Solutions Section 1.1

Section 1.1

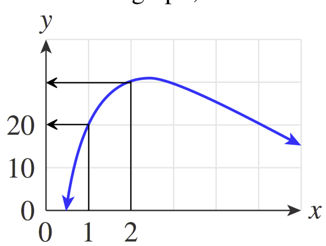
1. Using the table: **a.** $f(0) = 2$ **b.** $f(2) = -0.5$

2. Using the table: **a.** $f(-1) = 4$ **b.** $f(1) = -1$

3. Using the table: **a.** $f(2) - f(-2) = -0.5 - 2 = -2.5$ **b.** $f(-1)f(-2) = (4)(2) = 8$
c. $-2f(-1) = -2(4) = -8$

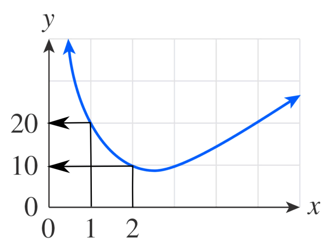
4. Using the table: **a.** $f(1) - f(-1) = -1 - 4 = -5$ **b.** $f(1)f(-2) = (-1)(2) = -2$ **c.** $3f(-2) = 3(2) = 6$

5. From the graph, we estimate: **a.** $f(1) = 20$ **b.** $f(2) = 30$



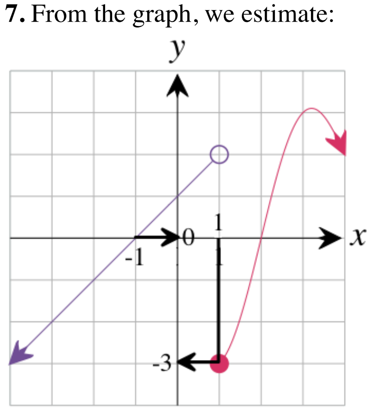
In a similar way, we find: **c.** $f(3) = 30$ **d.** $f(5) = 20$ **e.** $f(3) - f(2) = 30 - 30 = 0$ **f.** $f(3-2) = f(1) = 20$

6. From the graph, we estimate: **a.** $f(1) = 20$ **b.** $f(2) = 10$



In a similar way, we find: **c.** $f(3) = 10$ **d.** $f(5) = 20$ **e.** $f(3) - f(2) = 10 - 10 = 0$ **f.** $f(3-2) = f(1) = 20$

7. From the graph, we estimate: **a.** $f(-1) = 0$ **b.** $f(1) = -3$ since the solid dot is on $(1, -3)$.



In a similar way, we estimate **c.** $f(3) = 3$ **d.** Since $f(3) = 3$ and $f(1) = -3$, $\dfrac{f(3) - f(1)}{3 - 1} = \dfrac{3 - (-3)}{3 - 1} = 3.$

8. From the graph, we estimate: **a.** $f(-3) = 3$ **b.** $f(-1) = -2$ since the solid dot is on $(-1, -2)$.

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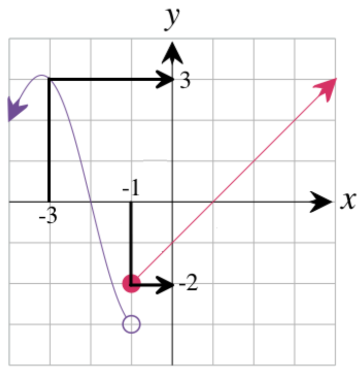

In a similar way, we estimate **c.** $f(1) = 0$ **d.** Since $f(3) = 2$ and $f(1) = 0$, $\dfrac{f(3) - f(1)}{3 - 1} = \dfrac{2 - 0}{3 - 1} = 1$.

9. $f(x) = x - \dfrac{1}{x^2}$, with its natural domain.

The natural domain consists of all x for which $f(x)$ makes sense: all real numbers other than 0.

a. Since 4 is in the natural domain, $f(4)$ is defined, and $f(4) = 4 - \dfrac{1}{4^2} = 4 - \dfrac{1}{16} = \dfrac{63}{16}$.

b. Since 0 is not in the natural domain, $f(0)$ is not defined.

c. Since -1 is in the natural domain, $f(-1) = -1 - \dfrac{1}{(-1)^2} = -1 - \dfrac{1}{1} = -2$.

10. $f(x) = \dfrac{2}{x} - x^2$, with domain $[2, +\infty)$

a. Since 4 is in $[2, +\infty)$, $f(4)$ is defined, and $f(4) = \dfrac{2}{4} - 4^2 = \dfrac{1}{2} - 16 = -\dfrac{31}{2}$.

b. Since 0 is not in $[2, +\infty)$, $f(0)$ is not defined. **c.** Since 1 is not in $[2, +\infty)$, $f(1)$ is not defined.

11. $f(x) = \sqrt{x + 10}$, with domain $[-10, 0)$

a. Since 0 is not in $[-10, 0)$, $f(0)$ is not defined. **b.** Since 9 is not in $[-10, 0)$, $f(9)$ is not defined.

c. Since -10 is in $[-10, 0)$, $f(-10)$ is defined, and $f(-10) = \sqrt{-10 + 10} = \sqrt{0} = 0$

12. $f(x) = \sqrt{9 - x^2}$, with domain $(-3, 3)$

a. Since 0 is in $(-3, 3)$, $f(0)$ is defined, and $f(0) = \sqrt{9 - 0} = 3$.

b. Since 3 is not in $(-3, 3)$, $f(3)$ is not defined. **c.** Since -3 is not in $(-3, 3)$, $f(-3)$ is not defined.

13. $f(x) = 4x - 3$

a. $f(-1) = 4(-1) - 3 = -4 - 3 = -7$ **b.** $f(0) = 4(0) - 3 = 0 - 3 = -3$

c. $f(1) = 4(1) - 3 = 4 - 3 = 1$ **d.** Substitute y for x to obtain $f(y) = 4y - 3$

e. Substitute $(a + b)$ for x to obtain $f(a + b) = 4(a + b) - 3$.

14. $f(x) = -3x + 4$

a. $f(-1) = -3(-1) + 4 = 3 + 4 = 7$ **b.** $f(0) = -3(0) + 4 = 0 + 4 = 4$

c. $f(1) = -3(1) + 4 = -3 + 4 = 1$ **d.** Substitute y for x to obtain $f(y) = -3y + 4$

e. Substitute $(a + b)$ for x to obtain $f(a + b) = -3(a + b) + 4$.

15. $f(x) = x^2 + 2x + 3$

a. $f(0) = (0)^2 + 2(0) + 3 = 0 + 0 + 3 = 3$ **b.** $f(1) = 1^2 + 2(1) + 3 = 1 + 2 + 3 = 6$

c. $f(-1) = (-1)^2 + 2(-1) + 3 = 1 - 2 + 3 = 2$ **d.** $f(-3) = (-3)^2 + 2(-3) + 3 = 9 - 6 + 3 = 6$

e. Substitute a for x to obtain $f(a) = a^2 + 2a + 3$. **f.** Substitute $(x + h)$ for x to obtain $f(x + h) = (x + h)^2 + 2(x + h) + 3$.

16. $g(x) = 2x^2 - x + 1$

a. $g(0) = 2(0)^2 - 0 + 1 = 0 - 0 + 1 = 1$ **b.** $g(-1) = 2(-1)^2 - (-1) + 1 = 2 + 1 + 1 = 4$

c. Substitute r for x to obtain $g(r) = 2r^2 - r + 1$.

d. Substitute $(x + h)$ for x to obtain $g(x + h) = 2(x + h)^2 - (x + h) + 1$.

17. $g(s) = s^2 + \frac{1}{s}$

a. $g(1) = 1^2 + \frac{1}{1} = 1 + 1 = 2$ **b.** $g(-1) = (-1)^2 + \frac{1}{-1} = 1 - 1 = 0$

c. $g(4) = 4^2 + \frac{1}{4} = 16 + \frac{1}{4} = \frac{65}{4}$ or 16.25 **d.** Substitute x for s to obtain $g(x) = x^2 + \frac{1}{x}$

e. Substitute $(s + h)$ for s to obtain $g(s + h) = (s + h)^2 + \frac{1}{s + h}$

f. $g(s + h) - g(s) =$ Answer to part (e) $-$ Original function $= \left((s + h)^2 + \frac{1}{s + h}\right) - \left(s^2 + \frac{1}{s}\right)$

18. $h(r) = \frac{1}{r + 4}$

a. $h(0) = \frac{1}{0 + 4} = \frac{1}{4}$ **b.** $h(-3) = \frac{1}{(-3) + 4} = \frac{1}{1} = 1$

c. $h(-5) = \frac{1}{(-5) + 4} = \frac{1}{(-1)} = -1$ **d.** Substitute x^2 for r to obtain $h(x^2) = \frac{1}{x^2 + 4}$.

e. Substitute $(x^2 + 1)$ for r to obtain $h(x^2 + 1) = \frac{1}{(x^2 + 1) + 4} = \frac{1}{x^2 + 5}$.

f. $h(x^2) + 1 =$ Answer to part (d) $+ 1 = \frac{1}{x^2 + 4} + 1$

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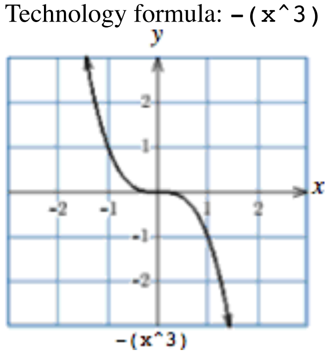
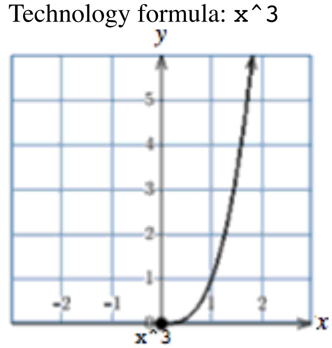
19. $f(x) = -x^3$ (domain $(-\infty, +\infty)$)

Technology formula: `-(x^3)`



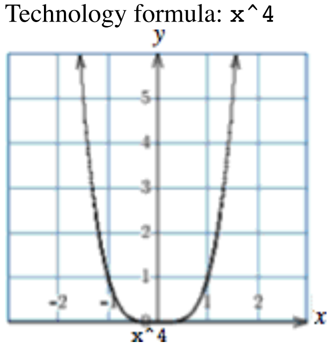
20. $f(x) = x^3$ (domain $[0, +\infty)$)

Technology formula: `x^3`



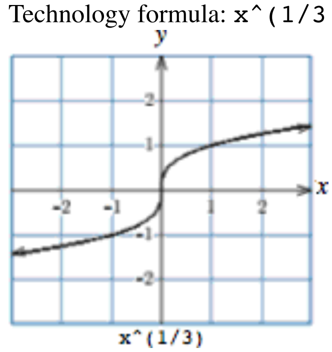
21. $f(x) = x^4$ (domain $(-\infty, +\infty)$)

Technology formula: `x^4`



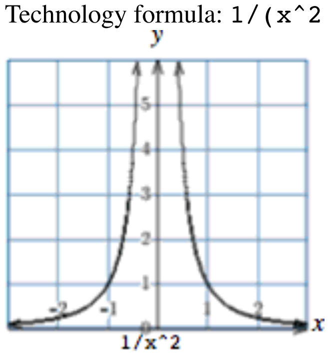
22. $f(x) = \sqrt[3]{x}$ (domain $(-\infty, +\infty)$)

Technology formula: `x^(1/3)`



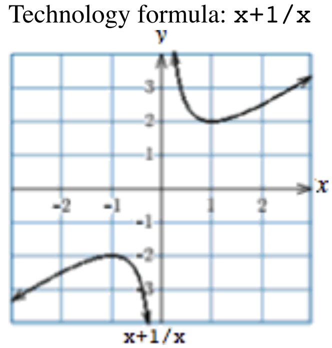
23. $f(x) = \dfrac{1}{x^2}$ $(x \neq 0)$

Technology formula: `1/(x^2)`



24. $f(x) = x + \dfrac{1}{x}$ $(x \neq 0)$

Technology formula: `x+1/x`



25. a. $f(x) = x$ $(-1 \le x \le 1)$

Since the graph of $f(x) = x$ is a diagonal 45° line through the origin inclining up from left to right, the correct graph is (A).

b. $f(x) = -x$ $(-1 \le x \le 1)$

Since the graph of $f(x) = -x$ is a diagonal 45° line through the origin inclining down from left to right, the correct graph is (D).

c. $f(x) = \sqrt{x}$ $(0 < x < 4)$

Since the graph of $f(x) = \sqrt{x}$ is the top half of a sideways parabola, the correct graph is (E).

d. $f(x) = x + \dfrac{1}{x} - 2$ $(0 < x < 4)$

If we plot a few points like $x = 1/2,\ 1, 2,$ and 3, we find that the correct graph is (F).

e. $f(x) = |x|\ \ (-1 \le x \le 1)$

Since the graph of $f(x) = |x|$ is a "V"-shape with its vertex at the origin, the correct graph is (C).

f. $f(x) = x - 1\ \ (-1 \le x \le 1)$

Since the graph of $f(x) = x - 1$ is a straight line through $(0, -1)$ and $(1, 0)$, the correct graph is (B).

26. a. $f(x) = -x + 3\ \ (0 < x \le 3)$

Since the graph of $f(x) = -x + 3$ is a straight line inclining down from left to right, the correct graph must be (D).

b. $f(x) = 2 - |x|\ \ (-2 < x \le 2)$

Since $f(x) = 2 - |x|$ is obtained from the graph of $y = |x|$ by flipping it vertically (the minus sign in front of $|x|$) and then moving it 2 units vertically up (adding 2 to all the values), the correct graph is (F).

c. $f(x) = \sqrt{x+2}\ \ (-2 < x \le 2)$

The graph of $f(x) = \sqrt{x+2}$ is similar to that of $y = \sqrt{x}$, which is half a parabola on its side, and the correct graph is (A).

d. $f(x) = -x^2 + 2\ \ (-2 < x \le 2)$

The graph of $f(x) = -x^2 + 2$ is a parabola opening down, so the correct graph is (C).

e. $f(x) = \dfrac{1}{x} - 1$

The graph of $f(x) = \dfrac{1}{x} - 1\ \ (0 < x \le 3)$ is part of a hyperbola, and the correct graph is (E).

f. $f(x) = x^2 - 1\ \ (-2 < x \le 2)$

The graph of $f(x) = x^2 - 1$ is a parabola opening up, so the correct graph is (B).

27. Technology formula: `0.1*x^2 - 4*x+5`
Table of values:

x	0	1	2	3	4	5	6	7	8	9	10
$f(x)$	5	1.1	−2.6	−6.1	−9.4	−12.5	−15.4	−18.1	−20.6	−22.9	−25

28. Technology formula: `0.4*x^2-6*x-0.1`
Table of values:

x	−5	−4	−3	−2	−1	0	1	2	3	4	5
$g(x)$	39.9	30.3	21.5	13.5	6.3	−0.1	−5.7	−10.5	−14.5	−17.7	−20.1

29. Technology formula: `(x^2-1)/(x^2+1)`
Table of values:

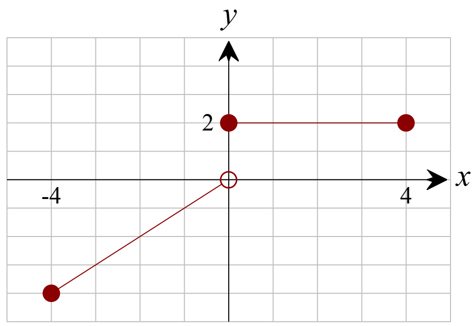
x	0.5	1.5	2.5	3.5	4.5	5.5	6.5	7.5	8.5	9.5	10.5
$h(x)$	−0.6000	0.3846	0.7241	0.8491	0.9059	0.9360	0.9538	0.9651	0.9727	0.9781	0.9820

30. Technology formula: `(2*x^2+1)/(2*x^2-1)`
Table of values:

x	−1	0	1	2	3	4	5	6	7	8	9
$r(x)$	3.0000	−1.0000	3.0000	1.2857	1.1176	1.0645	1.0408	1.0282	1.0206	1.0157	1.0124

31. $f(x) = \begin{cases} x & \text{if } -4 \le x < 0 \\ 2 & \text{if } 0 \le x \le 4 \end{cases}$

Technology formula: `x*(x<0)+2*(x>=0)` (For a graphing calculator, use ≥ instead of >=.)



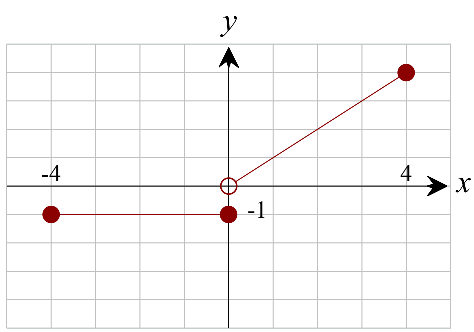
a. $f(-1) = -1$. We used the first formula, since -1 is in $[-4, 0)$.

b. $f(0) = 2$. We used the second formula, since 0 is in $[0, 4]$.

c. $f(1) = 2$. We used the second formula, since 1 is in $[0, 4]$.

32. $f(x) = \begin{cases} -1 & \text{if } -4 \le x \le 0 \\ x & \text{if } 0 < x \le 4 \end{cases}$

Technology formula: `(-1)*(x<=0)+x*(x>0)` (For a graphing calculator, use ≤ instead of <=.)



a. $f(-1) = -1$. We used the first formula, since -1 is in $[-4, 0]$.

b. $f(0) = -1$. We used the first formula, since 0 is in $[-4, 0]$.

c. $f(1) = 1$. We used the second formula, since 1 is in $(0, 4]$.

33. $f(x) = \begin{cases} x^2 & \text{if } -2 < x \le 0 \\ 1/x & \text{if } 0 < x \le 4 \end{cases}$

Technology formula: `(x^2)*(x<=0)+(1/x)*(0<x)` (For a graphing calculator, use ≤ instead of <=.)

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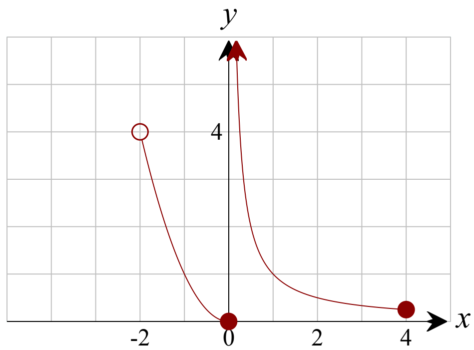


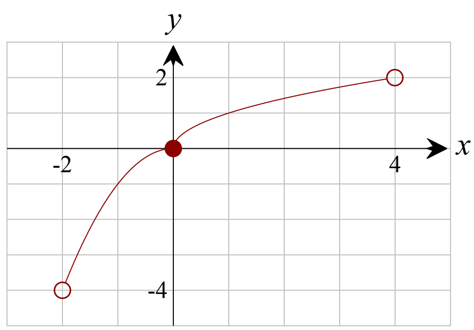
a. $f(-1) = 1^2 = 1$. We used the first formula, since -1 is in $(-2, 0]$.

b. $f(0) = 0^2 = 0$. We used the first formula, since 0 is in $(-2, 0]$.

c. $f(1) = 1/1 = 1$. We used the second formula, since 1 is in $(0, 4]$.

34. $f(x) = \begin{cases} -x^2 & \text{if } -2 < x \leq 0 \\ \sqrt{x} & \text{if } 0 < x < 4 \end{cases}$

Technology formula: Excel: `(-1*x^2)*(x<=0)+SQRT(ABS(x))*(x>0)`
TI-83/84 Plus: `(-1*x^2)*(x\leq0)+ √(x)*(x>0)`



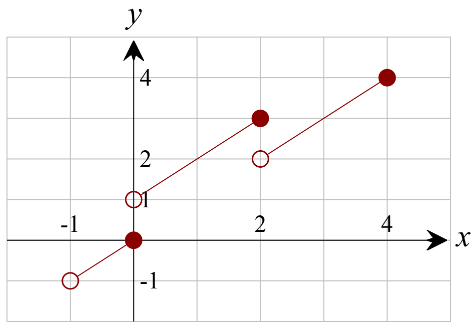
a. $f(-1) = -(-1)^2 = -1$. We used the first formula, since -1 is in $(-2, 0]$.

b. $f(0) = -0^2 = 0$. We used the first formula, since 0 is in $(-2, 0]$.

c. $f(1) = \sqrt{1} = 1$. We used the second formula, since 1 is in $(0, 4)$.

35. $f(x) = \begin{cases} x & \text{if } -1 < x \leq 0 \\ x+1 & \text{if } 0 < x \leq 2 \\ x & \text{if } 2 < x \leq 4 \end{cases}$

Technology formula: `x*(x<=0)+(x+1)*(0<x)*(x<=2)+x*(2<x)` (For a graphing calculator, use ≤ instead of <=.)



a. $f(0) = 0$. We used the first formula, since 0 is in $(-1, 0]$.

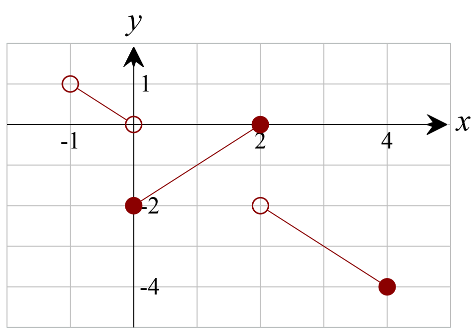
b. $f(1) = 1 + 1 = 2$. We used the second formula, since 1 is in $(0, 2]$.

c. $f(2) = 2 + 1 = 3$. We used the second formula, since 2 is in $(0, 2]$.

d. $f(3) = 3$. We used the third formula, since 3 is in $(2, 4]$.

36. $f(x) = \begin{cases} -x & \text{if } -1 < x < 0 \\ x - 2 & \text{if } 0 \le x \le 2 \\ -x & \text{if } 2 < x \le 4 \end{cases}$

Technology formula: `x*(x<0)+(x-2)*(0<=x)*(x<=2)+(-x)*(2<x)` (For a graphing calculator, use ≤ instead of <=.)



a. $f(0) = 0 - 2 = -2$. We used the second formula, since 0 is in $[0, 2]$.

b. $f(1) = 1 - 2 = -1$. We used the second formula, since 1 is in $[0, 2]$.

c. $f(2) = 2 - 2 = 0$. We used the second formula, since 2 is in $[0, 2]$.

d. $f(3) = -3$. We used the third formula, since 3 is in $(2, 4]$.

37. $f(x) = x^2$

a. $f(x + h) = (x + h)^2$ Therefore,

$$\begin{aligned} f(x+h) - f(x) &= (x+h)^2 - x^2 \\ &= x^2 + 2xh + h^2 - x^2 \\ &= 2xh + h^2 = h(2x + h) \end{aligned}$$

b. Using the answer to part (a),

$$\frac{f(x+h)-f(x)}{h}=\frac{h(2x+h)}{h}=2x+h$$

38. $f(x)=3x-1$

a. $f(x+h)=3(x+h)-1=3x+3h-1$ Therefore,

$$\begin{aligned}f(x+h)-f(x)&=3x+3h-1-(3x-1)\\&=3x+3h-1-3x+1=3h\end{aligned}$$

b. Using the answer to part (a),

$$\frac{f(x+h)-f(x)}{h}=\frac{3h}{h}=3$$

39. $f(x)=2-x^2$

a. $f(x+h)=2-(x+h)^2$ Therefore,

$$\begin{aligned}f(x+h)-f(x)&=2-(x+h)^2-(2-x^2)\\&=2-x^2-2xh-h^2-2+x^2\\&=-2xh-h^2=-h(2x+h)\end{aligned}$$

b. Using the answer to part (a),

$$\frac{f(x+h)-f(x)}{h}=\frac{-h(2x+h)}{h}=-(2x+h)$$

40. $f(x)=x^2+x$

a. $f(x+h)=(x+h)^2+(x+h)$ Therefore,

$$\begin{aligned}f(x+h)-f(x)&=(x+h)^2+(x+h)-(x^2+x)\\&=x^2+2xh+h^2+x+h-x^2-x\\&=2xh+h^2+h=h(2x+h+1)\end{aligned}$$

b. Using the answer to part (a),

$$\frac{f(x+h)-f(x)}{h}=\frac{h(2x+h+1)}{h}=2x+h+1$$

Applications

41. From the table,

a. $p(2)=2.95$; Pemex produced 2.95 million barrels of crude oil per day in 2010 ($t=2$).

$p(3)=2.94$; Pemex produced 2.94 million barrels of crude oil per day in 2011 ($t=3$).

$p(6)=2.79$; Pemex produced 2.79 million barrels of crude oil per day in 2014 ($t=6$).

b. $p(4)-p(2)=2.91-2.95=-0.04$; Crude oil production by Pemex decreased by 0.04 million barrels/day from 2010 ($t=2$) to 2012 ($t=4$).

42. From the table,

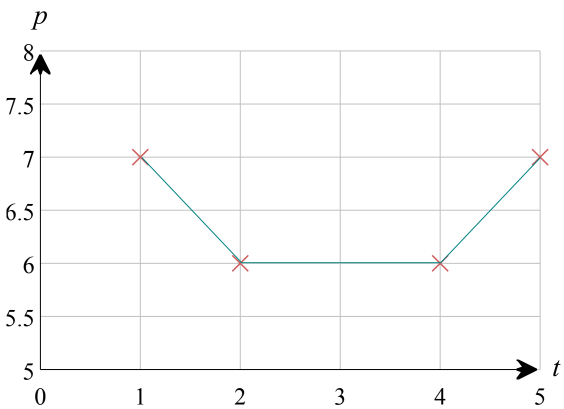
a. $s(0)=2.25$; Pemex produced 2.25 million barrels of offshore crude oil per day in 2008 ($t=0$).

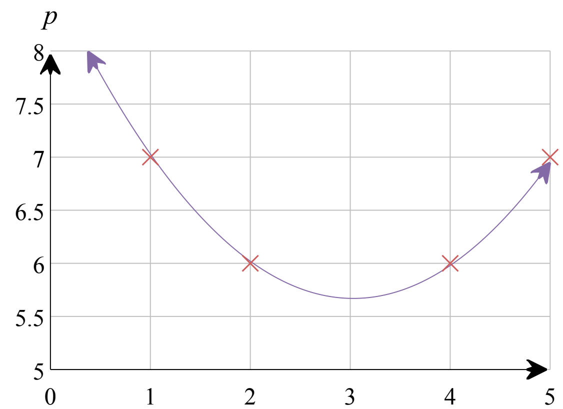
$s(2)=1.94$; Pemex produced 1.94 million barrels of offshore crude oil per day in 2010 ($t=2$).

$s(4)=1.90$; Pemex produced 1.90 million barrels of offshore crude oil per day in 2012 ($t=4$).

b. $s(4) - s(0) = 1.90 - 2.25 = -0.35$; Offshore crude oil production by Pemex decreased by 0.35 million barrels/day from 2008 ($t = 0$) to 2012 ($t = 4$).

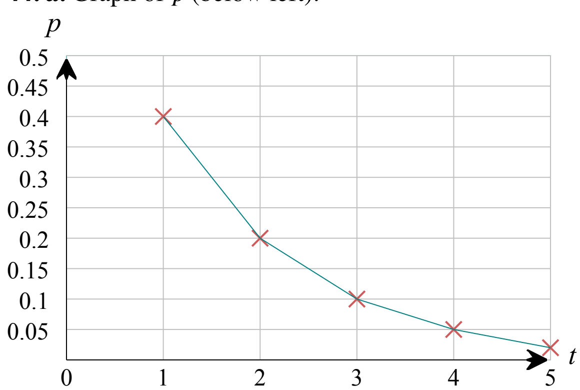
43. a. Graph of p (below left):

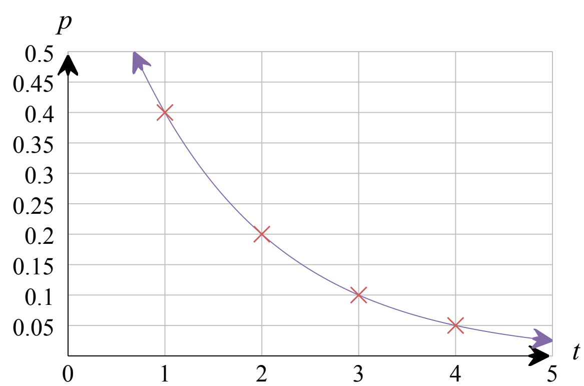




From the graph, $p(4.5) \approx 6.5$. Interpretation: $t = 4.5$ represents 4.5 years since the start of 2008, or midway through 2012. Thus, we interpret the answer as follows: The popularity of Twitter midway through 2012 was about 6.5%.
b. The four points suggest a "u"-shaped curve such as a parabola, and only Choice (D) is of this type. (Choice (B) gives an "upside-down" (concave down) parabola.)

44. a. Graph of p (below left):





From the graph, $p(3.5) \approx 0.075$. Interpretation: $t = 3.5$ represents 3.5 years since the start of 2008, or midway through 2011. Thus, we interpret the answer as follows: The popularity of Delicious midway through 2011 was about 0.075%.
b. Referring to the table of common functions at the end of Section 1.1, we see that the plotted points suggest an exponential curve that decreases with increaing t, and only Choice (A) is of this type. (Choice (B) gives an exponential curve that increases with t.) Moreover, Choice (A) gives an almost exact fit to the data.

45. From the graph, $f(7) \approx 1{,}000$. Because f is the number of thousands of housing starts in year t, we interpret the result as follows: Approximately 1,000,000 homes were started in 2007.
Similarly, $f(14) \approx 600$: Approximately 600,000 homes were started in 2014.
Also, we estimate $f(9.5) \approx 450$. Because $t = 9.5$ is midway between 2009 and 2010, we interpret the result as follows: 450,000 homes were started in the year beginning July 2009.

46. From the graph, $f(3) \approx 1{,}500$, $f(6) \approx 1{,}500$, and $f(8.5) \approx 500$. Because f is the number of thousands of housing starts in year t, we interpret the result as follows:

$f(3) \approx 1{,}500$: 1.5 million homes were started in 2003.

$f(6) \approx 1{,}500$: 1.5 million homes were started in 2006.

$f(8.5) \approx 500$: 500,000 homes were started in the year beginning July 2008.

