, and so the roots of the equation are $x = 0$, $x = -3$, and $x = 1$.

28. $2x^4 + x^2 = 1$. If we let $y = x^2$, we can write the equation as $2y^2 + y - 1 = (2y-1)\,(y+1) = 0$, giving $y = \frac{1}{2}$ or $y = -1$. We reject the second root since $y = x^2$ must be nonnegative. Therefore, $x^2 = \frac{1}{2}$, and so $x = \pm\frac{1}{\sqrt{2}} = \pm\frac{\sqrt{2}}{2}$.

29. Factoring the given expression, we have $(2x-1)\,(x+2) \le 0$. From the sign diagram, we conclude that the given inequality is satisfied when $-2 \le x \le \frac{1}{2}$.



30. $\dfrac{1}{x+2} > 2$ gives $\dfrac{1}{x+2} - 2 > 0$, $\dfrac{1-2x-4}{x+2} > 0$, and finally $\dfrac{-2x-3}{x+2} > 0$. From the sign diagram, we see that the given inequality is satisfied when $-2 < x < -\frac{3}{2}$.



31. The given inequality is equivalent to $|2x-3| < 5$ or $-5 < 2x - 3 < 5$. Thus, $-2 < 2x < 8$, or $-1 < x < 4$.

32. The given equation implies that either $\dfrac{x+1}{x-1} = 5$ or $\dfrac{x+1}{x-1} = -5$. Solving the first equality, we have $x + 1 = 5\,(x-1) = 5x - 5$, $-4x = -6$, and $x = \frac{3}{2}$. Similarly, we solve the second equality and obtain $x + 1 = -5\,(x-1) = -5x + 5$, $6x = 4$, and $x = \frac{2}{3}$. Thus, the two values of x that satisfy the equation are $x = \frac{3}{2}$ and $x = \frac{2}{3}$.

33. We use the quadratic formula to solve the equation $x^2 - 2x - 5 = 0$. Here $a = 1$, $b = -2$, and $c = -5$, so

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-2) \pm \sqrt{(-2)^2 - 4\,(1)\,(-5)}}{2\,(1)} = \frac{2 \pm \sqrt{24}}{2} = 1 \pm \sqrt{6}.$$

34. We use the quadratic formula to solve the equation $2x^2 + 8x + 7 = 0$. Here $a = 2$, $b = 8$, and $c = 7$, so

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-8 \pm \sqrt{8^2 - 4\,(2)\,(7)}}{4} = \frac{-8 \pm 2\sqrt{2}}{4} = -2 \pm \tfrac{1}{2}\sqrt{2}.$$

35. $$\frac{(t+6)\,(60) - (60t + 180)}{(t+6)^2} = \frac{60t + 360 - 60t - 180}{(t+6)^2} = \frac{180}{(t+6)^2}.$$

36. $$\frac{6x}{2\left(3x^2+2\right)} + \frac{1}{4\,(x+2)} = \frac{(6x)\,(2)\,(x+2) + \left(3x^2+2\right)}{4\left(3x^2+2\right)(x+2)} = \frac{12x^2 + 24x + 3x^2 + 2}{4\left(3x^2+2\right)(x+2)} = \frac{15x^2 + 24x + 2}{4\left(3x^2+2\right)(x+2)}.$$

37. $$\frac{2}{3}\left(\frac{4x}{2x^2-1}\right) + 3\left(\frac{3}{3x-1}\right) = \frac{8x}{3\left(2x^2-1\right)} + \frac{9}{3x-1} = \frac{8x\,(3x-1) + 27\left(2x^2-1\right)}{3\left(2x^2-1\right)(3x-1)} = \frac{78x^2 - 8x - 27}{3\left(2x^2-1\right)(3x-1)}.$$

38. $$\frac{-2x}{\sqrt{x+1}} + 4\sqrt{x+1} = \frac{-2x + 4\,(x+1)}{\sqrt{x+1}} = \frac{2\,(x+2)}{\sqrt{x+1}}.$$

39. $$\frac{\sqrt{x}-1}{x-1} = \frac{\sqrt{x}-1}{x-1} \cdot \frac{\sqrt{x}+1}{\sqrt{x}+1} = \frac{\left(\sqrt{x}\right)^2 - 1}{(x-1)\left(\sqrt{x}+1\right)} = \frac{x-1}{(x-1)\left(\sqrt{x}+1\right)} = \frac{1}{\sqrt{x}+1}.$$

40. $\dfrac{\sqrt{x}-1}{2\sqrt{x}}=\dfrac{\sqrt{x}-1}{2\sqrt{x}}\cdot\dfrac{\sqrt{x}}{\sqrt{x}}=\dfrac{x-\sqrt{x}}{2x}$.

41. The distance is $d=\sqrt{[1-(-2)]^2+[-7-(-3)]^2}=\sqrt{3^2+(-4)^2}=\sqrt{9+16}=\sqrt{25}=5$.

42. The distance is $d=\sqrt{(6-9)^2+(2-6)^2}=\sqrt{9+16}=\sqrt{25}=5$.

43. The distance is $d=\sqrt{\left(-\frac{1}{2}-\frac{1}{2}\right)^2+\left(2\sqrt{3}-\sqrt{3}\right)^2}=\sqrt{1+3}=\sqrt{4}=2$.

44. An equation is $x=-2$.

45. An equation is $y=4$.

46. The slope of L is $m=\dfrac{\frac{7}{2}-4}{3-(-2)}=-\dfrac{1}{10}$, and an equation of L is $y-4=-\frac{1}{10}[x-(-2)]=-\frac{1}{10}x-\frac{1}{5}$, or $y=-\frac{1}{10}x+\frac{19}{5}$. The general form of this equation is $x+10y-38=0$.

47. The line passes through the points $(-2,4)$ and $(3,0)$, so its slope is $m=\dfrac{4-0}{-2-3}=-\dfrac{4}{5}$. An equation is $y-0=-\frac{4}{5}(x-3)$, or $y=-\frac{4}{5}x+\frac{12}{5}$.

48. Writing the given equation in the form $y=\frac{5}{2}x-3$, we see that the slope of the given line is $\frac{5}{2}$. Thus, an equation is $y-4=\frac{5}{2}(x+2)$, or $y=\frac{5}{2}x+9$. The general form of this equation is $5x-2y+18=0$.

49. Writing the given equation in the form $y=-\frac{4}{3}x+2$, we see that the slope of the given line is $-\frac{4}{3}$. Therefore, the slope of the required line is $\frac{3}{4}$ and an equation of the line is $y-4=\frac{3}{4}(x+2)$ or $y=\frac{3}{4}x+\frac{11}{2}$.

50. Rewriting the given equation in slope-intercept form, we have $4y=-3x+8$ or $y=-\frac{3}{4}x+2$. We conclude that the slope of the required line is $-\frac{3}{4}$. Using the point-slope form of the equation of a line with the point $(2,3)$ and slope $-\frac{3}{4}$, we obtain $y-3=-\frac{3}{4}(x-2)$, and so $y=-\frac{3}{4}x+\frac{6}{4}+3=-\frac{3}{4}x+\frac{9}{2}$. The general form of this equation is $3x+4y-18=0$.

51. The slope of the line joining the points $(-3,4)$ and $(2,1)$ is $m=\dfrac{1-4}{2-(-3)}=-\dfrac{3}{5}$. Using the point-slope form of the equation of a line with the point $(-1,3)$ and slope $-\frac{3}{5}$, we have $y-3=-\frac{3}{5}[x-(-1)]$. Therefore, $y=-\frac{3}{5}(x+1)+3=-\frac{3}{5}x+\frac{12}{5}$.

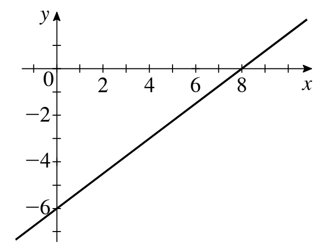
52. The slope of the line passing through $(-2,-4)$ and $(1,5)$ is $m=\dfrac{5-(-4)}{1-(-2)}=\dfrac{9}{3}=3$, so the required line is $y-(-2)=3[x-(-3)]$. Simplifying, this is equivalent to $y+2=3x+9$, or $y=3x+7$.

53. Rewriting the given equation in the slope-intercept form $y=\frac{2}{3}x-8$, we see that the slope of the line with this equation is $\frac{2}{3}$. The slope of a line perpendicular to this line is thus $-\frac{3}{2}$. Using the point-slope form of the equation of a line with the point $(-2,-4)$ and slope $-\frac{3}{2}$, we have $y-(-4)=-\frac{3}{2}[x-(-2)]$ or $y=-\frac{3}{2}x-7$. The general form of this equation is $3x+2y+14=0$.

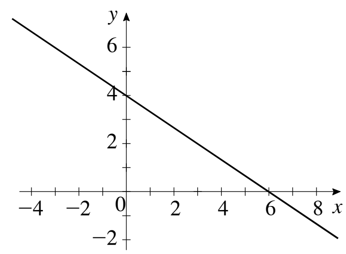
54. Substituting $x = -1$ and $y = -\frac{5}{4}$ into the left-hand side of the equation gives $6(-1) - 8\left(-\frac{5}{4}\right) - 16 = -12$. The equation is not satisfied, and so we conclude that the point $\left(-1, -\frac{5}{4}\right)$ does not lie on the line $6x - 8y - 16 = 0$.

55. Substituting $x = 2$ and $y = -4$ into the equation, we obtain $2(2) + k(-4) = -8$, so $-4k = -12$ and $k = 3$.

56. Setting $x = 0$ gives $y = -6$ as the y-intercept. Setting $y = 0$ gives $x = 8$ as the x-intercept. The graph of $3x - 4y = 24$ is shown.



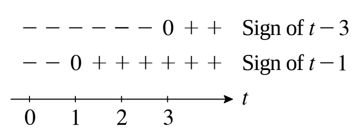
57. Using the point-slope form of an equation of a line, we have $y - 2 = -\frac{2}{3}(x - 3)$ or $y = -\frac{2}{3}x + 4$. If $y = 0$, then $x = 6$, and if $x = 0$, then $y = 4$. A sketch of the line is shown.



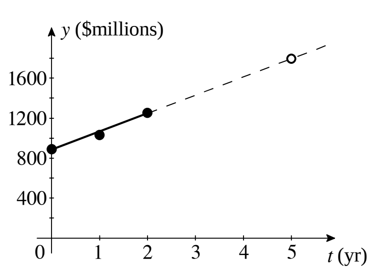
58. Simplifying $2(1.5C + 80) \le 2(2.5C - 20)$, we obtain $1.5C + 80 \le 2.5C - 20$, so $C \ge 100$ and the minimum cost is \$100.

59. $3(2R - 320) \le 3R + 240$ gives $6R - 960 \le 3R + 240$, $3R \le 1200$ and finally $R \le 400$. We conclude that the maximum revenue is \$400.

60. We solve the inequality $-16t^2 + 64t + 80 \ge 128$, obtaining $-16t^2 + 64t - 48 \ge 0$, $t^2 - 4t + 3 \le 0$, and $(t - 3)(t - 1) \le 0$. From the sign diagram, we see that the required solution is $[1, 3]$. Thus, the stone is 128 ft or higher off the ground between 1 and 3 seconds after it was thrown.



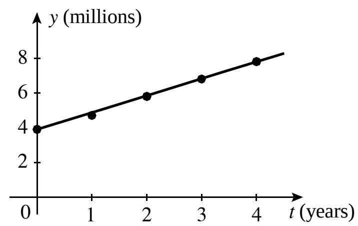
61. a, b.



c. The slope of L is $\dfrac{1251 - 887}{2 - 0} = 182$, so an equation of L is $y - 887 = 182(t - 0)$ or $y = 182t + 887$.

d. The amount consumers are projected to spend on Cyber Monday, 2016 ($t = 7$) is $182(7) + 887$, or \$2.161 billion.

62. a, b.



c. $P_1\,(0, 3.9)$ and $P_2\,(4, 7.8)$, so $m = \dfrac{7.8 - 3.9}{4 - 0} = \dfrac{3.9}{4} = 0.975$. Thus, $y - 3.9 = 0.975\,(t - 0)$, or $y = 0.975t + 3.9$.

d. If $t = 3$, then $y = 0.975\,(3) + 3.9 = 6.825$. Thus, the number of systems installed in 2005 (when $t = 3$) is 6,825,000, which is close to the projected value of 6.8 million.

CHAPTER 1 Before Moving On... page 50

1. a. $\left|\pi - 2\sqrt{3}\right| - \left|\sqrt{3} - \sqrt{2}\right| = -\left(\pi - 2\sqrt{3}\right) - \left(\sqrt{3} - \sqrt{2}\right) = \sqrt{3} + \sqrt{2} - \pi$.

b. $\left[\left(-\frac{1}{3}\right)^{-3}\right]^{1/3} = \left(-\frac{1}{3}\right)^{(-3)\left(\frac{1}{3}\right)} = \left(-\frac{1}{3}\right)^{-1} = -3$.

2. a. $\sqrt[3]{64x^6} \cdot \sqrt{9y^2x^6} = \left(4x^2\right)\left(3yx^3\right) = 12x^5y$.

b. $\left(\dfrac{a^{-3}}{b^{-4}}\right)^2 \left(\dfrac{b}{a}\right)^{-3} = \dfrac{a^{-6}}{b^{-8}} \cdot \dfrac{b^{-3}}{a^{-3}} = \dfrac{b^8}{a^6} \cdot \dfrac{a^3}{b^3} = \dfrac{b^5}{a^3}$.

3. a. $\dfrac{2x}{3\sqrt{y}} \cdot \dfrac{\sqrt{y}}{\sqrt{y}} = \dfrac{2x\sqrt{y}}{3y}$.

b. $\dfrac{x}{\sqrt{x} - 4} \cdot \dfrac{\sqrt{x} + 4}{\sqrt{x} + 4} = \dfrac{x\left(\sqrt{x} + 4\right)}{x - 16}$.

4. a. $\dfrac{\left(x^2 + 1\right)\left(\frac{1}{2}x^{-1/2}\right) - x^{1/2}\,(2x)}{\left(x^2 + 1\right)^2} = \dfrac{\frac{1}{2}x^{-1/2}\left[\left(x^2 + 1\right) - 4x^2\right]}{\left(x^2 + 1\right)^2} = \dfrac{1 - 3x^2}{2x^{1/2}\left(x^2 + 1\right)^2}$.

b. $-\dfrac{3x}{\sqrt{x + 2}} + 3\sqrt{x + 2} = \dfrac{-3x + 3\,(x + 2)}{\sqrt{x + 2}} = \dfrac{6}{\sqrt{x + 2}} = \dfrac{6\sqrt{x + 2}}{x + 2}$.

5. $\dfrac{\sqrt{x} + \sqrt{y}}{\sqrt{x} - \sqrt{y}} = \dfrac{\sqrt{x} + \sqrt{y}}{\sqrt{x} - \sqrt{y}} \cdot \dfrac{\sqrt{x} - \sqrt{y}}{\sqrt{x} - \sqrt{y}} = \dfrac{x - y}{\left(\sqrt{x} - \sqrt{y}\right)^2}$.

6. a. $12x^3 - 10x^2 - 12x = 2x\left(6x^2 - 5x - 6\right) = 2x\,(2x - 3)\,(3x + 2)$.

b. $2bx - 2by + 3cx - 3cy = 2b\,(x - y) + 3c\,(x - y) = (2b + 3c)\,(x - y)$

7. a. $12x^2 - 9x - 3 = 0$, so $3\left(4x^2 - 3x - 1\right) = 0$ and $3\,(4x + 1)\,(x - 1) = 0$. Thus, $x = -\frac{1}{4}$ or $x = 1$.

b. $3x^2 - 5x + 1 = 0$. Using the quadratic formula with $a = 3$, $b = -5$, and $c = 1$, we have $x = \dfrac{-(-5) \pm \sqrt{25 - 12}}{2\,(3)} = \dfrac{5 \pm \sqrt{13}}{6}$.

8. $d = \sqrt{[6 - (-2)]^2 + (8 - 4)^2} = \sqrt{64 + 16} = \sqrt{80} = 4\sqrt{5}$.

9. $m = \dfrac{5 - (-2)}{4 - (-1)} = \dfrac{7}{5}$, so $y - (-2) = \frac{7}{5}\,[x - (-1)]$, $y + 2 = \frac{7}{5}x + \frac{7}{5}$, or $y = \frac{7}{5}x - \frac{3}{5}$.

10. $m = -\frac{1}{3}$ and $b = \frac{4}{3}$, so an equation is $y = -\frac{1}{3}x + \frac{4}{3}$.

CHAPTER 1 Explore & Discuss

Page 27

1. Let $P_1 = (2, 6)$ and $P_2 = (-4, 3)$. Then we have $x_1 = 2$, $y_1 = 6$, $x_2 = -4$, and $y_2 = 3$. Using Formula (1), we have $d = \sqrt{(-4-2)^2 + (3-6)^2} = \sqrt{36+9} = \sqrt{45} = 3\sqrt{5}$, as obtained in Example 1.

2. Let $P_1\,(x_1, y_1)$ and $P_2\,(x_2, y_2)$ be any two points in the plane. Then the result follows from the equality $\sqrt{(x_2-x_1)^2 + (y_2-y_1)^2} = \sqrt{(x_1-x_2)^2 + (y_1-y_2)^2}$.

Page 28

1. **a.** All points on and inside the circle with center (h, k) and radius r.

b. All points inside the circle with center (h, k) and radius r.

c. All points on and outside the circle with center (h, k) and radius r.

d. All points outside the circle with center (h, k) and radius r.

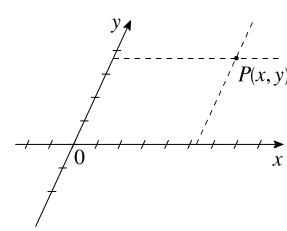
2. **a.** $y^2 = 4 - x^2$, and so $y = \pm\sqrt{4 - x^2}$.

b. **(i)** The upper semicircle with center at the origin and radius 2.

(ii) The lower semicircle with center at the origin and radius 2.

Page 29

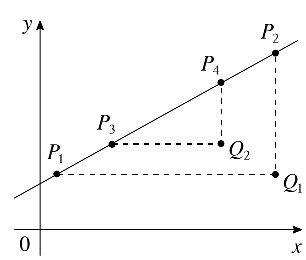
1. Let $P\,(x, y)$ be any point in the plane. Draw a line through P parallel to the y-axis and a line through P parallel to the x-axis (see the figure). The x-coordinate of P is the number corresponding to the point on the x-axis at which the line through P crosses the x-axis. Similarly, y is the number that corresponds to the point on the y-axis at which the line parallel to the x-axis crosses the y-axis. To show the converse, reverse the process.



2. You can use the Pythagorean Theorem in the Cartesian coordinate system. This greatly simplifies the computations.

Page 35

1. Refer to the accompanying figure. Observe that triangles $\Delta P_1Q_1P_2$ and $\Delta P_3Q_2P_4$ are similar. From this we conclude that $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{y_4 - y_3}{x_4 - x_3}$. Because P_3 and P_4 are arbitrary, the conclusion follows.



Page 39

1. We obtain a family of parallel lines each having slope m.

2. We obtain a family of straight lines all of which pass through the point $(0, b)$.

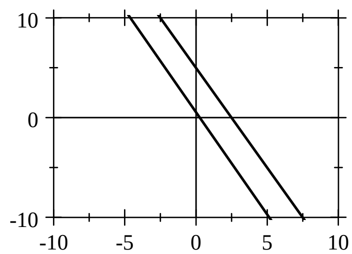
Page 40

1. In Example 11, we are told that the object is expected to appreciate in value at a given rate for the next five years, and the equation obtained in that example is based on this fact. Thus, the equation may not be used to predict the value of the object very much beyond five years from the date of purchase.

CHAPTER 1 Exploring with Technology

Page 38

1.

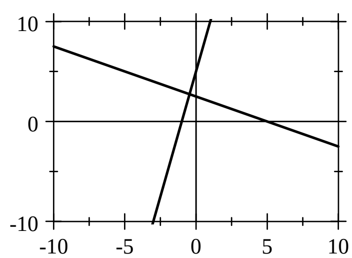


The straight lines L_1 and L_2 are shown in the figure.

a. L_1 and L_2 seem to be parallel.

b. Writing each equation in the slope-intercept form gives $y = -2x + 5$ and $y = -\frac{41}{20}x + \frac{11}{20}$, from which we see that the slopes of L_1 and L_2 are -2 and $-\frac{41}{20} = -2.05$, respectively. This shows that L_1 and L_2 are not parallel.

2.



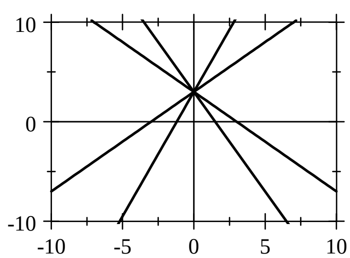
The straight lines L_1 and L_2 are shown in the figure.

a. L_1 and L_2 seem to be perpendicular.

b. The slopes of L_1 and L_2 are $m_1 = -\frac{1}{2}$ and $m_2 = 5$, respectively. Because $m_1 = -\dfrac{1}{2} \neq -\dfrac{1}{5} = -\dfrac{1}{m_2}$, we see that L_1 and L_2 are not perpendicular.

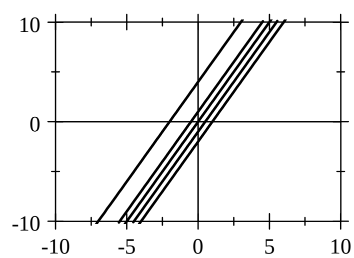
Page 39

1.



The straight lines with the given equations are shown in the figure. Changing the value of m in the equation $y = mx + b$ changes the slope of the line and thus rotates it.

2.



The straight lines of interest are shown in the figure. Changing the value of b in the equation $y = mx + b$ changes the y-intercept of the line and thus translates it (upward if $b > 0$ and downward if $b < 0$).

3. Changing both m and b in the equation $y = mx + b$ both rotates and translates the line.

2 FUNCTIONS, LIMITS, AND THE DERIVATIVE

2.1 Functions and Their Graphs

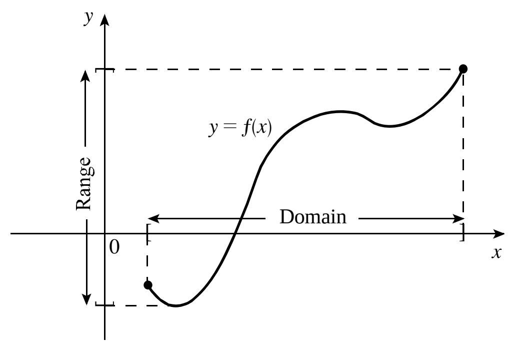
Concept Questions page 59

1. a. A function is a rule that associates with each element in a set A exactly one element in a set B.

b. The domain of a function f is the set of all elements x in the set such that $f(x)$ is an element in B. The range of f is the set of all elements $f(x)$ whenever x is an element in its domain.

c. An independent variable is a variable in the domain of a function f. The dependent variable is $y = f(x)$.

2. a. The graph of a function f is the set of all ordered pairs (x, y) such that $y = f(x)$, x being an element in the domain of f.



b. Use the vertical line test to determine if every vertical line intersects the curve in at most one point. If so, then the curve is the graph of a function.

3. a. Yes, every vertical line intersects the curve in at most one point.

b. No, a vertical line intersects the curve at more than one point.

c. No, a vertical line intersects the curve at more than one point.

d. Yes, every vertical line intersects the curve in at most one point.

4. The domain is $[1, 3) \cup [3, 5)$ and the range is $\left[\frac{1}{2}, 2\right) \cup (2, 4]$.

Exercises page 59

1. $f(x) = 5x + 6$. Therefore $f(3) = 5(3) + 6 = 21$, $f(-3) = 5(-3) + 6 = -9$, $f(a) = 5(a) + 6 = 5a + 6$, $f(-a) = 5(-a) + 6 = -5a + 6$, and $f(a + 3) = 5(a + 3) + 6 = 5a + 15 + 6 = 5a + 21$.

2. $f(x) = 4x - 3$. Therefore, $f(4) = 4(4) - 3 = 16 - 3 = 13$, $f\left(\frac{1}{4}\right) = 4\left(\frac{1}{4}\right) - 3 = 1 - 3 = -2$, $f(0) = 4(0) - 3 = -3$, $f(a) = 4(a) - 3 = 4a - 3$, $f(a + 1) = 4(a + 1) - 3 = 4a + 1$.

3. $g(x) = 3x^2 - 6x - 3$, so $g(0) = 3(0) - 6(0) - 3 = -3$, $g(-1) = 3(-1)^2 - 6(-1) - 3 = 3 + 6 - 3 = 6$, $g(a) = 3(a)^2 - 6(a) - 3 = 3a^2 - 6a - 3$, $g(-a) = 3(-a)^2 - 6(-a) - 3 = 3a^2 + 6a - 3$, and $g(x+1) = 3(x+1)^2 - 6(x+1) - 3 = 3\left(x^2 + 2x + 1\right) - 6x - 6 - 3 = 3x^2 + 6x + 3 - 6x - 9 = 3x^2 - 6$.

4. $h(x) = x^3 - x^2 + x + 1$, so $h(-5) = (-5)^3 - (-5)^2 + (-5) + 1 = -125 - 25 - 5 + 1 = -154$, $h(0) = (0)^3 - (0)^2 + 0 + 1 = 1$, $h(a) = a^3 - (a)^2 + a + 1 = a^3 - a^2 + a + 1$, and $h(-a) = (-a)^3 - (-a)^2 + (-a) + 1 = -a^3 - a^2 - a + 1$.

5. $f(x) = 2x + 5$, so $f(a+h) = 2(a+h) + 5 = 2a + 2h + 5$, $f(-a) = 2(-a) + 5 = -2a + 5$, $f\left(a^2\right) = 2\left(a^2\right) + 5 = 2a^2 + 5$, $f(a - 2h) = 2(a - 2h) + 5 = 2a - 4h + 5$, and $f(2a - h) = 2(2a - h) + 5 = 4a - 2h + 5$

6. $g(x) = -x^2 + 2x$, $g(a+h) = -(a+h)^2 + 2(a+h) = -a^2 - 2ah - h^2 + 2a + 2h$, $g(-a) = -(-a)^2 + 2(-a) = -a^2 - 2a = -a(a+2)$, $g\left(\sqrt{a}\right) = -\left(\sqrt{a}\right)^2 + 2\left(\sqrt{a}\right) = -a + 2\sqrt{a}$, $a + g(a) = a - a^2 + 2a = -a^2 + 3a = -a(a-3)$, and $\dfrac{1}{g(a)} = \dfrac{1}{-a^2 + 2a} = -\dfrac{1}{a(a-2)}$.

7. $s(t) = \dfrac{2t}{t^2 - 1}$. Therefore, $s(4) = \dfrac{2(4)}{(4)^2 - 1} = \dfrac{8}{15}$, $s(0) = \dfrac{2(0)}{0^2 - 1} = 0$, $s(a) = \dfrac{2(a)}{a^2 - 1} = \dfrac{2a}{a^2 - 1}$; $s(2+a) = \dfrac{2(2+a)}{(2+a)^2 - 1} = \dfrac{2(2+a)}{a^2 + 4a + 4 - 1} = \dfrac{2(2+a)}{a^2 + 4a + 3}$, and $s(t+1) = \dfrac{2(t+1)}{(t+1)^2 - 1} = \dfrac{2(t+1)}{t^2 + 2t + 1 - 1} = \dfrac{2(t+1)}{t(t+2)}$.

8. $g(u) = (3u - 2)^{3/2}$. Therefore, $g(1) = [3(1) - 2]^{3/2} = (1)^{3/2} = 1$, $g(6) = [3(6) - 2]^{3/2} = 16^{3/2} = 4^3 = 64$, $g\left(\frac{11}{3}\right) = \left[3\left(\frac{11}{3}\right) - 2\right]^{3/2} = (9)^{3/2} = 27$, and $g(u+1) = [3(u+1) - 2]^{3/2} = (3u+1)^{3/2}$.

9. $f(t) = \dfrac{2t^2}{\sqrt{t-1}}$. Therefore, $f(2) = \dfrac{2\left(2^2\right)}{\sqrt{2-1}} = 8$, $f(a) = \dfrac{2a^2}{\sqrt{a-1}}$, $f(x+1) = \dfrac{2(x+1)^2}{\sqrt{(x+1)-1}} = \dfrac{2(x+1)^2}{\sqrt{x}}$, and $f(x-1) = \dfrac{2(x-1)^2}{\sqrt{(x-1)-1}} = \dfrac{2(x-1)^2}{\sqrt{x-2}}$.

10. $f(x) = 2 + 2\sqrt{5-x}$. Therefore, $f(-4) = 2 + 2\sqrt{5-(-4)} = 2 + 2\sqrt{9} = 2 + 2(3) = 8$, $f(1) = 2 + 2\sqrt{5-1} = 2 + 2\sqrt{4} = 2 + 4 = 6$, $f\left(\frac{11}{4}\right) = 2 + 2\left(5 - \frac{11}{4}\right)^{1/2} = 2 + 2\left(\frac{9}{4}\right)^{1/2} = 2 + 2\left(\frac{3}{2}\right) = 5$, and $f(x+5) = 2 + 2\sqrt{5-(x+5)} = 2 + 2\sqrt{-x}$.

11. Because $x = -2 \leq 0$, we calculate $f(-2) = (-2)^2 + 1 = 4 + 1 = 5$. Because $x = 0 \leq 0$, we calculate $f(0) = (0)^2 + 1 = 1$. Because $x = 1 > 0$, we calculate $f(1) = \sqrt{1} = 1$.

12. Because $x = -2 < 2$, $g(-2) = -\frac{1}{2}(-2) + 1 = 1 + 1 = 2$. Because $x = 0 < 2$, $g(0) = -\frac{1}{2}(0) + 1 = 0 + 1 = 1$. Because $x = 2 \geq 2$, $g(2) = \sqrt{2-2} = 0$. Because $x = 4 \geq 2$, $g(4) = \sqrt{4-2} = \sqrt{2}$.

13. Because $x = -1 < 1$, $f(-1) = -\frac{1}{2}(-1)^2 + 3 = \frac{5}{2}$. Because $x = 0 < 1$, $f(0) = -\frac{1}{2}(0)^2 + 3 = 3$. Because $x = 1 \geq 1$, $f(1) = 2\left(1^2\right) + 1 = 3$. Because $x = 2 \geq 1$, $f(2) = 2\left(2^2\right) + 1 = 9$.

14. Because $x = 0 \le 1$, $f(0) = 2 + \sqrt{1-0} = 2 + 1 = 3$. Because $x = 1 \le 1$, $f(1) = 2 + \sqrt{1-1} = 2 + 0 = 2$. Because $x = 2 > 1$, $f(2) = \frac{1}{1-2} = \frac{1}{-1} = -1$.

15. **a.** $f(0) = -2$.

b. **(i)** $f(x) = 3$ when $x \approx 2$. **(ii)** $f(x) = 0$ when $x = 1$.

c. $[0, 6]$

d. $[-2, 6]$

16. **a.** $f(7) = 3$. **b.** $x = 4$ and $x = 6$. **c.** $x = 2$; 0. **d.** $[-1, 9]$; $[-2, 6]$.

17. $g(2) = \sqrt{2^2 - 1} = \sqrt{3}$, so the point $\left(2, \sqrt{3}\right)$ lies on the graph of g.

18. $f(3) = \frac{3+1}{\sqrt{3^2+7}} + 2 = \frac{4}{\sqrt{16}} + 2 = \frac{4}{4} + 2 = 3$, so the point $(3, 3)$ lies on the graph of f.

19. $f(-2) = \frac{|-2-1|}{-2+1} = \frac{|-3|}{-1} = -3$, so the point $(-2, -3)$ does lie on the graph of f.

20. $h(-3) = \frac{|-3+1|}{(-3)^3+1} = \frac{2}{-27+1} = -\frac{2}{26} = -\frac{1}{13}$, so the point $\left(-3, -\frac{1}{13}\right)$ does lie on the graph of h.

21. Because the point $(1, 5)$ lies on the graph of f it satisfies the equation defining f. Thus, $f(1) = 2(1)^2 - 4(1) + c = 5$, or $c = 7$.

22. Because the point $(2, 4)$ lies on the graph of f it satisfies the equation defining f. Thus, $f(2) = 2\sqrt{9-(2)^2} + c = 4$, or $c = 4 - 2\sqrt{5}$.

23. Because $f(x)$ is a real number for any value of x, the domain of f is $(-\infty, \infty)$.

24. Because $f(x)$ is a real number for any value of x, the domain of f is $(-\infty, \infty)$.

25. $f(x)$ is not defined at $x = 0$ and so the domain of f is $(-\infty, 0) \cup (0, \infty)$.

26. $g(x)$ is not defined at $x = 1$ and so the domain of g is $(-\infty, 1) \cup (1, \infty)$.

27. $f(x)$ is a real number for all values of x. Note that $x^2 + 1 \ge 1$ for all x. Therefore, the domain of f is $(-\infty, \infty)$.

28. Because the square root of a number is defined for all real numbers greater than or equal to zero, we have $x - 5 \ge 0$ or $x \ge 5$, and the domain is $[5, \infty)$.

29. Because the square root of a number is defined for all real numbers greater than or equal to zero, we have $5 - x \ge 0$, or $-x \ge -5$ and so $x \le 5$. (Recall that multiplying by -1 reverses the sign of an inequality.) Therefore, the domain of f is $(-\infty, 5]$.

30. Because $2x^2 + 3$ is always greater than zero, the domain of g is $(-\infty, \infty)$.

31. The denominator of f is zero when $x^2 - 1 = 0$, or $x = \pm 1$. Therefore, the domain of f is $(-\infty, -1) \cup (-1, 1) \cup (1, \infty)$.

32. The denominator of f is equal to zero when $x^2 + x - 2 = (x+2)(x-1) = 0$; that is, when $x = -2$ or $x = 1$. Therefore, the domain of f is $(-\infty, -2) \cup (-2, 1) \cup (1, \infty)$.

33. f is defined when $x + 3 \geq 0$, that is, when $x \geq -3$. Therefore, the domain of f is $[-3, \infty)$.

34. g is defined when $x - 1 \geq 0$; that is when $x \geq 1$. Therefore, the domain of f is $[1, \infty)$.

35. The numerator is defined when $1 - x \geq 0$, $-x \geq -1$ or $x \leq 1$. Furthermore, the denominator is zero when $x = \pm 2$. Therefore, the domain is the set of all real numbers in $(-\infty, -2) \cup (-2, 1]$.

36. The numerator is defined when $x - 1 \geq 0$, or $x \geq 1$, and the denominator is zero when $x = -2$ and when $x = 3$. So the domain is $[1, 3) \cup (3, \infty)$.

37. a. The domain of f is the set of all real numbers.

b. $f(x) = x^2 - x - 6$, so

$f(-3) = (-3)^2 - (-3) - 6 = 9 + 3 - 6 = 6$,

$f(-2) = (-2)^2 - (-2) - 6 = 4 + 2 - 6 = 0$,

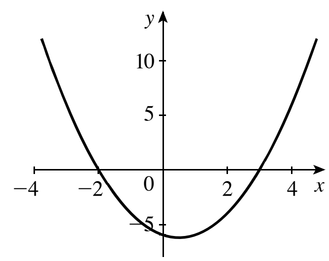
$f(-1) = (-1)^2 - (-1) - 6 = 1 + 1 - 6 = -4$,

$f(0) = (0)^2 - (0) - 6 = -6$,

$f\left(\frac{1}{2}\right) = \left(\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right) - 6 = \frac{1}{4} - \frac{2}{4} - \frac{24}{4} = -\frac{25}{4}$, $f(1) = (1)^2 - 1 - 6 = -6$,

$f(2) = (2)^2 - 2 - 6 = 4 - 2 - 6 = -4$, and $f(3) = (3)^2 - 3 - 6 = 9 - 3 - 6 = 0$.

c.



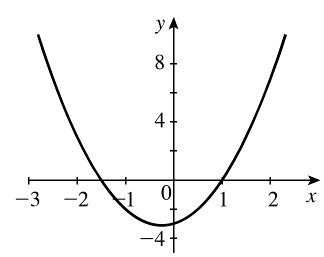
38. $f(x) = 2x^2 + x - 3$.

a. Because $f(x)$ is a real number for all values of x, the domain of f is $(-\infty, \infty)$.

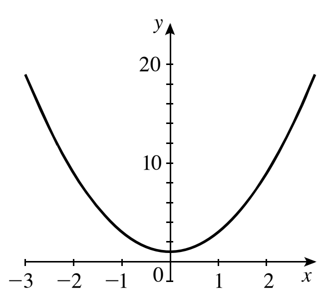
b.

x	-3	-2	-1	$-\frac{1}{2}$	0	1	2	3
y	12	3	-2	-3	-3	0	7	18

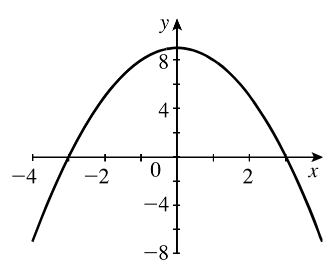
c.



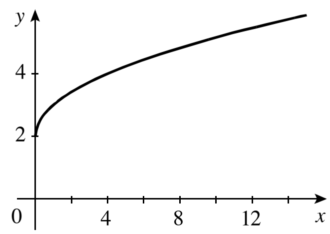
39. $f(x) = 2x^2 + 1$ has domain $(-\infty, \infty)$ and range $[1, \infty)$.



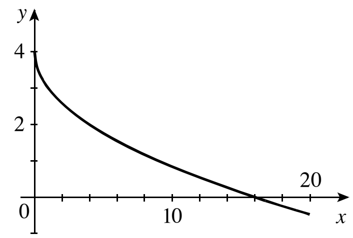
40. $f(x) = 9 - x^2$ has domain $(-\infty, \infty)$ and range $(-\infty, 9]$.



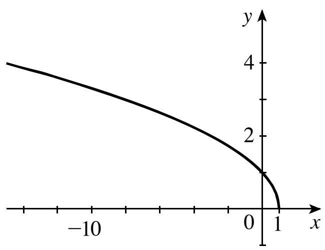
41. $f(x) = 2 + \sqrt{x}$ has domain $[0, \infty)$ and range $[2, \infty)$.



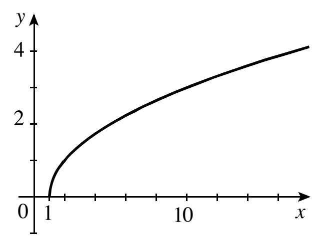
42. $g(x) = 4 - \sqrt{x}$ has domain $[0, \infty)$ and range $(-\infty, 4]$.



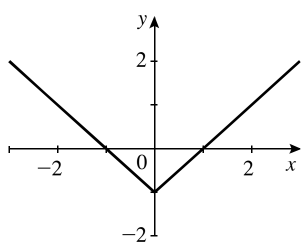
43. $f(x) = \sqrt{1 - x}$ has domain $(-\infty, 1]$ and range $[0, \infty)$



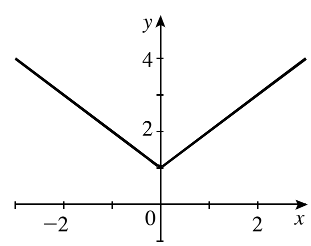
44. $f(x) = \sqrt{x - 1}$ has domain $(1, \infty)$ and range $[0, \infty)$.



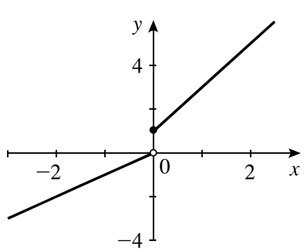
45. $f(x) = |x| - 1$ has domain $(-\infty, \infty)$ and range $[-1, \infty)$.



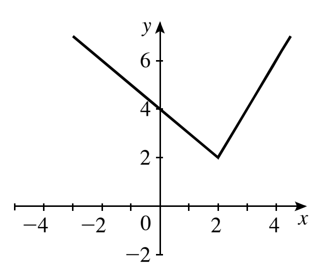
46. $f(x) = |x| + 1$ has domain $(-\infty, \infty)$ and range $[1, \infty)$



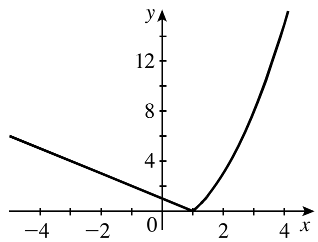
47. $f(x) = \begin{cases} x & \text{if } x < 0 \\ 2x + 1 & \text{if } x \geq 0 \end{cases}$ has domain $(-\infty, \infty)$ and range $(-\infty, 0) \cup [1, \infty)$.



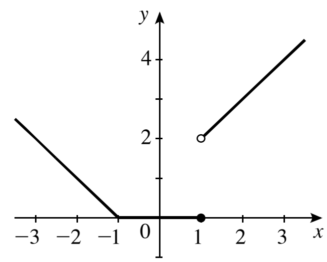
48. For $x < 2$, the graph of f is the half-line $y = 4 - x$. For $x \geq 2$, the graph of f is the half-line $y = 2x - 2$. f has domain $(-\infty, \infty)$ and range $[2, \infty)$.



49. If $x \leq 1$, the graph of f is the half-line $y = -x + 1$. For $x > 1$, we calculate a few points: $f(2) = 3$, $f(3) = 8$, and $f(4) = 15$. f has domain $(-\infty, \infty)$ and range $[0, \infty)$.



50. If $x < -1$ the graph of f is the half-line $y = -x - 1$. For $-1 \leq x \leq 1$, the graph consists of the line segment $y = 0$. For $x > 1$, the graph is the half-line $y = x + 1$. f has domain $(-\infty, \infty)$ and range $[0, \infty)$.



51. Each vertical line cuts the given graph at exactly one point, and so the graph represents y as a function of x.

52. Because the y-axis, which is a vertical line, intersects the graph at two points, the graph does not represent y as a function of x.

53. Because there is a vertical line that intersects the graph at three points, the graph does not represent y as a function of x.

54. Each vertical line intersects the graph of f at exactly one point, and so the graph represents y as a function of x.

55. Each vertical line intersects the graph of f at exactly one point, and so the graph represents y as a function of x.

56. The y-axis intersects the circle at *two* points, and this shows that the circle is not the graph of a function of x.

57. Each vertical line intersects the graph of f at exactly one point, and so the graph represents y as a function of x.

58. A vertical line containing a line segment comprising the graph cuts it at infinitely many points and so the graph does not define y as a function of x.

59. The circumference of a circle with a 5-inch radius is given by $C(5) = 2\pi(5) = 10\pi$, or 10π inches.

60. $V(2.1) = \frac{4}{3}\pi(2.1)^3 \approx 38.79$, $V(2) = \frac{4}{3}\pi(8) \approx 33.51$, and so $V(2.1) - V(2) = 38.79 - 33.51 = 5.28$ is the amount by which the volume of a sphere of radius 2.1 exceeds the volume of a sphere of radius 2.

61. $C(0) = 6$, or 6 billion dollars; $C(50) = 0.75(50) + 6 = 43.5$, or 43.5 billion dollars; and $C(100) = 0.75(100) + 6 = 81$, or 81 billion dollars.

62. The child should receive $D(4) = \frac{2}{25}(500)(4) = 160$, or 160 mg.

63. **a.** From $t = 0$ through $t = 5$, that is, from the beginning of 2001 until the end of 2005.

b. From $t = 5$ through $t = 9$, that is, from the beginning of 2006 until the end of 2010.

c. The average expenditures were the same at approximately $t = 5.2$, that is, in the year 2006. The level of expenditure on each service was approximately $900.

64. a. The slope of the straight line passing through $(0, 0.61)$ and $(10, 0.59)$ is $m_1 = \frac{0.59 - 0.61}{10 - 0} = -0.002$. Therefore, an equation of the straight line passing through the two points is $y - 0.61 = -0.002\,(t - 0)$ or $y = -0.002t + 0.61$. Next, the slope of the straight line passing through $(10, 0.59)$ and $(20, 0.60)$ is $m_2 = \frac{0.60 - 0.59}{20 - 10} = 0.001$, and so an equation of the straight line passing through the two points is $y - 0.59 = 0.001\,(t - 10)$ or $y = 0.001t + 0.58$. The slope of the straight line passing through $(20, 0.60)$ and $(30, 0.66)$ is $m_3 = \frac{0.66 - 0.60}{30 - 20} = 0.006$, and so an equation of the straight line passing through the two points is $y - 0.60 = 0.006\,(t - 20)$ or $y = 0.006t + 0.48$. The slope of the straight line passing through $(30, 0.66)$ and $(40.0, 0.78)$ is $m_4 = \frac{0.78 - 0.66}{40 - 30} = 0.012$, and so an equation of the straight line passing through the two points is $y = 0.012t + 0.30$. Therefore, a rule for f is $f(t) = \begin{cases} -0.002t + 0.61 & \text{if } 0 \le t \le 10 \\ 0.001t + 0.58 & \text{if } 10 < t \le 20 \\ 0.006t + 0.48 & \text{if } 20 < t \le 30 \\ 0.012t + 0.30 & \text{if } 30 < t \le 40 \end{cases}$

b. The gender gap was expanding between 1960 and 1970 and shrinking between 1970 and 2000.

c. The gender gap was expanding at the rate of 0.002/yr between 1960 and 1970, shrinking at the rate of 0.001/yr between 1970 and 1980, shrinking at the rate of 0.006/yr between 1980 and 1990, and shrinking at the rate of 0.012/yr between 1990 and 2000.

65. a. The slope of the straight line passing through the points $(0, 0.58)$ and $(20, 0.95)$ is $m_1 = \frac{0.95 - 0.58}{20 - 0} = 0.0185$, so an equation of the straight line passing through these two points is $y - 0.58 = 0.0185\,(t - 0)$ or $y = 0.0185t + 0.58$. Next, the slope of the straight line passing through the points $(20, 0.95)$ and $(30, 1.1)$ is $m_2 = \frac{1.1 - 0.95}{30 - 20} = 0.015$, so an equation of the straight line passing through the two points is $y - 0.95 = 0.015\,(t - 20)$ or $y = 0.015t + 0.65$. Therefore, a rule for f is

$$f(t) = \begin{cases} 0.0185t + 0.58 & \text{if } 0 \le t \le 20 \\ 0.015t + 0.65 & \text{if } 20 < t \le 30 \end{cases}$$

b. The ratios were changing at the rates of 0.0185/yr from 1960 through 1980 and 0.015/yr from 1980 through 1990.

c. The ratio was 1 when $t \approx 20.3$. This shows that the number of bachelor's degrees earned by women equaled the number earned by men for the first time around 1983.

66. a. $T(x) = 0.06x$

b. $T(200) = 0.06\,(200) = 12$, or \$12.00 and $T(5.65) = 0.06\,(5.65) = 0.34$, or \$0.34.

67. a. $I(x) = 1.053x$

b. $I(1520) = 1.053\,(1520) = 1600.56$, or \$1600.56.

68. a. The function is linear with y-intercept 1.44 and slope 0.058, so we have $f(t) = 0.058t + 1.44$, $0 \le t \le 9$.

b. The projected spending in 2018 will be $f(9) = 0.058\,(9) + 1.44 = 1.962$, or \$1.962 trillion.

69. $S(r) = 4\pi r^2$.

70. $\frac{4}{3}(\pi)(2r)^3 = \frac{4}{3}\pi 8r^3 = 8\left(\frac{4}{3}\pi r^3\right)$. Therefore, the volume of the tumor is increased by a factor of 8.

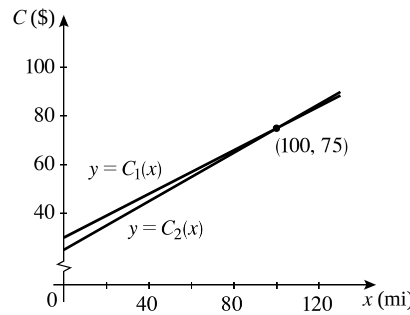
71. **a.** The median age was changing at the rate of 0.3 years/year.

b. The median age in 2011 was $M(11) = 0.3(11) + 37.9 = 41.2$ (years).

c. The median age in 2015 is projected to be $M(5) = 0.3(15) + 37.9 = 42.4$ (years).

72. **a.** The daily cost of leasing from Ace is $C_1(x) = 30 + 0.45x$, while the daily cost of leasing from Acme is $C_2(x) = 25 + 0.50x$, where x is the number of miles driven.

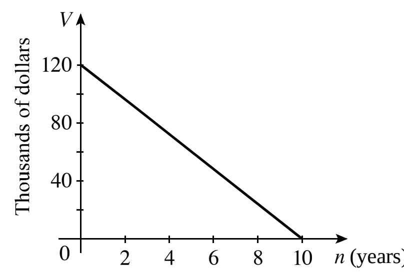
b.



c. The costs are the same when $C_1(x) = C_2(x)$, that is, when $30 + 0.45x = 25 + 0.50x$, $-0.05x = -5$, or $x = 100$. Because $C_1(70) = 30 + 0.45(70) = 61.5$ and $C_2(70) = 25 + 0.50(70) = 60$, and the customer plans to drive less than 70 miles, she should rent from Acme.

73. **a.** The graph of the function is a straight line passing through $(0, 120000)$ and $(10, 0)$. Its slope is $m = \dfrac{0 - 120{,}000}{10 - 0} = -12{,}000$. The required equation is $V = -12{,}000n + 120{,}000$.

b.



c. $V = -12{,}000(6) + 120{,}000 = 48{,}000$, or \$48,000.

d. This is given by the slope, that is, \$12,000 per year.

74. Here $V = -20{,}000n + 1{,}000{,}000$. The book value in 2012 is given by $V = -20{,}000(15) + 1{,}000{,}000$, or \$700,000. The book value in 2016 is given by $V = -20{,}000(19) + 1{,}000{,}000$, or \$620,000. The book value in 2021 is $V = -20{,}000(24) + 1{,}000{,}000$, or \$520,000.

75. **a.** The number of incidents in 2009 was $f(0) = 0.46$ (million).

b. The number of incidents in 2013 was $f(4) = 0.2\left(4^2\right) - 0.14(4) + 0.46 = 3.1$ (million).

76. **a.** The number of passengers in 1995 was $N(0) = 4.6$ (million).

b. The number of passengers in 2010 was $N(15) = 0.011(15)^2 + 0.521(15) + 4.6 = 14.89$ (million).

77. **a.** The life expectancy of a male whose current age is 65 is $f(65) = 0.0069502(65)^2 - 1.6357(65) + 93.76 \approx 16.80$, or approximately 16.8 years.

b. The life expectancy of a male whose current age is 75 is $f(75) = 0.0069502(75)^2 - 1.6357(75) + 93.76 \approx 10.18$, or approximately 10.18 years.

78. **a.** $N(t) = 0.00445t^2 + 0.2903t + 9.564$. $N(0) = 9.564$, or 9.6 million people; $N(12) = 0.00445(12)^2 + 0.2903(12) + 9.564 \approx 13.6884$, or approximately 13.7 million people.

b. $N(14) = 0.00445(14)^2 + 0.2903(14) + 9.564 \approx 14.5004$, or approximately 14.5 million people.

79. The projected number in 2030 is $P(20) = -0.0002083(20)^3 + 0.0157(20)^2 - 0.093(20) + 5.2 = 7.9536$, or approximately 8 million.

The projected number in 2050 is $P(40) = -0.0002083(40)^3 + 0.0157(40)^2 - 0.093(40) + 5.2 = 13.2688$, or approximately 13.3 million.

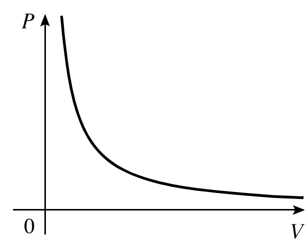
80. $N(t) = -t^3 + 6t^2 + 15t$. Between 8 a.m. and 9 a.m., the average worker can be expected to assemble $N(1) - N(0) = (-1 + 6 + 15) - 0 = 20$, or 20 walkie-talkies. Between 9 a.m. and 10 a.m., we expect that $N(2) - N(1) = \left[-2^3 + 6\left(2^2\right) + 15(2)\right] - (-1 + 6 + 15) = 46 - 20 = 26$, or 26 walkie-talkies can be assembled by the average worker.

81. When the proportion of popular votes won by the Democratic presidential candidate is 0.60, the proportion of seats in the House of Representatives won by Democratic candidates is given by

$$s(0.6) = \frac{(0.6)^3}{(0.6)^3 + (1 - 0.6)^3} = \frac{0.216}{0.216 + 0.064} = \frac{0.216}{0.280} \approx 0.77.$$

82. The amount spent in 2004 was $S(0) = 5.6$, or \$5.6 billion. The amount spent in 2008 was $S(4) = -0.03(4)^3 + 0.2(4)^2 + 0.23(4) + 5.6 = 7.8$, or \$7.8 billion.

83. The domain of the function f is the set of all real positive numbers where $V \neq 0$; that is, $(0, \infty)$.

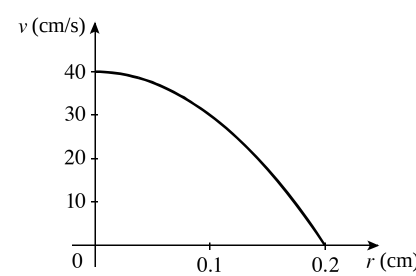


84. **a.** We require that $0.04 - r^2 \geq 0$ and $r \geq 0$. This is true if $0 \leq r \leq 0.2$. Therefore, the domain of v is $[0, 0.2]$.

b. We compute $v(0) = 1000\left[0.04 - (0)^2\right] = 1000(0.04) = 40$,

$$v(0.1) = 1000\left[0.04 - (0.1)^2\right] = 1000(0.04 - 0.01)$$
$$= 1000(0.03) = 30, \text{ and}$$
$$v(0.2) = 1000\left[0.04 - (0.2)^2\right] = 1000(0.04 - 0.04) = 0.$$

c.



d. As the distance r increases, the velocity of the blood decreases.

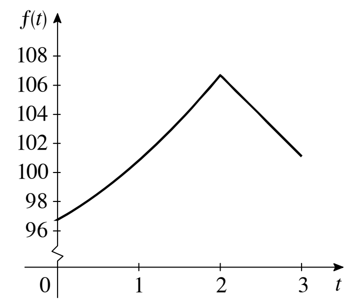
85. **a.** The assets at the beginning of 2002 were \$0.6 trillion. At the beginning of 2003, they were $f(1) = 0.6$, or \$0.6 trillion.

b. The assets at the beginning of 2005 were $f(3) = 0.6(3)^{0.43} \approx 0.96$, or \$0.96 trillion. At the beginning of 2007, they were $f(5) = 0.6(5)^{0.43} \approx 1.20$, or \$1.2 trillion.

86. a. We compute $f(0) = 0.88(0)^2 + 3.21(0) + 96.75 = 96.75$, $f(1) = 0.88(1)^2 + 3.21(1) + 96.75 = 100.84$, and $f(2) = -5.58(2) + 117.85 = 106.69$. We summarize these results in a table.

Year	2006	2007	2008
Rate	96.75	100.84	106.69

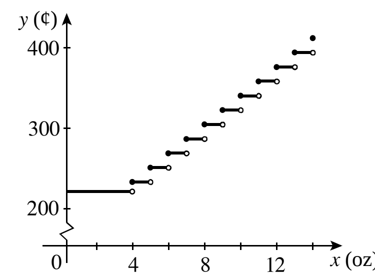
b.



87. a. The domain of f is $(0, 13]$.

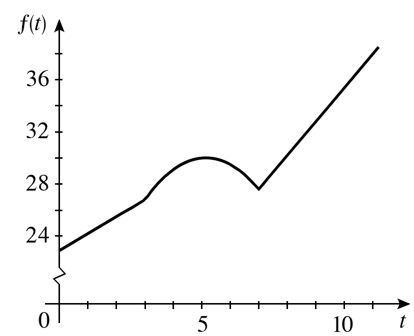
$$f(x) = \begin{cases} 2.32 & \text{if } 0 < x < 4 \\ 2.50 & \text{if } 4 \le x < 5 \\ 2.68 & \text{if } 5 \le x < 6 \\ 2.86 & \text{if } 6 \le x < 7 \\ 3.04 & \text{if } 7 \le x < 8 \\ 3.22 & \text{if } 8 \le x < 9 \\ 3.40 & \text{if } 9 \le x < 10 \\ 3.58 & \text{if } 10 \le x < 11 \\ 3.76 & \text{if } 11 \le x < 12 \\ 3.94 & \text{if } 12 \le x < 13 \\ 4.12 & \text{if } x = 13 \end{cases}$$

b.



88. a. The median age of the U.S. population at the beginning of 1900 was $f(0) = 22.9$, or 22.9 years; at the beginning of 1950 it was $f(5) = -0.7(5)^2 + 7.2(5) + 11.5 = 30$, or 30 years; and at the beginning of 2000 it was $f(10) = 2.6(10) + 9.4 = 35.4$, or 35.4 years.

b.



89. a. The passenger ship travels a distance given by $14t$ miles east and the cargo ship travels a distance of $10(t-2)$ miles north. After two hours have passed, the distance between the two ships is given by $\sqrt{[10(t-2)]^2 + (14t)^2} = \sqrt{296t^2 - 400t + 400}$ miles, so $D(t) = \begin{cases} 14t & \text{if } 0 \le t \le 2 \\ 2\sqrt{74t^2 - 100t + 100} & \text{if } t > 2 \end{cases}$

b. Three hours after the cargo ship leaves port the value of t is 5. Therefore, $D = 2\sqrt{74(5)^2 - 100(5) + 100} \approx 76.16$, or 76.16 miles.

90. True, by definition of a function (page 52).

91. False. Take $f(x) = x^2$, $a = 1$, and $b = -1$. Then $f(1) = 1 = f(-1)$, but $a \ne b$.

92. False. Let $f(x) = x^2$, then take $a = 1$ and $b = 2$. Then $f(a) = f(1) = 1$, $f(b) = f(2) = 4$, and $f(a) + f(b) = 1 + 4 \ne f(a+b) = f(3) = 9$.

93. False. It intersects the graph of a function in at most one point.

94. True. We have $x + 2 \ge 0$ and $2 - x \ge 0$ simultaneously; that is $x \ge -2$ and $x \le 2$. These inequalities are satisfied if $-2 \le x \le 2$.

95. False. Take $f(x) = x^2$ and $k = 2$. Then $f(x) = (2x)^2 = 4x^2 \neq 2x^2 = 2f(x)$.

96. False. Take $f(x) = 2x + 3$ and $c = 2$. Then $f(2x + y) = 2(2x + y) + 3 = 4x + 2y + 3$, but $cf(x) + f(y) = 2(2x + 3) + (2y + 3) = 4x + 2y + 9 \neq f(2x + y)$.

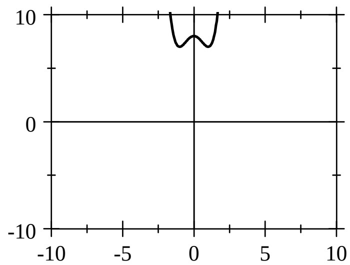
97. False. They are equal everywhere except at $x = 0$, where g is not defined.

98. False. The rule suggests that R takes on the values 0 and 1 when $x = 1$. This violates the uniqueness property that a function must possess.

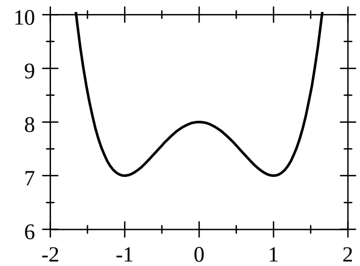
Using Technology

page 68

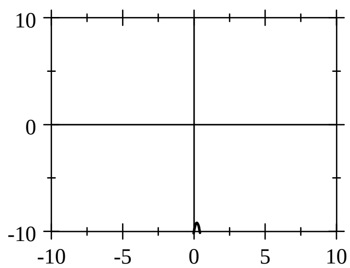
1. a.



b.



2. a.



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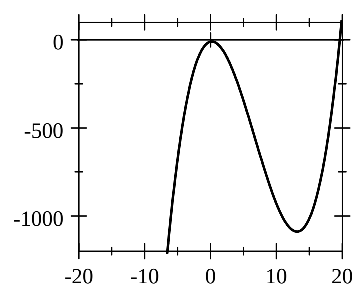

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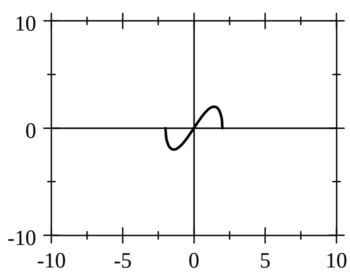

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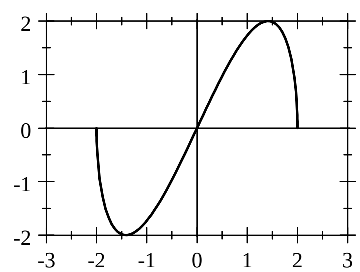
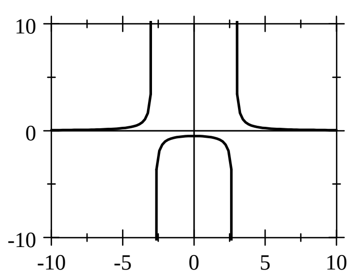
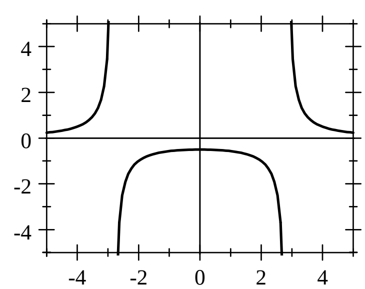


4. a.



b.



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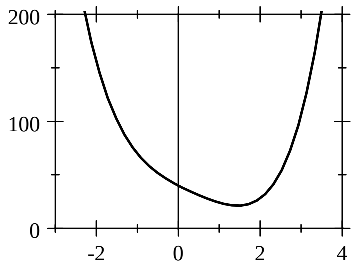

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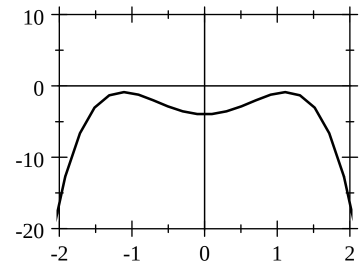

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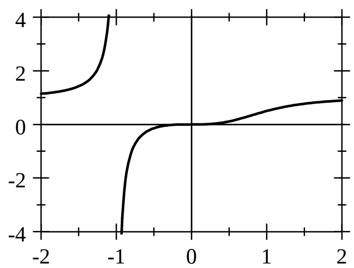
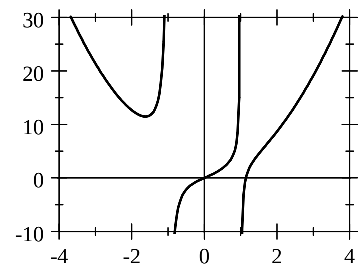


8.



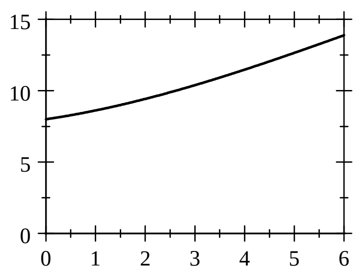
9. $f(2.145) \approx 18.5505$.

10. $f(1.28) \approx 17.3850$.

11. $f(2.41) \approx 4.1616$.

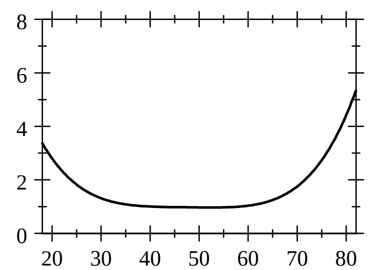
12. $f(0.62) \approx 1.7214$.

13. a.



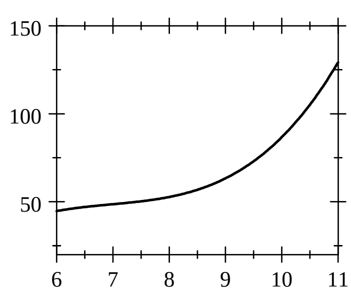
b. The amount spent in the year 2005 was $f(2) \approx 9.42$, or approximately \$9.4 billion. In 2009, it was $f(6) \approx 13.88$, or approximately \$13.9 billion.

