

# An Introduction to Stochastic Modeling

## Fourth Edition

*Instructor Solutions Manual*

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# Contents

<b>Chapter 1</b> .....	<b>1</b>
<b>Chapter 2</b> .....	<b>7</b>
<b>Chapter 3</b> .....	<b>12</b>
<b>Chapter 4</b> .....	<b>24</b>
<b>Chapter 5</b> .....	<b>31</b>
<b>Chapter 6</b> .....	<b>41</b>
<b>Chapter 7</b> .....	<b>52</b>
<b>Chapter 8</b> .....	<b>57</b>
<b>Chapter 9</b> .....	<b>61</b>
<b>Chapter 10</b> .....	<b>65</b>
<b>Chapter 11</b> .....	<b>68</b>



# Chapter 1

**2.1**  $E[\mathbf{1}\{A_1\}] = Pr\{A_1\} = \frac{1}{13}$ . Similarly,  $E[\mathbf{1}\{A_k\}] = Pr\{A_k\} = \frac{1}{13}$  for  $k = 1, \dots, 13$ . Then, because the expected value of a sum is always the sum of the expected values,  $E[N] = E[\mathbf{1}\{A_1\}] + \dots + E[\mathbf{1}\{A_{13}\}] = \frac{1}{13} + \dots + \frac{1}{13} = 1$ .

**2.2** Let  $X$  be the first number observed and let  $Y$  be the second. We use the identity  $(\sum x_i)^2 = \sum x_i^2 + \sum_{i \neq j} x_i x_j$  several times.

$$E[X] = E[Y] = \frac{1}{N} \sum x_i;$$

$$Var[X] = Var[Y] = \frac{1}{N} \sum x_i^2 - \left(\frac{1}{N} \sum x_i\right)^2 = \frac{(N-1) \sum x_i^2 - \sum_{i \neq j} x_i x_j}{N^2};$$

$$E[XY] = \frac{\sum_{i \neq j} x_i x_j}{N(N-1)};$$

$$Cov[X, Y] = E[XY] - E[X]E[Y] = \frac{\sum_{i \neq j} x_i x_j - (N-1) \sum x_i^2}{N^2(N-1)}$$

$$\tilde{n}_{X,Y} = \frac{Cov[X, Y]}{\sigma_X \sigma_Y} = -\frac{1}{N-1}.$$

**2.3** Write  $S_r = \xi_1 + \dots + \xi_r$  where  $\xi_k$  is the number of additional samples needed to observe  $k$  distinct elements, assuming that  $k-1$  distinct elements have already been observed. Then, defining  $p_k = Pr\{\xi_k = 1\} = 1 - \frac{k-1}{N}$  we have  $Pr\{\xi_k = n\} = p_k(1-p_k)^{n-1}$  for  $n = 1, 2, \dots$  and  $E[\xi_k] = \frac{1}{p_k}$ . Finally,  $E[S_r] = E[\xi_1] + \dots + E[\xi_r] = \frac{1}{p_1} + \dots + \frac{1}{p_r}$  will verify the given formula.

**2.4** Using an obvious notation, the event  $\{N = n\}$  is equivalent to either  $\overbrace{HTH \dots HTT}^{n-1}$  or  $\overbrace{THT \dots THH}^{n-1}$  so  $Pr\{N = n\} = 2 \times \left(\frac{1}{2}\right)^{n-1} \times \frac{1}{2} = \left(\frac{1}{2}\right)^{n-1}$  for  $n = 2, 3, \dots$ ;  $Pr\{N \text{ is even}\} = \sum_{n=2,4,\dots} \left(\frac{1}{2}\right)^{n-1} = \frac{2}{3}$  and  $Pr\{N \leq 6\} = \sum_{n=2}^6 \left(\frac{1}{2}\right)^{n-1} = \frac{31}{32}$ .  
 $Pr\{N \text{ is even and } N \leq 6\} = 5 \sum_{m=2,4,6} \left(\frac{1}{2}\right)^{m-1} = \frac{21}{32}$ .

