Preface

This manual contains solutions/answers to all exercises in the text *Algebra and Trigonometry with Analytic Geometry, Thirteenth Edition*, by Earl W. Swokowski and Jeffery A. Cole. A *Student's Solutions Manual* is also available; it contains solutions for the odd-numbered exercises in each section and for the Discussion Exercises, as well as solutions for all the exercises in the Review Sections and for the Chapter Tests.

For most problems, a reasonably detailed solution is included. It is my hope that by merely browsing through the solutions, professors will save time in determining appropriate assignments for their particular class.

I appreciate feedback concerning errors, solution correctness or style, and manual style—comments from professors using previous editions have greatly strengthened the ancillary package as well as the text. Any comments may be sent directly to me at the address below, at jeff.cole@anokaramsey.edu, or in care of the publisher: Brooks/Cole | Cengage Learning, 20 Davis Drive, Belmont, CA 94002-3098.

I would like to thank: Marv Riedesel and Mary Johnson for accuracy checking of the new exercises; Andrew Bulman-Fleming, for manuscript preparation; and Brian Morris and the late George Morris, of Scientific Illustrators, for creating the mathematically precise art package. I dedicate this book to my children, Becky and Brad.

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Table of Contents

To the Instructor

In the chapter review sections, the solutions are abbreviated since more detailed solutions were given in chapter sections. In easier groups of exercises, representative solutions are shown. When appropriate, only the answer is listed.

All figures have been plotted using computer software, offering a high degree of precision. The calculator graphs are from various TI screens. When possible, we tried to make each piece of art with the same scale to show a realistic and consistent graph.

This manual was done using $\mathbb{EXP}$: *The Scientific Word Processor*.

The following notations are used in the manual.

Note: Notes to the instructor/student pertaining to hints on instruction or conventions to follow.

{ }	{ comments to the reader are in braces }
LS	{ Left Side of an equation }
RS	{ Right Side of an equation }
$\Rightarrow$	{ implies, next equation, logically follows }
$\Leftrightarrow$	{ if and only if, is equivalent to }
•	{ bullet, used to separate problem statement from solution or explanation }
★	{ used to identify the answer to the problem }
§	{ *section* references }
$\forall$	{ For all, i.e., $\forall x$ means "for all x". }
$\mathbb{R} - \{a\}$	{ The set of all real numbers except a. }
$\therefore$	{ therefore }
QI–QIV	{ quadrants I, II, III, IV }

Chapter 1: Fundamental Concepts of Algebra

1.1 Exercises

1 (a) Since x and y have opposite signs, the product xy is negative.

(b) Since $x^2 > 0$ and $y > 0$, the product x^2y is positive.

(c) Since $x < 0$ {x is negative} and $y > 0$ {y is positive}, the quotient $\dfrac{x}{y}$ is negative.

Thus, $\dfrac{x}{y} + x$ is the sum of two negatives, which is negative.

(d) Since $y > 0$ and $x < 0$, $y - x > 0$.

2 (a) Since x and y have opposite signs, the quotient $\dfrac{x}{y}$ is negative.

(b) Since $x < 0$ and $y^2 > 0$, the product xy^2 is negative.

(c) Since $x - y < 0$ and $xy < 0$, $\dfrac{x-y}{xy} > 0$.

(d) Since $y > 0$ and $y - x > 0$, $y(y - x) > 0$.

3 (a) Since -7 is to the left of -4 on a coordinate line, $-7 \boxed{<} -4$.

(b) Using a calculator, we see that $\frac{\pi}{2} \approx 1.57$. Hence, $\frac{\pi}{2} \boxed{>} 1.5$.

(c) $\sqrt{225} \boxed{=} 15$ ***Note:*** $\sqrt{225} \neq \pm 15$

4 (a) Since -3 is to the right of -6 on a coordinate line, $-3 \boxed{>} -6$.

(b) Using a calculator, we see that $\frac{\pi}{4} \approx 0.79$. Hence, $\frac{\pi}{4} \boxed{<} 0.8$.

(c) $\sqrt{289} \boxed{=} 17$ ***Note:*** $\sqrt{289} \neq \pm 17$

5 (a) Since $\frac{1}{11} = 0.\overline{09} = 0.0909\ldots$, $\frac{1}{11} \boxed{>} 0.09$.

(b) Since $\frac{2}{3} = 0.\overline{6} = 0.6666\ldots$, $\frac{2}{3} \boxed{>} 0.666$.

(c) Since $\frac{22}{7} = 3.\overline{142857}$ and $\pi \approx 3.141593$, $\frac{22}{7} \boxed{>} \pi$.

6 (a) Since $\frac{1}{7} = 0.\overline{142857}$, $\frac{1}{7} \boxed{<} 0.143$.

(b) Since $\frac{5}{6} = 0.8\overline{3} = 0.8333\ldots$, $\frac{5}{6} \boxed{>} 0.833$.

(c) Since $\sqrt{2} \approx 1.414$, $\sqrt{2} \boxed{>} 1.4$.

7 (a) "x is negative" is equivalent to $x < 0$. We symbolize this by writing "x is negative $\Leftrightarrow$ $x < 0$."

(b) y is nonnegative $\Leftrightarrow$ $y \geq 0$

(c) q is less than or equal to π $\Leftrightarrow$ $q \leq \pi$

(d) d is between 4 and 2 $\Leftrightarrow$ $2 < d < 4$

(e) t is not less than 5 $\Leftrightarrow$ $t \geq 5$

(f) The negative of z is not greater than 3 $\Leftrightarrow$ $-z \leq 3$

(g) The quotient of p and q is at most 7 $\Leftrightarrow$ $\dfrac{p}{q} \leq 7$

(h) The reciprocal of w is at least 9 $\Leftrightarrow$ $\dfrac{1}{w} \geq 9$

(i) The absolute value of x is greater than 7 $\Leftrightarrow$ $|x| > 7$

Note: An informal definition of absolute value that may be helpful is

$$|\textit{something}| = \begin{cases} \textit{itself} & \text{if } \textit{itself} \text{ is positive or zero} \\ -(\textit{itself}) & \text{if } \textit{itself} \text{ is negative} \end{cases}$$

8 (a) b is positive $\Leftrightarrow$ $b > 0$

(b) s is nonpositive $\Leftrightarrow$ $s \leq 0$

(c) w is greater than or equal to -4 $\Leftrightarrow$ $w \geq -4$

(d) c is between $\frac{1}{5}$ and $\frac{1}{3}$ $\Leftrightarrow$ $\frac{1}{5} < c < \frac{1}{3}$

(e) p is not greater than -2 $\Leftrightarrow$ $p \leq -2$

(f) The negative of m is not less than -2 $\Leftrightarrow$ $-m \geq -2$

(g) The quotient of r and s is at least $\frac{1}{5}$ $\Leftrightarrow$ $\frac{r}{s} \geq \frac{1}{5}$

(h) The reciprocal of f is at most 14 $\Leftrightarrow$ $\frac{1}{f} \leq 14$

(i) The absolute value of x is less than 4 $\Leftrightarrow$ $|x| < 4$

9 (a) $|-3-4| = |-7| = -(-7)$ {since $-7 < 0$} $= 7$

(b) $|-5| - |2| = -(-5) - 2 = 5 - 2 = 3$

(c) $|7| + |-4| = 7 + [-(-4)] = 7 + 4 = 11$

10 (a) $|-11+1| = |-10| = -(-10)$ { since $-10 < 0$ } $= 10$

(b) $|6| - |-3| = 6 - [-(-3)] = 6 - 3 = 3$

(c) $|8| + |-9| = 8 + [-(-9)] = 8 + 9 = 17$

11 (a) $(-5)|3-6| = (-5)|-3| = (-5)[-(-3)] = (-5)(3) = -15$

(b) $|-6|/(-2) = -(-6)/(-2) = 6/(-2) = -3$

(c) $|-7| + |4| = -(-7) + 4 = 7 + 4 = 11$

12 (a) $(4)|6-7| = (4)|-1| = (4)[-(-1)] = (4)(1) = 4$

(b) $5/|-2| = 5/[-(-2)] = 5/2$

(c) $|-1| + |-9| = -(-1) + [-(-9)] = 1 + 9 = 10$

13 (a) Since $(4-\pi)$ is positive, $|4-\pi| = 4-\pi$.

(b) Since $(\pi-4)$ is negative, $|\pi-4| = -(\pi-4) = 4-\pi$.

(c) Since $\left(\sqrt{2}-1.5\right)$ is negative, $\left|\sqrt{2}-1.5\right| = -\left(\sqrt{2}-1.5\right) = 1.5-\sqrt{2}$.

14 (a) Since $\left(\sqrt{3}-1.7\right)$ is positive, $\left|\sqrt{3}-1.7\right| = \sqrt{3}-1.7$.

(b) Since $\left(1.7-\sqrt{3}\right)$ is negative, $\left|1.7-\sqrt{3}\right| = -\left(1.7-\sqrt{3}\right) = \sqrt{3}-1.7$.

(c) $\left|\frac{1}{5}-\frac{1}{3}\right| = \left|\frac{3}{15}-\frac{5}{15}\right| = \left|-\frac{2}{15}\right| = -\left(-\frac{2}{15}\right) = \frac{2}{15}$

15 (a) $d(A,B) = |7-3| = |4| = 4$

(b) $d(B,C) = |-5-7| = |-12| = 12$

(c) $d(C,B) = d(B,C) = 12$

(d) $d(A,C) = |-5-3| = |-8| = 8$

16 (a) $d(A,B) = |-2-(-6)| = |4| = 4$

(b) $d(B,C) = |4-(-2)| = |6| = 6$

(c) $d(C,B) = d(B,C) = 6$

(d) $d(A,C) = |4-(-6)| = |10| = 10$

17 (a) $d(A,B) = |1-(-9)| = |10| = 10$

(b) $d(B,C) = |10-1| = |9| = 9$

(c) $d(C,B) = d(B,C) = 9$

(d) $d(A,C) = |10-(-9)| = |19| = 19$

18 (a) $d(A,B) = |-4-8| = |-12| = 12$

(b) $d(B,C) = |-1-(-4)| = |3| = 3$

(c) $d(C,B) = d(B,C) = 3$

(d) $d(A,C) = |-1-8| = |-9| = 9$

Note: Because $|a| = |-a|$, the answers to Exercises 19–24 could have a different form. For example, $|-3-x| \geq 8$ is equivalent to $|x+3| \geq 8$.

19 $A = x$ and $B = 7$, so $d(A,B) = |7-x|$. Thus, "$d(A,B)$ is less than 2" can be written as $|7-x| < 2$.

20 $d(A,B) = \left|-\sqrt{2}-x\right| \Rightarrow \left|-\sqrt{2}-x\right| > 1$

21 $d(A,B) = |-3-x| \Rightarrow |-3-x| \geq 8$

22 $d(A,B) = |4-x| \Rightarrow |4-x| \leq 5$

23 $d(A, B) = |x - 4| \quad \Rightarrow \quad |x - 4| \leq 3$

24 $d(A, B) = |x - (-2)| = |x + 2| \quad \Rightarrow \quad |x + 2| \geq 4$

Note: In Exercises 25–32, you may want to substitute a permissible value for the variable to first test if the expression inside the absolute value symbol is positive or negative.

25 Pick an arbitrary value for x that is less than -3, say -5.
Since $3 + (-5) = -2$ is negative, we conclude that if $x < -3$, then $3 + x$ is negative.
Hence, $|3 + x| = -(3 + x) = -x - 3$.

26 If $x > 5$, then $5 - x < 0$, and $|5 - x| = -(5 - x) = x - 5$.

27 If $x < 2$, then $2 - x > 0$, and $|2 - x| = 2 - x$.

28 If $x \geq -7$, then $7 + x \geq 0$, and $|7 + x| = 7 + x$.

29 If $a < b$, then $a - b < 0$, and $|a - b| = -(a - b) = b - a$.

30 If $a > b$, then $a - b > 0$, and $|a - b| = a - b$.

31 Since $x^2 + 4 > 0$ for every x, $|x^2 + 4| = x^2 + 4$.

32 Since $-x^2 - 1 < 0$ for every x, $|-x^2 - 1| = -(-x^2 - 1) = x^2 + 1$.

33 $\text{LS} = \dfrac{ab + ac}{a} = \dfrac{ab}{a} + \dfrac{ac}{a} = b + c \boxed{\neq} \text{RS}$ (which is $b + ac$).

34 $\text{LS} = \dfrac{ab + ac}{a} = \dfrac{ab}{a} + \dfrac{ac}{a} = b + c \boxed{=} \text{RS}$.

35 $\text{LS} = \dfrac{b + c}{a} = \dfrac{b}{a} + \dfrac{c}{a} \boxed{=} \text{RS}$.

36 $\text{LS} = \dfrac{a + c}{b + d} = \dfrac{a}{b + d} + \dfrac{c}{b + d} \boxed{\neq} \text{RS} \left(\text{which is } \dfrac{a}{b} + \dfrac{c}{d}\right)$.

37 $\text{LS} = (a \div b) \div c = \dfrac{a}{b} \cdot \dfrac{1}{c} = \dfrac{a}{bc}$. $\text{RS} = a \div (b \div c) = a \div \dfrac{b}{c} = a \cdot \dfrac{c}{b} = \dfrac{ac}{b}$. $\text{LS} \boxed{\neq} \text{RS}$

38 $\text{LS} = (a - b) - c = a - b - c$. $\text{RS} = a - (b - c) = a - b + c$. $\text{LS} \boxed{\neq} \text{RS}$

39 $\text{LS} = \dfrac{a - b}{b - a} = \dfrac{-(b - a)}{b - a} = -1 \boxed{=} \text{RS}$.

40 $\text{LS} = -(a + b) = -a - b \boxed{\neq} \text{RS}$ (which is $-a + b$).

41 (a) On the TI-83/4 Plus, the absolute value function is choice 1 under MATH, NUM.
Enter abs(3.2^2-$\sqrt{}$(4.27)). $\left|3.2^2 - \sqrt{4.27}\right| \approx 8.1736$

(b) $\sqrt{(15.6 - 1.5)^2 + (4.3 - 5.4)^2} \approx 14.1428$

42 (a) $\dfrac{3.42 - 1.29}{5.83 + 2.64} \approx 0.2515$ (b) $\pi^3 \approx 31.0063$

43 (a) $\dfrac{1.2 \times 10^3}{3.1 \times 10^2 + 1.52 \times 10^3} \approx 0.6557 = 6.557 \times 10^{-1}$

Note: For the TI-83/4 Plus, use 1.2E3/(3.1E2 + 1.52E3), where E is obtained by pressing [2nd] [EE].

(b) $\left(1.23 \times 10^{-4}\right) + \sqrt{4.5 \times 10^3} \approx 67.08 = 6.708 \times 10^1$

44 (a) $\sqrt{|3.45 - 1.2 \times 10^4| + 10^5} \approx 334.7 = 3.347 \times 10^2$

(b) $(1.79 \times 10^2) \times (9.84 \times 10^3) = 1{,}761{,}360 \approx 1.761 \times 10^6$

45 Construct a right triangle with sides of lengths $\sqrt{2}$ and 1. The hypotenuse will have length $\sqrt{\left(\sqrt{2}\right)^2 + 1^2} = \sqrt{3}$. Next construct a right triangle with sides of lengths $\sqrt{3}$ and $\sqrt{2}$. The hypotenuse will have length $\sqrt{\left(\sqrt{3}\right)^2 + \left(\sqrt{2}\right)^2} = \sqrt{5}$.

46 Use $C = 2\pi r$ with $r = 1$, 2, and 10 to obtain 2π, 4π, and 20π units from the origin.

47 The large rectangle has area = width × length = $a(b + c)$. The sum of the areas of the two small rectangles is $ab + ac$. Since the areas are the same, we have $a(b + c) = ab + ac$.

48 $x_1 = \frac{3}{2}$ and $n = 2 \quad \Rightarrow \quad x_2 = \frac{1}{2}\left(x_1 + \frac{n}{x_1}\right) = \frac{1}{2}\left(\frac{3}{2} + \frac{2}{\frac{3}{2}}\right) = \frac{1}{2}\left(\frac{3}{2} + \frac{4}{3}\right) = \frac{1}{2}\left(\frac{17}{6}\right) = \frac{17}{12}$.

$$x_3 = \frac{1}{2}\left(x_2 + \frac{2}{x_2}\right) = \frac{1}{2}\left(\frac{17}{12} + \frac{2}{\frac{17}{12}}\right) = \frac{1}{2}\left(\frac{17}{12} + \frac{24}{17}\right) = \frac{1}{2}\left(\frac{577}{204}\right) = \frac{577}{408}$$

49 (a) Since the decimal point is 5 places to the right of the first nonzero digit, $427{,}000 = 4.27 \times 10^5$.

(b) Since the decimal point is 8 places to the left of the first nonzero digit, $0.000\,000\,093 = 9.3 \times 10^{-8}$.

(c) Since the decimal point is 8 places to the right of the first nonzero digit, $810{,}000{,}000 = 8.1 \times 10^8$.

50 (a) $85{,}200 = 8.52 \times 10^4$ (b) $0.000\,005\,4 = 5.4 \times 10^{-6}$

(c) $24{,}900{,}000 = 2.49 \times 10^7$

51 (a) Moving the decimal point 5 places to the right, we have $8.3 \times 10^5 = 830{,}000$.

(b) Moving the decimal point 12 places to the left, we have $2.9 \times 10^{-12} = 0.000\,000\,000\,002\,9$.

(c) Moving the decimal point 8 places to the right, we have $5.64 \times 10^8 = 564{,}000{,}000$.

52 (a) $2.3 \times 10^7 = 23{,}000{,}000$ (b) $7.01 \times 10^{-9} = 0.000\,000\,007\,01$

(c) $1.25 \times 10^{10} = 12{,}500{,}000{,}000$

53 Since the decimal point is 24 places to the left of the first nonzero digit,

$$0.000\,000\,000\,000\,000\,000\,000\,001\,7 = 1.7 \times 10^{-24}.$$

54 $9.1 \times 10^{-31} = 0.000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,91$

55 It is helpful to write the units of any fraction, and then "cancel" those units to determine the units of the final answer. $\frac{186{,}000 \text{ miles}}{\text{second}} \cdot \frac{60 \text{ seconds}}{1 \text{ minute}} \cdot \frac{60 \text{ minutes}}{1 \text{ hour}} \cdot \frac{24 \text{ hours}}{1 \text{ day}} \cdot \frac{365 \text{ days}}{1 \text{ year}} \cdot 1 \text{ year} \approx 5.87 \times 10^{12}$ mi

56 (a) 100 billion $= 100{,}000{,}000{,}000 = 1 \times 10^{11}$

(b) $d \approx (100{,}000 \text{ yr})\left(5.87 \times 10^{12} \frac{\text{mi}}{\text{yr}}\right) = 5.87 \times 10^{17}$ mi

57 $$\frac{\frac{1.01 \text{ grams}}{\text{mole}}}{\frac{6.02 \times 10^{23} \text{ atoms}}{\text{mole}}} \cdot 1 \text{ atom} = \frac{1.01 \text{ grams}}{6.02 \times 10^{23}} \approx 0.1678 \times 10^{-23} \text{ g} = 1.678 \times 10^{-24} \text{ g}$$

58 $(2.5 \text{ million})(0.00035\%) = (2.5 \times 10^6)(3.5 \times 10^{-6}) = 8.75 \approx 9$ halibut

59 $\dfrac{24 \text{ frames}}{\text{second}} \cdot \dfrac{60 \text{ seconds}}{1 \text{ minute}} \cdot \dfrac{60 \text{ minutes}}{1 \text{ hour}} \cdot 48 \text{ hours} = 4.1472 \times 10^6 \text{ frames}$

60 $\dfrac{2 \times 10^{11} \text{ calculations}}{\text{second}} \cdot \dfrac{60 \text{ seconds}}{1 \text{ minute}} \cdot \dfrac{60 \text{ minutes}}{1 \text{ hour}} \cdot \dfrac{24 \text{ hours}}{1 \text{ day}} \cdot 60 \text{ days} = 1.0368 \times 10^{18} \text{ calculations}$

61 (a) $1 \text{ ft}^2 = 144 \text{ in}^2$, so the force on one square foot of a wall is $144 \text{ in}^2 \times 1.4 \text{ lb/in}^2 = 201.6$ lb.

(b) The area of the wall is $40 \times 8 = 320 \text{ ft}^2$, or $320 \text{ ft}^2 \times 144 \text{ in}^2/\text{ft}^2 = 46{,}080 \text{ in}^2$.
The total force is $46{,}080 \text{ in}^2 \times 1.4 \text{ lb/in}^2 = 64{,}512$ lb.
Converting to tons, we have $64{,}512 \text{ lb}/(2000 \text{ lb/ton}) = 32.256$ tons.

62 (a) We start with 400 adults, 150 yearlings, and 200 calves {total = 750}

Number of Adults = surviving adults + surviving yearlings
$= (0.90)(400) + (0.80)(150) = \underline{480}$

Number of Yearlings = surviving calves
$= (0.75)(200) = \underline{150}$

Number of Calves = number of female adults
$= (0.50)(480) = \underline{240}$

(b) 75% of last spring's calves equal the number of this year's yearlings (150), so the number of calves is $\underline{200}$.
The number of calves is equal to the number of adult females and this is one-half of the number of adults, so the number of adults is $\underline{400}$.
90% of these (360) are part of the 400 adults this year. The other 40 adults represent 80% of last year's yearlings, so the number of yearlings is $\underline{50}$.

1.2 Exercises

1 $\left(-\frac{2}{3}\right)^4 = \left(-\frac{2}{3}\right) \cdot \left(-\frac{2}{3}\right) \cdot \left(-\frac{2}{3}\right) \cdot \left(-\frac{2}{3}\right) = \frac{16}{81}$

Note: Do not confuse $(-x)^4$ and $-x^4$ since $(-x)^4 = x^4$ and $-x^4$ is the negative of x^4.

2 $(-3)^3 = -27 = \frac{-27}{1}$

3 $\dfrac{2^{-3}}{3^{-2}} = \dfrac{3^2}{2^3} = \dfrac{9}{8}$

Note: Remember that negative exponents don't necessarily give negative results—that is, $2^{-3} = \dfrac{1}{2^3} = \dfrac{1}{8}$, not $-\dfrac{1}{8}$.

4 $\dfrac{2^0 + 0^2}{2 + 0} = \dfrac{1 + 0}{2} = \dfrac{1}{2}$

5 $-2^4 + 3^{-1} = -16 + \frac{1}{3} = -\frac{48}{3} + \frac{1}{3} = -\frac{47}{3}$

6 $\left(-\frac{3}{2}\right)^4 - 2^{-4} = \frac{81}{16} - \frac{1}{16} = \frac{80}{16} = \frac{5}{1}$

7 $9^{5/2} = \left(\sqrt{9}\right)^5 = 3^5 = \frac{243}{1}$

8 $16^{-3/4} = 1/16^{3/4} = 1/\left(\sqrt[4]{16}\right)^3 = 1/2^3 = \frac{1}{8}$

9 $(-0.008)^{2/3} = \left(\sqrt[3]{-0.008}\right)^2 = (-0.2)^2 = 0.04 = \frac{4}{100} = \frac{1}{25}$

10 $(0.008)^{-2/3} = 1/(0.008)^{2/3} = 1/\left(\sqrt[3]{0.008}\right)^2 = 1/(0.2)^2 = 1/(0.04) = \frac{25}{1}$

11 $\left(\frac{1}{2}x^4\right)(16x^5) = \left(\frac{1}{2} \cdot 16\right)x^{4+5} = 8x^9$

12 $(-3x^{-2})(4x^4) = (-3 \cdot 4)x^{-2+4} = -12x^2$

13 A common mistake is to write $x^3x^2 = x^6$, and another is to write $(x^2)^3 = x^5$.

The following solution illustrates the proper use of the exponent rules.

$$\frac{(2x^3)(3x^2)}{(x^2)^3} = \frac{(2\cdot 3)x^{3+2}}{x^{2\cdot 3}} = \frac{6x^5}{x^6} = 6x^{5-6} = 6x^{-1} = \frac{6}{x}$$

14 $\frac{(2x^2)^3y^2}{4x^4y^2} = \frac{8x^6}{4x^4} = 2x^2$

15 $\left(\frac{1}{6}a^5\right)(-3a^2)(4a^7) = \frac{1}{6}\cdot(-3)\cdot 4\cdot a^{5+2+7} = -2a^{14}$

16 $(-4b^3)\left(\frac{1}{6}b^2\right)(-9b^4) = (-4)\cdot\frac{1}{6}\cdot(-9)\cdot b^{3+2+4} = 6b^9$

17 $\frac{(6x^3)^2}{(2x^2)^3}\cdot(3x^2)^0 = \frac{6^2x^{3\cdot 2}}{2^3x^{2\cdot 3}}\cdot 1$ {an expression raised to the zero power is equal to 1} $= \frac{36x^6}{8x^6} = \frac{36}{8} = \frac{9}{2}$

18 $\frac{(3y^3)(2y^2)^2}{(y^4)^3}\cdot(5y^3)^0 = \frac{(3y^3)(4y^4)}{y^{12}}\cdot 1 = \frac{12y^7}{y^{12}} = \frac{12}{y^5}$

19 $(3u^7v^3)(4u^4v^{-5}) = 12u^{7+4}v^{3+(-5)} = 12u^{11}v^{-2} = \frac{12u^{11}}{v^2}$

20 $(x^2yz^3)(-2xz^2)(x^3y^{-2}) = -2x^{2+1+3}y^{1-2}z^{3+2} = -2x^6y^{-1}z^5 = \frac{-2x^6z^5}{y}$

21 $(8x^4y^{-3})\left(\frac{1}{2}x^{-5}y^2\right) = 4x^{4-5}y^{-3+2} = 4x^{-1}y^{-1} = \frac{4}{xy}$

22 $\left(\frac{4a^2b}{a^3b^2}\right)\left(\frac{5a^2b}{2b^4}\right) = \frac{20a^{2+2}b^{1+1}}{2a^3b^{2+4}} = \frac{20a^4b^2}{2a^3b^6} = \frac{10a^{4-3}b^{2-6}}{1} = \frac{10a}{b^4}$

23 $\left(\frac{1}{3}x^4y^{-3}\right)^{-2} = \left(\frac{1}{3}\right)^{-2}(x^4)^{-2}(y^{-3})^{-2} = \left(\frac{3}{1}\right)^2x^{-8}y^6 = 3^2x^{-8}y^6 = \frac{9y^6}{x^8}$

24 $(-2xy^2)^5\left(\frac{x^7}{8y^3}\right) = (-32x^5y^{10})\left(\frac{x^7}{8y^3}\right) = -4x^{12}y^7$

25 $(3y^3)^4(4y^2)^{-3} = 3^4y^{12}\cdot 4^{-3}y^{-6} = 81y^6\cdot\frac{1}{4^3} = \frac{81}{64}y^6$

26 $(-3a^2b^{-5})^3 = -27a^6b^{-15} = -\frac{27a^6}{b^{15}}$

27 $(-2r^4s^{-3})^{-2} = (-2)^{-2}r^{-8}s^6 = \frac{s^6}{(-2)^2r^8} = \frac{s^6}{4r^8}$

28 $(2x^2y^{-5})(6x^{-3}y)\left(\frac{1}{3}x^{-1}y^3\right) = 4x^{-2}y^{-1} = \frac{4}{x^2y}$

29 $(5x^2y^{-3})(4x^{-5}y^4) = 20x^{2-5}y^{-3+4} = 20x^{-3}y^1 = \frac{20y}{x^3}$

30 $(-2r^2s)^5(3r^{-1}s^3)^2 = (-32r^{10}s^5)(9r^{-2}s^6) = -288r^8s^{11}$

31 $\left(\frac{3x^5y^4z}{x^0y^{-3}z}\right)^2$ {remember that $x^0 = 1$, cancel z} $= \frac{9x^{10}y^8}{y^{-6}} = 9x^{10}y^{8-(-6)} = 9x^{10}y^{14}$

32 $(4a^2b)^4\left(\frac{-a^3}{2b}\right)^2 = (256a^8b^4)\left(\frac{a^6}{4b^2}\right) = 64a^{14}b^2$

33 $(-5a^{3/2})(2a^{1/2}) = -5\cdot 2a^{(3/2)+(1/2)} = -10a^{4/2} = 8a^2$

34 $\left(-6x^{7/5}\right)\left(2x^{8/5}\right) = -6 \cdot 2x^{(7/5)+(8/5)} = -12x^{15/5} = -12x^3$

35 $\left(3x^{5/6}\right)\left(8x^{2/3}\right) = 3 \cdot 8x^{(5/6)+(4/6)} = 24x^{9/6} = 24x^{3/2}$

36 $(8r)^{1/3}\left(2r^{1/2}\right) = \left(2r^{1/3}\right)\left(2r^{1/2}\right) = 4r^{(2/6)+(3/6)} = 4r^{5/6}$

37 $\left(27a^6\right)^{-2/3} = 27^{-2/3}a^{-12/3} = \dfrac{a^{-4}}{27^{2/3}} = \dfrac{1}{\left(\sqrt[3]{27}\right)^2 a^4} = \dfrac{1}{3^2\,a^4} = \dfrac{1}{9a^4}$

38 $\left(25z^4\right)^{-3/2} = 25^{-3/2}z^{-12/2} = \dfrac{z^{-6}}{25^{3/2}} = \dfrac{1}{\left(\sqrt{25}\right)^3 z^6} = \dfrac{1}{5^3\,z^6} = \dfrac{1}{125z^6}$

39 $\left(8x^{-2/3}\right)x^{1/6} = 8x^{(-4/6)+(1/6)} = 8x^{-3/6} = \dfrac{8}{x^{1/2}}$

40 $\left(3x^{1/2}\right)\left(-2x^{5/2}\right) = -6x^{(1/2)+(5/2)} = -6x^3$

41 $\left(\dfrac{-8x^3}{y^{-6}}\right)^{2/3} = \dfrac{(-8)^{2/3}(x^3)^{2/3}}{(y^{-6})^{2/3}} = \dfrac{\left(\sqrt[3]{-8}\right)^2 x^{(3)(2/3)}}{y^{(-6)(2/3)}} = \dfrac{(-2)^2x^2}{y^{-4}} = \dfrac{4x^2}{y^{-4}} = 4x^2y^4$

42 $\left(\dfrac{-y^{3/2}}{y^{-1/3}}\right)^3 = \dfrac{-y^{9/2}}{y^{-1}} = -y^{11/2}$

43 $\left(\dfrac{x^6}{16y^{-4}}\right)^{-1/2} = \dfrac{x^{-3}}{16^{-1/2}y^2} = \dfrac{16^{1/2}}{x^3y^2} = \dfrac{4}{x^3y^2}$

44 $\left(\dfrac{c^{-4}}{81d^8}\right)^{3/4} = \dfrac{c^{-3}}{\left(\sqrt[4]{81}\right)^3 d^6} = \dfrac{c^{-3}}{3^3d^6} = \dfrac{1}{27c^3d^6}$

45 $\dfrac{(x^6y^3)^{-1/3}}{(x^4y^2)^{-1/2}} = \dfrac{(x^6)^{-1/3}(y^3)^{-1/3}}{(x^4)^{-1/2}(y^2)^{-1/2}} = \dfrac{x^{-2}y^{-1}}{x^{-2}y^{-1}} = 1$

46 $a^{4/3}a^{-3/2}a^{1/6} = a^{(8/6)-(9/6)+(1/6)} = a^{0/6} = a^0 = 1$

47 $\sqrt[4]{x^4+y} = \left(x^4+y\right)^{1/4}$

48 $\sqrt[3]{x^3+y^2} = (x^3+y^2)^{1/3}$

49 $\sqrt[3]{(a+b)^2} = \left[(a+b)^2\right]^{1/3} = (a+b)^{2/3}$

50 $\sqrt{a+\sqrt{b}} = \left(a+b^{1/2}\right)^{1/2}$

51 $\sqrt{x^2+y^2} = (x^2+y^2)^{1/2}$ ***Note:*** $\sqrt{x^2+y^2} \neq x+y$

52 $\sqrt[3]{r^3-s^3} = (r^3-s^3)^{1/3}$

53 (a) $4x^{3/2} = 4x^1x^{1/2} = 4x\sqrt{x}$ (b) $(4x)^{3/2} = (4x)^1(4x)^{1/2} = (4x)^1\,4^{1/2}x^{1/2} = 4x \cdot 2 \cdot x^{1/2} = 8x\sqrt{x}$

54 (a) $4+x^{3/2} = 4+x^1x^{1/2} = 4+x\sqrt{x}$ (b) $(4+x)^{3/2} = (4+x)^1(4+x)^{1/2} = (4+x)\sqrt{4+x}$

55 (a) $8-y^{1/3} = 8-\sqrt[3]{y}$ (b) $(8-y)^{1/3} = \sqrt[3]{8-y}$

56 (a) $64y^{1/3} = 64\sqrt[3]{y}$ (b) $(64y)^{1/3} = 64^{1/3}y^{1/3} = 4\sqrt[3]{y}$

57 $\sqrt{81} = \sqrt{9^2} = 9$

58 $\sqrt[3]{-216} = \sqrt[3]{(-6)^3} = -6$

59 $\sqrt[5]{-64} = \sqrt[5]{-32}\,\sqrt[5]{2} = \sqrt[5]{(-2)^5}\,\sqrt[5]{2} = -2\,\sqrt[5]{2}$

60 $\sqrt[4]{512} = \sqrt[4]{256}\,\sqrt[4]{2} = \sqrt[4]{4^4}\,\sqrt[4]{2} = 4\sqrt[4]{2}$

61 In the denominator, you would like to have $\sqrt[3]{2^3}$. How do you get it? Multiply by $\sqrt[3]{2^2}$, or, equivalently, $\sqrt[3]{4}$. Of course, we have to multiply the numerator by the same value so that we don't change the value of the given fraction.

$$\frac{1}{\sqrt[3]{2}} = \frac{1}{\sqrt[3]{2}} \cdot \frac{\sqrt[3]{4}}{\sqrt[3]{4}} = \frac{\sqrt[3]{4}}{\sqrt[3]{2\cdot 4}} = \frac{\sqrt[3]{4}}{\sqrt[3]{8}} = \frac{\sqrt[3]{4}}{2} = \frac{1}{2}\sqrt[3]{4}$$

62 $\sqrt{\dfrac{1}{5}} = \dfrac{\sqrt{1}}{\sqrt{5}} = \dfrac{1}{\sqrt{5}} \cdot \dfrac{\sqrt{5}}{\sqrt{5}} = \dfrac{1}{5}\sqrt{5}$

63 $\sqrt{9x^{-4}y^6} = \left(9x^{-4}y^6\right)^{1/2} = 9^{1/2}\left(x^{-4}\right)^{1/2}\left(y^6\right)^{1/2} = 3x^{-2}y^3 = \dfrac{3y^3}{x^2}$

64 $\sqrt{16a^8b^{-2}} = 4a^4b^{-1} = \dfrac{4a^4}{b}$

65 $\sqrt[3]{8a^6b^{-3}} = 2a^2b^{-1} = \dfrac{2a^2}{b}$

66 $\sqrt[4]{81r^5s^8} = \sqrt[4]{3^4r^4s^8}\,\sqrt[4]{r} = 3rs^2\sqrt[4]{r}$

Note: For exercises similar to numbers 67–74, pick a multiplier that will make all of the exponents of the terms in the denominator a multiple of the index.

67 The index is 2. Choose the multiplier to be $\sqrt{2y}$ so that the denominator contains only terms with even exponents.
$\sqrt{\dfrac{3x}{2y^3}} = \sqrt{\dfrac{3x}{2y^3}} \cdot \dfrac{\sqrt{2y}}{\sqrt{2y}} = \dfrac{\sqrt{6xy}}{\sqrt{4y^4}} = \dfrac{\sqrt{6xy}}{2y^2}$, or $\dfrac{1}{2y^2}\sqrt{6xy}$

68 $\sqrt{\dfrac{1}{3x^3y}} = \sqrt{\dfrac{1}{3x^3y}} \cdot \dfrac{\sqrt{3xy}}{\sqrt{3xy}} = \dfrac{\sqrt{3xy}}{\sqrt{9x^4y^2}} = \dfrac{1}{3x^2y}\sqrt{3xy}$

69 The index is 3. Choose the multiplier to be $\sqrt[3]{3x^2}$ so that the denominator contains only terms with exponents that are multiples of 3.

$$\sqrt[3]{\frac{2x^4y^4}{9x}} = \sqrt[3]{\frac{2x^4y^4}{9x}} \cdot \frac{\sqrt[3]{3x^2}}{\sqrt[3]{3x^2}} = \frac{\sqrt[3]{6x^6y^4}}{\sqrt[3]{27x^3}} = \frac{\sqrt[3]{x^6y^3}\,\sqrt[3]{6y}}{3x} = \frac{x^2y\,\sqrt[3]{6y}}{3x} = \frac{xy}{3}\sqrt[3]{6y}$$

70 $\sqrt[3]{\dfrac{3x^2y^5}{4x}} = \sqrt[3]{\dfrac{3x^2y^5}{4x}} \cdot \dfrac{\sqrt[3]{2x^2}}{\sqrt[3]{2x^2}} = \dfrac{\sqrt[3]{6x^4y^5}}{\sqrt[3]{8x^3}} = \dfrac{\sqrt[3]{x^3y^3}\,\sqrt[3]{6xy^2}}{2x} = \dfrac{xy\,\sqrt[3]{6xy^2}}{2x} = \dfrac{y}{2}\sqrt[3]{6xy^2}$

71 The index is 4. Choose the multiplier to be $\sqrt[4]{3x^2}$ so that the denominator contains only terms with exponents that are multiples of 4.

$$\sqrt[4]{\frac{5x^8y^3}{27x^2}} = \sqrt[4]{\frac{5x^8y^3}{27x^2}} \cdot \frac{\sqrt[4]{3x^2}}{\sqrt[4]{3x^2}} = \frac{\sqrt[4]{15x^{10}y^3}}{\sqrt[4]{81x^4}} = \frac{\sqrt[4]{x^8}\,\sqrt[4]{15x^2y^3}}{3x} = \frac{x^2\,\sqrt[4]{15x^2y^3}}{3x} = \frac{x}{3}\sqrt[4]{15x^2y^3}$$

72 $\sqrt[4]{\dfrac{x^7y^{12}}{125x}} = \sqrt[4]{\dfrac{x^7y^{12}}{125x}} \cdot \dfrac{\sqrt[4]{5x^3}}{\sqrt[4]{5x^3}} = \dfrac{\sqrt[4]{5x^{10}y^{12}}}{\sqrt[4]{625x^4}} = \dfrac{\sqrt[4]{x^8y^{12}}\,\sqrt[4]{5x^2}}{5x} = \dfrac{x^2y^3\,\sqrt[4]{5x^2}}{5x} = \dfrac{xy^3}{5}\sqrt[4]{5x^2}$

73 The index is 5. Choose the multiplier to be $\sqrt[5]{4x^2}$ so that the denominator contains only terms with exponents that are multiples of 5.

$$\sqrt[5]{\frac{5x^7y^2}{8x^3}} = \sqrt[5]{\frac{5x^7y^2}{8x^3}} \cdot \frac{\sqrt[5]{4x^2}}{\sqrt[5]{4x^2}} = \frac{\sqrt[5]{20x^9y^2}}{\sqrt[5]{32x^5}} = \frac{\sqrt[5]{x^5}\,\sqrt[5]{20x^4y^2}}{2x} = \frac{x\,\sqrt[5]{20x^4y^2}}{2x} = \frac{1}{2}\sqrt[5]{20x^4y^2}$$

74 $\sqrt[5]{\dfrac{3x^{11}y^3}{9x^2}} = \sqrt[5]{\dfrac{3x^{11}y^3}{9x^2}} \cdot \dfrac{\sqrt[5]{27x^3}}{\sqrt[5]{27x^3}} = \dfrac{\sqrt[5]{81x^{14}y^3}}{\sqrt[5]{243x^5}} = \dfrac{\sqrt[5]{x^{10}}\,\sqrt[5]{81x^4y^3}}{3x} = \dfrac{x^2\,\sqrt[5]{81x^4y^3}}{3x} = \dfrac{x}{3}\sqrt[5]{81x^4y^3}$

75 $\sqrt[4]{\left(5x^5y^{-2}\right)^4} = 5x^5y^{-2} = \dfrac{5x^5}{y^2}$

76 $\sqrt[6]{\left(7u^{-3}v^4\right)^6} = 7u^{-3}v^4 = \dfrac{7v^4}{u^3}$

77 $\sqrt[5]{\dfrac{8x^3}{y^4}}\sqrt[5]{\dfrac{4x^4}{y^2}} = \sqrt[5]{\dfrac{8x^3}{y^4}}\sqrt[5]{\dfrac{4x^4}{y^2}} \cdot \dfrac{\sqrt[5]{y^4}}{\sqrt[5]{y^4}} = \dfrac{\sqrt[5]{32x^5}\,\sqrt[5]{x^2y^4}}{\sqrt[5]{y^{10}}} = \dfrac{\sqrt[5]{32x^5}\,\sqrt[5]{x^2y^4}}{y^2} = \dfrac{2x}{y^2}\sqrt[5]{x^2y^4}$

78 $\sqrt{5xy^7}\,\sqrt{15x^3y^3} = \sqrt{25x^4y^{10}}\,\sqrt{3} = 5x^2y^5\sqrt{3}$

79 $\sqrt[3]{3t^4v^2}\sqrt[3]{-9t^{-1}v^4} = \sqrt[3]{-27t^3v^6} = -3tv^2$

80 $\sqrt[3]{(2r-s)^3} = 2r - s$

81 $\sqrt{x^6y^4} = \sqrt{(x^3)^2(y^2)^2} = \sqrt{(x^3)^2}\sqrt{(y^2)^2} = |x^3||y^2| = |x^3|y^2$ since y^2 is always nonnegative.

Note: $|x^3|$ could be written as $x^2|x|$.

82 $\sqrt{x^4y^{10}} = \sqrt{(x^2)^2(y^5)^2} = |x^2||y^5| = x^2|y^5|$

83 $\sqrt[4]{x^8(y-3)^{12}} = \sqrt[4]{(x^2)^4\left((y-3)^3\right)^4} = |x^2|\left|(y-3)^3\right| = x^2\left|(y-3)^3\right|$, or $x^2(y-3)^2|(y-3)|$

84 $\sqrt[4]{(x+2)^{12}y^4} = \sqrt[4]{\left((x+2)^3\right)^4y^4} = \left|(x+2)^3\right||y|$, or $(x+2)^2|(x+2)y|$

85 $(a^r)^2 = a^{2r}\ \boxed{\neq}\ a^{(r^2)}$ since $2r \neq r^2$ for all values of r; for example, let $r = 1$.

86 Squaring the right side gives us $(a+1)^2 = a^2 + 2a + 1$. Squaring the left side gives us $a^2 + 1$.
$a^2 + 2a + 1\ \boxed{\neq}\ a^2 + 1$ for all values of a; for example, let $a = 1$.

87 $(ab)^{xy} = a^{xy}b^{xy}\ \boxed{\neq}\ a^xb^y$ for all values of x and y; for example, let $x = 1$ and $y = 2$.

88 $\sqrt{a^r} = (a^r)^{1/2} = \left(a^{1/2}\right)^r\ \boxed{=}\ \left(\sqrt{a}\right)^r$

89 $\sqrt[n]{\dfrac{1}{c}} = \left(\dfrac{1}{c}\right)^{1/n} = \dfrac{1^{1/n}}{c^{1/n}}\ \boxed{=}\ \dfrac{1}{\sqrt[n]{c}}$

90 $\dfrac{1}{a^k} = a^{-k}\ \boxed{\neq}\ a^{1/k}$ since $-k \neq 1/k$ for all values of k; for example, let $k = 1$.

91 (a) $(-3)^{2/5} = \left[(-3)^2\right]^{1/5} = 9^{1/5} \approx 1.5518$ (b) $(-7)^{4/3} = \left[(-7)^4\right]^{1/3} = 2401^{1/3} \approx 13.3905$

92 (a) $(-1.2)^{3/7} = \left[(-1.2)^3\right]^{1/7} = (-1.728)^{1/7} \approx -1.0813$

(b) $(-5.08)^{7/3} = \left[(-5.08)^7\right]^{1/3} \approx (-87{,}306.38)^{1/3} \approx -44.3624$

93 (a) $\sqrt{\pi + 1} \approx 2.0351$ (b) $\sqrt[3]{17.1} + 5^{1/4} \approx 4.0717$

94 (a) $(2.6 - 1.3)^{-2} \approx 0.5917$ (b) $5^{\sqrt{7}} \approx 70.6807$

95 $\$200(1.04)^{180} \approx \$232{,}825.78$

96 $h = 1454$ ft $\Rightarrow$ $d = 1.2\sqrt{h} = 1.2\sqrt{1454} \approx 45.8$ mi

97 $W = 230$ kg $\Rightarrow$ $L = 0.46\sqrt[3]{W} = 0.46\sqrt[3]{230} \approx 2.82$ m

98 $L = 25$ ft $\Rightarrow$ $W = 0.0016L^{2.43} = 0.0016(25)^{2.43} \approx 3.99$ tons

99 $b = 75$ and $w = 180$ $\Rightarrow$ $W = \dfrac{w}{\sqrt[3]{b-35}} = \dfrac{180}{\sqrt[3]{75-35}} \approx 52.6.$

$b = 120$ and $w = 250$ $\Rightarrow$ $W = \dfrac{w}{\sqrt[3]{b-35}} = \dfrac{250}{\sqrt[3]{120-35}} \approx 56.9.$

It is interesting to note that the 75-kg lifter can lift 2.4 times his/her body weight and the 120-kg lifter can lift approximately 2.08 times his/her body weight, but the formula ranks the 120-kg lifter as the superior lifter.

100 (a) $h = 72$ in. and $w = 175$ lb $\Rightarrow$ $S = (0.1091)w^{0.425}h^{0.725} = (0.1091)(175)^{0.425}(72)^{0.725} \approx 21.76\ \text{ft}^2$.

(b) $h = 66$ in. $\Rightarrow$ $S_1 = (0.1091)w^{0.425}(66)^{0.725}$. A 10% increase in weight would be represented by $1.1w$ and thus $S_2 = (0.1091)(1.1w)^{0.425}(66)^{0.725}$. $S_2/S_1 = (1.1)^{0.425} \approx 1.04$, which represents a 4% increase in S.

101 $W = 0.1166h^{1.7}$

Height	64	65	66	67	68	69	70	71
Weight	137	141	145	148	152	156	160	164
Height	72	73	74	75	76	77	78	79
Weight	168	172	176	180	184	188	192	196

102 $W = 0.1049h^{1.7}$

Height	60	61	62	63	64	65	66	67
Weight	111	114	117	120	123	127	130	133
Height	68	69	70	71	72	73	74	75
Weight	137	140	144	147	151	154	158	162

1.3 Exercises

1 $(3x^3 + 4x^2 - 7x + 1) + (9x^3 - 4x^2 - 6x) = 12x^3 - 13x + 1$

2 $(7x^3 + 2x^2 - 11x) + (-3x^3 - 2x^2 + 4x - 3) = 4x^3 - 7x - 3$

3 $$\begin{aligned}\left(4x^3 + 5x - 3\right) - \left(3x^3 + 2x^2 + 5x - 8\right) &= 4x^3 + 5x - 3 - 3x^3 - 2x^2 - 5x + 8 \\ &= \left(4x^3 - 3x^3\right) - 2x^2 + (5x - 5x) + (-3 + 8) \\ &= x^3 - 2x^2 + 5\end{aligned}$$

4 $(6x^3 - 2x^2 + x - 3) - (8x^2 - x - 3) = 6x^3 - 2x^2 + x - 3 - 8x^2 + x + 3 = 6x^3 - 10x^2 + 2x$

5 $$\begin{aligned}(2x + 5)(3x - 7) &= (2x)(3x) + (2x)(-7) + (5)(3x) + (5)(-7) \\ &= 6x^2 - 14x + 15x - 35 \\ &= 6x^2 + x - 35\end{aligned}$$

6 $(3x - 4)(2x + 9) = (3x)(2x) + (3x)(9) + (-4)(2x) + (-4)(9) = 6x^2 + 27x - 8x - 36 = 6x^2 + 19x - 36$

7 $$\begin{aligned}(5x + 4y)(3x + 2y) &= (5x)(3x) + (5x)(2y) + (4y)(3x) + (4y)(2y) \\ &= 15x^2 + 10xy + 12xy + 8y^2 = 15x^2 + 22xy + 8y^2\end{aligned}$$

8 $$\begin{aligned}(4x - 3y)(x - 5y) &= (4x)(x) + (4x)(-5y) + (-3y)(x) + (-3y)(-5y) \\ &= 4x^2 - 20xy - 3xy + 15y^2 = 4x^2 - 23xy + 15y^2\end{aligned}$$

9 $(2u + 3)(u - 4) + 4u(u - 2) = (2u^2 - 5u - 12) + (4u^2 - 8u) = 6u^2 - 13u - 12$

10 $(3u - 1)(u + 2) + 7u(u + 1) = (3u^2 + 5u - 2) + (7u^2 + 7u) = 10u^2 + 12u - 2$

11 $$\begin{aligned}(3x + 5)\left(2x^2 + 9x - 5\right) &= 3x\left(2x^2 + 9x - 5\right) + 5\left(2x^2 + 9x - 5\right) \\ &= \left(6x^3 + 27x^2 - 15x\right) + \left(10x^2 + 45x - 25\right) \\ &= 6x^3 + 37x^2 + 30x - 25\end{aligned}$$

12 $$\begin{aligned}(7x - 4)\left(x^3 - x^2 + 6\right) &= 7x\left(x^3 - x^2 + 6\right) + (-4)\left(x^3 - x^2 + 6\right) \\ &= \left(7x^4 - 7x^3 + 42x\right) + \left(-4x^3 + 4x^2 - 24\right) \\ &= 7x^4 - 11x^3 + 4x^2 + 42x - 24\end{aligned}$$

13 $$\begin{aligned}\left(t^2 + 2t - 5\right)\left(3t^2 - t + 2\right) &= t^2\left(3t^2 - t + 2\right) + 2t\left(3t^2 - t + 2\right) + (-5)\left(3t^2 - t + 2\right) \\ &= \left(3t^4 - t^3 + 2t^2\right) + \left(6t^3 - 2t^2 + 4t\right) + \left(-15t^2 + 5t - 10\right) \\ &= 3t^4 + 5t^3 - 15t^2 + 9t - 10\end{aligned}$$

14 $(r^2-8r-2)(-r^2+3r-5) = r^2(-r^2+3r-5)+(-8r)(-r^2+3r-5)+(-2)(-r^2+3r-5)$
$= (-r^4+3r^3-5r^2)+(8r^3-24r^2+40r)+(2r^2-6r+10)$
$= -r^4+11r^3-27r^2+34r+10$

15 $(x+1)(2x^2-2)(x^3+5) = 2\left[(x+1)(x^2-1)\right](x^3+5)$
$= 2(x^3+x^2-x-1)(x^3+5)$
$= 2(x^6+x^5-x^4+4x^3+5x^2-5x-5)$
$= 2x^6+2x^5-2x^4+8x^3+10x^2-10x-10$

16 $(2x-1)(x^2-5)(x^3-1) = (2x^3-x^2-10x+5)(x^3-1) = 2x^6-x^5-10x^4+3x^3+x^2+10x-5$

17 $\dfrac{8x^2y^3-6x^3y}{2x^2y} = \dfrac{8x^2y^3}{2x^2y} - \dfrac{6x^3y}{2x^2y} = 4y^2-3x$

18 $\dfrac{6a^3b^3-9a^2b^2\ +\ 3ab^4}{3ab^2} = \dfrac{6a^3b^3}{3ab^2} - \dfrac{9a^2b^2}{3ab^2} + \dfrac{3ab^4}{3ab^2} = 2a^2b-3a+b^2$

19 $\dfrac{3u^3v^4-2u^5v^2\ +\ (u^2v^2)^2}{u^3v^2} = \dfrac{3u^3v^4}{u^3v^2} - \dfrac{2u^5v^2}{u^3v^2} + \dfrac{u^4v^4}{u^3v^2} = 3v^2-2u^2+uv^2$

20 $\dfrac{6x^2yz^3-xy^2z}{xyz} = \dfrac{6x^2yz^3}{xyz} - \dfrac{xy^2z}{xyz} = 6xz^2-y$

21 We recognize this product as the difference of two squares.

$$(2x+7y)(2x-7y) = (2x)^2-(7y)^2 = 4x^2-49y^2$$

22 $(5x+3y)(5x-3y) = (5x)^2-(3y)^2 = 25x^2-9y^2$

23 $(x^2+5y)(x^2-5y) = (x^2)^2-(5y)^2 = x^4-25y^2$

24 $(3x+y^3)(3x-y^3) = (3x)^2-(y^3)^2 = 9x^2-y^6$

25 $(x^2+9)(x^2-4) = x^4-4x^2+9x^2-36 = x^4+5x^2-36$

26 $(x^2+1)(x^2-8) = x^4-8x^2+x^2-8 = x^4-7x^2-8$

27 $(3x+2y)^2 = (3x)^2+2(3x)(2y)+(2y)^2 = 9x^2+12xy+4y^2$

28 $(5x-4y)^2 = (5x)^2-2(5x)(4y)+(4y)^2 = 25x^2-40xy+16y^2$

29 $(x^2-5y^2)^2 = (x^2)^2-2(x^2)(5y^2)+(5y^2)^2 = x^4-10x^2y^2+25y^4$

30 $(2x^2+5y^2)^2 = (2x^2)^2+2(2x^2)(5y^2)+(5y^2)^2 = 4x^4+20x^2y^2+25y^4$

31 We could expand $(x+2)^2$ and $(x-2)^2$ and then multiply the resulting expressions, but the following solution is simpler.

$$\begin{aligned}(x+2)^2(x-2)^2 &= [(x+2)(x-2)]^2 && \{a^2b^2=(ab)^2\}\\ &= (x^2-4^2)^2 && \{\text{difference of two squares}\}\\ &= (x^2)^2-2(x^2)(4)+(4)^2 && \{\text{square of a binomial}\}\\ &= x^4-8x^2+16 && \{\text{simplify}\}\end{aligned}$$

32 $(x+y)^2(x-y)^2 = [(x+y)(x-y)]^2 = (x^2-y^2)^2 = (x^2)^2-2(x^2)(y^2)+(y^2)^2 = x^4-2x^2y^2+y^4$

33 $(\sqrt{x}+\sqrt{y})(\sqrt{x}-\sqrt{y}) = (\sqrt{x})^2-(\sqrt{y})^2 = x-y$

34 $(\sqrt{x}+\sqrt{y})^2(\sqrt{x}-\sqrt{y})^2 = [(\sqrt{x}+\sqrt{y})(\sqrt{x}-\sqrt{y})]^2 = (x-y)^2 = x^2-2xy+y^2$

35 $$\begin{aligned}(x^{1/3}-2y^{1/3})(x^{2/3}+2x^{1/3}y^{1/3}+4y^{2/3}) &= x^{1/3}(x^{2/3}+2x^{1/3}y^{1/3}+4y^{2/3}) - 2y^{1/3}(x^{2/3}+2x^{1/3}y^{1/3}+4y^{2/3})\\ &= x+2x^{2/3}y^{1/3}+4x^{1/3}y^{2/3}-2x^{2/3}y^{1/3}-4x^{1/3}y^{2/3}-8y = x-8y\end{aligned}$$

This exercise illustrates how the difference of any two terms can be factored as the difference of cubes. Another example of this concept is

$$x-5 = \left(\sqrt[3]{x}-\sqrt[3]{5}\right)\left(\sqrt[3]{x^2}+\sqrt[3]{5x}+\sqrt[3]{25}\right)$$

36 $$\begin{aligned}(x^{1/3}+y^{1/3})(x^{2/3}-x^{1/3}y^{1/3}+y^{2/3}) &= x^{1/3}(x^{2/3}-x^{1/3}y^{1/3}+y^{2/3}) + y^{1/3}(x^{2/3}-x^{1/3}y^{1/3}+y^{2/3})\\ &= x-x^{2/3}y^{1/3}+x^{1/3}y^{2/3}+x^{2/3}y^{1/3}-x^{1/3}y^{2/3}+y = x+y\end{aligned}$$

37 Use Product Formula (3) on page 32 of the text.

$$(x-2y)^3 = (x)^3-3(x)^2(2y)+3(x)(2y)^2-(2y)^3 = x^3-6x^2y+12xy^2-8y^3$$

38 $(x+3y)^3 = (x)^3+3(x)^2(3y)+3(x)(3y)^2+(3y)^3 = x^3+9x^2y+27xy^2+27y^3$

39 $$\begin{aligned}(2x+3y)^3 &= (2x)^3+3(2x)^2(3y)+3(2x)(3y)^2+(3y)^3\\ &= 8x^3+3(4x^2)(3y)+3(2x)(9y^2)+27y^3 = 8x^3+36x^2y+54xy^2+27y^3\end{aligned}$$

40 $(3x-4y)^3 = (3x)^3-3(3x)^2(4y)+3(3x)(4y)^2-(4y)^3 = 27x^3-108x^2y+144xy^2-64y^3$

Note: Treat Exercises 41–44 as "the sum of the squares plus twice the product of all possible pairs of terms," that is,

$$(x+y+z)^2 = x^2+y^2+z^2+2xy+2xz+2yz.$$

41 $(a+b-c)^2 = [a\ +\ b+(-c)]^2 = a^2+b^2+c^2+2ab-2ac-2bc$

42 $(x^2+x+1)^2 = (x^2)^2+x^2+1^2+2x^2(x)+2x^2(1)+2x(1) = x^4+2x^3+3x^2+2x+1$

43 $$\begin{aligned}(y^2-y+2)^2 &= (y^2)^2+(-y)^2+(2)^2+2(y^2)(-y)+2(y^2)(2)+2(-y)(2)\\ &= y^4+y^2+4-2y^3+4y^2-4y\\ &= y^4-2y^3+5y^2-4y+4\end{aligned}$$

44 $$\begin{aligned}(x-2y+3z)^2 &= (x)^2+(-2y)^2+(3z)^2+2(x)(-2y)+2(x)(3z)+2(-2y)(3z)\\ &= x^2+4y^2+9z^2-4xy+6xz-12yz\end{aligned}$$

45 Always factor out the greatest common factor {**gcf**} first. $rs+4st =$ {gcf is s} $s(r+4t)$

46 $4u^2-2uv =$ {gcf is $2u$} $2u(2u-v)$

47 $3a^2b^2-6a^2b =$ {gcf is $3a^2b$} $3a^2b(b-2)$

48 $12xy+18xy^2 =$ {gcf is $6xy$} $6xy(2+3y)$

49 $3x^2y^3-9x^3y^2 =$ {gcf is $3x^2y^2$} $3x^2y^2(y-3x)$

50 $16x^5y^2+8x^3y^3 =$ {gcf is $8x^3y^2$} $8x^3y^2(2x^2+y)$

51 $15x^3y^5-25x^4y^2+10x^6y^4 =$ {gcf is $5x^3y^2$} $5x^3y^2(3y^3-5x+2x^3y^2)$

52 $121r^3s^4+77r^2s^4-55r^4s^3 =$ {gcf is $11r^2s^3$} $11r^2s^3(11rs+7s-5r^2)$

53 We recognize $8x^2 - 17x - 21$ as a trinomial that may be able to be factored into the product of two binomials. Using trial and error, we obtain $8x^2 - 17x - 21 = (8x + 7)(x - 3)$. If you are interested in a sure-fire method for factoring trinomials, see Example 10 on page 81 of the text.

54 Using trial and error, we obtain $7x^2 + 10x - 8 = (7x - 4)(x + 2)$.

55 The factors for $x^2 + 4x + 5$ would have to be of the form $(x + \_\_)$ and $(x + \_\_)$.
The factors of 5 are 1 and 5, but their sum is 6 (*not* 4). Thus, $x^2 + 4x + 5$ is irreducible.

56 $3x^2 - 4x + 2$ is irreducible.

57 $6x^2 + 7x - 20 = (3x - 4)(2x + 5)$

58 $12x^2 - x - 6 = (3x + 2)(4x - 3)$

59 $12x^2 - 29x + 15 = (3x - 5)(4x - 3)$

60 $21x^2 + 41x + 10 = (3x + 5)(7x + 2)$

61 $36x^2 - 60x + 25 = (6x - 5)(6x - 5) = (6x - 5)^2$

62 $9x^2 + 24x + 16 = (3x + 4)(3x + 4) = (3x + 4)^2$

63 $25z^2 + 30z + 9 = (5z + 3)(5z + 3) = (5z + 3)^2$

64 $16z^2 - 56z + 49 = (4z - 7)(4z - 7) = (4z - 7)^2$

65 $45x^2 + 38xy + 8y^2 = (5x + 2y)(9x + 4y)$

66 $50x^2 + 45xy - 18y^2 = (5x + 6y)(10x - 3y)$

67 $64r^2 - 25t^2 = (8r)^2 - (5t)^2 = (8r + 5t)(8r - 5t)$

68 $81r^2 - 16t^2 = (9r)^2 - (4t)^2 = (9r + 4t)(9r - 4t)$

69 $z^4 - 64w^2 = (z^2)^2 - (8w)^2 = (z^2 + 8w)(z^2 - 8w)$

70 $9y^4 - 121x^2 = (3y^2)^2 - (11x)^2 = (3y^2 + 11x)(3y^2 - 11x)$

71 $x^4 - 4x^2 = x^2(x^2 - 4) = x^2(x^2 - 2^2) = x^2(x + 2)(x - 2)$

72 $x^3 - 16x = x(x^2 - 16) = x(x^2 - 4^2) = x(x + 4)(x - 4)$

73 $x^2 + 169$ is irreducible.
Note: A common mistake is to confuse the sum of two squares with the difference of two squares.

74 $4x^2 + 9$ is irreducible.

75 $75x^2 - 48y^2 = 3(25x^2 - 16y^2) = 3\left[(5x)^2 - (4y)^2\right] = 3(5x + 4y)(5x - 4y)$

76 $64x^2 - 36y^2 = 4(16x^2 - 9y^2) = 4\left[(4x)^2 - (3y)^2\right] = 4(4x + 3y)(4x - 3y)$

77 We recognize $64x^3 + 27$ as the sum of two cubes.

$$\begin{aligned} 64x^3 + 27 = (4x)^3 + (3)^3 &= (4x + 3)\left[(4x)^2 - (4x)(3) + (3)^2\right] \\ &= (4x + 3)\left(16x^2 - 12x + 9\right) \end{aligned}$$

78 $125x^3 - 8 = (5x)^3 - (2)^3 = (5x - 2)\left[(5x)^2 + (5x)(2) + (2)^2\right] = (5x - 2)(25x^2 + 10x + 4)$

79 We recognize $8x^3 - y^6$ as the difference of two cubes.

$$\begin{aligned} 8x^3 - y^6 = (2x)^3 - \left(y^2\right)^3 &= \left(2x - y^2\right)\left[(2x)^2 + (2x)\left(y^2\right) + \left(y^2\right)^2\right] \\ &= \left(2x - y^2\right)\left(4x^2 + 2xy^2 + y^4\right) \end{aligned}$$

80 $$\begin{aligned} 216x^9 + 125y^3 = \left(6x^3\right)^3 + (5y)^3 &= \left(6x^3 + 5y\right)\left[\left(6x^3\right)^2 - \left(6x^3\right)(5y) + (5y)^2\right] \\ &= \left(6x^3 + 5y\right)\left(36x^6 - 30x^3y + 25y^2\right) \end{aligned}$$

81 We recognize $343x^3 + y^9$ as the sum of two cubes.

$$343x^3 + y^9 = (7x)^3 + \left(y^3\right)^3 = \left(7x + y^3\right)\left[(7x)^2 - (7x)\left(y^3\right) + \left(y^3\right)^2\right]$$
$$= \left(7x + y^3\right)\left(49x^2 - 7xy^3 + y^6\right)$$

82 $x^6 - 27y^3 = \left(x^2\right)^3 - (3y)^3 = \left(x^2 - 3y\right)\left[\left(x^2\right)^2 + \left(x^2\right)(3y) + (3y)^2\right] = \left(x^2 - 3y\right)\left(x^4 + 3x^2y + 9y^2\right)$

83 We recognize $125 - 27x^3$ as the difference of two cubes.

$$125 - 27x^3 = (5)^3 - (3x)^3 = (5 - 3x)\left[(5)^2 + (5)(3x) + (3x)^2\right]$$
$$= (5 - 3x)\left(25 + 15x + 9x^2\right)$$

84 $x^3 + 64 = (x)^3 + (4)^3 = (x + 4)\left[(x)^2 - (x)(4) + (4)^2\right] = (x + 4)(x^2 - 4x + 16)$

85 Since there are more than 3 terms, we will try to factor by grouping first.

$$2ax - 6bx + ay - 3by = 2x(a - 3b) + y(a - 3b)$$
$$= (2x + y)(a - 3b) \qquad \{\text{factor out } (a - 3b)\}$$

86 $2by^2 - bxy + 6xy - 3x^2 = by(2y - x) + 3x(2y - x) = (by + 3x)(2y - x)$

87 $3x^3 + 3x^2 - 27x - 27 = 3\left(x^3 + x^2 - 9x - 9\right)$ {gcf = 3}

$= 3\left[x^2(x + 1) - 9(x + 1)\right]$ {factor by grouping}

$= 3\left(x^2 - 9\right)(x + 1)$ {factor out $(x + 1)$}

$= 3(x + 3)(x - 3)(x + 1)$ {difference of two squares}

88 $5x^3 + 10x^2 - 20x - 40 = 5\left(x^3 + 2x^2 - 4x - 8\right) = 5\left[x^2(x + 2) - 4(x + 2)\right]$

$= 5\left(x^2 - 4\right)(x + 2) = 5(x + 2)(x - 2)(x + 2) = 5(x + 2)^2(x - 2)$

89 Since there are more than 3 terms, we will try to factor by grouping first.

$$x^4 + 2x^3 - x - 2 = x^3(x + 2) - 1(x + 2) = \left(x^3 - 1\right)(x + 2)$$

Now recognize $x^3 - 1$ as the difference of two cubes.

$$\left(x^3 - 1\right)(x + 2) = \left[(x - 1)\left(x^2 + x + 1\right)\right](x + 2) = (x - 1)(x + 2)\left(x^2 + x + 1\right)$$

90 $x^4 - 3x^3 + 8x - 24 = x^3(x - 3) + 8(x - 3) = (x^3 + 8)(x - 3) = (x + 2)(x - 3)(x^2 - 2x + 4)$

91 $a^3 - a^2b + ab^2 - b^3 = a^2(a - b) + b^2(a - b) = (a^2 + b^2)(a - b)$

92 $6w^8 + 17w^4 + 12 = \left(2w^4 + 3\right)\left(3w^4 + 4\right)$

93 We could treat $a^6 - b^6$ as the difference of two squares or the difference of two cubes. Factoring $a^6 - b^6$ as the difference of two squares and then factoring as the sum and difference of two cubes leads to the following:

$$a^6 - b^6 = \left(a^3\right)^2 - \left(b^3\right)^2 = \left(a^3 + b^3\right)\left(a^3 - b^3\right)$$
$$= (a + b)(a - b)\left(a^2 - ab + b^2\right)\left(a^2 + ab + b^2\right)$$

94 $x^8 - 16 = \left(x^4\right)^2 - 4^2 = \left(x^4 + 4\right)\left(x^4 - 4\right) = \left(x^4 + 4\right)(x^2 + 2)(x^2 - 2)$

95 We might first try to factor $x^2 + 4x + 4 - 9y^2$ by grouping since it has more than 3 terms, but this would prove to be unsuccessful. Instead, we will group the terms containing x and the constant term together, and then proceed as in Example 10(c).

$$x^2 + 4x + 4 - 9y^2 = (x+2)^2 - (3y)^2 = (x+2+3y)(x+2-3y)$$

96 $x^2 - 4y^2 - 6x + 9 = (x^2 - 6x + 9) - 4y^2 = (x-3)^2 - (2y)^2 = (x-3+2y)(x-3-2y)$

97 We will group the terms containing y and the constant term together, and then proceed as in Example 10(c).

$$\begin{aligned} y^2 - x^2 + 8y + 16 &= \left(y^2 + 8y + 16\right) - x^2 \\ &= (y+4)^2 - (x)^2 \\ &= (y+4+x)(y+4-x) \end{aligned}$$

98 $y^2 + 9 - 6y - 4x^2 = (y^2 - 6y + 9) - 4x^2 = (y-3)^2 - (2x)^2 = (y-3+2x)(y-3-2x)$

99 We should first note that one of the two variable terms, y^6, is the square of the other, y^3. Thus, we may treat this expression as a simple trinomial that can be factored into the product of two binomials.

$$y^6 + 7y^3 - 8 = \left(y^3+8\right)\left(y^3-1\right) = (y+2)\left(y^2-2y+4\right)(y-1)\left(y^2+y+1\right)$$

100 $8c^6 + 19c^3 - 27 = (8c^3+27)(c^3-1) = (2c+3)(4c^2-6c+9)(c-1)(c^2+c+1)$

101 $$\begin{aligned} x^{16} - 1 = \left(x^8+1\right)\left(x^8-1\right) &= \left(x^8+1\right)\left(x^4+1\right)\left(x^4-1\right) \\ &= \left(x^8+1\right)\left(x^4+1\right)\left(x^2+1\right)\left(x^2-1\right) \\ &= \left(x^8+1\right)\left(x^4+1\right)\left(x^2+1\right)(x+1)(x-1) \end{aligned}$$

102 $4x^3 + 4x^2 + x = x(4x^2+4x+1) = x(2x+1)(2x+1) = x(2x+1)^2$

103 In the second figure, the dimensions of area I are (x) and $(x-y)$. The area of I is $(x-y)x$, and the area of II is $(x-y)y$. The area $A = \underline{x^2 - y^2}$ {in the first figure}

$= (x-y)x + (x-y)y$ {in the second figure}

$= \underline{(x-y)(x+y)}.$ {in the third figure}

104 Volume of I is $x^2(x-y)$, volume of II is $xy(x-y)$, and volume of III is $y^2(x-y)$.

$$V = \underline{x^3 - y^3} = x^2(x-y) + xy(x-y) + y^2(x-y) = \underline{(x-y)(x^2+xy+y^2)}.$$

105 (a) For the 25-year-old female, use

$C_f = 66.5 + 13.8w + 5h - 6.8y$ with $w = 59$, $h = 163$, and $y = 25$.

$C_f = 66.5 + 13.8(59) + 5(163) - 6.8(25) = 1525.7$ calories

For the 55-year-old male, use

$C_m = 655 + 9.6w + 1.9h - 4.7y$ with $w = 75$, $h = 178$, and $y = 55$.

$C_m = 655 + 9.6(75) + 1.9(178) - 4.7(55) = 1454.7$ calories

(b) As people age they require fewer calories. The coefficients of w and h are positive because large people require more calories.

1.4 Exercises

1 $$\frac{3}{50} + \frac{7}{30} = \frac{3}{2\cdot5^2} + \frac{7}{2\cdot3\cdot5} = \frac{3\cdot3 + 7\cdot5}{2\cdot3\cdot5^2} = \frac{9+35}{2\cdot3\cdot5^2} = \frac{44}{2\cdot3\cdot5^2} = \frac{22}{3\cdot5^2} = \frac{22}{75}$$

2 $\dfrac{8}{63}+\dfrac{5}{42}=\dfrac{8}{3^2\cdot 7}+\dfrac{5}{2\cdot 3\cdot 7}=\dfrac{8\cdot 2+5\cdot 3}{2\cdot 3^2\cdot 7}=\dfrac{16+15}{2\cdot 3^2\cdot 7}=\dfrac{31}{2\cdot 3^2\cdot 7}=\dfrac{31}{126}$

3 $\dfrac{5}{24}-\dfrac{3}{20}=\dfrac{5}{2^3\cdot 3}-\dfrac{3}{2^2\cdot 5}=\dfrac{5\cdot 5-3(2\cdot 3)}{2^3\cdot 3\cdot 5}=\dfrac{25-18}{2^3\cdot 3\cdot 5}=\dfrac{7}{2^3\cdot 3\cdot 5}=\dfrac{7}{120}$

4 $\dfrac{7}{54}-\dfrac{5}{72}=\dfrac{7}{2\cdot 3^3}-\dfrac{5}{2^3\cdot 3^2}=\dfrac{7\cdot 2^2-5\cdot 3}{2^3\cdot 3^3}=\dfrac{28-15}{2^3\cdot 3^3}=\dfrac{13}{2^3\cdot 3^3}=\dfrac{13}{216}$

5 $\dfrac{2x^2+7x+3}{2x^2-7x-4}=\dfrac{(2x+1)(x+3)}{(2x+1)(x-4)}=\dfrac{x+3}{x-4}$

6 $\dfrac{2x^2+7x-15}{3x^2+17x+10}=\dfrac{(x+5)(2x-3)}{(x+5)(3x+2)}=\dfrac{2x-3}{3x+2}$

7 $\dfrac{y^2-25}{y^3-125}=\dfrac{(y+5)(y-5)}{(y-5)(y^2+5y+25)}=\dfrac{y+5}{y^2+5y+25}$

8 $\dfrac{y^2-9}{y^3+27}=\dfrac{(y+3)(y-3)}{(y+3)(y^2-3y+9)}=\dfrac{y-3}{y^2-3y+9}$

9 $\dfrac{12+r-r^2}{r^3+3r^2}=\dfrac{(3+r)(4-r)}{r^2(r+3)}=\dfrac{4-r}{r^2}$

10 $\dfrac{10+3r-r^2}{r^4+2r^3}=\dfrac{(2+r)(5-r)}{r^3(r+2)}=\dfrac{5-r}{r^3}$

11 $\dfrac{9x^2-4}{3x^2-5x+2}\cdot\dfrac{9x^4-6x^3+4x^2}{27x^4+8x}=\dfrac{(3x+2)(3x-2)}{(3x-2)(x-1)}\cdot\dfrac{x^2(9x^2-6x+4)}{x(27x^3+8)}$

$$=\frac{(3x+2)(3x-2)x^2(9x^2-6x+4)}{(3x-2)(x-1)x(3x+2)(9x^2-6x+4)}=\frac{x}{x-1}$$

12 $\dfrac{4x^2-9}{2x^2+7x+6}\cdot\dfrac{4x^4+6x^3+9x^2}{8x^7-27x^4}=\dfrac{(2x+3)(2x-3)}{(2x+3)(x+2)}\cdot\dfrac{x^2(4x^2+6x+9)}{x^4(2x-3)(4x^2+6x+9)}=\dfrac{1}{x^2(x+2)}$

13 $\dfrac{5a^2+12a+4}{a^4-16}\div\dfrac{25a^2+20a+4}{a^2-2a}=\dfrac{(5a+2)(a+2)}{(a^2+4)(a+2)(a-2)}\cdot\dfrac{a(a-2)}{(5a+2)(5a+2)}=\dfrac{a}{(a^2+4)(5a+2)}$

14 $\dfrac{a^3-8}{a^2-4}\div\dfrac{a}{a^3+8}=\dfrac{(a-2)(a^2+2a+4)}{(a+2)(a-2)}\cdot\dfrac{(a+2)(a^2-2a+4)}{a}=\dfrac{(a^2+2a+4)(a^2-2a+4)}{a}$

15 $\dfrac{6}{x^2-4}-\dfrac{3x}{x^2-4}=\dfrac{6-3x}{x^2-4}=\dfrac{3(2-x)}{(x+2)(x-2)}=\dfrac{-3}{x+2}$. Since $2-x=-(x-2)$, we canceled the two factors, $2-x$ and $x-2$, and replaced them with -1. In general, you may do this whenever you encounter factors of the form $a-b$ and $b-a$ in the numerator and the denominator, respectively, of a fractional expression.

16 $\dfrac{15}{x^2-9}-\dfrac{5x}{x^2-9}=\dfrac{15-5x}{x^2-9}=\dfrac{5(3-x)}{(x+3)(x-3)}=\dfrac{-5}{x+3}$

17 $\dfrac{4}{3s+1}-\dfrac{11}{(3s+1)^2}=\dfrac{4(3s+1)}{(3s+1)^2}-\dfrac{11}{(3s+1)^2}=\dfrac{12s+4-11}{(3s+1)^2}=\dfrac{12s-7}{(3s+1)^2}$

18 $\dfrac{4}{(5s-2)^2}+\dfrac{s}{5s-2}=\dfrac{4}{(5s-2)^2}+\dfrac{s(5s-2)}{(5s-2)^2}=\dfrac{4+5s^2-2s}{(5s-2)^2}=\dfrac{5s^2-2s+4}{(5s-2)^2}$

19 $\dfrac{2}{x}+\dfrac{3x+1}{x^2}-\dfrac{x-2}{x^3}=\dfrac{2x^2}{x^3}+\dfrac{(3x+1)x}{x^3}-\dfrac{(x-2)}{x^3}=\dfrac{2x^2+3x^2+x-x+2}{x^3}=\dfrac{5x^2+2}{x^3}$

20 $\dfrac{5}{x}-\dfrac{2x-1}{x^2}+\dfrac{x+7}{x^3}=\dfrac{5x^2}{x^3}-\dfrac{(2x-1)x}{x^3}+\dfrac{x+7}{x^3}=\dfrac{5x^2-2x^2+x+x+7}{x^3}=\dfrac{3x^2+2x+7}{x^3}$

21 $\dfrac{3t}{t+2}+\dfrac{5t}{t-2}-\dfrac{40}{t^2-4}=\dfrac{3t}{t+2}+\dfrac{5t}{t-2}-\dfrac{40}{(t+2)(t-2)}$

$$=\frac{3t(t-2)}{(t+2)(t-2)}+\frac{5t(t+2)}{(t+2)(t-2)}-\frac{40}{(t+2)(t-2)}$$

$$=\frac{3t^2-6t+5t^2+10t-40}{(t+2)(t-2)}$$

$$=\frac{8t^2+4t-40}{(t+2)(t-2)}=\frac{4(2t^2+t-10)}{(t+2)(t-2)}=\frac{4(2t+5)(t-2)}{(t+2)(t-2)}=\frac{4(2t+5)}{t+2}$$

22 $\dfrac{t}{t+3}+\dfrac{4t}{t-3}-\dfrac{18}{t^2-9}=\dfrac{t(t-3)+4t(t+3)-18}{t^2-9}=\dfrac{5t^2+9t-18}{t^2-9}=\dfrac{(5t-6)(t+3)}{(t+3)(t-3)}=\dfrac{5t-6}{t-3}$

23 $\dfrac{4x}{3x-4}+\dfrac{8}{3x^2-4x}+\dfrac{2}{x}=\dfrac{4x(x)+8+2(3x-4)}{x(3x-4)}=\dfrac{4x^2+6x}{x(3x-4)}=\dfrac{2x(2x+3)}{x(3x-4)}=\dfrac{2(2x+3)}{3x-4}$

24 $\dfrac{12x}{2x+1}-\dfrac{3}{2x^2+x}+\dfrac{5}{x}=\dfrac{12x(x)-3+5(2x+1)}{x(2x+1)}=\dfrac{12x^2+10x+2}{x(2x+1)}=\dfrac{2(6x^2+5x+1)}{x(2x+1)}$

$=\dfrac{2(2x+1)(3x+1)}{x(2x+1)}=\dfrac{2(3x+1)}{x}$

25 $\dfrac{2x}{x+2}-\dfrac{8}{x^2+2x}+\dfrac{3}{x}=\dfrac{2x(x)-8+3(x+2)}{x(x+2)}=\dfrac{2x^2+3x-2}{x(x+2)}=\dfrac{(2x-1)(x+2)}{x(x+2)}=\dfrac{2x-1}{x}$

26 $\dfrac{5x}{2x+3}-\dfrac{6}{2x^2+3x}+\dfrac{2}{x}=\dfrac{5x(x)-6+2(2x+3)}{x(2x+3)}=\dfrac{5x^2+4x}{x(2x+3)}=\dfrac{x(5x+4)}{x(2x+3)}=\dfrac{5x+4}{2x+3}$

27 $\dfrac{p^4+3p^3-8p-24}{p^3-2p^2-9p+18}=\dfrac{p^3(p+3)-8(p+3)}{p^2(p-2)-9(p-2)}$ {factor by grouping}

$=\dfrac{(p^3-8)(p+3)}{(p^2-9)(p-2)}$ {factor out $p+3$, factor out $p-2$}

$=\dfrac{(p-2)(p^2+2p+4)(p+3)}{(p+3)(p-3)(p-2)}$ {difference of two cubes, difference of two squares}

$=\dfrac{p^2+2p+4}{p-3}$ {cancel $p+3$ and $p-2$}

28 $\dfrac{2ac+bc-6ad-3bd}{6ac+2ad+3bc+bd}=\dfrac{c(2a+b)-3d(2a+b)}{2a(3c+d)+b(3c+d)}=\dfrac{(c-3d)(2a+b)}{(2a+b)(3c+d)}=\dfrac{c-3d}{3c+d}$

29 $3+\dfrac{5}{u}+\dfrac{2u}{3u+1}=\dfrac{3u(3u+1)+5(3u+1)+2u(u)}{u(3u+1)}$ {common denominator}

$=\dfrac{9u^2+3u+15u+5+2u^2}{u(3u+1)}$ {multiply terms}

$=\dfrac{11u^2+18u+5}{u(3u+1)}$ {add like terms}

30 $6+\dfrac{2}{u}-\dfrac{3u}{u+5}=\dfrac{6u(u+5)+2(u+5)-3u(u)}{u(u+5)}=\dfrac{3u^2+32u+10}{u(u+5)}$

31 $\dfrac{2x+1}{x^2+4x+4}-\dfrac{6x}{x^2-4}+\dfrac{3}{x-2}=\dfrac{2x+1}{(x+2)^2}-\dfrac{6x}{(x+2)(x-2)}+\dfrac{3}{x-2}$

$=\dfrac{(2x+1)(x-2)-6x(x+2)+3(x^2+4x+4)}{(x+2)^2(x-2)}$

$=\dfrac{2x^2-3x-2-6x^2-12x+3x^2+12x+12}{(x+2)^2(x-2)}$

$=\dfrac{-x^2-3x+10}{(x+2)^2(x-2)}=-\dfrac{x^2+3x-10}{(x+2)^2(x-2)}=-\dfrac{(x+5)(x-2)}{(x+2)^2(x-2)}=-\dfrac{x+5}{(x+2)^2}$

32 $\dfrac{4x+12}{x^2+6x+9}+\dfrac{5x}{x^2-9}+\dfrac{7}{x-3}=\dfrac{4}{x+3}+\dfrac{5x}{x^2-9}+\dfrac{7}{x-3}=\dfrac{4(x-3)+5x+7(x+3)}{x^2-9}=\dfrac{16x+9}{x^2-9}$

33 The lcd of the entire expression is ab. Thus, we will multiply both the numerator and denominator by ab.

$$\dfrac{\dfrac{b}{a}-\dfrac{a}{b}}{\dfrac{1}{a}-\dfrac{1}{b}}=\dfrac{\left(\dfrac{b}{a}-\dfrac{a}{b}\right)\cdot ab}{\left(\dfrac{1}{a}-\dfrac{1}{b}\right)\cdot ab}=\dfrac{b^2-a^2}{b-a}=\dfrac{(b+a)(b-a)}{b-a}=a+b$$

34 $\dfrac{\dfrac{1}{x+2}-5}{\dfrac{4}{x}-x}=\dfrac{\dfrac{1-5(x+2)}{x+2}}{\dfrac{4-x(x)}{x}}=\dfrac{\dfrac{-5x-9}{x+2}}{\dfrac{4-x^2}{x}}=\dfrac{-(5x+9)x}{(x+2)(2+x)(2-x)}=\dfrac{x(5x+9)}{(x-2)(x+2)^2}$

35 The lcd of the entire expression is x^2y^2. Thus, we will multiply both the numerator and denominator by x^2y^2.

$$\dfrac{\dfrac{x}{y^2}-\dfrac{y}{x^2}}{\dfrac{1}{y^2}-\dfrac{1}{x^2}}=\dfrac{\left(\dfrac{x}{y^2}-\dfrac{y}{x^2}\right)\cdot x^2y^2}{\left(\dfrac{1}{y^2}-\dfrac{1}{x^2}\right)\cdot x^2y^2}=\dfrac{x^3-y^3}{x^2-y^2}=\dfrac{(x-y)(x^2+xy+y^2)}{(x+y)(x-y)}=\dfrac{x^2+xy+y^2}{x+y}$$

36 $\dfrac{\dfrac{r}{s}+\dfrac{s}{r}}{\dfrac{r^2}{s^2}-\dfrac{s^2}{r^2}}=\dfrac{\left(\dfrac{r}{s}+\dfrac{s}{r}\right)\cdot r^2s^2}{\left(\dfrac{r^2}{s^2}-\dfrac{s^2}{r^2}\right)\cdot r^2s^2}=\dfrac{r^3s+rs^3}{r^4-s^4}=\dfrac{rs(r^2+s^2)}{(r^2+s^2)(r^2-s^2)}=\dfrac{rs}{r^2-s^2}$

37 The lcd of the entire expression is xy. Thus, we will multiply both the numerator and denominator by xy.

$$\dfrac{y^{-1}+x^{-1}}{(xy)^{-1}}=\dfrac{\dfrac{1}{y}+\dfrac{1}{x}}{\dfrac{1}{xy}}=\dfrac{\left(\dfrac{1}{y}+\dfrac{1}{x}\right)\cdot xy}{\left(\dfrac{1}{xy}\right)\cdot xy}=\dfrac{x+y}{1}=x+y$$

38 $\dfrac{y^{-2}-x^{-2}}{y^{-2}+x^{-2}}=\dfrac{\dfrac{1}{y^2}-\dfrac{1}{x^2}}{\dfrac{1}{y^2}+\dfrac{1}{x^2}}=\dfrac{\left(\dfrac{1}{y^2}-\dfrac{1}{x^2}\right)\cdot x^2y^2}{\left(\dfrac{1}{y^2}+\dfrac{1}{x^2}\right)\cdot x^2y^2}=\dfrac{x^2-y^2}{x^2+y^2}$

39 $\dfrac{\dfrac{5}{x+1}+\dfrac{2x}{x+3}}{\dfrac{x}{x+1}+\dfrac{7}{x+3}}=\dfrac{\dfrac{5(x+3)+2x(x+1)}{(x+1)(x+3)}}{\dfrac{x(x+3)+7(x+1)}{(x+1)(x+3)}}=\dfrac{5x+15+2x^2+2x}{x^2+3x+7x+7}=\dfrac{2x^2+7x+15}{x^2+10x+7}$

40 $\dfrac{\dfrac{2}{w}-\dfrac{4}{2w+1}}{\dfrac{5}{w}+\dfrac{8}{2w+1}}=\dfrac{\dfrac{2(2w+1)-4w}{w(2w+1)}}{\dfrac{5(2w+1)+8w}{w(2w+1)}}=\dfrac{4w+2-4w}{10w+5+8w}=\dfrac{2}{18w+5}$

41 $\dfrac{\dfrac{5}{x-1}-\dfrac{5}{a-1}}{x-a}=\dfrac{\dfrac{5(a-1)-5(x-1)}{(x-1)(a-1)}}{x-a}=\dfrac{5a-5x}{(x-1)(a-1)(x-a)}=\dfrac{5(a-x)}{(x-1)(a-1)(x-a)}$

$=-\dfrac{5}{(x-1)(a-1)}$

42 $\dfrac{\dfrac{x+2}{x}-\dfrac{a+2}{a}}{x-a}=\dfrac{\dfrac{a(x+2)-x(a+2)}{ax}}{x-a}=\dfrac{2a-2x}{ax(x-a)}=\dfrac{2(a-x)}{ax(x-a)}=-\dfrac{2}{ax}$

43 $\dfrac{(x+h)^2-3(x+h)-(x^2-3x)}{h}=\dfrac{x^2+2xh+h^2-3x-3h-x^2+3x}{h}$

$=\dfrac{2xh+h^2-3h}{h}=\dfrac{h(2x+h-3)}{h}=2x+h-3$

44 $\dfrac{(x+h)^3+5(x+h)-(x^3+5x)}{h}=\dfrac{x^3+3x^2h+3xh^2+h^3+5x+5h-x^3-5x}{h}$

$=\dfrac{3x^2h+3xh^2+h^3+5h}{h}$

$=\dfrac{h(3x^2+3xh+h^2+5)}{h}=3x^2+3xh+h^2+5$

45 $\dfrac{\dfrac{1}{(x+h)^3}-\dfrac{1}{x^3}}{h}=\dfrac{\dfrac{x^3-(x+h)^3}{(x+h)^3x^3}}{h}$

$=\dfrac{x^3-(x+h)^3}{hx^3(x+h)^3}=\dfrac{[x-(x+h)]\left[x^2+x(x+h)+(x+h)^2\right]}{hx^3(x+h)^3}$ {difference of two cubes}

$=\dfrac{-h[x^2+x^2+xh+x^2+2xh+h^2]}{hx^3(x+h)^3}=\dfrac{-h(3x^2+3xh+h^2)}{hx^3(x+h)^3}=-\dfrac{3x^2+3xh+h^2}{x^3(x+h)^3}$

46 $\dfrac{\dfrac{1}{x+h}-\dfrac{1}{x}}{h}=\dfrac{\dfrac{x-(x+h)}{(x+h)x}}{h}=\dfrac{-h}{hx(x+h)}=\dfrac{-1}{x(x+h)}$

47 $\dfrac{\dfrac{4}{3x+3h-1}-\dfrac{4}{3x-1}}{h}=\dfrac{\dfrac{4(3x-1)-4(3x+3h-1)}{(3x+3h-1)(3x-1)}}{h}=\dfrac{12x-4-12x-12h+4}{h(3x+3h-1)(3x-1)}$

$=\dfrac{-12h}{h(3x+3h-1)(3x-1)}=\dfrac{-12}{(3x+3h-1)(3x-1)}$

48 $\dfrac{\dfrac{7}{2x+2h+3}-\dfrac{7}{2x+3}}{h}=\dfrac{\dfrac{7(2x+3)-7(2x+2h+3)}{(2x+2h+3)(2x+3)}}{h}=\dfrac{14x+21-14x-14h-21}{h(2x+2h+3)(2x+3)}$

$=\dfrac{-14h}{h(2x+2h+3)(2x+3)}=\dfrac{-14}{(2x+2h+3)(2x+3)}$

49 The conjugate of $\sqrt{t}-5$ is $\sqrt{t}+5$. Multiply the numerator and the denominator by the conjugate of the denominator. This will eliminate the radical in the denominator.

$$\frac{\sqrt{t}+5}{\sqrt{t}-5}=\frac{\sqrt{t}+5}{\sqrt{t}-5}\cdot\frac{\sqrt{t}+5}{\sqrt{t}+5}=\frac{\left(\sqrt{t}\right)^2+2\cdot 5\sqrt{t}+5^2}{\left(\sqrt{t}\right)^2-5^2}=\frac{t+10\sqrt{t}+25}{t-25}$$

50 $\dfrac{\sqrt{t}-7}{\sqrt{t}+7}=\dfrac{\sqrt{t}-7}{\sqrt{t}+7}\cdot\dfrac{\sqrt{t}-7}{\sqrt{t}-7}=\dfrac{\left(\sqrt{t}\right)^2-2\cdot 7\sqrt{t}+7^2}{\left(\sqrt{t}\right)^2-7^2}=\dfrac{t-14\sqrt{t}+49}{t-49}$

51 $\dfrac{81x^2-16y^2}{3\sqrt{x}-2\sqrt{y}}=\dfrac{81x^2-16y^2}{3\sqrt{x}-2\sqrt{y}}\cdot\dfrac{3\sqrt{x}+2\sqrt{y}}{3\sqrt{x}+2\sqrt{y}}=\dfrac{(9x+4y)(9x-4y)\left(3\sqrt{x}+2\sqrt{y}\right)}{9x-4y}=(9x+4y)\left(3\sqrt{x}+2\sqrt{y}\right)$

52 $\dfrac{16x^2-y^2}{2\sqrt{x}-\sqrt{y}}=\dfrac{16x^2-y^2}{2\sqrt{x}-\sqrt{y}}\cdot\dfrac{2\sqrt{x}+\sqrt{y}}{2\sqrt{x}+\sqrt{y}}=\dfrac{(4x+y)(4x-y)\left(2\sqrt{x}+\sqrt{y}\right)}{4x-y}=(4x+y)\left(2\sqrt{x}+\sqrt{y}\right)$

53 We must recognize $\sqrt[3]{a}-\sqrt[3]{b}$ as the first factor of the product formula for the difference of two cubes, $x^3-y^3=(x-y)(x^2+xy+y^2)$. The second factor is then

$$\left(\sqrt[3]{a}\right)^2+\left(\sqrt[3]{a}\right)\left(\sqrt[3]{b}\right)+\left(\sqrt[3]{b}\right)^2=\sqrt[3]{a^2}+\sqrt[3]{ab}+\sqrt[3]{b^2}.$$

$$\frac{1}{\sqrt[3]{a}-\sqrt[3]{b}}=\frac{1}{\sqrt[3]{a}-\sqrt[3]{b}}\cdot\frac{\sqrt[3]{a^2}+\sqrt[3]{ab}+\sqrt[3]{b^2}}{\sqrt[3]{a^2}+\sqrt[3]{ab}+\sqrt[3]{b^2}}=\frac{\sqrt[3]{a^2}+\sqrt[3]{ab}+\sqrt[3]{b^2}}{a-b}$$

54 $\dfrac{1}{\sqrt[3]{x}+\sqrt[3]{y}}=\dfrac{1}{\sqrt[3]{x}+\sqrt[3]{y}}\cdot\dfrac{\sqrt[3]{x^2}-\sqrt[3]{xy}+\sqrt[3]{y^2}}{\sqrt[3]{x^2}-\sqrt[3]{xy}+\sqrt[3]{y^2}}=\dfrac{\sqrt[3]{x^2}-\sqrt[3]{xy}+\sqrt[3]{y^2}}{x+y}$

55 $\dfrac{\sqrt{a}-\sqrt{b}}{a^2-b^2}=\dfrac{\sqrt{a}-\sqrt{b}}{a^2-b^2}\cdot\dfrac{\sqrt{a}+\sqrt{b}}{\sqrt{a}+\sqrt{b}}=\dfrac{a-b}{(a+b)(a-b)\left(\sqrt{a}+\sqrt{b}\right)}=\dfrac{1}{(a+b)\left(\sqrt{a}+\sqrt{b}\right)}$

56 $\dfrac{\sqrt{b}+\sqrt{c}}{b^2-c^2}=\dfrac{\sqrt{b}+\sqrt{c}}{b^2-c^2}\cdot\dfrac{\sqrt{b}-\sqrt{c}}{\sqrt{b}-\sqrt{c}}=\dfrac{b-c}{(b+c)(b-c)\left(\sqrt{b}-\sqrt{c}\right)}=\dfrac{1}{(b+c)\left(\sqrt{b}-\sqrt{c}\right)}$

57
$$\begin{aligned}\frac{\sqrt{2(x+h)+1}-\sqrt{2x+1}}{h}&=\frac{\sqrt{2(x+h)+1}-\sqrt{2x+1}}{h}\cdot\frac{\sqrt{2(x+h)+1}+\sqrt{2x+1}}{\sqrt{2(x+h)+1}+\sqrt{2x+1}}\\&=\frac{(2x+2h+1)-(2x+1)}{h\left(\sqrt{2(x+h)+1}+\sqrt{2x+1}\right)}\\&=\frac{2h}{h\left(\sqrt{2(x+h)+1}+\sqrt{2x+1}\right)}=\frac{2}{\sqrt{2(x+h)+1}+\sqrt{2x+1}}\end{aligned}$$

58
$$\begin{aligned}\frac{\sqrt{x}-\sqrt{x+h}}{h\sqrt{x}\sqrt{x+h}}&=\frac{\sqrt{x}-\sqrt{x+h}}{h\sqrt{x}\sqrt{x+h}}\cdot\frac{\sqrt{x}+\sqrt{x+h}}{\sqrt{x}+\sqrt{x+h}}=\frac{x-(x+h)}{h\sqrt{x}\sqrt{x+h}\left(\sqrt{x}+\sqrt{x+h}\right)}\\&=\frac{-h}{h\sqrt{x}\sqrt{x+h}\left(\sqrt{x}+\sqrt{x+h}\right)}=\frac{-1}{\sqrt{x}\sqrt{x+h}\left(\sqrt{x}+\sqrt{x+h}\right)}\end{aligned}$$

59
$$\begin{aligned}\frac{\sqrt{1-x-h}-\sqrt{1-x}}{h}&=\frac{\sqrt{1-x-h}-\sqrt{1-x}}{h}\cdot\frac{\sqrt{1-x-h}+\sqrt{1-x}}{\sqrt{1-x-h}+\sqrt{1-x}}=\frac{(1-x-h)-(1-x)}{h\left(\sqrt{1-x-h}+\sqrt{1-x}\right)}\\&=\frac{-h}{h\left(\sqrt{1-x-h}+\sqrt{1-x}\right)}=\frac{-1}{\sqrt{1-x-h}+\sqrt{1-x}}\end{aligned}$$

60
$$\begin{aligned}\frac{\sqrt[3]{x+h}-\sqrt[3]{x}}{h}&=\frac{\sqrt[3]{x+h}-\sqrt[3]{x}}{h}\cdot\frac{\sqrt[3]{(x+h)^2}+\sqrt[3]{x(x+h)}+\sqrt[3]{x^2}}{\sqrt[3]{(x+h)^2}+\sqrt[3]{x(x+h)}+\sqrt[3]{x^2}}\\&=\frac{(x+h)-x}{h\left(\sqrt[3]{(x+h)^2}+\sqrt[3]{x(x+h)}+\sqrt[3]{x^2}\right)}=\frac{1}{\sqrt[3]{(x+h)^2}+\sqrt[3]{x(x+h)}+\sqrt[3]{x^2}}\end{aligned}$$

61 $\dfrac{3x^2-x+7}{x^{2/3}}=\dfrac{3x^2}{x^{2/3}}-\dfrac{x}{x^{2/3}}+\dfrac{7}{x^{2/3}}=3x^{4/3}-x^{1/3}+7x^{-2/3}$

62 $\dfrac{x^2+4x-6}{\sqrt{x}}=\dfrac{x^2}{\sqrt{x}}+\dfrac{4x}{\sqrt{x}}-\dfrac{6}{\sqrt{x}}=x^{3/2}+4x^{1/2}-6x^{-1/2}$

63 $\dfrac{(x^2+2)^2}{x^5}=\dfrac{x^4+4x^2+4}{x^5}=\dfrac{x^4}{x^5}+\dfrac{4x^2}{x^5}+\dfrac{4}{x^5}=x^{-1}+4x^{-3}+4x^{-5}$

64 $\dfrac{(\sqrt{x}-3)^2}{x^3}=\dfrac{x-6\sqrt{x}+9}{x^3}=\dfrac{x}{x^3}-\dfrac{6\sqrt{x}}{x^3}+\dfrac{9}{x^3}=x^{-2}-6x^{-5/2}+9x^{-3}$

Note: You may wish to demonstrate the three techniques shown in Example 9 with one of these simpler expressions in 65–68.