14. a.



b. $f(18) = 3.3709$, $f(50) = 0.971$, and $f(80) = 4.4078$.

15. a.



b. $f(6) = 44.7$, $f(8) = 52.7$, and $f(11) = 129.2$.

2.2 The Algebra of Functions

Concept Questions page 73

1. a. $P(x_1) = R(x_1) - C(x_1)$ gives the profit if x_1 units are sold.

b. $P(x_2) = R(x_2) - C(x_2)$. Because $P(x_2) < 0$, $|R(x_2) - C(x_2)| = -[R(x_2) - C(x_2)]$ gives the loss sustained if x_2 units are sold.

2. a. $(f+g)(x) = f(x) + g(x)$, $(f-g)(x) = f(x) - g(x)$, and $(fg)(x) = f(x)g(x)$; all have domain $A \cap B$. $(f/g)(x) = \dfrac{f(x)}{g(x)}$ has domain $A \cap B$ excluding $x \in A \cap B$ such that $g(x) = 0$.

b. $(f+g)(2) = f(2) + g(2) = 3 + (-2) = 1$, $(f-g)(2) = f(2) - g(2) = 3 - (-2) = 5$, $(fg)(2) = f(2)g(2) = 3(-2) = -6$, and $(f/g)(2) = \dfrac{f(2)}{g(2)} = \dfrac{3}{-2} = -\dfrac{3}{2}$

3. a. $y = (f+g)(x) = f(x) + g(x)$

b. $y = (f-g)(x) = f(x) - g(x)$

c. $y = (fg)(x) = f(x)g(x)$

d. $y = \left(\dfrac{f}{g}\right)(x) = \dfrac{f(x)}{g(x)}$

4. a. The domain of $(f \circ g)(x) = f(g(x))$ is the set of all x in the domain of g such that $g(x)$ is in the domain of f. The domain of $(g \circ f)(x) = g(f(x))$ is the set of all x in the domain of f such that $f(x)$ is in the domain of g.

b. $(g \circ f)(2) = g(f(2)) = g(3) = 8$. We cannot calculate $(f \circ g)(3)$ because $(f \circ g)(3) = f(g(3)) = f(8)$, and we don't know the value of $f(8)$.

5. No. Let $A = (-\infty, \infty)$, $f(x) = x$, and $g(x) = \sqrt{x}$. Then $a = -1$ is in A, but $(g \circ f)(-1) = g(f(-1)) = g(-1) = \sqrt{-1}$ is not defined.

6. The required expression is $P = g(f(p))$.

Exercises page 74

1. $(f+g)(x) = f(x) + g(x) = \left(x^3 + 5\right) + \left(x^2 - 2\right) = x^3 + x^2 + 3$.

2. $(f-g)(x) = f(x) - g(x) = \left(x^3 + 5\right) - \left(x^2 - 2\right) = x^3 - x^2 + 7$.

3. $fg(x) = f(x)g(x) = \left(x^3 + 5\right)\left(x^2 - 2\right) = x^5 - 2x^3 + 5x^2 - 10$.

4. $gf(x) = g(x)f(x) = \left(x^2 - 2\right)\left(x^3 + 5\right) = x^5 - 2x^3 + 5x^2 - 10$.

5. $\dfrac{f}{g}(x) = \dfrac{f(x)}{g(x)} = \dfrac{x^3 + 5}{x^2 - 2}$.

6. $\dfrac{f-g}{h}(x) = \dfrac{f(x) - g(x)}{h(x)} = \dfrac{x^3 + 5 - \left(x^2 - 2\right)}{2x + 4} = \dfrac{x^3 - x^2 + 7}{2x + 4}$.

7. $\dfrac{fg}{h}(x) = \dfrac{f(x)g(x)}{h(x)} = \dfrac{\left(x^3 + 5\right)\left(x^2 - 2\right)}{2x + 4} = \dfrac{x^5 - 2x^3 + 5x^2 - 10}{2x + 4}$.

8. $fgh(x) = f(x)\,g(x)\,h(x) = \left(x^3+5\right)\left(x^2-2\right)(2x+4) = \left(x^5-2x^3+5x^2-10\right)(2x+4)$

$= 2x^6-4x^4+10x^3-20x+4x^5-8x^3+20x^2-40 = 2x^6+4x^5-4x^4+2x^3+20x^2-20x-40.$

9. $(f+g)(x) = f(x)+g(x) = x-1+\sqrt{x+1}.$

10. $(g-f)(x) = g(x)-f(x) = \sqrt{x+1}-(x-1) = \sqrt{x+1}-x+1.$

11. $(fg)(x) = f(x)\,g(x) = (x-1)\sqrt{x+1}.$

12. $(gf)(x) = g(x)\,f(x) = \sqrt{x+1}\,(x-1).$

13. $\dfrac{g}{h}(x) = \dfrac{g(x)}{h(x)} = \dfrac{\sqrt{x+1}}{2x^3-1}.$

14. $\dfrac{h}{g}(x) = \dfrac{h(x)}{g(x)} = \dfrac{2x^3-1}{\sqrt{x+1}}.$

15. $\dfrac{fg}{h}(x) = \dfrac{(x-1)\left(\sqrt{x+1}\right)}{2x^3-1}.$

16. $\dfrac{fh}{g}(x) = \dfrac{(x-1)\left(2x^3-1\right)}{\sqrt{x+1}} = \dfrac{2x^4-2x^3-x+1}{\sqrt{x+1}}.$

17. $\dfrac{f-h}{g}(x) = \dfrac{x-1-\left(2x^3-1\right)}{\sqrt{x+1}} = \dfrac{x-2x^3}{\sqrt{x+1}}.$

18. $\dfrac{gh}{g-f}(x) = \dfrac{\sqrt{x+1}\left(2x^3-1\right)}{\sqrt{x+1}-(x-1)} = \dfrac{\sqrt{x+1}\left(2x^3-1\right)}{\sqrt{x+1}-x+1}.$

19. $(f+g)(x) = x^2+5+\sqrt{x}-2 = x^2+\sqrt{x}+3$, $(f-g)(x) = x^2+5-\left(\sqrt{x}-2\right) = x^2-\sqrt{x}+7$, $(fg)(x) = \left(x^2+5\right)\left(\sqrt{x}-2\right)$, and $\left(\dfrac{f}{g}\right)(x) = \dfrac{x^2+5}{\sqrt{x}-2}.$

20. $(f+g)(x) = \sqrt{x-1}+x^3+1$, $(f-g)(x) = \sqrt{x-1}-x^3-1$, $(fg)(x) = \sqrt{x-1}\left(x^3+1\right)$, and $\left(\dfrac{f}{g}\right)(x) = \dfrac{\sqrt{x-1}}{x^3+1}.$

21. $(f+g)(x) = \sqrt{x+3}+\dfrac{1}{x-1} = \dfrac{(x-1)\sqrt{x+3}+1}{x-1}$, $(f-g)(x) = \sqrt{x+3}-\dfrac{1}{x-1} = \dfrac{(x-1)\sqrt{x+3}-1}{x-1}$, $(fg)(x) = \sqrt{x+3}\left(\dfrac{1}{x-1}\right) = \dfrac{\sqrt{x+3}}{x-1}$, and $\left(\dfrac{f}{g}\right) = \sqrt{x+3}\,(x-1).$

22. $(f+g)(x) = \dfrac{1}{x^2+1}+\dfrac{1}{x^2-1} = \dfrac{x^2-1+x^2+1}{\left(x^2+1\right)\left(x^2-1\right)} = \dfrac{2x^2}{\left(x^2+1\right)\left(x^2-1\right)}$,

$(f-g)(x) = \dfrac{1}{x^2+1}-\dfrac{1}{x^2-1} = \dfrac{x^2-1-x^2-1}{\left(x^2+1\right)\left(x^2-1\right)} = -\dfrac{2}{\left(x^2+1\right)\left(x^2-1\right)}$, $(fg)(x) = \dfrac{1}{\left(x^2+1\right)\left(x^2-1\right)}$, and $\left(\dfrac{f}{g}\right)(x) = \dfrac{x^2-1}{x^2+1}.$

23. $(f+g)(x) = \dfrac{x+1}{x-1} + \dfrac{x+2}{x-2} = \dfrac{(x+1)(x-2)+(x+2)(x-1)}{(x-1)(x-2)} = \dfrac{x^2-x-2+x^2+x-2}{(x-1)(x-2)}$
$= \dfrac{2x^2-4}{(x-1)(x-2)} = \dfrac{2\left(x^2-2\right)}{(x-1)(x-2)}$,
$(f-g)(x) = \dfrac{x+1}{x-1} - \dfrac{x+2}{x-2} = \dfrac{(x+1)(x-2)-(x+2)(x-1)}{(x-1)(x-2)} = \dfrac{x^2-x-2-x^2-x+2}{(x-1)(x-2)}$
$= \dfrac{-2x}{(x-1)(x-2)}$,
$(fg)(x) = \dfrac{(x+1)(x+2)}{(x-1)(x-2)}$, and $\left(\dfrac{f}{g}\right)(x) = \dfrac{(x+1)(x-2)}{(x-1)(x+2)}$.

24. $(f+g)(x) = x^2+1+\sqrt{x+1}$, $(f-g)(x) = x^2+1-\sqrt{x+1}$, $(fg)(x) = \left(x^2+1\right)\sqrt{x+1}$, and $\left(\dfrac{f}{g}\right)(x) = \dfrac{x^2+1}{\sqrt{x+1}}$.

25. $(f\circ g)(x) = f(g(x)) = f\left(x^2\right) = \left(x^2\right)^2 + x^2 + 1 = x^4 + x^2 + 1$ and
$(g\circ f)(x) = g(f(x)) = g\left(x^2+x+1\right) = \left(x^2+x+1\right)^2$.

26. $(f\circ g)(x) = f(g(x)) = 3\left[g(x)\right]^2 + 2g(x) + 1 = 3(x+3)^2 + 2(x+3) + 1 = 3x^2 + 20x + 34$ and
$(g\circ f)(x) = g(f(x)) = f(x) + 3 = 3x^2 + 2x + 1 + 3 = 3x^2 + 2x + 4$.

27. $(f\circ g)(x) = f(g(x)) = f\left(x^2-1\right) = \sqrt{x^2-1} + 1$ and
$(g\circ f)(x) = g(f(x)) = g\left(\sqrt{x}+1\right) = \left(\sqrt{x}+1\right)^2 - 1 = x + 2\sqrt{x} + 1 - 1 = x + 2\sqrt{x}$.

28. $(f\circ g)(x) = f(g(x)) = 2\sqrt{g(x)} + 3 = 2\sqrt{x^2+1} + 3$ and
$(g\circ f)(x) = g(f(x)) = \left[f(x)\right]^2 + 1 = \left(2\sqrt{x}+3\right)^2 + 1 = 4x + 12\sqrt{x} + 10$.

29. $(f\circ g)(x) = f(g(x)) = f\left(\dfrac{1}{x}\right) = \dfrac{1}{x} \div \left(\dfrac{1}{x^2}+1\right) = \dfrac{1}{x}\cdot\dfrac{x^2}{x^2+1} = \dfrac{x}{x^2+1}$ and
$(g\circ f)(x) = g(f(x)) = g\left(\dfrac{x}{x^2+1}\right) = \dfrac{x^2+1}{x}$.

30. $(f\circ g)(x) = f(g(x)) = f\left(\dfrac{1}{x-1}\right) = \sqrt{\dfrac{x}{x-1}}$ and
$(g\circ f)(x) = g(f(x)) = g\left(\sqrt{x+1}\right) = \dfrac{1}{\sqrt{x+1}-1}\cdot\dfrac{\sqrt{x+1}+1}{\sqrt{x+1}+1} = \dfrac{\sqrt{x+1}+1}{x}$.

31. $h(2) = g(f(2))$. But $f(2) = 2^2 + 2 + 1 = 7$, so $h(2) = g(7) = 49$.

32. $h(2) = g(f(2))$. But $f(2) = \left(2^2-1\right)^{1/3} = 3^{1/3}$, so $h(2) = g\left(3^{1/3}\right) = 3\left(3^{1/3}\right)^3 + 1 = 3(3) + 1 = 10$.

33. $h(2) = g(f(2))$. But $f(2) = \dfrac{1}{2(2)+1} = \dfrac{1}{5}$, so $h(2) = g\left(\dfrac{1}{5}\right) = \dfrac{1}{\sqrt{5}} = \dfrac{\sqrt{5}}{5}$.

34. $h(2) = g(f(2))$. But $f(2) = \dfrac{1}{2-1} = 1$, so $g(1) = 1^2 + 1 = 2$.

35. $f(x) = 2x^3 + x^2 + 1$, $g(x) = x^5$.

36. $f(x) = 3x^2 - 4$, $g(x) = x^{-3}$.

37. $f(x) = x^2 - 1$, $g(x) = \sqrt{x}$.

38. $f(x) = (2x - 3)$, $g(x) = x^{3/2}$.

39. $f(x) = x^2 - 1$, $g(x) = \dfrac{1}{x}$.

40. $f(x) = x^2 - 4$, $g(x) = \dfrac{1}{\sqrt{x}}$.

41. $f(x) = 3x^2 + 2$, $g(x) = \dfrac{1}{x^{3/2}}$.

42. $f(x) = \sqrt{2x + 1}$, $g(x) = \dfrac{1}{x} + x$.

43. $f(a + h) - f(a) = [3(a + h) + 4] - (3a + 4) = 3a + 3h + 4 - 3a - 4 = 3h$.

44. $f(a + h) - f(a) = -\frac{1}{2}(a + h) + 3 - \left(-\frac{1}{2}a + 3\right) = -\frac{1}{2}a - \frac{1}{2}h + 3 + \frac{1}{2}a - 3 = -\frac{1}{2}h$.

45. $f(a + h) - f(a) = 4 - (a + h)^2 - \left(4 - a^2\right) = 4 - a^2 - 2ah - h^2 - 4 + a^2 = -2ah - h^2 = -h(2a + h)$.

46.
$$\begin{aligned} f(a + h) - f(a) &= \left[(a + h)^2 - 2(a + h) + 1\right] - \left(a^2 - 2a + 1\right) \\ &= a^2 + 2ah + h^2 - 2a - 2h + 1 - a^2 + 2a - 1 = h(2a + h - 2). \end{aligned}$$

47.
$$\begin{aligned} \frac{f(a + h) - f(a)}{h} &= \frac{\left[(a + h)^2 + 1\right] - \left(a^2 + 1\right)}{h} = \frac{a^2 + 2ah + h^2 + 1 - a^2 - 1}{h} = \frac{2ah + h^2}{h} \\ &= \frac{h(2a + h)}{h} = 2a + h. \end{aligned}$$

48.
$$\begin{aligned} \frac{f(a + h) - f(a)}{h} &= \frac{\left[2(a + h)^2 - (a + h) + 1\right] - \left(2a^2 - a + 1\right)}{h} \\ &= \frac{2a^2 + 4ah + 2h^2 - a - h + 1 - 2a^2 + a - 1}{h} = \frac{4ah + 2h^2 - h}{h} = 4a + 2h - 1. \end{aligned}$$

49.
$$\begin{aligned} \frac{f(a + h) - f(a)}{h} &= \frac{\left[(a + h)^3 - (a + h)\right] - \left(a^3 - a\right)}{h} = \frac{a^3 + 3a^2h + 3ah^2 + h^3 - a - h - a^3 + a}{h} \\ &= \frac{3a^2h + 3ah^2 + h^3 - h}{h} = 3a^2 + 3ah + h^2 - 1. \end{aligned}$$

50.
$$\begin{aligned} \frac{f(a + h) - f(a)}{h} &= \frac{\left[2(a + h)^3 - (a + h)^2 + 1\right] - \left(2a^3 - a^2 + 1\right)}{h} \\ &= \frac{2a^3 + 6a^2h + 6ah^2 + 2h^3 - a^2 - 2ah - h^2 + 1 - 2a^3 + a^2 - 1}{h} \\ &= \frac{6a^2h + 6ah^2 + 2h^3 - 2ah - h^2}{h} = 6a^2 + 6ah + 2h^2 - 2a - h. \end{aligned}$$

51.
$$\frac{f(a + h) - f(a)}{h} = \frac{\dfrac{1}{a + h} - \dfrac{1}{a}}{h} = \frac{\dfrac{a - (a + h)}{a(a + h)}}{h} = -\frac{1}{a(a + h)}.$$

52.
$$\frac{f(a + h) - f(a)}{h} = \frac{\sqrt{a + h} - \sqrt{a}}{h} \cdot \frac{\sqrt{a + h} + \sqrt{a}}{\sqrt{a + h} + \sqrt{a}} = \frac{(a + h) - a}{h\left(\sqrt{a + h} + \sqrt{a}\right)} = \frac{1}{\sqrt{a + h} + \sqrt{a}}.$$

53. $F(t)$ represents the total revenue for the two restaurants at time t.

54. $F(t)$ represents the net rate of growth of the species of whales in year t.

55. $f(t)g(t)$ represents the dollar value of Nancy's holdings at time t.

56. $f(t)/g(t)$ represents the unit cost of the commodity at time t.

57. $g \circ f$ is the function giving the amount of carbon monoxide pollution from cars in parts per million at time t.

58. $f \circ g$ is the function giving the revenue at time t.

59. $C(x) = 0.6x + 12{,}100$.

60. **a.** $h(t) = f(t) - g(t) = (3t + 69) - (-0.2t + 13.8) = 3.2t + 55.2$, $0 \le t \le 5$.

b. $f(5) = 3(5) + 69 = 84$, $g(5) = -0.2(5) + 13.8 = 12.8$, and $h(5) = 3.2(5) + 55.2 = 71.2$. Since $f(5) - g(5) = 84 - 12.8 = 71.2$, we see that $h(5)$ is indeed equal to $f(5) - g(5)$.

61. $$D(t) = (D_2 - D_1)(t) = D_2(t) - D_1(t) = \left(0.035t^2 + 0.21t + 0.24\right) - \left(0.0275t^2 + 0.081t + 0.07\right)$$
$$\approx 0.0075t^2 + 0.129t + 0.17.$$

The function D gives the difference in year t between the deficit without the \$160 million rescue package and the deficit with the rescue package.

62. **a.** $(g \circ f)(0) = g(f(0)) = g(0.64) = 26$, so the mortality rate of motorcyclists in the year 2000 was 26 per 100 million miles traveled.

b. $(g \circ f)(6) = g(f(6)) = g(0.51) = 42$, so the mortality rate of motorcyclists in 2006 was 42 per 100 million miles traveled.

c. Between 2000 and 2006, the percentage of motorcyclists wearing helmets had dropped from 64 to 51, and as a consequence, the mortality rate of motorcyclists had increased from 26 million miles traveled to 42 million miles traveled.

63. **a.** $(g \circ f)(1) = g(f(1)) = g(406) = 23$. So in 2002, the percentage of reported serious crimes that end in arrests or in the identification of suspects was 23.

b. $(g \circ f)(6) = g(f(6)) = g(326) = 18$. In 2007, 18% of reported serious crimes ended in arrests or in the identification of suspects.

c. Between 2002 and 2007, the total number of detectives had dropped from 406 to 326 and as a result, the percentage of reported serious crimes that ended in arrests or in the identification of suspects dropped from 23 to 18.

64. **a.** $C(x) = 0.000003x^3 - 0.03x^2 + 200x + 100{,}000$.

b. $$P(x) = R(x) - C(x) = -0.1x^2 + 500x - \left(0.000003x^3 - 0.03x^2 + 200x + 100{,}000\right)$$
$$= -0.000003x^3 - 0.07x^2 + 300x - 100{,}000.$$

c. $P(1500) = -0.000003(1500)^3 - 0.07(1500)^2 + 300(1500) - 100{,}000 = 182{,}375$, or \$182,375.

65. a. $C(x) = V(x) + 20000 = 0.000001x^3 - 0.01x^2 + 50x + 20000 = 0.000001x^3 - 0.01x^2 + 50x + 20{,}000.$

b. $P(x) = R(x) - C(x) = -0.02x^2 + 150x - 0.000001x^3 + 0.01x^2 - 50x - 20{,}000$

$= -0.000001x^3 - 0.01x^2 + 100x - 20{,}000.$

c. $P(2000) = -0.000001(2000)^3 - 0.01(2000)^2 + 100(2000) - 20{,}000 = 132{,}000$, or \$132,000.

66. a. $D(t) = R(t) - S(t)$

$= \left(0.023611t^3 - 0.19679t^2 + 0.34365t + 2.42\right) - \left(-0.015278t^3 + 0.11179t^2 + 0.02516t + 2.64\right)$

$= 0.038889t^3 - 0.30858t^2 + 0.31849t - 0.22,\ 0 \le t \le 6.$

b. $S(3) = 3.309084$, $R(3) = 2.317337$, and $D(3) = -0.991747$, so the spending, revenue, and deficit are approximately \$3.31 trillion, \$2.32 trillion, and \$0.99 trillion, respectively.

c. Yes: $R(3) - S(3) = 2.317337 - 3.308841 = -0.991504 = D(3)$.

67. a. $h(t) = f(t) + g(t) = \left(4.389t^3 - 47.833t^2 + 374.49t + 2390\right) + \left(13.222t^3 - 132.524t^2 + 757.9t + 7481\right)$

$= 17.611t^3 - 180.357t^2 + 1132.39t + 9871,\ 1 \le t \le 7.$

b. $f(6) = 3862.976$ and $g(6) = 10{,}113.488$, so $f(6) + g(6) = 13{,}976.464$. The worker's contribution was approximately \$3862.98, the employer's contribution was approximately \$10,113.49, and the total contributions were approximately \$13,976.46.

c. $h(6) = 13{,}976 = f(6) + g(6)$, as expected.

68. a. $N(r(t)) = \dfrac{7}{1 + 0.02\left(\dfrac{5t + 75}{t + 10}\right)^2}.$

b. $N(r(0)) = \dfrac{7}{1 + 0.02\left(\dfrac{5 \cdot 0 + 75}{0 + 10}\right)^2} = \dfrac{7}{1 + 0.02\left(\dfrac{75}{10}\right)^2} \approx 3.29$, or 3.29 million units.

$N(r(12)) = \dfrac{7}{1 + 0.02\left(\dfrac{5 \cdot 12 + 75}{12 + 10}\right)^2} = \dfrac{7}{1 + 0.02\left(\dfrac{135}{22}\right)^2} \approx 3.99$, or 3.99 million units.

$N(r(18)) = \dfrac{7}{1 + 0.02\left(\dfrac{5 \cdot 18 + 75}{18 + 10}\right)^2} = \dfrac{7}{1 + 0.02\left(\dfrac{165}{28}\right)^2} \approx 4.13$, or 4.13 million units.

69. a. The occupancy rate at the beginning of January is $r(0) = \frac{10}{81}(0)^3 - \frac{10}{3}(0)^2 + \frac{200}{9}(0) + 55 = 55$, or 55%.
$r(5) = \frac{10}{81}(5)^3 - \frac{10}{3}(5)^2 + \frac{200}{9}(5) + 55 \approx 98.2$, or approximately 98.2%.

b. The monthly revenue at the beginning of January is $R(55) = -\frac{3}{5000}(55)^3 + \frac{9}{50}(55)^2 \approx 444.68$, or approximately \$444,700.

The monthly revenue at the beginning of June is $R(98.2) = -\frac{3}{5000}(98.2)^3 + \frac{9}{50}(98.2)^2 \approx 1167.6$, or approximately \$1,167,600.

70. $N(t) = 1.42 \cdot x(t) = \dfrac{1.42 \cdot 7(t+10)^2}{(t+10)^2 + 2(t+15)^2} = \dfrac{9.94(t+10)^2}{(t+10)^2 + 2(t+15)^2}$. The number of jobs created 6 months from now will be $N(6) = \dfrac{9.94(16)^2}{(16)^2 + 2(21)^2} \approx 2.24$, or approximately 2.24 million jobs. The number of jobs created 12 months from now will be $N(12) = \dfrac{9.94(22)^2}{(22)^2 + 2(27)^2} \approx 2.48$, or approximately 2.48 million jobs.

71. **a.** $s = f + g + h = (f + g) + h = f + (g + h)$. This suggests we define the sum s by $s(x) = (f + g + h)(x) = f(x) + g(x) + h(x)$.

b. Let f, g, and h define the revenue (in dollars) in week t of three branches of a store. Then its total revenue (in dollars) in week t is $s(t) = (f + g + h)(t) = f(t) + g(t) + h(t)$.

72. **a.** $(h \circ g \circ f)(x) = h(g(f(x)))$

b. Let t denote time. Suppose f gives the number of people at time t in a town, g gives the number of cars as a function of the number of people in the town, and h gives the amount of carbon monoxide in the atmosphere. Then $(h \circ g \circ f)(t) = h(g(f(t)))$ gives the amount of carbon monoxide in the atmosphere at time t.

73. True. $(f + g)(x) = f(x) + g(x) = g(x) + f(x) = (g + f)(x)$.

74. False. Let $f(x) = x + 2$ and $g(x) = \sqrt{x}$. Then $(g \circ f)(x) = \sqrt{x + 2}$ is defined at $x = -1$, But $(f \circ g)(x) = \sqrt{x} + 2$ is not defined at $x = -1$.

75. False. Take $f(x) = \sqrt{x}$ and $g(x) = x + 1$. Then $(g \circ f)(x) = \sqrt{x} + 1$, but $(f \circ g)(x) = \sqrt{x + 1}$.

76. False. Take $f(x) = x + 1$. Then $(f \circ f)(x) = f(f(x)) = x + 2$, but $f^2(x) = [f(x)]^2 = (x + 1)^2 = x^2 + 2x + 1$.

77. True. $(h \circ (g \circ f))(x) = h((g \circ f)(x)) = h(g(f(x)))$ and $((h \circ g) \circ f)(x) = (h \circ g)(f(x)) = h(g(f(x)))$.

78. False. Take $h(x) = \sqrt{x}$, $g(x) = x$, and $f(x) = x^2$. Then $(h \circ (g + f))(x) = h(x + x^2) = \sqrt{x + x^2} \neq ((h \circ g) + (h \circ f))(x) = h(g(x)) + h(f(x)) = \sqrt{x} + \sqrt{x^2}$.

2.3 Functions and Mathematical Models

Concept Questions

page 88

1. See page 78 of the text. Answers will vary.

2. **a.** $P(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_0$, where $a_n \neq 0$ and n is a positive integer. An example is $P(x) = 4x^3 - 3x^2 + 2$.

b. $R(x) = \dfrac{P(x)}{Q(x)}$, where P and Q are polynomials with $Q(x) \neq 0$. An example is $R(x) = \dfrac{3x^4 - 2x^2 + 1}{x^2 + 3x + 5}$.

3. **a.** A demand function $p = D(x)$ gives the relationship between the unit price of a commodity p and the quantity x demanded. A supply function $p = S(x)$ gives the relationship between the unit price of a commodity p and the quantity x the supplier will make available in the marketplace.

b. Market equilibrium occurs when the quantity produced is equal to the quantity demanded. To find the market equilibrium, we solve the equations $p = D(x)$ and $p = S(x)$ simultaneously.

Exercises page 88

1. Yes. $2x + 3y = 6$ and so $y = -\frac{2}{3}x + 2$.

2. Yes. $4y = 2x + 7$ and so $y = \frac{1}{2}x + \frac{7}{4}$.

3. Yes. $2y = x + 4$ and so $y = \frac{1}{2}x + 2$.

4. Yes. $3y = 2x - 8$ and so $y = \frac{2}{3}x - \frac{8}{3}$.

5. Yes. $4y = 2x + 9$ and so $y = \frac{1}{2}x + \frac{9}{4}$.

6. Yes. $6y = 3x + 7$ and so $y = \frac{1}{2}x + \frac{7}{6}$.

7. No, because of the term x^2.

8. No, because of the term $\sqrt{x}$.

9. f is a polynomial function in x of degree 6.

10. f is a rational function.

11. Expanding $G(x) = 2(x^2 - 3)^3$, we have $G(x) = 2x^6 - 18x^4 + 54x^2 - 54$, and we conclude that G is a polynomial function of degree 6 in x.

12. We can write $H(x) = \dfrac{2}{x^3} + \dfrac{5}{x^2} + 6 = \dfrac{2 + 5x + 6x^3}{x^3}$ and conclude that H is a rational function.

13. f is neither a polynomial nor a rational function.

14. f is a rational function.

15. $f(0) = 2$ gives $f(0) = m(0) + b = b = 2$. Next, $f(3) = -1$ gives $f(3) = m(3) + b = -1$. Substituting $b = 2$ in this last equation, we have $3m + 2 = -1$, or $3m = -3$, and therefore, $m = -1$ and $b = 2$.

16. $f(2) = 4$ gives $f(2) = 2m + b = 4$. We also know that $m = -1$. Therefore, we have $2(-1) + b = 4$ and so $b = 6$.

17. a. $C(x) = 8x + 40{,}000$.

b. $R(x) = 12x$.

c. $P(x) = R(x) - C(x) = 12x - (8x + 40{,}000) = 4x - 40{,}000$.

d. $P(8000) = 4(8000) - 40{,}000 = -8000$, or a loss of \$8000. $P(12{,}000) = 4(12{,}000) - 40{,}000 = 8000$, or a profit of \$8000.

18. a. $C(x) = 14x + 100{,}000$.

b. $R(x) = 20x$.

c. $P(x) = R(x) - C(x) = 20x - (14x + 100{,}000) = 6x - 100{,}000$.

d. $P(12{,}000) = 6(12{,}000) - 100{,}000 = -28{,}000$, or a loss of \$28,000. $P(20{,}000) = 6(20{,}000) - 100{,}000 = 20{,}000$, or a profit of \$20,000.

19. The individual's disposable income is $D = (1 - 0.28) \cdot 60{,}000 = 43{,}200$, or \$43,200.

20. The child should receive $D(0.4) = \dfrac{(0.4)(500)}{1.7} \approx 117.65$, or approximately 118 mg.

21. The child should receive $D(4) = \left(\dfrac{4 + 1}{24}\right)(500) \approx 104.17$, or approximately 104 mg.

22. a. The graph of f passes through the points $P_1(0, 17.5)$ and $P_2(10, 10.3)$. Its slope is $\dfrac{10.3 - 17.5}{10 - 0} = -0.72$. An equation of the line is $y - 17.5 = -0.72(t - 0)$ or $y = -0.72t + 17.5$, so the linear function is $f(t) = -0.72t + 17.5$.

b. The rate was decreasing at 0.72% per year.

c. The percentage of high school students who drink and drive at the beginning of 2014 is projected to be $f(13) = -0.72(13) + 17.5 = 8.14$, or 8.14%.

23. a. The slope of the graph of f is a line with slope -13.2 passing through the point $(0, 400)$, so an equation of the line is $y - 400 = -13.2(t - 0)$ or $y = -13.2t + 400$, and the required function is $f(t) = -13.2t + 400$.

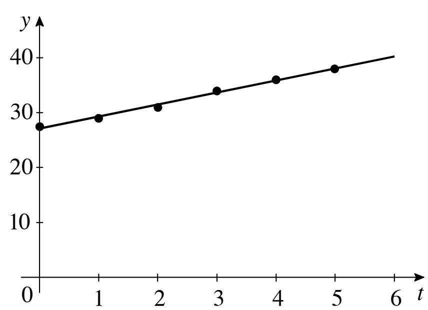
b. The emissions cap is projected to be $f(2) = -13.2(2) + 400 = 373.6$, or 373.6 million metric tons of carbon dioxide equivalent.

24. a. The graph of f is a line through the points $P_1(0, 0.7)$ and $P_2(20, 1.2)$, so it has slope $\dfrac{1.2 - 0.7}{20 - 0} = 0.025$. Its equation is $y - 0.7 = 0.025(t - 0)$ or $y = 0.025t + 0.7$. The required function is thus $f(t) = 0.025t + 0.7$.

b. The projected annual rate of growth is the slope of the graph of f, that is, 0.025 billion per year, or 25 million per year.

c. The projected number of boardings per year in 2022 is $f(10) = 0.025(10) + 0.7 = 0.95$, or 950 million boardings per year.

25. a.



b. The projected revenue in 2010 is $f(6) = 2.19(6) + 27.12 = 40.26$, or \$40.26 billion.

c. The rate of increase is the slope of the graph of f, that is, 2.19 (billion dollars per year).

26. Two hours after starting work, the average worker will be assembling at the rate of $f(2) = -\frac{3}{2}(2)^2 + 6(2) + 10 = 16$, or 16 phones per hour.

27. $P(28) = -\frac{1}{8}(28)^2 + 7(28) + 30 = 128$, or \$128,000.

28. a. The amount paid out in 2010 was $S(0) = 0.72$, or \$0.72 trillion (or \$720 billion).

b. The amount paid out in 2030 is projected to be $S(3) = 0.1375(3)^2 + 0.5185(3) + 0.72 = 3.513$, or \$3.513 trillion.

29. a. The average time spent per day in 2009 was $f(0) = 21.76$ (minutes).

b. The average time spent per day in 2013 is projected to be $f(4) = 2.25(4)^2 + 13.41(4) + 21.76 = 111.4$ (minutes).

30. a. The GDP in 2011 was $G(0) = 15$, or \$15 trillion.

b. The projected GDP in 2015 is $G(4) = 0.064(4)^2 + 0.473(4) + 15.0 = 17.916$, or \$17.916 trillion.

31. a. The GDP per capita in 2000 was $f(10) = 1.86251(10)^2 - 28.08043(10) + 884 = 789.4467$, or \$789.45.

b. The GDP per capita in 2030 is projected to be $f(40) = 1.86251(40)^2 - 28.08043(40) + 884 = 2740.7988$, or \$2740.80.

32. **a.** The number of enterprise IM accounts in 2006 is given by $N(0) = 59.7$, or 59.7 million.

b. The number of enterprise IM accounts in 2010, assuming a continuing trend, is given by $N(4) = 2.96(4)^2 + 11.37(4) + 59.7 = 152.54$ million.

33. $S(6) = 0.73(6)^2 + 15.8(6) + 2.7 = 123.78$ million kilowatt-hr.
$S(8) = 0.73(8)^2 + 15.8(8) + 2.7 = 175.82$ million kilowatt-hr.

34. The U.S. public debt in 2005 was $f(0) = 8.246$, or \$8.246 trillion. The public debt in 2008 was $f(3) = -0.03817(3)^3 + 0.4571(3)^2 - 0.1976(3) + 8.246 = 10.73651$, or approximately \$10.74 trillion.

35. The percentage who expected to work past age 65 in 1991 was $f(0) = 11$, or 11%. The percentage in 2013 was $f(22) = 0.004545(22)^3 - 0.1113(22)^2 + 1.385(22) + 11 = 35.99596$, or approximately 36%.

36. $N(0) = 0.7$ per 100 million vehicle miles driven. $N(7) = 0.0336(7)^3 - 0.118(7)^2 + 0.215(7) + 0.7 = 7.9478$ per 100 million vehicle miles driven.

37. **a.** Total global mobile data traffic in 2009 was $f(0) = 0.06$, or 60,000 terabytes.

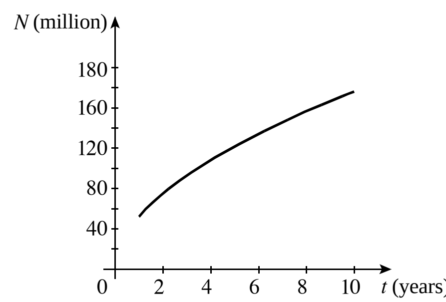
b. The total in 2014 will be $f(5) = 0.021(5)^3 + 0.015(5)^2 + 0.12(5) + 0.06 = 3.66$, or 3.66 million terabytes.

38. $L = \dfrac{1 + 0.05D}{D}$. If $D = 20$, then $L = \dfrac{1 + 0.05(20)}{20} = 0.10$, or 10%.

39. **a.** We first construct a table.

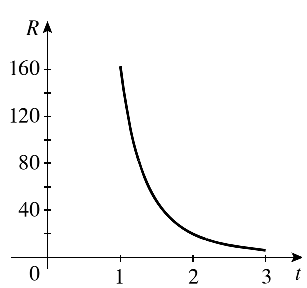
t	$N(t)$
1	52
2	75
3	93
4	109
5	122

t	$N(t)$
6	135
7	146
8	157
9	167
10	177



b. The number of viewers in 2012 is given by $N(10) = 52(10)^{0.531} \approx 176.61$, or approximately 177 million viewers.

40. **a.**



$R(1) = 162.8(1)^{-3.025} = 162.8$, $R(2) = 162.8(2)^{-3.025} \approx 20.0$, and $R(3) = 162.8(3)^{-3.025} \approx 5.9$.

b. The infant mortality rates in 1900, 1950, and 2000 are 162.8, 20.0, and 5.9 per 1000 live births, respectively.

41. $N(5) = 0.0018425(10)^{2.5} \approx 0.58265$, or approximately 0.583 million. $N(13) = 0.0018425(18)^{2.5} \approx 2.5327$, or approximately 2.5327 million.

42. **a.** $S(0) = 4.3(0 + 2)^{0.94} \approx 8.24967$, or approximately \$8.25 billion.

b. $S(8) = 4.3(8 + 2)^{0.94} \approx 37.45$, or approximately \$37.45 billion.

43. a. We are given that $f(1) = 5240$ and $f(4) = 8680$. This leads to the system of equations $a + b = 5240$, $11a + b = 8680$. Solving, we find $a = 344$ and $b = 4896$.

b. From part (a), we have $f(t) = 344t + 4896$, so the approximate per capita costs in 2005 were $f(5) = 344(5) + 4896 = 6616$, or \$6616.

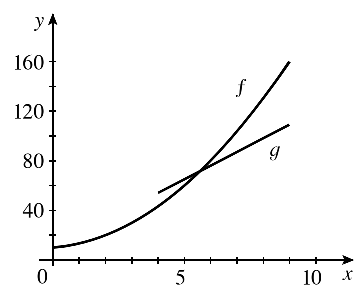
44. a. The given data imply that $R(40) = 50$, that is, $\dfrac{100(40)}{b + 40} = 50$, so $50(b + 40) = 4000$, or $b = 40$. Therefore, the required response function is $R(x) = \dfrac{100x}{40 + x}$.

b. The response will be $R(60) = \dfrac{100(60)}{40 + 60} = 60$, or approximately 60 percent.

45. a. $f(0) = 6.85$, $g(0) = 16.58$. Because $g(0) > f(0)$, we see that more film cameras were sold in 2001 (when $t = 0$).

b. We solve the equation $f(t) = g(t)$, that is, $3.05t + 6.85 = -1.85t + 16.58$, so $4.9t = 9.73$ and $t = 1.99 \approx 2$. So sales of digital cameras first exceed those of film cameras in approximately 2003.

46. a.



b. $5x^2 + 5x + 30 = 33x + 30$, so $5x^2 - 28x = 0$, $x(5x - 28) = 0$, and $x = 0$ or $x = \frac{28}{5} = 5.6$, representing 5.6 mi/h. $g(x) = 11(5.6) + 10 = 71.6$, or 71.6 mL/lb/min.

c. The oxygen consumption of the walker is greater than that of the runner.

47. a. We are given that $T = aN + b$ where a and b are constants to be determined. The given conditions imply that $70 = 120a + b$ and $80 = 160a + b$. Subtracting the first equation from the second gives $10 = 40a$, or $a = \frac{1}{4}$. Substituting this value of a into the first equation gives $70 = 120\left(\frac{1}{4}\right) + b$, or $b = 40$. Therefore, $T = \frac{1}{4}N + 40$.

b. Solving the equation in part (a) for N, we find $\frac{1}{4}N = T - 40$, or $N = f(t) = 4T - 160$. When $T = 102$, we find $N = 4(102) - 160 = 248$, or 248 times per minute.

48. a. $f(0) = 3173$ gives $c = 3173$, $f(4) = 6132$ gives $16a + 4b + c = 6132$, and $f(6) = 7864$ gives $36a + 6b + c = 1864$. Solving, we find $a \approx 21.0417$, $b \approx 655.5833$, and $c = 3173$.

b. From part (a), we have $f(t) = 21.0417t^2 + 655.5833t + 3173$, so the number of farmers' markets in 2014 is projected to be $f(8) = 21.0417(8)^2 + 655.5833(8) + 3173 = 9764.3352$, or approximately 9764.

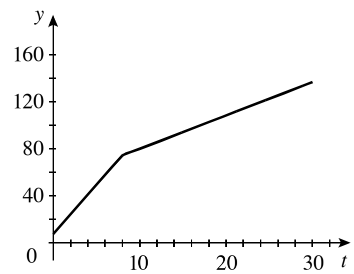
49. a. We have $f(0) = c = 1547$, $f(2) = 4a + 2b + c = 1802$, and $f(4) = 16a + 4b + c = 2403$. Solving this system of equations gives $a = 43.25$, $b = 41$, and $c = 1547$.

b. From part (a), we have $f(t) = 43.25t^2 + 41t + 1547$, so the number of craft-beer breweries in 2014 is projected to be $f(6) = 43.25(6)^2 + 41(6) + 1547 = 3350$.

50. The slope of the line is $m = \dfrac{S - C}{n}$. Therefore, an equation of the line is $y - C = \dfrac{S - C}{n}(t - 0)$. Letting $y = V(t)$, we have $V(t) = C - \dfrac{C - S}{n}t$.

51. Using the formula given in Exercise 50, we have
$V(2) = 100{,}000 - \frac{100{,}000 - 30{,}000}{5}(2) = 100{,}000 - \frac{70{,}000}{5}(2) = 72{,}000$, or \$72,000.

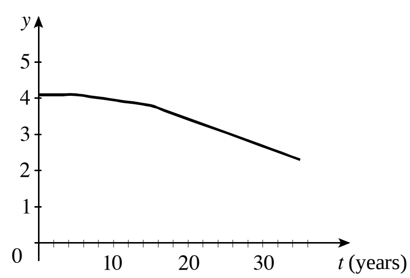
52. a.



b. $f(0) = 8.37(0) + 7.44 = 7.44$, or \$7.44/kilo.
$f(20) = 2.84(20) + 51.68 = 108.48$, or \$108.48/kilo.

53. The total cost by 2011 is given by $f(1) = 5$, or \$5 billion. The total cost by 2015 is given by $f(5) = -0.5278\left(5^3\right) + 3.012\left(5^2\right) + 49.23(5) - 103.29 = 152.185$, or approximately \$152 billion.

54. a.

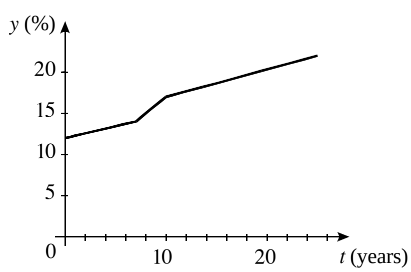


b. At the beginning of 2005, the ratio will be $f(10) = -0.03(10) + 4.25 = 3.95$. At the beginning of 2020, the ratio will be $f(25) = -0.075(25) + 4.925 = 3.05$.

c. The ratio is constant from 1995 to 2000.

d. The decline of the ratio is greatest from 2010 through 2030. It is $\frac{f(35) - f(15)}{35 - 15} = \frac{2.3 - 3.8}{20} = -0.075$.

55. a.

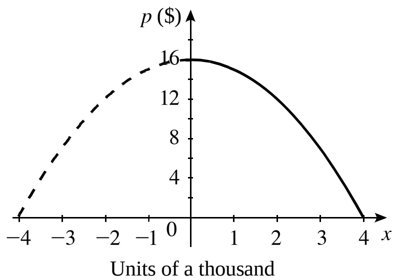


b. $f(5) = \frac{2}{7}(5) + 12 = \frac{10}{7} + 12 \approx 13.43$, or approximately 13.43%. $f(25) = \frac{1}{3}(25) + \frac{41}{3} = 22$, or 22%.

56. a. $f(0) = 5.6$ and $g(0) = 22.5$. Because $g(0) > f(0)$, we conclude that more VCRs than DVD players were sold in 2001.

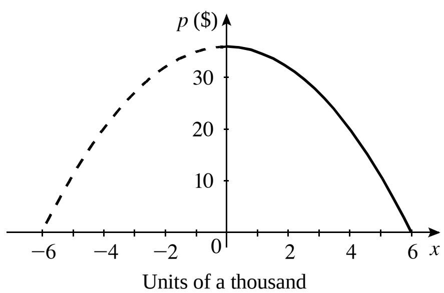
b. We solve the equations $f(t) = g(t)$ over each of the subintervals. $5.6 + 5.6t = -9.6t + 22.5$ for $0 \le t \le 1$. We solve to find $15.2t = 16.9$, so $t \approx 1.11$. This is outside the range for t, so we reject it. $5.6 + 5.6t = -0.5t + 13.4$ for $1 < t \le 2$, so $6.1t = 7.8$, and thus $t \approx 1.28$. So sales of DVD players first exceed those of VCRs at $t \approx 1.3$, or in early 2002.

57. a.



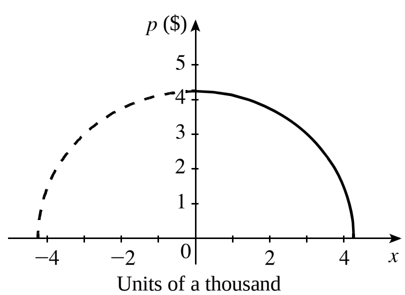
b. If $p = 7$, we have $7 = -x^2 + 16$, or $x^2 = 9$, so that $x = \pm 3$. Therefore, the quantity demanded when the unit price is \$7 is 3000 units.

58. a.



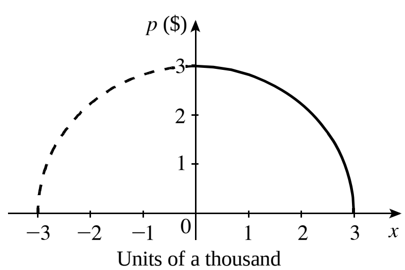
b. If $p = 11$, we have $11 = -x^2 + 36$, or $x^2 = 25$, so that $x = \pm 5$. Therefore, the quantity demanded when the unit price is \$11 is 5000 units.

59. a.



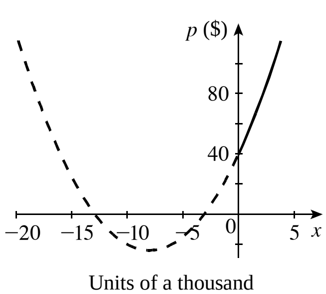
b. If $p = 3$, then $3 = \sqrt{18 - x^2}$, and $9 = 18 - x^2$, so that $x^2 = 9$ and $x = \pm\, 3$. Therefore, the quantity demanded when the unit price is \$3 is 3000 units.

60. a.



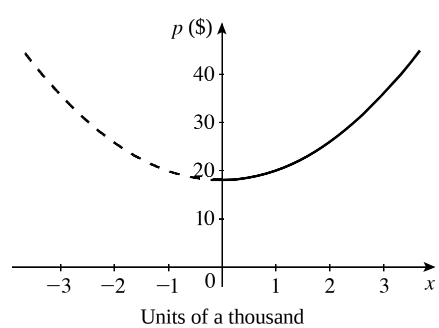
b. If $p = 2$, then $2 = \sqrt{9 - x^2}$, and $4 = 9 - x^2$, so that $x^2 = 5$ and and $x = \pm\sqrt{5}$, or $x \approx \pm 2.236$, Therefore, the quantity demanded when the unit price is \$2 is approximately 2236 units.

61. a.



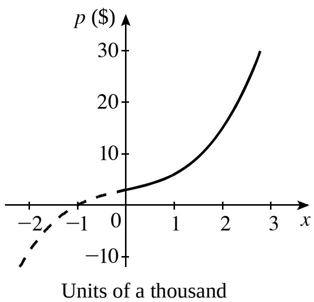
b. If $x = 2$, then $p = 2^2 + 16\,(2) + 40 = 76$, or \$76.

62. a.



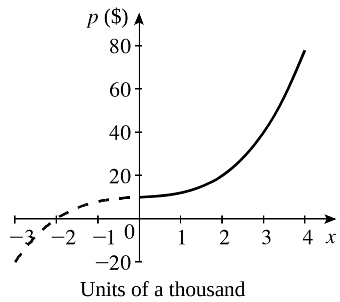
b. If $x = 2$, then $p = 2\,(2)^2 + 18 = 26$, or \$26.

63. a.



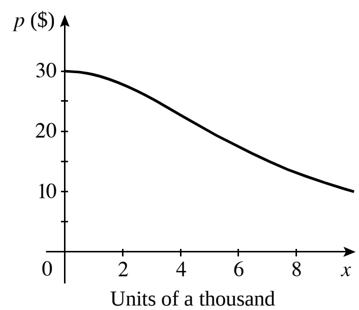
b. $p = 2^3 + 2(2) + 3 = 15$, or \$15.

64. a.



b. $p = 2^3 + 2 + 10 = 20$, or \$20.

65. a.

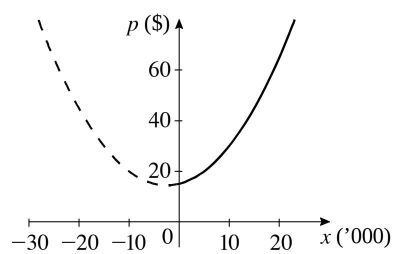


b. Substituting $x = 10$ into the demand function, we have

$$p = \frac{30}{0.02(10)^2 + 1} = \frac{30}{3} = 10, \text{ or } \$10.$$

66. Substituting $x = 6$ and $p = 8$ into the given equation gives $8 = \sqrt{-36a + b}$, or $-36a + b = 64$. Next, substituting $x = 8$ and $p = 6$ into the equation gives $6 = \sqrt{-64a + b}$, or $-64a + b = 36$. Solving the system $\begin{cases} -36a + b = 64 \\ -64a + b = 36 \end{cases}$ for a and b, we find $a = 1$ and $b = 100$. Therefore the demand equation is $p = \sqrt{-x^2 + 100}$. When the unit price is set at \$7.50, we have $7.5 = \sqrt{-x^2 + 100}$, or $56.25 = -x^2 + 100$ from which we deduce that $x \approx \pm 6.614$. Thus, the quantity demanded is approximately 6614 units.

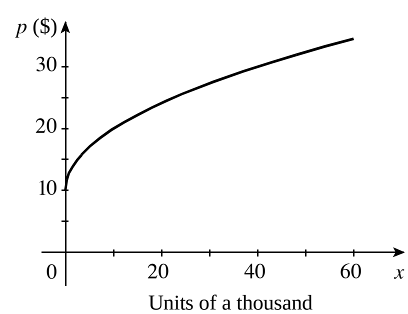
67. a.



b. If $x = 5$, then

$p = 0.1(5)^2 + 0.5(5) + 15 = 20$, or \$20.

68. Substituting $x = 10{,}000$ and $p = 20$ into the given equation yields $20 = a\sqrt{10{,}000} + b = 100a + b$. Next, substituting $x = 62{,}500$ and $p = 35$ into the equation yields $35 = a\sqrt{62{,}500} + b = 250a + b$. Subtracting the first equation from the second yields $15 = 150a$, or $a = \frac{1}{10}$. Substituting this value of a into the first equation gives $b = 10$. Therefore, the required equation is $p = \frac{1}{10}\sqrt{x} + 10$. Substituting $x = 40{,}000$ into the supply equation yields $p = \frac{1}{10}\sqrt{40{,}000} + 10 = 30$, or \$30.



69. a. We solve the system of equations $p = cx + d$ and $p = ax + b$. Substituting the first equation into the second gives $cx + d = ax + d$, so $(c - a)x = b - d$ and $x = \frac{b-d}{c-a}$. Because $a < 0$ and $c > 0$, $c - a \neq 0$ and x is well-defined. Substituting this value of x into the second equation, we obtain $p = a\left(\frac{b-d}{c-a}\right) + b = \frac{ab - ad + bc - ab}{c-a} = \frac{bc - ad}{c-a}$. Therefore, the equilibrium quantity is $\frac{b-d}{c-a}$ and the equilibrium price is $\frac{bc-ad}{c-a}$.

b. If c is increased, the denominator in the expression for x increases and so x gets smaller. At the same time, the first term in the first equation for p decreases and so p gets larger. This analysis shows that if the unit price for producing the product is increased, then the equilibrium quantity decreases while the equilibrium price increases.

c. If b is decreased, the numerator of the expression for x decreases while the denominator stays the same. Therefore, x decreases. The expression for p also shows that p decreases. This analysis shows that if the (theoretical) upper bound for the unit price of a commodity is lowered, then both the equilibrium quantity and the equilibrium price drop.

70. We solve the system of equations $p = -x^2 - 2x + 100$ and $p = 8x + 25$. Thus, $-x^2 - 2x + 100 = 8x + 25$, or $x^2 + 10x - 75 = 0$. Factoring this equation, we have $(x + 15)(x - 5) = 0$. Therefore, $x = -15$ or $x = 5$. Rejecting the negative root, we have $x = 5$, and the corresponding value of p is $p = 8(5) + 25 = 65$. We conclude that the equilibrium quantity is 5000 and the equilibrium price is \$65.

71. We solve the equation $-2x^2 + 80 = 15x + 30$, or $2x^2 + 15x - 50 = 0$ for x. Thus, $(2x - 5)(x + 10) = 0$, and so $x = \frac{5}{2}$ or $x = -10$. Rejecting the negative root, we have $x = \frac{5}{2}$. The corresponding value of p is $p = -2\left(\frac{5}{2}\right)^2 + 80 = 67.5$. We conclude that the equilibrium quantity is 2500 and the equilibrium price is \$67.50.

72. We solve the system $\begin{cases} p = 60 - 2x^2 \\ p = x^2 + 9x + 30 \end{cases}$ Equating the right-hand sides, we have $x^2 + 9x + 30 = 60 - 2x^2$, so $3x^2 + 9x - 30 = 0$, $x^2 + 3x - 10 = 0$, and $(x + 5)(x - 2) = 0$, giving $x = -5$ or $x = 2$. We take $x = 2$. The corresponding value of p is 52, so the equilibrium quantity is 2000 and the equilibrium price is \$52.

73. Solving both equations for x, we have $x = -\frac{11}{3}p + 22$ and $x = 2p^2 + p - 10$. Equating the right-hand sides, we have $-\frac{11}{3}p + 22 = 2p^2 + p - 10$, or $-11p + 66 = 6p^2 + 3p - 30$, and so $6p^2 + 14p - 96 = 0$. Dividing this last equation by 2 and then factoring, we have $(3p + 16)(p - 3) = 0$, so $p = 3$ is the only valid solution. The corresponding value of x is $2(3)^2 + 3 - 10 = 11$. We conclude that the equilibrium quantity is 11,000 and the equilibrium price is \$3.

74. Equating the right-hand sides of the two equations, we have $0.1x^2 + 2x + 20 = -0.1x^2 - x + 40$, so $0.2x^2 + 3x - 20 = 0$, $2x^2 + 30x - 200 = 0$, $x^2 + 15x - 100 = 0$, and $(x + 20)(x - 5) = 0$. Therefore the only valid solution is $x = 5$. Substituting $x = 5$ into the first equation gives $p = -0.1(25) - 5 + 40 = 32.5$. Therefore, the equilibrium quantity is 500 tents (x is measured in hundreds) and the equilibrium price is \$32.50.

75. Equating the right-hand sides of the two equations, we have $144 - x^2 = 48 + \frac{1}{2}x^2$, so $288 - 2x^2 = 96 + x^2$, $3x^2 = 192$, and $x^2 = 64$. Therefore, $x = \pm 8$. We take $x = 8$, and the corresponding value of p is $144 - 8^2 = 80$. We conclude that the equilibrium quantity is 8000 tires and the equilibrium price is \$80.

76. Because there is 80 feet of fencing available, $2x + 2y = 80$, so $x + y = 40$ and $y = 40 - x$. Then the area of the garden is given by $f = xy = x(40 - x) = 40x - x^2$. The domain of f is $[0, 40]$.

77. The area of Juanita's garden is 250 ft^2. Therefore $xy = 250$ and $y = \frac{250}{x}$. The amount of fencing needed is given by $2x + 2y$. Therefore, $f = 2x + 2\left(\frac{250}{x}\right) = 2x + \frac{500}{x}$. The domain of f is $x > 0$.

78. The volume of the box is given by area of the base times the height of the box. Thus, $V = f(x) = (15 - 2x)(8 - 2x)x$.

79. Because the volume of the box is the area of the base times the height of the box, we have $V = x^2y = 20$. Thus, we have $y = \frac{20}{x^2}$. Next, the amount of material used in constructing the box is given by the area of the base of the box, plus the area of the four sides, plus the area of the top of the box; that is, $A = x^2 + 4xy + x^2$. Then, the cost of constructing the box is given by $f(x) = 0.30x^2 + 0.40x \cdot \frac{20}{x^2} + 0.20x^2 = 0.5x^2 + \frac{8}{x}$, where $f(x)$ is measured in dollars and $f(x) > 0$.

80. Because the perimeter of a circle is $2\pi r$, we know that the perimeter of the semicircle is πx. Next, the perimeter of the rectangular portion of the window is given by $2y + 2x$, so the perimeter of the Norman window is $\pi x + 2y + 2x$ and $\pi x + 2y + 2x = 28$, or $y = \frac{1}{2}(28 - \pi x - 2x)$. Because the area of the window is given by $2xy + \frac{1}{2}\pi x^2$, we see that $A = 2xy + \frac{1}{2}\pi x^2$. Substituting the value of y found earlier, we see that

$$A = f(x) = x(28 - \pi x - 2x) + \tfrac{1}{2}\pi x^2 = \tfrac{1}{2}\pi x^2 + 28x - \pi x^2 - 2x^2 = 28x - \tfrac{\pi}{2}x^2 - 2x^2$$
$$= 28x - \left(\frac{\pi}{2} + 2\right)x^2.$$

81. The average yield of the apple orchard is 36 bushels/tree when the density is 22 trees/acre. Let x be the unit increase in tree density beyond 22. Then the yield of the apple orchard in bushels/acre is given by $(22 + x)(36 - 2x)$.

82. $xy = 50$ and so $y = \frac{50}{x}$. The area of the printed page is $A = (x - 1)(y - 2) = (x - 1)\left(\frac{50}{x} - 2\right) = -2x + 52 - \frac{50}{x}$, so the required function is $f(x) = -2x + 52 - \frac{50}{x}$. We must have $x > 0$, $x - 1 \geq 0$, and and $\frac{50}{x} - 2 \geq 2$. The last inequality is solved as follows: $\frac{50}{x} \geq 4$, so $\frac{x}{50} \leq \frac{1}{4}$, so $x \leq \frac{50}{4} = \frac{25}{2}$. Thus, the domain is $\left[1, \frac{25}{2}\right]$.

83. a. Let x denote the number of bottles sold beyond 10,000 bottles. Then $P(x) = (10{,}000 + x)(5 - 0.0002x) = -0.0002x^2 + 3x + 50{,}000$.

b. He can expect a profit of $P(6000) = -0.0002\left(6000^2\right) + 3(6000) + 50{,}000 = 60{,}800$, or \$60,800.

84. a. Let x denote the number of people beyond 20 who sign up for the cruise. Then the revenue is $R(x) = (20 + x)(600 - 4x) = -4x^2 + 520x + 12{,}000$.

b. $R(40) = -4\left(40^2\right) + 520(40) + 12{,}000 = 26{,}400$, or \$26,400.

c. $R(60) = -4\left(60^2\right) + 520(60) + 12{,}000 = 28{,}800$, or \$28,800.

85. False. $f(x) = 3x^{3/4} + x^{1/2} + 1$ is not a polynomial function. The powers of x must be nonnegative integers.

86. True. If $P(x)$ is a polynomial function, then $P(x) = \frac{P(x)}{1}$ and so it is a rational function. The converse is false. For example, $R(x) = \frac{x+1}{x-1}$ is a rational function that is not a polynomial.

87. False. $f(x) = x^{1/2}$ is not defined for negative values of x.

88. False. A power function has the form x^r, where r is a real number.

Using Technology page 98

1. $(-3.0414, 0.1503)$, $(3.0414, 7.4497)$.

2. $(-5.3852, 9.8007)$, $(5.3852, -4.2007)$.

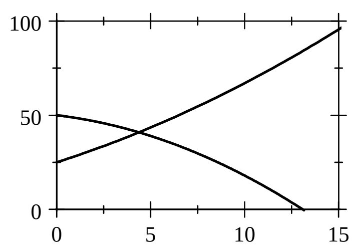
3. $(-2.3371, 2.4117)$, $(6.0514, -2.5015)$.

4. $(-2.5863, -0.3585)$, $(6.1863, -4.5694)$.

5. $(-1.0219, -6.3461)$, $(1.2414, -1.5931)$, and $(5.7805, 7.9391)$.

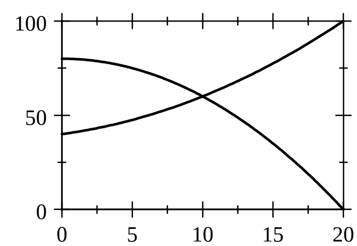
6. $(-0.0484, 2.0609)$, $(2.0823, 2.8986)$, and $(4.9661, 1.1405)$.

7. a.



b. 438 wall clocks; \$40.92.

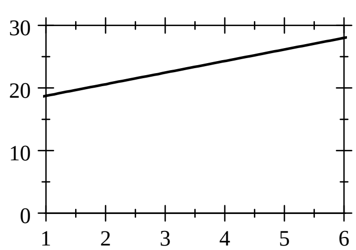
8. a.



b. 1000 cameras; \$60.00.

9. a. $f(t) = 1.85t + 16.9$.

b.



c.

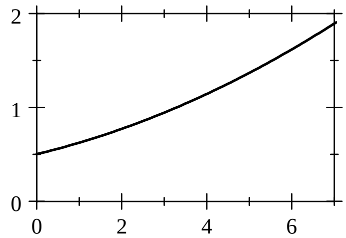
t	y
1	18.8
2	20.6
3	22.5
4	24.3
5	26.2
6	28.0

These values are close to the given data.

d. $f(8) = 1.85(8) + 16.9 = 31.7$ gallons.

10. **a.** $f(t) = 0.0128t^2 + 0.109t + 0.50$.

b.



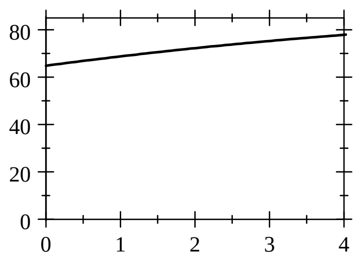
c.

t	y
0	0.50
3	0.94
6	1.61
7	1.89

These values are close to the given data.

11. **a.** $f(t) = -0.221t^2 + 4.14t + 64.8$.

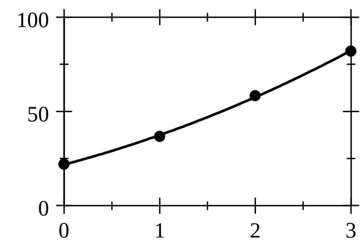
b.



c. 77.8 million

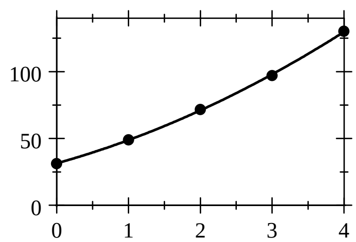
12. **a.** $f(t) = 2.25x^2 + 13.41x + 21.76$.

b.



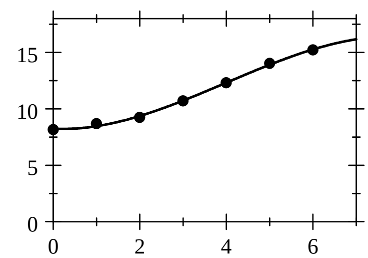
13. **a.** $f(t) = 2.4t^2 + 15t + 31.4$.

b.



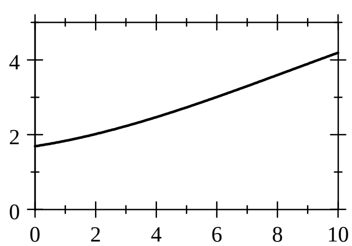
14. **a.** $f(t) = -0.038167t^3 + 0.45713t^2 - 0.19758t + 8.2457$.

b.



15. **a.** $f(t) = -0.00081t^3 + 0.0206t^2 + 0.125t + 1.69$.

b.