47. $f(7-3) = f(4) \approx 1{,}600$ Interpretation: 1,600,000 homes were started in 2004 ($t = 4$).

$f(7) - f(3) = 1{,}000 - 1{,}500 = -500$

Interpretation:

$f(7) - f(3)$ is the change in the number of housing starts (in thousands) from 2003 to 2007; there were 500,000 fewer housing starts in 2007 than in 2003.

48. $f(13-3) = f(10) \approx 500$ Interpretation: 500,000 homes were started in 2010 ($t = 10$).

$f(13) - f(3) = 600 - 1{,}500 = -900$

Interpretation:

$f(13) - f(3)$ is the change in the number of housing starts (in thousands) from 2003 to 2012; there were 900,000 fewer housing starts in 2012 than in 2003.

49. $f(t+5) - f(t)$ measures the change from year t to the year five years later. It is greatest when the the line segment from year t to year $t + 5$ is steepest upward-sloping. From the graph, this occurs when $t = 0$, for a change of $1{,}700 - 1{,}200 = 500$. Interpretation: The greatest five-year increase in the number of housing starts occurred in 2000–2005.

50. $f(t) - f(t-1)$ measures the change from year $t - 1$ to the following year. It is least when the the line segment from year $t - 1$ to year t is steepest downward-sloping. From the graph, this occurs when $t = 7$, for a change of $1{,}000 - 1{,}500 = -500$. Interpretation: The greatest annual decrease in the number of housing starts occurred in 2006–2007.

51. a. From the graph, $n(2) \approx 400$, $n(4) \approx 400$, $n(4.5) \approx 350$. Because $n(t)$ is Abercrombie's net income in the year ending $t + 2004$, we interpret the results as follows:

Abercrombie's net income was $400 million in 2006.

Abercrombie's net income was $400 million in 2008.

Abercrombie's net income was $350 million in the year ending June 2009 (because $t = 4.5$ represents June 2009).

b. Increasing most rapidly at $t \approx 8$ (over the interval $[3, 8]$ the graph is steepest upward-sloping at around $t = 8$.) Interpretation: Between Dec. 2007 and Dec. 2012 Abercrombie's net income was increasing most rapidly in December 2012.

c. Decreasing most rapidly at $t \approx 5$ (over the interval $[3, 8]$ the graph is steepest downward-sloping at around $t = 5$.) Interpretation: Between Dec. 2007 and Dec. 2012, Abercrombie's net income was decreasing most rapidly in Dec. 2009.

52. a. From the graph,

$n(0) \approx 100, n(4) \approx 0, n(5.5) \approx -75$

Because $n(t)$ is Pacific Sunwear's net income in the year ending $t + 2004$, we interpret the results as follows:

Pacific Sunwear's net income was $100 million in 2004.

Pacific Sunwear's net income was zero in 2008.

Pacific Sunwear lost $75 million in the year ending June 2010 ($t = 5.5$ represents June 2010).

b. Increasing most rapidly at $t = 9$ (the graph is steepest upward-sloping at $t = 9$.) Interpretation: Pacific Sunwear's net income was increasing most rapidly in 2013.

c. decreasing most rapidly at $t = 4$ (the graph is steepest downward-sloping at $t = 4$.) Interpretation: Pacific Sunwear's net income was decreasing most rapidly in 2008.

Solutions Section 1.1

53. a. The model is valid for the range 1958 ($t = 0$) through 1966 ($t = 8$). Thus, an appropriate domain is $[0, 8]$. $t \geq 0$ is not an appropriate domain because it would predict federal funding of NASA beyond 1966, whereas the model is based only on data up to 1966.

b. $p(t) = \dfrac{4.5}{1.07^{(t-8)^2}}$

$$\Rightarrow \qquad p(5) = \frac{4.5}{1.07^{(5-8)^2}} \approx 2.4$$

Technology formula: `4.5/(1.07^((t-8)^2))`

$t = 5$ represents $1958 + 5 = 1963$, and therefore we interpret the result as follows: In 1963, 2.4% of the U.S. federal budget was allocated to NASA.

c. $p(t)$ is increasing most rapidly when the graph is steepest upward-sloping from left to right, and, among the given values of t, this occurs when $t = 5$. Thus, the percentage of the budget allocated to NASA was increasing most rapidly in 1963.

54. a. $[1, 50]$; $[0, 50]$ is not an appropriate domain because p is undefined at 0.

b. $p(t) = 0.03 + \dfrac{5}{t^{0.6}}$

$$\Rightarrow \qquad p(40) = 0.03 + \frac{5}{40^{0.6}} \approx 0.58$$

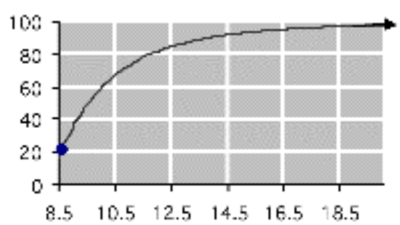
Technology formula: `0.03+5/t^0.6`

$t = 40$ represents $1965 + 40 = 2005$, and therefore we interpret the result as follows: In 2005, 0.58% of the US federal budget was allocated to NASA.

c. If we evaluate $p(t)$ for $t = 100, 1{,}000, 100{,}000, 1{,}000{,}000...,$ we find values of $p(t)$ decreasing toward 0.03. Thus, in the (very) long term, the percentage of the budget allocated to NASA is predicted to approach 0.03%

55. $p(t) = 100\left(1 - \dfrac{12{,}200}{t^{4.48}}\right)$ $\quad (t \geq 8.5)$ **a.** Technology formula: `100*(1-12200/t^4.48)`

b. Graph:



c. Table of values:

t	9	10	11	12	13	14	15	16	17	18	19	20
$p(t)$	35.2	59.6	73.6	82.2	87.5	91.1	93.4	95.1	96.3	97.1	97.7	98.2

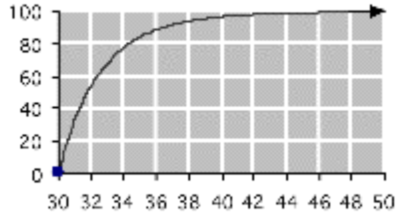
d. From the table, $p(12) = 82.2$, so that 82.2% of children are able to speak in at least single words by the age of 12 months.

e. We seek the first value of t such that $p(t)$ is at least 90. Since $t = 14$ has this property ($p(14) = 91.1$) we conclude that, at 14 months, 90% or more children are able to speak in at least single words.

56. $p(t) = 100\left(1 - \dfrac{5.27 \times 10^{17}}{t^{12}}\right)$ $\quad (t \geq 30)$

a. Technology formula: `100*(1-5.27*10^17/t^12)`

b. Graph:



c. Table of values:

t	30	31	32	33	34	35	36	37	38	39	40
$p(t)$	0.8	33.1	54.3	68.4	77.9	84.4	88.9	92.0	94.2	95.7	96.9

d. From the table, $p(36) = 88.9$, so that 88.9% of children are able to speak in sentences of five or more words by the age of 36 months.

e. We seek the first value of t such that $p(t)$ is at least 75. Since $t = 34$ has this property ($p(34) = 77.9$) we conclude that, at 34 months, 75% or more children are able to speak in sentences of five or more words.

57. $v(t) = \begin{cases} 8(1.22)^t & \text{if } 0 \le t < 16 \\ 400t - 6{,}200 & \text{if } 16 \le t < 25 \\ 3800 & \text{if } 25 \le t \le 30. \end{cases}$

a. $v(10) = 8(1.22)^{10} \approx 58$. We used the first formula, since 10 is in [0, 16).

$v(16) = 400(16) - 6,200 = 200$. We used the second formula, since 16 is in [16, 25).

$v(28) = 3,800$. We used the third formula, since 28 is in [25, 30].

Interpretation: Processor speeds were about 58 MHz in 1990, 200 MHz in 1996, and 3800 MHz in 2008.

b. Technology formula (using x as the independent variable):

```
(8*(1.22)^x)*(x<16)+(400*x-6200)*(x>=16)*(x<25)+3800*(x>=25)
```

(For a graphing calculator, use ≤ instead of <=.)

c. Using the above technology formula (for instance, on the Function Evaluator and Grapher on the Web site) we obtain the graph and table of values. Graph:

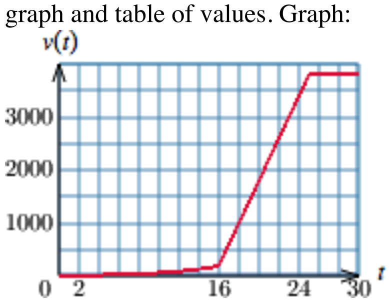


Table of values:

t	0	2	4	6	8	10	12	14	16	18	20	22	24	26	28	30
$v(t)$	8	12	18	26	39	58	87	129	200	1,000	1,800	2,600	3,400	3,800	3,800	3,800

d. From either the graph or the table, we see that the speed reached 3,000 MHz around $t = 23$. We can obtain a more precise answer algebraically by using the formula for the corresponding portion of the graph:

$$3,000 = 400t - 6,200$$

giving

$$t = \frac{9,200}{400} = 23$$

Since t is time since 1980, $t = 23$ corresponds to 2003.

58. $v(t) = \begin{cases} 0.12t^2 + 0.04t + 0.2 & \text{if } 0 \le t < 12 \\ 1.1(1.22)^t & \text{if } 12 \le t < 26 \\ 400t - 10{,}200 & \text{if } 26 \le t \le 30 \end{cases}$

a. $v(2) = 0.12(2)^2 + 0.04(2) + 0.2 = 0.76$. We used the first formula, since 2 is in $[0, 12)$.

$v(12) = 1.1(1.22)^{12} \approx 12$. We used the second formula, since 12 is in $[12, 26)$.

$v(28) = 400(28) - 10,200 = 1,000$. We used the third formula, since 28 is in $[26, 30]$.

Interpretation: Processor speeds were about 0.76 MHz in 1972, 12 MHz in 1982, and 1,000 MHz in 1998.

b. Technology formula (using x as the independent variable):

```
(0.12*x^2+0.04*x+0.2)*(x<12)+(1.1*(1.22)^x)*(x>=12)*(x<26)+(400*x-10200)*(x>=26)
```

(For a graphing calculator, use ≤ instead of <=.)

c. Using the above technology formula (for instance, on the Function Evaluator and Grapher on the Web site) we obtain the graph and table of values.

Graph:

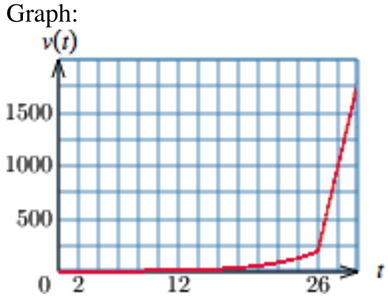


Table of values:

t	0	2	4	6	8	10	12	14	16	18	20	22	24	26	28	30
$v(t)$	0.20	0.76	2.3	4.8	8.2	13	12	18	26	39	59	87	130	200	1,000	1,800

d. From either the graph or the table, we see that the speed reached 500 MHz around $t = 27$. We can obtain a more precise answer algebraically by using the formula for the corresponding portion of the graph:

$500 = 400t - 10,200$

giving

$$t = \frac{10,700}{400} = 26.75 \approx 27 \text{ to the nearest year}$$

Since t is time since 1970, $t = 27$ corresponds to 1997.

59. a. Each row of the table gives us a formula with a condition:
First row in words: 10% of the amount over \$0 if your income is over \$0 and not over \$9,225.
Translation to formula:

$0.10x \quad \text{if } 0 < x \leq 9{,}225.$

Second row in words: \$922.50 + 15% of the amount over \$9,225 if your income is over \$9,225 and not over \$37,450.
Translation to formula:

$922.50 + 0.15(x - 9,225) \quad \text{if } 9{,}225 < x \leq 37{,}450.$

Continuing in this way leads to the following piecewise-defined function:

$$T(x) = \begin{cases} 0.10x & \text{if } 0 < x \leq 9{,}225 \\ 922.50 + 0.15(x - 9{,}225) & \text{if } 9{,}225 < x \leq 37{,}450 \\ 5{,}156.25 + 0.25(x - 37{,}450) & \text{if } 37{,}450 < x \leq 90{,}750 \\ 18{,}481.25 + 0.28(x - 90{,}750) & \text{if } 90{,}750 < x \leq 189{,}300 \\ 46{,}075.25 + 0.33(x - 189{,}300) & \text{if } 189{,}300 < x \leq 411{,}500 \\ 119{,}401.25 + 0.35(x - 411{,}500) & \text{if } 411{,}500 < x \leq 413{,}200 \\ 119{,}996.25 + 0.396(x - 413{,}200) & \text{if } 413{,}200 < x \end{cases}$$

b. A taxable income of \$45,000 falls in the bracket $37{,}450 < x \leq 90{,}750$ and so we use the formula

$5{,}156.25 + 0.25(x - 37{,}450)$:

$$5{,}156.25 + 0.25(45{,}000 - 37{,}450) = 5{,}156.25 + 0.25(7{,}550) = \$7{,}043.75.$$

60. a. Each row of the table gives us a formula with a condition:
First row in words: 10% of the amount over \$0 if your income is over \$0 and not over \$8,700.
Translation to formula:

$0.10x \quad \text{if } 0 < x \leq 8{,}700.$

Second row in words: \$870.00 + 15% of the amount over \$8,700 if your income is over \$8,700 and not over \$35,350.
Translation to formula:

$870.00 + 0.15(x - 8{,}700) \quad \text{if } 8{,}700 < x \leq 35{,}350.$

Continuing in this way leads to the following piecewise-defined function:

$$T(x) = \begin{cases} 0.10x & \text{if } 0 < x \leq 8{,}700 \\ 870.00 + 0.15(x - 8{,}700) & \text{if } 8{,}700 < x \leq 35{,}350 \\ 4{,}867.50 + 0.25(x - 35{,}350) & \text{if } 35{,}350 < x \leq 85{,}650 \\ 17{,}442.50 + 0.28(x - 85{,}650) & \text{if } 85{,}650 < x \leq 178{,}650 \\ 43{,}482.50 + 0.33(x - 178{,}650) & \text{if } 178{,}650 < x \leq 388{,}350 \\ 112{,}683.50 + 0.35(x - 388{,}350) & \text{if } 388{,}350 < x \end{cases}$$

b. A taxable income of \$45,000 falls in the bracket $35{,}350 < x \leq 85{,}650$ and so we use the formula

$4{,}867.50 + 0.25(x - 35{,}350)$:

$$4{,}867.50 + 0.25(45{,}000 - 35{,}350) = 4{,}867.50 + 0.25(9{,}650) = \$7{,}280.00.$$

Communication and reasoning exercises

61. The dependent variable is a function of the independent variable. Here, the market price of gold m is a function of time t. Thus, the independent variable is t and the dependent variable is m.

62. The dependent variable is a function of the independent variable. Here, the weekly profit P is a function of the selling price s. Thus, the independent variable is s and the dependent variable is P.

63. To obtain the function notation, write the dependent variable as a function of the independent variable. Thus $y = 4x^2 - 2$ can be written as

$f(x) = 4x^2 - 2$ or $y(x) = 4x^2 - 2$

64. To obtain the equation notation, introduce a dependent variable instead of the function notation. Thus $C(t) = -0.34t^2 + 0.1t$ can be written as

$c = -0.34t^2 + 0.1t$ or $y = -0.34t^2 + 0.1t$

65. False. A graph usually gives infinitely many values of the function while a numerical table will give only a finite number of values.

66. True. An algebraically specified function f is specified by algebraic formulas for $f(x)$. Given such formulas, we can construct the graph of f by plotting the points $(x, f(x))$ for values of x in the domain of f.

67. False. In a numerically specified function, only certain values of the function are specified so we cannot know its value on every real number in $[0, 10]$, whereas an algebraically specified function would give values for every real number in $[0, 10]$.

68. False. A graphically specified function is specified by a graph. However, we cannot always expect to find an algebraic formula whose graph is exactly the graph that is given.

69. Functions with infinitely many points in their domain (such as $f(x) = x^2$) cannot be specified numerically. So, the assertion is false.

70. A numerical model supplies only the values of a function at specific values of the independent variable, whereas an algebraic model supplies the value of a function at every point in its domain. Thus, an algebraic model supplies more information.

71. As the text reminds us: to evaluate f of a quantity (such as $x + h$) replace x everywhere by the *whole quantity* $x + h$:

$$f(x) = x^2 - 1$$
$$f(x + h) = (x + h)^2 - 1.$$

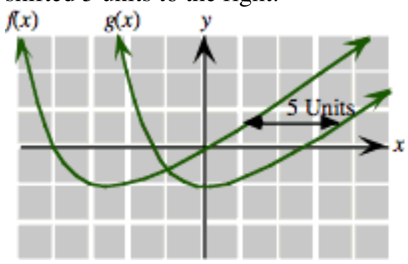
72. Knowing $f(x)$ for two values of x does not convey any information about $f(x)$ at any other value of x. Interpolation is only a way of *estimating* $f(x)$ at values of x not given.

73. If two functions are specified by the same formula $f(x)$, say, their graphs must follow the same curve $y = f(x)$. However, it is the domain of the function that specifies what portion of the curve appears on the graph. Thus, if the functions have different domains, their graphs will be different portions of the curve $y = f(x)$.

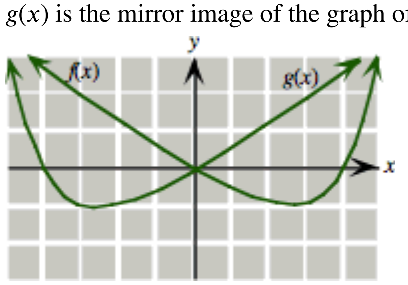
Solutions Section 1.1

74. If we plot points of the graphs $y = f(x)$ and $y = g(x)$, we see that, since $g(x) = f(x) + 10$, we must add 10 to the y-coordinate of each point in the graph of f to get a point on the graph of g. Thus, the graph of g is 10 units higher up than the graph of f.

75. Suppose we already have the graph of f and want to construct the graph of g. We can plot a point of the graph of g as follows: Choose a value for x ($x = 7$, say) and then "look back" 5 units to read off $f(x-5)$ ($f(2)$ in this instance). This value gives the y-coordinate we want. In other words, points on the graph of g are obtained by "looking back 5 units" to the graph of f and then copying that portion of the curve. Put another way, the graph of g is the same as the graph of f, but shifted 5 units to the right:



76. Suppose we already have the graph of f and want to construct the graph of g. We can plot a point of the graph of g as follows: Choose a value for x ($x = 7$, say) and then look on the other side of the y-axis to read off $f(-x)$ ($f(-7)$ in this instance). This value gives the y-coordinate we want. In other words, points on the graph of g are obtained by "looking back" to the graph of f on the opposite side of the y-axis and then copying that portion of the curve. Put another way, the graph of $g(x)$ is the mirror image of the graph of $f(x)$ in the y-axis:



Solutions Section 1.2

Section 1.2

1. $f(x) = x^2 + 1$ with domain $(-\infty, +\infty)$

$g(x) = x - 1$ with domain $(-\infty, +\infty)$

a. $s(x) = f(x) + g(x) = (x^2 + 1) + (x - 1) = x^2 + x$

b. Since both functions are defined for every real number x, the domain of s is the set of all real numbers: $(-\infty, +\infty)$.

c. $s(-3) = (-3)^2 + (-3) = 9 - 3 = 6$

2. $f(x) = x^2 + 1$ with domain $(-\infty, +\infty)$

$g(x) = x - 1$ with domain $(-\infty, +\infty)$

a. $d(x) = g(x) - f(x) = (x - 1) - (x^2 + 1) = -x^2 + x - 2$

b. Since both functions are defined for every real number x, the domain of d is the set of all real numbers: $(-\infty, +\infty)$.

c. $d(-1) = -(-1)^2 + (-1) - 2 = -4$

3. $g(x) = x - 1$ with domain $(-\infty, +\infty)$

$u(x) = \sqrt{x + 10}$ with domain $[-10, 0)$

a. $p(x) = g(x)u(x) = (x - 1)\sqrt{x + 10}$

b. The domain of p consists of all real numbers x simultaneously in the domains of g and u; that is, $[-10, 0)$.

c. $p(-6) = (-6 - 1)\sqrt{-6 + 10} = (-7)(2) = -14$

4. $h(x) = x + 4$ with domain $[10, +\infty)$

$v(x) = \sqrt{10 - x}$ with domain $[0, 10]$

a. $p(x) = h(x)v(x) = (x + 4)\sqrt{10 - x}$

b. The domain of p consists of all real numbers x simultaneously in the domains of h and v; that is, the single point $x = 10$.

c. As 1 is not in the domain of p, $p(1)$ is not defined.

5. $g(x) = x - 1$ with domain $(-\infty, +\infty)$

$v(x) = \sqrt{10 - x}$ with domain $[0, 10]$

a. $q(x) = \dfrac{v(x)}{g(x)} = \dfrac{\sqrt{10 - x}}{x - 1}$

b. The domain of q consists of all real numbers x simultaneously in the domains of v and g such that $g(x) \neq 0$. Since

$g(x) = 0$ when $x - 1 = 0$, or $x = 1$

we exclude $x = 1$ from the domain of the quotient. Thus, the domain consists of all x in $[0, 10]$ excluding $x = 1$ (since $g(1) = 0$), or $0 \leq x \leq 10$; $x \neq 1$.

c. As 1 is not in the domain of q, $q(1)$ is not defined.

6. $g(x) = x - 1$ with domain $(-\infty, +\infty)$

$v(x) = \sqrt{10 - x}$ with domain $[0, 10]$

Solutions Section 1.2

a. $q(x) = \dfrac{g(x)}{v(x)} = \dfrac{x-1}{\sqrt{10-x}}$

b. The domain of q consists of all real numbers x simultaneously in the domains of v and g such that $v(x) \neq 0$. Since

$v(x) = 0$ when $\sqrt{10-x} = 0$, or $x = 10$

we exclude $x = 10$ from the domain of the quotient. Thus, the domain consists of all x in $[0, 10]$ excluding $x = 10$; that is, $[0, 10)$.

c. 1 is in the domain of q, and $q(1) = \dfrac{1-1}{\sqrt{10-1}} = 0$

7. $f(x) = x^2 + 1$ with domain $(-\infty, +\infty)$

a. $m(x) = 5f(x) = 5(x^2 + 1)$

b. The domain of m is the same as the domain of f: $(-\infty, +\infty)$.

c. $m(1) = 5f(1) = 5(1^2 + 1) = 10$

8. $u(x) = \sqrt{x+10}$ with domain $[-10, 0)$

a. $m(x) = 3u(x) = 3\sqrt{x+10}$

b. The domain of m is the same as the domain of u: $[-10, 0)$

c. $m(-1) = 3u(-1) = 3\sqrt{-1+10} = 9$

Applications

9. Number of music files = Starting number + New files = 200 + 10 × Number of days

So, $N(t) = 200 + 10t$ (N = number of music files, t = time in days)

10. Free space left = Current amount − Decrease = 50 − 5 × Number of months

So, $S(t) = 50 - 5t$ (S = space on your HD, t = time in months)

11. Take y to be the width. Since the length is twice the width,

$x = 2y$, so $y = x/2$.

The area is therefore

$A(x) = xy = x(x/2) = x^2/2$.

12. Take y to be the length, so the perimeter is $x + y + x + y = 2(x + y)$

The area is 100 sq. ft., so

$100 = xy$, giving $y = \dfrac{100}{x}$

Thus, the perimeter is

$P(x) = 2(x + y) = 2(x + 100/x)$ or $2x + 200/x$

13. Since the patch is square the width and length are both equal to x. The costs are:

East and West sides: $4x + 4x = 8x$

North and South Sides: $2x + 2x = 4x$

Total cost $C(x) = 8x + 4x = 12x$

14. Since the garden is square the width and length are both equal to x. The costs are:

East and West sides: $2x + 2x = 4x$

South Side: $4x$

Total cost $C(x) = 4x + 4x = 8x$

15. The number of hours you study, $h(n)$, equals 4 on Sunday through Thursday and equals 0 on the remaining days. Since Sunday corresponds to $n = 1$ and Thursday to $n = 5$, we get

$$h(n) = \begin{cases} 4 & \text{if } 1 \le n \le 5 \\ 0 & \text{if } n > 5 \end{cases}.$$

16. The number of hours you watch movies, $h(n)$, equals 5 on Saturday ($n = 7$) and Sunday ($n = 1$) and equals 2 on the remaining days.

$$h(n) = \begin{cases} 5 & \text{if } n = 1 \text{ or } n = 7 \\ 2 & \text{otherwise} \end{cases}.$$

17. For a linear cost function, $C(x) = mx + b$. Here, m = marginal cost = \$1,500 per piano, b = fixed cost = \$1,000. Thus, the daily cost function is

$C(x) = 1{,}500x + 1{,}000.$

a. The cost of manufacturing 3 pianos is

$C(3) = 1{,}500(3) + 1{,}000 = 4{,}500 + 1{,}000 = \$5{,}500.$

b. The cost of manufacturing each additional piano (such as the third one or the 11th one) is the marginal cost, $m = \$1{,}500$.

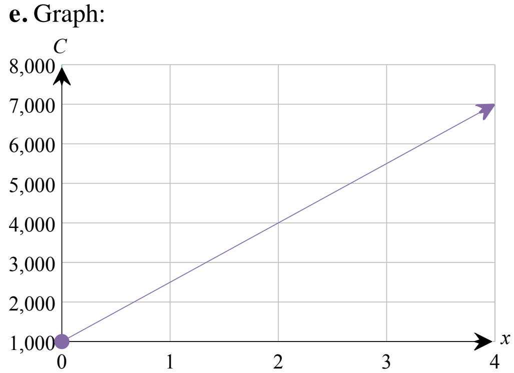
c. Same answer as (b).

d. Variable cost = part of the cost function that depends on $x = \$1{,}500x$

Fixed cost = constant summand of the cost function = \$1,000

Marginal cost = slope of the cost function = \$1,500 per piano

e. Graph:



18. For a linear cost function, $C(x) = mx + b$. Here, m = marginal cost = \$88 per tuxedo, b = fixed cost = \$20. Thus, the cost function is

$C(x) = 88x + 20.$

a. The cost of renting 2 tuxes is

$C(2) = 88(2) + 20 = \$196$

b. The cost of each additional tux is the marginal cost $m = \$88$.

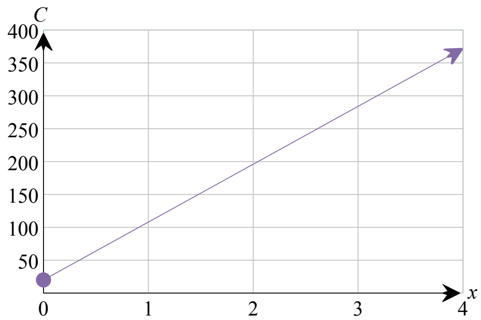
c. Same answer as (b).

d. Variable cost = part of the cost function that depends on x = \$88x

Fixed cost = constant summand of the cost function = \$20

Marginal cost = slope of the cost function = \$88 per tuxedo

e. Graph:



19. a. For a linear cost function, $C(x) = mx + b$. Here, m = marginal cost = \$0.40 per copy, b = fixed cost = \$70.

Thus, the cost function is $C(x) = 0.4x + 70$.

The revenue function is $R(x) = 0.50x$. (x copies at 50¢ per copy)

The profit function is

$$\begin{aligned} P(x) &= R(x) - C(x) \\ &= 0.5x - (0.4x + 70) \\ &= 0.5x - 0.4x - 70 \\ &= 0.1x - 70 \end{aligned}$$

b. $P(500) = 0.1(500) - 70 = 50 - 70 = -20$

Since P is negative, this represents a loss of \$20.

c. For breakeven, $P(x) = 0$:

$$0.1x - 70 = 0$$

$$0.1x = 70$$

$$x = \frac{70}{0.1} = 700 \text{ copies}$$

20. a. For a linear cost function, $C(x) = mx + b$. Here, m = marginal cost = \$0.15 per serving, b = fixed cost = \$350.

Thus, the cost function is $C(x) = 0.15x + 350$.

The revenue function is $R(x) = 0.50x$.

The profit function is

$$\begin{aligned} P(x) &= R(x) - C(x) \\ &= 0.50x - (0.15x + 350) \\ &= 0.35x - 350 \end{aligned}$$

b. For break-even, $P(x) = 0$:

$$0.35x - 350 = 0$$

$$0.35x = 350$$

$$x = 1{,}000 \text{ servings}$$

c. $P(1{,}500) = 0.35(1{,}500) - 350 = 525 - 350 = \$175,$ representing a profit of \$175.

21. The revenue per jersey is \$100. Therefore, Revenue $R(x) = \$100x$.
Profit = Revenue − Cost

$$\begin{aligned} P(x) &= R(x) - C(x) \\ &= 100x - (2{,}000 + 10x + 0.2x^2) \\ &= -2{,}000 + 90x - 0.2x^2 \end{aligned}$$

To break even, $P(x) = 0,$ so $-2{,}000 + 90x - 0.2x^2 = 0.$

This is a quadratic equation with $a = -0.2,\ b = 90,\ c = -2{,}000$ and solution

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-90 \pm \sqrt{(90)^2 - 4(-2{,}000)(-0.2)}}{2(-0.2)} \approx \ 23.44 \text{ or } 426.56 \text{ jerseys.} \end{aligned}$$

Since the second value is outside the domain, we use the first: $x = 23.44$ jerseys. To make a profit, x should be larger than this value: at least 24 jerseys.

22. The revenue per pair is \$120. Therefore, Revenue $R(x) = \$120x$.
Profit = Revenue − Cost

$$\begin{aligned} P(x) &= R(x) - C(x) \\ &= 120x - (3{,}000 + 8x + 0.1x^2) \\ &= -3{,}000 + 112 - 0.1x^2 \end{aligned}$$

To break even, $P(x) = 0,$ so $-3{,}000 + 112 - 0.1x^2 = 0.$

This is a quadratic equation with $a = -0.1,\ b = 112,\ c = -3{,}000$ and solution

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-90 \pm \sqrt{(112)^2 - 4(-3{,}000)(-0.1)}}{2(-0.1)} \approx \ 27.46 \text{ or } 1092.54 \text{ jerseys.} \end{aligned}$$

Since the second value is outside the domain, we use the first: $x = 27.46$ jerseys. To make a profit, x should be larger than this value: at least 28 pairs of cleats.

