Chapter 2
An Introduction to Linear Programming

Case Problem 1: Workload Balancing

1.

	Production Rate (minutes per printer)		
Model	Line 1	Line 2	Profit Contribution ($)
DI-910	3	4	42
DI-950	6	2	87

Capacity: 8 hours × 60 minutes/hour = 480 minutes per day

Let D_1 = number of units of the DI-910 produced
D_2 = number of units of the DI-950 produced

$$
\begin{array}{lllll}
\text{Max} & 42D_1 + 87D_2 & & & \\
\text{s.t.} & & & & \\
& 3D_1 + 6D_2 & \leq & 480 & \text{Line 1 Capacity} \\
& 4D_1 + 2D_2 & \leq & 480 & \text{Line 2 Capacity} \\
& D_1, D_2 \geq 0 & & &
\end{array}
$$

Using *The Management Scientist*, the optimal solution is $D_1 = 0$, $D_2 = 80$. The value of the optimal solution is $6960.

Management would not implement this solution because no units of the DI-910 would be produced.

2.

Adding the constraint $D_1 \geq D_2$ and resolving the linear program results in the optimal solution $D_1 = 53.333$, $D_2 = 53.333$. The value of the optimal solution is $6880.

3.

Time spent on Line 1: 3(53.333) + 6(53.333) = 480 minutes

Time spent on Line 2: 4(53.333) + 2(53.333) = 320 minutes

Thus, the solution does not balance the total time spent on Line 1 and the total time spent on Line 2. This might be a concern to management if no other work assignments were available for the employees on Line 2.

4.

Let T_1 = total time spent on Line 1
T_2= total time spent on Line 2

Whatever the value of T_2 is,

$$T_1 \leq T_2 + 30$$
$$T_1 \geq T_2 - 30$$

Thus, with $T_1 = 3D_1 + 6D_2$ and $T_2 = 4D_1 + 2D_2$

$$3D_1 + 6D_2 \leq 4D_1 + 2D_2 + 30$$
$$3D_1 + 6D_2 \geq 4D_1 + 2D_2 - 30$$

Hence,

$$-1D_1 + 4D_2 \leq 30$$
$$-1D_1 + 4D_2 \geq -30$$

Rewriting the second constraint by multiplying both sides by -1, we obtain

$$-1D_1 + 4D_2 \leq 30$$
$$1D_1 - 4D_2 \leq 30$$

Adding these two constraints to the linear program formulated in part (2) and re-solving using *The Management Scientist*, we obtain the optimal solution $D_1 = 96.667$, $D_2 = 31.667$. The value of the optimal solution is $6815. Line 1 is scheduled for 480 minutes and Line 2 for 450 minutes. The effect of workload balancing is to reduce the total contribution to profit by $6880 - $6815 = $65 per shift.

5.

The optimal solution is $D_1 = 106.667$, $D_2 = 26.667$. The total profit contribution is

$$42(106.667) + 87(26.667) = \$6800$$

Comparing the solutions to part (4) and part (5), maximizing the number of printers produced (106.667 + 26.667 = 133.33) has increased the production by 133.33 - (96.667 + 31.667) = 5 printers but has reduced profit contribution by $6815 - $6800 = $15. But, this solution results in perfect workload balancing because the total time spent on each line is 480 minutes.

Case Problem 2: Production Strategy

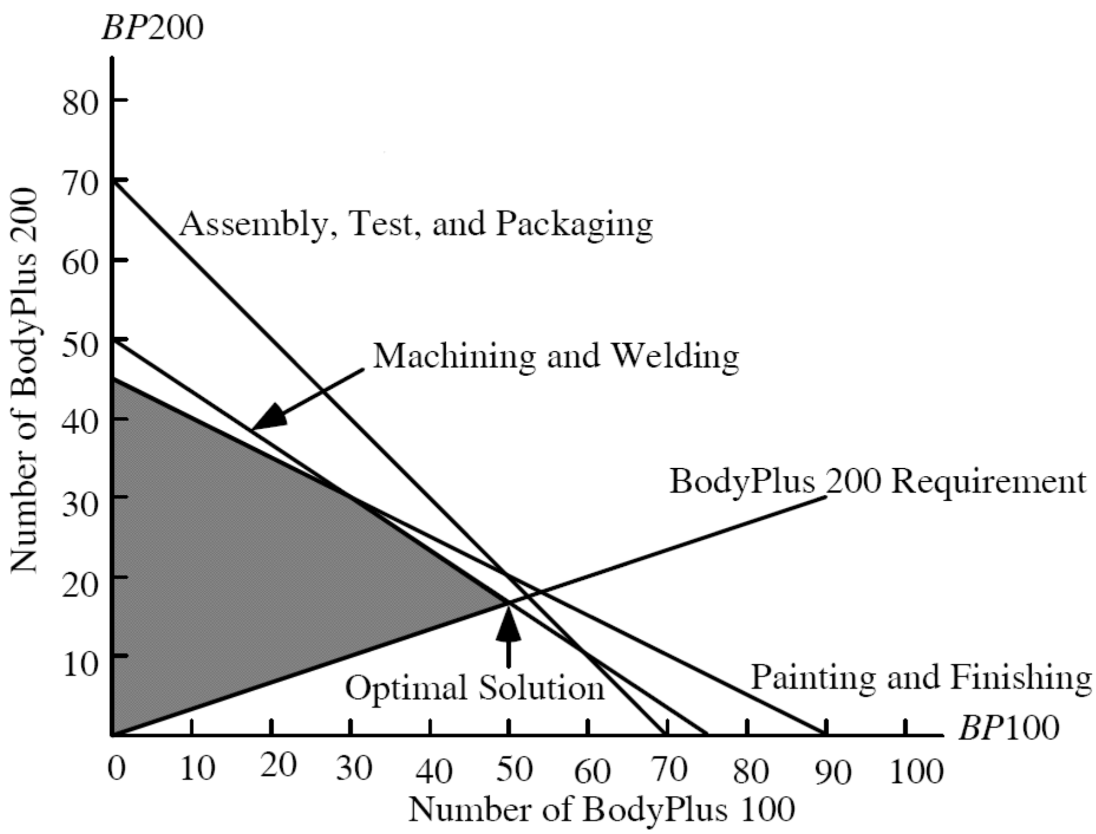
Let $BP100$ = the number of BodyPlus 100 machines produced
$BP200$ = the number of BodyPlus 200 machines produced

