# INSTRUCTOR'S SOLUTIONS MANUAL

JAMES LAPP

# A SURVEY OF MATHEMATICS WITH APPLICATIONS

**ELEVENTH EDITION** 

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# Table of Contents

r 1: Critical Thinking Skills	
Inductive and Deductive Reasoning  Estimation Techniques  Problem-Solving Procedures  Review Exercises  Chapter Test	1
r 2: Sets	
Set Concepts Subsets Venn Diagrams and Set Operations Venn Diagrams with Three Sets and Verification of Equality of Sets Applications of Sets Infinite Sets Review Exercises Chapter Test	
r 3: Logic	
Statements and Logical Connectives  Truth Tables for Negation, Conjunction, and Disjunction  Truth Tables for the Conditional and Biconditional  Equivalent Statements  Symbolic Arguments  Euler Diagrams and Syllogistic Arguments  Switching Circuits  Review Exercises  Chapter Test	
r 4: Systems of Numeration	
Additive, Multiplicative, and Ciphered Systems of Numeration  Place-Value or Positional-Value Numeration Systems  Other Bases  Perform Computations in Other Bases  Early Computational Methods  Review Exercises  Chapter Test	9101011
r 5: Number Theory and the Real Number System	
Number Theory The Integers The Rational Numbers The Irrational Numbers Real Numbers and Their Properties Rules of Exponents and Scientific Notation Arithmetic and Geometric Sequences Fibonacci Sequence	
Review Exercises	14
	Inductive and Deductive Reasoning.  Estimation Techniques. Problem-Solving Procedures.  Review Exercises. Chapter Test.  **7 2: Set**  Set Concepts. Subsets. Venn Diagrams and Set Operations. Venn Diagrams and Set Operations. Venn Diagrams with Three Sets and Verification of Equality of Sets. Applications of Sets. Infinite Sets. Review Exercises. Chapter Test.  **7 3: Logic**  Statements and Logical Connectives. Truth Tables for Negation, Conjunction, and Disjunction. Truth Tables for the Conditional and Biconditional. Equivalent Statements. Symbolic Arguments. Switching Circuits. Review Exercises. Chapter Test.  **7 4: Systems of Numeration**  Additive, Multiplicative, and Ciphered Systems of Numeration. Place-Value or Positional-Value Numeration Systems. Other Bases. Early Computations in Other Bases. Early Computations Methods. Review Exercises. Chapter Test.  **7 5: Number Theory and the Real Number System** Number Theory. The Integers. The Rational Numbers. The Rational Numbers. Real Numbers and Their Properties. Rules of Exponents and Scientific Notation. Arithmetic and Geometric Sequence.

	r 6: Algebra, Graphs, and Functions	
	Order of Operations and Solving Equations	
	Formulas	
	Applications of Algebra	
	Variation	
	Solving Linear Inequalities	
	Graphing Linear Equations	
	Solving Systems of Linear Equations	
	Linear Inequalities in Two Variables and Systems of Linear Inequalities	
	Solving Quadratic Equations by Using Factoring and by Using the Quadratic Formula	
6.10	: Functions and Their Graphs	
	Review Exercises	
	Chapter Test	227
Chapte	r 7: The Metric System	
	Basic Terms and Conversions Within the Metric System	
7.2:	Length, Area, and Volume	232
	Mass and Temperature	
7.4:	Dimensional Analysis and Conversions to and from the Metric System	
	Review Exercises	
	Chapter Test	243
Chapte	r 8: Geometry	
8.1:	Points, Lines, Planes, and Angles	245
	Polygons	
8.3:	Perimeter and Area	252
8.4:	Volume and Surface Area	256
8.5:	Transformational Geometry, Symmetry, and Tessellations	260
8.6:	Topology	265
8.7:	Non-Euclidean Geometry and Fractal Geometry	266
	Review Exercises	267
	Chapter Test	270
Chapte	r 9: Mathematical Systems	
9.1:	Groups	273
9.2:	Finite Mathematical Systems	274
9.3:	Modular Arithmetic	277
9.4:	Matrices	282
	Review Exercises	288
	Chapter Test	290
Chapte	r 10: Consumer Mathematics	
	Percent	
	Personal Loans and Simple Interest	
	Compound Interest	
	Installment Buying	
	Buying a House with a Mortgage	
10.6:	Ordinary Annuities, Sinking Funds, and Retirement Investments	
	Review Exercises	
	Chapter Test	322

Chapte	er 11: Probability	
	Empirical and Theoretical Probabilities	325
11.2:	Odds	328
11.3:	Expected Value (Expectation)	332
11.4:	Tree Diagrams	336
11.5:	OR and AND Problems	340
11.6:	Conditional Probability	344
11.7:	The Fundamental Counting Principle and Permutations	347
	Combinations	
11.9:	Solving Probability Problems by Using Combinations	351
11.10	: Binomial Probability Formula	353
	Review Exercises	355
	Chapter Test	359
Chapte	er 12: Statistics	
12.1:	Sampling Techniques and Misuses of Statistics	361
	Frequency Distributions and Statistical Graphs	
	Measures of Central Tendency and Position	
	Measures of Dispersion	
12.5:	The Normal Curve	379
12.6:	Linear Correlation and Regression	385
	Review Exercises	395
	Chapter Test	400
Chapte	er 13: Graph Theory	
13.1:	Graphs, Paths, and Circuits	403
	Euler Paths and Euler Circuits	
	Hamilton Paths and Hamilton Circuits	
13.4:	Trees	411
	Review Exercises	415
	Chapter Test	417
Chapte	er 14: Voting and Apportionment	
14.1:	Voting Methods	419
14.2:	Flaws of the Voting Methods	423
14.3:	Apportionment Methods	428
	Flaws of the Apportionment Methods	
	Review Exercises	438
	Chapter Test	445