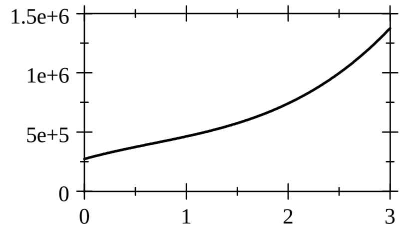
c.

t	y
1	1.8
5	2.7
10	4.2

The revenues were \$1.8 trillion in 2001, \$2.7 trillion in 2005, and \$4.2 trillion in 2010.

16. **a.** $y = 44{,}560t^3 - 89{,}394t^2 + 234{,}633t + 273{,}288$.

b.

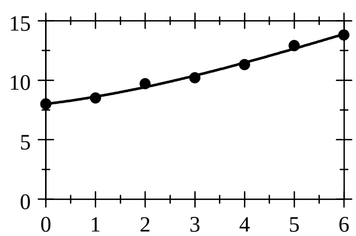


c.

t	$f(t)$
0	273,288
1	463,087
2	741,458
3	1,375,761

17. **a.** $f(t) = -0.0056t^3 + 0.112t^2 + 0.51t + 8$.

b.

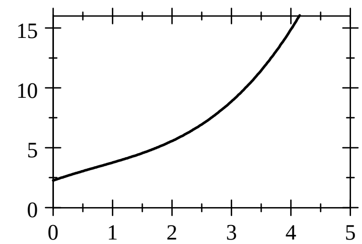


c.

t	0	3	6
$f(t)$	8	10.4	13.9

18. **a.** $f(t) = 0.2t^3 - 0.45t^2 + 1.75t + 2.26$.

b.

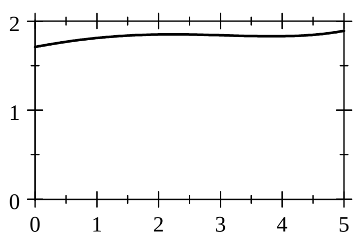


c.

t	0	1	2	3	4
$f(t)$	2.3	3.8	5.6	8.9	14.9

19. **a.** $f(t) = 0.00125t^4 - 0.0051t^3 - 0.0243t^2 + 0.129t + 1.71$.

b.



c.

t	0	1	2	3	4	5
$f(t)$	1.71	1.81	1.85	1.84	1.83	1.89

d. The average amount of nicotine in 2005 is $f(6) = 2.128$, or approximately 2.13 mg/cigarette.

20. $A(t) = 0.000008140t^4 - 0.00043833t^3 - 0.0001305t^2 + 0.02202t + 2.612$.

2.4 Limits

Concept Questions

page 115

1. The values of $f(x)$ can be made as close to 3 as we please by taking x sufficiently close to $x = 2$.

2. **a.** Nothing. Whether $f(3)$ is defined or not does not depend on $\lim_{x \to 3} f(x)$.

b. Nothing. $\lim_{x \to 2} f(x)$ has nothing to do with the value of f at $x = 2$.

3. **a.** $\lim_{x\to 4}\sqrt{x}\left(2x^2+1\right) = \lim_{x\to 4}\left(\sqrt{x}\right)\lim_{x\to 4}\left(2x^2+1\right)$ (Property 4)

$= \sqrt{4}\left[2(4)^2+1\right]$ (Properties 1 and 3)

$= 66$

b. $\lim_{x\to 1}\left(\frac{2x^2+x+5}{x^4+1}\right)^{3/2} = \left(\lim_{x\to 1}\frac{2x^2+x+5}{x^4+1}\right)^{3/2}$ (Property 1)

$= \left(\frac{2+1+5}{1+1}\right)^{3/2}$ (Properties 2, 3, and 5)

$= 4^{3/2} = 8$

4. A limit that has the form $\lim_{x\to a}\frac{f(x)}{g(x)} = \frac{0}{0}$. For example, $\lim_{x\to 3}\frac{x^2-9}{x-3}$.

5. $\lim_{x\to\infty} f(x) = L$ means $f(x)$ can be made as close to L as we please by taking x sufficiently large. $\lim_{x\to -\infty} f(x) = M$ means $f(x)$ can be made as close to M as we please by taking negative x as large as we please in absolute value.

Exercises page 115

1. $\lim_{x\to -2} f(x) = 3$.

2. $\lim_{x\to 1} f(x) = 2$.

3. $\lim_{x\to 3} f(x) = 3$.

4. $\lim_{x\to 1} f(x)$ does not exist. If we consider any value of x to the right of $x = 1$, we find that $f(x) = 3$. On the other hand, if we consider values of x to the left of $x = 1$, $f(x) \le 1.5$, and we conclude that $f(x)$ does not approach a fixed number as x approaches 1.

5. $\lim_{x\to -2} f(x) = 3$.

6. $\lim_{x\to -2} f(x) = 3$.

7. The limit does not exist. If we consider any value of x to the right of $x = -2$, $f(x) \le 0$. If we consider values of x to the left of $x = -2$, $f(x) \ge 0$. Because $f(x)$ does not approach any one number as x approaches $x = -2$, we conclude that the limit does not exist.

8. The limit does not exist.

9.

x	1.9	1.99	1.999	2.001	2.01	2.1
$f(x)$	4.61	4.9601	4.9960	5.004	5.0401	5.41

$\lim_{x\to 2}\left(x^2+1\right) = 5$.

10.

x	0.9	0.99	0.999	1.001	1.01	1.1
$f(x)$	0.62	0.9602	0.996002	1.004002	1.0402	1.42

$\lim_{x\to 1}\left(2x^2 - 1\right) = 1.$

11.

x	−0.1	−0.01	−0.001	0.001	0.01	0.1
$f(x)$	−1	−1	−1	1	1	1

The limit does not exist.

12.

x	0.9	0.99	0.999	1.001	1.01	1.1
$f(x)$	−1	−1	−1	1	1	1

The limit does not exist.

13.

x	0.9	0.99	0.999	1.001	1.01	1.1
$f(x)$	100	10,000	1,000,000	1,000,000	10,000	100

The limit does not exist.

14.

x	1.9	1.99	1.999	2.001	2.01	2.1
$f(x)$	−10	−100	−1000	1000	100	10

The limit does not exist.

15.

x	0.9	0.99	0.999	1.001	1.01	1.1
$f(x)$	2.9	2.99	2.999	3.001	3.01	3.1

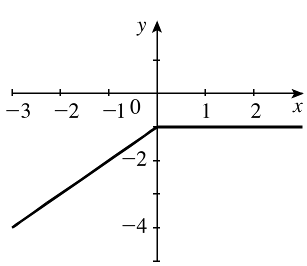
$\lim_{x\to 1}\dfrac{x^2 + x - 2}{x - 1} = 3.$

16.

x	0.9	0.99	0.999	1.001	1.01	1.1
$f(x)$	1	1	1	1	1	1

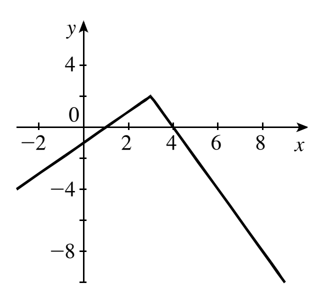
$\lim_{x\to 1}\dfrac{x - 1}{x - 1} = 1.$

17.

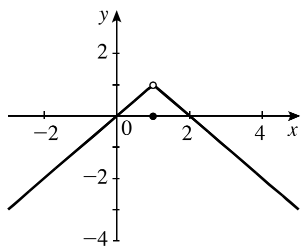


$\lim_{x\to 0} f(x) = -1.$

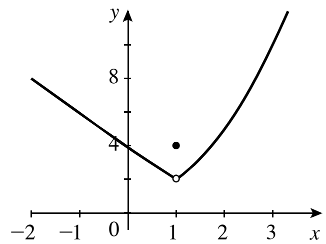
18.



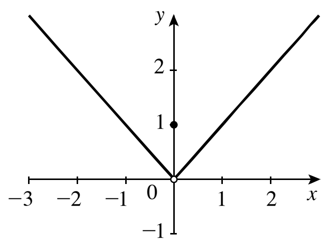
$\lim_{x\to 3} f(x) = 2.$

19. 

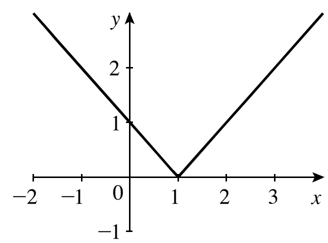
$\lim_{x \to 1} f(x) = 1.$

20. 

$\lim_{x \to 1} f(x) = 2.$

21. 

$\lim_{x \to 0} f(x) = 0.$

22. 

$\lim_{x \to 1} f(x) = 0.$

23. $\lim_{x \to 2} 3 = 3.$

24. $\lim_{x \to -2} -3 = -3.$

25. $\lim_{x \to 3} x = 3.$

26. $\lim_{x \to -2} -3x = -3(-2) = 6.$

27. $\lim_{x \to 1} \left(1 - 2x^2\right) = 1 - 2(1)^2 = -1.$

28. $\lim_{t \to 3} \left(4t^2 - 2t + 1\right) = 4(3)^2 - 2(3) + 1 = 31.$

29. $\lim_{x \to 1} \left(2x^3 - 3x^2 + x + 2\right) = 2(1)^3 - 3(1)^2 + 1 + 2 = 2.$

30. $\lim_{x \to 0} \left(4x^5 - 20x^2 + 2x + 1\right) = 4(0)^5 - 20(0)^2 + 2(0) + 1 = 1.$

31. $\lim_{s \to 0} \left(2s^2 - 1\right)(2s + 4) = (-1)(4) = -4.$

32. $\lim_{x \to 2} \left(x^2 + 1\right)\left(x^2 - 4\right) = \left(2^2 + 1\right)\left(2^2 - 4\right) = 0.$

33. $\lim_{x \to 2} \frac{2x + 1}{x + 2} = \frac{2(2) + 1}{2 + 2} = \frac{5}{4}.$

34. $\lim_{x \to 1} \frac{x^3 + 1}{2x^3 + 2} = \frac{1^3 + 1}{2\left(1^3\right) + 2} = \frac{2}{4} = \frac{1}{2}.$

35. $\lim_{x \to 2} \sqrt{x + 2} = \sqrt{2 + 2} = 2.$

36. $\lim_{x \to -2} \sqrt[3]{5x + 2} = \sqrt[3]{5(-2) + 2} = \sqrt[3]{-8} = -2.$

37. $\lim_{x \to -3} \sqrt{2x^4 + x^2} = \sqrt{2(-3)^4 + (-3)^2} = \sqrt{162 + 9} = \sqrt{171} = 3\sqrt{19}.$

38. $\lim_{x \to 2} \sqrt{\frac{2x^3 + 4}{x^2 + 1}} = \sqrt{\frac{2(8) + 4}{4 + 1}} = 2.$

39. $\lim_{x \to -1} \frac{\sqrt{x^2 + 8}}{2x + 4} = \frac{\sqrt{(-1)^2 + 8}}{2(-1) + 4} = \frac{\sqrt{9}}{2} = \frac{3}{2}.$

40. $\lim_{x \to 3} \frac{x\sqrt{x^2 + 7}}{2x - \sqrt{2x + 3}} = \frac{3\sqrt{3^2 + 7}}{2(3) - \sqrt{2(3) + 3}} = \frac{12}{3} = 4.$

41. $\lim_{x\to a}\left[f(x)-g(x)\right]=\lim_{x\to a}f(x)-\lim_{x\to a}g(x)$
$=3-4=-1.$

42. $\lim_{x\to a}2f(x)=2(3)=6.$

43. $\lim_{x\to a}\left[2f(x)-3g(x)\right]=\lim_{x\to a}2f(x)-\lim_{x\to a}3g(x)$
$=2(3)-3(4)=-6.$

44. $\lim_{x\to a}\left[f(x)g(x)\right]=\lim_{x\to a}f(x)\cdot\lim_{x\to a}g(x)=3\cdot 4$
$=12.$

45. $\lim_{x\to a}\sqrt{g(x)}=\lim_{x\to a}\sqrt{4}=2.$

46. $\lim_{x\to a}\sqrt[3]{5f(x)+3g(x)}=\sqrt[3]{5(3)+3(4)}=\sqrt[3]{27}=3.$

47. $\lim_{x\to a}\dfrac{2f(x)-g(x)}{f(x)g(x)}=\dfrac{2(3)-(4)}{(3)(4)}=\dfrac{2}{12}=\dfrac{1}{6}.$

48. $\lim_{x\to a}\dfrac{g(x)-f(x)}{f(x)+\sqrt{g(x)}}=\dfrac{4-3}{3+2}=\dfrac{1}{5}.$

49. $\lim_{x\to 1}\dfrac{x^2-1}{x-1}=\lim_{x\to 1}\dfrac{(x-1)(x+1)}{x-1}=\lim_{x\to 1}(x+1)$
$=1+1=2.$

50. $\lim_{x\to -2}\dfrac{x^2-4}{x+2}=\lim_{x\to -2}\dfrac{(x-2)(x+2)}{x+2}$
$=\lim_{x\to -2}(x-2)=-2-2=-4.$

51. $\lim_{x\to 0}\dfrac{x^2-x}{x}=\lim_{x\to 0}\dfrac{x(x-1)}{x}=\lim_{x\to 0}(x-1)$
$=0-1=-1.$

52. $\lim_{x\to 0}\dfrac{2x^2-3x}{x}=\lim_{x\to 0}\dfrac{x(2x-3)}{x}=\lim_{x\to 0}(2x-3)$
$=-3.$

53. $\lim_{x\to -5}\dfrac{x^2-25}{x+5}=\lim_{x\to -5}\dfrac{(x+5)(x-5)}{x+5}$
$=\lim_{x\to -5}(x-5)=-10.$

54. $\lim_{b\to -3}\dfrac{b+1}{b+3}$ does not exist.

55. $\lim_{x\to 1}\dfrac{x}{x-1}$ does not exist.

56. $\lim_{x\to 2}\dfrac{x+2}{x-2}$ does not exist.

57. $\lim_{x\to -2}\dfrac{x^2-x-6}{x^2+x-2}=\lim_{x\to -2}\dfrac{(x-3)(x+2)}{(x+2)(x-1)}=\lim_{x\to -2}\dfrac{x-3}{x-1}=\dfrac{-2-3}{-2-1}=\dfrac{5}{3}.$

58. $\lim_{z\to 2}\dfrac{z^3-8}{z-2}=\lim_{z\to 2}\dfrac{(z-2)\left(z^2+2z+4\right)}{z-2}=\lim_{z\to 2}\left(z^2+2z+4\right)=2^2+2(2)+4=12.$

59. $\lim_{x\to 1}\dfrac{\sqrt{x}-1}{x-1}=\lim_{x\to 1}\dfrac{\sqrt{x}-1}{x-1}\cdot\dfrac{\sqrt{x}+1}{\sqrt{x}+1}=\lim_{x\to 1}\dfrac{x-1}{(x-1)\left(\sqrt{x}+1\right)}=\lim_{x\to 1}\dfrac{1}{\sqrt{x}+1}=\dfrac{1}{2}.$

60. $\lim_{x\to 4}\dfrac{x-4}{\sqrt{x}-2}=\lim_{x\to 4}\dfrac{x-4}{\sqrt{x}-2}\cdot\dfrac{\sqrt{x}+2}{\sqrt{x}+2}=\lim_{x\to 4}\sqrt{x}+2=2+2=4.$

61. $\lim_{x\to 1}\dfrac{x-1}{x^3+x^2-2x}=\lim_{x\to 1}\dfrac{x-1}{x(x-1)(x+2)}=\lim_{x\to 1}\dfrac{1}{x(x+2)}=\dfrac{1}{3}.$

62. $\lim_{x\to -2}\dfrac{4-x^2}{2x^2+x^3}=\lim_{x\to -2}\dfrac{(2-x)(2+x)}{x^2(2+x)}=\lim_{x\to -2}\dfrac{2-x}{x^2}=\dfrac{2-(-2)}{(-2)^2}=1.$

63. $\lim_{x\to\infty}f(x)=\infty$ (does not exist) and $\lim_{x\to -\infty}f(x)=\infty$ (does not exist).

64. $\lim_{x\to\infty} f(x) = \infty$ (does not exist) and $\lim_{x\to-\infty} f(x) = -\infty$ (does not exist).

65. $\lim_{x\to\infty} f(x) = 0$ and $\lim_{x\to-\infty} f(x) = 0$.

66. $\lim_{x\to\infty} f(x) = 1$ and $\lim_{x\to-\infty} f(x) = 1$.

67. $\lim_{x\to\infty} f(x) = -\infty$ (does not exist) and $\lim_{x\to-\infty} f(x) = -\infty$ (does not exist).

68. $\lim_{x\to\infty} f(x) = 1$ and $\lim_{x\to-\infty} f(x) = \infty$ (does not exist).

69. $f(x) = \dfrac{1}{x^2+1}$.

x	1	10	100	1000
$f(x)$	0.5	0.009901	0.0001	0.000001

x	-1	-10	-100	-1000
$f(x)$	0.5	0.009901	0.0001	0.000001

$\lim_{x\to\infty} f(x) = \lim_{x\to-\infty} f(x) = 0$.

70. $f(x) = \dfrac{2x}{x+1}$.

x	1	10	100	1000
$f(x)$	1	1.818	1.980	1.998

x	-5	-10	-100	-1000
$f(x)$	2.5	2.222	2.020	2.002

$\lim_{x\to\infty} f(x) = \lim_{x\to-\infty} f(x) = 2$.

71. $f(x) = 3x^3 - x^2 + 10$.

x	1	5	10	100	1000
$f(x)$	12	360	2910	2.99×10^6	2.999×10^9

x	-1	-5	-10	-100	-1000
$f(x)$	6	-390	-3090	-3.01×10^6	-3.0×10^9

$\lim_{x\to\infty} f(x) = \infty$ (does not exist) and $\lim_{x\to-\infty} f(x) = -\infty$ (does not exist).

72. $f(x) = \dfrac{|x|}{x}$

x	1	10	100
$f(x)$	1	1	1

x	-1	-10	-100
$f(x)$	-1	-1	-1

$\lim_{x\to\infty} f(x) = 1$ and $\lim_{x\to-\infty} f(x) = -1$.

73. $\displaystyle\lim_{x\to\infty} \frac{3x+2}{x-5} = \lim_{x\to\infty} \frac{3+\frac{2}{x}}{1-\frac{5}{x}} = \frac{3}{1} = 3.$

74. $\displaystyle\lim_{x\to-\infty}\frac{4x^2-1}{x+2}=\lim_{x\to-\infty}\frac{4x-\frac{1}{x}}{1+\frac{2}{x}}=-\infty$; that is, the limit does not exist.

75. $\displaystyle\lim_{x\to-\infty}\frac{3x^3+x^2+1}{x^3+1}=\lim_{x\to-\infty}\frac{3+\frac{1}{x}+\frac{1}{x^3}}{1+\frac{1}{x^3}}=3.$

76. $\displaystyle\lim_{x\to\infty}\frac{2x^2+3x+1}{x^4-x^2}=\lim_{x\to\infty}\frac{\frac{2}{x^2}+\frac{3}{x^3}+\frac{1}{x^4}}{1-\frac{1}{x^2}}=0.$

77. $\displaystyle\lim_{x\to-\infty}\frac{x^4+1}{x^3-1}=\lim_{x\to-\infty}\frac{x+\frac{1}{x^3}}{1-\frac{1}{x^3}}=-\infty$; that is, the limit does not exist.

78. $\displaystyle\lim_{x\to\infty}\frac{4x^4-3x^2+1}{2x^4+x^3+x^2+x+1}=\lim_{x\to\infty}\frac{4-\frac{3}{x^2}+\frac{1}{x^4}}{2+\frac{1}{x}+\frac{1}{x^2}+\frac{1}{x^3}+\frac{1}{x^4}}=2.$

79. $\displaystyle\lim_{x\to\infty}\frac{x^5-x^3+x-1}{x^6+2x^2+1}=\lim_{x\to\infty}\frac{\frac{1}{x}-\frac{1}{x^3}+\frac{1}{x^5}-\frac{1}{x^6}}{1+\frac{2}{x^4}+\frac{1}{x^6}}=0.$

80. $\displaystyle\lim_{x\to\infty}\frac{2x^2-1}{x^3+x^2+1}=\lim_{x\to\infty}\frac{\frac{2}{x}-\frac{1}{x^3}}{1+\frac{1}{x}+\frac{1}{x^3}}=0.$

81. a. The cost of removing 50% of the pollutant is $C(50)=\dfrac{0.5(50)}{100-50}=0.5$, or \$500,000. Similarly, we find that the cost of removing 60%, 70%, 80%, 90%, and 95% of the pollutant is \$750,000, \$1,166,667, \$2,000,000, \$4,500,000, and \$9,500,000, respectively.

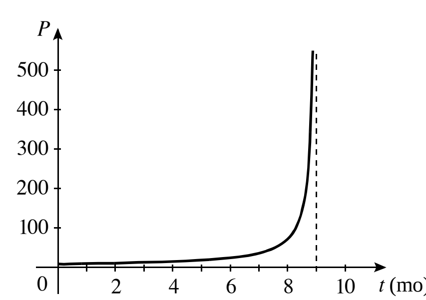
b. $\displaystyle\lim_{x\to100}\frac{0.5x}{100-x}=\infty$, which means that the cost of removing the pollutant increases without bound if we wish to remove almost all of the pollutant.

82. a. The number present initially is given by $P(0)=\dfrac{72}{9-0}=8.$

b. As t approaches 9 (remember that $0<t<9$), the denominator approaches 0 while the numerator remains constant at 72. Therefore, $P(t)$ gets larger and larger. Thus,

$$\lim_{t\to9}P(t)=\lim_{t\to9}\frac{72}{9-t}=\infty.$$

c.



83. $\lim_{x\to\infty} \overline{C}(x) = \lim_{x\to\infty}\left(2.2 + \frac{2500}{x}\right) = 2.2$, or \$2.20 per DVD. In the long run, the average cost of producing x DVDs approaches \$2.20/disc.

84. $\lim_{t\to\infty} C(t) = \lim_{t\to\infty} \frac{0.2t}{t^2+1} = \lim_{t\to\infty} \frac{\frac{0.2}{t}}{1+\frac{1}{t^2}} = 0$, which says that the concentration of drug in the bloodstream eventually decreases to zero.

85. **a.** $T(1) = \frac{120}{1+4} = 24$, or \$24 million. $T(2) = \frac{120(2)^2}{8} = 60$, or \$60 million. $T(3) = \frac{120(3)^2}{13} = 83.1$, or \$83.1 million.

b. In the long run, the movie will gross $\lim_{x\to\infty} \frac{120x^2}{x^2+4} = \lim_{x\to\infty} \frac{120}{1+\frac{4}{x^2}} = 120$, or \$120 million.

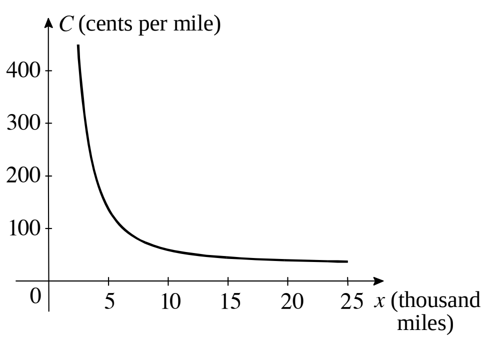
86. **a.** The current population is $P(0) = \frac{200}{40} = 5$, or 5000.

b. The population in the long run will be $\lim_{t\to\infty} \frac{25t^2+125t+200}{t^2+5t+40} = \lim_{t\to\infty} \frac{25+\frac{125}{t}+\frac{200}{t^2}}{1+\frac{5}{t}+\frac{40}{t^2}} = 25$, or 25,000.

87. **a.** The average cost of driving 5000 miles per year is $C(5) = \frac{2410}{5^{1.95}} + 32.8 \approx 137.28$, or 137.3 cents per mile. Similarly, we see that the average costs of driving 10, 15, 20, and 25 thousand miles per year are 59.8, 45.1, 39.8, and 37.3 cents per mile, respectively.

c. It approaches 32.8 cents per mile.

b.

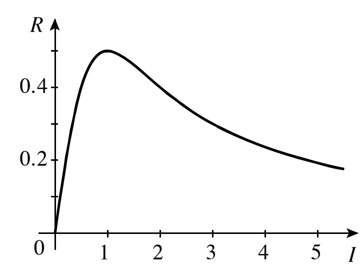


88. **a.** $R(I) = \frac{I}{1+I^2}$

I	0	1	2	3	4	5
$R(I)$	0	$\frac{1}{2}$	$\frac{2}{5}$	$\frac{3}{10}$	$\frac{4}{17}$	$\frac{5}{26}$

b. $\lim_{I\to\infty} R(I) = \lim_{I\to\infty} \frac{I}{1+I^2} = 0$.

c.



89. False. Let $f(x) = \begin{cases} -1 & \text{if } x < 0 \\ 1 & \text{if } x \geq 0 \end{cases}$ Then $\lim_{x\to 0} f(x) = 1$, but $f(1)$ is not defined.

90. True.

91. True. Division by zero is not permitted.

92. False. Let $f(x) = (x-3)^2$ and $g(x) = x - 3$. Then $\lim_{x\to 3} f(x) = 0$ and $\lim_{x\to 3} g(x) = 0$, but

$$\lim_{x\to 3} \frac{f(x)}{g(x)} = \lim_{x\to 3} \frac{(x-3)^2}{x-3} = \lim_{x\to 3} (x-3) = 0.$$

93. True. Each limit in the sum exists. Therefore, $\lim_{x\to 2} \left(\frac{x}{x+1} + \frac{3}{x-1}\right) = \lim_{x\to 2} \frac{x}{x+1} + \lim_{x\to 2} \frac{3}{x-1} = \frac{2}{3} + \frac{3}{1} = \frac{11}{3}$.

94. False. Neither of the limits $\lim_{x\to 1} \frac{2x}{x-1}$ and $\lim_{x\to 1} \frac{2}{x-1}$ exists.

95. $\lim_{x\to\infty} \frac{ax}{x+b} = \lim_{x\to\infty} \frac{a}{1+\frac{b}{x}} = a$. As the amount of substrate becomes very large, the initial speed approaches the constant a moles per liter per second.

96. Consider the functions $f(x) = 1/x$ and $g(x) = -1/x$. Observe that $\lim_{x\to 0} f(x)$ and $\lim_{x\to 0} g(x)$ do not exist, but $\lim_{x\to 0} [f(x) + g(x)] = \lim_{x\to 0} 0 = 0$. This example does not contradict Theorem 1 because the hypothesis of Theorem 1 is that $\lim_{x\to 0} f(x)$ and $\lim_{x\to 0} g(x)$ both exist. It does not say anything about the situation where one or both of these limits fails to exist.

97. Consider the functions $f(x) = \begin{cases} -1 & \text{if } x < 0 \\ 1 & \text{if } x \geq 0 \end{cases}$ and $g(x) = \begin{cases} 1 & \text{if } x < 0 \\ -1 & \text{if } x \geq 0 \end{cases}$ Then $\lim_{x\to 0} f(x)$ and $\lim_{x\to 0} g(x)$ do not exist, but $\lim_{x\to 0} [f(x)\, g(x)] = \lim_{x\to 0} (-1) = -1$. This example does not contradict Theorem 1 because the hypothesis of Theorem 1 is that $\lim_{x\to 0} f(x)$ and $\lim_{x\to 0} g(x)$ both exist. It does not say anything about the situation where one or both of these limits fails to exist.

98. Take $f(x) = \frac{1}{x}$, $g(x) = \frac{1}{x^2}$, and $a = 0$. Then $\lim_{x\to 0} f(x)$ and $\lim_{x\to 0} g(x)$ do not exist, but $\lim_{x\to 0} \frac{f(x)}{g(x)} = \lim_{x\to 0} \frac{1}{x} \cdot \frac{x^2}{1} = \lim_{x\to 0} x = a$ exists. This example does not contradict Theorem 1 because the hypothesis of Theorem 1 is that $\lim_{x\to 0} f(x)$ and $\lim_{x\to 0} g(x)$ both exist. It does not say anything about the situation where one or both of these limits fails to exist.

Using Technology page 121

1. 5

2. 11

3. 3

4. 0

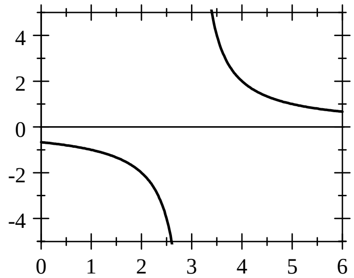
5. $\frac{2}{3}$

6. $\frac{1}{2}$

7. $e^2 \approx 7.38906$

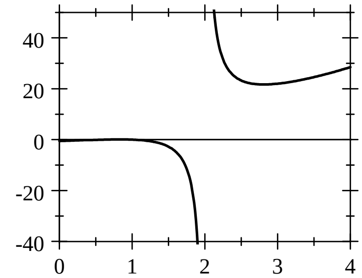
8. $\ln 2 \approx 0.693147$

9.



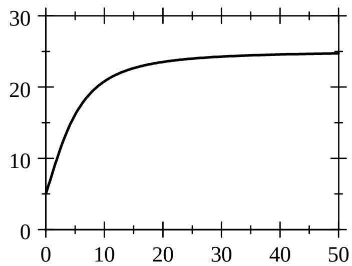
From the graph we see that $f(x)$ does not approach any finite number as x approaches 3.

10.



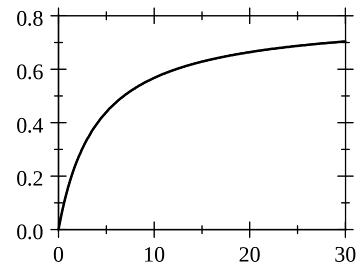
From the graph, we see that $f(x)$ does not approach any finite number as x approaches 2.

11. a.



b. $\lim_{t\to\infty} \dfrac{25t^2 + 125t + 200}{t^2 + 5t + 40} = 25$, so in the long run the population will approach 25,000.

12. a.



b. $\lim_{t\to\infty} \dfrac{0.8t}{t + 4.1} = \lim_{t\to\infty} \dfrac{0.8}{1 + \dfrac{4.1}{t}} = 0.8.$

2.5 One-Sided Limits and Continuity

Concept Questions

page 129

1. $\lim_{x\to 3^-} f(x) = 2$ means $f(x)$ can be made as close to 2 as we please by taking x sufficiently close to but to the left of $x = 3$. $\lim_{x\to 3^+} f(x) = 4$ means $f(x)$ can be made as close to 4 as we please by taking x sufficiently close to but to the right of $x = 3$.

2. a. $\lim_{x\to 1} f(x)$ does not exist because the left- and right-hand limits at $x = 1$ are different.

b. Nothing, because the existence or value of f at $x = 1$ does not depend on the existence (or nonexistence) of the left- or right-hand, or two-sided, limits of f.

3. a. f is continuous at a if $\lim_{x\to a} f(x) = f(a)$.

b. f is continuous on an interval I if f is continuous at each point in I.

4. $f(a) = L = M$.

5. a. f is continuous because the plane does not suddenly jump from one point to another.

b. f is continuous.

c. f is discontinuous because the fare “jumps” after the cab has covered a certain distance or after a certain amount of time has elapsed.

d. f is discontinuous because the rates "jump" by a certain amount (up or down) when it is adjusted at certain times.

6. Refer to page 127 in the text. Answers will vary.

Exercises page 130

1. $\lim_{x\to 2^-} f(x) = 3$ and $\lim_{x\to 2^+} f(x) = 2$, so $\lim_{x\to 2} f(x)$ does not exist.

2. $\lim_{x\to 3^-} f(x) = 3$ and $\lim_{x\to 3^+} f(x) = 5$, so $\lim_{x\to 3} f(x)$ does not exist.

3. $\lim_{x\to -1^-} f(x) = \infty$ and $\lim_{x\to -1^+} f(x) = 2$, so $\lim_{x\to -1} f(x)$ does not exist.

4. $\lim_{x\to 1^-} f(x) = 3$ and $\lim_{x\to 1^+} f(x) = 3$, so $\lim_{x\to 1} f(x) = 3$.

5. $\lim_{x\to 1^-} f(x) = 0$ and $\lim_{x\to 1^+} f(x) = 2$, so $\lim_{x\to 1} f(x)$ does not exist.

6. $\lim_{x\to 0^-} f(x) = 2$ and $\lim_{x\to 0^+} f(x) = \infty$, so $\lim_{x\to 0} f(x)$ does not exist.

7. $\lim_{x\to 0^-} f(x) = -2$ and $\lim_{x\to 0^+} f(x) = 2$, so $\lim_{x\to 0} f(x)$ does not exist.

8. $\lim_{x\to 0^-} f(x) = \lim_{x\to 0^+} f(x) = \lim_{x\to 0} f(x) = 2$.

9. True. **10.** True. **11.** True. **12.** True. **13.** False. **14.** True.

15. True. **16.** True. **17.** False. **18.** True. **19.** True. **20.** False

21. $\lim_{x\to 1^+} (2x+4) = 6$.

22. $\lim_{x\to 1^-} (3x-4) = -1$.

23. $\lim_{x\to 2^-} \frac{x-3}{x+2} = \frac{2-3}{2+2} = -\frac{1}{4}$.

24. $\lim_{x\to 1^+} \frac{x+2}{x+1} = \frac{1+2}{1+1} = \frac{3}{2}$.

25. $\lim_{x\to 0^+} \frac{1}{x}$ does not exist because $\frac{1}{x} \to \infty$ as $x \to 0$ from the right.

26. $\lim_{x\to 0^-} \frac{1}{x} = -\infty$; that is, the limit does not exist.

27. $\lim_{x\to 0^+} \frac{x-1}{x^2+1} = \frac{-1}{1} = -1$.

28. $\lim_{x\to 2^+} \frac{x+1}{x^2-2x+3} = \frac{2+1}{4-4+3} = 1$.

29. $\lim_{x\to 0^+} \sqrt{x} = \sqrt{\lim_{x\to 0^+} x} = 0$.

30. $\lim_{x\to 2^+} 2\sqrt{x-2} = 2\cdot 0 = 0$.

31. $\lim_{x\to -2^+} \left(2x+\sqrt{2+x}\right) = \lim_{x\to -2^+} 2x + \lim_{x\to -2^+} \sqrt{2+x} = -4+0 = -4$.

32. $\lim_{x\to -5^+} x\left(1+\sqrt{5+x}\right) = -5\left[1+\sqrt{5+(-5)}\right] = -5$.

33. $\lim_{x\to 1^-} \frac{1+x}{1-x} = \infty$, that is, the limit does not exist.

34. $\lim_{x\to 1^+} \frac{1+x}{1-x} = -\infty.$

35. $\lim_{x\to 2^-} \frac{x^2-4}{x-2} = \lim_{x\to 2^-} \frac{(x+2)(x-2)}{x-2} = \lim_{x\to 2^-} (x+2) = 4.$

36. $\lim_{x\to -3^+} \frac{\sqrt{x+3}}{x^2+1} = \frac{0}{10} = 0.$

37. $\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} x^2 = 0$ and $\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} 2x = 0.$

38. $\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} (2x+3) = 3$ and $\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} (-x+1) = 1.$

39. The function is discontinuous at $x = 0$. Conditions 2 and 3 are violated.

40. The function is not continuous because condition 3 for continuity is not satisfied.

41. The function is continuous everywhere.

42. The function is continuous everywhere.

43. The function is discontinuous at $x = 0$. Condition 3 is violated.

44. The function is not continuous at $x = -1$ because condition 3 for continuity is violated.

45. f is continuous for all values of x.

46. f is continuous for all values of x.

47. f is continuous for all values of x. Note that $x^2 + 1 \geq 1 > 0$.

48. f is continuous for all values of x. Note that $2x^2 + 1 \geq 1 > 0$.

49. f is discontinuous at $x = \frac{1}{2}$, where the denominator is 0. Thus, f is continuous on $\left(-\infty, \frac{1}{2}\right)$ and $\left(\frac{1}{2}, \infty\right)$.

50. f is discontinuous at $x = 1$, where the denominator is 0. Thus, f is continuous on $(-\infty, 1)$ and $(1, \infty)$.

51. Observe that $x^2 + x - 2 = (x+2)(x-1) = 0$ if $x = -2$ or $x = 1$, so f is discontinuous at these values of x. Thus, f is continuous on $(-\infty, -2)$, $(-2, 1)$, and $(1, \infty)$.

52. Observe that $x^2 + 2x - 3 = (x+3)(x-1) = 0$ if $x = -3$ or $x = 1$, so, f is discontinuous at these values of x. Thus, f is continuous on $(-\infty, -3)$, $(-3, 1)$, and $(1, \infty)$.

53. f is continuous everywhere since all three conditions are satisfied.

54. f is continuous everywhere since all three conditions are satisfied.

55. f is continuous everywhere since all three conditions are satisfied.

56. f is not defined at $x = 1$ and is discontinuous there. It is continuous everywhere else.

57. Because the denominator $x^2 - 1 = (x-1)(x+1) = 0$ if $x = -1$ or 1, we see that f is discontinuous at -1 and 1.

58. The function f is not defined at $x = 1$ and $x = 2$. Therefore, f is discontinuous at 1 and 2.

59. Because $x^2 - 3x + 2 = (x - 2)(x - 1) = 0$ if $x = 1$ or 2, we see that the denominator is zero at these points and so f is discontinuous at these numbers.

60. The denominator of the function f is equal to zero when $x^2 - 2x = x(x - 2) = 0$; that is, when $x = 0$ or $x = 2$. Therefore, f is discontinuous at $x = 0$ and $x = 2$.

61. The function f is discontinuous at $x = 4, 5, 6, \ldots, 13$ because the limit of f does not exist at these points.

62. f is discontinuous at $t = 20$, 40, and 60. When $t = 0$, the inventory stands at 750 reams. The level drops to about 250 reams by the twentieth day at which time a new order of 500 reams arrives to replenish the supply. A similar interpretation holds for the other values of t.

63. Having made steady progress up to $x = x_1$, Michael's progress comes to a standstill at that point. Then at $x = x_2$ a sudden breakthrough occurs and he then continues to solve the problem.

64. The total deposits of Franklin make a jump at each of these points as the deposits of the ailing institutions become a part of the total deposits of the parent company.

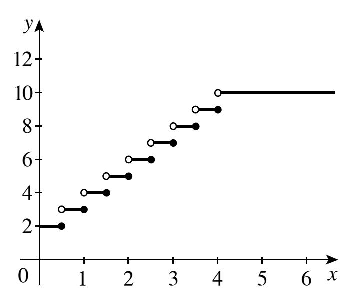
65. Conditions 2 and 3 are not satisfied at any of these points.

66. The function P is discontinuous at $t = 12$, 16, and 28. At $t = 12$, the prime interest rate jumped from $3\frac{1}{2}\%$ to 4%, at $t = 16$ it jumped to $4\frac{1}{2}\%$, and at $t = 28$ it jumped back down to 4%.

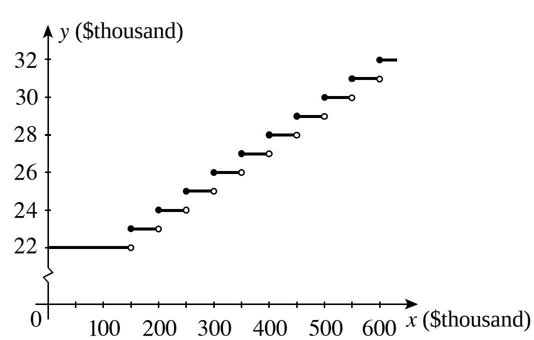
67.

$$f(x) = \begin{cases} 2 & \text{if } 0 < x \le \frac{1}{2} \\ 3 & \text{if } \frac{1}{2} < x \le 1 \\ \vdots & \vdots \\ 10 & \text{if } x > 4 \end{cases}$$

f is discontinuous at $x = \frac{1}{2}, 1, 1\frac{1}{2}, \ldots, 4$.

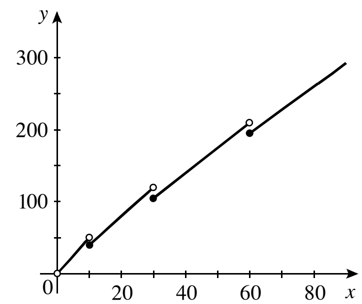


68.



f is discontinuous at $x = 150{,}000$, at $x = 200{,}000$, at $x = 250{,}000$, and so on.

69.



C is discontinuous at $x = 0$, 10, 30, and 60.

70. a. $\lim_{v \to u^+} \frac{aLv^3}{v - u} = \infty$. This reflects the fact that when the speed of the fish is very close to that of the current, the energy expended by the fish will be enormous.

b. $\lim_{v \to \infty} \frac{aLv^3}{v - u} = \infty$. This says that if the speed of the fish increases greatly, so does the amount of energy required to swim a distance of L ft.

71. a. $\lim_{t \to 0^+} S(t) = \lim_{t \to 0^+} \frac{a}{t} + b = \infty$. As the time taken to excite the tissue is made shorter and shorter, the electric current gets stronger and stronger.

b. $\lim_{t \to \infty} \frac{a}{t} + b = b$. As the time taken to excite the tissue is made longer and longer, the electric current gets weaker and weaker and approaches b.

72. a. $\lim_{D \to 0^+} L = \lim_{D \to 0^+} \frac{Y - (1 - D) R}{D} = \infty$, so if the investor puts down next to nothing to secure the loan, the leverage approaches infinity.

b. $\lim_{D \to 1^-} L = \lim_{D \to 1^-} \frac{Y - (1 - D) R}{D} = Y$, so if the investor puts down all of the money to secure the loan, the leverage is equal to the yield.

73. We require that $f(1) = 1 + 2 = 3 = \lim_{x \to 1^+} kx^2 = k$, so $k = 3$.

74. Because $\lim_{x \to -2} \frac{x^2 - 4}{x + 2} = \lim_{x \to -2} \frac{(x - 2)(x + 2)}{x + 2} = \lim_{x \to -2} (x - 2) = -4$, we define $f(-2) = k = -4$, that is, take $k = -4$.

75. a. f is a polynomial of degree 2 and is therefore continuous everywhere, including the interval $[1, 3]$.

b. $f(1) = 3$ and $f(3) = -1$ and so f must have at least one zero in $(1, 3)$.

76. a. f is a polynomial of degree 3 and is thus continuous everywhere.

b. $f(0) = 14$ and $f(1) = -23$ and so f has at least one zero in $(0, 1)$.

77. a. f is a polynomial of degree 3 and is therefore continuous on $[-1, 1]$.

b. $f(-1) = (-1)^3 - 2(-1)^2 + 3(-1) + 2 = -1 - 2 - 3 + 2 = -4$ and $f(1) = 1 - 2 + 3 + 2 = 4$. Because $f(-1)$ and $f(1)$ have opposite signs, we see that f has at least one zero in $(-1, 1)$.

78. f is continuous on $[14, 16]$, $f(14) = 2(14)^{5/3} - 5(14)^{4/3} \approx -6.06$, and $f(16) = 2(16)^{5/3} - 5(16)^{4/3} \approx 1.60$. Thus, f has at least one zero in $(14, 16)$.

79. $f(0) = 6$, $f(3) = 3$, and f is continuous on $[0, 3]$. Thus, the Intermediate Value Theorem guarantees that there is at least one value of x for which $f(x) = 4$. Solving $f(x) = x^2 - 4x + 6 = 4$, we find $x^2 - 4x + 2 = 0$. Using the quadratic formula, we find that $x = 2 \pm \sqrt{2}$. Because $2 + \sqrt{2}$ does not lie in $[0, 3]$, we see that $x = 2 - \sqrt{2} \approx 0.59$.

80. Because $f(-1) = 3$, $f(4) = 13$, and f is continuous on $[-1, 4]$, the Intermediate Value Theorem guarantees that there is at least one value of x for which $f(x) = 7$ because $3 < 7 < 13$. Solving $f(x) = x^2 - x + 1 = 7$, we find $x^2 - x - 6 = (x - 3)(x + 2) = 0$, that is, $x = -2$ or 3. Because -2 does not lie in $[-1, 4]$, the required solution is 3.

81. $x^5 + 2x - 7 = 0$

Step	Interval in which a root lies
1	(1, 2)
2	(1, 1.5)
3	(1.25, 1.5)
4	(1.25, 1.375)
5	(1.3125, 1.375)
6	(1.3125, 1.34375)
7	(1.328125, 1.34375)
8	(1.3359375, 1.34375)

We see that a root is approximately 1.34.

82. $x^3 - x + 1 = 0$

Step	Interval in which a root lies
1	$(-2, -1)$
2	$(-1.5, -1)$
3	$(-1.5, -1.25)$
4	$(-1.375, -1.25)$
5	$(-1.375, -1.3125)$
6	$(-1.34375, -1.3125)$
7	$(-1.328125, -1.3125)$
8	$(-1.328125, -1.3203125)$
9	$(-1.32421875, -1.3203125)$

We see that a root is approximately -1.32.

83. a. $h(0) = 4 + 64(0) - 16(0) = 4$ and $h(2) = 4 + 64(2) - 16(4) = 68$.

b. The function h is continuous on $[0, 2]$. Furthermore, the number 32 lies between 4 and 68. Therefore, the Intermediate Value Theorem guarantees that there is at least one value of t in $(0, 2]$ such that $h(t) = 32$, that is, Joan must see the ball at least once during the time the ball is in the air.

c. We solve $h(t) = 4 + 64t - 16t^2 = 32$, obtaining $16t^2 - 64t + 28 = 0$, $4t^2 - 16t + 7 = 0$, and $(2t - 1)(2t - 7) = 0$. Thus, $t = \frac{1}{2}$ or $t = \frac{7}{2}$. Joan sees the ball on its way up half a second after it was thrown and again 3 seconds later when it is on its way down. Note that the ball hits the ground when $t \approx 4.06$, but Joan sees it approximately half a second before it hits the ground.

84. a. $f(0) = 100\left(\dfrac{0 + 0 + 100}{0 + 0 + 100}\right) = 100$ and $f(10) = 100\left(\dfrac{100 + 100 + 100}{100 + 200 + 100}\right) = \dfrac{30{,}000}{400} = 75$.

b. Because 80 lies between 75 and 100 and f is continuous on $[75, 100]$, we conclude that there exists some t in $[0, 10]$ such that $f(t) = 80$.

c. We solve $f(t) = 80$; that is, $100\left[\dfrac{t^2 + 10t + 100}{t^2 + 20t + 100}\right] = 80$, obtaining $5\left(t^2 + 10t + 100\right) = 4\left(t^2 + 20t + 100\right)$, and $t^2 - 30t + 100 = 0$. Thus, $t = \dfrac{30 \pm \sqrt{900 - 400}}{2} = \dfrac{30 \pm \sqrt{500}}{2} \approx 3.82$ or 26.18. Because 26.18 lies outside the interval of interest, we reject it. Thus, the oxygen content is 80% approximately 3.82 seconds after the organic waste has been dumped into the pond.

85. False. Take $f(x) = \begin{cases} -1 & \text{if } x < 2 \\ 4 & \text{if } x = 2 \\ 1 & \text{if } x > 2 \end{cases}$ Then $f(2) = 4$, but $\lim_{x\to 2} f(x)$ does not exist.

86. False. Take $f(x) = \begin{cases} x+3 & \text{if } x \neq 0 \\ 1 & \text{if } x = 0 \end{cases}$ Then $\lim_{x\to 0} f(x) = 3$, but $f(0) = 1$.

87. False. Consider $f(x) = \begin{cases} 0 & \text{if } x < 2 \\ 3 & \text{if } x \geq 2 \end{cases}$ Then $\lim_{x\to 2^+} f(x) = f(2) = 3$, but $\lim_{x\to 2^-} f(x) = 0$.

88. False. Consider $f(x) = \begin{cases} 2 & \text{if } x < 3 \\ 1 & \text{if } x = 3 \\ 4 & \text{if } x \geq 3 \end{cases}$ Then $\lim_{x\to 3^-} f(x) = 2$ and $\lim_{x\to 3^+} f(x) = 4$, so $\lim_{x\to 3} f(x)$ does not exist.

89. False. Consider $f(x) = \begin{cases} 2 & \text{if } x < 5 \\ 3 & \text{if } x > 5 \end{cases}$ Then $f(5)$ is not defined, but $\lim_{x\to 5^-} f(x) = 2$.

90. False. Consider the function $f(x) = x^2 - 1$ on the interval $[-2, 2]$. Here $f(-2) = f(2) = 3$, but f has zeros at $x = -1$ and $x = 1$.

91. False. Let $f(x) = \begin{cases} x & \text{if } x \neq 0 \\ 1 & \text{if } x = 0 \end{cases}$ Then $\lim_{x\to 0^+} f(x) = \lim_{x\to 0^-} f(x)$, but $f(0) = 1$.

92. False. Let $f(x) = x$ and let $g(x) = \begin{cases} x & \text{if } x \neq 1 \\ 2 & \text{if } x = 1 \end{cases}$ Then $\lim_{x\to 1} f(x) = 1 = L$, $g(1) = 2 = M$, $\lim_{x\to 1} g(x) = 1$, and $\lim_{x\to 1} f(x)\, g(x) = \left[\lim_{x\to 1} f(x)\right]\left[\lim_{x\to 1} g(x)\right] = (1)(1) = 1 \neq 2 = LM$.

93. False. Let $f(x) = \begin{cases} 1/x & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$ Then f is continuous for all $x \neq 0$ and $f(0) = 0$, but $\lim_{x\to 0} f(x)$ does not exist.

94. False. Consider $f(x) = \begin{cases} -1 & \text{if } -1 \leq x \leq 0 \\ 1 & \text{if } 0 < x \leq 1 \end{cases}$ and $g(x) = \begin{cases} 1 & \text{if } -1 \leq x \leq 0 \\ -1 & \text{if } 0 < x \leq 1 \end{cases}$

95. False. Consider $f(x) = \begin{cases} -1 & \text{if } -1 \leq x \leq 0 \\ 1 & \text{if } 0 < x \leq 1 \end{cases}$ and $g(x) = \begin{cases} 1 & \text{if } -1 \leq x \leq 0 \\ -1 & \text{if } 0 < x \leq 1 \end{cases}$

96. False. Let $f(x) = \begin{cases} -1 & \text{if } x < 0 \\ 1 & \text{if } x \geq 0 \end{cases}$ and $g(x) = x^2$.

97. False. Consider $f(x) = \begin{cases} -1 & \text{if } x < 0 \\ 1 & \text{if } x \geq 0 \end{cases}$ and $g(x) = \begin{cases} x+1 & \text{if } x < 0 \\ x-1 & \text{if } x \geq 0 \end{cases}$

98. False. There is no contradiction, because the Intermediate Value Theorem says that there is at least one number c in $[a, b]$ such that $f(c) = M$ if M is a number between $f(a)$ and $f(b)$.

99. a. f is a rational function whose denominator is never zero, and so it is continuous for all values of x.

b. Because the numerator x^2 is nonnegative and the denominator is $x^2 + 1 \geq 1$ for all values of x, we see that $f(x)$ is nonnegative for all values of x.

c. $f(0) = \dfrac{0}{0+1} = \dfrac{0}{1} = 0$, and so f has a zero at $x = 0$. This does not contradict Theorem 5.

100. a. Both $g(x) = x$ and $h(x) = \sqrt{1-x^2}$ are continuous on $[-1, 1]$ and so $f(x) = x - \sqrt{1-x^2}$ is continuous on $[-1, 1]$.

b. $f(-1) = -1$ and $f(1) = 1$, and so f has at least one zero in $(-1, 1)$.

c. Solving $f(x) = 0$, we have $x = \sqrt{1-x^2}$, $x^2 = 1 - x^2$, and $2x^2 = 1$, so $x = \frac{\pm\sqrt{2}}{2}$.

101. a. (i) Repeated use of Property 3 shows that $g(x) = x^n = \underbrace{x \cdot x \cdot \cdots \cdot x}_{n \text{ times}}$ is a continuous function, since $f(x) = x$ is continuous by Property 1.

(ii) Properties 1 and 5 combine to show that $c \cdot x^n$ is continuous using the results of part (a)(i).

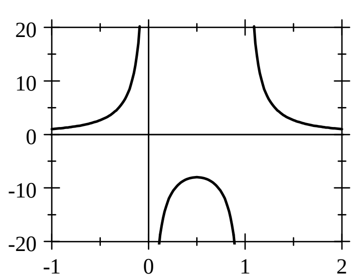
(iii) Each of the terms of $p(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_0$ is continuous and so Property 4 implies that p is continuous.

b. Property 6 now shows that $R(x) = \dfrac{p(x)}{q(x)}$ is continuous if $q(a) \neq 0$, since p and q are continuous at $x = a$.

102. Consider the function f defined by $f(x) = \begin{cases} -1 & \text{if } -1 \leq x < 0 \\ 1 & \text{if } 0 \leq x < 1 \end{cases}$ Then $f(-1) = -1$ and $f(1) = 1$, but if we take the number $\frac{1}{2}$, which lies between $y = -1$ and $y = 1$, there is no value of x such that $f(x) = \frac{1}{2}$.

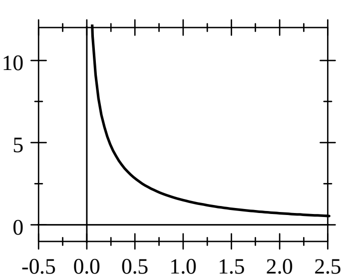
Using Technology page 136

1.



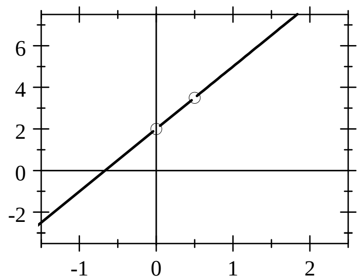
The function is discontinuous at $x = 0$ and $x = 1$.

2.



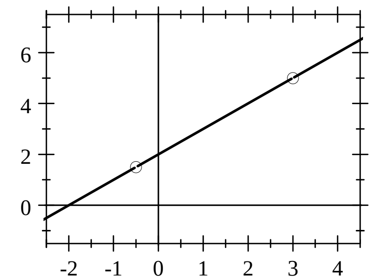
The function is undefined for $x \leq 0$.

3.



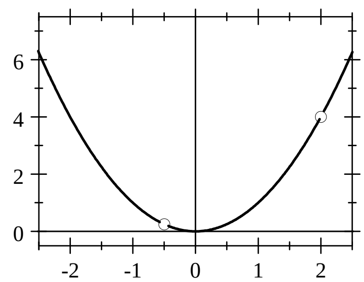
The function is discontinuous at $x = 0$ and $\frac{1}{2}$.

4.



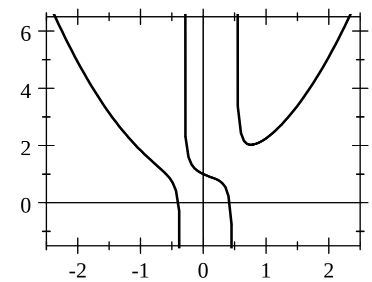
The function is discontinuous at $x = -\frac{1}{2}$ and 3.

5.



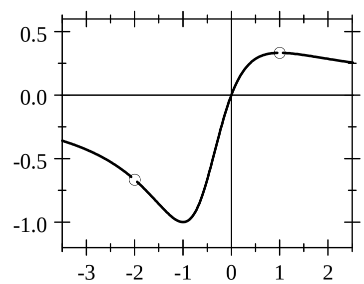
The function is discontinuous at $x = -\frac{1}{2}$ and 2.

6.



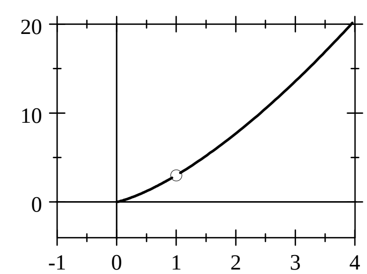
The function is discontinuous at $x = -\frac{1}{3}$ and $\frac{1}{2}$.

7.



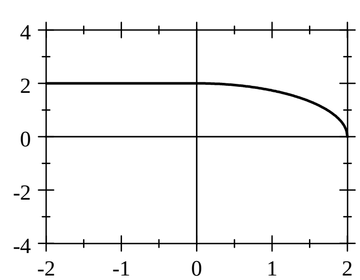
The function is discontinuous at $x = -2$ and 1.

8.

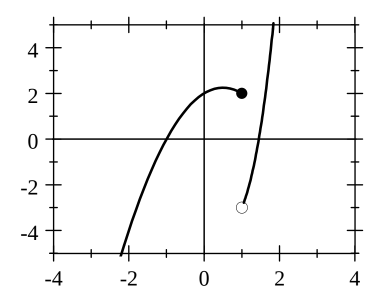


The function is discontinuous only at $x = -1$ and 1.

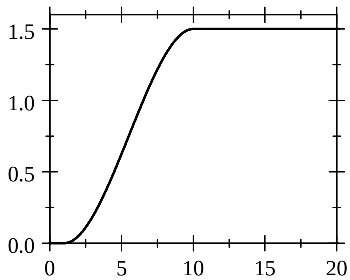
9.



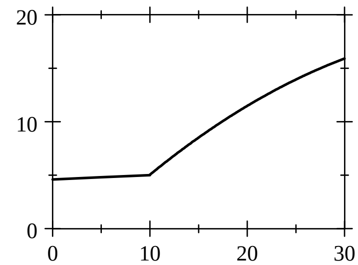
10.



11.



12. a.



b. 4.6%; 8.5%; 15.9%

2.6 The Derivative

Concept Questions

page 148

1. a. $m = \dfrac{f(2+h) - f(2)}{h}$

b. The slope of the tangent line is $\lim\limits_{h \to 0} \dfrac{f(2+h) - f(2)}{h}$.

2. a. The average rate of change is $\dfrac{f(2+h) - f(2)}{h}$.

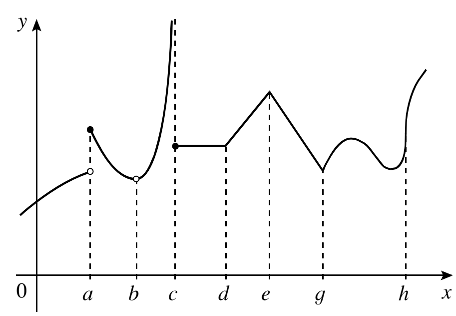
b. The instantaneous rate of change of f at 2 is $\lim\limits_{h \to 0} \dfrac{f(2+h) - f(2)}{h}$.

c. The expression for the slope of the secant line is the same as that for the average rate of change. The expression for the slope of the tangent line is the same as that for the instantaneous rate of change.

3. a. The expression $\dfrac{f(x+h) - f(x)}{h}$ gives (i) the slope of the secant line passing through the points $(x, f(x))$ and $(x+h, f(x+h))$, and (ii) the average rate of change of f over the interval $[x, x+h]$.

b. The expression $\lim\limits_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$ gives (i) the slope of the tangent line to the graph of f at the point $(x, f(x))$, and (ii) the instantaneous rate of change of f at x.

4. Loosely speaking, a function f does not have a derivative at a if the graph of f does not have a tangent line at a, or if the tangent line does exist, but is vertical. In the figure, the function fails to be differentiable at $x = a$, b, and c because it is discontinuous at each of these numbers. The derivative of the function does not exist at $x = d$, e, and g because it has a kink at each point on the graph corresponding to these numbers. Finally, the function is not differentiable at $x = h$ because the tangent line is vertical at $(h, f(h))$.



5. a. $C(500)$ gives the total cost incurred in producing 500 units of the product.

b. $C'(500)$ gives the rate of change of the total cost function when the production level is 500 units.

6. a. $P(5)$ gives the population of the city (in thousands) when $t = 5$.

b. $P'(5)$ gives the rate of change of the city's population (in thousands/year) when $t = 5$.

Exercises page 149

1. The rate of change of the average infant's weight when $t = 3$ is $\frac{7.5}{5}$, or 1.5 lb/month. The rate of change of the average infant's weight when $t = 18$ is $\frac{3.5}{6}$, or approximately 0.58 lb/month. The average rate of change over the infant's first year of life is $\frac{22.5-7.5}{12}$, or 1.25 lb/month.

2. The rate at which the wood grown is changing at the beginning of the 10th year is $\frac{4}{12}$, or $\frac{1}{3}$ cubic meter per hectare per year. At the beginning of the 30th year, it is $\frac{10}{8}$, or 1.25 cubic meters per hectare per year.

3. The rate of change of the percentage of households watching television at 4 p.m. is $\frac{12.3}{4}$, or approximately 3.1 percent per hour. The rate at 11 p.m. is $\frac{-42.3}{2} = -21.15$, that is, it is dropping off at the rate of 21.15 percent per hour.

4. The rate of change of the crop yield when the density is 200 aphids per bean stem is $\frac{-500}{300}$, a decrease of approximately 1.7 kg/4000 m^2 per aphid per bean stem. The rate of change when the density is 800 aphids per bean stem is $\frac{-150}{300}$, a decrease of approximately 0.5 kg/4000 m^2 per aphid per bean stem.

5. a. Car A is travelling faster than Car B at t_1 because the slope of the tangent line to the graph of f is greater than the slope of the tangent line to the graph of g at t_1.

b. Their speed is the same because the slope of the tangent lines are the same at t_2.

c. Car B is travelling faster than Car A.

d. They have both covered the same distance and are once again side by side at t_3.

6. a. At t_1, the velocity of Car A is greater than that of Car B because $f(t_1) > g(t_1)$. However, Car B has greater acceleration because the slope of the tangent line to the graph of g is increasing, whereas the slope of the tangent line to f is decreasing as you move across t_1.

b. Both cars have the same velocity at t_2, but the acceleration of Car B is greater than that of Car A because the slope of the tangent line to the graph of g is increasing, whereas the slope of the tangent line to the graph of f is decreasing as you move across t_2.

7. a. P_2 is decreasing faster at t_1 because the slope of the tangent line to the graph of g at t_1 is greater than the slope of the tangent line to the graph of f at t_1.

b. P_1 is decreasing faster than P_2 at t_2.

c. Bactericide B is more effective in the short run, but bactericide A is more effective in the long run.

8. a. The revenue of the established department store is decreasing at the slowest rate at $t = 0$.

b. The revenue of the established department store is decreasing at the fastest rate at t_3.

c. The revenue of the discount store first overtakes that of the established store at t_1.

d. The revenue of the discount store is increasing at the fastest rate at t_2 because the slope of the tangent line to the graph of f is greatest at the point $(t_2, f(t_2))$.

9. $f(x) = 13$.

Step 1 $f(x+h) = 13$.

Step 2 $f(x+h) - f(x) = 13 - 13 = 0$.

Step 3 $\dfrac{f(x+h) - f(x)}{h} = \dfrac{0}{h} = 0$.

Step 4 $f'(x) = \lim\limits_{h\to 0} \dfrac{f(x+h) - f(x)}{h} = \lim\limits_{h\to 0} 0 = 0$.

10. $f(x) = -6$.

Step 1 $f(x+h) = -6$.

Step 2 $f(x+h) - f(x) = -6 - (-6) = 0$.

Step 3 $\dfrac{f(x+h) - f(x)}{h} = \dfrac{0}{h} = 0$.

Step 4 $f'(x) = \lim\limits_{h\to 0} \dfrac{f(x+h) - f(x)}{h} = \lim\limits_{h\to 0} 0 = 0$.

11. $f(x) = 2x + 7$.

Step 1 $f(x+h) = 2(x+h) + 7$.

Step 2 $f(x+h) - f(x) = 2(x+h) + 7 - (2x+7) = 2h$.

Step 3 $\dfrac{f(x+h) - f(x)}{h} = \dfrac{2h}{h} = 2$.

Step 4 $f'(x) = \lim\limits_{h\to 0} \dfrac{f(x+h) - f(x)}{h} = \lim\limits_{h\to 0} 2 = 2$.

12. $f(x) = 8 - 4x$.

Step 1 $f(x+h) = 8 - 4(x+h) = 8 - 4x - 4h$.

Step 2 $f(x+h) - f(x) = (8 - 4x - 4h) - (8 - 4x) = -4h$.

Step 3 $\dfrac{f(x+h) - f(x)}{h} = -\dfrac{4h}{h} = -4$.

Step 4 $f'(x) = \lim\limits_{h\to 0} \dfrac{f(x+h) - f(x)}{h} = \lim\limits_{h\to 0} (-4) = -4$.