Max $371BP100 + 461BP200$

s.t.

$8BP100$	$+ 12BP200$	≤ 600 Machining and Welding
$5BP100$	$+ 10BP200$	≤ 450 Painting and Finishing
$2BP100$	$+ 2BP200$	≤ 140 Assembly, Test, and Packaging
$-0.25BP100$	$+ 0.75BP200$	≥ 0 BodyPlus 200 Requirement

$BP100, BP200 \geq 0$



Optimal solution: *BP*100 = 50, *BP*200 = 50/3, profit = €26,233.33. Note: If the optimal solution is rounded to *BP*100 = 50, *BP*200 = 16.67, the value of the optimal solution will differ from the value shown. The value we show for the optimal solution is the same as the value that will be obtained if the problem is solved using a linear programming software package such as *The Management Scientist.*

2.

In the short run the requirement reduces profits. For instance, if the requirement were reduced to at least 24% of total production, the new optimal solution is *BP*100 = 1425/28, *BP*200 = 225/14, with a total profit of €26,290.18; thus, total profits would increase by €56.85. Note: If the optimal solution is rounded to *BP*100 = 50.89, *BP*200 = 16.07, the value of the optimal solution will differ from the value shown. The value we show for the optimal solution is the same as the value that will be obtained if the problem is solved using a linear programming software package such as *The Management Scientist.*

3.

If management really believes that the BodyPlus 200 can help position BFI as one of the leader's in high-end exercise equipment, the constraint requiring that the number of units of the BodyPlus 200 produced be at least 25% of total production should not be changed. Since the optimal solution uses all of the available machining and welding time, management should try to obtain additional hours of this resource.

Case Problem 3: Blending

1.
The problem facing the café is how many of the two types of veggie burger to make each day in order to maximise profit but at the same time stay within the limits of the available ingredients and within the limits of likely sales.

This is a typical business problem concerned with finding an optimal solution (in this case optimum profit) and there is a standard procedure for finding such a solution.

The procedure requires us to translate the business problem into a mathematical form. We do this first of all by using two variables to stand for the two types of burger. We shall use L to show the number of large burgers produced and S the number of standard burgers. We can then show each part of the café's problem using simple mathematics. First let's look at profit. The total profit earned from the burgers will be:

Profit (€) $0.85L + 0.55S$

since we get 85c for each Large burger and 55c for each Standard
We want to achieve maximum possible profit.

We know that the café never sells more than 100 burgers so we have:

$$L + S \leq 100$$

That is, the total of Large plus Standard burgers cannot exceed 100.
Similarly we know that we sell at least 25 Large burgers each day and at least 30 Standard burgers so we have:

$$L \geq 25$$

and

$$S \geq 30$$

Each burger is made up of 3 ingredients and we have limited supplies of each. For the Vegetable mix, for example, a Large burger needs 30g, a Standard burger also needs 30g and we have a total of 4 kilos (or 4000g) available. We show this mathematically as:

$$30L + 30S \leq 4000$$

that is, the Vegetable mix used in making Large and Standard burgers cannot be more than the supply we have of 4000g (or 4 kilos). We can do exactly the same for the other two ingredients. For the Soybean mix we have:

$$150L + 100S \leq 10000$$

and for the Crumb mix we have:

$$20L + 20S \leq 1500$$

We have now put each part of the café's problem into mathematics. If we pull all this together we then have:

Maximise Profit $0.85L + 0.55S$

but making sure that:

$L + S \leq 100$
$L \geq 25$
$S \geq 30$
$30L + 30S \leq 4000$
$150L + 100S \leq 10000$
$20L + 20S \leq 1500$

and because we're dealing in mathematics and not business we have to force the procedure to give us a sensible business solution by adding

$L \geq 0$
$S \geq 0$

that is, we can't allow negative values for the two types of burger.

2. Following the standard solution procedure, our advice in order to maximise profit is as follows:

Produce 45 Large Burgers and 30 Standard burgers each day for a total daily profit of €54.75.

By doing this:

Total combined sales are less than the 100 units required
We are selling 20 more Large burgers than the minimum required
We are selling the minimum required quantity of Standard burgers
We need 2.25 kilos of the 4 kilos of Veggie mix available
We need 9.75 kilos of the 10 kilos of Soybean mix available
We need all of the 1.5 kilos of Crumb mix available.

3. We are assuming by making this recommendation that the information about the problem is accurate and also constant from day to day.