23. The revenue from one thousand square feet ($x = 1$) is \$0.1 million. Therefore, Revenue $R(x) = \$0.1x$. Profit = Revenue − Cost

$$\begin{aligned} P(x) &= R(x) - C(x) \\ &= 0.1x - (1.7 + 0.12x - 0.0001x^2) \\ &= -1.7 - 0.02x + 0.0001x^2 \end{aligned}$$

To break even, $P(x) = 0,$ so $-1.7 - 0.02x + 0.0001x^2 = 0.$

This is a quadratic equation with $a = 0.0001, b = -0.02, c = -1.7$ and solution

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{0.02 \pm \sqrt{(-0.02)^2 - 4(0.0001)(-1.7)}}{2(0.0001)} = \frac{0.02 \pm 0.03286}{0.0002} \approx 264 \text{ thousand square feet}$$

24. The revenue from one thousand square feet ($x = 1$) is \$0.2 million. Therefore, Revenue $R(x) = \$0.2x$.

Profit = Revenue − Cost

$$P(x) = R(x) - C(x)$$

$$= 0.2x - (1.7 + 0.14x - 0.0001x^2)$$

$$= -1.7 + 0.06x + 0.0001x^2$$

To break even, $P(x) = 0$, so $-1.7 + 0.06x + 0.0001x^2 = 0$.

This is a quadratic equation with $a = 0.0001$, $b = 0.06$, $c = -1.7$ and solution

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-0.06 \pm \sqrt{(-0.06)^2 - 4(0.0001)(-1.7)}}{2(0.0001)} = \frac{-0.06 \pm 0.0654}{0.0002} \approx 27 \text{ thousand square feet}$$

25. The hourly profit function is given by

Profit = Revenue − Cost

$P(x) = R(x) - C(x)$

(Hourly) cost function: This is a fixed cost of \$5,132 only:

$C(x) = 5{,}132$

(Hourly) revenue function: This is a variable of \$100 per passenger cost only:

$R(x) = 100x$

Thus, the profit function is

$P(x) = R(x) - C(x)$

$P(x) = 100x - 5{,}132$

For the domain of $P(x)$, the number of passengers x cannot exceed the capacity: 405. Also, x cannot be negative.

Thus, the domain is given by $0 \le x \le 405$, or $[0, 405]$.

For breakeven, $P(x) = 0$

$100x - 5{,}132 = 0$

$100x = 5{,}132$, or $x = \dfrac{5{,}132}{100} = 51.32$

If x is larger than this, then the profit function is positive, and so there should be at least 52 passengers (note that x must be a whole number); $x \ge 52$, for a profit.

26. The hourly profit function is given by

Profit = Revenue − Cost

$P(x) = R(x) - C(x)$

(Hourly) cost function: This is a fixed cost of \$3,885 only:

$C(x) = 3{,}885$

(Hourly) revenue function: This is a variable of \$100 per passenger cost only:

$R(x) = 100x$

Thus, the profit function is

$P(x) = R(x) - C(x)$

$P(x) = 100x - 3{,}885$

For the domain of $P(x)$, the number of passengers x cannot exceed the capacity: 295. Also, x cannot be negative.

Thus, the domain is given by $0 \le x \le 295$, or $[0, 295]$.

For breakeven, $P(x) = 0$

$100x - 3{,}885 = 0$

$100x = 3{,}885$, or $x = \frac{3{,}885}{100} = 38.85$

If x is larger than this, then the profit function is positive, and so there should be at least 39 passengers (note that x must be a whole number); $x \ge 39$, for a profit.

27. To compute the break-even point, we use the profit function: Profit = Revenue − Cost

$P(x) = R(x) - C(x)$

$R(x) = 2x$ \$2 per unit

$C(x)$ = Variable Cost + Fixed Cost

= 40% of Revenue + 6,000 = $0.4(2x) + 6{,}000 = 0.8x + 6{,}000$

Thus, $P(x) = R(x) - C(x)$

$P(x) = 2x - (0.8x + 6{,}000) = 1.2x - 6{,}000$

For breakeven, $P(x) = 0$

$1.2x - 6{,}000 = 0$

$1.2x = 6{,}000$, so $x = \frac{6{,}000}{1.2x} = 5{,}000$

Therefore, 5,000 units should be made to break even.

28. To compute the break-even point, we use the profit function: Profit = Revenue − Cost

$P(x) = R(x) - C(x)$ n $R(x) = 5x$ \$5 per unit

$C(x)$ = Variable Cost + Fixed Cost

= 30% of Revenue + 7,000 = $0.3(5x) + 7{,}000 = 1.5x + 7{,}000$

Thus, $P(x) = R(x) - C(x)$

$P(x) = 5x - (1.5x + 7{,}000) = 3.5x - 7{,}000$

For breakeven, $P(x) = 0$

$3.5x = 7{,}000$, so $x = 2{,}000$ units.

29. To compute the break-even point, we use the revenue and cost functions:

$R(x)$ = Selling price × Number of units = SPx

$C(x)$ = Variable Cost + Fixed Cost = $VCx + FC$

(Note that "variable cost per unit" is marginal cost.) For breakeven

$R(x) = C(x)$

$SPx = VCx + FC$

Solve for x:

$SPx - VCx = FC$ n $\quad x(SP - VC) = FC,$ so $x = \dfrac{FC}{SP - VC}$.

30. To compute the break-even point, we use the revenue and cost functions:

$R(x) = SPx; C(x) = VCx + FC$

At breakeven

$R(BE) = C(BE)$

$SP(BE) = VC(BE) + FC$

Thus, $FC = SP(BE) - VC(BE) = BE(SP - VC)$.

31. Take x to be the number of grams of perfume he buys and sells. The profit function is given by Profit = Revenue − Cost: that is, $P(x) = R(x) - C(x)$

Cost function $C(x)$:

Fixed costs:	20,000
Cheap perfume @ $20 per g:	$20x$
Transportation @ $30 per 100 g:	$0.3x$

Thus the cost function is

$C(x) = 20x + 0.3x + 20{,}000 = 20.3x + 20{,}000$

Revenue function R(x)

$R(x) = 600x$ $600 per gram

Thus, the profit function is

$P(x) = R(x) - C(x)$

$P(x) = 600x - (20.3x + 20{,}000) = 579.7x - 20{,}000,$ with domain $x \geq 0$.

For breakeven, $P(x) = 0$

$579.7x - 20{,}000 = 0$

$579.7x = 20{,}000,$ so $x = \dfrac{20{,}000}{579.7} \approx 34.50$

Thus, he should buy and sell 34.50 grams of perfume per day to break even.

32. Take x to be the number of grams of perfume he buys and sells. The profit function is given by

Profit = Revenue − Cost

Cost function: $C(x) = 400x + 30x = 430x$

Revenue: $R(x) = 420x$

Thus, the profit function is $P(x) = R(x) - C(x) = 420x - 430x = -10x,$ with domain $x \geq 0$.

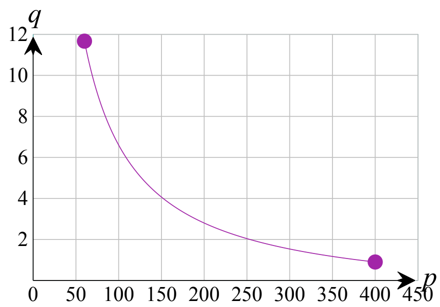
For breakeven, $-10x = 0,$ so $x = 0$ grams per day; he should shut down the operation.

33. a. To graph the demand function we use technology with the formula

```
760/x-1
```

and with xMin = 60 and xMax = 400.

Graph:



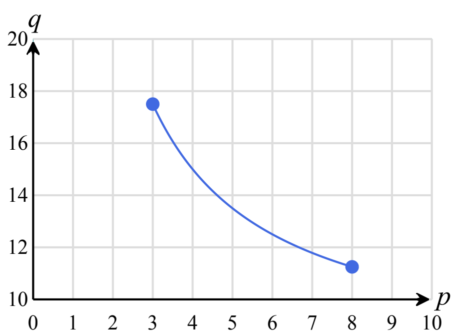
b. $q(100) = \frac{760}{100} - 1 = 7.6 - 1 = 6.6, \quad q(200) = \frac{760}{200} - 1 = 3.8 - 1 = 2.8$

So, the change in demand is $2.8 - 6.6 = -3.8$ million units, which means that demand decreases by about 3.8 million units per year.

c. The value of q on the graph decreases by smaller and smaller amounts as we move to the right on the graph, indicating that the demand decreases at a smaller and smaller rate (Choice (D)).

34. a. To graph the demand function we use technology with the formula `7.5+30/x` and with xMin = 3 and xMax = 8.

Graph:



b. $q(5) = 7.5 + \frac{30}{5} = 13.5 \qquad q(3) = 7.5 + \frac{30}{3} = 17.5$

So, the change in demand is $17.5 - 13.5 = 4$ million rides, which says that ridership increases by about 4 million per day.

c. The graph levels off toward the $q = 7.5$ line, suggesting that ridership decreases toward 7.5 million rides per day.

35. $q(p) = 0.17p^2 - 63p + 5{,}900$

a. Setting $p = 110$ gives

$$q(110) = 0.17(110)^2 - 63(110) + 5{,}900 = 1{,}027 \text{ million units.}$$

b. Setting $p = 90$ gives

$$q(90) = 0.17(90)^2 - 63(90) + 5{,}900 = 1{,}607 \text{ million units.}$$

c. Revenue = Price × Quantity

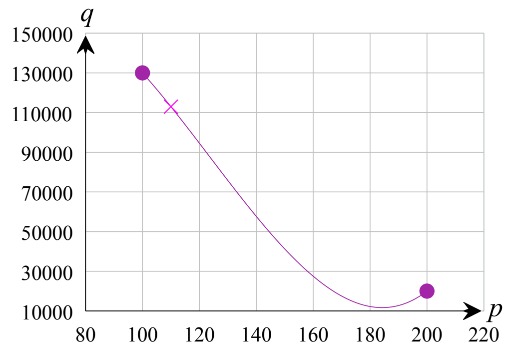
$$R(p) = pq(p) = p(0.17p^2 - 63p + 5{,}900)$$

$$= 0.17p^3 - 63p^2 + 5{,}900p \text{ million dollars/year} \qquad \text{Revenue function}$$

$$R(110) = 0.17(110)^3 - 63(110)^2 + 5{,}900(110)$$

$$= 112{,}970 \approx 113{,}000 \text{ million dollars/year, or \$113 billion/year}$$

d. Graph: Technology formula: `0.17x^3-63x^2+5900x`



The graph shows revenue increasing as p decreases past \$110.

36. $q(p) = 36{,}900(0.968^p)$

a. Setting $p = 120$ gives

$q(120) = 36{,}900(0.968^{120}) \approx 745$ million units.

b. Setting $p = 210$ gives

$q(180) = 36{,}900(0.968^{210}) \approx 40$ million units.

c. Revenue = Price × Quantity

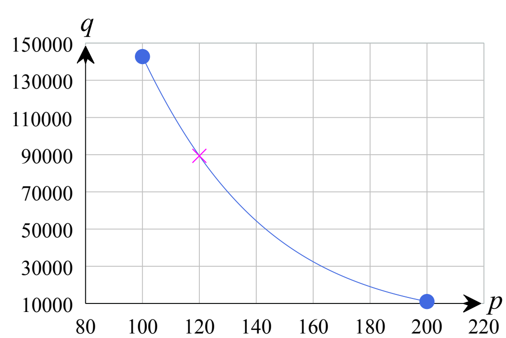
$$R(p) = pq(p) = p(36{,}900(0.968^p))$$

$$= 36{,}900p(0.968^p) \text{ million dollars/year} \qquad \text{Revenue function}$$

$$R(120) = 36{,}900(120)(0.968^{120})$$

$$\approx 89{,}382 \approx 89{,}000 \text{ million dollars/year, or \$89 billion/year}$$

d. Graph: Technology formula: `36900x*.968^x`



As the graph shows revenue increasing as p decreases from \$120, we conclude that increased worldwide revenue result from decreasing the price beyond \$120 dollars.

37. The price at which there is neither a shortage nor surplus is the equilibrium price, which occurs when demand = supply:

$-3p + 700 = 2p - 500$

$5p = 1200$

$p = \$240$ per skateboard

38. The price at which there is neither a shortage nor surplus is the equilibrium price, which occurs when demand = supply:

$-5p + 50 = 3p - 30$

$8p = 80$

$p = \$10$ per skateboard

39. a. The equilibrium price occurs when demand = supply:

$-p + 156 = 4p - 394$

$5p = 550$

$p = \$110$ per phone. **b.** Since \$105 is below the equilibrium price, there would be a shortage at that price. To calculate it, compute demand and supply:

Demand: $q = -105 + 156 = 51$ million phones

Supply: $q = 4(105) - 394 = 26$ million phones

Shortage = Demand − Supply = $51 - 26 = 25$ million phones

40. a. The equilibrium price occurs when demand = supply:

$-10p + 1{,}600 = 14p - 800$

$24p = 2{,}400$

$p = \$100$ per phone.

b. Since \$80 is below the equilibrium price, there would be a shortage at that price. To calculate it, compute demand and supply:

Demand: $q = -10(80) + 1{,}600 = 800$ million phones

Supply: $q = 14(80) - 800 = 320$ million phones

Shortage = Demand − Supply = $800 - 320 = 480$ million phones

41. a. For equilibrium, Demand = Supply:

$$\frac{760}{p} - 1 = 0.019p - 1$$

$$\frac{760}{p} = 0.019p$$

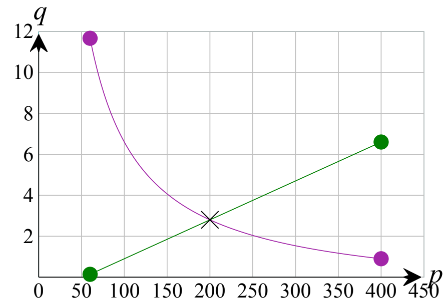
Cross-multiply:

$$0.019p^2 = 760 \quad \Rightarrow \quad p^2 = \frac{760}{0.019} = 40{,}000$$

So $\quad p = \sqrt{40{,}000} = \$200.$

Thus, the equilibrium price is \$200, and the equilibrium demand (or supply) is $760/200 - 1 = 2.8$ million e-readers

b. Graph:



Technology formulas:
Demand: `y = 760/x-1`
Supply: `y = 0.019x-1`

The graphs cross at (200, 2.8) confirming the calclation in part (a)

c. Since \$72 is below the equilibrium price, there would be a shortage at that price. To calculate it, compute demand and supply:

Demand: $q = \dfrac{760}{72} - 1 \approx 9.556$ million e-readers

Supply: $0.019(72) - 1 = 0.368$ million e-readers

Shortage = Demand − Supply $\approx 9.556 - 0.368 \approx 9.2$ million e-readers, or 9,200,000 e-readers.

42. a. For equilibrium, Demand = Supply:

$$7.5 + \frac{30}{p} = 1.2p + 7.5$$

$$\frac{30}{p} = 1.2p$$

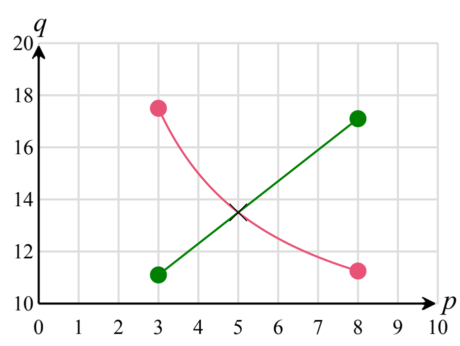
Cross-multiply:

$$1.2p^2 = 30 \quad \Rightarrow \quad p^2 = \frac{30}{1.2} = 25$$

So $p = \sqrt{25} = 5$ zonars.

Thus, the equilibrium price is Z5, and the equlibrium demand (or supply) is $7.5 + 30/5 = 13.5$ million rides

b. Graph:



Technology formulas:
Demand: `y = 7.5+30/x`
Supply: `y = 1.2x+7.5`

The graphs cross at (5, 13.5) confirming the calclation in part (a)

b. Since Z6 is above the equilibrium price, there would be a surplus at that price. To calculate it, compute demand and supply:

Demand: $q = 7.5 + \frac{30}{6} = 12.5$ million rides

Supply: $q = 1.2(6) + 7.5 = 14.7$ million rides

Surplus = Supply − Demand = 14.7 − 12.5 = 2.2 million rides.

43. $C(q) = 2{,}000 + 100q^2$

a. $C(10) = 2{,}000 + 100(10)^2 = 2{,}000 + 10{,}000 = \$12{,}000$

b. $N = C - S$, so

$$N(q) = C(q) - S(q) = 2{,}000 + 100q^2 - 500q$$

This is the cost of removing q lb of PCPs per day after the subsidy is taken into account.

c. $N(20) = 2{,}000 + 100(20)^2 - 500(20) = 2{,}000 + 40{,}000 - 10{,}000 = \$32{,}000$

44. $C(q) = 1{,}000 + 100\sqrt{q}$

a. $C(100) = 1{,}000 + 100\sqrt{100} = 1{,}000 + 100(10) = \$2{,}000$

b. $N = C - S$, so

$$N(q) = C(q) - S(q) = 1{,}000 + 100\sqrt{q} - 200q$$

This is the cost for dental coverage to the company if it has q employees, after the subsidy is taken into account.

c. $N(100) = 1{,}000 + 100\sqrt{100} - 200(100) = 1000 + 100(10) - 20{,}000 = -\$18{,}000$

The company makes $18,000 from the government for dental coverage if it employs 100 people.

45. The technology formulas are:

(A) `-0.2*t^2+t+16`

(B) `0.2*t^2+t+16`

(C) `t+16`

The following table shows the values predicted by the three models:

t	0	2	4	6	7
$S(t)$	16	18	22	28	30
(A)	16	17.2	16.8	14.8	13.2
(B)	16	18.8	23.2	29.2	32.8
(C)	16	18	20	22	23

As shown in the table, the values predicted by model (B) are much closer to the observed values $S(t)$ than those predicted by the other models.

b. Since 1998 corresponds to $t = 8$,

$$S(t) = 0.2t^2 + t + 16$$

$$S(8) = 0.2(8)^2 + 8 + 16 = 36.8$$

So the spending on corrections in 1998 was predicted to be approximately $37 billion.

46. The technology formulas are:

(A) `16+2*t`

(B) `16+t+0.5*t^2`

(C) `16+t-0.5*t^2`

The following table shows the values predicted by the three models:

t	0	2	4	6	7
$S(t)$	16	18	22	28	30
(A)	16	20	24	28	30
(B)	16	20	28	40	47.5
(C)	16	16	12	4	-1.5

As shown in the table, the values predicted by model (A) are much closer to the observed values $S(t)$ than those predicted by the other models.

b. Since 1998 corresponds to $t = 8$,

$S(t) = 16 + 2t$

$S(8) = 16 + 2(8) = 32$

So the spending on corrections in 1998 was predicted to be $32 billion.

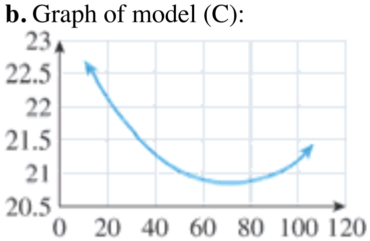
47. The technology formulas are:
(A) `0.005*x+2.75`
(B) `0.01*x+20+25/x`
(C) `0.0005*x^2-0.07*x+23.25`
(D) `25.5*1.08^(x-5)`

The following table shows the values predicted by the four models:

x	5	25	40	100	125
$A(x)$	22.91	21.81	21.25	21.25	22.31
(A)	20.775	20.875	20.95	21.25	21.375
(B)	25.05	21.25	21.025	21.25	21.45
(C)	22.913	21.813	21.25	21.25	22.313
(D)	25.5	118.85	377.03	38,177	261,451

Model (C) fits the data almost perfectly—more closely than any of the other models.

b. Graph of model (C):



`0.0005*x^2-0.07*x+23.25`

The lowest point on the graph occurs at $x = 70$ with a y-coordinate of 20.8. Thus, the lowest cost per shirt is $20.80, which the team can obtain by buying 70 shirts.

48. The technology formulas are:
(A) `0.05*x+20.75`
(B) `0.1*x+20+25/x`
(C) `0.0008*x^2-0.07*x+23.25`

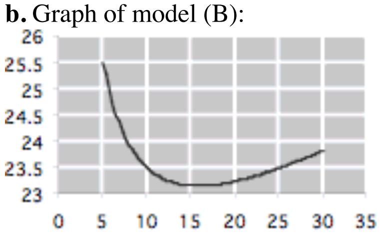
(D) `25.5*1.08^(x-5)`

The following table shows the values predicted by the four models:

x	5	25	40	100	125
$A(x)$	25.50	23.50	24.63	30.25	32.70
(A)	21	22	22.75	25.75	27
(B)	25.5	23.5	24.625	30.25	32.7
(C)	22.92	22	21.73	24.25	27
(D)	25.5	118.85	377.03	38,177	261,451

Model (B) fits the data almost perfectly—more closely than any of the other models.

b. Graph of model (B):



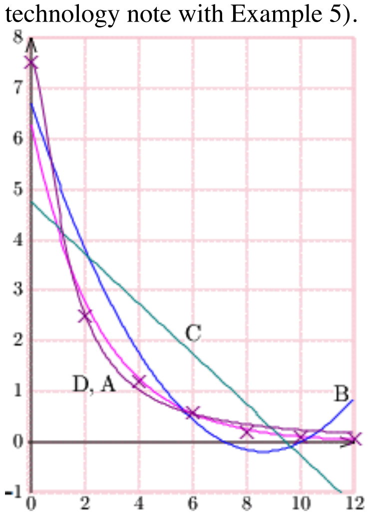
`0.1*x+20+25/x`

The lowest point on the graph occurs at x = 16 with a y-coordinate of 23.1625. Thus, the lowest cost per hat is \$23.16, which the team can obtain by buying 16 hats.

49. Here are the technology formulas as entered in the online Function Evaluator and Grapher at the Web Site:

(A) `6.3*0.67^x`
(B) `0.093*x^2-1.6*x+6.7`
(C) `4.75-0.50*x`
(D) `12.8/(x^1.7+1.7)`

The following graph shows all three curves together with the plotted points (entered as shown in the margin technology note with Example 5).



As shown in the graph, the values predicted by models (A) and (D) are much closer to the observed values than those predicted by the other models.

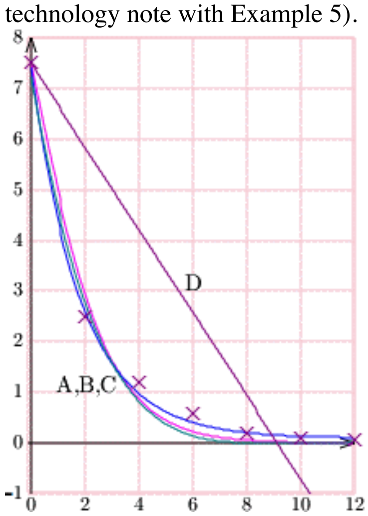
b. Since 2020 corresponds to t = 20,

Model (A): $c(20) = 6.3(0.67)^{20} \approx \$0.0021$

Model (D): $c(20) = \dfrac{12.8}{10^{1.7} + 1.7} \approx \$0.0778$

So model (A) gives the lower price: approximately \$0.0021.

50. Here are the technology formulas as entered in the online Function Evaluator and Grapher at the Web Site:
(A) `15/(1+2^x)`
(B) `(7.32)*0.59^x+0.10`
(C) `0.00085*(x-9.6)^4`
(D) `7.5-0.82x`
The following graph shows all three curves together with the plotted points (entered as shown in the margin technology note with Example 5).



As shown in the graph, the values predicted by models (A), (B), and (C) are much closer to the observed values than those predicted by (D).

b. Since 2020 corresponds to $t = 20$,

Model (A): $c(20) = \dfrac{15}{1 + 2^{20}} \approx \$0.000014$

Model (B): $c(20) = (7.32)0.59^{20} + 0.10 \approx \$0.10019$

Model (C): $c(20) = 0.00085(20 - 9.6)^4 \approx \$9.94$

Model (C) makes the unreasonable prediction that the price in 2020 will be \$9.94—a great deal higher and a sharp reversal of the downward trend.

51. A plot of the given points gives a straight line (Option (A)). Options (B) and (C) give curves, so (A) is the best choice.

52. A linear model would predict perpetually increasing or decreasing popularity of Twitter (depending on whether the slope is positive or negative) and an exponential model $p(t) = Ab^t$ would also be perpetually increasing or decreasing (depending whether b is larger than 1 or less than 1). This leaves a quadratic model as the only possible choice. In fact, a quadratic can always be found that passes through any three points not on the same straight line with different x-coordinates. Therefore, a quadratic model would give an exact fit.

53. A plot of the given data suggests a concave-down curve that becomes steeper downward as the price p increases, suggesting Model (D). Model (A) would predict increasing demand with increasing price, Model (B) would correspond to a descending curve that becomes less steep as p increases (a concave up curve), and Model (C) would give a concave-up parabola.

54. Model (B) is the best choice; Model (A) would predict increasing demand with increasing price, Model (D) would correspond to a concave down parabola, and Model (C), would predict demand that, rather than flattening out as the price increases, would begin to climb again.

55. Apply the formula

$$A(t) = P\left(1 + \frac{r}{m}\right)^{mt}$$

with $P = 5{,}000, r = 0.05/100 = 0.0005,$ and $m = 12$. We get the model

$$A(t) = 5{,}000(1 + 0.0005/12)^{12t}$$

In August 2020 ($t = 7$), the deposit would be worth $5{,}000(1 + 0.0005/12)^{12(7)} \approx \$5{,}018$.

56. Apply the formula

$$A(t) = P\left(1 + \frac{r}{m}\right)^{mt}$$

with $P = 4{,}000, r = 0.0061,$ and $m = 365$. We get the model

$$A(t) = 4{,}000(1 + 0.0061/365)^{365t}$$

In August 2021 ($t = 8$), the deposit would be worth $4{,}000(1 + 0.0061/365)^{365(8)} \approx \$4{,}200$.

57. From the answer to Exercise 55, the value of the investment after t years is

$$A(t) = 5{,}000(1 + 0.0005/12)^{12t}.$$

TI-83/84 Plus: Enter

`Y1 = 5000*(1+0.0005/12)^(12*X)`, Press [2nd] [TBLSET], and set `Indpnt` to Ask. (You do this once and for all; it will permit you to specify values for x in the table screen.) Then, press [2nd] [TABLE], and you will be able to evaluate the function at several values of x. Here are some values of x and the resulting values of Y1.

x	18	19	20	21
Y1	5045.20	5047.73	5050.25	5052.78

Notice that Y1 first exceeds 5,050 when $x = 20$. Since $x = 0$ represents August 2013, $x = 20$ represents August 2033, so the investment will first exceed \$5,050 in August 2033.

58. From the answer to Exercise 56, the value of the investment after t years is

$$A(t) = 4{,}000(1 + 0.0061/365)^{365t}$$

TI-83/84 Plus: Enter

`Y1 = 4000*(1+0.0061/365)^(365*X)`, Press [2nd] [TBLSET], and set `Indpnt` to Ask. (You do this once and for all; it will permit you to specify values for x in the table screen.) Then, press [2nd] [TABLE], and you will be able to evaluate the function at several values of x. Here are some values of x and the resulting values of Y1.

x	14	15	16	17
Y1	4356.61	4383.26	4410.08	4437.07

Notice that Y1 first exceeds 4,400 when $x = 16$. Since $x = 0$ represents August 2013, $x = 16$ represents August 2029, so the investment will first exceed \$4,400 in August 2029.

59. $C(t) = 104(0.999879)^t$, so $C(10{,}000) \approx 31.0$ g, $C(20{,}000) \approx 9.25$ g, and $C(30{,}000) \approx 2.76$ g.

60. $A(t) = 4.06(0.999879)^{-t}$, t years ago, so $A(10{,}000) \approx 13.6$ g, $A(20{,}000) \approx 45.7$ g, and $A(30{,}000) \approx 153$ g.

61. We are looking for t such that $4.06 = C(t) = 46(0.999879)^t$. Among the values suggested we find that $C(15{,}000) \approx 7.5$, $C(20{,}000) \approx 4.09$ and $C(25{,}000) \approx 2.23$. Thus, the answer is 20,000 years to the nearest 5,000 years.

62. We are looking for t such that $104 = A(t) = 2.8(.999879)^{-t}$. Among the values suggested we find that $A(25{,}000) \approx 57.67$, $A(30{,}000) \approx 105.62$, and $A(35{,}000) \approx 193.42$. Thus, the answer is 30,000 years to the nearest 5,000 years.

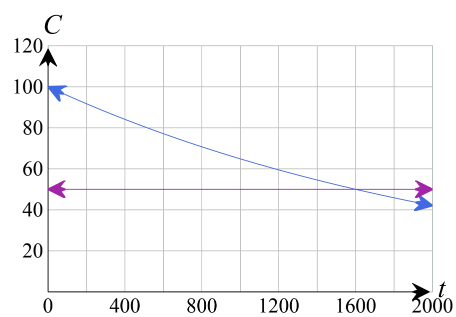
63. a. Amount left after 1,000 years: $C(1{,}000) = A(0.999567)^{1{,}000} \approx 0.6485A$, or about 65% of the original amount.

Amount left after 2,000 years: $C(2{,}000) = A(0.999567)^{2{,}000} \approx 0.4206A$, or about 42% of the original amount.

Amount left after 3,000 years: $C(3{,}000) = A(0.999567)^{3{,}000} \approx 0.2727A$, or about 27% of the original amount.

b. For a sample of 100 g, $C(t) = 100(0.999567)^t$.

Here is the graph, together with the line $y = 50$ (one half the original sample):
Technology: `100*(0.999567)^x`



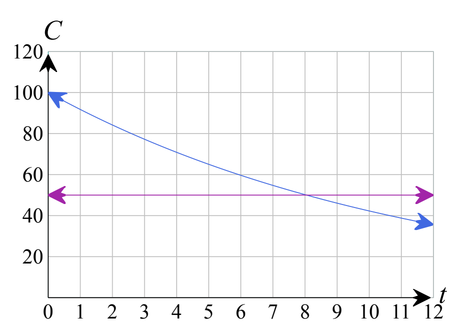
Since the graphs intersect close to $x = 1{,}600$, we conclude that half the sample will have decayed after about 1,600 years.