**2.5** Using an obvious notation, the probability that A wins on the  $2n+1$  trial is  $Pr\left\{\overbrace{A^c B^c \dots A^c B^c A}^{n \text{ losses}}\right\} = [(1-p)(1-q)]^n p$ ,  
 $n = 0, 1, \dots$ .  $Pr\{A \text{ wins}\} = \sum_{n=0}^{\infty} [(1-p)(1-q)]^n p = \frac{p}{1-(1-p)(1-q)}$ .  $Pr\{A \text{ wins on } 2n+1 \text{ play} | A \text{ wins}\} = (1-\pi)\pi^n$  where  $\pi = (1-p)(1-q)$ .  $E[\#\text{trials} | A \text{ wins}] = \sum_{n=0}^{\infty} (2n+1)(1-\pi)\pi^n = 1 + \frac{2\pi}{1-\pi} = \frac{1+(1-p)(1-q)}{1-(1-p)(1-q)} = \frac{2}{1-(1-p)(1-q)} - 1$ .

**2.6** Let  $N$  be the number of losses and let  $S$  be the sum. Then  $Pr\{N = n, S = k\} = \left(\frac{1}{6}\right)^{n-1} \left(\frac{5}{6} p_k\right)$  where  $p_3 = p_{11} = p_4 = p_{10} = \frac{1}{15}$ ;  $p_5 = p_9 = p_6 = p_8 = \frac{2}{15}$  and  $p_7 = \frac{3}{15}$ . Finally  $Pr\{S = k\} = \sum_{n=1}^{\infty} Pr\{N = n, S = k\} = p_k$ . (It is *not* a correct argument to simply say  $Pr\{S = k\} = Pr\{\text{Sum of 2 dice} = k | \text{Dice differ}\}$ . Compare with Exercise II, 2.1.)

**2.7** We are given that (\*)  $Pr\{U > u, W > w\} = [1 - F_U(u)][1 - F_W(w)]$  for all  $u, w$ . According to the definition for independence we wish to show that  $Pr\{U \leq u, W \leq w\} = F_U(u)F_W(w)$  for all  $u, w$ . Taking complements and using the addition law

$$Pr\{U \leq u, W \leq w\} = 1 - Pr\{U > u \text{ or } W > w\}$$

$$= 1 - [Pr\{U > u\} + Pr\{W > w\} - Pr\{U > u, W > w\}]$$

$$= 1 - [(1 - F_U(u)) + (1 - F_W(w)) - (1 - F_U(u))(1 - F_W(w))]$$

$$= F_U(u)F_W(w) \text{ after simplification.}$$

**2.8 (a)**  $E[Y] = E[a + bX] = \int (a + bx)dF_X(x) = a \int dF_X(x) + b \int x dF_X(x) = a + bE[X] = a + b\mu$ . In words, (a) implies that the expected value of a constant times a random variable is the constant times the expected value of the random variable. So  $E[b^2(X - \mu)^2] = b^2E[(X - \mu)^2]$ .

$$(b) \text{Var}[Y] = E[(Y - E\{Y\})^2] = E[(a + bX - a - b\mu)^2] = E[b^2(X - \mu)^2] = b^2E[(X - \mu)^2] = b^2\sigma^2$$

**2.9** Use the usual sums of numbers formula (See I, 6 if necessary) to establish

$$\sum_{k=1}^n k(n-k) = \frac{1}{6}n(n+1)(n-1); \text{ and}$$

$$\sum_{k=1}^n k^2(n-k) = n \sum k^2 - \sum k^3 = \frac{1}{12}n^2(n+1)(n-1), \text{ so}$$

$$E[X] = \frac{2}{n(n-1)} \sum k(n-k) = \frac{1}{3}(n+1)$$

$$E[X^2] = \frac{3}{n(n-1)} \sum k^2(n-k) = \frac{1}{6}n(n+1), \text{ and}$$

$$\text{Var}[X] = E[X^2] - (E[X])^2 = \frac{1}{18}(n+1)(n-2).$$

**2.10** Observe, for example,  $Pr\{Z = 4\} = Pr\{X = 3, Y = 1\} = \left(\frac{1}{2}\right)\left(\frac{1}{6}\right)$ , using independence. Continuing in this manner,

$z$	1	2	3	4	5	6
$Pr\{Z = z\}$	$\frac{1}{12}$	$\frac{1}{6}$	$\frac{1}{4}$	$\frac{1}{12}$	$\frac{1}{6}$	$\frac{1}{4}$