## **Chapter One: Critical Thinking Skills**

### **Section 1.1: Inductive and Deductive Reasoning**

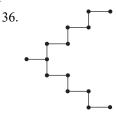
- 1. Natural
- 2. Divisible
- 3. Counterexample
- 4. Hypothesis
- 5. Inductive
- 6. Deductive
- 7. Deductive
- 8. Inductive
- 9.  $5 \times 3 = 15$
- 10.  $19 \times 10 = 190$
- 12.  $100,000 = 10^5$
- 23. 21, 28, 36 (15+6=21, 21+7=28, 28+8=36)
- 24. 16, 22, 29 (Add 1, then 2, then 3, ...)
- 25. 34, 55, 89 (Each number in the sequence is the sum of the previous two numbers.)
- 26. 76, 123, 199 (Each number in the sequence is the sum of the previous two numbers.)
- 27. There are three letters in the pattern.  $39 \times 3 = 117$ , so, the 117th entry is the second R in the pattern. Therefore, the 118th entry is Y.
- 28. a) Answers will vary.
  - b) The sum of the digits is 9.
  - c) The sum of the digits in the product when a one- or two-digit number is multiplied by 9 is divisible by 9.
- 29. a) 36, 49, 64
  - b) Square the numbers 6, 7, 8, 9 and 10.
  - c)  $8 \times 8 = 64$ ,  $9 \times 9 = 81$ ; 72 is not a square number since it falls between the two square numbers 64 and 81.
- 30. a) 28 and 36
  - b) To find the 7th triangular number, add 7 to the 6th triangular number.
    - To find the 8th triangular number, add 8 to the 7th triangular number.
    - To find the 9th triangular number, add 9 to the 8th triangular number.
    - To find the 10th triangular number, add 10 to the 9th triangular number.
    - To find the 11th triangular number, add 11 to the 10th triangular number.
  - c) 36+9=45, 45+10=55, 55+11=66; 66+12=78; 72 is not a triangular number since it falls between the consecutive triangular numbers 66 and 78.
- 31. Blue: 1, 5, 7, 10, 12; Purple: 2, 4, 6, 9, 11; Yellow: 3, 8
- 32. a) 19 (Each new row has two additional triangles.)
  - b) 1+3+5+7+9+11+13+15+17+19=100
- 33. a) \$3700
  - b) We are using observation of specific cases to make a prediction.

- 13.  $\langle \circ \rangle$

- 17. 9, 11, 13 (Add 2 to previous number.)
- 18. 16, 19, 22 (Add 3 to the previous number.)
- 19. 5, -5, 5 (Alternate 5 and -5.)
- 20. 16, -32, 64 (Multiply previous number by -2.)
- 21.  $\frac{1}{5}$ ,  $\frac{1}{6}$ ,  $\frac{1}{7}$  (Increase denominator value by 1.)
- 22.  $\frac{1}{9}$ ,  $\frac{1}{11}$ ,  $\frac{1}{13}$  (Increase denominator value by 2.)

- 34. a) \$410
  - b) We are using observation of specific cases to make a prediction.





- 37. a) You should obtain the original number.
  - b) You should obtain the original number.
  - c) Conjecture: The result is always the original number.

d) 
$$n \to 3n \to 3n + 6 \to \frac{3n+6}{3} = \frac{3n}{3} + \frac{6}{3} = n+2 \to n+2-2 = n$$

- 38. a) You should obtain twice the original number.
  - b) You should obtain twice the original number.
  - c) Conjecture: The result is always twice the original number.

d) 
$$n \to 4n \to 4n + 6 \to \frac{4n+6}{2} = \frac{4n}{2} + \frac{6}{2} = 2n+3 \to 2n+3-3 = 2n$$

- 39. a) You should obtain the number 5.
  - b) You should obtain the number 5.
  - c) Conjecture: The result is always the number 5.

d) 
$$n \to n+1 \to n+(n+1) = 2n+1 \to 2n+1+9 = 2n+10 \to \frac{2n+10}{2} = \frac{2n}{2} + \frac{10}{2} = n+5 \to n+5-n=5$$

- 40. a) You should obtain the number 0.
  - b) You should obtain the number 0.
  - c) Conjecture: The result is always the number 0.

d) 
$$n \to n+10 \to \frac{n+10}{5} \to 5\left(\frac{n+10}{5}\right) = n+10 \to n+10-10 = n \to n-n = 0$$

- 41.  $3 \times 5 = 15$  is one counterexample.
- 42. 10+11+12=33, which is not a three digit number.
- 43. Two is a counting number. The sum of 2 and 3 is 5. Five divided by two is 5/2, which is not an even number.
- 44. 900 is a three-digit number. The product of 900 and 900 is 810,000, which is not a five-digit number.
- 45. One and two are counting numbers. The difference of 1 and 2 is 1-2=-1, which is not a counting number.
- 46. The sum of the odd numbers 1 and 5 is 6, which is not divisible by 4.
- 47. a) The sum of the measures of the interior angles should be 180°.
  - b) Yes, the sum of the measures of the interior angles should be 180°.
  - c) Conjecture: The sum of the measures of the interior angles of a triangle is 180°.
- 48. a) The sum of the measures of the interior angles should be 360°.
  - b) Yes, the sum of the measures of the interior angles should be 360°.
  - c) Conjecture: The sum of the measures of the interior angles of a quadrilateral is 360°.
- 49. Inductive reasoning: a general conclusion is obtained from observation of specific cases.

51. 129; The numbers in the positions of each inner  $4 \times 4$  square are determined as follows.

$$a$$
  $b$   $c$   $a+b+c$ 

- 52. 1881, 8008, 8118 (They look the same when looked at in a mirror.)
- 53. c

#### **Section 1.2: Estimation Techniques**

Answers in this section will vary depending on how you round your numbers. The answers may differ from the answers in the back of the textbook. However, your answers should be something near the answers given. All answers are approximate.)

- 1. Estimation
- 2. Equal
- 3. 26.9 + 67.3 + 219 + 143.3 $\approx 30 + 67 + 220 + 143 = 460$
- 4. 86 + 47.2 + 289.8 + 532.4 + 12.8 $\approx 90 + 50 + 290 + 530 + 10 = 970$
- 5.  $197,500 \div 4.063 \approx 200,000 \div 4.000 = 50,000$
- 6.  $\frac{405}{0.049} \approx \frac{400}{0.05} = 8000$
- 7.  $1776 \times 0.0098 \approx 1800 \times 0.01 = 18$
- 8.  $0.63 \times 1523 \approx 0.6 \times 1500 = 900$
- 9.  $23.97 7.05 \approx 24 7 = 17$