13. $f(x) = 3x^2$.

Step 1 $f(x+h) = 3(x+h)^2 = 3x^2 + 6xh + 3h^2$.

Step 2 $f(x+h) - f(x) = \left(3x^2 + 6xh + 3h^2\right) - 3x^2 = 6xh + 3h^2 = h(6x + 3h)$.

Step 3 $\dfrac{f(x+h) - f(x)}{h} = \dfrac{h(6x+3h)}{h} = 6x + 3h$.

Step 4 $f'(x) = \lim\limits_{h\to 0} \dfrac{f(x+h) - f(x)}{h} = \lim\limits_{h\to 0} (6x + 3h) = 6x$.

14. $f(x) = -\frac{1}{2}x^2$.

Step 1 $f(x+h) = -\frac{1}{2}(x+h)^2$.

Step 2 $f(x+h) - f(x) = -\frac{1}{2}x^2 - xh - \frac{1}{2}h^2 + \frac{1}{2}x^2 = -h\left(x + \frac{1}{2}h\right)$.

Step 3 $\dfrac{f(x+h) - f(x)}{h} = \dfrac{-h\left(x + \frac{1}{2}h\right)}{h} = -\left(x + \frac{1}{2}h\right)$.

Step 4 $f'(x) = \lim\limits_{h\to 0} \dfrac{f(x+h) - f(x)}{h} = \lim\limits_{h\to 0} -\left(x + \frac{1}{2}h\right) = -x$.

15. $f(x) = -x^2 + 3x$.

Step 1 $f(x+h) = -(x+h)^2 + 3(x+h) = -x^2 - 2xh - h^2 + 3x + 3h$.

Step 2 $f(x+h) - f(x) = \left(-x^2 - 2xh - h^2 + 3x + 3h\right) - \left(-x^2 + 3x\right) = -2xh - h^2 + 3h$

$$= h(-2x - h + 3).$$

Step 3 $\dfrac{f(x+h) - f(x)}{h} = \dfrac{h(-2x - h + 3)}{h} = -2x - h + 3$.

Step 4 $f'(x) = \lim_{h \to 0} \dfrac{f(x+h) - f(x)}{h} = \lim_{h \to 0} (-2x - h + 3) = -2x + 3$.

16. $f(x) = 2x^2 + 5x$.

Step 1 $f(x+h) = 2(x+h)^2 + 5(x+h) = 2x^2 + 4xh + 2h^2 + 5x + 5h$.

Step 2 $f(x+h) - f(x) = 2x^2 + 4xh + 2h^2 + 5x + 5h - 2x^2 - 5x = h(4x + 2h + 5)$.

Step 3 $\dfrac{f(x+h) - f(x)}{h} = \dfrac{h(4x + 2h + 5)}{h} = 4x + 2h + 5$.

Step 4 $f'(x) = \lim_{h \to 0} \dfrac{f(x+h) - f(x)}{h} = \lim_{h \to 0} (4x + 2h + 5) = 4x + 5$.

17. $f(x) = 2x + 7$.

Step 1 $f(x+h) = 2(x+h) + 7 = 2x + 2h + 7$.

Step 2 $f(x+h) - f(x) = 2x + 2h + 7 - 2x - 7 = 2h$.

Step 3 $\dfrac{f(x+h) - f(x)}{h} = \dfrac{2h}{h} = 2$.

Step 4 $f'(x) = \lim_{h \to 0} \dfrac{f(x+h) - f(x)}{h} = \lim_{h \to 0} 2 = 2$.

Therefore, $f'(x) = 2$. In particular, the slope at $x = 2$ is 2. Therefore, an equation of the tangent line is $y - 11 = 2(x - 2)$ or $y = 2x + 7$.

18. $f(x) = -3x + 4$. First, we find $f'(x) = -3$ using the four-step process. Thus, the slope of the tangent line is $f'(-1) = -3$ and an equation is $y - 7 = -3(x + 1)$ or $y = -3x + 4$.

19. $f(x) = 3x^2$.We first compute $f'(x) = 6x$ (see Exercise 13). Because the slope of the tangent line is $f'(1) = 6$, we use the point-slope form of the equation of a line and find that an equation is $y - 3 = 6(x - 1)$, or $y = 6x - 3$.

20. $f(x) = 3x - x^2$.

Step 1 $f(x+h) = 3(x+h) - (x+h)^2 = 3x + 3h - x^2 - 2xh - h^2$.

Step 2 $f(x+h) - f(x) = 3x + 3h - x^2 - 2xh - h^2 - 3x + x^2 = 3h - 2xh - h^2 = h(3 - 2x - h)$.

Step 3 $\dfrac{f(x+h) - f(x)}{h} = \dfrac{h(3 - 2x - h)}{h} = 3 - 2x - h$.

Step 4 $f'(x) = \lim_{h \to 0} \dfrac{f(x+h) - f(x)}{h} = \lim_{h \to 0} (3 - 2x - h) = 3 - 2x$.

Therefore, $f'(x) = 3 - 2x$. In particular, $f'(-2) = 3 - 2(-2) = 7$. Using the point-slope form of an equation of a line, we find $y + 10 = 7(x + 2)$, or $y = 7x + 4$.

21. $f(x) = -1/x$. We first compute $f'(x)$ using the four-step process:

Step 1 $f(x+h) = -\dfrac{1}{x+h}$.

Step 2 $f(x+h) - f(x) = -\dfrac{1}{x+h} + \dfrac{1}{x} = \dfrac{-x+(x+h)}{x(x+h)} = \dfrac{h}{x(x+h)}$.

Step 3 $\dfrac{f(x+h)-f(x)}{h} = \dfrac{\dfrac{h}{x(x+h)}}{h} = \dfrac{1}{x(x+h)}$.

Step 4 $f'(x) = \lim\limits_{h\to 0} \dfrac{f(x+h)-f(x)}{h} = \lim\limits_{h\to 0} \dfrac{1}{x(x+h)} = \dfrac{1}{x^2}$.

The slope of the tangent line is $f'(3) = \frac{1}{9}$. Therefore, an equation is $y - \left(-\frac{1}{3}\right) = \frac{1}{9}(x-3)$, or $y = \frac{1}{9}x - \frac{2}{3}$.

22. $f(x) = \dfrac{3}{2x}$. First use the four-step process to find $f'(x) = -\dfrac{3}{2x^2}$. (This is similar to Exercise 21.) The slope of the tangent line is $f'(1) = -\frac{3}{2}$. Therefore, an equation is $y - \frac{3}{2} = -\frac{3}{2}(x-1)$ or $y = -\frac{3}{2}x + 3$.

23. a. $f(x) = 2x^2 + 1$.

Step 1 $f(x+h) = 2(x+h)^2 + 1 = 2x^2 + 4xh + 2h^2 + 1$.

Step 2 $f(x+h) - f(x) = (2x^2 + 4xh + 2h^2 + 1) - (2x^2 + 1)$

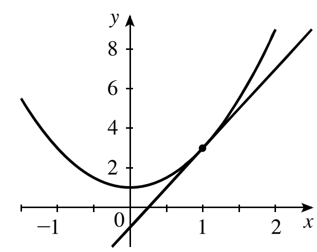
$= 4xh + 2h^2 = h(4x + 2h)$.

Step 3 $\dfrac{f(x+h)-f(x)}{h} = \dfrac{h(4x+2h)}{h} = 4x + 2h$.

Step 4 $f'(x) = \lim\limits_{h\to 0} \dfrac{f(x+h)-f(x)}{h} = \lim\limits_{h\to 0}(4x+2h) = 4x$.

b. The slope of the tangent line is $f'(1) = 4(1) = 4$. Therefore, an equation is $y - 3 = 4(x-1)$ or $y = 4x - 1$.

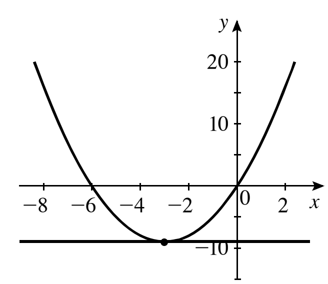
c.



24. a. $f(x) = x^2 + 6x$. Using the four-step process, we find that $f'(x) = 2x + 6$.

b. At a point on the graph of f where the tangent line to the curve is horizontal, $f'(x) = 0$. Then $2x + 6 = 0$, or $x = -3$. Therefore, $y = f(-3) = (-3)^2 + 6(-3) = -9$. The required point is $(-3, -9)$.

c.



25. a. $f(x) = x^2 - 2x + 1$. We use the four-step process:

Step 1 $f(x+h) = (x+h)^2 - 2(x+h) + 1 = x^2 + 2xh + h^2 - 2x - 2h + 1$.

Step 2 $f(x+h) - f(x) = (x^2 + 2xh + h^2 - 2x - 2h + 1) - (x^2 - 2x + 1) = 2xh + h^2 - 2h$

$= h(2x + h - 2)$.

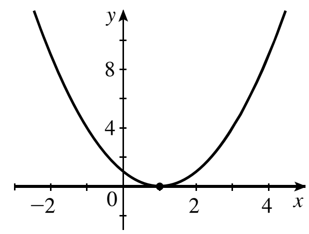
Step 3 $\dfrac{f(x+h)-f(x)}{h}=\dfrac{h(2x+h-2)}{h}=2x+h-2.$

Step 4 $f'(x)=\lim_{h\to 0}\dfrac{f(x+h)-f(x)}{h}=\lim_{h\to 0}(2x+h-2)$

$=2x-2.$

b. At a point on the graph of f where the tangent line to the curve is horizontal, $f'(x)=0$. Then $2x-2=0$, or $x=1$. Because $f(1)=1-2+1=0$, we see that the required point is $(1,0)$.

c.



d. It is changing at the rate of 0 units per unit change in x.

26. a. $f(x)=\dfrac{1}{x-1}.$

Step 1 $f(x+h)=\dfrac{1}{(x+h)-1}=\dfrac{1}{x+h-1}.$

Step 2 $f(x+h)-f(x)=\dfrac{1}{x+h-1}-\dfrac{1}{x-1}=\dfrac{x-1-(x+h-1)}{(x+h-1)(x-1)}=-\dfrac{h}{(x+h-1)(x-1)}.$

Step 3 $\dfrac{f(x+h)-f(x)}{h}=-\dfrac{1}{(x+h-1)(x-1)}.$

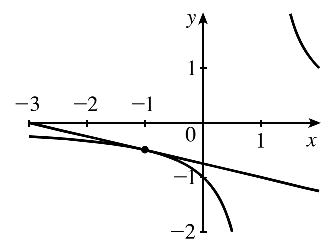
Step 4 $f'(x)=\lim_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$

$=\lim_{h\to 0}-\dfrac{1}{(x+h-1)(x-1)}=-\dfrac{1}{(x-1)^2}.$

b. The slope is $f'(-1)=-\frac{1}{4}$, so, an equation is

$y-\left(-\frac{1}{2}\right)=-\frac{1}{4}(x+1)$ or $y=-\frac{1}{4}x-\frac{3}{4}.$

c.



27. a. $f(x)=x^2+x$, so $\dfrac{f(3)-f(2)}{3-2}=\dfrac{(3^2+3)-(2^2+2)}{1}=6,$

$\dfrac{f(2.5)-f(2)}{2.5-2}=\dfrac{(2.5^2+2.5)-(2^2+2)}{0.5}=5.5$, and $\dfrac{f(2.1)-f(2)}{2.1-2}=\dfrac{(2.1^2+2.1)-(2^2+2)}{0.1}=5.1.$

b. We first compute $f'(x)$ using the four-step process.

Step 1 $f(x+h)=(x+h)^2+(x+h)=x^2+2xh+h^2+x+h.$

Step 2 $f(x+h)-f(x)=\left(x^2+2xh+h^2+x+h\right)-\left(x^2+x\right)=2xh+h^2+h=h(2x+h+1).$

Step 3 $\dfrac{f(x+h)-f(x)}{h}=\dfrac{h(2x+h+1)}{h}=2x+h+1.$

Step 4 $f'(x)=\lim_{h\to 0}\dfrac{f(x+h)-f(x)}{h}=\lim_{h\to 0}(2x+h+1)=2x+1.$

The instantaneous rate of change of y at $x=2$ is $f'(2)=2(2)+1$, or 5 units per unit change in x.

c. The results of part (a) suggest that the average rates of change of f at $x=2$ approach 5 as the interval $[2,2+h]$ gets smaller and smaller ($h=1$, 0.5, and 0.1). This number is the instantaneous rate of change of f at $x=2$ as computed in part (b).

28. a. $f(x)=x^2-4x$, so $\dfrac{f(4)-f(3)}{4-3}=\dfrac{(16-16)-(9-12)}{1}=3,$

$\dfrac{f(3.5)-f(3)}{3.5-3}=\dfrac{(12.25-14)-(9-12)}{0.5}=2.5$, and $\dfrac{f(3.1)-f(3)}{3.1-3}=\dfrac{(9.61-12.4)-(9-12)}{0.1}=2.1.$

b. We first compute $f'(x)$ using the four-step process:

Step 1 $f(x+h) = (x+h)^2 - 4(x+h) = x^2 + 2xh + h^2 - 4x - 4h.$

Step 2 $f(x+h) - f(x) = \left(x^2 + 2xh + h^2 - 4x - 4h\right) - \left(x^2 - 4x\right) = 2xh + h^2 - 4h = h(2x + h - 4).$

Step 3 $\dfrac{f(x+h) - f(x)}{h} = \dfrac{h(2x+h-4)}{h} = 2x + h - 4.$

Step 4 $f'(x) = \lim_{h\to 0} \dfrac{f(x+h) - f(x)}{h} = \lim_{h\to 0}(2x + h - 4) = 2x - 4.$

The instantaneous rate of change of y at $x = 3$ is $f'(3) = 6 - 4 = 2$, or 2 units per unit change in x.

c. The results of part (a) suggest that the average rates of change of f over smaller and smaller intervals containing $x = 3$ approach the instantaneous rate of change of 2 units per unit change in x obtained in part (b).

29. a. $f(t) = 2t^2 + 48t$. The average velocity of the car over the time interval [20, 21] is $\dfrac{f(21) - f(20)}{21 - 20} = \dfrac{\left[2(21)^2 + 48(21)\right] - \left[2(20)^2 + 48(20)\right]}{1} = 130\ \dfrac{\text{ft}}{\text{s}}$. Its average velocity over [20, 20.1] is $\dfrac{f(20.1) - f(20)}{20.1 - 20} = \dfrac{\left[2(20.1)^2 + 48(20.1)\right] - \left[2(20)^2 + 48(20)\right]}{0.1} = 128.2\ \dfrac{\text{ft}}{\text{s}}$. Its average velocity over [20, 20.01] is $\dfrac{f(20.01) - f(20)}{20.01 - 20} = \dfrac{\left[2(20.01)^2 + 48(20.01)\right] - \left[2(20)^2 + 48(20)\right]}{0.01} = 128.02\ \dfrac{\text{ft}}{\text{s}}$.

b. We first compute $f'(t)$ using the four-step process.

Step 1 $f(t+h) = 2(t+h)^2 + 48(t+h) = 2t^2 + 4th + 2h^2 + 48t + 48h.$

Step 2 $f(t+h) - f(t) = \left(2t^2 + 4th + 2h^2 + 48t + 48h\right) - \left(2t^2 + 48t\right) = 4th + 2h^2 + 48h$

$$= h(4t + 2h + 48).$$

Step 3 $\dfrac{f(t+h) - f(t)}{h} = \dfrac{h(4t + 2h + 48)}{h} = 4t + 2h + 48.$

Step 4 $f'(t) = \lim_{t\to 0} \dfrac{f(t+h) - f(t)}{h} = \lim_{t\to 0}(4t + 2h + 48) = 4t + 48.$

The instantaneous velocity of the car at $t = 20$ is $f'(20) = 4(20) + 48$, or 128 ft/s.

c. Our results show that the average velocities do approach the instantaneous velocity as the intervals over which they are computed decreases.

30. a. The average velocity of the ball over the time interval [2, 3] is $\dfrac{s(3) - s(2)}{3 - 2} = \dfrac{\left[128(3) - 16(3)^2\right] - \left[128(2) - 16(2)^2\right]}{1} = 48$, or 48 ft/s. Over the time interval [2, 2.5], it is $\dfrac{s(2.5) - s(2)}{2.5 - 2} = \dfrac{\left[128(2.5) - 16(2.5)^2\right] - \left[128(2) - 16(2)^2\right]}{0.5} = 56$, or 56 ft/s. Over the time interval [2, 2.1], it is $\dfrac{s(2.1) - s(2)}{2.1 - 2} = \dfrac{\left[128(2.1) - 16(2.1)^2\right] - \left[128(2) - 16(2)^2\right]}{0.1} = 62.4$, or 62.4 ft/s.

b. Using the four-step process, we find that the instantaneous velocity of the ball at any time t is given by $v(t) = 128 - 32t$. In particular, the velocity of the ball at $t = 2$ is $v(2) = 128 - 32(2) = 64$, or 64 ft/s.

c. At $t = 5$, $v(5) = 128 - 32(5) = -32$, so the speed of the ball at $t = 5$ is 32 ft/s and it is falling.

d. The ball hits the ground when $s(t) = 0$, that is, when $128t - 16t^2 = 0$, whence $t(128 - 16t) = 0$, so $t = 0$ or $t = 8$. Thus, it will hit the ground when $t = 8$.

31. a. We solve the equation $16t^2 = 400$ and find $t = 5$, which is the time it takes the screwdriver to reach the ground.

b. The average velocity over the time interval [0, 5] is $\dfrac{f(5)-f(0)}{5-0}=\dfrac{16(25)-0}{5}=80$, or 80 ft/s.

c. The velocity of the screwdriver at time t is

$$v(t)=\lim_{h\to 0}\frac{f(t+h)-f(t)}{h}=\lim_{h\to 0}\frac{16(t+h)^2-16t^2}{h}=\lim_{h\to 0}\frac{16t^2+32th+16h^2-16t^2}{h}$$
$$=\lim_{h\to 0}\frac{(32t+16h)h}{h}=32t.$$

In particular, the velocity of the screwdriver when it hits the ground (at $t=5$) is $v(5)=32(5)=160$, or 160 ft/s.

32. a. We write $f(t)=\frac{1}{2}t^2+\frac{1}{2}t$. The height after 40 seconds is $f(40)=\frac{1}{2}(40)^2+\frac{1}{2}(40)=820$.

b. Its average velocity over the time interval [0, 40] is $\dfrac{f(40)-f(0)}{40-0}=\dfrac{820-0}{40}=20.5$, or 20.5 ft/s.

c. Its velocity at time t is

$$v(t)=\lim_{h\to 0}\frac{f(t+h)-f(t)}{h}=\lim_{h\to 0}\frac{\frac{1}{2}(t+h)^2+\frac{1}{2}(t+h)-\left(\frac{1}{2}t^2+\frac{1}{2}t\right)}{h}$$
$$=\lim_{h\to 0}\frac{\frac{1}{2}t^2+th+\frac{1}{2}h^2+\frac{1}{2}t+\frac{1}{2}h-\frac{1}{2}t^2-\frac{1}{2}t}{h}=\lim_{h\to 0}\frac{th+\frac{1}{2}h^2+\frac{1}{2}h}{h}=\lim_{h\to 0}\left(t+\frac{1}{2}h+\frac{1}{2}\right)=t+\frac{1}{2}.$$

In particular, the velocity at the end of 40 seconds is $v(40)=40+\frac{1}{2}$, or $40\frac{1}{2}$ ft/s.

33. a. We write $V=f(p)=\dfrac{1}{p}$. The average rate of change of V is $\dfrac{f(3)-f(2)}{3-2}=\dfrac{\frac{1}{3}-\frac{1}{2}}{1}=-\dfrac{1}{6}$, a decrease of $\frac{1}{6}$ liter/atmosphere.

b. $V'(t)=\lim_{h\to 0}\dfrac{\dfrac{f(p+h)-f(p)}{h}}{h}=\lim_{h\to 0}\dfrac{\dfrac{1}{p+h}-\dfrac{1}{p}}{h}=\lim_{h\to 0}\dfrac{p-(p+h)}{hp(p+h)}=\lim_{h\to 0}-\dfrac{1}{p(p+h)}=-\dfrac{1}{p^2}$. In particular, the rate of change of V when $p=2$ is $V'(2)=-\dfrac{1}{2^2}$, a decrease of $\frac{1}{4}$ liter/atmosphere.

34. $C(x)=-10x^2+300x+130$.

a. Using the four-step process, we find

$$C'(x)=\lim_{h\to 0}\frac{C(x+h)-C(x)}{h}=\lim_{h\to 0}\frac{h(-20x-10h+300)}{h}=-20x+300.$$

b. The rate of change is $C'(10)=-20(10)+300=100$, or \$100/surfboard.

35. a. $P(x)=-\frac{1}{3}x^2+7x+30$. Using the four-step process, we find that

$$P'(x)=\lim_{h\to 0}\frac{P(x+h)-P(x)}{h}=\lim_{h\to 0}\frac{-\frac{1}{3}\left(x^2+2xh+h^2\right)+7x+7h+30-\left(-\frac{1}{3}x^2+7x+30\right)}{h}$$
$$=\lim_{h\to 0}\frac{-\frac{2}{3}xh-\frac{1}{3}h^2+7h}{h}=\lim_{h\to 0}\left(-\frac{2}{3}x-\frac{1}{3}h+7\right)=-\frac{2}{3}x+7.$$

b. $P'(10)=-\frac{2}{3}(10)+7\approx 0.333$, or approximately \$333 per \$1000 spent on advertising.
$P'(30)=-\frac{2}{3}(30)+7=-13$, a decrease of \$13,000 per \$1000 spent on advertising.

36. a. $f(x) = -0.1x^2 - x + 40$, so $\dfrac{f(5.05) - f(5)}{5.05 - 5} = \dfrac{\left[-0.1(5.05)^2 - 5.05 + 40\right] - \left[-0.1(5)^2 - 5 + 40\right]}{0.05} = -2.005$, or approximately $-\$2.01$ per 1000 tents. $\dfrac{f(5.01) - f(5)}{5.01 - 5} = \dfrac{\left[-0.1(5.01)^2 - 5.01 + 40\right] - \left[-0.1(5)^2 - 5 + 40\right]}{0.01} = -2.001$, or approximately $-\$2.00$ per 1000 tents.

b. We compute $f'(x)$ using the four-step process, obtaining $f'(x) = \lim\limits_{h \to 0} \dfrac{f(x+h) - f(x)}{h} = \dfrac{h(-0.2x - 0.1h - 1)}{h} = \lim\limits_{h \to 0} (-0.2x - 0.1h - 1) = -0.2x - 1$. The rate of change of the unit price if $x = 5000$ is $f'(5) = -0.2(5) - 1 = -2$, a decrease of \$2 per 1000 tents.

37. $N(t) = t^2 + 2t + 50$. We first compute $N'(t)$ using the four-step process.

Step 1 $N(t+h) = (t+h)^2 + 2(t+h) + 50 = t^2 + 2th + h^2 + 2t + 2h + 50$.

Step 2 $N(t+h) - N(t) = \left(t^2 + 2th + h^2 + 2t + 2h + 50\right) - \left(t^2 + 2t + 50\right) = 2th + h^2 + 2h = h(2t + h + 2)$.

Step 3 $\dfrac{N(t+h) - N(t)}{h} = 2t + h + 2$.

Step 4 $N'(t) = \lim\limits_{h \to 0} (2t + h + 2) = 2t + 2$.

The rate of change of the country's GNP two years from now is $N'(2) = 2(2) + 2 = 6$, or \$6 billion/yr. The rate of change four years from now is $N'(4) = 2(4) + 2 = 10$, or \$10 billion/yr.

38. $f(t) = 3t^2 + 2t + 1$. Using the four-step process, we obtain $f'(t) = \lim\limits_{t \to 0} \dfrac{f(t+h) - f(t)}{h} = \lim\limits_{t \to 0} \dfrac{h(6t + 3h + 2)}{h} = \lim\limits_{t \to 0} (6t + 3h + 2) = 6t + 2$. Next, $f'(10) = 6(10) + 2 = 62$, and we conclude that the rate of bacterial growth at $t = 10$ is 62 bacteria per minute.

39. a. $f'(h)$ gives the instantaneous rate of change of the temperature with respect to height at a given height h, in °F per foot.

b. Because the temperature decreases as the altitude increases, the sign of $f'(h)$ is negative.

c. Because $f'(1000) = -0.05$, the change in the air temperature as the altitude changes from 1000 ft to 1001 ft is approximately $-0.05°$ F.

40. a. $\dfrac{f(b) - f(a)}{b - a}$ measures the average rate of change in revenue as the advertising expenditure changes from a thousand dollars to b thousand dollars. The units of measurement are thousands of dollars per thousands of dollars.

b. $f'(x)$ gives the instantaneous rate of change in the revenue when x thousand dollars is spent on advertising. It is measured in thousands of dollars per thousands of dollars.

c. Because $f'(20) \cdot (21 - 20) = 3 \cdot 1 = 3$, the approximate change in revenue is \$3000.

41. $\dfrac{f(a+h) - f(a)}{h}$ gives the average rate of change of the seal population over the time interval $[a, a+h]$. $\lim\limits_{h \to 0} \dfrac{f(a+h) - f(a)}{h}$ gives the instantaneous rate of change of the seal population at $x = a$.

42. $\dfrac{f(a+h) - f(a)}{h}$ gives the average rate of change of the prime interest rate over the time interval $[a, a+h]$. $\lim\limits_{h \to 0} \dfrac{f(a+h) - f(a)}{h}$ gives the instantaneous rate of change of the prime interest rate at $x = a$.

43. $\dfrac{f(a+h)-f(a)}{h}$ gives the average rate of change of the country's industrial production over the time interval $[a, a+h]$. $\lim\limits_{h\to 0}\dfrac{f(a+h)-f(a)}{h}$ gives the instantaneous rate of change of the country's industrial production at $x=a$.

44. $\dfrac{f(a+h)-f(a)}{h}$ gives the average rate of change of the cost incurred in producing the commodity over the production level $[a, a+h]$. $\lim\limits_{h\to 0}\dfrac{f(a+h)-f(a)}{h}$ gives the instantaneous rate of change of the cost of producing the commodity at $x=a$.

45. $\dfrac{f(a+h)-f(a)}{h}$ gives the average rate of change of the atmospheric pressure over the altitudes $[a, a+h]$. $\lim\limits_{h\to 0}\dfrac{f(a+h)-f(a)}{h}$ gives the instantaneous rate of change of the atmospheric pressure with respect to altitude at $x=a$.

46. $\dfrac{f(a+h)-f(a)}{h}$ gives the average rate of change of the fuel economy of a car over the speeds $[a, a+h]$. $\lim\limits_{h\to 0}\dfrac{f(a+h)-f(a)}{h}$ gives the instantaneous rate of change of the fuel economy at $x=a$.

47. **a.** f has a limit at $x=a$.
b. f is not continuous at $x=a$ because $f(a)$ is not defined.
c. f is not differentiable at $x=a$ because it is not continuous there.

48. **a.** f has a limit at $x=a$.
b. f is continuous at $x=a$.
c. f is differentiable at $x=a$.

49. **a.** f has a limit at $x=a$.
b. f is continuous at $x=a$.
c. f is not differentiable at $x=a$ because f has a kink at the point $x=a$.

50. **a.** f does not have a limit at $x=a$ because the left-hand and right-hand limits are not equal.
b. f is not continuous at $x=a$ because the limit does not exist there.
c. f is not differentiable at $x=a$ because it is not continuous there.

51. **a.** f does not have a limit at $x=a$ because it is unbounded in the neighborhood of a.
b. f is not continuous at $x=a$.
c. f is not differentiable at $x=a$ because it is not continuous there.

52. **a.** f does not have a limit at $x=a$ because the left-hand and right-hand limits are not equal.
b. f is not continuous at $x=a$ because the limit does not exist there.
c. f is not differentiable at $x=a$ because it is not continuous there.

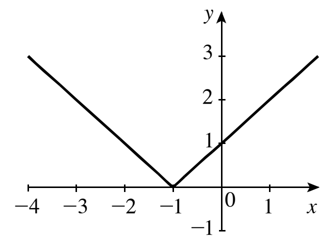
53. $s(t)=-0.1t^3+2t^2+24t$. Our computations yield the following results: 32.1, 30.939, 30.814, 30.8014, 30.8001, and 30.8000. The motorcycle's instantaneous velocity at $t=2$ is approximately 30.8 ft/s.

54. $C(x) = 0.000002x^3 + 5x + 400$. Our computations yield the following results: 5.060602, 5.06006002, 5.060006, 5.0600006, and 5.0600001. The rate of change of the total cost function when the level of production is 100 cases a day is approximately \$5.06.

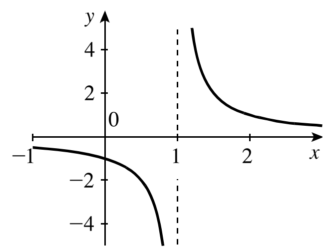
55. False. Let $f(x) = |x|$. Then f is continuous at $x = 0$, but is not differentiable there.

56. True. If g is differentiable at $x = a$, then it is continuous there. Therefore, the product fg is continuous, and so $\lim_{x \to a} f(x)g(x) = \left[\lim_{x \to a} f(x)\right]\left[\lim_{x \to a} g(x)\right] = f(a)g(a)$.

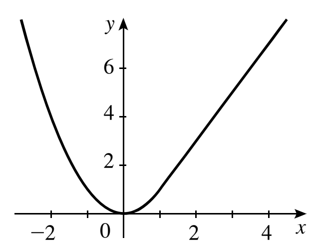
57. Observe that the graph of f has a kink at $x = -1$. We have $\dfrac{f(-1+h) - f(-1)}{h} = 1$ if $h > 0$, and -1 if $h < 0$, so that $\lim_{h \to 0} \dfrac{f(-1+h) - f(-1)}{h}$ does not exist.



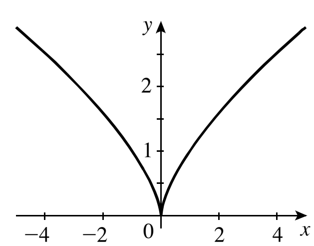
58. f does not have a derivative at $x = 1$ because it is not continuous there.



59. For continuity, we require that $f(1) = 1 = \lim_{x \to 1^+} (ax + b) = a + b$, or $a + b = 1$. Next, using the four-step process, we have $f'(x) = \begin{cases} 2x & \text{if } x < 1 \\ a & \text{if } x > 1 \end{cases}$ In order that the derivative exist at $x = 1$, we require that $\lim_{x \to 1^-} 2x = \lim_{x \to 1^+} a$, or $2 = a$. Therefore, $b = -1$ and so $f(x) = \begin{cases} x^2 & \text{if } x \leq 1 \\ 2x - 1 & \text{if } x > 1 \end{cases}$



60. f is continuous at $x = 0$, but $f'(0)$ does not exist because the graph of f has a vertical tangent line at $x = 0$.



61. We have $f(x) = x$ if $x > 0$ and $f(x) = -x$ if $x < 0$. Therefore, when $x > 0$, $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{x+h-x}{h} = \lim_{h \to 0} \frac{h}{h} = 1$, and when $x < 0$, $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{-x-h-(-x)}{h} = \lim_{h \to 0} \frac{-h}{h} = -1$. Because the right-hand limit does not equal the left-hand limit, we conclude that $\lim_{h \to 0} f(x)$ does not exist.

62. From $f(x) - f(a) = \left[\frac{f(x) - f(a)}{x-a}\right](x-a)$, we see that $\lim_{x \to a} [f(x) - f(a)] = \lim_{x \to a} \left[\frac{f(x) - f(a)}{x-a}\right] \lim_{x \to a} (x-a) = f'(a) \cdot 0 = 0$, and so $\lim_{x \to a} f(x) = f(a)$. This shows that f is continuous at $x = a$.

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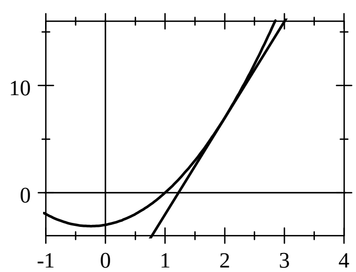
1. a. 9

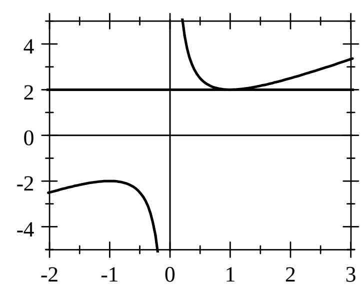
b.



c. $y = 9x - 11$

2. a. 0

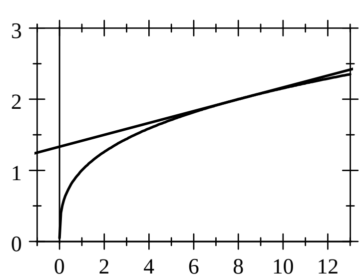
b.



c. $y = 2$

3. a. 0.08

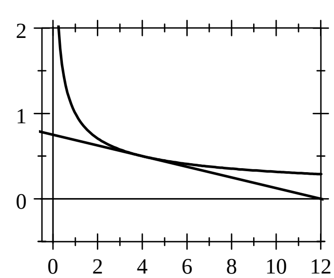
b.



c. $y = \frac{1}{12}x + \frac{4}{3}$

4. a. -0.06

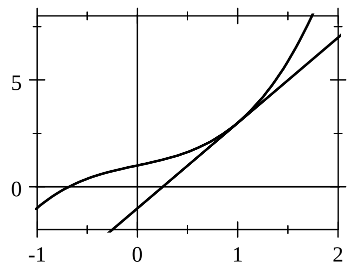
b.



c. $y = -\frac{1}{16}x + \frac{3}{4}$

5. **a.** 4

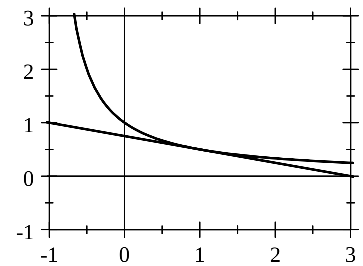
b.



c. $y = 4x - 1$

6. **a.** -0.25

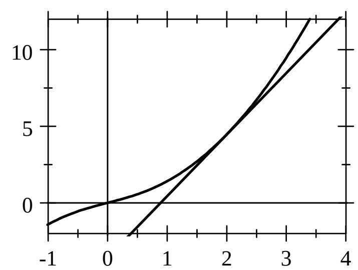
b.



c. $y = -\frac{1}{4}x + \frac{3}{4}$

7. **a.** 4.02

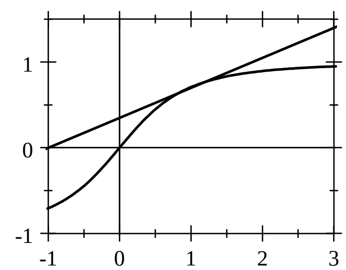
b.



c. $y = 4.02x - 3.57$

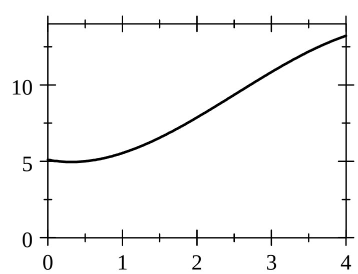
8. **a.** 0.35

b.



c. $y = 0.35x + 0.36$

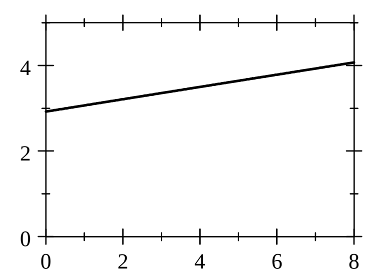
9. **a.**



b. $f'(3) = 2.8826$ (million per decade)

10. **a.** $S(t) = -0.000114719t^2 + 0.144618t + 2.92202$

b.



c. \$3.786 billion

d. \$143 million/yr

CHAPTER 2 Concept Review Questions page 156

1. domain, range, B

2. domain, $f(x)$, vertical, point

3. $f(x) \pm g(x)$, $f(x)g(x)$, $\dfrac{f(x)}{g(x)}$, $A \cap B$, $A \cap B$, 0

4. $g(f(x))$, f, $f(x)$, g

5. a. $P(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$, where $a_n \neq 0$ and n is a positive integer

b. linear, quadratic, cubic **c.** quotient, polynomials **d.** x^r, where r is a real number

6. $f(x), L, a$

7. a. L^r **b.** $L \pm M$ **c.** LM **d.** $\dfrac{L}{M}, M \neq 0$

8. a. L, x **b.** M, negative, absolute

9. a. right **b.** left **c.** L, L

10. a. continuous **b.** discontinuous **c.** every

11. a. $a, a, g(a)$ **b.** everywhere **c.** Q

12. a. $[a, b],\ f(c) = M$ **b.** $f(x) = 0, (a, b)$

13. a. $f'(a)$ **b.** $y - f(a) = m(x - a)$

14. a. $\dfrac{f(a+h) - f(a)}{h}$ **b.** $\lim_{h \to 0} \dfrac{f(a+h) - f(a)}{h}$

CHAPTER 2 Review Exercises page 157

1. a. $9 - x \geq 0$ gives $x \leq 9$, and the domain is $(-\infty, 9]$.

b. $2x^2 - x - 3 = (2x-3)(x+1)$, and $x = \frac{3}{2}$ or -1. Because the denominator of the given expression is zero at these points, we see that the domain of f cannot include these points and so the domain of f is $(-\infty, -1) \cup \left(-1, \frac{3}{2}\right) \cup \left(\frac{3}{2}, \infty\right)$.

2. a. We must have $2 - x \geq 0$ and $x + 3 \neq 0$. This implies $x \leq 2$ and $x \neq -3$, so the domain of f is $(-\infty, -3) \cup (-3, 2]$.

b. The domain is $(-\infty, \infty)$.

3. a. $f(-2) = 3(-2)^2 + 5(-2) - 2 = 0$.

b. $f(a+2) = 3(a+2)^2 + 5(a+2) - 2 = 3a^2 + 12a + 12 + 5a + 10 - 2 = 3a^2 + 17a + 20$.

c. $f(2a) = 3(2a)^2 + 5(2a) - 2 = 12a^2 + 10a - 2$.

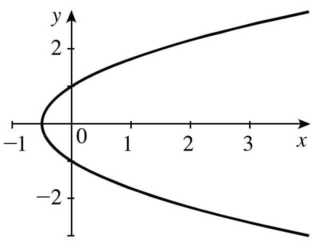
d. $f(a+h) = 3(a+h)^2 + 5(a+h) - 2 = 3a^2 + 6ah + 3h^2 + 5a + 5h - 2$.

4. a. $f(x-1) + f(x+1) = \left[2(x-1)^2 - (x-1) + 1\right] + \left[2(x+1)^2 - (x+1) + 1\right]$

$$= \left(2x^2 - 4x + 2 - x + 1 + 1\right) + \left(2x^2 + 4x + 2 - x - 1 + 1\right) = 4x^2 - 2x + 6.$$

b. $f(x+2h) = 2(x+2h)^2 - (x+2h) + 1 = 2x^2 + 8xh + 8h^2 - x - 2h + 1$.

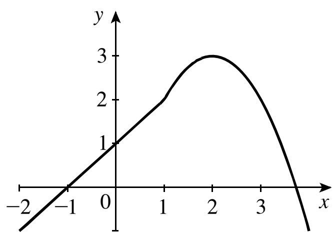
5. a.



b. For each value of $x > 0$, there are two values of y. We conclude that y is not a function of x. (We could also note that the function fails the vertical line test.)

c. Yes. For each value of y, there is only one value of x.

6.



7. a. $f(x)\,g(x) = \dfrac{2x+3}{x}$.

b. $\dfrac{f(x)}{g(x)} = \dfrac{1}{x(2x+3)}$.

c. $f(g(x)) = \dfrac{1}{2x+3}$.

d. $g(f(x)) = 2\left(\dfrac{1}{x}\right) + 3 = \dfrac{2}{x} + 3$.

8. a. $(f \circ g)(x) = f(g(x)) = 2g(x) - 1 = 2\left(x^2+4\right) - 1 = 2x^2 + 7$ and
$(g \circ f)(x) = g(f(x)) = \left[f(x)\right]^2 + 4 = (2x-1)^2 + 4 = 4x^2 - 4x + 5$.

b. $(f \circ g)(x) = f(g(x)) = 1 - g(x) = 1 - \dfrac{1}{3x+4} = \dfrac{3x+3}{3x+4} = \dfrac{3(x+1)}{3x+4}$ and
$(g \circ f)(x) = g(f(x)) = \dfrac{1}{3f(x)+4} = \dfrac{1}{3(1-x)+4} = \dfrac{1}{7-3x}$.

c. $(f \circ g)(x) = f(g(x)) = g(x) - 3 = \dfrac{1}{\sqrt{x+1}} - 3$ and
$(g \circ f)(x) = g(f(x)) = \dfrac{1}{\sqrt{f(x)+1}} = \dfrac{1}{\sqrt{(x-3)+1}} = \dfrac{1}{\sqrt{x-2}}$.

9. a. Take $f(x) = 2x^2 + x + 1$ and $g(x) = \dfrac{1}{x^3}$.

b. Take $f(x) = x^2 + x + 4$ and $g(x) = \sqrt{x}$.

10. We have $c(4)^2 + 3(4) - 4 = 2$, so $16c + 12 - 4 = 2$, or $c = -\dfrac{6}{16} = -\dfrac{3}{8}$.

11. $\lim\limits_{x\to 0}(5x-3) = 5(0) - 3 = -3$.

12. $\lim\limits_{x\to 1}\left(x^2+1\right) = (1)^2 + 1 = 1 + 1 = 2$.

13. $\lim\limits_{x\to -1}\left(3x^2+4\right)(2x-1) = \left[3(-1)^2+4\right]\left[2(-1)-1\right] = -21$.

14. $\lim\limits_{x\to 3}\dfrac{x-3}{x+4} = \dfrac{3-3}{3+4} = 0$

15. $\lim\limits_{x\to 2}\dfrac{x+3}{x^2-9} = \dfrac{2+3}{4-9} = -1$.

16. $\lim\limits_{x\to -2}\dfrac{x^2-2x-3}{x^2+5x+6}$ does not exist. (The denominator is 0 at $x = -2$.)

17. $\lim_{x\to 3}\sqrt{2x^3-5}=\sqrt{2(27)-5}=7.$

18. $\lim_{x\to 3}\frac{4x-3}{\sqrt{x+1}}=\frac{12-3}{\sqrt{4}}=\frac{9}{2}.$

19. $\lim_{x\to 1^+}\frac{x-1}{x(x-1)}=\lim_{x\to 1^+}\frac{1}{x}=1.$

20. $\lim_{x\to 1^-}\frac{\sqrt{x}-1}{x-1}=\lim_{x\to 1^-}\frac{(\sqrt{x}-1)(\sqrt{x}+1)}{(x-1)(\sqrt{x}+1)}=\lim_{x\to 1^-}\frac{x-1}{(x-1)(\sqrt{x}+1)}=\lim_{x\to 1^-}\frac{1}{\sqrt{x}+1}=\frac{1}{2}.$

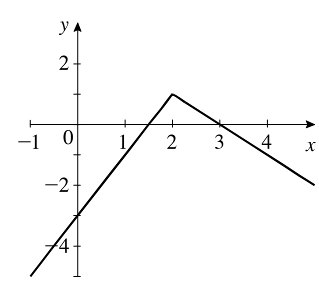
21. $\lim_{x\to\infty}\frac{x^2}{x^2-1}=\lim_{x\to\infty}\frac{1}{1-\frac{1}{x^2}}=1.$

22. $\lim_{x\to-\infty}\frac{x+1}{x}=\lim_{x\to-\infty}\left(1+\frac{1}{x}\right)=1.$

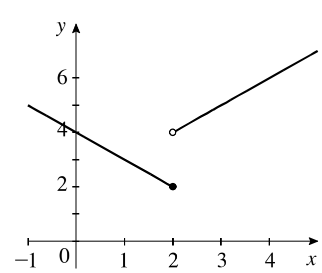
23. $\lim_{x\to\infty}\frac{3x^2+2x+4}{2x^2-3x+1}=\lim_{x\to\infty}\frac{3+\frac{2}{x}+\frac{4}{x^2}}{2-\frac{3}{x}+\frac{1}{x^2}}=\frac{3}{2}.$

24. $\lim_{x\to-\infty}\frac{x^2}{x+1}=\lim_{x\to-\infty}\left(x\cdot\frac{1}{1+\frac{1}{x}}\right)=-\infty$, so the limit does not exist.

25. $\lim_{x\to 2^+}f(x)=\lim_{x\to 2^+}(-x+3)=-2+3=1$ and
$\lim_{x\to 2^-}f(x)=\lim_{x\to 2^-}(2x-3)=2(2)-3=4-3=1.$
Therefore, $\lim_{x\to 2}f(x)=1.$



26. $\lim_{x\to 2^+}f(x)=\lim_{x\to 2^+}(x+2)=4$ and
$\lim_{x\to 2^-}f(x)=\lim_{x\to 2^-}(4-x)=2$. Therefore, $\lim_{x\to 2}f(x)$ does not exist.



27. The function is discontinuous at $x=2$.

28. Because the denominator $4x^2-2x-2=2\left(2x^2-x-1\right)=2(2x+1)(x-1)=0$ if $x=-\frac{1}{2}$ or 1, we see that f is discontinuous at these points.

29. Because $\lim_{x\to-1} f(x) = \lim_{x\to-1} \dfrac{1}{(x+1)^2} = \infty$ (does not exist), we see that f is discontinuous at $x = -1$.

30. The function is discontinuous at $x = 0$.

31. a. Let $f(x) = x^2 + 2$. Then the average rate of change of y over $[1, 2]$ is $\dfrac{f(2) - f(1)}{2 - 1} = \dfrac{(4+2) - (1+2)}{1} = 3$.

Over $[1, 1.5]$, it is $\dfrac{f(1.5) - f(1)}{1.5 - 1} = \dfrac{(2.25+2) - (1+2)}{0.5} = 2.5$. Over $[1, 1.1]$, it is $\dfrac{f(1.1) - f(1)}{1.1 - 1} = \dfrac{(1.21+2) - (1+2)}{0.1} = 2.1$.

b. Computing $f'(x)$ using the four-step process., we obtain $f'(x) = \lim_{h\to0} \dfrac{f(x+h) - f(x)}{h} = \lim_{h\to0} \dfrac{h(2x+h)}{h} = \lim_{h\to0} (2x+h) = 2$. Therefore, the instantaneous rate of change of f at $x = 1$ is $f'(1) = 2$, or 2 units per unit change in x.

32. $f(x) = 4x + 5$. We use the four-step process:

Step 1 $f(x+h) = 4(x+h) + 5 = 4x + 4h + 5$.

Step 2 $f(x+h) - f(x) = 4x + 4h + 5 - 4x - 5 = 4h$.

Step 3 $\dfrac{f(x+h) - f(x)}{h} = \dfrac{4h}{h} = 4$.

Step 4 $f'(x) = \lim_{h\to0} \dfrac{f(x+h) - f(x)}{h} = \lim_{h\to0} (4) = 4$.

33. $f(x) = \frac{3}{2}x + 5$. We use the four-step process:

Step 1 $f(x+h) = \frac{3}{2}(x+h) + 5 = \frac{3}{2}x + \frac{3}{2}h + 5$.

Step 2 $f(x+h) - f(x) = \frac{3}{2}x + \frac{3}{2}h + 5 - \frac{3}{2}x - 5 = \frac{3}{2}h$.

Step 3 $\dfrac{f(x+h) - f(x)}{h} = \dfrac{3}{2}$.

Step 4 $f'(x) = \lim_{h\to0} \dfrac{f(x+h) - f(x)}{h} = \lim_{h\to0} \dfrac{3}{2} = \dfrac{3}{2}$.

Therefore, the slope of the tangent line to the graph of the function f at the point $(-2, 2)$ is $\frac{3}{2}$. To find the equation of the tangent line to the curve at the point $(-2, 2)$, we use the point-slope form of the equation of a line, obtaining $y - 2 = \frac{3}{2}[x - (-2)]$ or $y = \frac{3}{2}x + 5$.

34. $f(x) = -x^2$. We use the four-step process:

Step 1 $f(x+h) = -(x+h)^2 = -x^2 - 2xh - h^2$.

Step 2 $f(x+h) - f(x) = \left(-x^2 - 2xh - h^2\right) - \left(-x^2\right) = -2xh - h^2 = h(-2x - h)$.

Step 3 $\dfrac{f(x+h) - f(x)}{h} = -2x - h$.

Step 4 $f'(x) = \lim_{h\to0} \dfrac{f(x+h) - f(x)}{h} = \lim_{h\to0} (-2x - h) = -2x$.

The slope of the tangent line is $f'(2) = -2(2) = -4$. An equation of the tangent line is $y - (-4) = -4(x - 2)$, or $y = -4x + 4$.

35. $f(x) = -\frac{1}{x}$. We use the four-step process:

Step 1 $f(x+h) = -\frac{1}{x+h}$.

Step 2 $f(x+h) - f(x) = -\frac{1}{x+h} - \left(-\frac{1}{x}\right) = -\frac{1}{x+h} + \frac{1}{x} = \frac{h}{x(x+h)}$.

Step 3 $\frac{f(x+h) - f(x)}{h} = \frac{1}{x(x+h)}$.

Step 4 $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{1}{x(x+h)} = \frac{1}{x^2}$.

36. **a.** f is continuous at $x = a$ because the three conditions for continuity are satisfied at $x = a$; that is, 1. $f(x)$ is defined. 2. $\lim_{x \to a} f(x)$ exists. 3. $\lim_{x \to a} f(x) = f(a)$.

b. f is not differentiable at $x = a$ because the graph of f has a kink at $x = a$.

37. $S(4) = 6000(4) + 30{,}000 = 54{,}000$.

38. **a.** The line passes through $(0, 2.4)$ and $(5, 7.4)$ and has slope $m = \frac{7.4 - 2.4}{5 - 0} = 1$. Letting y denote the sales, we see that an equation of the line is $y - 2.4 = 1(t - 0)$, or $y = t + 2.4$. We can also write this in the form $S(t) = t + 2.4$.

b. The sales in 2011 are $S(3) = 3 + 2.4 = 5.4$, or \$5.4 million.

39. **a.** $C(x) = 6x + 30{,}000$.

b. $R(x) = 10x$.

c. $P(x) = R(x) - C(x) = 10x - (6x + 30{,}000) = 4x - 30{,}000$.

d. $P(6000) = 4(6000) - 30{,}000 = -6000$, or a loss of \$6000. $P(8000) = 4(8000) - 30{,}000 = 2000$, or a profit of \$2000. $P(12{,}000) = 4(12{,}000) - 30{,}000 = 18{,}000$, or a profit of \$18,000.

40. Substituting the first equation into the second yields $3x - 2\left(\frac{3}{4}x + 6\right) + 3 = 0$, so $\frac{3}{2}x - 12 + 3 = 0$ and $x = 6$. Substituting this value of x into the first equation then gives $y = \frac{21}{2}$, so the point of intersection is $\left(6, \frac{21}{2}\right)$.

41. The profit function is given by $P(x) = R(x) - C(x) = 20x - (12x + 20{,}000) = 8x - 20{,}000$.

42. We solve the system $\begin{cases} 3x + p - 40 = 0 \\ 2x - p + 10 = 0 \end{cases}$ Adding these two equations, we obtain $5x - 30 = 0$, or $x = 6$. Thus, $p = 2x + 10 = 12 + 10 = 22$. Therefore, the equilibrium quantity is 6000 and the equilibrium price is \$22.

43. The child should receive $D(35) = \frac{500(35)}{150} \approx 117$, or approximately 117 mg.

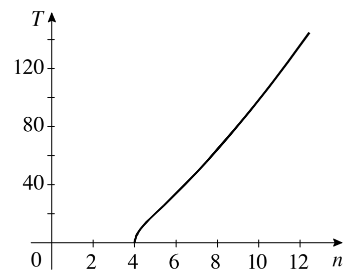
44. When 1000 units are produced, $R(1000) = -0.1(1000)^2 + 500(1000) = 400{,}000$, or \$400,000.

45. $R(30) = -\frac{1}{2}(30)^2 + 30(30) = 450$, or \$45,000.

46. $N(0) = 200(4 + 0)^{1/2} = 400$, and so there are 400 members initially. $N(12) = 200(4 + 12)^{1/2} = 800$, and so there are 800 members after one year.

47. The population will increase by $P(9) - P(0) = \left[50{,}000 + 30(9)^{3/2} + 20(9)\right] - 50{,}000$, or 990, during the next 9 months. The population will increase by $P(16) - P(0) = \left[50{,}000 + 30(16)^{3/2} + 20(16)\right] - 50{,}000$, or 2240 during the next 16 months.

48. $T = f(n) = 4n\sqrt{n-4}$. $f(4) = 0$, $f(5) = 20\sqrt{1} = 20$, $f(6) = 24\sqrt{2} \approx 33.9$, $f(7) = 28\sqrt{3} \approx 48.5$, $f(8) = 32\sqrt{4} = 64$, $f(9) = 36\sqrt{5} \approx 80.5$, $f(10) = 40\sqrt{6} \approx 98$, $f(11) = 44\sqrt{7} \approx 116$, and $f(12) = 48\sqrt{8} \approx 135.8$.



49. We need to find the point of intersection of the two straight lines representing the given linear functions. We solve the equation $2.3 + 0.4t = 1.2 + 0.6t$, obtaining $1.1 = 0.2t$ and thus $t = 5.5$. This tells us that the annual sales of the Cambridge Drug Store first surpasses that of the Crimson Drug store $5\frac{1}{2}$ years from now.

50. We solve $-1.1x^2 + 1.5x + 40 = 0.1x^2 + 0.5x + 15$, obtaining $1.2x^2 - x - 25 = 0$, $12x^2 - 10x - 250 = 0$, $6x^2 - 5x - 125 = 0$, and $(x-5)(6x+25) = 0$. Therefore, $x = 5$. Substituting this value of x into the second supply equation, we have $p = 0.1(5)^2 + 0.5(5) + 15 = 20$. So the equilibrium quantity is 5000 and the equilibrium price is \$20.

51. The life expectancy of a female whose current age is 65 is $C(65) = 0.0053694(65)^2 - 1.4663(65) + 92.74 \approx 20.12$ (years). The life expectancy of a female whose current age is 75 is $C(75) = 0.0053694(75)^2 - 1.4663(75) + 92.74 \approx 12.97$ (years).

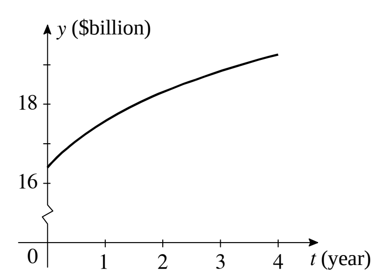
52. **a.** The amount of Medicare benefits paid out in 2010 is $B(0) = 0.25$, or \$250 billion.

b. The amount of Medicare benefits projected to be paid out in 2040 is $B(3) = 0.09(3)^2 + (0.102)(3) + 0.25 = 1.366$, or \$1.366 trillion.

53. $N(0) = 648$, or 648,000, $N(1) = -35.8 + 202 + 87.7 + 648 \approx 902$ or 902,000, $N(2) = -35.8(2)^3 + 202(2)^2 + 87.8(2) + 648 = 1345.2$ or 1,345,200, and $N(3) = -35.8(3)^3 + 202(3)^2 + 87.8(3) + 648 = 1762.8$ or 1,762,800.

54. **a.** $A(0) = 16.4$, or \$16.4 billion; $A(1) = 16.4(1+1)^{0.1} \approx 17.58$, or \$17.58 billion; $A(2) = 16.4(2+1)^{0.1} \approx 18.30$, or \$18.3 billion; $A(3) = 16.4(3+1)^{0.1} \approx 18.84$, or \$18.84 billion; and $A(4) = 16.4(4+1)^{0.1} \approx 19.26$, or \$19.26 billion. The nutritional market grew over the years 1999 to 2003.

b.



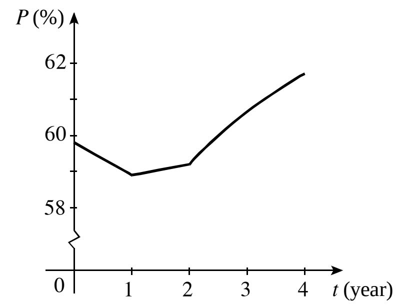
55. **a.** $f(t) = 267$; $g(t) = 2t^2 + 46t + 733$.

b. $h(t) = (f+g)(t) = f(t) + g(t) = 267 + \left(2t^2 + 46t + 733\right) = 2t^2 + 46t + 1000$.

c. $h(13) = 2(13)^2 + 46(13) + 1000 = 1936$, or 1936 tons.

56. **a.** $P(0) = 59.8$, $P(1) = 0.3(1) + 58.6 = 58.9$, $P(2) = 56.79(2)^{0.06} \approx 59.2$, $P(3) = 56.79(3)^{0.06} \approx 60.7$, and $P(4) = 56.79(4)^{0.06} \approx 61.7$.

b.



c. $P(3) \approx 60.7$, or 60.7%.

57. **a.** $f(r) = \pi r^2$.

b. $g(t) = 2t$.

c. $h(t) = (f \circ g)(t) = f(g(t)) = \pi [g(t)]^2 = 4\pi t^2$.

d. $h(30) = 4\pi (30^2) = 3600\pi$, or 3600π ft^2.

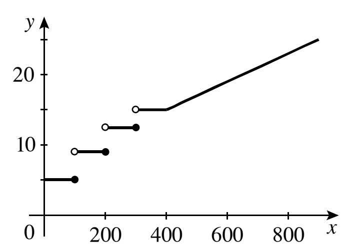
58. Measured in inches, the sides of the resulting box have length $20 - 2x$ and the height is x, so its volume is $V = x(20 - 2x)^2$ in^3.

59. Let h denote the height of the box. Then its volume is $V = (x)(2x)h = 30$, so that $h = \dfrac{15}{x^2}$. Thus, the cost is

$$\begin{aligned} C(x) &= 30(x)(2x) + 15[2xh + 2(2x)h] + 20(x)(2x) \\ &= 60x^2 + 15(6xh) + 40x^2 = 100x^2 + (15)(6)x\left(\frac{15}{x^2}\right) \\ &= 100x^2 + \frac{1350}{x}. \end{aligned}$$

60.

$$C(x) = \begin{cases} 5 & \text{if } 1 \le x \le 100 \\ 9 & \text{if } 100 < x \le 200 \\ 12.50 & \text{if } 200 < x \le 300 \\ 15.00 & \text{if } 300 < x \le 400 \\ 7 + 0.02x & \text{if } x > 400 \end{cases}$$



61. $\lim_{x\to\infty} \overline{C}(x) = \lim_{x\to\infty} \left(20 + \dfrac{400}{x}\right) = 20$. As the level of production increases without bound, the average cost of producing the commodity steadily decreases and approaches \$20 per unit.

62. **a.** $C'(x)$ gives the instantaneous rate of change of the total manufacturing cost c in dollars when x units of a certain product are produced.

b. Positive

c. Approximately \$20.

63. True. If $x < 0$, then $\sqrt{x}$ is not defined, and if $x > 0$, then $\sqrt{-x}$ is not defined. Therefore $f(x)$ is defined nowhere, and is not a function.

64. False. Let $f(x) = x^{1/3} + 1$. Then $f'(x) = \frac{1}{3}x^{-2/3}$, so $f'(1) = \frac{1}{3}$ and an equation of the tangent line to the graph of f at the point $(1, 2)$ is $y - 2 = \frac{1}{3}(x - 1)$ or $y = \frac{1}{3}x + \frac{5}{3}$. This tangent line intersects the graph of f at the point $(-8, -1)$, as can be easily verified.

CHAPTER 2 Before Moving On... page 160

1. a. $f(-1) = -2(-1) + 1 = 3$.

b. $f(0) = 2$.

c. $f\left(\frac{3}{2}\right) = \left(\frac{3}{2}\right)^2 + 2 = \frac{17}{4}$.

2. a. $(f + g)(x) = f(x) + g(x) = \dfrac{1}{x+1} + x^2 + 1$.

b. $(fg)(x) = f(x)g(x) = \dfrac{x^2+1}{x+1}$.

c. $(f \circ g)(x) = f(g(x)) = \dfrac{1}{g(x)+1} = \dfrac{1}{x^2+2}$.

d. $(g \circ f)(x) = g(f(x)) = [f(x)]^2 + 1 = \dfrac{1}{(x+1)^2} + 1$.

3. $4x + h = 108$, so $h = 108 - 4x$. The volume is $V = x^2h = x^2(108 - 4x) = 108x^2 - 4x^3$.

4. $\displaystyle\lim_{x\to-1} \frac{x^2+4x+3}{x^2+3x+2} = \lim_{x\to-1} \frac{(x+3)(x+1)}{(x+2)(x+1)} = 2$.

5. a. $\displaystyle\lim_{x\to1^-} f(x) = \lim_{x\to1^-} \left(x^2 - 1\right) = 0$.

b. $\displaystyle\lim_{x\to1^+} f(x) = \lim_{x\to1^+} x^3 = 1$.

Because $\displaystyle\lim_{x\to1^-} f(x) \neq \lim_{x\to1^+} f(x)$, f is not continuous at 1.

6. The slope of the tangent line at any point is

$$\begin{aligned}\lim_{h\to0} \frac{f(x+h) - f(x)}{h} &= \lim_{h\to0} \frac{(x+h)^2 - 3(x+h) + 1 - \left(x^2 - 3x + 1\right)}{h} \\ &= \lim_{h\to0} \frac{x^2 + 2xh + h^2 - 3x - 3h - x^2 + 3x - 1}{h} \\ &= \lim_{h\to0} \frac{h(2x+h-3)}{h} = \lim_{h\to0}(2x+h-3) = 2x - 3.\end{aligned}$$

Therefore, the slope at 1 is $2(1) - 3 = -1$. An equation of the tangent line is $y - (-1) = -1(x - 1)$, or $y + 1 = -x + 1$, or $y = -x$.

CHAPTER 2 Explore & Discuss

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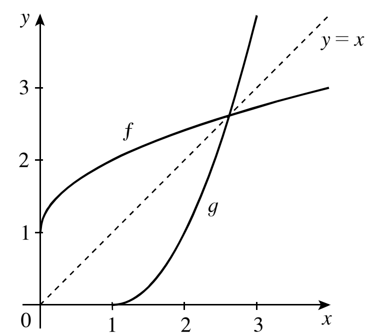
1. $(g \circ f)(x) = g(f(x)) = [f(x) - 1]^2 = [(\sqrt{x} + 1) - 1]^2 = (\sqrt{x})^2 = x$ and
$(f \circ g)(x) = f(g(x)) = \sqrt{g(x)} + 1 = \sqrt{(x-1)^2} + 1 = (x-1) + 1 = x$.

2. Refer to the figure at right. If a mirror is placed along the line $y = x$, then the graphs are reflections of each other.



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1. As x approaches 0 from either direction, $h(x)$ oscillates more and more rapidly between -1 and 1 and therefore cannot approach a specific number. But this says $\lim_{x \to 0} h(x)$ does not exist.

2. The function f fails to have a limit at $x = 0$ because $f(x)$ approaches 1 from the right but -1 from the left. The function g fails to have a limit at $x = 0$ because $g(x)$ is unbounded on either side of $x = 0$. The function h here does not approach any number from either the right or the left and has no limit at 0, as explained earlier.

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1. $\lim_{x \to \infty} f(x)$ does not exist because no matter how large x is, $f(x)$ takes on values between -1 and 1. In other words, $f(x)$ does not approach a definite number as x approaches infinity. Similarly, $\lim_{x \to -\infty} f(x)$ fails to exist.

2. The function of Example 10 fails to have a limit at infinity (negative infinity) because $f(x)$ increases (decreases) without bound as x approaches infinity (negative infinity). On the other hand, the function whose graph is depicted here, though bounded (its values lie between -1 and 1), does not approach any specific number as x increases (decreases) without bound and this is the reason it fails to have a limit at infinity or negative infinity.

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The average rate of change of a function f is measured over an interval. Thus, the average rate of change of f over the interval $[a, b]$ is the number $\dfrac{f(b) - f(a)}{b - a}$. On the other hand, the instantaneous rate of change of a function measures the rate of change of the function at a point. As we have seen, this quantity can be found by taking the limit of an appropriate difference quotient. Specifically, the instantaneous rate of change of f at $x = a$ is $\lim_{h \to 0} \dfrac{f(a+h) - f(a)}{h}$.