**2.11** Observe, for example,  $Pr\{W = z\} = Pr\{U = 0, V = 2\} + Pr\{U = 1, V = 1\} = \frac{1}{6} + \frac{1}{6} + \frac{1}{3}$ . Continuing in this manner, arrive at

$w$	1	2	3	4
$Pr\{W = w\}$	$\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{6}$

**2.12** Changing any of the random variables by adding or subtracting a constant will not affect the covariance. Therefore, by replacing  $U$  with  $U - E[U]$ , if necessary, etc, we may assume, without loss of generality that all of the means are zero. Because the means are zero,

$$\text{Cov}[X, Y] = E[XY] - E[X]E[Y] = E[XY] = E[UV - UW + VW - W^2] = -E[W^2] = -\sigma^2. (E[UV] = E[U]E[V] = 0, \text{ etc.})$$

**2.13**  $Pr\{v < V, U \leq u\} = Pr\{v < X \leq u, v < Y \leq u\}$

$$= Pr\{v < X \leq u\} Pr\{v < Y \leq u\} \text{ (by independence)}$$

$$= (u - v)^2$$

$$= \iint_{(u', v') \mid v < v' \leq u' \leq u} f_{u,v}(u', v') du' dv'$$

$$= \int_v^u \left\{ \int_{v'}^u f_{u,v}(u', v') du' \right\} dv'.$$

The integrals are removed from the last expression by successive differentiation, first w.r.t.  $v$  (changing sign because  $v$  is a lower limit) than w.r.t.  $u$ . This tells us

$$f_{u,v}(u, v) = -\frac{d}{du} \frac{d}{dv} (u - v)^2 = 2 \text{ for } 0 < v \leq u \leq 1.$$

**3.1**  $Z$  has a discrete uniform distribution on  $0, 1, \dots, 9$ .

**3.2** In maximizing a continuous function, we often set the derivative equal to zero. In maximizing a function of a discrete variable, we equate the ratio of successive terms to one. More precisely,  $k^*$  is the smallest  $k$  for which  $\frac{p(k+1)}{p(k)} < 1$ , or, the smallest  $k$  for which  $\frac{n-k}{k+1} \left( \frac{p}{1-p} \right) < 1$ . Equivalently, (b)  $k^* = \lceil (n+1)p \rceil$  where  $\lceil x \rceil =$  greatest integer  $\leq x$ . for (a) let  $n \rightarrow \infty, p \rightarrow 0, \lambda = np$ . Then  $k^* = \lceil \lambda \rceil$ .

**3.3** Recall that  $e^\lambda = 1 + \lambda + \frac{\lambda^2}{2!} + \frac{\lambda^3}{3!} + \dots$  and  $e^{-\lambda} = 1 - \lambda + \frac{\lambda^2}{2!} - \frac{\lambda^3}{3!} + \dots$  so that  $\sinh \lambda \equiv \frac{1}{2}(e^\lambda - e^{-\lambda}) = \lambda + \frac{\lambda^3}{3!} + \frac{\lambda^5}{5!} + \dots$ . Then  $\Pr\{X \text{ is odd}\} = \sum_{k=1,3,5,\dots} \frac{\lambda^k e^{-\lambda}}{k!} = e^{-\lambda} \sinh(\lambda) = \frac{1}{2}(1 - e^{-2\lambda})$ .

$$\begin{aligned} \mathbf{3.4} \quad E[V] &= \sum_{k=0}^{\infty} \frac{1}{k+1} \frac{\lambda^k e^{-\lambda}}{k!} = \frac{e^{-\lambda}}{\lambda} \sum_{k=0}^{\infty} \frac{\lambda^{k+1}}{(k+1)!} \\ &= \frac{1}{\lambda} e^{-\lambda} (e^\lambda - 1) = \frac{1}{\lambda} (1 - e^{-\lambda}). \end{aligned}$$

$$\begin{aligned} \mathbf{3.5} \quad E[XY] &= E[X(N-X)] = NE[X] - E[X^2] \\ &= N^2 p - [Np(1-p) + N^2 p^2] = N^2 p(1-p) - Np(1-p) \end{aligned}$$

$$\text{Cov}[X, Y] = E[XY] - E[X]E[Y] = -Np(1-p).$$

**3.6** Your intuition should suggest the correct answers: (a)  $X_1$  is binomially distributed with parameters  $M$  and  $\pi_1$ ; (b)  $N$  is binomial with parameters  $M$  and  $\pi_1 + \pi_2$ ; and (c)  $X_1$ , given  $N = n$ , is conditionally binomial with parameters  $n$  and  $p = \pi_1 / (\pi_1 + \pi_2)$ . To derive these correct answers formally, begin with

$$\Pr\{X_1 = i, X_2 = j, X_3 = k\} = \frac{M!}{i!j!k!} \pi_1^i \pi_2^j \pi_3^k; i+j+k = M.$$