Note: Exercises 65–82 are worked using the factoring concept given as the third method of simplification in Example 9.

65 The smallest exponent that appears on the variable x is -3.

$$x^{-3}+x^2\ \{\text{factor out } x^{-3}\}\ =x^{-3}\left(1+x^{2-(-3)}\right)=x^{-3}\left(1+x^5\right)=\frac{1+x^5}{x^3}$$

66 $x^{-5} - x$ {factor out x^{-5}} $= x^{-5}\left(1 - x^{1-(-5)}\right) = x^{-5}(1 - x^6) = \dfrac{1 - x^6}{x^5}$

67 $x^{-1/2} - x^{3/2}$ {factor out $x^{-1/2}$} $= x^{-1/2}\left(1 - x^{3/2-(-1/2)}\right) = x^{-1/2}(1 - x^2) = \dfrac{1 - x^2}{x^{1/2}}$

68 $x^{-2/3} + x^{7/3}$ {factor out $x^{-2/3}$} $= x^{-2/3}\left(1 + x^{7/3-(-2/3)}\right) = x^{-2/3}(1 + x^3) = \dfrac{1 + x^3}{x^{2/3}}$

69 $\left(2x^2 - 3x + 1\right)(4)(3x+2)^3(3) + (3x+2)^4(4x-3)$

$= (3x+2)^3\left[12\left(2x^2 - 3x + 1\right) + (3x+2)(4x-3)\right]$ {factor out the gcf of $(3x+2)^3$}

$= (3x+2)^3\left(24x^2 - 36x + 12 + 12x^2 - x - 6\right)$

$= (3x+2)^3\left(36x^2 - 37x + 6\right)$

70 $(6x-5)^3(2)\left(x^2+4\right)(2x) + \left(x^2+4\right)^2(3)(6x-5)^2(6) = 2(6x-5)^2\left(x^2+4\right)\left[2x(6x-5) + 9\left(x^2+4\right)\right]$

$= 2(6x-5)^2\left(x^2+4\right)\left(12x^2 - 10x + 9x^2 + 36\right)$

$= 2\left(x^2+4\right)(6x-5)^2\left(21x^2 - 10x + 36\right)$

71 The smallest exponent that appears on the factor (x^2-4) is $-\frac{1}{2}$ and the smallest exponent that appears on the factor $(2x+1)$ is 2. Thus, we will factor out $(x^2-4)^{-1/2}(2x+1)^2$.

$(x^2-4)^{1/2}(3)(2x+1)^2(2) + (2x+1)^3\left(\frac{1}{2}\right)(x^2-4)^{-1/2}(2x) = (x^2-4)^{-1/2}(2x+1)^2[6(x^2-4) + x(2x+1)]$

If you are unsure of this factoring, it is easy to visually check at this stage by merely multiplying the expression—that is, we mentally add the exponents on the factor (x^2-4), $-\frac{1}{2}$ and 1, and we get $\frac{1}{2}$, which is the exponent we started with.

Proceeding: $\left(x^2-4\right)^{-1/2}(2x+1)^2\left[6\left(x^2-4\right) + x(2x+1)\right] = \left(x^2-4\right)^{-1/2}(2x+1)^2\left(6x^2 - 24 + 2x^2 + x\right)$

$= \dfrac{(2x+1)^2(8x^2 + x - 24)}{(x^2-4)^{1/2}}$

72 $(3x+2)^{1/3}(2)(4x-5)(4) + (4x-5)^2\left(\dfrac{1}{3}\right)(3x+2)^{-2/3}(3) = (3x+2)^{-2/3}(4x-5)[8(3x+2) + (4x-5)]$

$= \dfrac{(4x-5)(28x+11)}{(3x+2)^{2/3}}$

73 $(3x+1)^6\left(\frac{1}{2}\right)(2x-5)^{-1/2}(2) + (2x-5)^{1/2}(6)(3x+1)^5(3)$

$= (3x+1)^5(2x-5)^{-1/2}[(3x+1) + 18(2x-5)]$ $\left\{\text{factor out } (3x+1)^5(2x-5)^{-1/2}\right\}$

$= \dfrac{(3x+1)^5(3x + 1 + 36x - 90)}{(2x-5)^{1/2}} = \dfrac{(3x+1)^5(39x - 89)}{(2x-5)^{1/2}}$

74 $\left(x^2+9\right)^4\left(-\frac{1}{3}\right)(x+6)^{-4/3} + (x+6)^{-1/3}(4)\left(x^2+9\right)^3(2x)$

$= \left(\frac{1}{3}\right)\left(x^2+9\right)^3(x+6)^{-4/3}\left[-\left(x^2+9\right) + 24x(x+6)\right] = \dfrac{(x^2+9)^3(23x^2 + 144x - 9)}{3(x+6)^{4/3}}$

75 $\dfrac{(6x+1)^3(27x^2+2) - (9x^3+2x)(3)(6x+1)^2(6)}{(6x+1)^6} = \dfrac{(6x+1)^2[(6x+1)(27x^2+2) - 18(9x^3+2x)]}{(6x+1)^6}$

$= \dfrac{(6x+1)^2(162x^3 + 27x^2 + 12x + 2 - 162x^3 - 36x)}{(6x+1)^6}$

$= \dfrac{27x^2 - 24x + 2}{(6x+1)^4}$

76 $\dfrac{(x^2-1)^4(2x) - x^2(4)(x^2-1)^3(2x)}{(x^2-1)^8} = \dfrac{(2x)(x^2-1)^3[(x^2-1) - 4x^2]}{(x^2-1)^8} = \dfrac{2x(-3x^2-1)}{(x^2-1)^5} = \dfrac{-2x(3x^2+1)}{(x^2-1)^5}$

77 $\dfrac{(x^2+2)^3(2x)-x^2(3)(x^2+2)^2(2x)}{\left[(x^2+2)^3\right]^2}=\dfrac{(x^2+2)^2(2x)\left[(x^2+2)^1-x^2(3)\right]}{(x^2+2)^6}=$

$$\frac{2x(x^2+2-3x^2)}{(x^2+2)^4}=\frac{2x(2-2x^2)}{(x^2+2)^4}=\frac{4x(1-x^2)}{(x^2+2)^4}$$

78 $\dfrac{(x^2-5)^4(3x^2)-x^3(4)(x^2-5)^3(2x)}{\left[(x^2-5)^4\right]^2}=\dfrac{(x^2-5)^3(x^2)\left[(x^2-5)^1(3)-(x)(4)(2x)\right]}{(x^2-5)^8}=$

$$\frac{x^2(3x^2-15-8x^2)}{(x^2-5)^5}=\frac{x^2(-5x^2-15)}{(x^2-5)^5}=-\frac{5x^2(x^2+3)}{(x^2-5)^5}$$

79 $\dfrac{(x^2+4)^{1/3}(3)-(3x)\left(\frac{1}{3}\right)(x^2+4)^{-2/3}(2x)}{\left[(x^2+4)^{1/3}\right]^2}=\dfrac{(x^2+4)^{-2/3}[3(x^2+4)-2x^2]}{(x^2+4)^{2/3}}=\dfrac{3x^2+12-2x^2}{(x^2+4)^{4/3}}=\dfrac{x^2+12}{(x^2+4)^{4/3}}$

80 $\dfrac{(1-x^2)^{1/2}(2x)-x^2\left(\frac{1}{2}\right)(1-x^2)^{-1/2}(-2x)}{\left[(1-x^2)^{1/2}\right]^2}=\dfrac{x(1-x^2)^{-1/2}[2(1-x^2)+x^2]}{(1-x^2)^1}=\dfrac{x(2-2x^2+x^2)}{(1-x^2)^{3/2}}=\dfrac{x(2-x^2)}{(1-x^2)^{3/2}}$

81 $\dfrac{(4x^2+9)^{1/2}(2)-(2x+3)\left(\frac{1}{2}\right)(4x^2+9)^{-1/2}(8x)}{\left[(4x^2+9)^{1/2}\right]^2}=\dfrac{(4x^2+9)^{-1/2}[2(4x^2+9)-4x(2x+3)]}{(4x^2+9)^1}$

$$=\frac{8x^2+18-8x^2-12x}{(4x^2+9)^{3/2}}=\frac{18-12x}{(4x^2+9)^{3/2}}=\frac{6(3-2x)}{(4x^2+9)^{3/2}}$$

82 $\dfrac{(3x+2)^{1/2}\left(\frac{1}{3}\right)(2x+3)^{-2/3}(2)-(2x+3)^{1/3}\left(\frac{1}{2}\right)(3x+2)^{-1/2}(3)}{\left[(3x+2)^{1/2}\right]^2}$

$$=\frac{\left(\frac{1}{3}\right)\left(\frac{1}{2}\right)(3x+2)^{-1/2}(2x+3)^{-2/3}[4(3x+2)-9(2x+3)]}{(3x+2)^1}$$

$$=\frac{\left(\frac{1}{6}\right)(12x+8-18x-27)}{(3x+2)^{3/2}(2x+3)^{2/3}}=\frac{\left(\frac{1}{6}\right)(-6x-19)}{(3x+2)^{3/2}(2x+3)^{2/3}}=-\frac{6x+19}{6(3x+2)^{3/2}(2x+3)^{2/3}}$$

83 Table $Y_1=\dfrac{113x^3+280x^2-150x}{22x^3+77x^2-100x-350}$ and $Y_2=\dfrac{3x}{2x+7}+\dfrac{4x^2}{1.1x^2-5}$.

x	Y_1	Y_2
1	−0.6923	−0.6923
2	−26.12	−26.12
3	8.0392	8.0392
4	5.8794	5.8794
5	5.3268	5.3268

The values for Y_1 and Y_2 agree. Therefore, the two expressions might be equal.

84 Table $Y_1=\dfrac{20x^2+41x+31}{10x^3+10x^2}$ and $Y_2=\dfrac{1}{x}+\dfrac{1}{x+1}+\dfrac{3.2}{x^2}$.

x	Y_1	Y_2
1	4.6	4.7
2	1.6083	1.6333
3	0.92778	0.93889
4	0.64375	0.65
5	0.49067	0.49467

The values for Y_1 and Y_2 do not agree. Therefore, the two expressions are not equal.

Chapter 1 Review Exercises

1 (a) $\left(\frac{2}{3}\right)\left(-\frac{5}{8}\right) = -\frac{1}{3} \cdot \frac{5}{4} = -\frac{5}{12}$ (b) $\frac{3}{4} + \frac{6}{5} = \frac{15}{20} + \frac{24}{20} = \frac{39}{20}$

(c) $\frac{5}{8} - \frac{9}{7} = \frac{35}{56} - \frac{72}{56} = -\frac{37}{56}$ (d) $\frac{3}{4} \div \frac{6}{5} = \frac{3}{4} \cdot \frac{5}{6} = \frac{1}{4} \cdot \frac{5}{2} = \frac{5}{8}$

2 (a) Since -0.1 is to the left of -0.01 on a coordinate line, $-0.1 \boxed{<} -0.01$.

(b) Since $\sqrt{9} = 3$ and 3 is to the right of -3 on a coordinate line, $\sqrt{9} \boxed{>} -3$.

(c) Since $\frac{1}{6} = 0.1\overline{6} = 0.1666\ldots$, $\frac{1}{6} \boxed{>} 0.166$.

3 (a) x is negative $\Leftrightarrow$ $x < 0$ (b) a is between $\frac{1}{2}$ and $\frac{1}{3}$ $\Leftrightarrow$ $\frac{1}{3} < a < \frac{1}{2}$

(c) The absolute value of x is not less than 4 $\Leftrightarrow$ $|x| \geq 4$

4 (a) $|-4| = -(-4) = 4$ (b) $\dfrac{|-5|}{-5} = \dfrac{-(-5)}{-5} = \dfrac{5}{-5} = -1$

(c) $|3^{-1} - 2^{-1}| = \left|\frac{1}{3} - \frac{1}{2}\right| = \left|\frac{2}{6} - \frac{3}{6}\right| = \left|-\frac{1}{6}\right| = -\left(-\frac{1}{6}\right) = \frac{1}{6}$

5 (a) $d(A, C) = |-3 - (-8)| = |5| = 5$ (b) $d(C, A) = d(A, C) = 5$

(c) $d(B, C) = |-3 - 4| = |-7| = -(-7) = 7$

6 (a) $d(x, -2)$ is at least 7 $\Rightarrow$ $|-2 - x| \geq 7$ (b) $d(4, x)$ is less than 4 $\Rightarrow$ $|x - 4| < 4$

7 If $x \leq -3$, then $x + 3 \leq 0$, and $|x + 3| = -(x + 3) = -x - 3$.

8 If $2 < x < 3$, then $x - 2 > 0$ $\{x - 2$ is positive$\}$ and $x - 3 < 0$ $\{x - 3$ is negative$\}$. Thus, $(x - 2)(x - 3) < 0$ {positive times negative is negative}, and since the absolute value of an expression that is negative is the negative of the expression, $|(x - 2)(x - 3)| = -(x - 2)(x - 3)$, or, equivalently, $(2 - x)(x - 3)$.

9 (a) $(x + y)^2 = x^2 + 2xy + y^2 \boxed{\neq} x^2 + y^2$ for every nonzero x and nonzero y.

(b) $\dfrac{1}{\sqrt{x + y}} = \dfrac{1}{\sqrt{x}} + \dfrac{1}{\sqrt{y}}$ is not true if $x = y = 1$

{we need only find one set of values of the variables for which the expression is false}.

(c) $\dfrac{1}{\sqrt{c} - \sqrt{d}} = \dfrac{1}{\sqrt{c} - \sqrt{d}} \cdot \dfrac{\sqrt{c} + \sqrt{d}}{\sqrt{c} + \sqrt{d}} \boxed{=} \dfrac{\sqrt{c} + \sqrt{d}}{c - d}$

10 (a) $93{,}700{,}000{,}000 = 9.37 \times 10^{10}$ (b) $0.000\,004\,02 = 4.02 \times 10^{-6}$

11 (a) $6.8 \times 10^7 = 68{,}000{,}000$ (b) $7.3 \times 10^{-4} = 0.000\,73$

12 (a) $\left|\sqrt{5} - 17^2\right| \approx 286.7639$, which is 2.867639×10^2 in scientific notation.

(b) Expressed to four *significant* figures, we have 2.868×10^2.

13 $-3^2 + 3^0 + 27^{-2/3} = -9 + 1 + \dfrac{1}{27^{2/3}} = -8 + \dfrac{1}{\left(\sqrt[3]{27}\right)^2} = -8 + \dfrac{1}{3^2} = -\dfrac{72}{9} + \dfrac{1}{9} = \dfrac{-71}{9}$

14 $\left(\frac{1}{2}\right)^0 - 1^2 + 16^{-3/4} = 1 - 1 + \dfrac{1}{16^{3/4}} = 0 + \dfrac{1}{\left(\sqrt[4]{16}\right)^3} = \dfrac{1}{2^3} = \dfrac{1}{8}$

15 $(3a^2b)^2(2ab^3) = \left(9a^4b^2\right)(2ab^3) = 18a^5b^5$

16 $\dfrac{6r^3y^2z}{2r^5yz} = \dfrac{3y}{r^2}$

17 $\dfrac{(3x^2y^{-3})^{-2}}{x^{-5}y} = \dfrac{3^{-2}x^{-4}y^6}{x^{-5}y} = \dfrac{x^5y^5}{3^2x^4} = \dfrac{xy^5}{9}$

18 $\left(\dfrac{a^{2/3}b^{3/2}}{a^2b}\right)^6 = \dfrac{a^4b^9}{a^{12}b^6} = \dfrac{b^3}{a^8}$

19 $(-2p^2q)^3\left(\dfrac{p}{4q^2}\right)^2 = (-8p^6q^3)\left(\dfrac{p^2}{16q^4}\right) = -\dfrac{p^8}{2q}$

20 $c^{-4/3}c^{3/2}c^{1/6} = c^{-8/6}c^{9/6}c^{1/6} = c^{(-8+9+1)/6} = c^{2/6} = c^{1/3}$

21 $\left(\dfrac{xy^{-1}}{\sqrt{z}}\right)^4 \div \left(\dfrac{x^{1/3}y^2}{z}\right)^3 = \dfrac{x^4y^{-4}}{z^2}\cdot\dfrac{z^3}{xy^6} = \dfrac{x^3z}{y^{10}}$

22 $\left(\dfrac{-64x^3}{z^6y^9}\right)^{2/3} = \dfrac{\left(\sqrt[3]{-64}\right)^2x^2}{z^4y^6} = \dfrac{16x^2}{z^4y^6}$

23 $\left[\left(a^{2/3}b^{-2}\right)^3\right]^{-1} = (a^2b^{-6})^{-1} = a^{-2}b^6 = \dfrac{b^6}{a^2}$

24 $\dfrac{\left(3u^2v^5w^{-4}\right)^3}{\left(2uv^{-3}w^2\right)^4} = \dfrac{3^3u^6v^{15}w^{-12}}{2^4u^4v^{-12}w^8} = \dfrac{27u^2v^{27}}{16w^{20}}$

25 $\dfrac{r^{-1}+s^{-1}}{(rs)^{-1}} = \left(\dfrac{1}{r}+\dfrac{1}{s}\right)\div\dfrac{1}{rs} = \left(\dfrac{1}{r}+\dfrac{1}{s}\right)\cdot rs = s + r$

26 Do not expand $(u+v)^3$ since it can be combined with $(u+v)^{-2}$.

$$(u+v)^3(u+v)^{-2} = (u+v)^1 = u + v$$

27 $s^{5/2}s^{-4/3}s^{-1/6} = s^{(15-8-1)/6} = s^{6/6} = s^1 = s$

28 $x^{-2} - y^{-1} = \dfrac{1}{x^2} - \dfrac{1}{y} = \dfrac{y - x^2}{x^2y}$

29 $\sqrt[3]{(x^4y^{-1})^6} = \left(x^4y^{-1}\right)^{6/3} = \left(x^4y^{-1}\right)^2 = x^8y^{-2} = \dfrac{x^8}{y^2}$

30 $\sqrt[3]{27x^5y^3z^4} = \sqrt[3]{27x^3y^3z^3}\sqrt[3]{x^2z} = 3xyz\sqrt[3]{x^2z}$

31 Since $\sqrt[3]{4} = \sqrt[3]{2^2}$, we need to multiply the numerator and the denominator by $\sqrt[3]{2}$ to obtain a cube in the radicand of the denominator. $\dfrac{1}{\sqrt[3]{4}} = \dfrac{1}{\sqrt[3]{4}}\cdot\dfrac{\sqrt[3]{2}}{\sqrt[3]{2}} = \dfrac{\sqrt[3]{2}}{\sqrt[3]{8}} = \dfrac{\sqrt[3]{2}}{2}$, or $\dfrac{1}{2}\sqrt[3]{2}$

32 $\sqrt{\dfrac{a^2b^3}{c}} = \dfrac{\sqrt{a^2b^3}}{\sqrt{c}}\cdot\dfrac{\sqrt{c}}{\sqrt{c}} = \dfrac{\sqrt{a^2b^2}\sqrt{bc}}{c} = \dfrac{ab}{c}\sqrt{bc}$

33 $\sqrt[3]{4x^2y}\,\sqrt[3]{2x^5y^2} = \sqrt[3]{8x^7y^3} = \sqrt[3]{8x^6y^3}\,\sqrt[3]{x} = 2x^2y\,\sqrt[3]{x}$

34 $\sqrt[4]{(-4a^3b^2c)^2} = \sqrt[4]{16a^6b^4c^2} = \sqrt[4]{2^4a^4b^4}\,\sqrt[4]{a^2c^2} = 2ab\,\sqrt[4]{(ac)^2} = 2ab\,\sqrt{ac}$

35 $\dfrac{1}{\sqrt{t}}\left(\dfrac{1}{\sqrt{t}} - 1\right) = \dfrac{1}{\sqrt{t}}\left(\dfrac{1}{\sqrt{t}} - \dfrac{\sqrt{t}}{\sqrt{t}}\right) = \dfrac{1}{\sqrt{t}}\left(\dfrac{1-\sqrt{t}}{\sqrt{t}}\right) = \dfrac{1-\sqrt{t}}{t}$

36 $\sqrt{\sqrt[3]{(c^3d^6)^4}} = \sqrt[6]{c^{12}d^{24}} = c^2d^4$

37 $\dfrac{\sqrt{12x^4y}}{\sqrt{3x^2y^7}} = \sqrt{\dfrac{12x^4y}{3x^2y^7}} = \sqrt{\dfrac{4x^2}{y^6}} = \dfrac{2x}{y^3}$

38 $\sqrt[3]{(a+2b)^3} = a + 2b$

39 $\sqrt[3]{\dfrac{1}{2\pi^2}} = \dfrac{1}{\sqrt[3]{2\pi^2}}\cdot\dfrac{\sqrt[3]{4\pi}}{\sqrt[3]{4\pi}} = \dfrac{\sqrt[3]{4\pi}}{\sqrt[3]{8\pi^3}} = \dfrac{\sqrt[3]{4\pi}}{2\pi}$, or $\dfrac{1}{2\pi}\sqrt[3]{4\pi}$

40 $\sqrt[3]{\dfrac{x^2}{9y}} = \sqrt[3]{\dfrac{x^2}{9y}}\cdot\dfrac{\sqrt[3]{3y^2}}{\sqrt[3]{3y^2}} = \dfrac{\sqrt[3]{3x^2y^2}}{\sqrt[3]{27y^3}} = \dfrac{1}{3y}\sqrt[3]{3x^2y^2}$

41 $\dfrac{1-\sqrt{x}}{1+\sqrt{x}} = \dfrac{1-\sqrt{x}}{1+\sqrt{x}}\cdot\dfrac{1-\sqrt{x}}{1-\sqrt{x}} = \dfrac{1-2\sqrt{x}+x}{1-x}$

42 $\dfrac{1}{\sqrt{a}+\sqrt{a-2}} = \dfrac{1}{\sqrt{a}+\sqrt{a-2}}\cdot\dfrac{\sqrt{a}-\sqrt{a-2}}{\sqrt{a}-\sqrt{a-2}} = \dfrac{\sqrt{a}-\sqrt{a-2}}{a-(a-2)} = \dfrac{\sqrt{a}-\sqrt{a-2}}{2}$

43 $\dfrac{81x^2-y^2}{3\sqrt{x}+\sqrt{y}} = \dfrac{81x^2-y^2}{3\sqrt{x}+\sqrt{y}}\cdot\dfrac{3\sqrt{x}-\sqrt{y}}{3\sqrt{x}-\sqrt{y}} = \dfrac{(9x+y)(9x-y)\left(3\sqrt{x}-\sqrt{y}\right)}{9x-y} = (9x+y)\left(3\sqrt{x}-\sqrt{y}\right)$

44 $\dfrac{3+\sqrt{x}}{3-\sqrt{x}} = \dfrac{3+\sqrt{x}}{3-\sqrt{x}} \cdot \dfrac{3+\sqrt{x}}{3+\sqrt{x}} = \dfrac{x+6\sqrt{x}+9}{9-x}$

45 $(3x^3-4x^2+x-6)+(x^4-2x^3+3x^2+5) = x^4+x^3-x^2+x-1$

46 $(4z^4-3z^2+1)-z(z^3+4z^2-4) = 4z^4-3z^2+1-z^4-4z^3+4z = 3z^4-4z^3-3z^2+4z+1$

47 $(x+4)(x+3)-(2x-3)(x-5) = (x^2+7x+12)-(2x^2-13x+15) = -x^2+20x-3$

48 $(4x-3)(2x^2+5x-7) = (4x)(2x^2+5x-7)+(-3)(2x^2+5x-7)$
$= (8x^3+20x^2-28x)+(-6x^2-15x+21) = 8x^3+14x^2-43x+21$

49 $(3y^3-2y^2+y+4)(y^2-3) = (3y^3-2y^2+y+4)(y^2)+(3y^3-2y^2+y+4)(-3)$
$= (3y^5-2y^4+y^3+4y^2)+(-9y^3+6y^2-3y-12)$
$= 3y^5-2y^4-8y^3+10y^2-3y-12$

50 $(3x+2)(x-5)(5x+4) = (3x+2)(5x^2-21x-20)$
$= (3x)(5x^2-21x-20)+(2)(5x^2-21x-20)$
$= (15x^3-63x^2-60x)+(10x^2-42x-40) = 15x^3-53x^2-102x-40$

51 $(a-b)(a^3+a^2b+ab^2+b^3) = (a)(a^3+a^2b+ab^2+b^3)+(-b)(a^3+a^2b+ab^2+b^3)$
$= (a^4+a^3b+a^2b^2+ab^3)-(a^3b+a^2b^2+ab^3+b^4) = a^4-b^4$

52 $\dfrac{9p^4q^3-6p^2q^4+5p^3q^2}{3p^2q^2} = \dfrac{9p^4q^3}{3p^2q^2}-\dfrac{6p^2q^4}{3p^2q^2}+\dfrac{5p^3q^2}{3p^2q^2} = 3p^2q-2q^2+\dfrac{5}{3}p$

53 $(3a-5b)(4a+7b) = 12a^2+21ab-20ab-35b^2 = 12a^2+ab-35b^2$

54 $(4r^2-3s)^2 = (4r^2)^2-2(4r^2)(3s)+(3s)^2 = 16r^4-24r^2s+9s^2$

55 $(13a^2+5b)(13a^2-5b) = (13a^2)^2-(5b)^2 = 169a^4-25b^2$

56 $(a^3-a^2)^2 = (a^3)^2-2(a^3)(a^2)+(a^2)^2 = a^6-2a^5+a^4$.

Alternatively, we could factor out a^2 first, as follows:

$$(a^3-a^2)^2 = [a^2(a-1)]^2 = (a^2)^2(a-1)^2 = a^4(a^2-2a+1) = a^6-2a^5+a^4$$

57 $(3y+x)^2 = (3y)^2+2(3y)(x)+x^2 = 9y^2+6xy+x^2$

58 $(c^2-d^2)^3 = (c^2)^3-3(c^2)^2(d^2)+3(c^2)(d^2)^2-(d^2)^3 = c^6-3c^4d^2+3c^2d^4-d^6$

59 $(2a+b)^3 = (2a)^3+3(2a)^2(b)+3(2a)(b)^2+(b)^3 = 8a^3+12a^2b+6ab^2+b^3$

60 $(x^2-2x+3)^2 = (*)(x^2)^2+(-2x)^2+(3)^2+2(x^2)(-2x)+2(x^2)(3)+2(-2x)(3)$
$= x^4+4x^2+9-4x^3+6x^2-12x = x^4-4x^3+10x^2-12x+9$

(*) See the note before the solutions to Exercises 41–44 in Section 1.3.

61 $(3x+2y)^2(3x-2y)^2 = [(3x+2y)(3x-2y)]^2 = (9x^2-4y^2)^2 = 81x^4-72x^2y^2+16y^4$

62 $(a+b+c+d)^2 = a^2+b^2+c^2+d^2+2(ab+ac+ad+bc+bd+cd)$

63 $60xw+50w = 10w(6x+5)$

64 $3r^4s^3-12r^2s^5 = 3r^2s^3(r^2-4s^2) = 3r^2s^3(r+2s)(r-2s)$

65 $28x^2-4x-5 = (14x+5)(2x-1)$

66 $16a^4 + 24a^2b^2 + 9b^4 = (4a^2 + 3b^2)(4a^2 + 3b^2) = (4a^2 + 3b^2)^2$

67 $2wy + 3yx - 8wz - 12zx = y(2w + 3x) - 4z(2w + 3x) = (y - 4z)(2w + 3x)$

68 $2c^3 - 12c^2 + 3c - 18 = 2c^2(c - 6) + 3(c - 6) = (2c^2 + 3)(c - 6)$

69 $8x^3 + 64y^3 = 8(x^3 + 8y^3) = 8\left[(x)^3 + (2y)^3\right] = 8(x + 2y)(x^2 - 2xy + 4y^2)$

70 $u^3v^4 - u^6v = u^3v(v^3 - u^3) = u^3v(v - u)(v^2 + uv + u^2)$

71 $p^8 - q^8 = \left(p^4\right)^2 - \left(q^4\right)^2 = \left(p^4 + q^4\right)\left(p^4 - q^4\right) = \left(p^4 + q^4\right)\left(p^2 + q^2\right)\left(p^2 - q^2\right)$
$= \left(p^4 + q^4\right)\left(p^2 + q^2\right)(p + q)(p - q)$

72 $x^4 - 12x^3 + 36x^2 = x^2(x^2 - 12x + 36) = x^2(x - 6)(x - 6) = x^2(x - 6)^2$

73 $w^6 + 1 = (w^2)^3 + (1)^3 = (w^2 + 1)\left(w^4 - w^2 + 1\right)$, which cannot be factored any further.

74 $5x + 20 = 5(x + 4)$

75 $x^2 + 49$ is irreducible.

76 $x^2 - 49y^2 - 14x + 49 = (x^2 - 14x + 49) - 49y^2 = (x - 7)^2 - (7y)^2 = (x - 7 + 7y)(x - 7 - 7y)$

77 $x^5 - 4x^3 + 8x^2 - 32 = x^3\left(x^2 - 4\right) + 8\left(x^2 - 4\right) = \left(x^3 + 8\right)\left(x^2 - 4\right)$
$= \left[(x + 2)\left(x^2 - 2x + 4\right)\right]\left[(x + 2)(x - 2)\right] = (x - 2)(x + 2)^2\left(x^2 - 2x + 4\right)$

78 $4x^4 + 12x^3 + 20x^2 = 4x^2(x^2 + 3x + 5)$

79 $\dfrac{6x^2 - 7x - 5}{4x^2 + 4x + 1} = \dfrac{(3x - 5)(2x + 1)}{(2x + 1)(2x + 1)} = \dfrac{3x - 5}{2x + 1}$

80 $\dfrac{r^3 - t^3}{r^2 - t^2} = \dfrac{(r - t)(r^2 + rt + t^2)}{(r + t)(r - t)} = \dfrac{r^2 + rt + t^2}{r + t}$

81 $\dfrac{6x^2 - 5x - 6}{x^2 - 4} \div \dfrac{2x^2 - 3x}{x + 2} = \dfrac{(3x + 2)(2x - 3)}{(x + 2)(x - 2)} \cdot \dfrac{x + 2}{x(2x - 3)} = \dfrac{3x + 2}{x(x - 2)}$

82 $\dfrac{6}{4x - 5} - \dfrac{15}{10x + 1} = \dfrac{6(10x + 1) - 15(4x - 5)}{(4x - 5)(10x + 1)} = \dfrac{60x + 6 - 60x + 75}{(4x - 5)(10x + 1)} = \dfrac{81}{(4x - 5)(10x + 1)}$

83 $\dfrac{7}{x + 2} + \dfrac{3x}{(x + 2)^2} - \dfrac{5}{x} = \dfrac{7(x)(x + 2) + 3x(x) - 5(x + 2)^2}{x(x + 2)^2} = \dfrac{7x^2 + 14x + 3x^2 - 5x^2 - 20x - 20}{x(x + 2)^2} =$
$\dfrac{5x^2 - 6x - 20}{x(x + 2)^2}$

84 $\dfrac{x + x^{-2}}{1 + x^{-2}} = \dfrac{x + \dfrac{1}{x^2}}{1 + \dfrac{1}{x^2}} = \dfrac{\left(x + \dfrac{1}{x^2}\right) \cdot x^2}{\left(1 + \dfrac{1}{x^2}\right) \cdot x^2} = \dfrac{x^3 + 1}{x^2 + 1}$. We could factor the numerator,
but since it doesn't lead to a reduction of the fraction, we leave it in this form.

85 $\dfrac{1}{x} - \dfrac{2}{x^2 + x} - \dfrac{3}{x + 3} = \dfrac{1(x + 1)(x + 3) - 2(x + 3) - 3x(x + 1)}{x(x + 1)(x + 3)} = \dfrac{x^2 + 4x + 3 - 2x - 6 - 3x^2 - 3x}{x(x + 1)(x + 3)} =$
$\dfrac{-2x^2 - x - 3}{x(x + 1)(x + 3)}$

86 $(a^{-1} + b^{-1})^{-1} = \left(\dfrac{1}{a} + \dfrac{1}{b}\right)^{-1} = \left(\dfrac{b + a}{ab}\right)^{-1} = \left(\dfrac{ab}{a + b}\right)^{1} = \dfrac{ab}{a + b}$

87 $\dfrac{x+2-\dfrac{3}{x+4}}{\dfrac{x}{x+4}+\dfrac{1}{x+4}} = \dfrac{\dfrac{(x+2)(x+4)-3}{x+4}}{\dfrac{x+1}{x+4}} = \dfrac{(x^2+6x+8)-3}{x+1} = \dfrac{x^2+6x+5}{x+1} = \dfrac{(x+1)(x+5)}{x+1} = x+5$

88 $\dfrac{\dfrac{x}{x+2}-\dfrac{4}{x+2}}{x-3-\dfrac{6}{x+2}} = \dfrac{\dfrac{x-4}{x+2}}{\dfrac{(x-3)(x+2)-6}{x+2}} = \dfrac{x-4}{(x^2-x-6)-6} = \dfrac{x-4}{x^2-x-12} = \dfrac{x-4}{(x+3)(x-4)} = \dfrac{1}{x+3}$

89 $(x^2+1)^{3/2}(4)(x+5)^3+(x+5)^4\left(\frac{3}{2}\right)(x^2+1)^{1/2}(2x) = (x^2+1)^{1/2}(x+5)^3\left[4(x^2+1)+3x(x+5)\right]$
$= (x^2+1)^{1/2}(x+5)^3(7x^2+15x+4)$

90 $\dfrac{(4-x^2)\left(\frac{1}{3}\right)(6x+1)^{-2/3}(6)-(6x+1)^{1/3}(-2x)}{(4-x^2)^2} = \dfrac{2(6x+1)^{-2/3}[(4-x^2)+x(6x+1)]}{(4-x^2)^2}$
$= \dfrac{2(4-x^2+6x^2+x)}{(6x+1)^{2/3}(4-x^2)^2} = \dfrac{2(5x^2+x+4)}{(6x+1)^{2/3}(4-x^2)^2}$

91 $\dfrac{(x+5)^2}{\sqrt{x}} = \dfrac{x^2+10x+25}{\sqrt{x}} = \dfrac{x^2}{\sqrt{x}}+\dfrac{10x}{\sqrt{x}}+\dfrac{25}{\sqrt{x}} = \dfrac{x^2}{x^{1/2}}+\dfrac{10x}{x^{1/2}}+\dfrac{25}{x^{1/2}} = x^{3/2}+10x^{1/2}+25x^{-1/2}$

92 $x^3+x^{-1} = x^{-1}\left(x^{3-(-1)}+1\right) = \dfrac{x^4+1}{x}$ **OR** $x^3+x^{-1} = x^3+\dfrac{1}{x} = \dfrac{x^4}{x}+\dfrac{1}{x} = \dfrac{x^4+1}{x}$

93 $(5.5 \text{ liters})\left(10^6\ \dfrac{\text{mm}^3}{\text{liter}}\right)\left(5\times10^6\ \dfrac{\text{cells}}{\text{mm}^3}\right) = 2.75\times10^{13}$ red blood cells {27.5 trillion}

94 $\dfrac{70 \text{ (or 90) beats}}{\text{minute}}\cdot\dfrac{60 \text{ minutes}}{1 \text{ hour}}\cdot\dfrac{24 \text{ hours}}{1 \text{ day}}\cdot\dfrac{365 \text{ days}}{1 \text{ year}}\cdot 80 \text{ years} = 2.94336\times10^9$ (or 3.78432×10^9) beats

95 $h = 91.2$ cm and $w = 13.7$ kg $\Rightarrow$ $S = (0.007184)w^{0.425}h^{0.725} = (0.007184)(13.7)^{0.425}(91.2)^{0.725} \approx 0.58 \text{ m}^2$.

96 $p = 40 \text{ dyne/cm}^2$ and $v = 60 \text{ cm}^3$ $\Rightarrow$ $c = pv^{-1.4} = 40(60)^{-1.4} \approx 0.13$ dyne-cm.

Chapter 1 Discussion Exercises

1 $\dfrac{\$1 \text{ in cash back}}{100 \text{ points}}\times\dfrac{1 \text{ point}}{\$10 \text{ charged}} = \dfrac{\$1 \text{ in cash back}}{\$1000 \text{ charged}} = 0.001$, or 0.1%.

2 Squaring the right side gives us $(a+b)^2 = a^2+2ab+b^2$. Squaring the left side gives us a^2+b^2. Now $a^2+2ab+b^2$ will equal a^2+b^2 only if $2ab = 0$. The expression $2ab$ equals zero only if either $a = 0$ or $b = 0$.

3 We first need to determine the term that needs to be added and subtracted. Since $25 = \underline{5}^2$, it makes sense to add and subtract $2\cdot\underline{5}x = 10x$. Then we will obtain the square of a binomial—i.e.,
$(x^2+10x+25)-10x = (x+5)^2-10x$. We can now factor this expression as the difference of two squares,

$$(x+5)^2-10x = (x+5)^2-\left(\sqrt{10x}\right)^2 = \left(x+5+\sqrt{10x}\right)\left(x+5-\sqrt{10x}\right).$$

4 The expression $\dfrac{1}{x+1}$ can be evaluated at $x = 1$, whereas the expression $\dfrac{x-1}{x^2-1}$ is undefined at $x = 1$.

5 Try $\dfrac{3x^2 - 4x + 7}{8x^2 + 9x - 100}$ with $x = 10^3$, 10^4, and 10^5. You get approximately 0.374, 0.3749, and 0.37499. The numbers seem to be getting closer to 0.375, which is the decimal representation for $\frac{3}{8}$, which is the ratio of the coefficients of the x^2 terms. In general, the quotients of this form get close to the ratio of leading coefficients as x gets larger.

6 $\dfrac{3x^2 - 5x - 2}{x^2 - 4} = \dfrac{(3x+1)(x-2)}{(x+2)(x-2)} = \dfrac{3x+1}{x+2}$. Evaluating the original expression and the simplified expression with any $x \neq \pm 2$ gives us the same value. This evaluation does not prove that the expressions are equal for any value of x other than the one selected. The simplification proves that the expressions are equal for all values of x except $x = 2$.

7 Follow the algebraic simplification given.

1) Write down his/her age.	Denote the age by x.
2) Multiply it by 2.	$2x$
3) Add 5.	$2x + 5$
4) Multiply this sum by 50.	$50(2x + 5) = 100x + 250$
5) Subtract 365.	$(100x + 250) - 365 = 100x - 115$
6) Add his/her height (in inches).	$100x - 115 + y$, where y is the height
7) Add 115.	$100x - 115 + y + 115 = 100x + y$

As a specific example, suppose the age is 21 and the height is 68. The number obtained by following the steps is $100x + y = 2168$ and we can see that the first two digits of the result equal the age and the last two digits equal the height.

8
$$
\begin{aligned}
V_{\text{out}} &= I_{\text{in}}\left(-\frac{RXi}{R - Xi}\right) = \frac{V_{\text{in}}}{Z_{\text{in}}}\left(-\frac{RXi}{R - Xi}\right) && \{\text{definition of } I_{\text{in}}\}\\
&= \frac{V_{\text{in}}}{\dfrac{R^2 - X^2 - 3RXi}{R - Xi}}\left(-\frac{RXi}{R - Xi}\right) && \{\text{definition of } Z_{\text{in}}\}\\
&= \frac{V_{\text{in}}(R - Xi)}{R^2 - X^2 - 3RXi}\left(-\frac{RXi}{R - Xi}\right)\\
&= -\frac{RXi}{R^2 - X^2 - 3RXi}(V_{\text{in}})\\
&= -\frac{RRi}{R^2 - R^2 - 3RRi}(V_{\text{in}}) && \{\text{let } X = R\}\\
&= -\frac{R^2 i}{-3R^2 i}(V_{\text{in}}) = \tfrac{1}{3}V_{\text{in}}
\end{aligned}
$$

9 (a) $S = 975$, $A = 599$, and $x = 1.83 \Rightarrow$ winning percentage $= \dfrac{S^x}{S^x + A^x} \approx 0.709\,206$. Since they played 154 games $(110 + 44)$, the number of wins using the estimated winning percentage would be $0.709(154) \approx 109$. Hence, the Pythagorean win-loss record of the 1927 Yankees is 109–45 (only one game off their actual record).

(b) The actual winning percentage is $\frac{110}{154} \approx 0.714\,286$. For an estimate of x, we'll assign $\dfrac{975^x}{975^x + 599^x}$ to Y_1 and look at a table of values of x starting with $x = 1.80$ and incrementing by 0.01. From the table, we see that $x = 1.88$ corresponds to $\mathrm{Y}_1 \approx 0.714\,204$, which is the closest value to the actual winning percentage. Thus, the value of x is 1.88.

Chapter 1 Test

1 y^{99} is negative since it is a negative number raised to an odd power. $y - x$ is negative since it is a negative number made even more negative by subtracting a positive number. The quotient of two negatives is a *positive* number.

2 The quotient of x and y is not greater than 5 $\Leftrightarrow$ $\dfrac{x}{y} \leq 5$.

3 Since $-x^2 - 3 < 0$ for every x (it doesn't matter that x is negative), $|-x^2 - 3| = -(-x^2 - 3) = x^2 + 3$.

4 Using distance = rate × time, we get $t = \dfrac{d}{r} = \dfrac{91{,}500{,}000 \text{ miles}}{186{,}000 \text{ miles per second}} \approx 492$ seconds.

5 $\dfrac{x^2y^{-3}}{z}\left(\dfrac{3x^0}{zy^2}\right)^{-2} = \dfrac{x^2}{y^3z}\left(\dfrac{zy^2}{3x^0}\right)^2 = \dfrac{x^2}{y^3z} \cdot \dfrac{z^2y^4}{3^2} = \dfrac{x^2yz}{9}$

6 $x^{-2/3}x^{3/4} = x^{-8/12}x^{9/12} = x^{(-8/12)+(9/12)} = x^{1/12} = \sqrt[12]{x}$

7 $\sqrt[3]{\dfrac{x^2y}{3}} = \dfrac{\sqrt[3]{x^2y}}{\sqrt[3]{3}} \cdot \dfrac{\sqrt[3]{xy^2}}{\sqrt[3]{xy^2}} = \dfrac{\sqrt[3]{x^3y^3}}{\sqrt[3]{3xy^2}} = \dfrac{xy}{\sqrt[3]{3xy^2}}$

8
$$\begin{aligned}(x+2)(x^2-3x+5) &= x(x^2) + x(-3x) + x(5) + 2(x^2) + 2(-3x) + 2(5)\\ &= x^3 - 3x^2 + 5x + 2x^2 - 6x + 10\\ &= x^3 - x^2 - x + 10\end{aligned}$$

9 The leading term of $2x^2(2x+3)^4$ will be determined by multiplying $2x^2$ times $(2x)^4$. The " + 3" will affect other terms, but not the leading term. Hence, $2x^2(2x)^4 = 2x^2(16x^4) = 32x^6$.

10 By trial and error, $2x^2 + 7x - 15 = (2x-3)(x+5)$.

11 $3x^3 - 27x = 3x(x^2-9) = 3x(x+3)(x-3)$

12 Recognizing this polynomial as a sum of cubes, we get

$$64x^3 + 1 = (4x)^3 + 1^3 = (4x+1)[(4x)^2 - (4x)(1) + 1^2] = (4x+1)(16x^2 - 4x + 1).$$

13 We must recognize that $(\sqrt[3]{x})^3 = x$, and then factor as we would any other difference of cubes.

$$x - 5 = (\sqrt[3]{x})^3 - (\sqrt[3]{5})^3 = (\sqrt[3]{x} - \sqrt[3]{5})[(\sqrt[3]{x})^2 + (\sqrt[3]{x})(\sqrt[3]{5}) + (\sqrt[3]{5})^2] = (\sqrt[3]{x} - \sqrt[3]{5})(\sqrt[3]{x^2} + \sqrt[3]{5x} + \sqrt[3]{25})$$

14 Factor by grouping. $2x^2 + 4x - 3xy - 6y = 2x(x+2) - 3y(x+2) = (2x-3y)(x+2)$

15 Recognizing this polynomial as a difference of cubes, we get

$$x^{93} - 1 = (x^{31})^3 - 1^3 = (x^{31}-1)[(x^{31})^2 + (x^{31})(1) + 1^2] = (x^{31}-1)(x^{62} + x^{31} + 1).$$

16
$$\begin{aligned}\frac{3x}{x-2} + \frac{5}{x} - \frac{12}{x^2-2x} = \frac{3x(x) + 5(x-2) - 12}{x(x-2)} &= \frac{3x^2 + 5x - 10 - 12}{x(x-2)}\\ &= \frac{3x^2+5x-22}{x(x-2)} = \frac{(3x+11)(x-2)}{x(x-2)} = \frac{3x+11}{x}\end{aligned}$$

17 Multiply numerator and denominator by xy.

$$\frac{\dfrac{x^2}{y} - \dfrac{y^2}{x}}{\dfrac{x}{y} + 1 + \dfrac{y}{x}} = \frac{\left(\dfrac{x^2}{y} - \dfrac{y^2}{x}\right)\cdot xy}{\left(\dfrac{x}{y} + 1 + \dfrac{y}{x}\right)\cdot xy} = \frac{x^3 - y^3}{x^2 + xy + y^2} = \frac{(x-y)(x^2+xy+y^2)}{x^2+xy+y^2} = x - y$$

18 $$\frac{(x+h)^2+7(x+h)-(x^2+7x)}{h} = \frac{x^2+2xh+h^2+7x+7h-x^2-7x}{h} = \frac{2xh+h^2+7h}{h}$$
$$= \frac{h(2x+h+7)}{h} = 2x+h+7$$

19 $$\frac{6h^2}{\sqrt{x+h}-\sqrt{x}} = \frac{6h^2}{\sqrt{x+h}-\sqrt{x}} \cdot \frac{\sqrt{x+h}+\sqrt{x}}{\sqrt{x+h}+\sqrt{x}} = \frac{6h^2(\sqrt{x+h}+\sqrt{x})}{(x+h)-x} = \frac{6h^2(\sqrt{x+h}+\sqrt{x})}{h}$$
$$= 6h(\sqrt{x+h}+\sqrt{x})$$

20 $$(x+2)^3(4)(x-3)^3+(x-3)^4(3)(x+2)^2 = (x+2)^2(x-3)^3[4(x+2)+3(x-3)]$$
$$= (x+2)^2(x-3)^3(4x+8+3x-9) = (x+2)^2(x-3)^3(7x-1)$$

21 $$\frac{(x^2-3)^2(2x)-x^2(2)(x^2-3)(2x)}{[(x^2-3)^2]^2} = \frac{(x^2-3)(2x)[(x^2-3)-2x^2]}{(x^2-3)^4} = \frac{2x(-3-x^2)}{(x^2-3)^3}$$

Chapter 2: Equations and Inequalities

2.1 Exercises

1 $-3x+4=-1 \quad\Rightarrow\quad -3x=-5 \quad\Rightarrow\quad x=\frac{5}{3}$

2 $2x-4=-9 \quad\Rightarrow\quad 2x=-5 \quad\Rightarrow\quad x=-\frac{5}{2}$

3 $4x-3=-5x+6 \quad\Rightarrow\quad 4x+5x=6+3 \quad\Rightarrow\quad 9x=9 \quad\Rightarrow\quad x=1$

4 $5x-4=2(x-2) \quad\Rightarrow\quad 5x-4=2x-4 \quad\Rightarrow\quad 3x=0 \quad\Rightarrow\quad x=0$

5
$$\begin{aligned} 4(2y+5) &= 3(5y-2) && \{\text{given}\} \\ 8y+20 &= 15y-6 && \{\text{multiply terms}\} \\ 26 &= 7y && \{\text{get constants on one side, variables on the other}\} \\ y &= \tfrac{26}{7} && \{\text{divide by 7 to solve for } y\} \end{aligned}$$

6 $6(2y+3)-4(y-5)=0 \quad\Rightarrow\quad 12y+18-4y+20=0 \quad\Rightarrow\quad 8y=-38 \quad\Rightarrow\quad y=-\frac{19}{4}$

7
$$\begin{aligned} \tfrac{1}{5}x+4 &= 5-\tfrac{2}{7}x && \{\text{given}\} \\ \left(\tfrac{1}{5}x+4\right)\cdot 35 &= \left(5-\tfrac{2}{7}x\right)\cdot 35 && \{\text{multiply each side by the lcd, 35}\} \\ 7x+140 &= 175-10x && \{\text{distributive property}\} \\ 17x &= 35 && \{\text{get constants on one side, variables on the other}\} \\ x &= \tfrac{35}{17} && \{\text{divide by 17 to solve for } x\} \end{aligned}$$

8 $\frac{5}{3}x-1=4+\frac{2}{3}x \quad\Rightarrow\quad \frac{5}{3}x-\frac{2}{3}x=4+1 \quad\Rightarrow\quad x=5$

9 Decimal values can be thought of as equivalent rationals, e.g., $0.3=\frac{3}{10}$. We then multiply all terms by the lcd, 10 in this case, just as we multiplied by 35 in Exercise 7. A common mistake is to multiply 10 times a term such as $0.3(x+1)$ and get $3(10x+10)$. However, this is just like multiplying 10 times ab, which is $10ab$. Thus, $10[0.3(x+1)]=10(0.3)(x+1)=3(x+1)$. With that in mind, we proceed with the solution:
$[0.3(3+2x)+1.2x=3.2]\cdot 10 \quad\Rightarrow\quad 3(3+2x)+12x=32 \quad\Rightarrow\quad 9+6x+12x=32 \quad\Rightarrow\quad 18x=23 \quad\Rightarrow\quad x=\frac{23}{18}$

10 $[1.5x-0.7=0.6(3-5x)]\cdot 10 \quad\Rightarrow\quad 15x-7=6(3-5x) \quad\Rightarrow\quad 15x-7=18-30x \quad\Rightarrow\quad 45x=25 \quad\Rightarrow\quad x=\frac{25}{45}=\frac{5}{9}$

11 $\left[\dfrac{3+5x}{5}=\dfrac{4-x}{8}\right]\cdot 40 \quad\Rightarrow\quad 8(3+5x)=5(4-x) \quad\Rightarrow\quad 24+40x=20-5x \quad\Rightarrow\quad 45x=-4 \quad\Rightarrow\quad x=-\frac{4}{45}$

Note: You may have solved equations such as those in Exercise 11 using a process called *cross-multiplication* in the past. This method is sufficient for problems of the form $\dfrac{P}{Q}=\dfrac{R}{S}$, but the guidelines for solving an equation containing rational expressions (page 60 in the text) apply to rational equations of a more complex form.

12 $\left[\dfrac{2x-9}{4}=2+\dfrac{x}{12}\right]\cdot 12 \quad\Rightarrow\quad 3(2x-9)=24+x \quad\Rightarrow\quad 6x-27=24+x \quad\Rightarrow\quad 5x=51 \quad\Rightarrow\quad x=\frac{51}{5}$

13 The lcd is $4(4x+1)$ and we need to remember that x cannot equal $-\frac{1}{4}$.

$\left[\dfrac{13+2x}{4x+1}=\dfrac{3}{4}\right]\cdot 4(4x+1) \quad\Rightarrow\quad 4(13+2x)=3(4x+1) \quad\Rightarrow\quad 52+8x=12x+3 \quad\Rightarrow$

$49=4x \quad\Rightarrow\quad x=\frac{49}{4}$. Since $x\neq -\frac{1}{4}$, $x=\frac{49}{4}$ is a solution.

14 $\left[\dfrac{3}{7x-2}=\dfrac{9}{3x+1}\right]\cdot(7x-2)(3x+1) \quad\Rightarrow\quad 9x+3=63x-18 \quad\Rightarrow\quad 21=54x \quad\Rightarrow\quad x=\frac{7}{18}$

15 The lcd is x and we need to remember that x cannot equal 0.

$\left[6-\dfrac{5}{x}=4+\dfrac{3}{x}\right]\cdot x \quad\Rightarrow\quad 6x-5=4x+3 \quad\Rightarrow\quad 2x=8 \quad\Rightarrow\quad x=4$

16 $\left[\dfrac{3}{y}+\dfrac{6}{y}-\dfrac{1}{y}=11\right]\cdot y \quad\Rightarrow\quad 3+6-1=11y \quad\Rightarrow\quad 8=11y \quad\Rightarrow\quad y=\frac{8}{11}$

17 $(3x-2)^2=(x-5)(9x+4) \quad\Rightarrow\quad 9x^2-12x+4=9x^2-41x-20 \quad\Rightarrow\quad 29x=-24 \quad\Rightarrow\quad x=-\frac{24}{29}$

18 $(x+5)^2+3=(x-2)^2 \quad\Rightarrow\quad x^2+10x+25+3=x^2-4x+4 \quad\Rightarrow\quad 14x=-24 \quad\Rightarrow\quad x=-\frac{12}{7}$

19 $(4x-7)(2x+3)-8x(x-4)=0 \quad\Rightarrow\quad 8x^2-2x-21-8x^2+32x=0 \quad\Rightarrow\quad 30x=21 \quad\Rightarrow\quad x=\frac{7}{10}$

20 $(2x+9)(4x-3)=8x^2-12 \quad\Rightarrow\quad 8x^2+30x-27=8x^2-12 \quad\Rightarrow\quad 30x=15 \quad\Rightarrow\quad x=\frac{1}{2}$

21 $\left[\dfrac{3x+1}{6x-2}=\dfrac{2x+5}{4x-13}\right]\cdot(6x-2)(4x-13) \quad\Rightarrow\quad (3x+1)(4x-13)=(2x+5)(6x-2) \quad\Rightarrow$

$12x^2-35x-13=12x^2+26x-10 \quad\Rightarrow\quad -3=61x \quad\Rightarrow\quad x=-\frac{3}{61}$ $\left\{\text{note that } x\neq\frac{1}{3},\frac{13}{4}\right\}$

22 $\left[\dfrac{7x+2}{14x-3}=\dfrac{x-8}{2x+3}\right]\cdot(14x-3)(2x+3) \quad\Rightarrow\quad (7x+2)(2x+3)=(x-8)(14x-3) \quad\Rightarrow$

$14x^2+25x+6=14x^2-115x+24 \quad\Rightarrow\quad 140x=18 \quad\Rightarrow\quad x=\frac{9}{70}$

23 $\left[\dfrac{2}{5}+\dfrac{4}{10x+5}=\dfrac{7}{2x+1}\right]\cdot 5(2x+1) \quad\Rightarrow\quad 2(2x+1)+4=7(5) \quad\Rightarrow\quad (4x+2)+4=35 \quad\Rightarrow$

$4x=29 \quad\Rightarrow\quad x=\frac{29}{4}$

24 $\left[\dfrac{-5}{3x-9}+\dfrac{4}{x-3}=\dfrac{5}{6}\right]\cdot 6(x-3) \quad\Rightarrow\quad -5(2)+4(6)=5(x-3) \quad\Rightarrow\quad -10+24=5x-15 \quad\Rightarrow$

$29=5x \quad\Rightarrow\quad x=\frac{29}{5}$

25 Since $2x-4=2(x-2)$ and $3x-6=3(x-2)$, the lcd is $2\cdot 3\cdot 5(x-2)=30(x-2)$.

$\left[\dfrac{3}{2x-4}-\dfrac{5}{3x-6}=\dfrac{3}{5}\right]\cdot 30(x-2) \quad\Rightarrow\quad 3(15)-5(10)=3(6)(x-2) \quad\Rightarrow\quad 45-50=18x-36 \quad\Rightarrow$

$31=18x \quad\Rightarrow\quad x=\frac{31}{18}$

26 Since $2x+6=2(x+3)$ and $5x+15=5(x+3)$, the lcd is $2\cdot 3\cdot 5(x+3)=30(x+3)$.

$\left[\dfrac{9}{2x+6}-\dfrac{7}{5x+15}=\dfrac{2}{3}\right]\cdot 30(x+3) \quad\Rightarrow\quad 9(15)-7(6)=2(10)(x+3) \quad\Rightarrow\quad 135-42=20x+60 \quad\Rightarrow$

$33=20x \quad\Rightarrow\quad x=\frac{33}{20}$

27 $4-\dfrac{5}{3x-7}=4 \quad\Rightarrow\quad \dfrac{5}{3x-7}=0 \quad\Rightarrow$ **no solution** since the numerator is never 0.

28 $\dfrac{6}{2x+11}+5=5 \quad\Rightarrow\quad \dfrac{6}{2x+11}=0 \quad\Rightarrow$ **no solution** since the numerator is never 0.

29 $\dfrac{1}{2x-1}=\dfrac{4}{8x-4} \quad\Rightarrow\quad \dfrac{1}{2x-1}=\dfrac{4}{4(2x-1)} \quad\Rightarrow\quad \dfrac{1}{2x-1}=\dfrac{1}{2x-1}$.

This is an identity, and the solutions consist of every number in the domains of the given expressions.

Thus, the solutions are all real numbers except $\frac{1}{2}$, which we denote by $\mathbb{R}-\left\{\frac{1}{2}\right\}$.

30 $\dfrac{4}{5x+2} - \dfrac{12}{15x+6} = 0 \Rightarrow \dfrac{4}{5x+2} = \dfrac{12}{3(5x+2)} \Rightarrow \dfrac{4}{5x+2} = \dfrac{4}{5x+2}$, an identity.

Thus, the solutions are $\mathbb{R} - \left\{-\frac{2}{5}\right\}$.

31 $\left[\dfrac{7}{y^2-4} - \dfrac{4}{y+2} = \dfrac{5}{y-2}\right] \cdot (y+2)(y-2) \Rightarrow 7 - 4(y-2) = 5(y+2) \Rightarrow$

$7 - 4y + 8 = 5y + 10 \Rightarrow 5 = 9y \Rightarrow y = \frac{5}{9}$

32 $\left[\dfrac{4}{2u-3} + \dfrac{10}{4u^2-9} = \dfrac{1}{2u+3}\right] \cdot (2u+3)(2u-3) \Rightarrow 4(2u+3) + 10 = 2u - 3 \Rightarrow$

$8u + 12 + 10 = 2u - 3 \Rightarrow 6u = -25 \Rightarrow u = -\frac{25}{6}$

33 $(x+3)^3 - (3x-1)^2 = x^3 + 4 \Rightarrow (x^3 + 9x^2 + 27x + 27) - (9x^2 - 6x + 1) = x^3 + 4 \Rightarrow$

$27x + 27 + 6x - 1 = 4 \Rightarrow 33x = -22 \Rightarrow x = -\frac{2}{3}$

34 $(x-1)^3 = (x+1)^3 - 6x^2 \Rightarrow x^3 - 3x^2 + 3x - 1 = (x^3 + 3x^2 + 3x + 1) - 6x^2 \Rightarrow$

$-3x^2 + 3x - 1 = -3x^2 + 3x + 1 \Rightarrow -1 = 1$. This is a contradiction and there is **no solution**.

35 $\left[\dfrac{9x}{3x-1} = 2 + \dfrac{3}{3x-1}\right] \cdot (3x-1) \Rightarrow 9x = 2(3x-1) + 3 \Rightarrow 9x = 2(3x-1) + 3 \Rightarrow$

$9x = 6x - 2 + 3 \Rightarrow 3x = 1 \Rightarrow x = \frac{1}{3}$, which is not in the domain of the given expressions. **No solution**

36 $\left[\dfrac{2x}{2x+3} + \dfrac{6}{4x+6} = 5\right] \cdot 2(2x+3) \Rightarrow 2x(2) + 6 = 5(2)(2x+3) \Rightarrow 4x + 6 = 20x + 30 \Rightarrow$

$-24 = 16x \Rightarrow x = -\frac{3}{2}$, which is not in the domain of the given expressions. **No solution**

37 $\left[\dfrac{1}{x+4} + \dfrac{3}{x-4} = \dfrac{3x+8}{x^2-16}\right] \cdot (x+4)(x-4) \Rightarrow x - 4 + 3(x+4) = 3x + 8 \Rightarrow$

$x - 4 + 3x + 12 = 3x + 8 \Rightarrow 4x + 8 = 3x + 8 \Rightarrow x = 0$

38 $\left[\dfrac{3}{2x+3} + \dfrac{5}{2x-3} = \dfrac{4x+6}{4x^2-9}\right] \cdot (2x+3)(2x-3) \Rightarrow 3(2x-3) + 5(2x+3) = 4x + 6 \Rightarrow$

$6x - 9 + 10x + 15 = 4x + 6 \Rightarrow 12x = 0 \Rightarrow x = 0$

39 $\left[\dfrac{4}{x+2} + \dfrac{1}{x-2} = \dfrac{5x-6}{x^2-4}\right] \cdot (x+2)(x-2) \Rightarrow 4(x-2) + 1(x+2) = 5x - 6 \Rightarrow$

$4x - 8 + x + 2 = 5x - 6 \Rightarrow 5x - 6 = 5x - 6$ {or $0 = 0$}, indicating an identity. The solution is $\mathbb{R} - \{\pm 2\}$.

40 $\left[\dfrac{2}{2x+5} + \dfrac{3}{2x-5} = \dfrac{10x+5}{4x^2-25}\right] \cdot (2x+5)(2x-5) \Rightarrow 2(2x-5) + 3(2x+5) = 10x + 5 \Rightarrow$

$4x - 10 + 6x + 15 = 10x + 5 \Rightarrow 10x + 5 = 10x + 5$ {or $0 = 0$}, indicating an identity.

The solution is $\mathbb{R} - \left\{\pm\frac{5}{2}\right\}$.

41 $\left[\dfrac{2}{2x+1} - \dfrac{3}{2x-1} = \dfrac{-2x+7}{4x^2-1}\right] \cdot (2x+1)(2x-1) \Rightarrow 2(2x-1) - 3(2x+1) = -2x + 7 \Rightarrow$

$4x - 2 - 6x - 3 = -2x + 7 \Rightarrow -2x - 5 = -2x + 7 \Rightarrow -5 = 7$, a contradiction. **No solution**

42 $\left[\dfrac{3}{2x+5} + \dfrac{4}{2x-5} = \dfrac{14x+3}{4x^2-25}\right] \cdot (2x+5)(2x-5) \Rightarrow 3(2x-5) + 4(2x+5) = 14x + 3 \Rightarrow$

$6x - 15 + 8x + 20 = 14x + 3 \Rightarrow 14x + 5 = 14x + 3 \Rightarrow 2 = 0$, a contradiction. **No solution**

43 $\left[\dfrac{5}{2x+3} + \dfrac{4}{2x-3} = \dfrac{14x+3}{4x^2-9}\right] \cdot (2x+3)(2x-3) \Rightarrow 5(2x-3) + 4(2x+3) = 14x + 3 \Rightarrow$

$10x - 15 + 8x + 12 = 14x + 3 \Rightarrow 18x - 3 = 14x + 3 \Rightarrow 4x = 6 \Rightarrow$

$x = \frac{3}{2}$, which is not in the domain of the given expressions. **No solution**

44 $\left[\dfrac{-3}{x+4} + \dfrac{7}{x-4} = \dfrac{-5x+4}{x^2-16}\right] \cdot (x+4)(x-4) \Rightarrow -3(x-4) + 7(x+4) = -5x+4 \Rightarrow$
$-3x + 12 + 7x + 28 = -5x + 4 \Rightarrow 4x + 40 = -5x + 4 \Rightarrow 9x = -36 \Rightarrow$
$x = -4$, which is not in the domain of the given expressions. **No solution**

Note: For Exercises 45–50, we must show that LS = RS.

45 LS $= (4x-3)^2 - 16x^2 = (16x^2 - 24x + 9) - 16x^2 = 9 - 24x =$ RS

46 LS $= (3x-4)(2x+1) + 5x = (6x^2 - 5x - 4) + 5x = 6x^2 - 4 =$ RS

47 LS $= \dfrac{x^2-16}{x+4} = \dfrac{(x+4)(x-4)}{x+4} = x - 4 =$ RS, $\forall x$ except $x = -4$.

48 LS $= \dfrac{x^3+8}{x+2} = \dfrac{(x+2)(x^2-2x+4)}{x+2} = x^2 - 2x + 4 =$ RS, $\forall x$ except $x = -2$.

49 LS $= \dfrac{5x^2+8}{x} = \dfrac{5x^2}{x} + \dfrac{8}{x} = \dfrac{8}{x} + 5x =$ RS, $\forall x$ except $x = 0$.

50 LS $= \dfrac{49x^2-25}{7x-5} = \dfrac{(7x+5)(7x-5)}{7x-5} = 7x + 5 =$ RS, $\forall x$ except $x = \dfrac{5}{7}$.

51 Substituting -2 for x in $4x + 1 + 2c = 5c - 3x + 6$ yields $-7 + 2c = 5c + 12 \Rightarrow -19 = 3c \Rightarrow c = -\frac{19}{3}$.

52 Substituting 4 for x in $3x - 2 + 6c = 2c - 5x + 1$ yields $10 + 6c = 2c - 19 \Rightarrow 4c = -29 \Rightarrow c = -\frac{29}{4}$.

53 (a) $\dfrac{7x}{x-5} = \dfrac{42}{x-5} \Rightarrow 7x = 42 \Rightarrow x = 6$ {the second equation},
and the two equations are equivalent since they have the same solution.

(b) $\dfrac{3x}{x-5} = \dfrac{15}{x-5}$ • No, 5 is not a solution of the first equation since it is undefined if $x = 5$.

54 (a) $\dfrac{6x}{x-7} = \dfrac{54}{x-7} \Rightarrow 6x = 54 \Rightarrow x = 9$ {the second equation},
and the two equations are equivalent since they have the same solution.

(b) $\dfrac{8x}{x-7} = \dfrac{56}{x-7}$ • No, 7 is not a solution of the first equation since it is undefined if $x = 7$.

55 Substituting $\frac{5}{3}$ for x in $ax + b = 0$ yields $\frac{5}{3}a + b = 0$, or, equivalently, $b = -\frac{5}{3}a$.
Choose any a and b such that $b = -\frac{5}{3}a$. For example, let $a = 3$ and $b = -5$.

56 Substituting $\frac{5}{3}$ for x in $ax^2 + bx = 0$ yields $\frac{25}{9}a + \frac{5}{3}b = 0$, or, equivalently, $\frac{5}{3}b = -\frac{25}{9}a \Rightarrow b = -\frac{5}{3}a$.
Choose any a and b such that $b = -\frac{5}{3}a$. For example, let $a = 3$ and $b = -5$.

57 Going from the second line to the third line, we must remember that division by the variable expression $x - 2$ is not allowed. ★ $x + 1 = x + 2$

58 Going from the third line to the fourth line, we must remember that division by the variable expression $x + 3$ is not allowed. ★ $x + 2 = x + 1$

59
$EK + L = D - TK$ {given equation, solve for K}
$EK + TK = D - L$ {get K-terms on one side, everything else on the other}
$K(E + T) = D - L$ {factor out K}
$K = \dfrac{D - L}{E + T}$ {divide by $(E + T)$ to solve for K}

60 $CD + C = PC + R \Rightarrow CD + C - PC = R \Rightarrow C(D + 1 - P) = R \Rightarrow C = \dfrac{R}{D + 1 - P}$

61 $N = \dfrac{Q + 1}{Q} \Rightarrow NQ = Q + 1 \Rightarrow NQ - Q = 1 \Rightarrow Q(N - 1) = 1 \Rightarrow Q = \dfrac{1}{N - 1}$

62 $\beta = \dfrac{\alpha}{1 - \alpha} \Rightarrow \beta(1 - \alpha) = \alpha \Rightarrow \beta - \beta\alpha = \alpha \Rightarrow \beta = \alpha + \beta\alpha \Rightarrow \beta = \alpha(1 + \beta) \Rightarrow \alpha = \dfrac{\beta}{1 + \beta}$

63 $I = Prt \Rightarrow \dfrac{I}{rt} = \dfrac{Prt}{rt} \Rightarrow P = \dfrac{I}{rt}$

64 $C = 2\pi r \Rightarrow \dfrac{C}{2\pi} = \dfrac{2\pi r}{2\pi} \Rightarrow r = \dfrac{C}{2\pi}$

65 $A = \dfrac{1}{2}bh \Rightarrow 2A = bh \Rightarrow h = \dfrac{2A}{b}$

66 $V = \dfrac{1}{3}\pi r^2 h \Rightarrow 3V = \pi r^2 h \Rightarrow h = \dfrac{3V}{\pi r^2}$

67 $F = g\dfrac{mM}{d^2} \Rightarrow Fd^2 = gmM \Rightarrow m = \dfrac{Fd^2}{gM}$

68 $R = \dfrac{V}{I} \Rightarrow RI = V \Rightarrow I = \dfrac{V}{R}$

69 $P = 2l + 2w \Rightarrow P - 2l = 2w \Rightarrow w = \dfrac{P - 2l}{2}$

70 $A = P + Prt \Rightarrow A - P = Prt \Rightarrow r = \dfrac{A - P}{Pt}$

71 $A = \dfrac{1}{2}(b_1 + b_2)h \Rightarrow 2A = (b_1 + b_2)h \Rightarrow \dfrac{2A}{h} = b_1 + b_2 \Rightarrow b_1 = \dfrac{2A}{h} - b_2$, or $b_1 = \dfrac{2A - hb_2}{h}$

72 $s = \dfrac{1}{2}gt^2 + v_0 t \Rightarrow 2s = gt^2 + 2v_0 t \Rightarrow 2s - gt^2 = 2v_0 t \Rightarrow v_0 = \dfrac{2s - gt^2}{2t}$

73

$S = \dfrac{p}{q + p(1 - q)}$	{given equation, solve for q}
$S[q + p(1 - q)] = p$	{eliminate the fraction}
$Sq + Sp(1 - q) = p$	{multiply terms}
$Sq + Sp - Spq = p$	{multiply terms}
$Sq - Spq = p - Sp$	{isolate terms containing q}
$Sq(1 - p) = p(1 - S)$	{factor our Sq}
$q = \dfrac{p(1 - S)}{S(1 - p)}$	{divide by $S(1 - p)$ to solve for q}

74 $S = 2(lw + hw + hl) \Rightarrow S = 2lw + 2hw + 2hl \Rightarrow S - 2lw = 2h(w + l) \Rightarrow h = \dfrac{S - 2lw}{2(w + l)}$

75 $\dfrac{1}{f} = \dfrac{1}{p} + \dfrac{1}{q}$ {multiply by the lcd, fpq} $\Rightarrow$

$pq = fq + fp \Rightarrow pq - fq = fp \Rightarrow q(p - f) = fp \Rightarrow q = \dfrac{fp}{p - f}$

76 $\dfrac{1}{R} = \dfrac{1}{R_1} + \dfrac{1}{R_2} + \dfrac{1}{R_3}$ {multiply by the lcd, $RR_1R_2R_3$} $\Rightarrow R_1R_2R_3 = RR_2R_3 + RR_1R_3 + RR_1R_2 \Rightarrow$

$R_1R_2R_3 - RR_2R_3 - RR_1R_2 = RR_1R_3 \Rightarrow R_2(R_1R_3 - RR_3 - RR_1) = RR_1R_3 \Rightarrow$

$R_2 = \dfrac{RR_1R_3}{R_1R_3 - RR_3 - RR_1}$

77 The y-value decreases 1.2 units for each 1 unit increase in the x-value. The data are best described by equation (1), $y = -1.2x + 2$.

78 The y-values are increasing rapidly and can best be described by equation (4), $y = x^3 - x^2 + x - 10$.

2.2 Exercises

1 Let x denote the score on the next test.

$$\frac{75+82+71+84+x}{5}=80 \quad\Rightarrow\quad 312+x=5(80) \quad\Rightarrow\quad x=400-312=88$$

2 The pre-final average is $\dfrac{72+83+65+73+62}{5}=71$. If x denotes the score on the final exam, then

$$\tfrac{2}{3}(71)+\tfrac{1}{3}(x)=76 \quad\Rightarrow\quad \tfrac{1}{3}x=\tfrac{228-142}{3} \quad\Rightarrow\quad \tfrac{1}{3}x=\tfrac{86}{3} \quad\Rightarrow\quad x=86.$$

3 Let x denote the gross pay. Gross pay $-$ deductions $=$ Net (take-home) pay $\Rightarrow$

$$x-0.40x=504 \quad\Rightarrow\quad x(1-0.40)=504 \quad\Rightarrow\quad 0.60x=504 \quad\Rightarrow\quad x=\tfrac{504}{0.60} \quad\Rightarrow\quad x=840.$$

4 Let x denote the amount of the bill before the tax and tip are added. Bill $+$ Tax $+$ Tip $=70$ $\Rightarrow$

$x+0.08x+0.15(x+0.08x)=70 \quad\Rightarrow\quad 1.08x+0.15(1.08x)=70 \quad\Rightarrow\quad 1.15(1.08x)=70 \quad\Rightarrow$

$$1.242x=70 \quad\Rightarrow\quad x\approx 56.36.$$

5 (a) $\text{IQ}=\dfrac{\text{mental age (MA)}}{\text{chronological age (CA)}}\times 100=\dfrac{15}{10}\times 100=150.$

(b) $\text{CA}=15$ and $\text{IQ}=140 \quad\Rightarrow\quad 140=\dfrac{\text{MA}}{15}\times 100 \quad\Rightarrow\quad \dfrac{140}{100}=\dfrac{\text{MA}}{15} \quad\Rightarrow\quad \text{MA}=\dfrac{15(140)}{100}=21.$

6 Let S denote the surface area of Earth.

$$\text{Then, } 0.708S=361\times 10^6 \quad\Rightarrow\quad S=\frac{361\times 10^6}{0.708} \quad\Rightarrow\quad S\approx 510\times 10^6\ \text{km}^2.$$

7 Let x denote the number of months needed to recover the cost of the insulation. The savings in one month is 10% of \$200 $=$ \$20, so the savings in x months is $20x$. $\qquad 20x=2400 \quad\Rightarrow\quad x=120$ months (or 10 yr).

8 Let x denote the number of hours the workman made $\$12\times 1.5=\$18$ per hour.

$$40(\$12)+x(\$18)=\$714 \quad\Rightarrow\quad 480+18x=714 \quad\Rightarrow\quad 18x=234 \quad\Rightarrow\quad x=13\ \text{hr}.$$

9 Let x denote the amount invested in the 4% account and $500{,}000-x$ the amount invested in the 3.2% account.

Use the formula $I=Prt$ (interest $=$ principal $\times$ rate $\times$ time) with $t=1$.

$\text{Interest}_{\text{first account}}+\text{Interest}_{\text{second account}}=\text{Interest}_{\text{total}} \quad\Rightarrow\quad x(0.04)+(500{,}000-x)(0.032)=18{,}500 \quad\Rightarrow$

$0.04x+16{,}000-0.032x=18{,}500 \quad\Rightarrow\quad 0.008x=2500 \quad\Rightarrow\quad x=312{,}500.$ Since only \$250,000 can be insured in the 4% account, we cannot fully insure the money and earn annual interest of \$18,500.

10 Let x denote the amount (in millions) invested in bonds.

$x(0.06)+(800-x)(0.05)=42 \quad\Rightarrow\quad 0.06x+40-0.05x=42 \quad\Rightarrow\quad 0.01x=2 \quad\Rightarrow\quad x=200.$

The arena should be financed by selling \$200 million in bonds and borrowing \$600 million.

11 Let x denote the number of children and $600-x$ the number of adults.

$\text{Receipts}_{\text{children}}+\text{Receipts}_{\text{adults}}=\text{Receipts}_{\text{total}} \quad\Rightarrow\quad x(6)+(600-x)(9)=4800 \quad\Rightarrow$

$$6x+5400-9x=4800 \quad\Rightarrow\quad -3x=-600 \quad\Rightarrow\quad x=200\ \text{children}.$$

12 Let x denote the engineer's hours. $\text{Bill}_{\text{engineer}}+\text{Bill}_{\text{assistant}}=\text{Bill}_{\text{total}} \quad\Rightarrow\quad 60(x)+20(x-5)=580 \quad\Rightarrow$

$80x=680 \quad\Rightarrow\quad x=8.5.$ The engineer spent 8.5 hr on the job and the assistant spent $8.5-5=3.5$ hr.

13 Let x denote the number of ounces of glucose solution and $7-x$ the number of ounces of water.

$\text{Glucose}_{30\%}+\text{Water}_{0\%}=\text{Glucose}_{20\%} \quad\Rightarrow\quad x(0.30)+(7-x)(0)=7(0.20) \quad\Rightarrow$

$0.3x=1.4 \quad\Rightarrow\quad x=\frac{14}{3}$. Use $\frac{14}{3}$ oz of the 30% glucose solution and $7-\frac{14}{3}=\frac{7}{3}$ oz of water.

14 Let x denote the number of mL of 1% solution.
$x(1)+(15-x)(10)=15(2)$ {all in %} $\Rightarrow$ $x+150-10x=30$ $\Rightarrow$ $-9x=-120$ $\Rightarrow$ $x=\frac{120}{9}=\frac{40}{3}$.
Use $\frac{40}{3}$ mL of the 1% solution and $15-\frac{40}{3}=\frac{5}{3}$ mL of the 10% solution.

15 Let x denote the number of grams of British sterling silver and $200-x$ the number of grams of pure copper. We will compare the amounts of pure copper. {The percentages of pure copper are 7.5%, 100%, and 10% or, equivalently, 0.075, 1, and 0.10.} $\text{Copper}_{\text{British sterling silver}}+\text{Copper}_{\text{pure}}=\text{Copper}_{\text{alloy}}$ $\Rightarrow$
$(0.075)x+1(200-x)=(0.10)(200)$ $\Rightarrow$ $0.075x+200-x=20$ $\Rightarrow$ $180=0.925x$ $\Rightarrow$ $x=194.6$.
Use 194.6 g of British sterling silver and $200-194.6=5.4$ g of pure copper.

16 Let x denote the number of mL of the elixir. $x(5)+(100-x)(0)=100(2)$ $\Rightarrow$ $5x=200$ $\Rightarrow$ $x=40$.
Use 40 mL of the elixir and $100-40=60$ mL of the cherry-flavored syrup.

17 (a) They will meet when the sum of their distances is 224. Let t denote the desired number of seconds.
Using distance = rate × time, we have $1.5t+2t=224$ $\Rightarrow$ $3.5t=224$ $\Rightarrow$ $t=64$ sec.

(b) The children will have walked $64(1.5)=96$ m and $64(2)=128$ m, respectively.

18 Let t denote the number of seconds that the second runner has been running. The first runner has been running for 5 minutes, so the distance for the first runner at time t is $6\left(\frac{5}{60}\right)+6t$. The distance for the second runner is $8t$.
Equating yields $\frac{1}{2}+6t=8t$ $\Rightarrow$ $\frac{1}{2}=2t$ $\Rightarrow$ $t=\frac{1}{4}$ hr, or 15 min.

19 Let r denote the rate of the snowplow. At 8:30 A.M., the car has traveled 15 miles and the snowplow has been traveling for $2\frac{1}{2}=\frac{5}{2}$ hours. Using the relationship time × rate = distance, $\frac{5}{2}r=15$ $\Rightarrow$ $r=6$ mi/hr.

20 Let t denote the time in hours after 1:15 P.M. The first child has walked 1 mile (4 mi/hr for 15 minutes), so the first child's distance is $1+4t$ miles and the second child's distance is $6t$.
$(1+4t)+6t=2$ $\Rightarrow$ $10t=1$ $\Rightarrow$ $t=\frac{1}{10}$ hr, or 6 min. after 1:21 P.M.

21 (a) Let x denote the rate of the river's current. The rates of the boat upstream and downstream are $5-x$ and $5+x$, respectively. Use distance = rate × time.
$\text{Distance}_{\text{upstream}}=\text{Distance}_{\text{downstream}}$ $\Rightarrow$ $(5-x)\frac{15}{60}=(5+x)\frac{12}{60}$ $\Rightarrow$ $(5-x)\frac{1}{4}=(5+x)\frac{1}{5}$ $\Rightarrow$
$5(5-x)=4(5+x)$ $\Rightarrow$ $25-5x=20+4x$ $\Rightarrow$ $5=9x$ $\Rightarrow$ $x=\frac{5}{9}$ mi/hr.

(b) The distance upstream is $\left(5-\frac{5}{9}\right)\frac{1}{4}=\frac{40}{9}\cdot\frac{1}{4}=\frac{10}{9}$. The total distance is $2\cdot\frac{10}{9}=\frac{20}{9}$, or $2\frac{2}{9}$ mi.

22 Let x denote the number of gallons used in the city. $\text{Miles}_{\text{city}}+\text{Miles}_{\text{highway}}=\text{Miles}_{\text{total}}$ $\Rightarrow$
$x(27)+(51-x)(38)=1762$ $\Rightarrow$ $27x+1938-38x=1762$ $\Rightarrow$ $176=11x$ $\Rightarrow$ $x=16$.
The number of miles driven in the city is $16\cdot 27=432$ mi.

23 Let x denote the distance to the target. We know the total time involved and need a formula for time. Solving $d=rt$ for t gives us $t=d/r$.
$\text{Time}_{\text{to target}}+\text{Time}_{\text{from target}}=\text{Time}_{\text{total}}$ $\Rightarrow$ $\dfrac{x}{3300}+\dfrac{x}{1100}=1.5$ {multiply by the lcd, 3300} $\Rightarrow$
$x+3x=1.5(3300)$ $\Rightarrow$ $4x=4950$ $\Rightarrow$ $x=1237.5$ ft.

24 Let x denote the miles in one direction. A 6-minute-mile pace is equivalent to a rate of $\frac{1}{6}$ mile/min. Solving $d=rt$ for t gives us $t=d/r$. $\text{Minutes}_{\text{north}}+\text{Minutes}_{\text{south}}=\text{Minutes}_{\text{total}}$ $\Rightarrow$ $\dfrac{x}{1/6}+\dfrac{x}{1/7}=47$ $\Rightarrow$
$6x+7x=47$ $\Rightarrow$ $x=\frac{47}{13}$. The total distance is $2\cdot\frac{47}{13}=\frac{94}{13}$, or $7\frac{3}{13}$ mi.

25 Let l denote the length of the side parallel to the river bank. $P = 2w + l$

(a) $l = 2w \Rightarrow P = 2w + 2w = 4w$. $4w = 180 \Rightarrow w = 45$ ft and $A = (45)(90) = 4050$ ft^2.

(b) $l = \frac{1}{2}w \Rightarrow P = 2w + \frac{1}{2}w = \frac{5}{2}w$. $\frac{5}{2}w = 180 \Rightarrow w = 72$ ft and $A = (72)(36) = 2592$ ft^2.

(c) $l = w \Rightarrow P = 2w + w = 3w$. $3w = 180 \Rightarrow w = 60$ ft and $A = (60)(60) = 3600$ ft^2.

26 The first story has cross-sectional area $8 \times 30 = 240$.
The second story has cross-sectional area $(30 \times 3) + 2\left(\frac{1}{2}\right)(15)(h - 3) = 90 + 15h - 45 = 15h + 45$.
Equating yields $15h + 45 = 240 \Rightarrow 15h = 195 \Rightarrow h = 13$ ft.

27 $\text{Area}_{\text{semicircle}} + \text{Area}_{\text{rectangle}} = \text{Area}_{\text{total}} \Rightarrow \frac{1}{2}\pi r^2 + lw = 24 \Rightarrow \frac{1}{2}\pi\left(\frac{3}{2}\right)^2 + \left(h - \frac{3}{2}\right)3 = 24 \Rightarrow$
$\left(h - \frac{3}{2}\right)3 = 24 - \frac{9\pi}{8} \Rightarrow h - \frac{3}{2} = 8 - \frac{3\pi}{8} \Rightarrow h = \frac{19}{2} - \frac{3\pi}{8} \approx 8.32$ ft.

28 Let b_2 denote the larger base. $A = \frac{1}{2}(b_1 + b_2)h \Rightarrow 5 = \frac{1}{2}(3 + b_2)(1) \Rightarrow 10 = 3 + b_2 \Rightarrow b_2 = 7$ ft.

29 Let h_1 denote the height of the cylinder. $V = \frac{2}{3}\pi r^3 + \pi r^2 h_1 = 11{,}250\pi$ and $r = 15 \Rightarrow$
$2250\pi + 225\pi h_1 = 11{,}250\pi \Rightarrow 225\pi h_1 = 9000\pi \Rightarrow h_1 = 40$. The total height is 40 ft + 15 ft = 55 ft.

30 $V = \frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3$, $r = 1$, and $V = 8 \Rightarrow 8 = \frac{\pi}{3}h + \frac{2\pi}{3} \Rightarrow h = \frac{3}{\pi}\left(8 - \frac{2\pi}{3}\right) = \frac{24}{\pi} - 2 \approx 5.64$ in.

31 Let x denote the desired time. Using the rates (in minutes), $\dfrac{1}{90} + \dfrac{1}{60} = \dfrac{1}{x} \Rightarrow$
{multiply by the lcd, $180x$} $2x + 3x = 180 \Rightarrow 5x = 180 \Rightarrow x = 36$ min.

32 Let x denote the desired time. Using the hourly rates, $\dfrac{1}{8} + \dfrac{1}{5} = \dfrac{1}{x} \Rightarrow$
{multiply by the lcd, $40x$} $5x + 8x = 40 \Rightarrow 13x = 40 \Rightarrow x = \frac{40}{13}$ hr.

33 Let x denote the desired time. Using the rates (in minutes), $\dfrac{1}{45} + \dfrac{1}{x} = \dfrac{1}{20} \Rightarrow$
{multiply by the lcd, $180x$} $4x + 180 = 9x \Rightarrow 180 = 5x \Rightarrow x = 36$ min.

34 The larger pump will empty $\frac{4}{5}$ of the tank in 4 hours (1 P.M. to 5 P.M.). The smaller pump can empty the remaining $\frac{1}{5}$ tank in $\frac{1}{5}$ of 8 hours, or 1 hr 36 min. Start the smaller pump 1 hr 36 min before 5 P.M., or 3:24 P.M.

35 First, a simple example of calculating a GPA. Suppose a student gets a 3-credit A (worth 4 honor points) and a 4-credit C (worth 2 honor points). Then,

$$\text{GPA} = \frac{\text{total weighted honor points}}{\text{total credit hours}} = \frac{3(4.0) + 4(2.0)}{3 + 4} = \frac{12 + 8}{7} = \frac{20}{7} \approx 2.86.$$

Let x denote the number of additional credit hours.
$\text{GPA} = 3.2 \Rightarrow \dfrac{48(2.75) + x(4.0)}{48 + x} = 3.2 \Rightarrow 132 + 4x = 3.2(48 + x) \Rightarrow$
$132 + 4x = 153.6 + 3.2x \Rightarrow 21.6 = 0.8x \Rightarrow x = \frac{21.6}{0.8} = 27.$

36 Let x denote the numerical amount to be added to V and R.
$I = \dfrac{V}{R} = \dfrac{110}{50}$. Thus, $2I = \dfrac{110 + x}{50 + x}$ and $2I = \dfrac{2 \cdot 110}{50} \Rightarrow \dfrac{110 + x}{50 + x} = \dfrac{220}{50} \Rightarrow$
$220(50 + x) = 50(110 + x) \Rightarrow 11{,}000 + 220x = 5500 + 50x \Rightarrow 170x = -5500 \Rightarrow$
$x = -\frac{550}{17}$. *Decrease* both V and R by $\frac{550}{17} \approx 32.35$.

37 (a) $T = T_0 - \left(\frac{5.5}{1000}\right)h$ • $h = 5280$ ft and $T_0 = 70$°F $\Rightarrow T = 70 - \left(\frac{5.5}{1000}\right)5280 = 40.96$°F.

(b) $T = 32$°F $\Rightarrow 32 = 70 - \left(\frac{5.5}{1000}\right)h \Rightarrow \left(\frac{5.5}{1000}\right)h = 38 \Rightarrow h = 38\left(\frac{1000}{5.5}\right) \approx 6909$ ft.

38 (a) $h = 227(T - D)$ • $T = 70°\text{F}$ and $D = 55°\text{F}$ $\Rightarrow$ $h = 227(70 - 55) = 3405$ ft.

(b) $h = 3500$ ft and $D = 65°\text{F}$ $\Rightarrow$ $3500 = 227(T - 65)$ $\Rightarrow$ $T = \frac{3500}{227} + 65 \approx 80.4°\text{F}$

39 $T = B - \left(\frac{3}{1000}\right)h$ • $B = 55°\text{F}$ and $h = 10{,}000 - 4000 = 6000$ ft $\Rightarrow$ $T = 55 - \left(\frac{3}{1000}\right)(6000) = 37°\text{F}$.

40 (a) $h = 65 + 3.14x$ • $x = 30$ cm $\Rightarrow$ $h = 65 + 3.14(30) = 159.2$ cm.

(b) $h = 73.6 + 3.0x$ • $x = 34$ $\Rightarrow$ $h = 73.6 + 3(34) = 175.6$ cm. The height of the skeleton has decreased by $175.6 - 174 = 1.6$ cm due to aging after age 30. $\frac{1.6}{0.06} \approx 27$ years. The male was approximately $30 + 27 = 57$ years old at death.