- 10.  $400.15 297.87 \approx 400 300 = 100$
- 11.  $22\% \times 9116 \approx 20\% \times 9000 = 0.20 \times 9000$ = 1800
- 12. 11% of  $8221 \approx 10\%$  of  $8000 = 0.10 \times 8000$ = 800
- 13.  $\frac{\$91.35}{3} \approx \frac{\$90}{3} = \$30$
- 14.  $\frac{\$210}{8} \approx \frac{\$200}{8} = \$25$
- 15. 12 months  $\times$  \$47  $\approx$  12  $\times$  \$50 = \$600
- 16. 19 gallons  $\times$  \$3.11  $\approx$  20  $\times$  \$3 = \$60
- 17.  $\$7.99 + \$4.23 + \$16.82 + \$3.51 + \$20.12 \approx \$8 + \$4 + \$17 + \$4 + \$20 = \$53$
- 18.  $\$1.29 + \$6.86 + \$12.43 + \$25.62 + \$8.99 \approx \$1 + \$7 + \$12 + \$26 + \$9 = \$55$
- 19.  $951b + 1271b + 2101b \approx 100 + 100 + 200 = 4001b$
- 20.  $16.1 \text{ lb} + 7.9 \text{ lb} + 21.4 \text{ lb} + 11.6 \text{ lb} + 3.7 \text{ lb} \approx 16 + 8 + 21 + 12 + 4 = 61 \text{ lb}$
- 21. 15% of \$26.32  $\approx$  15% of \$26 = 0.15 \times \$26 = \$3.90

22. 
$$\frac{\$400}{\$23} \approx \frac{\$400}{\$25} = 16$$

- 23.  $\$595 + \$289 + \$120 + \$110 + 230 \approx \$600 + \$300 + \$100 + \$100 + \$200 = \$1300$
- 24. Team A:  $189 + 172 + 191 \approx 190 + 170 + 190 = 550$ Team B:  $183 + 229 + 167 \approx 180 + 230 + 170 = 580$ 580 - 550 = 30 lb
- 25.  $11 \times 8 \times \$1.50 \approx 10 \times 8 \times \$1.50 = 10 \times \$12 = \$120$
- 26. 9 min, 55 sec/mi × 26.2 mi ≈ 10 min × 26 mi = 260 min/mi;  $\frac{260 \text{ min}}{60 \text{ min}} \approx 4\frac{1}{3} \text{ hours}$
- 27.  $\frac{599 \text{ Mexican pesos}}{20.14 \text{ Mexican pesos / U.S. dollars}} \approx \frac{600}{20} = \$30$

28. 
$$\$973 + 6(\$61) + 6(\$97) + 6(\$200) \approx \$970 + 6(\$60) + 6(\$100) + 6(\$200)$$
  
=  $\$970 + \$360 + \$600 + \$1200$   
=  $\$3130$ 

- 29. 90 miles
- 30. 75 miles

31. a) 
$$23\%$$
 of  $700 \approx 25\%$  of  $700 = 0.25 \times 700 = 175$ 

b) 
$$12\%$$
 of  $700 \approx 10\%$  of  $700 = 0.10 \times 700 = 70$ 

c) 
$$21\% \text{ of } 700 \approx 20\% \text{ of } 700 = 0.20 \times 700 = 140$$

32. a) 
$$18\%$$
 of  $2987 \approx 18\%$  of  $3000 = 0.18 \times 3000 = 540$ 

b) 
$$42\%$$
 of  $2987 \approx 42\%$  of  $3000 = 0.42 \times 3000 = 1260$ 

c) 
$$7\% \text{ of } 2987 \approx 7\% \text{ of } 3000 = 0.07 \times 3000 = 210$$

- 33. a) 5 million
  - b) 98 million
  - c) 98 million 33 million = 65 million
  - d) 19 million + 79 million + 84 million + 65 million + 33 million = 280 million
- 34. a) 19%
  - b) 25%
  - c) 20% of 179 lb  $\approx$  20% of 180 = 0.2×180 = 36 lb
- 35. a) 85%
  - b) 68% 53% = 15%
  - c) 85% of 70 million acres = 59,500,000 acres
  - d) No, since we are not given the area of each state.

36. a) 
$$2(410) + 4(545) \approx 2(400) + 4(500) = 800 + 2000 = 2800$$
 calories

b) Running: 
$$4(920) \approx 4(900) = 3600$$
 calories

Walking: 
$$4(330) \approx 4(300) = 1200$$
 calories

Difference: 
$$3600 - 1200 = 2400$$
 calories

c) 
$$3(545) + 3(545) \approx 3(550) + 3(550) = 1650 + 1650 = 3300$$
 calories per week  $3300$  calories per week  $(52 \text{ weeks}) \approx 3000 \times 50 = 150,000$  calories

- 37. 20
- 38. 25
- 39. 120 bananas
- 40. 100 grapes
- 41. 150°
- 42. 315°
- 43. 10%

- 44. 25%
- 45. 9 square units
- 46. 12 square units
- 47. 160 feet
- 48. 4(60) = 240 in. or  $\frac{240}{12} \approx 20$  ft
- 49. 57. Answers will vary.

Cost of one fish oil tablet: 
$$\frac{\$47.99}{120} \approx \frac{\$48}{120} = \$0.40$$

Cost of one vitamin D tablet: 
$$\frac{$17.99}{180} \approx \frac{$18}{180} = $0.10$$

Cost of one multivitamin tablet: 
$$\frac{$19.99}{80} \approx \frac{$20}{80} = $0.25$$

Roberto's daily cost: 
$$5 \times \$0.05 + 4 \times \$0.40 + 1 \times \$0.10 + 1 \times \$0.25 = \$1.40$$

- 59. There are 118 ridges around the edge.
- 60. a) Answers will vary.
  - b) 11.6 days. There are 24.60.60 = 86,400 seconds in a day, and  $1,000,000/86,400 = 11.5\overline{740}$ .
- 62. Answers will vary.
- 63. Answers will vary.