Since  $k = M - (i+j)$

$$\Pr\{X_1 = i, X_2 = j\} = \frac{M!}{i!j!(M-i-j)!} \pi_1^i \pi_2^j \pi_3^{M-i-j}; 0 \leq i+j \leq M.$$

$$\begin{aligned} \mathbf{(a)} \quad \Pr\{X_1 = i\} &= \sum_j \Pr\{X_1 = i, X_2 = j\} \\ &= \frac{M!}{i!(M-i)!} \pi_1^i \sum_{j=0}^{M-i} \frac{(M-i)!}{j!(M-i-j)!} \pi_2^j \pi_3^{M-i-j} \\ &= \binom{M}{i} \pi_1^i (\pi_2 + \pi_3)^{M-i}, i = 0, 1, \dots, M. \end{aligned}$$

**(b)** Observe that  $N = n$  if and only if  $X_3 = M - n$ . Apply the results of (a) to  $X_3$ :

$$\Pr\{N = n\} = \Pr\{X_3 = M - n\} = \frac{M!}{n!(M-n)!} (\pi_1 + \pi_2)^n \pi_3^{M-n}$$

$$\begin{aligned} \mathbf{(c)} \quad \Pr\{X_1 = k | N = n\} &= \frac{\Pr\{X_1 = k, X_2 = n-k\}}{\Pr\{N = n\}} \\ &= \frac{\frac{M!}{k!(M-n)!(n-k)!} \pi_1^k \pi_2^{n-k} \pi_3^{M-n}}{\frac{M!}{n!(M-n)!} (\pi_1 + \pi_2)^n \pi_3^{M-n}} \\ &= \frac{n!}{k!(n-k)!} \left( \frac{\pi_1}{\pi_1 + \pi_2} \right)^k \left( \frac{\pi_2}{\pi_1 + \pi_2} \right)^{n-k}, k = 0, 1, \dots, n. \end{aligned}$$

$$\begin{aligned}
 3.7 \Pr\{Z = n\} &= \sum_{k=0}^n \Pr\{X = k\}\Pr\{Y = n - k\} \\
 &= \sum_{k=0}^n \frac{\mu^k e^{-\mu} \nu^{(n-k)} e^{-\nu}}{k!(n-k)!} = e^{-(\mu+\nu)} \frac{1}{n!} \sum_{k=0}^n \frac{n!}{k!(n-k)!} \mu^k \nu^{n-k} \\
 &= \frac{e^{-(\mu+\nu)} (\mu + \nu)^n}{n!} \quad (\text{Using binomial formula.})
 \end{aligned}$$

$Z$  is Poisson distributed, parameter  $\mu + \nu$ .

**3.8 (a)**  $X$  is the sum of  $N$  independent Bernoulli random variables, each with parameter  $p$ , and  $Y$  is the sum of  $M$  independent Bernoulli random variables each with the same parameter  $p$ .  $Z$  is the sum of  $M + N$  independent Bernoulli random variables, each with parameter  $p$ .

**(b)** By considering the ways in which a committee of  $n$  people may be formed from a group comprised of  $M$  men and  $N$  women, establish the identity  $\binom{M+N}{n} = \sum_{k=0}^n \binom{N}{k} \binom{M}{n-k}$ .