64. a. Amount left after 2 days: $C(2) = A(0.9175)^2 \approx 0.8418A$, or about 84% of the original amount.

Amount left after 4 days: $C(4) = A(0.9175)^4 \approx 0.7086A$, or about 71% of the original amount.

Amount left after 6 days: $C(6) = A(0.9175)^6 \approx 0.5965A$, or about 60% of the original amount.

b. For a sample of 100 g, $C(t) = 100(0.9175)^t$. Here is the graph, together with the line $y = 50$ (one half the original sample):
Technology: `100*(0.9175)^x`



Since the graphs intersect close to $x = 8$, we conclude that half the sample will have decayed after about 8 days.

Communication and reasoning exercises

65. $P(0) = 200$, $P(1) = 230$, $P(2) = 260$, ... and so on. Thus, the population is increasing by 30 per year.

66. $B(0) = 5000$, $B(1) = 4800$, $B(2) = 4600$, ... and so on. Thus, the balance is decreasing by \$200 per day.

67. Curve fitting. The model is based on fitting a curve to a given set of observed data.

68. Analytical. The model is obtained by analyzing the situation being modeled.

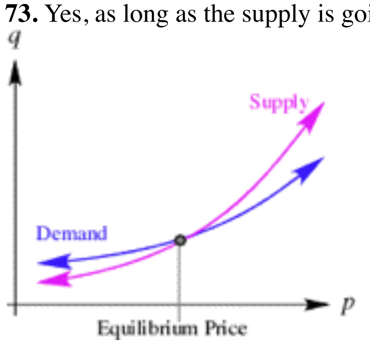
69. The given model is $c(t) = 4 - 0.2t$. This tells us that c is \$4 at time $t = 0$ (January) and is decreasing by \$0.20 per month. So, the cost of downloading a movie was \$4 in January and is decreasing by 20¢ per month.

70. The given model is $c(t) = 4 - 0.2t$ and therefore passes through the points $(t, c) = (0, 4)$ and $(1, 3.8)$. So, the cost of downloading a movie was \$4 in January and \$3.80 in February.

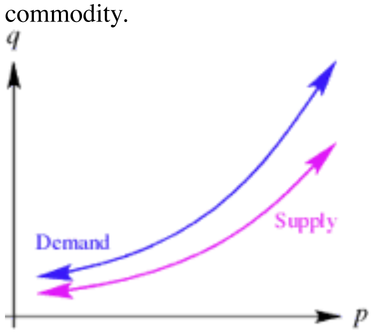
71. In a linear cost function, the variable cost is x times the marginal cost.

72. In a linear cost function, the marginal cost is the additional (or incremental) cost per item.

73. Yes, as long as the supply is going up at a faster rate, as illustrated by the following graph:



74. There would be a shortage at any given price. Therefore, consumers would be willing to pay more for a scarce commodity and sellers would naturally oblige by charging more, resulting in an upward spiral of prices for the commodity.



75. Extrapolate both models and choose the one that gives the most reasonable predictions.

76. No; as long as a is negative, the value of $s(t)$ for large t will be negative, making the model unreasonable for large value of t.

77. The value of $f - g$ at x is $f(x) - g(x)$. Since $f(x) \geq g(x)$ for every x, it follows that $f(x) - g(x) \geq 0$ for every x.

78. The value of $\frac{f}{g}$ at x is $\frac{f(x)}{g(x)}$. Since $f(x) > g(x) > 0$ for every x, it follows that $\frac{f(x)}{g(x)} > 1$ for every x.

79. Since the values of $\frac{f}{g}$ at x are ratios $\frac{f(x)}{g(x)}$, it follows that the units of measurement of $\frac{f}{g}$ are units of f per unit of g; that is, books per person.

80. Write $f(x) = mx + b$ and $g(x) = nx + c$. Then

$$f(x) - g(x) = mx + b - (nx + c) = (m - n)x + (b - c),$$

also a linear function.

Section 1.3

1.

x	-1	0	1
y	5	8	

We calculate the slope m first. The first two points shown give changes in x and y of

$\Delta x = 0 - (-1) = 1$

$\Delta y = 8 - 5 = 3$ This gives a slope of

$$m = \frac{\Delta y}{\Delta x} = \frac{3}{1} = 3.$$

Now look at the second and third points: The change in x is again

$\Delta x = 1 - 0 = 1$

and so Δy must be given by the formula

$\Delta y = m\Delta x$

$\Delta y = 3(1) = 3$

This means that the missing value of y is

$8 + \Delta y = 8 + 3 = 11.$

2.

x	-1	0	1
y	-1	-3	

We calculate the slope m first. The first two points shown give changes in x and y of

$\Delta x = 0 - (-1) = 1$

$\Delta y = -3 - (-1) = -2$

This gives a slope of

$$m = \frac{\Delta y}{\Delta x} = \frac{-2}{1} = -2.$$

Now look at the second and third points: The change in x is again

$\Delta x = 1 - 0 = 1$

and so Δy must be given by the formula

$\Delta y = m\Delta x$

$\Delta y = -2(1) = -2$

This means that the missing value of y is

$-3 + \Delta y = -3 + (-2) = -5.$

3.

x	2	3	5
y	-1	-2	

We calculate the slope m first. The first two points shown give changes in x and y of

$\Delta x = 3 - 2 = 1$

$\Delta y = -2 - (-1) = -1$

This gives a slope of

$$m = \frac{\Delta y}{\Delta x} = \frac{-1}{1} = -1.$$

Now look at the second and third points: The change in x is

$$\Delta x = 5 - 3 = 2$$

and so Δy must be given by the formula

$$\Delta y = m\Delta x$$

$$\Delta y = (-1)(2) = -2$$

This means that the missing value of y is

$$-2 + \Delta y = -2 + (-2) = -4.$$

4.

x	2	4	5
y	−1	−2	

We calculate the slope m first. The first two points shown give changes in x and y of

$$\Delta x = 4 - 2 = 2$$

$$\Delta y = -2 - (-1) = -1$$

This gives a slope of

$$m = \frac{\Delta y}{\Delta x} = \frac{-1}{2} = -\frac{1}{2}.$$

Now look at the second and third points: The change in x is

$$\Delta x = 5 - 4 = 1$$

and so Δy must be given by the formula

$$\Delta y = m\Delta x$$

$$\Delta y = \left(-\frac{1}{2}\right)(1) = -\frac{1}{2}$$

This means that the missing value of y is

$$-2 + \Delta y = -2 + \left(-\frac{1}{2}\right) = -\frac{5}{2} \text{ or } -2.5.$$

5.

x	−2	0	2
y	4		10

We calculate the slope m first. The first and third points shown give changes in x and y of

$$\Delta x = 2 - (-2) = 4$$

$$\Delta y = 10 - 4 = 6$$

This gives a slope of

$$m = \frac{\Delta y}{\Delta x} = \frac{6}{4} = \frac{3}{2}.$$

Now look at the first and second points: The change in x is

$$\Delta x = 0 - (-2) = 2$$

and so Δy must be given by the formula

$$\Delta y = m\Delta x$$

$$\Delta y = \left(-\frac{3}{2}\right)(2) = 3$$

This means that the missing value of y is

$$4 + \Delta y = 4 + 3 = 7.$$

6.

x	0	3	6
y	−1		−5

We calculate the slope m first. The first and third points shown give changes in x and y of

$$\Delta x = 6 - 0 = 6$$

$$\Delta y = -5 - (-1) = -4$$

This gives a slope of

$$m = \frac{\Delta y}{\Delta x} = \frac{-4}{6} = -\frac{2}{3}.$$

Now look at the first and second points: The change in x is

$$\Delta x = 3 - 0 = 3$$

and so Δy must be given by the formula

$$\Delta y = m\Delta x$$

$$\Delta y = \left(-\frac{2}{3}\right)(3) = -2$$

This means that the missing value of y is

$$-1 + \Delta y = -1 + (-2) = -3$$

7. From the table, $b = f(0) = -2$.
The slope (using the first two points) is

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-2 - (-1)}{0 - (-2)} = \frac{-1}{2} = -\frac{1}{2}.$$

Thus, the linear equation is

$$f(x) = mx + b = -\frac{1}{2}x - 2, \quad \text{or } f(x) = -\frac{x}{2} - 2.$$

8. From the table, $b = f(0) = 3$.
The slope (using the first two points) is

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2 - 1}{(-3) - (-6)} = \frac{1}{3}$$

Thus, the linear equation is

$$f(x) = mx + b = \frac{1}{3}x + 3, \quad \text{or } f(x) = \frac{x}{3} + 3.$$

9. The slope (using the first two points) is

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-2 - (-1)}{-3 - (-4)} = \frac{-1}{1} = -1.$$

To obtain $f(0) = b$, use the formula for b:

$$f(0) = b = y_1 - mx_1 = -1 - (-1)(-4) = -5$$ Using the point $(x_1, y_1) = (-4, -1)$

This gives

$$f(x) = mx + b = -x - 5.$$

10. The slope (using the first two points) is

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{6-4}{2-1} = \frac{2}{1} = 2$$

To obtain $f(0) = b$, use the formula for b:

$$f(0) = b = y_1 - mx_1 = 4 - (2)(1) = 2 \qquad \text{Using the point } (x_1, y_1) = (1, 4)$$

This gives

$$f(x) = mx + b = 2x + 2.$$

11. In the table, x increases in steps of 1 and f increases in steps of 4, showing that f is linear with slope

$$m = \frac{\Delta y}{\Delta x} = \frac{4}{1} = 4$$

and intercept

$$b = f(0) = 6$$

giving

$$f(x) = mx + b = 4x + 6.$$

The function g does not increase in equal steps, so g is not linear.

12. In the table, x increases in steps of 10 and g increases in steps of 5, showing that g is linear with slope

$$m = \frac{\Delta y}{\Delta x} = \frac{5}{10} = \frac{1}{2}$$

and intercept

$$b = g(0) = -4$$

giving

$$g(x) = mx + b = \frac{1}{2}x - 4.$$

The function f does not increase in equal steps, so f is not linear.

13. In the first three points listed in the table, x increases in steps of 3, but f does not increase in equal steps, whereas g increases in steps of 6. Thus, based on the first three points, only g could possibly be linear, with slope

$$m = \frac{\Delta y}{\Delta x} = \frac{6}{3} = 2$$

and intercept

$$b = g(0) = -1$$

giving

$$g(x) = mx + b = 2x - 1.$$

We can now check that the remaining points in the table fit the formula $g(x) = 2x - 1$, showing that g is indeed linear.

14. In the first and last pairs of points listed in the table, x increases in steps of 3, but f does not increase in equal steps, whereas g increases in steps of 9. Thus, based on those points, only g could possibly be linear, with slope

$$m = \frac{\Delta y}{\Delta x} = \frac{9}{3} = 3$$

and intercept

$b = g(0) = -1$

giving

$g(x) = mx + b = 3x - 1.$

We can now check that the remaining points in the table fit the formula $g(x) = 3x - 1$, showing that g is indeed linear.

15. Slope = coefficient of $x = -\frac{3}{2}$

16. Slope = coefficient of $x = \frac{2}{3}$

17. Slope = coefficient of $x = \frac{1}{6}$

18. Write the equation as $y = -\frac{2x}{3} + \frac{1}{3}$

Slope = coefficient of $x = -\frac{2}{3}$

19. If we solve for x we find that the given equation represents the vertical line $x = -1/3$, and so its slope is infinite (undefined).

20. $8x - 2y = 1$. Solving for y:

$2y = 8x - 1$

$y = 4x - \frac{1}{2}$

Slope = coefficient of $x = 4$

21. $3y + 1 = 0$. Solving for y:

$3y = -1$

$y = -\frac{1}{3}$

Slope = coefficient of $x = 0$

22. If we solve for x we find that the given equation represents the vertical line $x = -3/2$, and so its slope is infinite (undefined).

23. $4x + 3y = 7$. Solve for y:

$3y = -4x + 7$

$y = -\frac{4}{3}x + \frac{7}{3}$

Slope = coefficient of $x = -\frac{4}{3}$

Solutions Section 1.3

24. $2y + 3 = 0$. Solve for y:

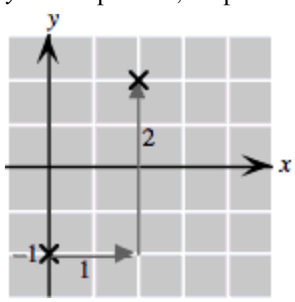
$$2y = -3$$

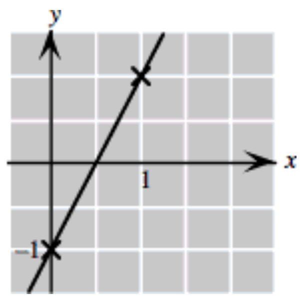
$$y = -\frac{3}{2}$$

Slope = coefficient of $x = 0$

25. $y = 2x - 1$

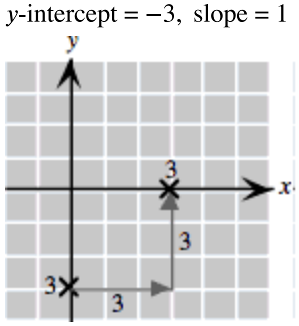
y-intercept $= -1$, slope $= 2$

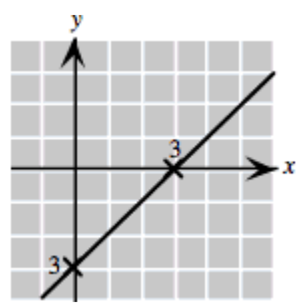




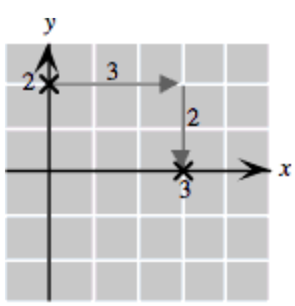
26. $y = x - 3$

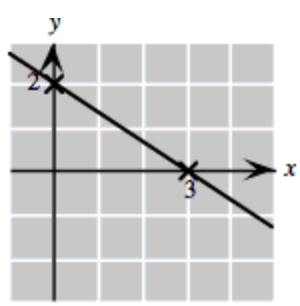
y-intercept $= -3$, slope $= 1$





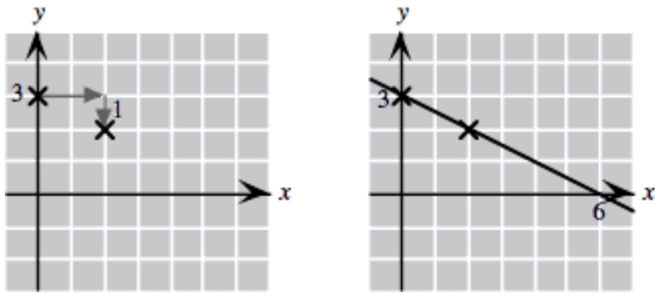
27. y-intercept $= 2$, slope $= -\frac{2}{3}$





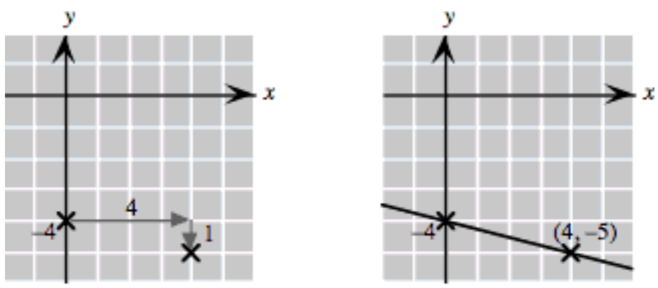
28. $y = -\frac{1}{2}x + 3$

y-intercept $= 3$, slope $= -\frac{1}{2}$



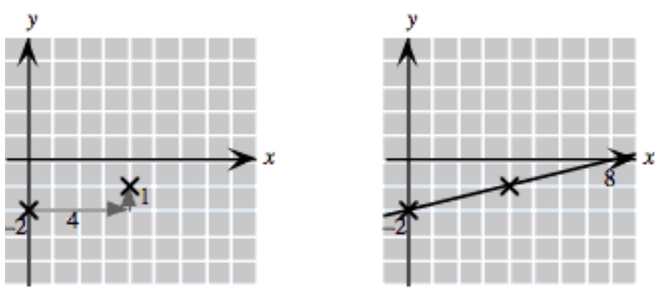
29. $y+\frac{1}{4}x=-4$. Solve for y to obtain $y=-\frac{1}{4}x-4$

y-intercept = -4, slope = $-\frac{1}{4}$



30. $y-\frac{1}{4}x=-2$. Solve for y to obtain $y=\frac{1}{4}x-2$

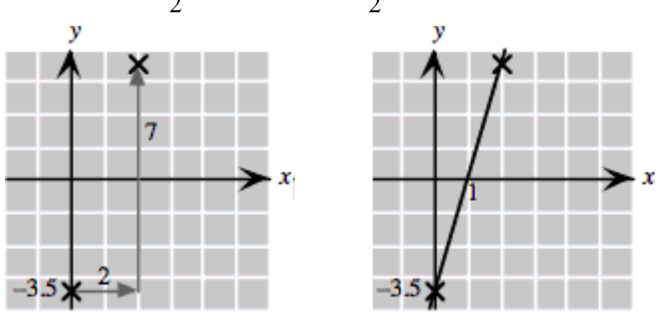
y-intercept = -2, slope = $\frac{1}{4}$



31. $7x-2y=7$. Solve for y:

$-2y=-7x+7$, so $y=\frac{7}{2}x-\frac{7}{2}$

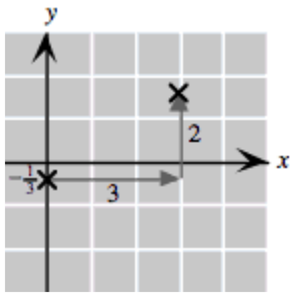
y-intercept = $-\frac{7}{2}=-3.5$, slope = $\frac{7}{2}=3.5$

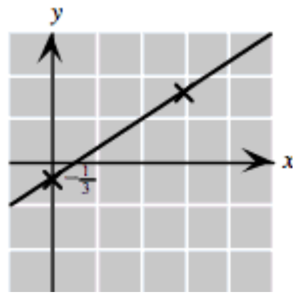


32. $2x - 3y = 1$. Solve for y:

$$-3y = -2x + 1, \text{ so } y = \frac{2}{3}x - \frac{1}{3}$$

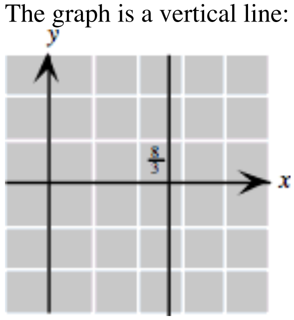
y-intercept $= -\frac{1}{3}$, slope $= \frac{2}{3}$





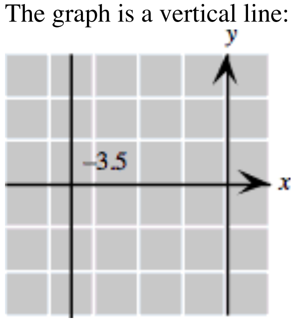
33. $3x = 8$. Solve for x to obtain $x = \frac{8}{3}$.

The graph is a vertical line:



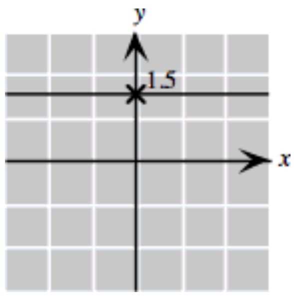
34. $2x = -7$. Solve for x to obtain $x = -\frac{7}{2} = -3.5$.

The graph is a vertical line:



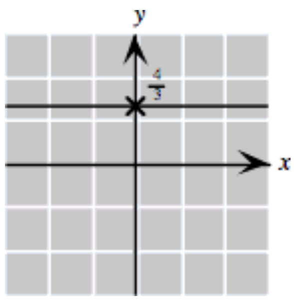
35. $6y = 9$. Solve for y to obtain $y = \frac{9}{6} = \frac{3}{2} = 1.5$

y-intercept $= \frac{3}{2} = 1.5$, slope $= 0$. The graph is a horizontal line:



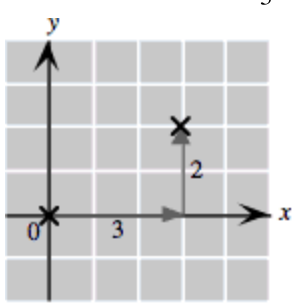
36. $3y = 4$. Solve for y to obtain $y = \frac{4}{3}$

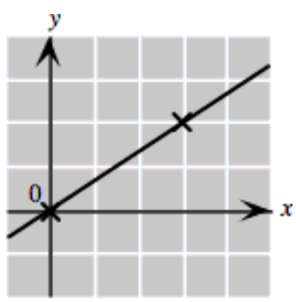
y-intercept = $\frac{4}{3}$, slope = 0. The graph is a horizontal line:



37. $2x = 3y$. Solve for y to obtain $y = \frac{2}{3}x$

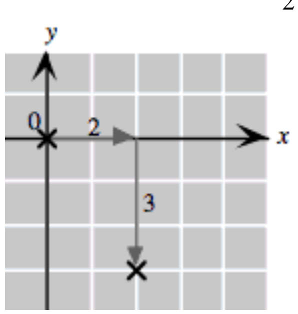
y-intercept = 0, slope = $\frac{2}{3}$

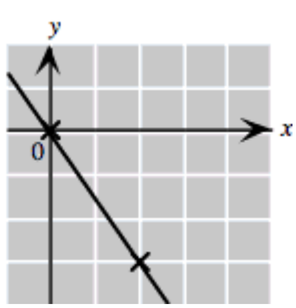




38. $3x = -2y$. Solve for y to obtain $y = -\frac{3}{2}x$

y-intercept = 0, slope = $-\frac{3}{2}$





39. $(0, 0)$ and $(1, 2)$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2 - 0}{1 - 0} = 2$$

40. $(0, 0)$ and $(-1, 2)$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2 - 0}{-1 - 0} = -2$$

41. $(-1, -2)$ and $(0, 0)$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0 - (-2)}{0 - (-1)} = 2$$

42. $(2, 1)$ and $(0, 0)$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0 - 1}{0 - 2} = \frac{1}{2}$$

43. $(4, 3)$ and $(5, 1)$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1 - 3}{5 - 4} = \frac{-2}{1} = -2$$

44. $(4, 3)$ and $(4, 1)$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1 - 3}{4 - 4} \text{ Undefined}$$

45. $(1, -1)$ and $(1, -2)$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-2 - (-1)}{1 - 1} \text{ Undefined}$$

46. $(-2, 2)$ and $(-1, -1)$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1 - 2}{-1 - (-2)} = \frac{-3}{1} = -3$$

47. $(2, 3.5)$ and $(4, 6.5)$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{6.5 - 3.5}{4 - 2} = \frac{3}{2} = 1.5$$

48. $(10, -3.5)$ and $(0, -1.5)$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1.5 - (-3.5)}{0 - 10} = \frac{2}{-10} = -0.2$$

49. $(300, 20.2)$ and $(400, 11.2)$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{11.2 - 20.2}{400 - 300} = \frac{-9}{100} = -0.09$$

50. $(1, -20.2)$ and $(2, 3.2)$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3.2 - (-20.2)}{2 - 1} = \frac{23.4}{1} = 23.4$$

51. $(0, 1)$ and $\left(-\frac{1}{2}, \frac{3}{4}\right)$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3/4 - 1}{-1/2 - 0} = \frac{-1/4}{-1/2} = \frac{2}{4} = \frac{1}{2}$$

52. $\left(\frac{1}{2}, 1\right)$ and $\left(-\frac{1}{2}, \frac{3}{4}\right)$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3/4 - 1}{-1/2 - 1/2} = \frac{-1/4}{-1} = \frac{1}{4}$$

53. (a, b) and (c, d) $(a \neq c)$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{d - b}{c - a}$$

54. (a, b) and (c, b) $(a \neq c)$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{b - b}{c - a} = 0$$

55. (a, b) and (a, d) $(b \neq d)$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{d - b}{a - a} \text{ Undefined}$$

56. (a, b) and $(-a, -b)$ $(a \neq 0)$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-b - b}{-a - a} = \frac{-2b}{-2a} = \frac{b}{a}$$

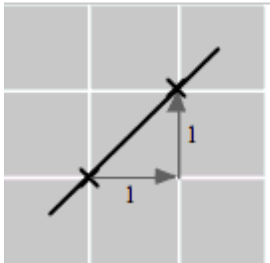
57. $(-a, b)$ and $(a, -b)$ $(a \neq 0)$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-b - b}{a - (-a)} = \frac{-2b}{2a} = -\frac{b}{a}$$

58. (a, b) and (b, a) $(a \neq b)$

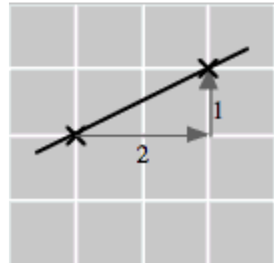
$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{a - b}{b - a} = -1$$

59. a.



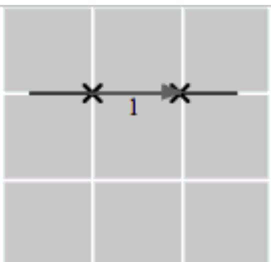
$m = \frac{\Delta y}{\Delta x} = \frac{1}{1} = 1$

b.



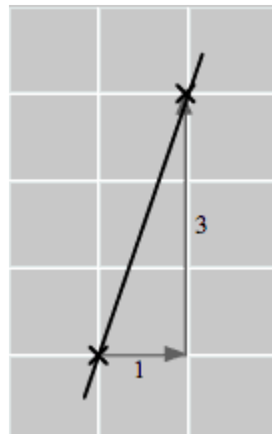
$m = \frac{\Delta y}{\Delta x} = \frac{1}{2}$

c.



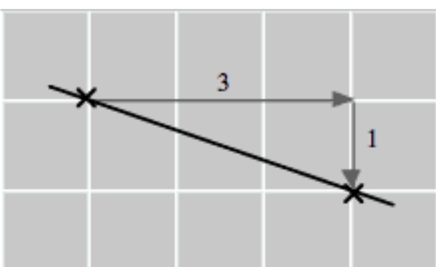
$m = \frac{\Delta y}{\Delta x} = \frac{0}{1} = 0$

d.



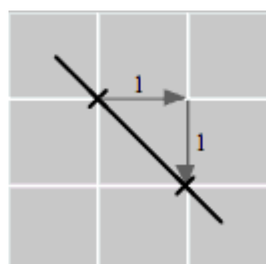
$m = \frac{\Delta y}{\Delta x} = \frac{3}{1} = 3$

e.



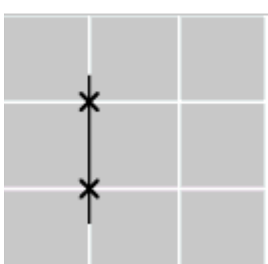
$m = \frac{\Delta y}{\Delta x} = \frac{-1}{3} = -\frac{1}{3}$

f.



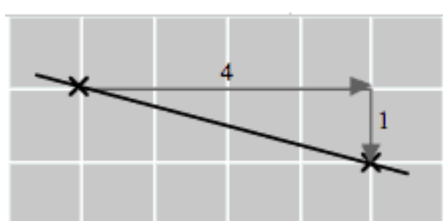
$m = \frac{\Delta y}{\Delta x} = \frac{-1}{1} = -1$

g.



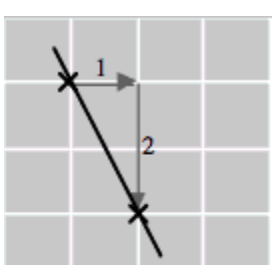
Vertical line; undefined slope

h.



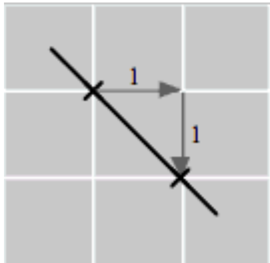
$m = \frac{\Delta y}{\Delta x} = \frac{-1}{4} = -\frac{1}{4}$

i.



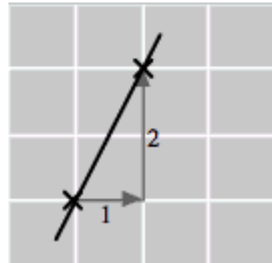
$m = \frac{\Delta y}{\Delta x} = \frac{-2}{1} = -2$

60. **a.**



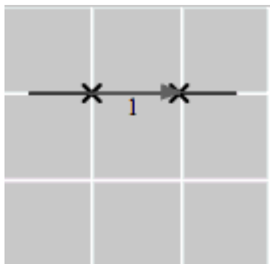
$$m = \frac{\Delta y}{\Delta x} = \frac{-1}{1} = -1$$

b.



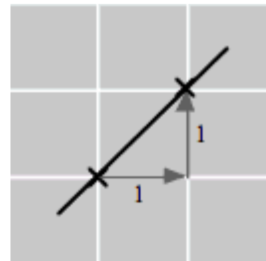
$$m = \frac{\Delta y}{\Delta x} = \frac{2}{1} = 2$$

c.



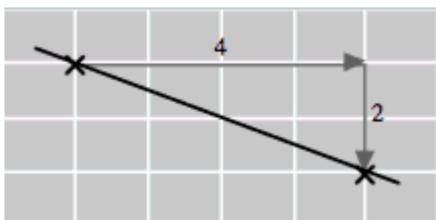
$$m = \frac{\Delta y}{\Delta x} = \frac{0}{1} = 0$$

d.



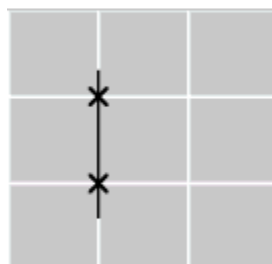
$$m = \frac{\Delta y}{\Delta x} = \frac{1}{1} = 1$$

e.



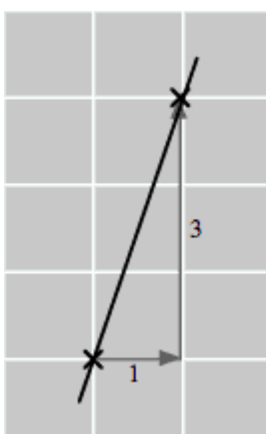
$$m = \frac{\Delta y}{\Delta x} = \frac{-2}{4} = -\frac{1}{2}$$

f.



