INSTRUCTOR'S SOLUTIONS MANUAL

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ALGEBRA & TRIGONOMETRY
GRAPHS AND MODELS
SIXTH EDITION

PRECALCULUS
GRAPHS AND MODELS, A RIGHT TRIANGLE APPROACH
SIXTH EDITION

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Contents

Just-in-Time Review

1. Real Numbers

1. Rational numbers: $\frac{2}{3}$, 6, -2.45, $18.\overline{4}$, -11, $\sqrt[3]{27}$, $5\frac{1}{6}$, $-\frac{8}{7}$, 0, $\sqrt{16}$
2. Rational numbers but not integers: $\frac{2}{3}$, -2.45, $18.\overline{4}$, $5\frac{1}{6}$, $-\frac{8}{7}$
3. Irrational numbers: $\sqrt{3}$, $\sqrt[6]{26}$, $7.151551555\ldots$, $-\sqrt{35}$, $\sqrt[5]{3}$ (Although there is a pattern in $7.151551555\ldots$, there is no repeating block of digits.)
4. Integers: 6, -11, $\sqrt[3]{27}$, 0, $\sqrt{16}$
5. Whole numbers: 6, $\sqrt[3]{27}$, 0, $\sqrt{16}$
6. Real numbers: All of them

2. Properties of Real Numbers

1. $-24 + 24 = 0$ illustrates the additive inverse property.
2. $7(xy) = (7x)y$ illustrates the associative property of multiplication.
3. $9(r - s) = 9r - 9s$ illustrates a distributive property.
4. $11 + z = z + 11$ illustrates the commutative property of addition.
5. $-20 \cdot 1 = -20$ illustrates the multiplicative identity property.
6. $5(x + y) = (x + y)5$ illustrates the commutative property of multiplication.
7. $q + 0 = q$ illustrates the additive identity property.
8. $75 \cdot \frac{1}{75} = 1$ illustrates the multiplicative inverse property.
9. $(x+y)+w = x+(y+w)$ illustrates the associative property of addition.
10. $8(a + b) = 8a + 8b$ illustrates a distributive property.

3. Order on the Number Line

1. 9 is to the right of -9 on the number line, so it is false that $9 < -9$.
2. -10 is to the left of -1 on the number line, so it is true that $-10 \leq -1$.
3. $-5 = -\sqrt{25}$, and $-\sqrt{26}$ is to the left of $-\sqrt{25}$, or -5, on the number line. Thus it is true that $-\sqrt{26} < -5$.
4. $\sqrt{6} = \sqrt{6}$, so it is true that $\sqrt{6} \leq \sqrt{6}$.
5. -30 is to the left of -25 on the number line, so it is false that $-30 > -25$.
6. $-\frac{4}{5} = -\frac{16}{20}$ and $-\frac{5}{4} = -\frac{25}{20}$; $-\frac{16}{20}$ is to the right of $-\frac{25}{20}$, so it is true that $-\frac{4}{5} > -\frac{5}{4}$.

4. Absolute Value

1. $|-98| = 98 \quad (|a| = -a, \text{ if } a < 0.)$
2. $|0| = 0 \quad (|a| = a, \text{ if } a \geq 0.)$
3. $|4.7| = 4.7 \quad (|a| = a, \text{ if } a \geq 0.)$
4. $\left|-\frac{2}{3}\right| = \frac{2}{3} \quad (|a| = -a, \text{ if } a < 0.)$
5. $|-7 - 13| = |-20| = 20$, or $|13 - (-7)| = |13 + 7| = |20| = 20$
6. $|2 - 14.6| = |-12.6| = 12.6$, or $|14.6 - 2| = |12.6| = 12.6$
7. $|-39 - (-28)| = |-39 + 28| = |-11| = 11$, or $|-28 - (-39)| = |-28 + 39| = |11| = 11$
8. $\left|-\frac{3}{4} - \frac{15}{8}\right| = \left|-\frac{6}{8} - \frac{15}{8}\right| = \left|-\frac{21}{8}\right| = \frac{21}{8}$, or $\left|\frac{15}{8} - \left(-\frac{3}{4}\right)\right| = \left|\frac{15}{8} + \frac{6}{8}\right| = \left|\frac{21}{8}\right| = \frac{21}{8}$

5. Operations with Real Numbers

1. $8 - (-11) = 8 + 11 = 19$
2. $-\frac{3}{10} \cdot \left(-\frac{1}{3}\right) = \frac{3 \cdot 1}{10 \cdot 3} = \frac{3}{3} \cdot \frac{1}{10} = 1 \cdot \frac{1}{10} = \frac{1}{10}$
3. $15 \div (-3) = -5$
4. $-4 - (-1) = -4 + 1 = -3$
5. $7 \cdot (-50) = -350$
6. $-0.5 - 5 = -0.5 + (-5) = -5.5$
7. $-3 + 27 = 24$
8. $-400 \div -40 = 10$
9. $4.2 \cdot (-3) = -12.6$

10. $-13-(-33)=-13+33=20$

11. $-60+45=-15$

12. $\frac{1}{2}-\frac{2}{3}=\frac{1}{2}+\left(-\frac{2}{3}\right)=\frac{3}{6}+\left(-\frac{4}{6}\right)=-\frac{1}{6}$

13. $-24\div 3=-8$

14. $-6+(-16)=-22$

15. $-\frac{1}{2}\div\left(-\frac{5}{8}\right)=-\frac{1}{2}\cdot\left(-\frac{8}{5}\right)=\frac{1\cdot 8}{2\cdot 5}=\frac{1\cdot \cancel{2}\cdot 4}{\cancel{2}\cdot 5}=\frac{4}{5}$

6. Interval Notation

1. This is a closed interval, so we use brackets. Interval notation is $[-5,5]$.

2. This is a half-open interval. We use a parenthesis on the left and a bracket on the right. Interval notation is $(-3,-1]$.

3. This interval is of unlimited extent in the negative direction, and the endpoint -2 is included. Interval notation is $(-\infty,-2]$.

4. This interval is of unlimited extent in the positive direction, and the endpoint 3.8 is not included. Interval notation is $(3.8,\infty)$.

5. $\{x|7<x\}$, or $\{x|x>7\}$.

This interval is of unlimited extent in the positive direction and the endpoint 7 is not included. Interval notation is $(7,\infty)$.

6. The endpoints -2 and 2 are not included in the interval, so we use parentheses. Interval notation is $(-2,2)$.

7. The endpoints -4 and 5 are not included in the interval, so we use parentheses. Interval notation is $(-4,5)$.

8. The interval is of unlimited extent in the positive direction, and the endpoint 1.7 is included. Internal notation is $[1.7,\infty)$.

9. The endpoint -5 is not included in the interval, so we use a parenthesis before -5. The endpoint -2 is included in the interval, so we use a bracket after -2. Interval notation is $(-5,-2]$.

10. This interval is of unlimited extent in the negative direction, and the endpoint $\sqrt{5}$ is not included. Interval notation is $(-\infty,\sqrt{5})$.

7. Integers as Exponents

1. $3^{-6}=\frac{1}{3^6}$ Using $a^{-m}=\frac{1}{a^m}$

2. $\frac{1}{(0.2)^{-5}}=(0.2)^5$ Using $a^{-m}=\frac{1}{a^m}$

3. $\frac{w^{-4}}{z^{-9}}=\frac{z^9}{w^4}$ Using $\frac{a^{-m}}{b^{-n}}=\frac{b^n}{a^m}$

4. $\left(\frac{z}{y}\right)^2=\frac{z^2}{y^2}$ Raising a quotient to a power

5. $100^0=1$ Using $a^0=1$, $a\neq 0$

6. $\frac{a^5}{a^{-3}}=a^{5-(-3)}=a^{5+3}=a^8$ Using the quotient rule

7.
$$\begin{aligned}(2xy^3)(-3x^{-5}y)&=2(-3)x\cdot x^{-5}\cdot y^3\cdot y\\&=-6x^{1+(-5)}y^{3+1}\\&=-6x^{-4}y^4,\text{ or }-\frac{6y^4}{x^4}\end{aligned}$$

8. $x^{-4}\cdot x^{-7}=x^{-4+(-7)}=x^{-11}$, or $\frac{1}{x^{11}}$

9. $(mn)^{-6}=m^{-6}n^{-6}$, or $\frac{1}{m^6n^6}$

10. $(t^{-5})^4=t^{-5\cdot 4}=t^{-20}$, or $\frac{1}{t^{20}}$

8. Scientific Notation

1. Convert 18,500,000 to scientific notation.

We want the decimal point to be positioned between the 1 and the 8, so we move it 7 places to the left. Since 18,500,000 is greater than 10, the exponent must be positive.

$$18,500,000=1.85\times 10^7$$

2. Convert 0.000786 to scientific notation.

We want the decimal point to be positioned between the 7 and the 8, so we move it 4 places to the right. Since 0.000786 is between 0 and 1, the exponent must be negative.

$$0.000786=7.86\times 10^{-4}$$

3. Convert 0.0000000023 to scientific notation.

We want the decimal point to be positioned between the 2 and the 3, so we move it 9 places to the right. Since 0.0000000023 is between 0 and 1, the exponent must be negative.

$$0.0000000023=2.3\times 10^{-9}$$

4. Convert 8,927,000,000 to scientific notation.

We want the decimal point to be positioned between the 8 and the 9, so we move it 9 places to the left. Since 8,927,000,000 is greater than 10, the exponent must be positive.

$$8,927,000,000=8.927\times 10^9$$

5. Convert 4.3×10^{-8} to decimal notation.

The exponent is negative, so the number is between 0 and 1. We move the decimal point 8 places to the left.

$$4.3\times 10^{-8}=0.000000043$$

6. Convert 5.17×10^6 to decimal notation.

The exponent is positive, so the number is greater than 10. We move the decimal point 6 places to the right.

$$5.17 \times 10^6 = 5,170,000$$

7. Convert 6.203×10^{11} to decimal notation.

The exponent is positive, so the number is greater than 10. We move the decimal point 11 places to the right.

$$6.203 \times 10^{11} = 620,300,000,000$$

8. Convert 2.94×10^{-5} to scientific notation.

The exponent is negative, so the number is between 0 and 1. We move the decimal point 5 places to the left.

$$2.94 \times 10^{-5} = 0.0000294$$

9. Order of Operations

1. $3 + 18 \div 6 - 3 = 3 + 3 - 3$ Dividing

$= 6 - 3 = 3$ Adding and subtracting

2. $= 5 \cdot 3 + 8 \cdot 3^2 + 4(6 - 2)$

$= 5 \cdot 3 + 8 \cdot 3^2 + 4 \cdot 4$ Working inside parentheses

$= 5 \cdot 3 + 8 \cdot 9 + 4 \cdot 4$ Evaluating 3^2

$= 15 + 72 + 16$ Multiplying

$= 87 + 16$ Adding in order

$= 103$ from left to right

3.
$$\begin{aligned} &5[3 - 8 \cdot 3^2 + 4 \cdot 6 - 2] \\ &= 5[3 - 8 \cdot 9 + 4 \cdot 6 - 2] \\ &= 5[3 - 72 + 24 - 2] \\ &= 5[-69 + 24 - 2] \\ &= 5[-45 - 2] \\ &= 5[-47] \\ &= -235 \end{aligned}$$

4. $16 \div 4 \cdot 4 \div 2 \cdot 256$

$= 4 \cdot 4 \div 2 \cdot 256$ Multiplying and dividing in order from left to right

$= 16 \div 2 \cdot 256$

$= 8 \cdot 256$

$= 2048$

5.
$$\begin{aligned} &2^6 \cdot 2^{-3} \div 2^{10} \div 2^{-8} \\ &= 2^3 \div 2^{10} \div 2^{-8} \\ &= 2^{-7} \div 2^{-8} \\ &= 2 \end{aligned}$$

6. $\dfrac{4(8-6)^2 - 4 \cdot 3 + 2 \cdot 8}{3^1 + 19^0}$

$= \dfrac{4 \cdot 2^2 - 4 \cdot 3 + 2 \cdot 8}{3 + 1}$ Calculating in the numerator and in the denominator

$= \dfrac{4 \cdot 4 - 4 \cdot 3 + 2 \cdot 8}{4}$

$= \dfrac{16 - 12 + 16}{4}$

$= \dfrac{4 + 16}{4}$

$= \dfrac{20}{4}$

$= 5$

7. $64 \div [(-4) \div (-2)] = 64 \div 2 = 32$

8.
$$\begin{aligned} &6[9 - (3 - 2)] + 4(2 - 3) \\ &= 6[9 - 1] + 4(2 - 3) \\ &= 6 \cdot 8 + 4(-1) \\ &= 48 - 4 \\ &= 44 \end{aligned}$$

10. Introduction to Polynomials

1. $5 - x^6$

The term of highest degree is $-x^6$, so the degree of the polynomial is 6.

2. $x^2y^5 - x^7y + 4$

The degree of x^2y^5 is $2 + 5$, or 7; the degree of $-x^7y$ is $7 + 1$, or 8; the degree of 4 is 0 ($4 = 4x^0$). Thus the degree of the polynomial is 8.

3. $2a^4 - 3 + a^2$

The term of highest degree is $2a^4$, so the degree of the polynomial is 4.

4. $-41 = -41x^0$, so the degree of the polynomial is 0.

5. $4x - x^3 + 0.1x^8 - 2x^5$

The term of highest degree is $0.1x^8$, so the degree of the polynomial is 8.

6. $x - 3$ has two terms. It is a binomial.

7. $14y^5$ has one term. It is a monomial.

8. $2y - \dfrac{1}{4}y^2 + 8$ has three terms. It is a trinomial.

11. Add and Subtract Polynomials

1.
$$\begin{aligned} &(8y - 1) - (3 - y) \\ &= (8y - 1) + (-3 + y) \\ &= (8 + 1)y + (-1 - 3) \\ &= 9y - 4 \end{aligned}$$

2. $(3x^2-2x-x^3+2)-(5x^2-8x-x^3+4)$
$=(3x^2-2x-x^3+2)+(-5x^2+8x+x^3-4)$
$=(3-5)x^2+(-2+8)x+(-1+1)x^3+(2-4)$
$=-2x^2+6x-2$

3. $(2x+3y+z-7)+(4x-2y-z+8)+$
$(-3x+y-2z-4)$
$=(2+4-3)x+(3-2+1)y+(1-1-2)z+$
$(-7+8-4)$
$=3x+2y-2z-3$

4. $(3ab^2-4a^2b-2ab+6)+$
$(-ab^2-5a^2b+8ab+4)$
$=(3-1)ab^2+(-4-5)a^2b+(-2+8)ab+(6+4)$
$=2ab^2-9a^2b+6ab+10$

5. $(5x^2+4xy-3y^2+2)-(9x^2-4xy+2y^2-1)$
$=(5x^2+4xy-3y^2+2)+(-9x^2+4xy-2y^2+1)$
$=(5-9)x^2+(4+4)xy+(-3-2)y^2+(2+1)$
$=-4x^2+8xy-5y^2+3$

12. Multiply Polynomials

1. $(3a^2)(-7a^4)=[3(-7)](a^2\cdot a^4)$
$=-21a^6$

2. $(y-3)(y+5)$
$=y^2+5y-3y-15$ Using FOIL
$=y^2+2y-15$ Collecting like terms

3. $(x+6)(x+3)$
$=x^2+3x+6x+18$ Using FOIL
$=x^2+9x+18$ Collecting like terms

4. $(2a+3)(a+5)$
$=2a^2+10a+3a+15$ Using FOIL
$=2a^2+13a+15$ Collecting like terms

5. $(2x+3y)(2x+y)$
$=4x^2+2xy+6xy+3y^2$ Using FOIL
$=4x^2+8xy+3y^2$

6. $(11t-1)(3t+4)$
$=33t^2+44t-3t-4$ Using FOIL
$=33t^2+41t-4$

13. Special Products of Binomials

1. $(x+3)^2$
$=x^2+2\cdot x\cdot 3+3^2$
$[(A+B)^2=A^2+2AB+B^2]$
$=x^2+6x+9$

2. $(5x-3)^2$
$=(5x)^2-2\cdot 5x\cdot 3+3^2$
$[(A-B)^2=A^2-2AB+B^2]$
$=25x^2-30x+9$

3. $(2x+3y)^2$
$=(2x)^2+2(2x)(3y)+(3y)^2$
$[(A+B)^2=A^2+2AB+B^2]$
$=4x^2+12xy+9y^2$

4. $(a-5b)^2$
$=a^2-2\cdot a\cdot 5b+(5b)^2$
$[(A-B)^2=A^2-2AB+B^2]$
$=a^2-10ab+25b^2$

5. $(n+6)(n-6)$
$=n^2-6^2$ $[(A+B)(A-B)=A^2-B^2]$
$=n^2-36$

6. $(3y+4)(3y-4)$
$=(3y)^2-4^2$ $[(A+B)(A-B)=A^2-B^2]$
$=9y^2-16$

14. Factor Polynomials; The FOIL Method

1. $3x+18=3\cdot x+3\cdot 6=3(x+6)$

2. $2z^3-8z^2=2z^2\cdot z-2z^2\cdot 4=2z^2(z-4)$

3. $3x^3-x^2+18x-6$
$=x^2(3x-1)+6(3x-1)$
$=(3x-1)(x^2+6)$

4. $t^3+6t^2-2t-12$
$=t^2(t+6)-2(t+6)$
$=(t+6)(t^2-2)$

5. $w^2-7w+10$

We look for two numbers with a product of 10 and a sum of -7. By trial, we determine that they are -5 and -2.

$$w^2-7w+10=(w-5)(w-2)$$

6. $t^2+8t+15$

We look for two numbers with a product of 15 and a sum of 8. By trial, we determine that they are 3 and 5.

$$t^2+8t+15=(t+3)(t+5)$$

7. $2n^2-20n-48=2(n^2-10n-24)$

Now factor $n^2-10n-24$. We look for two numbers with a product of -24 and a sum of -10. By trial, we determine that they are 2 and -12. Then $n^2-10n-24=(n+2)(n-12)$. We must include the common factor, 2, to have a factorization of the original trinomial.

$$2n^2-20n-48=2(n+2)(n-12)$$

8. $y^4 - 9y^3 + 14y^2 = y^2(y^2 - 9y + 14)$

Now factor $y^2 - 9y + 14$. Look for two numbers with a product of 14 and a sum of -9. The numbers are -2 and -7. Then $y^2 - 9y + 14 = (y-2)(y-7)$. We must include the common factor, y^2, in order to have a factorization of the original trinomial.

$$y^4 - 9y^3 + 14y^2 = y^2(y-2)(y-7)$$

9. $2n^2 + 9n - 56$

1. There is no common factor other than 1 or -1.
2. The factorization must be of the form $(2n+\quad)(n+\quad)$.
3. Factor the constant term, -56. The possibilities are $-1\cdot 56$, $1(-56)$, $-2\cdot 28$, $2(-28)$, $-4\cdot 16$, $4(-16)$, $-7\cdot 8$, and $7(-8)$. The factors can be written in the opposite order as well: $56(-1)$, $-56\cdot 1$, $28(-2)$, $-28\cdot 2$, $16(-4)$, $-16\cdot 4$, $8(-7)$, and $-8\cdot 7$.
4. Find a pair of factors for which the sum of the outer and the inner products is the middle term, $9n$. By trial, we determine that the factorization is $(2n-7)(n+8)$.

10. $2y^2 + y - 6$

1. There is no common factor other than 1 or -1.
2. The factorization must be of the form $(2y+\quad)(y+\quad)$.
3. Factor the constant term, -6. The possibilities are $-1\cdot 6$, $1(-6)$, $-2\cdot 3$, and $2(-3)$. The factors can be written in the opposite order as well: $6(-1)$, $-6\cdot 1$, $3(-2)$ and $-3\cdot 2$.
4. Find a pair of factors for which the sum of the outer and the inner products is the middle term, y. By trial, we determine that the factorization is $(2y-3)(y+2)$.

11. $b^2 - 6bt + 5t^2$

We look for two numbers with a product of 5 and a sum of -6. By trial, we determine that they are -1 and -5.

$$b^2 - 6bt + 5t^2 = (b-t)(b-5t)$$

12. $x^4 - 7x^2 - 30 = (x^2)^2 - 7x^2 - 30$

We look for two numbers with a product of -30 and a sum of -7. By trial, we determine that they are 3 and -10.

$$x^4 - 7x^2 - 30 = (x^2+3)(x^2-10)$$

15. Factoring Polynomials; The ac-Method

1. $8x^2 - 6x - 9$

1. There is no common factor other than 1 or -1.
2. Multiply the leading coefficient and the constant: $8(-9) = -72$.
3. Try to factor -72 so that the sum of the factors is the coefficient of the middle term, -6. The factors we want are -12 and 6.
4. Split the middle term using the numbers found in step (3):
   $-6x = -12x + 6x$
5. Factor by grouping.
$$\begin{aligned} 8x^2 - 6x - 9 &= 8x^2 - 12x + 6x - 9 \\ &= 4x(2x-3) + 3(2x-3) \\ &= (2x-3)(4x+3) \end{aligned}$$

2. $10t^2 + 4t - 6$

1. Factor out the largest common factor, 2.
   $10t^2 + 4t - 6 = 2(5t^2 + 2t - 3)$
   Now factor $5t^2 + 2t - 3$.
2. Multiply the leading coefficient and the constant: $5(-3) = -15$.
3. Try to factor -15 so that the sum of the factors is the coefficient of the middle term, 2. The factors we want are 5 and -3.
4. Split the middle term using the numbers found in step (3):
   $2t = 5t - 3t$.
5. Factor by grouping.
$$\begin{aligned} 5t^2 + 2t - 3 &= 5t^2 + 5t - 3t - 3 \\ &= 5t(t+1) - 3(t+1) \\ &= (t+1)(5t-3) \end{aligned}$$

Include the largest common factor in the final factorization.

$$10t^2 + 4t - 6 = 2(t+1)(5t-3)$$

3. $18a^2 - 51a + 15$

1. Factor out the largest common factor, 3.
   $18a^2 - 51a + 15 = 3(6a^2 - 17a + 5)$
   Now factor $6a^2 - 17a + 5$.
2. Multiply the leading coefficient and the constant: $6(5) = 30$.
3. Try to factor 30 so that the sum of the factors is the coefficient of the middle term, -17. The factors we want are -2 and -15.
4. Split the middle term using the numbers found in step (3):
   $-17a = -2a - 15a$.
5. Factor by grouping.
$$\begin{aligned} 6a^2 - 17a + 5 &= 6a^2 - 2a - 15a + 5 \\ &= 2a(3a-1) - 5(3a-1) \\ &= (3a-1)(2a-5) \end{aligned}$$

Include the largest common factor in the final factorization.

$$18a^2 - 51a + 15 = 3(3a-1)(2a-5)$$

16. Special Factorizations

1. $z^2 - 81 = z^2 - 9^2 = (z+9)(z-9)$

2. $16x^2 - 9 = (4x)^2 - 3^2 = (4x+3)(4x-3)$

3. $$\begin{aligned} 7pq^4 - 7py^4 &= 7p(q^4 - y^4) \\ &= 7p[(q^2)^2 - (y^2)^2] \\ &= 7p(q^2+y^2)(q^2-y^2) \\ &= 7p(q^2+y^2)(q+y)(q-y) \end{aligned}$$

4. $$\begin{aligned} x^2 + 12x + 36 &= x^2 + 2\cdot x\cdot 6 + 6^2 \\ &= (x+6)^2 \end{aligned}$$

5. $9z^2 - 12z + 4 = (3z)^2 - 2\cdot 3z\cdot 2 + 2^2 = (3z-2)^2$

6. $$\begin{aligned} &a^3 + 24a^2 + 144a \\ &= a(a^2 + 24a + 144) \\ &= a(a^2 + 2\cdot a\cdot 12 + 12^2) \\ &= a(a+12)^2 \end{aligned}$$

7. $$\begin{aligned} x^3 + 64 &= x^3 + 4^3 \\ &= (x+4)(x^2 - 4x + 16) \end{aligned}$$

8. $$\begin{aligned} m^3 - 216 &= m^3 - 6^3 \\ &= (m-6)(m^2 + 6m + 36) \end{aligned}$$

9. $$\begin{aligned} 3a^5 - 24a^2 &= 3a^2(a^3 - 8) \\ &= 3a^2(a^3 - 2^3) \\ &= 3a^2(a-2)(a^2 + 2a + 4) \end{aligned}$$

10. $$\begin{aligned} t^6 + 1 &= (t^2)^3 + 1^3 \\ &= (t^2+1)(t^4 - t^2 + 1) \end{aligned}$$

17. Equation-Solving Principles

1. $$\begin{aligned} 7t &= 70 \\ t &= 10 \quad \text{Dividing by 7} \end{aligned}$$
The solution is 10.

2. $$\begin{aligned} x - 5 &= 7 \\ x &= 12 \quad \text{Adding 5} \end{aligned}$$
The solution is 12.

3. $$\begin{aligned} 3x + 4 &= -8 \\ 3x &= -12 \quad \text{Subtracting 4} \\ x &= -4 \quad \text{Dividing by 3} \end{aligned}$$
The solution is -4.

4. $$\begin{aligned} 6x - 15 &= 45 \\ 6x &= 60 \quad \text{Adding 15} \\ x &= 10 \quad \text{Dividing by 6} \end{aligned}$$
The solution is 10.

5. $$\begin{aligned} 7y - 1 &= 23 - 5y \\ 12y - 1 &= 23 \quad \text{Adding } 5y \\ 12y &= 24 \quad \text{Adding 1} \\ y &= 2 \quad \text{Dividing by 12} \end{aligned}$$
The solution is 2.

6. $$\begin{aligned} 3m - 7 &= -13 + m \\ 2m - 7 &= -13 \quad \text{Subtracting } m \\ 2m &= -6 \quad \text{Adding 7} \\ m &= -3 \quad \text{Dividing by 2} \end{aligned}$$
The solution is -3.

7. $$\begin{aligned} 2(x+7) &= 5x + 14 \\ 2x + 14 &= 5x + 14 \\ -3x + 14 &= 14 \quad \text{Subtracting } 5x \\ -3x &= 0 \quad \text{Subtracting 14} \\ x &= 0 \end{aligned}$$
The solution is 0.

8. $$\begin{aligned} 5y - (2y - 10) &= 25 \\ 5y - 2y + 10 &= 25 \\ 3y + 10 &= 25 \quad \text{Collecting like terms} \\ 3y &= 15 \quad \text{Subtracting 10} \\ y &= 5 \quad \text{Dividing by 3} \end{aligned}$$
The solution is 5.

18. Inequality-Solving Principles

1. $$\begin{aligned} p + 25 &\geq -100 \\ p &\geq -125 \quad \text{Subtracting 25} \end{aligned}$$
The solution set is $[-125, \infty)$.

2. $$\begin{aligned} -\frac{2}{3}x &> 6 \\ x &< -\frac{3}{2}\cdot 6 \quad \text{Multiplying by } -\frac{3}{2} \text{ and reversing the inequality symbol} \\ x &< -9 \end{aligned}$$
The solution set is $(-\infty, -9)$.

3. $$\begin{aligned} 9x - 1 &< 17 \\ 9x &< 18 \quad \text{Adding 1} \\ x &< 2 \quad \text{Dividing by 9} \end{aligned}$$
The solution set is $(-\infty, 2)$.

4. $$\begin{aligned} -x - 16 &\geq 40 \\ -x &\geq 56 \quad \text{Adding 6} \\ x &\leq -56 \quad \text{Multiplying by } -1 \text{ and reversing the inequality symbol} \end{aligned}$$
The solution set is $(-\infty, -56]$.

5. $\frac{1}{3}y - 6 < 3$

$\frac{1}{3}y < 9$ Adding 6

$y < 27$ Multiplying by 3

The solution set is $(-\infty, 27)$.

6. $8 - 2w \leq -14$

$-2w \leq -22$ Subtracting 8

$w \geq 11$ Dividing by -2 and reversing the inequality symbol

The solution set is $[11, \infty)$.

19. The Principle of Zero Products

1. $2y^2 + 42y = 0$

$2y(y + 21) = 0$

$2y = 0$ *or* $y + 21 = 0$

$y = 0$ *or* $y = -21$

The solutions are 0 and -21.

2. $(a + 7)(a - 1) = 0$

$a + 7 = 0$ *or* $a - 1 = 0$

$a = -7$ *or* $a = 1$

The solutions are -7 and 1.

3. $(5y + 3)(y - 4) = 0$

$5y + 3 = 0$ *or* $y - 4 = 0$

$5y = -3$ *or* $y = 4$

$y = -\frac{3}{5}$ *or* $y = 4$

The solutions are $-\frac{3}{5}$ and 4.

4. $6x^2 + 7x - 5 = 0$

$(3x + 5)(2x - 1) = 0$

$3x + 5 = 0$ *or* $2x - 1 = 0$

$3x = -5$ *or* $2x = 1$

$x = -\frac{5}{3}$ *or* $x = \frac{1}{2}$

The solutions are $-\frac{5}{3}$ and $\frac{1}{2}$.

5. $t(t - 8) = 0$

$t = 0$ *or* $t - 8 = 0$

$t = 0$ *or* $t = 8$

The solutions are 0 and 8.

6. $x^2 - 8x - 33 = 0$

$(x + 3)(x - 11) = 0$

$x + 3 = 0$ *or* $x - 11 = 0$

$x = -3$ *or* $x = 11$

The solutions are -3 and 11.

7. $x^2 + 13x = 30$

$x^2 + 13x - 30 = 0$

$(x + 15)(x - 2) = 0$

$x + 15 = 0$ *or* $x - 2 = 0$

$x = -15$ *or* $x = 2$

The solutions are -15 and 2.

8. $12x^2 - 7x - 12 = 0$

$(4x + 3)(3x - 4) = 0$

$4x + 3 = 0$ *or* $3x - 4 = 0$

$4x = -3$ *or* $3x = 4$

$x = -\frac{3}{4}$ *or* $x = \frac{4}{3}$

The solutions are $-\frac{3}{4}$ and $\frac{4}{3}$.

20. The Principle of Square Roots

1. $x^2 - 36 = 0$

$x^2 = 36$

$x = \sqrt{36}$ *or* $x = -\sqrt{36}$

$x = 6$ *or* $x = -6$

The solutions are 6 and -6, or ± 6.

2. $2y^2 - 20 = 0$

$2y^2 = 20$

$y^2 = 10$

$y = \sqrt{10}$ *or* $y = -\sqrt{10}$

The solutions are $\sqrt{10}$ and $-\sqrt{10}$, or $\pm\sqrt{10}$.

3. $6z^2 = 18$

$z^2 = 3$

$z = \sqrt{3}$ *or* $z = -\sqrt{3}$

The solutions are $\sqrt{3}$ and $-\sqrt{3}$, or $\pm\sqrt{3}$.

4. $3t^2 - 15 = 0$

$3t^2 = 15$

$t^2 = 5$

$t = \sqrt{5}$ *or* $t = -\sqrt{5}$

The solutions are $\sqrt{5}$ and $-\sqrt{5}$, or $\pm\sqrt{5}$.

5. $z^2 - 1 = 24$

$z^2 = 25$

$z = \sqrt{25}$ *or* $z = -\sqrt{25}$

The solutions are 5 and -5, or ± 5.

6. $5x^2 - 75 = 0$

$5x^2 = 75$

$x^2 = 15$

$x = \sqrt{15}$ *or* $x = -\sqrt{15}$

The solutions are $\sqrt{15}$ and $-\sqrt{15}$, or $\pm\sqrt{15}$.

21. Simplify Rational Expressions

1. $\dfrac{3x-3}{x(x-1)}$

The denominator is 0 when the factor $x=0$ and also when $x-1=0$, or $x=1$. The domain is the set of all real numbers except 0 and 1.

2. $\dfrac{y+6}{y^2+4y-21}=\dfrac{y+6}{(y+7)(y-3)}$

The denominator is 0 when $y=-7$ or $y=3$. The domain is the set of all real numbers except -7 and 3.

3. $\dfrac{x^2-4}{x^2-4x+4}=\dfrac{(x+2)\cancel{(x-2)}}{(x-2)\cancel{(x-2)}}=\dfrac{x+2}{x-2}$

4. $\dfrac{x^2+2x-3}{x^2-9}=\dfrac{(x-1)\cancel{(x+3)}}{\cancel{(x+3)}(x-3)}=\dfrac{x-1}{x-3}$

5.
$$\frac{x^3-6x^2+9x}{x^3-3x^2}=\frac{x(x^2-6x+9)}{x^2(x-3)}$$
$$=\frac{\cancel{x}\cancel{(x-3)}(x-3)}{\cancel{x}\cdot x\cancel{(x-3)}}$$
$$=\frac{x-3}{x}$$

6.
$$\frac{6y^2+12y-48}{3y^2-9y+6}=\frac{6(y^2+2y-8)}{3(y^2-3y+2)}$$
$$=\frac{2\cdot\cancel{3}\cdot(y+4)\cancel{(y-2)}}{\cancel{3}(y-1)\cancel{(y-2)}}$$
$$=\frac{2(y+4)}{y-1}$$

22. Multiply and Divide Rational Expressions

1.
$$\frac{r-s}{r+s}\cdot\frac{r^2-s^2}{(r-s)^2}=\frac{(r-s)(r^2-s^2)}{(r+s)(r-s)^2}$$
$$=\frac{\cancel{(r-s)}\cancel{(r-s)}\cancel{(r+s)}\cdot 1}{\cancel{(r+s)}\cancel{(r-s)}\cancel{(r-s)}}$$
$$=1$$

2.
$$\frac{m^2-n^2}{r+s}\div\frac{m-n}{r+s}$$
$$=\frac{m^2-n^2}{r+s}\cdot\frac{r+s}{m-n}$$
$$=\frac{(m+n)\cancel{(m-n)}\cancel{(r+s)}}{\cancel{(r+s)}\cancel{(m-n)}}$$
$$=m+n$$

3.
$$\frac{4x^2+9x+2}{x^2+x-2}\cdot\frac{x^2-1}{3x^2+x-2}$$
$$=\frac{(4x+1)\cancel{(x+2)}\cancel{(x+1)}\cancel{(x-1)}}{\cancel{(x+2)}\cancel{(x-1)}(3x-2)\cancel{(x+1)}}$$
$$=\frac{4x+1}{3x-2}$$

4.
$$\frac{3x+12}{2x-8}\div\frac{(x+4)^2}{(x-4)^2}$$
$$=\frac{3x+12}{2x-8}\cdot\frac{(x-4)^2}{(x+4)^2}$$
$$=\frac{3\cancel{(x+4)}\cancel{(x-4)}(x-4)}{2\cancel{(x-4)}\cancel{(x+4)}(x+4)}$$
$$=\frac{3(x-4)}{2(x+4)}$$

5.
$$\frac{a^2-a-2}{a^2-a-6}\div\frac{a^2-2a}{2a+a^2}$$
$$=\frac{a^2-a-2}{a^2-a-6}\cdot\frac{2a+a^2}{a^2-2a}$$
$$=\frac{\cancel{(a-2)}(a+1)\cancel{(a)}\cancel{(2+a)}}{(a-3)\cancel{(a+2)}\cancel{(a)}\cancel{(a-2)}}$$
$$=\frac{a+1}{a-3}$$

6.
$$\frac{x^2-y^2}{x^3-y^3}\cdot\frac{x^2+xy+y^2}{x^2+2xy+y^2}$$
$$=\frac{(x+y)(x-y)(x^2+xy+y^2)}{(x-y)(x^2+xy+y^2)(x+y)(x+y)}$$
$$=\frac{1}{x+y}\cdot\frac{(x+y)(x-y)(x^2+xy+y^2)}{(x+y)(x-y)(x^2+xy+y^2)}$$
$$=\frac{1}{x+y}\cdot 1 \quad \text{Removing a factor of 1}$$
$$=\frac{1}{x+y}$$

23. Add and Subtract Rational Expressions

1.
$$\frac{a-3b}{a+b}+\frac{a+5b}{a+b}=\frac{2a+2b}{a+b}$$
$$=\frac{2\cancel{(a+b)}}{1\cdot\cancel{(a+b)}}$$
$$=2$$

2.
$$\frac{x^2-5}{3x^2-5x-2}+\frac{x+1}{3x-6}$$
$$=\frac{x^2-5}{(3x+1)(x-2)}+\frac{x+1}{3(x-2)}$$
$$=\frac{x^2-5}{(3x+1)(x-2)}\cdot\frac{3}{3}+\frac{x+1}{3(x-2)}\cdot\frac{3x+1}{3x+1}$$
$$=\frac{3(x^2-5)+(x+1)(3x+1)}{3(3x+1)(x-2)}$$
$$=\frac{3x^2-15+3x^2+4x+1}{3(3x+1)(x-2)}$$
$$=\frac{6x^2+4x-14}{3(3x+1)(x-2)}$$

3. $$\frac{a^2+1}{a^2-1}-\frac{a-1}{a+1}$$
$$=\frac{a^2+1}{(a+1)(a-1)}-\frac{a-1}{a+1},\text{ LCD is }(a+1)(a-1)$$
$$=\frac{a^2+1-(a-1)(a-1)}{(a+1)(a-1)}$$
$$=\frac{a^2+1-a^2+2a-1}{(a+1)(a-1)}$$
$$=\frac{2a}{(a+1)(a-1)}$$

4. $$\frac{9x+2}{3x^2-2x-8}+\frac{7}{3x^2+x-4}$$
$$=\frac{9x+2}{(3x+4)(x-2)}+\frac{7}{(3x+4)(x-1)},$$
LCD is $(3x+4)(x-2)(x-1)$
$$=\frac{9x+2}{(3x+4)(x-2)}\cdot\frac{x-1}{x-1}+\frac{7}{(3x+4)(x-1)}\cdot\frac{x-2}{x-2}$$
$$=\frac{9x^2-7x-2}{(3x+4)(x-2)(x-1)}+\frac{7x-14}{(3x+4)(x-1)(x-2)}$$
$$=\frac{9x^2-16}{(3x+4)(x-2)(x-1)}$$
$$=\frac{\cancel{(3x+4)}(3x-4)}{\cancel{(3x+4)}(x-2)(x-1)}$$
$$=\frac{3x-4}{(x-2)(x-1)}$$

5. $$\frac{y}{y^2-y-20}-\frac{2}{y+4}$$
$$=\frac{y}{(y+4)(y-5)}-\frac{2}{y+4},\text{ LCD is }(y+4)(y-5)$$
$$=\frac{y}{(y+4)(y-5)}-\frac{2}{y+4}\cdot\frac{y-5}{y-5}$$
$$=\frac{y}{(y+4)(y-5)}-\frac{2y-10}{(y+4)(y-5)}$$
$$=\frac{y-(2y-10)}{(y+4)(y-5)}$$
$$=\frac{y-2y+10}{(y+4)(y-5)}$$
$$=\frac{-y+10}{(y+4)(y-5)}$$

6. $$\frac{3y}{y^2-7y+10}-\frac{2y}{y^2-8y+15}$$
$$=\frac{3y}{(y-2)(y-5)}-\frac{2y}{(y-5)(y-3)},$$
LCD is $(y-2)(y-5)(y-3)$
$$=\frac{3y(y-3)-2y(y-2)}{(y-2)(y-5)(y-3)}$$
$$=\frac{3y^2-9y-2y^2+4y}{(y-2)(y-5)(y-3)}$$
$$=\frac{y^2-5y}{(y-2)(y-5)(y-3)}$$
$$=\frac{y\cancel{(y-5)}}{(y-2)\cancel{(y-5)}(y-3)}$$
$$=\frac{y}{(y-2)(y-3)}$$

24. Simplify Complex Rational Expressions

1. $$\frac{\frac{x}{y}-\frac{y}{x}}{\frac{1}{y}+\frac{1}{x}}=\frac{\frac{x}{y}-\frac{y}{x}}{\frac{1}{y}+\frac{1}{x}}\cdot\frac{xy}{xy},\text{ LCM is }xy$$
$$=\frac{\left(\frac{x}{y}-\frac{y}{x}\right)(xy)}{\left(\frac{1}{y}+\frac{1}{x}\right)(xy)}$$
$$=\frac{x^2-y^2}{x+y}$$
$$=\frac{\cancel{(x+y)}(x-y)}{\cancel{(x+y)}\cdot 1}$$
$$=x-y$$

2. $$\frac{\frac{a-b}{b}}{\frac{a^2-b^2}{ab}}=\frac{a-b}{b}\cdot\frac{ab}{a^2-b^2}$$
$$=\frac{a-b}{b}\cdot\frac{ab}{(a+b)(a-b)}$$
$$=\frac{a\cancel{b}\cancel{(a-b)}}{\cancel{b}(a+b)\cancel{(a-b)}}$$
$$=\frac{a}{a+b}$$

3. $$\frac{w+\dfrac{8}{w^2}}{1+\dfrac{2}{w}} = \frac{w\cdot\dfrac{w^2}{w^2}+\dfrac{8}{w^2}}{1\cdot\dfrac{w}{w}+\dfrac{2}{w}} = \frac{\dfrac{w^3+8}{w^2}}{\dfrac{w+2}{w}} = \frac{w^3+8}{w^2}\cdot\frac{w}{w+2} = \frac{\cancel{(w+2)}(w^2-2w+4)\cancel{w}}{\cancel{w}\cdot w\cancel{(w+2)}} = \frac{w^2-2w+4}{w}$$

4. $$\frac{\dfrac{x^2-y^2}{xy}}{\dfrac{x-y}{y}} = \frac{x^2-y^2}{xy}\cdot\frac{y}{x-y} = \frac{(x+y)\cancel{(x-y)}\cancel{y}}{x\cancel{y}\cancel{(x-y)}} = \frac{x+y}{x}$$

5. $$\frac{\dfrac{a}{b}-\dfrac{b}{a}}{\dfrac{1}{a}-\dfrac{1}{b}} = \frac{a^2-b^2}{b-a} \quad \text{Multiplying by } \frac{ab}{ab}$$
$$= \frac{(a+b)(a-b)}{b-a} = \frac{(a+b)\cancel{(a-b)}}{-1\cdot\cancel{(a-b)}} = -a-b$$

25. Simplify Radical Expressions

1. $\sqrt{(-21)^2} = |-21| = 21$

2. $\sqrt{9y^2} = \sqrt{(3y)^2} = |3y| = 3y$

3. $\sqrt{(a-2)^2} = a-2$

4. $\sqrt[3]{-27x^3} = \sqrt[3]{(-3x)^3} = -3x$

5. $\sqrt[4]{81x^8} = \sqrt[4]{(3x^2)^4} = 3x^2$

6. $\sqrt[5]{32} = \sqrt[5]{2^5} = 2$

7. $\sqrt[4]{48x^6y^4} = \sqrt[4]{16x^4y^4\cdot 3x^2} = 2xy\sqrt[4]{3x^2} =$
$2xy\sqrt[4]{3x^2}$

8. $\sqrt{15}\sqrt{35} = \sqrt{15\cdot 35} = \sqrt{3\cdot 5\cdot 5\cdot 7} = \sqrt{5^2\cdot 3\cdot 7} =$
$\sqrt{5^2}\cdot\sqrt{3\cdot 7} = 5\sqrt{21}$

9. $\dfrac{\sqrt{40xy}}{\sqrt{8x}} = \sqrt{\dfrac{40xy}{8x}} = \sqrt{5y}$

10. $\dfrac{\sqrt[3]{3x^2}}{\sqrt[3]{24x^5}} = \sqrt[3]{\dfrac{3x^2}{24x^5}} = \sqrt[3]{\dfrac{1}{8x^3}} = \dfrac{1}{2x}$

11. $\sqrt{x^2-4x+4} = \sqrt{(x-2)^2} = x-2$

12. $\sqrt{2x^3y}\sqrt{12xy} = \sqrt{24x^4y^2} = \sqrt{4x^4y^2\cdot 6} = 2x^2y\sqrt{6}$

13. $\sqrt[3]{3x^2y}\sqrt[3]{36x} = \sqrt[3]{108x^3y} = \sqrt[3]{27x^3\cdot 4y} = 3x\sqrt[3]{4y}$

14. $$5\sqrt{2}+3\sqrt{32} = 5\sqrt{2}+3\sqrt{16\cdot 2} = 5\sqrt{2}+3\cdot 4\sqrt{2} = 5\sqrt{2}+12\sqrt{2} = (5+12)\sqrt{2} = 17\sqrt{2}$$

15. $7\sqrt{12}-2\sqrt{3} = 7\cdot 2\sqrt{3}-2\sqrt{3} = 14\sqrt{3}-2\sqrt{3} = 12\sqrt{3}$

16. $2\sqrt{32}+3\sqrt{8}-4\sqrt{18} = 2\cdot 4\sqrt{2}+3\cdot 2\sqrt{2}-4\cdot 3\sqrt{2} =$
$8\sqrt{2}+6\sqrt{2}-12\sqrt{2} = 2\sqrt{2}$

17. $$6\sqrt{20}-4\sqrt{45}+\sqrt{80} = 6\sqrt{4\cdot 5}-4\sqrt{9\cdot 5}+\sqrt{16\cdot 5} = 6\cdot 2\sqrt{5}-4\cdot 3\sqrt{5}+4\sqrt{5} = 12\sqrt{5}-12\sqrt{5}+4\sqrt{5} = (12-12+4)\sqrt{5} = 4\sqrt{5}$$

18. $$(2+\sqrt{3})(5+2\sqrt{3}) = 2\cdot 5+2\cdot 2\sqrt{3}+\sqrt{3}\cdot 5+\sqrt{3}\cdot 2\sqrt{3} = 10+4\sqrt{3}+5\sqrt{3}+3\cdot 2 = 10+9\sqrt{3}+6 = 16+9\sqrt{3}$$

19. $$\left(\sqrt{8}+2\sqrt{5}\right)\left(\sqrt{8}-2\sqrt{5}\right) = \left(\sqrt{8}\right)^2-\left(2\sqrt{5}\right)^2 = 8-4\cdot 5 = 8-20 = -12$$

20. $$(1+\sqrt{3})^2 = 1^2+2\cdot 1\cdot\sqrt{3}+(\sqrt{3})^2 = 1+2\sqrt{3}+3 = 4+2\sqrt{3}$$

26. Rationalizing Denominators

1. $\dfrac{4}{\sqrt{11}} = \dfrac{4}{\sqrt{11}}\cdot\dfrac{\sqrt{11}}{\sqrt{11}} = \dfrac{4\sqrt{11}}{11}$

2. $\sqrt{\dfrac{3}{7}} = \sqrt{\dfrac{3}{7}\cdot\dfrac{7}{7}} = \sqrt{\dfrac{21}{49}} = \dfrac{\sqrt{21}}{\sqrt{49}} = \dfrac{\sqrt{21}}{7}$

3. $\dfrac{\sqrt[3]{7}}{\sqrt[3]{2}} = \dfrac{\sqrt[3]{7}}{\sqrt[3]{2}}\cdot\dfrac{\sqrt[3]{4}}{\sqrt[3]{4}} = \dfrac{\sqrt[3]{28}}{\sqrt[3]{8}} = \dfrac{\sqrt[3]{28}}{2}$

4. $\sqrt[3]{\dfrac{16}{9}} = \sqrt[3]{\dfrac{16}{9}\cdot\dfrac{3}{3}} = \sqrt[3]{\dfrac{48}{27}} = \dfrac{\sqrt[3]{48}}{\sqrt[3]{27}} =$
$\dfrac{\sqrt[3]{8\cdot 6}}{3} = \dfrac{2\sqrt[3]{6}}{3}$

5. $$\frac{3}{\sqrt{30}-4} = \frac{3}{\sqrt{30}-4}\cdot\frac{\sqrt{30}+4}{\sqrt{30}+4} = \frac{3\sqrt{30}+12}{(\sqrt{30})^2-4^2} = \frac{3\sqrt{30}+12}{30-16} = \frac{3\sqrt{30}+12}{14}$$

6. $$\frac{4}{\sqrt{7}-\sqrt{3}} = \frac{4}{\sqrt{7}-\sqrt{3}}\cdot\frac{\sqrt{7}+\sqrt{3}}{\sqrt{7}+\sqrt{3}} = \frac{4\sqrt{7}+4\sqrt{3}}{(\sqrt{7})^2-(\sqrt{3})^2} = \frac{4\sqrt{7}+4\sqrt{3}}{7-3} = \frac{4\sqrt{7}+4\sqrt{3}}{4} = \frac{4(\sqrt{7}+\sqrt{3})}{4} = \sqrt{7}+\sqrt{3}$$

7. $$\frac{6}{\sqrt{m}-\sqrt{n}} = \frac{6}{\sqrt{m}-\sqrt{n}}\cdot\frac{\sqrt{m}+\sqrt{n}}{\sqrt{m}+\sqrt{n}} = \frac{6(\sqrt{m}+\sqrt{n})}{(\sqrt{m})^2-(\sqrt{n})^2} = \frac{6\sqrt{m}+6\sqrt{n}}{m-n}$$

8. $$\frac{1-\sqrt{2}}{\sqrt{3}-\sqrt{6}} = \frac{1-\sqrt{2}}{\sqrt{3}-\sqrt{6}}\cdot\frac{\sqrt{3}+\sqrt{6}}{\sqrt{3}+\sqrt{6}} = \frac{\sqrt{3}+\sqrt{6}-\sqrt{6}-\sqrt{12}}{3-6} = \frac{\sqrt{3}+\sqrt{6}-\sqrt{6}-2\sqrt{3}}{3-6} = \frac{-\sqrt{3}}{-3} = \frac{\sqrt{3}}{3}$$

27. Rational Exponents

1. $y^{5/6} = \sqrt[6]{y^5}$

2. $x^{2/3} = \sqrt[3]{x^2}$

3. $16^{3/4} = (16^{1/4})^3 = \left(\sqrt[4]{16}\right)^3 = 2^3 = 8$

4. $4^{7/2} = (\sqrt{4})^7 = 2^7 = 128$

5. $125^{-1/3} = \dfrac{1}{125^{1/3}} = \dfrac{1}{\sqrt[3]{125}} = \dfrac{1}{5}$

6. $32^{-4/5} = \left(\sqrt[5]{32}\right)^{-4} = 2^{-4} = \dfrac{1}{16}$

7. $\sqrt[12]{y^4} = y^{4/12} = y^{1/3}$

8. $\sqrt{x^5} = x^{5/2}$

9. $x^{1/2}\cdot x^{2/3} = x^{1/2+2/3} = x^{3/6+4/6} = x^{7/6} = \sqrt[6]{x^7} = x\sqrt[6]{x}$

10. $(a-2)^{9/4}(a-2)^{-1/4} = (a-2)^{9/4+(-1/4)} = (a-2)^{8/4} = (a-2)^2$

11. $(m^{1/2}n^{5/2})^{2/3} = m^{\frac{1}{2}\cdot\frac{2}{3}}n^{\frac{5}{2}\cdot\frac{2}{3}} = m^{1/3}n^{5/3} = \sqrt[3]{m}\sqrt[3]{n^5} = \sqrt[3]{mn^5} = n\sqrt[3]{mn^2}$

28. The Pythagorean Theorem

1. $$\begin{aligned} a^2+b^2 &= c^2\\ 8^2+15^2 &= c^2\\ 64+225 &= c^2\\ 289 &= c^2\\ 17 &= c \end{aligned}$$

2. $$\begin{aligned} a^2+b^2 &= c^2\\ 4^2+4^2 &= c^2\\ 16+16 &= c^2\\ 32 &= c^2\\ \sqrt{32} &= c\\ 5.657 &\approx c \end{aligned}$$

3. $$\begin{aligned} a^2+b^2 &= c^2\\ 5^2+b^2 &= 13^2\\ 25+b^2 &= 169\\ b^2 &= 144\\ b &= 12 \end{aligned}$$

4. $$\begin{aligned} a^2+b^2 &= c^2\\ a^2+12^2 &= 13^2\\ a^2+144 &= 169\\ a^2 &= 25\\ a &= 5 \end{aligned}$$

5. $$\begin{aligned} a^2+b^2 &= c^2\\ (\sqrt{5})^2+b^2 &= 6^2\\ 5+b^2 &= 36\\ b^2 &= 31\\ b &= \sqrt{31}\approx 5.568 \end{aligned}$$

Chapter 1

Graphs, Functions, and Models

Exercise Set 1.1

1. Point A is located 5 units to the left of the y-axis and 4 units up from the x-axis, so its coordinates are $(-5, 4)$.

Point B is located 2 units to the right of the y-axis and 2 units down from the x-axis, so its coordinates are $(2, -2)$.

Point C is located 0 units to the right or left of the y-axis and 5 units down from the x-axis, so its coordinates are $(0, -5)$.

Point D is located 3 units to the right of the y-axis and 5 units up from the x-axis, so its coordinates are $(3, 5)$.

Point E is located 5 units to the left of the y-axis and 4 units down from the x-axis, so its coordinates are $(-5, -4)$.

Point F is located 3 units to the right of the y-axis and 0 units up or down from the x-axis, so its coordinates are $(3, 0)$.

2. G: $(2, 1)$; H: $(0, 0)$; I: $(4, -3)$; J: $(-4, 0)$; K: $(-2, 3)$; L: $(0, 5)$

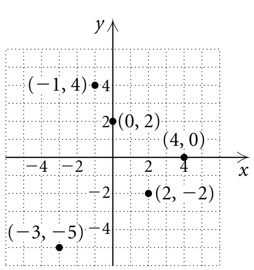
3. To graph $(4, 0)$ we move from the origin 4 units to the right of the y-axis. Since the second coordinate is 0, we do not move up or down from the x-axis.

To graph $(-3, -5)$ we move from the origin 3 units to the left of the y-axis. Then we move 5 units down from the x-axis.

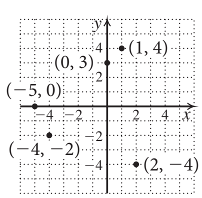
To graph $(-1, 4)$ we move from the origin 1 unit to the left of the y-axis. Then we move 4 units up from the x-axis.

To graph $(0, 2)$ we do not move to the right or the left of the y-axis since the first coordinate is 0. From the origin we move 2 units up.

To graph $(2, -2)$ we move from the origin 2 units to the right of the y-axis. Then we move 2 units down from the x-axis.



4.



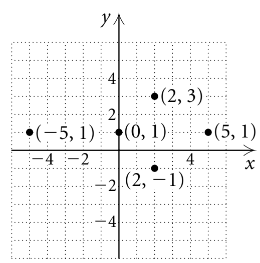
5. To graph $(-5, 1)$ we move from the origin 5 units to the left of the y-axis. Then we move 1 unit up from the x-axis.

To graph $(5, 1)$ we move from the origin 5 units to the right of the y-axis. Then we move 1 unit up from the x-axis.

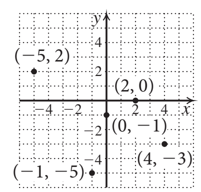
To graph $(2, 3)$ we move from the origin 2 units to the right of the y-axis. Then we move 3 units up from the x-axis.

To graph $(2, -1)$ we move from the origin 2 units to the right of the y-axis. Then we move 1 unit down from the x-axis.

To graph $(0, 1)$ we do not move to the right or the left of the y-axis since the first coordinate is 0. From the origin we move 1 unit up.



6.



7. The first coordinate represents the year and the corresponding second coordinate represents the number of cities served by Southwest Airlines. The ordered pairs are (1971, 3), (1981, 15), (1991, 32), (2001, 59), (2011, 72), and (2014, 96).

8. The first coordinate represents the year and the second coordinate represents the percent of Marines who are women. The ordered pairs are (1960, 1%), (1970, 0.9%), (1980, 3.6%), (1990, 4.9%), (2000, 6.1%), (2011, 6.8%), and (2014, 7.6%).

9. To determine whether $(-1, -9)$ is a solution, substitute -1 for x and -9 for y.

$$\begin{array}{c|l} \multicolumn{2}{c}{y = 7x - 2} \\ \hline -9 \; ? & 7(-1) - 2 \\ & -7 - 2 \\ -9 & -9 \qquad \text{TRUE} \end{array}$$

The equation $-9 = -9$ is true, so $(-1, -9)$ is a solution.

To determine whether $(0,2)$ is a solution, substitute 0 for x and 2 for y.

$$\begin{array}{r|ll}
\multicolumn{2}{c}{y = 7x - 2} \\ \hline
2 \ ? & 7\cdot 0 - 2 & \\
 & 0 - 2 & \\
2 & -2 & \text{FALSE}
\end{array}$$

The equation $2 = -2$ is false, so $(0,2)$ is not a solution.

10. For $\left(\frac{1}{2}, 8\right)$:

$$\begin{array}{r|ll}
\multicolumn{2}{c}{y = -4x + 10} \\ \hline
8 \ ? & -4\cdot\frac{1}{2} + 10 & \\
 & -2 + 10 & \\
8 & 8 & \text{TRUE}
\end{array}$$

$\left(\frac{1}{2}, 8\right)$ is a solution.

For $(-1,6)$:

$$\begin{array}{r|ll}
\multicolumn{2}{c}{y = -4x + 10} \\ \hline
6 \ ? & -4(-1) + 10 & \\
 & 4 + 10 & \\
6 & 14 & \text{FALSE}
\end{array}$$

$(-1,6)$ is not a solution.

11. To determine whether $\left(\frac{2}{3}, \frac{3}{4}\right)$ is a solution, substitute $\frac{2}{3}$ for x and $\frac{3}{4}$ for y.

$$\begin{array}{r|ll}
\multicolumn{2}{c}{6x - 4y = 1} \\ \hline
6\cdot\frac{2}{3} - 4\cdot\frac{3}{4} \ ? & 1 & \\
4 - 3 & & \\
1 & 1 & \text{TRUE}
\end{array}$$

The equation $1 = 1$ is true, so $\left(\frac{2}{3}, \frac{3}{4}\right)$ is a solution.

To determine whether $\left(1, \frac{3}{2}\right)$ is a solution, substitute 1 for x and $\frac{3}{2}$ for y.

$$\begin{array}{r|ll}
\multicolumn{2}{c}{6x - 4y = 1} \\ \hline
6\cdot 1 - 4\cdot\frac{3}{2} \ ? & 1 & \\
6 - 6 & & \\
0 & 1 & \text{FALSE}
\end{array}$$

The equation $0 = 1$ is false, so $\left(1, \frac{3}{2}\right)$ is not a solution.

12. For $(1.5, 2.6)$:

$$\begin{array}{r|ll}
\multicolumn{2}{c}{x^2 + y^2 = 9} \\ \hline
(1.5)^2 + (2.6)^2 \ ? & 9 & \\
2.25 + 6.76 & & \\
9.01 & 9 & \text{FALSE}
\end{array}$$

$(1.5, 2.6)$ is not a solution.

For $(-3,0)$:

$$\begin{array}{r|ll}
\multicolumn{2}{c}{x^2 + y^2 = 9} \\ \hline
(-3)^2 + 0^2 \ ? & 9 & \\
9 + 0 & & \\
9 & 9 & \text{TRUE}
\end{array}$$

$(-3,0)$ is a solution.

13. To determine whether $\left(-\frac{1}{2}, -\frac{4}{5}\right)$ is a solution, substitute $-\frac{1}{2}$ for a and $-\frac{4}{5}$ for b.

$$\begin{array}{r|ll}
\multicolumn{2}{c}{2a + 5b = 3} \\ \hline
2\left(-\frac{1}{2}\right) + 5\left(-\frac{4}{5}\right) \ ? & 3 & \\
-1 - 4 & & \\
-5 & 3 & \text{FALSE}
\end{array}$$

The equation $-5 = 3$ is false, so $\left(-\frac{1}{2}, -\frac{4}{5}\right)$ is not a solution.

To determine whether $\left(0, \frac{3}{5}\right)$ is a solution, substitute 0 for a and $\frac{3}{5}$ for b.

$$\begin{array}{r|ll}
\multicolumn{2}{c}{2a + 5b = 3} \\ \hline
2\cdot 0 + 5\cdot\frac{3}{5} \ ? & 3 & \\
0 + 3 & & \\
3 & 3 & \text{TRUE}
\end{array}$$

The equation $3 = 3$ is true, so $\left(0, \frac{3}{5}\right)$ is a solution.

14. For $\left(0, \frac{3}{2}\right)$:

$$\begin{array}{r|ll}
\multicolumn{2}{c}{3m + 4n = 6} \\ \hline
3\cdot 0 + 4\cdot\frac{3}{2} \ ? & 6 & \\
0 + 6 & & \\
6 & 6 & \text{TRUE}
\end{array}$$

$\left(0, \frac{3}{2}\right)$ is a solution.

For $\left(\frac{2}{3}, 1\right)$:

$$\begin{array}{r|ll}
\multicolumn{2}{c}{3m + 4n = 6} \\ \hline
3\cdot\frac{2}{3} + 4\cdot 1 \ ? & 6 & \\
2 + 4 & & \\
6 & 6 & \text{TRUE}
\end{array}$$

The equation $6 = 6$ is true, so $\left(\frac{2}{3}, 1\right)$ is a solution.

15. To determine whether $(-0.75, 2.75)$ is a solution, substitute -0.75 for x and 2.75 for y.

$$\begin{array}{r|ll}
\multicolumn{2}{c}{x^2 - y^2 = 3} \\ \hline
(-0.75)^2 - (2.75)^2 \ ? & 3 & \\
0.5625 - 7.5625 & & \\
-7 & 3 & \text{FALSE}
\end{array}$$

The equation $-7 = 3$ is false, so $(-0.75, 2.75)$ is not a solution.

To determine whether $(2, -1)$ is a solution, substitute 2 for x and -1 for y.

$$\begin{array}{c|l} x^2 - y^2 & = 3 \\ \hline 2^2 - (-1)^2 & ?\ 3 \\ 4 - 1 & \\ 3 & 3 \quad \text{TRUE} \end{array}$$

The equation $3 = 3$ is true, so $(2, -1)$ is a solution.

16. For $(2, -4)$:

$$\begin{array}{c|l} 5x + 2y^2 & = 70 \\ \hline 5 \cdot 2 + 2(-4)^2 & ?\ 70 \\ 10 + 2 \cdot 16 & \\ 10 + 32 & \\ 42 & 70 \quad \text{FALSE} \end{array}$$

$(2, -4)$ is not a solution.

For $(4, -5)$:

$$\begin{array}{c|l} 5x + 2y^2 & = 70 \\ \hline 5 \cdot 4 + 2(-5)^2 & ?\ 70 \\ 20 + 2 \cdot 25 & \\ 20 + 50 & \\ 70 & 70 \quad \text{TRUE} \end{array}$$

$(4, -5)$ is a solution.

17. Graph $5x - 3y = -15$.

To find the x-intercept we replace y with 0 and solve for x.

$$5x - 3 \cdot 0 = -15$$
$$5x = -15$$
$$x = -3$$

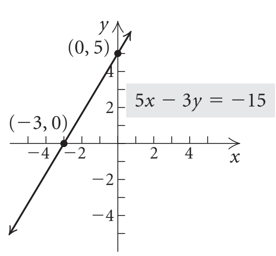
The x-intercept is $(-3, 0)$.

To find the y-intercept we replace x with 0 and solve for y.

$$5 \cdot 0 - 3y = -15$$
$$-3y = -15$$
$$y = 5$$

The y-intercept is $(0, 5)$.

We plot the intercepts and draw the line that contains them. We could find a third point as a check that the intercepts were found correctly.



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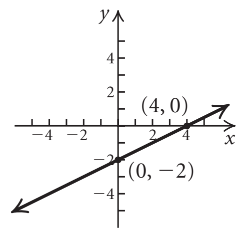


$2x - 4y = 8$

19. Graph $2x + y = 4$.

To find the x-intercept we replace y with 0 and solve for x.

$$2x + 0 = 4$$
$$2x = 4$$
$$x = 2$$

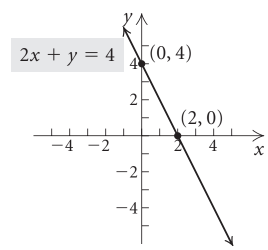
The x-intercept is $(2, 0)$.

To find the y-intercept we replace x with 0 and solve for y.

$$2 \cdot 0 + y = 4$$
$$y = 4$$

The y-intercept is $(0, 4)$.

We plot the intercepts and draw the line that contains them. We could find a third point as a check that the intercepts were found correctly.



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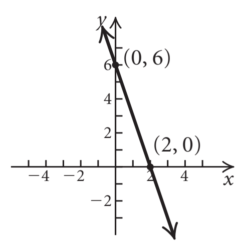


$3x + y = 6$

21. Graph $4y - 3x = 12$.

To find the x-intercept we replace y with 0 and solve for x.

$$4 \cdot 0 - 3x = 12$$
$$-3x = 12$$
$$x = -4$$

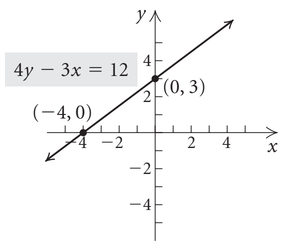
The x-intercept is $(-4, 0)$.

To find the y-intercept we replace x with 0 and solve for y.

$$4y - 3 \cdot 0 = 12$$
$$4y = 12$$
$$y = 3$$

The y-intercept is $(0, 3)$.

We plot the intercepts and draw the line that contains them. We could find a third point as a check that the intercepts were found correctly.



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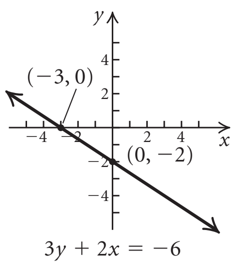


23. Graph $y = 3x + 5$.

We choose some values for x and find the corresponding y-values.

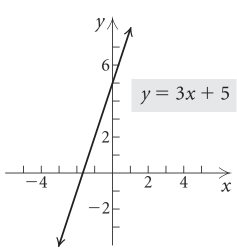
When $x = -3$, $y = 3x + 5 = 3(-3) + 5 = -9 + 5 = -4$.

When $x = -1$, $y = 3x + 5 = 3(-1) + 5 = -3 + 5 = 2$.

When $x = 0$, $y = 3x + 5 = 3 \cdot 0 + 5 = 0 + 5 = 5$

We list these points in a table, plot them, and draw the graph.

x	y	(x, y)
-3	-4	$(-3, -4)$
-1	2	$(-1, 2)$
0	5	$(0, 5)$



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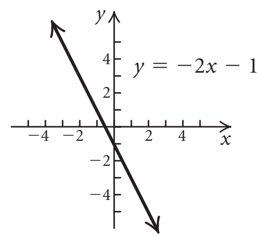
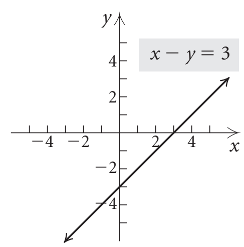


25. Graph $x - y = 3$.

Make a table of values, plot the points in the table, and draw the graph.

x	y	(x, y)
-2	-5	$(-2, -5)$
0	-3	$(0, -3)$
3	0	$(3, 0)$



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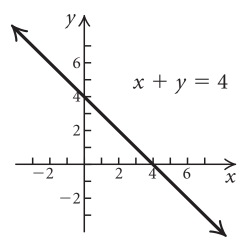
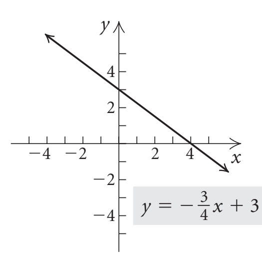


27. Graph $y = -\dfrac{3}{4}x + 3$.

By choosing multiples of 4 for x, we can avoid fraction values for y. Make a table of values, plot the points in the table, and draw the graph.

x	y	(x, y)
-4	6	$(-4, 6)$
0	3	$(0, 3)$
4	0	$(4, 0)$



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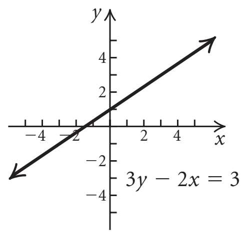


29. Graph $5x - 2y = 8$.

We could solve for y first.