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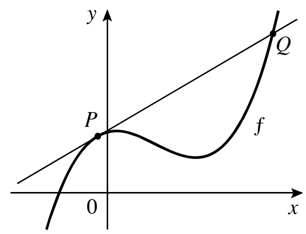
Yes. Here the line tangent to the graph of f at P also intersects the graph at the point Q lying on the graph of f.



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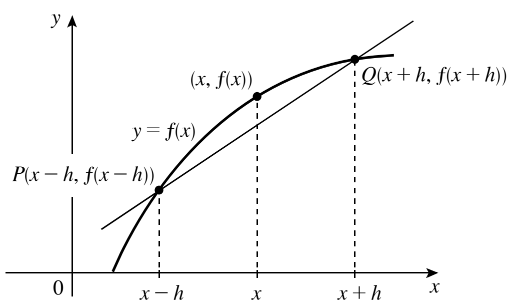
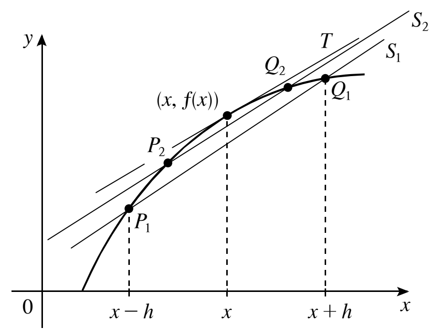
1. The quotient gives the slope of the secant line passing through $P\,(x-h,\ f\,(x-h))$ and $Q\,(x+h,\ f\,(x+h))$. It also gives the average rate of change of f over the interval $[x-h, x+h]$.



2. The limit gives the slope of the tangent line to the graph of f at the point $(x,\ f\,(x))$. It also gives the instantaneous rate of change of f at the point $(x,\ f\,(x))$. As h gets smaller and smaller, the secant lines approach the tangent line T.



3. The observation in part (b) suggests that this definition makes sense. We can also justify this observation as follows: From the definition of $f'(x)$, we have $f'(x) = \lim_{h\to 0} \dfrac{f(x+h) - f(x)}{h}$. Replacing h by $-h$ gives $f'(x) = \lim_{h\to 0} \dfrac{f(x-h) - f(x)}{-h} = \lim_{h\to 0} \dfrac{f(x) - f(x-h)}{h}$. Thus, $2f'(x) = \lim_{h\to 0} \left[\dfrac{f(x+h) - f(x)}{h} + \dfrac{f(x) - f(x-h)}{h}\right]$, and so $f'(x) = \lim_{h\to 0} \dfrac{f(x+h) - f(x-h)}{2h}$, in agreement with the result of Example 3.

4. **Step 1** Compute $f(x+h)$ and $f(x-h)$.
   Step 2 Form the difference $f(x+h) - f(x-h)$.
   Step 3 Form the quotient $\dfrac{f(x+h) - f(x-h)}{2h}$.
   Step 4 Compute $f'(x) = \lim_{h\to 0} \dfrac{f(x+h) - f(x-h)}{2h}$.
   For the function $f(x) = x^2$, we have the following:
   Step 1 $f(x+h) = (x+h)^2 = x^2 + 2xh + h^2$ and $f(x-h) = (x-h)^2 = x^2 - 2xh + h^2$.
   Step 2 $f(x+h) - f(x-h) = \left(x^2 + 2xh + h^2\right) - \left(x^2 - 2xh + h^2\right) = 4xh$.
   Step 3 $\dfrac{f(x+h) - f(x)}{2h} = \dfrac{4xh}{2h} = 2x$.
   Step 4 $f'(x) = \lim_{h\to 0} \dfrac{f(x+h) - f(x)}{2h} = \lim_{h\to 0} 2x = 2x$, in agreement with the result of Example 3.

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No. The slope of the tangent line to the graph of f at $(a, f(a))$ is defined by $f'(a) = \lim_{h\to 0} \frac{f(a+h) - f(a)}{h}$, and because the limit must be unique (see the definition of a limit), there is only one number $f'(a)$ giving the slope of the tangent line. Furthermore, since there can only be one straight line with a given slope, $f'(a)$, passing through a given point, $(a, f(a))$, our conclusion follows.

CHAPTER 2 Exploring with Technology

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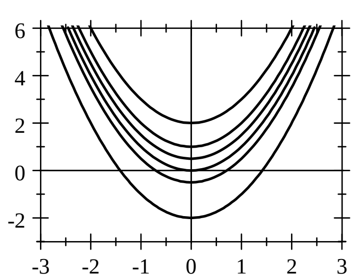
1.



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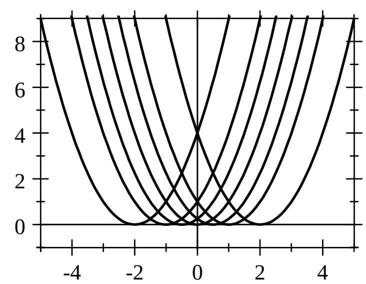

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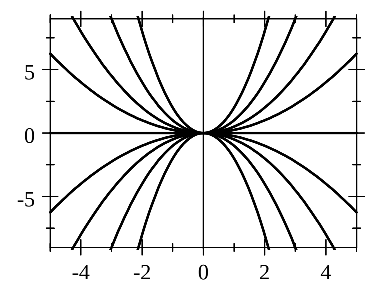

4. The graph of $f(x) + c$ is obtained by translating the graph of f along the y-axis by c units. The graph of $f(x + c)$ is obtained by translating the graph of f along the x-axis by c units. Finally, the graph of cf is obtained from that of f by "expanding"(if $c > 1$) or "contracting"(if $0 < c < 1$) that of f. If $c < 0$, the graph of cf is obtained from that of f by reflecting it with respect to the x-axis as well as expanding or contracting it.

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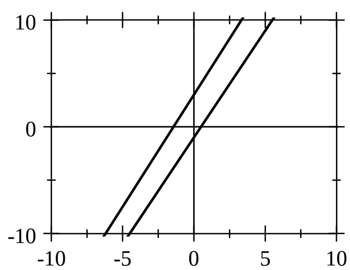
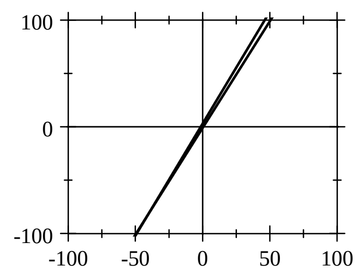
1. a.



The lines seem to be parallel to each other and do not appear to intersect.

b.

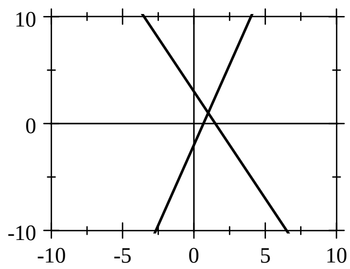


They appear to intersect. But finding the point of intersection using TRACE and ZOOM with any degree of accuracy seems to be an impossible task. Using the intersection feature of the graphing utility yields the point of intersection $(-40, -81)$ immediately.

c. Substituting the first equation into the second gives $2x - 1 = 2.1x + 3$, $-4 = 0.1x$, and thus $x = -40$. The corresponding y-value is -81.

d. Using TRACE and ZOOM is not effective. The intersection feature gives the desired result immediately. The algebraic method also yields the answer with little effort and without the use of a graphing utility.

2. a.



Plotting the straight lines L_1 and L_2 and using TRACE and ZOOM repeatedly, you will see that the iterations approach the answer $(1, 1)$. Using the intersection feature of the graphing utility gives the result $x = 1$ and $y = 1$, immediately.

b. Substituting the first equation into the second yields $3x - 2 = -2x + 3$, so $5x = 5$ and $x = 1$. Substituting this value of x into either equation gives $y = 1$.

c. The iterations obtained using TRACE and ZOOM converge to the solution $(1, 1)$. The use of the intersection feature is clearly superior to the first method. The algebraic method also yields the desired result easily.

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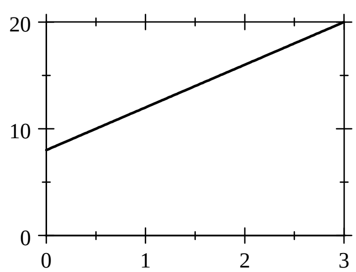
1.



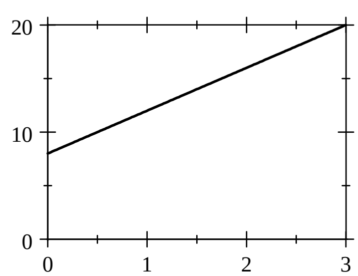
2. Using TRACE and ZOOM repeatedly, we find that $g(x)$ approaches 16 as x approaches 2.

3. If we try to use the evaluation function of the graphing utility to find $g(2)$ it will fail. This is because $x = 2$ is not in the domain of g.

4. The results obtained here confirm those obtained in the preceding example.

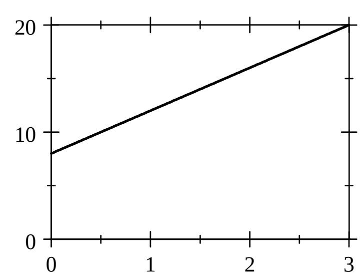
Page 109 (First Box)

1.



Using TRACE, we find $\lim_{x\to 2} \frac{4(x^2 - 4)}{x - 2} = 16$.

2.



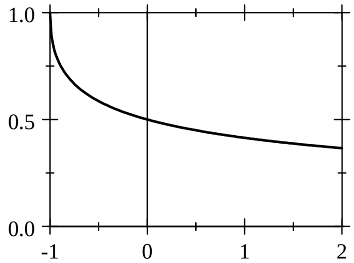
Using TRACE, we find $\lim_{x\to 2} 4(x + 2) = 16$. When $x = 2$, $y = 16$. The function $f(x) = 4(x + 2)$ is defined at $x = 2$ and so $f(2) = 16$ is defined.

3. No.

4. As we saw in Example 5, the function f is not defined at $x = 2$, but g is defined there.

Page 109 (Second Box)

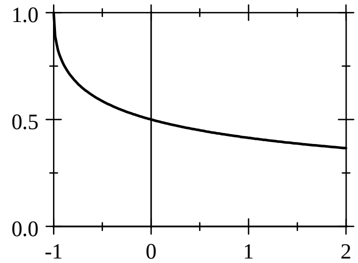
1.



Using TRACE and ZOOM, we see that

$$\lim_{x\to 0}\frac{\sqrt{1+x}-1}{x}=0.5.$$

2.



The graph of f is the same as that of g except that the domain of f includes $x=0$. (This is not evident from simply looking at the graphs!) Using the evaluation function to find the value of y, we obtain $y=0.5$ when $x=0$. This is to be expected since $x=0$ lies in the domain of g.

3. As mentioned in part 2, the graphs are indistinguishable even though $x=0$ is in the domain of g but not in the domain of f.

4. The functions f and g are the same everywhere except at $x=0$ and so $\lim_{x\to 0}\frac{\sqrt{1+x}-1}{x}=\lim_{x\to 0}\frac{1}{\sqrt{1+x}+1}=\frac{1}{2}$, as seen in Example 6.

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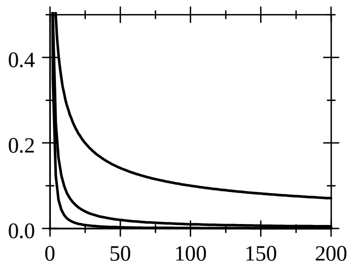
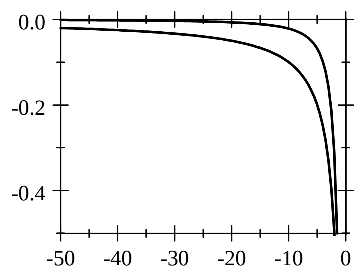
1.



The results suggest that $\frac{1}{x^n}$ goes to zero (as x increases) with increasing rapidity as n gets larger, as predicted by Theorem 2.

2.



The results suggest that $\frac{1}{x^n}$ goes to zero (as negative x increases in absolute value) with increasing rapidity as n gets larger, as predicted by Theorem 2.

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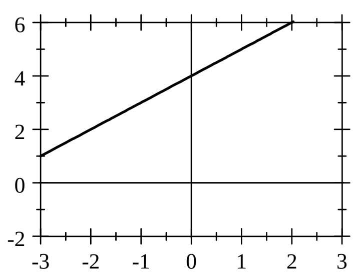
1.



2. Using ZOOM repeatedly, we find $\lim_{x\to 0} g(x)=4$.

3. The fact that the limit found in part 2 is $f'(2)$ is an illustration of the definition of a derivative.

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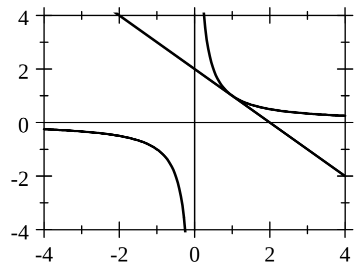

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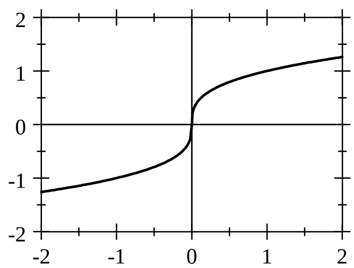
1.



2. The graphing utility will indicate an error when you try to draw the tangent line to the graph of f at $(0, 0)$. This happens because the slope of the tangent line to the graph of $f(x)$ is not defined at $x = 0$.

3 DIFFERENTIATION

3.1 Basic Rules of Differentiation

Concept Questions page 169

1. **a.** The derivative of a constant is zero.

   b. The derivative of $f(x) = x^n$ is n times x raised to the $(n-1)$th power.

   c. The derivative of a constant times a function is the constant times the derivative of the function.

   d. The derivative of a sum is the sum of the derivatives.

2. **a.** $h'(x) = 2f'(x)$, so $h'(2) = 2f'(2) = 2(3) = 6$.

   b. $F'(x) = 3f'(x) - 4g'(x)$, so $F'(2) = 3f'(2) - 4g'(2) = 3(3) - 4(-2) = 17$.

3. **a.** $F'(x) = \frac{d}{dx}\left[af(x) + bg(x)\right] = \frac{d}{dx}\left[af(x)\right] + \frac{d}{dx}\left[bg(x)\right] = af'(x) + bg'(x)$.

   b. $F'(x) = \frac{d}{dx}\left[\frac{f(x)}{a}\right] = \frac{1}{a}\frac{d}{dx}\left[f(x)\right] = \frac{f'(x)}{a}$.

4. No. The expression on the left is the derivative at a of the function f, whereas the expression on the right is the derivative of a constant obtained by evaluating f at a. For example, if $f(x) = x^2$ and $a = 1$, then $\left[f'(x)\right](a) = 2x|_{x=1} = 2$, but $\frac{d}{dx}\left[f(a)\right] = \frac{d}{dx}(1^2) = 0$.

Exercises page 169

1. $f'(x) = \frac{d}{dx}(-3) = 0$.

2. $f'(x) = \frac{d}{dx}(365) = 0$.

3. $f'(x) = \frac{d}{dx}\left(x^5\right) = 5x^4$.

4. $f'(x) = \frac{d}{dx}\left(x^7\right) = 7x^6$.

5. $f'(x) = \frac{d}{dx}\left(x^{3.1}\right) = 3.1x^{2.1}$.

6. $f'(x) = \frac{d}{dx}\left(x^{0.8}\right) = 0.8x^{-0.2}$.

7. $f'(x) = \frac{d}{dx}\left(3x^2\right) = 6x$.

8. $f'(x) = \frac{d}{dx}\left(-2x^3\right) = -6x^2$.

9. $f'(r) = \frac{d}{dr}\left(\pi r^2\right) = 2\pi r$.

10. $f'(r) = \frac{d}{dr}\left(\frac{4}{3}\pi r^3\right) = 4\pi r^2$.

11. $f'(x) = \frac{d}{dx}\left(9x^{1/3}\right) = \frac{1}{3}(9)\,x^{(1/3-1)} = 3x^{-2/3}$.

12. $f'(x) = \frac{d}{dx}\left(\frac{5}{4}x^{4/5}\right) = \left(\frac{4}{5}\right)\left(\frac{5}{4}\right)x^{-1/5} = x^{-1/5}$.

13. $f'(x) = \frac{d}{dx}\left(3\sqrt{x}\right) = \frac{d}{dx}\left(3x^{1/2}\right) = \frac{1}{2}(3)\,x^{-1/2} = \frac{3}{2}x^{-1/2} = \frac{3}{2\sqrt{x}}$.

14. $f'(u) = \frac{d}{du}\left(\frac{2}{\sqrt{u}}\right) = \frac{d}{du}\left(2u^{-1/2}\right) = -\frac{1}{2}(2)\,u^{-3/2} = -u^{-3/2}$.

15. $f'(x) = \frac{d}{dx}\left(7x^{-12}\right) = (-12)(7)\,x^{-12-1} = -84x^{-13}$.

16. $f'(x) = \frac{d}{dx}\left(0.3x^{-1.2}\right) = (0.3)(-1.2)\,x^{-2.2} = -0.36x^{-2.2}$.

17. $f'(x) = \frac{d}{dx}\left(5x^2 - 3x + 7\right) = 10x - 3$.

18. $f'(x) = \frac{d}{dx}\left(x^3 - 3x^2 + 1\right) = 3x^2 - 6x$.

19. $f'(x) = \frac{d}{dx}\left(-x^3 + 2x^2 - 6\right) = -3x^2 + 4x$.

20. $f'(x) = \frac{d}{dx}\left[\left(1 + 2x^2\right)^2 + 2x^3\right] = \frac{d}{dx}\left(1 + 4x^2 + 4x^4 + 2x^3\right) = 8x + 16x^3 + 6x^2 = 2x\left(8x^2 + 3x + 4\right)$.

21. $f'(x) = \frac{d}{dx}\left(0.03x^2 - 0.4x + 10\right) = 0.06x - 0.4$.

22. $f'(x) = \frac{d}{dx}\left(0.002x^3 - 0.05x^2 + 0.1x - 20\right) = 0.006x^2 - 0.1x + 0.1$.

23. $f(x) = \dfrac{2x^3 - 4x^2 + 3}{x} = 2x^2 - 4x + \dfrac{3}{x}$, so $f'(x) = \dfrac{d}{dx}\left(2x^2 - 4x + 3x^{-1}\right) = 4x - 4 - \dfrac{3}{x^2}$.

24. $f(x) = \dfrac{x^3 + 2x^2 + x - 1}{x} = x^2 + 2x + 1 - x^{-1}$, so $f'(x) = \frac{d}{dx}\left(x^2 + 2x + 1 - x^{-1}\right) = 2x + 2 + x^{-2}$.

25. $f'(x) = \frac{d}{dx}\left(4x^4 - 3x^{5/2} + 2\right) = 16x^3 - \frac{15}{2}x^{3/2}$.

26. $f'(x) = \frac{d}{dx}\left(5x^{4/3} - \frac{2}{3}x^{3/2} + x^2 - 3x + 1\right) = \frac{20}{3}x^{1/3} - x^{1/2} + 2x - 3$.

27. $f'(x) = \frac{d}{dx}\left(5x^{-1} + 4x^{-2}\right) = -5x^{-2} - 8x^{-3} = \dfrac{-5}{x^2} - \dfrac{8}{x^3}$.

28. $f'(x) = \frac{d}{dx}\left[-\frac{1}{3}\left(x^{-3} - x^6\right)\right] = -\frac{1}{3}\left(-3x^{-4} - 6x^5\right) = x^{-4} + 2x^5$.

29. $f'(t) = \frac{d}{dt}\left(4t^{-4} - 3t^{-3} + 2t^{-1}\right) = -16t^{-5} + 9t^{-4} - 2t^{-2} = -\dfrac{16}{t^5} + \dfrac{9}{t^4} - \dfrac{2}{t^2}$.

30. $f'(x) = \frac{d}{dx}\left(5x^{-3} - 2x^{-2} - x^{-1} + 200\right) = -15x^{-4} + 4x^{-3} + x^{-2} = -\dfrac{15}{x^4} + \dfrac{4}{x^3} + \dfrac{1}{x^2}$.

31. $f'(x) = \frac{d}{dx}\left(3x - 5x^{1/2}\right) = 3 - \frac{5}{2}x^{-1/2} = 3 - \dfrac{5}{2\sqrt{x}}$.

32. $f'(t) = \frac{d}{dt}\left(2t^2 + t^{3/2}\right) = 4t + \frac{3}{2}t^{1/2}$.

33. $f'(x) = \dfrac{d}{dx}\left(2x^{-2} - 3x^{-1/3}\right) = -4x^{-3} + x^{-4/3} = -\dfrac{4}{x^3} + \dfrac{1}{x^{4/3}}$.

34. $f'(x) = \dfrac{d}{dx}\left(\dfrac{3}{x^3} + \dfrac{4}{\sqrt{x}} + 1\right) = \dfrac{d}{dx}\left(3x^{-3} + 4x^{-1/2} + 1\right) = -9x^{-4} - 2x^{-3/2} = -\dfrac{9}{x^4} - \dfrac{2}{x^{3/2}}$.

35. $f'(x) = \frac{d}{dx}\left(2x^3 - 4x\right) = 6x^2 - 4$.

a. $f'(-2) = 6(-2)^2 - 4 = 20$. **b.** $f'(0) = 6(0) - 4 = -4$. **c.** $f'(2) = 6(2)^2 - 4 = 20$.

36. $f'(x) = \frac{d}{dx}\left(4x^{5/4} + 2x^{3/2} + x\right) = 5x^{1/4} + 3x^{1/2} + 1.$

a. $f'(4) = 5(4)^{1/4} + 3(4)^{1/2} + 1 = 5(4)^{1/4} + 6 + 1 = 5(4)^{1/4} + 7 = 5\sqrt{2} + 7.$

b. $f'(16) = 5(16)^{1/4} + 3(16)^{1/2} + 1 = 10 + 12 + 1 = 23.$

37. The given limit is $f'(1)$, where $f(x) = x^3$. Because $f'(x) = 3x^2$, we have $\lim_{h\to 0}\dfrac{(1+h)^3 - 1}{h} = f'(1) = 3.$

38. Letting $h = x - 1$ or $x = h + 1$ and observing that $h \to 0$ as $x \to 1$, we find $\lim_{x\to 1}\dfrac{x^5 - 1}{x - 1} = \lim_{h\to 0}\dfrac{(h+1)^5 - 1}{h} = f'(1)$, where $f(x) = x^5$. Because $f'(x) = 5x^4$, we have $f'(1) = 5$, the value of the limit; that is, $\lim_{x\to 1}\dfrac{x^5 - 1}{x - 1} = 5.$

39. Let $f(x) = 3x^2 - x$. Then $\lim_{h\to 0}\dfrac{3(2+h)^2 - (2+h) - 10}{h} = \lim_{h\to 0}\dfrac{f(2+h) - f(2)}{h}$ because $f(2+h) - f(2) = 3(2+h)^2 - (2+h) - [3(4) - 2] = 3(2+h)^2 - (2+h) - 10$. But the last limit is simply $f'(2)$. Because $f'(x) = 6x - 1$, we have $f'(2) = 11$. Therefore, $\lim_{h\to 0}\dfrac{3(2+h)^2 - (2+h) - 10}{h} = 11.$

40. Write $\lim_{t\to 0}\dfrac{1 - (1+t)^2}{t(1+t)^2} = \lim_{t\to 0}\dfrac{1}{(1+t)^2} \cdot \lim_{t\to 0}\dfrac{1 - (1+t)^2}{t}$. Now let $f(t) = -t^2$. Then $\lim_{t\to 0}\dfrac{1 - (1+t)^2}{t} = \lim_{t\to 0}\dfrac{f(1+t) - f(1)}{t} = f'(1)$. Because $f'(t) = -2t$, we find $f'(1) = -2$. Therefore, $\lim_{t\to 0}\dfrac{1 - (1+t)^2}{t(1+t)^2} = \lim_{t\to 0}\dfrac{1}{(1+t)^2} \cdot f'(1) = 1 \cdot (-2) = -2.$

41. $f(x) = 2x^2 - 3x + 4$. The slope of the tangent line at any point $(x, f(x))$ on the graph of f is $f'(x) = 4x - 3$. In particular, the slope of the tangent line at the point $(2, 6)$ is $f'(2) = 4(2) - 3 = 5$. An equation of the required tangent line is $y - 6 = 5(x - 2)$ or $y = 5x - 4$.

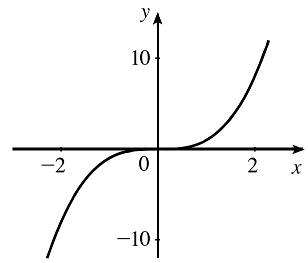
42. $f(x) = -\frac{5}{3}x^2 + 2x + 2$, so $f'(x) = -\frac{10}{3}x + 2$. The slope is $f'(-1) = \frac{10}{3} + 2 = \frac{16}{3}$. An equation of the tangent line is $y + \frac{5}{3} = \frac{16}{3}(x + 1)$ or $y = \frac{16}{3}x + \frac{11}{3}$.

43. $f(x) = x^4 - 3x^3 + 2x^2 - x + 1$, so $f'(x) = 4x^3 - 9x^2 + 4x - 1$. The slope is $f'(2) = 4(2)^3 - 9(2)^2 + 4(2) - 1 = 3$. An equation of the tangent line is $y - (-1) = 3(x - 2)$ or $y = 3x - 7$.

44. $f(x) = \sqrt{x} + 1/\sqrt{x}$. The slope of the tangent line at any point $(x, f(x))$ on the graph of f is $f'(x) = \frac{1}{2}x^{-1/2} - \frac{1}{2}x^{-3/2} = \dfrac{1}{2\sqrt{x}} - \dfrac{1}{2\sqrt{x^3}}$. In particular, the slope of the tangent line at the point $\left(4, \frac{5}{2}\right)$ is $f'(4) = \frac{1}{4} - \frac{1}{16} = \frac{3}{16}$. An equation of the required tangent line is $y - \frac{5}{2} = \frac{3}{16}(x - 4)$ or $y = \frac{3}{16}x + \frac{7}{4}$.

45. **a.** $f(x) = x^3$, so $f'(x) = 3x^2$. At a point where the tangent line is horizontal, $f'(x) = 0$, or $3x^2 = 0$, and so $x = 0$. Therefore, the point is $(0, 0)$.

b.

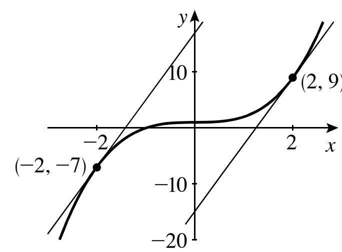


46. $f(x) = x^3 - 4x^2$, so $f'(x) = 3x^2 - 8x = x(3x - 8)$. Thus, $f'(x) = 0$ if $x = 0$ or $x = \frac{8}{3}$. Therefore, the points are $(0, 0)$ and $\left(\frac{8}{3}, -\frac{256}{27}\right)$.

47. **a.** $f(x) = x^3 + 1$. The slope of the tangent line at any point $(x, f(x))$ on the graph of f is $f'(x) = 3x^2$. At the point(s) where the slope is 12, we have $3x^2 = 12$, so $x = \pm 2$. The required points are $(-2, -7)$ and $(2, 9)$.

b. The tangent line at $(-2, -7)$ has equation $y - (-7) = 12[x - (-2)]$, or $y = 12x + 17$, and the tangent line at $(2, 9)$ has equation $y - 9 = 12(x - 2)$, or $y = 12x - 15$.

c.



48. $f(x) = \frac{2}{3}x^3 + x^2 - 12x + 6$, so $f'(x) = 2x^2 + 2x - 12$.

a. $f'(x) = -12$ gives $2x^2 + 2x - 12 = -12$, $2x^2 + 2x = 0$, $2x(x + 1) = 0$; that is, $x = 0$ or $x = -1$.

b. $f'(x) = 0$ gives $2x^2 + 2x - 12 = 0$, $2\left(x^2 + x - 6\right) = 2(x + 3)(x - 2) = 0$, and so $x = -3$ or $x = 2$.

c. $f'(x) = 12$ gives $2x^2 + 2x - 12 = 12$, $2\left(x^2 + x - 12\right) = 2(x + 4)(x - 3) = 0$, and so $x = -4$ or $x = 3$.

49. $f(x) = \frac{1}{4}x^4 - \frac{1}{3}x^3 - x^2$, so $f'(x) = x^3 - x^2 - 2x$.

a. $f'(x) = x^3 - x^2 - 2x = -2x$ implies $x^3 - x^2 = 0$, so $x^2(x - 1) = 0$. Thus, $x = 0$ or $x = 1$. $f(1) = \frac{1}{4}(1)^4 - \frac{1}{3}(1)^3 - (1)^2 = -\frac{13}{12}$ and $f(0) = \frac{1}{4}(0)^4 - \frac{1}{3}(0)^3 - (0)^2 = 0$. We conclude that the corresponding points on the graph are $\left(1, -\frac{13}{12}\right)$ and $(0, 0)$.

b. $f'(x) = x^3 - x^2 - 2x = 0$ implies $x\left(x^2 - x - 2\right) = 0$, $x(x - 2)(x + 1) = 0$, and so $x = 0, 2$, or -1. $f(0) = 0$, $f(2) = \frac{1}{4}(2)^4 - \frac{1}{3}(2)^3 - (2)^2 = 4 - \frac{8}{3} - 4 = -\frac{8}{3}$, and $f(-1) = \frac{1}{4}(-1)^4 - \frac{1}{3}(-1)^3 - (-1)^2 = \frac{1}{4} + \frac{1}{3} - 1 = -\frac{5}{12}$. We conclude that the corresponding points are $(0, 0)$, $\left(2, -\frac{8}{3}\right)$, and $\left(-1, -\frac{5}{12}\right)$.

c. $f'(x) = x^3 - x^2 - 2x = 10x$ implies $x^3 - x^2 - 12x = 0$, $x\left(x^2 - x - 12\right) = 0$, $x(x - 4)(x + 3) = 0$, so $x = 0, 4$, or -3. $f(0) = 0$, $f(4) = \frac{1}{4}(4)^4 - \frac{1}{3}(4)^3 - (4)^2 = 48 - \frac{64}{3} = \frac{80}{3}$, and $f(-3) = \frac{1}{4}(-3)^4 - \frac{1}{3}(-3)^3 - (-3)^2 = \frac{81}{4} + 9 - 9 = \frac{81}{4}$. We conclude that the corresponding points are $(0, 0)$, $\left(4, \frac{80}{3}\right)$, and $\left(-3, \frac{81}{4}\right)$.

50. $y = x^3 - 3x + 1$, so $\frac{dy}{dx} = 3x^2 - 3$. The slope of the tangent line to the given graph is $\left.\frac{dy}{dx}\right|_{x=2} = 3(4) - 3 = 9$. Therefore, an equation of the tangent line at $(2, 3)$ is $y - 3 = 9(x - 2)$, or $y = 9x - 15$. The slope of the normal line through the point $(2, 3)$ is $-\frac{1}{9}$. Therefore, an equation of the required normal line is $y - 3 = -\frac{1}{9}(x - 2)$ or $y = -\frac{1}{9}x + \frac{29}{9}$.

51. $V(r) = \frac{4}{3}\pi r^3$, so $V'(r) = 4\pi r^2$.

a. $V'\left(\frac{2}{3}\right) = 4\pi\left(\frac{4}{9}\right) = \frac{16}{9}\pi \text{ cm}^3/\text{cm}$.

b. $V'\left(\frac{5}{4}\right) = 4\pi\left(\frac{25}{16}\right) = \frac{25}{4}\pi \text{ cm}^3/\text{cm}$.

52. $v(r) = k\left(R^2 - r^2\right) = 1000\left(0.04 - r^2\right)$, so $v(0.1) = 1000\left(0.04 - 0.1^2\right) = 1000(0.03) = 30$. This says that the velocity of blood 0.1 cm from the central axis is 30 cm/sec. Next, $v'(r) = -2000r$, and so $v'(0.1) = -200$. This says that at a point 0.1 cm from the central axis, the velocity of blood is decreasing at the rate of 200 cm/sec per cm along a line perpendicular to the central axis.

53. a. The number of tablets and smartphones in use in 2011 was $f(2) = 128.1(2)^{1.94} \approx 491.5269$, or approximately 491.5 million.

b. $f'(t) = 128.1 \cdot 1.94t^{0.94} = 248.514t^{0.94}$, so the number of tablets and smartphones in 2011 was changing at the rate of $f'(2) = 248.514(2)^{0.94} \approx 476.7811$, or approximately 476.8 million/year.

54. a. The percentage of the UK population that is expected to watch video content on mobile phones in 2015 is $P(4) = 13.86(4)^{0.535} \approx 29.098$, or approximately 29.10%.

b. $P'(t) = 13.86 \cdot 0.535t^{-0.465}$, so the percentage of mobile phone video viewers in 2015 is projected to be changing at the rate of $P'(4) = 13.86 \cdot 0.535(4)^{-0.465} \approx 3.8919$, or approximately 3.89%/year.

55. a. $P(t) = \dfrac{49.6}{t^{0.27}}$.

1970 ($t = 1$)	1990 ($t = 3$)	2010 ($t = 5$)
49.6%	36.9%	32.1%

b. $P'(t) = (49.6)\left(-0.27t^{-1.27}\right) = -\dfrac{13.392}{t^{1.27}}$. In 1990, $P'(3) \approx -3.3$, or decreasing at 3.3%/decade. In 2000, $P'(4) \approx -2.3$, or decreasing at 2.3%/decade.

56. $A = \dfrac{26.5}{x^{0.45}}$, so $\dfrac{dA}{dx} = 26.5\dfrac{d}{dx}\left(x^{-0.45}\right) = 26.5(-0.45)x^{-1.45} = -\dfrac{11.925}{x^{1.45}}$. Therefore, $\left.\dfrac{dA}{dx}\right|_{x=0.25} = -\dfrac{11.925}{(0.25)^{1.45}} \approx -89.01$ and $\left.\dfrac{dA}{dx}\right|_{x=2} = -\dfrac{11.925}{(2)^{1.45}} \approx -4.36$. Our computations reveal that if you make 0.25 stops per mile, your average speed decreases at the rate of approximately 89.01 mph per stop per mile. If you make 2 stops per mile, your average speed decreases at the rate of approximately 4.36 mph per stop per mile.

57. a. $P(t) = 50.3t^{-0.09}$. The percentage of households with annual incomes within 50 percent of the median in 2010 was $P(4) = 50.3(4)^{-0.09} \approx 44.40$ (percent).

b. $P'(t) = 50.3\left(-0.09t^{-1.09}\right) \approx -4.527t^{-1.09}$, so the percentage of households with annual incomes within 50 percent of the median in 2010 was changing at the rate of $P'(4) = -4.527(4)^{-1.09} \approx -1.00$; that is, decreasing at the rate of approximately 1%/decade.

58. **a.** $f(x) = -0.1x^2 - 0.4x + 35$, so $f'(x) = -0.2x - 0.4$.

b. $f'(10) = -0.2(10) - 0.4 = -2.4$; that is, it is decreasing at the rate of \$2.40 per 1000 lamps. The unit price at this level of demand is $f(10) = -0.1\left(10^2\right) - 0.4(10) + 35 = 21$, or \$21.

59. **a.** $f(t) = 120t - 15t^2$, so $v = f'(t) = 120 - 30t$.

b. $v(0) = 120$ ft/sec

c. Setting $v = 0$ gives $120 - 30t = 0$, or $t = 4$. Therefore, the stopping distance is $f(4) = 120(4) - 15(16)$ or 240 ft.

60. **a.** The total percentage was $P(2) = 0.257\left(2^2\right) + 0.57(2) + 3.9 = 6.068$, or approximately 6.1%.

b. In 2008, the percentage was changing at the rate of $P'(2) = (0.514t + 0.57)\big|_{t=2} = 0.514(2) + 0.57 = 1.598$, or approximately 1.6%/year.

61. **a.** The approximate average medical cost for a family of four in 2010 was $C(10) = 22.9883(10)^2 + 830.358(10) + 7513 = 18{,}115.41$, or \$18,115.41.

b. $C'(t) = 45.9766t + 830.358$, so the rate at which the average medical cost for a family of four was increasing in 2010 was approximately $C'(10) = 45.9766(10) + 830.358 = 1290.124$, or \$1290.12/year.

62. **a.** $P(t) = 0.27t^2 + 1.4t + 2.2$, so $P'(t) = 0.54t + 1.4$. In 2010, $P'(1) = 0.54(1) + 1.4 = 1.94$ or 1.94%/decade. In 2020, $P'(2) = 0.54(2) + 1.4 = 2.48$, or 2.48%/decade.

b. In 2010, $P(1) = 0.27\left(1^2\right) + 1.4(1) + 2.2 = 3.87$, or 3.87%. In 2020, $P(2) = 0.27\left(2^2\right) + 1.4(2) + 2.2 = 6.08$, or 6.08%.

63. $I(t) = -0.2t^3 + 3t^2 + 100$, so $I'(t) = -0.6t^2 + 6t$.

a. In 2008, it was changing at a rate of $I'(5) = -0.6(25) + 6(5)$, or 15 points/yr. In 2010, it is $I'(7) = -0.6(49) + 6(7)$, or 12.6 points/yr. In 2013, it is $I'(10) = -0.6(100) + 6(10)$, or 0 points/yr.

b. The average rate of increase of the CPI over the period from 2008 to 2013 is

$$\frac{I(10) - I(5)}{5} = \frac{[-0.2(1000) + 3(100) + 100] - [-0.2(125) + 3(25) + 100]}{5} = \frac{200 - 150}{5} = 10, \text{ or}$$

10 points/yr.

64. **a.** $N(t) = -t^3 + 6t^2 + 15t$. The rate is given by $N'(t) = -3t^2 + 12t + 15$.

b. The rate at which the average worker is assembling walkie-talkies at 10 a.m. is $N'(2) = -3(2)^2 + 12(2) + 15 = 27$, or 27 walkie-talkies/hour. At 11 a.m., we have $N'(3) = -3(3)^2 + 12(3) + 15 = 24$, or 24 walkie-talkies/hour.

c. The number will be $N(3) - N(2) = (-27 + 54 + 45) - (-8 + 24 + 30) = 26$, or 26 walkie-talkies.

65. $P(t) = -\frac{1}{3}t^3 + 64t + 3000$, so $P'(t) = -t^2 + 64$. The rates of change at the end of years one, two, three and four are $P'(1) = -1 + 64 = 63$, or 63,000 people/yr; $P'(2) = -4 + 64 = 60$, or 60,000 people/yr; $P'(3) = -9 + 64 = 55$, or 55,000 people/yr; and $P'(4) = -16 + 64 = 48$, or 48,000 people/yr. It appears that the plan is working.

66. $N(t) = 2t^3 + 3t^2 - 4t + 1000$, so $N'(t) = 6t^2 + 6t - 4$. $N'(2) = 6(4) + 6(2) - 4 = 32$, or 32 turtles/yr; and $N'(8) = 6(64) + 6(8) - 4 = 428$, or 428 turtles/yr. The population ten years after implementation of the conservation measures will be $N(10) = 2\left(10^3\right) + 3\left(10^2\right) - 4(10) + 1000$, or 3260 turtles.

67. a. $f(t) = -2t^3 + 12t^2 + 5$, so $v = f'(t) = -6t^2 + 24t$.

b. $f'(0) = 0$, or 0 ft/sec; $f'(2) = -6(4) + 24(2) = 24$, or 24 ft/sec; $f'(4) = -6(16) + 24(4) = 0$, or 0 ft/sec; and $f'(6) = -6(36) + 24(6) = -72$, or -72 ft/sec. The rocket starts out at an initial velocity of 0 ft/sec. It climbs upward until a maximum altitude is attained 4 seconds into flight. It then descends until it hits the ground.

c. At the highest point, $v = 0$. But this occurs when $t = 4$ (see part (b)). The maximum altitude is $f(4) = -2(4)^3 + 12(4)^2 + 5 = 69$, or 69 feet.

68. a. $f'(x) = \frac{d}{dx}\left[0.0001x^{5/4} + 10\right] = \frac{5}{4}\left(0.0001x^{1/4}\right) = 0.000125x^{1/4}$.

b. $f'(10{,}000) = 0.000125(10{,}000)^{1/4} = 0.00125$, or \$0.00125/radio.

69. $P(t) = 50{,}000 + 30t^{3/2} + 20t$. The rate at which the population is increasing at any time t is $P'(t) = 45t^{1/2} + 20$. Nine months from now the population will be increasing at the rate of $P'(9) = 45(9)^{1/2} + 20$, or 155 people/month. Sixteen months from now the population will be increasing at the rate of $P'(16) = 45(16)^{1/2} + 20$, or 200 people/month.

70. a. $f(t) = 20t - 40\sqrt{t} + 50$, so $f'(t) = 20 - 40\left(\frac{1}{2}\right)t^{-1/2} = 20\left(1 - \frac{1}{\sqrt{t}}\right)$.

b. $f(0) = 20(0) - 40\sqrt{0} + 50 = 50$, so $f(1) = 20(1) - 40\sqrt{1} + 50 = 30$ and $f(2) = 20(2) - 40\sqrt{2} + 50 \approx 33.43$. The average velocities at 6, 7, and 8 a.m. are 50, 30, and 33.43 mph, respectively.

c. $f'\left(\frac{1}{2}\right) = 20 - 20\left(\frac{1}{2}\right)^{-1/2} \approx -8.28$, $f'(1) = 20 - 20(1)^{-1/2} \approx 0$, and $f'(2) = 20 - 20(2)^{-1/2} \approx 5.86$. At 6:30 a.m. the average velocity is decreasing at the rate of 8.28 mph/hr, at 7 a.m. it is not changing, and at 8 a.m. it is increasing at the rate of 5.86 mph.

71. $S(x) = -0.002x^3 + 0.6x^2 + x + 500$, so $S'(x) = -0.006x^2 + 1.2x + 1$.

a. When $x = 100$, $S'(100) = -0.006(100)^2 + 1.2(100) + 1 = 61$, or \$61,000 per thousand dollars.

b. When $x = 150$, $S'(150) = -0.006(150)^2 + 1.2(150) + 1 = 46$, or \$46,000 per thousand dollars. We conclude that the company's total sales increase at a faster rate with option (a); that is, when \$100,000 is spent on advertising.

72. a. The per capita health spending in 2010 was $C(10) = -1.1708(10)^3 + 7.029(10)^2 + 389.69(10) + 4780 = 8209$, or \$8209.

b. $C'(t) = -3.5124t^2 + 14.058t + 389.69$, so the per capita health spending in 2010 was changing at the rate of $C'(10) = -3.5124(10)^2 + 14.058(10) + 389.69 = 179.03$; that is, it was increasing at \$179.03/year.

73. a. $P(t) = 0.0004t^3 + 0.0036t^2 + 0.8t + 12$. At the beginning of 1991, $P(0) = 12\%$. At the beginning of 2010, $P(19) = 0.0004(19)^3 + 0.0036(19)^2 + 0.8(19) + 12 \approx 31.2$, or approximately 31.2%.

b. $P'(t) = 0.0012t^2 + 0.0072t + 0.8$. At the beginning of 1991, $P'(0) = 0.8$, or 0.8%/yr. At the beginning of 2010, $P'(19) = 0.0012(19)^2 + 0.0072(19) + 0.8 \approx 1.4$, or approximately 1.4%/yr.

74. a. At any time t, the function $D = g + f$ at t, $D(t) = (g + f)(t) = g(t) + f(t)$, gives the total population aged 65 and over of the developed and the underdeveloped/emerging countries.

b. $D(t) = g(t) + f(t) = \left(0.46t^2 + 0.16t + 287.8\right) + (3.567t + 175.2) = 0.46t^2 + 3.727t + 463$, so $D'(t) = 0.92t + 3.727$. Therefore, $D'(10) = 0.92(10) + 3.727 = 12.927$, which says that the combined population is growing at the rate of approximately 13 million people per year in 2010.

75. a. $$G(t) = J(t) - N(t) = \begin{cases} -0.0002t^2 + 0.032t + 0.1 & \text{if } 0 \le t < 5 \\ 0.0002t^2 - 0.006t + 0.28 & \text{if } 5 \le t < 10 \\ -0.0012t^2 + 0.082t - 0.46 & \text{if } 10 \le t < 15 \end{cases}$$

b. In 2008, when $t = 8$, the gap is changing at a rate of $G'(8) = \left[\frac{d}{dt}\left(0.0002t^2 - 0.006t + 0.28\right)\right]_{t=8} = (0.0004t - 0.006)|_{t=8} = -0.0028$; that is, the gap is narrowing at a rate of 2800 jobs/yr. In 2012, when $t = 12$, the gap is changing at a rate of $G'(12) = \left[\frac{d}{dt}\left(-0.0012t^2 + 0.082t - 0.46\right)\right]_{t=12} = (-0.0024t + 0.082)|_{t=12} = 0.0532$; that is, the gap is increasing at a rate of 53,200 jobs/yr.

76. True. $\frac{d}{dx}\left[2f(x) - 5g(x)\right] = \frac{d}{dx}\left[2f(x)\right] - \frac{d}{dx}\left[5g(x)\right] = 2f'(x) - 5g'(x)$.

77. False. f is not a power function.

78. $$\frac{d}{dx}\left(x^3\right) = \lim_{h \to 0} \frac{(x+h)^3 - x^3}{h} = \lim_{h \to 0} \frac{x^3 + 3x^2h + 3xh^2 + h^3 - x^3}{h} = \lim_{h \to 0} \frac{h\left(3x^2 + 3xh + h^2\right)}{h}$$
$$= \lim_{h \to 0} \left(3x^2 + 3xh + h^2\right) = 3x^2.$$

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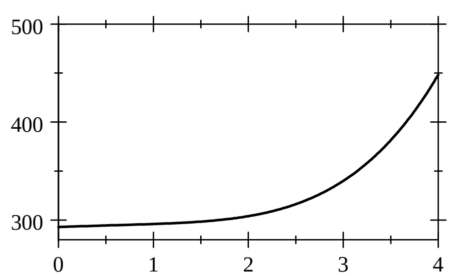
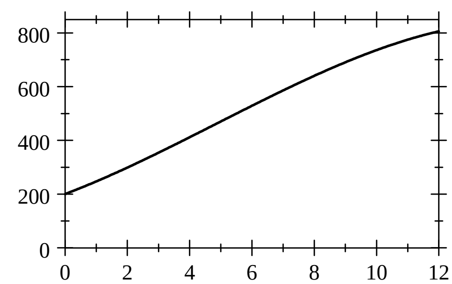
1. 1 **2.** 3.0720 **3.** 0.4226 **4.** 0.0732 **5.** 0.1613 **6.** 3.9730

7. a.



b. 3.4295 parts/million per 40 years; 164.239 parts/million per 40 years

8. a.



b. 42,272 cases/year

3.2 The Product and Quotient Rules

Concept Questions

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1. **a.** The derivative of the product of two functions is equal to the first function times the derivative of the second function plus the second function times the derivative of the first function.

   b. The derivative of the quotient of two functions is equal to the quotient whose numerator is given by the denominator times the derivative of the numerator minus the numerator times the derivative of the denominator, and whose denominator is the square of the denominator of the quotient.

2. **a.** $h'(x) = f(x)g'(x) + f'(x)g(x)$, so $h'(1) = f(1)g'(1) + f'(1)g(1) = (3)(4) + (-1)(2) = 10$.

   b. $F'(x) = \dfrac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$, so $F'(1) = \dfrac{g(1)f'(1) - f(1)g'(1)}{[g(1)]^2} = \dfrac{2(-1) - 3(4)}{2^2} = -\dfrac{7}{2}$.

Exercises

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1. $f(x) = 2x(x^2+1)$, so $f'(x) = 2x\frac{d}{dx}(x^2+1) + (x^2+1)\frac{d}{dx}(2x) = 2x(2x) + (x^2+1)(2) = 6x^2+2$.

2. $f(x) = 3x^2(x-1)$, so $f'(x) = 3x^2\frac{d}{dx}(x-1) + (x-1)\frac{d}{dx}(3x^2) = 3x^2 + (x-1)(6x) = 9x^2 - 6x$.

3. $f(t) = (t-1)(2t+1)$, so
   $f'(t) = (t-1)\frac{d}{dt}(2t+1) + (2t+1)\frac{d}{dt}(t-1) = (t-1)(2) + (2t+1)(1) = 4t-1$.

4. $f(x) = (2x+3)(3x-4)$, so
   $f'(x) = (2x+3)\frac{d}{dx}(3x-4) + (3x-4)\frac{d}{dx}(2x+3) = (2x+3)(3) + (3x-4)(2) = 12x+1$.

5. $f(x) = (3x+1)(x^2-2)$, so
   $f'(x) = (3x+1)\frac{d}{dx}(x^2-2) + (x^2-2)\frac{d}{dx}(3x+1) = (3x+1)(2x) + (x^2-2)(3) = 9x^2+2x-6$.

6. $f(x) = (x+1)(2x^2-3x+1)$, so
   $$f'(x) = (x+1)\frac{d}{dx}(2x^2-3x+1) + (2x^2-3x+1)\frac{d}{dx}(x+1) = (x+1)(4x-3) + (2x^2-3x+1)(1)$$
   $$= 4x^2-3x+4x-3+2x^2-3x+1 = 6x^2-2x-2 = 2(3x^2-x-1).$$

7. $f(x) = (x^3-1)(x+1)$, so
   $f'(x) = (x^3-1)\frac{d}{dx}(x+1) + (x+1)\frac{d}{dx}(x^3-1) = (x^3-1)(1) + (x+1)(3x^2) = 4x^3+3x^2-1$.

8. $f(x) = (x^3-12x)(3x^2+2x)$, so
   $$f'(x) = (x^3-12x)\frac{d}{dx}(3x^2+2x) + (3x^2+2x)\frac{d}{dx}(x^3-12x)$$
   $$= (x^3-12x)(6x+2) + (3x^2+2x)(3x^2-12)$$
   $$= 6x^4+2x^3-72x^2-24x+9x^4+6x^3-36x^2-24x = 15x^4+8x^3-108x^2-48x.$$

9. $f(w) = \left(w^3 - w^2 + w - 1\right)\left(w^2 + 2\right)$, so

$$\begin{aligned} f'(w) &= \left(w^3 - w^2 + w - 1\right)\frac{d}{dw}\left(w^2 + 2\right) + \left(w^2 + 2\right)\frac{d}{dw}\left(w^3 - w^2 + w - 1\right) \\ &= \left(w^3 - w^2 + w - 1\right)(2w) + \left(w^2 + 2\right)\left(3w^2 - 2w + 1\right) \\ &= 2w^4 - 2w^3 + 2w^2 - 2w + 3w^4 - 2w^3 + w^2 + 6w^2 - 4w + 2 = 5w^4 - 4w^3 + 9w^2 - 6w + 2. \end{aligned}$$

10. $f(x) = \frac{1}{5}x^5 + \left(x^2 + 1\right)\left(x^2 - x - 1\right) + 28$, so

$$\begin{aligned} f'(x) &= x^4 + \left(x^2 + 1\right)(2x - 1) + 2x\left(x^2 - x - 1\right) = x^4 + 2x^3 - x^2 + 2x - 1 + 2x^3 - 2x^2 - 2x \\ &= x^4 + 4x^3 - 3x^2 - 1. \end{aligned}$$

11. $f(x) = \left(5x^2 + 1\right)\left(2\sqrt{x} - 1\right)$, so

$$\begin{aligned} f'(x) &= \left(5x^2 + 1\right)\tfrac{d}{dx}\left(2x^{1/2} - 1\right) + \left(2x^{1/2} - 1\right)\tfrac{d}{dx}\left(5x^2 + 1\right) = \left(5x^2 + 1\right)\left(x^{-1/2}\right) + \left(2x^{1/2} - 1\right)(10x) \\ &= 5x^{3/2} + x^{-1/2} + 20x^{3/2} - 10x = \frac{25x^2 - 10x\sqrt{x} + 1}{\sqrt{x}}. \end{aligned}$$

12. $f(t) = \left(1 + \sqrt{t}\right)\left(2t^2 - 3\right)$, so

$$\begin{aligned} f'(t) &= \left(1 + t^{1/2}\right)(4t) + \left(2t^2 - 3\right)\left(\tfrac{1}{2}t^{-1/2}\right) = 4t + 4t^{3/2} + t^{3/2} - \tfrac{3}{2}t^{-1/2} = 5t^{3/2} + 4t - \tfrac{3}{2}t^{-1/2} \\ &= \frac{10t^2 + 8t\sqrt{t} - 3}{2\sqrt{t}}. \end{aligned}$$

13. $f(x) = \left(x^2 - 5x + 2\right)\left(x - \dfrac{2}{x}\right)$, so

$$\begin{aligned} f'(x) &= \left(x^2 - 5x + 2\right)\frac{d}{dx}\left(x - \frac{2}{x}\right) + \left(x - \frac{2}{x}\right)\frac{d}{dx}\left(x^2 - 5x + 2\right) \\ &= \frac{\left(x^2 - 5x + 2\right)\left(x^2 + 2\right)}{x^2} + \frac{\left(x^2 - 2\right)(2x - 5)}{x} = \frac{\left(x^2 - 5x + 2\right)\left(x^2 + 2\right) + x\left(x^2 - 2\right)(2x - 5)}{x^2} \\ &= \frac{x^4 + 2x^2 - 5x^3 - 10x + 2x^2 + 4 + 2x^4 - 5x^3 - 4x^2 + 10x}{x^2} = \frac{3x^4 - 10x^3 + 4}{x^2}. \end{aligned}$$

14. $f(x) = \left(x^3 + 2x + 1\right)\left(2 + \dfrac{1}{x^2}\right) = 2x^3 + 4x + 2 + x + \dfrac{2}{x} + \dfrac{1}{x^2}$, so

$$f'(x) = \frac{d}{dx}\left(2x^3 + 5x + 2 + \frac{2}{x} + \frac{1}{x^2}\right) = 6x^2 + 5 - \frac{2}{x^2} - \frac{2}{x^3} = \frac{6x^5 + 5x^3 - 2x - 2}{x^3}.$$

15. $f(x) = \dfrac{1}{x - 2}$, so $f'(x) = \dfrac{(x - 2)\frac{d}{dx}(1) - (1)\frac{d}{dx}(x - 2)}{(x - 2)^2} = \dfrac{0 - 1(1)}{(x - 2)^2} = -\dfrac{1}{(x - 2)^2}$.

16. $g(x) = \dfrac{3}{2x + 4} + 2x^2$, so $g'(x) = \dfrac{d}{dx}\left(\dfrac{3}{2x + 4}\right) + \dfrac{d}{dx}\left(2x^2\right) = \dfrac{(2x + 4)(0) - 3(2)}{(2x + 4)^2} + 4x = -\dfrac{6}{(2x + 4)^2} + 4x$.

17. $f(x) = \dfrac{2x - 1}{2x + 1}$, so

$$f'(x) = \frac{(2x + 1)\frac{d}{dx}(2x - 1) - (2x - 1)\frac{d}{dx}(2x + 1)}{(2x + 1)^2} = \frac{2(2x + 1) - (2x - 1)(2)}{(2x + 1)^2} = \frac{4}{(2x + 1)^2}.$$

18. $f(t) = \dfrac{1-2t}{1+3t}$, so $f'(t) = \dfrac{(1+3t)(-2)-(1-2t)(3)}{(1+3t)^2} = \dfrac{-5}{(1+3t)^2}$.

19. $f(x) = \dfrac{1}{x^2+x+2}$, so $f'(x) = \dfrac{\left(x^2+x+2\right)(0)-(1)(2x+1)}{\left(x^2+x+2\right)^2} = -\dfrac{2x+1}{\left(x^2+x+2\right)^2}$.

20. $f(u) = \dfrac{u}{u^2+1}$, so $f'(u) = \dfrac{\left(u^2+1\right)\dfrac{d}{du}(u) - u\dfrac{d}{du}\left(u^2+1\right)}{\left(u^2+1\right)^2} = \dfrac{\left(u^2+1\right)(1)-u(2u)}{\left(u^2+1\right)^2} = \dfrac{1-u^2}{\left(u^2+1\right)^2}$.

21. $f(s) = \dfrac{s^2-4}{s+1}$, so

$$f'(s) = \frac{(s+1)\dfrac{d}{ds}\left(s^2-4\right)-\left(s^2-4\right)\dfrac{d}{ds}(s+1)}{(s+1)^2} = \frac{(s+1)(2s)-\left(s^2-4\right)(1)}{(s+1)^2} = \frac{s^2+2s+4}{(s+1)^2}.$$

22. $f(x) = \dfrac{x^3-2}{x^2+1}$, so

$$f'(x) = \frac{\left(x^2+1\right)\frac{d}{dx}\left(x^3-2\right)-\left(x^3-2\right)\frac{d}{dx}\left(x^2+1\right)}{\left(x^2+1\right)^2} = \frac{\left(x^2+1\right)\left(3x^2\right)-\left(x^3-2\right)(2x)}{\left(x^2+1\right)^2} = \frac{x\left(x^3+3x+4\right)}{\left(x^2+1\right)^2}.$$

23. $f(x) = \dfrac{\sqrt{x}+1}{x^2+1}$, so

$$f'(x) = \frac{\left(x^2+1\right)\frac{d}{dx}\left(x^{1/2}\right)-\left(x^{1/2}+1\right)\frac{d}{dx}\left(x^2+1\right)}{\left(x^2+1\right)^2} = \frac{\left(x^2+1\right)\left(\frac{1}{2}x^{-1/2}\right)-\left(x^{1/2}+1\right)(2x)}{\left(x^2+1\right)^2}$$

$$= \frac{\left(\frac{1}{2}x^{-1/2}\right)\left[\left(x^2+1\right)-\left(x^{1/2}+1\right)4x^{3/2}\right]}{\left(x^2+1\right)^2} = \frac{1-3x^2-4x^{3/2}}{2\sqrt{x}\left(x^2+1\right)^2}.$$

24. $f(x) = \dfrac{x}{\sqrt{x}+2} = \dfrac{x}{x^{1/2}+2}$, so

$$f'(x) = \frac{\left(x^{1/2}+2\right)(1)-x\left(\frac{1}{2}x^{-1/2}\right)}{\left(x^{1/2}+2\right)^2} = \frac{x^{1/2}+2-\frac{1}{2}x^{1/2}}{\left(x^{1/2}+2\right)^2} = \frac{\frac{1}{2}x^{1/2}+2}{\left(x^{1/2}+2\right)^2} = \frac{\frac{1}{2}\left(x^{1/2}+4\right)}{\left(x^{1/2}+2\right)^2} = \frac{\sqrt{x}+4}{2\left(\sqrt{x}+2\right)^2}.$$

25. $f(x) = \dfrac{x^2+2}{x^2+x+1}$, so

$$f'(x) = \frac{\left(x^2+x+1\right)\frac{d}{dx}\left(x^2+2\right)-\left(x^2+2\right)\frac{d}{dx}\left(x^2+x+1\right)}{\left(x^2+x+1\right)^2}$$

$$= \frac{\left(x^2+x+1\right)(2x)-\left(x^2+2\right)(2x+1)}{\left(x^2+x+1\right)^2} = \frac{2x^3+2x^2+2x-2x^3-x^2-4x-2}{\left(x^2+x+1\right)^2} = \frac{x^2-2x-2}{\left(x^2+x+1\right)^2}.$$

26. $f(x) = \dfrac{x+1}{2x^2+2x+3}$, so

$$f'(x) = \frac{\left(2x^2+2x+3\right)(1)-(x+1)(4x+2)}{\left(2x^2+2x+3\right)^2} = \frac{2x^2+2x+3-4x^2-2x-4x-2}{\left(2x^2+2x+3\right)^2} = \frac{-2x^2-4x+1}{\left(2x^2+2x+3\right)^2}.$$

27. $f(x) = \dfrac{(x+1)(x^2+1)}{x-2} = \dfrac{(x^3+x^2+x+1)}{x-2}$, so

$$f'(x) = \frac{(x-2)\frac{d}{dx}(x^3+x^2+x+1) - (x^3+x^2+x+1)\frac{d}{dx}(x-2)}{(x-2)^2}$$
$$= \frac{(x-2)(3x^2+2x+1) - (x^3+x^2+x+1)}{(x-2)^2}$$
$$= \frac{3x^3+2x^2+x-6x^2-4x-2-x^3-x^2-x-1}{(x-2)^2} = \frac{2x^3-5x^2-4x-3}{(x-2)^2}.$$

28. $f(x) = (3x^2-1)\left(x^2-\dfrac{1}{x}\right)$, so

$$f'(x) = 6x\left(x^2-\frac{1}{x}\right) + (3x^2-1)\left(2x+\frac{1}{x^2}\right) = 6x^3-6+6x^3+3-2x-\frac{1}{x^2} = 12x^3-2x-3-\frac{1}{x^2}.$$

29. $f(x) = \dfrac{x}{x^2-4} - \dfrac{x-1}{x^2+4} = \dfrac{x(x^2+4)-(x-1)(x^2-4)}{(x^2-4)(x^2+4)} = \dfrac{x^2+8x-4}{(x^2-4)(x^2+4)}$, so

$$f'(x) = \frac{(x^2-4)(x^2+4)\frac{d}{dx}(x^2+8x-4) - (x^2+8x-4)\frac{d}{dx}(x^4-16)}{(x^2-4)^2(x^2+4)^2}$$
$$= \frac{(x^2-4)(x^2+4)(2x+8) - (x^2+8x-4)(4x^3)}{(x^2-4)^2(x^2+4)^2}$$
$$= \frac{2x^5+8x^4-32x-128-4x^5-32x^4+16x^3}{(x^2-4)^2(x^2+4)^2} = \frac{-2x^5-24x^4+16x^3-32x-128}{(x^2-4)^2(x^2+4)^2}.$$

30. $f(x) = \dfrac{x+\sqrt{3x}}{3x-1}$, so

$$f'(x) = \frac{(3x-1)\left(1+\frac{1}{2}\sqrt{3}x^{-1/2}\right) - \left(x+\sqrt{3}x^{1/2}\right)(3)}{(3x-1)^2}$$
$$= \frac{3x+\frac{3}{2}\sqrt{3}x^{1/2}-1-\frac{1}{2}\sqrt{3}x^{-1/2}-3x-3\sqrt{3}x^{1/2}}{(3x-1)^2} = -\frac{3\sqrt{3}x+2\sqrt{x}+\sqrt{3}}{2\sqrt{x}(3x-1)^2}.$$

31. $h(x) = f(x)g(x)$, so $h'(x) = f(x)g'(x) + f'(x)g(x)$ by the Product Rule. Therefore, $h'(1) = f(1)g'(1) + f'(1)g(1) = (2)(3) + (-1)(-2) = 8$.

32. $h(x) = (x^2+1)g(x)$, so $h'(x) = (x^2+1)g'(x) + \frac{d}{dx}(x^2+1)\cdot g(x) = (x^2+1)g'(x) + 2xg(x)$. Therefore, $h'(1) = 2g'(1) + 2g(1) = (2)(3) + 2(-2) = 2$.