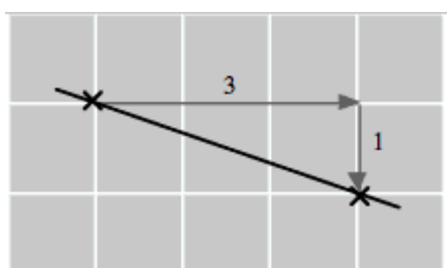
Vertical line; undefined slope

g.



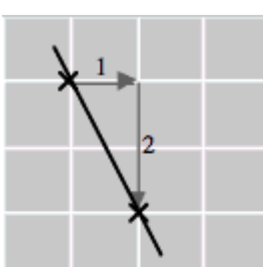
$$m = \frac{\Delta y}{\Delta x} = \frac{3}{1} = 3$$

h.



$$m = \frac{\Delta y}{\Delta x} = \frac{-1}{3} = -\frac{1}{3}$$

i.



$$m = \frac{\Delta y}{\Delta x} = \frac{-2}{1} = -2$$

61. Through $(1, 3)$ with slope 3

Point: $(1, 3)$ **Slope:** $m = 3$ **Intercept:** $b = y_1 - mx_1 = 3 - 3(1) = 0$

Thus, the equation is $y = mx + b = 3x + 0$, or $y = 3x$

62. Through (2, 1) with slope 2

Point: (2, 1) **Slope:** $m = 2$ **Intercept:** $b = y_1 - mx_1 = 1 - 2(2) = -3$

Thus, the equation is $y = mx + b = 2x - 3$

63. Through $\left(1, -\frac{3}{4}\right)$ with slope $\frac{1}{4}$

Point: $\left(1, -\frac{3}{4}\right)$ **Slope:** $m = \frac{1}{4}$ **Intercept:** $b = y_1 - mx_1 = -\frac{3}{4} - \frac{1}{4}(1) = -1$

Thus, the equation is $y = mx + b = \frac{1}{4}x - 1$

64. Through $\left(0, -\frac{1}{3}\right)$ with slope $\frac{1}{3}$

Point: $\left(0, -\frac{1}{3}\right)$ **Slope:** $m = \frac{1}{3}$ **Intercept:** $b = y_1 - mx_1 = -\frac{1}{3} - \frac{1}{3}(0) = -\frac{1}{3}$

Thus, the equation is $y = mx + b = \frac{1}{3}x - \frac{1}{3}$

65. Through (20, −3.5) and increasing at a rate of 10 units of y per unit of x

Point: (20, −3.5) **Slope:** $m = \frac{\Delta y}{\Delta x} = \frac{10}{1} = 10$

Intercept: $b = y_1 - mx_1 = -3.5 - (10)(20) = -3.5 - 200 = -203.5$

Thus, the equation is $y = mx + b = 10x - 203.5$

66. Through (3.5, −10) and increasing at a rate of 1 unit of y per 2 units of x

Point: (3.5, −10) **Slope:** $m = \frac{\Delta y}{\Delta x} = \frac{1}{2} = 0.5$ **Intercept:** $b = y_1 - mx_1 = -10 - (0.5)(3.5) = -11.75$

Thus, the equation is $y = mx + b = 0.5x - 11.75$

67. Through (2, −4) and (1, 1)

Point:(2, −4) **Slope:** $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5}{-1} = -5$ **Intercept:** $b = y_1 - mx_1 = -4 - (-5)(2) = 6$

Thus, the equation is $y = mx + b = -5x + 6$

68. Through (1, −4) and (−1, −1)

Point: (1, −4) **Slope:** $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3}{-2} = -1.5$ **Intercept:** $b = y_1 - mx_1 = -4 - (-1.5)(1) = -2.5$

Thus, the equation is $y = mx + b = -1.5x - 2.5$

69. Through (1, −0.75) and (0.5, 0.75)

Point: $(1, -0.75)$ **Slope:** $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{0.75 - (-0.75)}{0.5 - 1} = \dfrac{1.5}{-0.5} = -3$

Intercept: $b = y_1 - mx_1 = -0.75 - (-3)(1) = -0.75 + 3 = 2.25$

Thus, the equation is $y = mx + b = -3x + 2.25$

70. Through $(0.5, -0.75)$ and $(1, -3.75)$

Point: $(0.5, -0.75)$ **Slope:** $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{-3.75 - (-0.75)}{1 - 0.5} = \dfrac{-3}{0.5} = -6$

Intercept: $b = y_1 - mx_1 = -0.75 - (-6)(0.5) = -0.75 + 3 = 2.25$

Thus, the equation is $y = mx + b = -6x + 2.25$

71. Through $(6, 6)$ and parallel to the line $x + y = 4$

Point: $(6, 6)$

Slope: Same as slope of $x + y = 4$. To find the slope, solve for y, getting $y = -x + 4$. Thus, $m = -1$.

Intercept: $b = y_1 - mx_1 = 6 - (-1)(6) = 6 + 6 = 12$

Thus, the equation is $y = mx + b = -x + 12$

72. Through $(1/3, -1)$ and parallel to the line $3x - 4y = 8$

Point: $(1/3, -1)$

Slope: Same as slope of $3x - 4y = 8$. To find the slope, solve for y, getting $y = \dfrac{3}{4}x - 2$. Thus, $m = \dfrac{3}{4}$.

Intercept: $b = y_1 - mx_1 = (-1) - \left(\dfrac{3}{4}\right)\left(\dfrac{1}{3}\right) = -1 - \dfrac{1}{4} = -\dfrac{5}{4}$

Thus, the equation is $y = mx + b = \dfrac{3}{4}x - \dfrac{5}{4}$

73. Through $(0.5, 5)$ and parallel to the line $4x - 2y = 11$

Point: $(0.5, 5)$

Slope: Same as slope of $4x - 2y = 11$. To find the slope, solve for y, getting $y = 2x - \dfrac{11}{2}$. Thus, $m = 2$.

Intercept: $b = y_1 - mx_1 = 5 - (2)(0.5) = 5 - 1 = 4$

Thus, the equation is $y = mx + b = 2x + 4$

74. Through $(1/3, 0)$ and parallel to the line $6x - 2y = 11$

Point: $(1/3, 0)$

Slope: Same as slope of $6x - 2y = 11$. To find the slope, solve for y, getting $y = 3x - \dfrac{11}{2}$. Thus, $m = 2$.

Intercept: $b = y_1 - mx_1 = 0 - (3)\left(\dfrac{1}{3}\right) = -1$

Thus, the equation is $y = mx + b = 3x - 1$

75. Through $(0, 0)$ and (p, q)

Point: $(0, 0)$ **Slope:** $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{q - 0}{p - 0} = \frac{q}{p}$ **Intercept:** $b = y_1 - mx_1 = 0 - \frac{q}{p}(0) = 0$

Thus, the equation is $y = mx + b = \frac{q}{p}x$

76. Through (p, q) parallel to $y = rx + s$

Point: (p, q) **Slope:** Since the line has the same slope as $y = rx + s$, $m = r$ **Intercept:** $b = y_1 - mx_1 = q - rp$

Thus, the equation is $y = mx + b = rx + q - rp$ or $y = r(x - p) + q$

77. Through (p, q) and (r, q) $(p \neq r)$

Point: (p, q) **Slope:** $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{q - q}{r - p} = 0$

Intercept: $b = y_1 - mx_1 = q - (0)p = q$

Thus, the equation is $y = mx + b$; that is, $y = q$

78. Through (p, q) and $(-p, -q)$ $(p \neq 0)$

Point: (p, q) **Slope:** $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-q - q}{-p - p} = \frac{-2q}{-2p} = \frac{q}{p}$

Intercept: $b = y_1 - mx_1 = q - \frac{q}{p}p = q - q = 0$

Thus, the equation is $y = mx + b = \frac{q}{p}x$

79. Through $(-p, q)$ and $(p, -q)$ $(p \neq 0)$

Point: $(-p, q)$ **Slope:** $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-q - q}{p - (-p)} = \frac{-2q}{2p} = -\frac{q}{p}$

Intercept: $b = y_1 - mx_1 = q - \left(-\frac{q}{p}\right)(-p) = q - q = 0$

Thus, the equation is $y = mx + b = -\frac{q}{p}x$

80. Through (p, q) and (r, s) $(p \neq r)$

Point: (p, q) **Slope:** $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{s - q}{r - p}$

Intercept: $b = y_1 - mx_1 = q - \left(\frac{s - q}{r - p}\right)p$

Thus, the equation is $y = mx + b = \left(\frac{s - q}{r - p}\right)x + q - \left(\frac{s - q}{r - p}\right)p$, or $y = \left(\frac{s - q}{r - p}\right)(x - p) + q$

Applications

81. We are given two points on the graph of the linear cost function: (100, 10,500) and (120, 11,000) (x is the number of bicycles, and the second coordinate is the cost C).

Marginal cost:

$$m = \frac{C_2 - C_1}{x_2 - x_1} = \frac{11{,}000 - 10{,}500}{120 - 100} = \frac{500}{20} = \$25 \text{ per bicycle}$$

Fixed cost:

$$b = C_1 - mx_1 = 10{,}500 - (25)(100) = 10{,}500 - 2{,}500 = \$8{,}000$$

82. We are given two points on the graph of the linear cost function: (1,000, 6,000) and (1,500, 8,500) (x is the number of cases, and the second coordinate is the cost C).

Marginal cost:

$$m = \frac{C_2 - C_1}{x_2 - x_1} = \frac{8{,}500 - 6{,}000}{1{,}500 - 1{,}000} = \frac{2{,}500}{500} = \$5 \text{ per case.}$$

Fixed cost:

$$b = C_1 - mx_1 = 6{,}000 - (5)(1{,}000) = \$1{,}000$$

83. We are given two points on the graph of the linear cost function: (10, 2,070) and (20, 4,120) (x is the number of iPhones, and the second coordinate is the cost C).

Marginal cost:

$$m = \frac{C_2 - C_1}{x_2 - x_1} = \frac{4{,}120 - 2{,}070}{20 - 10} = \frac{2{,}050}{10} = \$205 \text{ per iPhone}$$

Fixed cost:

$$b = C_1 - mx_1 = 2{,}070 - (205)(10) = \$20$$

Thus, the cost equation is $C = mx + b = 205x + 20$.

The cost to manufacture each additional iPhone is the marginal cost: \$205.

The cost to manufacture 40 iPhones is obtained by setting $x = 40$ in the cost equation:

$$C(40) = 205(40) + 20 = \$8{,}220$$

84. We are given two points on the graph of the linear cost function: (8, 1,230) and (16, 2,430) (x is the number of Kinects, and the second coordinate is the cost C).

Marginal cost:

$$m = \frac{C_2 - C_1}{x_2 - x_1} = \frac{2{,}430 - 1{,}230}{16 - 8} = \frac{1{,}200}{8} = \$150 \text{ per unit}$$

Fixed cost:

$$b = C_1 - mx_1 = 1{,}230 - (150)(8) = \$30$$

Thus, the cost equation is $C = mx + b = 150x + 30$.

The cost to manufacture each additional Kinect is the marginal cost: \$150.

The cost to manufacture 30 Kinects is obtained by setting $x = 30$ in the cost equation:

$$C(30) = 150(30) + 30 = \$4{,}530$$

85. A linear demand function has the form

$$q = mp + b$$

(p is the price, and q is the demand). We are given two points on its graph: (1, 1,960) and (5, 1,800).

Slope:

$$m = \frac{q_2 - q_1}{p_2 - p_1} = \frac{1{,}800 - 1{,}960}{5 - 1} = \frac{-160}{4} = -40$$

Intercept:

$$b = q_1 - mp_1 = 1{,}960 - (-40)(1) = 1{,}960 + 40 = 2{,}000$$

Thus, the demand equation is

$$q = mp + b = -40p + 2{,}000$$

86. A linear demand function has the form

$$q = mp + b.$$

(p is the price, and q is the demand). We are given two points on its graph: (5, 3,950) and (10, 3,700).

Slope:

$$m = \frac{q_2 - q_1}{p_2 - p_1} = \frac{3{,}700 - 3{,}950}{10 - 5} = \frac{-250}{5} = -50$$

Intercept:

$$b = q_1 - mp_1 = 3{,}950 - (-50)5 = 4{,}200$$

Thus, the demand equation is

$$q = mp + b = -50p + 4{,}200$$

87. a. A linear demand function has the form $q = mp + b$. (p is the price, and q is the demand). We are given two points on its graph:
2012: (385, 720)
2013: (335, 1,010)

Slope: $m = \frac{q_2 - q_1}{p_2 - p_1} = \frac{1{,}010 - 720}{335 - 385} = \frac{290}{-50} = -5.8$

Intercept: $b = q_1 - mp_1 = 720 - (-5.8)385 = 2{,}953$

Thus, the demand equation is $q = mp + b = -5.8p + 2{,}953$.

If $p = \$265$, then $q = -5.8(265) + 2{,}953 = 1{,}416$ million phones.

b. Since the slope is -5.8 million phones per unit increase in price, we interpret of the slope as follows: For every $1 increase in price, sales of smartphones decrease by 5.8 million units.

88. a. A linear demand function has the form $q = mp + b$. (p is the price, and q is the demand). We are given two points on its graph:
2013: (335, 1,010)
2017: (265, 1,710)

Slope: $m = \frac{q_2 - q_1}{p_2 - p_1} = \frac{1{,}710 - 1{,}010}{265 - 335} = \frac{700}{-70} = -10$

Intercept: $b = q_1 - mp_1 = 1{,}010 - (-10)335 = 4{,}360$

Thus, the demand equation is $q = mp + b = -10p + 4{,}360$.

If $p = \$385$, then $q = -10(385) + 4{,}360 = 510$ million phones.

b. Since the slope is -10 million phones per unit increase in price, we interpret of the slope as follows: For every $1 increase in price, sales of smartphones decrease by 10 million units.

89. a. A linear demand function has the form $q = mp + b$. (p is the price, and q is the demand).

We are given two points on its graph: (3, 28,000) and (5, 19,000).

Slope: $m = \frac{q_2 - q_1}{p_2 - p_1} = \frac{19{,}000 - 28{,}000}{5 - 3} = \frac{-9{,}000}{2} = -4{,}500$

Intercept: $b = q_1 - mp_1 = 28{,}000 - (-4{,}500)3 = 28{,}000 + 13{,}500 = 41{,}500$

Thus, the demand equation is $q = mp + b = -4{,}500p + 41{,}500$.

b. The units of measurement of the slope are generally units of y per unit of x. In this case: Units of q per unit of p. That is,

Rides per day per \$1 increase in the fare.

Since the slope is −4,500 rides/day per \$1 increase in the price, we interpret it as saying that ridership decreases by 4,500 rides per day for every \$1 increase in the fare.

c. From part (a), the demand equation is

$q = -4{,}500p + 41{,}500$

If the fare is \$6, we have $p = 6$, so

$q = -4500(6) + 41{,}500 = -27{,}000 + 41{,}500 = 14{,}500$ rides/day.

90. a. A linear demand function has the form $q = mp + b$. (p is the price, and q is the demand).

We are given two points on its graph: (5, 14) and (3, 18).

Slope: $m = \frac{q_2 - q_1}{p_2 - p_1} = \frac{18 - 14}{3 - 5} = \frac{4}{-2} = -2$

Intercept: $b = q_1 - mp_1 = 14 - (-2)(5) = 24$

Thus, the demand equation is $q = mp + b = -2p + 24$.

b. The units of measurement of the slope are generally units of y per unit of x. In this case: Units of q per unit of p. That is,

Millions of rides/day per Z̲1 increase in the fare.

Since the slope is −2 million rides/day per Z̲1 increase in the price, we interpret it as saying that ridership decreases by 2 million rides per day for every Z̲1 increase in the fare.

c. From part (a), the demand equation is

$q = -2p + 24$

If the fare is Z̲10, we have $p = 10$, so

$q = -2(10) + 24 = 4$ million rides/day.

91. a. In a linear model of y versus time t, the slope is the number of units of y per unit time, and we are given this quantity: 40 million pounds/year. Thus, working in millions of pounds, we can take $m = 40$. We are also given the y -intercept (the value of y at $t = 0$) as $b = 290$. Thus, the model is

$y = 40t + 290$ million pounds of pasta

b. In 2005, $t = 15$, and so $y(15) = 40(15) + 290 = 890$ million pounds.

92. a. In a linear model of y versus time t, the slope is the number of units of y per unit time, and we are given this quantity: 60 million kg/year. Thus, working in millions of kilograms, we can take $m = 60$. We are also given the y -intercept (the value of y at $t = 0$) as $b = 550$. Thus, the model is

$y = 60t + 550$ million kg of mercury

b. The year 2230 corresponds to $t = 20$, and so $y = 60(20) + 550 = 1{,}750$ million kg.

93. a. The desired linear model has the form $N = mt + b$, where t is time in years since 2010. We are given two points on its graph: 2011 data: $(1, 0.63)$; 2014 data: $(4, -0.24)$

Slope: $m = \dfrac{N_2 - N_1}{t_2 - t_1} = \dfrac{-0.24 - 0.63}{4 - 1} = \dfrac{-0.87}{3} = -0.29$

Intercept: $b = N_1 - mt_1 = 0.63 - (-0.29)(1) = 0.92$

Thus, the linear model is $N = mp + b = -0.29t + 0.92$.

b. The units of measurement of the slope are units of N per unit of t; that is, billions of dollars per year. Amazon's net income decreased at a rate of \$0.29 billion per year.

c. The year 2013 corresponds to $t = 3$, and so

$$N = -0.29(3) + 0.92 = \$0.05 \text{ billion},$$

which differs quite significantly from the actual net income.

94. a. The desired linear model has the form $E = mt + b$, where t is time in years since 2010. We are given two points on its graph: 2008 data: $(-2, 3.6)$; 2012 data: $(2, 16.3)$

Slope: $m = \dfrac{E_2 - E_1}{t_2 - t_1} = \dfrac{16.3 - 3.6}{2 - (-2)} = \dfrac{12.7}{4} = 3.175$

Intercept: $b = E_1 - mt_1 = 3.6 - (3.175)(-2) = 9.95$

Thus, the linear model is $E = mp + b = 3.175t + 9.95$.

b. The units of measurement of the slope are units of E per unit of t; that is, billions of dollars per year. Amazon's operating expenses grew at a rate of \$3.175 billion per year.

c. The year 2011 corresponds to $t = 1$, and so

$$E = 3.175(1) + 9.95 = \$13.125 \text{ billion},$$

reasonably close to the actual operating expenses.

95. $s(t) = 2.5t + 10$

a. Velocity = slope = 2.5 feet/sec.

b. After 4 seconds, $t = 4$, so

$$s(4) = 2.5(4) + 10 = 10 + 10 = 20$$

Thus the model train has moved 20 feet along the track.

c. The train will be 25 feet along the track when $s = 25$. Substituting gives

$$25 = 2.5t + 10$$

Solving for time t gives

$$2.5t = 25 - 10 = 15$$

$$t = \frac{15}{2.5} = 6 \text{ seconds}$$

96. $s(t) = -1.8t + 9$

a. Velocity = slope = -1.8 feet/sec.

b. $s(4) = -1.8(4) + 9 = 1.8$ feet from the ground.

c. $0 = -1.8t + 9$, giving $t = 5$ seconds

97. a. Take s to be displacement from Jones Beach, and t to be time in hours. We are given two points:

$(t, s) = (10, 0)$ $s = 0$ for Jones Beach.

$(t, s) = (10.1, 13)$ 6 minutes = 0.1 hours

We are asked for the speed, which equals the magnitude of the slope.

$$m = \frac{s_2 - s_1}{t_2 - t_1} = \frac{13 - 0}{10.1 - 10} = \frac{13}{0.1} = 130$$

Units of slope = units of s per unit of t = miles per hour

Thus, the police car was traveling at 130 mph.

b. For the displacement from Jones Beach at time t, we want to express s as a linear function of t; namely, $s = mt + b$.

We already know $m = 130$ from part (a). For the intercept, use

$$b = s_1 - mt_1 = 0 - 130(10) = -1{,}300$$

Therefore, the displacement at time t is

$$s = mt + b = 130t - 1{,}300$$

98. a. Take s to be displacement from Jones Beach, and t to be time in hours. We are given two points:

$(t, s) = (9.9, 0)$ and $(10.1, 13)$

We are asked for the speed, which equals the magnitude of the slope.

$$m = \frac{s_2 - s_1}{t_2 - t_1} = \frac{13 - 0}{10.1 - 9.9} = 65 \text{ miles per hour}$$

Thus, the perp was traveling at 65 mph.

b. $s = mt + b$, where $m = 65$ from part (a), and

$$b = s_1 - mt_1 = 0 - 65(9.9) = -643.5$$

Therefore,

$$s = mt + b = 65t - 643.5$$

99. a. The desired linear model has the form $L = mn + b$. We are given two points on its graph: Second edition data: $(2, 585)$; Sixth edition data: $(6, 755)$

Slope: $m = \frac{L_2 - L_1}{n_2 - n_1} = \frac{755 - 585}{6 - 2} = \frac{170}{4} = 42.5$

Intercept: $b = L_1 - mn_1 = 585 - (42.5)(2) = 500$

Thus, the linear model is $L = mn + b = 42.5n + 500$.

b. The units of measurement of the slope are units of L per unit of n; that is, pages per edition; *Applied Calculus* is growing at a rate of 42.5 pages per edition.

c. The length L will equal 1,500 when $42.5n + 500 = 1{,}500$. Solving for n gives

$$42.5n = 1{,}500 - 500 = 1{,}000$$

$$n = \frac{1{,}000}{42.5} \approx 23.5$$

Thus, by the 24th edition, the book will be over 1,500 pages long.

100. a. The desired linear model has the form $L = mn + b$. We are given two points on its graph: Second edition data: $(2, 603)$; Fifth edition data: $(5, 690)$

Slope: $m = \frac{L_2 - L_1}{n_2 - n_1} = \frac{690 - 603}{5 - 2} = \frac{87}{3} = 29$

Intercept: $b = L_1 - mn_1 = 603 - (29)(2) = 545$

Thus, the linear model is $L = mn + b = 29n + 545$.

b. The units of measurement of the slope are units of L per unit of n; that is, pages per edition; *Finite Mathematics* is growing at a rate of 29 pages per edition.

c. $L = 29n + 545 = 1{,}000$ when $n = \frac{1{,}000 - 545}{29} \approx 15.7$. Thus, by the 16th edition, the book will be over 1,000 pages long.

101. F = Fahrenheit temperature, C = Celsius temperature, and we want F as a linear function of C. That is,

$F = mC + b.$

(F plays the role of y and C plays the role of x.)
We are given two points:

$(C, F) = (0, 32)$	Freezing point
$(C, F) = (100, 212)$	Boiling point

Slope: $m = \frac{F_2 - F_1}{C_2 - C_1} = \frac{212 - 32}{100 - 0} = \frac{180}{100} = 1.8$

Intercept: $b = F_1 - mC_1 = 32 - 1.8(0) = 32$
Thus, the linear relation is

$F = mC + b = 1.8C + 32$

When $C = 30°$

$F = 1.8(30) + 32 = 54 + 32 = 86°$

When $C = 22°$

$F = 1.8(22) + 32 = 39.6 + 32 = 71.6°$. Rounding to the nearest degree gives 72°F.

When $C = -10°$, $F = 1.8(-10) + 32 = -18 + 32 = 14°$.

When $C = -14°$, $F = 1.8(-14) + 32 = -25.2 + 32 = 6.8°$. Rounding to the nearest degree gives 7°F.

102. F = Fahrenheit temperature, C = Celsius temperature, and we want F as a linear function of C. That is,

$F = mC + b.$

(F plays the role of y and C plays the role of x.)
We are given two points:

$(F, C) = (32, 0)$ and $(212, 100)$

Slope: $m = \frac{C_2 - C_1}{F_2 - F_1} = \frac{100 - 0}{212 - 32} = \frac{100}{180} = \frac{5}{9}$

Intercept: $b = C_1 - mF_1 = 0 - \frac{5}{9}(32) = -\frac{160}{9}$

Thus, the linear relation is

$C = mF + b = \frac{5}{9}F - \frac{160}{9}$

$C(104) = \frac{5}{9}(104) - \frac{160}{9} = \frac{360}{9} = 40°$

$C(77) = \frac{5}{9}(77) - \frac{160}{9} = \frac{225}{9} = 25°$

$C(14) = \frac{5}{9}(14) - \frac{160}{9} = \frac{-90}{9} = -10°$

$$C(-40) = \frac{5}{9}(-40) - \frac{160}{9} = \frac{-360}{9} = -40°$$

103. a. S = Southwest Airlines net income (in $ millions), J = JetBlue Airways net income (in $ millions), and we want J as a linear function of S. That is,

$J = mS + b$ J plays the role of y and S plays the role of x.

We are given two points:

$(S, J) = (400, 130)$ 2012 data

$(S, J) = (900, 400)$ 2014 data

Slope: $m = \frac{J_2 - J_1}{S_2 - S_1} = \frac{400 - 130}{900 - 400} = \frac{270}{500} = 0.54$

Intercept: $b = J_1 - mS_1 = 130 - (0.54)(400) = -86$

Thus, the linear relation is

$J = mS + b = 0.54S - 86.$

b. In 2010, Southwest Airlines' net income was $S = 450$, so

$J = 0.54(450) - 86 = 157,$

predicting a $157 million net income for JetBlue, $57 million higher than the actual $100 million net income JetBlue earned in 2010.

c. The units of measurement of the slope are units of J per unit of S; that is, millions of dollars of JetBlue Airways net income per million dollars of Southwest Airlines net income; JetBlue Airways earned an additional net income of $0.54 per $1 additional net income earned by Southwest Airlines.

104. a. A = Alaska Air Group net income (in $ millions), J = JetBlue Airways net income (in $ millions), and we want J as a linear function of A. That is,

$J = mA + b$ J plays the role of y and A plays the role of x.

We are given two points:

$(A, J) = (250, 100)$ 2010 data

$(A, J) = (500, 170)$ 2013 data

Slope: $m = \frac{J_2 - J_1}{A_2 - A_1} = \frac{170 - 100}{500 - 250} = \frac{70}{250} = 0.28$

Intercept: $b = J_1 - mA_1 = 100 - (0.28)(250) = 30$

Thus, the linear relation is

$J = mA + b = 0.28A + 30.$

b. 2011: $A = 250$; $J = 0.28(250) + 30 = \$100$ million

2012: $A = 300$; $J = 0.28(300) + 30 = \$114$ million

2014: $A = 600$; $J = 0.28(600) + 30 = \$198$ million

comparing these values with those in the table shows that the model gives the best prediction for 2011 ($10 million higher than the actual JetBlue income).

c. The units of measurement of the slope are units of J per unit of A; that is, millions of dollars of JetBlue Airways net income per million dollars of Southwest Airlines net income; JetBlue Airways earned an additional net income of $0.28 per $1 additional net income earned by Southwest Airlines.

105. Income = royalties + screen rights

I = 5% of net profits + 50,000

$I = 0.05N + 50{,}000$ Equation notation

$I(N) = 0.05N + 50{,}000$ Function notation

For an income of $100,000,

$100{,}00 = 0.05N + 50{,}000$

$0.05N = 50{,}000$

$N = \frac{50{,}000}{0.05} = \$1{,}000{,}000$

Her marginal income is her increase in income per $1 increase in net profit. This is the slope, $m = 0.05$ dollars of income per dollar of net profit, or 5¢ per dollar of net profit.

106. $I = mN + b$

$b = 100{,}000, m = 0.08,$ and so $I = 0.08N + 100{,}000$

For an income of $1,000,000,

$1{,}000{,}000 = 0.08N + 100{,}000$

$0.08N = 900{,}000$

$N = \$11{,}250{,}000$

Marginal income is $m =$ 8¢ per dollar of net profit.

107. The year 2000 corresponds to $t = 10,$ which is in the range $6 \leq t < 15,$ so we use the first equation: $v(t) = 400t - 2{,}200$. The slope is 400 MHz/year, telling us that the speed of a processor was increasing by 400 MHz/year.

108. The year 2000 corresponds to $t = 25,$ which is in the range $20 \leq t \leq 30,$ so we use the second equation: $v(t) = 174t - 3{,}420$. The slope is 174 MHz/year, telling us that the speed of a processor was increasing by 174 MHz/year.

109. The data are

t	0	20	40
y	78	2,100	2,950

a. 1970–1990 (first two data points):

Slope: $m = \frac{y_2 - y_1}{t_2 - t_1} = \frac{700 - 78}{20 - 0} = 31.1$

Intercept: $b = 78,$ specified in first data point

Thus, the linear model is

$y = mt + b = 31.1t + 78.$

b. 1990–2010 (second and third data points):

Slope: $m = \frac{y_2 - y_1}{t_2 - t_1} = \frac{2{,}950 - 700}{40 - 20} = 112.5$

Intercept: $b = y_1 - mt_1 = 700 - (112.5)20 = -1{,}550$

Thus, the linear model is

$y = mt + b = 112.5t - 1{,}550.$

c. Since the first model is valid for $0 \le t \le 20$ and the second one for $20 \le t \le 40$, we put them together as

$$y = \begin{cases} 31.1t + 78 & \text{if } 0 \le t < 20 \\ 112.5t - 1{,}550 & \text{if } 20 \le t \le 40. \end{cases}$$

Notice that, since both formulas agree at $t = 20$, we can also say

$$y = \begin{cases} 31.1t + 78 & \text{if } 0 \le t \le 20 \\ 112.5t - 1{,}550 & \text{if } 20 < t \le 40. \end{cases}$$

d. Since 2004 is represented by $t = 34$, we use the second formula to obtain

$$y = 112.5(34) - 1{,}550 = 2{,}275,$$

or $2,275,000, in good agreement with the actual value shown in the graph.