2.3 Exercises

1 $6x^2 + x - 12 = 0$ $\Rightarrow$ $(2x + 3)(3x - 4) = 0$ $\Rightarrow$ $2x + 3 = 0$ or $3x - 4 = 0$ $\Rightarrow$ $x = -\frac{3}{2}, \frac{4}{3}$

2 $4x^2 + 13x - 35 = 0$ $\Rightarrow$ $(x + 5)(4x - 7) = 0$ $\Rightarrow$ $x = -5, \frac{7}{4}$

3 $15x^2 - 6 = -13x$ {get *zero* on one side of the equals sign} $\Rightarrow$ $15x^2 + 13x - 6 = 0$ {factor} $\Rightarrow$
$(5x + 6)(3x - 1) = 0$ {zero factor theorem} $\Rightarrow$ $x = -\frac{6}{5}, \frac{1}{3}$

4 $15x^2 - 14 = 29x$ $\Rightarrow$ $15x^2 - 29x - 14 = 0$ $\Rightarrow$ $(5x + 2)(3x - 7) = 0$ $\Rightarrow$ $x = -\frac{2}{5}, \frac{7}{3}$

5 A common mistake for this exercise is to write $2x = 27$ or $4x + 15 = 27$.
However, remember that you want to get 0 on one side of the equals sign.
$2x(4x + 15) = 27$ $\Rightarrow$ $8x^2 + 30x - 27 = 0$ $\Rightarrow$ $(2x + 9)(4x - 3) = 0$ $\Rightarrow$ $x = -\frac{9}{2}, \frac{3}{4}$

6 $x(3x + 10) = 77$ $\Rightarrow$ $3x^2 + 10x - 77 = 0$ $\Rightarrow$ $(x + 7)(3x - 11) = 0$ $\Rightarrow$ $x = -7, \frac{11}{3}$

7 Divide both sides by a nonzero constant whenever possible. In this case, 5 divides evenly into both sides.
$75x^2 + 35x - 10 = 0$ {divide by 5} $\Rightarrow$ $15x^2 + 7x - 2 = 0$ {factor} $\Rightarrow$
$(3x + 2)(5x - 1) = 0$ {zero factor theorem} $\Rightarrow$ $x = -\frac{2}{3}, \frac{1}{5}$

8 $48x^2 + 12x - 90 = 0$ {divide by 6} $\Rightarrow$ $8x^2 + 2x - 15 = 0$ $\Rightarrow$ $(2x + 3)(4x - 5) = 0$ $\Rightarrow$ $x = -\frac{3}{2}, \frac{5}{4}$

9 $12x^2 + 60x + 75 = 0$ {divide by 3} $\Rightarrow$ $4x^2 + 20x + 25 = 0$ $\Rightarrow$ $(2x + 5)^2 = 0$ $\Rightarrow$ $x = -\frac{5}{2}$

10 $4x^2 - 72x + 324 = 0$ {divide by 4} $\Rightarrow$ $x^2 - 18x + 81 = 0$ $\Rightarrow$ $(x - 9)^2 = 0$ $\Rightarrow$ $x = 9$

11 We will use the same process for solving rational equations as outlined in Section 2.1.
Here, the lcd is $x(x + 3)$ and we need to remember that $x \neq 0, -3$.
$\left[\dfrac{x}{x + 3} + \dfrac{1}{x} - 4 = \dfrac{9}{x^2 + 3x}\right] \cdot x(x + 3)$ $\Rightarrow$ $x(x) + 1(x + 3) - 4(x^2 + 3x) = 9$ $\Rightarrow$
$x^2 + x + 3 - 4x^2 - 12x = 9$ $\Rightarrow$ $0 = 3x^2 + 11x + 6$ $\Rightarrow$ $(3x + 2)(x + 3) = 0$ $\Rightarrow$
$x = -\frac{2}{3}$ {-3 is not in the domain of the given expressions}

12 $\left[\dfrac{5x}{x - 2} + \dfrac{3}{x} + 2 = \dfrac{-6}{x^2 - 2x}\right] \cdot x(x - 2)$ $\Rightarrow$ $5x(x) + 3(x - 2) + 2(x^2 - 2x) = -6$ $\Rightarrow$
$5x^2 + 3x - 6 + 2x^2 - 4x = -6$ $\Rightarrow$ $7x^2 - x = 0$ $\Rightarrow$ $x(7x - 1) = 0$ $\Rightarrow$
$x = \frac{1}{7}$ {0 is not in the domain of the given expressions}

13 $\left[\dfrac{5x}{x-3}+\dfrac{4}{x+3}=\dfrac{90}{x^2-9}\right]\cdot(x+3)(x-3) \Rightarrow 5x(x+3)+4(x-3)=90 \Rightarrow$
$5x^2+15x+4x-12=90 \Rightarrow 5x^2+19x-102=0 \Rightarrow (5x+34)(x-3)=0 \Rightarrow$
$x=-\frac{34}{5}$ {3 is not in the domain of the given expressions}

14 $\left[\dfrac{3x}{x-2}+\dfrac{1}{x+2}=\dfrac{-4}{x^2-4}\right]\cdot(x+2)(x-2) \Rightarrow 3x(x+2)+1(x-2)=-4 \Rightarrow$
$3x^2+6x+x-2=-4 \Rightarrow 3x^2+7x+2=0 \Rightarrow (3x+1)(x+2)=0 \Rightarrow$
$x=-\frac{1}{3}$ {-2 is not in the domain of the given expressions}

15 (a) The first equation, $x^2=16$, has solutions $x=\pm 4$. The equations are not equivalent since -4 is not a solution of the second equation, $x=4$.

(b) First note that $x=\sqrt{49}=7$. Thus, the equations are equivalent since they have exactly the same solutions.

16 (a) The first equation, $x^2=25$, has solutions $x=\pm 5$. The equations are not equivalent since -5 is not a solution of the second equation, $x=5$.

(b) First note that $x=\sqrt{64}=8$. Thus, the equations are equivalent since they have exactly the same solutions.

17 Using the special quadratic equation in this section, $x^2=225 \Rightarrow x=\pm\sqrt{225} \Rightarrow x=\pm 15$.
Note that this is *not* the same as saying $\sqrt{225}=\pm 15$.

18 $x^2=361 \Rightarrow x=\pm\sqrt{361} \Rightarrow x=\pm 19$

19 $25x^2=9 \Rightarrow x^2=\frac{9}{25} \Rightarrow x=\pm\sqrt{\frac{9}{25}} \Rightarrow x=\pm\frac{3}{5}$

20 $64x^2=49 \Rightarrow x^2=\frac{49}{64} \Rightarrow x=\pm\sqrt{\frac{49}{64}} \Rightarrow x=\pm\frac{7}{8}$

21 $(x-3)^2=17 \Rightarrow x-3=\pm\sqrt{17} \Rightarrow x=3\pm\sqrt{17}$

22 $(x+5)^2=29 \Rightarrow x+5=\pm\sqrt{29} \Rightarrow x=-5\pm\sqrt{29}$

23 $4(x+7)^2=13 \Rightarrow (x+7)^2=\frac{13}{4} \Rightarrow x+7=\pm\sqrt{\frac{13}{4}} \Rightarrow x=-7\pm\frac{1}{2}\sqrt{13}$

24 $9(x-1)^2=7 \Rightarrow (x-1)^2=\frac{7}{9} \Rightarrow x-1=\pm\sqrt{\frac{7}{9}} \Rightarrow x=1\pm\frac{1}{3}\sqrt{7}$

25 For this exercise, consider the general expression x^2+bx+c.

(a) In general, $d=\left(\frac{1}{2}b\right)^2$. In this case, $d=\left[\frac{1}{2}(9)\right]^2=\frac{81}{4}$.

(b) As in part (a), $d=\left(\frac{1}{2}b\right)^2=\left[\frac{1}{2}(-12)\right]^2=36$. ***Note:*** It is appropriate to use 12 or -12.

(c) In general, $d=2\left(\pm\sqrt{c}\right)$ for $c>0$. In this case, $c=36 \Rightarrow \sqrt{c}=6$, and $d=2(\pm 6)=\pm 12$.

(d) $c=\frac{49}{4} \Rightarrow \sqrt{c}=\frac{7}{2}$, and $d=2\left(\pm\frac{7}{2}\right)=\pm 7$.

26 For this exercise, consider the general expression x^2+bx+c.

(a) In general, $d=\left(\frac{1}{2}b\right)^2$. In this case, $d=\left[\frac{1}{2}(17)\right]^2=\frac{289}{4}$.

(b) As in part (a), $d=\left(\frac{1}{2}b\right)^2=\left[\frac{1}{2}(-6)\right]^2=9$. ***Note:*** It is appropriate to use 6 or -6.

(c) In general, $d=2\left(\pm\sqrt{c}\right)$ for $c>0$. In this case, $c=25 \Rightarrow \sqrt{c}=5$, and $d=2(\pm 5)=\pm 10$.

(d) $c=\frac{81}{4} \Rightarrow \sqrt{c}=\frac{9}{2}$, and $d=2\left(\pm\frac{9}{2}\right)=\pm 9$.

27 $x^2+6x-4=0$ {add 9 to both sides to complete the square with x^2+6x} $\Rightarrow$

$$x^2+6x+\underline{9}=4+\underline{9} \quad\Rightarrow\quad (x+3)^2=13 \quad\Rightarrow\quad x+3=\pm\sqrt{13} \quad\Rightarrow\quad x=-3\pm\sqrt{13}$$

28 $x^2-10x+20=0$ {add 25 to both sides to complete the square with x^2-10x} $\Rightarrow$

$$x^2-10x+\underline{25}=-20+\underline{25} \quad\Rightarrow\quad (x-5)^2=5 \quad\Rightarrow\quad x-5=\pm\sqrt{5} \quad\Rightarrow\quad x=5\pm\sqrt{5}$$

29

$4x^2-12x-11=0$	{given}
$x^2-3x-\frac{11}{4}=0$	{divide by 4}
$x^2-3x=\frac{11}{4}$	{isolate x^2 and x terms}
$x^2-3x+\frac{9}{4}=\frac{11}{4}+\frac{9}{4}$	{add $\left(\frac{1}{2}\cdot 3\right)^2=\frac{9}{4}$}
$\left(x-\frac{3}{2}\right)^2=5$	{factor and simplify}
$x-\frac{3}{2}=\pm\sqrt{5}$	{take the square root}
$x=\frac{3}{2}\pm\sqrt{5}$	{solve for x}

30 $4x^2+20x+13=0 \quad\Rightarrow\quad x^2+5x+\frac{13}{4}=0 \quad\Rightarrow\quad x^2+5x+\frac{25}{4}=-\frac{13}{4}+\frac{25}{4} \quad\Rightarrow$

$$\left(x+\tfrac{5}{2}\right)^2=3 \quad\Rightarrow\quad x+\tfrac{5}{2}=\pm\sqrt{3} \quad\Rightarrow\quad x=-\tfrac{5}{2}\pm\sqrt{3}$$

31 $6x^2-x=2 \quad\Rightarrow\quad 6x^2-x-2=0.$

Use the quadratic formula, $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$, with $a=6$, $b=-1$, and $c=-2$.

$$x=\frac{-(-1)\pm\sqrt{(-1)^2-4(6)(-2)}}{2(6)}=\frac{1\pm\sqrt{1+48}}{12}=\frac{1\pm 7}{12}=-\frac{1}{2},\frac{2}{3}$$

32 $5x^2+13x=6 \quad\Rightarrow\quad 5x^2+13x-6=0 \quad\Rightarrow\quad x=\dfrac{-13\pm\sqrt{169+120}}{2(5)}=\dfrac{-13\pm 17}{10}=-3,\dfrac{2}{5}$

33 $x^2+6x+3=0 \quad\Rightarrow\quad x=\dfrac{-6\pm\sqrt{36-12}}{2(1)}=\dfrac{-6\pm\sqrt{24}}{2}=\dfrac{-6\pm 2\sqrt{6}}{2}=-3\pm\sqrt{6}$

34 $x^2-4x-2=0 \quad\Rightarrow\quad x=\dfrac{4\pm\sqrt{16+8}}{2(1)}=\dfrac{4\pm\sqrt{24}}{2}=\dfrac{4\pm 2\sqrt{6}}{2}=2\pm\sqrt{6}$

35 $2x^2-3x-4=0 \quad\Rightarrow\quad x=\dfrac{-(-3)\pm\sqrt{(-3)^2-4(2)(-4)}}{2(2)}=\dfrac{3\pm\sqrt{9+32}}{4}=\dfrac{3\pm\sqrt{41}}{4}=\dfrac{3}{4}\pm\dfrac{1}{4}\sqrt{41}$

Note: A common mistake is to not divide 4 into *both* terms of the numerator.

36 $3x^2+5x+1=0 \quad\Rightarrow\quad x=\dfrac{-5\pm\sqrt{25-12}}{2(3)}=\dfrac{-5\pm\sqrt{13}}{6}=-\dfrac{5}{6}\pm\dfrac{1}{6}\sqrt{13}$

37 $\frac{3}{2}z^2-4z-1=0$ {multiply by 2} $\Rightarrow\quad 3z^2-8z-2=0 \quad\Rightarrow$

$$z=\frac{8\pm\sqrt{64+24}}{2(3)}=\frac{8\pm\sqrt{88}}{2(3)}=\frac{8\pm 2\sqrt{22}}{2(3)}=\frac{2\left(4\pm\sqrt{22}\right)}{2(3)}=\frac{4}{3}\pm\frac{1}{3}\sqrt{22}$$

38 $\frac{5}{3}s^2+3s+1=0 \quad\Rightarrow\quad 5s^2+9s+3=0 \quad\Rightarrow\quad s=\dfrac{-9\pm\sqrt{81-60}}{10}=-\dfrac{9}{10}\pm\dfrac{1}{10}\sqrt{21}$

39 Multiply by the lcd, w^2. $\left[\dfrac{5}{w^2}-\dfrac{10}{w}+2=0\right]\cdot w^2 \quad\Rightarrow\quad 5-10w+2w^2=0 \quad\Rightarrow$

$$w=\frac{10\pm\sqrt{100-40}}{2(2)}=\frac{10\pm\sqrt{60}}{2(2)}=\frac{10\pm 2\sqrt{15}}{2(2)}=\frac{2\left(5\pm\sqrt{15}\right)}{2(2)}=\frac{5}{2}\pm\frac{1}{2}\sqrt{15}$$

40 $\left[\dfrac{x+1}{3x+2}=\dfrac{x-2}{2x-3}\right]\cdot(3x+2)(2x-3) \Rightarrow (x+1)(2x-3)=(x-2)(3x+2) \Rightarrow$

$$2x^2-x-3=3x^2-4x-4 \Rightarrow 0=x^2-3x-1 \Rightarrow x=\frac{3\pm\sqrt{9+4}}{2}=\frac{3}{2}\pm\frac{1}{2}\sqrt{13}$$

41 $4x^2+81=36x \Rightarrow 4x^2-36x+81=0 \Rightarrow x=\dfrac{36\pm\sqrt{1296-1296}}{8}=\dfrac{36}{8}=\dfrac{9}{2}$

42 $30x+9=-25x^2 \Rightarrow 25x^2+30x+9=0 \Rightarrow x=\dfrac{-30\pm\sqrt{900-900}}{50}=-\dfrac{30}{50}=-\dfrac{3}{5}$

43 $\dfrac{3x}{x^2+9}=-2 \Rightarrow 3x=-2x^2-18 \Rightarrow 2x^2+3x+18=0 \Rightarrow x=\dfrac{-3\pm\sqrt{9-144}}{4}$.

Since the discriminant is negative, there are **no real solutions**.

44 $\frac{1}{7}x^2+1=\frac{4}{7}x \Rightarrow x^2+7=4x \Rightarrow x^2-4x+7=0 \Rightarrow x=\dfrac{4\pm\sqrt{16-28}}{2}$.

Since the discriminant is negative, there are **no real solutions**.

45 The *expression* is x^2+x-30. The associated quadratic *equation* is $x^2+x-30=0$.

Using the quadratic formula, $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$, to solve for x with $a=1$, $b=1$, and $c=-30$ gives us:

$$x=\frac{-1\pm\sqrt{1^2-4(1)(-30)}}{2(1)}=\frac{-1\pm\sqrt{1+120}}{2}=\frac{-1\pm\sqrt{121}}{2}=\frac{-1\pm 11}{2}=\frac{10}{2},\frac{-12}{2}=5,-6$$

Write the equation as a product of linear factors: $[x-(5)]\,[x-(-6)]=0$

Now simplify: $(x-5)(x+6)=0$

So the final factored form of x^2+x-30 is $(x-5)(x+6)$.

46 $x^2-11x=0$ $\{a=1, b=-11, c=0\}$, so $x=\dfrac{11\pm\sqrt{121-0}}{2}=\dfrac{11\pm 11}{2}=11, 0$.

Thus, $x^2-11x=(x-11)(x-0)=x(x-11)$.

47 $12x^2-16x-3=0$ $\{a=12, b=-16, c=-3\}$, so $x=\dfrac{16\pm\sqrt{256+144}}{24}=\dfrac{16\pm 20}{24}=\dfrac{3}{2},-\dfrac{1}{6}$.

Write the equation as a product of linear factors: $\left[x-\left(\frac{3}{2}\right)\right]\left[x-\left(-\frac{1}{6}\right)\right]=0$

Now multiply the first factor by 2 and the second factor by 6. $(2x-3)(6x+1)=0$

So the final factored form of $12x^2-16x-3$ is $(2x-3)(6x+1)$.

48 $15x^2+34x-16=0$ $\{a=15, b=34, c=-16\}$, so $x=\dfrac{-34\pm\sqrt{1156+960}}{30}=\dfrac{-34\pm 46}{30}=\dfrac{2}{5},-\dfrac{8}{3}$.

Thus, $15x^2+34x-16=5\left[x-\left(\frac{2}{5}\right)\right]\cdot 3\left[x-\left(-\frac{8}{3}\right)\right]=(5x-2)(3x+8)$.

49 (a) For this exercise, we must recognize the equation as a quadratic in x, that is,

$$Ax^2+Bx+C=0$$

where A is the coefficient of x^2, B is the coefficient of x, and C is the collection of all terms that do not contain x^2 or x.

$4x^2-4xy+1-y^2=0 \Rightarrow (4)x^2+(-4y)x+(1-y^2)=0 \Rightarrow$

$$x=\frac{4y\pm\sqrt{16y^2-16(1-y^2)}}{2(4)}=\frac{4y\pm\sqrt{16\left[y^2-(1-y^2)\right]}}{2(4)}=\frac{4y\pm 4\sqrt{2y^2-1}}{2(4)}=\frac{y\pm\sqrt{2y^2-1}}{2}$$

(b) Similar to part (a), we must now recognize the equation as a quadratic equation in y.

$4x^2 - 4xy + 1 - y^2 = 0 \quad \Rightarrow \quad (-1)y^2 + (-4x)y + (4x^2 + 1) = 0 \quad \Rightarrow$

$$y = \frac{4x \pm \sqrt{16x^2 + 4(4x^2+1)}}{2(-1)} = \frac{4x \pm \sqrt{4[4x^2 + (4x^2+1)]}}{-2} = \frac{4x \pm 2\sqrt{8x^2+1}}{-2} = -2x \pm \sqrt{8x^2+1}$$

50 (a) $2x^2 - xy = 3y^2 + 1 \quad \Rightarrow \quad (2)x^2 + (-y)x + (-3y^2 - 1) = 0 \quad \Rightarrow$

$$x = \frac{y \pm \sqrt{y^2 - 8(-3y^2-1)}}{2(2)} = \frac{y \pm \sqrt{25y^2+8}}{4}$$

(b) $2x^2 - xy = 3y^2 + 1 \quad \Rightarrow \quad (-3)y^2 + (-x)y + (2x^2 - 1) = 0 \quad \Rightarrow$

$$y = \frac{x \pm \sqrt{x^2 + 12(2x^2-1)}}{2(-3)} = \frac{x \pm \sqrt{25x^2 - 12}}{-6}$$

51 $K = \frac{1}{2}mv^2 \quad \Rightarrow \quad v^2 = \frac{2K}{m} \quad \Rightarrow \quad v = \pm\sqrt{\frac{2K}{m}} \quad \Rightarrow \quad v = \sqrt{\frac{2K}{m}}$ since $v > 0$.

52 $F = g\frac{mM}{d^2} \quad \Rightarrow \quad d^2 = \frac{gmM}{F} \quad \Rightarrow \quad d = \pm\sqrt{\frac{gmM}{F}} \quad \Rightarrow \quad d = \sqrt{\frac{gmM}{F}}$ since $d > 0$.

53 $A = 2\pi r(r+h) \quad \Rightarrow \quad A = 2\pi r^2 + 2\pi rh \quad \Rightarrow \quad (2\pi)r^2 + (2\pi h)r - A = 0$ {a quadratic equation in r} $\Rightarrow$

$$r = \frac{-(2\pi h) \pm \sqrt{(2\pi h)^2 - 4(2\pi)(-A)}}{2(2\pi)} = \frac{-2\pi h \pm \sqrt{4\pi^2h^2 + 8\pi A}}{2(2\pi)}$$

$$= \frac{-2\pi h \pm 2\sqrt{\pi^2h^2 + 2\pi A}}{2(2\pi)} = \frac{-\pi h \pm \sqrt{\pi^2h^2 + 2\pi A}}{2\pi}$$

Since $r > 0$, we must use the plus sign, and $r = \frac{-\pi h + \sqrt{\pi^2h^2 + 2\pi A}}{2\pi}$.

54 $s = \frac{1}{2}gt^2 + v_0t \quad \Rightarrow \quad \left(\frac{1}{2}g\right)t^2 + (v_0)t - s = 0 \quad \Rightarrow \quad t = \frac{-v_0 \pm \sqrt{v_0^2 + 2gs}}{g}$.

Since $t > 0$, we must use the plus sign, and $t = \frac{-v_0 + \sqrt{v_0^2 + 2gs}}{g}$.

55 $V = V_{\text{max}}\left[1 - \left(\frac{r}{r_0}\right)^2\right] \quad \Rightarrow \quad \frac{V}{V_{\text{max}}} = 1 - \left(\frac{r}{r_0}\right)^2 \quad \Rightarrow \quad \left(\frac{r}{r_0}\right)^2 = 1 - \frac{V}{V_{\text{max}}} \quad \Rightarrow$

$$r^2 = r_0^2\left[1 - (V/V_{\text{max}})\right] \;\; \{r > 0\} \quad \Rightarrow \quad r = r_0\sqrt{1 - (V/V_{\text{max}})}$$

56 $D = 0.74 \quad \Rightarrow \quad 0.74 = 1.225 - (1.12 \times 10^{-4})h + (3.24 \times 10^{-9})h^2 \quad \Rightarrow$

$(3.24 \times 10^{-9})h^2 - (1.12 \times 10^{-4})h + 0.485 = 0 \quad \Rightarrow \quad h \approx 5076$ and 29,492 by the quadratic formula.

Since the formula is valid only for $0 \le h \le 10{,}000$, $h \approx 5076$ m.

57 Using $V = \pi r^2h$ with $V = 3000$ and $h = 20$ gives us:

$$3000 = \pi r^2(20) \quad \Rightarrow \quad r^2 = 150/\pi \quad \Rightarrow \quad r = \sqrt{150/\pi} \approx 6.9 \text{ cm}$$

58 Let x denote the original width, $2x$ the length. $V = lwh \quad \Rightarrow \quad 60 = (2x-6)(x-6)(3) \quad \Rightarrow$

$10 = (x-3)(x-6) \quad \Rightarrow \quad x^2 - 9x + 8 = 0 \quad \Rightarrow \quad (x-1)(x-8) = 0 \quad \Rightarrow \quad x = 8$ for $x > 6$.

The sheet should be 8 in. by 16 in.

59 (a) $s = 48 \quad \Rightarrow \quad -16t^2 + 64t = 48 \quad \Rightarrow \quad -16t^2 + 64t - 48 = 0 \quad \Rightarrow \quad t^2 - 4t + 3 = 0 \quad \Rightarrow$

$(t-1)(t-3) = 0 \quad \Rightarrow \quad t = 1, 3.$ After 1 seconds and after 3 seconds

(b) It will hit the ground when $s = 0$.

$s = 0 \quad \Rightarrow \quad -16t^2 + 64t = 0 \quad \Rightarrow \quad t^2 - 4t = 0 \quad \Rightarrow \quad t(t-4) = 0 \quad \Rightarrow \quad t = 0, 4.$ After 4 seconds

60 (a) $v = 55 \Rightarrow d = v + (v^2/20) = 55 + (55^2/20) = 206.25$ ft

(b) $d = 120 \Rightarrow 120 = v + (v^2/20) \Rightarrow 2400 = 20v + v^2 \Rightarrow v^2 + 20v - 2400 = 0 \Rightarrow$
$(v + 60)(v - 40) = 0 \Rightarrow v = 40$ mi/hr

61 (a) $T = 98 \Rightarrow h = 1000(100 - T) + 580(100 - T)^2 = 1000(2) + 580(2)^2 = 4320$ m.

(b) If $x = 100 - T$ and $h = 8840$, then $8840 = 1000x + 580x^2 \Rightarrow$
$29x^2 + 50x - 442 = 0 \Rightarrow x = \dfrac{-50 \pm \sqrt{2500 + 51{,}272}}{2(29)} = \dfrac{-25 \pm \sqrt{13{,}443}}{29} \approx -4.86, 3.14.$
$x = -4.86 \Rightarrow T = 100 - x = 104.86$°C, which is outside the allowable range of T.
$x = 3.14 \Rightarrow T = 100 - x = 96.86$°C for $95 \le T \le 100$.

62 $F = 0 \Rightarrow \dfrac{2k}{(2-x)^2} = \dfrac{k}{(x+2)^2} \Rightarrow 2k(x+2)^2 = k(2-x)^2 \Rightarrow 2(x+2)^2 = (2-x)^2 \Rightarrow$
$2x^2 + 8x + 8 = x^2 - 4x + 4 \Rightarrow x^2 + 12x + 4 = 0 \Rightarrow$
$x = \dfrac{-12 \pm \sqrt{128}}{2} = \dfrac{-12 \pm 8\sqrt{2}}{2} = -6 + 4\sqrt{2} \approx -0.34$ for $-2 \le x \le 2$.

63 Let x denote the width of the walk. The area (including the walk) has dimensions $(26 + x + x)$ by $(30 + x + x)$ or, equivalently, $(26 + 2x)$ by $(30 + 2x)$. $\text{Area}_{\text{plot}} + \text{Area}_{\text{walk}} = \text{Area}_{\text{total}} \Rightarrow$
$26 \cdot 30 + 240 = (26 + 2x)(30 + 2x) \Rightarrow 26 \cdot 30 + 240 = 26 \cdot 30 + 52x + 60x + 4x^2 \Rightarrow$
$240 = 4x^2 + 112x \Rightarrow x^2 + 28x - 60 = 0 \Rightarrow (x + 30)(x - 2) = 0 \Rightarrow x = 2$ ft, since x is positive.

64 Let x denote the width of the side or top margin, $2x$ the bottom.
Printed area $= lw \Rightarrow 661.5 = (24 - 2x)(36 - 3x) \Rightarrow 661.5 = 864 - 72x - 72x + 6x^2 \Rightarrow$
$6x^2 - 144x + 202.5 = 0 \Rightarrow x = \dfrac{144 \pm \sqrt{15{,}876}}{2(6)} = \dfrac{144 \pm 126}{12} = 12 \pm 10.5 = 1.5$ {22.5 is too large}.
The margins are 1.5 in. for the sides and the top, and 3 in. for the bottom.

65 Let x denote the length of one side. The area of the garden is x^2 and the perimeter is $4x$.
$\text{Cost}_{\text{preparation}} + \text{Cost}_{\text{fence}} = \text{Cost}_{\text{total}} \Rightarrow x^2(\$0.50) + 4x(\$1) = \$120 \Rightarrow \frac{1}{2}x^2 + 4x = 120 \Rightarrow$
$x^2 + 8x - 240 = 0 \Rightarrow (x + 20)(x - 12) = 0 \Rightarrow x = 12.$ The size of the garden should be 12 ft by 12 ft.

66 Let x denote the length of an adjacent side, $2x$ the parallel side. $A = lw \Rightarrow 128 = (2x)(x) \Rightarrow$
$2x^2 = 128 \Rightarrow x^2 = 64 \Rightarrow x = 8.$ The farmer should purchase $8 + 8 + 16 = 32$ ft of fencing.

67 Let $d(A, P) = x$ and $d(P, B) = 6 - x$. $x^2 + (6 - x)^2 = 5^2 \Rightarrow x^2 + 36 - 12x + x^2 = 25 \Rightarrow$
$2x^2 - 12x + 11 = 0 \Rightarrow x = \dfrac{12 \pm \sqrt{56}}{2(2)} = \dfrac{12 \pm 2\sqrt{14}}{2(2)} = 3 \pm \dfrac{1}{2}\sqrt{14} \approx 4.9, 1.1$ mi.
There are 4 possible roads since P could be on either side of segment AB.

68 Let r denote the city's original radius (its current radius is $\frac{1}{2} \cdot 10 = 5$). $\text{Area}_{\text{original}} + \text{Area}_{\text{growth}} = \text{Area}_{\text{current}} \Rightarrow$
$\pi r^2 + 16\pi = \pi(5)^2 \Rightarrow r^2 + 16 = 25 \Rightarrow r^2 = 9 \Rightarrow r = 3$, and $5 - r = 2$ miles.

69 (a) The northbound plane travels $\frac{1}{2} \cdot 200 = 100$ miles from 2 P.M. to 2:30 P.M., so the distances of the northbound and eastbound planes are $100 + 200t$ and $400t$, respectively. Using the Pythagorean theorem,

$$d = \sqrt{(100 + 200t)^2 + (400t)^2} = \sqrt{100^2(1 + 2t)^2 + 100^2(4t)^2} = \sqrt{100^2\left[(1 + 2t)^2 + (4t)^2\right]}$$
$$= 100\sqrt{1 + 4t + 4t^2 + 16t^2} = 100\sqrt{20t^2 + 4t + 1}.$$

(b) $d = 500 \quad \Rightarrow \quad 500 = 100\sqrt{20t^2 + 4t + 1} \quad \Rightarrow \quad 5 = \sqrt{20t^2 + 4t + 1} \quad \Rightarrow \quad 5^2 = 20t^2 + 4t + 1 \quad \Rightarrow$
$20t^2 + 4t - 24 = 0 \quad \Rightarrow \quad 5t^2 + t - 6 = 0 \quad \Rightarrow \quad (5t + 6)(t - 1) = 0 \quad \Rightarrow$
$t = 1$ hour after 2:30 P.M., or 3:30 P.M.

70 Let t denote the desired time. Using the Pythagorean theorem, $(4t)^2 + (3t)^2 = 2^2 \quad \Rightarrow \quad 25t^2 = 4 \quad \Rightarrow$
$t^2 = \frac{4}{25} \quad \Rightarrow \quad t = \frac{2}{5}$ hr, or 24 min. They will be in range until 9:24 A.M.

71 Let x denote the outer width of the box, so $x - 2$ is the inner width.
Since the base is square, $(x - 2)^2 = 144 \quad \Rightarrow \quad x - 2 = \pm 12 \quad \Rightarrow \quad x = 14 \ \{x > 0\}$.
The length of the box is $3(1) + 2(14) = 27$. Thus, the size is 14 in. by 27 in.

72 Let x denote the length of one side of the larger frame. $4x$ and $(100 - 4x)$ are the perimeters.
Larger area $= 2 \times$ (smaller area) $\quad \Rightarrow \quad \left(\dfrac{4x}{4}\right)^2 = 2\left(\dfrac{100 - 4x}{4}\right)^2 \quad \Rightarrow$
$(x)^2 = 2(25 - x)^2 \quad \Rightarrow \quad x^2 = 2(625 - 50x + x^2) \quad \Rightarrow \quad x^2 = 1250 - 100x + 2x^2 \quad \Rightarrow$
$x^2 - 100x + 1250 = 0 \quad \Rightarrow \quad x = \dfrac{100 \pm \sqrt{5000}}{2(1)} = \dfrac{100 \pm 50\sqrt{2}}{2(1)} = 50 - 25\sqrt{2} \approx 14.64$ in. for $x < 25$.
The length of a side for the smaller frame is $25 - x = 25\sqrt{2} - 25 \approx 10.36$ in.

73 Let x denote the rate of the canoeist in still water. $x - 5$ is the rate upstream and $x + 5$ is the rate downstream.
$\text{Time}_{\text{up}} = \text{Time}_{\text{down}} + \dfrac{1}{2} \quad \Rightarrow \quad \left\{t = \dfrac{d}{r}\right\} \ \dfrac{1.2}{x - 5} = \dfrac{1.2}{x + 5} + \dfrac{1}{2}$ {multiply by lcd $= 2(x + 5)(x - 5)$} $\quad \Rightarrow$
$2.4(x + 5) = 2.4(x - 5) + x^2 - 25 \quad \Rightarrow \quad 2.4x + 12 = 2.4x - 12 + x^2 - 25 \quad \Rightarrow \quad x^2 = 49 \quad \Rightarrow \quad x = 7$ mi/hr.

74 Let t denote the number of seconds the rock falls, so that $4 - t$ is the number of seconds for the sound to travel.
$\text{Distance}_{\text{down}} = \text{Distance}_{\text{up}} \quad \Rightarrow \quad 16t^2 = 1100(4 - t) \ \{d = rt\} \quad \Rightarrow \quad 4t^2 = 275(4 - t) \quad \Rightarrow$
$4t^2 + 275t - 1100 = 0 \quad \Rightarrow \quad t = \dfrac{-275 \pm \sqrt{93{,}225}}{2(4)} = \dfrac{-275 + 5\sqrt{3729}}{8} \approx 3.79.$
The height is $16t^2 \approx 229.94$, or 230 ft.

75 Let x denote the number of pairs ordered. The price per pair is the discount subtracted from \$40.
Since the discount is \$0.04 times the number ordered, x, the cost per pair is $40 - 0.04x$.
Cost = (# of pairs)(cost per pair) $\quad \Rightarrow \quad 8400 = x(40 - 0.04x) \quad \Rightarrow \quad 8400 = 40x - 0.04x^2 \quad \Rightarrow$
$\frac{1}{25}x^2 - 40x + 8400 = 0$ {multiply by 25} $\quad \Rightarrow \quad x^2 - 1000x + 210{,}000 = 0 \quad \Rightarrow \quad (x - 300)(x - 700) = 0 \quad \Rightarrow$
$x = 300$ for $0 \le x \le 600$.

76 Let x denote the number of \$10 reductions in price.
Revenue = (unit price) $\times$ (# of units) $\quad \Rightarrow \quad 7000 = (300 - 10x)(15 + 2x) \quad \Rightarrow$
$7000 = 10(30 - x)(15 + 2x) \quad \Rightarrow \quad 700 = -2x^2 + 45x + 450 \quad \Rightarrow \quad 2x^2 - 45x + 250 = 0 \quad \Rightarrow$
$(2x - 25)(x - 10) = 0 \quad \Rightarrow \quad x = 10$ or 12.5.
The selling price is \$300 − \$10(10) = \$200, or \$300 − \$10(12.5) = \$175.

77 The total surface area is the sum of the surface area of the cylinder and that of the top and bottom.
$S = 2\pi rh + 2\pi r^2$ and $h = 4 \quad \Rightarrow \quad 10\pi = 8\pi r + 2\pi r^2$ {divide by 2π} $\quad \Rightarrow \quad 5 = 4r + r^2 \quad \Rightarrow$
$r^2 + 4r - 5 = 0 \quad \Rightarrow \quad (r + 5)(r - 1) = 0 \quad \Rightarrow \quad r = 1$, and the diameter is 2 ft.

78 (a) $\textbf{Area}_{\textbf{capsule}} = \text{Area}_{\text{sphere}}$ {the two ends are hemispheres} $+ \text{Area}_{\text{cylinder}}$

$$= 4\pi r^2 + 2\pi rh = 4\pi\left(\tfrac{1}{4}\right)^2 + 2\pi\left(\tfrac{1}{4}\right)\left(2 - \tfrac{1}{2}\right) = \tfrac{\pi}{4} + \tfrac{3\pi}{4} = \pi \text{ cm}^2.$$

$\textbf{Area}_{\textbf{tablet}} = \text{Area}_{\text{top and bottom}} + \text{Area}_{\text{cylinder}} = 2\pi r^2 + 2\pi r\left(\tfrac{1}{2}\right) = 2\pi r^2 + \pi r.$

Equating the two surface areas yields $2\pi r^2 + \pi r = \pi \quad \Rightarrow$

$2r^2 + r - 1 = 0 \quad \Rightarrow \quad (2r-1)(r+1) = 0 \quad \Rightarrow \quad r = \tfrac{1}{2}$, and the diameter is 1 cm.

(b) $\textbf{Volume}_{\textbf{capsule}} = \text{Volume}_{\text{sphere}} + \text{Volume}_{\text{cylinder}}$

$$= \tfrac{4}{3}\pi r^3 + \pi r^2 h = \tfrac{4}{3}\pi\left(\tfrac{1}{4}\right)^3 + \pi\left(\tfrac{1}{4}\right)^2 \tfrac{3}{2} = \tfrac{\pi}{48} + \tfrac{3\pi}{32} = \tfrac{11\pi}{96} \approx 0.360 \text{ cm}^3.$$

$\textbf{Volume}_{\textbf{tablet}} = \text{Volume}_{\text{cylinder}} = \pi r^2 h = \pi\left(\tfrac{1}{2}\right)^2 \tfrac{1}{2} = \tfrac{\pi}{8} \approx 0.393 \text{ cm}^3.$

79 V is 95% of $V_0 \quad \Rightarrow \quad V = 0.95V_0 \quad \Rightarrow \quad \dfrac{V}{V_0} = 0.95.$ $0.95 = 0.8197 + 0.007752t + 0.0000281t^2 \quad \Rightarrow$

$0.281t^2 + 77.52t - 1303 = 0 \quad \Rightarrow \quad t = \dfrac{-77.52 \pm \sqrt{(77.52)^2 - 4(0.281)(-1303)}}{2(0.281)} \approx -291.76, 15.89.$ Thus, the volume of the fireball will be 95% of the maximum volume approximately 15.89 seconds after the explosion.

80 As in the previous solution, $0.95 = 0.831 + 0.00598t + 0.0000919t^2 \quad \Rightarrow \quad 0.919t^2 + 59.8t - 1190 = 0 \quad \Rightarrow$

$t \approx -81.05, 15.98.$ Approximately 15.98 seconds after the explosion.

81 (a) $x = \dfrac{-4{,}500{,}000 \pm \sqrt{4{,}500{,}000^2 - 4(1)(-0.96)}}{2} \approx 0$ and $-4{,}500{,}000$

(b) $$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \cdot \frac{-b \mp \sqrt{b^2 - 4ac}}{-b \mp \sqrt{b^2 - 4ac}} = \frac{b^2 - (b^2 - 4ac)}{2a\left(-b \mp \sqrt{b^2 - 4ac}\right)}$$

$$= \frac{4ac}{2a\left(-b \mp \sqrt{b^2 - 4ac}\right)} = \frac{2c}{-b \mp \sqrt{b^2 - 4ac}}$$

The root near zero was obtained in part (a) using the plus sign. In the second formula, it corresponds to the minus sign. $x = \dfrac{2(-0.96)}{-4{,}500{,}000 - \sqrt{4{,}500{,}000^2 - 4(1)(-0.96)}} \approx 2.13 \times 10^{-7}$

82 (a) $x = \dfrac{73{,}000{,}000 \pm \sqrt{(-73{,}000{,}000)^2 - 4(1)(2.01)}}{2} \approx 73{,}000{,}000$ and 0

(b) The root near zero was obtained in part (a) using the minus sign. In the second formula, it corresponds to the plus sign. $x = \dfrac{2(2.01)}{73{,}000{,}000 + \sqrt{(-73{,}000{,}000)^2 - 4(1)(2.01)}} \approx 2.75 \times 10^{-8}$

83 (a) Let $\text{Y}_1 = T_1 = -1.09L + 96.01$ and $\text{Y}_2 = T_2 = -0.011L^2 - 0.126L + 81.45$. Table each equation and compare them to the actual temperatures.

x (L)	85	75	65	55	45	35	25	15	5
Y_1	3.36	14.26	25.16	36.06	46.96	57.86	68.76	75.66	90.56
Y_2	−8.74	10.13	26.79	41.25	53.51	63.57	71.43	77.09	80.55
S. Hem.	−5	10	27	42	53	65	75	78	79

Comparing Y_1 (T_1) with Y_2 (T_2), we can see that the linear equation T_1 is not as accurate as the quadratic equation T_2.

(b) $L = 50 \quad \Rightarrow \quad T_2 = -0.011(50)^2 - 0.126(50) + 81.45 = 47.65°\text{F}.$

2.4 Exercises

1 $(5-2i)+(-3+6i)=[5+(-3)]+(-2+6)i=2+4i$

2 $(-5+4i)+(3+9i)=(-5+3)+(4+9)i=-2+13i$

3 $(7-8i)-(-5-3i)=(7+5)+(-8+3)i=12-5i$

4 $(-3+8i)-(2+3i)=(-3-2)+(8-3)i=-5+5i$

5 $(3+5i)(2-7i)=(3+5i)2+(3+5i)(-7i)$ {distributive property}
$=6+10i-21i-35i^2$ {multiply terms}
$=6-11i-35(-1)$ {combine i-terms, $i^2=-1$}
$=6-11i+35$
$=41-11i$

6 $(-2+3i)(8-i)=(-16-3i^2)+(2+24)i=(-16+3)+26i=-13+26i$

7 $(4-3i)(2+7i)=(8-21i^2)+(28-6)i=(8+21)+22i=29+22i$

8 $(8+2i)(7-3i)=(56-6i^2)+(-24+14)i=(56+6)-10i=62-10i$

9 Use the *special product formula* for $(x-y)^2$ on the inside front cover of the text.
$$(5-2i)^2=5^2-2(5)(2i)+(2i)^2=25-20i+4i^2=(25-4)-20i=21-20i$$

10 $(6+7i)^2=6^2+2(6)(7i)+(7i)^2=(36-49)+84i=-13+84i$

11 $i(3+4i)^2=i\left[3^2+2(3)(4i)+(4i)^2\right]=i\left[(9-16)+24i\right]=i(-7+24i)=-24-7i$

12 $i(2-7i)^2=i\left[2^2+2(2)(-7i)+(-7i)^2\right]=i\left[(4-49)-28i\right]=i(-45-28i)=28-45i$

13 $(3+4i)(3-4i)$ {note that this difference of squares ...}
$$=3^2-(4i)^2=9-(-16)=\{\ldots \text{ becomes a "sum of squares"}\}\ 9+16=25$$

14 $(4+7i)(4-7i)=4^2-(7i)^2=16-(-49)=16+49=65$

15 (a) Since $i^k=1$ if k is a multiple of 4, we will write i^{43} as $i^{40}i^3$, knowing that i^{40} will reduce to 1.
$$i^{43}=i^{40}i^3=\left(i^4\right)^{10}(-i)=1^{10}(-i)=-i$$

(b) As in Example 3(e), choose $b=20$. $i^{-20}\cdot i^{20}=i^0=1$.

16 (a) $i^{68}=\left(i^4\right)^{17}=1^{17}=1$

(b) As in Example 3(e), choose $b=36$. $i^{-33}\cdot i^{36}=i^3=-i$.

17 (a) Since $i^k=1$ if k is a multiple of 4, we will write i^{73} as $i^{72}i^1$, knowing that i^{72} will reduce to 1.
$$i^{73}=i^{72}i=\left(i^4\right)^{18}i=1^{18}i=i$$

(b) As in Example 3(e), choose $b=48$. $i^{-46}\cdot i^{48}=i^2=-1$.

18 (a) $i^{66}=i^{64}i^2=\left(i^4\right)^{16}(-1)=1^{16}(-1)=-1$

(b) As in Example 3(e), choose $b=56$. $i^{-55}\cdot i^{56}=i^1=i$.

19 Multiply both the numerator and the denominator by the conjugate of the denominator to eliminate all i's in the denominator. The new denominator is the sum of the squares of the coefficients—in this case, 2^2 and 4^2.

$$\frac{3}{2+4i}\cdot\frac{2-4i}{2-4i}=\frac{3(2-4i)}{4-(-16)}=\frac{6-12i}{20}=\frac{6}{20}-\frac{12}{20}i=\frac{3}{10}-\frac{3}{5}i$$

20 $\frac{5}{3-7i}\cdot\frac{3+7i}{3+7i}=\frac{5(3+7i)}{9-(-49)}=\frac{15+35i}{58}=\frac{15}{58}+\frac{35}{58}i$

21 Multiply both the numerator and the denominator by the conjugate of the denominator to eliminate all i's in the denominator. The new denominator is the sum of the squares of the coefficients—in this case, 6^2 and 2^2.

$$\frac{1-7i}{6-2i}\cdot\frac{6+2i}{6+2i}=\frac{(6+14)+(2-42)i}{36-(-4)}=\frac{20-40i}{40}=\frac{20}{40}-\frac{40}{40}i=\frac{1}{2}-i$$

22 $\frac{2+9i}{-3-i}\cdot\frac{-3+i}{-3+i}=\frac{(-6-9)+(2-27)i}{9-(-1)}=\frac{-15-25i}{10}=-\frac{3}{2}-\frac{5}{2}i$

23 $\frac{-4+6i}{2+7i}\cdot\frac{2-7i}{2-7i}=\frac{(-8+42)+(28+12)i}{4-(-49)}=\frac{34+40i}{53}=\frac{34}{53}+\frac{40}{53}i$

24 $\frac{-3-2i}{5+2i}\cdot\frac{5-2i}{5-2i}=\frac{(-15-4)+(6-10)i}{25-(-4)}=\frac{-19-4i}{29}=-\frac{19}{29}-\frac{4}{29}i$

25 Multiplying the denominator by i will eliminate the i's in the denominator.

$$\frac{4-2i}{-7i}=\frac{4-2i}{-7i}\cdot\frac{i}{i}=\frac{4i-2i^2}{-7i^2}=\frac{2+4i}{7}=\frac{2}{7}+\frac{4}{7}i$$

26 $\frac{-2+6i}{3i}=\frac{-2+6i}{3i}\cdot\frac{-i}{-i}=\frac{2i-6i^2}{-3i^2}=\frac{6+2i}{3}=2+\frac{2}{3}i$

27 Use the *special product formula* for $(x+y)^3$ on the inside front cover of the text.

$$\begin{aligned}(2+5i)^3&=(2)^3+3(2)^2(5i)+3(2)(5i)^2+(5i)^3\\&=8+60i+6(25i^2)+125i^3\\&=(8+150i^2)+(60i+125i^3)=(8-150)+(60-125)i=-142-65i\end{aligned}$$

28 $(3-2i)^3=(3)^3+3(3)^2(-2i)+3(3)(-2i)^2+(-2i)^3=27-54i+9(4i^2)-8i^3$
$=(27+36i^2)+(-54i-8i^3)=(27-36)+(-54+8)i=-9-46i$

29 A common mistake is to multiply $\sqrt{-4}\sqrt{-16}$ and obtain $\sqrt{64}$, or 8.
The correct procedure is $\sqrt{-4}\sqrt{-16}=\sqrt{4}\,i\cdot\sqrt{16}\,i=(2i)(4i)=8i^2=-8$.

$$\left(2-\sqrt{-4}\right)\left(3-\sqrt{-16}\right)=(2-2i)(3-4i)=(6-8)+(-6i-8i)=-2-14i$$

30 $\left(-3+\sqrt{-25}\right)\left(8-\sqrt{-36}\right)=(-3+5i)(8-6i)=(-24+30)+(40i+18i)=6+58i$

31 $\frac{4+\sqrt{-81}}{2-\sqrt{-9}}=\frac{4+9i}{2-3i}\cdot\frac{2+3i}{2+3i}=\frac{(8-27)+(12+18)i}{4-(-9)}=\frac{-19+30i}{13}=-\frac{19}{13}+\frac{30}{13}i$

32 $\frac{5-\sqrt{-121}}{1+\sqrt{-25}}=\frac{5-11i}{1+5i}\cdot\frac{1-5i}{1-5i}=\frac{(5-55)+(-25-11)i}{1-(-25)}=\frac{-50-36i}{26}=-\frac{25}{13}-\frac{18}{13}i$

33 $\frac{\sqrt{-36}\sqrt{-49}}{\sqrt{-16}}=\frac{(6i)(7i)}{4i}=\frac{42i^2}{4i}=\frac{-21}{2i}=\frac{-21}{2i}\cdot\frac{-i}{-i}=\frac{21i}{-2i^2}=\frac{21i}{2}=\frac{21}{2}i$

34 $\frac{\sqrt{-25}}{\sqrt{-16}\sqrt{-81}}=\frac{5i}{(4i)(9i)}=\frac{5i}{36i^2}=\frac{5i}{-36}=-\frac{5}{36}i$

35 We need to equate the real parts and the imaginary parts on each side of "$=$".

$4+(x+2y)i = x+2i \Rightarrow 4 = x$ and $x+2y=2 \Rightarrow$

$x=4$ and $4+2y=2 \Rightarrow 2y=-2 \Rightarrow y=-1$, so $x=4$ and $y=-1$.

36 $(x-y)+3i = 4+yi \Rightarrow 3=y$ and $x-y=4 \Rightarrow x-3=4 \Rightarrow x=7, y=3$

37 $(2x-y)-16i = 10+4yi \Rightarrow 2x-y=10$ and $-16=4y \Rightarrow y=-4$ and $2x-(-4)=10 \Rightarrow$

$2x+4=10 \Rightarrow 2x=6 \Rightarrow x=3$, so $x=3$ and $y=-4$.

38 $8+(3x+y)i = 2x-4i \Rightarrow 2x=8$ and $3x+y=-4 \Rightarrow x=4$ and $12+y=-4 \Rightarrow x=4, y=-16$

39 $x^2-6x+13=0 \Rightarrow x = \dfrac{-(-6)\pm\sqrt{(-6)^2-4(1)(13)}}{2(1)} = \dfrac{6\pm\sqrt{36-52}}{2} = \dfrac{6\pm\sqrt{-16}}{2} = \dfrac{6\pm 4i}{2} = 3\pm 2i$

40 $x^2-2x+26=0 \Rightarrow$

$$x = \frac{-(-2)\pm\sqrt{(-2)^2-4(1)(26)}}{2(1)} = \frac{2\pm\sqrt{4-104}}{2} = \frac{2\pm\sqrt{-100}}{2} = \frac{2\pm 10i}{2} = 1\pm 5i$$

41 $x^2+12x+37=0 \Rightarrow$

$$x = \frac{-12\pm\sqrt{(-12)^2-4(1)(37)}}{2(1)} = \frac{-12\pm\sqrt{144-148}}{2} = \frac{-12\pm\sqrt{-4}}{2} = \frac{-12\pm 2i}{2} = -6\pm i$$

42 $x^2+8x+17=0 \Rightarrow x = \dfrac{-8\pm\sqrt{64-68}}{2(1)} = \dfrac{-8\pm\sqrt{-4}}{2} = \dfrac{-8\pm 2i}{2} = -4\pm i$

43 $x^2-5x+20=0 \Rightarrow x = \dfrac{-(-5)\pm\sqrt{(-5)^2-4(1)(20)}}{2(1)} = \dfrac{5\pm\sqrt{25-80}}{2} = \dfrac{5\pm\sqrt{-55}}{2} = \dfrac{5}{2}\pm\dfrac{1}{2}\sqrt{55}\,i$

44 $x^2+3x+6=0 \Rightarrow x = \dfrac{-3\pm\sqrt{9-24}}{2(1)} = \dfrac{-3\pm\sqrt{-15}}{2} = -\dfrac{3}{2}\pm\dfrac{1}{2}\sqrt{15}\,i$

45 $4x^2+x+3=0 \Rightarrow x = \dfrac{-1\pm\sqrt{1^2-4(4)(3)}}{2(4)} = \dfrac{-1\pm\sqrt{1-48}}{8} = \dfrac{-1\pm\sqrt{-47}}{8} = -\dfrac{1}{8}\pm\dfrac{1}{8}\sqrt{47}\,i$

46 $-3x^2+x-5=0 \Rightarrow x = \dfrac{-1\pm\sqrt{1^2-4(-3)(-5)}}{2(-3)} = \dfrac{-1\pm\sqrt{1-60}}{-6} = \dfrac{1}{6}\pm\dfrac{1}{6}\sqrt{59}\,i$

47 Solving $x^3=-64$ by taking the cube root of both sides would only give us the solution $x=-4$, so we need to factor x^3+64 as the sum of cubes. $x^3+64=0 \Rightarrow (x+4)(x^2-4x+16)=0 \Rightarrow$

$x=-4$ or $x = \dfrac{4\pm\sqrt{16-64}}{2} = \dfrac{4\pm 4\sqrt{3}\,i}{2}$. The three solutions are $-4, 2\pm 2\sqrt{3}\,i$.

48 $x^3-27=0 \Rightarrow (x-3)(x^2+3x+9)=0 \Rightarrow$

$x=3$ or $x = \dfrac{-3\pm\sqrt{9-36}}{2} = \dfrac{-3\pm 3\sqrt{3}\,i}{2}$. The three solutions are $3, -\dfrac{3}{2}\pm\dfrac{3}{2}\sqrt{3}\,i$.

49 $27x^3 = (x+5)^3 \Rightarrow (3x)^3-(x+5)^3=0 \Rightarrow$

{difference of cubes} $[3x-(x+5)]\left[(3x)^2+3x(x+5)+(x+5)^2\right]=0 \Rightarrow$

$(3x-x-5)(9x^2+3x^2+15x+x^2+10x+25)=0 \Rightarrow$

$(2x-5)(13x^2+25x+25)=0 \Rightarrow x=\dfrac{5}{2}$ or $x = \dfrac{-25\pm\sqrt{625-1300}}{2(13)} = \dfrac{-25\pm 15\sqrt{3}\,i}{26}$.

The three solutions are $\frac{5}{2}, -\frac{25}{26}\pm\frac{15}{26}\sqrt{3}\,i$.

50 $16x^4 = (x-4)^4 \;\Rightarrow\; (4x^2)^2 - \left[(x-4)^2\right]^2 = 0 \;\Rightarrow\;$ {difference of squares}
$\left[4x^2 + (x-4)^2\right]\left[4x^2 - (x-4)^2\right] = 0 \;\Rightarrow\; (5x^2 - 8x + 16)(3x^2 + 8x - 16) = 0 \;\Rightarrow$
$5x^2 - 8x + 16 = 0$ or $3x^2 + 8x - 16 = (x+4)(3x-4) = 0$.
$5x^2 - 8x + 16 = 0 \;\Rightarrow\; x = \dfrac{8 \pm \sqrt{64-320}}{10} = \dfrac{8 \pm 16i}{10} = \dfrac{4 \pm 8i}{5}$. The four solutions are $-4, \frac{4}{3}, \frac{4}{5} \pm \frac{8}{5}i$.

51 $x^4 = 625 \;\Rightarrow\; x^4 - 625 = 0 \;\Rightarrow\; (x^2-25)(x^2+25) = 0 \;\Rightarrow$
$x^2 = 25, -25 \;\Rightarrow\; x = \pm\sqrt{25}, \pm\sqrt{-25} \;\Rightarrow\; x = \pm 5, \pm 5i$

52 $x^4 = 81 \;\Rightarrow\; x^4 - 81 = 0 \;\Rightarrow\; (x^2-9)(x^2+9) = 0 \;\Rightarrow$
$x^2 = 9, -9 \;\Rightarrow\; x = \pm\sqrt{9}, \pm\sqrt{-9} \;\Rightarrow\; x = \pm 3, \pm 3i$

53 $4x^4 + 25x^2 + 36 = 0 \;\Rightarrow\; (x^2+4)(4x^2+9) = 0 \;\Rightarrow$
$x^2 = -4, -\frac{9}{4} \;\Rightarrow\; x = \pm\sqrt{-4}, \pm\sqrt{-\frac{9}{4}} \;\Rightarrow\; x = \pm 2i, \pm\frac{3}{2}i$

54 $27x^4 + 21x^2 + 4 = 0 \;\Rightarrow\; (9x^2+4)(3x^2+1) = 0 \;\Rightarrow$
$x^2 = -\frac{4}{9}, -\frac{1}{3} \;\Rightarrow\; x = \pm\sqrt{-\frac{4}{9}}, \pm\sqrt{-\frac{1}{3}} \;\Rightarrow\; x = \pm\frac{2}{3}i, \pm\frac{1}{3}\sqrt{3}\,i$

55 $x^3 + 3x^2 + 4x = 0 \;\Rightarrow\; x(x^2+3x+4) = 0 \;\Rightarrow$
$x = 0$ or $x = \dfrac{-3 \pm \sqrt{9-16}}{2} = \dfrac{-3 \pm \sqrt{7}\,i}{2}$. The three solutions are $0, -\dfrac{3}{2} \pm \dfrac{1}{2}\sqrt{7}\,i$.

56 $8x^3 - 12x^2 + 2x - 3 = 0 \;\Rightarrow\; 4x^2(2x-3) + 1(2x-3) = 0 \;\Rightarrow\; (4x^2+1)(2x-3) = 0 \;\Rightarrow$
$x^2 = -\frac{1}{4}$ or $x = \frac{3}{2} \;\Rightarrow\; x = \pm\sqrt{-\frac{1}{4}}$ or $x = \frac{3}{2} \;\Rightarrow\; x = \frac{3}{2}, \pm\frac{1}{2}i$

Note: In Exercises 57–62: Let $z = a + bi$ and $w = c + di$.

57
$$\begin{aligned}
\overline{z+w} &= \overline{(a+bi)+(c+di)} && \{\text{definition of } z \text{ and } w\}\\
&= \overline{(a+c)+(b+d)i} && \{\text{write in complex number form}\}\\
&= (a+c)-(b+d)i && \{\text{definition of conjugate}\}\\
&= (a-bi)+(c-di) && \{\text{rearrange terms } (*)\}\\
&= \overline{z}+\overline{w} && \{\text{definition of conjugates of } z \text{ and } w\}
\end{aligned}$$

(*) We are really looking ahead to the terms we want to obtain, $\overline{z}$ and $\overline{w}$.

58 $\overline{z-w} = \overline{(a+bi)-(c+di)} = \overline{(a-c)+(b-d)i} = (a-c)-(b-d)i = (a-bi)-(c-di) = \overline{z}-\overline{w}$.

59 $\overline{z \cdot w} = \overline{(a+bi)\cdot(c+di)} = \overline{(ac-bd)+(ad+bc)i}$
$= (ac-bd)-(ad+bc)i = ac - adi - bd - bci = a(c-di) - bi(c-di) = (a-bi)\cdot(c-di) = \overline{z}\cdot\overline{w}$

60
$$\overline{\left(\frac{z}{w}\right)} = \overline{\left(\frac{a+bi}{c+di}\right)} = \overline{\left(\frac{a+bi}{c+di}\cdot\frac{c-di}{c-di}\right)} = \overline{\left(\frac{(ac+bd)+(bc-ad)i}{c^2+d^2}\right)} = \overline{\left(\frac{ac+bd}{c^2+d^2}+\frac{bc-ad}{c^2+d^2}i\right)}$$
$$= \frac{ac+bd}{c^2+d^2} - \frac{bc-ad}{c^2+d^2}i = \frac{(ac+bd)+(ad-bc)i}{c^2+d^2} = \frac{a-bi}{c-di}\cdot\frac{c+di}{c+di} = \frac{a-bi}{c-di} = \frac{\overline{(a+bi)}}{\overline{(c+di)}} = \frac{\overline{z}}{\overline{w}}$$

61 **(1)** If $\overline{z} = z$, then $a - bi = a + bi$ and hence $-bi = bi$, or $2bi = 0$. Thus, $b = 0$ and $z = a$ is real.
(2) Conversely, if z is real, then $b = 0$ and hence $\overline{z} = \overline{a+0i} = a - 0i = a + 0i = z$.
Thus, by **(1)** and **(2)**, $\overline{z} = z$ if and only if z is real.

62 $\overline{z^2} = \overline{(a+bi)^2} = \overline{a^2+2abi-b^2} = \overline{(a^2-b^2)+2abi} = (a^2-b^2)-2abi = a^2-2abi-b^2 = (a-bi)^2 = (\overline{z})^2$

2.5 Exercises

1 $|x+4| = 11 \Rightarrow x+4 = 11$ or $x+4 = -11 \Rightarrow x = 7$ or $x = -15$

2 $|x-7| = 3 \Rightarrow x-7 = 3$ or $x-7 = -3 \Rightarrow x = 10$ or $x = 4$

3 We must first isolate the absolute value term before proceeding.
$|3x-2| + 3 = 7 \Rightarrow |3x-2| = 4 \Rightarrow 3x-2 = 4$ or $3x-2 = -4 \Rightarrow$
$3x = 6$ or $3x = -2 \Rightarrow x = 2$ or $x = -\frac{2}{3}$

4 $2|5x+2| - 1 = 5 \Rightarrow 2|5x+2| = 6 \Rightarrow |5x+2| = 3 \Rightarrow$
$5x+2 = 3$ or $5x+2 = -3 \Rightarrow 5x = 1$ or $5x = -5 \Rightarrow x = \frac{1}{5}$ or $x = -1$

5 $3|x+1| - 5 = -11 \Rightarrow 3|x+1| = -6 \Rightarrow |x+1| = -2$.
Since the absolute value of an expression is nonnegative, $|x+1| = -2$ has no solution.

6 $|x-3| + 6 = 6 \Rightarrow |x-3| = 0$. Since the absolute value of an expression can only equal 0 if the expression itself is 0, $|x-3| = 0 \Rightarrow x-3 = 0 \Rightarrow x = 3$.

7 Since there are four terms, we first try factoring by grouping.
$9x^3 - 18x^2 - 4x + 8 = 0 \Rightarrow 9x^2(x-2) - 4(x-2) = 0 \Rightarrow$
$(9x^2-4)(x-2) = 0 \Rightarrow x^2 = \frac{4}{9}$ or $x = 2 \Rightarrow x = \pm\frac{2}{3}, 2$

8 $3x^3 - 5x^2 - 12x + 20 = 0 \Rightarrow x^2(3x-5) - 4(3x-5) = 0 \Rightarrow (x^2-4)(3x-5) = 0 \Rightarrow x = \pm 2, \frac{5}{3}$

9 Notice that we can factor an x out of each term, and then factor by grouping.
$4x^4 + 10x^3 = 6x^2 + 15x \Rightarrow 4x^4 + 10x^3 - 6x^2 - 15x = 0 \Rightarrow x(4x^3 + 10x^2 - 6x - 15) = 0 \Rightarrow$
$x\left[2x^2(2x+5) - 3(2x+5)\right] = 0 \Rightarrow x(2x^2-3)(2x+5) = 0 \Rightarrow$
$x = 0$ or $x^2 = \frac{3}{2}$ or $x = -\frac{5}{2} \Rightarrow x = 0, \pm\frac{1}{2}\sqrt{6}, -\frac{5}{2}$

Note: $x^2 = \frac{3}{2} \Rightarrow x = \pm\sqrt{\frac{3}{2}} = \pm\frac{\sqrt{3}}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \pm\frac{\sqrt{6}}{2} = \pm\frac{1}{2}\sqrt{6}$.

There are several ways to write this answer—your professor may have a preference.

10 $15x^5 - 20x^4 = 6x^3 - 8x^2 \Rightarrow 15x^5 - 20x^4 - 6x^3 + 8x^2 = 0 \Rightarrow x^2(15x^3 - 20x^2 - 6x + 8) = 0 \Rightarrow$
$x^2\left[5x^2(3x-4) - 2(3x-4)\right] = 0 \Rightarrow x^2(5x^2-2)(3x-4) = 0 \Rightarrow x = 0, \pm\frac{1}{5}\sqrt{10}, \frac{4}{3}$

11 $y^{3/2} = 5y \Rightarrow y^{3/2} - 5y = 0 \Rightarrow y\left(y^{1/2} - 5\right) = 0 \Rightarrow y = 0$ or $y^{1/2} = 5$.
$y^{1/2} = 5 \Rightarrow \left(y^{1/2}\right)^2 = 5^2 \Rightarrow y = 25$. The solutions are $y = 0$ and $y = 25$.

Note: The following guidelines may be helpful when solving radical equations.

Guidelines for Solving a Radical Equation

(1) Isolate the radical. If we cannot get the radical isolated on one side of the equals sign because there is more than one radical, then we will split up the radical terms as evenly as possible on each side of the equals sign. For example, if there are two radicals, we put one on each side; if there are three radicals, we put two on one side and one on the other.

(2) Raise both sides to the same power as the root index. ***Note:*** Remember here that

$$\boxed{\left(a+b\sqrt{n}\right)^2 = a^2 + 2ab\sqrt{n} + b^2 n}$$

and that $\left(a+b\sqrt{n}\right)^2$ is **not** $a^2 + b^2 n$.

(3) If your equation contains no radicals, proceed to part (4). If there are still radicals in the equation, go back to part (1).

(4) Solve the resulting equation.

(5) Check the answers found in part (4) in the original equation to determine the valid solutions.
Note: You may check the solutions in any equivalent equation of the original equation, that is, an equation which occurs prior to raising both sides to a power. Also, extraneous real number solutions are introduced when raising both sides to an even power. Hence, all solutions *must* be checked in this case. Checking solutions when raising each side to an odd power is up to the individual professor.

12 $y^{4/3} = -4y \Rightarrow y^{4/3} + 4y = 0 \Rightarrow y\left(y^{1/3} + 4\right) = 0 \Rightarrow y = 0$ or $y^{1/3} = -4$.
$y^{1/3} = -4 \Rightarrow \left(y^{1/3}\right)^3 = (-4)^3 \Rightarrow y = -64$. $y = 0, -64$

13 $\sqrt{7-5x} = 8 \Rightarrow \left(\sqrt{7-5x}\right)^2 = 8^2 \Rightarrow 7 - 5x = 64 \Rightarrow -57 = 5x \Rightarrow x = -\frac{57}{5}$

14 $\sqrt{2x-9} = \frac{1}{2} \Rightarrow \left(\sqrt{2x-9}\right)^2 = \left(\frac{1}{2}\right)^2 \Rightarrow 2x - 9 = \frac{1}{4} \Rightarrow 2x = \frac{37}{4} \Rightarrow x = \frac{37}{8}$

15 $4 + \sqrt[3]{1-5t} = 0 \Rightarrow \sqrt[3]{1-5t} = -4 \Rightarrow \left(\sqrt[3]{1-5t}\right)^3 = (-4)^3 \Rightarrow$
$1 - 5t = -64 \Rightarrow -5t = -65 \Rightarrow t = 13$

16 $\sqrt[3]{6-s^2} + 5 = 0 \Rightarrow \left(\sqrt[3]{6-s^2}\right)^3 = (-5)^3 \Rightarrow 6 - s^2 = -125 \Rightarrow 131 = s^2 \Rightarrow s = \pm\sqrt{131}$

17 $\sqrt[5]{2x^2+1} - 2 = 0 \Rightarrow \sqrt[5]{2x^2+1} = 2 \Rightarrow \left(\sqrt[5]{2x^2+1}\right)^5 = 2^5 \Rightarrow$
$2x^2 + 1 = 32 \Rightarrow 2x^2 = 31 \Rightarrow x^2 = \frac{31}{2} \Rightarrow x = \pm\sqrt{\frac{31}{2}} \Rightarrow x = \pm\frac{1}{2}\sqrt{62}$

18 $\sqrt[4]{6x^2-9} = x \Rightarrow \left(\sqrt[4]{6x^2-9}\right)^4 = x^4 \Rightarrow 6x^2 - 9 = x^4 \Rightarrow 0 = x^4 - 6x^2 + 9 \Rightarrow$
$(x^2-3)^2 = 0 \Rightarrow \left[\left(x+\sqrt{3}\right)\left(x-\sqrt{3}\right)\right]^2 \Rightarrow x = \sqrt{3}$ and $-\sqrt{3}$ is an extraneous solution.

19 $\sqrt{7-x} = x - 5$ {square both sides} $\Rightarrow 7 - x = x^2 - 10x + 25$ {set equal to zero} $\Rightarrow$
$x^2 - 9x + 18 = 0$ {factor} $\Rightarrow (x-3)(x-6) = 0 \Rightarrow x = 3, 6$.
Check $x = 6$: LS $= \sqrt{7-6} = 1$; RS $= 6 - 5 = 1$.
Since both sides have the same value, $x = 6$ is a valid solution.
Check $x = 3$: LS $= \sqrt{7-3} = 2$; RS $= 3 - 5 = -2$.
Since both sides do not have the same value, $x = 3$ is an extraneous solution.

20 $\sqrt{3-x} - x = 3 \Rightarrow \left(\sqrt{3-x}\right)^2 = (x+3)^2 \Rightarrow 3 - x = x^2 + 6x + 9 \Rightarrow x^2 + 7x + 6 = 0 \Rightarrow$
$(x+1)(x+6) = 0 \Rightarrow x = -1$ and -6 is an extraneous solution.

21 $3\sqrt{2x-3}+2\sqrt{7-x}=11$ {given}

$3\sqrt{2x-3}=11-2\sqrt{7-x}$ {split radicals evenly}

$9(2x-3)=121-44\sqrt{7-x}+4(7-x)$ {square both sides}

$18x-27=121-44\sqrt{7-x}+28-4x$ {multiply}

$44\sqrt{7-x}=176-22x$ {isolate radical again, simplify}

$2\sqrt{7-x}=8-x$ {divide by gcf, 22}

$4(7-x)=64-16x+x^2$ {square both sides}

$28-4x=64-16x+x^2$ {multiply}

$0=x^2-12x+36$ {collect terms on one side}

$0=(x-6)^2$ {factor}

$0=x-6$ {take the square root}

$6=x$ {solve for x}

Check $x=6$: LS $=3(3)+2(1)=11=$ RS, so $x=6$ is the solution.

22 $\sqrt{2x+15}-2=\sqrt{6x+1} \Rightarrow 2x+15-4\sqrt{2x+15}+4=6x+1 \Rightarrow 4\sqrt{2x+15}=-4x+18 \Rightarrow$
$2\sqrt{2x+15}=-2x+9 \Rightarrow 4(2x+15)=4x^2-36x+81 \Rightarrow 4x^2-44x+21=0 \Rightarrow$
$(2x-1)(2x-21)=0 \Rightarrow x=\frac{1}{2}$ and $\frac{21}{2}$ is an extraneous solution.

23 Remember to isolate the radical term first.
$x=4+\sqrt{4x-19} \Rightarrow x-4=\sqrt{4x-19} \Rightarrow (x-4)^2=\left(\sqrt{4x-19}\right)^2 \Rightarrow$
$x^2-8x+16=4x-19 \Rightarrow x^2-12x+35=0 \Rightarrow (x-5)(x-7)=0 \Rightarrow x=5,7$

24 $x=3+\sqrt{5x-9} \Rightarrow x-3=\sqrt{5x-9} \Rightarrow x^2-6x+9=5x-9 \Rightarrow$
$x^2-11x+18=0 \Rightarrow (x-2)(x-9)=0 \Rightarrow x=9$ and 2 is an extraneous solution.

25 $x-\sqrt{-7x-24}=-2 \Rightarrow x+2=\sqrt{-7x-24} \Rightarrow x^2+4x+4=-7x-24 \Rightarrow$
$x^2+11x+28=0 \Rightarrow (x+4)(x+7)=0 \Rightarrow$ there is no solution since -4 and -7 are extraneous.

26 $x+\sqrt{5x+19}=-1 \Rightarrow \sqrt{5x+19}=-x-1 \Rightarrow 5x+19=x^2+2x+1 \Rightarrow$
$x^2-3x-18=0 \Rightarrow (x-6)(x+3)=0 \Rightarrow x=-3,6.$

Check $x=-3$: LS $=-3+2=-1=$ RS $\Rightarrow x=-3$ is a solution.

Check $x=6$: LS $=6+7=13\neq$ RS $\Rightarrow x=6$ is an extraneous solution.

27 $\sqrt{7-2x}-\sqrt{5+x}=\sqrt{4+3x}$ {given}

$(7-2x)-2\sqrt{(7-2x)(5+x)}+(5+x)=4+3x$ {square both sides}

$-4x+8=2\sqrt{-2x^2-3x+35}$ {isolate the radical}

$-2x+4=\sqrt{-2x^2-3x+35}$ {divide by 2}

$4x^2-16x+16=-2x^2-3x+35$ {square both sides}

$6x^2-13x-19=0$ {simplify}

$(x+1)(6x-19)=0 \Rightarrow x=-1,\frac{19}{6}$ {factor, solve for x}

Check $x=-1$: LS $=3-2=1=$ RS $\Rightarrow x=-1$ is a solution.

Check $x=\frac{19}{6}$: LS $=\sqrt{\frac{2}{3}}-\sqrt{\frac{49}{6}}$ {note that this is negative} $\neq\sqrt{\frac{27}{2}}=$ RS $\Rightarrow$
$x=\frac{19}{6}$ is an extraneous solution.

28 $4\sqrt{1+3x}+\sqrt{6x+3}=\sqrt{-6x-1} \Rightarrow 16(1+3x)+8\sqrt{(3x+1)(6x+3)}+(6x+3)=-6x-1 \Rightarrow$
$8\sqrt{18x^2+15x+3}=-60x-20 \Rightarrow 2\sqrt{18x^2+15x+3}=-15x-5 \Rightarrow$
$4(18x^2+15x+3)=225x^2+150x+25 \Rightarrow 0=153x^2+90x+13 \Rightarrow$
$(3x+1)(51x+13)=0 \Rightarrow x=-\frac{1}{3}$ and $-\frac{13}{51}$ is an extraneous solution.

29 $\sqrt{11+8x}+1=\sqrt{9+4x} \Rightarrow (11+8x)+2\sqrt{11+8x}+1=9+4x \Rightarrow$
$2\sqrt{8x+11}=-4x-3 \Rightarrow 4(8x+11)=16x^2+24x+9 \Rightarrow 32x+44=16x^2+24x+9 \Rightarrow$
$16x^2-8x-35=0 \Rightarrow (4x-7)(4x+5)=0 \Rightarrow x=-\frac{5}{4},\frac{7}{4}$.
Check $x=-\frac{5}{4}$: LS $=1+1=2=$ RS $\Rightarrow x=-\frac{5}{4}$ is a solution.
Check $x=\frac{7}{4}$: LS $=5+1=6\neq 4=$ RS $\Rightarrow x=\frac{7}{4}$ is an extraneous solution.

30 $2\sqrt{x}-\sqrt{x-3}=\sqrt{5+x} \Rightarrow 4x-4\sqrt{x(x-3)}+(x-3)=5+x \Rightarrow$
$4x-8=4\sqrt{x^2-3x} \Rightarrow x-2=\sqrt{x^2-3x} \Rightarrow x^2-4x+4=x^2-3x \Rightarrow x=4$

31 $\sqrt{2\sqrt{x+1}}=\sqrt{3x-5} \Rightarrow 2\sqrt{x+1}=3x-5 \Rightarrow 4(x+1)=9x^2-30x+25 \Rightarrow$
$4x+4=9x^2-30x+25 \Rightarrow 9x^2-34x+21=0 \Rightarrow (x-3)(9x-7)=0 \Rightarrow x=3,\frac{7}{9}$.
Check $x=3$: LS $=2=$ RS $\Rightarrow x=3$ is a solution.
Check $x=\frac{7}{9}$: LS $=\sqrt{2\cdot\frac{4}{3}}=\sqrt{\frac{8}{3}}\neq\sqrt{-\frac{8}{3}}=$ RS $\Rightarrow x=\frac{7}{9}$ is an extraneous solution.

32 $\sqrt{5\sqrt{x}}=\sqrt{2x-3} \Rightarrow 5\sqrt{x}=2x-3 \Rightarrow 25x=4x^2-12x+9 \Rightarrow 4x^2-37x+9=0 \Rightarrow$
$(4x-1)(x-9)=0 \Rightarrow x=9$ and $\frac{1}{4}$ is an extraneous solution.

33 $\sqrt{1+4\sqrt{x}}=\sqrt{x}+1 \Rightarrow 1+4\sqrt{x}=x+2\sqrt{x}+1 \Rightarrow 2\sqrt{x}=x \Rightarrow$
$4x=x^2 \Rightarrow 4x-x^2=0 \Rightarrow x(4-x)=0 \Rightarrow x=0,4$

34 $\sqrt{x+2}=\sqrt{x-2} \Rightarrow x+2=x-2 \Rightarrow 2=-2 \Rightarrow$ No solution

35 $x^4-34x^2+225=0 \Rightarrow (x^2-9)(x^2-25)=0 \Rightarrow x^2=9,25 \Rightarrow x=\pm\sqrt{9},\pm\sqrt{25} \Rightarrow$
$x=\pm 3,\pm 5$

36 $2x^4-10x^2+8=0 \Rightarrow 2\left(x^4-5x^2+4\right)=0 \Rightarrow 2(x^2-1)(x^2-4)=0 \Rightarrow x=\pm 1,\pm 2$

Note: Substitution could be used instead of factoring for the following exercises.

37 We recognize this equation as a quadratic equation in y^2 and apply the quadratic formula, solving for y^2, *not* y.
$5y^4-7y^2+1.5=0 \Rightarrow y^2=\dfrac{7\pm\sqrt{19}}{10}\cdot\dfrac{10}{10}=\dfrac{70\pm 10\sqrt{19}}{100} \Rightarrow y=\pm\dfrac{1}{10}\sqrt{70\pm 10\sqrt{19}}$
Alternatively, let $u=y^2$ and solve $5u^2-7u+1.5=0$.

38 $3y^4-5y^2+1.5=0 \Rightarrow y^2=\dfrac{5\pm\sqrt{7}}{6}\cdot\dfrac{6}{6}=\dfrac{30\pm 6\sqrt{7}}{36} \Rightarrow y=\pm\dfrac{1}{6}\sqrt{30\pm 6\sqrt{7}}$

39 $36x^{-4}-13x^{-2}+1=0 \Rightarrow (4x^{-2}-1)(9x^{-2}-1)=0 \Rightarrow x^{-2}=\frac{1}{4},\frac{1}{9} \Rightarrow x^2=4,9 \Rightarrow x=\pm 2,\pm 3$
Alternatively, let $u=x^{-2}$ and solve $36u^2-13u+1=0$.

40 $x^{-2}-2x^{-1}-35=0 \Rightarrow (x^{-1}-7)(x^{-1}+5)=0 \Rightarrow x^{-1}=7,-5 \Rightarrow x=\frac{1}{7},-\frac{1}{5}$

41 $3x^{2/3}+4x^{1/3}-4=0 \Rightarrow \left(3x^{1/3}-2\right)\left(x^{1/3}+2\right)=0 \Rightarrow$
$\sqrt[3]{x}=\frac{2}{3},-2 \Rightarrow x=\left(\frac{2}{3}\right)^3,(-2)^3 \Rightarrow x=\frac{8}{27},-8$
Alternatively, let $u=x^{1/3}$ and solve $3u^2+4u-4=0$.

42 $2y^{1/3} - 3y^{1/6} + 1 = 0 \Rightarrow \left(2y^{1/6} - 1\right)\left(y^{1/6} - 1\right) = 0 \Rightarrow \sqrt[6]{y} = \frac{1}{2}, 1 \Rightarrow y = \frac{1}{64}, 1$

43 $6w + 7w^{1/2} - 20 = 0 \Rightarrow \left(2w^{1/2} + 5\right)\left(3w^{1/2} - 4\right) = 0 \Rightarrow$
$\sqrt{w} = -\frac{5}{2}, \frac{4}{3} \Rightarrow w = \left(\frac{4}{3}\right)^2 = \frac{16}{9}$ $\{\sqrt{w}$ cannot be negative$\}$

Alternatively, let $u = w^{1/2}$ and solve $6u^2 + 7u - 20 = 0$.

44 $8t + 6t^{1/2} - 35 = 0 \Rightarrow \left(2t^{1/2} + 5\right)\left(4t^{1/2} - 7\right) = 0 \Rightarrow$
$\sqrt{t} = -\frac{5}{2}, \frac{7}{4} \Rightarrow t = \left(\frac{7}{4}\right)^2 = \frac{49}{16}$ $\{\sqrt{t}$ cannot be negative$\}$

45 $2x^{-2/3} - 7x^{-1/3} - 15 = 0 \Rightarrow \left(2x^{-1/3} + 3\right)\left(x^{-1/3} - 5\right) = 0 \Rightarrow x^{-1/3} = -\frac{3}{2}, 5 \Rightarrow$
$x^{1/3} = -\frac{2}{3}, \frac{1}{5} \Rightarrow \sqrt[3]{x} = -\frac{2}{3}, \frac{1}{5} \Rightarrow x = \left(-\frac{2}{3}\right)^3, \left(\frac{1}{5}\right)^3 \Rightarrow x = -\frac{8}{27}, \frac{1}{125}$

Alternatively, let $u = x^{-1/3}$ and solve $2u^2 - 7u - 15 = 0$.

46 $6u^{-1/2} - 13u^{-1/4} + 6 = 0 \Rightarrow \left(2u^{-1/4} - 3\right)\left(3u^{-1/4} - 2\right) \Rightarrow u^{-1/4} = \frac{3}{2}, \frac{2}{3} \Rightarrow$
$u^{1/4} = \frac{2}{3}, \frac{3}{2} \Rightarrow \sqrt[4]{u} = \frac{2}{3}, \frac{3}{2} \Rightarrow x = \left(\frac{2}{3}\right)^4, \left(\frac{3}{2}\right)^4 \Rightarrow u = \frac{16}{81}, \frac{81}{16}$

47 $\left(\dfrac{t}{t+1}\right)^2 - \dfrac{2t}{t+1} - 8 = 0 \Rightarrow \left(\dfrac{t}{t+1} - 4\right)\left(\dfrac{t}{t+1} + 2\right) = 0 \Rightarrow \dfrac{t}{t+1} = 4$ or $-2 \Rightarrow$
$t = 4(t+1)$ or $-2(t+1) \Rightarrow t = 4t + 4$ or $t = -2t - 2 \Rightarrow -3t = 4$ or $3t = -2 \Rightarrow t = -\frac{4}{3}, -\frac{2}{3}$

Alternatively, let $u = \dfrac{t}{t+1}$ and solve $u^2 - 2u - 8 = 0$.

48 $\left(\dfrac{x}{x-2}\right)^2 - \dfrac{2x}{x-2} - 15 = 0 \Rightarrow \left(\dfrac{x}{x-2} - 5\right)\left(\dfrac{x}{x-2} + 3\right) = 0 \Rightarrow \dfrac{x}{x-2} = 5$ or $-3 \Rightarrow$
$x = 5(x-2)$ or $x = -3(x-2) \Rightarrow x = 5x - 10$ or $x = -3x + 6 \Rightarrow -4x = -10$ or $4x = 6 \Rightarrow$
$x = \frac{5}{2}, \frac{3}{2}$

49 The least common multiple of 3 and 4 is 12—so by raising both sides to the 12th power we will eliminate the radicals. $\sqrt[3]{x} = 2\sqrt[4]{x} \Rightarrow \left(\sqrt[3]{x}\right)^{12} = \left(2\sqrt[4]{x}\right)^{12} \Rightarrow$
$x^4 = 2^{12}x^3 \Rightarrow x^4 - 4096x^3 = 0 \Rightarrow x^3(x - 4096) = 0 \Rightarrow x = 0, 4096.$

Check $x = 0$: LS $= 0 =$ RS $\Rightarrow x = 0$ is a solution.

Check $x = 4096 = 2^{12}$: LS $= \sqrt[3]{2^{12}} = 2^4$; RS $= 2\sqrt[4]{2^{12}} = 2 \cdot 2^3 = 2^4 \Rightarrow x = 4096$ is a solution.

50 $\sqrt{x+3} = \sqrt[4]{2x+6} \Rightarrow \left(\sqrt{x+3}\right)^4 = \left(\sqrt[4]{2x+6}\right)^4 \Rightarrow (x+3)^2 = 2x + 6 \Rightarrow$
$x^2 + 6x + 9 = 2x + 6 \Rightarrow x^2 + 4x + 3 = 0 \Rightarrow (x+1)(x+3) = 0 \Rightarrow x = -1, -3$

51 See the illustrations and discussion on text page 96 for help on solving equations by raising both sides to a reciprocal power. Note that if $x^{m/n}$ is in the given equation and m is even, we have to use the $\pm$ symbol for the solutions. Here are a few more examples:

Equation	**Solution**
$x^{1/2} = 4$	$\left(x^{1/2}\right)^{2/1} = 4^{2/1} \Rightarrow x = 16$
$x^{-1/2} = 5$	$\left(x^{-1/2}\right)^{-2/1} = 5^{-2/1} \Rightarrow x = \frac{1}{25}$
$x^{3/4} = 8$	$\left(x^{3/4}\right)^{4/3} = 8^{4/3} \Rightarrow x = 16$
$x^{4/3} = 81$	$\left(x^{4/3}\right)^{3/4} = 81^{3/4} \Rightarrow x = \pm 27$ $\{\pm$ since 4 is even$\}$

(a) $x^{5/3} = 32 \Rightarrow \left(x^{5/3}\right)^{3/5} = (32)^{3/5} \Rightarrow x = \left(\sqrt[5]{32}\right)^3 = 2^3 = 8$

(b) $x^{4/3} = 16 \Rightarrow \left(x^{4/3}\right)^{3/4} = \pm(16)^{3/4} \Rightarrow x = \pm\left(\sqrt[4]{16}\right)^3 = \pm 2^3 = \pm 8$

(c) $x^{2/3} = -64 \Rightarrow \left(x^{2/3}\right)^{3/2} = \pm(-64)^{3/2} \Rightarrow x = \pm\left(\sqrt{-64}\right)^3$, which are not real numbers.

No real solutions

(d) $x^{3/4} = 125 \Rightarrow \left(x^{3/4}\right)^{4/3} = (125)^{4/3} \Rightarrow x = \left(\sqrt[3]{125}\right)^4 = 5^4 = 625$

(e) $x^{3/2} = -27 \Rightarrow \left(x^{3/2}\right)^{2/3} = (-27)^{2/3} \Rightarrow x = \left(\sqrt[3]{-27}\right)^2 = (-3)^2 = 9$,

which is an extraneous solution. **No real solutions**

52 (a) $x^{3/5} = -27 \Rightarrow \left(x^{3/5}\right)^{5/3} = (-27)^{5/3} \Rightarrow x = \left(\sqrt[3]{-27}\right)^5 = (-3)^5 = -243$

(b) $x^{2/3} = 25 \Rightarrow \left(x^{2/3}\right)^{3/2} = \pm(25)^{3/2} \Rightarrow x = \pm\left(\sqrt{25}\right)^3 = \pm 5^3 = \pm 125$

(c) $x^{4/3} = -49 \Rightarrow \left(x^{4/3}\right)^{3/4} = \pm(-49)^{3/4} \Rightarrow x = \pm\left(\sqrt[4]{-49}\right)^3$, which are not real numbers.

No real solutions

(d) $x^{3/2} = 64 \Rightarrow \left(x^{3/2}\right)^{2/3} = (64)^{2/3} \Rightarrow x = \left(\sqrt[3]{64}\right)^2 = 4^2 = 16$

(e) $x^{3/4} = -8 \Rightarrow \left(x^{3/4}\right)^{4/3} = (-8)^{4/3} \Rightarrow x = \left(\sqrt[3]{-8}\right)^4 = (-2)^4 = 16$, which is an extraneous solution.

No real solutions

53 $T = 2\pi\sqrt{\dfrac{l}{g}} \Rightarrow \dfrac{T}{2\pi} = \sqrt{\dfrac{l}{g}} \Rightarrow \left(\dfrac{T}{2\pi}\right)^2 = \left(\sqrt{\dfrac{l}{g}}\right)^2 \Rightarrow \dfrac{T^2}{4\pi^2} = \dfrac{l}{g} \Rightarrow l = \dfrac{gT^2}{4\pi^2}$

54 $d = \frac{1}{2}\sqrt{4R^2 - C^2} \Rightarrow 2d = \sqrt{4R^2 - C^2} \Rightarrow 4d^2 = 4R^2 - C^2 \Rightarrow C^2 = 4R^2 - 4d^2 \Rightarrow$
$C^2 = 4(R^2 - d^2) \Rightarrow C = \pm 2\sqrt{R^2 - d^2} \Rightarrow C = 2\sqrt{R^2 - d^2}$ since $C > 0$

55 $S = \pi r\sqrt{r^2 + h^2} \Rightarrow \dfrac{S}{\pi r} = \sqrt{r^2 + h^2} \Rightarrow \left(\dfrac{S}{\pi r}\right)^2 = \left(\sqrt{r^2 + h^2}\right)^2 \Rightarrow \dfrac{S^2}{\pi^2 r^2} = r^2 + h^2 \Rightarrow$
$\dfrac{S^2}{\pi^2 r^2} - r^2 = h^2 \Rightarrow \dfrac{S^2}{\pi^2 r^2} - \dfrac{\pi^2 r^4}{\pi^2 r^2} = h^2 \Rightarrow h^2 = \dfrac{1}{\pi^2 r^2}\left(S^2 - \pi^2 r^4\right) \Rightarrow h = \pm\dfrac{1}{\pi r}\sqrt{S^2 - \pi^2 r^4} \Rightarrow$
$h = \dfrac{1}{\pi r}\sqrt{S^2 - \pi^2 r^4}$ since $h > 0$

56 $\omega = \dfrac{1}{\sqrt{LC}} \Rightarrow (\omega)^2 = \left(\dfrac{1}{\sqrt{LC}}\right)^2 \Rightarrow \omega^2 = \dfrac{1}{LC} \Rightarrow C\omega^2 = \dfrac{1}{L} \Rightarrow C = \dfrac{1}{L\omega^2}$

57 From the Pythagorean theorem, $d^2 + h^2 = L^2$. Since d is to be 25% of L, we have
$d = \frac{1}{4}L$, so $\left(\frac{1}{4}L\right)^2 + h^2 = L^2 \Rightarrow h^2 = L^2 - \left(\frac{1}{4}L\right)^2 \Rightarrow h^2 = 1L^2 - \frac{1}{16}L^2 \Rightarrow$
$h^2 = \frac{15}{16}L^2 \Rightarrow h = \sqrt{\frac{15}{16}L^2}$ {since $h > 0$} $= \frac{\sqrt{15}}{4}L \approx 0.97L$. Thus, $h \approx 97\%L$.

58 $A = k\sqrt{\dfrac{t}{T}} \Rightarrow \dfrac{A}{k} = \sqrt{\dfrac{t}{T}} \Rightarrow \left(\dfrac{A}{k}\right)^2 = \left(\sqrt{\dfrac{t}{T}}\right)^2 \Rightarrow \dfrac{A^2}{k^2} = \dfrac{t}{T} \Rightarrow t = \dfrac{TA^2}{k^2}$

59 $P = 0.31ED^2V^3 \Rightarrow V^3 = \dfrac{P}{0.31ED^2} \Rightarrow V = \left(\dfrac{P}{0.31ED^2}\right)^{1/3} = \left(\dfrac{10{,}000}{(0.31)(0.42)10^2}\right)^{1/3} \approx 9.16$ ft/sec.

Multiplying by $\frac{60}{88}$ $\left(\text{or } \frac{15}{22}\right)$ to convert to mi/hr gives us approximately 6.24 mi/hr.

60 $P = 15{,}700S^{5/2}RD \Rightarrow S^{5/2} = \dfrac{P}{15{,}700RD} \Rightarrow$

$$S = \left(\frac{P}{15{,}700RD}\right)^{2/5} = \left[\frac{380}{(15{,}700)(0.113/2)(2)}\right]^{2/5} \approx 0.54$$

61 $k = 10^5$ and $c = \frac{1}{2} \Rightarrow Q = kP^{-c} = 10^5P^{-1/2} \Rightarrow Q = \dfrac{10^5}{\sqrt{P}} \Rightarrow \sqrt{P} = \dfrac{10^5}{Q} \Rightarrow$

$$P = \left(\frac{10^5}{Q}\right)^2 = \left(\frac{100{,}000}{5000}\right)^2 = (20)^2 = 400 \text{ cents, or } \$4.00.$$

62 $T = 0.25P^{1/4}/\sqrt{v} \quad\Rightarrow\quad P^{1/4} = 4T\sqrt{v} \quad\Rightarrow\quad P = (4T)^4v^2 = 4^4 3^4 5^2 = 518{,}400$

63 $V = 144$ and $V = \frac{1}{3}\pi r^2 h \quad\Rightarrow\quad 144 = \frac{1}{3}\pi r^3$ {since $r = h$} $\Rightarrow$

$$r^3 = \frac{3 \cdot 144}{\pi} \quad\Rightarrow\quad r = \sqrt[3]{\frac{432}{\pi}}, \text{ and the diameter is } 2 \cdot \sqrt[3]{\frac{432}{\pi}} \approx 10.3 \text{ cm.}$$

64 **Original:** $V = \dfrac{4}{3}\pi r^3 \quad\Rightarrow\quad \dfrac{32}{3} = \dfrac{4}{3}\pi r^3 \quad\Rightarrow\quad r^3 = \dfrac{8}{\pi} \quad\Rightarrow\quad r = \dfrac{2}{\sqrt[3]{\pi}}$ and $d = \dfrac{4}{\sqrt[3]{\pi}}$

Inflated: $V = 25\dfrac{1}{3} + 10\dfrac{2}{3} \quad\Rightarrow\quad \dfrac{4}{3}\pi r^3 = 36 \quad\Rightarrow\quad r^3 = \dfrac{27}{\pi} \quad\Rightarrow\quad r = \dfrac{3}{\sqrt[3]{\pi}}$ and $d = \dfrac{6}{\sqrt[3]{\pi}}$

The change in the diameter is $\dfrac{6}{\sqrt[3]{\pi}} - \dfrac{4}{\sqrt[3]{\pi}} = \dfrac{2}{\sqrt[3]{\pi}} \approx 1.37$ ft.

65 $y = \dfrac{x^3}{x^3 + (1-x)^3}$ • $y = 60\%\left\{= \dfrac{3}{5}\right\} \quad\Rightarrow\quad \dfrac{x^3}{x^3 + (1-x)^3} = \dfrac{3}{5} \quad\Rightarrow\quad 5x^3 = 3x^3 + 3(1-x)^3 \quad\Rightarrow$

$2x^3 = 3(1-x)^3 \quad\Rightarrow\quad \left(\dfrac{x}{1-x}\right)^3 = \dfrac{3}{2} \quad\Rightarrow\quad \dfrac{x}{1-x} = \sqrt[3]{1.5} \quad\Rightarrow\quad x = \sqrt[3]{1.5} - \sqrt[3]{1.5}\,x \quad\Rightarrow$

$x + \sqrt[3]{1.5}\,x = \sqrt[3]{1.5} \quad\Rightarrow\quad \left(1 + \sqrt[3]{1.5}\right)x = \sqrt[3]{1.5} \quad\Rightarrow\quad x = \dfrac{\sqrt[3]{1.5}}{1 + \sqrt[3]{1.5}} \approx 0.534$, or 53.4%.

66 $S = \pi r\sqrt{r^2 + h^2}$ with $S = 6\pi$ in.^{2} and $h = 3$ in. $\Rightarrow\quad 6\pi = \pi r\sqrt{r^2 + 9} \quad\Rightarrow\quad 6 = r\sqrt{r^2 + 9} \quad\Rightarrow$

$36 = r^2(r^2 + 9) \quad\Rightarrow\quad r^4 + 9r^2 - 36 = 0 \quad\Rightarrow\quad (r^2 + 12)(r^2 - 3) = 0 \quad\Rightarrow\quad r = \sqrt{3}$ in.

67 $\text{Cost}_{\text{underwater}} + \text{Cost}_{\text{overland}} = \text{Cost}_{\text{total}} \quad\Rightarrow\quad 7500 \cdot (\text{underwater miles}) + 6000 \cdot (\text{overland miles}) = 35{,}000 \quad\Rightarrow$

$7500\sqrt{x^2 + 1} + 6000(5 - x) = 35{,}000 \quad\Rightarrow\quad 7500\sqrt{x^2 + 1} + 30{,}000 - 6000x = 35{,}000 \quad\Rightarrow$

$7500\sqrt{x^2 + 1} = 6000x + 5{,}000 \quad\Rightarrow\quad 15\sqrt{x^2 + 1} = 12x + 10$ {divide by 500} $\Rightarrow$

$225(x^2 + 1) = 144x^2 + 240x + 100 \quad\Rightarrow\quad 225x^2 + 225 = 144x^2 + 240x + 100 \quad\Rightarrow$

$81x^2 - 240x + 125 = 0 \quad\Rightarrow\quad x = \dfrac{240 \pm \sqrt{17{,}100}}{162} = \dfrac{6 \cdot 40 \pm \sqrt{900 \cdot 19}}{6 \cdot 27} = \dfrac{6 \cdot 40 \pm 30\sqrt{19}}{6 \cdot 27} = \dfrac{40 \pm 5\sqrt{19}}{27} \approx$

2.2887, 0.6743 mi. There are two possible routes.

68 (a) $h + \dfrac{h}{h_M - h} = at + \dfrac{h_0 + t}{1 + \frac{4}{3}t}$ • $h_0 = 0.5$, $h_M = 1.684$, $a = 0.545$, and $t = 12 \quad\Rightarrow$

$h + \dfrac{h}{1.684 - h} = 0.545(12) + \dfrac{0.5 + 12}{1 + \frac{4}{3}(12)} \quad\Rightarrow\quad \dfrac{1.684h - h^2 + h}{1.684 - h} = 6.54 + \dfrac{12.5}{17} \quad\Rightarrow$

$\dfrac{h^2 - 2.684h}{h - 1.684} = \dfrac{123.68}{17} \quad\Rightarrow\quad 17h^2 - 45.628h = 123.68h - 208.27712 \quad\Rightarrow$

$17h^2 - 169.308h + 208.27712 = 0 \quad\Rightarrow\quad h = \dfrac{169.308 \pm \sqrt{14{,}502.3547}}{34} \quad\Rightarrow$

$h \approx 8.5216, 1.4377$; only 1.4377 meters (≈ 56.60 inches) makes sense.