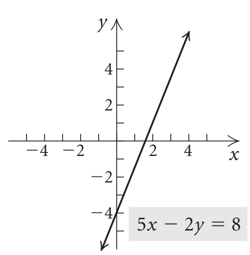
$5x - 2y = 8$

$-2y = -5x + 8$ Subtracting $5x$ on both sides

$y = \frac{5}{2}x - 4$ Multiplying by $-\frac{1}{2}$ on both sides

By choosing multiples of 2 for x we can avoid fraction values for y. Make a table of values, plot the points in the table, and draw the graph.

x	y	(x, y)
0	−4	(0, −4)
2	1	(2, 1)
4	6	(4, 6)



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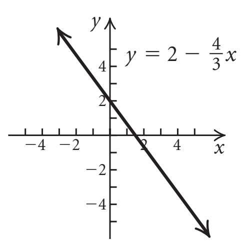
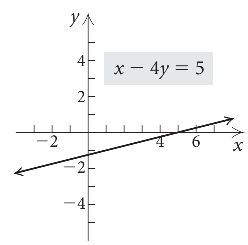


31. Graph $x - 4y = 5$.

Make a table of values, plot the points in the table, and draw the graph.

x	y	(x, y)
−3	−2	(−3, −2)
1	−1	(1, −1)
5	0	(5, 0)



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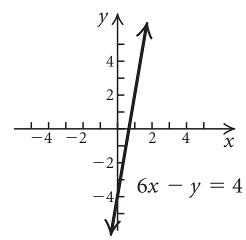
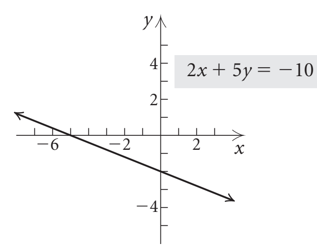


33. Graph $2x + 5y = -10$.

In this case, it is convenient to find the intercepts along with a third point on the graph. Make a table of values, plot the points in the table, and draw the graph.

x	y	(x, y)
−5	0	(−5, 0)
0	−2	(0, −2)
5	−4	(5, −4)



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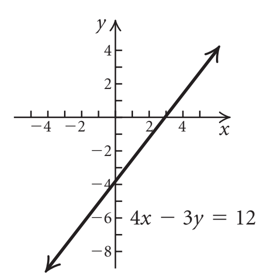
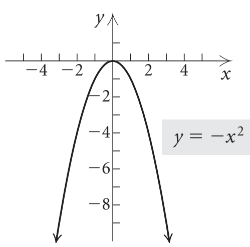


35. Graph $y = -x^2$.

Make a table of values, plot the points in the table, and draw the graph.

x	y	(x, y)
-2	-4	$(-2, -4)$
-1	-1	$(-1, -1)$
0	0	$(0, 0)$
1	-1	$(1, -1)$
2	-4	$(2, -4)$



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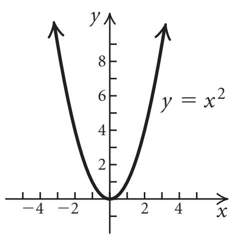
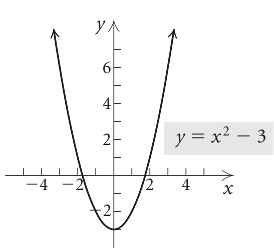


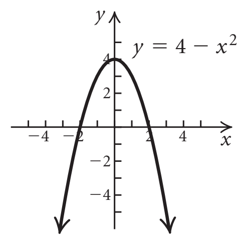
37. Graph $y = x^2 - 3$.

Make a table of values, plot the points in the table, and draw the graph.

x	y	(x, y)
-3	6	$(-3, 6)$
-1	-2	$(-1, -2)$
0	-3	$(0, -3)$
1	-2	$(1, -2)$
3	6	$(3, 6)$



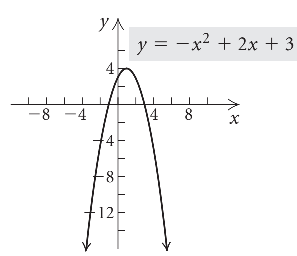
38.



39. Graph $y = -x^2 + 2x + 3$.

Make a table of values, plot the points in the table, and draw the graph.

x	y	(x, y)
-2	-5	$(-2, -5)$
-1	0	$(-1, 0)$
0	3	$(0, 3)$
1	4	$(1, 4)$
2	3	$(2, 3)$
3	0	$(3, 0)$
4	-5	$(4, -5)$



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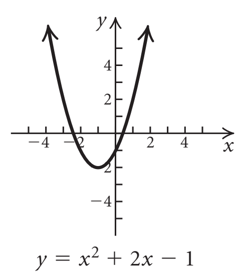


41. Graph (b) is the graph of $y = 3 - x$.

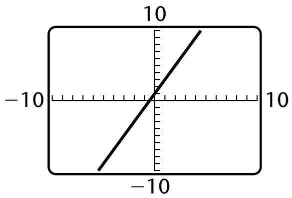
42. Graph (d) is the graph of $2x - y = 6$.

43. Graph (a) is the graph of $y = x^2 + 2x + 1$.

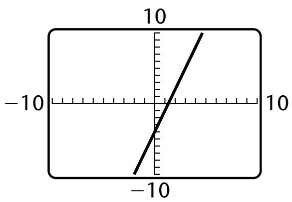
44. Graph (c) is the graph of $y = 8 - x^2$.

45. Enter the equation, select the standard window, and graph the equation.

$y = 2x + 1$

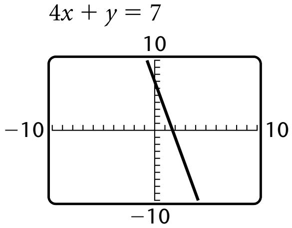


46. $y = 3x - 4$



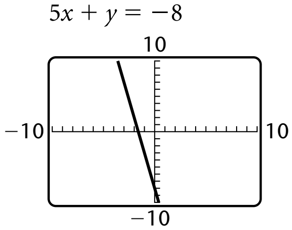
47. First solve the equation for y: $y = -4x + 7$. Enter the equation in this form, select the standard window, and graph the equation.

$4x + y = 7$



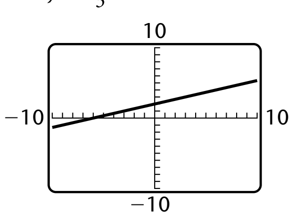
48. $5x + y = -8$, so $y = -5x - 8$.

$5x + y = -8$

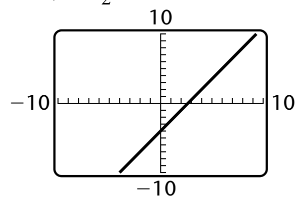


49. Enter the equation, select the standard window, and graph the equation.

$y = \frac{1}{3}x + 2$



50. $y = \frac{3}{2}x - 4$

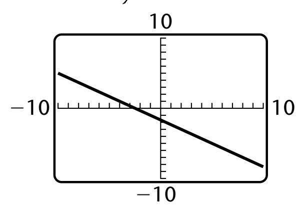


51. First solve the equation for y.

$$2x + 3y = -5$$
$$3y = -2x - 5$$
$$y = \frac{-2x - 5}{3}, \text{ or } \frac{1}{3}(-2x - 5)$$

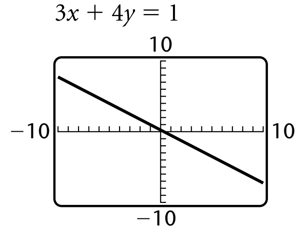
Enter the equation in "$y =$" form, select the standard window, and graph the equation.

$2x + 3y = -5$



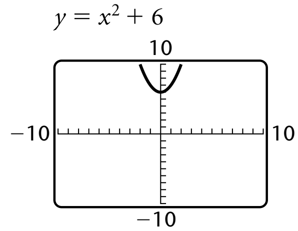
52. $3x + 4y = 1$, so $y = \frac{-3x + 1}{4}$, or $y = -\frac{3}{4}x + \frac{1}{4}$

$3x + 4y = 1$

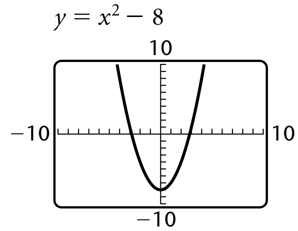


53. Enter the equation, select the standard window, and graph the equation.

$y = x^2 + 6$

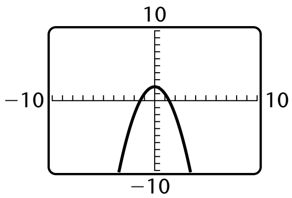


54. $y = x^2 - 8$

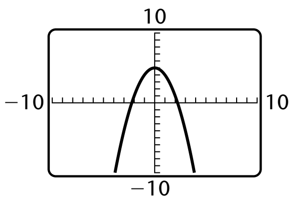


55. Enter the equation, select the standard window, and graph the equation.

$y = 2 - x^2$

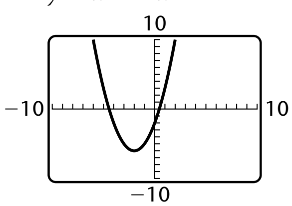


56. $y = 5 - x^2$

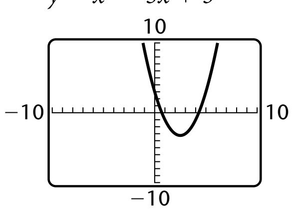


57. Enter the equation, select the standard window, and graph the equation.

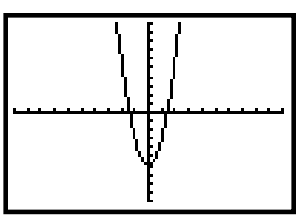
$y = x^2 + 4x - 2$



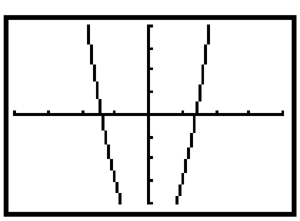
58. $y = x^2 - 5x + 3$



59. Standard window:

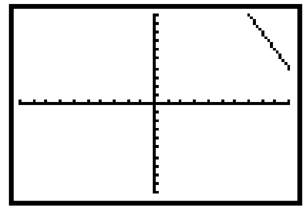


$[-4, 4, -4, 4]$

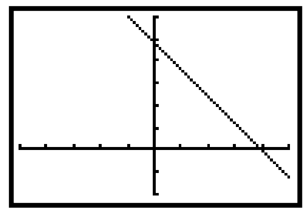


We see that the standard window is a better choice for this graph.

60. Standard window:

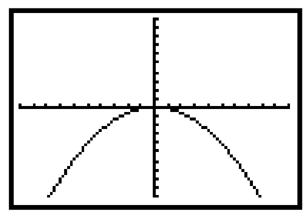


$[-15, 15, -10, 30]$, Xscl = 3, Yscl = 5

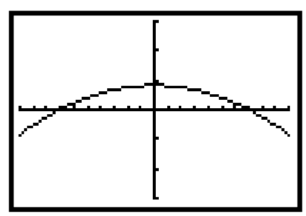


We see that $[-15, 15, -10, 30]$ is a better choice for this graph.

61. Standard window:

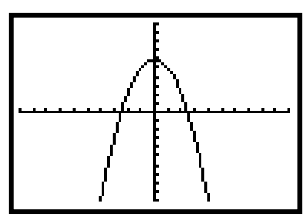


$[-1, 1, -0.3, 0.3]$, Xscl = 0.1, Yscl = 0.1

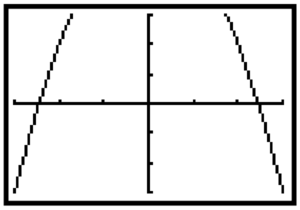


We see that $[-1, 1, -0.3, 0.3]$ is a better choice for this graph.

62. Standard window:



$[-3, 3, -3, 3]$



We see that the standard window is a better choice for this graph.

63. Either point can be considered as (x_1, y_1).

$$d = \sqrt{(4-5)^2 + (6-9)^2}$$
$$= \sqrt{(-1)^2 + (-3)^2} = \sqrt{10} \approx 3.162$$

64. $d = \sqrt{(-3-2)^2 + (7-11)^2} = \sqrt{41} \approx 6.403$

65. Either point can be considered as (x_1, y_1).

$$d = \sqrt{(-13-(-8))^2 + (1-(-11))^2}$$
$$= \sqrt{(-5)^2 + 12^2} = \sqrt{169} = 13$$

66. $d = \sqrt{(-20-(-60))^2 + (35-5)^2} = \sqrt{2500} = 50$

67. Either point can be considered as (x_1, y_1).

$$d = \sqrt{(6-9)^2 + (-1-5)^2}$$
$$= \sqrt{(-3)^2 + (-6)^2} = \sqrt{45} \approx 6.708$$

68. $d = \sqrt{(-4-(-1))^2 + (-7-3)^2} = \sqrt{109} \approx 10.440$

69. Either point can be considered as (x_1, y_1).

$$d = \sqrt{(-8-8)^2 + \left(\frac{7}{11} - \frac{7}{11}\right)^2}$$
$$= \sqrt{(-16)^2 + 0^2} = 16$$

70. $d = \sqrt{\left(\frac{1}{2} - \frac{1}{2}\right)^2 + \left(-\frac{4}{25} - \left(-\frac{13}{25}\right)\right)^2} = \sqrt{\left(\frac{9}{25}\right)^2} = \frac{9}{25}$

71. $d = \sqrt{\left[-\frac{3}{5} - \left(-\frac{3}{5}\right)\right]^2 + \left(-4 - \frac{2}{3}\right)^2}$

$$= \sqrt{0^2 + \left(-\frac{14}{3}\right)^2} = \frac{14}{3}$$

72. $d = \sqrt{\left(-\frac{11}{3} - \frac{1}{3}\right)^2 + \left(-\frac{1}{2} - \frac{5}{2}\right)^2} = \sqrt{16+9} = \sqrt{25} = 5$

73. Either point can be considered as (x_1, y_1).

$$d = \sqrt{(-4.2-2.1)^2 + [3-(-6.4)]^2}$$
$$= \sqrt{(-6.3)^2 + (9.4)^2} = \sqrt{128.05} \approx 11.316$$

74. $d = \sqrt{[0.6-(-8.1)]^2 + [-1.5-(-1.5)]^2} = \sqrt{(8.7)^2} = 8.7$

75. Either point can be considered as (x_1, y_1).

$$d = \sqrt{(0-a)^2 + (0-b)^2} = \sqrt{a^2 + b^2}$$

76. $d = \sqrt{[r-(-r)]^2 + [s-(-s)]^2} = \sqrt{4r^2 + 4s^2} = 2\sqrt{r^2 + s^2}$

77. First we find the length of the diameter:

$$d = \sqrt{(-3-9)^2 + (-1-4)^2}$$
$$= \sqrt{(-12)^2 + (-5)^2} = \sqrt{169} = 13$$

The length of the radius is one-half the length of the diameter, or $\frac{1}{2}(13)$, or 6.5.

78. Radius $= \sqrt{(-3-0)^2 + (5-1)^2} = \sqrt{25} = 5$

Diameter $= 2 \cdot 5 = 10$

79. First we find the distance between each pair of points.

For $(-4, 5)$ and $(6, 1)$:

$$d = \sqrt{(-4-6)^2 + (5-1)^2}$$
$$= \sqrt{(-10)^2 + 4^2} = \sqrt{116}$$

For $(-4, 5)$ and $(-8, -5)$:

$$d = \sqrt{(-4-(-8))^2 + (5-(-5))^2}$$
$$= \sqrt{4^2 + 10^2} = \sqrt{116}$$

For $(6, 1)$ and $(-8, -5)$:

$$d = \sqrt{(6-(-8))^2 + (1-(-5))^2}$$
$$= \sqrt{14^2 + 6^2} = \sqrt{232}$$

Since $(\sqrt{116})^2 + (\sqrt{116})^2 = (\sqrt{232})^2$, the points could be the vertices of a right triangle.

80. For $(-3, 1)$ and $(2, -1)$:

$$d = \sqrt{(-3-2)^2 + (1-(-1))^2} = \sqrt{29}$$

For $(-3, 1)$ and $(6, 9)$:

$$d = \sqrt{(-3-6)^2 + (1-9)^2} = \sqrt{145}$$

For $(2, -1)$ and $(6, 9)$:

$$d = \sqrt{(2-6)^2 + (-1-9)^2} = \sqrt{116}$$

Since $(\sqrt{29})^2 + (\sqrt{116})^2 = (\sqrt{145})^2$, the points could be the vertices of a right triangle.

81. First we find the distance between each pair of points.

For $(-4, 3)$ and $(0, 5)$:

$$d = \sqrt{(-4-0)^2 + (3-5)^2}$$
$$= \sqrt{(-4)^2 + (-2)^2} = \sqrt{20}$$

For $(-4, 3)$ and $(3, -4)$:

$$d = \sqrt{(-4-3)^2 + [3-(-4)]^2}$$
$$= \sqrt{(-7)^2 + 7^2} = \sqrt{98}$$

For $(0, 5)$ and $(3, -4)$:

$$d = \sqrt{(0-3)^2 + [5-(-4)]^2}$$
$$= \sqrt{(-3)^2 + 9^2} = \sqrt{90}$$

The greatest distance is $\sqrt{98}$, so if the points are the vertices of a right triangle, then it is the hypotenuse. But $(\sqrt{20})^2 + (\sqrt{90})^2 \neq (\sqrt{98})^2$, so the points are not the vertices of a right triangle.

82. See the graph of this rectangle in Exercise 93.

The segments with endpoints $(-3,4)$, $(2,-1)$ and $(5,2)$, $(0,7)$ are one pair of opposite sides. We find the length of each of these sides.

For $(-3,4)$, $(2,-1)$:

$$d = \sqrt{(-3-2)^2 + (4-(-1))^2} = \sqrt{50}$$

For $(5,2)$, $(0,7)$:

$$d = \sqrt{(5-0)^2 + (2-7)^2} = \sqrt{50}$$

The segments with endpoints $(2,-1)$, $(5,2)$ and $(0,7)$, $(-3,4)$ are the second pair of opposite sides. We find their lengths.

For $(2,-1)$, $(5,2)$:

$$d = \sqrt{(2-5)^2 + (-1-2)^2} = \sqrt{18}$$

For $(0,7)$, $(-3,4)$:

$$d = \sqrt{(0-(-3))^2 + (7-4)^2} = \sqrt{18}$$

The endpoints of the diagonals are $(-3,4)$, $(5,2)$ and $(2,-1)$, $(0,7)$. We find the length of each.

For $(-3,4)$, $(5,2)$:

$$d = \sqrt{(-3-5)^2 + (4-2)^2} = \sqrt{68}$$

For $(2,-1)$, $(0,7)$:

$$d = \sqrt{(2-0)^2 + (-1-7)^2} = \sqrt{68}$$

The opposite sides of the quadrilateral are the same length and the diagonals are the same length, so the quadrilateral is a rectangle.

83. We use the midpoint formula.

$$\left(\frac{4+(-12)}{2}, \frac{-9+(-3)}{2}\right) = \left(-\frac{8}{2}, -\frac{12}{2}\right) = (-4,-6)$$

84. $\left(\frac{7+9}{2}, \frac{-2+5}{2}\right) = \left(8, \frac{3}{2}\right)$

85. We use the midpoint formula.

$$\left(\frac{0+\left(-\frac{2}{5}\right)}{2}, \frac{\frac{1}{2}-0}{2}\right) = \left(\frac{-\frac{2}{5}}{2}, \frac{\frac{1}{2}}{2}\right) = \left(-\frac{1}{5}, \frac{1}{4}\right)$$

86. $\left(\frac{0+\left(-\frac{7}{13}\right)}{2}, \frac{0+\frac{2}{7}}{2}\right) = \left(-\frac{7}{26}, \frac{1}{7}\right)$

87. We use the midpoint formula.

$$\left(\frac{6.1+3.8}{2}, \frac{-3.8+(-6.1)}{2}\right) = \left(\frac{9.9}{2}, -\frac{9.9}{2}\right) =$$

$(4.95, -4.95)$

88. $\left(\frac{-0.5+4.8}{2}, \frac{-2.7+(-0.3)}{2}\right) = (2.15, -1.5)$

89. We use the midpoint formula.

$$\left(\frac{-6+(-6)}{2}, \frac{5+8}{2}\right) = \left(-\frac{12}{2}, \frac{13}{2}\right) = \left(-6, \frac{13}{2}\right)$$

90. $\left(\frac{1+(-1)}{2}, \frac{-2+2}{2}\right) = (0,0)$

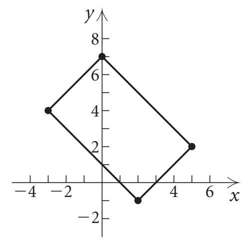
91. We use the midpoint formula.

$$\left(\frac{-\frac{1}{6}+\left(-\frac{2}{3}\right)}{2}, \frac{-\frac{3}{5}+\frac{5}{4}}{2}\right) = \left(\frac{-\frac{5}{6}}{2}, \frac{\frac{13}{20}}{2}\right) =$$

$$\left(-\frac{5}{12}, \frac{13}{40}\right)$$

92. $\left(\frac{\frac{2}{9}+\left(-\frac{2}{5}\right)}{2}, \frac{\frac{1}{3}+\frac{4}{5}}{2}\right) = \left(-\frac{4}{45}, \frac{17}{30}\right)$

93.



For the side with vertices $(-3,4)$ and $(2,-1)$:

$$\left(\frac{-3+2}{2}, \frac{4+(-1)}{2}\right) = \left(-\frac{1}{2}, \frac{3}{2}\right)$$

For the side with vertices $(2,-1)$ and $(5,2)$:

$$\left(\frac{2+5}{2}, \frac{-1+2}{2}\right) = \left(\frac{7}{2}, \frac{1}{2}\right)$$

For the side with vertices $(5,2)$ and $(0,7)$:

$$\left(\frac{5+0}{2}, \frac{2+7}{2}\right) = \left(\frac{5}{2}, \frac{9}{2}\right)$$

For the side with vertices $(0,7)$ and $(-3,4)$:

$$\left(\frac{0+(-3)}{2}, \frac{7+4}{2}\right) = \left(-\frac{3}{2}, \frac{11}{2}\right)$$

For the quadrilateral whose vertices are the points found above, the diagonals have endpoints

$\left(-\frac{1}{2}, \frac{3}{2}\right)$, $\left(\frac{5}{2}, \frac{9}{2}\right)$ and $\left(\frac{7}{2}, \frac{1}{2}\right)$, $\left(-\frac{3}{2}, \frac{11}{2}\right)$.

We find the length of each of these diagonals.

For $\left(-\frac{1}{2}, \frac{3}{2}\right)$, $\left(\frac{5}{2}, \frac{9}{2}\right)$:

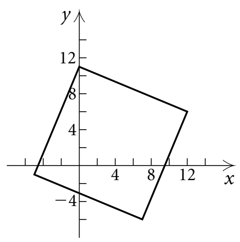
$$d = \sqrt{\left(-\frac{1}{2}-\frac{5}{2}\right)^2 + \left(\frac{3}{2}-\frac{9}{2}\right)^2}$$
$$= \sqrt{(-3)^2 + (-3)^2} = \sqrt{18}$$

For $\left(\frac{7}{2}, \frac{1}{2}\right)$, $\left(-\frac{3}{2}, \frac{11}{2}\right)$:

$$d = \sqrt{\left(\frac{7}{2}-\left(-\frac{3}{2}\right)\right)^2 + \left(\frac{1}{2}-\frac{11}{2}\right)^2}$$
$$= \sqrt{5^2 + (-5)^2} = \sqrt{50}$$

Since the diagonals do not have the same lengths, the midpoints are not vertices of a rectangle.

94.



For the side with vertices $(-5,-1)$ and $(7,-6)$:
$$\left(\frac{-5+7}{2},\frac{-1+(-6)}{2}\right)=\left(1,-\frac{7}{2}\right)$$
For the side with vertices $(7,-6)$ and $(12,6)$:
$$\left(\frac{7+12}{2},\frac{-6+6}{2}\right)=\left(\frac{19}{2},0\right)$$
For the side with vertices $(12,6)$ and $(0,11)$:
$$\left(\frac{12+0}{2},\frac{6+11}{2}\right)=\left(6,\frac{17}{2}\right)$$
For the side with vertices $(0,11)$ and $(-5,-1)$:
$$\left(\frac{0+(-5)}{2},\frac{11+(-1)}{2}\right)=\left(-\frac{5}{2},5\right)$$
For the quadrilateral whose vertices are the points found above, one pair of opposite sides has endpoints $\left(1,-\frac{7}{2}\right)$, $\left(\frac{19}{2},0\right)$ and $\left(6,\frac{17}{2}\right)$, $\left(-\frac{5}{2},5\right)$. The length of each of these sides is $\frac{\sqrt{338}}{2}$. The other pair of opposite sides has endpoints $\left(\frac{19}{2},0\right)$, $\left(6,\frac{17}{2}\right)$ and $\left(-\frac{5}{2},5\right)$, $\left(1,-\frac{7}{2}\right)$. The length of each of these sides is also $\frac{\sqrt{338}}{2}$. The endpoints of the diagonals of the quadrilateral are $\left(1,-\frac{7}{2}\right)$, $\left(6,\frac{17}{2}\right)$ and $\left(\frac{19}{2},0\right)$, $\left(-\frac{5}{2},5\right)$. The length of each diagonal is 13. Since the four sides of the quadrilateral are the same length and the diagonals are the same length, the midpoints are vertices of a square.

95. We use the midpoint formula.
$$\left(\frac{\sqrt{7}+\sqrt{2}}{2},\frac{-4+3}{2}\right)=\left(\frac{\sqrt{7}+\sqrt{2}}{2},-\frac{1}{2}\right)$$

96. $\left(\frac{-3+1}{2},\frac{\sqrt{5}+\sqrt{2}}{2}\right)=\left(-1,\frac{\sqrt{5}+\sqrt{2}}{2}\right)$

97. Square the viewing window. For the graph shown, one possibility is $[-12,9,-4,10]$.

98. Square the viewing window. For the graph shown, one possibility is $[-10,20,-15,5]$.

99.
$$(x-h)^2+(y-k)^2=r^2$$
$$(x-2)^2+(y-3)^2=\left(\frac{5}{3}\right)^2 \quad \text{Substituting}$$
$$(x-2)^2+(y-3)^2=\frac{25}{9}$$

100.
$$(x-4)^2+(y-5)^2=(4.1)^2$$
$$(x-4)^2+(y-5)^2=16.81$$

101. The length of a radius is the distance between $(-1,4)$ and $(3,7)$:
$$r=\sqrt{(-1-3)^2+(4-7)^2}$$
$$=\sqrt{(-4)^2+(-3)^2}=\sqrt{25}=5$$

$$(x-h)^2+(y-k)^2=r^2$$
$$[x-(-1)]^2+(y-4)^2=5^2$$
$$(x+1)^2+(y-4)^2=25$$

102. Find the length of a radius:
$$r=\sqrt{(6-1)^2+(-5-7)^2}=\sqrt{169}=13$$

$$(x-6)^2+[y-(-5)]^2=13^2$$
$$(x-6)^2+(y+5)^2=169$$

103. The center is the midpoint of the diameter:
$$\left(\frac{7+(-3)}{2},\frac{13+(-11)}{2}\right)=(2,1)$$
Use the center and either endpoint of the diameter to find the length of a radius. We use the point $(7,13)$:
$$r=\sqrt{(7-2)^2+(13-1)^2}$$
$$=\sqrt{5^2+12^2}=\sqrt{169}=13$$

$$(x-h)^2+(y-k)^2=r^2$$
$$(x-2)^2+(y-1)^2=13^2$$
$$(x-2)^2+(y-1)^2=169$$

104. The points $(-9,4)$ and $(-1,-2)$ are opposite vertices of the square and hence endpoints of a diameter of the circle. We use these points to find the center and radius.

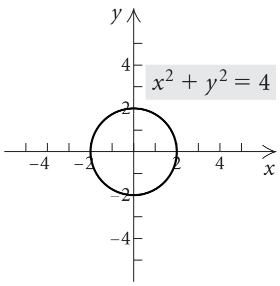
Center: $\left(\frac{-9+(-1)}{2},\frac{4+(-2)}{2}\right)=(-5,1)$

Radius: $\frac{1}{2}\sqrt{(-9-(-1))^2+(4-(-2))^2}=\frac{1}{2}\cdot 10=5$
$$[x-(-5)]^2+(y-1)^2=5^2$$
$$(x+5)^2+(y-1)^2=25$$

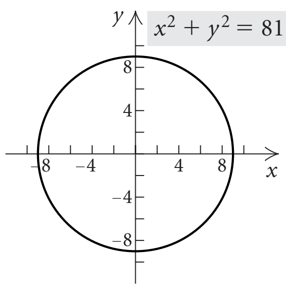
105. Since the center is 2 units to the left of the y-axis and the circle is tangent to the y-axis, the length of a radius is 2.
$$(x-h)^2+(y-k)^2=r^2$$
$$[x-(-2)]^2+(y-3)^2=2^2$$
$$(x+2)^2+(y-3)^2=4$$

106. Since the center is 5 units below the x-axis and the circle is tangent to the x-axis, the length of a radius is 5.
$$(x-4)^2+[y-(-5)]^2=5^2$$
$$(x-4)^2+(y+5)^2=25$$

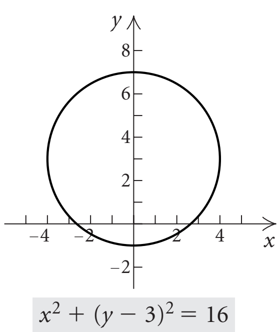
107.
$$x^2 + y^2 = 4$$
$$(x-0)^2 + (y-0)^2 = 2^2$$
Center: $(0, 0)$; radius: 2



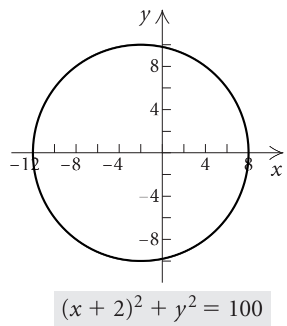
108.
$$x^2 + y^2 = 81$$
$$(x-0)^2 + (y-0)^2 = 9^2$$
Center: $(0, 0)$; radius: 9



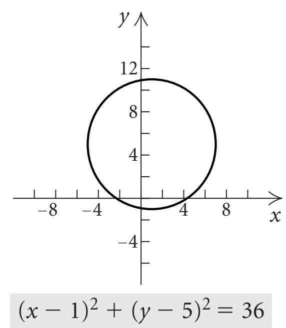
109.
$$x^2 + (y-3)^2 = 16$$
$$(x-0)^2 + (y-3)^2 = 4^2$$
Center: $(0, 3)$; radius: 4



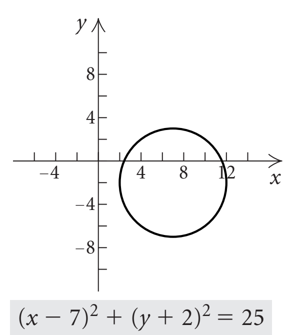
110.
$$(x+2)^2 + y^2 = 100$$
$$[x-(-2)]^2 + (y-0)^2 = 10^2$$
Center: $(-2, 0)$; radius: 10



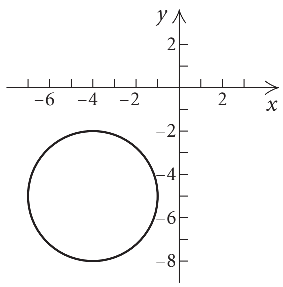
111.
$$(x-1)^2 + (y-5)^2 = 36$$
$$(x-1)^2 + (y-5)^2 = 6^2$$
Center: $(1, 5)$; radius: 6



112.
$$(x-7)^2 + (y+2)^2 = 25$$
$$(x-7)^2 + [y-(-2)]^2 = 5^2$$
Center: $(7, -2)$; radius: 5

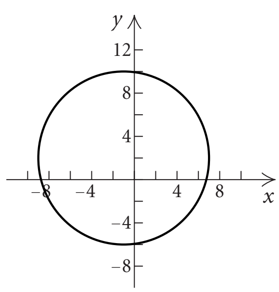


113.
$$(x+4)^2+(y+5)^2=9$$
$$[x-(-4)]^2+[y-(-5)]^2=3^2$$
Center: $(-4,-5)$; radius: 3



$(x + 4)^2 + (y + 5)^2 = 9$

114.
$$(x+1)^2+(y-2)^2=64$$
$$[x-(-1)]^2+(y-2)^2=8^2$$
Center: $(-1,2)$; radius: 8



$(x + 1)^2 + (y - 2)^2 = 64$

115. From the graph we see that the center of the circle is $(-2,1)$ and the radius is 3. The equation of the circle is $[x-(-2)]^2+(y-1)^2=3^2$, or $(x+2)^2+(y-1)^2=3^2$.

116. Center: $(3,-5)$, radius: 4

Equation: $(x-3)^2+[y-(-5)]^2=4^2$, or

$(x-3)^2+(y+5)^2=4^2$

117. From the graph we see that the center of the circle is $(5,-5)$ and the radius is 15. The equation of the circle is $(x-5)^2+[y-(-5)]^2=15^2$, or $(x-5)^2+(y+5)^2=15^2$.

118. Center: $(-8,2)$, radius: 4

Equation: $[x-(-8)]^2+(y-2)^2=4^2$, or

$(x+8)^2+(y-2)^2=4^2$

119. If the point (p,q) is in the fourth quadrant, then $p>0$ and $q<0$. If $p>0$, then $-p<0$ so both coordinates of the point $(q,-p)$ are negative and $(q,-p)$ is in the third quadrant.

120. Use the distance formula:
$$d=\sqrt{(a+h-a)^2+\left(\frac{1}{a+h}-\frac{1}{a}\right)^2}=$$
$$\sqrt{h^2+\left(\frac{-h}{a(a+h)}\right)^2}=\sqrt{h^2+\frac{h^2}{a^2(a+h)^2}}=$$
$$\sqrt{\frac{h^2a^2(a+h)^2+h^2}{a^2(a+h)^2}}=\sqrt{\frac{h^2(a^2(a+h)^2+1)}{a^2(a+h)^2}}=$$
$$\left|\frac{h}{a(a+h)}\right|\sqrt{a^2(a+h)^2+1}$$
Find the midpoint:
$$\left(\frac{a+a+h}{2},\frac{\frac{1}{a}+\frac{1}{a+h}}{2}\right)=\left(\frac{2a+h}{2},\frac{2a+h}{2a(a+h)}\right)$$

121. Use the distance formula. Either point can be considered as (x_1,y_1).
$$d=\sqrt{(a+h-a)^2+(\sqrt{a+h}-\sqrt{a})^2}$$
$$=\sqrt{h^2+a+h-2\sqrt{a^2+ah}+a}$$
$$=\sqrt{h^2+2a+h-2\sqrt{a^2+ah}}$$
Next we use the midpoint formula.
$$\left(\frac{a+a+h}{2},\frac{\sqrt{a}+\sqrt{a+h}}{2}\right)=\left(\frac{2a+h}{2},\frac{\sqrt{a}+\sqrt{a+h}}{2}\right)$$

122.
$$C=2\pi r$$
$$10\pi=2\pi r$$
$$5=r$$
Then $[x-(-5)]^2+(y-8)^2=5^2$, or $(x+5)^2+(y-8)^2=25$.

123. First use the formula for the area of a circle to find r^2:
$$A=\pi r^2$$
$$36\pi=\pi r^2$$
$$36=r^2$$
Then we have:
$$(x-h)^2+(y-k)^2=r^2$$
$$(x-2)^2+[y-(-7)]^2=36$$
$$(x-2)^2+(y+7)^2=36$$

124. Let the point be $(x,0)$. We set the distance from $(-4,-3)$ to $(x,0)$ equal to the distance from $(-1,5)$ to $(x,0)$ and solve for x.
$$\sqrt{(-4-x)^2+(-3-0)^2}=\sqrt{(-1-x)^2+(5-0)^2}$$
$$\sqrt{16+8x+x^2+9}=\sqrt{1+2x+x^2+25}$$
$$\sqrt{x^2+8x+25}=\sqrt{x^2+2x+26}$$
$$x^2+8x+25=x^2+2x+26$$
Squaring both sides
$$8x+25=2x+26$$
$$6x=1$$
$$x=\frac{1}{6}$$
The point is $\left(\frac{1}{6},0\right)$.

125. Let $(0, y)$ be the required point. We set the distance from $(-2, 0)$ to $(0, y)$ equal to the distance from $(4, 6)$ to $(0, y)$ and solve for y.

$$\sqrt{[0-(-2)]^2+(y-0)^2} = \sqrt{(0-4)^2+(y-6)^2}$$
$$\sqrt{4+y^2} = \sqrt{16+y^2-12y+36}$$
$$4+y^2 = 16+y^2-12y+36 \quad \text{Squaring both sides}$$
$$-48 = -12y$$
$$4 = y$$

The point is $(0, 4)$.

126. We first find the distance between each pair of points.

For $(-1, -3)$ and $(-4, -9)$:

$$d_1 = \sqrt{[-1-(-4)]^2+[-3-(-9)]^2} = \sqrt{3^2+6^2} = \sqrt{9+36} = \sqrt{45} = 3\sqrt{5}$$

For $(-1, -3)$ and $(2, 3)$:

$$d_2 = \sqrt{(-1-2)^2+(-3-3)^2} = \sqrt{(-3)^2+(-6)^2} = \sqrt{9+36} = \sqrt{45} = 3\sqrt{5}$$

For $(-4, -9)$ and $(2, 3)$:

$$d_3 = \sqrt{(-4-2)^2+(-9-3)^2} = \sqrt{(-6)^2+(-12)^2} = \sqrt{36+144} = \sqrt{180} = 6\sqrt{5}$$

Since $d_1 + d_2 = d_3$, the points are collinear.

127. a) When the circle is positioned on a coordinate system as shown in the text, the center lies on the y-axis and is equidistant from $(-4, 0)$ and $(0, 2)$.

Let $(0, y)$ be the coordinates of the center.

$$\sqrt{(-4-0)^2+(0-y)^2} = \sqrt{(0-0)^2+(2-y)^2}$$
$$4^2+y^2 = (2-y)^2$$
$$16+y^2 = 4-4y+y^2$$
$$12 = -4y$$
$$-3 = y$$

The center of the circle is $(0, -3)$.

b) Use the point $(-4, 0)$ and the center $(0, -3)$ to find the radius.

$$(-4-0)^2+[0-(-3)]^2 = r^2$$
$$25 = r^2$$
$$5 = r$$

The radius is 5 ft.

128. The coordinates of P are $\left(\frac{b}{2}, \frac{h}{2}\right)$ by the midpoint formula. By the distance formula, each of the distances from P to $(0, h)$, from P to $(0, 0)$, and from P to $(b, 0)$ is $\frac{\sqrt{b^2+h^2}}{2}$.

129.

$$x^2+y^2 = 1$$
$$\left(\frac{\sqrt{3}}{2}\right)^2+\left(-\frac{1}{2}\right)^2 \ ? \ 1$$
$$\frac{3}{4}+\frac{1}{4}$$
$$1 \ | \ 1 \quad \text{TRUE}$$

$\left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$ lies on the unit circle.

130.

$$x^2+y^2 = 1$$
$$0^2+(-1)^2 \ ? \ 1$$
$$1 \ | \ 1 \quad \text{TRUE}$$

$(0, -1)$ lies on the unit circle.

131.

$$x^2+y^2 = 1$$
$$\left(-\frac{\sqrt{2}}{2}\right)^2+\left(\frac{\sqrt{2}}{2}\right)^2 \ ? \ 1$$
$$\frac{2}{4}+\frac{2}{4}$$
$$1 \ | \ 1 \quad \text{TRUE}$$

$\left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$ lies on the unit circle.

132.

$$x^2+y^2 = 1$$
$$\left(\frac{1}{2}\right)^2+\left(-\frac{\sqrt{3}}{2}\right)^2 \ ? \ 1$$
$$\frac{1}{4}+\frac{3}{4}$$
$$1 \ | \ 1 \quad \text{TRUE}$$

$\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$ lies on the unit circle.

133. See the answer section in the text.

Exercise Set 1.2

1. This correspondence is a function, because each member of the domain corresponds to exactly one member of the range.

2. This correspondence is a function, because each member of the domain corresponds to exactly one member of the range.

3. This correspondence is a function, because each member of the domain corresponds to exactly one member of the range.

4. This correspondence is not a function, because there is a member of the domain (1) that corresponds to more than one member of the range (4 and 6).

5. This correspondence is not a function, because there is a member of the domain (m) that corresponds to more than one member of the range (A and B).

6. This correspondence is a function, because each member of the domain corresponds to exactly one member of the range.

7. This correspondence is a function, because each member of the domain corresponds to exactly one member of the range.

8. This correspondence is not a function, because there is a member of the domain that corresponds to more than one member of the range. In fact, Elton John, Claude-Michel Schönberg, and Andrew Lloyd Webber all correspond to two members of the range.

9. This correspondence is a function, because each car has exactly one license number.

10. This correspondence is not a function, because we can safely assume that at least one person uses more than one doctor.

11. This correspondence is a function, because each integer less than 9 corresponds to exactly one multiple of 5.

12. This correspondence is not a function, because we can safely assume that at least one band member plays more than one instrument.

13. This correspondence is not a function, because at least one student will have more than one neighboring seat occupied by another student.

14. This correspondence is a function, because each bag has exactly one weight.

15. The relation is a function, because no two ordered pairs have the same first coordinate and different second coordinates.

The domain is the set of all first coordinates: $\{2, 3, 4\}$.

The range is the set of all second coordinates: $\{10, 15, 20\}$.

16. The relation is a function, because no two ordered pairs have the same first coordinate and different second coordinates.

Domain: $\{3, 5, 7\}$

Range: $\{1\}$

17. The relation is not a function, because the ordered pairs $(-2, 1)$ and $(-2, 4)$ have the same first coordinate and different second coordinates.

The domain is the set of all first coordinates: $\{-7, -2, 0\}$.

The range is the set of all second coordinates: $\{3, 1, 4, 7\}$.

18. The relation is not a function, because each of the ordered pairs has the same first coordinate and different second coordinates.

Domain: $\{1\}$

Range: $\{3, 5, 7, 9\}$

19. The relation is a function, because no two ordered pairs have the same first coordinate and different second coordinates.

The domain is the set of all first coordinates: $\{-2, 0, 2, 4, -3\}$.

The range is the set of all second coordinates: $\{1\}$.

20. The relation is not a function, because the ordered pairs $(5, 0)$ and $(5, -1)$ have the same first coordinates and different second coordinates. This is also true of the pairs $(3, -1)$ and $(3, -2)$.

Domain: $\{5, 3, 0\}$

Range: $\{0, -1, -2\}$

21. $g(x) = 3x^2 - 2x + 1$

a) $g(0) = 3 \cdot 0^2 - 2 \cdot 0 + 1 = 1$

b) $g(-1) = 3(-1)^2 - 2(-1) + 1 = 6$

c) $g(3) = 3 \cdot 3^2 - 2 \cdot 3 + 1 = 22$

d) $g(-x) = 3(-x)^2 - 2(-x) + 1 = 3x^2 + 2x + 1$

e) $g(1 - t) = 3(1 - t)^2 - 2(1 - t) + 1 =$
$3(1-2t+t^2)-2(1-t)+1 = 3-6t+3t^2-2+2t+1 =$
$3t^2 - 4t + 2$

22. $f(x) = 5x^2 + 4x$

a) $f(0) = 5 \cdot 0^2 + 4 \cdot 0 = 0 + 0 = 0$

b) $f(-1) = 5(-1)^2 + 4(-1) = 5 - 4 = 1$

c) $f(3) = 5 \cdot 3^2 + 4 \cdot 3 = 45 + 12 = 57$

d) $f(t) = 5t^2 + 4t$

e) $f(t - 1) = 5(t - 1)^2 + 4(t - 1) = 5t^2 - 6t + 1$

23. $g(x) = x^3$

a) $g(2) = 2^3 = 8$

b) $g(-2) = (-2)^3 = -8$

c) $g(-x) = (-x)^3 = -x^3$

d) $g(3y) = (3y)^3 = 27y^3$

e) $g(2 + h) = (2 + h)^3 = 8 + 12h + 6h^2 + h^3$

24. $f(x) = 2|x| + 3x$

a) $f(1) = 2|1| + 3 \cdot 1 = 2 + 3 = 5$

b) $f(-2) = 2| - 2| + 3(-2) = 4 - 6 = -2$

c) $f(-x) = 2| - x| + 3(-x) = 2|x| - 3x$

d) $f(2y) = 2|2y| + 3 \cdot 2y = 4|y| + 6y$

e) $f(2 - h) = 2|2 - h| + 3(2 - h) =$
$2|2 - h| + 6 - 3h$

25. $g(x) = \dfrac{x - 4}{x + 3}$

a) $g(5) = \dfrac{5 - 4}{5 + 3} = \dfrac{1}{8}$

b) $g(4) = \dfrac{4 - 4}{4 + 7} = 0$

c) $g(-3) = \dfrac{-3-4}{-3+3} = \dfrac{-7}{0}$

Since division by 0 is not defined, $g(-3)$ does not exist.

d) $g(-16.25) = \dfrac{-16.25-4}{-16.25+3} = \dfrac{-20.25}{-13.25} = \dfrac{81}{53} \approx 1.5283$

e) $g(x+h) = \dfrac{x+h-4}{x+h+3}$

26. $f(x) = \dfrac{x}{2-x}$

a) $f(2) = \dfrac{2}{2-2} = \dfrac{2}{0}$

Since division by 0 is not defined, $f(2)$ does not exist.

b) $f(1) = \dfrac{1}{2-1} = 1$

c) $f(-16) = \dfrac{-16}{2-(-16)} = \dfrac{-16}{18} = -\dfrac{8}{9}$

d) $f(-x) = \dfrac{-x}{2-(-x)} = \dfrac{-x}{2+x}$

e) $f\left(-\dfrac{2}{3}\right) = \dfrac{-\frac{2}{3}}{2-\left(-\frac{2}{3}\right)} = \dfrac{-\frac{2}{3}}{\frac{8}{3}} = -\dfrac{1}{4}$

27. $g(x) = \dfrac{x}{\sqrt{1-x^2}}$

$g(0) = \dfrac{0}{\sqrt{1-0^2}} = \dfrac{0}{\sqrt{1}} = \dfrac{0}{1} = 0$

$g(-1) = \dfrac{-1}{\sqrt{1-(-1)^2}} = \dfrac{-1}{\sqrt{1-1}} = \dfrac{-1}{\sqrt{0}} = \dfrac{-1}{0}$

Since division by 0 is not defined, $g(-1)$ does not exist.

$g(5) = \dfrac{5}{\sqrt{1-5^2}} = \dfrac{5}{\sqrt{1-25}} = \dfrac{5}{\sqrt{-24}}$

Since $\sqrt{-24}$ is not defined as a real number, $g(5)$ does not exist as a real number.

$$g\left(\frac{1}{2}\right) = \frac{\frac{1}{2}}{\sqrt{1-\left(\frac{1}{2}\right)^2}} = \frac{\frac{1}{2}}{\sqrt{1-\frac{1}{4}}} = \frac{\frac{1}{2}}{\sqrt{\frac{3}{4}}} =$$

$$\frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{2}\cdot\frac{2}{\sqrt{3}} = \frac{1\cdot 2}{2\sqrt{3}} = \frac{1}{\sqrt{3}}, \text{ or } \frac{\sqrt{3}}{3}$$

28. $h(x) = x + \sqrt{x^2-1}$

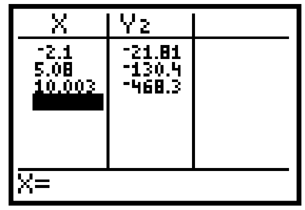
$h(0) = 0 + \sqrt{0^2-1} = 0 + \sqrt{-1}$

Since $\sqrt{-1}$ is not defined as a real number, $h(0)$ does not exist as a real number.

$h(2) = 2 + \sqrt{2^2-1} = 2 + \sqrt{3}$

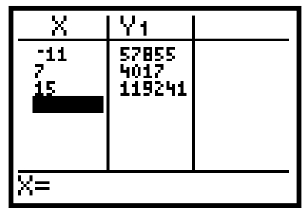
$h(-x) = -x + \sqrt{(-x)^2-1} = -x + \sqrt{x^2-1}$

29.



Rounding to the nearest tenth, we see that $g(-2.1) \approx -21.8$, $g(5.08) \approx -130.4$, and $g(10.003) \approx -468.3$.

30.

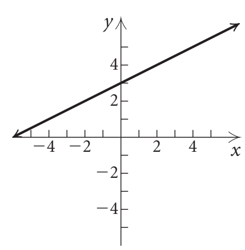


We see that $h(-11) = 57,885$, $h(7) = 4017$, and $h(15) = 119,241$.

31. Graph $f(x) = \dfrac{1}{2}x + 3$.

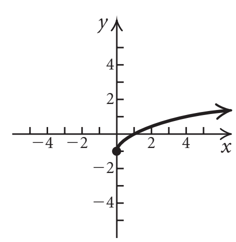
We select values for x and find the corresponding values of $f(x)$. Then we plot the points and connect them with a smooth curve.

x	$f(x)$	$(x, f(x))$
-4	1	$(-4, 1)$
0	3	$(0, 3)$
2	4	$(2, 4)$



$f(x) = \frac{1}{2}x + 3$

32.

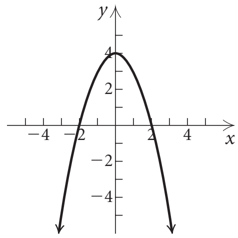


$f(x) = \sqrt{x} - 1$

33. Graph $f(x) = -x^2 + 4$.

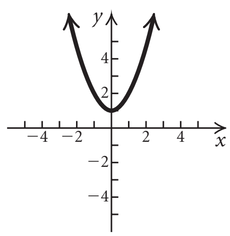
We select values for x and find the corresponding values of $f(x)$. Then we plot the points and connect them with a smooth curve.

x	$f(x)$	$(x, f(x))$
-3	-5	$(-3, -5)$
-2	0	$(-2, 0)$
-1	3	$(-1, 3)$
0	4	$(0, 4)$
1	3	$(1, 3)$
2	0	$(2, 0)$
3	-5	$(3, -5)$



$f(x) = -x^2 + 4$

34.

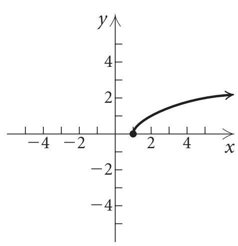


$f(x) = x^2 + 1$

35. Graph $f(x) = \sqrt{x-1}$.

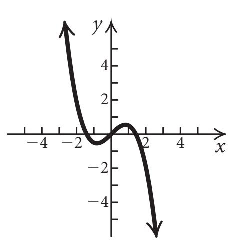
We select values for x and find the corresponding values of $f(x)$. Then we plot the points and connect them with a smooth curve.

x	$f(x)$	$(x, f(x))$
1	0	$(1, 0)$
2	1	$(2, 1)$
4	1.7	$(4, 1.7)$
5	2	$(5, 2)$



$f(x) = \sqrt{x - 1}$

36.



$f(x) = x - \frac{1}{2}x^3$

37. From the graph we see that, when the input is 1, the output is -2, so $h(1) = -2$. When the input is 3, the output is 2, so $h(3) = 2$. When the input is 4, the output is 1, so $h(4) = 1$.

38. $t(-4) = 3$; $t(0) = 3$; $t(3) = 3$

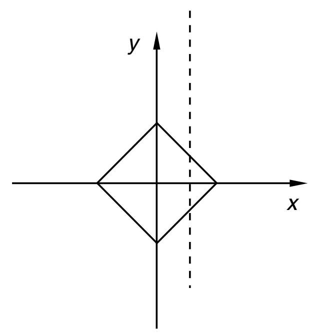
39. From the graph we see that, when the input is -4, the output is 3, so $s(-4) = 3$. When the input is -2, the output is 0, so $s(-2) = 0$. When the input is 0, the output is -3, so $s(0) = -3$.

40. $g(-4) = \frac{3}{2}$; $g(-1) = -3$; $g(0) = -\frac{5}{2}$

41. From the graph we see that, when the input is -1, the output is 2, so $f(-1) = 2$. When the input is 0, the output is 0, so $f(0) = 0$. When the input is 1, the output is -2, so $f(1) = -2$.

42. $g(-2) = 4$; $g(0) = -4$; $g(2.4) = -2.6176$

43. This is not the graph of a function, because we can find a vertical line that crosses the graph more than once.



44. This is not the graph of a function, because we can find a vertical line that crosses the graph more than once.

45. This is the graph of a function, because there is no vertical line that crosses the graph more than once.

46. This is the graph of a function, because there is no vertical line that crosses the graph more than once.

47. This is the graph of a function, because there is no vertical line that crosses the graph more than once.

48. This is the graph of a function, because there is no vertical line that crosses the graph more than once.

49. This is not the graph of a function, because we can find a vertical line that crosses the graph more than once.

50. This is not the graph of a function, because we can find a vertical line that crosses the graph more than once.

51. We can substitute any real number for x. Thus, the domain is the set of all real numbers, or $(-\infty, \infty)$.

52. We can substitute any real number for x. Thus, the domain is the set of all real numbers, or $(-\infty, \infty)$.

53. We can substitute any real number for x. Thus, the domain is the set of all real numbers, or $(-\infty, \infty)$.

54. The input 0 results in a denominator of 0. Thus, the domain is $\{x|x \neq 0\}$, or $(-\infty, 0) \cup (0, \infty)$.

55. The input 0 results in a denominator of 0. Thus, the domain is $\{x|x \neq 0\}$, or $(-\infty, 0) \cup (0, \infty)$.

56. We can substitute any real number for x. Thus, the domain is the set of all real numbers, or $(-\infty, \infty)$.

57. We can substitute any real number in the numerator, but we must avoid inputs that make the denominator 0. We find these inputs.

$$2 - x = 0$$
$$2 = x$$

The domain is $\{x|x \neq 2\}$, or $(-\infty, 2) \cup (2, \infty)$.

58. We find the inputs that make the denominator 0:

$$x + 4 = 0$$
$$x = -4$$

The domain is $\{x|x \neq -4\}$, or $(-\infty, -4) \cup (-4, \infty)$.

59. We find the inputs that make the denominator 0:

$$x^2 - 4x - 5 = 0$$
$$(x-5)(x+1) = 0$$
$$x - 5 = 0 \quad or \quad x + 1 = 0$$
$$x = 5 \quad or \quad x = -1$$

The domain is $\{x|x \neq 5 \text{ and } x \neq -1\}$, or $(-\infty, -1) \cup (-1, 5) \cup (5, \infty)$.

60. We can substitute any real number in the numerator, but the input 0 makes the denominator 0. Thus, the domain is $\{x|x \neq 0\}$, or $(-\infty, 0) \cup (0, \infty)$.

61. We can substitute any real number for x. Thus, the domain is the set of all real numbers, or $(-\infty, \infty)$.

62. We can substitute any real number for x. Thus, the domain is the set of all real numbers, or $(-\infty, \infty)$.

63. We can substitute any real number in the numerator, but we must avoid inputs that make the denominator 0. We find these inputs.

$$x^2 - 7x = 0$$
$$x(x-7) = 0$$
$$x = 0 \quad or \quad x - 7 = 0$$
$$x = 0 \quad or \quad x = 7$$

The domain is $\{x|x \neq 0 \text{ and } x \neq 7\}$, or $(-\infty, 0) \cup (0, 7) \cup (7, \infty)$.

64. We can substitute any real number in the numerator, but we must avoid inputs that make the denominator 0. We find these inputs.

$$3x^2 - 10x - 8 = 0$$
$$(3x+2)(x-4) = 0$$
$$3x + 2 = 0 \quad or \quad x - 4 = 0$$
$$3x = -2 \quad or \quad x = 4$$
$$x = -\frac{2}{3} \quad or \quad x = 4$$

The domain is $\left\{x \middle| x \neq -\frac{2}{3} \text{ and } x \neq 4\right\}$, or $\left(-\infty, -\frac{2}{3}\right) \cup \left(-\frac{2}{3}, 4\right) \cup (4, \infty)$.

65. We can substitute any real number for x. Thus, the domain is the set of all real numbers, or $(-\infty, \infty)$.

66. We can substitute any real number for x. Thus, the domain is the set of all real numbers, or $(-\infty, \infty)$.

67. The inputs on the x-axis that correspond to points on the graph extend from 0 to 5, inclusive. Thus, the domain is $\{x|0 \leq x \leq 5\}$, or $[0, 5]$.

The outputs on the y-axis extend from 0 to 3, inclusive. Thus, the range is $\{y|0 \leq y \leq 3\}$, or $[0, 3]$.

68. The inputs on the x-axis that correspond to points on the graph extend from -3 up to but not including 5. Thus, the domain is $\{x| -3 \leq x < 5\}$, or $[-3, 5)$.

The outputs on the y-axis extend from -4 up to but not including 1. Thus, the range is $\{y| -4 \leq y < 1\}$, or $[-4, 1)$.

69. The inputs on the x-axis that correspond to points on the graph extend from -2π to 2π inclusive. Thus, the domain is $\{x| -2\pi \leq x \leq 2\pi\}$, or $[-2\pi, 2\pi]$.

The outputs on the y-axis extend from -1 to 1, inclusive. Thus, the range is $\{y| -1 \leq y \leq 1\}$, or $[-1, 1]$.

70. The inputs on the x-axis that correspond to points on the graph extend from -2 to 1, inclusive. Thus, the domain is $\{x| -2 \leq x \leq 1\}$, or $[-2, 1]$.

The outputs on the y-axis extend from -1 to 4, inclusive. Thus, the range is $\{y| -1 \leq y \leq 4\}$, or $[-1, 4]$.

71. The graph extends to the left and to the right without bound. Thus, the domain is the set of all real numbers, or $(-\infty, \infty)$.

The only output is -3, so the range is $\{-3\}$.

72. The graph extends to the left and to the right without bound. Thus, the domain is the set of all real numbers, or $(-\infty, \infty)$.

The outputs on the y-axis start at -3 and increase without bound. Thus, the range is $[-3, \infty)$.

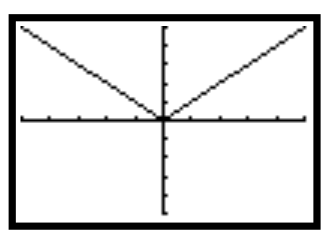
73. The inputs on the x-axis extend from -5 to 3, inclusive. Thus, the domain is $[-5, 3]$.

The outputs on the y-axis extend from -2 to 2, inclusive. Thus, the range is $[-2, 2]$.

74. The inputs on the x-axis extend from -2 to 4, inclusive. Thus, the domain is $[-2, 4]$.

The only output is 4. Thus, the range is $\{4\}$.

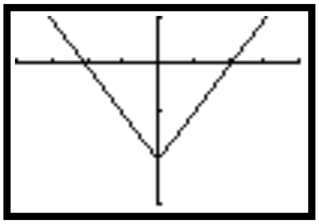
75.



To find the domain we look for the inputs on the x-axis that correspond to a point on the graph. We see that each point on the x-axis corresponds to a point on the graph so the domain is the set of all real numbers, or $(-\infty, \infty)$.

To find the range we look for outputs on the y-axis. The number 0 is the smallest output, and every number greater than 0 is also an output. Thus, the range is $[0, \infty)$.

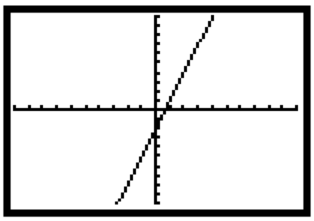
76.



Domain: all real numbers, or $(-\infty, \infty)$

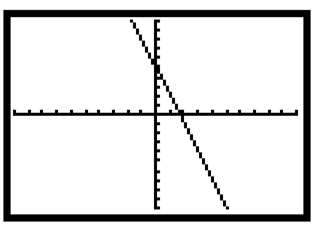
Range: $[-2, \infty)$

77.



We see that each point on the x-axis corresponds to a point on the graph so the domain is the set of all real numbers, or $(-\infty, \infty)$. We also see that each point on the y-axis corresponds to an output so the range is the set of all real numbers, or $(-\infty, \infty)$.

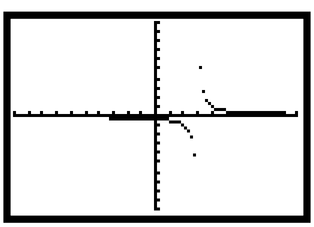
78.



Domain: all real numbers, or $(-\infty, \infty)$

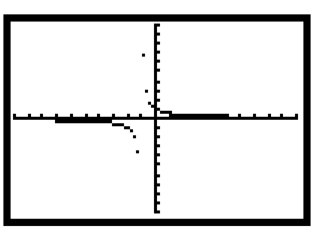
Range: all real numbers, or $(-\infty, \infty)$

79.



We see that each point on the x-axis except 3 corresponds to a point on the graph, so the domain is $(-\infty, 3) \cup (3, \infty)$. We also see that each point on the y-axis except 0 corresponds to an output, so the range is $(-\infty, 0) \cup (0, \infty)$.

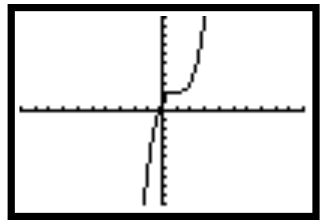
80.



Domain: $(-\infty, -1) \cup (-1, \infty)$

Range: $(-\infty, 0) \cup (0, \infty)$

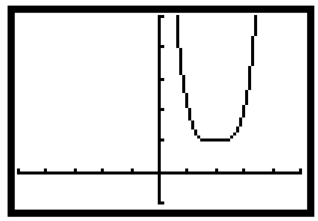
81.



Each point on the x-axis corresponds to a point on the graph, so the domain is the set of all real numbers, or $(-\infty, \infty)$.

Each point on the y-axis also corresponds to a point on the graph, so the range is the set of all real numbers, $(-\infty, \infty)$.

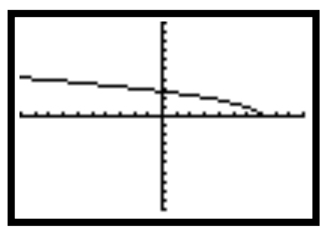
82.



Domain: all real numbers, or $(-\infty, \infty)$

Range: $[1, \infty)$

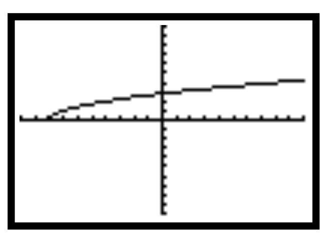
83.



The largest input on the x-axis is 7 and every number less than 7 is also an input. Thus, the domain is $(-\infty, 7]$.

The number 0 is the smallest output, and every number greater than 0 is also an output. Thus, the range is $[0, \infty)$.

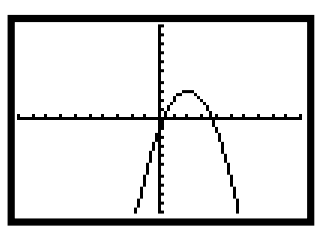
84.



Domain: $[-8, \infty)$

Range: $[0, \infty)$

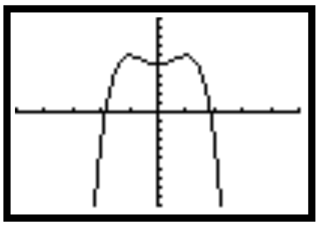
85.



Each point on the x-axis corresponds to a point on the graph, so the domain is the set of all real numbers, or $(-\infty, \infty)$.

The largest output is 3 and every number less than 3 is also an output. Thus, the range is $(-\infty, 3]$.

86.



Domain: all real numbers, or $(-\infty, \infty)$

Range: $(-\infty, 6]$

87. $E(t) = 1000(100 - t) + 580(100 - t)^2$

a) $E(99.5) = 1000(100-99.5)+580(100-99.5)^2$
$= 1000(0.5) + 580(0.5)^2$
$= 500 + 580(0.25) = 500 + 145$
$= 645$ m above sea level

b) $E(100) = 1000(100 - 100) + 580(100 - 100)^2$
$= 1000 \cdot 0 + 580(0)^2 = 0 + 0$
$= 0$ m above sea level, or at sea level

88. $P(15) = 0.015(15)^3 = 50.625$ watts per hour
$P(35) = 0.015(35)^3 = 643.125$ watts per hour

89. a) $V(33) = 0.4306(33) + 11.0043 \approx \$25.21$
$V(40) = 0.4306(40) + 11.0043 \approx \$28.23$

b) Substitute 32 for $V(x)$ and solve for x.

$$32 = 0.4306x + 11.0043$$
$$20.9957 = 0.4306x$$
$$49 \approx x$$

It will take approximately \$32 to equal the value of \$1 in 1913 about 49 years after 1985, or in 2034.

90. a) $P(30) = 2,511,040(30) + 151,143,509 = 226,474,709$
$P(70) = 2,511,040(70) + 151,143,509 = 326,916,309$

b) $400,000,000 = 2,511,143x + 151,143,509$
$248,856,491 = 2,511,143x$
$99 \approx x$

The population will be approximately 400,000,000 about 99 years after 1950, or in 2049.

91. For $(-3, -2)$:

$y^2 - x^2 = -5$	
$(-2)^2 - (-3)^2$?	-5
$4 - 9$	
-5	-5 TRUE

$(-3, -2)$ is a solution.

For $(2, -3)$:

$y^2 - x^2 = -5$	
$(-3)^2 - 2^2$?	-5
$9 - 4$	
5	-5 FALSE

$(2, -3)$ is not a solution.

92. For $\left(\frac{4}{5}, -2\right)$:

$15x - 10y = 32$	
$15 \cdot \frac{4}{5} - 10(-2)$?	32
$12 + 20$	
32	32 TRUE

$\left(\frac{4}{5}, -2\right)$ is a solution.

For $\left(\frac{11}{5}, \frac{1}{10}\right)$:

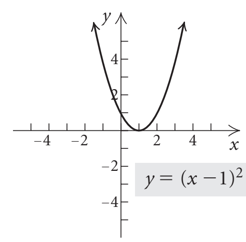
$15x - 10y = 32$	
$15 \cdot \frac{11}{5} - 10 \cdot \frac{1}{10}$?	32
$33 - 1$	
32	32 TRUE

$\left(\frac{11}{5}, \frac{1}{10}\right)$ is a solution.

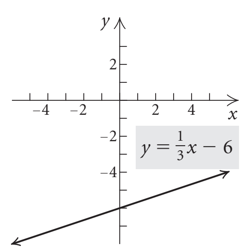
93. Graph $y = (x - 1)^2$.

Make a table of values, plot the points in the table, and draw the graph.

x	y	(x, y)
-1	4	$(-1, 4)$
0	1	$(0, 1)$
1	0	$(1, 0)$
2	1	$(2, 1)$
3	4	$(3, 4)$



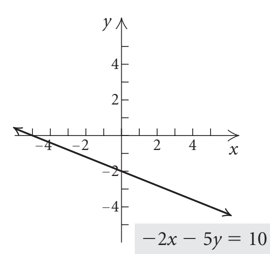
94.



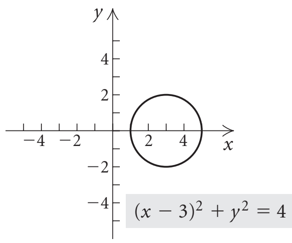
95. Graph $-2x - 5y = 10$.

Make a table of values, plot the points in the table, and draw the graph.

x	y	(x, y)
-5	0	$(-5, 0)$
0	-2	$(0, -2)$
5	-4	$(5, -4)$



96.



97. We find the inputs for which $2x+5$ is nonnegative.

$$2x+5 \geq 0$$
$$2x \geq -5$$
$$x \geq -\frac{5}{2}$$

Thus, the domain is $\left\{x \middle| x \geq -\frac{5}{2}\right\}$, or $\left[-\frac{5}{2}, \infty\right)$.

98. In the numerator we can substitute any real number for which the radicand is nonnegative. We see that $x+1 \geq 0$ for $x \geq -1$. The denominator is 0 when $x = 0$, so 0 cannot be an input. Thus the domain is $\{x|x \geq -1 \textit{ and } x \neq 0\}$, or $[-1, 0) \cup (0, \infty)$.

99. $\sqrt{x+6}$ is not defined for values of x for which $x+6$ is negative. We find the inputs for which $x+6$ is nonnegative.

$$x+6 \geq 0$$
$$x \geq -6$$

We must also avoid inputs that make the denominator 0.

$$(x+2)(x-3) = 0$$
$$x+2 = 0 \quad or \quad x-3 = 0$$
$$x = -2 \quad or \quad x = 3$$

Then the domain is $\{x|x \geq -6 \textit{ and } x \neq -2 \textit{ and } x \neq 3\}$, or $[-6, -2) \cup (-2, 3) \cup (3, \infty)$.

100. $\sqrt{x}$ is defined for $x \geq 0$.

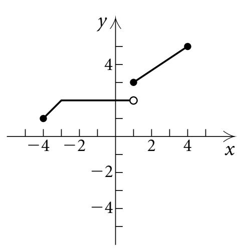
We find the inputs for which $4-x$ is nonnegative.

$$4-x \geq 0$$
$$4 \geq x, \text{ or } x \leq 4$$

The domain is $\{x|0 \leq x \leq 4\}$, or $[0, 4]$.