Then

$$\begin{aligned}
 \Pr\{Z = n\} &= \sum_{k=0}^n \Pr\{X = k\}\Pr\{Y = n - K\} \\
 &= \sum_{k=0}^n \binom{N}{k} p^k (1-p)^{N-k} \binom{M}{n-k} p^{n-k} (1-p)^{M-n+k} \\
 &= \binom{M+N}{n} p^n (1-p)^{M+N-n} \text{ for } n = 0, 1, \dots, M+N.
 \end{aligned}$$

Note:

$$\binom{N}{k} = 0 \text{ for } k > N.$$

$$\begin{aligned}
 3.9 \Pr\{X + Y = n\} &= \sum_{k=0}^n \Pr\{X = k, Y = n - k\} = \sum_{k=0}^n (1-\pi)\pi^k (1-\pi)\pi^{n-k} \\
 &= (1-\pi)^2 \pi^n \sum_{k=0}^n 1 = (n+1)(1-\pi)^2 \pi^n \text{ for } n \geq 0.
 \end{aligned}$$

**3.10**

$k$	Binomial $n = 10 \ p = .1$	Binomial $n = 100 \ p = .01$	Poisson $\lambda = 1$
0	.349	.366	.368
1	.387	.370	.368
2	.194	.185	.184

$$3.11 \Pr\{U = u, W = 0\} = \Pr\{X = u, Y = u\} = (1-\pi)^2 \pi^{2u}, u \geq 0.$$

$$\Pr\{U = u, W = w > 0\} = \Pr\{X = u, Y = u + w\} + \Pr\{Y = u, X = u + w\} = 2(1-\pi)^2 \pi^{2u+w}$$

$$\Pr\{U = u\} = \sum_{w=0}^{\infty} \Pr\{U = u, W = w\} = \pi^{2u} (1-\pi^2).$$

$$\Pr\{W = 0\} = \sum_{w=0}^{\infty} \Pr\{U = u, W = 0\} = (1-\pi)^2 / (1-\pi^2).$$

$$\Pr\{W = w > 0\} = 2 \left[ (1-\pi)^2 / (1-\pi^2) \right] \pi^w, \text{ and}$$

$$\Pr\{U = u, W = w\} = \Pr\{U = u\}\Pr\{W = w\} \text{ for all } u, w.$$



**3.12** Let  $X$  = number of calls to switch board in a minute.  $Pr\{X \geq 7\} = 1 - \sum_{k=0}^6 \frac{4^k e^{-4}}{k!} = .111$ .

**3.13** Assume that inspected items are independently defective or good. Let  $X$  = # of defects in sample.

$$Pr\{X = 0\} = (.95)^{10} = .599$$

$$Pr\{X = 1\} = 10(.95)^9(.05) = .315$$

$$Pr\{X \geq 2\} = 1 - (.599 + .315) = .086.$$

**3.14 (a)**  $E[Z] = \frac{1-p}{p} = 9$ ,  $Var[Z] = \frac{1-p}{p^2} = 90$

**(b)**  $Pr\{Z > 10\} = (.9)^{10} = .349$ .

**3.15**  $Pr\{X \leq 2\} = \left(1 + 2 + \frac{2^2}{2}\right)e^{-2} = 5e^{-2} = .677$ .

**3.16 (a)**  $p_0 = 1 - b \sum_{k=1}^{\infty} (1-p)^k = 1 - b\left(\frac{1-p}{p}\right)$ .

**(b)** When  $b = p$ , then  $p_k$  is given by (3.4).

When  $b = \frac{p}{1-p}$ , then  $p_k$  is given by (3.5).

**(c)**  $Pr\{N = n > 0\} = Pr\{X = 0, Z = n\} + Pr\{X = 1, Z = n - 1\}$   
 $= (1 - \alpha)p(1 - p)^n + \alpha p(1 - p)^{n-1}$   
 $= [(1 - \alpha)p + \alpha p / (1 - p)](1 - p)^n$

So  $b = (1 - \alpha)p + \alpha p / (1 - p)$ .

$$4.1 \ E[e^{\lambda Z}] = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} e^{\frac{1}{2}z^2 + \lambda z} dz = e^{\frac{1}{2}\lambda^2} \left\{ \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} e^{\frac{1}{2}(z-\lambda)^2} dz \right\} = e^{\frac{1}{2}\lambda^2}.$$

**4.2 (a)**  $PrW > \frac{1}{\theta} = e^{-\theta/\theta} = e^{-1} = .368 \dots$

**(b)** Mode = 0.

**4.3**  $X - \theta$  and  $Y - \theta$  are both uniform over  $\left[-\frac{1}{2}, \frac{1}{2}\right]$ , independent of  $\theta$ , and  $W = X - Y = (X - \theta) - (Y - \theta)$ . Therefore the distribution of  $W$  is independent of  $\theta$  and we may determine it assuming  $\theta = 0$ . Also, the density of  $W$  is symmetric since that of both  $X$  and  $Y$  are.