110. The data are

t	0	20	30
y	222	1,100	2,500

a. 1980–2000 (first two data points):

Slope: $m = \dfrac{y_2 - y_1}{t_2 - t_1} = \dfrac{2{,}100 - 222}{20 - 0} = 93.9$

Intercept: $b = 222$, specified in first data point

Thus, the linear model is

$$y = mt + b = 93.9t + 222.$$

b. 2000–2010 (second and third data points):

Slope: $m = \dfrac{y_2 - y_1}{t_2 - t_1} = \dfrac{2{,}950 - 2{,}100}{30 - 20} = 85$

Intercept: $b = y_1 - mt_1 = 2{,}100 - (85)20 = 400$

Thus, the linear model is

$$y = mt + b = 85t + 400.$$

c. Since the first model is valid for $0 \le t \le 20$ and the second one for $20 \le t \le 30$, we put them together as

$$y = \begin{cases} 93.9t + 222 & \text{if } 0 \le t < 20 \\ 85t + 400 & \text{if } 20 \le t \le 30. \end{cases}$$

Notice that, since both formulas agree at $t = 20$, we can also say

$$y = \begin{cases} 93.9t + 222 & \text{if } 0 \le t \le 20 \\ 85t + 400 & \text{if } 20 < t \le 30. \end{cases}$$

d. Since 1992 is represented by $t = 12$, we use the first formula to obtain

$$y = 93.9(12) + 222 = 1348.8,$$

or $1,348,800, considerably higher than the actual value shown on the graph. The actual cost is not linear in the range 1980–2000.

111. 1995–2000: Points:

1995 data: $(t, N) = (0, 3)$

2000 data: $(t, N) = (5, 4.1)$

Slope: $m = \dfrac{N_2 - N_1}{t_2 - t_1} = \dfrac{4.1 - 3}{5 - 0} = 0.22$

Intercept: $b = 3$, specified in first data point

Thus, the linear model is

$$N = mt + b = 0.22t + 3.$$

2000–2004: Points:

2000 data: $(t, N) = (5, 4.1)$

2004 data: $(t, N) = (9, 3.5)$

Slope: $m = \dfrac{N_2 - N_1}{t_2 - t_1} = \dfrac{3.5 - 4.1}{9 - 5} = -0.15$

Intercept: $b = N_1 - mt_1 = 4.1 - (-0.15)(5) = 4.85$

Thus, the linear model is

$$N = mt + b = -0.15t + 4.85.$$

Putting them together gives

$$N = \begin{cases} 0.22t + 3 & \text{if } 0 \le t \le 5 \\ -0.15t + 4.85 & \text{if } 5 < t \le 9 \end{cases}$$

The number of manufacturing jobs in Mexico in 2002 is $N(7)$, so we use the second formula to obtain

$$N(7) = -0.15(7) + 4.85 = 3.8 \text{ million jobs}$$

112. 2001–2004: Points:

2001 data: $(t, P) = (0, 9.7)$

2004 data: $(t, P) = (3, 4.3)$

Slope: $m = \dfrac{P_2 - P_1}{t_2 - t_1} = \dfrac{4.3 - 9.7}{3 - 0} = -1.8$

Intercept: $b = 9.7$, specified in first data point

Thus, the linear model is

$$N = mt + b = -1.8t + 9.7.$$

2004–2007: Points:

2004 data: $(t, P) = (3, 4.3)$

2007 data: $(t, P) = (6, 10.3)$

Slope: $m = \dfrac{P_2 - P_1}{t_2 - t_1} = \dfrac{10.3 - 4.3}{6 - 3} = 2$

Intercept: $b = P_1 - mt_1 = 4.3 - (2)(3) = -1.7$

Thus, the linear model is

$$N = mt + b = 2t - 1.7.$$

Putting them together gives

$$P = \begin{cases} -1.8t + 9.7 & \text{if } 0 \le t \le 3 \\ 2t - 1.7 & \text{if } 3 < t \le 6 \end{cases}$$

The percentage of delinquent borrowers in 2006 is $P(5)$, so we use the second formula to obtain

$P(5) = 2(5) - 1.7 = 8.3\%$.

Communication and reasoning exercises

113. Compute the corresponding successive changes Δx in x and Δy in y, and compute the ratios $\Delta y/\Delta x$. If the answer is always the same number, then the values in the table come from a linear function.

114. The desired equation has the form $y = mx + b$. The slope m is given by $m = \Delta y/\Delta x$, where Δx and Δy are corresponding changes in x and y. The intercept b is given by the y-value corresponding to $x = 0$, if supplied. If it is not supplied, choose any point (x_1, y_1) and use the formula $b = y_1 - mx_1$.

115. To find the linear function, solve the equation $ax + by = c$ for y:

$$by = -ax + c$$

$$y = -\frac{a}{b}x + \frac{c}{b}$$

Thus, the desired function is $f(x) = -\frac{a}{b}x + \frac{c}{b}$.

If $b = 0$, then $\frac{a}{b}$ and $\frac{c}{b}$ are undefined, and y cannot be specified as a function of x. (The graph of the resulting equation would be a vertical line.)

116. The slope of the line with equation $y = mx + b$ is the number of units that _y_ increases per unit increase in _x_.

117. The slope of the line is $m = \frac{\Delta y}{\Delta x} = \frac{3}{1} = 3$. Therefore, if, in a straight line, y is increasing three times as fast as x, then its slope is 3.

118. The slope is $m = -4/3$ units of y per unit of x. We do not have enough information to compute the intercept.

119. If m is positive, then y will increase as x increases; if m is negative, then y will decrease as x increases; if m is zero, then y will not change as x changes.

120. Since $\Delta y = -\Delta x$, the function is linear with slope

$$m = \frac{\Delta y}{\Delta x} = \frac{-\Delta x}{\Delta x} = -1.$$

121.

◇	A	B	C	D
1	x	y	m	b
2	1	2	=(B3-B2)/(A3-A2)	=B2-C2*A2
3	3	-1	Slope	Intercept

The slope computed in cell C2 is given by

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1 - 2}{3 - 1} = -1.5$$

If we increase the y-coordinate in cell B3, this increases y_2, and thus increases the numerator $\Delta y = y_2 - y_1$ without affecting the denominator Δx. Thus the slope will increase.

122.

◇	A	B	C	D
1	x	y	m	b
2	1	2	=(B3-B2)/(A3-A2)	=B2-C2*A2
3	3	-1	Slope	Intercept

The slope increases: Δy is negative, and stays the same, while Δx becomes a larger positive number, so the (negative) slope decreases in absolute value, meaning that it actually *increases*.

Solutions Section 1.3

123. The units of the slope m are units of y (bootlags) per unit of x (zonars). The intercept b is on the y-axis, and is thus measured in units of y (bootlags). Thus, m is measured in bootlags per zonar and b is measured in bootlags.

124. Units of slope = units of y per unit of x = miles per dollar. Thus, the independent variable is measured in dollars and the dependent variable is measured in miles.

125. If a quantity changes linearly with time, it must change by the same amount for every unit change in time. Thus, since it increases by 10 units in the first day, it must increase by 10 units each day, including the third.

126. Since $Q(0)$ is positive, b is positive. Since Q decreases with increasing T, m is negative.

127. Since the slope is 0.1, the velocity is increasing at a rate of 0.1 m/sec every second. Since the velocity is increasing, the object is accelerating (choice B).

128. Velocity = slope = 0.2 units of position per unit time. Thus, the object is moving with constant speed (choice A).

129. Write $f(x) = mx + b$ and $g(x) = nx + c$. Then

$$f(x) + g(x) = mx + b + (nx + c) = (m + n)x + (b + c),$$

also a linear function with slope $m + n$.

130. Not necessarily; for instance, if $f(x) = 2x + 1$ and $g(x) = x$, then their ratio has values $\frac{2x + 1}{x} = 2 + \frac{1}{x}$, not a linear function of x. (Only if g is a nonzero constant will the ratio will be linear.)

131. Answers may vary. For example, $f(x) = x^{1/3}, g(x) = x^{2/3}$ gives

$$f(x)g(x) = x^{1/3}x^{2/3} = x.$$

132. Answers may vary. For example, $f(x) = x^3 + 2x^2,\quad g(x) = x^2$ gives

$$\frac{f(x)}{g(x)} = \frac{x^3 + 2x^2}{x^2} = x + 2 \text{ (with domain all real numbers other than 0).}$$

133. Increasing the number of items from the break-even number results in a profit: Because the slope of the revenue graph is larger than the slope of the cost graph, it is higher than the cost graph to the right of the point of intersection, and hence corresponds to a profit.

134. You should solve your equation for q to obtain q as a function of p. Simply switching p and q will result in the wrong equation, and starting from scratch would be a less efficient way of obtaining the result you want.

Section 1.4

1. $(1, 1),\ (2, 2),\ (3, 4);\ \ y = x - 1$

x	y	$\hat{y} = x - 1$	$y - \hat{y}$	$(y - \hat{y})^2$
1	1	0	1	1
2	2	1	1	1
3	4	2	2	4

SSE = Sum of squares of residuals (last column) = 1 + 1 + 4 = 6

2. $(0, 1),\ (1, 1),\ (2, 2);\ \ y = x + 1$

x	y	$\hat{y} = x + 1$	$y - \hat{y}$	$(y - \hat{y})^2$
0	1	1	0	0
1	1	2	−1	1
2	2	3	−1	1

SSE = Sum of squares of residuals (last column) = 0 + 1 + 1 = 2

3. $(0, -1),\ (1, 3),\ (4, 6),\ (5, 0);\ \ y = -x + 2$

x	y	$\hat{y} = -x + 2$	$y - \hat{y}$	$(y - \hat{y})^2$
0	−1	2	−3	9
1	3	1	2	4
4	6	−2	8	64
5	0	−3	3	9

SSE = Sum of squares of residuals (last column) = 9 + 4 + 64 + 9 = 86

4. $(2, 4),\ (6, 8),\ (8, 12),\ (10, 0);\ \ y = 2x - 8$

x	y	$\hat{y} = 2x - 8$	$y - \hat{y}$	$(y - \hat{y})^2$
2	4	−4	8	64
6	8	4	4	16
8	12	8	4	16
10	0	12	−12	144

SSE = Sum of squares of residuals (last column) = 240

5. $(1, 1),\ (2, 2),\ (3, 4)$

a. $y = 1.5x - 1$

x	y	$\hat{y} = 1.5x - 1$	$y - \hat{y}$	$(y - \hat{y})^2$
1	1	0.5	0.5	0.25
2	2	2	0	0
3	4	3.5	0.5	0.25

SSE = Sum of squares of residuals = 0.5

b. $y = 2x - 1.5$

x	y	$\hat{y} = 2x - 1.5$	$y - \hat{y}$	$(y - \hat{y})^2$
1	1	0.5	0.5	0.25
2	2	2.5	−0.5	0.25
3	4	4.5	−0.5	0.25

SSE = Sum of squares of residuals = 0.75
The model that gives the better fit is (a) because it gives the smaller value of SSE.

6. $(0, 1),\ (1, 1),\ (2, 2)$

a. $y = 0.4x + 1.1$

x	y	$\hat{y} = 0.4x + 1.1$	$y - \hat{y}$	$(y - \hat{y})^2$
0	1	1.1	−0.1	0.01
1	1	1.5	−0.5	0.25
2	2	1.9	0.1	0.01

SSE = Sum of squares of residuals = 0.27

b. $y = 0.5x + 0.9$

x	y	$\hat{y} = 0.5x + 0.9$	$y - \hat{y}$	$(y - \hat{y})^2$
0	1	0.9	0.1	0.01
1	1	1.4	−0.4	0.16
2	2	1.9	0.1	0.01

SSE = Sum of squares of residuals = 0.18
The model that gives the better fit is (b) because it gives the smaller value of SSE.

7. $(0, -1),\ (1, 3),\ (4, 6),\ (5, 0)$

a. $y = 0.3x + 1.1$

x	y	$\hat{y} = 0.3x + 1.1$	$y - \hat{y}$	$(y - \hat{y})^2$
0	−1	1.1	−2.1	4.41
1	3	1.4	1.6	2.56
4	6	2.3	3.7	13.69
5	0	2.6	−2.6	6.76

SSE = Sum of squares of residuals = 27.42

b. $y = 0.4x + 0.9$

x	y	$\hat{y} = 0.4x + 0.9$	$y - \hat{y}$	$(y - \hat{y})^2$
0	−1	0.9	−1.9	3.61
1	3	1.3	1.7	2.89
4	6	2.5	3.5	12.25
5	0	2.9	−2.9	8.41

SSE = Sum of squares of residuals = 27.16
The model that gives the better fit is (b) because it gives the smaller value of SSE.

8. $(2, 4)$, $(6, 8)$, $(8, 12)$, $(10, 0)$

a. $y = -0.1x + 7$

x	y	$\hat{y} = -0.1x + 7$	$y - \hat{y}$	$(y - \hat{y})^2$
2	4	6.8	−2.8	7.84
6	8	6.4	1.6	2.56
8	12	6.2	5.8	33.64
10	0	6	−6	36

SSE = Sum of squares of residuals = 80.04

b. $y = -0.2x + 6$

x	y	$\hat{y} = -0.2x + 6$	$y - \hat{y}$	$(y - \hat{y})^2$
2	4	5.6	−1.6	2.56
6	8	4.8	3.2	10.24
8	12	4.4	7.6	57.76
10	0	4	−4	16

SSE = Sum of squares of residuals = 86.56
The model that gives the better fit is (a) because it gives the smaller value of SSE.

9. Data points (x, y): $(1, 1)$, $(2, 2)$, $(3, 4)$

x	y	xy	x^2
1	1	1	1
2	2	4	4
3	4	12	9
6	7	17	14

(The bottom row contains the column sums.)

$n = 3$ (number of data points)

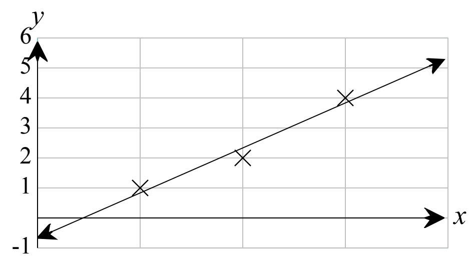
Slope: $m = \dfrac{n(\sum xy) - (\sum x)(\sum y)}{n(\sum x^2) - (\sum x)^2} = \dfrac{3(17) - (6)(7)}{3(14) - 6^2} = \dfrac{9}{6} = 1.5$

Intercept: $b = \dfrac{\sum y - m(\sum x)}{n} = \dfrac{7 - 1.5(6)}{3} = \dfrac{-2}{3} \approx -0.6667$

Thus, the regression line is

$y = mx + b \approx 1.5x - 0.6667.$

Graph:



10. Data points (x, y): $(0, 1)$, $(1, 1)$, $(2, 2)$

x	y	xy	x^2
0	1	0	0
1	1	1	1
2	2	4	4
3	4	5	5

(The bottom row contains the column sums.)

$n = 3$ (number of data points)

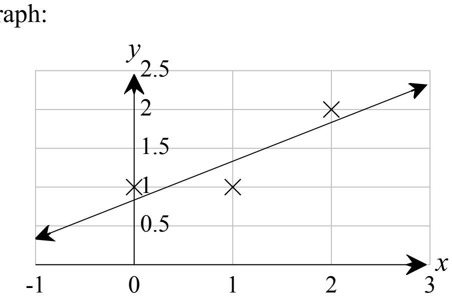
Slope: $m = \dfrac{n(\sum xy) - (\sum x)(\sum y)}{n(\sum x^2) - (\sum x)^2} = \dfrac{3(5) - (3)(4)}{3(5) - 3^2} = \dfrac{3}{6} = 0.5$

Intercept: $b = \dfrac{\sum y - m(\sum x)}{n} = \dfrac{4 - 0.5(3)}{3} = \dfrac{2.5}{3} \approx 0.8333$

Thus, the regression line is

$y = mx + b \approx 0.5x + 0.8333.$

Graph:



11. Data points (x, y): $(0, -1)$, $(1, 3)$, $(3, 6)$, $(4, 1)$

x	y	xy	x^2
0	−1	0	0
1	3	3	1
3	6	18	9
4	1	4	16
8	9	25	26

(The bottom row contains the column sums.)

$n = 4$ (number of data points)

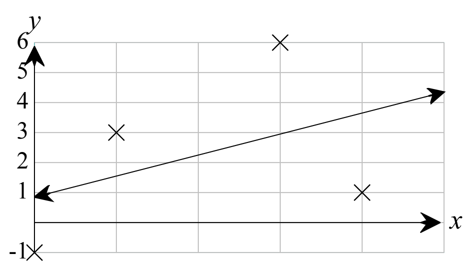
Slope: $$m = \frac{n(\sum xy) - (\sum x)(\sum y)}{n(\sum x^2) - (\sum x)^2} = \frac{4(25) - (8)(9)}{4(26) - 8^2} = \frac{28}{40} = 0.7$$

Intercept: $$b = \frac{\sum y - m(\sum x)}{n} = \frac{9 - 0.7(8)}{4} = \frac{3.4}{4} = 0.85$$

Thus, the regression line is

$y = mx + b = 0.7x + 0.85.$

Graph:



12. Data points (x, y): $(2, 4)$, $(4, 8)$, $(8, 12)$, $(10, 0)$

x	y	xy	x^2
2	4	8	4
4	8	32	16
8	12	96	64
10	0	0	100
24	24	136	184

(The bottom row contains the column sums.)

$n = 4$ (number of data points)

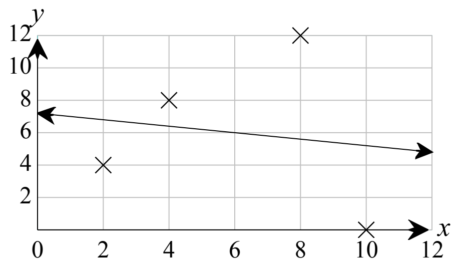
Slope: $m = \dfrac{n(\sum xy) - (\sum x)(\sum y)}{n(\sum x^2) - (\sum x)^2} = \dfrac{4(136) - (24)(24)}{4(184) - 24^2} = \dfrac{-32}{160} = -0.2$

Intercept: $b = \dfrac{\sum y - m(\sum x)}{n} = \dfrac{24 - (-0.2)(24)}{4} = \dfrac{28.8}{4} = 7.2$

Thus, the regression line is

$y = mx + b = -0.2x + 7.2.$

Graph:



13. a. (1, 3), (2, 4), (5, 6)

x	y	xy	x^2	y^2
1	3	3	1	9
2	4	8	4	16
5	6	30	25	36
8	13	41	30	61

(The bottom row contains the column sums.)

$n = 3$ (number of data points)

$$r = \frac{n(\sum xy) - (\sum x)(\sum y)}{\sqrt{n(\sum x^2) - (\sum x)^2} \cdot \sqrt{n(\sum y^2) - (\sum y)^2}} = \frac{3(41) - (8)(13)}{\sqrt{3(30) - 8^2}\sqrt{3(61) - 13^2}} \approx \frac{19}{19.078784} \approx 0.9959$$

b. (0, −1), (2, 1), (3, 4)

x	y	xy	x^2	y^2
0	−1	0	0	1
2	1	2	4	1
3	4	12	9	16
5	4	14	13	18

(The bottom row contains the column sums.)

$n = 3$ (number of data points)

$$r = \frac{n(\sum xy) - (\sum x)(\sum y)}{\sqrt{n(\sum x^2) - (\sum x)^2} \cdot \sqrt{n(\sum y^2) - (\sum y)^2}} = \frac{3(14) - (5)(4)}{\sqrt{3(13) - 5^2}\sqrt{3(18) - 4^2}} \approx \frac{22}{23.0651252} \approx 0.9538$$

c. (4, −3), (5, 5), (0, 0)

x	y	xy	x^2	y^2
4	−3	−12	16	9
5	5	25	25	25
0	0	0	0	0
9	2	13	41	34

(The bottom row contains the column sums.)

$n = 3$ (number of data points)

$$r = \frac{n(\sum xy) - (\sum x)(\sum y)}{\sqrt{n(\sum x^2) - (\sum x)^2} \cdot \sqrt{n(\sum y^2) - (\sum y)^2}} = \frac{3(13) - (9)(2)}{\sqrt{3(41) - 9^2}\sqrt{3(34) - 2^2}} \approx \frac{21}{64.1560597} \approx 0.3273$$

The value of r in part (a) has the largest absolute value. Therefore, the regression line for the data in part (a) is the best fit.

The value of r in part (c) has the smallest absolute value. Therefore, the regression line for the data in part (c) is the worst fit.

Since r is not ± 1 for any of these lines, none of them is a perfect fit.

14. a. $(1, 3),\ (-2, 9),\ (2, 1)$

x	y	xy	x^2	y^2
1	3	3	1	9
-2	9	−18	4	81
2	1	2	4	1
1	13	-13	9	91

(The bottom row contains the column sums.)

$n = 3$ (number of data points)

$$r = \frac{n(\sum xy) - (\sum x)(\sum y)}{\sqrt{n(\sum x^2) - (\sum x)^2} \cdot \sqrt{n(\sum y^2) - (\sum y)^2}} = \frac{3(-13) - (1)(13)}{\sqrt{3(9) - 1^2}\sqrt{3(91) - 13^2}} = \frac{-52}{52} = -1 \quad \text{(Best, perfect fit)}$$

b. $(0, 1),\ (1, 0),\ (2, 1)$

x	y	xy	x^2	y^2
0	1	0	0	1
1	0	0	1	0
2	1	2	4	1
3	2	2	5	2

(The bottom row contains the column sums.)

$n = 3$ (number of data points)

$$r = \frac{n(\sum xy) - (\sum x)(\sum y)}{\sqrt{n(\sum x^2) - (\sum x)^2} \cdot \sqrt{n(\sum y^2) - (\sum y)^2}} = \frac{3(2) - (3)(2)}{\sqrt{3(5) - 3^2}\sqrt{3(2) - 2^2}} = 0 \quad \text{(Worst)}$$

c. $(0, 0),\ (5, -5),\ (2, -2.1)$

x	y	xy	x^2	y^2
0	0	0	0	0
5	−5	−25	25	25
2	−2.1	−4.2	4	4.41
7	-7.1	-29.2	29	29.41

(The bottom row contains the column sums.)

$n = 3$ (number of data points)

$$r = \frac{n(\sum xy) - (\sum x)(\sum y)}{\sqrt{n(\sum x^2) - (\sum x)^2} \cdot \sqrt{n(\sum y^2) - (\sum y)^2}} = \frac{3(-29.2) - (7)(-7.1)}{\sqrt{3(29) - 7^2}\sqrt{3(29.41) - (-7.1)^2}} \approx \frac{-37.9}{37.9098932} \approx -0.9997$$

Applications

15. The entries in the xy column are obtained by multiplying the entries in the x column by the corresponding entries in the y column. The entries in the x^2 column are the squares of the entries in the x column. The entries in the last row are the sums of the respective columns.

Data points (x, y): $(0, 800)$, $(2, 1{,}600)$, $(4, 2{,}300)$

x	y	xy	x^2
0	800	0	0
2	1,600	3,200	4
4	2,300	9,200	16
6	4,700	12,400	20

(The bottom row contains the column sums.)

$n = 3$ (number of data points)

Slope: $$m = \frac{n(\sum xy) - (\sum x)(\sum y)}{n(\sum x^2) - (\sum x)^2} = \frac{3(12{,}400) - (6)(4{,}700)}{3(20) - 6^2} = \frac{9{,}000}{24} = 375$$

Intercept: $$b = \frac{\sum y - m(\sum x)}{n} = \frac{4{,}700 - 375(6)}{3} = \frac{2{,}450}{3} \approx 816.7$$

Thus, the regression line is

$y = mx + b \approx 375x + 816.7.$

Since 2016 corresponds to $x = 6$, the prediction for 2016 is

$y = 375(6) + 816.7 \approx 3{,}066.7$ million subscribers.

16. The entries in the xy column are obtained by multiplying the entries in the x column by the corresponding entries in the y column. The entries in the x^2 column are the squares of the entries in the x column. The entries in the last row are the sums of the respective columns.

Data points (x, y): $(0, 35)$, $(5, 34)$, $(14, 33)$

x	y	xy	x^2
0	35	0	0
5	34	170	25
14	33	462	196
19	102	632	221

(The bottom row contains the column sums.)

$n = 3$ (number of data points)

Slope: $m = \dfrac{n(\sum xy) - (\sum x)(\sum y)}{n(\sum x^2) - (\sum x)^2} = \dfrac{3(632) - (19)(102)}{3(221) - 19^2} = \dfrac{-42}{302} \approx -0.1$

Intercept: $b = \dfrac{\sum y - m(\sum x)}{n} \approx \dfrac{102 - (-0.139)(19)}{3} = \dfrac{104.642}{3} \approx 34.9$

Thus, the regression line is

$y = mx + b \approx -0.1x + 34.9.$

Since 2015 corresponds to $x = 15$, the prediction for 2015 is

$y = -0.1(15) + 34.9. \approx 33.4$ million subscribers.

17. A linear demand function has the form $q = mp + b$. (p is the price, and q is the demand).

Data points (p, q): $(4, 0.7)$, $(3, 1)$, $(2, 2)$

p	q	pq	p^2
4	0.7	2.8	16
3	1	3	9
2	2	4	4
9	3.7	9.8	29

(The bottom row contains the column sums.)

$n = 3$ (number of data points)

Slope: $m = \dfrac{n(\sum pq) - (\sum p)(\sum q)}{n(\sum p^2) - (\sum p)^2} = \dfrac{3(9.8) - (9)(3.7)}{3(29) - 9^2} = \dfrac{-3.9}{6} \approx -0.7$

Intercept: $b = \dfrac{\sum q - m(\sum p)}{n} = \dfrac{3.7 - (-0.65)(9)}{3} = \dfrac{9.55}{3} \approx 3.2$

Thus, the regression line is

$q = mp + b \approx -0.7p + 3.2.$

When the selling price is \$350, $p = 3.5$, and so $q \approx -0.7(3.5) + 3.2 = 0.75$ billion, or 750 million smartphones.

18. A linear demand function has the form $q = mp + b$. (p is the price, and q is the demand).

Data points (p, q): $(5, 0.3)$, $(4, 0.7)$, $(3, 1)$

p	q	pq	p^2
5	0.3	1.5	25
4	0.7	2.8	16
3	1	3	9
12	2	7.3	50

(The bottom row contains the column sums.)

$n = 3$ (number of data points)

Slope: $$m = \frac{n(\sum pq) - (\sum p)(\sum q)}{n(\sum p^2) - (\sum p)^2} = \frac{3(7.3) - (12)(2)}{3(50) - 12^2} = \frac{-2.1}{6} \approx -0.4$$

Intercept: $$b = \frac{\sum q - m(\sum p)}{n} = \frac{2 - (-0.35)(12)}{3} = \frac{6.2}{3} \approx 2.1$$

Thus, the regression line is

$q = mp + b \approx -0.4p + 2.1.$

When the selling price is \$450, $p = 4.5,$ and so $q \approx -0.4(4.5) + 2.1 = 0.3$ billion, or 300 million smartphones.

19. Following is the table we use to compute the regression line:

x	y	xy	x^2
20	3	60	400
40	6	240	1,600
80	9	720	6,400
100	15	1,500	10,000
240	33	2,520	18,400

(The bottom row contains the column sums.)

$n = 3$ (number of data points)

Slope: $$m = \frac{n(\sum xy) - (\sum x)(\sum y)}{n(\sum x^2) - (\sum x)^2} = \frac{4(2{,}520) - (240)(33)}{4(18{,}400) - 240^2} = \frac{2{,}160}{16{,}000} = 0.135$$

Intercept: $$b = \frac{\sum y - m(\sum x)}{n} = \frac{33 - (0.135)(240)}{4} = \frac{0.6}{4} = 0.15$$

The regression model is therefore $y = mx + b = 0.135x + 0.15.$

$y(50) = 0.135(50) + 0.15 = 6.9$ million jobs

20. Following is the table we use to compute the regression line:

x	y	xy	x^2
10	200	2,000	100
40	900	36,000	1,600
50	1,000	50,000	2,500
80	2,000	160,000	6,400
180	4,100	248,000	10,600

(The bottom row contains the column sums.)

$n = 3$ (number of data points)

Slope: $$m = \frac{n(\sum xy) - (\sum x)(\sum y)}{n(\sum x^2) - (\sum x)^2} = \frac{4(248,000) - (180)(4,100)}{4(10,600) - 180^2} = \frac{254,000}{10,000} = 25.4$$

Intercept: $$b = \frac{\sum y - m(\sum x)}{n} = \frac{4100 - (25.4)(180)}{4} = \frac{-472}{4} = -118$$

The regression model is therefore $y = mx + b = 25.4x - 118$.

$y(70) = 25.4(70) - 118 = \$1,660$ billion

21. a. Data points (S, I): $(50, 0.6)$, $(60, 0.1)$, $(70, 0.3)$, $(80, 0.3)$

S	I	SI	S^2
50	0.6	30	2,500
60	0.1	6	3,600
70	0.3	21	4,900
80	0.3	24	6,400
260	1.3	81	17,400

(The bottom row contains the column sums.)

$n = 4$ (number of data points)

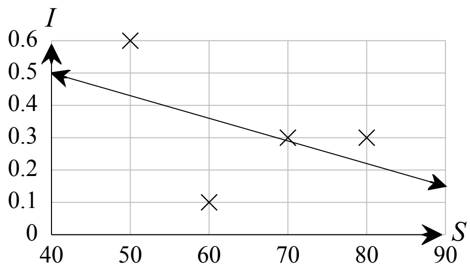
Slope: $$m = \frac{n(\sum SI) - (\sum S)(\sum I)}{n(\sum S^2) - (\sum S)^2} = \frac{4(81) - (260)(1.3)}{4(17,400) - 260^2} = \frac{-14}{2,000} = -0.007$$

Intercept: $$b = \frac{\sum I - m(\sum S)}{n} = \frac{1.3 - (-0.007)(260)}{4} = \frac{3.12}{4} = 0.78$$

Thus, the regression line is

$I = mS + b = -0.007S + 0.78$.

Graph:



Independent variable is *S* and dependent variable is *I*.

b. The units of measurement of the slope are units of net income per unit of net sales: millions of dollars of net income per billion dollars of net sales. Thus, Amazon *lost* \$0.007 billion (\$7 million) in net income per additional billion dollars in net sales.

c. $I = -0.007S + 0.78,$ and we are given $I = 0.5$. Substituting gives

$$0.5 = -0.007S + 0.78$$

Solving for S gives

$$S = \frac{-0.28}{-0.007} = 40$$

Thus, the company would need to earn \$40 billion in net sales.

d. The graph shows a poor fit, so the linear model does not seem reasonable.