101. Answers may vary. Two possibilities are $f(x) = x$, $g(x) = x+1$ and $f(x) = x^2$, $g(x) = x^2 - 4$.

102.



103. First find the value of x for which $x+3 = -1$.

$$x+3 = -1$$
$$x = -4$$

Then we have:

$$g(x+3) = 2x+1$$
$$g(-1) = g(-4+3) = 2(-4)+1 = -8+1 = -7$$

104. $f(x) = |x+3| - |x-4|$

a) If x is in the interval $(-\infty, -3)$, then $x+3 < 0$ and $x-4 < 0$. We have:

$$\begin{aligned} f(x) &= |x+3| - |x-4| \\ &= -(x+3) - [-(x-4)] \\ &= -(x+3) - (-x+4) \\ &= -x-3+x-4 \\ &= -7 \end{aligned}$$

b) If x is in the interval $[-3, 4)$, then $x+3 \geq 0$ and $x-4 < 0$. We have:

$$\begin{aligned} f(x) &= |x+3| - |x-4| \\ &= x+3 - [-(x-4)] \\ &= x+3 - (-x+4) \\ &= x+3+x-4 \\ &= 2x-1 \end{aligned}$$

c) If x is in the interval $[4, \infty)$, then $x+3 > 0$ and $x-4 \geq 0$. We have:

$$\begin{aligned} f(x) &= |x+3| - |x-4| \\ &= x+3 - (x-4) \\ &= x+3-x+4 \\ &= 7 \end{aligned}$$

Exercise Set 1.3

1. a) Yes. Each input is 1 more than the one that precedes it.

b) Yes. Each output is 3 more than the one that precedes it.

c) Yes. Constant changes in inputs result in constant changes in outputs.

2. a) Yes. Each input is 10 more than the one that precedes it.

b) No. The change in the outputs varies.

c) No. Constant changes in inputs do not result in constant changes in outputs.

3. a) Yes. Each input is 15 more than the one that precedes it.

b) No. The change in the outputs varies.

c) No. Constant changes in inputs do not result in constant changes in outputs.

4. a) Yes. Each input is 2 more than the one that precedes it.

b) Yes. Each output is 4 less than the one that precedes it.

c) Yes. Constant changes in inputs result in constant changes in outputs.

5. Two points on the line are $(-4, -2)$ and $(1, 4)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4-(-2)}{1-(-4)} = \frac{6}{5}$$

6. $m = \dfrac{-5-1}{3-(-3)} = \dfrac{-6}{6} = -1$

7. Two points on the line are $(0, 3)$ and $(5, 0)$.

$m = \dfrac{y_2-y_1}{x_2-x_1} = \dfrac{0-3}{5-0} = \dfrac{-3}{5}$, or $-\dfrac{3}{5}$

8. $m = \dfrac{0-(-3)}{-2-(-2)} = \dfrac{3}{0}$

The slope is not defined.

9. $m = \dfrac{y_2-y_1}{x_2-x_1} = \dfrac{3-3}{3-0} = \dfrac{0}{3} = 0$

10. $m = \dfrac{1-(-4)}{5-(-3)} = \dfrac{5}{8}$

11. $m = \dfrac{y_2-y_1}{x_2-x_1} = \dfrac{2-4}{-1-9} = \dfrac{-2}{-10} = \dfrac{1}{5}$

12. $m = \dfrac{-1-7}{5-(-3)} = \dfrac{-8}{8} = -1$

13. $m = \dfrac{y_2-y_1}{x_2-x_1} = \dfrac{6-(-9)}{4-4} = \dfrac{15}{0}$

Since division by 0 is not defined, the slope is not defined.

14. $m = \dfrac{-13-(-1)}{2-(-6)} = \dfrac{-12}{8} = -\dfrac{3}{2}$

15. $m = \dfrac{y_2-y_1}{x_2-x_1} = \dfrac{-0.4-(-0.1)}{-0.3-0.7} = \dfrac{-0.3}{-1} = 0.3$

16. $m = \dfrac{-\frac{5}{7}-\left(-\frac{1}{4}\right)}{\frac{2}{7}-\left(-\frac{3}{4}\right)} = \dfrac{-\frac{20}{28}+\frac{7}{28}}{\frac{8}{28}+\frac{21}{28}} = \dfrac{-\frac{13}{28}}{\frac{29}{28}} =$

$-\dfrac{13}{28} \cdot \dfrac{28}{29} = -\dfrac{13}{29}$

17. $m = \dfrac{y_2-y_1}{x_2-x_1} = \dfrac{-2-(-2)}{4-2} = \dfrac{0}{2} = 0$

18. $m = \dfrac{-6-8}{7-(-9)} = \dfrac{-14}{16} = -\dfrac{7}{8}$

19. $m = \dfrac{y_2-y_1}{x_2-x_1} = \dfrac{\frac{3}{5}-\left(-\frac{3}{5}\right)}{-\frac{1}{2}-\frac{1}{2}} = \dfrac{\frac{6}{5}}{-1} = -\dfrac{6}{5}$

20. $m = \dfrac{-2.16-4.04}{3.14-(-8.26)} = \dfrac{-6.2}{11.4} = -\dfrac{62}{114} = -\dfrac{31}{57}$

21. $m = \dfrac{y_2-y_1}{x_2-x_1} = \dfrac{-5-(-13)}{-8-16} = \dfrac{8}{-24} = -\dfrac{1}{3}$

22. $m = \dfrac{y_2-y_1}{x_2-x_1} = \dfrac{2-(-3)}{\pi-\pi} = \dfrac{5}{0}$

The slope is not defined.

23. $m = \dfrac{7-(-7)}{-10-(-10)} = \dfrac{14}{0}$

Since division by 0 is not defined, the slope is not defined.

24. $m = \dfrac{-4-(-4)}{0.56-\sqrt{2}} = \dfrac{0}{0.56-\sqrt{2}} = 0$

25. We have the points $(4, 3)$ and $(-2, 15)$.

$m = \dfrac{y_2-y_1}{x_2-x_1} = \dfrac{15-3}{-2-4} = \dfrac{12}{-6} = -2$

26. $m = \dfrac{-5-1}{-4-4} = \dfrac{-6}{-8} = \dfrac{3}{4}$

27. We have the points $\left(\dfrac{1}{5}, \dfrac{1}{2}\right)$ and $\left(-1, -\dfrac{11}{2}\right)$.

$m = \dfrac{y_2-y_1}{x_2-x_1} = \dfrac{-\frac{11}{2}-\frac{1}{2}}{-1-\frac{1}{5}} = \dfrac{-6}{-\frac{6}{5}} = -6 \cdot \left(-\dfrac{5}{6}\right) = 5$

28. $m = \dfrac{\frac{10}{3}-(-1)}{-\frac{2}{3}-8} = \dfrac{\frac{13}{3}}{-\frac{26}{3}} = \dfrac{13}{3} \cdot \left(-\dfrac{3}{26}\right) = -\dfrac{1}{2}$

29. We have the points $\left(-6, \dfrac{4}{5}\right)$ and $\left(0, \dfrac{4}{5}\right)$.

$m = \dfrac{y_2-y_1}{x_2-x_1} = \dfrac{\frac{4}{5}-\frac{4}{5}}{-6-0} = \dfrac{0}{-6} = 0$

30. $m = \dfrac{-\frac{5}{2}-\frac{2}{9}}{\frac{2}{5}-\left(-\frac{9}{2}\right)} = \dfrac{-\frac{49}{18}}{\frac{49}{10}} = -\dfrac{49}{18} \cdot \dfrac{10}{49} = -\dfrac{5}{9}$

31. $y = 1.3x - 5$ is in the form $y = mx + b$ with $m = 1.3$, so the slope is 1.3.

32. $-\dfrac{2}{5}$

33. The graph of $x = -2$ is a vertical line, so the slope is not defined.

34. 4

35. $f(x) = -\dfrac{1}{2}x + 3$ is in the form $y = mx + b$ with $m = -\dfrac{1}{2}$, so the slope is $-\dfrac{1}{2}$.

36. The graph of $y = \dfrac{3}{4}$ is a horizontal line, so the slope is 0. (We also see this if we write the equation in the form $y = 0x + \dfrac{3}{4}$.)

37. $y = 9 - x$ can be written as $y = -x + 9$, or $y = -1 \cdot x + 9$. Now we have an equation in the form $y = mx + b$ with $m = -1$, so the slope is -1.

38. The graph of $x = 8$ is a vertical line, so the slope is not defined.

39. The graph of $y = 0.7$ is a horizontal line, so the slope is 0. (We also see this if we write the equation in the form $y = 0x + 0.7$).

40. $y = \dfrac{4}{5} - 2x$, or $y = -2x + \dfrac{4}{5}$

The slope is -2.

41. We have the points (1946, 1.20) and (2012, 110). We find the average rate of change, or slope.

$m = \frac{110 - 1.20}{2012 - 1946} = \frac{108.8}{66} \approx 1.65$

The average rate of change in the lowest price of a World Series ticket from 1946 to 2012 was about \$1.65 per year.

42. $m = \frac{688,701 - 1,027,974}{2013 - 1990} = \frac{-339,273}{23} = -14,751$

The average rate of change in the population in Detroit, Michigan, over the 23-year period was $-14,751$ people per year.

43. We have the data points (2000, 478,403) and (2013, 390,113). We find the average rate of change, or slope.

$m = \frac{390\,113 - 478,403}{2013 - 2000} = \frac{-88,290}{13} \approx -6792$

The average rate of change in the population of Cleveland, Ohio, over the 13-year period was about -6792 people per year.

44. $m = \frac{87.7 - 96.6}{2014 - 2006} = \frac{-8.9}{8} \approx -1.1$

The average rate of change in the number of cattle in the U.S. from 2006 to 2014 was a decrease of about 1.1 million per year. (We could also say that the average rate of change was about -1.1 million per year.)

45. We have the data points (1970, 25.3) and (2011, 5.5). We find the average rate of change, or slope.

$m = \frac{5.5 - 25.3}{2011 - 1970} = \frac{-19.8}{41} \approx -0.5$

The average rate of change in the per capita consumption of whole milk from 1970 to 2011 was about -0.5 gallons per year.

46. $m = \frac{58.4 - 42.5}{2011 - 1990} = \frac{15.9}{21} \approx 0.8$

The average rate of change in per capita consumption of chicken from 1990 to 2011 was about 0.8 lb per year.

47. We have the data points (2003, 550,000) and (2012, 810,000). We find the average rate of change, or slope.

$m = \frac{810,000 - 550,000}{2012 - 2003} = \frac{260,000}{9} \approx 28,889$

The average rate of change in the number of acres used for growing almonds in California from 2003 to 2012 was about 28,889 acres per year.

48. $m = \frac{4.35 - 2.66}{2014 - 2004} = \frac{1.69}{10} \approx 0.17$

The average rate of change in the average fee to use an out-of-network ATM from 2004 to 2014 was about \$0.17 per year.

49. $y = \frac{3}{5}x - 7$

The equation is in the form $y = mx + b$ where $m = \frac{3}{5}$ and $b = -7$. Thus, the slope is $\frac{3}{5}$, and the y-intercept is $(0, -7)$.

50. $f(x) = -2x + 3$

Slope: -2; y-intercept: $(0, 3)$

51. $x = -\frac{2}{5}$

This is the equation of a vertical line $\frac{2}{5}$ unit to the left of the y-axis. The slope is not defined, and there is no y-intercept.

52. $y = \frac{4}{7} = 0 \cdot x + \frac{4}{7}$

Slope: 0; y-intercept: $\left(0, \frac{4}{7}\right)$

53. $f(x) = 5 - \frac{1}{2}x$, or $f(x) = -\frac{1}{2}x + 5$

The second equation is in the form $y = mx + b$ where $m = -\frac{1}{2}$ and $b = 5$. Thus, the slope is $-\frac{1}{2}$ and the y-intercept is $(0, 5)$.

54. $y = 2 + \frac{3}{7}x$

Slope: $\frac{3}{7}$; y-intercept: $(0, 2)$

55. Solve the equation for y.

$$\begin{aligned} 3x + 2y &= 10 \\ 2y &= -3x + 10 \\ y &= -\frac{3}{2}x + 5 \end{aligned}$$

Slope: $-\frac{3}{2}$; y-intercept: $(0, 5)$

56.
$$\begin{aligned} 2x - 3y &= 12 \\ -3y &= -2x + 12 \\ y &= \frac{2}{3}x - 4 \end{aligned}$$

Slope: $\frac{2}{3}$; y-intercept: $(0, -4)$

57. $y = -6 = 0 \cdot x - 6$

Slope: 0; y-intercept: $(0, -6)$

58. $x = 10$

This is the equation of a vertical line 10 units to the right of the y-axis. The slope is not defined, and there is no y-intercept.

59. Solve the equation for y.

$$\begin{aligned} 5y - 4x &= 8 \\ 5y &= 4x + 8 \\ y &= \frac{4}{5}x + \frac{8}{5} \end{aligned}$$

Slope: $\frac{4}{5}$; y-intercept: $\left(0, \frac{8}{5}\right)$

60.
$$\begin{aligned} 5x - 2y + 9 &= 0 \\ -2y &= -5x - 9 \\ y &= \frac{5}{2}x + \frac{9}{2} \end{aligned}$$

Slope: $\frac{5}{2}$; y-intercept: $\left(0, \frac{9}{2}\right)$

61. Solve the equation for y.

$$4y - x + 2 = 0$$
$$4y = x - 2$$
$$y = \frac{1}{4}x - \frac{1}{2}$$

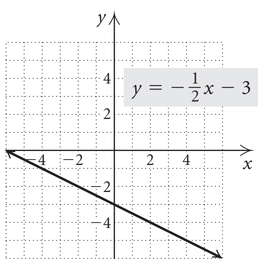
Slope: $\frac{1}{4}$; y-intercept: $\left(0, -\frac{1}{2}\right)$

62. $f(x) = 0.3 + x$; or $f(x) = x + 0.3$

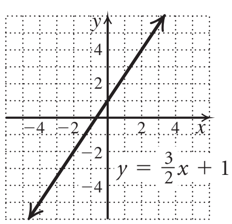
Slope: 1; y-intercept: $(0, 0.3)$

63. Graph $y = -\frac{1}{2}x - 3$.

Plot the y-intercept, $(0, -3)$. We can think of the slope as $\frac{-1}{2}$. Start at $(0, -3)$ and find another point by moving down 1 unit and right 2 units. We have the point $(2, -4)$. We could also think of the slope as $\frac{1}{-2}$. Then we can start at $(0, -3)$ and get another point by moving up 1 unit and left 2 units. We have the point $(-2, -2)$. Connect the three points to draw the graph.

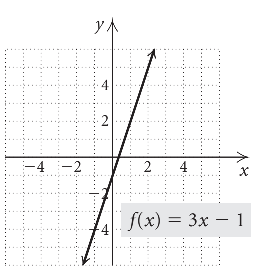


64.

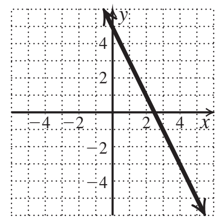


65. Graph $f(x) = 3x - 1$.

Plot the y-intercept, $(0, -1)$. We can think of the slope as $\frac{3}{1}$. Start at $(0, -1)$ and find another point by moving up 3 units and right 1 unit. We have the point $(1, 2)$. We can move from the point $(1, 2)$ in a similar manner to get a third point, $(2, 5)$. Connect the three points to draw the graph.



66.

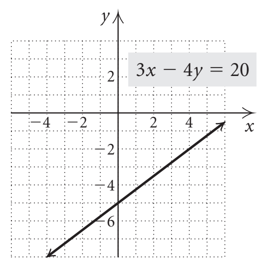


$f(x) = -2x + 5$

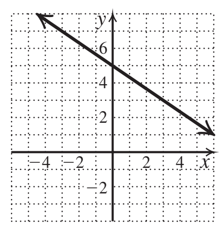
67. First solve the equation for y.

$$3x - 4y = 20$$
$$-4y = -3x + 20$$
$$y = \frac{3}{4}x - 5$$

Plot the y-intercept, $(0, -5)$. Then using the slope, $\frac{3}{4}$, start at $(0, -5)$ and find another point by moving up 3 units and right 4 units. We have the point $(4, -2)$. We can move from the point $(4, -2)$ in a similar manner to get a third point, $(8, 1)$. Connect the three points to draw the graph.



68.

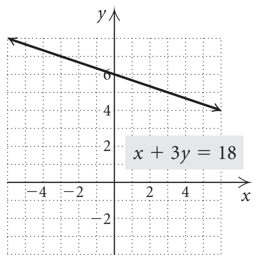


$2x + 3y = 15$

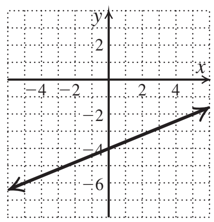
69. First solve the equation for y.

$$x + 3y = 18$$
$$3y = -x + 18$$
$$y = -\frac{1}{3}x + 6$$

Plot the y-intercept, $(0, 6)$. We can think of the slope as $\frac{-1}{3}$. Start at $(0, 6)$ and find another point by moving down 1 unit and right 3 units. We have the point $(3, 5)$. We can move from the point $(3, 5)$ in a similar manner to get a third point, $(6, 4)$. Connect the three points and draw the graph.

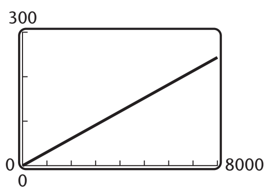


70.



$5y - 2x = -20$

71. a) $y = \frac{1}{33}x + 1$



b) $P(0) = \frac{1}{33} \cdot 0 + 1 = 1$ atm

$P(33) = \frac{1}{33} \cdot 33 + 1 = 2$ atm

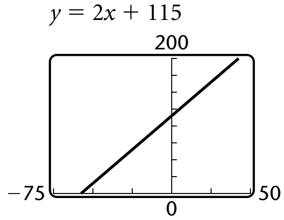
$P(1000) = \frac{1}{33} \cdot 1000 + 1 = 31\frac{10}{33}$ atm

$P(5000) = \frac{1}{33} \cdot 5000 + 1 = 152\frac{17}{33}$ atm

$P(7000) = \frac{1}{33} \cdot 7000 + 1 = 213\frac{4}{33}$ atm

72. $D(F) = 2F + 115$

a) $y = 2x + 115$



b) $D(0) = 2 \cdot 0 + 115 = 115$ ft

$D(-20) = 2(-20) + 115 = -40 + 115 = 75$ ft

$D(10) = 2 \cdot 10 + 115 = 20 + 115 = 135$ ft

$D(32) = 2 \cdot 32 + 115 = 64 + 115 = 179$ ft

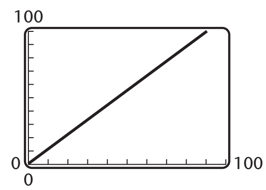
c) Below $-57.5°$, stopping distance is negative; above $32°$, ice doesn't form. The domain should be restricted to $[-57.5°, 32°]$.

73. a) $D(r) = \frac{11}{10}r + \frac{1}{2}$

The slope is $\frac{11}{10}$.

For each mph faster the car travels, it takes $\frac{11}{10}$ ft longer to stop.

b) $y = \frac{11}{10}x + \frac{1}{2}$



c) $D(5) = \frac{11}{10} \cdot 5 + \frac{1}{2} = \frac{11}{2} + \frac{1}{2} = \frac{12}{2} = 6$ ft

$D(10) = \frac{11}{10} \cdot 10 + \frac{1}{2} = 11 + \frac{1}{2} = 11\frac{1}{2}$, or 11.5 ft

$D(20) = \frac{11}{10} \cdot 20 + \frac{1}{2} = 22 + \frac{1}{2} = 22\frac{1}{2}$, or 22.5 ft

$D(50) = \frac{11}{10} \cdot 50 + \frac{1}{2} = 55 + \frac{1}{2} = 55\frac{1}{2}$, or 55.5 ft

$D(65) = \frac{11}{10} \cdot 65 + \frac{1}{2} = \frac{143}{2} + \frac{1}{2} = \frac{144}{2} = 72$ ft

d) The speed cannot be negative. $D(0) = \frac{1}{2}$ which says that a stopped car travels $\frac{1}{2}$ ft before stopping. Thus, 0 is not in the domain. The speed can be positive, so the domain is $\{r|r > 0\}$, or $(0, \infty)$.

74. $V(t) = \$38,000 - \$4300t$

a) $V(0) = \$38,000 - \$4300 \cdot 0 = \$38,000$

$V(1) = \$38,000 - \$4300 \cdot 1 = \$33,700$

$V(2) = \$38,000 - \$4300 \cdot 2 = \$29,400$

$V(3) = \$38,000 - \$4300 \cdot 3 = \$25,100$

$V(5) = \$38,000 - \$4300 \cdot 5 = \$16,500$

b) Since the time must be nonnegative and not more than 5 years, the domain is $[0, 5]$. The value starts at \$38,000 and declines to \$16,500, so the range is $[16,500,\ 38,000]$.

75. $C(t) = 2250 + 3380t$

$C(20) = 2250 + 3380 \cdot 20 = \$69,850$

76. $C(t) = 95 + 125t$

$C(18) = 95 + 125(18) = \$2345$

77. $C(x) = 750 + 15x$

$C(32) = 750 + 15 \cdot 32 = \$1230$

78. $C(x) = 1250 + 4.25x$

$C(85) = 1250 + 4.25(85) = \$1611.25$

79. $f(x) = x^2 - 3x$

$f\left(\frac{1}{2}\right) = \left(\frac{1}{2}\right)^2 - 3 \cdot \frac{1}{2} = \frac{1}{4} - \frac{3}{2} = -\frac{5}{4}$

80. $f(5) = 5^2 - 3 \cdot 5 = 10$

81. $f(x) = x^2 - 3x$

$f(-5) = (-5)^2 - 3(-5) = 25 + 15 = 40$

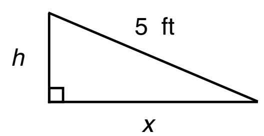
82. $f(x) = x^2 - 3x$

$f(-a) = (-a)^2 - 3(-a) = a^2 + 3a$

83. $f(x) = x^2 - 3x$

$f(a+h) = (a+h)^2 - 3(a+h) = a^2 + 2ah + h^2 - 3a - 3h$

84. We make a drawing and label it. Let h = the height of the triangle, in feet.



Using the Pythagorean theorem we have:

$$x^2 + h^2 = 25$$
$$x^2 = 25 - h^2$$
$$x = \sqrt{25 - h^2}$$

We know that the grade of the treadmill is 8%, or 0.08. Then we have

$$\frac{h}{x} = 0.08$$
$$\frac{h}{\sqrt{25-h^2}} = 0.08 \quad \text{Substituting } \sqrt{25-h^2} \text{ for } x$$
$$\frac{h^2}{25-h^2} = 0.0064 \quad \text{Squaring both sides}$$
$$h^2 = 0.16 - 0.0064h^2$$
$$1.0064h^2 = 0.16$$
$$h^2 = \frac{0.16}{1.0064}$$
$$h \approx 0.4 \text{ ft}$$

85. $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{(a+h)^2 - a^2}{a+h-a} = \dfrac{a^2+2ah+h^2-a^2}{h} =$

$\dfrac{2ah+h^2}{h} = \dfrac{h(2a+h)}{h} = 2a + h$

86. $m = \dfrac{s-(s+t)}{r-r} = \dfrac{s-s-t}{0}$

The slope is not defined.

87. False. For example, let $f(x) = x + 1$. Then $f(c-d) = c - d + 1$, but $f(c) - f(d) = c + 1 - (d+1) = c - d$.

88. False. For example, let $f(x) = x+1$. Then $f(kx) = kx+1$, but $kf(x) = k(x+1) = kx + k \neq kx + 1$ for $k \neq 1$.

89.

$$f(x) = mx + b$$
$$f(x+2) = f(x) + 2$$
$$m(x+2) + b = mx + b + 2$$
$$mx + 2m + b = mx + b + 2$$
$$2m = 2$$
$$m = 1$$

Thus, $f(x) = 1 \cdot x + b$, or $f(x) = x + b$.

90.

$$3mx + b = 3(mx + b)$$
$$3mx + b = 3mx + 3b$$
$$b = 3b$$
$$0 = 2b$$
$$0 = b$$

Thus, $f(x) = mx + 0$, or $f(x) = mx$.

Chapter 1 Mid-Chapter Mixed Review

1. The statement is false. The x-intercept of a line that passes through the origin is $(0, 0)$.

2. The statement is true. See the definitions of a function and a relation on pages 17 and 19, respectively.

3. The statement is false. The line parallel to the y-axis that passes through $(-5, 25)$ is $x = -5$.

4. To find the x-intercept we replace y with 0 and solve for x.

$$-8x + 5y = -40$$
$$-8x + 5 \cdot 0 = -40$$
$$-8x = -40$$
$$x = 5$$

The x-intercept is $(5, 0)$.

To find the y-intercept we replace x with 0 and solve for y.

$$-8x + 5y = -40$$
$$-8 \cdot 0 + 5y = -40$$
$$5y = -40$$
$$y = -8$$

The y-intercept is $(0, -8)$.

5. Distance:

$$d = \sqrt{(-8-3)^2 + (-15-7)^2}$$
$$= \sqrt{(-11)^2 + (-22)^2}$$
$$= \sqrt{121 + 484}$$
$$= \sqrt{605} \approx 24.6$$

Midpoint: $\left(\dfrac{-8+3}{2}, \dfrac{-15+7}{2}\right) = \left(\dfrac{-5}{2}, \dfrac{-8}{2}\right) =$

$\left(-\dfrac{5}{2}, -4\right)$

6. Distance:

$$d = \sqrt{\left(-\frac{3}{4} - \frac{1}{4}\right)^2 + \left[\frac{1}{5} - \left(-\frac{4}{5}\right)\right]^2}$$
$$= \sqrt{(-1)^2 + 1^2} = \sqrt{1+1}$$
$$= \sqrt{2} \approx 1.4$$

Midpoint: $\left(\dfrac{-\frac{3}{4} + \frac{1}{4}}{2}, \dfrac{\frac{1}{5} + \left(-\frac{4}{5}\right)}{2}\right) = \left(\dfrac{-\frac{1}{2}}{2}, \dfrac{-\frac{3}{5}}{2}\right) =$

$\left(-\dfrac{1}{4}, -\dfrac{3}{10}\right)$

7.

$$(x-h)^2+(y-k)^2=r^2$$
$$(x-(-5))^2+(y-2)^2=13^2$$
$$(x+5)^2+(y-2)^2=169$$

8.

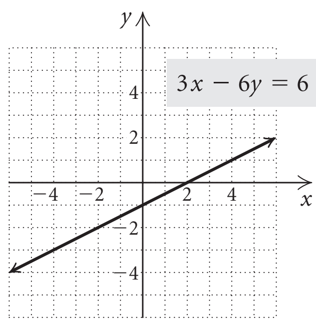
$$(x-3)^2+(y+1)^2=4$$
$$(x-3)^2+(y-(-1))^2=2^2$$

Center: $(3,-1)$; radius: 2

9. Graph $3x-6y=6$.

We will find the intercepts along with a third point on the graph. Make a table of values, plot the points, and draw the graph.

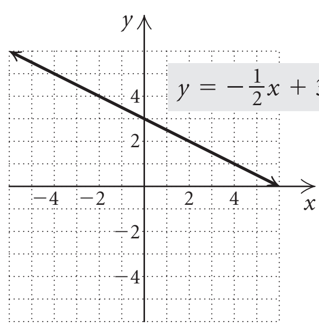
x	y	(x,y)
2	0	$(2,0)$
0	-1	$(0,-1)$
4	1	$(4,1)$



10. Graph $y=-\dfrac{1}{2}x+3$.

We choose some values for x and find the corresponding y-values. We list these points in a table, plot them, and draw the graph.

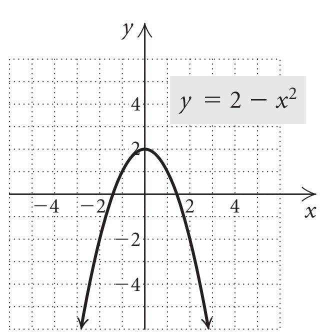
x	y	(x,y)
-2	4	$(-2,4)$
0	3	$(0,3)$
2	2	$(2,2)$



11. Graph $y=2-x^2$.

We choose some values for x and find the corresponding y-values. We list these points in a table, plot them, and draw the graph.

x	y	(x,y)
-2	-2	$(-2,-2)$
-1	1	$(-1,1)$
0	2	$(0,2)$
1	1	$(1,1)$
2	-2	$(2,-2)$

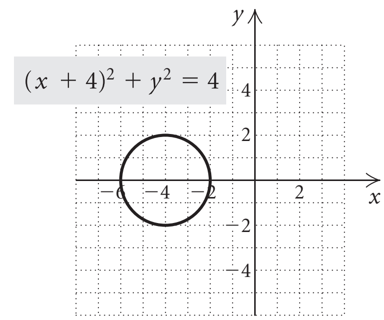


12. Graph $(x+4)^2+y^2=4$.

This is an equation of a circle. We write it in standard form.

$$(x-(-4))^2+(y-0)^2=2^2$$

The center is $(-4,0)$, and the radius is 2. We draw the graph.



13. $f(x)=x-2x^2$

$f(-4)=-4-2(-4)^2=-4-2\cdot 16=-4-32=-36$

$f(0)=0-2\cdot 0^2=0-0=0$

$f(1)=1-2\cdot 1^2=1-2\cdot 1=1-2=-1$

14. $g(x)=\dfrac{x+6}{x-3}$

$g(-6)=\dfrac{-6+6}{-6-3}=\dfrac{0}{-9}=0$

$g(0)=\dfrac{0+6}{0-3}=\dfrac{6}{-3}=-2$

$g(3)=\dfrac{3+6}{3-3}=\dfrac{9}{0}$

Since division by 0 is not defined, $g(3)$ does not exist.

15. We can substitute any real number for x. Thus, the domain is the set of all real numbers, or $(-\infty,\infty)$.

16. We find the inputs for which the denominator is 0.

$$x+5=0$$
$$x=-5$$

The domain is $\{x|x\neq -5\}$, or $(-\infty,-5)\cup(-5,\infty)$.

17. We find the inputs for which the denominator is 0.

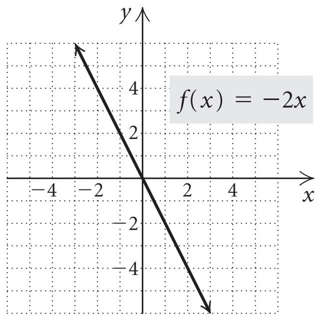
$$x^2+2x-3=0$$
$$(x+3)(x-1)=0$$
$$x+3=0 \quad \textit{or} \quad x-1=0$$
$$x=-3 \quad \textit{or} \quad x=1$$

The domain is $\{x|x\neq -3 \textit{ and } x\neq 1\}$, or $(-\infty,-3)\cup(-3,1)\cup(1,\infty)$.

18. Graph $f(x)=-2x$.

Make a table of values, plot the points in the table, and draw the graph.

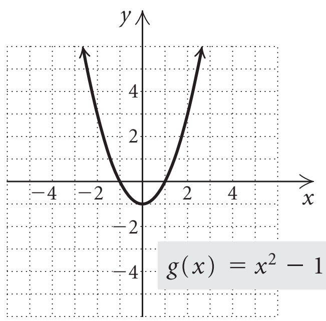
x	$f(x)$	$(x, f(x))$
-2	4	$(-2, 4)$
0	0	$(0, 0)$
2	-4	$(2, -4)$



19. Graph $g(x) = x^2 - 1$.

Make a table of values, plot the points in the table, and draw the graph.

x	$g(x)$	$(x, g(x))$
-2	3	$(-2, 3)$
-1	0	$(-1, 0)$
0	-1	$(0, -1)$
1	0	$(1, 0)$
2	3	$(2, 3)$



20. The inputs on the x-axis that correspond to points on the graph extend from -4 to 3, not including 3. Thus the domain is $[-4, 3)$.

The outputs on the y-axis extend from -4 to 5, not including 5. Thus, the range is $[-4, 5)$.

21. $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{-5 - 13}{-2 - (-2)} = \dfrac{-18}{0}$

Since division by 0 is not defined, the slope is not defined.

22. $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{3 - (-1)}{-6 - 10} = \dfrac{4}{-16} = -\dfrac{1}{4}$

23. $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{\frac{1}{3} - \frac{1}{3}}{\frac{2}{7} - \frac{5}{7}} = \dfrac{0}{-\frac{3}{7}} = 0$

24. $f(x) = -\dfrac{1}{9}x + 12$ is in the form $y = mx + b$ with $m = -\dfrac{1}{9}$ and $b = 12$, so the slope is $-\dfrac{1}{9}$ and the y-intercept is $(0, 12)$.

25. We can write $y = -6$ as $y = 0x - 6$, so the slope is 0 and the y-intercept is $(0, -6)$.

26. The graph of $x = 2$ is a vertical line 2 units to the right of the y-axis. The slope is not defined and there is no y-intercept.

27.
$$3x - 16y + 1 = 0$$
$$3x + 1 = 16y$$
$$\frac{3}{16}x + \frac{1}{16} = y$$

Slope: $\dfrac{3}{16}$; y-intercept: $\left(0, \dfrac{1}{16}\right)$

28. The sign of the slope indicates the slant of a line. A line that slants up from left to right has positive slope because corresponding changes in x and y have the same sign. A line that slants down from left to right has negative slope, because corresponding changes in x and y have opposite signs. A horizontal line has zero slope, because there is no change in y for a given change in x. A vertical line has undefined slope, because there is no change in x for a given change in y and division by 0 is undefined. The larger the absolute value of slope, the steeper the line. This is because a larger absolute value corresponds to a greater change in y, compared to the change in x, than a smaller absolute value.

29. A vertical line $(x = a)$ crosses the graph more than once.

30. The domain of a function is the set of all inputs of the function. The range is the set of all outputs. The range depends on the domain.

31. Let $A = (a, b)$ and $B = (c, d)$. The coordinates of a point C one-half of the way from A to B are $\left(\dfrac{a+c}{2}, \dfrac{b+d}{2}\right)$. A point D that is one-half of the way from C to B is $\dfrac{1}{2} + \dfrac{1}{2} \cdot \dfrac{1}{2}$, or $\dfrac{3}{4}$ of the way from A to B. Its coordinates are $\left(\dfrac{\frac{a+c}{2} + c}{2}, \dfrac{\frac{b+d}{2} + d}{2}\right)$, or $\left(\dfrac{a+3c}{4}, \dfrac{b+3d}{4}\right)$. Then a point E that is one-half of the way from D to B is $\dfrac{3}{4} + \dfrac{1}{2} \cdot \dfrac{1}{4}$, or $\dfrac{7}{8}$ of the way from A to B. Its coordinates are $\left(\dfrac{\frac{a+3c}{4} + c}{2}, \dfrac{\frac{b+3d}{4} + d}{2}\right)$, or $\left(\dfrac{a+7c}{8}, \dfrac{b+7d}{8}\right)$.

Exercise Set 1.4

1. We see that the y-intercept is $(0, -2)$. Another point on the graph is $(1, 2)$. Use these points to find the slope.

$m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{2 - (-2)}{1 - 0} = \dfrac{4}{1} = 4$

We have $m = 4$ and $b = -2$, so the equation is $y = 4x - 2$.

2. We see that the y-intercept is $(0, 2)$. Another point on the graph is $(4, -1)$.

$m = \dfrac{-1 - 2}{4 - 0} = -\dfrac{3}{4}$

The equation is $y = -\dfrac{3}{4}x + 2$.

3. We see that the y-intercept is $(0, 0)$. Another point on the graph is $(3, -3)$. Use these points to find the slope.

$m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{-3 - 0}{3 - 0} = \dfrac{-3}{3} = -1$

We have $m = -1$ and $b = 0$, so the equation is
$y = -1 \cdot x + 0$, or $y = -x$.

4. We see that the y-intercept is $(0, -1)$. Another point on the graph is $(3, 1)$.

$m = \dfrac{1-(-1)}{3-0} = \dfrac{2}{3}$

The equation is $y = \dfrac{2}{3}x - 1$.

5. We see that the y-intercept is $(0, -3)$. This is a horizontal line, so the slope is 0. We have $m = 0$ and $b = -3$, so the equation is $y = 0 \cdot x - 3$, or $y = -3$.

6. We see that the y-intercept is $(0, 0)$. Another point on the graph is $(3, 3)$.

$m = \dfrac{3-0}{3-0} = \dfrac{3}{3} = 1$

The equation is $y = 1 \cdot x + 0$, or $y = x$.

7. We substitute $\dfrac{2}{9}$ for m and 4 for b in the slope-intercept equation.

$$y = mx + b$$
$$y = \frac{2}{9}x + 4$$

8. $y = -\dfrac{3}{8}x + 5$

9. We substitute -4 for m and -7 for b in the slope-intercept equation.

$$y = mx + b$$
$$y = -4x - 7$$

10. $y = \dfrac{2}{7}x - 6$

11. We substitute -4.2 for m and $\dfrac{3}{4}$ for b in the slope-intercept equation.

$$y = mx + b$$
$$y = -4.2x + \frac{3}{4}$$

12. $y = -4x - \dfrac{3}{2}$

13. Using the point-slope equation:

$$y - y_1 = m(x - x_1)$$
$$y - 7 = \frac{2}{9}(x - 3) \quad \text{Substituting}$$
$$y - 7 = \frac{2}{9}x - \frac{2}{3}$$
$$y = \frac{2}{9}x + \frac{19}{3} \quad \text{Slope-intercept equation}$$

Using the slope-intercept equation:

Substitute $\dfrac{2}{9}$ for m, 3 for x, and 7 for y in the slope-intercept equation and solve for b.

$$y = mx + b$$
$$7 = \frac{2}{9} \cdot 3 + b$$
$$7 = \frac{2}{3} + b$$
$$\frac{19}{3} = b$$

Now substitute $\dfrac{2}{9}$ for m and $\dfrac{19}{3}$ for b in $y = mx + b$.

$$y = \frac{2}{9}x + \frac{19}{3}$$

14. Using the point-slope equation:

$$y - 6 = -\frac{3}{8}(x - 5)$$
$$y = -\frac{3}{8}x + \frac{63}{8}$$

Using the slope-intercept equation:

$$6 = -\frac{3}{8} \cdot 5 + b$$
$$\frac{63}{8} = b$$

We have $y = -\dfrac{3}{8}x + \dfrac{63}{8}$.

15. The slope is 0 and the second coordinate of the given point is 8, so we have a horizontal line 8 units above the x-axis. Thus, the equation is $y = 8$.

We could also use the point-slope equation or the slope-intercept equation to find the equation of the line.

Using the point-slope equation:

$$y - y_1 = m(x - x_1)$$
$$y - 8 = 0(x - (-2)) \quad \text{Substituting}$$
$$y - 8 = 0$$
$$y = 8$$

Using the slope-intercept equation:

$$y = mx + b$$
$$y = 0(-2) + 8$$
$$y = 8$$

16. Using the point-slope equation:

$$y - 1 = -2(x - (-5))$$
$$y = -2x - 9$$

Using the slope-intercept equation:

$$1 = -2(-5) + b$$
$$-9 = b$$

We have $y = -2x - 9$.

17. Using the point-slope equation:

$$y - y_1 = m(x - x_1)$$
$$y - (-1) = -\frac{3}{5}(x - (-4))$$
$$y + 1 = -\frac{3}{5}(x + 4)$$
$$y + 1 = -\frac{3}{5}x - \frac{12}{5}$$
$$y = -\frac{3}{5}x - \frac{17}{5} \quad \text{Slope-intercept equation}$$

Using the slope-intercept equation:

$$y = mx + b$$
$$-1 = -\frac{3}{5}(-4) + b$$
$$-1 = \frac{12}{5} + b$$
$$-\frac{17}{5} = b$$

Then we have $y = -\frac{3}{5}x - \frac{17}{5}$.

18. Using the point-slope equation:

$$y - (-5) = \frac{2}{3}(x - (-4))$$
$$y = \frac{2}{3}x - \frac{7}{3}$$

Using the slope-intercept equation:

$$-5 = \frac{2}{3}(-4) + b$$
$$-\frac{7}{3} = b$$

We have $y = \frac{2}{3}x - \frac{7}{3}$.

19. First we find the slope.

$$m = \frac{-4-5}{2-(-1)} = \frac{-9}{3} = -3$$

Using the point-slope equation:

Using the point $(-1, 5)$, we get

$y - 5 = -3(x - (-1))$, or $y - 5 = -3(x + 1)$.

Using the point $(2, -4)$, we get

$y - (-4) = -3(x - 2)$, or $y + 4 = -3(x - 2)$.

In either case, the slope-intercept equation is $y = -3x + 2$.

Using the slope-intercept equation and the point $(-1, 5)$:

$$y = mx + b$$
$$5 = -3(-1) + b$$
$$5 = 3 + b$$
$$2 = b$$

Then we have $y = -3x + 2$.

20. First we find the slope:

$$m = \frac{\frac{1}{2} - \frac{1}{2}}{-3-1} = \frac{0}{-4} = 0$$

We have a horizontal line $\frac{1}{2}$ unit above the x-axis. The equation is $y = \frac{1}{2}$.

(We could also have used the point-slope equation or the slope-intercept equation.)

21. First we find the slope.

$$m = \frac{4-0}{-1-7} = \frac{4}{-8} = -\frac{1}{2}$$

Using the point-slope equation:

Using the point $(7, 0)$, we get

$$y - 0 = -\frac{1}{2}(x - 7).$$

Using the point $(-1, 4)$, we get

$$y - 4 = -\frac{1}{2}(x - (-1)), \text{ or}$$
$$y - 4 = -\frac{1}{2}(x + 1).$$

In either case, the slope-intercept equation is $y = -\frac{1}{2}x + \frac{7}{2}$.

Using the slope-intercept equation and the point $(7, 0)$:

$$0 = -\frac{1}{2} \cdot 7 + b$$
$$\frac{7}{2} = b$$

Then we have $y = -\frac{1}{2}x + \frac{7}{2}$.

22. First we find the slope.

$$m = \frac{-5-7}{-1-(-3)} = \frac{-12}{2} = -6$$

Using the point-slope equation:

Using $(-3, 7)$: $y - 7 = -6(x - (-3))$, or

$$y - 7 = -6(x + 3)$$

Using $(-1, -5)$: $y - (-5) = -6(x - (-1))$, or

$$y + 5 = -6(x + 1)$$

In either case, we have $y = -6x - 11$.

Using the slope-intercept equation and the point $(-1, -5)$:

$$-5 = -6(-1) + b$$
$$-11 = b$$

We have $y = -6x - 11$.

23. First we find the slope.

$$m = \frac{-4-(-6)}{3-0} = \frac{2}{3}$$

We know the y-intercept is $(0, -6)$, so we substitute in the slope-intercept equation.

$$y = mx + b$$
$$y = \frac{2}{3}x - 6$$

24. First we find the slope.

$$m = \frac{\frac{4}{5} - 0}{0 - (-5)} = \frac{\frac{4}{5}}{5} = \frac{4}{25}$$

We know the y-intercept is $\left(0, \frac{4}{5}\right)$, so we substitute in the slope-intercept equation.

$$y = \frac{4}{25}x + \frac{4}{5}$$

25. First we find the slope.

$$m = \frac{7.3-7.3}{-4-0} = \frac{0}{-4} = 0$$

We know the y-intercept is $(0,\ 7.3)$, so we substitute in the slope-intercept equation.

$$y = mx + b$$
$$y = 0 \cdot x + 7.3$$
$$y = 7.3$$

26. First we find the slope.

$$m = \frac{-5-0}{-13-0} = \frac{5}{13}$$

We know the y-intercept is $(0, 0)$, so we substitute in the slope intercept equation.

$$y = \frac{5}{13}x + 0$$
$$y = \frac{5}{13}x$$

27. The equation of the horizontal line through $(0, -3)$ is of the form $y = b$ where b is -3. We have $y = -3$.

The equation of the vertical line through $(0, -3)$ is of the form $x = a$ where a is 0. We have $x = 0$.

28. Horizontal line: $y = 7$

Vertical line: $x = -\frac{1}{4}$

29. The equation of the horizontal line through $\left(\frac{2}{11}, -1\right)$ is of the form $y = b$ where b is -1. We have $y = -1$.

The equation of the vertical line through $\left(\frac{2}{11}, -1\right)$ is of the form $x = a$ where a is $\frac{2}{11}$. We have $x = \frac{2}{11}$.

30. Horizontal line: $y = 0$

Vertical line: $x = 0.03$

31. We have the points $(1, 4)$ and $(-2, 13)$. First we find the slope.

$$m = \frac{13-4}{-2-1} = \frac{9}{-3} = -3$$

We will use the point-slope equation, choosing $(1, 4)$ for the given point.

$$y - 4 = -3(x - 1)$$
$$y - 4 = -3x + 3$$
$$y = -3x + 7, \text{ or}$$
$$h(x) = -3x + 7$$

Then $h(2) = -3 \cdot 2 + 7 = -6 + 7 = 1$.

32. $m = \dfrac{3-(-6)}{2-\left(-\frac{1}{4}\right)} = \dfrac{9}{\frac{9}{4}} = 9 \cdot \dfrac{4}{9} = 4$

Using the point-slope equation and the point $(2, 3)$:

$$y - 3 = 4(x - 2)$$
$$y - 3 = 4x - 8$$
$$y = 4x - 5, \text{ or}$$
$$g(x) = 4x - 5$$

Then $g(-3) = 4(-3) - 5 = -12 - 5 = -17$.

33. We have the points $(5, 1)$ and $(-5, -3)$. First we find the slope.

$$m = \frac{-3-1}{-5-5} = \frac{-4}{-10} = \frac{2}{5}$$

We will use the slope-intercept equation, choosing $(5, 1)$ for the given point.

$$y = mx + b$$
$$1 = \frac{2}{5} \cdot 5 + b$$
$$1 = 2 + b$$
$$-1 = b$$

Then we have $f(x) = \frac{2}{5}x - 1$.

Now we find $f(0)$.

$$f(0) = \frac{2}{5} \cdot 0 - 1 = -1.$$

34. $m = \dfrac{2-3}{0-(-3)} = \dfrac{-1}{3} = -\dfrac{1}{3}$

Using the slope-intercept equation and the point $(0, 2)$, which is the y-intercept, we have $h(x) = -\frac{1}{3}x + 2$.

Then $h(-6) = -\frac{1}{3}(-6) + 2 = 2 + 2 = 4$.

35. The slopes are $\frac{26}{3}$ and $-\frac{3}{26}$. Their product is -1, so the lines are perpendicular.

36. The slopes are -3 and $-\frac{1}{3}$. The slopes are not the same and their product is not -1, so the lines are neither parallel nor perpendicular.

37. The slopes are $\frac{2}{5}$ and $-\frac{2}{5}$. The slopes are not the same and their product is not -1, so the lines are neither parallel nor perpendicular.

38. The slopes are the same $\left(\frac{3}{2} = 1.5\right)$ and the y-intercepts, -8 and 8, are different, so the lines are parallel.

39. We solve each equation for y.

$$x + 2y = 5 \qquad\qquad 2x + 4y = 8$$
$$y = -\frac{1}{2}x + \frac{5}{2} \qquad\qquad y = -\frac{1}{2}x + 2$$

We see that $m_1 = -\frac{1}{2}$ and $m_2 = -\frac{1}{2}$. Since the slopes are the same and the y-intercepts, $\frac{5}{2}$ and 2, are different, the lines are parallel.

40.

$$2x - 5y = -3 \qquad\qquad 2x + 5y = 4$$
$$y = \frac{2}{5}x + \frac{3}{5} \qquad\qquad y = -\frac{2}{5}x + \frac{4}{5}$$

$m_1 = \frac{2}{5}$, $m_2 = -\frac{2}{5}$; $m_1 \neq m_2$; $m_1 m_2 = -\frac{4}{25} \neq -1$

The lines are neither parallel nor perpendicular.

41. We solve each equation for y.

$y = 4x - 5 \qquad 4y = 8 - x$

$y = -\frac{1}{4}x + 2$

We see that $m_1 = 4$ and $m_2 = -\frac{1}{4}$. Since $m_1 m_2 = 4\left(-\frac{1}{4}\right) = -1$, the lines are perpendicular.

42. $y = 7 - x$,

$y = x + 3$

$m_1 = -1$, $m_2 = 1$; $m_1 m_2 = -1 \cdot 1 = -1$

The lines are perpendicular.

43. $y = \frac{2}{7}x + 1$; $m = \frac{2}{7}$

The line parallel to the given line will have slope $\frac{2}{7}$. We use the point-slope equation for a line with slope $\frac{2}{7}$ and containing the point $(3, 5)$:

$$y - y_1 = m(x - x_1)$$
$$y - 5 = \frac{2}{7}(x - 3)$$
$$y - 5 = \frac{2}{7}x - \frac{6}{7}$$
$$y = \frac{2}{7}x + \frac{29}{7} \quad \text{Slope-intercept form}$$

The slope of the line perpendicular to the given line is the opposite of the reciprocal of $\frac{2}{7}$, or $-\frac{7}{2}$. We use the point-slope equation for a line with slope $-\frac{7}{2}$ and containing the point $(3, 5)$:

$$y - y_1 = m(x - x_1)$$
$$y - 5 = -\frac{7}{2}(x - 3)$$
$$y - 5 = -\frac{7}{2}x + \frac{21}{2}$$
$$y = -\frac{7}{2}x + \frac{31}{2} \quad \text{Slope-intercept form}$$

44. $f(x) = 2x + 9$

$m = 2$, $-\frac{1}{m} = -\frac{1}{2}$

Parallel line: $y - 6 = 2(x - (-1))$

$y = 2x + 8$

Perpendicular line: $y - 6 = -\frac{1}{2}(x - (-1))$

$y = -\frac{1}{2}x + \frac{11}{2}$

45. $y = -0.3x + 4.3$; $m = -0.3$

The line parallel to the given line will have slope -0.3. We use the point-slope equation for a line with slope -0.3 and containing the point $(-7, 0)$:

$$y - y_1 = m(x - x_1)$$
$$y - 0 = -0.3(x - (-7))$$
$$y = -0.3x - 2.1 \quad \text{Slope-intercept form}$$

The slope of the line perpendicular to the given line is the opposite of the reciprocal of -0.3, or $\frac{1}{0.3} = \frac{10}{3}$.

We use the point-slope equation for a line with slope $\frac{10}{3}$ and containing the point $(-7, 0)$:

$$y - y_1 = m(x - x_1)$$
$$y - 0 = \frac{10}{3}(x - (-7))$$
$$y = \frac{10}{3}x + \frac{70}{3} \quad \text{Slope-intercept form}$$

46. $2x + y = -4$

$y = -2x - 4$

$m = -2$, $-\frac{1}{m} = \frac{1}{2}$

Parallel line: $y - (-5) = -2(x - (-4))$

$y = -2x - 13$

Perpendicular line: $y - (-5) = \frac{1}{2}(x - (-4))$

$y = \frac{1}{2}x - 3$

47. $3x + 4y = 5$

$4y = -3x + 5$

$y = -\frac{3}{4}x + \frac{5}{4}$; $m = -\frac{3}{4}$

The line parallel to the given line will have slope $-\frac{3}{4}$. We use the point-slope equation for a line with slope $-\frac{3}{4}$ and containing the point $(3, -2)$:

$$y - y_1 = m(x - x_1)$$
$$y - (-2) = -\frac{3}{4}(x - 3)$$
$$y + 2 = -\frac{3}{4}x + \frac{9}{4}$$
$$y = -\frac{3}{4}x + \frac{1}{4} \quad \text{Slope-intercept form}$$

The slope of the line perpendicular to the given line is the opposite of the reciprocal of $-\frac{3}{4}$, or $\frac{4}{3}$. We use the point-slope equation for a line with slope $\frac{4}{3}$ and containing the point $(3, -2)$:

$$y - y_1 = m(x - x_1)$$
$$y - (-2) = \frac{4}{3}(x - 3)$$
$$y + 2 = \frac{4}{3}x - 4$$
$$y = \frac{4}{3}x - 6 \quad \text{Slope-intercept form}$$

48. $y = 4.2(x - 3) + 1$

$y = 4.2x - 11.6$

$m = 4.2$; $-\frac{1}{m} = -\frac{1}{4.2} = -\frac{5}{21}$

Parallel line: $y-(-2)=4.2(x-8)$
$$y=4.2x-35.6$$
Perpendicular line: $y-(-2)=-\frac{5}{21}(x-8)$
$$y=-\frac{5}{21}x-\frac{2}{21}$$

49. $x=-1$ is the equation of a vertical line. The line parallel to the given line is a vertical line containing the point $(3,-3)$, or $x=3$.

The line perpendicular to the given line is a horizontal line containing the point $(3,-3)$, or $y=-3$.

50. $y=-1$ is a horizontal line.

Parallel line: $y=-5$

Perpendicular line: $x=4$

51. $x=-3$ is a vertical line and $y=5$ is a horizontal line, so it is true that the lines are perpendicular.

52. The slope of $y=2x-3$ is 2, and the slope of $y=-2x-3$ is -2. Since $2(-2)=-4\neq-1$, it is false that the lines are perpendicular.

53. The lines have the same slope, $\frac{2}{5}$, and different y-intercepts, $(0,4)$ and $(0,-4)$, so it is true that the lines are parallel.

54. $y=2$ is a horizontal line 2 units above the x-axis; $x=-\frac{3}{4}$ is a vertical line $\frac{3}{4}$ unit to the left of the y-axis. Thus it is true that their intersection is the point $\frac{3}{4}$ unit to the left of the y-axis and 2 units above the x-axis, or $\left(-\frac{3}{4},2\right)$.

55. $x=-1$ and $x=1$ are both vertical lines, so it is false that they are perpendicular.

56. The slope of $2x+3y=4$, or $y=-\frac{2}{3}x+\frac{4}{3}$ is $-\frac{2}{3}$; the slope of $3x-2y=4$, or $y=\frac{3}{2}x-2$, is $\frac{3}{2}$. Since $-\frac{2}{3}\cdot\frac{3}{2}=-1$, it is true that the lines are perpendicular.

57. No. The data points fall faster from 0 to 2 than after 2 (that is, the rate of change is not constant), so they cannot be modeled by a linear function.

58. Yes. The rate of change seems to be constant, so the data points might be modeled by a linear function.

59. Yes. The rate of change seems to be constant, so the data points might be modeled by a linear function.

60. No. The data points rise, fall, and then rise again in a way that cannot be modeled by a linear function.

61. a) Answers may vary depending on the data points used. We will use (1, 333) and (4, 380).
$$m=\frac{380-333}{4-1}=\frac{47}{3}\approx 15.67$$
We will use the point-slope equation, letting $(x_1,y_1)=(1,\ 333)$.
$$y-333=15.67(x-1)$$
$$y-333=15.67x-15.67$$
$$y=15.67x+317.33,$$
where x is the number of years after 2009.

b) In 2018, $x=2018-2009=9$.

$y=15.67(9)+317.33=458.36$

We estimate the average monthly cost to workers for family health insurance to be \$458.36 in 2018.

In 2023, $x=2023-2009=14$.

$y=15.67(14)+317.33=536.71$

We estimate the average monthly cost to workers for family health insurance to be \$536.71 in 2023.

62. a) Answers may vary depending on the data points used. We will use (2, 8185) and (6, 6950).
$$m=\frac{6950-8185}{6-2}=\frac{-1235}{4}=-308.75$$
We will use the point-slope equation, letting $(x_1,y_1)=(2,\ 8185)$.
$$y-8185=-308.75(x-2)$$
$$y-8185=-308.75x+617.5$$
$$y=-308.75x+8802.5,$$
where x is the number of years after 2007.

b) 2017: $y=-308.75(10)+8802.5=5715$ banks

2020: $y=-308.75(13)+8802.5\approx 4789$ banks

63. Answers may vary depending on the data points used. We will use (1, 32.5) and (4, 49.9).
$$m=\frac{49.9-32.5}{4-1}=\frac{17.4}{3}=5.8$$
We will use the slope-intercept equation with (1, 32.5).
$$32.5=5.8(1)+b$$
$$32.5=5.8+b$$
$$26.7=b$$
We have $y=5.8x+26.7$ where x is the number of years after 2010 and y is in billions.

In 2019, $x=2019-2010=9$.

$y=5.8(9)+26.7=\$78.9$ billion

64. Answers may vary depending on the data points used. We will use (10, 167.015) and (32, 211.815).
$$m=\frac{211.815-167.015}{32-10}=\frac{44.8}{22}\approx 2.036$$
We will use the slope-intercept equation with (10, 167.015).
$$167.015=2.036(10)+b$$
$$167.015=20.36+b$$
$$146.655=b$$
We have $y=2.036x+146.655$, where x is the number of years after 1980 and y is in millions.

2005: $y = 2.036(25) + 146.655 = 197.555$ million licensed drivers, or 197,555,000 licensed drivers

2021: $y = 2.036(41) + 146.655 = 230.131$ million licensed drivers, or 230,131,000 licensed drivers

65. Answers may vary depending on the data points used. We will use (1, 28.3) and (3, 30.8).

$$m = \frac{30.8 - 28.3}{3 - 1} = \frac{2.5}{2} = 1.25$$

We will use the point-slope equation, letting $(x_1, y_1) = (1,\ 28.3)$

$$y - 28.3 = 1.25(x - 1)$$
$$y - 28.3 = 1.25x - 1.25$$
$$y = 1.25x + 27.05,$$

where x is the number of years after 2009 and y is in gallons.

In 2017, $x = 2017 - 2009 = 8$.

$y = 1.25(8) + 27.05 \approx 37.1$ gallons

66. Answers may vary depending on the data points used. We will use (0, 11,504) and (3, 10,819).

$$m = \frac{10,819 - 11,504}{3 - 0} = \frac{-685}{3} \approx -228$$

We see that the y-intercept is (0, 11,504), so using the slope-intercept equation, we have $y = -228x + 11,504$, where x is the number of years after 2010 and y is in kilowatt-hours.

In 2019, $x = 2019 - 2010 = 9$.

$y = -228(9) + 11,504 = 9452$ kilowatt-hours

67. a) Using the linear regression feature on a graphing calculator, we have $y = 20.05714286x + 301.8571429$, where x is the number of years after 2009.

b) In 2018, $x = 2018 - 2009 = 9$.

$y = 20.05714286(9) + 301.8571429 \approx \$482.37$

This is \$24.01 more than the cost found in Exercise 61.

c) $r \approx 0.9851$; the line fits the data fairly well.

68. a) $y = -286.8452381x + 8685.083333$, where x is the number of years after 2007.

b) For $x = 13$, $y \approx 4956$ banks; this number is 167 more than the number found in Exercise 62.

c) $r \approx -0.9980$; the line fits the data well.

69. a) Using the linear regression feature on a graphing calculator, we have $y = 6.47x + 23.8$, where x is the number of years after 2010 and y is in billions.

b) In 2019, $x = 2019 - 2010 = 9$.

$y = 6.47(9) + 23.8 = \$82.03$ billion

This is \$3.13 billion more than the value found in Exercise 63.

c) $r \approx 0.9915$; the line fits the data well.

70. a) $y = 2.1152407x + 146.0525711$, where x is the number of years after 1980 and y is in millions.

b) For $x = 41$, $y \approx 232,777,440$ licensed drivers; this number is 2,646,440 more than the number found in Exercise 64.

c) $r \approx 0.9985$; the line fits the data well.

71. a) Using the linear regression feature on a graphing calculator, we get $M = 0.2H + 156$.

b) For $H = 40$: $M = 0.2(40) + 156 = 164$ beats per minute

For $H = 65$: $M = 0.2(65) + 156 = 169$ beats per minute

For $H = 76$: $M = 0.2(76) + 156 \approx 171$ beats per minute

For $H = 84$: $M = 0.2(84) + 156 \approx 173$ beats per minute

c) $r = 1$; all the data points are on the regression line so it should be a good predictor.

72. a) $y = 0.072050673x + 81.99920823$

b) For $x = 24$:
$y = 0.072050673(24) + 81.99920823 \approx 84\%$

For $x = 6$:
$y = 0.072050673(6) + 81.99920823 \approx 82\%$

For $x = 18$:
$y = 0.072050673(18) + 81.99920823 \approx 83\%$

c) $r = 0.0636$; since there is a very low correlation, the regression line is not a good predictor.

73.
$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1 - (-8)}{-5 - 2} = \frac{-1 + 8}{-7} = \frac{7}{-7} = -1$$

74. $m = \dfrac{-7 - 7}{5 - 5} = \dfrac{-14}{0}$

The slope is not defined.

75.
$$(x - h)^2 + (y - k)^2 = r^2$$
$$[x - (-7)]^2 + [y - (-1)]^2 = \left(\frac{9}{5}\right)^2$$
$$(x + 7)^2 + (y + 1)^2 = \frac{81}{25}$$

76. $r = \dfrac{d}{2} = \dfrac{5}{2}$

$$(x - 0)^2 + (y - 3)^2 = \left(\frac{5}{2}\right)^2$$
$$x^2 + (y - 3)^2 = \frac{25}{4}, \text{ or}$$
$$x^2 + (y - 3)^2 = 6.25$$

77. The slope of the line containing $(-3,k)$ and $(4,8)$ is
$$\frac{8-k}{4-(-3)}=\frac{8-k}{7}.$$
The slope of the line containing $(5,3)$ and $(1,-6)$ is
$$\frac{-6-3}{1-5}=\frac{-9}{-4}=\frac{9}{4}.$$
The slopes must be equal in order for the lines to be parallel:
$$\frac{8-k}{7}=\frac{9}{4}$$
$$32-4k=63 \quad \text{Multiplying by 28}$$
$$-4k=31$$
$$k=-\frac{31}{4},\ \text{or}\ -7.75$$

78. $m=\dfrac{920.58}{13,740}=0.067$

The road grade is 6.7%.

We find an equation of the line with slope 0.067 and containing the point $(13,740,920.58)$:
$$y-920.58=0.067(x-13,740)$$
$$y-920.58=0.067x-920.58$$
$$y=0.067x$$

79. The slope of the line containing $(-1,3)$ and $(2,9)$ is
$$\frac{9-3}{2-(-1)}=\frac{6}{3}=2.$$
Then the slope of the desired line is $-\frac{1}{2}$. We find the equation of that line:
$$y-5=-\frac{1}{2}(x-4)$$
$$y-5=-\frac{1}{2}x+2$$
$$y=-\frac{1}{2}x+7$$

Exercise Set 1.5

1. $4x+5=21$
$$4x=16 \quad \text{Subtracting 5 on both sides}$$
$$x=4 \quad \text{Dividing by 4 on both sides}$$
The solution is 4.

2. $2y-1=3$
$$2y=4$$
$$y=2$$
The solution is 2.

3. $23-\frac{2}{5}x=-\frac{2}{5}x+23$
$$23=23 \qquad \text{Adding } \frac{2}{5}x \text{ on both sides}$$
We get an equation that is true for any value of x, so the solution set is the set of real numbers,
$\{x|x \text{ is a real number}\}$, or $(-\infty,\infty)$.

4. $\frac{6}{5}y+3=\frac{3}{10}$ The LCD is 10.
$$10\left(\frac{6}{5}y+3\right)=10\cdot\frac{3}{10}$$
$$12y+30=3$$
$$12y=-27$$
$$y=-\frac{9}{4}$$
The solution is $-\frac{9}{4}$.

5. $4x+3=0$
$$4x=-3 \quad \text{Subtracting 3 on both sides}$$
$$x=-\frac{3}{4} \quad \text{Dividing by 4 on both sides}$$
The solution is $-\frac{3}{4}$.

6. $3x-16=0$
$$3x=16$$
$$x=\frac{16}{3}$$
The solution is $\frac{16}{3}$.

7. $3-x=12$
$$-x=9 \quad \text{Subtracting 3 on both sides}$$
$$x=-9 \quad \text{Multiplying (or dividing) by } -1 \text{ on both sides}$$
The solution is -9.

8. $4-x=-5$
$$-x=-9$$
$$x=9$$
The solution is 9.

9. $3-\frac{1}{4}x=\frac{3}{2}$ The LCD is 4.
$$4\left(3-\frac{1}{4}x\right)=4\cdot\frac{3}{2} \quad \text{Multiplying by the LCD to clear fractions}$$
$$12-x=6$$
$$-x=-6 \quad \text{Subtracting 12 on both sides}$$
$$x=6 \quad \text{Multiplying (or dividing) by } -1 \text{ on both sides}$$
The solution is 6.

10. $10x-3=8+10x$
$$-3=8 \qquad \text{Subtracting } 10x \text{ on both sides}$$
We get a false equation. Thus, the original equation has no solution.

11. $\frac{2}{11}-4x=-4x+\frac{9}{11}$
$$\frac{2}{11}=\frac{9}{11} \qquad \text{Adding } 4x \text{ on both sides}$$
We get a false equation. Thus, the original equation has no solution.

12. $8 - \frac{2}{9}x = \frac{5}{6}$ The LCD is 18.

$18\left(8 - \frac{2}{9}x\right) = 18 \cdot \frac{5}{6}$

$144 - 4x = 15$

$-4x = -129$

$x = \frac{129}{4}$

The solution is $\frac{129}{4}$.

13. $8 = 5x - 3$

$11 = 5x$ Adding 3 on both sides

$\frac{11}{5} = x$ Dividing by 5 on both sides

The solution is $\frac{11}{5}$.

14. $9 = 4x - 8$

$17 = 4x$

$\frac{17}{4} = x$

The solution is $\frac{17}{4}$.

15. $\frac{2}{5}y - 2 = \frac{1}{3}$ The LCD is 15.

$15\left(\frac{2}{5}y - 2\right) = 15 \cdot \frac{1}{3}$ Multiplying by the LCD to clear fractions

$6y - 30 = 5$

$6y = 35$ Adding 30 on both sides

$y = \frac{35}{6}$ Dividing by 6 on both sides

The solution is $\frac{35}{6}$.

16. $-x + 1 = 1 - x$

$1 = 1$ Adding x on both sides

We get an equation that is true for any value of x, so the solution set is the set of real numbers, $\{x|x$ is a real number$\}$, or $(-\infty, \infty)$.

17. $y + 1 = 2y - 7$

$1 = y - 7$ Subtracting y on both sides

$8 = y$ Adding 7 on both sides

The solution is 8.

18. $5 - 4x = x - 13$

$18 = 5x$

$\frac{18}{5} = x$

The solution is $\frac{18}{5}$.

19. $2x + 7 = x + 3$

$x + 7 = 3$ Subtracting x on both sides

$x = -4$ Subtracting 7 on both sides

The solution is -4.

20. $5x - 4 = 2x + 5$

$3x - 4 = 5$

$3x = 9$

$x = 3$

The solution is 3.

21. $3x - 5 = 2x + 1$

$x - 5 = 1$ Subtracting $2x$ on both sides

$x = 6$ Adding 5 on both sides

The solution is 6.

22. $4x + 3 = 2x - 7$

$2x = -10$

$x = -5$

The solution is -5.

23. $4x - 5 = 7x - 2$

$-5 = 3x - 2$ Subtracting $4x$ on both sides

$-3 = 3x$ Adding 2 on both sides

$-1 = x$ Dividing by 3 on both sides

The solution is -1.

24. $5x + 1 = 9x - 7$

$8 = 4x$

$2 = x$

The solution is 2.

25. $5x - 2 + 3x = 2x + 6 - 4x$

$8x - 2 = 6 - 2x$ Collecting like terms

$8x + 2x = 6 + 2$ Adding $2x$ and 2 on both sides

$10x = 8$ Collecting like terms

$x = \frac{8}{10}$ Dividing by 10 on both sides

$x = \frac{4}{5}$ Simplifying

The solution is $\frac{4}{5}$.

26. $5x - 17 - 2x = 6x - 1 - x$

$3x - 17 = 5x - 1$

$-2x = 16$

$x = -8$

The solution is -8.

27. $7(3x + 6) = 11 - (x + 2)$

$21x + 42 = 11 - x - 2$ Using the distributive property

$21x + 42 = 9 - x$ Collecting like terms

$21x + x = 9 - 42$ Adding x and subtracting 42 on both sides

$22x = -33$ Collecting like terms

$x = -\frac{33}{22}$ Dividing by 22 on both sides

$x = -\frac{3}{2}$ Simplifying

The solution is $-\frac{3}{2}$.

28. $4(5y+3) = 3(2y-5)$

$$20y + 12 = 6y - 15$$
$$14y = -27$$
$$y = -\frac{27}{14}$$

The solution is $-\frac{27}{14}$.