$$Pr\{W > w\} = Pr\{X > Y + w\} = \frac{1}{2}(1 - w)^2, \quad w > 0$$

So  $f_w(w) = 1 - w$  for  $0 \leq w \leq 1$  and  $f_w(w) = 1 - |w|$  for  $-1 \leq w \leq +1$

**4.4**  $\mu_c = .010$ ;  $\sigma_c^2 = (.005)^2$ ,  $Pr\{C < 0\} = Pr\left\{\frac{C-.010}{.005} < \frac{-.010}{.005}\right\} = Pr\{Z < -2\} = .0228$ .

**4.5**  $Pr\{Z < Y\} = \int_0^{\infty} \left\{ \int_x^{\infty} 3e^{-3y} dy \right\} 2e^{-2x} dx = \frac{2}{5}$ .

**5.1**  $Pr\{N > k\} = Pr\{X_1 \leq \xi, \dots, X_k \leq \xi\} = [F(\xi)]^k, k = 0, 1, \dots$

$Pr\{N = k\} = Pr\{N > k - 1\} - Pr\{N > k\} = [1 - F(\xi)]F(\xi)^{k-1}, k = 1, 2, \dots$

**5.2**  $Pr\{Z > z\} = Pr\{X_1 > z, \dots, X_n > z\} = Pr\{X_1 > z\} \dots Pr\{X_n > z\}$   
 $= e^{-\lambda z} \dots e^{-\lambda z} = e^{-n\lambda z}, z > 0$ .

$Z$  is exponentially distributed, parameter  $n\lambda$ .

**5.3**  $Pr\{X > k\} = \sum_{l=k+1}^{\infty} p(1-p)^l = p(1-p)^{k+1}, k = 0, 1, \dots$

$$E[X] = \sum_{k=0}^{\infty} Pr\{X > k\} = \frac{1-p}{p}.$$

**5.4** Write  $V = V^+ - V^-$  when  $V^+ = \max\{V, 0\}$  and  $V^- = \max\{-V, 0\}$ . Then  $Pr\{V^+ > v\} = 1 - F_v(v)$  and  $Pr\{V^- > v\} = F_v(-v)$  for  $v > 0$ . Use (5.3) on  $V^+$  and  $V^-$  together with  $E[V] = E[V^+] - E[V^-]$ . Mean does not exist if  $E[V^+] = E[V^-] = \infty$ .

**5.5**  $E[W^2] = \int_0^\infty P\{W^2 > t\} dt = \int_0^\infty [1 - F_w(\sqrt{t})] dt = \int_0^\infty 2y[1 - F_w(y)] dy$  by letting  $y = \sqrt{t}$ .

**5.6**  $Pr\{V > t\} = \int_t^\infty \lambda e^{-\lambda v} dv = e^{-\lambda t}$ ;  $E[V] = \int_0^\infty Pr\{V > t\} dt = \frac{1}{\lambda} \int_0^\infty \lambda e^{-\lambda t} dt = \frac{1}{\lambda}$ .

**5.7**  $Pr\{V > v\} = Pr\{X_1 > v, \dots, X_n > v\} = Pr\{X_1 > v\} \cdots Pr\{X_n > v\}$   
 $= e^{-\lambda_1 v} \cdots e^{-\lambda_n v} = e^{-(\lambda_1 + \cdots + \lambda_n)v}$ ,  $v > 0$ .

$V$  is exponentially distributed with parameter  $\sum_i \lambda_i$ .

**5.8**

<b>Spares</b>	3	2	1	0
<b>A</b>				
<b>B</b>				
<b>Mean</b>	$\frac{1}{2\lambda}$	$\frac{1}{2\lambda}$	$\frac{1}{2\lambda}$	$\frac{1}{2\lambda}$

Expected flash light operating duration =  $\frac{1}{2\lambda} + \frac{1}{2\lambda} + \frac{1}{2\lambda} + \frac{1}{2\lambda} = \frac{2}{\lambda} = 2$  Expected battery operating durations!