22. a. Data points (S, E): $(50, 11),\ (60, 16),\ (70, 23),\ (80, 25)$

S	E	SE	S^2
50	11	550	2,500
60	16	960	3,600
70	23	1,610	4,900
80	25	2,000	6,400
260	75	5,120	17,400

(The bottom row contains the column sums.)

$n = 4$ (number of data points)

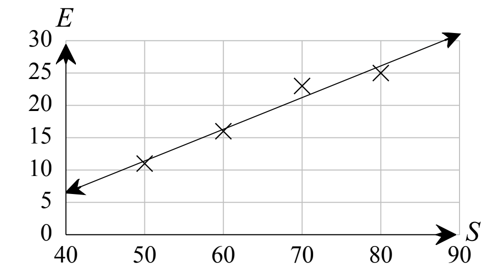
Slope: $$m = \frac{n(\sum SE) - (\sum S)(\sum E)}{n(\sum S^2) - (\sum S)^2} = \frac{4(5{,}120) - (260)(75)}{4(17{,}400) - 260^2} = \frac{980}{2{,}000} = 0.49$$

Intercept: $$b = \frac{\sum E - m(\sum S)}{n} = \frac{75 - 0.49(260)}{4} = \frac{-52.4}{4} = -13.1$$

Thus, the regression line is

$$E = mS + b = 0.49S - 13.1.$$

Graph:



Independent variable is *S* and dependent variable is *E*.

b. The units of measurement of the slope are units of operating expenses per unit of net sales: billions of dollars of operating expenses per billion dollars of net sales. Thus, Amazon had operating expenses of \$0.49 billion per \$billion in net sales; that is 49¢ per \$1 of net sales.

c. $E = 0.49S - 13.1$, and we are given $E = 5$. Substituting gives

$$5 = 0.49S - 13.1$$

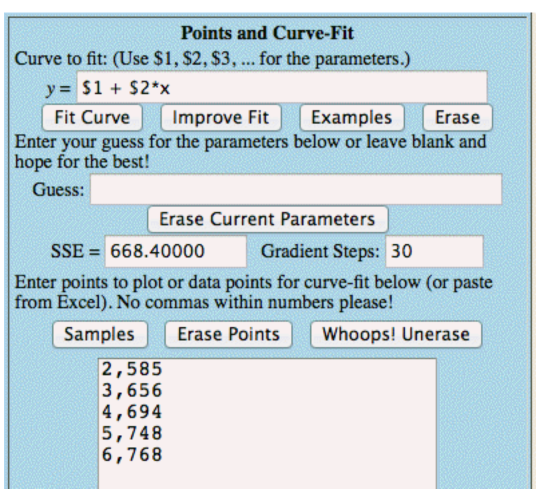
Solving for S gives

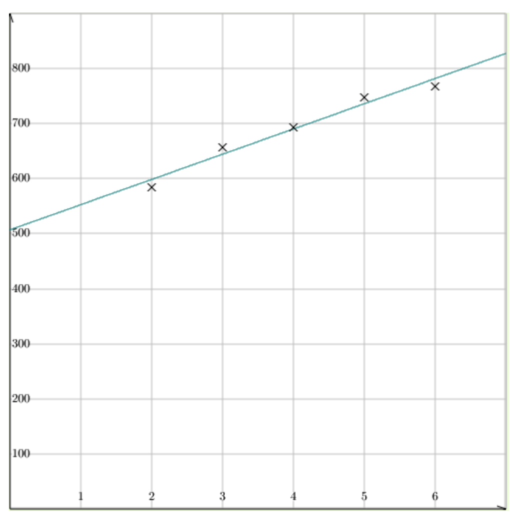
$$S = \frac{18.1}{0.49} \approx 37$$

Thus, the company would need to earn approximately \$37 billion in net sales.

d. The graph shows a close fit, so the linear model seems reasonable.

23. a. See the technology note accompanying Example 2 for the use of technology to obtain regression lines. The following result and plot were obtained using the Function Evaluator and Grapher on the Web site with the setup shown:



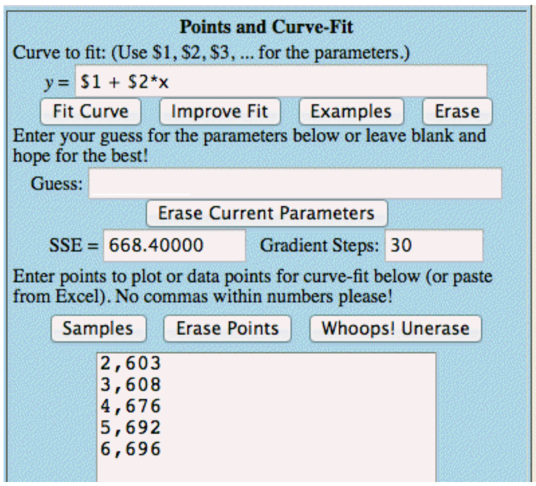


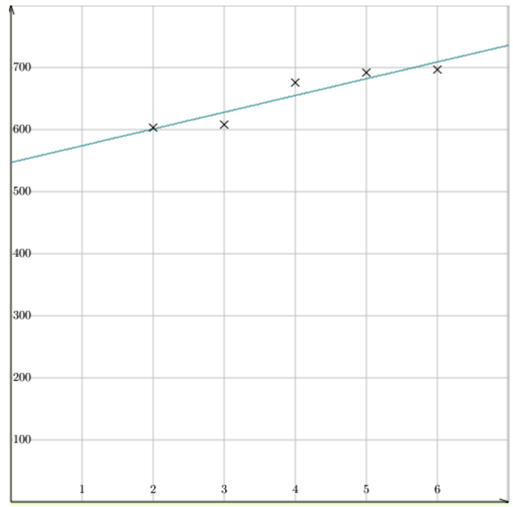
Regression equation: $L = 45.8n + 507$

b. The units of measurement of the slope are units of L per unit of n; that is, pages per edition; *Applied Calculus* is growing at a rate of 45.8 pages per edition.

24. b. See the technology note accompanying Example 2 for the use of technology to obtain regression lines. The following result and plot were obtained using the Function Evaluator and Grapher on the Web site with the setup shown:

Solutions Section 1.4





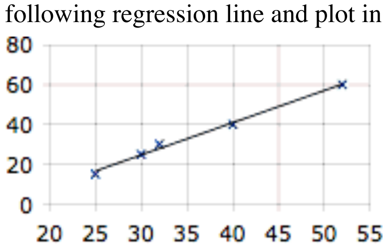
Regression equation: $L = 27n + 547$

b. The units of measurement of the slope are units of L per unit of n; that is, pages per edition; *Finite Mathematics* is growing at a rate of 27 pages per edition.

25. a. Since production is a function of cultivated area, we take x as cultivated area, and y as production:

x	25	30	32	40	52
y	15	25	30	40	60

See the technology note accompanying Example 2 for the use of technology to obtain regression lines. We obtained the following regression line and plot in Excel. (coefficients rounded to two decimal places): $y = 1.62x - 23.87$



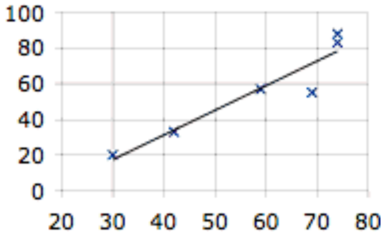
b. To interpret the slope $m = 1.62$, recall that units of m are units of y per unit of x; that is, millions of tons of production of soybeans per million acres of cultivated land. Thus, production increases by 1.62 million tons of soybeans per million acres of cultivated land. More simply, each acre of cultivated land produces about 1.62 tons of soybeans.

26. a. Since production is a function of cultivated area, we take x as cultivated area, and y as production:

x	30	42	69	59	74	74
y	20	33	55	57	83	88

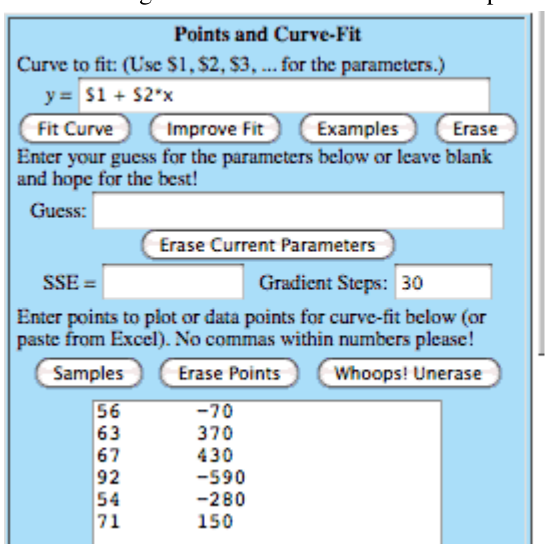
See the technology note accompanying Example 2 for the use of technology to obtain regression lines. We obtained the following regression line and plot in Excel. (coefficients rounded to two decimal places): $y = 1.38x - 24.04$

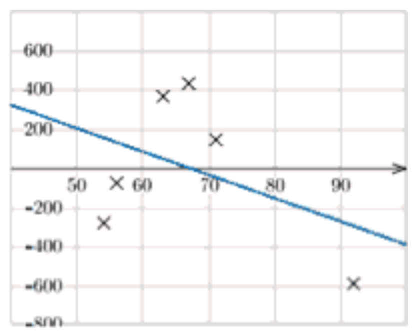
Solutions Section 1.4



b. To interpret the slope $m = 1.38$, recall that units of m are units of y per unit of x; That is, millions of tons of production of soybeans per million acres of cultivated land. Thus, production increases by 1.38 million tons of soybeans per million acres of cultivated land. More simply, each acre of cultivated land produces about 1.38 tons of soybeans.

27. a. y = Continental net income as a function of x = Price of oil. See the technology notes accompanying Example 2 and 3 for the use of technology to obtain regression lines and correlation coefficients. The following result and plot were obtained using the Function Evaluator and Grapher on the Web site with the setup shown:



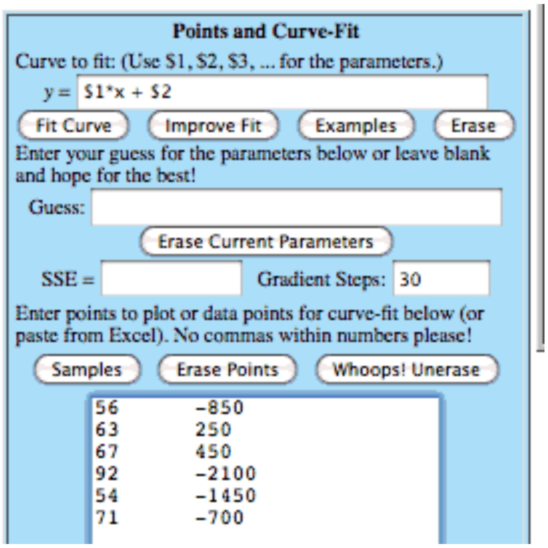


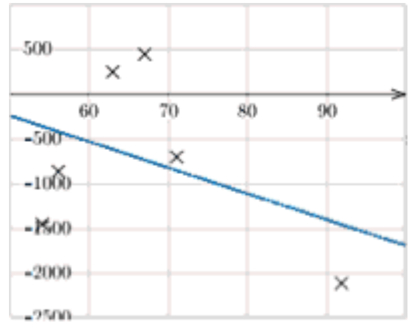
Regression equation: $y = -11.85x + 797.71$

Correlation coefficient: $r \approx -0.414$

b. As $|r| \approx 0.414$ is significantly less than 0.8, the values of x and y are not strongly correlated, so that Continental's net income does not appear correlated to the price of oil.c. The points in the graph are nowhere near the regression line, confirming the conclusion in (b).

28. a. y = Continental net income as a function of x = Price of oil. See the technology notes accompanying Examples 2 and 3 for the use of technology to obtain regression lines and correlation coefficients. The following result and plot were obtained using the Function Evaluator and Grapher on the Web site with the setup shown:





Regression equation: $\$y = -28.90x + 1208.01$

Correlation coefficient: $r \approx -0.408$

b. As $|r| \approx 0.408$ is significantly less than 0.8, the values of x and y are not strongly correlated, so that American's net income does not appear correlated to the price of oil.c. The points in the graph are nowhere near the regression line, confirming the conclusion in (b).

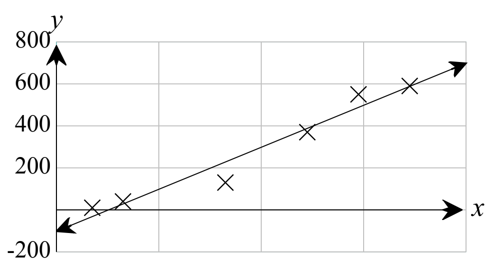
29. a. Using x = Number of natural science doctorates and y = Number of engineering doctorates gives us the following table of values:

x	70	130	330	490	590	690
y	10	40	130	370	550	590

Using a technology method of Example 2 (see the marginal note on using technology), we obtain the following regression line and plot (coefficients rounded to three significant digits):

$y = x - 102$

Graph:



b. To interpret the slope, recall that units of the slope are units of y (engineering doctorates) per unit of x (natural science doctorates). Thus, $m \approx 1$ engineering doctorate per natural science doctorate, indicating that there is about one additional doctorate in engineering per additional doctorate in the natural sciences.

c. Using the technology method of Example 3, we can use technology to show the value of r^2:

$$r^2 \approx 0.9525$$

$$r = \sqrt{r^2} \approx \sqrt{0.9525} \approx 0.976$$

Since r is close to 1, the correlation between x and y is a strong one.

d. Yes; the graph suggests a linear relationship; the data points are close to the regression line and show no obvious pattern (such as a curve).

Solutions Section 1.4

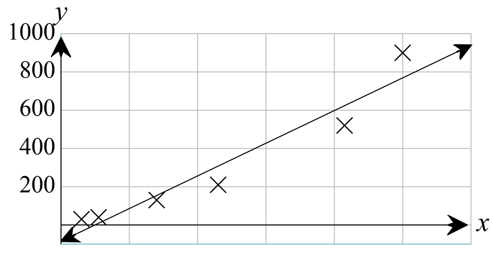
30. a. Using x = Number of social science doctorates and y = Number of education doctorates gives us the following table of values:

x	60	110	280	460	830	1,000
y	30	40	130	210	520	900

Using a technology method of Example 2 (see the marginal note on using technology), we obtain the following regression line and plot (coefficients rounded to three significant digits):

$y = 0.854x - 85.2$

Graph:



b. To interpret the slope, recall that units of the slope are units of y (education doctorates) per unit of x (social science doctorates). Thus, $m \approx 0.85$ education doctorates per social science doctorate, indicating that there are about 0.85 additional doctorates in education per additional doctorate in the social sciences.

c. Using the technology method of Example 3, we can use technology to show the value of r^2:

$$r^2 \approx 0.9264$$

$$r = \sqrt{r^2} \approx \sqrt{0.9264} \approx 0.962$$

Since r is close to 1, the correlation between x and y is a strong one.

d. No; the graph suggests a concave-up curve rather than a straight line. (On the left, the points begin above the regression line, then drop below it, and then climb above it again.)

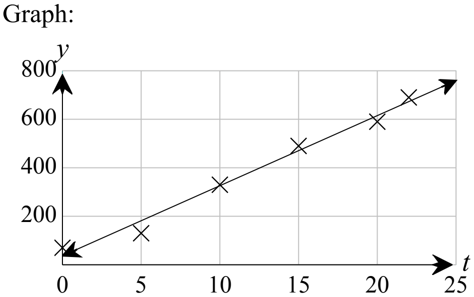
31. a. As t is time in years since 1990, we use the following set of data for the regression:

t	0	5	10	15	20	22
y	70	130	330	490	590	690

Using a technology method of Example 2 (see the marginal note on using technology), we obtain the following regression line and plot (coefficients rounded to three significant digits):

$y = 28.9t + 37.0$

Graph:



$r \approx \sqrt{0.9839} \approx 0.992$

b. Units of the slope are units of y (natural science doctorates) per unit of t (years); thus, doctorates per year. So, the number of natural science doctorates has been increasing at a rate of about 28.9 per year.
c. The slopes of successive pairs of points do not show an increasing nor decreasing trend as we go from left to right, so the number of natural science doctorates is increasing at a more-or-less constant rate.

d. Yes: If r had been equal to 1, then the points would lie exactly on the regression line, which would indicate that the number of doctorates is growing at a constant rate.

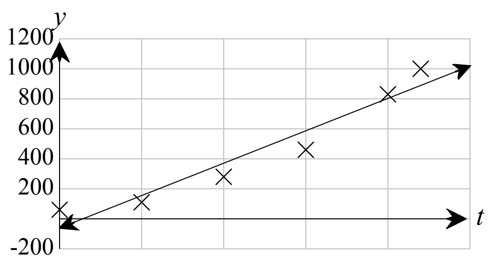
32. a. As t is time in years since 1990, we use the following set of data for the regression:

t	0	5	10	15	20	22
y	60	110	280	460	830	1,000

Using a technology method of Example 2 (see the marginal note on using technology), we obtain the following regression line and plot (coefficients rounded to three significant digits):

$y = 43.2t - 61.3$

Graph:



$r \approx \sqrt{0.9273} \approx 0.963$

b. Units of the slope are units of y (social science doctorates) per unit of t (years); thus, doctorates per year. Thus, the number of social science doctorates has been increasing at a rate of about 43.2 per year.
c. The data points suggest a concave-up curve rather than a straight line, indicating that the number of doctorates has been growing at a faster and faster rate (the slopes of successive pairs of points increase as we go from left to right).

d. No: If r had been equal to 1, then the points would lie exactly on the regression line, which would indicate that the number of doctorates is growing at a constant rate.

33. a. More-or-less constant rate; Exercise 29 suggests a roughly linear relationship between the number of natural science doctorates and the number of engineering doctorates, and Exercise 31 suggests that the number of natural science doctorates has been increasing at a more-or less constant rate. Therefore, the number of engineering doctorates is also increasing at a more-or-less constant rate.

b. No; $r = 1$ in Exercise 29 would indicate an exactly linear relationship between the number of natural science doctorates and the number of engineering doctorates, and so the conclusion would be the same.

c. No; $r = 1$ in Exercise 31 would indicate that the number of natural science doctorates has been increasing at a constant rate, and so the conclusion would be the same.

34. a. Faster and faster rate; The graph in Exercise 30 suggests that the number of education doctorates is increasing at a faster and faster rate with respect to the number of social science doctorates, and Exercise 32 suggests that the number of social science doctorates has been increasing at a faster and faster rate. Therefore, the number of education doctorates is also increasing at a faster and faster rate.

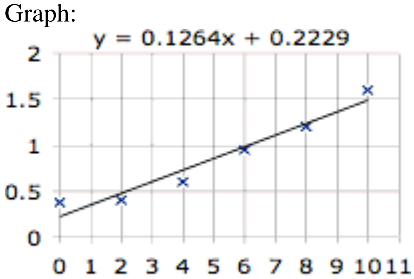
b. No; $r = 1$ in Exercise 30 would indicate an exactly linear relationship between the number of social science doctorates and the number of education doctorates, and so the conclusion would be the same.

c. No; $r = 1$ in Exercise 32 would indicate that the number of social science doctorates has been increasing at a constant rate, and so the conclusion would be the same.

Solutions Section 1.4

35. a. Using the method of Example 3, we obtain the following regression line and plot (coefficients rounded to two decimal places): $p = 0.13t + 0.22$; $r \approx 0.97$

Graph:



b. The first and last points lie above the regression line, while the central points lie below it, suggesting a curve.
c. Here is a worksheet showing the computation of the residuals (based on Example 1 in the text):

	A	B	C	D
1	t	p (observed)	p (predicted)	Residual
2	0	0.38	=0.13*A2+0.22	=B2-C2
3	2	0.4		
4	4	0.6		
5	6	0.95		
6	8	1.2		
7	10	1.6		

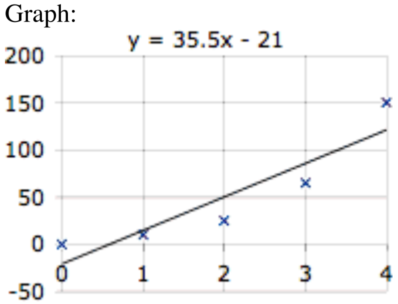
↓

	A	B	C	D
1	t	p (observed)	p (predicted)	Residual
2	0	0.38	0.22	0.16
3	2	0.4	0.48	-0.08
4	4	0.6	0.74	-0.14
5	6	0.95	1	-0.05
6	8	1.2	1.26	-0.06
7	10	1.6	1.52	0.08

Notice that the residuals are positive at first, become negative, and then become positive, confirming the impression from the graph.

36. a. Using the method of Example 3, we obtain the following regression line and plot (coefficients rounded to two decimal places): $c = 35.5t - 21$; $r \approx 0.92$

Graph:



b. The first and last points lie above the regression line, while the central points lie below it, suggesting a curve.
c. Here is a worksheet showing the computation of the residuals (based on Example 1 in the text):

	A	B	C	D
1	t	c (observed)	c (predicted)	Residual
2	0	0	=35.5*A2-21	=B2-C2
3	1	10		
4	2	25		
5	3	65		
6	4	150		

↓

◇	A	B	C	D
1	t	c (observed)	c (predicted)	Residual
2	0	0	-21	21
3	1	10	14.5	-4.5
4	2	25	50	-25
5	3	65	85.5	-20.5
6	4	150	121	29

Notice that the residuals are positive at first, become negative, and then become positive, confirming the impression from the graph.

Communication and reasoning exercises

37. The regression line is defined to be the line that gives the lowest sum-of-squares error, SSE. If we are given two points, (a, b) and (c, d) with $a \neq c$, then there is a line that passes through these two points, giving SSE = 0. Since 0 is the smallest value possible, this line must be the regression line.

38. SSE = 0; the straight line passing through the given points has predicted values equal to the observed values. Hence, the residuals are zero, giving SSE = 0.

39. If the points (x_1, y_1), (x_2, y_2), ..., (x_n, y_n) lie on a straight line, then the sum-of-squares error, SSE, for this line is zero. Since 0 is the smallest value possible, this line must be the regression line.

40. No. The regression line may pass through none of the given points.

41. Calculation of the regression line:

x	y	xy	x^2
0	0	0	0
$-a$	a	$-a^2$	a^2
a	a	a^2	a^2
0	$2a$	0	$2a^2$

(The bottom row contains the column sums.)

$n = 3$ (number of data points)

Slope: $$m = \frac{n(\sum xy) - (\sum x)(\sum y)}{n(\sum x^2) - (\sum x)^2} = \frac{3(0) - (0)(2a)}{3(2a^2) - 0^2} = 0$$

Correlation coefficient $$r = \frac{n(\sum xy) - (\sum x)(\sum y)}{\sqrt{n(\sum x^2) - (\sum x)^2} \cdot \sqrt{n(\sum y^2) - (\sum y)^2}}$$ has the same numerator as m, and we have just seen that this numerator is zero. Hence, $r = 0$.

42. Calculation of the regression line:

x	y	xy	x^2
0	a	0	0
0	$-a$	0	0
a	0	0	a^2
a	0	0	a^2

(The bottom row contains the column sums.)

$n = 3$ (number of data points)

Slope: $m = \dfrac{n(\sum xy) - (\sum x)(\sum y)}{n(\sum x^2) - (\sum x)^2} = \dfrac{3(0) - (a)(0)}{3a^2 - 0^2} = 0$

Correlation coefficient $r = \dfrac{n(\sum xy) - (\sum x)(\sum y)}{\sqrt{n(\sum x^2) - (\sum x)^2} \cdot \sqrt{n(\sum y^2) - (\sum y)^2}}$ has the same numerator as m, and we have just seen that this numerator is zero. Hence, $r = 0$.

43. No. The regression line through $(-1, 1)$, $(0, 0)$, and $(1, 1)$ passes through none of these points.

44. A mathematical model may only be valid for a limited range of values of the variables concerned, and extrapolation can lead to absurd results.

45. (Answers may vary.) The data in Exercise 35 give $r \approx 0.97$, yet the plotted points suggest a curve, not a straight line.

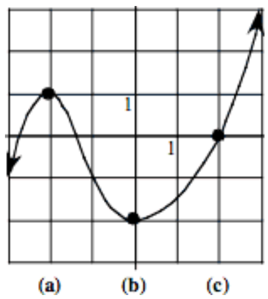
46. (Answers may vary.) If r is not close to 1, then the points are not close to the regression line; they may be scattered randomly above and below the line in a manner not suggesting a parabola.

Solutions Chapter 1 Review

Chapter 1 Review

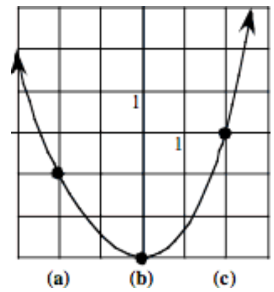
1. a. 1 **b.** -2 **c.** 0

d. $f(2) - f(-2) = 0 - 1 = -1$



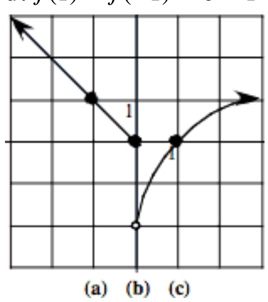
2. a. -1 **b.** -3 **c.** 0

d. $f(2) - f(-2) = 0 - (-1) = 1$



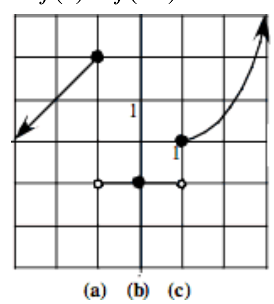
3. a. 1 **b.** 0 **c.** 0

d. $f(1) - f(-1) = 0 - 1 = -1$



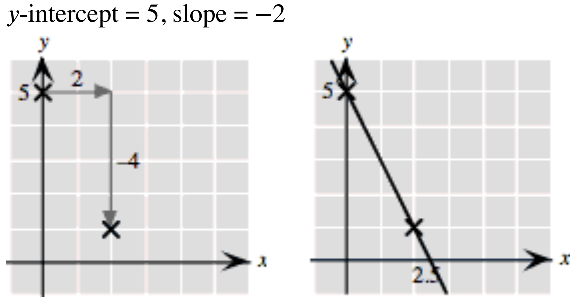
4. a. 2 **b.** -1 **c.** 0

d. $f(1) - f(-1) = 0 - 2 = -2$



5. $y = -2x + 5$

y-intercept = 5, slope = -2



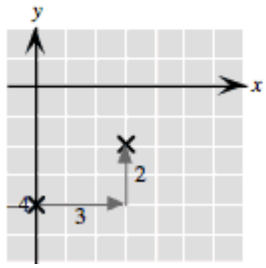
6. $2x - 3y = 12$

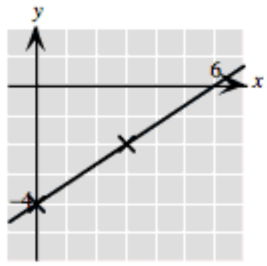
Solving for y gives

$$-3y = -2x + 12$$

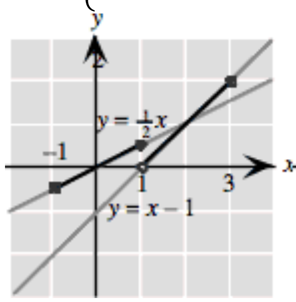
$$y = \frac{2}{3}x - 4\text{: } y\text{-intercept} = -4, \text{ slope} = \frac{2}{3}$$

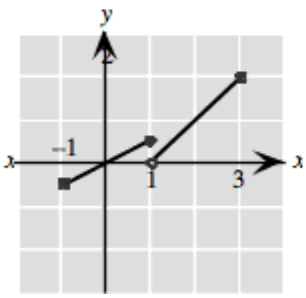
Solutions Chapter 1 Review





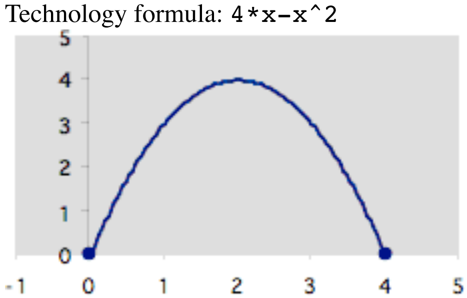
7. $y = \begin{cases} x/2 & \text{if} \quad -1 \le x \le 1 \\ x-1 & \text{if} \quad 1 < x \le 3 \end{cases}$



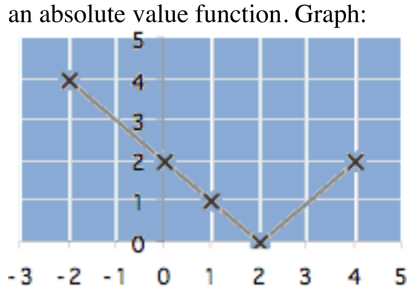


8. $f(x) = 4x - x^2$ with domain $[0, 4]$

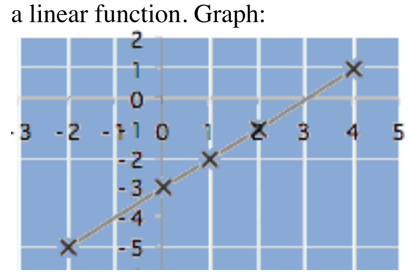
Technology formula: `4*x-x^2`



9. The graph of the function has a V-shape, indicating an absolute value function. Graph:

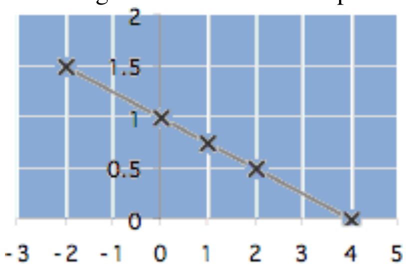


10. The graph of the function is a straight line, indicating a linear function. Graph:

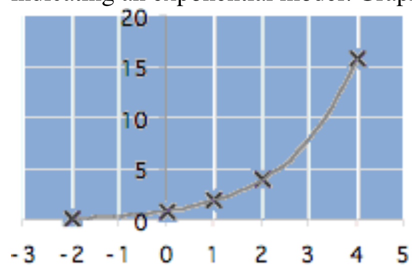


Solutions Chapter 1 Review

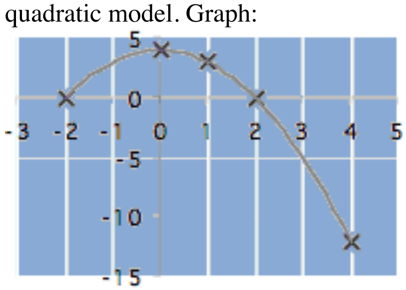
11. The graph of the function is a straight line, indicating a linear function. Graph:



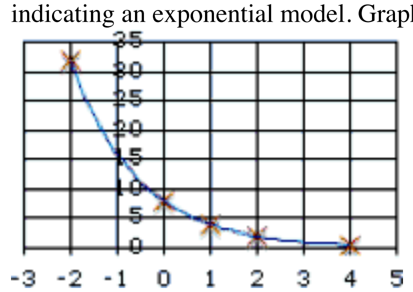
12. In the graph, y doubles for each 1-unit increase in x, indicating an exponential model. Graph:



13. The parabolic shape of the graph indicates a quadratic model. Graph:



14. In the graph, y is halved for each 1-unit increase in x, indicating an exponential model. Graph:



15. Through $(3, 2)$ with slope -3

Point: $(3, 2)$ **Slope:** $m = -3$ **Intercept:** $b = y_1 - mx_1 = 2 - (-3)(3) = 2 + 9 = 11$

Thus, the equation is $y = mx + b = -3x + 11$.