#### **Section 1.3: Problem-Solving Procedures**

1. 
$$\frac{1 \text{ in.}}{12 \text{ mi}} = \frac{4.25 \text{ in.}}{x \text{ mi}}$$
  
 $x = 12(4.25)$   
 $x = 51 \text{ mi}$ 

2. 
$$\frac{1 \text{ in.}}{2.5 \text{ yd}} = \frac{22.4 \text{ in.}}{x \text{ yd}}$$
  
 $1x = 2.5(22.4)$   
 $x = 56 \text{ yd}$ 

3. 
$$\frac{3 \text{ ft}}{1.2 \text{ ft}} = \frac{x \text{ ft}}{15.36 \text{ ft}}$$
$$3(15.36) = 1.2x$$
$$\frac{46.03}{1.2} = \frac{1.2x}{1.2}$$
$$x = \frac{46.03}{1.2} = 38.4 \text{ ft}$$

4. 
$$\frac{1 \text{ bag}}{4000 \text{ ft}^2} = \frac{x \text{ bags}}{35,000 \text{ ft}^2}$$
$$4000x = 1(35,000)$$
$$\frac{4000x}{4000} = \frac{35,000}{4000}$$
$$x = \frac{35,000}{4000} = 8.75 \text{ bags}$$

5. 
$$4 \times \$113 = \$452$$
  
 $9.25\%$  of  $\$452 = 0.0925 \times \$452 = \$41.81$   
 $\$452 + \$41.81 = \$493.81$ 

7. a) Entertainment/Miscellaneous: 19.1% of  $$1950 = 0.191 \times $1950 = $372.45$ 

Food: 
$$12.7\%$$
 of  $$1950 = 0.127 \times $1950 = $247.65$ 

$$$372.45 - $247.65 = $124.80$$

b) Housing: 34.4% of  $$1950 = 0.344 \times $1950 = $670.80$ 

Transportation: 16% of  $$1950 = 0.16 \times $1750 = $312.00$ 

$$\$670.80 - \$312.00 = \$358.80$$

8. a) Business: 19% of  $2,895,000 = 0.19 \times 2,895,000 = 550,050$ 

Engineering: 5% of  $2,895,000 = 0.05 \times 2,895,000 = 144,750$ 

550,050-144,750 = 405,300 degrees

- 8. (continued)
  - b) Health Professions: 11% of 2,895,000 = 0.11×2,895,000 = 318,450 Psychology: 6% of 2,895,000 = 0.06×2,895,000 = 173,700 318,450-173,700 = 144,750 degrees
- 9. 43 rides × \$2.00 per ride = \$86.00; In order for the cost of rides with the \$84.50 MetroCard to be less than the cost of the rides without the CharlieCard, Marcelo would have to take 43 bus rides per month.

Note that 
$$\frac{\$84.50}{43 \text{ rides}} \approx \$1.97 \text{ per ride.}$$

10. 45 rides × \$2.75 per ride = \$123.75; In order for the cost of rides with the \$121.50 MetroCard to be less than the cost of the rides without the MetroCard, Karissa would have to take 45 subway rides per month.

Note that 
$$\frac{$121.50}{45 \text{ rides}} = $2.70 \text{ per ride}$$

11. \$250 + \$130(18) = \$250 + \$2340 = \$2590

Savings: 
$$$2590 - $2500 = $90$$

- 12. \$349.72 \$32.39(4) = \$349.72 129.56 = \$220.16
- 13. 15-year mortgage: \$887.63(12)(15) = \$159,773.40

30-year mortgage: 
$$$572.90(12)(30) = $206,244.00$$

14. Points needed for 80 average: 80(5) = 400 points

Wallace's points so far: 
$$79+93+91+68=331$$
 points

Grade needed on fifth exam: 
$$400 - 331 = 69$$

15. a) 
$$10 \times 10 \times 10 \times 10 = 10,000$$

b) 1 in 10,000

16. a) 
$$\frac{460}{25} = 18.4 \text{ min}$$

b) 
$$\frac{1550}{50} = 31 \text{ min}$$

c) 
$$\frac{1400}{35} = 40 \text{ min}$$

d) 
$$\frac{1550}{80} + \frac{2200}{80} = \frac{3750}{80} \approx 47 \text{ min}$$

- 17.  $38,687.0 \text{ mi} 38,451.4 \text{ mi} = 235.6 \text{ mi}; \frac{235.6 \text{ mi}}{12.6 \text{ gal}} \approx 18.7 \text{ mpg}$
- 18. a)  $40 \times \$8.50 \times 52 = \$17,680$ 
  - b) Each week he makes  $40 \times \$8.50 = \$340$ , it will take him  $\frac{\$1275}{\$340} = 3.75$  weeks.

19. By mail: 
$$(\$52.80 + \$5.60 + \$8.56) \times 4 = \$66.96 \times 4 = \$267.84$$

Tire store: 
$$$324 + 0.08 \times $324 = $324 + $25.92 = $349.92$$

Savings: 
$$$349.92 - $267.84 = $82.08$$

20. 
$$\$1750 - \$50(15) = \$1750 - \$750 = \$1000$$
 and  $\frac{\$1000}{\$40} = 25$  hours

- 21. 15,000 ft 3000 ft = 12,000 ft decrease in elevation. Temperature increases 2.4°F for every 1000 ft decrease in elevation.
  - $-6^{\circ}$  F + 28.8° F = 22.8° F and  $-6^{\circ}$  F + 28.8° F = 22.8° F; The precipitation at the airport will be snow.
- 22. a) \$620(0.12) = \$74.40
  - b) \$1200(0.22) = \$264
  - c) The store lost \$1200 \$1000 = \$200 on the purchase, so the store's profit is \$264 \$200 = \$64.
- 23. a) Since \$876 is less than  $10\% \times \$9525 = \$952.50$ , Shenile's tax was 10% of her taxable income. Her taxable income was  $\frac{$876}{0.10} = $8760$ .
  - b) Since \$2017.50 is greater than \$952.50 and less than \$4453.50, Logan's taxable income was in the 12% bracket. Since \$2017.50 - \$962.50 = \$1065 was 12% of his taxable income above \$9525, he made an additional  $\frac{\$1065}{0.12} = \$8875$ . His taxable income was \$9625 + \$8875 = \$18,400.
- 24. a) Since \$4541.50 is greater than \$4453.50 and less than \$14,089.50, Lewis' taxable income was in the 22% bracket. Since \$4541.50 - \$4453.50 = \$88 was 22% of his taxable income above \$38,700, he made an additional  $\frac{$88}{0.22}$  = \$400. His taxable income was \$38,700 + \$400 = \$39,100.
  - b) Since \$14,593.50.50 is greater than \$14,089.50 and less than \$32,089, Leticia's taxable income was in the 24% bracket. Since \$14,593.50 - \$14,089.50 = \$504 was 24% of his taxable income above \$82,500, she made an additional  $\frac{$504}{0.24}$  = \$2100. Her taxable income was \$82,500 + \$2100 = \$84,600.
- 25. a)  $1 \times 60 \times 24 \times 365 = 525,600$  oz and  $\frac{525,600}{128} = 4106.25$  gal
  - b)  $\frac{4106.25}{1000} \times \$11.20 = 4.10625 \times \$11.20 = \$45.99$
- 26. a)  $0.1 \times 60 \times 60 \times 24 \times 365 = 3.153,600 \text{ cm}^3$ 
  - b)  $\frac{\text{Volume of basin}}{\text{Daily water loss}} = \frac{(30 \text{ cm})(20 \text{ cm})(20 \text{ cm})}{0.1 \times 60 \times 60 \times 24} = \frac{12,000 \text{ cm}^3}{8640 \text{ cm}^3} \approx 1.4$ ; It would take about 1.4 days.
- 27. a)  $\frac{20,000}{20.8} \frac{20,000}{21.6} \approx 961.538 925.926 = 35.612 \approx 35.61 \text{ gal}$ 
  - b)  $35.61 \times \$3.00 = \$106.83$
  - c)  $140,000,000 \times 35.61 = 4,985,400,000$  gal
- 28. a) Parking:  $$30 \times 6 = $180$