33. $h(x) = \dfrac{xf(x)}{x+g(x)}$. Using the Quotient Rule followed by the Product Rule, we obtain

$$h'(x) = \frac{[x+g(x)]\frac{d}{dx}[xf(x)] - xf(x)\frac{d}{dx}[x+g(x)]}{[x+g(x)]^2} = \frac{[x+g(x)][xf'(x)+f(x)] - xf(x)[1+g'(x)]}{[x+g(x)]^2}.$$

Therefore,

$$h'(1) = \frac{[1+g(1)][f'(1)+f(1)] - f(1)[1+g'(1)]}{[1+g(1)]^2} = \frac{(1-2)(-1+2)-2(1+3)}{(1-2)^2} = \frac{-1-8}{1} = -9.$$

34. $h(x) = \dfrac{f(x)\,g(x)}{f(x) - g(x)}$. Using the Quotient Rule followed by the Product Rule and the Sum Rule, we obtain

$$h'(x) = \frac{\left[f(x) - g(x)\right]\frac{d}{dx}\left[f(x)\,g(x)\right] - f(x)\,g(x)\frac{d}{dx}\left[f(x) - g(x)\right]}{\left[f(x) - g(x)\right]^2}$$

$$= \frac{\left[f(x) - g(x)\right]\left[f(x)\,g'(x) + f'(x)\,g(x)\right] - f(x)\,g(x)\left[f'(x) - g'(x)\right]}{\left[f(x) - g(x)\right]^2}.$$

Therefore,

$$h'(1) = \frac{\left[f(1) - g(1)\right]\left[f(1)\,g'(1) + f'(1)\,g(1)\right] - f(1)\,g(1)\left[f'(1) - g'(1)\right]}{\left[f(1) - g(1)\right]^2}$$

$$= \frac{[2 - (-2)]\,[(2)(3) + (-1)(-2)] - (2)(-2)\,[(-1) - 3]}{[2 - (-2)]^2} = \frac{(4)(8) - (-4)(-4)}{4^2} = \frac{16}{16} = 1.$$

35. $f(x) = (2x - 1)\left(x^2 + 3\right)$, so

$$f'(x) = (2x - 1)\frac{d}{dx}\left(x^2 + 3\right) + \left(x^2 + 3\right)\frac{d}{dx}(2x - 1) = (2x - 1)(2x) + \left(x^2 + 3\right)(2)$$

$$= 6x^2 - 2x + 6 = 2\left(3x^2 - x + 3\right).$$

At $x = 1$, $f'(1) = 2\left[3(1)^2 - (1) + 3\right] = 2(5) = 10$.

36. $f(x) = \dfrac{2x + 1}{2x - 1}$, so

$$f'(x) = \frac{(2x - 1)\frac{d}{dx}(2x + 1) - (2x + 1)\frac{d}{dx}(2x - 1)}{(2x - 1)^2} = \frac{(2x - 1)(2) - (2x + 1)(2)}{(2x - 1)^2}$$

$$= \frac{4x - 2 - 4x - 2}{(2x - 1)^2} = -\frac{4}{(2x - 1)^2}.$$

At $x = 2$, $f'(2) = \dfrac{-4}{[2(2) - 1]^2} = -\dfrac{4}{9}$.

37. $f(x) = \dfrac{x}{x^4 - 2x^2 - 1}$, so

$$f'(x) = \frac{\left(x^4 - 2x^2 - 1\right)\frac{d}{dx}(x) - x\frac{d}{dx}\left(x^4 - 2x^2 - 1\right)}{\left(x^4 - 2x^2 - 1\right)^2} = \frac{\left(x^4 - 2x^2 - 1\right)(1) - x\left(4x^3 - 4x\right)}{\left(x^4 - 2x^2 - 1\right)^2}$$

$$= \frac{-3x^4 + 2x^2 - 1}{\left(x^4 - 2x^2 - 1\right)^2}.$$

Therefore, $f'(-1) = \dfrac{-3 + 2 - 1}{(1 - 2 - 1)^2} = -\dfrac{2}{4} = -\dfrac{1}{2}$.

38. $f(x) = \left(x^{1/2} + 2x\right)\left(x^{3/2} - x\right) = x^2 - x^{3/2} + 2x^{5/2} - 2x^2 = 2x^{5/2} - x^2 - x^{3/2}$, so $f'(x) = 5x^{3/2} - 2x - \frac{3}{2}x^{1/2}$. At $x = 4$, $f'(4) = 5(4)^{3/2} - 2(4) - \frac{3}{2}(4)^{1/2} = 5(8) - 2(4) - \frac{3}{2}(2) = 29$.

39. $f(x) = \left(x^3 + 1\right)\left(x^2 - 2\right)$, so

$f'(x) = \left(x^3 + 1\right)\frac{d}{dx}\left(x^2 - 2\right) + \left(x^2 - 2\right)\frac{d}{dx}\left(x^3 + 1\right) = \left(x^3 + 1\right)(2x) + \left(x^2 - 2\right)\left(3x^2\right)$. The slope of the tangent line at $(2, 18)$ is $f'(2) = (8 + 1)(4) + (4 - 2)(12) = 60$. An equation of the tangent line is $y - 18 = 60(x - 2)$, or $y = 60x - 102$.

40. $f(x) = \dfrac{x^2}{x+1}$, so $f'(x) = \dfrac{(x+1)(2x) - x^2(1)}{(x+1)^2} = \dfrac{x^2+2x}{(x+1)^2}$. The slope of the tangent line at $x = 2$ is $f'(2) = \frac{8}{9}$. An equation of the line is $y - \frac{4}{3} = \frac{8}{9}(x-2)$, or $y = \frac{8}{9}x - \frac{4}{9}$.

41. $f(x) = \dfrac{x+1}{x^2+1}$, so

$$f'(x) = \frac{\left(x^2+1\right)\frac{d}{dx}(x+1) - (x+1)\frac{d}{dx}\left(x^2+1\right)}{\left(x^2+1\right)^2} = \frac{\left(x^2+1\right)(1) - (x+1)(2x)}{\left(x^2+1\right)^2} = \frac{-x^2-2x+1}{\left(x^2+1\right)^2}.$$ At $x = 1$, $f'(1) = \dfrac{-1-2+1}{4} = -\dfrac{1}{2}$. Therefore, the slope of the tangent line at $x = 1$ is $-\frac{1}{2}$ and an equation is $y - 1 = -\frac{1}{2}(x-1)$ or $y = -\frac{1}{2}x + \frac{3}{2}$.

42. $f(x) = \dfrac{1+2x^{1/2}}{1+x^{3/2}}$, so $f'(x) = \dfrac{\left(1+x^{3/2}\right)\left(x^{-1/2}\right) - \left(1+2x^{1/2}\right)\left(\frac{3}{2}x^{1/2}\right)}{\left(1+x^{3/2}\right)^2}$. The slope of the tangent line at $x = 4$ is $f'(4) = \dfrac{(1+8)\left(\frac{1}{2}\right) - (1+4)(3)}{9^2} = -\dfrac{7}{54}$ and an equation is $y - \frac{5}{9} = -\frac{7}{54}(x-4)$, or $y = -\frac{7}{54}x + \frac{29}{27}$.

43. Using the Product Rule, we find $g'(x) = \frac{d}{dx}\left[x^2 f(x)\right] = x^2 \frac{d}{dx}\left[f(x)\right] + f(x)\frac{d}{dx}\left(x^2\right) = x^2 f'(x) + 2xf(x)$. Therefore, $g'(2) = 2^2 f'(2) + 2(2)f(2) = (4)(-1) + 4(3) = 8$.

44. Using the Product Rule, we find $g'(x) = \frac{d}{dx}\left[\left(x^2+1\right)f(x)\right] = \left(x^2+1\right)\frac{d}{dx}\left[f(x)\right] + f(x)\frac{d}{dx}\left(x^2+1\right) = \left(x^2+1\right)f'(x) + 2xf(x)$. Therefore, $g'(2) = \left(2^2+1\right)f'(2) + 2(2)f(2) = (5)(-1) + 4(3) = 7$.

45. $f(x) = \left(x^3+1\right)\left(3x^2-4x+2\right)$, so

$$\begin{aligned} f'(x) &= \left(x^3+1\right)\frac{d}{dx}\left(3x^2-4x+2\right) + \left(3x^2-4x+2\right)\frac{d}{dx}\left(x^3+1\right) \\ &= \left(x^3+1\right)(6x-4) + \left(3x^2-4x+2\right)\left(3x^2\right) \\ &= 6x^4 + 6x - 4x^3 - 4 + 9x^4 - 12x^3 + 6x^2 = 15x^4 - 16x^3 + 6x^2 + 6x - 4. \end{aligned}$$

At $x = 1$, $f'(1) = 15(1)^4 - 16(1)^3 + 6(1) + 6(1) - 4 = 7$. Thus, the slope of the tangent line at the point $x = 1$ is 7 and an equation is $y - 2 = 7(x-1)$, or $y = 7x - 5$.

46. $f(x) = \dfrac{3x}{x^2-2}$. The slope of the tangent line at any point $(x, f(x))$ lying on the graph of f is

$$f'(x) = \frac{\left(x^2-2\right)\frac{d}{dx}(3x) - (3x)\frac{d}{dx}\left(x^2-2\right)}{\left(x^2-2\right)^2} = \frac{\left(x^2-2\right)(3) - 3x(2x)}{\left(x^2-2\right)^2} = \frac{-3x^2-6}{\left(x^2-2\right)^2} = \frac{-3\left(x^2+2\right)}{\left(x^2-2\right)^2}.$$

In particular, the slope of the tangent line at $(2, 3)$ is $f'(2) = \dfrac{-3(4+2)}{4} = -\dfrac{9}{2}$. Therefore, an equation of the tangent line is $y - 3 = -\frac{9}{2}(x-2)$ or $y = -\frac{9}{2}x + 12$.

47. $f(x) = (x^2+1)(2-x)$, so
$f'(x) = (x^2+1)\frac{d}{dx}(2-x) + (2-x)\frac{d}{dx}(x^2+1) = (x^2+1)(-1) + (2-x)(2x) = -3x^2+4x-1$. At a point where the tangent line is horizontal, we have $f'(x) = -3x^2+4x-1 = 0$ or $3x^2-4x+1 = (3x-1)(x-1) = 0$, giving $x = \frac{1}{3}$ or $x = 1$. Because $f\left(\frac{1}{3}\right) = \left(\frac{1}{9}+1\right)\left(2-\frac{1}{3}\right) = \frac{50}{27}$ and $f(1) = 2(2-1) = 2$, we see that the required points are $\left(\frac{1}{3}, \frac{50}{27}\right)$ and $(1, 2)$.

48. $f(x) = \dfrac{x}{x^2+1}$, so $f'(x) = \dfrac{(x^2+1)(1) - x(2x)}{(x^2+1)^2} = \dfrac{1-x^2}{(x^2+1)^2}$. At a point where the tangent line is horizontal, we have $f'(x) = 0$ or $1-x^2 = 0$, giving $x = \pm 1$. Therefore, the required points are $\left(-1, -\frac{1}{2}\right)$ and $\left(1, \frac{1}{2}\right)$.

49. $f(x) = (x^2+6)(x-5)$, so

$$f'(x) = (x^2+6)\frac{d}{dx}(x-5) + (x-5)\frac{d}{dx}(x^2+6) = (x^2+6)(1) + (x-5)(2x)$$
$$= x^2+6+2x^2-10x = 3x^2-10x+6.$$

At a point where the slope of the tangent line is -2, we have $f'(x) = 3x^2-10x+6 = -2$. This gives $3x^2-10x+8 = (3x-4)(x-2) = 0$, so $x = \frac{4}{3}$ or $x = 2$. Because $f\left(\frac{4}{3}\right) = \left(\frac{16}{9}+6\right)\left(\frac{4}{3}-5\right) = -\frac{770}{27}$ and $f(2) = (4+6)(2-5) = -30$, the required points are $\left(\frac{4}{3}, -\frac{770}{27}\right)$ and $(2, -30)$.

50. $f(x) = \dfrac{x+1}{x-1}$. The slope of the tangent line at any point $(x, f(x))$ on the graph of f is $f'(x) = \dfrac{(x-1)\frac{d}{dx}(x+1) - (x+1)\frac{d}{dx}(x-1)}{(x-1)^2} = \dfrac{(x-1)(1)-(x+1)(1)}{(x-1)^2} = -\dfrac{2}{(x-1)^2}$. At the point(s) where the slope is equal to $-\frac{1}{2}$, we have $-\dfrac{2}{(x-1)^2} = -\dfrac{1}{2}$, so $(x-1)^2 = 4$ and $x = 1\pm 2 = -1$ or 3. Therefore, the required points are $(-1, 0)$ and $(3, 2)$.

51. $y = \dfrac{1}{1+x^2}$, so $y' = \dfrac{(1+x^2)\frac{d}{dx}(1) - (1)\frac{d}{dx}(1+x^2)}{(1+x^2)^2} = \dfrac{-2x}{(1+x^2)^2}$. Thus, the slope of the tangent line at $\left(1, \frac{1}{2}\right)$ is $y'\big|_{x=1} = \dfrac{-2x}{(1+x^2)^2}\bigg|_{x=1} = \dfrac{-2}{4} = -\dfrac{1}{2}$ and an equation of the tangent line is $y - \frac{1}{2} = -\frac{1}{2}(x-1)$, or $y = -\frac{1}{2}x + 1$. Next, the slope of the required normal line is 2 and its equation is $y - \frac{1}{2} = 2(x-1)$, or $y = 2x - \frac{3}{2}$.

52. $C(t) = \dfrac{0.2t}{t^2+1}$.

a. $$C'(t) = \frac{(t^2+1)\dfrac{d}{dt}(0.2t) - (0.2t)\dfrac{d}{dt}(t^2+1)}{(t^2+1)^2} = \frac{(t^2+1)(0.2) - (0.2t)(2t)}{(t^2+1)^2} = \frac{-0.2t^2+0.2}{(t^2+1)^2}$$
$$= \frac{0.2(1-t^2)}{(t^2+1)^2}.$$

b. The rate of change of the concentration of the drug one-half hour after injection is

$C'\left(\frac{1}{2}\right) = \frac{0.2\left(1-\frac{1}{4}\right)}{\left(\frac{1}{4}+1\right)^2} = \frac{0.2\,(0.75)}{(1.25)^2} = 0.096$, or 0.096%/hr. The rate of change of the concentration of the drug one hour after injection is $C'(1) = \frac{0.2\,(1-1)}{(1+1)^2} = 0$, or 0%/hr. The rate of change of the concentration of the drug 2 hours after injection is $C'(2) = \frac{0.2\left(1-2^2\right)}{\left(2^2+1\right)^2} = \frac{0.2\,(-3)}{25} = -0.024$, or -0.024%/hr.

53. $C(x) = \frac{0.5x}{100-x}$, so $C'(x) = \frac{(100-x)(0.5) - 0.5x(-1)}{(100-x)^2} = \frac{50}{(100-x)^2}$. $C'(80) = \frac{50}{20^2} = 0.125$, $C'(90) = \frac{50}{10^2} = 0.5$, $C'(95) = \frac{50}{5^2} = 2$, and $C'(99) = \frac{50}{1} = 50$. The rates of change of the cost of removing 80%, 90%, 95%, and 99% of the toxic waste are 0.125, 0.5, 2, and 50 million dollars per 1% increase in waste removed. It is too costly to remove all of the pollutant.

54. $D(t) = \frac{500t}{t+12}$, so $D'(t) = \frac{(t+12)\,500 - 500t}{(t+12)^2} = \frac{6000}{(t+12)^2}$. The rates of change for a six-year-old child and a ten-year-old child are $D'(6) = \frac{6000}{(18)^2} \approx 18.5$ mg/yr and $D'(10) = \frac{6000}{(22)^2} \approx 12.4$ mg/yr.

55. $N(t) = \frac{10{,}000}{1+t^2} + 2000$, so $N'(t) = \frac{d}{dt}\left[10{,}000\left(1+t^2\right)^{-1} + 2000\right] = -\frac{10{,}000}{\left(1+t^2\right)^2}(2t) = -\frac{20{,}000t}{\left(1+t^2\right)^2}$. The rates of change after 1 minute and 2 minutes are $N'(1) = -\frac{20{,}000}{\left(1+1^2\right)^2} = -5000$ and $N'(2) = -\frac{20{,}000\,(2)}{\left(1+2^2\right)^2} = -1600$.

The population of bacteria after one minute is $N(1) = \frac{10{,}000}{1+1} + 2000 = 7000$, and the population after two minutes is $N(2) = \frac{10{,}000}{1+4} + 2000 = 4000$.

56. a. $d(x) = \frac{50}{0.01x^2+1}$, so $d'(x) = \frac{\left(0.01x^2+1\right)(0) - 50\,(0.02x)}{\left(0.01x^2+1\right)^2} = -\frac{x}{\left(0.01x^2+1\right)^2}$.

b. $d'(5) = -\frac{5}{(0.25+1)^2} = -3.2$, $d'(10) = -\frac{10}{(2)^2} = -2.5$, and $d'(15) = -\frac{15}{(3.25)^2} \approx -1.4$, so the rates of change of the price when the demand is 5,000, 10,000, and 15,000 units are decreasing at the rates of \$3200, \$2500, and \$1400 per 1000 watches, respectively.

57. a. $R(x) = xd(x) = \frac{50x}{0.01x^2+1}$.

b. $R'(x) = \frac{d}{dx}\left(\frac{50x}{0.01x^2+1}\right) = 50\frac{d}{dx}\left(\frac{x}{0.01x^2+1}\right) = 50 \cdot \frac{\left(0.01x^2+1\right)(1) - x\,(0.02x)}{\left(0.01x^2+1\right)^2} = \frac{50\left(1-0.01x^2\right)}{\left(0.01x^2+1\right)^2}$.

c. $R'(8) \approx 6.69$, $R'(10) = 0$, and $R'(12) \approx -3.70$, so the revenue is increasing at the rate of approximately \$6700 per thousand watches at a sales level of 8000 watches per week, the revenue is stable at a sales level of 10,000 watches per week, and the revenue is decreasing by approximately \$3700 per thousand watches at a sales level of 12,000 watches per week.

58. a. $P(x) = \frac{50x}{0.01x^2+1} - 0.025x^3 + 0.35x^2 - 10x - 30$. $P(0) = -30$, indicating that the company loses \$30,000 per week if no watches are sold.

b. Using the result of Exercise 57, we find $P'(x) = \frac{50(1-0.01x^2)}{(0.01x^2+1)^2} - 0.075x^2 + 0.7x - 10$, so $P'(5) = 15.625$ and $P'(10) = -10.5$. Thus, the profit increases by approximately \$15,625 per thousand watches at a sales level of 5000 watches per week, and the profit decreases by approximately 10,500 per thousand watches at a sales level of 10,000 watches per week.

59. a. The average 30-year fixed mortgage rate in the first week of May in 2010 was $M(1) = \frac{55.9}{1-0.31+11.2} \approx 4.701$, or approximately 4.7% per year.

b. $M'(t) = \frac{(t^2-0.31t+11.2)(0)-55.9(2t-0.31)}{(t^2-0.31t+11.2)^2} = \frac{-55.9(2t-0.31)}{(t^2-0.31t+11.2)^2}$. Thus, the 30-year fixed mortgage rate was changing at the rate of $M'(1) = \frac{-55.9(2-0.31)}{(1-0.31+11.2)^2} \approx -0.668$ in the first week of May in 2010. That is, it was decreasing at approximately 0.67% per year per year.

60. $T(x) = \frac{120x^2}{x^2+4}$, so

$$T'(x) = \frac{(x^2+4)\frac{d}{dx}(120x^2) - (120x^2)\frac{d}{dx}(x^2+4)}{(x^2+4)^2} = \frac{(x^2+4)(240x)-(120x^2)(2x)}{(x^2+4)^2} = \frac{960x}{(x^2+4)^2}.$$

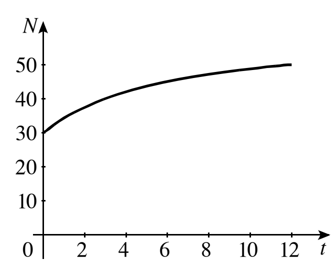
$T'(1) = \frac{960}{(1+4)^2} = \frac{960}{25} = 38.4$, or \$38.4 million per year; $T'(3) = \frac{960(3)}{(9+4)^2} \approx 17.04$, or approximately \$17.04 million per year; and $T'(5) = \frac{960(5)}{(25+4)^2} \approx 5.71$ or approximately \$5.71 million per year.

61. a. $N(t) = \frac{60t+180}{t+6}$, so

$$N'(t) = \frac{(t+6)\frac{d}{dt}(60t+180) - (60t+180)\frac{d}{dt}(t+6)}{(t+6)^2} = \frac{(t+6)(60)-(60t+180)(1)}{(t+6)^2} = \frac{180}{(t+6)^2}.$$

b. $N'(1) = \frac{180}{(1+6)^2} \approx 3.7$, $N'(3) = \frac{180}{(3+6)^2} \approx 2.2$, $N'(4) = \frac{180}{(4+6)^2} = 1.8$, and $N'(7) = \frac{180}{(7+6)^2} \approx 1.1$. We conclude that the rates at which the average student is increasing his or her speed one week, three weeks, four weeks, and seven weeks into the course are approximately 3.7, 2.2, 1.8, and 1.1 words per minute, respectively.

c. Yes.



d. $N(12) = \frac{60(12)+180}{12+6} = 50$, or 50 words/minute.

62. $f(t) = \dfrac{5t+300}{t^2+25}$, so

$$f'(t) = \frac{(t^2+25)(5) - (5t+300)(2t)}{(t^2+25)^2} = \frac{5t^2+125-10t^2-600t}{(t^2+25)^2} = \frac{-5(t^2+120t-25)}{(t^2+25)^2} \text{ and}$$

$$f'(3) = \frac{-5\left[3^2+120(3)-25\right]}{(3^2+25)^2} \approx -1.488, \text{ or approximately } -1.49\% \text{ per year.}$$

63. $f(t) = \dfrac{0.055t+0.26}{t+2}$, so $f'(t) = \dfrac{(t+2)(0.055)-(0.055t+0.26)(1)}{(t+2)^2} = -\dfrac{0.15}{(t+2)^2}$. At the beginning, the formaldehyde level is changing at the rate of $f'(0) = -\dfrac{0.15}{4} = -0.0375$; that is, it is decreasing at the rate of 0.0375 parts per million per year. Next, $f'(3) = -\dfrac{0.15}{5^2} = -0.006$, and so the level is decreasing at the rate of 0.006 parts per million per year at the beginning of the fourth year (when $t = 3$).

64. **a.** $P(t) = \dfrac{25t^2+125t+200}{t^2+5t+40}$. The rate at which Glen Cove's population is changing with respect to time is

$$\begin{aligned} P'(t) &= \frac{(t^2+5t+40)\dfrac{d}{dt}(25t^2+125t+200) - (25t^2+125t+200)\dfrac{d}{dt}(t^2+5t+40)}{(t^2+5t+40)^2} \\ &= \frac{(t^2+5t+40)(50t+125)-(25t^2+125t+200)(2t+5)}{(t^2+5t+40)^2} \\ &= \frac{25(2t+5)(t^2+5t+40-t^2-5t-8)}{(t^2+5t+40)^2} = \frac{(25)(32)(2t+5)}{(t^2+5t+40)^2} = \frac{800(2t+5)}{(t^2+5t+40)^2}. \end{aligned}$$

b. After ten years the population will be $P(10) = \dfrac{25(10)^2+125(10)+200}{(10)^2+5(10)+40} = \dfrac{3950}{190} = 20.789$, or 20,789. After ten years the population will be increasing at the rate of $P'(10) = \dfrac{800[2(10)+5]}{\left[(10)^2+5(10)+40\right]^2} = \dfrac{20{,}000}{190^2} \approx 0.554$, or 554 people per year.

65. **a.** $R'(x) = \frac{d}{dx}[xD(x)] = xD'(x) + (1)D(x) = xD'(x) + D(x)$.

b. Here $p = D(x) = a - bx$, so $D'(x) = -b$. Therefore, $R'(x) = x(-b) + (a - bx) = a - 2bx$.

c. $R(x) = xD(x) = x(a - bx) = ax - bx^2$, so $R'(x) = a - 2bx$, as obtained in part (a).

66. The per capita income of the country at time t is $P(t) = \dfrac{g(t)}{f(t)} = \dfrac{60+4t}{3+0.06t}$. Thus, $P'(t) = \dfrac{(3+0.06t)(4)-(60+4t)(0.06)}{(3+0.06t)^2} = \dfrac{8.4}{(3+0.06t)^2}$. The per capita income in two years' time is projected to be changing at $P'(2) = \dfrac{8.4}{(3+0.06\cdot 2)^2} \approx 0.8629$, or approximately \$0.86 billion/year.

67. **a.** If there is no substrate present, then the relative growth rate is $R(0) = 0$.

b. $\lim\limits_{s\to\infty} R(s) = \lim\limits_{s\to\infty} \dfrac{cs}{k+s} = \lim\limits_{s\to\infty} \dfrac{c}{\frac{k}{s}+1} = c$. Thus, the relative growth rate approaches c when the substrate is present in great excess.

c. $R'(s) = \dfrac{(k+s)(c) - cs(1)}{(k+s)^2} = \dfrac{kc}{(k+s)^2}$.

68. **a.** $\frac{1}{f} = \frac{1}{p} + \frac{1}{q}$ with $q = 2.5$ and $p = 50$. Thus, $\frac{1}{f} = \frac{1}{2.5} + \frac{1}{50} = 0.42$, and so $f = \frac{1}{0.42} \approx 2.38$, or approximately 2.38 cm.

b. $\frac{1}{f} = \frac{1}{p} + \frac{1}{q}$, so $f = \frac{1}{\frac{1}{p} + \frac{1}{q}} = \frac{pq}{p+q}$. Differentiating with respect to p, we have

$\frac{df}{dp} = \frac{(p+q)\,q - pq\,(1)}{(p+q)^2} = \left(\frac{q}{p+q}\right)^2$. When $p = 50$, we have $\frac{df}{dp} = \left(\frac{2.5}{50+2.5}\right)^2 \approx 0.00227$, or 0.00227 cm/cm.

69. False. Take $f(x) = x$ and $g(x) = x$. Then $f(x)\,g(x) = x^2$, so $\frac{d}{dx}\left[f(x)\,g(x)\right] = \frac{d}{dx}\left(x^2\right) = 2x \neq f'(x)\,g'(x) = 1$.

70. True. Using the Product Rule, $\frac{d}{dx}\left[x f(x)\right] = f(x)\,\frac{d}{dx}(x) + x\frac{d}{dx}\left[f(x)\right] = f(x)\,(1) + x f'(x)$.

71. False. Let $f(x) = x^3$. Then $\frac{d}{dx}\left[\frac{f(x)}{x^2}\right] = \frac{d}{dx}\left(\frac{x^3}{x^2}\right) = \frac{d}{dx}(x) = 1 \neq \frac{f'(x)}{2x} = \frac{3x^2}{2x} = \frac{3}{2}x$.

72. True. Using the Quotient Rule followed by the Product Rule,

$$\frac{d}{dx}\left[\frac{f(x)\,g(x)}{h(x)}\right] = \frac{h(x)\,\frac{d}{dx}\left[f(x)\,g(x)\right] - f(x)\,g(x)\,\frac{d}{dx}\left[h(x)\right]}{\left[h(x)\right]^2} = \frac{h(x)\left[f'(x)\,g(x) + f(x)\,g'(x)\right] - f(x)\,g(x)\,h'(x)}{\left[h(x)\right]^2}.$$

73. Let $f(x) = u(x)\,v(x)$ and $g(x) = w(x)$. Then $h(x) = f(x)\,g(x)$. Therefore, $h'(x) = f'(x)\,g(x) + f(x)\,g'(x)$. But $f'(x) = u(x)\,v'(x) + u'(x)\,v(x)$, so

$$h'(x) = \left[u(x)\,v'(x) + u'(x)\,v(x)\right]g(x) + u(x)\,v(x)\,w'(x) = u(x)\,v(x)\,w'(x) + u(x)\,v'(x)\,w(x) + u'(x)\,v(x)\,w(x).$$

74. Let $k(x) = \frac{f(x)}{g(x)}$.

a. $\frac{k(x+h) - k(x)}{h} = \frac{\frac{f(x+h)}{g(x+h)} - \frac{f(x)}{g(x)}}{h} = \frac{f(x+h)\,g(x) - f(x)\,g(x+h)}{hg(x+h)\,g(x)}$.

b. By adding $-f(x)\,g(x) + f(x)\,g(x)$ (which is equal to zero) to the numerator and simplifying, we have

$$\frac{k(x+h) - k(x)}{h} = \frac{1}{g(x+h)\,g(x)}\left\{\left[\frac{f(x+h) - f(x)}{h}\right]g(x) - \left[\frac{g(x+h) - g(x)}{h}\right]f(x)\right\}.$$

c. Taking the limit and using the definition of the derivative, we find

$$k'(x) = \lim_{h\to 0}\frac{k(x+h) - k(x)}{h} = \frac{1}{\left[g(x)\right]^2}\left[f'(x)\,g(x) - g'(x)\,f(x)\right] = \frac{f'(x)\,g(x) - g'(x)\,f(x)}{\left[g(x)\right]^2}.$$

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1. 0.8750 **2.** 16.7980 **3.** 0.0774 **4.** -0.1314 **5.** -0.5000 **6.** 2.875

7. 31,312 per year **8.** **a.** 20,790 **b.** 554/year

3.3 The Chain Rule

Concept Questions page 194

1. The derivative of $h(x) = g(f(x))$ is equal to the derivative of g evaluated at $f(x)$ times the derivative of f.

2. $h'(x) = \dfrac{d}{dx}\left[f(x)\right]^n = n\left[f(x)\right]^{n-1} f'(x)$.

3. $(g \circ f)'(t) = \left[(g \circ f)(t)\right]' = g'(f(t))\, f'(t)$ describes the rate of change of the revenue as a function of time.

4. $(f \circ g)'(t)$ gives the rate of change of the air temperature.

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1. $f(x) = (2x-1)^3$, so $f'(x) = 3(2x-1)^2 \frac{d}{dx}(2x-1) = 3(2x-1)^2(2) = 6(2x-1)^2$.

2. $f(x) = (1-x)^4$, so $f'(x) = 4(1-x)^3(-1) = -4(1-x)^3$.

3. $f(x) = \left(x^2+2\right)^5$, so $f'(x) = 5\left(x^2+2\right)^4(2x) = 10x\left(x^2+2\right)^4$.

4. $f(t) = 2\left(t^3-1\right)^5$, so $f'(t) = (2)(5)\left(t^3-1\right)^4\left(3t^2\right) = 30t^2\left(t^3-1\right)^4$.

5. $f(x) = \left(2x-x^2\right)^3$, so $f'(x) = 3\left(2x-x^2\right)^2 \frac{d}{dx}\left(2x-x^2\right) = 3\left(2x-x^2\right)^2(2-2x) = 6x^2(1-x)(2-x)^2$.

6. $f(x) = 3\left(x^3-x\right)^4$, so $f'(x) = (3)(4)\left(x^3-x\right)^3\left(3x^2-1\right) = 12\left(3x^2-1\right)\left(x^3-x\right)^3$.

7. $f(x) = (2x+1)^{-2}$, so $f'(x) = -2(2x+1)^{-3} \frac{d}{dx}(2x+1) = -2(2x+1)^{-3}(2) = -4(2x+1)^{-3}$.

8. $f(t) = \frac{1}{2}\left(2t^2+t\right)^{-3}$, so $f'(t) = \frac{1}{2}(-3)\left(2t^2+t\right)^{-4}(4t+1) = -\dfrac{3(1+4t)}{2\left(2t^2+t\right)^4}$.

9. $f(x) = \left(x^2-4\right)^{5/2}$, so $f'(x) = \frac{5}{2}\left(x^2-4\right)^{3/2} \frac{d}{dx}\left(x^2-4\right) = \frac{5}{2}\left(x^2-4\right)^{3/2}(2x) = 5x\left(x^2-4\right)^{3/2}$.

10. $f(t) = \left(3t^2-2t+1\right)^{3/2}$, so $f'(t) = \frac{3}{2}\left(3t^2-2t+1\right)^{1/2}(6t-2) = 3(3t-1)\left(3t^2-2t+1\right)^{1/2}$.

11. $f(x) = \sqrt{3x-2} = (3x-2)^{1/2}$, so $f'(x) = \frac{1}{2}(3x-2)^{-1/2}(3) = \frac{3}{2}(3x-2)^{-1/2} = \dfrac{3}{2\sqrt{3x-2}}$.

12. $f(t) = \sqrt{3t^2-t} = \left(3t^2-t\right)^{1/2}$, so $f'(t) = \frac{1}{2}\left(3t^2-t\right)^{-1/2}(6t-1) = \dfrac{6t-1}{2\sqrt{3t^2-t}}$.

13. $f(x) = \sqrt[3]{1-x^2}$, so

$$f'(x) = \frac{d}{dx}\left(1-x^2\right)^{1/3} = \frac{1}{3}\left(1-x^2\right)^{-2/3} \frac{d}{dx}\left(1-x^2\right) = \frac{1}{3}\left(1-x^2\right)^{-2/3}(-2x) = -\frac{2}{3}x\left(1-x^2\right)^{-2/3}$$
$$= \frac{-2x}{3\left(1-x^2\right)^{2/3}}.$$

14. $f(x) = \sqrt{2x^2-2x+3}$, so $f'(x) = \frac{1}{2}\left(2x^2-2x+3\right)^{-1/2}(4x-2) = (2x-1)\left(2x^2-2x+3\right)^{-1/2}$.

15. $f(x) = \dfrac{1}{(2x+3)^3} = (2x+3)^{-3}$, so $f'(x) = -3(2x+3)^{-4}(2) = -6(2x+3)^{-4} = -\dfrac{6}{(2x+3)^4}$.

16. $f(x) = \dfrac{2}{\left(x^2-1\right)^4}$, so $f'(x) = 2\frac{d}{dx}\left(x^2-1\right)^{-4} = 2(-4)\left(x^2-1\right)^{-5}(2x) = -16x\left(x^2-1\right)^{-5}$.

17. $f(t) = \dfrac{1}{\sqrt{2t-4}}$, so $f'(t) = \frac{d}{dt}(2t-4)^{-1/2} = -\frac{1}{2}(2t-4)^{-3/2}(2) = -(2t-4)^{-3/2} = -\dfrac{1}{(2t-4)^{3/2}}$.

18. $f(x) = \dfrac{1}{\sqrt{2x^2-1}} = \left(2x^2-1\right)^{-1/2}$, so $f'(x) = -\frac{1}{2}\left(2x^2-1\right)^{-3/2}(4x) = -\dfrac{2x}{\sqrt{\left(2x^2-1\right)^3}}$.

19. $y = \dfrac{1}{\left(4x^4+x\right)^{3/2}}$, so $\frac{dy}{dx} = \frac{d}{dx}\left(4x^4+x\right)^{-3/2} = -\frac{3}{2}\left(4x^4+x\right)^{-5/2}\left(16x^3+1\right) = -\frac{3}{2}\left(16x^3+1\right)\left(4x^4+x\right)^{-5/2}$.

20. $f(t) = \dfrac{4}{\sqrt[3]{2t^2+t}}$, so $f'(t) = 4\frac{d}{dt}\left(2t^2+t\right)^{-1/3} = -\frac{4}{3}\left(2t^2+t\right)^{-4/3}(4t+1) = -\frac{4}{3}(4t+1)\left(2t^2+t\right)^{-4/3}$.

21. $f(x) = \left(3x^2+2x+1\right)^{-2}$, so

$$f'(x) = -2\left(3x^2+2x+1\right)^{-3}\frac{d}{dx}\left(3x^2+2x+1\right) = -2\left(3x^2+2x+1\right)^{-3}(6x+2)$$
$$= -4(3x+1)\left(3x^2+2x+1\right)^{-3}.$$

22. $f(t) = \left(5t^3+2t^2-t+4\right)^{-3}$, so $f'(t) = -3\left(5t^3+2t^2-t+4\right)^{-4}\left(15t^2+4t-1\right)$.

23. $f(x) = \left(x^2+1\right)^3 - \left(x^3+1\right)^2$, so

$$f'(x) = 3\left(x^2+1\right)^2\frac{d}{dx}\left(x^2+1\right) - 2\left(x^3+1\right)\frac{d}{dx}\left(x^3+1\right) = 3\left(x^2+1\right)^2(2x) - 2\left(x^3+1\right)\left(3x^2\right)$$
$$= 6x\left[\left(x^2+1\right)^2 - x\left(x^3+1\right)\right] = 6x\left(2x^2-x+1\right).$$

24. $f(t) = (2t-1)^4 + (2t+1)^4$, so $f'(t) = 4(2t-1)^3(2) + 4(2t+1)^3(2) = 8\left[(2t-1)^3 + (2t+1)^3\right]$.

25. $f(t) = \left(t^{-1}-t^{-2}\right)^3$, so $f'(t) = 3\left(t^{-1}-t^{-2}\right)^2\frac{d}{dt}\left(t^{-1}-t^{-2}\right) = 3\left(t^{-1}-t^{-2}\right)^2\left(-t^{-2}+2t^{-3}\right)$.

26. $f(v) = \left(v^{-3}+4v^{-2}\right)^3$, so $f'(v) = 3\left(v^{-3}+4v^{-2}\right)^2\left(-3v^{-4}-8v^{-3}\right)$.

27. $f(x) = \sqrt{x+1} + \sqrt{x-1} = (x+1)^{1/2} + (x-1)^{1/2}$, so

$f'(x) = \frac{1}{2}(x+1)^{-1/2}(1) + \frac{1}{2}(x-1)^{-1/2}(1) = \frac{1}{2}\left[(x+1)^{-1/2} + (x-1)^{-1/2}\right]$.

28. $f(u) = (2u+1)^{3/2} + \left(u^2-1\right)^{-3/2}$, so

$f'(u) = \frac{3}{2}(2u+1)^{1/2}(2) - \frac{3}{2}\left(u^2-1\right)^{-5/2}(2u) = 3(2u+1)^{1/2} - 3u\left(u^2-1\right)^{-5/2}$.

29. $f(x) = 2x^2(3-4x)^4$, so

$$f'(x) = 2x^2(4)(3-4x)^3(-4) + (3-4x)^4(4x) = 4x(3-4x)^3(-8x+3-4x)$$
$$= 4x(3-4x)^3(-12x+3) = (-12x)(4x-1)(3-4x)^3.$$

30. $h(t) = t^2(3t+4)^3$, so
$$h'(t) = 2t(3t+4)^3 + t^2(3)(3t+4)^2(3) = t(3t+4)^2[2(3t+4)+9t] = t(15t+8)(3t+4)^2.$$

31. $f(x) = (x-1)^2(2x+1)^4$, so
$$\begin{aligned} f'(x) &= (x-1)^2 \tfrac{d}{dx}(2x+1)^4 + (2x+1)^4 \tfrac{d}{dx}(x-1)^2 \quad \text{(by the Product Rule)} \\ &= (x-1)^2(4)(2x+1)^3 \tfrac{d}{dx}(2x+1) + (2x+1)^4(2)(x-1)\tfrac{d}{dx}(x-1) \\ &= 8(x-1)^2(2x+1)^3 + 2(x-1)(2x+1)^4 = 2(x-1)(2x+1)^3(4x-4+2x+1) \\ &= 6(x-1)(2x-1)(2x+1)^3. \end{aligned}$$

32. $g(u) = (u+1)^{1/2}\left(1-2u^2\right)^8$, so
$$\begin{aligned} g'(u) &= (u+1)^{1/2}(8)\left(1-2u^2\right)^7(-4u) + \left(1-2u^2\right)^8\left(\tfrac{1}{2}\right)(u+1)^{-1/2} \\ &= -\tfrac{1}{2}(u+1)^{-1/2}\left(1-2u^2\right)^7\left[64u(u+1) - \left(1-2u^2\right)\right] \\ &= -\frac{\left(66u^2+64u-1\right)\left(1-2u^2\right)^7}{2\sqrt{u+1}} = \frac{\left(2u^2-1\right)^7\left(66u^2+64u-1\right)}{2\sqrt{u+1}}. \end{aligned}$$

33. $f(x) = \left(\dfrac{x+3}{x-2}\right)^3$, so
$$\begin{aligned} f'(x) &= 3\left(\frac{x+3}{x-2}\right)^2 \frac{d}{dx}\left(\frac{x-3}{x-2}\right) = 3\left(\frac{x+3}{x-2}\right)^2\left[\frac{(x-2)(1)-(x+3)(1)}{(x-2)^2}\right] \\ &= 3\left(\frac{x+3}{x-2}\right)^2\left[-\frac{5}{(x-2)^2}\right] = -\frac{15(x+3)^2}{(x-2)^4}. \end{aligned}$$

34. $f(x) = \left(\dfrac{x+1}{x-1}\right)^5$, so $f'(x) = 5\left(\dfrac{x+1}{x-1}\right)^4\left[\dfrac{(x-1)(1)-(x+1)(1)}{(x-1)^2}\right] = -\dfrac{10(x+1)^4}{(x-1)^6}$.

35. $s(t) = \left(\dfrac{t}{2t+1}\right)^{3/2}$, so
$$\begin{aligned} s'(t) &= \frac{3}{2}\left(\frac{t}{2t+1}\right)^{1/2}\frac{d}{dt}\left(\frac{t}{2t+1}\right) = \frac{3}{2}\left(\frac{t}{2t+1}\right)^{1/2}\left[\frac{(2t+1)(1)-t(2)}{(2t+1)^2}\right] \\ &= \frac{3}{2}\left(\frac{t}{2t+1}\right)^{1/2}\left[\frac{1}{(2t+1)^2}\right] = \frac{3t^{1/2}}{2(2t+1)^{5/2}}. \end{aligned}$$

36. $g(s) = \left(s^2+\dfrac{1}{s}\right)^{3/2} = \left(s^2+s^{-1}\right)^{3/2}$, so
$$g'(s) = \tfrac{3}{2}\left(s^2+s^{-1}\right)^{1/2}\left(2s-s^{-2}\right) = \frac{3}{2}\left(s^2+\frac{1}{s}\right)^{1/2}\left(2s-\frac{1}{s^2}\right) = \frac{3}{2}\left(\frac{s^3+1}{s}\right)^{1/2}\left(\frac{2s^3-1}{s^2}\right).$$

37. $g(u) = \left(\dfrac{u+1}{3u+2}\right)^{1/2}$, so
$$\begin{aligned} g'(u) &= \frac{1}{2}\left(\frac{u+1}{3u+2}\right)^{-1/2}\frac{d}{du}\left(\frac{u+1}{3u+2}\right) = \frac{1}{2}\left(\frac{u+1}{3u+2}\right)^{-1/2}\left[\frac{(3u+2)(1)-(u+1)(3)}{(3u+2)^2}\right] \\ &= -\frac{1}{2\sqrt{u+1})(3u+2)^{3/2}}. \end{aligned}$$

38. $g(x)=\left(\dfrac{2x+1}{2x-1}\right)^{1/2}$, so

$$g'(x)=\frac{1}{2}\left(\frac{2x+1}{2x-1}\right)^{-1/2}\left[\frac{(2x-1)(2)-(2x+1)(2)}{(2x-1)^2}\right]=\frac{1}{2}\left(\frac{2x+1}{2x-1}\right)^{-1/2}\left(-\frac{4}{(2x-1)^2}\right)$$
$$=-\frac{2}{(2x+1)^{1/2}(2x-1)^{3/2}}.$$

39. $f(x)=\dfrac{x^2}{\left(x^2-1\right)^4}$, so

$$f'(x)=\frac{\left(x^2-1\right)^4\dfrac{d}{dx}\left(x^2\right)-\left(x^2\right)\dfrac{d}{dx}\left(x^2-1\right)^4}{\left[\left(x^2-1\right)^4\right]^2}=\frac{\left(x^2-1\right)^4(2x)-x^2(4)\left(x^2-1\right)^3(2x)}{\left(x^2-1\right)^8}$$
$$=\frac{\left(x^2-1\right)^3(2x)\left(x^2-1-4x^2\right)}{\left(x^2-1\right)^8}=\frac{(-2x)\left(3x^2+1\right)}{\left(x^2-1\right)^5}.$$

40. $g(u)=\dfrac{2u^2}{\left(u^2+u\right)^3}$, so

$$g'(u)=\frac{\left(u^2+u\right)^3(4u)-\left(2u^2\right)3\left(u^2+u\right)^2(2u+1)}{\left(u^2+u\right)^6}$$
$$=\frac{2u\left(u^2+u\right)^2\left[2\left(u^2+u\right)-3u(2u+1)\right]}{\left(u^2+u\right)^6}=\frac{-2u\left(4u^2+u\right)}{\left(u^2+u\right)^4}=\frac{-2u^2(4u+1)}{u^4(u+1)^4}=-\frac{2(4u+1)}{u^2(u+1)^4}.$$

41. $h(x)=\dfrac{\left(3x^2+1\right)^3}{\left(x^2-1\right)^4}$, so

$$h'(x)=\frac{\left(x^2-1\right)^4(3)\left(3x^2+1\right)^2(6x)-\left(3x^2+1\right)^3(4)\left(x^2-1\right)^3(2x)}{\left(x^2-1\right)^8}$$
$$=\frac{2x\left(x^2-1\right)^3\left(3x^2+1\right)^2\left[9\left(x^2-1\right)-4\left(3x^2+1\right)\right]}{\left(x^2-1\right)^8}=-\frac{2x\left(3x^2+13\right)\left(3x^2+1\right)^2}{\left(x^2-1\right)^5}.$$

42. $g(t)=\dfrac{(2t-1)^2}{(3t+2)^4}$, so

$$g'(t)=\frac{(3t+2)^4(2)(2t-1)(2)-(2t-1)^2(4)(3t+2)^3(3)}{(3t+2)^8}$$
$$=\frac{4(3t+2)^3(2t-1)[(3t+2)-3(2t-1)]}{(3t+2)^8}=\frac{4(2t-1)(5-3t)}{(3t+2)^5}.$$

43. $f(x)=\dfrac{\sqrt{2x+1}}{x^2-1}$, so

$$f'(x)=\frac{\left(x^2-1\right)\left(\frac{1}{2}\right)(2x+1)^{-1/2}(2)-(2x+1)^{1/2}(2x)}{\left(x^2-1\right)^2}=\frac{(2x+1)^{-1/2}\left[\left(x^2-1\right)-(2x+1)(2x)\right]}{\left(x^2-1\right)^2}$$
$$=-\frac{3x^2+2x+1}{\sqrt{2x+1}\left(x^2-1\right)^2}.$$

44. $f(t) = \dfrac{4t^2}{\sqrt{2t^2+2t-1}} = \dfrac{4t^2}{\left(2t^2+2t-1\right)^{1/2}}$, so

$$f'(t) = \frac{\left(2t^2+2t-1\right)^{1/2}\dfrac{d}{dt}\left(4t^2\right) - 4t^2\dfrac{d}{dt}\left(2t^2+2t-1\right)^{1/2}}{\left[\left(2t^2+2t-1\right)^{1/2}\right]^2}$$

$$= \frac{\left(2t^2+2t-1\right)^{1/2}(8t) - 4t^2\left(\frac{1}{2}\right)\left(2t^2+2t-1\right)^{-1/2}(4t+2)}{2t^2+2t-1}$$

$$= \frac{4t\left(2t^2+2t-1\right)^{-1/2}\left[2\left(2t^2+2t-1\right) - t(2t+1)\right]}{2t^2+2t-1} = \frac{4t\left(2t^2+3t-2\right)}{\left(\sqrt{2t^2+2t-1}\right)^3}.$$

45. $g(t) = \dfrac{(t+1)^{1/2}}{\left(t^2+1\right)^{1/2}}$, so

$$g'(t) = \frac{\left(t^2+1\right)^{1/2}\dfrac{d}{dt}(t+1)^{1/2} - (t+1)^{1/2}\dfrac{d}{dt}\left(t^2+1\right)^{1/2}}{t^2+1}$$

$$= \frac{\left(t^2+1\right)^{1/2}\left(\frac{1}{2}\right)(t+1)^{-1/2}(1) - (t+1)^{1/2}\left(\frac{1}{2}\right)\left(t^2+1\right)^{-1/2}(2t)}{t^2+1}$$

$$= \frac{\frac{1}{2}(t+1)^{-1/2}\left(t^2+1\right)^{-1/2}\left[\left(t^2+1\right) - 2t(t+1)\right]}{t^2+1} = -\frac{t^2+2t-1}{2\sqrt{t+1}\left(t^2+1\right)^{3/2}}.$$

46. $f(x) = \dfrac{\left(x^2+1\right)^{1/2}}{\left(x^2-1\right)^{1/2}}$, so

$$f'(x) = \frac{\left(x^2-1\right)^{1/2}\frac{d}{dx}\left(x^2+1\right)^{1/2} - \left(x^2+1\right)^{1/2}\frac{d}{dx}\left(x^2-1\right)^{1/2}}{\left(x^2-1\right)}$$

$$= \frac{\left(x^2-1\right)^{1/2}\left(\frac{1}{2}\right)\left(x^2+1\right)^{-1/2}(2x) - \left(x^2+1\right)^{1/2}\left(\frac{1}{2}\right)\left(x^2-1\right)^{-1/2}(2x)}{x^2-1}$$

$$= \frac{x\left(x^2-1\right)^{-1/2}\left(x^2+1\right)^{-1/2}\left[\left(x^2-1\right) - \left(x^2+1\right)\right]}{x^2-1} = -\frac{2x}{\sqrt{x^2+1}\left(x^2-1\right)^{3/2}}.$$

47. $f(x) = (3x+1)^4\left(x^2-x+1\right)^3$, so

$$f'(x) = (3x+1)^4\frac{d}{dx}\left(x^2-x+1\right)^3 + \left(x^2-x+1\right)^3\frac{d}{dx}(3x+1)^4$$

$$= (3x+1)^4\cdot 3\left(x^2-x+1\right)^2(2x-1) + \left(x^2-x+1\right)^3\cdot 4(3x+1)^3\cdot 3$$

$$= 3(3x+1)^3\left(x^2-x+1\right)^2\left[(3x+1)(2x-1) + 4\left(x^2-x+1\right)\right]$$

$$= 3(3x+1)^3\left(x^2-x+1\right)^2\left(6x^2-3x+2x-1+4x^2-4x+4\right)$$

$$= 3(3x+1)^3\left(x^2-x+1\right)^2\left(10x^2-5x+3\right).$$

48. $g(t) = (2t+3)^2\left(3t^2-1\right)^{-3}$, so

$$\begin{aligned} g'(t) &= (2t+3)^2 \frac{d}{dt}\left(3t^2-1\right)^{-3} + \left(3t^2-1\right)^{-3}\frac{d}{dt}(2t+3)^2 \\ &= (2t+3)^2(-3)\left(3t^2-1\right)^{-4}(6t) + \left(3t^2-1\right)^{-3}(2)(2t+3)(2) \\ &= 2(2t+3)\left(3t^2-1\right)^{-4}\left[-9t(2t+3)+2\left(3t^2-1\right)\right] \\ &= 2(2t+3)\left(3t^2-1\right)^{-4}\left(-18t^2-27t+6t^2-2\right) \\ &= -2\left(12t^2+27t+2\right)(2t+3)\left(3t^2-1\right)^{-4}. \end{aligned}$$

49. $y = g(u) = u^{4/3}$, so $\frac{dy}{du} = \frac{4}{3}u^{1/3}$, and $u = f(x) = 3x^2-1$, so $\dfrac{du}{dx} = 6x$. Thus,
$\dfrac{dy}{dx} = \dfrac{dy}{du}\cdot\dfrac{du}{dx} = \frac{4}{3}u^{1/3}(6x) = \frac{4}{3}\left(3x^2-1\right)^{1/3}6x = 8x\left(3x^2-1\right)^{1/3}$.

50. $y = \sqrt{u}$ and $u = 7x-2x^2$, so $\dfrac{dy}{du} = \frac{1}{2}u^{-1/2}$ and $\dfrac{du}{dx} = 7-4x$. Thus, $\dfrac{dy}{dx} = \dfrac{dy}{du}\cdot\dfrac{du}{dx} = \dfrac{7-4x}{2\sqrt{u}} = \dfrac{7-4x}{2\sqrt{7x-2x^2}}$.

51. $y = u^{-2/3}$ and $u = 2x^3-x+1$, so $\dfrac{dy}{du} = -\frac{2}{3}u^{-5/3} = -\dfrac{2}{3u^{5/3}}$ and $\dfrac{du}{dx} = 6x^2-1$. Thus,
$\dfrac{dy}{dx} = \dfrac{dy}{du}\cdot\dfrac{du}{dx} = -\dfrac{2\left(6x^2-1\right)}{3u^{5/3}} = -\dfrac{2\left(6x^2-1\right)}{3\left(2x^3-x+1\right)^{5/3}}$.

52. $y = 2u^2+1$ and $u = x^2+1$, so $\dfrac{dy}{du} = 4u$ and $\dfrac{du}{dx} = 2x$. Thus, $\dfrac{dy}{dx} = \dfrac{dy}{du}\cdot\dfrac{du}{dx} = 4u(2x) = 8xu = 8x\left(x^2+1\right)$.

53. $y = \sqrt{u} + \dfrac{1}{\sqrt{u}}$ and $u = x^3-x$, so $\dfrac{dy}{du} = \frac{1}{2}u^{-1/2} - \frac{1}{2}u^{-3/2}$ and $\dfrac{du}{dx} = 3x^2-1$. Thus,
$\dfrac{dy}{dx} = \dfrac{dy}{du}\cdot\dfrac{du}{dx} = \left[\dfrac{1}{2\sqrt{x^3-x}} - \dfrac{1}{2\left(x^3-x\right)^{3/2}}\right]\left(3x^2-1\right) = \dfrac{\left(3x^2-1\right)\left(x^3-x-1\right)}{2\left(x^3-x\right)^{3/2}}$.

54. $y = \dfrac{1}{u}$ and $u = \sqrt{x}+1$, so $\dfrac{dy}{du} = -\dfrac{1}{u^2}$ and $\dfrac{du}{dx} = \dfrac{1}{2}x^{-1/2}$. Thus,
$\dfrac{dy}{dx} = \dfrac{dy}{du}\cdot\dfrac{du}{dx} = -\dfrac{1}{u^2}\cdot\left(\dfrac{1}{2}x^{-1/2}\right) = -\dfrac{1}{2\sqrt{x}\left(\sqrt{x}+1\right)^2}$.

55. $g(x) = f(2x+1)$. Let $u = 2x+1$, so $\dfrac{du}{dx} = 2$. Using the Chain Rule, we have
$g'(x) = f'(u)\dfrac{du}{dx} = f'(2x+1)\cdot 2 = 2f'(2x+1)$.

56. $h(x) = f\left(-x^3\right)$. Let $u = -x^3$, so $\dfrac{du}{dx} = -3x^2$. Using the Chain Rule, we have
$h'(x) = f'(u)\dfrac{du}{dx} = f'\left(-x^3\right)\left(-3x^2\right) = -3x^2 f'\left(-x^3\right)$.

57. $F(x) = g(f(x))$, so $F'(x) = g'(f(x))\,f'(x)$. Thus, $F'(2) = g'(3)(-3) = (4)(-3) = -12$.

58. $h = f(g(x))$, so $h'(0) = f'(g(0))\,g'(0) = f'(5)\cdot 3 = -2\cdot 3 = -6$.

59. Let $g(x) = x^2 + 1$. Then $F(x) = f(g(x))$. Next, $F'(x) = f'(g(x))g'(x)$ and $F'(1) = f'(2)(2x) = (3)(2) = 6$.

60. No. Let $F(x) = f(f(x))$. Then $F'(x) = f'(f(x))f'(x)$. Then let $f(x) = x^2$. $f(f(x)) = f(x^2) = x^4$ and $F'(x) = 4x^3$, but $f'(x) = 2x$, so $[f'(x)]^2 = 4x^2 \neq 4x^3$.

61. No. Suppose $h = g(f(x))$. Let $f(x) = x$ and $g(x) = x^2$. Then $h = g(f(x)) = g(x) = x^2$ and $h'(x) = 2x \neq g'(f'(x)) = g'(1) = 2(1) = 2$.

62. $h = f(g(x))$, so $h' = f'(g(x))g'(x)$ and $f' \circ g = f'(g(x))$, but $(f' \circ g)g' = f'(g(x))g'(x)$.

63. $f(x) = (1-x)(x^2-1)^2$, so $f'(x) = (1-x)2(x^2-1)(2x) + (-1)(x^2-1)^2 = (x^2-1)(4x - 4x^2 - x^2 + 1) = (x^2-1)(-5x^2+4x+1)$. Therefore, the slope of the tangent line at $(2, -9)$ is $f'(2) = [(2)^2 - 1][-5(2)^2 + 4(2) + 1] = -33$. Then an equation of the line is $y + 9 = -33(x-2)$, or $y = -33x + 57$.

64. $f(x) = \left(\frac{x+1}{x-1}\right)^2$, so $f'(x) = 2\left(\frac{x+1}{x-1}\right)\left[\frac{(x-1)(1)-(x+1)(1)}{(x-1)^2}\right] = 2\left(\frac{x+1}{x-1}\right)\left[-\frac{2}{(x-1)^2}\right]$. The slope of the tangent line at $x = 3$ is $f'(3) = 2\left(\frac{4}{2}\right)\left(-\frac{2}{4}\right) = -2$, so an equation is $y - 4 = -2(x-3)$, or $y = -2x + 10$.

65. $f(x) = x\sqrt{2x^2+7}$, so $f'(x) = \sqrt{2x^2+7} + x\left(\frac{1}{2}\right)(2x^2+7)^{-1/2}(4x)$. The slope of the tangent line at $x = 3$ is $f'(3) = \sqrt{25} + \left(\frac{3}{2}\right)(25)^{-1/2}(12) = \frac{43}{5}$, so an equation is $y - 15 = \frac{43}{5}(x-3)$, or $y = \frac{43}{5}x - \frac{54}{5}$.

66. $f(x) = \frac{8}{\sqrt{x^2+6x}} = 8(x^2+6x)^{-1/2}$, so $f'(x) = -\frac{1}{2}(8)(x^2+6x)^{-3/2}(2x+6) = -4(2x+6)(x^2+6x)^{-3/2}$. Therefore, the slope of the tangent line at $(2, 2)$ is $f'(2) = -4(10)(4+12)^{-3/2} = -4(10)(16)^{-3/2} = -40\left(\frac{1}{64}\right) = -\frac{5}{8}$, and an equation is $y - 2 = -\frac{5}{8}(x-2)$, or $y = -\frac{5}{8}x + \frac{13}{4}$.

67. $N(t) = (60+2t)^{2/3}$, so $N'(t) = \frac{2}{3}(60+2t)^{-1/3}\frac{d}{dt}(60+2t) = \frac{4}{3}(60+2t)^{-1/3}$. The rate of increase at the end of the second week is $N'(2) = \frac{4}{3}(64)^{-1/3} = \frac{1}{3}$, or $\frac{1}{3}$ million/week. At the end of the 12th week, $N'(12) = \frac{4}{3}(84)^{-1/3} \approx 0.3$, or 0.3 million/week. The number of viewers in the 2nd week is $N(2) = (60+4)^{2/3} = 16$, or 16 million, and the number of viewers in the 24th week is $N(24) = (60+48)^{2/3} \approx 22.7$, or approximately 22.7 million.

68. **a.** The number of gigabytes of information being created monthly at the beginning of 2008 was $f(0) = 400\left(\frac{0}{12}+1\right)^{1.09} = 400$ billion.

b. $f'(t) = 400(1.09)\left(\frac{t}{12}+1\right)^{0.09}\left(\frac{1}{12}\right) = \frac{109}{3}\left(\frac{t}{12}+1\right)^{0.09}$, so the rate at which the rate of digital information creation was changing at the beginning of 2010 was approximately $f'(24) = \frac{109}{3}\left(\frac{24}{12}+1\right)^{0.09} \approx 40.1094$, or approximately 40.109 billion gigabytes per month.

69. $N(t) = -0.05(t+1.1)^{2.2} + 0.7t + 0.9$, so in 2008, the cumulative number of jobs that were outsourced was changing at the rate of $N'(3) = \left[-0.05(2.2)(t+1.1)^{1.2} + 0.7\right]_{t=3} = -0.05(2.2)(3+1.1)^{1.2} + 0.7 \approx 0.101956$, or approximately 102,000 per year.

70. $A'(t) = 699\dfrac{d}{dt}(t+1)^{-0.94} = -657.06(t+1)^{-1.94}$. At the beginning of 2002, $A'(0) = -657.06$, so it is falling at the rate of \$657.06 per year. At the beginning of 2012, $A'(10) = -6.27$, so it is falling at the rate of \$6.27 per year.

71. a. $P(1) = \dfrac{100}{(1+0.14)^{9.2}} \approx 30.0$ and $P(2) = \dfrac{100}{(1+0.28)^{9.2}} \approx 10.3$. The probability of survival at the moment of diagnosis is 100%, the probability of survival 1 year after diagnosis is approximately 30%, and the probability after 2 years is approximately 10.3%.

b. $P'(t) = \dfrac{d}{dt}\left[100(1+0.14t)^{-9.2}\right] = 100(-9.2)(1+0.14t)^{-10.2}(0.14) = -\dfrac{920 \cdot 0.14}{(1+0.14t)^{10.2}} = -\dfrac{128.8}{(1+0.14t)^{10.2}}$.

Thus, $P'(1) = -\dfrac{128.8}{(1+0.14)^{10.2}} \approx -33.84$ and $P'(2) = -\dfrac{128.8}{(1+0.28)^{10.2}} \approx -10.38$. After 1 year, the probability of survival is dropping at the rate of approximately 34% per year, and after 2 years, it is dropping at approximately 10.4% per year.

72. $f(t) = 10.72(0.9t+10)^{0.3}$. The rate of change at any time t is given by $f'(t) = 10.72(0.3)(0.9t+10)^{-0.7}(0.9) = 2.8944(0.9t+10)^{-0.7}$. At the beginning of 2000, we find $f'(0) = 2.8944(10)^{-0.7} \approx 0.5775$, or 0.6%/yr. At the beginning of 2015, we have $f'(15) = 2.8944(13.5+10)^{-0.7} \approx 0.3175$, or 0.3%/yr. The percent of the population of Americans age 55 or over in 2015 is $f(15) = 10.72(13.5+10)^{0.3} \approx 27.64$, or 27.6%.

73. $C(t) = 0.01\left(0.2t^2 + 4t + 64\right)^{2/3}$.

a. $C'(t) = 0.01\left(\frac{2}{3}\right)\left(0.2t^2+4t+64\right)^{-1/3}\dfrac{d}{dt}\left(0.2t^2+4t+64\right)$

$= (0.01)(0.667)(0.4t+4)\left(0.2t^2+4t+64\right)^{-1/3} \approx 0.027(0.1t+1)\left(0.2t^2+4t+64\right)^{-1/3}$.

b. $C'(5) = 0.027(0.5+1)[0.2(25)+4(5)+64]^{-1/3} \approx 0.009$, or 0.009 parts per million per year.

74. $N(t) = -\dfrac{20{,}000}{\sqrt{1+0.2t}} + 21{,}000$, so $N'(t) = -20{,}000\left(-\frac{1}{2}\right)(1+0.2t)^{-3/2}(0.2) = \dfrac{2000}{(1+0.2t)^{3/2}}$.

$N'(1) = \dfrac{2000}{[1+0.2(1)]^{3/2}} \approx 1521$, or 1521 students/yr. $N'(5) = \dfrac{2000}{[1+0.2(5)]^{3/2}} \approx 707$, or 707 students/yr.

75. a. $A(t) = 0.03t^3(t-7)^4 + 60.2$, so $A'(t) = 0.03\left[3t^2(t-7)^4 + t^3(4)(t-7)^3\right] = 0.03t^2(t-7)^3[3(t-7)+4t] = 0.21t^2(t-3)(t-7)^3$.

b. $A'(1) = 0.21(-2)(-6)^3 = 90.72$, $A'(3) = 0$, and $A'(4) = 0.21(16)(1)(-3)^3 = -90.72$. The amount of pollutant is increasing at the rate of 90.72 units/hr at 8 a.m. The rate of change is 0 units/hr at 10 a.m. and -90.72 units/hr at 11 a.m.

76. $N(x) = \left(10{,}000 - 40x - 0.02x^2\right)^{1/2}$, so $N'(x) = \frac{1}{2}\left(10{,}000 - 40x - 0.02x^2\right)^{-1/2}(-40 - 0.04x)$. Thus, $N'(10) = \frac{1}{2}(10{,}000 - 400 - 2)^{-1/2}(-40 - 0.4) = \frac{1}{2}(9598)^{-1/2}(-40.4) \approx -0.2062$, representing a 20.6% drop in consumption per 10% tax increase. Similarly, $N'(100) \approx -0.2889$, a drop of 28.9%, and $N'(150) \approx -0.3860$, a drop of 38.6%.