16. Through $(-2, 4)$ with slope -1

Point: $(-2, 4)$ **Slope:** $m = -2$ **Intercept:** $b = y_1 - mx_1 = 4 - (-1)(-2) = 2$

Thus, the equation is $y = mx + b = -x + 2$.

17. Through $(1, -3)$ and $(5, 2)$

Point: $(1, -3)$ **Slope:** $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{2 - (-3)}{5 - 1} = \dfrac{5}{4} = 1.25$

Intercept: $b = y_1 - mx_1 = -3 - (1.25)(1) = -4.25$

Thus, the equation is $y = mx + b = 1.25x - 4.25$.

18. Through $(-1, 2)$ and $(1, 0)$

Point: $(-1, 2)$ **Slope:** $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{0 - 2}{1 - (-1)} = \dfrac{-2}{2} = -1$ **Intercept:** $b = y_1 - mx_1 = 2 - (-1)(-1) = 1$

Thus, the equation is $y = mx + b = -x + 1$.

19. Through $(1, 2)$ parallel to $x - 2y = 2$

Point: $(1, 2)$

Slope: Same as slope of $x - 2y = 2$. To find the slope, solve for y:

$$-2y = -x - 2$$

$y = \frac{1}{2}x + 1$, so that $m = \frac{1}{2}$.

Intercept: $b = y_1 - mx_1 = 2 - \frac{1}{2}(1) = \frac{3}{2}$

Thus, the equation is $y = mx + b = \frac{1}{2}x + \frac{3}{2}$.

20. Through $(-3, 1)$ parallel to $-2x - 4y = 5$

Point: $(-3, 1)$

Slope: Same as slope of $-2x - 4y = 5$. To find the slope, solve for y, getting $y = -\frac{1}{2}x - \frac{5}{4}$.

Thus, $m = -\frac{1}{2}$.

Intercept: $b = y_1 - mx_1 = 1 + \frac{1}{2}(-3) = -\frac{1}{2}$

Thus, the equation is $y = mx + b = -\frac{1}{2}x - \frac{1}{2}$.

21. With slope 4 crossing $2x - 3y = 6$ at its x-intercept

We need the x-intercept of $2x - 3y = 6$. This is given by setting $y = 0$ and solving for x:

$2x - 0 = 6$

$x = 3$

Thus, the point is $(3, 0)$ because $y = 0$ on the x-axis.

Slope: $m = 4$

Intercept: $b = y_1 - mx_1 = 0 - 4(3) = -12$

Thus, the equation is

$y = mx + b = 4x - 12$

22. With slope 1/2 crossing $3x + y = 6$ at its x-intercept

We need the x-intercept of $3x + y = 6$. This is given by setting $y = 0$ and solving for x:

$3x + 0 = 6$

$x = 2$

Thus, the point is $(2, 0)$ because $y = 0$ on the x-axis.

Slope: $m = \frac{1}{2}$

Intercept: $b = y_1 - mx_1 = 0 - \frac{1}{2}(2) = -1$

Thus, the equation is

$y = mx + b = \frac{1}{2}x - 1$

23.

$y = -x/2 + 1$:

x	**Observed** y	**Predicted** y	**Residual**^{2}
−1	1	1.5	0.25
1	2	0.5	2.25
2	0	0	0
		SSE:	2.5

$y = -x/4 + 1$:

x	**Observed** y	**Predicted** y	**Residual**^{2}
−1	1	1.25	0.0625
1	2	0.75	1.5625
2	0	0.5	0.25
		SSE:	1.875

The second line, $y = -x/4 + 1$, is a better fit.

24.

$y = x + 1$:

x	**Observed** y	**Predicted** y	**Residual**^{2}
−2	−1	−1	0
−1	1	0	1
0	1	1	0
1	2	2	0
2	4	3	1
3	3	4	1
		SSE:	3

$y = x/2 + 1$:

x	**Observed** y	**Predicted** y	**Residual**^{2}
−2	−1	0	1
−1	1	0.5	0.25
0	1	1	0
1	2	1.5	0.25
2	4	2	4
3	3	2.5	0.25
		SSE:	5.75

The first line, $y = x + 1$, is the better fit.

25.

x	y	xy	x^2	y^2
−1	1	−1	1	1
1	2	2	1	4
2	0	0	4	0
2	3	1	6	5

(The bottom row contains the column sums.)

$n = 3$ (number of data points)

Slope: $$m = \frac{n(\sum xy) - (\sum x)(\sum y)}{n(\sum x^2) - (\sum x)^2} = \frac{3(1) - (2)(3)}{3(6) - 2^2} = \frac{-3}{14} \approx -0.214$$

Intercept: $$b = \frac{\sum y - m(\sum x)}{n} = \frac{3 - (-0.214)(2)}{3} \approx 1.14$$

Thus, the regression line is $y = mx + b = -0.214x + 1.14$.

$$r = \frac{n(\sum xy) - (\sum x)(\sum y)}{\sqrt{n(\sum x^2) - (\sum x)^2} \cdot \sqrt{n(\sum y^2) - (\sum y)^2}} = \frac{3(1) - (2)(3)}{\sqrt{3(6) - 2^2}\sqrt{3(5) - 3^2}} \approx -0.33$$

26.

x	y	xy	x^2	y^2
−2	−1	2	4	1
−1	1	−1	1	1
0	1	0	0	1
1	2	2	1	4
2	4	8	4	16
3	3	9	9	9
3	10	20	19	32

(The bottom row contains the column sums.)

$n = 6$ (number of data points)

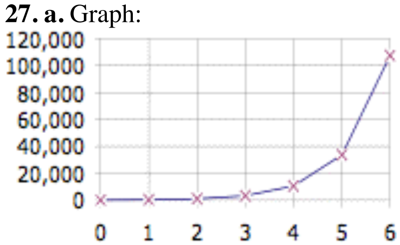
Slope: $m = \dfrac{n(\sum xy) - (\sum x)(\sum y)}{n(\sum x^2) - (\sum x)^2} = \dfrac{6(20) - (3)(10)}{6(19) - 3^2} = \dfrac{90}{105} \approx 0.857$

Intercept: $b = \dfrac{\sum y - m(\sum x)}{n} = \dfrac{10 - (0.857)(3)}{6} \approx 1.24$

Thus, the regression line is $y = mx + b = 0.857x + 1.24$.

$$r = \frac{n(\sum xy) - (\sum x)(\sum y)}{\sqrt{n(\sum x^2) - (\sum x)^2} \cdot \sqrt{n(\sum y^2) - (\sum y)^2}} = \frac{6(20) - (3)(10)}{\sqrt{6(19) - 3^2}\sqrt{6(32) - 10^2}} \approx 0.92$$

27. a. Graph:



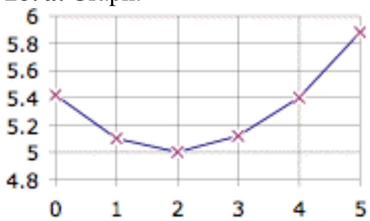
Since the data definitely suggests a curve, we rule out a linear function, leaving us with a choice of quadratic or exponential. Of the two, an exponential function would fit best, given the leveling off we see on the left; the graph of a quadratic function would not flatten out, but instead form a low point and begin rising again toward the left.b. The ratios (rounded to 1 decimal place) are:

$V(1)/V(0)$	$V(2)/V(1)$	$V(3)/V(2)$	$V(4)/V(3)$	$V(5)/V(4)$	$V(6)/V(5)$
$\frac{300}{100} = 3$	$\frac{1,000}{300} \approx 3.3$	$\frac{3,300}{1,000} = 3.3$	$\frac{10,500}{3,300} \approx 3.2$	$\frac{33,600}{10,500} \approx 3.2$	$\frac{107,400}{33,600} \approx 3.2$

They are close to 3.2.

c. The data suggest that website traffic is increasing by a factor of around 3.2 per year, so the prediction for next year (year 6) would be around $3.2 \times 107,400 \approx 343,700$ visits per day.

28. a. Graph:



Since the data definitely suggest a curve, we rule out a linear function, leaving us with a choice of quadratic or exponential. Of the two, a quadratic function would fit best, given the parabolic shape of the graph.

b. The differences (rounded to 1 decimal place) are:

$C(1)-C(0)$	$C(2)-C(1)$	$C(3)-C(2)$	$C(4)-C(3)$	$C(5)-C(4)$
$-0.32 \approx -0.3$	-0.1	$0.12 \approx 0.1$	$0.28 \approx 0.3$	$0.48 \approx 0.5$

The rounded differences increase linearly with slope 0.2.

c. Assuming the linear trend of differences continue, the next difference $C(6)-C(5)$ will be around 0.7, so that the cost of a paperback will be about $\$5.88 + 0.70 = \$6.58$.

29. a. $c(x) = \begin{cases} 0.03x + 2 & \text{if } 0 \le x \le 50 \\ 0.05x + 1 & \text{if } x > 50 \end{cases},$

Notice that x is *thousands* of visit per day, so 10,000 visits corresponds to $x = 10$, and the servers will crash an average of

$$c(10) = 0.03(10) + 2 = 2.3 \text{ times per day.}$$

(We used the first formula because 10 is in the interval $[0, 50]$.) For 50,000 visitors,

$$c(50) = 0.03(50) + 2 = 3.5 \text{ crashes per day.}$$

(We again used the first formula because 50 is still in the interval $[0, 50]$.) For 100,000 visitors,

$$c(100) = 0.05(100) + 1 = 6 \text{ crashes per day.}$$

(We used the second formula because 100 is in the interval $(50, +\infty)$.)

b. The coefficient 0.03 is the slope of the first formula, indicating that, for Web site traffic of up to 50,000 visits per day $(0 \le x \le 50)$, the number of crashes is increasing by 0.03 per additional thousand visits.

c. To experience 8 crashes in a day, we desire $c(x) = 8$. If we try the first formula, we get

$$0.03x + 2 = 8$$

giving $x = (8-2)/0.03 = 200$, which is not in the domain of the first formula. So, we try the second formula:

$$0.05x + 1 = 8$$

$$0.05x = 7, \text{ so } x = \frac{7}{0.05} = 140,$$

which is in the domain of the second formula. Thus, we estimate that there were 140,000 visitors that day.

30. a. $s(x) = \begin{cases} 1.55x & \text{if } 0 \le x \le 100 \\ 1.75x - 20 & \text{if } 100 < x \le 250 \end{cases}$

$s(60) = 1.55(60) = 93$ books per day (We used the first formula, since 60 is in $[0, 100]$.)

$s(100) = 1.55(100) = 155$ books per day (We used the first formula, since 100 is in $[0, 100]$.)

$s(160) = 1.75(160) - 20 = 260$ books per day (We used the second formula, since 160 is in $(100, 250]$.)

b. The coefficient 1.75 is the slope of the second formula, measured in books sold per thousand visitors. Thus, book sales are increasing at a rate of 1.75 books per thousand new visitors when the number of visitors is between 100,000 and 250,000 per day.

c. To sell an average of 300 books per day, we desire $n(x) = 300$. If we try the first formula, we get

$1.55x = 300,$

giving $x \approx 194,$ which is not in the domain of the first formula. So, we try the second formula:

$1.75x - 20 = 300$

$1.75x = 320,$ so $x = \frac{320}{1.75} \approx 182.9$ thousand visitors,

which is in the domain of the second formula. Thus, about 182,900 visitors per day will result in average sales of 300 books per day.

31.

t	1	2	3	4	5	6
$n(t)$	12.5	37.5	62.5	72.0	74.5	75.0

(a) Technology formulas:

(A): `300/(4+100*5^(-t))` (B): `13.3*t+8.0`

(C): `-2.3*t^2+30.0*t-3.3` (D): `7*3^(0.5*t)`

Here are the values for the four given models (rounded to 1 decimal place):

t	1	2	3	4	5	6
(A)	12.5	37.5	62.5	72.1	74.4	74.9
(B)	21.3	34.6	47.9	61.2	74.5	87.8
(C)	24.4	47.5	66.0	79.9	89.2	93.9
(D)	12.1	21.0	36.4	63.0	109.1	189.0

Model (A) gives an almost perfect fit, whereas the other models are not even close.

b. Looking at the table, we see the following behavior as t increases:

(A) Leveling off (B) Rising (C) Rising (begins to fall after 7 months, however) (D) Rising

32.

t	1	2	3	4	5
$n(t)$	1,330	520	520	1,340	2,980

(a) Technology formulas:

(A): `3000/(1+12*2^(-t))` (B): `2000/(4.2-0.7*t)`

(C): `300*1.6^t` (D): `100*(4.1*t^2-20.4*t+29.5)`

Here are the values for the three given models (rounded to the nearest integer):

t	1	2	3	4	5
(A)	429	750	1,200	1,714	2,182
(B)	571	714	952	1,429	2,857
(C)	480	768	1,229	1,966	3,146
(D)	1,320	510	520	1,350	3,000

Model (D) gives a close fit, whereas the other models are not even close.

b. If you extrapolate the models, you find the following behavior:

(A) Leveling off (B) Becomes undefined and then negative (C) Rising (D) Rising

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33. a. Using $v(c) = -0.000005c^2 + 0.085c + 1{,}750$, we get

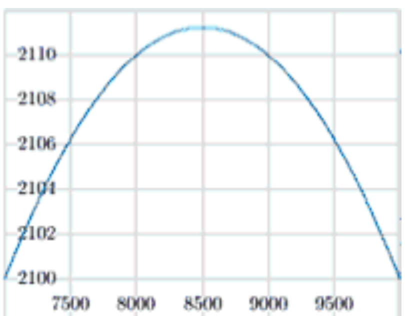
$$v(5{,}000) = -0.000005(5{,}000)^2 + 0.085(5{,}000) + 1{,}750 = -125 + 425 + 1{,}750 = 2{,}050$$

$$v(6{,}000) = -0.000005(6{,}000)^2 + 0.085(6{,}000) + 1{,}750 = -180 + 510 + 1{,}750 = 2{,}080$$

Thus, increasing monthly advertising from $5,000 to $6,000 per month would result in 2,080 − 2,050 = 30 more visits per day.
b. The following table shows the result of increasing expenditure by steps of $1,000:
Tech formula: `-0.000005*x^2+0.085*x+1750`

c	5,000	6,000	7,000	8,000	9,000	10,000
$v(c)$	2,050	2,080	2,100	2,110	2,110	2,100

The successive changes in the numbers of visits are:
2,080 − 2,050 = 30;2,100 − 2,080 = 20;2,110 − 2,100 = 10;2,110 − 2,110 = 0;2,100 − 2,110 = −10,
showing that the numbers of visits would increase at a slower and slower rate and then begin to decrease.
c. Here is a portion of the graph of v:



For c = 8,500 or larger, we see that Web site traffic is projected to decrease as advertising increases, and then drop toward zero. Thus, the model does not appear to give a reasonable prediction of traffic at expenditures larger than $8,500 per month.

34. a. Using $c(n) = 0.0008n^2 - 72n + 2{,}000{,}000$, we get

$$c(20{,}000) = 0.0008(20{,}000)^2 - 72(20{,}000) + 2{,}000{,}000 = 880{,}000$$

$$c(30{,}000) = 0.0008(30{,}000)^2 - 72(30{,}000) + 2{,}000{,}000 = 560{,}000$$

Thus, increasing the run size from 20,000 to 30,000 per month would result in a savings of $880{,}000 - 560{,}000 = 320{,}000$ dollars.
b. The following table shows the result of increasing run size in steps of 10,000:
Tech formula: `0.0008*x^2-72*x+2000000`

n	20,000	30,000	40,000	50,000	60,000	70,000
$c(n)$	880,000	560,000	400,000	400,000	560,000	880,000
Change		-320,000	-160,000	0	160,000	320,000

The table shows that the cost decreases at a slower and slower rate and then begins to increase.
Going from 30,000 to 40,000 decreases the cost by $160,000—considerably less than going from 20,000 to 30,000.
c. Here is a portion of the graph of v:

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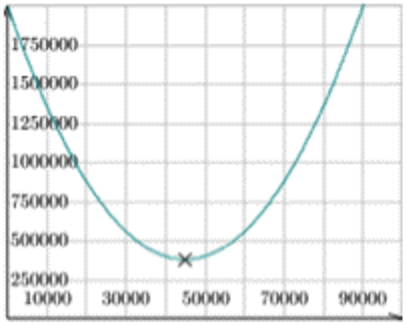

The graph shows that the cost is a minimum for a print run size of around 45,000.

35. a. Point: (5,000, 2,050) **Slope:** $m = \dfrac{v_2 - v_1}{c_2 - c_1} = \dfrac{2{,}100 - 2{,}050}{6{,}000 - 5{,}000} = \dfrac{50}{1{,}000} = 0.05$

Intercept: $b = v_1 - mc_1 = 2{,}050 - (0.05)(5{,}000) = 1{,}800$

Thus, the equation is $v = mc + b = 0.05c + 1{,}800$.

b. A budget of \$7,000 per month for banner ads corresponds to $v = 7{,}000$.

$v(7{,}000) = 0.05(7{,}000) + 1{,}800 = 2{,}150$ new visitors per day

c. We are given $v = 2{,}500$ and want c.

$2{,}500 = 0.05c + 1{,}800$

$0.05c = 2{,}500 - 1{,}800 = 700$

Thus, $c = \dfrac{700}{0.05} = \$14{,}000$ per month.

36. a. Point: (20,000, 880,000) **Slope:** $m = \dfrac{c_2 - c_1}{n_2 - n_1} = \dfrac{550{,}000 - 880{,}000}{40{,}000 - 20{,}000} = -\dfrac{330{,}000}{20{,}000} = -16.5$

Intercept: $b = c_1 - mn_1 = 880{,}000 - (-16.5)(20{,}000) = 1{,}210{,}000$

Thus, the equation is $c = mn + b = -16.5n + 1{,}210{,}000$.

b. $c(25{,}000) = -16.5(25{,}000) + 1{,}210{,}000 = \$797{,}500$

c. We are given $c = 418{,}000$ and want n.

$418{,}000 = -16.5n + 1{,}210{,}000$

$-16.5n = 418{,}000 - 1{,}210{,}000 = -792{,}000$

Thus, $n = \dfrac{-792{,}000}{-16.5} = 48{,}000$.

37. Point: $(w, d) = (70, 74.5)$ **Slope:** $m = \dfrac{d_2 - d_1}{w_2 - w_1} = \dfrac{93.5 - 74.5}{90 - 70} = \dfrac{19}{20} = 0.95$

Intercept: $b = d_1 - mw_1 = 74.5 - (0.95)(70) = 8$

Thus, the equation is $d = mw + b = 0.95w + 8$.

OHagan dropped 90 m, so $d = 90$, and we want w.

$90 = 0.95w + 8$

$0.95w = 90 - 8 = 82$

Thus, $w = \dfrac{82}{.95} \approx 86$ kg.

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38. Point: $(r, T) = (140, 80)$ **Slope:** $m = \dfrac{T_2 - T_1}{r_2 - r_1} = \dfrac{75 - 80}{120 - 140} = \dfrac{-5}{-20} = 0.25$

Intercept: $b = T_1 - mr_1 = 80 - (0.25)(140) = 45$

Thus, the equation is $T = mr + b = 0.25r + 45$.

$T = 65$, and we want r.

$65 = 0.25r + 45$

$0.25r = 65 - 45 = 20$

Thus, $r = \dfrac{20}{0.25} = 80$ chirps/min.

39. a. Cost function: $C = mx + b$, where b = fixed cost = \$500 per week, and m = marginal cost = \$5.50 per album

Thus, the linear cost function is $C = 5.5x + 500$.

Revenue function: $R = mx + b$, where b = fixed revenue = 0, and m = marginal revenue = \$9.50 per album

Thus, the linear revenue function is $R = 9.5x$.

Profit function: $P = R - C$

$$P = 9.5x - (5.5x + 500)$$
$$= 4x - 500$$

b. For breakeven, $P = 0$

$4x - 500 = 0$

$4x = 500$

$x = \dfrac{500}{4} = 125$ albums per week

To make a profit, the company should sell more than this number.

c. $R = 8.00x$

$P = 8x - (5.5x + 500) = 2.5x - 500$

For breakeven,

$2.5x - 500 = 0$, so $x = \dfrac{500}{2.5} = 200$

To make a profit, the company should sell more than this number.

40. a. Cost function: $C = mx + b$, where b = fixed cost = \$900 per month, and m = marginal cost = \$4 per novel

Thus, the linear cost function is $C = 4x + 900$.

Revenue function: $R = mx + b$, where b = fixed revenue = 0, and m = marginal revenue = \$5.50 per novel

Thus, the linear revenue function is $R = 5.50x$.

Profit function: $P = R - C$

$$P = 5.50x - (4x + 900)$$
$$= 5.50x - 4x - 900$$
$$= 1.50x - 900$$

b. For breakeven, $P = 0$

$1.50x - 900 = 0$

$$1.50x = 900$$

$$x = \frac{900}{1.50} = 600 \text{ novels per month}$$

c. $R = 5.00x$

$$P = 5.00x - (4x + 900) = 5x - 4x - 900 = x - 900$$

For breakeven,

$x - 900 = 0,$ so $x = 900$ novels per month

41. a. Demand: We are given two points: $(p, q) = (7, 500)$ and $(9.5, 300)$

Slope: $m = \frac{q_2 - q_1}{p_2 - p_1} = \frac{300 - 500}{9.5 - 7} = \frac{-200}{2.5} = -80$

Intercept: $b = q_1 - mp_1 = 500 - (-80)(7) = 500 + 560 = 1{,}060$

Thus, the demand equation is $q = mp + b = -80p + 1{,}060$.

b. When $p = \$12,$ the demand is

$q = -80(12) + 1{,}060 = 100$ albums per week

c. From Exercise 39, the cost function is

$C = 5.5q + 500$ We use q for the monthly sales rather than x.

$= 5.5(-80p + 1{,}060) + 500$ We want everything expressed in terms of $p,$ so we used the demand equation.

$= -440p + 5{,}830 + 500$

$C = -440p + 6{,}330$

To compute the profit in terms of price, we need the revenue as well:

$R = pq = p(-80p + 1{,}060) = -80p^2 + 1{,}060p$**Profit:** $P = R - C$

$P = -80p^2 + 1{,}060p - (-440p + 6{,}330) = -80p^2 + 1{,}500p - 6{,}330$

Now we compare profits for the three prices:

$$P(7.00) = -80(7)^2 + 1{,}500(7) - 6{,}330 = \$250$$

$$P(9.50) = -80(9.5)^2 + 1{,}500(9.5) - 6{,}330 = \$700$$

$$P(12) = -80(12)^2 + 1{,}500(12) - 6{,}330 = \$150$$

Thus, charging \$9.50 will result in the largest weekly profit of \$700.

42. a. Demand: We are given two points: $(p, q) = (10, 350)$ and $(5.5, 620)$

Slope: $m = \frac{q_2 - q_1}{p_2 - p_1} = \frac{620 - 350}{5.5 - 10} = \frac{270}{-4.5} = -60$

Intercept: $b = q_1 - mp_1 = 350 - (-60)(10) = 350 + 600 = 950$

Thus, the demand equation is $q = mp + b = -60p + 950$.

b. When $p = \$15,$ the demand is

$q = -60(15) + 950 = -900 + 950 = 50$ novels per month

c. From Exercise 40, the cost function is

$C = 4q + 900$ We use q for the monthly sales rather than x.

$= 4(-60p + 950) + 900$ We want everything expressed in terms of p, so we used the demand equation.

$= -240p + 3{,}800 + 900$

$C = -240p + 4{,}700$

To compute the profit in terms of price, we need the revenue as well:

$R = pq = p(-60p + 950) = -60p^2 + 950p$

Profit: $P = R - C$

$P = -60p^2 + 950p - (-240p + 4{,}700) = -60p^2 + 1{,}190p - 4{,}700$

Now we compare profits for the three prices:

$P(5.50) = -60(5.5)^2 + 1{,}190(5.5) - 4{,}700 = \$30$

$P(10) = -60(10)^2 + 1{,}190(10) - 4{,}700 = \$1{,}200$

$P(15) = -60(15)^2 + 1{,}190(15) - 4{,}700 = -\$350$ (loss)

Thus, charging \$10 will result in the largest monthly profit of \$1,200.

43. a. Calculation of the regression line:

x	y	xy	x^2
8	440	3,520	64
8.5	380	3,230	72.25
10	250	2,500	100
11.5	180	2,070	132.25
38	1,250	11,320	368.5

(The bottom row contains the column sums.)

$n = 4$ (number of data points)

Slope: $$m = \frac{n(\sum xy) - (\sum x)(\sum y)}{n(\sum x^2) - (\sum x)^2} = \frac{4(11{,}320) - (38)(1{,}250)}{4(368.5) - 38^2} = \frac{-2{,}220}{30} = -74$$

Intercept: $$b = \frac{\sum y - m(\sum x)}{n} = \frac{1{,}250 - (-74)(38)}{4} = 1{,}015.5$$

Thus, the regression line is $y = mx + b = -74x + 1{,}015.5$. Using the variable names p and q makes this equation $q = -74p + 1{,}015.5$.

b. $q(10.50) = -74(10.50) + 1{,}015.5 = 238.5 \approx 239$ albums per week

44. a. Calculation of the regression line:

x	y	xy	x^2
5.5	620	3,410	30.25
10	350	3,500	100
11.5	350	4,025	132.25
12	300	3,600	144
39	1,620	14,535	406.5

(The bottom row contains the column sums.)

$n = 4$ (number of data points)

Slope: $$m = \frac{n(\sum xy) - (\sum x)(\sum y)}{n(\sum x^2) - (\sum x)^2} = \frac{4(14{,}535) - (39)(1{,}620)}{4(406.5) - 39^2} = \frac{-5{,}040}{105} = -48$$

Intercept: $$b = \frac{\sum y - m(\sum x)}{n} = \frac{1{,}620 - (-48)(39)}{4} = 873$$

Thus, the regression line is $y = mx + b = -48x + 873$. Using the variable names p and q makes this equation $q = -48p + 873$.

b. $q(8) = -48(8) + 873 = 489$ novels per month

Solutions Chapter 1 Case Study

Chapter 1 Case Study

1. Here is the given data, with $t = 0$ representing 2010:

t	0	1	2	3	4	5	6
y	0	0.3	1.5	2.6	3.4	4.3	5.0

Using technology, we obtain the following linear regression model:

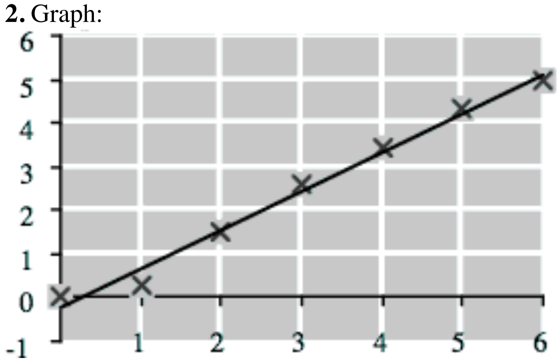
$y = 0.8893t - 0.225$; $r \approx 0.9948$

Since $m \approx 0.889$, spending on mobile advertising is increasing at a rate of about \$0.889 billion per year. Since Impact Advertising has 0.25% of the mobile advertising market, this translates to an increase in revenues of about

$0.0025 \times 0.889 \approx \$0.00222$ billion,

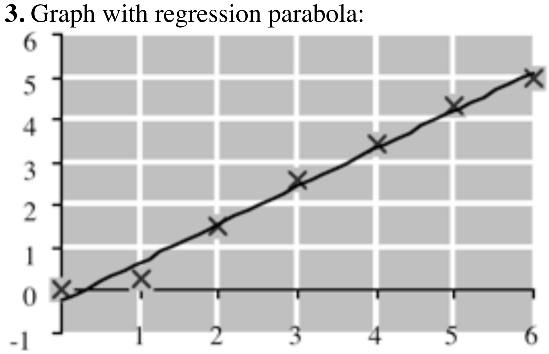
or \$2.22 million per year.

2. Graph:



The graph does not suggest a quadratic model because the plotted points do not suggest a parabola.

3. Graph with regression parabola:



Regression equation: $y = -0.001190t^2 + 0.8964t - 0.2310$

The parabola appears to fit no better than the regression line, suggesting that the quadratic model is not appropriate.

Solutions Chapter 1 Case Study

4. Here is the tabulated data together with the result of `=LINEST(A2:A8,B2:C8,,TRUE)`:

◊	A	B	C
1	y	x	x^2
2	0	0	0
3	0.3	1	1
4	1.5	2	4
5	2.6	3	9
6	3.4	4	16
7	4.3	5	25
8	5	6	36

→

---	E	F	G
1	-0.0012	0.8964	-0.2310
2	0.0264	0.1647	0.2110
3	0.9896	0.2418	#N/A
4	189.4134	4	#N/A
5	22.1433	0.2338	#N/A

p is computed using `=TDIST(ABS(E1/E2),F4,2)`, and returns the value $p \approx 0.9662$. There is a very low degree of confidence, $1 - p \approx 0.0338$, or 3.38%, we can have in asserting that the coefficient of x^2 is *not* zero (or that a quadratic model is needed) and so a quadratic model is not appropriate.