Tax: 9.5% of  $$180 = 0.095 \times $180 = $17.10$ 

Total: \$180 + \$17.10 = \$197.10

b) Parking:  $$17.95 \times 6 = $107.70$ 

Tax: 8.75% of  $$107.70 = 0.0875 \times $107.70 \approx $9.42$ 

Total: \$107.70 + \$9.42 = \$117.12

c) She can save \$197.10 - \$117.12 = \$79.98.

29. Cost after 1 year: \$999 + 0.02(\$999) = \$999 + \$19.98 = \$1018.98

Cost after second year: \$1018.98 + 0.02(\$1018.98) = \$1018.98 + \$20.38 = \$1039.36

30. Value after first year: \$1000 + 0.10(\$1000) = \$1000 + \$100 = \$1100

Value after second year: \$1100 - 0.10(\$1100) = \$1100 - \$110 = \$990

\$990 is less than the initial investment of \$1000.

31. After paying the \$100 deductible, Yungchen must pay 20% of the cost of x-rays.

First x-ray: \$100 + 0.20(\$620) = \$100 + \$124 = \$224

Second x-ray: 0.20(\$980) = \$196

Total: \$224 + \$196 = \$420

32. \$3000 is the difference between one-fourth of the cost and one-fifth of the cost.

 $\frac{1}{4} - \frac{1}{5} = \frac{1}{20}$  and  $20 \times \$3000 = \$60,000$ 

33. a) water/milk: 3(1) = 3 cups

salt:  $3\left(\frac{1}{8}\right) = \frac{3}{8} \operatorname{tsp}$ 

Cream of wheat: 3(3) = 9 tbsp =  $\frac{9}{16}$  cup (because 16 tbsp = 1 cup)

b) water/milk:  $\frac{2+3\frac{3}{4}}{2} = \frac{\frac{23}{4}}{2} = \frac{23}{8} = 2\frac{7}{8}$  cups

salt:  $\frac{\frac{1}{4} + \frac{1}{2}}{2} = \frac{\frac{3}{4}}{2} = \frac{3}{8} \text{tsp}$ 

cream of wheat:  $\frac{\frac{1}{2} + \frac{3}{4}}{2} = \frac{\frac{5}{4}}{\frac{2}{2}} = \frac{5}{8}$  cups = 5 tbsp

c) water/milk:  $3\frac{3}{4} - 1 = \frac{15}{4} - \frac{4}{4} = \frac{11}{4} = 2\frac{3}{4}$  cups

salt:  $\frac{1}{2} - \frac{1}{8} = \frac{4}{8} - \frac{1}{8} = \frac{3}{8}$  tsp

cream of wheat:  $\frac{3}{4} - \frac{3}{16} = \frac{12}{16} - \frac{3}{16} = \frac{9}{16}$  cup = 9 tbsp

d) Differences exist in water/milk because the amount for 4 servings is not twice that for 2 servings.

Differences also exist in Cream of Wheat because  $\frac{1}{2}$  cup is not twice 3 tbsp.

water: 
$$1\frac{1}{3}(4) = \frac{4}{3}(4) = \frac{16}{3} = 5\frac{1}{3}$$
 cups

salt: 
$$\frac{1}{4}(4) = 1$$
 tsp

butter/margarine: 1(4) = 4 tsp

b) rice: 1(2) = 2 cups

water: 
$$2\frac{1}{4}(2) = \frac{9}{4}(2) = \frac{18}{4} = 4\frac{2}{4} = 4\frac{1}{2}$$
 cups

salt: 
$$\frac{1}{2}(2) = 1$$
 tsp

butter/margarine: 2(2) = 4 tsp

c) rice: 
$$\frac{1}{2} + 1\frac{1}{2} = \frac{1}{2} + \frac{3}{2} = \frac{4}{2} = 2$$
 cups

water: 
$$1\frac{1}{3} + 3\frac{1}{3} = \frac{4}{3} + \frac{10}{3} = \frac{14}{3} = 4\frac{2}{3}$$
 cups

salt: 
$$\frac{1}{4} + \frac{3}{4} = \frac{4}{4} = 1$$
 tsp

butter/margarine: 1 tsp + 1 tbsp = 1 tsp + 3 tsp = 4 tsp

d) rice: 3-1=2 cups

water: 
$$6 - 2\frac{1}{4} = \frac{24}{4} - \frac{9}{4} = \frac{15}{4} = 3\frac{3}{4}$$
 cups

salt: 
$$1\frac{1}{2} - \frac{1}{2} = 1$$
 tsp

butter/margarine: 2 tbsp - 2 tsp = 2(3tsp) - 2 tsp = 6 tsp - 2 tsp = 4 tsp

- e) Differences exist in water because the amount for 4 servings is not twice that for 2 servings.
- 35. a)

20 DVDs	12 DVDs	Total Number of DVDs
1	1	20 + 12 = 32
0	2	12 + 12 = 24

b) 
$$$240 + $180 = $420$$

36. Mark will win.

37. 
$$1 \text{ ft}^2 = 12 \text{ in.} \times 12 \text{ in.} = 144 \text{ square inches}$$

38. 
$$1 \text{ ft}^3 = 12 \text{ in.} \times 12 \text{ in.} \times 12 \text{ in.} = 1728 \text{ cubic inches}$$

39. Area of original rectangle: lw

Area of new rectangle: (2l)(2w) = 4(lw)

Thus, if the length and width of a rectangle are doubled, the area is 4 times as large.