77. $P(t) = \dfrac{300\sqrt{\frac{1}{2}t^2+2t+25}}{t+25} = \dfrac{300\left(\frac{1}{2}t^2+2t+25\right)^{1/2}}{t+25}$, so

$$P'(t) = 300\left[\frac{(t+25)\frac{1}{2}\left(\frac{1}{2}t^2+2t+25\right)^{-1/2}(t+2) - \left(\frac{1}{2}t^2+2t+25\right)^{1/2}(1)}{(t+25)^2}\right]$$

$$= 300\left[\frac{\left(\frac{1}{2}t^2+2t+25\right)^{-1/2}\left[(t+25)(t+2) - 2\left(\frac{1}{2}t^2+2t+25\right)\right]}{2(t+25)^2}\right] = \frac{3450t}{(t+25)^2\sqrt{\frac{1}{2}t^2+2t+25}}.$$

Ten seconds into the run, the athlete's pulse rate is increasing at $P'(10) = \dfrac{3450(10)}{(35)^2\sqrt{50+20+25}} \approx 2.9$, or approximately 2.9 beats per minute per second. Sixty seconds into the run, it is increasing at $P'(60) = \dfrac{3450(60)}{(85)^2\sqrt{1800+120+25}} \approx 0.65$, or approximately 0.7 beats per minute per second. Two minutes into the run, it is increasing at $P'(120) = \dfrac{3450(120)}{(145)^2\sqrt{7200+240+25}} \approx 0.23$, or approximately 0.2 beats per minute per second. The pulse rate two minutes into the run is given by $P(120) = \dfrac{300\sqrt{7200+240+25}}{120+25} \approx 178.8$, or approximately 179 beats per minute.

78. a. $\dfrac{dT}{dn} = A(n-b)^{1/2} + An\left(\frac{1}{2}\right)(n-b)^{-1/2} = \frac{1}{2}A(n-b)^{-1/2}[2(n-b)+n] = \dfrac{(3n-2b)A}{2\sqrt{n-b}}$, and this gives the rate of change of the learning time with respect to the length of the list.

b. $\dfrac{dT}{dn} = \dfrac{(3n-8)4}{2\sqrt{n-4}}$ if $A=4$ and $b=4$. $f'(13) = \dfrac{(39-8)4}{2\sqrt{9}} \approx 20.7$, or approximately 21 units of time per unit increase in the word list. $f'(29) = \dfrac{(87-8)4}{2\sqrt{25}} \approx 31.6$, or approximately 32 units of time per unit increase in the word list.

79. The area is given by $A = \pi r^2$. The rate at which the area is increasing is given by dA/dt, that is, $\dfrac{dA}{dt} = \dfrac{d}{dt}\left(\pi r^2\right) = \dfrac{d}{dt}\left(\pi r^2\right)\dfrac{dr}{dt} = 2\pi r\dfrac{dr}{dt}$. If $r=40$ and $dr/dt=2$, then $\dfrac{dA}{dt} = 2\pi(40)(2) = 160\pi$, that is, it is increasing at the rate of 160π, or approximately 503 ft^2/sec.

80. $g(t) = 0.5t^2\left(t^2+10\right)^{-1}$, so

$g'(t) = 0.5(2t)\left(t^2+10\right)^{-1} + 0.5t^2(-1)\left(t^2+10\right)^{-2}(2t) = t\left(t^2+10\right)^{-2}\left[\left(t^2+10\right)-t^2\right] = \dfrac{10t}{\left(t^2+10\right)^2}$. In particular, $g'(5) = \dfrac{50}{(35)^2} \approx 0.04$ cm/yr.

81. $N(x) = 1.42x$ and $x(t) = \dfrac{7t^2 + 140t + 700}{3t^2 + 80t + 550}$. The number of construction jobs as a function of time is $n(t) = N(x(t))$. Using the Chain Rule,

$$n'(t) = \frac{dN}{dx} \cdot \frac{dx}{dt} = 1.42\frac{dx}{dt} = (1.42)\left[\frac{\left(3t^2 + 80t + 550\right)(14t + 140) - \left(7t^2 + 140t + 700\right)(6t + 80)}{\left(3t^2 + 80t + 550\right)^2}\right]$$

$$= \frac{1.42\left(140t^2 + 3500t + 21000\right)}{\left(3t^2 + 80t + 550\right)^2}.$$

$n'(12) = \dfrac{1.42\left[140(12)^2 + 3500(12) + 21000\right]}{\left[3(12)^2 + 80(12) + 550\right]^2} \approx 0.0313115$, or approximately 31,312 jobs/year/month.

82. $r(t) = \frac{10}{81}t^3 - \frac{10}{3}t^2 + \frac{200}{9}t + 56.2$ and $R(r) = -\frac{3}{5000}r^3 + \frac{9}{50}r^2$.

a. The rate of change of Wonderland's occupancy rate with respect to time is given by $r'(t) = \frac{10}{27}t^2 - \frac{20}{3}t + \frac{200}{9}$.

b. The rate of change of Wonderlands' monthly revenue with respect to the occupancy rate is given by $R'(r) = -\frac{9}{5000}r^2 + \frac{9}{25}r$.

c. When $t = 0$, $r(0) = \frac{10}{81}(0)^3 - \frac{10}{3}(0)^2 + \frac{200}{9}(0) + 56.2 = 56.2$, $r'(0) = \frac{10}{27}(0)^2 - \frac{20}{3}(0) + \frac{200}{9} = \frac{200}{9}$, $R'(56.2) = -\frac{9}{5000}(56.2)^2 + \frac{9}{25}(56.2) \approx 14.55$, and $R'(r(0))\,r'(0) \approx 14.55\left(\frac{200}{9}\right) \approx 323.3$. The rate of change of Wonderland's monthly revenue with respect to time at the beginning of January is approximately \$323,300/month. Next, when $t = 6$, $r(6) = \frac{10}{81}(6)^3 - \frac{10}{3}(6)^2 + \frac{200}{9}(6) + 56.2 = 96.2$, $r'(6) = \frac{30}{81}(6)^2 - \frac{20}{3}(6) + \frac{200}{9} = -4.44$, $R'(96.2) = -\frac{9}{5000}(96.2)^2 + \frac{9}{25}(96.2) \approx 17.97$, and $R'(r(6))\,r'(6) \approx (17.97)(-4.44) \approx -79.79$, so the rate of change of Wonderland's monthly revenue with respect to time at the beginning of July is approximately $-$\$79,790/month; that is, the revenue is decreasing at the rate of \$79,790/month.

83. $x = f(p) = \frac{100}{9}\sqrt{810{,}000 - p^2}$ and $p(t) = \dfrac{400}{1 + \frac{1}{8}\sqrt{t}} + 200$. We want to find $\dfrac{dx}{dt} = \dfrac{dx}{dp} \cdot \dfrac{dp}{dt}$. But $\dfrac{dx}{dp} = \dfrac{100}{9}\left(\dfrac{1}{2}\right)\left(810{,}000 - p^2\right)^{-1/2}(-2p) = -\dfrac{100p}{9\sqrt{810{,}000 - p^2}}$ and

$$\frac{dp}{dt} = 400\frac{d}{dt}\left(1 + \tfrac{1}{8}t^{1/2}\right)^{-1} + \frac{d}{dt}(200) = -400\left(1 + \tfrac{1}{8}t^{1/2}\right)^{-2}\left(\tfrac{1}{8}\right)\left(\tfrac{1}{2}t^{-1/2}\right) = -\frac{25}{\sqrt{t}\left(1 + \frac{1}{8}\sqrt{t}\right)^2},$$

so $\dfrac{dx}{dt} = \dfrac{2500p}{9\sqrt{t}\sqrt{810{,}000 - p^2}\left(1 + \frac{1}{8}\sqrt{t}\right)^2}$ and when $t = 16$, $p = \dfrac{400}{1 + \frac{1}{8}\sqrt{16}} + 200 = \frac{1400}{3}$. Therefore,

$\dfrac{dx}{dt} = \dfrac{2500\left(\frac{1400}{3}\right)}{9\sqrt{16}\sqrt{810{,}000 - \left(\frac{1400}{3}\right)^2}\left(1 + \frac{1}{8}\sqrt{16}\right)^2} \approx 18.7$. The quantity demanded will be changing at the rate of approximately 19 computers/month.

84. $p = f(t) = 50\left(\dfrac{t^2+2t+4}{t^2+4t+8}\right)$ and $R(p) = 1000\left(\dfrac{p+4}{p+2}\right)$. We want $\dfrac{dR}{dt} = \dfrac{dR}{dp}\cdot\dfrac{dp}{dt}$. Now $\dfrac{dR}{dp} = 1000\left[\dfrac{(p+2)(1)-(p+4)(1)}{(p+2)^2}\right] = -\dfrac{2000}{(p+2)^2}$ and $\dfrac{dp}{dt} = 50\left[\dfrac{\left(t^2+4t+8\right)(2t+2)-\left(t^2+2t+4\right)(2t+4)}{\left(t^2+4t+8\right)^2}\right] = \dfrac{100t(t+4)}{\left(t^2+4t+8\right)^2}$. When $t=2$, $p = 50\left(\dfrac{4+4+4}{4+8+8}\right) = 30$, and $\dfrac{dR}{dt} = -\dfrac{2000}{(p+2)^2}\cdot\dfrac{100t(t+4)}{\left(t^2+4t+8\right)^2}\bigg|_{t=2} = -\dfrac{2000}{(32)^2}\cdot\dfrac{100(2)(6)}{(4+8+8)^2} \approx -5.86$; that is, the passage will decrease at the rate of approximately \$5.86 per passenger per year.

85. True. This is just the statement of the Chain Rule.

86. True. $\frac{d}{dx}\left[f(cx)\right] = f'(cx)\frac{d}{dx}(cx) = f'(cx)\cdot c$.

87. True. $\frac{d}{dx}\sqrt{f(x)} = \frac{d}{dx}\left[f(x)\right]^{1/2} = \frac{1}{2}\left[f(x)\right]^{-1/2} f'(x) = \dfrac{f'(x)}{2\sqrt{f(x)}}$.

88. False. Let $f(x) = x$. Then $f\left(\dfrac{1}{x}\right) = \dfrac{1}{x}$ and so $f'(x) = -\dfrac{1}{x^2}$. But $f'(x) = 1$, so $f'\left(\dfrac{1}{x}\right) = 1$.

89. Let $f(x) = x^{1/n}$ so that $\left[f(x)\right]^n = x$. Differentiating both sides with respect to x, we get $n\left[f(x)\right]^{n-1} f'(x) = 1$, so $f'(x) = \dfrac{1}{n\left[f(x)\right]^{n-1}} = \dfrac{1}{n\left[x^{1/n}\right]^{n-1}} = \dfrac{1}{nx^{1-(1/n)}} = \dfrac{1}{n}x^{(1/n)-1}$, as was to be shown.

90. Let $f(x) = x^r = x^{m/n} = (x^m)^{1/n}$. Then $\left[f(x)\right]^n = x^m$. Therefore,

$$n\left[f(x)\right]^{n-1} f'(x) = \frac{m}{n}\left[f(x)\right]^{-n+1}x^{m-1} = \frac{m}{n}\left(x^{m/n}\right)^{-n+1}x^{m-1} = \frac{m}{n}x^{[m(-n+1)/n]+m-1}$$
$$= \frac{m}{n}x^{(m-n)/n} = \frac{m}{n}x^{(m/n)-1} = rx^{r-1}.$$

Using Technology page 198

1. 0.5774

2. 1.4364

3. 0.9390

4. 3.9051

5. −4.9498

6. 0.1056

7. a. Using the numerical derivative operation, we find that $N'(0) = 5.41450$, so the rate of change of the number of people watching TV on mobile phones at the beginning of 2007 is approximately 5.415 million/year.

b. $N'(4) \approx 2.5136$, so the corresponding rate of change at the beginning of 2011 is approximately 2.5136 million/year.

8. a. 43.6 million

b. 0.432745 million/year

3.4 Marginal Functions in Economics

Concept Questions page 209

1. **a.** The marginal cost function is the derivative of the cost function.

   b. The average cost function is equal to the total cost function divided by the total number of the commodity produced.

   c. The marginal average cost function is the derivative of the average cost function.

   d. The marginal revenue function is the derivative of the revenue function.

   e. The marginal profit function is the derivative of the profit function.

2. **a.** The elasticity of demand at a price P is $E(p) = -\dfrac{pf'(p)}{f(p)}$, where f is the demand function $x = f(p)$.

   b. The elasticity of demand is elastic if $E(p) > 1$, unitary if $E(p) = 1$, and inelastic if $E(p) < 1$. If $E(p) > 1$, then an increase in the unit price will cause the revenue to decrease, whereas a decrease in the unit price will cause the revenue to increase. If $E(p) = 1$, then an increase in the unit price will have no effect on the revenue. If $E(p) < 1$, then an increase in the unit price will cause the revenue to increase, and a decrease in the unit price will cause the revenue to decrease.

3. $P(x) = R(x) - C(x)$, so $P'(x) = R'(x) - C'(x)$. Using the given information, we find $P'(500) = R'(500) - C'(500) = 3 - 2.8 = 0.2$. Thus, if the level of production is 500, then the marginal profit is \$0.20 per unit. This tells us that the proprietor should increase production in order to increase the company's profit.

Exercises page 209

1. **a.** $C(x)$ is always increasing because as x, the number of units produced, increases, the amount of money that must be spent on production also increases.

   b. This occurs at $x = 4$, a production level of 4000. You can see this by looking at the slopes of the tangent lines for x less than, equal to, and a little larger then $x = 4$.

2. **a.** If very few units of the commodity are produced then the cost per unit of production will be very large. If x is very large, the typical total cost is very large due to overtime, excessive cost of raw material, machinery breakdown, etc., so that $A(x)$ is very large as well; in fact, for a typical total cost function $C(x)$, $A(x)$ ultimately grows faster than x, that is, $\lim_{x\to\infty} \dfrac{C(x)}{x} = \infty$.

   b. The average cost per unit is smallest (\$$y_0$) when the level of production is x_0 units.

3. **a.** The actual cost incurred in the production of the 1001st disc is given by

   $$C(1001) - C(1000) = \left[2000 + 2(1001) - 0.0001(1001)^2\right] - \left[2000 + 2(1000) - 0.0001(1000)^2\right]$$
   $$= 3901.7999 - 3900 = 1.7999, \text{ or approximately } \$1.80.$$

   The actual cost incurred in the production of the 2001st disc is given by

   $$C(2001) - C(2000) = \left[2000 + 2(2001) - 0.0001(2001)^2\right] - \left[2000 + 2(2000) - 0.0001(2000)^2\right]$$
   $$= 5601.5999 - 5600 = 1.5999, \text{ or approximately } \$1.60.$$

   b. The marginal cost is $C'(x) = 2 - 0.0002x$. In particular, $C'(1000) = 2 - 0.0002(1000) = 1.80$ and $C'(2000) = 2 - 0.0002(2000) = 1.60$.

4. a. $C(101) - C(100) = \left[0.0002(101)^3 - 0.06(101)^2 + 120(101) + 5000\right] - \left[0.0002(100)^3 - 0.06(100)^2 + 120(100) + 5000\right] \approx 114$, or approximately \$114.

Similarly, we find $C(201) - C(200) \approx \$120.06$ and $C(301) - C(300) \approx \$138.12$.

b. We compute $C'(x) = 0.0006x^2 - 0.12x + 120$. Thus, the required quantities are $C'(100) = 0.0006(100)^2 - 0.12(100) + 120 = 114$, or \$114; $C'(200) = 0.0006(200)^2 - 0.12(200) + 120 = 120$, or \$120; and $C'(300) = 0.0006(300)^2 - 0.12(300) + 120 = 138$, or \$138.

5. a. $\overline{C}(x) = \dfrac{C(x)}{x} = \dfrac{100x + 200{,}000}{x} = 100 + \dfrac{200{,}000}{x}$.

b. $\overline{C}'(x) = \dfrac{d}{dx}(100) + \dfrac{d}{dx}\left(200{,}000x^{-1}\right) = -200{,}000x^{-2} = -\dfrac{200{,}000}{x^2}$.

c. $\lim\limits_{x\to\infty} \overline{C}(x) = \lim\limits_{x\to\infty}\left(100 + \dfrac{200{,}000}{x}\right) = 100$. This says that the average cost approaches \$100 per unit if the production level is very high.

6. a. $\overline{C}(x) = \dfrac{C(x)}{x} = \dfrac{5000}{x} + 2$.

b. $\overline{C}'(x) = -\dfrac{5000}{x^2}$.

c. Because the marginal average cost function is negative for $x > 0$, the rate of change of the average cost function is negative for all $x > 0$.

7. $\overline{C}(x) = \dfrac{C(x)}{x} = \dfrac{2000 + 2x - 0.0001x^2}{x} = \dfrac{2000}{x} + 2 - 0.0001x$, so $\overline{C}'(x) = -\dfrac{2000}{x^2} + 0 - 0.0001 = -\dfrac{2000}{x^2} - 0.0001$.

8. $\overline{C}(x) = \dfrac{C(x)}{x} = \dfrac{0.0002x^3 - 0.06x^2 + 120x + 5000}{x} = 0.0002x^2 - 0.06x + 120 + \dfrac{5000}{x}$, so $\overline{C}'(x) = 0.0004x - 0.06 - \dfrac{5000}{x^2}$.

9. a. $R'(x) = \dfrac{d}{dx}\left(8000x - 100x^2\right) = 8000 - 200x$.

b. $R'(39) = 8000 - 200(39) = 200$, $R'(40) = 8000 - 200(40) = 0$, and $R'(41) = 8000 - 200(41) = -200$.

c. This suggests the total revenue is maximized if the price charged per passenger is \$40.

10. a. $R(x) = px = x(-0.04x + 800) = -0.04x^2 + 800x$.

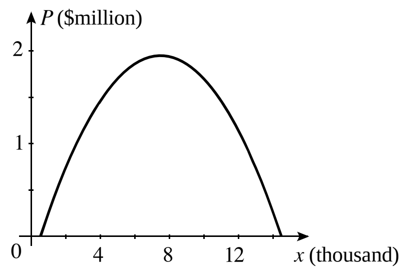
b. $R'(x) = -0.08x + 800$.

c. $R'(5000) = -0.08(5000) + 800 = 400$. This says that when the level of production is 5000 units, the production of the next speaker system will bring an additional revenue of \$400.

11. a. $P(x) = R(x) - C(x) = \left(-0.04x^2 + 800x\right) - (200x + 300{,}000) = -0.04x^2 + 600x - 300{,}000$.

b. $P'(x) = -0.08x + 600$.

c. $P'(5000) = -0.08(5000) + 600 = 200$ and $P'(8000) = -0.08(8000) + 600 = -40$.

d. 

The profit realized by the company increases as production increases, peaking at a production level of 7500 units. Beyond this level of production, the profit begins to fall.

12. a. $P(x) = -10x^2 + 1760x - 50{,}000$. To find the actual profit realized from renting the 51st unit, assuming that 50 units have already been rented, we calculate

$$P(51) - P(50) = \left[-10(51)^2 + 1760(51) - 50{,}000\right] - \left[-10(50)^2 + 1760(50) - 50{,}000\right]$$
$$= -26{,}010 + 89{,}760 - 50{,}000 + 25{,}000 - 88{,}000 + 50{,}000 = 750, \text{ or } \$750.$$

b. The marginal profit is given by $P'(x) = -20x + 1760$. When $x = 50$, $P'(50) = -20(50) + 1760 = 760$, or \$760.

13. a. The revenue function is $R(x) = px = (600 - 0.05x)x = 600x - 0.05x^2$ and the profit function is

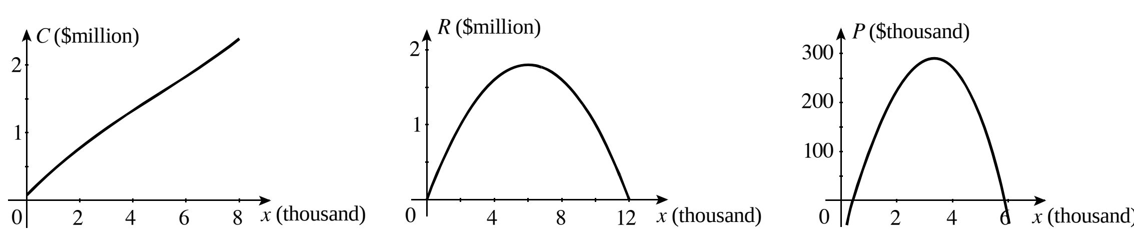
$$P(x) = R(x) - C(x) = \left(600x - 0.05x^2\right) - \left(0.000002x^3 - 0.03x^2 + 400x + 80{,}000\right)$$
$$= -0.000002x^3 - 0.02x^2 + 200x - 80{,}000.$$

b. $C'(x) = \dfrac{d}{dx}\left(0.000002x^3 - 0.03x^2 + 400x + 80{,}000\right) = 0.000006x^2 - 0.06x + 400$,

$R'(x) = \dfrac{d}{dx}\left(600x - 0.05x^2\right) = 600 - 0.1x$, and

$P'(x) = \dfrac{d}{dx}\left(-0.000002x^3 - 0.02x^2 + 200x - 80{,}000\right) = -0.000006x^2 - 0.04x + 200$.

c. $C'(2000) = 0.000006(2000)^2 - 0.06(2000) + 400 = 304$, and this says that at a production level of 2000 units, the cost for producing the 2001st unit is \$304. $R'(2000) = 600 - 0.1(2000) = 400$, and this says that the revenue realized in selling the 2001st unit is \$400. $P'(2000) = R'(2000) - C'(2000) = 400 - 304 = 96$, and this says that the revenue realized in selling the 2001st unit is \$96.

d. 

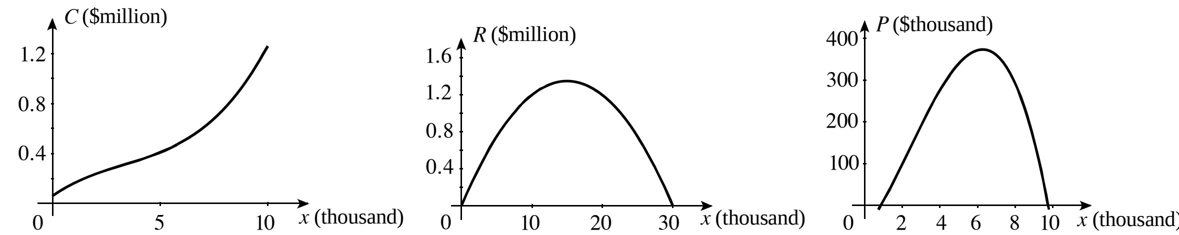
14. a. $R(x) = xp(x) = -0.006x^2 + 180x$ and

$$P(x) = R(x) - C(x) = \left(-0.006x^2 + 180x\right) - \left(0.000002x^3 - 0.02x^2 + 120x + 60{,}000\right)$$
$$= -0.000002x^3 + 0.014x^2 + 60x - 60{,}000.$$

b. $C'(x) = 0.000006x^2 - 0.04x + 120$, $R'(x) = -0.012x + 180$, and $P'(x) = -0.000006x^2 + 0.028x + 60$.

c. $C'(2000) = 0.000006(2000)^2 - 0.04(2000) + 120 = 64$, $R'(2000) = -0.012(2000) + 180 = 156$, and $P'(2000) = -0.000006(2000)^2 + 0.028(2000) + 60 = 92$.

d.



15. a. $\overline{C}(x) = \dfrac{C(x)}{x} = \dfrac{0.000002x^3 - 0.03x^2 + 400x + 80{,}000}{x} = 0.000002x^2 - 0.03x + 400 + \dfrac{80{,}000}{x}$.

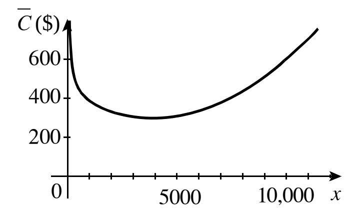
b. $\overline{C}'(x) = 0.000004x - 0.03 - \dfrac{80{,}000}{x^2}$.

c. $\overline{C}'(5000) = 0.000004(5000) - 0.03 - \dfrac{80{,}000}{5000^2} \approx -0.0132$, and this says that at a production level of 5000 units, the average cost of production is dropping at the rate of approximately a penny per unit.

$\overline{C}'(10{,}000) = 0.000004(10000) - 0.03 - \dfrac{80{,}000}{10{,}000^2} \approx 0.0092$,

and this says that, at a production level of 10,000 units, the average cost of production is increasing at the rate of approximately a penny per unit.

d.



16. a. $C(x) = 0.000002x^3 - 0.02x^2 + 120x + 60{,}000$, so $\overline{C}(x) = 0.000002x^2 - 0.02x + 120 + \dfrac{60{,}000}{x}$.

b. The marginal average cost function is given by $\overline{C}'(x) = 0.000004x - 0.02 - \dfrac{60{,}000}{x^2}$.

c. $\overline{C}'(5000) = 0.000004(5000) - 0.02 - \dfrac{60{,}000}{(5000)^2}$

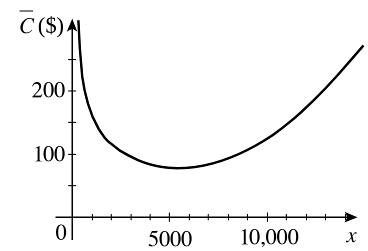
$= 0.02 - 0.02 - 0.0024 = -0.0024$ and

$\overline{C}'(10{,}000) = 0.000004(10{,}000) - 0.02 - \dfrac{60{,}000}{(10000)^2}$

$= 0.04 - 0.02 - 0.0006 = 0.0194$.

We conclude that the average cost is decreasing when 5000 TV sets are produced and increasing when 10,000 units are produced.

d.



17. a. $R(x) = px = \dfrac{50x}{0.01x^2 + 1}$.

b. $R'(x) = \dfrac{(0.01x^2 + 1)\,50 - 50x\,(0.02x)}{(0.01x^2 + 1)^2} = \dfrac{50 - 0.5x^2}{(0.01x^2 + 1)^2}$.

c. $R'(2) = \dfrac{50 - 0.5(4)}{[0.01(4) + 1]^2} \approx 44.379$. This result says that at a sales level of 2000 units, the revenue increases at the rate of approximately $44,379 per 1000 units.

18. $\dfrac{dC}{dx} = \dfrac{d}{dx}(0.712x + 95.05) = 0.712$.

19. $C(x) = 0.873x^{1.1} + 20.34$, so $C'(x) = 0.873(1.1)x^{0.1}$. $C'(10) = 0.873(1.1)(10)^{0.1} = 1.21$.

20. $\dfrac{dS}{dx} = \dfrac{d}{dx}\left[x - C(x)\right] = 1 - \dfrac{dC}{dx}$.

21. The consumption function is given by $C(x) = 0.712x + 95.05$. The marginal propensity to consume is given by $\frac{dC}{dx} = 0.712$. The marginal propensity to save is given by $\frac{dS}{dx} = 1 - \frac{dC}{dx} = 1 - 0.712 = 0.288$.

22. Here $C(x) = 0.873x^{1.1} + 20.34$, so $C'(x) = 0.9603x^{0.1}$ and $\frac{dS}{dx} = 1 - \frac{dC}{dx} = 1 - 0.9603x^{0.1}$. When $x = 10$, we have $\frac{dS}{dx} = 1 - 0.9603(10)^{0.1} \approx -0.209$, or approximately $-\$0.209$ billion per billion dollars.

23. $f(x) = 2x^2 + x + 1$, so $f'(x) = 4x + 1$. The percentage rate of change of f at $x = 2$ is $100\dfrac{f'(2)}{f(2)} = 100\left[\dfrac{4x+1}{2x^2+x+1}\right]_{x=2} = 100\left(\dfrac{8+1}{8+2+1}\right) \approx 81.82$ (percent per unit change in x).

24. $f(x) = \left(2x^2+7\right)^{1/2}$, so $f'(x) = \frac{1}{2}\left(2x^2+7\right)^{-1/2}(4x)$. Then $f(3) = 25^{1/2}$ and $f'(3) = 6(25)^{-1/2}$, so the percentage rate of change of f at $x = 3$ is $100\dfrac{f'(3)}{f(3)} = 100\cdot\dfrac{6(25)^{-1/2}}{25^{1/2}} = \dfrac{100\cdot 6}{25} = 24$ (percent per unit change in x).

25. $f(x) = \dfrac{x+1}{x^3+x+1}$, so $f'(x) = \dfrac{\left(x^3+x+1\right)(1) - (x+1)\left(3x^2+1\right)}{\left(x^3+x+1\right)^2}$. Then $f(2) = \frac{3}{11}$ and $f'(2) = -\frac{28}{121}$, so the percentage rate of change of f at $x = 2$ is $100\cdot\dfrac{-\frac{28}{121}}{\frac{3}{11}} \approx -84.85$ (percent per unit change in x).

26. $f(x) = \left(\dfrac{x}{x^2+3x+12}\right)^{3/2}$, so

$$f'(x) = \frac{3}{2}\left(\frac{x}{x^2+3x+12}\right)^{1/2}\frac{d}{dx}\left(\frac{x}{x^2+3x+12}\right)$$
$$= \frac{3}{2}\left(\frac{x}{x^2+3x+12}\right)^{1/2}\left[\frac{\left(x^2+3x-12\right)(1) - x(2x+3)}{\left(x^2+3x+12\right)^2}\right].$$

Then $f(1) = \left(\frac{1}{16}\right)^{3/2} = \frac{1}{64}$ and $f'(1) = \frac{3}{2}\left(\frac{1}{16}\right)^{1/2}\left(\frac{11}{256}\right) = \frac{33}{2048}$, so the percentage rate of change of f at $x = 1$ is $100\cdot\dfrac{\frac{33}{2048}}{\frac{1}{64}} = 103.125$ (percent per unit change in x).

27. Here $x = f(p) = -\frac{5}{4}p + 20$ and so $f'(p) = -\frac{5}{4}$. Therefore, $E(p) = -\dfrac{pf'(p)}{f(p)} = -\dfrac{p\left(-\frac{5}{4}\right)}{-\frac{5}{4}p+20} = \dfrac{5p}{80-5p}$. $E(10) = \dfrac{5(10)}{80-5(10)} = \dfrac{50}{30} = \dfrac{5}{3} > 1$, and so the demand is elastic.

28. $f(p) = -\frac{3}{2}p + 9$, so $f'(p) = -\frac{3}{2}$. Then the elasticity of demand is given by $E(p) = -\dfrac{pf'(p)}{f(p)} = -\dfrac{p\left(-\frac{3}{2}\right)}{-\frac{3}{2}p+9}$. Therefore, when $p = 2$, $E(2) = -\dfrac{2\left(-\frac{3}{2}\right)}{-\frac{3}{2}(2)+9} = \dfrac{3}{6} = \dfrac{1}{2} < 1$, and we conclude that the demand is inelastic at this price.

29. $f(p) = -\frac{1}{3}p + 20$, so $f'(p) = -\frac{1}{3}$. Then the elasticity of demand is given by $E(p) = -\dfrac{p\left(-\frac{1}{3}\right)}{-\frac{1}{3}p + 20}$, and

$E(30) = -\dfrac{30\left(-\frac{1}{3}\right)}{-\frac{1}{3}(30) + 20} = 1$, and we conclude that the demand is unitary at this price.

30. Solving the demand equation for x, we find $x^2 = 144 - p$, or $x = \sqrt{144 - p}$ (because x must be nonnegative). With $x = f(p) = (144 - p)^{1/2}$, we have $f'(p) = \frac{1}{2}(144 - p)^{-1/2}(-1) = -\dfrac{1}{2\sqrt{144 - p}}$. Therefore,

$$E(p) = -\frac{pf'(p)}{f(p)} = -\frac{p\left(-\frac{1}{2\sqrt{144-p}}\right)}{\sqrt{144 - p}} = \frac{p}{2(144 - p)}.$$

$E(96) = \frac{96}{2(48)} = 1$, and so the demand equation is unitary.

31. $x^2 = 169 - p$ and $f(p) = (169 - p)^{1/2}$. Next, $f'(p) = \frac{1}{2}(169 - p)^{-1/2}(-1) = -\frac{1}{2}(169 - p)^{-1/2}$. Then the elasticity of demand is given by $E(p) = -\dfrac{pf'(p)}{f(p)} = -\dfrac{p\left(-\frac{1}{2}\right)(169 - p)^{-1/2}}{(169 - p)^{1/2}} = \dfrac{\frac{1}{2}p}{169 - p}$. Therefore, when $p = 29$, $E(p) = \dfrac{\frac{1}{2}(29)}{169 - 29} = \dfrac{14.5}{140} \approx 0.104$. Because $E(p) < 1$, we conclude that demand is inelastic at this price.

32. $I(t) = -0.02t^3 + 0.4t^2 + 120$, so $I'(t) = -0.06t^2 + 0.8t$. Then $I(1) = -0.02 + 0.4 + 120 = 120.38$ and $I'(1) = -0.06 + 0.8 = 0.74$, so the annual percentage rate of inflation in the CPI of the country at the beginning of 2014 ($t = 1$) is $100 \cdot \dfrac{I'(1)}{I(1)} = 100 \cdot \dfrac{0.74}{120.38} \approx 0.6147$, or approximately 0.615%.

33. a. The percentage rate of change in per capita income in year t is

$$100\frac{C'(t)}{C(t)} = 100 \cdot \frac{\frac{d}{dt}\left[\frac{I(t)}{P(t)}\right]}{C(t)} = 100 \cdot \frac{P(t)\,I'(t) - I(t)\,P'(t)}{[P(t)]^2} \cdot \frac{P(t)}{I(t)} = 100 \cdot \frac{P(t)\,I'(t) - I(t)\,P'(t)}{P(t)\,I(t)}.$$

b. Here $I(t) = 10^9(300 + 12t)$ and $P(t) = 2 \times 10^7 e^{0.02t}$, so $I'(t) = 12 \times 10^9$ and $P'(t) = 4 \cdot 10^5 e^{0.02t}$. Therefore, the percentage rate of change in per capita income in year t is

$$100 \cdot \frac{2 \times 10^7 e^{0.02t}\left(12 \times 10^9\right) - 10^9(300 + 12t)\left(4 \cdot 10^5 e^{0.02t}\right)}{\left(2 \cdot 10^7 e^{0.02t}\right)10^9(300 + 12t)} = \frac{2400 \times 10^{16} e^{0.02t} - 48 \times 10^{16} e^{0.02t}(25 + t)}{24 \cdot 10^{16} e^{0.02t}(25 + t)} = \frac{50 - 2t}{25 + t}.$$

c. The percentage rate of change in per capita income 2 years from now is projected to be $\dfrac{50 - 2(2)}{25 + 2} = \dfrac{46}{27}$, or approximately 1.7%/yr.

34. The percentage growth rate at $t = a$ is $100 \cdot \dfrac{R'(a)}{R(a)} = \dfrac{100\left[f'(a) + g'(a)\right]}{f(a) + g(a)}$. Taking $f'(a) = 0.24$, $g'(a) = 0.30$, $f(a) = 3.2$, and $g(a) = 2.6$, we have $100 \cdot \dfrac{R'(a)}{R(a)} = \dfrac{100(0.24 + 0.30)}{3.2 + 2.6} \approx 9.31$. Thus, the percentage growth rate is approximately 9.31% per year.

35. $f(p) = \frac{1}{5}(225 - p^2)$, so $f'(p) = \frac{1}{5}(-2p) = -\frac{2}{5}p$. Then the elasticity of demand is given by

$$E(p) = -\frac{pf'(p)}{f(p)} = -\frac{p\left(-\frac{2}{5}p\right)}{\frac{1}{5}(225 - p^2)} = \frac{2p^2}{225 - p^2}.$$

a. When $p = 8$, $E(8) = \dfrac{2(64)}{225 - 64} \approx 0.8 < 1$ and the demand is inelastic. When $p = 10$,

$E(10) = \dfrac{2(100)}{225 - 100} = 1.6 > 1$ and the demand is elastic.

b. The demand is unitary when $E = 1$. Solving $\dfrac{2p^2}{225 - p^2} = 1$, we find $2p^2 = 225 - p^2$, $3p^2 = 225$, and $p \approx 8.66$. So the demand is unitary when $p \approx 8.66$.

c. Because demand is elastic when $p = 10$, lowering the unit price will cause the revenue to increase.

d. Because the demand is inelastic at $p = 8$, a slight increase in the unit price will cause the revenue to increase.

36. $f(p) = (144 - p)^{1/2}$, so $f'(p) = \frac{1}{2}(144 - p)^{-1/2}(-1)$. Then the elasticity of demand is given by

$$E(p) = -\frac{pf'(p)}{f(p)} = -\frac{p\left(-\frac{1}{2}\right)(144 - p)^{-1/2}}{(144 - p)^{1/2}} = \frac{p}{2(144 - p)}.$$

a. $E(63) = \dfrac{63}{2(144 - 63)} \approx 0.39$, $E(96) = \dfrac{96}{2(144 - 96)} = 1$, and $E(108) = \dfrac{108}{2(144 - 108)} = 1.5$.

b. At a unit price of \$63, a unit price increase of \$1 will result in a decrease of approximately 0.39% in demand as well as increased revenue. When the unit price is set at \$96, a price increase of \$1 will not cause any change in demand or revenue. When the price is set at \$108, a price increase of \$1 will cause a decrease of approximately 1.5% in demand as well as decreased revenue.

c. The demand is inelastic when $p = 63$, unitary when $p = 96$, and elastic when $p = 108$.

37. $f(p) = \frac{2}{3.}(36 - p^2)^{1/2}$. $f'(p) = \frac{2}{3}\left(\frac{1}{2}\right)(36 - p^2)^{-1/2}(-2p) = -\frac{2}{3}p(36 - p^2)^{-1/2}$. Then the elasticity of demand is given by $E(p) = -\dfrac{pf'(p)}{f(p)} = -\dfrac{-\frac{2}{3}p(36 - p^2)^{-1/2}p}{\frac{2}{3}(36 - p^2)^{1/2}} = \dfrac{p^2}{36 - p^2}$.

a. When $p = 2$, $E(2) = \dfrac{4}{36 - 4} = \dfrac{1}{8} < 1$, and we conclude that the demand is inelastic.

b. Because the demand is inelastic, the revenue will increase when the rental price is increased.

38. Here $x = f(p) = \sqrt{400 - 5p} = (400 - 5p)^{1/2}$. Therefore, $f'(p) = \frac{1}{2}(400 - 5p)^{-1/2}(-5) = -\dfrac{5}{2\sqrt{400 - 5p}}$,

and so $E(p) = -\dfrac{pf'(p)}{f(p)} = -\dfrac{p\left(-\frac{5}{2\sqrt{400-5p}}\right)}{\sqrt{400 - 5p}} = \dfrac{5p}{2(400 - 5p)}$.

a. $E(40) = \dfrac{5(40)}{2[400 - 5(40)]} = 0.5$, and so the demand is inelastic when $p = 40$.

$E(60) = \dfrac{5(60)}{2[400 - 5(60)]} = 1.5$, and so the demand is elastic when $p = 60$.

b. The demand is unitary if $\dfrac{5p}{2(400 - 5p)} = 1$, or $5p = 800 - 10p$, that is, when $p = 53\frac{1}{3}$. (This also follows from part (a).)

c. Because the demand is elastic at $p = 60$, lowering the unit price a little will cause the revenue to increase.

d. Because the demand is inelastic at $p = 40$, a slight increase of the unit price will cause the revenue to increase.

39. We first solve the demand equation for x in terms of p. Thus, $p = \sqrt{9 - 0.02x}$, and $p^2 = 9 - 0.02x$, or $x = -50p^2 + 450$. With $f(p) = -50p^2 + 450$, we find $E(p) = -\frac{pf'(p)}{f(p)} = -\frac{p(-100p)}{-50p^2 + 450} = \frac{2p^2}{9 - p^2}$. Setting $E(p) = 1$ gives $2p^2 = 9 - p^2$, so $p = \sqrt{3}$. So the demand is inelastic in $\left(0, \sqrt{3}\right)$, unitary when $p = \sqrt{3}$, and elastic in $\left(\sqrt{3}, 3\right)$.

40. $f(p) = 10\left(\frac{50 - p}{p}\right)^{1/2} = 10\left(\frac{50}{p} - 1\right)^{1/2}$, so

$f'(p) = 10\left(\frac{1}{2}\right)\left(\frac{50}{p} - 1\right)^{-1/2}\left(-\frac{50}{p^2}\right) = -\frac{250}{p^2}\left(\frac{50}{p} - 1\right)^{-1/2}$. Then the elasticity of demand is given by

$$E(p) = -\frac{pf'(p)}{f(p)} = -\frac{p\left(-\frac{250}{p^2}\right)\left(\frac{50}{p} - 1\right)^{-1/2}}{10\left(\frac{50}{p} - 1\right)^{1/2}} = -\frac{\frac{250}{p}}{10\left(\frac{50}{p} - 1\right)} = \frac{25}{p\left(\frac{50 - p}{p}\right)} = \frac{25}{50 - p}.$$ Setting $E = 1$ gives $1 = \frac{25}{50 - p}$, and so $25 = 50 - p$, and $p = 25$. Thus, if $p > 25$, then $E > 1$, and the demand is elastic; if $p = 25$, then $E = 1$ and the demand is unitary; and if $p < 25$, then $E < 1$ and the demand is inelastic.

41. True. $\overline{C}'(x) = \frac{d}{dx}\left[\frac{C(x)}{x}\right] = \frac{xC'(x) - C(x)\frac{d}{dx}(x)}{x^2} = \frac{xC'(x) - C(x)}{x^2}$.

42. False. In fact, it makes good sense to *increase* the level of production since, in this instance, the profit increases by $f'(a)$ units per unit increase in x.

3.5 Higher-Order Derivatives

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1. a. The second derivative of f is the derivative of f'.

b. To find the second derivative of f, we differentiate f'.

2. $f'(t)$ measures its velocity at time t, and $f''(t)$ measures its acceleration at time t.

3. a. $f'(t) > 0$ and $f''(t) > 0$ in (a, b).

b. $f'(t) > 0$ and $f''(t) < 0$ in (a, b).

c. $f'(t) < 0$ and $f''(t) > 0$ in (a, b).

d. $f'(t) < 0$ and $f''(t) < 0$ in (a, b).

4. a. $f'(t) > 0$ and $f''(t) = 0$ in (a, b).

b. $f'(t) < 0$ and $f''(t) = 0$ in (a, b).

c. $f'(t) = 0$ and $f''(t) = 0$ in (a, b).

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1. $f(x) = 4x^2 - 2x + 1$, so $f'(x) = 8x - 2$ and $f''(x) = 8$.

2. $f(x) = -0.2x^2 + 0.3x + 4$, so $f'(x) = -0.4x + 0.3$ and $f''(x) = -0.4$.

3. $f(x) = 2x^3 - 3x^2 + 1$, so $f'(x) = 6x^2 - 6x$ and $f''(x) = 12x - 6 = 6(2x - 1)$.

4. $g(x) = -3x^3 + 24x^2 + 6x - 64$, so $g'(x) = -9x^2 + 48x + 6$ and $g''(x) = -18x + 48$.

5. $h(t) = t^4 - 2t^3 + 6t^2 - 3t + 10$, so $h'(t) = 4t^3 - 6t^2 + 12t - 3$ and $h''(t) = 12t^2 - 12t + 12 = 12\left(t^2 - t + 1\right)$.

6. $f(x) = x^5 - x^4 + x^3 - x^2 + x - 1$, so $f'(x) = 5x^4 - 4x^3 + 3x^2 - 2x + 1$ and $f''(x) = 20x^3 - 12x^2 + 6x - 2$.

7. $f(x) = \left(x^2 + 2\right)^5$, so $f'(x) = 5\left(x^2 + 2\right)^4 (2x) = 10x\left(x^2 + 2\right)^4$ and
$f''(x) = 10\left(x^2 + 2\right)^4 + 10x(4)\left(x^2 + 2\right)^3 (2x) = 10\left(x^2 + 2\right)^3\left[\left(x^2 + 2\right) + 8x^2\right] = 10\left(9x^2 + 2\right)\left(x^2 + 2\right)^3$.

8. $g(t) = t^2(3t + 1)^4$, so
$g'(t) = 2t(3t + 1)^4 + t^2(4)(3t + 1)^3(3) = 2t(3t + 1)^3[(3t + 1) + 6t] = (3t + 1)^3\left(18t^2 + 2t\right)$ and
$g''(t) = 2t(9t + 1)(3)(3t + 1)^2(3) + (3t + 1)^3(36t + 2) = 2(3t + 1)^2[9t(9t + 1) + (3t + 1)(18t + 1)]$
$= 2(3t + 1)^2\left(81t^2 + 9t + 54t^2 + 3t + 18t + 1\right) = 2\left(135t^2 + 30t + 1\right)(3t + 1)^2$.

9. $g(t) = \left(2t^2 - 1\right)^2\left(3t^2\right)$, so
$g'(t) = 2\left(2t^2 - 1\right)(4t)\left(3t^2\right) - \left(2t^2 - 1\right)^2(6t) = 6t\left(2t^2 - 1\right)\left[4t^2 + \left(2t^2 - 1\right)\right] = 6t\left(2t^2 - 1\right)\left(6t^2 - 1\right)$
$= 6t\left(12t^4 - 8t^2 + 1\right) = 72t^5 - 48t^3 + 6t$
and $g''(t) = 360t^4 - 144t^2 + 6 = 6\left(60t^4 - 24t^2 + 1\right)$.

10. $h(x) = \left(x^2 + 1\right)^2(x - 1)$, so $h'(x) = 2\left(x^2 + 1\right)(2x)(x - 1) +$
$\left(x^2 + 1\right)^2(1) = \left(x^2 + 1\right)\left[4x(x - 1) + \left(x^2 + 1\right)\right] = \left(x^2 + 1\right)\left(5x^2 - 4x + 1\right)$ and
$h''(x) = 2x\left(5x^2 - 4x + 1\right) + \left(x^2 + 1\right)(10x - 4) = 10x^3 - 8x^2 + 2x + 10x^3 - 4x^2 + 10x - 4$
$= 20x^3 - 12x^2 + 12x - 4 = 4\left(5x^3 - 3x^2 + 3x - 1\right)$.

11. $f(x) = \left(2x^2 + 2\right)^{7/2}$, so $f'(x) = \frac{7}{2}\left(2x^2 + 2\right)^{5/2}(4x) = 14x\left(2x^2 + 2\right)^{5/2}$ and
$f''(x) = 14\left(2x^2 + 2\right)^{5/2} + 14x\left(\frac{5}{2}\right)\left(2x^2 + 2\right)^{3/2}(4x) = 14\left(2x^2 + 2\right)^{3/2}\left[\left(2x^2 + 2\right) + 10x^2\right]$
$= 28\left(6x^2 + 1\right)\left(2x^2 + 2\right)^{3/2}$.

12. $h(w) = \left(w^2 + 2w + 4\right)^{5/2}$, so $h'(w) = \frac{5}{2}\left(w^2 + 2w + 4\right)^{3/2}(2w + 2) = 5(w + 1)\left(w^2 + 2w + 4\right)^{3/2}$ and
$h''(w) = 5\left(w^2 + 2w + 4\right)^{3/2} + 5(w + 1)\left(\frac{3}{2}\right)\left(w^2 + 2w + 4\right)^{1/2}(2w + 2)$
$= 5\left(w^2 + 2w + 4\right)^{1/2}\left[\left(w^2 + 2w + 4\right) + 3(w + 1)^2\right] = 5\left(4w^2 + 8w + 7\right)\left(w^2 + 2w + 4\right)^{1/2}$.

13. $f(x) = x\left(x^2 + 1\right)^2$, so
$f'(x) = \left(x^2 + 1\right)^2 + x(2)\left(x^2 + 1\right)(2x) = \left(x^2 + 1\right)\left[\left(x^2 + 1\right) + 4x^2\right] = \left(x^2 + 1\right)\left(5x^2 + 1\right)$ and
$f''(x) = 2x\left(5x^2 + 1\right) + \left(x^2 + 1\right)(10x) = 2x\left(5x^2 + 1 + 5x^2 + 5\right) = 4x\left(5x^2 + 3\right)$.

14. $g(u) = u(2u - 1)^3$, so $g'(u) = (2u - 1)^3 + u(3)(2u - 1)^2(2) = (2u - 1)^2[(2u - 1) + 6u] = (8u - 1)(2u - 1)^2$
and $g''(u) = 8(2u - 1)^2 + (8u - 1)(2)(2u - 1)(2) = 4(2u - 1)[2(2u - 1) + (8u - 1)] = 12(2u - 1)(4u - 1)$.

15. $f(x) = \dfrac{x}{2x+1}$, so $f'(x) = \dfrac{(2x+1)(1) - x(2)}{(2x+1)^2} = \dfrac{1}{(2x+1)^2}$ and

$f''(x) = \dfrac{d}{dx}(2x+1)^{-2} = -2(2x+1)^{-3}(2) = -\dfrac{4}{(2x+1)^3}$.

16. $g(t) = \dfrac{t^2}{t-1}$, so $g'(t) = \dfrac{(t-1)(2t) - t^2(1)}{(t-1)^2} = \dfrac{t^2-2t}{(t-1)^2} = \dfrac{t(t-2)}{(t-1)^2}$ and

$g''(t) = \dfrac{(t-1)^2(2t-2) - t(t-2)2(t-1)}{(t-1)^4} = \dfrac{2(t-1)\left[(t-1)^2 - t(t-2)\right]}{(t-1)^4} = \dfrac{2}{(t-1)^3}$.

17. $f(s) = \dfrac{s-1}{s+1}$, so $f'(s) = \dfrac{(s+1)(1) - (s-1)(1)}{(s+1)^2} = \dfrac{2}{(s+1)^2}$ and

$f''(s) = 2\dfrac{d}{ds}(s+1)^{-2} = -4(s+1)^{-3} = -\dfrac{4}{(s+1)^3}$.

18. $f(u) = \dfrac{u}{u^2+1}$, so $f'(u) = \dfrac{\left(u^2+1\right)(1) - (u)(2u)}{\left(u^2+1\right)^2} = \dfrac{-u^2+1}{\left(u^2+1\right)^2}$ and

$f''(u) = \dfrac{\left(u^2+1\right)^2(-2u) - \left(-u^2+1\right)(2)\left(u^2+1\right)(2u)}{\left(u^2+1\right)^4} = \dfrac{2u\left(u^2+1\right)\left(-u^2-1+2u^2-2\right)}{\left(u^2+1\right)^4} = \dfrac{2u\left(u^2-3\right)}{\left(u^2+1\right)^3}$.

19. $f(u) = \sqrt{4-3u} = (4-3u)^{1/2}$, so $f'(u) = \frac{1}{2}(4-3u)^{-1/2}(-3) = -\dfrac{3}{2\sqrt{4-3u}}$ and

$f''(u) = -\dfrac{3}{2}\cdot\dfrac{d}{du}(4-3u)^{-1/2} = -\dfrac{3}{2}\left(-\dfrac{1}{2}\right)(4-3u)^{-3/2}(-3) = -\dfrac{9}{4(4-3u)^{3/2}}$.

20. $f(x) = \sqrt{2x-1} = (2x-1)^{1/2}$, so $f'(x) = \frac{1}{2}(2x-1)^{-1/2}(2) = (2x-1)^{-1/2} = \dfrac{1}{\sqrt{2x-1}}$ and

$f''(x) = -\frac{1}{2}(2x-1)^{-3/2}(2) = -(2x-1)^{-3/2} = -\dfrac{1}{\sqrt{(2x-1)^3}}$.

21. $f(x) = 3x^4 - 4x^3$, so $f'(x) = 12x^3 - 12x^2$, $f''(x) = 36x^2 - 24x$, and $f'''(x) = 72x - 24$.

22. $f(x) = 3x^5 - 6x^4 + 2x^2 - 8x + 12$, so $f'(x) = 15x^4 - 24x^3 + 4x - 8$, $f''(x) = 60x^3 - 72x^2 + 4$, and $f'''(x) = 180x^2 - 144x$.

23. $f(x) = \dfrac{1}{x}$, so $f'(x) = \dfrac{d}{dx}\left(x^{-1}\right) = -x^{-2} = -\dfrac{1}{x^2}$, $f''(x) = 2x^{-3} = \dfrac{2}{x^3}$, and $f'''(x) = -6x^{-4} = -\dfrac{6}{x^4}$.

24. $f(x) = \dfrac{2}{x^2}$, so $f'(x) = 2\dfrac{d}{dx}\left(x^{-2}\right) = -4x^{-3} = -\dfrac{4}{x^3}$, $f''(x) = 12x^{-4} = \dfrac{12}{x^4}$, and $f'''(x) = -48x^{-5} = -\dfrac{48}{x^5}$.

25. $g(s) = (3s-2)^{1/2}$, so $g'(s) = \frac{1}{2}(3s-2)^{-1/2}(3) = \dfrac{3}{2(3s-2)^{1/2}}$,

$g''(s) = \frac{3}{2}\left(-\frac{1}{2}\right)(3s-2)^{-3/2}(3) = -\frac{9}{4}(3s-2)^{-3/2} = -\dfrac{9}{4(3s-2)^{3/2}}$, and

$g'''(s) = \frac{27}{8}(3s-2)^{-5/2}(3) = \frac{81}{8}(3s-2)^{-5/2} = \dfrac{81}{8(3s-2)^{5/2}}$.

26. $g(t)=\sqrt{2t+3}$, so $g'(t)=\frac{1}{2}(2t+3)^{-1/2}(2)=(2t+3)^{-1/2}$, $g''(t)=-\frac{1}{2}(2t+3)^{-3/2}(2)=-(2t+3)^{-3/2}$, and $g'''(t)=\frac{3}{2}(2t+3)^{-5/2}(2)=\dfrac{3}{(2t+3)^{5/2}}$.

27. $f(x)=(2x-3)^4$, so $f'(x)=4(2x-3)^3(2)=8(2x-3)^3$, $f''(x)=24(2x-3)^2(2)=48(2x-3)^2$, and $f'''(x)=96(2x-3)(2)=192(2x-3)$.

28. $g(t)=\left(\frac{1}{2}t^2-1\right)^5$, so $g'(t)=5\left(\frac{1}{2}t^2-1\right)^4(t)=5t\left(\frac{1}{2}t^2-1\right)^4$,

$g''(t)=5\left(\frac{1}{2}t^2-1\right)^4+5t(4)\left(\frac{1}{2}t^2-1\right)^3(t)=5\left(\frac{1}{2}t^2-1\right)^3\left[\left(\frac{1}{2}t^2-1\right)+4t^2\right]=\frac{5}{2}\left(9t^2-2\right)\left(\frac{1}{2}t^2-1\right)^3$, and

$$g'''(t)=\frac{5}{2}\left[18t\left(\frac{1}{2}t^2-1\right)^3+\left(9t^2-2\right)3\left(\frac{1}{2}t^2-1\right)^2(t)\right]=\frac{15}{2}t\left(\frac{1}{2}t^2-1\right)^2\left[6\left(\frac{1}{2}t^2-1\right)+\left(9t^2-2\right)\right]$$

$$=30t\left(3t^2-2\right)\left(\frac{1}{2}t^2-1\right)^2.$$

29. Its velocity at any time t is $v(t)=\frac{d}{dt}\left(16t^2\right)=32t$. The hammer strikes the ground when $16t^2=256$ or $t=4$ (we reject the negative root). Therefore, its velocity at the instant it strikes the ground is $v(4)=32(4)=128$ ft/sec. Its acceleration at time t is $a(t)=\frac{d}{dt}(32t)=32$. In particular, its acceleration at $t=4$ is 32 ft/sec^2.

30. $s(t)=20t+8t^2-t^3$, so $s'(t)=20+16t-3t^2$ and $s''(t)=16-6t$. In particular, $s''\left(\frac{8}{3}\right)=16-6\left(\frac{8}{3}\right)=16-\frac{48}{3}=0$. We conclude that the acceleration of the car at $t=\frac{8}{3}$ seconds is zero and that the car will start to decelerate at that point in time.

31. $P(t)=0.38t^2+1.3t+3$.

a. The projected percentage is $P(5)=0.38(5)^2+1.3(5)+3=19$, or 19%.

b. $P'(t)=0.76t+1.3$, so the percentage of vehicles with transmissions that have 7 or more speeds is projected to be changing at the rate of $P'(5)=0.76(5)+1.3=5.1$, or 5.1% per year (in 2015).

c. $P''(5)=0.76$, so the rate of increase in vehicles with such transmissions is itself increasing at the rate of 0.76% per year per year in 2015.

32. $N(t)=0.00525t^2+0.075t+4.7$.

a. The projected number of people of age 65 and over with Alzheimer's disease in the U.S. is $N(2)=0.00525(2)^2+0.075(2)+4.7=4.871$, or 4.871 million, in 2030.

b. $N'(t)=0.0105t+0.075$, so the number of patients is projected to be growing at the rate of $N'(2)=0.0105(2)+0.075=0.096$, or 96,000 per decade, in 2030.

c. $N''(t)=0.0105$, so the rate of growth is projected to be growing at the rate of 10,500 per decade per decade.

33. $N(t)=-0.1t^3+1.5t^2+100$.

a. $N'(t)=-0.3t^2+3t=0.3t(10-t)$. Because $N'(t)>0$ for $t=0,1,2,\ldots,8$, it is evident that $N(t)$ (and therefore the crime rate) was increasing from 2006 through 2014.

b. $N''(t)=-0.6t+3=0.6(5-t)$. Now $N''(4)=0.6>0$, $N''(5)=0$, $N''(6)=-0.6<0$, $N''(7)=-1.2<0$, and $N''(8)=-1.8<0$. This shows that the rate of the rate of change was decreasing beyond $t=5$ (in the year 2011). This indicates that the program was working.

34. $G(t) = -0.2t^3 + 2.4t^2 + 60.$

a. $G'(t) = -0.6t^2 + 4.8t = 0.6t(8 - t)$, so $G'(1) = 4.2$, $G'(2) = 7.2$, $G'(3) = 9$, $G'(4) = 9.6$, $G'(5) = 9$, $G'(6) = 7.2$, $G'(7) = 4.2$, and $G'(8) = 0$.

b. $G''(t) = -1.2t + 4.8 = 1.2(4 - t)$, so $G''(1) = 3.6$, $G''(2) = 2.4$, $G''(3) = 1.2$, $G''(4) = 0$, $G''(5) = -1.2$, $G''(6) = -2.4$, $G''(7) = -3.6$, and $G''(8) = -4.8$.

c. Our calculations show that the GDP is increasing at an increasing rate in the first five years. Even though the GDP continues to rise from that point on, the negativity of $G''(t)$ shows that the rate of increase is slowing down.

35. $S(t) = 4t^3 + 2t^2 + 300t$, so $S(6) = 4(6)^3 + 2(6)^2 + 300(6) = 2736$. This says that 6 months after the grand opening of the store, monthly LP sales are projected to be 2736 units.
$S'(t) = 12t^2 + 4t + 300$, so $S'(6) = 12(6)^2 + 4(6) + 300 = 756$. Thus, monthly sales are projected to be increasing by 756 units per month.
$S''(t) = 24t + 4$, so $S''(6) = 24(6) + 4 = 148$. This says that the rate of increase of monthly sales is itself increasing at the rate of 148 units per month per month.

36. a. $f(t) = -0.2176t^3 + 1.962t^2 - 2.833t + 29.4$, so the median age of the population in the year 2000 was $f(4) = -0.2176(4)^3 + 1.962(4)^2 - 2.833(4) + 29.4 \approx 35.53$, or approximately 35.5 years old.

b. $f'(t) = -0.6528t^2 + 3.924t - 2.833$, so the median age of the population in the year 2000 was changing at the rate of $-0.6528(4)^2 + 3.924(4) - 2.833 = 2.4182$; that is, it was increasing at the rate of approximately 2.4 years of age per decade.

c. $f''(t) = -1.3056t + 3.924$, so the rate at which the median age of the population was changing in the year was $f''(4) = -1.3056(4) + 3.924 = -1.2984$. That is, the rate of change was decreasing at the rate of approximately 1.3 years of age per decade per decade.

37. $P(t) = 0.0004t^3 + 0.0036t^2 + 0.8t + 12$, so $P'(t) = 0.00012t^2 + 0.0072t + 0.8$. Thus, $P'(t) \geq 0.8$ for $0 \leq t \leq 13$. $P''(t) = 0.00024t + 0.0072$, and for $0 \leq t \leq 13$, $P''(t) > 0$. This means that the proportion of the U.S. population that was obese was increasing at an increasing rate from 1991 through 2004.

38. a. $h(t) = \frac{1}{16}t^4 - t^3 + 4t^2$, so $h'(t) = \frac{1}{4}t^3 - 3t^2 + 8t$.

b. $h'(0) = 0$, or 0 ft/sec. $h'(4) = \frac{1}{4}(64) - 3(16) + 8(4) = 0$, or 0 ft/sec, and $h'(8) = \frac{1}{4}(8)^3 - 3(64) + 8(8) = 0$, or 0 ft/sec.

c. $h''(t) = \frac{3}{4}t^2 - 6t + 8$.

d. $h''(0) = 8\text{ ft/sec}^2$, $h''(4) = \frac{3}{4}(16) - 6(4) + 8 = -4\text{ ft/sec}^2$, and $h''(8) = \frac{3}{4}(64) - 6(8) + 8 = 8\text{ ft/sec}^2$.

e. $h(0) = 0$ ft, $h(4) = \frac{1}{16}(4)^4 - (4)^3 + 4(4)^2 = 16$ ft, and $h(8) = \frac{1}{16}(8)^4 - (8)^3 + 4(8)^2 = 0$ ft.

39. $A(t) = -0.00006t^5 + 0.00468t^4 - 0.1316t^3 + 1.915t^2 - 17.63t + 100$, so $A'(t) = -0.0003t^4 + 0.01872t^3 - 0.3948t^2 + 3.83t - 17.63$ and $A''(t) = -0.0012t^3 + 0.05616t^2 - 0.7896t + 3.83$. Thus, $A'(10) = -3.09$ and $A''(10) = 0.35$. Our calcutations show that 10 minutes after the start of the test, the smoke remaining is decreasing at a rate of 3.09% per minute, but the rate at which the rate of smoke is decreasing is decreasing at the rate of 0.35 percent per minute per minute.

40. $P(t) = 33.55(t+5)^{0.205}$, so $P'(t) = 33.55(0.205)(t+5)^{-0.795} = 6.87775(t+5)^{-0.795}$ and $P''(t) = 6.87775(-0.795)(t+5)^{-1.795} = -5.46781125(t+5)^{-1.795}$. Thus, $P''(20) = -5.46781125(20+5)^{-1.795} \approx -0.017$, which says that the rate of the rate of change of such mothers is decreasing at the rate of 0.02%/yr^2.

41. $f(t) = 10.72(0.9t+10)^{0.3}$, so $f'(t) = 10.72(0.3)(0.9t+10)^{-0.7}(0.9) = 2.8944(0.9t+10)^{-0.7}$ and $f''(t) = 2.8944(-0.7)(0.9t+10)^{-1.7}(0.9) = -1.823472(0.9t+10)^{-1.7}$. Thus, $f''(10) = -1.823472(19)^{-1.7} \approx -0.01222$, which says that the rate of the rate of change of the population is decreasing at the rate of 0.01%/yr^2.

42. False. If f has derivatives of order two at $x = a$, then $f''(a) = \left[f'(x)\right]'\Big|_{x=a}$.

43. True. If $h = fg$ where f and g have derivatives of order 2, then $h''(x) = f''(x)g(x) + 2f'(x)g'(x) + f(x)g''(x)$.

44. True. If $f(x)$ is a polynomial function of degree n, then $f^{(n+1)}(x) = 0$.

45. True. Suppose $P(t)$ represents the population of bacteria at time t and suppose $P'(t) > 0$ and $P''(t) < 0$. Then the population is increasing at time t, but at a decreasing rate.

46. True. Using the Chain Rule, $h'(x) = f'(2x) \cdot \frac{d}{dx}(2x) = f'(2x) \cdot 2 = 2f'(2x)$. Using the Chain Rule again, $h''(x) = 2f''(2x) \cdot 2 = 4f''(2x)$.

47. $\overline{C}(x) = \dfrac{C(x)}{x}$, so $\overline{C}'(x) = \dfrac{xC'(x) - C(x) \cdot 1}{x^2} = \dfrac{xC'(x) - C(x)}{x^2}$ and

$$\overline{C}''(x) = \frac{x^2\left[xC''(x) + C'(x) - C'(x)\right] - \left[xC'(x) - C(x)\right]2x}{x^4} = \frac{x^3C''(x) - 2x^2C'(x) + 2xC(x)}{x^4}$$
$$= \frac{C''(x)}{x} - \frac{2C'(x)}{x^2} + \frac{2C(x)}{x^3}.$$

48. $f'(x) = \frac{7}{3}x^{5/3}$ and $f''(x) = \frac{35}{9}x^{2/3}$, so f' and f''exist everywhere. However, $f'''(x) = \frac{70}{27}x^{-1/3} = \dfrac{70}{27x^{1/3}}$ is not defined at $x = 0$.