(b) Let $h = \frac{1}{2}h_{\text{M}} = \frac{1}{2}(1.684) = 0.842$. $0.842 + \dfrac{0.842}{1.684 - 0.842} = 0.545t + \dfrac{0.5 + t}{1 + \frac{4}{3}t} \quad\Rightarrow$

$0.842 + 1 = 0.545t + \dfrac{0.5 + t}{1 + \frac{4}{3}t} \cdot \dfrac{6}{6} \quad\Rightarrow\quad 1.842 - 0.545t = \dfrac{3 + 6t}{6 + 8t} \quad\Rightarrow$

$(1842 - 545t)(6 + 8t) = 1000(3 + 6t) \quad\Rightarrow\quad 4360t^2 + 11{,}466t + 11{,}052 = 3000 + 6000t \quad\Rightarrow$

$4360t^2 - 5466t - 8052 = 0 \quad\Rightarrow\quad t = \dfrac{5466 \pm \sqrt{170{,}304{,}036}}{8720} \quad\Rightarrow$

$t \approx -0.8697, 2.1234$; only 2.1234 years (≈ 25.5 months) makes sense.

69 (a) Let $Y_1 = D_1 = 6.096L + 685.7$ and $Y_2 = D_2 = 0.00178L^3 - 0.072L^2 + 4.37L + 719$.

Table each equation and compare them to the actual values.

x (L)	0	10	20	30	40	50	60
Y_1	686	747	808	869	930	991	1051
Y_2	719	757	792	833	893	980	1106
Summer	720	755	792	836	892	978	1107

Comparing Y_1 (D_1) with Y_2 (D_2) we can see that the linear equation D_1 is not as accurate as the cubic equation D_2.

(b) $L = 35 \quad \Rightarrow \quad D_2 = 0.00178(35)^3 - 0.072(35)^2 + 4.37(35) + 719 \approx 860$ min.

70 (a) The volume of the box is given by $V = x(24 - 2x)(36 - 2x)$.

(b) Let $Y_1 = x(24 - 2x)(36 - 2x)$. The maximum V is $1825.292 \approx 1825.3$ in^2 when $x = 4.7$ in.

x	4.5	4.6	4.7	4.8	4.9	5.0
V	1822.5	1824.5	1825.3	1824.8	1823.0	1820.0

71 The volume of the box is $V = hw^2 = 25$, where h is the height and w is the length of a side of the square base. The amount of cardboard will be minimized when the surface area of the box is a minimum. The surface area is given by $S = w^2 + 4wh$. Since $h = 25/w^2$, we have $S = w^2 + 100/w$. Form a table for w and S.

w	3.4	3.5	3.6	3.7	3.8	3.9
S	40.972	40.821	40.738	40.717	40.756	40.851

The minimum surface area is $S \approx 40.717$ when $w \approx 3.7$ and $h = 25/w^2 \approx 1.8$.

2.6 Exercises

1 (a) 5 is added to both sides: $-7 + 5 < -3 + 5 \quad \Rightarrow \quad -2 < 2$

(b) 2 is subtracted from both sides: $-7 - 2 < -3 - 2 \quad \Rightarrow \quad -9 < -5$

(c) both sides are multiplied by $\frac{1}{3}$: $-7 \cdot \frac{1}{3} < -3 \cdot \frac{1}{3} \quad \Rightarrow \quad -\frac{7}{3} < -1$

(d) both sides are multiplied by $-\frac{1}{3}$: $-7 \cdot \left(-\frac{1}{3}\right) > -3 \cdot \left(-\frac{1}{3}\right) \quad \Rightarrow \quad \frac{7}{3} > 1 \quad \Rightarrow \quad 1 < \frac{7}{3}$

2 (a) 5 is added to both sides: $4 + 5 > -5 + 5 \quad \Rightarrow \quad 9 > 0$

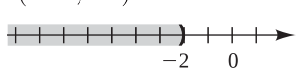
(b) -3 is subtracted from both sides: $4 - (-3) > -5 - (-3) \quad \Rightarrow \quad 7 > -2$

(c) both sides are divided by 6: $\frac{4}{6} > -\frac{5}{6} \quad \Rightarrow \quad \frac{2}{3} > -\frac{5}{6}$

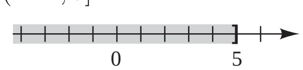
(d) both sides are divided by -6: $\frac{4}{-6} < -\frac{5}{-6} \quad \Rightarrow \quad -\frac{2}{3} < \frac{5}{6}$

Note: Brackets, "[" and "]", are used with $\leq$ or $\geq$ and indicate that the endpoint of the interval is part of the solution. Parentheses, "(" and ")", are used with $<$ or $>$ and indicate that the endpoint is *not* part of the solution.

3 $x < -2 \quad \Leftrightarrow \quad (-\infty, -2)$



4 $x \leq 5 \quad \Leftrightarrow \quad (-\infty, 5]$

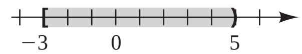


5 $x \geq 4 \quad \Leftrightarrow \quad [4, \infty)$

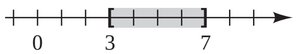
6 $x > -3 \quad \Leftrightarrow \quad (-3, \infty)$

7 $-2 < x \le 4 \;\Leftrightarrow\; (-2, 4]$

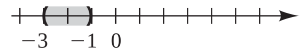
8 $-3 \le x < 5 \;\Leftrightarrow\; [-3, 5)$



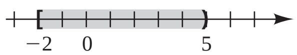
9 $3 \le x \le 7 \;\Leftrightarrow\; [3, 7]$



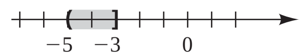
10 $-3 < x < -1 \;\Leftrightarrow\; (-3, -1)$



11 $5 > x \ge -2 \;\Leftrightarrow\; -2 \le x < 5 \;\Leftrightarrow\; [-2, 5)$



12 $-3 \ge x > -5 \;\Leftrightarrow\; -5 < x \le -3 \;\Leftrightarrow\; (-5, -3]$



13 $(-5, 4] \;\Leftrightarrow\; -5 < x \le 4$

14 $[0, 4) \;\Leftrightarrow\; 0 \le x < 4$

15 $[-8, -1] \;\Leftrightarrow\; -8 \le x \le -1$

16 $(3, 7) \;\Leftrightarrow\; 3 < x < 7$

17 $[4, \infty) \;\Leftrightarrow\; x \ge 4$

18 $(-6, \infty) \;\Leftrightarrow\; x > -6$

19 $(-\infty, -7) \;\Leftrightarrow\; x < -7$

20 $(-\infty, 2] \;\Leftrightarrow\; x \le 2$

21 $3x - 2 > 12 \;\Rightarrow\; 3x > 14$ {add 2} $\Rightarrow\; x > \frac{14}{3}$ {divide by 3} $\Leftrightarrow\; \left(\frac{14}{3}, \infty\right)$

22 $2x + 5 \le 8 \;\Rightarrow\; 2x \le 3$ {subtract 5} $\Rightarrow\; x \le \frac{3}{2}$ {divide by 2} $\Leftrightarrow\; \left(-\infty, \frac{3}{2}\right]$

23 $-2 - 3x \ge 2 \;\Rightarrow\; -3x \ge 4$ {Remember to change the direction of the inequality when multiplying or dividing by a negative value.} $\Rightarrow\; x \le -\frac{4}{3} \;\Leftrightarrow\; \left(-\infty, -\frac{4}{3}\right]$

24 $3 - 5x < 11 \;\Rightarrow\; -5x < 8 \;\Rightarrow\; x > -\frac{8}{5}$ {change inequality} $\Leftrightarrow\; \left(-\frac{8}{5}, \infty\right)$

25 $2x + 5 < 3x - 7 \;\Rightarrow\; -x < -12 \;\Rightarrow\; x > 12$ {change inequality} $\Leftrightarrow\; (12, \infty)$

26 $x - 6 > 5x + 3 \;\Rightarrow\; -4x > 9 \;\Rightarrow\; x < -\frac{9}{4}$ {change inequality} $\Leftrightarrow\; \left(-\infty, -\frac{9}{4}\right)$

27 $\left[\frac{1}{4}x + 7 \le \frac{1}{3}x - 2\right] \cdot 12$ {multiply both sides by the lcd, 12} $\Rightarrow\; 3x + 84 \le 4x - 24 \;\Rightarrow\; -x \le -108 \;\Rightarrow\; x \ge 108 \;\Leftrightarrow\; [108, \infty)$

28 $\left[9 + \frac{1}{3}x \ge 4 - \frac{1}{2}x\right] \cdot 6$ {multiply both sides by the lcd, 6} $\Rightarrow\; 54 + 2x \ge 24 - 3x \;\Rightarrow\; 5x \ge -30 \;\Rightarrow\; x \ge -6 \;\Leftrightarrow\; [-6, \infty)$

29 $-3 < 2x - 5 < 7 \;\Rightarrow\; 2 < 2x < 12$ {add 5 to all three expressions} $\Rightarrow\; 1 < x < 6$ {divide all three expressions by 2} $\Leftrightarrow\; (1, 6)$

30 $4 \ge 3x + 5 > -1 \;\Leftrightarrow\; -1 < 3x + 5 \le 4 \;\Rightarrow\; -6 < 3x \le -1$ {subtract 5 from all three expressions} $\Rightarrow\; -2 < x \le -\frac{1}{3}$ {divide all three expressions by 3} $\Leftrightarrow\; \left(-2, -\frac{1}{3}\right]$

31 $\left[3 \le \dfrac{2x - 9}{5} < 7\right] \cdot 5$ {multiply by the lcd, 5} $\Rightarrow\; 15 \le 2x - 9 < 35 \;\Rightarrow$
$24 \le 2x < 44$ {add 9 to all three parts} $\Rightarrow\; 12 \le x < 22$ {divide all three parts by 2} $\Leftrightarrow$
$[12, 22)$ {equivalent interval notation}

32 $\left[-2 < \dfrac{4x + 1}{3} \le 0\right] \cdot 3 \;\Rightarrow\; -6 < 4x + 1 \le 0 \;\Rightarrow\; -7 < 4x \le -1 \;\Rightarrow\; -\frac{7}{4} < x \le -\frac{1}{4} \;\Leftrightarrow\; \left(-\frac{7}{4}, -\frac{1}{4}\right]$

33 $4 > \dfrac{2-3x}{7} \geq -2$ {multiply all three expressions by 7} $\Rightarrow$ $28 > 2-3x \geq -14$ $\Rightarrow$

$26 > -3x \geq -16$ {divide by -3 and change directions of *both* inequality signs} $\Rightarrow$

$-\frac{26}{3} < x \leq \frac{16}{3}$ $\Leftrightarrow$ $\left(-\frac{26}{3}, \frac{16}{3}\right]$

34 $5 \geq \dfrac{6-5x}{3} > 2$ $\Rightarrow$ $15 \geq 6-5x > 6$ $\Rightarrow$ $9 \geq -5x > 0$ $\Rightarrow$ $-\frac{9}{5} \leq x < 0$ $\Leftrightarrow$ $\left[-\frac{9}{5}, 0\right)$

35 $0 \leq 4 - \frac{1}{3}x < 2$ $\Rightarrow$ $-4 \leq -\frac{1}{3}x < -2$ $\Rightarrow$ $12 \geq x > 6$ {multiply by -3} $\Rightarrow$ $6 < x \leq 12$ $\Leftrightarrow$ $(6, 12]$

36 $-2 < 3 + \frac{1}{4}x \leq 5$ $\Rightarrow$ $-5 < \frac{1}{4}x \leq 2$ $\Rightarrow$ $-20 < x \leq 8$ $\Leftrightarrow$ $(-20, 8]$

37 $(2x-3)(4x+5) \leq (8x+1)(x-7)$ $\Rightarrow$ $8x^2 - 2x - 15 \leq 8x^2 - 55x - 7$ $\Rightarrow$

$53x \leq 8$ $\Rightarrow$ $x \leq \frac{8}{53}$ $\Leftrightarrow$ $\left(-\infty, \frac{8}{53}\right]$

38 $(x-3)(x+3) \geq (x+5)^2$ $\Rightarrow$ $x^2 - 9 \geq x^2 + 10x + 25$ $\Rightarrow$ $-34 \geq 10x$ $\Rightarrow$

$10x \leq -34$ $\Rightarrow$ $x \leq -\frac{17}{5}$ $\Leftrightarrow$ $\left(-\infty, -\frac{17}{5}\right]$

39 $(x-4)^2 > x(x+12)$ $\Rightarrow$ $x^2 - 8x + 16 > x^2 + 12x$ $\Rightarrow$ $-20x > -16$ $\Rightarrow$ $x < \frac{4}{5}$ $\Leftrightarrow$ $\left(-\infty, \frac{4}{5}\right)$

40 $2x(6x+5) < (3x-2)(4x+1)$ $\Rightarrow$ $12x^2 + 10x < 12x^2 - 5x - 2$ $\Rightarrow$ $15x < -2$ $\Rightarrow$

$x < -\frac{2}{15}$ $\Leftrightarrow$ $\left(-\infty, -\frac{2}{15}\right)$

41 By the law of signs, a quotient is positive if the sign of the numerator and the sign of the denominator are the same. Since the numerator is positive, $\dfrac{4}{3x+2} > 0$ $\Rightarrow$ $3x+2 > 0$ $\Rightarrow$ $x > -\frac{2}{3}$ $\Leftrightarrow$ $\left(-\frac{2}{3}, \infty\right)$.

The expression is never equal to 0 since the numerator is never 0. Thus, the solution of $\dfrac{4}{3x+2} \geq 0$ is $\left(-\frac{2}{3}, \infty\right)$.

42 $\dfrac{3}{2x+5} \leq 0$ $\Rightarrow$ $2x+5 < 0$ {denominator must be negative} $\Rightarrow$ $2x < -5$ $\Rightarrow$ $x < -\frac{5}{2}$ $\Leftrightarrow$ $\left(-\infty, -\frac{5}{2}\right)$

43 $\dfrac{-7}{4-3x} > 0$ $\Rightarrow$ $4 - 3x < 0$ {denominator must also be negative} $\Rightarrow$

$4 < 3x$ $\Rightarrow$ $3x > 4$ $\Rightarrow$ $x > \frac{4}{3}$ $\Leftrightarrow$ $\left(\frac{4}{3}, \infty\right)$

44 $\dfrac{-3}{2-x} < 0$ $\Rightarrow$ $2 - x > 0$ $\Rightarrow$ $2 > x$ $\Rightarrow$ $x < 2$ $\Leftrightarrow$ $(-\infty, 2)$

45 $(1-x)^2 > 0$ $\forall x$ *except* 1. Thus, $\dfrac{5}{(1-x)^2} > 0$ has solution $\mathbb{R} - \{1\}$.

46 $x^2 + 4 > 0$ $\forall x$. Hence, $\dfrac{3}{x^2+4} > 0$ $\forall x$, and $\dfrac{3}{x^2+4} < 0$ has no solution.

47 When reading this inequality, think of the concept "I want all the numbers that lie less than 8 units from the origin," and your answer should make some common sense.

$$|x| < 8 \quad \Rightarrow \quad -8 < x < 8 \quad \Leftrightarrow \quad (-8, 8)$$

48 $|x| \leq 7$ $\Rightarrow$ $-7 \leq x \leq 7$ $\Leftrightarrow$ $[-7, 7]$

49 When reading this inequality, think of the concept "I want all the numbers that lie at least 5 units from the origin," and your answer should make some common sense.

$$|x| \geq 5 \quad \Rightarrow \quad x \geq 5 \quad \text{or} \quad x \leq -5 \quad \Leftrightarrow \quad (-\infty, -5] \cup [5, \infty)$$

50 $|-x| > 6$ $\Rightarrow$ $-x > 6$ or $-x < -6$ {or else first use $|-x| = |x|$} $\Rightarrow$

$x < -6$ or $x > 6$ $\Leftrightarrow$ $(-\infty, -6) \cup (6, \infty)$

51 $|x+3| < 0.01 \;\Rightarrow\; -0.01 < x+3 < 0.01 \;\Rightarrow\; -3.01 < x < -2.99 \;\Leftrightarrow\; (-3.01, -2.99)$

52 $|x-4| \le 0.03 \;\Rightarrow\; -0.03 \le x-4 \le 0.03 \;\Rightarrow\; 3.97 \le x \le 4.03 \;\Leftrightarrow\; [3.97, 4.03]$

53 $|x+2| + 0.1 \ge 0.2$ {isolate the absolute value expression} $\Rightarrow\; |x+2| \ge 0.1 \;\Rightarrow$
$x+2 \ge 0.1$ or $x+2 \le -0.1 \;\Rightarrow\; x \ge -1.9$ or $x \le -2.1 \;\Leftrightarrow\; (-\infty, -2.1] \cup [-1.9, \infty)$

54 $|x-3| - 0.3 > 0.1 \;\Rightarrow\; |x-3| > 0.4 \;\Rightarrow\; x-3 > 0.4$ or $x-3 < -0.4 \;\Rightarrow$
$x > 3.4$ or $x < 2.6 \;\Leftrightarrow\; (-\infty, 2.6) \cup (3.4, \infty)$

55 $|2x+5| < 4 \;\Rightarrow\; -4 < 2x+5 < 4 \;\Rightarrow\; -9 < 2x < -1 \;\Rightarrow\; -\frac{9}{2} < x < -\frac{1}{2} \;\Leftrightarrow\; \left(-\frac{9}{2}, -\frac{1}{2}\right)$

56 $|3x-7| \ge 5 \;\Rightarrow\; 3x-7 \ge 5$ or $3x-7 \le -5 \;\Rightarrow\; 3x \ge 12$ or $3x \le 2 \;\Rightarrow$
$x \ge 4$ or $x \le \frac{2}{3} \;\Leftrightarrow\; \left(-\infty, \frac{2}{3}\right] \cup [4, \infty)$

57 $-\frac{1}{3}|6-5x| + 2 \ge 1 \;\Rightarrow\; -\frac{1}{3}|6-5x| \ge -1 \;\Rightarrow\; |6-5x| \le 3 \;\Rightarrow$
$-3 \le 6-5x \le 3 \;\Rightarrow\; -9 \le -5x \le -3 \;\Rightarrow\; \frac{9}{5} \ge x \ge \frac{3}{5} \;\Leftrightarrow\; \left[\frac{3}{5}, \frac{9}{5}\right]$

58 $2|-11-7x| - 2 > 10 \;\Rightarrow\; 2|-11-7x| > 12 \;\Rightarrow\; |-11-7x| > 6 \;\Rightarrow$
$-11-7x > 6$ or $-11-7x < -6 \;\Rightarrow\; -7x > 17$ or $-7x < 5 \;\Rightarrow$
$x < -\frac{17}{7}$ or $x > -\frac{5}{7} \;\Leftrightarrow\; \left(-\infty, -\frac{17}{7}\right) \cup \left(-\frac{5}{7}, \infty\right)$

59 Since $|7x+2| \ge 0 \;\forall x$, $|7x+2| > -2$ has solution $(-\infty, \infty)$.

60 Since $|6x-5| \ge 0 \;\forall x$, $|6x-5| \le -2$ has no solution.

61 $|3x-9| > 0 \;\forall x$ except when $3x-9=0$, or $x=3$. The solution is $(-\infty, 3) \cup (3, \infty)$.

62 $|5x+2| = 0$ if $x = -\frac{2}{5}$, but is never less than 0. Thus, $|5x+2| \le 0$ has solution $x = -\frac{2}{5}$.

63 Since $|2x-5| \ge 0 \;\forall x$, $|2x-5| < -1$ has no solution.

64 $|4x+7| = 0$ if $x = -\frac{7}{4}$, but is never less than 0. Thus, $|4x+7| \le 0$ has solution $x = -\frac{7}{4}$.

65 Since $|2x-11| \ge 0 \;\forall x$, $|2x-11| \ge -3$ has solution $(-\infty, \infty)$.

66 $|x-5| > 0 \;\forall x$ except when $x-5=0$, or $x=5$. The solution is $(-\infty, 5) \cup (5, \infty)$.

67 $\left|\dfrac{2-3x}{5}\right| \ge 2 \;\Rightarrow\; \dfrac{|2-3x|}{|5|} \ge 2 \;\Rightarrow\; |2-3x| \ge 10 \;\Rightarrow\; 2-3x \ge 10$ or $2-3x \le -10 \;\Rightarrow$
$-3x \ge 8$ or $-3x \le -12 \;\Rightarrow\; x \le -\frac{8}{3}$ or $x \ge 4 \;\Leftrightarrow\; \left(-\infty, -\frac{8}{3}\right] \cup [4, \infty)$

68 $\left|\dfrac{2x+5}{3}\right| < 1 \;\Rightarrow\; \dfrac{|2x+5|}{|3|} < 1 \;\Rightarrow\; |2x+5| < 3 \;\Rightarrow\; -3 < 2x+5 < 3 \;\Rightarrow$
$-8 < 2x < -2 \;\Rightarrow\; -4 < x < -1 \;\Leftrightarrow\; (-4, -1)$

69 Since $|5-2x| \ge 0 \;\forall x$, we can multiply the inequality by $|5-2x|$ without changing the direction of the inequality sign. We must exclude $x = \frac{5}{2}$ from the solution since it makes the original inequality undefined.
$\dfrac{3}{|5-2x|} < 2 \;\Rightarrow\; |5-2x| > \frac{3}{2} \;\Rightarrow\; 5-2x > \frac{3}{2}$ or $5-2x < -\frac{3}{2} \;\Rightarrow\; -2x > -\frac{7}{2}$ or $-2x < -\frac{13}{2} \;\Rightarrow$
$x < \frac{7}{4}$ or $x > \frac{13}{4}$ $\left\{\frac{5}{2}\right.$ doesn't fall in this region$\left.\right\}$ $\Leftrightarrow\; \left(-\infty, \frac{7}{4}\right) \cup \left(\frac{13}{4}, \infty\right)$

70 $\dfrac{2}{|2x+3|} \ge 5 \;\Rightarrow\; |2x+3| \le \frac{2}{5} \;\Rightarrow\; -\frac{2}{5} \le 2x+3 \le \frac{2}{5} \;\Rightarrow\; -\frac{17}{5} \le 2x \le -\frac{13}{5} \;\Rightarrow$
$-\frac{17}{10} \le x \le -\frac{13}{10}$ $\left\{x \ne -\frac{3}{2}\right\}$ $\Leftrightarrow\; \left[-\frac{17}{10}, -\frac{3}{2}\right) \cup \left(-\frac{3}{2}, -\frac{13}{10}\right]$

71 $-2 < |x| < 4 \Rightarrow -2 < |x|$ *and* $|x| < 4$. Since -2 is always less than $|x|$ {because $|x| \geq 0$}, we only need to consider $|x| < 4$. $|x| < 4 \Rightarrow -4 < x < 4 \Leftrightarrow (-4, 4)$

72 $1 < |x| < 5 \Rightarrow 1 < x < 5$ *or* $1 < -x < 5 \Rightarrow 1 < x < 5$ or $-1 > x > -5 \Rightarrow$ $1 < x < 5$ or $-5 < x < -1 \Leftrightarrow (-5, -1) \cup (1, 5)$

73 From the definition of absolute value, $|x - 2|$ equals either $x - 2$ or $-(x - 2)$.
Thus, $1 < |x - 2| < 4 \Rightarrow 1 < x - 2 < 4$ or $1 < -(x - 2) < 4 \Rightarrow$
$1 < x - 2 < 4$ or $-1 > x - 2 > -4 \Rightarrow 3 < x < 6$ or $1 > x > -2 \Leftrightarrow (-2, 1) \cup (3, 6)$.
An alternative method is to rewrite the inequality as $|x - 2| > 1$ *and* $|x - 2| < 4$. Solving independently gives us $x - 2 > 1$ or $x - 2 < -1 \Rightarrow x > 3$ or $x < 1$ *and* $-4 < x - 2 < 4 \Rightarrow -2 < x < 6$.
Taking the *intersection* of these intervals gives $(-2, 1) \cup (3, 6)$.

74 $2 < |2x - 1| < 3 \Rightarrow 2 < 2x - 1 < 3$ or $2 < -(2x - 1) < 3 \Rightarrow$
$2 < 2x - 1 < 3$ or $-2 > 2x - 1 > -3 \Rightarrow 3 < 2x < 4$ or $-1 > 2x > -2 \Rightarrow$
$\frac{3}{2} < x < 2$ or $-\frac{1}{2} > x > -1 \Leftrightarrow \left(-1, -\frac{1}{2}\right) \cup \left(\frac{3}{2}, 2\right)$

75 (a) $|x + 5| = 3 \Rightarrow x + 5 = 3$ or $x + 5 = -3 \Rightarrow x = -2$ or $x = -8$.

(b) $|x + 5| < 3$ has solutions between the values found in part (a), that is, $(-8, -2)$.

(c) The solutions of $|x + 5| > 3$ are the portions of the real line that are not in parts (a) and (b), that is, $(-\infty, -8) \cup (-2, \infty)$.

76 (a) $|x - 4| < 3 \Rightarrow -3 < x - 4 < 3 \Rightarrow 1 < x < 7 \Leftrightarrow (1, 7)$.

(b) $|x - 4| = 3$ has solutions at the endpoints of the interval in part (a); that is, at $x = 1$ and $x = 7$.

(c) As in Exercise 75(c), $|x - 4| > 3$ has solutions in $(-\infty, 1) \cup (7, \infty)$.

77 We could think of this statement as "the difference between w and 141 is at most 2." In symbols, we have $|w - 141| \leq 2$. Intuitively, we know that this inequality must describe the weights from 139 to 143.

78 "r must be within 0.01 centimeter of 1 centimeter" is written as $|r - 1| \leq 0.01$.

79 The difference of two temperatures T_1 and T_2 can be represented by $T_1 - T_2$.
Since there is no indication as to whether T_1 is larger than T_2, or vice versa,
we will use $|T_1 - T_2|$. Thus, the inequality is $5 < |T_1 - T_2| < 10$.

80 5 minutes is $\frac{1}{12}$ of an hour. $|t - 4| \geq \frac{1}{12}$

81 $30 \leq C \leq 40 \Rightarrow 30 \leq \frac{5}{9}(F - 32) \leq 40 \Rightarrow 30\left(\frac{9}{5}\right) \leq (F - 32) \leq 40\left(\frac{9}{5}\right) \Rightarrow$
$54 \leq F - 32 \leq 72 \Rightarrow 86 \leq F \leq 104$

82 $10 \leq F \leq 18 \Rightarrow 10 \leq (4.5)x \leq 18 \Rightarrow 10 \leq \frac{9}{2}x \leq 18 \Rightarrow 10\left(\frac{2}{9}\right) \leq x \leq 18\left(\frac{2}{9}\right) \Rightarrow \frac{20}{9} \leq x \leq 4$

83 $R = \dfrac{V}{I}$ • Since $V = 110$, $R = \dfrac{110}{I}$, or equivalently, $I = \dfrac{110}{R}$. If the current is not to exceed 10, we want to solve the inequality $I \leq 10$. $I \leq 10 \Rightarrow \dfrac{110}{R} \leq 10 \Rightarrow 110 \leq 10R \Rightarrow$
{$R > 0$, so we may multiply by R without changing the direction of the inequality} $10R \geq 110 \Rightarrow R \geq 11$

84 $\dfrac{1}{R} = \dfrac{1}{R_1} + \dfrac{1}{R_2}$ • $R_1 = 10 \Rightarrow \dfrac{1}{R} = \dfrac{1}{10} + \dfrac{1}{R_2} \Rightarrow \dfrac{1}{R} = \dfrac{R_2 + 10}{10R_2} \Rightarrow R = \dfrac{10R_2}{R_2 + 10}$.

$R < 5 \Rightarrow \dfrac{10R_2}{R_2+10} < 5 \Rightarrow 10R_2 < 5R_2 + 50\ \{R_2 + 10 > 0\} \Rightarrow 5R_2 < 50 \Rightarrow$

$R_2 < 10 \Rightarrow 0 < R_2 < 10$ {since $R_2 > 0$}

85 $M = \dfrac{f}{f-p}$ • We want to know what condition will assure us that an object's image is at least 3 times as large as the object, or, equivalently, when $M \geq 3$. $M \geq 3$ { and $f = 6$} $\Rightarrow \dfrac{6}{6-p} \geq 3 \Rightarrow 6 \geq 18 - 3p$ {since $6 - p > 0$, we can multiply by $6 - p$ and not change the direction of the inequality} $\Rightarrow 3p \geq 12 \Rightarrow p \geq 4$, but $p < 6$ since $p < f$. Thus, $4 \leq p < 6$.

86 $c = \dfrac{3.5t}{t+1}$ • $c > 1.5 \Rightarrow \dfrac{3.5t}{t+1} > 1.5 \Rightarrow \{t+1>0\}\ 3.5t > 1.5t + 1.5 \Rightarrow 2t > \dfrac{3}{2} \Rightarrow t > \dfrac{3}{4}$ hr

87 Let x denote the number of years before A becomes more economical than B.

The costs are the initial costs plus the yearly costs times the number of years.

$\text{Cost}_\text{A} < \text{Cost}_\text{B} \Rightarrow 100{,}000 + 8000x < 80{,}000 + 11{,}000x \Rightarrow 20{,}000 < 3000x \Rightarrow x > \frac{20}{3}$, or $6\frac{2}{3}$ yr.

88 Let t denote the time in years from the present. $\text{Cost}_\text{B} < \text{Cost}_\text{A} \Rightarrow$

$\text{Purchase}_\text{B} + \text{Insurance}_\text{B} + \text{Gas}_\text{B} < \text{Purchase}_\text{A} + \text{Insurance}_\text{A} + \text{Gas}_\text{A} \Rightarrow$

$24{,}000 + 1200t + \dfrac{15{,}000}{50} \cdot 3t < 20{,}000 + 1000t + \dfrac{15{,}000}{30} \cdot 3t \Rightarrow$

$24{,}000 + 2100t < 20{,}000 + 2500t \Rightarrow 4000 < 400t \Rightarrow t > 10$ yr.

89 (a) 5 ft 9 in $= 69$ in. In a 40 year period, a person's height will decrease by $40 \times 0.024 = 0.96$ in ≈ 1 in. The person will be approximately one inch shorter, or 5 ft 8 in. at age 70.

(b) 5 ft 6 in $= 66$ in. In 20 years, a person's height $(h = 66)$ will change by $0.024 \times 20 = 0.48$ in. Thus, $66 - 0.48 \leq h \leq 66 + 0.48 \Rightarrow 65.52 \leq h \leq 66.48$.

2.7 Exercises

1 (a) Since $x^2 \geq 0$, $x^2 + 4 \geq 4$, so the solution for $x^2 + 4 > 0$ is all real numbers, or $\mathbb{R}$.

(b) Since $x^2 \geq 0$, $x^2 + 4 \geq 4$, so there is no solution for $x^2 + 4 < 0$.

2 (a) Since $x^2 \geq 0$, $x^2 + 9 \geq 9$, so the solution for $x^2 + 9 > 0$ is all real numbers, or $\mathbb{R}$.

(b) Since $x^2 \geq 0$, $x^2 + 9 \geq 9$, so there is no solution for $x^2 + 9 < 0$.

Note: Many solutions for exercises involving inequalities contain a sign chart. You may want to read Example 2 again if you have trouble interpreting the sign charts.

3 $(3x+1)(5-10x) > 0$ • See the sign chart for details concerning the signs of the individual factors and the resulting sign. The given inequality has solutions in the interval $\left(-\frac{1}{3}, \frac{1}{2}\right)$, which corresponds to the positive values for the **Resulting sign**.

Interval	$\left(-\infty, -\frac{1}{3}\right)$	$\left(-\frac{1}{3}, \frac{1}{2}\right)$	$\left(\frac{1}{2}, \infty\right)$
Sign of $5 - 10x$	$+$	$+$	$-$
Sign of $3x + 1$	$-$	$+$	$+$
Resulting sign	$-$	$+$	$-$

4 $(2-3x)(4x-7)\geq 0$ ★ $[\frac{2}{3},\frac{7}{4}]$

Interval	$(-\infty,\frac{2}{3})$	$(\frac{2}{3},\frac{7}{4})$	$(\frac{7}{4},\infty)$
Sign of $4x-7$	$-$	$-$	$+$
Sign of $2-3x$	$+$	$-$	$-$
Resulting sign	$-$	$+$	$-$

5 $(x+2)(x-1)(4-x)\leq 0$ • From the chart, we see the product is negative for $x\in(-2,1)\cup(4,\infty)$. Since we want to also include the values that make the product equal to zero $\{-2, 1, \text{and } 4\}$, the solution is $[-2,1]\cup[4,\infty)$.

Interval	$(-\infty,-2)$	$(-2,1)$	$(1,4)$	$(4,\infty)$
Sign of $4-x$	$+$	$+$	$+$	$-$
Sign of $x-1$	$-$	$-$	$+$	$+$
Sign of $x+2$	$-$	$+$	$+$	$+$
Resulting sign	$+$	$-$	$+$	$-$

6 $(x-6)(x+3)(-2-x)<0$ ★ $(-3,-2)\cup(6,\infty)$

Interval	$(-\infty,-3)$	$(-3,-2)$	$(-2,6)$	$(6,\infty)$
Sign of $-2-x$	$+$	$+$	$-$	$-$
Sign of $x-6$	$-$	$-$	$-$	$+$
Sign of $x+3$	$-$	$+$	$+$	$+$
Resulting sign	$+$	$-$	$+$	$-$

7 $x^2-x-6<0 \quad\Rightarrow\quad (x-3)(x+2)<0$ ★ $(-2,3)$

Interval	$(-\infty,-2)$	$(-2,3)$	$(3,\infty)$
Sign of $x-3$	$-$	$-$	$+$
Sign of $x+2$	$-$	$+$	$+$
Resulting sign	$+$	$-$	$+$

8 $x^2+4x+3\geq 0 \quad\Rightarrow\quad (x+1)(x+3)\geq 0$ ★ $(-\infty,-3]\cup[-1,\infty)$

Interval	$(-\infty,-3)$	$(-3,-1)$	$(-1,\infty)$
Sign of $x+1$	$-$	$-$	$+$
Sign of $x+3$	$-$	$+$	$+$
Resulting sign	$+$	$-$	$+$

9 $x^2-2x-7>1 \quad\Rightarrow\quad x^2-2x-8>0 \quad\Rightarrow\quad (x-4)(x+2)>0$ ★ $(-\infty,-2)\cup(4,\infty)$

Interval	$(-\infty,-2)$	$(-2,4)$	$(4,\infty)$
Sign of $x-4$	$-$	$-$	$+$
Sign of $x+2$	$-$	$+$	$+$
Resulting sign	$+$	$-$	$+$

10 $x^2-4x-15\leq 6 \quad\Rightarrow\quad x^2-4x-21\leq 0 \quad\Rightarrow\quad (x-7)(x+3)\leq 0$ ★ $[-3,7]$

Interval	$(-\infty,-3)$	$(-3,7)$	$(7,\infty)$
Sign of $x-7$	$-$	$-$	$+$
Sign of $x+3$	$-$	$+$	$+$
Resulting sign	$+$	$-$	$+$

11 $x(2x+3) \geq 5 \Rightarrow 2x^2+3x-5 \geq 0 \Rightarrow (2x+5)(x-1) \geq 0$ ★ $(-\infty, -\frac{5}{2}] \cup [1, \infty)$

Interval	$(-\infty, -\frac{5}{2})$	$(-\frac{5}{2}, 1)$	$(1, \infty)$
Sign of $x-1$	$-$	$-$	$+$
Sign of $2x+5$	$-$	$+$	$+$
Resulting sign	$+$	$-$	$+$

12 $x(3x-1) \leq 4 \Rightarrow 3x^2-x-4 \leq 0 \Rightarrow (3x-4)(x+1) \leq 0$ ★ $[-1, \frac{4}{3}]$

Interval	$(-\infty, -1)$	$(-1, \frac{4}{3})$	$(\frac{4}{3}, \infty)$
Sign of $3x-4$	$-$	$-$	$+$
Sign of $x+1$	$-$	$+$	$+$
Resulting sign	$+$	$-$	$+$

13 $8x-15 > x^2 \Rightarrow x^2-8x+15 < 0 \Rightarrow (x-3)(x-5) < 0$ ★ $(3, 5)$

Interval	$(-\infty, 3)$	$(3, 5)$	$(5, \infty)$
Sign of $x-5$	$-$	$-$	$+$
Sign of $x-3$	$-$	$+$	$+$
Resulting sign	$+$	$-$	$+$

Note: Solving $x^2 < a^2$ or $x^2 > a^2$ for $a > 0$ may be achieved using factoring, that is, $x^2 - a^2 < 0 \Rightarrow (x+a)(x-a) < 0 \Rightarrow -a < x < a$; or by taking the square root of each side, that is, $\sqrt{x^2} < \sqrt{a^2} \Rightarrow |x| < a \Rightarrow -a < x < a$.

14 $x+20 \leq x^2 \Rightarrow x^2-x-20 \geq 0 \Rightarrow (x-5)(x+4) \geq 0$ ★ $(-\infty, -4] \cup [5, \infty)$

Interval	$(-\infty, -4)$	$(-4, 5)$	$(5, \infty)$
Sign of $x-5$	$-$	$-$	$+$
Sign of $x+4$	$-$	$+$	$+$
Resulting sign	$+$	$-$	$+$

15 ***Note:*** The most common mistake is forgetting that $\sqrt{x^2} = |x|$.

$$x^2 < 16 \Rightarrow \sqrt{x^2} < \sqrt{16} \Rightarrow |x| < 4 \Rightarrow -4 < x < 4 \Leftrightarrow (-4, 4).$$

16 $x^2 > 64 \Rightarrow \sqrt{x^2} > \sqrt{64} \Rightarrow |x| > 8 \Rightarrow x > 8$ or $x < -8 \Leftrightarrow (-\infty, -8) \cup (8, \infty)$

17 $25x^2 - 16 < 0 \Rightarrow x^2 < \frac{16}{25} \Rightarrow |x| < \frac{4}{5} \Rightarrow -\frac{4}{5} < x < \frac{4}{5} \Leftrightarrow (-\frac{4}{5}, \frac{4}{5})$

18 $25x^2 - 16x < 0 \Rightarrow x(25x-16) < 0$ ★ $(0, \frac{16}{25})$

Interval	$(-\infty, 0)$	$(0, \frac{16}{25})$	$(\frac{16}{25}, \infty)$
Sign of $25x-16$	$-$	$-$	$+$
Sign of x	$-$	$+$	$+$
Resulting sign	$+$	$-$	$+$

19 $16x^2 \geq 9x \Rightarrow 16x^2 - 9x \geq 0 \Rightarrow x(16x-9) \geq 0$ ★ $(-\infty, 0] \cup [\frac{9}{16}, \infty)$

Interval	$(-\infty, 0)$	$(0, \frac{9}{16})$	$(\frac{9}{16}, \infty)$
Sign of $16x-9$	$-$	$-$	$+$
Sign of x	$-$	$+$	$+$
Resulting sign	$+$	$-$	$+$

20 $16x^2 > 9 \Rightarrow x^2 > \frac{9}{16} \Rightarrow |x| > \frac{3}{4} \Rightarrow x > \frac{3}{4}$ or $x < -\frac{3}{4} \Leftrightarrow (-\infty, -\frac{3}{4}) \cup (\frac{3}{4}, \infty)$

21 $x^4 + 5x^2 \geq 36 \Rightarrow x^4 + 5x^2 - 36 \geq 0 \Rightarrow (x^2+9)(x^2-4) \geq 0 \Rightarrow$
$x^2 - 4 \geq 0$ $\{x^2 + 9 > 0$, so we don't have to consider its sign$\}$ $\Rightarrow$
$x^2 \geq 4 \Rightarrow |x| \geq 2 \Rightarrow x \geq 2$ or $x \leq -2 \Leftrightarrow (-\infty, -2] \cup [2, \infty)$

22 $x^4 + 15x^2 < 16 \Rightarrow x^4 + 15x^2 - 16 < 0 \Rightarrow (x^2+16)(x^2-1) < 0 \Rightarrow$
$x^2 - 1 < 0 \{x^2 + 16 > 0\} \Rightarrow x^2 < 1 \Rightarrow |x| < 1 \Rightarrow -1 < x < 1 \Leftrightarrow (-1, 1)$

23 $x^3 + 2x^2 - 4x - 8 \geq 0 \Rightarrow x^2(x+2) - 4(x+2) \geq 0 \Rightarrow (x^2-4)(x+2) \geq 0 \Rightarrow$
$(x-2)(x+2)(x+2) \geq 0 \Rightarrow (x-2)(x+2)^2 \geq 0$. The expression $(x+2)^2$ is positive except when $x = -2$. The sign is determined by the sign of $x - 2$, which is positive if $x > 2$. Since $x = \pm 2$ make the expression zero, the solution is $x \geq 2$ or $x = -2 \Leftrightarrow \{-2\} \cup [2, \infty)$.

24 $2x^3 - 3x^2 - 2x + 3 \leq 0 \Rightarrow x^2(2x-3) - 1(2x-3) \leq 0 \Rightarrow (x^2-1)(2x-3) \leq 0 \Rightarrow$
$(x+1)(x-1)(2x-3) \leq 0$ ★ $(-\infty, -1] \cup [1, \frac{3}{2}]$

Interval	$(-\infty, -1)$	$(-1, 1)$	$(1, \frac{3}{2})$	$(\frac{3}{2}, \infty)$
Sign of $2x - 3$	$-$	$-$	$-$	$+$
Sign of $x - 1$	$-$	$-$	$+$	$+$
Sign of $x + 1$	$-$	$+$	$+$	$+$
Resulting sign	$-$	$+$	$-$	$+$

25 $\frac{x^2(x+2)}{(x+2)(x+1)} \leq 0 \Rightarrow \frac{x^2}{x+1} \leq 0$ {we will exclude $x = -2$ since it makes the original expression undefined} $\Rightarrow \frac{1}{x+1} \leq 0$ {we can divide by x^2 since $x^2 \geq 0$ and we will include $x = 0$ since it makes x^2 equal to zero and we want all solutions less than *or equal to* zero} $\Rightarrow x + 1 < 0$ {the fraction cannot equal zero and $x + 1$ must be negative so that the fraction is negative} $\Rightarrow x < -1$ ★ $(-\infty, -2) \cup (-2, -1) \cup \{0\}$

26 $\frac{(x^2+1)(x-3)}{x^2-9} \geq 0 \Rightarrow \frac{x-3}{(x+3)(x-3)} \geq 0 \{x^2 + 1 > 0\} \Rightarrow \frac{1}{x+3} \geq 0$ {exclude 3} $\Rightarrow$
$x + 3 > 0$ {exclude -3} $\Rightarrow x > -3$ ★ $(-3, 3) \cup (3, \infty)$

27 $\frac{x^2-x}{x^2+2x} \leq 0 \Rightarrow \frac{x(x-1)}{x(x+2)} \leq 0 \Rightarrow \frac{x-1}{x+2} \leq 0$ {we will exclude $x = 0$ from the solution} ★ $(-2, 0) \cup (0, 1]$

Interval	$(-\infty, -2)$	$(-2, 1)$	$(1, \infty)$
Sign of $x - 1$	$-$	$-$	$+$
Sign of $x + 2$	$-$	$+$	$+$
Resulting sign	$+$	$-$	$+$

28 $\frac{(x+3)^2(2-x)}{(x+4)(x^2-4)} \leq 0 \Rightarrow \frac{2-x}{(x+4)(x+2)(x-2)} \leq 0$ {include -3} $\Rightarrow \frac{1}{(x+4)(x+2)} \geq 0$ {cancel, change inequality, exclude 2} ★ $(-\infty, -4) \cup \{-3\} \cup (-2, 2) \cup (2, \infty)$

Interval	$(-\infty, -4)$	$(-4, -2)$	$(-2, \infty)$
Sign of $x + 2$	$-$	$-$	$+$
Sign of $x + 4$	$-$	$+$	$+$
Resulting sign	$+$	$-$	$+$

29 $\dfrac{x-2}{x^2-3x-10} \geq 0 \Rightarrow \dfrac{x-2}{(x-5)(x+2)} \geq 0$ {$x = 2$ is a solution since it makes the fraction equal to zero, $x = 5$ and $x = -2$ are excluded since these values make the fraction undefined} ★ $[-2, 2) \cup (5, \infty)$

Interval	$(-\infty, -2)$	$(-2, 2)$	$(2, 5)$	$(5, \infty)$
Sign of $x-5$	$-$	$-$	$-$	$+$
Sign of $x-2$	$-$	$-$	$+$	$+$
Sign of $x+2$	$-$	$+$	$+$	$+$
Resulting sign	$-$	$+$	$-$	$+$

30 $\dfrac{x+6}{x^2-7x+12} \leq 0 \Rightarrow \dfrac{x+6}{(x-3)(x-4)} \leq 0$ ★ $(-\infty, -6] \cup (3, 4)$

Interval	$(-\infty, -6)$	$(-6, 3)$	$(3, 4)$	$(4, \infty)$
Sign of $x-4$	$-$	$-$	$-$	$+$
Sign of $x-3$	$-$	$-$	$+$	$+$
Sign of $x+6$	$-$	$+$	$+$	$+$
Resulting sign	$-$	$+$	$-$	$+$

31 $\dfrac{-3x}{x^2-9} > 0 \Rightarrow \dfrac{x}{(x+3)(x-3)} < 0$ {divide by -3} ★ $(-\infty, -3) \cup (0, 3)$

Interval	$(-\infty, -3)$	$(-3, 0)$	$(0, 3)$	$(3, \infty)$
Sign of $x-3$	$-$	$-$	$-$	$+$
Sign of x	$-$	$-$	$+$	$+$
Sign of $x+3$	$-$	$+$	$+$	$+$
Resulting sign	$-$	$+$	$-$	$+$

32 $\dfrac{5x}{16-x^2} < 0 \Rightarrow \dfrac{x}{(4+x)(4-x)} < 0$ {divide by 5} ★ $(-4, 0) \cup (4, \infty)$

Interval	$(-\infty, -4)$	$(-4, 0)$	$(0, 4)$	$(4, \infty)$
Sign of $4-x$	$+$	$+$	$+$	$-$
Sign of x	$-$	$-$	$+$	$+$
Sign of $4+x$	$-$	$+$	$+$	$+$
Resulting sign	$+$	$-$	$+$	$-$

33 $\dfrac{x+1}{2x-3} > 2 \Rightarrow \dfrac{x+1}{2x-3} - 2 > 0 \Rightarrow \dfrac{x+1-2(2x-3)}{2x-3} > 0 \Rightarrow \dfrac{x+1-4x+6}{2x-3} > 0 \Rightarrow$ $\dfrac{-3x+7}{2x-3} > 0$. From the sign chart, the solution is $\left(\frac{3}{2}, \frac{7}{3}\right)$. Note that you should *not* multiply by the factor $2x-3$ as we did with rational *equations* because $2x-3$ may be positive or negative, and multiplying by it would require solving two inequalities. This method of solution tends to be more difficult than the sign chart method.

Interval	$\left(-\infty, \frac{3}{2}\right)$	$\left(\frac{3}{2}, \frac{7}{3}\right)$	$\left(\frac{7}{3}, \infty\right)$
Sign of $-3x+7$	$+$	$+$	$-$
Sign of $2x-3$	$-$	$+$	$+$
Resulting sign	$-$	$+$	$-$

34 $\dfrac{x-2}{3x+5} \leq 4 \Rightarrow \dfrac{x-2-4(3x+5)}{3x+5} \leq 0 \Rightarrow \dfrac{-11x-22}{3x+5} \leq 0$ ★ $(-\infty, -2] \cup \left(-\frac{5}{3}, \infty\right)$

Interval	$(-\infty, -2)$	$\left(-2, -\frac{5}{3}\right)$	$\left(-\frac{5}{3}, \infty\right)$
Sign of $3x+5$	$-$	$-$	$+$
Sign of $-11x-22$	$+$	$-$	$-$
Resulting sign	$-$	$+$	$-$

35 $\dfrac{1}{x-2} \geq \dfrac{3}{x+1} \Rightarrow \dfrac{1}{x-2} - \dfrac{3}{x+1} \geq 0 \Rightarrow \dfrac{1(x+1)-3(x-2)}{(x-2)(x+1)} \geq 0 \Rightarrow$
$\dfrac{x+1-3x+6}{(x-2)(x+1)} \geq 0 \Rightarrow \dfrac{-2x+7}{(x-2)(x+1)} \geq 0$ ★ $(-\infty,-1) \cup \left(2, \frac{7}{2}\right]$

Interval	$(-\infty,-1)$	$(-1,2)$	$\left(2,\frac{7}{2}\right)$	$\left(\frac{7}{2},\infty\right)$
Sign of $-2x+7$	+	+	+	−
Sign of $x-2$	−	−	+	+
Sign of $x+1$	−	+	+	+
Resulting sign	+	−	+	−

36 $\dfrac{2}{2x+3} \leq \dfrac{2}{x-5} \Rightarrow \dfrac{2(x-5)-2(2x+3)}{(2x+3)(x-5)} \leq 0 \Rightarrow \dfrac{-2x-16}{(2x+3)(x-5)} \leq 0$ ★ $\left[-8,-\frac{3}{2}\right) \cup (5,\infty)$

Interval	$(-\infty,-8)$	$\left(-8,-\frac{3}{2}\right)$	$\left(-\frac{3}{2},5\right)$	$(5,\infty)$
Sign of $x-5$	−	−	−	+
Sign of $2x+3$	−	−	+	+
Sign of $-2x-16$	+	−	−	−
Resulting sign	+	−	+	−

37 $\dfrac{4}{3x-2} \leq \dfrac{2}{x+2} \Rightarrow \dfrac{4}{3x-2} - \dfrac{2}{x+2} \leq 0 \Rightarrow \dfrac{4(x+2)-2(3x-2)}{(3x-2)(x+2)} \leq 0 \Rightarrow$
$\dfrac{4x+8-6x+4}{(3x-2)(x+2)} \leq 0 \Rightarrow \dfrac{-2x+12}{(3x-2)(x+2)} \leq 0$ ★ $\left(-1,\frac{2}{3}\right) \cup [6,\infty)$

Interval	$(-\infty,-2)$	$\left(-2,\frac{2}{3}\right)$	$\left(\frac{2}{3},6\right)$	$(6,\infty)$
Sign of $-2x+12$	+	+	+	−
Sign of $3x-2$	−	−	+	+
Sign of $x+2$	−	+	+	+
Resulting sign	+	−	+	−

38 $\dfrac{3}{5x+1} \geq \dfrac{1}{x-3} \Rightarrow \dfrac{3(x-3)-1(5x+1)}{(5x+1)(x-3)} \geq 0 \Rightarrow \dfrac{-2x-10}{(5x+1)(x-3)} \geq 0$ ★ $(-\infty,-5] \cup \left(-\frac{1}{5},3\right)$

Interval	$(-\infty,-5)$	$\left(-5,-\frac{1}{5}\right)$	$\left(-\frac{1}{5},3\right)$	$(3,\infty)$
Sign of $x-3$	−	−	−	+
Sign of $5x+1$	−	−	+	+
Sign of $-2x-10$	+	−	−	−
Resulting sign	+	−	+	−

39 $\dfrac{x}{3x-5} \leq \dfrac{2}{x-1} \Rightarrow \dfrac{x}{3x-5} - \dfrac{2}{x-1} \leq 0 \Rightarrow \dfrac{x(x-1)-2(3x-5)}{(3x-5)(x-1)} \leq 0 \Rightarrow$
$\dfrac{x^2-x-6x+10}{(3x-5)(x-1)} \leq 0 \Rightarrow \dfrac{x^2-7x+10}{(3x-5)(x-1)} \leq 0 \Rightarrow \dfrac{(x-2)(x-5)}{(3x-5)(x-1)} \leq 0$ ★ $\left(1,\frac{5}{3}\right) \cup [2,5]$

Interval	$(-\infty,1)$	$\left(1,\frac{5}{3}\right)$	$\left(\frac{5}{3},2\right)$	$(2,5)$	$(5,\infty)$
Sign of $x-5$	−	−	−	−	+
Sign of $x-2$	−	−	−	+	+
Sign of $3x-5$	−	−	+	+	+
Sign of $x-1$	−	+	+	+	+
Resulting sign	+	−	+	−	+

40 $\dfrac{x}{2x-1} \geq \dfrac{3}{x+2} \Rightarrow \dfrac{x}{2x-1} - \dfrac{3}{x+2} \geq 0 \Rightarrow \dfrac{x(x+2)-3(2x-1)}{(2x-1)(x+2)} \geq 0 \Rightarrow$
$\dfrac{x^2+2x-6x+3}{(2x-1)(x+2)} \geq 0 \Rightarrow \dfrac{x^2-4x+3}{(2x-1)(x+2)} \geq 0 \Rightarrow \dfrac{(x-1)(x-3)}{(2x-1)(x+2)} \geq 0$ ★ $(-\infty,-2) \cup \left(\frac{1}{2},1\right] \cup [3,\infty)$

Interval	$(-\infty,-2)$	$\left(-2,\frac{1}{2}\right)$	$\left(\frac{1}{2},1\right)$	$(1,3)$	$(3,\infty)$
Sign of $x-3$	$-$	$-$	$-$	$-$	$+$
Sign of $x-1$	$-$	$-$	$-$	$+$	$+$
Sign of $2x-1$	$-$	$-$	$+$	$+$	$+$
Sign of $x+2$	$-$	$+$	$+$	$+$	$+$
Resulting sign	$+$	$-$	$+$	$-$	$+$

41 $x^3 > x \Rightarrow x^3 - x > 0 \Rightarrow x(x^2-1) > 0 \Rightarrow x(x+1)(x-1) > 0$ ★ $(-1,0) \cup (1,\infty)$

Interval	$(-\infty,-1)$	$(-1,0)$	$(0,1)$	$(1,\infty)$
Sign of $x-1$	$-$	$-$	$-$	$+$
Sign of x	$-$	$-$	$+$	$+$
Sign of $x+1$	$-$	$+$	$+$	$+$
Resulting sign	$-$	$+$	$-$	$+$

42 $x^4 \geq x^2 \Rightarrow x^4 - x^2 \geq 0 \Rightarrow x^2(x^2-1) \geq 0 \Rightarrow x^2(x+1)(x-1) \geq 0$. Since $x^2 \geq 0$, x^2 does not need to be included in the sign chart, but 0 must be included in the answer because of the equality.

★ $(-\infty,-1] \cup \{0\} \cup [1,\infty)$

Interval	$(-\infty,-1)$	$(-1,1)$	$(1,\infty)$
Sign of $x-1$	$-$	$-$	$+$
Sign of $x+1$	$-$	$+$	$+$
Resulting sign	$+$	$-$	$+$

43 $v \geq k \Rightarrow t^3 - 3t^2 - 4t + 20 \geq 8 \Rightarrow t^3 - 3t^2 - 4t + 12 \geq 0 \Rightarrow t^2(t-3) - 4(t-3) \geq 0 \Rightarrow$
$(t^2-4)(t-3) \geq 0 \Rightarrow (t+2)(t-2)(t-3) \geq 0$. For $[0,5]$, we have the solution $[0,2] \cup [3,5]$.

Interval	$(-\infty,-2)$	$(-2,2)$	$(2,3)$	$(3,\infty)$
Sign of $t-3$	$-$	$-$	$-$	$+$
Sign of $t-2$	$-$	$-$	$+$	$+$
Sign of $t+2$	$-$	$+$	$+$	$+$
Resulting sign	$-$	$+$	$-$	$+$

44 $v \geq k \Rightarrow t^4 - 4t^2 + 10 \geq 10 \Rightarrow t^4 - 4t^2 \geq 0 \Rightarrow t^2(t^2-4) \geq 0 \Rightarrow t^2(t+2)(t-2) \geq 0$.

For $[1,6]$, we have the solution $[2,6]$.

Interval	$(-\infty,-2)$	$(-2,2)$	$(2,\infty)$
Sign of $t-2$	$-$	$-$	$+$
Sign of $t+2$	$-$	$+$	$+$
Resulting sign	$+$	$-$	$+$

45 $s > 9 \Rightarrow -16t^2 + 24t + 1 > 9 \Rightarrow -16t^2 + 24t - 8 > 0 \Rightarrow 2t^2 - 3t + 1 < 0$ {divide by -8} $\Rightarrow$
$(2t-1)(t-1) < 0$ {use a sign chart} $\Rightarrow \frac{1}{2} < t < 1$.

The dog is more than 9 ft off the ground for $1 - \frac{1}{2} = \frac{1}{2}$ sec.

46 $s \geq 1536 \Rightarrow -16t^2 + 320t \geq 1536 \Rightarrow -16t^2 + 320t - 1536 \geq 0 \Rightarrow$
$t^2 - 20t + 96 \leq 0$ {divide by -16} $\Rightarrow (t-8)(t-12) \leq 0$ {use a sign chart} $\Leftrightarrow 8 \leq t \leq 12$

47 $d < 75 \Rightarrow v + \frac{1}{20}v^2 < 75$ {multiply by 20} $\Rightarrow 20v + v^2 < 1500 \Rightarrow v^2 + 20v - 1500 < 0 \Rightarrow$
$(v+50)(v-30) < 0$ {use a sign chart} $\Rightarrow -50 < v < 30 \Rightarrow 0 \le v < 30$ {since $v \ge 0$}

48 $M \ge 45 \Rightarrow -\frac{1}{30}v^2 + \frac{5}{2}v \ge 45$ {multiply by -30} $\Rightarrow v^2 - 75v \le -1350 \Rightarrow$
$v^2 - 75v + 1350 \le 0 \Rightarrow (v-30)(v-45) \le 0$ {use a sign chart} $\Leftrightarrow 30 \le v \le 45$

49 $R > S \Rightarrow \frac{4500\,S}{S+500} > S \Rightarrow \frac{4500\,S}{S+500} - S > 0 \Rightarrow \frac{4500\,S - S(S+500)}{S+500} > 0 \Rightarrow$
$\frac{4500\,S - S^2 - 500S}{S+500} > 0 \Rightarrow \frac{-S^2 + 4000S}{S+500} > 0 \Rightarrow \frac{S^2 - 4000S}{S+500} < 0$ {multiply by -1, change inequality} $\Rightarrow \frac{S(S-4000)}{S+500} < 0$ {use a sign chart} $\Rightarrow$
$S < -500$ or $0 < S < 4000 \Rightarrow 0 < S < 4000$ {since $S > 0$}

50 $D > 400 \Rightarrow \frac{5000x}{x^2+36} > 400 \Rightarrow 25x > 2(x^2+36)$ $\left\{\text{multiply by } \frac{x^2+36}{200}, \text{ which is positive}\right\}$ $\Rightarrow$
$25x > 2x^2 + 72 \Rightarrow 2x^2 - 25x + 72 < 0 \Rightarrow (2x-9)(x-8) < 0$ {use a sign chart} $\Rightarrow 4.5 < x < 8$

51 $W < 5 \Rightarrow 125\left(\frac{6400}{6400+x}\right)^2 < 5 \Rightarrow \left(\frac{6400}{6400+x}\right)^2 < \frac{1}{25} \Rightarrow \left(\frac{6400}{6400+x}\right)^2 < \left(\frac{1}{5}\right)^2 \Rightarrow$
$\frac{6400}{6400+x} < \frac{1}{5}$ $\left\{\text{take square roots, no } \pm \text{ needed since } \frac{6400}{6400+x} > 0\right\}$ $\Rightarrow$
$5(6400) < x + 6400 \Rightarrow 32{,}000 < x + 6400 \Rightarrow x > 25{,}600$ km.

52 $L < \frac{1}{2}L_0 \Rightarrow L^2 < \frac{1}{4}L_0^2 \Rightarrow L_0^2\left(1 - \frac{v^2}{c^2}\right) < \frac{1}{4}L_0^2 \Rightarrow 1 - \frac{v^2}{c^2} < \frac{1}{4} \Rightarrow \frac{3}{4} < \frac{v^2}{c^2} \Rightarrow$
$v^2 > \frac{3}{4}c^2 \Rightarrow v > \frac{1}{2}\sqrt{3}\,c$ since $v > 0$ and $c > 0$

53 $7500 \le W \le 10{,}000 \Rightarrow 7500 \le 0.00334V^2S \le 10{,}000 \Rightarrow 7500 \le 0.00334(210)V^2 \le 10{,}000 \Rightarrow$
$\frac{7500}{0.7014} \le V^2 \le \frac{10{,}000}{0.7014} \Rightarrow \sqrt{\frac{7500}{0.7014}} \le V \le \sqrt{\frac{10{,}000}{0.7014}} \Rightarrow 103.4 \le V \le 119.4$ {in ft/sec}. To convert ft/sec to mi/hr, multiply by $\frac{60}{88}$ or $\frac{15}{22}$, which are reduced forms of $\frac{1 \text{ foot}}{1 \text{ second}} \times \frac{3600 \text{ seconds}}{1 \text{ hour}} \times \frac{1 \text{ mile}}{5280 \text{ feet}}$. Using the approximations in ft/sec, we get $103.4 \le V \le 119.4$ {in ft/sec} $\Rightarrow 70.5 \le V \le 81.4$ {in mi/hr}.

54 The numerator is equal to zero when $x = 2, 3$ and the denominator is equal to zero when $x = \pm 1$. From the table, the expression $Y_1 = \frac{(2-x)(3x-9)}{(1-x)(x+1)}$ is positive when $x \in [-2,-1) \cup (1,2) \cup (3,3.5]$. See the table on the left.

x	Y_1	x	Y_1
−2.0	20	1.0	ERROR
−1.5	37.8	1.5	1.8
−1.0	ERROR	2.0	0
−0.5	−35	2.5	−0.1429
0.0	−18	3.0	0
0.5	−15	3.5	0.2

x	Y_1	x	Y_1
−3.5	30.938	1.0	36
−3.0	0	1.5	19.688
−2.5	−7.313	2.0	0
−2.0	0	2.5	−18.56
−1.5	14.438	3.0	−30
−1.0	30	3.5	−26.81
−0.5	42.188	4.0	0
0.0	48	4.5	60.938
0.5	45.938	5.0	168

55 By using a table it can be shown that the expression is equal to zero when $x = -3, -2, 2, 4$.
The expression $Y_1 = x^4 - x^3 - 16x^2 + 4x + 48$ is negative when $x \in (-3,-2) \cup (2,4)$. See the table on the right.

Chapter 2 Review Exercises

1 $\left[\dfrac{3x+1}{5x+7}=\dfrac{6x+11}{10x-3}\right]\cdot(5x+7)(10x-3) \Rightarrow (3x+1)(10x-3)=(6x+11)(5x+7) \Rightarrow$

$30x^2+x-3=30x^2+97x+77 \Rightarrow -96x=80 \Rightarrow x=-\frac{5}{6}$

2 $\left[2-\dfrac{1}{x}=1+\dfrac{3}{x}\right]\cdot x \Rightarrow 2x-1=1x+3 \Rightarrow x=4$

3 $\left[\dfrac{2}{x+5}-\dfrac{3}{2x+1}=\dfrac{5}{6x+3}\right]\cdot 3(x+5)(2x+1) \Rightarrow 6(2x+1)-9(x+5)=5(x+5) \Rightarrow$

$12x+6-9x-45=5x+25 \Rightarrow 3x-39=5x+25 \Rightarrow -2x=64 \Rightarrow x=-32$

4 $\left[\dfrac{7}{x-2}-\dfrac{6}{x^2-4}=\dfrac{3}{2x+4}\right]\cdot 2(x+2)(x-2) \Rightarrow 14(x+2)-12=3(x-2) \Rightarrow$

$14x+28-12=3x-6 \Rightarrow 11x=-22 \Rightarrow x=-2$, which is not in the domain of the given expressions.

No solution

5 LS $=\dfrac{1}{\sqrt{x}}-3=\dfrac{1}{\sqrt{x}}-\dfrac{3\sqrt{x}}{\sqrt{x}}=\dfrac{1-3\sqrt{x}}{\sqrt{x}}=$ RS, an identity. The given equation is true for every $x>0$.

6 $2x^2+7x-15=0 \Rightarrow (x+5)(2x-3)=0 \Rightarrow x=-5, \frac{3}{2}$

7 $x(3x+4)=2 \Rightarrow 3x^2+4x-2=0 \Rightarrow x=\dfrac{-4\pm\sqrt{16+24}}{6}=\dfrac{-4\pm2\sqrt{10}}{6}=-\dfrac{2}{3}\pm\dfrac{1}{3}\sqrt{10}$

8 $\left[\dfrac{x}{3x+1}=\dfrac{x-1}{2x+3}\right]\cdot(3x+1)(2x+3) \Rightarrow x(2x+3)=(x-1)(3x+1) \Rightarrow$

$2x^2+3x=3x^2-2x-1 \Rightarrow x^2-5x-1=0 \Rightarrow x=\dfrac{5\pm\sqrt{25+4}}{2}=\dfrac{5}{2}\pm\dfrac{1}{2}\sqrt{29}$

9 $(x-2)(x+1)=6 \Rightarrow x^2-x-2=6 \Rightarrow x^2-x-8=0 \Rightarrow x=\dfrac{1\pm\sqrt{1+32}}{2}=\dfrac{1}{2}\pm\dfrac{1}{2}\sqrt{33}$

10 $4x^4-37x^2+75=0 \Rightarrow (4x^2-25)(x^2-3) \Rightarrow x^2=\frac{25}{4},3 \Rightarrow x=\pm\frac{5}{2},\pm\sqrt{3}$

11 $x^{2/3}-2x^{1/3}-15=0 \Rightarrow \left(x^{1/3}+3\right)\left(x^{1/3}-5\right)=0 \Rightarrow \sqrt[3]{x}=-3,5 \Rightarrow$

$x=(-3)^3,5^3 \Rightarrow x=-27,125$

12 $20x^3+8x^2-55x-22=0 \Rightarrow 4x^2(5x+2)-11(5x+2)=0 \Rightarrow$

$(4x^2-11)(5x+2)=0 \Rightarrow x^2=\frac{11}{4}$ or $x=-\frac{2}{5} \Rightarrow x=\pm\frac{1}{2}\sqrt{11},-\frac{2}{5}$

13 $5x^2=2x-3 \Rightarrow 5x^2-2x+3=0 \Rightarrow x=\dfrac{2\pm\sqrt{4-60}}{10}=\dfrac{2\pm2\sqrt{14}i}{10}=\dfrac{1}{5}\pm\dfrac{1}{5}\sqrt{14}i$

14 $x^2+\frac{1}{3}x+2=0 \Rightarrow 3x^2+x+6=0 \Rightarrow x=\dfrac{-1\pm\sqrt{1-72}}{6}=-\dfrac{1}{6}\pm\dfrac{1}{6}\sqrt{71}i$

15 $6x^4+29x^2+28=0 \Rightarrow (2x^2+7)(3x^2+4)=0 \Rightarrow x^2=-\frac{7}{2},-\frac{4}{3} \Rightarrow x=\pm\frac{1}{2}\sqrt{14}i,\pm\frac{2}{3}\sqrt{3}i$

16 $x^4-3x^2+1=0 \Rightarrow x^2=\dfrac{3\pm\sqrt{5}}{2}\cdot\dfrac{2}{2}=\dfrac{6\pm2\sqrt{5}}{4} \Rightarrow x=\pm\sqrt{\dfrac{6\pm2\sqrt{5}}{4}} \Rightarrow$

$x=\pm\frac{1}{2}\sqrt{6\pm2\sqrt{5}}\approx\pm1.62,\pm0.62$

17 $|4x-1|=7 \Rightarrow 4x-1=7$ or $4x-1=-7 \Rightarrow 4x=8$ or $4x=-6 \Rightarrow x=2$ or $x=-\frac{3}{2}$

18 $2|2x+1|+1=15 \Rightarrow 2|2x+1|=14 \Rightarrow |2x+1|=7 \Rightarrow$
$2x+1=7$ or $2x+1=-7 \Rightarrow 2x=6$ or $2x=-8 \Rightarrow x=3$ or $x=-4$

19 $\left[\dfrac{1}{x}+6=\dfrac{5}{\sqrt{x}}\right]\cdot x \Rightarrow 1+6x=5\sqrt{x} \Rightarrow$
$6x-5\sqrt{x}+1=0$ {factoring or substituting would be appropriate} $\Rightarrow$
$(2\sqrt{x}-1)(3\sqrt{x}-1)=0 \Rightarrow \sqrt{x}=\frac{1}{2},\frac{1}{3} \Rightarrow x=\left(\frac{1}{2}\right)^2,\left(\frac{1}{3}\right)^2 \Rightarrow x=\frac{1}{4},\frac{1}{9}$

Check $x=\frac{1}{4}$: LS $=4+6=10$; RS $=5/\frac{1}{2}=10 \Rightarrow x=\frac{1}{4}$ is a solution.

Check $x=\frac{1}{9}$: LS $=9+6=15$; RS $=5/\frac{1}{3}=15 \Rightarrow x=\frac{1}{9}$ is a solution.

20 $\sqrt[3]{4x-5}-3=0 \Rightarrow \left(\sqrt[3]{4x-5}\right)^3=3^3 \Rightarrow 4x-5=27 \Rightarrow 4x=32 \Rightarrow x=8$

21 $\sqrt{7x+2}+x=6 \Rightarrow \left(\sqrt{7x+2}\right)^2=(6-x)^2 \Rightarrow 7x+2=36-12x+x^2 \Rightarrow$
$x^2-19x+34=0 \Rightarrow (x-2)(x-17)=0 \Rightarrow x=2$ and 17 is an extraneous solution.

22 $\sqrt{x+4}=\sqrt[4]{6x+19} \Rightarrow \left(\sqrt{x+4}\right)^4=\left(\sqrt[4]{6x+19}\right)^4 \Rightarrow (x+4)^2=6x+19 \Rightarrow$
$x^2+8x+16=6x+19 \Rightarrow x^2+2x-3=0 \Rightarrow (x+3)(x-1)=0 \Rightarrow x=-3,1$

23 $\sqrt{3x+1}-\sqrt{x+4}=1 \Rightarrow \sqrt{3x+1}=1+\sqrt{x+4} \Rightarrow \left(\sqrt{3x+1}\right)^2=\left(1+\sqrt{x+4}\right)^2 \Rightarrow$
$3x+1=1+2\sqrt{x+4}+x+4 \Rightarrow 2\sqrt{x+4}=2x-4 \Rightarrow \sqrt{x+4}=x-2 \Rightarrow$
$\left(\sqrt{x+4}\right)^2=(x-2)^2 \Rightarrow x+4=x^2-4x+4 \Rightarrow x^2-5x=0 \Rightarrow x(x-5)=0 \Rightarrow x=0,5.$

Check $x=0$: LS $=1-2=-1\neq$ RS $\Rightarrow x=0$ is an extraneous solution.

Check $x=5$: LS $=4-3=1=$ RS $\Rightarrow x=5$ is a solution.

24 $x^{4/3}=16 \Rightarrow \left(x^{4/3}\right)^{3/4}=\pm 16^{3/4} \Rightarrow x=\pm\left(\sqrt[4]{16}\right)^3=\pm 2^3=\pm 8$

25 $3x^2-12x+3=0 \Rightarrow x^2-4x+1=0 \Rightarrow x^2-4x+4=-1+4 \Rightarrow$
$(x-2)^2=3 \Rightarrow x-2=\pm\sqrt{3} \Rightarrow x=2\pm\sqrt{3}$

26 $x^2+10x+36=0 \Rightarrow x^2+10x+25=-36+25 \Rightarrow (x+5)^2=-11 \Rightarrow$
$x+5=\pm\sqrt{-11} \Rightarrow x=-5\pm\sqrt{11}i$

27 The expression $(x-5)^2$ is never less than 0, but it is equal to 0 when $x=5$.
Thus, $(x-5)^2\leq 0$ has solution $x=5$.

28 $10-7x<4+8x \Rightarrow -15x<-6 \Rightarrow x>\frac{2}{5} \Leftrightarrow \left(\frac{2}{5},\infty\right)$

29 $\left[-\dfrac{1}{2}<\dfrac{2x+3}{5}<\dfrac{3}{2}\right]\cdot 10 \Rightarrow -5<4x+6<15 \Rightarrow -11<4x<9 \Rightarrow -\frac{11}{4}<x<\frac{9}{4} \Leftrightarrow \left(-\frac{11}{4},\frac{9}{4}\right)$

30 $(3x-1)(10x+4)\geq(6x-5)(5x-7) \Rightarrow 30x^2+2x-4\geq 30x^2-67x+35 \Rightarrow$
$69x\geq 39 \Rightarrow x\geq\frac{13}{23} \Leftrightarrow \left[\frac{13}{23},\infty\right)$

31 $\dfrac{7}{10x+3}<0 \Rightarrow 10x+3<0$ {since $7>0$} $\Rightarrow x<-\frac{3}{10} \Leftrightarrow \left(-\infty,-\frac{3}{10}\right)$

32 $|4x+7|<21 \Rightarrow -21<4x+7<21 \Rightarrow -28<4x<14 \Rightarrow -7<x<\frac{7}{2} \Leftrightarrow \left(-7,\frac{7}{2}\right)$

33 $2|3-x|+1>5 \Rightarrow 2|3-x|>4 \Rightarrow |3-x|>2 \Rightarrow 3-x>2$ or $3-x<-2 \Rightarrow$
$1>x$ or $5<x \Rightarrow x<1$ or $x>5 \Leftrightarrow (-\infty,1)\cup(5,\infty)$

34 $-2|x-3|+1\geq -5 \Rightarrow -2|x-3|\geq -6 \Rightarrow |x-3|\leq 3 \Rightarrow -3\leq x-3\leq 3 \Rightarrow$
$0\leq x\leq 6 \Leftrightarrow [0,6]$

35 $|16-3x| \geq 5 \;\Rightarrow\; 16-3x \geq 5$ or $16-3x \leq -5 \;\Rightarrow\; -3x \geq -11$ or $-3x \leq -21 \;\Rightarrow$
$x \leq \frac{11}{3}$ or $x \geq 7 \;\Leftrightarrow\; \left(-\infty, \frac{11}{3}\right] \cup [7, \infty)$

36 $2 < |x-6| < 4 \;\Rightarrow\; 2 < x-6 < 4$ or $2 < -(x-6) < 4 \;\Rightarrow$
$8 < x < 10$ or $-2 > x-6 > -4 \;\Rightarrow\; 8 < x < 10$ or $4 > x > 2 \;\Leftrightarrow\; (2,4) \cup (8,10)$

37 $10x^2 + 11x > 6 \;\Rightarrow\; 10x^2 + 11x - 6 > 0 \;\Rightarrow\; (2x+3)(5x-2) > 0$ ★ $\left(-\infty, -\frac{3}{2}\right) \cup \left(\frac{2}{5}, \infty\right)$

Interval	$\left(-\infty, -\frac{3}{2}\right)$	$\left(-\frac{3}{2}, \frac{2}{5}\right)$	$\left(\frac{2}{5}, \infty\right)$
Sign of $5x-2$	$-$	$-$	$+$
Sign of $2x+3$	$-$	$+$	$+$
Resulting sign	$+$	$-$	$+$

38 $x(x-3) \leq 18 \;\Rightarrow\; x^2 - 3x - 18 \leq 0 \;\Rightarrow\; (x-6)(x+3) \leq 0$ ★ $[-3, 6]$

Interval	$(-\infty, -3)$	$(-3, 6)$	$(6, \infty)$
Sign of $x-6$	$-$	$-$	$+$
Sign of $x+3$	$-$	$+$	$+$
Resulting sign	$+$	$-$	$+$

39 $\dfrac{x^2(3-x)}{x+2} \leq 0 \;\Rightarrow\; \dfrac{3-x}{x+2} \leq 0 \;\{x^2 \geq 0, \text{ include } 0\}$ ★ $(-\infty, -2) \cup \{0\} \cup [3, \infty)$

Interval	$(-\infty, -2)$	$(-2, 3)$	$(3, \infty)$
Sign of $3-x$	$+$	$+$	$-$
Sign of $x+2$	$-$	$+$	$+$
Resulting sign	$-$	$+$	$-$

40 $\dfrac{x^2 - x - 2}{x^2 + 4x + 3} \leq 0 \;\Rightarrow\; \dfrac{(x-2)(x+1)}{(x+1)(x+3)} \leq 0 \;\Rightarrow\; \dfrac{x-2}{x+3} \leq 0 \;\{\text{exclude } -1\}$ ★ $(-3, -1) \cup (-1, 2]$

Interval	$(-\infty, -3)$	$(-3, 2)$	$(2, \infty)$
Sign of $x-2$	$-$	$-$	$+$
Sign of $x+3$	$-$	$+$	$+$
Resulting sign	$+$	$-$	$+$

41 $\dfrac{3}{2x+3} < \dfrac{1}{x-2} \;\Rightarrow\; \dfrac{3}{2x+3} - \dfrac{1}{x-2} < 0 \;\Rightarrow\; \dfrac{3(x-2) - 1(2x+3)}{(2x+3)(x-2)} < 0 \;\Rightarrow$
$\dfrac{3x-6-2x-3}{(2x+3)(x-2)} < 0 \;\Rightarrow\; \dfrac{x-9}{(2x+3)(x-2)} < 0$ ★ $\left(-\infty, -\frac{3}{2}\right) \cup (2, 9)$

Interval	$\left(-\infty, -\frac{3}{2}\right)$	$\left(-\frac{3}{2}, 2\right)$	$(2, 9)$	$(9, \infty)$
Sign of $x-9$	$-$	$-$	$-$	$+$
Sign of $x-2$	$-$	$-$	$+$	$+$
Sign of $2x+3$	$-$	$+$	$+$	$+$
Resulting sign	$-$	$+$	$-$	$+$

42 $\dfrac{x+2}{x^2 - 25} \leq 0 \;\Rightarrow\; \dfrac{x+2}{(x+5)(x-5)} \leq 0$ ★ $(-\infty, -5) \cup [-2, 5)$

Interval	$(-\infty, -5)$	$(-5, -2)$	$(-2, 5)$	$(5, \infty)$
Sign of $x-5$	$-$	$-$	$-$	$+$
Sign of $x+2$	$-$	$-$	$+$	$+$
Sign of $x+5$	$-$	$+$	$+$	$+$
Resulting sign	$-$	$+$	$-$	$+$

43 $x^3 > x^2 \quad \Rightarrow \quad x^2(x-1) > 0 \; \{x^2 \geq 0\} \quad \Rightarrow \quad x - 1 > 0 \quad \Rightarrow \quad x > 1 \quad \Leftrightarrow \quad (1, \infty)$

44 $(x^2 - x)(x^2 - 5x + 6) < 0 \quad \Rightarrow \quad x(x-1)(x-2)(x-3) < 0$ ★ $(0,1) \cup (2,3)$

Interval	$(-\infty, 0)$	$(0,1)$	$(1,2)$	$(2,3)$	$(3,\infty)$
Sign of $x-3$	$-$	$-$	$-$	$-$	$+$
Sign of $x-2$	$-$	$-$	$-$	$+$	$+$
Sign of $x-1$	$-$	$-$	$+$	$+$	$+$
Sign of x	$-$	$+$	$+$	$+$	$+$
Resulting sign	$+$	$-$	$+$	$-$	$+$

45 $P + N = \dfrac{C+2}{C} \quad \Rightarrow \quad C(P+N) = C + 2 \quad \Rightarrow \quad CP + CN = C + 2 \quad \Rightarrow$

$$CP + CN - C = 2 \quad \Rightarrow \quad C(P + N - 1) = 2 \quad \Rightarrow \quad C = \frac{2}{P+N-1}$$

46 $A = B\sqrt[3]{\dfrac{C}{D}} - E \quad \Rightarrow \quad A + E = B\sqrt[3]{\dfrac{C}{D}} \quad \Rightarrow \quad \dfrac{A+E}{B} = \sqrt[3]{\dfrac{C}{D}} \quad \Rightarrow \quad \left(\dfrac{A+E}{B}\right)^3 = \dfrac{C}{D} \quad \Rightarrow$

$$D \cdot \frac{(A+E)^3}{B^3} = C \quad \Rightarrow \quad D(A+E)^3 = C \cdot B^3 \quad \Rightarrow \quad D = \frac{CB^3}{(A+E)^3}$$

47 $V = \dfrac{4}{3}\pi r^3 \quad \Rightarrow \quad r^3 = \dfrac{3V}{4\pi} \quad \Rightarrow \quad r = \sqrt[3]{\dfrac{3V}{4\pi}}$

48 $F = \dfrac{\pi P R^4}{8VL} \quad \Rightarrow \quad R^4 = \dfrac{8FVL}{\pi P} \quad \Rightarrow \quad R = \pm\sqrt[4]{\dfrac{8FVL}{\pi P}} \quad \Rightarrow \quad R = \sqrt[4]{\dfrac{8FVL}{\pi P}}$ since $R > 0$

49 $c = \sqrt{4h(2R-h)} \quad \Rightarrow \quad c^2 = 8Rh - 4h^2 \quad \Rightarrow \quad 4h^2 - 8Rh + c^2 = 0 \quad \Rightarrow$

$$h = \frac{8R \pm \sqrt{64R^2 - 16c^2}}{8} = \frac{8R \pm 4\sqrt{4R^2 - c^2}}{8} = R \pm \frac{1}{2}\sqrt{4R^2 - c^2}$$

50 $V = \frac{1}{3}\pi h(r^2 + R^2 + rR) \quad \Rightarrow \quad 3V = \pi h(r^2 + R^2 + rR) \quad \Rightarrow \quad (\pi h)r^2 + (\pi h R)r + (\pi h R^2 - 3V) = 0 \quad \Rightarrow$

$$r = \frac{-(\pi h R) \pm \sqrt{(\pi h R)^2 - 4(\pi h)(\pi h R^2 - 3V)}}{2(\pi h)} = \frac{-\pi h R \pm \sqrt{12\pi h V - 3\pi^2 h^2 R^2}}{2\pi h}.$$

Since $r > 0$, we must use the plus sign, and $r = \dfrac{-\pi h R + \sqrt{12\pi h V - 3\pi^2 h^2 R^2}}{2\pi h}$.

51 $(7 + 5i) - (-2 + 3i) = 7 + 5i + 2 - 3i = (7 + 2) + (5 - 3)i = 9 + 2i$

52 $(4 + 2i)(-5 + 4i) = -20 + 16i - 10i + 8i^2 = (-20 - 8) + (16 - 10)i = -28 + 6i$

53 $(5 + 8i)^2 = 5^2 + 2(5)(8i) + (8i)^2 = (25 - 64) + 80i = -39 + 80i$

54 $\dfrac{1}{9 - \sqrt{-4}} = \dfrac{1}{9 - 2i} = \dfrac{1}{9 - 2i} \cdot \dfrac{9 + 2i}{9 + 2i} = \dfrac{9 + 2i}{81 + 4} = \dfrac{9}{85} + \dfrac{2}{85}i$

55 $\dfrac{6 - 3i}{2 + 7i} = \dfrac{6 - 3i}{2 + 7i} \cdot \dfrac{2 - 7i}{2 - 7i} = \dfrac{(12 - 21) + (-42 - 6)i}{4 + 49} = -\dfrac{9}{53} - \dfrac{48}{53}i$

56 $\dfrac{24 - 8i}{4i} = \dfrac{4(6 - 2i)}{4i} = \dfrac{6 - 2i}{i} \cdot \dfrac{-i}{-i} = \dfrac{-6i + 2i^2}{-i^2} = \dfrac{-2 - 6i}{1} = -2 - 6i$

57 Let x denote the score of the third game. Average score $= 250 \quad \Rightarrow$

$$\frac{x + 267 + 225}{3} = 250 \quad \Rightarrow \quad x + 492 = 3(250) \quad \Rightarrow \quad x = 750 - 492 = 258$$

58 Let x denote the presale price. Presale price $-$ discount $= 50 \quad \Rightarrow$

$$x - 0.37x = 50 \quad \Rightarrow \quad 0.63x = 50 \quad \Rightarrow \quad x = \tfrac{50}{0.63} \approx \$79.37$$

59 Let x denote the number of years from now to retirement eligibility.
Age + service $\geq 90 \Rightarrow (37+x)+(15+x)\geq 90 \Rightarrow 2x+52\geq 90 \Rightarrow 2x\geq 38 \Rightarrow x\geq 19.$
The teacher will be eligible to retire at age $37+19=56$.

60 $\frac{1}{R}=\frac{1}{R_1}+\frac{1}{R_2}$ • $R=2$ and $R_1=5 \Rightarrow \left[\frac{1}{2}=\frac{1}{5}+\frac{1}{R_2}\right]\cdot 10R_2 \Rightarrow 5R_2=2R_2+10 \Rightarrow$
$3R_2=10 \Rightarrow R_2=\frac{10}{3}$ ohms

61 Let P denote the principal that will be invested, and r the yield rate of the stock fund.
$\text{Income}_{\text{stocks}}-28\%$ federal tax $-\ 7\%$ state tax $=\text{Income}_{\text{bonds}} \Rightarrow$
$(Pr)-0.28(Pr)-0.07(Pr)=0.07186P$ {divide by P} $\Rightarrow 1r-0.28r-0.07r=0.07186 \Rightarrow$
$0.65r=0.07186 \Rightarrow r=\frac{0.07186}{0.65} \Rightarrow r\approx 0.11055$, or, 11.055%.

62 Let x denote the amount invested in the first fund, so $216{,}000-x$ is the amount invested in the second fund.
$\text{Interest}_{\text{Fund 1}}+\text{Interest}_{\text{Fund 2}}=12{,}000 \Rightarrow 0.045x+0.0925(216{,}000-x)=12{,}000 \Rightarrow$
$0.045x+19{,}980-0.0925x=12{,}000 \Rightarrow -0.0475x=-7980 \Rightarrow x=\frac{-7980}{-0.0475}=\$168{,}000$

63 Let x denote the amount of time it would take to clear the driveway if they worked together.
45 minutes $=\frac{3}{4}$ hour, so comparing the amount of the job per hour rates, we get
$\frac{1}{3/4}+\frac{1}{2}=\frac{1}{x} \Rightarrow \left[\frac{4}{3}+\frac{1}{2}=\frac{1}{x}\right](6x) \Rightarrow 8x+3x=6 \Rightarrow 11x=6 \Rightarrow x=\frac{6}{11}$ hr or ≈ 32.7 min

64 Let x denote the number of cm^3 of gold. $\text{Grams}_{\text{gold}}+\text{Grams}_{\text{silver}}=\text{Grams}_{\text{total}} \Rightarrow$
$x(19.3)+(5-x)(10.5)=80 \Rightarrow 19.3x+52.5-10.5x=80 \Rightarrow 8.8x=27.5 \Rightarrow x=3.125.$
The number of grams of gold is $19.3x=60.3125\approx 60.3$.

65 Let x denote the number of ounces of the vegetable portion, $10-x$ the number of ounces of meat.
$\text{Protein}_{\text{vegetable}}+\text{Protein}_{\text{meat}}=\text{Protein}_{\text{total}} \Rightarrow \frac{1}{2}(x)+1(10-x)=7 \Rightarrow \frac{1}{2}x+10-x=7 \Rightarrow$
$-\frac{1}{2}x=-3 \Rightarrow x=6$. Use 6 oz of vegetables and 4 oz of meat.

66 Let x denote the number of grams of 95% ethyl alcohol solution used, $400-x$ the number of grams of water.
$95(x)+0(400-x)=75(400)$ {all in %} $\Rightarrow 95x=75(400) \Rightarrow$
$x=\frac{6000}{19}\approx 315.8$. Use 315.8 g of ethyl alcohol and 84.2 g of water.

67 Let x denote the number of gallons of 20% solution, $120-x$ the number of gallons of 50% solution.
$20(x)+50(120-x)=30(120)$ {all in %} $\Rightarrow 20x+6000-50x=3600 \Rightarrow 2400=30x \Rightarrow x=80.$
Use 80 gal of the 20% solution and 40 gal of the 50% solution.

68 Let $x=$ the amount of copper they have to mix with 140 kg of zinc to make brass.
$\text{Copper}_{\text{amount put in}}=\text{Copper}_{\text{amount in final product}} \Rightarrow$
$x=0.65(x+140) \Rightarrow x=0.65x+91 \Rightarrow 0.35x=91 \Rightarrow x=\frac{91}{0.35}=260$ kg

69 Let x denote the distance upstream. 10 gallons of gas @ 16 mi/gal = 160 miles.
At 20 mi/hr, there is enough fuel for 8 hours of travel.
The rate of the boat upstream is 15 mi/hr and the rate downstream is 25 mi/hr.
$\text{Time}_{\text{up}}+\text{Time}_{\text{down}}=\text{Time}_{\text{total}} \Rightarrow$
$\left[\frac{x}{15}+\frac{x}{25}=8\right]\cdot 75 \Rightarrow 5x+3x=600 \Rightarrow 8x=600 \Rightarrow x=75$ mi.

70 Let x denote the number of hours spent traveling in the smaller cities, $5\frac{1}{2} - x$ the number of hours in the country.
$\text{Distance}_{\text{country}} + \text{Distance}_{\text{cities}} = \text{Distance}_{\text{total}} \Rightarrow 100\left(5\frac{1}{2} - x\right) + 25(x) = 400 \Rightarrow$
$550 - 100x + 25x = 400 \Rightarrow 150 = 75x \Rightarrow x = 2$ hr.

71 Let x denote the speed of the wind.
$\text{Distance}_{\text{with wind}} = \text{Distance}_{\text{against wind}} \Rightarrow (320 + x)\frac{1}{2} = (320 - x)\frac{3}{4}\ \{d = rt\} \Rightarrow$
$(320 + x)(2) = (320 - x)(3) \Rightarrow 640 + 2x = 960 - 3x \Rightarrow 5x = 320 \Rightarrow x = 64$ mi/hr

72 Let $50 + r$ denote the rate the automobile, that is, r is the rate over 50 mi/hr.
The automobile must travel $40 + 20 = 60$ ft more than the truck (traveling at 50 mi/hr) in 5 seconds.
Since 1 mi/hr $= \frac{5280}{3600} = \frac{22}{15}$ ft/sec, the automobile's rate in *excess* of 50 mi/hr is $\frac{22}{15}r$.
Thus, $d = rt \Rightarrow 60 = \left(\frac{22}{15}r\right)(5) \Rightarrow r = \frac{90}{11}$. The rate is $50 + \frac{90}{11} = \frac{640}{11} \approx 58.2$ mi/hr.

73 Let x denote the time (in hours) the first speedboat travels, so $x - \frac{1}{3}$ is the time the second speedboat travels.
$\text{Distance}_{\text{East}} + \text{Distance}_{\text{West}} = \text{Distance}_{\text{Total}} \Rightarrow$
$30x + 24\left(x - \frac{1}{3}\right) = 37 \Rightarrow 30x + 24x - 8 = 37 \Rightarrow 54x = 45 \Rightarrow x = \frac{45}{54} = \frac{5}{6}$ hour or 50 minutes

74 Let x denote her jogging rate in miles per hour. Using the formula $d = rt$, we get $7 - 5 = x\left(\frac{24}{60}\right) \Rightarrow$
$2 = \frac{2}{5}x \Rightarrow x = 5$ mph. *Another solution:* Using $t = \frac{d}{r}$, $\text{time}_{\text{shorter jog}} + \text{time}_{\text{extra}} = \text{time}_{\text{longer jog}} \Rightarrow$
$\frac{5}{x} + \frac{24}{60} = \frac{7}{x} \Rightarrow \left[\frac{5}{x} + \frac{2}{5} = \frac{7}{x}\right](5x) \Rightarrow 25 + 2x = 35 \Rightarrow 2x = 10 \Rightarrow x = 5$ mph.

75 Let x denote the number of hours needed to fill an empty bin.
Using the hourly rates, $\left[\dfrac{1}{2} - \dfrac{1}{5} = \dfrac{1}{x}\right] \cdot 10x \Rightarrow 5x - 2x = 10 \Rightarrow 3x = 10 \Rightarrow$
$x = \frac{10}{3}$ hr. Since the bin was half-full at the start, $\frac{1}{2}x = \frac{1}{2} \cdot \frac{10}{3} = \frac{5}{3}$ hr, or, 1 hr 40 min.

76 Let x denote the number of gallons used in the city, $24 - x$ the number on the highway.
$\text{Distance}_{\text{city}} + \text{Distance}_{\text{highway}} = \text{Distance}_{\text{total}} \Rightarrow 22x + 28(24 - x) = 627 \Rightarrow 22x + 672 - 28x = 627 \Rightarrow$
$-6x = -45 \Rightarrow x = \frac{15}{2}$. The number of miles in the city is $22x = 22\left(\frac{15}{2}\right) = 165$.

77 Let d denote the distance from the center of the city to a corner and $2x$ denote the length of one side of the city.
$x^2 + x^2 = d^2 \Rightarrow 2x^2 = d^2 \Rightarrow d = \sqrt{2}x$. The area A of the city is $A = (2x)^2 = 4x^2$, or $2d^2$.
Currently: $d = 10 \Rightarrow A = 2(10)^2 = 200$.
One decade ago: $A = 150 \Rightarrow 2d^2 = 150 \Rightarrow d = \sqrt{75} = 5\sqrt{3}$. The change in d is $10 - 5\sqrt{3} \approx 1.34$ mi.

78 Let x denote the change in the radius. New surface area = 125% of original surface area $\Rightarrow$
$4\pi(6 + x)^2 = 1.25\left[4\pi(6)^2\right] \Rightarrow (x + 6)^2 = 45 \Rightarrow x + 6 = \sqrt{45} \Rightarrow x = 3\sqrt{5} - 6 \approx 0.71$ micron.

79 (a) The eastbound car has distance $20t$ and the southbound car has distance $(-2 + 50t)$.
$d^2 = (20t)^2 + (-2 + 50t)^2 \Rightarrow d^2 = 400t^2 + 4 - 200t + 2500t^2 \Rightarrow d = \sqrt{2900t^2 - 200t + 4}$

(b) $104 = \sqrt{2900t^2 - 200t + 4} \Rightarrow 10{,}816 = 2900t^2 - 200t + 4 \Rightarrow 2900t^2 - 200t - 10{,}812 = 0 \Rightarrow$
$725t^2 - 50t - 2703 = 0$ {divide by 4} $\Rightarrow$
$$t = \frac{50 \pm \sqrt{7{,}841{,}200}}{1450}\ \{t > 0\} = \frac{5 + 2\sqrt{19{,}603}}{145} \approx 1.97, \text{ or approximately 11:58 A.M.}$$

80 Let l and w denote the length and width, respectively. $3l + 6w = 270 \Rightarrow 6w = 270 - 3l \Rightarrow w = 45 - \frac{1}{2}l$.
The total area is to be $10 \cdot 100 = 1000$ ft^2. Area $= lw \Rightarrow 1000 = l\left(45 - \frac{1}{2}l\right) \Rightarrow 1000 = 45l - \frac{1}{2}l^2 \Rightarrow$

$l^2 - 90l + 2000 = 0 \Rightarrow (l-40)(l-50) = 0 \Rightarrow l = 40, 50$ and $w = 25, 20$.

There are two arrangements: 40 ft × 25 ft and 50 ft × 20 ft.