29. $3(x+1) = 5 - 2(3x+4)$

$3x + 3 = 5 - 6x - 8$ Removing parentheses

$3x + 3 = -6x - 3$ Collecting like terms

$9x + 3 = -3$ Adding $6x$

$9x = -6$ Subtracting 3

$x = -\frac{2}{3}$ Dividing by 9

The solution is $-\frac{2}{3}$.

30. $4(3x+2) - 7 = 3(x-2)$

$$12x + 8 - 7 = 3x - 6$$
$$12x + 1 = 3x - 6$$
$$9x + 1 = -6$$
$$9x = -7$$
$$x = -\frac{7}{9}$$

The solution is $-\frac{7}{9}$.

31. $2(x-4) = 3 - 5(2x+1)$

$2x - 8 = 3 - 10x - 5$ Using the distributive property

$2x - 8 = -10x - 2$ Collecting like terms

$12x = 6$ Adding $10x$ and 8 on both sides

$x = \frac{1}{2}$ Dividing by 12 on both sides

The solution is $\frac{1}{2}$.

32. $3(2x-5) + 4 = 2(4x+3)$

$$6x - 15 + 4 = 8x + 6$$
$$6x - 11 = 8x + 6$$
$$-2x = 17$$
$$x = -\frac{17}{2}$$

The solution is $-\frac{17}{2}$.

33. ***Familiarize***. Let w = the number of new words that appeared in the English language in the seventeenth century. Then the number of new words that appeared in the nineteenth century is $w + 46.9\%$ of w, or $w + 0.469w$, or $1.469w$.

Translate. The number of new words that appeared in the nineteenth century is 75,029, so we have

$$75,029 = 1.469w.$$

Carry out.

$75,029 = 1.469w$

$51,075 \approx w$ Dividing by 1.469

Check. 46.9% of $51,075 = 0.469(51,075) \approx 23,954$, and $51,075 + 23,954 = 75,029$. This is the number of new words that appeared in the nineteenth century, so the answer checks.

State. In the seventeenth century about 51,075 new words appeared in the English language.

34. Let d = the daily caloric intake per person in Haiti.

Solve: $3688 = 1.864d$

$d \approx 1979$ calories

35. ***Familiarize***. Let P = the amount Kea borrowed. We will use the formula $I = Prt$ to find the interest owed. For $r = 5\%$, or 0.05, and $t = 1$, we have $I = P(0.05)(1)$, or $0.05P$.

Translate.

Amount borrowed plus interest is \$1365.

$$\underbrace{\text{Amount borrowed}}_{P} \quad \text{plus} \to + \quad \text{interest} \to 0.05P \quad \text{is} \to = \quad \$1365 \to 1365$$

$$P + 0.05P = 1365$$

Carry out. We solve the equation.

$P + 0.05P = 1365$

$1.05P = 1365$ Adding

$P = 1300$ Dividing by 1.05

Check. The interest due on a loan of \$1300 for 1 year at a rate of 5% is \$1300(0.05)(1), or \$65, and \$1300 + \$65 = \$1365. The answer checks.

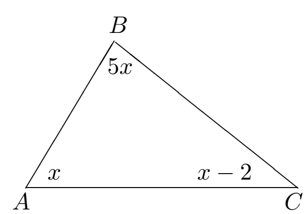
State. Kea borrowed \$1300.

36. Let P = the amount invested.

Solve: $P + 0.04P = \$1560$

$P = \$1500$

37. ***Familiarize***. We make a drawing.



We let x = the measure of angle A. Then $5x$ = the measure of angle B, and $x - 2$ = the measure of angle C. The sum of the angle measures is 180°.

Translate.

Measure of angle A + Measure of angle B + Measure of angle C = 180.

$$x + 5x + (x - 2) = 180$$

Carry out. We solve the equation.

$$\begin{aligned} x+5x+x-2 &= 180 \\ 7x-2 &= 180 \\ 7x &= 182 \\ x &= 26 \end{aligned}$$

If $x=26$, then $5x=5\cdot 26$, or 130, and $x-2=26-2$, or 24.

Check. The measure of angle B, 130°, is five times the measure of angle A, 26°. The measure of angle C, 24°, is 2° less than the measure of angle A, 26°. The sum of the angle measures is 26° + 130° + 24°, or 180°. The answer checks.

State. The measure of angles A, B, and C are 26°, 130°, and 24°, respectively.

38. Let x = the measure of angle A.

Solve: $x+2x+x+20=180$

$x=40°$, so the measure of angle A is 40°; the measure of angle B is $2\cdot 40°$, or 80°; and the measure of angle C is $40°+20°$, or 60°.

39. ***Familiarize***. Let c = the amount of apparel and clothing accessories exports from the United States in 2013, in billions of dollars.

Translate.

$$\underbrace{\text{Clothing imports in 2013}}_{\downarrow\ 87.924}\ \underbrace{\text{were}}_{=}\ \underbrace{25}_{25}\ \underbrace{\text{times}}_{\cdot}\ \underbrace{\text{clothing exports in 2013}}_{\downarrow\ c}\ \underbrace{\text{less}}_{-}\ \underbrace{\text{\$2.299 billion.}}_{\downarrow\ 2.299.}$$

Carry out.

$$\begin{aligned} 87.924 &= 25c-2.299 \\ 90.223 &= 25c \\ 3.609 &\approx c \end{aligned}$$

Check. $25(3.609)-2.299=90.225-2.299=87.926$. This is about the amount of apparel and clothing accessories imports, so the answer checks. (Recall that we rounded the value of c.)

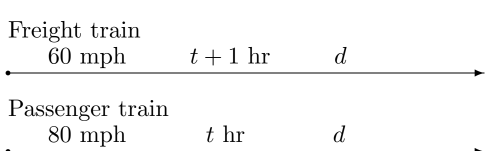
State. In 2013, apparel and clothing accessories exports from the United States were about \$3.609 billion.

40. Let v = the value of imports to the United States in 2013.

Solve: $1,579,593,000,000=\frac{1}{2}v+445,432,000,000$

$v=\$2,268,322,000,000$

41. ***Familiarize***. We make a drawing. Let t = the number of hours the passenger train travels before it overtakes the freight train. Then $t+1$ = the number of hours the freight train travels before it is overtaken by the passenger train. Also let d = the distance the trains travel.



We can also organize the information in a table.

$$d \quad = \quad r \quad \cdot \quad t$$

	Distance	Rate	Time
Freight train	d	60	$t+1$
Passenger train	d	80	t

Translate. Using the formula $d=rt$ in each row of the table, we get two equations.

$d=60(t+1)$ and $d=80t$.

Since the distances are the same, we have the equation

$60(t+1)=80t$.

Carry out. We solve the equation.

$$\begin{aligned} 60(t+1) &= 80t \\ 60t+60 &= 80t \\ 60 &= 20t \\ 3 &= t \end{aligned}$$

When $t=3$, then $t+1=3+1=4$.

Check. In 4 hr the freight train travels $60\cdot 4$, or 240 mi. In 3 hr the passenger train travels $80\cdot 3$, or 240 mi. Since the distances are the same, the answer checks.

State. It will take the passenger train 3 hr to overtake the freight train.

42. Let t = the time the private airplane travels.

	Distance	Rate	Time
Private airplane	d	180	t
Jet	d	900	$t-2$

From the table we have the following equations:

$d=180t \quad$ and $\quad d=900(t-2)$

Solve: $180t=900(t-2)$

$t=2.5$

In 2.5 hr the private airplane travels 180(2.5), or 450 km. This is the distance from the airport at which it is overtaken by the jet.

43. ***Familiarize***. Let s = the number of tweets about the State of the Union address in 2014, in millions.

Translate.

$$\underbrace{\text{Number of tweets about Grammy Awards}}_{\downarrow\ 13.8}\ \underbrace{\text{were}}_{=}\ \underbrace{\text{Number of tweets about State of the Union address}}_{\downarrow\ s}\ \underbrace{\text{plus}}_{+}\ \underbrace{\text{11.7 million.}}_{\downarrow\ 11.7}$$

Carry out. We solve the equation.

$$\begin{aligned} 13.8 &= s+11.7 \\ 2.1 &= s \end{aligned}$$

Check. $2.1 + 11.7 = 13.8$, so the answer checks.

State. In 2014, there were 2.1 million tweets about the State of the Union address.

44. Let $a =$ the average hourly earnings of a purchasing manager.

Solve: $a - 33.05 = 16.81$

$a = \$49.86$

45. ***Familiarize***. Let $a =$ the amount of sales for which the two choices will be equal.

Translate.

$$\underbrace{\$1800}_{1800}\ \underbrace{\text{equals}}_{=}\ \underbrace{\$1600}_{1600}\ \underbrace{\text{plus}}_{+}\ \underbrace{4\%}_{0.04}\ \underbrace{\text{of}}_{\cdot}\ \underbrace{\text{amount sold.}}_{a}$$

Carry out.

$$1800 = 1600 + 0.04a$$
$$200 = 0.04a$$
$$5000 = a$$

Check. $\$1600 + 4\%$ of $\$5000 = \$1600 + 0.04 \cdot \$5000 = \$1600 + \$200 = \$1800$, so the answer checks.

State. For sales of \$5000, the two choices will be equal.

46. Let $s =$ Edward's sales for the month.

Solve: $1270 + 0.06s = 3154$

$s = \$31,400$

47. ***Familiarize***. Let $s =$ the number of U.S. students who studied abroad during the 2012-2013 school year.

Translate.

$$\underbrace{\text{Number of U.S. students abroad}}_{s}\ \underbrace{\text{was}}_{=}\ \underbrace{\text{seven-twentieths}}_{\frac{7}{20}}\ \underbrace{\text{of}}_{\cdot}\ \underbrace{\text{number of foreign students in U.S.}}_{820,000}$$

Carry out.

$$s = \frac{7}{20} \cdot 820,000$$
$$s = 287,000$$

Check. We repeat the calculation. The answer checks.

State. About 287,000 U.S. students studied abroad during the 2012-2013 school year.

48. Let $p =$ the population density in the United States.

Solve: $p = \frac{1}{4} \cdot 365.3$

$p \approx 91.3$ persons per square mile

49. ***Familiarize***. Let $l =$ the length of the soccer field and $l - 35 =$ the width, in yards.

Translate. We use the formula for the perimeter of a rectangle. We substitute 330 for P and $l - 35$ for w.

$$P = 2l + 2w$$
$$330 = 2l + 2(l - 35)$$

Carry out. We solve the equation.

$$330 = 2l + 2(l - 35)$$
$$330 = 2l + 2l - 70$$
$$330 = 4l - 70$$
$$400 = 4l$$
$$100 = l$$

If $l = 100$, then $l - 35 = 100 - 35 = 65$.

Check. The width, 65 yd, is 35 yd less than the length, 100 yd. Also, the perimeter is

$$2 \cdot 100\text{ yd} + 2 \cdot 65\text{ yd} = 200\text{ yd} + 130\text{ yd} = 330\text{ yd}.$$

The answer checks.

State. The length of the field is 100 yd, and the width is 65 yd.

50. Let $h =$ the height of the poster and $\frac{2}{3}h =$ the width, in inches.

Solve: $100 = 2 \cdot h + 2 \cdot \frac{2}{3}h$

$h = 30$, so the height is 30 in. and the width is $\frac{2}{3} \cdot 30$, or 20 in.

51. ***Familiarize***. Using the labels on the drawing in the text, we let $w =$ the width of the test plot and $w + 25 =$ the length, in meters. Recall that for a rectangle, Perimeter $= 2 \cdot$ length $+ 2 \cdot$ width.

Translate.

$$\underbrace{\text{Perimeter}}_{322}\ \underbrace{=}_{=}\ \underbrace{2 \cdot\ \text{length}}_{2(w+25)}\ \underbrace{+}_{+}\ \underbrace{2 \cdot\ \text{width}}_{2 \cdot w}$$

Carry out. We solve the equation.

$$322 = 2(w + 25) + 2 \cdot w$$
$$322 = 2w + 50 + 2w$$
$$322 = 4w + 50$$
$$272 = 4w$$
$$68 = w$$

When $w = 68$, then $w + 25 = 68 + 25 = 93$.

Check. The length is 25 m more than the width: $93 = 68 + 25$. The perimeter is $2 \cdot 93 + 2 \cdot 68$, or $186 + 136$, or 322 m. The answer checks.

State. The length is 93 m; the width is 68 m.

52. Let $w =$ the width of the garden.

Solve: $2 \cdot 2w + 2 \cdot w = 39$

$w = 6.5$, so the width is 6.5 m, and the length is $2(6.5)$, or 13 m.

53. ***Familiarize***. Let $t =$ the number of hours it will take the plane to travel 1050 mi into the wind. The speed into the headwind is $450 - 30$, or 420 mph.

Translate. We use the formula $d = rt$.

$$1050 = 420 \cdot t$$

Carry out. We solve the equation.

$$1050 = 420 \cdot t$$
$$2.5 = t$$

Check. At a rate of 420 mph, in 2.5 hr the plane travels 420(2.5), or 1050 mi. The answer checks.

State. It will take the plane 2.5 hr to travel 1050 mi into the wind.

54. Let t = the number of hours it will take the plane to travel 700 mi with the wind. The speed with the wind is 375+25, or 400 mph.

Solve: $700 = 400t$

$t = 1.75$ hr

55. ***Familiarize***. Let x = the amount invested at 3% interest. Then $5000 - x$ = the amount invested at 4%. We organize the information in a table, keeping in mind the simple interest formula, $I = Prt$.

	Amount invested	Interest rate	Time	Amount of interest
3% investment	x	3%, or 0.03	1 yr	$x(0.03)(1)$, or $0.03x$
4% investment	$5000-x$	4%, or 0.04	1 yr	$(5000-x)(0.04)(1)$, or $0.04(5000-x)$
Total	5000			176

Translate.

$$\underbrace{\text{Interest on 3\% investment}}_{0.03x} \text{ plus } \underbrace{\text{interest on 4\% investment}}_{0.04(5000-x)} \text{ is } \$176.$$
$$0.03x + 0.04(5000 - x) = 176$$

Carry out. We solve the equation.

$$0.03x + 0.04(5000 - x) = 176$$
$$0.03x + 200 - 0.04x = 176$$
$$-0.01x + 200 = 176$$
$$-0.01x = -24$$
$$x = 2400$$

If $x = 2400$, then $5000 - x = 5000 - 2400 = 2600$.

Check. The interest on \$2400 at 3% for 1 yr is \$2400(0.03)(1) = \$72. The interest on \$2600 at 4% for 1 yr is \$2600(0.04)(1) = \$104. Since \$72 + \$104 = \$176, the answer checks.

State. \$2400 was invested at 3%, and \$2600 was invested at 4%.

56. Let x = the amount borrowed at 5%. Then $9000 - x$ = the amount invested at 6%.

Solve: $0.05x + 0.06(9000 - x) = 492$

$x = 4800$, so \$4800 was borrowed at 5% and \$9000 − \$4800 = \$4200 was borrowed at 6%.

57. ***Familiarize***. Let p = the number of patents Samsung received in 2013. Then $p + 2133$ = the number of patents IBM received.

Translate.

$$\underbrace{\text{Samsung patents}}_{p} \text{ plus } \underbrace{\text{IBM patents}}_{(p+2133)} \text{ is } \underbrace{\text{total number of patents.}}_{11,485}$$
$$p + (p + 2133) = 11,485$$

Carry out.

$$p + (p + 2133) = 11,485$$
$$2p + 2133 = 11,485$$
$$2p = 9352$$
$$p = 4676$$

Check. If $p = 4676$, then $p + 2133 = 4676 + 2133 = 6809$, and $4676 + 6809 = 11,485$, the total number of patents, so the answer checks.

State. In 2013, Samsung received 4676 patents, and IBM received 6809 patents.

58. Let b = the number of books written about George Washington. Then $b + 1675$ = the number of books written about Abraham Lincoln.

Solve: $b + (b + 1675) = 5493$

$b = 1909$, so 1909 books are written about George Washington and $1909 + 1675$, or 3584, books are written about Abraham Lincoln.

59. ***Familiarize***. Let p = the income to the estate of Elvis Presley, in millions of dollars, during the given time period. The $2p+30$ = the income to the estate of Michael Jackson.

Translate. The total income to the two estates was \$195 million, so we have

$$p + 2p + 30 = 195.$$

Carry out. We solve the equation.

$$p + 2p + 30 = 195$$
$$3p + 30 = 195$$
$$3p = 165$$
$$p = 55$$

If $p = 55$, then $2p + 30 = 2 \cdot 55 + 30 = 110 + 30 = 140$

Check. \$140 million is \$30 million more than twice \$55 million, and \$55 million + \$140 million = \$195 million. The answer checks.

State. The income to the estate of Elvis Presley was \$55 million, and the income to the estate of Michael Jackson was \$140 million.

60. Let d = the average depth of the Atlantic Ocean, in feet. Then $\frac{4}{5}d - 272$ = the average depth of the Indian Ocean.

Solve. $14,040 = d + \frac{4}{5}d = 272 - 8890$

$d = 12,890$, so the average depth of the Indian Ocean is $\frac{4}{5} \cdot 12,890 - 272 = 10,040$ ft.

61. ***Familiarize***. Let w = the number of pounds of Lily's body weight that is water.

Translate.

55%	of	body weight	is	water.
↓	↓	↓	↓	↓
0.55	×	135	=	w

Carry out. We solve the equation.

$$0.55 \times 135 = w$$
$$74.25 = w$$

Check. Since 55% of 135 is 74.25, the answer checks.

State. 74.25 lb of Lily's body weight is water.

62. Let $w =$ the number of pounds of Jake's body weight that is water.

Solve: $0.6 \times 186 = w$

$w = 111.6$ lb

63. ***Familiarize***. Let $t =$ the number of hours it takes the kayak to travel 36 mi upstream. The kayak travels upstream at a rate of $12 - 4$, or 8 mph.

Translate. We use the formula $d = rt$.

$$36 = 8 \cdot t$$

Carry out. We solve the equation.

$$36 = 8 \cdot t$$
$$4.5 = t$$

Check. At a rate of 8 mph, in 4.5 hr the kayak travels 8(4.5), or 36 mi. The answer checks.

State. It takes the kayak 4.5 hr to travel 36 mi upstream.

64. Let $t =$ the number of hours it will take Angelo to travel 20 km downstream. The kayak travels downstream at a rate of $14 + 2$, or 16 km/h.

Solve: $20 = 16t$

$t = 1.25$ hr

65. ***Familiarize***. Let $m =$ the number of miles Diego traveled in the cab.

Translate.

Pickup fee	plus	\$1.50	times	miles traveled	is	\$19.75
↓	↓	↓	↓	↓	↓	↓
1.75	+	1.50	·	m	=	19.75

Carry out. We solve the equation.

$$1.75 + 1.50 \cdot m = 19.75$$
$$1.50m = 18$$
$$m = 12$$

Check. $\$1.75 + \$1.50 \cdot 12 = \$1.75 + \$18 = \$19.75$, so the answer checks.

State. Diego traveled 12 mi in the cab.

66. ***Familiarize***. Let $w =$ Rosalyn's regular hourly wage. She earned $40w$ for working the first 40 hr. She worked $48-40$, or 8 hr, of overtime. She earned $8(1.5w)$ for working 8 hr of overtime.

Solve: $40w + 8(1.5w) = 1066$

$w = \$20.50$

67. ***Familiarize***. Let $p =$ the percent of the world's olive oil consumed in the United States. Then the percent consumed in Italy is $3\frac{3}{4} \cdot p$, or $\frac{15}{4}p$, and the percent consumed in Spain is $\frac{2}{3} \cdot \frac{15}{4}p$, or $\frac{5}{2}p$.

Translate.

Percent of olive oil consumed in Italy, Spain, and the U.S.	is	58%.
↓	↓	↓
$p + \frac{15}{4}p + \frac{5}{2}p$	=	58

Carry out.

$$p + \frac{15}{4}p + \frac{5}{2}p = 58$$
$$4\left(p + \frac{15}{4}p + \frac{5}{2}p\right) = 4 \cdot 58$$
$$4p + 15p + 10p = 232$$
$$29p = 232$$
$$p = 8$$

If $p = 8$, then $\frac{15}{4}p = \frac{15}{4} \cdot 8 = 30$ and $\frac{5}{2}p = \frac{5}{2} \cdot 8 = 20$.

Check. 30% is $3\frac{3}{4}$ times 8%, and 20% is $\frac{2}{3}$ of 30%. Also, $8\% + 30\% + 20\% = 58\%$, so the answer checks.

State. Italy, Spain, and the United States consume 30%, 20%, and 8% of the world's olive oil, respectively.

68. Let $s =$ the elevation of Lucas Oil Stadium.

Solve: $5280 = 7s + 275$

$s = 715$ ft

69.

$x + 5 = 0$	Setting $f(x) = 0$
$x + 5 - 5 = 0 - 5$	Subtracting 5 on both sides
$x = -5$	

The zero of the function is -5.

70.

$$5x + 20 = 0$$
$$5x = -20$$
$$x = -4$$

71.

$-2x + 11 = 0$	Setting $f(x) = 0$
$-2x + 11 - 11 = 0 - 11$	Subtracting 11 on both sides
$-2x = -11$	
$x = \frac{11}{2}$	Dividing by -2 on both sides

The zero of the function is $\frac{11}{2}$.

72.

$$8 + x = 0$$
$$x = -8$$

73.

$16 - x = 0$	Setting $f(x) = 0$
$16 - x + x = 0 + x$	Adding x on both sides
$16 = x$	

The zero of the function is 16.

74. $-2x + 7 = 0$
$-2x = -7$
$x = \frac{7}{2}$

75. $x + 12 = 0$ Setting $f(x) = 0$
$x + 12 - 12 = 0 - 12$ Subtracting 12 on both sides
$x = -12$

The zero of the function is -12.

76. $8x + 2 = 0$
$8x = -2$
$x = -\frac{1}{4}$, or -0.25

77. $-x + 6 = 0$ Setting $f(x) = 0$
$-x + 6 + x = 0 + x$ Adding x on both sides
$6 = x$

The zero of the function is 6.

78. $4 + x = 0$
$x = -4$

79. $20 - x = 0$ Setting $f(x) = 0$
$20 - x + x = 0 + x$ Adding x on both sides
$20 = x$

The zero of the function is 20.

80. $-3x + 13 = 0$
$-3x = -13$
$x = \frac{13}{3}$, or $4.\overline{3}$

81. $\frac{2}{5}x - 10 = 0$ Setting $f(x) = 0$
$\frac{2}{5}x = 10$ Adding 10 on both sides
$\frac{5}{2} \cdot \frac{2}{5}x = \frac{5}{2} \cdot 10$ Multiplying by $\frac{5}{2}$ on both sides
$x = 25$

The zero of the function is 25.

82. $3x - 9 = 0$
$3x = 9$
$x = 3$

83. $-x + 15 = 0$ Setting $f(x) = 0$
$15 = x$ Adding x on both sides

The zero of the function is 15.

84. $4 - x = 0$
$4 = x$

85. a) The graph crosses the x-axis at $(4, 0)$. This is the x-intercept.

b) The zero of the function is the first coordinate of the x-intercept. It is 4.

86. a) $(5, 0)$

b) 5

87. a) The graph crosses the x-axis at $(-2, 0)$. This is the x-intercept.

b) The zero of the function is the first coordinate of the x-intercept. It is -2.

88. a) $(2, 0)$

b) 2

89. a) The graph crosses the x-axis at $(-4, 0)$. This is the x-intercept.

b) The zero of the function is the first coordinate of the x-intercept. It is -4.

90. a) $(-2, 0)$

b) -2

91. First find the slope of the given line.

$3x + 4y = 7$
$4y = -3x + 7$
$y = -\frac{3}{4}x + \frac{7}{4}$

The slope is $-\frac{3}{4}$. Now write a slope-intercept equation of the line containing $(-1, 4)$ with slope $-\frac{3}{4}$.

$y - 4 = -\frac{3}{4}[x - (-1)]$
$y - 4 = -\frac{3}{4}(x + 1)$
$y - 4 = -\frac{3}{4}x - \frac{3}{4}$
$y = -\frac{3}{4}x + \frac{13}{4}$

92. $m = \frac{4 - (-2)}{-5 - 3} = \frac{6}{-8} = -\frac{3}{4}$
$y - 4 = -\frac{3}{4}(x - (-5))$
$y - 4 = -\frac{3}{4}x - \frac{15}{4}$
$y = -\frac{3}{4}x + \frac{1}{4}$

93. $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
$= \sqrt{(-10 - 2)^2 + (-3 - 2)^2}$
$= \sqrt{144 + 25} = \sqrt{169} = 13$

94. $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{-\frac{1}{2} + \left(-\frac{3}{2}\right)}{2}, \frac{\frac{2}{5} + \frac{3}{5}}{2}\right) =$
$\left(-\frac{2}{2}, \frac{1}{2}\right) = \left(-1, \frac{1}{2}\right)$

95. $f(x) = \dfrac{x}{x-3}$

$f(-3) = \dfrac{-3}{-3-3} = \dfrac{-3}{-6} = \dfrac{1}{2}$

$f(0) = \dfrac{0}{0-3} = \dfrac{0}{-3} = 0$

$f(3) = \dfrac{3}{3-3} = \dfrac{3}{0}$

Since division by 0 is not defined, $f(3)$ does not exist.

96.
$$\begin{aligned} 7x - y &= \frac{1}{2} \\ -y &= -7x + \frac{1}{2} \\ y &= 7x - \frac{1}{2} \end{aligned}$$

With the equation in the form $y = mx + b$, we see that the slope is 7 and the y-intercept is $\left(0, -\dfrac{1}{2}\right)$.

97. $f(x) = 7 - \dfrac{3}{2}x = -\dfrac{3}{2}x + 7$

The function can be written in the form $y = mx + b$, so it is a linear function.

98. $f(x) = \dfrac{3}{2x} + 5$ cannot be written in the form $f(x) = mx + b$, so it is not a linear function.

99. $f(x) = x^2 + 1$ cannot be written in the form $f(x) = mx + b$, so it is not a linear function.

100. $f(x) = \dfrac{3}{4}x - (2.4)^2$ is in the form $f(x) = mx + b$, so it is a linear function.

101.
$$\begin{aligned} 2x - \{x - [3x - (6x + 5)]\} &= 4x - 1 \\ 2x - \{x - [3x - 6x - 5]\} &= 4x - 1 \\ 2x - \{x - [-3x - 5]\} &= 4x - 1 \\ 2x - \{x + 3x + 5\} &= 4x - 1 \\ 2x - \{4x + 5\} &= 4x - 1 \\ 2x - 4x - 5 &= 4x - 1 \\ -2x - 5 &= 4x - 1 \\ -6x - 5 &= -1 \\ -6x &= 4 \\ x &= -\frac{2}{3} \end{aligned}$$

The solution is $-\dfrac{2}{3}$.

102.
$$\begin{aligned} 14 - 2[3 + 5(x - 1)] &= 3\{x - 4[1 + 6(2 - x)]\} \\ 14 - 2[3 + 5x - 5] &= 3\{x - 4[1 + 12 - 6x]\} \\ 14 - 2[5x - 2] &= 3\{x - 4[13 - 6x]\} \\ 14 - 10x + 4 &= 3\{x - 52 + 24x\} \\ 18 - 10x &= 3\{25x - 52\} \\ 18 - 10x &= 75x - 156 \\ 174 &= 85x \\ \frac{174}{85} &= x \end{aligned}$$

103. The size of the cup was reduced 8 oz − 6 oz, or 2 oz, and $\dfrac{2 \text{ oz}}{8 \text{ oz}} = 0.25$, so the size was reduced 25%. The price per ounce of the 8 oz cup was $\dfrac{89¢}{8 \text{ oz}}$, or 11.125¢/oz. The price per ounce of the 6 oz cup is $\dfrac{71¢}{6 \text{ oz}}$, or $11.8\overline{3}$¢/oz. Since the price per ounce was not reduced, it is clear that the price per ounce was not reduced by the same percent as the size of the cup. The price was increased by $11.8\overline{3} - 11.125$¢, or $0.708\overline{3}$¢ per ounce. This is an increase of $\dfrac{0.708\overline{3}¢}{11.125¢} \approx 0.064$, or about 6.4% per ounce.

104. Let x = the number of copies of *Unbroken* that were sold. Then $11,371 - x$ = the number of copies of *American Sniper* that were sold.

Solve: $\dfrac{x}{11,371 - x} = \dfrac{10}{3.7}$

$x = 8300$, so 8300 copies of *Unbroken* were sold, and $11,371 - 8300$, or 3071, copies of *American Sniper* were sold.

105. We use a proportion to determine the number of calories c burned running for 75 minutes, or 1.25 hr.
$$\begin{aligned} \frac{720}{1} &= \frac{c}{1.25} \\ 720(1.25) &= c \\ 900 &= c \end{aligned}$$

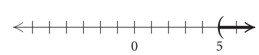
Next we use a proportion to determine how long the person would have to walk to use 900 calories. Let t represent this time, in hours. We express 90 min as 1.5 hr.
$$\begin{aligned} \frac{1.5}{480} &= \frac{t}{900} \\ \frac{900(1.5)}{480} &= t \\ 2.8125 &= t \end{aligned}$$

Then, at a rate of 4 mph, the person would have to walk 4(2.8125), or 11.25 mi.

Exercise Set 1.6

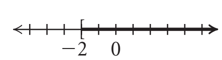
1.
$$\begin{aligned} 4x - 3 &> 2x + 7 & \\ 2x - 3 &> 7 & \text{Subtracting } 2x \\ 2x &> 10 & \text{Adding } 3 \\ x &> 5 & \text{Dividing by } 2 \end{aligned}$$

The solution set is $\{x|x > 5\}$, or $(5, \infty)$. The graph is shown below.



2.
$$\begin{aligned} 8x + 1 &\geq 5x - 5 \\ 3x &\geq -6 \\ x &\geq -2 \end{aligned}$$

The solution set is $\{x|x \geq -2\}$, or $[-2, \infty)$. The graph is shown below.



3. $x+6<5x-6$

$6+6<5x-x$ Subtracting x and adding 6 on both sides

$12<4x$

$\frac{12}{4}<x$ Dividing by 4 on both sides

$3<x$

This inequality could also be solved as follows:

$x+6<5x-6$

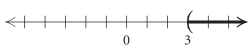
$x-5x<-6-6$ Subtracting $5x$ and 6 on both sides

$-4x<-12$

$x>\frac{-12}{-4}$ Dividing by -4 on both sides and reversing the inequality symbol

$x>3$

The solution set is $\{x|x>3\}$, or $(3,\infty)$. The graph is shown below.

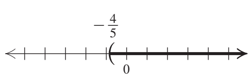


4. $3-x<4x+7$

$-5x<4$

$x>-\frac{4}{5}$

The solution set is $\left\{x\middle|x>-\frac{4}{5}\right\}$, or $\left(-\frac{4}{5},\infty\right)$. The graph is shown below.



5. $4-2x\le 2x+16$

$4-4x\le 16$ Subtracting $2x$

$-4x\le 12$ Subtracting 4

$x\ge -3$ Dividing by -4 and reversing the inequality symbol

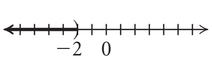
The solution set is $\{x|x\ge -3\}$, or $[-3,\infty)$. The graph is shown below.

6. $3x-1>6x+5$

$-3x>6$

$x<-2$

The solution set is $\{x|x<-2\}$, or $(-\infty,-2)$. The graph is shown below.



7. $14-5y\le 8y-8$

$14+8\le 8y+5y$

$22\le 13y$

$\frac{22}{13}\le y$

This inequality could also be solved as follows:

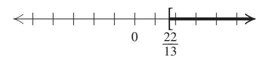
$14-5y\le 8y-8$

$-5y-8y\le -8-14$

$-13y\le -22$

$y\ge \frac{22}{13}$ Dividing by -13 on both sides and reversing the inequality symbol

The solution set is $\left\{y\middle|y\ge\frac{22}{13}\right\}$, or $\left[\frac{22}{13},\infty\right)$. The graph is shown below.

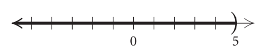


8. $8x-7<6x+3$

$2x<10$

$x<5$

The solution set is $\{x|x<5\}$, or $(-\infty,5)$. The graph is shown below.



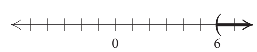
9. $7x-7>5x+5$

$2x-7>5$ Subtracting $5x$

$2x>12$ Adding 7

$x>6$ Dividing by 2

The solution set is $\{x|x>6\}$, or $(6,\infty)$. The graph is shown below.

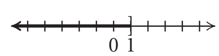


10. $12-8y\ge 10y-6$

$-18y\ge -18$

$y\le 1$

The solution set is $\{y|y\le 1\}$, or $(-\infty,1]$. The graph is shown below.



11. $3x-3+2x\ge 1-7x-9$

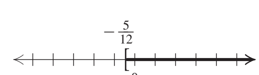
$5x-3\ge -7x-8$ Collecting like terms

$5x+7x\ge -8+3$ Adding $7x$ and 3 on both sides

$12x\ge -5$

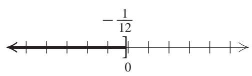
$x\ge -\frac{5}{12}$ Dividing by 12 on both sides

The solution set is $\left\{x\middle|x\ge-\frac{5}{12}\right\}$, or $\left[-\frac{5}{12},\infty\right)$. The graph is shown below.



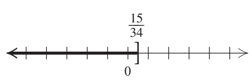
12. $$\begin{aligned} 5y-5+y &\le 2-6y-8 \\ 6y-5 &\le -6y-6 \\ 12y &\le -1 \\ y &\le -\frac{1}{12} \end{aligned}$$

The solution set is $\left\{y\middle|y \le -\frac{1}{12}\right\}$, or $\left(-\infty, -\frac{1}{12},\right]$. The graph is shown below.



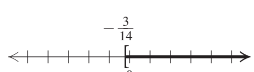
13. $$\begin{aligned} -\frac{3}{4}x &\ge -\frac{5}{8}+\frac{2}{3}x \\ \frac{5}{8} &\ge \frac{3}{4}x+\frac{2}{3}x \\ \frac{5}{8} &\ge \frac{9}{12}x+\frac{8}{12}x \\ \frac{5}{8} &\ge \frac{17}{12}x \\ \frac{12}{17}\cdot\frac{5}{8} &\ge \frac{12}{17}\cdot\frac{17}{12}x \\ \frac{15}{34} &\ge x \end{aligned}$$

The solution set is $\left\{x\middle|x \le \frac{15}{34}\right\}$, or $\left(-\infty, \frac{15}{34}\right]$. The graph is shown below.



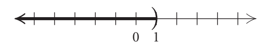
14. $$\begin{aligned} -\frac{5}{6}x &\le \frac{3}{4}+\frac{8}{3}x \\ -\frac{21}{6}x &\le \frac{3}{4} \\ x &\ge -\frac{3}{14} \end{aligned}$$

The solution set is $\left\{x\middle|x \ge -\frac{3}{14}\right\}$, or $\left[-\frac{3}{14}, \infty\right)$. The graph is shown below.



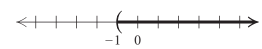
15. $$\begin{aligned} 4x(x-2) &< 2(2x-1)(x-3) \\ 4x(x-2) &< 2(2x^2-7x+3) \\ 4x^2-8x &< 4x^2-14x+6 \\ -8x &< -14x+6 \\ -8x+14x &< 6 \\ 6x &< 6 \\ x &< \frac{6}{6} \\ x &< 1 \end{aligned}$$

The solution set is $\{x|x<1\}$, or $(-\infty, 1)$. The graph is shown below.



16. $$\begin{aligned} (x+1)(x+2) &> x(x+1) \\ x^2+3x+2 &> x^2+x \\ 2x &> -2 \\ x &> -1 \end{aligned}$$

The solution set is $\{x|x>-1\}$, or $(-1, \infty)$. The graph is shown below.



17. The radicand must be nonnegative, so we solve the inequality $x-7 \ge 0$.

$$\begin{aligned} x-7 &\ge 0 \\ x &\ge 7 \end{aligned}$$

The domain is $\{x|x \ge 7\}$, or $[7, \infty)$.

18. $$\begin{aligned} x+8 &\ge 0 \\ x &\ge -8 \end{aligned}$$

The domain is $\{x|x \ge -8\}$, or $[-8, \infty)$.

19. The radicand must be nonnegative, so we solve the inequality $1-5x \ge 0$.

$$\begin{aligned} 1-5x &\ge 0 \\ 1 &\ge 5x \\ \frac{1}{5} &\ge x \end{aligned}$$

The domain is $\left\{x\middle|x \le \frac{1}{5}\right\}$, or $\left(-\infty, \frac{1}{5}\right]$.

20. $$\begin{aligned} 2x+3 &\ge 0 \\ 2x &\ge -3 \\ x &\ge -\frac{3}{2} \end{aligned}$$

The domain is $\left\{x\middle|x \ge -\frac{3}{2}\right\}$, or $\left[-\frac{3}{2}, \infty\right)$.

21. The radicand must be positive, so we solve the inequality $4+x>0$.

$$\begin{aligned} 4+x &> 0 \\ x &> -4 \end{aligned}$$

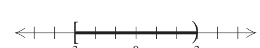
The domain is $\{x|x>-4\}$, or $(-4, \infty)$.

22. $$\begin{aligned} 8-x &> 0 \\ 8 &> x \end{aligned}$$

The domain is $\{x|x<8\}$, or $(-\infty, 8)$.

23. $$\begin{aligned} -2 &\le x+1 < 4 \\ -3 &\le x < 3 \quad \text{Subtracting 1} \end{aligned}$$

The solution set is $[-3, 3)$. The graph is shown below.



24. $-3 < x + 2 \leq 5$

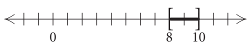
$-5 < x \leq 3$

$(-5, 3]$

25. $5 \leq x - 3 \leq 7$

$8 \leq x \leq 10$ Adding 3

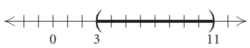
The solution set is $[8, 10]$. The graph is shown below.



26. $-1 < x - 4 < 7$

$3 < x < 11$

$(3, 11)$



27. $-3 \leq x + 4 \leq 3$

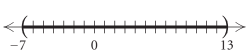
$-7 \leq x \leq -1$ Subtracting 4

The solution set is $[-7, -1]$. The graph is shown below.

28. $-5 < x + 2 < 15$

$-7 < x < 13$

$(-7, 13)$

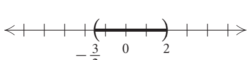


29. $-2 < 2x + 1 < 5$

$-3 < 2x < 4$ Adding -1

$-\frac{3}{2} < x < 2$ Multiplying by $\frac{1}{2}$

The solution set is $\left(-\frac{3}{2}, 2\right)$. The graph is shown below.

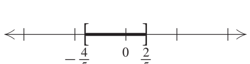


30. $-3 \leq 5x + 1 \leq 3$

$-4 \leq 5x \leq 2$

$-\frac{4}{5} \leq x \leq \frac{2}{5}$

$\left[-\frac{4}{5}, \frac{2}{5}\right]$



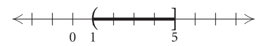
31. $-4 \leq 6 - 2x < 4$

$-10 \leq -2x < -2$ Adding -6

$5 \geq x > 1$ Multiplying by $-\frac{1}{2}$

or $1 < x \leq 5$

The solution set is $(1, 5]$. The graph is shown below.

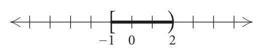


32. $-3 < 1 - 2x \leq 3$

$-4 < -2x \leq 2$

$2 > x \geq -1$

$[-1, 2)$



33. $-5 < \frac{1}{2}(3x + 1) < 7$

$-10 < 3x + 1 < 14$ Multiplying by 2

$-11 < 3x < 13$ Adding -1

$-\frac{11}{3} < x < \frac{13}{3}$ Multiplying by $\frac{1}{3}$

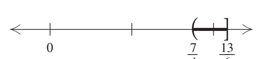
The solution set is $\left(-\frac{11}{3}, \frac{13}{3}\right)$. The graph is shown below.

34. $\frac{2}{3} \leq -\frac{4}{5}(x - 3) < 1$

$-\frac{5}{6} \geq x - 3 > -\frac{5}{4}$

$\frac{13}{6} \geq x > \frac{7}{4}$

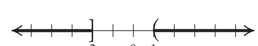
$\left(\frac{7}{4}, \frac{13}{6}\right]$



35. $3x \leq -6$ *or* $x - 1 > 0$

$x \leq -2$ *or* $x > 1$

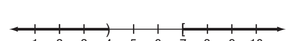
The solution set is $(-\infty, -2] \cup (1, \infty)$. The graph is shown below.



36. $2x < 8$ *or* $x + 3 \geq 10$

$x < 4$ *or* $x \geq 7$

$(-\infty, 4) \cup [7, \infty)$

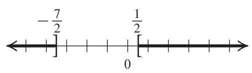


37. $2x + 3 \le -4$ *or* $2x + 3 \ge 4$

$2x \le -7$ *or* $2x \ge 1$

$x \le -\frac{7}{2}$ *or* $x \ge \frac{1}{2}$

The solution set is $\left(-\infty, -\frac{7}{2}\right] \cup \left[\frac{1}{2}, \infty\right)$. The graph is shown below.

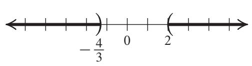


38. $3x - 1 < -5$ *or* $3x - 1 > 5$

$3x < -4$ *or* $3x > 6$

$x < -\frac{4}{3}$ *or* $x > 2$

$\left(-\infty, -\frac{4}{3}\right) \cup (2, \infty)$



39. $2x - 20 < -0.8$ *or* $2x - 20 > 0.8$

$2x < 19.2$ *or* $2x > 20.8$

$x < 9.6$ *or* $x > 10.4$

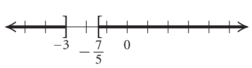
The solution set is $(-\infty, 9.6) \cup (10.4, \infty)$. The graph is shown below.

40. $5x + 11 \le -4$ *or* $5x + 11 \ge 4$

$5x \le -15$ *or* $5x \ge -7$

$x \le -3$ *or* $x \ge -\frac{7}{5}$

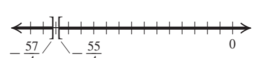
$(-\infty, -3] \cup \left[-\frac{7}{5}, \infty\right)$



41. $x + 14 \le -\frac{1}{4}$ *or* $x + 14 \ge \frac{1}{4}$

$x \le -\frac{57}{4}$ *or* $x \ge -\frac{55}{4}$

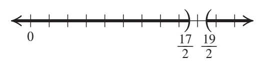
The solution set is $\left(-\infty, -\frac{57}{4}\right] \cup \left[-\frac{55}{4}, \infty\right)$. The graph is shown below.



42. $x - 9 < -\frac{1}{2}$ *or* $x - 9 > \frac{1}{2}$

$x < \frac{17}{2}$ *or* $x > \frac{19}{2}$

$\left(-\infty, \frac{17}{2}\right) \cup \left(\frac{19}{2}, \infty\right)$



43. ***Familiarize and Translate***. World rice production is given by the equation $y = 9.06x + 410.81$. We want to know when production will be more than 820 million metric tons, so we have

$9.06x + 410.81 > 820.$

Carry out. We solve the equation.

$9.06x + 410.81 > 820$

$9.06x > 409.19$

$x > 45$ Rounding

Check. When $x \approx 45$, $y = 9.06(45) + 410.81 = 818.51 \approx 820$. As a partial check, we could try a value of x less than 45 and one greater than 45. When $x = 44.8$, we have $y = 9.06(44.8) + 410.81 = 816.698 < 820$; when $x = 45.2$, we have $y = 9.06(45.2) + 410.81 = 820.322 > 820$. Since $y \approx 820$ when $x = 45$ and $y > 820$ when $x > 45$, the answer is probably correct.

State. World rice production will exceed 820 million metric tons more than 45 years after 1980.

44. Solve: $0.326x + 7.148 > 12$

$x > 15$, so more than 12 million people will be collecting Social Security disability payments more than 15 years after 2007.

45. ***Familiarize***. Let t = the number of hours worked. Then Acme Movers charge $200 + 45t$ and Leo's Movers charge $65t$.

Translate.

$$\underbrace{\text{Leo's charge}}_{\displaystyle 65t} \quad \underbrace{\text{is less than}}_{\displaystyle <} \quad \underbrace{\text{Acme's charge.}}_{\displaystyle 200 + 45t}$$

Carry out. We solve the inequality.

$65t < 200 + 45t$

$20t < 200$

$t < 10$

Check. When $t = 10$, Leo's Movers charge $65 \cdot 10$, or \$650 and Acme Movers charge $200 + 45 \cdot 10$, or \$650, so the charges are the same. As a partial check, we find the charges for a value of $t < 10$. When $t = 9.5$, Leo's Movers charge $65(9.5) = \$617.50$ and Acme Movers charge $200 + 45(9.5) = \$627.50$. Since Leo's charge is less than Acme's, the answer is probably correct.

State. For times less than 10 hr it costs less to hire Leo's Movers.

46. Let x = the amount invested at 4%. Then $12,000 - x$ = the amount invested at 6%.

Solve: $0.04x + 0.06(12,000 - x) \ge 650$

$x \le 3500$, so at most \$3500 can be invested at 4%.

47. ***Familiarize***. Let x = the amount invested at 4%. Then $7500 - x$ = the amount invested at 5%. Using the simple-interest formula, $I = Prt$, we see that in one year the

4% investment earns $0.04x$ and the 5% investment earns $0.05(7500 - x)$.

Translate.

$$\underbrace{\text{Interest at 4\%}}_{0.04x}\ \underset{+}{\text{plus}}\ \underbrace{\text{interest at 5\%}}_{0.05(7500-x)}\ \underbrace{\text{is at least}}_{\geq}\ \underset{325}{\$325.}$$

Carry out. We solve the inequality.

$$\begin{aligned}0.04x + 0.05(7500 - x) &\geq 325\\ 0.04x + 375 - 0.05x &\geq 325\\ -0.01x + 375 &\geq 325\\ -0.01x &\geq -50\\ x &\leq 5000\end{aligned}$$

Check. When \$5000 is invested at 4%, then \$7500−\$5000, or \$2500, is invested at 5%. In one year the 4% investment earns 0.04(\$5000), or \$200, in simple interest and the 5% investment earns 0.05(\$2500), or \$125, so the total interest is \$200 + \$125, or \$325. As a partial check, we determine the total interest when an amount greater than \$5000 is invested at 4%. Suppose \$5001 is invested at 4%. Then \$2499 is invested at 5%, and the total interest is 0.04(\$5001) + 0.05(\$2499), or \$324.99. Since this amount is less than \$325, the answer is probably correct.

State. The most that can be invested at 4% is \$5000.

48. Let x = the amount invested at 7%. Then $2x$ = the amount invested at 4%, and $150,000 - x - 2x$, or $150,000 - 3x$ = the amount invested at 5.5%. The interest earned is $0.07x + 0.04 \cdot 2x + 0.055(150,000 - 3x)$, or $0.07x + 0.08x + 8250 - 0.165x$, or $-0.015x + 8250$.

Solve: $-0.015x + 8250 \geq 7575$

$x \leq 45,000$, so $2x \leq 90,000$

Thus the most that can be invested at 4% is \$90,000.

49. ***Familiarize and Translate.*** Let x = the amount invested at 5%. Then

$\frac{1}{2}x$ = the amount invested at 3.5%, and

$1,400,000 - x - \frac{1}{2}x$, or $1,400,000 - \frac{3}{2}x$ = the amount invested at 5.5%. The interest earned is

$0.05x + 0.035\left(\frac{1}{2}x\right) + 0.055\left(1,400,000 - \frac{3}{2}x\right)$, or $0.05x + 0.0175x + 77,000 - 0.0825x$, or $-0.015x + 77,000$. The foundation wants the interest to be at least \$68,000, so we have

$$-0.015x + 77,000 \geq 68,000.$$

Carry out. We solve the inequality.

$$\begin{aligned}-0.015x + 77,000 &\geq 68,000\\ -0.015x &\geq -9000\\ x &\leq 600,000\end{aligned}$$

If $x \leq 600,000$ then $\frac{1}{2}x \leq 300,000$.

Check. If \$600,000 is invested at 5% and \$300,000 is invested at 3.5%, then the amount invested at 5.5% is \$1,400,000 − \$600,000 − \$300,000 = \$500,000. The interest earned is 0.05(\$600,000) + 0.035(\$300,000) + 0.055(\$500,000), or \$30,000 + \$10,500 + \$27,500, or \$68,000. As a partial check, we can determine if the total interest earned when more than \$300,000 is invested at 3.5% is less than \$68,000. This is the case, so the answer is probably correct.

State. The most than can be invested at 3.5% is \$300,000.

50. Let s = the monthly sales.

Solve: $750 + 0.1s > 1000 + 0.08(s - 2000)$

$s > 4500$, so Plan A is better for monthly sales greater than \$4500.

51. ***Familiarize.*** Let s = the monthly sales. Then the amount of sales in excess of \$8000 is $s - 8000$.

Translate.

$$\underbrace{\text{Income from plan B}}_{1200 + 0.15(s - 8000)}\ \underbrace{\text{is greater than}}_{>}\ \underbrace{\text{income from plan A.}}_{900 + 0.1s}$$

Carry out. We solve the inequality.

$$\begin{aligned}1200 + 0.15(s - 8000) &> 900 + 0.1s\\ 1200 + 0.15s - 1200 &> 900 + 0.1s\\ 0.15s &> 900 + 0.1s\\ 0.05s &> 900\\ s &> 18,000\end{aligned}$$

Check. For sales of \$18,000 the income from plan A is \$900+0.1(\$18,000), or \$2700, and the income from plan B is 1200 + 0.15(18,000 − 8000), or \$2700 so the incomes are the same. As a partial check we can compare the incomes for an amount of sales greater than \$18,000. For sales of \$18,001, for example, the income from plan A is \$900 + 0.1(\$18,001), or \$2700.10, and the income from plan B is \$1200 + 0.15(\$18,001 − \$8000), or \$2700.15. Since plan B is better than plan A in this case, the answer is probably correct.

State. Plan B is better than plan A for monthly sales greater than \$18,000.

52. Solve: $200 + 12n > 20n$

$n < 25$

53. Function; domain; range; domain; exactly one; range

54. Midpoint formula

55. x-intercept

56. Constant; identity

57. $2x \leq 5 - 7x < 7 + x$

$$\begin{aligned}&2x \leq 5 - 7x \quad \textit{and} \quad 5 - 7x < 7 + x\\ &9x \leq 5 \quad \textit{and} \quad -8x < 2\\ &x \leq \frac{5}{9} \quad \textit{and} \quad x > -\frac{1}{4}\end{aligned}$$

The solution set is $\left(-\frac{1}{4}, \frac{5}{9}\right]$.

58. $x \le 3x - 2 \le 2 - x$

$$\begin{aligned} x \le 3x - 2 \quad &\textit{and} \quad 3x - 2 \le 2 - x \\ -2x \le -2 \quad &\textit{and} \quad 4x \le 4 \\ x \ge 1 \quad &\textit{and} \quad x \le 1 \end{aligned}$$

The solution is 1.

59.
$$\begin{aligned} 3y &< 4 - 5y < 5 + 3y \\ 0 &< 4 - 8y < 5 && \text{Subtracting } 3y \\ -4 &< -8y < 1 && \text{Subtracting } 4 \\ \frac{1}{2} &> y > -\frac{1}{8} && \text{Dividing by } -8 \text{ and reversing the inequality symbols} \end{aligned}$$

The solution set is $\left(-\frac{1}{8}, \frac{1}{2}\right)$.

60.
$$\begin{aligned} y - 10 &< 5y + 6 \le y + 10 \\ -10 &< 4y + 6 \le 10 && \text{Subtracting } y \\ -16 &< 4y \le 4 \\ -4 &< y \le 1 \end{aligned}$$

The solution set is $(-4, 1]$.

Chapter 1 Review Exercises

1. First we solve each equation for y.

$$\begin{aligned} ax + y &= c & x - by &= d \\ y &= -ax + c & -by &= -x + d \\ & & y &= \frac{1}{b}x - \frac{d}{b} \end{aligned}$$

If the lines are perpendicular, the product of their slopes is -1, so we have $-a \cdot \frac{1}{b} = -1$, or $-\frac{a}{b} = -1$, or $\frac{a}{b} = 1$. The statement is true.

2. For the lines $y = \frac{1}{2}$ and $x = -5$, the x-coordinate of the point of intersection is -5 and the y-coordinate is $\frac{1}{2}$, so the statement is true.

3. $f(-3) = \frac{\sqrt{3-(-3)}}{-3} = \frac{\sqrt{6}}{-3}$, so -3 is in the domain of $f(x)$. Thus, the statement is false.

4. The line parallel to the x-axis that passes through $\left(-\frac{1}{4}, 7\right)$ is 7 units above the x-axis. Thus, its equation is $y = 7$. The given statement is false.

5. The statement is true. See page 133 in the text.

6. The statement is false. See page 139 in the text.

7. For $\left(3, \frac{24}{9}\right)$:

$$\begin{array}{r|l} 2x - 9y & = -18 \\ \hline 2 \cdot 3 - 9 \cdot \frac{24}{9} & ?\ -18 \\ 6 - 24 & \\ -18 & \ -18 \quad \text{TRUE} \end{array}$$

$\left(3, \frac{24}{9}\right)$ is a solution.

For $(0, -9)$:

$$\begin{array}{r|l} 2x - 9y & = -18 \\ \hline 2(0) - 9(-9) & ?\ -18 \\ 0 + 81 & \\ 81 & \ -18 \quad \text{FALSE} \end{array}$$

$(0, -9)$ is not a solution.

8. For $(0, 7)$:

$$\begin{array}{r|l} y & = 7 \\ \hline 7 & ?\ 7 \quad \text{TRUE} \end{array}$$

$(0, 7)$ is a solution.

For $(7, 1)$:

$$\begin{array}{r|l} y & = 7 \\ \hline 1 & ?\ 7 \quad \text{FALSE} \end{array}$$

$(7, 1)$ is not a solution.

9. $2x - 3y = 6$

To find the x-intercept we replace y with 0 and solve for x.

$$\begin{aligned} 2x - 3 \cdot 0 &= 6 \\ 2x &= 6 \\ x &= 3 \end{aligned}$$

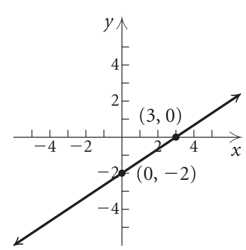
The x-intercept is $(3, 0)$.

To find the y-intercept we replace x with 0 and solve for y.

$$\begin{aligned} 2 \cdot 0 - 3y &= 6 \\ -3y &= 6 \\ y &= -2 \end{aligned}$$

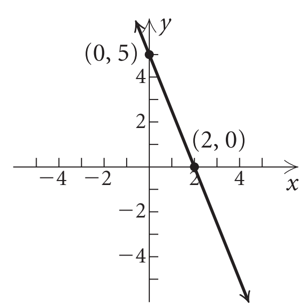
The y-intercept is $(0, -2)$.

We plot the intercepts and draw the line that contains them. We could find a third point as a check that the intercepts were found correctly.



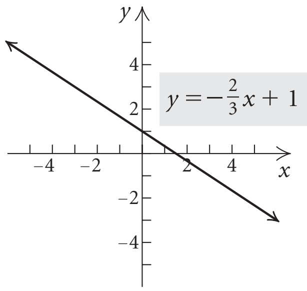
$2x - 3y = 6$

10.

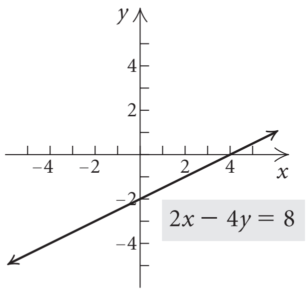


$10 - 5x = 2y$

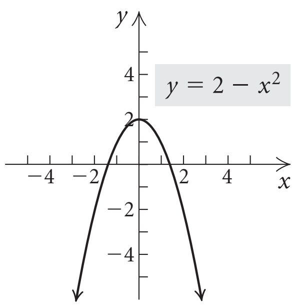
11.



12.



13.



14. $d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

$= \sqrt{(3 - (-2))^2 + (7 - 4)^2}$

$= \sqrt{5^2 + 3^2} = \sqrt{34} \approx 5.831$

15. $m = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

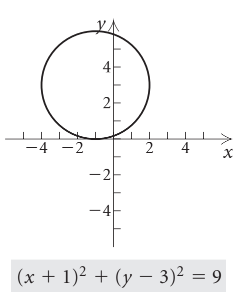
$= \left(\frac{3 + (-2)}{2}, \frac{7 + 4}{2}\right)$

$= \left(\frac{1}{2}, \frac{11}{2}\right)$

16. $(x + 1)^2 + (y - 3)^2 = 9$

$[x - (-1)]^2 + (y - 3)^2 = 3^2$ Standard form

The center is $(-1, 3)$ and the radius is 3.



17. $(x - h)^2 + (y - k)^2 = r^2$

$(x - 0)^2 + [y - (-4)]^2 = \left(\frac{3}{2}\right)^2$ Substituting

$x^2 + (y + 4)^2 = \frac{9}{4}$

18. $(x - h)^2 + (y - k)^2 = r^2$

$[x - (-2)]^2 + (y - 6)^2 = (\sqrt{13})^2$

$(x + 2)^2 + (y - 6)^2 = 13$

19. The center is the midpoint of the diameter:

$\left(\frac{-3 + 7}{2}, \frac{5 + 3}{2}\right) = \left(\frac{4}{2}, \frac{8}{2}\right) = (2, 4)$

Use the center and either endpoint of the diameter to find the radius. We use the point $(7, 3)$.

$r = \sqrt{(7 - 2)^2 + (3 - 4)^2} = \sqrt{5^2 + (-1)^2} = \sqrt{25 + 1} = \sqrt{26}$

The equation of the circle is $(x-2)^2 + (y-4)^2 = (\sqrt{26})^2$, or $(x - 2)^2 + (y - 4)^2 = 26$.

20. The correspondence is not a function because one member of the domain, 2, corresponds to more than one member of the range.

21. The correspondence is a function because each member of the domain corresponds to exactly one member of the range.

22. The relation is not a function, because the ordered pairs $(3, 1)$ and $(3, 5)$ have the same first coordinate and different second coordinates.

Domain: $\{3, 5, 7\}$

Range: $\{1, 3, 5, 7\}$

23. The relation is a function, because no two ordered pairs have the same first coordinate and different second coordinates. The domain is the set of first coordinates: $\{-2, 0, 1, 2, 7\}$. The range is the set of second coordinates: $\{-7, -4, -2, 2, 7\}$.

24. $f(x) = x^2 - x - 3$

a) $f(0) = 0^2 - 0 - 3 = -3$

b) $f(-3) = (-3)^2 - (-3) - 3 = 9 + 3 - 3 = 9$

c) $f(a - 1) = (a - 1)^2 - (a - 1) - 3$
$= a^2 - 2a + 1 - a + 1 - 3$
$= a^2 - 3a - 1$

d) $f(-x) = (-x)^2 - (-x) - 3$
$= x^2 + x - 3$

25. $f(x) = \frac{x - 7}{x + 5}$

a) $f(7) = \frac{7 - 7}{7 + 5} = \frac{0}{12} = 0$

b) $f(x + 1) = \frac{x + 1 - 7}{x + 1 + 5} = \frac{x - 6}{x + 6}$

c) $f(-5) = \frac{-5 - 7}{-5 + 5} = \frac{-12}{0}$

Since division by 0 is not defined, $f(-5)$ does not exist.

d) $f\left(-\dfrac{1}{2}\right) = \dfrac{-\dfrac{1}{2}-7}{-\dfrac{1}{2}+5} = \dfrac{-\dfrac{15}{2}}{\dfrac{9}{2}} = -\dfrac{15}{2}\cdot\dfrac{2}{9} =$

$-\dfrac{\cancel{3}\cdot 5\cdot \cancel{2}}{\cancel{2}\cdot\cancel{3}\cdot 3} = -\dfrac{5}{3}$

26. From the graph we see that when the input is 2, the output is -1, so $f(2) = -1$. When the input is -4, the output is -3, so $f(-4) = -3$. When the input is 0, the output is -1, so $f(0) = -1$.

27. This is not the graph of a function, because we can find a vertical line that crosses the graph more than once.

28. This is the graph of a function, because there is no vertical line that crosses the graph more than once.

29. This is not the graph of a function, because we can find a vertical line that crosses the graph more than once.

30. This is the graph of a function, because there is no vertical line that crosses the graph more than once.

31. We can substitute any real number for x. Thus, the domain is the set of all real numbers, or $(-\infty, \infty)$.

32. The input 0 results in a denominator of zero. Thus, the domain is $\{x|x \neq 0\}$, or $(-\infty, 0) \cup (0, \infty)$.

33. Find the inputs that make the denominator zero:

$$x^2 - 6x + 5 = 0$$
$$(x-1)(x-5) = 0$$
$$x - 1 = 0 \quad or \quad x - 5 = 0$$
$$x = 1 \quad or \quad x = 5$$

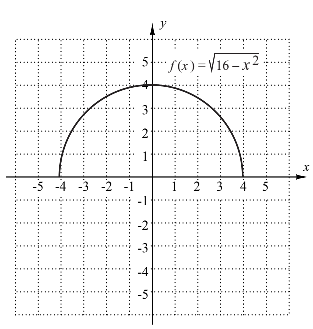
The domain is $\{x|x \neq 1 \text{ and } x \neq 5\}$, or $(-\infty, 1) \cup (1, 5) \cup (5, \infty)$.

34. Find the inputs that make the denominator zero:

$$|16 - x^2| = 0$$
$$16 - x^2 = 0$$
$$(4+x)(4-x) = 0$$
$$4 + x = 0 \quad or \quad 4 - x = 0$$
$$x = -4 \quad or \quad 4 = x$$

The domain is $\{x|x \neq -4 \text{ and } x \neq 4\}$, or $(-\infty, -4) \cup (-4, 4) \cup (4, \infty)$.

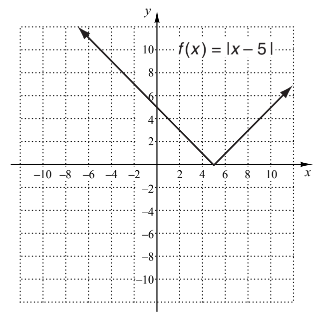
35.



The inputs on the x axis extend from -4 to 4, so the domain is $[-4, 4]$.

The outputs on the y-axis extend from 0 to 4, so the range is $[0, 4]$.

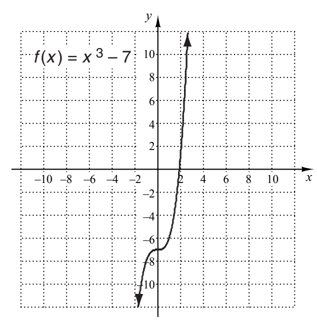
36.



Each point on the x-axis corresponds to a point on the graph, so the domain is the set of all real numbers, or $(-\infty, \infty)$.

The number 0 is the smallest output on the y-axis and every number greater than 0 is also an output, so the range is $[0, \infty)$.

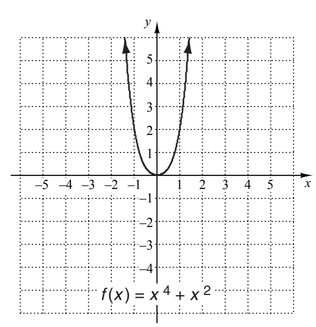
37.



Every point on the x-axis corresponds to a point on the graph, so the domain is the set of all real numbers, or $(-\infty, \infty)$.

Each point on the y-axis also corresponds to a point on the graph, so the range is the set of all real numbers, or $(-\infty, \infty)$.

38.



Each point on the x-axis corresponds to a point on the graph, so the domain is the set of all real numbers, or $(-\infty, \infty)$.

The number 0 is the smallest output on the y-axis and every number greater than 0 is also an output, so the range is $[0, \infty)$.

39. a) Yes. Each input is 1 more than the one that precedes it.

b) No. The change in the output varies.

c) No. Constant changes in inputs do not result in constant changes in outputs.

40. a) Yes. Each input is 10 more than the one that precedes it.

b) Yes. Each output is 12.4 more than the one that precedes it.

c) Yes. Constant changes in inputs result in constant changes in outputs.

41. $$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-6-(-11)}{5-2} = \frac{5}{3}$$

42. $m = \dfrac{4-4}{-3-5} = \dfrac{0}{-8} = 0$

43. $$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0-3}{\frac{1}{2} - \frac{1}{2}} = \frac{-3}{0}$$

The slope is not defined.

44. We have the data points (1990, 26.8) and (2012, 24.7). We find the average rate of change, or slope.

$$m = \frac{24.7 - 26.8}{2012 - 1990} = \frac{-2.1}{22} \approx -0.1$$

The average rate of change in per capita coffee consumption from 1990 to 2012 was about -0.1 gallons per year.

45. $y = -\dfrac{7}{11}x - 6$

The equation is in the form $y = mx + b$. The slope is $-\dfrac{7}{11}$, and the y-intercept is $(0, -6)$.

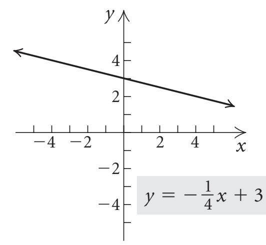
46.
$$-2x - y = 7$$
$$-y = 2x + 7$$
$$y = -2x - 7$$

Slope: -2; y-intercept: $(0, -7)$

47. Graph $y = -\dfrac{1}{4}x + 3$.

Plot the y-intercept, $(0, 3)$. We can think of the slope as $\dfrac{-1}{4}$. Start at $(0, 3)$ and find another point by moving down 1 unit and right 4 units. We have the point $(4, 2)$.

We could also think of the slope as $\dfrac{1}{-4}$. Then we can start at $(0, 3)$ and find another point by moving up 1 unit and left 4 units. We have the point $(-4, 4)$. Connect the three points and draw the graph.



48. Let t = number of months of basic service.

$C(t) = 110 + 85t$

$C(12) = 110 + 85 \cdot 12 = \$1130$

49. a) $T(d) = 10d + 20$

$T(5) = 10(5) + 20 = 70°\text{C}$

$T(20) = 10(20) + 20 = 220°\text{C}$

$T(1000) = 10(1000) + 20 = 10,020°\text{C}$

b) 5600 km is the maximum depth. Domain: $[0, 5600]$.

50. $y = mx + b$

$y = -\dfrac{2}{3}x - 4$ Substituting $-\dfrac{2}{3}$ for m and -4 for b

51.
$$y - y_1 = m(x - x_1)$$
$$y - (-1) = 3(x - (-2))$$
$$y + 1 = 3(x + 2)$$
$$y + 1 = 3x + 6$$
$$y = 3x + 5$$

52. First we find the slope.

$$m = \frac{-1-1}{-2-4} = \frac{-2}{-6} = \frac{1}{3}$$

Use the point-slope equation:

Using $(4, 1)$: $y - 1 = \dfrac{1}{3}(x - 4)$

Using $(-2, -1)$: $y - (-1) = \dfrac{1}{3}(x - (-2))$, or

$y + 1 = \dfrac{1}{3}(x + 2)$

In either case, we have $y = \dfrac{1}{3}x - \dfrac{1}{3}$.

53. The horizontal line that passes through $\left(-4, \dfrac{2}{5}\right)$ is $\dfrac{2}{5}$ unit above the x-axis. An equation of the line is $y = \dfrac{2}{5}$.

The vertical line that passes through $\left(-4, \dfrac{2}{5}\right)$ is 4 units to the left of the y-axis. An equation of the line is $x = -4$.

54. Two points on the line are $(-2, -9)$ and $(4, 3)$. First we find the slope.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3-(-9)}{4-(-2)} = \frac{12}{6} = 2$$

Now we use the point-slope equation with the point $(4, 3)$.

$$y - y_1 = m(x - x_1)$$
$$y - 3 = 2(x - 4)$$
$$y - 3 = 2x - 8$$
$$y = 2x - 5, \text{ or}$$
$$h(x) = 2x - 5$$

Then $h(0) = 2 \cdot 0 - 5 = -5$.

55. $3x - 2y = 8$ $\qquad$ $6x - 4y = 2$

$y = \frac{3}{2}x - 4$ $\qquad$ $y = \frac{3}{2}x - \frac{1}{2}$

The lines have the same slope, $\frac{3}{2}$, and different y-intercepts, $(0, -4)$ and $\left(0, -\frac{1}{2}\right)$, so they are parallel.