49. Consider the function $f(x) = x^{(2n+1)/2} = x^{n+(1/2)}$. We calculate $f'(x) = \left(n + \frac{1}{2}\right)x^{n-(1/2)}$, $f''(x) = \left(n + \frac{1}{2}\right)\left(n - \frac{1}{2}\right)x^{n-(3/2)}, \ldots, f^{(n)}(x) = \left(n + \frac{1}{2}\right)\left(n - \frac{1}{2}\right)\cdots\frac{3}{2}x^{1/2}$, and $f^{(n+1)}(x) = \left(n + \frac{1}{2}\right)\left(n - \frac{1}{2}\right)\cdots\frac{1}{2}x^{-1/2}$. The first n derivatives exist at $x = 0$, but the $(n+1)$st derivative fails to be defined there.

50. Let $P(x) = a_nx^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \cdots + a_1x + a_0$. Then $P'(x) = na_nx^{n-1} + (n-1)a_{n-1}x^{n-2} + \cdots a_1$. Eventually, we calculate $P^{(n)}(x) = a_n$, and so $P^{(n+1)}(x) = P^{(n+2)}(x) = P^{(n+3)}(x) = \cdots = 0$. Thus, P has derivatives of all orders.

Using Technology

page 221

1. -18 **2.** 425.25 **3.** 15.2762 **4.** 128.7540

5. -0.6255 **6.** -13.9463 **7.** 0.1973 **8.** -0.0163

3.6 Implicit Differentiation and Related Rates

Concept Questions page 230

1. a. We differentiate both sides of $F(x, y) = 0$ with respect to x, then solve for dy/dx.

b. The Chain Rule is used to differentiate any expression involving the dependent variable y.

2. $xg(y) + yf(x) = 0$. Differentiating both sides with respect to x gives $xg'(y)y' + g(y) + y'f(x) + yf'(x) = 0$, so $\left[xg'(y) + f(x)\right]y' = -\left[g(y) + yf'(x)\right]$, and finally $y' = -\dfrac{g(y) + yf'(x)}{f(x) + xg'(y)}$

3. Suppose x and y are two variables that are related by an equation. Furthermore, suppose x and y are both functions of a third variable t. (Normally t represents time.) Then a related rates problem involves finding dx/dt or dy/dt.

4. See page 228 in the text.

Exercises page 231

1. a. $x + 2y = 5$. Solving for y in terms of x, we have $y = -\frac{1}{2}x + \frac{5}{2}$. Therefore, $y' = -\frac{1}{2}$.

b. Next, differentiating $x + 2y = 5$ implicitly, we have $1 + 2y' = 0$, or $y' = -\frac{1}{2}$.

2. a. $3x + 4y = 6$. Solving for y in terms of x, we have $y = -\frac{3}{4}x + \frac{3}{2}$. Therefore, $y' = -\frac{3}{4}$.

b. Next, differentiating $3x + 4y = 6$ implicitly, we obtain $3 + 4y' = 0$, or $y' = -\frac{3}{4}$.

3. a. $xy = 1$, $y = \dfrac{1}{x}$, and $\dfrac{dy}{dx} = -\dfrac{1}{x^2}$.

b. $x\dfrac{dy}{dx} + y = 0$, so $x\dfrac{dy}{dx} = -y$ and $\dfrac{dy}{dx} = -\dfrac{y}{x} = \dfrac{-1/x}{x} = -\dfrac{1}{x^2}$.

4. a. $xy - y - 1 = 0$. Solving for y, we have $y(x-1) = 1$ or $y = (x-1)^{-1}$. Therefore, $y' = -(x-1)^{-2} = -\dfrac{1}{(x-1)^2}$.

b. Next, differentiating $xy - y - 1 = 0$ implicitly, we obtain $y + xy' - y' = 0$, or $y'(x-1) = -y$, so $y' = -\dfrac{y}{x-1} = -\dfrac{1}{(x-1)^2}$.

5. $x^3 - x^2 - xy = 4$.

a. $-xy = 4 - x^3 + x^2$, so $y = -\dfrac{4}{x} + x^2 - x$ and $\dfrac{dy}{dx} = \dfrac{4}{x^2} + 2x - 1$.

b. $x^3 - x^2 - xy = 4$, so $-x\dfrac{dy}{dx} = -3x^2 + 2x + y$, and therefore

$$\frac{dy}{dx} = 3x - 2 - \frac{y}{x} = 3x - 2 - \frac{1}{x}\left(-\frac{4}{x} + x^2 - x\right) = 3x - 2 + \frac{4}{x^2} - x + 1 = \frac{4}{x^2} + 2x - 1.$$

6. $x^2y - x^2 + y - 1 = 0$.

a. $\left(x^2 + 1\right)y = 1 + x^2$, or $y = \dfrac{1 + x^2}{1 + x^2} = 1$. Therefore, $\dfrac{dy}{dx} = 0$.

b. Differentiating implicitly, $x^2y' + 2xy - 2x + y' = 0$, so $\left(x^2+1\right)y' = 2x(1-y)$, and thus $y' = \dfrac{2x(1-y)}{x^2+1}$. But from part (a), we know that $y = 1$, so $y' = \dfrac{2x(1-1)}{x^2+1} = 0$.

7. a. $\dfrac{x}{y} - x^2 = 1$ is equivalent to $\dfrac{x}{y} = x^2+1$, or $y = \dfrac{x}{x^2+1}$. Therefore, $y' = \dfrac{\left(x^2+1\right) - x(2x)}{\left(x^2+1\right)^2} = \dfrac{1-x^2}{\left(x^2+1\right)^2}$.

b. Next, differentiating the equation $x - x^2y = y$ implicitly, we obtain $1 - 2xy - x^2y' = y'$, $y'\left(1+x^2\right) = 1-2xy$, and thus $y' = \dfrac{1-2xy}{\left(1+x^2\right)}$. This may also be written in the form $-2y^2 + \dfrac{y}{x}$. To show that this is equivalent to the results obtained earlier, use the earlier value of y to get $y' = \dfrac{1-2x\left(\dfrac{x}{x^2+1}\right)}{1+x^2} = \dfrac{x^2+1-2x^2}{\left(1+x^2\right)^2} = \dfrac{1-x^2}{\left(1+x^2\right)^2}$.

8. a. $\dfrac{y}{x} - 2x^3 = 4$ is equivalent to $y = 2x^4 + 4x$. Therefore, $y' = 8x^3 + 4$.

b. Next, differentiating the equation $y - 2x^4 = 4x$ implicitly, we obtain $y' - 8x^3 = 4$, and so $y' = 8x^3 + 4$, as obtained earlier.

9. $x^2 + y^2 = 16$. Differentiating both sides of the equation implicitly, we obtain $2x + 2yy' = 0$, and so $y' = -x/y$.

10. $2x^2 + y^2 = 16$, $4x + 2y\dfrac{dy}{dx} = 0$ and $\dfrac{dy}{dx} = -\dfrac{2x}{y}$.

11. $x^2 - 2y^2 = 16$. Differentiating implicitly with respect to x, we have $2x - 4y\dfrac{dy}{dx} = 0$, and so $\dfrac{dy}{dx} = \dfrac{x}{2y}$.

12. $x^3 + y^3 + y - 4 = 0$. Differentiating both sides of the equation implicitly, we obtain $3x^2 + 3y^2y' + y' = 0$ or $y'\left(3y^2+1\right) = -3x^2$. Therefore, $y' = -\dfrac{3x^2}{3y^2+1}$.

13. $x^2 - 2xy = 6$. Differentiating both sides of the equation implicitly, we obtain $2x - 2y - 2xy' = 0$ and so $y' = \dfrac{x-y}{x} = 1 - \dfrac{y}{x}$.

14. $x^2 + 5xy + y^2 = 10$. Differentiating both sides of the equation implicitly, we obtain $2x + 5y + 5xy' + 2yy' = 0$, $2x + 5y + y'(5x+2y) = 0$, and so $y' = -\dfrac{2x+5y}{5x+2y}$.

15. $x^2y^2 - xy = 8$. Differentiating both sides of the equation implicitly, we obtain $2xy^2 + 2x^2yy' - y - xy' = 0$, $2xy^2 - y + y'\left(2x^2y - x\right) = 0$, and so $y' = \dfrac{y(1-2xy)}{x(2xy-1)} = -\dfrac{y}{x}$.

16. $x^2y^3 - 2xy^2 = 5$. Differentiating both sides of the equation implicitly, we obtain $2xy^3 + 3x^2y^2y' - 2y^2 - 4xyy' = 0$, $2y^2(xy-1) + xy(3xy-4)y' = 0$, and so $y' = \dfrac{2y(1-xy)}{x(3xy-4)}$.

17. $x^{1/2} + y^{1/2} = 1$. Differentiating implicitly with respect to x, we have $\frac{1}{2}x^{-1/2} + \frac{1}{2}y^{-1/2}\frac{dy}{dx} = 0$. Therefore, $\frac{dy}{dx} = -\frac{x^{-1/2}}{y^{-1/2}} = -\frac{\sqrt{y}}{\sqrt{x}}$.

18. $x^{1/3} + y^{1/3} = 1$. Differentiating both sides of the equation implicitly, we obtain $\frac{1}{3}x^{-2/3} + \frac{1}{3}y^{-2/3}y' = 0$, so $y' = -\frac{x^{-2/3}}{y^{-2/3}} = -\frac{y^{2/3}}{x^{2/3}} = -\left(\frac{y}{x}\right)^{2/3}$.

19. $\sqrt{x+y} = x$. Differentiating both sides of the equation implicitly, we obtain $\frac{1}{2}(x+y)^{-1/2}(1+y') = 1$, $1 + y' = 2(x+y)^{1/2}$, and so $y' = 2\sqrt{x+y} - 1$.

20. $(2x+3y)^{1/3} = x^2$. Differentiating both sides of the equation implicitly, we obtain $\frac{1}{3}(2x+3y)^{-2/3}(2+3y') = 2x$, $2 + 3y' = 6x(2x+3y)^{2/3}$, and so $y' = \frac{2}{3}\left[3x(2x+3y)^{2/3} - 1\right]$.

21. $\frac{1}{x^2} + \frac{1}{y^2} = 1$. Differentiating both sides of the equation implicitly, we obtain $-\frac{2}{x^3} - \frac{2}{y^3}y' = 0$, or $y' = -\frac{y^3}{x^3}$.

22. $\frac{1}{x^3} + \frac{1}{y^3} = 5$. Differentiating both sides of the equation implicitly, we obtain $-\frac{3}{x^4} - \frac{3}{y^4}y' = 0$, or $y' = -\frac{y^4}{x^4}$.

23. $\sqrt{xy} = x + y$. Differentiating both sides of the equation implicitly, we obtain $\frac{1}{2}(xy)^{-1/2}(xy' + y) = 1 + y'$, so $xy' + y = 2\sqrt{xy}(1+y')$, $y'\left(x - 2\sqrt{xy}\right) = 2\sqrt{xy} - y$, and so $y' = -\frac{2\sqrt{xy} - y}{2\sqrt{xy} - x} = \frac{2\sqrt{xy} - y}{x - 2\sqrt{xy}}$.

24. $\sqrt{xy} = 2x + y^2$. Differentiating both sides of the equation implicitly, we obtain $\frac{1}{2}(xy)^{-1/2}(xy' + y) = 2 + 2yy'$, $xy' + y = 4\sqrt{xy} + 4\sqrt{xy}yy'$, $y'\left(x - 4y\sqrt{xy}\right) = 4\sqrt{xy} - y$, and so $y' = \frac{4\sqrt{xy} - y}{x - 4y\sqrt{xy}}$.

25. $\frac{x+y}{x-y} = 3x$, or $x + y = 3x^2 - 3xy$. Differentiating both sides of the equation implicitly, we obtain $1 + y' = 6x - 3xy' - 3y$, so $y' + 3xy' = 6x - 3y - 1$ and $y' = \frac{6x - 3y - 1}{3x + 1}$.

26. $\frac{x-y}{2x+3y} = 2x$, or $x - y = 4x^2 + 6xy$. Differentiating both sides of the equation implicitly, we have $1 - y' = 8x + 6y + 6xy'$, so $y' + 6xy' = -8x - 6y + 1$ and $y' = -\frac{8x + 6y - 1}{6x + 1}$.

27. $xy^{3/2} = x^2 + y^2$. Differentiating implicitly with respect to x, we obtain $y^{3/2} + x\left(\frac{3}{2}\right)y^{1/2}\frac{dy}{dx} = 2x + 2y\frac{dy}{dx}$. Multiply both sides by 2 to get $2y^{3/2} + 3xy^{1/2}\frac{dy}{dx} = 4x + 4y\frac{dy}{dx}$. Then $\left(3xy^{1/2} - 4y\right)\frac{dy}{dx} = 4x - 2y^{3/2}$, so $\frac{dy}{dx} = \frac{2\left(2x - y^{3/2}\right)}{3xy^{1/2} - 4y}$.

28. $x^2y^{1/2} = x + 2y^3$. Differentiating implicitly with respect to x, we have $2xy^{1/2} + \frac{1}{2}x^2y^{-1/2}y' = 1 + 6y^2y'$, so $4xy + x^2y' = 2y^{1/2} + 12y^{5/2}y'$, $y'\left(x^2 - 12y^{5/2}\right) = -4xy + 2y^{1/2}$, and $y' = \frac{2\sqrt{y} - 4xy}{x^2 - 12y^{5/2}}$.

29. $(x+y)^3+x^3+y^3=0$. Differentiating implicitly with respect to x, we obtain $3(x+y)^2\left(1+\frac{dy}{dx}\right)+3x^2+3y^2\frac{dy}{dx}=0$, $(x+y)^2+(x+y)^2\frac{dy}{dx}+x^2+y^2\frac{dy}{dx}=0$, $\left[(x+y)^2+y^2\right]\frac{dy}{dx}=-\left[(x+y)^2+x^2\right]$, and thus $\frac{dy}{dx}=-\frac{2x^2+2xy+y^2}{x^2+2xy+2y^2}$.

30. $\left(x+y^2\right)^{10}=x^2+25$. Differentiating both sides of this equation with respect to x, we obtain $10\left(x+y^2\right)^9\left(1+2yy'\right)=2x$, so $1+2yy'=\frac{2x}{10\left(x+y^2\right)^9}$, $2yy'=\frac{2x}{10\left(x+y^2\right)^9}-1$, and $y'=\frac{x-5\left(x+y^2\right)^9}{10y\left(x+y^2\right)^9}$.

31. $4x^2+9y^2=36$. Differentiating the equation implicitly, we obtain $8x+18yy'=0$. At the point $(0,2)$, we have $0+36y'=0$, and the slope of the tangent line is 0. Therefore, an equation of the tangent line is $y=2$.

32. $y^2-x^2=16$. Differentiating both sides of this equation implicitly, we obtain $2yy'-2x=0$. At the point $\left(2,2\sqrt{5}\right)$, we have $4\sqrt{5}y'-4=0$, or $y'=m=\frac{1}{\sqrt{5}}=\frac{\sqrt{5}}{5}$. Using the point-slope form of an equation of a line, we have $y=\frac{\sqrt{5}}{5}x+\frac{8\sqrt{5}}{5}$.

33. $x^2y^3-y^2+xy-1=0$. Differentiating implicitly with respect to x, we have $2xy^3+3x^2y^2\frac{dy}{dx}-2y\frac{dy}{dx}+y+x\frac{dy}{dx}=0$. At $(1,1)$, $2+3\frac{dy}{dx}-2\frac{dy}{dx}+1+\frac{dy}{dx}=0$, and so $2\frac{dy}{dx}=-3$ and $\frac{dy}{dx}=-\frac{3}{2}$. Using the point-slope form of an equation of a line, we have $y-1=-\frac{3}{2}(x-1)$, and the equation of the tangent line to the graph of the function f at $(1,1)$ is $y=-\frac{3}{2}x+\frac{5}{2}$.

34. $(x-y-1)^3=x$. Differentiating both sides of the given equation implicitly, we obtain $3(x-y-1)^2\left(1-y'\right)=1$. At the point $(1,-1)$, $3(1+1-1)^2\left(1-y'\right)=1$ or $y'=\frac{2}{3}$. Using the point-slope form of an equation of a line, we have $y+1=\frac{2}{3}(x-1)$ or $y=\frac{2}{3}x-\frac{5}{3}$.

35. $xy=1$. Differentiating implicitly, we have $xy'+y=0$, or $y'=-\frac{y}{x}$. Differentiating implicitly once again, we have $xy''+y'+y'=0$. Therefore, $y''=-\frac{2y'}{x}=\frac{2\left(\frac{y}{x}\right)}{x}=\frac{2y}{x^2}$.

36. $x^3+y^3=28$. Differentiating implicitly, we have $3x^2+3y^2y'=0$. Differentiating again, we have $6x+3y^2y''+6y\left(y'\right)^2=0$. Thus, $y''=-\frac{2y\left(y'\right)^2+2x}{y^2}$. But $\frac{dy}{dx}=-\frac{x^2}{y^2}$, and therefore,

$$y''=-\frac{2y\left(\frac{x^4}{y^4}\right)+2x}{y^2}=-\frac{2\left(\frac{x^4}{y^3}+x\right)}{y^2}=-\frac{2x\left(x^3+y^3\right)}{y^5}.$$

37. $y^2 - xy = 8$. Differentiating implicitly we have $2yy' - y - xy' = 0$, and so $y' = \dfrac{y}{2y - x}$. Differentiating implicitly again, we have $2(y')^2 + 2yy'' - y' - y' - xy'' = 0$, so $y'' = \dfrac{2y' - 2(y')^2}{2y - x} = \dfrac{2y'(1 - y')}{2y - x}$. Then

$$y'' = \frac{2\left(\dfrac{y}{2y - x}\right)\left(1 - \dfrac{y}{2y - x}\right)}{2y - x} = \frac{2y(2y - x - y)}{(2y - x)^3} = \frac{2y(y - x)}{(2y - x)^3}.$$

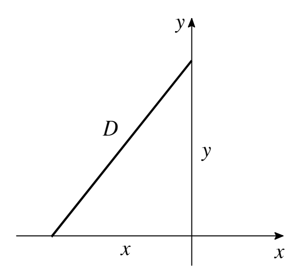
38. Differentiating $x^{1/3} + y^{1/3} = 1$ implicitly, we have $\frac{1}{3}x^{-2/3} + \frac{1}{3}y^{-2/3}y' = 0$ and $y' = -\dfrac{y^{2/3}}{x^{2/3}}$. Differentiating implicitly once again, we have

$$y'' = -\frac{x^{2/3}\left(\frac{2}{3}\right)y^{-1/3}y' - y^{2/3}\left(\frac{2}{3}\right)x^{-1/3}}{x^{4/3}} = \frac{-\dfrac{2}{3}x^{2/3}y^{-1/3}\left(-\dfrac{y^{2/3}}{x^{2/3}}\right) + \dfrac{2}{3}y^{2/3}x^{-1/3}}{x^{4/3}} = \frac{2}{3}\left(\frac{y^{1/3} + y^{2/3}x^{-1/3}}{x^{4/3}}\right)$$
$$= \frac{2y^{1/3}\left(x^{1/3} + y^{1/3}\right)}{3x^{4/3}x^{1/3}} = \frac{2y^{1/3}}{3x^{5/3}}.$$

39. a. Differentiating the given equation with respect to t, we obtain $\frac{dV}{dt} = \pi r^2 \frac{dh}{dt} + 2\pi r h \frac{dr}{dt} = \pi r \left(r \frac{dh}{dt} + 2h \frac{dr}{dt}\right)$.

b. Substituting $r = 2$, $h = 6$, $\frac{dr}{dt} = 0.1$, and $\frac{dh}{dt} = 0.3$ into the expression for $\frac{dV}{dt}$, we obtain $\frac{dV}{dt} = \pi(2)[2(0.3) + 2(6)(0.1)] = 3.6\pi$, and so the volume is increasing at the rate of 3.6π in^3/sec.

40. Let $(x, 0)$ and $(0, y)$ denote the position of the two cars. Then $D^2 = x^2 + y^2$. Differentiating with respect to t, we obtain $2D\frac{dD}{dt} = 2x\frac{dx}{dt} + 2y\frac{dy}{dt}$, so $D\frac{dD}{dt} = x\frac{dx}{dt} + y\frac{dy}{dt}$. When $t = 4$, $x = -20$, and $y = 28$, $\frac{dx}{dt} = -9$ and $\frac{dy}{dt} = 11$. Therefore, $\left(\sqrt{(-20)^2 + (28)^2}\right)\frac{dD}{dt} = (-20)(-9) + (28)(11) = 488$, and so $\frac{dD}{dt} = \frac{488}{\sqrt{1184}} = 14.18$ ft/sec. Thus, the distance is changing at the rate of 14.18 ft/sec.



41. We are given $\frac{dp}{dt} = 2$ and wish to find $\frac{dx}{dt}$ when $x = 9$ and $p = 63$. Differentiating the equation $p + x^2 = 144$ with respect to t, we obtain $\frac{dp}{dt} + 2x\frac{dx}{dt} = 0$. When $x = 9$, $p = 63$, and $\frac{dp}{dt} = 2$, we have $2 + 2(9)\frac{dx}{dt} = 0$, and so and $\frac{dx}{dt} = -\frac{1}{9} \approx -0.111$. Thus, the quantity demanded is decreasing at the rate of approximately 111 tires per week.

42. $p = \frac{1}{2}x^2 + 48$. Differentiating implicitly, we have $\frac{dp}{dt} - x\frac{dx}{dt} = 0$, so $-x\frac{dx}{dt} = -\frac{dp}{dt}$, and thus $\frac{dx}{dt} = \frac{dp/dt}{x}$. When $x = 6$, $p = 66$, and $\frac{dp}{dt} = -3$, we have $\frac{dx}{dt} = -\frac{3}{6} = -\frac{1}{2}$, or $\left(-\frac{1}{2}\right)(1000) = -500$ tires/week.

43. $100x^2 + 9p^2 = 3600$. Differentiating the given equation implicitly with respect to t, we have $200x\frac{dx}{dt} + 18p\frac{dp}{dt} = 0$. Next, when $p = 14$, the given equation yields $100x^2 + 9(14)^2 = 3600$, so $100x^2 = 1836$, or $x \approx 4.2849$. When $p = 14$, $\frac{dp}{dt} = -0.15$, and $x \approx 4.2849$, we have $200(4.2849)\frac{dx}{dt} + 18(14)(-0.15) = 0$, and so $\frac{dx}{dt} \approx 0.0441$. Thus, the quantity demanded is increasing at the rate of approximately 44 headphones per week.

44. $625p^2 - x^2 = 100$. Differentiating the given equation implicitly with respect to t, we have $1250p\frac{dp}{dt} - 2x\frac{dx}{dt} = 0$. To find p when $x = 25$, we solve the equation $625p^2 - 625 = 100$, obtaining $p = \sqrt{\frac{725}{625}} \approx 1.0770$. Therefore, $1250(1.077)(-0.02) - 2(25)\frac{dx}{dt} = 0$, and so $\frac{dx}{dt} = -0.5385$. We conclude that the supply is falling at the rate of 539 dozen eggs per week.

45. Differentiating $625p^2 - x^2 = 100$ implicitly, we have $1250p\frac{dp}{dt} - 2x\frac{dx}{dt} = 0$. When $p = 1.0770$, $x = 25$, and $\frac{dx}{dt} = -1$, we find that $1250(1.077)\frac{dp}{dt} - 2(25)(-1) = 0$, and so $\frac{dp}{dt} = -\frac{50}{1250(1.077)} = -0.037$. We conclude that the price is decreasing at the rate of 3.7 cents per carton.

46. $p = -0.01x^2 - 0.1x + 6$. Differentiating the given equation with respect to p, we obtain $1 = -0.02x\frac{dx}{dp} - 0.1\frac{dx}{dp} = -(0.02x + 0.1)\frac{dx}{dp}$. When $x = 10$, we have $1 = -[0.02(10) + 0.1]\frac{dx}{dp}$, so $\frac{dx}{dp} = -\frac{1}{0.3} = -\frac{10}{3}$. Also, for this value of x, $p = -0.01(100) - 0.1(10) + 6 = 4$. Therefore, for these values of x and p, $E(p) = -\dfrac{pf'(p)}{f(p)} = -\dfrac{p\frac{dx}{dp}}{f(p)} = -\dfrac{4\left(-\frac{10}{3}\right)}{10} = \dfrac{4}{3} > 1$, and so the demand is elastic.

47. $p = -0.01x^2 - 0.2x + 8$. Differentiating the given equation implicitly with respect to p, we have $1 = -0.02x\dfrac{dx}{dp} - 0.2\dfrac{dx}{dp} = -[0.02x + 0.2]\dfrac{dx}{dp}$, so $\dfrac{dx}{dp} = -\dfrac{1}{0.02x + 0.2}$. When $x = 15$, $p = -0.01(15)^2 - 0.2(15) + 8 = 2.75$, and so and $\dfrac{dx}{dp} = -\dfrac{1}{0.02(15) + 0.2} = -2$. Therefore, $E(p) = -\dfrac{pf'(p)}{f(p)} = -\dfrac{(2.75)(-2)}{15} \approx 0.37 < 1$, and the demand is inelastic.

48. **a.** The required output is $Q(16, 81) = 5\left(16^{1/4}\right)\left(81^{3/4}\right) = 270$, or \$270,000.

b. Rewriting $5x^{1/4}y^{3/4} = 270$ as $x^{1/4}y^{3/4} = 54$ and differentiating implicitly, we have $\frac{1}{4}x^{-3/4}y^{3/4} + x^{1/4}\left(\frac{3}{4}y^{-1/4}\dfrac{dy}{dx}\right) = 0$, so $\dfrac{dy}{dx} = -\frac{1}{4}x^{-3/4}y^{3/4}\left(\frac{4}{3}x^{-1/4}y^{1/4}\right) = -\dfrac{y}{3x}$. If $x = 16$ and $y = 81$, then $\dfrac{dy}{dx} = -\dfrac{81}{3 \cdot 16} = -1.6875$. Thus, to keep the output constant at \$270,000, the amount spent on capital should decrease by \$1687.50 per \$1000 in labor spending. The MRTS is \$1687.50 per thousand dollars.

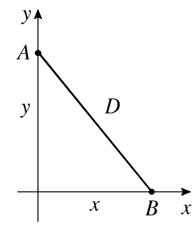
49. **a.** The required output is $Q(32, 243) = 20\left(32^{3/5}\right)\left(243^{2/5}\right) = 1440$, or \$1440 billion.

b. Differentiating $20x^{3/5}y^{2/5} = 1440$ implicitly with respect to x, we have $20\left(\frac{3}{5}x^{-2/5}y^{2/5}\right) + 20\left(x^{3/5}\right)\left(\frac{2}{5}y^{-3/5}\dfrac{dy}{dx}\right) = 0$, so $\dfrac{dy}{dx} = -\frac{3}{5}x^{-2/5}y^{2/5}\left(\frac{5}{2}x^{-3/5}y^{3/5}\right) = -\dfrac{3y}{2x}$. If $x = 32$ and $y = 243$, then $\dfrac{dy}{dx} = -\dfrac{3 \cdot 243}{2 \cdot 32} \approx -11.39$, so the amount spent on capital should decrease by approximately \$11.4 billion. The MRTS is \$11.4 billion per billion dollars.

50. $V = x^3$, so $\frac{dV}{dt} = 3x^2\frac{dx}{dt}$. When $x = 5$ and $\frac{dx}{dt} = 0.1$, we have $\frac{dV}{dt} = 3(25)(0.1) = 7.5$ in^3/sec.

51. $A = \pi r^2$. Differentiating with respect to t, we obtain $\frac{dA}{dt} = 2\pi r\frac{dr}{dt}$. When the radius of the circle is 60 ft and increasing at the rate of $\frac{1}{2}$ ft/sec, $\frac{dA}{dt} = 2\pi(60)\left(\frac{1}{2}\right) = 60\pi$ ft^2/sec. Thus, the area is increasing at a rate of approximately 188.5 ft^2/sec.

52. Let D denote the distance between the two ships, x the distance that ship A traveled north, and y the distance that ship B traveled east. Then $D^2 = x^2 + y^2$. Differentiating implicitly, we have $2D\frac{dD}{dt} = 2x\frac{dx}{dt} + 2y\frac{dy}{dt}$, so $D\frac{dD}{dt} = x\frac{dx}{dt} + y\frac{dy}{dt}$. At 1 p.m., $x = 12$ and $y = 15$, so $\sqrt{144+225}\frac{dD}{dt} = (12)(12) + (15)(15)$. Thus, $\frac{dD}{dt} = \frac{369}{\sqrt{369}} \approx 19.21$ ft/sec.

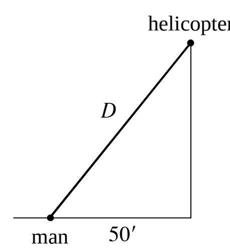


53. $A = \pi r^2$, so $r = \left(\frac{A}{\pi}\right)^{1/2}$. Differentiating with respect to t, we obtain $\frac{dr}{dt} = \frac{1}{2}\left(\frac{A}{\pi}\right)^{-1/2}\frac{dA}{dt}$. When the area of the spill is 1600π ft^2 and increasing at the rate of 80π ft^2/sec, $\frac{dr}{dt} = \frac{1}{2}\left(\frac{1600\pi}{\pi}\right)^{-1/2}(80\pi) = \pi$ ft/sec. Thus, the radius is increasing at the rate of approximately 3.14 ft/sec.

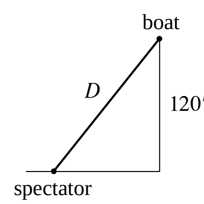
54. Let D denote the distance between the two cars, x the distance traveled by the car heading east, and y the distance traveled by the car heading north. Then $D^2 = x^2 + y^2$. Differentiating with respect to t, we have $2D\frac{dD}{dt} = 2x\frac{dx}{dt} + 2y\frac{dy}{dt}$, so $D\frac{dD}{dt} = x\frac{dx}{dt} + y\frac{dy}{dt}$. Notice also that $\frac{dx}{dt} = 2t + 1$ and $\frac{dy}{dt} = 2t + 3$. When $t = 5$, $x = 5^2 + 5 = 30$, $y = 5^2 + 3(5) = 40$, $\frac{dx}{dt} = 2(5) + 1 = 11$, and $\frac{dy}{dt} = 2(5) + 3 = 13$, so $\frac{dD}{dt} = \frac{(30)(11) + (40)(13)}{\sqrt{900+1600}} = 17$ ft/sec.

55. Let $(x, 0)$ and $(0, y)$ denote the position of the two cars at time t. Then $y = t^2 + 2t$. Now $D^2 = x^2 + y^2$ so $2D\frac{dD}{dt} = 2x\frac{dx}{dt} + 2y\frac{dy}{dt}$ and thus $D\frac{dD}{dt} = x\frac{dx}{dt} + \left(t^2+2t\right)(2t+2)$. When $t = 4$, we have $x = -20$, $\frac{dx}{dt} = -9$, and $y = 24$, so $\sqrt{(-20)^2 + (24)^2}\frac{dD}{dt} = (-20)(-9) + (24)(10)$, and therefore $\frac{dD}{dt} = \frac{420}{\sqrt{976}} \approx 13.44$. That is, the distance is changing at approximately 13.44 ft/sec.

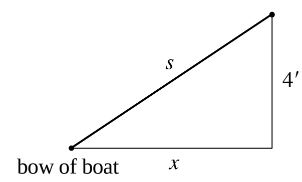
56. $D^2 = x^2 + (50)^2 = x^2 + 2500$. Differentiating implicitly with respect to t, we have $2D\frac{dD}{dt} = 2x\frac{dx}{dt}$, so $\frac{dD}{dt} = \frac{x\frac{dx}{dt}}{D}$. When $x = 120$ and $\frac{dx}{dt} = 44$, $\frac{dD}{dt} = \frac{(120)(44)}{\sqrt{(120)^2 + (50)^2}} \approx 40.6$, and so the distance between the helicopter and the man is increasing at the rate of 40.6 ft/sec.



57. Referring to the diagram, we see that $D^2 = 120^2 + x^2$. Differentiating this last equation with respect to t, we have $2D\frac{dD}{dt} = 2x\frac{dx}{dt}$, and so $\frac{dD}{dt} = \frac{x\frac{dx}{dt}}{D}$. When $x = 50$ and $\frac{dx}{dt} = 20$, $D = \sqrt{120^2 + 50^2} = 130$ and $\frac{dD}{dt} = \frac{(20)(50)}{130} \approx 7.69$, or 7.69 ft/sec.



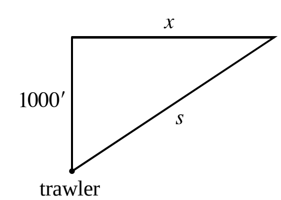
58. By the Pythagorean Theorem, $s^2 = x^2 + 4^2 = x^2 + 16$. We want to find $\frac{dx}{dt}$ when $x = 25$, given that $\frac{ds}{dt} = -3$. Differentiating both sides of the equation with respect to t yields $2s\dfrac{ds}{dt} = 2x\dfrac{dx}{dt}$, or $\dfrac{dx}{dt} = \dfrac{s\frac{ds}{dt}}{x}$. Now when $x = 25$, $s^2 = 25^2 + 16 = 641$ and $s = \sqrt{641}$. Therefore, when $x = 25$, we have $\dfrac{dx}{dt} = \frac{\sqrt{641}(-3)}{25} \approx -3.04$; that is, the boat is approaching the dock at the rate of approximately 3.04 ft/sec.



59. Let V and S denote its volume and surface area. Then we are given that $\frac{dV}{dt} = kS$, where k is the constant of proportionality. But from $V = \frac{4}{3}\pi r^3$, we find, upon differentiating both sides with respect to t, that $\frac{dV}{dt} = \frac{d}{dt}\left(\frac{4}{3}\pi r^3\right) = 4\pi r^2 \frac{dr}{dt} = kS = k\left(4\pi r^2\right)$. Therefore, $\frac{dr}{dt} = k$ a constant.

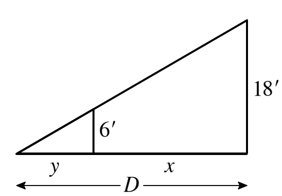
60. Let V denote the volume of the soap bubble and r its radius. Then, we are given $\frac{dV}{dt} = 8$. Differentiating the formula $V = \frac{4}{3}\pi r^3$ with respect to t, we find $\frac{dV}{dt} = \left(\frac{4}{3}\right)\left(3\pi r^2 \frac{dr}{dt}\right) = 4\pi r^2 \frac{dr}{dt}$, so $\frac{dr}{dt} = \frac{dV/dt}{4\pi r^2}$. When $r = 10$, we have $\dfrac{dr}{dt} = \dfrac{8}{4\pi\left(10^2\right)} \approx 0.0064$. Thus, the radius is increasing at the rate of approximately 0.0064 cm/sec. From $s = 4\pi r^2$, we find $\frac{ds}{dt} = 4\pi(2r)\frac{dr}{dt} = 8\pi r\frac{dr}{dt}$. Therefore, when $r = 10$, we have $\frac{ds}{dt} = 8\pi(10)(0.0064) \approx 1.6$. Thus, the surface area is increasing at the rate of approximately 1.6 cm^2/sec.

61. We are given that $\frac{dx}{dt} = 264$. Using the Pythagorean Theorem, $s^2 = x^2 + 1000^2 = x^2 + 1{,}000{,}000$. We want to find $\dfrac{ds}{dt}$ when $s = 1500$. Differentiating both sides of the equation with respect to t, we have $2s\dfrac{ds}{dt} = 2x\dfrac{dx}{dt}$ and so $\dfrac{ds}{dt} = \dfrac{x\frac{dx}{dt}}{s}$. When $s = 1500$, we have $1500^2 = x^2 + 1{,}000{,}000$, or $x = \sqrt{1{,}250{,}000}$. Therefore, $\dfrac{ds}{dt} = \dfrac{\sqrt{1{,}250{,}000}\cdot(264)}{1500} \approx 196.8$, that is, the aircraft is receding from the trawler at the speed of approximately 196.8 ft/sec.



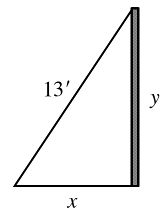
62. The volume V of the water in the pot is $V = \pi r^2 h = \pi(16)h = 16\pi h$. Differentiating with respect to t, we obtain $\frac{dV}{dt} = 16\pi\frac{dh}{dt}$. Therefore, with $\frac{dh}{dt} = 0.4$, we find $\frac{dV}{dt} = 16\pi(0.4) \approx 20.1$; that is, water is being poured into the pot at the rate of approximately 20.1 cm^3/sec.

63. $\dfrac{y}{6} = \dfrac{y+x}{18}$, $18y = 6(y+x)$, so $3y = y + x$, $2y = x$, and $y = \frac{1}{2}x$. Thus, $D = y + x = \frac{3}{2}x$. Differentiating implicitly, we have $\frac{dD}{dt} = \frac{3}{2}\cdot\frac{dx}{dt}$, and when $\frac{dx}{dt} = 6$, $\frac{dD}{dt} = \frac{3}{2}(6) = 9$, or 9 ft/sec.



64. Differentiating $x^2 + y^2 = 400$ with respect to t gives $2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$. When $x = 12$, we have $144 + y^2 = 400$, or $y = \sqrt{256} = 16$. Therefore, with $x = 12$, $\frac{dx}{dt} = 5$, and $y = 16$, we find $2(12)(5) + 2(16)\frac{dy}{dt} = 0$, or $\frac{dy}{dt} = -3.75$. Thus, the top of the ladder is sliding down the wall at the rate of 3.75 ft/sec.

65. Differentiating $x^2 + y^2 = 13^2 = 169$ with respect to t gives $2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$. When $x = 12$, we have $144 + y^2 = 169$, or $y = 5$. Therefore, with $x = 12$, $y = 5$, and $\frac{dx}{dt} = 8$, we find $2(12)(8) + 2(5)\frac{dy}{dt} = 0$, or $\frac{dy}{dt} = -19.2$. Thus, the top of the ladder is sliding down the wall at the rate of 19.2 ft/sec.



66. Differentiating the equation $2h^{1/2} + \frac{1}{25}t - 2\sqrt{20} = 0$ with respect to t gives $2\left(\frac{1}{2}h^{-1/2}\right)\frac{dh}{dt} + \frac{1}{25} = 0$, or $\frac{dh}{dt} = -\frac{\sqrt{h}}{25}$. Therefore, with $h = 8$, we have $\frac{dh}{dt} = \frac{\sqrt{8}}{25} \approx -0.113$. Thus, the height of the water is decreasing at the rate of approximately 0.11 ft/sec.

67. $P^5V^7 = C$, so $V^7 = CP^{-5}$ and $7V^6\dfrac{dV}{dt} = -5CP^{-6}\dfrac{dP}{dt}$. Therefore, $\dfrac{dV}{dt} = -\dfrac{5C}{7P^6V^6}\dfrac{dP}{dt} = -\dfrac{5P^5V^7}{7P^6V^6}\dfrac{dP}{dt} = -\dfrac{5}{7}\dfrac{V}{P}\dfrac{dP}{dt}$. When $V = 4$ L, $P = 100$ kPa, and $\dfrac{dP}{dt} = -5\ \dfrac{\text{kPa}}{\text{sec}}$, we have $\dfrac{dV}{dt} = -\dfrac{5}{7}\cdot\dfrac{4}{100}(-5) = \dfrac{1}{7}\left(\dfrac{\text{L}}{\text{kPa}}\cdot\dfrac{\text{kPa}}{\text{s}}\right) = \dfrac{1}{7}\ \dfrac{\text{L}}{\text{s}}$.

68. $m = \dfrac{m_0}{\sqrt{1 - v^2/c^2}}$. When $v = 2.92 \times 10^8$ and $\dfrac{dv}{dt} = a = 2.42 \times 10^5$,

$$\frac{dm}{dt} = \frac{\left(9.11 \times 10^{-31}\right)\left(2.92 \times 10^8\right)\left(2.42 \times 10^5\right)}{\left(2.98 \times 10^8\right)^2\left[1 - \left(\frac{2.92\times10^8}{2.98\times10^8}\right)^2\right]^{3/2}} \approx 9.1 \times 10^{-32},$$

so the mass is increasing at the rate of approximately 9.1×10^{-32} kg/sec.

69. False. There are no real numbers x and y such that $x^2 + y^2 = -1$.

70. True. If $-1 \le x < 0$, then $y^2 = \left(\sqrt{1 - x^2}\right)^2 = 1 - x^2$, so $x^2 + y^2 = 1$. If $0 \le x \le 1$, then $y^2 = \left(-\sqrt{1 - x^2}\right)^2 = 1 - x^2$, so $x^2 + y^2 = 1$.

71. True. Differentiating both sides of the equation with respect to x, we have $\frac{d}{dx}\left[f(x)g(y)\right] = \frac{d}{dx}(0)$, so $f(x)g'(y)\dfrac{dy}{dx} + f'(x)g(y) = 0$, and therefore $\frac{dy}{dx} = -\frac{f'(x)g(y)}{f(x)g'(y)}$, provided $f(x) \ne 0$ and $g'(y) \ne 0$.

72. True. Differentiating both sides of the equation with respect to x, $\frac{d}{dx}\left[f(x) + g(y)\right] = \frac{d}{dx}(0)$, so $f'(x) + g'(y)\frac{dy}{dx} = 0$, and therefore $\frac{dy}{dx} = -\frac{f'(x)}{g'(y)}$.

73. True. If $y = f(x)$, then $\Delta y = f(x + \Delta x) - f(x) \approx f'(x)\Delta x$, from which it follows that $f(x + \Delta x) \approx f(x) + f'(x)\Delta x$.

74. True. Let $y = f(x) = x^{1/3}$. Then $y' = f'(x) = \frac{1}{3}x^{-2/3} = \dfrac{1}{3x^{2/3}}$. At $x = a$, $\Delta y = f(a + \Delta x) - f(a) \approx f'(a)\Delta x$, so $f(a + \Delta x) \approx f(a) + f'(a)\Delta x = a^{1/3} + \dfrac{\Delta x}{3a^{2/3}}$. Letting $\Delta x = h$, we have $(a + h)^{1/3} = f(a + h) \approx a^{1/3} + \dfrac{h}{3a^{2/3}}$.

3.7 Differentials

Concept Questions

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1. The differential of x is dx. The differential of y is $dy = f'(x)\,dx$.

2. a. $A = \Delta x$, $B = \Delta y$, and $C = dy$.

b. $f'(x) = \dfrac{dy}{\Delta x}$.

c. From part (b), we see that $dy = f'(x)\,\Delta x$. Because $B \approx C$, $\Delta y \approx f'(x)\,\Delta x = f'(x)\,dx$.

3. Because $\Delta P = P(t_0 + \Delta t) - P(t_0) \approx P'(t_0)\,\Delta t$, we see that $P'(t_0)\,\Delta t$ is an approximation of the change in the population from time t_0 to time $t_0 + \Delta t$.

4. $P(t) = P(t_0 + \Delta t) \approx P(t_0) + P'(t_0)\,\Delta t$.

Exercises

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1. $f(x) = 2x^2$ and $dy = 4x\,dx$.

2. $f(x) = 3x^2 + 1$ and $dy = 6x\,dx$.

3. $f(x) = x^3 - x$ and $dy = \left(3x^2 - 1\right)dx$.

4. $f(x) = 2x^3 + x$ and $dy = \left(6x^2 + 1\right)dx$.

5. $f(x) = \sqrt{x+1} = (x+1)^{1/2}$ and $dy = \frac{1}{2}(x+1)^{-1/2}\,dx = \dfrac{dx}{2\sqrt{x+1}}$.

6. $f(x) = 3x^{-1/2}$ and $dy = -\dfrac{3}{2x^{3/2}}\,dx$.

7. $f(x) = 2x^{3/2} + x^{1/2}$ and $dy = \left(3x^{1/2} + \frac{1}{2}x^{-1/2}\right)dx = \frac{1}{2}x^{-1/2}(6x+1)\,dx = \dfrac{6x+1}{2\sqrt{x}}\,dx$.

8. $f(x) = 3x^{5/6} + 7x^{2/3}$ and $dy = \left(\frac{5}{2}x^{-1/6} + \frac{14}{3}x^{-1/3}\right)dx$.

9. $f(x) = x + \dfrac{2}{x}$ and $dy = \left(1 - \dfrac{2}{x^2}\right)dx = \dfrac{x^2-2}{x^2}dx$.

10. $f(x) = \dfrac{3}{x-1}$ and $dy = -\dfrac{3}{(x-1)^2}dx$.

11. $f(x) = \dfrac{x-1}{x^2+1}$ and $dy = \dfrac{x^2+1-(x-1)\,2x}{\left(x^2+1\right)^2}\,dx = \dfrac{-x^2+2x+1}{\left(x^2+1\right)^2}\,dx$.

12. $f(x) = \dfrac{2x^2+1}{x+1}$ and $dy = \dfrac{(x+1)(4x) - \left(2x^2+1\right)}{(x+1)^2}\,dx = \dfrac{2x^2+4x-1}{(x+1)^2}\,dx$.

13. $f(x) = \sqrt{3x^2 - x} = \left(3x^2 - x\right)^{1/2}$ and $dy = \frac{1}{2}\left(3x^2 - x\right)^{-1/2}(6x-1)\,dx = \dfrac{6x-1}{2\sqrt{3x^2-x}}\,dx$.

14. $f(x) = \left(2x^2+3\right)^{1/3}$ and $dy = \frac{1}{3}\left(2x^2+3\right)^{-2/3}(4x)\,dx = \dfrac{4x}{3\left(2x^2+3\right)^{2/3}}\,dx$.

15. $f(x) = x^2 - 1$.

a. $dy = 2x\,dx$.

b. $dy \approx 2(1)(0.02) = 0.04$.

c. $\Delta y = \left[(1.02)^2 - 1\right] - (1-1) = 0.0404$.

16. $f(x) = 3x^2 - 2x + 6$

a. $dy = (6x-2)\,dx$.

b. $dy \approx 10(-0.03) = -0.3$.

c. $\Delta y = \left[3(1.97)^2 - 2(1.97) + 6\right] - \left[3(2)^2 - 2(2) + 6\right] = -0.2973$.

17. $f(x) = \dfrac{1}{x}$.

a. $dy = -\dfrac{dx}{x^2}$.

b. $dy \approx -0.05$.

c. $\Delta y = \frac{1}{-0.95} - \frac{1}{-1} \approx -0.05263$.

18. $f(x) = \sqrt{2x+1} = (2x+1)^{1/2}$.

a. $dy = \frac{1}{2}(2x+1)^{-1/2}(2)\,dx = \dfrac{dx}{\sqrt{2x+1}}$.

b. $dy \approx \frac{0.1}{\sqrt{9}} \approx 0.03333$.

c. $\Delta y = [2(4.1)+1]^{1/2} - [2(4)+1]^{1/2} \approx 0.03315$.

19. $y = \sqrt{x}$ and $dy = \dfrac{dx}{2\sqrt{x}}$. Therefore, $\sqrt{10} \approx 3 + \dfrac{1}{2\cdot\sqrt{9}} \approx 3.167$.

20. $y = \sqrt{x}$ and $dy = \dfrac{dx}{2\sqrt{x}}$. Therefore, $\sqrt{17} \approx 4 + \dfrac{1}{2\cdot 4} = 4.125$.

21. $y = \sqrt{x}$ and $dy = \dfrac{dx}{2\sqrt{x}}$. Therefore, $\sqrt{49.5} \approx 7 + \dfrac{0.5}{2\cdot 7} \approx 7.0357$.

22. $y = \sqrt{x}$ and $dy = \dfrac{dx}{2\sqrt{x}}$. Therefore, $\sqrt{99.7} \approx 10 - \dfrac{0.3}{2\cdot 10} = 9.985$.

23. $y = x^{1/3}$ and $dy = \frac{1}{3}x^{-2/3}\,dx$. Therefore, $\sqrt[3]{7.8} \approx 2 - \dfrac{0.2}{3\cdot 4} \approx 1.983$.

24. $y = x^{1/4}$ and $dy = \frac{1}{4}x^{-3/4}\,dx$. Therefore, $\sqrt[4]{81.6} \approx 3 + \dfrac{0.6}{4\cdot 27} \approx 3.0056$.

25. $y = \sqrt{x}$ and $dy = \dfrac{dx}{2\sqrt{x}}$. Therefore, $\sqrt{0.089} = \frac{1}{10}\sqrt{8.9} \approx \frac{1}{10}\left(3 - \frac{0.1}{2\cdot 3}\right) \approx 0.298$.

26. $y = \sqrt[3]{x}$ and $dy = \dfrac{dx}{3x^{2/3}}$. Therefore, $\sqrt[3]{0.00096} = \dfrac{1}{100}\sqrt[3]{960} \approx \dfrac{1}{100}\left[10 - \dfrac{40}{3(100)}\right] \approx 0.0987$.

27. $y = f(x) = \sqrt{x} + \dfrac{1}{\sqrt{x}} = x^{1/2} + x^{-1/2}$. Therefore, $\dfrac{dy}{dx} = \dfrac{1}{2}x^{-1/2} - \dfrac{1}{2}x^{-3/2}$, so $dy = \left(\dfrac{1}{2x^{1/2}} - \dfrac{1}{2x^{3/2}}\right)dx$. Letting $x = 4$ and $dx = 0.02$, we find $\sqrt{4.02} + \dfrac{1}{\sqrt{4.02}} - f(4) = f(4.02) - f(4) = \Delta y \approx dy$, so $\sqrt{4.02} + \dfrac{1}{\sqrt{4.02}} \approx f(4) + dy \approx 2 + \dfrac{1}{2} + \left(\dfrac{1}{2 \cdot 2} - \dfrac{1}{16}\right)(0.02) = 2.50375$.

28. Let $y = f(x) = \dfrac{2x}{x^2+1}$. Then $\dfrac{dy}{dx} = \dfrac{\left(x^2+1\right)(2) - 2x(2x)}{\left(x^2+1\right)^2} = \dfrac{2\left(1-x^2\right)}{\left(x^2+1\right)^2}$ and $dy = \dfrac{2\left(1-x^2\right)}{\left(x^2+1\right)^2}dx$. Letting $x = 5$ and $dx = -0.02$, we find $f(5) - f(4.98) = \dfrac{2(5)}{5^2+1} - \dfrac{2(4.98)}{(4.98)^2+1} = \Delta y \approx dy$, so $\dfrac{2(4.98)}{(4.98)^2+1} \approx \dfrac{10}{26} - \dfrac{2\left(1-5^2\right)}{\left(5^2+1\right)^2}(-0.02) \approx 0.3832$.

29. The volume of the cube is given by $V = x^3$. Then $dV = 3x^2\,dx$, and when $x = 12$ and $dx = 0.02$, $dV = 3(144)(\pm 0.02) = \pm\,8.64$. The possible error that might occur in calculating the volume is $\pm 8.64\ \text{cm}^3$.

30. The area of the cube of side x cm is $S = 6x^2$. Thus, the amount of paint required is approximately $\Delta S = 6(x + \Delta x)^2 - 6x^2 \approx dS = 12x\,dx$. With $x = 30$ and $dx = \Delta x = 0.05$, $\Delta S \approx 12(30)(0.05) = 18$, or approximately $18\ \text{cm}^3$.

31. The volume of the hemisphere is given by $V = \frac{2}{3}\pi r^3$. The amount of rust-proofer needed is $\Delta V = \frac{2}{3}\pi(r + \Delta r)^3 - \frac{2}{3}\pi r^3 \approx dV = \frac{2}{3}\left(3\pi r^2\right)dr$. Thus, with $r = 60$ and $dr = \dfrac{1}{12}(0.01)$, we have $\Delta V \approx 2\pi\left(60^2\right)\left(\frac{1}{12}\right)(0.01) \approx 18.85$. So we need approximately $18.85\ \text{ft}^3$ of rust-proofer.

32. The volume of the tumor is given by $V = \frac{4}{3}\pi r^3$. Then $dV = 4\pi r^2\,dr$. When $r = 1.1$ and $dr = 0.005$, $dV = 4\pi(1.1)^2(\pm 0.005) = \pm 0.076\ \text{cm}^3$.

33. $dR = \dfrac{d}{dr}\left(k\ell r^{-4}\right)dr = -4k\ell r^{-5}\,dr$. With $\dfrac{dr}{r} = 0.1$, we find $\dfrac{dR}{R} = -\dfrac{4k\ell r^{-5}}{k\ell r^{-4}}\,dr = -4\dfrac{dr}{r} = -4(0.1) = -0.4$. In other words, the resistance will drop by 40%.

34. $f(x) = 640x^{1/5}$ and $df = 128x^{-4/5}\,dx$. When $x = 243$ and $dx = 5$, we have $df = 128(243)^{-4/5}(5) = 128\left(\frac{5}{81}\right) \approx 7.9$, or approximately \$7.9 billion.

35. $f(n) = 4n\sqrt{n-4} = 4n(n-4)^{1/2}$, so $df = 4\left[(n-4)^{1/2} + \frac{1}{2}n(n-4)^{-1/2}\right]dn$. When $n = 85$ and $dn = 5$, $df = 4\left(9 + \dfrac{85}{2 \cdot 9}\right)5 \approx 274$ seconds.

36. $P(x) = -\frac{1}{8}x^2 + 7x + 30$ and $dP = \left(-\frac{1}{4}x + 7\right)dx$. To estimate the increase in profits when the amount spent on advertising each quarter is increased from \$24,000 to \$26,000, we set $x = 24$ and $dx = 2$ and compute $dP = \left(-\frac{24}{4} + 7\right)(2) = 2$, or \$2000.

37. $N(r) = \dfrac{7}{1+0.02r^2}$ and $dN = -\dfrac{0.28r}{\left(1+0.02r^2\right)^2}\,dr$. To estimate the decrease in the number of housing starts when the mortgage rate is increased from 6% to 6.5%, we set $r = 6$ and $dr = 0.5$ and compute $dN = -\dfrac{(0.28)(6)(0.5)}{(1.72)^2} \approx -0.283937$, or 283,937 fewer housing starts.

38. $s(x) = 0.3\sqrt{x} + 10$ and $s' = \dfrac{0.15}{\sqrt{x}}\,dx$. To estimate the change in price when the quantity supplied is increased from 10,000 units to 10,500 units, we compute $ds = \dfrac{(0.15)\,500}{100} = 0.75$, or 75 cents.

39. $p = \dfrac{30}{0.02x^2+1}$ and $dp = -\dfrac{1.2x}{\left(0.02x^2+1\right)^2}\,dx$. To estimate the change in the price p when the quantity demanded changes from 5000 to 5500 units per week (that is, x changes from 5 to 5.5), we compute $dp = \dfrac{(-1.2)(5)(0.5)}{[0.02(25)+1]^2} \approx -1.33$, a decrease of \$1.33.

40. $S = kW^{2/3}$ and $dS = \dfrac{0.2}{3W^{1/3}}\,dW$. To determine the percentage error in the calculation of the surface area of a horse that weighs 300 kg when the maximum error in measurement is 0.6 kg and $k = 0.1$, we compute $\dfrac{dS}{S} = \dfrac{0.2}{3W^{1/3}}\,dW \cdot \dfrac{1}{0.1W^{2/3}} = \dfrac{2}{3W}\,dW = \dfrac{2(0.6)}{3(300)} \approx 0.00133$, or 0.133%.

41. $P(x) = -0.000032x^3 + 6x - 100$ and $dP = \left(-0.000096x^2 + 6\right)dx$. To determine the error in the estimate of Trappee's profits corresponding to a maximum error in the forecast of 15 percent [that is, $dx = \pm 0.15\,(200)$], we compute $dP = \left[(-0.000096)(200)^2 + 6\right](\pm 30) = (2.16)(30) = \pm 64.80$, or \$64,800.

42. $p = \dfrac{55}{2x^2+1}$ and $dp = -\dfrac{220x}{\left(2x^2+1\right)^2}\,dx$. To find the error corresponding to a possible error of 15% in a forecast of 1.8 billion bushels, we compute $dp = -\dfrac{(220)(1.8)(\pm 0.27)}{\left[2(1.8)^2+1\right]^2} \approx \pm 1.91$, or approximately \$1.91/bushel.

43. The approximate change in the quantity demanded is given by $\Delta x \approx dx = f'(p)\,\Delta p = \dfrac{d}{dp}(144-p)^{1/2}\ \Delta p = -\dfrac{1}{2}\cdot\dfrac{1}{\sqrt{144-p}}\cdot\Delta p$. When $\Delta p = 110 - 108 = 2$, we find $\Delta x = -\dfrac{1}{2}\cdot\dfrac{1}{\sqrt{144-108}}(2) = -\frac{1}{6} \approx -0.1667$. Thus, the quantity demanded decreases by approximately 167 tires/week.

44. The change is given by

$$\Delta A \approx dA = A'(t)\,dt = 136\frac{d}{dt}\left\{\left[1+0.25(t-4.5)^2\right]^{-1} + 28\right\}\Delta t$$

$$= 136\left[1+0.25(t-4.5)^2\right]^{-2}(0.25)(2)(t-4.5)\,\Delta t = \frac{68(t-4.5)}{\left[1+0.25(t-4.5)^2\right]^2}\,\Delta t.$$

When $t = 8$ and $\Delta t = 8.05 - 8 = 0.05$, we find $A \approx 0.7210$, so the change in the amount of nitrogen dioxide is approximately 0.72 PSI.

45. $N(x) = \dfrac{500(400+20x)^{1/2}}{(5+0.2x)^2}$ and

$N'(x) = \dfrac{(5+0.2x)^2\, 250(400+20x)^{-1/2}(20) - 500(400+20x)^{1/2}(2)(5+0.2x)(0.2)}{(5+0.2x)^4}\,dx$. To estimate the change in the number of crimes if the level of reinvestment changes from 20 cents to 22 cents per dollar deposited, we compute

$$dN = \frac{(5+4)^2(250)(800)^{-1/2}(20) - 500(400+400)^{1/2}(2)(9)(0.2)}{(5+4)^4}(2) \approx \frac{(14318.91 - 50911.69)}{9^4}(2)$$

≈ -11, a decrease of approximately 11 crimes per year.

46. a. $P = \dfrac{20{,}000r}{1-\left(1+\frac{r}{12}\right)^{-360}}$ and

$$dP = \frac{\left[1-\left(1+\frac{r}{12}\right)^{-360}\right]20{,}000 - 20{,}000r(360)\left(1+\frac{r}{12}\right)^{-361}\left(\frac{1}{12}\right)}{\left[1-\left(1+\frac{r}{12}\right)^{-360}\right]^2}dr$$

$$= \frac{20{,}000\left\{\left[1-\left(1+\frac{r}{12}\right)^{-360}\right] - 30r\left(1+\frac{r}{12}\right)^{-361}\right\}}{\left[1-\left(1+\frac{r}{12}\right)^{-360}\right]^2}dr$$

b. When $r = 0.05$, $dP \approx \dfrac{20{,}000(0.776173404 - 0.334346782)}{(0.776173404)^2} \approx 14{,}667.77912\,dr$. When the interest rate increases from 5% to 5.2% per year, $dP = 14{,}667.77912(0.002) \approx 29.34$, or approximately \$29.34. When the interest rate increases from 5% to 5.3% per year, $dP = 14{,}667.77912(0.003) \approx 44.00$, or approximately \$44.00. When the interest rate increases from 5% to 5.4% per year, $dP = 14{,}667.77912(0.004) \approx 58.67$, or approximately \$58.67. When the interest rate increases from 5% to 5.5% per year, $dP = 14{,}667.77912(0.005) \approx 73.34$, or approximately \$73.34.

47. $A = 10{,}000\left(1+\frac{r}{12}\right)^{120}$.

a. $dA = 10{,}000(120)\left(1+\frac{r}{12}\right)^{119}\left(\frac{1}{12}\right)dr = 100{,}000\left(1+\frac{r}{12}\right)^{119}dr$.

b. At 3.1%, it will be worth $100{,}000\left(1+\frac{0.03}{12}\right)^{119}(0.001)$, or approximately \$134.60 more. At 3.2%, it will be worth $100{,}000\left(1+\frac{0.03}{12}\right)^{119}(0.002)$, or approximately \$269.20 more. At 3.3%, it will be worth $100{,}000\left(1+\frac{0.03}{12}\right)^{119}(0.003)$, or approximately \$403.80 more.

48. $S = \dfrac{24{,}000\left[\left(1+\frac{r}{12}\right)^{300} - 1\right]}{r}$

a. $dS = 24{,}000\left[\dfrac{(r)300\left(1+\frac{r}{12}\right)^{299}\left(\frac{1}{12}\right) - \left(1+\frac{r}{12}\right)^{300} + 1}{r^2}\right]dr$.

b. With $r = 0.04$, we find $dS = 14{,}864{,}762.53\,dr$. Therefore, if John's account earned 4.1%, it would be worth $dS = 14{,}864{,}762.53(0.001) \approx \$14{,}864.76$ more; if it earned 4.2%, it would be worth $dS = 14{,}864{,}762.53(0.002) \approx \$29{,}729.53$ more, and if it earned 4.3%, it would be worth $dS = 14{,}864{,}762.53(0.003) \approx \$44{,}594.29$ more.

49. True. $dy = f'(x)\,dx = \frac{d}{dx}(ax+b)\,dx = a\,dx$. On the other hand, $\Delta y = f(x+\Delta x) - f(x) = [a(x+\Delta x)+b] - (ax+b) = a\,\Delta x = a\,dx$.

50. True. The percentage change in A is approximately $\dfrac{100\left[f(x+\Delta x) - f(x)\right]}{f(x)} \approx \dfrac{100 f'(x)\,dx}{f(x)}$.

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1. $dy = f'(3)\,dx = 757.87\,(0.01) \approx 7.5787$.

2. $dy = f'(2)\,dx = -0.125639152666\,(-0.04) \approx -0.0050256$.

3. $dy = f'(1)\,dx = 1.04067285926\,(0.03) \approx 0.031220$.

4. $dy = f'(2)\,(-0.02) \approx 9.66379267622\,(-0.02) = -0.19328$.

5. $dy = f'(4)\,(0.1) \approx -0.198761598\,(0.1) = -0.01988$.

6. $dy = f'(3)\,(-0.05) \approx 12.3113248654\,(-0.05) = -0.6155662$.

7. If the interest rate changes from 5% to 5.3% per year, the monthly payment will increase by $dP = f'(0.05)\,(0.003) \approx 44.00$, or approximately \$44.00 per month. If the rate changes from 5% to 5.4% per year, the payment will increase by \$58.67 per month, and if it changes from 5% to 5.5% per year, the payment will increase by \$73.34 per month.

8. $A = \pi r^2$, so $dA = 2\pi r\,dr$. The area of the ring is approximately $dA = 2\pi\,(53{,}200)\,(15)$, or 5,013,982 km^2.

9. $dx = f'(40)\,(2) \approx -0.625$. That is, the quantity demanded will decrease by 625 watches per week.

10. $T'(22{,}000) = 0.0000570472$, so $\Delta T \approx T'(22{,}000)\,\Delta d \approx -0.0285236$. The period changes by $(-0.0285236)\,(24) \approx -0.6845664$, a decrease of approximately 0.69 hours.

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1. a. 0 **b.** nx^{n-1} **c.** $cf'(x)$. **d.** $f'(x) \pm g'(x)$

2. a. $f(x)\,g'(x) + g(x)\,f'(x)$ **b.** $\dfrac{g(x)\,f'(x) - f(x)\,g'(x)}{[g(x)]^2}$

3. a. $g'(f(x))\,f'(x)$ **b.** $n\left[f(x)\right]^{n-1} f'(x)$

4. Marginal cost, marginal revenue, marginal profit, marginal average cost

5. a. $-\dfrac{pf'(p)}{f(p)}$ **b.** Elastic, unitary, inelastic

6. Both sides, dy/dx **7.** $y, dy/dt, a$ **8.** $-\dfrac{f(t)\,f'(t)}{g(t)}, -\dfrac{f(t)\,g'(t)}{g(t)}$