81 Let x denote the length of one side of an end.

(a) $V = lwh \Rightarrow 48 = 6 \cdot x \cdot x \Rightarrow x^2 = 8 \Rightarrow x = 2\sqrt{2}$ ft

(b) $S = lw + 2wh + 2lh \Rightarrow 44 = 6x + 2(x^2) + 2(6x) \Rightarrow 44 = 2x^2 + 18x \Rightarrow$
$x^2 + 9x - 22 = 0 \Rightarrow (x+11)(x-2) = 0 \Rightarrow x = 2$ ft

82 Let x and $4x$ denote the width and length of the pool, respectively.
$A = lw \Rightarrow 1440 = (x+12)(4x+12) \Rightarrow 1440 = 4x^2 + 60x + 144 \Rightarrow 4x^2 + 60x - 1296 = 0 \Rightarrow$
$x^2 + 15x - 324 = 0 \Rightarrow (x+27)(x-12) = 0 \Rightarrow x = 12\ \{x > 0\}$, and $4x = 48$.

The dimensions of the pool are 12 ft by 48 ft.

83 Let x and $2x$ denote the width and length of the tiled area, respectively.
The bathing area has measurements $x - 2$ and $2x - 2$. For the bathing area, width · length = area $\Rightarrow$
$(x-2)(2x-2) = 40 \Rightarrow 2x^2 - 6x + 4 = 40 \Rightarrow x^2 - 3x + 2 = 20 \Rightarrow x^2 - 3x - 18 = 0 \Rightarrow$
$(x-6)(x+3) = 0 \Rightarrow x = 6\ \{x > 0\}$. The tiled area is 12 ft by 6 ft and the bathing area is 10 ft by 4 ft.

84 $P = 15 + \sqrt{3t+2}$ • $P = 20 \Rightarrow 15 + \sqrt{3t+2} = 20 \Rightarrow \sqrt{3t+2} = 5 \Rightarrow 3t + 2 = 25 \Rightarrow$
$3t = 23 \Rightarrow t = \frac{23}{3}$, or after $7\frac{2}{3}$ yr.

85 $pv = 200 \Rightarrow v = \dfrac{200}{p}$. $25 \le v \le 50 \Rightarrow 25 \le \dfrac{200}{p} \le 50 \Rightarrow \dfrac{1}{25} \ge \dfrac{p}{200} \ge \dfrac{1}{50} \Rightarrow 8 \ge p \ge 4 \Rightarrow$
$4 \le p \le 8$

86 Let x denote the amount of yearly business. $\text{Pay}_{\text{B}} > \text{Pay}_{\text{A}} \Rightarrow \$40{,}000 + 0.20x > \$50{,}000 + 0.10x \Rightarrow$
$0.10x > \$10{,}000 \Rightarrow x > \$100{,}000$

87 $v > 1100 \Rightarrow 1087\sqrt{\dfrac{T}{273}} > 1100 \Rightarrow \sqrt{\dfrac{T}{273}} > \dfrac{1100}{1087} \Rightarrow \dfrac{T}{273} > \dfrac{1100^2}{1087^2} \Rightarrow T > \dfrac{273 \cdot 1100^2}{1087^2} \Rightarrow$
$T > 279.57$ K

88 $T = 2\pi\sqrt{\dfrac{l}{980}} \Rightarrow T^2 = 4\pi^2\left(\dfrac{l}{980}\right) \Rightarrow l = \dfrac{980\,T^2}{4\pi^2}$.
$98 \le l \le 100 \Rightarrow 98 \le \dfrac{980\,T^2}{4\pi^2} \le 100 \Rightarrow \dfrac{2\pi^2}{5} \le T^2 \le \dfrac{20\pi^2}{49} \Rightarrow \dfrac{10\pi^2}{25} \le T^2 \le \dfrac{20\pi^2}{49} \Rightarrow$
$\dfrac{\pi}{5}\sqrt{10} \le T \le \dfrac{2\pi}{7}\sqrt{5}\ \{T \ge 0\}$, or, approximately, $1.987 \le T \le 2.007$ sec.

89 $v = \dfrac{626.4}{\sqrt{h + 6372}} \Rightarrow v^2 = \dfrac{(626.4)^2}{h + 6372} \Rightarrow h + 6372 = \dfrac{(626.4)^2}{v^2} \Rightarrow h = \dfrac{(626.4)^2}{v^2} - 6372$.
$h > 100 \Rightarrow \dfrac{(626.4)^2}{v^2} - 6372 > 100 \Rightarrow \dfrac{(626.4)^2}{v^2} > 6472 \Rightarrow v^2 < \dfrac{(626.4)^2}{6472}\ \{v > 0\} \Rightarrow$
$0 < v < \dfrac{626.4}{\sqrt{6472}} \approx 7.786$ km/sec

90 $P = 2l + 2w \Rightarrow 100 = 2l + 2w \Rightarrow l = 50 - w$. $A \ge 600 \Rightarrow lw \ge 600 \Rightarrow$
$(50 - w)w \ge 600 \Rightarrow -w^2 + 50w - 600 \ge 0 \Rightarrow w^2 - 50w + 600 \le 0 \Rightarrow$
$(w-20)(w-30) \le 0 \Rightarrow 20 \le w \le 30$. If w is greater than 25, it would be the length.

Hence, the desired values of w are between 20 and 25, that is, $20 \le w \le 25$.

91 Let x denote the number of trees *over* 24. Then $24 + x$ represents the total number of trees planted per acre, and $600 - 12x$ represents the number of apples per tree.

$$\textbf{Total apples} = (\text{number of trees})(\text{number of apples per tree})$$
$$= (24 + x)(600 - 12x) = -12x^2 + 312x + 14{,}400$$

Apples $\geq 16{,}416 \Rightarrow -12x^2 + 312x + 14{,}400 \geq 16{,}416 \Rightarrow -12x^2 + 312x - 2016 \geq 0 \Rightarrow$
$x^2 - 26x + 168 \leq 0 \Rightarrow (x - 12)(x - 14) \leq 0 \Rightarrow 12 \leq x \leq 14 \Rightarrow 36 \leq 24 + x \leq 38$

Hence, 36 to 38 trees per acre should be planted.

92 Let x denote the number of \$25 increases in rent. Then the number of occupied apartments is $218 - 5x$ and the rent per apartment is $940 + 25x$.

$$\textbf{Total income} = (\text{number of occupied apartments})(\text{rent per apartment})$$
$$= (218 - 5x)(940 + 25x) = -125x^2 + 750x + 204{,}920$$

Income $\geq 205{,}920 \Rightarrow -125x^2 + 750x + 204{,}920 \geq 205{,}920 \Rightarrow -125x^2 + 750x - 1000 \geq 0 \Rightarrow$
$x^2 - 6x + 8 \leq 0 \Rightarrow (x - 2)(x - 4) \leq 0 \Rightarrow 2 \leq x \leq 4 \Rightarrow 990 \leq 940 + 25x \leq 1040$

Hence, the rent charged should be \$990 to \$1040.

93 The y-values are increasing slowly and can best be described by equation (3), $y = 3\sqrt{x - 0.5}$.

Chapter 2 Discussion Exercises

1 We need to solve the equation $x^2 - xy + y^2 = 0$ for x.
Use the quadratic formula with $a = 1$, $b = -y$, and $c = y^2$.

$$x = \frac{-(-y) \pm \sqrt{(-y)^2 - 4(1)(y^2)}}{2(1)} = \frac{y \pm \sqrt{y^2 - 4y^2}}{2} = \frac{y \pm \sqrt{-3y^2}}{2} = \frac{y \pm |y|\sqrt{3}\,i}{2}.$$

Since this equation has imaginary solutions, $x^2 - xy + y^2$ is not factorable over the reals.

A similar argument holds for $x^2 + xy + y^2$.

2 The solutions are $x_1 = \dfrac{-b + \sqrt{b^2 - 4ac}}{2a}$ and $x_2 = \dfrac{-b - \sqrt{b^2 - 4ac}}{2a}$.
The average is $\dfrac{x_1 + x_2}{2} = \dfrac{-2b/2a}{2} = -\dfrac{b}{2a}$. Suppose you solve the equation $-x^2 + 4x + 7 = 0$ and obtain the solutions $x_1 \approx -1.32$ and $x_2 \approx 5.32$. Averaging these numbers gives us the value 2, which we can easily see is equal to $-b/(2a)$.

3 (a) $$\frac{1}{\dfrac{a+bi}{c+di}} = \frac{c+di}{a+bi} \cdot \frac{a-bi}{a-bi} = \frac{ac+bd+(ad-bc)i}{a^2+b^2} = \frac{ac+bd}{a^2+b^2} + \frac{ad-bc}{a^2+b^2}\,i = p + qi$$

(b) Yes, try an example such as $\frac{3}{4}$. Let $a = 3$, $b = 0$, $c = 4$, and $d = 0$. Then, from part (a),
$p + qi = \frac{12}{9} + \frac{0}{9}i = \frac{12}{9} = \frac{4}{3}$, which is the multiplicative inverse of $\frac{3}{4}$.

(c) a and b cannot both be 0 because then the denominator would be 0.

4 Since we don't know the value of x, we don't know the sign of $x-2$, and hence we are unsure of whether or not to reverse the direction of the inequality sign.

5 *Hint*: Try these examples to help you get to the general solution.

(1) $x^2+1\geq 0$ {In this case, $a>0$, $D=-4<0$, and by examining a sign chart with x^2+1 as the only factor, we see that the solution is $x\in\mathbb{R}$.}

(2) $x^2-2x-3\geq 0$

(3) $-x^2-4\geq 0$

(4) $-x^2-2x-1\geq 0$

(5) $-x^2+2x+3\geq 0$

General solutions categorized by a and D:

(1) $a>0$, $D\leq 0$: solution is $x\in\mathbb{R}$

(2) $a>0$, $D>0$: let $x_1=\dfrac{-b-\sqrt{D}}{2a}$ and $x_2=\dfrac{-b+\sqrt{D}}{2a}$ $\Rightarrow$ solution is $(-\infty,x_1]\cup[x_2,\infty)$

(3) $a<0$, $D<0$: no solution

(4) $a<0$, $D=0$: solution is $x=-\dfrac{b}{2a}$

(5) $a<0$, $D>0$: solution is $[x_1,x_2]$

6 (a) This problem is solved in three steps.

(i) First, we must determine the height of the cloud base using the formula in Exercise 38 in Section 2.2, $h=227(T-D)=227(80-68)=2724$ ft.

(ii) Next, we must determine the temperature T at the cloud base. From (i), the height of the cloud base is $h=2724$ and $T=T_0-\frac{5.5}{1000}h=80-\frac{5.5}{1000}\cdot 2724=65.018$°F.

(iii) Finally, we must solve the equation $T=B-\frac{3}{1000}h$ for h, when $T=32$°F and $B=65.018$°F.

$$32=65.018-\frac{3}{1000}h \quad\Rightarrow\quad h=(65.018-32)\frac{1000}{3}=11{,}006 \text{ ft.}$$

(b) Following the procedure in part (a) and changing a few variable names, we obtain the following:

(i) $H=227(G-D)=227G-227D$

(ii) $T=T_0-\frac{5.5}{1000}h \quad\Rightarrow\quad B=G-\frac{11}{2000}H \quad\Rightarrow\quad B=G-\frac{11}{2000}(227G-227D) \quad\Rightarrow$
$B=G-\frac{2497}{2000}G+\frac{2497}{2000}D=\frac{2497}{2000}D-\frac{497}{2000}G$

(iii) $h=(B-32)\frac{1000}{3} \quad\Rightarrow\quad h=\left(\frac{2497}{2000}D-\frac{497}{2000}G-32\right)\frac{1000}{3} \quad\Rightarrow\quad h=\frac{2497}{6}D-\frac{497}{6}G-\frac{32{,}000}{3} \quad\Rightarrow$
$h=\frac{2497}{6}D-\frac{497}{6}G-\frac{64{,}000}{6} \quad\Rightarrow\quad h=\frac{1}{6}(2497D-497G-64{,}000)$

7 The first equation, $\sqrt{2x-3}+\sqrt{x+5}=0$, is a sum of square roots that is equal to 0. The only way this could be true is if both radicals are actually equal to 0. It is easy to see that $\sqrt{x+5}$ is equal to 0 only if $x=-5$, but -5 will not make $\sqrt{2x-3}$ equal to 0, so there is no reason to try to solve the first equation.

On the other hand, the second equation, $\sqrt[3]{2x-3}+\sqrt[3]{x+5}=0$, can be written as $\sqrt[3]{2x-3}=-\sqrt[3]{x+5}$. This just says that one cube root is equal to the negative of another cube root, which could happen since a cube root can be negative. Solving this equation gives us $2x-3=-(x+5) \quad\Rightarrow\quad 3x=-2 \quad\Rightarrow\quad x=-\frac{2}{3}$.

8 $\sqrt{x} = cx - 2/c \quad \Rightarrow \quad c\sqrt{x} = c^2x - 2 \quad \Rightarrow \quad c^2x = c^4x^2 - 4c^2x + 4 \quad \Rightarrow \quad 0 = c^4x^2 - 5c^2x + 4 \quad \Rightarrow$
$0 = (c^2x - 1)(c^2x - 4) \quad \Rightarrow \quad x_{1,2} = \dfrac{1}{c^2}, \dfrac{4}{c^2}.$

Check $x_1 = \dfrac{1}{c^2} = \dfrac{1}{(2 \times 10^{500})^2} = \dfrac{1}{4 \times 10^{1000}}.$

$\text{LS} = \sqrt{x_1} = \dfrac{1}{2 \times 10^{500}}$

$\text{RS} = cx_1 - \dfrac{2}{c} = \dfrac{2 \times 10^{500}}{4 \times 10^{1000}} - \dfrac{2}{2 \times 10^{500}} = \dfrac{1}{2 \times 10^{500}} - \dfrac{2}{2 \times 10^{500}} = -\dfrac{1}{2 \times 10^{500}}$

Check $x_2 = \dfrac{4}{c^2} = \dfrac{4}{(2 \times 10^{500})^2} = \dfrac{4}{4 \times 10^{1000}} = \dfrac{1}{10^{1000}}.$

$\text{LS} = \sqrt{x_2} = \dfrac{1}{10^{500}}$

$\text{RS} = cx_2 - \dfrac{2}{c} = \dfrac{2 \times 10^{500}}{10^{1000}} - \dfrac{2}{2 \times 10^{500}} = \dfrac{2}{10^{500}} - \dfrac{1}{10^{500}} = \dfrac{1}{10^{500}}$

So x_2 is a valid solution. The right side of the original equation, $cx - 2/c$, must be nonnegative since it is equal to a square root. Note that the right side equals a negative number when $x = x_1$.

9 1 gallon ≈ 0.13368 ft^3 is a conversion factor that would help.
The volume of the tank is 10,000 gallons ≈ 1336.8 ft^3. Use $V = \frac{4}{3}\pi r^3$ to determine the radius.
$1336.8 = \dfrac{4}{3}\pi r^3 \quad \Rightarrow \quad r^3 = \dfrac{1002.6}{\pi} \quad \Rightarrow \quad r \approx 6.83375$ ft. Then use $S = 4\pi r^2$ to find the surface area.
$S = 4\pi(6.83375)^2 \approx 586.85$ ft^2.

Chapter 2 Test

1 $\left[\dfrac{5x}{x-3} + \dfrac{7}{x} = \dfrac{45}{x^2 - 3x}\right] \cdot x(x-3) \quad \Rightarrow \quad 5x^2 + 7(x-3) = 45 \quad \Rightarrow \quad 5x^2 + 7x - 66 = 0 \quad \Rightarrow$
$(5x + 22)(x - 3) = 0 \quad \Rightarrow \quad x = -\frac{22}{5}, 3$. But x cannot equal 3 since it would make denominators in the original equation equal to 0, so $x = -\frac{22}{5}$.

2 $A = \dfrac{3B}{2B - 5} \quad \Rightarrow \quad A(2B - 5) = 3B \quad \Rightarrow \quad 2AB - 5A = 3B \quad \Rightarrow \quad 2AB - 3B = 5A \quad \Rightarrow$
$B(2A - 3) = 5A \quad \Rightarrow \quad B = \dfrac{5A}{2A - 3}$

3 Let x denote the original value of the stock. Then $x + 0.2x$ is the value after the first year and $x + 0.3(x + 0.2x)$ is the value after the next year, so an equation that describes the problem is $x + 0.3(x + 0.2x) = 2720$. Solving gives us $x + 0.3(x + 0.2x) = 2720 \quad \Rightarrow \quad x + 0.3x + 0.06x = 2720 \quad \Rightarrow \quad 1.36x = 2720 \quad \Rightarrow \quad x = \frac{2720}{1.36} = 2000$.
The original value was \$2000.

4 $3x^2 + \sqrt{60}xy + 5y^2 = 0 \quad \Rightarrow \quad 3x^2 + \left(\sqrt{60}y\right)x + 5y^2 = 0 \quad \Rightarrow$

$$x = \frac{-\sqrt{60}y \pm \sqrt{\left(\sqrt{60}y\right)^2 - 4(3)(5y^2)}}{2(3)} = \frac{-\sqrt{4}\sqrt{15}y \pm \sqrt{60y^2 - 60y^2}}{2(3)} = \frac{-2\sqrt{15}y \pm 0}{2(3)} = \frac{-\sqrt{15}y}{3}$$

5 $(x - y + z)^2 = 9 \quad \Rightarrow \quad x - y + z = \pm 3 \quad \Rightarrow \quad x = y - z \pm 3$

6 $h = 1584 \Rightarrow -16t^2 + 320t = 1584 \Rightarrow -16t^2 + 320t - 1584 = 0 \Rightarrow$
$t^2 - 20t + 99 = 0$ {divide by -16} $\Rightarrow (t-9)(t-11) = 0 \Rightarrow t = 9$ or 11.
Thus, the object is 1584 feet above the ground after 9 seconds and after 11 seconds.

7 $i^{4x+3} = (i^{4x})(i^3) = (i^4)^x(i^3) = (1)^x(-i) = (1)(-i) = -i = 0 - i$, so $a = 0$ and $b = -1$.

8 $x^3 - 64 = 0 \Rightarrow (x-4)(x^2+4x+16) = 0 \Rightarrow x = 4$ or $x^2 + 4x + 16 = 0$.
By the quadratic formula, $x = \dfrac{-4 \pm \sqrt{4^2 - 4(1)(16)}}{2(1)} = \dfrac{-4 \pm \sqrt{16-64}}{2} = \dfrac{-4 \pm \sqrt{-48}}{2}$ ★ $4, -2 \pm 2\sqrt{3}i$
$= \dfrac{-4 \pm \sqrt{16}\sqrt{-3}}{2} = \dfrac{-4 \pm 4\sqrt{3}i}{2} = -2 \pm 2\sqrt{3}i.$

9 $A = B\sqrt{x^2 + r^2} \Rightarrow \dfrac{A}{B} = \sqrt{x^2+r^2} \Rightarrow \left(\dfrac{A}{B}\right)^2 = x^2 + r^2 \Rightarrow \dfrac{A^2}{B^2} - r^2 = x^2 \Rightarrow$
$x^2 = \dfrac{1}{B^2}(A^2 - B^2r^2) \Rightarrow x = \pm\dfrac{1}{B}\sqrt{A^2 - B^2r^2}$

10 $3x^{32}(x+2)^{65}(x-5)^{13}(x^{2/3} - 4) = 0 \Rightarrow x^{32} = 0$ or $(x+2)^{65} = 0$ or $(x-5)^{13} = 0$ or $x^{2/3} - 4 = 0 \Rightarrow$
$x = 0$ or $x = -2$ or $x = 5$ or $x^{2/3} = 4$. Now $x^{2/3} = 4 \Rightarrow (x^{2/3})^{3/2} = \pm(4)^{3/2} \Rightarrow$
$x = \pm\left(\sqrt{4}\right)^3 = \pm 2^3 = \pm 8$. Thus, the solutions of the equation are $0, -2, 5$, and ± 8.

11 $20{,}000 = \frac{4}{3}\pi r_1^3 \Rightarrow r_1^3 = 15{,}000/\pi \Rightarrow r_1 = \sqrt[3]{15{,}000/\pi}$.
Similarly, $25{,}000 = \frac{4}{3}\pi r_2^3 \Rightarrow r_2 = \sqrt[3]{18{,}750/\pi}$. The radius increased $(r_2 - r_1)/r_1 \approx 0.077$, or about 7.7%.

12 Plan A pays out \$3300 per month for 10 years before plan B starts, so its total payout is $(10)(12)(3300) + 3300x$, where x is the number of months that plan B has paid out. Plan B's total payout is $4200x$.
Plan B $\geq$ Plan A $\Rightarrow 4200x \geq 396{,}000 + 3300x \Rightarrow 900x \geq 396{,}000 \Rightarrow x \geq 440$.
It will take plan B 440 months (36 years, 8 months) to have a total payout at least as large as plan A.

13 $-\frac{1}{4}|3-2x| + 6 \geq 2 \Rightarrow -\frac{1}{4}|3-2x| \geq -4 \Rightarrow |3-2x| \leq 16 \Rightarrow -16 \leq 3 - 2x \leq 16 \Rightarrow$
$-19 \leq -2x \leq 13 \Rightarrow \frac{19}{2} \geq x \geq -\frac{13}{2}$. The solution in interval notation is $\left[-\frac{13}{2}, \frac{19}{2}\right]$.

14 $x(2x+1) \geq 3 \Rightarrow 2x^2 + x - 3 \geq 0 \Rightarrow (2x+3)(x-1) \geq 0 \Rightarrow$ the solution is $(-\infty, -\frac{3}{2}] \cup [1, \infty)$.

15 $\dfrac{(x+1)^2(x-7)}{(7-x)(x-4)} \leq 0 \Rightarrow \dfrac{x-7}{(7-x)(x-4)} \leq 0$ {include -1} $\Rightarrow$
$\dfrac{1}{x-4} \geq 0$ {cancel, change inequality, exclude 7} $\Rightarrow x - 4 > 0$ {exclude 4} $\Rightarrow x > 4 \Rightarrow$
the solution is $\{-1\} \cup (4, 7) \cup (7, \infty)$.

16 $\dfrac{2}{x-3} \leq \dfrac{2}{x+1} \Rightarrow \dfrac{2}{x-3} - \dfrac{2}{x+1} \leq 0 \Rightarrow \dfrac{2(x+1) - 2(x-3)}{(x-3)(x+1)} \leq 0 \Rightarrow$
$\dfrac{2x + 2 - 2x + 6}{(x-3)(x+1)} \leq 0 \Rightarrow \dfrac{8}{(x-3)(x+1)} \leq 0 \Rightarrow (x-3)(x+1) < 0$ ★ $(-1, 3)$

Interval	$(-\infty, -1)$	$(-1, 3)$	$(3, \infty)$
Sign of $x - 3$	$-$	$-$	$+$
Sign of $x + 1$	$-$	$+$	$+$
Resulting sign	$+$	$-$	$+$

17 Let L and W denote the length and width of the rectangle. Then $L + W = 14$, so $L = 14 - W$ and the area is $A = LW = (14 - W)W$. Since $A \geq 45$, we have $(14 - W)W \geq 45 \quad \Rightarrow \quad -W^2 + 14W \geq 45 \quad \Rightarrow$ $-W^2 + 14W - 45 \geq 0 \quad \Rightarrow \quad W^2 - 14W + 45 \leq 0 \quad \Rightarrow \quad (W - 5)(W - 9) \leq 0$.

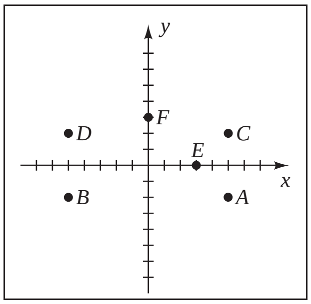
Interval	$(-\infty, 5)$	$(5, 9)$	$(9, \infty)$
Sign of $W - 5$	$-$	$+$	$+$
Sign of $W - 9$	$-$	$-$	$+$
Resulting sign	$+$	$-$	$+$

From the sign chart, we see that the inequality is satisfied for $5 \leq W \leq 9$. Of course, once the width passes 7, it becomes the length, but that's not the point of the problem.

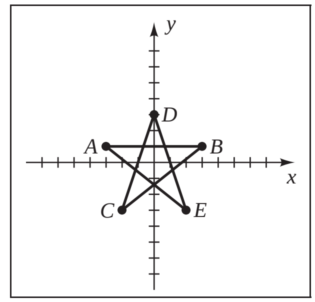
Chapter 3: Functions and Graphs

3.1 Exercises

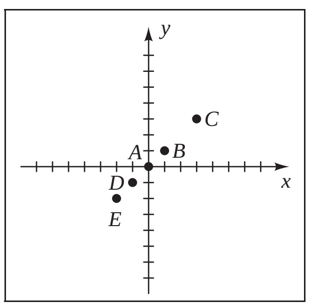
1 The points $A(5, -2)$, $B(-5, -2)$, $C(5, 2)$, $D(-5, 2)$, $E(3, 0)$, and $F(0, 3)$ are plotted in the figure.



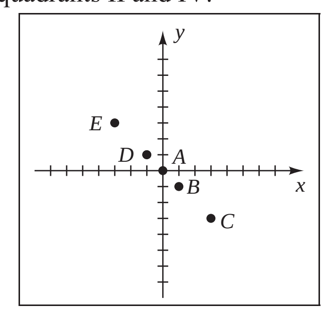
2 The points $A(-3, 1)$, $B(3, 1)$, $C(-2, -3)$, $D(0, 3)$, and $E(2, -3)$, as well as $\overline{AB}$, $\overline{BC}$, $\overline{CD}$, $\overline{DE}$, $\overline{EA}$, are plotted in the figure.



3 The points $A(0, 0)$, $B(1, 1)$, $C(3, 3)$, $D(-1, -1)$, and $E(-2, -2)$ are plotted in the figure. The set of all points of the form (a, a) is the line bisecting quadrants I and III.



4 The points $A(0, 0)$, $B(1, -1)$, $C(3, -3)$, $D(-1, 1)$, and $E(-3, 3)$ are plotted in the figure. The set of all points of the form $(a, -a)$ is the line bisecting quadrants II and IV.



5 The points are $A(3, 3)$, $B(-3, 3)$, $C(-3, -3)$, $D(3, -3)$, $E(1, 0)$, and $F(0, 3)$.

6 The points are $A(0, 4)$, $B(-4, 0)$, $C(0, -4)$, $D(4, 0)$, $E(2, 2)$, and $F(-2, -2)$.

7 (a) $x = -2$ is the line parallel to the y-axis that intersects the x-axis at $(-2, 0)$.

(b) $y = 5$ is the line parallel to the x-axis that intersects the y-axis at $(0, 5)$.

(c) $x \geq 0$ $\{x$ is zero or positive$\}$ is the set of all points to the right of and on the y-axis.

(d) $xy > 0$ $\{x$ and y have the same sign, that is, either both are positive or both are negative$\}$ is the set of all points in quadrants I and III.

(e) $y < 0$ $\{y$ is negative$\}$ is the set of all points below the x-axis.

(f) $x = 0$ is the set of all points on the y-axis.

8 (a) $y = -2$ is the line parallel to the x-axis that intersects the y-axis at $(0, -2)$.

(b) $x = 4$ is the line parallel to the y-axis that intersects the x-axis at $(4, 0)$.

(c) $x/y < 0$ is the set of all points in quadrants II and IV {x and y have opposite signs}.

(d) $xy = 0$ is the set of all points on the x-axis or y-axis.

(e) $y > 1$ is the set of all points above the line parallel to the x-axis which intersects the y-axis at $(0, 1)$.

(f) $y = 0$ is the set of all points on the x-axis.

9 (a) $A(4, -3), B(6, 2) \Rightarrow d(A, B) = \sqrt{(6-4)^2 + [2-(-3)]^2} = \sqrt{4+25} = \sqrt{29}$

(b) $A(4, -3), B(6, 2) \Rightarrow M_{AB} = \left(\frac{4+6}{2}, \frac{-3+2}{2}\right) = \left(5, -\frac{1}{2}\right)$

10 (a) $A(-2, -5), B(4, 6) \Rightarrow d(A, B) = \sqrt{[4-(-2)]^2 + [6-(-5)]^2} = \sqrt{36+121} = \sqrt{157}$

(b) $A(-2, -5), B(4, 6) \Rightarrow M_{AB} = \left(\frac{-2+4}{2}, \frac{-5+6}{2}\right) = \left(1, \frac{1}{2}\right)$

11 (a) $A(-7, 0), B(-2, -4) \Rightarrow d(A, B) = \sqrt{[-2-(-7)]^2 + (-4-0)^2} = \sqrt{25+16} = \sqrt{41}$

(b) $A(-7, 0), B(-2, -4) \Rightarrow M_{AB} = \left(\frac{-7+(-2)}{2}, \frac{0+(-4)}{2}\right) = \left(-\frac{9}{2}, -2\right)$

12 (a) $A(5, 2), B(5, -2) \Rightarrow d(A, B) = \sqrt{(5-5)^2 + (-2-2)^2} = \sqrt{0+16} = 4$

(b) $A(5, 2), B(5, -2) \Rightarrow M_{AB} = \left(\frac{5+5}{2}, \frac{2+(-2)}{2}\right) = (5, 0)$

13 (a) $A(7, -3), B(3, -3) \Rightarrow d(A, B) = \sqrt{(3-7)^2 + [-3-(-3)]^2} = \sqrt{16+0} = 4$

(b) $A(7, -3), B(3, -3) \Rightarrow M_{AB} = \left(\frac{7+3}{2}, \frac{-3+(-3)}{2}\right) = (5, -3)$

14 (a) $A(-4, 7), B(0, -8) \Rightarrow d(A, B) = \sqrt{[0-(-4)]^2 + (-8-7)^2} = \sqrt{16+225} = \sqrt{241}$

(b) $A(-4, 7), B(0, -8) \Rightarrow M_{AB} = \left(\frac{-4+0}{2}, \frac{7+(-8)}{2}\right) = \left(-2, -\frac{1}{2}\right)$

15 The points are $A(6, 3)$, $B(1, -2)$, and $C(-3, 2)$. We need to show that the sides satisfy the Pythagorean theorem. Finding the distances, we have $d(A, B) = \sqrt{50}$, $d(B, C) = \sqrt{32}$, and $d(A, C) = \sqrt{82}$. Since $d(A, C)$ is the largest of the three values, it must be the hypotenuse, hence, we need to check if $d(A, C)^2 = d(A, B)^2 + d(B, C)^2$. Since $\left(\sqrt{82}\right)^2 = \left(\sqrt{50}\right)^2 + \left(\sqrt{32}\right)^2$, we know that $\triangle ABC$ is a right triangle. The area of a triangle is given by $A = \frac{1}{2}$(base)(height). We can use $d(B, C)$ for the base and $d(A, B)$ for the height.
Hence, area $= \frac{1}{2}bh = \frac{1}{2}\left(\sqrt{32}\right)\left(\sqrt{50}\right) = \frac{1}{2}\left(4\sqrt{2}\right)\left(5\sqrt{2}\right) = \frac{1}{2}(20)(2) = 20$.

16 The points are $A(-6, 3)$, $B(3, -5)$, and $C(-1, 5)$.
Show that $d(A, B)^2 = d(A, C)^2 + d(B, C)^2$; that is, $\left(\sqrt{145}\right)^2 = \left(\sqrt{29}\right)^2 + \left(\sqrt{116}\right)^2$.
Area $= \frac{1}{2}bh = \frac{1}{2} \cdot d(A, C) \cdot d(B, C) = \frac{1}{2}\left(\sqrt{29}\right)\left(\sqrt{116}\right) = \frac{1}{2}\left(\sqrt{29}\right)\left(2\sqrt{29}\right) = 29$.

17 The points are $A(-4, 2)$, $B(1, 4)$, $C(3, -1)$, and $C(-2, -3)$. We need to show that all four sides are the same length. Checking, we find that $d(A, B) = d(B, C) = d(C, D) = d(D, A) = \sqrt{29}$. This guarantees that we have a rhombus {a parallelogram with 4 equal sides}. Thus, we also need to show that adjacent sides meet at right angles. This can be done by showing that two adjacent sides and a diagonal form a right triangle. Using $\triangle ABC$, we see that $d(A, C) = \sqrt{58}$ and hence $d(A, C)^2 = d(A, B)^2 + d(B, C)^2$. We conclude that $ABCD$ is a square.

18 The points are $A(-4,-1)$, $B(0,-2)$, $C(6,1)$, and $C(2,2)$.
Show that $d(A,D) = d(B,C) = \sqrt{45}$ and $d(A,B) = d(C,D) = \sqrt{17}$.

19 Let $B = (x,y)$. $A(-3,8) \Rightarrow M_{AB} = \left(\dfrac{-3+x}{2}, \dfrac{8+y}{2}\right)$.
Since $M_{AB} = C(5,-10)$, we must have $\dfrac{-3+x}{2} = 5$ and $\dfrac{8+y}{2} = -10 \Rightarrow$
$-3 + x = 2(5)$ and $8 + y = 2(-10) \Rightarrow x = 13$ and $y = -28$. Thus, $B = (13,-28)$.

20 If Q is the midpoint of segment AB, then the midpoint of QB is the point that is three-fourths of the way from $A(5,-8)$ to $B(-6,2)$.
$Q = M_{AB} = \left(\dfrac{5+(-6)}{2}, \dfrac{-8+2}{2}\right) = \left(-\dfrac{1}{2}, -3\right)$. $M_{QB} = \left(\dfrac{(-1/2)+(-6)}{2}, \dfrac{-3+2}{2}\right) = \left(-\dfrac{13}{4}, -\dfrac{1}{2}\right)$.

21 The perpendicular bisector of AB is the line that passes through the midpoint of segment AB and intersects segment AB at a right angle. The points on the perpendicular bisector are all equidistant from A and B. Thus, we need to show that $d(A,C) = d(B,C)$, where $A = (-4,-3)$, $B = (6,1)$, and $C = (3,-6)$. Since each of these distances is $\sqrt{58}$, we conclude that C is on the perpendicular bisector of AB.

22 Show that $d(A,C) = d(B,C) = \sqrt{125}$, where $A = (-3,2)$, $B = (5,-4)$, and $C = (7,7)$.

23 The points are $A(-4,-3)$, $B(6,1)$, and $P(x,y)$. We must have $d(A,P) = d(B,P)$.
$\sqrt{(x+4)^2 + (y+3)^2} = \sqrt{(x-6)^2 + (y-1)^2} \Rightarrow$
$x^2 + 8x + 16 + y^2 + 6y + 9 = x^2 - 12x + 36 + y^2 - 2y + 1$ {square both sides} $\Rightarrow$
$8x + 6y + 25 = -12x - 2y + 37 \Rightarrow 20x + 8y = 12 \Rightarrow 5x + 2y = 3$

24 The points are $A(-3,2)$, $B(5,-4)$, and $P(x,y)$. We must have $d(A,P) = d(B,P)$.
$\sqrt{(x+3)^2 + (y-2)^2} = \sqrt{(x-5)^2 + (y+4)^2} \Rightarrow$
$x^2 + 6x + 9 + y^2 - 4y + 4 = x^2 - 10x + 25 + y^2 + 8y + 16$ {square both sides} $\Rightarrow$
$6x - 4y + 13 = -10x + 8y + 41 \Rightarrow 16x - 12y = 28 \Rightarrow 4x - 3y = 7$

25 Let $O(0,0)$ represent the origin. Applying the distance formula with O and $P(x,y)$, we have
$d(O,P) = 5 \Rightarrow \sqrt{(x-0)^2 + (y-0)^2} = 5 \Rightarrow \sqrt{x^2 + y^2} = 5$.
This formula represents a circle of radius 5 with center at the origin.

26 $d(C,P) = r \Rightarrow \sqrt{(x-h)^2 + (y-k)^2} = r$. This formula represents a circle of radius r and center (h,k).

27 Let $Q(0,y)$ be an arbitrary point on the y-axis. Applying the distance formula with Q and $P(5,3)$, we have
$6 = d(P,Q) \Rightarrow 6 = \sqrt{(0-5)^2 + (y-3)^2} \Rightarrow 36 = 25 + y^2 - 6y + 9 \Rightarrow$
$y^2 - 6y - 2 = 0 \Rightarrow y = 3 \pm \sqrt{11}$. The points are $\left(0, 3 + \sqrt{11}\right)$ and $\left(0, 3 - \sqrt{11}\right)$.

28 Let $Q(x,0)$ be an arbitrary point on the x-axis. Applying the distance formula with Q and $P(-2,4)$, we have
$5 = d(P,Q) \Rightarrow 5 \Rightarrow \sqrt{(x+2)^2 + (0-4)^2} \Rightarrow 25 = x^2 + 4x + 4 + 16 \Rightarrow$
$x^2 + 4x - 5 = 0 \Rightarrow (x+5)(x-1) = 0 \Rightarrow x = -5, 1$. The points are $(1,0)$ and $(-5,0)$.

29 $d = 5$ with points $(2a, a)$ and $(1,3) \Rightarrow 5 = \sqrt{(2a-1)^2 + (a-3)^2} \Rightarrow$
$25 = 4a^2 - 4a + 1 + a^2 - 6a + 9 \Rightarrow 5a^2 - 10a - 15 = 0 \Rightarrow a^2 - 2a - 3 = 0 \Rightarrow$

$(a-3)(a+1)=0 \quad \Rightarrow \quad a=3,-1.$

Since the y-coordinate is negative in the third quadrant, $a=-1$, and $(2a,a)=(-2,-1)$.

30 $d=3$ with points (a,a) and $(-2,1)$ $\Rightarrow$ $3=\sqrt{(a+2)^2+(a-1)^2}$ $\Rightarrow$
$9=a^2+4a+4+a^2-2a+1$ $\Rightarrow$ $0=2a^2+2a-4$ $\Rightarrow$ $a^2+a-2=0$ $\Rightarrow$ $(a+2)(a-1)=0$ $\Rightarrow$
$a=-2,1$. The points are $(-2,-2)$ and $(1,1)$.

31 With $P(a,3)$ and $Q(5,2a)$, we get $d(P,Q)>\sqrt{26}$ $\Rightarrow$ $\sqrt{(5-a)^2+(2a-3)^2}>\sqrt{26}$ $\Rightarrow$
$25-10a+a^2+4a^2-12a+9>26$ $\Rightarrow$ $5a^2-22a+8>0$ $\Rightarrow$ $(5a-2)(a-4)>0$.

Interval	$\left(-\infty,\frac{2}{5}\right)$	$\left(\frac{2}{5},4\right)$	$(4,\infty)$
Sign of $5a-2$	$-$	$+$	$+$
Sign of $a-4$	$-$	$-$	$+$
Resulting sign	$+$	$-$	$+$

From the sign chart, we see that $a<\frac{2}{5}$ or $a>4$ will assure us that $d(P,Q)>\sqrt{26}$.

32 $A(-2,0)$, $B(2,0)$, $P(x,y)$ • $d(A,P)+d(B,P)=5$ $\Rightarrow$ $\sqrt{(x+2)^2+y^2}+\sqrt{(x-2)^2+y^2}=5$ $\Rightarrow$
$\sqrt{x^2+4x+4+y^2}=5-\sqrt{x^2-4x+4+y^2}$ $\Rightarrow$
$x^2+4x+4+y^2=25-10\sqrt{x^2-4x+4+y^2}+x^2-4x+4+y^2$ $\Rightarrow$
$10\sqrt{x^2-4x+4+y^2}=25-8x$ $\Rightarrow$ $100(x^2-4x+4+y^2)=625-400x+64x^2$ $\Rightarrow$
$100x^2-400x+400+100y^2=625-400x+64x^2$ $\Rightarrow$ $36x^2+100y^2=225$

33 Let M be the midpoint of the hypotenuse. Then $M=\left(\frac{1}{2}a,\frac{1}{2}b\right)$. Since $A=(a,0)$, we have

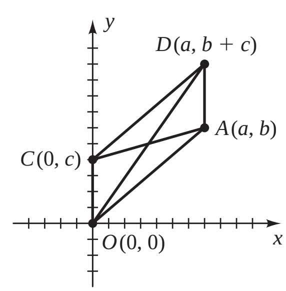
$$d(A,M)=\sqrt{\left(a-\tfrac{1}{2}a\right)^2+\left(0-\tfrac{1}{2}b\right)^2}=\sqrt{\left(\tfrac{1}{2}a\right)^2+\left(-\tfrac{1}{2}b\right)^2}=\sqrt{\tfrac{1}{4}a^2+\tfrac{1}{4}b^2}=\sqrt{\tfrac{1}{4}(a^2+b^2)}=\tfrac{1}{2}\sqrt{a^2+b^2}$$

In a similar fashion, show that $d(B,M)=d(O,M)=\frac{1}{2}\sqrt{a^2+b^2}$.

34 Let $D(a,b+c)$ be the fourth vertex as shown in the figure. We need to show that the midpoint of OD is the same as the midpoint of AC.

$M_{OD}=\left(\dfrac{0+a}{2},\dfrac{0+b+c}{2}\right)=\left(\dfrac{a}{2},\dfrac{b+c}{2}\right)$ and

$M_{CA}=\left(\dfrac{0+a}{2},\dfrac{c+b}{2}\right)=\left(\dfrac{a}{2},\dfrac{b+c}{2}\right)$.



35 Plot the points $A(-5,-3.5)$, $B(-2,2)$, $C(1,0.5)$, $D(4,1)$, and $E(7,2.5)$.

$[-10,10]$ by $[-10,10]$

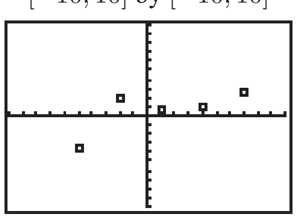


Figure 35

$[-12,12]$ by $[-8,8]$

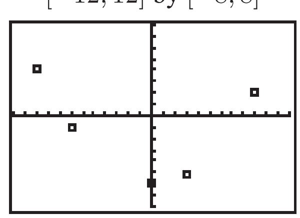


Figure 36

36 Plot the points $A(-10,4)$, $B(-7,-1.1)$, $C(0,-6)$, $D(3,-5.1)$, and $E(9,2.1)$.

37 (a) Plot (1984, 87,073), (1993, 98,736), (2003, 113,126), and (2009, 119,296).

(b) The number of U.S. households with a computer is increasing each year.

$[1982, 2012]$ by $[80\text{E}3, 120\text{E}3, 10\text{E}3]\{80\text{E}3 = 80 \times 10^3\}$

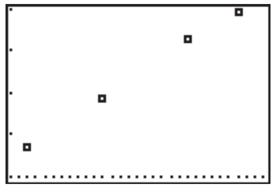


Figure 37

$[1895, 2005, 10]$ by $[0, 3000, 1000]$

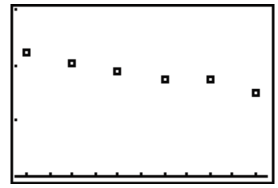


Figure 38

38 (a) Plot $(1900, 2226)$, $(1920, 2042)$, $(1940, 1878)$, $(1960, 1763)$, $(1980, 1745)$, and $(2000, 1480)$.

(b) Find the midpoint of $(1920, 2042)$ and $(1940, 1878)$. $\left(\dfrac{1920+1940}{2}, \dfrac{2042+1878}{2}\right) = (1930, 1960)$.

The midpoint formula predicts 1960 daily newspapers published in the year 1930 compared to the actual value of 1942 daily newspapers.

3.2 Exercises

1 As in Example 1, we expect the graph of $y = 2x - 3$ to be a line.

Creating a table of values similar to those in the text, we have:

x	-2	-1	0	1	2
y	-7	-5	-3	-1	1

By plotting these points and connecting them, we obtain the figure.

To find the x-intercept, let $y = 0$ in $y = 2x - 3$, and solve for x to get 1.5.

To find the y-intercept, let $x = 0$ in $y = 2x - 3$, and solve for y to get -3.

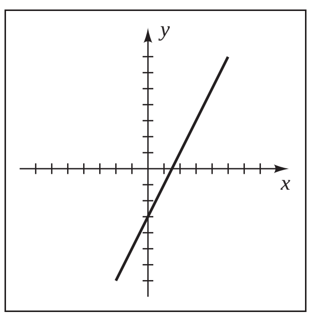


Figure 1

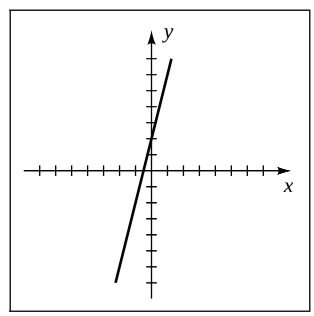


Figure 2

2 $y = 4x + 2$ • $y = 0$ gives the x-intercept $-\frac{1}{2}$; $x = 0$ gives the y-intercept 2

3 $y = -x + 2$ • x-intercept: $y = 0 \Rightarrow 0 = -x + 2 \Rightarrow x = 2$

y-intercept: $x = 0 \Rightarrow y = -0 + 2 = 2$

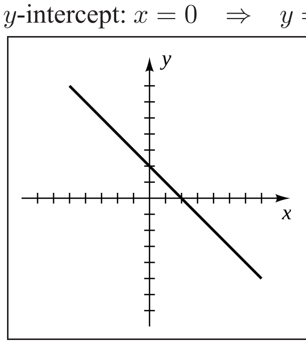


Figure 3

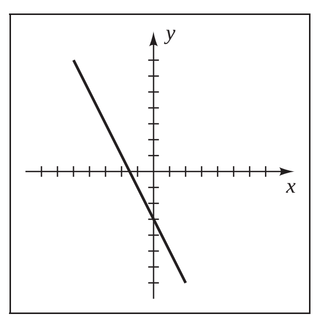


Figure 4

4 $y = -2x - 3$ • x-intercept: $y = 0 \Rightarrow x = -1.5$; y-intercept: $x = 0 \Rightarrow y = -3$

5 $y = -2x^2$ • Multiplying the y-values of $y = x^2$ by -2 gives us all negative y-values for $y = -2x^2$. The vertex of the parabola is at $(0, 0)$, so its x-intercept is 0 and its y-intercept is 0.

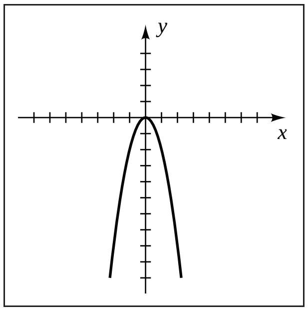


Figure 5

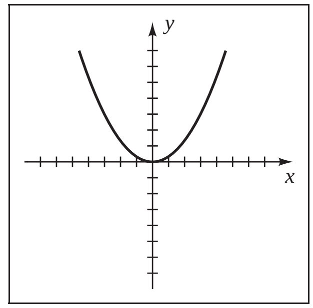


Figure 6

6 $y = \frac{1}{3}x^2$ • x-intercept 0; y-intercept 0

7 $y = 2x^2 - 1$ • x-intercepts: $y = 0 \Rightarrow 0 = 2x^2 - 1 \Rightarrow 1 = 2x^2 \Rightarrow x^2 = \frac{1}{2} \Rightarrow x = \pm\sqrt{\frac{1}{2}}$, which can be written as $\pm\frac{1}{\sqrt{2}}$ or $\pm\frac{\sqrt{2}}{2}$ or $\pm\frac{1}{2}\sqrt{2}$; y-intercept: $x = 0 \Rightarrow y = -1$

Since we can substitute $-x$ for x in the equation and obtain an equivalent equation, we know the graph is symmetric with respect to the y-axis. We will make use of this fact when constructing our table. As in Example 2, we obtain a parabola.

x	± 2	$\pm\frac{3}{2}$	± 1	$\pm\frac{1}{2}$	0
y	7	$\frac{7}{2}$	1	$-\frac{1}{2}$	-1

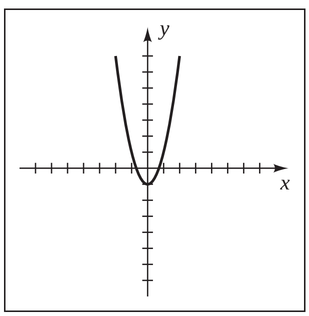


Figure 7

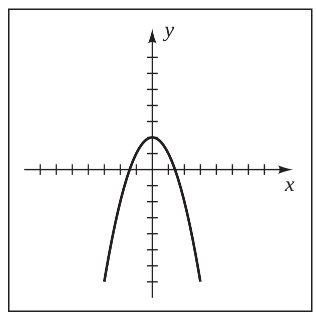


Figure 8

8 $y = -x^2 + 2$ • x-intercepts: $y = 0 \Rightarrow x^2 = 2 \Rightarrow x = \pm\sqrt{2}$; y-intercept 2

9 $x = \frac{1}{4}y^2$ • This graph is similar to the one in Example 5—multiplying by $\frac{1}{4}$ narrows the parabola $x = y^2$. The x-intercept is 0 and the y-intercept is 0.

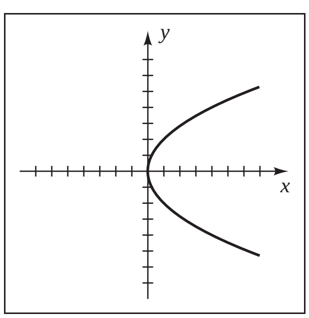


Figure 9

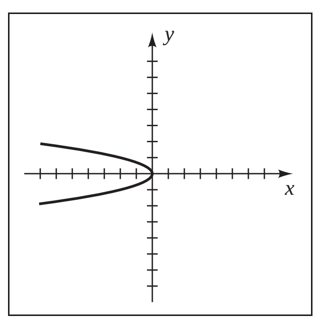


Figure 10

10 $x = -2y^2$ • x-intercept 0; y-intercept 0

11 $x = -y^2 + 5$ • x-intercept: $y = 0 \Rightarrow x = 5$;
y-intercepts: $x = 0 \Rightarrow 0 = -y^2 + 5 \Rightarrow y^2 = 5 \Rightarrow y = \pm\sqrt{5}$
Since we can substitute $-y$ for y in the equation and obtain an equivalent equation, we know the graph is symmetric with respect to the x-axis. We will make use of this fact when constructing our table. As in Example 5, we obtain a parabola.

x	-11	-4	1	4	5
y	± 4	± 3	± 2	± 1	0

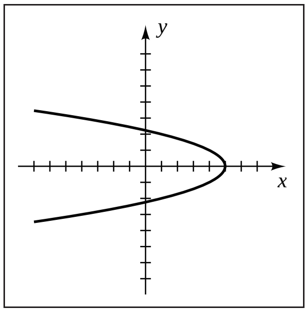


Figure 11

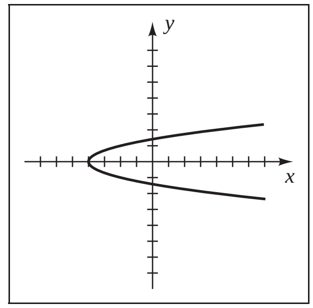


Figure 12

12 $x = 2y^2 - 4$ • x-intercept -4; y-intercepts: $x = 0 \Rightarrow y^2 = 4 \Rightarrow y = \pm\sqrt{2}$

13 $y = -\frac{1}{4}x^3$ • The graph is similar to the graph of $y = \frac{1}{4}x^3$ in Example 6. The negative has the effect of "flipping" the graph about the x-axis. The x-intercept is 0 and the y-intercept is 0.

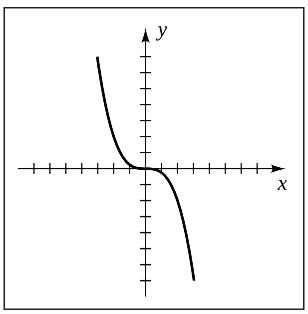


Figure 13

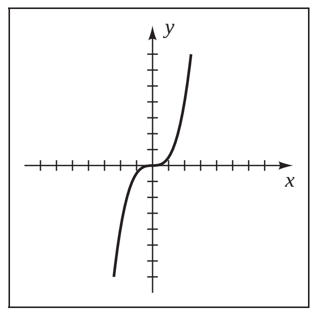


Figure 14

14 $y = \frac{1}{2}x^3$ • x-intercept 0; y-intercept 0

15 $y = x^3 - 8$ • x-intercept: $y = 0 \Rightarrow 0 = x^3 - 8 \Rightarrow x^3 = 8 \Rightarrow x = \sqrt[3]{8} = 2$
y-intercept: $x = 0 \Rightarrow y = -8$
The effect of the -8 is to shift the graph of $y = x^3$ down 8 units.

x	-2	-1	0	1	2
y	-16	-9	-8	-7	0

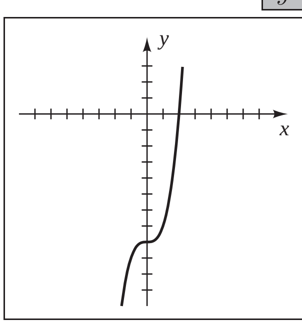


Figure 15

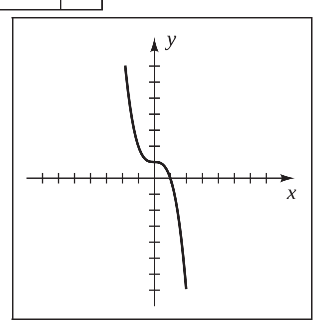


Figure 16

16 $y = -x^3 + 1$ • x-intercept: $y = 0 \Rightarrow x^3 = 1 \Rightarrow x = \sqrt[3]{1} = 1$; y-intercept 1

17 $y = \sqrt{x}$ • x-intercept 0; y-intercept 0. This is the top half of the parabola $x = y^2$. An equation of the bottom half is $y = -\sqrt{x}$.

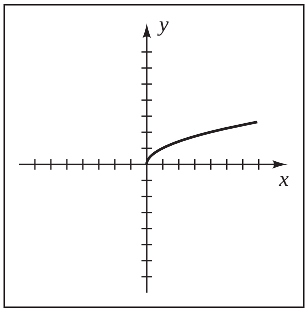


Figure 17

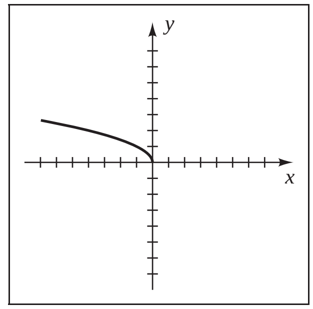


Figure 18

18 $y = \sqrt{-x}$ • The negative has the effect of "flipping" the graph of $y = \sqrt{x}$ about the y-axis. The x-intercept is 0 and the y-intercept is 0.

19 $y = \sqrt{x} - 4$ • x-intercept: $y = 0 \Rightarrow 0 = \sqrt{x} - 4 \Rightarrow 4 = \sqrt{x} \Rightarrow x = 16$ (not on graph)
y-intercept: $x = 0 \Rightarrow y = -4$

x	0	1	4	9	16
y	-4	-3	-2	-1	0

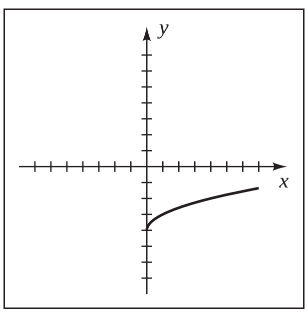


Figure 19

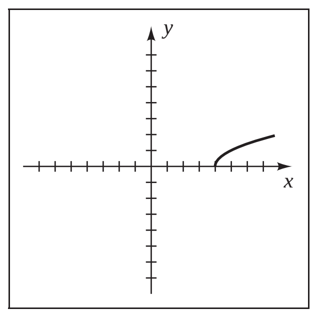


Figure 20

20 $y = \sqrt{x - 4}$ • x-intercept 4; y-intercept: None

21 You may be able to do this exercise mentally. For example (using Exercise 1 with $y = 2x - 3$), we see that substituting $-x$ for x gives us $y = -2x - 3$; substituting $-y$ for y gives us $-y = 2x - 3$ or, equivalently, $y = -2x + 3$; and substituting $-x$ for x and $-y$ for y gives us $-y = -2x - 3$ or, equivalently, $y = 2x + 3$. None of the resulting equations are equivalent to the original equation, so there is no symmetry with respect to the y-axis, x-axis, or the origin.

(a) The graphs of the equations in Exercises 5 and 7 are symmetric with respect to the y-axis.

(b) The graphs of the equations in Exercises 9 and 11 are symmetric with respect to the x-axis.

(c) The graph of the equation in Exercise 13 is symmetric with respect to the origin.

22 (a) $6, 8$ (b) $10, 12$ (c) 14

23 (a) As $x \to -1^-$, $f(x) \to \underline{2}$ (b) As $x \to 2^+$, $f(x) \to \underline{1}$ (c) As $x \to 3$, $f(x) \to \underline{4}$

(d) As $x \to \infty$, $f(x) \to \underline{\infty}$ (e) As $x \to -\infty$, $f(x) \to \underline{-\infty}$

24 (a) As $x \to 2^-$, $f(x) \to \underline{3}$ (b) As $x \to -1^+$, $f(x) \to \underline{-3}$ (c) As $x \to 0$, $f(x) \to \underline{-1}$

(d) As $x \to \infty$, $f(x) \to \underline{-\infty}$ (e) As $x \to -\infty$, $f(x) \to \underline{\infty}$

25 $x^2 + y^2 = 11 \quad \Leftrightarrow \quad (x-0)^2 + (y-0)^2 = \left(\sqrt{11}\right)^2$ is a circle of radius $\sqrt{11}$ with center at the origin.

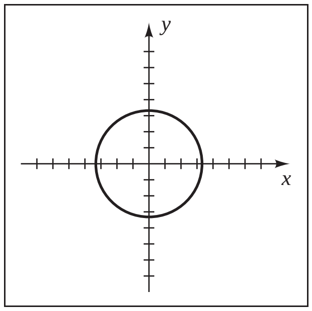


Figure 25

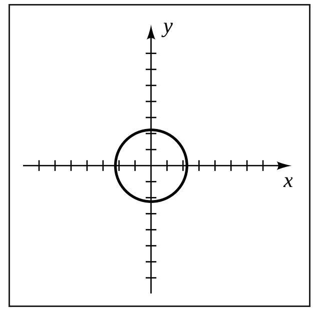


Figure 26

26 $x^2 + y^2 = 5 \quad \Leftrightarrow \quad (x-0)^2 + (y-0)^2 = \left(\sqrt{5}\right)^2$ is a circle of radius $\sqrt{5}$ with center at the origin.

27 $(x+3)^2 + (y-2)^2 = 9$ is a circle of radius $r = \sqrt{9} = 3$ with center $C(-3, 2)$. To determine the center from the given equation, it may help to ask yourself "What values makes the expressions $(x+3)$ and $(y-2)$ equal to zero?" The answers are -3 and 2.

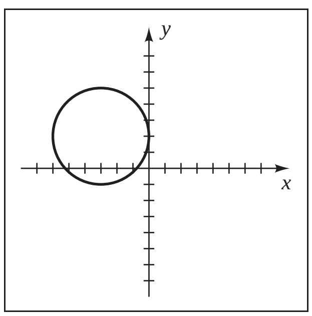


Figure 27

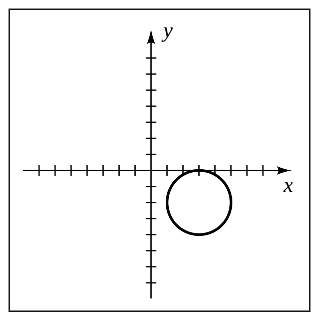


Figure 28

28 $(x-3)^2 + (y+2)^2 = 4$ is a circle of radius $r = \sqrt{4} = 2$ with center $C(3, -2)$.

29 $(x+3)^2 + y^2 = 16$ is a circle of radius $r = \sqrt{16} = 4$ with center $C(-3, 0)$.

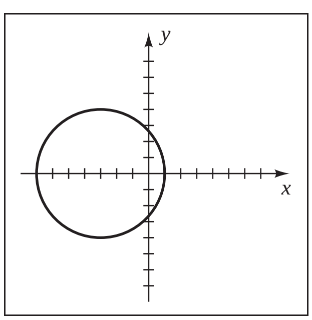


Figure 29

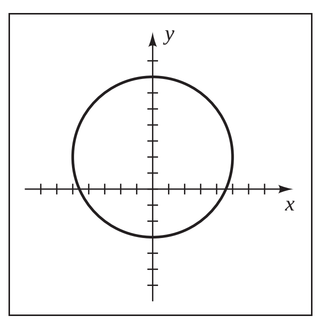


Figure 30

30 $x^2 + (y-2)^2 = 25$ is a circle of radius $r = \sqrt{25} = 5$ with center $C(0, 2)$.

31 $4x^2 + 4y^2 = 1 \quad \Rightarrow \quad x^2 + y^2 = \frac{1}{4}$ is a circle of radius $r = \sqrt{\frac{1}{4}} = \frac{1}{2}$ with center $C(0, 0)$.

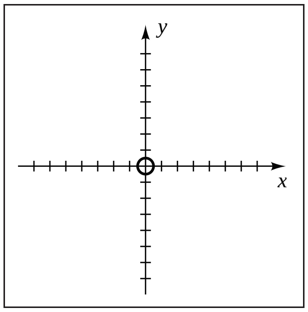


Figure 31

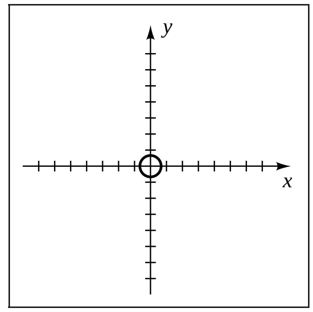


Figure 32

32 $9x^2 + 9y^2 = 4 \quad \Rightarrow \quad x^2 + y^2 = \frac{4}{9}$ is a circle of radius $r = \sqrt{\frac{4}{9}} = \frac{2}{3}$ with center $C(0, 0)$.

33 As in Example 9, $y = -\sqrt{16 - x^2}$ is the lower half of the circle $x^2 + y^2 = 16$.

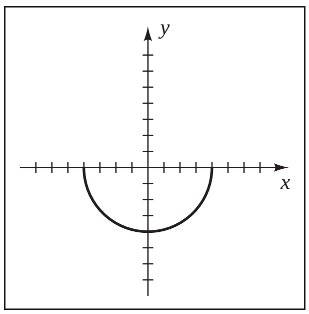


Figure 33

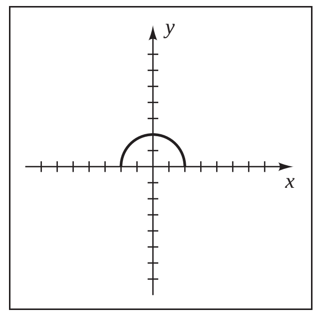


Figure 34

34 $y = \sqrt{4 - x^2}$ is the upper half of the circle $x^2 + y^2 = 4$.

35 $x = \sqrt{9 - y^2}$ is the right half of the circle $x^2 + y^2 = 9$.

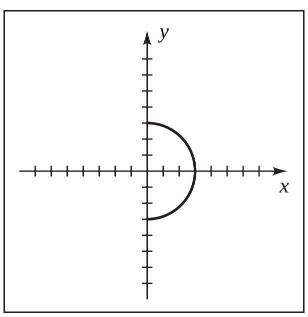


Figure 35

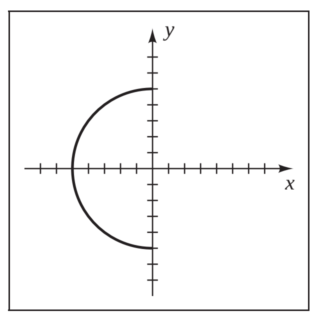


Figure 36

36 $x = -\sqrt{25 - y^2}$ is the left half of the circle $x^2 + y^2 = 25$.

37 Center $C(2, -3)$, radius 5 • $(x - 2)^2 + [y - (-3)]^2 = 5^2 \quad \Leftrightarrow \quad (x - 2)^2 + (y + 3)^2 = 25$

38 Center $C(-5, 1)$, radius 3 • $[x - (-5)]^2 + (y - 1)^2 = 3^2 \quad \Leftrightarrow \quad (x + 5)^2 + (y - 1)^2 = 9$

39 Center $C\left(\frac{1}{4}, 0\right)$, radius $\sqrt{5}$ • $\left(x - \frac{1}{4}\right)^2 + (y - 0)^2 = \left(\sqrt{5}\right)^2 \quad \Leftrightarrow \quad \left(x - \frac{1}{4}\right)^2 + y^2 = 5$

40 Center $C\left(\frac{3}{4}, -\frac{2}{3}\right)$, radius $3\sqrt{2}$ • $\left(x - \frac{3}{4}\right)^2 + \left[y - \left(-\frac{2}{3}\right)\right]^2 = \left(3\sqrt{2}\right)^2 \quad \Leftrightarrow \quad \left(x - \frac{3}{4}\right)^2 + \left(y + \frac{2}{3}\right)^2 = 18$

41 An equation of a circle with center $C(-4, 6)$ is $(x + 4)^2 + (y - 6)^2 = r^2$.

Since the circle passes through $P(3, 1)$, we know that $x = 3$ and $y = 1$ is one solution of the general equation.

Letting $x = 3$ and $y = 1$ yields $7^2 + (-5)^2 = r^2 \quad \Rightarrow \quad r^2 = 74$. An equation is $(x + 4)^2 + (y - 6)^2 = 74$.

42 An equation of a circle with center at the origin is $x^2 + y^2 = r^2$.

Letting $x = 4$ and $y = -7$ yields $4^2 + (-7)^2 = r^2 \Rightarrow r^2 = 65$. $x^2 + y^2 = 65$

43 "Tangent to the y-axis" means that the circle will intersect the y-axis at exactly one point. The distance from the center $C(-3, 6)$ to this point of tangency is 3 units—this is the length of the radius of the circle.

An equation is $(x+3)^2 + (y-6)^2 = 9$.

44 The circle is tangent to the x-axis and has center $C(4, -3)$.

Its radius, 3, is the distance from the x-axis to the y-value of the center. An equation is $(x-4)^2 + (y+3)^2 = 9$.

45 Since the radius is 2 and $C(h, k)$ is in QII, $h = -2$ and $k = 2$. An equation is $(x+2)^2 + (y-2)^2 = 4$.

46 Since the radius is 3 and $C(h, k)$ is in QIV, $h = 3$ and $k = -3$. An equation is $(x-3)^2 + (y+3)^2 = 9$.

47 The center of the circle is the midpoint M of $A(4, -3)$ and $B(-2, 7)$. $M_{AB} = (1, 2)$. The radius of the circle is $\frac{1}{2} \cdot d(A, B) = \frac{1}{2}\sqrt{[4-(-2)]^2 + (-3-7)^2} = \frac{1}{2}\sqrt{36+100} = \frac{1}{2}\sqrt{136} = \frac{1}{2}\sqrt{4 \cdot 34} = \sqrt{34}$.

An equation is $(x-1)^2 + (y-2)^2 = 34$.

Alternatively, once we know the center, $(1, 2)$, we also know that the equation has the form $(x-1)^2 + (y-2)^2 = r^2$. Now substitute 4 for x and -3 for y to obtain

$3^2 + (-5)^2 = r^2 \Rightarrow 9 + 25 = r^2$, or $r^2 = 34$.

48 As in the solution to Exercise 47, $M_{AB} = (-1, 4)$ and $r = \frac{1}{2} \cdot d(A, B) = \frac{1}{2}\sqrt{80} = \sqrt{20}$.

An equation is $(x+1)^2 + (y-4)^2 = 20$.

49 $x^2 + y^2 - 4x + 6y - 36 = 0$ {complete the square on x and y} $\Rightarrow$

$x^2 - 4x + \underline{4} + y^2 + 6y + \underline{9} = 36 + \underline{4} + \underline{9} \Rightarrow (x-2)^2 + (y+3)^2 = 49$.

This is a circle with center $C(2, -3)$ and radius $r = 7$.

50 $x^2 + y^2 + 8x - 10y + 37 = 0 \Rightarrow x^2 + 8x + \underline{16} + y^2 - 10y + \underline{25} = -37 + \underline{16} + \underline{25} \Rightarrow$

$(x+4)^2 + (y-5)^2 = 4$. $C(-4, 5)$; $r = 2$

51 $x^2 + y^2 + 4y - 7 = 0$ {complete the square on x and y} $\Rightarrow x^2 + y^2 + 4y + \underline{4} = 7 + \underline{4} \Rightarrow$

$x^2 + (y+2)^2 = 11$. This is a circle with center $C(0, -2)$ and radius $r = \sqrt{11}$.

52 $x^2 + y^2 - 10x + 18 = 0 \Rightarrow x^2 - 10x + \underline{25} + y^2 = -18 + \underline{25} \Rightarrow (x-5)^2 + y^2 = 7$. $C(5, 0)$; $r = \sqrt{7}$

53 $2x^2 + 2y^2 - 12x + 4y - 15 = 0$ {add 15 to both sides and divide by 2} $\Rightarrow$

$x^2 + y^2 - 6x + 2y = \frac{15}{2}$ {complete the square on x and y} $\Rightarrow$

$x^2 - 6x + \underline{9} + y^2 + 2y + \underline{1} = \frac{15}{2} + \underline{9} + \underline{1} \Rightarrow (x-3)^2 + (y+1)^2 = \frac{35}{2}$.

This is a circle with center $C(3, -1)$ and radius $r = \frac{1}{2}\sqrt{70}$.

54 $4x^2 + 4y^2 + 16x + 24y + 31 = 0 \Rightarrow x^2 + y^2 + 4x + 6y = -\frac{31}{4} \Rightarrow$

$x^2 + 4x + \underline{4} + y^2 + 6y + \underline{9} = -\frac{31}{4} + \underline{4} + \underline{9} \Rightarrow (x+2)^2 + (y+3)^2 = \frac{21}{4}$. $C(-2, -3)$; $r = \frac{1}{2}\sqrt{21}$

55 $x^2 + y^2 + 4x - 2y + 5 = 0 \Rightarrow x^2 + 4x + \underline{4} + y^2 - 2y + \underline{1} = -5 + \underline{4} + \underline{1} \Rightarrow$

$(x+2)^2 + (y-1)^2 = 0$. $C(-2, 1)$; $r = 0$ (a point)

56 $x^2 + y^2 - 6x - 4y + 13 = 0 \Rightarrow x^2 - 6x + \underline{9} + y^2 - 4y + \underline{4} = -13 + \underline{9} + \underline{4} \Rightarrow$

$(x-3)^2 + (y-2)^2 = 0$. $C(3, 2)$; $r = 0$ (a point)

57 $x^2+y^2-2x-8y+21=0 \quad\Rightarrow\quad x^2-2x+\underline{1}+y^2-8y+\underline{16}=-21+\underline{1}+\underline{16} \quad\Rightarrow$
$(x-1)^2+(y-4)^2=-4$. This is not a circle since r^2 cannot equal -4.

58 $x^2+y^2+4x+6y+16=0 \quad\Rightarrow\quad x^2+4x+\underline{4}+y^2+6y+\underline{9}=-16+\underline{4}+\underline{9} \quad\Rightarrow$
$(x+2)^2+(y+3)^2=-3$. This is not a circle since r^2 cannot equal -3.

59 To obtain equations for the upper and lower halves, we solve the given equation for y in terms of x.
$x^2+y^2=25 \quad\Rightarrow\quad y^2=25-x^2 \quad\Rightarrow\quad y=\pm\sqrt{25-x^2}$.
The upper half is $y=\sqrt{25-x^2}$ and the lower half is $y=-\sqrt{25-x^2}$.
To obtain equations for the right and left halves, we solve the given equation for x in terms of y.
$x^2+y^2=25 \quad\Rightarrow\quad x^2=25-y^2 \quad\Rightarrow\quad x=\pm\sqrt{25-y^2}$.
The right half is $x=\sqrt{25-y^2}$ and the left half is $x=-\sqrt{25-y^2}$.

60 $(x+3)^2+y^2=64 \quad\Rightarrow\quad y^2=64-(x+3)^2 \quad\Rightarrow\quad y=\pm\sqrt{64-(x+3)^2}$.
$(x+3)^2+y^2=64 \quad\Rightarrow\quad (x+3)^2=64-y^2 \quad\Rightarrow\quad x+3=\pm\sqrt{64-y^2} \quad\Rightarrow\quad x=-3\pm\sqrt{64-y^2}$.

61 To obtain equations for the upper and lower halves, we solve the given equation for y in terms of x.
$(x-2)^2+(y+1)^2=49 \quad\Rightarrow\quad (y+1)^2=49-(x-2)^2 \quad\Rightarrow\quad y+1=\pm\sqrt{49-(x-2)^2} \quad\Rightarrow$
$y=-1\pm\sqrt{49-(x-2)^2}$.
The upper half is $y=-1+\sqrt{49-(x-2)^2}$ and the lower half is $y=-1-\sqrt{49-(x-2)^2}$.
To obtain equations for the right and left halves, we solve the given equation for x in terms of y.
$(x-2)^2+(y+1)^2=49 \quad\Rightarrow\quad (x-2)^2=49-(y+1)^2 \quad\Rightarrow\quad x-2=\pm\sqrt{49-(y+1)^2} \quad\Rightarrow$
$x=2\pm\sqrt{49-(y+1)^2}$. The right half is $x=2+\sqrt{49-(y+1)^2}$ and the left half is $x=2-\sqrt{49-(y+1)^2}$.

62 $(x-3)^2+(y-5)^2=4 \quad\Rightarrow\quad (y-5)^2=4-(x-3)^2 \quad\Rightarrow\quad y-5=\pm\sqrt{4-(x-3)^2} \quad\Rightarrow$
$y=5\pm\sqrt{4-(x-3)^2}$.
$(x-3)^2+(y-5)^2=4 \quad\Rightarrow\quad (x-3)^2=4-(y-5)^2 \quad\Rightarrow\quad x-3=\pm\sqrt{4-(y-5)^2} \quad\Rightarrow$
$x=3\pm\sqrt{4-(y-5)^2}$.

63 From the figure, we see that the diameter {look at the x-values of the rightmost and leftmost points} of the circle is $1-(-7)=8$ units, so the radius is $\frac{1}{2}(8)=4$. The center (h,k) is at the average of the extreme values; that is, $h=\dfrac{-7+1}{2}=-3$, and similarly, $k=\dfrac{-2+6}{2}=2$. Using the standard form of an equation of a circle, $(x-h)^2+(y-k)^2=r^2$, we have $[x-(-3)]^2+(y-2)^2=4^2$, or $(x+3)^2+(y-2)^2=4^2$ {or use 16}.

64 diameter $=4-(-2)=6$; radius $=\frac{1}{2}(6)=3$; $h=\dfrac{-2+4}{2}=1$ and $k=\dfrac{-5+1}{2}=-2$
An equation is $(x-1)^2+[y-(-2)]^2=3^2$, or $(x-1)^2+(y+2)^2=3^2$ {or use 9}.

65 The figure shows the lower semicircle of a circle centered at the origin having radius 4. The circle has equation $x^2+y^2=4^2$. We want the lower semicircle, so we must have *negative* y-values, indicating that we should solve for y and use the negative sign. $x^2+y^2=4^2 \quad\Rightarrow\quad y^2=4^2-x^2 \quad\Rightarrow\quad y=\pm\sqrt{4^2-x^2}$, so $y=-\sqrt{4^2-x^2}$ is the desired equation.

66 $x^2+y^2=3^2 \quad\Rightarrow\quad x^2=3^2-y^2 \quad\Rightarrow\quad x=\pm\sqrt{3^2-y^2}$, so $x=-\sqrt{3^2-y^2}$ is the desired equation since we want *negative* x-values.

67 We need to determine if the distance from P to C is *less than* r, *greater than* r, or *equal to* r and hence, P will be *inside* the circle, *outside* the circle, or *on* the circle, respectively.

(a) $P(2,3), C(4,6) \quad\Rightarrow\quad d(P,C) = \sqrt{4+9} = \sqrt{13} < r \;\{r=4\} \quad\Rightarrow\quad P$ is *inside* C.

(b) $P(4,2), C(1,-2) \quad\Rightarrow\quad d(P,C) = \sqrt{9+16} = 5 = r \;\{r=5\} \quad\Rightarrow\quad P$ is *on* C.

(c) $P(-3,5), C(2,1) \quad\Rightarrow\quad d(P,C) = \sqrt{25+16} = \sqrt{41} > r \;\{r=6\} \quad\Rightarrow\quad P$ is *outside* C.

68 (a) $P(3,8), C(-2,-4) \quad\Rightarrow\quad d(P,C) = \sqrt{25+144} = 13 = r \;\{r=13\} \quad\Rightarrow\quad P$ is *on* C.

(b) $P(-2,5), C(3,7) \quad\Rightarrow\quad d(P,C) = \sqrt{25+4} = \sqrt{29} < r \;\{r=6\} \quad\Rightarrow\quad P$ is *inside* C.

(c) $P(1,-2), C(6,-7) \quad\Rightarrow\quad d(P,C) = \sqrt{25+25} = \sqrt{50} > r \;\{r=7\} \quad\Rightarrow\quad P$ is *outside* C.

69 (a) To find the x-intercepts of $x^2+y^2-4x-6y+4=0$, let $y=0$ and solve the resulting equation for x.

$$x^2-4x+4=0 \quad\Rightarrow\quad (x-2)^2=0 \quad\Rightarrow\quad x-2=0 \quad\Rightarrow\quad x=2.$$

(b) To find the y-intercepts of $x^2+y^2-4x-6y+4=0$, let $x=0$ and solve the resulting equation for y.

$$y^2-6y+4=0 \quad\Rightarrow\quad y=\frac{6\pm\sqrt{36-16}}{2}=\frac{6\pm2\sqrt{5}}{2}=3\pm\sqrt{5}.$$

70 (a) $x^2+y^2-10x+4y+13=0 \;\bullet\; y=0 \quad\Rightarrow\quad x^2-10x+13=0 \quad\Rightarrow\quad x=\dfrac{10\pm\sqrt{100-52}}{2}=5\pm2\sqrt{3}$

(b) $x^2+y^2-10x+4y+13=0 \;\bullet\; x=0 \quad\Rightarrow\quad y^2+4y+13=0 \quad\Rightarrow\quad y=\dfrac{-4\pm\sqrt{16-52}}{2}$.

The negative discriminant implies that there are no real solutions to the equation and hence, no y-intercepts.

71 $x^2+y^2+4x-6y+4=0 \;\Leftrightarrow\; (x+2)^2+(y-3)^2=9$. This is a circle with center $C(-2,3)$ and radius 3. The circle we want has the same center, $C(-2,3)$, and radius that is equal to the distance from C to $P(2,6)$.

$d(P,C)=\sqrt{16+9}=5$, so an equation is $(x+2)^2+(y-3)^2=25$.

72 By the Pythagorean theorem, the two stations are $d=\sqrt{100^2+80^2}\approx 128.06$ miles apart. The sum of their radii, $80+50=130$, is greater than d, indicating that the circles representing their broadcast ranges do overlap.

73 The equation of circle C_2 is $(x-h)^2+(y-2)^2=2^2$. If we draw a line from the origin to the center of C_2, we form a right triangle with hypotenuse $5-2$ $\{C_2$ radius $-$ C_1 radius$\}=3$ and sides of length 2 and h. Thus, $h^2+2^2=3^2 \quad\Rightarrow\quad h=\sqrt{5}$.

74 The equation of circle C_2 is $(x-h)^2+(y-3)^2=2^2$. If we draw a line from the origin to the center of C_2, we form a right triangle with hypotenuse $5+2$ $\{C_2$ radius $+$ C_1 radius$\}=7$ and sides of length 3 and h. Thus, $h^2+3^2=7^2 \quad\Rightarrow\quad h=\sqrt{40}$.

75 The graph of y_1 is *below* the graph of y_2 to the left of $x=-3$ and to the right of $x=2$. Writing the x-values in interval notation gives us $(-\infty,-3)\cup(2,\infty)$.

Note that the y-values play no role in writing the answer in interval notation.

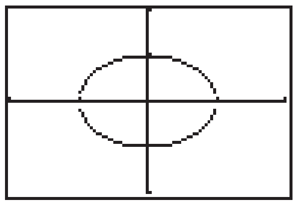
76 $y_1<y_2$ between $x=-8$ and $x=8$, so the interval is $(-8,8)$.

77 $y_1<y_2$ between $x=-1$ and $x=1$, excluding $x=0$ since $y_1=y_2$ at that value, so the interval is $(-1,0)\cup(0,1)$.

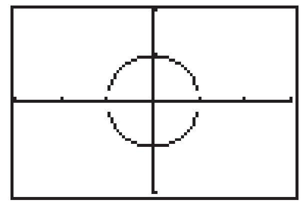
78 $y_1<y_2$ to the left of $x=-8$, between $x=-1$ and $x=1$, and to the right of $x=8$, so the interval notation is $(-\infty,-8)\cup(-1,1)\cup(8,\infty)$.

79 The viewing rectangles significantly affect the shape of the circle. The second viewing rectangle results in a graph that most looks like a circle.

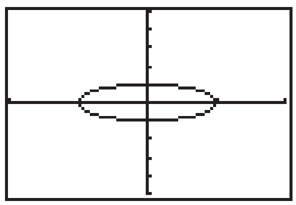
(1) $[-2, 2]$ by $[-2, 2]$



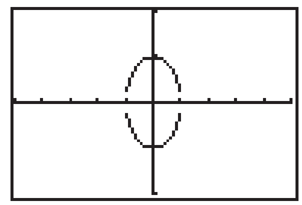
(2) $[-3, 3]$ by $[-2, 2]$



(3) $[-2, 2]$ by $[-5, 5]$

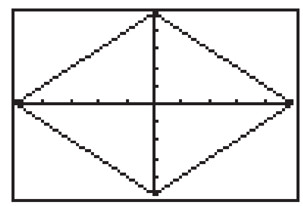


(4) $[-5, 5]$ by $[-2, 2]$



80 (a) From the graph, there are two x–intercepts and two y–intercepts.

(b) Using the free–moving cursor, one can conclude that $|x| + |y| < 5$ is true whenever the point (x, y) is located inside the diamond shape.



81 Assign $x^3 - \frac{9}{10}x^2 - \frac{43}{25}x + \frac{24}{25}$ to Y_1. After trying a standard viewing rectangle, we see that the x-intercepts are near the origin and we choose the viewing rectangle $[-6, 6]$ by $[-4, 4]$. This is simply one choice, not necessarily the best choice. For most graphing calculator exercises, we have selected viewing rectangles that are in a $3:2$ proportion (horizontal : vertical) to maintain a true proportion. From the graph, there are three x-intercepts. Use a root feature to determine that they are -1.2, 0.5, and 1.6.

$[-6, 6]$ by $[-4, 4]$

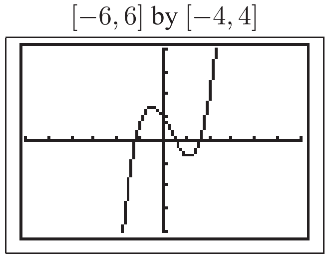


Figure 81

$[-6, 6]$ by $[-4, 4]$

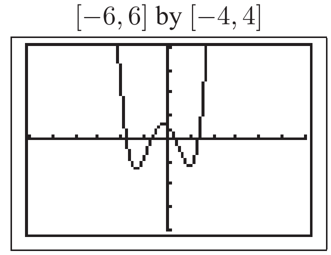


Figure 82

82 From the graph, there are four x-intercepts. They are approximately $-1.8, -0.7, 0.3$ and 1.35.

83 Make the assignments $\text{Y}_1 = x^3 + x$, $\text{Y}_2 = \sqrt{1 - x^2}$, and $\text{Y}_3 = -\text{Y}_2$. From the graph, there are two points of intersection. They are approximately $(0.6, 0.8)$ and $(-0.6, -0.8)$.

$[-3, 3]$ by $[-2, 2]$

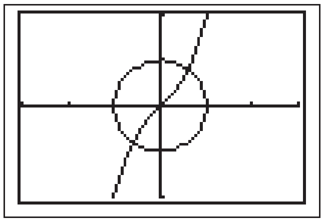


Figure 83

$[-3, 3]$ by $[-2, 2]$

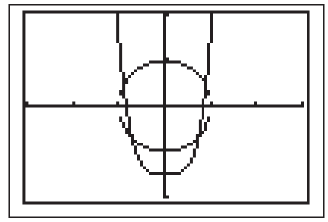


Figure 84

84 Make the assignments $Y_1 = 3x^4 - \frac{3}{2}$, $Y_2 = \sqrt{1 - x^2}$, and $Y_3 = -Y_2$. From the graph, there are four points of intersection. They are approximately $(\pm 0.9, 0.4)$ and $(\pm 0.7, -0.7)$.

85 Depending on the type of graphing utility used, you may need to solve for y first.

$x^2 + (y-1)^2 = 1 \quad \Rightarrow \quad y = 1 \pm \sqrt{1 - x^2}$; $\left(x - \frac{5}{4}\right)^2 + y^2 = 1 \quad \Rightarrow \quad y = \pm\sqrt{1 - \left(x - \frac{5}{4}\right)^2}$.

Make the assignments $Y_1 = \sqrt{1 - x^2}$, $Y_2 = 1 + Y_1$, $Y_3 = 1 - Y_1$, $Y_4 = \sqrt{1 - \left(x - \frac{5}{4}\right)^2}$, and $Y_5 = -Y_4$.

Be sure to "turn off" Y_1 before graphing. From the graph, there are two points of intersection.

They are approximately $(0.999, 0.968)$ and $(0.251, 0.032)$.

$[-3, 3]$ by $[-2, 2]$

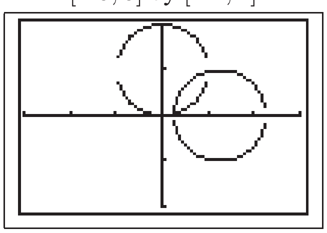


Figure 85

$[-3, 3]$ by $[-2, 2]$

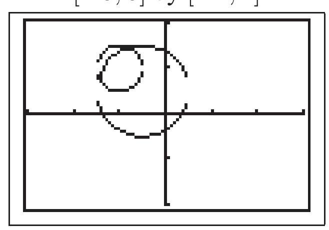


Figure 86

86 $(x+1)^2 + (y-1)^2 = \frac{1}{4} \quad \Rightarrow \quad y = 1 \pm \sqrt{\frac{1}{4} - (x+1)^2}$; $\left(x + \frac{1}{2}\right)^2 + \left(y - \frac{1}{2}\right)^2 = 1 \quad \Rightarrow$

$y = \frac{1}{2} \pm \sqrt{1 - \left(x + \frac{1}{2}\right)^2}$. From the graph, there are two points of intersection.

They are approximately $(-0.79, 1.46)$ and $(-1.46, 0.79)$.

87 The cars are initially 4 miles apart. The distance between them decreases to 0 when they meet on the highway after 2 minutes. Then, the distance between them starts to increase until it is 4 miles after a total of 4 minutes.

$[0, 4]$ by $[0, 4]$

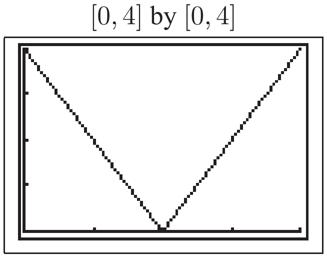


Figure 87

$[0, 6]$ by $[0, 20{,}000, 5000]$

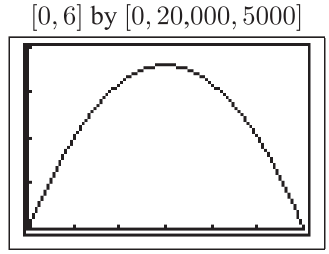


Figure 88

88 At noon on Sunday the pool is empty since when $x = 0$, $A = 0$. It is then filled with water, until at noon on Wednesday $(x = 3)$, it contains 18,000 gallons. It is then drained until at noon on Saturday $(x = 6)$, it is empty again.

89 (a) $v = 1087\sqrt{\dfrac{T + 273}{273}}$ • $T = 20°\text{C} \quad \Rightarrow \quad v = 1087\sqrt{\dfrac{20 + 273}{273}} \approx 1126$ ft/sec.

(b) *Algebraically:* $v = 1000 \Rightarrow 1000 = 1087\sqrt{\dfrac{T+273}{273}} \Rightarrow \sqrt{\dfrac{T+273}{273}} = \dfrac{1000}{1087} \Rightarrow$

$\dfrac{T+273}{273} = \dfrac{1000^2}{1087^2} \Rightarrow T + 273 = \dfrac{273 \cdot 1000^2}{1087^2} \Rightarrow T = \dfrac{273 \cdot 1000^2}{1087^2} - 273 \approx -42°\text{C}.$

Graphically: Graph $Y_1 = 1087\sqrt{(T+273)/273}$ and $Y_2 = 1000$.

At the point of their intersection, $T \approx -42°\text{C}$.

$[-50, 50, 10]$ by $[900, 1200, 100]$

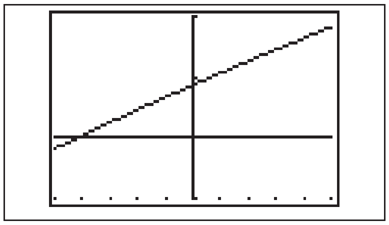


Figure 89

$[14, 16]$ by $[95, 105]$

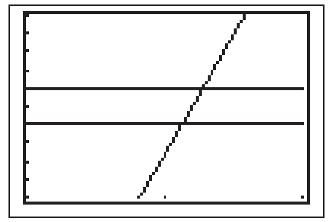


Figure 90

90 The horizontal lines ($y = 99$ and $y = 101$) intersect the graph of $A = \left(\sqrt{3}/4\right)s^2$ at $s \approx 15.12, 15.27$.

Thus, if $15.12 \le s \le 15.27$, then $99 \le A \le 101$.

3.3 Exercises

1 $A(-3, 2), B(5, -4) \Rightarrow m_{AB} = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{(-4) - 2}{5 - (-3)} = \dfrac{-6}{8} = -\dfrac{3}{4}$

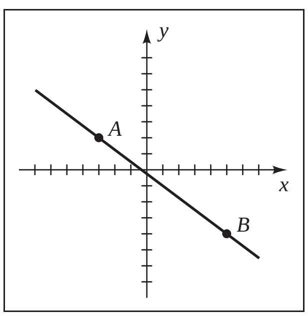


Figure 1

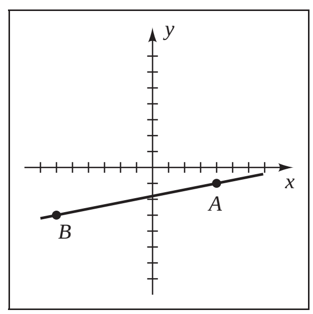


Figure 2

2 $A(4, -1), B(-6, -3) \Rightarrow m_{AB} = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{-3 - (-1)}{-6 - 4} = \dfrac{-2}{-10} = \dfrac{1}{5}$

3 $A(3, 4), B(-6, 4) \Rightarrow m_{AB} = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{4 - 4}{-6 - 3} = \dfrac{0}{-9} = 0$ {horizontal line}

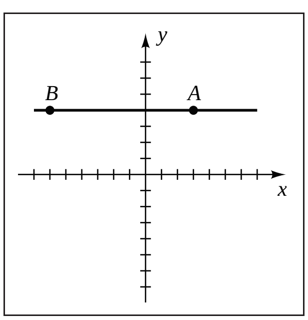


Figure 3

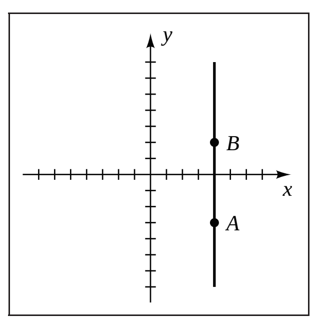


Figure 4

4 $A(4, -3), B(4, 2) \Rightarrow m_{AB} = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{2 - (-3)}{4 - 4} = \dfrac{5}{0} \Rightarrow m$ is undefined {vertical line}

5 $A(-3,2), B(-3,5) \quad \Rightarrow \quad m_{AB} = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{5-2}{-3-(-3)} = \dfrac{3}{0} \quad \Rightarrow \quad m$ is undefined {vertical line}

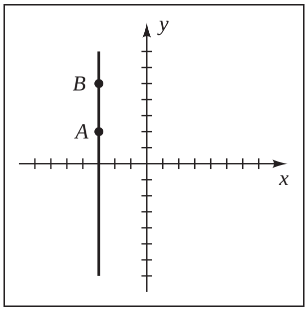


Figure 5

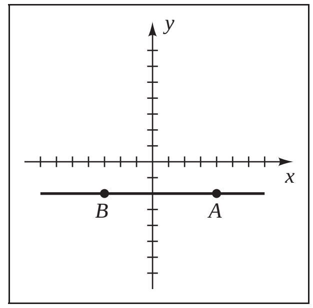


Figure 6

6 $A(4,-2), B(-3,-2) \quad \Rightarrow \quad m_{AB} = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{-2-(-2)}{-3-4} = \dfrac{0}{-7} = 0$ {horizontal line}

7 To show that the polygon is a parallelogram, we must show that the slopes of opposite sides are equal.
$A(-2,1), B(6,3), C(4,0), D(-4,-2) \quad \Rightarrow \quad m_{AB} = \frac{1}{4} = m_{DC}$ and $m_{DA} = \frac{3}{2} = m_{CB}$.

8 To show that the polygon is a trapezoid, we must show that the slopes of one pair of opposite sides are equal.
$A(0,3), B(3,-1), C(-2,-6), D(-8,2) \quad \Rightarrow \quad m_{AB} = -\frac{4}{3} = m_{CD}$.

9 To show that the polygon is a rectangle, we must show that the slopes of opposite sides are equal (parallel lines) and the slopes of two adjacent sides are negative reciprocals (perpendicular lines).
$A(6,15), B(11,12), C(-1,-8), D(-6,-5) \quad \Rightarrow \quad m_{DA} = \frac{5}{3} = m_{CB}$ and $m_{AB} = -\frac{3}{5} = m_{DC}$.

10 To show that the polygon is a right triangle, we must show that the slopes of two adjacent sides are negative reciprocals (perpendicular lines). $A(1,4), B(6,-4), C(-15,-6) \quad \Rightarrow \quad m_{AB} = -\frac{8}{5}$ and $m_{AC} = \frac{5}{8}$.

11 $A(-1,-3)$ is 5 units to the left and 5 units down from $B(4,2)$. The fourth vertex D will have the same relative position from $C(-7,5)$, that is, 5 units to the left and 5 units down from C. Its coordinates are $(-7-5, 5-5) = (-12, 0)$.

12 Let $E = M_{AB} = \left(\dfrac{x_1+x_2}{2}, \dfrac{y_1+y_2}{2}\right)$, $F = M_{BC} = \left(\dfrac{x_2+x_3}{2}, \dfrac{y_2+y_3}{2}\right)$, $G = M_{CD} = \left(\dfrac{x_3+x_4}{2}, \dfrac{y_3+y_4}{2}\right)$, and $H = M_{AD} = \left(\dfrac{x_1+x_4}{2}, \dfrac{y_1+y_4}{2}\right)$. The slopes of opposite sides are equal (or lines are vertical).
$m_{EF} = m_{GH} = \dfrac{y_3 - y_1}{x_3 - x_1}$ and $m_{FG} = m_{EH} = \dfrac{y_4 - y_2}{x_4 - x_2}$.