56. $y - 2x = 4$ $\qquad$ $2y - 3x = -7$

$y = 2x + 4$ $\qquad$ $y = \frac{3}{2}x - \frac{7}{2}$

The lines have different slopes, 2 and $\frac{3}{2}$, so they are not parallel. The product of the slopes, $2 \cdot \frac{3}{2}$, or 3, is not -1, so the lines are not perpendicular. Thus the lines are neither parallel nor perpendicular.

57. The slope of $y = \frac{3}{2}x + 7$ is $\frac{3}{2}$ and the slope of $y = -\frac{2}{3}x - 4$ is $-\frac{2}{3}$. Since $\frac{3}{2}\left(-\frac{2}{3}\right) = -1$, the lines are perpendicular.

58. $2x + 3y = 4$

$$3y = -2x + 4$$
$$y = -\frac{2}{3}x + \frac{4}{3};\ m = -\frac{2}{3}$$

The slope of a line parallel to the given line is $-\frac{2}{3}$.

We use the point-slope equation.

$$y - y_1 = m(x - x_1)$$
$$y - (-1) = -\frac{2}{3}(x - 1)$$
$$y = -\frac{2}{3}x - \frac{1}{3}$$

59. From Exercise 58 we know that the slope of the given line is $-\frac{2}{3}$. The slope of a line perpendicular to this line is the negative reciprocal of $-\frac{2}{3}$, or $\frac{3}{2}$.

We use the slope-intercept equation to find the y-intercept.

$$y = mx + b$$
$$-1 = \frac{3}{2} \cdot 1 + b$$
$$-1 = \frac{3}{2} + b$$
$$-\frac{5}{2} = b$$

Then the equation of the desired line is $y = \frac{3}{2}x - \frac{5}{2}$.

60. a) Answers may vary depending on the data points used and when rounding is done. We will use $(2,\ 7969)$ and $(8,\ 8576)$.

$$m = \frac{8576 - 7969}{8 - 2} = \frac{607}{6} \approx 101.17$$

We will use the point-slope equation with $(2,\ 7969)$.

$$W(x) - 7969 = 101.17(x - 2)$$
$$W(x) - 7969 = 101.17x - 202.34$$
$$W(x) = 101.17x + 7766.66$$

Now we find $W(7)$.

$W(7) = 101.17(7) + 7766.66 \approx 8475$ female graduates

b) $y = 98.9x + 7747.8$

When $x = 7$, $y \approx 8440$ female graduates.

$r \approx 0.9942$; the line fits the data well.

61. $4y - 5 = 1$

$$4y = 6$$
$$y = \frac{3}{2}$$

The solution is $\frac{3}{2}$.

62. $3x - 4 = 5x + 8$

$$-12 = 2x$$
$$-6 = x$$

63. $5(3x + 1) = 2(x - 4)$

$$15x + 5 = 2x - 8$$
$$13x = -13$$
$$x = -1$$

The solution is -1.

64. $2(n - 3) = 3(n + 5)$

$$2n - 6 = 3n + 15$$
$$-21 = n$$

65. $\frac{3}{5}y - 2 = \frac{3}{8}$ $\qquad$ The LCD is 40

$40\left(\frac{3}{5}y - 2\right) = 40 \cdot \frac{3}{8}$ $\qquad$ Multiplying to clear fractions

$$24y - 80 = 15$$
$$24y = 95$$
$$y = \frac{95}{24}$$

The solution is $\frac{95}{24}$.

66. $5 - 2x = -2x + 3$

$5 = 3$ $\qquad$ False equation

The equation has no solution.

67. $x - 13 = -13 + x$

$-13 = -13$ Subtracting x

We have an equation that is true for any real number, so the solution set is the set of all real numbers, $\{x|x \text{ is a real number}\}$, or $(-\infty, \infty)$.

68. Let $q =$ the number of quarters produced in 2012, in millions.

Solve: $q + 1.56q = 1455$

$q \approx 568$ million quarters

69. ***Familiarize***. Let $a =$ the amount originally invested. Using the simple interest formula, $I = Prt$, we see that the interest earned at 5.2% interest for 1 year is $a(0.052) \cdot 1 = 0.052a$.

Translate.

$$\underbrace{\text{Amount invested}}_{\downarrow\; a} \;\; \underset{\downarrow\; +}{\text{plus}} \;\; \underbrace{\text{interest earned}}_{\downarrow\; 0.052a} \;\; \underset{\downarrow\; =}{\text{is}} \;\; \underset{\downarrow\; 2419.60}{\$2419.60}$$

Carry out. We solve the equation.

$$a + 0.052a = 2419.60$$
$$1.052a = 2419.60$$
$$a = 2300$$

Check. 5.2% of \$2300 is 0.052(\$2300), or \$119.60, and \$2300 + \$119.60 = \$2419.60. The answer checks.

State. \$2300 was originally invested.

70. Let $t =$ the time it will take the plane to travel 1802 mi.

Solve: $1802 = (550 - 20)t$

$t = 3.4$ hr

71. $6x - 18 = 0$

$6x = 18$

$x = 3$

The zero of the function is 3.

72. $x - 4 = 0$

$x = 4$

The zero of the function is 4.

73. $2 - 10x = 0$

$-10x = -2$

$x = \frac{1}{5}$, or 0.2

The zero of the function is $\frac{1}{5}$, or 0.2.

74. $8 - 2x = 0$

$-2x = -8$

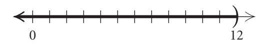
$x = 4$

The zero of the function is 4.

75. $2x - 5 < x + 7$

$x < 12$

The solution set is $\{x|x < 12\}$, or $(-\infty, 12)$.



76. $3x + 1 \geq 5x + 9$

$-2x + 1 \geq 9$ Subtracting $5x$

$-2x \geq 8$ Subtracting 1

$x \leq -4$ Dividing by -2 and reversing the inequality symbol

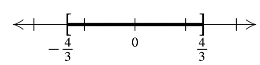
The solution set is $\{x|x \leq -4\}$, or $(-\infty, -4]$.

77. $-3 \leq 3x + 1 \leq 5$

$-4 \leq 3x \leq 4$

$-\frac{4}{3} \leq x \leq \frac{4}{3}$

$\left[-\frac{4}{3}, \frac{4}{3}\right]$

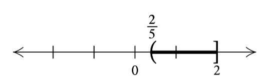


78. $-2 < 5x - 4 \leq 6$

$2 < 5x \leq 10$ Adding 4

$\frac{2}{5} < x \leq 2$ Dividing by 5

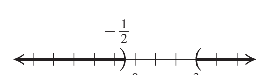
The solution set is $\left(\frac{2}{5}, 2\right]$.



79. $2x < -1$ *or* $x - 3 > 0$

$x < -\frac{1}{2}$ *or* $x > 3$

The solution set is $\left\{x \middle| x < -\frac{1}{2} \text{ or } x > 3\right\}$, or $\left(-\infty, -\frac{1}{2}\right) \cup (3, \infty)$.

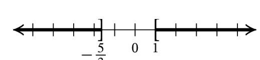


80. $3x + 7 \leq 2$ *or* $2x + 3 \geq 5$

$3x \leq -5$ *or* $2x \geq 2$

$x \leq -\frac{5}{3}$ *or* $x \geq 1$

The solution set is $\left(-\infty, -\frac{5}{3}\right] \cup [1, \infty)$.



81. ***Familiarize and Translate***. The number of homeschooled children in the U.S., in millions, is estimated by the equation $y = 0.073x + 0.848$, where x is the number of years after 1999. We want to know for what year this number will exceed 2.3 million, so we have

$$0.073x + 0.848 > 2.3.$$

Carry out. We solve the inequality.

$$0.073x + 0.848 > 2.3$$
$$0.073x > 1.452$$
$$x > 20 \quad \text{Rounding}$$

Check. When $x = 20$, $y = 0.073(20) + 0.848 = 2.308 \approx 2.3$. As a partial check, we could try a value less than 20 and a value greater than 20. When $x = 19$, we have $y = 0.073(19) + 0.848 = 2.235 < 2.3$; when $x = 21$, we have $y = 0.073(21) + 0.848 = 2.381 > 2.3$. Since $y \approx 2.3$ when $x = 20$ and $y > 2.3$ when $x = 21 > 20$, the answer is probably correct.

State. In years more than about 20 years after 1999, or in years after 2019, the number of homeschooled children will exceed 2.3 million.

82. Solve: $\frac{5}{9}(F - 32) < 45$

$F < 113°$

83. $f(x) = \frac{x+3}{8-4x}$

When $x = 2$, the denominator is 0, so 2 is not in the domain of the function. Thus, the domain is $(-\infty, 2) \cup (2, \infty)$ and answer B is correct.

84. $(x-1)^2 + y^2 = 9$

$(x-1)^2 + (y-0)^2 = 3^2$

The center is $(1, 0)$, so answer B is correct.

85. The graph of $f(x) = -\frac{1}{2}x - 2$ has slope $-\frac{1}{2}$, so it slants down from left to right. The y-intercept is $(0, -2)$. Thus, graph C is the graph of this function.

86. Let $(x, 0)$ be the point on the x-axis that is equidistant from the points $(1, 3)$ and $(4, -3)$. Then we have:

$$\sqrt{(x-1)^2+(0-3)^2} = \sqrt{(x-4)^2+(0-(-3))^2}$$
$$\sqrt{x^2-2x+1+9} = \sqrt{x^2-8x+16+9}$$
$$\sqrt{x^2-2x+10} = \sqrt{x^2-8x+25}$$
$$x^2-2x+10 = x^2-8x+25 \quad \text{Squaring both sides}$$
$$6x = 15$$
$$x = \frac{5}{2}$$

The point is $\left(\frac{5}{2}, 0\right)$.

87. $f(x) = \frac{\sqrt{1-x}}{x-|x|}$

We cannot find the square root of a negative number, so $x \le 1$. Division by zero is undefined, so $x < 0$.

Domain of f is $\{x|x < 0\}$, or $(-\infty, 0)$.

88. $f(x) = (x - 9x^{-1})^{-1} = \frac{1}{x - \frac{9}{x}}$

Division by zero is undefined, so $x \ne 0$. Also, note that we can write the function as $f(x) = \frac{x}{x^2-9}$, so $x \ne -3, 0, 3$.

Domain of f is $\{x|x \ne -3 \text{ and } x \ne 0 \text{ and } x \ne 3\}$, or $(-\infty, -3) \cup (-3, 0) \cup (0, 3) \cup (3, \infty)$.

89. Think of the slopes as $\frac{-3/5}{1}$ and $\frac{1/2}{1}$. The graph of $f(x)$ changes $\frac{3}{5}$ unit vertically for each unit of horizontal change while the graph of $g(x)$ changes $\frac{1}{2}$ unit vertically for each unit of horizontal change. Since $\frac{3}{5} > \frac{1}{2}$, the graph of $f(x) = -\frac{3}{5}x + 4$ is steeper than the graph of $g(x) = \frac{1}{2}x - 6$.

90. If an equation contains no fractions, using the addition principle before using the multiplication principle eliminates the need to add or subtract fractions.

91. The solution set of a disjunction is a union of sets, so it is not possible for a disjunction to have no solution.

92. The graph of $f(x) = mx + b$, $m \ne 0$, is a straight line that is not horizontal. The graph of such a line intersects the x-axis exactly once. Thus, the function has exactly one zero.

93. By definition, the notation $3 < x < 4$ indicates that $3 < x$ *and* $x < 4$. The disjunction $x < 3$ *or* $x > 4$ cannot be written $3 > x > 4$, or $4 < x < 3$, because it is not possible for x to be greater than 4 *and* less than 3.

94. A function is a correspondence between two sets in which each member of the first set corresponds to exactly one member of the second set.

Chapter 1 Test

1.

$5y - 4$	$= x$
$5 \cdot \frac{9}{10} - 4$	? $\frac{1}{2}$
$\frac{9}{2} - 4$	
$\frac{1}{2}$	$\frac{1}{2}$ TRUE

$\left(\frac{1}{2}, \frac{9}{10}\right)$ is a solution.

2. $5x - 2y = -10$

To find the x-intercept we replace y with 0 and solve for x.

$$5x - 2 \cdot 0 = -10$$
$$5x = -10$$
$$x = -2$$

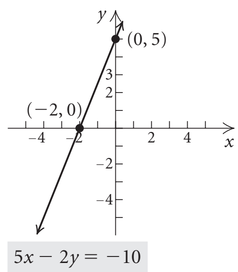
The x-intercept is $(-2, 0)$.

To find the y-intercept we replace x with 0 and solve for y.

$$\begin{aligned} 5 \cdot 0 - 2y &= -10 \\ -2y &= -10 \\ y &= 5 \end{aligned}$$

The y-intercept is $(0, 5)$.

We plot the intercepts and draw the line that contains them. We could find a third point as a check that the intercepts were found correctly.



3. $d = \sqrt{(5-(-1))^2 + (8-5)^2} = \sqrt{6^2+3^2} = \sqrt{36+9} = \sqrt{45} \approx 6.708$

4. $m = \left(\dfrac{-2+(-4)}{2}, \dfrac{6+3}{2}\right) = \left(\dfrac{-6}{2}, \dfrac{9}{2}\right) = \left(-3, \dfrac{9}{2}\right)$

5.
$$\begin{aligned} (x+4)^2 + (y-5)^2 &= 36 \\ [x-(-4)]^2 + (y-5)^2 &= 6^2 \end{aligned}$$

Center: $(-4, 5)$; radius: 6

6.
$$\begin{aligned} [x-(-1)]^2 + (y-2)^2 &= (\sqrt{5})^2 \\ (x+1)^2 + (y-2)^2 &= 5 \end{aligned}$$

7. a) The relation is a function, because no two ordered pairs have the same first coordinate and different second coordinates.

b) The domain is the set of first coordinates: $\{-4, 0, 1, 3\}$.

c) The range is the set of second coordinates: $\{0, 5, 7\}$.

8. $f(x) = 2x^2 - x + 5$

a) $f(-1) = 2(-1)^2 - (-1) + 5 = 2 + 1 + 5 = 8$

b)
$$\begin{aligned} f(a+2) &= 2(a+2)^2 - (a+2) + 5 \\ &= 2(a^2+4a+4) - (a+2) + 5 \\ &= 2a^2 + 8a + 8 - a - 2 + 5 \\ &= 2a^2 + 7a + 11 \end{aligned}$$

9. $f(x) = \dfrac{1-x}{x}$

a) $f(0) = \dfrac{1-0}{0} = \dfrac{1}{0}$

Since the division by 0 is not defined, $f(0)$ does not exist.

b) $f(1) = \dfrac{1-1}{1} = \dfrac{0}{1} = 0$

10. From the graph we see that when the input is -3, the output is 0, so $f(-3) = 0$.

11. a) This is not the graph of a function, because we can find a vertical line that crosses the graph more than once.

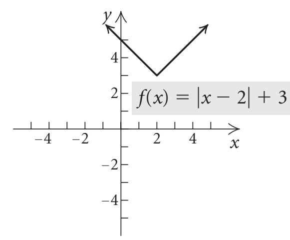
b) This is the graph of a function, because there is no vertical line that crosses the graph more than once.

12. The input 4 results in a denominator of 0. Thus the domain is $\{x|x \neq 4\}$, or $(-\infty, 4) \cup (4, \infty)$.

13. We can substitute any real number for x. Thus the domain is the set of all real numbers, or $(-\infty, \infty)$.

14. We cannot find the square root of a negative number. Thus $25 - x^2 \geq 0$ and the domain is $\{x| -5 \leq x \leq 5\}$, or $[-5, 5]$.

15. a)



b) Each point on the x-axis corresponds to a point on the graph, so the domain is the set of all real numbers, or $(-\infty, \infty)$.

c) The number 3 is the smallest output on the y-axis and every number greater than 3 is also an output, so the range is $[3, \infty)$.

16. $m = \dfrac{5 - \frac{2}{3}}{-2-(-2)} = \dfrac{\frac{13}{3}}{0}$

The slope is not defined.

17. $m = \dfrac{12-(-10)}{-8-4} = \dfrac{22}{-12} = -\dfrac{11}{6}$

18. $m = \dfrac{6-6}{\frac{3}{4} - (-5)} = \dfrac{0}{\frac{23}{4}} = 0$

19. We have the data points (1960, 72) and (2012, 51).

$m = \dfrac{51-72}{2012-1960} = \dfrac{-21}{52} \approx -0.4$

The average rate of change in the percent of adults who are married for the years 1960 to 2012 was about -0.4% per year.

20.
$$\begin{aligned} -3x + 2y &= 5 \\ 2y &= 3x + 5 \\ y &= \frac{3}{2}x + \frac{5}{2} \end{aligned}$$

Slope: $\dfrac{3}{2}$; y-intercept: $\left(0, \dfrac{5}{2}\right)$

21. $C(t) = 80 + 49.95t$

2 yr = $2 \cdot 1$ yr = $2 \cdot 12$ months = 24 months

$C(24) = 80 + 49.95(24) = \$1278.80$

22. $y = mx + b$

$y = -\frac{5}{8}x - 5$

23. First we find the slope:

$m = \frac{-2-4}{3-(-5)} = \frac{-6}{8} = -\frac{3}{4}$

Use the point-slope equation.

Using $(-5, 4)$: $y - 4 = -\frac{3}{4}(x - (-5))$, or

$y - 4 = -\frac{3}{4}(x + 5)$

Using $(3, -2)$: $y - (-2) = -\frac{3}{4}(x - 3)$, or

$y + 2 = -\frac{3}{4}(x - 3)$

In either case, we have $y = -\frac{3}{4}x + \frac{1}{4}$.

24. The vertical line that passes through $\left(-\frac{3}{8}, 11\right)$ is $\frac{3}{8}$ unit to the left of the y-axis. An equation of the line is $x = -\frac{3}{8}$.

25.
$$\begin{aligned} 2x + 3y &= -12 & 2y - 3x &= 8 \\ y &= -\frac{2}{3}x - 4 & y &= \frac{3}{2}x + 4 \end{aligned}$$

$m_1 = -\frac{2}{3}$, $m_2 = \frac{3}{2}$; $m_1 m_2 = -1$.

The lines are perpendicular.

26. First find the slope of the given line.

$$\begin{aligned} x + 2y &= -6 \\ 2y &= -x - 6 \\ y &= -\frac{1}{2}x - 3;\ m = -\frac{1}{2} \end{aligned}$$

A line parallel to the given line has slope $-\frac{1}{2}$. We use the point-slope equation.

$$\begin{aligned} y - 3 &= -\frac{1}{2}(x - (-1)) \\ y - 3 &= -\frac{1}{2}(x + 1) \\ y - 3 &= -\frac{1}{2}x - \frac{1}{2} \\ y &= -\frac{1}{2}x + \frac{5}{2} \end{aligned}$$

27. First we find the slope of the given line.

$$\begin{aligned} x + 2y &= -6 \\ 2y &= -x - 6 \\ y &= -\frac{1}{2}x - 3,\ m = -\frac{1}{2} \end{aligned}$$

The slope of a line perpendicular to this line is the negative reciprocal of $-\frac{1}{2}$, or 2. Now we find an equation of the line with slope 2 and containing $(-1, 3)$.

Using the slope-intercept equation:

$$\begin{aligned} y &= mx + b \\ 3 &= 2(-1) + b \\ 3 &= -2 + b \\ 5 &= b \end{aligned}$$

The equation is $y = 2x + 5$.

Using the point-slope equation.

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - 3 &= 2(x - (-1)) \\ y - 3 &= 2(x + 1) \\ y - 3 &= 2x + 2 \\ y &= 2x + 5 \end{aligned}$$

28. a) Answers may vary depending on the data points used. We will use (2, 544.05) and (8, 653.19).

$m = \frac{653.19 - 544.05}{8 - 2} = \frac{109.14}{6} = 18.19$

We will use the point-slope equation with (2, 544.05).

$$\begin{aligned} y - 544.05 &= 18.19(x - 2) \\ y - 544.05 &= 18.19x - 36.38 \\ y &= 18.19x + 507.67, \end{aligned}$$

where x is the number of years after 2003

In 2017, $x = 2017 - 2003 = 14$.

$y = 18.19(14) + 507.67 = \$762.33$

b) $y = 16.47728571x + 517.2819048$, where x is the number of years after 2003.

In 2017, $y \approx \$747.96$.

$r \approx 0.9971$

29.
$$\begin{aligned} 6x + 7 &= 1 \\ 6x &= -6 \\ x &= -1 \end{aligned}$$

The solution is -1.

30.
$$\begin{aligned} 2.5 - x &= -x + 2.5 \\ 2.5 &= 2.5 \quad \text{True equation} \end{aligned}$$

The solution set is $\{x | x \text{ is a real number}\}$, or $(-\infty, \infty)$.

31.
$$\begin{aligned} \frac{3}{2}y - 4 &= \frac{5}{3}y + 6 \quad \text{The LCD is 6.} \\ 6\left(\frac{3}{2}y - 4\right) &= 6\left(\frac{5}{3}y + 6\right) \\ 9y - 24 &= 10y + 36 \\ -24 &= y + 36 \\ -60 &= y \end{aligned}$$

The solution is -60.

32. $2(4x+1) = 8-3(x-5)$

$$8x+2 = 8-3x+15$$
$$8x+2 = 23-3x$$
$$11x+2 = 23$$
$$11x = 21$$
$$x = \frac{21}{11}$$

The solution is $\frac{21}{11}$.

33. ***Familiarize***. Let l = the length, in meters. Then $\frac{3}{4}l$ = the width. Recall that the formula for the perimeter P of a rectangle with length l and width w is $P = 2l+2w$.

Translate.

The perimeter is 210 m.

$$2l + 2\cdot\frac{3}{4}l = 210$$

Carry out. We solve the equation.

$$2l + 2\cdot\frac{3}{4}l = 210$$
$$2l + \frac{3}{2}l = 210$$
$$\frac{7}{2}l = 210$$
$$l = 60$$

If $l = 60$, then $\frac{3}{4}l = \frac{3}{4}\cdot 60 = 45$.

Check. The width, 45 m, is three-fourths of the length, 60 m. Also, $2\cdot 60$ m $+ 2\cdot 45$ m $= 210$ m, so the answer checks.

State. The length is 60 m and the width is 45 m.

34. ***Familiarize***. Let p = the wholesale price of the juice.

Translate. We express 25¢ as \$0.25.

Wholesale price	plus	50% of wholesale price	plus	\$0.25	is	\$2.95.
↓	↓	↓	↓	↓	↓	↓
p	$+$	$0.5p$	$+$	0.25	$=$	2.95

Carry out. We solve the equation.

$$p + 0.5p + 0.25 = 2.95$$
$$1.5p + 0.25 = 2.95$$
$$1.5p = 2.7$$
$$p = 1.8$$

Check. 50% of \$1.80 is \$0.90 and \$1.80 + \$0.90 + \$0.25 = \$2.95, so the answer checks.

State. The wholesale price of a bottle of juice is \$1.80.

35. $3x+9 = 0$ Setting $f(x) = 0$

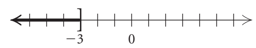
$$3x = -9$$
$$x = -3$$

The zero of the function is -3.

36. $5-x \geq 4x+20$

$$5-5x \geq 20$$
$$-5x \geq 15$$
$$x \leq -3$$ Dividing by -5 and reversing the inequality symbol

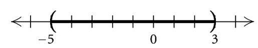
The solution set is $\{x|x \leq -3\}$, or $(-\infty, -3]$.



37. $-7 < 2x+3 < 9$

$$-10 < 2x < 6$$ Subtracting 3
$$-5 < x < 3$$ Dividing by 2

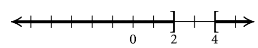
The solution set is $(-5, 3)$.



38. $2x-1 \leq 3$ *or* $5x+6 \geq 26$

$$2x \leq 4 \text{ or } 5x \geq 20$$
$$x \leq 2 \text{ or } x \geq 4$$

The solution set is $(-\infty, 2] \cup [4, \infty)$.



39. ***Familiarize***. Let t = the number of hours a move requires. Then Morgan Movers charges $90+25t$ to make a move and McKinley Movers charges $40t$.

Translate.

Morgan Movers' charge is less than McKinley Movers' charge.

$$90+25t < 40t$$

Carry out. We solve the inequality.

$$90+25t < 40t$$
$$90 < 15t$$
$$6 < t$$

Check. For $t = 6$, Morgan Movers charge $90+25\cdot 6$, or \$240, and McKinley Movers charge $40\cdot 6$, or \$240, so the charge is the same for 6 hours. As a partial check, we can find the charges for a value of t greater than 6. For instance, for 6.5 hr Morgan Movers charge $90+25(6.5)$, or \$252.50, and McKinley Movers charge $40(6.5)$, or \$260. Since Morgan Movers cost less for a value of t greater than 6, the answer is probably correct.

State. It costs less to hire Morgan Movers when a move takes more than 6 hr.

40. The slope is $-\frac{1}{2}$, so the graph slants down from left to right. The y-intercept is $(0, 1)$. Thus, graph B is the graph of $g(x) = 1 - \frac{1}{2}x$.

41. First we find the value of x for which $x+2 = -2$:

$$x+2 = -2$$
$$x = -4$$

Now we find $h(-4+2)$, or $h(-2)$.

$$h(-4+2) = \frac{1}{2}(-4) = -2$$

Chapter 2

More on Functions

Exercise Set 2.1

1. a) For x-values from -5 to 1, the y-values increase from -3 to 3. Thus the function is increasing on the interval $(-5, 1)$.

b) For x-values from 3 to 5, the y-values decrease from 3 to 1. Thus the function is decreasing on the interval $(3, 5)$.

c) For x-values from 1 to 3, y is 3. Thus the function is constant on $(1, 3)$.

2. a) For x-values from 1 to 3, the y-values increase from 1 to 2. Thus, the function is increasing on the interval $(1, 3)$.

b) For x-values from -5 to 1, the y-values decrease from 4 to 1. Thus the function is decreasing on the interval $(-5, 1)$.

c) For x-values from 3 to 5, y is 2. Thus the function is constant on $(3, 5)$.

3. a) For x-values from -3 to -1, the y-values increase from -4 to 4. Also, for x-values from 3 to 5, the y-values increase from 2 to 6. Thus the function is increasing on $(-3, -1)$ and on $(3, 5)$.

b) For x-values from 1 to 3, the y-values decrease from 3 to 2. Thus the function is decreasing on the interval $(1, 3)$.

c) For x-values from -5 to -3, y is 1. Thus the function is constant on $(-5, -3)$.

4. a) For x-values from 1 to 2, the y-values increase from 1 to 2. Thus the function is increasing on the interval $(1, 2)$.

b) For x-values from -5 to -2, the y-values decrease from 3 to 1. For x-values from -2 to 1, the y-values decrease from 3 to 1. And for x-values from 3 to 5, the y-values decrease from 2 to 1. Thus the function is decreasing on $(-5, -2)$, on $(-2, 1)$, and on $(3, 5)$.

c) For x-values from 2 to 3, y is 2. Thus the function is constant on $(2, 3)$.

5. a) For x-values from $-\infty$ to -8, the y-values increase from $-\infty$ to 2. Also, for x-values from -3 to -2, the y-values increase from -2 to 3. Thus the function is increasing on $(-\infty, -8)$ and on $(-3, -2)$.

b) For x-values from -8 to -6, the y-values decrease from 2 to -2. Thus the function is decreasing on the interval $(-8, -6)$.

c) For x-values from -6 to -3, y is -2. Also, for x-values from -2 to ∞, y is 3. Thus the function is constant on $(-6, -3)$ and on $(-2, \infty)$.

6. a) For x-values from 1 to 4, the y-values increase from 2 to 11. Thus the function is increasing on the interval $(1, 4)$.

b) For x-values from -1 to 1, the y-values decrease from 6 to 2. Also, for x-values from 4 to ∞, the y-values decrease from 11 to $-\infty$. Thus the function is decreasing on $(-1, 1)$ and on $(4, \infty)$.

c) For x-values from $-\infty$ to -1, y is 3. Thus the function is constant on $(-\infty, -1)$.

7. The x-values extend from -5 to 5, so the domain is $[-5, 5]$.

The y-values extend from -3 to 3, so the range is $[-3, 3]$.

8. Domain: $[-5, 5]$; range: $[1, 4]$

9. The x-values extend from -5 to -1 and from 1 to 5, so the domain is $[-5, -1] \cup [1, 5]$.

The y-values extend from -4 to 6, so the range is $[-4, 6]$.

10. Domain: $[-5, 5]$; range: $[1, 3]$

11. The x-values extend from $-\infty$ to ∞, so the domain is $(-\infty, \infty)$.

The y-values extend from $-\infty$ to 3, so the range is $(-\infty, 3]$.

12. Domain: $(-\infty, \infty)$; range: $(-\infty, 11]$

13. From the graph we see that a relative maximum value of the function is 3.25. It occurs at $x = 2.5$. There is no relative minimum value.

The graph starts rising, or increasing, from the left and stops increasing at the relative maximum. From this point, the graph decreases. Thus the function is increasing on $(-\infty, 2.5)$ and is decreasing on $(2.5, \infty)$.

14. From the graph we see that a relative minimum value of 2 occurs at $x = 1$. There is no relative maximum value.

The graph starts falling, or decreasing, from the left and stops decreasing at the relative minimum. From this point, the graph increases. Thus the function is increasing on $(1, \infty)$ and is decreasing on $(-\infty, 1)$.

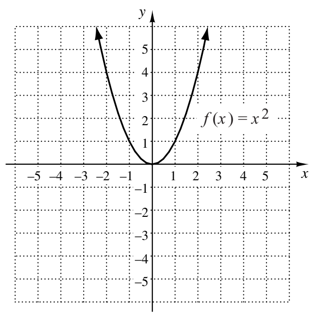
15. From the graph we see that a relative maximum value of the function is 2.370. It occurs at $x = -0.667$. We also see that a relative minimum value of 0 occurs at $x = 2$.

The graph starts rising, or increasing, from the left and stops increasing at the relative maximum. From this point it decreases to the relative minimum and then increases again. Thus the function is increasing on $(-\infty, -0.667)$ and on $(2, \infty)$. It is decreasing on $(-0.667, 2)$.

16. From the graph we see that a relative maximum value of 2.921 occurs at $x = 3.601$. A relative minimum value of 0.995 occurs at $x = 0.103$.

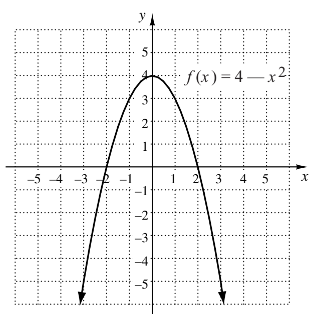
The graph starts decreasing from the left and stops decreasing at the relative minimum. From this point it increases to the relative maximum and then decreases again. Thus the function is increasing on $(0.103, 3.601)$ and is decreasing on $(-\infty, 0.103)$ and on $(3.601, \infty)$.

17.



The function is increasing on $(0, \infty)$ and decreasing on $(-\infty, 0)$. We estimate that the minimum is 0 at $x = 0$. There are no maxima.

18.



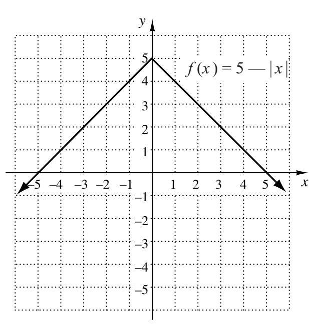
Increasing: $(-\infty, 0)$

Decreasing: $(0, \infty)$

Maximum: 4 at $x = 0$

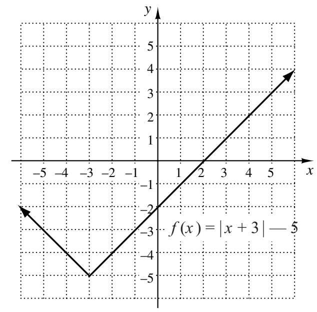
Minima: none

19.



The function is increasing on $(-\infty, 0)$ and decreasing on $(0, \infty)$. We estimate that the maximum is 5 at $x = 0$. There are no minima.

20.



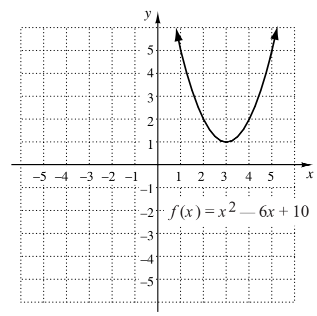
Increasing: $(-3, \infty)$

Decreasing: $(-\infty, -3)$

Maxima: none

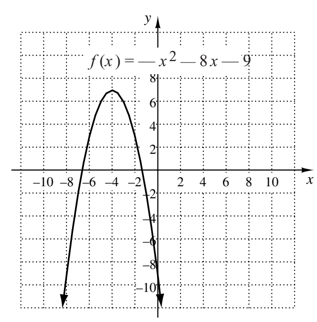
Minimum: -5 at $x = -3$

21.



The function is decreasing on $(-\infty, 3)$ and increasing on $(3, \infty)$. We estimate that the minimum is 1 at $x = 3$. There are no maxima.

22.



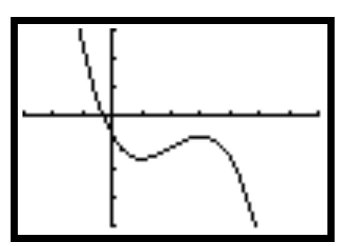
Increasing: $(-\infty, -4)$

Decreasing: $(-4, \infty)$

Maximum: 7 at $x = -4$

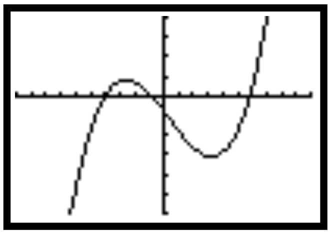
Minima: none

23.



Beginning at the left side of the window, the graph first drops as we move to the right. We see that the function is decreasing on $(-\infty, 1)$. We then find that the function is increasing on $(1, 3)$ and decreasing again on $(3, \infty)$. The MAXIMUM and MINIMUM features also show that the relative maximum is -4 at $x = 3$ and the relative minimum is -8 at $x = 1$.

24.



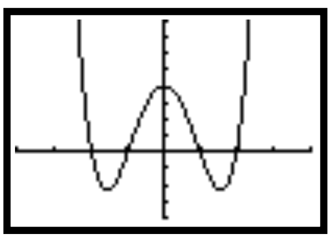
Increasing: $(-\infty, -2.573)$, $(3.239, \infty)$

Decreasing: $(-2.573, 3.239)$

Relative maximum: 4.134 at $x = -2.573$

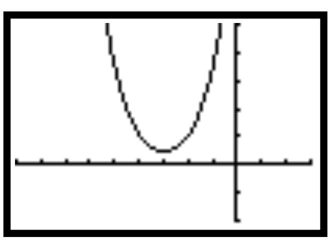
Relative minimum: -15.497 at $x = 3.239$

25.



We find that the function is increasing on $(-1.552, 0)$ and on $(1.552, \infty)$ and decreasing on $(-\infty, -1.552)$ and on $(0, 1.552)$. The relative maximum is 4.07 at $x = 0$ and the relative minima are -2.314 at $x = -1.552$ and -2.314 at $x = 1.552$.

26.



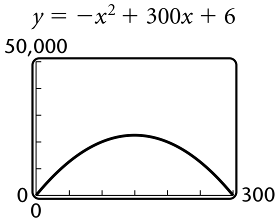
Increasing: $(-3, \infty)$

Decreasing: $(-\infty, -3)$

Relative maxima: none

Relative minimum: 9.78 at $x = -3$

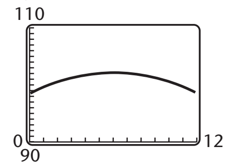
27. a) $y = -x^2 + 300x + 6$



b) $22,506$ at $a = 150$

c) The greatest number of fruit trees will be sold when \$150 thousand is spent on advertising. For that amount, 22,506 fruit trees will be sold.

28. a) $y = -0.1x^2 + 1.2x + 98.6$



b) Using the MAXIMUM feature we find that the relative maximum is 102.2 at $t = 6$. Thus, we know that the patient's temperature was the highest at $t = 6$, or 6 days after the onset of the illness and that the highest temperature was 102.2°F.

29. Graph $y = \dfrac{8x}{x^2 + 1}$.

Increasing: $(-1, 1)$

Decreasing: $(-\infty, -1)$, $(1, \infty)$

30. Graph $y = \dfrac{-4}{x^2 + 1}$.

Increasing: $(0, \infty)$

Decreasing: $(-\infty, 0)$

31. Graph $y = x\sqrt{4 - x^2}$, for $-2 \le x \le 2$.

Increasing: $(-1.414, 1.414)$

Decreasing: $(-2, -1.414)$, $(1.414, 2)$

32. Graph $y = -0.8x\sqrt{9 - x^2}$, for $-3 \le x \le 3$.

Increasing: $(-3, -2.121)$, $(2.121, 3)$

Decreasing: $(-2.121, 2.121)$

33. If x = the length of the rectangle, in meters, then the width is $\dfrac{480 - 2x}{2}$, or $240 - x$. We use the formula Area = length $\times$ width:

$$A(x) = x(240 - x), \text{ or}$$
$$A(x) = 240x - x^2$$

34. Let h = the height of the scarf, in inches. Then the length of the base $= 2h - 7$.

$$A(h) = \frac{1}{2}(2h - 7)(h)$$
$$A(h) = h^2 - \frac{7}{2}h$$

35. We use the Pythagorean theorem.

$$[h(d)]^2 + 3500^2 = d^2$$
$$[h(d)]^2 = d^2 - 3500^2$$
$$h(d) = \sqrt{d^2 - 3500^2}$$

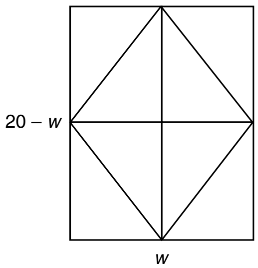
We considered only the positive square root since distance must be nonnegative.

36. After t minutes, the balloon has risen $120t$ ft. We use the Pythagorean theorem.

$$[d(t)]^2 = (120t)^2 + 400^2$$
$$d(t) = \sqrt{(120t)^2 + 400^2}$$

We considered only the positive square root since distance must be nonnegative.

37. Let w = the width of the rectangle. Then the length $= \dfrac{40-2w}{2}$, or $20-w$. Divide the rectangle into quadrants as shown below.



In each quadrant there are two congruent triangles. One triangle is part of the rhombus and both are part of the rectangle. Thus, in each quadrant the area of the rhombus is one-half the area of the rectangle. Then, in total, the area of the rhombus is one-half the area of the rectangle.

$$A(w) = \frac{1}{2}(20-w)(w)$$
$$A(w) = 10w - \frac{w^2}{2}$$

38. Let w = the width, in feet. Then the length $= \dfrac{46-2w}{2}$, or $23-w$.

$$A(w) = (23-w)w$$
$$A(w) = 23w - w^2$$

39. We will use similar triangles, expressing all distances in feet. $\left(6 \text{ in.} = \frac{1}{2} \text{ ft},\ s \text{ in.} = \frac{s}{12} \text{ ft, and } d \text{ yd} = 3d \text{ ft}\right)$ We have

$$\frac{3d}{7} = \frac{\frac{1}{2}}{\frac{s}{12}}$$
$$\frac{s}{12}\cdot 3d = 7\cdot\frac{1}{2}$$
$$\frac{sd}{4} = \frac{7}{2}$$
$$d = \frac{4}{s}\cdot\frac{7}{2}, \text{ so}$$
$$d(s) = \frac{14}{s}.$$

40. The volume of the tank is the sum of the volume of a sphere with radius r and a right circular cylinder with radius r and height 6 ft.

$$V(r) = \frac{4}{3}\pi r^3 + 6\pi r^2$$

41. a) After 4 pieces of float line, each of length x ft, are used for the sides perpendicular to the beach, there remains $(240-4x)$ ft of float line for the side parallel to the beach. Thus we have a rectangle with length $240-4x$ and width x. Then the total area of the three swimming areas is

$$A(x) = (240-4x)x, \text{ or } 240x - 4x^2.$$

b) The length of the sides labeled x must be positive and their total length must be less than 240 ft, so $4x < 240$, or $x < 60$. Thus the domain is $\{x|0 < x < 60\}$, or $(0, 60)$.

c) We see from the graph that the maximum value of the area function on the interval $(0, 60)$ appears to be 3600 when $x = 30$. Thus the dimensions that yield the maximum area are 30 ft by $240 - 4\cdot 30$, or $240-120$, or 120 ft.

42. a) If the length = x feet, then the width = $24-x$ feet.

$$A(x) = x(24-x)$$
$$A(x) = 24x - x^2$$

b) The length of the rectangle must be positive and less than 24 ft, so the domain of the function is $\{x|0 < x < 24\}$, or $(0, 24)$.

c) We see from the graph that the maximum value of the area function on the interval $(0, 24)$ appears to be 144 when $x = 12$. Then the dimensions that yield the maximum area are length = 12 ft and width = $24-12$, or 12 ft.

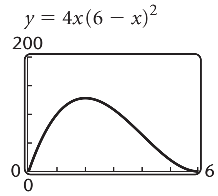
43. a) When a square with sides of length x is cut from each corner, the length of each of the remaining sides of the piece of cardboard is $12-2x$. Then the dimensions of the box are x by $12-2x$ by $12-2x$. We use the formula Volume = length × width × height to find the volume of the box:

$$V(x) = (12-2x)(12-2x)(x)$$
$$V(x) = (144 - 48x + 4x^2)(x)$$
$$V(x) = 144x - 48x^2 + 4x^3$$

This can also be expressed as $V(x) = 4x(x-6)^2$, or $V(x) = 4x(6-x)^2$.

b) The length of the sides of the square corners that are cut out must be positive and less than half the length of a side of the piece of cardboard. Thus, the domain of the function is $\{x|0 < x < 6\}$, or $(0, 6)$.

c) $y = 4x(6-x)^2$



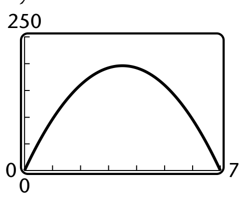
d) Using the MAXIMUM feature, we find that the maximum value of the volume occurs when $x = 2$. When $x = 2$, $12-2x = 12-2\cdot 2 = 8$, so the dimensions that yield the maximum volume are 8 cm by 8 cm by 2 cm.

44. a) If the height of the file is x inches, then the width is $14-2x$ inches and the length is 8 in. We use the formula Volume = length × width × height to find the volume of the file.

$$V(x) = 8(14-2x)x, \text{ or}$$
$$V(x) = 112x - 16x^2$$

b) The height of the file must be positive and less than half of the measure of the long side of the piece of plastic. Thus, the domain is $\left\{x \middle| 0 < x < \frac{14}{2}\right\}$, or $\{x|0 < x < 7\}$.

c) $y = 112x - 16x^2$



d) Using the MAXIMUM feature, we find that the maximum value of the volume function occurs when $x = 3.5$, so the file should be 3.5 in. tall.

45. a) The length of a diameter of the circle (and a diagonal of the rectangle) is $2 \cdot 8$, or 16 ft. Let $l =$ the length of the rectangle. Use the Pythagorean theorem to write l as a function of x.

$$x^2 + l^2 = 16^2$$
$$x^2 + l^2 = 256$$
$$l^2 = 256 - x^2$$
$$l = \sqrt{256 - x^2}$$

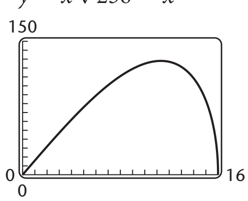
Since the length must be positive, we considered only the positive square root.

Use the formula Area = length × width to find the area of the rectangle:

$$A(x) = x\sqrt{256 - x^2}$$

b) The width of the rectangle must be positive and less than the diameter of the circle. Thus, the domain of the function is $\{x|0 < x < 16\}$, or $(0, 16)$.

c) $y = x\sqrt{256 - x^2}$



d) Using the MAXIMUM feature, we find that the maximum area occurs when x is about 11.314. When $x \approx 11.314$, $\sqrt{256 - x^2} \approx \sqrt{256 - (11.314)^2} \approx 11.313$. Thus, the dimensions that maximize the area are about 11.314 ft by 11.313 ft. (Answers may vary slightly due to rounding differences.)

46. a) Let $h(x) =$ the height of the box.

$$320 = x \cdot x \cdot h(x)$$
$$\frac{320}{x^2} = h(x)$$

Area of the bottom: x^2

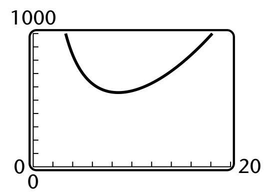
Area of each side: $x\left(\frac{320}{x^2}\right)$, or $\frac{320}{x}$

Area of the top: x^2

$$C(x) = 1.5x^2 + +4(2.5)\left(\frac{320}{x}\right) + 1 \cdot x^2$$
$$C(x) = 2.5x^2 + \frac{3200}{x}$$

b) The length of the base must be positive, so the domain of the function is $\{x|x > 0\}$, or $(0, \infty)$.

c) $y = 2.5x^2 + \frac{3200}{x}$



d) Using the MIMIMUM feature, we find that the minimum cost occurs when $x \approx 8.618$. Thus, the dimensions that minimize the cost are about 8.618 ft by 8.618 ft by $\frac{320}{(8.618)^2}$, or about 4.309 ft.

47. $g(x) = \begin{cases} x + 4, & \text{for } x \le 1, \\ 8 - x, & \text{for } x > 1 \end{cases}$

Since $-4 \le 1$, $g(-4) = -4 + 4 = 0$.

Since $0 \le 1$, $g(0) = 0 + 4 = 4$.

Since $1 \le 1$, $g(1) = 1 + 4 = 5$.

Since $3 > 1$, $g(3) = 8 - 3 = 5$.

48. $f(x) = \begin{cases} 3, & \text{for } x \le -2, \\ \frac{1}{2}x + 6, & \text{for } x > -2 \end{cases}$

$f(-5) = 3$

$f(-2) = 3$

$f(0) = \frac{1}{2} \cdot 0 + 6 = 6$

$f(2) = \frac{1}{2} \cdot 2 + 6 = 7$

49. $h(x) = \begin{cases} -3x - 18, & \text{for } x < -5, \\ 1, & \text{for } -5 \le x < 1, \\ x + 2, & \text{for } x \ge 1 \end{cases}$

Since -5 is in the interval $[-5, 1)$, $h(-5) = 1$.

Since 0 is in the interval $[-5, 1)$, $h(0) = 1$.

Since $1 \ge 1$, $h(1) = 1 + 2 = 3$.

Since $4 \ge 1$, $h(4) = 4 + 2 = 6$.

50. $f(x) = \begin{cases} -5x - 8, & \text{for } x < -2, \\ \frac{1}{2}x + 5, & \text{for } -2 \le x \le 4, \\ 10 - 2x, & \text{for } x > 4 \end{cases}$

Since $-4 < -2$, $f(-4) = -5(-4) - 8 = 12$.

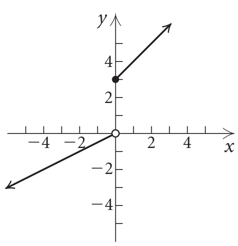
Since -2 is in the interval $[-2, 4]$, $f(-2) = \frac{1}{2}(-2) + 5 = 4$.

Since 4 is in the interval $[-2, 4]$, $f(4) = \frac{1}{2} \cdot 4 + 5 = 7$.

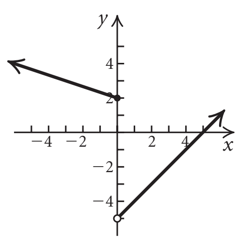
Since $6 > 4$, $f(6) = 10 - 2 \cdot 6 = -2$.

51. $f(x) = \begin{cases} \frac{1}{2}x, & \text{for } x < 0, \\ x + 3, & \text{for } x \ge 0 \end{cases}$

We create the graph in two parts. Graph $f(x) = \frac{1}{2}x$ for inputs x less than 0. Then graph $f(x) = x + 3$ for inputs x greater than or equal to 0.

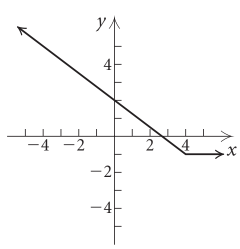


52. $f(x) = \begin{cases} -\frac{1}{3}x + 2, & \text{for } x \le 0, \\ x - 5, & \text{for } x > 0 \end{cases}$

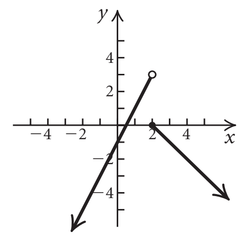


53. $f(x) = \begin{cases} -\frac{3}{4}x + 2, & \text{for } x < 4, \\ -1, & \text{for } x \ge 4 \end{cases}$

We create the graph in two parts. Graph $f(x) = -\frac{3}{4}x + 2$ for inputs x less than 4. Then graph $f(x) = -1$ for inputs x greater than or equal to 4.

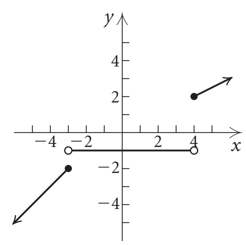


54. $h(x) = \begin{cases} 2x - 1, & \text{for } x < 2 \\ 2 - x, & \text{for } x \ge 2 \end{cases}$

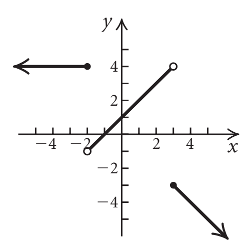


55. $f(x) = \begin{cases} x + 1, & \text{for } x \le -3, \\ -1, & \text{for } -3 < x < 4 \\ \frac{1}{2}x, & \text{for } x \ge 4 \end{cases}$

We create the graph in three parts. Graph $f(x) = x + 1$ for inputs x less than or equal to -3. Graph $f(x) = -1$ for inputs greater than -3 and less than 4. Then graph $f(x) = \frac{1}{2}x$ for inputs greater than or equal to 4.

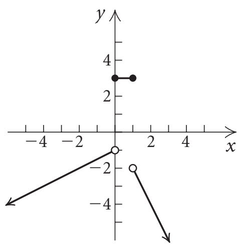


56. $f(x) = \begin{cases} 4, & \text{for } x \le -2, \\ x + 1, & \text{for } -2 < x < 3 \\ -x, & \text{for } x \ge 3 \end{cases}$

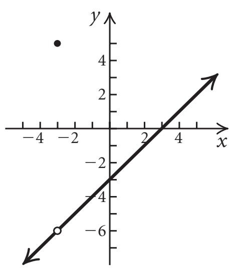


57. $g(x) = \begin{cases} \frac{1}{2}x - 1, & \text{for } x < 0, \\ 3, & \text{for } 0 \le x \le 1 \\ -2x, & \text{for } x > 1 \end{cases}$

We create the graph in three parts. Graph $g(x) = \frac{1}{2}x - 1$ for inputs less than 0. Graph $g(x) = 3$ for inputs greater than or equal to 0 and less than or equal to 1. Then graph $g(x) = -2x$ for inputs greater than 1.



58. $f(x) = \begin{cases} \dfrac{x^2-9}{x+3}, & \text{for } x \neq -3, \\ 5, & \text{for } x = -3 \end{cases}$



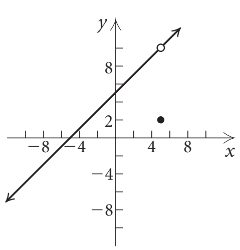
59. $f(x) = \begin{cases} 2, & \text{for } x = 5, \\ \dfrac{x^2-25}{x-5}, & \text{for } x \neq 5 \end{cases}$

When $x \neq 5$, the denominator of $(x^2 - 25)/(x - 5)$ is nonzero so we can simplify:

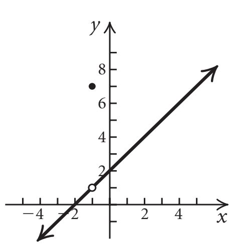
$$\frac{x^2-25}{x-5} = \frac{(x+5)(x-5)}{x-5} = x+5.$$

Thus, $f(x) = x + 5$, for $x \neq 5$.

The graph of this part of the function consists of a line with a "hole" at the point $(5, 10)$, indicated by an open dot. At $x = 5$, we have $f(5) = 2$, so the point $(5, 2)$ is plotted below the open dot.

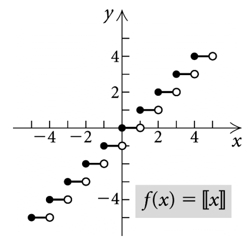


60. $f(x) = \begin{cases} \dfrac{x^2+3x+2}{x+1}, & \text{for } x \neq -1, \\ 7, & \text{for } x = -1 \end{cases}$



61. $f(x) = [[x]]$

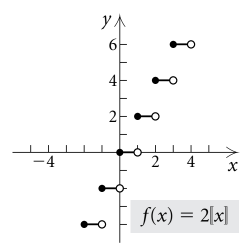
See Example 9.



62. $f(x) = 2[[x]]$

This function can be defined by a piecewise function with an infinite number of statements:

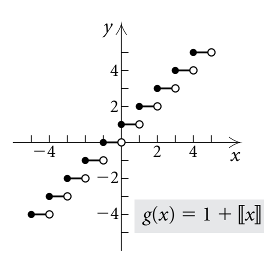
$$f(x) = \begin{cases} . \\ . \\ . \\ -4, & \text{for } -2 \le x < -1, \\ -2, & \text{for } -1 \le x < 0, \\ 0, & \text{for } 0 \le x < 1, \\ 2, & \text{for } 1 \le x < 2, \\ . \\ . \\ . \end{cases}$$



63. $f(x) = 1 + [[x]]$

This function can be defined by a piecewise function with an infinite number of statements:

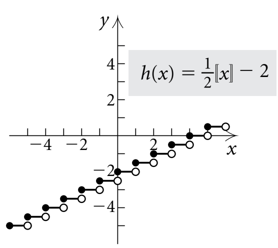
$$f(x) = \begin{cases} . \\ . \\ . \\ -1, & \text{for } -2 \le x < -1, \\ 0, & \text{for } -1 \le x < 0, \\ 1, & \text{for } 0 \le x < 1, \\ 2, & \text{for } 1 \le x < 2, \\ . \\ . \\ . \end{cases}$$



64. $f(x) = \frac{1}{2}[[x]] - 2$

This function can be defined by a piecewise function with an infinite number of statements:

$$f(x) = \begin{cases} . \\ . \\ . \\ -2\frac{1}{2}, & \text{for } -1 \le x < 0, \\ -2, & \text{for } 0 \le x < 1, \\ -1\frac{1}{2}, & \text{for } 1 \le x < 2, \\ -1, & \text{for } 2 \le x < 3, \\ . \\ . \\ . \end{cases}$$



65. From the graph we see that the domain is $(-\infty, \infty)$ and the range is $(-\infty, 0) \cup [3, \infty)$.

66. Domain: $(-\infty, \infty)$; range: $(-5, \infty)$

67. From the graph we see that the domain is $(-\infty, \infty)$ and the range is $[-1, \infty)$.

68. Domain: (∞, ∞); range: $(-\infty, 3)$

69. From the graph we see that the domain is $(-\infty, \infty)$ and the range is $\{y | y \le -2 \text{ or } y = -1 \text{ or } y \ge 2\}$.

70. Domain: $(-\infty, \infty)$; range: $(-\infty, -3] \cup (-1, 4]$

71. From the graph we see that the domain is $(-\infty, \infty)$ and the range is $\{-5, -2, 4\}$. An equation for the function is:

$$f(x) = \begin{cases} -2, & \text{for } x < 2, \\ -5, & \text{for } x = 2, \\ 4, & \text{for } x > 2 \end{cases}$$

72. Domain: $(-\infty, \infty)$; range: $\{y | y = -3 \text{ or } y \ge 0\}$

$$g(x) = \begin{cases} -3, & \text{for } x < 0, \\ x, & \text{for } x \ge 0 \end{cases}$$

73. From the graph we see that the domain is $(-\infty, \infty)$ and the range is $(-\infty, -1] \cup [2, \infty)$. Finding the slope of each segment and using the slope-intercept or point-slope formula, we find that an equation for the function is:

$$g(x) = \begin{cases} x, & \text{for } x \le -1, \\ 2, & \text{for } -1 < x \le 2, \\ x, & \text{for } x > 2 \end{cases}$$

This can also be expressed as follows:

$$g(x) = \begin{cases} x, & \text{for } x \le -1, \\ 2, & \text{for } -1 < x < 2, \\ x, & \text{for } x \ge 2 \end{cases}$$

74. Domain: $(-\infty, \infty)$; range: $\{y | y = -2 \text{ or } y \ge 0\}$. An equation for the function is:

$$h(x) = \begin{cases} |x|, & \text{for } x < 3, \\ -2, & \text{for } x \ge 3 \end{cases}$$

This can also be expressed as follows:

$$h(x) = \begin{cases} -x, & \text{for } x \le 0, \\ x, & \text{for } 0 < x < 3, \\ -2, & \text{for } x \ge 3 \end{cases}$$

It can also be expressed as follows:

$$h(x) = \begin{cases} -x, & \text{for } x < 0, \\ x, & \text{for } 0 \le x < 3, \\ -2, & \text{for } x \ge 3 \end{cases}$$

75. From the graph we see that the domain is $[-5, 3]$ and the range is $(-3, 5)$. Finding the slope of each segment and using the slope-intercept or point-slope formula, we find that an equation for the function is:

$$h(x) = \begin{cases} x + 8, & \text{for } -5 \le x < -3, \\ 3, & \text{for } -3 \le x \le 1, \\ 3x - 6, & \text{for } 1 < x \le 3 \end{cases}$$

76. Domain: $[-4, \infty)$; range: $[-2, 4]$

$$f(x) = \begin{cases} -2x - 4, & \text{for } -4 \le x \le -1, \\ x - 1, & \text{for } -1 < x < 2, \\ 2, & \text{for } x \ge 2 \end{cases}$$

This can also be expressed as:

$$f(x) = \begin{cases} -2x - 4, & \text{for } -4 \le x < -1, \\ x - 1, & \text{for } -1 \le x < 2, \\ 2, & \text{for } x \ge 2 \end{cases}$$

77. $f(x) = 5x^2 - 7$

a) $f(-3) = 5(-3)^2 - 7 = 5 \cdot 9 - 7 = 45 - 7 = 38$

b) $f(3) = 5 \cdot 3^2 - 7 = 5 \cdot 9 - 7 = 45 - 7 = 38$

c) $f(a) = 5a^2 - 7$

d) $f(-a) = 5(-a)^2 - 7 = 5a^2 - 7$

78. $f(x) = 4x^3 - 5x$

a) $f(2) = 4 \cdot 2^3 - 5 \cdot 2 = 4 \cdot 8 - 5 \cdot 2 = 32 - 10 = 22$

b) $f(-2) = 4(-2)^3 - 5(-2) = 4(-8) - 5(-2) = -32 + 10 = -22$

c) $f(a) = 4a^3 - 5a$

d) $f(-a) = 4(-a)^3 - 5(-a) = 4(-a^3) - 5(-a) = -4a^3 + 5a$

79. First find the slope of the given line.

$$8x - y = 10$$
$$8x = y + 10$$
$$8x - 10 = y$$

The slope of the given line is 8. The slope of a line perpendicular to this line is the opposite of the reciprocal of 8, or $-\frac{1}{8}$.

$$y - y_1 = m(x - x_1)$$
$$y - 1 = -\frac{1}{8}[x - (-1)]$$
$$y - 1 = -\frac{1}{8}(x + 1)$$
$$y - 1 = -\frac{1}{8}x - \frac{1}{8}$$
$$y = -\frac{1}{8}x + \frac{7}{8}$$

80. $2x - 9y + 1 = 0$
$$2x + 1 = 9y$$
$$\frac{2}{9}x + \frac{1}{9} = y$$

Slope: $\frac{2}{9}$; y-intercept: $\left(0, \frac{1}{9}\right)$

81. Graph $y = x^4 + 4x^3 - 36x^2 - 160x + 400$

Increasing: $(-5, -2)$, $(4, \infty)$

Decreasing: $(-\infty, -5)$, $(-2, 4)$

Relative maximum: 560 at $x = -2$

Relative minima: 425 at $x = -5$, -304 at $x = 4$

82. Graph $y = 3.22x^5 - 5.208x^3 - 11$

Increasing: $(-\infty, -0.985)$, $(0.985, \infty)$

Decreasing: $(-0.985, 0.985)$

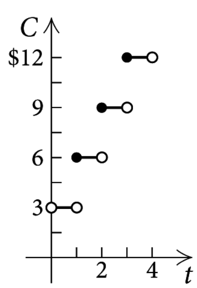
Relative maximum: -9.008 at $x = -0.985$

Relative minimum: -12.992 at $x = 0.985$

83. a) The function $C(t)$ can be defined piecewise.

$$C(t) = \begin{cases} 3, & \text{for } 0 < t < 1, \\ 6, & \text{for } 1 \le t < 2, \\ 9, & \text{for } 2 \le t < 3, \\ . \\ . \\ . \end{cases}$$

We graph this function.



b) From the definition of the function in part (a), we see that it can be written as

$$C(t) = 3[[t]] + 1,\ t > 0.$$

84. If $[[x + 2]] = -3$, then $-3 \le x + 2 < -2$, or $-5 \le x < -4$. The possible inputs for x are $\{x| - 5 \le x < -4\}$.

85. If $[[x]]^2 = 25$, then $[[x]] = -5$ or $[[x]] = 5$. For $-5 \le x < -4$, $[[x]] = -5$. For $5 \le x < 6$, $[[x]] = 5$. Thus, the possible inputs for x are $\{x| - 5 \le x < -4 \text{ or } 5 \le x < 6\}$.

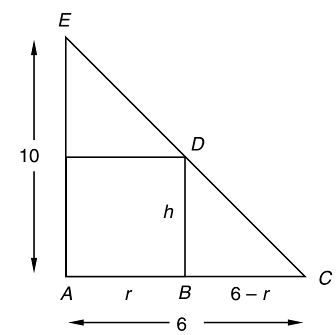
86. a) The distance from A to S is $4 - x$.

Using the Pythagorean theorem, we find that the distance from S to C is $\sqrt{1 + x^2}$.

Then $C(x) = 3000(4-x) + 5000\sqrt{1 + x^2}$, or $12,000 - 3000x + 5000\sqrt{1 + x^2}$.

b) Use a graphing calculator to graph $y = 12,000 - 3000x + 5000\sqrt{1 + x^2}$ in a window such as $[0, 5, 10,000, 20,000]$, Xscl = 1, Yscl = 1000. Using the MINIMUM feature, we find that cost is minimized when $x = 0.75$, so the line should come to shore 0.75 mi from B.

87. a) We add labels to the drawing in the text.



We write a proportion involving the lengths of the sides of the similar triangles BCD and ACE. Then we solve it for h.

$$\frac{h}{6 - r} = \frac{10}{6}$$
$$h = \frac{10}{6}(6 - r) = \frac{5}{3}(6 - r)$$
$$h = \frac{30 - 5r}{3}$$

Thus, $h(r) = \dfrac{30 - 5r}{3}$.

b) $V = \pi r^2 h$

$V(r) = \pi r^2 \left(\dfrac{30 - 5r}{3}\right)$ Substituting for h

c) We first express r in terms of h.

$$h = \frac{30-5r}{3}$$
$$3h = 30-5r$$
$$5r = 30-3h$$
$$r = \frac{30-3h}{5}$$
$$V = \pi r^2 h$$
$$V(h) = \pi\left(\frac{30-3h}{5}\right)^2 h$$

Substituting for r

We can also write $V(h) = \pi h\left(\frac{30-3h}{5}\right)^2$.

Exercise Set 2.2

1. $(f+g)(5) = f(5)+g(5)$
$= (5^2-3)+(2\cdot 5+1)$
$= 25-3+10+1$
$= 33$

2. $(fg)(0) = f(0)\cdot g(0)$
$= (0^2-3)(2\cdot 0+1)$
$= -3(1) = -3$

3. $(f-g)(-1) = f(-1)-g(-1)$
$= ((-1)^2-3)-(2(-1)+1)$
$= -2-(-1) = -2+1$
$= -1$

4. $(fg)(2) = f(2)\cdot g(2)$
$= (2^2-3)(2\cdot 2+1)$
$= 1\cdot 5 = 5$

5. $(f/g)\left(-\frac{1}{2}\right) = \frac{f\left(-\frac{1}{2}\right)}{g\left(-\frac{1}{2}\right)}$
$= \frac{\left(-\frac{1}{2}\right)^2-3}{2\left(-\frac{1}{2}\right)+1}$
$= \frac{\frac{1}{4}-3}{-1+1}$
$= \frac{-\frac{11}{4}}{0}$

Since division by 0 is not defined, $(f/g)\left(-\frac{1}{2}\right)$ does not exist.

6. $(f-g)(0) = f(0)-g(0)$
$= (0^2-3)-(2\cdot 0+1)$
$= -3-1 = -4$

7. $(fg)\left(-\frac{1}{2}\right) = f\left(-\frac{1}{2}\right)\cdot g\left(-\frac{1}{2}\right)$
$= \left[\left(-\frac{1}{2}\right)^2-3\right]\left[2\left(-\frac{1}{2}\right)+1\right]$
$= -\frac{11}{4}\cdot 0 = 0$

8. $(f/g)(-\sqrt{3}) = \frac{f(-\sqrt{3})}{g(-\sqrt{3})}$
$= \frac{(-\sqrt{3})^2-3}{2(-\sqrt{3})+1}$
$= \frac{0}{-2\sqrt{3}+1} = 0$

9. $(g-f)(-1) = g(-1)-f(-1)$
$= [2(-1)+1]-[(-1)^2-3]$
$= (-2+1)-(1-3)$
$= -1-(-2)$
$= -1+2$
$= 1$

10. $(g/f)\left(-\frac{1}{2}\right) = \frac{g\left(-\frac{1}{2}\right)}{f\left(-\frac{1}{2}\right)}$
$= \frac{2\left(-\frac{1}{2}\right)+1}{\left(-\frac{1}{2}\right)^2-3}$
$= \frac{0}{-\frac{11}{4}}$
$= 0$

11. $(h-g)(-4) = h(-4)-g(-4)$
$= (-4+4)-\sqrt{-4-1}$
$= 0-\sqrt{-5}$

Since $\sqrt{-5}$ is not a real number, $(h-g)(-4)$ does not exist.

12. $(gh)(10) = g(10)\cdot h(10)$
$= \sqrt{10-1}(10+4)$
$= \sqrt{9}(14)$
$= 3\cdot 14 = 42$

13. $(g/h)(1) = \frac{g(1)}{h(1)}$
$= \frac{\sqrt{1-1}}{1+4}$
$= \frac{\sqrt{0}}{5}$
$= \frac{0}{5} = 0$

14. $(h/g)(1) = \frac{h(1)}{g(1)}$
$= \frac{1+4}{\sqrt{1-1}}$
$= \frac{5}{0}$

Since division by 0 is not defined, $(h/g)(1)$ does not exist.

15. $(g+h)(1) = g(1)+h(1)$
$= \sqrt{1-1}+(1+4)$
$= \sqrt{0}+5$
$= 0+5 = 5$

16. $(hg)(3) = h(3)\cdot g(3)$
$= (3+4)\sqrt{3-1}$
$= 7\sqrt{2}$

17. $f(x) = 2x+3$, $g(x) = 3-5x$

a) The domain of f and of g is the set of all real numbers, or $(-\infty,\infty)$. Then the domain of $f+g$, $f-g$, ff, and fg is also $(-\infty,\infty)$. For f/g we must exclude $\frac{3}{5}$ since $g\left(\frac{3}{5}\right) = 0$. Then the domain of f/g is $\left(-\infty,\frac{3}{5}\right)\cup\left(\frac{3}{5},\infty\right)$. For g/f we must exclude $-\frac{3}{2}$ since $f\left(-\frac{3}{2}\right) = 0$. The domain of g/f is $\left(-\infty,-\frac{3}{2}\right)\cup\left(-\frac{3}{2},\infty\right)$.

b) $(f+g)(x) = f(x)+g(x) = (2x+3)+(3-5x) = -3x+6$

$(f-g)(x) = f(x)-g(x) = (2x+3)-(3-5x) = 2x+3-3+5x = 7x$

$(fg)(x) = f(x)\cdot g(x) = (2x+3)(3-5x) = 6x-10x^2+9-15x = -10x^2-9x+9$

$(ff)(x) = f(x)\cdot f(x) = (2x+3)(2x+3) = 4x^2+12x+9$

$(f/g)(x) = \dfrac{f(x)}{g(x)} = \dfrac{2x+3}{3-5x}$

$(g/f)(x) = \dfrac{g(x)}{f(x)} = \dfrac{3-5x}{2x+3}$

18. $f(x) = -x+1$, $g(x) = 4x-2$

a) The domain of f, g, $f+g$, $f-g$, fg, and ff is $(-\infty,\infty)$. Since $g\left(\frac{1}{2}\right) = 0$, the domain of f/g is $\left(-\infty,\frac{1}{2}\right)\cup\left(\frac{1}{2},\infty\right)$. Since $f(1) = 0$, the domain of g/f is $(-\infty,1)\cup(1,\infty)$.

b) $(f+g)(x) = (-x+1)+(4x-2) = 3x-1$

$(f-g)(x) = (-x+1)-(4x-2) = -x+1-4x+2 = -5x+3$

$(fg)(x) = (-x+1)(4x-2) = -4x^2+6x-2$

$(ff)(x) = (-x+1)(-x+1) = x^2-2x+1$

$(f/g)(x) = \dfrac{-x+1}{4x-2}$

$(g/f)(x) = \dfrac{4x-2}{-x+1}$

19. $f(x) = x-3$, $g(x) = \sqrt{x+4}$

a) Any number can be an input in f, so the domain of f is the set of all real numbers, or $(-\infty,\infty)$.