- 40.  $20 \,\text{ft} \times 20 \,\text{ft} = 400 \,\text{ft}^2$ ;  $5 \,\text{ft} \times 5 \,\text{ft} = 25 \,\text{ft}^2$ ;  $\frac{400 \,\text{ft}^2}{25 \,\text{ft}^2} = 16 \,\text{squares}$
- 41. Volume of original cube: lwh

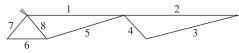
Volume of new cube: (2l)(2w)(2h) = 8(lwh)

Thus, if the length, width, and height of a cube are doubled, the volume is 8 times as large.

- 42. 11 ft is one-sixth of the pole, so the length is  $6 \times 11$  ft = 66 ft.
- 43.  $\frac{10 \text{ pieces}}{x} = \frac{1000 \text{ pieces}}{\$10}$ 1000x = 10(10) $\frac{1000x}{1000} = \frac{100}{1000}$  $x = \frac{100}{1000} = \$0.10 = 10 \text{ } \text{¢}$
- 44. Left side Right side  $\frac{1(-6) = -6}{2(-2) = -4}$   $\frac{2(-2) = -4}{(-6) = 6}$   $\frac{(6) = 6}{(-6 + (-4) = -10)}$

Place it at -1, so the left side would total -10 + (-1) = -1.

- 45. You have three ties, each a different color.
- 46. There are 10 palindromes: 2002, 2112, 2222, 2332, 2442, 2552, 2662, 2772, 2882, 2992.
- 47. a) refresh
  - b) workout
- 48. Answers will vary.



- 51. 8 6 16 18 10 2 4 14 12
- 52. 10 5 6 3 7 11 8 9 4

- 49. <u>4</u> <u>3 2</u> <u>5 1 6</u>
- 50. Eight pieces
- 53. 8+6+2+4=20; 3+7+5+1=16; 10+14+12+8=44

The sum of the four corner entries is 4 times the number in the center of the middle row.

- 54. 15,12,33; Multiply the number in the center of the middle row by 3.
- 55. 45,36,99; Multiply the number in the center of the middle row by 9.
- 56. 35-15=20 cubes
- 57.  $3 \times 2 \times 1 = 6$  ways
- 58. Each salesperson shakes hands with four people.

61. Other answers are possible.

5 4 4 5

2 1

2 1 3

2 3

5

59. Other answers are possible, but 1 and 8 must appear in the center.

	7	
3	1	4
5	8	6
	2	

60. The diagram shows the number of times each part is used.

-	8	
6	7	8
4		9
	6	

62. With umbrella policy:

Mustang reduced premium: \$1648 - \$90 = \$1558

Focus reduced premium: \$1530 - 0.12(\$1530) = \$1530 - \$183.60 = \$1346.40

Total for umbrella policy: \$1558 + \$1346.40 + \$450 = \$3354.40

Without umbrella policy: \$1648 + \$1530 = \$3178

Net amount for umbrella policy: \$3354.40 - \$3178 = \$176.40

63. Mark plays the drums.

64. 16+16+4+4+4=44

65. The areas of the colored regions are  $1\times1$ ,  $1\times1$ ,  $2\times2$ ,  $3\times3$ ,  $5\times5$ ,  $8\times8$ ,  $13\times13$ ,  $21\times21$ .

$$1+1+4+9+25+64+169+441=714$$
 square units

66. Let x be the amount Samantha had to start.

After first store: 
$$x - \frac{1}{2}x - 20 = \frac{1}{2}x - 20$$

After second store:  $\frac{1}{2} \left( \frac{1}{2} x - 20 \right) - 20 = \frac{1}{4} x - 30$ 

$$\frac{1}{4}x - 30 = 0$$
$$x = 120$$

The original amount was \$120.

67. Thomas would have opened the box labeled grapes and cherries. Because all the boxes are labeled incorrectly, whichever fruit he pulls from the box of grapes and cherries, will be the only fruit in that box. If he pulled a grape, he labeled the box grape. If he pulled a cherry, he labeled the box cherries. That left two boxes whose original labels were incorrect. Because all labels must be changed, there was only one way for Thomas to assign the two remaining labels.

#### **Review Exercises**

1. 23, 28, 33 (Add 5 to previous number.)

2. 16, 13, 10 (Subtract 3 from previous number)

3. 64, -128, 256 (Multiply previous number by -2.)

4. 25, 32, 40 (19+6=25, 25+7=32, 32+8=40)

5. 10, 4, -3 (Subtract 1, then 2, then 3, ...)

6.  $\frac{3}{8}$ ,  $\frac{3}{16}$ ,  $\frac{3}{32}$  (Multiply previous number by  $\frac{1}{2}$ .)

7.

9. c



- 10. a) The final number is twice the original number.
  - b) The final number is twice the original number.
  - c) Conjecture: The final number is twice the original number.

d) 
$$n \to 10n \to 10n + 5 \to \frac{10n+5}{5} = \frac{10n}{5} + \frac{5}{5} = 2n+1 \to 2n+1-1 = 2n$$

11. This process will always result in an answer of 3.

$$n \to n+5 \to 6(n+5) = 6n+30 \to 6n+30-12 = 6n+18$$
$$\to \frac{6n+18}{2} = \frac{6n}{2} + \frac{18}{2} = 3n+9 \to \frac{3n+9}{3} = \frac{3n}{3} + \frac{9}{3} = n+3 \to n+3 = 3$$

12.  $1^2 + 2^2 = 5$  and 5 is an odd number. Other answers are possible.

Answers for Exercises 13–25 will vary depending on how you round your numbers. The answers may differ from the answers in the back of the textbook. However, your answers should be something near the answers given. All answers are approximate.