13 $m = 3, -2, \frac{2}{3}, -\frac{1}{4}$ • Lines with equation $y = mx$ pass through the origin. Draw lines through the origin with slopes 3 {rise = 3, run = 1}, −2 {rise = −2, run = 1}, $\frac{2}{3}$ {rise = 2, run = 3}, and $-\frac{1}{4}$ {rise = −1, run = 4}. A negative "rise" can be thought of as a "drop."

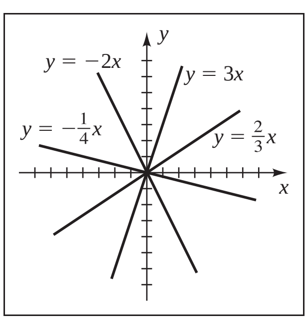


Figure 13

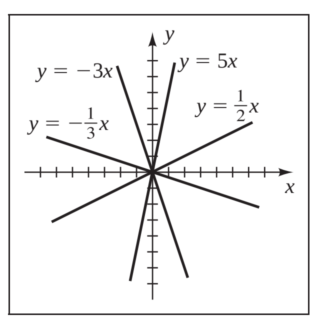


Figure 14

14 $m = 5, -3, \frac{1}{2}, -\frac{1}{3}$ •

15 Draw lines through the point $P(3,1)$ with slopes $\frac{1}{2}$ {rise $= 1$, run $= 2$}, -1 {rise $= -1$, run $= 1$}, and $-\frac{1}{5}$ {rise $= -1$, run $= 5$}.

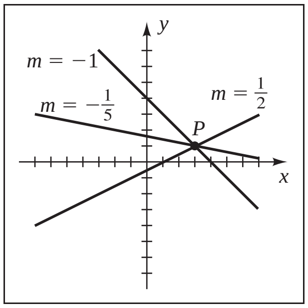


Figure 15

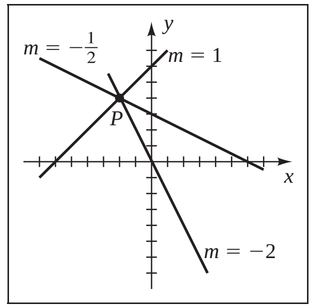


Figure 16

16 $P(-2,4)$; $m = 1, -2, -\frac{1}{2}$ •

17 From the figure, the slope of one of the lines is $\dfrac{\Delta y}{\Delta x} = \dfrac{5}{4}$, so the slopes are $\pm\dfrac{5}{4}$.

Using the point-slope form for the equation of a line with slope $m = \pm\frac{5}{4}$ and point $(x_1, y_1) = (2, -3)$ gives us

$y - (-3) = \pm\frac{5}{4}(x - 2)$, or $y + 3 = \pm\frac{5}{4}(x - 2)$.

18 $m = \dfrac{\Delta y}{\Delta x} = \dfrac{3}{4}$, so the slopes are $\pm\dfrac{3}{4}$.

Using the point $(x_1, y_1) = (-1, 2)$ gives us $y - 2 = \pm\frac{3}{4}[x - (-1)]$, or $y - 2 = \pm\frac{3}{4}(x + 1)$.

19 The line $y = 1x + 3$ has slope 1 and y-intercept 3. The line $y = 1x + 1$ has slope 1 and y-intercept 1.

The line $y = -1x + 1$ has slope -1 and y-intercept 1. Check your graph for parallel and perpendicular lines.

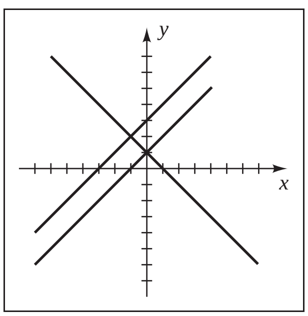


Figure 19

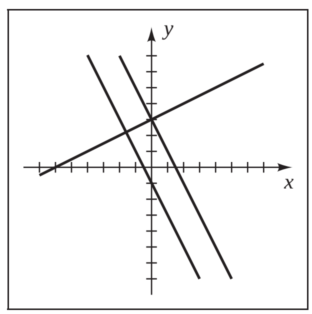


Figure 20

20 $y = -2x - 1$, $y = -2x + 3$, $y = \frac{1}{2}x + 3$ • Check your graph for parallel and perpendicular lines.

21 (a) "Parallel to the y-axis" implies the equation is of the form $x = k$.

The x-value of $A(3, -1)$ is 3, hence $x = 3$ is the equation.

(b) "Perpendicular to the y-axis" implies the equation is of the form $y = k$.

The y-value of $A(3, -1)$ is -1, hence $y = -1$ is the equation.

22 (a) The line through $A(-4, 2)$ and parallel to the x-axis is $y = 2$.

(b) The line through $A(-4, 2)$ and perpendicular to the x-axis is $x = -4$.

23 Using the point-slope form, the equation of the line through $A(5, -3)$ with slope -4 is

$$y - (-3) = -4(x - 5) \quad\Rightarrow\quad y + 3 = -4x + 20 \quad\Rightarrow\quad 4x + y = 17.$$

24 Using the point-slope form, the equation of the line through $A(-1,4)$ with slope $\frac{2}{5}$ is
$$y-4=\tfrac{2}{5}[x-(-1)] \;\Rightarrow\; 5(y-4)=2(x+1) \;\Rightarrow\; 5y-20=2x+2 \;\Rightarrow\; 2x-5y=-22.$$

25 $A(4,1)$; slope $-\frac{1}{3}$ {use the point-slope form of a line} $\Rightarrow$
$$y-1=-\tfrac{1}{3}(x-4) \;\Rightarrow\; 3(y-1)=-1(x-4) \;\Rightarrow\; 3y-3=-x+4 \;\Rightarrow\; x+3y=7.$$

26 $A(0,-2)$; slope 5 $\Rightarrow$ $y+2=5(x-0)$ $\Rightarrow$ $y+2=5x$ $\Rightarrow$ $5x-y=2$.

27 $A(4,-5)$, $B(-3,6)$ $\Rightarrow$ $m_{AB}=-\frac{11}{7}$. By the point-slope form, with $A(4,-5)$, an equation of the line is
$$y+5=-\tfrac{11}{7}(x-4) \;\Rightarrow\; 7(y+5)=-11(x-4) \;\Rightarrow\; 7y+35=-11x+44 \;\Rightarrow\; 11x+7y=9.$$

28 $A(-1,6)$, $B(5,0)$ $\Rightarrow$ $m_{AB}=-1$. $y-0=-1(x-5)$ $\Rightarrow$ $y=-x+5$ $\Rightarrow$ $x+y=5$.

29 $5x-2y=4$ $\Leftrightarrow$ $5x-4=2y$ $\Leftrightarrow$ $y=\frac{5}{2}x-2$. Using the same slope, $\frac{5}{2}$, with $A(3,-1)$, gives us
$$y+1=\tfrac{5}{2}(x-3) \;\Rightarrow\; 2(y+1)=5(x-3) \;\Rightarrow\; 2y+2=5x-15 \;\Rightarrow\; 5x-2y=17.$$

30 $x+3y=1$ $\Leftrightarrow$ $3y=-x+1$ $\Leftrightarrow$ $y=-\frac{1}{3}x+\frac{1}{3}$. Using the same slope, $-\frac{1}{3}$, with $A(-3,5)$, gives us
$$y-5=-\tfrac{1}{3}(x+3) \;\Rightarrow\; 3y-15=-x-3 \;\Rightarrow\; x+3y=12.$$

31 $2x-5y=8$ $\Leftrightarrow$ $2x-8=5y$ $\Leftrightarrow$ $y=\frac{2}{5}x-\frac{8}{5}$. The slope of this line is $\frac{2}{5}$, so we'll use the negative reciprocal, $-\frac{5}{2}$, for the slope of the new line, with $A(7,-3)$.
$$y+3=-\tfrac{5}{2}(x-7) \;\Rightarrow\; 2(y+3)=-5(x-7) \;\Rightarrow\; 2y+6=-5x+35 \;\Rightarrow\; 5x+2y=29.$$

32 $3x+2y=7$ $\Leftrightarrow$ $y=-\frac{3}{2}x+\frac{7}{2}$. Using the negative reciprocal of $-\frac{3}{2}$ for the slope, with $A(5,4)$,
$$y-4=\tfrac{2}{3}(x-5) \;\Rightarrow\; 3y-12=2x-10 \;\Rightarrow\; 2x-3y=-2.$$

33 $A(4,0)$, $B(0,-3)$ $\Rightarrow$ $m_{AB}=\frac{3}{4}$.
Since B *is* the y-intercept, we use the slope-intercept form with $b=-3$ to get $y=\frac{3}{4}x-3$.

34 $A(-6,0)$, $B(0,-1)$ $\Rightarrow$ $m_{AB}=-\frac{1}{6}$. Using the slope-intercept form with $b=-1$ gives us $y=-\frac{1}{6}x-1$.

35 $A(5,2)$, $B(-1,4)$ $\Rightarrow$ $m_{AB}=-\frac{1}{3}$. By the point-slope form, with $A(5,2)$, an equation of the line is
$$y-2=-\tfrac{1}{3}(x-5) \;\Rightarrow\; y=-\tfrac{1}{3}x+\tfrac{5}{3}+2 \;\Rightarrow\; y=-\tfrac{1}{3}x+\tfrac{11}{3}.$$

36 $A(-3,1)$, $B(2,7)$ $\Rightarrow$ $m_{AB}=\frac{6}{5}$. By the point-slope form, with $A(-3,1)$, an equation of the line is
$$y-1=\tfrac{6}{5}(x+3) \;\Rightarrow\; y=\tfrac{6}{5}x+\tfrac{18}{5}+1 \;\Rightarrow\; y=\tfrac{6}{5}x+\tfrac{23}{5}.$$

37 We need the line through the midpoint M of segment AB that is perpendicular to segment AB.
$A(3,-1)$, $B(-2,6)$ $\Rightarrow$ $M_{AB}=\left(\frac{1}{2},\frac{5}{2}\right)$ and $m_{AB}=-\frac{7}{5}$. Use M_{AB} and $m=\frac{5}{7}$ in the point-slope form.
$$y-\tfrac{5}{2}=\tfrac{5}{7}\left(x-\tfrac{1}{2}\right) \;\Rightarrow\; 7\left(y-\tfrac{5}{2}\right)=5\left(x-\tfrac{1}{2}\right) \;\Rightarrow\; 7y-\tfrac{35}{2}=5x-\tfrac{5}{2} \;\Rightarrow\; 5x-7y=-15.$$

38 $A(4,2)$, $B(-2,-6)$ $\Rightarrow$ $M_{AB}=(1,-2)$ and $m_{AB}=\frac{4}{3}$. Use M_{AB} and $m=-\frac{3}{4}$ in the point-slope form.
$$y+2=-\tfrac{3}{4}(x-1) \;\Rightarrow\; 4y+8=-3x+3 \;\Rightarrow\; 3x+4y=-5.$$

39 An equation of the line with slope -1 through the origin is $y-0=-1(x-0)$, or $y=-x$.

40 An equation of the line with slope 1 through the origin is $y-0=1(x-0)$, or $y=x$.

41 We can solve the given equation for y to obtain the slope-intercept form, $y = mx + b$.

$2x = 15 - 3y \quad \Rightarrow \quad 3y = -2x + 15 \quad \Rightarrow \quad y = -\frac{2}{3}x + 5;\ m = -\frac{2}{3},\ b = 5$

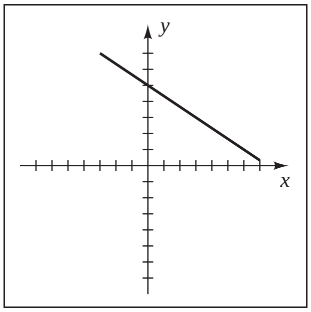


Figure 41

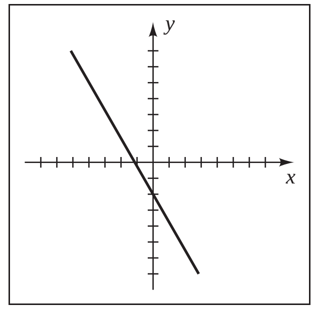


Figure 42

42 $7x = -4y - 8 \quad \Rightarrow \quad -4y = 7x + 8 \quad \Rightarrow \quad y = -\frac{7}{4}x - 2;\ m = -\frac{7}{4},\ b = -2$

43 $4x - 3y = 9 \quad \Rightarrow \quad -3y = -4x + 9 \quad \Rightarrow \quad y = \frac{4}{3}x - 3;\ m = \frac{4}{3},\ b = -3$

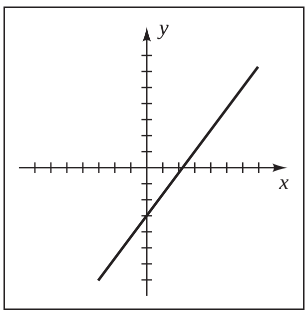


Figure 43

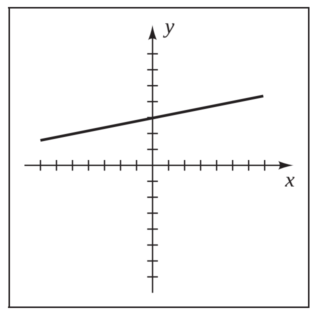


Figure 44

44 $x - 5y = -15 \quad \Rightarrow \quad -5y = -x - 15 \quad \Rightarrow \quad y = \frac{1}{5}x + 3;\ m = \frac{1}{5},\ b = 3$

45 (a) An equation of the horizontal line with y-intercept 3 is $y = 3$.

(b) An equation of the line through the origin with slope $-\frac{1}{2}$ is $y = -\frac{1}{2}x$.

(c) An equation of the line with slope $-\frac{3}{2}$ and y-intercept 1 is $y = -\frac{3}{2}x + 1$.

(d) An equation of the line through $(3, -2)$ with slope -1 is $y + 2 = -(x - 3)$.

Alternatively, we have a slope of -1 and a y-intercept of 1, i.e., $y = -x + 1$.

46 (a) An equation of the vertical line with x-intercept -2 is $x = -2$.

(b) An equation of the line through the origin with slope $\frac{4}{3}$ is $y = \frac{4}{3}x$.

(c) An equation of the line with slope $\frac{1}{3}$ and y-intercept -2 is $y = \frac{1}{3}x - 2$.

(d) An equation of the line through $(-2, -5)$ with slope 3 is $y + 5 = 3(x + 2)$.

Alternatively, we have a slope of 3 and a y-intercept of 1, i.e., $y = 3x + 1$.

47 Since we want to obtain a "1" on the right side of the equation, we will divide by 6.

$[4x - 2y = 6] \cdot \frac{1}{6} \quad \Rightarrow \quad \frac{4x}{6} - \frac{2y}{6} = \frac{6}{6} \quad \Rightarrow \quad \frac{2x}{3} - \frac{y}{3} = 1 \quad \Rightarrow \quad \frac{x}{\frac{3}{2}} + \frac{y}{-3} = 1.$

The x-intercept is $\frac{3}{2}$ and the y-intercept is -3.

48 $[x - 3y = -2] \cdot \frac{1}{-2} \quad \Rightarrow \quad \frac{x}{-2} + \frac{3y}{2} = 1 \quad \Rightarrow \quad \frac{x}{-2} + \frac{y}{\frac{2}{3}} = 1.$ The x-intercept is -2 and the y-intercept is $\frac{2}{3}$.

49 The radius of the circle is the vertical distance from the center of the circle to the line $y = 5$, that is,
$r = 5 - (-2) = 7$. With $C(3, -2)$, an equation is $(x-3)^2 + (y+2)^2 = 49$.

50 The line through the origin and P is perpendicular to the desired line.
This line has equation $y = \frac{4}{3}x$, so the desired line has slope $-\frac{3}{4}$.
With $P(3, 4)$, an equation is $y - 4 = -\frac{3}{4}(x-3) \Rightarrow y = -\frac{3}{4}x + \frac{9}{4} + 4 \Rightarrow y = -\frac{3}{4}x + \frac{25}{4}$.

51 $L = 1.53t - 6.7$ • $L = 28 \Rightarrow 1.53t - 6.7 = 28 \Rightarrow t = \dfrac{28 + 6.7}{1.53} \approx 22.68$, or approximately 23 weeks.

52 $S = 0.03 + 1.805C$ • $S = 0.35 \Rightarrow 0.03 + 1.805C = 0.35 \Rightarrow C = \dfrac{0.35 - 0.03}{1.805} \approx 0.177$.

53 (a) $W = 1.70L - 42.8$ • $L = 40 \Rightarrow W = 1.70(40) - 42.8 = 25.2$ tons

(b) Error in $L = \pm 2 \Rightarrow$ Error in $W = 1.70(\pm 2) = \pm 3.4$ tons

54 (a) $L = at + b$ and $L = 24$ when $t = 0 \Rightarrow L = at + 24$.
$L = 53$ when $t = 7 \Rightarrow 53 = 7a + 24 \Rightarrow a = \frac{29}{7}$ and $L = \frac{29}{7}t + 24$.

(b) From part (a), the slope is $\frac{29}{7}$ ft/month $= \frac{29}{210}$ ft/day ≈ 1.657 inches/day.

(c) $W = at + b$ and $W = 3$ when $t = 0 \Rightarrow W = at + 3$.
$W = 23$ when $t = 7 \Rightarrow 23 = 7a + 3 \Rightarrow a = \frac{20}{7}$ and $W = \frac{20}{7}t + 3$.

(d) From part (c), the slope is $\frac{20}{7}$ tons/month $= \frac{2}{21}$ tons/day ≈ 190.476 pounds/day.

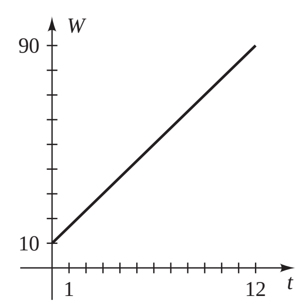
55 (a) $y = mx = \dfrac{\text{change in } y \text{ from the beginning of the season}}{\text{change in } x \text{ from the beginning of the season}}(x) = \dfrac{5-0}{14-0}x = \dfrac{5}{14}x$.

(b) $x = 162 \Rightarrow y = \frac{5}{14}(162) \approx 58$.

56 (a) $y = mx = \dfrac{\text{change in } y \text{ for the year}}{\text{change in } x \text{ for the year}}(x) = \dfrac{18{,}000 - 0}{(31 + 28 + 24) - 0}x = \dfrac{18{,}000}{83}x$.

(b) $x = 365 \Rightarrow y = \frac{18{,}000}{83}(365) \approx 79{,}157$.

57 (a) Using the slope-intercept form, $W = mt + b = mt + 10$.
$W = 30$ when $t = 3 \Rightarrow 30 = 3m + 10 \Rightarrow$
$m = \frac{20}{3}$ and $W = \frac{20}{3}t + 10$.

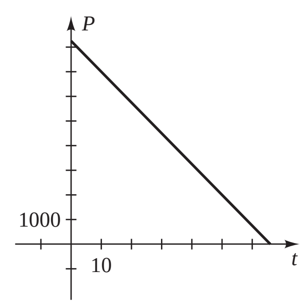


(b) $t = 6 \Rightarrow W = \frac{20}{3}(6) + 10 \Rightarrow W = 50$ lb

(c) $W = 70 \Rightarrow 70 = \frac{20}{3}t + 10 \Rightarrow 60 = \frac{20}{3}t \Rightarrow$
$t = 9$ years old

(d) The graph has endpoints at $(0, 10)$ and $(12, 90)$.

58 (a) Using the slope-intercept form, $P = mt + b = mt + 8250$.
$P = 8125$ when $t = 1$ {after one payment} $\Rightarrow$
$8125 = m + 8250 \Rightarrow m = -125$ and $P = -125t + 8250$.



(b) $P = 5000 \Rightarrow -125\text{t} + 8250 = 5000 \Rightarrow$
$-125t = -3250 \Rightarrow t = 26$ months

(c) $\frac{8250}{125} = 66$ payments. The graph has endpoints at $(0, 8250)$ and $(66, 0)$.

59 Using $(10, 2480)$ and $(25, 2440)$ {since an increase of 15°C lowers H by 40}, we have

$$H - 2440 = \frac{2440 - 2480}{25 - 10}(T - 25) \Rightarrow H - 2440 = \frac{-40}{15}(T - 25) \Rightarrow H - 2440 = -\frac{8}{3}(T - 25) \Rightarrow$$

$$H - \tfrac{7320}{3} = -\tfrac{8}{3}T + \tfrac{200}{3} \Rightarrow H = -\tfrac{8}{3}T + \tfrac{7520}{3}.$$

60 (a) Using $(1800, 100)$ and $(5000, 40)$, we have $P - 40 = \dfrac{40 - 100}{5000 - 1800}(h - 5000) \Rightarrow$

$$P - 40 = -\tfrac{3}{160}(h - 5000) \Rightarrow P - \tfrac{6400}{160} = -\tfrac{3}{160}h + \tfrac{15{,}000}{160} \Rightarrow P = -\tfrac{3}{160}h + \tfrac{21{,}400}{160} \Rightarrow$$

$$P = -\tfrac{3}{160}h + \tfrac{535}{4} \text{ for } 1800 \le h \le 5000.$$

(b) $h = 2400 \Rightarrow P = -\frac{3}{160}(2400) + \frac{535}{4} = -\frac{180}{4} + \frac{535}{4} = \frac{355}{4}$, or 88.75%.

61 (a) Using the slope-intercept form with $m = 0.032$ and $b = 13.5$, we have $T = 0.032t + 13.5$.

(b) $t = 2020 - 1915 = 105 \Rightarrow T = 0.032(105) + 13.5 = 16.86$°C.

62 (a) Using $(1870, 11.8)$ and $(1969, 13.5)$, we have

$$T - 13.5 = \frac{13.5 - 11.8}{1969 - 1870}(t - 1969), \text{ or } T = \frac{1.7}{99}(t - 1969) + 13.5.$$

(b) $T = 12.5 \Rightarrow 12.5 - 13.5 = \dfrac{1.7}{99}(t - 1969) \Rightarrow -1 = \dfrac{1.7}{99}(t - 1969) \Rightarrow -\dfrac{99}{1.7} = t - 1969 \Rightarrow$

$t \approx 1910.76$, or during the year 1910.

63 (a) Expenses (E) = (\$1000) + (5% of R) + (\$2600) + (50% of R) $\Rightarrow$

$$E = 1000 + 0.05R + 2600 + 0.50R \Rightarrow E = 0.55R + 3600.$$

(b) Profit (P) = Revenue (R) − Expenses (E) $\Rightarrow P = R - (0.55R + 3600) \Rightarrow$

$$P = R - 0.55R - 3600 \Rightarrow P = 0.45R - 3600.$$

(c) *Break even* means P would be 0. $P = 0 \Rightarrow 0 = 0.45R - 3600 \Rightarrow$

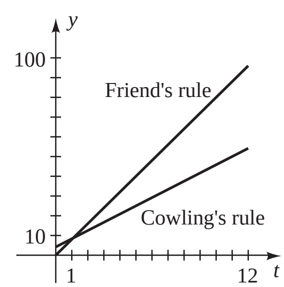
$$0.45R = 3600 \Rightarrow \tfrac{45}{100}R = 3600 \Rightarrow R = 3600\left(\tfrac{100}{45}\right) = \$8000/\text{month}$$

64 (a) $a = 100$ gives us $y = \frac{25}{6}(t + 1)$ with endpoints $\left(0, \frac{25}{6}\right)$ and $\left(12, \frac{325}{6}\right)$, and $y = 8t$ with endpoints $(0, 0)$ and $(12, 96)$.

(b) $\frac{1}{24}(t + 1)a = \frac{2}{25}ta \Rightarrow \frac{1}{24}(t + 1) = \frac{2}{25}t \Rightarrow$

$25(t + 1) = 24(2t) \Rightarrow 25t + 25 = 48t \Rightarrow$

$25 = 23t \Rightarrow t = \frac{25}{23}$ yr ≈ 13 months



65 The targets are on the x-axis {which is the line $y = 0$}. To determine if a target is hit, set $y = 0$ and solve for x.

(a) $y - 2 = -1(x - 1) \Rightarrow x + y = 3$. $y = 0 \Rightarrow x = 3$ and a creature is hit.

(b) $y - \frac{5}{3} = -\frac{4}{9}\left(x - \frac{3}{2}\right) \Rightarrow 4x + 9y = 21$. $y = 0 \Rightarrow x = 5.25$ and no creature is hit.

66 (a) $C = F$ and $C = \frac{5}{9}(F - 32) \Rightarrow F = \frac{5}{9}(F - 32) \Rightarrow 9F = 5F - 160 \Rightarrow 4F = -160 \Rightarrow$

$F = -40$.

(b) $F = 2C$ and $C = \frac{5}{9}(F - 32) \Rightarrow C = \frac{5}{9}(2C - 32) \Rightarrow 9C = 10C - 160 \Rightarrow$

$C = 160$ and hence, $F = 320$.

67 $s = \dfrac{v_2 - v_1}{h_2 - h_1} \Rightarrow 0.07 = \dfrac{v_2 - 22}{185 - 0} \Rightarrow 0.07(185) = v_2 - 22 \Rightarrow v_2 = 22 + 12.95 = 34.95$ mi/hr.

68 From Exercise 67, the average wind shear is $s = \dfrac{v_2 - v_1}{h_2 - h_1}$.

We know $v_1 = 32$ at $h_1 = 20$. We need to find v_2 at $h_2 = 200$. $\dfrac{v_1}{v_2} = \left(\dfrac{h_1}{h_2}\right)^P \;\Rightarrow\; \dfrac{v_2}{v_1} = \left(\dfrac{h_2}{h_1}\right)^P \;\Rightarrow$

$$v_2 = v_1\left(\frac{h_2}{h_1}\right)^P = 32\left(\frac{200}{20}\right)^{0.13}. \text{ Thus, } s = \frac{32(10^{0.13}) - 32}{200 - 20} \approx 0.062 \text{ (mi/hr)/ft.}$$

69 The slope of AB is $\dfrac{-1.11905 - (-1.3598)}{-0.55 - (-1.3)} = 0.321$. Similarly, the slopes of BC and CD are also 0.321.

Therefore, the four points all lie on the same line. Since the common slope is 0.321, let $a = 0.321$.

$y = 0.321x + b \;\Rightarrow\; -1.3598 = 0.321(-1.3) + b \;\Rightarrow\; b = -0.9425$.

Thus, the points are linearly related by the equation $y = 0.321x - 0.9425$.

70 The slopes of AB and BC are both -0.44, whereas the slope of CD is approximately -1.107.

Thus, the points do not lie on the same line.

71 $x - 3y = -58 \;\Leftrightarrow\; y = (x + 58)/3$ and $3x - y = -70 \;\Leftrightarrow\; y = 3x + 70$. Assign $(x + 58)/3$ to Y_1 and $3x + 70$ to Y_2. Using a standard viewing rectangle, we don't see the lines. Zooming out gives us an indication where the lines intersect and by using an intersect feature, we find that the lines intersect at $(-19, 13)$.

$[-30, 3, 2]$ by $[-2, 20, 2]$

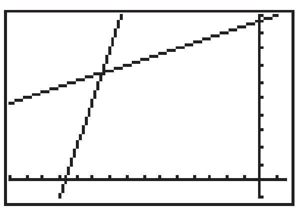


Figure 71

$[-12, 12]$ by $[-2, 14]$

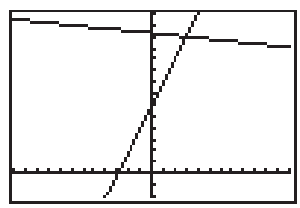


Figure 72

72 $x + 10y = 123 \;\Leftrightarrow\; y = (-x + 123)/10$ and $2x - y = -6 \;\Leftrightarrow\; y = 2x + 6$. Assign $(-x + 123)/10$ to Y_1 and $y = 2x + 6$ to Y_2. Similarly to Exercise 71, the lines intersect at $(3, 12)$.

73 From the graph, we can see that the points of intersection are $A(-0.8, -0.6)$, $B(4.8, -3.4)$, and $C(2, 5)$. The lines intersecting at A are perpendicular since they have slopes of 2 and $-\frac{1}{2}$. Since $d(A, B) = \sqrt{39.2}$ and $d(A, C) = \sqrt{39.2}$, the triangle is isosceles. Thus, the polygon is a right isosceles triangle.

$[-15, 15]$ by $[-10, 10]$

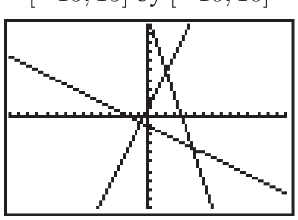


Figure 73

$[-3, 3]$ by $[-2, 2]$

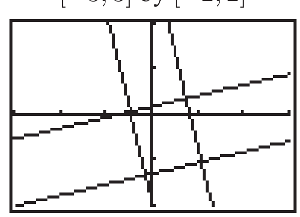


Figure 74

74 The equations of the lines can be rewritten as $y = \frac{1}{4.2}x + 0.17$, $y = -4.2x - 1.9$, $y = \frac{1}{4.2}x - 1.3$, and $y = -4.2x + 3.5$. From the graph, we can see that the points of intersection are approximately $A(0.75, 0.35)$, $B(1.08, -1.04)$, $C(-0.14, -1.33)$, and $D(-0.47, 0.059)$. The first and third lines are parallel as are the second and fourth lines. In addition, these pairs of lines are perpendicular to each other since their slopes are -4.2 and $\frac{1}{4.2}$. Since $d(A, B) \approx 1.43$ and $d(A, D) \approx 1.25$, it is not a square. Thus, the polygon is a rectangle.

75 The data appear to be linear. Using the two arbitrary points $(0.6, 1.3)$ and $(4.6, 8.5)$, the slope of the line is $\dfrac{8.5-1.3}{4.6-0.6} = 1.8$. An equation of the line is $y - 1.3 = 1.8(x - 0.6) \quad \Rightarrow \quad y = 1.8x + 0.22$.
If we find the regression line on a calculator, we get the model $y \approx 1.84589x + 0.21027$.

$[0, 5]$ by $[0, 10]$

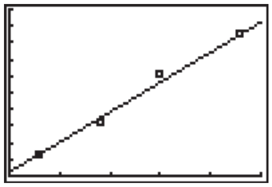


Figure 75

$[0, 5]$ by $[0, 3.5]$

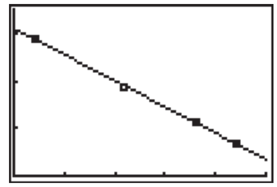


Figure 76

76 The data appear to be linear. Using the two arbitrary points $(0.4, 2.88)$ and $(4.4, 0.68)$, the slope of the line is $\dfrac{0.68-2.88}{4.4-0.4} = -0.55$. An equation of the line is $y - 2.88 = -0.55(x - 0.4) \quad \Rightarrow \quad y = -0.55x + 3.1$.
If we find the regression line on a calculator, we get the model $y \approx -0.54951x + 3.09621$.

77 (a) Plot the points with the form (Year, Distance): $(1911, 15.52)$, $(1932, 15.72)$, $(1955, 16.56)$, $(1975, 17.89)$, and $(1995, 18.29)$.

(b) To find a first approximation for the line use the arbitrary points $(1911, 15.52)$ and $(1995, 18.29)$. The resulting line is $D_1 \approx 0.033Y - 47.545$. Adjustments may be made to this equation. If we find the regression line on a calculator, we get the model $D \approx 0.03648Y - 54.47409$.

(c) Using D_1 from part (b), when $x = 1985$, $y = 17.96$ (so the distance is 17.96 meters)—almost equal to the actual record of 17.97 meters.

$[1900, 2010, 20]$ by $[15, 20]$

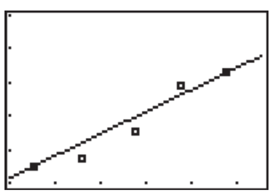


Figure 77

$[1900, 2010, 20]$ by $[210, 260, 10]$

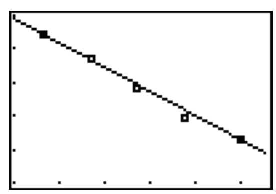


Figure 78

78 (b) For an approximation for the line, we'll use the first and last points given; that is, $(1913, 254.4)$ and $(1999, 223.1)$. These points determine the line $T - 254.4 = \dfrac{223.1 - 254.4}{1999 - 1913}(Y - 1913) \quad \Rightarrow$
$T \approx -0.36395Y + 950.643$. The regression line is $y \approx -0.37455x + 970.58262$.

(c) $T = -0.36395(1985) + 950.643 \approx 228.2$ seconds, which is off by 1.9 seconds.

(d) The slope of the line is approximately -0.4. This means that *on the average*, the record time for the mile has decreased by 0.4 sec/yr.

3.4 Exercises

1 $f(x) = -x^2 - x - 4 \quad \Rightarrow \quad f(-2) = -4 + 2 - 4 = -6$, $f(0) = -4$, and $f(4) = -16 - 4 - 4 = -24$.

2 $f(x) = -x^3 - x^2 + 3 \quad \Rightarrow \quad f(-3) = 27 - 9 + 3 = 21$, $f(0) = 3$, and $f(2) = -8 - 4 + 3 = -9$.

3 $f(x) = \sqrt{x-2} + 3x \Rightarrow f(3) = \sqrt{3-2} + 3(3) = \sqrt{1} + 9 = 1 + 9 = 10.$

Similarly, $f(6) = \sqrt{6-2} + 3(6) = \sqrt{4} + 18 = 2 + 18 = 20$

and $f(11) = \sqrt{11-2} + 3(11) = \sqrt{9} + 33 = 3 + 33 = 36.$

Note that $f(a)$, with $a < 2$, would be undefined.

4 $f(x) = \dfrac{x}{x-3} \Rightarrow f(-2) = \dfrac{2}{5}$, $f(0) = 0$, and $f(3)$ is undefined.

5 (a) $f(x) = 5x - 2 \Rightarrow f(a) = 5(a) - 2 = 5a - 2$ (b) $f(-a) = 5(-a) - 2 = -5a - 2$

(c) $-f(a) = -1 \cdot (5a - 2) = -5a + 2$ (d) $f(a+h) = 5(a+h) - 2 = 5a + 5h - 2$

(e) $f(a) + f(h) = (5a - 2) + (5h - 2) = 5a + 5h - 4$

(f) Using parts (d) and (a), $\dfrac{f(a+h) - f(a)}{h} = \dfrac{(5a + 5h - 2) - (5a - 2)}{h} = \dfrac{5h}{h} = 5.$

6 (a) $f(x) = 1 - 4x \Rightarrow f(a) = 1 - 4(a) = 1 - 4a$ (b) $f(-a) = 1 - 4(-a) = 1 + 4a$

(c) $-f(a) = -1 \cdot (1 - 4a) = 4a - 1$ (d) $f(a+h) = 1 - 4(a+h) = 1 - 4a - 4h$

(e) $f(a) + f(h) = (1 - 4a) + (1 - 4h) = 2 - 4a - 4h$

(f) Using parts (d) and (a), $\dfrac{f(a+h) - f(a)}{h} = \dfrac{(1 - 4a - 4h) - (1 - 4a)}{h} = \dfrac{-4h}{h} = -4.$

7 (a) $f(x) = -x^2 + 3 \Rightarrow f(a) = -(a)^2 + 3 = -a^2 + 3$

(b) $f(-a) = -(-a)^2 + 3 = -a^2 + 3$ (c) $-f(a) = -1 \cdot (-a^2 + 3) = a^2 - 3$

(d) $f(a+h) = -(a+h)^2 + 3 = -(a^2 + 2ah + h^2) + 3 = -a^2 - 2ah - h^2 + 3$

(e) $f(a) + f(h) = (-a^2 + 3) + (-h^2 + 3) = -a^2 - h^2 + 6$

(f) $\dfrac{f(a+h) - f(a)}{h} = \dfrac{(-a^2 - 2ah - h^2 + 3) - (-a^2 + 3)}{h} = \dfrac{-2ah - h^2}{h} = \dfrac{h(-2a - h)}{h} = -2a - h$

8 (a) $f(x) = 3 - x^2 \Rightarrow f(a) = 3 - (a)^2 = 3 - a^2$ (b) $f(-a) = 3 - (-a)^2 = 3 - a^2$

(c) $-f(a) = -1 \cdot (3 - a^2) = -3 + a^2$

(d) $f(a+h) = 3 - (a+h)^2 = 3 - (a^2 + 2ah + h^2) = 3 - a^2 - 2ah - h^2$

(e) $f(a) + f(h) = (3 - a^2) + (3 - h^2) = 6 - a^2 - h^2$

(f) $\dfrac{f(a+h) - f(a)}{h} = \dfrac{(3 - a^2 - 2ah - h^2) - (3 - a^2)}{h} = \dfrac{-2ah - h^2}{h} = \dfrac{h(-2a - h)}{h} = -2a - h$

9 (a) $f(x) = x^2 - x + 3 \Rightarrow f(a) = (a)^2 - (a) + 3 = a^2 - a + 3$

(b) $f(-a) = (-a)^2 - (-a) + 3 = a^2 + a + 3$ (c) $-f(a) = -1 \cdot (a^2 - a + 3) = -a^2 + a - 3$

(d) $f(a+h) = (a+h)^2 - (a+h) + 3 = a^2 + 2ah + h^2 - a - h + 3$

(e) $f(a) + f(h) = (a^2 - a + 3) + (h^2 - h + 3) = a^2 + h^2 - a - h + 6$

(f) $\dfrac{f(a+h) - f(a)}{h} = \dfrac{(a^2 + 2ah + h^2 - a - h + 3) - (a^2 - a + 3)}{h} = \dfrac{2ah + h^2 - h}{h} = \dfrac{h(2a + h - 1)}{h}$

$= 2a + h - 1$

10 (a) $f(x) = 2x^2 + 3x - 7 \Rightarrow f(a) = 2(a)^2 + 3(a) - 7 = 2a^2 + 3a - 7$

(b) $f(-a) = 2(-a)^2 + 3(-a) - 7 = 2a^2 - 3a - 7$

(c) $-f(a) = -1 \cdot (2a^2 + 3a - 7) = -2a^2 - 3a + 7$

(d) $f(a+h) = 2(a+h)^2 + 3(a+h) - 7 = 2(a^2 + 2ah + h^2) + 3a + 3h - 7 =$

$2a^2 + 4ah + 2h^2 + 3a + 3h - 7$

(e) $f(a) + f(h) = (2a^2 + 3a - 7) + (2h^2 + 3h - 7) = 2a^2 + 2h^2 + 3a + 3h - 14$

(f) $$\frac{f(a+h) - f(a)}{h} = \frac{(2a^2 + 4ah + 2h^2 + 3a + 3h - 7) - (2a^2 + 3a - 7)}{h}$$

$$= \frac{4ah + 2h^2 + 3h}{h} = \frac{h(4a + 2h + 3)}{h} = 4a + 2h + 3$$

11 (a) $g(x) = 4x^2 \Rightarrow g\left(\frac{1}{a}\right) = 4\left(\frac{1}{a}\right)^2 = 4 \cdot \frac{1}{a^2} = \frac{4}{a^2}$ (b) $g(a) = 4a^2 \Rightarrow \frac{1}{g(a)} = \frac{1}{4a^2}$

(c) $g(\sqrt{a}) = 4(\sqrt{a})^2 = 4a$ (d) $\sqrt{g(a)} = \sqrt{4a^2} = 2|a| = 2a$ since $a > 0$

12 (a) $g(x) = 2x - 7 \Rightarrow g\left(\frac{1}{a}\right) = 2\left(\frac{1}{a}\right) - 7 = \frac{2}{a} - 7 = \frac{2 - 7a}{a}$ (b) $\frac{1}{g(a)} = \frac{1}{2(a) - 7} = \frac{1}{2a - 7}$

(c) $g(\sqrt{a}) = 2(\sqrt{a}) - 7 = 2\sqrt{a} - 7$ (d) $\sqrt{g(a)} = \sqrt{2a - 7}$

13 (a) $g(x) = \frac{2x}{x^2 + 1} \Rightarrow g\left(\frac{1}{a}\right) = \frac{2(1/a)}{(1/a)^2 + 1} = \frac{2/a}{1/a^2 + 1} \cdot \frac{a^2}{a^2} = \frac{2a}{1 + a^2} = \frac{2a}{a^2 + 1}$

(b) $\frac{1}{g(a)} = \frac{1}{\frac{2a}{a^2 + 1}} = \frac{a^2 + 1}{2a}$ (c) $g(\sqrt{a}) = \frac{2\sqrt{a}}{(\sqrt{a})^2 + 1} = \frac{2\sqrt{a}}{a + 1}$

(d) $\sqrt{g(a)} = \sqrt{\frac{2a}{a^2 + 1}} \cdot \frac{\sqrt{a^2 + 1}}{\sqrt{a^2 + 1}} = \frac{\sqrt{2a(a^2 + 1)}}{a^2 + 1}$, or, equivalently, $\frac{\sqrt{2a^3 + 2a}}{a^2 + 1}$

14 (a) $g(x) = \frac{x^2}{x + 1} \Rightarrow g\left(\frac{1}{a}\right) = \frac{\left(\frac{1}{a}\right)^2}{\frac{1}{a} + 1} = \frac{\frac{1}{a^2}}{\frac{1 + a}{a}} \cdot \frac{a^2}{a^2} = \frac{1}{a(a + 1)}$ (b) $\frac{1}{g(a)} = \frac{1}{\frac{a^2}{a + 1}} = \frac{a + 1}{a^2}$

(c) $g(\sqrt{a}) = \frac{(\sqrt{a})^2}{\sqrt{a} + 1} \cdot \frac{\sqrt{a} - 1}{\sqrt{a} - 1} = \frac{a(\sqrt{a} - 1)}{a - 1}$ (d) $\sqrt{g(a)} = \sqrt{\frac{a^2}{a + 1}} \cdot \sqrt{\frac{a + 1}{a + 1}} = \frac{a\sqrt{a + 1}}{a + 1}$

15 All vertical lines intersect the graph in at most one point,

so the graph *is* the graph of a function because it passes the Vertical Line Test.

16 At least one vertical line intersects the graph in more than one point,

so the graph *is not* the graph of a function because it fails the Vertical Line Test.

17 The domain D is the set of x-values; that is, $D = [-4, 1] \cup [2, 4)$. Note that the solid dots on the figure correspond to using brackets {including}, whereas the open dot corresponds to using parentheses {excluding}. The range R is the set of y-values; that is, $R = [-3, 3)$.

18 Domain $D = \{x\text{-values}\} = [-4, 4)$. Range $R = \{y\text{-values}\} = (-3, -1) \cup [1, 3]$.

19 (a) The domain of a function f is the set of all x-values for which the function is defined. In this case, the graph extends from $x = -3$ to $x = 4$. Hence, the domain is $[-3, 4]$.

(b) The range of a function f is the set of all y-values that the function takes on. In this case, the graph includes all values from $y = -2$ to $y = 2$. Hence, the range is $[-2, 2]$.

(c) $f(1)$ is the y-value of f corresponding to $x = 1$. In this case, $f(1) = 0$.

(d) If we were to draw the horizontal line $y = 1$ on the same coordinate plane, it would intersect the graph at $x = -1, \frac{1}{2}$, and 2. Hence, $f(x) = 1 \Rightarrow x = -1, \frac{1}{2}$, and 2.

(e) The function is above 1 between $x = -1$ and $x = \frac{1}{2}$, and also to the right of $x = 2$. Hence, $f(x) > 1 \Rightarrow x \in \left(-1, \frac{1}{2}\right) \cup (2, 4]$.

20 (a) $[-5, 7]$ (b) $[-1, 2]$ (c) $f(1) = -1$

(d) $f(x) = 1 \Rightarrow x = -3, -1, 3, 5$ (e) $f(x) > 1 \Rightarrow x \in (-3, -1) \cup (3, 5)$

Note: In Exercises 21–32, we need to make sure that the radicand {the expression under the radical sign} is greater than or equal to zero and that the denominator is not equal to zero.

21 $f(x) = \sqrt{2x + 7}$ • $2x + 7 \geq 0 \Rightarrow 2x \geq -7 \Rightarrow x \geq -\frac{7}{2} \Leftrightarrow \left[-\frac{7}{2}, \infty\right)$

22 $f(x) = \sqrt{4 - 3x}$ • $4 - 3x \geq 0 \Rightarrow 4 \geq 3x \Rightarrow x \leq \frac{4}{3} \Leftrightarrow \left(-\infty, \frac{4}{3}\right]$

23 $f(x) = \sqrt{16 - x^2}$ • $16 - x^2 \geq 0 \Rightarrow 16 \geq x^2 \Rightarrow x^2 \leq 16 \Rightarrow |x| \leq 4 \Rightarrow$
$-4 \leq x \leq 4$, or $[-4, 4]$ in interval notation.

24 $f(x) = \sqrt{x^2 - 25}$ • $x^2 - 25 \geq 0 \Rightarrow |x| \geq 5 \Rightarrow x \geq 5$ or $x \leq -5 \Leftrightarrow (-\infty, -5] \cup [5, \infty)$

25 $f(x) = \dfrac{x + 1}{x^3 - 9x}$ • For this function we must have the denominator not equal to 0. The denominator is $x^3 - 9x = x(x^2 - 9) = x(x + 3)(x - 3)$, so $x \neq 0, -3, 3$. The solution is then all real numbers *except* $0, -3, 3$. In interval notation, we have $(-\infty, -3) \cup (-3, 0) \cup (0, 3) \cup (3, \infty)$. We could also denote this solution as $\mathbb{R} - \{\pm 3, 0\}$.

26 $f(x) = \dfrac{4x}{6x^2 + 13x - 5}$ • $6x^2 + 13x - 5 = 0 \Rightarrow (2x + 5)(3x - 1) = 0 \Rightarrow \mathbb{R} - \left\{-\frac{5}{2}, \frac{1}{3}\right\}$

27 $f(x) = \dfrac{\sqrt{2x - 5}}{x^2 - 5x + 4}$ • For this function we must have the radicand greater than or equal to 0 *and* the denominator not equal to 0. The radicand is greater than or equal to 0 if $2x - 5 \geq 0$, or, equivalently, $x \geq \frac{5}{2}$. The denominator is $(x - 1)(x - 4)$, so $x \neq 1, 4$. The solution is then all real numbers greater than or equal to $\frac{5}{2}$, excluding 4. In interval notation, we have $\left[\frac{5}{2}, 4\right) \cup (4, \infty)$.

28 $f(x) = \dfrac{\sqrt{4x - 3}}{x^2 - 4}$ • $x^2 - 4 = 0 \Rightarrow (x + 2)(x - 2) = 0 \Rightarrow x = \pm 2$;
$4x - 3 \geq 0 \Rightarrow x \geq \frac{3}{4}$, so the domain is $\left[\frac{3}{4}, 2\right) \cup (2, \infty)$

29 $f(x) = \dfrac{x - 4}{\sqrt{x - 2}}$ • For this function we must have $x - 2 > 0 \Rightarrow x > 2$.
Note that " $>$ " must be used since the denominator cannot *equal* 0. In interval notation, we have $(2, \infty)$.

30 $f(x) = \dfrac{1}{(x - 3)\sqrt{x + 3}}$ • $x + 3 > 0 \Rightarrow x > -3$ $\{x \neq 3\}$, so the domain is $(-3, 3) \cup (3, \infty)$

31 $f(x) = \sqrt{x + 3} + \sqrt{3 - x}$ • We must have $x + 3 \geq 0 \Rightarrow x \geq -3$ and $3 - x \geq 0 \Rightarrow x \leq 3$.
The domain is the intersection of $x \geq -3$ and $x \leq 3$, that is, $[-3, 3]$.

32 $f(x) = \sqrt{(x - 2)(x - 6)}$ • $(x - 2)(x - 6) \geq 0 \Rightarrow x \leq 2$ or $x \geq 6$ {use a sign chart} $\Rightarrow$
$(-\infty, 2] \cup [6, \infty)$

33 (a) $D = \{x\text{-values}\} = [-5, -3) \cup (-1, 1] \cup (2, 4]$; $R = \{y\text{-values}\} = \{-3\} \cup [-1, 4]$.
Note that the notation for including the single value -3 in R uses braces.

(b) ***f* is increasing** on an interval if it goes up as we move from left to right, so f is increasing on $[-4, -3) \cup [3, 4]$.

***f* is decreasing** on an interval if it goes down as we move from left to right, so f is decreasing on $[-5, -4] \cup (2, 3]$. Note that the values $x = -4$ and $x = 3$ are in intervals that are listed as increasing and in intervals that are listed as decreasing.

***f* is constant** on an interval if the y-values do not change, so f is constant on $(-1, 1]$.

34 (a) $D = \{x\text{-values}\} = [-5, -3) \cup (-2, -1] \cup 0, 2) \cup (3, 5]$; $R = \{y\text{-values}\} = [-3, 3] \cup \{4\}$

(b) ***f* is increasing** on $[-5, -3) \cup (3, 4]$. ***f* is decreasing** on $[0, 2)$. ***f* is constant** on $(-2, -1] \cup [4, 5]$.

35 The graph of the function is increasing on $(-\infty, -3]$ and is decreasing on $[-3, 2]$, so there must be a high point at $x = -3$. Now the graph of the function is increasing on $[2, \infty)$, so there must be a low point at $x = 2$.

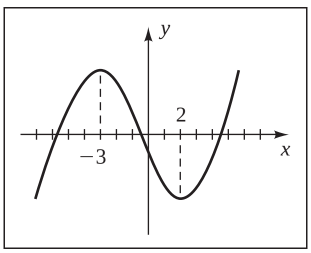


Figure 35

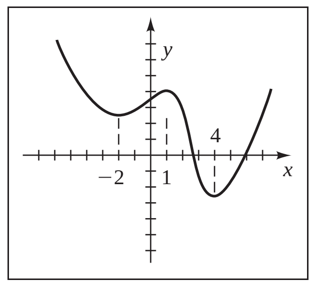


Figure 36

36 The graph of the function is decreasing on $(-\infty, -2]$ and $[1, 4]$, and is increasing on $[-2, 1]$ and $[4, \infty)$.

37 (a) $f(x) = -2x + 1$ • This is a line with slope -2 and y-intercept 1.

(b) The domain D and the range R are equal to $(-\infty, \infty)$.

(c) f is decreasing on its entire domain, that is, $(-\infty, \infty)$.

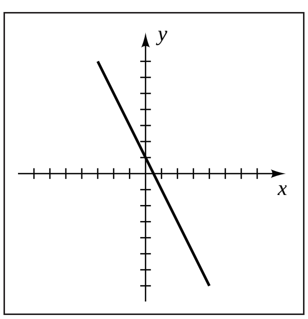


Figure 37

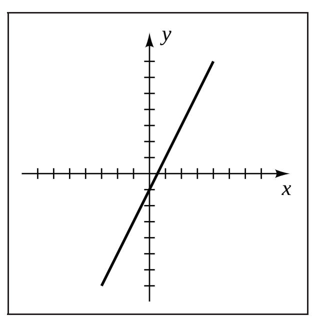


Figure 38

38 (a) $f(x) = 2x - 1$ • This is a line with slope 2 and y-intercept -1.

(b) $D = (-\infty, \infty)$, $R = (-\infty, \infty)$

(c) Increasing on $(-\infty, \infty)$

39 (a) To sketch the graph of $f(x) = 4 - x^2$, we can make use of the symmetry with respect to the y-axis.

x	± 4	± 3	± 2	± 1	0
y	-12	-5	0	3	4

(b) Since we can substitute any number for x, the domain is all real numbers, that is, $D = \mathbb{R}$. By examining the figure, we see that the values of y are at most 4. Hence, the range of f is all reals less than or equal to 4, that is, $R = (-\infty, 4]$.

(c) A common mistake is to confuse the function values, the y's, with the input values, the x's. We are not interested in the specific y-values for determining if the function is increasing, decreasing, or constant. We are only interested if the y-values are going up, going down, or staying the same. For the function $f(x) = 4 - x^2$, we say f *is increasing on* $(-\infty, 0]$ since the y-values are getting larger as we move from left to right over the x-values from $-\infty$ to 0. Also, f *is decreasing on* $[0, \infty)$ since the y-values are getting smaller as we move from left to right over the x-values from 0 to ∞. Note that this answer would have been the same if the function was $f(x) = 500 - x^2$, $f(x) = -300 - x^2$, or any function of the form $f(x) = a - x^2$, where a is any real number.

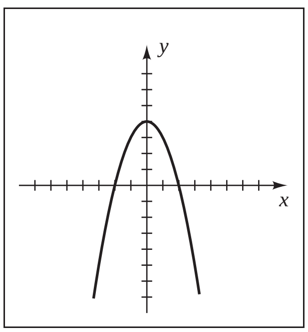


Figure 39

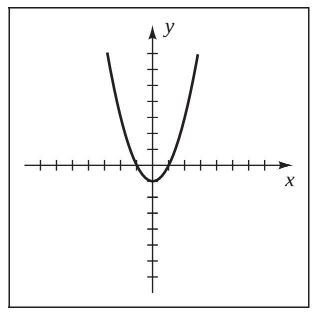


Figure 40

40 (b) $D = (-\infty, \infty)$, $R = [-1, \infty)$

(c) Decreasing on $(-\infty, 0]$, increasing on $[0, \infty)$

41 (a) $f(x) = \sqrt{x - 1}$ • This is half of a parabola opening to the right. It has an x-intercept of 1 and no y-intercept.

(b) $x - 1 \geq 0 \quad \Rightarrow \quad x \geq 1$, so the domain is $D = [1, \infty)$.
The y-values are all positive or zero, so the range is $R = [0, \infty)$.

(c) f is increasing on its entire domain, that is, on $[1, \infty)$.

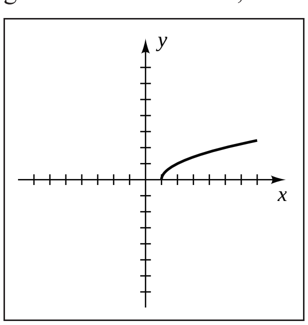


Figure 41

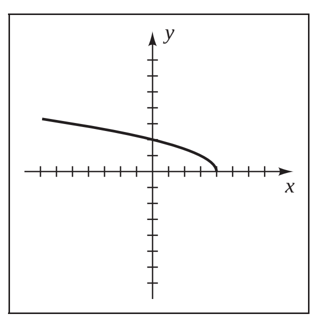


Figure 42

42 (a) $f(x) = x^2 - 1$ • (b) $D = (-\infty, 4]$, $R = [0, \infty)$ (c) Decreasing on $(-\infty, 4]$

43 (a) $f(x) = -4$ • This is a horizontal line with y-intercept -4.

(b) The domain is $D = (-\infty, \infty)$ and the range consists of a single value, so $R = \{-4\}$.

(c) f is constant on its entire domain, that is, $(-\infty, \infty)$.

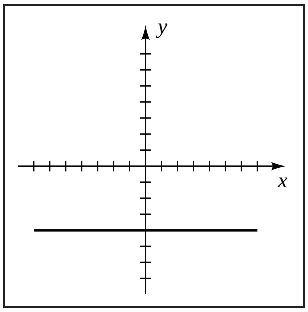


Figure 43

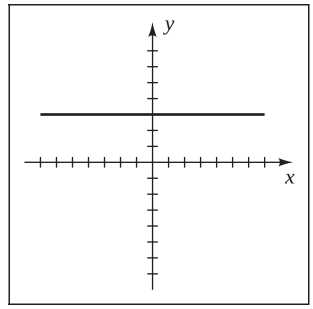


Figure 44

44 (a) $f(x) = 3$ • (b) $D = (-\infty, \infty)$, $R = \{3\}$ (c) Constant on $(-\infty, \infty)$

45 (a) We recognize $y = f(x) = -\sqrt{36 - x^2}$ as the lower half of the circle $x^2 + y^2 = 36$.

(b) To find the domain, we solve $36 - x^2 \geq 0$. $36 - x^2 \geq 0 \Rightarrow x^2 \leq 36 \Rightarrow |x| \leq 6 \Rightarrow D = [-6, 6]$. From the figure, we see that the y-values vary from $y = -6$ to $y = 0$. Hence, the range R is $[-6, 0]$.

(c) As we move from left to right, for $x = -6$ to $x = 0$, the y-values are decreasing. From $x = 0$ to $x = 6$, the y-values increase. Hence, f is decreasing on $[-6, 0]$ and increasing on $[0, 6]$.

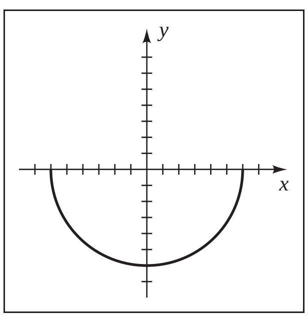


Figure 45

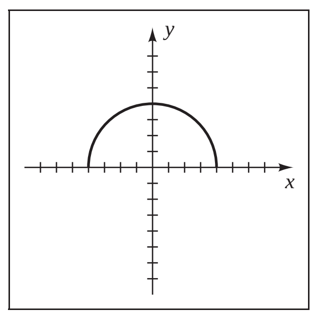


Figure 46

46 (a) $f(x) = \sqrt{16 - x^2}$ • (b) $D = [-4, 4]$, $R = [0, 4]$ (c) Increasing on $[-4, 0]$, decreasing on $[0, 4]$

47 $f(x) = x^2 - 6x$, so $f(2) = 2^2 - 6(2) = 4 - 12 = -8$.

$$\frac{f(2+h) - f(2)}{h} = \frac{\left[(2+h)^2 - 6(2+h)\right] - (-8)}{h} = \frac{4 + 4h + h^2 - 12 - 6h + 8}{h} = \frac{h - 2h}{h} = \frac{h(h-2)}{h}$$
$$= h - 2$$

48 $f(x) = -2x^2 + 5$, so $f(2) = -2(2)^2 + 5 = -2 \cdot 4 + 5 = -8 + 5 = -3$.

$$\frac{f(2+h) - f(2)}{h} = \frac{\left[-2(2+h)^2 + 5\right] - (-3)}{h} = \frac{-2(4 + 4h + h^2) + 5 + 3}{h} = \frac{-8 - 8h - 2h^2 + 8}{h}$$
$$= \frac{-8h - 2h^2}{h} = \frac{h(-8 - 2h)}{h} = -2h - 8$$

49 $$\frac{f(x+h) - f(x)}{h} = \frac{\left[(x+h)^2 + 5\right] - \left[x^2 + 5\right]}{h} = \frac{(x^2 + 2xh + h^2 + 5) - (x^2 + 5)}{h} = \frac{2xh + h^2}{h} = \frac{h(2x+h)}{h}$$
$$= 2x + h$$

50 $\dfrac{f(x+h)-f(x)}{h} = \dfrac{\dfrac{1}{(x+h)^2} - \dfrac{1}{x^2}}{h} = \dfrac{\dfrac{x^2-(x+h)^2}{(x+h)^2 x^2}}{h} = \dfrac{x^2-(x^2+2xh+h^2)}{hx^2(x+h)^2}$

$$= \frac{-2xh-h^2}{hx^2(x+h)^2} = -\frac{h(2x+h)}{hx^2(x+h)^2} = -\frac{2x+h}{x^2(x+h)^2}$$

51 $\dfrac{f(x)-f(a)}{x-a} = \dfrac{\sqrt{x-3}-\sqrt{a-3}}{x-a} = \dfrac{\sqrt{x-3}-\sqrt{a-3}}{x-a} \cdot \dfrac{\sqrt{x-3}+\sqrt{a-3}}{\sqrt{x-3}+\sqrt{a-3}}$

$$= \frac{(x-3)-(a-3)}{(x-a)\left(\sqrt{x-3}+\sqrt{a-3}\right)} = \frac{x-a}{(x-a)\left(\sqrt{x-3}+\sqrt{a-3}\right)} = \frac{1}{\sqrt{x-3}+\sqrt{a-3}}$$

52 $\dfrac{f(x)-f(a)}{x-a} = \dfrac{(x^3-2)-(a^3-2)}{x-a} = \dfrac{x^3-a^3}{x-a} = \dfrac{(x-a)(x^2+ax+a^2)}{x-a} = x^2+ax+a^2$

53 As in Example 7, $a = \dfrac{2-1}{3-(-3)} = \dfrac{1}{6}$ and f has the form $f(x) = \dfrac{1}{6}x + b$.

$f(3) = \frac{1}{6}(3) + b = \frac{1}{2} + b$. But $f(3) = 2$, so $\frac{1}{2} + b = 2 \Rightarrow b = \frac{3}{2}$, and $f(x) = \frac{1}{6}x + \frac{3}{2}$.

54 As in Example 7, $a = \dfrac{-2-7}{4-(-2)} = -\dfrac{9}{6} = -\dfrac{3}{2}$ and f has the form $f(x) = -\dfrac{3}{2}x + b$.

$f(-2) = -\frac{3}{2}(-2) + b = 3 + b$. But $f(-2) = 7$, so $3 + b = 7 \Rightarrow b = 4$, and $f(x) = -\frac{3}{2}x + 4$.

Note: For Exercises 55–64, a good question to consider is, "Given a particular value of x, can a unique value of y be found?" If the answer is yes, the value of y (general formula) is given. If not, two ordered pairs satisfying the relation having x in the first position are given.

55 $3y = x^2 + 7 \Rightarrow y = \dfrac{x^2+7}{3}$, which is a function

56 $x = 3y + 2 \Rightarrow x - 2 = 3y \Rightarrow y = \dfrac{x-2}{3}$, which is a function

57 $x^2 + y^2 = 4 \Rightarrow y^2 = 4 - x^2 \Rightarrow y = \pm\sqrt{4-x^2}$, not a function: $(0, \pm 2)$

58 $y^2 - x^2 = 4 \Rightarrow y^2 = 4 + x^2 \Rightarrow y = \pm\sqrt{4+x^2}$, not a function: $(0, \pm 2)$

59 $y = 5$ is a function since for any x, $(x, 5)$ is the only ordered pair in W having x in the first position.

60 $x = 3$ is not a function: $(3, 0)$ and $(3, 1)$

61 Any ordered pair with x-coordinate 0 satisfies $xy = 0$. Two such ordered pairs are $(0, 0)$ and $(0, 1)$. Not a function

62 $x + y = 0 \Rightarrow y = -x$, which is a function

63 $|y| = |x| \Rightarrow \pm y = \pm x \Rightarrow y = \pm x$, not a function: $(1, \pm 1)$

64 Many ordered pairs with x-coordinate 3 (or any other number) satisfy $y < x$.

Two such ordered pairs are $(3, 1)$ and $(3, 2)$. Not a function

65 $V = lwh = (30 - x - x)(20 - x - x)(x)$

$$= (30-2x)(20-2x)(x) = 2(15-x) \cdot 2(10-x)(x) = 4x(15-x)(10-x)$$

66 $S = 2\pi rh + 2(2\pi r^2) = 2\pi r(10) + 4\pi r^2 = 20\pi r + 4\pi r^2 = 4\pi r(5 + r)$

67 (a) The formula for the area of a rectangle is $A = lw$ {Area = length × width}.

$$A = 500 \quad \Rightarrow \quad xy = 500 \quad \Rightarrow \quad y = \frac{500}{x}$$

(b) We need to determine the number of linear feet (P) first. There are two walls of length y, two walls of length $(x-3)$, and one wall of length x, so P = Linear feet of wall $= x + 2(y) + 2(x-3) = 3x + 2\left(\frac{500}{x}\right) - 6$.

The cost C is 100 times P, so $C = 100P = 300x + \frac{100{,}000}{x} - 600$.

68 (a) $V = lwh \quad \Rightarrow \quad 6 = xy(1.5) \quad \Rightarrow \quad xy = 4 \quad \Rightarrow \quad y = \frac{4}{x}$

(b) Surface area $S = xy + 2(1.5)x + 2(1.5)y = x\left(\frac{4}{x}\right) + 3x + 3\left(\frac{4}{x}\right) = 4 + 3x + \frac{12}{x}$

69 The expression $(h - 25)$ represents the number of feet *above* 25 feet.

$S(h) = 6(h-25) + 100 = 6h - 150 + 100 = 6h - 50$.

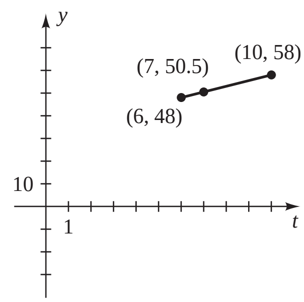
70 $T(x) = \frac{125{,}000 \text{ BTUs}}{1 \text{ gallon of gas}} \cdot x \text{ gallons} \cdot \frac{\$0.342}{1{,}000{,}000 \text{ BTU}} = \$0.04275x$.

71 (a) Using $(6, 48)$ and $(7, 50.5)$, we have

$y - 48 = \frac{50.5 - 48}{7 - 6}(t - 6)$, or $y = 2.5t + 33$.

(b) The slope represents the yearly increase in height, 2.5 in./yr.

(c) $t = 10 \quad \Rightarrow \quad y = 2.5(10) + 33 = 58$ in.



72 Let A denote the area of the contamination. A is linearly related to t, so $A = at + b$.

$A = 0$ when $t = 0 \quad \Rightarrow \quad b = 0$. $A = 40{,}000$ when $t = 40 \quad \Rightarrow \quad a = 1000$.

Thus, $A = 1000t$. Since the contamination is circular, $A = \pi r^2$. Hence, $\pi r^2 = 1000t \quad \Rightarrow \quad r = \sqrt{\frac{1000t}{\pi}}$.

73 The height of the balloon is $2t$. Using the Pythagorean theorem,

$$d^2 = 100^2 + (2t)^2 = 10{,}000 + 4t^2 \quad \Rightarrow \quad d = \sqrt{4(2500 + t^2)} \quad \Rightarrow \quad d = 2\sqrt{t^2 + 2500}.$$

74 (a) By the Pythagorean theorem, $x^2 + y^2 = 15^2 \quad \Rightarrow \quad y = \sqrt{225 - x^2}$.

(b) $\mathcal{A} = \frac{1}{2}bh = \frac{1}{2}xy = \frac{1}{2}x\sqrt{225 - x^2}$.

The domain of this function is $-15 \le x \le 15$; however, only $0 < x < 15$ will form triangles.

75 (a) CTP forms a right angle, so the Pythagorean theorem may be applied.

$(CT)^2 + (PT)^2 = (PC)^2 \quad \Rightarrow \quad r^2 + y^2 = (h + r)^2 \quad \Rightarrow$

$$r^2 + y^2 = h^2 + 2hr + r^2 \quad \Rightarrow \quad y^2 = h^2 + 2hr \; \{y > 0\} \quad \Rightarrow \quad y = \sqrt{h^2 + 2hr}$$

(b) $y = \sqrt{(200)^2 + 2(4000)(200)} = \sqrt{(200)^2(1 + 40)} = 200\sqrt{41} \approx 1280.6$ mi

76 (a) The dimensions L, 50, and $x - 2$ form a right triangle 2 feet off the ground.

Thus, $L^2 = 50^2 + (x-2)^2 \quad \Rightarrow \quad L = \sqrt{2500 + (x-2)^2}$.

(b) $L = 75 \quad \Rightarrow \quad 75^2 = 50^2 + (x-2)^2 \quad \Rightarrow \quad x - 2 = \pm\sqrt{3125} \quad \Rightarrow \quad x = 25\sqrt{5} + 2 \approx 57.9$ ft

77 Form a right triangle with the control booth and the beginning of the runway. Let y denote the distance from the control booth to the beginning of the runway and apply the Pythagorean theorem. $y^2 = 300^2 + 20^2 \quad \Rightarrow \quad y^2 = 90{,}400$. Now form a right triangle, in a different plane, with sides y and x and hypotenuse d. Then $d^2 = y^2 + x^2 \quad \Rightarrow \quad d^2 = 90{,}400 + x^2 \quad \Rightarrow \quad d = \sqrt{90{,}400 + x^2}$.

78 $\text{Time}_{\text{total}} = \text{Time}_{\text{rowing}} + \text{Time}_{\text{walking}}$ {use $t = d/r$ and $d(A, P) = 6 - x$} $\Rightarrow$

$$T = \frac{\sqrt{2^2 + (6-x)^2}}{3} + \frac{x}{5} \quad \Rightarrow \quad T = \frac{\sqrt{x^2 - 12x + 40}}{3} + \frac{x}{5}$$

79 (b) The maximum y-value of 0.75 occurs when $x \approx 0.55$ and the minimum y-value of -0.75 occurs when $x \approx -0.55$. Therefore, the range of f is approximately $[-0.75, 0.75]$.

(c) f is decreasing on $[-2, -0.55]$ and on $[0.55, 2]$. f is increasing on $[-0.55, 0.55]$.

$[-2, 2]$ by $[-2, 2]$

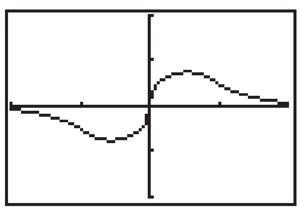


Figure 79

$[-1, 1]$ by $[-1, 1]$

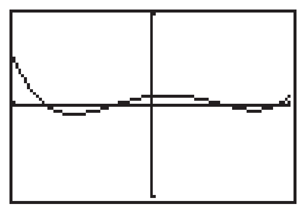


Figure 80

80 (b) The maximum y-value of 0.5 occurs when $x = -1$ and the minimum y-value of -0.094 occurs when $x \approx -0.56$. Therefore, the range of f is approximately $[-0.094, 0.5]$.

(c) f is decreasing on $[-1, -0.56]$ and on $[0.12, 0.75]$. f is increasing on $[-0.56, 0.12]$ and on $[0.75, 1]$.

81 (b) The maximum y-value of 1 occurs when $x = 0$ and the minimum y-value of -1.03 occurs when $x \approx 1.06$. Therefore, the range of f is approximately $[-1.03, 1]$.

(c) f is decreasing on $[0, 1.06]$. f is increasing on $[-0.7, 0]$ and on $[1.06, 1.4]$.

$[-0.7, 1.4]$ by $[-1.1, 1]$

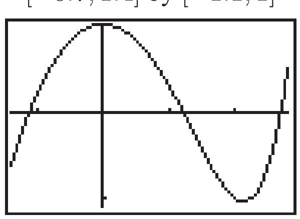


Figure 81

$[-4, -4]$ by $[-4, -4]$

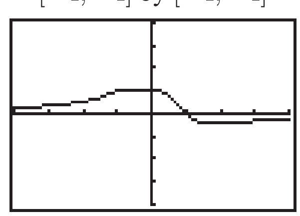


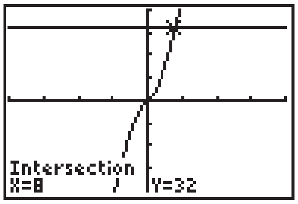
Figure 82

82 (b) The maximum y-value of 1.08 occurs when $x \approx -0.69$ and the minimum y-value of -0.42 occurs when $x \approx 1.78$. Therefore, the range of f is approximately $[-0.42, 1.08]$.

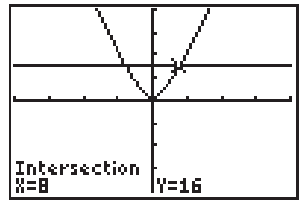
(c) f is decreasing on $[-0.69, 1.78]$. f is increasing on $[-4, -0.69]$ and on $[1.78, 4]$.

83 For each of (a)–(e), an assignment to Y_1, an appropriate viewing rectangle and the solution(s) are listed.

(a) $Y_1 = (x\hat{\ }5)\hat{\ }(1/3) = 32$,
VR: $[-40, 40, 10]$ by $[-40, 40, 10]$, $x = 8$



(b) $Y_1 = (x\hat{\ }4)\hat{\ }(1/3) = 16$,
VR: $[-40, 40, 10]$ by $[-40, 40, 10]$, $x = \pm 8$



(c) $Y_1 = (x\hat{\ }2)\hat{\ }(1/3) = -64$, VR: $[-40, 40, 10]$ by $[-40, 40, 10]$, no real solutions

(d) $Y_1 = (x\hat{\ }3)\hat{\ }(1/4) = 125$, VR: $[0, 800, 100]$ by $[0, 200, 100]$, $x = 625$

(e) $Y_1 = (x\hat{\ }3)\hat{\ }(1/2) = -27$, VR: $[-30, 30, 10]$ by $[-30, 30, 10]$, no real solutions

84 (a) $Y_1 = (x\hat{\ }3)\hat{\ }(1/5) = -27$, VR: $[-250, 250, 100]$ by $[-30, 30, 10]$, $x = -243$

(b) $Y_1 = (x\hat{\ }2)\hat{\ }(1/3) = 25$, VR: $[-130, 130, 50]$ by $[0, 30, 10]$, $x = \pm 125$

(c) $Y_1 = (x\hat{\ }4)\hat{\ }(1/3) = -49$, VR: $[-50, 50, 10]$ by $[-50, 50, 10]$, no real solutions

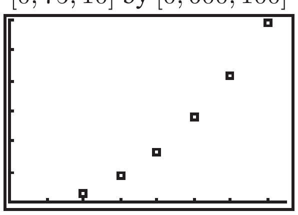
(d) $Y_1 = (x\hat{\ }3)\hat{\ }(1/2) = 64$, VR: $[-5, 30, 5]$ by $[-5, 30, 5]$, $x = 16$

(e) $Y_1 = (x\hat{\ }3)\hat{\ }(1/4) = -8$, VR: $[-10, 10,]$ by $[-10, 10]$, no real solutions

85 (a) There are $95 \times 63 = 5985$ total pixels in the screen.

(b) If a function is graphed in dot mode, only one pixel in each column of pixels on the screen can be darkened. Therefore, there are at most 95 pixels darkened. ***Note:*** In connected mode this may not be true.

86 (a) $[0, 75, 10]$ by $[0, 600, 100]$

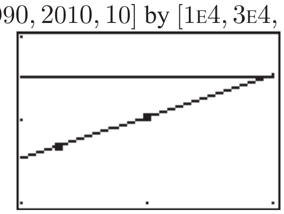


(b) The data and plot show that stopping distance is not a linear function of the speed. The distance required to stop a car traveling at 30 mi/hr is 86 ft whereas the distance required to stop a car traveling at 60 mi/hr is 414 ft. $\frac{414}{86} \approx 4.81$ rather than double.

(c) If you double the speed of a car, it requires almost *five times* the stopping distance. If stopping distance were a linear function of speed, doubling the speed would require twice the stopping distance.

87 (a) First, we must determine an equation of the line that passes through the points $(1993, 16{,}871)$ and $(2000, 20{,}356)$.
$y - 16{,}871 = \frac{20{,}356-16{,}871}{2000-1993}(x - 1993) = \frac{3485}{7}(x - 1993) \Rightarrow$
$y = \frac{3485}{7}x - \frac{6{,}827{,}508}{7}$. Thus, let $f(x) = \frac{3485}{7}x - \frac{6{,}827{,}508}{7}$ and graph f.

$[1990, 2010, 10]$ by $[1\text{E}4, 3\text{E}4, 1\text{E}4]$



(b) The average annual increase in the price paid for a new car is equal to the slope: $\frac{3485}{7} \approx \$497.86$.

(c) Graph $y = \frac{3485}{7}x - \frac{6{,}827{,}508}{7}$ and $y = 25{,}000$ on the same coordinate axes. Their point of intersection is approximately $(2009.33, 25{,}000)$. Thus, according to this model, in the year 2009 the average price paid for a new car will be \$25,000.

3.5 Exercises

1 Since f is an even function, $f(-x) = f(x)$. From the table, we see that $f(2) = 7$, so $f(-2) = 7$. In general, if f is an even function, and (a, b) is a point on the graph of f, then the point $(-a, b)$ is also on the graph.

Since g is an odd function, $g(-x) = -g(x)$. From the table, we see that $g(2) = -6$, so $g(-2) = -(-6) = 6$. In general, if f is an odd function, and (a, b) is a point on the graph of f, then the point $(-a, -b)$ is also on the graph.

2 Since f is an even function, $f(-x) = f(x)$. From the table, we see that $f(3) = -5$, so $f(-3) = -5$.

Since g is an odd function, $g(-x) = -g(x)$. From the table, we see that $g(3) = 6$, so $g(-3) = -6$.

3 $f(x) = 5x^3 + 2x \quad \Rightarrow \quad f(-x) = 5(-x)^3 + 2(-x) = -5x^3 - 2x$ and

$-f(x) = -1 \cdot f(x) = -(5x^3 + 2x) = -5x^3 - 2x$.

Since $f(-x) = -f(x)$, f is odd and its graph is symmetric with respect to the origin.

Note that this means if (a, b) is a point on the graph of f, then the point $(-a, -b)$ is also on the graph.

4 $f(x) = |x| - 3 \quad \Rightarrow \quad f(-x) = |-x| - 3 = |x| - 3 = f(x)$, so f is even

5 $f(x) = 3x^4 - 6x^2 - 5 \quad \Rightarrow \quad f(-x) = 3(-x)^4 - 6(-x)^2 - 5 = 3x^4 - 6x^2 - 5 = f(x)$.

Since $f(-x) = f(x)$, f is even and its graph is symmetric with respect to the y-axis.

Note that this means if (a, b) is a point on the graph of f, then the point $(-a, b)$ is also on the graph.

6 $f(x) = 7x^5 + 2x^3 \quad \Rightarrow \quad f(-x) = 7(-x)^5 + 2(-x)^3 = -7x^5 - 2x^3 = -(7x^5 + 2x^3) = -f(x)$, so f is odd

7 $f(x) = 8x^3 - 3x^2 \quad \Rightarrow \quad f(-x) = 8(-x)^3 - 3(-x)^2 = -8x^3 - 3x^2$

$-f(x) = -1 \cdot f(x) = -1(8x^3 - 3x^2) = -8x^3 + 3x^2$

Since $f(-x) \neq f(x)$ and $f(-x) \neq -f(x)$, f is neither even nor odd.

8 $f(x) = \sqrt[3]{5} \quad \Rightarrow \quad f(-x) = \sqrt[3]{5} = f(x)$, so f is even

9 $f(x) = \sqrt{x^2 + 4} \quad \Rightarrow \quad f(-x) = \sqrt{(-x)^2 + 4} = \sqrt{x^2 + 4} = f(x)$.

Since $f(-x) = f(x)$, f is even and its graph is symmetric with respect to the y-axis.

10 $f(x) = 3x^2 + 2x - 4 \quad \Rightarrow \quad f(-x) = 3(-x)^2 + 2(-x) - 4 = 3x^2 - 2x - 4 \neq \pm f(x)$,

so f is neither even nor odd

11 $f(x) = \sqrt[3]{x^3 - x} \quad \Rightarrow$

$f(-x) = \sqrt[3]{(-x)^3 - (-x)} = \sqrt[3]{-x^3 + x} = \sqrt[3]{-1(x^3 - x)} = \sqrt[3]{-1}\,\sqrt[3]{x^3 - x} = -\sqrt[3]{x^3 - x}$.

$-f(x) = -1 \cdot f(x) = -1 \cdot \sqrt[3]{x^3 - x} = -\sqrt[3]{x^3 - x}$.

Since $f(-x) = -f(x)$, f is odd and its graph is symmetric with respect to the origin.

12 $f(x) = x^3 - \dfrac{1}{x} \quad \Rightarrow \quad f(-x) = (-x)^3 - \dfrac{1}{-x} = -x^3 + \dfrac{1}{x} = -\left(x^3 - \dfrac{1}{x}\right) = -f(x)$, so f is odd

13 $f(x) = |x| + c,\ c = -3, 1, 3$ • Shift $g(x) = |x|$ {in Figure 1 in the text} down 3, up 1, and up 3 units, respectively.

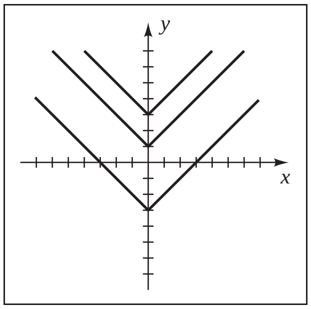


Figure 13

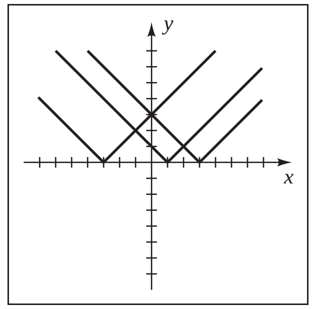


Figure 14

14 $f(x) = |x - c|, c = -3, 1, 3$ • Shift $g(x) = |x|$ left 3, right 1, and right 3 units, respectively.

15 $f(x) = -x^2 + c, c = -4, 2, 4$ • Shift $g(x) = -x^2$ {in Figure 5 in the text} down 4, up 2, up 4, respectively.

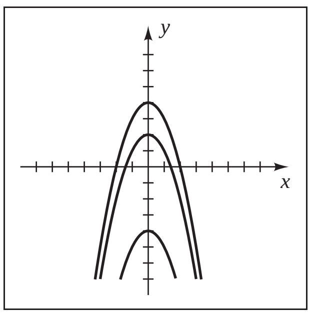


Figure 15

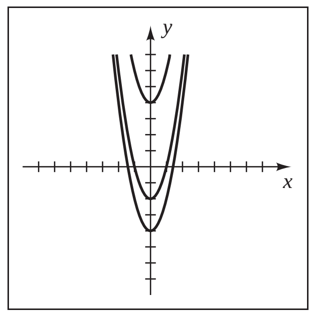


Figure 16

16 $f(x) = 2x^2 - c, c = -4, 2, 4$ • Shift $g(x) = 2x^2$ up 4, down 2, down 4, respectively.

17 $f(x) = 2\sqrt{x} + c, c = -3, 0, 2$ • The graph of $y^2 = x$ is shown in Figure 3 in Section 3.2 of the text. The top half of this graph is the graph of the square root function, $h(x) = \sqrt{x}$. The second value of c, 0, gives us the graph of $g(x) = 2\sqrt{x}$, which is a vertical stretching of h by a factor of 2. The effect of adding -3 and 2 is to vertically shift g down 3 units and up 2 units, respectively.

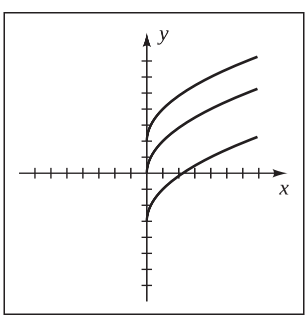


Figure 17

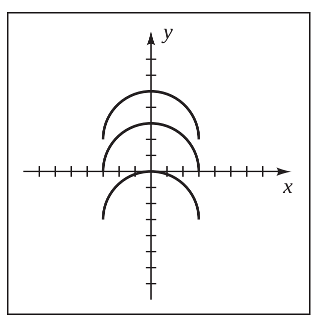


Figure 18

18 $f(x) = \sqrt{9 - x^2} + c, c = -3, 0, 2$ • Shift $g(x) = \sqrt{9 - x^2}$ down 3, up 2, respectively.

19 $f(x) = \frac{1}{2}\sqrt{x - c}$, $c = -3, 0, 4$ • The graph of $g(x) = \frac{1}{2}\sqrt{x}$ is a vertical compression of the square root function by a factor of $1/(1/2) = 2$. The effect of subtracting -3 and 4 from x will be to horizontally shift g left 3 units and right 4 units, respectively. If you forget which way to shift the graph, it is helpful to find the domain of the function. For example, if $h(x) = \sqrt{x - 2}$, then $x - 2$ must be nonnegative. $x - 2 \geq 0 \quad \Rightarrow \quad x \geq 2$, which also indicates a shift of 2 units to the right.

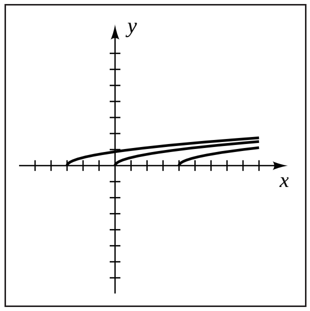


Figure 19

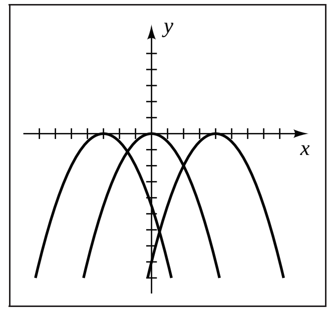


Figure 20

20 $f(x) = -\frac{1}{2}(x - c)^2$, $c = -3, 0, 4$ • Shift $g(x) = -\frac{1}{2}x^2$ left 3, right 4, respectively.

21 $f(x) = c\sqrt{4 - x^2}$, $c = -2, 1, 3$ • For $c = 1$, the graph of $g(x) = \sqrt{4 - x^2}$ is the upper half of the circle $x^2 + y^2 = 4$. For $c = -2$, reflect g through the x-axis and vertically stretch it by a factor of 2. For $c = 3$, vertically stretch g by a factor of 3.

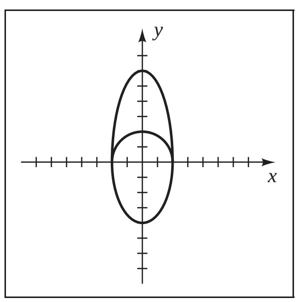


Figure 21

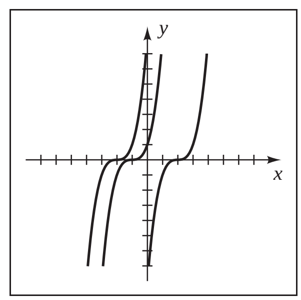


Figure 22

22 $f(x) = (x + c)^3$, $c = -2, 1, 2$ • Shift $g(x) = x^3$ right 2, left 1, and left 2, respectively.

23 $f(x) = cx^3$, $c = -\frac{1}{3}, 1, 2$ • For $c = 1$, see a graph of the cubing function in Appendix I of the text. For $c = -\frac{1}{3}$, reflect $g(x) = x^3$ through the x-axis and vertically compress it by a factor of $1/(1/3) = 3$. For $c = 2$, vertically stretch g by a factor of 2.

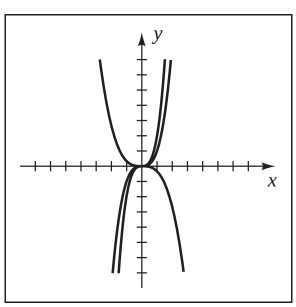


Figure 23

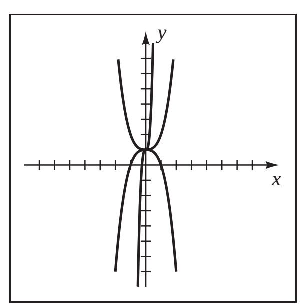


Figure 24

24 $f(x) = (cx)^3 + 1$, $c = -1, 1, 4$ • For $c = -1$, reflect $g(x) = x^3 + 1$ through the y-axis. For $c = 4$, horizontally compress g by a factor of 4 {this could also be considered as a vertical stretch by a factor of $4^3 = 64$}.

25 $f(x) = \sqrt{cx} - 1,\ c = -1,\ \frac{1}{9},\ 4$ • If $c = 1$, then the graph of $g(x) = \sqrt{x} - 1$ is the graph of the square root function vertically shifted down one unit. For $c = -1$, reflect g through the y-axis. For $c = \frac{1}{9}$, horizontally stretch g by a factor $1/(1/9) = 9$ {x-intercept changes from 1 to 9}. For $c = 4$, horizontally compress g by a factor 4 {x-intercept changes from 1 to $\frac{1}{4}$}.

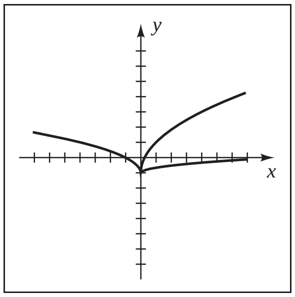


Figure 25

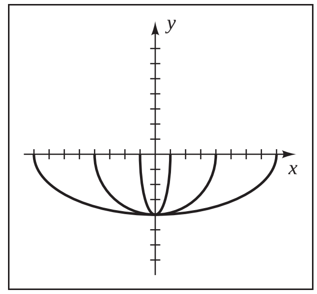


Figure 26

26 $f(x) = -\sqrt{16 - (cx)^2},\ c = 1,\ \frac{1}{2},\ 4$ • For $c = \frac{1}{2}$, horizontally stretch $g(x) = -\sqrt{16 - x^2}$ by a factor of $1/(1/2) = 2$ {x-intercepts change from ± 4 to ± 8}. For $c = 4$, horizontally compress g by a factor of 4 {x-intercepts change from ± 4 to ± 1}.

27 You know that $y = f(x+2)$ is the graph of $y = f(x)$ shifted to the left 2 units, so the point $P(0,5)$ would move to the point $(-2,5)$. The graph of $y = f(x+2) - 1$ is the graph of $y = f(x+2)$ shifted down 1 unit, so the point $(-2,5)$ moves to $(-2,4)$. Summarizing these steps gives us the following:

$P(0,5)$	{$x + 2$ [subtract 2 from the x-coordinate]}	$\to (-2,5)$
	{-1 [subtract 1 from the y-coordinate]}	$\to (-2,4)$

28 $y = 2f(x) + 4$ • $P(3,-1)$

{$\times 2$ [multiply the y-coordinate by 2]}	$\to (3,-2)$
{$+4$ [add 4 to the y-coordinate]}	$\to (3,2)$

29 To determine what happens to a point P under this transformation, think of how you would evaluate $y = 2f(x-4) + 1$ for a particular value of x. You would first subtract 4 from x and then put that value into the function, obtaining a corresponding y-value. Next, you would multiply that y-value by 2 and finally, add 1. Summarizing these steps using the given point P, we have the following:

$P(3,-2)$	{$x - 4$ [add 4 to the x-coordinate]}	$\to (7,-2)$
	{$\times 2$ [multiply the y-coordinate by 2]}	$\to (7,-4)$
	{$+1$ [add 1 to the y-coordinate]}	$\to (7,-3)$

30 $y = \frac{1}{2}f(x-3) + 3$ • $P(-5,8)$

{$x - 3$ [add 3 to the x-coordinate]}	$\to (-2,8)$
{$\times \frac{1}{2}$ [multiply the y-coordinate by $\frac{1}{2}$]}	$\to (-2,4)$
{$+3$ [add 3 to the y-coordinate]}	$\to (-2,7)$

31 $y = \frac{1}{3}f\left(\frac{1}{2}x\right) - 1$ • $P(4,9)$

{$\frac{1}{2}x$ [multiply the x-coordinate by 2]}	$\to (8,9)$
{$\times \frac{1}{3}$ [multiply the y-coordinate by $\frac{1}{3}$]}	$\to (8,3)$
{-1 [subtract 1 from the y-coordinate]}	$\to (8,2)$

32 $y = -3f(2x) - 5$ • $P(-2,1)$

{$2x$ [divide the x-coordinate by 2]}	$\to (-1,1)$
{$\times (-3)$ [multiply the y-coordinate by -3]}	$\to (-1,-3)$
{-5 [subtract 5 from the y-coordinate]}	$\to (-1,-8)$

33 For $y = f(x-2) + 3$, the graph of f is shifted 2 units to the right and 3 units up.

34 For $y = 3f(x-1)$, the graph of f is shifted 1 unit to the right and stretched vertically by a factor of 3.

35 For $y = f(-x) - 4$, the graph of f is reflected through the y-axis and shifted 4 units down.

36 For $y = -f(x+2)$, the graph of f is shifted 2 units to the left and reflected through the x-axis.

37 For $y = -\frac{1}{2}f(x)$, the graph of f is compressed vertically by a factor of 2 and reflected through the x-axis.

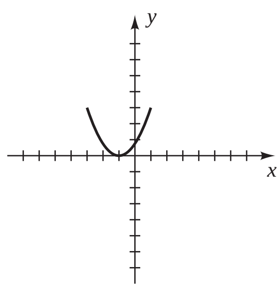
38 For $y = f\left(\frac{1}{2}x\right) - 3$, the graph of f is stretched horizontally by a factor of 2 and shifted 3 units down.

39 For $y = -2f\left(\frac{1}{3}x\right)$, the graph of f is stretched horizontally by a factor of 3, stretched vertically by a factor of 2, and reflected through the x-axis.

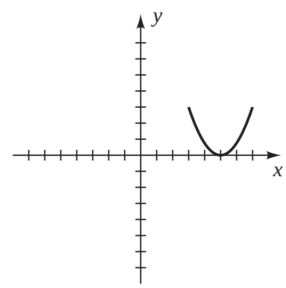
40 For $y = \frac{1}{3}|f(x)|$, the part of the graph of f below the x-axis is reflected through the x-axis and that graph is compressed vertically by a factor of 3.

41 When graphing the parts in this exercise, it may help to keep track of at least one of the points $(0,3)$, $(2,0)$, or $(4,3)$, on the given graph.