The domain of g consists of all values of x for which $x+4$ is nonnegative, so we have $x+4\geq 0$, or $x\geq -4$. Thus, the domain of g is $[-4,\infty)$.

The domain of $f+g$, $f-g$, and fg is the set of all numbers in the domains of both f and g. This is $[-4,\infty)$.

The domain of ff is the domain of f, or $(-\infty,\infty)$.

The domain of f/g is the set of all numbers in the domains of f and g, excluding those for which $g(x) = 0$. Since $g(-4) = 0$, the domain of f/g is $(-4,\infty)$.

The domain of g/f is the set of all numbers in the domains of g and f, excluding those for which $f(x) = 0$. Since $f(3) = 0$, the domain of g/f is $[-4,3)\cup(3,\infty)$.

b) $(f+g)(x) = f(x)+g(x) = x-3+\sqrt{x+4}$

$(f-g)(x) = f(x)-g(x) = x-3-\sqrt{x+4}$

$(fg)(x) = f(x)\cdot g(x) = (x-3)\sqrt{x+4}$

$(ff)(x) = \left[f(x)\right]^2 = (x-3)^2 = x^2-6x+9$

$(f/g)(x) = \dfrac{f(x)}{g(x)} = \dfrac{x-3}{\sqrt{x+4}}$

$(g/f)(x) = \dfrac{g(x)}{f(x)} = \dfrac{\sqrt{x+4}}{x-3}$

20. $f(x) = x+2$, $g(x) = \sqrt{x-1}$

a) The domain of f is $(-\infty,\infty)$. The domain of g consists of all the values of x for which $x-1$ is nonnegative, or $[1,\infty)$. Then the domain of $f+g$, $f-g$, and fg is $[1,\infty)$. The domain of ff is $(-\infty,\infty)$. Since $g(1) = 0$, the domain of f/g is $(1,\infty)$. Since $f(-2) = 0$ and -2 is not in the domain of g, the domain of g/f is $[1,\infty)$.

b) $(f+g)(x) = x+2+\sqrt{x-1}$

$(f-g)(x) = x+2-\sqrt{x+1}$

$(fg)(x) = (x+2)\sqrt{x-1}$

$(ff)(x) = (x+2)(x+2) = x^2+4x+4$

$(f/g)(x) = \dfrac{x+2}{\sqrt{x-1}}$

$(g/f)(x) = \dfrac{\sqrt{x-1}}{x+2}$

21. $f(x) = 2x-1$, $g(x) = -2x^2$

a) The domain of f and of g is $(-\infty,\infty)$. Then the domain of $f+g$, $f-g$, fg, and ff is $(-\infty,\infty)$. For f/g, we must exclude 0 since $g(0) = 0$. The domain of f/g is $(-\infty,0)\cup(0,\infty)$. For g/f, we must exclude $\frac{1}{2}$ since $f\left(\frac{1}{2}\right) = 0$. The domain of g/f is $\left(-\infty,\frac{1}{2}\right)\cup\left(\frac{1}{2},\infty\right)$.

b) $(f+g)(x) = f(x)+g(x) = (2x-1)+(-2x^2) = -2x^2+2x-1$

$(f-g)(x) = f(x)-g(x) = (2x-1)-(-2x^2) = 2x^2+2x-1$

$(fg)(x) = f(x)\cdot g(x) = (2x-1)(-2x^2) = -4x^3+2x^2$

$(ff)(x) = f(x)\cdot f(x) = (2x-1)(2x-1) = 4x^2-4x+1$

$(f/g)(x) = \dfrac{f(x)}{g(x)} = \dfrac{2x-1}{-2x^2}$

$(g/f)(x) = \dfrac{g(x)}{f(x)} = \dfrac{-2x^2}{2x-1}$

22. $f(x) = x^2-1$, $g(x) = 2x+5$

a) The domain of f and of g is the set of all real numbers, or $(-\infty,\infty)$. Then the domain of $f+g$, $f-g$, fg and ff is $(-\infty,\infty)$. Since $g\left(-\dfrac{5}{2}\right) = 0$, the domain of f/g is $\left(-\infty,-\dfrac{5}{2}\right)\cup\left(-\dfrac{5}{2},\infty\right)$. Since $f(1) = 0$ and $f(-1) = 0$, the domain of g/f is $(-\infty,-1)\cup(-1,1)\cup(1,\infty)$.

b) $(f+g)(x) = x^2-1+2x+5 = x^2+2x+4$

$(f-g)(x) = x^2-1-(2x+5) = x^2-2x-6$

$(fg)(x) = (x^2-1)(2x+5) = 2x^3+5x^2-2x-5$

$(ff)(x) = (x^2-1)^2 = x^4-2x^2+1$

$(f/g)(x) = \dfrac{x^2-1}{2x+5}$

$(g/f)(x) = \dfrac{2x+5}{x^2-1}$

23. $f(x) = \sqrt{x-3}$, $g(x) = \sqrt{x+3}$

a) Since $f(x)$ is nonnegative for values of x in $[3,\infty)$, this is the domain of f. Since $g(x)$ is nonnegative for values of x in $[-3,\infty)$, this is the domain of g. The domain of $f+g$, $f-g$, and fg is the intersection of the domains of f and g, or $[3,\infty)$. The domain of ff is the same as the domain of f, or $[3,\infty)$. For f/g, we must exclude -3 since $g(-3) = 0$. This is not in $[3,\infty)$, so the domain of f/g is $[3,\infty)$. For g/f, we must exclude 3 since $f(3) = 0$. The domain of g/f is $(3,\infty)$.

b) $(f+g)(x) = f(x)+g(x) = \sqrt{x-3}+\sqrt{x+3}$

$(f-g)(x) = f(x)-g(x) = \sqrt{x-3}-\sqrt{x+3}$

$(fg)(x) = f(x)\cdot g(x) = \sqrt{x-3}\cdot\sqrt{x+3} = \sqrt{x^2-9}$

$(ff)(x) = f(x)\cdot f(x) = \sqrt{x-3}\cdot\sqrt{x-3} = |x-3|$

$(f/g)(x) = \dfrac{\sqrt{x-3}}{\sqrt{x+3}}$

$(g/f)(x) = \dfrac{\sqrt{x+3}}{\sqrt{x-3}}$

24. $f(x) = \sqrt{x}$, $g(x) = \sqrt{2-x}$

a) The domain of f is $[0,\infty)$. The domain of g is $(-\infty,2]$. Then the domain of $f+g$, $f-g$, and fg is $[0,2]$. The domain of ff is the same as the domain of f, $[0,\infty)$. Since $g(2) = 0$, the domain of f/g is $[0,2)$. Since $f(0) = 0$, the domain of g/f is $(0,2]$.

b) $(f+g)(x) = \sqrt{x}+\sqrt{2-x}$

$(f-g)(x) = \sqrt{x}-\sqrt{2-x}$

$(fg)(x) = \sqrt{x}\cdot\sqrt{2-x} = \sqrt{2x-x^2}$

$(ff)(x) = \sqrt{x}\cdot\sqrt{x} = \sqrt{x^2} = |x|$

$(f/g)(x) = \dfrac{\sqrt{x}}{\sqrt{2-x}}$

$(g/f)(x) = \dfrac{\sqrt{2-x}}{\sqrt{x}}$

25. $f(x) = x+1$, $g(x) = |x|$

a) The domain of f and of g is $(-\infty,\infty)$. Then the domain of $f+g$, $f-g$, fg, and ff is $(-\infty,\infty)$. For f/g, we must exclude 0 since $g(0) = 0$. The domain of f/g is $(-\infty,0)\cup(0,\infty)$. For g/f, we must exclude -1 since $f(-1) = 0$. The domain of g/f is $(-\infty,-1)\cup(-1,\infty)$.

b) $(f+g)(x) = f(x)+g(x) = x+1+|x|$

$(f-g)(x) = f(x)-g(x) = x+1-|x|$

$(fg)(x) = f(x)\cdot g(x) = (x+1)|x|$

$(ff)(x) = f(x)\cdot f(x) = (x+1)(x+1) = x^2+2x+1$

$(f/g)(x) = \dfrac{x+1}{|x|}$

$(g/f)(x) = \dfrac{|x|}{x+1}$

26. $f(x) = 4|x|$, $g(x) = 1-x$

a) The domain of f and of g is $(-\infty,\infty)$. Then the domain of $f+g$, $f-g$, fg, and ff is $(-\infty,\infty)$. Since $g(1) = 0$, the domain of f/g is $(-\infty,1)\cup(1,\infty)$. Since $f(0) = 0$, the domain of g/f is $(-\infty,0)\cup(0,\infty)$.

b) $(f+g)(x) = 4|x|+1-x$

$(f-g)(x) = 4|x|-(1-x) = 4|x|-1+x$

$(fg)(x) = 4|x|(1-x) = 4|x|-4x|x|$

$(ff)(x) = 4|x|\cdot 4|x| = 16x^2$

$(f/g)(x) = \dfrac{4|x|}{1-x}$

$(g/f)(x) = \dfrac{1-x}{4|x|}$

27. $f(x) = x^3$, $g(x) = 2x^2+5x-3$

a) Since any number can be an input for either f or g, the domain of f, g, $f+g$, $f-g$, fg, and ff is the set of all real numbers, or $(-\infty,\infty)$.

Since $g(-3) = 0$ and $g\left(\dfrac{1}{2}\right) = 0$, the domain of f/g is $(-\infty,-3)\cup\left(-3,\dfrac{1}{2}\right)\cup\left(\dfrac{1}{2},\infty\right)$.

Since $f(0) = 0$, the domain of g/f is $(-\infty, 0) \cup (0, \infty)$.

b) $(f+g)(x) = f(x) + g(x) = x^3 + 2x^2 + 5x - 3$

$(f-g)(x) = f(x) - g(x) = x^3 - (2x^2 + 5x - 3) = x^3 - 2x^2 - 5x + 3$

$(fg)(x) = f(x) \cdot g(x) = x^3(2x^2 + 5x - 3) = 2x^5 + 5x^4 - 3x^3$

$(ff)(x) = f(x) \cdot f(x) = x^3 \cdot x^3 = x^6$

$(f/g)(x) = \dfrac{f(x)}{g(x)} = \dfrac{x^3}{2x^2 + 5x - 3}$

$(g/f)(x) = \dfrac{g(x)}{f(x)} = \dfrac{2x^2 + 5x - 3}{x^3}$

28. $f(x) = x^2 - 4$, $g(x) = x^3$

a) The domain of f and of g is $(-\infty, \infty)$. Then the domain of $f+g$, $f-g$, fg, and ff is $(-\infty, \infty)$. Since $g(0) = 0$, the domain of f/g is $(-\infty, 0) \cup (0, \infty)$. Since $f(-2) = 0$ and $f(2) = 0$, the domain of g/f is $(-\infty, -2) \cup (-2, 2) \cup (2, \infty)$.

b) $(f+g)(x) = x^2 - 4 + x^3$, or $x^3 + x^2 - 4$

$(f-g)(x) = x^2 - 4 - x^3$, or $-x^3 + x^2 - 4$

$(fg)(x) = (x^2 - 4)(x^3) = x^5 - 4x^3$

$(ff)(x) = (x^2 - 4)(x^2 - 4) = x^4 - 8x^2 + 16$

$(f/g)(x) = \dfrac{x^2 - 4}{x^3}$

$(g/f)(x) = \dfrac{x^3}{x^2 - 4}$

29. $f(x) = \dfrac{4}{x+1}$, $g(x) = \dfrac{1}{6-x}$

a) Since $x + 1 = 0$ when $x = -1$, we must exclude -1 from the domain of f. It is $(-\infty, -1) \cup (-1, \infty)$. Since $6 - x = 0$ when $x = 6$, we must exclude 6 from the domain of g. It is $(-\infty, 6) \cup (6, \infty)$. The domain of $f+g$, $f-g$, and fg is the intersection of the domains of f and g, or $(-\infty, -1) \cup (-1, 6) \cup (6, \infty)$. The domain of ff is the same as the domain of f, or $(-\infty, -1) \cup (-1, \infty)$. Since there are no values of x for which $g(x) = 0$ or $f(x) = 0$, the domain of f/g and g/f is $(-\infty, -1) \cup (-1, 6) \cup (6, \infty)$.

b) $(f+g)(x) = f(x) + g(x) = \dfrac{4}{x+1} + \dfrac{1}{6-x}$

$(f-g)(x) = f(x) - g(x) = \dfrac{4}{x+1} - \dfrac{1}{6-x}$

$(fg)(x) = f(x) \cdot g(x) = \dfrac{4}{x+1} \cdot \dfrac{1}{6-x} = \dfrac{4}{(x+1)(6-x)}$

$(ff)(x) = f(x) \cdot f(x) = \dfrac{4}{x+1} \cdot \dfrac{4}{x+1} = \dfrac{16}{(x+1)^2}$, or $\dfrac{16}{x^2 + 2x + 1}$

$(f/g)(x) = \dfrac{\frac{4}{x+1}}{\frac{1}{6-x}} = \dfrac{4}{x+1} \cdot \dfrac{6-x}{1} = \dfrac{4(6-x)}{x+1}$

$(g/f)(x) = \dfrac{\frac{1}{6-x}}{\frac{4}{x+1}} = \dfrac{1}{6-x} \cdot \dfrac{x+1}{4} = \dfrac{x+1}{4(6-x)}$

30. $f(x) = 2x^2$, $g(x) = \dfrac{2}{x-5}$

a) The domain of f is $(-\infty, \infty)$. Since $x - 5 = 0$ when $x = 5$, the domain of g is $(-\infty, 5) \cup (5, \infty)$. Then the domain of $f+g$, $f-g$, and fg is $(-\infty, 5) \cup (5, \infty)$. The domain of ff is $(-\infty, \infty)$. Since there are no values of x for which $g(x) = 0$, the domain of f/g is $(-\infty, 5) \cup (5, \infty)$. Since $f(0) = 0$, the domain of g/f is $(-\infty, 0) \cup (0, 5) \cup (5, \infty)$.

b) $(f+g)(x) = 2x^2 + \dfrac{2}{x-5}$

$(f-g)(x) = 2x^2 - \dfrac{2}{x-5}$

$(fg)(x) = 2x^2 \cdot \dfrac{2}{x-5} = \dfrac{4x^2}{x-5}$

$(ff)(x) = 2x^2 \cdot 2x^2 = 4x^4$

$(f/g)(x) = \dfrac{2x^2}{\frac{2}{x-5}} = 2x^2 \cdot \dfrac{x-5}{2} = x^2(x-5) = x^3 - 5x^2$

$(g/f)(x) = \dfrac{\frac{2}{x-5}}{2x^2} = \dfrac{2}{x-5} \cdot \dfrac{1}{2x^2} = \dfrac{1}{x^2(x-5)} = \dfrac{1}{x^3 - 5x^2}$

31. $f(x) = \dfrac{1}{x}$, $g(x) = x - 3$

a) Since $f(0)$ is not defined, the domain of f is $(-\infty, 0) \cup (0, \infty)$. The domain of g is $(-\infty, \infty)$. Then the domain of $f+g$, $f-g$, fg, and ff is $(-\infty, 0) \cup (0, \infty)$. Since $g(3) = 0$, the domain of f/g is $(-\infty, 0) \cup (0, 3) \cup (3, \infty)$. There are no values of x for which $f(x) = 0$, so the domain of g/f is $(-\infty, 0) \cup (0, \infty)$.

b) $(f+g)(x) = f(x) + g(x) = \dfrac{1}{x} + x - 3$

$(f-g)(x) = f(x) - g(x) = \dfrac{1}{x} - (x-3) = \dfrac{1}{x} - x + 3$

$(fg)(x) = f(x) \cdot g(x) = \dfrac{1}{x} \cdot (x-3) = \dfrac{x-3}{x}$, or $1 - \dfrac{3}{x}$

$(ff)(x) = f(x) \cdot f(x) = \dfrac{1}{x} \cdot \dfrac{1}{x} = \dfrac{1}{x^2}$

$(f/g)(x) = \dfrac{f(x)}{g(x)} = \dfrac{\frac{1}{x}}{x-3} = \dfrac{1}{x} \cdot \dfrac{1}{x-3} = \dfrac{1}{x(x-3)}$

$(g/f)(x) = \dfrac{g(x)}{f(x)} = \dfrac{x-3}{\frac{1}{x}} = (x-3) \cdot \dfrac{x}{1} = x(x-3)$, or $x^2 - 3x$

32. $f(x) = \sqrt{x+6}$, $g(x) = \dfrac{1}{x}$

a) The domain of $f(x)$ is $[-6, \infty)$. The domain of $g(x)$ is $(-\infty, 0) \cup (0, \infty)$. Then the domain of $f + g$, $f - g$, and fg is $[-6, 0) \cup (0, \infty)$. The domain of ff is $[-6, \infty)$. Since there are no values of x for which $g(x) = 0$, the domain of f/g is $[-6, 0) \cup (0, \infty)$. Since $f(-6) = 0$, the domain of g/f is $(-6, 0) \cup (0, \infty)$.

b) $(f + g)(x) = \sqrt{x+6} + \dfrac{1}{x}$

$(f - g)(x) = \sqrt{x+6} - \dfrac{1}{x}$

$(fg)(x) = \sqrt{x+6} \cdot \dfrac{1}{x} = \dfrac{\sqrt{x+6}}{x}$

$(ff)(x) = \sqrt{x+6} \cdot \sqrt{x+6} = |x+6|$

$(f/g)(x) = \dfrac{\sqrt{x+6}}{\frac{1}{x}} = \sqrt{x+6} \cdot \dfrac{x}{1} = x\sqrt{x+6}$

$(g/f)(x) = \dfrac{\frac{1}{x}}{\sqrt{x+6}} = \dfrac{1}{x} \cdot \dfrac{1}{\sqrt{x+6}} = \dfrac{1}{x\sqrt{x+6}}$

33. $f(x) = \dfrac{3}{x-2}$, $g(x) = \sqrt{x-1}$

a) Since $f(2)$ is not defined, the domain of f is $(-\infty, 2) \cup (2, \infty)$. Since $g(x)$ is nonnegative for values of x in $[1, \infty)$, this is the domain of g. The domain of $f + g$, $f - g$, and fg is the intersection of the domains of f and g, or $[1, 2) \cup (2, \infty)$. The domain of ff is the same as the domain of f, or $(-\infty, 2) \cup (2, \infty)$. For f/g, we must exclude 1 since $g(1) = 0$, so the domain of f/g is $(1, 2) \cup (2, \infty)$. There are no values of x for which $f(x) = 0$, so the domain of g/f is $[1, 2) \cup (2, \infty)$.

b) $(f + g)(x) = f(x) + g(x) = \dfrac{3}{x-2} + \sqrt{x-1}$

$(f - g)(x) = f(x) - g(x) = \dfrac{3}{x-2} - \sqrt{x-1}$

$(fg)(x) = f(x) \cdot g(x) = \dfrac{3}{x-2}(\sqrt{x-1})$, or $\dfrac{3\sqrt{x-1}}{x-2}$

$(ff)(x) = f(x) \cdot f(x) = \dfrac{3}{x-2} \cdot \dfrac{3}{x-2} \cdot \dfrac{9}{(x-2)^2}$

$(f/g)(x) = \dfrac{f(x)}{g(x)} = \dfrac{\frac{3}{x-2}}{\sqrt{x-1}} = \dfrac{3}{(x-2)\sqrt{x-1}}$

$(g/f)(x) = \dfrac{g(x)}{f(x)} = \dfrac{\sqrt{x-1}}{\frac{3}{x-2}} = \dfrac{(x-2)\sqrt{x-1}}{3}$

34. $f(x) = \dfrac{2}{4-x}$, $g(x) = \dfrac{5}{x-1}$

a) The domain of f is $(-\infty, 4) \cup (4, \infty)$. The domain of g is $(-\infty, 1) \cup (1, \infty)$. The domain of $f + g$, $f - g$, and fg is $(-\infty, 1) \cup (1, 4) \cup (4, \infty)$. The domain of ff is $(-\infty, 4) \cup (4, \infty)$. The domain of f/g and of g/f is $(-\infty, 1) \cup (1, 4) \cup (4, \infty)$.

b) $(f + g)(x) = \dfrac{2}{4-x} + \dfrac{5}{x-1}$

$(f - g)(x) = \dfrac{2}{4-x} - \dfrac{5}{x-1}$

$(fg)(x) = \dfrac{2}{4-x} \cdot \dfrac{5}{x-1} = \dfrac{10}{(4-x)(x-1)}$

$(ff)(x) = \dfrac{2}{4-x} \cdot \dfrac{2}{4-x} = \dfrac{4}{(4-x)^2}$

$(f/g)(x) = \dfrac{\frac{2}{4-x}}{\frac{5}{x-1}} = \dfrac{2(x-1)}{5(4-x)}$

$(g/f)(x) = \dfrac{\frac{5}{x-1}}{\frac{2}{4-x}} = \dfrac{5(4-x)}{2(x-1)}$

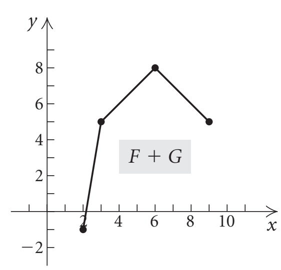
35. From the graph we see that the domain of F is $[2, 11]$ and the domain of G is $[1, 9]$. The domain of $F + G$ is the set of numbers in the domains of both F and G. This is $[2, 9]$.

36. The domain of $F - G$ and FG is the set of numbers in the domains of both F and G. (See Exercise 33.) This is $[2, 9]$.

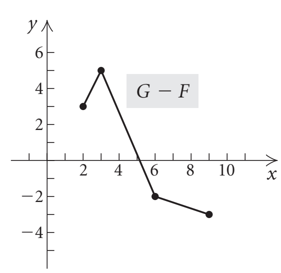
The domain of F/G is the set of numbers in the domains of both F and G, excluding those for which $G = 0$. Since $G > 0$ for all values of x in its domain, the domain of F/G is $[2, 9]$.

37. The domain of G/F is the set of numbers in the domains of both F and G (See Exercise 33.), excluding those for which $F = 0$. Since $F(3) = 0$, the domain of G/F is $[2, 3) \cup (3, 9]$.

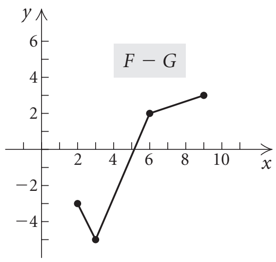
38.



39.



40.



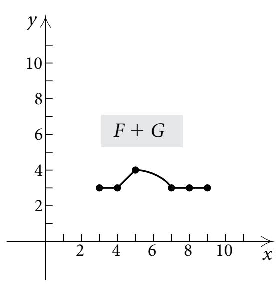
41. From the graph, we see that the domain of F is $[0, 9]$ and the domain of G is $[3, 10]$. The domain of $F+G$ is the set of numbers in the domains of both F and G. This is $[3, 9]$.

42. The domain of $F-G$ and FG is the set of numbers in the domains of both F and G. (See Exercise 39.) This is $[3, 9]$.

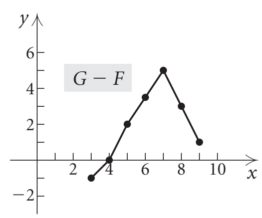
The domain of F/G is the set of numbers in the domains of both F and G, excluding those for which $G = 0$. Since $G > 0$ for all values of x in its domain, the domain of F/G is $[3, 9]$.

43. The domain of G/F is the set of numbers in the domains of both F and G (See Exercise 39.), excluding those for which $F = 0$. Since $F(6) = 0$ and $F(8) = 0$, the domain of G/F is $[3, 6) \cup (6, 8) \cup (8, 9]$.

44. $(F+G)(x) = F(x) + G(x)$



45.



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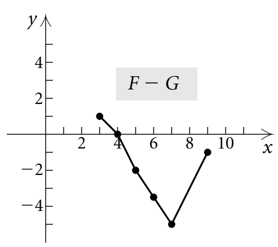


47. a) $P(x) = R(x) - C(x) = 60x - 0.4x^2 - (3x + 13) =$
$60x - 0.4x^2 - 3x - 13 = -0.4x^2 + 57x - 13$

b) $R(100) = 60 \cdot 100 - 0.4(100)^2 = 6000 - 0.4(10,000) =$
$6000 - 4000 = 2000$
$C(100) = 3 \cdot 100 + 13 = 300 + 13 = 313$
$P(100) = R(100) - C(100) = 2000 - 313 = 1687$

48. a) $P(x) = 200x - x^2 - (5000 + 8x) =$
$200x - x^2 - 5000 - 8x = -x^2 + 192x - 5000$

b) $R(175) = 200(175) - 175^2 = 4375$
$C(175) = 5000 + 8 \cdot 175 = 6400$
$P(175) = R(175) - C(175) = 4375 - 6400 = -2025$
(We could also use the function found in part (a) to find $P(175)$.)

49. $f(x) = 3x - 5$
$f(x+h) = 3(x+h) - 5 = 3x + 3h - 5$

$$\frac{f(x+h) - f(x)}{h} = \frac{3x + 3h - 5 - (3x - 5)}{h} = \frac{3x + 3h - 5 - 3x + 5}{h} = \frac{3h}{h} = 3$$

50. $f(x) = 4x - 1$

$$\frac{f(x+h) - f(x)}{h} = \frac{4(x+h) - 1 - (4x - 1)}{h} = \frac{4x + 4h - 1 - 4x + 1}{h} = \frac{4h}{h} = 4$$

51. $f(x) = 6x + 2$
$f(x+h) = 6(x+h) + 2 = 6x + 6h + 2$

$$\frac{f(x+h) - f(x)}{h} = \frac{6x + 6h + 2 - (6x + 2)}{h} = \frac{6x + 6h + 2 - 6x - 2}{h} = \frac{6h}{h} = 6$$

52. $f(x) = 5x + 3$

$$\frac{f(x+h) - f(x)}{h} = \frac{5(x+h) + 3 - (5x + 3)}{h} = \frac{5x + 5h + 3 - 5x - 3}{h} = \frac{5h}{h} = 5$$

53. $f(x) = \frac{1}{3}x + 1$

$f(x+h) = \frac{1}{3}(x+h) + 1 = \frac{1}{3}x + \frac{1}{3}h + 1$

$$\frac{f(x+h) - f(x)}{h} = \frac{\frac{1}{3}x + \frac{1}{3}h + 1 - \left(\frac{1}{3}x + 1\right)}{h} = \frac{\frac{1}{3}x + \frac{1}{3}h + 1 - \frac{1}{3}x - 1}{h} = \frac{\frac{1}{3}h}{h} = \frac{1}{3}$$

54. $f(x) = -\frac{1}{2}x + 7$

$$\frac{f(x+h)-f(x)}{h} = \frac{-\frac{1}{2}(x+h)+7-\left(-\frac{1}{2}x+7\right)}{h} =$$

$$\frac{-\frac{1}{2}x-\frac{1}{2}h+7+\frac{1}{2}-7}{h} = \frac{-\frac{1}{2}h}{h} = -\frac{1}{2}$$

55. $f(x) = \frac{1}{3x}$

$f(x+h) = \frac{1}{3(x+h)}$

$$\frac{f(x+h)-f(x)}{h} = \frac{\frac{1}{3(x+h)}-\frac{1}{3x}}{h}$$

$$= \frac{\frac{1}{3(x+h)}\cdot\frac{x}{x}-\frac{1}{3x}\cdot\frac{x+h}{x+h}}{h}$$

$$= \frac{\frac{x}{3x(x+h)}-\frac{x+h}{3x(x+h)}}{h}$$

$$= \frac{\frac{x-(x+h)}{3x(x+h)}}{h} = \frac{\frac{x-x-h}{3x(x+h)}}{h}$$

$$= \frac{\frac{-h}{3x(x+h)}}{h} = \frac{-h}{3x(x+h)}\cdot\frac{1}{h}$$

$$= \frac{-h}{3x(x+h)\cdot h} = \frac{-1\cdot \cancel{h}}{3x(x+h)\cdot\cancel{h}}$$

$$= \frac{-1}{3x(x+h)}, \text{ or } -\frac{1}{3x(x+h)}$$

56. $f(x) = \frac{1}{2x}$

$$\frac{f(x+h)-f(x)}{h} = \frac{\frac{1}{2(x+h)}-\frac{1}{2x}}{h} = \frac{\frac{1}{2(x+h)}\cdot\frac{x}{x}-\frac{1}{2x}\cdot\frac{x+h}{x+h}}{h} =$$

$$\frac{\frac{x}{2x(x+h)}-\frac{x+h}{2x(x+h)}}{h} = \frac{\frac{x-x-h}{2x(x+h)}}{h} = \frac{\frac{-h}{2x(x+h)}}{h} =$$

$$\frac{-h}{2x(x+h)}\cdot\frac{1}{h} = \frac{-1}{2x(x+h)}, \text{ or } -\frac{1}{2x(x+h)}$$

57. $f(x) = -\frac{1}{4x}$

$f(x+h) = -\frac{1}{4(x+h)}$

$$\frac{f(x+h)-f(x)}{h} = \frac{-\frac{1}{4(x+h)}-\left(-\frac{1}{4x}\right)}{h}$$

$$= \frac{-\frac{1}{4(x+h)}\cdot\frac{x}{x}-\left(-\frac{1}{4x}\right)\cdot\frac{x+h}{x+h}}{h}$$

$$= \frac{-\frac{x}{4x(x+h)}+\frac{x+h}{4x(x+h)}}{h}$$

$$= \frac{\frac{-x+x+h}{4x(x+h)}}{h} = \frac{\frac{h}{4x(x+h)}}{h}$$

$$= \frac{h}{4x(x+h)}\cdot\frac{1}{h} = \frac{\cancel{h}\cdot 1}{4x(x+h)\cdot\cancel{h}} = \frac{1}{4x(x+h)}$$

58. $f(x) = -\frac{1}{x}$

$$\frac{f(x+h)-f(x)}{h} = \frac{-\frac{1}{x+h}-\left(-\frac{1}{x}\right)}{h} =$$

$$\frac{-\frac{1}{x+h}\cdot\frac{x}{x}-\left(-\frac{1}{x}\right)\cdot\frac{x+h}{x+h}}{h} = \frac{-\frac{x}{x(x+h)}+\frac{x+h}{x(x+h)}}{h} =$$

$$\frac{\frac{-x+x+h}{x(x+h)}}{h} = \frac{\frac{h}{x(x+h)}}{h} = \frac{h}{x(x+h)}\cdot\frac{1}{h} = \frac{1}{x(x+h)}$$

59. $f(x) = x^2 + 1$

$f(x+h) = (x+h)^2 + 1 = x^2 + 2xh + h^2 + 1$

$$\frac{f(x+h)-f(x)}{h} = \frac{x^2+2xh+h^2+1-(x^2+1)}{h}$$

$$= \frac{x^2+2xh+h^2+1-x^2-1}{h}$$

$$= \frac{2xh+h^2}{h}$$

$$= \frac{h(2x+h)}{h}$$

$$= \frac{h}{h}\cdot\frac{2x+h}{1}$$

$$= 2x+h$$

60. $f(x) = x^2 - 3$

$$\frac{f(x+h)-f(x)}{h} = \frac{(x+h)^2-3-(x^2-3)}{h} =$$

$$\frac{x^2+2xh+h^2-3-x^2+3}{h} = \frac{2xh+h^2}{h} = \frac{h(2x+h)}{h} =$$

$$2x+h$$

61. $f(x) = 4 - x^2$

$f(x+h) = 4 - (x+h)^2 = 4 - (x^2 + 2xh + h^2) =$

$4 - x^2 - 2xh - h^2$

$$\frac{f(x+h) - f(x)}{h} = \frac{4 - x^2 - 2xh - h^2 - (4 - x^2)}{h}$$

$$= \frac{4 - x^2 - 2xh - h^2 - 4 + x^2}{h}$$

$$= \frac{-2xh - h^2}{h} = \frac{\not{h}(-2x - h)}{\not{h}}$$

$$= -2x - h$$

62. $f(x) = 2 - x^2$

$$\frac{f(x+h) - f(x)}{h} = \frac{2 - (x+h)^2 - (2 - x^2)}{h} =$$

$$\frac{2 - x^2 - 2xh - h^2 - 2 + x^2}{h} = \frac{-2xh - h^2}{h} =$$

$$\frac{h(-2x - h)}{h} = -2x - h$$

63. $f(x) = 3x^2 - 2x + 1$

$f(x+h) = 3(x+h)^2 - 2(x+h) + 1 =$

$3(x^2 + 2xh + h^2) - 2(x+h) + 1 =$

$3x^2 + 6xh + 3h^2 - 2x - 2h + 1$

$f(x) = 3x^2 - 2x + 1$

$$\frac{f(x+h) - f(x)}{h} =$$

$$\frac{(3x^2 + 6xh + 3h^2 - 2x - 2h + 1) - (3x^2 - 2x + 1)}{h} =$$

$$\frac{3x^2 + 6xh + 3h^2 - 2x - 2h + 1 - 3x^2 + 2x - 1}{h} =$$

$$\frac{6xh + 3h^2 - 2h}{h} = \frac{h(6x + 3h - 2)}{h \cdot 1} =$$

$$\frac{h}{h} \cdot \frac{6x + 3h - 2}{1} = 6x + 3h - 2$$

64. $f(x) = 5x^2 + 4x$

$$\frac{f(x+h) - f(x)}{h} = \frac{(5x^2 + 10xh + 5h^2 + 4x + 4h) - (5x^2 + 4x)}{h} =$$

$$\frac{10xh + 5h^2 + 4h}{h} = 10x + 5h + 4$$

65. $f(x) = 4 + 5|x|$

$f(x+h) = 4 + 5|x+h|$

$$\frac{f(x+h) - f(x)}{h} = \frac{4 + 5|x+h| - (4 + 5|x|)}{h}$$

$$= \frac{4 + 5|x+h| - 4 - 5|x|}{h}$$

$$= \frac{5|x+h| - 5|x|}{h}$$

66. $f(x) = 2|x| + 3x$

$$\frac{f(x+h) - f(x)}{h} = \frac{(2|x+h| + 3x + 3h) - (2|x| + 3x)}{h} =$$

$$\frac{2|x+h| - 2|x| + 3h}{h}$$

67. $f(x) = x^3$

$f(x+h) = (x+h)^3 = x^3 + 3x^2h + 3xh^2 + h^3$

$f(x) = x^3$

$$\frac{f(x+h) - f(x)}{h} = \frac{x^3 + 3x^2h + 3xh^2 + h^3 - x^3}{h} =$$

$$\frac{3x^2h + 3xh^2 + h^3}{h} = \frac{h(3x^2 + 3xh + h^2)}{h \cdot 1} =$$

$$\frac{h}{h} \cdot \frac{3x^2 + 3xh + h^2}{1} = 3x^2 + 3xh + h^2$$

68. $f(x) = x^3 - 2x$

$$\frac{f(x+h) - f(x)}{h} = \frac{(x+h)^3 - 2(x+h) - (x^3 - 2x)}{h} =$$

$$\frac{x^3 + 3x^2h + 3xh^2 + h^3 - 2x - 2h - x^3 + 2x}{h} =$$

$$\frac{3x^2h + 3xh^2 + h^3 - 2h}{h} = \frac{h(3x^2 + 3xh + h^2 - 2)}{h} =$$

$$3x^2 + 3xh + h^2 - 2$$

69. $f(x) = \dfrac{x - 4}{x + 3}$

$$\frac{f(x+h) - f(x)}{h} = \frac{\frac{x + h - 4}{x + h + 3} - \frac{x - 4}{x + 3}}{h} =$$

$$\frac{\frac{x + h - 4}{x + h + 3} - \frac{x - 4}{x + 3}}{h} \cdot \frac{(x + h + 3)(x + 3)}{(x + h + 3)(x + 3)} =$$

$$\frac{(x + h - 4)(x + 3) - (x - 4)(x + h + 3)}{h(x + h + 3)(x + 3)} =$$

$$\frac{x^2 + hx - 4x + 3x + 3h - 12 - (x^2 + hx + 3x - 4x - 4h - 12)}{h(x + h + 3)(x + 3)} =$$

$$\frac{x^2 + hx - x + 3h - 12 - x^2 - hx + x + 4h + 12}{h(x + h + 3)(x + 3)} =$$

$$\frac{7h}{h(x + h + 3)(x + 3)} = \frac{h}{h} \cdot \frac{7}{(x + h + 3)(x + 3)} =$$

$$\frac{7}{(x + h + 3)(x + 3)}$$

70. $f(x) = \dfrac{x}{2 - x}$

$$\frac{f(x+h) - f(x)}{h} = \frac{\frac{x + h}{2 - (x + h)} - \frac{x}{2 - x}}{h} =$$

$$\frac{\frac{(x + h)(2 - x) - x(2 - x - h)}{(2 - x - h)(2 - x)}}{h} =$$

$$\frac{\frac{2x - x^2 + 2h - hx - 2x + x^2 + hx}{(2 - x - h)(2 - x)}}{h} =$$

$$\frac{\frac{2h}{(2 - x - h)(2 - x)}}{h} =$$

$$\frac{2h}{(2 - x - h)(2 - x)} \cdot \frac{1}{h} = \frac{2}{(2 - x - h)(2 - x)}$$

71. Graph $y = 3x - 1$.

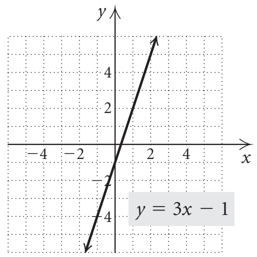
We find some ordered pairs that are solutions of the equation, plot these points, and draw the graph.

When $x = -1$, $y = 3(-1) - 1 = -3 - 1 = -4$.

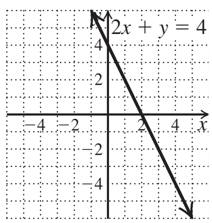
When $x = 0$, $y = 3 \cdot 0 - 1 = 0 - 1 = -1$.

When $x = 2$, $y = 3 \cdot 2 - 1 = 6 - 1 = 5$.

x	y
-1	-4
0	-1
2	5



72.



73. Graph $x - 3y = 3$.

First we find the x- and y-intercepts.

$$x - 3 \cdot 0 = 3$$
$$x = 3$$

The x-intercept is $(3, 0)$.

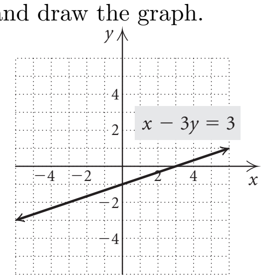
$$0 - 3y = 3$$
$$-3y = 3$$
$$y = -1$$

The y-intercept is $(0, -1)$.

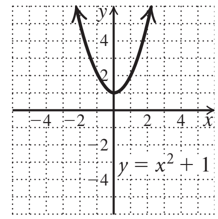
We find a third point as a check. We let $x = -3$ and solve for y.

$$-3 - 3y = 3$$
$$-3y = 6$$
$$y = -2$$

Another point on the graph is $(-3, -2)$. We plot the points and draw the graph.



74.



75. Answers may vary; $f(x) = \dfrac{1}{x+7}$, $g(x) = \dfrac{1}{x-3}$

76. The domain of $h+f$, $h-f$, and hf consists of all numbers that are in the domain of both h and f, or $\{-4, 0, 3\}$.

The domain of h/f consists of all numbers that are in the domain of both h and f, excluding any for which the value of f is 0, or $\{-4, 0\}$.

77. The domain of $h(x)$ is $\left\{x \middle| x \neq \dfrac{7}{3}\right\}$, and the domain of $g(x)$ is $\{x|x \neq 3\}$, so $\dfrac{7}{3}$ and 3 are not in the domain of $(h/g)(x)$. We must also exclude the value of x for which $g(x) = 0$.

$$\frac{x^4 - 1}{5x - 15} = 0$$
$$x^4 - 1 = 0 \quad \text{Multiplying by } 5x - 15$$
$$x^4 = 1$$
$$x = \pm 1$$

Then the domain of $(h/g)(x)$ is $\left\{x \middle| x \neq \dfrac{7}{3} \textit{ and } x \neq 3 \textit{ and } x \neq -1 \textit{ and } x \neq 1\right\}$, or $(-\infty, -1) \cup (-1, 1) \cup \left(1, \dfrac{7}{3}\right) \cup \left(\dfrac{7}{3}, 3\right) \cup (3, \infty)$.

Exercise Set 2.3

1. $(f \circ g)(-1) = f(g(-1)) = f((-1)^2 - 2(-1) - 6) = f(1 + 2 - 6) = f(-3) = 3(-3) + 1 = -9 + 1 = -8$

2. $(g \circ f)(-2) = g(f(-2)) = g(3(-2) + 1) = g(-5) = (-5)^2 - 2(-5) - 6 = 25 + 10 - 6 = 29$

3. $(h \circ f)(1) = h(f(1)) = h(3 \cdot 1 + 1) = h(3 + 1) = h(4) = 4^3 = 64$

4. $(g \circ h)\left(\dfrac{1}{2}\right) = g\left(h\left(\dfrac{1}{2}\right)\right) = g\left(\left(\dfrac{1}{2}\right)^3\right) = g\left(\dfrac{1}{8}\right) = \left(\dfrac{1}{8}\right)^2 - 2\left(\dfrac{1}{8}\right) - 6 = \dfrac{1}{64} - \dfrac{1}{4} - 6 = -\dfrac{399}{64}$

5. $(g \circ f)(5) = g(f(5)) = g(3 \cdot 5 + 1) = g(15 + 1) = g(16) = 16^2 - 2 \cdot 16 - 6 = 218$

6. $(f \circ g)\left(\dfrac{1}{3}\right) = f\left(g\left(\dfrac{1}{3}\right)\right) = f\left(\left(\dfrac{1}{3}\right)^2 - 2\left(\dfrac{1}{3}\right) - 6\right) = f\left(\dfrac{1}{9} - \dfrac{2}{3} - 6\right) = f\left(-\dfrac{59}{9}\right) = 3\left(-\dfrac{59}{9}\right) + 1 = -\dfrac{56}{3}$

7. $(f \circ h)(-3) = f(h(-3)) = f((-3)^3) = f(-27) = 3(-27) + 1 = -81 + 1 = -80$

8. $(h\circ g)(3) = h(g(3)) = h(3^2 - 2\cdot 3 - 6) =$
$h(9-6-6) = h(-3) = (-3)^3 = -27$

9. $(g\circ g)(-2) = g(g(-2)) = g((-2)^2 - 2(-2) - 6) =$
$g(4+4-6) = g(2) = 2^2 - 2\cdot 2 - 6 = 4 - 4 - 6 = -6$

10. $(g\circ g)(3) = g(g(3)) = g(3^2 - 2\cdot 3 - 6) = g(9-6-6) =$
$g(-3) = (-3)^2 - 2(-3) - 6 = 9 + 6 - 6 = 9$

11. $(h\circ h)(2) = h(h(2)) = h(2^3) = h(8) = 8^3 = 512$

12. $(h\circ h)(-1) = h(h(-1)) = h((-1)^3) = h(-1) = (-1)^3 = -1$

13. $(f\circ f)(-4) = f(f(-4)) = f(3(-4)+1) = f(-12+1) =$
$f(-11) = 3(-11) + 1 = -33 + 1 = -32$

14. $(f\circ f)(1) = f(f(1)) = f(3\cdot 1 + 1) = f(3+1) = f(4) =$
$3\cdot 4 + 1 = 12 + 1 = 13$

15. $(h\circ h)(x) = h(h(x)) = h(x^3) = (x^3)^3 = x^9$

16. $(f\circ f)(x) = f(f(x)) = f(3x+1) = 3(3x+1) + 1 =$
$9x + 3 + 1 = 9x + 4$

17. $(f\circ g)(x) = f(g(x)) = f(x-3) = x - 3 + 3 = x$
$(g\circ f)(x) = g(f(x)) = g(x+3) = x + 3 - 3 = x$

The domain of f and of g is $(-\infty,\infty)$, so the domain of $f\circ g$ and of $g\circ f$ is $(-\infty,\infty)$.

18. $(f\circ g)(x) = f\left(\frac{5}{4}x\right) = \frac{4}{5}\cdot\frac{5}{4}x = x$

$(g\circ f)(x) = g\left(\frac{4}{5}x\right) = \frac{5}{4}\cdot\frac{4}{5}x = x$

The domain of f and of g is $(-\infty,\infty)$, so the domain of $f\circ g$ and of $g\circ f$ is $(-\infty,\infty)$.

19. $(f\circ g)(x) = f(g(x)) = f(3x^2-2x-1) = 3x^2-2x-1+1 =$
$3x^2 - 2x$
$(g\circ f)(x) = g(f(x)) = g(x+1) = 3(x+1)^2 - 2(x+1) - 1 =$
$3(x^2+2x+1) - 2(x+1) - 1 = 3x^2+6x+3-2x-2-1 =$
$3x^2 + 4x$

The domain of f and of g is $(-\infty,\infty)$, so the domain of $f\circ g$ and of $g\circ f$ is $(-\infty,\infty)$.

20. $(f\circ g)(x) = f(x^2+5) = 3(x^2+5) - 2 = 3x^2 + 15 - 2 =$
$3x^2 + 13$
$(g\circ f)(x) = g(3x-2) = (3x-2)^2 + 5 = 9x^2 - 12x + 4 + 5 =$
$9x^2 - 12x + 9$

The domain of f and of g is $(-\infty,\infty)$, so the domain of $f\circ g$ and of $g\circ f$ is $(-\infty,\infty)$.

21. $(f\circ g)(x) = f(g(x)) = f(4x-3) = (4x-3)^2 - 3 =$
$16x^2 - 24x + 9 - 3 = 16x^2 - 24x + 6$
$(g\circ f)(x) = g(f(x)) = g(x^2-3) = 4(x^2-3) - 3 =$
$4x^2 - 12 - 3 = 4x^2 - 15$

The domain of f and of g is $(-\infty,\infty)$, so the domain of $f\circ g$ and of $g\circ f$ is $(-\infty,\infty)$.

22. $(f\circ g)(x) = f(2x-7) = 4(2x-7)^2 - (2x-7) + 10 =$
$4(4x^2 - 28x + 49) - (2x-7) + 10 =$
$16x^2 - 112x + 196 - 2x + 7 + 10 = 16x^2 - 114x + 213$
$(g\circ f)(x) = g(4x^2 - x + 10) = 2(4x^2 - x + 10) - 7 =$
$8x^2 - 2x + 20 - 7 = 8x^2 - 2x + 13$

The domain of f and of g is $(-\infty,\infty)$, so the domain of $f\circ g$ and of $g\circ f$ is $(-\infty,\infty)$.

23. $(f\circ g)(x) = f(g(x)) = f\left(\frac{1}{x}\right) = \dfrac{4}{1 - 5\cdot\frac{1}{x}} = \dfrac{4}{1 - \frac{5}{x}} =$

$\dfrac{4}{\frac{x-5}{x}} = 4\cdot\dfrac{x}{x-5} = \dfrac{4x}{x-5}$

$(g\circ f)(x) = g(f(x)) = g\left(\dfrac{4}{1-5x}\right) = \dfrac{1}{\frac{4}{1-5x}} =$

$1\cdot\dfrac{1-5x}{4} = \dfrac{1-5x}{4}$

The domain of f is $\left\{x\middle|x \neq \frac{1}{5}\right\}$ and the domain of g is $\{x|x\neq 0\}$. Consider the domain of $f\circ g$. Since 0 is not in the domain of g, 0 is not in the domain of $f\circ g$. Since $\frac{1}{5}$ is not in the domain of f, we know that $g(x)$ cannot be $\frac{1}{5}$. We find the value(s) of x for which $g(x) = \frac{1}{5}$.

$\frac{1}{x} = \frac{1}{5}$
$5 = x$ Multiplying by $5x$

Thus 5 is also not in the domain of $f\circ g$. Then the domain of $f\circ g$ is $\{x|x\neq 0 \text{ and } x\neq 5\}$, or $(-\infty,0)\cup(0,5)\cup(5,\infty)$.

Now consider the domain of $g\circ f$. Recall that $\frac{1}{5}$ is not in the domain of f, so it is not in the domain of $g\circ f$. Now 0 is not in the domain of g but $f(x)$ is never 0, so the domain of $g\circ f$ is $\left\{x\middle|x\neq\frac{1}{5}\right\}$, or $\left(-\infty,\frac{1}{5}\right)\cup\left(\frac{1}{5},\infty\right)$.

24. $(f\circ g)(x) = f\left(\dfrac{1}{2x+1}\right) = \dfrac{6}{\frac{1}{2x+1}} = 6\cdot\dfrac{2x+1}{1} =$

$6(2x+1)$, or $12x+6$

$(g\circ f)(x) = g\left(\dfrac{6}{x}\right) = \dfrac{1}{2\cdot\frac{6}{x}+1} = \dfrac{1}{\frac{12}{x}+1} = \dfrac{1}{\frac{12+x}{x}} =$

$1\cdot\dfrac{x}{12+x} = \dfrac{x}{12+x}$

The domain of f is $\{x|x\neq 0\}$ and the domain of g is $\left\{x\middle|x\neq -\frac{1}{2}\right\}$. Consider the domain of $f\circ g$. Since $-\frac{1}{2}$ is not in the domain of g, $-\frac{1}{2}$ is not in the domain of $f\circ g$. Now 0 is not in the domain of f but $g(x)$ is never 0, so the domain of $f\circ g$ is $\left\{x\middle|x\neq -\frac{1}{2}\right\}$, or $\left(-\infty,-\frac{1}{2}\right)\cup\left(-\frac{1}{2},\infty\right)$.

Now consider the domain of $g\circ f$. Since 0 is not in the domain of f, then 0 is not in the domain of $g\circ f$. Also, since $-\frac{1}{2}$ is not in the domain of g, we find the value(s) of x for which $f(x) = -\frac{1}{2}$.

$$\frac{6}{x} = -\frac{1}{2}$$
$$-12 = x$$

Then the domain of $g \circ f$ is $\left\{x \middle| x \neq -12 \ \ and \ \ x \neq 0\right\}$, or $(-\infty, -12) \cup (-12, 0) \cup (0, \infty)$.

25. $(f \circ g)(x) = f(g(x)) = f\left(\frac{x+7}{3}\right) =$

$$3\left(\frac{x+7}{3}\right) - 7 = x + 7 - 7 = x$$

$(g \circ f)(x) = g(f(x)) = g(3x-7) = \frac{(3x-7)+7}{3} =$

$$\frac{3x}{3} = x$$

The domain of f and of g is $(-\infty, \infty)$, so the domain of $f \circ g$ and of $g \circ f$ is $(-\infty, \infty)$.

26. $(f \circ g)(x) = f(1.5x + 1.2) = \frac{2}{3}(1.5x + 1.2) - \frac{4}{5}$

$$x + 0.8 - \frac{4}{5} = x$$

$(g \circ f)(x) = g\left(\frac{2}{3}x - \frac{4}{5}\right) = 1.5\left(\frac{2}{3}x - \frac{4}{5}\right) + 1.2 =$

$$x - 1.2 + 1.2 = x$$

The domain of f and of g is $(-\infty, \infty)$, so the domain of $f \circ g$ and of $g \circ f$ is $(-\infty, \infty)$.

27. $(f \circ g)(x) = f(g(x)) = f(\sqrt{x}) = 2\sqrt{x} + 1$

$(g \circ f)(x) = g(f(x)) = g(2x+1) = \sqrt{2x+1}$

The domain of f is $(-\infty, \infty)$ and the domain of g is $\{x|x \geq 0\}$. Thus the domain of $f \circ g$ is $\{x|x \geq 0\}$, or $[0, \infty)$.

Now consider the domain of $g \circ f$. There are no restrictions on the domain of f, but the domain of g is $\{x|x \geq 0\}$. Since $f(x) \geq 0$ for $x \geq -\frac{1}{2}$, the domain of $g \circ f$ is $\left\{x \middle| x \geq -\frac{1}{2}\right\}$, or $\left[-\frac{1}{2}, \infty\right)$.

28. $(f \circ g)(x) = f(2 - 3x) = \sqrt{2 - 3x}$

$(g \circ f)(x) = g(\sqrt{x}) = 2 - 3\sqrt{x}$

The domain of f is $\{x|x \geq 0\}$ and the domain of g is $(-\infty, \infty)$. Since $g(x) \geq 0$ when $x \leq \frac{2}{3}$, the domain of $f \circ g$ is $\left(-\infty, \frac{2}{3}\right]$.

Now consider the domain of $g \circ f$. Since the domain of f is $\{x|x \geq 0\}$ and the domain of g is $(-\infty, \infty)$, the domain of $g \circ f$ is $\{x|x \geq 0\}$, or $[0, \infty)$.

29. $(f \circ g)(x) = f(g(x)) = f(0.05) = 20$

$(g \circ f)(x) = g(f(x)) = g(20) = 0.05$

The domain of f and of g is $(-\infty, \infty)$, so the domain of $f \circ g$ and of $g \circ f$ is $(-\infty, \infty)$.

30. $(f \circ g)(x) = (\sqrt[4]{x})^4 = x$

$(g \circ f)(x) = \sqrt[4]{x^4} = |x|$

The domain of f is $(-\infty, \infty)$ and the domain of g is $\{x|x \geq 0\}$, so the domain of $f \circ g$ is $\{x|x \geq 0\}$, or $[0, \infty)$.

Now consider the domain of $g \circ f$. There are no restrictions on the domain of f and $f(x) \geq 0$ for all values of x, so the domain is $(-\infty, \infty)$.

31. $(f \circ g)(x) = f(g(x)) = f(x^2 - 5) =$

$$\sqrt{x^2 - 5 + 5} = \sqrt{x^2} = |x|$$

$(g \circ f)(x) = g(f(x)) = g(\sqrt{x+5}) =$

$$(\sqrt{x+5})^2 - 5 = x + 5 - 5 = x$$

The domain of f is $\{x|x \geq -5\}$ and the domain of g is $(-\infty, \infty)$. Since $x^2 \geq 0$ for all values of x, then $x^2 - 5 \geq -5$ for all values of x and the domain of $g \circ f$ is $(-\infty, \infty)$.

Now consider the domain of $f \circ g$. There are no restrictions on the domain of g, so the domain of $f \circ g$ is the same as the domain of f, $\{x|x \geq -5\}$, or $[-5, \infty)$.

32. $(f \circ g)(x) = (\sqrt[5]{x+2})^5 - 2 = x + 2 - 2 = x$

$(g \circ f)(x) = \sqrt[5]{x^5 - 2 + 2} = \sqrt[5]{x^5} = x$

The domain of f and of g is $(-\infty, \infty)$, so the domain of $f \circ g$ and of $g \circ f$ is $(-\infty, \infty)$.

33. $(f \circ g)(x) = f(g(x)) = f(\sqrt{3-x}) = (\sqrt{3-x})^2 + 2 =$

$$3 - x + 2 = 5 - x$$

$(g \circ f)(x) = g(f(x)) = g(x^2 + 2) = \sqrt{3 - (x^2 + 2)} =$

$$\sqrt{3 - x^2 - 2} = \sqrt{1 - x^2}$$

The domain of f is $(-\infty, \infty)$ and the domain of g is $\{x|x \leq 3\}$, so the domain of $f \circ g$ is $\{x|x \leq 3\}$, or $(-\infty, 3]$.

Now consider the domain of $g \circ f$. There are no restrictions on the domain of f and the domain of g is $\{x|x \leq 3\}$, so we find the values of x for which $f(x) \leq 3$. We see that $x^2 + 2 \leq 3$ for $-1 \leq x \leq 1$, so the domain of $g \circ f$ is $\{x|-1 \leq x \leq 1\}$, or $[-1, 1]$.

34. $(f \circ g)(x) = f(\sqrt{x^2 - 25}) = 1 - (\sqrt{x^2 - 25})^2 =$

$$1 - (x^2 - 25) = 1 - x^2 + 25 = 26 - x^2$$

$(g \circ f)(x) = g(1 - x^2) = \sqrt{(1 - x^2)^2 - 25} =$

$$\sqrt{1 - 2x^2 + x^4 - 25} = \sqrt{x^4 - 2x^2 - 24}$$

The domain of f is $(-\infty, \infty)$ and the domain of g is $\{x|x \leq -5 \ \ or \ \ x \geq 5\}$, so the domain of $f \circ g$ is $\{x|x \leq -5 \ \ or \ \ x \geq 5\}$, or $(-\infty, -5] \cup [5, \infty)$.

Now consider the domain of $g \circ f$. There are no restrictions on the domain of f and the domain of g is $\{x|x \leq -5 \ \ or \ \ x \geq 5\}$, so we find the values of x for which $f(x) \leq -5$ or $f(x) \geq 5$. We see that $1 - x^2 \leq -5$ when $x \leq -\sqrt{6}$ or $x \geq \sqrt{6}$ and $1 - x^2 \geq 5$ has no solution, so the domain of $g \circ f$ is $\{x|x \leq -\sqrt{6} \ \ or \ \ x \geq \sqrt{6}\}$, or $(-\infty, -\sqrt{6}] \cup [\sqrt{6}, \infty)$.

35. $(f \circ g)(x) = f(g(x)) = f\left(\frac{1}{1+x}\right) =$

$$\frac{1-\left(\frac{1}{1+x}\right)}{\frac{1}{1+x}} = \frac{\frac{1+x-1}{1+x}}{\frac{1}{1+x}} =$$

$$\frac{x}{1+x} \cdot \frac{1+x}{1} = x$$

$(g \circ f)(x) = g(f(x)) = g\left(\frac{1-x}{x}\right) =$

$$\frac{1}{1+\left(\frac{1-x}{x}\right)} = \frac{1}{\frac{x+1-x}{x}} =$$

$$\frac{1}{\frac{1}{x}} = 1 \cdot \frac{x}{1} = x$$

The domain of f is $\{x|x \neq 0\}$ and the domain of g is $\{x|x \neq -1\}$, so we know that -1 is not in the domain of $f \circ g$. Since 0 is not in the domain of f, values of x for which $g(x) = 0$ are not in the domain of $f \circ g$. But $g(x)$ is never 0, so the domain of $f \circ g$ is $\{x|x \neq -1\}$, or $(-\infty, -1) \cup (-1, \infty)$.

Now consider the domain of $g \circ f$. Recall that 0 is not in the domain of f. Since -1 is not in the domain of g, we know that $g(x)$ cannot be -1. We find the value(s) of x for which $f(x) = -1$.

$$\frac{1-x}{x} = -1$$

$1 - x = -x$ Multiplying by x

$1 = 0$ False equation

We see that there are no values of x for which $f(x) = -1$, so the domain of $g \circ f$ is $\{x|x \neq 0\}$, or $(-\infty, 0) \cup (0, \infty)$.

36. $(f \circ g)(x) = f\left(\frac{x+2}{x}\right) = \frac{1}{\frac{x+2}{x} - 2}$

$$= \frac{1}{\frac{x+2-2x}{x}} = \frac{1}{\frac{-x+2}{x}}$$

$$= 1 \cdot \frac{x}{-x+2} = \frac{x}{-x+2}, \text{ or } \frac{x}{2-x}$$

$(g \circ f)(x) = g\left(\frac{1}{x-2}\right) = \frac{\frac{1}{x-2} + 2}{\frac{1}{x-2}}$

$$= \frac{\frac{1+2x-4}{x-2}}{\frac{1}{x-2}} = \frac{\frac{2x-3}{x-2}}{\frac{1}{x-2}}$$

$$= \frac{2x-3}{x-2} \cdot \frac{x-2}{1} = 2x - 3$$

The domain of f is $\{x|x \neq 2\}$ and the domain of g is $\{x|x \neq 0\}$, so 0 is not in the domain of $f \circ g$. We find the value of x for which $g(x) = 2$.

$$\frac{x+2}{x} = 2$$

$$x + 2 = 2x$$

$$2 = x$$

Then the domain of $f \circ g$ is $(-\infty, 0) \cup (0, 2) \cup (2, \infty)$.

Now consider the domain of $g \circ f$. Since the domain of f is $\{x|x \neq 2\}$, we know that 2 is not in the domain of $g \circ f$. Since the domain of g is $\{x|x \neq 0\}$, we find the value of x for which $f(x) = 0$.

$$\frac{1}{x-2} = 0$$

$$1 = 0$$

We get a false equation, so there are no such values. Then the domain of $g \circ f$ is $(-\infty, 2) \cup (2, \infty)$.

37. $(f \circ g)(x) = f(g(x)) = f(x + 1) =$
$(x + 1)^3 - 5(x + 1)^2 + 3(x + 1) + 7 =$
$x^3 + 3x^2 + 3x + 1 - 5x^2 - 10x - 5 + 3x + 3 + 7 =$
$x^3 - 2x^2 - 4x + 6$

$(g \circ f)(x) = g(f(x)) = g(x^3 - 5x^2 + 3x + 7) =$
$x^3 - 5x^2 + 3x + 7 + 1 = x^3 - 5x^2 + 3x + 8$

The domain of f and of g is $(-\infty, \infty)$, so the domain of $f \circ g$ and of $g \circ f$ is $(-\infty, \infty)$.

38. $(g \circ f)(x) = x^3 + 2x^2 - 3x - 9 - 1 =$
$x^3 + 2x^2 - 3x - 10$
$(g \circ f)(x) = (x - 1)^3 + 2(x - 1)^2 - 3(x - 1) - 9 =$
$x^3 - 3x^2 + 3x - 1 + 2x^2 - 4x + 2 - 3x + 3 - 9 =$
$x^3 - x^2 - 4x - 5$

The domain of f and of g is $(-\infty, \infty)$, so the domain of $f \circ g$ and of $g \circ f$ is $(-\infty, \infty)$.

39. $h(x) = (4 + 3x)^5$

This is $4 + 3x$ to the 5th power. The most obvious answer is $f(x) = x^5$ and $g(x) = 4 + 3x$.

40. $f(x) = \sqrt[3]{x}$, $g(x) = x^2 - 8$

41. $h(x) = \frac{1}{(x-2)^4}$

This is 1 divided by $(x - 2)$ to the 4th power. One obvious answer is $f(x) = \frac{1}{x^4}$ and $g(x) = x - 2$. Another possibility is $f(x) = \frac{1}{x}$ and $g(x) = (x - 2)^4$.

42. $f(x) = \frac{1}{\sqrt{x}}$, $g(x) = 3x + 7$

43. $f(x) = \frac{x-1}{x+1}$, $g(x) = x^3$

44. $f(x) = |x|$, $g(x) = 9x^2 - 4$

45. $f(x) = x^6$, $g(x) = \frac{2+x^3}{2-x^3}$

46. $f(x) = x^4$, $g(x) = \sqrt{x} - 3$

47. $f(x) = \sqrt{x}$, $g(x) = \dfrac{x-5}{x+2}$

48. $f(x) = \sqrt{1+x}$, $g(x) = \sqrt{1+x}$

49. $f(x) = x^3 - 5x^2 + 3x - 1$, $g(x) = x + 2$

50. $f(x) = 2x^{5/3} + 5x^{2/3}$, $g(x) = x - 1$, or
$f(x) = 2x^5 + 5x^2$, $g(x) = (x-1)^{1/3}$

51. a) Use the distance formula, distance = rate × time. Substitute 3 for the rate and t for time.

$r(t) = 3t$

b) Use the formula for the area of a circle.

$A(r) = \pi r^2$

c) $(A \circ r)(t) = A(r(t)) = A(3t) = \pi(3t)^2 = 9\pi t^2$

This function gives the area of the ripple in terms of time t.

52. a) $h = 2r$

$S(r) = 2\pi r(2r) + 2\pi r^2$
$S(r) = 4\pi r^2 + 2\pi r^2$
$S(r) = 6\pi r^2$

b) $r = \dfrac{h}{2}$

$$S(h) = 2\pi\left(\frac{h}{2}\right)h + 2\pi\left(\frac{h}{2}\right)^2$$
$$S(h) = \pi h^2 + \frac{\pi h^2}{2}$$
$$S(h) = \frac{3}{2}\pi h^2$$

53. $f(x) = (t \circ s)(x) = t(s(x)) = t(x-3) = x - 3 + 4 = x + 1$

We have $f(x) = x + 1$.

54. The manufacturer charges $m+6$ per drill. The chain store sells each drill for $150\%(m+6)$, or $1.5(m+6)$, or $1.5m+9$. Thus, we have $P(m) = 1.5m + 9$.

55. Equations $(a) - (f)$ are in the form $y = mx + b$, so we can read the y-intercepts directly from the equations. Equations (g) and (h) can be written in this form as $y = \dfrac{2}{3}x - 2$ and $y = -2x + 3$, respectively. We see that only equation (c) has y-intercept $(0, 1)$.

56. None (See Exercise 55.)

57. If a line slopes down from left to right, its slope is negative. The equations $y = mx + b$ for which m is negative are (b), (d), (f), and (h). (See Exercise 55.)

58. The equation for which $|m|$ is greatest is the equation with the steepest slant. This is equation (b). (See Exercise 55.)

59. The only equation that has $(0, 0)$ as a solution is (a).

60. Equations (c) and (g) have the same slope. (See Exercise 55.)

61. Only equations (c) and (g) have the same slope and different y-intercepts. They represent parallel lines.

62. The only equations for which the product of the slopes is -1 are (a) and (f).

63. Only the composition $(c \circ p)(a)$ makes sense. It represents the cost of the grass seed required to seed a lawn with area a.

64. Answers may vary. One example is $f(x) = 2x + 5$ and $g(x) = \dfrac{x-5}{2}$. Other examples are found in Exercises 17, 18, 25, 26, 32 and 35.

Chapter 2 Mid-Chapter Mixed Review

1. The statement is true. See page 96 in the text.

2. The statement is false. See page 110 in the text.

3. The statement is true. See Examples 1 and 2 in Section 2.3.

4. a) For x-values from 2 to 4, the y-values increase from 2 to 4. Thus the function is increasing on the interval $(2, 4)$.

b) For x-values from -5 to -3, the y-values decrease from 5 to 1. Also, for x-values from 4 to 5, the y-values decrease from 4 to -3. Thus the function is decreasing on $(-5, -3)$ and on $(4, 5)$.

c) For x-values from -3 to -1, y is 3. Thus the function is constant on $(-3, -1)$.

5. From the graph we see that a relative maximum value of 6.30 occurs at $x = -1.29$. We also see that a relative minimum value of -2.30 occurs at $x = 1.29$.

The graph starts rising, or increasing, from the left and stops increasing at the relative maximum. From this point it decreases to the relative minimum and then increases again. Thus the function is increasing on $(-\infty, -1.29)$ and on $(1.29, \infty)$. It is decreasing on $(-1.29, 1.29)$.

6. The x-values extend from -5 to -1 and from 2 to 5, so the domain is $[-5, -1] \cup [2, 5]$. The y-values extend from -3 to 5, so the range is $[-3, 5]$.

7. $A(h) = \dfrac{1}{2}(h+4)h$

$A(h) = \dfrac{h^2}{2} + 2h$

8. $$f(x) = \begin{cases} x - 5, & \text{for } x \le -3, \\ 2x + 3, & \text{for } -3 < x \le 0, \\ \frac{1}{2}x, & \text{for } x > 0, \end{cases}$$

Since $-5 \le -3$, $f(-5) = -5 - 5 = -10$.

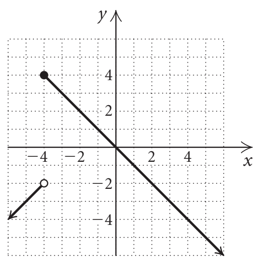
Since $-3 \le -3$, $f(-3) = -3 - 5 = -8$.