13. 
$$205,123 \times 4002 \approx 200,000 \times 4000$$
  
= 800,000,000

14. 
$$215.9 + 128.752 + 3.6 + 861 + 792$$
  
 $\approx 200 + 100 + 0 + 900 + 800 = 2000$ 

15. 21% of 2095 
$$\approx$$
 20% of 2000 =  $0.20 \times 2000 = 400$ 

- 16. Answers will vary.
- 17. 48 bricks  $\times$ \$3.97  $\approx$  50  $\times$  4 = \$200

18. 8% of \$21,000 
$$\approx$$
 8% of 20,000 =  $0.08 \times 20,000 = $1600$ 

19. 
$$\frac{1.1 \text{ mi}}{22 \text{ min}} \approx \frac{1 \text{ mi}}{20 \text{ min}} = \frac{3 \text{ mi}}{60 \text{ min}} = 3 \text{ mph}$$

20. 
$$\$2.49 + \$0.79 + \$1.89 + \$0.10 + \$2.19 + \$6.75$$
  
 $\approx \$2 + \$1 + \$2 + \$0 + \$2 + \$7 = \$14.00$ 

21. 5 in. = 
$$\frac{20}{4}$$
 in. =  $20\left(\frac{1}{4}\right)$  in. =  $20(0.1)$  mi  
= 2 mi

- 22. Approximately  $70^{\circ}F 30^{\circ}F = 40^{\circ}F$
- 23. Approximately  $90^{\circ}F 80^{\circ}F = 10^{\circ}F$
- 24. 13 square units
- 25. Length: 1.75 in.,  $1.75(1.25) = 21.875 \approx 22$  ft Height: 0.625 in.,  $0.625(12.5) = 7.8125 \approx 8$  ft

28. Freemac: 
$$$15 \times 4 \times 2 = $120$$
  
Sylvan:  $$25 \times 4 \text{ hours} = $100$   
 $$120 - $100 = $20$   
Sylvan Rental is less expensive by \$20.

29. Cost per person with 5 people:  $\frac{$445}{5} = $89$ 

Cost per person with 5 people:  $\frac{\$510}{6} = \$85$ 

Savings: 
$$\$89 - \$85 = \$4$$

30. a) 
$$\frac{30 \text{ lb}}{2500 \text{ ft}^2} = \frac{x}{24,000 \text{ ft}^2}$$
$$x = \frac{30 \times 24,000}{2500}$$
$$x = 288 \text{ lb}$$

b) 
$$\frac{150 \text{ lb}}{30 \text{ lb/bag}} = 5 \text{ bags}; \ 5 \times 2500 = 12,500 \text{ ft}^2$$

31. 
$$10\%$$
 of  $$1030 = 0.10 \times $1030 = $103$   
 $7 \times $103 = $721$ 

Savings: 
$$$721 - $60 = $661$$

32. 
$$\frac{1.5 \text{ mg}}{10 \text{ lb}} = \frac{x \text{ mg}}{52 \text{ lb}}$$
$$10x = 52(1.5)$$
$$\frac{10x}{10} = \frac{78}{10}$$
$$x = 7.8 \text{ mg}$$

33. 
$$\$5500 - 0.30(\$5500) = \$5500 - \$1650 = \$3850$$
 take-home; 28% of  $\$3850 = 0.28 \times \$3850 = \$1078$ 

35. 
$$3 \text{ P.M.} - 4 \text{ hr} = 11 \text{ A.M.}$$
; July 26, 11:00 A.M.

36. a) 
$$\frac{65 \text{ mi}}{1 \text{ hr}} \times \frac{1.6 \text{ km}}{\text{mi}} = \frac{104 \text{ km}}{1 \text{ hr}} \approx 104 \text{ km/hr}$$

b) 
$$\frac{90 \text{ km}}{1 \text{ hr}} \times \frac{1 \text{ mi}}{1.6 \text{ km}} = \frac{90 \text{ mi}}{1.6 \text{ km}} \approx 56.25 \text{ mi/hr}$$

37. Each figure has an additional two dots. To get the hundredth figure, 97 more figures must be drawn, 97(2) = 194 dots added to the third figure. Thus, 194 + 7 = 201.

- 40. 59 min 59 sec; Since it doubles every second, the jar was half full 1 second earlier than 1 hour.
- 41. 6
- 42. Nothing. Each friend paid \$9 for a total of \$27; \$25 to the hotel, \$2 to the clerk. \$25 for the room + \$3 for each friend + \$2 for the clerk = \$30
- 43. Let x = the score on the fifth exam

$$\frac{93+88+81+86+x}{5} = 80$$

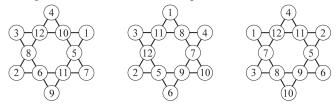
$$\frac{348+x}{5} = 80,$$

$$348+x = 400,$$

$$x = 52$$

- 44. Yes; 3 quarters and 4 dimes, or 1 half dollar, 1 quarter and 4 dimes, or 1 quarter and 9 dimes.
- 45.  $6 \text{ cm} \times 6 \text{ cm} \times 6 \text{ cm} = 216 \text{ cm}^3$
- 46. Place six coins in each pan with one coin off to the side. If it balances, the heavier coin is the one on the side. If the pan does not balance, take the six coins on the heavier side and split them into two groups of three. Select the three heavier coins and weigh two coins. If the pan balances, it is the third coin. If the pan does not balance, you can identify the heavier coin.
- 47. 1+500=501, 2+499=501, ...; There are 250 such pairs and 250(501)=125,250.
- 48. 4 green weigh the same as 8 blue, 2 yellow weigh the same as 5 blue, 2 white weigh the same as 3 blue, 8+5+3=16 blue

- 49. There are 90 such numbers: 101, 111, 121, 131, 141, 151, 161, 171, 181, 191, 202,h ..., 292, 303, ..., 393, 404, ..., 494, 505, ..., 595, 606, ..., 696, 707, ..., 797, 808, ..., 898, 919, ..., 999.
- 50. The fifth figure will be an octagon with sides of equal length. Inside the octagon will be a seven-sided figure with sides of equal length. The figure will have one antenna.
- 51. 61 orange tiles will be required. The sixth figure will have 6 rows of 6 tiles and 5 rows of 5 tiles  $(6 \times 6 + 5 \times 5 = 36 + 25 = 61)$ .
- 52. Some possible answers are given below. There are other possibilities.



- 53. a) 2
  - b) There are 3 choices for the first spot. Once that person is standing, there are 2 choices for the second spot and 1 for the third. Thus,  $3 \times 2 \times 1 = 6$ .
  - c)  $4 \times 3 \times 2 \times 1 = 24$
  - d)  $5 \times 4 \times 3 \times 2 \times 1 = 120$
  - e)  $n(n-1)(n-2)\cdots 1$ , (or n!), where n= the number of people in line.