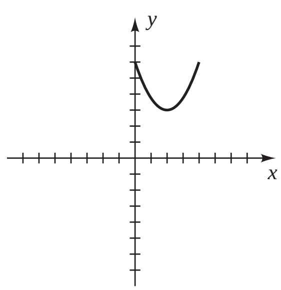
(a) $y = f(x+3)$ • shift f left 3 units



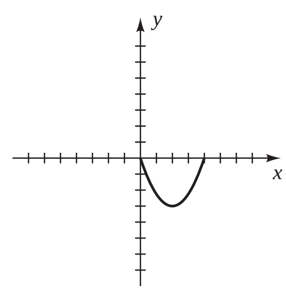
(b) $y = f(x-3)$ • shift f right 3 units



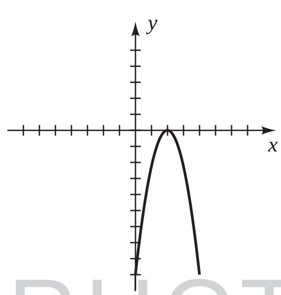
(c) $y = f(x) + 3$ • shift f up 3 units



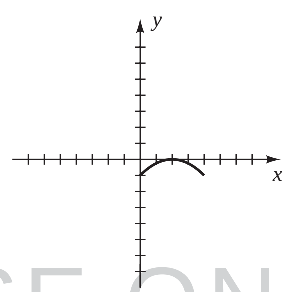
(d) $y = f(x) - 3$ • shift f down 3 units



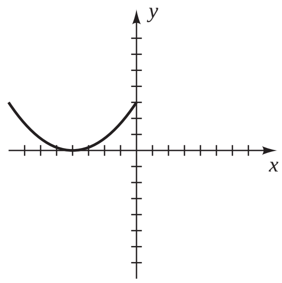
(e) $y = -3f(x)$ • reflect f through the x-axis {the effect of the negative sign in front of 3} and vertically stretch it by a factor of 3



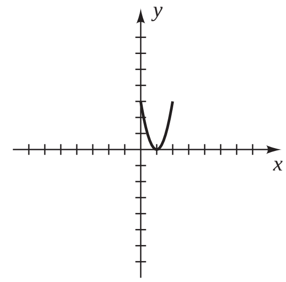
(f) $y = -\frac{1}{3}f(x)$ • reflect f through the x-axis {the effect of the negative sign in front of $\frac{1}{3}$} and vertically compress it by a factor of $1/(1/3) = 3$



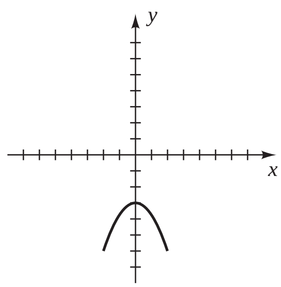
(g) $y = f\left(-\frac{1}{2}x\right)$ • reflect f through the y-axis {the effect of the negative sign inside the parentheses} and horizontally stretch it by a factor of $1/(1/2) = 2$



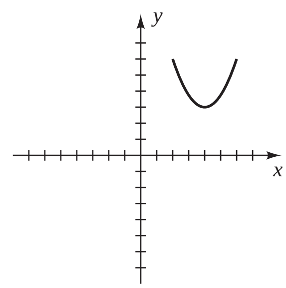
(h) $y = f(2x)$ • horizontally compress f by a factor of 2



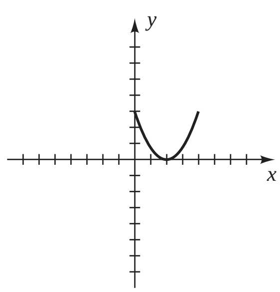
(i) $y = -f(x+2) - 3$ • shift f left 2 units, reflect it through the x-axis, and then shift it down 3 units



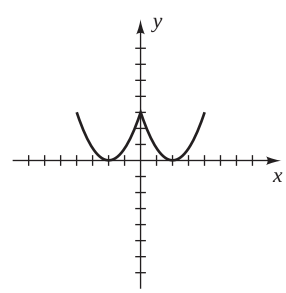
(j) $y = f(x-2) + 3$ • shift f right 2 units and up 3



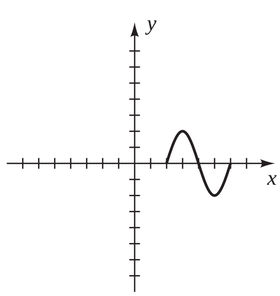
(k) $y = |f(x)|$ • since no portion of the graph lies below the x-axis, the graph is unchanged



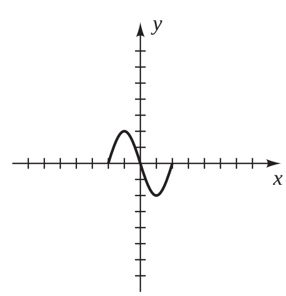
(l) $y = f(|x|)$ • include the reflection of the given graph through the y-axis since all points have positive x-coordinates



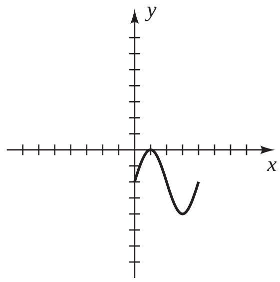
42 (a) $y = f(x-2)$ • shift f right 2 units



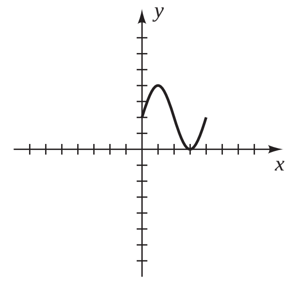
(b) $y = f(x+2)$ • shift f left 2 units



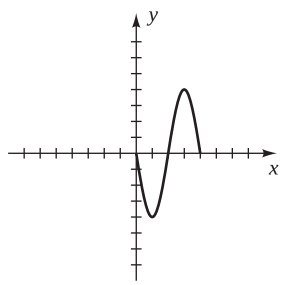
(c) $y = f(x) - 2$ • shift f down 2 units



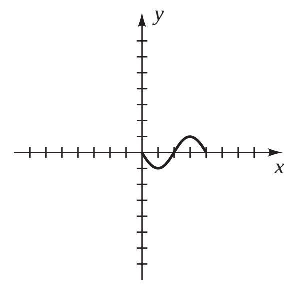
(d) $y = f(x) + 2$ • shift f up 2 units



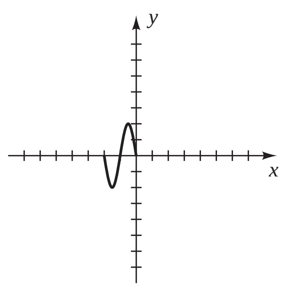
(e) $y = -2f(x)$ • reflect f through the x-axis and vertically stretch it by a factor of 2



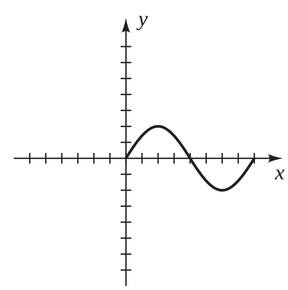
(f) $y = -\frac{1}{2}f(x)$ • reflect f through the x-axis and vertically compress it by a factor of $1/(1/2) = 2$



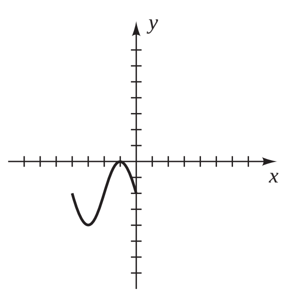
(g) $y = f(-2x)$ • reflect f through the y-axis and horizontally compress it by a factor of 2



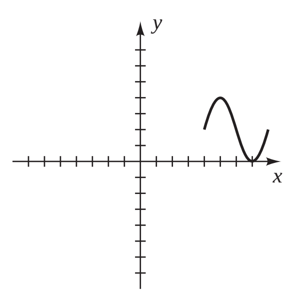
(h) $y = f\left(\frac{1}{2}x\right)$ • horizontally stretch f by a factor of $1/(1/2) = 2$



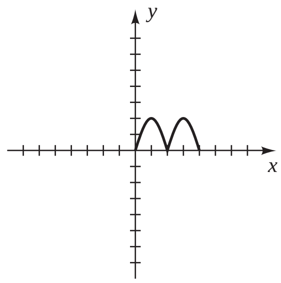
(i) $y = -f(x+4) - 2$ • reflect f about the x-axis, shift it left 4 units and down 2



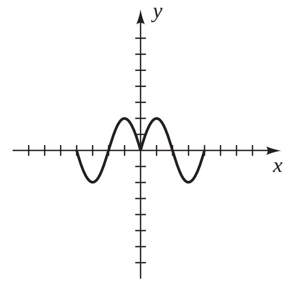
(j) $y = f(x-4) + 2$ • shift f right 4 units and up 2



(k) $y = |f(x)|$ • reflect the portion of the graph below the x-axis through the x-axis.



(l) $y = f(|x|)$ • include the reflection of the given graph through the y-axis since all points have positive x-coordinates



43 (a) The minimum point on $y = f(x)$ is $(2, -1)$. On the graph labeled (a), the minimum point is $(-7, 0)$. It has been shifted left 9 units and up 1. Hence, $y = f(x + 9) + 1$.

(b) f is reflected through the x-axis $\Rightarrow$ $y = -f(x)$

(c) f is reflected through the x-axis and shifted left 7 units and down 1 $\Rightarrow$ $y = -f(x + 7) - 1$

44 (a) f is shifted left 1 unit and up 1 $\Rightarrow$ $y = f(x + 1) + 1$

(b) f is reflected through the x-axis $\Rightarrow$ $y = -f(x)$ {or $y = f(-x)$}

(c) f is reflected through the x-axis and shifted right 2 units $\Rightarrow$ $y = -f(x - 2)$

45 (a) f is shifted left 4 units $\Rightarrow$ $y = f(x + 4)$

(b) f is shifted up 1 unit $\Rightarrow$ $y = f(x) + 1$

(c) f is reflected through the y-axis $\Rightarrow$ $y = f(-x)$

46 (a) f is shifted right 2 units and up 2 $\Rightarrow$ $y = f(x - 2) + 2$

(b) f is reflected through the x-axis $\Rightarrow$ $y = -f(x)$

(c) f is reflected through the x-axis and shifted left 4 units and up 2 $\Rightarrow$ $y = -f(x + 4) + 2$

47 $f(x) = \begin{cases} 3 & \text{if } x \leq -1 \\ -2 & \text{if } x > -1 \end{cases}$ • We can think of f as 2 functions: If $x \leq -1$, then $y = 3$ {include the point $(-1, 3)$}, and if $x > -1$, then $y = -2$ {exclude the point $(-1, -2)$}.

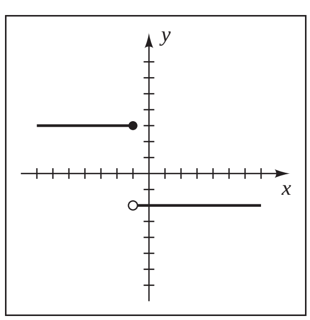


Figure 47

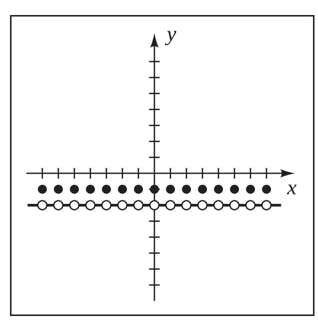


Figure 48

48 $f(x) = \begin{cases} -1 & \text{if } x \text{ is an integer} \\ -2 & \text{if } x \text{ is not an integer} \end{cases}$ • Solid dots at every integer
The line $y = -2$ with holes at every integer

49 $f(x) = \begin{cases} 3 & \text{if } x < -2 \\ -x+1 & \text{if } |x| \le 2 \\ -4 & \text{if } x > 2 \end{cases}$ • For the second part of the function, we have $|x| \le 2$, or, equivalently, $-2 \le x \le 2$. On this part of the domain, we want to graph $f(x) = -x + 1$, a line with slope -1 and y-intercept 1. Include both endpoints, $(-2, 3)$ and $(2, -1)$.

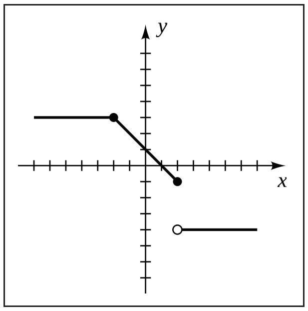


Figure 49

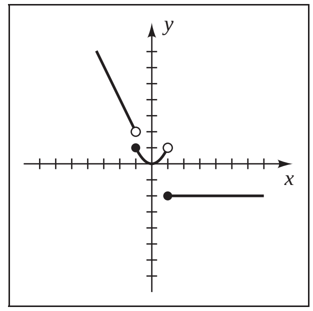


Figure 50

50 $f(x) = \begin{cases} -2x & \text{if } x < -1 \\ x^2 & \text{if } -1 \le x < 1 \\ -2 & \text{if } x \ge 1 \end{cases}$ •
A line with slope -2 and endpoint $(-1, 2)$
A parabola portion with open endpoint $(-1, 1)$ and endpoint $(1, 1)$
A horizontal line with endpoint $(1, -2)$

51 $f(x) = \begin{cases} x+2 & \text{if } x \le -1 \\ x^3 & \text{if } |x| < 1 \\ -x+3 & \text{if } x \ge 1 \end{cases}$ • If $x \le -1$, we want the graph of $y = x + 2$. To determine the endpoint of this part of the graph, merely substitute $x = -1$ in $y = x + 2$, obtaining $y = 1$. If $|x| < 1$, or, equivalently, $-1 < x < 1$, we want the graph of $y = x^3$. We do not include the endpoints $(-1, -1)$ and $(1, 1)$. If $x \ge 1$, we want the graph of $y = -x + 3$ and include its endpoint $(1, 2)$.

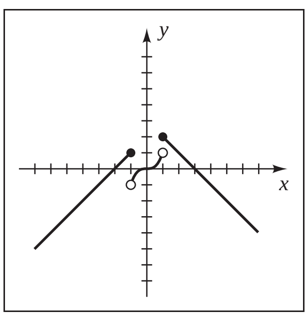


Figure 51

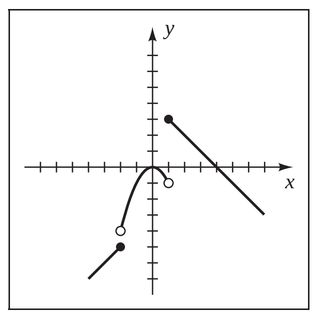


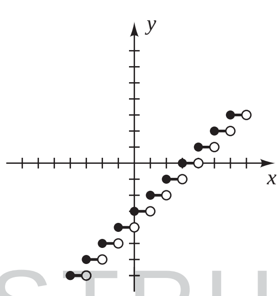
Figure 52

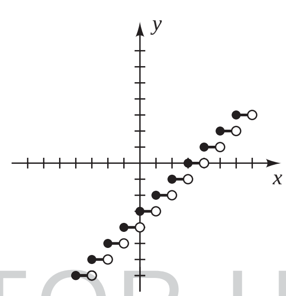
52 $f(x) = \begin{cases} x-3 & \text{if } x \le -2 \\ -x^2 & \text{if } -2 < x < 1 \\ -x+4 & \text{if } x \ge 1 \end{cases}$ •
A line with slope 1 and endpoint $(-2, -5)$
A parabola portion with open endpoints $(-2, -4)$ and $(1, -1)$
A line with slope -1 and endpoint $(1, 3)$

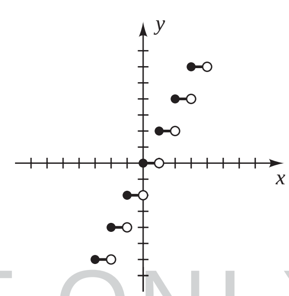
53 (a) $f(x) = [\![x - 3]\!]$ • shift $g(x) = [\![x]\!]$ right 3 units {see Figure 10 on page 190 of the text}

(b) $f(x) = [\![x]\!] - 3$ • shift $g(x) = [\![x]\!]$ down 3 units, which is the same graph as in part (a).

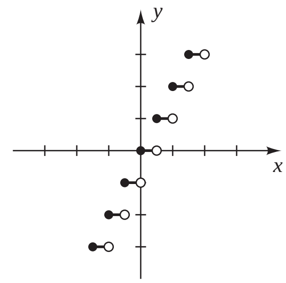
(c) $f(x) = 2[\![x]\!]$ • vertically stretch $g(x) = [\![x]\!]$ by a factor of 2



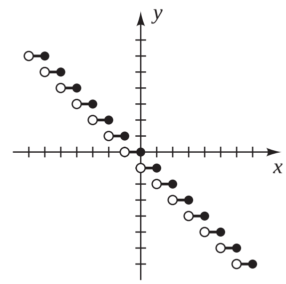




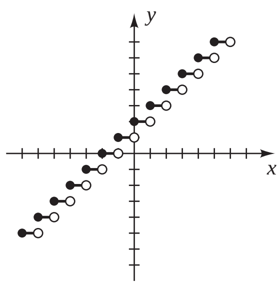
(d) $f(x) = [\![2x]\!]$ • horizontally compress $g(x) = [\![x]\!]$ by a factor of 2
Alternatively, we could determine the pattern of "steps" for this function by finding the values of x that make $f(x)$ change from 0 to 1, then from 1 to 2, etc. If $2x = 0$, then $x = 0$, and if $2x = 1$, then $x = \frac{1}{2}$.
Thus, the function will equal 0 from $x = 0$ to $x = \frac{1}{2}$ and then jump to 1 at $x = \frac{1}{2}$. If $2x = 2$, then $x = 1$. The pattern is established: each step will be $\frac{1}{2}$ unit long.



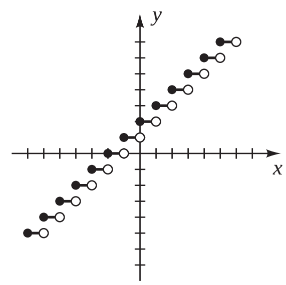
(e) $f(x) = [\![-x]\!]$ • reflect $g(x) = [\![x]\!]$ through the y-axis



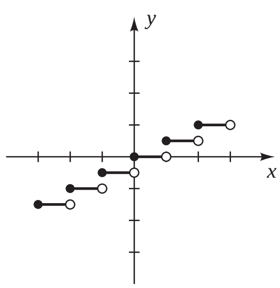
54 (a) $f(x) = [\![x + 2]\!]$ • shift $g(x) = [\![x]\!]$ left 2 units



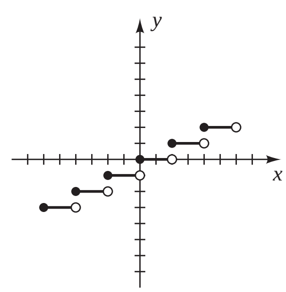
(b) $f(x) = [\![x]\!] + 2$ • shift $g(x) = [\![x]\!]$ up 2 units, which is the same graph as in part (a).



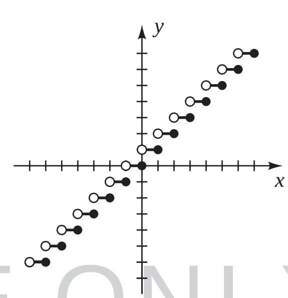
(c) $f(x) = \frac{1}{2}[\![x]\!]$ • vertically compress $g(x) = [\![x]\!]$ by a factor of $1/(1/2) = 2$



(d) $f(x) = [\![\frac{1}{2}x]\!]$ • horizontally stretch $g(x) = [\![x]\!]$ by a factor of $1/(1/2) = 2$



(e) $f(x) = -[\![-x]\!]$ • reflect $g(x) = [\![x]\!]$ through the y-axis and through the x-axis



55 (a) As $x \to 1^-$, $f(x) \to \underline{0}$ (b) As $x \to 1^+$, $f(x) \to \underline{2}$ (c) As $x \to -2$, $f(x) \to \underline{1}$

(d) As $x \to \infty$, $f(x) \to \underline{4}$ (e) As $x \to -\infty$, $f(x) \to \underline{-\infty}$

56 (a) As $x \to 2^-$, $f(x) \to \underline{3}$ (b) As $x \to 2^+$, $f(x) \to \underline{1}$ (c) As $x \to -1$, $f(x) \to \underline{-2}$

(d) As $x \to \infty$, $f(x) \to \underline{-\infty}$ (e) As $x \to -\infty$, $f(x) \to \underline{4}$

57 A question you can ask to help determine if a relationship is a function is "If x is a particular value, can I find a *unique* y-value?" In this case, if x was 16, then $16 = y^2 \Rightarrow y = \pm 4$. Since we cannot find a *unique* y-value, this is not a function. Graphically, {see Figure 3 in Section 3.2 in the text} given any x-value greater than 0, there are two points on the graph and a vertical line intersects the graph in more than one point.

58 The graph of $x = -|y|$ is not the graph of a function because if $x < 0$, two different points on the graph have x-coordinate x.

59 Reflect each portion of the graph that is below the x-axis through the x-axis.

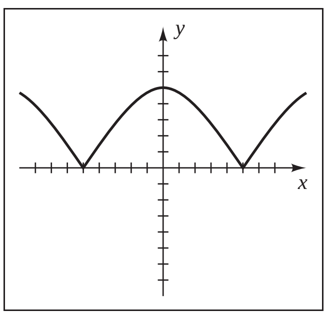


Figure 59

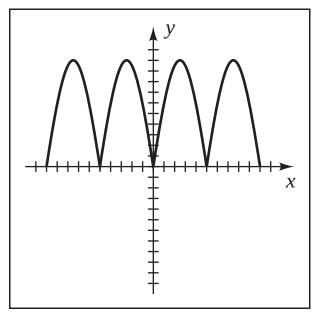


Figure 60

60 Reflect each portion of the graph that is below the x-axis through the x-axis.

61 $y = |4 - x^2|$ • First sketch $y = 4 - x^2$, then reflect the portions of the graph below the x-axis through the x-axis.

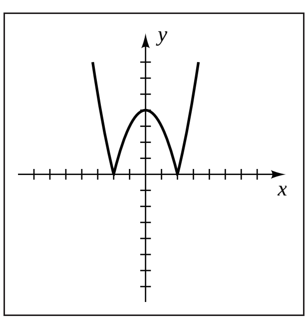


Figure 61

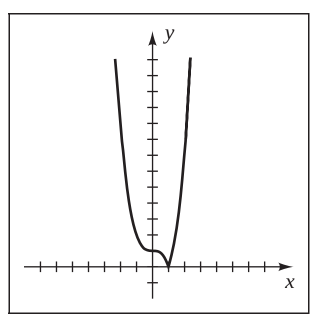


Figure 62

62 $y = |x^3 - 1|$ • First sketch $y = x^3 - 1$, then reflect the portions of the graph below the x-axis through the x-axis.

63 $y = \left|\sqrt{x} - 2\right|$ • First sketch $y = \sqrt{x} - 2$, which is the graph of the square root function shifted down 2 units. Then reflect the portions of the graph below the x-axis through the x-axis.

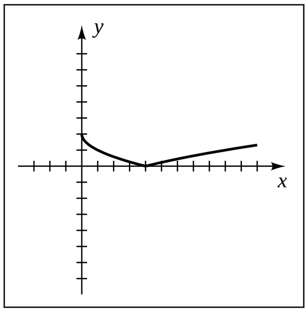


Figure 63

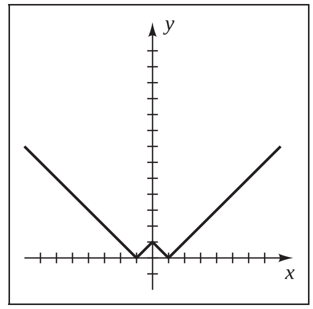


Figure 64

64 $y = \left||x| - 1\right|$ • First sketch $y = |x| - 1$, which is the graph of the absolute value function shifted down 1 unit. Then reflect the portions of the graph below the x-axis through the x-axis.

65 (a) For $y = -2f(x)$, multiply the y-coordinates by -2. So the given range, $[-4, 8]$, becomes $[-16, 8]$ {note that we changed the order so that -16 appears first—listing the range as $[8, -16]$ is incorrect}. The domain {x-coordinates} remains the same. $D = [-2, 6]$, $R = [-16, 8]$

(b) For $y = f\left(\frac{1}{2}x\right)$, multiply the x-coordinates by 2. The range {y-coordinates} remains the same. $D = [-4, 12]$, $R = [-4, 8]$

(c) For $y = f(x - 3) + 1$, add 3 to the x-coordinates and add 1 to the y-coordinates. $D = [1, 9]$, $R = [-3, 9]$

(d) For $y = f(x + 2) - 3$, subtract 2 from the x-coordinates and subtract 3 from the y-coordinates. $D = [-4, 4]$, $R = [-7, 5]$

(e) For $y = f(-x)$, multiply all x-coordinates by -1. $D = [-6, 2]$, $R = [-4, 8]$

(f) For $y = -f(x)$, multiply all y-coordinates by -1. $D = [-2, 6]$, $R = [-8, 4]$

(g) $y = f(|x|)$ • Graphically, we can reflect all points with positive x-coordinates through the y-axis, so the domain $[-2, 6]$ becomes $[-6, 6]$. Algebraically, we are replacing x with $|x|$, so $-2 \le x \le 6$ becomes $-2 \le |x| \le 6$, which is equivalent to $|x| \le 6$, or, equivalently, $-6 \le x \le 6$. The range stays the same because of the given assumptions: $f(2) = 8$ and $f(6) = -4$; that is, the full range is taken on for $x \ge 0$. Note that the range could not be determined if $f(-2)$ was equal to 8. $D = [-6, 6]$, $R = [-8, 4]$

(h) $y = |f(x)|$ • The points with y-coordinates having values from -4 to 0 will have values from 0 to 4, so the range will be $[0, 8]$. $D = [-2, 6]$, $R = [0, 8]$

66 (a) For $y = \frac{1}{2}f(x)$, multiply the y-coordinates by $\frac{1}{2}$. $D = [-6, -2]$, $R = [-5, -2]$

(b) For $y = f(2x)$, multiply the x-coordinates by $\frac{1}{2}$. $D = [-3, -1]$, $R = [-10, -4]$

(c) For $y = f(x - 2) + 5$, add 2 to the x-coordinates and add 5 to the y-coordinates. $D = [-4, 0]$, $R = [-5, 1]$

(d) For $y = f(x + 4) - 1$, subtract 4 from the x-coordinates and subtract 1 from the y-coordinates. $D = [-10, -6]$, $R = [-11, -5]$

(e) For $y = f(-x)$, negate all x-coordinates. $D = [2, 6]$, $R = [-10, -4]$

(f) For $y = -f(x)$, negate all y-coordinates. $D = [-6, -2]$, $R = [4, 10]$

(g) For $y = f(|x|)$, there is no graph since the domain of f consists of only negative values, -6 to -2, and $|x|$ is never negative.

(h) For $y = |f(x)|$, the negative y-coordinates having values from -10 to -4 will have values from 4 to 10.
$D = [-6, -2]$, $R = [4, 10]$

67 If $x \leq 20{,}000$, then $T(x) = 0.15x$. If $x > 20{,}000$, then the tax is 15% of the first 20,000, which is 3000, plus 20% of the amount over 20,000, which is $(x - 20{,}000)$. We may summarize and simplify as follows:

$$T(x) = \begin{cases} 0.15x & \text{if } 0 \leq x \leq 20{,}000 \\ 3000 + 0.20(x - 20{,}000) & \text{if } x > 20{,}000 \end{cases} = \begin{cases} 0.15x & \text{if } 0 \leq x \leq 20{,}000 \\ 3000 + 0.20x - 4000 & \text{if } x > 20{,}000 \end{cases}$$
$$= \begin{cases} 0.15x & \text{if } 0 \leq x \leq 20{,}000 \\ 0.20 - 1000 & \text{if } x > 20{,}000 \end{cases}$$

68 If $0 \leq x \leq 600{,}000$, then $T(x) = 0.01x$. If $x > 600{,}000$, then the tax is 1% of the first 600,000, which is 6000, plus 1.25% of the amount over 600,000.

$$T(x) = \begin{cases} 0.01x & \text{if } 0 \leq x \leq 600{,}000 \\ 6000 + 0.0125(x - 600{,}000) & \text{if } x > 600{,}000 \end{cases} = \begin{cases} 0.01x & \text{if } 0 \leq x \leq 600{,}000 \\ 0.0125x - 1500 & \text{if } x > 600{,}000 \end{cases}$$

69 The author receives \$1.20 on the first 10,000 copies, \$1.50 on the next 5000, and \$1.80 on each additional copy.

$$R(x) = \begin{cases} 1.20x & \text{if } 0 \leq x \leq 10{,}000 \\ 12{,}000 + 1.50(x - 10{,}000) & \text{if } 10{,}000 < x \leq 15{,}000 \\ 19{,}500 + 1.80(x - 15{,}000) & \text{if } x > 15{,}000 \end{cases} = \begin{cases} 1.20x & \text{if } 0 \leq x \leq 10{,}000 \\ 1.50x - 3000 & \text{if } 10{,}000 < x \leq 15{,}000 \\ 1.80x - 7500 & \text{if } x > 15{,}000 \end{cases}$$

70 The cost for 1000 kWh is \$57.70 and the cost for 5000 kWh is $\$57.70 + 4000(\$0.0532) = \$270.50$.

$$C(x) = \begin{cases} 0.0577x & \text{if } 0 \leq x \leq 1000 \\ 57.70 + 0.0532(x - 1000) & \text{if } 1000 < x \leq 5000 \\ 270.50 + 0.0511(x - 5000) & \text{if } x > 5000 \end{cases} = \begin{cases} 0.0577x & \text{if } 0 \leq x \leq 1000 \\ 4.50 + 0.0532x & \text{if } 1000 < x \leq 5000 \\ 15.00 + 0.0511x & \text{if } x > 5000 \end{cases}$$

71 Assign ABS$(1.3x + 2.8)$ to Y_1 and $1.2x + 5$ to Y_2. The viewing rectangle $[-10, 50, 10]$ by $[-10, 50, 10]$ shows intersection points at exactly -3.12 and -22. The solution of $|1.3x + 2.8| < 1.2x + 5$ is the interval $(-3.12, 22)$.

72 The solutions of $|0.3x| - 2 = 2.5 - 0.63x^2$ are approximately ± 2.45 {by symmetry}.
The solution of $|0.3x| - 2 > 2.5 - 0.63x^2$ is $(-\infty, -2.45) \cup (2.45, \infty)$.

73 Assign ABS$(1.2x^2 - 10.8)$ to Y_1 and $1.36x + 4.08$ to Y_2. The standard viewing rectangle $[-15, 15]$ by $[-10, 10]$ shows intersection points at exactly -3 and at approximately 1.87 and 4.13.
The solution of $|1.2x^2 - 10.8| > 1.36x + 4.08$ is $(-\infty, -3) \cup (-3, 1.87) \cup (4.13, \infty)$.

74 The solutions of $\left|\sqrt{16 - x^2} - 3\right| = 0.12x^2 - 0.3$ are approximately ± 3.60, ± 2.25 {by symmetry}.
The solution of $\left|\sqrt{16 - x^2} - 3\right| < 0.12x^2 - 0.3$ is $(-3.60, -2.25) \cup (2.25, 3.60)$.

75 $f(x) = (0.5x^3 - 4x - 5)$ and $g(x) = (0.5x^3 - 4x - 5) + 4 = f(x) + 4$, so the graph of g can be obtained by shifting the graph of f upward a distance of 4.

$[-12, 12]$ by $[-8, 8]$

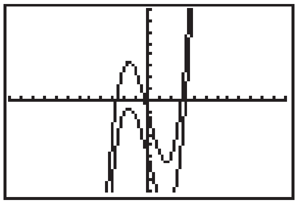


Figure 75

$[-12, 12]$ by $[-8, 8]$

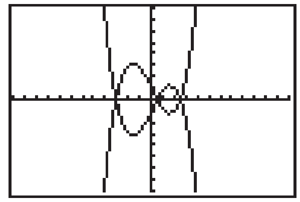


Figure 76

76 $f(x) = (0.25x^3 - 2x + 1)$ and $g(x) = (-0.25x^3 + 2x - 1) = -f(x)$, so the graph of g can be obtained reflecting the graph of f through the x–axis.

77 $f(x) = x^2 - 5$ and $g(x) = \left(\frac{1}{2}x\right)^2 - 5 = f\left(\frac{1}{2}x\right)$, so the graph of g can be obtained by stretching the graph of f horizontally by a factor of 2.

$[-12, 12]$ by $[-8, 8]$

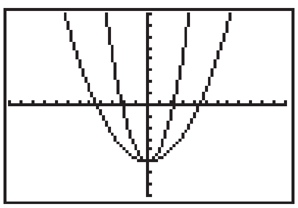


Figure 77

$[-12, 12]$ by $[-8, 8]$

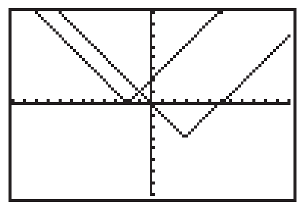


Figure 78

78 $f(x) = |x + 2|$ and $g(x) = |(x + 2) - 5| - 3 = f(x - 5) - 3$, so the graph of g can be obtained by shifting the graph of f horizontally to the right a distance of 5 and vertically downward a distance of 3.

79 $f(x) = x^3 - 5x$ and $g(x) = |x^3 - 5x| = |f(x)|$, so the graph of g is the same as the graph of f if f is nonnegative. If $f(x) < 0$, then the graph of f will be reflected through the x–axis.

$[-12, 12]$ by $[-8, 8]$

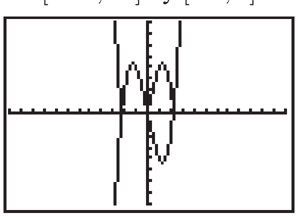


Figure 79

$[-12, 12]$ by $[-8, 8]$

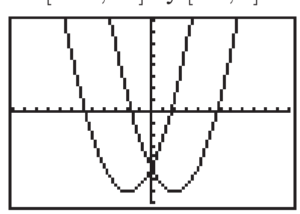


Figure 80

80 $f(x) = (0.5x^2 - 2x - 5)$ and $g(x) = (0.5x^2 + 2x - 5) = f(-x)$, so the graph of g can be obtained from the graph of f by reflecting the graph of f through the y–axis.

81 (a) Option I gives $C_1 = 4(\$45.00) + \$0.40(500 - 200) = 180.00 + 120.00 = \$300.00$.
Option II gives $C_2 = 4(\$58.75) + \$0.25(500) = 235.00 + 125.00 = \$360.00$.

(b) Let x represent the mileage. The cost function for Option I is the piecewise linear function

$$C_1(x) = \begin{cases} 180.00 & \text{if } 0 \le x \le 200 \\ 180.00 + 0.40(x - 200) & \text{if } x > 200 \end{cases}$$

Option II is the linear function $C_2(x) = 235.00 + 0.25x$ for $x \ge 0$.

(c) Let $C_1 = \text{Y}_1$ and $C_2 = \text{Y}_2$. Table $\text{Y}_1 = 180.00 + 0.40(x - 200)*(x > 200)$ and $\text{Y}_2 = 235.00 + 0.25x$

x	100	200	300	400	500	600	700	800	900	1000	1100	1200
Y_1	180	180	220	260	300	340	380	420	460	500	540	580
Y_2	260	285	310	335	360	385	410	435	460	485	510	535

(d) From the table, we see that the options are equal in cost for $x = 900$ miles. Option I is preferable if $x \in [0, 900)$ and Option II is preferable if $x > 900$.

82 (a) Since 1 mi = 5,280 ft and each car requires $(12 + d)$ ft, it follows directly that the bridge can hold $[\![5280/(12 + d)]\!]$ cars. The greatest integer function is necessary since a fraction of a car is not allowed.

(b) Since the bridge is 1 mile long, the car "density" is $\dfrac{5280}{12 + d}$ cars/mi. If each car is moving at v mi/hr, then the flow rate is $F = [\![5280v/(12 + d)]\!]$ cars/hr.

3.6 Exercises

1 Using the standard equation of a parabola with a vertical axis having vertex $V(-3, 1)$, we have $y = a[x - (-3)]^2 + 1$. The coefficient a determines whether the parabola opens upward {if a is positive} or opens downward {if a is negative}. If $|a| > 1$, then the parabola is narrower {steeper} than the graph of $y = x^2$; if $|a| < 1$, then the parabola is wider {flatter} than the graph of $y = x^2$. Simplifying the equation gives us $y = a(x + 3)^2 + 1$.

2 $V(5, -4)$ • $y = a(x - h)^2 + k \quad \Rightarrow \quad y = a(x - 5)^2 - 4$

3 $V(0, -2)$ • We can use the standard equation of a parabola with a vertical axis.

$$y = a(x - h)^2 + k \quad \Rightarrow \quad y = a(x - 0)^2 - 2 \quad \Rightarrow \quad y = ax^2 - 2.$$

4 $V(-7, 0)$ • $y = a(x - h)^2 + k \quad \Rightarrow \quad y = a\,[x - (-7)]^2 + 0 \quad \Rightarrow \quad y = a(x + 7)^2$

5 The approach shown here {like Solution 1 in Example 2} requires us to *factor out the leading coefficient*.

$$\begin{aligned} f(x) &= -x^2 - 4x - 5 && \text{\{given\}} \\ &= -\left(x^2 + 4x + \underline{\quad}\right) - 5 + \underline{\quad} && \text{\{factor out } -1 \text{ from } -x^2 - 4x\text{\}} \\ &= -\left(x^2 + 4x + \underline{\ 4\ }\right) - 5 + \underline{\ 4\ } && \text{\{complete the square for } x^2 + 4x\text{\}} \\ &= -(x + 2)^2 - 1 && \text{\{equivalent equation\}} \end{aligned}$$

6 $f(x) = x^2 - 6x + 11 = x^2 - 6x + \underline{\ 9\ } + 11 - \underline{\ 9\ } = (x - 3)^2 + 2$

7 The approach shown here {like Solution 2 in Example 2} requires us to *divide both sides by the leading coefficient*—remember to multiply both sides by the same coefficient in the end. Either method is fine.

$$\begin{aligned} f(x) &= 2x^2 - 16x + 35 && \text{\{given\}} \\ \tfrac{1}{2}f(x) &= (x^2 - 8x + \underline{\quad}) + \tfrac{35}{2} - \underline{\quad} && \text{\{divide the equation by 2\}} \\ &= \left(x^2 - 8x + \underline{\ 16\ }\right) + \tfrac{35}{2} - \underline{\ 16\ } && \text{\{complete the square for } x^2 - 8x\text{\}} \\ &= (x - 4)^2 + \tfrac{3}{2} && \text{\{equivalent equation\}} \\ 2 \cdot \tfrac{1}{2}f(x) &= 2 \cdot \left[(x - 4)^2 + \tfrac{3}{2}\right] && \text{\{multiply the equation by 2\}} \\ f(x) &= 2(x - 4)^2 + 3 && \text{\{the desired standard form\}} \end{aligned}$$

8 $f(x) = 5x^2 + 20x + 14 \quad\Rightarrow\quad \frac{1}{5}f(x) = x^2 + 4x + \underline{4} + \frac{14}{5} - \underline{4} = (x+2)^2 - \frac{6}{5} \quad\Rightarrow$
$5 \cdot \frac{1}{5}f(x) = 5 \cdot \left[(x+2)^2 - \frac{6}{5}\right] \quad\Rightarrow\quad f(x) = 5(x+2)^2 - 6$

9 $f(x) = -3x^2 - 6x - 5$ {divide by -3 and proceed as in Exercise 7} $\Rightarrow$
$-\frac{1}{3}f(x) = x^2 + 2x + \underline{1} + \frac{5}{3} - \underline{1} = (x+1)^2 + \frac{2}{3} \quad\Rightarrow$
$-3 \cdot \left(-\frac{1}{3}\right)f(x) = -3 \cdot \left[(x+1)^2 + \frac{2}{3}\right] \quad\Rightarrow\quad f(x) = -3(x+1)^2 - 2$

10 $f(x) = -4x^2 + 16x - 13 \quad\Rightarrow\quad -\frac{1}{4}f(x) = x^2 - 4x + \underline{4} + \frac{13}{4} - \underline{4} = (x-2)^2 - \frac{3}{4} \quad\Rightarrow$
$-4 \cdot \left(-\frac{1}{4}\right)f(x) = -4 \cdot \left[(x-2)^2 - \frac{3}{4}\right] \quad\Rightarrow\quad f(x) = -4(x-2)^2 + 3$

11 $f(x) = -\frac{3}{4}x^2 + 9x - 34$ {divide by $-\frac{3}{4}$ and proceed as in Exercise 7} $\Rightarrow$
$-\frac{4}{3}f(x) = x^2 - 12x + \frac{136}{3} = x^2 - 12x + \underline{36} + \frac{136}{3} - \underline{36} = (x-6)^2 + \frac{28}{3} \quad\Rightarrow$
$\left(-\frac{3}{4}\right)\left(-\frac{4}{3}\right)f(x) = \left(-\frac{3}{4}\right)\left[(x-6)^2 + \frac{28}{3}\right] \quad\Rightarrow\quad f(x) = -\frac{3}{4}(x-6)^2 - 7$

12 $f(x) = \frac{2}{5}x^2 - \frac{12}{5}x + \frac{23}{5} \quad\Rightarrow\quad \frac{5}{2}f(x) = x^2 - 6x + \frac{23}{2} = x^2 - 6x + \underline{9} + \frac{23}{2} - \underline{9} = (x-3)^2 + \frac{5}{2} \quad\Rightarrow$
$\frac{2}{5} \cdot \frac{5}{2}f(x) = \frac{2}{5} \cdot \left[(x-3)^2 + \frac{5}{2}\right] \quad\Rightarrow\quad f(x) = \frac{2}{5}(x-3)^2 + 1$

13 (a) $x^2 - 6x = 0$ $\{a = 1, b = -6, c = 0\} \quad\Rightarrow\quad x = \dfrac{6 \pm \sqrt{36 - 0}}{2} = \dfrac{6 \pm 6}{2} = 0, 6$

(b) Using the theorem for locating the vertex of a parabola with $y = x^2 - 6x$ gives us x-coordinate $-\dfrac{b}{2a} = -\dfrac{-6}{2(1)} = 3$. The parabola opens upward since $a = 1 > 0$, so there is a minimum value of y. To find the minimum, substitute 3 for x in $f(x) = x^2 - 6x$ to get $f(3) = -9$.

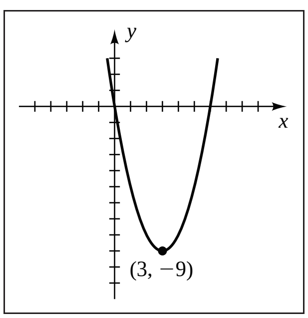


Figure 13

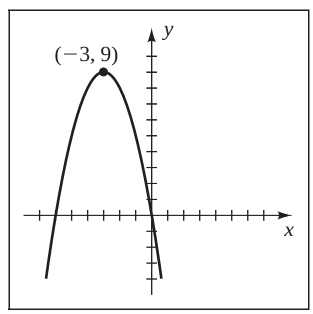


Figure 14

14 (a) $-x^2 - 6x = 0$ $\{a = -1, b = -6, c = 0\} \quad\Rightarrow\quad x = \dfrac{6 \pm \sqrt{36 - 0}}{-2} = -6, 0$

(b) $f(x) = -x^2 - 6x \quad\Rightarrow\quad -\dfrac{b}{2a} = -\dfrac{-6}{2(-1)} = -3$. $f(-3) = 9$ is a maximum since $a < 0$.

15 (a) $f(x) = -12x^2 + 11x + 15 = 0 \quad\Rightarrow\quad x = \dfrac{-11 \pm \sqrt{121 + 720}}{-24} = \dfrac{-11 \pm 29}{-24} = -\dfrac{3}{4}, \dfrac{5}{3}$

(b) The x-coordinate of the vertex is given by $x = -\dfrac{b}{2a} = -\dfrac{11}{2(-12)} = \dfrac{11}{24}$. Note that this value is easily obtained from part (a). The y-coordinate of the vertex is then $f\left(\frac{11}{24}\right) = \frac{841}{48} \approx 17.52$. This is a maximum since $a = -12 < 0$.

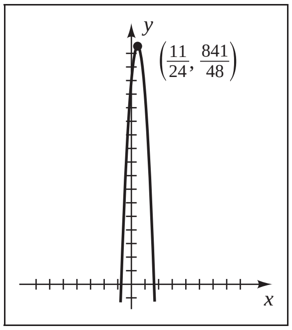


Figure 15

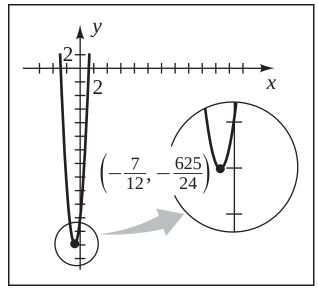


Figure 16

16 (a) $6x^2 + 7x - 24 = 0 \quad \Rightarrow \quad x = \dfrac{-7 \pm \sqrt{49 + 576}}{12} = \dfrac{-7 \pm 25}{12} = -\dfrac{8}{3}, \dfrac{3}{2}$

(b) $f(x) = 6x^2 + 7x - 24 \quad \Rightarrow \quad -\dfrac{b}{2a} = -\dfrac{7}{2(6)} = -\dfrac{7}{12}$. $f\left(-\frac{7}{12}\right) = -\frac{625}{24} \approx -26.04$ is a minimum since $a > 0$.

17 (a) $9x^2 + 24x + 16 = 0 \quad \Rightarrow \quad x = \dfrac{-24 \pm \sqrt{576 - 576}}{18} = \dfrac{-24}{18} = -\dfrac{4}{3}$

(b) $f(x) = 9x^2 + 24x + 16 \quad \Rightarrow \quad -\dfrac{b}{2a} = -\dfrac{24}{2(9)} = -\dfrac{4}{3}$. $f\left(-\frac{4}{3}\right) = 0$ is a minimum since $a > 0$.

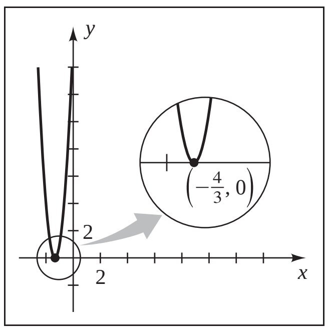


Figure 17

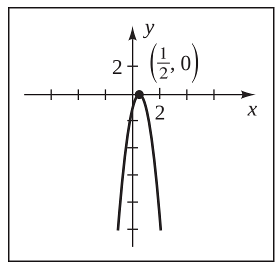


Figure 18

18 (a) $-4x^2 + 4x - 1 = 0 \quad \Rightarrow \quad x = \dfrac{-4 \pm \sqrt{16 - 16}}{-8} = \dfrac{-4}{-8} = \dfrac{1}{2}$

(b) $f(x) = -4x^2 + 4x - 1 \quad \Rightarrow \quad -\dfrac{b}{2a} = -\dfrac{4}{2(-4)} = \dfrac{1}{2}$. $f\left(\frac{1}{2}\right) = 0$ is a maximum since $a < 0$.

19 (a) $x^2 + 4x + 9 = 0 \quad \Rightarrow \quad x = \dfrac{-4 \pm \sqrt{16 - 36}}{2} = \dfrac{-4 \pm \sqrt{-20}}{2} = -2 \pm \sqrt{5}\, i$.

The imaginary part indicates that there are no x-intercepts.

(b) $f(x) = x^2 + 4x + 9 \quad \Rightarrow \quad -\dfrac{b}{2a} = -\dfrac{4}{2(1)} = -2$. $f(-2) = 5$ is a minimum since $a = 1 > 0$.

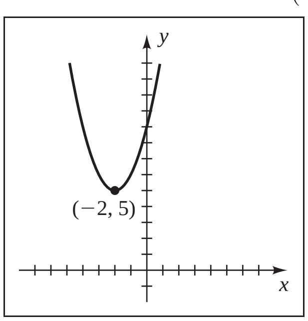


Figure 19

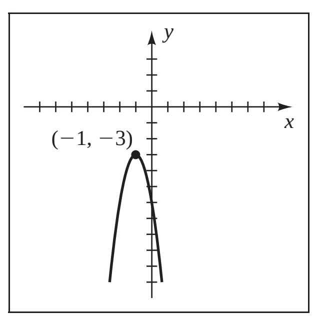


Figure 20

20 (a) $-3x^2 - 6x - 6 = 0 \;\Rightarrow\; x = \dfrac{6 \pm \sqrt{36 - 72}}{-6} = \dfrac{6 \pm \sqrt{-36}}{-6} = \dfrac{6 \pm 6i}{-6} = -1 \pm i.$

The imaginary part indicates that there are no x-intercepts. See *Figure 20* on the preceding page.

(b) $f(x) = -3x^2 - 6x - 6 \;\Rightarrow\; -\dfrac{b}{2a} = -\dfrac{-6}{2(-3)} = -1.$ $f(-1) = -3$ is a maximum since $a < 0$.

21 (a) $-2x^2 + 16x - 26 = 0 \;\Rightarrow\; x = \dfrac{-16 \pm \sqrt{256 - 208}}{-4} = 4 \pm \dfrac{1}{4}\sqrt{48} = 4 \pm \sqrt{3} \approx 5.73, 2.27$

(b) $f(x) = -2x^2 + 16x - 26 \;\Rightarrow\; -\dfrac{b}{2a} = -\dfrac{16}{2(-2)} = 4.$ $f(4) = 6$ is a maximum since $a < 0$.

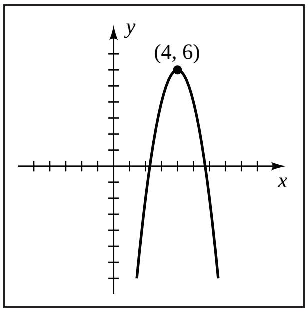


Figure 21

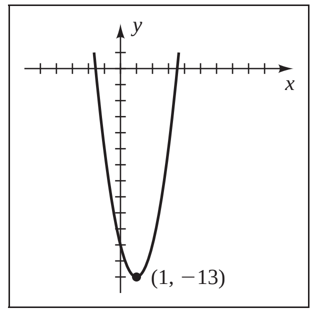


Figure 22

22 (a) $2x^2 - 4x - 11 = 0 \;\Rightarrow\; x = \dfrac{4 \pm \sqrt{16 + 88}}{4} = 1 \pm \dfrac{1}{2}\sqrt{26} \approx 3.55, -1.55$

(b) $f(x) = 2x^2 - 4x - 11 \;\Rightarrow\; -\dfrac{b}{2a} = -\dfrac{-4}{2(2)} = 1.$ $f(1) = -13$ is a minimum since $a > 0$.

23 $V(4, -1) \;\Rightarrow\; y = a(x-4)^2 - 1$. Substituting 0 for x and 1 for y gives us

$1 = a(0-4)^2 - 1 \;\Rightarrow\; 2 = 16a \;\Rightarrow\; a = \frac{1}{8}$. Hence, $y = \frac{1}{8}(x-4)^2 - 1$.

24 $V(2, 4) \;\Rightarrow\; y = a(x-2)^2 + 4$. Substituting 0 for x and 0 for y gives us

$0 = a(0-2)^2 + 4 \;\Rightarrow\; -4 = 4a \;\Rightarrow\; a = -1$. Hence, $y = -(x-2)^2 + 4$.

25 $V(-2, 5) \;\Rightarrow\; y = a(x+2)^2 + 5$. Substituting 2 for x and 0 for y gives us

$0 = a(2+2)^2 + 5 \;\Rightarrow\; -5 = 16a \;\Rightarrow\; a = -\frac{5}{16}$. Hence, $y = -\frac{5}{16}(x+2)^2 + 5$.

26 $V(-1, -2) \;\Rightarrow\; y = a(x+1)^2 - 2$. Substituting 2 for x and 3 for y gives us

$3 = a(2+1)^2 - 2 \;\Rightarrow\; 5 = 9a \;\Rightarrow\; a = \frac{5}{9}$. Hence, $y = \frac{5}{9}(x+1)^2 - 2$.

27 From the figure, the x-intercepts are -2 and 4, so the equation must have the form

$y = a[x - (-2)](x - 4) = a(x+2)(x-4)$. To find the value of a, use the point $(2, 4)$.

$4 = a(2+2)(2-4) \;\Rightarrow\; 4 = a(4)(-2) \;\Rightarrow\; -8a = 4 \;\Rightarrow\; a = -\frac{1}{2}$,

so the equation is $y = -\frac{1}{2}(x+2)(x-4)$.

28 $y = a(x+3)(x-7)$ • $x = 5, y = -4 \;\Rightarrow\; -4 = a(8)(-2) \;\Rightarrow\; a = \frac{1}{4}$,

so the equation is $y = \frac{1}{4}(x+3)(x-7)$.

29 $V(0, -2) \;\Rightarrow\; (h, k) = (0, -2)$. $x = 3, y = 25 \;\Rightarrow\; 25 = a(3-0)^2 - 2 \;\Rightarrow\; 27 = 9a \;\Rightarrow\; a = 3$.

Hence, $y = 3(x-0)^2 - 2$, or $y = 3x^2 - 2$.

30 $V(0, 7) \;\Rightarrow\; (h, k) = (0, 7)$. $x = 2, y = -1 \;\Rightarrow\; -1 = a(2-0)^2 + 7 \;\Rightarrow\; -8 = 4a \;\Rightarrow\; a = -2$.

Hence, $y = -2(x-0)^2 + 7$, or $y = -2x^2 + 7$.

31 $V(3,1) \Rightarrow y = a(x-3)^2 + 1$ **(*)**. If the x-intercept is 0, then the point $(0,0)$ is on the parabola. Substituting $x = 0$ and $y = 0$ into **(*)** gives us $0 = a(0-3)^2 + 1 \Rightarrow -1 = 9a \Rightarrow a = -\frac{1}{9}$. Hence, $y = -\frac{1}{9}(x-3)^2 + 1$.

32 $V(4,-7) \Rightarrow y = a(x-4)^2 - 7$. $x = -4, y = 0 \Rightarrow 0 = a(-4-4)^2 - 7 \Rightarrow 7 = 64a \Rightarrow a = \frac{7}{64}$.
Hence, $y = \frac{7}{64}(x-4)^2 - 7$.

33 Since the x-intercepts are -3 and 5, the x-coordinate of the vertex of the parabola is 1 {the average of the x-intercept values}. Since the highest point has y-coordinate 4, the vertex is $(1,4)$.
$V(1,4) \Rightarrow y = a(x-1)^2 + 4$. We can use the point $(-3,0)$ since there is an x-intercept of -3.
$x = -3, y = 0 \Rightarrow 0 = a(-3-1)^2 + 4 \Rightarrow -4 = 16a \Rightarrow a = -\frac{1}{4}$. Hence, $y = -\frac{1}{4}(x-1)^2 + 4$.

34 Since the x-intercepts are 8 and 0, the x-coordinate of the vertex of the parabola is 4 {the average of the x-intercept values}. Since the lowest point has y-coordinate -48, the vertex is $(4,-48)$.
$V(4,-48) \Rightarrow y = a(x-4)^2 - 48$. $x = 0, y = 0 \Rightarrow 0 = a(0-4)^2 - 48 \Rightarrow 48 = 16a \Rightarrow a = 3$.
Hence, $y = 3(x-4)^2 - 48$.

35 Let d denote the distance between the parabola and the line.

$$d = (\text{parabola}) - (\text{line}) = \left(-2x^2 + 4x + 3\right) - (x-2) = -2x^2 + 3x + 5.$$

This relation is quadratic and the x-value of its maximum value is $-\dfrac{b}{2a} = -\dfrac{3}{2(-2)} = \dfrac{3}{4}$.

Thus, maximum $d = -2\left(\frac{3}{4}\right)^2 + 3\left(\frac{3}{4}\right) + 5 = \frac{49}{8} = 6.125$. Note that the maximum value *of the parabola* is $f(1) = 5$, which is not the same as the maximum value of the distance d.

36 As in Exercise 35, $d = (\text{line}) - (\text{parabola}) = (-x+3) - (2x^2 + 8x + 4) = -2x^2 - 9x - 1$, and
$-\dfrac{b}{2a} = -\dfrac{-9}{2(-2)} = -\dfrac{9}{4}$. Thus, maximum $d = -2\left(-\frac{9}{4}\right)^2 - 9\left(-\frac{9}{4}\right) - 1 = \frac{73}{8} = 9.125$.

Note: We may find the vertex of a parabola in the following problems using either:

1) the complete the square method,
2) the formula method, $-b/(2a)$, or
3) the fact that the vertex lies halfway between the x-intercepts.

37 For $D(h) = -0.058h^2 + 2.867h - 24.239$, the vertex is located at $h = \dfrac{-b}{2a} = \dfrac{-2.867}{2(-0.058)} \approx 24.72$ km.
Since $a < 0$, this will produce a maximum value.

38 The vertex is located at $h = \dfrac{-b}{2a} = \dfrac{-3.811}{2(-0.078)} \approx 24.43$ km. Since $a < 0$, this will produce a maximum value.

39 Since the x-intercepts of $y = cx(21-x)$ are 0 and 21,
the maximum will occur halfway between them, that is, when the infant weighs 10.5 lb.

40 (a) $M = -\frac{1}{30}v^2 + \frac{5}{2}v$ • M will be a maximum when $v = \dfrac{-b}{2a} = \dfrac{-5/2}{2(-1/30)} = \dfrac{75}{2}$, or 37.5 mi/hr.

(b) $v = \frac{75}{2} \Rightarrow M = -\frac{1}{30}\left(\frac{75}{2}\right)^2 + \frac{5}{2}\left(\frac{75}{2}\right) = -\frac{375}{8} + \frac{375}{4} = \frac{375}{8} = 46.875$ mi/gal.

41 (a) $s(t) = -16t^2 + 144t + 100$ • s will be a maximum when $t = \dfrac{-b}{2a} = \dfrac{-144}{2(-16)} = \dfrac{9}{2}$.
$s\left(\frac{9}{2}\right) = -16\left(\frac{9}{2}\right)^2 + 144\left(\frac{9}{2}\right) + 100 = -324 + 648 + 100 = 424$ ft.

(b) When $t = 0$, $s(t) = 100$ ft, which is the height of the building.

42 (a) $s(t) = -16t^2 + v_0 t$ • $s = 0$ when $t = 12$ $\Rightarrow$ $0 = -16(12)^2 + v_0(12)$ $\Rightarrow$ $v_0 = 192$ ft/sec.

(b) Since the total flight is 12 seconds, the maximum height will occur when $t = 6$.

$s(6) = -16(6)^2 + 192(6) = 576$ ft.

43 Let x and $40 - x$ denote the numbers. Their product P is $P = x(40 - x) = -x^2 + 40x$.

P has zeros at 0 and 40 and is a maximum (since $a < 0$) when $x = \dfrac{0+40}{2} = 20$.

The product will be a maximum when both numbers are 20.

44 Let x and $x - 60$ denote the numbers. Their product P is $P = x(x - 60) = x^2 - 60x$.

P has zeros at 0 and 60 and is a minimum (since $a > 0$) when $x = \dfrac{0+60}{2} = 30$.

The product will be a minimum for $x = 30$ and $x - 60 = -30$.

45 (a) The 1000 ft of fence is made up of 3 sides of length x and 4 sides of length y.

To express y as a function of x, we need to solve $3x + 4y = 1000$ for y.

$3x + 4y = 1000$ $\Rightarrow$ $4y = 1000 - 3x$ $\Rightarrow$ $y = 250 - \frac{3}{4}x$.

(b) Using the value of y from part (a), $A = xy = x\left(250 - \frac{3}{4}x\right) = -\frac{3}{4}x^2 + 250x$.

(c) A will be a maximum when $x = \dfrac{-b}{2a} = \dfrac{-250}{2(-3/4)} = \dfrac{500}{3} = 166\frac{2}{3}$ ft.

Using part (a) to find the corresponding value of y, $y = 250 - \frac{3}{4}\left(\frac{500}{3}\right) = 250 - 125 = 125$ ft.

46 Represent the perimeter of the field by $2y + 4x = 1000$. $A = xy = x(500 - 2x)$.

A has zeros at 0 and 250 and will be a maximum when $x = \dfrac{0+250}{2} = 125$ yd. $y = 500 - 2(125) = 250$ yd.

The dimensions should be 125 yd by 250 yd with intermediate fences parallel to the short side.

47 The parabola has vertex $V\left(\frac{9}{2}, 3\right)$. Hence, the equation has the form $y = a\left(x - \frac{9}{2}\right)^2 + 3$.

Using the point $(9, 0)$ {or $(0, 0)$}, we have $0 = a\left(9 - \frac{9}{2}\right)^2 + 3$ $\Rightarrow$ $a = -\frac{4}{27}$.

Thus, the path may be described by $y = -\frac{4}{27}\left(x - \frac{9}{2}\right)^2 + 3$.

48 (a) $y = ax^2 + x + c$ • Since $(0, 15)$ is on the graph, $c = 15$. Substituting $(175, 0)$ for (x, y) yields

$0 = a(175)^2 + 175 + 15$ $\Rightarrow$ $a = -\dfrac{190}{175^2}$ and $y = -\dfrac{190}{175^2}x^2 + x + 15$.

(b) y will be a maximum when $x = \dfrac{-b}{2a} = \dfrac{-1}{2(-190/175^2)} = \dfrac{175^2}{380} \approx 80.59$.

The corresponding y-value is $y = -\dfrac{190}{175^2}\left(\dfrac{175^2}{380}\right)^2 + \dfrac{175^2}{380} + 15 = \dfrac{8405}{152} \approx 55.3$ ft.

49 (a) Since the vertex is at $(0, 10)$, an equation for the parabola is $y = ax^2 + 10$.

The points $(200, 90)$ and $(-200, 90)$ are on the parabola. Substituting $(200, 90)$ for (x, y) yields

$90 = a(200)^2 + 10$ $\Rightarrow$ $80 = 40{,}000a$ $\Rightarrow$ $a = \frac{80}{40{,}000} = \frac{1}{500}$. Hence, $y = \frac{1}{500}x^2 + 10$.

(b) The cables are spaced 40 ft apart. Using $y = \frac{1}{500}x^2 + 10$ with $x = 40$, 80, 120, and 160, we get $y = \frac{66}{5}$, $\frac{114}{5}$, $\frac{194}{5}$, and $\frac{306}{5}$, respectively. There is one cable of length 10 ft and 2 cables of each of the other lengths. Thus, the total length is $10 + 2\left(\frac{66}{5} + \frac{114}{5} + \frac{194}{5} + \frac{306}{5}\right) = 282$ ft.

50 (a) Substituting $x = 0$ and $m = 0$ into $m = 2ax + b$ yields $b = 0$.

Substituting $x = 800$ and $m = \frac{1}{5}$ into $m = 2ax$ yields $a = \frac{1}{8000}$, hence, $y = \frac{1}{8000}x^2$.

(b) Substitute $x = 800$ into $y = \frac{1}{8000}x^2$ to get $y = 80$. Thus, $B = (800, 80)$.

51 An equation describing the doorway is $y = ax^2 + 9$. Since the doorway is 6 feet wide at the base, $x = 3$ when $y = 0 \Rightarrow 0 = a(3)^2 + 9 \Rightarrow 0 = 9a + 9 \Rightarrow 9a = -9 \Rightarrow a = -1$. Thus, the equation is $y = -x^2 + 9$. To fit an 8 foot high box through the doorway, we must find x when $y = 8$. If $y = 8$, then $8 = -x^2 + 9 \Rightarrow x^2 = 1 \Rightarrow x = \pm 1$. Hence, the box can only be $1 - (-1) = 2$ feet wide.

52 (a) The maximum height of the baseball occurs at the vertex of the parabola.
The horizontal (x) coordinate of the vertex is $x = -\frac{b}{2a} = -\frac{3/10}{-3/4000} = 200$, so the corresponding vertical (y) coordinate is $y = -\frac{3}{4000}(200)^2 + \frac{3}{10}(200) + 3 = 33$. The maximum height is 33 feet.

(b) The height of the baseball when $x = 385$ is $y = -\frac{3}{4000}(385)^2 + \frac{3}{10}(385) + 3 \approx 7.33$, which is less than 8, so no, the baseball does not clear an 8-foot fence that is 385 feet from home plate.

53 Let x denote the number of pairs of shoes that are ordered. If $x < 50$, then the amount A of money that the company makes is $40x$. If $50 \le x \le 600$, then each pair of shoes is discounted $0.04x$, so the price per pair is $40 - 0.04x$, and the amount of money that the company makes is $(40 - 0.04x)x$. In piecewise form, we have

$$A(x) = \begin{cases} 40x & \text{if } x < 50 \\ (40 - 0.04x)x & \text{if } 50 \le x \le 600 \end{cases}$$

The maximum value of the first part of A is $(\$40)(49) = \$1960$. For the second part of A, $A = -0.04x^2 + 40x$ has a maximum when $x = \dfrac{-b}{2a} = \dfrac{-40}{2(-0.04)} = 500$ pairs.

$A(500) = 10{,}000 > 1960$, so $x = 500$ produces a maximum for both parts of A.

54 Let x denote the number of people in the group. The discount per person is $0.50(x - 30)$. The amount of money taken in by the agency may be expressed as:

$$A(x) = \begin{cases} 60x & \text{if } x \le 30 \\ [60 - 0.50(x - 30)]x & \text{if } 30 < x \le 90 \end{cases}$$

The maximum value of the first part of A is $(\$60)(30) = \$1800$. For the second part of A, $A = \left(75 - \frac{1}{2}x\right)x$ has a maximum when $x = 75$ {the x-intercepts are 0 and 150}. This value corresponds to each person receiving a discount of \$22.50 and hence, paying \$37.50 for the tour. $A(75) = 2812.50 > 1800$, so $x = 75$ produces a maximum for both parts of A.

55 (a) Let y denote the number of \$5 decreases in the monthly charge.

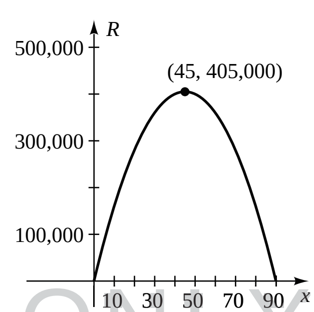
$$R(y) = (\text{\# of customers})(\text{monthly charge per customer}) = (8000 + 1000y)(50 - 5y)$$

Now let x denote the monthly charge, which is $50 - 5y$. $x = 50 - 5y \Rightarrow 5y = 50 - x \Rightarrow y = \dfrac{50 - x}{5}$.

R becomes

$$\left[8000 + 1000\left(\frac{50 - x}{5}\right)\right](x) = [8000 + 200(50 - x)](x) = 200x[40 + (50 - x)] = 200x(90 - x).$$

(b) R has x-intercepts at 0 and 90, and must have its vertex halfway between them at $x = 45$.
Note that this gives us $y = \dfrac{50 - 45}{5} = 1$, and we have $8000 + 1000(1) = 9000$ customers for a revenue of $(9000)(\$45) = \$405{,}000$.



56 Let x denote the number of \$25 increases in rent and $M(x)$ the monthly income. The number of occupied apartments is $218 - 5x$ and the rent per apartment is $940 + 25x$.

$$M(x) = (\text{\# of occupied apartments})(\text{rent per apartment}) = (218 - 5x)(940 + 25x) = 5(218 - 5x)(188 + 5x).$$

The x-intercepts of M are $\frac{218}{5}$ and $-\frac{188}{5}$. Hence, the maximum of M will occur when

$x = \frac{1}{2}\left(-\frac{188}{5} + \frac{218}{5}\right) = \frac{1}{2}(6) = 3$. The rent charged should be \$940 + \$25(3) = \$1015.

57 From the graph of $f(x) = x^2 - x - \frac{1}{4}$ and $y = x^3 - x^{1/3}$, there are three points of intersection. Their coordinates are approximately $(-0.57, 0.64)$, $(0.02, -0.27)$, and $(0.81, -0.41)$.

$[-3, 3]$ by $[-2, 2]$

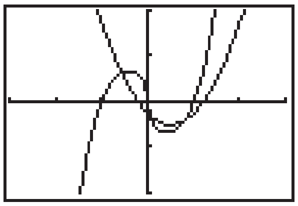


Figure 57

$[-6, 6]$ by $[-4, 4]$

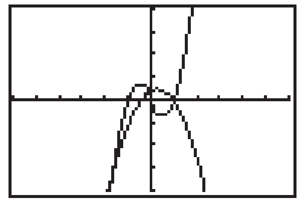


Figure 58

58 From the graph of $f(x) = -x^2 + 0.5x + 0.4$ and $y = x^3 - x^{1/3}$, there are three points of intersection. Their coordinates are approximately $(-1.61, -2.99)$, $(-0.05, 0.37)$, and $(0.98, -0.06)$.

59 $y = ax^2 + x + 1$ for $a = \frac{1}{4}, \frac{1}{2}, 1, 2$, and 4 • Since $a > 0$, all parabolas open upward. From the graph, we can see that smaller values of a result in the parabola opening wider while larger values of a result in the parabola becoming narrower.

$[-8, 4]$ by $[-1, 7]$

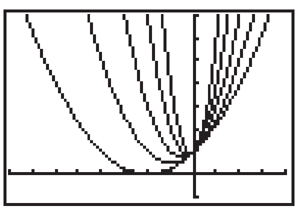


Figure 59

$[-6, 6]$ by $[-2, 6]$

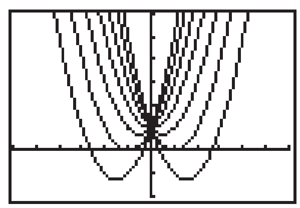


Figure 60

60 $y = x^2 + bx + 1$ for $b = 0, \pm 1, \pm 2$, and ± 3 • As $|b|$ increases, the graph of each parabola shifts downward. Negative values of b shift the parabola to the right while positive values of b shift the parabola to the left.

61 (a) Let January correspond to 1, February to 2, … , and December to 12.

(b) Let $f(x) = a(x - h)^2 + k$. The vertex appears to occur near $(7, 0.77)$ Thus, $h = 7$ and $k = 0.77$. Using trial and error, a reasonable value for a is 0.17. Thus, let $f(x) = 0.17(x - 7)^2 + 0.77$. {From the TI-83/4 Plus, the quadratic regression equation is $y \approx 0.1676x^2 - 2.1369x + 7.9471$.}

(c) $f(4) \approx 2.3$, compared to the actual value of 2.55 in.

$[0, 13]$ by $[0, 8]$

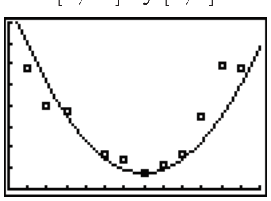


Figure 61

$[1980, 1995]$ by $[5, 15]$

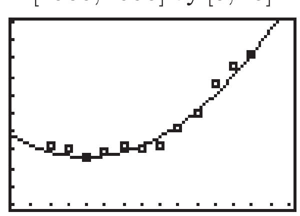


Figure 62

62 (a) Overall, the data decrease slightly and then start to increase, although there is an unexpected decrease in 1987.

(b) Start by choosing a vertex. The lowest data point is $(1984, 7.6)$, so let $h = 1984$ and $k = 7.6$ in the equation $f(x) = a(x - h)^2 + k$. Choosing one more point will determine the parabola.

$f(1993) = 13.3 \quad \Rightarrow \quad 13.3 = a(1993 - 1984)^2 + 7.6 \quad \Rightarrow \quad a \approx 0.07.$

Let $f(x) = 0.07(x - 1984)^2 + 7.6$. f can be adjusted to give a slightly better fit.

63 (a) The equation of the line passing through $A(-800, -48)$ and $B(-500, 0)$ is $y = \frac{4}{25}x + 80$.

The equation of the line passing through $D(500, 0)$ and $E(800, -48)$ is $y = -\frac{4}{25}x + 80$.

Let $y = a(x - h)^2 + k$ be the equation of the parabola passing through the points $B(-500, 0)$, $C(0, 40)$, and $D(500, 0)$. The vertex is located at $(0, 40)$ so $y = a(x - 0)^2 + 40$. Since $D(500, 0)$ is on the graph, $0 = a(500 - 0)^2 + 40 \quad \Rightarrow \quad a = -\frac{1}{6250}$ and $y = -\frac{1}{6250}x^2 + 40$. Thus, let

$$f(x) = \begin{cases} \frac{4}{25}x + 80 & \text{if } -800 \le x < -500 \\ -\frac{1}{6250}x^2 + 40 & \text{if } -500 \le x \le 500 \\ -\frac{4}{25}x + 80 & \text{if } 500 < x \le 800 \end{cases}$$

(b) Graph the equations: $\text{Y}_1 = (4/25{*}x + 80)/(x < -500)$,

$\text{Y}_2 = (-1/6250{*}x\hat{\ }2 + 40)/(x \ge -500 \text{ and } x \le 500)$, $\text{Y}_3 = (-4/25{*}x + 80)/(x > 500)$

$[-800, 800, 100]$ by $[-100, 200, 100]$

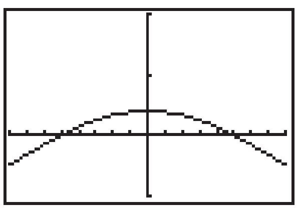


Figure 63

$[-500, 2000, 500]$ by $[0, 800, 100]$

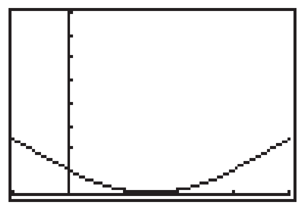


Figure 64

64 (a) The equation of the line passing through $A\left(-500, 243\frac{1}{3}\right)$ and $B(0, 110)$ is $y = -\frac{4}{15}x + 110$.

The equation of the line passing through $D(1500, 110)$ and $E\left(2000, 243\frac{1}{3}\right)$ is $y = \frac{4}{15}x - 290$.

Let $y = a(x - h)^2 + k$ be the equation of the parabola passing through the points $B(0, 110)$, $C(750, 10)$, and $D(1500, 110)$. The vertex is located at $(750, 10)$ so $y = a(x - 750)^2 + 10$. Since $D(1500, 110)$ is on the graph, $110 = a(1500 - 750)^2 + 10 \quad \Rightarrow \quad a = \frac{1}{5625}$; $y = \frac{1}{5625}(x - 750)^2 + 10$. Thus, let

$$f(x) = \begin{cases} -\frac{4}{15}x + 110 & \text{if } -500 \le x < 0 \\ \frac{1}{5625}(x - 750)^2 + 10 & \text{if } 0 \le x \le 1500 \\ \frac{4}{15}x - 290 & \text{if } 1500 < x \le 2000 \end{cases}$$

(b) Graph the equations: $\text{Y}_1 = (-4/15{*}x + 110)/(x < 0)$,

$\text{Y}_2 = (1/5625{*}(x - 750)\hat{\ }2 + 10)/(x \ge 0 \text{ and } x \le 1500)$, $\text{Y}_3 = (4/15{*}x - 290)/(x > 1500)$

65 (a) f must have zeros of 0 and 150. Thus, $f(x) = a(x - 0)(x - 150)$.

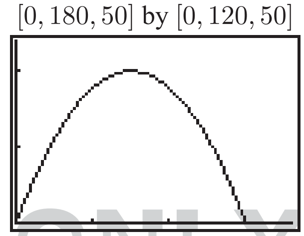
Also, f will have a maximum of 100 occurring at $x = 75$.

(The vertex will be midway between the zeros of f.)

$a(75 - 0)(75 - 150) = 100 \quad \Rightarrow \quad a = \dfrac{100}{(75)(-75)} = -\dfrac{4}{225}.$

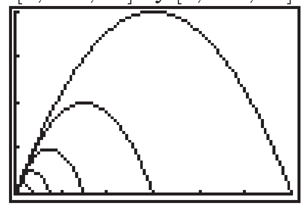
$f(x) = -\frac{4}{225}(x)(x - 150) = -\frac{4}{225}x^2 + \frac{8}{3}x$, so $a = -\frac{4}{225}$ and $b = \frac{8}{3}$.

(b) $[0, 180, 50]$ by $[0, 120, 50]$



(c) $y = kax^2 + bx$ for $k = \frac{1}{4}, \frac{1}{2}, 1, 2,$ and 4 • The value of k affects both the distance and the height traveled by the object. The distance and height decrease by a factor of $\dfrac{1}{k}$ when $k > 1$ and increase by a factor of $\dfrac{1}{k}$ when $0 < k < 1$.

$[0, 600, 50]$ by $[0, 400, 50]$



3.7 Exercises

1 $f(x) = x + 3, g(x) = x^2$ • $f(3) = 3 + 3 = 6, g(3) = 3^2 = 9$

(a) $(f + g)(3) = f(3) + g(3) = 6 + 9 = 15$

(b) $(f - g)(3) = f(3) - g(3) = 6 - 9 = -3$

(c) $(fg)(3) = f(3) \cdot g(3) = 6 \cdot 9 = 54$

(d) $(f/g)(3) = \dfrac{f(3)}{g(3)} = \dfrac{6}{9} = \dfrac{2}{3}$

2 $f(x) = -x^2, g(x) = 2x - 1$ • $f(3) = -3^2 = -9, g(3) = 2(3) - 1 = 5$

(a) $(f + g)(3) = f(3) + g(3) = -9 + 5 = -4$

(b) $(f - g)(3) = f(3) - g(3) = -9 - 5 = -14$

(c) $(fg)(3) = f(3) \cdot g(3) = -9 \cdot 5 = -45$

(d) $(f/g)(3) = \dfrac{f(3)}{g(3)} = \dfrac{-9}{5} = -\dfrac{9}{5}$

3 (a) $(f + g)(x) = f(x) + g(x) = (x^2 + 2) + (2x^2 - 1) = 3x^2 + 1$;
$(f - g)(x) = f(x) - g(x) = (x^2 + 2) - (2x^2 - 1) = 3 - x^2$;
$(fg)(x) = f(x) \cdot g(x) = (x^2 + 2) \cdot (2x^2 - 1) = 2x^4 + 3x^2 - 2$; $\left(\dfrac{f}{g}\right)(x) = \dfrac{f(x)}{g(x)} = \dfrac{x^2 + 2}{2x^2 - 1}$

(b) The domain of $f + g$, $f - g$, and fg is the set of all real numbers, $\mathbb{R}$.

(c) The domain of f/g is the same as in (b), except we must exclude the zeros of g.
$2x^2 - 1 = 0 \quad \Rightarrow \quad x^2 = \frac{1}{2} \quad \Rightarrow \quad x = \pm\sqrt{\frac{1}{2}} = \pm\frac{\sqrt{2}}{2}$ or $\pm\frac{1}{2}\sqrt{2}$.

Hence, the domain of f/g is all real numbers except $\pm\frac{1}{2}\sqrt{2}$.

4 (a) $(f + g)(x) = f(x) + g(x) = (x^2 + x) + (x^2 - 4) = 2x^2 + x - 4$;
$(f - g)(x) = f(x) - g(x) = (x^2 + x) - (x^2 - 4) = x + 4$;
$(fg)(x) = f(x) \cdot g(x) = (x^2 + x) \cdot (x^2 - 4) = x^4 + x^3 - 4x^2 - 4x$; $\left(\dfrac{f}{g}\right)(x) = \dfrac{f(x)}{g(x)} = \dfrac{x^2 + x}{x^2 - 4}$

(b) The domain of $f + g$, $f - g$, and fg is the set of all real numbers, $\mathbb{R}$.

(c) The domain of f/g is all real numbers except ± 2.

5 (a) $(f + g)(x) = f(x) + g(x) = \sqrt{x + 5} + \sqrt{x + 5} = 2\sqrt{x + 5}$;
$(f - g)(x) = f(x) - g(x) = \sqrt{x + 5} - \sqrt{x + 5} = 0$; $\quad (fg)(x) = f(x) \cdot g(x) = \sqrt{x + 5} \cdot \sqrt{x + 5} = x + 5$;
$\left(\dfrac{f}{g}\right)(x) = \dfrac{f(x)}{g(x)} = \dfrac{\sqrt{x + 5}}{\sqrt{x + 5}} = 1$

(b) The radicand, $x + 5$, must be nonnegative; that is $x + 5 \geq 0 \quad \Rightarrow \quad x \geq -5$.

Thus, the domain of $f + g$, $f - g$, and fg is $[-5, \infty)$.

(c) Now the radicand must be positive {can't have zero in the denominator}. Thus, the domain of f/g is $(-5, \infty)$.

6 (a) $(f+g)(x) = f(x) + g(x) = \sqrt{5-2x} + \sqrt{x+3}$; $(f-g)(x) = f(x) - g(x) = \sqrt{5-2x} - \sqrt{x+3}$;
$(fg)(x) = f(x) \cdot g(x) = \sqrt{5-2x} \cdot \sqrt{x+3} = \sqrt{(5-2x)(x+3)}$;
$\left(\dfrac{f}{g}\right)(x) = \dfrac{f(x)}{g(x)} = \dfrac{\sqrt{5-2x}}{\sqrt{x+3}} = \sqrt{\dfrac{5-2x}{x+3}}$

(b) The radicands must be nonnegative. $5-2x \geq 0 \Rightarrow 2x \leq 5 \Rightarrow x \leq \frac{5}{2}$ and $x+3 \geq 0 \Rightarrow x \geq -3$. Thus, the domain of $f+g$, $f-g$, and fg is $\left[-3, \frac{5}{2}\right]$.

(c) Now $x+3$ must be positive {can't have zero in the denominator}. Thus, the domain of f/g is $\left(-3, \frac{5}{2}\right]$.

7 (a) $(f+g)(x) = f(x) + g(x) = \dfrac{2x}{x-4} + \dfrac{x}{x+5} = \dfrac{2x(x+5) + x(x-4)}{(x-4)(x+5)} = \dfrac{3x^2+6x}{(x-4)(x+5)}$;
$(f-g)(x) = f(x) - g(x) = \dfrac{2x}{x-4} - \dfrac{x}{x+5} = \dfrac{2x(x+5) - x(x-4)}{(x-4)(x+5)} = \dfrac{x^2+14x}{(x-4)(x+5)}$;
$(fg)(x) = f(x) \cdot g(x) = \dfrac{2x}{x-4} \cdot \dfrac{x}{x+5} = \dfrac{2x^2}{(x-4)(x+5)}$; $\left(\dfrac{f}{g}\right)(x) = \dfrac{f(x)}{g(x)} = \dfrac{2x/(x-4)}{x/(x+5)} = \dfrac{2(x+5)}{x-4}$

(b) The domain of f is $\mathbb{R} - \{4\}$ and the domain of g is $\mathbb{R} - \{-5\}$. The intersection of these two domains, $\mathbb{R} - \{-5, 4\}$, is the domain of the three functions.

(c) To determine the domain of the quotient f/g, we also exclude any values that make the denominator g equal to zero. Hence, we exclude $x = 0$ and the domain of the quotient is all real numbers except -5, 0, and 4, that is, $\mathbb{R} - \{-5, 0, 4\}$.

8 (a) $(f+g)(x) = f(x) + g(x) = \dfrac{x}{x-2} + \dfrac{7x}{x+4} = \dfrac{x(x+4) + 7x(x-2)}{(x-2)(x+4)} = \dfrac{8x^2-10x}{(x-2)(x+4)}$;
$(f-g)(x) = f(x) - g(x) = \dfrac{x}{x-2} - \dfrac{7x}{x+4} = \dfrac{x(x+4) - 7x(x-2)}{(x-2)(x+4)} = \dfrac{-6x^2+18x}{(x-2)(x+4)}$;
$(fg)(x) = f(x) \cdot g(x) = \dfrac{x}{x-2} \cdot \dfrac{7x}{x+4} = \dfrac{7x^2}{(x-2)(x+4)}$; $\left(\dfrac{f}{g}\right)(x) = \dfrac{f(x)}{g(x)} = \dfrac{x/(x-2)}{7x/(x+4)} = \dfrac{x+4}{7(x-2)}$

(b) All real numbers except -4 and 2

(c) All real numbers except -4, 0, and 2

9 (a) $f(x) = 2x - 1$, $g(x) = -x^2$ • $(f \circ g)(x) = f(g(x)) = f(-x^2) = 2(-x^2) - 1 = -2x^2 - 1$

(b) $(g \circ f)(x) = g(f(x)) = g(2x-1) = -(2x-1)^2 = -(4x^2 - 4x + 1) = -4x^2 + 4x - 1$

(c) $(f \circ f)(x) = f(f(x)) = f(2x-1) = 2(2x-1) - 1 = (4x-2) - 1 = 4x - 3$

(d) $(g \circ g)(x) = g(g(x)) = g(-x^2) = -(-x^2)^2 = -(x^4) = -x^4$

10 (a) $(f \circ g)(x) = f(g(x)) = f(x-2) = 3(x-2)^2 = 3(x^2 - 4x + 4) = 3x^2 - 12x + 12$

(b) $(g \circ f)(x) = g(f(x)) = g(3x^2) = (3x^2) - 2 = 3x^2 - 2$

(c) $(f \circ f)(x) = f(f(x)) = f(3x^2) = 3(3x^2)^2 = 3(9x^4) = 27x^4$

(d) $(g \circ g)(x) = g(g(x)) = g(x-2) = (x-2) - 2 = x - 4$

Note: In Exercises 11–34, let $h(x) = (f \circ g)(x) = f(g(x))$ and $k(x) = (g \circ f)(x) = g(f(x))$.
$h(-2)$ and $k(3)$ could be worked two ways, as in Example 3(c) in the text.

11 (a) $h(x) = f(3x+4) = 2(3x+4) - 5 = 6x + 3$ (b) $k(x) = g(2x-5) = 3(2x-5) + 4 = 6x - 11$

(c) Using the result from part (a), $h(-2) = 6(-2) + 3 = -12 + 3 = -9$.

(d) Using the result from part (b), $k(3) = 6(3) - 11 = 18 - 11 = 7$.

12 (a) $h(x) = f(6x - 3) = 5(6x - 3) + 2 = 30x - 13$ (b) $k(x) = g(5x + 2) = 6(5x + 2) - 3 = 30x + 9$

(c) Using the result from part (a), $h(-2) = 30(-2) - 13 = -60 - 13 = -73$.

(d) Using the result from part (b), $k(3) = 30(3) + 9 = 90 + 9 = 99$.

13 (a) $h(x) = f(5x) = 3(5x)^2 + 4 = 75x^2 + 4$ (b) $k(x) = g(3x^2 + 4) = 5(3x^2 + 4) = 15x^2 + 20$

(c) $h(-2) = 75(-2)^2 + 4 = 300 + 4 = 304$ (d) $k(3) = 15(3)^2 + 20 = 135 + 20 = 155$

14 (a) $h(x) = f(4x^2) = 3(4x^2) - 1 = 12x^2 - 1$

(b) $k(x) = g(3x - 1) = 4(3x - 1)^2 = 4(9x^2 - 6x + 1) = 36x^2 - 24x + 4$

(c) $h(-2) = 12(-2)^2 - 1 = 48 - 1 = 47$

(d) $k(3) = 36(3)^2 - 24(3) + 4 = 324 - 72 + 4 = 256$

15 (a) $h(x) = f(2x - 1) = 2(2x - 1)^2 + 3(2x - 1) - 4 = 8x^2 - 2x - 5$

(b) $k(x) = g(2x^2 + 3x - 4) = 2(2x^2 + 3x - 4) - 1 = 4x^2 + 6x - 9$

(c) $h(-2) = 8(-2)^2 - 2(-2) - 5 = 32 + 4 - 5 = 31$

(d) $k(3) = 4(3)^2 + 6(3) - 9 = 36 + 18 - 9 = 45$

16 (a) $h(x) = f(3x^2 - x + 2) = 5(3x^2 - x + 2) - 7 = 15x^2 - 5x + 3$

(b) $k(x) = g(5x - 7) = 3(5x - 7)^2 - (5x - 7) + 2 = 75x^2 - 215x + 156$

(c) $h(-2) = 15(-2)^2 - 5(-2) + 3 = 60 + 10 + 3 = 73$

(d) $k(3) = 75(3)^2 - 215(3) + 156 = 675 - 645 + 156 = 186$

17 (a) $h(x) = f(2x^3 - 5x) = 4(2x^3 - 5x) = 8x^3 - 20x$ (b) $k(x) = g(4x) = 2(4x)^3 - 5(4x) = 128x^3 - 20x$

(c) $h(-2) = 8(-2)^3 - 20(-2) = -64 + 40 = -24$ (d) $k(3) = 128(3)^3 - 20(3) = 3456 - 60 = 3396$

18 (a) $h(x) = f(3x) = (3x)^3 + 2(3x)^2 = 27x^3 + 18x^2$ (b) $k(x) = g(x^3 + 2x^2) = 3(x^3 + 2x^2) = 3x^3 + 6x^2$

(c) $h(-2) = 27(-2)^3 + 18(-2)^2 = -216 + 72 = -144$ (d) $k(3) = 3(3)^3 + 6(3)^2 = 81 + 54 = 135$

19 (a) $h(x) = f(-7) = |-7| = 7$ (b) $k(x) = g(|x|) = -7$

(c) $h(-2) = 7$ since $h(\text{any value}) = 7$ (d) $k(3) = -7$ since $k(\text{any value}) = -7$

20 (a) $h(x) = f(x^2) = -5$ (b) $k(x) = g(-5) = (-5)^2 = 25$

(c) $h(-2) = -5$ since $h(\text{any value}) = -5$ (d) $k(3) = 25$ since $k(\text{any value}) = 25$

21 (a) $h(x) = f(\sqrt{x+2}) = (\sqrt{x+2})^2 - 3(\sqrt{x+2}) = x + 2 - 3\sqrt{x+2}$. The domain of $f \circ g$ is the set of all x in the domain of g, $x \geq -2$, such that $g(x)$ is in the domain of f. Since the domain of f is $\mathbb{R}$, any value of $g(x)$ is in its domain. Thus, the domain is all x such that $x \geq -2$. Note that *both* $g(x)$ *and* $f(g(x))$ are defined for x in $[-2, \infty)$.

(b) $k(x) = g(x^2 - 3x) = \sqrt{(x^2 - 3x) + 2} = \sqrt{x^2 - 3x + 2}$. The domain of $g \circ f$ is the set of all x in the domain of f, $\mathbb{R}$, such that $f(x)$ is in the domain of g. Since the domain of g is $x \geq -2$, we must solve $f(x) \geq -2$.
$x^2 - 3x \geq -2 \;\Rightarrow\; x^2 - 3x + 2 \geq 0 \;\Rightarrow\; (x - 1)(x - 2) \geq 0$

Interval	$(-\infty, 1)$	$(1, 2)$	$(2, \infty)$
Sign of $x-2$	$-$	$-$	$+$
Sign of $x-1$	$-$	$+$	$+$
Resulting sign	$+$	$-$	$+$

From the sign chart, $(x-1)(x-2) \geq 0 \quad \Rightarrow \quad x \in (-\infty, 1] \cup [2, \infty)$.

Thus, the domain is all x such that $x \in (-\infty, 1] \cup [2, \infty)$.

Note that *both* $f(x)$ *and* $g(f(x))$ are defined for x in $(-\infty, 1] \cup [2, \infty)$.

22 (a) $h(x) = f(x^2 + 2x) = \sqrt{(x^2 + 2x) - 15} = \sqrt{x^2 + 2x - 15}$.

Domain of $g = \mathbb{R}$. Domain of $f = [15, \infty)$. $g(x) \geq 15 \quad \Rightarrow \quad x^2 + 2x \geq 15 \quad \Rightarrow \quad x^2 + 2x - 15 \geq 0 \quad \Rightarrow$

$(x+5)(x-3) \geq 0 \quad \Rightarrow \quad x \in (-\infty, -5] \cup [3, \infty)$.

(b) $k(x) = g\left(\sqrt{x-15}\right) = \left(\sqrt{x-15}\right)^2 + 2\left(\sqrt{x-15}\right) = x - 15 + 2\sqrt{x-15}$.

Domain of $f = [15, \infty)$. Domain of $g = \mathbb{R}$. Since $f(x)$ is always in the domain of g, the domain of $g \circ f$ is the same as the domain of f, $[15, \infty)$.

23 (a) $h(x) = f\left(\sqrt{3x}\right) = \left(\sqrt{3x}\right)^2 - 4 = 3x - 4$. Domain of $g = [0, \infty)$. Domain of $f = \mathbb{R}$.

Since $g(x)$ is always in the domain of f, the domain of $f \circ g$ is the same as the domain of g, $[0, \infty)$.

Note that *both* $g(x)$ *and* $f(g(x))$ are defined for x in $[0, \infty)$.

(b) $k(x) = g(x^2 - 4) = \sqrt{3(x^2 - 4)} = \sqrt{3x^2 - 12}$. Domain of $f = \mathbb{R}$. Domain of $g = [0, \infty)$.

$f(x) \geq 0 \quad \Rightarrow \quad x^2 - 4 \geq 0 \quad \Rightarrow \quad x^2 \geq 4 \quad \Rightarrow \quad |x| \geq 2 \quad \Rightarrow \quad x \in (-\infty, -2] \cup [2, \infty)$.

Note that *both* $f(x)$ *and* $g(f(x))$ are defined for x in $(-\infty, -2] \cup [2, \infty)$.

24 (a) $h(x) = f\left(\sqrt{x}\right) = -\left(\sqrt{x}\right)^2 + 1 = -x + 1$. Domain of $g = [0, \infty)$. Domain of $f = \mathbb{R}$.

Since $g(x)$ is always in the domain of f, the domain of $f \circ g$ is the same as the domain of g, $[0, \infty)$.

(b) $k(x) = g(-x^2 + 1) = \sqrt{-x^2 + 1}$. Domain of $f = \mathbb{R}$. Domain of $g = [0, \infty)$.

$f(x) \geq 0 \quad \Rightarrow \quad -x^2 + 1 \geq 0 \quad \Rightarrow \quad x^2 \leq 1 \quad \Rightarrow \quad |x| \leq 1 \quad \Rightarrow \quad x \in [-1, 1]$.

25 (a) $h(x) = f\left(\sqrt{x+5}\right) = \sqrt{\sqrt{x+5} - 2}$. Domain of $g = [-5, \infty)$. Domain of $f = [2, \infty)$.

$g(x) \geq 2 \quad \Rightarrow \quad \sqrt{x+5} \geq 2 \quad \Rightarrow \quad x + 5 \geq 4 \quad \Rightarrow \quad x \geq -1$ or $x \in [-1, \infty)$.

(b) $k(x) = g\left(\sqrt{x-2}\right) = \sqrt{\sqrt{x-2} + 5}$. Domain of $f = [2, \infty)$. Domain of $g = [-5, \infty)$.

$f(x) \geq -5 \quad \Rightarrow \quad \sqrt{x-2} \geq -5$. This is always true since the result of a square root is nonnegative.

The domain is $[2, \infty)$.

26 (a) $h(x) = f\left(\sqrt{x+2}\right) = \sqrt{3 - \sqrt{x+2}}$. Domain of $g = [-2, \infty)$. Domain of $f = (-\infty, 3]$.

$g(x) \leq 3 \quad \Rightarrow \quad \sqrt{x+2} \leq 3 \quad \Rightarrow \quad x + 2 \leq 9 \quad \Rightarrow \quad x \leq 7$.

We must remember that $x \geq -2$, hence, $-2 \leq x \leq 7$.

(b) $k(x) = g\left(\sqrt{3-x}\right) = \sqrt{\sqrt{3-x} + 2}$. Domain of $f = (-\infty, 3]$. Domain of $g = [-2, \infty)$.

$f(x) \geq -2 \quad \Rightarrow \quad \sqrt{3-x} \geq -2$. This is always true since the result of a square root is nonnegative.

The domain is $(-\infty, 3]$.

27 (a) $h(x) = f\left(\sqrt{x^2-16}\right) = \sqrt{3-\sqrt{x^2-16}}$. Domain of $g = (-\infty,-4] \cup [4,\infty)$. Domain of $f = (-\infty, 3]$.

$g(x) \le 3 \quad \Rightarrow \quad \sqrt{x^2-16} \le 3 \quad \Rightarrow \quad x^2-16 \le 9 \quad \Rightarrow \quad x^2 \le 25 \quad \Rightarrow \quad x \in [-5,5]$.

But $|x| \ge 4$ from the domain of g, so the domain of $f \circ g$ is $[-5,-4] \cup [4,5]$.

(b) $k(x) = g\left(\sqrt{3-x}\right) = \sqrt{\left(\sqrt{3-x}\right)^2 - 16} = \sqrt{3-x-16} = \sqrt{-x-13}$. Domain of $f = (-\infty,3]$.

Domain of $g = (-\infty,-4] \cup [4,\infty)$. $f(x) \ge 4$ $\{f(x)$ cannot be less than $0\}$ $\Rightarrow$

$\sqrt{3-x} \ge 4 \quad \Rightarrow \quad 3-x \ge 16 \quad \Rightarrow \quad x \le -13$.

28 (a) $h(x) = f\left(\sqrt[3]{x-5}\right) = \left(\sqrt[3]{x-5}\right)^3 + 5 = x-5+5 = x$. Domain of $g = \mathbb{R}$. Domain of $f = \mathbb{R}$.

All values of $g(x)$ are in the domain of f. Hence, the domain of $f \circ g$ is $\mathbb{R}$.

(b) $k(x) = g(x^3+5) = \sqrt[3]{(x^3+5)-5} = \sqrt[3]{x^3} = x$. Domain of $f = \mathbb{R}$. Domain of $g = \mathbb{R}$.

All values of $f(x)$ are in the domain of g. Hence, the domain of $g \circ f$ is $\mathbb{R}$.

29 (a) $h(x) = f\left(\dfrac{5x-3}{2}\right) = \dfrac{2\left(\dfrac{5x-3}{2}\right)+3}{5} = \dfrac{5x-3+3}{5} = \dfrac{5x}{5} = x$. Domain of $g = \mathbb{R}$. Domain of $f = \mathbb{R}$.

All values of $g(x)$ are in the domain of f. Hence, the domain of $f \circ g$ is $\mathbb{R}$.

(b) $k(x) = g\left(\dfrac{2x+3}{5}\right) = \dfrac{5\left(\dfrac{2x+3}{5}\right)-3}{2} = \dfrac{2x+3-3}{2} = \dfrac{2x}{2} = x$. Domain of $f = \mathbb{R}$. Domain of $g = \mathbb{R}$.