Since $-3 < -1 \le 0$, $f(-1) = 2(-1) + 3 = -2 + 3 = 1$.

Since $6 > 0$, $f(6) = \dfrac{1}{2} \cdot 6 = 3$.

9. $g(x) = \begin{cases} x+2, & \text{for } x < -4, \\ -x, & \text{for } x \geq -4 \end{cases}$

We create the graph in two parts. Graph $g(x) = x + 2$ for inputs less than -4. Then graph $g(x) = -x$ for inputs greater than or equal to -4.



10. $(f+g)(-1) = f(-1) + g(-1)$
$= [3(-1) - 1] + [(-1)^2 + 4]$
$= -3 - 1 + 1 + 4$
$= 1$

11. $(fg)(0) = f(0) \cdot g(0)$
$= (3 \cdot 0 - 1) \cdot (0^2 + 4)$
$= -1 \cdot 4$
$= -4$

12. $(g-f)(3) = g(3) - f(3)$
$= (3^2 + 4) - (3 \cdot 3 - 1)$
$= 9 + 4 - (9 - 1)$
$= 9 + 4 - 9 + 1$
$= 5$

13. $$(g/f)\left(\frac{1}{3}\right) = \frac{g\left(\frac{1}{3}\right)}{f\left(\frac{1}{3}\right)} = \frac{\left(\frac{1}{3}\right)^2 + 4}{3 \cdot \frac{1}{3} - 1} = \frac{\frac{1}{9} + 4}{1 - 1} = \frac{\frac{37}{9}}{0}$$

Since division by 0 is not defined, $(g/f)\left(\frac{1}{3}\right)$ does not exist.

14. $f(x) = 2x + 5$, $g(x) = -x - 4$

a) The domain of f and of g is the set of all real numbers, or $(-\infty, \infty)$. Then the domain of $f+g$, $f-g$, fg, and ff is also $(-\infty, \infty)$.

For f/g we must exclude -4 since $g(-4) = 0$. Then the domain of f/g is $(-\infty, -4) \cup (-4, \infty)$.

For g/f we must exclude $-\frac{5}{2}$ since $f\left(-\frac{5}{2}\right) = 0$. Then the domain of g/f is

$$\left(-\infty, -\frac{5}{2}\right) \cup \left(-\frac{5}{2}, \infty\right).$$

b) $(f+g)(x) = f(x)+g(x) = (2x+5)+(-x-4) = x+1$

$(f-g)(x) = f(x) - g(x) = (2x+5) - (-x-4) = 2x + 5 + x + 4 = 3x + 9$

$(fg)(x) = f(x) \cdot g(x) = (2x+5)(-x-4) = -2x^2 - 8x - 5x - 20 = -2x^2 - 13x - 20$

$(ff)(x) = f(x) \cdot f(x) = (2x+5) \cdot (2x+5) = 4x^2 + 10x + 10x + 25 = 4x^2 + 20x + 25$

$(f/g)(x) = \dfrac{f(x)}{g(x)} = \dfrac{2x+5}{-x-4}$

$(g/f)(x) = \dfrac{g(x)}{f(x)} = \dfrac{-x-4}{2x+5}$

15. $f(x) = x - 1$, $g(x) = \sqrt{x+2}$

a) Any number can be an input for f, so the domain of f is the set of all real numbers, or $(-\infty, \infty)$.

The domain of g consists of all values for which $x+2$ is nonnegative, so we have $x + 2 \geq 0$, or $x \geq -2$, or $[-2, \infty)$. Then the domain of $f+g$, $f-g$, and fg is $[-2, \infty)$.

The domain of ff is $(-\infty, \infty)$.

Since $g(-2) = 0$, the domain of f/g is $(-2, \infty)$.

Since $f(1) = 0$, the domain of g/f is $[-2, 1) \cup (1, \infty)$.

b) $(f+g)(x) = f(x) + g(x) = x - 1 + \sqrt{x+2}$

$(f-g)(x) = f(x) - g(x) = x - 1 - \sqrt{x+2}$

$(fg)(x) = f(x) \cdot g(x) = (x-1)\sqrt{x+2}$

$(ff)(x) = f(x) \cdot f(x) = (x-1)(x-1) = x^2 - x - x + 1 = x^2 - 2x + 1$

$(f/g)(x) = \dfrac{f(x)}{g(x)} = \dfrac{x-1}{\sqrt{x+2}}$

$(g/f)(x) = \dfrac{g(x)}{f(x)} = \dfrac{\sqrt{x+2}}{x-1}$

16. $f(x) = 4x - 3$

$$\frac{f(x+h) - f(x)}{h} = \frac{4(x+h) - 3 - (4x - 3)}{h} = \frac{4x + 4h - 3 - 4x + 3}{h} = \frac{4h}{h} = 4$$

17. $f(x) = 6 - x^2$

$$\frac{f(x+h)-f(x)}{h} = \frac{6-(x+h)^2-(6-x^2)}{h} =$$

$$\frac{6-(x^2+2xh+h^2)-6+x^2}{h} = \frac{6-x^2-2xh-h^2-6+x^2}{h} =$$

$$\frac{-2xh-h^2}{h} = \frac{\not{h}(-2x-h)}{\not{h}\cdot 1} = -2x-h$$

18. $(f\circ g)(1) = f(g(1)) = f(1^3+1) = f(1+1) = f(2) = 5\cdot 2 - 4 = 10 - 4 = 6$

19. $(g\circ h)(2) = g(h(2)) = g(2^2 - 2\cdot 2 + 3) = g(4-4+3) = g(3) = 3^3 + 1 = 27 + 1 = 28$

20. $(f\circ f)(0) = f(f(0)) = f(5\cdot 0 - 4) = f(-4) = 5(-4) - 4 = -20 - 4 = -24$

21. $(h\circ f)(-1) = h(f(-1)) = h(5(-1)-4) = h(-5-4) = h(-9) = (-9)^2 - 2(-9) + 3 = 81 + 18 + 3 = 102$

22. $(f\circ g)(x) = f(g(x)) = f(6x+4) = \frac{1}{2}(6x+4) = 3x+2$

$(g\circ f)(x) = g(f(x)) = g\left(\frac{1}{2}x\right) = 6\cdot\frac{1}{2}x + 4 = 3x + 4$

The domain of f and g is $(-\infty,\infty)$, so the domain of $f\circ g$ and $g\circ f$ is $(-\infty,\infty)$.

23. $(f\circ g)(x) = f(g(x)) = f(\sqrt{x}) = 3\sqrt{x} + 2$

$(g\circ f)(x) = g(f(x)) = g(3x+2) = \sqrt{3x+2}$

The domain of f is $(-\infty,\infty)$ and the domain of g is $[0,\infty)$.

Consider the domain of $f\circ g$. Since any number can be an input for f, the domain of $f\circ g$ is the same as the domain of g, $[0,\infty)$.

Now consider the domain of $g\circ f$. Since the inputs of g must be nonnegative, we must have $3x+2\ge 0$, or $x\ge -\frac{2}{3}$.

Thus the domain of $g\circ f$ is $\left[-\frac{2}{3},\infty\right)$.

24. The graph of $y = (h-g)(x)$ will be the same as the graph of $y = h(x)$ moved down b units.

25. Under the given conditions, $(f+g)(x)$ and $(f/g)(x)$ have different domains if $g(x) = 0$ for one or more real numbers x.

26. If f and g are linear functions, then any real number can be an input for each function. Thus, the domain of $f\circ g =$ the domain of $g\circ f = (-\infty,\infty)$.

27. This approach is not valid. Consider Exercise 23 on page 120 in the text, for example. Since $(f\circ g)(x) = \frac{4x}{x-5}$, an examination of only this composed function would lead to the incorrect conclusion that the domain of $f\circ g$ is $(-\infty,5)\cup(5,\infty)$. However, we must also exclude from the domain of $f\circ g$ those values of x that are not in the domain of g. Thus, the domain of $f\circ g$ is $(-\infty,0)\cup(0,5)\cup(5,\infty)$.

Exercise Set 2.4

1. If the graph were folded on the x-axis, the parts above and below the x-axis would not coincide, so the graph is not symmetric with respect to the x-axis.

If the graph were folded on the y-axis, the parts to the left and right of the y-axis would coincide, so the graph is symmetric with respect to the y-axis.

If the graph were rotated 180°, the resulting graph would not coincide with the original graph, so it is not symmetric with respect to the origin.

2. If the graph were folded on the x-axis, the parts above and below the x-axis would not coincide, so the graph is not symmetric with respect to the x-axis.

If the graph were folded on the y-axis, the parts to the left and right of the y-axis would coincide, so the graph is symmetric with respect to the y-axis.

If the graph were rotated 180°, the resulting graph would not coincide with the original graph, so it is not symmetric with respect to the origin.

3. If the graph were folded on the x-axis, the parts above and below the x-axis would coincide, so the graph is symmetric with respect to the x-axis.

If the graph were folded on the y-axis, the parts to the left and right of the y-axis would not coincide, so the graph is not symmetric with respect to the y-axis.

If the graph were rotated 180°, the resulting graph would not coincide with the original graph, so it is not symmetric with respect to the origin.

4. If the graph were folded on the x-axis, the parts above and below the x-axis would not coincide, so the graph is not symmetric with respect to the x-axis.

If the graph were folded on the y-axis, the parts to the left and right of the y-axis would not coincide, so the graph is not symmetric with respect to the y-axis.

If the graph were rotated 180°, the resulting graph would coincide with the original graph, so it is symmetric with respect to the origin.

5. If the graph were folded on the x-axis, the parts above and below the x-axis would not coincide, so the graph is not symmetric with respect to the x-axis.

If the graph were folded on the y-axis, the parts to the left and right of the y-axis would not coincide, so the graph is not symmetric with respect to the y-axis.

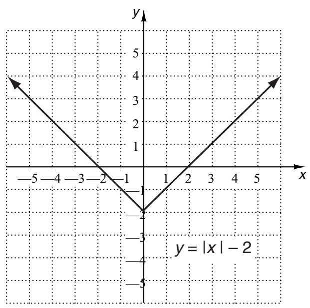
If the graph were rotated 180°, the resulting graph would coincide with the original graph, so it is symmetric with respect to the origin.

6. If the graph were folded on the x-axis, the parts above and below the x-axis would coincide, so the graph is symmetric with respect to the x-axis.

If the graph were folded on the y-axis, the parts to the left and right of the y-axis would coincide, so the graph is symmetric with respect to the y-axis.

If the graph were rotated 180°, the resulting graph would coincide with the original graph, so it is symmetric with respect to the origin.

7.



The graph is symmetric with respect to the y-axis. It is not symmetric with respect to the x-axis or the origin.

Test algebraically for symmetry with respect to the x-axis:

$y = |x| - 2$ Original equation

$-y = |x| - 2$ Replacing y by $-y$

$y = -|x| + 2$ Simplifying

The last equation is not equivalent to the original equation, so the graph is not symmetric with respect to the x-axis.

Test algebraically for symmetry with respect to the y-axis:

$y = |x| - 2$ Original equation

$y = |-x| - 2$ Replacing x by $-x$

$y = |x| - 2$ Simplifying

The last equation is equivalent to the original equation, so the graph is symmetric with respect to the y-axis.

Test algebraically for symmetry with respect to the origin:

$y = |x| - 2$ Original equation

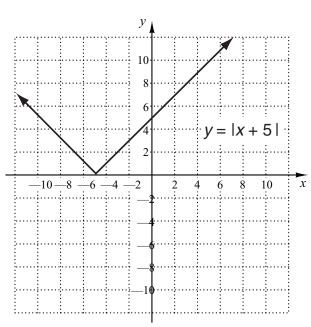
$-y = |-x| - 2$ Replacing x by $-x$ and y by $-y$

$-y = |x| - 2$ Simplifying

$y = -|x| + 2$

The last equation is not equivalent to the original equation, so the graph is not symmetric with respect to the origin.

8.



The graph is not symmetric with respect to the x-axis, the y-axis, or the origin.

Test algebraically for symmetry with respect to the x-axis:

$y = |x + 5|$ Original equation

$-y = |x + 5|$ Replacing y by $-y$

$y = -|x + 5|$ Simplifying

The last equation is not equivalent to the original equation, so the graph is not symmetric with respect to the x-axis.

Test algebraically for symmetry with respect to the y-axis:

$y = |x + 5|$ Original equation

$y = |-x + 5|$ Replacing x by $-x$

The last equation is not equivalent to the original equation, so the graph is not symmetric with respect to the y-axis.

Test algebraically for symmetry with respect to the origin:

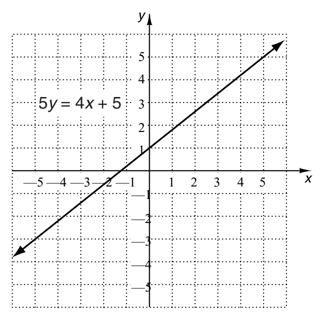
$y = |x + 5|$ Original equation

$-y = |-x + 5|$ Replacing x by $-x$ and y by $-y$

$y = -|-x + 5|$ Simplifying

The last equation is not equivalent to the original equation, so the graph is not symmetric with respect to the origin.

9.



The graph is not symmetric with respect to the x-axis, the y-axis, or the origin.

Test algebraically for symmetry with respect to the x-axis:

$5y = 4x + 5$ Original equation

$5(-y) = 4x + 5$ Replacing y by $-y$

$-5y = 4x + 5$ Simplifying

$5y = -4x - 5$

The last equation is not equivalent to the original equation, so the graph is not symmetric with respect to the x-axis.

Test algebraically for symmetry with respect to the y-axis:

$5y = 4x + 5$ Original equation

$5y = 4(-x) + 5$ Replacing x by $-x$

$5y = -4x + 5$ Simplifying

The last equation is not equivalent to the original equation, so the graph is not symmetric with respect to the y-axis.

Test algebraically for symmetry with respect to the origin:

$5y = 4x + 5$ Original equation

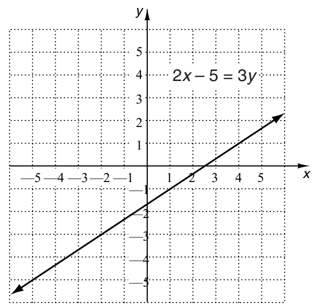
$5(-y) = 4(-x) + 5$ Replacing x by $-x$ and y by $-y$

$-5y = -4x + 5$ Simplifying

$5y = 4x - 5$

The last equation is not equivalent to the original equation, so the graph is not symmetric with respect to the origin.

10.



The graph is not symmetric with respect to the x-axis, the y-axis, or the origin.

Test algebraically for symmetry with respect to the x-axis:

$$\begin{aligned} 2x - 5 &= 3y && \text{Original equation} \\ 2x - 5 &= 3(-y) && \text{Replacing } y \text{ by } -y \\ -2x + 5 &= 3y && \text{Simplifying} \end{aligned}$$

The last equation is not equivalent to the original equation, so the graph is not symmetric with respect to the x-axis.

Test algebraically for symmetry with respect to the y-axis:

$$\begin{aligned} 2x - 5 &= 3y && \text{Original equation} \\ 2(-x) - 5 &= 3y && \text{Replacing } x \text{ by } -x \\ -2x - 5 &= 3y && \text{Simplifying} \end{aligned}$$

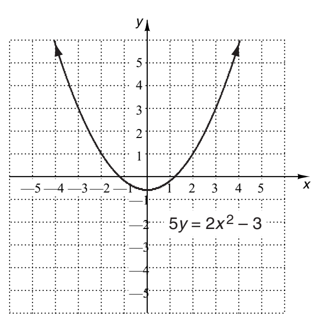
The last equation is not equivalent to the original equation, so the graph is not symmetric with respect to the y-axis.

Test algebraically for symmetry with respect to the origin:

$$\begin{aligned} 2x - 5 &= 3y && \text{Original equation} \\ 2(-x) - 5 &= 3(-y) && \text{Replacing } x \text{ by } -x \text{ and } y \text{ by } -y \\ -2x - 5 &= -3y && \text{Simplifying} \\ 2x + 5 &= 3y \end{aligned}$$

The last equation is not equivalent to the original equation, so the graph is not symmetric with respect to the origin.

11.



The graph is symmetric with respect to the y-axis. It is not symmetric with respect to the x-axis or the origin.

Test algebraically for symmetry with respect to the x-axis:

$$\begin{aligned} 5y &= 2x^2 - 3 && \text{Original equation} \\ 5(-y) &= 2x^2 - 3 && \text{Replacing } y \text{ by } -y \\ -5y &= 2x^2 - 3 && \text{Simplifying} \\ 5y &= -2x^2 + 3 \end{aligned}$$

The last equation is not equivalent to the original equation, so the graph is not symmetric with respect to the x-axis.

Test algebraically for symmetry with respect to the y-axis:

$$\begin{aligned} 5y &= 2x^2 - 3 && \text{Original equation} \\ 5y &= 2(-x)^2 - 3 && \text{Replacing } x \text{ by } -x \\ 5y &= 2x^2 - 3 \end{aligned}$$

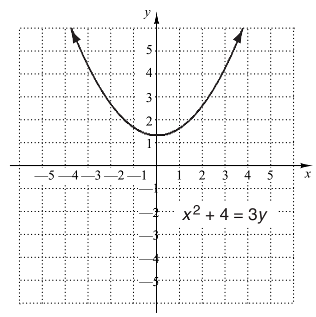
The last equation is equivalent to the original equation, so the graph is symmetric with respect to the y-axis.

Test algebraically for symmetry with respect to the origin:

$$\begin{aligned} 5y &= 2x^2 - 3 && \text{Original equation} \\ 5(-y) &= 2(-x)^2 - 3 && \text{Replacing } x \text{ by } -x \text{ and } y \text{ by } -y \\ -5y &= 2x^2 - 3 && \text{Simplifying} \\ 5y &= -2x^2 + 3 \end{aligned}$$

The last equation is not equivalent to the original equation, so the graph is not symmetric with respect to the origin.

12.



The graph is symmetric with respect to the y-axis. It is not symmetric with respect to the x-axis or the origin.

Test algebraically for symmetry with respect to the x-axis:

$$\begin{aligned} x^2 + 4 &= 3y && \text{Original equation} \\ x^2 + 4 &= 3(-y) && \text{Replacing } y \text{ by } -y \\ -x^2 - 4 &= 3y && \text{Simplifying} \end{aligned}$$

The last equation is not equivalent to the original equation, so the graph is not symmetric with respect to the x-axis.

Test algebraically for symmetry with respect to the y-axis:

$$\begin{aligned} x^2 + 4 &= 3y && \text{Original equation} \\ (-x)^2 + 4 &= 3y && \text{Replacing } x \text{ by } -x \\ x^2 + 4 &= 3y \end{aligned}$$

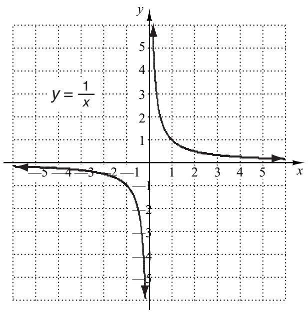
The last equation is equivalent to the original equation, so the graph is symmetric with respect to the y-axis.

Test algebraically for symmetry with respect to the origin:

$$\begin{aligned} x^2 + 4 &= 3y && \text{Original equation} \\ (-x)^2 + 4 &= 3(-y) && \text{Replacing } x \text{ by } -x \text{ and } y \text{ by } -y \\ x^2 + 4 &= -3y && \text{Simplifying} \\ -x^2 - 4 &= 3y \end{aligned}$$

The last equation is not equivalent to the original equation, so the graph is not symmetric with respect to the origin.

13.



The graph is not symmetric with respect to the x-axis or the y-axis. It is symmetric with respect to the origin.

Test algebraically for symmetry with respect to the x-axis:

$$y = \frac{1}{x} \quad \text{Original equation}$$
$$-y = \frac{1}{x} \quad \text{Replacing } y \text{ by } -y$$
$$y = -\frac{1}{x} \quad \text{Simplifying}$$

The last equation is not equivalent to the original equation, so the graph is not symmetric with respect to the x-axis.

Test algebraically for symmetry with respect to the y-axis:

$$y = \frac{1}{x} \quad \text{Original equation}$$
$$y = \frac{1}{-x} \quad \text{Replacing } x \text{ by } -x$$
$$y = -\frac{1}{x} \quad \text{Simplifying}$$

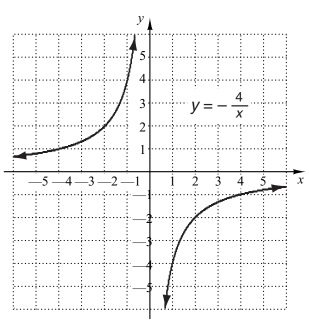
The last equation is not equivalent to the original equation, so the graph is not symmetric with respect to the y-axis.

Test algebraically for symmetry with respect to the origin:

$$y = \frac{1}{x} \quad \text{Original equation}$$
$$-y = \frac{1}{-x} \quad \text{Replacing } x \text{ by } -x \text{ and } y \text{ by } -y$$
$$y = \frac{1}{x} \quad \text{Simplifying}$$

The last equation is equivalent to the original equation, so the graph is symmetric with respect to the origin.

14.



The graph is not symmetric with respect to the x-axis or the y-axis. It is symmetric with respect to the origin.

Test algebraically for symmetry with respect to the x-axis:

$$y = -\frac{4}{x} \quad \text{Original equation}$$
$$-y = -\frac{4}{x} \quad \text{Replacing } y \text{ by } -y$$
$$y = \frac{4}{x} \quad \text{Simplifying}$$

The last equation is not equivalent to the original equation, so the graph is not symmetric with respect to the x-axis.

Test algebraically for symmetry with respect to the y-axis:

$$y = -\frac{4}{x} \quad \text{Original equation}$$
$$y = -\frac{4}{-x} \quad \text{Replacing } x \text{ by } -x$$
$$y = \frac{4}{x} \quad \text{Simplifying}$$

The last equation is not equivalent to the original equation, so the graph is not symmetric with respect to the y-axis.

Test algebraically for symmetry with respect to the origin:

$$y = -\frac{4}{x} \quad \text{Original equation}$$
$$-y = -\frac{4}{-x} \quad \text{Replacing } x \text{ by } -x \text{ and } y \text{ by } -y$$
$$y = -\frac{4}{x} \quad \text{Simplifying}$$

The last equation is equivalent to the original equation, so the graph is symmetric with respect to the origin.

15. Test for symmetry with respect to the x-axis:

$$5x - 5y = 0 \quad \text{Original equation}$$
$$5x - 5(-y) = 0 \quad \text{Replacing } y \text{ by } -y$$
$$5x + 5y = 0 \quad \text{Simplifying}$$

The last equation is not equivalent to the original equation, so the graph is not symmetric with respect to the x-axis.

Test for symmetry with respect to the y-axis:

$$5x - 5y = 0 \quad \text{Original equation}$$
$$5(-x) - 5y = 0 \quad \text{Replacing } x \text{ by } -x$$
$$-5x - 5y = 0 \quad \text{Simplifying}$$
$$5x + 5y = 0$$

The last equation is not equivalent to the original equation, so the graph is not symmetric with respect to the y-axis.

Test for symmetry with respect to the origin:

$$5x - 5y = 0 \quad \text{Original equation}$$
$$5(-x) - 5(-y) = 0 \quad \text{Replacing } x \text{ by } -x \text{ and } y \text{ by } -y$$
$$-5x + 5y = 0 \quad \text{Simplifying}$$
$$5x - 5y = 0$$

The last equation is equivalent to the original equation, so the graph is symmetric with respect to the origin.

16. Test for symmetry with respect to the x-axis:

$$6x + 7y = 0 \quad \text{Original equation}$$
$$6x + 7(-y) = 0 \quad \text{Replacing } y \text{ by } -y$$
$$6x - 7y = 0 \quad \text{Simplifying}$$

The last equation is not equivalent to the original equation, so the graph is not symmetric with respect to the x-axis.

Test for symmetry with respect to the y-axis:

$6x + 7y = 0$ Original equation
$6(-x) + 7y = 0$ Replacing x by $-x$
$6x - 7y = 0$ Simplifying

The last equation is not equivalent to the original equation, so the graph is not symmetric with respect to the y-axis.

Test for symmetry with respect to the origin:

$6x + 7y = 0$ Original equation
$6(-x) + 7(-y) = 0$ Replacing x by $-x$ and y by $-y$
$6x + 7y = 0$ Simplifying

The last equation is equivalent to the original equation, so the graph is symmetric with respect to the origin.

17. Test for symmetry with respect to the x-axis:

$3x^2 - 2y^2 = 3$ Original equation
$3x^2 - 2(-y)^2 = 3$ Replacing y by $-y$
$3x^2 - 2y^2 = 3$ Simplifying

The last equation is equivalent to the original equation, so the graph is symmetric with respect to the x-axis.

Test for symmetry with respect to the y-axis:

$3x^2 - 2y^2 = 3$ Original equation
$3(-x)^2 - 2y^2 = 3$ Replacing x by $-x$
$3x^2 - 2y^2 = 3$ Simplifying

The last equation is equivalent to the original equation, so the graph is symmetric with respect to the y-axis.

Test for symmetry with respect to the origin:

$3x^2 - 2y^2 = 3$ Original equation
$3(-x)^2 - 2(-y)^2 = 3$ Replacing x by $-x$ and y by $-y$
$3x^2 - 2y^2 = 3$ Simplifying

The last equation is equivalent to the original equation, so the graph is symmetric with respect to the origin.

18. Test for symmetry with respect to the x-axis:

$5y = 7x^2 - 2x$ Original equation
$5(-y) = 7x^2 - 2x$ Replacing y by $-y$
$5y = -7x^2 + 2x$ Simplifying

The last equation is not equivalent to the original equation, so the graph is not symmetric with respect to the x-axis.

Test for symmetry with respect to the y-axis:

$5y = 7x^2 - 2x$ Original equation
$5y = 7(-x)^2 - 2(-x)$ Replacing x by $-x$
$5y = 7x^2 + 2x$ Simplifying

The last equation is not equivalent to the original equation, so the graph is not symmetric with respect to the y-axis.

Test for symmetry with respect to the origin:

$5y = 7x^2 - 2x$ Original equation
$5(-y) = 7(-x)^2 - 2(-x)$ Replacing x by $-x$ and y by $-y$
$-5y = 7x^2 + 2x$ Simplifying
$5y = -7x^2 - 2x$

The last equation is not equivalent to the original equation, so the graph is not symmetric with respect to the origin.

19. Test for symmetry with respect to the x-axis:

$y = |2x|$ Original equation
$-y = |2x|$ Replacing y by $-y$
$y = -|2x|$ Simplifying

The last equation is not equivalent to the original equation, so the graph is not symmetric with respect to the x-axis.

Test for symmetry with respect to the y-axis:

$y = |2x|$ Original equation
$y = |2(-x)|$ Replacing x by $-x$
$y = |-2x|$ Simplifying
$y = |2x|$

The last equation is equivalent to the original equation, so the graph is symmetric with respect to the y-axis.

Test for symmetry with respect to the origin:

$y = |2x|$ Original equation
$-y = |2(-x)|$ Replacing x by $-x$ and y by $-y$
$-y = |-2x|$ Simplifying
$-y = |2x|$
$y = -|2x|$

The last equation is not equivalent to the original equation, so the graph is not symmetric with respect to the origin.

20. Test for symmetry with respect to the x-axis:

$y^3 = 2x^2$ Original equation
$(-y)^3 = 2x^2$ Replacing y by $-y$
$-y^3 = 2x^2$ Simplifying
$y^3 = -2x^2$

The last equation is not equivalent to the original equation, so the graph is not symmetric with respect to the x-axis.

Test for symmetry with respect to the y-axis:

$y^3 = 2x^2$ Original equation
$y^3 = 2(-x)^2$ Replacing x by $-x$
$y^3 = 2x^2$ Simplifying

The last equation is equivalent to the original equation, so the graph is symmetric with respect to the y-axis.

Test for symmetry with respect to the origin:

$y^3 = 2x^2$ Original equation
$(-y)^3 = 2(-x)^2$ Replacing x by $-x$ and y by $-y$
$-y^3 = 2x^2$ Simplifying
$y^3 = -2x^2$

The last equation is not equivalent to the original equation, so the graph is not symmetric with respect to the origin.

21. Test for symmetry with respect to the x-axis:

$2x^4 + 3 = y^2$ Original equation
$2x^4 + 3 = (-y)^2$ Replacing y by $-y$
$2x^4 + 3 = y^2$ Simplifying

The last equation is equivalent to the original equation, so the graph is symmetric with respect to the x-axis.

Test for symmetry with respect to the y-axis:

$2x^4 + 3 = y^2$ Original equation
$2(-x)^4 + 3 = y^2$ Replacing x by $-x$
$2x^4 + 3 = y^2$ Simplifying

The last equation is equivalent to the original equation, so the graph is symmetric with respect to the y-axis.

Test for symmetry with respect to the origin:

$2x^4 + 3 = y^2$ Original equation
$2(-x)^4 + 3 = (-y)^2$ Replacing x by $-x$ and y by $-y$
$2x^4 + 3 = y^2$ Simplifying

The last equation is equivalent to the original equation, so the graph is symmetric with respect to the origin.

22. Test for symmetry with respect to the x-axis:

$2y^2 = 5x^2 + 12$ Original equation
$2(-y)^2 = 5x^2 + 12$ Replacing y by $-y$
$2y^2 = 5x^2 + 12$ Simplifying

The last equation is equivalent to the original equation, so the graph is symmetric with respect to the x-axis.

Test for symmetry with respect to the y-axis:

$2y^2 = 5x^2 + 12$ Original equation
$2y^2 = 5(-x)^2 + 12$ Replacing x by $-x$
$2y^2 = 5x^2 + 12$ Simplifying

The last equation is equivalent to the original equation, so the graph is symmetric with respect to the y-axis.

Test for symmetry with respect to the origin:

$2y^2 = 5x^2 + 12$ Original equation
$2(-y)^2 = 5(-x)^2 + 12$ Replacing x by $-x$ and y by $-y$
$2y^2 = 5x^2 + 12$ Simplifying

The last equation is equivalent to the original equation, so the graph is symmetric with respect to the origin.

23. Test for symmetry with respect to the x-axis:

$3y^3 = 4x^3 + 2$ Original equation
$3(-y)^3 = 4x^3 + 2$ Replacing y by $-y$
$-3y^3 = 4x^3 + 2$ Simplifying
$3y^3 = -4x^3 - 2$

The last equation is not equivalent to the original equation, so the graph is not symmetric with respect to the x-axis.

Test for symmetry with respect to the y-axis:

$3y^3 = 4x^3 + 2$ Original equation
$3y^3 = 4(-x)^3 + 2$ Replacing x by $-x$
$3y^3 = -4x^3 + 2$ Simplifying

The last equation is not equivalent to the original equation, so the graph is not symmetric with respect to the y-axis.

Test for symmetry with respect to the origin:

$3y^3 = 4x^3 + 2$ Original equation
$3(-y)^3 = 4(-x)^3 + 2$ Replacing x by $-x$ and y by $-y$
$-3y^3 = -4x^3 + 2$ Simplifying
$3y^3 = 4x^3 - 2$

The last equation is not equivalent to the original equation, so the graph is not symmetric with respect to the origin.

24. Test for symmetry with respect to the x-axis:

$3x = |y|$ Original equation
$3x = |-y|$ Replacing y by $-y$
$3x = |y|$ Simplifying

The last equation is equivalent to the original equation, so the graph is symmetric with respect to the x-axis.

Test for symmetry with respect to the y-axis:

$3x = |y|$ Original equation
$3(-x) = |y|$ Replacing x by $-x$
$-3x = |y|$ Simplifying

The last equation is not equivalent to the original equation, so the graph is not symmetric with respect to the y-axis.

Test for symmetry with respect to the origin:

$3x = |y|$ Original equation
$3(-x) = |-y|$ Replacing x by $-x$ and y by $-y$
$-3x = |y|$ Simplifying

The last equation is not equivalent to the original equation, so the graph is not symmetric with respect to the origin.

25. Test for symmetry with respect to the x-axis:

$xy = 12$ Original equation
$x(-y) = 12$ Replacing y by $-y$
$-xy = 12$ Simplifying
$xy = -12$

The last equation is not equivalent to the original equation, so the graph is not symmetric with respect to the x-axis.

Test for symmetry with respect to the y-axis:

$xy = 12$ Original equation
$-xy = 12$ Replacing x by $-x$
$xy = -12$ Simplifying

The last equation is not equivalent to the original equation, so the graph is not symmetric with respect to the y-axis.

Test for symmetry with respect to the origin:

$xy = 12$ Original equation
$-x(-y) = 12$ Replacing x by $-x$ and y by $-y$
$xy = 12$ Simplifying

The last equation is equivalent to the original equation, so the graph is symmetric with respect to the origin.

26. Test for symmetry with respect to the x-axis:

$$\begin{aligned} xy - x^2 &= 3 && \text{Original equation} \\ x(-y) - x^2 &= 3 && \text{Replacing } y \text{ by } -y \\ xy + x^2 &= -3 && \text{Simplifying} \end{aligned}$$

The last equation is not equivalent to the original equation, so the graph is not symmetric with respect to the x-axis.

Test for symmetry with respect to the y-axis:

$$\begin{aligned} xy - x^2 &= 3 && \text{Original equation} \\ -xy - (-x)^2 &= 3 && \text{Replacing } x \text{ by } -x \\ xy + x^2 &= -3 && \text{Simplifying} \end{aligned}$$

The last equation is not equivalent to the original equation, so the graph is not symmetric with respect to the y-axis.

Test for symmetry with respect to the origin:

$$\begin{aligned} xy - x^2 &= 3 && \text{Original equation} \\ -x(-y) - (-x)^2 &= 3 && \text{Replacing } x \text{ by } -x \text{ and } y \text{ by } -y \\ xy - x^2 &= 3 && \text{Simplifying} \end{aligned}$$

The last equation is equivalent to the original equation, so the graph is symmetric with respect to the origin.

27. x-axis: Replace y with $-y$; $(-5, -6)$

y-axis: Replace x with $-x$; $(5, 6)$

Origin: Replace x with $-x$ and y with $-y$; $(5, -6)$

28. x-axis: Replace y with $-y$; $\left(\frac{7}{2}, 0\right)$

y-axis: Replace x with $-x$; $\left(-\frac{7}{2}, 0\right)$

Origin: Replace x with $-x$ and y with $-y$; $\left(-\frac{7}{2}, 0\right)$

29. x-axis: Replace y with $-y$; $(-10, 7)$

y-axis: Replace x with $-x$; $(10, -7)$

Origin: Replace x with $-x$ and y with $-y$; $(10, 7)$

30. x-axis: Replace y with $-y$; $\left(1, -\frac{3}{8}\right)$

y-axis: Replace x with $-x$; $\left(-1, \frac{3}{8}\right)$

Origin: Replace x with $-x$ and y with $-y$; $\left(-1, -\frac{3}{8}\right)$

31. x-axis: Replace y with $-y$; $(0, 4)$

y-axis: Replace x with $-x$; $(0, -4)$

Origin: Replace x with $-x$ and y with $-y$; $(0, 4)$

32. x-axis: Replace y with $-y$; $(8, 3)$

y-axis: Replace x with $-x$; $(-8, -3)$

Origin: Replace x with $-x$ and y with $-y$; $(-8, 3)$

33. The graph is symmetric with respect to the y-axis, so the function is even.

34. The graph is symmetric with respect to the y-axis, so the function is even.

35. The graph is symmetric with respect to the origin, so the function is odd.

36. The graph is not symmetric with respect to either the y-axis or the origin, so the function is neither even nor odd.

37. The graph is not symmetric with respect to either the y-axis or the origin, so the function is neither even nor odd.

38. The graph is not symmetric with respect to either the y-axis or the origin, so the function is neither even nor odd.

39.
$$\begin{aligned} f(x) &= -3x^3 + 2x \\ f(-x) &= -3(-x)^3 + 2(-x) = 3x^3 - 2x \\ -f(x) &= -(-3x^3 + 2x) = 3x^3 - 2x \end{aligned}$$
$f(-x) = -f(x)$, so f is odd.

40.
$$\begin{aligned} f(x) &= 7x^3 + 4x - 2 \\ f(-x) &= 7(-x)^3 + 4(-x) - 2 = -7x^3 - 4x - 2 \\ -f(x) &= -(7x^3 + 4x - 2) = -7x^3 - 4x + 2 \end{aligned}$$
$f(x) \neq f(-x)$, so f is not even.

$f(-x) \neq -f(x)$, so f is not odd.

Thus, $f(x) = 7x^3 + 4x - 2$ is neither even nor odd.

41.
$$\begin{aligned} f(x) &= 5x^2 + 2x^4 - 1 \\ f(-x) &= 5(-x)^2 + 2(-x)^4 - 1 = 5x^2 + 2x^4 - 1 \end{aligned}$$
$f(x) = f(-x)$, so f is even.

42.
$$\begin{aligned} f(x) &= x + \frac{1}{x} \\ f(-x) &= -x + \frac{1}{-x} = -x - \frac{1}{x} \\ -f(x) &= -\left(x + \frac{1}{x}\right) = -x - \frac{1}{x} \end{aligned}$$
$f(-x) = -f(x)$, so f is odd.

43.
$$\begin{aligned} f(x) &= x^{17} \\ f(-x) &= (-x)^{17} = -x^{17} \\ -f(x) &= -x^{17} \end{aligned}$$
$f(-x) = -f(x)$, so f is odd.

44.
$$\begin{aligned} f(x) &= \sqrt[3]{x} \\ f(-x) &= \sqrt[3]{-x} = -\sqrt[3]{x} \\ -f(x) &= -\sqrt[3]{x} \end{aligned}$$
$f(-x) = -f(x)$, so f is odd.

45.
$$\begin{aligned} f(x) &= x - |x| \\ f(-x) &= (-x) - |(-x)| = -x - |x| \\ -f(x) &= -(x - |x|) = -x + |x| \end{aligned}$$
$f(x) \neq f(-x)$, so f is not even.

$f(-x) \neq -f(x)$, so f is not odd.

Thus, $f(x) = x - |x|$ is neither even nor odd.

46. $f(x) = \dfrac{1}{x^2}$

$f(-x) = \dfrac{1}{(-x)^2} = \dfrac{1}{x^2}$

$f(x) = f(-x)$, so f is even.

47. $f(x) = 8$

$f(-x) = 8$

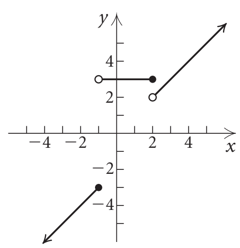
$f(x) = f(-x)$, so f is even.

48. $f(x) = \sqrt{x^2+1}$

$f(-x) = \sqrt{(-x)^2+1} = \sqrt{x^2+1}$

$f(x) = f(-x)$, so f is even.

49.



50. Let v = the number of volunteers from the University of Wisconsin - Madison. Then $v + 464$ = the number of volunteers from the University of California - Berkeley.

Solve: $v + (v + 464) = 6688$

$v = 3112$, so there were 3112 volunteers from the University of Wisconsin - Madison and $3112 + 464$, or 3576 volunteers from the University of California - Berkeley.

51. $f(x) = x\sqrt{10-x^2}$

$f(-x) = -x\sqrt{10-(-x)^2} = -x\sqrt{10-x^2}$

$-f(x) = -x\sqrt{10-x^2}$

Since $f(-x) = -f(x)$, f is odd.

52. $f(x) = \dfrac{x^2+1}{x^3+1}$

$f(-x) = \dfrac{(-x)^2+1}{(-x)^3+1} = \dfrac{x^2+1}{-x^3+1}$

$-f(x) = -\dfrac{x^2+1}{x^3+1}$

Since $f(x) \neq f(-x)$, f is not even.

Since $f(-x) \neq -f(x)$, f is not odd.

Thus, $f(x) = \dfrac{x^2+1}{x^3+1}$ is neither even nor odd.

53. If the graph were folded on the x-axis, the parts above and below the x-axis would coincide, so the graph is symmetric with respect to the x-axis.

If the graph were folded on the y-axis, the parts to the left and right of the y-axis would not coincide, so the graph is not symmetric with respect to the y-axis.

If the graph were rotated 180°, the resulting graph would not coincide with the original graph, so it is not symmetric with respect to the origin.

54. If the graph were folded on the x-axis, the parts above and below the x-axis would not coincide, so the graph is not symmetric with respect to the x-axis.

If the graph were folded on the y-axis, the parts to the left and right of the y-axis would not coincide, so the graph is not symmetric with respect to the y-axis.

If the graph were rotated 180°, the resulting graph would coincide with the original graph, so it is symmetric with respect to the origin.

55. See the answer section in the text.

56. $O(-x) = \dfrac{f(-x) - f(-(-x))}{2} = \dfrac{f(-x) - f(x)}{2}$,

$-O(x) = -\dfrac{f(x) - f(-x)}{2} = \dfrac{f(-x) - f(x)}{2}$. Thus,

$O(-x) = -O(x)$ and O is odd.

57. a), b) See the answer section in the text.

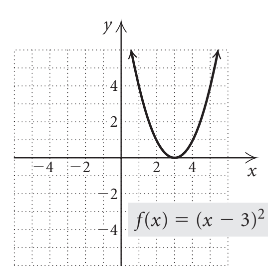
58. Let $f(x) = g(x) = x$. Now f and g are odd functions, but $(fg)(x) = x^2 = (fg)(-x)$. Thus, the product is even, so the statement is false.

59. Let $f(x)$ and $g(x)$ be even functions. Then by definition, $f(x) = f(-x)$ and $g(x) = g(-x)$. Thus, $(f+g)(x) = f(x) + g(x) = f(-x) + g(-x) = (f+g)(-x)$ and $f+g$ is even. The statement is true.

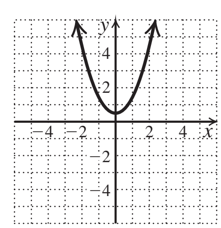
60. Let $f(x)$ be an even function, and let $g(x)$ be an odd function. By definition $f(x) = f(-x)$ and $g(-x) = -g(x)$, or $g(x) = -g(-x)$. Then $fg(x) = f(x) \cdot g(x) = f(-x) \cdot [-g(-x)] = -f(-x) \cdot g(-x) = -fg(-x)$, and fg is odd. The statement is true.

Exercise Set 2.5

1. Shift the graph of $f(x) = x^2$ right 3 units.

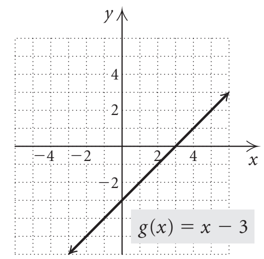


2. Shift the graph of $g(x) = x^2$ up $\frac{1}{2}$ unit.

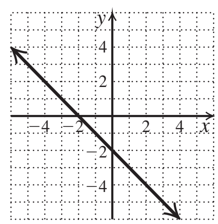


$g(x) = x^2 + \frac{1}{2}$

3. Shift the graph of $g(x) = x$ down 3 units.

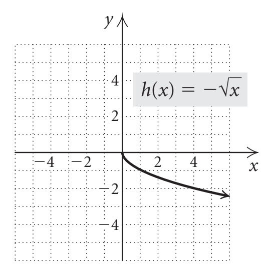


4. Reflect the graph of $g(x) = x$ across the x-axis and then shift it down 2 units.

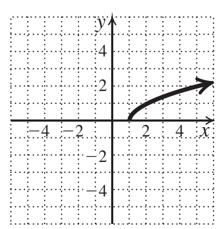


$g(x) = -x - 2$

5. Reflect the graph of $h(x) = \sqrt{x}$ across the x-axis.

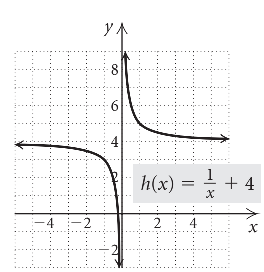


6. Shift the graph of $g(x) = \sqrt{x}$ right 1 unit.

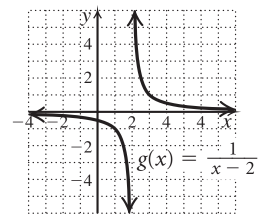


$g(x) = \sqrt{x - 1}$

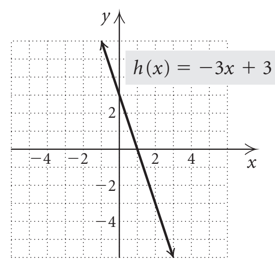
7. Shift the graph of $h(x) = \dfrac{1}{x}$ up 4 units.



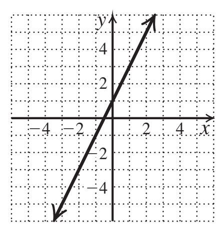
8. Shift the graph of $g(x) = \dfrac{1}{x}$ right 2 units.



9. First stretch the graph of $h(x) = x$ vertically by multiplying each y-coordinate by 3. Then reflect it across the x-axis and shift it up 3 units.

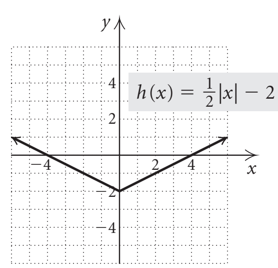


10. First stretch the graph of $f(x) = x$ vertically by multiplying each y-coordinate by 2. Then shift it up 1 unit.

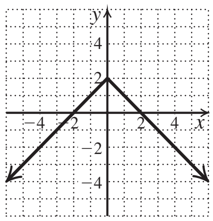


$f(x) = 2x + 1$

11. First shrink the graph of $h(x) = |x|$ vertically by multiplying each y-coordinate by $\dfrac{1}{2}$. Then shift it down 2 units.

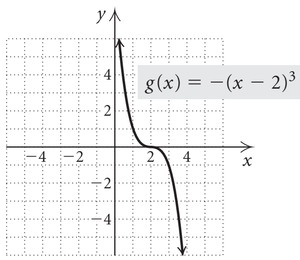


12. Reflect the graph of $g(x) = |x|$ across the x-axis and shift it up 2 units.

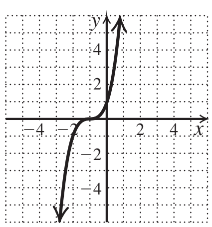


$g(x) = -|x| + 2$

13. Shift the graph of $g(x) = x^3$ right 2 units and reflect it across the x-axis.

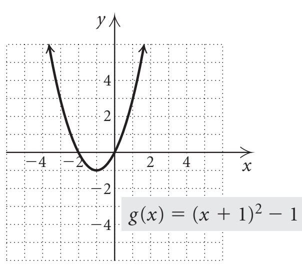


14. Shift the graph of $f(x) = x^3$ left 1 unit.

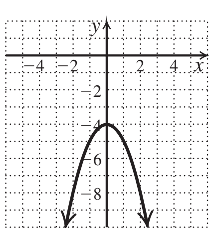


$f(x) = (x + 1)^3$

15. Shift the graph of $g(x) = x^2$ left 1 unit and down 1 unit.

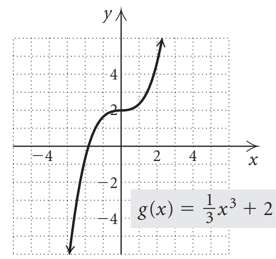


16. Reflect the graph of $h(x) = x^2$ across the x-axis and down 4 units.

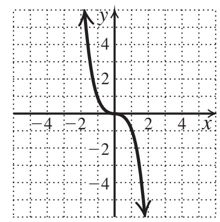


$h(x) = -x^2 - 4$

17. First shrink the graph of $g(x) = x^3$ vertically by multiplying each y-coordinate by $\frac{1}{3}$. Then shift it up 2 units.

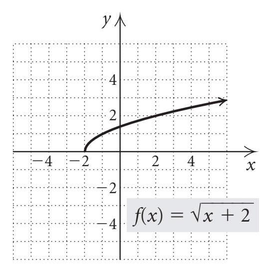


18. Reflect the graph of $h(x) = x^3$ across the y-axis.

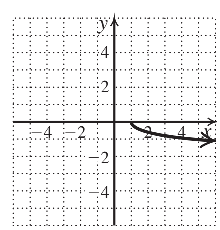


$h(x) = (-x)^3$

19. Shift the graph of $f(x) = \sqrt{x}$ left 2 units.

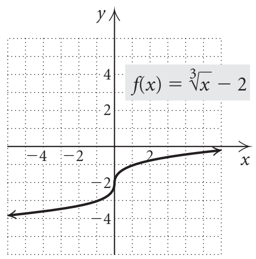


20. First shift the graph of $f(x) = \sqrt{x}$ right 1 unit. Shrink it vertically by multiplying each y-coordinate by $\frac{1}{2}$ and then reflect it across the x-axis.

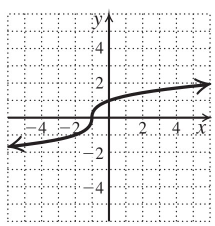


$f(x) = -\frac{1}{2}\sqrt{x - 1}$

21. Shift the graph of $f(x) = \sqrt[3]{x}$ down 2 units.



22. Shift the graph of $h(x) = \sqrt[3]{x}$ left 1 unit.



$h(x) = \sqrt[3]{x+1}$

23. Think of the graph of $f(x) = |x|$. Since $g(x) = f(3x)$, the graph of $g(x) = |3x|$ is the graph of $f(x) = |x|$ shrunk horizontally by dividing each x-coordinate by 3 $\left(\text{or multiplying each } x\text{-coordinate by } \frac{1}{3}\right)$.

24. Think of the graph of $g(x) = \sqrt[3]{x}$. Since $f(x) = \frac{1}{2}g(x)$, the graph of $f(x) = \frac{1}{2}\sqrt[3]{x}$ is the graph of $g(x) = \sqrt[3]{x}$ shrunk vertically by multiplying each y-coordinate by $\frac{1}{2}$.

25. Think of the graph of $f(x) = \frac{1}{x}$. Since $h(x) = 2f(x)$, the graph of $h(x) = \frac{2}{x}$ is the graph of $f(x) = \frac{1}{x}$ stretched vertically by multiplying each y-coordinate by 2.

26. Think of the graph of $g(x) = |x|$. Since $f(x) = g(x-3) - 4$, the graph of $f(x) = |x-3| - 4$ is the graph of $g(x) = |x|$ shifted right 3 units and down 4 units.

27. Think of the graph of $g(x) = \sqrt{x}$. Since $f(x) = 3g(x) - 5$, the graph of $f(x) = 3\sqrt{x} - 5$ is the graph of $g(x) = \sqrt{x}$ stretched vertically by multiplying each y-coordinate by 3 and then shifted down 5 units.

28. Think of the graph of $g(x) = \frac{1}{x}$. Since $f(x) = 5 - g(x)$, or $f(x) = -g(x) + 5$, the graph of $f(x) = 5 - \frac{1}{x}$ is the graph of $g(x) = \frac{1}{x}$ reflected across the x-axis and then shifted up 5 units.

29. Think of the graph of $f(x) = |x|$. Since $g(x) = f\left(\frac{1}{3}x\right) - 4$, the graph of $g(x) = \left|\frac{1}{3}x\right| - 4$ is the graph of $f(x) = |x|$ stretched horizontally by multiplying each x-coordinate by 3 and then shifted down 4 units.

30. Think of the graph of $g(x) = x^3$. Since $f(x) = \frac{2}{3}g(x) - 4$, the graph of $f(x) = \frac{2}{3}x^3 - 4$ is the graph of $g(x) = x^3$ shrunk vertically by multiplying each y-coordinate by $\frac{2}{3}$ and then shifted down 4 units.

31. Think of the graph of $g(x) = x^2$. Since $f(x) = -\frac{1}{4}g(x-5)$, the graph of $f(x) = -\frac{1}{4}(x-5)^2$ is the graph of $g(x) = x^2$ shifted right 5 units, shrunk vertically by multiplying each y-coordinate by $\frac{1}{4}$, and reflected across the x-axis.

32. Think of the graph of $g(x) = x^3$. Since $f(x) = g(-x) - 5$, the graph of $f(x) = (-x)^3 - 5$ is the graph of $g(x) = x^3$ reflected across the y-axis and shifted down 5 units.

33. Think of the graph of $g(x) = \frac{1}{x}$. Since $f(x) = g(x+3) + 2$, the graph of $f(x) = \frac{1}{x+3} + 2$ is the graph of $g(x) = \frac{1}{x}$ shifted left 3 units and up 2 units.

34. Think of the graph of $f(x) = \sqrt{x}$. Since $g(x) = f(-x) + 5$, the graph of $g(x) = \sqrt{-x} + 5$ is the graph of $f(x) = \sqrt{x}$ reflected across the y-axis and shifted up 5 units.

35. Think of the graph of $f(x) = x^2$. Since $h(x) = -f(x-3) + 5$, the graph of $h(x) = -(x-3)^2 + 5$ is the graph of $f(x) = x^2$ shifted right 3 units, reflected across the x-axis, and shifted up 5 units.

36. Think of the graph of $g(x) = x^2$. Since $f(x) = 3g(x+4) - 3$, the graph of $f(x) = 3(x+4)^2 - 3$ is the graph of $g(x) = x^2$ shifted left 4 units, stretched vertically by multiplying each y-coordinate by 3, and then shifted down 3 units.

37. The graph of $y = g(x)$ is the graph of $y = f(x)$ shrunk vertically by a factor of $\frac{1}{2}$. Multiply the y-coordinate by $\frac{1}{2}$: $(-12, 2)$.

38. The graph of $y = g(x)$ is the graph of $y = f(x)$ shifted right 2 units. Add 2 to the x-coordinate: $(-10, 4)$.

39. The graph of $y = g(x)$ is the graph of $y = f(x)$ reflected across the y-axis, so we reflect the point across the y-axis: $(12, 4)$.

40. The graph of $y = g(x)$ is the graph of $y = f(x)$ shrunk horizontally. The x-coordinates of $y = g(x)$ are $\frac{1}{4}$ the corresponding x-coordinates of $y = f(x)$, so we divide the x-coordinate by 4 $\left(\text{or multiply it by } \frac{1}{4}\right)$: $(-3, 4)$.

41. The graph of $y = g(x)$ is the graph of $y = f(x)$ shifted down 2 units. Subtract 2 from the y-coordinate: $(-12, 2)$.

42. The graph of $y = g(x)$ is the graph of $y = f(x)$ stretched horizontally. The x-coordinates of $y = g(x)$ are twice the corresponding x-coordinates of $y = f(x)$, so we multiply the x-coordinate by 2 $\left(\text{or divide it by } \frac{1}{2}\right)$: $(-24, 4)$.

43. The graph of $y = g(x)$ is the graph of $y = f(x)$ stretched vertically by a factor of 4. Multiply the y-coordinate by 4: $(-12, 16)$.

44. The graph of $y = g(x)$ is the graph $y = f(x)$ reflected across the x-axis. Reflect the point across the x-axis: $(-12, -4)$.

45. $g(x) = x^2 + 4$ is the function $f(x) = x^2 + 3$ shifted up 1 unit, so $g(x) = f(x) + 1$. Answer B is correct.

46. If we substitute $3x$ for x in f, we get $9x^2 + 3$, so $g(x) = f(3x)$. Answer D is correct.

47. If we substitute $x - 2$ for x in f, we get $(x - 2)^3 + 3$, so $g(x) = f(x - 2)$. Answer A is correct.

48. If we multiply $x^2 + 3$ by 2, we get $2x^2 + 6$, so $g(x) = 2f(x)$. Answer C is correct.

49. Shape: $h(x) = x^2$

Turn $h(x)$ upside-down (that is, reflect it across the x-axis): $g(x) = -h(x) = -x^2$

Shift $g(x)$ right 8 units: $f(x) = g(x - 8) = -(x - 8)^2$

50. Shape: $h(x) = \sqrt{x}$

Shift $h(x)$ left 6 units: $g(x) = h(x + 6) = \sqrt{x + 6}$

Shift $g(x)$ down 5 units: $f(x) = g(x) - 5 = \sqrt{x + 6} - 5$

51. Shape: $h(x) = |x|$

Shift $h(x)$ left 7 units: $g(x) = h(x + 7) = |x + 7|$

Shift $g(x)$ up 2 units: $f(x) = g(x) + 2 = |x + 7| + 2$

52. Shape: $h(x) = x^3$

Turn $h(x)$ upside-down (that is, reflect it across the x-axis): $g(x) = -h(x) = -x^3$

Shift $g(x)$ right 5 units: $f(x) = g(x - 5) = -(x - 5)^3$

53. Shape: $h(x) = \dfrac{1}{x}$

Shrink $h(x)$ vertically by a factor of $\dfrac{1}{2}$ $\Bigl($that is, multiply each function value by $\dfrac{1}{2}\Bigr)$:

$g(x) = \dfrac{1}{2}h(x) = \dfrac{1}{2} \cdot \dfrac{1}{x}$, or $\dfrac{1}{2x}$

Shift $g(x)$ down 3 units: $f(x) = g(x) - 3 = \dfrac{1}{2x} - 3$

54. Shape: $h(x) = x^2$

Shift $h(x)$ right 6 units: $g(x) = h(x - 6) = (x - 6)^2$

Shift $g(x)$ up 2 units: $f(x) = g(x) + 2 = (x - 6)^2 + 2$

55. Shape: $m(x) = x^2$

Turn $m(x)$ upside-down (that is, reflect it across the x-axis): $h(x) = -m(x) = -x^2$

Shift $h(x)$ right 3 units: $g(x) = h(x - 3) = -(x - 3)^2$

Shift $g(x)$ up 4 units: $f(x) = g(x) + 4 = -(x - 3)^2 + 4$

56. Shape: $h(x) = |x|$

Stretch $h(x)$ horizontally by a factor of 2 $\Bigl($that is, multiply each x-value by $\dfrac{1}{2}\Bigr)$: $g(x) = h\left(\dfrac{1}{2}x\right) = \left|\dfrac{1}{2}x\right|$

Shift $g(x)$ down 5 units: $f(x) = g(x) - 5 = \left|\dfrac{1}{2}x\right| - 5$

57. Shape: $m(x) = \sqrt{x}$

Reflect $m(x)$ across the y-axis: $h(x) = m(-x) = \sqrt{-x}$

Shift $h(x)$ left 2 units: $g(x) = h(x + 2) = \sqrt{-(x + 2)}$

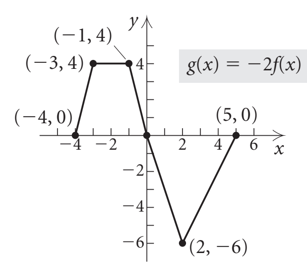
Shift $g(x)$ down 1 unit: $f(x) = g(x) - 1 = \sqrt{-(x + 2)} - 1$

58. Shape: $h(x) = \dfrac{1}{x}$

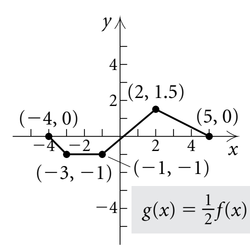
Reflect $h(x)$ across the x-axis: $g(x) = -h(x) = -\dfrac{1}{x}$

Shift $g(x)$ up 1 unit: $f(x) = g(x) + 1 = -\dfrac{1}{x} + 1$

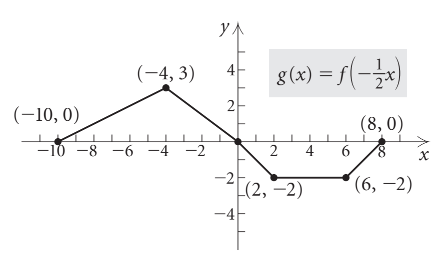
59. Each y-coordinate is multiplied by -2. We plot and connect $(-4, 0)$, $(-3, 4)$, $(-1, 4)$, $(2, -6)$, and $(5, 0)$.



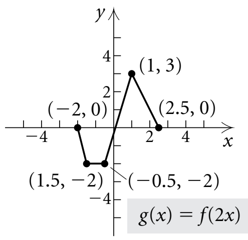
60. Each y-coordinate is multiplied by $\dfrac{1}{2}$. We plot and connect $(-4, 0)$, $(-3, -1)$, $(-1, -1)$, $(2, 1.5)$, and $(5, 0)$.



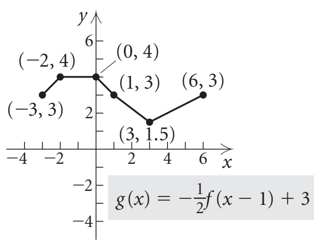
61. The graph is reflected across the y-axis and stretched horizontally by a factor of 2. That is, each x-coordinate is multiplied by -2 $\Bigl($or divided by $-\dfrac{1}{2}\Bigr)$. We plot and connect $(8, 0)$, $(6, -2)$, $(2, -2)$, $(-4, 3)$, and $(-10, 0)$.



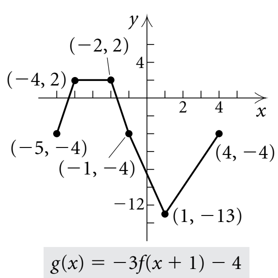
62. The graph is shrunk horizontally by a factor of 2. That is, each x-coordinate is divided by 2 $\left(\text{or multiplied by } \dfrac{1}{2}\right)$. We plot and connect $(-2,0)$, $(-1.5,-2)$, $(-0.5,-2)$, $(1,3)$, and $(2.5,0)$.



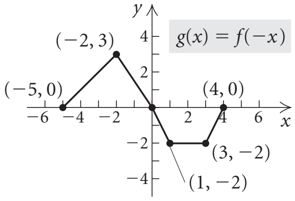
63. The graph is shifted right 1 unit so each x-coordinate is increased by 1. The graph is also reflected across the x-axis, shrunk vertically by a factor of 2, and shifted up 3 units. Thus, each y-coordinate is multiplied by $-\dfrac{1}{2}$ and then increased by 3. We plot and connect $(-3,3)$, $(-2,4)$, $(0,4)$, $(3,1.5)$, and $(6,3)$.



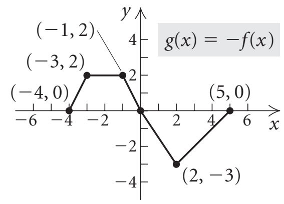
64. The graph is shifted left 1 unit so each x-coordinate is decreased by 1. The graph is also reflected across the x-axis, stretched vertically by a factor of 3, and shifted down 4 units. Thus, each y-coordinate is multiplied by -3 and then decreased by 4. We plot and connect $(-5,-4)$, $(-4,2)$, $(-2,2)$, $(1,-13)$, and $(4,-4)$.



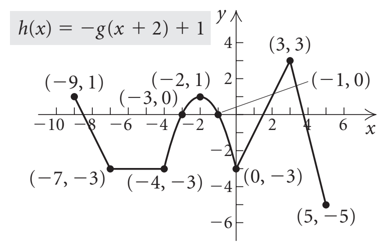
65. The graph is reflected across the y-axis so each x-coordinate is replaced by its opposite.



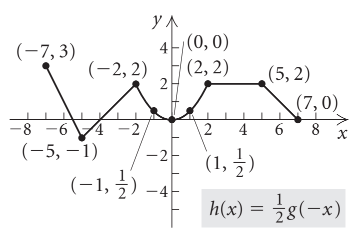
66. The graph is reflected across the x-axis so each y-coordinate is replaced by its opposite.



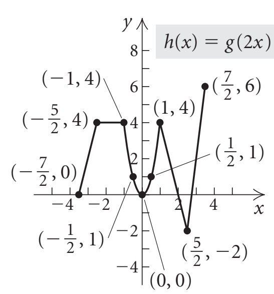
67. The graph is shifted left 2 units so each x-coordinate is decreased by 2. It is also reflected across the x-axis so each y-coordinate is replaced with its opposite. In addition, the graph is shifted up 1 unit, so each y-coordinate is then increased by 1.



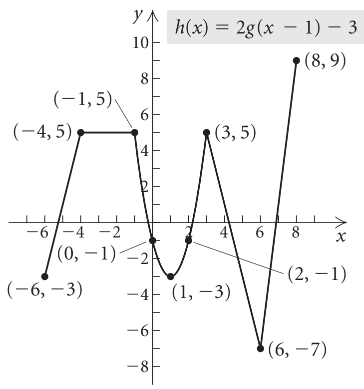
68. The graph is reflected across the y-axis so each x-coordinate is replaced with its opposite. It is also shrunk vertically by a factor of $\dfrac{1}{2}$, so each y-coordinate is multiplied by $\dfrac{1}{2}$ (or divided by 2).



69. The graph is shrunk horizontally. The x-coordinates of $y=h(x)$ are one-half the corresponding x-coordinates of $y=g(x)$.



70. The graph is shifted right 1 unit, so each x-coordinate is increased by 1. It is also stretched vertically by a factor of 2, so each y-coordinate is multiplied by 2 (or divided by $\left.\frac{1}{2}\right)$. In addition, the graph is shifted down 3 units, so each y-coordinate is decreased by 3.



71. $g(x) = f(-x) + 3$

The graph of $g(x)$ is the graph of $f(x)$ reflected across the y-axis and shifted up 3 units. This is graph (f).

72. $g(x) = f(x) + 3$

The graph of $g(x)$ is the graph of $f(x)$ shifted up 3 units. This is graph (h).

73. $g(x) = -f(x) + 3$

The graph of $g(x)$ is the graph of $f(x)$ reflected across the x-axis and shifted up 3 units. This is graph (f).

74. $g(x) = -f(-x)$

The graph of $g(x)$ is the graph of $f(x)$ reflected across the x-axis and the y-axis. This is graph (a).

75. $g(x) = \frac{1}{3}f(x-2)$

The graph of $g(x)$ is the graph of $f(x)$ shrunk vertically by a factor of 3 $\left(\text{that is, each } y\text{-coordinate is multiplied by } \frac{1}{3}\right)$ and then shifted right 2 units. This is graph (d).

76. $g(x) = \frac{1}{3}f(x) - 3$

The graph of $g(x)$ is the graph of $f(x)$ shrunk vertically by a factor of 3 $\left(\text{that is, each } y\text{-coordinate is multiplied by } \frac{1}{3}\right)$ and then shifted down 3 units. This is graph (e).

77. $g(x) = \frac{1}{3}f(x+2)$

The graph of $g(x)$ is the graph of $f(x)$ shrunk vertically by a factor of 3 $\left(\text{that is, each } y\text{-coordinate is multiplied by } \frac{1}{3}\right)$ and then shifted left 2 units. This is graph (c).

78. $g(x) = -f(x+2)$

The graph of $g(x)$ is the graph $f(x)$ reflected across the x-axis and shifted left 2 units. This is graph (b).

79. $f(-x) = 2(-x)^4 - 35(-x)^3 + 3(-x) - 5 =$
$2x^4 + 35x^3 - 3x - 5 = g(x)$

80. $f(-x) = \frac{1}{4}(-x)^4 + \frac{1}{5}(-x)^3 - 81(-x)^2 - 17 =$

$\frac{1}{4}x^4 - \frac{1}{5}x^3 - 81x^2 - 17 \neq g(x)$

81. The graph of $f(x) = x^3 - 3x^2$ is shifted up 2 units. A formula for the transformed function is $g(x) = f(x) + 2$, or $g(x) = x^3 - 3x^2 + 2$.

82. Each y-coordinate of the graph of $f(x) = x^3 - 3x^2$ is multiplied by $\frac{1}{2}$. A formula for the transformed function is $h(x) = \frac{1}{2}f(x)$, or $h(x) = \frac{1}{2}(x^3 - 3x^2)$.

83. The graph of $f(x) = x^3 - 3x^2$ is shifted left 1 unit. A formula for the transformed function is $k(x) = f(x+1)$, or $k(x) = (x+1)^3 - 3(x+1)^2$.

84. The graph of $f(x) = x^3 - 3x^2$ is shifted right 2 units and up 1 unit. A formula for the transformed function is $t(x) = f(x-2) + 1$, or $t(x) = (x-2)^3 - 3(x-2)^2 + 1$.

85. Test for symmetry with respect to the x-axis.

$y = 3x^4 - 3$ Original equation
$-y = 3x^4 - 3$ Replacing y by $-y$
$y = -3x^4 + 3$ Simplifying

The last equation is not equivalent to the original equation, so the graph is not symmetric with respect to the x-axis.

Test for symmetry with respect to the y-axis.

$y = 3x^4 - 3$ Original equation
$y = 3(-x)^4 - 3$ Replacing x by $-x$
$y = 3x^4 - 3$ Simplifying

The last equation is equivalent to the original equation, so the graph is symmetric with respect to the y-axis.

Test for symmetry with respect to the origin:

$y = 3x^4 - 3$
$-y = 3(-x)^4 - 3$ Replacing x by $-x$ and y by $-y$
$-y = 3x^4 - 3$
$y = -3x^4 + 3$ Simplifying

The last equation is not equivalent to the original equation, so the graph is not symmetric with respect to the origin.