#### **Chapter Test**

- 1. 26, 32, 38 (Add 6 to previous number.)
- 2.  $\frac{1}{5}$ ,  $\frac{1}{6}$ ,  $\frac{1}{7}$  (Add 1 to the denominator of the previous number.)
- 3. a) The result is the original number plus 1.
  - b) The result is the original number plus 1.
  - c) Conjecture: The result will always be the original number plus 1.

d) 
$$n \to 5n \to 5n + 10 \to \frac{5n+10}{5} = \frac{5n}{5} + \frac{10}{5} = n+2 \to n+2-1 = n+1$$

4.  $0.51 \times 96,000 \approx 0.5 \times 100,000 = 50,000$ 

5. 
$$\frac{188,000}{0.11} \approx \frac{200,000}{0.1} \approx 2,000,000$$

- 6. 9 square units
- 8. a) 325,000 visitors
  - b) 100,000 visitors

7. a) 
$$\frac{130 \text{ lb}}{63 \text{ in.}} \approx 2.0635$$
;  $\frac{2.0635}{63 \text{ in.}} = 0.032754$ ;  $0.032754 \times 703 \approx 23.03$ 

b) He is in the at-risk range.

9. 
$$$50.40 - $35.00 = $15.40;$$
  $\frac{$15.40}{$0.20} = 77$  miles

10. Since  $\frac{\$15}{\$2.59} \approx 5.79$ , a maximum of 5 six-packs can be purchased. There will be  $\$15.00 - (5 \times \$2.59)$ 

= \$15.00 - \$12.95 = \$2.05 remaining, so  $\frac{$2.05}{$0.80} = 2.5625$ , or two additional individual cans can be purchased.

The following table was created using similar calculations for the other possible number of six-packs.

Six-packs	Individual cans	Number of cans
5	2	$6 \times 5 + 2 = 32$
4	5	$6 \times 4 + 5 = 29$
3	9	$6 \times 3 + 9 = 27$
2	12	$6 \times 2 + 12 = 24$
1	15	$6 \times 1 + 15 = 21$
0	18	$6 \times 0 + 18 = 18$

The maximum number of cans is 32.

11. 3 cuts 
$$\times \frac{2.5 \text{ min}}{1 \text{ cut}} = 7\frac{1}{2} = 7.5 \text{ min}$$

12. 
$$10 \times \frac{3}{4} = 10 \times 0.75 = 7.5$$
 miles

13. 
$$$12.75 \times 40 = $510$$

$$12.75 \times 1.5 \times 10 = 191.25$$

$$$652.25 - $701.25 = -$49.00$$

She was underpaid by \$49.00.

15. Mary drove the first 15 miles at 60 mph which took  $\frac{15}{60} = \frac{1}{4}$  hour and drove the second 15 miles at 30 mph took  $\frac{15}{20} = \frac{1}{2}$  hour for a total time of  $\frac{3}{4}$  hour, or 45 minutes. If she drove the entire 30 miles at 45 mph, the trip would take  $\frac{30}{45} = \frac{2}{3}$  hour, or 40 minutes, which is less than  $\frac{3}{4}$  hour.

16. 
$$\frac{61b}{21b} = 3$$
;  $3 \times \frac{1}{2} tsp = \frac{3}{2} tsp$  or  $1\frac{1}{2} tsp = \frac{1}{2} tbsp$ 

17. Area of lawn including walkway:  $(10+2)\times(12+2) = 12\times14 = 168 \,\text{m}^2$ 

Area of lawn only:  $10 \times 12 = 120 \,\mathrm{m}^2$ 

Area of walkway:  $168-120=48 \,\text{m}^2$ 

- 18. 243 jelly beans; 260-17=243, 234+9=243, and 274-31=243
- 19. a)  $3 \times \$3.99 = \$11.97$ 
  - b)  $9(\$1.75 \times 0.75) = 11.8125 \approx \$11.81$
  - c) \$11.97 \$11.81 = \$0.16; Using the coupon is least expensive by \$0.16.
- 20.  $4 \times 3 \times 2 \times 1 = 24$ ; The first position can hold any of four letters, the second any of the three remaining letters, and so on.

### **Chapter Two: Sets**

#### **Section 2.1: Set Concepts**

1. Set

2. Ellipsis

3. Description, Roster form, Set-builder notation

4. Finite

5. Infinite

6. Equivalent

7. Equal

8. Cardinal

9. Empty or null

10.  $\{ \},\emptyset$ 

11. Not well defined, "best" is interpreted differently by different people.

12. Not well defined, "most interesting" is interpreted differently by different people.

13. Well defined, the contents can be clearly determined.

14. Well defined, the contents can be clearly determined.

15. Well defined, the contents can be clearly determined.

16. Not well defined, "most interesting" is interpreted differently by different people.

17. Infinite, the number of elements in the set is not a natural number.

18. Finite, the number of elements in the set is a natural number.

19. Infinite, the number of elements in the set is not a natural number.

20. Infinite, the number of elements in the set is not a natural number.

21. Infinite, the number of elements in the set is not a natural number.

22. Finite, the number of elements in the set is a natural number.

23. { Hawaii }

24. { January, June, July }

25. {11,12,13,14,...,177}

26.  $C = \{4\}$ 

27.  $B = \{ 2, 4, 6, 8, ... \}$ 

28.  $\{\}$  or  $\emptyset$ 

29.  $\{\}$  or  $\emptyset$ 

30. {Idaho, Oregon}

31.  $E = \{14, 15, 16, 17, \dots, 84\}$ 

32. { Alaska, Hawaii }

33. {Google Play Games, Pokémon Go}

34. {ApplsLoading}

35. {Solitaire by MobilityWare, Candy Crush Soda Saga, ApplsLoading}

36.  $\{\}$  or  $\emptyset$ 

37. {2015, 2016, 2017, 2018}

38. {2011, 2012, 2013, 2014}

39.  $\{\}$  or  $\emptyset$ 

40. {2013, 2014}

41.  $B = \{x \mid x \in N \text{ and } 6 < x < 15\} \text{ or } B = \{x \mid x \in N \text{ and } 7 \le x \le 14\}$ 

42.  $A = \{x \mid x \in N \text{ and } x < 10\} \text{ or } A = \{x \mid x \in N \text{ and } x \le 9\}$ 

43.  $C = \{x \mid x \in N \text{ and } x \text{ is a multiple of 3} \}$ 

46.  $A = \{x \mid x \text{ is Independence Day}\}$ 

44.  $D = \{x \mid x \in N \text{ and } x \text{ is a multiple of 5}\}$ 

47.  $C = \{x \mid x \text{ is February}\}$ 

45.  $E = \{x \mid x \in N \text{ and } x \text{ is odd}\}$