All values of $f(x)$ are in the domain of g. Hence, the domain of $g \circ f$ is $\mathbb{R}$.

30 (a) $h(x) = f(x-1) = \dfrac{1}{(x-1)-1} = \dfrac{1}{x-2}$. Domain of $g = \mathbb{R}$. Domain of $f = \mathbb{R} - \{1\}$.

$g(x) \ne 1 \quad \Rightarrow \quad x-1 \ne 1 \quad \Rightarrow \quad x \ne 2$. Hence, the domain of $f \circ g$ is $\mathbb{R} - \{2\}$.

(b) $k(x) = g\left(\dfrac{1}{x-1}\right) = \dfrac{1}{x-1} - 1 = \dfrac{1-(x-1)}{x-1} = \dfrac{2-x}{x-1}$. Domain of $f = \mathbb{R} - \{1\}$. Domain of $g = \mathbb{R}$.

All values of $f(x)$ are in the domain of g. The domain of $g \circ f$ is $\mathbb{R} - \{1\}$.

31 (a) $h(x) = f\left(\dfrac{1}{x^3}\right) = \left(\dfrac{1}{x^3}\right)^2 = \dfrac{1}{x^6}$. Domain of $g = \mathbb{R} - \{0\}$. Domain of $f = \mathbb{R}$.

All values of $g(x)$ are in the domain of f. Hence, the domain of $f \circ g$ is $\mathbb{R} - \{0\}$.

(b) $k(x) = g(x^2) = \dfrac{1}{(x^2)^3} = \dfrac{1}{x^6}$. Domain of $f = \mathbb{R}$. Domain of $g = \mathbb{R} - \{0\}$.

All values of $f(x)$ are in the domain of g except for 0. Since f is 0 when x is 0, the domain of $f \circ g$ is $\mathbb{R} - \{0\}$.

32 (a) $h(x) = f\left(\dfrac{3}{x}\right) = \dfrac{3/x}{(3/x)-2} \cdot \dfrac{x}{x} = \dfrac{3}{3-2x}$. Domain of $g = \mathbb{R} - \{0\}$. Domain of $f = \mathbb{R} - \{2\}$.

$g(x) \ne 2 \quad \Rightarrow \quad \dfrac{3}{x} \ne 2 \quad \Rightarrow \quad x \ne \frac{3}{2}$. Hence, the domain of $f \circ g$ is $\mathbb{R} - \left\{0, \frac{3}{2}\right\}$.

(b) $k(x) = g\left(\dfrac{x}{x-2}\right) = \dfrac{3}{x/(x-2)} = \dfrac{3x-6}{x}$. Domain of $f = \mathbb{R} - \{2\}$. Domain of $g = \mathbb{R} - \{0\}$.

$f(x) \ne 0 \quad \Rightarrow \quad \dfrac{x}{x-2} \ne 0 \quad \Rightarrow \quad x \ne 0$. Hence, the domain of $g \circ f$ is $\mathbb{R} - \{0, 2\}$.

33 (a) $h(x) = f\left(\dfrac{x-3}{x-4}\right) = \dfrac{\dfrac{x-3}{x-4} - 1}{\dfrac{x-3}{x-4} - 2} \cdot \dfrac{x-4}{x-4} = \dfrac{x-3-1(x-4)}{x-3-2(x-4)} = \dfrac{1}{5-x}$.

Domain of $g = \mathbb{R} - \{4\}$. Domain of $f = \mathbb{R} - \{2\}$.

$g(x) \ne 2 \quad \Rightarrow \quad \dfrac{x-3}{x-4} \ne 2 \quad \Rightarrow \quad x-3 \ne 2x-8 \quad \Rightarrow \quad x \ne 5$. The domain is $\mathbb{R} - \{4, 5\}$.

(b) $k(x) = g\left(\dfrac{x-1}{x-2}\right) = \dfrac{\dfrac{x-1}{x-2} - 3}{\dfrac{x-1}{x-2} - 4} \cdot \dfrac{x-2}{x-2} = \dfrac{x-1-3(x-2)}{x-1-4(x-2)} = \dfrac{-2x+5}{-3x+7}$.

Domain of $f = \mathbb{R} - \{2\}$. Domain of $g = \mathbb{R} - \{4\}$.

$f(x) \neq 4 \;\Rightarrow\; \dfrac{x-1}{x-2} \neq 4 \;\Rightarrow\; x - 1 \neq 4x - 8 \;\Rightarrow\; x \neq \frac{7}{3}$. The domain is $\mathbb{R} - \left\{2, \frac{7}{3}\right\}$.

34 (a) $h(x) = f\left(\dfrac{x-5}{x+4}\right) = \dfrac{\dfrac{x-5}{x+4} + 2}{\dfrac{x-5}{x+4} - 1} \cdot \dfrac{x+4}{x+4} = \dfrac{x-5+2(x+4)}{x-5-1(x+4)} = \dfrac{3x+3}{-9} = \dfrac{-x-1}{3}$.

Domain of $g = \mathbb{R} - \{-4\}$. Domain of $f = \mathbb{R} - \{1\}$. $g(x) \neq 1 \;\Rightarrow\; \dfrac{x-5}{x+4} \neq 1 \;\Rightarrow\; x - 5 \neq x + 4$.

This is always true—so no additional values need to be excluded, and thus, the domain is $\mathbb{R} - \{-4\}$.

(b) $k(x) = g\left(\dfrac{x+2}{x-1}\right) = \dfrac{\dfrac{x+2}{x-1} - 5}{\dfrac{x+2}{x-1} + 4} \cdot \dfrac{x-1}{x-1} = \dfrac{x+2-5(x-1)}{x+2+4(x-1)} = \dfrac{-4x+7}{5x-2}$.

Domain of $f = \mathbb{R} - \{1\}$. Domain of $g = \mathbb{R} - \{-4\}$. $f(x) \neq -4 \;\Rightarrow\; \dfrac{x+2}{x-1} \neq -4 \;\Rightarrow$

$x + 2 \neq -4x + 4 \;\Rightarrow\; x \neq \frac{2}{5}$. The domain is $\mathbb{R} - \left\{\frac{2}{5}, 1\right\}$.

35 $f(x) = x^2 - 2$, $g(x) = x + 3$ • $(f \circ g)(x) = f(g(x)) = f(x+3) = (x+3)^2 - 2$.

$(f \circ g)(x) = 0 \;\Rightarrow\; (x+3)^2 - 2 = 0 \;\Rightarrow\; (x+3)^2 = 2 \;\Rightarrow\; x + 3 = \pm\sqrt{2} \;\Rightarrow\; x = -3 \pm \sqrt{2}$

36 $f(x) = x^2 - x - 2$, $g(x) = 2x - 5$ •

$(f \circ g)(x) = f(g(x)) = f(2x-5) = (2x-5)^2 - (2x-5) - 2 = 4x^2 - 22x + 28$.

$(f \circ g)(x) = 0 \;\Rightarrow\; 4x^2 - 22x + 28 = 0 \;\Rightarrow\; (2x-4)(2x-7) = 0 \;\Rightarrow\; x = 2, \frac{7}{2}$

37 (a) $(f \circ g)(6) = f(g(6)) = f(8) = 5$

(b) $(g \circ f)(6) = g(f(6)) = g(7) = 6$

(c) $(f \circ f)(6) = f(f(6)) = f(7) = 6$

(d) $(g \circ g)(6) = g(g(6)) = g(8) = 5$

(e) $(f \circ g)(9) = f(g(9)) = f(4)$, but $f(4)$ cannot be determined from the table.

38 (a) $(T \circ S)(1) = T(S(1)) = T(0) = 2$

(b) $(S \circ T)(1) = S(T(1)) = S(3) = 2$

(c) $(T \circ T)(1) = T(T(1)) = T(3) = 0$

(d) $(S \circ S)(1) = S(S(1)) = S(0) = 1$

(e) $(T \circ S)(4) = T(S(4)) = T(5)$, but $T(5)$ cannot be determined from the table.

39 $D(t) = \sqrt{400 + t^2}$, $R(x) = 20x$ • $(D \circ R)(x) = D(R(x)) = D(20x) = \sqrt{400 + (20x)^2} = \sqrt{400 + 400x^2}$

$= \sqrt{400(1 + x^2)} = \sqrt{400}\,\sqrt{x^2 + 1} = 20\sqrt{x^2 + 1}$

40 $S(r) = 4\pi r^2$, $D(t) = 2t + 5$ • $(S \circ D)(t) = S(D(t)) = S(2t+5) = 4\pi(2t+5)^2$

41 We need to examine $(fg)(-x)$—if we obtain $(fg)(x)$, then fg is an even function,

whereas if we obtain $-(fg)(x)$, then fg is an odd function.

$(fg)(-x) = f(-x)g(-x)$ {definition of the product of two functions}

$= -f(x)g(x)$ {f is odd, so $f(-x) = -f(x)$; g is even, so $g(-x) = g(x)$}

$= -(fg)(x)$ {definition of the product of two functions}

Since $(fg)(-x) = -(fg)(x)$, fg is an odd function.

42 If f is even and odd, then $f(-x) = f(x)$ and $f(-x) = -f(x)$.
Thus, $f(x) = -f(x) \Rightarrow 2f(x) = 0 \Rightarrow f(x) = 0$. Hence, $f(x) = 0$ is a function that is both even and odd.

43 $$\begin{aligned}(\text{ROUND2} \circ \text{SSTAX})(525.00) &= \text{ROUND2}(\text{SSTAX}(525.00)) \\ &= \text{ROUND2}(0.0765 \cdot 525.00) \\ &= \text{ROUND2}(40.1625) = 40.16\end{aligned}$$

44 (a) (CHR ∘ ORD)(“C") = CHR(ORD(“C")) = CHR(67) = “C”

(b) CHR(ORD(“A") + 3) = CHR(65 + 3) = CHR(68) = “D”

45 $r = 5t$ and $A = \pi r^2 \Rightarrow A = \pi(5t)^2 = 25\pi t^2$ ft^2.

46 $V = \dfrac{4}{3}\pi r^3 \Rightarrow r^3 = \dfrac{3V}{4\pi} \Rightarrow r = \sqrt[3]{\dfrac{3V}{4\pi}}$. $V = \dfrac{9}{2}\pi t \Rightarrow r(t) = \sqrt[3]{\dfrac{3\left(\frac{9}{2}\pi t\right)}{4\pi}} = \sqrt[3]{\dfrac{27t}{8}} = \dfrac{3}{2}\sqrt[3]{t}$ ft.

47 The formula for the volume of a cone is $V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi r^3$ {since $h = r$} $\Rightarrow$
$$r^3 = \frac{3V}{\pi} \Rightarrow r = \sqrt[3]{\frac{3V}{\pi}}.\ V = 243\pi t \Rightarrow r = \sqrt[3]{\frac{3 \cdot 243\pi t}{\pi}} = \sqrt[3]{729t} = 9\sqrt[3]{t} \text{ ft.}$$

48 $x^2 + x^2 = y^2 \Rightarrow y^2 = 2x^2 \Rightarrow y = \sqrt{2}x$. $y^2 + x^2 = d^2 \Rightarrow d^2 = 3x^2 \Rightarrow d(x) = \sqrt{3}x$.

49 Let l denote the length of the rope. At $t = 0$, $l = 20$. At $t = 1$, $l = 25$.
At time t, $l = 20 + 5t$, *not* just $5t$. We have a right triangle with sides 20, h, and l. $h^2 + 20^2 = l^2 \Rightarrow$
$$h = \sqrt{(20 + 5t)^2 - 20^2} = \sqrt{25t^2 + 200t} = \sqrt{25(t^2 + 8t)} = 5\sqrt{t^2 + 8t}.$$

50 The triangle has sides of length 28, 50, and $\sqrt{28^2 + 50^2} = \sqrt{3284} = 2\sqrt{821}$.
Let $y = h - 2$. Using similar triangles and the fact that $d = 2t$,
$$\frac{y}{d} = \frac{28}{2\sqrt{821}} \Rightarrow y = \frac{14}{\sqrt{821}}d \Rightarrow h(t) = \frac{28}{\sqrt{821}}t + 2.$$

51 From Exercise 77 of Section 3.4, $d(x) = \sqrt{90{,}400 + x^2}$. The distance x of the plane from the control tower is 500 feet plus 150 feet per second, that is, $x(t) = 500 + 150t$. Thus,
$$\begin{aligned}d(t) &= \sqrt{90{,}400 + (500 + 150t)^2} = \sqrt{90{,}400 + 250{,}000 + 150{,}000t + 22{,}500t^2} \\ &= \sqrt{22{,}500t^2 + 150{,}000t + 340{,}400} = 10\sqrt{225t^2 + 1500t + 3404}\end{aligned}$$

52 Consider the cable to be a right circular cylinder.
$A = 2\pi rh = \pi dh = 1200\pi d$ {$h = 1200$ inches} $\Rightarrow d = \dfrac{A}{1200\pi}$.
$$A = -750t, \text{ so } d(t) = \text{original value} + \text{change} = 4 - \frac{750t}{1200\pi} = 4 - \frac{5}{8\pi}t \text{ inches.}$$

53 $y = (x^2 + 5x)^{1/3}$ • Suppose you were to find the value of y if x was equal to 3. Using a calculator, you might compute the value of $x^2 + 5x$ first, and then raise that result to the $\frac{1}{3}$ power. Thus, we would choose $y = u^{1/3}$ and $u = x^2 + 5x$.

54 For $y = \sqrt[4]{x^4 - 64}$, choose $u = x^4 - 64$ and $y = \sqrt[4]{u}$.

55 For $y = \dfrac{1}{(x-3)^6}$, choose $u = x - 3$ and $y = \dfrac{1}{u^6} = u^{-6}$.

56 For $y = 4 + \sqrt{x^2 + 1}$, choose $u = x^2 + 1$ and $y = 4 + \sqrt{u}$.

57 For $y = \left(x^4 - 2x^2 + 5\right)^5$, choose $u = x^4 - 2x^2 + 5$ and $y = u^5$.

58 For $y = \dfrac{1}{(x^2+3x-5)^3}$, choose $u = x^2+3x-5$ and $y = \dfrac{1}{u^3} = u^{-3}$.

59 For $y = \dfrac{\sqrt{x+4}-2}{\sqrt{x+4}+2}$, there is not a "simple" choice for y as in previous exercises.

One choice for u is $u = x+4$. Then y would be $\dfrac{\sqrt{u}-2}{\sqrt{u}+2}$.

Another choice for u is $u = \sqrt{x+4}$. Then y would be $\dfrac{u-2}{u+2}$.

60 For $y = \dfrac{\sqrt[3]{x}}{1+\sqrt[3]{x}}$, choose $u = \sqrt[3]{x}$ and $y = \dfrac{u}{1+u}$.

61 $(f \circ g)(x) = f(g(x)) = f(x^3+1) = \sqrt{x^3+1} - 1$. We will multiply this expression by $\dfrac{\sqrt{x^3+1}+1}{\sqrt{x^3+1}+1}$, treating it as though it was one factor of a difference of two squares.

$$\left(\sqrt{x^3+1}-1\right) \times \frac{\sqrt{x^3+1}+1}{\sqrt{x^3+1}+1} = \frac{\left(\sqrt{x^3+1}\right)^2 - 1^2}{\sqrt{x^3+1}+1} = \frac{x^3}{\sqrt{x^3+1}+1}$$

Thus, $(f \circ g)(0.0001) = f\left(g\left(10^{-4}\right)\right) \approx \dfrac{\left(10^{-4}\right)^3}{2} = 5 \times 10^{-13}$.

62 $f(1.12) \approx 0.321170$, $g(1.12) \approx 0.280105$, $f(5.2) \approx 4.106542$, and $f(f(5.2)) \approx 3.014835 \quad \Rightarrow$

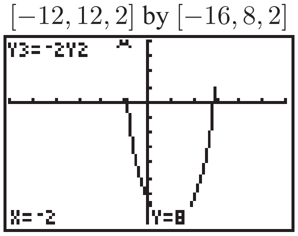
$$\frac{(f+g)(1.12) - (f/g)(1.12)}{[(f \circ f)(5.2)]^2} = \frac{[f(1.12)+g(1.12)] - f(1.12)/g(1.12)}{\{f[f(5.2)]\}^2} \approx -0.059997$$

63 ***Note:*** If we relate this problem to the composite function $g(f(u))$, then Y_2 can be considered to be f, Y_1 {which can be considered to be u} is the function of x we are substituting into f, and Y_3 {which can be considered to be g} is accepting Y_2's output values as its input values. Hence, Y_3 is a function of a function of a function.

Note: The graphs often do not show the correct endpoints—you need to change the viewing rectangle, or zoom in to actually view them on the screen.

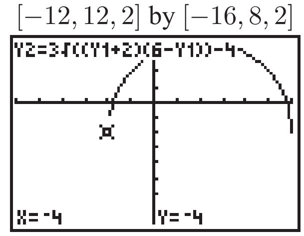
(a) $y = -2f(x)$; $\mathrm{Y}_1 = x$, $\mathrm{Y}_2 = 3\sqrt{(\mathrm{Y}_1+2)(6-\mathrm{Y}_1)} - 4$, graph $\mathrm{Y}_3 = -2\mathrm{Y}_2$ {turn off Y_1 and Y_2, leaving only Y_3 on}; $D = [-2, 6]$, $R = [-16, 8]$

$[-12, 12, 2]$ by $[-16, 8, 2]$



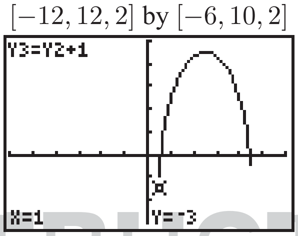
(b) $y = f\left(\frac{1}{2}x\right)$; $\mathrm{Y}_1 = 0.5x$, graph Y_2; $D = [-4, 12]$, $R = [-4, 8]$

$[-12, 12, 2]$ by $[-16, 8, 2]$



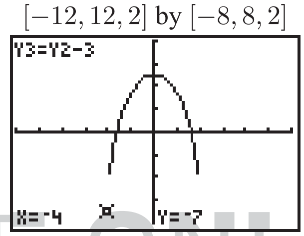
(c) $y = f(x-3)+1$; $\mathrm{Y}_1 = x-3$, graph $\mathrm{Y}_3 = \mathrm{Y}_2 + 1$; $D = [1, 9]$, $R = [-3, 9]$

$[-12, 12, 2]$ by $[-6, 10, 2]$



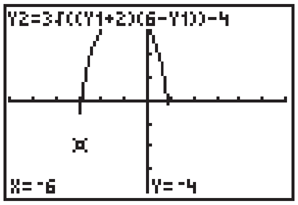
(d) $y = f(x+2)-3$; $\mathrm{Y}_1 = x+2$, graph $\mathrm{Y}_3 = \mathrm{Y}_2 - 3$; $D = [-4, 4]$, $R = [-7, 5]$

$[-12, 12, 2]$ by $[-8, 8, 2]$



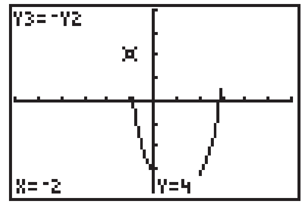
(e) $y = f(-x)$; $\mathrm{Y}_1 = -x$, graph Y_2; $D = [-6, 2]$, $R = [-4, 8]$

$[-12, 12, 2]$ by $[-8, 8, 2]$



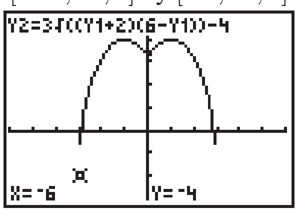
(f) $y = -f(x)$; $\mathrm{Y}_1 = x$, graph $\mathrm{Y}_3 = -\mathrm{Y}_2$; $D = [-2, 6]$, $R = [-8, 4]$

$[-12, 12, 2]$ by $[-8, 8, 2]$



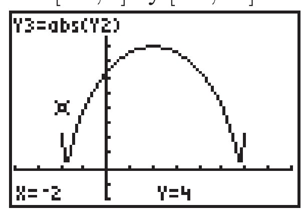
(g) $y = f(|x|)$; $\mathrm{Y}_1 = \mathrm{abs}\, x$, graph Y_2; $D = [-6, 6]$, $R = [-4, 8]$

$[-12, 12, 2]$ by $[-6, 10, 2]$



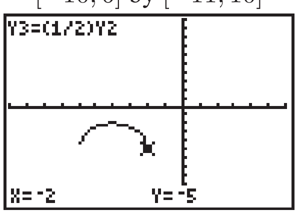
(h) $y = |f(x)|$; $\mathrm{Y}_1 = x$, graph $\mathrm{Y}_3 = \mathrm{abs}\, \mathrm{Y}_2$; $D = [-2, 6]$, $R = [0, 8]$

$[-4, 8]$ by $[-2, 10]$



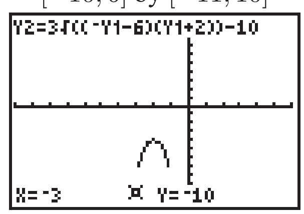
64 (a) $y = \frac{1}{2}f(x)$; $\mathrm{Y}_1 = x$, graph $\mathrm{Y}_3 = 0.5\mathrm{Y}_2$; $D = [-6, -2]$, $R = [-5, -2]$

$[-10, 6]$ by $[-11, 10]$



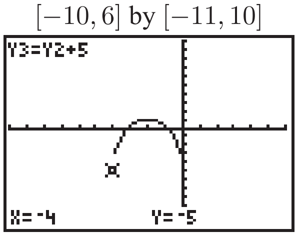
(b) $y = f(2x)$; $\mathrm{Y}_1 = 2x$, graph Y_2; $D = [-3, -1]$, $R = [-10, -4]$

$[-10, 6]$ by $[-11, 10]$



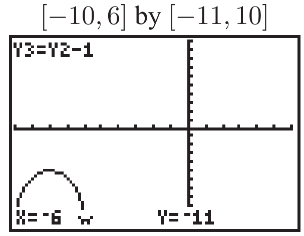
(c) $y = f(x-2) + 5$; $\mathrm{Y}_1 = x - 2$, graph $\mathrm{Y}_3 = \mathrm{Y}_2 + 5$; $D = [-4, 0]$, $R = [-5, 1]$

$[-10, 6]$ by $[-11, 10]$



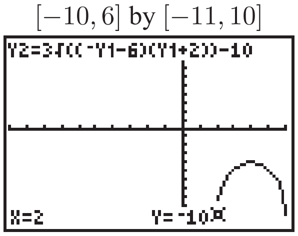
(d) $y = f(x+4) - 1$; $\mathrm{Y}_1 = x + 4$, graph $\mathrm{Y}_3 = \mathrm{Y}_2 - 1$; $D = [-10, -6]$, $R = [-11, -5]$

$[-10, 6]$ by $[-11, 10]$



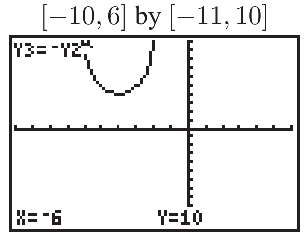
(e) $y = f(-x)$; $\mathrm{Y}_1 = -x$, graph Y_2; $D = [2, 6]$, $R = [-10, -4]$

$[-10, 6]$ by $[-11, 10]$



(f) $y = -f(x)$; $\mathrm{Y}_1 = x$, graph $\mathrm{Y}_3 = -\mathrm{Y}_2$; $D = [-6, -2]$, $R = [4, 10]$

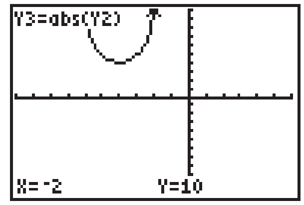
$[-10, 6]$ by $[-11, 10]$



(g) $y = f(|x|)$; No graph

(h) $y = |f(x)|$; $Y_1 = x$, graph $Y_3 = \text{abs}\, Y_2$;
$D = [-6, -2]$, $R = [4, 10]$

$[-10, 6]$ by $[-11, 10]$



Chapter 3 Review Exercises

1 If $y/x < 0$, then y and x must have opposite signs, and hence, the set consists of all points in quadrants II and IV.

2 The points are $A(3, 1)$, $B(-5, -3)$, and $C(4, -1)$.
Show that $d(A, B)^2 + d(A, C)^2 = d(B, C)^2$; that is, $\left(\sqrt{80}\right)^2 + \left(\sqrt{5}\right)^2 = \left(\sqrt{85}\right)^2$.

$$\text{Area} = \tfrac{1}{2}bh = \tfrac{1}{2}\left(\sqrt{80}\right)\left(\sqrt{5}\right) = \tfrac{1}{2}\left(4\sqrt{5}\right)\left(\sqrt{5}\right) = 10.$$

3 (a) $P(-5, 9), Q(-8, -7) \quad\Rightarrow\quad d(P, Q) = \sqrt{[-8-(-5)]^2 + (-7-9)^2} = \sqrt{9 + 256} = \sqrt{265}$.

(b) $P(-5, 9), Q(-8, -7) \quad\Rightarrow\quad M_{PQ} = \left(\dfrac{-5 + (-8)}{2}, \dfrac{9 + (-7)}{2}\right) = \left(-\dfrac{13}{2}, 1\right)$.

(c) Let $R = (x, y)$. $Q = M_{PR} \quad\Rightarrow\quad (-8, -7) = \left(\dfrac{-5 + x}{2}, \dfrac{9 + y}{2}\right) \quad\Rightarrow\quad -8 = \dfrac{-5 + x}{2}$ and
$-7 = \dfrac{9 + y}{2} \quad\Rightarrow\quad -5 + x = -16$ and $9 + y = -14 \quad\Rightarrow\quad x = -11$ and $y = -23 \quad\Rightarrow\quad R = (-11, -23)$.

4 Let $Q(0, y)$ be an arbitrary point on the y-axis.
$13 = d(P, Q)$ {with $P = (12, 8)$} $\quad\Rightarrow\quad 13 = \sqrt{(0-12)^2 + (y-8)^2} \quad\Rightarrow\quad 169 = 144 + y^2 - 16y + 64 \quad\Rightarrow$
$y^2 - 16y + 39 = 0 \quad\Rightarrow\quad (y-3)(y-13) = 0 \quad\Rightarrow\quad y = 3, 13$. The points are $(0, 3)$ and $(0, 13)$.

5 With $P(a, 1)$ and $Q(-2, a)$, $d(P, Q) < 3 \quad\Rightarrow\quad \sqrt{(-2-a)^2 + (a-1)^2} < 3 \quad\Rightarrow$
$4 + 4a + a^2 + a^2 - 2a + 1 < 9 \quad\Rightarrow\quad 2a^2 + 2a - 4 < 0 \quad\Rightarrow\quad a^2 + a - 2 < 0 \quad\Rightarrow\quad (a+2)(a-1) < 0$.

Interval	$(-\infty, -2)$	$(-2, 1)$	$(1, \infty)$
Sign of $a + 2$	$-$	$+$	$+$
Sign of $a - 1$	$-$	$-$	$+$
Resulting sign	$+$	$-$	$+$

From the chart, we see that $-2 < a < 1$ will assure us that $d(P, Q) < 3$.

6 The equation of a circle with center $C(7, -4)$ is $(x-7)^2 + (y+4)^2 = r^2$.
Letting $x = -2$ and $y = 5$ yields $(-9)^2 + 9^2 = r^2 \quad\Rightarrow\quad r^2 = 162$. An equation is $(x-7)^2 + (y+4)^2 = 162$.

7 The center of the circle is the midpoint of $A(8, 10)$ and $B(-2, -14)$, so the center is
$M_{AB} = \left(\dfrac{8 + (-2)}{2}, \dfrac{10 + (-14)}{2}\right) = (3, -2)$. The radius of the circle is
$\frac{1}{2} \cdot d(A, B) = \frac{1}{2}\sqrt{(-2-8)^2 + (-14-10)^2} = \frac{1}{2}\sqrt{100 + 576} = \frac{1}{2} \cdot 26 = 13$.

An equation is $(x-3)^2 + (y+2)^2 = 13^2 = 169$.

8 We need to solve the equation for x. $(x+2)^2+y^2=7 \quad\Rightarrow\quad (x+2)^2=7-y^2 \quad\Rightarrow\quad x+2=\pm\sqrt{7-y^2} \quad\Rightarrow$
$x=-2\pm\sqrt{7-y^2}$. Choose the term with the minus sign for the left half. $x=-2-\sqrt{7-y^2}$

9 $C(11,-5), D(-6,8) \quad\Rightarrow\quad m_{CD}=\dfrac{8-(-5)}{-6-11}=\dfrac{13}{-17}=-\dfrac{13}{17}$.

10 Show that the slopes of one pair of opposite sides are equal.
$A(-3,1), B(1,-1), C(4,1)$, and $D(3,5) \quad\Rightarrow\quad m_{AD}=\frac{2}{3}=m_{BC}$.

11 (a) $6x+2y+5=0 \quad\Leftrightarrow\quad y=-3x-\frac{5}{2}$. Using the same slope, -3, with $A\left(\frac{1}{2},-\frac{1}{3}\right)$, we have
$y+\frac{1}{3}=-3\left(x-\frac{1}{2}\right) \quad\Rightarrow\quad 6\left(y+\frac{1}{3}\right)=-18\left(x-\frac{1}{2}\right) \quad\Rightarrow\quad 6y+2=-18x+9 \quad\Rightarrow\quad 18x+6y=7$.

(b) Using the negative reciprocal of -3 for the slope,
$y+\frac{1}{3}=\frac{1}{3}\left(x-\frac{1}{2}\right) \quad\Rightarrow\quad 6\left(y+\frac{1}{3}\right)=2\left(x-\frac{1}{2}\right) \quad\Rightarrow\quad 6y+2=2x-1 \quad\Rightarrow\quad 2x-6y=3$.

12 Solving for y gives us: $8x+3y-15=0 \quad\Leftrightarrow\quad 3y=-8x+15 \quad\Leftrightarrow\quad y=-\frac{8}{3}x+5$

13 The radius of the circle is the distance from the line $x=4$ to the x-value of the
center $C(-5,-1)$; $r=4-(-5)=9$. An equation is $(x+5)^2+(y+1)^2=81$.

14 $x^2+y^2-4x+10y+26=0 \quad\Rightarrow\quad x^2-4x+\underline{\ 4\ }+y^2+10y+\underline{\ 25\ }=-26+\underline{\ 4\ }+\underline{\ 25\ } \quad\Rightarrow$
$(x-2)^2+(y+5)^2=3 \quad\Rightarrow\quad C(2,-5)$. We want the equation of the line through $(-3,0)$ and $(2,-5)$.
$y-0=\dfrac{-5-0}{2+3}(x+3) \quad\Rightarrow\quad y=-1(x+3) \quad\Rightarrow\quad x+y=-3$.

15 $P(3,-7)$ with $m=4 \quad\Rightarrow\quad y+7=4(x-3) \quad\Rightarrow\quad y+7=4x-12 \quad\Rightarrow\quad 4x-y=19$.

16 $A(-1,2)$ and $B(3,-4) \quad\Rightarrow\quad M_{AB}=(1,-1)$ and $m_{AB}=-\frac{3}{2}$. We want the equation of the line through $(1,-1)$ with slope $\frac{2}{3}$ $\left\{\text{the negative reciprocal of } -\frac{3}{2}\right\}$. $y+1=\frac{2}{3}(x-1) \quad\Rightarrow\quad 3y+3=2x-2 \quad\Rightarrow\quad 2x-3y=5$.

17 $x^2+y^2-12y+31=0 \quad\Rightarrow\quad x^2+y^2-12y+\underline{\ 36\ }=-31+\underline{\ 36\ } \quad\Rightarrow\quad x^2+(y-6)^2=5$. $C(0,6)$; $r=\sqrt{5}$

18 $4x^2+4y^2+24x-16y+41=0 \quad\Rightarrow\quad x^2+y^2+6x-4y+\frac{41}{4}=0 \quad\Rightarrow$
$x^2+6x+\underline{\ 9\ }+y^2-4y+\underline{\ 4\ }=-\frac{41}{4}+\underline{\ 9\ }+\underline{\ 4\ } \quad\Rightarrow\quad (x+3)^2+(y-2)^2=\frac{11}{4}$. $C(-3,2)$; $r=\frac{1}{2}\sqrt{11}$

19 (a) $f(x)=\dfrac{x}{\sqrt{x+3}} \quad\Rightarrow\quad f(1)=\dfrac{1}{\sqrt{4}}=\dfrac{1}{2}$

(b) $f(-1)=\dfrac{-1}{\sqrt{-1+3}}=-\dfrac{1}{\sqrt{2}}$

(c) $f(0)=\dfrac{0}{\sqrt{3}}=0$

(d) $f(-x)=\dfrac{-x}{\sqrt{-x+3}}=-\dfrac{x}{\sqrt{3-x}}$

(e) $-f(x)=-1\cdot f(x)=-\dfrac{x}{\sqrt{x+3}}$

(f) $f\left(x^2\right)=\dfrac{x^2}{\sqrt{x^2+3}}$

(g) $[f(x)]^2=\left(\dfrac{x}{\sqrt{x+3}}\right)^2=\dfrac{x^2}{x+3}$

20 $f(x)=\dfrac{-32(x^2-4)}{(9-x^2)^{5/3}} \quad\Rightarrow\quad f(4)=\dfrac{(-)(+)}{(-)}=+$. $f(4)$ is positive.

Note that the factor -32 always contributes one negative sign to the quotient.

21 $f(x) = \dfrac{-2(x^2-20)(3-x)}{(6-x^2)^{4/3}} \quad \Rightarrow \quad f(4) = \dfrac{(-)(-)(-)}{(+)} = (-).$ $f(4)$ is negative.

Note that the factor -2 always contributes one negative sign to the quotient and that the denominator is always positive if $x \neq \pm\sqrt{6}$.

22 (a) $y = f(x) = \sqrt{3x-4}$ • $3x-4 \geq 0 \quad \Rightarrow \quad x \geq \frac{4}{3}$; $D = [\frac{4}{3}, \infty)$.

Since y is the result of a square root, $y \geq 0$; $R = [0, \infty)$.

(b) $y = f(x) = \dfrac{1}{(x+4)^2}$ • $D =$ All real numbers except -4.

Since y is the square of the nonzero term $\dfrac{1}{x+4}$, $y > 0$; $R = (0, \infty)$.

23
$$\begin{aligned}\frac{f(a+h)-f(a)}{h} &= \frac{\left[-(a+h)^2+(a+h)+5\right]-\left[-a^2+a+5\right]}{h} \\ &= \frac{-a^2-2ah-h^2+a+h+5+a^2-a-5}{h} \\ &= \frac{-2ah-h^2+h}{h} = \frac{h(-2a-h+1)}{h} = -2a-h+1\end{aligned}$$

24
$$\frac{f(a+h)-f(a)}{h} = \frac{\dfrac{1}{a+h+4}-\dfrac{1}{a+4}}{h} = \frac{\dfrac{(a+4)-(a+h+4)}{(a+h+4)(a+4)}}{h} = \frac{-h}{(a+h+4)(a+4)h} = -\frac{1}{(a+h+4)(a+4)}$$

25 $f(x) = ax+b$ is the desired form. $f(1) = 3$ and $f(4) = 8 \quad \Rightarrow \quad a = \text{slope} = \dfrac{8-3}{4-1} = \dfrac{5}{3}$.

$f(x) = \frac{5}{3}x + b \quad \Rightarrow \quad f(1) = \frac{5}{3} + b$, but $f(1) = 3$, so $\frac{5}{3} + b = 3$, and $b = \frac{4}{3}$. Thus, $f(x) = \frac{5}{3}x + \frac{4}{3}$.

26 (a) $f(x) = \sqrt[3]{x^3+4x} \quad \Rightarrow \quad f(-x) = \sqrt[3]{(-x)^3+4(-x)} = \sqrt[3]{-1(x^3+4x)} = -\sqrt[3]{x^3+4x} = -f(x)$,

so f is odd

(b) $f(x) = \sqrt[3]{\pi x^2 - x^3} \quad \Rightarrow \quad f(-x) = \sqrt[3]{\pi(-x)^2-(-x)^3} = \sqrt[3]{\pi x^2 + x^3} \neq \pm f(x)$,

so f is neither even nor odd

(c) $f(x) = \sqrt[3]{x^4-3x^2+5} \quad \Rightarrow \quad f(-x) = \sqrt[3]{(-x)^4-3(-x)^2+5} = \sqrt[3]{x^4-3x^2+5} = f(x)$, so f is even

27 $x+5=0 \quad \Leftrightarrow \quad x = -5$, a vertical line; x-intercept -5; y-intercept: None

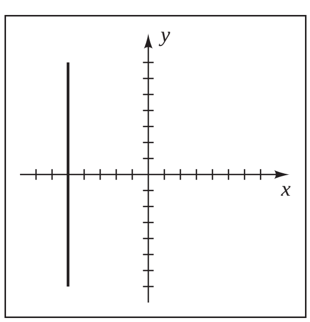


Figure 27

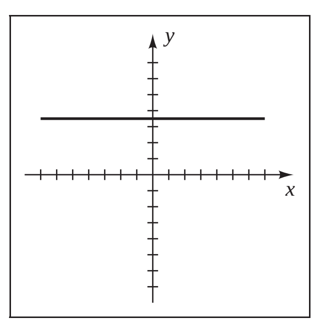


Figure 28

28 $2y-7=0 \quad \Leftrightarrow \quad y = \frac{7}{2}$, a horizontal line; x-intercept: None; y-intercept 3.5

29 $2y + 5x - 8 = 0 \quad \Leftrightarrow \quad y = -\frac{5}{2}x + 4$, a line with slope $-\frac{5}{2}$ and y-intercept 4.

x-intercept: $y = 0 \quad \Rightarrow \quad 2(0) + 5x - 8 = 0 \quad \Rightarrow \quad 5x = 8 \quad \Rightarrow \quad x = 1.6$

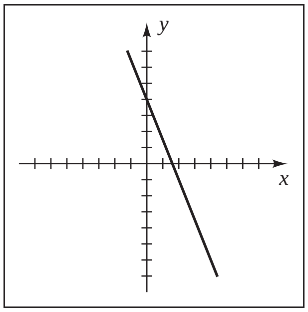


Figure 29

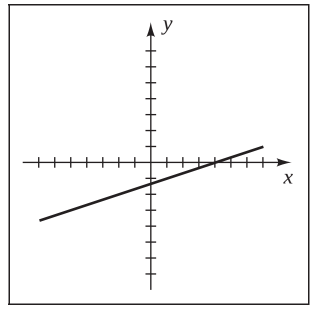


Figure 30

30 $x = 3y + 4 \quad \Leftrightarrow \quad y = \frac{1}{3}x - \frac{4}{3}$, a line with slope $\frac{1}{3}$ and y-intercept $-\frac{4}{3}$; x-intercept 4

31 $9y + 2x^2 = 0 \quad \Leftrightarrow \quad y = -\frac{2}{9}x^2$, a parabola opening down; x- and y-intercept 0

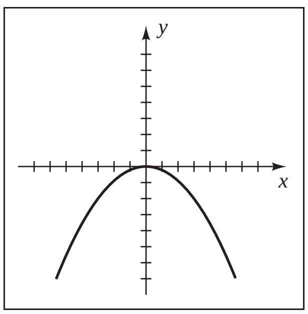


Figure 31

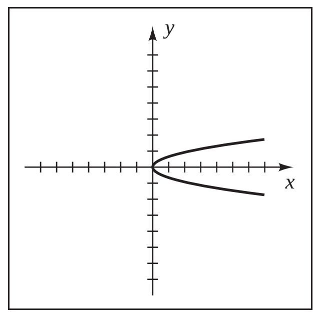


Figure 32

32 $3x - 7y^2 = 0 \quad \Leftrightarrow \quad x = \frac{7}{3}y^2$, a parabola opening to the right; x- and y-intercept 0

33 $y = \sqrt{1 - x}$ • The radicand must be nonnegative for the radical to be defined. $1 - x \geq 0 \quad \Rightarrow \quad 1 \geq x$, or equivalently, $x \leq 1$.

The domain is $(-\infty, 1]$ and the range is $[0, \infty)$.

x-intercept: $y = 0 \quad \Rightarrow \quad 0 = \sqrt{1 - x} \quad \Rightarrow \quad 0 = 1 - x \quad \Rightarrow \quad x = 1$

y-intercept: $x = 0 \quad \Rightarrow \quad y = \sqrt{1 - 0} = 1$

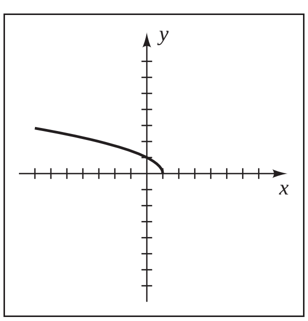


Figure 33

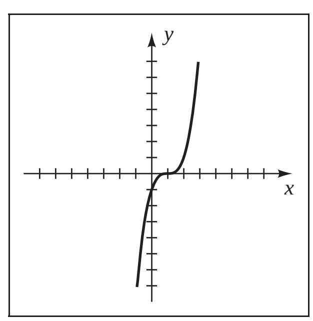


Figure 34

34 $y = (x - 1)^3$ • shift $y = x^3$ right one unit; x-intercept 1; y-intercept -1

35 $y^2 = 16 - x^2 \quad \Leftrightarrow \quad x^2 + y^2 = 16$; x-intercepts ± 4; y-intercepts ± 4

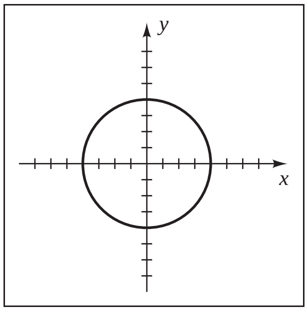


Figure 35

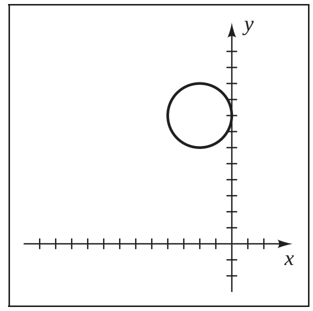


Figure 36

36 $x^2 + y^2 + 4x - 16y + 64 = 0 \quad \Rightarrow \quad x^2 + 4x + \underline{\ 4\ } + y^2 - 16y + \underline{\ 64\ } = -64 + \underline{\ 4\ } + \underline{\ 64\ } \quad \Rightarrow$
$(x+2)^2 + (y-8)^2 = 4$; $C(-2, 8)$, $r = \sqrt{4} = 2$; x-intercept: None; y-intercept 8

37 $x^2 + y^2 - 8x = 0 \quad \Leftrightarrow \quad x^2 - 8x + \underline{\ 16\ } + y^2 = \underline{\ 16\ } \quad \Leftrightarrow \quad (x-4)^2 + y^2 = 16$. This is a circle with center $C(4, 0)$ and radius $r = \sqrt{16} = 4$. x-intercepts: $y = 0 \quad \Rightarrow \quad x^2 - 8x = 0 \quad \Rightarrow \quad x(x-8) = 0 \quad \Rightarrow \quad x = 0$ and 8;
y-intercept: 0

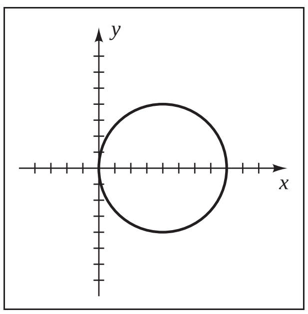


Figure 37

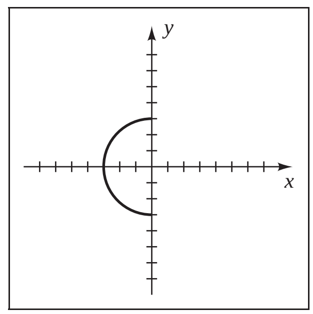


Figure 38

38 $x = -\sqrt{9 - y^2}$ is the left half of the circle $x^2 + y^2 = 9$; x-intercept -3; y-intercepts ± 3

39 $y = (x-3)^2 - 2$ is a parabola that opens upward and has vertex $(3, -2)$.
x-intercepts: $y = 0 \quad \Rightarrow \quad 0 = (x-3)^2 - 2 \quad \Rightarrow \quad (x-3)^2 = 2 \quad \Rightarrow \quad x - 3 = \pm\sqrt{2} \quad \Rightarrow \quad x = 3 \pm \sqrt{2}$
y-intercept: $x = 0 \quad \Rightarrow \quad y = (0-3)^2 - 2 = 9 - 2 = 7$

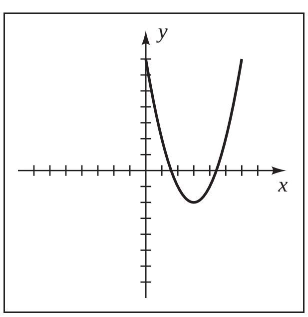


Figure 39

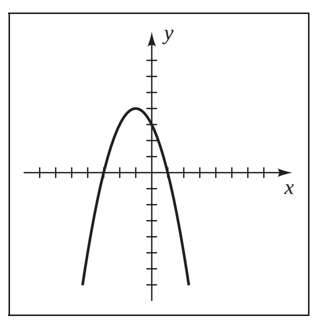


Figure 40

40 $y = -x^2 - 2x + 3 = -(x^2 + 2x + \underline{\ 1\ }) + 3 + \underline{\ 1\ } = -(x+1)^2 + 4$; $V(-1, 4)$; x-intercepts -3 and 1; y-intercept 3

41 The radius of the large circle is 3 and the radius of the small circle is 1. Since the center of the small circle is 4 units from the origin and lies on the line $y = x$, we must have $x^2 + y^2 = 4^2 \Rightarrow x^2 + x^2 = 16 \Rightarrow 2x^2 = 16 \Rightarrow x^2 = 8 \Rightarrow x = \sqrt{8}$. The center of the small circle is $\left(\sqrt{8}, \sqrt{8}\right)$.

42 The graph of $y = -f(x-2)$ is the graph of $y = f(x)$ shifted to the right 2 units and reflected through the x-axis.

43 (a) The graph of $f(x) = \dfrac{1-3x}{2} = -\dfrac{3}{2}x + \dfrac{1}{2}$ is a line with slope $-\dfrac{3}{2}$ and y-intercept $\dfrac{1}{2}$.

(b) The function is defined for all x, so the domain D is the set of all real numbers. The range is the set of all real numbers, so $D = \mathbb{R}$ and $R = \mathbb{R}$.

(c) The function f is decreasing on $(-\infty, \infty)$.

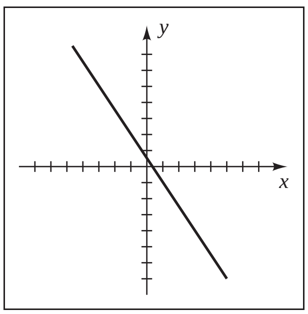


Figure 43

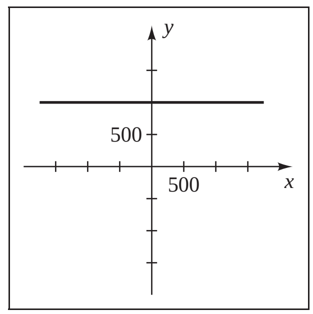


Figure 44

44 $f(x) = 1000$ • (b) $D = \mathbb{R}$; $R = \{1000\}$ (c) Constant on $(-\infty, \infty)$

45 (a) The graph of $f(x) = |x+3|$ can be thought of as the graph of $g(x) = |x|$ shifted left 3 units.

(b) The function is defined for all x, so the domain D is the set of all real numbers. The range is the set of all nonnegative numbers, that is, $R = [0, \infty)$.

(c) The function f is decreasing on $(-\infty, -3]$ and is increasing on $[-3, \infty)$.

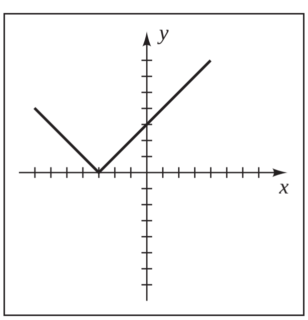


Figure 45

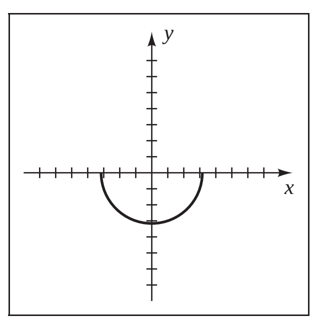


Figure 46

46 $f(x) = -\sqrt{10 - x^2}$ • (b) $D = \left[-\sqrt{10}, \sqrt{10}\right]$; $R = \left[-\sqrt{10}, 0\right]$

(c) Decreasing on $\left[-\sqrt{10}, 0\right]$, increasing on $\left[0, \sqrt{10}\right]$

47 (a) $f(x) = 1 - \sqrt{x+1} = -\sqrt{x+1} + 1$ • We can think of this graph as the graph of $y = \sqrt{x}$ shifted left 1 unit, reflected through the x-axis, and shifted up 1 unit.

(b) For f to be defined, we must have $x + 1 \geq 0 \Rightarrow x \geq -1$, so $D = [-1, \infty)$. The y-values of f are all the values less than or equal to 1, so $R = (-\infty, 1]$.

(c) The function f is decreasing on $[-1, \infty)$.

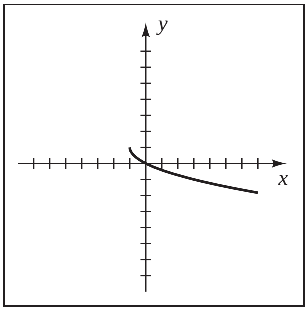


Figure 47

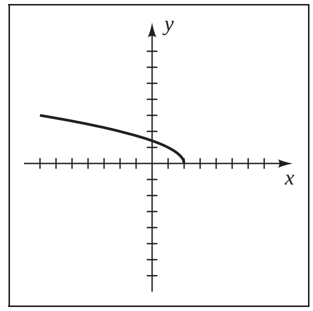


Figure 48

48 $f(x) = \sqrt{2 - x}$ • (b) $D = (-\infty, 2]$; $R = [0, \infty)$

(c) Decreasing on $(-\infty, 2]$

49 (a) $f(x) = 9 - x^2 = -x^2 + 9$ • We can think of this graph as the graph of $y = x^2$ reflected through the x-axis and shifted up 9 units.

(b) The function is defined for all x, so the domain D is the set of all real numbers. The y-values of f are all the values less than or equal to 9, so $R = (-\infty, 9]$.

(c) The function f is increasing on $(-\infty, 0]$ and is decreasing on $[0, \infty)$.

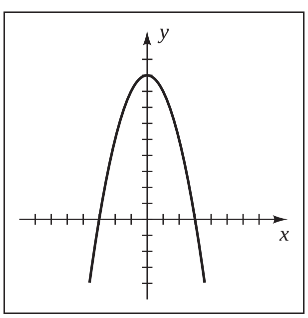


Figure 49

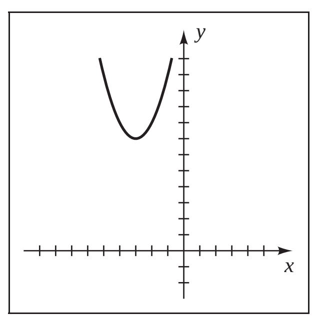


Figure 50

50 $f(x) = x^2 + 6x + 16 = x^2 + 6x + 9 + 7 = (x+3)^2 + 7$. The vertex is $(-3, 7)$.

(b) $D = \mathbb{R}$; $R = [7, \infty)$

(c) Decreasing on $(-\infty, -3]$, increasing on $[-3, \infty)$

51 (a) $f(x) = \begin{cases} x^2 & \text{if } x < 0 \\ 3x & \text{if } 0 \le x < 2 \\ 6 & \text{if } x \ge 2 \end{cases}$

If $x < 0$, we want the graph of the parabola $y = x^2$. The endpoint of this part of the graph is $(0,0)$, but it is not included. If $0 \le x < 2$, we want the graph of $y = 3x$, a line with slope 3 and y-intercept 0. Now we do include the endpoint $(0,0)$, but we don't include the endpoint $(2,6)$. If $x \ge 2$, we want the graph of the horizontal line $y = 6$ and include its endpoint $(2,6)$, so there are no open endpoints on the graph of f.

(b) The function is defined for all x, so the domain D is the set of all real numbers. The y-values of f are all the values greater than or equal to 0, so $R = [0, \infty)$.

(c) f is decreasing on $(-\infty, 0]$, increasing on $[0, 2]$, and constant on $[2, \infty)$.

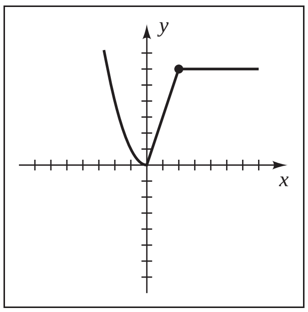


Figure 51

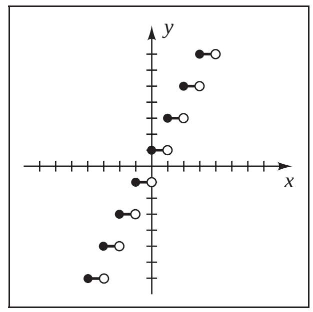


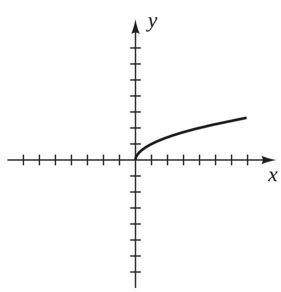
Figure 52

52 (a) $f(x) = 1 + 2[\![x]\!]$ • The "2" in front of $[\![x]\!]$ has the effect of doubling all the y-values of $g(x) = [\![x]\!]$. The "$+1$" has the effect of vertically shifting the graph of $h(x) = 2[\![x]\!]$ up 1 unit.

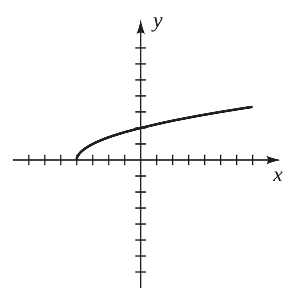
(b) The function is defined for all x, so the domain D is the set of all real numbers. The range of $g(x) = [\![x]\!]$ is the set of integers, that is, $\{\ldots, -2, -1, 0, 1, 2, \ldots\}$. The range of $h(x) = 2[\![x]\!]$ is the set of even integers since we are doubling the values of g—that is, $\{\ldots, -4, -2, 0, 2, 4, \ldots\}$. Since $f(x) = 1 + h(x)$, the range R of f is $\{\ldots, -3, -1, 1, 3, \ldots\}$.

(c) The function f is constant on intervals such as $[0, 1)$, $[1, 2)$, and $[2, 3)$. In general, f is constant on $[n, n+1)$, where n is any integer.

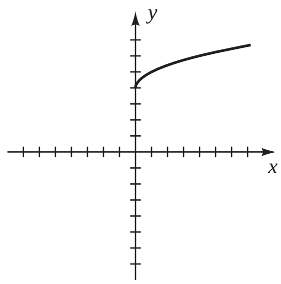
53 (a) $y = \sqrt{x}$ • This is the square root function, whose graph is a half-parabola.



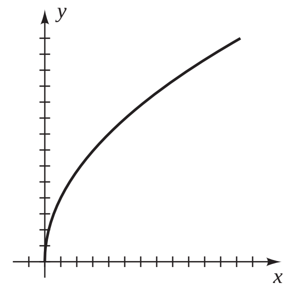
(b) $y = \sqrt{x+4}$ • shift $y = \sqrt{x}$ left 4 units



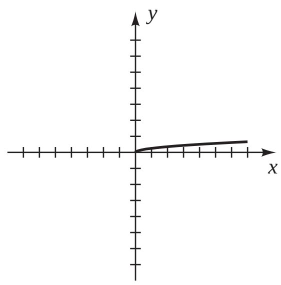
(c) $y = \sqrt{x} + 4$ • shift $y = \sqrt{x}$ up 4 units



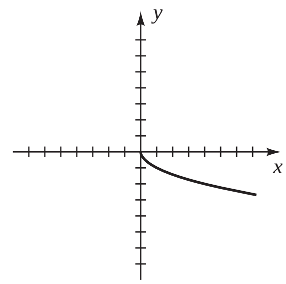
(d) $y = 4\sqrt{x}$ • vertically stretch the graph of $y = \sqrt{x}$ by a factor of 4



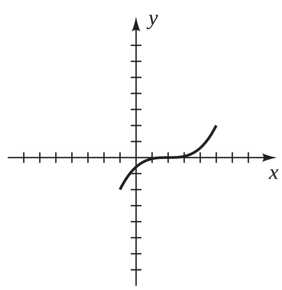
(e) $y = \frac{1}{4}\sqrt{x}$ • vertically compress the graph of $y = \sqrt{x}$ by a factor of $1/\frac{1}{4} = 4$



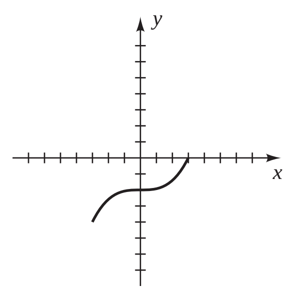
(f) $y = -\sqrt{x}$ • reflect the graph of $y = \sqrt{x}$ through the x-axis



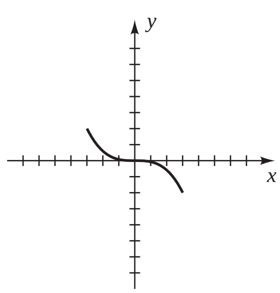
54 (a) $y = f(x - 2)$ • shift f right 2 units



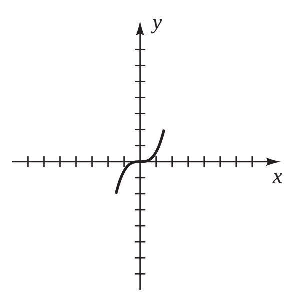
(b) $y = f(x) - 2$ • shift f down 2 units



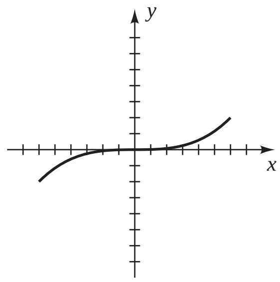
(c) $y = f(-x)$ • reflect f through the y-axis



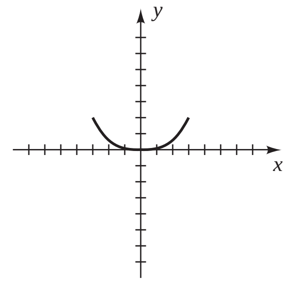
(d) $y = f(2x)$ • horizontally compress f by a factor of 2



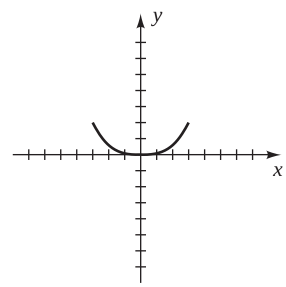
(e) $y = f\left(\frac{1}{2}x\right)$ • horizontally stretch f by a factor of $1/(1/2) = 2$



(f) $y = |f(x)|$ • reflect the portion of the graph below the x-axis through the x-axis.



(g) $y = f(|x|)$ • include the reflection of all points with positive x-coordinates through the y-axis—results in the same graph as in part (f).



55 Using the intercept form of a line with x-intercept 5 and y-intercept -2, an equation of the line is $\frac{x}{5} + \frac{y}{-2} = 1$.
Multiplying by 10 gives us $\left[\frac{x}{5} + \frac{y}{-2} = 1\right] \cdot 10$, or, equivalently, $2x - 5y = 10$.

56 The midpoint of $(-7, 1)$ and $(3, 1)$ is $M(-2, 1)$. This point is 5 units from either of the given points.
An equation of the circle is $(x+2)^2 + (y-1)^2 = 5^2 = 25$.

57 $V(2, -4)$ and $P(-2, 4)$ with $y = a(x-h)^2 + k \Rightarrow 4 = a(-2-2)^2 - 4 \Rightarrow 8 = 16a \Rightarrow a = \frac{1}{2}$.
An equation of the parabola is $y = \frac{1}{2}(x-2)^2 - 4$.

58 The graph could be made by taking the graph of $y = |x|$, reflecting it through the x-axis $\{y = -|x|\}$, then shifting that graph to the right 2 units $\{y = -|x-2|\}$, and then shifting that graph down 1 unit, resulting in the graph of the equation $y = -|x-2| - 1$.

59 $f(x) = 3x^2 - 24x + 46 \Rightarrow -\frac{b}{2a} = -\frac{-24}{2(3)} = 4$. $f(4) = -2$ is a minimum since $a = 3 > 0$.

60 $f(x) = -2x^2 - 12x - 24 \Rightarrow -\frac{b}{2a} = -\frac{-12}{2(-2)} = -3$. $f(-3) = -6$ is a maximum since $a = -2 < 0$.

61 $f(x) = -12(x+4)^2 + 20$ is in the standard form. $f(-4) = 20$ is a maximum since $a = -12 < 0$.

62 $f(x) = 3(x+2)(x-10)$ has x-intercepts at -2 and 10. The vertex is halfway between them at $x = 4$.
$f(4) = 3 \cdot 6 \cdot (-6) = -108$ is a minimum since $a = 3 > 0$.

63 $f(x) = -2x^2 + 12x - 14 = -2\left(x^2 - 6x + \underline{\quad}\right) - 14 + \underline{\quad}$ {complete the square}
$= -2\left(x^2 - 6x + \underline{9}\right) - 14 + 18 = -2(x-3)^2 + 4$.

64 $V(3, -2) \Rightarrow (h, k) = (3, -2)$ in $y = a(x-h)^2 + k$.
$x = 1, y = -5 \Rightarrow -5 = a(1-3)^2 - 2 \Rightarrow -3 = 4a \Rightarrow a = -\frac{3}{4}$. Hence, $y = -\frac{3}{4}(x-3)^2 - 2$.

65 The domain of $f(x) = \sqrt{9 - x^2}$ is $[-3, 3]$. The domain of $g(x) = \sqrt{x}$ is $[0, \infty)$.

(a) The domain of fg is the intersection of those two domains, $[0, 3]$.

(b) The domain of f/g is the same as that of fg, excluding any values that make g equal to 0.

Thus, the domain of f/g is $(0, 3]$.

66 (a) $f(x) = 8x - 3$ and $g(x) = \sqrt{x - 2} \Rightarrow (f \circ g)(2) = f(g(2)) = f(0) = -3$

(b) $(g \circ f)(2) = g(f(2)) = g(13) = \sqrt{11}$

67 (a) $f(x) = 2x^2 - 5x + 1$ and $g(x) = 3x + 2 \Rightarrow$
$(f \circ g)(x) = f(g(x)) = 2(3x + 2)^2 - 5(3x + 2) + 1 = 2(9x^2 + 12x + 4) - 15x - 10 + 1 = 18x^2 + 9x - 1$

(b) $(g \circ f)(x) = g(f(x)) = 3(2x^2 - 5x + 1) + 2 = 6x^2 - 15x + 3 + 2 = 6x^2 - 15x + 5$

68 (a) $f(x) = \sqrt{3x + 2}$ and $g(x) = \dfrac{1}{x^2} \Rightarrow (f \circ g)(x) = f(g(x)) = \sqrt{3\left(\dfrac{1}{x^2}\right) + 2} = \sqrt{\dfrac{3 + 2x^2}{x^2}}$

(b) $(g \circ f)(x) = g(f(x)) = \dfrac{1}{\left(\sqrt{3x + 2}\right)^2} = \dfrac{1}{3x + 2}$

69 (a) $(f \circ g)(x) = f(g(x)) = f\left(\sqrt{x - 3}\right) = \sqrt{25 - \left(\sqrt{x - 3}\right)^2} = \sqrt{25 - (x - 3)} = \sqrt{28 - x}$.
Domain of $g(x) = \sqrt{x - 3} = [3, \infty)$. Domain of $f(x) = \sqrt{25 - x^2} = [-5, 5]$.
$g(x) \leq 5$ $\{g(x)$ cannot be less than $0\}$ $\Rightarrow \sqrt{x - 3} \leq 5 \Rightarrow x - 3 \leq 25 \Rightarrow x \leq 28$.

$[3, \infty) \cap (-\infty, 28] = [3, 28]$

(b) $(g \circ f)(x) = g(f(x)) = g\left(\sqrt{25 - x^2}\right) = \sqrt{\sqrt{25 - x^2} - 3}$. Domain of $f = [-5, 5]$. Domain of $g = [3, \infty)$.

$f(x) \geq 3 \Rightarrow \sqrt{25 - x^2} \geq 3 \Rightarrow 25 - x^2 \geq 9 \Rightarrow x^2 \leq 16 \Rightarrow x \in [-4, 4]$.

70 (a) $(f \circ g)(x) = f(g(x)) = f\left(\dfrac{2}{x}\right) = \dfrac{2/x}{3(2/x) + 2} \cdot \dfrac{x}{x} = \dfrac{2}{6 + 2x} = \dfrac{1}{x + 3}$. Domain of $g = \mathbb{R} - \{0\}$.
Domain of $f = \mathbb{R} - \{-\frac{2}{3}\}$. $g(x) \neq -\frac{2}{3} \Rightarrow \dfrac{2}{x} \neq -\dfrac{2}{3} \Rightarrow x \neq -3$.

Hence, the domain of $f \circ g$ is $\mathbb{R} - \{-3, 0\}$.

(b) $(g \circ f)(x) = g(f(x)) = g\left(\dfrac{x}{3x + 2}\right) = \dfrac{2}{x/(3x + 2)} = \dfrac{6x + 4}{x}$. Domain of $f = \mathbb{R} - \left\{-\frac{2}{3}\right\}$. Domain of $g = \mathbb{R} - \{0\}$. $f(x) \neq 0 \Rightarrow \dfrac{x}{3x + 2} \neq 0 \Rightarrow x \neq 0$. Hence, the domain of $g \circ f$ is $\mathbb{R} - \left\{-\frac{2}{3}, 0\right\}$.

71 For $y = \sqrt[3]{x^2 - 5x}$, choose $u = x^2 - 5x$ and $y = \sqrt[3]{u}$.

72 The slope of the ramp should be between $\frac{1}{12}$ and $\frac{1}{20}$.
If the rise of the ramp is 3 feet, then the run should be between $3 \times 12 = 36$ ft and $3 \times 20 = 60$ ft.

The range of the ramp lengths should be from $L = \sqrt{3^2 + 36^2} \approx 36.1$ ft to $L = \sqrt{3^2 + 60^2} \approx 60.1$ ft.

73 (a) The year 2016 corresponds to $t = 2016 - 1948 = 68$. $d = 181 + 1.065(68) = 253.42 \approx 253$ ft

(b) $d = 265 \Rightarrow 265 = 181 + 1.065t \Rightarrow 84 = 1.065t \Rightarrow t = \frac{84}{1.065} \approx 78.9$ yr,

which corresponds to $79 + 1948 = 2027$ or Olympic year 2028.

74 (a) $V = at + b$ is the desired form. $V = 179{,}000$ when $t = 0 \Rightarrow V = at + 179{,}000$.
$V = 215{,}000$ when $t = 6 \Rightarrow 215{,}000 = 6a + 179{,}000 \Rightarrow a = \frac{36{,}000}{6} = 6000$ and hence,
$V = 6000t + 179{,}000$.

(b) $V = 193{,}000 \Rightarrow 193{,}000 = 6000t + 179{,}000 \Rightarrow t = \frac{14{,}000}{6000} = \frac{7}{3}$, or $2\frac{1}{3}$.

75 (a) $F = aC + b$ is the desired form. $F = 32$ when $C = 0 \Rightarrow F = aC + 32$. $F = 212$ when $C = 100 \Rightarrow$
$212 = 100a + 32 \Rightarrow a = \frac{180}{100} = \frac{9}{5}$ and hence, $F = \frac{9}{5}C + 32$.

(b) If C increases $1°$, F increases $\left(\frac{9}{5}\right)°$, or $1.8°$.

76 (a) $C_1(x) = \left(3.00 \dfrac{\text{dollars}}{\text{gallon}}\right) \div \left(20 \dfrac{\text{miles}}{\text{gallon}}\right) \cdot x \text{ miles} = \dfrac{3}{20}x$, or $0.15x$.

(b) After the tune-up, the gasoline mileage will be 10% more than 20 mi/gal; that is, 22 mi/gal.
$C_2(x) = \frac{3}{22}x + 120 \approx 0.136x + 120$.

(c) $C_2 < C_1 \Rightarrow \left[\frac{3}{22}x + 120 < \frac{3}{20}x\right] \cdot 220 \Rightarrow 30x + 26{,}400 < 33x \Rightarrow 3x > 26{,}400 \Rightarrow$
$x > 8800$ miles.

77 (a) The length across the top of the pen is $3x$ and the length across the bottom is $2y$. These lengths are equal, so $2y = 3x \Rightarrow y = \frac{3}{2}x$, or $y(x) = \frac{3}{2}x$.

(b) There are 6 long lengths (y) and 9 short lengths (x), so the perimeter P is given by $P = 6y + 9x$ and $C = 10P = 10(6y + 9x) = 10\left[(6 \cdot \frac{3}{2}x) + 9x\right] = 10(9x + 9x) = 10(18x) = 180x$. Thus, $C(x) = 180x$.

78 The distances covered by cars A and B in t seconds are $88t$ and $66t$, respectively. The vertical distance between car A and car B after t seconds is $(20 + 88t) - 66t = 20 + 22t$. Using the Pythagorean theorem, we have $d(t) = \sqrt{10^2 + (20 + 22t)^2} = \sqrt{100 + 400 + 880t + 484t^2} = 2\sqrt{121t^2 + 220t + 125}$.

79 Surface area $S = 2(4)(x) + (4)(y) = 8x + 4y$. Cost $C = 2(8x) + 5(4y) = 16x + 20y$.

(a) $C = 400 \Rightarrow 16x + 20y = 400 \Rightarrow 20y = -16x + 400 \Rightarrow y = -\frac{4}{5}x + 20$

(b) $V = lwh = (y)(4)(x) = 4xy = 4x\left(-\frac{4}{5}x + 20\right)$

80 $V = \pi r^2 h$ and $V = 24\pi \Rightarrow h = \dfrac{24}{r^2}$. $S = \pi r^2 + 2\pi r h = \pi r^2 + 2\pi r \cdot \dfrac{24}{r^2} = \pi r^2 + \dfrac{48\pi}{r}$.

$$C = (0.30)(\pi r^2) + (0.10)\left(\frac{48\pi}{r}\right) = \frac{3\pi r^2}{10} + \frac{48\pi}{10r} = \frac{3\pi(r^3 + 16)}{10r}.$$

81 (a) $V = (10 \text{ ft}^3 \text{ per minute})(t \text{ minutes}) = 10t$

(b) The height and length of the bottom triangular region are in the proportion 6–60, or 1–10, and the length is 10 times the height. When $0 \le h \le 6$, the volume is

$$V = (\text{cross sectional area})(\text{pool width}) = \tfrac{1}{2}bh(40) = \tfrac{1}{2}(10h)(h)(40) = 200h^2 \text{ ft}^3.$$

When $6 < h \le 9$, the triangular region is full and $V = 200(6)^2 + (h - 6)(80)(40) = 7200 + 3200(h - 6)$.

(c) For $0 \le h \le 6$: $10t = 200h^2 \Rightarrow h^2 = \dfrac{10t}{200} \Rightarrow h = \sqrt{\dfrac{t}{20}}$;

$$0 \le h \le 6 \Rightarrow 0 \le \sqrt{\frac{t}{20}} \le 6 \Rightarrow 0 \le \frac{t}{20} \le 36 \Rightarrow 0 \le t \le 720.$$

For $6 < h \le 9$: $10t = 7200 + 3200(h-6) \Rightarrow h - 6 = \dfrac{10t - 7200}{3200} \Rightarrow h = 6 + \dfrac{t-720}{320}$;

$6 < h \le 9 \Rightarrow 6 < 6 + \dfrac{t-720}{320} \le 9 \Rightarrow 0 < \dfrac{t-720}{320} \le 3 \Rightarrow 0 < t - 720 \le 960 \Rightarrow$

$$720 < t \le 1680.$$

82 (a) Using similar triangles, $\dfrac{r}{x} = \dfrac{2}{4} \Rightarrow r = \dfrac{1}{2}x$.

(b) $\text{Volume}_{\text{cone}} + \text{Volume}_{\text{cup}} = \text{Volume}_{\text{total}} \Rightarrow \frac{1}{3}\pi r^2 h + \pi r^2 h = 5 \Rightarrow$

$$\tfrac{1}{3}\pi\left(\tfrac{1}{2}x\right)^2(x) + \pi(2)^2(y) = 5 \Rightarrow 4\pi y = 5 - \tfrac{\pi}{12}x^3 \Rightarrow y = \tfrac{5}{4\pi} - \tfrac{1}{48}x^3$$

83 (a) $\dfrac{y}{b} = \dfrac{y+h}{a} \Rightarrow ay = by + bh \Rightarrow ay - by = bh \Rightarrow y(a-b) = bh \Rightarrow y = \dfrac{bh}{a-b}$

(b) $V = \dfrac{1}{3}\pi a^2(y+h) - \dfrac{1}{3}\pi b^2 y = \dfrac{1}{3}\pi a^2 y + \dfrac{1}{3}\pi a^2 h - \dfrac{1}{3}\pi b^2 y = \dfrac{\pi}{3}\left[(a^2-b^2)y + a^2 h\right]$

$$= \frac{\pi}{3}\left[(a^2-b^2)\frac{bh}{a-b} + a^2 h\right] = \frac{\pi}{3}h\left[(a+b)b + a^2\right] = \frac{\pi}{3}h\left(a^2 + ab + b^2\right)$$

(c) $a = 6, b = 3, V = 600 \Rightarrow 600 = \frac{\pi}{3}h(6^2 + 6 \cdot 3 + 3^2) \Rightarrow h = \frac{1800}{63\pi} = \frac{200}{7\pi} \approx 9.1$ ft

84 If $0 \le x \le 5000$, then $B(x) = \$3.61(x/1000)$. If $x > 5000$, then the charge is $\$3.61(5000/1000)$ for the first 5000 gallons, which is \$18.05, plus $\$4.17/1000$ for the number of gallons over 5000, which is $(x - 5000)$. We may summarize and simplify as follows:

$$B(x) = \begin{cases} 3.61\left(\frac{x}{1000}\right) & \text{if } 0 \le x \le 5000 \\ 3.61(5) + 4.17\left(\frac{x-5000}{1000}\right) & \text{if } x > 5000 \end{cases} = \begin{cases} 0.00361x & \text{if } 0 \le x \le 5000 \\ 18.05 + 0.00417(x-5000) & \text{if } x > 5000 \end{cases}$$

$$= \begin{cases} 0.00361x & \text{if } 0 \le x \le 5000 \\ 0.00417x - 2.8 & \text{if } x > 5000 \end{cases}$$

85 The high point of the path occurs halfway through the jump, that is, at $\frac{1}{2}(8.95) = 4.475$ meters from the beginning of the jump. Using the form $y = a(x-h)^2 + k$, we have $y = a(x - 4.475)^2 + 1$. Substituting 0 for x and 0 for y {or equivalently, 8.95 for x}, we get $0 = a(0 - 4.475)^2 + 1 \Rightarrow -1 = a(4.475)^2 \Rightarrow$

$$a = -\frac{1}{4.475^2}\text{, and an equation is } y = -\frac{1}{4.475^2}(x - 4.475)^2 + 1.$$

86 (a) $P = 24 \Rightarrow 2x + 2y = 24 \Rightarrow y = 12 - x$ (b) $A = xy = x(12 - x)$

(c) A is zero at 0 and 12 and will be a maximum when $x = \dfrac{0+12}{2} = 6$.

Thus, the maximum value of A occurs if the rectangle is a square.

87 Let t denote the time (in hr) after 1:00 P.M. If the starting point for ship B is the origin, then the locations of A and B are $-30 + 15t$ and $-10t$, respectively. Using the Pythagorean theorem,

$$d^2 = (-30 + 15t)^2 + (-10t)^2 = 900 - 900t + 225t^2 + 100t^2 = 325t^2 - 900t + 900.$$

The time at which the distance between the ships is minimal is the same as the time at which the square of the distance between the ships is minimal. Thus, $t = -\dfrac{b}{2a} = -\dfrac{-900}{2(325)} = \dfrac{18}{13}$, or about 2:23 P.M.

88 Let r denote the radius of the semicircles and x the length of the rectangle. Perimeter = half-mile $\Rightarrow$ $2x + 2\pi r = \frac{1}{2} \Rightarrow x = -\pi r + \frac{1}{4}$. $A = 2rx = 2r\left(-\pi r + \frac{1}{4}\right) = -2\pi r^2 + \frac{1}{2}r$.

The maximum value of A occurs when $r = -\dfrac{b}{2a} = -\dfrac{1/2}{2(-2\pi)} = \dfrac{1}{8\pi}$ mi. $x = -\pi\left(\dfrac{1}{8\pi}\right) + \dfrac{1}{4} = \dfrac{1}{8}$ mi.

89 (a) $f(t) = -\frac{1}{2}gt^2 + 16t$ • $g = 32 \Rightarrow f(t) = -16t^2 + 16t$.

Solving $f(t) = 0$ gives us $-16t(t-1) \Rightarrow t = 0, 1$. The player is in the air for 1 second.

(b) $t = -\dfrac{b}{2a} = -\dfrac{16}{2(-16)} = \dfrac{1}{2}$. $f\left(\dfrac{1}{2}\right) = 4 \Rightarrow$ the player jumps 4 feet high.

(c) $g = \dfrac{32}{6} \Rightarrow f(t) = -\dfrac{8}{3}t^2 + 16t$. Solving $f(t) = 0$ yields $t = 0$ or 6.

The player would be in the air for 6 seconds on the moon.

$t = -\dfrac{b}{2a} = -\dfrac{16}{2(-8/3)} = 3$. $f(3) = 24 \Rightarrow$ the player jumps 24 feet high.

90 (a) Solving $-0.016x^2 + 1.6x = \frac{1}{5}x$ for x represents the intersection between the parabola and the line.

$-0.08x^2 + 8x = x$ {multiply by 5} $\Rightarrow 7x - \frac{8}{100}x^2 = 0 \Rightarrow x\left(7 - \frac{8}{100}x\right) = 0 \Rightarrow x = 0, \frac{175}{2}$.

The rocket lands at $\left(\frac{175}{2}, \frac{35}{2}\right) = (87.5, 17.5)$.

(b) The *difference* d between the parabola and the line is to be maximized here.

$d = (-0.016x^2 + 1.6x) - \left(\frac{1}{5}x\right) = -0.016x^2 + 1.4x$. d obtains a maximum when

$x = -\dfrac{b}{2a} = -\dfrac{1.4}{2(-0.016)} = 43.75$. The maximum height of the rocket *above the ground* is

$d = -0.016(43.75)^2 + 1.4(43.75) = 30.625$ units.

Chapter 3 Discussion Exercises

1 Graphs of equations of the form $y = x^{p/q}$, where $x \geq 0$, and p and q are positive integers all pass through $(0,0)$ and $(1,1)$. If $p/q < 1$, the graph is above $y = x$ for $0 \leq x \leq 1$ and below $y = x$ for $x \geq 1$. The closer p/q is to 1, the closer $y = x^{p/q}$ is to $y = x$. If $p/q > 1$, the graph is below $y = x$ for $0 \leq x \leq 1$ and above $y = x$ for $x \geq 1$.

2 (a) About the x-axis • replace y with $-y$: $-y = \frac{1}{2}x - 3 \Rightarrow g(x) = -\frac{1}{2}x + 3$

(b) About the y-axis • replace x with $-x$: $y = \frac{1}{2}(-x) - 3 \Rightarrow g(x) = -\frac{1}{2}x - 3$

(c) About the line $y = 2$ • by examining the graphs of $f(x) = \frac{1}{2}x - 3$ and $y = 2$, we observe that the slope of the reflected line should be $-\frac{1}{2}$ and the y-intercept should be 5 units above $y = 2$ (since the y-intercept of f is 5 units below 2). Hence, $g(x) = -\frac{1}{2}x + 7$.

(d) About the line $x = 3$ • similar to part (c), the slope of g is $-\frac{1}{2}$. The x-intercept of f, 6, is 3 units to the right of $x = 3$, so the x-intercept of g should be 3 units to the left of $x = 3$. Hence, $g(x) = -\frac{1}{2}x$.

Note: An interesting generalization can be made for problems of the form of those in parts (a)–(d) which would be a worthwhile exploratory exercise in itself. It goes as follows: If the graph of $y = f(x)$ is reflected about the line $y = k$ (or $x = k$), how can you obtain the equation of the new graph? Answer: For $y = k$, replace y with $2k - y$; for $x = k$, replace x with $2k - x$.

3 For the graph of $g(x) = \sqrt{f(x)}$, where $f(x) = ax^2 + bx + c$, consider 2 cases:

1) ($a > 0$) If f has 0 or 1 x-intercept(s), the domain of g is $\mathbb{R}$ and its range is $\left[\sqrt{k}, \infty\right)$, where k is the y-value of the vertex of f. If f has 2 x-intercepts (say x_1 and x_2 with $x_1 < x_2$), then the domain of g is $(-\infty, x_1] \cup [x_2, \infty)$ and its range is $[0, \infty)$. The general shape is similar to the v-shape of the graph of $y = \sqrt{a}|x|$.

2) ($a < 0$) If f has no x-intercepts, there is no graph of g. If f has 1 x-intercept, the graph of g consists of that point. If f has 2 x-intercepts, the domain of g is $[x_1, x_2]$ and the range is $\left[0, \sqrt{k}\right]$. The shape of g is that of the top half of an oval.

The main advantage of graphing g as a composition $\left(\text{say } \mathrm{Y}_1 = f \text{ and } \mathrm{Y}_2 = \sqrt{\mathrm{Y}_1} \text{ on a graphing calculator}\right)$ is to observe the relationship between the range of f and the domain of g.

4 $f(x) = ax^2 + bx + c \quad \Rightarrow$

$$\begin{aligned} \frac{f(x+h) - f(x)}{h} &= \frac{\left[a(x+h)^2 + b(x+h) + c\right] - [ax^2 + bx + c]}{h} \\ &= \frac{ax^2 + 2ahx + ah^2 + bx + bh + c - ax^2 - bx - c}{h} \\ &= \frac{2ahx + ah^2 + bh}{h} = \frac{h(2ax + ah + b)}{h} = 2ax + ah + b \end{aligned}$$

5 The expression $2x + h + 6$ represents the slope of the line between the points $P(x, f(x))$ and $Q(x+h, f(x+h))$. If $h = 0$, then $2x + 6$ represents the slope of the tangent line at the point $P(x, f(x))$.

6 To determine the x-coordinate of R, we want to start at x_1 and go $\frac{m}{n}$ of the way to x_2.

We could write this as $x_3 = x_1 + \frac{m}{n}\Delta x = x_1 + \frac{m}{n}(x_2 - x_1) = x_1 + \frac{m}{n}x_2 - \frac{m}{n}x_1 = \left(1 - \frac{m}{n}\right)x_1 + \frac{m}{n}x_2$.

Similarly, $y_3 = \left(1 - \frac{m}{n}\right)y_1 + \frac{m}{n}y_2$.

7 The values of the x-intercepts (if they exist) are found using the quadratic formula $x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$. Hence, the distance d from the axis of symmetry, $x = -\frac{b}{2a}$, to either x-intercept is $d = \frac{\sqrt{b^2 - 4ac}}{2|a|}$ and $d^2 = \frac{b^2 - 4ac}{4a^2}$. From page 199 of the text, the y-coordinate of the vertex is $h = c - \frac{b^2}{4a} = \frac{4ac - b^2}{4a}$, so

$$\frac{h}{d^2} = \frac{\frac{4ac - b^2}{4a}}{\frac{b^2 - 4ac}{4a^2}} = \frac{4a^2(4ac - b^2)}{4a(b^2 - 4ac)} = -a.$$

Thus, $h = -ad^2$. Note that this relationship also reveals a connection between the discriminant $D = b^2 - 4ac$ and the y-coordinate of the vertex, namely $h = -D/(4a)$.

8 The graph of $y = -[\![-x]\!]$ in the solution to Exercise 56(e) of Section 3.5 illustrates the concept of one of the most common billing methods with the open and closed endpoints reversed from those of the greatest integer function. Starting with $y = -[\![-x]\!]$ and adjusting for jumps every 15 minutes gives us $y = -[\![-x/15]\!]$. Since each quarter-hour charge is \$20, we multiply by 20 to obtain $y = -20[\![-x/15]\!]$. Because of the initial \$40 charge, we must add 40 to obtain the function $f(x) = 40 - 20[\![-x/15]\!]$.

9 $D = 0.0833x^2 - 0.4996x + 3.5491 \quad \Rightarrow \quad 0.0833x^2 - 0.4996x + (3.5491 - D) = 0.$

Solving for x with the quadratic formula yields $x = \dfrac{0.4996 \pm \sqrt{(-0.4996)^2 - 4(0.0833)(3.5491 - D)}}{2(0.0833)}$,

or, equivalently, $x = \dfrac{4996 \pm \sqrt{33{,}320{,}000D - 93{,}295{,}996}}{1666}$.

From the figure (a graph of D), we see that $3 \le x \le 15$ corresponds to the right half of the parabola.

Hence, we choose the plus sign in the equation for x.

$[-20, 20, 2]$ by $[0, 40, 2]$

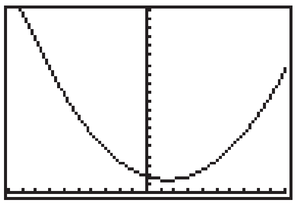


Figure 9

$[0.5, 12.5]$ by $[0, 5]$

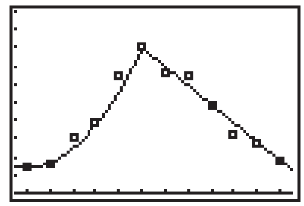


Figure 10

10 (a) Let January correspond to 1, February to 2, ... , and December to 12.

(b) The data points are (approximately) parabolic on the interval $[1, 6]$ and linear on $[6, 12]$.

Let $f_1(x) = a(x - h)^2 + k$ on $[1, 6]$ and $f_2(x) = mx + b$ on $[6, 12]$. On $[1, 6]$, let the vertex $(h, k) = (1, 0.7)$.

Since $(6, 4)$ is on the graph of f_1, $f_1(6) = a(6 - 1)^2 + 0.7 = 4 \quad \Rightarrow \quad a = 0.132$.

Thus, $f_1(x) = 0.132(x - 1)^2 + 0.7$ on $[1, 6]$. Now, let $f_2(x) = mx + b$ pass through the points

$(6, 4)$ and $(12, 0.9)$. An equation of this line is approximately $(y - 4) = -0.517(x - 6)$.

Thus, let $f_2(x) = -0.517x + 7.102$ on $[6, 12]$.

$$f(x) = \begin{cases} 0.132(x-1)^2 + 0.7 & \text{if } 1 \le x \le 6 \\ -0.517x + 7.102 & \text{if } 6 < x \le 12 \end{cases}$$

(c) To plot the piecewise function, let

$$\text{Y}_1 = (0.132(x - 1)\hat{\ }2 + 0.7)/(x \le 6) \quad \text{and} \quad \text{Y}_2 = (-0.517x + 7.102)/(x > 6).$$

These assignments use the concept of Boolean division. For example, when $(x \le 6)$ is false, the expression Y_1 will be undefined (division by 0) and the calculator will not plot any values.

Chapter 3 Test

1 To get to $B(1, 2)$ from $A(-3, 4)$, move 4 units right and 2 units down.

Since that represents one-tenth of the distance, 40 units right and 20 units down represents the whole distance to

$$P(x, y) = P(-3 + 40, 4 - 20) = P(37, -16).$$

2 $d(P, Q) > 5 \quad \Rightarrow \quad \sqrt{(6-2)^2 + (a-3)^2} > 5 \quad \Rightarrow \quad 4^2 + a^2 - 6a + 9 > 5^2 \quad \Rightarrow \quad a^2 - 6a > 0 \quad \Rightarrow$
$a(a - 6) > 0 \quad \Rightarrow \quad a < 0$ or $a > 6$. Make a sign chart to establish the final answer.

Interval	$(-\infty, 0)$	$(0, 6)$	$(6, \infty)$
Sign of a	$-$	$+$	$+$
Sign of $a - 6$	$-$	$-$	$+$
Resulting sign	$+$	$-$	$+$

3 The standard equation of the circle with center $(4, 5)$ has the form $(x-4)^2 + (y-5)^2 = r^2$. Since an x-intercept is 0, a y-intercept is 0, so $(0-4)^2 + (0-5)^2 = r^2 \Rightarrow 16 + 25 = r^2 \Rightarrow 41 = r^2$, and the desired equation is $(x-4)^2 + (y-5)^2 = 41$.

4 To go from the center $(4, 5)$ to the origin, it's 5 units down and 4 units left. By symmetry, to get to the other y-intercept from the center, move 5 units up and 4 units left to the point $(0, 10)$; and to get to the other x-intercept, move 5 units down and 4 units right to the point $(8, 0)$. Another method would be to find the standard equation of the circle, and then substitute 0 for x to find the y-intercept and then 0 for y to find the x-intercept.

5 The tangent line and the circle pass through the origin. The slope of the line from the origin to the center of the circle is $\frac{5}{4}$. Since the tangent line is perpendicular to that line, it has slope $-\frac{4}{5}$ and equation $y = -\frac{4}{5}x$.

6 $2x - 7y = 3 \Rightarrow -7y = -2x + 3 \Rightarrow y = \frac{2}{7}x - \frac{3}{7}$, so the slope of the given line is $\frac{2}{7}$ and the slope of the desired line is $-\frac{7}{2}$ $\left\{\text{the negative reciprocal of } \frac{2}{7}\right\}$. Since the desired line has x-intercept 4, its equation is $y - 0 = -\frac{7}{2}(x-4) \Rightarrow y = -\frac{7}{2}x + 14$.

7 The cost of the pizza plus the toppings is $9 + 0.80x$. Multiply by 1.10 to get the total cost including the tax. Thus, $T(x) = 1.10(9 + 0.80x) \Rightarrow T(x) = 9.9 + 0.88x$, or $T(x) = 0.88x + 9.9$.

8 $f(x) = \dfrac{\sqrt{-x}}{(x+2)(x-2)}$ • The numerator is defined for $x \le 0$. The denominator is defined for all real numbers, but we must exclude ± 2 since the denominator is zero for those values.

Thus, the domain of f is $(-\infty, -2) \cup (-2, 0]$.

9 For $f(x) = x^2 + 5x - 7$:

$$\begin{aligned}\frac{f(a+h) - f(a)}{h} &= \frac{[(a+h)^2 + 5(a+h) - 7] - (a^2 + 5a - 7)}{h} \\ &= \frac{[a^2 + 2ah + h^2 + 5a + 5h - 7] - (a^2 + 5a - 7)}{h} \\ &= \frac{2ah + h^2 + 5h}{h} = \frac{h(2a + h + 5)}{h} = 2a + h + 5\end{aligned}$$

Based on that answer, a prediction of $2a + h - 7$ seems reasonable for the second function, $f(x) = x^2 - 7x + 5$, since it appears that only the coefficient of x is part of the answer.

10 $V = 2 \times y \times y = 2y^2 \Rightarrow y^2 = V/2 \Rightarrow y = \sqrt{V/2}$. There are four sides of area $2 \times y$ and the top and bottom each have area $y \times y$, so $S(y) = 4(2y) + 2(y^2) \Rightarrow S(V) = 8\sqrt{V/2} + 2(V/2) = V + 8\sqrt{V/2}$ or $V + 4\sqrt{2V}$.

11 Start with the point $P(3, -2)$ on f and find the corresponding point on the graph of $y = 2|f(x-3)| - 1$.

$P(3, -2)$	{ $x - 3$ [add 3 to the x-coordinate] }	$\to (6, -2)$
	{ |absolute value| [make the y-coordinate positive] }	$\to (6, 2)$
	{ $\times 2$ [multiply the y-coordinate by 2] }	$\to (6, 4)$
	{ -1 [subtract 1 from the y-coordinate] }	$\to (6, 3)$

12 The salesman makes \$1.20 per cap on the first 1000 caps and \$1.80 on each additional cap.

$$C(x) = \begin{cases} 1.20x & \text{if } 0 \le x \le 1000 \\ 1.20(1000) + 1.80(x - 1000) & \text{if } x > 1000 \end{cases} = \begin{cases} 1.20x & \text{if } 0 \le x \le 1000 \\ 1.80x - 600 & \text{if } x > 1000 \end{cases}$$

13 The standard equation of a parabola with vertical axis and vertex $V(-2, 1)$ is $y = a(x+2)^2 + 1$.
If $a < 0$, then the graph will have two x-intercepts, so the desired restriction is $a > 0$.

14 A parabola that has x-intercepts -2 and 4 has equation $f(x) = a(x+2)(x-4)$.
Since $(3, -15)$ is on the parabola, $-15 = a(3+2)(3-4) \quad \Rightarrow \quad -15 = -5a \quad \Rightarrow \quad a = 3$.
The vertex has x-coordinate that is halfway between the x-intercepts, that is, $x = 1$.
The minimum value is of $f(x) = 3(x+2)(x-4)$ is $f(1) = 3(1+2)(1-4) = 3(3)(-3) = -27$.

15 Let x and $2x - 9$ denote the numbers, so that the product is $p = x(2x-9)$. The intercepts of the graph of p are 0 and $\frac{9}{2}$, so the vertex is halfway between them at $x = \frac{9}{4}$, and the minimum value is $\frac{9}{4}[2(\frac{9}{4}) - 9] = \frac{9}{4}(-\frac{9}{2}) = -\frac{81}{8}$. We could also use $p = 2x^2 - 9x$ and note that $-\frac{b}{2a} = -\frac{-9}{2(2)} = \frac{9}{4}$.

16 Let x denote the number of people in the group. The cost per person can be represented by $4 - 0.01(x - 100) = 4 - 0.01x + 1 = 5 - 0.01x$. The total cost T of the group is the product of these two expressions, so $T = x(5 - 0.01x)$. The intercepts of the graph of T are 0 and 500, so the vertex is at $x = 250$. $T(250) = 250(5 - 2.5) = 625$. The tour is only offered for 100 to 300 people at a time, so we check the endpoints and get $T(100) = 100(5-1) = 400$ and $T(300) = 300(5-3) = 600$. Thus, the maximum total cost for the group is \$625 when 250 people are in the group and the minimum total cost for the group is \$400 when 100 people are in the group. Note that group sizes from 251 to 300 have a smaller total cost than \$625.

17 $f(x) = x^2$ and $g(x) = \sqrt{x-3} \quad \Rightarrow \quad (f \circ g)(x) = f(g(x)) = (\sqrt{x-3})^2 = x - 3$, which is defined for all reals, but g is defined for only $x \geq 3$, so the domain of $(f \circ g)(x)$ is $[3, \infty)$.

18 $C = y^2 - 2y + 10$ and $y(t) = 5t$, so $(C \circ y)(t) = C(y(t)) = (5t)^2 - 2(5t) + 10 = 25t^2 - 10t + 10$.
The minimum cost occurs when $t = -\frac{b}{2a} = -\frac{-10}{2(25)} = \frac{1}{5}$.
This cost is $C(\frac{1}{5}) = 25(\frac{1}{25}) - 10(\frac{1}{5}) + 10 = 1 - 2 + 10 = 9$, which represents \$9000.