86. Test for symmetry with respect to the x-axis.

$y^2 = x$ Original equation
$(-y)^2 = x$ Replacing y by $-y$
$y^2 = x$ Simplifying

The last equation is equivalent to the original equation, so the graph is symmetric with respect to the x-axis.

Test for symmetry with respect to the y-axis:

$y^2 = x$ Original equation
$y^2 = -x$ Replacing x by $-x$

The last equation is not equivalent to the original equation, so the graph is not symmetric with respect to the y-axis.

Test for symmetry with respect to the origin:

$$\begin{aligned} y^2 &= x && \text{Original equation} \\ (-y)^2 &= -x && \text{Replacing } x \text{ by } -x \text{ and} \\ & && y \text{ by } -y \\ y^2 &= -x && \text{Simplifying} \end{aligned}$$

The last equation is not equivalent to the original equation, so the graph is not symmetric with respect to the origin.

87. Test for symmetry with respect to the x-axis:

$$\begin{aligned} 2x - 5y &= 0 && \text{Original equation} \\ 2x - 5(-y) &= 0 && \text{Replacing } y \text{ by } -y \\ 2x + 5y &= 0 && \text{Simplifying} \end{aligned}$$

The last equation is not equivalent to the original equation, so the graph is not symmetric with respect to the x-axis.

Test for symmetry with respect to the y-axis:

$$\begin{aligned} 2x - 5y &= 0 && \text{Original equation} \\ 2(-x) - 5y &= 0 && \text{Replacing } x \text{ by } -x \\ -2x - 5y &= 0 && \text{Simplifying} \end{aligned}$$

The last equation is not equivalent to the original equation, so the graph is not symmetric with respect to the y-axis.

Test for symmetry with respect to the origin:

$$\begin{aligned} 2x - 5y &= 0 && \text{Original equation} \\ 2(-x) - 5(-y) &= 0 && \text{Replacing } x \text{ by } -x \text{ and} \\ & && y \text{ by } -y \\ -2x + 5y &= 0 && \\ 2x - 5y &= 0 && \text{Simplifying} \end{aligned}$$

The last equation is equivalent to the original equation, so the graph is symmetric with respect to the origin.

88. Let $w =$ the average annual wages of a 64-year-old person with only a high school diploma.

Solve: $w + 0.537w = 67,735$

$w \approx \$44,070$

89. ***Familiarize***. Let $c =$ the cost of knee replacement surgery in India in 2014.

Translate.

Cost in U.S.	was	\$4000	more than	four	times	cost in India
↓	↓	↓	↓	↓	↓	↓
$34,000$	$=$	4000	$+$	4	$\cdot$	c

Carry out. We solve the equation.

$$\begin{aligned} 34,000 &= 4000 + 4 \cdot c \\ 30,000 &= 4c \\ 7500 &= c \end{aligned}$$

Check. \$4000 more than 4 times \$7500 is $\$4000 + 4 \cdot \$7500 = \$4000 + \$30,000 = \$34,000$. The answer checks.

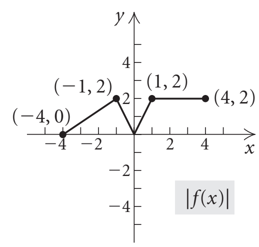
State. In 2014, the cost of knee replacement surgery in India was \$7500.

90. Let $c =$ the number of students Canada sent to the U.S. to study in universities in 2013-2014. Then $c + 25,615 =$ the number of students Saudi Arabia sent.

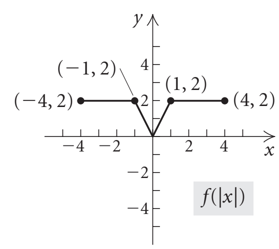
Solve: $c + (c + 25,615) = 82,223$

$c = 28,304$ students, and $c + 25,615 = 53,919$ students.

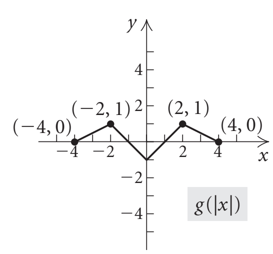
91. Each point for which $f(x) < 0$ is reflected across the x-axis.



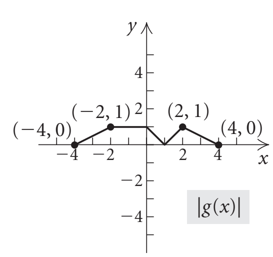
92. The graph of $y = f(|x|)$ consists of the points of $y = f(x)$ for which $x \geq 0$ along with their reflections across the y-axis.



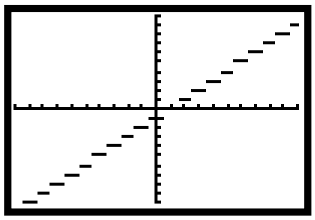
93. The graph of $y = g(|x|)$ consists of the points of $y = g(x)$ for which $x \geq 0$ along with their reflections across the y-axis.



94. Each point for which $g(x) < 0$ is reflected across the x-axis.



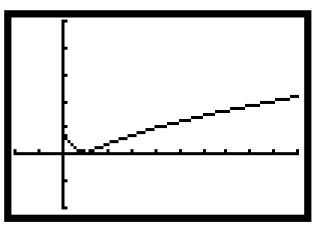
95. Think of the graph of $g(x) = \text{int}(x)$. Since $f(x) = g\left(x - \frac{1}{2}\right)$, the graph of $f(x) = \text{int}\left(x - \frac{1}{2}\right)$ is the graph of $g(x) = \text{int}(x)$ shifted right $\frac{1}{2}$ unit. The domain is the set of all real numbers; the range is the set of all integers.



96. This function can be defined piecewise as follows:

$$f(x) = \begin{cases} -(\sqrt{x}-1), & \text{for } 0 \le x < 1, \\ \sqrt{x}-1, & \text{for } x \ge 1, \end{cases}$$

Think of the graph of $g(x) = \sqrt{x}$. First shift it down 1 unit. Then reflect across the x-axis the portion of the graph for which $0 < x < 1$. The domain and range are both the set of nonnegative real numbers, or $[0, \infty)$.



97. On the graph of $y = 2f(x)$ each y-coordinate of $y = f(x)$ is multiplied by 2, so $(3, 4\cdot 2)$, or $(3, 8)$ is on the transformed graph.

On the graph of $y = 2+f(x)$, each y-coordinate of $y = f(x)$ is increased by 2 (shifted up 2 units), so $(3, 4+2)$, or $(3, 6)$ is on the transformed graph.

On the graph of $y = f(2x)$, each x-coordinate of $y = f(x)$ is multiplied by $\frac{1}{2}$ (or divided by 2), so $\left(\frac{1}{2}\cdot 3, 4\right)$, or $\left(\frac{3}{2}, 4\right)$ is on the transformed graph.

98. Using a graphing calculator we find that the zeros are -2.582, 0, and 2.582.

The graph of $y = f(x-3)$ is the graph of $y = f(x)$ shifted right 3 units. Thus we shift each of the zeros of $f(x)$ 3 units right to find the zeros of $f(x-3)$. They are $-2.582+3$, or 0.418; $0+3$, or 3; and $2.582+3$, or 5.582.

The graph of $y = f(x+8)$ is the graph of $y = f(x)$ shifted 8 units left. Thus we shift each of the zeros of $f(x)$ 8 units left to find the zeros of $f(x+8)$. They are $-2.582-8$, or -10.582; $0-8$, or -8; and $2.582-8$, or -5.418.

Exercise Set 2.6

1.
$$\begin{aligned} y &= kx \\ 54 &= k\cdot 12 \\ \frac{54}{12} &= k, \text{ or } k = \frac{9}{2} \end{aligned}$$

The variation constant is $\frac{9}{2}$, or 4.5. The equation of variation is $y = \frac{9}{2}x$, or $y = 4.5x$.

2.
$$\begin{aligned} y &= kx \\ 0.1 &= k(0.2) \\ \frac{1}{2} &= k \quad \text{Variation constant} \end{aligned}$$

Equation of variation: $y = \frac{1}{2}x$, or $y = 0.5x$.

3.
$$\begin{aligned} y &= \frac{k}{x} \\ 3 &= \frac{k}{12} \\ 36 &= k \end{aligned}$$

The variation constant is 36. The equation of variation is $y = \frac{36}{x}$.

4.
$$\begin{aligned} y &= \frac{k}{x} \\ 12 &= \frac{k}{5} \\ 60 &= k \quad \text{Variation constant} \end{aligned}$$

Equation of variation: $y = \frac{60}{x}$

5.
$$\begin{aligned} y &= kx \\ 1 &= k\cdot\frac{1}{4} \\ 4 &= k \end{aligned}$$

The variation constant is 4. The equation of variation is $y = 4x$.

6.
$$\begin{aligned} y &= \frac{k}{x} \\ 0.1 &= \frac{k}{0.5} \\ 0.05 &= k \quad \text{Variation constant} \end{aligned}$$

Equation of variation: $y = \frac{0.05}{x}$

7.
$$\begin{aligned} y &= \frac{k}{x} \\ 32 &= \frac{k}{\frac{1}{8}} \\ \frac{1}{8}\cdot 32 &= k \\ 4 &= k \end{aligned}$$

The variation constant is 4. The equation of variation is $y = \frac{4}{x}$.

8.
$$\begin{aligned} y &= kx \\ 3 &= k\cdot 33 \\ \frac{1}{11} &= k \quad \text{Variation constant} \end{aligned}$$

Equation of variation: $y = \frac{1}{11}x$

9.
$$y = kx$$
$$\frac{3}{4} = k \cdot 2$$
$$\frac{1}{2} \cdot \frac{3}{4} = k$$
$$\frac{3}{8} = k$$

The variation constant is $\frac{3}{8}$. The equation of variation is $y = \frac{3}{8}x$.

10.
$$y = \frac{k}{x}$$
$$\frac{1}{5} = \frac{k}{35}$$
$$7 = k \quad \text{Variation constant}$$

Equation of variation: $y = \frac{7}{x}$

11.
$$y = \frac{k}{x}$$
$$1.8 = \frac{k}{0.3}$$
$$0.54 = k$$

The variation constant is 0.54. The equation of variation is $y = \frac{0.54}{x}$.

12.
$$y = kx$$
$$0.9 = k(0.4)$$
$$\frac{9}{4} = k \quad \text{Variation constant}$$

Equation of variation: $y = \frac{9}{4}x$, or $y = 2.25x$

13. Let W = the weekly allowance and a = the child's age.
$$W = ka$$
$$5.50 = k \cdot 6$$
$$\frac{11}{12} = k$$
$$W = \frac{11}{12}x$$
$$W = \frac{11}{12} \cdot 9$$
$$W = \$8.25$$

14. Let S = the sales tax and p = the purchase price.
$$S = kp \quad S \text{ varies directly as } p.$$
$$7.14 = k \cdot 119 \quad \text{Substituting}$$
$$0.06 = k \quad \text{Variation constant}$$
$$S = 0.06p \quad \text{Equation of variation}$$
$$S = 0.06(21) \quad \text{Substituting}$$
$$S \approx 1.26$$

The sales tax is \$1.26.

15.
$$t = \frac{k}{r}$$
$$5 = \frac{k}{80}$$
$$400 = r$$
$$t = \frac{400}{r}$$
$$t = \frac{400}{70}$$
$$t = \frac{40}{7}, \text{ or } 5\frac{5}{7} \text{ hr}$$

16.
$$W = \frac{k}{L} \quad W \text{ varies inversely as } L.$$
$$1200 = \frac{k}{8} \quad \text{Substituting}$$
$$9600 = k \quad \text{Variation constant}$$
$$W = \frac{9600}{L} \quad \text{Equation of variation}$$
$$W = \frac{9600}{14} \quad \text{Substituting}$$
$$W \approx 686$$

A 14-m beam can support about 686 kg.

17. Let F = the number of grams of fat and w = the weight.
$$F = kw \quad F \text{ varies directly as } w.$$
$$60 = k \cdot 120 \quad \text{Substituting}$$
$$\frac{60}{120} = k, \text{ or} \quad \text{Solving for } k$$
$$\frac{1}{2} = k \quad \text{Variation constant}$$
$$F = \frac{1}{2}w \quad \text{Equation of variation}$$
$$F = \frac{1}{2} \cdot 180 \quad \text{Substituting}$$
$$F = 90$$

The maximum daily fat intake for a person weighing 180 lb is 90 g.

18.
$$N = kP$$
$$53 = k \cdot 38,333,000 \quad \text{Substituting}$$
$$\frac{53}{38,333,000} = k \quad \text{Variation constant}$$
$$N = \frac{53}{38,333,000}P$$
$$N = \frac{53}{38,333,000} \cdot 26,448,000 \quad \text{Substituting}$$
$$N \approx 37$$

Texas has 37 representatives.

19.
$$T = \frac{k}{P} \quad T \text{ varies inversely as } P.$$
$$5 = \frac{k}{7} \quad \text{Substituting}$$
$$35 = k \quad \text{Variation constant}$$

$T = \frac{35}{P}$ Equation of variation

$T = \frac{35}{10}$ Substituting

$T = 3.5$

It will take 10 bricklayers 3.5 hr to complete the job.

20.
$$t = \frac{k}{r}$$
$$45 = \frac{k}{600}$$
$$27,000 = k$$
$$t = \frac{27,000}{r}$$
$$t = \frac{27,000}{1000}$$
$$t = 27 \text{ min}$$

21. $d = km$ d varies directly as m.

$40 = k \cdot 3$ Substituting

$\frac{40}{3} = k$ Variation constant

$d = \frac{40}{3}m$ Equation of variation

$d = \frac{40}{3} \cdot 5 = \frac{200}{3}$ Substituting

$d = 66\frac{2}{3}$

A 5-kg mass will stretch the spring $66\frac{2}{3}$ cm.

22.
$$f = kF$$
$$6.3 = k \cdot 150$$
$$0.042 = k$$
$$f = 0.042F$$
$$f = 0.042(80)$$
$$f = 3.36$$

23. $P = \frac{k}{W}$ P varies inversely as W.

$330 = \frac{k}{3.2}$ Substituting

$1056 = k$ Variation constant

$P = \frac{1056}{W}$ Equation of variation

$550 = \frac{1056}{W}$ Substituting

$550W = 1056$ Multiplying by W

$W = \frac{1056}{550}$ Dividing by 550

$W = 1.92$ Simplifying

A tone with a pitch of 550 vibrations per second has a wavelength of 1.92 ft.

24. $M = kE$ M varies directly as E.

$35.9 = k \cdot 95$ Substituting

$0.378 \approx k$ Variation constant

$M = 0.378E$ Equation of variation

$M = 0.378 \cdot 100$ Substituting

$M = 37.8$

A 100-lb person would weigh about 37.8 lb on Mars.

25. $y = \frac{k}{x^2}$

$0.15 = \frac{k}{(0.1)^2}$ Substituting

$0.15 = \frac{k}{0.01}$

$0.15(0.01) = k$

$0.0015 = k$

The equation of variation is $y = \frac{0.0015}{x^2}$.

26.
$$y = \frac{k}{x^2}$$
$$6 = \frac{k}{3^2}$$
$$54 = k$$
$$y = \frac{54}{x^2}$$

27. $y = kx^2$

$0.15 = k(0.1)^2$ Substituting

$0.15 = 0.01k$

$\frac{0.15}{0.01} = k$

$15 = k$

The equation of variation is $y = 15x^2$.

28.
$$y = kx^2$$
$$6 = k \cdot 3^2$$
$$\frac{2}{3} = k$$
$$y = \frac{2}{3}x^2$$

29. $y = kxz$

$56 = k \cdot 7 \cdot 8$ Substituting

$56 = 56k$

$1 = k$

The equation of variation is $y = xz$.

30.
$$y = \frac{kx}{z}$$
$$4 = \frac{k \cdot 12}{15}$$
$$5 = k$$
$$y = \frac{5x}{z}$$

31.
$$y = kxz^2$$
$$105 = k \cdot 14 \cdot 5^2 \quad \text{Substituting}$$
$$105 = 350k$$
$$\frac{105}{350} = k$$
$$\frac{3}{10} = k$$

The equation of variation is $y = \frac{3}{10}xz^2$.

32.
$$y = k \cdot \frac{xz}{w}$$
$$\frac{3}{2} = k \cdot \frac{2 \cdot 3}{4}$$
$$1 = k$$
$$y = \frac{xz}{w}$$

33.
$$y = k\frac{xz}{wp}$$
$$\frac{3}{28} = k\frac{3 \cdot 10}{7 \cdot 8} \quad \text{Substituting}$$
$$\frac{3}{28} = k \cdot \frac{30}{56}$$
$$\frac{3}{28} \cdot \frac{56}{30} = k$$
$$\frac{1}{5} = k$$

The equation of variation is $y = \frac{1}{5}\frac{xz}{wp}$, or $\frac{xz}{5wp}$.

34.
$$y = k \cdot \frac{xz}{w^2}$$
$$\frac{12}{5} = k \cdot \frac{16 \cdot 3}{5^2}$$
$$\frac{5}{4} = k$$
$$y = \frac{5}{4}\frac{xz}{w^2}, \text{ or } \frac{5xz}{4w^2}$$

35.
$$I = \frac{k}{d^2}$$
$$90 = \frac{k}{5^2} \quad \text{Substituting}$$
$$90 = \frac{k}{25}$$
$$2250 = k$$

The equation of variation is $I = \frac{2250}{d^2}$.

Substitute 40 for I and find d.
$$40 = \frac{2250}{d^2}$$
$$40d^2 = 2250$$
$$d^2 = 56.25$$
$$d = 7.5$$

The distance from 5 m to 7.5 m is $7.5 - 5$, or 2.5 m, so it is 2.5 m further to a point where the intensity is 40 W/m^2.

36.
$$D = kAv$$
$$222 = k \cdot 37.8 \cdot 40$$
$$\frac{37}{252} = k$$
$$D = \frac{37}{252}Av$$
$$430 = \frac{37}{252} \cdot 51v$$
$$v \approx 57.4 \text{ mph}$$

37.
$$d = kr^2$$
$$200 = k \cdot 60^2 \quad \text{Substituting}$$
$$200 = 3600k$$
$$\frac{200}{3600} = k$$
$$\frac{1}{18} = k$$

The equation of variation is $d = \frac{1}{18}r^2$.

Substitute 72 for d and find r.
$$72 = \frac{1}{18}r^2$$
$$1296 = r^2$$
$$36 = r$$

A car can travel 36 mph and still stop in 72 ft.

38.
$$W = \frac{k}{d^2}$$
$$220 = \frac{k}{(3978)^2}$$
$$3,481,386,480 = k$$
$$W = \frac{3,481,386,480}{d^2}$$
$$W = \frac{3,481,386,480}{(3978 + 200)^2}$$
$$W \approx 199 \text{ lb}$$

39. $E = \frac{kR}{I}$

We first find k.
$$1.77 = \frac{k \cdot 39}{198.1} \quad \text{Substituting}$$
$$1.77\left(\frac{198.1}{39}\right) = k \quad \text{Multiplying by } \frac{198.1}{39}$$
$$9 \approx k$$

The equation of variation is $E = \frac{9R}{I}$.

Substitute 1.77 for E and 220 for I and solve for R.
$$1.77 = \frac{9R}{220}$$
$$\frac{1.77(220)}{9} = R \quad \text{Multiplying by } \frac{220}{9}$$
$$43 \approx R$$

Clayton Kershaw would have given up about 43 earned runs if he had pitched 220 innings.

40.
$$V = \frac{kT}{P}$$
$$231 = \frac{k \cdot 42}{20}$$
$$110 = k$$
$$V = \frac{110T}{P}$$
$$V = \frac{110 \cdot 30}{15}$$
$$V = 220 \text{ cm}^3$$

41. parallel

42. zero

43. relative minimum

44. odd function

45. inverse variation

46. a) $7xy = 14$

$$y = \frac{2}{x}$$
Inversely

b) $x - 2y = 12$

$$y = \frac{x}{2} - 6$$
Neither

c) $-2x + y = 0$

$$y = 2x$$
Directly

d) $x = \frac{3}{4}y$

$$y = \frac{4}{3}x$$
Directly

e) $\frac{x}{y} = 2$

$$y = \frac{1}{2}x$$
Directly

47. Let V represent the volume and p represent the price of a jar of peanut butter.

$V = kp$ — V varies directly as p.

$\pi\left(\frac{3}{2}\right)^2(5) = k(2.89)$ — Substituting

$3.89\pi = k$ — Variation constant

$V = 3.89\pi p$ — Equation of variation

$\pi(1.625)^2(5.5) = 3.89\pi p$ — Substituting

$3.73 \approx p$

If cost is directly proportional to volume, the larger jar should cost \$3.73.

Now let W represent the weight and p represent the price of a jar of peanut butter.

$W = kp$

$18 = k(2.89)$ — Substituting

$6.23 \approx k$ — Variation constant

$W = 6.23p$ — Equation of variation

$28 = 6.23p$ — Substituting

$4.49 \approx p$

If cost is directly proportional to weight, the larger jar should cost \$4.49. (Answers may vary slightly due to rounding differences.)

48. $Q = \frac{kp^2}{q^3}$

Q varies directly as the square of p and inversely as the cube of q.

49. We are told $A = kd^2$, and we know $A = \pi r^2$ so we have:

$kd^2 = \pi r^2$

$kd^2 = \pi\left(\frac{d}{2}\right)^2 \qquad r = \frac{d}{2}$

$kd^2 = \frac{\pi d^2}{4}$

$k = \frac{\pi}{4}$ — Variation constant

Chapter 2 Review Exercises

1. This statement is true by the definition of the greatest integer function.

2. Thes statement is false. See Example 2(b) in Section 2.3 in the text.

3. The graph of $y = f(x - d)$ is the graph of $y = f(x)$ shifted right d units, so the statement is true.

4. The graph of $y = -f(x)$ is the reflection of the graph of $y = f(x)$ across the x-axis, so the statement is true.

5. a) For x-values from -4 to -2, the y-values increase from 1 to 4. Thus the function is increasing on the interval $(-4, -2)$.

b) For x-values from 2 to 5, the y-values decrease from 4 to 3. Thus the function is decreasing on the interval $(2, 5)$.

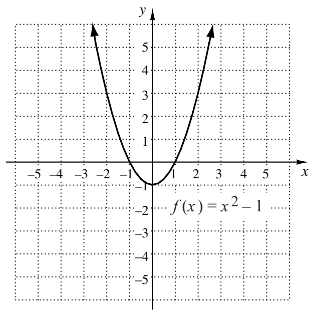
c) For x-values from -2 to 2, y is 4. Thus the function is constant on the interval $(-2, 2)$.

6. a) For x-values from -1 to 0, the y-values increase from 3 to 4. Also, for x-values from 2 to ∞, the y-values increase from 0 to ∞. Thus the function is increasing on the intervals $(-1, 0)$, and $(2, \infty)$.

b) For x-values from 0 to 2, the y-values decrease from 4 to 0. Thus, the function is decreasing on the interval $(0, 2)$.

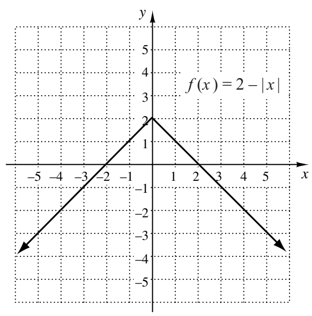
c) For x-values from $-\infty$ to -1, y is 3. Thus the function is constant on the interval $(-\infty, -1)$.

7.



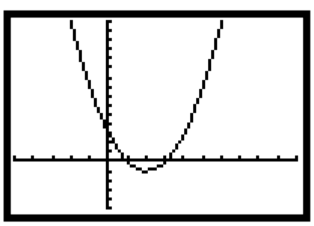
The function is increasing on $(0, \infty)$ and decreasing on $(-\infty, 0)$. We estimate that the minimum value is -1 at $x = 0$. There are no maxima.

8.



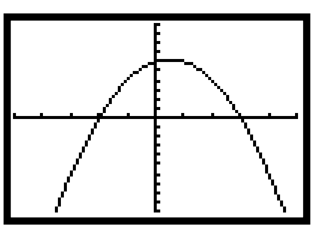
The function is increasing on $(-\infty, 0)$ and decreasing on $(0, \infty)$. We estimate that the maximum value is 2 at $x = 0$. There are no minima.

9.



We find that the function is increasing on $(2, \infty)$ and decreasing on $(-\infty, 2)$. The relative minimum is -1 at $x = 2$. There are no maxima.

10.



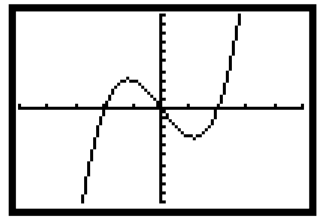
Increasing: $(-\infty, 0.5)$

Decreasing: $(0.5, \infty)$

Relative maximum: 6.25 at $x = 0.5$

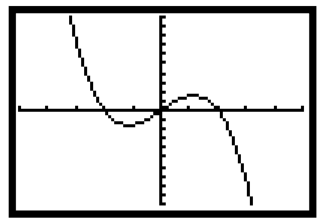
Relative minima: none

11.



We find that the function is increasing on $(-\infty, -1.155)$ and on $(1.155, \infty)$ and decreasing on $(-1.155, 1.155)$. The relative maximum is 3.079 at $x = -1.155$ and the relative minimum is -3.079 at $x = 1.155$.

12.



We find that the function is increasing on $(-1.155, 1.155)$ and decreasing on $(-\infty, -1.155)$ and on $(1.155, \infty)$. The relative maximum is 1.540 at $x = 1.155$ and the relative minimum is -1.540 at $x = -1.155$.

13. If two sides of the patio are each x feet, then the remaining side will be $(48 - 2x)$ ft. We use the formula Area = length × width.

$$A(x) = x(48 - 2x), \text{ or } 48x - 2x^2$$

14. The length of the rectangle is $2x$. The width is the second coordinate of the point (x, y) on the circle. The circle has center $(0, 0)$ and radius 2, so its equation is $x^2 + y^2 = 4$ and $y = \sqrt{4 - x^2}$. Thus the area of the rectangle is given by $A(x) = 2x\sqrt{4 - x^2}$.

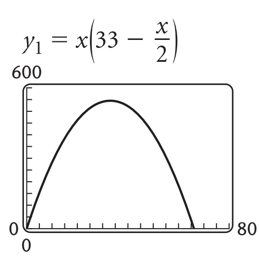
15. a) If the length of the side parallel to the garage is x feet long, then the length of each of the other two sides is $\dfrac{66 - x}{2}$, or $33 - \dfrac{x}{2}$. We use the formula Area = length × width.

$$A(x) = x\left(33 - \frac{x}{2}\right), \text{ or}$$

$$A(x) = 33x - \frac{x^2}{2}$$

b) The length of the side parallel to the garage must be positive and less than 66 ft, so the domain of the function is $\{x | 0 < x < 66\}$, or $(0, 66)$.

c)



d) By observing the graph or using the MAXIMUM feature, we see that the maximum value of the function occurs when $x = 33$. When $x = 33$, then $33 - \frac{x}{2} = 33 - \frac{33}{2} = 33 - 16.5 = 16.5$. Thus the dimensions that yield the maximum area are 33 ft by 16.5 ft.

16. a) Let h = the height of the box. Since the volume is 108 in^3, we have:

$$108 = x \cdot x \cdot h$$
$$108 = x^2 h$$
$$\frac{108}{x^2} = h$$

Now find the surface area.

$$S = x^2 + 4 \cdot x \cdot h$$
$$S(x) = x^2 + 4 \cdot x \cdot \frac{108}{x^2}$$
$$S(x) = x^2 + \frac{432}{x}$$

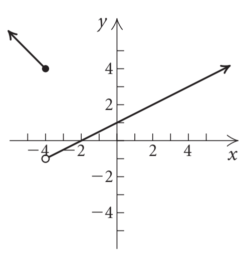
b) x must be positive, so the domain is $(0, \infty)$.

c) From the graph, we see that the minimum value of the function occurs when $x = 6$ in. For this value of x,

$$h = \frac{108}{x^2} = \frac{108}{6^2} = \frac{108}{36} = 3 \text{ in.}$$

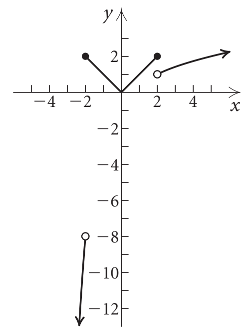
17. $f(x) = \begin{cases} -x, & \text{for } x \le -4, \\ \frac{1}{2}x + 1, & \text{for } x > -4 \end{cases}$

We create the graph in two parts. Graph $f(x) = -x$ for inputs less than or equal to -4. Then graph $f(x) = \frac{1}{2}x + 1$ for inputs greater than -4.



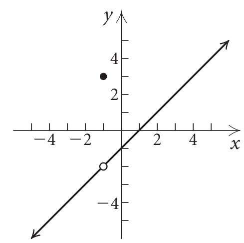
18. $f(x) = \begin{cases} x^3, & \text{for } x < -2, \\ |x|, & \text{for } -2 \le x \le 2, \\ \sqrt{x-1}, & \text{for } x > 2 \end{cases}$

We create the graph in three parts. Graph $f(x) = x^3$ for inputs less than -2. Then graph $f(x) = |x|$ for inputs greater than or equal to -2 and less than or equal to 2. Finally graph $f(x) = \sqrt{x-1}$ for inputs greater than 2.

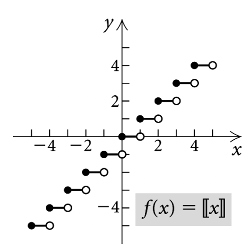


19. $f(x) = \begin{cases} \frac{x^2 - 1}{x + 1}, & \text{for } x \ne -1, \\ 3, & \text{for } x = -1 \end{cases}$

We create the graph in two parts. Graph $f(x) = \frac{x^2 - 1}{x + 1}$ for all inputs except -1. Then graph $f(x) = 3$ for $x = -1$.



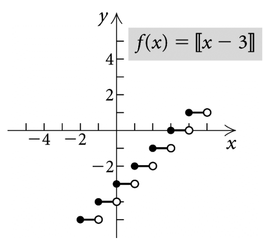
20. $f(x) = [[x]]$. See Example 9 on page 166 of the text.



21. $f(x) = [[x - 3]]$

This function could be defined by a piecewise function with an infinite number of statements.

$$f(x) = \begin{cases} \cdot \\ \cdot \\ \cdot \\ -4, & \text{for } -1 \le x < 0, \\ -3, & \text{for } 0 \le x < 1, \\ -2, & \text{for } 1 \le x < 2, \\ -1, & \text{for } 2 \le x < 3, \\ \cdot \\ \cdot \\ \cdot \end{cases}$$



22. $f(x) = \begin{cases} x^3, & \text{for } x < -2, \\ |x|, & \text{for } -2 \le x \le 2, \\ \sqrt{x-1}, & \text{for } x > 2 \end{cases}$

Since -1 is in the interval $[-2, 2]$, $f(-1) = |-1| = 1$.

Since $5 > 2$, $f(5) = \sqrt{5-1} = \sqrt{4} = 2$.

Since -2 is in the interval $[-2, 2]$, $f(-2) = |-2| = 2$.

Since $-3 < -2$, $f(-3) = (-3)^3 = -27$.

23. $f(x) = \begin{cases} \dfrac{x^2-1}{x+1}, & \text{for } x \neq -1, \\ 3, & \text{for } x = -1 \end{cases}$

Since $-2 \neq -1$, $f(-2) = \dfrac{(-2)^2-1}{-2+1} = \dfrac{4-1}{-1} = \dfrac{3}{-1} = -3$.

Since $x = -1$, we have $f(-1) = 3$.

Since $0 \neq -1$, $f(0) = \dfrac{0^2-1}{0+1} = \dfrac{-1}{1} = -1$.

Since $4 \neq -1$, $f(4) = \dfrac{4^2-1}{4+1} = \dfrac{16-1}{5} = \dfrac{15}{5} = 3$.

24.
$$\begin{aligned}(f-g)(6) &= f(6) - g(6) \\ &= \sqrt{6-2} - (6^2 - 1) \\ &= \sqrt{4} - (36-1) \\ &= 2 - 35 \\ &= -33\end{aligned}$$

25.
$$\begin{aligned}(fg)(2) &= f(2) \cdot g(2) \\ &= \sqrt{2-2} \cdot (2^2 - 1) \\ &= 0 \cdot (4-1) \\ &= 0\end{aligned}$$

26.
$$\begin{aligned}(f+g)(-1) &= f(-1) + g(-1) \\ &= \sqrt{-1-2} + ((-1)^2 - 1) \\ &= \sqrt{-3} + (1-1)\end{aligned}$$

Since $\sqrt{-3}$ is not a real number, $(f+g)(-1)$ does not exist.

27. $f(x) = \dfrac{4}{x^2}$, $g(x) = 3 - 2x$

a) Division by zero is undefined, so the domain of f is $\{x|x \neq 0\}$, or $(-\infty, 0) \cup (0, \infty)$. The domain of g is the set of all real numbers, or $(-\infty, \infty)$.

The domain of $f+g$, $f-g$ and fg is $\{x|x \neq 0\}$, or $(-\infty, 0) \cup (0, \infty)$. Since $g\left(\dfrac{3}{2}\right) = 0$, the domain of f/g is $\left\{x \middle| x \neq 0 \textit{ and } x \neq \dfrac{3}{2}\right\}$, or $(-\infty, 0) \cup \left(0, \dfrac{3}{2}\right) \cup \left(\dfrac{3}{2}, \infty\right)$.

b) $(f+g)(x) = \left(\dfrac{4}{x^2}\right) + (3-2x) = \dfrac{4}{x^2} + 3 - 2x$

$(f-g)(x) = \left(\dfrac{4}{x^2}\right) - (3-2x) = \dfrac{4}{x^2} - 3 + 2x$

$(fg)(x) = \left(\dfrac{4}{x^2}\right)(3-2x) = \dfrac{12}{x^2} - \dfrac{8}{x}$

$(f/g)(x) = \dfrac{\left(\dfrac{4}{x^2}\right)}{(3-2x)} = \dfrac{4}{x^2(3-2x)}$

28. a) The domain of f, g, $f+g$, $f-g$, and fg is all real numbers, or $(-\infty, \infty)$. Since $g\left(\dfrac{1}{2}\right) = 0$, the domain of f/g is $\left\{x \middle| x \neq \dfrac{1}{2}\right\}$, or $\left(-\infty, \dfrac{1}{2}\right) \cup \left(\dfrac{1}{2}, \infty\right)$.

b) $(f+g)(x) = (3x^2 + 4x) + (2x - 1) = 3x^2 + 6x - 1$

$(f-g)(x) = (3x^2 + 4x) - (2x - 1) = 3x^2 + 2x + 1$

$(fg)(x) = (3x^2 + 4x)(2x - 1) = 6x^3 + 5x^2 - 4x$

$(f/g)(x) = \dfrac{3x^2 + 4x}{2x - 1}$

29.
$$\begin{aligned}P(x) &= R(x) - C(x) \\ &= (120x - 0.5x^2) - (15x + 6) \\ &= 120x - 0.5x^2 - 15x - 6 \\ &= -0.5x^2 + 105x - 6\end{aligned}$$

30. $f(x) = 2x + 7$

$\dfrac{f(x+h) - f(x)}{h} = \dfrac{2(x+h) + 7 - (2x+7)}{h} =$

$\dfrac{2x + 2h + 7 - 2x - 7}{h} = \dfrac{2h}{h} = 2$

31. $f(x) = 3 - x^2$

$f(x+h) = 3 - (x+h)^2 = 3 - (x^2 + 2xh + h^2) =$
$3 - x^2 - 2xh - h^2$

$$\begin{aligned}\frac{f(x+h) - f(x)}{h} &= \frac{3 - x^2 - 2xh - h^2 - (3 - x^2)}{h} \\ &= \frac{3 - x^2 - 2xh - h^2 - 3 + x^2}{h} \\ &= \frac{-2xh - h^2}{h} = \frac{h(-2x - h)}{h} \\ &= \frac{h}{h} \cdot \frac{-2x - h}{1} = -2x - h\end{aligned}$$

32. $f(x) = \dfrac{4}{x}$

$\dfrac{f(x+h) - f(x)}{h} = \dfrac{\dfrac{4}{x+h} - \dfrac{4}{x}}{h} = \dfrac{\dfrac{4}{x+h} \cdot \dfrac{x}{x} - \dfrac{4}{x} \cdot \dfrac{x+h}{x+h}}{h} =$

$\dfrac{\dfrac{4x - 4(x+h)}{x(x+h)}}{h} = \dfrac{\dfrac{4x - 4x - 4h}{x(x+h)}}{h} = \dfrac{\dfrac{-4h}{x(x+h)}}{h} =$

$\dfrac{-4h}{x(x+h)} \cdot \dfrac{1}{h} = \dfrac{-4 \cdot \cancel{h}}{x(x+h) \cdot \cancel{h}} = \dfrac{-4}{x(x+h)}$, or $-\dfrac{4}{x(x+h)}$

33. $(f \circ g)(1) = f(g(1)) = f(1^2 + 4) = f(1+4) = f(5) =$
$2 \cdot 5 - 1 = 10 - 1 = 9$

34. $(g \circ f)(1) = g(f(1)) = g(2 \cdot 1 - 1) = g(2-1) = g(1) = 1^2 + 4 = 1 + 4 = 5$

35. $(h \circ f)(-2) = h(f(-2)) = h(2(-2) - 1) = h(-4-1) = h(-5) = 3 - (-5)^3 = 3 - (-125) = 3 + 125 = 128$

36. $(g \circ h)(3) = g(h(3)) = g(3 - 3^3) = g(3 - 27) = g(-24) = (-24)^2 + 4 = 576 + 4 = 580$

37. $(f \circ h)(-1) = f(h(-1)) = f(3 - (-1)^3) = f(3 - (-1)) = f(3+1) = f(4) = 2 \cdot 4 - 1 = 8 - 1 = 7$

38. $(h \circ g)(2) = h(g(2)) = h(2^2 + 4) = h(4+4)$
$h(8) = 3 - 8^3 = 3 - 512 = -509$

39. $(f \circ f)(x) = f(f(x)) = f(2x-1) = 2(2x-1) - 1 = 4x - 2 - 1 = 4x - 3$

40. $(h \circ h)(x) = h(h(x)) = h(3 - x^3) = 3 - (3 - x^3)^3 = 3 - (27 - 27x^3 + 9x^6 - x^9) = 3 - 27 + 27x^3 - 9x^6 + x^9 = -24 + 27x^3 - 9x^6 + x^9$

41. a) $f \circ g(x) = f(3 - 2x) = \dfrac{4}{(3-2x)^2}$

$g \circ f(x) = g\left(\dfrac{4}{x^2}\right) = 3 - 2\left(\dfrac{4}{x^2}\right) = 3 - \dfrac{8}{x^2}$

b) The domain of f is $\{x | x \neq 0\}$ and the domain of g is the set of all real numbers. To find the domain of $f \circ g$, we find the values of x for which $g(x) = 0$. Since $3 - 2x = 0$ when $x = \dfrac{3}{2}$, the domain of $f \circ g$ is $\left\{x \middle| x \neq \dfrac{3}{2}\right\}$, or $\left(-\infty, \dfrac{3}{2}\right) \cup \left(\dfrac{3}{2}, \infty\right)$. Since any real number can be an input for g, the domain of $g \circ f$ is the same as the domain of f, $\{x | x \neq 0\}$, or $(-\infty, 0) \cup (0, \infty)$.

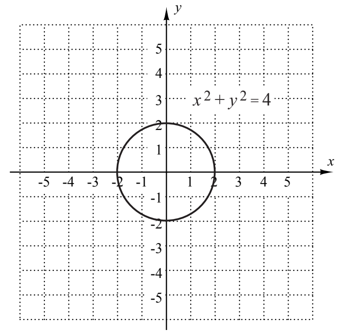
42. a)
$$\begin{aligned} f \circ g(x) &= f(2x-1) \\ &= 3(2x-1)^2 + 4(2x-1) \\ &= 3(4x^2 - 4x + 1) + 4(2x-1) \\ &= 12x^2 - 12x + 3 + 8x - 4 \\ &= 12x^2 - 4x - 1 \end{aligned}$$
$$\begin{aligned} (g \circ f)(x) &= g(3x^2 + 4x) \\ &= 2(3x^2 + 4x) - 1 \\ &= 6x^2 + 8x - 1 \end{aligned}$$

b) Domain of f = domain of g = all real numbers, so domain of $f \circ g$ = domain of $g \circ f$ = all real numbers, or $(-\infty, \infty)$.

43. $f(x) = \sqrt{x}$, $g(x) = 5x + 2$. Answers may vary.

44. $f(x) = 4x^2 + 9$, $g(x) = 5x - 1$. Answers may vary.

45. $x^2 + y^2 = 4$



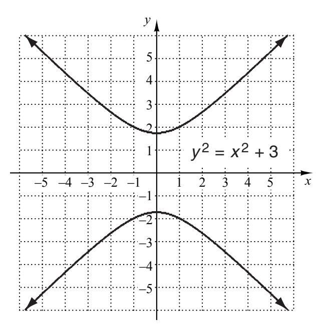
The graph is symmetric with respect to the x-axis, the y-axis, and the origin.

Replace y with $-y$ to test algebraically for symmetry with respect to the x-axis.
$$\begin{aligned} x^2 + (-y)^2 &= 4 \\ x^2 + y^2 &= 4 \end{aligned}$$
The resulting equation is equivalent to the original equation, so the graph is symmetric with respect to the x-axis.

Replace x with $-x$ to test algebraically for symmetry with respect to the y-axis.
$$\begin{aligned} (-x)^2 + y^2 &= 4 \\ x^2 + y^2 &= 4 \end{aligned}$$
The resulting equation is equivalent to the original equation, so the graph is symmetric with respect to the y-axis.

Replace x and $-x$ and y with $-y$ to test for symmetry with respect to the origin.
$$\begin{aligned} (-x)^2 + (-y)^2 &= 4 \\ x^2 + y^2 &= 4 \end{aligned}$$
The resulting equation is equivalent to the original equation, so the graph is symmetric with respect to the origin.

46. $y^2 = x^2 + 3$



The graph is symmetric with respect to the x-axis, the y-axis, and the origin.

Replace y with $-y$ to test algebraically for symmetry with respect to the x-axis.
$$\begin{aligned} (-y)^2 &= x^2 + 3 \\ y^2 &= x^2 + 3 \end{aligned}$$
The resulting equation is equivalent to the original equation, so the graph is symmetric with respect to the x-axis.

Replace x with $-x$ to test algebraically for symmetry with respect to the y-axis.

$$y^2 = (-x)^2 + 3$$
$$y^2 = x^2 + 3$$

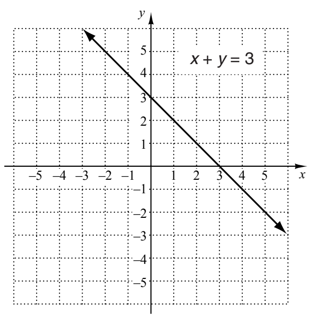
The resulting equation is equivalent to the original equation, so the graph is symmetric with respect to the y-axis.

Replace x and $-x$ and y with $-y$ to test for symmetry with respect to the origin.

$$(-y)^2 = (-x)^2 + 3$$
$$y^2 = x^2 + 3$$

The resulting equation is equivalent to the original equation, so the graph is symmetric with respect to the origin.

47. $x + y = 3$



The graph is not symmetric with respect to the x-axis, the y-axis, or the origin.

Replace y with $-y$ to test algebraically for symmetry with respect to the x-axis.

$$x - y = 3$$

The resulting equation is not equivalent to the original equation, so the graph is not symmetric with respect to the x-axis.

Replace x with $-x$ to test algebraically for symmetry with respect to the y-axis.

$$-x + y = 3$$

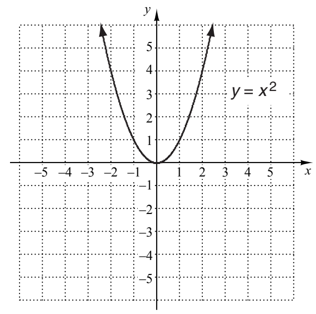
The resulting equation is not equivalent to the original equation, so the graph is not symmetric with respect to the y-axis.

Replace x and $-x$ and y with $-y$ to test for symmetry with respect to the origin.

$$-x - y = 3$$
$$x + y = -3$$

The resulting equation is not equivalent to the original equation, so the graph is not symmetric with respect to the origin.

48. $y = x^2$



The graph is symmetric with respect to the y-axis. It is not symmetric with respect to the x-axis or the origin.

Replace y with $-y$ to test algebraically for symmetry with respect to the x-axis.

$$-y = x^2$$
$$y = -x^2$$

The resulting equation is not equivalent to the original equation, so the graph is not symmetric with respect to the x-axis.

Replace x with $-x$ to test algebraically for symmetry with respect to the y-axis.

$$y = (-x)^2$$
$$y = x^2$$

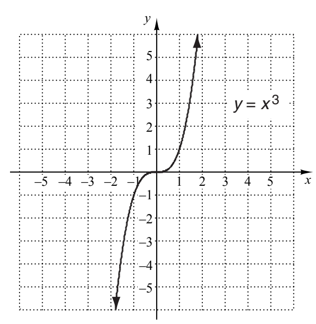
The resulting equation is equivalent to the original equation, so the graph is symmetric with respect to the y-axis.

Replace x and $-x$ and y with $-y$ to test for symmetry with respect to the origin.

$$-y = (-x)^2$$
$$-y = x^2$$
$$y = -x^2$$

The resulting equation is not equivalent to the original equation, so the graph is not symmetric with respect to the origin.

49. $y = x^3$



The graph is symmetric with respect to the origin. It is not symmetric with respect to the x-axis or the y-axis.

Replace y with $-y$ to test algebraically for symmetry with respect to the x-axis.

$$-y = x^3$$
$$y = -x^3$$

The resulting equation is not equivalent to the original equation, so the graph is not symmetric with respect to the x-axis.

Replace x with $-x$ to test algebraically for symmetry with respect to the y-axis.

$$y = (-x)^3$$
$$y = -x^3$$

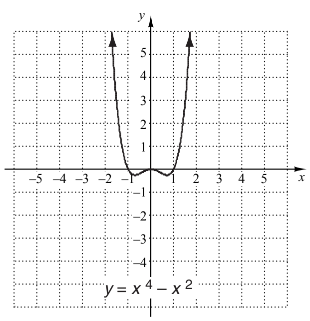
The resulting equation is not equivalent to the original equation, so the graph is not symmetric with respect to the y-axis.

Replace x and $-x$ and y with $-y$ to test for symmetry with respect to the origin.

$$-y = (-x)^3$$
$$-y = -x^3$$
$$y = x^3$$

The resulting equation is equivalent to the original equation, so the graph is symmetric with respect to the origin.

50. $y = x^4 - x^2$



The graph is symmetric with respect to the y-axis. It is not symmetric with respect to the x-axis or the origin.

Replace y with $-y$ to test algebraically for symmetry with respect to the x-axis.

$$-y = x^4 - x^2$$
$$y = -x^4 + x^2$$

The resulting equation is not equivalent to the original equation, so the graph is not symmetric with respect to the x-axis.

Replace x with $-x$ to test algebraically for symmetry with respect to the y-axis.

$$y = (-x)^4 - (-x)^2$$
$$y = x^4 - x^2$$

The resulting equation is equivalent to the original equation, so the graph is symmetric with respect to the y-axis.

Replace x and $-x$ and y with $-y$ to test for symmetry with respect to the origin.

$$-y = (-x)^4 - (-x)^2$$
$$-y = x^4 - x^2$$
$$y = -x^4 + x^2$$

The resulting equation is not equivalent to the original equation, so the graph is not symmetric with respect to the origin.

51. The graph is symmetric with respect to the y-axis, so the function is even.

52. The graph is symmetric with respect to the y-axis, so the function is even.

53. The graph is symmetric with respect to the origin, so the function is odd.

54. The graph is symmetric with respect to the y-axis, so the function is even.

55. $f(x) = 9 - x^2$

$f(-x) = 9 - (-x^2) = 9 - x^2$

$f(x) = f(-x)$, so f is even.

56. $f(x) = x^3 - 2x + 4$

$f(-x) = (-x)^3 - 2(-x) + 4 = -x^3 + 2x + 4$

$f(x) \neq f(-x)$, so f is not even.

$-f(x) = -(x^3 - 2x + 4) = -x^3 + 2x - 4$

$f(-x) \neq -f(x)$, so f is not odd.

Thus, $f(x) = x^3 - 2x + 4$ is neither even or odd.

57. $f(x) = x^7 - x^5$

$f(-x) = (-x)^7 - (-x)^5 = -x^7 + x^5$

$f(x) \neq f(-x)$, so f is not even.

$-f(x) = -(x^7 - x^5) = -x^7 + x^5$

$f(-x) = -f(x)$, so f is odd.

58. $f(x) = |x|$

$f(-x) = |-x| = |x|$

$f(x) = f(-x)$, so f is even.

59. $f(x) = \sqrt{16 - x^2}$

$f(-x) = \sqrt{16 - (-x^2)} = \sqrt{16 - x^2}$

$f(x) = f(-x)$, so f is even.

60. $f(x) = \dfrac{10x}{x^2 + 1}$

$f(-x) = \dfrac{10(-x)}{(-x)^2 + 1} = -\dfrac{10x}{x^2 + 1}$

$f(x) \neq f(-x)$, so $f(x)$ is not even.

$-f(x) = -\dfrac{10x}{x^2 + 1}$

$f(-x) = -f(x)$, so f is odd.

61. Shape: $g(x) = x^2$

Shift $g(x)$ left 3 units: $f(x) = g(x + 3) = (x + 3)^2$

62. Shape: $t(x) = \sqrt{x}$

Turn $t(x)$ upside down (that is, reflect it across the x-axis): $h(x) = -t(x) = -\sqrt{x}$.

Shift $h(x)$ right 3 units: $g(x) = h(x - 3) = -\sqrt{x - 3}$.

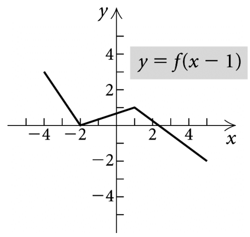
Shift $g(x)$ up 4 units: $f(x) = g(x) + 4 = -\sqrt{x - 3} + 4$.

63. Shape: $h(x) = |x|$

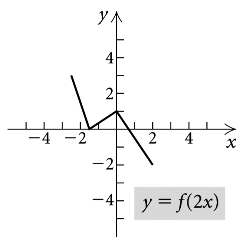
Stretch $h(x)$ vertically by a factor of 2 (that is, multiply each function value by 2): $g(x) = 2h(x) = 2|x|$.

Shift $g(x)$ right 3 units: $f(x) = g(x - 3) = 2|x - 3|$.

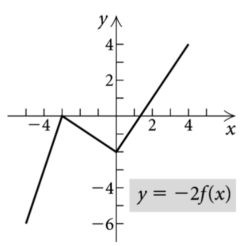
64. The graph is shifted right 1 unit so each x-coordinate is increased by 1. We plot and connect $(-4, 3)$, $(-2, 0)$, $(1, 1)$ and $(5, -2)$.



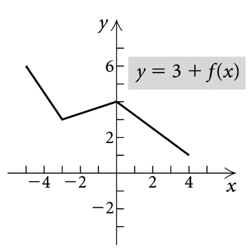
65. The graph is shrunk horizontally by a factor of 2. That is, each x-coordinate is divided by 2. We plot and connect $\left(-\frac{5}{2}, 3\right)$, $\left(-\frac{3}{2}, 0\right)$, $(0, 1)$ and $(2, -2)$.



66. Each y-coordinate is multiplied by -2. We plot and connect $(-5, -6)$, $(-3, 0)$, $(0, -2)$ and $(4, 4)$.



67. Each y-coordinate is increased by 3. We plot and connect $(-5, 6)$, $(-3, 3)$, $(0, 4)$ and $(4, 1)$.



68.
$$\begin{aligned} y &= kx \\ 100 &= 25x \\ 4 &= x \end{aligned}$$

Equation of variation: $y = 4x$

69.
$$\begin{aligned} y &= kx \\ 6 &= 9x \\ \frac{2}{3} &= x \quad \text{Variation constant} \end{aligned}$$

Equation of variation: $y = \frac{2}{3}x$

70.
$$\begin{aligned} y &= \frac{k}{x} \\ 100 &= \frac{k}{25} \\ 2500 &= k \end{aligned}$$

Equation of variation: $y = \frac{2500}{x}$

71.
$$\begin{aligned} y &= \frac{k}{x} \\ 6 &= \frac{k}{9} \\ 54 &= k \quad \text{Variation constant} \end{aligned}$$

Equation of variation: $y = \frac{54}{x}$

72.
$$\begin{aligned} y &= \frac{k}{x^2} \\ 12 &= \frac{k}{2^2} \\ 48 &= k \\ y &= \frac{48}{x^2} \end{aligned}$$

73.
$$\begin{aligned} y &= \frac{kxz^2}{w} \\ 2 &= \frac{k(16)\left(\frac{1}{2}\right)^2}{0.2} \\ 2 &= \frac{k(16)\left(\frac{1}{4}\right)}{0.2} \\ 2 &= \frac{4k}{0.2} \\ 2 &= 20k \\ \frac{1}{10} &= k \\ y &= \frac{1}{10}\frac{xz^2}{w} \end{aligned}$$

74.
$$\begin{aligned} t &= \frac{k}{r} \\ 35 &= \frac{k}{800} \\ 28{,}000 &= k \\ t &= \frac{28{,}000}{r} \\ t &= \frac{28{,}000}{1400} \\ t &= 20 \text{ min} \end{aligned}$$

75.
$$N = ka$$
$$87 = k \cdot 29$$
$$3 = k$$
$$N = 3a$$
$$N = 3 \cdot 25$$
$$N = 75$$

Sam's score would have been 75 if he had answered 25 questions correctly.

76.
$$P = kC^2$$
$$180 = k \cdot 6^2$$
$$5 = k \quad \text{Variation constant}$$
$$P = 5C^2 \quad \text{Variation equation}$$
$$P = 5 \cdot 10^2$$
$$P = 500 \text{ watts}$$

77. $f(x) = x + 1$, $g(x) = \sqrt{x}$

The domain of f is $(-\infty, \infty)$, and the domain of g is $[0, \infty)$. To find the domain of $(g \circ f)(x)$, we find the values of x for which $f(x) \geq 0$.
$$x + 1 \geq 0$$
$$x \geq -1$$

Thus the domain of $(g \circ f)(x)$ is $[-1, \infty)$. Answer A is correct.

78. For $b > 0$, the graph of $y = f(x)+b$ is the graph of $y = f(x)$ shifted up b units. Answer C is correct.

79. The graph of $g(x) = -\frac{1}{2}f(x) + 1$ is the graph of $y = f(x)$ shrunk vertically by a factor of $\frac{1}{2}$, then reflected across the x-axis, and shifted up 1 unit. The correct graph is B.

80. Let $f(x)$ and $g(x)$ be odd functions. Then by definition, $f(-x) = -f(x)$, or $f(x) = -f(-x)$, and $g(-x) = -g(x)$, or $g(x) = -g(-x)$. Thus $(f + g)(x) = f(x) + g(x) = -f(-x) + [-g(-x)] = -[f(-x) + g(-x)] = -(f + g)(-x)$ and $f + g$ is odd.

81. Reflect the graph of $y = f(x)$ across the x-axis and then across the y-axis.

82. $f(x) = 4x^3 - 2x + 7$

a) $f(x) + 2 = 4x^3 - 2x + 7 + 2 = 4x^3 - 2x + 9$

b)
$$\begin{aligned} f(x + 2) &= 4(x + 2)^3 - 2(x + 2) + 7 \\ &= 4(x^3 + 6x^2 + 12x + 8) - 2(x + 2) + 7 \\ &= 4x^3 + 24x^2 + 48x + 32 - 2x - 4 + 7 \\ &= 4x^3 + 24x^2 + 46x + 35 \end{aligned}$$

c)
$$\begin{aligned} f(x) + f(2) &= 4x^3 - 2x + 7 + 4 \cdot 2^3 - 2 \cdot 2 + 7 \\ &= 4x^3 - 2x + 7 + 32 - 4 + 7 \\ &= 4x^3 - 2x + 42 \end{aligned}$$

$f(x) + 2$ adds 2 to each function value; $f(x + 2)$ adds 2 to each input before the function value is found; $f(x) + f(2)$ adds the output for 2 to the output for x.

83. In the graph of $y = f(cx)$, the constant c stretches or shrinks the graph of $y = f(x)$ horizontally. The constant c in $y = cf(x)$ stretches or shrinks the graph of $y = f(x)$ vertically. For $y = f(cx)$, the x-coordinates of $y = f(x)$ are divided by c; for $y = cf(x)$, the y-coordinates of $y = f(x)$ are multiplied by c.

84. The graph of $f(x) = 0$ is symmetric with respect to the x-axis, the y-axis, and the origin. This function is both even and odd.

85. If all of the exponents are even numbers, then $f(x)$ is an even function. If $a_0 = 0$ and all of the exponents are odd numbers, then $f(x)$ is an odd function.

86. Let $y(x) = kx^2$. Then $y(2x) = k(2x)^2 = k \cdot 4x^2 = 4 \cdot kx^2 = 4 \cdot y(x)$. Thus, doubling x causes y to be quadrupled.

87. Let $y = k_1x$ and $x = \frac{k_2}{z}$. Then $y = k_1 \cdot \frac{k_2}{z}$, or $y = \frac{k_1k_2}{z}$, so y varies inversely as z.

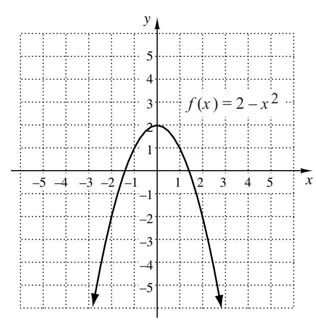
Chapter 2 Test

1. a) For x-values from -5 to -2, the y-values increase from -4 to 3. Thus the function is increasing on the interval $(-5, -2)$.

b) For x-values from 2 to 5, the y-values decrease from 2 to -1. Thus the function is decreasing on the interval $(2, 5)$.

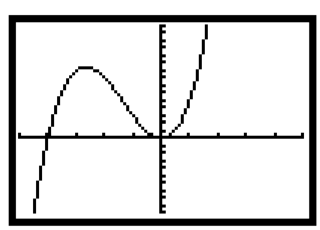
c) For x-values from -2 to 2, y is 2. Thus the function is constant on the interval $(-2, 2)$.

2.



The function is increasing on $(-\infty, 0)$ and decreasing on $(0, \infty)$. The relative maximum is 2 at $x = 0$. There are no minima.

3.

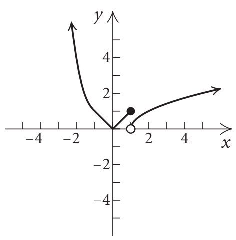


We find that the function is increasing on $(-\infty, -2.667)$ and on $(0, \infty)$ and decreasing on $(-2.667, 0)$. The relative maximum is 9.481 at -2.667 and the relative minimum is 0 at $x = 0$.

4. If b = the length of the base, in inches, then the height = $4b-6$. We use the formula for the area of a triangle, $A=\frac{1}{2}bh$.

$$A(b)=\frac{1}{2}b(4b-6),\text{ or}$$
$$A(b)=2b^2-3b$$

5. $f(x)=\begin{cases} x^2, & \text{for } x<-1, \\ |x|, & \text{for } -1\le x\le 1, \\ \sqrt{x-1}, & \text{for } x>1 \end{cases}$



6. Since $-1\le -\frac{7}{8}\le 1$, $f\left(-\frac{7}{8}\right)=\left|-\frac{7}{8}\right|=\frac{7}{8}$.

Since $5>1$, $f(5)=\sqrt{5-1}=\sqrt{4}=2$.

Since $-4<-1$, $f(-4)=(-4)^2=16$.

7. $(f+g)(-6)=f(-6)+g(-6)=$
$(-6)^2-4(-6)+3+\sqrt{3-(-6)}=$
$36+24+3+\sqrt{3+6}=63+\sqrt{9}=63+3=66$

8. $(f-g)(-1)=f(-1)-g(-1)=$
$(-1)^2-4(-1)+3-\sqrt{3-(-1)}=$
$1+4+3-\sqrt{3+1}=8-\sqrt{4}=8-2=6$

9. $(fg)(2)=f(2)\cdot g(2)=(2^2-4\cdot 2+3)(\sqrt{3-2})=$
$(4-8+3)(\sqrt{1})=-1\cdot 1=-1$

10. $(f/g)(1)=\dfrac{f(1)}{g(1)}=\dfrac{1^2-4\cdot 1+3}{\sqrt{3-1}}=\dfrac{1-4+3}{\sqrt{2}}=\dfrac{0}{\sqrt{2}}=0$

11. Any real number can be an input for $f(x)=x^2$, so the domain is the set of real numbers, or $(-\infty,\infty)$.

12. The domain of $g(x)=\sqrt{x-3}$ is the set of real numbers for which $x-3\ge 0$, or $x\ge 3$. Thus the domain is $\{x|x\ge 3\}$, or $[3,\infty)$.

13. The domain of $f+g$ is the intersection of the domains of f and g. This is $\{x|x\ge 3\}$, or $[3,\infty)$.

14. The domain of $f-g$ is the intersection of the domains of f and g. This is $\{x|x\ge 3\}$, or $[3,\infty)$.

15. The domain of fg is the intersection of the domains of f and g. This is $\{x|x\ge 3\}$, or $[3,\infty)$.

16. The domain of f/g is the intersection of the domains of f and g, excluding those x-values for which $g(x)=0$. Since $x-3=0$ when $x=3$, the domain is $(3,\infty)$.

17. $(f+g)(x)=f(x)+g(x)=x^2+\sqrt{x-3}$

18. $(f-g)(x)=f(x)-g(x)=x^2-\sqrt{x-3}$

19. $(fg)(x)=f(x)\cdot g(x)=x^2\sqrt{x-3}$

20. $(f/g)(x)=\dfrac{f(x)}{g(x)}=\dfrac{x^2}{\sqrt{x-3}}$

21. $f(x)=\frac{1}{2}x+4$

$$f(x+h)=\frac{1}{2}(x+h)+4=\frac{1}{2}x+\frac{1}{2}h+4$$

$$\frac{f(x+h)-f(x)}{h}=\frac{\frac{1}{2}x+\frac{1}{2}h+4-\left(\frac{1}{2}x+4\right)}{h}$$
$$=\frac{\frac{1}{2}x+\frac{1}{2}h+4-\frac{1}{2}x-4}{h}$$
$$=\frac{\frac{1}{2}h}{h}=\frac{1}{2}h\cdot\frac{1}{h}=\frac{1}{2}\cdot\frac{h}{h}=\frac{1}{2}$$

22. $f(x)=2x^2-x+3$
$f(x+h)=2(x+h)^2-(x+h)+3=2(x^2+2xh+h^2)-x-h+3=$
$2x^2+4xh+2h^2-x-h+3$

$$\frac{f(x+h)-f(x)}{h}=\frac{2x^2+4xh+2h^2-x-h+3-(2x^2-x+3)}{h}$$
$$=\frac{2x^2+4xh+2h^2-x-h+3-2x^2+x-3}{h}$$
$$=\frac{4xh+2h^2-h}{h}$$
$$=\frac{\not{h}(4x+2h-1)}{\not{h}}$$
$$=4x+2h-1$$

23. $(g\circ h)(2)=g(h(2))=g(3\cdot 2^2+2\cdot 2+4)=$
$g(3\cdot 4+4+4)=g(12+4+4)=g(20)=4\cdot 20+3=$
$80+3=83$

24. $(f\circ g)(-1)=f(g(-1))=f(4(-1)+3)=f(-4+3)=$
$f(-1)=(-1)^2-1=1-1=0$

25. $(h\circ f)(1)=h(f(1))=h(1^2-1)=h(1-1)=h(0)=$
$3\cdot 0^2+2\cdot 0+4=0+0+4=4$

26. $(g\circ g)(x)=g(g(x))=g(4x+3)=4(4x+3)+3=$
$16x+12+3=16x+15$

27. $(f\circ g)(x)=f(g(x))=f(x^2+1)=\sqrt{x^2+1-5}=$
$\sqrt{x^2-4}$
$(g\circ f)(x)=g(f(x))=g(\sqrt{x-5})=(\sqrt{x-5})^2+1=$
$x-5+1=x-4$

28. The inputs for $f(x)$ must be such that $x-5\ge 0$, or $x\ge 5$. Then for $(f\circ g)(x)$ we must have $g(x)\ge 5$, or $x^2+1\ge 5$, or $x^2\ge 4$. Then the domain of $(f\circ g)(x)$ is $(-\infty,-2]\cup[2,\infty)$.

Since we can substitute any real number for x in g, the domain of $(g\circ f)(x)$ is the same as the domain of $f(x)$, $[5,\infty)$.

29. Answers may vary. $f(x) = x^4$, $g(x) = 2x - 7$

30. $y = x^4 - 2x^2$

Replace y with $-y$ to test for symmetry with respect to the x-axis.

$$-y = x^4 - 2x^2$$
$$y = -x^4 + 2x^2$$

The resulting equation is not equivalent to the original equation, so the graph is not symmetric with respect to the x-axis.

Replace x with $-x$ to test for symmetry with respect to the y-axis.

$$y = (-x)^4 - 2(-x)^2$$
$$y = x^4 - 2x^2$$

The resulting equation is equivalent to the original equation, so the graph is symmetric with respect to the y-axis.

Replace x with $-x$ and y with $-y$ to test for symmetry with respect to the origin.

$$-y = (-x)^4 - 2(-x)^2$$
$$-y = x^4 - 2x^2$$
$$y = -x^4 + 2x^2$$

The resulting equation is not equivalent to the original equation, so the graph is not symmetric with respect to the origin.

31. $f(x) = \dfrac{2x}{x^2 + 1}$

$f(-x) = \dfrac{2(-x)}{(-x)^2 + 1} = -\dfrac{2x}{x^2 + 1}$

$f(x) \neq f(-x)$, so f is not even.

$-f(x) = -\dfrac{2x}{x^2 + 1}$

$f(-x) = -f(x)$, so f is odd.

32. Shape: $h(x) = x^2$

Shift $h(x)$ right 2 units: $g(x) = h(x - 2) = (x - 2)^2$

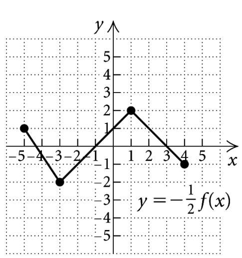
Shift $g(x)$ down 1 unit: $f(x) = (x - 2)^2 - 1$

33. Shape: $h(x) = x^2$

Shift $h(x)$ left 2 units: $g(x) = h(x + 2) = (x + 2)^2$

Shift $g(x)$ down 3 units: $f(x) = (x + 2)^2 - 3$

34. Each y-coordinate is multiplied by $-\frac{1}{2}$. We plot and connect $(-5, 1)$, $(-3, -2)$, $(1, 2)$ and $(4, -1)$.



35.
$$y = \frac{k}{x}$$
$$5 = \frac{k}{6}$$
$$30 = k \quad \text{Variation constant}$$

Equation of variation: $y = \dfrac{30}{x}$

36.
$$y = kx$$
$$60 = k \cdot 12$$
$$5 = k \quad \text{Variation constant}$$

Equation of variation: $y = 5x$

37.
$$y = \frac{kxz^2}{w}$$
$$100 = \frac{k(0.1)(10)^2}{5}$$
$$100 = 2k$$
$$50 = k \quad \text{Variation constant}$$
$$y = \frac{50xz^2}{w} \quad \text{Equation of variation}$$

38.
$$d = kr^2$$
$$200 = k \cdot 60^2$$
$$\frac{1}{18} = k \quad \text{Variation constant}$$
$$d = \frac{1}{18}r^2 \quad \text{Equation of variation}$$
$$d = \frac{1}{18} \cdot 30^2$$
$$d = 50 \text{ ft}$$

39. The graph of $g(x) = 2f(x) - 1$ is the graph of $y = f(x)$ stretched vertically by a factor of 2 and shifted down 1 unit. The correct graph is C.

40. Each x-coordinate on the graph of $y = f(x)$ is divided by 3 on the graph of $y = f(3x)$. Thus the point $\left(\dfrac{-3}{3}, 1\right)$, or $(-1, 1)$ is on the graph of $f(3x)$.